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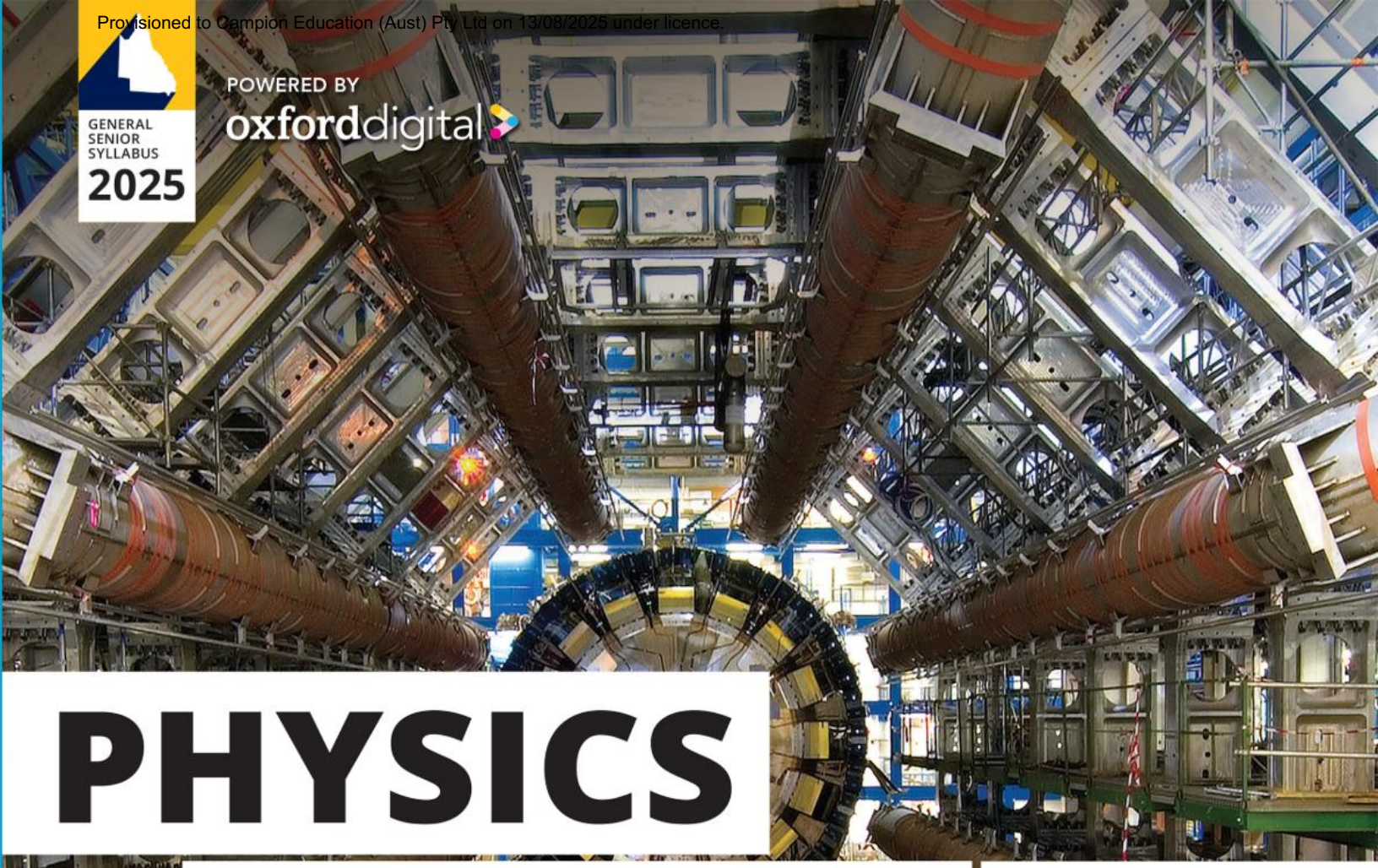

PHYSICS

FOR QUEENSLAND

UNITS 3&4

RICHARD WALDING

FOURTH EDITION

OXFORD

GENERAL SENIOR SYLLABUS 2025

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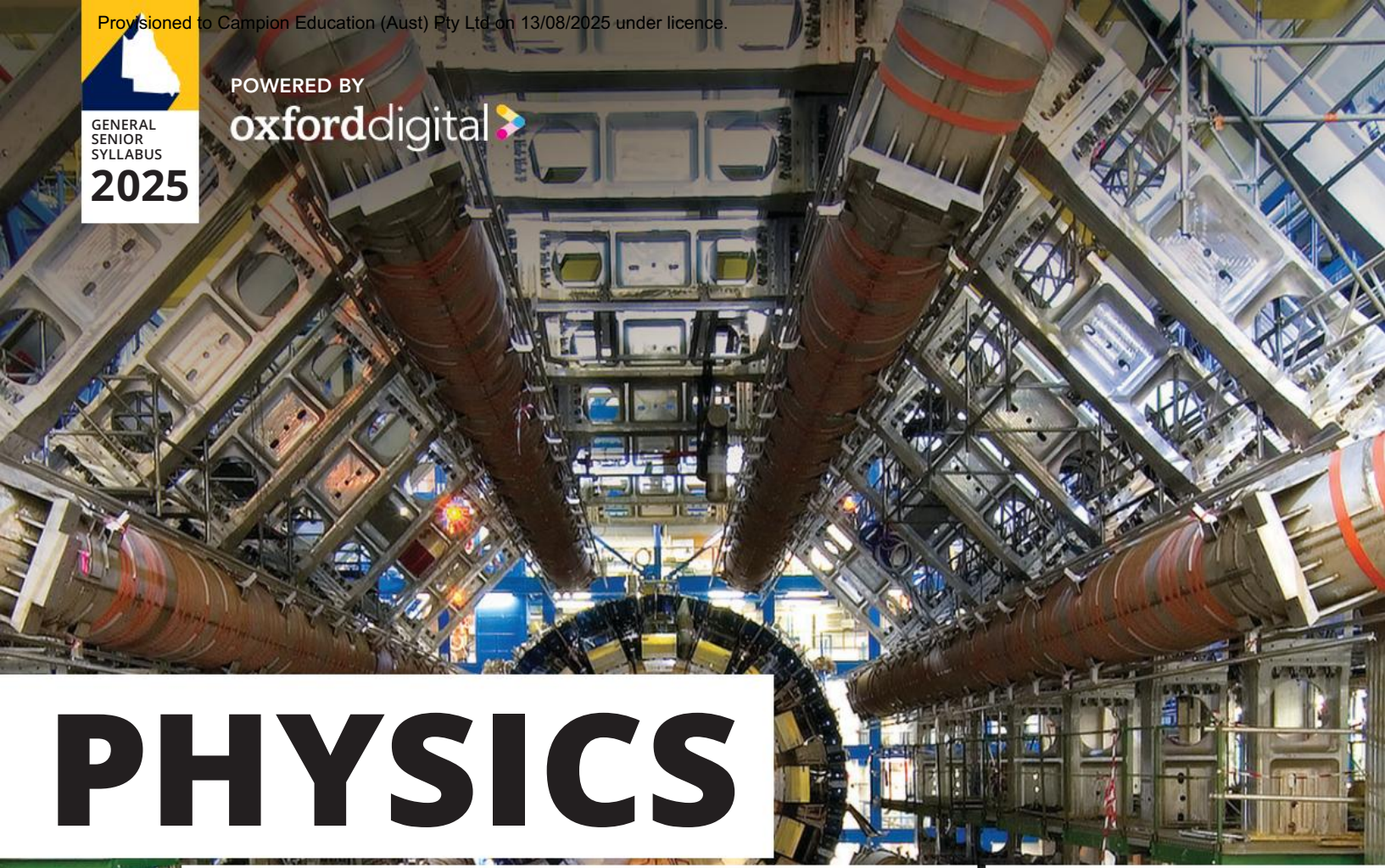

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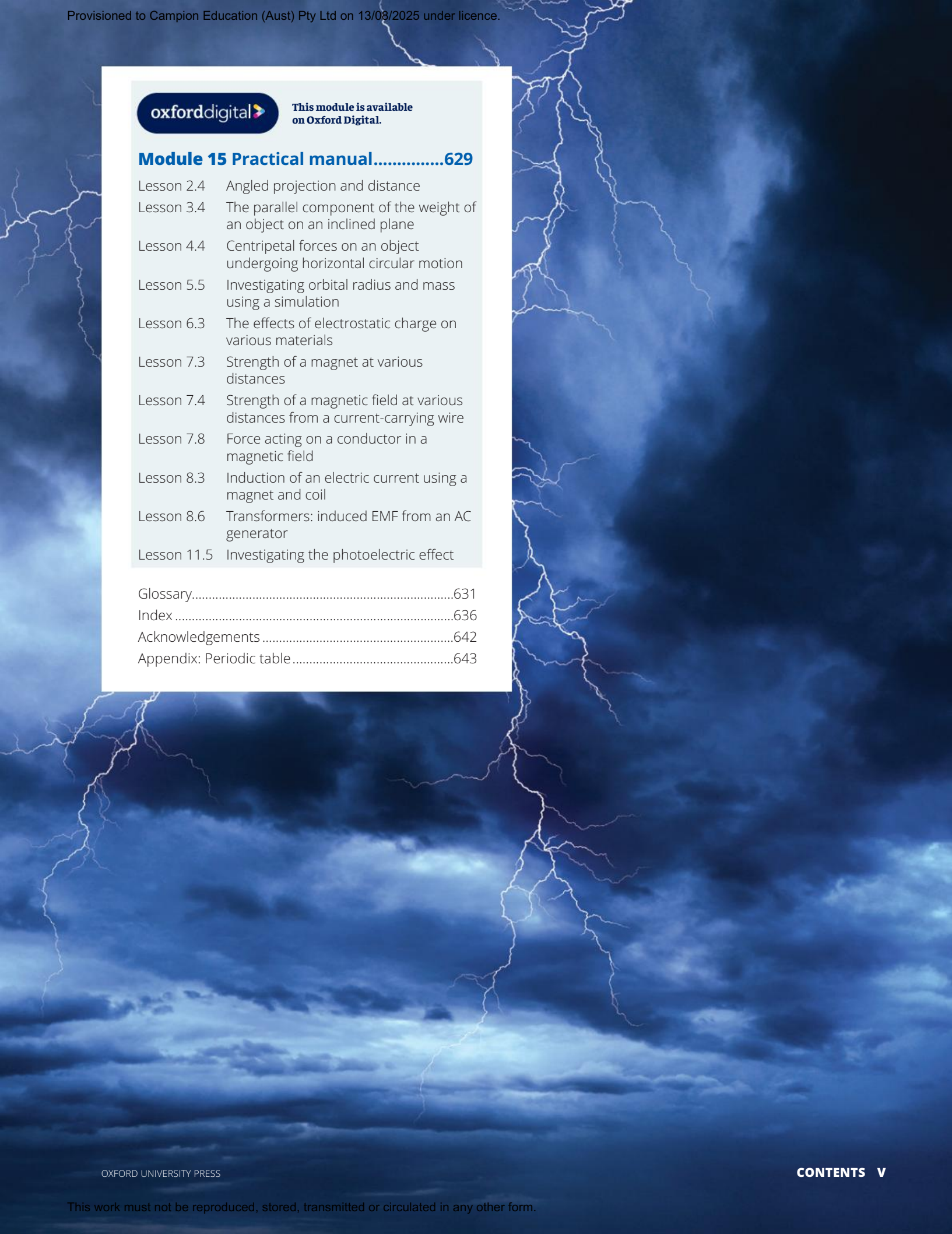
Warning to First Nations Australians
Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

Contents



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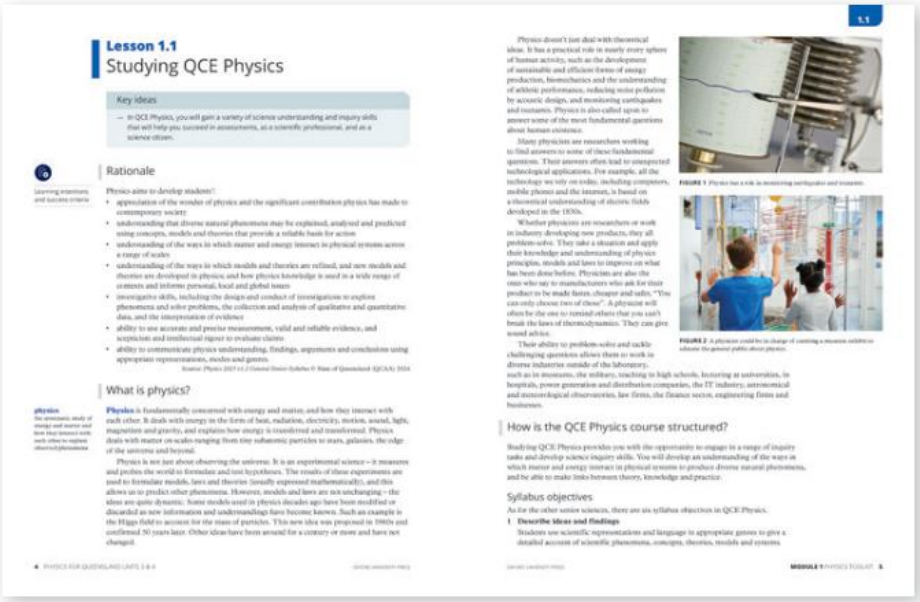
Introducing *Physics for Queensland Units 3 & 4* (Fourth edition)

Congratulations on choosing *Physics for Queensland Units 3 & 4* as part of your studies this year!

Physics for Queensland Units 3 & 4 has been purpose-written to meet the requirements of the QCAA Physics 2025 General senior syllabus. It includes a range of flexible print and digital products to suit your school and incorporates a wide variety of features designed to make learning fun, purposeful and accessible for all students!

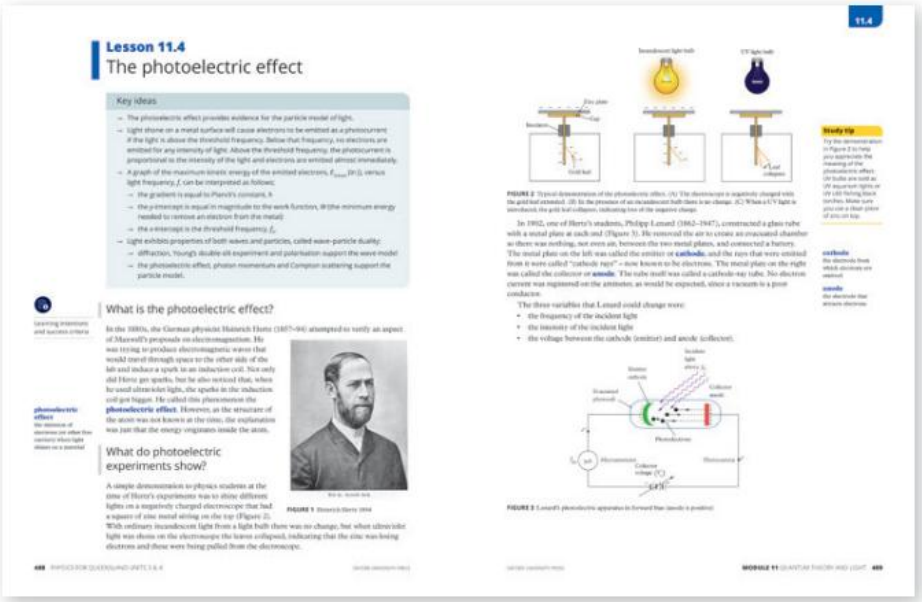
Key features of the Student Books

The Physics toolkit module provides an overview of the syllabus, student-friendly guidance for every science inquiry skill and tips for success on assessment tasks.



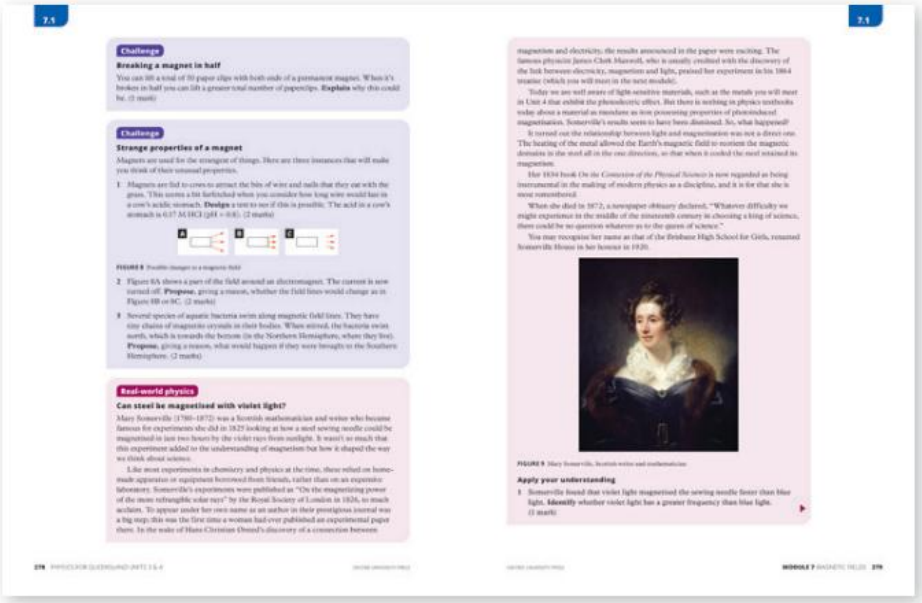
Each module begins with a module opener that includes:

- QCAA subject matter
- reference to a supporting prior knowledge quiz that assesses and informs student understanding of pre-requisite concepts
- a list of practical lessons that support science inquiry



Each lesson includes:

- **learning intentions and success criteria**
- clearly structured content written in clear, concise language
- definitions for all key terms on the page
- engaging, relevant and informative images and illustrations
- a range of tips and features designed to bring course content to life including **study tips, worked examples, skill drills** and examples of **real-world science applications**
- references to supporting **digital resources**
- **Check your learning** activities organised according to **Marzano and Kendall's taxonomy** and incorporating **cognitive verbs**.



Each module contains a range of practical activities designed to meet the requirements of science understanding and science inquiry subject matter and develop science inquiry skills.

Find out more

For a complete overview of all the features and benefits of this Student Book:

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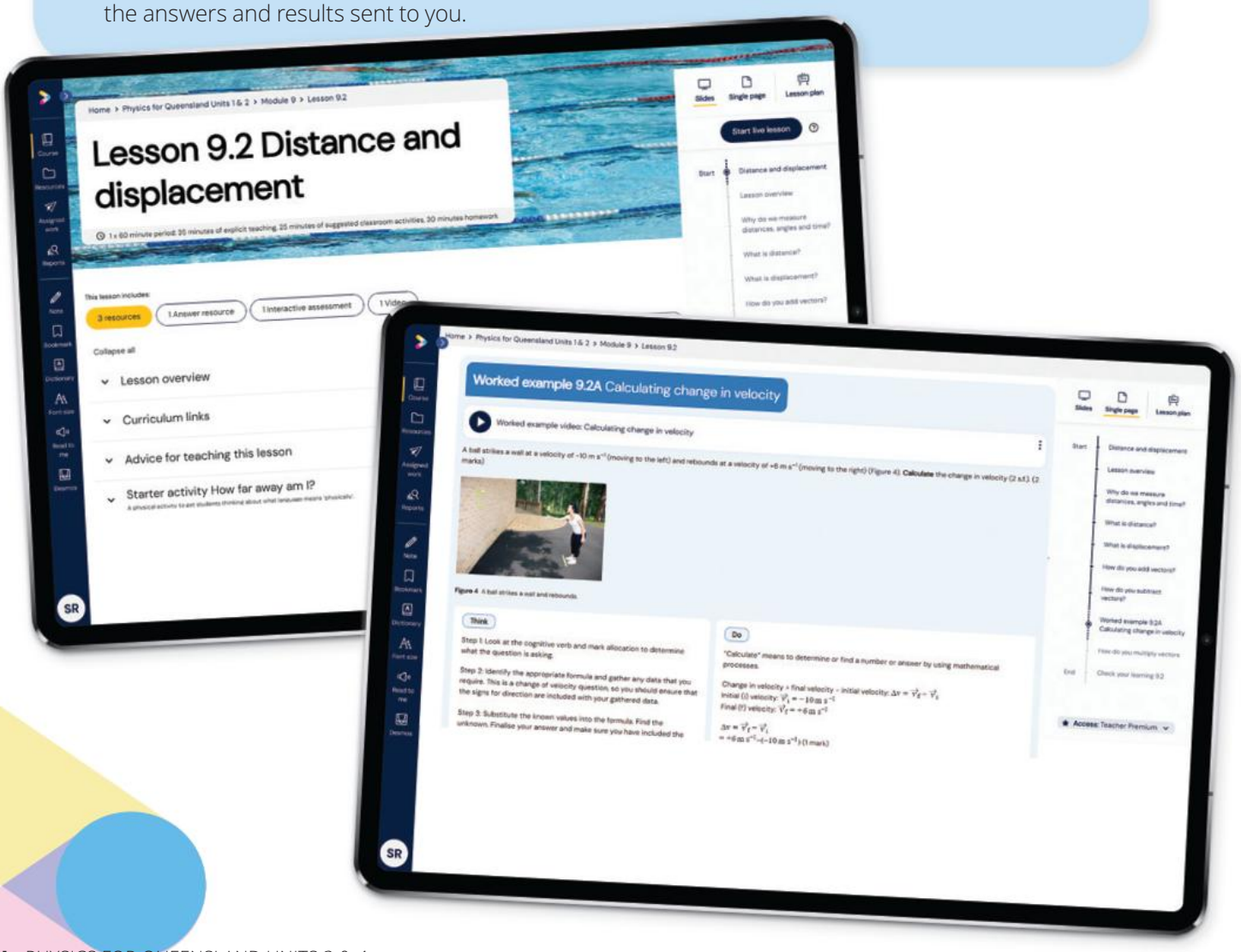

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- **personalise** learning for every student and **differentiate** content based on student strengths and weaknesses. Assign support or extension resources to any student using a range of differentiation resources
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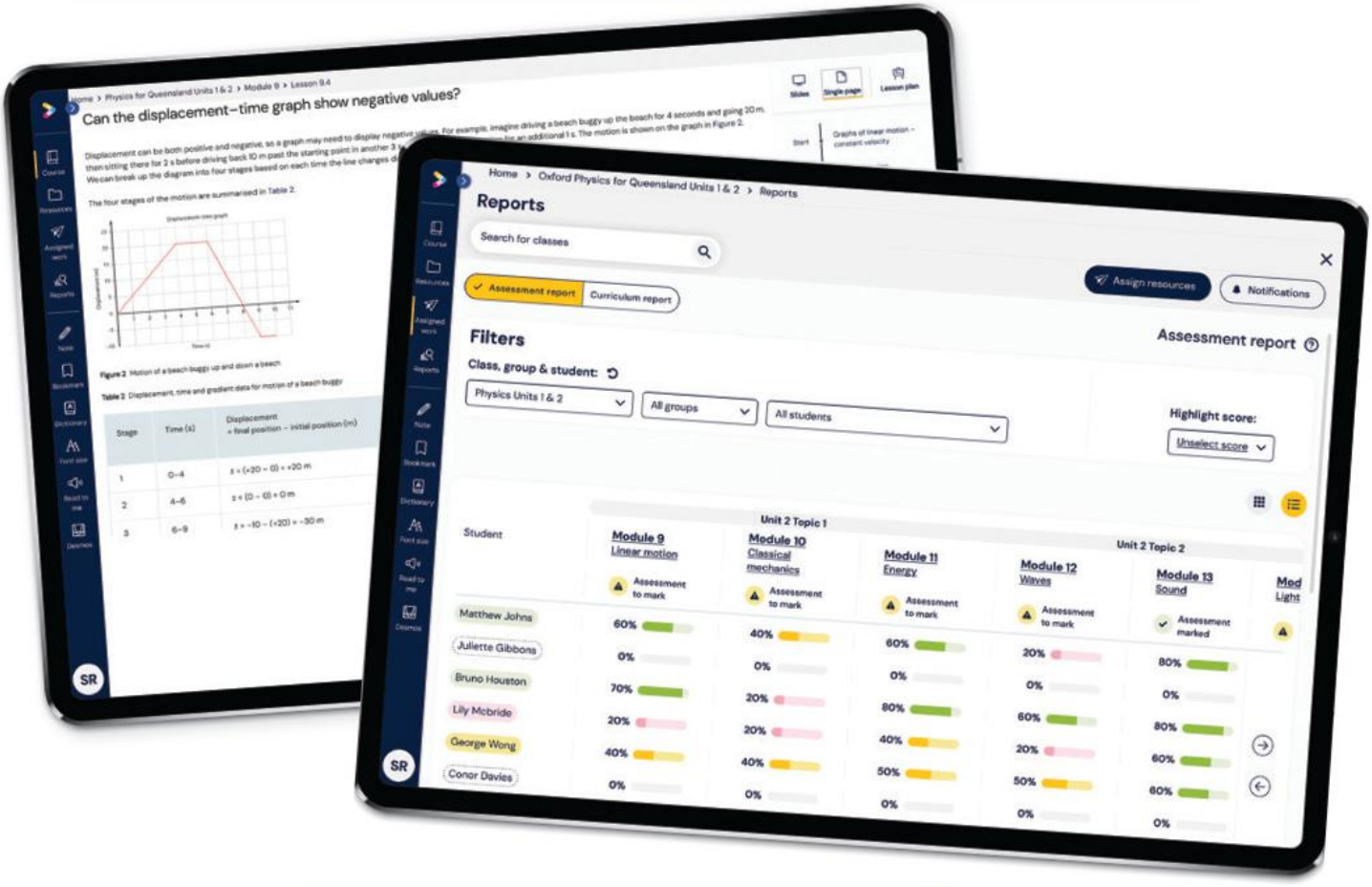

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Meet the authors & reviewers

Richard Walding

Author

Dr Richard Walding began his career as a scientist and has extensive experience teaching both Physics and Chemistry, including two decades as Head of Science. His doctoral studies on gender differences in physics and chemistry assessment helped inform the development of varied assessment tasks for the Queensland physics syllabus. Richard is also a research fellow at Griffith University and for the past decade has been researching the history and applications of electromagnetic induction particularly as related to harbour defence. Richard was elected Fellow of both the Australian Institute of Physics and the Royal Australian Chemical Institute for his lifetime contribution to these disciplines. He was also awarded a Peter Doherty Award for work in STEM education and in 2019 received the Australian Institute of Physics (QLD Branch) Excellence in Physics Teaching award. For the past decade Richard has freely given his time to present annual professional development seminars for the UQ-STAQ Senior Science Days and for the Griffith University Cutting Edge STEM program.

Bernice Zaro

First Nations consultant

Bernice is a proud Aboriginal and Torres Strait Islander Woman with a strong passion for educational greatness through culturally inclusive learning. Bernice identifies with parents who are Traditionally connected to Aboriginal Communities of South East Queensland and Mer in Torres Strait Islands (Murray Island) and she is inspired to share and learn continuously. Bernice, along with her husband Aicey Zaro, a recognised traditional Artist, has been educating schools and communities through Cultural Awareness art workshops for over 15 years during their time managing the Zaro Cultural Gallery in the Burdekin region. Bernice has a passion for learning through her ongoing studies in Community Development, Child Wellbeing, Cultural Diversity and also sharing personal experiences through family, community and business, which inspire her to take on new opportunities.

Matthew Flinders

Reviewer

Matthew completed a Bachelor of Science and Bachelor of Education at the University of Queensland. For the last 18 years, he has worked in both state and independent schools, enjoying a teaching load of mainly Science and Physics. He is the Physics Coordinator at Hillbrook Anglican School, where he is also involved in many other aspects of the school, such as data analysis.

Reeja Mathew

Reviewer

Since completing a Bachelor's degree in Education and a Master's degree in Physics at Mahatma Gandhi University (Kerala, India), Reeja Mathew has been teaching QCE Physics and Engineering in Queensland schools over the past 13 years. She also has a long-standing involvement with QCAA, contributing as a Physics panel member in the old QCE system and now, holding various assessor roles in the new QCE system.

We would also like to acknowledge and thank Adam Warren, Andrew Chung, Anja van Hooydonk, Michael Tomlin, Jade Abidi, Timothy Butler, Bruce Vaschina, Clinton Jackson, Michael Leask and Ian McCulloch for their contributions.

MODULE 1

Physics toolkit

Introduction

Physics, like the other sciences, is all about explaining the natural world. Measurement is at its very heart. Ever since humans have been thinking about their place in the universe, they have been making measurements. There are many different things that humans measure, and therefore there are different types of measurement and different ways of interpreting the measurements taken. As you study physics, you will learn about how different questions have been solved. Eventually, you will ask your own questions and make your own measurements.

This module will become a useful reference throughout Units 3 and 4 of Physics. Physics applies a rigorous scientific method to collect information and test theories, so in some ways this module tells you many of the most important things in all of physics. It *should* come first!

This module is set out in a way that makes each piece of information easy to access. It is not meant to be read from beginning to end. Rather, it's like a toolbox – you dip your hand into it, get the tool you need and then use it.

Prior knowledge

Prior knowledge quiz Check your understanding of the science inquiry skills before you start.

oxforddigital **Full lessons for this module are available on Oxford Digital.**

Online-only lessons

Lesson 1.2 Considering First Nations perspectives in Physics
Lesson 1.3 Understanding the scientific method
Lesson 1.4 Planning investigations
Lesson 1.5 Considering safety and ethics
Lesson 1.6 Collecting data
Lesson 1.8 Evaluating evidence
Lesson 1.9 Communicating scientifically
Lesson 1.10 Preparing for your data test
Lesson 1.11 Conducting your student experiment
Lesson 1.12 Conducting your research investigation
Lesson 1.13 Preparing for your exams

Lesson 1.1
Studying QCE Physics

Key ideas

→ In QCE Physics, you will gain a variety of science understanding and inquiry skills that will help you succeed in assessments, as a scientific professional, and as a science citizen.

Learning intentions and success criteria

Rationale

Physics aims to develop students':

- appreciation of the wonder of physics and the significant contribution physics has made to contemporary society
- understanding that diverse natural phenomena may be explained, analysed and predicted using concepts, models and theories that provide a reliable basis for action
- understanding of the ways in which matter and energy interact in physical systems across a range of scales
- understanding of the ways in which models and theories are refined, and new models and theories are developed in physics; and how physics knowledge is used in a wide range of contexts and informs personal, local and global issues
- investigative skills, including the design and conduct of investigations to explore phenomena and solve problems, the collection and analysis of qualitative and quantitative data, and the interpretation of evidence
- ability to use accurate and precise measurement, valid and reliable evidence, and scepticism and intellectual rigour to evaluate claims
- ability to communicate physics understanding, findings, arguments and conclusions using appropriate representations, modes and genres.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

What is physics?

physics
the systematic study of energy and matter and how they interact with each other to explain observed phenomena

Physics is fundamentally concerned with energy and matter, and how they interact with each other. It deals with energy in the form of heat, radiation, electricity, motion, sound, light, magnetism and gravity, and explains how energy is transferred and transformed. Physics deals with matter on scales ranging from tiny subatomic particles to stars, galaxies, the edge of the universe and beyond.

Physics is not just about observing the universe. It is an experimental science – it measures and probes the world to formulate and test hypotheses. The results of these experiments are used to formulate models, laws and theories (usually expressed mathematically), and this allows us to predict other phenomena. However, models and laws are not unchanging – the ideas are quite dynamic. Some models used in physics decades ago have been modified or discarded as new information and understandings have become known. Such an example is the Higgs field to account for the mass of particles. This new idea was proposed in 1960s and confirmed 50 years later. Other ideas have been around for a century or more and have not changed.

Physics doesn't just deal with theoretical ideas. It has a practical role in nearly every sphere of human activity, such as the development of sustainable and efficient forms of energy production, biomechanics and the understanding of athletic performance, reducing noise pollution by acoustic design, and monitoring earthquakes and tsunamis. Physics is also called upon to answer some of the most fundamental questions about human existence.

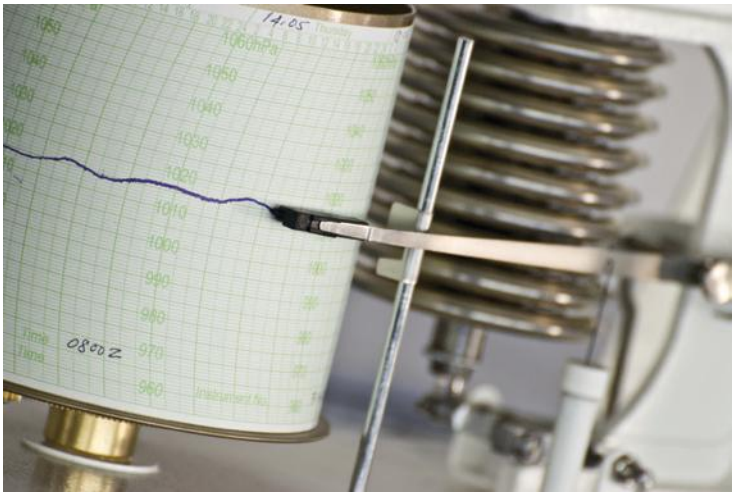

FIGURE 1 Physics has a role in monitoring earthquakes and tsunamis.

Many physicists are researchers working to find answers to some of these fundamental questions. Their answers often lead to unexpected technological applications. For example, all the technology we rely on today, including computers, mobile phones and the internet, is based on a theoretical understanding of electric fields developed in the 1830s.

Whether physicists are researchers or work in industry developing new products, they all problem-solve. They take a situation and apply their knowledge and understanding of physics principles, models and laws to improve on what has been done before. Physicists are also the ones who say to manufacturers who ask for their product to be made faster, cheaper and safer, "You can only choose two of those". A physicist will often be the one to remind others that you can't break the laws of thermodynamics. They can give sound advice.

FIGURE 2 A physicist could be in charge of curating a museum exhibit to educate the general public about physics.

Their ability to problem-solve and tackle challenging questions allows them to work in diverse industries outside of the laboratory, such as in museums, the military, teaching in high schools, lecturing at universities, in hospitals, power generation and distribution companies, the IT industry, astronomical and meteorological observatories, law firms, the finance sector, engineering firms and businesses.

How is the QCE Physics course structured?

Studying QCE Physics provides you with the opportunity to engage in a range of inquiry tasks and develop science inquiry skills. You will develop an understanding of the ways in which matter and energy interact in physical systems to produce diverse natural phenomena, and be able to make links between theory, knowledge and practice.

Syllabus objectives

As for the other senior sciences, there are six syllabus objectives in QCE Physics.

1 **Describe ideas and findings**

Students use scientific representations and language in appropriate genres to give a detailed account of scientific phenomena, concepts, theories, models and systems.

2 **Apply understanding**

Students use scientific concepts, theories, models and systems within their limitations. They use algebraic, visual and graphical representations of scientific relationships and data to determine unknown scientific quantities or features. They explain phenomena, concepts, theories, models, systems and modifications to methodologies.

3 **Analyse data**

Students consider scientific information from primary and secondary sources to identify trends, patterns, relationships, limitations and uncertainty. In qualitative data, they identify the essential elements, features or components. In quantitative data, they use mathematical processes and algorithms. They identify data to support ideas, conclusions or decisions.

4 **Interpret evidence**

Students use their understanding of scientific concepts, theories, models and systems and their limitations to draw conclusions and develop scientific arguments. They compare, deduce, extrapolate, infer, justify and make predictions based on their analysis of data.

5 **Evaluate conclusions, claims and processes**

Students critically reflect on the available evidence and make judgements about its application to research questions. They extrapolate findings to support or refute claims. They use the quality of the evidence to evaluate the validity and reliability of inquiry processes and suggest improvements and extensions for further investigation.

6 **Investigate phenomena**

Students develop rationales and research questions for experiments and investigations. They modify methodologies to collect primary data and select secondary sources. They manage risks, environmental and ethical issues, and acknowledge sources of information.

Study tip

Your assessments in QCE Physics are mapped to these syllabus objectives. The data test (Lesson 1.10) assesses objectives 2 to 4. The student experiment (Lesson 1.11) and research investigation (Lesson 1.12) assess objectives 1 to 6. The external exam (Lesson 1.13) assesses objectives 1 to 4.

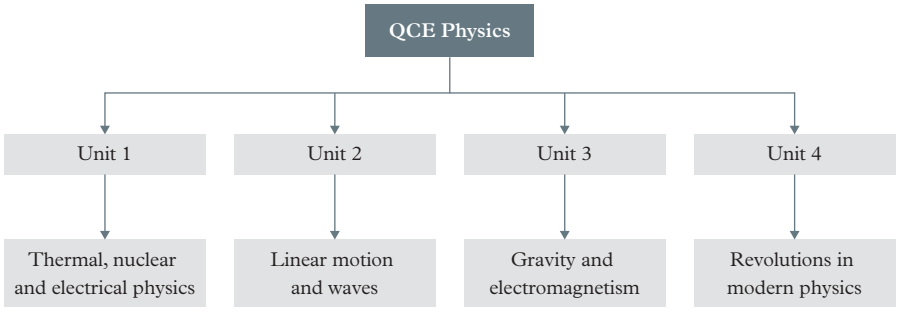

FIGURE 3 The structure of the QCE Physics course

Subject matter

The structure of the QCE Physics course is laid out in the Physics General Senior Syllabus. The course consists of four units. Units 1 and 2 are completed in the first year of the QCE Physics course and Units 3 and 4 in the second year. Each unit is divided into topics and each topic can include science understanding, science as a human endeavour and science inquiry subject matter. You should be familiar with these categories of understanding from your studies in Years 7 to 10.

An overview of the QCE Physics units is shown in Figure 3, and Units 3 and 4 are summarised in Table 1. Each unit has its own specific objectives, which are outlined in the Unit 3 and Unit 4 openers.

TABLE 1 Topics in Units 3 and 4 Physics

Unit 3 Gravity and electromagnetism	
Topic	**Description**
1. Gravity and motion	In this topic, you will learn about: • vectors and projectile motion • inclined planes and circular motion • orbital mechanics.
2. Electromagnetism	In this topic, you will learn about: • electrostatics • magnetic fields • electromagnetic induction and radiation.
Unit 4 Revolutions in modern physics	
Topic	**Description**
1. Special relativity	In this topic, you will learn about: • special relativity.
2. Quantum theory	In this topic, you will learn about: • quantum theory.
3. The Standard Model	In this topic, you will learn about: • the Standard Model • particle interactions.

Source: Adapted from *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Assessment in QCE Physics

In Units 3 and 4, schools develop three assessments using the assessment specifications and conditions provided in the syllabus.

The structure of your assessment is outlined in Table 2.

TABLE 2 Units 3 and 4 assessments

Unit and assessment type	Assessment description	Assessment objectives
Unit 3 Gravity and motion: Data test	Students respond to items using qualitative data and/or quantitative data derived from practicals, activities or case studies relevant to Unit 3 subject matter.	2. Apply understanding of gravity and motion, or electromagnetism to given algebraic, visual or graphical representations of scientific relationships and data to determine unknown scientific quantities or features. 3. Analyse data about gravity and motion, or electromagnetism to identify trends, patterns, relationships, limitations or uncertainty in datasets. 4. Interpret evidence about gravity and motion, or electromagnetism to draw conclusions based on analysis of datasets.
Unit 3 Gravity and motion: Student experiment	Students modify (i.e. refine, extend or redirect) an experiment relevant to Unit 3 subject matter to address their own related hypothesis or question. This assessment provides opportunities to assess science inquiry skills.	1. Describe ideas and experimental findings about gravity and motion, or electromagnetism. 2. Apply understanding of gravity and motion, or electromagnetism to modify experimental methodologies and process data. 3. Analyse experimental data about gravity and motion, or electromagnetism. 4. Interpret experimental evidence about gravity and motion, or electromagnetism. 5. Evaluate experimental processes and conclusions about gravity and motion, or electromagnetism. 6. Investigate phenomena associated with gravity and motion, or electromagnetism through an experiment.

Unit and assessment type	Assessment description	Assessment objectives
Unit 4 Revolutions in modern physics: Research investigation	Students gather evidence related to a research question to evaluate a claim relevant to Unit 4 subject matter. This assessment provides opportunities to assess science inquiry skills and science as a human endeavour (SHE) subject matter.	1. Describe ideas and findings about special relativity, quantum theory or the Standard Model. 2. Apply understanding of special relativity, quantum theory or the Standard Model to develop research questions. 3. Analyse research data about special relativity, quantum theory or the Standard Model. 4. Interpret research evidence about special relativity, quantum theory or the Standard Model. 5. Evaluate research processes, claims and conclusions about special relativity, quantum theory or the Standard Model. 6. Investigate phenomena associated with special relativity, quantum theory or the Standard Model through research.
Units 3 and 4 examination(s)		

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You can use Lesson 1.10 Preparing for your data test, Lesson 1.11 Conducting your student experiment, Lesson 1.12 Conducting your research investigation and Lesson 1.13 Preparing for your exams to guide you through these assessments. Note that science as a human endeavour content will not be directly assessed in your examinations.

What are the science inquiry skills?

In addition to developing your science understanding in physics (which we will cover in Modules 2 to 14), the QCE course requires you to develop and apply a range of science inquiry skills. These skills are specified in the QCE Physics General Senior Syllabus and are listed on the opening pages of this module. This module will help you develop these skills.

The science inquiry skills are applicable to all areas of study in Units 1 to 4 of the QCE Physics course. They are especially important for preparing and planning for your data test, student experiment and research investigation assessment tasks.

Check your learning 1.1

Check your learning 1.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Define** the term "physics" in 10 words or less. (1 mark)

Analytical processes

2 **Distinguish** between "problem-solving" and "developing technology" as applied to the use of physics. (1 mark)

3 **Judge** whether this is true: "Physics is said to be an experimental science, so all physics theories have to come from experiments." (2 marks)

Knowledge utilisation

4 **Investigate** what role physicists may play in the finance and business sectors. (2 marks)

5 **Evaluate** the statement "physics is a practical science". Does it mean that all research has to have a practical outcome such as saving energy or making more powerful satellites? (2 marks)

6 **Propose** a response to this question from a friend: "How can they say the Big Bang really occurred when no one was there?" (1 mark)

7 **Propose** how physics might be used in chemistry and biology, and how physics relies on mathematics. (3 marks)

Lesson 1.2
Considering First Nations perspectives in Physics

Key ideas

- → First Nations peoples have longstanding scientific knowledge.
- → First Nations peoples have developed knowledge about the world by observing using all the senses, predicting and hypothesising, testing (trial and error), and making generalisations within specific contexts such as the use of food, natural materials, navigation and sustainability of the environment.
- → Correctly acknowledging cultural and/or language groups, rejecting deficit discourse, avoiding Eurocentrism and critically evaluating sources of information can help you to respectfully engage with First Nations perspectives in QCE Physics.

Science inquiry skills

Learning intentions and success criteria

This lesson provides support for the following science inquiry skills:

- identify and implement strategies to manage risks, ethics and environmental impact, e.g.
  - cultural guidelines, protocols for working with the knowledges of First Nations peoples.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

This lesson is available on Oxford Digital.

Lesson 1.3
Understanding the scientific method

Key ideas

- → The scientific method is an iterative and cyclic process.
- → Research questions define the scope of an investigation. They can be used to develop hypotheses that predict the outcome of the investigation.

Science inquiry skills

Learning intentions and success criteria

This lesson provides support for the following science inquiry skills:

- identify, research and construct questions for investigation
- propose hypotheses and/or predict possible outcomes.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

This lesson is available on Oxford Digital.

Lesson 1.4
Planning investigations

Key ideas

→ A method outlines the steps followed in an experiment and lists all of the materials and equipment used.

→ Valid and reliable measurements can be obtained by carefully designing your investigation to collect sufficient data and minimise errors.

→ All measurements include errors or uncertainties, either systematic or random. It is important to consider these and implement strategies to minimise their effects when planning your experiments.

Learning intentions and success criteria

Science inquiry skills

This lesson provides support for the following science inquiry skills:

- design investigations, including the procedure/s to be followed, the materials required, and the type and amount of primary and/or secondary data required to obtain valid and reliable evidence, e.g.
  - consider replicates, number of data points, and quality of sources
  - identify the types of errors, extraneous variables or confounding factors that are likely to influence results and implement strategies to minimise systematic and random error
- use appropriate equipment, techniques, procedures and sources to systematically and safely collect primary and secondary data, e.g.
  - laboratory and field techniques: measurement, and equipment calibration
  - ICTs, scientific texts, databases, online sources
- suggest improvements and extensions to minimise uncertainty, address limitations and improve the overall quality of evidence, e.g.
  - analyse the impact of random error/measurement uncertainties and systematic errors in experimental work and determine how these errors/measurement uncertainties can be reduced
  - discriminate between random and systematic errors
  - identify that experimental design and procedure usually leads to systematic errors in measurement, which causes a deviation in a direction and that repeated trials and measurements will reduce random error but not systematic error
- use data and reasoning to discuss and evaluate the validity and reliability of evidence, e.g.
  - discriminate between validity and reliability

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

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Lesson 1.5
Considering safety and ethics

Key ideas

→ Laboratory safety is important to prevent harm to yourself and those sharing the lab space with you.

→ Safety and ethics extend beyond the lab; you should be considering the broader potential impacts of your experiment.

Science inquiry skills

Learning intentions and success criteria

This lesson provides support for the following science inquiry skills:

- identify and implement strategies to manage risks, ethics and environmental impact, e.g.
  - workplace health and safety guidelines
  - standard operating procedures.

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Lesson 1.6
Collecting data

Key ideas

- → Single experimental measurements are reported using best estimates, indicators of measurement uncertainty and units.
- → Scientific notation is used to easily express extremely large or small values.
- → Significant figures are digits in a number that are known with certainty and the first digit that is uncertain.
- → All measurements, information and observations should be recorded in your logbook.

Learning intentions and success criteria

Science inquiry skills

This lesson provides support for the following science inquiry skills:

- use scientific language and representations to systematically record information, observations, data and measurement error, e.g.
  - symbols, units and prefixes
  - tables, graphs and diagrams
  - logbooks
- translate information between graphical, numerical and/or algebraic forms, e.g.
  - units and measurement conversions
  - symbols and notation

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Worked examples

This lesson is supported by the following Worked examples:

- **Worked example 1.6A** Writing numbers using scientific notation
- **Worked example 1.6B** Converting between units for physical quantities

oxforddigital **This lesson is available on Oxford Digital.**

Lesson 1.7
Processing and analysing data

Key ideas

- → Data is processed and analysed to identify trends, patterns, relationships, limitations and uncertainty.
- → Absolute and percentage uncertainty give an idea of the precision of measurements.
- → Absolute and percentage error give an idea of the accuracy of measurements.
- → Data can be summarised in a variety of ways, such as in a table, a scatterplot, or a scientific diagram.
- → Linearising data helps you to identify the exact relationship between variables.

Science inquiry skills

Learning intentions and success criteria

This lesson provides support for the following science inquiry skills:

- use scientific language and representations to systematically record information, observations, data and measurement error, e.g.
  - indicators of measurement uncertainty and state measurement uncertainties as a range (±) to an appropriate precision, e.g. when adding or subtracting, the final answer should be given to the least number of decimal places, when multiplying or dividing, the final answer should be given to the least number of significant figures
- use mathematical techniques to summarise data in a way that allows for identification of relevant trends, patterns, relationships, limitations and uncertainty, e.g.
  - mean
  - gradient analysis
  - scatterplots (with maximum and minimum trendlines and R^2)
  - propagate random error in data processing to show the impact of measurement uncertainties on the final result
  - apply simple treatment of error analysis, e.g. for functions such as addition and subtraction, absolute uncertainties should be added, for multiplication, division and powers, percentage uncertainties should be added
  - calculate the measurement uncertainties in processed data, including the use of absolute uncertainties of the mean (Formula: $\Delta\bar{x} = \pm\frac{(x_{max} - x_{min})}{2}$ and percentage uncertainties (Formula: percentage uncertainty (%) $= \frac{\text{absolute uncertainty}}{\text{measurement}} \times 100\%$)
  - calculate the percentage error, when the experimental result can be compared with a theoretical or accepted result (value) (Formula: percentage error (%) $= \left|\frac{\text{measured value - true value}}{\text{true value}}\right| \times 100\%$)
  - discriminate between absolute uncertainty and percentage error
- select and construct appropriate representations to present data and communicate findings, e.g.
  - summary tables
  - scatterplots (with maximum and minimum trendlines and R^2)

  - scientific drawings
  - apply appropriate graphical representations to analyse data and draw conclusions
- discriminate between precision and accuracy
  - identify that all measurements have limits to their precision and accuracy that must be considered when evaluating experimental results
  - identify that quantitative data obtained from measurements is associated with random error/measurement uncertainties
- analyse data to identify trends, patterns and relationships; recognising error, uncertainty and limitations of evidence
- extrapolate findings to determine unknown values, predict outcomes and evaluate claims.
- interpret graphs in terms of the relationship between dependent and independent variables; draw and interpret best-fit lines or curves through data points, including evaluating when it can and cannot be considered as a linear function

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

What information does data analysis give us?

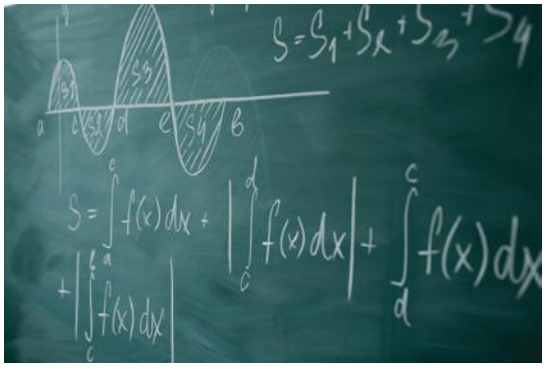


FIGURE 1 Analysing data, for example by using mathematical techniques, helps us to draw conclusions from an investigation.

Once we have obtained our raw data, we are ready to process it. This typically involves completing calculations and constructing scatterplots. By transforming the data into more meaningful forms, we can analyse it to quantify uncertainty, identify limitations, construct evidence-based arguments and draw conclusions.

What are precision and accuracy?

A part of any physics experiment is to record and analyse your measurements for quality. This is called an error analysis and is done by assessing both the precision and accuracy of the experimental observations. Precision and accuracy mean very different things, so it is important that these terms are used correctly. For a set of measurements:

precision
the range of values found around the observed (measured) value; how close two or more measurements of the same object or phenomenon are to each other (i.e. the "uncertainty" of the measurement); the higher the uncertainty the lower the precision

accuracy
the difference between the observed (measured) value and a currently accepted value of a quantity

- **precision** is the range of values found around the observed (measured) value, or how close two or more measurements of the same object or phenomenon are to each other. Precision may be represented as an absolute uncertainty or as a percentage uncertainty. The higher the uncertainty, the lower the precision.
- **accuracy** is the difference between the best estimate or observed (measured) value and the accepted (true) value of the observed quantity. Accuracy may be represented as an absolute error or as a percentage error.

FIGURE 2 Accuracy (closeness to the centre) and precision (closeness to each other) are different concepts.

A low systematic error is likely to make the results of an experiment accurate and therefore valid. A large systematic error would mean the experiment is not accurate and hence the results would not be valid. Results with a low random error, and hence low uncertainty, are said to be precise and thus reliable. Low precision (high uncertainty) means the result is imprecise and thus unreliable.

TABLE 1 Links between accuracy, precision and errors

Size of experimental error	Systematic error	Random error
Small	accurate, therefore valid	precise, therefore reliable
Large	not accurate, therefore lacks validity	imprecise (low precision), therefore unreliable

Systematic error affects accuracy and therefore validity. Random error affects precision and therefore reliability. Both these concepts should be carefully considered when you analyse your data. Understanding them helps you to analyse the uncertainty in your results. Let's look at some ways you can process your data to determine uncertainty.

How do you report repeated measurements?

Single measurements are reported using half-scale divisions. When you make repeated single measurements, you need to complete additional calculations (i.e. process the data) to obtain your best estimate. In this case, the best estimate is the **mean** (average) of your replicate measurements.

mean
the average of multiple values

The mean, $\bar{x}$, is calculated by dividing the sum of the measurements by the number of measurements taken:

$$\bar{x} = \frac{\sum x}{n}$$

where x is a measurement and n is the number of replicates.

Averaging your results gives you a single value (best estimate), but you need to also state the spread of results around this value. This is called the uncertainty of the results and can be expressed as absolute or percentage uncertainty.

How do you calculate absolute uncertainty?

Absolute uncertainty ($\delta\bar{x}$) is the dispersion (spread) of values around the mean, reported by half the range of the measurements. It gives you information about the precision of your results. It is calculated by subtracting the highest from the lowest value and dividing by two:

absolute uncertainty
the dispersion (spread) of values around the mean, $\delta\bar{x}$, reported by half the range of the measurements; an indicator of the precision of measurements

$$\delta\bar{x} = \pm\frac{(x_{max} - x_{min})}{2}$$

where x_{max} is the largest value and x_{min} is the smallest value.

As when reporting single measurements, you would report your result as the best estimate ± uncertainty. Say you measured the diameter of a piece of wire four times using a vernier calliper. We often do this because wire is not always perfectly round, so measurements are taken at different angles around a point on the wire. Let the readings be 0.60 mm, 0.62 mm, 0.65 mm and 0.62 mm. The best estimate is the average (mean) of 0.6225 mm and the range is 0.60 – 0.65 = 0.05 mm. Half the range of 0.05 mm is 0.025 mm, so $\delta\bar{x}$ = 0.025 mm. The results are 0.6225 ± 0.025 mm. However, using the following rules, this would be reported as 0.62 ± 0.03 mm.

Study tip

Absolute uncertainty can be represented by $\delta\bar{x}$, or just δx to keep it simple. Some sources use the capital delta and write $\Delta\bar{x}$.

How to quote uncertainties

The function of uncertainties is to quantify the interval in which the true value of the measurement lies. There is no point in quoting the uncertainty to more than one or two significant figures, and usually one significant figure is enough. Often your calculation for uncertainty will give you many digits. The rule is to round the uncertainty to one significant figure (or two if the first figure is a "1"). For example, say you calculated the mean of a set of distances to be 123.456 m and the uncertainty to be 0.789 m. You would round the uncertainty to 1 s.f. or 0.8 m, and then round the best estimate (mean) value to the same number of decimal places (1 d.p.). You would then state the measurement as 123.5 ± 0.8 m.

Where your best estimate (mean) has an uncertainty that starts with a "1", such as in 0.167 m, you would round this to two significant figures (0.17) and round your best estimate to the same number of decimal places (2 d.p.). For example, if a best estimate (mean) was 9.234 m s^{-1} and the uncertainty was 0.167 m s^{-1}, you would show your uncertainty to 2 s.f. as 0.17 m s^{-1} and then round the best estimate to 2 d.p. or 9.23 m s^{-1}. You would report your measurement as 9.23 ± 0.17 m s^{-1}.

The same rule applies to percentage uncertainty: round the uncertainty to one significant figure (or two if the first figure is a "1").

Worked example 1.7A

Calculating the mean value and the absolute uncertainty

The diameter of a wire was measured four times using a digital vernier calliper that has a resolution of 0.01 mm. The readings were 0.72 mm, 0.75 mm, 0.70 mm, 0.73 mm. **Calculate** the mean and the absolute uncertainty, and report your final answer correctly. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the mean and absolute uncertainty. The question is worth 2 marks, so we must correctly apply the formulas to complete the calculations and express the answer correctly.
Step 2: Select the appropriate equations and gather any data required.	$\bar{x} = \frac{\sum x}{n}$ $\delta\bar{x} = \pm \frac{(x_{max} - x_{min})}{2}$ $x_{max} = 0.70\,\text{mm}$; $x_{min} = 0.75\,\text{mm}$; $n = 4$
Step 3: Substitute the known values into the formulas and solve for the final answers.	$\bar{x} = \frac{0.72 + 0.75 + 0.70 + 0.73}{4}$ $= 0.725$ $\delta\bar{x} = \pm \frac{(0.75 - 0.70)}{2}$ $= \pm 0.025$
Step 4: Report your final answer as the best estimate (mean), absolute uncertainty, and the correct units. Make sure you use the correct number of significant figures in the mean and the same number of decimal places in the uncertainty. Note: marks are not awarded in the external exam for accuracy and uncertainty calculations. They may be awarded in your Data Test.	The absolute uncertainty is rounded to 1 s.f. (as it doesn't start with a "1"), so it is 0.03 mm (remembering that "5 rounds up"). The mean is then rounded to 2 d.p., the same number of decimal places as the uncertainty, so it becomes 0.73 mm. The final answer is 0.73 (1 mark) ± 0.03 mm (1 mark).

Your turn

The separation distance between the poles of two magnets was measured with a metre ruler marked in 1 mm increments. The distances were 52.0 mm, 58.5 mm, 49.5 mm, 52.0 mm, 56.5 mm. **Calculate** the mean distance and the absolute uncertainty, and report your final answer correctly. (2 marks)

Which uncertainty value do you report?

You have learnt that uncertainty for single measurements is reported as the half-scale division or resolution of the measurement instrument. When dealing with measurements that rely on reaction time, there is also the reaction time uncertainty to consider when reporting data. What if these values are larger than the absolute uncertainty? Which value should you use?

The rule is that the uncertainty value you report should be the largest one. This is because we don't want to underestimate the uncertainty of the result. This would make the result appear more precise and reliable than is justified or warranted.

Study tip

Like uncertainty for single measurements, the absolute uncertainty is reported to the same number of decimal places as the best estimate (mean).

Study tip

You should compare the uncertainty based on the size of half-scale division of the instrument with the uncertainty based on the calculation using half the maximum minus the minimum. When you report your final answer, always take the larger uncertainty of the two.

Uncertainty for periodic motion

In "Your turn" for Worked example 1.7B, the period (time) for one oscillation was measured. Often, with periodic motion such as this, we measure the time for 10 oscillations and divide by 10. The reaction time uncertainty of 0.2 s is spread over 10 periods of time, so instead of it being 1.28 ± 0.2 s for one oscillation, it would be 12.8 ± 0.2 s for 10 oscillations, or 1.28 ± 0.02 s for the one oscillation. This reduces the percentage uncertainty from 16% to 1.6%. You will use this technique if you do pendulum experiments, or in the "Gravity and motion" topic in Unit 3, in which you will perform circular motion experiments.

Worked example 1.7B

Calculating the mean and uncertainty for stopwatch measurements

A ball is projected horizontally from a set height and the time taken to reach the ground was measured using a digital stopwatch. The readings were 1.12 s, 1.05 s, 1.15 s, 0.95 s, 1.00 s and 1.20 s. The uncertainty for a hand-held stopwatch is 0.2 s. **Calculate** the mean and uncertainty and report your final answer correctly. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the mean and uncertainty. The question is worth 2 marks, so we must correctly apply the formulas to complete the calculations and express the answer correctly.
Step 2: Select the appropriate equations and gather any data required.	$\bar{x} = \frac{\sum x}{n}$ $\delta\bar{x} = \pm \frac{(x_{max} - x_{min})}{2}$ $x_{max} = 1.20\,\text{mm}$; $x_{min} = 0.95\,\text{mm}$; $n = 6$
Step 3: Substitute the known values into the formulas and solve for the final answers.	$\bar{x} = \frac{1.12 + 1.05 + 1.15 + 0.95 + 1.00 + 1.20}{6}$ $= 1.0783$ $\delta\bar{x} = \pm \frac{(1.20 - 0.95)}{2}$ $= \pm 0.125$

Think	Do
Step 4: Recognise that reaction time error is relevant and consider it when reporting your measurements. Select the largest value and adjust uncertainty as required.	Reaction time error = ± 0.2 s Digital stopwatch resolution = ± 0.01 s Absolute uncertainty = ± 0.125 The reaction time error is the largest.
Step 5: Report your final answer as the best estimate (mean), uncertainty, and the correct units. Make sure you use the correct number of significant figures in the mean.	Uncertainty is expressed to 2 s.f. as it starts with a "1", so it is 0.13 s. However, the reaction time uncertainty is 0.2 s, so we will just report this. The mean has to have the same number of d.p. as the uncertainty, so the final answer is 1.1 ± 0.2 s. (1 mark for correct mean; 1 mark for correct absolute uncertainty)

Your turn

A pendulum consisting of a brass bob on a 2.00 m string is pulled to one side and allowed to swing for one complete oscillation (back and forth). The duration of one oscillation was timed using a digital stopwatch. This was repeated and the readings were 1.28 s, 1.26 s, 1.29 s and 1.30 s. Use an uncertainty of 0.2 s for the stopwatch. **Calculate** the mean and uncertainty, and report your final answer correctly. (2 marks)

How do you calculate standard deviation?

standard deviation
a measure of the amount of variation of a set of data values

When we deal with a small set of values (as is typical in senior Physics), absolute uncertainty may not be so useful. A large spread could arise because a single reading is very different from the others. An alternative is to report the **standard deviation** (SD, or sigma, σ). This is calculated using the following formula or the "stdev.s" function in Excel:

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$$

For example, the SD of the four results for the diameter of a piece of wire (0.62 mm, 0.63 mm, 0.63 mm, 0.64 mm) using "stdev.s" is 0.008 mm. Therefore, the results could also be reported as 0.63 ± 0.008 mm. A low SD means the data points tend to be close to the mean of the dataset and your data is precise; a high SD indicates that the data points are spread out over a wider range of values.

outlier
a value data point that is much smaller or larger than most of the other values in a set of data; often quantified as being greater than three standard deviations from the mean

SD is highly unlikely to appear in your data test, but you could use it in your student experiment to identify errors and **outliers**, and to evaluate the reliability of your results. Expressing uncertainty using absolute uncertainty is more common in physics.

How do you calculate percentage uncertainty?

Once you have collected the data, you may need to transform it in some way or apply a formula to it to calculate other quantities. For example, if you gather mass, time or voltage data, you can calculate speed, acceleration, density or resistance, and compare these calculated values with accepted (true) values.

If you add, subtract, multiply or divide the data, you can propagate uncertainty through your calculations. First, you may need to get the data into the right form. Often, this can involve different units and scales, so absolute uncertainty may no longer be useful. You will need to make adjustments to take into account the calculations you have performed, and the value you use may instead be percentage uncertainty.

Percentage uncertainty (δ%) is calculated by dividing the absolute uncertainty by the observed measurement and multiplying the result by 100 to give a percentage:

$$\delta\% = \frac{\text{absolute uncertainty } (\delta\bar{x})}{\text{observed measurement } (x_O)} \times 100$$
$$= \frac{\delta\bar{x}}{x_O} \times 100$$

percentage uncertainty
an indicator of uncertainty in which the range of values for a measurement result (the absolute uncertainty) is expressed as a percentage of the observed measurement

Reporting percentage uncertainty

When percentage uncertainty is reported, the format is:

1. observed measurement (best estimate)
2. unit symbol
3. ± the percentage uncertainty.

Study tip

In the QCAA Physics formula and data book, this equation is shown as:
Percentage uncertainty (%)
$= \frac{\text{absolute uncertainty}}{\text{measurement}} \times 100\%$

Worked example 1.7C

Calculating percentage uncertainty

Calculate the percentage uncertainty for each measurement and report your final answer correctly.

a A digital multimeter reading of 0.818 V and an absolute uncertainty of ±0.001 V (2 marks)

b A voltmeter reading of 0.75 ± 0.05 V (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the percentage uncertainty. The questions are worth 2 marks each, so we must correctly apply the formula to complete the calculation and express the answer correctly.
Step 2: Select the appropriate equation and gather any data required.	$\delta\% = \frac{\delta\bar{x}}{x_O} \times 100$ **a** $x_O = 0.818$, $\delta\bar{x} = 0.001$ **b** $x_O = 0.75$, $\delta\bar{x} = 0.05$
Step 3: Substitute the known values into the formula and solve for the final answer.	**a** $\delta\% = \frac{0.001}{0.818} \times 100$ $= \pm 0.122\%$ **b** $\delta\% = \frac{0.05}{0.75} \times 100$ $= \pm 6.67\%$
Step 4: Report your final answer as the best estimate (mean), uncertainty, and the correct units.	**a** δ% = 0.122, so, using the rule, round to 2 s.f. to become 0.13, then express 0.812 to the same number of d.p. as $\delta\bar{x}$ (2 d.p). Report final answer as 0.82 V ± 0.13%. (1 mark for correct uncertainty; 1 mark for correct reporting) **b** δ% = 6.67, so, using the rule, round to 1 s.f. to become 7%, then express 0.75 to the same number of d.p. as $\delta\bar{x}$ (2 d.p.). Report final answer as 0.75 V ± 7%. (1 mark for correct uncertainty; 1 mark for correct reporting)

Your turn

Calculate the percentage uncertainty for each measurement and report your final answer correctly.

a A digital multimeter reads the resistance of nichrome wire as 24.5 Ω with an absolute uncertainty of 0.1 Ω. (2 marks)

b A spring balance gives a reading of 15.5 ± 1 N. (2 marks)

How do you calculate percentage uncertainty for temperature?

Study tip

When calculating percentage uncertainty for repeated measurements, "measured value", "observed measurement", "observed value", "experimental value" and "best estimate" are the same thing.

All percentage uncertainty calculations for temperature readings should be completed in kelvin (K). This means you need to convert all Celsius temperatures (T_C) to kelvin (T_K) before performing the calculation. The formula to convert Celsius to kelvin is $T_K = T_C + 273$.

To understand why, let's consider the following: what would the percentage uncertainty be for the temperature of an ice cube measured at 0.0°C? If the absolute uncertainty was ±0.5°C, the percentage uncertainty would be $\frac{0.5}{0.0} \times 100$ = infinity. This is clearly wrong and does not make sense.

Study tip

You can convert between absolute and percentage uncertainty by rearranging the δ% formula.

TABLE 2 Estimating uncertainty with temperature

Measurement	Celsius	Kelvin
Best estimate and absolute uncertainty	5.0 ± 0.5°C	278 ± 0.5K
Percentage uncertainty	$\frac{0.5}{5.0} \times 100 = 10\%$. This is incorrect.	$\frac{0.5}{278} \times 100 = 0.2\%$. This is correct.

How is uncertainty propagated in calculations?

Uncertainty can be propagated through calculations. Because you will be multiplying, dividing, adding and subtracting data, the uncertainty needs to have mathematical operations performed on it as well. Therefore, when you complete calculations with multiple measurements, you need to consider the uncertainties of each measurement. The following rules apply:

- When you add or subtract measurements, add **absolute** uncertainties (see Worked example 1.7D).
- When you multiply or divide measurements, add **percentage** uncertainties (see Worked examples 1.7E and 1.7F).

Worked example 1.7D

Calculating uncertainties involving subtraction

The output of a transformer was used to heat 150.0 g of water at 25.0°C, which rose to 33.5°C. The temperature was measured with an analogue thermometer with 1°C divisions. **Calculate** the change in temperature and the percentage uncertainty of the result, and report it correctly. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the change in temperature and percentage uncertainty. The question is worth 3 marks, so we must gather the correct information and correctly apply the formula to complete the calculations.
Step 2: Select the appropriate equation(s) and gather any data required. An analogue (print) scale is used, so the uncertainty is the half-scale division.	$\delta\% = \frac{\delta\bar{x}}{x_O} \times 100$ uncertainty = $\frac{\text{resolution}}{2} = \frac{1}{2} = \pm 0.5$°C $x_O = \Delta T = T_f - T_i$ $T_f = 35.5$°C, $T_i = 25.0$°C

Think	Do
Step 3: Substitute the known values into the formula(s) and solve for the final answer(s). Subtraction is used to obtain this value, so the absolute uncertainties must be added. The absolute and percentage uncertainties are rounded to 2 s.f. as they start with a "1".	$\Delta T = 35.5 - 25.0$ $= 8.5°C$ (1 mark) $\delta\bar{x} = 0.5 + 0.5$ $= \pm 1.0°C$ $\delta\% = \frac{1.0}{8.5} \times 100$ $= \pm 11.76\%$ rounded to 12% (1 mark)
Step 4: Report your final answer as the best estimate (mean), uncertainty, and the correct units. Note: the absolute uncertainty is to 2 s.f. (1.0°C) as it starts with a "1", which means it has 1 d.p. The best estimate must also then be expressed to 1 d.p. to match, so it is 9°C. The percentage uncertainty has the same number of s.f. (2) as the absolute uncertainty, so it is 12%.	$9 \pm 1.0°C$, which is expressed as a percentage as $9°C \pm 12\%$ (1 mark)

Your turn

A rectangular loop of wire measures 23.5 ± 0.5 mm long and 15.0 ± 0.5 mm wide. **Calculate** the perimeter (3 s.f.) and percentage uncertainty, and express your answer in "best estimate, percentage uncertainty, unit symbol" format. (3 marks)

Worked example 1.7E

Calculating uncertainties involving multiplication

A golf ball has a mass (as measured on a digital scale) of 46.93 ± 0.01 g. The golf ball is raised to a vertical height of 755.0 mm (as measured on a metre ruler that is calibrated in 1 mm divisions). **Calculate** the gravitational potential energy (E_P) of the ball ($E_P = mgh$ and $g = 9.807\,m\,s^{-2}$). Report your answer in the correct format, showing the percentage uncertainty. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the gravitational potential energy and percentage uncertainty. The question is worth 3 marks, so we must correctly apply the formulas to complete the calculations.
Step 2: Select the appropriate equation(s) and gather any data required. A mix of digital and analogue scales were used, so you will use the resolution and half-scale division.	$\delta\% = \frac{\delta\bar{x}}{x_O} \times 100$ uncertainty (digital scale) = resolution $= \pm 0.01$ g uncertainty (metre ruler) $= \frac{\text{resolution}}{2} = \frac{1}{2} = \pm 0.5$ mm $x_O = E_P = mgh$, $g = 9.807\,m\,s^{-2}$ $m = 46.93$ g $= 46.93 \times \frac{1\,kg}{1{,}000\,g}$ $= 0.04693$ kg $h = 755.0$ mm $= 755.0 \times \frac{1\,m}{1{,}000\,mm}$ $= 0.7550$ m

Think	Do
Step 3: Convert mass and height to SI units. In this case divide the mass in grams (g) by 1,000 to get kilograms (kg); and divide the height in millimetres (mm) by 1,000 to get metres (m). Substitute the known values into the formula(s) and solve for the final answer(s). Multiplication is used, so the percentage uncertainties must be added.	$E_p = 0.04693 \times 9.807 \times 0.7550$ $= 0.3475$ J (4 s.f.) (1 mark) $\delta\%$ (digital scale) $= \frac{0.01}{46.93} \times 100$ $= \pm 0.085\%$ $\delta\%$ (metre ruler) $= \frac{0.5}{755.0} \times 100$ $= \pm 0.066\%$ $\delta\% = 0.085 + 0.066$ $= \pm 0.15\%$ (1 mark)
Step 4: Report your final answer as the best estimate (mean), uncertainty, and the correct units. Note: this is a difficult one, as the uncertainty is so small. The absolute uncertainty would be $0.15\% \times 0.3475 = 0.00052$. This does not start with a "1" so use 2 s.f.; the % uncertainty would have 2 s.f. as well (= 0.15%). The best estimate should then have the same number of d.p. as the absolute uncertainty (5), but this is not reasonable as there are only 4 s.f. in each of the original measurements. The best we can do is leave the best estimate at 0.3475 and the % uncertainty as 0.15%.	0.3475 J ± 0.15% (1 mark)

Your turn

A rectangular loop of copper wire (measured using a metre ruler marked in 1 mm increments) is 34.5 mm long and 25.0 mm wide. **Calculate** its area and report the answer with the absolute uncertainty. Use the correct number of significant figures and units. (3 marks)

Worked example 1.7F

Calculating uncertainties involving division

A toboggan slides 100 m down a 30.0° incline in 9.81 s. For uncertainties in this measurement, assume the time is 9.81 ± 0.01 s and the distance is 100.0 ± 0.1 m. **Calculate** the average speed and absolute uncertainty, and report your answer in the correct format. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the speed and absolute uncertainty. The question is worth 3 marks, so we must correctly apply the formulas to complete the calculations.
Step 2: Select the appropriate equation(s) and gather any data required. The uncertainties for each measurement are provided.	$\delta\% = \frac{\delta\bar{x}}{x_o} \times 100$ uncertainty (time) = ± 0.01 s uncertainty (distance) = ± 0.1 m $x_o = v = \frac{d}{t}$ $d = 100.0$ m, $t = 9.81$ s

Think	Do
Step 3: Substitute the known values into the formula(s) and solve for the final answer(s). Division is used, so the percentage uncertainties must be added.	$v = \frac{100.0}{9.81}$ $= 10.2\,\text{m s}^{-1}$ (1 mark) $\delta\%\ (\text{time}) = \frac{0.015}{9.81} \times 100$ $= \pm\ 0.1\%$ $\delta\%\ (\text{distance}) = \frac{0.1}{100.0} \times 100$ $= \pm\ 0.1\%$ $\delta\% = 0.1 + 0.1$ $= \pm\ 0.2\%$
Step 4: The question asks for absolute uncertainty, so we will need to convert it from percentage uncertainty.	$\delta\bar{x} = \frac{\delta\%}{100} \times x_o$ $= \frac{0.2}{100} \times 10.2$ $= 0.02\,\text{m s}^{-1}$ (1 mark)
Step 5: Report your final answer as the best estimate (mean), uncertainty, and the correct units. Make sure you use the correct number of significant figures in the mean. The absolute uncertainty in velocity is 0.0204, which is expressed with 1 s.f. as 0.02, as the first number is not a "1". This means the velocity should have 2 d.p. but can't as there are just 3 s.f. in the original measurements, so we leave it as $10.2\,\text{m s}^{-1}$.	$10.2 \pm 0.02\,\text{m s}^{-1}$ (1 mark)

Your turn

A cube of iron, of 62.99 ± 0.01 g mass, measures 2.000 ± 0.001 cm along each side. **Calculate** its density ($\rho = \frac{m}{V}$) in g cm^{-3} and absolute uncertainty, and report your result in the correct format. (2 marks)

How do you calculate errors?

We have seen that uncertainty is used to express the precision of a set of measurements. However, students often find their results are different from the accepted result, even though they performed the experiment as carefully as possible and read the instruments as best they could. This measurement discrepancy is known by the term "error" and is a measurement of the accuracy of a result. Error can be represented as absolute error or percentage error.

Absolute error (E_a) is the difference between the observed (measured) value (x_O) and the accepted (true) value (x_A). It is a measure of the accuracy of a result.

absolute error = measured value − accepted value

$$E_a = |x_O - x_A|$$

The straight lines (modulus signs, | |) in the equation indicate the "absolute value", which means the sign (+/−) of the answer is ignored.

Percentage error ($E\%$) is the absolute error expressed as a percentage of the accepted (or true) value.

$$\text{percentage error} = \left|\frac{\text{measured value} - \text{true value}}{\text{true value}}\right| \times 100$$

$$E\% = \left|\frac{x_O - x_A}{x_A}\right| \times 100$$

$$= \left|\frac{E_a}{x_A} \times 100\right|$$

Study tip

The term "accepted result" is also called "true value". All this means is that it is accepted by the scientific community.

absolute error
the difference between the observed (measured) value and the accepted (true) value

percentage error
the absolute error expressed as a percentage of the accepted (true) value

Study tip

Absolute and percentage uncertainty are measures of precision. Absolute and percentage error are measures of accuracy.

When you express errors, you should use the least number of s.f. used in the observed or accepted value. You are not penalised for incorrect significant figures in the external exam. The number of significant figures will not be specified in the external exam for any calculations at all, so it will be up to you to decide what is appropriate. You are safe if you use 2 or 3 s.f. in your answer.

Worked example 1.7G

Calculating absolute and percentage error

A student measured the acceleration due to gravity as $9.73\,m\,s^{-2}$. The accepted value at their location is $9.813\,m\,s^{-2}$. **Calculate** the absolute and percentage errors. Express your answer to 1 s.f. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We are being asked to calculate the absolute and percentage errors. This question is worth 2 marks, so we must correctly apply the formulas to complete the calculations.
Step 2: Select the appropriate equation(s) and gather any data required.	$E_a = \lvert x_O - x_A \rvert$ $E\,\% = \frac{E_a}{x_A} \times 100$ $x_O = 9.73\,m\,s^{-2}$, $x_A = 9.813\,m\,s^{-2}$
Step 3: Substitute the known values into the formula(s) and solve for the final answer(s).	$E_a = \lvert 9.73 - 9.813 \rvert$ $= \lvert -0.083 \rvert$ $= 0.083$ $E\% = \frac{0.083}{9.813} \times 100$ $= 0.846\%$
Step 4: Finalise your answer(s) and make sure you use the correct number of s.f. and units.	$E_a = 0.08\,m\,s^{-2}$ (1 s.f.) (1 mark) $E\% = 0.9\%$ (1 s.f.) (1 mark)

Your turn

The force acting on a 1.00 m length of nichrome wire in a particular magnetic field was found to be 8.44 N, whereas the accepted value is 9.12 N. **Calculate** the percentage error (1 s.f.). (2 marks)

Challenge

A coil of wire has a stated resistance of 330 Ω and is manufactured to a tolerance of 5%. This is the maximum percentage error. **Determine** the range of resistances that a coil of wire marked 330 Ω may have. (2 marks)

How can you present your results?

Once you have finalised your measurements and uncertainties, you can use summary tables and graphs to present the data. Both are useful to help you clearly observe relationships between variables.

Summary tables

Whereas a data table states what has been measured and presents the raw data, a summary table can bring together processed data. Sometimes, summary tables for simpler investigations will present measurements, but they generally also show values such as mean, values derived from calculations and even absolute or percentage uncertainty. For example, the data table in Figure 7 of Lesson 1.6 could be further summarised by removing the observations and only including the mean and uncertainty values for each trial. Summary tables present the most important information for later analysis.

Scatterplots

Graphs are useful to show how one quantity depends on another. On a graph, the horizontal *x*-axis is where the IV (or the cause) is plotted. The DV (or the effect of that cause) is plotted on the vertical *y*-axis. All graphs should include:

- a graph title that succinctly describes what the graph is showing (typically includes the IV and DV)
- clearly labelled axes, including units of measurement
- equally spaced units of measurement along the axes (scaling)
- axes that start at zero (where possible)
- data plotted within the confines of each axis
- distinguishing symbols, colours or keys when more than one dataset is plotted on a single graph
- absolute uncertainty (where relevant) as error bars; do not use percentage uncertainty value for errors (uncertainty) bars
- a trendline that fits between all error bars.

By analysing the shape of your graphs, you can construct arguments about the relationship between variables.

Study tip

The term "error bars" implies that they are to do with the accuracy of a data point. They are not – they are about the uncertainty of results around that data point. Some teachers prefer to call them "uncertainty bars" although Excel calls them "error bars".

How do you graph linear relationships?

A linear relationship produces a straight line when graphed. The variables being measured can be **directly proportional** (i.e. the straight line goes through the origin (0, 0)) or not directly proportional (i.e. the straight line does not go through (0, 0)).

directly proportional relationship that is linear and the line goes through the origin (0, 0)

Consider an experiment to determine how much a certain rubber band stretches when masses are hung vertically from it. The single measurements are shown in Table 3.

TABLE 3 Data obtained from a rubber band experiment

Independent variable: mass, *m* (g)	0	20	40	60	80
Dependent variable: stretch, *d* (mm)	0	10	21	28	42

When the points are plotted, it is obvious that the relationship is linear and directly proportional. Note that each point is plotted as a dot (or cross) that is big enough to still be seen once a line of best fit or **trendline** is drawn. The trendline has as many points on the line as possible. There are usually some points that do not sit exactly on the line, so the line should be drawn with the same number of points below it as above it.

trendline a line of best fit which shows the general direction that a group of points seem to follow

Scientists and engineers use a complex mathematical procedure to determine where the trendline should be. Any point that is a long way out of place is called an outlier or an **anomaly**, and could be false. Outliers "lie outside" (are much smaller or larger than) most of the other values in a set of data. It is a legitimate data point originated from a real observation.

anomaly a false data point often the result of a faulty observation or wrong equipment

There are mathematical ways of deciding whether the value is within the acceptable range for that set of data points. In general, outliers are discarded if they are more than three standard deviations away from the mean. An anomaly is something that deviates from what is standard, normal, or expected. It is not a legitimate value and is produced by an artificial process (such as mistakes in the method, wrong equipment or a badly recorded result).

Any individual outlier or anomaly should be noted and the reasons for its existence can be discussed – but it should not be used for the trendline. If the trendline is straight and goes through the origin (as in Figure 3), it takes the general form of $y \propto x$. The proportional sign ($\propto$) can be replaced by an equals sign and a constant (m) to give:

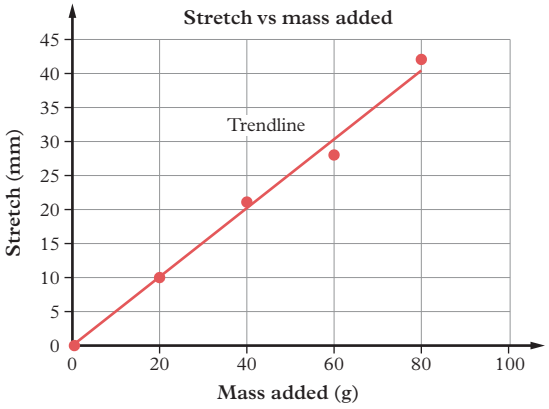


FIGURE 3 Graph showing a directly proportional relationship between stretch and mass added

$$y = mx \text{ or } y = mx + c$$

gradient
the slope of a graph

where x and y are the variables, m is the **gradient** or slope of the line, and c is the y-intercept or point where the line cuts the y-axis when $x = 0$.

Study tip

You might find it easier to remember the gradient as "rise over run", where "rise" refers to the change in the y-axis values and "run" refers to the change in the x-axis values.

In the graph of stretch versus mass (Figure 3), the intercept c is zero. The gradient is found by dividing the change in the y value by the change in the x value for the same section of the line. This can be written as:

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

The gradient of the graph in Figure 3 is given by $m = \frac{40 - 0}{80 - 0} = 0.5\,\text{mm}\,\text{g}^{-1}$, and the intercept c is zero. Thus, for every 1 g change in mass (x-axis), the rubber band changes by 0.5 mm in length (y-axis).

There are four important points to remember:

- Don't force the line to go through the origin; it should go through the middle of the data points, with as many points above it as below it.
- Use the x and y coordinates of points on the trendline to calculate the gradient.
- Use points as widely separated as possible to get the most accurate value for the gradient.
- Gradients have the y-axis unit divided by the x-axis unit.

Worked example 1.7H

Determining the relationship between variables

The position of a car rolling down an inclined road was noted every 5 seconds and the data in Table 4 was obtained.

TABLE 4 Data obtained from the experiment

Time, t (s)	0	5	10	15	20
Distance, s (m)	16	23	34	41	50

a **Sketch** a graph of the data with the independent variable on the x-axis. (3 marks)

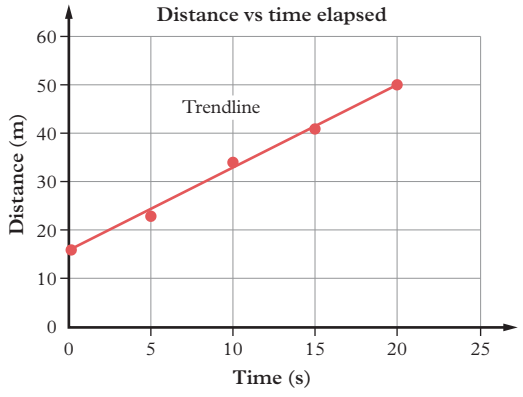
b **Calculate** the average velocity (gradient or slope). (2 marks)

c **Identify** the y-intercept. (1 mark)

d **Derive** the equation for the line. (1 mark)

e **Explain** whether or not the graph is directly proportional. (1 mark)

f **Determine** the distance travelled at 25 s, assuming the velocity is constant. (1 mark)

Think	Do
Step 1: Look at the cognitive verb(s) and mark allocation to determine what the question(s) are asking you to do.	A variety of cognitive verbs are used in this question. "Sketch" means to represent using a diagram or graph. "Calculate" means to determine or find a number or answer by using mathematical processes. "Identify" means to locate or recognise and name. "Derive" means manipulate a mathematical relationship to give a new equation. "Explain" means to describe in more detail. "Determine" means to establish after calculation. These questions are worth a variety of marks.
Step 2: To sketch the graph, draw and label the axes with the IV (time) on the x-axis and DV (distance) on the y-axis. Include units! Plot each data point and draw a trendline.	**a**  (3 marks)
Step 3: To find the gradient, select the appropriate equation. Determine the change in y and change in x for two widely separated points on the line. You can use $x = 0$ as one of the points.	**b** $m = \frac{\Delta y}{\Delta x}$ $\Delta y = 50 - 16$ $= 34$ $\Delta x = 20 - 0$ $= 20\,\text{s}$ (1 mark)
Step 4: Substitute the known values into the formula and solve for the gradient.	$m = \frac{34}{29}$ $= 1.7\,\text{m s}^{-1}$ (1 mark)
Step 5: Read the intercept off the y-axis and include the units. We can also obtain this value from Table 4.	**c** $c = 16\,\text{m}$ (1 mark)
Step 6: To find the equation for the straight line, select the appropriate equation. You can substitute y and x for DV and IV, respectively.	**d** $y = mx + c$ $s = 1.7t + 16$ (1 mark)
Step 7: Directly proportional means the graph has an intercept of zero ($c = 0$) and a relationship $y = mx$.	**e** The graph is linear but not directly proportional as it does not go through (0, 0). The y-intercept occurs at 16 m. (1 mark)
Step 8: Use the equation for the line and substitute in the value for t. Two s.f. are appropriate based on the readings from the graph.	**f** $s = 1.7t + 16$ $= (1.7 \times 25) + 16$ $= 58.5\,\text{m}$ (59 m to 2 s.f) (1 mark)

Your turn

Students performed an experiment in which they connected a power supply across the primary coil of a transformer. As the voltage (V) in volts (V) was increased, they recorded the current (I) in amperes (A) passing through the resistor. Their results are shown in Table 5.

TABLE 5 Data obtained from the experiment

Voltage, V (V)	2.0	4.0	6.0	8.0	10.0
Current, I (A)	7.7	13.6	20.8	26.2	33.0

a **Sketch** a graph of the data with the independent variable on the x-axis. (3 marks)

b **Calculate** the gradient and state its units. (2 marks)

c **Identify** the y-intercept. (1 mark)

d **Derive** the equation for the line using symbols V and I for the variables. (1 mark)

e **Explain** whether or not the graph is directly proportional. (1 mark)

f **Determine** the current at 9.0 V. (1 mark)

How do you graph non-linear relationships?

A non-linear relationship is not a straight line when graphed. The most common non-linear relationships you will meet in physics are power, exponential and logarithmic relationships, as outlined in Table 6.

TABLE 6 Examples of common non-linear relationships

Type of relationship	Equation(s)
Power	$y \propto x^a$ • inverse ($a = -1$): $y \propto x^{-1}$ or $y \propto \frac{1}{x}$ • inverse-square ($a = -2$): $y \propto x^{-2}$ or $y \propto \frac{1}{x^2}$ • square or parabolic ($a = 2$): $y \propto x^2$ • square root ($a = \frac{1}{2}$): $y \propto x^{\frac{1}{2}}$ or $y \propto \sqrt{x}$
Exponential	$y \propto e^{kt}$ or $y = y_0 e^{kt}$ (natural exponential) where y_0 is the y-intercept, e is the "natural" base (2.718) and k is a constant
Logarithmic	$y \propto 10^x$

Power relationships

Inverse relationship

In an inverse relationship, $y \propto x^{-1}$ or $y \propto \frac{1}{x}$, where the power a is –1. Consider a case in which the volume of gas in a syringe is measured as the pressure on the syringe is increased by adding weights to the plunger. This is an inverse relationship (also called an inversely proportional relationship), shown in Figure 4.

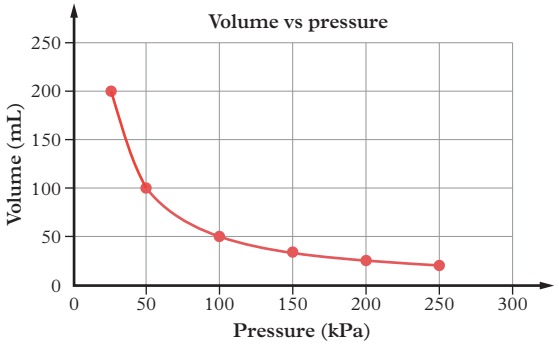


FIGURE 4 Graph showing an inverse relationship

Inverse-square relationship

In an inverse-square relationship, $y \propto x^{-2}$ or $y \propto \frac{1}{x^2}$, where the power a is –2. It looks similar to an inverse relationship but has a much sharper bend. This type of relationship is very common in physics. For example, the variation in gravitational force with distance is given by $F \propto \frac{1}{d^2}$.

There is also the inverse-cube relationship, $y \propto x^{-3}$ or $y \propto \frac{1}{x^3}$, where the power a is –3. This is not as common in physics, but you may find it explains the relationship for force between two magnets at certain distances. Figure 5 shows both the inverse square (solid line), and the inverse cube (dashed line) relationships, so you can see the inverse cube is sharper.

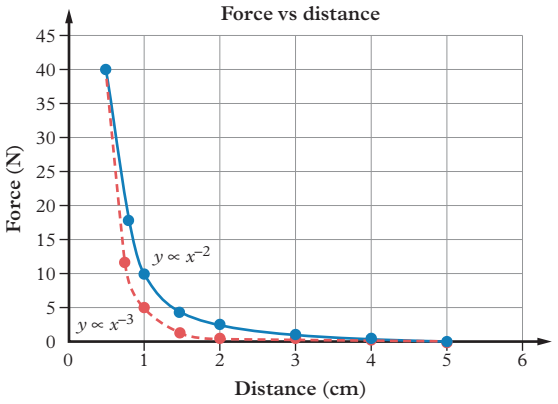


FIGURE 5 Graph showing inverse-square and inverse-cube relationships

Square (parabolic) relationship

In a square or parabolic relationship, $y \propto x^2$, where the power a is 2. It is the most common non-linear relationship you will encounter, along with inverse. An example is the relationship between the distance a rock has fallen from the top of a cliff and the time elapsed. The data in Table 7 shows these variables and Figure 6 shows a graph of the distance against the time.

TABLE 7 Parabolic data

Time, t (s)	0	1	2	3	4	5
Distance, s (m)	0	4	16	36	64	100

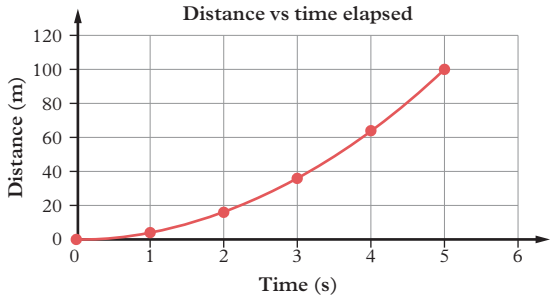


FIGURE 6 Graph showing a parabolic relationship

Other phenomena that exhibit parabolic relationships are the paths of comets (except Halley's Comet, which is elliptical), and the shape of curved mirrors in telescopes and projectiles (e.g. arrows in flight).

Square root relationship

In a square root relationship, $y \propto x^{\frac{1}{2}}$ or $y \propto \sqrt{x}$, where the power a is $\frac{1}{2}$. A square root relationship can be demonstrated by letting a ball roll down a ramp. The speed at the bottom of the ramp, v, varies with the square root of the height, h, of the ramp ($v \propto \sqrt{h}$), as shown in Figure 7.

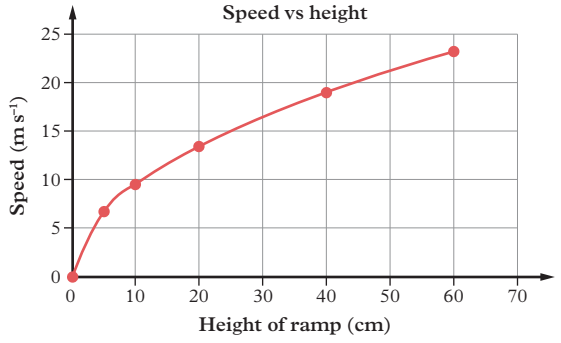


FIGURE 7 Graph showing a square root relationship

Exponential relationships

The exponential relationship is given by $y = a^x$. The "natural" exponential function is a version of this: $y \propto e^{kt}$ or $y = y_0 e^{kt}$, where the a value is e, the "natural" base (2.718), the symbol k is a constant ($-k$ for decay, $+k$ for growth), and t is time elapsed. Physics has some quantities that are related exponentially. For example, in a radioactive substance, the number of radioactive atoms remaining follows natural exponential decay (as long as the remaining number of atoms is large). The breakdown (decay) of a radioactive substance is given by intensity $I = e^{-kt}$. This is shown in Figure 8.

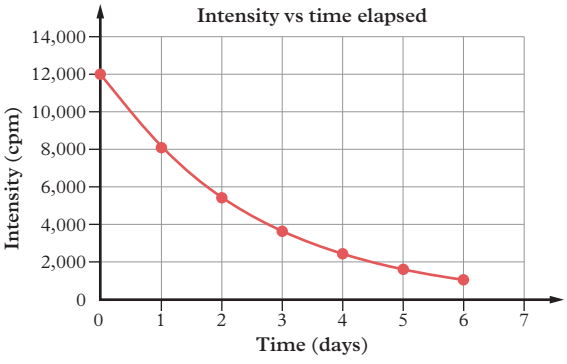


FIGURE 8 Graph showing a natural exponential decay relationship

Logarithmic relationships

Study tip

Make simple labelled drawings of each graph shape. This will help you to identify them quickly when you plot your own data or see them in an exam. These can also be found in the summary at the end of this module.

In a logarithmic relationship, $y \propto 10^x$ or $x = \log_{10} y$. Let's consider an example. When it is quiet in your laboratory, you could hear a pin drop onto a bench from the other side of the room. The amount of energy released is very small, and human hearing is very sensitive when things are quiet. The human ear can also hear noise, such as an explosion, that could be a trillion (10^{12}) times louder. This is impossible to capture meaningfully with a uniform scale. The sensitivity of human hearing is good for low power sounds but reduced at high power, as shown in Figure 9. Sound intensity (y-axis) is in decibels (dB), which describes the relative intensity of a sound based on a logarithmic scale containing values ranging from 0 to 194. So, the y-axis has intervals of 10^{88}, 10^{89} and so on.

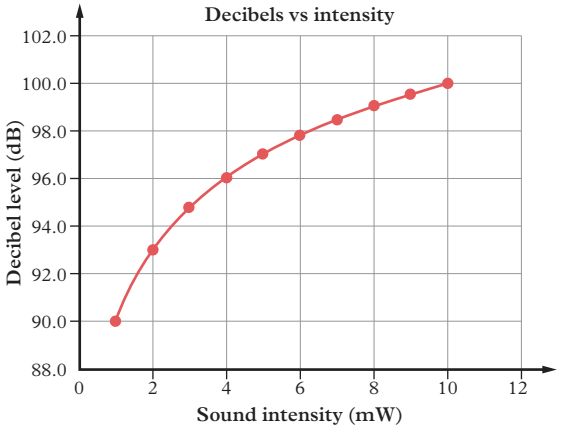


FIGURE 9 Graph showing a logarithmic relationship

How do you linearise a graph?

Even though a graph may have a particular shape, sometimes we cannot be sure of the exact relationship between variables. For example, $y \propto \frac{1}{x}$ and $y \propto \frac{1}{x^2}$ are very similar and therefore hard to distinguish. The relationship could also be in between them, such as $y \propto \frac{1}{x^{1.5}}$. The only way to tell is to plot them in the form of $y = mx$, where x is the not the original x-value but the transformed value (e.g. x^2). This is called **linearising** a graph.

linearising
the process of transforming non-linear data by applying a mathematical function to one of the variables so that the relationship between the variables becomes closer to a straight line. Each type of relationship has a different method for linearising. If the linearised graph is indeed linear, then we can say that the relationship has been correctly identified.

Linearising inverse relationships

If you already have a straight-line relationship such as $y \propto x$, then a graph of y versus x is a straight line. However, if you have $y \propto \frac{1}{x}$, then a graph of y versus x is non-linear. Let's look at this using the data in Table 8 and plotting y versus x. The graph of V versus P is non-linear (Figure 10) and instead looks like a $y \propto \frac{1}{x}$ graph.

TABLE 8 Data obtained for change in pressure with volume that has a suspected inverse relationship

Pressure, P (kPa)	x	1	2	3	4	5
Volume, V (mL)	y	3.0	1.5	1.0	0.8	0.6

You can try to linearise the graph ("undo" the inverse) by calculating $\frac{1}{x}$ (Table 9) and plot y versus $\frac{1}{x}$ (Figure 11). If it is a straight line, then you have confirmed the relationship $y \propto \frac{1}{x}$ (which is $V \propto \frac{1}{P}$ in this case). If the $y \propto \frac{1}{x}$ graph is still curved, then it may be a $y \propto \frac{1}{x^2}$ relationship.

TABLE 9 Data obtained for change in pressure with volume after inverse transformation

Pressure, P (kPa)	x	1	2	3	4	5
$\frac{1}{P}$ (kPa^{-1})	$\frac{1}{x}$	1.00	0.50	0.33	0.25	0.20
Volume, V (mL)	y	3.0	1.5	1.0	0.8	0.6

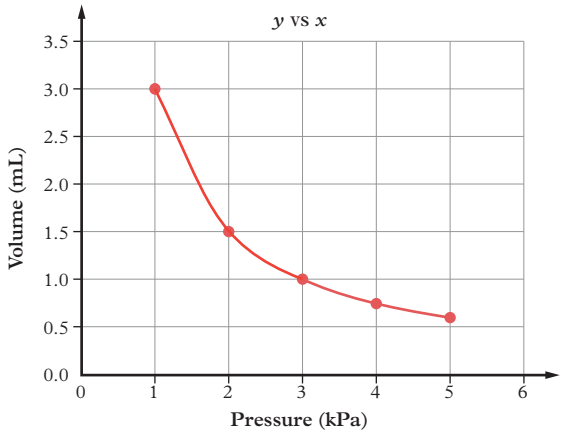


FIGURE 10 The graph of V vs P appears to show an inverse relationship.

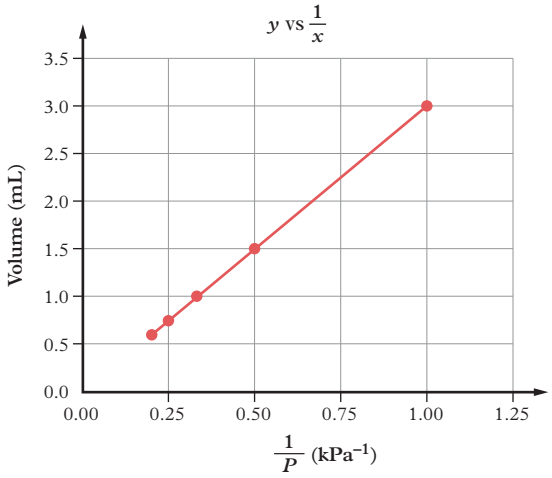


FIGURE 11 The V vs $\frac{1}{P}$ graph of the data gives a straight line, so it is a $y \propto \frac{1}{x}$ relationship.

Linearising inverse-square relationships

Consider the force between two magnets at various distances (Table 10). The graphs of y versus x and y versus $\frac{1}{x^2}$ are shown in Figure 12.

TABLE 10 Data obtained for force and distance between two magnets after inverse-square transformation

Distance (mm)	x	0.5	1.0	2.0	3.0	4.0	5.0
Force (N)	y	12.0	3.0	0.8	0.3	0.2	0.1
$\frac{1}{\text{distance}^2}$ (mm^{-2})	$\frac{1}{x^2}$	4.00	1.00	0.25	0.11	0.06	0.04

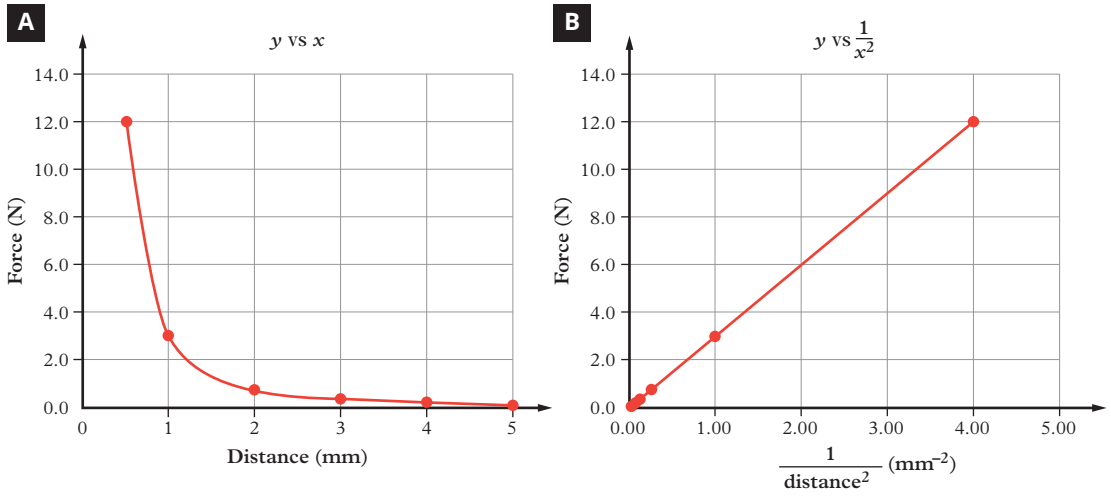


FIGURE 12 (A) A y vs x graph of the data does not give a straight line. (B) The straight line indicates it is a $y \propto \frac{1}{x^2}$ relationship.

A graph of y versus x is non-linear and looks more like a $y \propto \frac{1}{x^2}$ graph. However, a graph of y versus $\frac{1}{x^2}$ is a straight line, so this confirms it is a $y \propto \frac{1}{x^2}$ relationship.

Linearisation of parabolic relationships

Consider the data of a car accelerating from the traffic lights (Table 11).

TABLE 11 Data obtained for the speed of a car over time that has a suspected square relationship

Time (s)	x	0	1	2	3	4	5
Speed (m s^{-1})	y	0	2	8	18	32	50

The graph of y versus x is shown in Figure 13A. It looks very much like a $y \propto x^2$ relationship. To confirm this, we would linearise the graph by calculating x^2 and plotting y versus x^2 (Figure 13B).

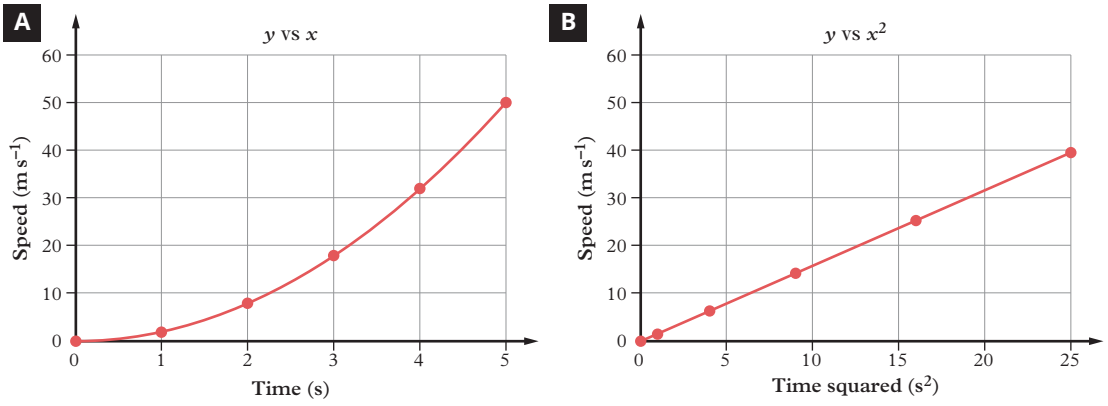


FIGURE 13 (A) The y vs x graph of the data does not give a straight line. (B) The straight line indicates it is a $y \propto x^2$ relationship.

Linearisation of exponential relationships

You will have met exponential relationships in the Unit 1 nuclear physics module. You will not meet them in the external exam as they are not a part of the subject matter for Units 3 and 4. However, they are included here for completeness as you may find them necessary in your student experiment or research investigation.

Radioactive substances decay in an exponential fashion, which means the rate of decrease in the y value changes with the x value. The change in y starts off fast at a low value of x but then decreases as x gets larger. Here is an example of a radioactive substance whose radiation intensity was measured over a period of 14 days and plotted in the graph (Figure 14).

Excel suggests it is an exponential relationship by the base e in the equation for the trendline. If we now plot the natural logarithm (ln) of intensity, we get a linearised graph (Figure 15).

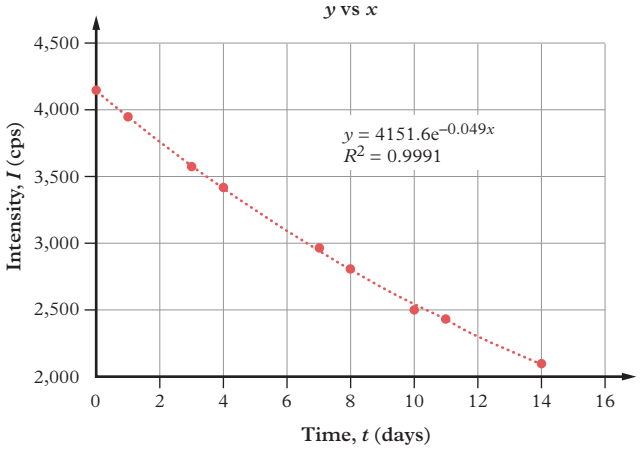


FIGURE 14 Graph of intensity vs time for a radioactive specimen

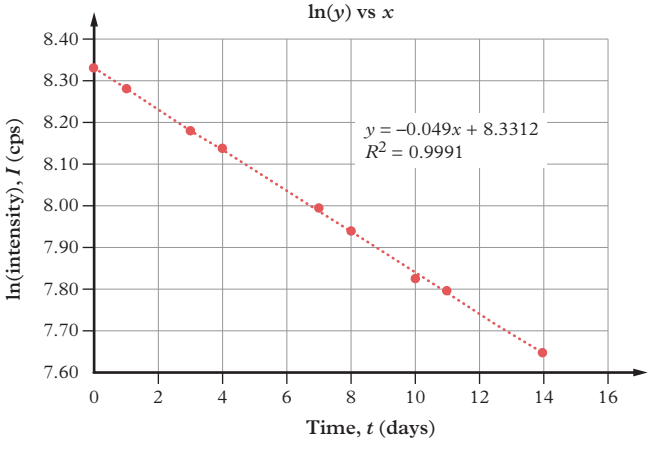


FIGURE 15 Linearised graph of radioactive decay

We can see it is linear because the equation for the trendline is in the pattern $y = mx + c$.

How can you infer data by extrapolation and interpolation?

Extending and reading a graph beyond the last plotted point is called **extrapolation**. Inferring a reading between plotted points is called **interpolation**. Although the data points in Figure 16 are only at 0, 27 kJ, 53 kJ, 81 kJ, 108 kJ and 135 kJ, you can extrapolate the trendline past 135 kJ to 160 kJ and read off the temperature as 66.5°C. You can also interpolate the data to read off values between points. For example, at 100 kJ, the temperature is 51°C. This is commonly done in Physics questions.

extrapolation
the prediction of values beyond the range of data points by extending the trendline

interpolation
the prediction of values between data points using a trendline

Study tip

Be careful not to extrapolate too far beyond the last point. The further out you go, the less valid and less reliable the estimate.

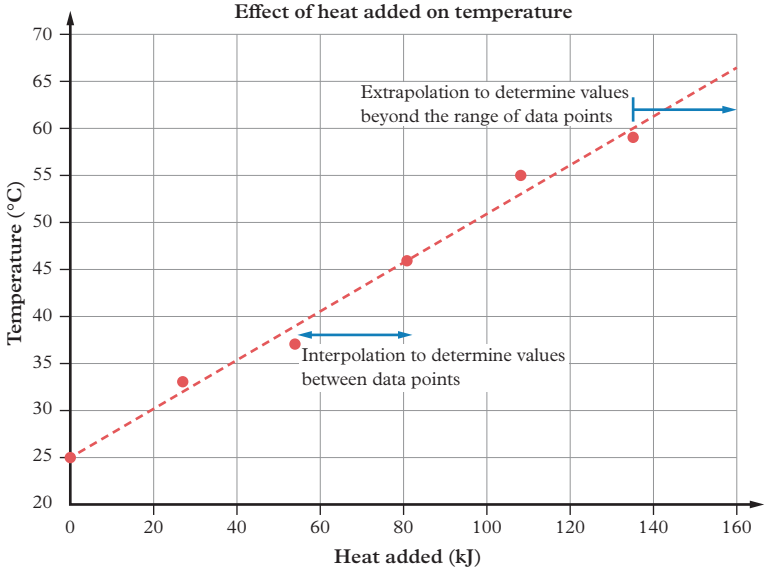


FIGURE 16 The temperature of water vs the amount of heat energy added can be extrapolated or interpolated to infer values.

How are error bars used to represent uncertainty?

The measurement uncertainty associated with a particular mean value can be easily displayed on a graph using error bars. These are vertical or horizontal lines that are added to each data point and represent the absolute uncertainty at that point. For example, an experiment is carried out in which a ball is dropped from a height of 5.0 m, and the displacement recorded every 0.20 s. This was then repeated two more times.

The graph in Figure 17A shows vertical error bars for the measurement of displacement (y-axis). For most experiments, the uncertainty in the IV (x-axis) will be very low and can be neglected. The trendline should be midway between the caps of the error bar. If you use zero as the IV for the first trial, the average would be $\frac{(0 + 0 + 0)}{3} = 0$. The uncertainty using the $\frac{(x_{max} - x_{min})}{2}$ formula would be $\frac{(0 + 0)}{2} = 0$. This says there is no uncertainty. However, there is still the scale reading uncertainty from the measuring instrument and that will be the value of a half-scale division for an analogue device, or the resolution for a digital device. In this experiment, a metre ruler graduated in 1 mm increments was used, so the uncertainty is ±0.5 mm or ± 0.005 m. This is too small to show up on the error bar for the (0, 0) point.

The graph in Figure 17B shows a graph of the velocity at each time interval, which was calculated using the formula $v = 2v_{av} = \frac{2s}{t}$. A trendline (red line) has been drawn and its calculated gradient is 9.9 m s^{-2}. Two other trendlines – the maximum and the minimum trendlines – have been added.

maximum trendline
a line of best fit of maximum gradient within the bounds of the error bars

The **maximum trendline** has the highest possible slope. It connects the bottom of the error bar of the first data point (0, 0) with the top of the error bar for the last data point (10.9, 1.00) but must not go beyond any of the error bars in between. It shows the maximum gradient.

The **minimum trendline** has the smallest possible slope. It connects the top of the error bar of the first data point (0, 0) with the bottom of the error bar of the last data point (8.9, 1.00), but must not go beyond any of the error bars in between. It shows the minimum gradient.

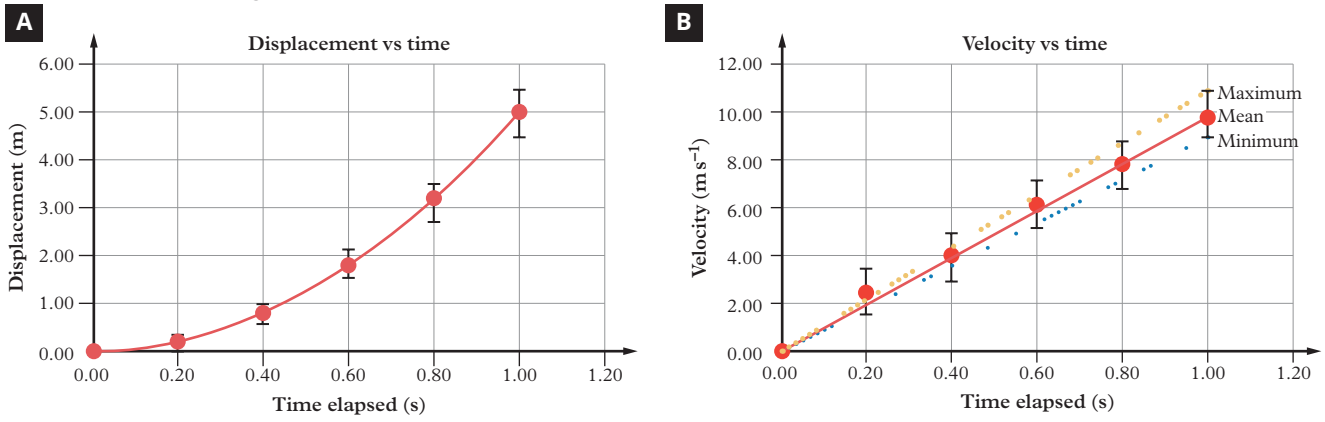


FIGURE 17 (A) Vertical error bars show the zone of uncertainty for the displacement values. (B) Maximum and minimum trendlines can be added to the graph to show the uncertainty in the gradient.

minimum trendline
a line of best fit of minimum gradient within the bounds of the error bars

You can then calculate the maximum and minimum gradients using any part of these trendlines. It is good practice that you use points spaced as widely apart as possible, otherwise you may lose a mark in the external exam. Ideally, try to use the full line between the origin (0 s) and 1.00 s. Remember that the error bar on the (0, 0) point at the origin is too small to be seen.

- The maximum trendline (yellow) gradient is $\frac{(10.9 - 0.0)}{(1.00 - 0.00)} = \frac{10.9}{1.00} = 10.9\,\text{m s}^{-2}$.
- The minimum trendline (red) gradient is $\frac{(8.9 - 0.0)}{(1.00 - 0.00)} = \frac{8.9}{1.00} = 8.9\,\text{m s}^{-2}$.

The uncertainty in the gradient is calculated using the following formula:

$$\delta\bar{x} = \pm\frac{(x_{max} - x_{min})}{2}$$
$$= \pm\frac{(10.9 - 8.9)}{2}$$
$$= \pm 1.0\,\text{m s}^{-2}$$

This can also be expressed as percentage uncertainty:

$$\delta\,\% = \frac{\delta\bar{x}}{x_0} \times 100$$
$$= \frac{1.0}{9.90} \times 100$$
$$= 10.1\%$$

Thus, the gradient can be stated using absolute uncertainty as $m = 9.9 \pm 1.0\,\text{m s}^{-2}$. If using percentage uncertainty, $m = 9.9\,\text{m s}^{-2} \pm 10\%$.

The gradient in this case is the experimental value for acceleration due to gravity of $9.9\,\text{m s}^{-2}$ with a minimum of $8.9\,\text{m s}^{-2}$ and a maximum of $10.9\,\text{m s}^{-2}$. We could say that the experiment was accurate, as the accepted value of $9.8\,\text{m s}^{-2}$ is within our experimental range. However, as the range is large, we could also say that there is a lot of uncertainty in the measurements (a lack of precision, or that the data is "imprecise") that detracts from the result. It is thus better to rely on absolute and percentage error to determine accuracy.

Check your learning 1.7

Check your learning 1.7: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Define** the following terms: absolute uncertainty, percentage uncertainty, precision, accuracy, absolute error, percentage error. (6 marks)

2 A student obtained a value of 9.74 m s^{-2} for the acceleration due to gravity. The accepted value is 9.81 m s^{-2} at that location. **Calculate** the

- **a** absolute error (2 marks)
- **b** percentage error. (2 marks)

3 The time for a cart to travel down an incline of length 80.0 cm was measured in triplicate and found to be 1.79 s, 1.85 s and 1.72 s. **Calculate** the

- **a** mean value for the time (1 mark)
- **b** absolute uncertainty (2 marks)
- **c** percentage uncertainty. (2 marks)

4 A student carries out an investigation to measure the time taken for 10 complete revolutions of a ball swung in a horizontal circle. The following values are obtained: 3.1 s, 3.8 s, 3.3 s, 4.1 s and 3.4 s. **Calculate** the absolute and percentage uncertainty for one swing. (5 marks)

5 **Explain** whether it is usual to plot the independent variable on the vertical axis or the horizontal axis. (2 marks)

6 **Define** the following terms: anomaly, outlier, relationship, trendline, line of best fit. (5 marks)

7 **Explain** what it means to "linearise" a relationship. (2 marks)

8 Students obtained the relationship $y \propto x$. **Explain** whether there is any need to linearise it. (1 mark)

Analytical processes

9 **Compare** "precision" and "accuracy". (2 marks)

10 A digital ammeter shows the current in a circuit to be 0.10 A. **Determine** the percentage uncertainty in the value of I^2. (2 marks)

11 A Physics textbook measures 1.98 ± 0.01 cm by 28.05 ± 0.05 cm by 21.70 ± 0.05 cm. Its mass is 1.205 ± 0.001 kg and it has 502 pages. Use these measurements to **determine** (including the percentage uncertainty of each result)

- **a** the surface area of the front cover in m^2 (4 marks)
- **b** the total external surface area in m^2 (4 marks)
- **c** the volume in m^3 (3 marks)
- **d** the thickness of one page in mm (2 marks)
- **e** the density in kg m^{-3}. (4 marks)

12 Two groups of students were measuring the acceleration due to gravity by projecting a ball from a measured height and timing how long it took to reach the ground. Their results are shown in the table.

Group	Height, s (m)	Time, t (s)		
		Trial 1	Trial 2	Trial 3
A	0.50	0.37	0.44	0.50
B	2.00	0.75	0.87	0.94

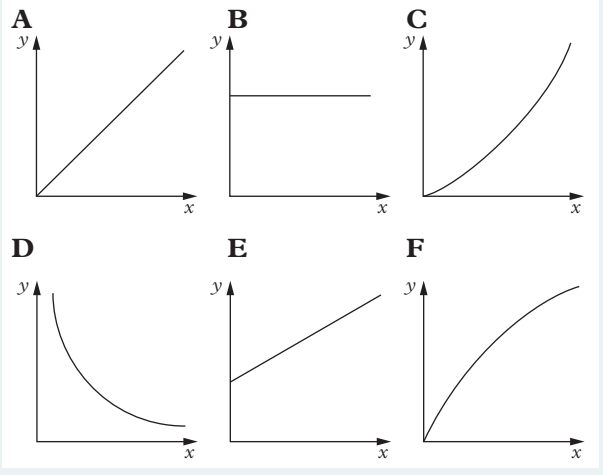
To calculate the vertical acceleration, the students used the formula $a = \frac{2 \times s}{t^2}$, where s is drop height and t is average time. The accepted value for g is 9.81 m s^{-2}.

For each group, **use** the average time to **determine** the percentage uncertainty of the vertical acceleration and the percentage error. (4 marks)

13 Deduce which of the following voltage measurements is the more precise: $V_1 = 0.55 \pm 0.01$ V or $V_2 = 6.4 \pm 0.1$ V. (3 marks)

14 For the graphs shown, **determine** the graph that best represents the following relationships.

- **a** y is directly proportional to x. (1 mark)
- **b** y is inversely proportional to x. (1 mark)
- **c** y is independent of x. (1 mark)
- **d** y is proportional to x^2. (1 mark)



15 a If $W = kV$, **determine** the effect on W of

i tripling V (1 mark)

ii halving V. (1 mark)

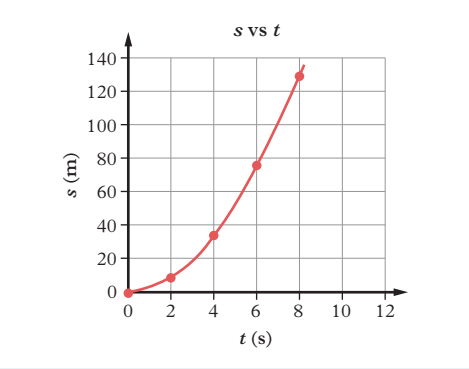
b **Explain** what a graph of W as a function of V would look like. (2 marks)

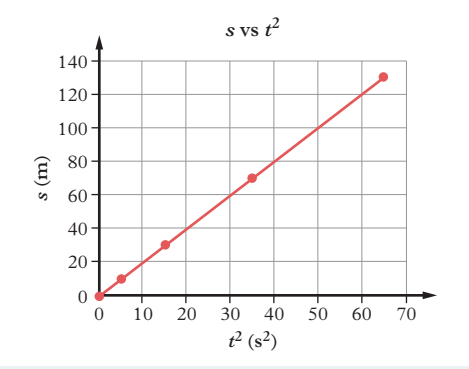
16 A plot of the variables F (vertical axis) and v (horizontal axis) gave a curve in the shape of $y \propto x^2$. **Determine** what you would need to plot to linearise the data. (1 mark)

17 A car is rolling down a steep hill. The displacement (s) in metres of the car at various times t (in seconds) is given in the table.

t (s)	0	2	4	6	8
s (m)	0	8	32	72	128

Graphs were drawn to show s vs t and s vs t^2.





Calculate the gradient and **determine** the relationship between s and t. (2 marks)

Knowledge utilisation

18 **Construct** a graph of each set of data in the following questions. For each, draw the trendlines, linearise if non-linear and calculate the gradient. (Note: the independent variable is listed first.)

a Force on a current-carrying wire vs current applied (4 marks)

Current, I (A)	1.0	2.0	3.0	4.0
Force, F (N)	0.32	0.66	1.00	1.30

b Diameter vs circumference of a circular loop of wire (4 marks)

Diameter of circle (cm)	0.0	4.0	8.0	12.0
Circumference of circle (cm)	0.0	12.5	25.4	37.3

19 **Construct** a conclusion about the validity and reliability of an experiment that has a large systematic error but a small random error, with reference to accuracy and precision. (2 marks)

20 "I can't linearise my data." **Discuss** if this means that your experiment was not valid. (3 marks)

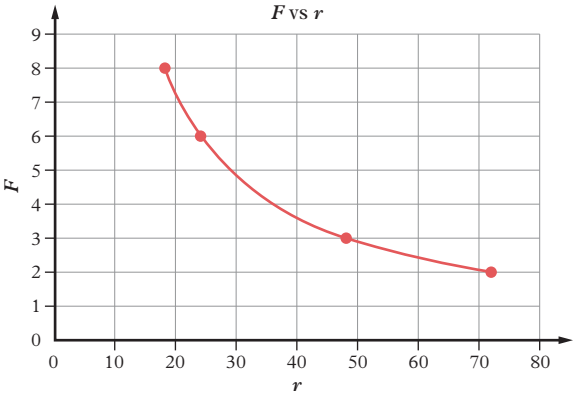
21 The table shows data from an experiment where force F and radial distance r are two related variables.

Force, F (N)	72	48	24	18
Radial distance, r (m)	2	3	6	8

The data was plotted to give the graph shown.

a **Predict** the mathematical relationship suggested by this graph. (1 mark)

b **Sketch** another graph to confirm your prediction. (5 marks)



Lesson 1.8
Evaluating evidence

Key ideas

- → Evidence is evaluated to determine its validity and reliability.
- → Goodness of fit (R^2) provides information about the precision of the data.
- → Random and systematic errors can be identified by assessing the trendline of a graph.
- → Not all secondary sources are reliable; they must be evaluated before use.

Science inquiry skills

Learning intentions and success criteria

This lesson provides support for the following science inquiry skills:

- identify and implement strategies to manage risks, ethics and environmental impact, e.g.
  - acknowledgement of sources and referencing
- use data and reasoning to discuss and evaluate the validity and reliability of evidence, e.g.
  - discuss ways in which measurement error, instrumental uncertainty, the nature of the methodology or other factors influence uncertainty and limitations in the data
  - evaluate information sources and compare ideas, information and opinions presented within and between texts, considering aspects such as acceptance, bias, status, appropriateness and reasonableness
  - compare findings to theoretical models or expected values
- analyse data to identify trends, patterns and relationships; recognising error, uncertainty and limitations of evidence
- appreciate the role of peer review in scientific research.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

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Lesson 1.9
Communicating scientifically

Key ideas

- → Different communication conventions are used depending on the target audience.
- → Evidence-based arguments bring together scientific ideas and primary and/or secondary data obtained from scientific investigation.
- → Sources can be acknowledged using in-text referencing and a bibliography.

Learning intentions and success criteria

Science inquiry skills

This lesson provides support for the following science inquiry skills:

- select, synthesise and use evidence to construct scientific arguments and draw conclusions
- communicate to specific audiences and for specific purposes using appropriate language, nomenclature, genres and modes
- acknowledge sources of information and use standard scientific referencing conventions.

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Lesson 1.10
Preparing for your data test

Key ideas

→ The data test is an assessment task in which you respond to items using qualitative data and/or quantitative data derived from practicals, activities or case studies.

→ In your data test, you will apply your understanding, analyse data, and interpret evidence.

Learning intentions and success criteria

Assessment objectives

This lesson provides support for achieving the assessment outcomes for the data test:

Objective 2. Apply understanding of gravity and motion, or electromagnetism to given algebraic, visual or graphical representations of scientific relationships and data to determine unknown scientific quantities or features.

Objective 3. Analyse data about gravity and motion, or electromagnetism to identify trends, patterns, relationships, limitations or uncertainty in datasets.

Objective 4. Interpret evidence about gravity and motion, or electromagnetism to draw conclusions based on analysis of datasets.

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Worked examples

This lesson is supported by the following Worked example:

- **Worked example 1.10A** Responding to the data test

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Lesson 1.11
Conducting your student experiment

Key ideas

→ Your student experiment requires you to modify an experiment you have conducted in Units 3 and 4 to address your own related hypothesis or question.

→ You will research and plan your experiment, analyse, interpret and evaluate your evidence, and communicate your findings.

Assessment objectives

Learning intentions and success criteria

This lesson provides support for achieving the assessment outcomes for the student experiment:

Objective 1. Describe ideas and experimental findings about gravity and motion, or electromagnetism.

Objective 2. Apply understanding of gravity and motion, or electromagnetism to modify experimental methodologies and process data.

Objective 3. Analyse experimental data about gravity and motion, or electromagnetism.

Objective 4. Interpret experimental evidence about gravity and motion, or electromagnetism.

Objective 5. Evaluate experimental processes and conclusions about gravity and motion, or electromagnetism.

Objective 6. Investigate phenomena associated with gravity and motion, or electromagnetism through an experiment..

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Lesson 1.12
Conducting your research investigation

Key ideas

→ The research investigation is an assessment task in which students gather evidence related to a research question to evaluate a claim in physics.

Assessment objectives

Learning intentions and success criteria

This lesson provides support for achieving the assessment outcomes for the research investigation:

Objective 1. Describe ideas and findings about special relativity, quantum theory or the Standard Model.

Objective 2. Apply understanding of special relativity, quantum theory or the Standard Model.

Objective 3. Analyse data about special relativity, quantum theory or the Standard Model.

Objective 4. Interpret evidence about special relativity, quantum theory or the Standard Model.

Objective 5. Evaluate processes, claims and conclusions about special relativity, quantum theory or the Standard Model.

Objective 6. Investigate phenomena associated with special relativity, quantum theory or the Standard Model.

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Lesson 1.13
Preparing for your exams

Key ideas

→ Your external exam requires you to demonstrate your understanding of the QCE Physics course by completing multiple-choice and short-answer questions.

→ Exam questions are typically written using cognitive verbs, which tell you what information you need to provide in your answer to the question.

Assessment objectives

Learning intentions and success criteria

This lesson provides support for achieving the assessment outcomes for the examination:

Objective 1. Describe ideas and findings about gravity and motion, electromagnetism, special relativity, quantum theory and the Standard Model.

Objective 2. Apply understanding of gravity and motion, electromagnetism, special relativity, quantum theory and the Standard Model.

Objective 3. Analyse data about gravity and motion, electromagnetism, special relativity, quantum theory and the Standard Model.

Objective 4. Interpret evidence about gravity and motion, electromagnetism, special relativity, quantum theory and the Standard Model.

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MODULE 1

Lesson 1.14
Review: Physics toolkit

Summary

1.1
- In QCE Physics, you will gain a variety of science understanding and inquiry skills that will help you succeed in assessments, as a scientific professional, and as a science citizen.

1.2
- First Nations peoples have longstanding scientific knowledge.
- First Nations peoples have developed knowledge about the world by observing using all the senses, predicting and hypothesising, testing (trial and error), and making generalisations within specific contexts such as the use of food, natural materials, navigation and sustainability of the environment.
- Correctly acknowledging cultural and/or language groups, rejecting deficit discourse, avoiding Eurocentrism and critically evaluating sources of information can help you to respectfully engage with First Nations perspectives in QCE Physics.

1.3
- The scientific method is an iterative and cyclic process.
- Research questions define the scope of an investigation. They can be used to develop hypotheses that predict the outcome of the investigation.

1.4
- A method outlines the steps followed in an experiment and lists all of the materials and equipment used.
- Valid and reliable measurements can be obtained by carefully designing your investigation to collect sufficient data and minimise errors.
- All measurements include errors or uncertainties, either systematic or random. It is important to consider these and implement strategies to minimise their effects when planning your experiments.

1.5
- Laboratory safety is important to prevent harm to yourself and those sharing the lab space with you.
- Safety and ethics extend beyond the lab; you should be considering the broader potential impacts of your experiment.

1.6
- Single experimental measurements are reported using best estimates, indicators of measurement uncertainty and units.
- Scientific notation is used to easily express extremely large or small values.
- Significant figures are digits in a number that are known with certainty and the first digit that is uncertain.
- All measurements, information and observations should be recorded in your logbook.

1.7
- Data is processed and analysed to identify trends, patterns, relationships, limitations and uncertainty.
- Absolute and percentage uncertainty give an idea of the precision of measurements.
- Absolute and percentage error give an idea of the accuracy of measurements.
- Data can be summarised in a variety of ways, such as in a table, a scatterplot, or a scientific diagram.
- Linearising data helps you to identify the exact relationship between variables.

1.8
- Evidence is evaluated to determine its validity and reliability.
- Goodness of fit (R^2) provides information about the precision of the data.
- Random and systematic errors can be identified by assessing the trendline of a graph.
- Not all secondary sources are reliable; they must be evaluated before use.

1.9
- Different communication conventions are used depending on the target audience.
- Evidence-based arguments bring together scientific ideas and primary and/or secondary data obtained from scientific investigation.
- Sources can be acknowledged using in-text referencing and a bibliography.

1.10
- The data test is an assessment task in which you respond to items using qualitative data and/or quantitative data derived from practicals, activities or case studies.
- In your data test, you will apply your understanding, analyse data, and interpret evidence.

1.10
- Your student experiment requires you to modify an experiment you have conducted in Units 3 and 4 to address your own related hypothesis or question.
- You will research and plan your experiment, analyse, interpret and evaluate your evidence, and communicate your findings.

1.12
- The research investigation is an assessment task in which students gather evidence related to a research question to evaluate a claim in physics.

1.13
- Your external exam requires you to demonstrate your understanding of the QCE Physics course by completing multiple-choice and short-answer questions.
- Exam questions are typically written using cognitive verbs, which tell you what information you need to provide in your answer to the question.

Key formulas

Mean (best estimate of repeated single measurements)	$\bar{x} = \frac{\Sigma x}{n}$
Absolute uncertainty	$\delta\bar{x} = \pm \frac{(x_{max} - x_{min})}{2}$
Standard deviation	$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$
Percentage uncertainty	$\delta\,\% = \frac{\delta\bar{x}}{x_O} \times 100$
Absolute error	$E_a = \lvert x_O - x_A \rvert$
Percentage error	$E\% = \frac{E_a}{x_A} \times 100$
Gradient	$m = \frac{\Delta y}{\Delta x}$

Review questions 1.14A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 Select the option that has the smallest magnitude.
- **A** -1×10^2
- **B** 1×10^2
- **C** 10
- **D** 100

2 Which one of the following is closest in area to $2.0\,m^2$?
- **A** $200\,cm^2$
- **B** $20{,}000\,cm^2$
- **C** $2 \times 10^7\,mm^2$
- **D** $200{,}000\,mm^2$

3 Calculate the magnitude of the following to the correct number of significant figures: $1.18\,cm \times 3.1416\,cm$.
- **A** 3.7
- **B** 3.71
- **C** 3.707
- **D** 3.7071

4 Calculate the following and express in scientific notation to the correct number of significant figures: $(2.0 \times 10^{-3}\,m) \times (2.00 \times 10^{-4}\,m)$.
- **A** $4.0 \times 10^{-7}\,m$
- **B** $4.0 \times 10^{-7}\,m^2$
- **C** $4.00 \times 10^{-7}\,m$
- **D** $4.0 \times 10^{-12}\,m^2$

5 Use scientific notation to express the number 0.000552.
- **A** 552×10^{-6}
- **B** 0.552×10^{-3}
- **C** 5.52×10^{-4}
- **D** 5.52×10^{-2}

6 Use the correct scientific notation to express the number 73,000,000.
- **A** 0.73×10^8
- **B** 7.3×10^7
- **C** 73×10^6
- **D** $73 \times (10^3)^2$

7 The force acting on a 2.0 cm length of wire in a magnetic field was measured four times as 15.5 N, 16.0 N, 15.7 N and 15.9 N. Calculate the absolute uncertainty of the mean.
- **A** 0.1
- **B** 0.3
- **C** 0.5
- **D** 15.8

8 The time for a ball to roll down an incline was measured as 1.05 s, 1.20 s, 1.13 s, 1.05 s. Calculate the percentage uncertainty of the mean.
- **A** 0.075
- **B** 1.11
- **C** 6.8
- **D** 7.2

9 An analogue thermometer has a resolution of 1°C. The initial and final temperatures were measured as 24.5°C and 54.0°C respectively. Calculate the change in temperature including the absolute uncertainty.
- **A** 29 ± 0.5°C
- **B** 29.5 ± 0.5°C
- **C** 29.5 ± 1.0°C
- **D** 30 ± 1°C

10 In the analysis of a motion experiment, the equation for the linear line of best fit was $y = 5.0x + 2.5$. The maximum line of best fit was $y = 6.5x$, and the minimum line of best fit was $y = 4.8x + 3.0$. Calculate the absolute uncertainty of the gradient.
- **A** 0.85
- **B** 1.7
- **C** 2.5
- **D** 5.5

11 The formula $\delta\bar{x} = \pm \frac{(x_{max} - x_{min})}{2}$ is used to calculate
- **A** absolute uncertainty.
- **B** best estimate.
- **C** absolute error.
- **D** mean.

12 A rectangular loop of copper wire measures 20.0 ± 0.05 mm by 30.0 ± 0.05 mm. Calculate the area of this loop.
- **A** $600 \pm 0.0025\,mm^2$
- **B** $600 \pm 0.10\,mm^2$
- **C** $600 \pm 0.42\,mm^2$
- **D** $600 \pm 2.5\,mm^2$

13 The cross-sectional area of a piece of nichrome wire was $0.1\,mm^2$. Calculate the area in square metres.
- **A** $1.0 \times 10^{-7}\,m^2$
- **B** $1.0 \times 10^{-6}\,m^2$
- **C** $1.0 \times 10^{-4}\,m^2$
- **D** $1.0 \times 10^{-3}\,m^2$

Review questions 1.14B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

14 **Classify** the following units as SI or non-SI: metres, kilograms, gauss, kelvin. (1 mark)

15 In terms of the analysis of data

a **explain** the difference between significant figures and scientific notation (2 marks)

b **explain** how a "systematic error" differs from a "random error". (2 marks)

16 **Explain** what is meant by "propagation of errors" when performing a calculation. (3 marks)

17 **Explain** why it may be necessary to calculate percentage error of a result rather than just leave it as absolute error. (3 marks)

18 **Identify** the number of significant figures in each number.

a 83.83 (1 mark)

b 20.0 (1 mark)

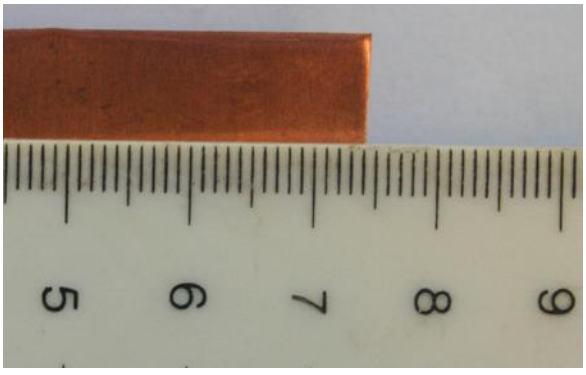
19 **Use** scientific notation to express each number.

a 0.000552 (1 mark)

b 58,000,000 (1 mark)

Analytical processes

20 **Determine** the reading and uncertainty of the measurement on the ruler. The numbers are centimetres and the small divisions are millimetres. (2 marks)



21 Complete the following questions.

a **Sketch** a graph of the data given in the table and draw a line of best fit. (Note: the independent variable is listed first.) (3 marks)

Time, t (s)	0.0	2.0	4.0	6.0
Distance, s (m)	0.0	12	23	37

b For the line plotted on the graph

i **calculate** the gradient (1 mark)

ii **determine** the distance after 8.0 s by extrapolation (1 mark)

iii **determine** the distance after 3.0 s by interpolation. (1 mark)

22 An analogue voltmeter with a scale division of 0.1 V reads 2.4 V when placed across a resistor of 47 Ω ± 5 %.

a **Determine** the absolute uncertainty in the voltage reading. (1 mark)

b If a calculation of $\frac{V^2}{R}$ was made, **calculate** the percentage uncertainty in the result. (2 marks)

23 German physicist Arnd Leike, from the University of Munich, found that the decay of foam height in beer with time was exponential: $y \propto \frac{1}{x^n}$, where y = height of the foam and x = time. He was awarded an "Ig Nobel" Prize by the science humour magazine *Annals of Improbable Research* for one of the world's most useless pieces of research. **Differentiate** between the shapes of the graphs when $n = 3$ (Leike's result) and $n = 2$ (inverse square). (2 marks)

Knowledge utilisation

24 An experiment using ball on a string was conducted to test the effect of different centripetal forces on the velocity of a ball swung in a horizontal circle. The results are shown in the table.

Force, F (N)	0.98	1.96	2.94	3.92	4.9
Velocity, v (m s^{-1})	5.49	7.42	9.34	10.58	11.86

a **Construct** a graph, with force on the x-axis, to show the relationship between velocity v, in m s^{-1}, and the centripetal force F in N. (3 marks)

b **Describe** the relationship between F and v. (1 mark)

c **Construct** a linearised graph to confirm the relationship between F and v. (4 marks)

25 As part of an investigation into motion of a falling object, a tennis ball was dropped from various heights as shown in the table. The heights were measured with a metre ruler marked in mm and the uncertainty was considered insignificant. Times for three trials were measured with a stopwatch.

Height, s (m)	Drop time, t (s)		
	Trial 1	Trial 2	Trial 3
0.50	0.37	0.44	0.50
1.00	0.62	0.56	0.56
1.50	0.75	0.75	0.69
2.00	0.75	0.87	0.94

a **Calculate** the mean ($\bar{x}$), and the absolute uncertainties of the mean for the times for each height. (8 marks)

b **Construct** a graph with height s on the horizontal axis, and the corresponding mean of the times on the vertical axis. (3 marks)

c **Construct** a linearised graph by plotting $\sqrt{s}$ on the horizontal axis and t on the vertical axis, including vertical error bars. (3 marks)

d **Determine** the gradients of the maximum and minimum trendlines and express these as an absolute uncertainty of the gradient. (4 marks)

26 The following sets of data represent common relationships in physics. **Predict** the relationship by graphical means for each dataset and confirm it where necessary by linearisation.

a Dataset 1 (4 marks)

x	1	2	3	5	6
y	2.00	1.10	0.67	0.41	0.33

b Dataset 2 (4 marks)

x	1	2	4	6	8
y	3.00	0.75	0.19	0.08	0.05

c Dataset 3 (4 marks)

x	0	2	3	5	7
y	0	20	45	125	245

27 The following sets of data represent common relationships in physics. **Predict** the relationship by graphical means for each dataset and confirm it where necessary by linearisation.

a Dataset 1 (4 marks)

x	0.50	1.70	2.10	3.60	7.00
y	0.54	0.99	1.10	1.44	2.01

b Dataset 2 (4 marks)

x	0	1.5	3.4	5.1	7
y	3.2	9.2	16.8	23.6	31.2

28 Students were investigating the swing of a pendulum as a function of the length of the string. Their results are shown in the table. The period is the time for one complete swing.

Length, L (m)	Period, T (s) ± 0.15 s
0.10	0.78
0.20	1.20
0.40	1.42
0.60	1.90
0.80	2.04
1.00	2.16
1.20	2.50
1.40	2.67

a **Construct** a graph with length on the horizontal axis and time on the vertical axis and draw a trendline. (3 marks)

b **Predict** the relationship between T and L. (1 mark)

c **Construct** a linearised graph to confirm the relationship. (3 marks)

d **Determine** the gradient of the trendline (2 s.f.). (1 mark)

e **Construct** error bars on the graph and **determine** the gradient of the maximum and the minimum trendlines. (3 marks)

f **Express** the gradient as a mean value ± absolute uncertainty. (1 mark)

Module 1 checklist: Physics toolkit

UNIT 3

Gravity and electromagnetism

Unit 3 overview

In Unit 3, students develop a deeper understanding of motion and its causes by using Newton's laws of motion and the gravitational field model to analyse motion on inclined planes, and the motion of projectiles and satellites. Field theories have enabled physicists to explain a vast array of natural phenomena and have contributed to the development of technologies that have changed the world, including electrical power generation and distribution systems, artificial satellites and modern communication systems. Students develop their understanding of field theories of gravity and electromagnetism through investigations of motion and electromagnetic phenomena. Finally, they investigate the production of electromagnetic waves.

Participation in a range of experiments and investigations will allow students to develop skills in relating graphical representations of data to quantitative relationships between variables, using lines of force to represent vector fields, and interpreting interactions in two and three dimensions.

Throughout the unit, students develop skills in planning and conducting investigations, interpreting results and evaluating the validity of primary and secondary data, as well as the communication of these evaluations to others in a range of formats.

Unit objectives

1 Describe ideas and findings about gravity and motion, and electromagnetism.
2 Apply understanding of gravity and motion, and electromagnetism.
3 Analyse data about gravity and motion, and electromagnetism.
4 Interpret evidence about gravity and motion, and electromagnetism.
5 Evaluate processes, claims and conclusions about gravity and motion, and electromagnetism.
6 Investigate phenomena associated with gravity and motion, and electromagnetism.

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Unit 3 Topics

Topic	Module	
Topic 1 Gravity and motion	**Module 2**	Vectors and projectile motion
	Module 3	Inclined planes
	Module 4	Circular motion
	Module 5	Orbital mechanics
Topic 2 Electromagnetism	**Module 6**	Electrostatics
	Module 7	Magnetic fields
	Module 8	Electromagnetic induction and radiation

MODULE 2

Vectors and projectile motion

Introduction

Projectiles are objects thrown forwards. The word comes from the Latin "pro" meaning "forwards" and "jacere" meaning "to throw". Golf, javelin, archery and rifle shooting all involve projectiles. So does a motorcyclist jumping a row of cars, a ballet dancer leaping across the stage, and a rock thrown off a cliff. In this module, you will learn to analyse the motion of projectiles using vectors in two dimensions.

Prior knowledge

Prior knowledge quiz

Check your understanding of concepts related to vectors and projectile motion before you start.

Subject matter

Science understanding

→ Apply vector analysis to resolve a vector into two perpendicular components.
→ Solve vector problems by resolving vectors into components, adding or subtracting the components and recombining them to determine the resultant vector.
→ Describe how horizontal and vertical components of a velocity vector are independent of each other.
→ Solve problems involving projectile motion in the absence of drag effects using $v_y = u_y + gt$, $s_y = u_y t + \frac{1}{2}gt^2$, $v_y^2 = u_y^2 + 2gs_y$, $v_x = u_x$, and $s_x = u_x t$.
→ Interpret data relating to the horizontal distance travelled by an object projected at various angles from the horizontal.

Science inquiry

→ Investigate the horizontal distance travelled by an object projected at various angles from the horizontal.

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Practicals

oxforddigital **This lesson is available on Oxford Digital.**

Lesson 2.4 Angled projection and distance

Lesson 2.1
Working with vectors

Key ideas

- → A vector has both magnitude and direction. It can be represented graphically or using symbols or words.
- → A vector can be resolved into two components at right angles.
- → Vectors or their components can be added and subtracted.
- → Vectors can be recombined to get a resultant vector.

Learning intentions and success criteria

vector
a variable quantity, such as velocity, force, or gravitational field strength, that has magnitude and direction

What are vectors?

In Unit 2 we worked with scalar quantities, such as distance, speed and energy, which just have magnitude. We also dealt with **vector** quantities such as displacement, velocity, acceleration, force and momentum. These need both magnitude (e.g. $20\,m\,s^{-1}$) and direction (e.g. at 45° to the ground) to describe them fully.

The vectors you learnt to analyse in Unit 2 were in one dimension – for example, for the displacement and velocity of an object falling vertically due to the gravitational force from Earth. However, not all motion is straight down. The motion of a projectile such as a basketball in flight, or a box sliding down an incline, has both vertical and horizontal components. In Unit 3 we will use vectors to analyse forces and motion in two dimensions. This includes vectors that describe the forces on, and motion of, objects, but also vectors that describe the strength and direction of fields, such as gravitational fields, electric fields and magnetic fields.

Vector analysis in two dimensions is hugely important in this unit and the groundwork begins here. Some of the questions that puzzle students about vectors are:

- How can something at constant speed be accelerating?
- Can a vector have zero magnitude if one of its components is not zero?
- Does a negative vector mean something is slowing down or getting weaker?

How are vectors represented?

Study tip

In external exams, vector diagrams may show vector symbols in italics, such as $A = 300$, or with an arrow above the symbol, such as $\vec{A} = 300$. Both methods are correct. You probably find it hard to show italics with handwriting, so we have used both methods here to give you experience with each.

As stated before, a vector is a quantity that has both magnitude and direction. A vector quantity can be represented graphically by an arrowed line segment, or symbolically by accents such as a bold typeface, tildes or overhead arrows.

The list below gives four different ways of representing a vector quantity – in this case, velocity. In this book we will use the first and third methods – arrowed line segments and overhead arrows.

1. Arrowed line segment: $\underrightarrow{v = 5\,m\,s^{-1}}$
2. Bold (accent): $\mathbf{v} = 5\,m\,s^{-1}$ to the right
3. Arrow (accent): $\vec{v} = 5\,m\,s^{-1}$ to the right
4. Tilde (accent): $\tilde{v} = 5\,m\,s^{-1}$ to the right

When using an arrow to represent a vector, the length of the arrow is used to show the magnitude. In this case, we are using the convention that "towards the right" or "up" are the positive direction, and "towards the left" and "down" are negative (Figure 1).

When vectors are not horizontal or vertical, angles need to be specified. Figure 2 shows how the direction is indicated. The vector on the left is $v = +14\,\text{m s}^{-1}$ at 35° above the horizontal. The vector on the right is $v = 12\,\text{m s}^{-1}$ to the left at 60° to the vertical.

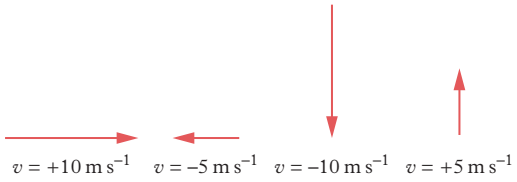


FIGURE 1 Horizontal and vertical conventions for representing vectors

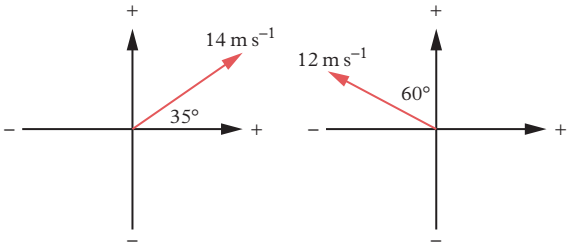


FIGURE 2 Vector direction can be represented at an angle to the horizontal or vertical.

Challenge

What does an odometer measure?

Does the odometer (kilometre counter) of a car measure a scalar quantity or a vector quantity? Make a similar statement for the car's speedometer. (2 marks)

Worked example 2.1A

Describing vectors

Identify the magnitude and direction of the vector shown in Figure 3. (2 marks)

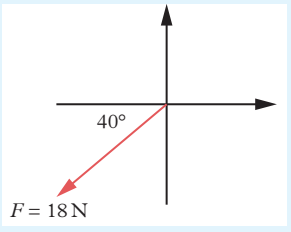


FIGURE 3 A vector pointing to the bottom left represented by a vector diagram

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Identify" means to recognise and state. This question is worth 2 marks so we must provide two features.
Step 2: Identify the magnitude of the vector, including the correct symbol, quantity and units. (1 mark for "Describes magnitude of the vector")	$F = 18\,\text{N}$ (1 mark)
Step 3: Recall that a vector has both magnitude and direction. Determine the direction. (1 mark for "Identifies direction of the vector")	The direction is to the left and 40° below the horizontal. (1 mark)

Your turn

Identify the magnitude and direction of the vector in Figure 4. (2 marks)

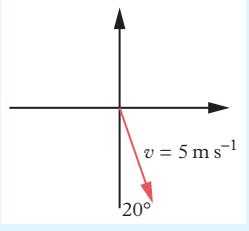


FIGURE 4 A vector pointing to the bottom right represented by a vector diagram

How are vectors resolved into components?

component
(of a vector) the part of a vector that acts in a given direction

resolving
determining the components of a vector, usually at right angles to each other. Also called "decomposition".

As you have just seen, sometimes the vectors will not be horizontal or vertical but at an angle to the horizontal. We can break up the vector into its **components** at right angles to each other. It is then sometimes easier to apply the laws of physics to these components. This process is called **resolving** the vector or "decomposition" of the vector. In maths, you may use the "unit vectors" i, j and k as components, but it is the same idea.

First, a reminder of the trigonometric ratios (Figure 5) that we will need for our work with vectors.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

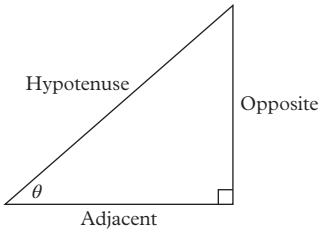


FIGURE 5 A vector (the hypotenuse) can be resolved into components at right angles.

Consider the diagram on the left in Figure 6. It shows a vector $\vec{A}$ of magnitude 150 units (e.g. a velocity of 150 m s^{-1}) pointing to the right at an angle of 50° above the horizontal. In the diagram on the right, this vector has been resolved into two components at right angles – the **horizontal component of velocity** and the **vertical component of velocity**. In Figure 6, be aware that vector $\vec{A}$ has been fully replaced by A_x and A_y. The vector is described by one or the other, but not both at the same time. This is usually clear when you are doing these problems.

horizontal component of velocity
the resolution of a projectile's velocity in the horizontal direction

vertical component of velocity
the resolution of a projectile's velocity in the vertical direction

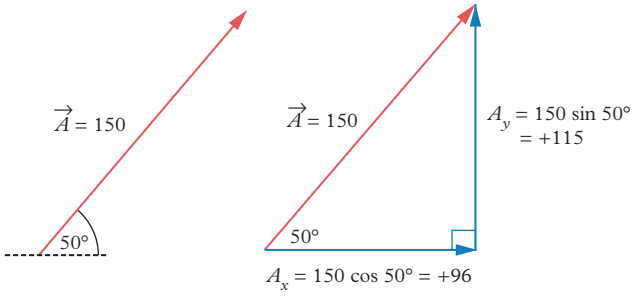


FIGURE 6 Resolving a vector into two components at right angles

Using the trigonometric ratios, we can resolve vector A into its x- and y-components as follows.

Horizontal (x) component:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\begin{aligned}\text{adjacent} &= \text{hypotenuse} \times \cos\theta \\ &= 150 \times \cos 50° \\ &= 96 \text{ units}\end{aligned}$$

Vertical (y) component:

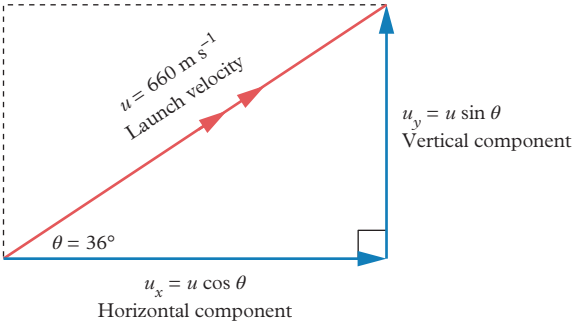
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\begin{aligned}\text{opposite} &= \text{hypotenuse} \times \sin\theta \\ &= 150 \times \sin 50° \\ &= 115 \text{ units}\end{aligned}$$

Worked example 2.1B

Resolving a velocity vector

A bullet is fired at a velocity of 660 m s^{-1} to the right (+) at an angle of 36.0° above the horizontal. **Construct** a diagram showing the horizontal and vertical components and **calculate** their magnitudes. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the question is asking you to do.	"Construct" means to display information in a diagrammatic or logical form, in this case to construct a vector diagram. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Construct a vector diagram by drawing the vector arrow and then drawing a rectangle around it using the vector as the diagonal. A double arrow is sometimes used to identify the vector being resolved. (1 mark for "Constructs vector diagram correctly")	 (1 mark)
Step 3: Label the horizontal and vertical components as u_x and u_y, respectively.	See the diagram for Step 2.
Step 4: Calculate the magnitude of the horizontal component using trigonometry: $\left(\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}\right)$ Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that the answer has the correct units (m s^{-1}). (1 mark for "Calculates the horizontal component correctly")	$u_x = u\cos\theta$ $= 660 \times \cos 36°$ $= 534$ m s^{-1} (3 s.f.) (1 mark)
Step 5: Calculate the magnitude of the vertical component using trigonometry. (1 mark for "Calculates the vertical component correctly")	$u_y = u\sin\theta$ $= 660 \times \sin 36°$ $= 388$ m s^{-1} (3 s.f.) (1 mark)

Your turn

A golf ball is struck off the ground at a velocity of 75 m s^{-1} to the right and an angle of 15° to the horizontal. **Construct** a diagram showing the horizontal and vertical components of the initial velocity and **calculate** their magnitudes. Give your answer to an appropriate number of significant figures. (3 marks)

How are vectors resolved when they are negative?

When a vector points in the negative direction, either horizontally (x-direction) or vertically (y-direction), the sign of that component must also be negative. By convention, "towards the left" (in the horizontal direction) and "down" (in the vertical direction) are usually negative, as shown in Figure 7.

Study tip

During assessments (particularly the data test and external exam) always check that your calculator is in degrees and not radians! This can be the difference between a correct and an incorrect answer.

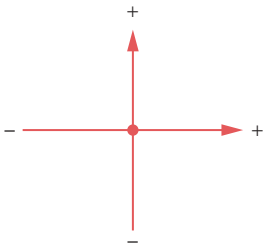


FIGURE 7 Typical convention for vector directions

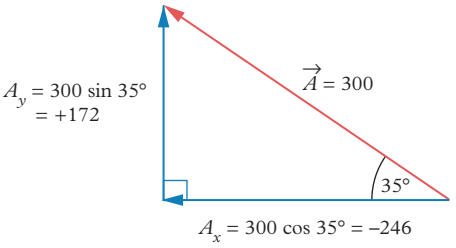


FIGURE 8 Vector $\vec{A}$ points in the negative x-direction so its x-component is also negative.

Consider a vector $\vec{A}$ that has a magnitude of 300 units (Figure 8). It points to the left (–) at an angle of 35° from the horizontal. Vector A has been resolved into horizontal and vertical components. The vertical component A_y points upwards so it is positive. The horizontal component A_x points to the left so it is negative. The magnitudes of A_y and A_x can be calculated using trigonometric ratios. In the diagram, we see that A_y is +172 units and A_x is –246 units.

This example uses the general term "units" rather than specifying any particular unit as we will be using a variety of units throughout our work (m, $m\,s^{-1}$, $m\,s^{-2}$, N, $N\,C^{-1}$, T, and so on).

Worked example 2.1C

Resolving vectors

Resolve the vector shown in Figure 9. **Calculate** the magnitude of the vertical and horizontal components. Give your answer to an appropriate number of significant figures. (3 marks)

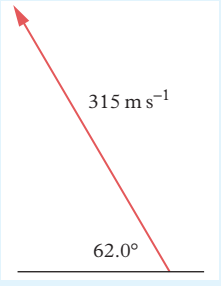
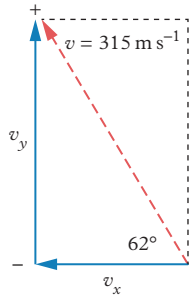


FIGURE 9 A vector with its magnitude and direction shown

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Construct a vector diagram by drawing the vector arrow and then drawing a rectangle around it using the vector as the diagonal. Draw two vector arrows head to tail along two sides, starting at the tail of the original vector arrow. (1 mark for "Constructs vector diagram showing vertical and horizontal components")	 (1 mark)
Step 3: Calculate the magnitude of the horizontal component using trigonometry. (1 mark for "Calculates the horizontal component")	$v_x = v\cos\theta$ $= 315 \times \cos 62.0°$ $= -148\,m\,s^{-1}$ (1 mark)
Step 4: Calculate the magnitude of the vertical component. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Calculates the horizontal component")	$v_y = v\sin\theta$ $= 315 \times \sin 62.0°$ $= 278\,m\,s^{-1}$ (1 mark)

Your turn

Resolve the vector shown in Figure 10. **Calculate** the magnitude of the vertical and horizontal components. Give your answer to an appropriate number of significant figures. (3 marks)

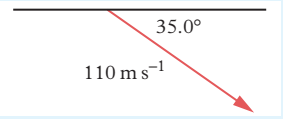


FIGURE 10 A vector with its magnitude and direction shown

How are vectors combined?

You have seen how a vector can be resolved or decomposed into its component vectors, but we can also do the reverse. You can **combine** (add or subtract) two or more vector quantities to produce a single **resultant vector**. Vectors in a straight line (in one dimension) can be combined, as you will recall from the linear motion module in Unit 2. Vectors at an angle to each other (in two dimensions) can also be combined, as we do in this unit.

combine (vectors) add two or more vectors to determine the resultant vector

resultant vector a single vector that is a combination of two or more other vectors

Adding vectors in one dimension (in a line)

Case 1: Vectors in the same direction

Consider you are in a boat that is rowed to the right at 16 m s^{-1} relative to the water, and the water is moving to the right relative to the ground at 9 m s^{-1}. Your velocity relative to the ground is then 16 + 9 = 25 m s^{-1} to the right relative to the ground. This can be found by drawing the two vector arrows head to tail. The resultant vector is a line drawn from the tail of the first arrow to the head of the second arrow (Figure 11). A double arrowhead is sometimes shown on the resultant.

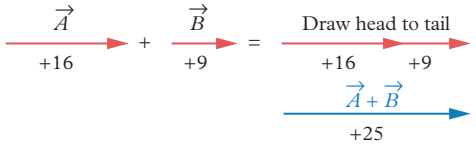


FIGURE 11 Adding vectors by drawing them head to tail

Study tip

The resultant vector is always drawn from the tail of the first vector (arrow) to the head of the second.

Case 2: Vectors in opposite directions

Consider the same boat being rowed against the current. In this case, the velocity of the river is 9 m s^{-1} to the left, or –9 m s^{-1}. This is in the direction opposite to the velocity of the boat, which is 16 m s^{-1} to the right or +16 m s^{-1}. It will therefore slow the boat down (Figure 12).

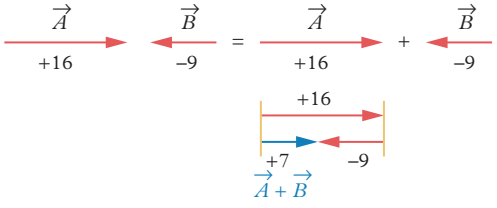


FIGURE 12 Adding vectors in opposite direction

The resultant velocity is +16 + (–9) = +7 m s^{-1}, or 7 m s^{-1} to the right. Note that when we add two vectors in the same line, the resultant is in the same direction as the larger vector.

Subtracting vectors in one dimension (in a line)

It is not often that you have to subtract vectors. The most obvious case is when you calculate a change in velocity, Δv or $v - u$, where the delta symbol, Δ, means a change or difference. When subtracting vectors in a line, you reverse the vector being subtracted to make it negative, and then just add it to the other one. To draw this, you draw the two vectors head to tail, but draw the negative vector in the opposite direction. You then draw the resultant by drawing an arrow from the tail of the first arrow to the head of the second arrow. For example, to subtract vector B from vector A ($\vec{A} - \vec{B}$), we just reverse the direction of vector B to make it $-\vec{B}$, and then add it to $\vec{A}$ to get $\vec{A} + (-\vec{B})$ (Figure 13).

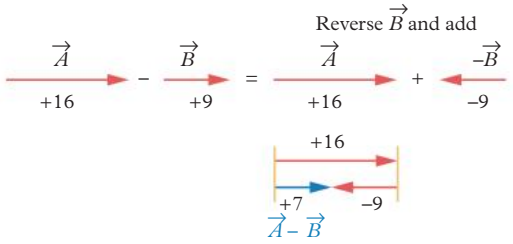


FIGURE 13 Subtraction of vectors can be turned into addition.

Combining vectors in two dimensions (at right angles)

Study tip

Pythagoras' theorem states that $c^2 = a^2 + b^2$, where c is the hypotenuse, and a and b are the other sides of the right-angled triangle.

In Unit 2 you dealt with vectors in a line, but in Unit 3 you will extend that to vectors in two dimensions – first at right angles, and in later lessons at any angle. Vectors will be combined in a horizontal plane or in a vertical plane. The approach is the same.

Case 1: Combining vectors in two dimensions in a horizontal plane

Consider a motorboat travelling to the right at $4\,m\,s^{-1}$ across a river whose current is flowing down at $3\,m\,s^{-1}$, as shown in Figure 14A. The boat would be dragged off course by the current and its resultant velocity would be $5\,m\,s^{-1}$ at an angle down the river. In a vector diagram, the two vectors are added head to tail, and the resultant is a line drawn from the tail of the first arrow to the head of the second arrow (Figure 14B).

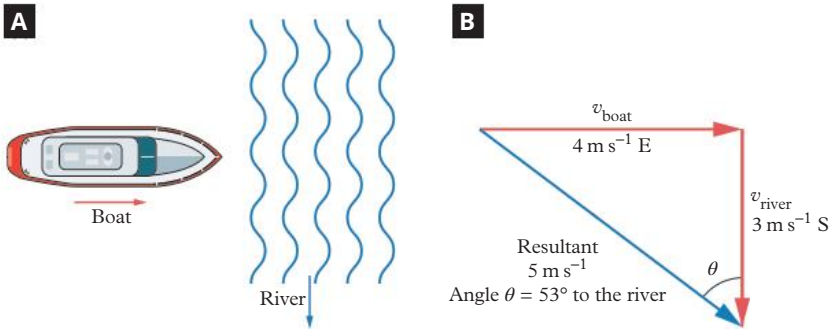


FIGURE 14 A practical example of adding vectors at right angles

The resultant velocity is found in two parts – the magnitude ($5\,m\,s^{-1}$) and the direction (53 to the direction of river flow), as follows.

1 **Magnitude.** Because the initial vectors for the boat and river are at right angles (right and down), Pythagoras' theorem can be used:

$$\begin{aligned}\text{Resultant} &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} \\ &= 5\,m\,s^{-1}\end{aligned}$$

2 **Direction.** Because the two vectors and the resultant form a right-angled triangle, trigonometry can be used:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{4}{3}$$
$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$
$$= 53°$$

This vector-analysis technique can be applied in the vertical plane as well.

Case 2: Combining vectors in two dimensions in a vertical plane

Consider a golf ball that is struck (Figure 15) and eventually lands back on the ground. Its final velocity as it strikes the ground is often called its **impact velocity**. Upon impact, it has two velocity vectors, v_x (in the horizontal or x-direction) and v_y (in the vertical or y-direction). The two vectors are combined (added) to get the resultant velocity (Figure 16).

impact velocity the velocity of a projectile as it strikes the ground; the magnitude of impact velocity is impact speed

FIGURE 15 A golf ball is hit off the tee with a driver. The ball has vertical and horizontal components of motion after it is hit and also when it lands.

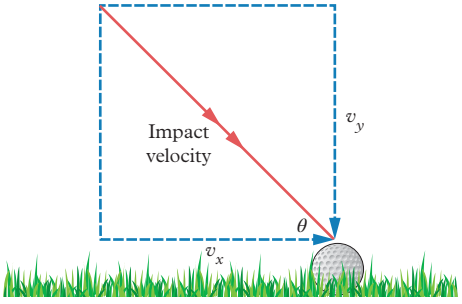


FIGURE 16 A projectile strikes the ground at an angle. Its resultant impact velocity is a combination of the horizontal (v_x) and vertical (v_y) impact velocities.

Study tip

Horizontal velocities are labelled u_x and v_x. Vertical velocities are labelled u_y and v_y.

Worked example 2.1D

Adding vectors in the vertical plane

A ball makes impact with the ground with a horizontal velocity of 3.0 m s^{-1} and a vertical velocity of 4.0 m s^{-1}, as shown in Figure 17.

a **Construct** a diagram showing the horizontal and vertical components of the velocity, and the resultant vector. (1 mark)

b **Calculate** the magnitude and direction of the resultant impact velocity. Give your answer to an appropriate number of significant figures. (3 marks)

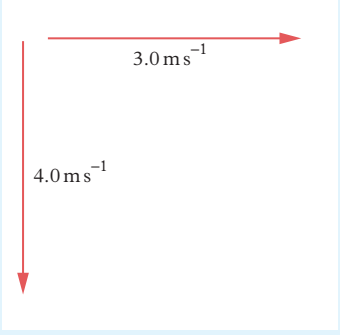
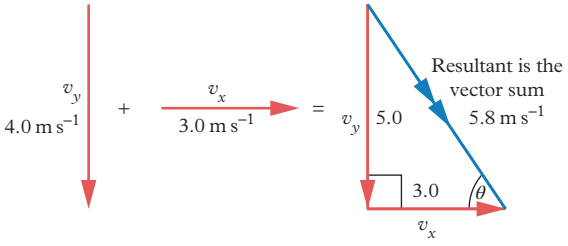


FIGURE 17 Vector diagram for a ball at impact with the ground

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Construct" means to display information in a diagrammatic or logical form – in this case to construct a vector diagram. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Construct a vector diagram by drawing the vector arrow and then drawing a rectangle around it using the vector as the diagonal. Draw two vector arrows head to tail along two sides, starting at the tail of the original vector arrow. (1 mark for "Constructs vector diagram correctly showing resultant vector")	**a**  (1 mark)
Step 3: Calculate the magnitude of the resultant vector. This can be found using Pythagoras' theorem: $c^2 = a^2 + b^2$ $c = \sqrt{a^2 + b^2}$ (1 mark for "Provides appropriate mathematical reasoning")	**b** Resultant impact velocity $= \sqrt{3^2 + 5^2}$ $= \sqrt{34}$ $= 5.8\,\text{m s}^{-1}$ (1 mark)
Step 4: Calculate the direction. This is given by the angle, θ, that the resultant vector makes with the ground. Use trigonometry to do this. (1 mark for "Calculates the angle correctly")	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $= \frac{5}{3}$ $\theta = \tan^{-1}\left(\frac{5}{3}\right)$ $= 59°$ to the horizontal (1 mark)
Step 5: State the magnitude and direction using the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$v = 5.8\,\text{m s}^{-1}$ at 59° to the horizontal (1 mark)

Your turn

A rock is thrown off a cliff and lands on the ground below with a horizontal velocity of 9.0 m s^{-1} and a vertical velocity of 3.0 m s^{-1}.

a **Construct** a diagram showing the horizontal and vertical components of the velocity, and the resultant vector. (1 mark)

b **Calculate** the magnitude and direction of the resultant impact velocity. Give your answer to an appropriate number of significant figures. (3 marks)

Study tip

In Worked example 2.1D, if you were asked to find the angle with respect to the vertical, it would just be 90° – θ; that is, 90° – 59°, which gives you 31° to the vertical. Take note of what you are asked in the question.

Challenge

As the crow flies

You plan a trip from one town to another by drawing a straight line between them on a map app. Is that always the quickest way there? (1 mark)

Check your learning 2.1

Check your learning 2.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Recall** the difference between a vector quantity and a scalar quantity. (1 mark)

2 **Calculate** the resultant velocity (magnitude and direction) for a ball with a horizontal component of 20.0 m s^{-1} to the right and a vertical component of 40.0 m s^{-1} downwards (3 s.f.). (3 marks)

3 An arrow is fired to the left at an angle of 25.0° to the horizontal and a speed of 46.0 m s^{-1}. **Calculate** its horizontal and vertical components (3 s.f.). (3 marks)

4 A 10.0 m s^{-1} horizontal velocity vector is represented by an arrow 2.0 cm long and pointing to the right. **Sketch** and label the vector that would represent a ball projected vertically upwards at 5.0 m s^{-1} (2 s.f.). (2 marks)

Analytical processes

5 A ball strikes the ground with a resultant velocity of magnitude 150 m s^{-1}. It is known that the horizontal component of the velocity is 62 m s^{-1} to the right. **Determine** the vertical component and the angle of impact relative to the ground. (3 marks)

6 A football is kicked to the right at a speed of 26.6 m s^{-1} and the horizontal component is measured as 25.0 m s^{-1}. **Determine** the vertical component of its speed and the angle above the horizontal (3 s.f.). (3 marks)

7 **Deduce** the angle above the horizontal of the velocity vector that would produce a vertical component of velocity that is twice its horizontal component. Give your answer to the nearest whole number. (3 marks)

Knowledge utilisation

8 Vector A has a magnitude of 100 units and is directed upwards towards the left at 15° from the vertical. **Prove** that the x and y components are approximately 97 and 26 units respectively. (3 marks)

9 Vector A has a magnitude of 100 units and points towards the left 45° above the horizontal. Vector B is twice the length and points towards the right at 45° above the horizontal. **Prove** that when these two vectors are added, the resultant is 224 units at an angle of 72° above the horizontal pointing towards the right. (5 marks)

10 **Evaluate** the following statement: "For a velocity vector at an angle above the horizontal of 45°, the horizontal and vertical components of any velocity are equal." (2 marks)

11 **Assess** whether the following statement is true: "When the angle of elevation of a velocity vector above the horizontal is doubled, the vertical component of the velocity is doubled." (2 marks)

12 A vector has only an x-component and a y-component. **Identify** whether the x- or y-component can be greater than the magnitude of the resultant vector. **Explain** your answer. (2 marks)

Lesson 2.2
Horizontal projection

Key ideas

- → The velocity vector of a projectile can be resolved into horizontal and vertical components.
- → The horizontal and vertical components of the velocity vector are independent of each other.
- → The horizontal component of velocity is constant; only the vertical component changes.

Learning intentions and success criteria

trajectory
the path taken by a projectile in flight

What types of projectile motion are there?

Projectile motion can be divided into two types: horizontal projection and projection at an angle, as shown in Figure 1. You can see each projectile has a different path or **trajectory** (from the Latin "trajectus" meaning "crossing" or "passage"). In both cases air resistance is assumed to be negligible. However, air can have a dramatic effect on the trajectory of projectiles. We will consider projection without air first and then discuss the effect of projection through a fluid (such as air) later.

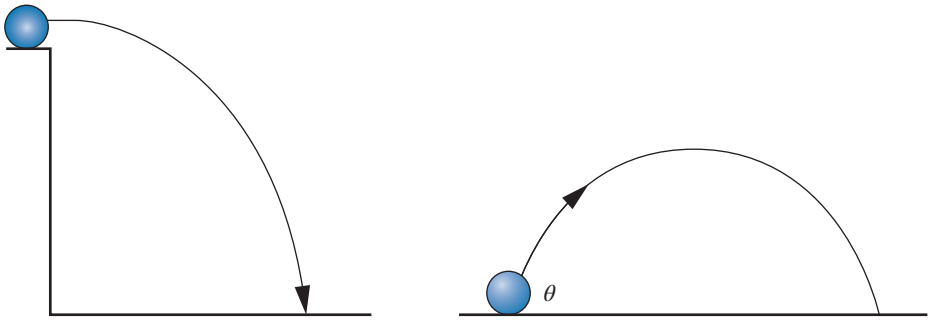


FIGURE 1 Horizontal projection and projection at an angle

FIGURE 2 The iconic closing scene from the film *Thelma and Louise* shows a car as a horizontal projectile with a hubcap just coming off. Which will reach the ground first – the car or the detached hubcap – and why?

What is free-fall acceleration?

If you drop a rock, it will accelerate downwards at $9.8\,m\,s^{-2}$ provided that no other external forces such as air resistance interfere with its motion. This is called its **free-fall acceleration**.

free-fall acceleration
the acceleration of a body falling freely in a vacuum near the surface of an astronomical body in the local gravitational field

On Earth, the acceleration due to the force of gravity, g, is equal to $9.8\,m\,s^{-2}$, with slight variations from location to location. The value of acceleration due to gravity on other astronomical bodies depends largely on the mass of the astronomical body. Some examples are shown in Table 1. This information is used in later modules.

TABLE 1 Mass and free-fall acceleration

Astronomical body	Free-fall acceleration ($m\,s^{-2}$) at the surface
Moon	1.6
Mars	3.7
Earth	9.8
Jupiter	24.8
Sun	274.4

What are the conventions for analysing motion?

Before you analyse the motion, you need to specify the symbols for various quantities. You will use the convention that the upwards direction is positive (+) and downwards is negative (–), and in the horizontal direction to the right is positive (+) and to the left is negative (–).

- s_x is the displacement in the horizontal (x) direction (also called the **range**).
- s_y is the displacement at impact in the vertical (y) direction.
- u_x is the initial velocity in the horizontal (x) direction.
- v_x is the final velocity in the horizontal (x) direction.
- u_y is the initial velocity in the vertical (y) direction.
- v_y is the final velocity in the vertical (y) direction.
- g is the acceleration due to gravity ($-9.8\,m\,s^{-2}$).

range
the horizontal displacement of a projectile from launch to impact

Horizontal projection is the simplest trajectory to analyse. An example is a rock thrown straight out off a cliff. There is no net force on the rock in the horizontal (x) direction to cause acceleration, so the final velocity equals the initial velocity ($v_x = u_x$). In the vertical direction, the initial vertical velocity equals zero ($u_y = 0$) but increases as time passes as the rock accelerates downwards due to gravity.

A graph of vertical displacement (s_y) versus horizontal range (s_x) of two balls in free-fall shows this clearly (Figure 3). The graph depicts the balls at equal time intervals of 0.1 s and shows that the balls fall a greater distance each time interval. It also shows that the balls fall at the same rate whether they are dropped (red ball) or projected horizontally (blue ball). Note that the blue ball moves the same distance horizontally every time interval because it is moving at constant speed in that direction.

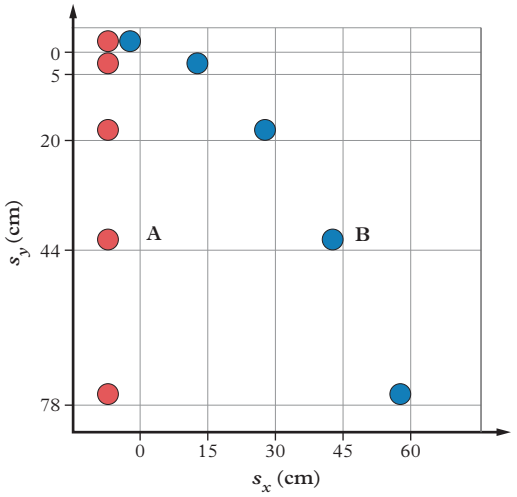


FIGURE 3 The positions of two balls in free-fall each second

The vertical and horizontal components of velocity vectors are independent of each other

Study tip

It may help you understand that the vertical and horizontal components of the velocity vectors are independent of each other by making a video of two balls being released simultaneously: one free-falling vertically, and the other projected horizontally.

We can represent the vertical and horizontal components of the velocity of the blue ball using vector arrows, as shown in Figure 4. The vector arrows for the horizontal velocity, v_x, are the same length throughout the motion. The arrows for the vertical velocity, v_y, get longer over the ball's motion. The vertical and horizontal components of the velocity vectors are independent of each other. That is, the horizontal component stays constant in magnitude and direction whereas the vertical component changes.

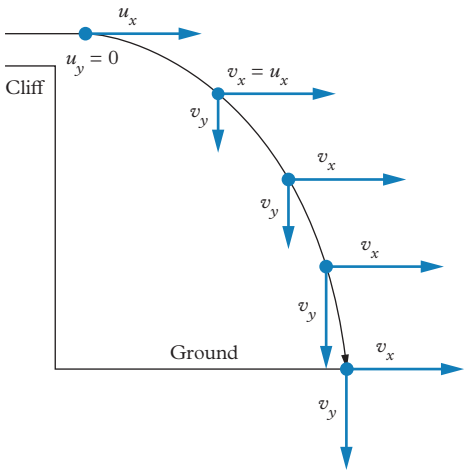


FIGURE 4 Vector arrows showing changes in the vertical and horizontal components of velocity of a projectile

Formulas for horizontal motion

In the horizontal direction, the acceleration (a) is zero, so the appropriate formulas for velocity and displacement are:

i $v_x = u_x + at$ (where acceleration, $a = 0$)

$v_x = u_x$

ii $s_x = u_x t + \frac{1}{2}at^2$ (where acceleration, $a = 0$)

$s_x = u_x t$

Formulas for vertical motion

In the vertical direction, an object will accelerate downwards. If it is given no initial downwards velocity, then it will start at rest in the vertical direction ($u_y = 0$), and its acceleration (g) will be $-9.8\,\text{m s}^{-2}$. The appropriate formulas for velocity and displacement are:

$v_y = u_y + gt$

$v_y^2 = u_y^2 + 2gs_y$

$s_y = u_y t + \frac{1}{2}gt^2$

Remember that the time taken in the vertical direction is the same as the time taken in the horizontal direction. The object doesn't split in two and have two different times. That's why you can just use t for time in both directions.

FIGURE 5 The impact velocity of a long jumper is the combination of the vertical and horizontal components of velocity.

Worked example 2.2A

Calculating horizontal displacement for a projectile

A golf ball is projected horizontally off a cliff at 20.0 m s^{-1} and it takes 4.0 s to reach the ground. **Calculate** how far out from the base of the cliff the ball will land. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather the relevant data.	$u_x = v_x = 20.0\,\text{m s}^{-1}$, $t = 4.0\,\text{s}$ Find s_x. $s_x = u_x t$
Step 3: Substitute in the known values. (1 mark for "Demonstrates correct substitution")	$s_x = u_x t$ $= 20.0 \times 4.0$ (1 mark) $= 80.0$
Step 4: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	$s_x = 80\,\text{m}$ (1 mark)

Your turn

A pot plant is thrown horizontally out of a third-floor window at a speed of 18 m s^{-1}, and it takes 1.2 s to strike the ground below. **Calculate** how far out from the base of the building the pot plant lands. Give your answer to an appropriate number of significant figures. (2 marks)

Worked example 2.2B

Calculating vertical velocity and displacement

For the golf ball in Worked example 2.2A, **calculate** (to an appropriate number of significant figures)

a the final (impact) velocity in the vertical direction (2 marks)

b the height of the cliff. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question.	$u_x = 20.0\,\text{m s}^{-1}$, $u_y = 0\,\text{m s}^{-1}$, $t = 4.0$ s, $g = -9.8\,\text{m s}^{-2}$
Step 3: Identify the quantities to be calculated.	v_y, s_y
Step 4: Select the correct formula and substitute in the known values. Recall that g is directed downwards so it is negative. (1 mark for "Demonstrates correct substitution")	**a** $v_y = u_y + gt$ $= 0 + -9.8 \times 4.0$ (1 mark)

Think	Do
Step 5: Perform the calculation and state the answer with the correct sign and units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Calculates the answer correctly")	$v_y = -39.2\,\text{m s}^{-1}$ The impact velocity is $39\,\text{m s}^{-1}$ in the downwards direction (2 s.f.). (1 mark)
Step 6: Select the correct formula and substitute in the known values. Recall that g is directed downwards so it is negative. (1 mark for "Correctly substitutes values")	**b** $s_y = u_y t + \frac{1}{2} g t^2$ $= 0 \times 4.0 + \frac{1}{2} \times -9.8 \times 4.0^2$ (1 mark)
Step 7: Perform the calculation and state the answer with the correct sign and units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Calculates the displacement")	$s_y = -78\,\text{m}$ (1 mark)
Step 8: The question asks for the height of the cliff, which is a scalar quantity and has no direction, rather than the displacement, which is a vector quantity and has direction (+/−). The displacement is −78 m (to 2 s.f.), so the cliff is 78 m high. (1 mark for "Provides correct answer")	$h = 78\,\text{m}$ (2 s.f.) (1 mark)

Your turn

A rock is thrown horizontally off a cliff at a speed of $18\,\text{m s}^{-1}$, and it crashes into the rocks below after 3.5 s have elapsed. **Calculate** (to an appropriate number of significant figures)

a the final (impact) velocity in the vertical direction (2 marks)

b the height of the cliff. (3 marks)

How is the resultant velocity on impact determined?

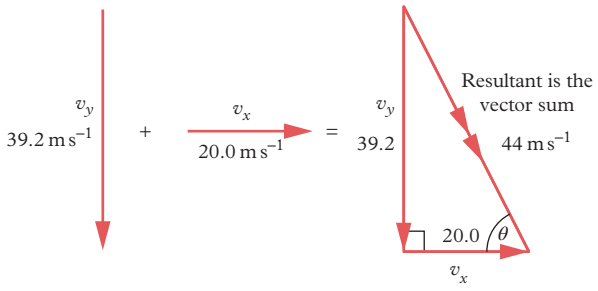
So far you have calculated the vertical and horizontal components of the velocity of a golf ball when it lands on the ground. This is the final, or impact, velocity. You need to combine these vectors to determine the resultant vector using some of the techniques you learnt in Lesson 2.1. We see it here in this Worked example 2.2C.

Worked example 2.2C

Calculating the resultant velocity of a projectile

A golf ball is projected horizontally off a cliff, and when it strikes the ground at the base of the cliff it has a final velocity in the vertical direction of $39.2\,\text{m s}^{-1}$ downwards, and a final velocity in the horizontal direction of $20.0\,\text{m s}^{-1}$ to the right. **Calculate** the magnitude and direction of the resultant velocity. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant facts from the question.	$v_x = 20.0\,\text{m s}^{-1}$, $v_y = -39.2\,\text{m s}^{-1}$

Think	Do
Step 3: Draw a vector diagram by placing the horizontal and vertical vectors head to tail and then draw the resultant vector. (1 mark for "Constructs vector diagram to find resultant")	 (1 mark)
Step 4: Calculate the magnitude of the resultant vector using Pythagoras' theorem. (1 mark for "Provides appropriate mathematical reasoning")	$c = \sqrt{a^2 + b^2}$ Resultant $= \sqrt{39.2^2 + 20.0^2}$ $= \sqrt{1{,}937}$ $= 44.0$ (1 mark)
Step 5: Mark the angle θ between the resultant vector and the horizontal vector. Calculate the value of θ. (1 mark for "Calculates the angle")	$\theta = \tan^{-1}\frac{39.2}{20.0}$ $= \tan^{-1} 1.96$ $= 63.0°$ (1 mark)
Step 6: State the answer, including the magnitude and direction of the resultant. Make sure you use the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	The impact velocity is 44 m s^{-1} at 63° to the horizontal. (1 mark for full description)

Your turn

An arrow is fired horizontally from a shoulder height of 1.50 m at a speed of 55.0 m s^{-1}. When it strikes the ground 30 m away, it has a horizontal component of velocity of 55.0 m s^{-1} and a vertical component of velocity of −5.42 m s^{-1}. **Calculate** its resultant velocity on impact. Give your answer to an appropriate number of significant figures. (4 marks)

How are the vertical and horizontal components of motion linked?

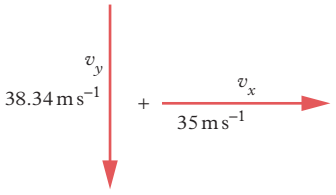
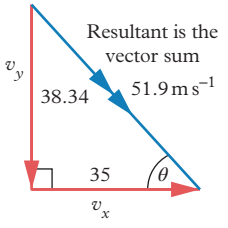
Projectile questions will not always be straightforward. You will be given different quantities, and it will require critical thinking to solve problems with the data. Worked example 2.2D shows you some more-complicated situations.

Worked example 2.2D

Calculating impact velocity for horizontal projection

A rock is thrown horizontally at a speed of 35 m s^{-1} off a cliff 75 m high. **Determine** the rock's velocity when it impacts the ground below. Give your answer to an appropriate number of significant figures. (6 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: Gather the relevant information from the question. The vertical displacement of the rock when it hits the ground is in the negative (downwards) direction, so the displacement, which is a vector quantity, is –75 m. Make sure you include the sign, and do not just use the height of 75 m.	$u_x = 35\,\text{m s}^{-1}$, $u_y = 0\,\text{m s}^{-1}$, $g = -9.8\,\text{m s}^{-2}$, $s_y = -75\,\text{m}$
Step 3: Identify what quantity you have to find.	You need to find the final velocity, v.
Step 4: To determine the final velocity, you need the final horizontal velocity and the final vertical velocity. First, determine the final horizontal velocity. The horizontal velocity is constant, so the final horizontal velocity equals the initial horizontal velocity. State this value and its units. (1 mark for "Identifies the equivalence of final and initial horizontal velocities")	$v_x = u_x$ $= 35\,\text{m s}^{-1}$ (1 mark)
Step 5: To find the final vertical velocity, you need to select the correct formula to solve for v_y using the other given information.	$v_y^2 = u_y^2 + 2gs_y$
Step 6: Substitute in the known values and solve for v_y. (1 mark for "Demonstrates correct substitution") Note: you may prefer to rearrange the equation first to make v_y the subject before making the substitutions. One less step can help avoid transcription errors.	$v_y^2 = 0^2 + 2 \times -9.8 \times -75$ $v_y^2 = 1{,}470$ $v_y = \sqrt{1{,}470}$ $= \pm 38.34\,\text{m s}^{-1}$ (1 mark)
Step 7: Determine the correct value for v_y. The square root of a number can be either positive or negative depending on the conventions. As downwards is the negative direction, the vertical velocity must also be negative. Write this value down.	$v_y = -38.34\,\text{m s}^{-1}$
Step 8: Draw the two vectors as directed line segments (arrows).	
Step 9: Place the two vectors head to tail. Draw a resultant vector from the tail of one to the head of the other, with a double arrowhead. (1 mark for "Constructs vector diagram")	 (1 mark)
Step 10: Calculate the length of the resultant vector using Pythagoras' theorem. (1 mark for "Provides appropriate mathematical reasoning")	$c = \sqrt{a^2 + b^2}$ Resultant $= \sqrt{38.34^2 + 35^2}$ $= \sqrt{2{,}695}$ $= 51.9\,\text{m s}^{-1}$ (1 mark)
Step 11: Determine the angle between the resultant and the horizontal using trigonometry. (1 mark for "Determines the angle correctly")	$\theta = \tan^{-1}\frac{38.34}{35}$ $= \tan^{-1} 1.095$ $= 47.6°$ (1 mark)
Step 12: State the magnitude and direction of the resultant vector using the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. You don't need to specify that the angle is above the horizontal if it is clear in your vector diagram. (1 mark for "Provides correct answer")	The impact velocity is $52\,\text{m s}^{-1}$ at 48° (above the horizontal) (2 s.f.). (1 mark)

Your turn

A ball is kicked horizontally off a 100.0 m high cliff at a speed of 10.0 m s^{-1}. **Determine** the velocity on impact with the ground. Give your answer to an appropriate number of significant figures. (6 marks)

What do graphs of horizontal projectile motion show?

Graphs are an important way to show the various relationships involved in projectile motion. Here we will look at the graphs for horizontal projection.

The most useful graphs are plots of the horizontal and vertical components of displacement and velocity for an object projected horizontally. First, we can look at displacement.

Displacement–time graphs

Consider a ball kicked horizontally at a speed of $10\,m\,s^{-1}$ off a cliff 100 m high. As the ball moves away from the cliff it will start to fall towards the ground. The horizontal displacement will increase by 10 m every second. This will be a constant 10 m every second as there are no net forces in the horizontal direction to cause it to speed up or slow down (Newton's first law). In the vertical direction, the ball will also have an increasing displacement, but the increase won't be regular because it is accelerating. This is shown in the two graphs in Figures 6 and 7.

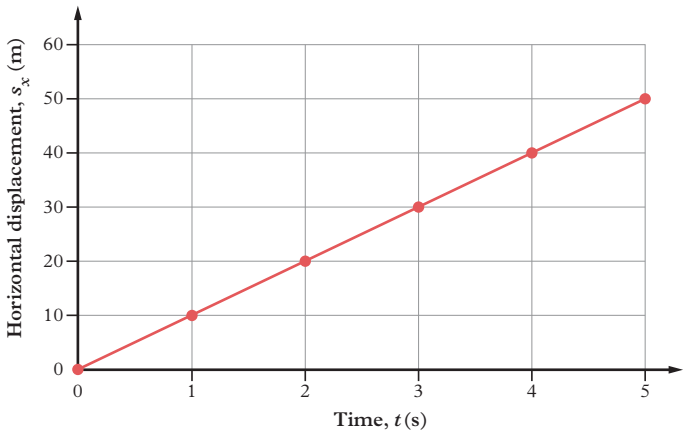


FIGURE 6 The horizontal displacement of the ball increases regularly with time.

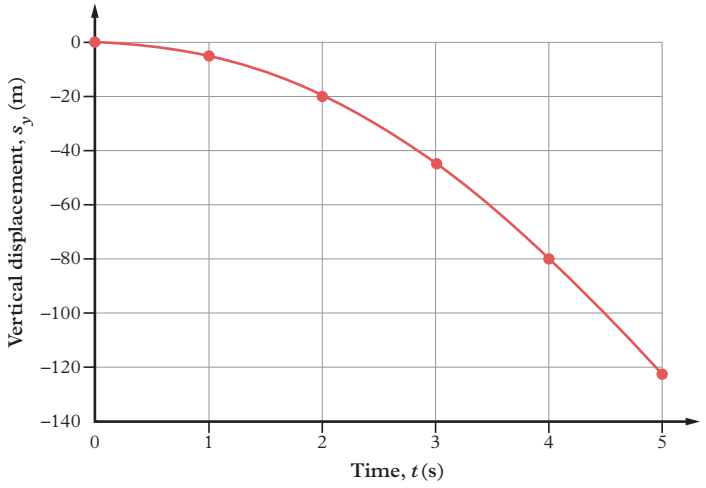


FIGURE 7 The vertical displacement of the ball does not increase regularly. The displacement–time graph shows a line that is getting more curved as time goes by.

In Figure 6, the displacement–time graph for horizontal motion, shows a regular increase. We know that the gradient of a displacement–time graph is velocity. The gradient of this graph is $10\,m\,s^{-1}$, which is the horizontal component of the ball's velocity.

In Figure 7, the displacement–time graph for vertical motion, you can see the displacement is in the negative direction, which is consistent with the "negative is down" convention we have chosen. The graph curves more over time because the ball is accelerating downwards due to gravity. The gradient is not constant but gets steeper as time progresses. The instantaneous velocity at any point is the gradient of the tangent to the curve at that point; this is also negative and gets steeper over time.

Velocity–time graphs

We have analysed displacement–time graphs to draw conclusions about the horizontal and vertical velocity of a ball. This tells us what the velocity–time graphs should look like for the horizontal and vertical components of motion of the ball.

The horizontal velocity is constant at $10\,m\,s^{-1}$. The gradient of the velocity–time graph (Figure 8) represents the acceleration. As there is no change in horizontal velocity, there is no acceleration in the horizontal direction and the graph is a horizontal line (i.e. $a = 0\,m\,s^{-2}$). The area under the graph is the horizontal displacement after 5 s ($10 \times 5 = 50$ m).

The vertical velocity increases in the negative direction by $9.8\,m\,s^{-1}$ every second. The gradient of the velocity–time graph (Figure 9) represents the acceleration, $a = -9.8\,m\,s^{-2}$, as you would expect for vertical motion. The area under the curve is the vertical displacement after 5 s (about −125 m).

This is the acceleration due to gravity, g.

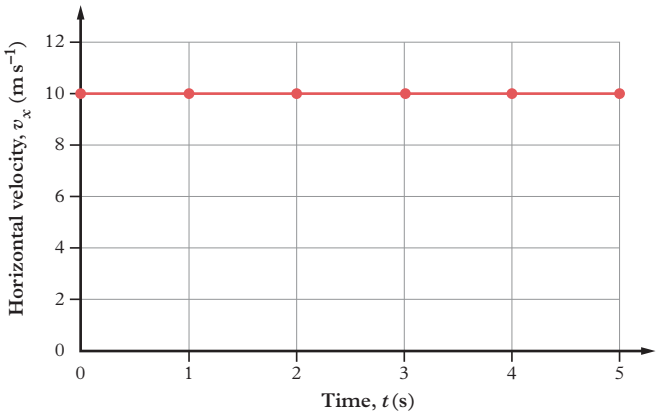


FIGURE 8 Velocity–time graph in the horizontal direction

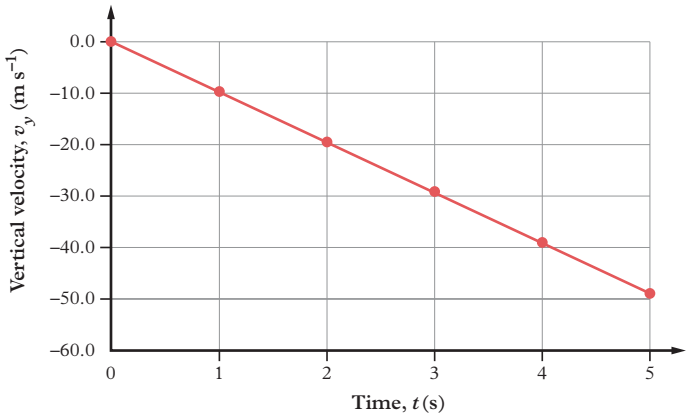


FIGURE 9 Velocity–time graph in the vertical direction

Skill drill

Analysing height and range data for a projectile

Science inquiry skills: Processing and analysing data (Lesson 1.7)

Being able to analyse data to identify trends is a key part of experimental physics.

A ball is projected horizontally at constant speed from different heights s_y above the floor, and its horizontal range s_x is measured (Figure 10).

The results are given in Table 2.

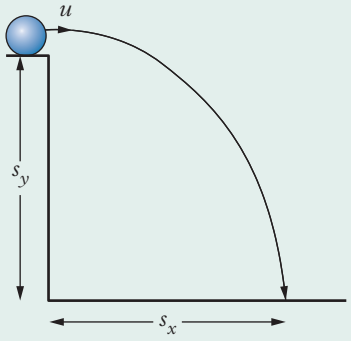


FIGURE 10 Setup for the "range versus height" experiment

TABLE 2 Data for the "range versus height" experiment

Vertical height, s_y (m)	Horizontal range, s_x (m)				Horizontal range squared, s_x^2 (m^2)
	Trial 1	Trial 2	Trial 3	Average	
0.10	0.19	0.18	0.17	0.18	0.03
0.20	0.23	0.25	0.24	0.24	0.06
0.40	0.33	0.34	0.35	0.34	0.12
0.60	0.42	0.42	0.42	0.42	0.18
0.80	0.47	0.48	0.48	0.48	0.23
1.00	0.57	0.54	0.52	0.54	0.30

Practise your skills

1 **Determine** the percentage uncertainty in the horizontal range for a vertical height of 1.00 m (1 s.f.). (2 marks)

2 **Deduce** the relationship between the horizontal range and vertical height and **justify** your answer. (2 marks)

3 **Explain** how you would linearise the horizontal range and vertical height data to confirm the relationship. (2 marks)

Check your learning 2.2

Check your learning 2.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Recall** whether the horizontal component of a projectile's velocity is independent of the vertical component. (1 mark)

2 The acceleration of a projectile is usually stated as $-9.8\,m\,s^{-2}$. **Explain** how it can be negative when the projectile gets faster and faster as it falls towards the ground. (2 marks)

3 A dropped ball and a ball projected horizontally from the same height both land at the same time. **Explain** how this can be when the projected ball has to travel a longer path. (2 marks)

4 **Clarify** how the initial and final components of horizontal velocity are related. (1 mark)

Analytical processes

5 A skier jumps off a mountaintop at a horizontal velocity of $25.0\,m\,s^{-1}$ and takes 5.00 s to reach the ground.

Determine (3 s.f.) the

- **a** height of the mountaintop (3 marks)
- **b** distance out from the base of the mountaintop that the skier lands (2 marks)
- **c** velocity on impact with the ground. (3 marks)

6 A rock is thrown horizontally at $8.00\,m\,s^{-1}$ off a 100.0 m high cliff. **Determine** (3 s.f.)

- **a** how long it takes the rock to hit the ground (3 marks)
- **b** the impact velocity of the rock (5 marks)
- **c** how far out from the cliff the rock lands. (1 mark)

7 A ball is thrown horizontally from the top of a cliff and hits the ground 6.0 s later at 30° to the vertical, moving at $67.9\,m\,s^{-1}$. **Determine**

- **a** the initial velocity of the projectile (4 marks)
- **b** its initial horizontal and vertical velocity (1 mark)
- **c** the range (1 mark)
- **d** the final vertical velocity. (2 marks)

Knowledge utilisation

8 A cricket player hits a ball horizontally on Earth at a speed of $50\,m\,s^{-1}$. Now imagine they hit the ball in the same way on the Moon. **Predict** and **justify** with calculations how the range and impact velocity would be different. (5 marks)

9 An astronaut on the surface of Mars decides to emulate the golf stroke of *Apollo 14* astronaut Alan Shepard, who hit some golf balls on the Moon while visiting in 1971. The Mars astronaut stands on the edge of the Endeavour Crater and hits a golf ball horizontally. It lands off in the distance. **Evaluate**, with reasoned evidence, this statement: "Under exactly the same conditions, the ball would travel horizontally 1.6 times further on Mars than on Earth." (4 marks)

Lesson 2.3
Projection at an angle

Key ideas

- → The velocity vector for projection at an angle can be resolved into horizontal and vertical components.
- → The horizontal and vertical components of the velocity vector are independent for projection at an angle.
- → The horizontal component of velocity is constant; only the vertical component changes.

Learning intentions and success criteria

How does projection at an angle differ from horizontal projection?

Not all projectiles are launched in the horizontal direction. Arrows, cannonballs, footballs and netballs, for example, are usually projected upwards at an angle. You can still resolve the initial velocity into horizontal and vertical components as you did for horizontal projection.

Consider an arrow being launched upwards at an angle of θ to the horizontal. This is called the **elevation angle** and can be represented in a vector diagram. The initial **launch velocity** u and the angle θ are shown in the vector diagram in Figure 1.

elevation angle the angle at which a projectile is launched with respect to the horizontal

launch velocity the velocity at which a projectile is launched; a vector quantity that has both magnitude and direction, usually specified as an angle of elevation to the horizontal

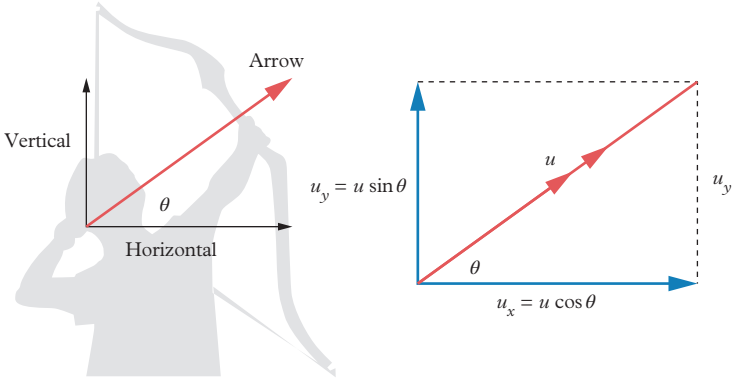


FIGURE 1 Vertical and horizontal components of the initial velocity of a projectile

The initial velocity can be resolved into two components at right angles (perpendicular) to each other. The initial velocity u is resolved into a vertical (y-axis) component u_y and a horizontal (x-axis) component u_x. All you need to do is make a rectangle with the initial vector as the diagonal. Using the trigonometric ratios, you get, as before:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{u_y}{u}$$
$$\text{or } u_y = u\sin\theta$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{u_x}{u}$$
$$\text{or } u_x = u\cos\theta$$

Motion of an object projected at an angle

The motion of the projectile is a **parabola** because the vertical displacement varies as a function of t^2 (i.e. $s_y = u_y t + \frac{1}{2} g t^2$), as it is uniformly accelerated motion; however, the horizontal displacement (the range) varies with just t (i.e. $s_x = u_x t$), as it is constant velocity.

parabola
a graph of a quadratic function for which the power a is 2, e.g. $y = x^2$

Case 1: What happens if the projectile lands at the same height as at launch?

For the simplest case of projection at an angle, the object will land at the same height (vertical displacement) from which it was launched, as shown in Figure 2. For this scenario, the following statements can be made.

- The **impact speed** will be equal to the launch speed. Recall from earlier in this module that for purely vertical motion, initial speed equals final speed.
- The impact velocity will have the same magnitude and angle to the ground as the launch velocity, but it will be in a general downwards direction, not upwards as at launch.
- The horizontal component of velocity remains constant for the duration of the flight, hence $v_x = u_x$.
- The vertical component of velocity at launch equals the vertical component of velocity at impact, but in the opposite direction: v_y (impact) $= -u_y$ (launch).

impact speed
the speed at which an object hits a surface; the final speed

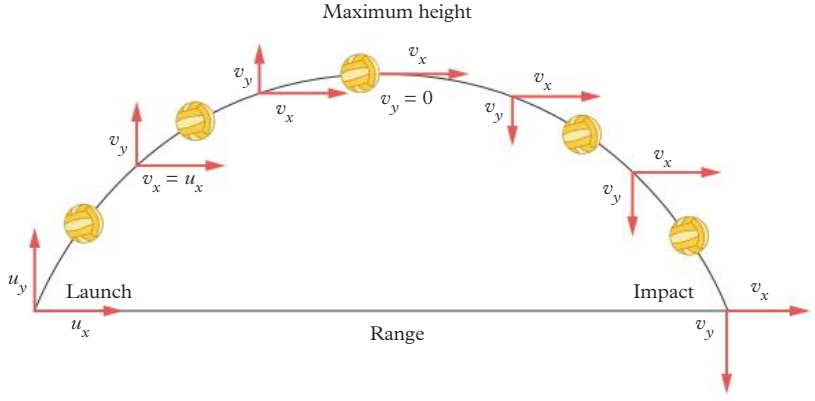


FIGURE 2 Horizontal and vertical components of velocity for the flight of a projectile

Worked example 2.3A

Calculating the motion of a projectile landing at the same height

A bullet was fired from a rifle at 200.0 m s^{-1} at an angle of 40.0° to the ground. Ignoring air resistance, **calculate** (to an appropriate number of significant figures) the

a initial vertical and horizontal components of the velocity (2 marks)

b maximum height reached (3 marks)

c time of flight (total time taken from start to finish) (2 marks)

d horizontal range. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question. Check for any information that is non-numerical such as "from rest".	**a** $u = 200.0\,\text{m s}^{-1}$, $\theta = 40.0°$, $g = -9.8\,\text{m s}^{-2}$ (the upwards direction is positive)
Step 3: Apply trigonometry to determine the formula for the vertical component and substitute in the given information. Calculate the value and check that the sign is appropriate (up is +, down is −). Check the units are appropriate. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides correct answer for vertical component")	$u_y = u\sin\theta$ $= 200.0 \times \sin 40.0°$ $= 128.6$ $= +129\,\text{m s}^{-1}$ (3 s.f.) or $129\,\text{m s}^{-1}$ upwards (1 mark)
Step 4: Apply trigonometry to determine the formula for the horizontal component and substitute in the given information. Calculate the value and check again that the sign is appropriate (up is +, down is −). Check the units are appropriate. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides correct answer for horizontal component")	$u_x = u\cos\theta$ $= 200.0 \times \cos 40.0°$ $= 153.2$ $= 153\,\text{m s}^{-1}$ (1 mark)
Step 5: Select the appropriate formula, $v_y^2 = u_y^2 + 2gs_y$, and rearrange to make s_y the subject. (1 mark for "Rearranges equation")	**b** $v_y^2 = u_y^2 + 2gs_y$ $s_y = \frac{v_y^2 - u_y^2}{2g}$ (1 mark)
Step 6: Substitute in the known values and solve for s_y. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$s_y = \frac{0^2 - (+129)^2}{2 \times (-9.8)}$ (1 mark) $= +844$ m (3 s.f.) (1 mark)
The time of flight can be calculated in three ways. All three methods are included so you can see the differences and decide what may suit you the best. Step 7: **Method 1 for time of flight** Determine the time taken to reach the maximum height ($v_y = 0$) and double it to get the time of flight. Select the appropriate equation $v_y = u_y + gt$ and rearrange to make t the subject.	**c** $v_y = u_y + gt$ $t = \frac{v_y - u_y}{g}$
Step 8: **Method 1 continued** Substitute in the known values and solve for t. Use the correct units. (1 mark for "Provides appropriate mathematical reasoning")	$t = \frac{0 - (+129)}{-9.8}$ $= 13.12\,\text{s}$ (1 mark)
Step 9: **Method 1 continued** To get the time of flight, double the time it takes to get to the maximum height. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides correct answer")	$t_{\text{total}} = 2 \times t_{\text{max height}}$ $= 2 \times 13.12$ $= 26.24$ $= 26.2$ (3 s.f.) (1 mark)
Method 2 for time of flight Determine the time taken until the final vertical velocity is equal and opposite to initial vertical velocity. Select the appropriate formula, $v_y = u_y + gt$, and rearrange to make t the subject.	$v_y = u_y + gt$ $t = \frac{v_y - u_y}{g}$

Think	Do
Method 2 continued Substitute in the known values and solve for t. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$t_{total} = \frac{-129 - (+129)}{-9.8}$ (1 mark) $= 26.24$ $= 26.2$ s (3 s.f.) (1 mark)
Method 3 for time of flight Determine the time until the vertical displacement, s_y, is zero again. Select the appropriate formula.	$s_y = u_y t + gt^2$
Method 3 continued Substitute in the known values and solve for t. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$s_y = u_y t + gt^2$ $0 = +128.6t + -4.9t^2$ (1 mark) $4.9t = 128.6$ $t = 26.24$ $= 26.2$ s (3 s.f.) (1 mark)
Step 10: Select the correct formula and substitute in the known values. Solve for the horizontal range. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	**d** $s_x = u_x \times t$ $= 153.2 \times 26.24$ (1 mark) $= 4{,}020$ m (3 s.f.) (1 mark)

Your turn

A golf ball was launched at 61.0 m s^{-1} at an angle of 22.0° to the ground. Ignoring air resistance, **calculate** (to an appropriate number of significant figures) the

a initial vertical and horizontal components of the velocity (2 marks)

b maximum height reached (3 marks)

c time of flight (total time taken from start to finish) (2 marks)

d horizontal range. (2 marks)

Challenge

Archery world speed record

The world speed record for an archery arrow shot over 100 m is 1.64 s (220 km h^{-1}). **Calculate** the elevation angle of the arrow so that it hits the bullseye at the same height as it was fired (shoulder height). (5 marks)

Case 2: What happens if the projectile lands at a lower height than at launch?

In some scenarios, an object will land at a lower height than at launch – for example, a person launching a shot-put at shoulder height and it hitting the ground, or a skier launching upwards off a ramp and landing further down the mountain (Figure 3). The main difference between this and Case 1, landing at the same height as at launch, is that the vertical displacement at impact is no longer zero but some negative quantity. This is explored in Worked example 2.3B.

FIGURE 3 For a skier jumping from a bump, their landing position is lower than their take-off position.

FIGURE 4 When shooting for the hoop, the velocity, angle and height of release have to be just right.

Worked example 2.3B

Calculating motion for a projectile landing at a lower height

A cannon is fired from the edge of a cliff, as shown in Figure 5. The tip of the cannon is 60.0 m above the surface of the sea.

The initial velocity of the cannonball is 88.3 m s^{-1} and it is fired at an upwards angle of 34.5° to the horizontal.

a **Determine** the time the ball is in the air. Give your answer to an appropriate number of significant figures. (4 marks)

b **Construct** a vector diagram, and determine the impact velocity when it strikes the water. Give your answer to an appropriate number of significant figures. (6 marks)

c **Determine** the horizontal distance from the base of the cliff to where the ball strikes the water. Give your answer to an appropriate number of significant figures. (2 marks)

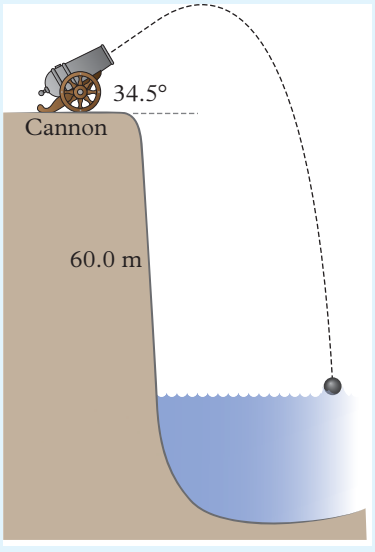
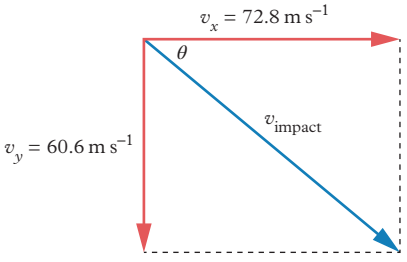


FIGURE 5 The cannonball lands lower than its starting position.

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after calculation. "Construct" means to display information in a diagrammatic or logical form. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question.	**a** $v = 88.3\,\text{m s}^{-1}$, $s_y = -60.0\,\text{m}$, $\theta = 34.5°$ to the horizontal, $g = -9.8\,\text{m s}^{-2}$
Step 3: Identify what is asked for in the question.	t_{total}, v_{final}, s_x
Step 4: To calculate the time of flight we need to calculate the vertical component of initial velocity using trigonometry. (1 mark for "Calculates the vertical component of initial velocity")	$u_y = u\sin\theta$ $= 88.3\sin 34.5°$ $= +50.0\,\text{m s}^{-1}$ (positive is up) (1 mark)

Think	Do
Step 5: You can calculate time of flight directly, but it would involve solving a quadratic equation (and all the possibility for errors this method seems to attract). To avoid this, calculate the vertical component of the impact velocity first. (1 mark for "Calculates the vertical component of impact velocity")	$v_y^2 = u_y^2 + 2gs_y$ $= +50^2 + 2 \times (-9.8 \times (-60.0))$ $= 2{,}500 + 1{,}176$ $= 3{,}676$ $v_y = \sqrt{3{,}676}$ $= \pm 60.6\,\text{m s}^{-1}$ $= -60.6\,\text{m s}^{-1}$ (discard the +) (1 mark)
Step 6: Calculate the time of flight by selecting the appropriate equation and rearranging it, before substituting in the values and making the calculation. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$v_y = u_y + gt$ $t = \frac{v_y - u_y}{g}$ $= \frac{-60.6 - (+50)}{-9.8}$ (1 mark) $= 11.3\,\text{s}$ (3 s.f.) (1 mark)
Step 7: Determine the impact velocity. First, determine the vertical component of the initial velocity. You may have already done this in Step 3. (1 mark for "Identifies the vertical component of initial velocity")	**b** $v_y = u_y + gt$ $= -60.6\,\text{m s}^{-1}$ (1 mark)
Step 8: We also need to calculate the horizontal component of the initial velocity, which we can do using trigonometry. (1 mark for "Identifies the horizontal component of initial velocity")	$u_x = u\cos\theta$ $= 88.3\cos 34.5°$ $= 72.8\,\text{m s}^{-1}$ (1 mark)
Step 9: The final horizontal velocity is the same as the initial horizontal velocity as there is no net force in the horizontal direction. Restate the value for u_x. (1 mark for "Identifies the equivalence of final and initial horizontal velocities")	$v_x = u_x$ $= 72.8\,\text{m s}^{-1}$ (1 mark)
Step 10: The impact velocity is calculated by the vector addition of the final vertical velocity and the final horizontal velocity. Draw a vector diagram showing the components of the final velocity and the resultant. (1 mark for "Constructs appropriate vector diagram")	 (1 mark)
Step 11: Calculate the magnitude of the resultant velocity using Pythagoras' theorem. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides correct answer for velocity")	$v = \sqrt{(-60.6)^2 + 72.8^2}$ $= 94.7\,\text{m s}^{-1}$ (1 mark)
Step 12: Calculate the angle θ. State the angle and that it is with respect to the horizontal (ground). Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides correct answer for angle")	$\theta = \tan^{-1}\frac{60.6}{72.8}$ $= 39.8°$ to the horizontal (1 mark)
Step 13: Select the correct formula to calculate the horizontal distance (the range). In this case: horizontal distance, s_x = horizontal velocity, v_x × time of flight, t. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**c** $s_x = v_x \times t$ $= 72.8 \times 11.3$ (1 mark) $= 823\,\text{m}$ out from the base (3 s.f.) (1 mark)

Your turn

A ball is thrown from a third story window 32.0 m above the ground. The initial velocity of the ball is 25.0 m s^{-1}, and it is thrown at an upwards angle of 22.0° to the horizontal. **Determine** (to an appropriate number of significant figures)

a the time the ball is in the air (3 marks)

b the impact velocity when it strikes the ground (4 marks)

c the horizontal distance from the base of the building to where the ball strikes the ground. (2 marks)

Case 3: What happens if a projectile lands higher than launch height?

The final type of projectile trajectory is where it lands higher than the launch height. A good example of this is a basketball player shooting a ball into a hoop.

For example, in Figure 6 the ball leaves the player's hand at an initial vertical displacement of +1.9 m and lands in the hoop at a final vertical displacement of +3.1 m. You can simplify this by saying the initial vertical displacement is 0 m (where it leaves the player's hands), and the final vertical displacement at the hoop is +1.2 m (higher than at the start).
For this example, s_y = +1.2 m, s_x (range) = 4.0 m, θ = 60° upwards, g = − 9.8 m s^{-2}.

Study tip

This type of question is a favourite for external exams. Pay particular attention to whether the final displacement is positive or negative.

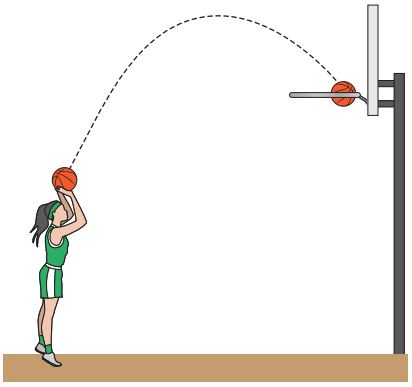


FIGURE 6 Schematic for a ball landing higher than at launch

Worked example 2.3C

Calculating motion for a projectile landing at a greater height

Consider the basketball scenario in Figure 6, and (to an appropriate number of significant figures)

a **determine** the initial velocity of the ball (6 marks)

b **calculate** the time of flight of the ball. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after calculation. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the facts provided in the question.	**a** s_y = +1.2 m, s_x (range) = 4.0 m, θ = 60° to the horizontal, g = −9.8 m s^{-2}
Step 3: Determine the quantities to be found.	u, t
Step 4: Determine the vertical component of the initial velocity. (1 mark for "Calculates the vertical component of velocity")	$u_y = u \sin 60°$ $= 0.866u$ (1 mark)
Step 5: Determine the horizontal component of the initial velocity. (1 mark for "Calculates the horizontal component of velocity")	$u_x = u \cos 60°$ $= 0.500u$ (1 mark)

Think	Do
Step 6: Derive a relationship between the horizontal velocity, distance and time. Replace u_x with $0.500u$ from Step 5. (1 mark for "Identifies the relationship between quantities")	$v_x = u_x = \frac{s_x}{t}$ $t = \frac{s_x}{u_x} = \frac{4.00}{u_x} = \frac{4.00}{0.500\,u}$ (1 mark)
Step 7: Use the formula for displacement in the vertical direction to derive a relationship between s_y, u and t. (1 mark for "Demonstrates correct substitution")	$s_y = u_y t + \frac{1}{2}gt^2$ $+1.2 = 0.866\,ut + \frac{1}{2} \times (-9.8)t^2$ (1 mark)
Step 8: Substitute in expressions for t from Step 6 and solve for u. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$1.2 = 0.866\,u \times \frac{4.0}{0.5u} - 4.9\left(\frac{4.0}{0.5u}\right)^2$ (1 mark) $1.2 = 0.866\,u \times \frac{8.0}{u} - 4.9\left(\frac{8.0}{u}\right)^2$ $1.2 = 6.928 - \frac{313.6}{u^2}$ $5.728 = -\frac{313.6}{u^2}$ $u^2 = \frac{313.6}{5.728}$ $u = 7.4\,\text{m s}^{-1}$ (2 s.f.) (1 mark)
Step 9: Calculate u_x from the value of u in Step 8. (1 mark for "Demonstrates correct substitution")	**b** $u_x = 0.50\,u$ $= 0.50 \times 7.4$ $= 3.7\,\text{m s}^{-1}$ (1 mark)
Step 10: Use the relationship for time from Step 6 and u_x to solve for t. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$t = \frac{4.0}{u_x} = \frac{4.0}{3.7} = 1.1$ s (1 mark)

Your turn

A cricketer hits a ball at a height of 1.50 m from the ground at an angle of 25.0° to the horizontal. The ball just passes over the top of a sightscreen 50.0 m from the cricketer, as shown in Figure 7.

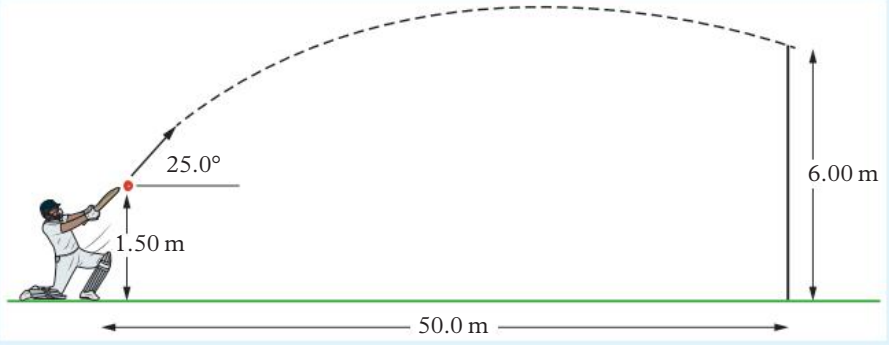


FIGURE 7 The ball is in a higher position than at the start.

a **Determine** the speed at which the ball leaves the bat. Give your answer to an appropriate number of significant figures. (5 marks)

b **Calculate** the time that the ball is in the air before it reaches the screen. Give your answer to an appropriate number of significant figures. (2 marks)

Challenge

Cricket

Which hits the ground faster: a cricket ball thrown horizontally at $10\,\text{m s}^{-1}$ or one thrown at $10\,\text{m s}^{-1}$ but at 45°? (5 marks)

Challenge

Cannonball horizontal distance

A cannonball is fired and, after travelling 5 m horizontally, reaches half its maximum height (h_{max}) (Figure 8). **Determine** the horizontal distance at which the ball will land. (Beware, this is difficult and the solution for this challenge is at least three pages long!) (5 marks)

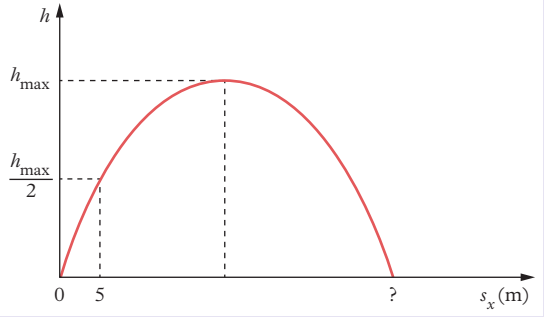


FIGURE 8 Trajectory of a cannonball

What are complementary angles of elevation?

complementary angles
angles that add to 90°; for example, 60° is the complementary angle (complement) of 30° and 75° is the complement of 15°.

If you have ever hosed the garden, you would know that you can direct the water to fall on a particular plant by using either a high or a low angle of elevation for the stream of water. In both cases, the range is the same. The two angles that give the same range are complements of each other. This means they add up to 90°. For example, 60° and 30° give the same range and they add up to 90°. Figure 9 shows a variety of **complementary angles** and the trajectory of a projectile (including a stream of water) for each angle. Practical 2.4 "Angled projection and distance" investigates this.

You can explore this further. Imagine hitting two baseballs with the same force so they both have the same initial speed, but one is at 60° and the other is at 30°. Worked example 2.3D shows how this can be approached.

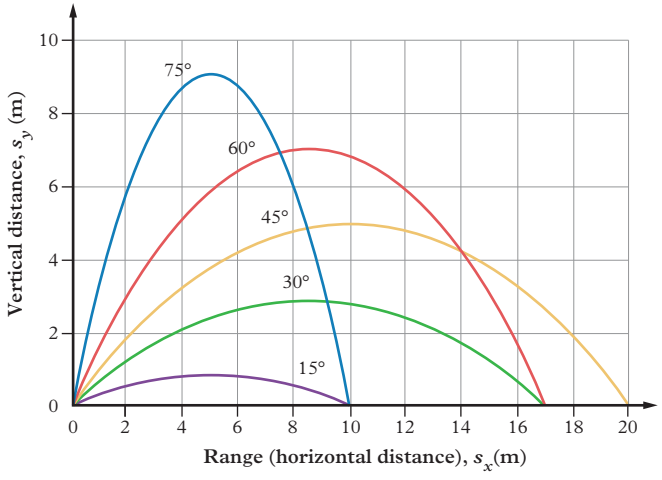


FIGURE 9 Complementary angles give the same range in the absence of air.

Worked example 2.3D

Calculating time and range for projection at complementary angles

A baseball is struck with an initial speed of $20\,m\,s^{-1}$ at angles of elevation of 60° and 30°. Assume air resistance to be negligible.

a **Determine** the time of flight for each angle. Give your answer to an appropriate number of significant figures. (6 marks)

b **Determine** the horizontal range for each angle. Give your answer to an appropriate number of significant figures. (2 marks)

c **Assess** (i) the times of flight, and (ii) the ranges for each of the two angles. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after calculation. "Assess" means to make a judgment about the outcomes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the facts and the quantities required for the answers.	**a** $u = 20\,m\,s^{-1}$, $\theta = 30°$ and $\theta = 60°$ Find t for each and the range for each.
Step 3: Calculate the vertical and horizontal components of the velocity for 60°. (1 mark for "Calculates the components of velocity correctly")	$u_y = 20\sin 60°$ $= 17.3\,m\,s^{-1}$ $u_x = 20\cos 60°$ $= 10\,m\,s^{-1}$ (1 mark)
Step 4: Select the appropriate equations to determine the time of flight for 60°.	$s_y = u_y t + \frac{1}{2}gt^2$
Step 5: Substitute into the equation and make the calculation. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$0 = +17.3t + -4.9t^2$ (1 mark) $17.3t = 4.9t^2$ $17.3 = 4.9t$ $t = 3.5$ (1 mark)
Step 6: Calculate the vertical and horizontal components of velocity for 30°. (1 mark for "Calculates the components of velocity correctly")	$u_y = 20\sin 30°$ $= 10\,m\,s^{-1}$ $u_x = 20\cos 30°$ $= 17.3\,m\,s^{-1}$ (1 mark)
Step 7: Substitute into the appropriate equation and make the calculation. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer") Note: for the higher angle of elevation, the ball is in the air for 3.5 s, compared to 2.0 s for the lower angle. You can decide which would be better in a baseball game: more time in the air means more time to run but also more time to get in position for a catch.	$s_y = u_y t + \frac{1}{2}gt^2$ $0 = +10t + -4.9t^2$ (1 mark) $10t = 4.9t^2$ $10 = 4.9t$ $t = 2.0\,s$ (1 mark)
Step 8: For 60° select the appropriate equation and substitute in values. Calculate the range. (1 mark for "Calculates the range for 60°")	**b** $s_x = u_x t$ $= 10 \times 3.5$ $= 35\,m$ (1 mark)
Step 9: For 30° select the appropriate equation and substitute values. Calculate the range. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Calculates the range for 30°")	$s_x = u_x t$ $= 17.3 \times 2.0$ $= 35\,m$ (1 mark)

Think	Do
Step 10: Compare the times and ranges. (1 mark for "Justifies the conclusion using calculated data for the time and for the range")	**c** (i) The time is longer for the higher angle ($t = 3.5\,s$) than for the lower angle ($t = 2.0\,s$). (1 mark) (ii) The ranges are identical at 35 m. (1 mark)

Your turn

An experiment in class was conducted to test the effect of varying the angle of elevation for projectiles. A steel ball was projected with an initial speed of $20\,m\,s^{-1}$ at angles of elevation of 65° and 25°. Assume air resistance to be negligible. **Determine** (to an appropriate number of significant figures) which angle gave

a the longer time of flight (5 marks)

b the longer horizontal range. (3 marks)

How does the angle of projection affect the horizontal range?

You saw in the previous section that the range of a projectile depends on its angle of projection, and that complementary angles have the same horizontal range. We can look at range as a function of the angle in another way. Figure 10 shows a graph of the results of an experiment in which this relationship is tested. Consider a steel ball with a mass of about 20 g being projected at a constant velocity at various angles of elevation. The initial and final positions of the ball are at the same vertical height.

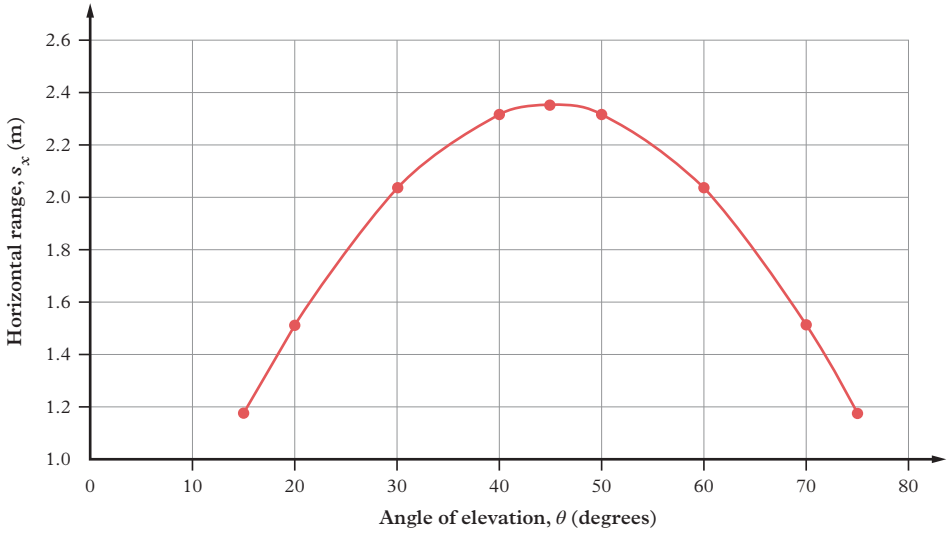


FIGURE 10 Theoretical horizontal range for various launch angles

We can see from the graph that the maximum theoretical range is for 45° and that complementary angles have the same range. However, the time of flight for complementary angles is not the same. The greater the angle of elevation, the longer it takes for the projectile to reach the ground (Figure 11).

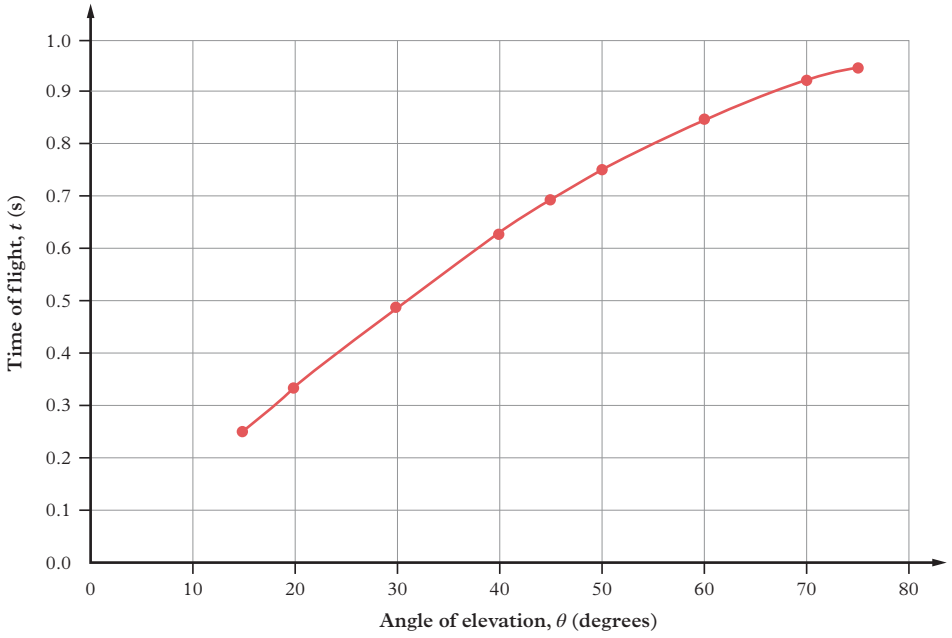


FIGURE 11 Theoretical time of flight for various launch angles

Challenge

How far will a flea go?

A flea can jump 18.4 cm high when jumping at 45°. How far horizontally will it go? (5 marks)

Challenge

Linearising time of flight vs angle data

Propose how you would linearise the relationship shown in Figure 11. **Justify** your proposal by drawing a linearised graph. Hint: you need to consider how the theoretical value of time is calculated from the angle data. (3 marks)

Real-world physics

The effect of air on projectiles

Aristotle argued that once a projectile ran out of a quantity he called "impetus" (what we might call "energy" or "momentum") it would fall vertically from the sky (Figure 12). Galileo argued that this was wrong, and that the trajectory would be parabolic. Galileo was right – or was he?

In the discussion so far, air resistance has been ignored. When air resistance is taken into account, the trajectory is different. Aristotle is almost right, but for the wrong reasons. At low speeds air resistance is negligible, but at greater speeds it becomes considerable.

For instance, a baseball hit at an angle of elevation of 60° at 45 m s^{-1} will have a different trajectory in air than in a vacuum.

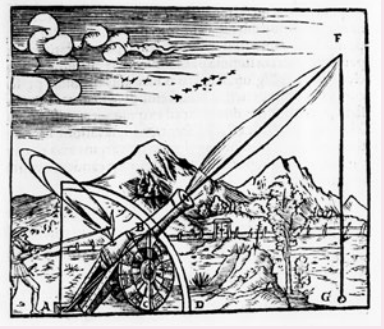


FIGURE 12 The path of a projectile according to Aristotelian physics, later contradicted by Galileo

TABLE 3 Computer model of the trajectory of a ball through air and a vacuum

	Path A (in air)	Path B (in a vacuum)
Range	100 m	177 m
Maximum height	53 m	77 m
Time of flight	6.6 s	7.9 s

Figure 13A shows the difference between the trajectory of a ball in air and in a vacuum, as predicted by a computer model, and Figure 13B shows that of a bullet as tracked by ballistics experts on a rifle range.

The differences come about because bullets have more complicated motion than a round ball in flight.

Trial-and-error has shown that the maximum range for a bullet fired in air is achieved at an elevation of 33°, a rough rule-of-thumb that works for most guns. As an approximation, the angle of descent at maximum range is about 80°, or very nearly vertical. Any greater elevation of the gun merely means that the bullet will actually drop vertically, and the last part of the flight will add nothing to the range. So, the war veterans were probably right – bullets did fall on them vertically from the sky (and were just as lethal).

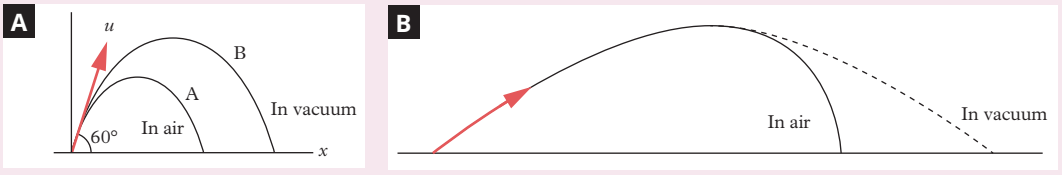


FIGURE 13 The trajectories of (A) a ball and (B) a bullet in air and in a vacuum

Apply your understanding

1 According to Newton's first law, an object continues its motion in a straight line if there is no net force acting on it. Once a cannonball leaves the cannon and there is no longer a pushing force, the ball should travel in a straight line forever. **Explain** why this is not the case. (2 marks)

2 **Explain** whether the air resistance on a projectile increases with speed. (1 mark)

Check your learning 2.3

Check your learning 2.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how initial velocity and final velocity are related for an object projected upwards at an angle and returning to the same height as at launch. (2 marks)

2 **Describe** the relationship between complementary angles of elevation and range for a projectile in the absence of air. (1 mark)

3 **Explain** how and why air resistance affects the range of a projectile. (1 mark)

Analytical processes

4 A tennis ball close to the ground is hit by a racquet with a velocity of 30.0 m s^{-1} at an angle of 25.0° to the horizontal. **Determine** (3 s.f.)

a the vertical and horizontal components of the initial velocity (2 marks)

b the maximum height reached by the ball (3 marks)

c the time of flight (2 marks)

d the horizontal range. (2 marks)

5 A football is kicked off the ground at an angle of 30.0° to the horizontal. It moves away at 23.0 m s^{-1}. **Determine** (3 s.f.)
   - **a** the vertical velocity after 1.00 s (2 marks)
   - **b** the velocity (magnitude and direction) of the ball after 1.00 s (4 marks)
   - **c** the maximum height reached (3 marks)
   - **d** the time of flight (2 marks)
   - **e** the range of the ball. (2 marks)

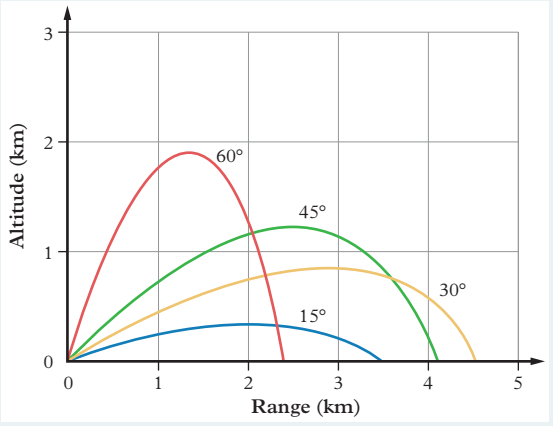
6 A rock is thrown off a 100.0 m cliff upwards at an angle of 20.0° to the horizontal with an initial velocity of 15.0 m s^{-1} and strikes the rocks below. **Determine** (3 s.f.)
   - **a** the time of flight (4 marks)
   - **b** the impact velocity (2 marks)
   - **c** how far out from the base of the cliff the rock strikes the ground. (3 marks)

Knowledge utilisation

7 **Propose** how you would prove theoretically that the range of a projectile that falls back to the same height as that from which it was launched is a maximum when the elevation angle is 45°. (1 mark)

8 On the Moon in February 1971, an astronaut hit a golf ball a distance of 180 m. **Evaluate** the claim that "if the astronaut hit the same ball on Earth with the same speed and angle (assumed to be 30°), it wouldn't go as far".
Note $g_{Moon} = -1.62\,m\,s^{-2}$. (3 marks)

9 The following graph shows how the range and maximum altitude (height) of a projectile change with elevation angle in the presence of air.
   - **a** **Evaluate** the relationship between maximum altitude and elevation angle with reference to the graph. (1 mark)
   - **b** **Predict** the maximum altitude for an angle of 90°. (1 mark)
   - **c** **Describe** the shape of a graph of range versus angle and draw a justified conclusion about the best angle for maximum range. (3 marks)



Practical

Lesson 2.4
Angled projection and distance

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

Learning intentions and success criteria

MODULE 2

Lesson 2.5
Review: Vectors and projectile motion

Summary

2.1
- A vector has both magnitude and direction. It can be represented graphically or using symbols or words.
- A vector can be resolved into two components at right angles.
- Vectors or their components can be added and subtracted.
- Vectors can be recombined to get a resultant vector.

2.2
- The velocity vector of a projectile can be resolved into horizontal and vertical components.
- The horizontal and vertical components of a velocity vector are independent of each other.
- The horizontal component of the velocity is constant; only the vertical component changes.
- The velocity vector for projection at an angle can be resolved into horizontal and vertical components.

2.3
- The horizontal and vertical components of the velocity vector are independent for projection at an angle.
- The horizontal component of the velocity is constant; only the vertical component changes.

2.4
- Practical: Angled projection and distance

Key formulas

Constancy of horizontal velocity	$v_x = u_x$
Horizontal displacement (range)	$s_x = u_x t$
Initial velocity: vertical component	$u_y = u \sin\theta$
Initial velocity: horizontal component	$u_x = u \cos\theta$
Vertical velocity as a function of time	$v_y = u_y + gt$
Vertical velocity as a function of (vertical) displacement	$v_y^2 = u_y^2 + 2gs_y$
Vertical displacement as a function of time	$s_y = u_y t + \frac{1}{2} g t^2$

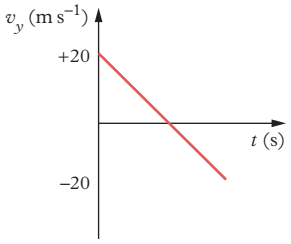
Review questions 2.5A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 The graph shows the vertical velocity of a projectile (a ball) with time.
Which one of the following is the best description of the ball's initial motion?

A Thrown vertically upwards
B Thrown vertically down
C Dropped from rest
D Projected horizontally off a cliff



2 Balls are fired horizontally off a cliff at different heights. Which one of the following best shows the relationship between time of fall (t) and height (h)?

A $t \propto \sqrt{h}$
B $t \propto h$
C $t \propto h^2$
D $t \propto \frac{1}{h}$

3 A golf ball is hit into the air at an angle, reaches maximum height and falls back to the ground. Take the upwards direction as positive. Which one of the following best describes its vertical acceleration at maximum height?

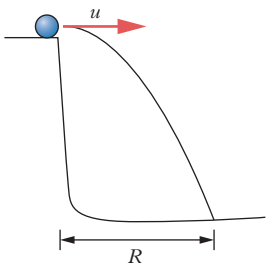
A $+9.8\,\text{m s}^{-2}$
B $-9.8\,\text{m s}^{-2}$
C $0\,\text{m s}^{-2}$
D $-9.8^2\,\text{m s}^{-2}$

4 An arrow is fired with the same initial speed at angles of elevation ranging from 0° to 90°. Which one of the following will increase with increasing angle?

A Time of flight
B Range
C Impact speed
D Impact velocity

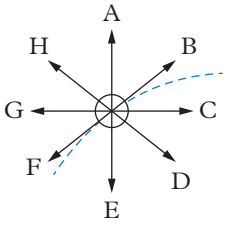
5 The diagram shows a ball being launched horizontally off a cliff at a speed of u, which gives the ball a horizontal range R and a time of flight t. If the speed is increased to $2u$, which one of the following best describes the new time of flight and the new range respectively?

A t, R
B t, $2R$
C $2t$, R
D $2t$, $2R$

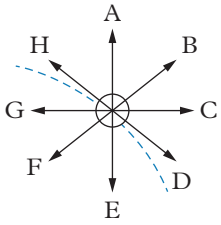


6 The three diagrams illustrate a projectile

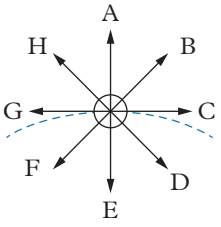
a shortly after it was launched



b at its highest point



c shortly before it hits the ground.

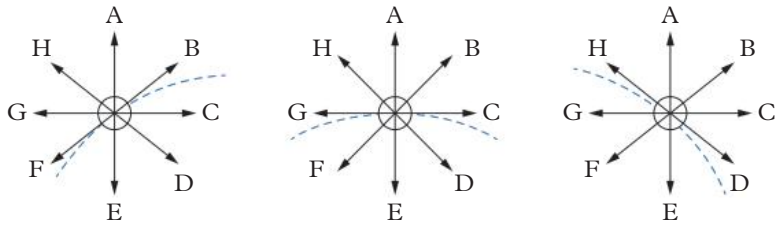


If air resistance is ignored, the only force acting on the projectile will be gravity.
Which labelled arrow in the diagrams best represents the resultant acceleration acting on the projectile?

A B **B** E **C** A **D** C

Use the following information to answer questions 7 to 10.

A batter hits a cricket ball so that it leaves the bat at an angle of 30° to the horizontal and with an initial velocity of $60\,m\,s^{-1}$, as shown in the diagram.



7 What is the magnitude of the initial vertical component of the cricket ball's velocity?

A $30\,m\,s^{-1}$

B $52\,m\,s^{-1}$

C $59\,m\,s^{-1}$

D $9.3\,m\,s^{-1}$

8 Shortly after leaving the bat, the cricket ball is travelling on the upwards portion of its trajectory. Which of the arrows best represents the direction of the cricket ball's acceleration at this time?

A A

B E

C F

D C

9 At the highest point on its trajectory, which of the arrows best represents the direction of the cricket ball's velocity?

A E

B B

C H

D C

10 At the highest point in its trajectory, which of the arrows best represents the direction of the cricket ball's acceleration, if air resistance cannot be ignored?

A D

B F

C E

D B

11 A projectile is fired at an angle and its time of flight is measured. Which of the following states the elevation angle in degrees that would give, in order, the minimum time of flight and the maximum time of flight?

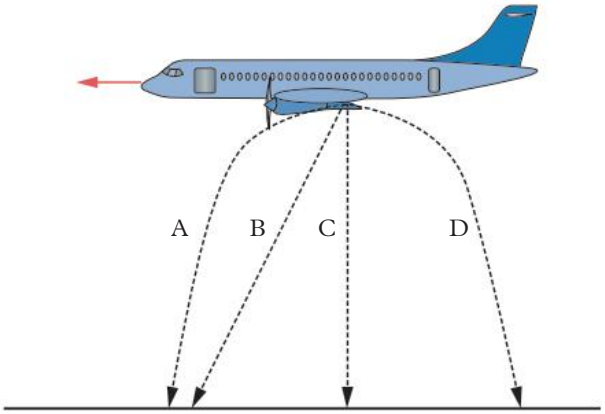
A 0, 90

B 45, 90

C 90, 45

D 90, 0

12 A box is dropped from an airplane while in flight, as shown in the diagram. Which path will the box follow with respect to the ground?



A Path A

B Path B

C Path C

D Path D

13 A 5 kg white ball is thrown horizontally at a speed of 8 m s^{-1} from a tall building. A 10 kg black ball is thrown horizontally from the same height at a speed of 16 m s^{-1}. How long will it take the white ball to reach the ground, compared to the time it takes the black ball to reach the ground?

A One-half as long
B The same time
C Twice as long
D Four times as long

14 A hose lying on the ground shoots a stream of water upwards at an angle of 40° to the horizontal and at a speed 20 m s^{-1} as it leaves the hose. It strikes a wall 8 m away. How high up the wall will the water strike?

A 5.4 m **B** 6.7 m
C 7.8 m **D** 8.3 m

15 A ball is projected at an angle θ and an initial speed u. Which one of the following shows the time of flight of the ball?

A $t = \frac{2u\sin\theta}{g}$ **B** $t = \frac{2u + \sin\theta}{g}$
C $t = \frac{u\sin\theta}{2g}$ **D** $t = \frac{2u}{g}$

Review questions 2.5B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 **Determine** whether or not the horizontal and vertical components of a projectile are dependent on each other. (1 mark)

17 Vector A has a magnitude of 30 units and is directed towards the right at 25° above the horizontal. **Calculate** the x and y components of the vector (2 s.f.). (2 marks)

Analytical processes

18 A projectile is fired horizontally. **Determine**, with justification, the part of its trajectory for which it has the

a highest speed (2 marks)
b lowest speed. (2 marks)

19 When a wedding ring is thrown horizontally out of a fifth-floor window that is 15 m off the ground, it lands 7.5 m out from the base of the building. **Determine** (2 s.f.) the

a throwing speed (3 marks)
b impact velocity. (3 marks)

20 A person sitting in a train carriage moving at constant velocity throws a ball straight up in the air. **Determine**, with justification

a whether the ball will fall behind them, in front of them or into their hands (2 marks)
b what will happen if the train accelerates while the ball is in the air (2 marks)
c what will happen if the train turns a corner while the ball is in the air. (2 marks)

21 A golf ball is hit by a club and moves off with a velocity of 30 m s^{-1} at an angle of 55° to the horizontal. **Determine** (2 s.f.) the

a initial vertical and horizontal components of the velocity (2 marks)
b maximum height reached (2 marks)
c time of flight (2 marks)
d horizontal range (2 marks)
e impact velocity. (3 marks)

22 A soccer ball is kicked off the ground at an angle of 20.0° to the horizontal. It moves away at 30.0 m s^{-1}. **Determine** the

a vertical velocity after 0.50 s (3 s.f.) (2 marks)
b velocity of the ball after 1.0 s (3 s.f.) (2 marks)
c maximum height reached (2 s.f.) (2 marks)
d time of flight (2 s.f.) (2 marks)
e range of the ball (2 s.f.). (2 marks)

23 The world record for throwing an uncooked hen's egg is 96.90 m, set in 1981. The egg was thrown at an angle of elevation of 45° and with negligible air resistance. **Determine** (3 s.f.) what would have been the

a throwing speed (5 marks)
b maximum height above the ground (3 marks)
c time of flight (3 marks)
d impact velocity. (2 marks)

24 A dart is thrown horizontally towards the bullseye of a dartboard, but it strikes the 3 on the bottom of the board directly underneath the bullseye 0.19 s after it left the player's hand (shown by the arrow in the diagram). **Determine** (2 s.f.) the distance from the bullseye to the 3. (2 marks)

25 A football is kicked off the ground and it lands on the roof of a house 57.3 m away at a height of 3.8 m. The time of flight was 3.0 s. **Determine** (2 s.f.)

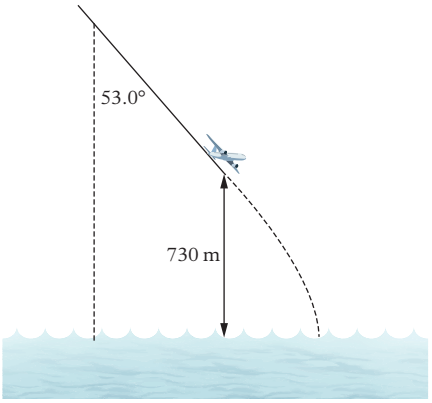
a its launch speed (5 marks)

b its initial elevation angle (2 marks)

c the maximum height reached (2 marks)

d the time of flight if the footballer kicked the ball at the complementary angle. (3 marks)

26 A plane diving at an angle of 53.0° to the vertical releases a rescue kit at an altitude of 730 m. This projectile hits the water 4.50 s after being released. **Determine** (3 s.f.)

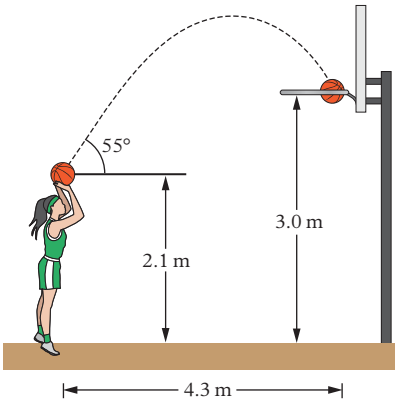


a the speed of the plane (4 marks)

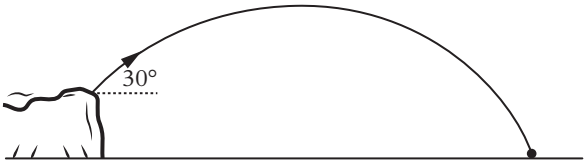
b how far the projectile travelled horizontally during its flight (2 marks)

c the impact velocity of the projectile. (5 marks)

27 A basketball player shoots a ball at an angle of 55° into a hoop on a post 4.3 m away. The ball is released from a height of 2.1 m and goes through the hoop, which is 3.0 m off the ground. **Determine** (2 s.f.) the initial speed of the ball for this shot to be successful. (4 marks)



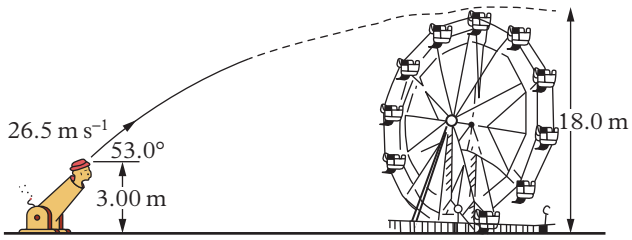
28 Astronauts on the Moon hit a golf ball to see how far it would go under the weaker effects of gravity. From the edge of a crater, they hit a ball that landed on the flat surface some distance away. The time it took from hitting the ball to it landing was 22.77 s. They radioed back to Earth that the equation for its vertical displacement was given by the formula $s_y = 18t - 0.80t^2$. **Determine** the horizontal distance the golf ball travelled before landing. (4 marks)



29 Emanuel Zacchini was a famous US "human cannonball". In 1940 he attempted to clear a Ferris wheel 18.0 m high after being launched from a cannon at an elevation angle of 53.0° and a muzzle velocity of 26.5 m s^{-1}. His initial point of projection from the cannon was 3.00 m above the ground. **Determine**, with reasons and a labelled diagram

a whether he cleared the Ferris wheel (3 s.f.) (5 marks)

b how far away from the cannon the net to catch him would have been placed (3 s.f.). (4 marks)



Knowledge utilisation

30 An experiment was done to investigate the range of a steel ball launched horizontally off a benchtop 0.76 m from the floor. The horizontal velocity and range were measured and are shown in the following table.

Horizontal velocity, v_x (m s^{-1})	0.30	0.53	0.99	1.34	1.50
Horizontal range, s_x (m)	0.12	0.22	0.40	0.53	0.59

a **Construct** a graph of range (on the vertical axis) versus horizontal velocity (on the horizontal axis). (3 marks)

b **Assess** whether the range is directly proportional to the horizontal velocity. (2 marks)

c **Determine** the time of flight from the graph (2 s.f.). (2 marks)

d **Determine** the acceleration due to gravity from the data (2 s.f.). (2 marks)

e **Calculate** the percentage error, given that $g_{(\text{accepted})} = -9.8\,\text{m s}^{-2}$ (2 s.f.). (4 marks)

31 Data were collected in an experiment to investigate the effect of launch angle on range, and are listed in the following table.

Launch angle (°)	Range (m)		
	Trial 1	Trial 2	Trial 3
15	1.30	1.06	1.31
30	2.11	2.15	2.11
45	2.39	2.50	2.47
60	2.11	2.15	2.11
75	1.30	1.06	1.31

Analyse evidence in the dataset to

a **identify** trends in the relationship (5 marks)

b **justify** a conclusion about the effect of angle on range. (2 marks)

32 In the 1968 Olympics in Mexico City, Bob Beamon shattered the world long-jump record with a jump of 8.90 m. His speed on take-off was measured at 9.50 m s^{-1}, about equal to that of a sprinter. At the time, newspapers questioned how close he came to achieving maximum range for this speed in the absence of air resistance (assuming maximum range is at an initial elevation of 45°). The value of g in Mexico City is 9.78 m s^{-2}. **Propose** an answer to their question and justify your response by showing calculations. (6 marks)

Data drill

Calculating horizontal range

An "onager" is a type of catapult that was used by the ancient Romans and during medieval times to attack forts and castles by firing rocks as projectiles. A simplified diagram of an onager is shown in Figure 1.

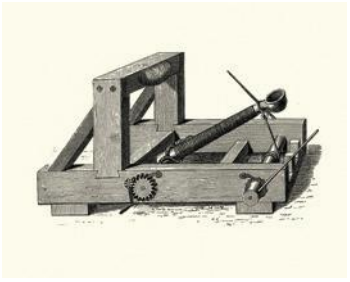


FIGURE 1 An onager

A group of researchers made a full-sized replica onager to launch 30 kg stone balls. They launched the balls at constant angle θ to the horizontal for five variations of the initial velocity, u. They did three trials (replicates) for each velocity. The data are presented in Table 1.

TABLE 1 Trial launch and range data for the onager

Initial velocity, u (m s^{-1})	Horizontal range, R (m)				
	Trial 1	Trial 2	Trial 3	Average, $\bar{x}$	$\delta\bar{x}$
63	331	354	367	351	**Q1**
67	419	397	379	398	20
71	440	462	418	440	22
75	472	522	497	497	25
85	635	672	**Q3**	638	32

The relationship between the initial velocity u and the horizontal range R was non-linear, so they linearised it by plotting range R (y-axis) versus the square of the initial velocity u^2 (x-axis) as shown in Figure 2. Error bars were also added.

The processed data for the trendlines is:

Equation for the linear trendline: $y = 0.0884x$

Equation for the maximum linear trendline: $y = 0.0900x$

Equation for the minimum linear trendline: $y = 0.0700x$

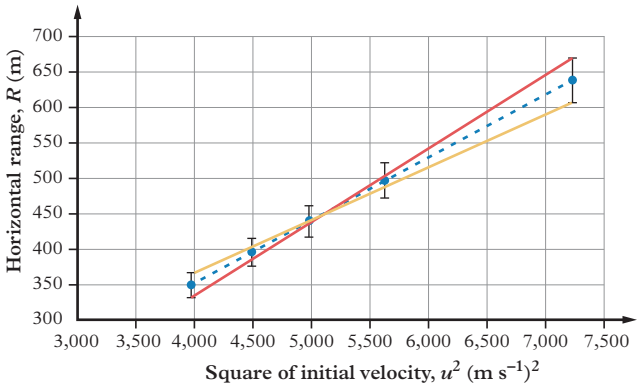


FIGURE 2 Linearised velocity and range data for the onager experiment

Apply understanding

1 **Calculate** the absolute uncertainty $\delta\bar{x}$ for the first set of trials at an initial velocity of 63 m s^{-1}. Express your answer to the correct number of significant figures. (1 mark)

2 **Determine** the percentage uncertainty δ% for the three trials at an initial velocity of 67 m s^{-1} (2 s.f.). (1 mark)

3 **Determine** the horizontal range for Trial 3 for an initial velocity of 85 m s^{-1} (3 s.f.). (1 mark)

Analyse data

4 **Identify** the percentage uncertainty for the gradient of the linear trendline (2 s.f.). (1 mark)

5 **Identify** a mathematical relationship between the initial velocity and the horizontal range of the projectiles, including the uncertainty of the gradient (1 s.f.). (2 marks)

Interpret evidence

Use the following information to answer questions 6 and 7.

The "range formula" can be used to determine the theoretical range R of a projectile on horizontal ground knowing its initial velocity u and launch angle θ above the horizontal:

$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

6 **Draw a conclusion** about the magnitude of the launch angle θ, including its percentage uncertainty. (3 marks)

7 **Predict** the range of possible launch angles given the percentage uncertainty. (1 mark)

Module 2 checklist: Vectors and projectile motion

MODULE 3

Inclined planes

Introduction

An inclined plane or ramp is a flat surface raised at one end, used as an aid for raising or lowering a load. Moving an object up an inclined plane requires less force than lifting it straight up, although it has to be moved through a longer distance. The incline used by the ancient Egyptians for building pyramids and the sloping roads built by the Romans are examples of early inclined planes. It shows that they understood the value of this technology for moving things uphill.

Prior knowledge

Prior knowledge quiz

Check your understanding of concepts related to inclined planes before you start.

Subject matter

Science understanding

- Solve problems involving force due to gravity (weight) and mass using $F_g = mg$.
- Describe the concept of normal force.
- Describe the forces acting on an object on an inclined plane (e.g. force due to gravity, normal force, tension, frictional force and applied force) through the use of free-body diagrams.
- Determine the net force acting on an object on an inclined plane using vector analysis.

Science inquiry

- Investigate the parallel component of the weight of an object down an inclined plane at various angles.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Practicals

This lesson is available on Oxford Digital.

Lesson 3.4 The parallel component of the weight of an object on an inclined plane

Lesson 3.1
Forces due to gravity

Key ideas

- → The force of gravity, F_g, on an object is the object's weight.
- → Mass, m, is a measure of an object's resistance to motion, or the amount of substance in an object. It is related to weight by $F_g = mg$.
- → When an object free-falls under gravity, it accelerates at g. Near the surface of Earth, $g = 9.8\ \text{m s}^{-2}$ downwards.
- → The normal force is the force exerted on an object by a surface against which it is pressed. The normal force is always perpendicular to the surface.

What is force?

Learning intentions and success criteria

Scientists have identified four fundamental types of **force**: gravitational, electromagnetic (involving both electrostatic and magnetic forces), weak nuclear and strong nuclear forces. All interactions between matter can be explained as the action of one or a combination of these four fundamental forces. There are other (non-fundamental) forces you will come across, such as weight, the normal force, tension and friction. Depending on how they are used, these forces can also be labelled as the applied force or net force.

force
a push or pull between objects, which may cause one or all objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape

How are weight and mass different?

Weight is different from mass. **Mass** is a measure of an object's resistance to motion (or inertia) and doesn't vary no matter where the object might be taken to in the universe. **Weight** is a force that depends on nearby astronomical bodies. In Unit 2 we used the symbol F_W for the gravitational weight force, but in this unit, we will use F_g as a more general symbol for all sorts of gravitational attraction.

FIGURE 1 Bathroom scales measure your weight but convert it to mass in kg for the display.

mass
an object's resistance to a change in motion; also commonly stated as the amount of matter in an object (symbol: m; SI unit: kilogram; unit symbol: kg)

weight
a measure of the force of gravity acting on an object (symbol: F_g; SI unit: newton; unit symbol: N)

Objects near or on the surface of Earth experience a gravitational attraction towards Earth's centre by a pulling force called the force of gravity. This pulling force causes freely falling objects to move downwards with an acceleration of $9.8\,\text{m s}^{-2}$. This is called Earth's free-fall acceleration, g.

On Earth, $g = 9.8\,\text{m s}^{-2}$, but on other planets and satellites it depends largely on the mass and size of the planet, as shown in Table 1.

Study tip

Free-fall acceleration, g, is also called acceleration due to gravity. In Module 5, you will learn how it is equivalent to gravitational field strength.

FIGURE 2 Your weight depends on the local gravitational attraction, not what you are doing – even when free-falling. Your mass is independent of gravity.

TABLE 1 Free-fall acceleration on selected astronomical bodies

Celestial body	Earth masses	Free-fall acceleration, g (m s^{-2})
Jupiter	317.8	24.8
Earth	1.00	9.8
Mars	0.11	3.71
Moon	0.0123	1.62

Weight, being the force due to gravity, is given the symbol F_g. For an object of mass m with free-fall acceleration g:

$$F_g = mg$$

For example, the weight of a 10 kg object on the surface of Earth, where free-fall acceleration g is 9.8 m s^{-2}, is given by:

$$\begin{aligned} F_g &= mg \\ &= 10 \times 9.8 \\ &= 98\,\text{N} \end{aligned}$$

On the surface of the Moon, this object of mass 10 kg would have a weight of 16 N. On Jupiter it would be nearly 250 N.

Study tip

Significant figures (s.f.) will not be examined in the QCAA External Exam, even though you may see that this was done in past papers. You should aim to express your answer to the same number of significant figures as the data given in the question, which is usually 2 s.f. or 3 s.f. Most questions in our Unit 3 modules state the number of significant figures to help you understand what is appropriate. Remember to round answers to the correct number of significant figures only at the very end, so you don't introduce rounding errors.

Worked example 3.1A

Mass and weight at different places

A person has a mass of 85.0 kg. **Calculate** their weight in newtons (to an appropriate number of significant figures) on

a Earth (1 mark)

b the Moon (g_{Moon} = one-sixth of g_{Earth}) (1 mark)

c Jupiter ($g = 24.8\,\text{m s}^{-2}$). (1 mark)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Select the correct formula for calculating gravitational force (weight) from mass.	$F_g = mg$
Step 3: Recall that the value of g on the surface of Earth is 9.8 m s^{-2}. Substitute values into the formula and calculate the weight. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that the correct units are used. (1 mark for "Provides correct answer")	**a** $F_g = mg$ $= 85.0 \times 9.8$ $= 833\,\text{N}$ (3 s.f.) (1 mark)

Think	Do
Step 4: Identify from the question data that the value of g on the Moon is $\frac{1}{6} \times 9.8\,\text{m s}^{-2}$. Substitute values into the equation and calculate the weight on the Moon. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that the correct units are used. (1 mark for "Provides correct answer")	**b** $F_g = mg$ $= 85.0 \times 1.62$ $= 138\,\text{N}$ (3 s.f.) (1 mark)
Step 5: Identify from the question data that the value of g on Jupiter is $24.8\,\text{m s}^{-2}$. Substitute values into the equation and calculate the weight on Jupiter. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that the correct units are used. (1 mark for "Provides correct answer")	**c** $F_g = mg$ $= 85.0 \times 24.8$ $= 2{,}110\,\text{N}$ (3 s.f.) (1 mark)

Your turn

A person has a weight of 650 N on Earth. **Calculate** (to an appropriate number of significant figures) their

a mass on Earth (1 mark)

b mass on Mars ($g = 0.11\,\text{m s}^{-2}$) (1 mark)

c weight on Mars. (1 mark)

Challenge

Which has the greater weight?

Which one of each pair has the greater weight?

a 1 kg of lead or 1 kg of feathers (on Earth) (1 mark)

b 1 cm^3 of lead or 1 cm^3 of feathers on Earth (1 mark)

c 1 tonne of lead or 1 tonne of feathers (both in free-fall) (1 mark)

d An adult or a child both floating in a swimming pool (1 mark)

Challenge

How heavy is cork?

Cork is a very lightweight substance, as you would know if you have ever put a cork in water. Imagine a 1.5 m diameter ball of cork. Do you think its mass is more or less than half a tonne (500 kg)? **Predict** whether the mass is more or less than half a tonne. **Calculate** the value to check your guess using the density of cork as $286\,\text{kg m}^{-3}$ and the volume of a sphere as $V_{\text{sphere}} = \frac{4}{3}\pi r^3$. (2 marks)

Challenge

Finger in water and the normal force

A beaker of water is placed on an electronic balance and the reading in grams is noted. You then stick your finger in the water (Figure 3). **Predict** whether the reading on the balance will increase, decrease or stay the same. (1 mark)
Propose how the reading in grams and the normal force are related. (1 mark)

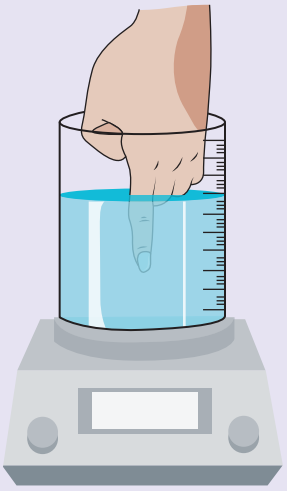


FIGURE 3 A finger in a beaker of water set on an electronic balance. Does the reading change?

What is the normal force?

When a box is resting on a table, gravity pulls the box towards Earth with a force F_g called its weight. The table pushes back with an equal and opposite force that is at right angles to the tabletop (Figure 4). A right angle is also called a perpendicular or "normal", so it is called the **normal force** (F_N).

normal force
the force acting along an imaginary line drawn perpendicular to the surface

Study tip

In the convention for writing subscripts, F_{AB} means "Force **on** A **by** B".

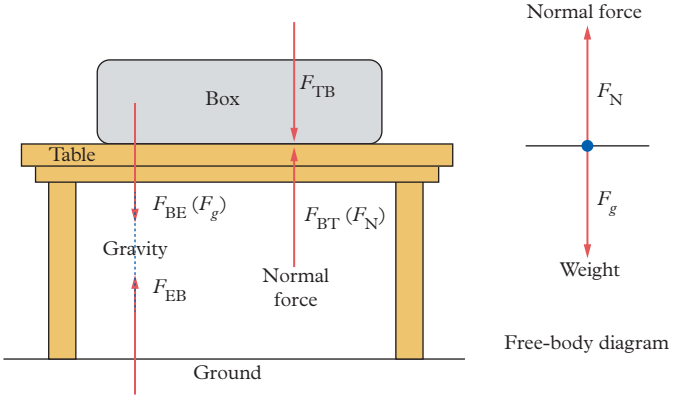


FIGURE 4 The normal force, F_N, is perpendicular to the surface.

Students often assume these two forces make up an action–reaction (or agent–receiver) pair consistent with Newton's third law. Even though these two forces are equal and opposite, they are not a third law pair. There are two third law pairs in the situation of the box on the table:

1. The force on the box by Earth (F_{BE}) is also known as the weight of the box (F_g). The paired force is the pulling force on Earth by the box (F_{EB}).
2. The downwards force on the table by the box (F_{TB}) is paired with the upwards force on the box by the table (F_{BT}) – also known as the normal force (F_N).

Each pair make up the action–reaction pair. The magnitudes are equal: $F_N = F_g$. This can be written in vector notation, which shows the forces are in opposite directions: $\vec{F}_N = -\vec{F}_g$.

For example, when you are standing on the ground, the gravitational attraction between you and Earth pulls you straight down towards the centre of Earth. This is called your weight, F_g. The surface of Earth pushes back on your feet, and this is the normal force, F_N.

FIGURE 5 When the pull of gravity downwards is stronger than the supporting surface can push back up, the inevitable happens.

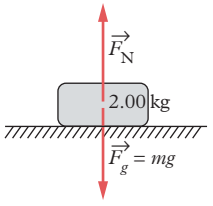
Worked example 3.1B

Calculating normal forces

A 2.00 kg box rests on a horizontal table. There are no applied forces.

a **Construct** a free-body diagram and **calculate** the normal force. Give your answer to an appropriate number of significant figures. (4 marks)

b A person then pushes down on the box with a 5.0 N force. **Calculate** the normal force now. Give your answer to an appropriate number of significant figures. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Construct" means to display information in a diagrammatic or logical form. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula, substitute values and use it to find an answer.
Step 2: Draw a free-body diagram showing the normal force F_N and weight F_g vector arrows as equal and opposite in length. Make sure they are correctly labelled. The horizontal arrow above the symbol for force is optional if the vertical arrows are correctly drawn. (1 mark for "Constructs a correct free-body diagram")	**a**  (1 mark)
Step 3: Select the correct formula and substitute in values. Calculate the value for F_g. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that the correct units are used. There is no need to say "down" if you have a correctly labelled free-body diagram. (1 mark for "Calculates the weight of the box")	$F_g = mg$ $= 2.00 \times 9.8$ (1 mark) $= 19.6$ N down (3 s.f.) (1 mark)
Step 4: State that the value of F_N is equal and opposite to F_g. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that the correct units are used. Again, there is no need to say "up" if you have a correctly labelled free-body diagram. (1 mark for "Identifies the relationship between weight and normal force")	$\vec{F}_N = -\vec{F}_g$ $= 19.6$ N up (3 s.f.) (1 mark)
Step 5: Determine that the total force acting downwards on the table is the weight, plus the added force downwards from the finger. The normal force is equal and opposite to the sum of these two forces. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Concludes that normal force will be 24.6 N up")	**b** $F_N = 19.6 + 5.0$ $= 24.6$ N up (3 s.f.) (1 mark)

Your turn

A 10.00 kg box is at rest on a horizontal table. There are no applied forces.

a **Construct** a free-body diagram and **calculate** the normal force. Give your answer to an appropriate number of significant figures. (4 marks)

b A person pulls up on the box with a piece of string with a 15.0 N force. **Calculate** the normal force now. Give your answer to an appropriate number of significant figures. (1 mark)

Check your learning 3.1

Check your learning 3.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Define** the terms "weight" and "normal force". (2 marks)

2 **Explain** how the mass and weight of an object differ on the surface of the Moon compared to the surface of Earth. (2 marks)

3 **Construct** a labelled free-body diagram to show the relationship between the normal force and weight of an object when it is resting on the ground. (2 marks)

4 **Explain** whether this statement is true: "For an object at rest on a horizontal surface and with no other forces acting, the weight and the normal force are equal in magnitude." (2 marks)

5 **Calculate** the weight of a 30 kg box of vegetables on Earth (2 s.f.). (1 mark)

6 A parent said their child had a weight of 36 N at birth. **Calculate** the child's mass (2 s.f.). (1 mark)

Analytical processes

7 **Distinguish** between weight and mass. (1 mark)

8 A bag of apples with a mass of 3.0 kg rests on a table. **Determine** the magnitude and direction of the normal force. (2 marks)

9 Imagine you could take a 1 kg rock to the centre of Earth. **Deduce** what its weight would be. (2 marks)

Knowledge utilisation

10 A student of mass 65 kg stands barefoot on the ground and says, "I am pulling Earth towards me with a force of 637 N." A friend says, "No, Earth is pulling on you with a force of 637 N." **Evaluate** both statements and **determine** which is correct. (2 marks)

11 A student stands barefoot on the ground and says, "Earth is providing the normal force to my weight." She then puts shoes on and says, "Now my shoes are providing the normal force and Earth is having a rest." **Evaluate** these statements for accuracy. (3 marks)

Lesson 3.2
Applied forces: Friction and tension

Key ideas

- → As well as force due to gravity (weight) and the normal force, the forces acting on an object include tension, frictional force and applied force.
- → A force is a push or pull between objects that may cause one or both objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape.
- → Friction is the resistance to motion of a surface moving relative to another. Unlike gravity, electromagnetism or the strong and weak forces, friction is not a fundamental force. It is the result of the electromagnetic attraction between charged particles in the two touching surfaces.
- → Tension is the pulling force transmitted along a rope, string, cable or chain on an object. It has the symbol F_T.
- → An applied force (F_A) is a force applied to an object by a person or another object. It can be a push or a pull.

What is friction?

Learning intentions and success criteria

You may have measured frictional force by dragging an object across a desk at constant speed and noting the reading (in newtons) on a spring balance (Figure 1).

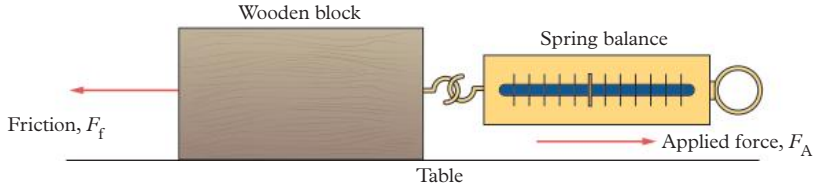


FIGURE 1 A laboratory spring balance is calibrated in both grams (g) and newtons (N). A reading of 100 g (0.1 kg) corresponds to a reading of about 1.0 N. You can use the formula $F_g = mg$ to show this is true.

Friction is the resistance to motion of a surface moving relative to another. It is the result of the electromagnetic attraction between the charged particles in two touching surfaces.

friction
the resistance to motion of a surface moving relative to another

Unit 2 discussed what life would be like without friction – it would be impossible. Friction is absolutely necessary, but it is also a hindrance. The search for ways of altering it has gone on for thousands of years, as has the search to understand it.

The properties of friction

In Unit 2, you also saw that friction has several properties, and these are worth reviewing:

- Friction is a force between two surfaces, and it opposes motion (Figures 2 and 3).
- Friction has the symbol F_f and is measured in units of newton, with the unit symbol N.
- When there is no acceleration along the surfaces, the frictional force equals the horizontal force.

FIGURE 2 Rubber tyres have good friction with a dry road surface but less so on a wet road. The grooves in a tyre serve to get rid of water between the tyre and road, but when the tyre is worn it can't do this effectively.

FIGURE 3 A lubricant eye drop is used to treat dryness and irritation in the eye by providing a low-friction interface between the eyeball and the eyelid.

For this section on inclined planes, one of the most important features of friction is its relationship to net forces and acceleration. If an object is being dragged along a horizontal surface by an applied force (F_H) at constant speed, there is no acceleration, so the net force (F_{net}) is zero. The applied force (F_H) must then be equal to the frictional force (F_f), as shown in Figure 4A. In contrast, when the horizontal force is greater than the frictional force (Figure 4B), the object will accelerate in the direction of the larger force.

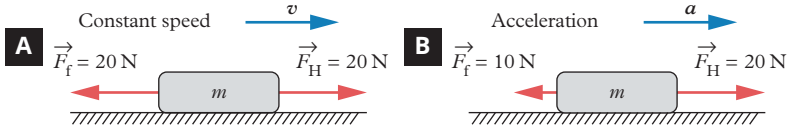


FIGURE 4 The effect of friction on (A) the net force and (B) acceleration

Increasing and decreasing friction

Sometimes you want to increase friction, such as for tyres on a road. They can be made with softer rubber or a pattern that has ridges and valleys. At other times you want to reduce friction, such as in an engine, and this is done with a lubricant such as oil.

In Worked example 3.2A, you will consider the effect of friction on acceleration.

Worked example 3.2A

Motion on a horizontal surface with friction

In Figure 4B, the horizontal force is 20 N (2.0×10^1 N) to the right and the frictional force is 10 N (1.0×10^1 N) to the left. The mass m of the block is 3.00 kg. **Determine** (to an appropriate number of significant figures)

a the weight of the block (1 mark)

b the normal force acting on the block (1 mark)

c the net force acting on the block in the horizontal direction (1 mark)

d the acceleration of the block in the horizontal direction (1 mark)

e the net force acting on the block in the vertical direction. (1 mark)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: To calculate the weight of the block, select the correct formula and substitute in the known values. Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. As weight is a vector, you need to add a direction (down). (1 mark for "Determines the correct answer")	**a** $F_g = mg$ $= 3.0 \times 9.8$ $= 29.4\,\text{N}$ $= 29\,\text{N}$ down (2 s.f.) (1 mark)
Step 3: Determine the normal force. There are no applied forces, so the normal force is equal and opposite to the weight. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Add a direction and units. (1 mark for "Determines the correct answer")	**b** $F_N = -F_g$ $= 29.4\,\text{N}$ up $= 29\,\text{N}$ up (2 s.f.) (1 mark)
Step 4: Determine the net force. The horizontal force is greater than friction, so the block moves in the direction of the larger force. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Add a direction and units. (1 mark for "Determines the correct answer")	**c** $F_{net} = F_H - F_f$ $= 20 - 10$ $= 10\,\text{N}$ to the right (2 s.f.) (1 mark) Alternatively, using vector notation: $\vec{F}_{net} = \vec{F}_H + \vec{F}_f$ $= 20 + (-10)$ $= +10\,\text{N}$ (to the right)
Step 5: Select the formula for Newton's second law and substitute in values for F_{net} and m. Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Add a direction and units. (1 mark for "Determines the correct answer")	**d** $F_{net} = ma$; therefore: $a = \frac{F_{net}}{m}$ $= \frac{10}{3.00}$ $= 3.3\,\text{m s}^{-2}$ to the right (2 s.f.) (1 mark)
Step 6: Identify that there is no vertical movement and explain that this means the net force must also be zero. (1 mark for "Justifies the conclusion using reference to Newton's second law")	**e** There is no movement in the vertical direction, so the vertical acceleration is zero. Therefore, according to Newton's second law, F_{net} (vertical) is zero. (1 mark)

Your turn

A block of wood of mass 650 g is being pushed along a horizontal surface by a force of 3.5 N to the left. A frictional force of 2.1 N acts. **Determine** (to an appropriate number of significant figures)

a the weight of the block (1 mark)

b the normal force acting on the block (1 mark)

c the net force acting on the block in the horizontal direction (1 mark)

d the acceleration of the block in the horizontal direction (1 mark)

e the net force acting on the block in the vertical direction. (1 mark)

What is tension?

tension
the pulling force transmitted along a rope, string, cable or chain on an object

Tension is the pulling force transmitted along a rope, string, cable or chain on an object. It comes from the Latin word "tensio" meaning "to stretch". The symbol F_T is used to show that it is a force, with the subscript T to signify that it is tension – a pulling force and not a compression or twisting force, or any other sort of force.

We will now look at some specific examples of tension.

FIGURE 5 Window cleaners hanging in a cradle suspended by high-tensile steel ropes

How does tension work?

Tension on an object hanging at rest

Consider a mass, m, that is suspended at rest from a scaffold such as an overhead beam. Gravity pulls downwards on the mass with a force F_g called the weight of the object (Figure 6). To support this weight, the rope has to pull upwards with an equal and opposite force. These two forces (the upwards pull of the rope and the weight) make up the action–reaction pair (or agent–receiver pair) you learnt about when you studied classical mechanics (Newton's laws) in Unit 2. As the object is not accelerating, the net force must be zero, so you can confidently say that the weight force and the tension force are equal and opposite, and their net sum is also zero. This would also apply to the object moving at constant speed (zero acceleration).

Note: assume for all examples that the rope is massless, or so small compared to the mass of the object that it can be neglected, and that the string is inextensible (i.e. not able to be stretched).

In Figure 6, you can see that at the top end the rope is pulling down on the scaffold with a force F_T that is the same magnitude as the tension in the rope pulling up at the lower end. These two tension forces have the same magnitude ($F_{T(upper)} = F_{T(lower)}$). At a microscopic level there are billions of these tension pairs acting on molecules in the rope for its entire length. Because they are all equal and opposite, you can just represent the ones at the ends.

The scaffold has to pull upwards on the rope with a force equal to this tension. The ultimate purpose of the rope is to transmit the weight of the block to the scaffold.

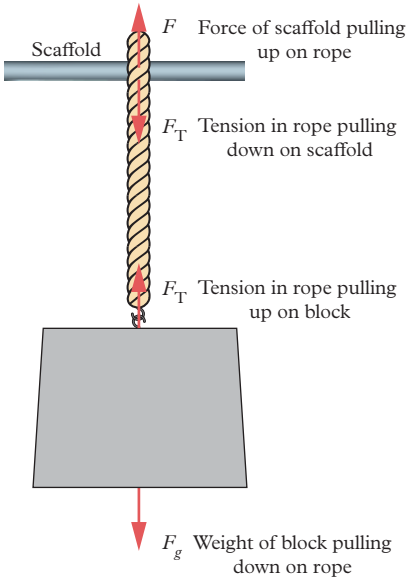


FIGURE 6 The rope is under tension from the force of gravity on a hanging object.

FIGURE 7 The rope is under tension from pulling forces. The tension is zero if it is pushed.

Of course, if the scaffold was accelerating upwards, the tension in the rope would increase, but when it is at rest or moving at a constant speed, the tension in the rope equals the weight of the object.

Worked example 3.2B shows you how to draw and calculate the tension in a rope from a hanging mass.

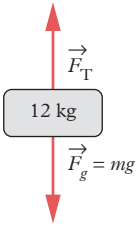
Worked example 3.2B

Tension in a vertical rope

A vertical rope is supporting a freely hanging mass of 12.0 kg.

a **Construct** a labelled free-body diagram. (1 mark)

b **Calculate** the tension in the rope. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Construct" means to display information in a diagrammatic or logical form. It is 1 mark, so no calculations need to be shown on the diagram. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Collect the given data and identify the quantity being determined.	$m = 12.0$ kg, $g = 9.8\,\text{m s}^{-2}$ We need to find the tension in the rope (F_T).
Step 3: Draw a correctly labelled free-body diagram. The arrows should be of equal length and opposite in direction. The horizontal arrow above the symbol for force (F) is optional if the vertical arrows are correctly drawn. (1 mark for "Constructs a correct free-body diagram identifying the forces")	**a**  (1 mark)
Step 4: Equate the magnitudes of the weight and the tension. (1 mark for "Identifies the equivalence of tension and weight")	**b** $F_g = F_T$ (the magnitudes are equal, as the mass is not accelerating, but are in opposite directions). Alternatively: $\vec{F}_T = -\vec{F}_g$ (1 mark)
Step 5: Select the correct formula, substitute in values and calculate the unknown (F_g). The direction is optional for this intermediate calculation. (1 mark for "Uses correct substitution to determine the weight")	$F_g = mg$ $= 12.0 \times 9.8$ (1 mark) $= 117.6$ N downwards
Step 6: Let tension be the same magnitude and in the opposite direction to the weight force from Step 5. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Determines the correct answer")	$F_T = 118$ N upwards (3 s.f.) (1 mark)

Your turn

A vertical string is supporting a freely hanging brass pendulum bob. The tension in the string is 0.62 N.

a **Construct** a labelled vector diagram. (1 mark)

b **Calculate** the mass of the brass bob. Give your answer to an appropriate number of significant figures. (2 marks)

Challenge

A pulley puzzle

If you passed a rope through an overhead pulley and tied it around your waist, could you pull on the other end and raise your body? (2 marks)

Tension on an object being accelerated

If you consider an object hanging at rest and imagine that the scaffold is a crane that is accelerating the object upwards, the tension in the rope would be greater as the net force would not be zero. The difference between the tension and the weight would be the net force (Figure 8). Flinging a rock upwards as in a trebuchet would be an example of this.

You could say that for the magnitudes of the forces:

$$F_{net} = F_T - F_g$$

which can be rearranged:

$$F_T = F_{net} - F_g$$

Alternatively, using vector symbols (remembering that the vector arrows point in opposite directions):

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_g$$

Worked example 3.2C shows you how to calculate tension when there is a net force.

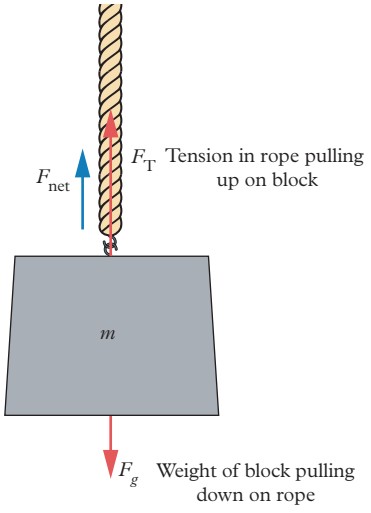


FIGURE 8 If the tension in the rope is greater than the weight, then the net force is upwards.

Worked example 3.2C

Tension in accelerated motion

A crane is lifting a 515 kg steel bin directly upwards using a chain hitched to the bin. The bin is accelerating upwards at 1.20 m s^{-2}. **Construct** a vector diagram and **calculate** the tension in the chain. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the question is asking you to do.	"Construct" means to display information in a diagrammatic or logical form. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Collect the relevant data from the question and state what quantity is to be found.	m = 515 kg, g = 9.8 m s^{-2} downwards or −9.8 m s^{-2} We need to find the tension in the chain, F_T.
Step 3: Draw a correctly labelled vector diagram, with the tension arrow longer than the weight arrow. You can show how the two vectors are added head to tail to produce a resultant, but this is optional. (1 mark for "Constructs a free-body diagram of the forces involved")	$\vec{F}_T$, $\vec{F}_g$ = $\vec{F}_T$, $\vec{F}_g$, $\vec{F}_{net}$ (resultant) Vector sum (1 mark)

Think	Do
Step 4: Write the equation for finding the net force. It can be in vector notation with the arrow over the symbol for force or it can be written as a subtraction of magnitudes. Replace F_{net} with $m \times a$ and F_g with $m \times g$. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Demonstrates correct substitution")	$F_{net} = F_T - F_g$ (for magnitudes) (1 mark) $F_T = F_{net} + F_g$ $= ma + mg$ $= 515 \times 1.20 + 515 \times 9.8$ (1 mark) $= 618 + 5{,}047$ $= 5{,}665\,\text{N}$
Step 5: Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Make sure you use the correct units and that you state the direction. (1 mark for "Provides correct answer")	$F_T = 5{,}670\,\text{N}$ upwards (3 s.f.) (1 mark)

Your turn

An elevator in a building has a mass of 4,850 kg and is designed to have a maximum upwards acceleration of $0.67\,\text{m s}^{-2}$. **Construct** a free-body diagram and **calculate** the tension in the supporting cable when the elevator is undergoing this acceleration. Give your answer to an appropriate number of significant figures. (4 marks)

Redirecting tension forces with a pulley

You don't always want to pull on a rope to move something towards the direction of the force. Sometimes you want to move it away from you, or sideways (Figure 9). A pulley can be used to redirect forces, particularly a weight force to a horizontally applied force – or through some other angle. A pulley is a frictionless wheel used for redirecting forces without loss of energy (Greek *polos* meaning "pivot" or "axis").

FIGURE 9 Pulleys are used to redirect forces.

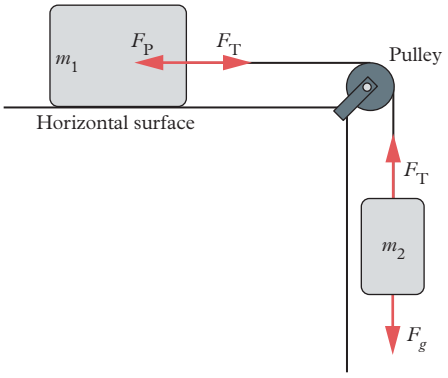


FIGURE 10 A single frictionless pulley redirects the force but doesn't change its magnitude.

Consider a block that rests on a surface and is attached to a hanging weight by a massless string (Figure 10). The block is being dragged along the horizontal surface by a weight hanging on an inextensible (non-stretching) massless cord passing over a frictionless pulley. The weight of the hanging mass is equal and opposite to the tension in the cord even though the cord is horizontal where it is attached to the block.

You can draw a free-body diagram of the arrangement showing the magnitude and names of the forces acting on the hanging mass (Figure 11).

You can do the same for the forces acting on the sliding mass (Figure 12). The magnitude of the tension force (F_T) is, of course, the same in both cases. The label F_P is used for the force parallel to the surface.

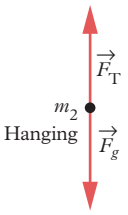


FIGURE 11 Free-body diagram for the vertical forces

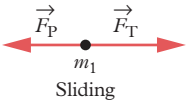


FIGURE 12 Free-body diagram for the horizontal forces

Challenge

What reaches the pulley first?

The monkey in Figure 13 has the same mass as the box. The monkey climbs a rope. Will the monkey or the box reach the pulley first? (2 marks)

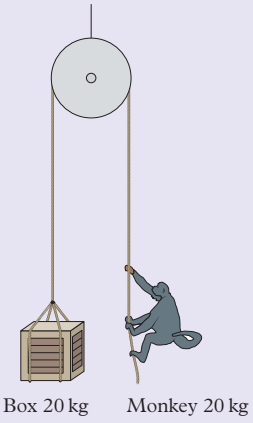


FIGURE 13 Will the monkey or the box reach the pulley first?

What is applied force?

applied force force applied to an object by a person or another object

An **applied force** (F_A) is a force applied to an object by a person or another object. It can be a push or a pull. For example, if you push a desk across the room, then your hands are providing an applied force on the desk. If you pull a cart by a rope, then you are providing an applied force through the tension in the rope on the cart.

We will now look at some specific examples of applied force.

How does applied force work?

Forces applied horizontally

Forces applied horizontally have no effect on the normal force of the object on the floor. All the forces go into horizontal movement of the object (Figure 14).

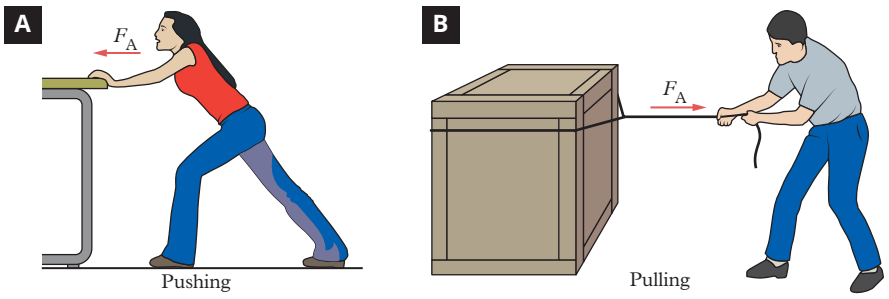


FIGURE 14 (A) Pushing or (B) pulling horizontally has no effect on the normal force.

Forces applied at an angle

Pulling

Have you noticed that is always easier to pull a pram or lawnmower backwards over rough ground? Likewise, it is easier to slide a box by pulling rather than pushing. In the case of pulling, the vertical component of the pulling force is directed upwards and tends to lift the object upwards. A free-body diagram (Figure 15) shows how the applied force ($\vec{F}_A$) has been resolved into two components at right angles: a vertical component ($\vec{F}_V$) and a horizontal component ($\vec{F}_H$).

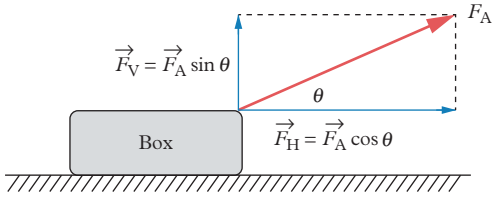


FIGURE 15 The applied force F_A is resolved into two components at right angles.

Pushing

If you have ever pushed a supermarket trolley or a lawnmower over bumpy ground, you know that it is hard to push because you seem to be pushing it into the ground as well. What is happening is that although some of your push is in the horizontal direction and moves the object along, a component of your pushing force is directed downward towards the ground, causing it to "dig in". For example, consider a person pushing a box across a floor (Figure 16). Some of the pushing force ($\vec{F}_A$) goes into moving the box across the floor and some of the force pushes the box down into the floor.

The free-body diagram in Figure 17 shows that the applied force has been resolved into two components at right angles: a vertical component (F_V) and a horizontal component (F_H).

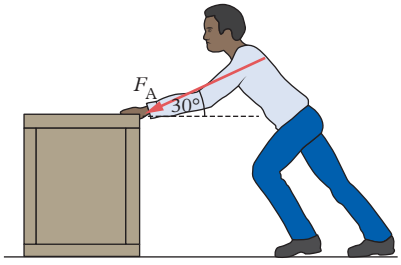


FIGURE 16 Pushing at an angle

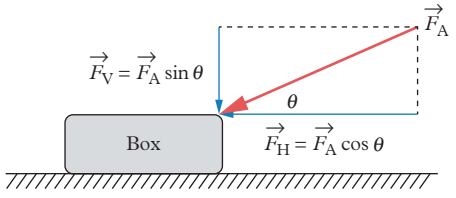


FIGURE 17 Free-body diagram showing vector components of the force on the box in Figure 16.

Worked examples 3.2D and 3.2E explore pulling and pushing an object horizontally.

Challenge

Navy destroyers

A World War II navy destroyer (ship) had a mass typically of about 2,000 tonnes. Imagine pushing away one moored at a dock with your hands. It sounds impossible, but your challenge is to work out how long it would take to shift it 30 cm with an applied force of 400 N (typical of a senior student). Assume that the water offers no friction. If you want to be more accurate, put a bathroom scale on the wall and see how hard you can press, and use this figure in your calculation. This will usually be about 40 kg. (3 marks)

Worked example 3.2D

Pulling an object at an angle

An 80.0 kg box is dragged across a concrete floor at constant speed by means of a 416 N (4.16×10^2 N) force in a rope at an angle of 30.0° to the floor (Figure 18). **Calculate** the (to an appropriate number of significant figures) the

a vertical component of the applied force (include a labelled free-body diagram) (2 marks)

b horizontal component of the applied force (3 marks)

c weight of the box (1 mark)

d normal force acting on the box (2 marks)

e frictional force. (1 mark)

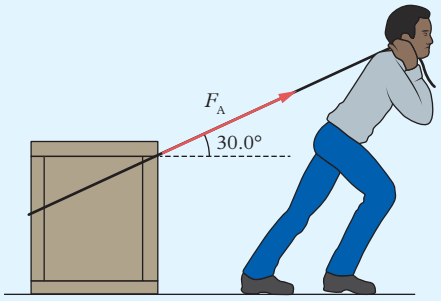
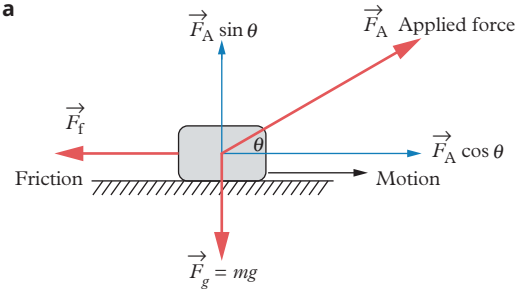


FIGURE 18 Pulling a box across the floor at an angle

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Draw a vector diagram showing the applied force resolved into two components at right angles. Derive formulas for the vertical and horizontal components. This is to help you solve the problem, but no marks are given for the diagram.	**a** 
Step 3: Substitute values into the correct formula to calculate the vertical component. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Include the units and direction. (1 mark for "Calculates the correct magnitude of the vertical component"; 1 mark for "Gives the correct direction")	$F_V = F_A \sin\theta$ $= 416 \sin 30°$ $= 416 \times 0.5$ $= 208$ N (1 mark) up (1 mark) (3 s.f.) $F_V = +208$ N (if up is +)
Step 4: Substitute values into the correct formula to calculate the horizontal component. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Include the units and direction. (1 mark for "Identifies the appropriate formula"; 1 mark for "Calculates the correct magnitude of the horizontal component correctly"; 1 mark for "Gives the correct direction")	**b** $F_H = F_A \cos\theta$ $= 400 \cos 30°$ (1 mark) $= 400 \times 0.866$ $= 346$ N (1 mark) to the right (1 mark) (3 s.f.)

Think	Do
Step 5: State the formula for weight, and substitute in the given values to calculate the weight. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Include the units and direction. (1 mark for "Calculates the weight correctly")	**c** $F_g = mg$ $= 80.0 \times 9.8$ $= 784\,\text{N down}$ $F_g = -784\,\text{N}$ (if up is +) (1 mark)
Step 6: State the formula for the normal force, remembering that the weight is downwards but the applied force is upwards. Substitute in the known values to calculate the weight. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Include the units and direction. (1 mark for "Identifies the relationship between the forces"; 1 mark for "Calculates the answer correctly")	**d** Forces are balanced in up and down direction so $F_{net} = 0$. $F_{net} = 0 = \vec{F}_N + \vec{F}_g + \vec{F}_V$ $0 = F_N + (-784) + 208$ (1 mark) $F_N = 784 - 208$ $= +576\,\text{N}$ or 576 N up (1 mark)
Step 7: The speed is constant so you can say that the net force is zero and thus the frictional force must equal the horizontal component of the applied force. No need to specify direction as it is shown on the diagram (i.e. to the left). (1 mark for "Provides correct answer")	**e** Frictional force $F_f = 346\,\text{N}$ (because $F_f = -F_H$ when speed is constant) (1 mark)

Your turn

A 650 g block of wood is dragged across a laboratory bench at constant speed by means of an applied force of 4.2 N in a string at an angle of 25.0° to the horizontal benchtop. **Calculate** (to an appropriate number of significant figures) the

a vertical component of the applied force (include a labelled vector diagram) (2 marks)
b horizontal component of the applied force (3 marks)
c weight of the block of wood (1 mark)
d normal force acting on the box (2 marks)
e frictional force. (1 mark)

Worked example 3.2E

Pushing an object horizontally

A person pushes an 80.0 kg box across a concrete floor at constant speed. They exert a force of 810 N through their arms at 30.0° to the floor (Figure 19). **Calculate** (to an appropriate number of significant figures) the

a vertical component of the applied force (include a labelled vector diagram) (2 marks)
b horizontal component of the applied force (2 marks)
c weight of the box (1 mark)
d normal force (2 marks)
e frictional force. (1 mark)

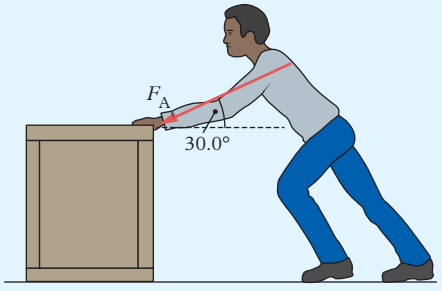
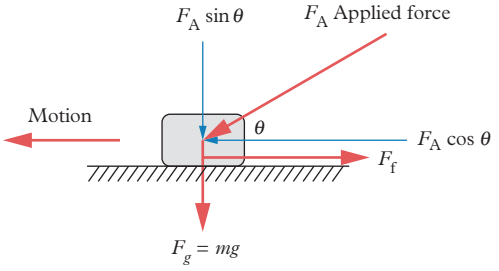


FIGURE 19 Pushing a box across the floor at an angle

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Collect the data given in the question.	**a** Applied force $F_A = 810\,\text{N}$ to the right at an angle θ of 30° to the floor.
Step 3: Draw a labelled vector diagram showing the applied force resolved into two components at right angles. This is to help you solve the problem, but no marks are given for the diagram.	
Step 4: Select the correct formula and calculate the vertical component of the applied force. State the direction. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Calculates the magnitude of the vertical component"; 1 mark for "Identifies the direction of the vertical component")	$F_V = F_A \sin\theta$ $= 810 \sin 30°$ $= 810 \times 0.5$ $= 405$ N (1 mark) down $= -405$ N (if up is +) (1 mark)
Step 5: Select the correct formula and calculate the horizontal component of the applied force. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Calculates the magnitude of the horizontal component"; 1 mark for "Identifies the direction of the horizontal component")	**b** $F_H = F_A \cos\theta$ $= 810 \cos 30°$ $= 810 \times 0.866$ $= 701$ N (1 mark) to the left (1 mark) (3 s.f.)
Step 6: Select the correct formula and calculate the weight of the box. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Calculates the weight correctly")	**c** $F_g = mg$ $= 80.0 \times 9.8$ $= 784\,\text{N}$ down $F_g = -784\,\text{N}$ (if down is $-$) (1 mark)
Step 7: Calculate the normal force. (1 mark for "Identifies the relationship between forces"; 1 mark for "Calculates the normal force correctly")	**d** Forces are balanced in up and down direction so $F_{net} = 0$. $F_{net} = 0 = \vec{F}_N + \vec{F}_g + \vec{F}_V$ $0 = F_N + (-784) + (-208)$ (1 mark) $F_N = 784 + 208$ $= +1{,}192\,\text{N}$ or 1,192 N up (1 mark)
Step 8: Calculate the frictional force. Give your answer to an appropriate number of significant figures, in this case 3 s.f. No need to specify direction as it is in the diagram (i.e. to the right). (1 mark for "Calculates the frictional force correctly")	**e** The frictional force is equal to the horizontal component as it is at constant speed. $F_f = 701\,\text{N}$ (1 mark)

Your turn

A person pushes a 650 g block of wood across a benchtop at constant speed. They exert a force of 8.37 N through their arms at 30.0° to the floor. **Calculate** (to an appropriate number of significant figures) the

a vertical component of the applied force (include a labelled vector diagram) (2 marks)

b horizontal component of the applied force (2 marks)

c weight of the block of wood (1 mark)

d normal force (2 marks)

e frictional force. (1 mark)

Skill drill

Sufficient and relevant data

Science inquiry skills: Evaluating evidence (Lesson 1.8)

In any experiment you need to ensure that you collect sufficient and relevant data to be able to analyse the data effectively and draw a justified conclusion. "Sufficient" means you have enough data points to be able to draw an accurate line of best fit, and sufficient replicates to reduce the random error for each data point.

In an experiment to measure the acceleration of a 950 g trolley across a horizontal table, hanging masses were attached to a string over a pulley as shown in Figure 20. Hanging masses of 25 g, 50 g and 75 g were used, and the time taken to travel 50 cm was measured for five repetitions.

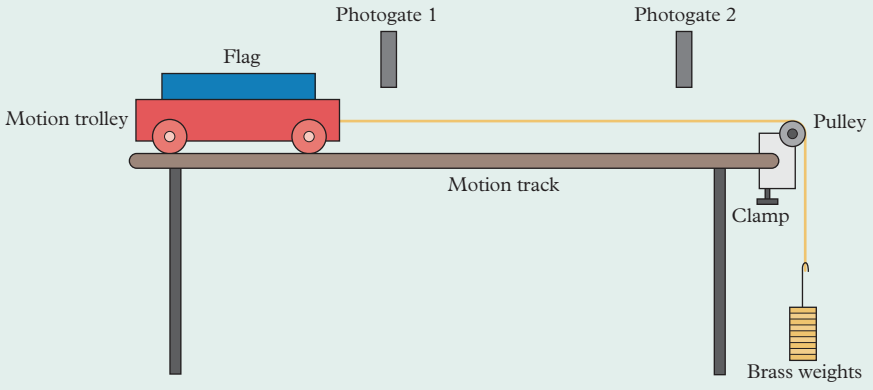


FIGURE 20 Experimental setup to measure acceleration of a trolley

Hanging mass (g)	Time to travel 50 cm, *t* (s)					
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
25	2.50	1.97	1.99	2.01	1.50	1.99
50	1.44	1.42	1.43	1.40	1.46	1.43
75	1.20	1.22	1.18	1.14	1.16	1.18

Practise your skills

1 **Identify** the independent and dependent variables. (2 marks)

2 **Judge** whether there is sufficient data to draw a justified conclusion. (2 marks)

3 **Assess** any limitations of the conclusion based on the uncertainty in the results. (2 marks)

Check your learning 3.2

Check your learning 3.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Define** "friction" (in one sentence). (1 mark)

2 **Recall** whether a tension force can be a push, a pull or both. (1 mark)

3 An object is being pulled along a horizontal surface by a rope at an upwards angle to the horizontal. **Explain** whether the normal force is greater or less than when the object is at rest. (2 marks)

4 **Describe** Newton's second law without using a formula. (1 mark)

5 A certain type of cotton thread will break when the tension force on it reaches 4.5 N. A 1.5 kg block is pulled horizontally along a frictionless surface by the string. **Calculate** the maximum acceleration possible (2 s.f.). (1 mark)

6 A 1.0 kg block of wood is dragged along a benchtop by a string to which a horizontal force of 15.0 N is applied. A frictional force of 10.0 N is acting.
   - **a** **Describe** the motion of the block (2 s.f.). (2 marks)
   - **b** **Calculate** the tension in the string (2 s.f.). (1 mark)

7 A 1.5 kg block is made to accelerate along a horizontal surface at 2.0 m s^{-2} to the right by a horizontal force of 5.0 N.
   - **a** **Construct** a free-body diagram. (2 marks)
   - **b** **Calculate** the frictional force that is acting to oppose this motion (2 s.f.). (2 marks)

8 A student uses a spring balance connected to a string at 30° to the horizontal to drag a wooden block along a laboratory bench at constant speed. The reading on the balance is 220 g.
   - **a** **Construct** a free-body diagram. (1 mark)
   - **b** **Calculate** the frictional force between the bench and the block (2 s.f.). (3 marks)

Analytical processes

9 A small block is dragged along a horizontal surface at constant speed by a force of 5.0 N. **Determine** the magnitude and direction of the frictional force (2 s.f.). (2 marks)

10 **Determine** if the following statement is true: "Friction only applies between two surfaces if they are accelerating past one another." (2 marks)

11 A man drags a 20 kg box across a horizontal concrete floor at constant speed using a rope at an angle of 30° to the floor. The applied force in the rope is 100 N. **Determine** (2 s.f.) the
   - **a** vertical component of the applied force (include a labelled vector diagram) (2 marks)
   - **b** horizontal component of the applied force (2 marks)
   - **c** weight of the box (1 mark)
   - **d** normal force acting on the box (2 marks)
   - **e** frictional force. (1 mark)

Knowledge utilisation

12 A set of brass masses rests on a laboratory balance that reads 452 g. A piece of string tied to the masses at an angle of 30° to the vertical is pulled gently, and the reading on the balance is reduced to 415 g.
   - **a** **Propose** a reason for this observation. (3 marks)
   - **b** **Determine** the tension in the string (2 s.f.). (2 marks)

13 A set of brass masses rests initially horizontal on an electronic laboratory balance that reads 494 g. The balance is tilted slightly, and the balance now reads 412 g.
   - **a** **Propose** a cause of this change in scale reading. (1 mark)
   - **b** **Calculate** the angle the balance makes with the benchtop (2 s.f.). (3 marks)

Lesson 3.3
Forces on an inclined plane

Key ideas

- → An inclined plane or ramp is a flat surface raised at one end, used as an aid for raising or lowering a load.
- → The force from gravity can be resolved into two vectors, one perpendicular to the plane and one parallel to the plane.
- → The component of the gravitational force in the perpendicular direction ($mg\cos\theta$) must be equal and opposite to normal force exerted by the plane, F_N, as there is no movement perpendicular to the plane.
- → The component of the force from gravity parallel to the surface is $mg\sin\theta$. If it is greater than the frictional force F_f then the body will accelerate down the inclined plane.

What is an inclined plane?

Learning intentions and success criteria

Inclined planes provided one of the first methods for studying accelerated motion. The Italian scientist Galileo Galilei (1564–1642) realised that measuring the rate of free-falling objects was difficult, as no accurate timing devices were available, especially for the short durations experienced in a laboratory. Galileo reasoned that free-fall was just a special case of an **inclined plane** for which the angle was 90°. He argued that acceleration still occurred on an inclined plane but that only the component of the acceleration along the track was effective in producing motion. The component perpendicular to the track was unable to produce motion as there was a surface in the way. By using a shallow angle, Galileo was able to study motion but at a greatly reduced rate. This gave the world an understanding of what he called "uniformly accelerated motion".

inclined plane
a flat surface raised at one end, used as an aid for raising or lowering a load

FIGURE 1 Using an inclined plane lessens the effort needed to raise an object, although the path length is greater.

Study tip

You can use any valid mathematical technique to get a solution, such as *i*, *j*, *k* vector matrices. However, if you use methods any more complicated than algebra and you make a mistake, the marker may not be able to follow your steps. In this case you may not be awarded part marks because they couldn't work out what you did.

How are forces on an incline resolved?

Galileo's analysis of the forces acting on an inclined plane are as good today as they were in 1589. When an object is placed on an inclined plane, its weight still acts vertically, but this is no longer perpendicular to the surface. The weight force can be resolved into components at right angles: one parallel and the other perpendicular to the surface of the incline (Figure 2).

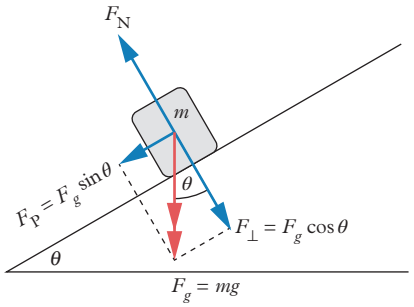


FIGURE 2 Components of forces on an incline

In the diagram, the weight of the object ($F_g = mg$) has been resolved into two components at right angles:

Parallel component: $F_P \left(\text{or } F_{||}\right) = F_g \sin\theta$
$= mg \sin\theta$

Perpendicular component: $F_\perp = F_g \cos\theta$
$= mg \cos\theta$

Note the use of the subscripts $\perp$ for perpendicular, and $||$ or P for parallel.

Note also that the normal force (F_N) is equal and opposite to the force perpendicular to the plane ($F_g \cos\theta$) because there is no acceleration in that direction; thus, $F_N = mg \cos\theta$.

Challenge

Initial acceleration

With the automatic gearbox in "drive", a 1,200 kg car will remain stationary facing uphill on a 5° slope. What would its initial acceleration be on the flat (assuming the driver's foot was not on the accelerator)? (2 marks)

Study tip

For an object on an incline, the two components of its weight are:
$F_P = mg \sin\theta$
$F_N = mg \cos\theta$.
It is handy to know these two equations off by heart. They are not formulas that will appear in the QCAA Formula and data book.

How do I perform vector analysis for inclined planes?

An object can move up or down an incline, and it can do so with or without friction. That gives four possible combinations, and these represent four situations found in everyday life. You will examine each of these for their particular features and approaches to problem-solving.

Object moving down an incline with no friction

An object placed on a smooth (frictionless) inclined plane will accelerate down the plane. The accelerating force is provided by the component of the object's weight in a direction down the plane, F_P (Figure 3). There are no frictional forces to slow its motion. It is difficult to avoid friction completely, but in factories trucks are often loaded by allowing boxes to slide down an incline made of zero-friction rollers. Objects can slide down them very fast. Rollercoasters have very low friction, especially when it rains. In fact, when it rains, amusement parks often have to stop rollercoasters as they get too fast (Figure 4).

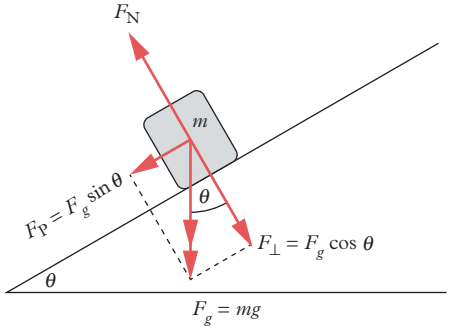


FIGURE 3 Free-body diagram – no friction

FIGURE 4 The roller-coaster is descending an almost frictionless incline.

Forces parallel to the plane: The net force F_{net} is the sum of all forces parallel to the plane. However, there is just the one force, F_P, so it is the net force:

$F_{net} = F_P$, and applying Newton's second law, $F_{net} = ma$, so:

$ma = mg\sin\theta$, and cancelling out the m term:

$a = g\sin\theta$

That is, acceleration down the incline equals the free-fall acceleration g multiplied by $\sin\theta$.

You can also say the acceleration is independent of the mass of the object in this very specific case (no friction).

In Worked example 3.3A, you will analyse an object sliding down an incline with no friction.

Study tip

When the forces on an object are equal (i.e. the net force is zero), there is no acceleration. This doesn't necessarily mean the object is at rest; it could be moving at a constant velocity.

Object moving up an incline with friction

It is more usual to have friction on an incline, and friction always opposes motion. A children's slippery slide is a good example of this.

The component of the child's weight down the incline, F_P, causes the child to move down the slide, but friction, F_f, acts upwards along the surface, opposing this motion. If $F_f = F_P$, there will be no net force and the child will not accelerate – the child will either remain stationary or travel down the slide at constant speed (Figure 5). Students often think that if the opposing forces on an object are equal, there is no motion. This is wrong – there is no acceleration, but the object may continue at constant speed or stay at rest (which is a constant speed of zero anyway).

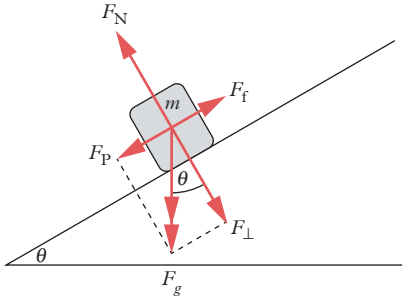


FIGURE 5 Free-body diagram of an object on an inclined plane with friction present

FIGURE 6 A slippery slide has very low friction but there is still some that acts up the incline as the child slides down.

Worked example 3.3A

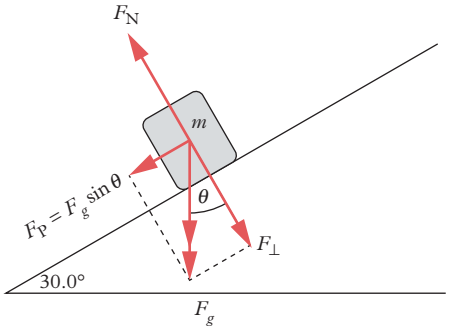
Sliding down an incline with no friction

A 15.0 kg bag of fertiliser is allowed to slide freely down a smooth (frictionless) incline that is at 30.0° to the horizontal.

a **Construct** a labelled free-body diagram of the forces. (2 marks)

b **Calculate** the net force parallel to the incline. Give your answer to an appropriate number of significant figures. (2 marks)

c **Calculate** the acceleration of the bag of fertiliser. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Construct" means to display information in a diagrammatic or logical form, in this case to construct a free-body diagram. "Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Collect the relevant data and draw a labelled free-body vector diagram with the weight resolved into its perpendicular components. Note: it is good to get into the habit of drawing a simple free-body diagram to show the forces. Often with these questions, a mark is awarded for a correctly drawn and annotated diagram. (1 mark for "Draws a free-body diagram of the forces involved"; 1 mark for "Forces are correctly identified or labelled")	**a** $m = 15.0\,\text{kg}$, $\theta = 30.0°$  (2 marks)
Step 3: Identify the appropriate formula and substitute in the known values. Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check that you have included the correct units. As force is a vector quantity you need to state the direction. (1 mark for "Calculates correct magnitude"; 1 mark for "Identifies the correct direction")	**b** $F_{net} = F_P$ $= mg\sin\theta$ $= 15.0 \times 9.8 \times \sin 30.0°$ $= 73.5$ N (3 s.f.) (1 mark) down the incline (1 mark)

Think	Do
Step 4: Use the Newton's second law formula to calculate the acceleration. Substitute the known values into the equation and solve for a. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. As acceleration is a vector quantity you normally need to state the direction; however, in this case the question states the direction so only the magnitude is required. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**c** $F_{net} = ma$ $73.5 = 15.0 \times a$ $a = \frac{73.5}{15.0}$ (1 mark) $= 4.90\,\text{m s}^{-2}$ (1 mark)

Your turn

A 650 g cart is placed on an incline that is at 15.0° to the horizontal.

a **Construct** a free-body diagram of the forces. (2 marks)

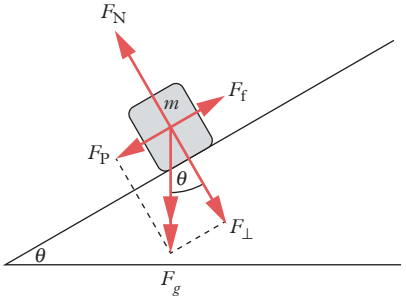
b **Calculate** the net force parallel to the incline. Give your answer to an appropriate number of significant figures. (2 marks)

c **Calculate** the acceleration of the cart. Give your answer to an appropriate number of significant figures. (2 marks)

Worked example 3.3B

Sliding down an incline with friction

A 34.0 kg child sits on a slippery slide that has an angle of elevation of 40.0° to the horizontal. If the frictional force is 110 N (1.10×10^2 N), **determine** the acceleration of the child down the incline. Show your working. Give your answer to an appropriate number of significant figures. (5 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For 5 marks there will be many steps so we must identify the appropriate formulas and relationships and use them to find an answer.
Step 2: Gather the relevant information and draw a labelled free-body vector diagram with the weight resolved into its perpendicular components. Include a vector arrow for friction up the plane. We don't yet know if friction is equal to, or less than, F_p, so we can just make the arrow the same length. Friction can never be greater than F_p as the child would move in the direction of friction, which is nonsensical. A diagram is necessary as a part of "show working". (1 mark for "Constructs a free-body diagram of the forces involved")	$m = 34.0\,\text{kg}$, $F_f = 110\,\text{N}$, $\theta = 40.0°$  (1 mark)
Step 3: Determine the parallel component of the weight down the incline, F_p. Identify the appropriate formula and substitute in the known values. Complete the calculation. (1 mark for "Determines the parallel component")	$F_p = mg\sin\theta$ $= 34.0 \times 9.8 \times \sin 40.0°$ $= 214$ N (1 mark)

Think	Do
Step 4: The other force acting along the plane is friction, which acts in the direction opposing motion. So, friction acts up the incline. Thus, the net force, F_{net}, equals the F_P component of weight minus the frictional force, F_f. Identify the appropriate formula and substitute in the known values. Complete the calculation. As force is a vector quantity you need to state the direction. (1 mark for "Calculates the net force and states the direction")	$F_{net} = F_P - F_f$ $= 214 - 110$ $= 104$ N down the incline (1 mark)
Step 5: Use the Newton's second law formula to calculate the acceleration. Substitute known values into the equation and solve for a. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. As acceleration is a vector quantity, a direction must be stated. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$F_{net} = ma$ $a = \frac{F_{net}}{m}$ $= \frac{104}{34.0}$ (1 mark) $= 3.06\,\text{m s}^{-2}$ down the incline (3 s.f.) (1 mark)

Your turn

A 650 g block of wood is placed on an incline that has an angle of elevation of 35.0° to the horizontal. If the frictional force is 3.03 N, **determine** the motion of the block. Show your working. Give your answer to an appropriate number of significant figures. (3 marks)

Real-world physics

Designing roads for steep hills

Road engineers are often faced with questions like "I have a client who would like to build their house on a site with a very steep access – about 16°. Is this too steep for a road?"

Queensland road design rules specify a maximum gradient of 9° for suburban roads, but it does depend on the average speed of traffic. Higher speed means lower gradient. Road gradients are designed so people can drive safely on them and to minimise the risk of accidents. The steepest public road in Queensland is Gower Street, Toowong, Brisbane, with an incline of 17.4°. The steepest public road in the world is Baldwin Street in Dunedin, New Zealand, at 22.5° (Figure 7).

FIGURE 7 The world's steepest public road is Baldwin St, Dunedin, in New Zealand.

For a car not to slide down a road, the component of the weight down the incline ($mg \sin\theta$) must be no greater than the friction (F_f) acting up the incline. The friction when sliding depends on the road surface and the tyres of the vehicle. This friction is proportional to the perpendicular component of the weight ($mg \cos\theta$) pressing the vehicle to the road. Typically, for slippery conditions such as rubber tyres on wet bitumen, the constant of proportionality (also known as the coefficient of friction) is about 0.4. This means the friction on a 1,500 kg car on a 9° gradient is: $0.4 \times 1{,}500 \times 9.8 \times \cos 9° = 5{,}800$ N. Let's compare that to the component of the weight down the incline, which is $1{,}500 \times 9.8 \times \sin 9° = 2{,}300$ N. So, theoretically friction is enough at this angle to stop the car sliding. This means the road would be suitable under most conditions.

If we try the same calculations for Gower Street in Brisbane:

$F_P = mg \sin\theta = 1{,}500 \times 9.8 \times \sin 17.4° = 4{,}400\,\text{N}$

$F_f = 0.4 \times 1{,}500 \times 9.8 \times \cos 17.4° = 5{,}600\,\text{N}$.

The friction is still greater than the parallel component of the weight of the car, but you would want to be careful in wet weather. In dry weather the coefficient is about 0.7, so $F_f = 0.7 \times 1{,}500 \times 9.8 \times \cos 17.4° = 9{,}800\,\text{N}$, which is perfectly safe.

Apply your understanding

1 **Determine** whether Baldwin Street in Dunedin is safe for a 1,500 kg car in wet weather where the frictional force is 5,400 N. Baldwin Street has a gradient of 22.5°. (3 marks)

2 **Determine** what amount of friction (in N) would be 100 N less than the parallel component of the weight, F_P. (2 marks)

Note: questions about the coefficient of friction will not appear on your QCAA external exam.

What forces act on an object pulled up an incline with no friction?

Objects can also be forced to move up an incline, such as when boxes are moved by trolley up a ramp into the back of a truck, or the carts in a coal mine are dragged up a rail line to the surface by a force applied to the end of a cable. The Swiss funicular is a train carriage that has a cable that pulls it up to the top of a mountain (Figure 8).

FIGURE 8 A Swiss funicular is used to reach the top of a mountain and uses a cable to pull the carriage up the incline.

In a coal mine, for instance, the applied force creates a tension (F_T) in the cable that overcomes the weight component of the cart acting down the plane. The cart will usually also have friction to oppose its motion, but first we will look at scenarios without friction (Figure 9).

If the cart is at rest or the velocity is constant, the net force acting up the plane will be zero so the tension in the cable, F_T, up the incline will equal the parallel component of the weight force, F_P, down the incline, i.e. $F_T = F_P$.

$F_{net} = F_T - F_P = 0$ (constant velocity, including stationary)

However, when the tension in the cable is greater than the weight force down the incline, i.e. $F_T > F_P$, there will be a net force on the cart, and it will accelerate up the incline:

$F_{net} = F_T - F_P = ma$ (accelerating up the incline)

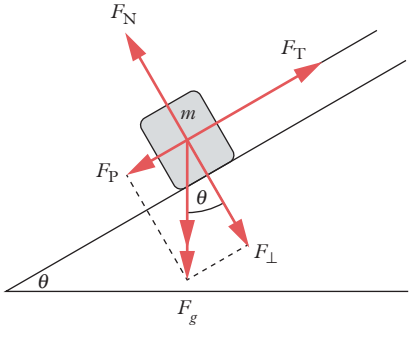


FIGURE 9 Free-body diagram of an object pulled up the incline (no friction)

Using a hanging or falling mass to provide the applied force

Instead of using an applied force such as a motor to provide tension in the cable to pull an object up an incline, the stored energy of a falling mass can do the same thing. The Egyptians developed a version of this to shift blocks of stone up inclines using a device called a "rope-roll". Weights are sometimes used to provide tension to overhead power lines for trams and railways using the same principle (Figure 10).

FIGURE 10 Counterweight blocks provide tension to overhead wires for electric trains.

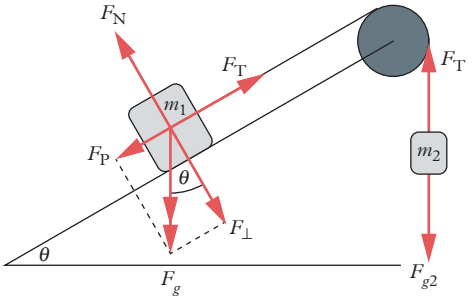


FIGURE 11 Free-body diagram of an object on an inclined slope with force applied using hanging masses

The free-body diagram in Figure 11 shows a cable connected to an object on an incline. The cable passes over a frictionless pulley at the top. A large hanging mass, m_2, provides a force up the incline. The balance between F_T and F_P will determine the direction in which the block moves, or if it moves at all.

The weight of the hanging mass m_2 can be calculated as $F_{g2} = m_2 g$. The symbol F_{g2} is used to distinguish it from F_{g1}, which is the weight of the object on the incline. The weight of the hanging mass provides the tension force, F_T, in the cable, and if it is large enough it will pull the block up the incline. If the weight of the hanging mass just equals the force on the block down the incline, F_P, the block will stay at rest, or if given a small push will continue to move at constant speed in that direction. If the weight of the hanging mass is less than the force down the incline, F_P, the block will accelerate down the incline, but not as fast as if the hanging mass wasn't there. These three scenarios can be summed up thus:

- tension force > force down the incline, $F_T > F_P$: object will accelerate up the incline
- tension force = force down the incline, $F_T = F_P$: object will remain stationary on the incline or continue moving at constant speed
- tension force < force down the incline, $F_T < F_P$: object will accelerate down the incline.

Study tip

Just because a block on an incline is heavier than the hanging mass, it doesn't mean the block will move down the incline. It depends on the angle too.

Angle of elevation and hanging mass

As you increase the angle of the incline (the angle of elevation), the normal force decreases but the parallel component of the object's weight, F_P, down the incline increases. That is, the steeper the incline, the greater the acceleration of the object. You can measure the downward force, F_P, by connecting a hanging mass over a pulley at the top and adjusting the hanging mass until the object on the incline moves neither up nor down. At this point, $F_P = F_{g2}$ (the weight of the hanging mass).

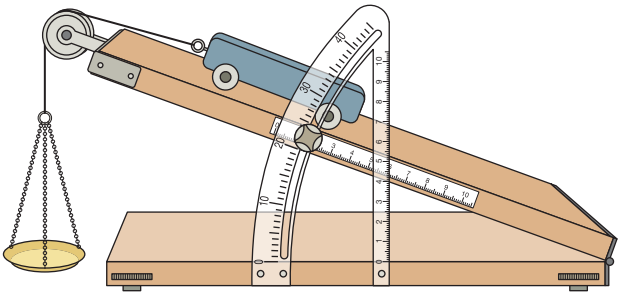


FIGURE 12 Apparatus for investigating forces on an inclined plane

This can be demonstrated using the apparatus shown in Figure 12. Masses are added to the hanging pan on the left to keep the frictionless trolley stationary on the incline. As the angle is increased, the component of the weight down the incline increases and thus more mass has to be added to the pan to keep the trolley stationary. For example, a 200 g trolley on an incline could have results like those in Table 1.

TABLE 1 Parallel component of weight on an inclined plane

Angle of elevation, θ (degrees)	10°	20°	30°	40°
$\sin\theta$	0.17	0.34	0.50	0.64
Mass of hanging mass, m_2 (g)	34	68	100	128
Weight of hanging mass, F_{g2} (N)	0.33	0.67	0.98	1.25
Force down incline, F_P (N)	0.33	0.67	0.98	1.25

It is immediately obvious that there is a relationship between the angle of elevation and the parallel component of the weight (equal to the weight of the hanging mass); that is, as you increase the angle, you increase the hanging mass needed. It looks almost linear, but it is not. The parallel component (the weight of the hanging mass) is proportional to the sine of the angle ($F_P \propto \sin\theta$). You can see this more clearly in Figure 13.

Study tip

The idea shown here forms the basis of one of the approaches in the practical "The parallel component of the weight of an object on an inclined plane". It uses freely hanging masses over a pulley to provide a force equal and opposite to the parallel component of weight down the incline. This would be a good candidate for modifying for a student experiment.

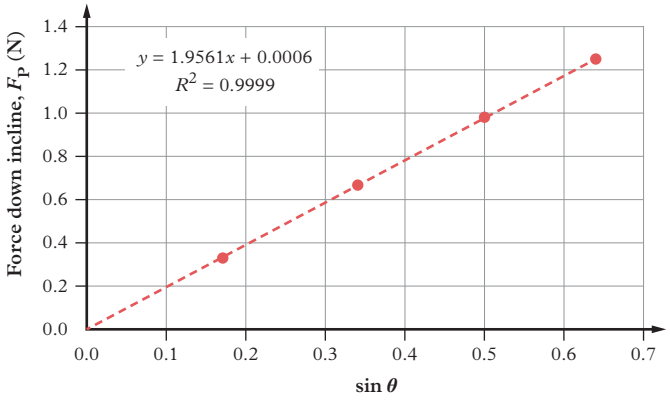


FIGURE 13 Relationship between the sine of the angle of an incline and the force down the incline

The gradient for the graph is 1.9561 and is equal to $\frac{F_P}{\sin\theta}$. From our theoretical equation $F_P = mg\sin\theta$, we see that this should equal mg. The gradient divided by the mass of 0.200 kg gives 9.78 N kg^{-1}, which is equivalent to 9.78 m s^{-2}. This is very close in magnitude to the accepted value of g of –9.81 m s^{-2} for Queensland. Remember, of course, that this is an experimental value for g. The QCAA Physics Formula and data book uses the value of g as –9.8 m s^{-2} and you should always use that value in your calculations.

Study tip

As θ gets bigger, $\sin\theta$ gets bigger but $\cos\theta$ gets smaller.

Challenge

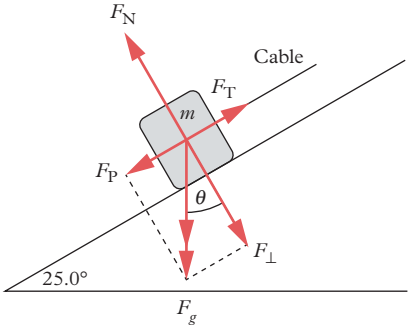
Graphing elevation

Construct a graph of an angle of elevation (from 0 to 90°) on the horizontal axis, and sin θ on the vertical axis. On the same graph, plot cos θ on the vertical axis. Note their shapes. (2 marks)

Worked example 3.3C

Object pulled up an incline with no friction

A 525 kg coal cart is pulled up a 25.0° incline by a cable. Ignoring any friction, if the cart moves up the track at constant speed, **calculate** the tension in the cable. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each of the three steps we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data and draw a labelled free-body vector diagram showing the weight resolved into its perpendicular components and tension acting up the incline. The truck is moving at constant speed, so there is no net force and the arrow for tension, F_T, will be the same length as the arrow for the parallel component of the weight, F_P. (1 mark for "Constructs a free-body diagram of the forces involved")	$F_{net} = 0$ (constant speed), $m = 525$ kg, $\theta = 25.0°$  (1 mark)
Step 3: Calculate the component of the weight acting down the incline. (1 mark for "Calculates the parallel component of the weight down the incline")	$F_P = mg\sin\theta$ $= 525 \times 9.8 \times \sin 25°$ $= 2{,}174$ N down the incline (1 mark)
Step 4: As the truck is moving at constant speed, there is no net force. State this and then equate the tension and F_P. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. As tension (a force) is a vector quantity, you need to give a direction (up the incline). (1 mark for "Identifies the relationship between tension and the parallel component and provides correct answer")	$F_{net} = 0$, so speed is constant. $F_T = F_P$ (equal and opposite) $F_T = 2{,}174$ N $= 2{,}170$ N (3 s.f.) up the incline (1 mark)

Your turn

In a laboratory experiment, a spring balance attached to a length of string was used to pull a 650 g cart up a 15.0° incline at a constant speed. Ignoring friction, **calculate** the tension in the string. Give your answer to an appropriate number of significant figures. (3 marks)

Worked example 3.3D

Object pulled up an incline by a hanging mass with no friction

A 525 kg coal cart, m_1, is pulled up a 25.0° incline by a cable over a pulley at the top. A hanging mass, m_2, of 300 kg (3.00×10^2 kg) is used to provide the applied force in the cable (Figure 14). Ignoring friction, **determine** (to an appropriate number of significant figures) the

a acceleration of the coal cart, m_1 (5 marks)

b tension, F_T, in the cable between m_1 and m_2. (2 marks)

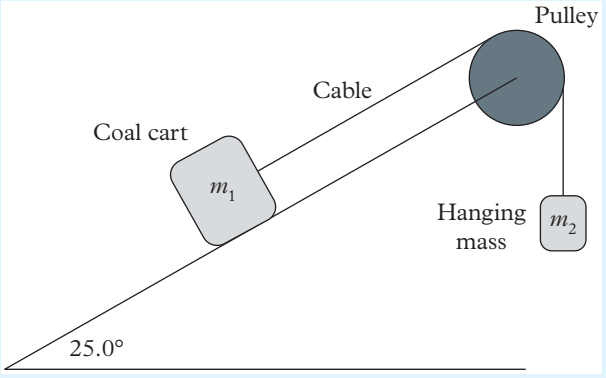
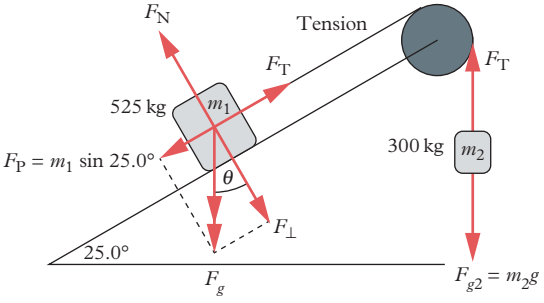


FIGURE 14 Coal cart pulled up an incline by a hanging mass attached to a cable

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking.	"Determine" means to establish after a calculation. So, for 5 marks in part **a**, we must consider the scenario and then find a numerical answer using mathematical processes and showing working.
Step 2: Gather the relevant information and draw a labelled free-body diagram showing any forces acting, including tension. Note that a labelled diagram can be worth marks, even if no further calculations are made. This is evidence of "showing working". (1 mark for "Constructs a free-body diagram of the forces involved")	**a**  (1 mark)
Step 3: Identify the relevant formula. Calculate the force down the incline due to the mass of the coal cart m_1. This is F_P. (1 mark for "Calculates the parallel component")	$F_P = m_1 g \sin\theta$ $= 525 \times 9.8 \times \sin 25°$ $= 2{,}174$ N (1 mark)
Step 4: Calculate the force on m_1 up the incline due to the hanging mass. This is force F_{g2}. This force is redirected by the pulley, and it pulls on m_1 through tension in the cable up the incline. (1 mark for "Calculates the pulling force")	$F_{g2} = m_2 g$ $= 300 \times 9.8$ $= 2{,}940$ N (1 mark)
Step 5: Calculate the net force. The force up the incline (2,940 N) is greater than the force down the incline (2,174 N), so the difference is the net force on the system. This force is on m_1 up the incline and on m_2 towards the ground. (1 mark for "Provides appropriate mathematical reasoning")	$F_{net} = F_{g2} - F_P$ $F_{net} = 2{,}940 - 2{,}174$ $= 766$ N up the incline (1 mark)

Think	Do
Step 6: This net force of 766 N provides the accelerating force of the system of two masses, so we need to consider the total mass of the system ($m_1 + m_2$ = 525 kg + 300 kg). Calculate the acceleration of the system of the two masses. Give your answer to an appropriate number of significant figures, in this case 3 s.f. State your answer with units and direction. (1 mark for "Provides correct answer")	$a = \frac{F_{net}}{m_{total}}$ $= \frac{766}{525 + 300}$ $= 0.928\,m\,s^{-2}$ up the incline (3 s.f.) (1 mark)
Step 7: The tension, F_T, in the cable acts on m_1 and m_2. Either can be used to find a solution, and in this case, we will use m_1. The force up the incline on m_1 is provided by the tension in the cable, F_T, and the force down the incline is from the parallel component of the weight of m_1 ($F_P = m_1 \times g \times \sin 25°$). The difference between F_T and F_P is the net force, which is equal to $m_1 \times a$ (Newton's second law) and is up the incline. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Identifies the relationship between the various forces"; 1 mark for "Provides correct answer")	**b** $F_{net} = F_T - m_1 g \sin\theta$ (1 mark) $m_1 a = F_T - m_1 g \sin\theta$ $F_T = m_1 a + m_1 g \sin\theta$ $= 525 \times 0.928 + 2{,}174$ $= 2{,}660$ N (3 s.f.) (1 mark)

Your turn

In a laboratory experiment with a setup as in Worked example 3.3D, students placed a cart of mass 650 g on a 30.0° incline. The cart was pulled up the incline by a light string over a pulley at the top. Attached to the string was a hanging mass of 436 g. Ignoring friction, **determine** (to an appropriate number of significant figures) the

a acceleration of the cart on the incline (5 marks)

b tension in the string. (2 marks)

What forces act on an object pulled up an incline with friction acting?

A more likely scenario is one in which friction is acting to oppose motion as an object is pulled up an incline. The forces acting down the incline will now be F_P and F_f, and the forces acting up the incline will be the applied forces via the tension in the cable (Figure 15). This is explored in Worked example 3.3E.

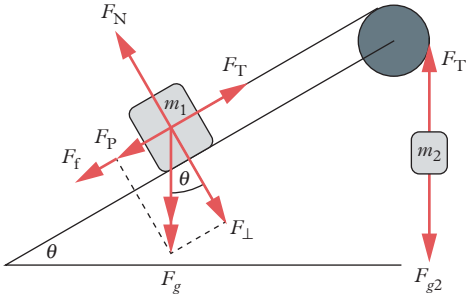


FIGURE 15 Free-body diagram. Friction acts to oppose motion – motion up, friction down.

Worked example 3.3E

Object pulled up an incline by a hanging mass with friction

A cart in a mine is pulled up a 20.0° incline by a cable attached to a hanging mass over a frictionless pulley system. The filled cart, m_1, has a mass of 750 kg, and the frictional force is 1,036 N. The hanging mass, m_2, has a mass of 383 kg (Figure 16).

a **Construct** a labelled free-body diagram to show the relevant forces. (2 marks)

b **Determine** the acceleration of the cart. Give your answer to an appropriate number of significant figures. (4 marks)

c **Determine** the tension in the cable. Give your answer to an appropriate number of significant figures. (2 marks)

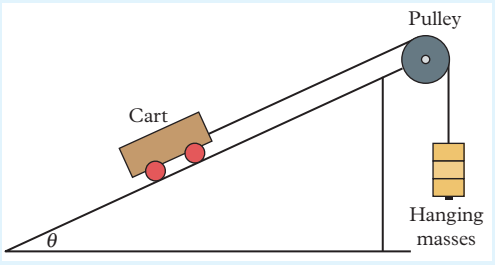
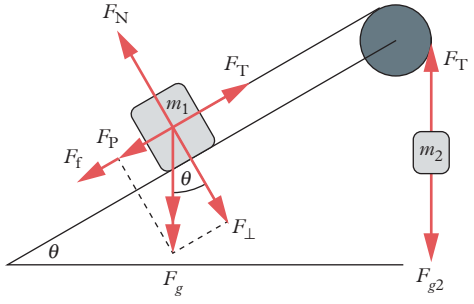


FIGURE 16 A hanging mass pulls a mine cart up an incline.

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Construct" means to display information in a diagrammatic or logical form – so for 2 marks it needs to be a free-body diagram with properly labelled force vectors. "Determine" means to establish after a calculation. So, for 6 marks, we must consider the scenario and then find a numerical answer to many steps using mathematical processes and showing working.
Step 2: Gather the relevant information and draw a free-body diagram similar to Figure 15 in this lesson. Your diagram should include the component of the cart's weight down the incline, the friction down the incline, the tension in the cable, and the gravitational force acting on the hanging mass. Forces should be labelled correctly. (1 mark for "Constructs a free-body diagram of the forces involved"; 1 mark for "Identifies the forces")	**a**  (2 marks)
Step 3: To determine the acceleration, we need to calculate the net force. First, determine the total force down the incline by adding F_P and F_f. (1 mark for "Determines the total force down")	**b** $F_{down} = F_P + F_f$ $= 2{,}514 + 1{,}036$ $= 3{,}550$ N down the incline (1 mark)
Step 4: Calculate the weight of the hanging mass using $F = mg$. (1 mark for "Calculates the weight of the hanging mass")	$F_{g2} = m_2 g$ $= 383 \times 9.8$ $= 3{,}753$ N (1 mark)
Step 5: Calculate the net force. This is the force up the incline due to the hanging mass minus the force down the incline. (1 mark for "Calculates the net force")	$F_{net} = F_{up} - F_{g2}$ $= 3{,}753 + 3{,}550$ $= 203$ N up the incline (1 mark)

Think	Do
Step 6: Calculate the acceleration using Newton's second law. The net force provides acceleration to the whole system made up of the two masses, so you need to use the total mass in your calculation. The total mass of the system is the sum of m_1 and m_2. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Include the correct units. Normally you would state the direction of the acceleration as it is a vector quantity, but in this case the question says it is up the incline, so you can omit the direction, although you will not be penalised if you provide it. (1 mark for "Provides correct answer")	$a = \frac{F_{net}}{m_{total}}$ $= \frac{203}{750 + 383}$ $= 0.179\,\text{m s}^{-2}$ up the incline (1 mark)
Step 7: The tension in the cable, F_T, acts on m_1 and m_2. Either can be used in arriving at a solution, and in this case we will use m_1. The force up the incline on m_1 is provided by the tension in the cable, F_T, and the force down the incline is from the parallel component of the weight, F_P, and the friction, F_f. The difference is the net force, which is equal to $m_1 \times a$ (Newton's second law of motion) and is up the incline. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	**c** $F_{net}(m_1) = F_T - F_{\text{down incline}}$ $m_1 a = F_T - F_{\text{down incline}}$ (1 mark) $F_T = m_1 a + F_{\text{down incline}}$ $= m_1 a + (F_P + F_f)$ $= 750 \times 0.179 + 2{,}514 + 1{,}036$ $= 3{,}684\,\text{N}$ $= 3{,}680\,\text{N}$ (3 s.f.) (1 mark)

Your turn

In an experiment similar to the setup in Worked example 3.3E, a trolley of mass 650 g was pulled up a 25.0° incline by an applied force provided by a hanging mass over a frictionless pulley system. Friction between the trolley and the incline was known to be 1.15 N. The hanging mass, m_2, has a mass of 404 g.

a **Construct** a free-body diagram to show relevant forces. (2 marks)

b **Determine** the acceleration of the trolley. Give your answer to an appropriate number of significant figures. (4 marks)

c **Determine** the tension in the cable. Give your answer to an appropriate number of significant figures. (2 marks)

Check your learning 3.3

Check your learning 3.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Identify** the practical purpose of an inclined plane. (1 mark)

2 **Explain** the purpose of resolving a weight vector into two components at right angles: one down the incline and one perpendicular to it. (2 marks)

3 **Recall** what happens to the magnitude of the parallel and perpendicular components of the weight as the angle of elevation of an incline increases. (1 mark)

Analytical processes

4 A 20 kg object rests on a 30° inclined plane. **Determine** (2 s.f.) the

a parallel component of the object's weight (1 mark)

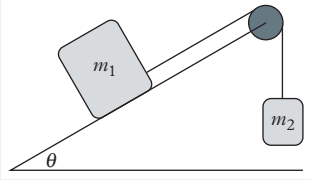
b perpendicular component of the object's weight (1 mark)

c normal force on the object by the surface of the plane. (1 mark)

5 An object of mass 10 kg is placed on a frictionless incline of 25°. **Determine** how many seconds it would take to travel down a 5.0 m incline from rest (2 s.f.). (3 marks)

6 A 14 kg toolbox is placed on a plank of wood. When one end of the plank is raised, the toolbox begins to slide down the incline at a uniform speed when the angle reaches 40°. **Determine** the frictional force acting on the box (2 s.f.). (3 marks)

7 An object of mass m_1 is placed on a frictionless incline as shown in the diagram. It is held at rest by a hanging mass, m_2, on a cord over a frictionless pulley at the top of the incline.



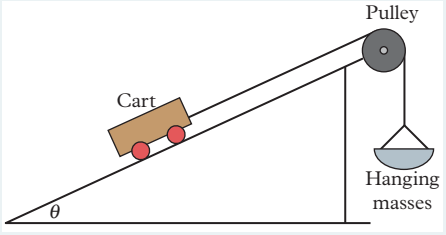
a **Determine** the angle of elevation if m_1 is 50 kg and m_2 is 23 kg (2 s.f.). (3 marks)

b **Calculate** the acceleration of the object m_1 if m_2 is increased to 30 kg (2 s.f.). (3 marks)

c **Calculate** the acceleration of the object m_1 if m_2 is reduced in mass to 20 kg (2 s.f.). (3 marks)

Knowledge utilisation

8 Students carried out an experiment to measure how the component of the weight parallel to the surface of an inclined plane varies with the angle of elevation. The setup is shown in the diagram.



They added small masses to the hanging cup to keep the frictionless cart (mass 279 g) in place on the incline. The table shows their results.

Angle θ (degrees)	10	20	30	40
Hanging mass (g)	52.4	96.4	144.5	179.3

a **Construct** an appropriate graph, linearised if necessary, and deduce the relationship between angle of elevation and the component of the weight parallel to the incline. (4 marks)

b **Propose** how the graph would appear if there was a small amount of friction present. **Justify** your answer. (2 marks)

c **Interpret** the methodology to state, with reasons, whether friction would be a systematic or random error. (2 marks)

9 A friend lives at the top of a road that has a 5° downhill slope. When she lets her car (of mass 2,000 kg) roll down the slope, it reaches 25 km h^{-1} by the time it gets to the bottom, 400 m away. **Determine** the frictional force that must be acting. (5 marks)

10 A crate of tiles (mass 14 kg) moves up a 30° incline at constant speed when pulled by a rope attached to a crate of cement of equal mass hanging over a frictionless pulley at the top. **Determine** the frictional force (2 s.f.). (4 marks)

Practical

Lesson 3.4

The parallel component of the weight of an object on an inclined plane

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

Learning intentions and success criteria

MODULE 3

Lesson 3.5
Review: Inclined planes

Summary

3.1
- The force of gravity, F_g, on an object is the object's weight.
- Mass, m, is a measure of an object's resistance to motion, or the amount of substance in an object. It is related to weight by $F_g = mg$.
- When an object free-falls under gravity, it accelerates at g. Near the surface of Earth, $g = 9.8\,\text{m}\,\text{s}^{-2}$ downwards.
- The normal force is the force exerted on an object by a surface against which it is pressed. The normal force is always perpendicular to the surface.

3.2
- As well as force due to gravity (weight) and the normal force, the forces acting on an object include tension, frictional force and applied force.
- A force is a push or pull between objects that may cause one or both objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape.
- Friction is the resistance to motion of a surface moving relative to another. Unlike gravity, electromagnetism or the strong and weak forces, friction is not a fundamental force. It is the result of the electromagnetic attraction between charged particles in the two touching surfaces.
- Tension is the pulling force transmitted along a rope, string, cable or chain on an object. It has the symbol F_T.
- An applied force (F_A) is the force applied to an object by a person or another object. It can be a push or a pull.

3.3
- An inclined plane or ramp is a flat surface raised at one end, used as an aid for raising or lowering a load.
- The force from gravity can be resolved into two vectors, one perpendicular to the plane and one parallel to the plane.
- The component of the gravitational force in the perpendicular direction ($mg\cos\theta$) must be equal and opposite to normal force exerted by the plane, F_N, as there is no movement perpendicular to the plane.
- The component of the force from gravity parallel to the surface is $mg\sin\theta$. If it is greater than the frictional force F_f then the body will accelerate down the inclined plane.

3.4
- Practical: The parallel component of the weight of an object on an inclined plane

Key formulas

Weight of an object	$F_g = mg$

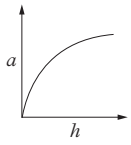
Review questions 3.5A Multiple choice

Review questions: Complete these questions online or in your workbook.

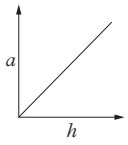
(1 mark each)

1 A frictionless inclined plane is raised at one end to a height h. A dynamics cart is allowed to roll down the plane, and its acceleration is measured for several different heights. Which option best shows the acceleration, a, of the cart as a function of height, h?

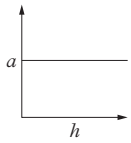
A



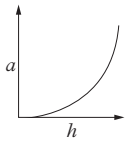
B



C

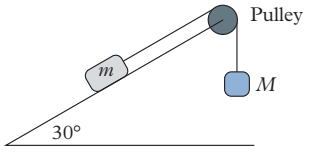


D



2 Determine which of the graphs in question 1 best shows the final velocity, v at the bottom of the incline, as a function of height, h.

3 A frictionless incline has a mass, m, resting on the surface and another mass, M, attached to a light string over a frictionless pulley, as shown in the diagram. Mass m equals mass M. Determine which one of the following is most likely to occur.



A Mass m moves at constant speed down the incline.

B Mass m moves at constant speed up the incline.

C Mass m accelerates down the incline.

D Mass m accelerates up the incline.

4 A block is placed on an incline, and it accelerates down the incline even though friction is present. Select the expression that best describes the relationship between friction, F_f, and the parallel component of the force on the block, F_P.

A $F_P > F_f$

B $F_P = F_f$

C $F_P < F_f$

D $F_P = F_f = 0$

5 A 2 kg object is placed on a frictionless incline set at an angle θ. Select the expression that best describes its acceleration.

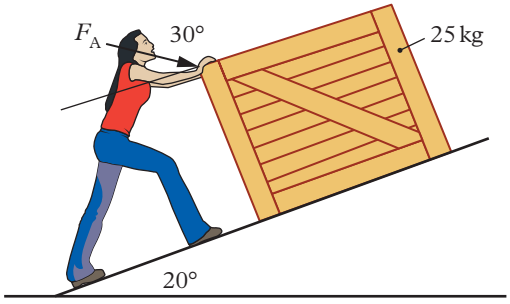
A $mg\sin\theta$

B $mg\cos\theta$

C $g\sin\theta$

D $g\cos\theta$

6 A furniture removalist pushes a 25 kg box up a frictionless incline at a steady speed. The incline is at an angle of 20° to the ground and the removalist's arms apply a force F_A to the box at an angle of 30° to the top, as shown in the diagram. Calculate the force F_A.



A 84 N

B 97 N

C 122 N

D 168 N

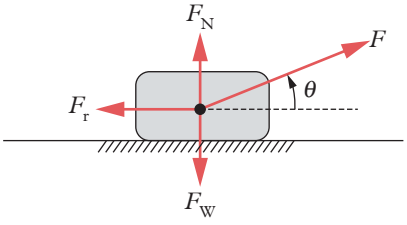
7 A block of mass m is being pulled by a force of F newtons acting at θ to the horizontal, as shown in the diagram. Select which statement correctly defines the resultant force acting on the block.

A $\Sigma F = F\tan\theta - F_f$

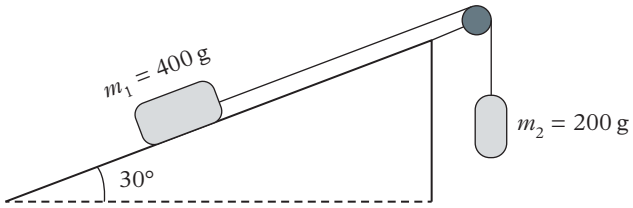
B $\Sigma F = F\sin\theta - F_f$

C $\Sigma F = F\cos\theta - F_f$

D $\Sigma F = F + F_N - F_g - F_f$



8 A frictionless incline has a 400 g mass, m_1, resting on its surface. A 200 g mass, m_2, is attached to it by a light, inextensible string over a frictionless pulley, as shown in the diagram.



Which of these events occurs?

A Mass m_1 moves at a constant speed down the incline.

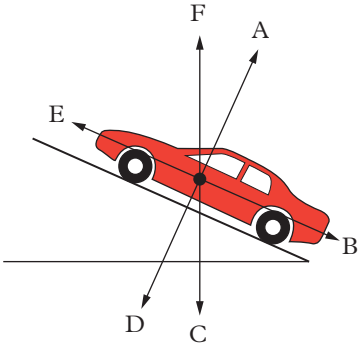
B Mass m_1 moves at a constant speed up the incline.

C Mass m_1 remains stationary.

D Mass m_1 accelerates down the incline.

Use the following information to answer questions 9 and 10.

A car of mass 1,500 kg is parked on a hill, as shown in the diagram.



9 Identify the arrow that correctly shows the direction of the normal force on the car.

A A **B** C **C** F **D** E

10 Select the arrow that correctly shows the direction of the frictional force acting on the stationary car.

A E **B** C **C** A **D** B

Review questions 3.5B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

11 **Explain** whether you are weightless when you float in a swimming pool. (2 marks)

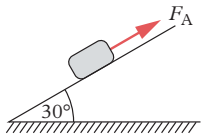
12 **Construct** a free-body diagram showing the forces acting on an object at rest on a frictionless incline, including the tension in a rope holding it there. (2 marks)

13 **Construct** a labelled free-body diagram showing the forces acting on an object accelerating down a frictionless incline. (2 marks)

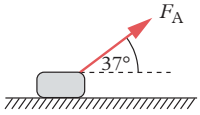
14 Diagrams **a** to **d** show a block on a horizontal or inclined plane. The block in each of parts **a** to **d** weighs 50 N. In parts **b** to **d** the applied force, F_A, is 20 N. **Calculate** the normal force in each case (2 s.f.). (2 marks each)

a

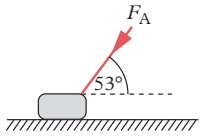
b



c



d



15 A computer of mass 2.5 kg rests on a table. **Calculate** the normal force exerted by the table on the computer. (2 marks)

Analytical processes

16 **Determine** how many times heavier by weight a person would be on Jupiter than on Earth. Note: $g_{Jupiter} = 25\,m\,s^{-2}$ (2 marks)

17 **Determine** the force necessary to uniformly accelerate the following objects (2 s.f.).

a A 6.4 kg mass at $2.4\,m\,s^{-2}$ (2 marks)

b An object weighing 25 N at $9.8\,m\,s^{-2}$ (2 marks)

c A 0.50 kg object from rest to $5.0\,m\,s^{-1}$ over 4.0 m (2 marks)

d A 75 kg object from $40\,m\,s^{-1}$ to $60\,m\,s^{-1}$ in 5 s (2 marks)

18 A butcher pulls on a 40 kg side of beef with a horizontal force of 220 N, and it slides across the boning table at constant speed.

a **Determine** the force of friction. (2 marks)

b The side of beef now hits a low friction area on the table. **Determine** what the butcher would notice about the motion of the beef if the applied force is kept the same. (1 mark)

19 A 30 kg box of vegetables moves down a 35° frictionless incline. **Determine** (2 s.f.) the

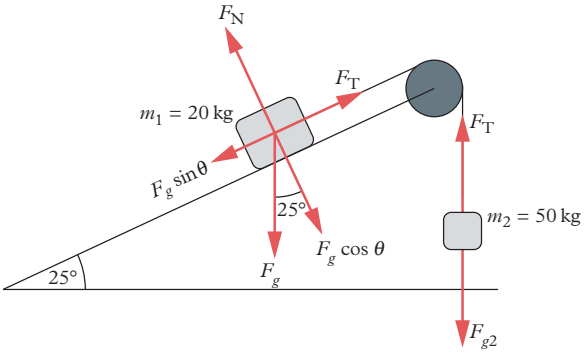
a component of the weight perpendicular to the incline (2 marks)

b component of the weight down the incline (2 marks)

c normal force (2 marks)

d acceleration down the incline. (4 marks)

20 A 20 kg object is attached by a thin cord to a 50 kg mass that hangs over a frictionless pulley at the top of a 25° incline, as shown in the diagram.

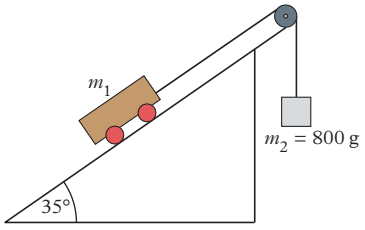


Determine (2 s.f.) the

a acceleration, if any, of the object (4 marks)

b tension in the string. (3 marks)

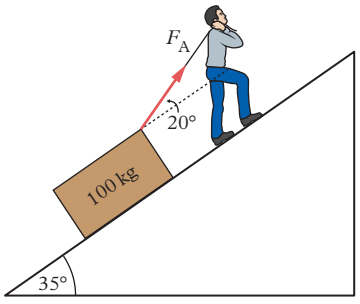
21 Consider the system shown in the diagram. The trolley has a mass of 1,000 g and moves up the incline at constant speed when the slope is 35° and under the conditions shown.



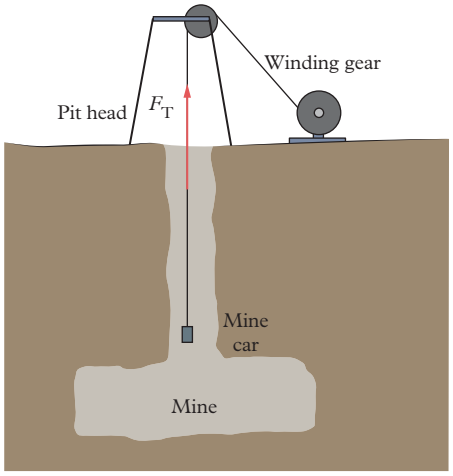
a **Determine** the frictional forces acting in this system (2 s.f.). (3 marks)

b **Calculate** the tension in the rope (2 s.f.). (2 marks)

22 A person hauls a box of mass 100 kg up a 35° incline by a rope attached to the top of the box, as shown in the diagram. The rope makes an angle of 20° to the incline and the friction between the box and the incline is 650 N. **Determine** the force applied by the person, F_A, to keep the box moving at constant speed (2 s.f.). (4 marks)



23 A cable used to pull mine cars vertically to the pit head has a breaking strain of 3.0×10^4 N. The mine shaft is 500 m deep, and a full mine car has a mass of 2.5 tonnes, as shown in the diagram.

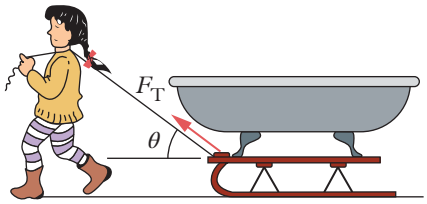


Determine (2 s.f.) the

a maximum acceleration of the car without breaking the cable (4 marks)

b shortest time in which the car can be pulled from rest to the surface in the event of an accident. (3 marks)

24 A person pulls a sled carrying a bathtub along a horizontal surface at constant speed, as shown in the diagram. The combined mass of the sled and bathtub is 75 kg, the frictional force is 70 N, and the angle θ of the rope to the horizonal is 42°. **Determine** the tension in the rope (2 s.f.). (3 marks)



Knowledge utilisation

25 A bag of cement is sliding down a 30° incline at constant speed. **Propose**, with reference to the scenario, whether each of the following relationships is true or false. **Justify** your responses.

a $F_g \sin 30° > F_f$ (2 marks)

b $F_f = F_g \sin 30°$ (2 marks)

c $F_N = mg \cos 30°$ (2 marks)

d $F_g = mg$ (2 marks)

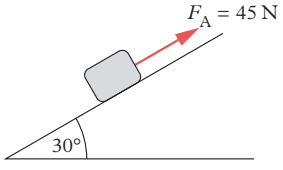
26 A horizontal steel cable is used to drag a bucket filled with coal along the ground at constant speed. The mass of the bucket and coal is 6.1 tonnes (6,100 kg) and the friction is 30,000 N.

a **Calculate** the normal force (3 s.f.). (2 marks)

b **Determine** the tension in the cable (3 s.f.). (1 mark)

c The tension in the cable is kept constant but the mass of the bucket increases as more coal is added. **Predict**, giving reasons, whether the bucket would continue at constant speed. (3 marks)

27 An 8 kg carton of soft drink is being pulled up a frictionless 30° incline using a rope and an applied force, F_A, of 45 N, as shown in the diagram. This applied force through the rope is the rope's tension, F_T.



a **Calculate** the acceleration, if any, up the incline (2 s.f.). (4 marks)

b **Decide** what the motion would be if the applied force was 40 N (2 s.f.). (2 marks)

28 Students were asked to investigate the relationship between the angle of elevation of a frictionless ramp and the parallel component of the weight of an object on the incline. They set up an experiment with a 1.2 m long aluminium ramp and varied its elevation through a range of angles from 10° to 50°. They used a Lego car of mass 75.5 g as the object on the ramp and measured the mass of a cup of coins and paper clips hanging freely over a pulley at the top of the ramp, which held the car stationary. The table shows their results.

Angle, θ (degrees)	10.0	20.0	30.0	40.0	50.0
Hanging mass, m_2 (g)	13.10	26.30	37.90	48.80	58.80

They hypothesised that the weight of the hanging mass would equal the parallel component of the car's weight down the incline for each angle. They planned to plot a graph to help confirm their thinking. **Analyse** their data and **evaluate** their hypothesis. (8 marks)

29 Students studying the acceleration of an object down a frictionless ramp used a Lego car of mass 56.0 g and let it run freely down a 1.0 m incline. They measured the time for the journey for several different angles of elevation. The table shows their results.

Angle, θ (degrees)	5	10	15	20	25
Time, t (s)	1.53	1.08	0.89	0.77	0.69

The students hypothesised that the acceleration down the incline should be as a result of the net force from the parallel component of the weight of the car. Draw a graph of acceleration (observed) on the vertical axis versus acceleration (theoretical) on the horizontal axis. If the formula for the trendline was $y = x$ with an R^2 of 1.00, they were going to claim their hypothesis had been confirmed. **Evaluate** their results. (2 marks)

Data drill

Effect of angle of elevation on acceleration

Students investigated how the angle of elevation of an inclined plane affects the acceleration of an object moving freely down it. They placed a toy car on an inclined plane 1.00 m long and allowed it to run freely down at various elevation angles. The motion was filmed and the time for each run was determined. Three trials were made for each angle. The results are shown in Table 1.

TABLE 1 Time and acceleration data for a toy car on an inclined plane at various angles

Angle, θ (degrees)	sin θ	Time taken for motion down the ramp, t (s)				Acceleration, a ($m\ s^{-2}$)
		Trial 1	Trial 2	Trial 3	Average	
10	0.17	1.18	1.22	1.23	1.21	**Q2**
30	0.50	0.70	0.70	0.69	0.70	4.1
50	0.77	0.56	0.55	0.56	0.56	6.5
70	0.94	0.51	0.51	0.52	0.51	7.6
90	1.00	0.50	0.50	0.50	0.50	8.0

A graph of acceleration versus the angle in degrees was found to be non-linear. A plot was then made of acceleration versus the sine of the angle (Figure 1).

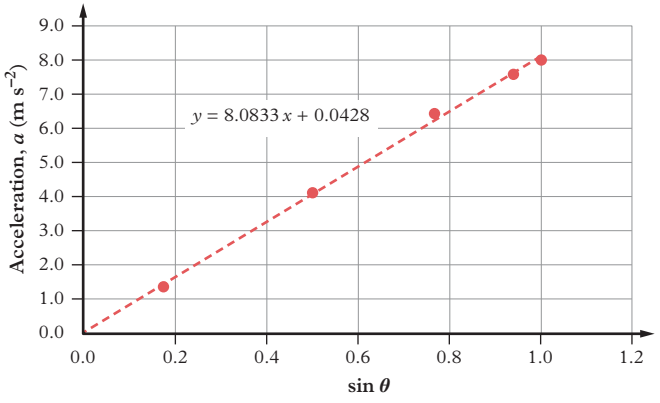


FIGURE 1 Plot of the data

Note: in the questions that follow, significant figures (s.f.) are mentioned as a guide but are not being examined.

Apply understanding

1 **Calculate** the absolute uncertainty for the time taken for an angle of 10° (1 s.f.). (1 mark)

2 **Determine** the acceleration for an angle of 10°. Show your working (1 s.f.). (2 marks)

Analyse data

3 **Identify** the relationship between the sin of the angle, θ, of the inclined plane to the horizontal and the acceleration, a, of the object down it. Use evidence from Figure 1 to support your answer. (3 marks)

Interpret evidence

4 **Draw a conclusion** that quantifies the value of the acceleration due to gravity, g, from the gradient of the graph. Show your reasoning. (2 marks)

5 **Deduce** a possible cause of the difference between the experimental value at 90° and the expected value. **Justify** your answer. (2 marks)

Module 3 checklist: Inclined planes

MODULE 4

Circular motion

Introduction

It really wasn't until the 1500s that people began to believe that Earth rotates on its own axis. Until then, the rate of rotation of objects was of little consequence. Today, rotation and its measurement is of fundamental importance to society, whether it is the rotation of microwave carousels, car tyres, engines, sewing machines, nuclei, planets or orbiting satellites.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to circular motion before you start.

Subject matter

Science understanding

- Apply vector analysis to resolve a vector into two perpendicular components.
- Solve vector problems by resolving vectors into components, adding or subtracting the components and recombining them to determine the resultant vector.
- Solve problems involving force due to gravity (weight) and mass using $F_g = mg$.
- Describe the concept of normal force.
- Describe the concept of uniform circular motion.
- Describe the concepts of average speed and period.
- Solve problems involving objects undergoing uniform circular motion at a constant speed using $v = \frac{2\pi r}{T}$ and $a_c = \frac{v^2}{r}$.
- Describe the concepts of centripetal acceleration and centripetal force.
- Solve problems involving forces acting on objects in uniform circular motion using $F_c = F_{net} = \frac{mv^2}{r}$.

Science inquiry

- Investigate the net forces acting on an object undergoing horizontal circular motion on a string.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Practicals

oxforddigital **This lesson is available on Oxford Digital.**

Lesson 4.4 Centripetal forces on an object undergoing horizontal circular motion

Lesson 4.1
Uniform circular motion

Key ideas

→ Uniform circular motion is the result of a force that acts on an object in a perpendicular direction to the velocity of the object.

What is uniform circular motion?

Learning intentions and success criteria

uniform circular motion
the motion of an object travelling at a constant speed in a circle

Circular motion is a simple idea – just moving around in a circle. To make it **uniform circular motion**, you need to be moving at constant speed as well. There are plenty of examples in daily life: whirling a toy plane around in a horizontal circle at constant speed (Figure 2), the carousel (platter) in a microwave oven, the spin dryer in a washing machine, a ceiling fan, and the second hand of a clock. It would be good to also add Earth orbiting the Sun, but it is not quite a circle and it is not quite constant in speed.

FIGURE 1 Uniform circular motion in a slowly rotating space station could be one way to simulate Earth's gravity in space.

Study tip

This module is concerned exclusively with uniform circular motion in a horizontal circle. Objects being whirled in vertical circles are outside the scope of this module.

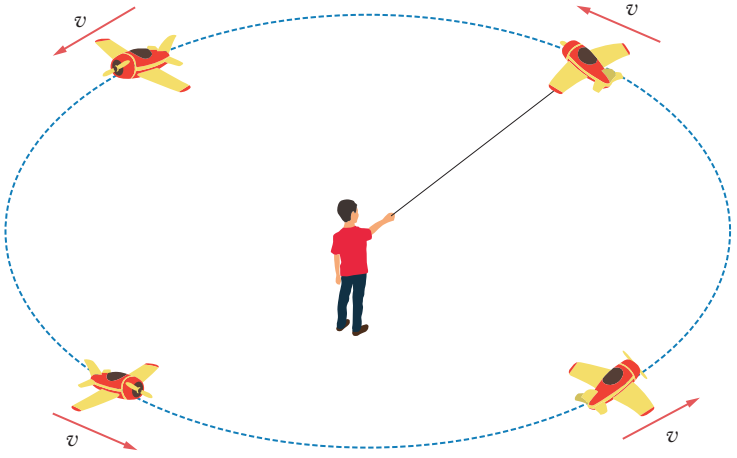


FIGURE 2 A child whirls a toy plane around in a horizontal circle.

Understanding what keeps an object in circular motion and what keeps the motion uniform relies on an understanding of the forces involved.

Challenge

Does a wheel ever rest?

When a wheel rolls along, is any point at rest? **Explain** your answer. (1 mark)

What is the difference between revolving and rotating?

Two similar terms are easy to get mixed up, and if you do this you may get the wrong answer. An object moving in a horizontal circle is said to be revolving or orbiting around the centre of the circle. So, a ball whirled on a string is said to revolve around the centre point at your hand. The similar term "rotating" means spinning around its own axis. So Earth rotates on its own axis once a day and revolves about the Sun once a year (Figure 3).

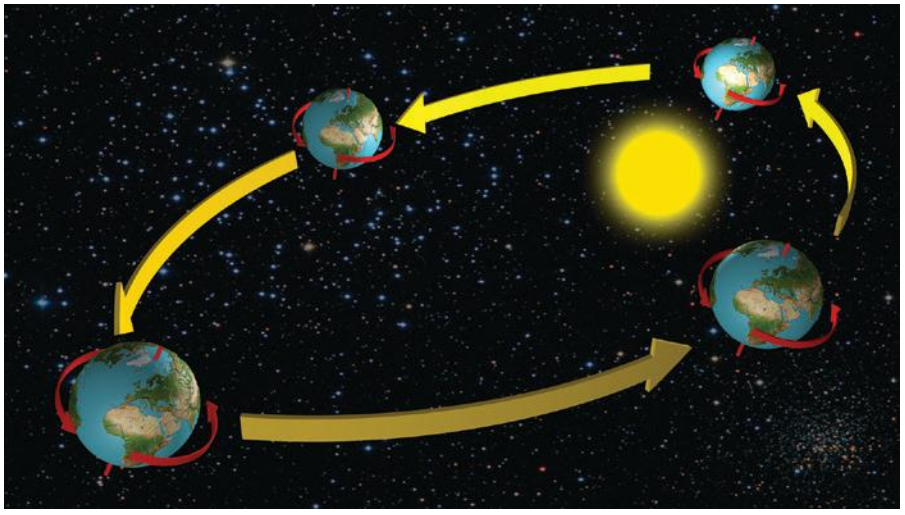


FIGURE 3 Earth revolves around the Sun, but it also rotates on its own axis. The viewpoint is from above the North Pole.

Challenge

Where did the term "clockwise" originate?

The hands of a clock go in a "clockwise" direction – but where did this term originate? The Sun moves across the sky in an anticlockwise direction as the day progresses, so why wasn't that direction "clockwise"? **Propose** a reason for this apparent contradiction. (2 marks)

How are forces and velocity involved in uniform circular motion?

Imagine you are whirling a ball in a horizontal circle above your head like the child with the toy plane shown in Figure 2. The ball is said to have **inertia**, and according to Newton's first law of motion, it would continue in a straight line with the same speed unless acted on by an unbalanced force.

inertia
the tendency of an object to resist change in its motion

centripetal force
the force acting on an object travelling in a circle that constantly either pulls or pushes the object towards the centre of motion (symbol: F_c; SI unit: newton; unit symbol: N)

centripetal acceleration
the acceleration experienced by any object moving in a circular path directed towards the centre of motion (symbol: a_c; SI unit: metres per second squared; unit symbol: $m\,s^{-2}$)

This force is provided by the pull on the string. As your hand is at the centre of the circle in which the ball moves, the force on the string, and hence on the ball, is always towards your hand and hence towards the centre. This force is called a **centripetal force** (from the Latin "centrum" meaning "centre" and "petere" meaning "to seek"), a centre-seeking force, and has the symbol F_c. If you are specifying it as a vector, you use the symbol $\vec{F}_c$.

The velocity at any point on the circle is a tangent to the path at that point. For instance, at position A in Figure 4, the velocity vector $\vec{v}_1$ points up the page. At point B, the velocity vector $\vec{v}_2$ points to the left but is drawn with the same length because the speed remains the same. As the direction of the velocity has changed, the ball is said to have **centripetal acceleration**. Remember that the length of the vector arrow indicates the magnitude, so the length of a velocity arrow gives the speed.

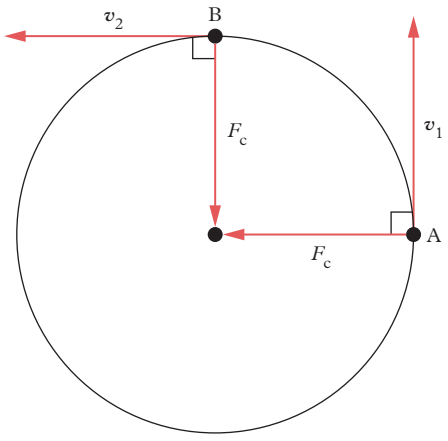


FIGURE 4 Direction of the velocity vectors in uniform circular motion

You should be able to observe that the centripetal force vector $\vec{F}_c$ is perpendicular to the direction of the velocity vector, $\vec{v}$, of the object.

Real-world physics

The Gravitron

In a Gravitron, or Rotor, at an amusement park, the riders feel "pressed" against the wall (Figure 5). Actually, each person is trying to travel in a straight line, but the wall pushes on them (the centripetal force) and the person pushes back. The centripetal force is the normal force on the rider from the wall directed radially inwards. At high speeds, this normal force is large enough to provide enough friction to stop the rider sliding down the wall under the influence of gravity. There is no force pressing the person against the wall. It is all because of inertia – the tendency for the person to travel in a straight line – as described by Newton's first law. For a Gravitron with a radius 8.0 m, a point on the edge travels 50 m ($2\pi r$) in one revolution. If that takes 2 seconds, then the person is going at $25\,m\,s^{-1}$, or $90\,km\,h^{-1}$.

FIGURE 5 The Gravitron uses Newton's first law to hold riders in place.

Apply your understanding

1 **Clarify** which force keeps you pressed against the wall of a Gravitron. (1 mark)
2 If you were covered in oil, would the Gravitron have to go faster or slower to keep you in position? **Explain** your answer. (2 marks)
3 A person in a Gravitron moves a distance of 137 m in 2.5 s in one revolution. **Determine** their speed in kilometres per hour. (3 marks)

Check your learning 4.1

Check your learning 4.1: Complete these questions online or in your workbook.

Retrieval and comprehension

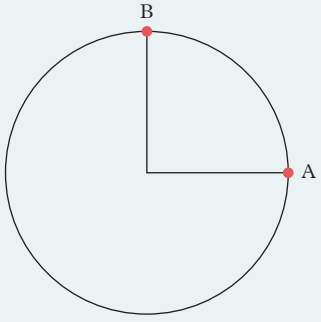
1 **Describe** uniform circular motion. (1 mark)

2 A ball is undergoing uniform circular motion on a fixed length of string. **Clarify** which of the following are zero after 100 revolutions: displacement, average velocity, average speed. (3 marks)

3 A ball is being whirled in a horizontal circle at uniform speed. This is shown from above in the diagram.

a **Identify** the direction of the ball's velocity vector at points A and B. (2 marks)

b **Identify** the direction of the centripetal force vectors at A and B. (2 marks)



4 The Moon moves in an almost uniform circular motion around Earth. **Identify** what is providing the centripetal force to keep the Moon revolving around Earth. (1 mark)

5 **Clarify** whether Earth's revolution around the Sun is considered to be uniform circular motion. (1 mark)

Analytical processes

6 A glass of water placed at the edge of the carousel (platter) in a microwave doesn't fly off as the carousel turns. **Consider** what keeps it there. (1 mark)

Knowledge utilisation

7 A spin dryer is an appliance used to remove water from clothes. It does this by spinning at high speeds, like the spin cycle of a washing machine.

a **Propose** why clothes that come out of a spin dryer still feel damp. (1 mark)

b **Predict** whether continued spinning at the same speed will get rid of more water. (1 mark)

c **Infer**, giving reasons, whether you could spin the clothes completely dry. (1 mark)

d **Identify** the force that pushes the water towards the outside of the tub and then out through the holes. (1 mark)

8 Small stones get caught in the treads of car tyres. **Propose** a reason why they stay there and are not flung out at high speeds. (1 mark)

9 Imagine you are measuring your weight on bathroom scales. It has been claimed that if Earth wasn't spinning, the reading on the scales would be different. **Assess** this claim with a reasoned explanation. (1 mark)

10 It is claimed that if Earth wasn't spinning, your height would be different. **Evaluate** this claim. (2 marks)

11 It has been said that if you measure your weight with bathroom scales at the equator, it would be less than if you measured it at the North or South Pole. The reason, it is said, is because there is less centripetal force at the poles due to the slower rotational speed. **Evaluate** this statement. (2 marks)

12 When going around a roundabout in a car, the front-seat passenger feels pressed against the door. People wonder whether they are moving towards the door or whether the door is moving towards them. **Assess** this situation and **propose** an answer. (2 marks)

Lesson 4.2
Objects undergoing uniform circular motion

Key ideas

- → The time for one revolution of an object in uniform circular motion is called the period, T.
- → The average speed for an object in uniform circular motion is the distance travelled in one period, T.
- → Problems involving objects undergoing uniform circular motion at constant speed can be solved using $v = \frac{2\pi r}{T}$.

What is average speed and period?

Learning intentions and success criteria

average speed
the rate of change of distance calculated by the formula average speed = $\frac{\text{distance}}{\text{time}}$; a scalar quantity (symbol: v; SI unit: metres per second; unit symbol: m s^{-1})

period
the time taken to complete one revolution calculated by the formula period = $\frac{\text{time}}{\text{no. of revolutions}}$ (symbol: T; SI unit: second; unit symbol: s)

tangential velocity
the linear velocity of an object undergoing circular motion, where the magnitude is the speed of the object, and the direction is a tangent to the circular path at that moment (directed towards the centre); for circular motion, the term is usually abbreviated to "velocity"

Calculating the **average speed**, v, of an object moving in a circle is no different from any other speed calculation: it is just the distance travelled per time of travel. The distance around a circle is equivalent to the circumference and calculated as $2\pi r$, where r is the radius of the circle. The time for one revolution around the circle is referred to as the **period** and has the symbol T. Thus, the average speed of an object in uniform circular motion is given by the expression:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}, \text{ or } v = \frac{2\pi r}{T}$$

This speed is sometimes called the "tangential speed" (Latin "tangere" meaning "to touch", as it touches the edge of the circle) to distinguish it from "rotational speed". It is also called **tangential velocity**.

FIGURE 1 A father spins his child in a horizontal circle. This is an example of uniform circular motion.

Worked example 4.2A

Calculating period, average speed and velocity in uniform circular motion

A student whirls a ball on a string at constant speed in a horizontal circle. The length of the string is 2.2 m and the time for one revolution is 3.1 s. **Calculate** (to an appropriate number of significant figures) the

a period (2 marks)

b average speed of the ball (2 marks)

c velocity of the ball. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather any data that you need.	$r = 2.2\,\text{m}$, $T = 3.1\,\text{s}$ $T = \frac{\text{time}}{\text{no. of revolutions}}$ $v = \frac{2\pi r}{T}$
Step 3: For part **a**, substitute values into the formula and solve for the period, T. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Determines the period")	**a** $T = \frac{\text{time}}{\text{no. of revolutions}}$ $= \frac{3.1}{1}$ (1 mark) $= 3.1\,\text{s}$ (1 mark)
Step 4: For part **b**, substitute values into the formula for velocity, v, and solve for speed (the magnitude of velocity). Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Calculates the speed")	**b** $v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 2.2}{3.1}$ (1 mark) $= 4.5\,\text{m s}^{-1}$ (1 mark)
Step 5: For part **c**, make a statement about the differences between speed and velocity for circular motion. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Describes the magnitude as being the same"; 1 mark for "Identifies the direction of speed and velocity")	**c** The magnitude of the velocity is the same the speed. The velocity is directed perpendicular to the string, in the direction of the motion at any instant. Therefore, the velocity, $v = 4.5\,\text{m s}^{-1}$ perpendicular to the string. (1 mark for magnitude; 1 mark for direction)

Your turn

A student whirls a ball on a string at constant speed in a horizontal circle. The time for one revolution is 1.8 s and the length of the string is 1.5 m. **Calculate** (to an appropriate number of significant figures) the

a period (1 mark)

b average speed of the ball (2 marks)

c velocity of the ball. (2 marks)

What are rotational speeds?

You probably don't know the speed of the Moon about Earth in metres per second or even kilometres per hour. But you know that it makes one revolution in just over 27 days. Internal combustion engine speeds too are usually expressed in a number of revolutions per minute (rpm). At idle, the engine might turn at 750 rpm and at cruising speed it may reach 4,000 to 6,000 rpm, which is shown on the vehicle's tachometer (Figure 2). These are called **rotational speeds**. An electric vehicle (EV) motor can reach 10,000 rpm or even 20,000 rpm, but there is no tachometer in an EV as it is irrelevant.

rotational speed the number of revolutions an object makes per second, as distinguished from the term "average speed", which is the linear speed

Study tip

For uniform circular motion, the term "average speed" means the linear speed in metres per second. The velocity has the same magnitude as the speed, with a direction tangential to the circle at that point.

However, when you say "average speed", you are talking about linear speeds in metres per second and not rotational speeds in revolutions per second.

Using rotational speeds

For now, rotational speeds are best converted into the time for one revolution. This is called the period, T. The simplest way to do this is:

$$T = \frac{\text{time}}{\text{no. of revolutions}}$$

This can then be used for any problem with rotational speeds, as shown in Worked examples 4.2B and 4.2C.

FIGURE 2 A tachometer in an internal combustion engine car is a "rev counter" that measures the engine speed in revolutions per minute (rpm). This one has a "red line" (maximum advisable engine speed) of 6,500 rpm.

Worked example 4.2B

Calculating period from revolutions per minute

Calculate the period in seconds for an object with a rotational speed of 30 rpm. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather any data that you need.	Rotational speed = 30 rpm $T = \frac{\text{time}}{\text{no. of revolutions}}$
Step 3: Substitute data into the formula and solve for the period, T. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Check that you have used the correct units. (1 mark for "Correctly substitutes values"; 1 mark for "Provides correct answer")	$T = \frac{\text{time}}{\text{no. of revolutions}}$ $= \frac{60\text{ s}}{30\text{ revolutions}}$ (1 mark) $= 2.0\text{ s}$ (1 mark)

Your turn

The "red line" of an engine is the maximum rpm at which it's safe to run. A particular motor bike engine "redlines" at 7,500 rpm. **Calculate** the period in seconds for this rotational speed. Give your answer to an appropriate number of significant figures. (2 marks)

Worked example 4.2C

Calculating the speeds of a rotating object

A car tyre has a diameter of 630 mm and is being rotated at 850 rpm on a tyre-balancing machine. **Determine** the average speed at which the outer edge of the tyre is moving. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather any data that you need. Note what quantity the question is asking for – in this case, average speed, which is in metres per second.	$d = 630\,\text{mm} = 0.63\,\text{m}$ Rotational speed = 850 rpm $T = \frac{\text{time}}{\text{no. of revolutions}}$ $v = \frac{2\pi r}{T}$
Step 3: Substitute the relevant data into the formula and solve for the period, T. (1 mark for "Demonstrates correct substitution"; 1 mark for "Calculates the period")	$T = \frac{\text{time}}{\text{no. of revolutions}}$ $= \frac{60\,\text{s}}{850\,\text{revolutions}}$ (1 mark) $= 0.07\,\text{s}$ (1 mark)
Step 4: Calculate the speed, v, of the outer edge of the tyre. The radius is half the diameter. (1 mark for "Demonstrates correct substitution")	$v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 0.315}{0.07}$ (1 mark) $= 28.3\,\text{m s}^{-1}$
Step 5: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	The average speed is $28\,\text{m s}^{-1}$ (2 s.f.). (1 mark)

Your turn

A circular power saw blade has a diameter of 184 mm and runs at a speed of 5,300 rpm. **Determine** the average speed in metres per second of the teeth on the outside of the blade. Give your answer to an appropriate number of significant figures. (4 marks)

Real-world physics

The centrifuge

The laboratory centrifuge is a device used in chemistry, biology, biochemistry and clinical medicine for isolating and separating suspensions and immiscible (non-mixing) liquids. A tube containing the suspension – for example, blood – is placed in a machine that spins it at a very high rate (Figure 3). Rotational speeds of 1,000–5,000 rpm are common. The centripetal acceleration causes denser substances and particles to move outwards to the bottom end of the tube, and at the same time, substances and particles that are less dense are displaced and move to the centre of rotation, which is the top of the tube. Blood, for instance, separates into cells, proteins and serum.

FIGURE 3 Blood samples are separated in a centrifuge. The red blood cells are heavier, so they move to the bottom.

Apply your understanding

1 Blood serum is the liquid component of blood. **Clarify** whether it moves towards the bottom or top of the tube. (1 mark)

2 **Calculate** the period, T, of a centrifuge tube spinning at 5,000 rpm (2 s.f.). (1 mark)

3 If blood cells accumulate in a position in a tube 13 cm from the centre, **determine** the speed of the cells in kilometres per hour (2 s.f.). (3 marks)

Worked example 4.2D

Factors affecting spin-dryer speeds

A spin dryer removes water from clothes by spinning at high speeds, like the spin cycle of a washing machine. It is suggested that you can increase the speed of a dryer by doubling the diameter or doubling the rotational speed. Dryer A has a tub radius of 50.0 cm and a rotational speed of 1,200 rpm, and dryer B has a radius of 100 cm and a speed of 600 rpm.

a **Determine** which dryer gives the higher speed. (4 marks)

b **Propose** which dryer would be preferred for a household laundry. **Justify** your response. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after a calculation. "Propose" means to put forward a point of view in a statement for consideration. "Justify" means to give reasons or evidence to support an answer. For the first part we must identify the appropriate formula and use it to find an answer.
Step 2: For part **a**, identify the appropriate formulas and gather any data that you require.	**a** $r_A = 0.500\,\text{m}$, rotational speed (A) = 1,200 rpm $r_B = 1.00\,\text{m}$, rotational speed (B) = 600 rpm $T = \frac{\text{time}}{\text{no. of revolutions}}$ $v = \frac{2\pi r}{T}$
Step 3: For spin dryer A, calculate the period, T_A, by substituting the known values into the formula. (1 mark for "Calculates the period")	$T_A = \frac{1}{1{,}200}$ minutes $= \frac{60}{1{,}200}$ seconds $= 0.050\,\text{s}$ (1 mark)
Step 4: For spin dryer A, calculate the speed, v_A, by substituting the known values into the formula. (1 mark for "Calculates the speed")	$v_A = \frac{2\pi r_A}{T_A}$ $= \frac{2\pi \times 0.50}{0.050}$ $= 62.8\,\text{m s}^{-1}$ (1 mark)
Step 5: For spin dryer B, calculate the period, T_B, by substituting the known values into the formula. No marks are awarded for calculating the period as it is similar to that for spin dryer A.	$T_B = \frac{1}{600}$ minutes $= \frac{60}{600}$ seconds $= 0.100\,\text{s}$
Step 6: For spin dryer B, calculate the speed, v_B, by substituting the known values into the formula. (1 mark for "Calculates the speed")	$v_B = \frac{2\pi r_B}{T_B}$ $= \frac{2\pi \times 1.00}{0.100}$ $= 62.8\,\text{m s}^{-1}$ (1 mark)
Step 7: Draw a conclusion that compares the results. (1 mark for "Provides correct answer")	They both have the same speed of $62.8\,\text{m s}^{-1}$. (1 mark)
Step 8: For part **b**, the only variable that is different is the radius, so we need to consider what effect the radius has. (1 mark for "Proposes a point of view based on evidence"; 1 mark for "Justifies the conclusion using reference to evidence about centripetal motion and forces")	**b** As they both have the same speed, the dryer with the smaller radius is preferred. It will have a greater change of velocity (centripetal acceleration) as it is moving in a smaller circle and so the change in direction will be greater in the same period of time. (1 mark) Thus, the clothes are being pulled away from the water with greater (centripetal) force, allowing the water to continue to move outward from the clothes. (1 mark)

Your turn

A car driver has tyres of diameter 531 mm that rotate at 1,028 rpm at highway speeds. The driver changes them for bigger ones with a diameter of 594 mm, thinking they would rotate at a lower speed and help the driver save money on petrol. **Calculate** (to the nearest whole number) how many rpm the new tyres would do at the same highway speed. (4 marks)

Challenge

Motion of a billy

An old camper's trick to make tea leaves settle to the bottom of a billy of tea is to swing the billy in a vertical circle, as shown in Figure 4.

Consider the following questions about this idea.

a **Explain** why this would make the leaves settle faster. (1 mark)

b **Explain** whether this is an example of uniform circular motion. (1 mark)

c **Deduce** whether this method would work if the billy was swung in a horizontal circle. (1 mark)

d **Sketch** an approximate graph of speed versus angle for one complete revolution of 360°, with directly overhead being 0°. (1 mark)

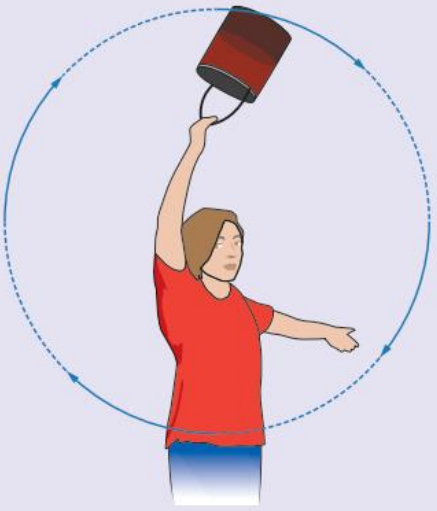


FIGURE 4 Swinging a billy of tea in a vertical circle is said to make the leaves settle faster.

Check your learning 4.2

Check your learning 4.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how to calculate the period of revolution from the number of revolutions per minute. (1 mark)

2 **Describe** one practical use of a device that utilises uniform circular motion, and the typical rotational speeds involved. (2 marks)

3 A Gravitron ride moves in a horizontal circle of 15.0 m radius and completes 24 turns every minute. **Calculate** its average speed. (2 marks)

Analytical processes

4 **Distinguish** between average speed and rotational speed. (1 mark)

5 A car of mass 2,250 kg is travelling around a circular track of radius 90.0 m at a constant speed of 30.0 $m\,s^{-1}$. **Determine** how many revolutions per minute the driver makes of the track. (2 marks)

6 The Large Hadron Collider (LHC) in Switzerland accelerates particles such as protons to very high speeds and then smashes them into each other. The LHC consists of a circular ring 4.25 km in diameter, as shown in the diagram, in which protons travel at close to the speed of light. **Determine** how many revolutions per minute a proton would make at a speed of $2.0 \times 10^8\,m\,s^{-1}$. (3 marks)

Knowledge utilisation

7 You are riding a bicycle at 20 $km\,h^{-1}$ and the wheels are spinning very fast. Bike tyres are typically 67 cm in diameter. **Assess** whether you have sufficient information to work out the rotational speed of your tyre and, if so, justify the logic used to calculate it. (3 marks)

8 Spin dryers go pretty fast – too fast to see with the naked eye. **Propose** a method to measure the speed of a spin dryer in revolutions per minute. You don't have to build it or have the parts at home or school; just design the procedure and instrumentation. (2 marks)

Lesson 4.3
Centripetal acceleration and force

Key ideas

- An object travelling in a circle at constant speed has an acceleration, called centripetal acceleration, a_c, directed towards the centre of the circular path and perpendicular to the velocity vector.
- Problems involving objects undergoing uniform circular motion at a constant speed can be solved using $a_c = \frac{v^2}{r}$.
- The centripetal force is the net force directed towards the centre of a circular path: $F_c = F_{net}$.
- Problems involving forces acting on objects in uniform circular motion can be solved using $F_c = F_{net} = \frac{mv^2}{r}$.

Learning intentions and success criteria

What is centripetal acceleration?

To keep a ball revolving in a horizontal circle you have to apply a force with your hand. This is an unbalanced force directed inwards so the ball must be accelerating inwards according to Newton's second law (Figure 1). If the ball is accelerating, then, by definition, it must be undergoing a change in velocity. It can change its velocity without changing its speed if it changes direction. This section is about how a centripetal force causes centripetal acceleration, which causes a change in velocity even though the speed remains constant.

The formulas relating centripetal acceleration and force to the average speed of an object travelling in uniform circular motion are quite complicated, as they involve vectors in two dimensions. First, you need to describe how subtracting vectors is different from adding them, which you did in the previous two modules. You need to subtract because acceleration is a change in velocity, and "change" means subtraction.

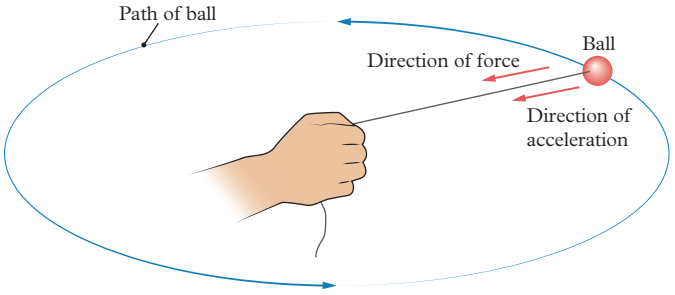


FIGURE 1 Path of a ball undergoing horizontal circular motion

How do you subtract vectors to find the change in velocity?

If you were riding your bicycle with an initial speed of $10\,km\,h^{-1}$ going downhill and had a final speed of $15\,km\,h^{-1}$, you would have gained $5\,km\,h^{-1}$ of speed. Your "change of speed" would be $+5\,km\,h^{-1}$:

$$\text{change in measurement} = \text{final measurement} - \text{initial measurement}$$

$$\text{change in speed} = \text{final speed} - \text{initial speed}$$

$$\Delta\vec{v} = \vec{v} - \vec{u}$$

Conversely, if you slowed down by $5\,\text{km h}^{-1}$ your change in speed would be $-5\,\text{km h}^{-1}$. The symbol for the Greek capital "D" (delta, Δ) is used to represent "difference". This is easy to remember as they both start with the letter "D".

This is simple for scalar quantities such as speed, mass, temperature and bank balances. But in physics it is also necessary to work with vector quantities. You have seen how to add vectors in two dimensions; now you will learn how to subtract vectors in two dimensions as well.

In maths, you will have learnt that subtraction is the same as adding a negative; that is, 15 – 10 is equivalent to 15 + –10 and the answer is +5 either way. Thus, when performing the subtraction vector A – vector B, the direction of vector B is changed to its opposite and then added to vector A head to tail; that is, $\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$ (Figure 2).

A more complicated situation arises when subtracting vectors in two dimensions, as you have with horizontal circular motion. You will start with the familiar example of a person on a bicycle changing direction (Worked example 4.3A).

You will need this technique for analysing the change in velocity vectors for uniform circular motion.

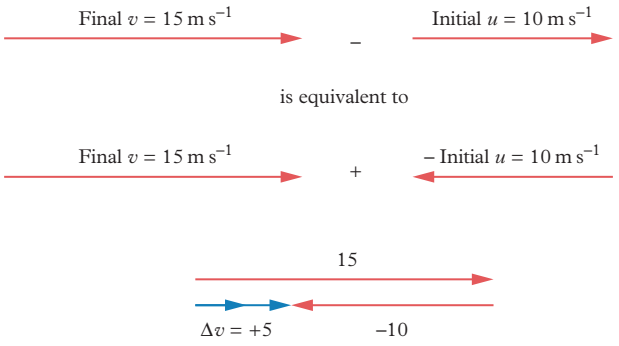
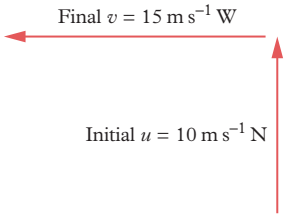


FIGURE 2 The change in velocity is found by subtracting vectors.

Worked example 4.3A

Calculating a change in velocity in 2D

A girl on a bicycle is travelling north at $10\,\text{m s}^{-1}$. She turns left and continues at $15\,\text{m s}^{-1}$ west. **Construct** a vector diagram and **calculate** the change in velocity. Give your answer to an appropriate number of significant figures. (7 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the question is asking you to do.	"Construct" means to display in a diagrammatic or logical form. "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather any data that you need.	$u = 10\,\text{m s}^{-1}\,\text{N}$; $v = 15\,\text{m s}^{-1}\,\text{W}$ $\Delta v = v - u$
Step 3: Substitute values into the formula. (1 mark for "Demonstrates correct substitution")	$\Delta v = v - u$ $= 15\,\text{m s}^{-1}\,\text{W} - 10\,\text{m s}^{-1}\,\text{N}$ (1 mark)
Step 4: Change the vector subtraction to an addition of the negative. (1 mark for "Provides appropriate mathematical reasoning")	$\Delta v = 15\,\text{m s}^{-1}\,\text{W} + (-10\,\text{m s}^{-1}\,\text{N})$ $= 15\,\text{m s}^{-1}\,\text{W} + 10\,\text{m s}^{-1}\,\text{S}$ (1 mark)
Step 5: Draw a vector diagram of the addition by putting the vectors head to tail. (1 mark for "Constructs a correct vector diagram of the relevant quantities being combined")	 (1 mark)

Think	Do
Step 6: Combine vectors to give a resultant by drawing an arrow from the tail of the first vector to the head of the second vector. (1 mark for "Constructs a correct vector diagram of the relevant quantities showing the resultant vector")	Final $v = 15\,\text{m s}^{-1}$ W θ − Initial $u = 10\,\text{m s}^{-1}$ S $\Delta v = v + (-u)$ $= 18\,\text{m s}^{-1}$ $\theta = 34°$ (1 mark)
Step 7: Calculate the magnitude of the resultant (the change in velocity) using Pythagoras' theorem. Mark the angle between the final vector and the resultant as θ and use trigonometry to solve for θ. (You could mark the other angle as θ instead, as long as it is clearly marked. In that case, θ would be the complement of 34°, i.e. 56°.) (1 mark for "Calculates the magnitude of the change in velocity"; 1 mark for "Calculates the angle correctly")	$\Delta v = \sqrt{10^2 + 15^2}$ $= 18\,\text{m s}^{-1}$ (1 mark) $\theta = \tan^{-1}\frac{10}{15}$ $= 34°$ (1 mark)
Step 8: State the value of the resultant vector (the change in velocity) in terms of direction (angle) and magnitude. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. If you have chosen to call the other interior angle θ, your answer would be 56° W of N. As long as this is shown clearly on the diagram and you describe the change in velocity correctly, your answer will be correct. (1 mark for "Provides correct answer")	The change in velocity is $18\,\text{m s}^{-1}$ at an angle of 34° S of W. (1 mark)

Your turn

A cricket ball is travelling at $40.0\,\text{m s}^{-1}$ south. After being struck by a bat, its velocity changes to $30.0\,\text{m s}^{-1}$ west. **Construct** a vector diagram and **calculate** the change in velocity. Give your answer to an appropriate number of significant figures. (6 marks)

How is centripetal acceleration calculated?

You will now use your techniques for dealing with "change of" velocity vectors to analyse uniform circular motion for acceleration (Worked example 4.3B). Consider a ball being whirled around in a horizontal circle (Figure 3). Again, you use the term "revolving" for this motion.

The velocity at any point on the circle is a tangent to the path at that point. For instance, at position A, the initial velocity vector $\vec{u}$ points up the page. At point B, the final velocity vector $\vec{v}$ points to the left but it has the same length as the initial velocity, as the speed remains the same. However, as the direction of the velocity has changed, the ball is said to be accelerating (centripetal acceleration).

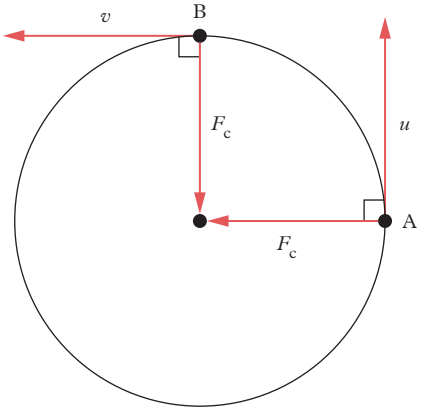


FIGURE 3 A free-body diagram showing the forces acting on a revolving object.

The formula for acceleration is, as always:

$$a = \frac{\text{change in velocity}}{\text{time}}$$
$$= \frac{v - u}{t}$$
$$= \frac{\Delta v}{t}$$

However, remember these velocities are in two dimensions now.

The magnitude and direction of this acceleration can be calculated by determining the change in velocity (Figure 4A):

Change in velocity (Δv) = final velocity (v) – initial velocity (u).

Reversing the direction of u and adding it to v: $\Delta v = v + (-u)$.

As can be seen in Figure 4B, the resultant is directed towards the centre of the circle, hence it is described as "centre seeking".

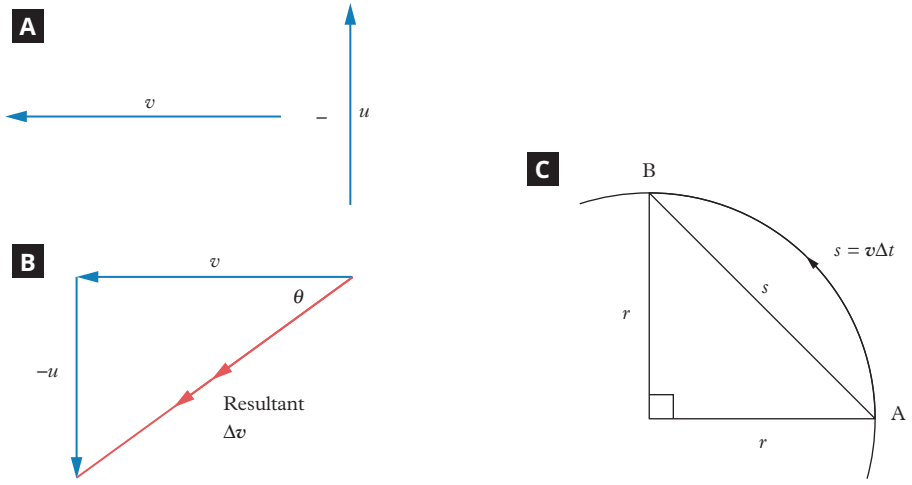


FIGURE 4 Subtraction of vectors gives a resultant that points towards the centre.

If the time taken for the ball to go from position A to position B is Δt, then the distance s is $v\Delta t$ (Figure 4C). Let the chord $s = v\Delta t$ (if s is short enough, this is a reasonable approximation). Using similar triangles:

$$\frac{\Delta v}{v} = \frac{s}{r}$$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Thus, the centripetal acceleration, a_c, is given by:

$$a_c = \frac{v^2}{r}$$

where r is the radius of the circular path in metres. The acceleration vector is towards the centre, whereas the velocity vector is perpendicular to this.

But it is a bit surprising that a_c is proportional to v^2. This implies, for example, that if you double the speed by going around a curve at 100 km h^{-1} instead of 50 km h^{-1}, it means there is four times the acceleration and hence four times the force required. As a_c is proportional to $\frac{1}{r}$, a small radius means a greater value of a_c, so a_c is greater for tighter turns.

Worked example 4.3B

Calculating speed and acceleration for circular motion

A rubber stopper of mass 20.0 g on a piece of string 1.5 m long is whirled in a horizontal circle at a uniform speed of 2.0 revolutions per second (rps). **Calculate** (to an appropriate number of significant figures) the

a average speed (3 marks)

b centripetal acceleration. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: For part **a**, identify the appropriate formulas and gather any data that you need.	**a** $r = 1.5\,\text{m}$, rotational speed = 2 rps $T = \frac{\text{time}}{\text{no. of revolutions}}$, $v = \frac{2\pi r}{T}$
Step 3: Substitute values into the formula and solve for the period, T. (1 mark for "Calculates the period")	$T = \frac{\text{time}}{\text{no. of revolutions}}$ $= \frac{1\,\text{s}}{2.0\text{ revolutions}}$ $= 0.50\,\text{s}$ (1 mark)
Step 4: Substitute values into the formula for velocity, v, and solve for v. (1 mark for "Calculates the velocity")	$v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 1.5}{0.50}$ $= 18.8\,\text{m s}^{-1}$ (1 mark)
Step 5: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	The average speed is $19\,\text{m s}^{-1}$ (2 s.f.). (1 mark)
Step 6: For part **b**, identify the formula for centripetal acceleration and substitute in the known values. Note that it is best to use the calculated value for the magnitude of the velocity before rounding. Make the calculation. (1 mark for "Demonstrates correct substitution"; 1 mark for "Calculates the magnitude of the acceleration")	**b** $a_c = \frac{v^2}{r}$ $= \frac{18.8^2}{1.5}$ (1 mark) $= 237\,\text{m s}^{-2}$ (1 mark)
Step 7: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	The centripetal acceleration is $240\,\text{m s}^{-2}$ towards the centre of the circle. (1 mark)

Your turn

A 50.0 g brass mass on a piece of string 75 cm long is whirled in a horizontal circle at a uniform speed of 5.0 revolutions per second (rps). **Calculate** (to an appropriate number of significant figures) the

a average speed (3 marks)

b centripetal acceleration. (3 marks)

Challenge

Coin revolution

How many revolutions will coin A make while rotating around coin B (Figure 5)? Try it. You will be surprised! The theory behind it is mind boggling so do not look it up – you might be sorry you did! (2 marks)

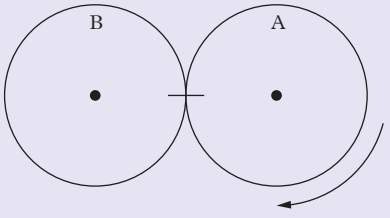


FIGURE 5 Coin A is revolving while it rotates around coin B.

Challenge

Radians and rotational speed

The radian per second (symbol: rad s^{-1}) is the SI unit of rotational speed, commonly denoted by the Greek letter omega (ω). From your maths classes, you may know that there are 2π radians in a circle of 360°. Engineers use the radians per second, for example, when working out the power of rotating shafts. **Determine** how many revolutions per second is 1 rad s^{-1}. (2 marks)

What factors affect centripetal force?

A ball whirled in a horizontal circle at uniform speed is experiencing a centripetal force (F_c) to keep it moving in a circle. This is provided by the tension (F_{net}) in the string. Using Newton's second law of motion ($F = ma$):

$$F_c = m \times a_c$$

$$F_c = F_{net} = \frac{mv^2}{r}$$

Note that $F_{net} = F_c$ only for horizontal circular motion. It doesn't apply for vertical circular motion – but you are not doing that here. Note also that the downward force of gravity on the ball is not considered for the simple introductory example in Worked example 4.3C. In reality, the ball would not travel in a circle in the same plane as the centre of motion. That is dealt with in Worked example 4.3D.

Worked example 4.3C

Calculating centripetal force and tension

A rubber stopper of mass 20.0 g is whirled in a horizontal circle at constant speed on a string that is 1.5 m long. The average speed is 18.8 m s^{-1}. **Calculate** the tension in the string due to centripetal motion. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather any data that you need. For horizontal circular motion, $F_{net} = F_c$.	$v = 18.8\,m\,s^{-1}$ (average speed), $r = 1.5\,m$, $m = 20.0\,g = 0.0200\,kg$ $F_c = F_{net} = \frac{mv^2}{r}$
Step 3: Substitute values into the formula for centripetal force, F_c, and solve for F_c. (1 mark for "Demonstrates correct substitution")	$F_c = F_{net}$ $= \frac{mv^2}{r}$ $= \frac{0.0200 \times 18.8^2}{1.5}$ (1 mark) $= 4.7\,N$
Step 4: The tension in the string provides the centripetal force on the rubber stopper. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	The centripetal force is 4.7 N directed towards the centre of the circle (the direction is needed as it is a vector). Therefore, the tension in the string is 4.7 N (no direction needed for tension). (1 mark)

Your turn

A 50.0 g brass mass on a piece of string 75 cm long is whirled in a horizontal circle at a uniform speed of 23.6 m s^{-1}. **Calculate** the tension in the string. Give your answer to an appropriate number of significant figures. (2 marks)

How do we analyse horizontal circular motion with gravity?

In Worked example 4.3C, we treated the ball as if it were rotating in a horizontal circle in the same plane as the centre of rotation (your hand). In practice, gravity pulls downwards on the ball so that it makes a small angle to the plane containing the centre of rotation. To make the ball stay in the same plane as the centre of rotation would require an infinite centripetal force from the tension in the string, so that doesn't happen.

In Worked example 4.3D, we analyse horizontal circular motion with the string at a downwards angle because of the force of gravity on the ball.

Worked example 4.3D

Analysing circular motion for a ball on a string angled downwards

During an experiment, a ball of mass 125 g tied to a string of length 2.00 m was swung in a horizontal circle at constant speed. The string made an angle of 75.0° to the vertical as shown in Figure 6. Draw a free-body diagram of the forces and **determine** (to an appropriate number of significant figures) the

a tension in the string (4 marks)
b centripetal force on the ball (2 marks)
c radius of the circle (2 marks)
d speed of the ball (2 marks)
e period of one revolution. (2 marks)

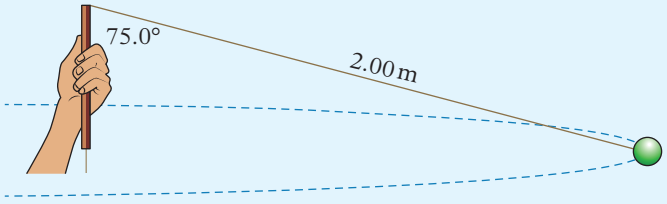
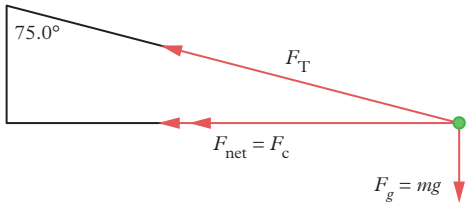
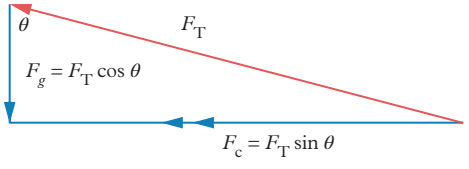


FIGURE 6 Ball on a string at an angle travelling in a horizontal circle

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We will need to find a numerical answer using mathematical processes and showing working. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: For part **a**, draw a labelled free-body diagram of all forces acting on the ball to help you resolve the forces. From the diagram, you can see that the vertical component of the tension, F_T, equals the weight, F_g, and the horizontal component of the tension is the net force, F_{net}, is the centripetal force, F_c, and points towards the centre of the circle. (1 mark for "Constructs a correctly labelled free-body diagram of the relevant quantities")	**a**  (1 mark)
Step 3: Draw a vector diagram showing the addition of forces. Resolve the tension into two components at right angles (horizontal and vertical). Use trigonometry to work out the relationships. (1 mark for "Constructs a correct vector diagram showing the vertical and horizontal components of tension")	 (1 mark)

Think	Do
Step 4: Select the formula that shows the relationship between the tension and the weight. Replace F_g with mg. We know all values except tension. Substitute values and solve for tension. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$F_g = F_T \cos\theta$ $mg = F_T \cos\theta$ $F_T = \frac{mg}{\cos\theta}$ $= \frac{0.125 \times 9.8}{\cos 75.0°}$ (1 mark) $= 4.733\,\text{N}$ $= 4.7\,\text{N}$ (2 s.f.) (1 mark)
Step 5: For part **b**, select the equation that shows the relationship between tension and the centripetal force. We know all values except F_c. Substitute values and solve for F_c. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**b** $F_c = F_T \sin\theta$ $= 4.733 \times \sin 75°$ (1 mark) $= 4.572\,\text{N}$ $= 4.6\,\text{N}$ (2 s.f.) (1 mark)
Step 6: For part **c**, look at the first diagram showing forces. Let the hypotenuse be the length of the string (2.00 m) and use trigonometry to solve for the radius. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**c** $r = 2.00 \times \sin 75°$ (1 mark) $= 1.932\,\text{m}$ $= 1.9\,\text{m}$ (2 s.f.) (1 mark)
Step 7: For part **d,** we have the value for centripetal force so we can substitute this value into the appropriate equation and solve for v. The question asks for speed, so you don't need to include the direction. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**d** $F_c = \frac{mv^2}{r}$ $v = \sqrt{\frac{F_c r}{m}}$ $= \sqrt{\frac{4.572 \times 1.932}{0.125}}$ (1 mark) $= 8.406\,\text{m s}^{-1}$ $= 8.4\,\text{m s}^{-1}$ (2 s.f.) (1 mark)
Step 8: One revolution around the centre traces out a path of length equal to the circumference of the circle ($2\pi r$). The time taken for one revolution is the period, T. Thus, velocity is the circumference divided by the period. Rearrange and solve for T. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**e** $v = \frac{2\pi r}{T}$ $T = \frac{2\pi r}{v}$ $= \frac{2\pi \times 1.932}{8.406}$ (1 mark) $= 1.444\,\text{s}$ $= 1.4\,\text{s}$ (2 s.f.) (1 mark)

Your turn

A ball of mass 150 g tied to a string of length 1.50 m was swung in a horizontal circle at constant speed. The string made an angle of 80.0° to the vertical. **Determine** (to an appropriate number of significant figures) the

a tension in the string (4 marks)

b centripetal force on the ball (2 marks)

c radius of the circle (2 marks)

d speed of the ball (2 marks)

e period of one revolution. (2 marks)

Skill drill

How many trials and repetitions are sufficient?

Science inquiry skills: Planning investigations (Lesson 1.4)

It is difficult to know how many replicates and data points are "sufficient" for reliable evidence. In the Skill drill for the previous module, you were asked to identify whether the data presented met the criteria for "sufficient". In this Skill drill we are building on this by considering how many trials and repetitions you should plan for different experiments.

Almost all the published studies on the subject (e.g. Taira et al., 2023) use a minimum of three data points (trials) as a general rule but agree that an experiment becomes more reliable only when the data points are five or more. In some cases where high reliability is required, they say you could use more, even as many as 18. However, five is the standard. In their words, you must collect a minimum of five data points to enable valid curve fitting.

If you can't get five for whatever reason, then you can make your experiment more reliable by increasing the number of repetitions. The advice is to do a minimum of three repetitions. You could then ask, "Do I have to do repetitions for all trials?" The researchers say it is still highly recommended to repeat the experiments for all trials, not just some. The key point when you design your own experiment is to plan 5 trials × 3 repetitions, but then you should consider all the factors about your experiment that may change that and justify this in your methodology.

Source: Taira, K., McInnes, D., & Zhang, L. (2023). How many data points and how large an *R*-squared value is essential for Arrhenius plots? *Journal of Catalysis, 419*, 26–36.

Practise your skills

1 A student wants to investigate centripetal motion and what effect changing the mass of the rubber stopper has. All they have is three rubber stoppers of different sizes, so their methodology calls for 3 trials × 3 repetitions. **Propose** a change to make this more reliable. (2 marks)

2 Students doing a centripetal motion experiment with a rubber stopper whirled horizontally on a string notice that the string makes an angle of less than 90° with the vertical. To overcome this problem, they do 10 repetitions instead of three for each trial. **Evaluate** whether this is a suitable approach for improving reliability. (2 marks)

3 In a magnetism experiment, a student has developed a research question that asks if Earth's magnetic field is greater at noon when the Sun is high than in the morning. They do two trials: one at 9 am and one at noon. To overcome the small number of trials, they decide to do 10 repetitions of each. **Propose** how this could be modified to make conclusions more reliable and valid. (2 marks)

How does a conical pendulum work?

Study tip

Sometimes, when drawing vector diagrams, you end up with a "wrong start" that does not give you any useful formulas to compare. Do not give up. Try other vector diagrams and you might get one that works.

You may have seen a swing ride at an amusement park where people sit in seats attached to long chains, and they are swung around in circles. You probably know it as a Chair-o-Plane. The photo at the start of this module shows one in action. It is similar to a conical pendulum. A conical pendulum is a mass attached to a nearly massless string that is held at the opposite end and swung in horizontal circles (Figure 7). During the 1800s, conical pendulums were used in clocks where a smooth motion was required, as opposed to the unavoidably jerky motion provided by ordinary pendulums.

If we want a particular period, T, and radius, r, for a pendulum, we can work out the velocity, v, the angle, θ, and the length of the string, L, required in this case. Questions like this (Worked example 4.3E) really test out your understanding of the vector nature of circular motion.

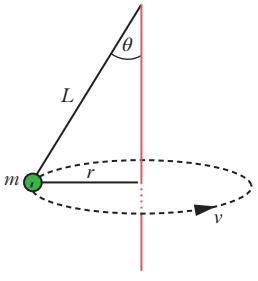


FIGURE 7 A conical pendulum of mass m travelling anticlockwise at constant speed v in a horizontal circle of radius r, on a string of length L making an angle θ with the supporting rod shown in red

Worked example 4.3E

Determining the motion of a conical pendulum

A conical pendulum is to be constructed with a radius of 25 cm and ball mass of 20.0 g travelling at constant speed in a horizontal circle that will produce a period of revolution of 1.00 s.

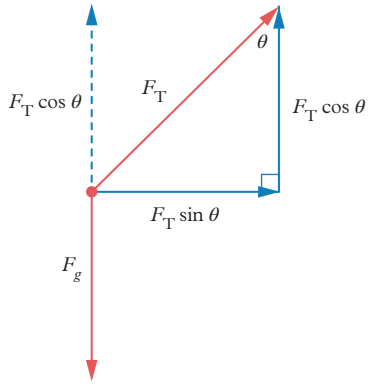
Determine, using free-body diagrams where appropriate (to an appropriate number of significant figures), the

a speed of the revolving ball (2 marks)

b centripetal force on the ball (2 marks)

c angle of the string with respect to the vertical (7 marks)

d length of the string. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We will need to find a numerical answer using mathematical processes and showing working. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Select the formula for the velocity of objects moving in a circle and substitute in the known values to solve for v, the magnitude of which is the speed. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Correctly substitutes values"; 1 mark for "Provides correct answer")	**a** $v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 0.25}{1.00}$ (1 mark) $= 1.257\,\text{m s}^{-1}$ $= 1.3\,\text{m s}^{-1}$ (2 s.f.) (1 mark)
Step 3: Select the formula for centripetal force, F_c, and substitute in the known values. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**b** $F_c = \frac{mv^2}{r}$ $= \frac{0.020 \times 1.571^2}{0.25}$ (1 mark) $= 0.197\,\text{N}$ $= 0.20\,\text{N}$ (2 s.f.) (1 mark)
Step 4: For part **c**, we need to find the angle θ. We need to draw a labelled free-body diagram showing the vertical and horizontal components of the tension force and gravitational force (weight) acting on the ball. This is called resolving or "decomposing" the forces into vertical and horizontal components. (1 mark for "Constructs a correct free-body diagram of the relevant quantities"; 1 mark for "Uses vector resolution to determine the components")	**c**  (1 mark for weight vector F_g) (1 mark for resolution of tension into correct vertical and horizontal components)
Step 5: We can see from the free-body diagram that the vertical vectors are equal as there is no acceleration in the vertical direction. Thus, we can make F_T the subject of the equation in terms of $\cos\theta$ and F_g (or mg). (1 mark for "Identifies the relationship between tension, weight and centripetal force")	$F_T \cos\theta = F_g$ $F_T \cos\theta = mg$ $F_T = \frac{mg}{\cos\theta}$ (1 mark)

Think	Do
Step 6: Now do the same with the horizontal vectors. This gives us F_T in two simultaneous equations. (1 mark for "Provides appropriate mathematical reasoning")	$F_T \sin\theta = F_c$ $F_T = \frac{F_c}{\sin\theta}$ (1 mark)
Step 7: Equate the two equations as they both are equal to F_T. We can use the trigonometric ratio $\frac{\sin\theta}{\cos\theta} = \tan\theta$ to simplify it, so we can now solve for θ. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$\frac{F_c}{\sin\theta} = \frac{mg}{\cos\theta}$ (1 mark) $\frac{\sin\theta}{\cos\theta} = \frac{F_c}{mg}$ $\tan\theta = \frac{0.197}{0.020 \times 9.8}$ (1 mark) $\theta = \tan^{-1} 1.005$ $= 45.1°$ $= 45°$ (2 s.f.) (1 mark)
Step 8: We can see from the diagram that as we have the angle, θ, and the opposite side of the right-angled triangle, r, we can work out the hypotenuse (length of string, L). Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "States correct relationship between length, radius and angle"; 1 mark for "Provides correct answer")	**d** $\sin\theta = \frac{r}{L}$ $L = \frac{r}{\sin\theta}$ (1 mark) $= \frac{0.25}{\sin 45.1}$ $= 0.353\,\text{m}$ $= 0.35\,\text{m}$ (2 s.f.) (1 mark)

Your turn

A conical pendulum similar to Figure 7 has a 35 g brass ball attached to string and revolves in a horizontal circle of radius 1.00 m at constant speed. It has a period of 2.00 s.

Determine, using free-body diagrams where appropriate (to an appropriate number of significant figures), the

a speed of the revolving ball (2 marks)

b centripetal force on the ball (2 marks)

c angle of the string with respect to the vertical (7 marks)

d length of the string. (2 marks)

What provides the centripetal force in a circular racetrack?

A racing car travelling around a circular track is similar to a ball being whirled around on a string. A vehicle travelling around a bend on a level road can be viewed also as going on a circular path. The sideways friction between the tyres and the road provides the force needed to prevent the car from just going straight ahead. The friction provides the centripetal force. If the car was to hit a wet patch all of a sudden, the friction would be reduced and insufficient centripetal force could be provided, so the car would tend to go straight ahead, possibly even spinning out of control.

FIGURE 8 The tyres on these racing cars have to provide the centripetal force to enable the cars to follow the circular path.

The maximum safe speed to go around a curve is when the car's average speed requires a centripetal force equal to the friction. For a car with rubber tyres on a dry bitumen road, friction is typically equal to 70 per cent (0.7 times) of the car's weight. (For a wet road the friction is closer to 50 per cent of the weight, and on an oil patch it is as low as 5 per cent.)

We can equate the centripetal force and frictional force, and use this to find an expression for the maximum safe speed:

$$F_c = F_f$$
$$\frac{mv^2}{r} = 0.7mg$$
$$v^2 = 0.7gr$$
$$v = \sqrt{0.7gr}$$

As you can see, the mass term, m, is on both sides of the equation, so it cancels out. Big cars have the same maximum safe speed as small cars. Note that the mathematical relationship between friction and weight is outside the scope of the syllabus. It is mentioned here as a practical example of horizontal circular motion.

Challenge

Cars on a racetrack have a choice of two paths to round a corner, as shown in Figure 9. Which path, 1 or 2, would allow the car to travel at a higher speed when doing so? **Justify** your answer. (2 marks)

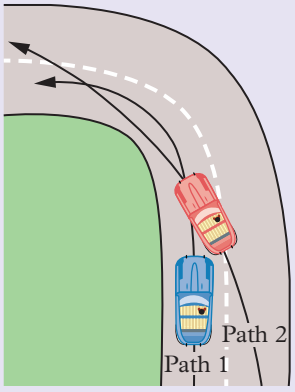


FIGURE 9 One of these paths will allow the car to travel through the curve at higher speed.

Worked example 4.3F

Calculating centripetal force and friction going through a curve

A motorcycle and rider with a total mass of 1,250 kg are travelling at a constant speed around a dry circular bitumen track of radius 56 m. **Calculate** the maximum safe speed without slipping. Give your answer to an appropriate number of significant figures. Assume friction is 0.7 times the weight of the motorcycle and rider. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify the appropriate formula and gather any data that you need.	$m = 1{,}250\,\text{kg}$, $r = 56\,\text{m}$; $F_f = 0.7F_g$ $F_c = \frac{mv^2}{r}$
Step 3: Equate the maximum centripetal force to the frictional force and simplify the equation. (1 mark for "Identifies the equivalence of centripetal force and friction" and 1 mark for "Provides appropriate mathematical reasoning")	$F_c = F_f$ (1 mark) $\frac{mv^2}{r} = 0.7mg$ $v^2 = 0.7gr$ $v = \sqrt{0.7gr}$ (1 mark)

Think	Do
Step 4: Substitute in the known values and solve for speed, v. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. The question asks for speed and not velocity, so a direction is not required. (1 mark for "Demonstrates correct substitution" and 1 mark for "Provides correct answer")	$v = \sqrt{0.7 \times 9.8 \times 56}$ (1 mark) $= 19.6\,\text{m s}^{-1}$ $= 20\,\text{m s}^{-1}$ (2 s.f.) (1 mark)

Your turn

A car and driver with a total mass of 1,950 kg are travelling at a constant speed around a wet circular bitumen track of radius 75.0 m. **Calculate** the maximum safe speed without slipping. Give your answer to an appropriate number of significant figures. Assume friction is 0.3 times the car's weight. (4 marks)

Why does a banked circular racetrack allow for more speed?

A banked curve is one that has its surface at an angle with respect to the horizontal. Racing cars use banked circular tracks to be able to go to high speeds without skidding off the track. When a racing car moves around a banked curve, two forces are acting on it: gravity, which pulls it downwards, and the normal force, which is the force exerted by the track that pushes the car upwards. The combination of these two forces provides the necessary centripetal force that keeps the car moving in a circular path. Without banking, the car would have to rely solely on friction between the tyres and the track to provide this centripetal force. However, there is a limit to how much friction can be generated, and beyond a certain speed, the car would skid off the track. The steeper the banking, the greater the component of the normal force that acts towards the centre of the circle, providing the necessary centripetal force. Let's consider this further.

FIGURE 10 A banked racetrack being used for the Daytona 500 in Florida, USA

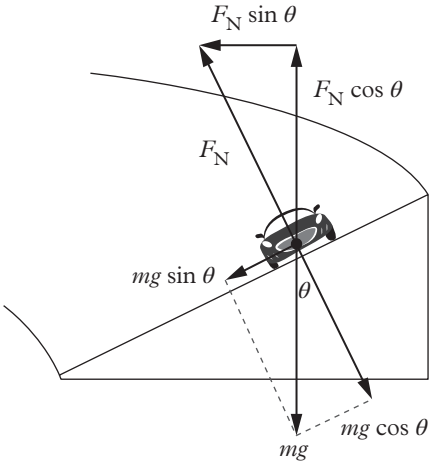


FIGURE 11 Resolution of forces on a banked racing track

To examine the various forces, we look at a coordinate system that points to the centre of a circle because this is about centripetal motion (Figure 11). You are familiar with the three lower vector arrows from your knowledge of inclined planes. The force F_g is shown as mg. This has been resolved into a parallel component, $mg\sin\theta$, and a perpendicular component, $mg\cos\theta$, which you will recall from the previous module on inclined planes.

We also have the uppermost forces. There is a normal force, F_N, from the surface of the track upwards against the car. Note that the F_N arrow is much longer than the $mg\cos\theta$ arrow. This may surprise you because you're used to making them the same length in inclined plane questions. But that was when the normal force was just responding to gravity alone. However, in this case the car is going in a circle and requires a centripetal force to make it do so. This centripetal force is supplied by the surface of the track. So, F_N is responding to components of gravity and the centripetal force. That's why it has a greater length.

The normal force can be resolved into two components: one vertical, $F_N\sin\theta$, and one horizontal, $F_N\cos\theta$. There is no acceleration in the vertical direction so $F_N\cos\theta$ must equal mg. You will notice how their vector arrows are equal in length and opposite in direction. We can rearrange this to give $F_N = \frac{mg}{\cos\theta}$. In the horizontal direction there is just $F_N\sin\theta$, and this must supply the centripetal force. In other words, $F_c = F_N\sin\theta$. Let's put all of this together. We should firstly note, though, that the development of the equation that follows is outside the scope of the syllabus and will not be tested in the external exam. It is included to make the point mathematically that a banked track allows for higher speeds.

$$F_c = F_N\sin\theta$$
$$\frac{mv^2}{r} = \frac{mg}{\cos\theta}\times\sin\theta$$
$$\frac{v^2}{r} = \frac{g\sin\theta}{\cos\theta}$$
$$\frac{v^2}{r} = g\tan\theta$$
$$\tan\theta = \frac{v^2}{rg}$$

Thus, for every single angle, there exists a velocity for which no friction is required at all to move the object along the curve. This means that the car will be able to turn even under the most slippery conditions (ice or water) at that angle and speed.

Worked example 4.3G

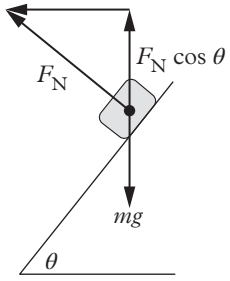
Angle of a banked track for high speed

A race car of mass 1,500 kg is travelling around a banked curve of radius 50 m at a maximum speed of 25 m s^{-1}. It is an icy track, so friction is zero. **Determine** (to an appropriate number of significant figures) the

a centripetal force on the car (4 marks)

b angle of the incline of the track. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We will need to find a numerical answer using mathematical processes and showing working. For each part we must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: For part **a**, identify the appropriate formula and gather any data that you need.	**a** $v = 25\,\text{m s}^{-1}$, $r = 50\,\text{m}$, $F_f = 0$ $F_c = \frac{mv^2}{r}$
Step 3: Draw a free-body diagram showing the relevant forces and their components. (1 mark for "Constructs a free-body diagram of the forces involved"; 1 mark for "Constructs a correctly labelled vector diagram of the relevant quantities")	$F_c = F_N \sin\theta$  (1 mark for diagram; 1 mark for correct labels)
Step 4: Select the correct formula to calculate the centripetal force, F_c. Make the calculation and give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Calculates the centripetal force")	$F_c = \frac{mv^2}{r}$ $= \frac{1{,}500 \times 25^2}{50}$ (1 mark) $= 18{,}750\,\text{N}$ $= 19{,}000\,\text{N}$ (2 s.f.) (1 mark)
Step 5: For part **b**, equate the weight (mg) and the vertical component of the normal force ($F_N \cos\theta$) as there is no acceleration in the vertical direction. Rearrange to make F_N the subject.	**b** $F_N \cos\theta = mg$ $F_N = \frac{mg}{\cos\theta}$
Step 6: Rearrange the centripetal force component ($F_N \sin\theta$) to make F_N the subject.	$F_c = F_N \sin\theta$ $F_N = \frac{F_c}{\sin\theta}$
Step 7: Equate the two formulas for F_N from steps 5 and 6. Substitute in the known values and solve for the angle θ. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Identifies the relationship between the normal force and other quantities"; 1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$F_N = \frac{mg}{\cos\theta} = \frac{F_c}{\sin\theta}$ (1 mark) $\frac{\sin\theta}{\cos\theta} = \frac{F_c}{mg}$ $\tan\theta = \frac{18{,}750}{1{,}500 \times 9.8}$ (1 mark) $= 1.2755$ $\theta = 51.90°$ $= 52°$ (2 s.f.) (1 mark)

Your turn

A racing car of mass 1,000 kg is travelling around a banked curve of average radius 125 m at a maximum speed of $25\,\text{m s}^{-1}$. It is an icy track, so friction is zero. **Determine** (to an appropriate number of significant figures) the

a centripetal force on the car (4 marks)

b angle of the incline of the track. (3 marks)

How do centripetal forces relate to the Sun, Moon and Earth?

Earth is held in orbit by a gravitational force between it and the Sun that provides enough centripetal force to keep it moving in a circle instead of continuing in a straight line. The same applies to Earth and the Moon (Figure 12). In the next module you will be making use of the relationships developed here to examine these solar and planetary forces.

FIGURE 12 The Sun, Earth and Moon's gravitational interactions supply the centripetal forces.

Check your learning 4.3

Check your learning 4.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Identify** what is "uniform" in uniform circular motion. (1 mark)

2 **Clarify** the following statements about uniform circular motion. (1 mark each)
- **a** Constant velocity means that an object is not accelerating.
- **b** High rotational velocity means high acceleration.
- **c** Whirling a weight on a string in a vertical circle is still "uniform".
- **d** The average velocity of an object in circular motion is zero.

3 **Describe** how the directions of the force and velocity vectors are related for an object undergoing uniform circular motion. (2 marks)

4 A ball is on a string 1.2 m in length and is swung in a horizontal circle at 3 revolutions per second. **Calculate** the centripetal acceleration. (3 marks)

5 A centripetal force of 64 N is acting on a brick of mass 2.8 kg being swung in a horizontal circle of radius 0.75 m. **Calculate**
- **a** the velocity of the brick (2 marks)
- **b** its centripetal acceleration (2 marks)
- **c** its rotational speed in rpm. (2 marks)

Analytical processes

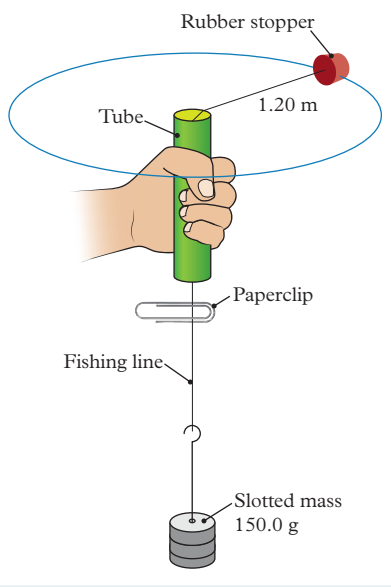
6 A 150 g ball is tied to a pole with a rope of length 1.5 m, and spins around the pole at 20.0 m s^{-1}. **Determine** the centripetal force acting on the ball. (2 marks)

7 An electron (m = 9.1093835 × 10^{-31} kg) moves in a circle of radius 0.02 m and a centripetal force of 4.60 × 10^{-14} N acts on the electron. **Determine** the average speed of the electron. (2 marks)

8 A 1,550 kg car follows a circular path of radius 69 m, at a speed of 20 m s^{-1}. **Determine** the minimum friction required between the car and the road so that the car does not skid. (2 marks)

9 A string can withstand a force of 120 N before breaking. A 1.8 kg brick is tied to the string and whirled in a horizontal circle of radius 90 cm. **Determine** the maximum speed of the brick before the string breaks. (2 marks)

10 In an investigation of uniform circular motion, a student whirled a 50.0 g rubber stopper above his head in a horizontal circle of radius 1.20 m. The string was passed through a piece of glass tubing, and a set of slotted brass masses was suspended from the end of the string. It required a hanging mass of 150.0 g to provide enough force to keep the rubber stopper whirling in a circle at a constant speed.



Determine the

a tension in the string provided by the hanging mass (2 marks)

b centripetal force (1 mark)

c velocity of the stopper (2 marks)

d period of the rubber stopper (2 marks)

e time taken for 10 revolutions of the stopper. (1 mark)

11 A car of mass 1,800 kg is attempting to go 90 km h^{-1} (9.0×10^{1} km h^{-1}) around a circular corner of radius 80 m (8.0×10^{1} m). The road is wet, and the friction is calculated to be 65 per cent of the car's weight.

a **Determine**, with supporting calculations, whether there will be enough friction for the car to travel successfully around the corner at that speed. (4 marks)

b **Determine** how much time it takes the car to complete one lap of the circular track. (2 marks)

Practical

Lesson 4.4

Centripetal forces on an object undergoing horizontal circular motion

Learning intentions and success criteria

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

MODULE 4

Lesson 4.5
Review: Circular motion

Summary

4.1
- Uniform circular motion is the result of a force that acts on an object in a perpendicular direction to the velocity of the object.

4.2
- The time for one revolution of an object in uniform circular motion is called the period, T.
- The average speed for an object in uniform circular motion is the distance travelled in one period, T.
- Problems involving objects undergoing uniform circular motion at constant speed can be solved using $v = \frac{2\pi r}{T}$.

4.3
- An object travelling in a circle at constant speed has an acceleration, called centripetal acceleration, a_c, directed towards the centre of the circular path and perpendicular to the velocity vector.
- Problems involving objects undergoing uniform circular motion at a constant speed can be solved using $a_c = \frac{v^2}{r}$.
- The centripetal force is the net force directed towards the centre of a circular path: $F_c = F_{net}$.
- Problems involving forces acting on objects in uniform circular motion can be solved using $F_c = F_{net} = \frac{mv^2}{r}$.

4.4
- Practical: Centripetal forces on an object undergoing horizontal circular motion

Key formulas

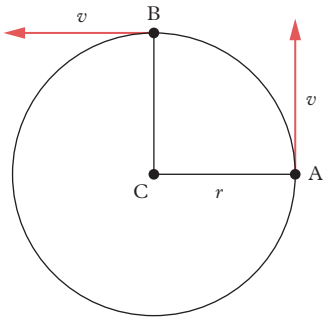
Velocity for circular motion	$v = \frac{2\pi r}{T}$
Centripetal acceleration	$a_c = \frac{v^2}{r}$
Centripetal force	$F_c = F_{net} = \frac{mv^2}{r}$
Force due to gravity (weight)	$F_g = mg$

Review questions 4.5A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 A ball is being whirled in a horizontal circle of radius r at constant speed. The velocity of the ball at positions A and B is shown in the diagram.



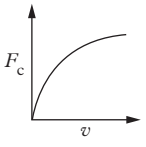
Which direction best describes the change in velocity of the ball from point A to point B?

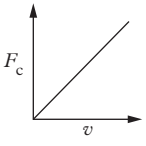
A ↖
B ↘
C ↗
D ↙

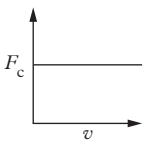
2 As part of a centripetal motion experiment, a student swung a rubber stopper around in an overhead horizontal circle at constant speed. In one particular trial she counts 20 revolutions in 10 seconds. Which expression best shows the velocity of the stopper?

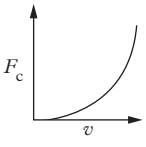
A $\frac{2\pi r \times 20}{10}$
B $\frac{2\pi r \times 10}{20}$
C $\frac{\pi r^2 \times 10}{20}$
D $\frac{\pi r^2 \times 20}{10}$

3 Which graph best shows the relationship between centripetal force (vertical axis) and the velocity of an object being whirled in a horizontal circle at constant speed?

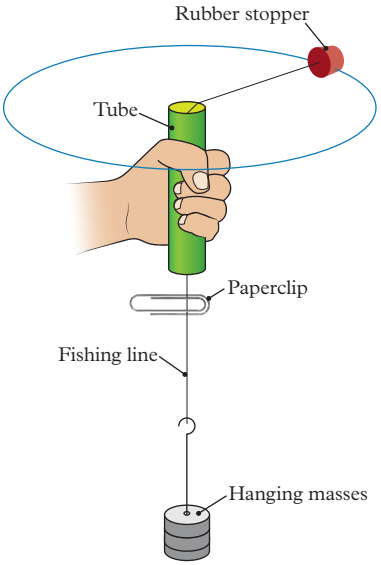
A


B


C


D


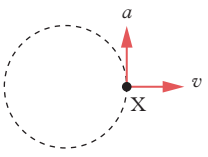
4 A student is investigating centripetal motion by whirling a rubber stopper above his head in a horizontal circle at constant speed.

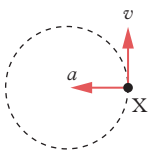


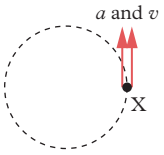
They increase the mass, m, of the hanging slotted masses but want to keep the radius, r, constant. To do this, they must

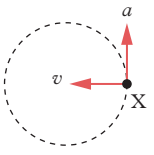
A increase m or v.
B increase m or decrease v.
C decrease m or increase v.
D decrease m or v.

5 A rubber stopper is moving in an anticlockwise horizontal circle at constant speed. Which diagram shows the direction of the velocity and acceleration at point X?

A


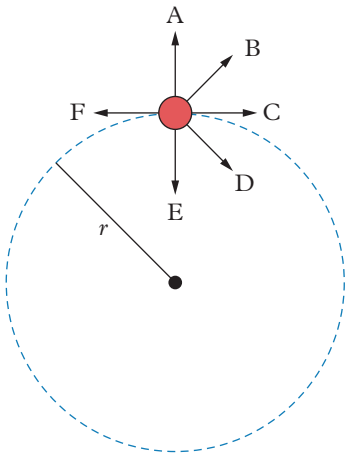
B


C


D


Use the following information to answer questions 6 to 8.

The object shown in the diagram is travelling clockwise on a smooth surface around a horizontal circle of radius r, with a constant speed of $v\,\text{m s}^{-1}$.



6 Which arrow best shows the velocity of the object at the instant shown in the diagram?

A C **B** D **C** A **D** F

7 Which arrow best shows the acceleration of the object at the instant shown in the diagram?

A A **B** E **C** C **D** D

8 If the force acting on the object shown in the diagram were to suddenly stop at the instant shown, in which direction would the object continue to move?

A C **B** A **C** F **D** D

9 A child stands at the edge of a rotating merry-go-round in a playground, as shown in the diagram. To keep the child moving in a circular path, the merry-go-round is applying a force on the child. Which statement best describes the reaction to this force?

A The normal reaction force on the child

B The static friction force exerted by the child on the merry-go-round

C The force of gravity exerted by Earth on the child

D The weight force exerted by the child on the merry-go-round

10 An object in horizontal circular motion makes 10 complete revolutions in a time of 8.0 s. What is its period of rotation?

A 8.0 s **B** 1.3 s

C 10 s **D** 0.80 s

11 A stone of mass of 4 kg is attached to a string 120 cm long and is whirled in a horizontal circle. The maximum tension the string can withstand is 8 N. What is the maximum velocity of revolution that can be given to the stone without breaking the string?

A $1.5\,\text{m s}^{-1}$

B $2.4\,\text{m s}^{-1}$

C $15.5\,\text{m s}^{-1}$

D $240\,\text{m s}^{-1}$

12 A ball on a string travels in a horizontal circular path of radius r at a constant speed v. What happens to the object's acceleration if the speed is doubled and the radius stays unchanged?

A It is reduced to a quarter

B It is reduced to a half

C It doubles

D It quadruples (×4)

13 A rubber stopper of mass m is being swung in a horizontal circle at constant speed. What other quantity is also constant?

A Acceleration

B Centripetal force

C Period

D Velocity

14 The maximum speed with which a car can take a circular turn of radius R is v. What is the maximum speed with which the same car, under the same conditions, can take a circular turn of radius $2R$?

A $2v$ **B** $\sqrt{2}\,v$

C $4v$ **D** $2\sqrt{v}$

15 A ball of mass m is being swung in a horizontal circle at constant speed v and radius r. What is the relationship between its centripetal force and its kinetic energy?

A $F_c = 2E_K$

B $F_c = \frac{E_K}{r}$

C $F_c = \frac{2E_K}{r}$

D $F_c = \frac{E_K}{2r}$

Review questions 4.5B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 **Identify** whether this is true: An object will travel the circumference of a circle in one period (T) of time if it is in uniform circular motion. (1 mark)

17 A ball on a string is swirled around in a horizontal circle at a constant speed of 2.5 m s^{-1} and experiences an acceleration of 3.9 m s^{-2}. **Calculate** the radius of its motion. (2 marks)

18 An object of mass 55 g undergoes uniform circular motion at a radius of 95 cm. It takes 18.8 s to complete 10 revolutions.

a **Calculate** the distance travelled in 20.0 s (2 s.f.). (2 marks)

b **Calculate** the centripetal force acting (2 s.f.). (2 marks)

19 A 5.0 kg mass experiences a centripetal force of 75 N when travelling at a constant speed around a horizontal circle at 6.5 m s^{-1}. **Calculate** the radius of the circle (2 s.f.). (3 marks)

20 A rubber stopper tied to a string is travelling at a constant speed of 3 m s^{-1} in a circle of radius 1.5 m.

a **Calculate** the magnitude of the centripetal acceleration of the stopper. (2 marks)

b **Identify** the direction of the acceleration. (1 mark)

Analytical processes

21 A car of mass 2,250 kg is travelling around a circular track of radius 80 m at a constant speed of 20 m s^{-1}. **Determine** how many revolutions of the track per minute the driver makes. (3 marks)

22 A microwave carousel platter of diameter 31 cm turns at 10 rpm. **Derive** the average speed for a point on the outer rim of the carousel. (3 marks)

23 A student is whirling a rubber stopper in a horizontal circle at a constant speed on a string of length 90 cm. The circle is 2.1 m from the ground. The student times the stopper and finds it makes 10 revolutions in 4.6 s. The string breaks and the stopper falls to the ground. **Determine** its horizontal displacement after being released (2 s.f.). (5 marks)

24 A 2.0 m long fishing line that has a breaking strain of 20 N is attached to a lead sinker of 120 g. **Determine** the maximum speed at which the sinker can be whirled in a horizontal circle without breaking the line. (2 marks)

25 A 540 kg satellite is in an orbit of radius of 32,000 km from the centre of Earth. The centripetal force experienced by the satellite is 197 N. **Determine** the

a average speed (2 marks)

b number of revolutions per day the satellite makes of Earth. (3 marks)

Knowledge utilisation

26 A coin is placed on a record turntable and the turntable is made to rotate. As the turntable picks up speed, the coin is flung off. **Propose** why this happens. (2 marks)

27 Students conducted a centripetal force practical to determine the effect of three variables on centripetal force. They investigated each variable and kept the other two constant. **Determine** the shape and **construct** a graph of the following pairs, with F_c on the y-axis

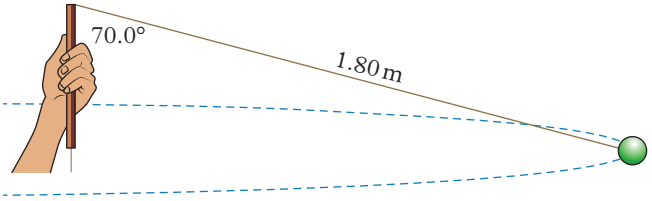
a F_c (vertical axis) versus v (1 mark)

b F_c versus r (1 mark)

c F_c versus m (mass of rotating mass). (1 mark)

28 A car of mass 1,300 kg is attempting to round a circular track of radius 60.0 m at a speed of 15 m s^{-1}. It is a wet track, and the friction is calculated to be 60 per cent of the car's weight. **Propose**, with justification, whether there will be sufficient friction for the car to travel the circular path successfully. (4 marks)

29 During an experiment, a ball of mass 175 g tied to a string of length 1.80 m was swung in a horizontal circle at constant speed. The string made an angle of 70.0° to the vertical, as shown in the diagram.



Determine (2 s.f.) the

a tension in the string (3 marks)

b centripetal force on the ball (2 marks)

c radius of the circle (2 marks)

d speed of the ball (2 marks)

e period of one revolution. (2 marks)

30 A conical pendulum has a 55 g brass ball attached to string and revolves in a horizontal circle of radius 75.0 cm at constant speed. It has a period of 1.00 s.

Determine (2 s.f.) the

a speed of the revolving ball (2 marks)

b centripetal force on the ball (2 marks)

c angle of the string with respect to the vertical (3 marks)

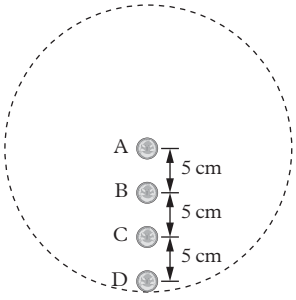
d length of the string. (2 marks)

31 A Formula 1 racing car of mass 798 kg is travelling around a banked curve of average radius 82.0 m at a maximum speed of 22.5 $m\,s^{-1}$. It is an icy track, so friction is zero. **Determine** (2 s.f.)

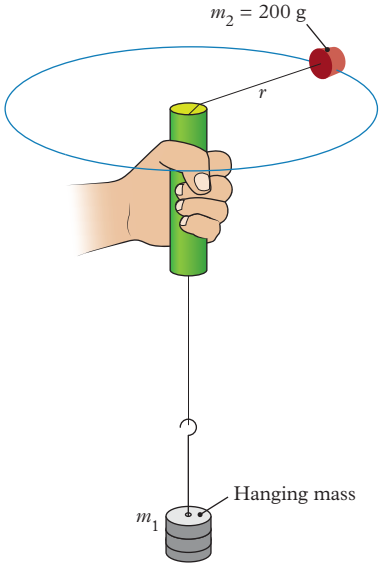
a the centripetal force (2 marks)

b the angle of the incline of the track. (3 marks)

32 Four $1 coins are spaced equally at 5 cm between their centres across the radius of a record turntable, as shown in the diagram. Coin A marks the centre. Each coin has a mass of 9.0 g. The turntable is set spinning at its normal rate of $33\frac{1}{3}$ rpm. The friction between a coin and the surface is estimated to be 1.24 per cent of the coin's weight. **Evaluate** this arrangement and deduce which coins, if any, will remain on the turntable while it is rotating. (6 marks)



33 Students performed a centripetal force experiment using the apparatus shown in the diagram. Their results are listed in the following table.



Hanging mass, m_1 (g)	Revolving mass, m_2 (g)	Radius, r (m)	Period, T (s)
50	200	1.00	4.11
100	200	1.20	3.18
150	200	1.40	2.81
200	200	1.60	2.60

a **Calculate** the centripetal acceleration and the centripetal force for each trial (use the weight of the hanging mass for F_c). (2 marks)

b **Construct** a graph of F_c on the vertical axis versus a_c on the horizontal axis. (3 marks)

c **Determine** the gradient and what it means. (2 marks)

d **Calculate** the observed value of the revolving mass from the gradient. (1 mark)

e **Compare** this to the actual value for m_2 by calculating absolute and percentage errors. (2 marks)

f **Assess** the result. (3 marks)

Data drill

Investigating the forces on an object undergoing uniform circular motion

An experiment was conducted to investigate the relationship between centripetal force and the period of circular motion. They used a set of hanging masses, m, to provide the centripetal force (Figure 1). A rubber stopper of mass M was swung in a horizontal circular path of radius 80.0 cm and the time for 10 revolutions was noted. Data was processed by calculating the period T and the inverse of the square of the period $\frac{1}{T^2}$.

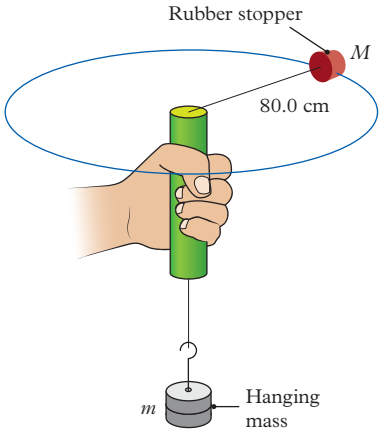


FIGURE 1 Setup for the centripetal force experiment

The results are shown in Table 1.

TABLE 1 Results for the centripetal motion experiment

Hanging mass, m (g)	Time for 10 revolutions, T_{10} (s)				Processed data	
	Trial 1	Trial 2	Trial 3	Average	Period, T (s)	$\frac{1}{T^2}$
10.0	33.3	34.5	33.8		3.4	0.087
30.0	19.5	19.4	19.6	19.5	2.0	0.263
50.0	15.0	15.2	15.0	15.1	1.5	0.439
70.0	12.7	12.8	12.9	12.8	1.3	0.611
90.0	11.1	11.4	11.2	11.3	1.1	0.787
110.0	10.2	10.1	10.1	10.1	1.0	0.971

Apply understanding

1 **Calculate** the average time for 10 revolutions at a hanging mass of 10.0 g. (1 mark)

2 **Identify** the uncertainty in the average time for 10 revolutions at a hanging mass of 10.0 g. (1 mark)

3 **Identify** a controlled variable for this experiment. (1 mark)

Analyse data

A linearised graph of the data is shown in Figure 2.

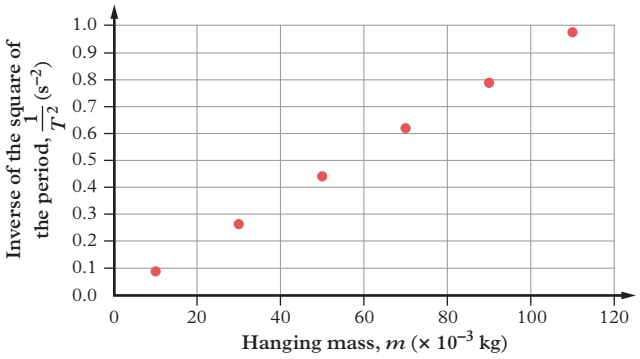


FIGURE 2 Linearised centripetal motion data

4 **Identify** a mathematical relationship between the inverse of the square of the period $\frac{1}{T^2}$ and the hanging mass, m, using the graph. (3 marks)

Interpret evidence

5 **Deduce** the mass, M, of the rubber stopper using the graph. (2 marks)

6 **Deduce** the period, T, of the rubber stopper for a hanging mass of 120 g using the mathematical relationship developed in question **4**. (2 marks)

Module 4 checklist: Circular motion

MODULE 5

Orbital mechanics

Introduction

From the earliest days, humans have looked into the sky and wondered what it's all about. They would have noticed the Sun travel across the sky during the day, the Moon and stars at night, and the strange motion of the planets doing loop-the-loops against the background of the stars.

Thousands of years ago priests in Babylon (present-day Iraq) developed theories that used myths and legends to explain the behaviour of the night skies. These always started and finished with Earth being at the centre of the universe – the "geocentric" model.

It wasn't until the work of Polish astronomer Nicolaus Copernicus in 1543 that the model changed to a "heliocentric" one, with the rotating Sun at the centre and the planets revolving around it. No-one could really explain the underlying principles of how it all worked, until Newton proposed his magnificent law of universal gravitation in 1686.

Newton built on the work of the German astronomer Johannes Kepler, who had outlined three laws of planetary motion 80 years earlier. Kepler himself was fortunate to have the very precise astronomical observations of the Danish astronomer Tycho Brahe. As Newton said of his law of gravitation, he was "standing on the shoulders of giants". This observation marks the work of astronomers over the centuries: better and better data is collected as time passes and models are adapted to fit the data.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to orbital mechanics.

Subject matter

Science understanding

→ Describe the Law of Universal Gravitation.
→ Solve problems involving the magnitude of the gravitational force between two masses using $F = \frac{GMm}{r^2}$.
→ Describe the concept of gravitational fields.
→ Solve problems involving the gravitational field strength at a distance from an object using $g = \frac{F}{m} = \frac{GM}{r^2}$.
→ State the three laws of planetary motion.
→ Describe the relationship between the Law of Universal Gravitation and uniform circular motion and recognise this as the third law of planetary motion.
→ Solve problems involving the third law of planetary motion using $\frac{T_a^{\ 2}}{r_a^{\ 3}} = \frac{T_b^{\ 2}}{r_b^{\ 3}} = \frac{4\pi^2}{GM}$.

Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Isaac Newton, Johannes Kepler, Émilie du Châtelet and Katherine Johnson who furthered our understanding of gravity and motion.
→ Appreciate how the accepted model of the solar system slowly shifted under the influence of carefully collected and analysed data.
→ Explore the difficulties experienced by scientists who supported a heliocentric model of the solar system and the hindrances to the acceptance of their discoveries by society.
→ Consider the international collaboration required to monitor the orbits of satellites, and the management of space debris.
→ Consider the factors that contribute to positioning of satellites used for observation of weather, natural phenomena, traffic and military movements.
→ Explore the international collaboration required in the discovery of gravity waves and associated technologies, e.g. Laser Interferometer Gravitational Wave Observatory (LIGO).

Science inquiry

→ Consider the difference between the heliocentric and geocentric models of the solar system.
→ Investigate the relationship between orbital radius and mass for orbiting objects using a simulation.

Practicals

This lesson is available on Oxford Digital.

Lesson 5.5 Investigating orbital radius and mass using a simulation

Lesson 5.1
What is gravity?

Key ideas

- → Gravity is the force of attraction between objects with mass.
- → Gravitational forces are transmitted by gravity waves.
- → International collaboration was required in the discovery of gravity waves and associated technologies, e.g. Laser Interferometer Gravitational Wave Observatory (LIGO).

How was gravity discovered?

Learning intentions and success criteria

Until 1687, the Latin word "gravitas" (gravity) meant "heavy" – i.e. the property of an object that made it hard to lift. No one thought that objects were being pulled to the ground. The Greek philosopher Aristotle, in about 330 BCE, proposed that things like metal and rock moved towards the ground because that was their natural place, and the heavier they were, the faster they fell. There was no experimental proof of this relationship; experiments were just not considered useful in the physical sciences of the time.

In 1604, Galileo Galilei experimented with balls rolling down inclines and found that their acceleration was independent of their mass, contrary to Aristotle's ideas. This created quite a sensation in scientific circles. Shortly after, in 1609, Johannes Kepler showed that the Sun was the centre of our solar system, contradicting Aristotle's idea that Earth was at the centre. Kepler believed that the Sun moved the planets by sending out rays like wheel spokes, which carried the planets around.

In 1687, English scientist Isaac Newton united the ideas of astronomers such as Kepler with Galileo's laws of falling bodies. Newton called the force between the Sun and the planets "gravity" and extended it to mean that gravity was a force that pulled objects towards Earth. He extended this idea in later work to mean a force between all bodies.

FIGURE 1 Cliff diving at sunset. The diver is attracted towards Earth, but Earth is also attracted to the diver. To Newton, "gravity" applied not only to the Sun and planets, but between all bodies.

Newton's laws of motion and gravity were confirmed repeatedly by experimentation and were regarded as the undisputed laws of nature until Albert Einstein proposed his general theory of relativity in 1915. It expanded his special theory of relativity to "the universe at large", including planets, stars and galaxies. His theory described gravity as a distortion of "space–time" and could explain how light is bent by massive astronomical bodies, which Newton's model could not. Again, experimental tests confirmed Einstein's predictions, culminating in the detection of gravitational waves in 2015.

gravity
the force of attraction between objects with mass

Whether gravity is modelled as a Newtonian force or an Einsteinian distortion of space–time, it can still be defined thus: **gravity** is a natural phenomenon by which all things with mass are attracted to one another.

Real-world physics

Detection of gravity waves

Scientists have long wondered how the gravitational force is transmitted over the vastness of space. Isaac Newton first described the nature of gravitational force but did not identify what was responsible for it. Eventually, the idea of gravity being some sort of wave took hold.

Gravitational waves were predicted by Einstein around 1916, as part of his theory of general relativity. The logic was that oscillating (accelerating) electric charges produced electromagnetic radiation, so an oscillating mass should produce gravitational waves. He predicted that gravitational waves would travel at the speed of light.

It was clearly a complicated concept, and it was well known that detection of these waves would be extremely difficult. It requires an enormous amount of accelerating mass to produce even a small gravitational wave. More recently, scientists thought that the collision of two black holes might produce waves big enough to detect.

When were they discovered? They were discovered on 15 September 2015 at the Laser Interferometer Gravitational-Wave Observatory (LIGO) facilities in the USA (Figure 2). Since then, international collaboration has seen these waves detected many times.

How are they detected? As a gravitational wave passes an observer, objects are stretched and shrunk by the wave in the direction of the wave but are unaffected in the direction parallel to the wave. If there are two detectors at right angles to each other, as in the LIGO setup, a gravitational wave will change the length of the detector that is parallel to the wave but leave the other unchanged. The trouble is that the changes are extremely small – a change of 1 part in 10^{22}. The only way to get this resolution is with a laser interferometer, which compares the phase of light beams in each detector. These still need to be very long (4 km) to ensure the percentage uncertainty is as low as possible. To improve this further, LIGO bounces the light between mirrors 300 times, so the effective length is 1,200 km.

Where did the waves come from? The gravitational waves detected in 2015 had been travelling for over a billion years, coming from a pair of colliding black holes each of about of 30 solar masses ($30M_S$).

Why do we want to detect them? Detecting and analysing the information carried by gravitational waves allows scientists to observe the universe in a way never before possible. In the first 380,000 years after the Big Bang, the universe was so hot that electromagnetic radiation could not pass through it, but gravitational waves could. There are now parts of the universe that can only be seen with gravitational wave detectors, and they may tell the story of those first 380,000 years.

FIGURE 2 The LIGO detector site in Livingston, Louisiana, USA

Apply your understanding

1 It takes the gravitational wave 0.007 s to travel from one LIGO detector in Louisiana to the other LIGO detector in the State of Washington, a distance of 3,030 km. **Analyse** these data to confirm that the wave must have been travelling at the speed of light. State whether this confirmed Einstein's prediction 100 years earlier. (3 marks)

2 **Deduce** what link Einstein was trying to make between oscillating electric charge and oscillating masses. (1 mark)

3 **Explain** why the LIGO tubes are 4 km long. (1 mark)

Challenge

Is Earth pulling on the Sun?

Which is greater, Earth's pull on the Sun, or the Sun's pull on Earth? (1 mark)

Challenge

What would gravity be like if Earth were flat?

How would gravity be different if Earth were flat instead of round? (1 mark)

Check your learning 5.1

Check your learning 5.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how Aristotle's explanation for the motion of heavy objects was disproved. (2 marks)

2 **Describe** how the word "gravity" changed meanings in Newton's writings. (1 mark)

Analytical processes

3 In his treatise on motion, Newton compared the gravitational force to electrostatic and magnetic forces. He said that gravity was an attractive force only. **Deduce** how this differs from electrostatic and magnetic forces. (2 marks)

Knowledge utilisation

4 Some people say gravity was invented by Newton, whereas others say he discovered it. **Evaluate** these two claims. (2 marks) (Note that this would not be an external exam question.)

5 **Propose**, with reasons, whether this statement is true: "Gravitational waves didn't exist until they were discovered in 2015." (2 marks)

Lesson 5.2
Newton's law of universal gravitation

Key ideas

- → Newton's law of universal gravitation quantifies the gravitational force between two objects.
- → Problems involving the magnitude of the gravitational force between two masses can be solved using $F = \frac{GMm}{r^2}$.

What is Newton's law of universal gravitation?

Learning intentions and success criteria

In his most famous work, *Philosophiae Naturalis Principia Mathematica* (often called just *Principia*), published in Latin in 1687, Newton wrote: "I deduced that the forces which keep the planets in their orbs must vary reciprocally as the squares of their distances from the centres about which they revolve and in direct proportion to their masses." This is known as **Newton's law of universal gravitation**. In the form of an equation, this is:

$$F = \frac{GMm}{r^2}$$

Newton's law of universal gravitation the force of attraction between each pair of point particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

where F is the force of gravitational attraction, M and m are the masses of the attracting objects, and r is the radial distance between the centres of the objects (Figure 1). "Radial" means a line radiating outwards from the object like the spokes of a wheel. G is a proportionality constant called the universal gravitational constant. It is more commonly known by astrophysicists as "big" G, to distinguish it from "little" g, the acceleration due to gravity. In SI units, G has the value $6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$.

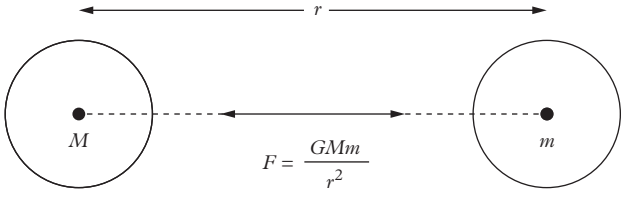


FIGURE 1 The meaning of the symbols for Newton's law of universal gravitation

The gravitational force has the following properties:

- This attractive force always points inwards, between pairs of objects; that is, there are two equal and opposite forces.
- It is a vector quantity as it has both magnitude and direction.
- The law applies to all objects with mass, big or small.
- If the distance between two objects is very large compared to their sizes, or if they are spherically symmetrical, you can treat them as point-like masses to simplify calculations. This means you can assume all of their mass is located at their centre of mass.

Although Newton discovered a law to describe gravitation, he couldn't explain what made it occur, as he noted in the *Principia*. This happens in science all the time. For example, the movement of Earth's tides was well understood and tables predicting tides for the year ahead were being produced for centuries, before their relationship to gravity had been worked out.

What effect does distance have on gravitational force calculations?

In Newton's law of universal gravitation, the separation distance r is between centres of mass, so it works well when the separation distance is very large compared to the diameter of the objects. When the two objects are close, their diameters must also be carefully considered. Let's consider how the formula applies at large and small distances.

Large distances

Large distances mean that the bodies are separated by thousands or millions of times their diameter, such as the distance between Earth and the Moon, other moons and their respective planets, and between stars, planets and asteroids.

If the distance between two objects is very large compared to their sizes, or if they are spherically symmetrical, you can treat them as point-like masses to simplify calculations. This means you can assume all of their mass is located at their centre of mass.

FIGURE 2 The Moon is far enough away from Earth that both can be regarded as point sources of mass. Likewise, the Sun and Earth can be regarded as point masses.

Order of magnitude

When talking about the distances between astronomical bodies such as planets, stars and galaxies it is sometimes said, for example, that a particular star is many orders of magnitude further away from us than another star. But what does "order of magnitude" mean, exactly?

When dealing with very large or very small numbers, we are often only interested in an approximate figure. We can use the order of magnitude (OM): the power of 10 following the coefficient. For example, the remotest object known is the JADES-GS-z13-0 galaxy, which is 3.18×10^{26} m away. The coefficient is 3.18 and the order of magnitude is the 10^{26} that follows. It is just as meaningful to say it is 10^{26} m away. Similarly, the mass of an electron is 9.109×10^{-31} kg, so its order of magnitude is 10^{-31} kg. Note that you don't need to round the coefficient up or down; you just ignore it. This is demonstrated in Worked example 5.2A.

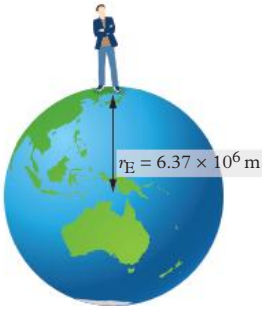


FIGURE 3 At close distances, the radius of the astronomical body must be considered.

Some examples of orders of magnitude are given in Table 1.

TABLE 1 Orders of magnitude for large and small values

Measurement	Dimension	Order of magnitude
Distance to Andromeda galaxy	1.9×10^{22} m	10^{22} m
Distance to nearest star	7.0×10^{16} m	10^{16} m
Weight of a proton on Earth	1.63×10^{-26} N	10^{-26} N
Thickness of a hair	6.1×10^{-5} m	10^{-5} m

Small distances

When you stand on the surface of Earth, your distance to Earth is not zero but 6.37×10^6 m (the mean radius of Earth, r_E, because that's how far away the centre of mass is (Figure 3). If you were in an aeroplane or space-station, you would need to add your altitude, r_A (i.e. your distance above the surface) to the radius of Earth, r_E, to calculate the separation distance, r. This is demonstrated in Worked example 5.2B.

Worked example 5.2A

Large separation distances

a **Calculate** the force of attraction between Earth ($M = 5.97 \times 10^{24}$ kg) and the Moon ($m = 7.35 \times 10^{22}$ kg), given that the Earth–Moon distance is 3.8×10^8 m. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** the order of magnitude of the force. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer. "Determine" means to establish after a calculation.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	**a** $m = 7.35 \times 10^{22}$ kg, $M = 5.97 \times 10^{24}$ kg, $r = 3.8 \times 10^8$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $F = \frac{GMm}{r^2}$
Step 3: Substitute values into the equation and solve for force, F. (1 mark for "Provides correct mathematical reasoning")	$F = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{(3.80 \times 10^8)^2}$ (1 mark)
Step 4: Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides correct answer")	$F = 2.03 \times 10^{20}$ N (3 s.f.) (1 mark)
Step 5: Determine the order of magnitude by identifying the power of 10 in the value found in Step 4. (1 mark for "Provides correct answer")	**b** Order of magnitude = 10^{20} N (1 mark)

Your turn

a **Calculate** the force of attraction between Earth ($m = 5.97 \times 10^{24}$ kg) and the Sun ($M = 2.00 \times 10^{30}$ kg), given that the Earth–Sun distance is 150.02 million km. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** the order of magnitude of the force. (1 mark)

Worked example 5.2B

Small separation distances

Determine the force of gravitational attraction between Earth ($m_{Earth} = 5.97 \times 10^{24}$ kg) and a 72.0 kg person (in an aeroplane) at an altitude of 10,000 m (1.000×10^4 m) above the Earth's surface. The radius of Earth (r_{Earth}) can be taken as 6.37×10^6 m. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$m_{Earth} = 5.97 \times 10^{24}$ kg, $m_P = 72.0$ kg, $r_A = 1.000 \times 10^4$ m, $r_{Earth} = 6.37 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $F = \frac{GMm}{r^2}$
Step 3: Calculate the distance between objects (Earth and the aeroplane). (1 mark for "Provides appropriate mathematical reasoning")	$r = r_{Earth} + r_A$ $= 6.37 \times 10^6 + 1.000 \times 10^4$ m $= 6.38 \times 10^6$ m (1 mark)
Step 4: Substitute known values into the equation and solve for F. (1 mark for "Provides appropriate mathematical reasoning")	$F = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 72.0}{(6.38 \times 10^6)^2}$ (1 mark)
Step 5: State the final answer with the correct units. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Even though force is a vector quantity, questions such as these only need a magnitude, not a direction. If it asked to specify a direction you could say "the force on the person acts downwards towards the centre of Earth and a force of the same magnitude acts on the Earth upwards towards the person". (1 mark for "Provides correct answer")	$F = 704$ N (3 s.f.) (1 mark)

Your turn

Determine the force of gravitational attraction between the Moon ($m_{Moon} = 7.34 \times 10^{22}$ kg) and a 1,846 kg lunar-orbiting satellite 50.0 km above Moon's surface. The radius of the Moon (r_{Moon}) can be taken as 1.74×10^6 m. Give your answer to an appropriate number of significant figures. (3 marks)

Challenge

The Great Attractor

Our galaxy is being drawn towards something called "The Great Attractor" at a rate of 610 km s^{-1}. Scientists agree that it is a beautifully named and very apt description of something that lies very far away and is pulling us ever faster, ever closer. What is it? (1 mark)

Challenge

How to make a swing go higher

Imagine you were writing an instruction manual for a child's swing. Without using a diagram, what instructions would you write on how to make it go higher? No one has ever written this successfully – maybe you will be the first! (2 marks)

How do you determine "big *G*"?

The gravitational constant G can be found experimentally by measuring the gravitational force between two spheres of known mass, separated by a known distance. The first person to do this was Henry Cavendish in 1798, more than a century after Newton proposed his law.

Cavendish fastened two small lead spheres, each of mass m, to the ends of a long wooden rod that was suspended from its midpoint by a fine fibre (Figure 4). Large lead balls (M) were brought up close to the small ones. The large and small lead balls attracted each other, which caused the fibre to twist. The amount of twist was proportional to the force between the spheres. Cavendish had standardised the device beforehand by determining how much force was needed to twist the fibre by certain amounts.

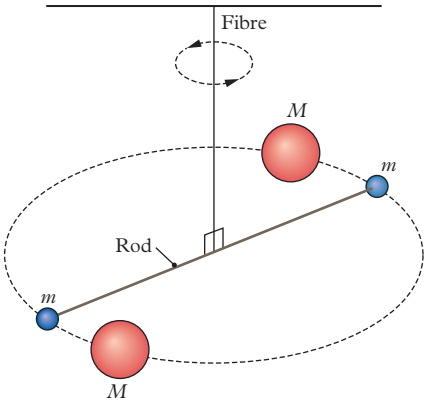


FIGURE 4 Cavendish's apparatus for determining the gravitational force between masses

The total force between the two pairs of lead masses was determined for various separation distances. Table 2 gives the results for the force on the fibre for each pair of masses (316 kg and 1.46 kg) at various distances. These are not Cavendish's results but are derived from them.

A graph of the data is shown in Figure 5. Note that distances have been converted to metres.

TABLE 2 Experimental data for Cavendish's apparatus

Separation distance, r (cm)	5.0	6.0	7.0	8.0	10.0	12.0	14.0
Total force, F ($\times 10^{-6}$ N)	12.2	8.5	6.2	4.8	3.1	2.1	1.6

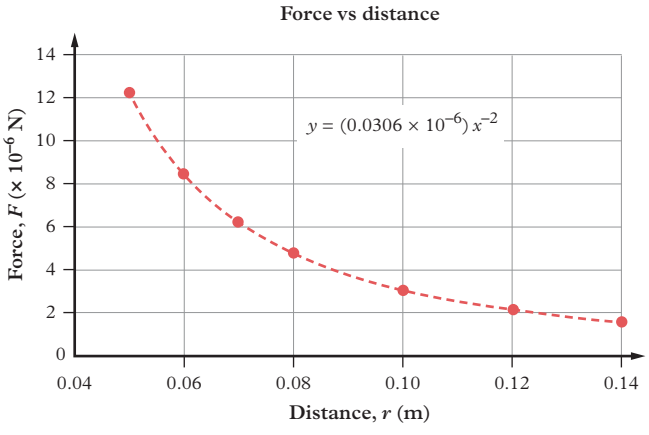


FIGURE 5 A force versus distance graph for Cavendish's apparatus

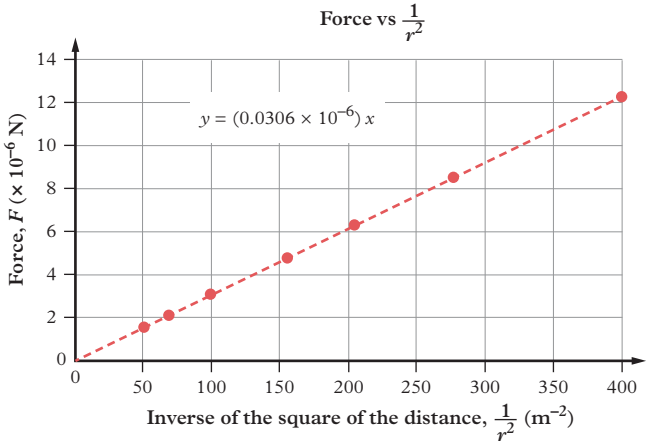


FIGURE 6 The data has been linearised by plotting F versus $\frac{1}{r^2}$.

The relationship appears to be inverse squared, so to linearise the data a graph of F versus $\frac{1}{r^2}$ can be plotted, which gives a linear graph (Figure 6). This implies that F divided by $\frac{1}{r^2}$ (that is, Fr^2) is a constant, which equals the gradient of the line.

$$Fr^2 = G \times M \times m$$

$$= 0.0306 \times 10^{-6} \text{ (the gradient)}$$

The total mass of the large spheres is 316 kg and the total mass of the small spheres is

2.92 kg. These can be substituted into the equation:

$$G = \frac{Fr^2}{Mm}$$
$$= \frac{\text{gradient}}{Mm}$$
$$= \frac{0.0306 \times 10^{-6}}{1.46 \times 316}$$
$$= 6.63 \times 10^{-11}\,\text{N kg}^2\,\text{m}^{-2}$$

Measurements of G have been made countless times using a variety of methods, but this value is very close to the accepted value of $6.67 \times 10^{-11}\,\text{N kg}^2\,\text{m}^{-2}$.

In the next lesson, gravitation is modelled in terms of a gravitational field similar to electric and magnetic fields.

Skill drill

Identifying relationships

Science inquiry skills: Processing and analysing data (Lesson 1.7)

In most experiments we manipulate the independent variable (IV) and measure the effect on the dependent variable (DV). For example, how does gravitational field strength (DV) vary as a function of distance (IV) from a star? One of the simplest ways to analyse your data is to draw a graph, usually with the IV on the x-axis and the DV on the y-axis. But sometimes you are not able to do that – in an exam, for instance.

Table 3 gives data showing examples of four different relationships between the IV and the DV.

TABLE 3 Independent variables and dependent variables for four types of relationships

IV	DV1	DV2	DV3	DV4
2	3	20	120	240
4	6	80	60	60
6	9	180	40	27
8	12	320	30	15
10	15	500	24	10
12	18	720	20	7

A good way to check for possible relationships is to double the IV and see what happens to the DV. In the first row of the table, doubling the IV from 2 to 4 also doubles DV1. We check again and double the IV from 4 to 8, and we see DV1 doubles from 6 to 12. This shows the relationship is linear with a constant $m = 2$. The equation is $y = 2x$.

For DV2, doubling the IV quadruples the DV, so this indicates the relationship is $y = mx^2$. For DV3, doubling the IV halves the DV, so the relationship is probably $y = \frac{m}{x}$. For DV4, doubling the IV quarters the DV, so this indicates an inverse square relationship $y = \frac{m}{x^2}$.

Practise your skills

Use Table 4 to answer the questions that follow.

TABLE 4 Independent variable and dependent variables for three types of relationships

IV	DV1	DV2	DV3
1	960	240	5
3	107	80	45
6	27	40	180
9	12	27	405
12	7	20	720
15	4	16	1125

1 **Propose** the relationship between DV1 and the IV. **Justify** your proposal. (2 marks)

2 **Propose** the relationship between DV2 and the IV. **Justify** your proposal. (2 marks)

3 **Propose** the relationship between DV3 and the IV. **Justify** your proposal. (2 marks)

Real-world physics

Gravity and your body

When astronauts arrive back on Earth after an extended stay in space, they are about 5 cm taller than when they left. This is a result of what's often called the "microgravity" of space, which allows the spine to elongate. When the astronauts are back on Earth, gravity (downwards) and the normal force from the surface of Earth (upwards) compress the spine's length back to normal.

FIGURE 7 The intervertebral discs of the spine are shown in pink. Gravity causes these discs to compress.

The main compression occurs in the cartilage discs between the vertebrae of the spine (Figure 7). This is the main function of these discs – to act as a springy insulator against compression when standing, walking or lifting. Of course, other parts of the body undergo compression, but not as much as the discs. If you measure your height in the morning it will be larger than in the afternoon because of the downwards pull of gravity. You probably shrink by a centimetre or so over the day.

Many tests have been done on the compression of the spine due to gravity. These tests are expressed using a quantity called Young's modulus of elasticity (E). The higher the value of Young's modulus, the less the object responds to compression.

For a person with an initial length when standing of L_0, a length when lying of L, and a change in length of $\Delta L = L - L_0$, Young's modulus, E, is:

$$E = \frac{FL_{\text{spine}}}{A\Delta L}$$

where F is the force due to gravity (your weight), A is the cross-sectional area at your waist, and L_{spine} is the length of your spine, assumed to be $\frac{L_0}{2}$.

This can be tested in class by measuring the height of a person standing (L_0), their height lying horizontally L, their waist circumference C, and mass m. You can calculate the force of gravity, as it is just the weight of the person, $F_g = mg$. Your cross-sectional area can be calculated from the waist circumference using $A = \frac{C^2}{4\pi}$, and $\Delta L = L - L_0$. The results will be in N m^{-2}, which are more commonly called pascal, Pa.

Table 5 provides some sample measurements for a Year 12 student.

TABLE 5 Measurements for a Year 12 student

Length (standing), L_0	Length (lying), L	Waist, C	Mass, m
170.5 cm	172.0 cm	70.0 cm	59.0 kg

The mean value and uncertainty from the literature for adult humans is 0.974 ± 0.408 MPa, where a megapascal, MPa, is 10^6 Pa.

The study of spinal compression with age, microgravity and injury is an ongoing field of research.

Apply your understanding

1 **Calculate** the length of the person's spine in Table 5 (4 s.f.). (1 mark)

2 Two astronauts have identical measurements of height, waist and mass. **Determine** who would have the higher Young's modulus: the one whose length changed by 5 cm or the other, whose length changed by 3 cm. (1 mark)

3 **Determine** Young's modulus for the student shown in Table 5 (3 s.f.). **Identify** whether it is in the range 0.974 ± 0.408 MPa. (4 marks)

Challenge

Drops of mercury

Two spherical drops of mercury each of mass 1.36 g are resting on a frictionless surface with their centres 1.00 cm apart and their surfaces 1.0 mm apart. Assume the only force between them is that of gravity. **Determine** how much time it would take for them to touch (that is, how long it would take each of them to move 0.5 mm). (6 marks)

Check your learning 5.2

Check your learning 5.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 Newton's law of gravitation is an inverse square law. **Explain** what that means and use an example. (2 marks)

2 **Explain** why Earth's radius needs to be taken into account for objects near the surface, when calculating gravitational forces. (1 mark)

3 **Calculate** the force between the Sun ($m_{Sun} = 1.99 \times 10^{30}$ kg) and Earth ($m_{Earth} = 5.97 \times 10^{24}$ kg), assuming their centres are 1.5×10^{8} km apart (2 s.f.). (3 marks)

4 A 10 kg rock rests on the ground on Earth. **Calculate** the gravitational force acting on it, using Newton's law of universal gravitation (2 s.f.). (3 marks)

Analytical processes

5 **Determine** what happens to the gravitational force when

a one of the masses is doubled (1 mark)

b the distance between the objects is halved. (1 mark)

6 Black holes are supermassive collapsed stars. The closest anything can get to one and still escape its gravitational force is called the "event horizon". **Determine** the force acting on a 15 tonne spacecraft at the event horizon 30 km from a black hole. The black hole has a mass equal to 10 times that of the Sun (2 s.f.). (3 marks)
Mass of Sun = 1.99×10^{30} kg; 1 tonne = 1,000 kg

7 When a star collapses to form a black hole, the size of the star is greatly reduced although the mass remains the same. **Deduce**, with justification, what would happen to the gravitational force of the Sun on our Earth if the Sun unexpectedly collapsed to form a black hole. (2 marks)

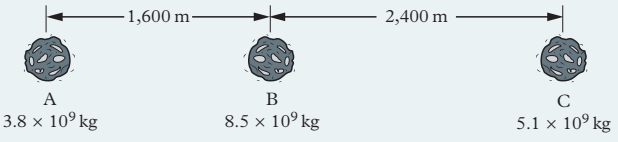
Knowledge utilisation

8 Jupiter is about 300 times more massive than Earth, so it would be easy to deduce that an object on the surface of Jupiter would weigh 300 times more than on the surface of Earth. For example, a rover (spacecraft) with a weight of 9,000 N on Earth might be expected to weigh 2,700,000 N on the surface of Jupiter. But this is not the case. A 9,000 N rover on Earth weighs only about 27,000 N on the surface of Jupiter. **Evaluate** this scenario and identify any misunderstandings. (3 marks)

9 A communications satellite is orbiting a planet. A student said that if you double the mass of the satellite or double the mass of the planet, the force between them will double. Another student said that doubling the mass of the planet has a bigger effect than doubling the mass of the satellite. **Evaluate** both claims and decide who is correct. (2 marks)

10 Three asteroids, A, B and C, are in a line as shown in the diagram. Their masses and separation distances are also shown. **Determine** the net force on B due to A and C (2 s.f.).

(3 marks)



Lesson 5.3
Gravitational fields

Key ideas

- → Gravitational fields are the region of space surrounding a body in which another body experiences a force of gravitational attraction.
- → Problems involving gravitational field strength at a distance from an object can be solved using $g = \frac{F}{m} = \frac{GM}{r^2}$.

Learning intentions and success criteria

What are gravitational fields?

You may have seen a demonstration in which iron filings are sprinkled over paper that has a magnet underneath, similar to the one in Figure 1. The patterns produced reveal lines of force and show the shape of the magnetic field – the region of space surrounding the magnet in which magnetic materials experience a force.

Study tip

It's worth making sure you understand the difference between a "scalar" field and a "vector" field at this point. These reappear in later modules when we look at the various fields that affect elementary particles.

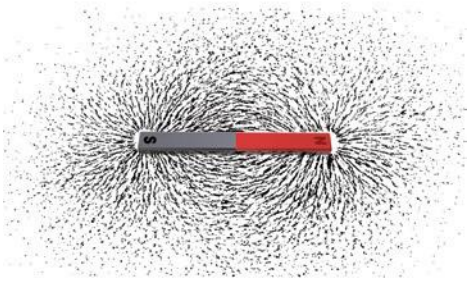


FIGURE 1 The magnetic field about a permanent magnet is shown as field lines.

A nice way to conceptualise a field is to imagine being in your physics laboratory and measuring the temperature of hundreds of points in the room and then mapping them on a diagram. This would be called a temperature field. Each point has a particular magnitude (e.g. 23.0°C, 24.5°C) but no direction, so it is called a scalar field. Given enough time, wisdom and a computer, someone may be able to develop an equation for the field.

Now, imagine you lit a Bunsen burner. The flame would be a source of temperature change that would affect your carefully mapped temperature field.

You could also map a vector field – for example, using compasses to map the magnetic field revealed by iron filings at the start of this lesson. In this case, the field has a direction (N, S, E, W). If you turned on an electromagnet nearby, the compasses would show that the field had changed.

gravitational field
the region of space surrounding a body in which another body experiences a force of gravitational attraction

In 1850, the English scientist Michael Faraday compared magnetic forces with electric and gravitational forces and conceptualised fields for them all. The use of a **gravitational field** as a way of depicting gravitational forces is now common.

In general, a field is a region of space where particular forces of nature are experienced. The force is described as mediated (brought about) by the field.

Challenge

Weighing less on the bathroom scales

Imagine you are standing on some bathroom scales, and you bend your knees quickly. **Predict** what would happen to the scale reading and try to explain why. (2 marks)

What is the direction of a gravitational field?

gravitational field direction
in the direction of the net gravitational force

Gravity is a vector field as it has both magnitude and direction. The **gravitational field direction** is in the direction of the net gravitational force. For the surface of Earth, the net force acts down towards the surface. "Down" is a local term and is from the perspective of the viewer on Earth's surface. A more general term would be "towards Earth's centre" (Figure 2).

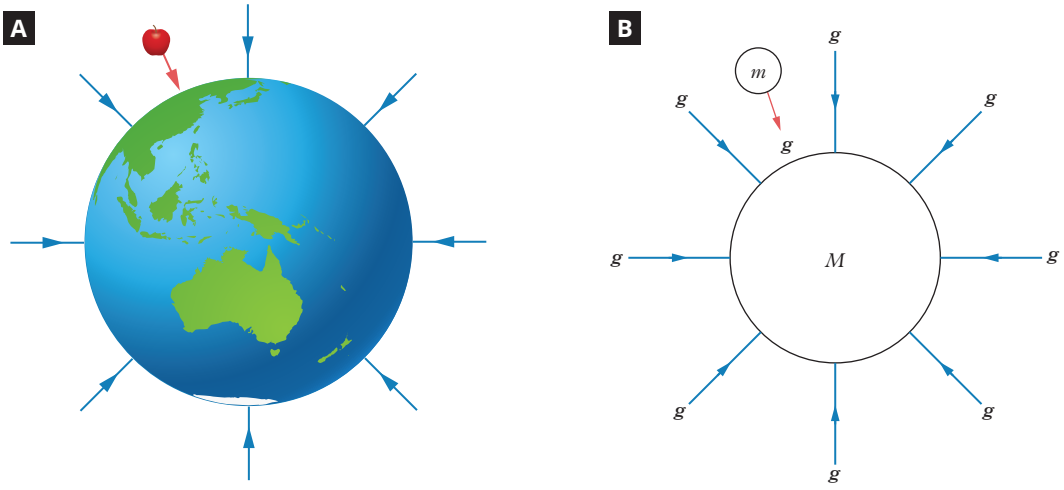


FIGURE 2 (A) All gravitational field lines point towards Earth's centre. A dropped apple would move in that direction. (B) In general, all gravitational field lines point towards the object's centre of mass. A small test mass would move in that direction.

Can we have antigravity?

Can the field arrows point the other way? In other words, can we have antigravity – a force that pushes two objects apart? The answer seems to be "no". Unlike electrostatic and magnetic forces (like charges/poles repel, unlike attract), physicists have never observed a repulsive gravitational force, only an attractive one, although it has been postulated that dark matter and dark energy may have repulsive properties.

Einstein produced a comprehensive theory of gravity, his general theory of relativity, in 1915. This theory argues that gravitational force is different from forces such as the magnetic and electrostatic forces, even though the mathematical relationships are identical. Einstein said that gravity is not so much something that happens in space; rather, it is a distortion or a warp of space itself. His theory encompasses all of Newton's laws and takes them further. Einstein's theory has been confirmed by research thousands of times, and physicists accept his theory as being the best current model for forces in the universe.

What is gravitational field strength?

We know that gravity is stronger on the surface of Earth than on the Moon, and that gravity at the event horizon of a black hole is so strong it would rip you apart. They have different field strengths as a measure of gravity. Field strength is a measure of the force acting on 1 unit of a given substance in the field. For an electric field, it is the force experienced by a 1 coulomb (+1 C) positive test charge in the field. Likewise, for a gravitational field, it is the force experienced by a 1 kg "test mass" of matter in the field. There's no such thing as negative mass, so a test charge is just 1 kg (not +1 kg).

Field forces

gravitational field strength
the net force, F_g, per unit mass, m, at a particular point in the gravitational field (symbol, g)

Gravitational field strength is described as force per unit mass, i.e. $\frac{F}{m}$. Newton's second law of motion for forces on an object in a gravitational field, $F_g = mg$, can be rearranged to give $g = \frac{F_g}{m}$. This says that the gravitational field strength (g) is the net force due to gravity (F_g) per unit mass (m) at a particular point in the gravitational field.

As F_g is a vector quantity and m is a scalar quantity, it follows that g is also a vector quantity and points in the same direction as the force. This is as you might expect: the force of gravity is towards Earth so objects accelerate in the direction of the field, which is also towards Earth's centre – the field points down.

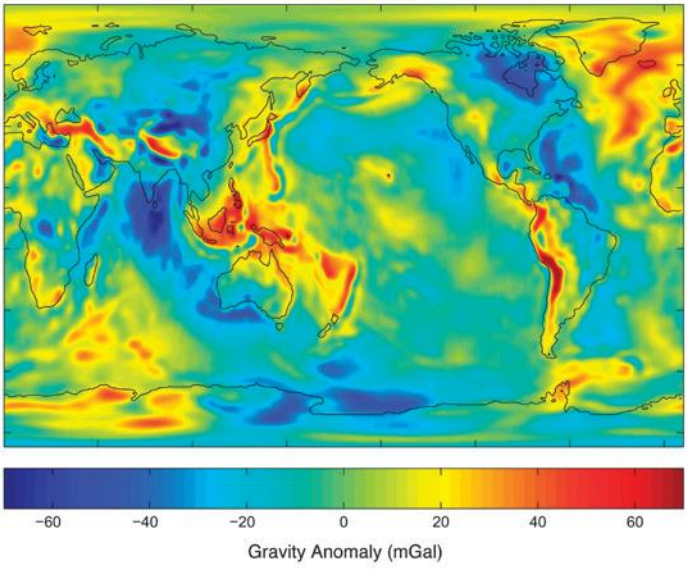


FIGURE 3 High-resolution gravity maps, such as this one of the world, were constructed from billions of calculations. Red indicates greater values of g; blue indicates lower values of g.

Worked example 5.3A

Gravitational field strength using weight

A 650 g mass is allowed to hang freely off a spring balance on the Moon. The scale is in newtons (N) and the weight reading on the scale is 1.06 N.

a **Calculate** the gravitational field strength on the Moon in $\mathrm{N\,C^{-1}}$. Give your answer to an appropriate number of significant figures. (2 marks)

b **Express** the gravitational field strength on the Moon as acceleration due to gravity on the Moon in $\mathrm{m\,s^{-2}}$. Give your answer to an appropriate number of significant figures. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer. "Express" means to convey, show or communicate something.

Think	Do
Step 2: Gather the relevant data from the questions and identify the appropriate formula.	**a** $m = 650\,\text{g} = 0.650\,\text{kg}$, $F_g = 1.06\,\text{N}$ $g = \frac{F_g}{m}$
Step 3: Substitute the values into the formula. (1 mark for "Provides sound mathematical reasoning")	$g = \frac{1.06}{0.650}$ (1 mark)
Step 4: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	$g = 1.6\,\text{N kg}^{-1}$ (2 s.f.) (1 mark)
Step 5: Restate the answer with the same magnitude but with the equivalent unit m s^{-2} (the acceleration due to gravity on the Moon). Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	**b** $g = 1.6\,\text{m s}^{-2}$ (1 mark)

Your turn

The gravitational field strength on Mars is $3.7\,\text{N C}^{-1}$. **Calculate** the gravitational force acting on a 2.5 kg object on Mars. Give your answer to an appropriate number of significant figures. (2 marks)

How do we work out *g* from theory?

The strength of the gravitational field can now be determined – not by measuring experimentally the force acting on a 1 kg test object, but by using Newton's law of universal gravitation to calculate the field strength g at a distance r from a body of mass M.

To do this, F_g in the formula $g = \frac{F_g}{m}$ is replaced with $\frac{GMm}{r^2}$, where G is the universal gravitational constant, $6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$, M (kg) is the mass of the larger object, m (kg) is the mass of the smaller object and r (m) is the distance between their centres. Using Newton's law:

$$g = \frac{F_g}{m}$$
$$= \frac{\frac{GMm}{r^2}}{m}$$
$$= \frac{GM}{r^2}$$

This allows g to be calculated for Earth if m_{Earth} and r are known. The units will be N kg^{-1} but these are equivalent to the units for acceleration, m s^{-2}.

Worked example 5.3B

Gravitational field strength using "big *G*"

Calculate the gravitational field strength at

a a point 10,000 km above Earth's centre (2 marks)

b Earth's surface (mean radius of Earth $r_E = 6.37 \times 10^6\,\text{m}$). (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number of answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: Gather the relevant data from the question and identify the appropriate formula.	**a** $r = 10{,}000\,\text{km} = 10{,}000 \times 10^3\,\text{m} = 10^7\,\text{m}$ Mass of Earth, $m_{\text{Earth}} = 5.97 \times 10^{24}\,\text{kg}$ Mean radius of Earth (r_{Earth}) $= 6.37 \times 10^6\,\text{m}$ $g = \frac{GM}{r^2}$
Step 3: Substitute the values into the formula and solve for g. (1 mark for "Provides sound mathematical reasoning")	$g = \frac{GM}{r^2}$ $= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(10^7)^2}$ $= 3.98\,\text{m s}^{-2}$ (or N kg^{-1}) (1 mark)
Step 4: Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. As g is a vector quantity, make sure you include the direction (downwards). (1 mark for "Calculates the (correct) field strength") Exam markers will likely be told that the answer must include the direction.	The field strength, g is $3.98\,\text{N kg}^{-1}$ (or m s^{-2}) directed downwards towards Earth. (1 mark)
Step 5: Substitute the values into the formula and solve for g. (1 mark for "Provides sound mathematical reasoning") Step 6: Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. Again, as g is a vector quantity, make sure you include the direction (downwards). (1 mark for "Calculates the (correct) field strength", including direction)	**b** $g = \frac{GM}{r^2}$ $= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2}$ $= 9.81\,\text{m s}^{-2}$ (1 mark) The field strength is $9.81\,\text{N kg}^{-1}$ (or m s^{-2}) directed downwards towards Earth. (1 mark)

Your turn

Calculate the gravitational field strength at

a a point 2,000 km above the centre of the Moon. Give your answer to an appropriate number of significant figures. (2 marks)

b Moon's surface (radius of the Moon, $r_{\text{Moon}} = 1.74 \times 10^6$ m). Give your answer to an appropriate number of significant figures. (2 marks)

How does gravitational field strength, *g*, vary with location and altitude?

In Worked example 5.3B, the value given for the radius of Earth is the mean radius of 6.37×10^6 m. However, Earth's radius varies from the equator (6.37×10^6 m) to the poles (6.36×10^6 m). Thus, g will vary across Earth; for example, in Peru it is $9.76\,\text{m s}^{-2}$ and at the Arctic it is $9.83\,\text{m s}^{-2}$.

Above Earth's surface, the value of g decreases with the distance from the centre of Earth following an inverse square relationship. At two Earth radii from the centre of Earth (i.e. one Earth radius above the surface), g would be one-quarter of $9.8\,\text{m s}^{-2}$ ($2.45\,\text{m s}^{-2}$), and at three Earth radii from the centre, g would be one-ninth of $9.8\,\text{m s}^{-2}$ ($1.09\,\text{m s}^{-2}$). Further values are shown in Table 1.

TABLE 1 Acceleration due to gravity at distances from the surface and centre of Earth

Acceleration due to gravity ($m\,s^{-2}$)	Distance from Earth's surface (km)	Distance from Earth's centre (km)
9.8	0	6,400
9.0	270	6,670
8.0	670	7,050
7.0	1,160	7,560
5.0	2,540	8,940
3.0	5,150	11,550
2.0	7,740	14,140
1.0	13,590	19,990

At the surface of Earth, on average g is given the nominal value of $9.8\,m\,s^{-2}$, and that's the value in the QCAA Physics Formula and data book that you will use for problem-solving. In experimental work you should use the value of g for your area.

Gravitational field strength inside Earth

In theory, at the centre of Earth the gravitational fields would all point outwards and cancel out. The field strength would be zero. Scientists have calculated that the gravitational field strength decreases linearly as you go from the surface to the centre. This is shown in Figure 4.

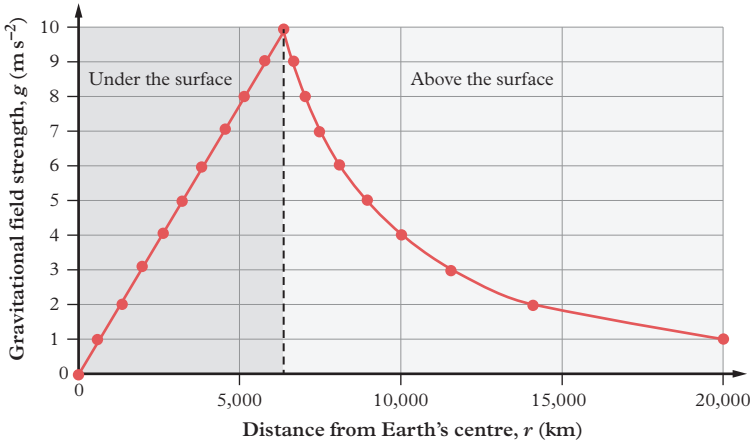


FIGURE 4 Variation in g from the centre of the Earth to the surface and upwards into nearby space

Challenge

Scale readings in a mine

If you took a set of bathroom scales to the bottom of a deep mine shaft, would your scale reading be less, the same or greater than on the surface? (1 mark)

Are you weightless in space?

In his 1867 novel *From the Earth to the Moon*, science fiction writer Jules Verne predicted that there would be a spot between Earth and the Moon where a space traveller would be weightless. We now know that you can achieve a state of so-called "weightlessness" in a space station close to Earth, or even just jumping off a diving board.

What Jules Verne was trying to say was that there was a point somewhere on the line between Earth and the Moon where the gravitational acceleration towards Earth and towards the Moon would cancel out, as shown in Figure 5.

In the figure, P is the point where the gravitational forces are equal and opposite. That is, the gravitational field strengths are equal and opposite at that point. The radius of the Moon's orbit around the Earth has the symbol r. If we let x be the distance from Earth to P, then the distance from P to the Moon will be $r - x$.

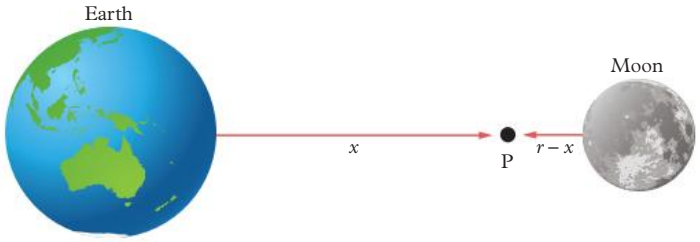


FIGURE 5 P is the point between Earth and the Moon where g is zero.

Then, point P can be found by equating $g = \frac{Gm_{\text{Earth}}}{r^2}$ for both bodies. It is known that Earth's mass, $m_{\text{Earth}} = 5.97 \times 0^{24}\text{kg}$, the Moon's mass, $m_{\text{Moon}} = 7.35 \times 10^{22}\text{kg}$, and the Earth–Moon distance, $r = 3.8 \times 10^{8}\text{m}$.

$$g_{\text{towards Earth}} = g_{\text{towards Moon}}$$
$$\frac{Gm_{\text{Earth}}}{x^2} = \frac{Gm_{\text{Moon}}}{(r-x)^2}$$
$$\frac{5.97 \times 10^{24}}{x^2} = \frac{7.35 \times 10^{22}}{(3.8 \times 10^{8} - x)^2}$$
$$\frac{x^2}{(3.8 \times 10^{8} - x)^2} = \frac{5.97 \times 10^{24}}{7.35 \times 10^{22}} \text{ (take square roots of both sides)}$$
$$\frac{x}{3.8 \times 10^{8} - x} = \sqrt{81.22}$$
$$x = 3.42 \times 10^{8}\text{m}$$

Thus, the distance from point P to Earth = 3.42×10^{8}m. This is where the gravitational field strength is zero.

Worked example 5.3C

Where does gravitational field strength equal zero?

Two supermassive stars A and B are 4.5×10^{13}m apart. Star A has a mass of 10 solar masses (the mass of the Sun, m_{Sun}). Star B has a mass of $40m_{\text{Sun}}$. There is a point P between the stars where the gravitational field strength is zero. **Determine** the distance from Star A to P. Give your answer to an appropriate number of significant figures. (5 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$m_{\text{A}} = 10m_{\text{Sun}}$, $m_{\text{B}} = 40m_{\text{Sun}}$, $d = 4.5 \times 10^{13}$m $g = \frac{GM}{r^2}$
Step 3: Draw a labelled diagram to show the situation. (1 mark for "Draws diagram to show relationships")	$10\,m_{\text{Sun}}$ A — P — B $40\,m_{\text{Sun}}$ r_{A} and $4.5 \times 10^{13} - r_{\text{A}}$ (1 mark)
Step 4: Let g at P due to star A be equal in magnitude to g at P due to star B. They will be in opposite directions. (1 mark for "Shows equivalence")	$g_{\text{PbyA}} = g_{\text{PbyB}}$ $\frac{Gm_A}{r_A^2} = \frac{Gm_B}{(4.5 \times 10^{13} - r_{\text{A}})^2}$ (1 mark)

Think	Do
Step 5: Substitute in the known values and rearrange so the unknowns are on one side. (1 mark for "Correct substitution")	$\frac{10}{{r_A}^2} = \frac{40}{(4.5 \times 10^{13} - r_A)^2}$ $\frac{10}{40} = \frac{{r_A}^2}{(4.5 \times 10^{13} - r_A)^2}$ (1 mark)
Step 6: Take the square roots of both sides and simplify. Solve for r_A. (1 mark for "Correct mathematical reasoning")	$\frac{1}{2} = \frac{r_A}{4.5 \times 10^{13} - r_A}$ (1 mark) $2r_A = 4.5 \times 10^{13} - r_A$ $3r_A = 4.5 \times 10^{13}$ $r_A = 1.5 \times 10^{13}$
Step 7: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. Note that the question says "between the stars"; otherwise, you could just say "at infinity". (1 mark for "Provides correct answer")	$r_A = 1.5 \times 10^{13}$ m (2 s.f.) (1 mark)

Your turn

Two stars, A and B, are 9.0×10^{13} m apart. Star A has a mass of 30 solar masses (the mass of the Sun, M_S). Star B has a mass of $60M_S$. There is a point P between the stars where the gravitational field strength is zero. **Determine** the distance from Star A to P. Give your answer to an appropriate number of significant figures. (5 marks)

What does weightless mean?

"Weightless" is a confusing term. Your weight is just your mass multiplied by the local value of gravitational field strength, g, i.e. $F_g = mg$. But the term "weightless" seems to imply your weight is zero. When you jump off a diving board or float in a swimming pool, your weight remains the same as if you were walking around. The same is true for astronauts who float around in the International Space Station. They still have the same mass, and while g is lower at that altitude, it isn't zero. Therefore, their weight can't be zero. They are just falling as if they were jumping off a diving tower. They are not without mass, so they are not really weightless; they are just in free-fall.

black hole
a region of space where the gravitational field is so intense that no matter or almost no radiation, except a type called Hawking radiation, can escape

Real-world physics

Gravitational fields and black holes

Our model of the solar system has developed as new technology has allowed us to see further into the cosmos. This has enabled us to gather and analyse greater amounts of data. Such data has helped scientists identify **black holes** in our universe.

A black hole is a term used to describe a region of space that contains matter so dense that even light cannot escape its grip. It was coined by John Wheeler of Princeton University (USA) in 1967. A black hole is thought to come from the gravitational collapse of a star. The first tentative identification of a black hole was announced in December 1972 in the binary-star X-ray source Cygnus X-1. It seems there are supermassive black holes at the centres of all currently known galaxies. They are hundreds of thousands to billions of solar masses (one solar mass m_{Sun} = mass of the Sun). In the case of our galaxy, the Milky Way, there appears to be a 4.3 million solar mass black hole in the region of Sagittarius A.

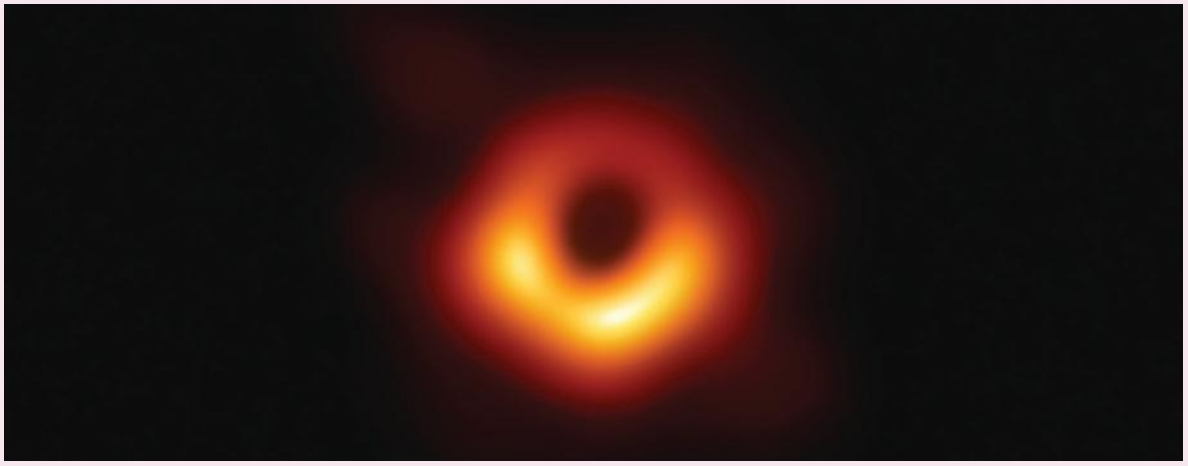


FIGURE 6 This image, released in April 2019, is the first ever image of a black hole. It was created using data collected by eight telescopes around the world.

For a black hole with a mass ten times that of our Sun, the point at which light cannot escape (i.e. the event horizon, the point at which it becomes "black") is within 30 km of the centre. This is called the **Schwarzschild radius** and is shown in Figure 7.

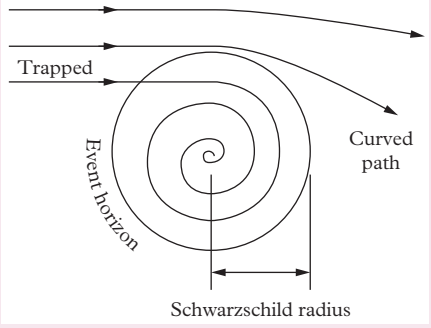


FIGURE 7 The event horizon and Schwarzschild radius of a black hole

Apply your understanding

1 **Calculate** the gravitational field strength at a distance of 30 km from a black hole of 10 solar masses ($m_{Sun} = 1.99 \times 10^{30}$ kg) (2 s.f.). (3 marks)

2 **Determine** the Schwarzschild radius r_S for the supermassive black hole Phoenix A, which has a mass of $1 \times 10^{11} m_{Sun}$. (3 marks)

3 **Calculate** the gravitational force between two black holes, each of which has a mass of $100 m_{Sun}$, and which are 1.5×10^{14} m apart (2 s.f.). (3 marks)

Schwarzschild radius
a point near a black hole where the gravity is so powerful that nothing, not even light, can escape; given by the formula $r_S = \frac{GM}{c^2}$ where M is the mass of the black hole and c is the speed of light (3×10^8 m s^{-1}).

How is gravitational field strength, *g*, calculated in 2D?

So far, we have considered the gravitational field in one dimension – that is, at a given distance from an object, or at a position in a line between two objects. We can also look at the gravitational field in two dimensions (2D). This is a more complex case because we end up having to add vectors in 2D. The syllabus makes it clear that adding vectors in 2D is a part of the Unit 3 and 4 course leading up to the external exam.

Such situations are not uncommon in astrophysics, where a planet's gravitational field influences another planet's orbit about the Sun. In fact, that's how Neptune was discovered. Astronomers noticed the odd motion of Uranus as it orbited the Sun, which could not be reconciled with Newton's law of universal gravitation for the Sun alone.

To learn how to calculate field strength in 2D, first let's look at a fairly straightforward example with the fields at right angles (Worked example 5.3D).

Worked example 5.3D

Gravitational fields in 2D at right angles

A spacecraft, S, is equidistant from two black holes that are 2.828×10^8 m apart. Black hole A on the left has a mass of 1×10^{26} kg and black hole B on the right has a mass of 2×10^{26} kg, twice that of A. From the spacecraft's point of view, the angle between the black holes is 90°, as shown in Figure 8.

Calculate the magnitude and direction of the gravitational field due to the two black holes that is experienced by the spacecraft at this position. Use a vector diagram. Give your answer to an appropriate number of significant figures. (7 marks)

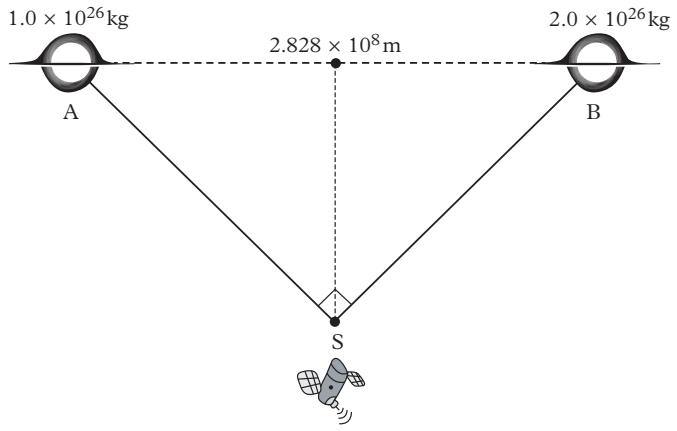
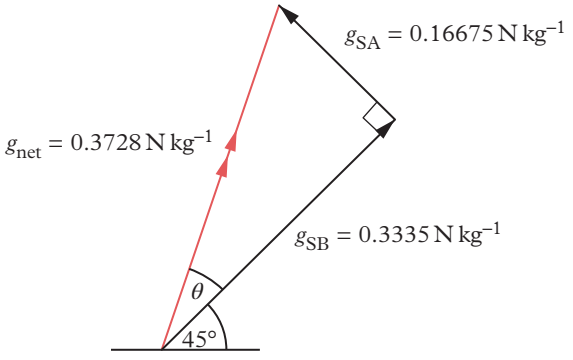


FIGURE 8 Field near a binary black hole

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question. Identify that this is a question about gravitational field strength and write down the appropriate formula.	$g = \frac{GM}{r^2}$
Step 3: Calculate the distance between the spacecraft and black hole A. The triangle ASB is an isosceles, right-angled triangle – that is, a triangle with two equal sides and an interior angle of 90°. The side AS is found using Pythagoras' theorem. (1 mark for "Calculates the length of AS")	$\text{AS} = \sqrt{\frac{(2.828 \times 10^8)^2}{2}} = 2 \times 10^8 \text{ m}$ (1 mark) The side BS has the same length.
Step 4: Calculate the gravitational field strength on S due to A using the appropriate formula. (1 mark for "Calculates field strength at S by A")	$g_{\text{on S due to A}} = \frac{GM}{r^2}$ $= \frac{6.67 \times 10^{-11} \times 1 \times 10^{26}}{(2 \times 10^8)^2}$ $= 0.16675 \text{ N kg}^{-1}$ (1 mark)
Step 5: Add the two field-strength vectors head to tail at right angles to each other. (1 mark for "Constructs correct vector diagram") Note: the resultant is not required for the mark at this stage.	 (1 mark)
Step 6: Calculate the magnitude of the gravitational field strength on S due to B. (1 mark for "Provides correct mathematical reasoning")	$g_{\text{on S due to B}} = \frac{GM}{r^2}$ $= \frac{6.67 \times 10^{-11} \times 2 \times 10^{26}}{(2 \times 10^8)^2}$ $= 0.3335 \text{ N kg}^{-1}$ (1 mark)

Think	Do
Step 7: Add the two field strength vectors head to tail at right angles to each other and draw the resultant. Use Pythagoras' theorem to find the magnitude of the resultant, g_{net}. (1 mark for "Provides correct mathematical reasoning")	$g_{net} = \sqrt{(0.3335)^2 + (0.16675)^2}$ $= 0.3728\,\text{N kg}^{-1}$ $= 0.37\,\text{N kg}^{-1}$ (2 s.f.) (1 mark)
Step 8: Solve for the angle, θ. (1 mark for "Provides mathematical reasoning")	$\theta = \tan^{-1}\left(\frac{0.16675}{0.3335}\right)$ $= 27°$ (1 mark)
Step 9: State the final answer, giving the magnitude and direction of the field. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	The net gravitational field strength is $0.37\,\text{N kg}^{-1}$ at an angle of 27° to the left of black hole B. (1 mark)

Your turn

Two stars, A and B, with equal masses of 5.0×10^{20} kg, are 8.0×10^{7} m apart. A spacecraft is located at point P, a distance of 5.6×10^{7} m from each star, and at a right angle to the stars (Figure 9). **Calculate** the magnitude and direction of the net gravitational field experienced by the spacecraft at P. Give your answer to an appropriate number of significant figures. (7 marks)

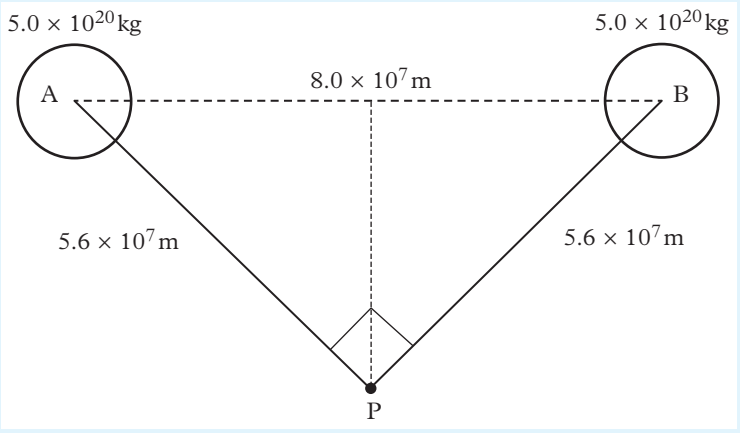


FIGURE 9 The gravitational field near two stars

How is the net gravitational field determined in more-complex 2D situations?

We have now looked at fields around two objects that make an isosceles right-angled triangle with a point in space. But this can be extended to more complex cases. This type of situation is introduced here because it will also apply to electromagnetic fields later in this unit.

Consider the gravitational field surrounding two astronomical objects. In this case we will use two black holes bound together as a binary system. Researchers in 2023 discovered a supermassive black hole binary system, one of only two such known systems. The two black holes, which orbit each other, are thought to be 100 million solar masses each and are expected to merge eventually. That event will release a massive amount of energy in the form of gravitational waves, causing ripples in space in every direction (and oscillations in matter) as the waves pass through.

Figure 10 shows two black holes, A and B, of 100 solar masses each (where $m_{Sun} = 1.99 \times 10^{30}$ kg). They are 1.5×10^{14} m apart. This is equivalent to 1,000 AU (astronomical units), where 1 AU is the distance between Earth and the Sun.

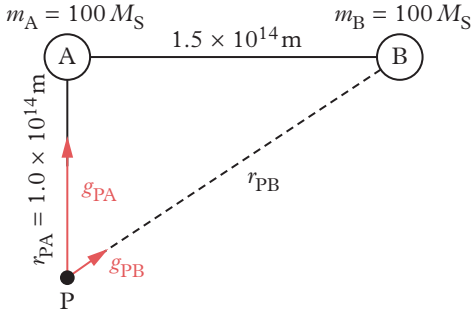


FIGURE 10 Binary black holes A and B and their fields at point P shown by the labelled arrows g_{PA} (g at P due to A) and g_{PB} (g at P due to B).

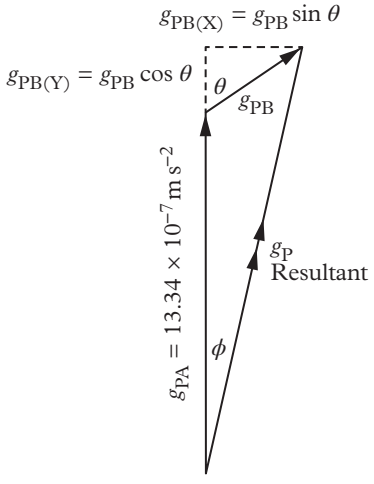
Consider the point in space, P, 1.0×10^{14} m away from black hole A and an unknown distance, r_{PB}, from black hole B. We can calculate the net gravitational field strength at point P by the vector addition of the contributions from black holes A and B.

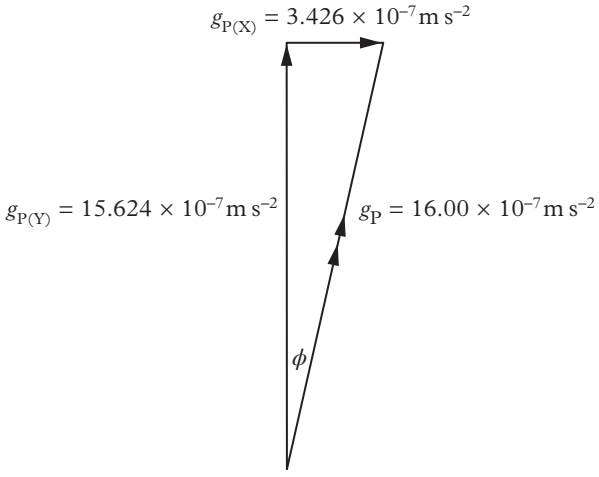
In Worked example 5.3E, the technique for adding vectors in 2D is shown as it relates to the binary black holes above.

Worked example 5.3E

More-complex gravitational fields in 2D

Using the data relating to the binary black holes in Figure 10, **calculate** the net gravitational field strength (magnitude and direction) at point P. Give your answer to an appropriate number of significant figures. (9 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$r_{PA} = 1.0 \times 10^{14}$ m, $d = 1.5 \times 10^{14}$ m, $m_A = m_B = 100M_S$ $g = \frac{GM}{r^2}$
Step 3: Calculate the distance from P to B using Pythagoras' theorem. (1 mark for "Provides correct mathematical reasoning")	$r_{PB} = \sqrt{(1 \times 10^{14})^2 + (1.5 \times 10^{14})^2}$ $= 1.803 \times 10^{14}$ m (1 mark)
Step 4: Calculate the angle, θ, between the directions PA and PB using trigonometry. (1 mark for "Identifies the correct angle")	$\theta = \tan^{-1}\left(\frac{1.5 \times 10^{14}}{1.0 \times 10^{14}}\right)$ $= 56.31°$ (1 mark)
Step 5: Calculate the gravitational field strength at P due to black hole A (g_{PA}). (1 mark for "Calculates the field at P due to A") Note: the field could be written as 13.34×10^{-7} m s^{-2} or 1.334×10^{-6} m s^{-2}.	$g_{PA} = \frac{GM_A}{r_{PA}^2}$ $= \frac{6.67 \times 10^{-11} \times 100 \times 2 \times 10^{30}}{(1 \times 10^{14})^2}$ $= 1.334 \times 10^{-6}$ m s^{-2} (1 mark)
Step 6: Calculate the gravitational field strength at P due to black hole B (g_{PB}). (1 mark for "Calculates the field at P due to B")	$g_{PB} = \frac{GM_A}{r_{PA}^2}$ $= \frac{6.67 \times 10^{-11} \times 100 \times 2 \times 10^{30}}{(1.803 \times 10^{14})^2}$ $= 0.4117 \times 10^{-6}$ m s^{-2} (1 mark)
Step 7: Add the two vectors g_{PA} and g_{PB} by joining the arrows head to tail. Draw in the resultant vector, g_P. (1 mark for "Draws correct vector diagram")	 (1 mark)

Think	Do
Step 8: Resolve the two vectors g_{PA} and g_{PB} into components along the x- and y-axes. (1 mark for "Calculates the components of the field on P due to B")	$g_{PB(x)} = g_{PB}\sin\theta$ $= 0.4117 \times 10^{-6}\sin 56.31°$ $= 3.426 \times 10^{-7}\ \text{m s}^{-2}$ $g_{PB(y)} = g_{PB}\cos\theta$ $= 0.4117 \times 10^{-6}\cos 56.31°$ $= 2.284 \times 10^{-7}\ \text{m s}^{-2}$ (1 mark)
Step 9: Add the x-components of the two vectors and add the y-components of the two vectors. (1 mark for "Calculates the sum of fields in both directions")	x-component: $g_{PA(x)} = 0$ $g_{PB(x)} = 3.426 \times 10^{-7}$ $g_x = g_{PA(x)} + g_{PB(x)}$ $= 3.426 \times 10^{-7}\ \text{m s}^{-2}$ y-component: $g_{PA(y)} = 13.34 \times 10^{-7}$ $g_{PB(y)} = 2.284 \times 10^{-7}$ $g_y = g_{PA(y)} + g_{PB(y)}$ $= 13.34 \times 10^{-7} + 2.284 \times 10^{-7}$ $= 15.624 \times 10^{-7}\ \text{m s}^{-2}$ (1 mark)
Step 10: Add the two vectors g_x and g_y by joining the arrows head to tail. Draw in the resultant vector, g_P. Solve for the resultant, g_P. Calculate the angle, ϕ. (1 mark for "Provides correct mathematical reasoning")	 $\phi = \tan^{-1}\left(\frac{3.426 \times 10^{-7}}{15.624 \times 10^{-7}}\right)$ $= 12.4°$ (1 mark)
Step 11: Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. Make sure you include the direction. (1 mark for "Provides correct answer")	The gravitational field at P is $16.0\ \text{m s}^{-2}$ (3 s.f.) at an angle of 12.4° towards B from the line joining A and P. (1 mark)

Your turn

A particular binary black hole consists of two black holes, A and B, each of 100 solar masses and separated by a distance of 1.5×10^{14} m. **Calculate** the net gravitational field strength at a point Q that is 1.5×10^{14} m away from black hole A, perpendicular to the line joining A and B, as shown in Figure 11. Give your answer to an appropriate number of significant figures. (9 marks). Note: 1 solar mass, $m_{Sun} = 1.99 \times 10^{30}$ kg.

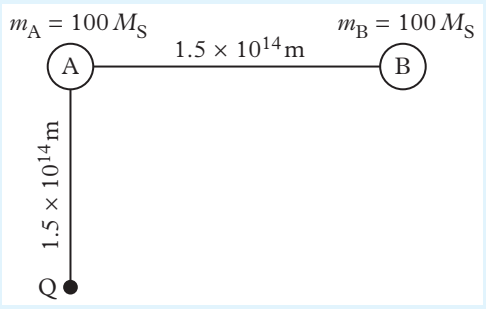


FIGURE 11 Black holes A and B form a binary system. Line QA is perpendicular to AB.

Check your learning 5.3

Check your learning 5.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how the unit symbol $N\,kg^{-1}$ is equivalent to $m\,s^{-2}$. (2 marks)

2 **Describe** a gravitational field. (1 mark)

3 **Define** "gravitational field strength" and state its units. (2 marks)

4 **Explain** what is meant by "the gravitational field strength does not depend on the mass of the object in the field". (2 marks)

5 **Calculate** the gravitational field strength at a point 100,000 km above Earth's centre. The mass of the Earth $m_{Earth} = 5.97 \times 10^{27}$ kg, mean radius of Earth $r_{Earth} = 6.37 \times 10^{6}$ m (3 s.f.). (3 marks)

6 **Clarify** how the direction of a gravitational field is defined. (1 mark)

Analytical processes

7 **Determine** the gravitational field strength at a point whose distance from Earth's surface is equal to three Earth radii (2 s.f.). (3 marks)

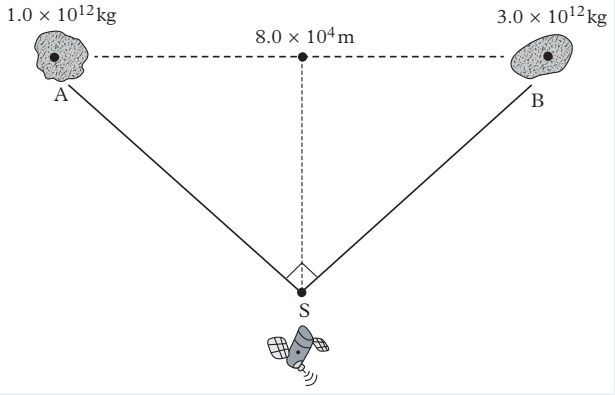
8 **Determine** the altitude in metres above Earth's surface where the gravitational field strength is one-eighth the value on the surface (2 s.f.). (3 marks)

Knowledge utilisation

9 **Derive** a location between the Sun and Earth where the gravitational field between them due to their mass is equal to zero (2 s.f.). (4 marks)
The mass of the Earth $m_{Earth} = 5.97 \times 10^{24}$ kg, mass of Sun $m_{Sun} = 1.99 \times 10^{30}$ kg, Earth–Sun distance = 150 million km.

11 When checking their work, a student realises they have calculated the gravitational field strength 100 km from Earth's centre as $3.99 \times 10^{4}\,m\,s^{-2}$. That's $40{,}000\,m\,s^{-2}$. **Deduce** why this nonsensical. (1 mark)

12 Asteroids A and B, of mass 1.0×10^{12} kg and 3.0×10^{12} kg respectively, are spotted by a satellite S in the Oort Cloud between Mars and Jupiter. The asteroids subtend an angle of 90° at S and are found to be 8.0×10^{4} m apart, as seen in the diagram. **Determine** the net gravitational field at S due to the two asteroids (2 s.f.). (8 marks)



Lesson 5.4
The three laws of planetary motion (Kepler's laws)

Key ideas

- → The three laws of planetary motion are the law of orbits (the first law), the law of areas (the second law), and the law of periods (the third law). These are also known as Kepler's laws.
- → The accepted model of the solar system slowly shifted under the influence of carefully collected and analysed data.
- → The third law of planetary motion (Kepler's third law) is recognised as the relationship between Newton's laws of universal gravitation and uniform circular motion.
- → Problems involving the third law of planetary motion can be solved using $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi^2}{GM}$.

Learning intentions and success criteria

What is a heliocentric solar system?

In 1618, Johannes Kepler published the first two of his three laws and was hailed as a hero by those who saw the defects of the old geocentric (Earth-centred) universe of Aristotle and Ptolemy – a model in which the Sun, Moon and planets revolved around a stationary Earth (Figure 1). Using observational data from sixteenth-century Danish astronomer Tycho Brahe, Kepler was able show that a heliocentric (from Greek "helios" meaning "the Sun") model fitted the data so much better than a geocentric one. In a heliocentric model, the planets (including Earth) revolve around the Sun. Scientists who supported a heliocentric model of the solar system faced many hindrances to the acceptance of their discoveries by society.

Later, however, Newton used the relationship between his law of universal gravitation and his laws governing circular motion to derive Kepler's laws independently. After this, the three laws garnered great esteem – so much so that they still hold true today. They are used to calculate the orbits of artificial satellites and make predictions about planetary systems in neighbouring galaxies. It is a truly wonderful conjunction of the work of many great scientists. We can now appreciate how the accepted model of the solar system slowly shifted under the influence of carefully collected and analysed data. Let's now look at the three laws of planetary motion.

FIGURE 1 Johannes Kepler

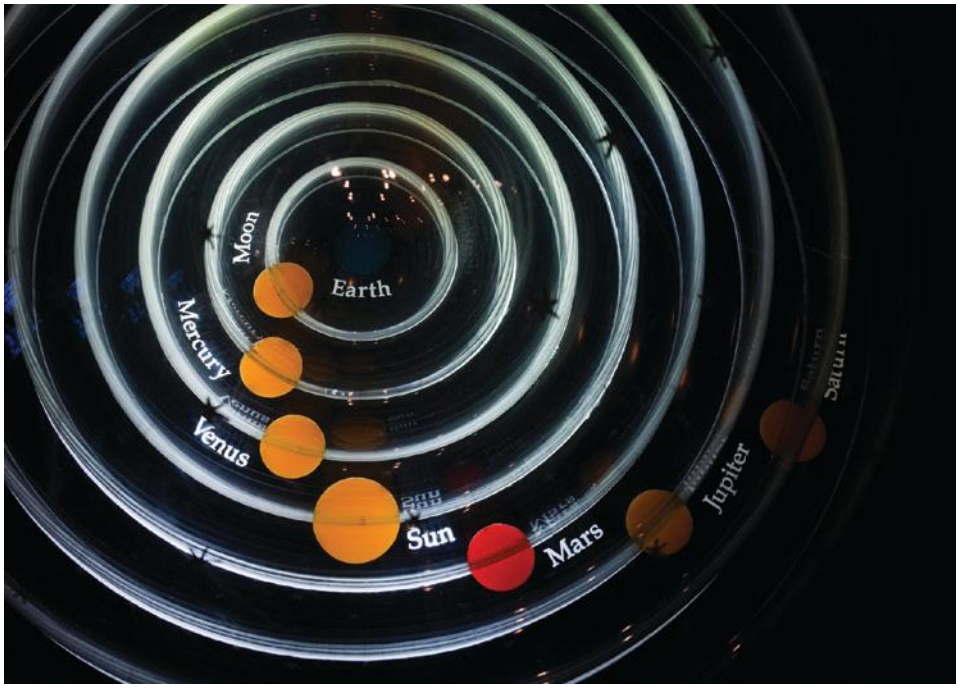


FIGURE 2 This early geocentric (Earth-centred) model of the solar system became less and less sustainable as astronomers collected more data.

What is the first law of planetary motion (the law of orbits)?

The orbit of Mars is quite irregular when viewed from Earth. It appears to track back on itself and do loop-the-loops. This wandering orbit of Mars could not be explained by astronomers using circular orbits for planets with Earth at the centre. Instead, Kepler used Copernicus's idea of the Sun at the centre of motion but proposed an elliptical orbit with the Sun as a focus.

The **first law of planetary motion (law of orbits)** states that all planets move about the Sun in elliptical orbits that have the Sun as one of the foci. It is also commonly referred to as "Kepler's first law".

first law of planetary motion (law of orbits)
all planets move about the Sun in elliptical orbits, having the Sun as one of their foci

Elliptical orbits

An ellipse looks like a slightly squashed or elongated circle. It is produced if you make a sloping cut through a conical pyramid (Figure 3).

A planet of mass m moves in an elliptical orbit around the Sun. The Sun, of mass M, is at one focus, F, of the ellipse (Figure 4). The other, or "empty", focus is F′. The point closest to the Sun is called the perihelion and the opposite point farthest from the Sun is called the aphelion. When referring to an elliptical orbit of the Moon or of artificial satellites about the Earth, these points are called the perigee and apogee respectively.

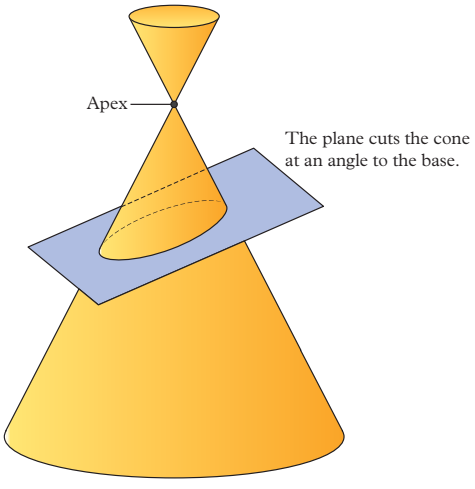


FIGURE 3 An ellipse is an angled cut by a plane through a conical pyramid.

Study tip

Terms such as perihelion, aphelion, perigee and apogee are not syllabus terms and won't appear on the external exam. They are used here to make the description clearer.

The shape of the ellipse is almost circular, and it has been exaggerated in Figure 4. At perihelion, Earth is 147,097,800 km from the Sun; at aphelion it is at a distance of 152,098,200 km, a difference of 5 million km. The difference in radius between the two is about
1.5 per cent, which means the orbit is almost circular. For example, consider an ellipse 152 cm long on its major axis. Its minor axis would be 147 cm – not that different to a circle of diameter 150 cm.

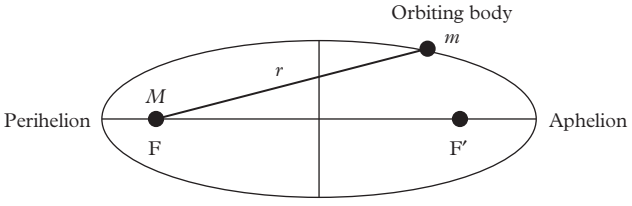


FIGURE 4 Earth's orbit around the Sun is an ellipse.

Challenge

What if gravity suddenly shut off?

If gravity suddenly switched off, what would happen to the motion of the planets? By way of comparison, imagine a marble is fired into an almost-circular tube lying flat on a table (Figure 5). When it exits at the end, which trajectory does it follow – A, B or C – and why? (2 marks)

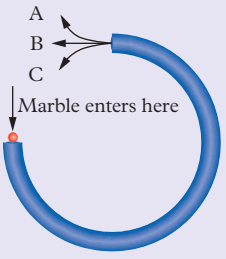


FIGURE 5 The trajectory of a marble

Challenge

Signal to Jupiter and back

Scientists can measure the distances to planets by timing how long it takes a radar beam to get there and back. By way of comparison, imagine that a marathon runner starts off at the same time as a radar signal leaves Earth for Jupiter, a distance of 588 million km. They stop running when the echo of the signal is received back on Earth. How many kilometres do they run? Note: radar signals are a form of electromagnetic radiation and travel at the speed of light ($3 \times 10^8\,m\,s^{-1}$), whereas marathon runners cover 40 km in about 2 hours. (3 marks)

Real-world physics

Katherine Johnson and orbital mechanics

There have been many significant contributions by scientists improving our understanding of gravity and motion: Newton and Kepler in particular. A more recent contribution is from Katherine Johnson (1918–2020) (Figure 6), whose work on orbital mechanics is legendary. They even made a movie about her work called *Hidden Figures* (2016), which alludes to the way she did computations in orbital mechanics.

Katherine Johnson was raised in West Virginia in the USA and in the early 1940s completed a mathematics degree at the state university. After graduation, she became a teacher, before getting a job at NASA doing calculations on the trajectory of rockets. Computations for trajectories use complex formulas involving rocket velocities, elevation angles, direction angles, oblateness (out-of-roundness) of the Earth, daily rotation of the earth, atmospheric drag and so on.

Before 1960 and the advent of modern computers, these calculations were done by hand by teams of "human computers" – 98 per cent of whom were women – using devices relying on gears, wheels and sprockets. To calculate the trajectory for a Moon landing would take 50,000 or so very involved computations, many so complex that they could only be solved by the iterative "trial-and-error" approach.

FIGURE 6 Katherine Johnson (1980), the NASA mathematician whose calculations of orbital mechanics were critical to the success of space missions

For example, for a Moon landing, the rocket must launch, orbit Earth a few times, catch up with the Moon (which is itself moving) and orbit the Moon a few times before landing, at an exact point decided before launch. Every step of this journey had to be calculated months before, and the smaller the steps, the more refined the trajectory needed to be. Johnson developed a new mathematical technique to shorten the process, and this revolutionised trajectory calculations.

She is renowned for making the calculations for astronaut Alan Shepard's 1961 *Freedom 7* mission to space – the first spaceflight for an American. Astronauts were reluctant to trust the newly invented mechanical computers, and John Glenn – the first American to orbit the Earth in 1962 – famously asked "Did the girl check it?" when told a computer had calculated his trajectory. She then did the calculations for the first Moon landing in 1969.

Katherine Johnson was the first woman to be named as a co-author on a paper for NASA. She met many other obstacles as an African American woman at that time, as one can imagine.

In 2015, President Barack Obama awarded her the Presidential Medal of Freedom, proclaiming, "Katherine G. Johnson refused to be limited by society's expectations of her gender and race while expanding the boundaries of humanity's reach."

Apply your understanding

1 **Describe** how the meaning of the term "computer" changed between early and late 1900s. (1 mark)

2 One of the formulas Johnson used to calculate the radial distance, r, to the Moon (in km) as it moved along its elliptical orbit was: $r = \frac{383{,}800\,(1 - e^2)}{1 + e\cos\theta}$. The value 383,800 is the radius in km of the minor axis of the Moon's elliptical path. The term e is a constant called the "eccentricity of the orbit" and for the Moon, $e = 0.0549$. **Decide** at what angle, θ, the value for r would be a maximum and at what angle r would be a minimum. (2 marks)

3 **Calculate** r for $\theta = 0°$ and $90°$ (in kilometres, to the nearest whole number). (2 marks)

4 The value e, the eccentricity of the orbit, is a ratio from 0 to 1 that describes the elongation of the ellipse. The bigger the value, the more elongated the ellipse. **Decide** what e would be for a circular orbit. (1 mark)

What is the second law of planetary motion (the law of areas)?

Kepler's analysis of Tycho Brahe's planetary data showed that planets changed speed during their orbit. When a planet was closer to the Sun it sped up and when it was further away it slowed down. Kepler developed a law that described this but could not explain it. Newton's explanation was still 60 years away.

This effect is used today when trying to speed up a spacecraft. It is called the "gravitational slingshot" effect or "gravity assist" (Figure 7). It increases the speed of the spacecraft without using any fuel. The gravitational attraction of the planet is used to speed up the spacecraft.

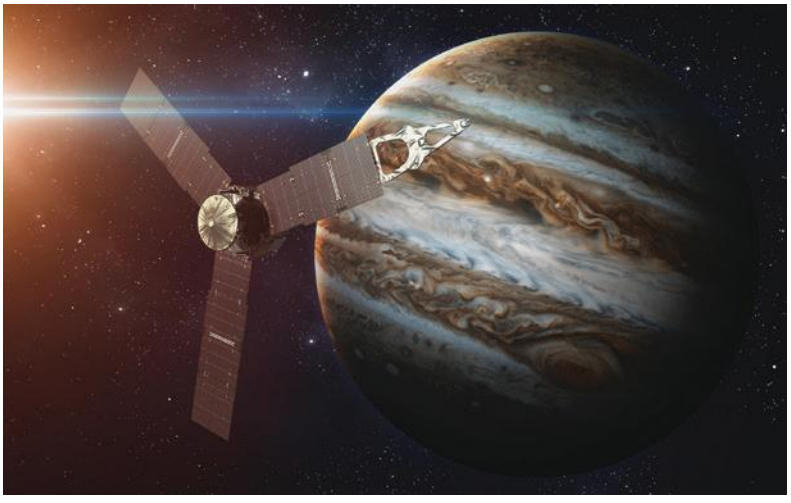


FIGURE 7 The gravity assist (slingshot) manoeuvre was used by the *Juno* mission to get to Jupiter. It used Earth's gravity to boost *Juno*'s speed, which allowed it to escape the Sun's orbit.

second law of planetary motion (law of areas)
a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time

Kepler found that a planet's higher speed at a short distance was equivalent to a slower speed at a larger distance, so the areas swept out were the same. He framed his law around this idea.

The **second law of planetary motion (law of areas)** states that a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time. A radius vector is any line from the Sun to the ellipse. It is also known as Kepler's second law.

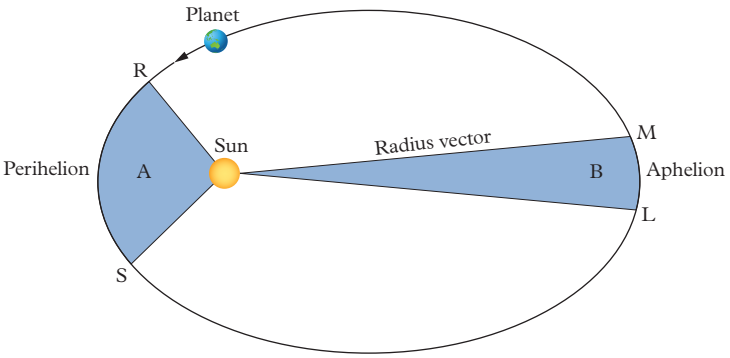


FIGURE 8 From the second law, if the time taken for the planet to travel from R to S is the same as that to travel from L to M, area A will equal area B.

In Figure 8, the planet takes the same time to move from L to M as it does to move from R to S. The shaded areas A and B are equal. This is the second law of planetary motion.

Both these laws contradicted conventional wisdom of the time.

What is the third law of planetary motion (the law of periods)?

In 1619, a more remarkable hypothesis followed. The **third law of planetary motion (law of periods)** states that the square of the sidereal periods of a planet is directly proportional to the cube of its mean distance from the Sun: $T^2 \propto r^3$.

The **sidereal period** is the true orbital period of a planet. It is the time it takes the planet to complete one full orbit of the Sun.

Kepler's third law takes the mathematical form: $T^2 \propto r^3$, or:

$$\frac{T^2}{r^3} = \text{constant (Kepler's ratio)}, \text{ or } \frac{T_a^{\,2}}{r_a^{\,3}} = \frac{T_b^{\,2}}{r_b^{\,3}}$$

where T_a is the period of planet A, T_b the period of planet B, r_a is the orbital radius of planet A (the distance from planet A to Sun), and r_b is the orbital radius of planet B.

Table 1 shows the $\frac{T^2}{r^3}$ values for Earth and its neighbouring planets. Note how constant Kepler's ratio is (average = 2.97×10^{-19} s^2 m^{-3}).

third law of planetary motion (law of periods) the square of the sidereal period of a planet is directly proportional to the cube of its mean distance from the Sun: $T^2 \propto r^3$

sidereal period the time it takes for a planet to complete one orbit of another body relative to the stars

TABLE 1 Law of periods data for selected planets

	Radius of orbit, r ($\times 10^{10}$ m)	Period of revolution, T ($\times 10^7$ s)	Kepler's ratio, ($\times 10^{-19}$ s^2 m^{-3})
Mercury	5.79	0.761	2.98
Venus	10.8	1.94	2.99
Earth	15.0	3.16	2.96
Mars	22.8	5.93	2.97
Jupiter	77.8	37.4	2.97
Saturn	143.0	93.1	2.96
Uranus	287.0	265.0	2.97
Neptune	450.0	521.0	2.98

Worked example 5.4A

The orbit of Neptune's moons

Triton and Nereid are the two moons of Neptune (Figure 9). Triton is r_t = 353,000 km from the surface of Neptune and has a period, T_t, of 5.87 Earth days. Nereid is r_n = 5,560,000 km from the surface of Neptune and its period, T_n, is 359.9 Earth days. **Determine** if these data are consistent with Kepler's third law. Neptune has a radius, r_N, of 24,750 km. (5 marks)

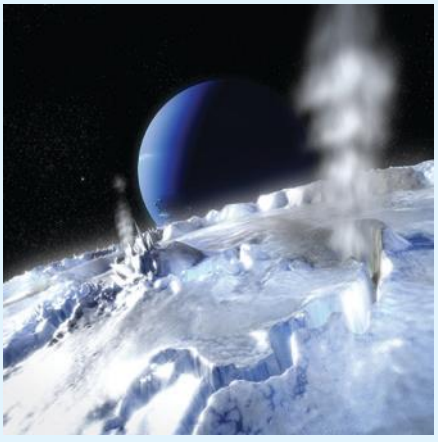


FIGURE 9 The planet Neptune (blue) as seen from Triton, one of its moons, with the moon Nereid in the distant background

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula. Use the symbols t for Triton, n for Nereid and N for Neptune. (1 mark for "Correctly calculates the orbital radii of Triton and Nereid") The radii can be left in kilometres or converted to metres as the conversion factors will later cancel out. Note that it is worth writing the formula down, even if you can't make further progress.	Radius of Neptune: r_N = 24,750 km Altitude of Triton from surface of Neptune: $r_{t(alt)}$ = 353,000 km Altitude of Nereid from surface of Neptune: $r_{n(alt)}$ = 5,560,000 km Orbital radius of Nereid: r_n = 5,569,000 + 24,750 = 5,584,750 km (1 mark) T_t = 5.87 days, T_n = 359.9 days $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3}$ = constant
Step 3: Rewrite the third law using the appropriate subscripts. (1 mark for "Identifies relationship between the two moons")	$\frac{T_t^2}{r_t^3} = \frac{T_n^2}{r_n^3}$ (1 mark)
Step 4: Calculate the value of Kepler's ratio for Triton. Substitute in the known values. (1 mark for "Provides correct substitution")	$\frac{T_t^2}{r_t^3} = \frac{5.87^2}{(353{,}000 + 24{,}750)^3}$ $= 6.39 \times 10^{-16}$ (1 mark)
Step 5: Calculate the value of Kepler's ratio for Nereid. Substitute in the known values. (1 mark for "Provides sound mathematical reasoning")	$\frac{T_n^2}{r_n^3} = \frac{359.9^2}{(5{,}560{,}000 + 24{,}750)^3}$ $= 7.346 \times 10^{-16}$ (1 mark)
Step 6: Draw a conclusion about whether the values are the same. A mark will be awarded if your argument is logical and applies to the comparison of data. (1 mark for "Draws logical conclusion")	The values are close, so Kepler's law is confirmed. (1 mark)

Your turn

Io and Europa are two moons of Jupiter. Io has an orbital radius of 4.22×10^8 m and a period of 1.77 days. Europa has an orbital radius of 6.71×10^8 m and a period of 3.55 days. **Determine** whether the value for Kepler's ratio $\frac{T^2}{r^3}$ is equal for the two moons. (5 marks)

How do we apply the third law to artificial satellites?

Irrespective of whether the orbiting mass is a natural satellite such as the Moon, or an artificial satellite such as the Hubble Space Telescope, or even chips of paint in orbit – they all tend to follow the third law of planetary motion. In Worked example 5.4B, we will look at how to make calculations about an artificial satellite.

Worked example 5.4B

Artificial satellites orbiting Earth

A satellite is launched into orbit around Earth. Knowing that the natural satellite of Earth (the Moon) has a period of 28 days and an orbiting radius of 3.8×10^8 m, **calculate** the desired orbital radius for the artificial satellite so that it has a period of 1 day. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$T_s = 1.0$ day, $T_M = 28$ days, $r_M = 3.8 \times 10^8$ m $r_s = ?$ $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \text{constant}$
Step 3: Rewrite the third law using the appropriate subscripts to identify the relationship between the Moon and the satellite. (1 mark for "Identifies relationship between Moon and satellite")	$\frac{T_M^2}{r_M^3} = \frac{T_s^2}{r_s^3}$ (1 mark)
Step 4: Substitute in the known values. (1 mark for "Provides correct substitution")	$r_s^3 = \frac{r_M^3 \times T_s^2}{T_M^2}$ $= \frac{(3.8 \times 10^8)^3 \times 1.0^2}{27.3^2}$ $= 7.4 \times 10^{22}$ (1 mark)
Step 5: Calculate the radius, r_s. (1 mark for "Provides sound mathematical reasoning")	$r_s = \sqrt[3]{7.4 \times 10^{22}}$ $= 4.2 \times 10^7$ m $= 42{,}000$ km (1 mark)
Step 6: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	The orbital radius for the artificial satellite should be 42,000 km. $r_s = 4.2 \times 10^4$ km or 4.2×10^7 m (2 s.f.) (1 mark)

Your turn

The period of the Moon is 28 days, and its orbital radius is 3.8×10^8 m. **Calculate** the period of a satellite 39,000 m from Earth's centre. The radius of the Earth is 6,371 km. Give your answer to an appropriate number of significant figures. (4 marks)

Challenge

A galactic year

The period of our solar system is called one galactic year. That is the time it takes for our solar system to return to the same place in the Milky Way. How long is a galactic year (in Earth years): 1 million, 230 million, 500 million or 5 billion? (1 mark)

How is the third law of planetary motion derived from Newton's laws?

Isaac Newton showed in 1687 that Kepler's third law from 1609 was a consequence of his own laws of motion and law of universal gravitation. Although planets have elliptical orbits around the Sun, we can assume that their paths are close enough to being circular. Some other motions are circular:

- The natural satellite of Earth (the Moon) has a circular path.
- Most Earth-orbiting artificial satellites have circular paths, which keeps their speed constant.
- Earth rotates on its own axis, so a point on its surface travels in a circular path.

The development of the third law starts by equating Newton's formula for universal gravitation with his formula for centripetal motion. This makes sense, as the centripetal force, F_c, needed to keep a planet in circular motion around the Sun is provided by the gravitational force, F_g, between the planet and the Sun. The logic is thus:

$$F_g = F_c$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Collecting m and r on the left-hand side and cancelling gives:

$$\frac{GMmr}{r^2 m} = v^2$$

$$\frac{GM}{r} = v^2$$

Multiplying both sides by r gives:

$$GM = v^2 r$$

As $v = \frac{s}{t} = \frac{2\pi r}{T}$ for a circle:

$$GM = \left(\frac{2\pi r}{T}\right)^2 r$$

Expanding the brackets and rearranging:

$$GM = \frac{4\pi^2 r^2 r}{T^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} = \text{constant}$$

Or more specifically, for two planets a and b:

$$\frac{T_a{}^2}{r_a{}^3} = \frac{T_b{}^2}{r_b{}^3} = \frac{4\pi^2}{GM}$$

This is the third law of planetary motion (Kepler's third law).

Study tip

A useful formula is $v = \sqrt{\frac{GM}{r}}$. You can work out the velocity, v, of an orbiting object if you know the mass, M, of the central body (planet or star) and the orbital radius, r.

Worked example 5.4C

Kepler's third law and Earth satellites

A satellite moves in a circular orbit around Earth at an altitude of 4,690 km. The radius of Earth is 6,378 km. **Determine** (to an appropriate number of significant figures) the satellite's

a period (3 marks)

b velocity. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	**a** Altitude, $r_A = 4{,}690\,\text{km}$, $r_E = 6{,}378\,\text{km}$ $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$
Step 3: Calculate the orbital radius of the satellite, r_o. (1 mark for "Calculates orbital radius")	Orbital radius = radius of Earth + altitude $r_o = r_E + r_A$ $= 6{,}378 + 4{,}690$ $= 11{,}068\,\text{km}$ $= 1.107 \times 10^7\,\text{m}$ (1 mark)
Step 4: Select the third law formula and substitute in the known values. Solve for T and convert to hours. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct substitution"; 1 mark for "Provides sound mathematical reasoning")	$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ $T^2 = \frac{4\pi^2 r^3}{GM}$ $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$ $= \sqrt{\frac{4\pi^2 (1.107 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}}$ (1 mark) $= 11{,}594\,\text{s}$ $= 3.22\,\text{h}$ $= 3.2\,\text{h}$ (2 s.f.) (1 mark)
Step 5: Select the velocity formula and substitute in the known values. Calculate the velocity and make sure you have the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct substitution"; 1 mark for "Provides correct answer")	**b** $v = \frac{s}{t}$ $= \frac{2\pi r}{T}$ $= \frac{2\pi \times 1.1068 \times 10^7}{11{,}594}$ (1 mark) $= 5{,}999\,\text{m s}^{-1}$ $= 6{,}000\,\text{m s}^{-1}$ or $6.0 \times 10^3\,\text{m s}^{-1}$ (2 s.f.) (1 mark)

Your turn

A GPS satellite moves in a circular orbit around Earth at an altitude of 20,000 km. The radius of Earth is 6,378 km. **Determine** (to an appropriate number of significant figures) the satellite's

a period (3 marks)

b velocity. (2 marks)

Real-world physics

The orbits of artificial satellites

The term "satellite" refers to an object in space that orbits around a bigger object. There are natural satellites such as the Moon, or artificial ones such as communications satellites.

Earlier in this module, the equation $v^2 = \frac{GM}{r}$ was developed, using Newton's law of gravity and his equation for centripetal force. This equation says that velocity is inversely related to the orbital radius of the satellite. In other words, to get a certain value of v, there is a corresponding value for orbital radius r. Remember that orbital radius = Earth's radius (6.37×10^3 km) + altitude (distance above Earth's surface).

Satellites' orbits are divided into three categories depending on their altitude: low Earth orbit (LEO), medium Earth orbit (MEO) and high Earth orbit (HEO). Satellites could orbit Earth on any elliptical or circular path, but they usually orbit around the equator or the poles. When a satellite's orbit matches the rotation of Earth, it is called a geosynchronous orbit. If its position over Earth remains fixed, it's called a geostationary orbit.

The actual satellite orbit that is chosen will depend on factors including its function, and the area it is to serve.

FIGURE 10 The Hubble Space Telescope orbits at 569 km. This is in low Earth orbit, which is high enough to avoid the distortions of the Earth's atmosphere but low enough to make communication with Earth straightforward.

Low Earth orbit (LEO), altitude 200 to 1,200 km, period 1.5 to 2 h

Uses: Vast majority of satellites: communications, Earth-monitoring, International Space Station, Hubble Space Telescope, Space-X Starlink (>6,000 satellites), military uses, satellite phones

Features: Short round-trip time (RTT) for signals; risks of damage from more than 128 million pieces of space debris (junk), particularly in the 600 to 1,200 km range; close, so easier to fix; passes overhead very quickly

Medium Earth orbit (MEO), altitude 1,200–35,790 km, period 2–24 h

Uses: Global positioning satellites (GPS) at 20,000 km, microgravity experiments

Features: Passes same spot twice a day; has short round-trip for signals; more solar radiation so it has a shorter life than LEOs

Geosynchronous orbit (GSO or GEO), altitude 35,790 km, period 24 h

Uses: Direct broadcast, communications relay

Features: Orbits once a day

Geostationary orbit (GEO), altitude 35,790 km, period 24 h

Uses: Radio, TV, weather; cover 42 per cent of Earth's surface; congested with several hundred satellites

Features: Orbits once a day and moves in the same direction as Earth so appears stationary above the same point on Earth's surface

High Earth orbit (HEO), altitude above 35,790 km, period varies

Uses: Astronomical observations

Features: High costs, long round-trip for signals

Apply your understanding

1 Starlink satellites are at 550 km altitude. **Calculate** the time in milliseconds for a signal to go from the ground to the satellite. (1 mark)

2 There are currently over 6,000 Starlink satellites in orbit, and they need to be replaced as they die. **Decide**, with justification, if it is better to leave the old ones in orbit as space junk or make them re-enter the atmosphere and break up in the sky. (2 marks)

3 **Deduce** why military satellites use a low Earth orbit. (2 marks)

4 **Propose** an argument for which type of orbit is best suited for transmitting live TV interviews. (2 marks)

Check your learning 5.4

Check your learning 5.4: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Describe** Kepler's three laws. (3 marks)

2 **Define** "elliptical" as applied to planetary orbits. (1 mark)

3 **Explain** why the idea of elliptical orbits was rejected for so long. (1 mark)

4 **Explain** the difference between the heliocentric and geocentric models of the solar system. (2 marks)

5 The average radius of the orbit of Uranus is 2.87×10^{12} m. **Calculate** the period of Uranus using an average value for $\frac{T^2}{r^3}$ of 2.97×10^{-19} s^2 m^{-3}. Give your answer (2 s.f.) in

a seconds (3 marks)

b years. (1 mark)

6 **Clarify** whether all orbits of natural astronomical bodies are elliptical. (1 mark)

7 **Identify** one of the contributions made by astronomer Johannes Kepler to the study of planetary motion. **Clarify** whether his explanation for the motion was correct. (2 marks)

Analytical processes

8 Neptune takes 164.8 years to orbit the Sun. **Determine** the average radius of Neptune's orbit using the average value of $\frac{T^2}{r^3}$ of $2.98 \times 10^{-19}\ s^2 m^{-3}$ (2 s.f.). (2 marks)

9 It was once thought that the planet Vulcan existed between Mercury and the Sun. **Determine** what the period would have been if it orbited at a mean radius of 40 million km from the Sun (2 s.f.). (3 marks)

10 The mean radius of the orbit of planet X in another solar system is 4×10^{12} m and the average value of $\frac{T^2}{r^3}$ for this system is $2.86 \times 10^{-19}\ s^2 m^{-3}$. **Determine** the period of planet Y in the same solar system, given it has an orbital radius of 3×10^{10} m (2 s.f.). (3 marks)

11 A satellite moves in a circular orbit around Earth with a speed of $5.0\ km\ s^{-1}$.

a **Determine** the satellite's altitude above Earth's surface (2 s.f.). (3 marks)

b **Calculate** the period of the satellite's orbit (2 s.f.). (3 marks)

Knowledge utilisation

12 In 1610, Galileo used his telescope to discover the four most prominent moons of Jupiter (the Galilean moons). Their mean orbital radii and periods are given in the table.

Name	r (× 10^8 m)	T (days)
Io	4.22	1.77
Europa	6.71	3.55
Ganymede	10.70	7.16
Callisto	18.80	16.70

a **Construct** a graph of r^3 (horizontal axis) against T^2 (vertical axis). (3 marks)

b **Comment** on what this graph shows about the relationship between r and T. (2 marks)

Practical

Lesson 5.5

Investigating orbital radius and mass using a simulation

Learning intentions and success criteria

This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.

MODULE 5

Lesson 5.6
Review: Orbital mechanics

Summary

5.1
- Gravity is the force of attraction between objects with mass.
- Gravitational forces are transmitted by gravity waves.
- International collaboration was required in the discovery of gravity waves and associated technologies, e.g. Laser Interferometer Gravitational Wave Observatory (LIGO).

5.2
- Newton's law of universal gravitation quantifies the gravitational force between two objects.
- Problems involving the magnitude of the gravitational force between two masses can be solved using $F = \frac{GMm}{r^2}$.

5.3
- Gravitational fields are the region of space surrounding a body in which another body experiences a force of gravitational attraction.
- Problems involving gravitational field strength at a distance from an object can be solved using $g = \frac{F}{m} = \frac{GM}{r^2}$.

5.4
- The three laws of planetary motion are the law of orbits (the first law), the law of areas (the second law), and the law of periods (the third law). These are also known as Kepler's laws.
- The accepted model of the solar system slowly shifted under the influence of carefully collected and analysed data.
- The third law of planetary motion (Kepler's third law) is recognised as the relationship between Newton's laws of universal gravitation and uniform circular motion.
- Problems involving the third law of planetary motion can be solved using $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi^2}{GM}$.

5.5
- Practical: Investigating orbital radius and mass using a simulation

Key formulas

Gravitational force	$F_g = mg$
Newton's law of universal gravitation	$F_g = \frac{GMm}{r^2}$
Gravitational field strength	$g = \frac{F}{m} = \frac{GM}{r^2}$
The third law of planetary motion (law of periods)	$\frac{T^2}{r^3} = \text{constant}$
Newton's derivation of the third law of planetary motion	$\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi r^2}{GM}$

Review questions 5.6A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

Use the following information to answer the questions in this review section.

Constant	Value
g (on surface of Earth) $= 9.8\,\text{m s}^{-2}$	r_{Mo} (radius of Moon's orbit around Earth) $= 3.8 \times 10^8\,\text{m}$
$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$	r_{Eo} (mean radius of Earth's orbit around Sun) $= 1.5 \times 10^{11}\,\text{m}$

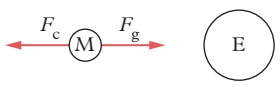
Body	Mass	Radius
Earth	$m_{Earth} = 5.97 \times 10^{24}\,\text{kg}$	$r_{Earth} = 6.38 \times 10^6\,\text{m}$
Moon	$m_{Moon} = 7.35 \times 10^{22}\,\text{kg}$	$r_{Moon} = 1.74 \times 10^6\,\text{m}$
Sun	$m_{Sun} = 1.99 \times 10^{30}\,\text{kg}$	$r_{Sun} = 6.96 \times 10^8\,\text{m}$

1 Two equal masses, m, separated by a distance, r, exert a force, F, on each other due to their gravitational attraction. What is the magnitude of the gravitational force between an object of mass m and an object of mass $2m$ separated by the same distance r?

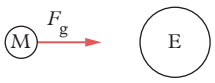
A $\frac{1}{4}F$ **B** $\frac{1}{2}F$ **C** $2F$ **D** $4F$

2 Which diagram best describes the forces acting on the Moon (M) due to Earth (E)?

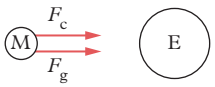
A



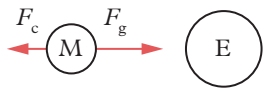
B



C



D



3 On the surface of a planet with a radius r_p, a mass experiences a gravitational force of attraction of g_p. At a height of $4r_p$ above the planet, the mass experiences a force of

A $\frac{g_p}{4}$ **B** $\frac{g_p}{5}$ **C** $\frac{g_p}{16}$ **D** $\frac{g_p}{25}$

4 The force of gravity on a 1 kg object on the surface of Earth is 9.8 N directed downwards. Which answer best states the force on a 1 kg object on the surface of Saturn, given that Saturn is 100 times the mass of Earth and 10 times the diameter?

A $\frac{1}{10}$ N **B** 1 N **C** 10 N **D** 100 N

5 Two planets have the same gravitational field strength at their surfaces. Which ratio must also be the same for the two planets?

A $\frac{\text{radius}^3}{\text{mass}}$ **B** $\frac{\text{radius}^2}{\text{mass}}$ **C** $\frac{\text{radius}}{\text{mass}}$ **D** radius

6 Planet X has mass M and radius R. Planet Y has mass $10M$ and radius $4R$. Determine the ratio $\frac{\text{gravitational field strength at the surface of planet X}}{\text{gravitational field strength at the surface of planet Y}}$.

A 0.5 **B** 1.6 **C** 2.0 **D** 2.5

7 Which statement best describes one of Kepler's laws of planetary motion?

A Planets move on elliptical orbits with the Sun at one focus.

B The gravitational force between two objects decreases with the distance squared.

C Gravitational field strength on the surface of inner planets is less than on the surface of outer planets.

D Inner planets orbit in the opposite direction to outer ones.

8 Suppose a planet has an elliptical orbit. The speed of the planet is $15\,\text{km s}^{-1}$ when it is at its average distance from its star. Which one of the following is most likely to be the planet's speed when it is nearest the star?

A $5\,\text{km s}^{-1}$ **B** $10\,\text{km s}^{-1}$
C $15\,\text{km s}^{-1}$ **D** $20\,\text{km s}^{-1}$

9 A comet from the outer part of the solar system has a very long orbit. Which one of the following is correct about the orbit according to Kepler's second law?

A As the comet approaches the Sun, it speeds up.

B As the comet approaches the Sun, it slows down.

C The speed of the comet is constant.

D The comet is moving in a circle.

10 The mean distance of Mars from the Sun is 1.524 times the distance of Earth from the Sun. What is the period of revolution of Mars around Sun?

A 1.23 year
B 1.32 year
C 1.88 year
D 3.53 years

11 Newton's laws of motion and law of gravity can be used to find the mass of Earth by using a relationship between period, T, and average distance, r, of the Moon from Earth's centre, and mass, M, of Earth. What is the theoretical mass of Earth, M, using $T = 27.3$ days and $r = 385{,}000\,\text{km}$?

A $6.1 \times 10^{15}\,\text{kg}$
B $4.5 \times 10^{25}\,\text{kg}$
C $6.1 \times 10^{24}\,\text{kg}$
D $4.5 \times 10^{30}\,\text{kg}$

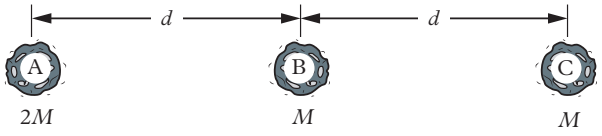
12 A satellite of mass m is in a circular orbit above Earth (mass m_E) at a distance h above the surface, where $h = r$ (the Earth's radius). Which expression best states the velocity the satellite must have in order to maintain its orbit?

A $v = \sqrt{\frac{Gm_E}{r}}$
B $v = \frac{Gm_E}{2r}$
C $v = \sqrt{\frac{Gm_E m}{2r}}$
D $v = \sqrt{\frac{Gm_E}{2r}}$

13 Imagine if the Sun became a black hole of the same mass but one-tenth of its diameter. Which statement could be said about Earth's orbit about the Sun?

A Earth would spiral into the Sun.
B Earth's orbit would be smaller.
C Earth's orbit would stay the same.
D Earth's orbit would get larger.

14 Three asteroids A, B and C are in a line. A and B are separated by a distance d, and asteroids B and C are separated by the same distance, d, as shown in the diagram. B and C each have mass M, and A has mass $2M$. Rank the magnitude of the net force on each asteroid due to the other two asteroids.



A $F_A > F_B > F_C$
B $F_B > F_A > F_C$
C $F_C > F_B > F_A$
D $F_C > F_A > F_B$

15 Three asteroids A, B and C are in a line. A and B are separated by a distance d, and asteroids B and C are separated by $4d$. Their masses are all M. There is no net force on B due to A and C.

The mass of C is

A M
B $4M$
C $8M$
D $16M$

Review questions 5.6B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 A satellite is in a circular orbit around Earth. **Explain** why an astronaut aboard the satellite feels weightless if the value of g at that altitude is $6.5\,\text{m s}^{-2}$. (2 marks)

17 **Calculate** the gravitational force between

a two 60 kg people 1.0 m apart (2 marks)
b two protons ($m_p = 1.7 \times 10^{-27}\,\text{kg}$) that are $5 \times 10^{-15}\,\text{m}$ apart. (2 marks)

18 **Calculate** the gravitational field strength in the region of a satellite orbiting 8,000 km above Earth's surface. (2 marks)

19 **Calculate** the gravitational field strength of the Sun ($m = 2.0 \times 10^{30}\,\text{kg}$) at a distance of $15 \times 10^{10}\,\text{m}$ from its centre. (2 marks)

20 **Describe** the difference between heliocentric and geocentric models of the solar system. (2 marks)

Analytical processes

21 An astronaut is on the Moon.

a **Calculate** the acceleration due to gravity on its surface. (2 marks)
b **Determine** the time it would take a spanner to fall from rest at a height of 1.5 m to the surface of the Moon. (2 marks)

22 **Determine** the distance between two objects of mass 5.0×10^4 kg and 2.5×10^4 kg respectively, if the gravitational force between them is 2.00×10^{-8} N. (2 marks)

23 **Determine** the gravitational field strength at the following points using the data table at the start of this Review section. (2 marks each)

a On Earth's surface

b 1.5 Earth radii above Earth's surface

c 3.0 Earth radii above Earth's surface

d 1,000 m above the Moon's surface

e On the surface of the Sun

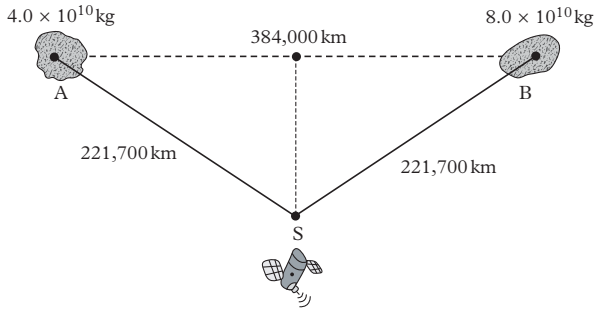
24 A satellite is in a circular orbit around Earth, at two Earth radii from Earth's surface.

a **Determine** the acceleration due to gravity aboard the satellite. (2 marks)

b **Calculate** the weight of a 70 kg astronaut aboard the satellite. (2 marks)

25 **Determine** the altitude above Earth's surface at which the gravitational field strength is $\frac{1}{25}$ the value on the surface of Earth. (3 marks)

26 A spacecraft locates two asteroids, A and B, that are both 221,700 km away, as shown in the diagram. The asteroids are 384,000 km apart. **Determine** the net gravitational field strength the spacecraft will experience from the asteroids (2 s.f.). (5 marks)



27 Io, one of the moons of Jupiter, orbits Jupiter every 1.8 Earth days at a distance of 4.2 units from its centre. Ganymede, another of Jupiter's moons, is 10.7 units from Jupiter's centre. **Determine** the period of Ganymede. (3 marks)

28 **Determine** the time taken for Venus to complete an orbit around the Sun, given that the radius of the orbit is 108.2 million km, the mass of Venus is 4.9×10^{24} kg, and the mass of the Sun is 1.99×10^{30} kg. (3 marks)

Knowledge utilisation

29 The orbital distance of Mars from the Sun is 1.52 times the orbital distance of Earth. **Predict** how many days it takes Mars to orbit the Sun. (3 marks)

30 In the early days of artificial Earth satellites, one particular satellite began with an elliptical orbit but slowly lost energy due to the effects of atmospheric friction and eventually crashed into the sea. Copy the table into your workbook and add columns as necessary to answer the following questions.

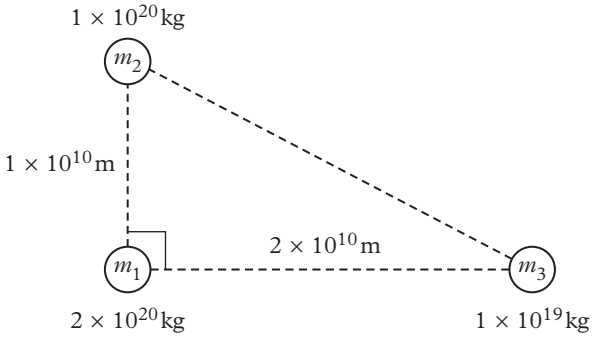
a **Calculate** the mean height (km), mean radius (km) and Kepler's ratio $\frac{T^2}{r^3}$ using the data in the table. (4 marks)

Data	January 1968	March 1968	May 1968
Orbital period (min)	96.2	95.4	91.0
Min. height (km)	219	216	196
Max. height (km)	941	866	463

b **Justify** the claim that the orbit became less elliptical as time passed by referring to the data. (2 marks)

c **Calculate** approximately how many orbits of Earth the satellite made in the 90 days it was in space. (2 marks)

31 In a galaxy far away, three planets, m_1, m_2 and m_3, are positioned at one moment in time as shown in the diagram.



Determine the net gravitational force acting on m_3 due to planets m_1 and m_2 (2 s.f.). (6 marks)

32 **Predict** how far you would have to travel upwards from Earth's surface to notice a 1 N kg^{-1} difference in gravitational field strength. (Earth has a radius of 6,378 km.) (3 marks)

33 **Predict** the height at which the force of gravity on a satellite will be half the force on it at ground level. (3 marks)

34 **Evaluate** whether the gravitational field strength of Earth at height h above the surface is given by $g = g_s\left(\frac{R}{R+h}\right)^2$, where g_s is the gravitational field strength at the surface and R is the radius of Earth. (4 marks)

35 Jupiter's four most significant moons are called the Galilean moons because they were discovered by Galileo in 1610. The orbital data for these moons is listed in the table. Copy the table into your workbook and add columns as necessary to answer the following questions.

Moon	Period (s)	Radius (m)
Io	1.5×10^5	4.2×10^8
Europa	3.1×10^5	6.7×10^8
Ganymede	6.2×10^5	10.7×10^8
Callisto	14.4×10^5	18.8×10^8

a **Calculate** Kepler's ratio $\frac{T^2}{r^3}$ for each moon. (4 marks)

b **Determine** the absolute uncertainty in Kepler's ratio. (2 marks)

c **Construct** a graph of r^3 (horizontal axis) versus T^2 (vertical axis) and determine the gradient. (4 marks)

d **Discuss** the relationship between the gradient of the graph and the average of Kepler's ratio. (2 marks)

e **Propose**, with justification, which one of Kepler's laws the data supports. (2 marks)

f **Determine** how the average for Kepler's ratio compares with Newton's derivation of the same quantity using the formula $\frac{T^2}{r^3} = \frac{4\pi}{GM}$, where M, the mass of Jupiter, is 1.9×10^{27} kg. (3 marks)

36 Another new planet outside our solar system was proposed earlier this century. It was thought to lie within the Oort Cloud and had an orbital radius of 0.5 ly, where 1 light-year (ly) = 9.46×10^{15} m. It has a mass 1.5 to 6 times of Jupiter and a period of 6 million years. **Evaluate** these data by analysing whether the $\frac{T^2}{r^3}$ ratio is the same as that of our solar system. (4 marks)

Data drill

The orbital characteristics of some of the moons of Neptune as measured by the Hubble Space Telescope are shown in Table 1.

TABLE 1 Orbital characteristics of four of Neptune's inner moons

Moon	Radius, r ($\times 10^7$ m)	Period, T ($\times 10^4$ s)
Naiad	4.82	2.54
Thalassa	5.01	2.69
Despina	5.25	2.89
Galatea	6.23	3.74
Proteus	11.8	-

The relationship is non-linear and can be linearised by plotting r^3 on the horizontal axis and T^2 on the vertical axis, as shown in Figure 1.

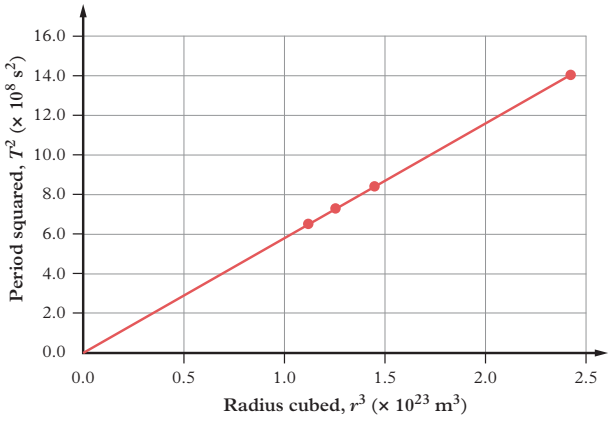


FIGURE 1 Linearised orbital data for Neptune's moons

Apply understanding

1 **Identify** how the relationship between radius and period in Table 1 is considered non-linear. (1 mark)

2 **Calculate** the value of Kepler's ratio, $\frac{T^2}{r^3}$, for the moon Galatea (3 s.f.). (2 marks)

Analyse data

3 **Compare** the gradient of the graph with Kepler's ratio for Galatea in the question above (2 s.f.). (1 mark)

4 **Identify** the period of the moon Proteus using the graph and relevant data from Table 1 (2 s.f.). (2 marks)

Interpret evidence

5 Using the graph and other formulas, **draw a conclusion** that quantifies the mass of Neptune (2 s.f.). (4 marks)

Module 5 checklist: Orbital mechanics

UNIT 3

Topic 1 review

Multiple choice

(1 mark each)

1 A projectile is fired at an angle and its horizontal range is measured. Select the option that states the elevation angle that would give the minimum range and the maximum range respectively.

A 0°, 90°
B 90°, 0°
C 45°, 90°
D 90°, 45°

2 Determine which of the following elevation angles give the same horizontal range to a projectile.

A 30°, 60°
B 35°, 65°
C 40°, 60°
D 45°, 145°

3 The time taken for a projectile launched at an angle of θ to the horizontal to reach its maximum height is

A $\frac{u\cos\theta}{g}$
B $\frac{u\sin\theta}{g}$
C $\frac{u}{g\cos\theta}$
D $\frac{u}{g\sin\theta}$

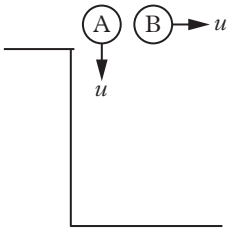
4 A ball moving horizontally with speed v falls off the edge of a vertical cliff and lands a distance d from the base of the cliff. If the height of the cliff was doubled, how far from the base of the cliff would the object land?

A d
B $2d$
C $4d$
D $\sqrt{2}d$

5 Police road-accident investigators find that a car hit the ground 60 m horizontally from the point where it left a cliff that is 45 m high. Determine at what speed the car was going when it left the cliff.

A $5\,\text{m s}^{-1}$
B $10\,\text{m s}^{-1}$
C $15\,\text{m s}^{-1}$
D $20\,\text{m s}^{-1}$

6 Two stones, A and B, are thrown from a cliff at the same speed. Stone A is thrown downwards, and stone B is thrown horizontally. Which stone will reach the ground first and which stone will have the greater speed on impact?

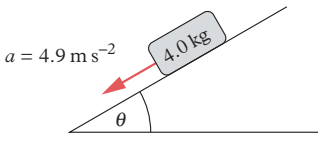


A The two stones will reach the ground at the same time with different speeds.
B Stone A will reach the ground first, but the two stones will have the same speed on impact.
C Stone A will reach the ground first, but stone B will have greater speed on impact.
D Stone A will reach the ground first and will have greater speed on impact.

7 On steep downhill highways there are sometimes uphill escape ramps for trucks whose brakes are not working properly. Consider a simple 15° upwards ramp being approached by a runaway truck travelling at $60\,\text{km h}^{-1}$ ($16.7\,\text{m s}^{-1}$). Determine the minimum stopping length.

A 14 m
B 27 m
C 55 m
D 110 m

8 The box shown in the diagram slides down the frictionless incline with an acceleration of $4.9\,\text{m s}^{-2}$. Determine the value of the angle of elevation θ.



A 15°
B 30°
C 45°
D 60°

9 An object has a mass of 65 kg on Earth. What would be its mass on Mars, where the acceleration due to gravity is $3.5\,m\,s^{-2}$?

A 65 kg

B 65×3.5 kg

C $65 \times \frac{3.5}{9.8}$ kg

D $65 \times \frac{9.8}{3.5}$ kg

10 A car travels in a horizontal circular path of radius r at a constant speed v. Decide what happens to the car's centripetal acceleration if it enters a section of the track where the radius of the circle is quadrupled ($4r$) and the driver doubles the speed.

A It is reduced to a quarter.

B It is reduced to a half.

C It stays the same.

D It doubles.

11 Select the option that correctly states the gravitational force of attraction between two spherical bodies, each of mass 100 kg, if the distance between their centres is 100 m. ($G = 6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$)

A 6.67×10^{-11} N

B 6.67×10^{-9} N

C 6.67 N

D 80 N

12 Each of the following lists two facts. Determine which pair can be used with Newton's version of the third law of planetary motion to determine the mass of the Sun. Note: 1 AU is 1 astronomical unit, the distance from Earth to the Sun.

A The mass of Earth is 5.97×10^{24} kg and Earth orbits the Sun in 1 year.

B Mercury is 0.387 AU from the Sun and Earth is 1 AU from the Sun.

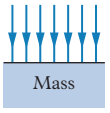
C Earth is 150 million km from the Sun and orbits the Sun in 1 year.

D Earth rotates in 1 day and orbits the Sun in 1 year.

13 Select the diagram that would represent a gravitational field if antigravity existed.

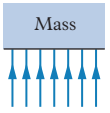
A

B



C

D



14 Jupiter's period of revolution around the Sun is 12 times that of Earth's. Determine how many times further the Sun is from Jupiter than it is from Earth.

A 2.29

B 3.46

C 5.24

D 41.57

15 Uranus's orbital radius around the Sun is 2.87 billion km, whereas Saturn's orbital radius is just 1.43 billion km. Saturn's orbital period is 29.5 years. Determine how long Uranus takes to go once around the Sun.

A 15 years

B 42 years

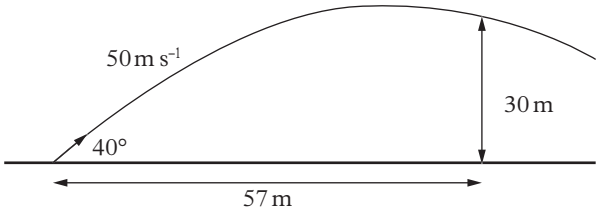
C 84 years

D 107 years

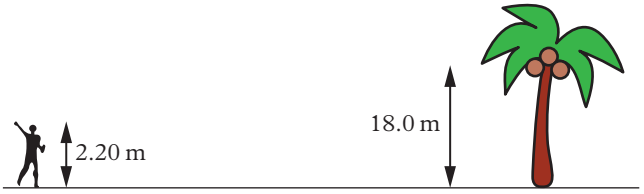
Short response

16 A javelin thrower can achieve a horizontal distance of 40.8 m when she throws her javelin at a speed of 20.0 m s^{-1} and at an angle of 45°. She believes she can throw faster and further at an angle of 30°. **Determine** the velocity required at this angle to reach a distance of 40.8 m. (4 marks)

17 A golfer hits a ball at a speed of 50 m s^{-1} and an angle of 40° in the direction of a 30 m high tree, which is 57 m from the golfer. **Determine** whether the ball will pass over the tree or hit it? (4 marks)



18 A hungry castaway wants to get a coconut from a tree. The coconut is 18.0 m above the ground. They decide the easiest way to get the coconut will be to throw a rock at it to knock it down. The castaway thinks that if they hit the coconut horizontally, it will be easier to break the stalk. They throw the rock with an initial speed of 20.0 m s^{-1} at 40.0° above the horizontal. The castaway and the coconut tree are shown in the diagram.

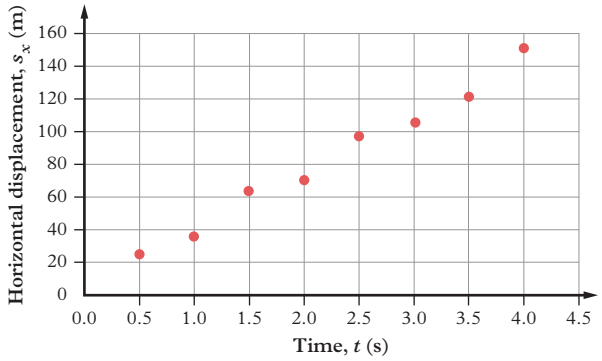


a If the castaway's hand is 2.20 m above the ground when the rock is released, **demonstrate** that the stone is not able to reach the coconut. (4 marks)

The castaway tries again and decides that they need to throw the rock at a larger angle to the horizontal in order to hit the coconut.

b **Calculate** the angle at which the stone must be thrown so that it will hit the coconut horizontally, if the rock has an initial speed of 20.0 m s^{-1}. (2 marks)

19 A golf ball was struck at an angle of 30° to the horizontal and landed some distance away at the same height. A video of the flight of the ball was analysed, and its horizontal displacement was measured every 0.5 s to just before it landed. The graph shows the results.

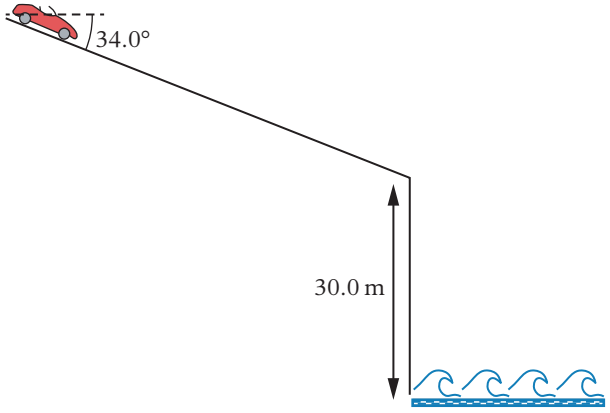


Analyse the graph to determine the maximum vertical height the ball reached. (5 marks)

20 An inclined plane is at an angle of 25° to the horizontal. A box placed on the incline experiences a frictional force of 20 N and slides down at a constant speed. **Determine** the mass of the box. (3 marks)

21 A car can decelerate at 5.0 m s^{-2} without skidding when coming to rest on a level road. **Determine** its deceleration if the road is inclined at 9.3° and the car moves uphill. Assume the same frictional force. (3 marks)

22 A 1.40 × 10^3 kg car is parked on a steep incline on a 30.0 m cliff overlooking the ocean. The road is inclined at 34.0° below the horizontal, as shown in the diagram. The driver steps out of the car to admire the view when the handbrake cable snaps. The car rolls 50.0 m along the road before it reaches the edge of the cliff.



a If there is a frictional force of 2.66 kN between the inclined road and the car, **calculate** the magnitude of the acceleration of the car. (3 marks)

b **Calculate** the speed of the car at the bottom of the road. (2 marks)

c **Determine** how far out from the bottom of the cliff the car landed. (4 marks)

23 A length of fishing line is rated as having a breaking strain of 55 N. A student spins a rubber stopper of mass 34 g on the fishing line in a horizontal circle of radius 85 cm at a constant speed. The stopper makes 10 revolutions in 1.25 s.

a **Determine** the magnitude of the velocity of the stopper. (3 marks)

b **Predict** the speed necessary to break the fishing line. (2 marks)

c **Calculate** the mass the fishing line would support if hung vertically. (1 mark)

24 A ball on a string travels in a horizontal circle at a constant speed of $7\,m\,s^{-1}$, and it takes 13.5 s for 10 revolutions. **Calculate** the radius of the circle. (3 marks)

25 A ball is swung in a horizontal circle on a 1.2 m length of string. It has a constant speed of $3.3\,m\,s^{-1}$. **Calculate** the centripetal acceleration of the ball. (2 marks)

26 You are driving a car around a corner, and you experience a sideways force of 450 N against your seatbelt. The radius of the curve is 30.0 m and your mass is 60.0 kg. **Determine** how fast you must be travelling in your car. (2 marks)

27 A deuterium atom consists of a nucleus made up of a proton and a neutron, and an orbital electron about 1.5 μm from the nucleus. **Determine** the gravitational force of attraction between the electron and the nucleus. (3 marks)

28 A distant star has a planet in a circular orbit of radius 5.0×10^9 m with a period of 50 days.

a **Determine** the order of magnitude of the mass of the star. (3 marks)

b **Predict** whether it is possible to determine the mass of this planet from the data given. Give a reason for your answer. (2 marks)

29 A satellite is in a circular orbit around the Earth at a distance of two Earth radii from the Earth's surface.

a **Determine** the acceleration due to gravity aboard the satellite. (2 marks)

b **Calculate** the weight of a 70 kg astronaut aboard the satellite. (2 marks)

30 Satellite X orbits a planet at a velocity of $12{,}000\,m\,s^{-1}$. Satellite Y is three times more massive than satellite X but orbits at only half the distance of X from the centre of the planet. **Determine** the speed of satellite Y. (3 marks)

31 Suppose there is a small planet orbiting our Sun at a distance 14 times the Sun–Earth distance of 1.5×10^{11} m. **Determine** the orbital period of such a planet, given Kepler's ratio is $2.97 \times 10^{-19}\,s^2\,m^{-3}$. (3 marks)

TOTAL MARKS

/80

MODULE 6

Electrostatics

Introduction

Electric charge is very important in nature. Electric forces and charges control many natural effects and are seen in dramatic circumstances such as lightning strikes and the magnificent southern lights display of the aurora australis.

Much of our modern technology relies on controlling electric charges, either trying to eliminate their effects or making use of their attracting or repelling properties. A knowledge of charge will help us to understand the operation of application devices such as electrostatic generators, photocopiers, lightning arrestors and even the various forms of biological electrostatic defences possessed by animals such as electric eels and rays.

Electric forces are responsible for most of the interactions in chemistry between atoms and molecules, either holding them together to form liquids and solids, or in reactions to make new substances.

Electric forces are also involved in the metabolic processes that occur within our cells. They have an important role to play in large biological molecules such as proteins and nucleic acids. These molecules – so important to life – are usually electrically charged.

Physicists regard the force of electricity as a fundamental force of nature that is ultimately responsible for other forces such as friction, contact pushes, adhesion and cohesion.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to electrostatics before you start.

Subject matter

Science understanding

→ Describe Coulomb's Law.

→ Solve problems using $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$.

→ Describe the concepts of electric fields, electric field strength and electrical potential energy.

→ Solve problems involving electric field strength using $E = \frac{F}{Q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$.

→ Solve problems involving the work done when an electric charge is moved in an electric field using $V = \frac{\Delta U}{q}$.

Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Charles-Augustin de Coulomb, Michael Faraday, Emil Lenz, Mary Somerville and James Clerk Maxwell who furthered our understanding of electromagnetism.

Science inquiry

→ Investigate the effects of electrostatic charge on various materials, e.g. on trickling water.

Practicals

oxforddigital **This lesson is available on Oxford Digital.**

Lesson 6.3 The effects of electrostatic charge on various materials

Lesson 6.1
Coulomb's law

Key ideas

- Coulomb's law states that like electric charges repel and opposite electric charges attract, with a force proportional to the product of the electric charges and inversely proportional to the square of the distance between them.
- Coulomb's law quantifies the force between electrically charged objects and their separation distances as $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$.
- Problems involving the force between charged objects in one and two dimensions can be solved using Coulomb's law.

Learning intentions and success criteria

What is Coulomb's law?

In Unit 1, you saw that when electrically charged objects are brought into close proximity, there is a force between them that is either attractive or repulsive, depending on the nature of the charge (Figure 1).

Many physicists in the past assumed the relationship between force and distance would be an inverse-square one like Newton's law of universal gravitation. It seemed reasonable to think the electrostatic effect would be like a spherical surface that spread out as it moved away from the source – and hence the effect would be inversely related to the square of the radius. In 1879, French military engineer and physicist Charles-Augustin de Coulomb used a very sensitive electrostatic torsion bar balance system to finally determine the nature of this force experimentally (Figure 2).

From careful measurements made of the quantity of charge, the distances between charges and the forces acting on the charges, Coulomb was able to show that the:

- magnitude of the force was proportional to the product of the charges
- magnitude of the force was inversely proportional to the square of the distance separating the charges
- direction of the force was along a line joining the centres of the charges
- magnitude of the force was dependent on the medium in which the charges were placed.

These points can be summarised mathematically as:

$$F \propto \frac{Qq}{r^2}$$

The constant of proportionality was found to be $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$. This was given the symbol k. Therefore:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$$

where Q and q are electric charges (C), r is the distance between their centres (m) and ε_0 is the permittivity of free space ($8.85 \times 10^{-12}\ \text{C V}^{-1}\ \text{m}^{-1}$).

This last equation is referred to as **Coulomb's law**. The force involved is a vector quantity.

FIGURE 1 A negatively charged rod deflects a stream of water.

FIGURE 2 A Coulomb torsion balance. Two charged spheres are visible on the right.

Coulomb's law states that like electric charges repel and opposite electric charges attract, with a force proportional to the product of the electric charges and inversely proportional to the square of the distance between them, expressed by the formula:
$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$
$= \frac{kQq}{r^2}$

Coulomb's constant, *k* a constant of proportionality relating the force between charged objects to the magnitude of their charge and separation distance;
$k = \frac{1}{4\pi\varepsilon_0}$
$= 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$

point charge a representation of a charged object as a single point in which all charge is concentrated rather than having dimensions in space; an idealised situation where the size of the point is considered negligible

Coulomb's constant, *k*, is equivalent to $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$ for electric charges in a vacuum. The symbol ε_0 (epsilon zero) is called the permittivity of free space (i.e. a vacuum) and has a magnitude of $8.85 \times 10^{-12}\ \text{C V}^{-1}\ \text{m}^{-1}$. For other media, such as plastic, the permittivity is double this. Air has almost the same permittivity as a vacuum, only differing in the fourth decimal place. You are not expected to use the permittivity value separately from the overall constant, *k*. Keep in mind, though, that the value of *k* has many significant figures (about 8) and is rounded to a value of $9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$ (1 s.f.) in the QCAA Formula and data book for convenience. Ignore it when determining how many significant figures to use.

It is important to mention that even though the charges accumulate on the surface of a sphere, mathematically we can consider the charges to be concentrated at the centre of the sphere as **point charges**. For that reason, diagrams showing distances between surfaces of charges are meant to represent the distance between their centres.

FIGURE 3 This coulomb meter can measure positive or negative charge up to 1.999 nC.

How are the forces described?

A pair of charged objects is a good example of Newton's third law in action. In Figure 4, the force on A due to B (F_{AB}) is equal and opposite to the force on B due to A (F_{BA}). The forces are shown as vectors of equal length and facing in opposite directions.

These vectors can also be shown using algebraic notation:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

The forces meet the conditions for Newton's third law. The forces are:

- equal in magnitude (vectors are equal length)
- opposite in direction (vectors point in opposite directions)
- the same type of force (both electrostatic)
- acting on different objects (not both acting on a third object).

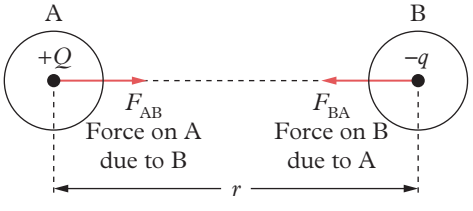


FIGURE 4 A system of two point charges, showing the forces between them. The opposite charges attract each other.

Several of the ideas developed in Unit 1 on electrical circuits are needed again. These include:

- charges can be positive or negative; unlike charges attract and like charges repel
- negative charge is an excess of electrons, and positive charge is a deficiency of electrons
- electric charge, q, is measured in coulomb, symbol C
- the charge on the electron, e, has a value of -1.6×10^{-19} C (QCAA Formula and data book)
- electrical potential difference, V, is measured in volts, symbol V.

Study tip

For two charges in isolation, the force will be repulsive if the charges are the same sign and attractive if the charges are opposite signs.

Real-world physics

How Coulomb discovered his law

Charles-Augustin de Coulomb (Figure 5) was born in France in 1736. After graduating from his local high school, he studied a variety of subjects at the College Mazarin in Paris, namely philosophy, language and literature. He also received a good education in mathematics, astronomy, chemistry and botany. But after his family ran short of money, he moved to the University of Montpellier in southern France, where he completed his science degree. This enabled him to enlist in the army as an engineer. During his time in the army, he was able to carry out many experiments – usually as part of his job, but also out of curiosity. His experiments on friction, electricity and magnetism began in 1779.

FIGURE 5 Charles-Augustin de Coulomb, in 1785. He is holding his famous torsion balance and a scientific paper.

Coulomb measured the electrostatic repulsion between two charged spheres as a function of the distance between them. He was well aware of Newton's law of universal gravitation from 1687, in which the gravitational force varies inversely with the square of the distance between the bodies. Coulomb published his results in 1785, stating that the attractive force between two oppositely charged spheres is proportional to the product of the quantities of charge on the spheres and inversely proportional to the square of the distance between them.

Coulomb also found the inverse square law held true for magnetic poles, but he didn't explore the link between electricity and magnetism as he thought they were two different "fluids". In a related area, the English physicist Henry Cavendish in 1798 wondered if the gravitational force could be confirmed using the same type of balance (a "torsion" balance) that Coulomb had developed, and Cavendish met with great success. Coulomb died at the age of 70 and left a legacy as a pioneer in the field of engineering and science. If you ever visit Paris, you will find his name inscribed on the Eiffel Tower. This would make a great photo.

Apply your understanding

1 Assuming Coulomb's law to hold true, **assess** whether a plot of force, F, on the y-axis and separation distance, r, on the x-axis would give a linear graph. (2 marks)

2 A graph is plotted of force, F, on the y-axis, and $\frac{1}{r^2}$ on the x-axis. **Propose** what value the gradient represents. (2 marks)

3 If both charges in Coulomb's experiment had a magnitude of q, **determine** whether the shape of a graph of F vs q^2 would be linear or non-linear. (2 marks)

Challenge

Which is the bigger charge?

Identify the bigger charge on each of the pairs below. (1 mark each)

a 3.5×10^{-6} C or 7.5×10^{-6} C

b 4×10^{-6} C or 7×10^{-5} C

c 1 mC or 1 μC

Challenge

The mass of a charged rod

When a rod becomes positively charged, does its mass increase or decrease? What if it became negatively charged? (2 marks)

How are forces calculated between charges in one dimension?

Study tip

If a question says two charges are the same (that is, $Q = q$), the product of $Q \times Q$ is Q^2 and not $2Q$. This is always a big mistake in exams.

The simplest problems involving Coulomb's law are for objects in one dimension – that is, in a straight line. This can be two charged objects such as two electrons near each other, or three or more charged objects in a straight line. Worked example 6.1A shows you how to calculate the force for two charges in a line.

Two-dimensional systems are more complicated. The vector nature of Coulomb's law is very important in these cases. You need to determine the magnitude and direction of the resultant force using vector addition techniques, which you can see in Worked example 6.1B.

In problems dealing with Coulomb's law, it is often convenient to consider the charges as point charges. This is possible whenever the separation distance of the charges is very large compared to the size of the charges themselves.

Labelling forces

Labelling the forces can be tricky, but it is good to be consistent. Consider a pair of charged objects, A and B (Figure 6). Each experiences a force due to the other. The force on A due to B is labelled as F_{AB}. It is a shortened version of $F_{\text{on A due to B}}$. Using this order for the subscripts helps you keep track of the various forces and what objects are being considered. It is important to recognise that both objects are responsible for the forces between them and the magnitudes of the forces are equal.

You can practise labelling forces as you calculate the forces between three charged objects in a line in Worked example 6.1B.

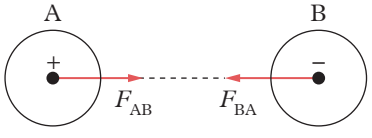


FIGURE 6 Labelling the forces between two charges

Worked example 6.1A

Calculating the force between two charges

Consider the system of two charges as shown in Figure 7. If $Q = +10\,\mu C$ and $q = -5.0\,\mu C$, and the separation distance is 30 cm, **calculate** the force between them to the nearest whole number. (3 marks)

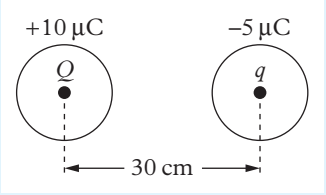


FIGURE 7 Two point charges, Q and q, 30 cm apart

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$Q = +10\,\mu C = +10 \times 10^{-6}\,C$, $q = -5\,\mu C = -5 \times 10^{-6}\,C$, $r = 30\text{ cm} = 0.30\,m$ $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$
Step 3: Substitute the known values into the formula and solve for F. (Note: k is from the QCAA Formula and data booklet, so you don't need to remember the value.) State the answer to the nearest whole number. (1 mark for "Provides appropriate conversion of units"; 1 mark for "Demonstrates correct substitution")	$F = \frac{kQq}{r^2}$ $= 9 \times 10^9 \times \frac{Qq}{r^2}$ $= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 5 \times 10^{-6}}{0.30^2}$ (1 mark) $= 5\,N$ (1 mark)
Step 4: Because force is a vector quantity, you need to state the direction. (1 mark for "Provides correct answer")	The charges are opposite, so the force is attractive. $F = 5\,N$ (attraction) (to the nearest whole number) (1 mark)

Your turn

Point sources with charges of −5.0 mC and −10 mC are placed 1.2 cm apart. **Calculate** the force between them. Give your answer to an appropriate number of significant figures. (3 marks)

Worked example 6.1B

Forces between multiple charges in a line

Three objects, A, B and C, are in a line and have charges $Q_A = +10\,\mu C$, $Q_B = -10\,\mu C$ and $Q_C = +10\,\mu C$. They are separated by distances to their centres as shown in Figure 8.

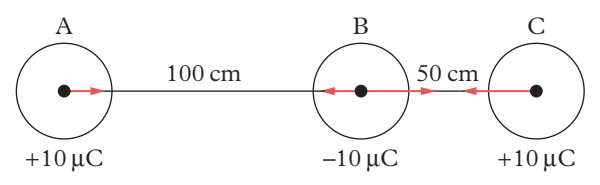


FIGURE 8 Diagram showing forces between charges A, B and C, in one dimension

Calculate the net force on B due to A and C. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$Q_A = +10\,\mu C$, $Q_B = -10\,\mu C$, and $Q_C = +10\,\mu C$ $r_{AB} = 100\text{ cm} = 1.00\text{ m}$, $r_{BC} = 50\text{ cm} = 0.50\text{ m}$ $F = \frac{kQq}{r^2}$
Step 3: Calculate the force on B due to A, F_{BA}. Substitute the known values into the formula and solve for F_{BA}. (1 mark for "Calculates the force between one of the pairs")	$F_{BA} = 9 \times 10^9 \times \frac{Q_A Q_B}{r^2}$ $= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{1.0^2}$ $= 0.90\text{ N}$ attraction (to the left) (1 mark)
Step 4: Calculate the force on B due to C, F_{BC}. Substitute the known values into the formula and solve for F_{BC}. (1 mark for "Calculates the force between the other pair")	$F_{BC} = 9 \times 10^9 \times \frac{Q_B Q_C}{r^2}$ $= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{0.50^2}$ $= 3.6\text{ N}$ attraction (to the right) (1 mark)
Step 5: Calculate the net force by adding vectors for F_{BA} and F_{BC}. Choose an appropriate sign convention such as "to the right is positive". (1 mark for "Provides appropriate mathematical reasoning")	$F_{net} = -0.90\text{ N} + 3.6\text{ N}$ $= +2.7\text{ N}$ (1 mark)
Step 6: Give your answer to an appropriate number of significant figures, in this case 2 s.f. State your answer with the correct units and direction. (1 mark for "Provides correct answer")	$F_{net} = 2.7\text{ N}$ to the right (2 s.f.) (1 mark)

Your turn

Three objects, A, B and C, are in a line and have charges $Q_A = -5.0\,mC$, $Q_B = +10\,mC$, and $Q_C = -5.0\,mC$. They are separated by distances to their centres as shown in Figure 9.

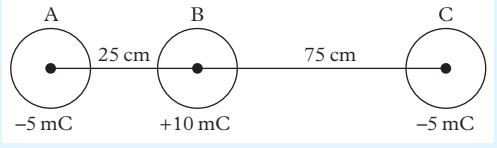


FIGURE 9 Diagram showing three charges, A, B and C, in one dimension

Calculate the net force on C due to A and B. Give your answer to an appropriate number of significant figures. (4 marks)

How are forces calculated between charges in two dimensions?

For three or more objects not in a line, the vector sum becomes more difficult. Worked examples 6.1C, 6.1D and 6.1E show you how to calculate forces for three charged objects in different non-linear arrangements.

Worked example 6.1C

Forces between equal non-linear charges with a right angle

Consider a system of three charges, A, B and C, which have charges $Q_A = +5.0\,\mu C$, $Q_B = +5.0\,\mu C$ and $Q_C = -5.0\,\mu C$. They are separated by the distances shown in Figure 10. The lines between the charges make a right-angled triangle with interior angles of 45°. **Calculate** the net force acting on object C due to A and B, and draw a vector diagram. Give your answer to an appropriate number of significant figures. (5 marks)

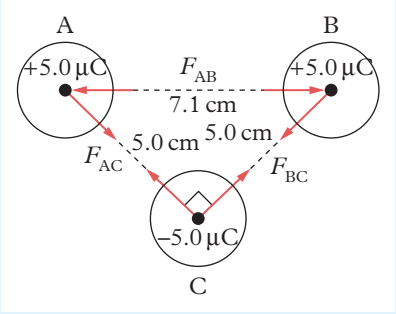


FIGURE 10 Diagram showing the forces between three charges, A, B and C, in two dimensions

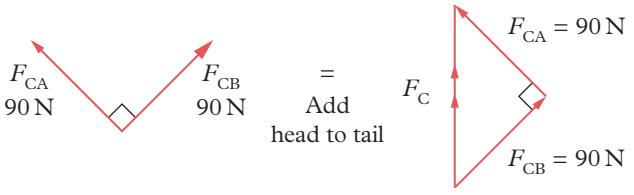
Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula. Based on the information given, you can identify the question as one relating to Coulomb's law.	$Q_A = +5.0\,\mu C$, $Q_B = +5.0\,\mu C$, and $Q_C = -5.0\,\mu C$ $r_{AB} = 7.1\,cm = 0.071\,m$, $r_{BC} = r_{AC} = 5.0\,cm = 0.050\,m$ $F = \frac{kQq}{r^2}$
Step 3: To calculate the force acting on C by A and B, we need to first calculate the force on C due to A, F_{CA}. (1 mark for "Calculates the force on C due to A")	$F_{CA} = 9 \times 10^9 \frac{Q_A Q_C}{r^2}\,Nm^2\,C^{-2}$ $= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 5 \times 10^{-6}}{0.05^2}\,Nm^2\,C^{-2}$ $= 90\,N$ attraction (1 mark)
Step 4: Then calculate the force on C due to B, F_{CB}. (1 mark for "Calculates the force on C due to B")	$F_{CB} = 9 \times 10^9 \times \frac{Q_B Q_C}{r^2}\,Nm^2\,C^{-2}$ $= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 5 \times 10^{-6}}{0.05^2}\,Nm^2\,C^{-2}$ $= 90\,N$ attraction (1 mark)

Study tip

You will be examined on a range of scenarios involving Coulomb's law, from a simple pair of charged objects up to the more difficult system of three objects not in a line. You can resolve vectors or use trigonometry (sin and cos) for difficult situations. If all else fails, you can try a graphical solution using a ruler. Protractors are not permitted in exams.

Study tip

Most problems involving forces in two dimensions may also be solved by dividing the diagram up into smaller right-angled triangles and using simpler Pythagorean calculations.

Think	Do
Step 5: Determine the net force by drawing a vector diagram and adding the vectors head to tail. Draw the resultant vector with a double arrow. As it is a right-angled triangle with interior angles of 45°, it is an isosceles triangle and CA = CB. (1 mark for "Constructs a correct vector diagram of the relevant quantities")	 (1 mark)
Step 6: Calculate the resultant using geometry. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer for magnitude"; 1 mark for "Provides correct answer for direction")	$F_C = \sqrt{90^2 + 90^2}$ $= 130\,\text{N}$ (1 mark) The direction is vertically up the page. (1 mark)

Your turn

Consider a system of three equal charges, A, B and C, which have charges $Q_A = -8.0\,\mu\text{C}$, $Q_B = -8.0\,\mu\text{C}$ and $Q_C = -8.0\,\mu\text{C}$. They are separated by the distances shown in Figure 11. **Calculate** the net force acting on object C due to A and B, and draw a vector diagram. Give your answer to an appropriate number of significant figures. (5 marks)

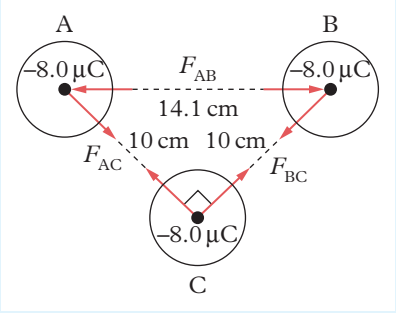
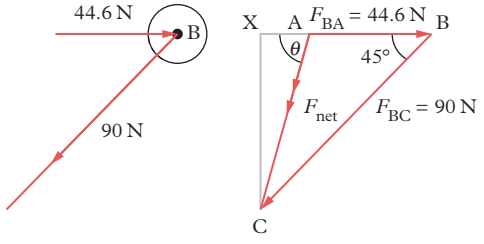


FIGURE 11 Diagram showing the forces between three charges, A, B and C, in two dimensions

Worked example 6.1D

Forces between unequal non-linear charges at a right angle

For the charges A, B and C in Worked example 6.1C (i.e. $Q_A = +5.0\,\mu C$, $Q_B = +5.0\,\mu C$ and $Q_C = -5.0\,\mu C$) and separation distances shown, **calculate** the net force on object B due to charges A and C, and draw a vector diagram. Give your answer to an appropriate number of significant figures. (7 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula. Based on the information given, you can identify the question as one relating to Coulomb's law.	$Q_A = +5.0\,\mu C$, $Q_B = +5.0\,\mu C$, and $Q_C = -5.0\,\mu C$ $r_{AB} = 7.1$ cm $= 0.071$ m, $r_{BC} = r_{AC} = 5.0$ cm $= 0.050$ m $F = \frac{kQq}{r^2}$
Step 3: The net force on B, $\vec{F}_{net}$, is the vector sum of the forces on B due to A and due to C. (1 mark for "Identifies the relationship between forces")	$\vec{F}_B$ (or $\vec{F}_{net}$) $= \vec{F}_{BC} + \vec{F}_{BA}$ (vector sum) (1 mark)
Step 4: You know that $F_{CB} = 90$ N from Worked example 6.1C. So, $F_{BC} = 90$ N but in the opposite direction (towards C). (1 mark for "Identifies the force on B due to C")	$F_{BC} = -90$ N (1 mark)
Step 5: Calculate the force on B due to A using Coulomb's law. (1 mark for "Calculates the magnitude and direction of the force on B due to A")	$F_{BA} = 9 \times 10^9 \times \frac{Q_B Q_A}{r^2}$ $= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 5 \times 10^{-6}}{0.071^2}$ $= 44.6$ N repulsion (1 mark)
Step 6: Determine the net force by drawing a vector diagram and adding the vectors head to tail. Draw the resultant vector with a double arrow. (1 mark for "Constructs a correct vector diagram of the relevant force values")	 (1 mark)
Step 7: Use geometry to solve for the resultant force shown by vector F_{AC}. Add a point X to the diagram in Step 6 that makes a right angle between B and C to help you solve it. Because the angle at B is 45°, you know that this is an isosceles triangle and XB and XC are the same length. This is one way to solve it (only because the angle is 45°) using Pythagoras' theorem. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides appropriate mathematical reasoning")	$(XB)^2 = \frac{1}{2} \times 90^2$ $XB = 63.6$ N $XC = 63.6$ N (45° triangle) $XA = 63.6 - 44.6$ $= 19.0$ N $F_{AC} = \sqrt{63.6^2 + 19.0^2}$ $= 66.4$ N (Pythagoras' theorem) (1 mark)

Think	Do
Step 8: Solve for the angle, θ. State the answer in full (with a magnitude and direction). The angle must be stated in reference to the horizontal, the vertical or some other line. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides correct answer for magnitude"; 1 mark for "Provides correct answer for direction")	$F_{net} = 66.4\,\text{N}$ $\theta = \tan^{-1}\left(\frac{63.6}{19.0}\right)$ $= 74.0°$ from the horizontal (1 mark for the magnitude and 1 mark for the angle)
Alternative method 1: Another way to solve the problem would be to resolve the two force vectors into horizontal (x) and vertical (y) components and add them separately. Then you can solve for the resultant (hypotenuse). This was covered in the section on vector addition, Lesson 2.1, in the Vectors and projectile motion module, and in the Orbital mechanics module.	x component: $F_{BA} = 44.63$ $F_{BC} = -90\sin 45°$ $= -63.64$ $F_{x\,total} = -19.00$ y component: $F_{BA} = 0$ $F_{BC} = -90\sin 45°$ $= -63.64$ $F_{y\,total} = -63.64$ $F_{net} = \sqrt{19.00^2 + 63.64^2}$ $= 66.4\,\text{N}$
Alternative method 2: An alternative solution is to use the cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, and then the sine rule to solve for the angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$F_{AC}{}^2 = F_{AB} + F_{BC}{}^2 - (2 \times F_{AB} \times F_{BC} \times \cos 45°)$ $F_{net}{}^2 = (44.6)^2 + (90)^2 - (2 \times 44.6 \times 90 \times 0.707)$ $= 4{,}412$ $F_{net} = 66.4\,\text{N}$

Your turn

For the three charges A, B and C shown in Worked example 6.1C (i.e. $Q_A = +5.0\,\mu\text{C}$, $Q_B = +5.0\,\mu\text{C}$ and $Q_C = -5.0\,\mu\text{C}$), with separation distances as shown, **calculate** the net force on object A due to the presence of B and C, and draw a vector diagram (2 s.f.). (7 marks)

Worked example 6.1E

Forces between equal non-linear charges — not at a right angle

Consider the system of charges in Figure 12. **Deduce** the magnitude and direction of the force on charge C due to the presence of charges A and B. Each charge is $+4.0 \times 10^{-8}\,\text{C}$ and they are arranged in an equilateral triangle of side 20 cm (2.0×10^{1} cm). Give your answer to an appropriate number of significant figures. (4 marks)

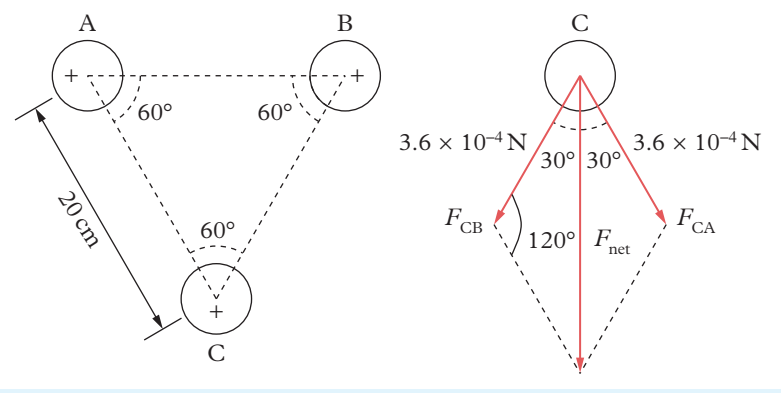
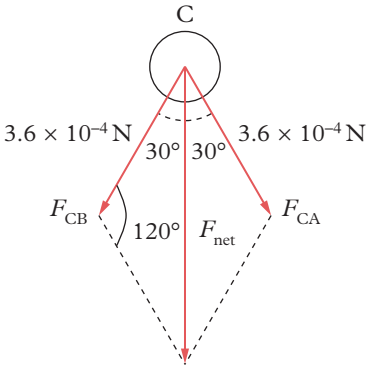


FIGURE 12 Charges A, B and C in an equilateral triangle

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Deduce" means to draw a logical conclusion. We will need to make calculations. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula. Based on the information given, you can identify the question as one relating to Coulomb's law.	$Q_A = Q_B = Q_C = +4.0 \times 10^{-8}$ C $r_{AB} = r_{BC} = r_{AC} = 20\,\text{cm} = 0.20\,\text{m}$ $F = \frac{kQq}{r^2}$
Step 3: The net force on charge C (F_C or F_{net}) is the vector sum (resultant) of forces F_{CA} and F_{CB}, which are equal to each other in magnitude. (1 mark for "Identifies the equivalence of F_{CA} and F_{CB}")	$F_{CA} = F_{CB}$ (magnitude) (1 mark)
Step 4: Calculate the magnitude of each force vector. (1 mark for "Calculates the magnitude and direction of F_{CA} and F_{CB}")	$F_{CA} = 9 \times 10^9 \times \frac{Q_A Q_C}{r^2}\,\text{Nm}^2\,\text{C}^{-2}$ $= 9 \times 10^9 \times \frac{4 \times 10^{-8} \times 4 \times 10^{-8}}{0.2^2}\,\text{Nm}^2\,\text{C}^{-2}$ $F_{CA} = 3.6 \times 10^{-4}$ N repulsion $F_{CB} = 3.6 \times 10^{-4}$ N repulsion (1 mark)
Step 5: Resolve the two vectors into their x and y components. Be careful to make the components that point left negative. (1 mark for "Calculates correctly the sum of components in each direction")	x component: $F_{CB} = 1.80 \times 10^{-3}$ $F_{CA} = -1.80 \times 10^{-3}$ $F_{x\,total} = 0$ $F_{net} = 6.24 \times 10^{-4}$N down (1 mark) y component: $F_{CB} = 3.12 \times 10^{-3}$ $F_{CA} = 3.12 \times 10^{-3}$ $F_{y\,total} = 6.24 \times 10^{-3}$ 
Step 6: State the answer in full (with a magnitude and direction). Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. State that the force is repulsive and will be directed vertically down the page or at an angle of 30° to the line of the force F_{CA}. (1 mark for "Provides correct answer")	$F_{net} = 6.2 \times 10^{-4}$N down (2 s.f.) repulsion The force is repulsive and is directed vertically down the page or at an angle of 30° to the line of the force F_{CA}. (1 mark)
Alternative method: Force (F_{net}) may also be calculated using the cosine rule for triangles. (1 mark for "Provides appropriate mathematical reasoning")	$F^2 = F_{CA}^2 + F_{CB}^2 - (2 \times F_{CA} \times F_{CB} \times \cos 120°)$ $F_{net}^2 = (3.6 \times 10^{-4})^2 + (3.6 \times 10^{-4})^2$ $-(2 \times 3.6 \times 10^{-4} \times 3.6 \times 10^{-4} \times -0.5)$ $= 3.89 \times 10^{-7}$ $F_{net} = 6.2 \times 10^{-4}$ N (1 mark)

Your turn

The charges shown in Figure 12 are moved to double their distance apart, i.e. from 20 cm to 40 cm apart. Students were wondering if this halved the net force on charge C. **Calculate** the net force on C due to A and B to check their proposition. Give your answer to an appropriate number of significant figures. (4 marks)

Challenge

The charge of table tennis balls

If table tennis balls each had a charge of +1 μC, and they could be brought together so that they were touching, what would be the charge on a suitcase containing 862 table tennis balls? (2 marks)

What experiments are done using Coulomb's law?

A great experiment to determine the charge on two spheres involves the use of two ping pong balls coated in conductive paint or wrapped in aluminium foil. The balls, each of mass m, are suspended from a common point by cotton threads of length L. When the balls are given equal negative charge, they move apart and finally settle in a rest position a distance r apart. This makes an angle θ to the vertical (Figure 13). Using vector diagrams and Coulomb's law calculations, we can work out the charge on the balls and compare that value to one measured by an electrometer.

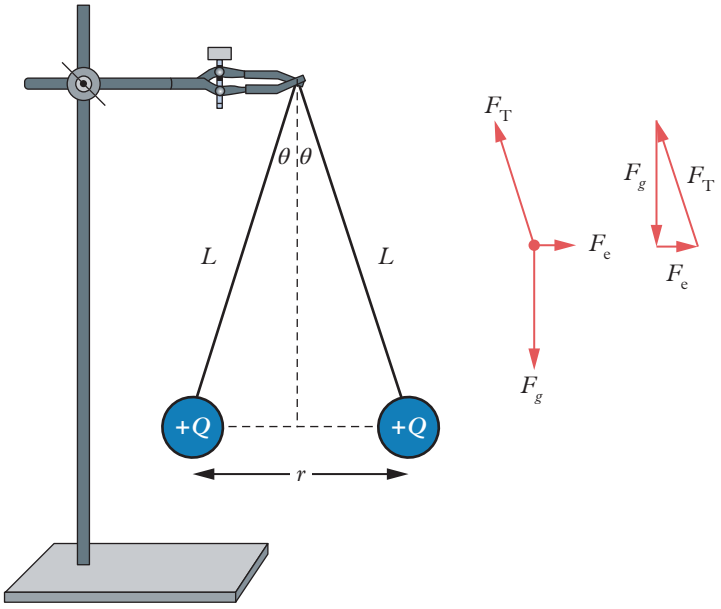


FIGURE 13 Diagram of the ping pong ball setup, along with a free-body diagram and a vector diagram of the forces

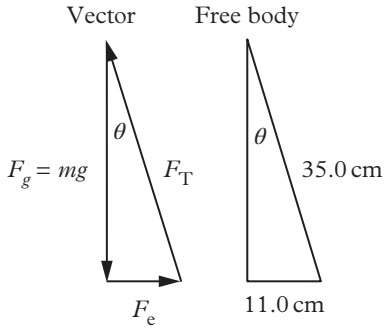
The three forces acting on the right-hand ball are: the weight of the ball, F_g (= mg), the electrostatic repulsion force, F_e, and the tension in the string, F_T. The vector addition of these three forces must be zero, as the ball is at rest so is not accelerating, and hence there is no net force acting on it. The vector diagram shows that the three forces form a closed loop, which means the sum is zero because the start of the arrows and the end of the arrows are in the same place. Another way to look at it is that the vector sum of the weight, F_g, and the electrostatic force, F_e, is equal in magnitude to the tension, F_T.

Worked example 6.1F gives an example of some experimental results and how to interpret them.

Worked example 6.1F

Experimental determination of charge

A pair of conductive ping pong balls of mass 3.15 g each are suspended from a common point by cotton thread of length 35.0 cm. The balls are given an equal negative charge and allowed to come to an equilibrium position 22.0 cm apart. **Calculate** the charge on each ball. Give your answer to an appropriate number of significant figures. (7 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$m = 3.15\,\text{g} = 3.15 \times 10^{-3}\,\text{kg}$ $L = 35.0\,\text{cm} = 0.350\,\text{m}$ $r = 22.0\,\text{cm} = 0.220\,\text{m}$ $F = \frac{kQq}{r^2}$
Step 3: Consider the forces acting on one of the balls. The question states that the balls are at equilibrium. Use this to make a statement about the forces acting on that ball. (1 mark for "Identifies the relationship between the motion of the ball and the net force")	As the ball is at rest, the vector sum of the forces must be zero. (1 mark)
Step 4: Identify the forces involved and draw a labelled vector and free-body diagram. (1 mark for "Constructs correct vector and free-body diagrams of the relevant quantities")	 (1 mark)
Step 5: Calculate the angle θ using trigonometry. (1 mark for "Calculates the angle θ correctly")	$\theta = \sin^{-1}\left(\frac{11.0}{35.0}\right)$ $= 18.32°$ (1 mark)
Step 6: Calculate the weight of the ball. (1 mark for "Calculates the weight of the ball, F_g") It doesn't matter how many significant figures you use for intermediate calculations, but it is wise to record 3 or 4 so you can round off more accurately at the end.	$F_g = mg$ $= 3.15 \times 10^{-3} \times 9.8$ $= 0.03087\,\text{N}$ (1 mark)
Step 7: Calculate the electrostatic (Coulomb) force using trigonometry. (1 mark for "Calculates the electrostatic force, F_e")	$\tan\theta = \frac{F_e}{F_g}$ $F_e = F_g \tan\theta$ $= 0.03087 \times \tan 18.32°$ $= 0.0102\,\text{N}$ (1 mark)
Step 8: Use Coulomb's law to calculate the charge on each ball (which are equal according to the question). The total distance of separation is 22.0 cm (0.220 m). (1 mark for "Provides appropriate mathematical reasoning")	$F_e = \frac{kQq}{r^2}$ $Q^2 = \frac{F_e r^2}{k}$ (1 mark) $= \frac{0.01022 \times 0.220^2}{9 \times 10^9}$ $Q = 2.34 \times 10^{-7}\,\text{C}$

Think	Do
Step 9: Give your answer to an appropriate number of significant figures, in this case 2 s.f. State the answer with the correct units. (1 mark for "Provides correct answer")	$Q = 2.3 \times 10^{-7}$ C (2 s.f.) (1 mark)

Your turn

A pair of conductive ping pong balls of mass 4.50 g each are suspended from a common point by cotton thread of length 42.0 cm as shown in Figure 13. The balls are given an equal negative charge and allowed to come to an equilibrium position 36.0 cm apart. **Calculate** the charge on each ball. Give your answer to an appropriate number of significant figures. (7 marks)

Challenge

Charges of increasing magnitude?

Arrange these electric charges in order of increasing magnitude (1 mark each)

a 4.3×10^{-6} C, 6 μC, 0.000005 C
b 500 μC, 400 nC, 4.5×10^{-6} C
c 0.001 C, 100 μC, 1.5 mC
d 10 mC, 15 μC, 12.5 nC.

Check your learning 6.1

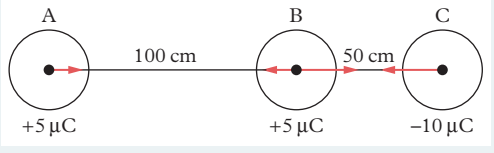
Check your learning 6.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Define** "Coulomb's law" and state the numerical value of the constant to 1 significant figure, including units. (2 marks)
2 **Describe** how you can predict whether two charged objects have like charges or opposite charges. (1 mark)
3 **Clarify** what is meant by an "elementary charge". (1 mark)
4 Two protons in a helium nucleus are separated by a distance of 1.0×10^{-14} m. The magnitude of the charge on a proton is the elementary charge.
   a **Calculate** the electrostatic force between the protons (2 s.f.). (3 marks)
   b **Identify** the order of magnitude of this force. (1 mark)

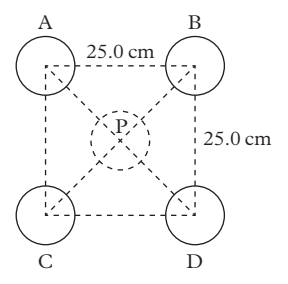
Analytical processes

5 Two point charges A and B, each with a charge $+Q$ C, are separated by a distance of r metres. The force acting between them is 6×10^{-4} N.
Determine
   a whether the force is attractive or repulsive (1 mark)
   b the magnitude of the force if distance r is doubled (2 marks)
   c the magnitude of the force if charge Q on both A and B is doubled (2 marks)
   d the magnitude of the force if charge Q on both is halved, and so is the distance between them. (3 marks)
6 Consider the system of charges in the diagram. **Determine** (2 s.f.) the net force (magnitude and direction on



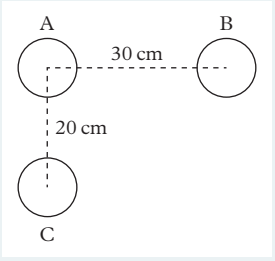
   a B (4 marks)
   b C. (4 marks)

7 Four point charges, A, B, C and D, are arranged on corners of a square of side 25.0 cm, as shown in the diagram. Charges A and B each have a charge of +1 μC, while C and D each have a charge of +2 μC. **Determine** the resultant force on a charge P of +1 μC that is placed at the centre of the square (3 s.f.). (10 marks)



8 An electron ($m_e = 9.1 \times 10^{-31}$ kg) is in a chamber on Earth's surface. Another electron is placed directly underneath it so that the weight of the first electron is balanced by the electrostatic force between the two. The charge on an electron is -1.60×10^{-19} C. **Determine** the distance between the electrons (2 s.f.). (4 marks)

9 Three charges, A (+20 μC), B (−20 μC) and C (+10 μC), are arranged as shown in the diagram. **Determine** the net force on A due to B and C. (6 marks)



Lesson 6.2
Electric fields and field strength

Key ideas

- → Electric fields are regions around a charged particle or object within which a force is exerted on other charged particles or objects.
- → Electric field strength is a measure of the intensity of an electric field at a particular location.
- → Problems on electric field strength can be solved using $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$.
- → Work is done when an electric charge is moved in an electric field.

What are electric fields?

Like other fields you have met in physics, such as gravitational fields, you can visualise an **electric field** as a region of space in which any electrified object will experience a force. This makes it similar to the concept of a mass experiencing a force in a gravitational field. The forces between charges that you studied in Lesson 6.1 exist because each charged object is surrounded by an electric field.

Learning intentions and success criteria

electric field the region of space near an electrically charged particle or object within which a force is exerted on other electrically charged particles or objects

Challenge

True or false?

Consider whether the following statement is true or false: "We are usually not aware of the electric field from everyday objects because there are as many positive charges in a substance as negative charges." (2 marks)

How are electric field lines drawn?

Michael Faraday introduced the idea of electric field lines in 1845, although he called them lines of force. They were a graphical way of depicting the field around a charged body, and at the time they were believed to be real things. Of course, they are not real, but they are a good way of visualising patterns about charges. It is worth remembering that electric fields are in three dimensions around a charged object but are represented as being in two dimensions on paper. This is called a cross-sectional electric field diagram.

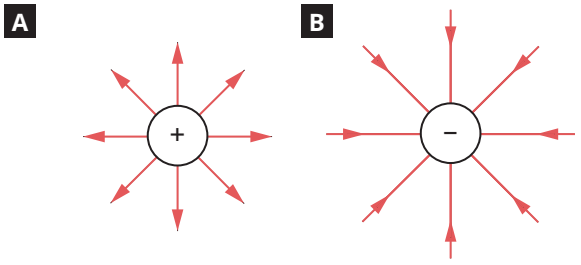


FIGURE 1 Electric field lines representing the fields about (A) a positive charge, and (B) a negative charge

FIGURE 2 Hair strands on a charged head follow the electric field lines radiating outwards.

The arrows point in the direction of the force on a positive test charge in the field. The test charge would move away from a positively charged object (Figure 1A) and would move inwards towards a negatively charged object (Figure 1B).

In Figure 3 you can see the field lines about several different objects. These electric field diagrams show the two-dimensional field lines present for one or more charges. It is important to realise that:

- electric fields radiate in three-dimensional space, but we can represent them in two dimensions on paper or on the screen using cross-sectional field diagrams
- around small, charged spheres, called point charges, the field lines radiate like spokes on a wheel
- electric field lines are directed away from positive charges and towards negative charges
- the direction of the field between two point charges is the vector sum of the fields from each of the point charges
- electric field lines enter or leave any charged object at right angles to the surface
- electric field lines never cross; if they did cross it would indicate that the field was in two directions at once, which isn't possible
- the number of lines per unit area represents the electric field strength at that point; that is, the closer the lines, the stronger the field
- a uniform field exists between two oppositely charged parallel plates. Straight field lines are drawn from the positive plate to the negative plate and are evenly spaced. At the ends, the field curves outwards. This is a non-uniform field sometimes called a "fringing field".

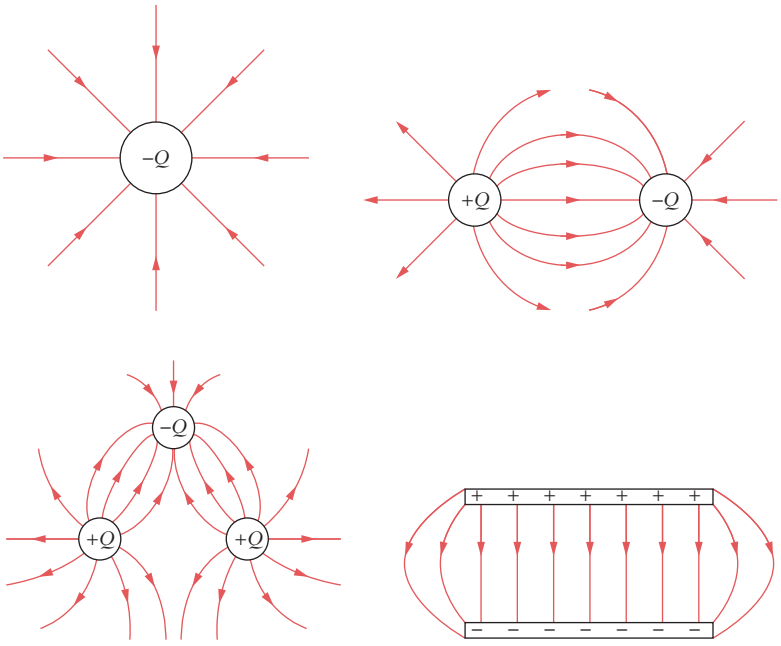


FIGURE 3 Cross-sectional electric field diagrams for one or more charges

Challenge

Which point in a field is stronger?

Figure 4 shows the electric field around a charged object. Are the following statements true or false about the field at point A, E_A, compared to the field at point B, E_B? (1 mark each)

a E_A is twice as strong as E_B because it is half as far.

b E_A is weaker than E_B because it is not on a field line.

c E_A is stronger than E_B because the field lines are closer at A.

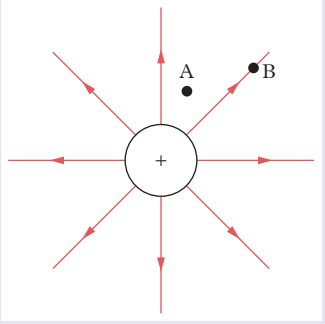


FIGURE 4 Electric field around a charged object

How is electric field strength defined?

We can have strong electric fields, such as a bolt of lightning, or weak electric fields, such as when you rub a plastic comb on a piece of wool. As force is a vector quantity, **electric field strength** is also a vector quantity. It has both magnitude and direction.

electric field strength
the intensity of an electric field at a particular location (symbol: E; SI unit: newtons per coulomb; unit symbol: NC^{-1}) often expressed as volts per metre, Vm^{-1}

One important difference between electric fields and gravitational fields is that gravitational fields have only attractive forces, whereas electric fields can provide both attractive and repulsive forces. The intensity of the electric field – its electric field strength – at any point is defined as the force acting on a test unit charge, q, placed at that point in the field. The direction of the electric field at any point is given by the direction of the resultant force acting on a positive test charge placed at that point in the field.

Thus, mathematically, electric field strength is defined by the following equation:

$$\vec{E} = \frac{\vec{F}}{q}$$

where $\vec{E}$ is the electric field strength in units of newton per coulomb ($\mathrm{N\,C^{-1}}$), $\vec{F}$ is the force acting on a charge in the field in newton (N), and q is the size of the test charge in coulomb (C).

Worked example 6.2A shows you how to use the electric field strength to calculate the force on a charged particle.

Worked example 6.2A

Force on a charged particle in an electric field

A short length of nichrome wire is held vertically under an overhead power wire where the field strength is $5{,}000\,\mathrm{N\,C^{-1}}$ downwards. **Calculate** the force experienced by an electron in the wire, $q_e = 1.60 \times 10^{-19}\,\mathrm{C}$. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$E = 5{,}000\,\mathrm{N\,C^{-1}}$, $q_e = 1.60 \times 10^{-19}\,\mathrm{C}$ $E = \frac{F}{q}$
Step 3: Substitute the known values into the equation and solve for F. (1 mark for "Demonstrates correct substitution")	$E = \frac{F}{q}$ $F = Eq$ $= 5{,}000 \times 1.60 \times 10^{-19}$ $= 8.0 \times 10^{-16}\,\mathrm{N}$ (1 mark)
Step 4: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. As the electric field is a vector quantity, the direction must be stated. (1 mark for "Provides correct answer for direction as upwards")	The field is directed downwards so the force on an electron (being a negative charge) is $8.0 \times 10^{-16}\,\mathrm{N}$ acting upwards. (1 mark)

Your turn

A particle consisting of a single positive elementary charge ($+1e$) is placed in an electric field of strength $12{,}000\,\mathrm{N\,C^{-1}}$ to the right. **Calculate** the force experienced by the charged particle wire, $q_e = 1.60 \times 10^{-19}\,\mathrm{C}$. Give your answer to an appropriate number of significant figures. (2 marks)

What are some examples of electric field strengths?

To give you a feel for the strength of an electric field, the fields about some common appliances and situations are listed in Table 1.

TABLE 1 Examples of electric field strengths

Electric field	Electric field strength ($N\,C^{-1}$)
Surface of a uranium nucleus	3×10^{21}
Electron orbit within a hydrogen atom	5×10^{11}
Electric breakdown in air	3×10^{6}
Surface of a charged drum in a photocopier	10^{5}
Charged plastic comb	1,000
275 kV transmission lines (at ground 25 m away)	500–1,000
11 kV transmission line on ground (10 m away)	50–200
Thunderstorm cloud (ground to base of cloud, no lightning)	50–1,000
Background (home/school/office)	3–50

FIGURE 5 A lightning discharge during a night thunderstorm can produce electric fields of $400\,kN\,C^{-1}$ ($kV\,m^{-1}$).

What types of electric fields are there?

There are two main types of fields we will consider:

- Radial fields – such as those surrounding a charged object
- Uniform fields – such as those between two parallel charged plates

Radial electric fields

The field about a charged object is called a "radial" field because the field lines radiate outwards like the spokes of a wheel (Figure 6). (In Roman times the spoke of a wheel was called a "radius".)

So, as you get further from the object, the field gets weaker. This is shown by the field lines moving further apart. The strength of the field follows an inverse square relationship with radial distance, similar to gravitational force.

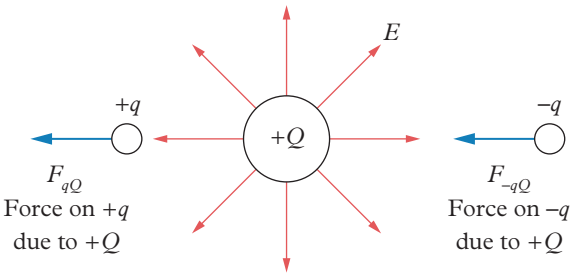


FIGURE 6 An electric field surrounds $+Q$ and test charges ($+q$ and $-q$) experience forces in the field.

Uniform electric fields

The field about a charged object is a radial field, but the field between two charged plates is said to be a "uniform field" (Figure 7).

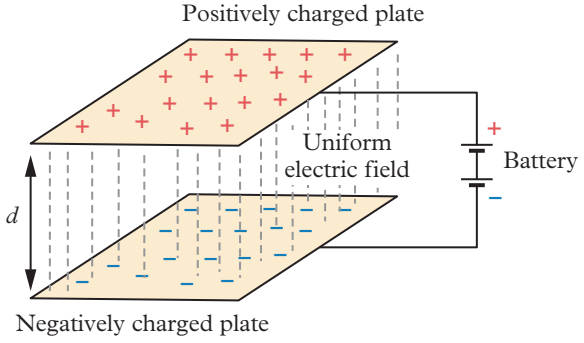


FIGURE 7 A uniform electric field between two oppositely charged plates

A simplified side view is shown in Figure 8.

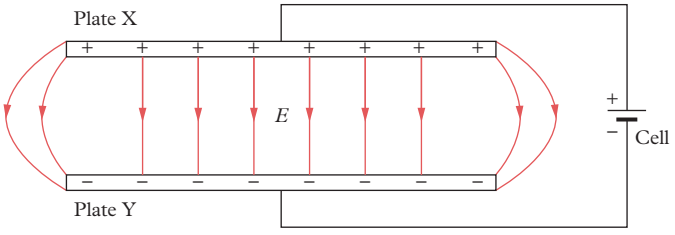


FIGURE 8 Side view of a uniform electric field between two oppositely charged parallel plates, showing field lines

uniform electric field
a field that has constant strength, as found between charged parallel plates

Notice how the electric field between the parallel charged plates is very regular or uniform. This **uniform electric field** will produce a constant force on any test charge held between the plates, independent of its position.

In 1909, the American physicist Robert Millikan used the very uniform electric field between charged parallel metallic plates to investigate the nature of electric charge itself. Charged oil droplets were sprayed between the plates and Millikan was able to balance the upwards electric force on the droplets with the droplets' own gravitational weight. By carefully measuring the mass of these oil droplets and changing their electric charge with X-rays, Millikan was able to calculate a value for the charge on an electron. He won the Nobel Prize in Physics in 1923 for his work on the electrostatics of elementary charges.

How is field strength determined?

Earlier in this lesson, we described electric field strength as the force acting on a charged particle in a field. We can now consider how to determine the strength of two different types of electric fields – radial and uniform. Notice that the cognitive verb is "determine", which means not only making a calculation but also drawing a conclusion based on the vector nature of the fields involved.

1 What is the field strength at a point in a radial field?

These are fields radiating out like spokes on a wheel from a charged object. Consider a point source of positive charge, $+Q$. If small positive test charges, $+q$, were near the charge $+Q$, then each would feel a force of repulsion in radial lines away from $+Q$. The further away $+q$ was moved along these lines, the less intense the electric field strength would be. At all times the magnitude of the force would be given by Coulomb's law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$$

Therefore, the electric field strength, E, at some distance, r, from the charge, $+Q$, would be force per unit charge:

$$\begin{aligned}\vec{E} &= \frac{\vec{F}}{q}\\ &= F \times \frac{1}{q}\\ &= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \times \frac{1}{q}\\ E &= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{kQ}{r^2}, \text{ where } k = 9 \times 10^9 \text{N m}^2 \text{ C}^{-2}\end{aligned}$$

Notice that we have used vector notation in the initial formula to highlight the vector nature of the electric field and the forces on particles in it. However, as the calculation is about the magnitude of the field strength, we don't need to keep showing them as vectors. Its direction points radially outwards.

In the syllabus, the charge is represented by small q, so we use that for consistency:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

This equation therefore describes the electric field strength at a point r from a large point charge $+Q$. The electric field strength, E, is defined as having a value of zero at an infinite distance from its source. Note that diagrams such as Figure 3, earlier in this lesson, show electric fields in a two-dimensional cross-section only. The actual field is always in three dimensions around point charges.

Worked example 6.2B

Field strength at a point in a radial field

Calculate the electric field strength 1 nm (1.0×10^{-9} m) from a proton ($q_p = +1.60 \times 10^{-19}$ C). Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$r = 1\text{ nm} = 1.0 \times 10^{-9}\text{m}$ $q_p = +1.60 \times 10^{-19}\text{C}$ $E = \frac{kq}{r^2}$
Step 3: Substitute the known values into the formula. Convert nanometres to metres ($\times 10^{-9}$). (1 mark for "Demonstrates correct substitution")	$E = \frac{9 \times 10^9 \times 1.60 \times 10^{-19}}{(1.0 \times 10^{-9})^2}$ (1 mark)
Step 4: Calculates answer. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Ensure you have the correct units. Field strength is a vector quantity, and as the question doesn't ask for "magnitude" only, you need to include direction for the mark. (1 mark for "Provides correct answer")	$E = 1.4 \times 10^9\text{NC}^{-1}$ directed away from the proton (1 mark)

Your turn

Calculate the electric field strength 120 cm from a Van de Graaff generator dome that has a net charge of +0.36 μC. Give your answer to an appropriate number of significant figures. (2 marks)

2 What is the field strength between two charges?

The field strength at a point on a line with two charges involves one-dimensional vector addition. The magnitude and direction of each field are used to find the vector sum. This calculation can be more complicated if you have to find a point with a specific field strength – for example a point where the field strength is zero.

Worked example 6.2C

Field strength between two charges

Figure 9 shows two charges A and B separated by a distance of 15.0 cm.

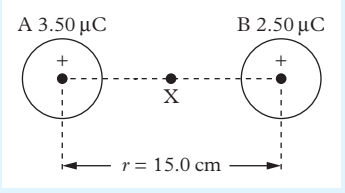
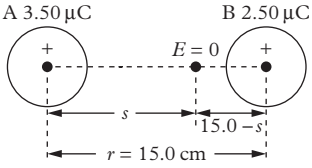


FIGURE 9 Charge A, point X and charge B are in a straight line.

a **Determine** the magnitude and direction of the electric field at point X midway between the two charges. Give your answer to an appropriate number of significant figures. (5 marks)

b **Determine** the point between the charges at which the electric field strength is zero. Give your answer to an appropriate number of significant figures. (5 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: Gather the relevant data from the question and identify the appropriate formula.	**a** $r = 15.0$ cm To point X, $r_X = \frac{r}{2} = 7.50$ cm $= 7.50 \times 10^{-2}$ m $q_A = 3.50$ μC, $q_B = 2.50$ μC $E_A = \frac{kq_A}{r_X^2}$
Step 3: Substitute values for object A into the equation and solve for field strength, E_A. (1 mark for "Calculates the field strength")	$E_A = \frac{9 \times 10^9 \times 3.50 \times 10^{-6}}{(7.50 \times 10^{-2})^2}$ $= 5.59 \times 10^6$ N C^{-1} (to the right) (1 mark)
Step 4: Identify the relevant information from the question and select the appropriate equation. No mark to be awarded.	$E_B = \frac{kq_B}{r_X^2}$
Step 5: Substitute values for object B into the equation and solve for field strength, E_B. (1 mark for "Calculates the field strength")	$E_B = \frac{9 \times 10^9 \times 2.50 \times 10^{-6}}{(7.50 \times 10^{-2})^2}$ $= 4.00 \times 10^6$ N C^{-1} (to the left) (1 mark)
Step 6: Add vectors to get net field strength. (1 mark for "Provides appropriate mathematical reasoning to determine E_{net}")	$\vec{E}_{net} = \vec{E}_A + \vec{E}_B$ (vector sum) (1 mark)
Step 7: Substitute values into the formula and solve for E_{net}. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Calculates the net field strength")	$E_{net} = 5.59 \times 10^6 - 4.00 \times 10^6$ (1 mark) $= 1.59 \times 10^6$ N C^{-1} (to the right, towards B) (1 mark)
Step 8: The point at which the electric field is zero is the point where the electric field E_A is equal in magnitude to the electric field E_B, but opposite in direction. Let this take place at a distance s from A. (1 mark for "Provides appropriate mathematical reasoning for location of point where $E = 0$")	**b**  (1 mark)
Step 9: Let E_A be equal in magnitude to E_B. Substitute the known values into formulas for both E_A and E_B. Solve for distance, s. (1 mark for "Identifies the equivalence of E_A and E_B"; 1 mark for "Demonstrates correct substitution"; 1 mark for "Provides appropriate mathematical reasoning") Note: you could also rearrange the starting equations before substituting. There is less chance of making transcription errors.	$\lvert E_A \rvert = \lvert E_B \rvert$ (1 mark) $\frac{9 \times 10^9 \times 3.50 \times 10^{-6}}{s^2} = \frac{9 \times 10^9 \times 2.50 \times 10^{-6}}{(15.0 - s)^2}$ (1 mark) $\frac{3.50}{s^2} = \frac{2.50}{(15.0 - s)^2}$ $\frac{3.50}{2.50} = \frac{s^2}{(15.0 - s)^2}$ $\sqrt{\frac{3.50}{2.50}} = \sqrt{\frac{s^2}{(15.0 - s)^2}}$ $1.183 = \frac{s}{(15.0 - s)}$ $1.183 \times 15.0 - 1.183s = s$ $17.745 = 2.183s$ $s = 8.13$ cm (1 mark)
Step 10: Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. Ensure the direction from A is made clear and not just left as 8.13 cm. (1 mark for "Provides correct answer")	$s = 8.13$ cm from A (1 mark)

Your turn

Two charges, A and B, of 7.5 μC and 3.5 μC respectively, are 50.0 cm apart.

a **Determine** the magnitude and direction of the electric field at point X midway between the two charges. Give your answer to an appropriate number of significant figures. (5 marks)

b **Determine** the point between the charges at which the electric field strength is zero. Give your answer to an appropriate number of significant figures. (5 marks)

3 What is the field strength near two charges?

Field strength near two charges but not in line with them involves two-dimensional vector addition. Magnitude and direction of each field is used to find the 2D vector sum. It can be complicated, but you have the skills for this after meeting similar situations with gravitational field strengths in the previous module.

FIGURE 10 Electric eels can generate an electric field of up to $800\,\text{N}\,\text{C}^{-1}$ along the length of their body to stun their prey.

Worked example 6.2D

Field strength near two charges (non-linear)

Figure 11 shows two charges A (+3.0 nC) and B (+34 nC) separated by a distance of 82 cm. There is a point C which is 30 cm directly below charge A. **Determine** the electric field strength experienced at C due to charges A and B. Give your answer to an appropriate number of significant figures. (9 marks)

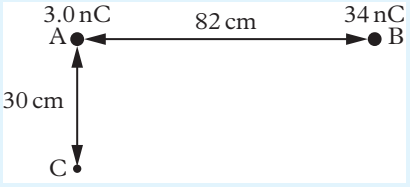
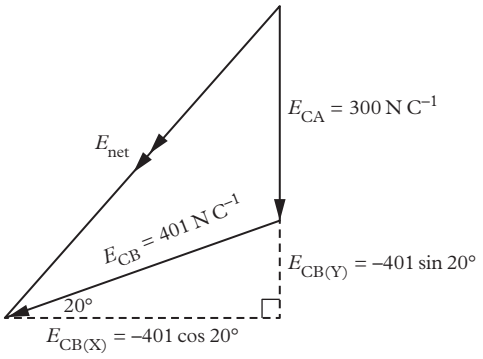
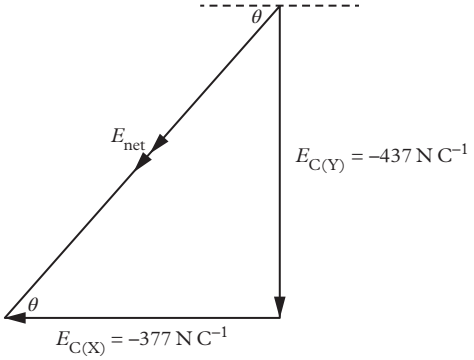


FIGURE 11 Three charges, A, B and C, are not in a straight line. Line AC is at right-angles to AB.

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula. Identify this as a question about electric field strength and write down the equation.	$q_A = +3.0\,\text{nC}$, $q_B = +34\,\text{nC}$ $\text{AB} = 82\,\text{cm} = 0.82\,\text{m}$ $\text{AC} = 30\,\text{cm} = 0.30\,\text{cm}$ $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$
Step 3: Calculate the distance between charge B and point C. The triangle BAC is a right triangle. The side BC is found using Pythagoras' theorem. (1 mark for "Calculates the length of BC")	$\text{BC} = \sqrt{0.82^2 + 0.30^2}$ $= 0.87\,\text{m}$ (1 mark)
Step 4: Calculate the field strength at C due to A using $E = \frac{kq}{r^2}$. Convert nm to m. (1 mark for "Calculates field strength at C by A")	$E_{CA} = \frac{9 \times 10^9 \times 3 \times 10^{-9}}{(0.30)^2}$ $= 300\,\text{N}\,\text{C}^{-1}$ (1 mark)
Step 5: Calculate the field strength at C due to A using $E = \frac{kq}{r^2}$. Convert nm to m. (1 mark for "Calculates field strength at C by A")	$E_{CB} = \frac{9 \times 10^9 \times 34 \times 10^{-9}}{(0.87)^2}$ $= 401\,\text{N}\,\text{C}^{-1}$ (1 mark)

Think	Do
Step 6: Add the two field strength vectors head to tail at the correct angle (E_{CB} is 110° anticlockwise from vertical). (1 mark for "Provides correct mathematical reasoning")	 (1 mark)
Step 7: Resolve the vector E_{CA} into components along the x- and y-axes. (1 mark for "Calculates the components of the field on C due to A")	$E_{CA(X)} = 0\,\text{N C}^{-1}$ $E_{CA(Y)} = 300\,\text{N C}^{-1}$ (1 mark)
Step 8: Decompose the vector E_{CB} into components along the x- and y-axes. (1 mark for "Calculates the components of the field on C due to B")	$E_{CB(X)} = E_{CB}\cos\theta$ $= -401\cos 20°$ $= -377\,\text{N C}^{-1}$ $E_{CB(Y)} = E_{CB}\sin\theta$ $= -401\sin 20°$ $= -137\,\text{N C}^{-1}$ (1 mark)
Step 9: Add the x components and add the y components of the vectors. (1 mark for "Calculates the sum of fields in both directions")	x component: $E_{CA(X)} = 0$ $E_{CB(X)} = -377$ $E_{C(X)} = E_{CA(X)} + E_{CB(X)}$ $= 0 + (-377)$ $= -377\,\text{N C}^{-1}$ y component: $E_{CA(Y)} = -300$ $E_{CB(Y)} = -137$ $E_{C(Y)} = E_{CA(Y)} + E_{CB(Y)}$ $= -300 + (-137)$ $= -437\,\text{N C}^{-1}$ (1 mark)
Step 10: Add the two vectors head to tail and draw a diagram to show the resultant. Solve for the resultant $E_{net} = E_C$ using Pythagoras' theorem. Calculate the angle, θ, using trigonometry. (1 mark for "Provides correct mathematical reasoning")	 $\theta = \tan^{-1}\left(\frac{437}{377}\right)$ $= 49°$ (2 s.f.) (1 mark)
Step 11: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. Be sure to include the direction. (1 mark for "Provides correct answer")	The electric field strength at C is $580\,\text{N C}^{-1}$ at an angle of 49° towards the left, below the line joining A and B. (1 mark)

Your turn

Two charges, A and B, of 10 nC and 100 nC respectively, are 60 cm apart.

There is a point C which is 30 cm directly below charge A (Figure 12). **Determine** the electric field strength experienced at C due to charges A and B. Give your answer to an appropriate number of significant figures. (9 marks)

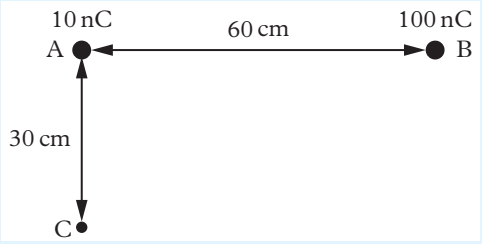


FIGURE 12 Charges A and B are in a horizontal line 60 cm apart. Point C is 30 cm directly below A.

4 What is the field strength between two charged plates?

You have seen that the electric field around a point charge is radial, so the field strength follows an inverse-square law. To find the resultant field strength from two point charges, you must consider the radial field from each point charge. However, unlike radial fields, the field between two oppositely charged parallel plates is uniform (Figure 13).

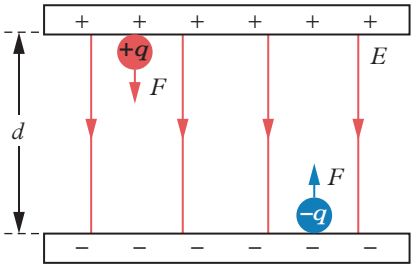


FIGURE 13 The field lines show the uniform field between parallel plates and run from + to –, as shown by the arrows.

If a charged particle is placed in a uniform electric field, it will experience a force that is constant everywhere in the field (between the plates). The sign of the charge (+/–) and direction of the field allow you to determine the direction of the force. A positive charge will experience a force in the direction of the field and, if free, would move "with the field". In Figure 13 the force is downwards and, if free, the charge $+q$ would move downwards. You can think of it as being repelled by the positive top plate. This is consistent with the definition of field direction, which is the direction of the force on a positive test charge. On the other hand, a negative charge such as an electron will experience a force upwards, in the opposite direction to the field. A free negative charge is said to move "against the field". In Figure 13 the charge $-q$ would move upwards.

If a net force acts on a charged particle, it will accelerate consistent with Newton's second law:

$$\vec{a} = \frac{\vec{F}}{m}$$

This is illustrated in Worked example 6.2E.

Worked example 6.2E

Force on a charged particle in a uniform field

A proton ($q = +1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) is introduced at rest into a uniform electric field of strength of 4.00×10^{3} N C^{-1}, as shown in Figure 14.

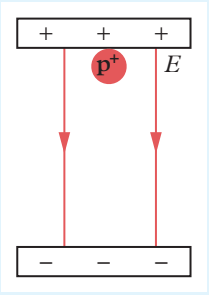


FIGURE 14 Proton in uniform electric field

Calculate (to an appropriate number of significant figures)

a the magnitude and direction of the force acting on the proton due to the electric field (2 marks)

b the acceleration of the proton in the direction of the field (2 marks)

c the time for the proton to reach a speed of 2.00×10^{5} m s^{-1} (2 marks)

d the distance travelled in going from rest to this speed. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Identify this as a scenario related to electric field strength. Recall the appropriate formula.	**a** $q = +1.60 \times 10^{-19}$ C $m = 1.67 \times 10^{-27}$ kg $E = 4.00 \times 10^{3}$ N C^{-1} $E = \frac{F}{q}$
Step 3: Substitute the known values into the formula and solve for force, F. (1 mark for "Demonstrates correct substitution")	$E = \frac{F}{q}$ $F = Eq$ $= 4.00 \times 10^{3} \times 1.60 \times 10^{-19}$ (1 mark)
Step 4: State the full answer with magnitude and direction. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Use the correct units. (1 mark for "Provides correct answer")	$F = 6.40 \times 10^{-16}$ N downwards (3 s.f.) (1 mark)
Step 5: Select the appropriate formula and substitute in the known values. Calculate the answer, making sure you have the correct number of significant figures and the correct units. (1 mark for "Demonstrates correct substitution")	**b** $a = \frac{F}{m}$ (Newton's second law) $= \frac{6.40 \times 10^{-16}}{1.67 \times 10^{-27}}$ (1 mark)
Step 6: State magnitude and direction of acceleration using the correct units. Give your answer to an appropriate number of significant figures, in this case 3 s.f. State the direction. (1 mark for "Provides correct answer")	$a = 3.83 \times 10^{11}$ m s^{-2} downwards (1 mark)
Step 7: Select the appropriate equation and substitute values into it. (1 mark for "Demonstrates correct substitution")	**c** $v = u + at$ $t = \frac{v - u}{a}$ $= \frac{2.00 \times 10^{5} - 0}{3.83 \times 10^{11}}$ (1 mark)
Step 8: Calculate the answer ensuring you have the correct number of significant figures and units. (1 mark for "Provides correct answer")	$t = 5.22 \times 10^{-7}$ s (3 s.f.) (1 mark)

Think	Do
Step 9: Select the appropriate equation and substitute values into it. (1 mark for "Demonstrates correct substitution")	**d** $s = ut + \frac{1}{2}at^2$ $= \frac{1}{2}at^2$ (as $u = 0$) $= 0.5 \times 3.83 \times 10^{11} \times (5.22 \times 10^{-7})^2$ (1 mark)
Step 10: Calculate the answer ensuring you have the correct units. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides correct answer") Distance is not a vector quantity, so a direction is not needed.	$s = 0.0522\,\text{m}$ (5.22 cm) (3 s.f.) (1 mark)

Your turn

An electron ($q = +1.60 \times 10^{-19}\,\text{C}$, $m = 9.109 \times 10^{-31}\,\text{kg}$) is introduced at rest into a uniform electric field of strength of $1{,}250\,\text{N C}^{-1}$, as shown in Figure 15.

Calculate (to an appropriate number of significant figures)

a the magnitude of force acting on the electron due to the electric field (2 marks)

b the acceleration of the electron in the direction opposite to the field (2 marks)

c the time for the electron to reach a speed of $3.00 \times 10^{7}\,\text{m s}^{-1}$ (2 marks)

d the distance travelled in going from rest to this speed. (2 marks)

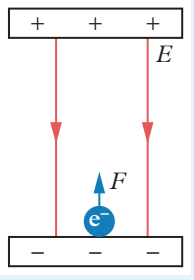


FIGURE 15 An electron in a uniform electric field

How are charged particles deflected in an electric field?

If a charged particle is fired into a uniform electric field from the left (Figure 16), it would continue at a constant velocity unless acted on by an outside force (Newton's first law). In this case, there is a force vertically up or down due to the field and it will affect the charges. A positive particle will deflect downwards, and a negative particle will deflect upwards. Their horizontal components of velocity will not change, as there are no net forces in the horizontal direction. The bigger the force, the greater the deflection away from their original horizontal paths.

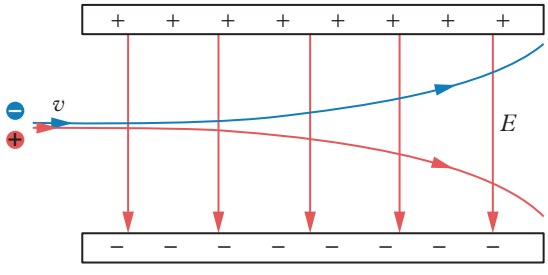


FIGURE 16 Charged particles travelling through parallel plates are deflected by the uniform field

The radius of the path initially can be found using the law for uniform circular motion, with the centripetal force being provided by the field. However, as the particle continues through the field, its motion is no longer perpendicular to the force vector, so this formula no longer works. The force will act only on the component of the particle's velocity that is perpendicular to the field lines (or force vector). The path of the particle becomes parabolic. The principle still holds, though, and can be summarised as:

- the greater the field strength, the more its path bends (smaller r)
- the greater the charge on the particle, the more its path bends (smaller r)
- the greater the mass of the particle, the less its path bends (larger r)
- the greater the velocity of the particle, the less its path bends (larger r).

This principle is used as the first stage in a mass spectrometer, where ions of different speeds can be sorted before they move to the next stage, which uses magnetic fields to sort them further. Magnetic effects on charged particles are discussed in Module 7.

Worked example 6.2F

Deflection of charged particles in an electric field

Particles are fired at right angles to the electric field between charged plates (Figure 16). For each pair that follows **deduce** which one has the smaller radius of curvature.

a A helium ion with a double net charge He^{2+}, or with a single net charge He^{+} (1 mark)

b A proton or an electron (1 mark)

c An electron travelling at $3.0 \times 10^{6}\,m\,s^{-1}$ or one travelling at $300{,}000\,m\,s^{-1}$ (1 mark)

d A helium atom or an alpha particle (1 mark)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Deduce" means to draw a logical conclusion. For each part we must identify the key characteristic and use it to find an answer.
Step 2: Decide which of the key characteristics is different between the two.	**a** Key factors are: charge, mass and speed. In this case charge is different.
Step 3: State which particle will be most affected by this characteristic. (1 mark for "Provides appropriate mathematical reasoning")	He^{2+} has double the charge so will have a smaller radius. (1 mark)
Step 4: Decide which of the key characteristics is different between the two.	**b** Key factors are: charge, mass and speed. In this case mass is different.
Step 5: State which particle will be most affected by this characteristic. (1 mark for "Provides appropriate mathematical reasoning")	The electron has a smaller mass so its path will have a smaller radius. (1 mark)
Step 6: Decide which of the key characteristics is different between the two.	**c** Key factors are: charge, mass and speed. In this case speed is different.
Step 7: State which particle will be most affected by this characteristic. (1 mark for "Provides appropriate mathematical reasoning")	The electron travelling at the slower speed ($300{,}000\,m\,s^{-1}$) will have a path with a smaller radius. (1 mark)
Step 8: Decide which of the key characteristics is different between the two.	**d** Key factors are: charge, mass and speed. In this case charge is different.
Step 9: State which particle will be most affected by this characteristic. (1 mark for "Provides appropriate mathematical reasoning")	A helium atom is neutral so it has no net charge. It will be undeflected by the field. The alpha particle is made up of two protons so it has a 2^{+} net charge. It will be deflected by the field, so its path has the smaller radius. (1 mark)

Your turn

Particles are fired at right angles to the electric field between charged plates (Figure 16). For each pair that follows **deduce** which one has the smaller radius of curvature.

a An oxygen ion with a double net charge O^{2-}, or with a single net charge O^{-} (1 mark)

b An alpha particle or a calcium ion Ca^{2+} (1 mark)

c A proton travelling at $1 \times 10^{7}\,m\,s^{-1}$ or one travelling at $70{,}000{,}000\,m\,s^{-1}$ (1 mark)

d A beta positive particle or a beta negative particle (1 mark)

Check your learning 6.2

Check your learning 6.2: Complete these questions online or in your workbook.

Retrieval and comprehension

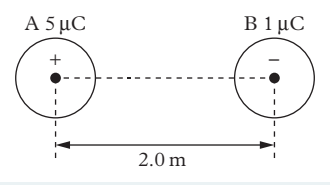
1 **Sketch** the cross-sectional electric field diagram for a system of three negative charges situated at the corners of an equilateral triangle. (2 marks)

2 **Calculate** the magnitude and direction of the electric field strength 0.200 m from a point charge of −6 μC (3 s.f.). (3 marks)

3 **Describe** the difference between electric field and electric field strength. (2 marks)

Analytical processes

4 Two charged spheres are placed 2.0 m apart.



Determine (2 s.f.)

a the electric field strength midway between the two charges (4 marks)

b the electric field strength at a point 50 cm to the right of object B (3 marks)

c the location of a point P (besides infinity) at which the field strength is zero. (5 marks)

5 An electron ($q_e = -1.60 \times 10^{-19}$C, $m_e = 9.109 \times 10^{-31}$ kg) is introduced at rest into an electric field with a constant electric field strength of 50 N C^{-1}. **Determine** (2 s.f.)

a the force acting on the electron due to the electric field (2 marks)

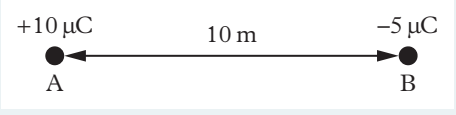
b the acceleration of the electron in the direction of the field (2 marks)

c the time taken for the electron to reach 80% of the speed of light (use $c = 3 \times 10^8$ m s^{-1} and treat the speed of light as having 2 s.f.) (2 marks)

d how far the electron moved in the time taken to go from rest to the final speed. (2 marks)

Knowledge utilisation

6 Charge A (+10 nC) and charge B (−5 nC) are 10 m apart. **Determine** the position of a point, P, where the field strength is zero, besides infinity (2 s.f.). (6 marks)



Practical

Lesson 6.3
The effects of electrostatic charge on various materials

Learning intentions and success criteria

This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.

Lesson 6.4
Electrical potential energy

Key ideas

- → Electrical potential energy is a measure of the energy of charged particles in a field and their ability to do work because of their position in the field.
- → Work is done when an electric charge is moved in an electric field.
- → Electrical potential is a measure of the electrical potential energy of charged particles in a field per unit of charge.
- → The change in electrical potential energy per unit charge between two defined points in an electric field is called electrical potential difference.
- → Equipotential lines join points in space that all have the same electrical potential.
- → Problems on electrical potential difference, electrical potential energy and work done on a charge can be solved using $V = \frac{\Delta U}{q}$.

How are energy and voltage related?

Learning intentions and success criteria

The concepts of electric field and electric field strength are just the beginning of our understanding of electrical phenomena. You are probably more familiar with other aspects of electricity, namely energy and voltage. Energy can be stored in common devices such as batteries, and those batteries have a voltage. But energy and voltage are different things. A computer battery and a car battery might be both 12 V, but you couldn't start a car with the computer battery as it would be too small. It is the total amount of electrical energy and the rate at which it can be delivered that makes the difference, not the voltage. In this section, you will learn about the energy and voltage of charged objects.

How is energy stored in fields?

The gravitational field analogy can be used again when discussing the energy stored in an electric field (Figure 1). When a mass moves in the direction of the gravitational field (towards the ground), it goes from high gravitational potential energy to low potential energy. It is said to go from a region of high potential to a region of low potential. Similarly, when a positive charge moves in the direction of an electric field, it too goes from high electrical potential energy to low potential energy.

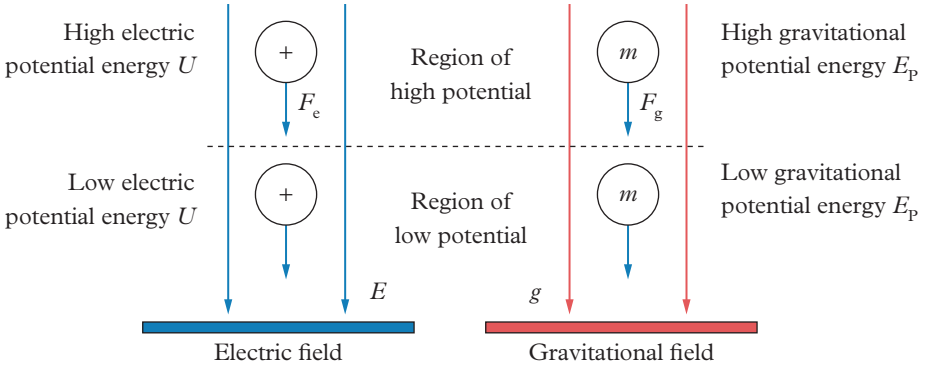


FIGURE 1 A gravitational field analogy for electric fields

Conversely, if a mass is lifted against the direction of a gravitational field, work must be done. As this work is done, gravitational potential energy is stored in the mass and the gravitational field. The energy stored is equivalent to the work done, W, and is calculated using $W = mgh$. Similarly, it requires work to be done to move a positive electric charge against the direction of the field. The charge and the field acquire **electrical potential energy** ($W = \Delta U$) from the work done. The charge then has the capacity to do work. For an electric field, the work done is given by $W = qEd$. You should be able to see the similarity in Table 1.

electrical potential energy the capacity of electric charge carriers to do work due to their position in an electric field (symbol: U; SI unit: joule; unit symbol: J)

TABLE 1 Gravitational and electric field quantities compared

	Gravitational field	Electric field
Work done	work, W (in J) $W = mgh$	work, W (in J) $W = qEd$
Property of the objects	mass, m (in kg)	charge, q (in C)
Field strength	gravitational, g (in $\mathrm{N\,kg^{-1}}$)	electric, E (in $\mathrm{N\,C^{-1}}$)
Distance moved	height, h (in m)	distance, d (in m)

If the charge is moved perpendicular to the field lines, then no work is done, just as rolling a ball on a frictionless horizontal desk requires no energy input and therefore no work is done.

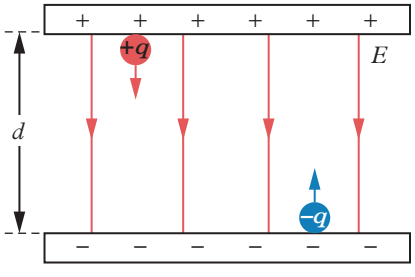


FIGURE 2 Positive and negative test charges freely moving in a uniform field

How are electrical potential energy and work related?

If you pushed two positively charged objects together you would increase their electrical potential energy. By the law of conservation of energy, the work done, W, would equal the change in their electrical potential energy, ΔU. That is, $W = \Delta U$. We use the symbol U for electrical potential energy, so ΔU is the change in electrical potential energy.

Consider the situation where you then release the charged object. In Figure 2, the positive test charge is close to the top, positive plate of the uniform field. It has high electrical potential energy and experiences a repulsive force from the positive plate. It will accelerate away from the plate in the direction of the electric field, gaining speed until it reaches the end of the field at the bottom plate, a distance, d, away. Here it will have low potential energy. The lost potential energy has been converted into kinetic energy. That is, $\Delta U = E_k$.

Likewise, if a negative test charge is at the bottom, negative plate, it has high electrical potential energy and experiences a repulsive force from the negative plate. It will accelerate away from the plate in the opposite direction to the electric field, gaining speed until it reaches the end of the field at the top plate. Here it will have low potential energy, and the lost potential energy has been converted into kinetic energy. The same relationship holds: $\Delta U = E_k$.

Here's an example that relates electric field strength, E, to electrical potential energy, ΔU, and kinetic energy, E_k.

Worked example 6.4A

A charged particle accelerating in an electric field

An alpha particle ($q = +3.2 \times 10^{-19}$ C) is accelerated from rest in a uniform electric field of strength 6,000 N C^{-1}. It travels 20.0 cm in the direction of the field. **Calculate** (to an appropriate number of significant figures)

a the force acting on the particle (2 marks)

b its acceleration (3 marks)

c its speed after travelling the 20.0 cm (3 marks)

d its kinetic energy after the acceleration (3 marks)

e the change in its electrical potential energy. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: **Calculating force.** Select the relevant equation.	**a** $q = +3.2 \times 10^{-19}$ C $E = 6{,}000$ N C^{-1} Distance, $s = 20.0$ cm $= 0.20$ m $E = \frac{F}{q}$
Step 3: Substitute in the known values and solve for F. (1 mark for "Correctly substitutes values")	$F = Eq$ $= 6{,}000 \times 3.2 \times 10^{-19}$ (1 mark)
Step 4: Complete the calculation and use the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. As it is a vector quantity, state the direction. (1 mark for "Provides correct answer")	$F = 1.920 \times 10^{-15}$ $= 1.9 \times 10^{-15}$ N (2 s.f.) in the direction of the field (1 mark)
Step 5: **Calculating acceleration.** Select the relevant equation and make the substitution. The mass of the alpha particle can be found in the QCAA Formula and data book. (1 mark for "Identifies the relationship between force and field strength"; 1 mark for "Demonstrates correct substitution")	**b** $F = ma$ $a = \frac{F}{m}$ (1 mark) $= \frac{1.920 \times 10^{-15}}{6.644 \times 10^{-27}}$ (1 mark)

Think	Do
Step 6: Complete the calculation and express the answer with the correct units and include the direction (if the answer is a vector). Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$a = 2.890 \times 10^{11}$ $= 2.9 \times 10^{11}\,\text{m s}^{-2}$ (2 s.f.) in the direction of the field (1 mark)
Step 7: **Calculating speed.** Select the relevant equation and make the substitution. Note that "at rest" means $u = 0\,\text{m s}^{-1}$. (1 mark for "Identifies the relationship between velocity and acceleration"; 1 mark for "Demonstrates correct substitution")	**c** $v^2 = u^2 + 2as$ $v = \sqrt{2as}$ (1 mark) $= \sqrt{2 \times 2.890 \times 10^{11} \times 0.200}$ (1 mark)
Step 8: Complete the calculation and check you have the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Direction is not required as speed is not a vector. (1 mark for "Provides correct answer")	$v = 340{,}000\,\text{m s}^{-1}$ $= 3.4 \times 10^{5}\,\text{m s}^{-1}$ (2 s.f.) (1 mark for either)
Step 9: **Calculating kinetic energy.** Select the relevant equation and make the substitution. (1 mark for "Identifies the relationship between kinetic energy and velocity"; 1 mark for "Demonstrates correct substitution")	**d** $E_k = \frac{1}{2}mv^2$ (1 mark) $= \frac{1}{2} \times 6.644 \times 10^{-27} \times (3.4 \times 10^{5})^2$ (1 mark)
Step 10: Complete the calculation and check units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Energy is a scalar quantity, so no direction is needed. (1 mark for "Provides correct answer")	$E_k = 3.840 \times 10^{-16}$ $= 3.8 \times 10^{-16}$ J (2 s.f.) (1 mark)
Step 11: **Calculating change in electrical potential energy.** Select the relevant relationship. (1 mark for "Identifies the relationship between electrical potential energy and kinetic energy")	**e** $\Delta U = \Delta E_k$ (1 mark)
Step 12: Complete the substitution and check units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Potential energy is a scalar, so no direction is required. (1 mark for "Provides correct answer")	$\Delta U = 3.8 \times 10^{-16}$ J (2 s.f.) (1 mark)

Your turn

A calcium ion Ca^{2+} has a 2+ elementary charge. It is accelerated from rest by an electric field of strength $2{,}500\,\text{N C}^{-1}$ and moves a distance of 35.0 cm in the direction of the field. The ion has a mass of 6.65×10^{-26} kg. **Calculate** (to an appropriate number of significant figures)

a the force acting on the particle (2 marks)

b its acceleration (3 marks)

c its speed after travelling the 35.0 cm (3 marks)

d its kinetic energy after the acceleration (3 marks)

e the change in its electrical potential energy. (2 marks)

How can formulas be combined to calculate work done?

Another relationship that comes out of the properties of electric fields is to do with work, force and distance. From your work on linear motion, you will have met the formula work, W, equals the net force, F, times the distance, d; that is, $W = F \times d$. We also know that $F = qE$. When we combine these two equations we get:

$$W = qEd$$

Or, in terms of electrical potential energy:

$$\Delta U = qEd$$

This equation is not in the syllabus so will not appear in the external exam. However, it can be useful for checking your answers to questions like these. For Worked example 6.4A:

$$\begin{aligned}\Delta U &= qEd \\ &= 3.2 \times 10^{-19} \times 6{,}000 \times 0.2 \\ &= 3.84 \times 10^{-16}\,\text{J}\end{aligned}$$

This is the same answer as we found for part **e** of Worked example 6.4A.

How is electrical potential related to energy?

So far, we've described an electric field in terms of its field strength, E. It would be useful if we could also describe it in terms of energy. We need to describe the field independently of any charge that may be present in the field. For this, we can use **electrical potential**. If we take a +1 C test charge from far away (infinity), where its electrical potential energy is zero ($U = 0$ J), and do work on it to move it to a certain point in the field, it will have an increase in electrical potential energy of ΔU. This point in the field is said to have an electrical potential, and this is quantified as the electrical potential energy per unit charge, V.

electrical potential
the electrical potential energy stored per unit charge at any given point (symbol: V; SI unit: volt; unit symbol: V)

For example, if you took a +1 C charge and you did 12 J of work on it to move it from infinity to a point in a field, the charge at that point would have an electrical potential energy of 12 J and an electrical potential of 12 J C^{-1} or 12 V. The change in electrical potential is from 0 V to +12 V. That is, $\Delta V = +12$ V. The delta sign is usually omitted, so we say $V = +12$ V.

At infinity, the electrical potential and electrical potential energy are defined as being zero ($V_\infty = 0$, $U_\infty = 0$), so ΔV and ΔU are the same as V and U respectively.

A +1 C charge is massive and not a realistic quantity to deal with, so in practice we would take a +1 μC test charge and measure the work needed to move it to some point in the field. It if took 2.3×10^{-4} J of work to do so, it would have a change of electrical potential energy of $\Delta U = 2.3 \times 10^{-4}$ J, and an electrical potential difference given by:

$$\begin{aligned}V &= \frac{\Delta U}{q} \\ &= \frac{2.3 \times 10^{-4}}{1 \times 10^{-6}} \\ &= 230\,\text{V}\end{aligned}$$

Worked example 6.4B

Calculating electrical potential energy from electrical potential

A helium nucleus has a +2 charge. It is accelerated from rest across a potential difference of +12 V. **Calculate** its change in electrical potential energy. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$q = +2e$ (elementary charges) $V = 12$ V Find ΔU. $V = \frac{\Delta U}{q}$ $\Delta U = qV$
Step 3: Substitute in the known values and solve for ΔU. (1 mark for "Demonstrates correct substitution")	$q = 2 \times 1.6 \times 10^{-19}$ C $\Delta U = (2 \times 1.6 \times 10^{-19}) \times 12$ (1 mark) $= 3.84 \times 10^{-18}$
Step 4: State the answer to the correct number with the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. It is not a vector quantity, so no direction is required. (1 mark for "Provides correct answer")	$\Delta U = 3.8 \times 10^{-18}$ J (2 s.f.) (1 mark)

Your turn

An alpha particle is accelerated from rest across a potential difference of +2,500 V. **Calculate** its change in electrical potential energy. Give your answer to an appropriate number of significant figures. (2 marks)

Is work done "on" or "by" a uniform electric field?

We can investigate the properties of charged particles by studying their motion across an electrical potential difference provided by parallel plates. In Figure 3, the plates are a distance d apart and are connected to a battery, which provides an electrical potential, V.

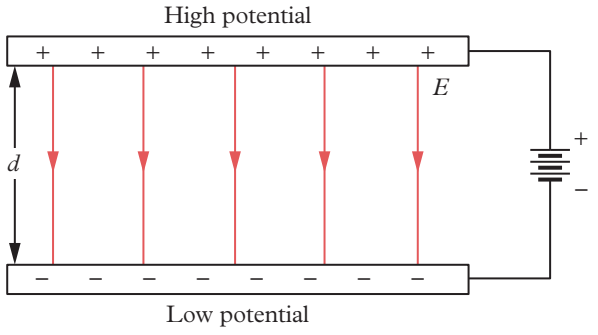


FIGURE 3 Areas of high and low electrical potential in a uniform field

The region of high potential is next to the positive plate, whereas the lowest potential (0 V) is next to the negative plate.

If a charged particle $+q$ is introduced next to the positive plate, it will have high electrical potential energy. It will accelerate towards the negative plate by losing potential energy and gaining kinetic energy (Figure 4).

The next worked example illustrates how we can work out the velocity of a particle accelerated across an electrical potential difference.

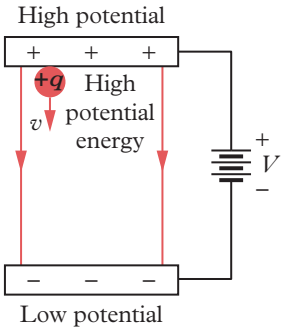


FIGURE 4 Particle accelerating across a uniform field as it moves from high potential energy to low potential energy

Worked example 6.4C

Calculating velocity gained across an electrical potential difference

A proton is accelerated from rest across an electrical potential difference of 75 V. **Calculate** its velocity at the end of the acceleration. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$V = 75\,\text{V}$ $V = \frac{\Delta U}{q}$ $\Delta U = Vq$
Step 3: Substitute in the known values and solve for the change in electrical potential energy, ΔU.	$\Delta U = 75 \times 1.6 \times 10^{-19}$ $= 7.95 \times 10^{-17}\,\text{J}$
Step 4: The work done on an object is equal to the change in kinetic energy. Locate the mass of the proton from the QCAA Formula and data book. (1 mark for "Identifies that work done on the charge equates to its kinetic energy")	$E_k = W = \Delta U$ $\frac{1}{2}mv^2 = \Delta U$ (1 mark)
Step 5: Substitute in the known values and solve for the velocity of the proton, v. (1 mark for "Provides appropriate mathematical reasoning")	$v = \sqrt{\frac{2 \times \Delta U}{m}}$ $= \sqrt{\frac{2 \times 7.95 \times 10^{-17}}{1.67 \times 10^{-27}}}$ (1 mark) $= 218{,}185\,\text{m s}^{-1}$
Step 6: State the answer in scientific notation, using the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Velocity is a vector quantity, so you need to give the direction. (1 mark for "Provides correct answer") Note: in their marking guide, QCAA doesn't always require a direction for vector quantities. Best to be safe.	$v = 2.2 \times 10^5\,\text{m s}^{-1}$ in the direction of the field (2 s.f.) (1 mark)

Your turn

A helium nucleus has a charge of $+3.2 \times 10^{-19}$ C. **Calculate** its velocity when it is accelerated from rest across an electrical potential difference of 120 V. Give your answer to an appropriate number of significant figures. (3 marks)

How are field strength and electrical potential related?

Field strength, E, is defined as force per unit charge $\left(\frac{F}{q}\right)$. If we multiply top and bottom by the distance a charge moves, we get:

$$E = \frac{F}{q} = \frac{Fd}{qd}$$

As work done, W = force, F × distance, d:

$$E = \frac{W}{qd}$$

We can replace $\frac{W}{q}$ by ΔV (potential difference, i.e. work done per unit charge) to give:

$$E = \frac{F}{q} = \frac{\Delta V}{d}$$

This is a "non-syllabus" formula, but it is useful for checking answers. Learn it as:

$$E = \frac{F}{q} = \frac{V}{d}$$

We can see that field strength, E, has the units newton per coulomb ($\mathrm{N\,C^{-1}}$) or volts per metre ($\mathrm{V\,m^{-1}}$). An electric field strength of $600\,\mathrm{N\,C^{-1}} = 600\,\mathrm{V\,m^{-1}}$. In the external exam you are likely to meet field strength expressed as $\mathrm{N\,C^{-1}}$. A simple example follows in Worked example 6.4D.

Worked example 6.4D

Using two different units for field strength

Calculate the electric field strength between a pair of charged parallel plates 50 mm apart with a 12 V potential difference across them. Give your answer to an appropriate number of significant figures in $\mathrm{V\,m^{-1}}$ and $\mathrm{N\,C^{-1}}$. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify this as a problem about electrical potential and field strength between parallel plates. Select the appropriate formula. (1 mark for "Provides appropriate mathematical reasoning")	$d = 50\,\mathrm{mm} = 0.050\,\mathrm{m}$ $V = 12\,\mathrm{V}$ $E = \frac{V}{d}$ $= \frac{12}{0.050}$ (1 mark) $= 24$
Step 3: State the answer with both type of units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Identifies the equivalence of $\mathrm{V\,m^{-1}}$ and $\mathrm{N\,C^{-1}}$")	$E = 24\,\mathrm{V\,m^{-1}}$ **or** $24\,\mathrm{N\,C^{-1}}$ (1 mark)

Your turn

Calculate the distance between two charged plates if a 6.0 V battery is connected across them and a field strength of $300\,\mathrm{V\,m^{-1}}$ is produced. Give your answer to an appropriate number of significant figures in $\mathrm{V\,m^{-1}}$ and $\mathrm{N\,C^{-1}}$. (2 marks)

What is electrical potential difference?

Electrical potential is defined as the energy needed to move a charge from infinity to a point in a field. It is a scalar quantity and is measured in units of joules per coulomb (JC^{-1}). One joule per coulomb is also known as a volt: 1 volt (V) = $1\,JC^{-1}$. Of course, moving unit positive charges from infinity to a point within the field of a point charge Q is not particularly realistic. The quantity **electrical potential difference** is more useful, because it describes the difference in electrical potential between two positions within the field (V_1 to V_2), neither of which is at infinity (Figure 5). Then the potential difference is the difference in potential energy (i.e. work done) per unit charge.

electrical potential difference
the work done in moving a unit charge between the final and the initial positions in an electric field (symbol: ΔV; SI unit: volt; unit symbol: V)

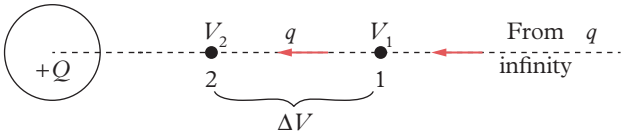


FIGURE 5 Electrical potential difference between two points in a field

$$\Delta V = V_2 - V_1 = \frac{\Delta U}{q}$$

The electrical potential difference ΔV is often given the symbol V, the same symbol as electrical potential. This is not strictly correct, but it is used for convenience. For example, a 9 V battery isn't labelled as a potential difference of 9 V, it just says 9 V and it is assumed that the negative terminal is 0 V.

The potential difference formula is usually written as:

$$V = \frac{\Delta U}{q}$$

but should be interpreted as:

$$\Delta V = \frac{\Delta U}{q}$$

This means the potential difference can be calculated from the work done on a charge, as we'll see in Worked example 6.4E.

Worked example 6.4E

Calculating electrical potential difference from work done

Calculate the magnitude of the potential difference when 15 J of work is done to move a particle with a charge of +2.5 C through an electric field. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Identify this as a question about electrical potential and potential energy. Select the appropriate formulas.	$W = 15\,J$, $q = +2.5\,C$ $W = \Delta U$ $V = \frac{\Delta U}{q}$
Step 3: Substitute the known values and make the calculation. (1 mark for "Demonstrates correct substitution")	$V = \frac{\Delta U}{q}$ $= \frac{15}{2.5}$ (1 mark) $= 6.0$

Think	Do
Step 4: State the answer with the correct units and give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$V = 6.0\,\text{V}$ (2 s.f.) (1 mark)

Your turn

An alpha particle moves through a field where the potential difference is 12 V. **Calculate** the work done on the particle. Give your answer to an appropriate number of significant figures. (2 marks)

Challenge

How do we work out electrical potential at a point?

The formula for electrical potential at a point has been worked out by physicists. It is $V = \frac{kQ}{r}$, where r is the radial distance from an object with a charge of Q (C). It is not a vector quantity but an algebraic one. Show that the electrical potential at a point P midway between a charge Q_1 of +10 μC and a charge Q_2 of −5 μC, which are 70 cm apart, is +128,571 V. (3 marks)

Challenge

Is this rocket launch possible?

You have invented a new way to shoot a rocket into space. Get 1 mole of hydrogen ions (6.02×10^{23} positive H^+ particles) and put them in the base of a spaceship standing on a launch pad. Get another mole of hydrogen ions (H^+) and put them in a wheelbarrow and wheel them under the rocket. The repulsive force will force the rocket into the air.

a **Calculate** the force between these two moles of H^+ ions when placed 1.0 m apart under a Saturn V rocket of mass 3.0 million kg. (2 marks)

b **Calculate** the initial acceleration. (2 marks)

c **Propose** what is implausible about this suggestion. (1 mark)

What do equipotential lines show?

Points in space that all have the same potential make up what is called an equipotential line, or, if they are in three dimensions, an equipotential surface. A series of equipotential lines with each line representing a different value of potential can be used to represent the electric field (Figure 6).

No net work is done on a charge by an electric field when the charge moves along an equipotential line. This is a consequence of $V_f - V_i = \frac{\Delta U}{q}$. That is, if there is no potential difference, no work is done when a charge is moved from one point to another. However, work is done if the charge moves from one equipotential line to another.

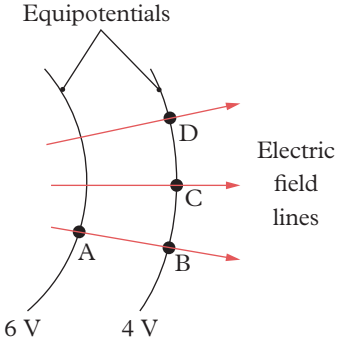


FIGURE 6 No work is done if a charge moves along an equipotential line (from B to C to D), but work is done if a charge moved from A to B.

Worked example 6.4F

Calculating potential difference

Figure 6 shows two equipotential lines – one at 4 V and one at 6 V. **Calculate** the potential difference experienced by a charge as it moves from

a B to A (1 mark)

b A to C (1 mark)

c B to C. (1 mark)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Identify that the scenario refers to electrical potential difference and select the appropriate formula. It is likely that you would only be expected to provide the magnitude of potential difference, but the signed value (+ or –) is calculated here.	$\Delta V = V_{final} - V_{initial}$
Step 3: Substitute the known values and make the calculation. Check that the sign and units are correct. (1 mark for "Provides correct answer")	**a** $\Delta V = V_A - V_B$ $= 6 - 4$ $= +2$V (1 mark)
Step 4: Repeat the calculation with values for C and A. (1 mark for "Provides correct answer")	**b** $\Delta V = V_C - V_A$ $= 4 - 6$ $= -2$V (1 mark)
Step 5: Repeat the calculation with values for C and B. (1 mark for "Provides correct answer")	**c** $\Delta V = V_C - V_B$ $= 4 - 4$ $= 0$V (1 mark)

Your turn

For the diagram in Figure 6, **calculate** the potential difference for a particle moving from

a A to D (1 mark)

b D to A (1 mark)

c C to D. (1 mark)

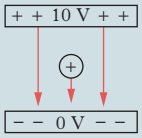
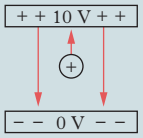
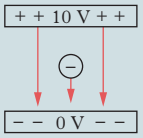
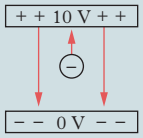
How is energy conserved by particles in fields?

For a charged particle in an electric field, work can be done either

- by the electric field on a charged object, which makes the object speed up
- on the electric field by the charged object, which makes the object slow down.

Table 2 shows the changes in energy and motion when a positive or negative charge moves with the field or against the field. That means there are four possible scenarios, which are given in the table.

TABLE 2 Energy changes in moving charges in an electric field

	CASE A Positive charge moving with the field	CASE B Positive charge moving against the field	CASE C Negative charge moving with the field	CASE D Negative charge moving against the field
				
Sign of charged particle	+	+	−	−
Motion of charge in field	with (down)	against (up)	with (down)	against (up)
Force on charge, F_{net}	down	down	up	up
Acceleration, a	down (speeds up)	down (slows down)	up (slows down)	up (speeds up)
Kinetic energy of object, ΔE_k	increases	decreases	decreases	increases
Potential energy, ΔU	decreases	increases	increases	decreases
Work done, W	by the field on the charge	on the field by the charge	on the field by the charge	by the field on the charge

- **Case A: A positive charge moving with the field**
  A positive charge will experience a net force in the direction of the field, so the direction of the acceleration will be down. It will speed up in that direction and its kinetic energy will increase. The charge's gain in kinetic energy is offset by a loss in potential energy by the field, so work has been done by the field on the charge.
- **Case B: A positive charge moving against the field**
  A positive charge will experience a net force in the direction of the field, so the direction of the acceleration will be down. Because the charge is moving upwards, against the field, it will slow down and its kinetic energy will decrease. The charge's loss in kinetic energy is offset by a gain in potential energy by the field, so work is done on the field by the charge.
- **Case C: A negative charge moving with the field**
  A negative charge experiences a net force in the opposite direction to the field, so the direction of the net force and hence acceleration will be up. Because the charge is moving downwards, against the field, it will slow down and its kinetic energy will decrease. The charge's loss in kinetic energy is offset by a gain in potential energy by the field, so work is done on the field by the charge.
- **Case D: A negative charge moving against the field**
  A negative charge experiences a net force upwards in the opposite direction to the field, so the direction of the acceleration will be up. It will speed up in that direction and its kinetic energy will increase. The charge's gain in kinetic energy is offset by a loss in potential energy by the field, so work is done by the field on the charge.

Skill drill

Relevant data

Science inquiry skills: Planning investigations (Lesson 1.4); Processing and analysing data (Lesson 1.7); Evaluating evidence (Lesson 1.8)

Validity and reliability

In a student experiment, you will be required to evaluate your experimental process. To evaluate your experiment, you need to make justified statements in your discussion of the reliability and validity of the experimental process. To do this effectively, you must first fully understand the different types of error and the concepts of reliability, validity, precision and accuracy.

Practise your skills

1 **Differentiate** between validity, reliability, precision and accuracy by completing the table. (1 mark for each cell)

	Validity	Reliability	Precision	Accuracy
What is it?				
What is it affected by?				
How can it be improved?				
How is it measured or what is it indicated by?				

2 **Differentiate** between random error and systematic error by completing the table. Provide an example of each. (1 mark for each cell)

	Random error	Systematic error
What is it?		
Example		
What is it affected by (validity, reliability, precision and/or accuracy) and how?		
How can it be improved?		

Check your learning 6.4

Check your learning 6.4: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Describe** what it means to say that when work is done on a charged particle, its value of U increases. (1 mark)

2 **Explain** whether an electron that is placed at rest in an electric field experiences a force with the field or against it. (1 mark)

3 An electron is accelerated from rest across a potential difference of +75 V. **Calculate** its velocity at the end of the acceleration (2 s.f.). (3 marks)

4 **Calculate** the work done by an electron ($q_e = -1.60 \times 10^{-19}$ C, $m_e = 9.109 \times 10^{-31}$ kg) in travelling from the +6 V end to the 0 V end of a piece of nichrome wire 20 cm long and connected across the terminals of a 6 V battery. (2 marks)

Analytical processes

5 **a** **Calculate** the kinetic energy of an electron as it enters an electric field at a speed of $0.6c$ (where the speed of light, $c = 3 \times 10^8$ m s^{-1}). (2 marks)

b **Determine** the potential difference the electron will experience when it is brought to rest by transferring all of its kinetic energy to the field. (2 marks)

6 Two points in space A and B are at electrical potentials of +18 V and −6 V respectively.

a **Calculate** the potential difference for a charge that moves from A and B. (1 mark)

b **Determine** the work done in moving a charge of 5.5 μC from A to B. (3 marks)

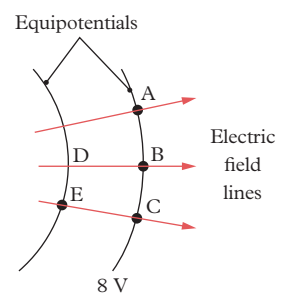
7 **Determine** the energy of a lightning flash when there is a potential difference of 2×10^9 V between the cloud and the ground, and 35 C of charge is transferred during the strike. (3 marks)

8 The diagram shows two equipotential lines about a charged object. It takes 0.2 J of work to move a +0.1 C charge at point C to point E. **Determine** the electrical potential in the electric field at the following points. (2 marks each)

a Point E

b Point D

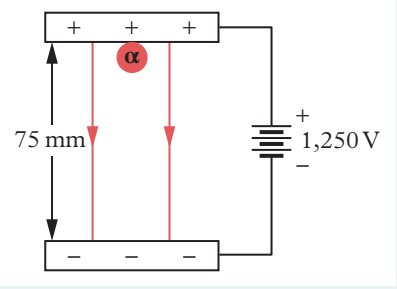
c Point B



Knowledge utilisation

9 An oxygen ion, O^{2-}, has a net charge of -3.2×10^{-19} C. It is accelerated from rest by an electric field of 2,500 N C^{-1} and moves a distance of 35 mm during acceleration. The ion has a mass of 2.60×10^{-26} kg. **Determine** its final velocity (2 s.f.). (4 marks)

10 Alpha particles, α, have a +2 charge. An alpha particle is accelerated from rest by a field between two charged plates 75 mm apart with a 1,250 V source connected across them, as shown in the diagram. **Assess** whether the alpha particle reached 0.2% of the speed of light by the time it reached the negative plate. Express your answer to 2 significant figures using scientific notation. (4 marks)



11 **Evaluate** whether the statement "electric field lines never cross" is true. (2 marks)

MODULE 6

Lesson 6.5
Review: Electrostatics

Summary

6.1
- Coulomb's law states that like electric charges repel and opposite electric charges attract, with a force proportional to the product of the electric charges and inversely proportional to the square of the distance between them.
- Coulomb's law quantifies the force between electrically charged objects and their separation distances as $F = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r^2} = \frac{kQq}{r^2}$.
- Problems involving the force between charged objects in one and two dimensions can be solved using Coulomb's law.

6.2
- Electric fields are regions around a charged particle or object within which a force is exerted on other charged particles or objects.
- Electric field strength is a measure of the intensity of an electric field at a particular location.
- Problems on electric field strength can be solved using $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$.
- Work is done when an electric charge is moved in an electric field.

6.3
- Practical: The effects of electrostatic charge on various materials

6.4
- Electrical potential energy is a measure of the energy of charged particles in a field and their ability to do work because of their position in the field.
- Work is done when an electric charge is moved in an electric field.
- Electrical potential is a measure of the electrical potential energy of charged particles in a field per unit of charge.
- The change in electrical potential energy per unit charge between two defined points in an electric field is called electrical potential difference.
- Equipotential lines join points in space that all have the same electrical potential.
- Problems on electrical potential difference, electrical potential energy and work done on a charge can be solved using $V = \frac{\Delta U}{q}$.

Key formulas

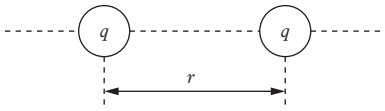
Coulomb's law	$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$
Coulomb's law constant	$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm}^2\,\text{C}^{-2}$
Electric field strength	$E = \frac{F}{Q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$
Electrical potential difference	$V = \frac{\Delta U}{q}$

Review questions 6.5A Multiple choice

Review questions: Complete these questions online or in your workbook.

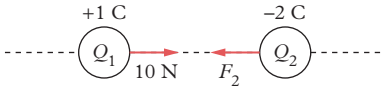
(1 mark each)

1 Two small spheres have equal charges q and are separated by a distance r. The force exerted on each sphere by the other has magnitude F. If the distance between them, r, is doubled, the force on each sphere has a magnitude



A $2F$ **B** $\frac{F}{2}$ **C** $\frac{F}{4}$ **D** $\frac{F}{8}$

2 Two oppositely charged particles experience an attractive force. If the force on Q_1 is 10 N, the force, F_2, acting on Q_2 (and the reason) is



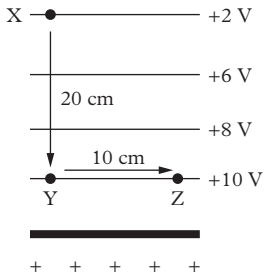
A 5 N, as Q_1 is half the charge and can only pull half as hard as Q_2.

B 10 N, as Newton's third law says action–reaction pairs are equal and opposite.

C 10 N, as they are the same distance apart.

D 20 N, as Q_2 has twice the charge of Q_1.

3 The electrical potentials near a charged plate are shown in the diagram.



A particle with a net charge of +10 μC is moved from point X to point Y and then to point Z. The work done in the process from X to Y and Y to Z respectively is

A 8×10^{-5} J, 0 J

B 8×10^{-5} J, 1×10^{-5} J

C 8 J, 0 J

D 8 J, 10 J

4 Electrical potential energy is best described as

A the capacity of electric charge carriers to do work.

B the energy per unit charge between two defined points.

C the intensity of an electric field at a particular location.

D the force exerted on other electrically charged particles or objects.

5 Two identical charges repel each other with a force of 16 N. When the distance between the charges is doubled, the force will be

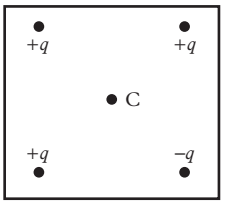
A 8 N

B 4 N

C 32 N

D 64 N

6 The diagram shows four charges placed at the corners of a square. Which arrow most accurately shows the direction of the force on a negatively charged particle at the centre point C?



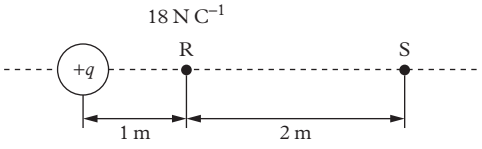
A ↑

B ↓

C ↖

D ↘

Use the following diagram to answer questions 7 and 8.



7 If the electric field strength is 18 N C^{-1} at point R in the diagram, what is it at point S?

A 9 N C^{-1}

B 4.5 N C^{-1}

C 3 N C^{-1}

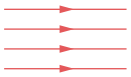
D 2 N C^{-1}

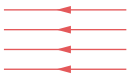
8 The direction of the electric field at point S in the diagram is

A up the page.
B to the right.
C down the page.
D to the left.

9 A negatively charged particle is fired into an electric field from the right and undergoes motion as shown in the diagram.

Which of the diagrams A–D best represents the electric field?

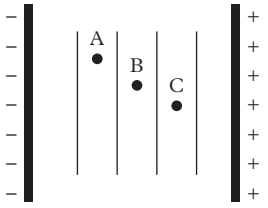
A


B


C


D


10 Three positions between a pair of positive and negative plates are shown in the diagram.



Which position has the highest electrical potential?

A A
B B
C C
D They are all at the same electrical potential.

11 Determine how much energy is required to move a +1.5 C charge in a uniform electric field generated by a 6 V battery.

A 2.25 J
B 4 J
C 6 J
D 9 J

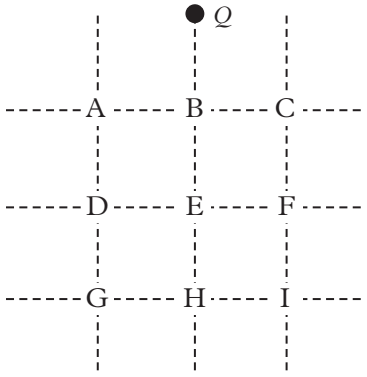
12 Two suspended spheres are 2.0 cm apart. Sphere A has a charge of $+2.0 \times 10^{-4}$ C and sphere B has a charge of -4.0×10^{-5} C. Determine which statement best describes the force between the spheres.

A Magnitude of 180 N and is repulsive
B Magnitude of 180 N and is attractive
C Magnitude of 1,800 N and is repulsive
D Magnitude of 1,800 N and is attractive

13 To move a +2.0 C charge from its initial position to a point X requires 4 J of work. Determine the electrical potential difference at point X.

A 2 V
B 4 V
C 8 V
D 16 V

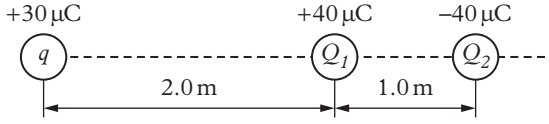
14 A point source, Q, is placed on a square grid.



Determine which points represent equal electric field strength.

A A and C
B B, E and H
C A, B and C
D A, E and C

15 The diagram shows three charges in a line and the distances between them.



Determine the force on q due to Q_1 and Q_2.

A 1.5 N
B 1.8 N
C 3.9 N
D 9.0 N

Review questions 6.5B Short answer

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 **Explain** the difference between U and ΔU. (2 marks)

17 **Identify** the difference between electric fields and gravitational fields. (2 marks)

18 The field strength near a +10 μC point charge is being measured.

a **Calculate** the field strength at a distance of 25 cm (2 s.f.). (2 marks)

b **Identify** the order of magnitude of this value. (1 mark)

19 Two point charges, A (+10 μC) and B (–5 μC), are 10 cm apart. **Calculate** the electric field strength at the mid-point between them (2 s.f.). (3 marks)

20 **Calculate** the magnitude of the charge on an oil droplet if the droplet experiences an electrostatic force of 5.6×10^{-14} N when placed into a uniform electric field of 4,000 N C^{-1}. (2 marks)

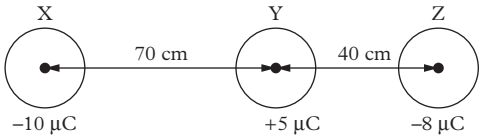
21 A pair of charged parallel plates are separated by a distance of 35 mm. The electric field strength between them is 600 V m^{-1}.

a **Identify** the field strength in units of N C^{-1}. (1 mark)

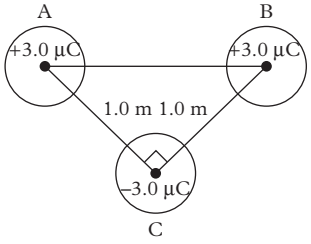
b **Calculate** the potential difference across the plates (2 s.f.). (2 marks)

Analytical processes

22 Three charged particles, X, Y and Z, are in a line, as shown in the diagram. **Calculate** the net force on particle Z (2 s.f.). (4 marks)



23 Three charged objects, A, B and C, are arranged in a triangle, as shown in the diagram. **Calculate** the net force on charged object B. (5 marks)



24 The diagram shows a particle of charge $+8.0 \times 10^{-19}$ C and mass 9.60×10^{-26} kg entering a uniform electric field of strength 20 N C^{-1} at a right angle to the field. The initial speed of the particle is 1,200 m s^{-1}.

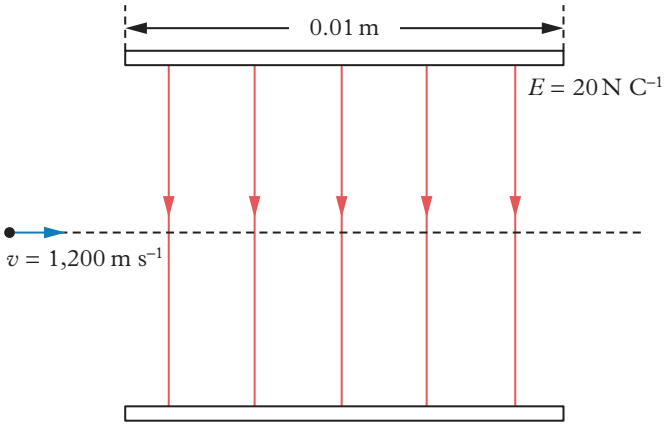
a **Determine** the magnitude and direction of the force on the charged particle. (2 marks)

b **Determine** the acceleration (magnitude and direction) of the particle in the field. (2 marks)

c **Determine** the time of travel of the particle in the field. (2 marks)

d **Determine** the final displacement and velocity in the direction of the field. (2 marks)

e **Describe** the probable path of the particle in the field. (1 mark)



25 It takes 2.30 J of work done on +0.5 C of charge to move it in an electric field.

a **Determine** the change in energy of the charged object. (1 mark)

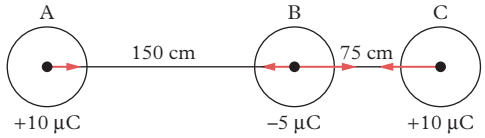
b **Calculate** the potential difference experience by the object. (1 mark)

c **Determine** the final electrical potential if the starting electrical potential was +6.6 V. (1 mark)

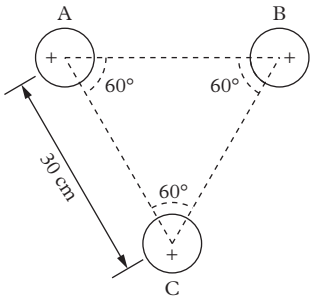
26 A system of three charges in a line, $Q_A = +10\,\mu C$, $Q_B = -5\,\mu C$ and $Q_C = +10\,\mu C$, are separated by distances to their centres as shown in the diagram. **Determine** the net force

a on B due to A and C (3 marks)

b on C due to A and B. (3 marks)

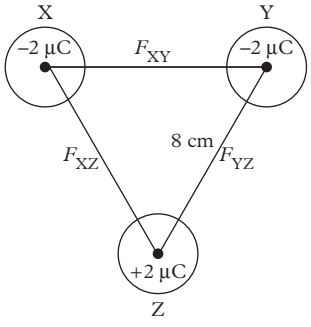


27 A system of charges A, B and C is shown in the diagram. Each charge is $+2.0 \times 10^{-6}$ C and they are arranged in an equilateral triangle of side 30 cm. **Deduce** the magnitude and direction of the force on charge C. (5 marks)



28 A lithium ion Li^{3+} has a net charge of 4.8×10^{-19} C and a mass of 4.5×10^{-27} kg. It is accelerated from rest across a potential of 125 V in a distance of 55 mm. **Determine** the velocity of the ion at the end of the acceleration. Express your answer to 2 significant figures in scientific notation. (4 marks)

29 A system of three charges X, Y and Z is shown in the following diagram. They are arranged in an equilateral triangle, so the inner angle at each corner is 60°. **Determine** the magnitude and direction of the force on charge Z (2 s.f.). (4 marks)



Knowledge utilisation

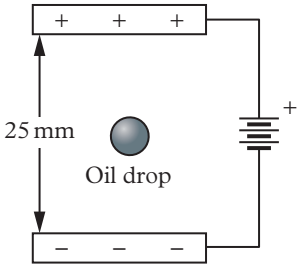
30 A proton ($q_p = +1.60 \times 10^{-19}$ C and $m = 1.67 \times 10^{-27}$ kg) experiences a force of 6.0×10^{-15} N due to an electric field.

a **Calculate** the electric field strength (2 s.f.). (2 marks)

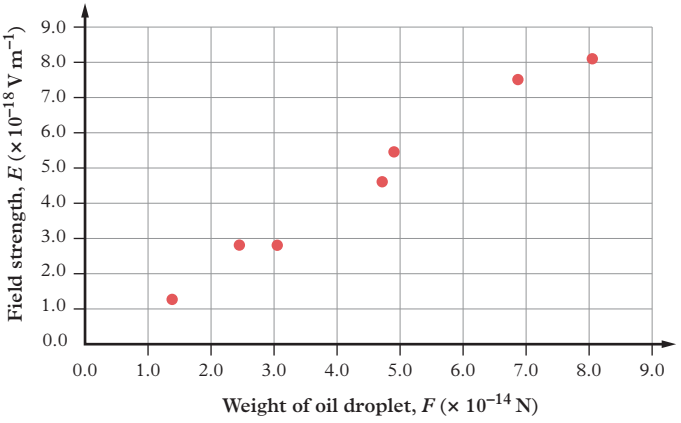
b **Determine** the acceleration of the proton (2 s.f.). (2 marks)

c **Propose** how doubling the force the proton experiences would affect its acceleration in the field. **Justify** your response. (2 marks)

31 Oil drops with the same net negative charge were placed in a region of uniform electric field between two parallel plates 25 mm apart. By varying the potential difference, V, across the plates the oil drop could be suspended in the air between the plates. Heavier drops required a stronger field strength, and hence voltage, to keep them suspended.



The graph shows the electric field strength required to suspend the oil drops of varying weight.

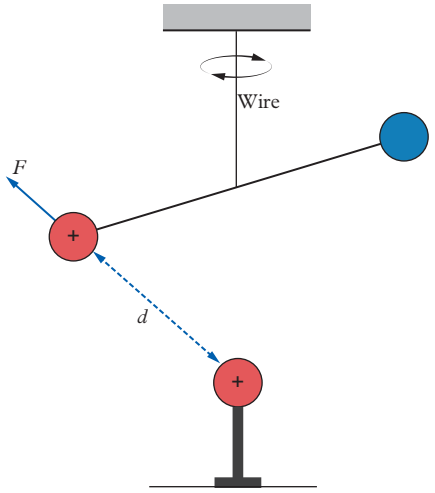


a **Calculate** the net charge on each drop (2 s.f.). (4 marks)

b **Calculate** how many elementary charges were on each drop. Express your answer to the nearest whole number. (4 marks)

c The oil drop suspended between the plates at a field strength of 7.5×10^{-18} V m^{-1} was moved from the top plate to the bottom plate, a distance of 25 mm. **Determine** the work done to do this (2 s.f.). Show your working. (4 marks)

32 Students conducted an experiment to replicate Coulomb's original experiment with electrostatic charges to derive a value for the electrostatic force of repulsion between the two spheres. They set up a torsional balance as shown in the diagram. The apparatus enabled the students to measure the twist in the suspension wire as the "free" charge tried to rotate away from the fixed charge. From the value of the twist, the students could determine a relationship between force and distance.



Part of the experimental data is shown in the table. Unfortunately, two of the data points were not recorded.

a **Construct** a graph to show the relationship between the electrostatic force (vertical axis) and the separation distance (horizontal axis). (3 marks)

b **Predict** the two missing readings. (2 marks)

c **Comment** on the relationship between the force and the distance as suggested by your graph. Explain your answer. (2 marks)

d **Construct** a linearised graph to confirm your answer. (3 marks)

Average force (units)	Separation (cm)
28.4	0.75
10.2	1.25
not recorded	1.50
not recorded	1.75
4.00	2.00
1.80	3.00
1.00	4.00

33 An oil drop with a mass of 2.48×10^{-15} kg is held suspended between two parallel plates 17 mm apart, maintained at a potential difference of 260 V. The lower plate is negative.

a **Calculate** the charge on the drop (2 s.f.). (4 marks)

b **Determine** how many elementary charges this is equivalent to. Give your answer to the nearest whole number. (2 marks)

Data drill

Effect of separation distance on force

A pair of lightweight conductive ping pong balls were suspended from a common point by cotton thread (Figure 1). The balls were given an equal negative charge and allowed to come to rest. The distance apart was varied and the force between the balls was measured.

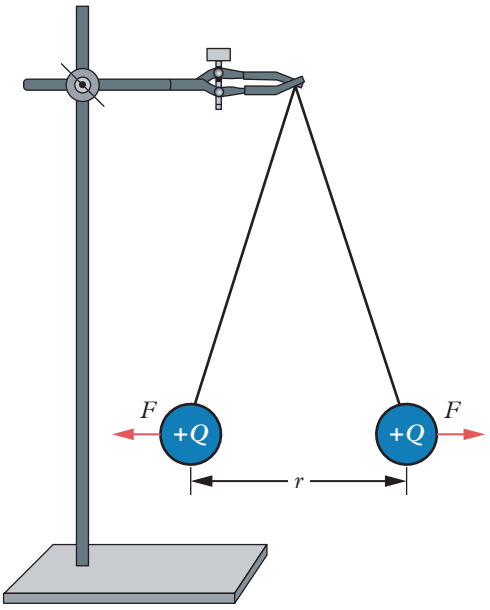


FIGURE 1 Experimental setup of two charged ping pong balls suspended from cotton thread

A graph of the results is shown in Figure 2:

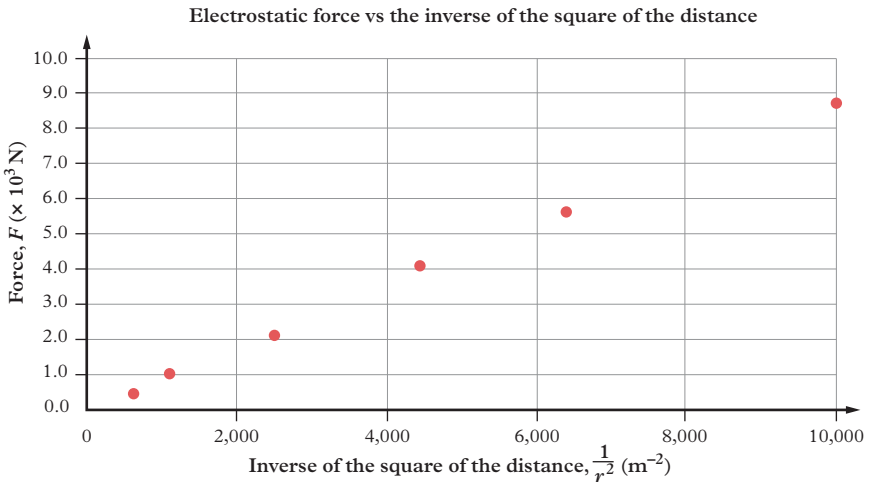


FIGURE 2 Graph of force vs the inverse of the square of the distance

Apply understanding

1 **Calculate** the separation distance in metres when the force is 8.8×10^{-3} N. (1 mark)

2 Draw a line of best fit and **calculate** the gradient for the line. (2 marks)

Analyse data

3 Use the graph to **identify** a mathematical relationship between the force, F, acting on the balls and the distance, r. (3 marks)

Interpret evidence

4 **Infer** what trend a graph of force, F (y-axis), versus distance, d (x-axis), would show. (1 mark)

5 In terms of Coulomb's law, **infer** what the gradient represents. (1 mark)

6 **Draw a conclusion** about the experimental value of the charge on each ping pong ball. (2 marks)

Module 6 checklist: Electrostatics

MODULE 7

Magnetic fields

Introduction

Magnetic rocks formed as the Earth cooled millions of years ago, and for a long time these magnets were just a natural curiosity whose properties could be put to good use. Thousands of years ago, Chinese, Arabic and Indian sailors are thought to have used chips of magnetic rocks as compasses for finding direction. Early Greeks knew that certain stones attracted pieces of iron, but the systematic study of magnets and their properties didn't begin until much later. From about 1300, various experiments were done by people trying to find out what this mysterious "magnetic" force is. In this module we will look at a modern understanding of magnets and answer questions like:

→ Why do compass needles always point north? Have they always done this?
→ How do long-distance migrating birds always find their way home?
→ Are all metals attracted to magnets or just steel?
→ Do magnetic fields from overhead wires cause medical problems?

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to magnetic fields.

Subject matter

Science understanding

→ Describe the concept of a magnetic field.
→ Sketch magnetic field lines due to a moving electric charge, electric currents and magnets.
→ Describe the generation of a magnetic field from a moving electric charge.
→ Solve problems involving the magnitude and direction of magnetic fields around a straight electric current-carrying wire and inside a solenoid using $B = \frac{\mu_0 I}{2\pi r}$ and $B = \mu_0 nI$.
→ Describe the force experienced by electric current-carrying conductors and moving electric charges when placed in a magnetic field.

→ Solve problems involving the magnetic force on an electric current-carrying wire and moving charge in a magnetic field using $F = BIL\sin\theta$ and $F = qvB\sin\theta$.
→ Interpret data relating to the force acting on a conductor in a magnetic field.
→ Interpret data relating to the strength of a magnet at various distances.

Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Charles-Augustin de Coulomb, Michael Faraday, Emil Lenz, Mary Somerville and James Clerk Maxwell who furthered our understanding of electromagnetism.

Science inquiry skills

→ Investigate the force acting on a conductor in a magnetic field.
→ Investigate the strength of a magnet at various distances.

Practicals

oxforddigital **These lessons are available on Oxford Digital.**

Lesson 7.3 Strength of a magnet at various distances
Lesson 7.4 Strength of a magnetic field at various distances from a current-carrying wire
Lesson 7.8 Force acting on a conductor in a magnetic field

Lesson 7.1
Magnetic fields

Key ideas

- → A magnetic field is a region of space in which magnetic effects are experienced.
- → Magnetic fields are produced by moving electric charges.
- → Magnetic fields around moving electric charges, electric currents and magnets can be depicted with directed line segments (arrows).

Learning intentions and success criteria

What are magnets?

Magnets are something you come across every day, in all shapes and sizes: the magnets that keep your refrigerator closed or that you stick on the door, or the magnets in your headphones, electric toothbrushes or motors of any sort. These magnets differ in the way they work and the formulas that represent their characteristics. There are three different types of magnets that we are concerned with.

- **Permanent magnets** – these hold their magnetic properties for a long time and don't need any external source of magnetism or electricity. These are usually made from rare-earth metals or alnico, an alloy of aluminium, nickel and cobalt.
- **Temporary magnets** – these behave as magnets when they are attached to or close to another magnet but lose this ability when the source is removed; examples are iron and iron alloys such as in paperclips and nails.
- **Electromagnets** – these are created by passing an electric current through a coil of wire that usually has a metal core.

Each end of a magnet behaves differently. If you bring one magnet close to another, you will find it will attract one end of your magnet and repel the other end. Each end is called a magnetic pole, and scientists have named them north pole (N) and south pole (S) based on how they align with the Earth when allowed to move freely. Their main feature is that like poles (N + N, or S + S) repel each other, whereas unlike poles (N + S) attract each other.

What is magnetism?

Magnetism describes the properties of magnets – for example, the property that like poles repel each other. It comes from the motion of electric charges. There are various motions of charge that give rise to magnetic fields. It can be electron flow in a wire, as in the case of an electromagnet, or charged particles moving through space or the motion of an electron in an atom. In the case of permanent magnets, picture the atom being made up of rotating electrons either spinning on their own axes (like a spinning top) or orbiting the nucleus. In classical physics, the motions spinning and orbiting turn the electrons and atoms into tiny bar magnets. In certain materials, such as iron, these tiny bar magnets all line up, and the result is a magnet. If certain alloys of iron are used, they will retain this magnetism and are said to be permanent magnets.

This model of spinning electrons isn't quite right – quantum physics makes it a bit more complicated, as you will see in Unit 4.

What makes a magnetic field?

Michael Faraday's ideas about magnetic fields and magnetic lines of force provided the world with a model for magnetism that enabled dramatic progress in science from 1830 onwards. The concept is that magnets transmit their effects through magnetic fields. We say that a **magnetic field** is the space in which magnetism is experienced or "mediated". You will see this word crop up a lot in the Unit 4 topic on particle physics. "Mediate" means to be in the middle – like a "median" strip on the road. A magnetic pole creates a magnetic field, and this field in turn exerts a **magnetic force** on other particles in the field. So, the field is the middle of this process – it mediates the magnetic effect from one object to another.

magnetic field
a region of space where a magnetic force is experienced by a moving charge, i.e. where magnetism is mediated

magnetic force
the force exerted by a magnetic field on other magnetic materials

In the previous modules, gravitational fields and electric fields were introduced. These have much in common with magnetic fields. They are all defined in terms of a force acting on an object in the field. For the gravitational field the object is a 1 kg mass, and for the electric field the object is a +1 C charge. Magnetic fields are a little more complicated, but the quantity is a combination of charge, Q, and velocity, v.

Electric and magnetic fields are both due to electric charges. Electric fields act between stationary, "static" charges, and magnetic fields act between moving charges.

What are the rules for sketching a magnetic field?

FIGURE 1 A simplified model of an atom is a spinning top with electrons rotating on their own axes.

You have probably seen a demonstration in which iron filings are sprinkled over a piece of paper that has a magnet underneath. The filings line up in patterns that represent the magnetic field of the magnet. Like the gravitational field and electric field of earlier modules, this magnetic field is a vector field that has both magnitude and direction. The magnitude is called the magnetic field strength, and arrows are used to indicate the direction. And like gravitational and electric fields, this field is three dimensional – the iron filings represent a cross-section through the full three-dimensional field.

Magnetic field diagrams are drawn with **magnetic field lines** and directional arrows indicating the direction of the force that a small test magnetic north pole would experience if placed into the field. However, magnetic poles come in pairs – for every north (N) pole there is a south (S) pole – and so the idea of an isolated test north pole is just an idea used to determine the direction of a magnetic field.

magnetic field line
the direction an isolated north pole would move in the field, or the direction a compass needle would point

Magnetic fields around a bar magnet

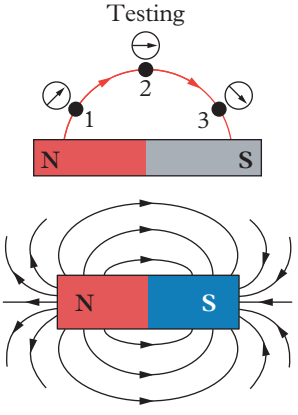


FIGURE 2 The field direction around a bar magnet is from the north to the south pole. This can be mapped with a small compass.

The field direction is determined using a north pole, and you know that like poles of magnets repel each other and unlike poles attract. Therefore, the magnetic field lines around a typical bar magnet will always be oriented away from the north pole and towards the south pole (Figure 2). The direction of the force at any point in a magnetic field diagram is given by the tangent to a field line at that point.

Magnetic fields around a pair of magnets

The field between two magnets will depend on whether they have like or unlike poles facing each other (Figure 3). Field lines around each magnet are from north to south. Note also that they don't cross paths and the closer they are to the magnet surface, the more closely they are bunched together.

Magnetic fields around a horseshoe magnet

A horseshoe magnet is like an ordinary bar magnet bent into a U-shape (Figure 4). It has north and south poles facing each other. As with a bar magnet, the magnetism cancels out in the middle (in this case at the bottom of the U).

Magnetic field around Earth

Earth has a magnetic field that comes from electromagnetic induction caused by swirling currents inside the Earth (you will learn more about this in Module 8).

A confusing situation arises when you consider the Earth's magnetic field lines: why do they point towards the geographic North Pole? If it is a north pole, shouldn't the field lines point away from it?

A small compass needle can be used to test the direction of Earth's magnetic field lines. The north pole of the compass used to be called "the north-seeking pole" but was shortened, hence the confusion. The north (seeking) end of the compass needle (the red end) points towards the geographic North Pole of the Earth – which is magnetically a south pole (Figure 5).

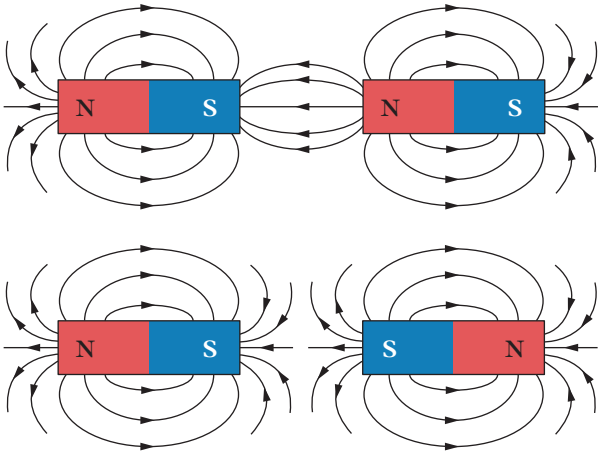


FIGURE 3 Field lines around two magnets with like poles facing, and with unlike poles facing

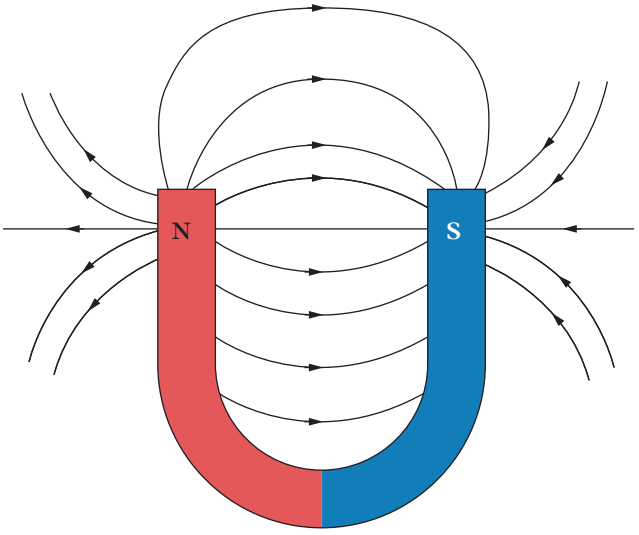


FIGURE 4 A horseshoe magnet has a very strong field in the gap between two unlike poles

In summary, field lines:

- indicate the direction a compass needle would point
- indicate the direction of the force on a theoretical isolated test north pole
- go from north to south, but don't cross
- are closer together for a stronger magnet
- represent a 3D field in 2D.

How else is field direction represented?

The other word for field lines is "flux" lines. The word flux comes from the Latin "fluere", meaning "to flow" (from an old idea about the flow of magnetism). Flux lines are just the same thing as field lines, so flux lines are shown as pointing away from the north pole or towards the south pole.

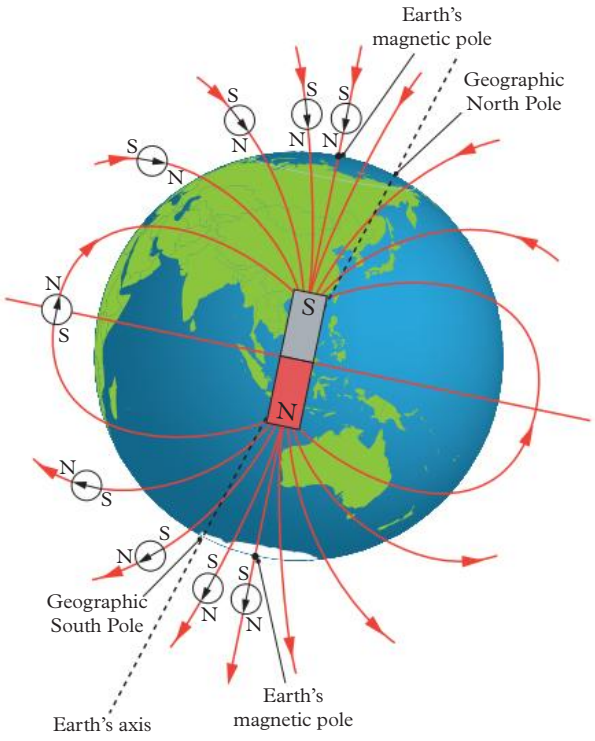


FIGURE 5 Earth's magnetic field

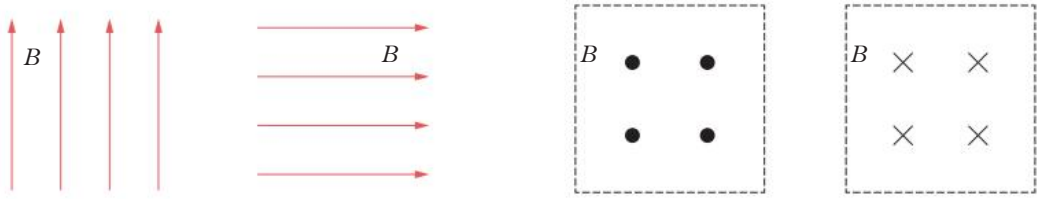


FIGURE 6 Representing field lines in two ways

The direction of a magnetic field can be shown in two ways (Figure 6):

- arrows along the page, where the arrows point from north to south.
- dots and crosses showing the lines perpendicular to the page. These can show flux lines coming towards you (drawn as a dot, like the tip of an arrow), or away from you (drawn as a cross representing the tail feathers of the arrow). Sometimes, for clarity, each dot or cross is drawn with a circle around it. It makes no difference.

How is field strength represented?

Magnets come in different strengths. Some are strong, some are weak. We need a way of specifying magnetic field strength, just as we did with electric field strength.

Magnetic field strength is a vector quantity and is represented by the vector symbol $\vec{B}$. The magnitude of the magnetic field strength can be represented by how close together the field lines are drawn, where closer together lines mean a stronger field (Figure 7).

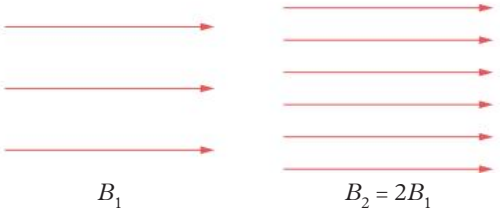


FIGURE 7 The strength of a magnetic field is shown by how close the field lines are together. The field on the right, B_2, is twice as strong as the field on the left, B_1.

Study tip

When you see dots, or circles with dots, you can imagine there is a north pole behind the page with flux lines coming towards you. When you see crosses, or circles with crosses, you can imagine you are the north pole, with the south pole behind the page and the flux lines heading away from you.

Study tip

Note that the symbol for magnetic field strength, B, is shown with an arrow over it, $\vec{B}$, when field strength as a vector (magnitude and direction) is considered. If you are just talking separately about the magnitude of the field strength, or the direction of the field, then the symbol is B.

Challenge

Breaking a magnet in half

You can lift a total of 50 paper clips with both ends of a permanent magnet. When it's broken in half you can lift a greater total number of paperclips. **Explain** why this could be. (1 mark)

Challenge

Strange properties of a magnet

Magnets are used for the strangest of things. Here are three instances that will make you think of their unusual properties.

1 Magnets are fed to cows to attract the bits of wire and nails that they eat with the grass. This seems a bit farfetched when you consider how long wire would last in a cow's acidic stomach. **Design** a test to see if this is possible. The acid in a cow's stomach is 0.17 M HCl (pH = 0.8). (2 marks)

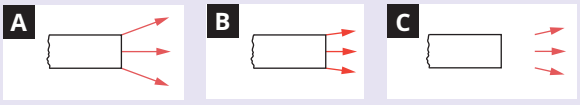


FIGURE 8 Possible changes to a magnetic field

2 Figure 8A shows a part of the field around an electromagnet. The current is now turned off. **Propose**, giving a reason, whether the field lines would change as in Figure 8B or 8C. (2 marks)

3 Several species of aquatic bacteria swim along magnetic field lines. They have tiny chains of magnetite crystals in their bodies. When stirred, the bacteria swim north, which is towards the bottom (in the Northern Hemisphere, where they live). **Propose**, giving a reason, what would happen if they were brought to the Southern Hemisphere. (2 marks)

Real-world physics

Can steel be magnetised with violet light?

Mary Somerville (1780–1872) was a Scottish mathematician and writer who became famous for experiments she did in 1825 looking at how a steel sewing needle could be magnetised in just two hours by the violet rays from sunlight. It wasn't so much that this experiment added to the understanding of magnetism but how it shaped the way we think about science.

Like most experiments in chemistry and physics at the time, these relied on home-made apparatus or equipment borrowed from friends, rather than on an expensive laboratory. Somerville's experiments were published as "On the magnetizing power of the more refrangible solar rays" by the Royal Society of London in 1826, to much acclaim. To appear under her own name as an author in their prestigious journal was a big step; this was the first time a woman had ever published an experimental paper there. In the wake of Hans Christian Ørsted's discovery of a connection between

magnetism and electricity, the results announced in the paper were exciting. The famous physicist James Clerk Maxwell, who is usually credited with the discovery of the link between electricity, magnetism and light, praised her experiment in his 1864 treatise (which you will meet in the next module).

Today we are well aware of light-sensitive materials, such as the metals you will meet in Unit 4 that exhibit the photoelectric effect. But there is nothing in physics textbooks today about a material as mundane as iron possessing properties of photoinduced magnetisation. Somerville's results seem to have been dismissed. So, what happened?

It turned out the relationship between light and magnetisation was not a direct one. The heating of the metal allowed the Earth's magnetic field to reorient the magnetic domains in the steel all in the one direction, so that when it cooled the steel retained its magnetism.

Her 1834 book *On the Connexion of the Physical Sciences* is now regarded as being instrumental in the making of modern physics as a discipline, and it is for that she is most remembered.

When she died in 1872, a newspaper obituary declared, "Whatever difficulty we might experience in the middle of the nineteenth century in choosing a king of science, there could be no question whatever as to the queen of science."

You may recognise her name as that of the Brisbane High School for Girls, renamed Somerville House in her honour in 1920.

FIGURE 9 Mary Somerville, Scottish writer and mathematician

Apply your understanding

1 Somerville found that violet light magnetised the sewing needle faster than blue light. **Identify** whether violet light has a greater frequency than blue light. (1 mark)

2 It is now believed that it was heat rather than the colour of light that resulted in her needles becoming magnetised. **Propose** a modification to Somerville's experiment that would have allowed her to investigate this. **Explain** your reasoning. (2 marks)

3 Somerville used a flint prism to disperse sunlight into its different colours on the principle that violet light was more "refrangible" than blue light. **Identify** the word we now use for "refrangible". (1 mark)

Check your learning 7.1

Check your learning 7.1: Complete these questions online or in your workbook.

Retrieval and comprehension

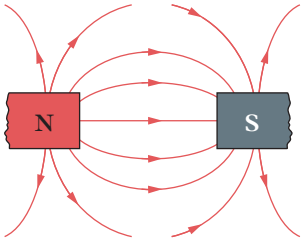
1 **Recall** whether this statement is true: magnetism is the result of a moving electric charge. (1 mark)

2 **Describe** how the direction of a magnetic field is determined. (1 mark)

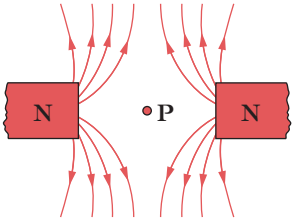
3 **Clarify** whether the direction of a field line is from N to S or from S to N. (1 mark)

4 **Describe** the concept of a magnetic field. (2 marks)

A

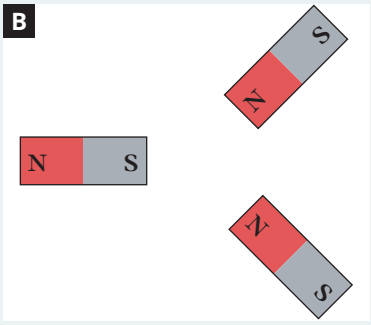


Unlike pole attraction



Like pole repulsion

B

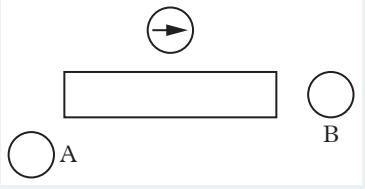


5 The magnetic field lines around two magnets with unlike poles together and with like poles together are shown in diagram A. **Sketch** the magnetic field lines around the arrangement of permanent bar magnets in diagram B. (1 mark)

6 **Sketch** a diagram showing four flux lines going "into the page" and another diagram showing a field twice as strong. (2 marks)

Analytical processes

7 A student is mapping the field around a bar magnet. The diagram shows the positions of a compass used in the procedure.



a **Determine** which end of the magnet (right or left) is the north pole. Explain your answer. (2 marks)

b **Determine** the positions of the compass needle in positions A and B. (2 marks)

Lesson 7.2
Magnetic fields around current-carrying wires

Key ideas

- → Moving electric charges, such as the current flowing in a wire, have an associated magnetic field.
- → The direction of the magnetic field around a current-carrying wire can be determined using Ampere's right-hand rule.
- → Problems involving the strength of the magnetic field around a straight current-carrying wire can be solved using $B = \frac{\mu_0 I}{2\pi r}$.

How can we model the magnetic field around moving charges?

Learning intentions and success criteria

The strength (or intensity) of a magnetic field can be defined by the force acting on charged particles moving in the field. The simplest way to do this is by considering charged particles moving in a length of wire – that is, an electric current. This is called a "current-carrying wire".

Module 6 was about electrostatics. The "statics" means stationary, so electric fields around stationary charges were discussed. You saw that a stationary electric charge produced an electric field around itself, and the strength of this field decreased with distance. So how does the strength of the magnetic field produced by moving electric charges (electric current) depend on distance from the current and the strength of the current?

How is the field around a current-carrying wire determined?

When you pass a current through a wire, a magnetic field develops perpendicular to the wire. This field can easily be shown by sprinkling iron filings on paper around a current-carrying wire (Figure 1).

As with any physical phenomenon, scientists want to know more about its properties. For this magnetic field, physicists experimented with and theorised about the factors affecting the magnitude and direction of this field.

Direction of the magnetic field surrounding a wire

The earliest recorded experience of this phenomenon was by the Danish physicist Hans Christian Ørsted (1777–1851). He was giving a lecture on electricity and noticed, quite by chance, that when a current flowed in a wire, a nearby compass needle would move.

In 1820 he published "Experiments on the effect of electricity on the magnetic needle". In this work he described the way the compass needle follows the almost circular pattern of magnetic field lines around the current-carrying wire (Figure 2).

FIGURE 1 Circular magnetic field lines make a pattern in iron filings around a current-carrying wire.

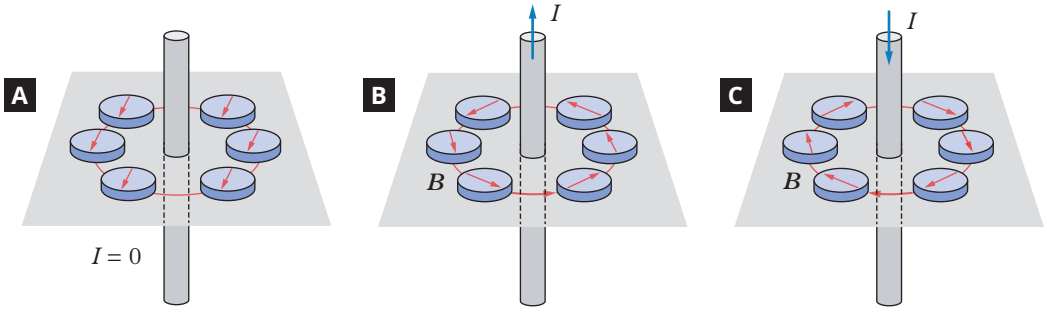


FIGURE 2 Compass directions around a wire. (A) There is no current, so the compasses point N–S. (B) Current up the page gives an anticlockwise circular field. (C) Current down the page gives a clockwise field.

Ampere's right-hand rule

The direction of a magnetic field can be easily found using Ampere's right-hand rule. Point the thumb of your right hand along the direction of the current in a wire and then curl your fingers around the wire. The direction in which your fingers are pointing represents the direction of the magnetic field lines (Figure 3).

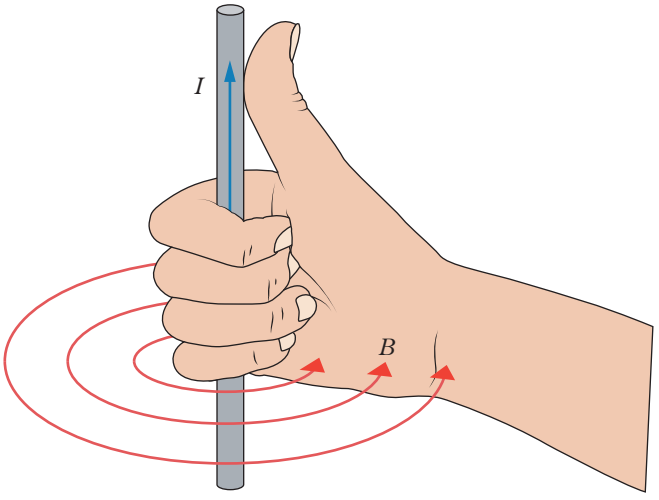


FIGURE 3 Applying Ampere's right-hand rule to find the direction of the magnetic field. The current is up the page (represented by your thumb), and the field is around the wire in an anticlockwise direction (your fingers).

Representing fields around a wire

The simplest way of representing the direction of a field around a wire is to imagine you are looking at the wire end on (Figure 4).

From this perspective, if the current is moving away from you, it is shown as a cross (or a circle with a cross) to represent the tail feathers of the arrow (Figure 5). This is how you would represent the current in the wire in Figure 4. However, if the current is coming towards you (out of the page) it is represented by a dot (or a circle with a dot) that resembles the pointy tip of the arrow.

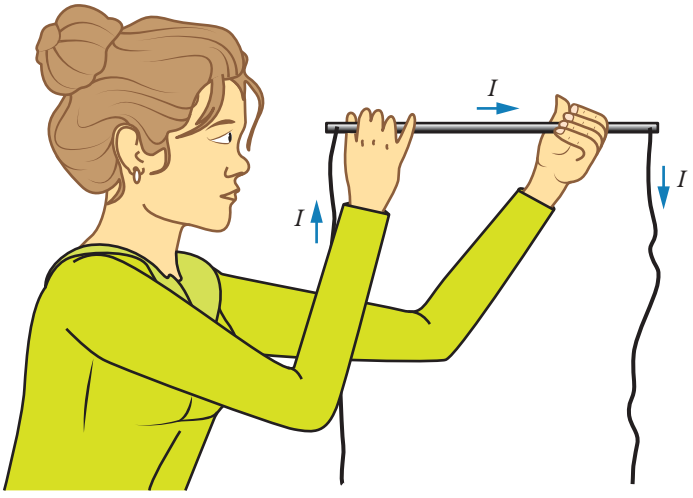


FIGURE 4 Looking at the end of a wire with the current moving away from you

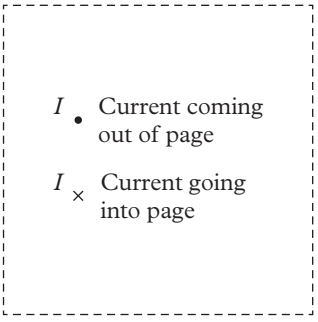


FIGURE 5 Convention for indicating the direction of the current when it is perpendicular to the page

The circular magnetic field lines around the wire are shown in Figure 6. The main characteristics of the magnetic field are:

- the field lines are circular and concentric around the wire
- the strength of the field decreases away from the wire, so the lines get further apart
- when the direction of the current is reversed, the direction of the field is also reversed.

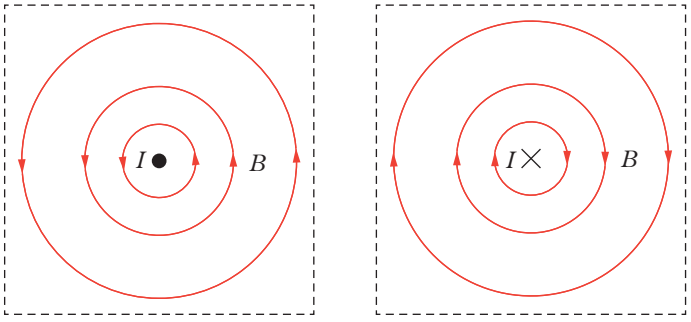


FIGURE 6 The circular field around a current-carrying wire. The symbol *B* shows the circular magnetic field lines. The symbol *I* shows the direction of the current in the wire.

Magnetic field strength

The unit of magnetic field strength is **tesla** (symbol T), named after the Serbian–American engineer and physicist Nikola Tesla (1856–1943). It is defined as the force acting on a moving charge, either as a particle or in a current-carrying wire, in a magnetic field. A tesla is a relatively large quantity.

tesla
the SI unit of magnetic field strength;
$1\,T = 1\,N\,C^{-1}\,m^{-1}\,s$

Some examples of magnetic field strengths are:

- 5×10^{-5} T – the Earth's magnetic field
- 5×10^{-3} T – fridge magnet
- 0.3 T – sunspot
- 3.25 T – surface of a neodymium magnet
- 3 T – magnetic resonance imaging (MRI) scanner
- 10^{10} T – magnetar (a special type of neutron star).

The formula relating the magnetic force to field strength will be explored in Lesson 7.5. For the time being just accept it and use a formula derived from these ideas.

TABLE 1 Electric and magnetic field constants

Electric fields	Magnetic fields
$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2} = \frac{kq}{r^2}$	$B = \frac{\mu_0 I}{2\pi r}$
Field strength is directly proportional to the charge.	Field strength is directly proportional to the charge.
Field strength is inversely proportional to the square of the distance.	Field strength is inversely proportional to the distance.
Uses the constants ε_0, the electric permittivity constant, and k, Coulomb's constant. $\varepsilon_0 = 8.85 \times 10^{-12}\,\text{C V}^{-1}\,\text{m}^{-1}$	Uses the constant μ_0, the permeability of free space. $\mu_0 = 4\pi \times 10^{-7}\,\text{T mA}^{-1}$ $= 1.26 \times 10^{-6}\,\text{T mA}^{-1}$
$k = \frac{1}{4\pi\varepsilon_0}$ $= 9 \times 10^9\ \text{N m}^2\text{C}^{-2}$	$k = \frac{\mu_0}{2\pi}$ $= \frac{4\pi \times 10^{-7}\,\text{T mA}^{-1}}{2\pi}$ $= 2 \times 10^{-7}\,\text{T mA}^{-1}$

Study tip

If you're worried about getting the k constants mixed up, you could call the electric one k_E and the magnetic one k_B.

You can use the formula to show, for instance, that the magnetic field strength at a distance of 5 cm from a wire carrying a current of 20 A is 8×10^{-5} T.

This is about twice Earth's magnetic field strength.

Worked example 7.2A

Magnetic field around a wire

Calculate the magnetic field (strength and direction) at point P, which is 5 cm directly above a wire carrying a current of 20 A to the left. Give your answer to an appropriate number of significant figures (Figure 7). (3 marks)

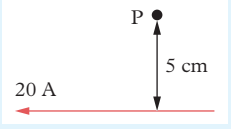
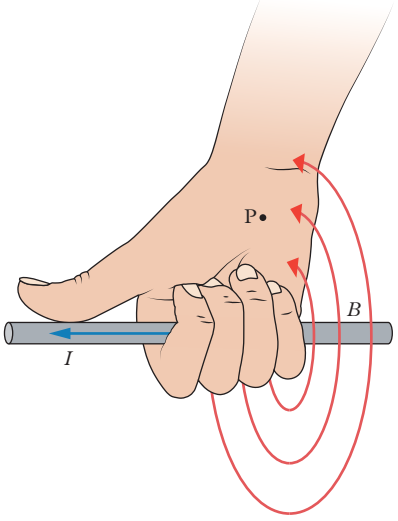


FIGURE 7 Field around a wire

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$r = 5$ cm, $I = 20$ A to the left $B = \frac{\mu_0 I}{2\pi r}$, where $\mu_0 = 4\pi \times 10^{-7}$
Step 3: Substitute the known values and solve for B (1 mark for "Substitutes correctly"; 1 mark for "Calculates field strength correctly"). You could also substitute 2×10^{-7} for $\frac{\mu_0}{2\pi}$ as mentioned earlier. You may find this more convenient and it has fewer chances of error. Give your answer to an appropriate number of significant figures, in this case 1 s.f.	$B_P = \frac{\mu_0 I}{2\pi r}$ (substitute μ_0) $= \frac{(4\pi \times 10^{-7}) \times I}{2\pi r}$ (cancel the 2π) (1 mark) $= \frac{2 \times 10^{-7} \times 20}{0.05}$ $= 8 \times 10^{-5}$ T (1 s.f.) (1 mark)

Think	Do
Step 4: Determine the direction of the field at P using the appropriate hand rule. (1 mark for "Identifies the relationship between current direction and field direction")	Using Ampere's right-hand rule, as shown in the diagram, the magnetic field is into the page.  (1 mark)

Your turn

Calculate the magnetic field (strength and direction) at point P, which is 35 mm directly to the right of a wire carrying a current of 15 A down the page. Give your answer to an appropriate number of significant figures (Figure 8). (3 marks)

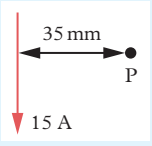


FIGURE 8 Field around a wire

How is the field around two wires determined?

Magnetic fields are vector fields, so they have magnitude and direction. To determine the field around more than one wire, the principle is the same as we used for more than one charged object in the previous module on electrostatics. You just determine the strength and direction of the field around each object separately and then add them together vectorially, as shown in Worked example 7.2B.

Worked example 7.2B

Magnetic field around two wires

In Figure 9, two separate parallel wires are in close proximity. **Calculate** the value of the magnetic field strength (magnitude and direction) at a point X between the two wires, given that wire A carries a current of 1.5 A and wire B carries a current of 2.5 A. (7 marks)

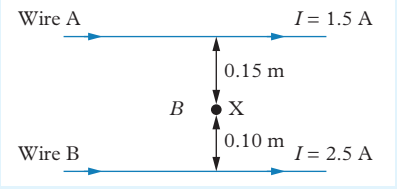


FIGURE 9 Field between two wires

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.

Think	Do
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$r_{XA} = 0.15\,\text{m}$, $r_{XB} = 0.10\,\text{m}$, $I_A = 1.5\,\text{A}$, $I_B = 2.5\,\text{A}$ $B = \frac{\mu_0 I}{2\pi r}$, where $\mu_0 = 4\pi \times 10^{-7}\,\text{T mA}^{-1}$
Step 3: The magnetic field at point X due to wire A is B_{XA}. Substitute into the equation and solve for B_{XA}. (1 mark for "Calculates field strength") Use Ampere's right-hand rule to determine the direction. (1 mark for "Determines the direction")	$B_{XA} = \frac{\mu_0 I}{2\pi r}$ $= \frac{1.26 \times 10^{-6}}{2\pi} \times \frac{1.5}{0.15}$ $= 2 \times 10^{-6}\,\text{T}$ (1 mark) into the page (1 mark)
Step 4: Calculate the magnetic field at point X due to wire B. Use Ampere's right-hand rule to determine the direction. (1 mark for "Calculates the field strength of the field"; 1 mark for "Determines the direction")	$B_{XB} = \frac{\mu_0 I}{2\pi r}$ $= \frac{1.26 \times 10^{-6}}{2\pi} \times \frac{2.5}{0.10}$ $= 5 \times 10^{-6}\,\text{T}$ (1 mark) out of the page (1 mark)
Step 5: The net field strength is the vector sum of the two fields. (1 mark for "Identifies the relationship between individual fields and the net field")	$\vec{B}_{net} = \vec{B}_{XA} + \vec{B}_{XB}$ (vector sum) (1 mark)
Step 6: Choose a convention for direction (e.g. "out of the page" is the positive direction). Substitute in the known values and calculate the net field strength. (1 mark for "Provides appropriate mathematical reasoning" and 1 mark for "Identifies the direction of the net field")	$B_{net} = -2 \times 10^{-6}\,\text{T} + +5 \times 10^{-6}\,\text{T}$ $= +3 \times 10^{-6}\,\text{T}$ (where + means out of the page) (2 marks)
Step 7: Alternatively, as the magnetic fields are in opposite directions, you can subtract one from the other. To determine the direction of the net field, remember that the greater value wins.	Alternatively: $B_{net} = 2 \times 10^{-6}\,\text{T}$ into the page + $5 \times 10^{-6}\,\text{T}$ out of the page $= 3.0 \times 10^{-6}\,\text{T}$ out of the page

Your turn

Two separate parallel wires are 35 cm apart, as shown in Figure 10. **Calculate** the value of the magnetic field strength and direction at a point X between the two wires, given that wire A carries a current of 13 mA and wire B carries a current of 25 mA. (7 marks)

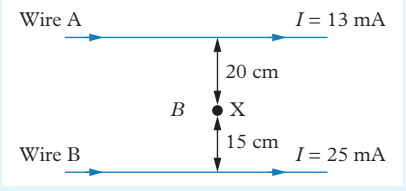


FIGURE 10 Field between two wires

FIGURE 11 The magnetic field strength at ground level underneath these 500 kV power lines is over the 0.1 μT safe level. You have to be 150 m away for it to be safe.

Combining magnetic fields in two dimensions

You also need to be able to calculate the resultant magnetic field in two dimensions, as you did for electrostatics. This is much more difficult than in one dimension, but the process is similar to the vector addition in two dimensions you did in electrostatics. Worked example 7.2C shows you how to do this.

Worked example 7.2C

Magnetic fields in two dimensions

Two long parallel wires X and Y are positioned 28.3 cm apart and perpendicular to the page, as shown in Figure 12. They carry currents of 8 A and 6 A out of the page respectively. There is a point P directly under the wires and 20 cm away from each that makes a 90° angle to them. **Calculate** the magnetic field strength (magnitude and angle) at point P. (5 marks)

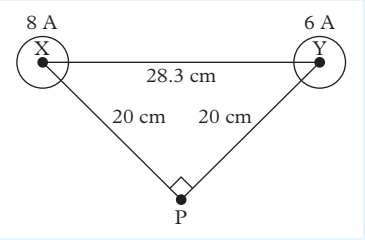
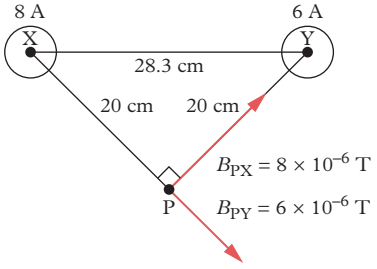
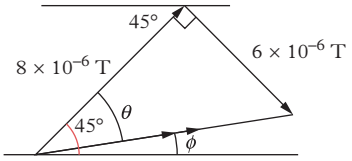


FIGURE 12 Calculating the resultant field in 2D

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: List the relevant facts from the question and select the appropriate formula.	$r_{PX} = 20\,\text{cm} = 0.20\,\text{m}$, $r_{PY} = 20\,\text{cm} = 0.20\,\text{m}$, $r_{XY} = 28.3\,\text{cm} = 0.283\,\text{m}$, angle XPY = 90° $I_X = 8\,\text{A}$, $I_Y = 6\,\text{A}$ $B = \frac{\mu_0 I}{2\pi r}$, where $\mu_0 = 4\pi \times 10^{-7}\,\text{T mA}^{-1}$
Step 3: Sketch a vector diagram to help with your analysis.	
Step 4: Calculate the field strength at point P due to X (B_{PX}), as shown in the diagram. (1 mark for "Calculates the field strength at P due to X")	$B_{PX} = \frac{\mu_0 I_X}{2\pi r_{PX}}$ $= \frac{(4\pi \times 10^{-7}) \times I_X}{2\pi r_{PX}}$ (substitute μ_0) $= \frac{2 \times 10^{-7} \times 8}{0.20}$ $= 8 \times 10^{-6}\,\text{T}$, perpendicular to line PX (1 mark)
Step 5: Calculate the field strength at point P due to Y (B_{PY}) as shown in the diagram. (1 mark for "Calculates the field strength at P due to Y")	$B_{PY} = \frac{\mu_0 I_Y}{2\pi r_{PY}}$ $= \frac{(4\pi \times 10^{-7}) \times I_Y}{2\pi r_{PY}}$ (substitute μ_0) $= \frac{2 \times 10^{-7} \times 6}{0.20}$ $= 6 \times 10^{-6}\,\text{T}$, perpendicular to line PY (1 mark)

Think	Do
Step 6: Place the two vectors head to tail and calculate the resultant vector, as shown in the diagram. (1 mark for "Constructs a correct vector diagram of the relevant quantities")	 (1 mark)
Step 7: Solve for θ and ϕ. (1 mark for "Determines the angle")	$\theta = \tan^{-1}\frac{6 \times 10^{-6}}{8 \times 10^{-6}}$ $= 36.9°$ $\phi = 45 - 36.9$ $= 8.1°$ (to the line joining the two wires) (1 mark)
Step 8: Calculate the magnitude of the resultant vector. (1 mark for "Determines the magnitude of the net field")	$B_{net} = \sqrt{(6 \times 10^{-6})^2 + (8 \times 10^{-6})^2}$ $= 1 \times 10^{-5}$T (1 mark)
Step 9: State the complete solution (magnitude and angle) for clarity.	$B_{net} = 1 \times 10^{-5}$T at 8.1° to the line XY.

Your turn

Two long parallel wires R and S are positioned 42.4 cm apart and perpendicular to the page (Figure 13). They carry currents of 3 A out of the page and 5 A into the page respectively. There is a point P directly under the wires and 30 cm away from each that makes a 90° angle to them. **Calculate** the magnetic field strength (magnitude and angle) at point P. (5 marks)

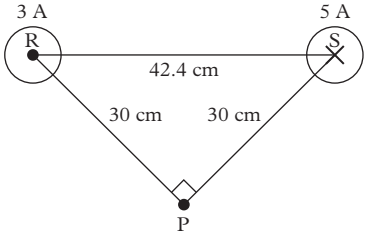


FIGURE 13 Field in two dimensions

Skill drill

Modifying an experiment

Science inquiry skills: Planning investigations (Lesson 1.4)

There are three ways to modify an experiment: redefine, extend or redirect. When you are deciding which modification to use, you need to consider which one will make your results have greater validity and reliability. Having a clear idea of your reasoning will be useful when you write up the methodology, because you will be able to justify your decision-making process.

Practise your skills

1 Refer to Practical 7.3 "Strength of a magnet at various distances". **Modify** this experiment to either redefine, extend or redirect. **Justify** your modification of the methodology by making suggested improvements and extensions that could be logically derived from the analysis of evidence. (3 marks)

Check your learning 7.2

Check your learning 7.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** whether this is true: "When you double the distance from a wire, you halve the field strength." (2 marks)

2 **Clarify** whether the field strength at a fixed distance from a wire is proportional to the current. (1 mark)

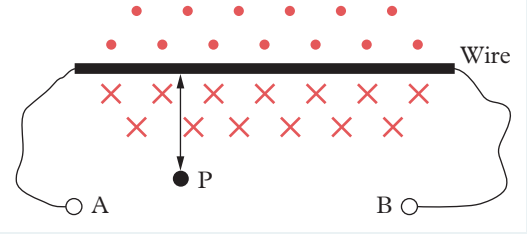
Analytical processes

3 **Determine** the magnetic field strength at a distance of 15 cm to the left of a wire that carries an electric current of

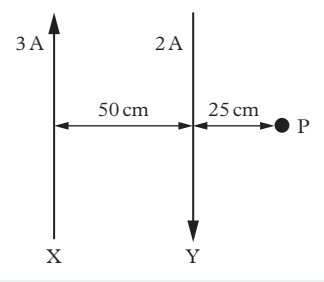
a 5.5 A north (1 mark)

b 25 A south. (1 mark)

4 **Determine** the magnitude and direction of the current in the wire AB shown in the diagram. The magnetic field at point P is 1.5×10^{-3} T and it is 1.0 cm from the wire. (3 marks)



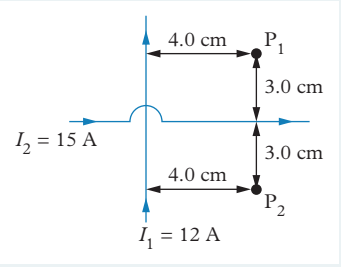
5 Two vertical wires X and Y are 50 cm apart and have currents flowing as shown in the diagram.



a **Calculate** the magnitude and direction of the magnetic field at point P, 25 cm to the right of wire Y. (4 marks)

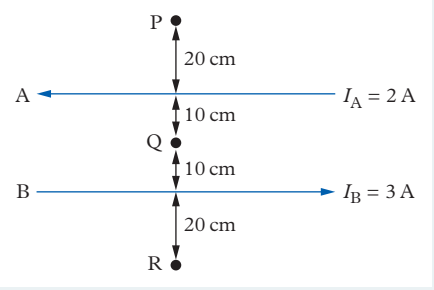
b **Determine** the order of magnitude of the magnetic field at P. (1 mark)

6 **Determine** the direction and magnitude of the magnetic field at points P_1 and P_2 in the diagram. (5 marks)



Knowledge utilisation

7 Consider two conducting wires in a guitar amplifier that are producing unwanted magnetic fields. The engineers are trying to work out the magnetic field strength at various positions near the wires. The wires carry 2 A and 3 A respectively, as shown in the diagram.



a **Calculate** the magnetic field strength due to both wires at these positions.

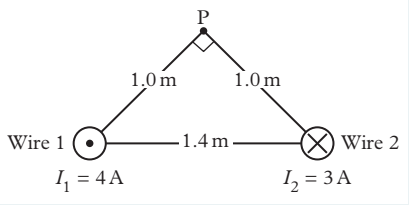
i P (3 marks)

ii Q (3 marks)

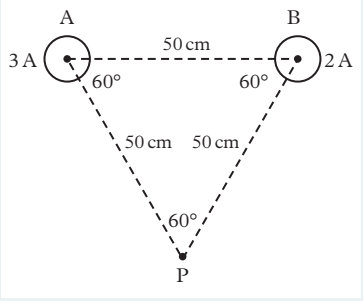
iii R (3 marks)

b **Determine** the position (besides infinity) where the field strength would be zero. (3 marks)

8 Sound recording studios are very sensitive to stray magnetic fields from current-carrying wires in their walls. In one studio two wires 1.4 m apart are running vertically up a wall, carrying currents in opposite directions. **Determine** the magnitude of the magnetic field at point P midway between the wires and 1.0 m from each, as shown in the diagram (2 s.f.). (4 marks)



9 Two wires A and B are 50 cm apart, as shown in the diagram. Currents of 3 A and 2 A respectively flow in the wires. **Determine** the magnetic field strength at point P (2 s.f.). (5 marks)



Practical

Lesson 7.3

Strength of a magnet at various distances

Learning intentions and success criteria

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

Practical

Lesson 7.4

Strength of a magnetic field at various distances from a current-carrying wire

Learning intentions and success criteria

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

Lesson 7.5
Magnetic fields in solenoids

Key ideas

- A solenoid is a long straight coil of wire used to produce a uniform magnetic field.
- The magnetic field direction in a solenoid can be determined using the right-hand rule.
- Problems on the magnitude of magnetic fields in a solenoid can be solved using $B = \mu_0 nI$ and $n = \frac{N}{L}$.

Learning intentions and success criteria

What are solenoids?

In your earlier studies of electricity and magnetism, you probably wound lots of turns of wire into a coil on a cardboard tube to make an electromagnet, which could pick up paperclips when connected to a power supply. What you made was a **solenoid** – a coil of many loops of wire that generates a magnetic field (Figure 1). The word solenoid comes from the Greek "solen", meaning "tube". Solenoids are very useful in home appliances and in industry. They are used for car door locks, water-pressure valves in washing machines (Figure 2), electromagnets, speakers and microphones, to name just a few examples.

solenoid
a long straight coil of wire used to generate a controlled and almost uniform magnetic field

FIGURE 1 A 20 cm long air-cored solenoid of 650 turns

FIGURE 2 Solenoids are used in many home appliances, such as in washing machines to control the flow of water.

A solenoid concentrates the magnetic field lines into a region of space that produces an almost perfectly uniform magnetic field within its hollow core. A solenoid can produce a stronger field if the hollow core contains a magnetically soft material. Magnetically soft materials are those that become magnetised when a current flows but lose their magnetic properties when the current is turned off. The core concentrates the lines of force and increases the magnetic strength through the induction principle. Iron–nickel alloys are the most commonly used materials in solenoid cores. They can generate a magnetic field that is several hundred times stronger than that of a solenoid with a hollow core. A big advantage of solenoids is that the magnetic field can be switched on or off simply by breaking the flow of current through the coil.

How is the direction of the field in a solenoid determined?

The properties of a solenoid can be understood by starting with a single loop, as shown in Figure 3. It shows a wire bent into a single circular loop. This loop can be thought of as many small, straight segments, each adding its individual magnetic field together at the centre of the loop. The magnetic field lines show this is where the field is the strongest and is directed through the loop.

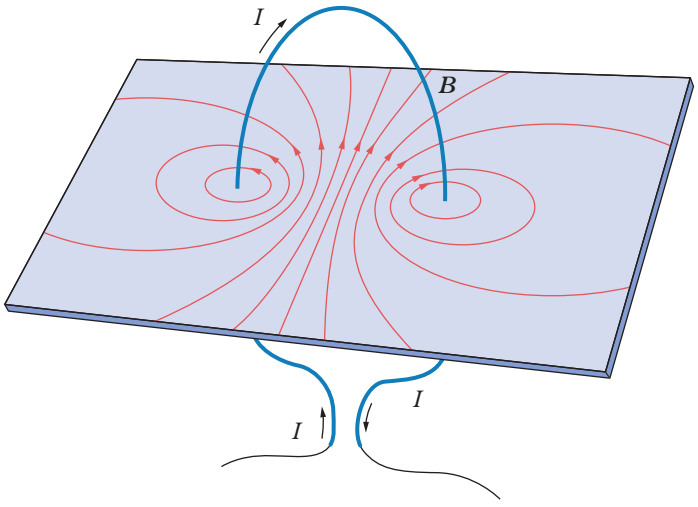


FIGURE 3 Representation of the field around a single loop

To determine the direction of the magnetic field, you can use Ampere's right-hand rule, as you did for a straight wire earlier in this module. However, in this case, you grip the solenoid in your right hand so that your fingers naturally curl around the solenoid in the direction of the conventional current, and then your extended thumb will point to the effective north pole of the solenoid's magnetic field (Figure 4). The field lines are then drawn outside the coil from the north pole of the coil towards the south pole at the coil's opposite end. Externally, the solenoid field has a very similar shape to that of a bar magnet. The magnetic field lines are continuous and extend through the centre of the solenoid, where there is a uniform field.

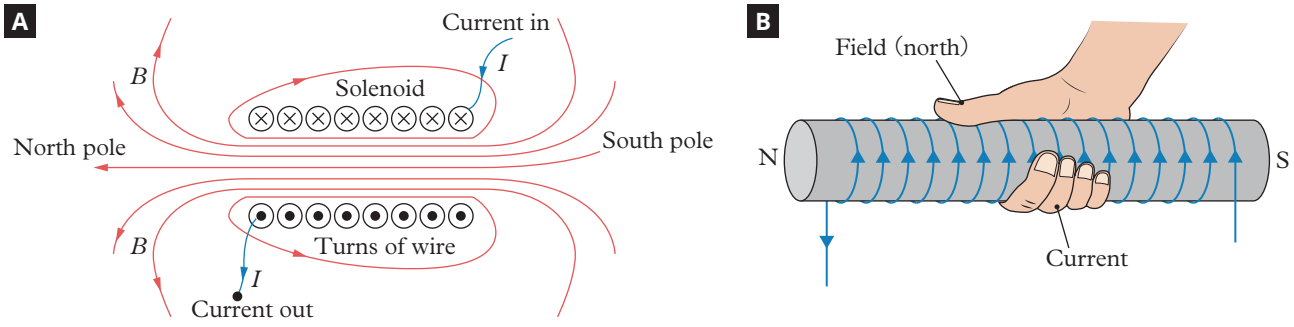


FIGURE 4 (A) Current flow and polarity for a solenoid; (B) using the right-hand rule for a solenoid

How is the field strength in a solenoid determined?

The magnetic field at the centre of a very long solenoid is constant. The strength of this field depends only on the current flowing in the coil and the number of turns of wire per unit length of the solenoid. The formula for the magnitude of the field strength in the solenoid's centre is:

$$B = \mu_0 nI$$

where B is the magnitude of the field strength in the solenoid's centre, μ_0 is the permeability constant of $1.26 \times 10^{-6}\,\text{T m A}^{-1}$, n is the number of turns per metre of length, and I is the current in ampere. To calculate the number of turns per metre, use the formula:

$$\text{number of turns per metre} = \frac{\text{total number of turns}}{\text{length of solenoid}}$$

$$n = \frac{N}{L}$$

Worked example 7.5A shows you how to apply these formulas.

Challenge

Modifying the hand rule for electron flow

Devise a hand rule for the magnetic field due to an electron flow in a solenoid. (1 mark)

Worked example 7.5A

Magnitude and direction of the field in a solenoid

A solenoid has a length of 25 cm and contains 600 turns, arranged as shown in Figure 5. It carries a current of 3.0 A.

a **Determine** the magnetic field strength in the centre of the solenoid. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** which end will be a north pole. (1 mark)

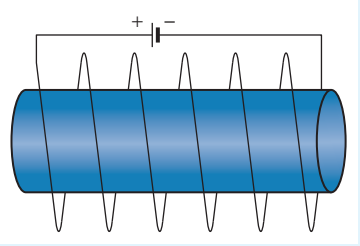
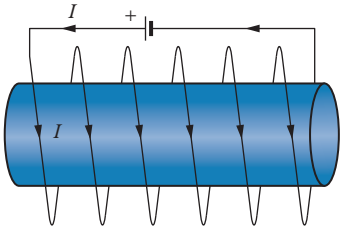
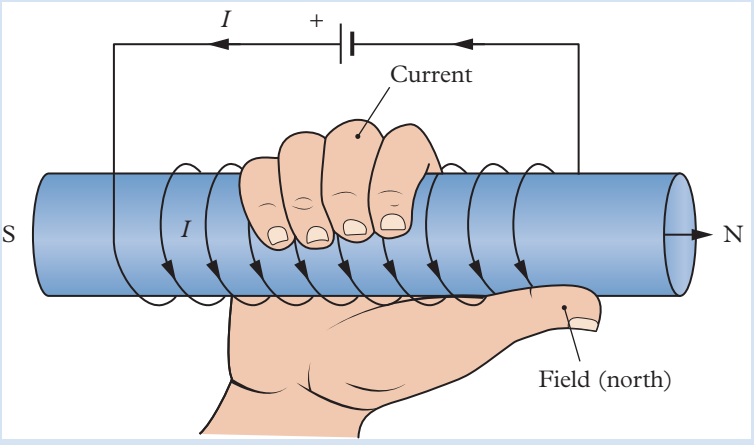


FIGURE 5 Current in a solenoid goes from the positive terminal (+) through the wire to the other side (–) of the cell.

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and hand rule and use them to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$L = 25\,\text{cm} = 0.25\,\text{m}$, $N = 600$, $I = 3.0\,\text{A}$ $n = \frac{N}{L}$, $B = \mu_0 nI$
Step 3: Substitute the known values into the formula to find the turns per metre. (1 mark for "Demonstrates correct substitution")	**a** $n = \frac{N}{L}$ $= \frac{600}{0.25}$ (1 mark) $= 2{,}400$ turns per metre
Step 4: Substitute the known values into the formula and solve for magnetic field strength, B. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Demonstrates correct substitution") Step 5: Use the right-hand rule to find the direction. First, identify the direction of the current from the information given.	$B = \mu_0 nI$ $= 1.26 \times 10^{-6} \times 2{,}400 \times 3.0$ (1 mark) $= 0.0091\,\text{T}$ **b** 

Think	Do
Step 6: Current is flowing down the front of the solenoid, so curl the fingers of your right hand that way. You thumb points in the direction of the field, to the north end. (1 mark for "Identifies the direction of the field and identifies the north end")  	The right end is a north pole. (1 mark)

Your turn

A solenoid 45 cm long and with 800 turns of wire arranged as in Figure 6 has a current of 2.5 A passed through it when switch K is closed.

a **Determine** the magnetic field strength inside the coil. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** the polarity (N or S) at end A. Show your working. (2 marks)

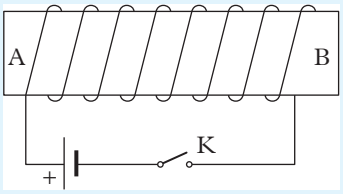


FIGURE 6 A solenoid connected to a cell in a circuit

Skill drill

Linearising data from a solenoid experiment

Science inquiry skills: Processing and analysing data (Lesson 1.7)

Experimental data is often non-linear, but there are techniques to help you find relationships between the variables. In the "Linear motion and force" topic of Unit 2 you will have measured the time for an object to fall certain distances. Those data were non-linear, and you will have linearised it to form a straight line. This made it easier to confirm or reject a suspected relationship.

Here is an example of the measurement of the magnetic field strength at a set distance from the end of a solenoid in which steel cores were added to concentrate the magnetic field. A plot of field strength versus the number of cores is non-linear, as shown in Figure 7.

The equation for the trendline, $y = 0.2649x^{0.505}$, suggests a "power" relationship, with the power value (exponent) being 0.505. This is approximately $\frac{1}{2}$, so it suggests a square root relationship: $y \propto \sqrt{x}$, or $B \propto \sqrt{N}$, where B is the field strength and N is the number of cores. To linearise this, we plot B on the y-axis and $\sqrt{N}$ on the x-axis, as in Figure 8. The graph is now linear, which confirms our prediction.

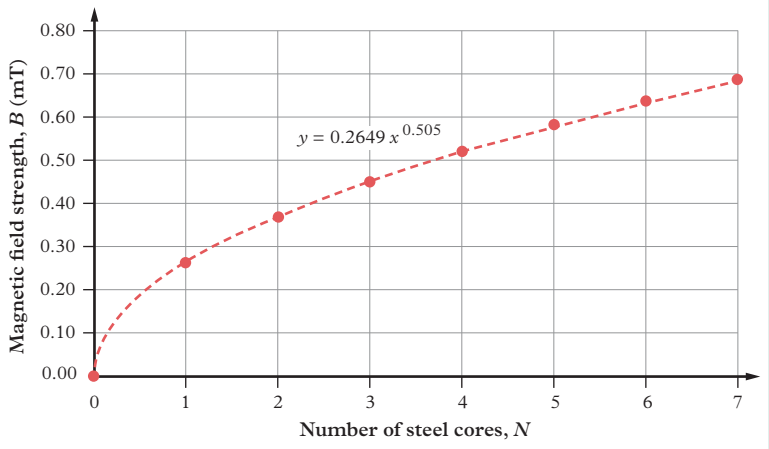


FIGURE 7 Non-linear graph: B versus N

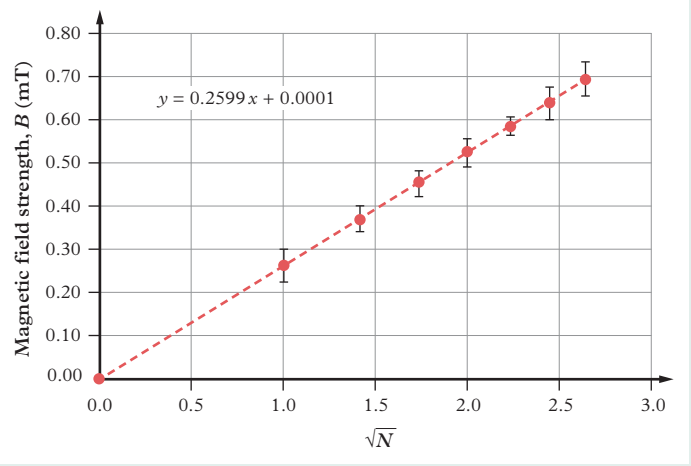


FIGURE 8 Linearised graph: B versus $\sqrt{N}$

Practise your skills

1 An experiment measured the magnetic field strength, B (mT), at a distance d (cm) from a rare-earth magnet. A graph of the raw data had a trendline with the equation $y = 5.344x^{-2}$. Note that the exponent is negative. **Identify** the likely power relationship in the form $y \propto x^a$. (1 mark)

2 **Describe** how you would linearise this relationship. That is, what would you plot on the x-axis and on the y-axis? (2 marks)

3 Suppose your linearised graph gave a trendline equation of $y = 5.344x$. **Draw a conclusion** about the relationship between magnetic field strength and distance. (2 marks)

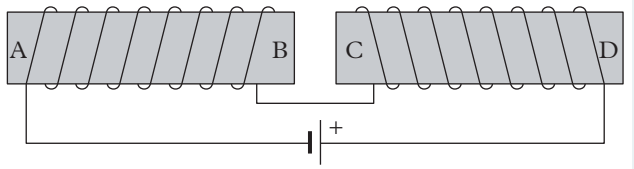
Check your learning 7.5

Check your learning 7.5: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Clarify** whether the field strength inside a solenoid is proportional to the current. (1 mark)

2 **Identify** the magnetic polarity of ends A and B, for the diagrams shown. (2 marks)



Analytical processes

3 **Determine** the magnetic field strength at the centre of a solenoid that has a length of 20 cm and contains 8,000 turns. It carries a current of 15 A (2 s.f.). (2 marks)

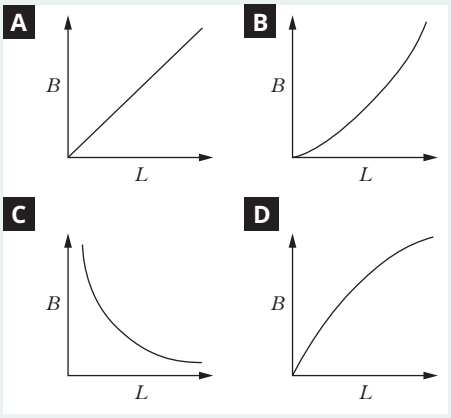
4 A 200-turn solenoid of length 50 cm has a magnetic field of 2.3×10^{-5} T. **Determine** the current flowing through it (2 s.f.). (2 marks)

5 A solenoid has a length of 70 cm and when a current of 12 A passes through it, a magnetic field strength of 6.91×10^{-4} T is produced. **Determine** the number of turns in the solenoid (to the nearest whole number). (3 marks)

Knowledge utilisation

6 Two solenoids are arranged in a line as shown in the diagram. **Decide** whether they will attract or repel each other. **Explain** your reasoning. (3 marks)

7 A metal Slinky (a spring) of 50 turns of wire is used as a solenoid and has a constant current passed through it. As it is stretched to different lengths, L, its magnetic field strength, B, is measured. **Decide** which one of the following graphs best represents the relationship between B and L. **Explain** your reasoning. (2 marks)



Lesson 7.6
Magnetic forces on moving charges

Learning intentions and success criteria

Key ideas

→ The direction of the force on a charged particle in a magnetic field can be determined using Fleming's left-hand rule or the right-hand slap (palm) rule.

→ Problems on the magnitude of the force on a moving charge in a magnetic field can be solved using $F = qvB \sin\theta$.

What forces act when a charge moves perpendicular to a magnetic field?

At the beginning of this module, we said that electric fields and magnetic fields both involved electric charges. However, electric fields act on stationary and moving charges, whereas magnetic fields act only on moving charges. Here's a summary of their properties (you will learn how to use the formulas for forces on charged particles in magnetic fields in this lesson).

TABLE 1 Summary of the properties of magnetic and electric fields

Field type	Acts on	Factors	Formula
Electric fields	Stationary charge	Electric field strength, E Charge, q	$F = Eq$
Magnetic fields	Moving charge	Magnetic field strength, B Charge, q Velocity, v	$F = qvB$ $F = qvB\sin\theta$

So far, we have looked at magnetic fields around moving charges and wires or coils. In this lesson, we will look at the forces charged particles experience because of magnetic fields.

Direction of the force on a moving charged particle in a magnetic field

Fleming's left-hand rule

To determine the direction of the force on a positively charged particle, we use Fleming's left-hand rule (Figure 1).

1. Hold your left hand so that your thumb, index finger and middle finger are at right angles to each other.
2. Move your hand so your index finger points in the direction of the magnetic field.
3. Move your hand so your middle finger points in the direction of the motion of a positive charge.
4. Your thumb will now be pointing in the direction of the force.

You can now apply the rule to a situation in which a positive charge is being fired from the left into a magnetic field, B, that is "out of the page" (as shown by the small dots in the circles in Figure 2). Fleming's left-hand rule tells you the force is down the page, so the charge moves down the page.

For a negative charge such as an electron, point your middle finger in the opposite direction to the movement of charge. In effect, you're just reversing the direction of the electron current and pretending it is a conventional current (Figure 2).

Alternatively, you can work out the direction of the force as if the charged particles are positive, in which case the force would be upwards. Then the force on electrons will be in the opposite direction, downwards, because they are negative.

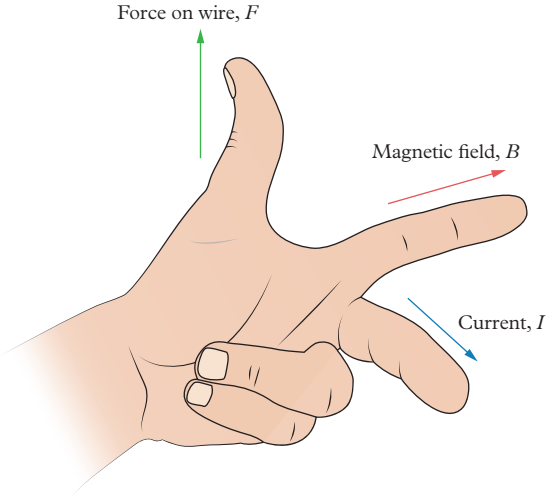


FIGURE 1 Fleming's left-hand rule

Study tip

When dealing with negative charges, you can still use Fleming's left-hand rule, but you should reverse the direction of the electron flow.

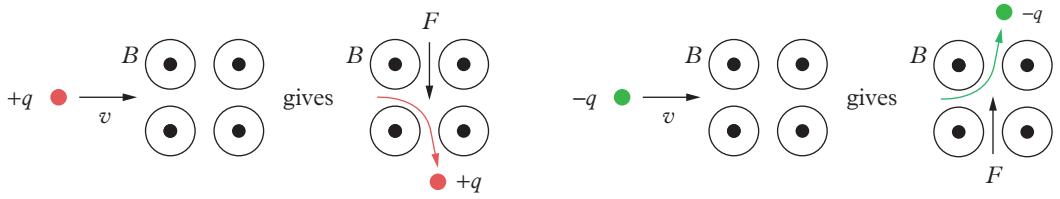


FIGURE 2 Use Fleming's left-hand rule to see how positive and negative charges move in a magnetic field out of the page.

Study tip

The syllabus doesn't require you to know about the use of hand rules. That means, in the external exam, you won't have to explain how a particular rule is applied. You may choose to use it, but you won't get marks for explaining it.

The right-hand palm rule

In this book we use Fleming's left-hand rule for forces on moving charges or an electric current in a wire. However, some teachers prefer to use another version of it called the "right-hand palm rule" (or "slap" rule) (Figure 3).

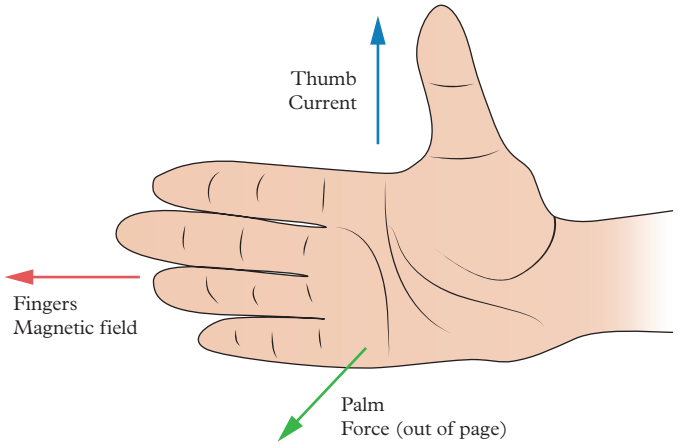


FIGURE 3 The right-hand palm rule can also be used to find the direction of the force on a moving charge in a magnetic field.

It gives the same results, and either is ok to use in external exams, but you need to decide on one or the other – and stick to it. The rule says:

- Hold your hand straight, so your fingers point away from you and your thumb points upwards at right angles to your hand.
- The thumb represents the direction of the current. The fingers represent the magnetic field. The direction of the force is the direction you would push an object with your palm. This is consistent with Ampere's right-hand rule for fields around a wire, where the fingers (curled) also represent the magnetic field.

Study tip

To remember which finger is which in Fleming's left-hand rule, start with the thumb and label them FBI (as in the US law enforcement agency): force, *F*, magnetic field, *B*, and current, *I*.

Applying Fleming's left-hand rule – cathode-ray tubes

A cathode-ray tube (like the one shown in Figure 4), produces a beam of electrons when a high voltage is placed across the terminals. "Cathode rays" was the name for electrons, before physicists knew what they were.

In a cathode-ray tube, electrons are produced at the negative terminal (the cathode) on the left in the diagram and are attracted to the positive terminal (the anode) on the right. The beam of electrons skims along a zinc sulfide screen, which emits green light when electrons strike it. A magnet brought up to the electron beam will deflect it.

In this case, Fleming's left-hand rule indicates that the beam will be deflected downwards. Remember that it is a beam of negatively charged particles, so you assume the "positive" current is from the right and thus the force is downwards. Alternatively, you can work out the direction of the force for positive charged particles travelling from left to right, which would be upwards. Then the force on the negatively charged electrons will be in the opposite direction, downwards. Both methods are valid but, again, it's best to pick the one that makes most sense to you and then stick to it.

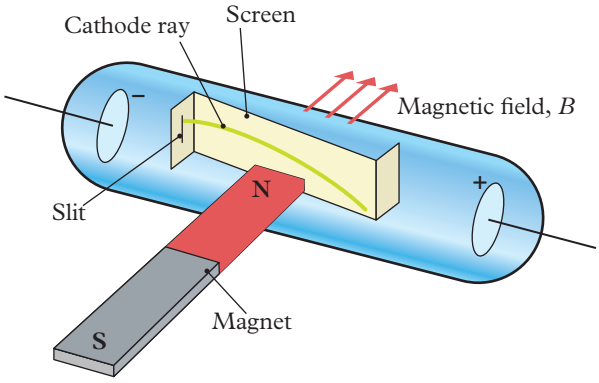


FIGURE 4 Cathode-ray tube showing the deflection of an electron beam (cathode ray) in a field that is facing into the page

Magnitude of the force on a moving charged particle in a magnetic field

If a moving particle with charge of q enters a magnetic field, it will experience a force that is dependent on the magnetic field strength, B, and the velocity of the particle, v.

As you learnt in Lesson 7.2, the SI unit of magnetic field strength is the tesla, symbol T. One tesla (1 T) is defined as the magnetic field strength generating one newton (N) of force per coulomb (C) of charge per metre per second (m s^{-1}) of speed. Thus, the force on a particle moving at right angles to the field is given by:

$$B = \frac{F}{qv}$$

$$F = qvB \text{ (at } 90° \text{ to the field)}$$

where B is the magnetic field strength in tesla (T), F is the force in newton (N), q is the charge in coulomb (C), and v is the velocity of the charge in m s^{-1}. We know that the force is a maximum when the motion is at right angles to the field ($\theta = 90°$) and the force is zero when the motion is parallel to the field ($\theta = 90°$). At all other angles in between, the equation is:

$$F = qvB \sin\theta$$

Representing the magnetic field in questions

In Worked examples 7.6A and 7.6B, magnetic fields are presented in two different ways:

- Worked example 7.6A has the field as an arrow perpendicular to the motion of the charge. The motion of the charged particle is shown by an arrow.
- Worked example 7.6B has the field as dots (the field is towards you) or as crosses (the field is away from you). The motion of the charged particle is shown by an arrow.

Worked example 7.6A

Force on a charged particle moving perpendicular to the field: field drawn as arrows

A +10 μC charge moves at a speed of 250 m s^{-1} perpendicular to a magnetic field of strength 1.5 T, as shown in Figure 5. **Calculate** the magnitude and direction of the force acting on the charge. Give your answer to an appropriate number of significant figures. (3 marks)

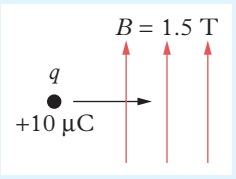
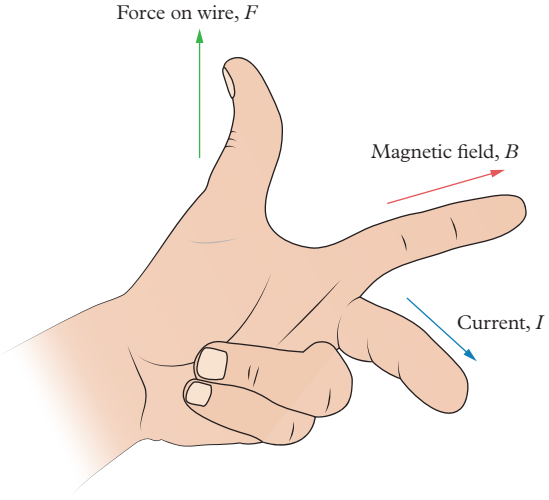


FIGURE 5 Charge moving perpendicular to a magnetic field

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formula.	$q = 10 \times 10^{-6}$ C, $v = 250$ m s^{-1}, $B = 1.5$ T, $\theta = 90°$ $F = qvB\sin 90°$
Step 3: Substitute the known values into the formula and solve for the force, F. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$F = qvB\sin 90°$ $= 10 \times 10^{-6} \times 250 \times 1.5 \times \sin 90°$ (1 mark) $= 0.00375$ N $= 3.8 \times 10^{-3}$ N (2 s.f.) (1 mark)
Step 4: Find the direction using Fleming's left-hand rule. Position your hand and fingers according to the rule. It may help to draw a diagram with the three directions shown at right angles.	
Step 5: Force is in the direction of the thumb, so the direction of the force is out of the page. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Identifies the direction of the force")	Direction: out of the page $F = 3.8 \times 10^{-3}$ N out of the page (2 s.f.) (1 mark)

Your turn

An electron, charge $q_e = -1.6 \times 10^{-19}$ C, moves at a speed of 1.3×10^7 m s^{-1} perpendicular to a magnetic field of strength $B = 1.5 \times 10^{-4}$ T (Figure 6). **Calculate** the magnitude and direction of the force acting on the electron. Give your answer to an appropriate number of significant figures. (3 marks)

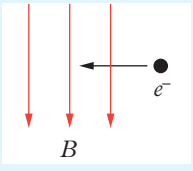


FIGURE 6 Electron moving perpendicular to a magnetic field

Worked example 7.6B

Force on a charged particle moving perpendicular to the field: field drawn as crosses

A +20 μC charge moves at a speed of 700 m s^{-1} in a magnetic field of strength 5.0 T, as shown in Figure 7. **Calculate** the magnitude and direction of the force acting on the charge. Give your answer to an appropriate number of significant figures. (3 marks)

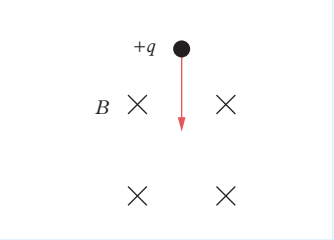
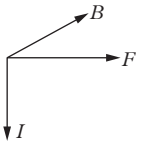


FIGURE 7 Positive charge moving perpendicular to a magnetic field

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formula. We can use the complete formula and substitute in a value for θ of 90°, as the motion is at right angles to the field.	$q = +20\,\mu C = +20 \times 10^{-6}$ C, $v = 700$ m s^{-1}, $B = 5.0$ T, $\theta = 90°$ $F = qvB\sin 90°$
Step 3: Substitute the known values into the equation and solve for F. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$F = qvB\sin 90°$ $= 20 \times 10^{-6} \times 700 \times 5.0 \times \sin 90°$ (1 mark) $= 0.07$ N $= 7.0 \times 10^{-2}$ N (2 s.f.) (1 mark)
Step 4: To find the direction, position your hand and fingers according to Fleming's left-hand rule. It may help to draw a diagram with the three directions shown at right angles.	
Step 5: Force is in the direction of your thumb, so it is to the right. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Identifies the direction of the force")	Direction: to the right $F = 7.0 \times 10^{-2}$ N to the right (2 s.f.) (1 mark)

Your turn

An electron, $q_e = -1.6 \times 10^{-19}$ C, moves at a speed of 1.3×10^{7} m s^{-1} perpendicular to a magnetic field of strength 1.5×10^{-4} T (Figure 8). **Calculate** the magnitude and direction of the force acting on the electron. Give your answer to an appropriate number of significant figures. (3 marks)

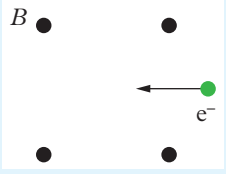


FIGURE 8 Electron moving perpendicular to a magnetic field

What happens when field and charge are not perpendicular?

We have considered the simplest case where the charged particle's direction and the field are at right angles to each other. Now we will consider what happens when they are not at right angles (Figure 9). In this case we resolve the magnetic field vector into two components at right angles. The component of the field that is perpendicular to the direction of motion of the charged particle is considered to be the part that interacts with the charge.

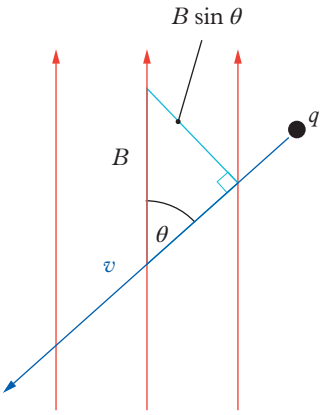


FIGURE 9 The angle θ is measured between the path of the charged particle and the field direction.

Calculating forces on charged particles moving at an angle to the field

For an angle θ between the direction of motion of the charged particle and the magnetic field, the perpendicular component of the field is $B\sin\theta$.

This means that the formula:

$$F = qvB$$

becomes

$$F = qvB\sin\theta$$

where F is the force in newtons (N), q is the charge in coulombs (C), v is the velocity of the charge in m s^{-1}, B is the magnetic field strength in teslas (T), and θ is the angle in degrees between the path of the charged particle and the magnetic field lines.

Note that:

- If the direction of motion is parallel to the field ($\theta = 0°$), $F = 0$.
- If the direction of motion is perpendicular to the field ($\theta = 90°$), F is a maximum.
- At angles between 0° and 90°, the factor $\sin\theta$ is used to allow for the weaker interaction of the field with the particle.
- The magnitude of the force is directly proportional to B, q, v and $\sin\theta$ – that is, the greater the value of any one of these, the bigger the force. Remember that as θ increases between 0° and 90°, $\sin\theta$ also increases between 0 and 1.

Alternatively, we could say that the $\sin\theta$ refers to the velocity vector instead of the magnetic field vector. In that case, $v\sin\theta$ represents the component of velocity that is perpendicular to the field, as in $F = qBv\sin\theta$. It makes no difference to the calculation. It takes a bit of geometry to work it out either way.

Study tip

The angle θ is the angle between the direction of motion (the velocity vector) and the magnetic field. Angles given in a question are not always the θ value to be used in the equation. Consider what any given angle represents before plugging it in. When looking for angle values, you could represent them using terms such as "perpendicular", "at right angles", or "parallel" to the field.

Worked example 7.6C

Force on a particle moving at an angle to the field

A +10 μC charge moves at a speed of 250 m s^{-1} in a magnetic field of strength 1.5 T at an angle of 60° to the field, as shown in Figure 10. **Calculate** the magnitude and direction of the force acting on the charge. Give your answer to an appropriate number of significant figures. (3 marks)

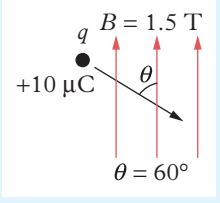
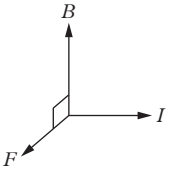


FIGURE 10 Charge moving at an angle to a magnetic field

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$q = 10 \times 10^{-6}$ C, $v = 250$ m s^{-1}, $B = 1.5$ T, $\theta = 60°$ $F = qvB\sin\theta$
Step 3: Substitute the known values into the formula and solve for F. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer") Step 4: Position your hand and fingers according to Fleming's left-hand rule. It may help to draw a diagram with the three directions shown at right angles. Check that it is a positive charge. If it was not, you would have to reverse the direction of I.	$F = qvB\sin 60°$ $= 10 \times 10^{-6} \times 250 \times 1.5 \times \sin 60°$ (1 mark) $= 0.003284$ N $= 3.2 \times 10^{-3}$ N (2 s.f.) (1 mark) 
Step 5: The force is in the direction of your thumb, so it is out of page. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Identifies the direction of the force")	Direction: out of the page (1 mark) $F = 3.2 \times 10^{-3}$ N out of the page (2 s.f.)

Your turn

A particle with a positive elementary charge moves at a speed of 2×10^{7} m s^{-1} in a magnetic field of strength 150 μT at an angle of 55° to the field, as shown in Figure 11. **Calculate** the magnitude and direction of the force acting on the charge. Give your answer to an appropriate number of significant figures. (3 marks)

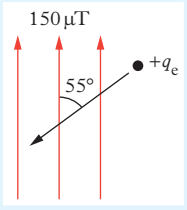


FIGURE 11 Charge moving at an angle to a magnetic field

How does a charged particle move in a magnetic field?

If a charged particle enters a magnetic field, it will be deflected by a force at right angles to its direction of motion. As it changes direction slightly, it is deflected at right angles to its new direction of motion, and so on. It is like whirling a ball on a string. The magnetic field acts like a string keeping a ball revolving in a circle. This motion, using an electron as an example, is shown in Figure 12. You can see that the electron is following a circular path. You can use Fleming's left-hand rule to confirm this.

The radius of the circle can be found by equating the force from two laws you have already met, namely the centripetal force, $F_c = \frac{mv^2}{r}$, and the force on a charged particle in a magnetic field, $F = qvB$:

$$\frac{mv^2}{r} = qvB$$
$$r = \frac{mv}{qB}$$

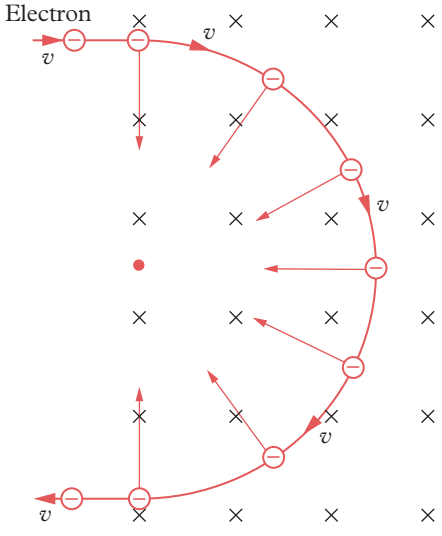


FIGURE 12 Circular motion of an electron in a magnetic field

This formula is not in the syllabus, but it is a combination of two well-known laws that are in the syllabus, so don't be surprised to see its use in exams. What it says is that:

- an increase in mass and/or velocity means the radius increases
- an increase in charge and/or magnetic field strength means the radius decreases.

Two particles with the same charge and velocity but with different masses – for example, ${}^{12}_{6}\text{C}$ and ${}^{14}_{6}\text{C}$ – that are shot into the same magnetic field will have different radii of curvature. ${}^{14}_{6}\text{C}$ will have the larger radius – in fact, $\frac{14}{12}$ times as much. For two particles with the same mass and velocity, a 1+ charge will have double the radius of a 2+ charge, and so on.

The mass spectrometer

A mass spectrometer (Figure 13) is a device based on this principle. It identifies gaseous charged particles according to their masses and their resulting deflection in a magnetic field. In this way, the molecules making up the gas can be identified and the concentration of each can be determined.

FIGURE 13 A mass spectrometer being opened for inspection

Worked example 7.6D

Calculating the mass of a charged particle

A chlorine ion ($|q| = 1.6 \times 10^{-19}$ C) enters a magnetic field of strength 0.910 T at right angles. It is deflected into a circular path with a radius of curvature of 27.0 cm, as shown in Figure 14. The velocity has been measured as 668 m s^{-1}.

a **Calculate** the mass of the particle in kg. Give your answer to an appropriate number of significant figures. (3 marks)

b **Determine** if the charge on the particle is positive or negative. **Explain** your reasoning. (2 marks)

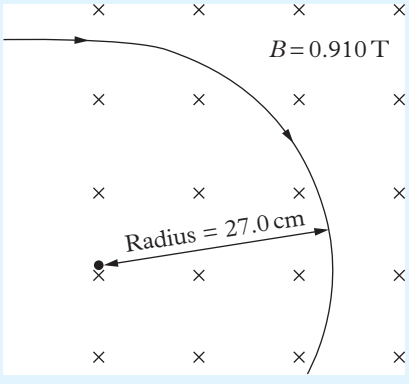


FIGURE 14 Path of a charged particle in a magnetic field

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	For part **a** "Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer. For part **b** "Determine" means to ascertain after consideration. "Explain" means to describe in more detail.
Step 2: Gather the relevant data from the question and identify the appropriate formulas. The question involves circular motion and the force on a charged particle in a magnetic field.	**a** $B = 0.910\,\text{T}$, $r = 0.270\,\text{m}$, $q = 1.6 \times 10^{-19}\,\text{C}$ $F_c = \frac{mv^2}{r}$, $F_B = qvB$
Step 3: Identify this as a scenario equating centripetal force to the force on a charged particle in a magnetic field. (1 mark for "Indicates an understanding of the physical scenario in relation to the two forces")	$F_c = F_B$ $\frac{mv^2}{r} = qvB\sin\theta$ $\theta = 90°$, therefore $\sin\theta = 1$ $m = \frac{qBr}{v}$ (1 mark)
Step 4: Substitute in the known values and make the calculation. (1 mark for "Provides pertinent mathematical operations correctly performed")	$m = \frac{1.6 \times 10^{-19} \times 0.910 \times 0.270}{668}$ $= 5.885 \times 10^{-24}\,\text{kg}$ (1 mark)
Step 5: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Determines the mass correctly")	$m = 5.9 \times 10^{-24}\,\text{kg}$ (2 s.f.) (1 mark)
Step 6: Use Fleming's left-hand rule to determine the direction of the force. (1 mark for "Identifies the correct sign of the charge"; 1 mark for "Demonstrates an understanding of the relationship between F, B and I")	**b** B (index finger) is into the page. At the top of the diagram where the particle enters the field, the F (thumb) is down towards the centre of the circle; therefore I (index finger) is to the left for a positive charge. (1 mark) As the charge is moving to the right, it must be a negative charge. (1 mark)

Your turn

A beta particle, β, with a mass of $9.109 \times 10^{-31}\,\text{kg}$ enters a magnetic field of strength 5.13 mT at right angles, as shown in Figure 15. It follows a circular path with a radius of 25.0 cm. The beta particle carries one elementary charge ($1q_e$).

a **Calculate** the speed of the beta particle. Give your answer to an appropriate number of significant figures. (3 marks)

b **Determine** whether the particle is a β^+ or β^- particle. **Explain** your reasoning. (2 marks)

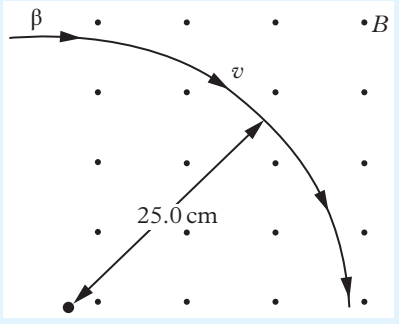


FIGURE 15 Path of a beta particle in a magnetic field

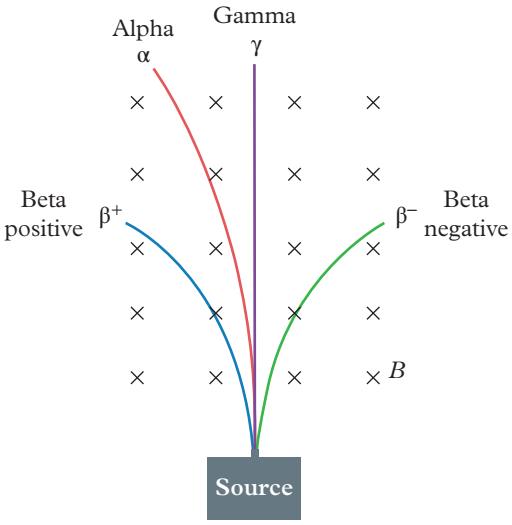


FIGURE 16 Deflection of radiation in a magnetic field (not to scale)

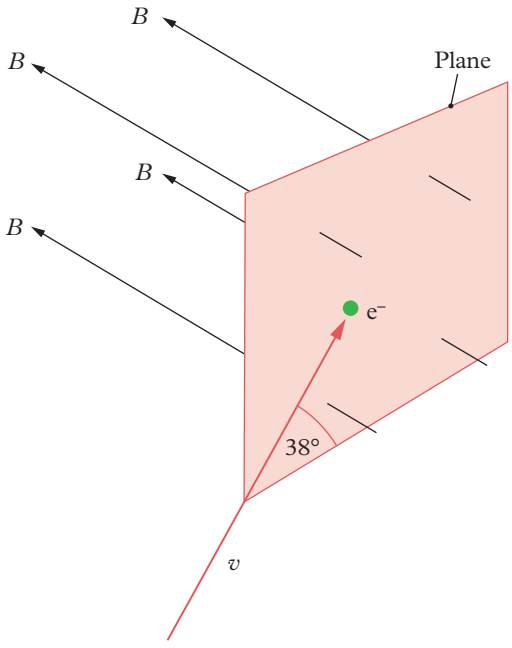


FIGURE 17 Motion of an electron in a plane at right angles to the field

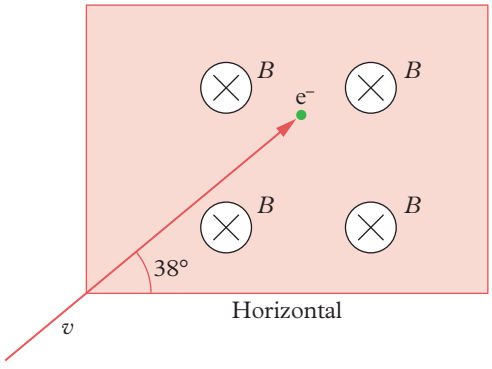


FIGURE 18 Motion of an electron viewed from the right of Figure 17, facing the plane

Nuclear radiation

Types of nuclear radiation deflect differently in a magnetic field because of their different charges and masses. Alpha particles have double the charge of beta particles, but they will deflect less than beta particles because they have 7,300 times the mass. Beta-positive and beta-negative particles will deflect the same amount as each other but in opposite directions. Neutrons, gamma rays and neutrinos are uncharged, so they won't deflect at all.

Figure 16 shows the differences in their behaviour – but note that it is not to scale. If the speeds of the particles were the same, the alpha particle would have a radius of curvature four times that of a proton, and 3,700 times that of the beta particle. You should be able to use Fleming's left-hand rule to verify the deflections to the right or left.

Other scenarios

A magnetic field exists in three dimensions and the interaction with charged particles has to be interpreted from a 2D drawing. This is difficult, but manageable, if you follow the conventions about field directions.

A further scenario is working out the direction of the force at a particular moment during the interaction. This may be too complex for an external exam but it is worth understanding how it could be approached.

Consider a magnetic field, B, that passes at right angles through a rectangular plane (i.e. the angle between the plane and the field lines is 90°), as shown in Figure 17. An electron is made to travel along the surface of the plane at an angle of 38° to its bottom horizontal edge. We can say that the path of the electron is in the plane of the rectangle at 38° above the horizontal edge.

If we move around to the right and view the plane straight on, it looks like Figure 18.

We can now consider the force acting on the electron. Using Fleming's left-hand rule, B (index finger into page) and I (middle finger downwards to the left opposite the path of the electron) gives F (thumb) as downwards towards the right at right angles to the path of the electron, as shown in Figure 19. The electron's path would thus start to curve down in a clockwise direction, as shown by the dotted path, as it passed through the field. We can't be more specific than this as the direction keeps changing and so does the direction of the force. But at this particular moment the force is as shown. If you were asked to indicate the electron's path, you would just draw a dotted line in a clockwise direction as shown.

As an example of a calculation to determine the magnitude of the force, if the magnetic field strength was 120 mT, and the speed of the electron was $2.5 \times 10^8\,\text{m s}^{-1}$, the force would be given by the following, where e (the charge on the electron) equals one elementary charge ($1 \times -1.6 \times 10^{-19}\,\text{C}$):

$$\begin{aligned} F &= qvB\sin\theta \\ &= 1.6 \times 10^{-19} \times 2.5 \times 10^8 \times 120 \times 10^{-3} \times \sin 90° \\ &= 4.8 \times 10^{-12}\,\text{N (2 s.f.)} \end{aligned}$$

Note that the angle of 38° mentioned earlier is not the angle θ between the direction of motion and the field. That is, it does not represent θ in the equation.

If, instead of an electron, an alpha particle, α, was used, the force would be in the opposite direction because the charge on an alpha particle is positive (+2), whereas the electron is negative (–1). The magnitude of the force would also be doubled, as q for an alpha particle is twice the magnitude of an elementary charge, or $2 \times 1.6 \times 10^{-19}$ C.

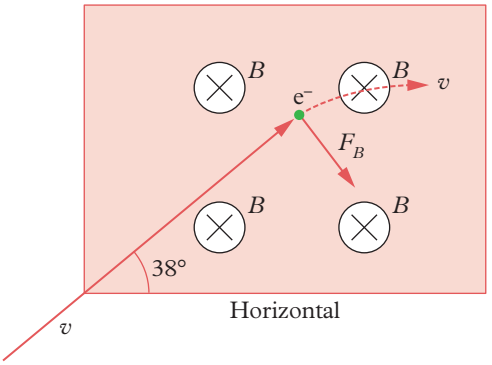


FIGURE 19 The forces on the electron in Figure 18

Worked example 7.6E

Direction of the force on a charged object in a field

An electron is fired at a speed of 2.2×10^8 m s^{-1} into a magnetic field of strength $B = 45\,\mu$T at an angle of 42° to the horizontal, as shown in Figure 20.

a **Calculate** the magnitude of the force on the electron. Give your answer to an appropriate number of significant figures. (4 marks)

b **Determine** the direction of the force. **Explain** your reasoning and show the direction of the force on the diagram. (2 marks)

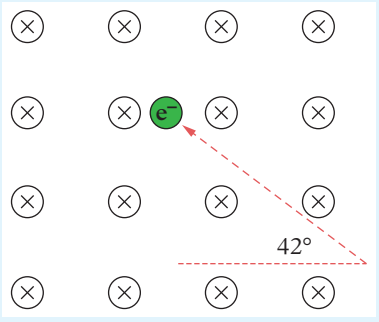
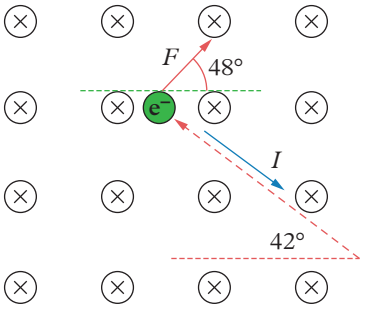


FIGURE 20 An electron fired into a magnetic field

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer. "Determine" means to ascertain after consideration. "Explain" means to describe in more detail. This question is worth 4 marks and 2 marks, so it requires you to show many statements, calculations or cognitions.
Step 2: Gather the relevant information from the question.	$q_e = 1.6 \times 10^{-19}$ C, $v = 2.2 \times 10^8$ m s^{-1} $B = 45\,\mu\text{T} = 4.5 \times 10^{-5}$ T Angle of velocity vector = 42° to horizontal
Step 3: Determine the angle the electron's path makes with the field. Note that the angle shown (42°) is irrelevant. (1 mark for "Recognises the value of θ and eliminates $\sin\theta$ from the relationship")	**a** The path of the electron is in a plane at right angles to the field direction, hence $\theta = 90°$ and $\sin\theta = 1$. (1 mark)
Step 4: Select the correct equation. (1 mark for "Recognises the scenario relates to motion in a magnetic field")	$F = qvB\sin\theta$ $= qvB$ (because $\sin 90° = 1$) (1 mark)
Step 5: Substitute the known values into the formula, including any unit conversions. The magnitude of the charge on 1 electron, q_e, is 1.6×10^{-19} C. (1 mark for "Determines the magnitude of the force")	$F = 1.6 \times 10^{-19} \times 2.2 \times 10^8 \times 45 \times 10^{-6}$ $= 1.58 \times 10^{-15}$ (1 mark)
Step 6: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	$F = 1.6 \times 10^{-15}$ N (1 mark)
Step 7: Determine the direction of the force using the appropriate rule (such as Fleming's left-hand rule). (1 mark for "Identifies the direction of the force")	**b** Using Fleming's left-hand rule, B (index finger into page), I (middle finger downwards opposite the path of the electron), F is shown by the thumb. (1 mark)

Think	Do
Step 8: State the direction of the force on the electron. For this particular moment in its travel, you could describe it as 48° above horizontal and to the right, as shown in the diagram. In general, it would be enough to say the force on the electron is at right angles to the direction of travel (the velocity vector) and to the right. (1 mark for "Provides correct answer") Note that the question asks for the direction of the force, not the direction of the path of the electron.	F (thumb) is upwards towards the right, at right angles to the path of the electron, as shown in the diagram. (1 mark for direction of force) 

Your turn

An alpha particle, ${}^{4}_{2}\mathrm{He}^{2+}$, is fired at a speed of $1.8 \times 10^{7}\,\mathrm{m\,s^{-1}}$ into a magnetic field B of magnitude 345 μT and at an angle of 35° to the horizontal, as shown in Figure 21.

a **Calculate** the magnitude of the force on the alpha particle. Give your answer to an appropriate number of significant figures. (4 marks)

b **Determine** the direction of the force on the alpha particle and show it on the diagram. (1 mark)

c **Sketch** the main parts of Figure 21 and show the path of the alpha particle in the field. (1 mark)

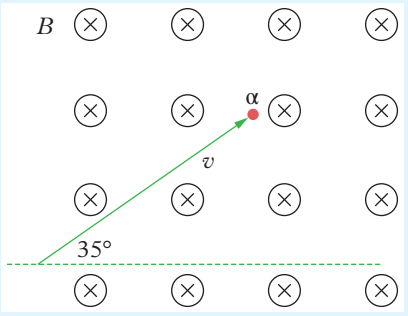


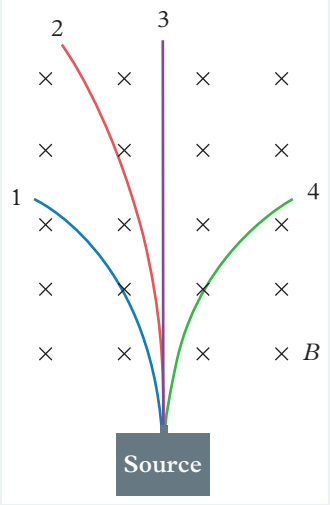
FIGURE 21 An alpha particle fired into a magnetic field

Check your learning 7.6

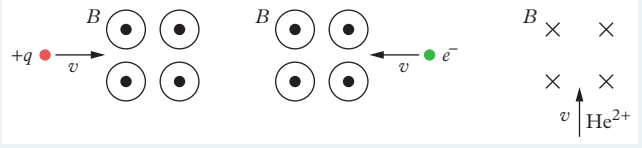
Check your learning 7.6: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Identify** the radiation labelled 1 to 4 in the diagram as alpha, beta-positive, beta-negative and gamma. (1 mark)

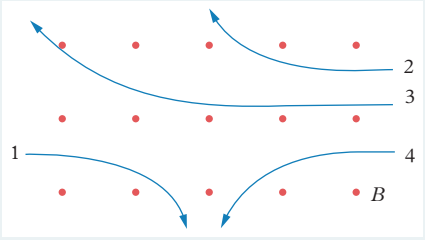


2 **Sketch** the paths of the charged particles as they pass through the fields shown in the diagrams. (3 marks)

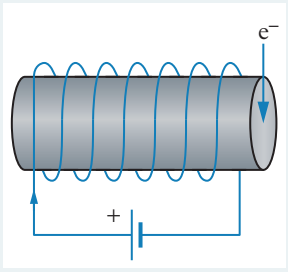


Analytical processes

3 Four charged particles are fired into a magnetic field as shown in the diagram. **Determine** which one is likely to be an electron if particle 1 is a positron. (1 mark)

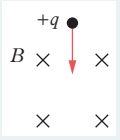


4 An electron passes by the end of a solenoid as shown in the diagram. **Determine** the path of the electron as it passes by. (1 mark)



Knowledge utilisation

5 A +5.0 μC charge moves at a speed of 1,600 m s^{-1} in a magnetic field of strength 2.0 T, as shown in the diagram. **Determine** the magnitude and direction of the force acting on the charge. (3 marks)

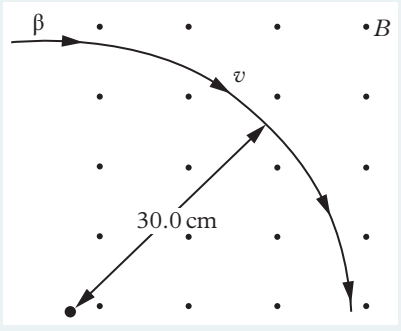


6 A beta particle, β, with a mass of 9.109×10^{-31} kg, enters a magnetic field of strength 5.50 mT at right angles, as shown in the diagram. It follows a circular path with a radius of 30.0 cm. If the beta particle carries one elementary charge ($1q_e$)

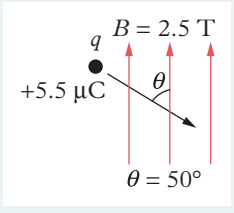
a **calculate** the speed of the beta particle (2 s.f.) (3 marks)

b **describe** the speed of the beta particle as a fraction of the speed of light (1 mark)

c **determine** whether the particle is a β^+ or a β^- particle. **Explain** your reasoning. (2 marks)



7 A +5.5 μC charge moves at a speed of 5,500 m s^{-1} in a magnetic field of strength 2.5 T at an angle of 50° to the field, as shown in the diagram. **Determine** the magnitude and direction of the force acting on the charge. (2 marks)



Lesson 7.7
Forces on current-carrying wires

Key ideas

- A current-carrying wire placed in a magnetic field will experience a force due to the interaction with the field.
- The direction of the force on a current-carrying wire in a magnetic field can be determined using Fleming's left-hand rule.
- Problems on the magnitude of the force on a current-carrying wire in a magnetic field can be solved using $F = BIL \sin\theta$.

Learning intentions and success criteria

What forces act on current-carrying wires in a magnetic field?

We have looked at the motion of charged particles in a magnetic field. Imagine now that these charges are constrained in a copper wire. We can find a second definition of magnetic field strength from simply considering the positive charges entering the field as being contained in a wire.

Determining the magnitude of the force

The formula can easily be developed from the $F = qvB\sin\theta$ formula. Let the length of the wire in the magnetic field be L, and the time taken for the charge to move this distance be t. Then the velocity, v, of the charge is L divided by t. If we substitute this into $F = qvB\sin\theta$:

$$F = qvB\sin\theta = \frac{qLB\sin\theta}{t}$$

then we can replace $\frac{q}{t}$ by I (the current in the wire):

$$F = BIL\sin\theta$$

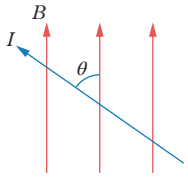


FIGURE 1 The orientation of a wire in a magnetic field

Magnetic field strength, B, can therefore also be defined in terms of force, length, current, velocity and angle: one tesla (1 T) is the magnetic field strength generating one newton (N) of force per ampere (A) of current per metre (m) of conductor.

The angle θ represents the orientation of the wire with the field. This is similar to the way we interpreted the direction of motion of charged particles in a magnetic field.

In Figure 1, the field is represented by the symbol B and the wire by the symbol I. The arrow indicates the direction of current in the wire. The wire is at a general angle of θ to the field, heading to the left.

Study tip

The direction of the current is the direction of flow of positive charge in the wire. When referring to motion of electrons, we call that "electron flow", not current.

Determining the direction of the force

As discussed earlier, Fleming's left-hand rule can be used to determine the direction of the force on charged particles moving in a magnetic field. The rule can also be used for charges in a wire. To do this, you simply use your middle finger to point in the direction of the current in the wire (Figure 2). If you point your index finger in the direction of the field, your thumb will show the direction of the force on the wire.

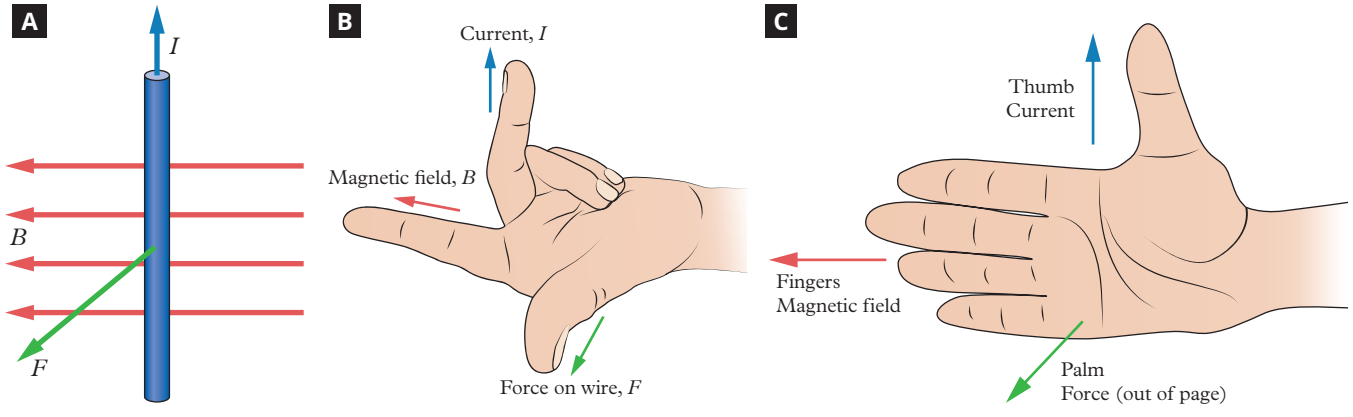


FIGURE 2 (A) and (B) Fleming's left-hand rule used to determine the direction of the force on a current-carrying wire; (C) right-hand palm rule

Worked example 7.7A

Magnitude and direction of the force on a current-carrying wire in a magnetic field

A 3.0 A current flows in a wire of length 1.5 m in a magnetic field of strength 5.0×10^{-5} T. **Determine** the magnitude and direction of the force on the wire, when

a the wire makes an angle of 55° to the field, as shown in Figure 3A (2 marks)

b the wire is at 90° to the field, as shown in Figure 3B. (2 marks)

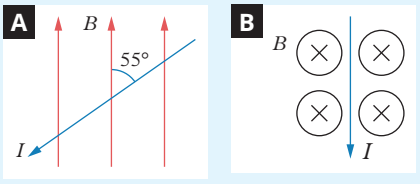
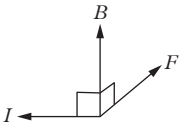
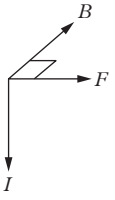


FIGURE 3 Current-carrying wire in magnetic field at (A) an angle of 55° to the field and (B) at 90° to the field

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$L = 1.5\,\text{m}$, $I = 3.0\,\text{A}$, $B = 5.0 \times 10^{-5}\,\text{T}$ For part **a**, $\theta = 55°$ For part **b**, $\theta = 90°$ $F = BIL\sin\theta$
Step 3: Substitute the known values into the formula and solve for F. (1 mark for "Demonstrates correct substitution")	**a** $F = BIL\sin\theta$ $= 5 \times 10^{-5} \times 3.0 \times 1.5 \times \sin 55°$ (1 mark) $= 1.8 \times 10^{-4}\,\text{N}$
Step 4: Use Fleming's left-hand rule to determine the direction. It can help to draw a diagram with F, B and I at right angles to each other. (1 mark for "Identifies the direction of the force") Note: you do not need to include a diagram. It is here to make the orientation of fingers and thumb in the hand rule clear. Step 5: Substitute the known values into the equation and solve for F. (1 mark for "Demonstrates correct substitution")	Using Fleming's left-hand rule, B (index finger upwards), I (middle finger downwards to the left, in the direction of the current), and F (thumb into the page). The force is into the page. (1 mark)  **b** $F = BIL\sin\theta$ $= 5 \times 10^{-5} \times 3.0 \times 1.5 \times \sin 90°$ (1 mark) $= 2.25 \times 10^{-4}\,N$
Step 6: Use Fleming's left-hand rule to determine the direction. It can help to draw a diagram with F, B and I at right angles to each other. (1 mark for "Identifies the direction of the force") Note: you do not need to include a diagram in your answer unless one is specifically asked for.	 Using Fleming's left-hand rule, B (index finger into the page), I (middle finger downwards, in the direction of the current), and F (thumb to the right). The force is to the right. (1 mark)

Your turn

A 5.0 A current flows in a wire of length 2.5 m in a magnetic field of strength 50 mT. **Determine** the magnitude and direction of the force on the wire when

a the wire makes an angle of 65° to the field, as shown in Figure 4A

b the wire is at 90° to the field, as shown in Figure 4B.

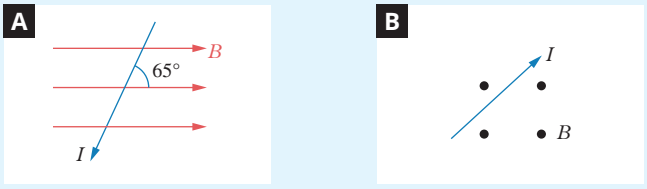


FIGURE 4 Current-carrying wire in magnetic field at (A) an angle of 65° to the field and (B) an angle of 90° to the field

What are some applications of magnetic fields?

Loudspeakers

A moving-coil loudspeaker, as is commonly found in small radios, headphones or home stereo systems, is designed to change electrical signals from the output of an amplifier back into sound waves. The speaker relies on the force produced by a current in a conductor within a magnetic field. A movable coil attached to a strengthened paper cone is placed over the central shaft of a permanent magnet. The magnetic field cuts the wire at right angles towards the centre of the inner pole, so that the movement of the coil (the "voice coil") produced will be backwards and forwards, as shown in Figure 5. The amplifier supplies currents of variable frequency into the loudspeaker, and as these currents flow through the speaker voice coil, it is forced to vibrate at the same rate as the current. The paper cone also vibrates backwards and forwards, moving the air and producing sound waves that match the amplitude and frequency of the original electric-current signals.

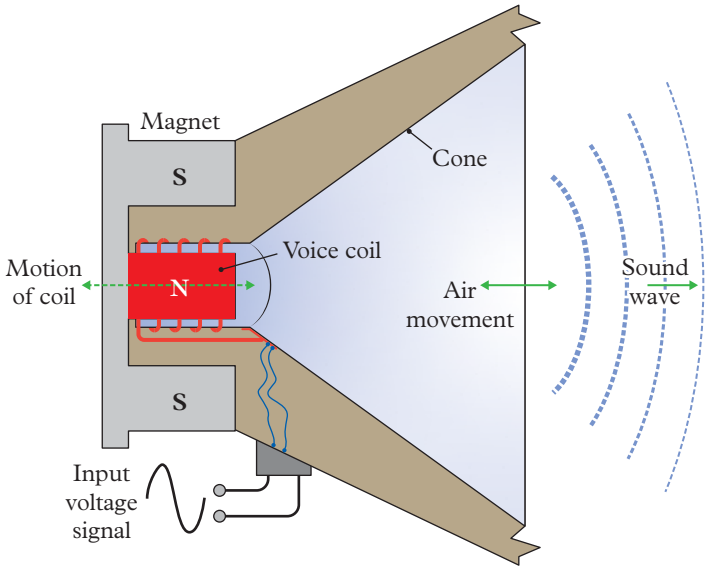


FIGURE 5 The interaction of an electric current and a magnetic field causes the motion of air particles.

Motors

Motors also work on the principle that a current-carrying wire or coil experiences a force when it is in a magnetic field. In the simplest case, the current in a coil is reversed every half rotation so there is a force that keeps the coil spinning in the same direction.

In Figure 6, the current is moving in a clockwise direction around the coil. If you apply Fleming's left-hand rule, you can see that the force on the left-hand side of the coil (labelled 1) is upwards and on the right-hand side (labelled 2) is downwards. This provides a turning force that rotates the coil. When the coil has moved through 180° and side 1 of the coil is near the north pole of the magnet, it will experience an upwards force, which would move the coil back in an anticlockwise direction. However, the direction of the current can be reversed by a "split-ring", which makes the coil keep rotating in the same direction. It is a clever idea and was first put into practice in 1837.

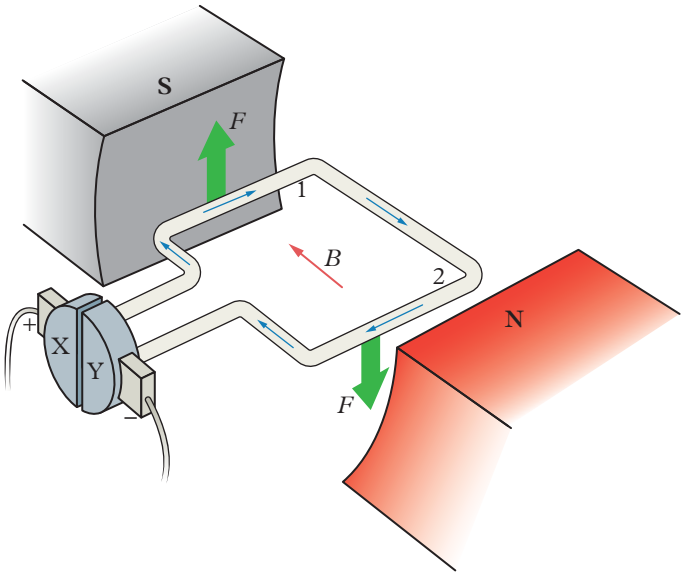


FIGURE 6 A simple DC motor

Study tip

When a wire is parallel to the field, the angle to the field is zero, so the force is zero. When the wire is at 90° to the field, the force is at a maximum.

Challenge

Magnetic crystals in tuna

In 1986, scientists discovered that the yellowfin tuna has 10 million magnetic crystals in its skull. **Propose** how you could test whether tuna use these crystals to aid navigation, as has been suggested. (2 marks)

Check your learning 7.7

Check your learning 7.7: Complete these questions online or in your workbook.

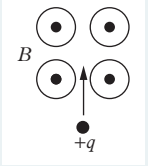
Retrieval and comprehension

1 **Explain** what it means when you say that "force, field and motion of charge are all perpendicular to each other". (1 mark)

Analytical processes

2 **Analyse** the diagram to establish the direction of the charged particle q after it enters the magnetic field in each case. (1 mark each)

a



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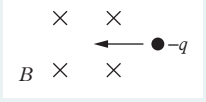

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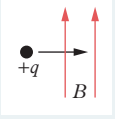
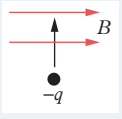
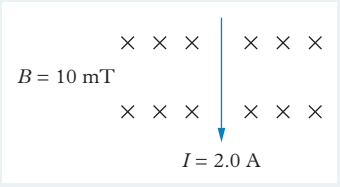


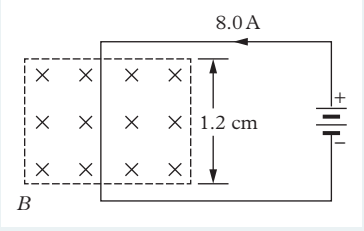
d



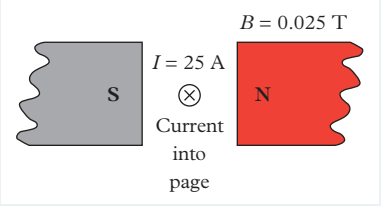
3 **Determine** the magnitude and direction of the force on a current-carrying wire placed in a uniform magnetic field, as shown in the diagram. The length of wire in the field is 50.0 cm. (3 marks)



4 **Determine** the magnitude and direction of the force on a current-carrying wire that passes perpendicularly through a magnetic field as shown in the diagram. The magnetic field strength is 1.5×10^{-3} T and the wire carries a current of 8.0 A. (2 marks)

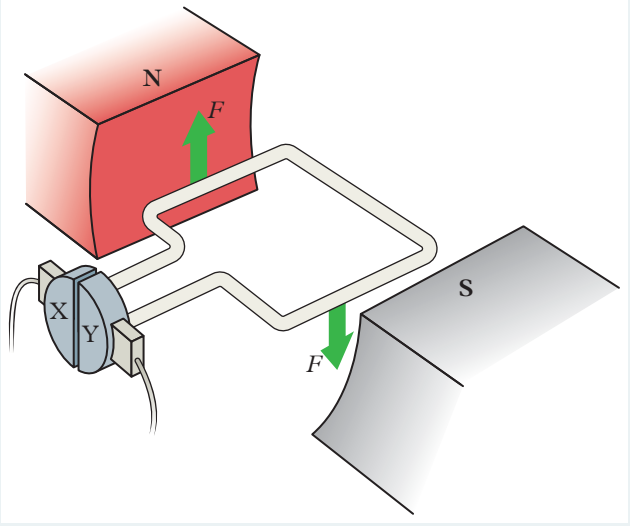


5 **Determine** the force on a conductor of length 8.5 cm that is placed between the poles of a large magnet, as shown in the diagram. The wire conductor carries a current of 25 A in the direction shown. (3 marks)



Knowledge utilisation

6 The diagram shows a motor consisting of a rotating coil between the poles of a permanent magnet. The forces acting on the coil are shown by the arrows labelled F on the sides of the coil. **Decide** whether terminal X or Y is the positive terminal and explain your reasoning. (3 marks)



7 Gaseous chlorine ions, all with a –1 charge, are injected at right angles into a 0.026 T magnetic field at a speed of 19,665 m s^{-1}. They trace out a circular path of radius 27.0 cm. **Determine** whether the ions are the ^{35}Cl nuclide or the ^{37}Cl nuclide. Note: 1 amu = 1.66×10^{-27} kg. (4 marks)

Practical

Lesson 7.8

Force acting on a conductor in a magnetic field

Learning intentions and success criteria

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

MODULE 7

Lesson 7.9
Review: Magnetic fields

Summary

7.1
- A magnetic field is a region of space in which magnetic effects are experienced.
- Magnetic fields are produced by moving electric charges.
- Magnetic fields around moving electric charges, electric currents and magnets can be depicted with directed line segments (arrows).

7.2
- Moving electric charges, such as the current flowing in a wire, have an associated magnetic field.
- The direction of the magnetic field around a current-carrying wire can be determined using Ampere's right-hand rule.
- Problems involving the strength of the magnetic field around a straight current-carrying wire can be solved using $B = \frac{\mu_0 I}{2\pi r}$.

7.3
- Practical: Strength of a magnet at various distances

7.4
- Practical: Strength of a magnetic field at various distances from a current-carrying wire

7.5
- A solenoid is a long straight coil of wire used to produce a uniform magnetic field.
- The magnetic field direction in a solenoid can be determined using the right-hand rule.
- Problems on the magnitude of magnetic fields in a solenoid can be solved using $B = \mu_0 nI$ and $n = \frac{N}{L}$.

7.6
- The direction of the force on a charged particle in a magnetic field can be determined using Fleming's left-hand rule or the right-hand slap (palm) rule.
- Problems on the magnitude of the force on a moving charge in a magnetic field can be solved using $F = qvB\sin\theta$.

7.7
- A current-carrying wire placed in a magnetic field will experience a force due to the interaction with the field.
- The direction of the force on a current-carrying wire in a magnetic field can be determined using Fleming's left-hand rule.
- Problems on the magnitude of the force on a current-carrying wire in a magnetic field can be solved using $F = BIL\sin\theta$.

7.8
- Practical: Force acting on a conductor in a magnetic field

Key formulas

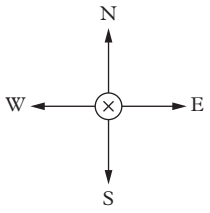
Force on a charged particle	$F = qvB\sin\theta$
Force on a wire	$F = BIL\sin\theta$
Field around a wire	$B = \frac{\mu_0 I}{2\pi r}$
Field inside a solenoid	$B = \mu_0 nI$
Magnetic constant	$\mu_0 = 4\pi \times 10^{-7}\,\text{T mA}^{-1}$

Revision questions 7.9A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 A straight wire carries a current into the page as shown in the diagram. What is the direction of the magnetic field at a point east of the wire?



A North **B** South
C West **D** East

2 A circular loop of wire is facing you and has an anticlockwise current running through it. What is the direction of the magnetic field inside the loop?

A North **B** South
C Into the page **D** Out of the page

3 A wire with mass m and length L has a current I flowing to the left of the page, as shown in the diagram.

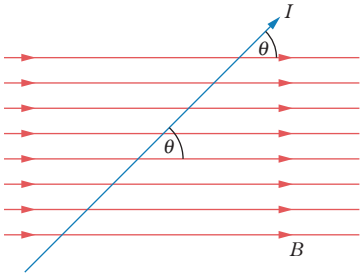
What would be the direction of a magnetic field that provides a magnetic force that could cancel out the gravitational force on the wire?

A Up **B** Down
C Out of the page **D** Into the page

4 If the magnitude of the magnetic field is B in question **3**, what must the magnitude of this field be to cancel out the gravitational force on the wire?

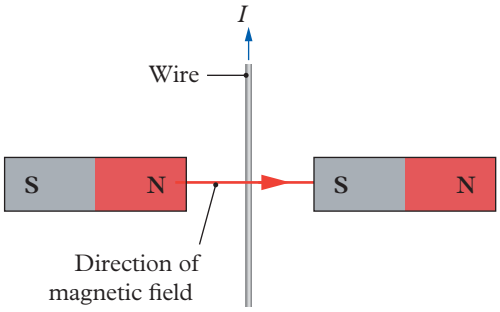
A $\frac{mg}{IL}$ **B** $\frac{m}{IL}$
C $\frac{IL}{mg}$ **D** $\frac{mgL}{I}$

5 The diagram shows a current-carrying wire in a magnetic field. Which one of the options best describes the direction of the force on the wire?



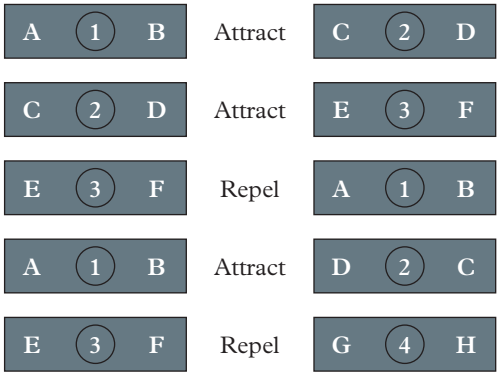
A Out of the page
B Into the page
C Perpendicular to angle θ
D At angle θ to the direction of the field

6 Analyse the diagram and determine in which direction the force on the wire acts.



A Into the page
B Out of page
C Left-hand side
D Right-hand side

7 A student investigated the pattern of attraction between four different bars of metal, shown in the diagram. One of the materials is not magnetised. **Analyse** the diagram and determine which bar is not magnetised.

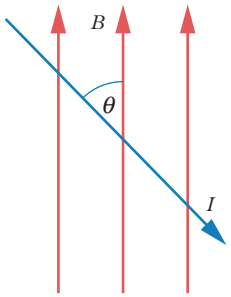


A Bar 1 **B** Bar 2
C Bar 3 **D** Bar 4

8 **Determine** the number of coils on a 150 cm solenoid that has a current of 10.0 A passing through it with a magnetic field strength of 0.00567 T.

A 400 **B** 450
C 500 **D** 550

9 A current-carrying wire is placed in a magnetic field, as shown in the diagram. Determine in which direction the force is acting on the wire.



A Perpendicular to angle θ
B At angle θ to the direction of the field
C Into the page
D Out of the page

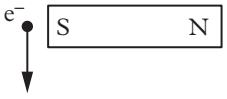
10 Determine the direction of the force acting on a moving positive charge that is fired from the right-hand side into a magnetic field, B, orientated into the page.

A Up the page
B Down the page
C To the right-hand side
D To the left-hand side

11 Three parallel wires carry equal currents in directions shown in the diagram (X and Z are into the page). In what direction does the magnetic force act on Z?

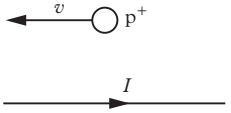
A To the right
B To the left
C Down the page
D Up the page

12 An electron passes the south pole of a bar magnet, as shown in the diagram. What is the direction of the magnetic force on the electron?



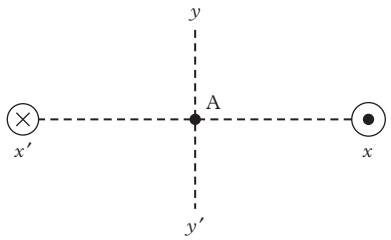
A Into the page
B Out of the page
C To the left
D To the right

13 A horizontal wire is carrying current I to the east and a proton is moving with a velocity v to the west, as shown in the diagram. What is the direction of the magnetic force acting on the proton due to the current-carrying wire?



A North
B South
C West
D East

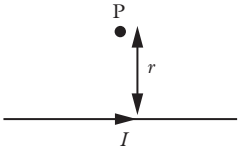
14 Two wires are in parallel with equal currents flowing in opposite directions, as shown in the diagram.



At what location would the magnetic fields cancel out?

A Anywhere on the line xx'
B Anywhere on the line yy'
C Anywhere on the line Ax'
D At no points on the diagram

15 A wire carries a current I to the right as shown in the diagram. Consider a point P at a distance r directly above the wire.



What would be the change to the magnetic field at P if the current I were doubled and the distance r were doubled?

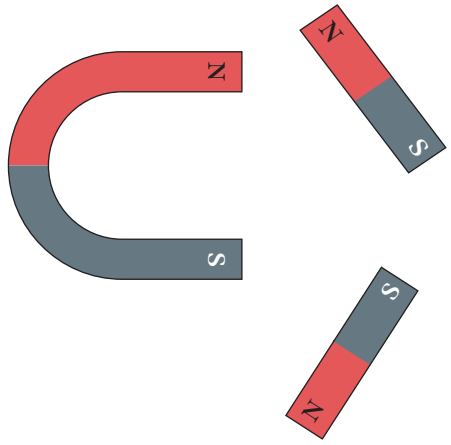
A There would be no change.
B It would double.
C It would halve.
D It would quadruple (×4).

Review questions 7.9B Short response

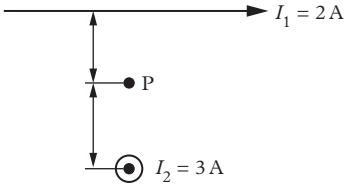
Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 **Sketch** the magnetic field lines for the U-shaped magnet and permanent bar magnets shown in the diagram. (3 marks)



17 Two wires are placed at right angles to each other and carry the currents shown in the diagram. **Calculate** the magnitude (only) of the magnetic field strength at point P vertically midway between them (2 s.f.) (3 marks)



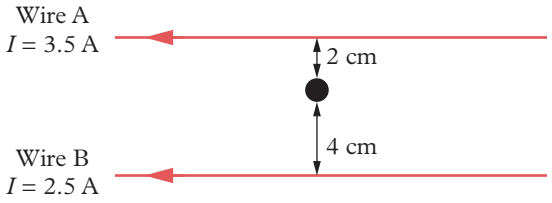
18 A straight wire in an electrical circuit carries a direct current I. The resulting magnetic field at a perpendicular distance of 18 mm from this wire is measured as 1.2×10^{-4} T. **Calculate** the current to the nearest whole number. (2 marks)

19 **Calculate** the field strength inside a solenoid made up of 800 turns of wire on a 20 cm length of plastic tube if the wire carries a current of 700 mA. (2 marks)

20 An electron ($q_e = -1.6 \times 10^{-19}$ C) is fired at right angles into a 1.2 T uniform magnetic field at 3×10^7 m s^{-1}. **Calculate** the magnitude of the force acting on it. (2 marks)

Analytical processes

21 **Determine** the magnetic field strength (magnitude and direction) at the point shown between wire A and wire B in the diagram, given wire A carries a current of 3.5 A and wire B carries a current of 2.5 A. Show your working and give your answer to 2 significant figures. (5 marks)



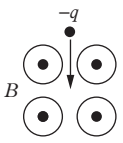
22 A proton of mass 1.67×10^{-27} kg and charge $+1.6 \times 10^{-19}$ C enters a magnetic field of strength 3.0×10^{-2} T at right angles to the field and with a velocity of 2.5×10^5 m s^{-1}.

a **Determine** the magnitude of the force on the proton (2 s.f.). (2 marks)

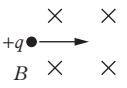
b **Determine** the magnitude of its acceleration (2 s.f.). (2 marks)

23 **Determine** the direction of an electric charge after it enters each of the magnetic fields shown in the diagram. (1 mark each)

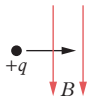
a



b



c

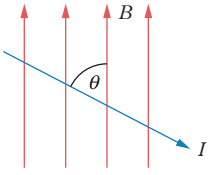


d

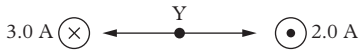
24 A wire of length 1.4 m is placed entirely in a magnetic field of strength 0.85 T and carries a current of 2.37 A, as shown in the diagram. The wire experiences a force of 1.19 N.

a **Determine** the angle θ it must make with the field. (2 marks)

b **Determine** the direction of the force. (1 mark)



25 Two parallel conductors 40.0 mm apart carry currents of 3.0 A (left wire) and 2.0 A (right wire), as shown in the diagram.

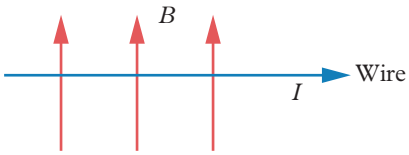


a **Calculate** the magnitude and direction of the magnetic field at point Y midway between the wires. (4 marks)

b **Determine** where, other than infinity, the magnetic field strength would be zero. (3 marks)

Knowledge utilisation

26 A current-carrying wire is placed perpendicular to a magnetic field, as shown in the diagram.

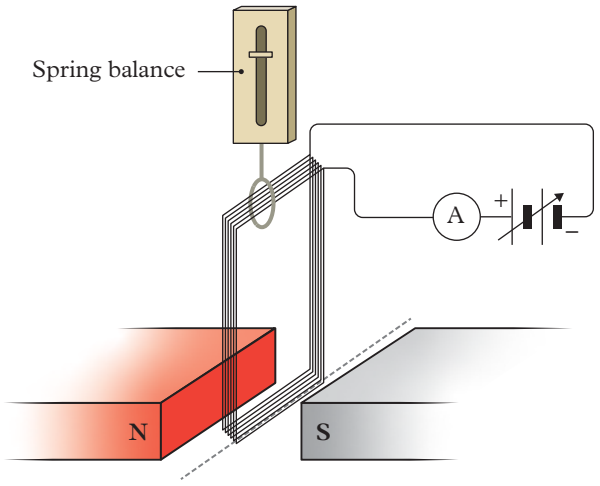


a **Calculate** the magnitude and direction of the force on the wire (2 s.f.). The magnetic field strength is 2.0×10^{-3} T and the 50.0 cm wire is carrying a current of 12.0 A. (3 marks)

b **Determine** the order of magnitude of the force calculated in **a**.

c **Propose** what would happen to the direction of the force if the current were reversed. **Justify** your answer. (2 marks)

27 A coil of wire is suspended from a spring balance between the poles of two magnets. The rectangular coil is 80 cm high and 10 cm wide and has 100 turns of wire. In an experiment, the spring balance readings were recorded for different currents. The apparatus is shown in the diagram and the results are in the table.



Current (A)	Force (N)
0.5	3.5
1.5	5.0
2.0	5.6
3.0	6.8
3.5	7.5
4.0	8.3
5.0	9.5

a **Construct** a graph of force (vertical axis) versus current (horizontal axis). (2 marks)

b **Determine** the weight of the coil. (1 mark)

c **Calculate** the magnetic field strength. (2 marks)

d **Determine** the current direction as clockwise or anticlockwise. (1 mark)

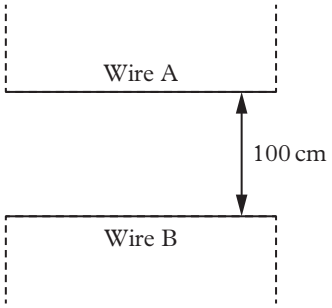
e The current is adjusted so that the balance reads zero. **Determine** the current that now flows in the coil and the direction in which it flows. (2 marks)

28 A solenoid with 500 turns and length of 20.0 cm carries a current of 2.0 A.

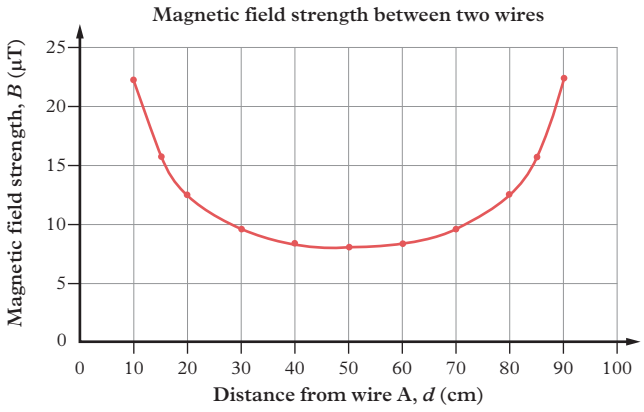
a **Calculate** the magnetic field strength at the centre of the solenoid. (2 marks)

b **Describe** how halving the number of turns but keeping the length constant would impact the magnetic field strength of the solenoid. **Justify** your response. (2 marks)

29 Two wires, A and B, lie parallel and horizontal on a bench as shown in the diagram. They are 100 cm apart and each carries the same magnitude current.



The net magnetic field due to the currents is measured at positions every 10 cm from wire A across to wire B. The results are plotted in the graph.

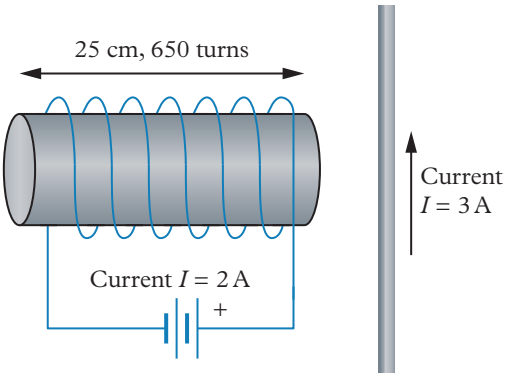


a **Determine** whether the currents in the wires are in the same direction or in opposite directions. **Explain** your answer. (2 marks)

b **Calculate** the magnitude of the current in the wires, which is assumed to be the same. (2 marks)

c An electron is fired into the space between the wires at right angles to the magnetic field. It experiences an initial force of 8.8×10^{-16} N. **Determine** the velocity of the electron (2 s.f.). (2 marks)

30 A 25 cm long solenoid has 650 turns of wire and carries a current of 2 A, as shown in the diagram. A copper wire carrying a current of 3 A up the page is placed near the end of the solenoid and at right angles to its axis as shown.

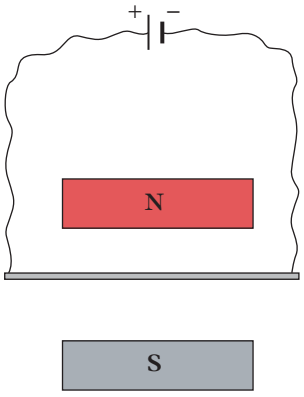


a **Determine** the magnetic field strength at the end of the coil (magnitude and direction). (2 marks)

b **Determine** the direction of the force on the wire due to the solenoid's magnetic field. (1 mark)

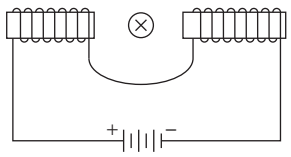
c Assume only a 5 cm segment of the wire is in the solenoid's field. **Determine** the magnitude of the force acting on the wire. (2 marks)

31 A wire is connected to a battery and allowed to hang between the poles of a horseshoe magnet, as shown in the diagram. **Predict** the motion, if any, of the wire. **Explain** your answer. (2 marks)



32 The diagrams show a current-carrying wire near the poles of an electromagnet. For each diagram, **predict** the direction of the induced force acting on the conductor if the direction of current flow is determined by the battery. (1 mark each)

a



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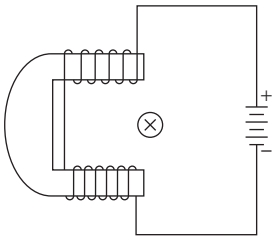
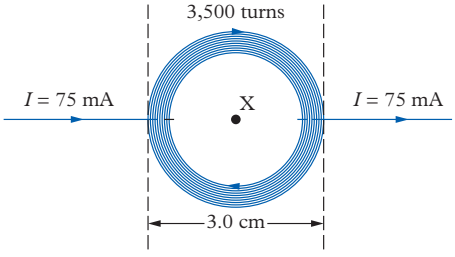
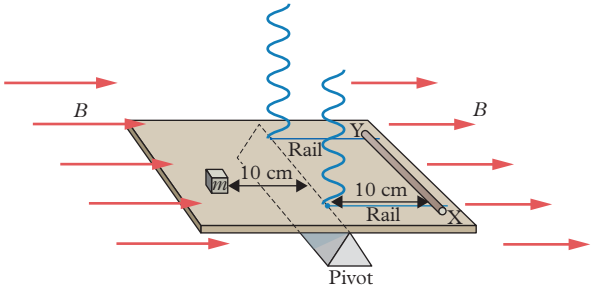


33 The diagram shows a 3,500 turn circular solenoid of diameter 3.0 cm and length 70.0 cm carrying an input current of 75 mA flowing east. The current circulates clockwise as shown. **Predict** the field strength (magnitude and direction) at point X, the centre of the loop. (3 marks)



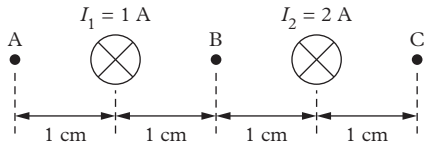
34 A metal rod XY is 5.0 cm long. It lies on two metal rails connected to a DC supply. The rod and rails are balanced on a flat insulator base in a magnetic field of strength 0.20 T. A current is then passed through the rod causing a downwards movement. A mass, m, of 1.0 g is needed to restore the system to a level position, as shown in the diagram. **Determine** the direction and magnitude of the current in rod XY. Use $g = 9.8\,\text{m s}^{-2}$. (3 marks)



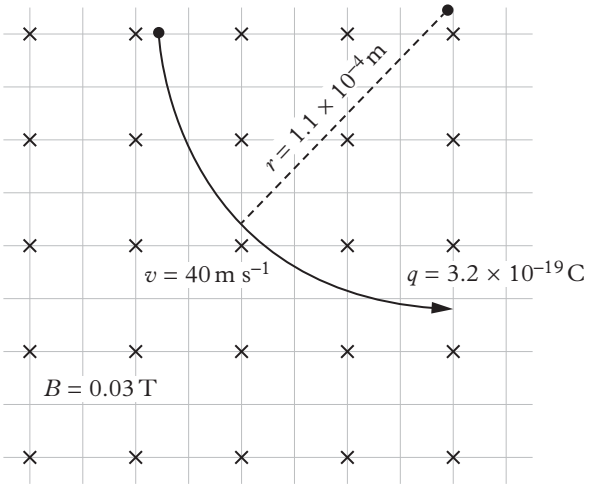
35 Two parallel wires carry the same current and in the same direction, as shown in the diagram.

a **Calculate** the magnetic field strength at positions A, B and C. (4 marks)

b **Determine** the position, if any, where the field strength would be zero. (2 marks)



36 The diagram shows a moving charged particle in a magnetic field.



Determine the mass of the charged particle using the information provided in in the diagram (2 s.f.). (3 marks)

37 An electron passes through the Earth's magnetic field of 35 μT with a velocity of $2.95 \times 10^{8}\,\text{m s}^{-1}$ at an angle of θ to the direction of the field. It experiences a force of 5.5×10^{-17} N due to the magnetic field. **Determine** the angle θ (2 s.f.). (2 marks)

Data drill

Force on a wire in a magnetic field

A student set up the apparatus shown in Figure 1 to address the following research question:

What is the relationship between the force on a wire in a constant uniform magnetic field and the current through it, when the current is varied?

A length of wire held rigidly in a cork and clamped was placed in a magnetic field between two button magnets, as shown in Figure 1. The wire is at 90° to the field and has an effective length of 15 mm.

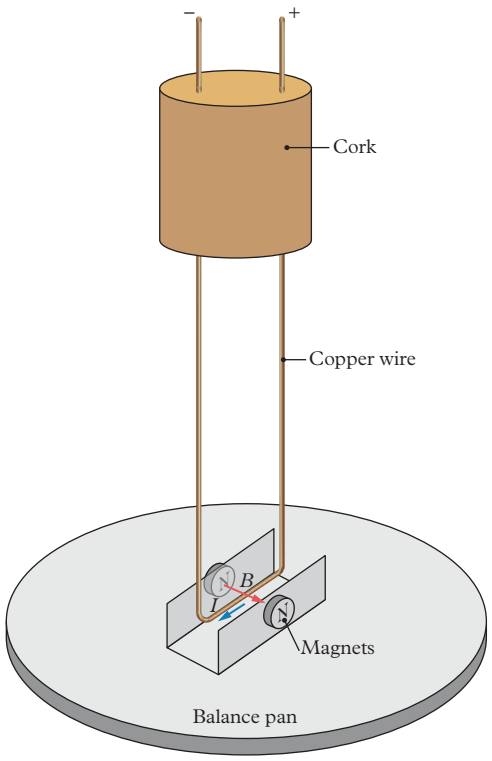


FIGURE 1 Apparatus used for experiment

The electronic balance was zeroed. As the current was increased, a scale reading was taken, and this was converted to a force (weight) in newton. The results were plotted in the graph shown in Figure 2.

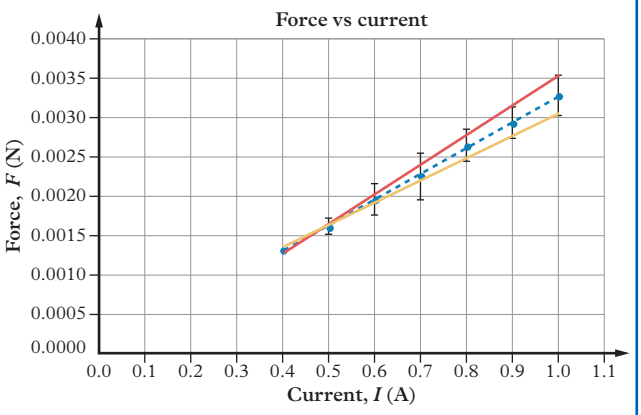


FIGURE 2 A plot of the raw data

Equation for the linear (dotted) trendline:

$y = 0.0033x - 0.0001$

Equation for the maximum linear trendline:

$y = 0.0037x - 0.0004$

Equation for the minimum trendline:

$y = 0.0029x + 0.0002$

Apply understanding

1 **Calculate** the scale reading in grams if the force was calculated to be 0.0013 N. (1 mark)

2 **Calculate** the percentage uncertainty for the force acting on the wire when the current, I, is 1.0 A. (2 marks)

Analyse data

3 **Identify** a mathematical relationship between the force acting on the conductor and the current passing through the conductor, showing the absolute uncertainty in the gradient and the absolute uncertainty in the y-intercept. (3 marks)

Interpret evidence

4 **Draw a conclusion** that expresses the magnitude of the magnetic field experienced by the wire between the two magnets and the percentage uncertainty. (2 marks)

5 **Draw a conclusion** by judging whether this experiment can be considered accurate, given the following information: the magnetic field strength has an accepted value of 0.22 T. (2 marks)

Module 7 checklist: Magnetic fields

MODULE 8

Electromagnetic induction and radiation

Introduction

One of the greatest achievements of physics was the discovery of electricity generation in 1830. The principles that were discovered then remain the basis for most of our electricity production today. The idea is quite simple: rotate a coil of wire in a magnetic field and the kinetic energy of the coil is transformed into electrical energy as a current is induced. You may have seen this principle in those little torches that you turn by hand and the bulb lights up or heard about hydroelectricity generators where falling water spins a magnet in a coil. You put kinetic energy in and get electrical energy out.

It really comes down to what Faraday discovered in 1830 – cut a magnetic field with a wire and you get an electric current. However, it was Faraday's idea of "lines of magnetic flux" that was so crucial to the successful development of motors and generators, which were to dominate the nineteenth century. Motors, generators and transformers are still used in household devices and large-scale power production and transmission today.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to electromagnetic induction and radiation before you start.

Subject matter

Science understanding

→ Describe the concepts of magnetic flux, magnetic flux density, electromagnetic induction, electromotive force (EMF), Faraday's Law and Lenz's Law.

→ Solve problems involving the magnetic flux in an electric current-carrying loop using $\phi = BA\cos\theta$.

→ Describe the process of inducing an EMF across a moving conductor in a magnetic field.

→ Explain how Lenz's Law is consistent with the principle of conservation of energy.

→ Explain how transformers work in terms of Faraday's Law and electromagnetic induction.

→ Solve problems involving electromagnetic induction using $\text{emf} = -\frac{N\Delta(BA_{\perp})}{\Delta t}$, $\text{emf} = -N\frac{\Delta\phi}{\Delta t}$, $I_p V_p = I_s V_s$ and $\frac{V_p}{V_s} = \frac{N_p}{N_s}$

→ Describe the concept of an electromagnetic wave.

→ Explain the relationship between oscillating electric charges and electromagnetic waves.

Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Charles-Augustin de Coulomb, Michael Faraday, Emil Lenz, Mary Somerville and James Clerk Maxwell who furthered our understanding of electromagnetism.

→ Explore how scientific knowledge has allowed the development of new methods for renewable energy production.

→ Consider the scientific evidence concerning the risks of electromagnetic phenomena and associated technologies (e.g. wi-fi and mobile phones) as reported in the media.

Science inquiry

→ Consider how electricity is made.

→ Investigate the induction of an electric current using a magnet and coil.

→ Investigate the induced EMF from an AC generator.

Practicals

oxforddigital **These lessons are available on Oxford Digital.**

Lesson 8.3 Induction of an electric current using a magnet and coil

Lesson 8.6 Transformers: induced EMF from an AC generator

Lesson 8.1
Magnetic flux

Key ideas

- → The characteristics of a magnetic field can be described in terms of magnetic flux and magnetic flux density.
- → Problems involving the magnetic flux in an electric current–carrying loop can be solved using $\phi = BA\cos\theta$.

What is magnetic flux?

Learning intentions and success criteria

When Michael Faraday developed his model for magnetic fields around current-carrying wires, he came up with the idea of lines of magnetic flux. Faraday thought of the wire as giving out a flow of magnetic force.

The term **magnetic flux** has stuck, even though it is now used as a visual representation of the magnetic field. This has the symbol ϕ (the Greek letter "phi", which rhymes with pie). You may recall from Module 7 that magnetic flux and magnetic fields are similar, and flux is a measure of the field lines that pass through a given area.

Magnetic field strength can be defined in terms of flux lines and how close they are together, which is called the **magnetic flux density**. This is the magnetic flux per unit area or the magnetic field strength.

magnetic flux
a measurement of the total magnetic field that passes through a given area; a measure of the number of magnetic field lines passing through the given area (symbol: ϕ; SI unit: weber; unit symbol: Wb)

magnetic flux density
the strength of a magnetic field or the number of magnetic field lines per unit area (symbol: B; SI unit: weber per square metre; unit symbol: Wb m^{-2} or T)

How is flux perpendicular to the field calculated?

Magnetic flux is measured in the unit weber (pronounced "vey-ber"), after the German physicist Wilhelm Weber (1804–91). Weber has the symbol Wb and is equal to the amount of flux flowing through an area of 1 m^2 in a magnetic field of strength 1 T.

$$\text{magnetic field strength} = \text{flux density}$$
$$1\,\text{T} = 1\,\text{Wb m}^{-2}$$

Note that we say that flux "flows" through an area, or flux "threads" an area such as a loop.

If a single arrow were used to represent 1 line of flux (1 Wb), then 4 arrows flowing through an area of 1 m^2 would be a flux density of 4 Wb m^{-2} or 4 T (Figure 1). The quantities "flux density" and "field strength" can be used interchangeably. The formula is:

$$\text{magnetic field strength} = \frac{\text{magnetic flux}}{\text{area}}$$
$$B = \frac{\phi}{A}$$

This is also stated as $\phi = BA$. When the loop is perpendicular to the magnetic flux we can specify the area of the loop as $A_\perp$ and the formula above could be written as $\phi = BA_\perp$.

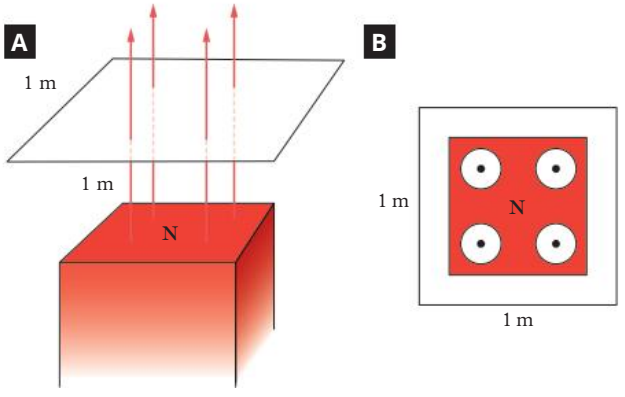


FIGURE 1 (A) Four lines of flux through an area of 1 m^2 represents a magnetic field strength of 4 Wb m^{-2} or 4 T. (B) B = 4 T as seen from above with flux lines pointing towards you.

Study tip

In Module 7, magnetic field strength (tesla, T) is defined in terms of the force acting on a moving charge, either as a particle in a magnetic field, or as a current-carrying wire in a field. The new definition given here (magnetic field lines per unit area, or weber per square metre, Wb m^{-2}) is just another way of representing the same quantity. You may find it easier to picture magnetic field lines rather than forces.

The flux from the Earth's magnetic field

The strength of Earth's magnetic field is about 5.3×10^{-5} T. In Brisbane, Earth's magnetic field is angled at 57° to the horizontal. An area the size of the Brisbane Cricket Ground (the Gabba) would contain just one flux line, as shown in Figure 2.

It is unrealistic to represent 1 Wb by one line, so the lines are used to show the relative amount of flux flowing through an area.

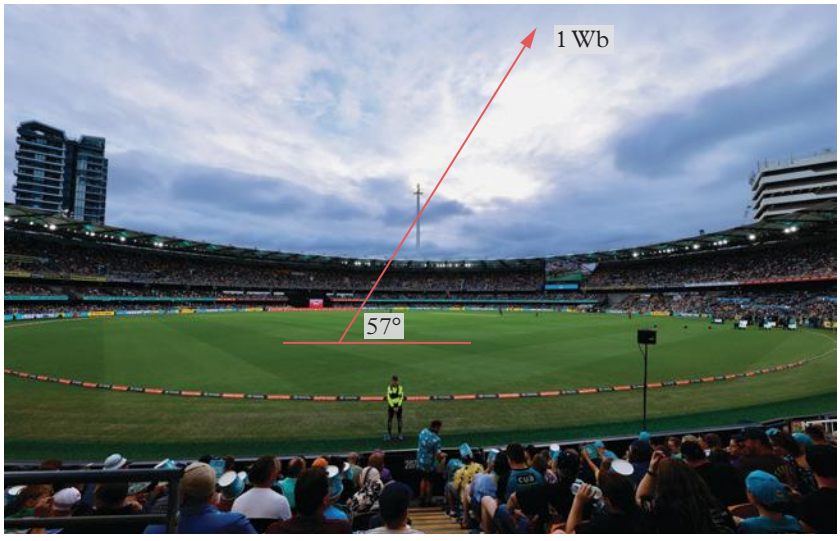


FIGURE 2 A typical sporting oval, like the Gabba shown here, has an area of 22,000 m^2, which encloses 1 Wb of flux due to Earth's magnetic field.

The characteristics of a loop

We have talked about flux through a "loop" but haven't described what a loop is yet. You probably think of a loop as something shaped like a circle, and this is correct. However, a loop can also be square, rectangular or oval. Simply speaking, a **loop** is a circuit where you start and end at the same point. Loops can be "open" or "closed" depending on whether they are a complete circuit or have a gap, as shown in Figure 3.

loop
a circuit, often a conducting wire, where you start and end at the same point; can be "open" or "closed"

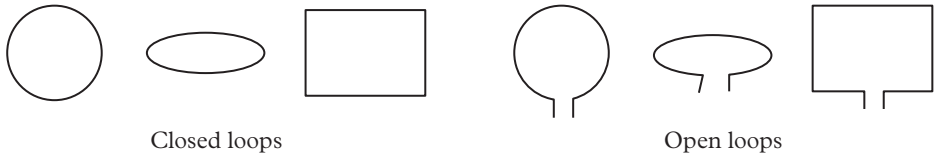


FIGURE 3 Two different types of loops

Current can flow in a closed loop, but there will be no EMF because there are no ends. You can have an EMF across the ends of an open loop, but no current can flow because of the gap. These are both types of conducting loop, and their properties are summarised in Table 1.

Loops can also be a region of space bounded by an imaginary line; for example, a magnetic field flowing through one square metre of a surface. You don't actually need anything to mark out the square meter – it is just imaginary.

TABLE 1 Properties of open and closed loops

Type of loop	Quantity that can be observed	Direction
Closed	Current	Clockwise or anticlockwise
Open	EMF	Positive to negative

Worked example 8.1A

Calculating flux through a loop perpendicular to the field

Calculate the flux threading a circular loop of diameter 24 cm with the plane of the loop at right angles to a field of strength 5.0 mT. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formulas.	Diameter, $d = 24\,\text{cm} = 0.24\,\text{m}$ $B = 5.0\,\text{mT}$ Area $= \pi r^2$, $\phi = BA$
Step 3: Calculate the area of the loop. (1 mark for "Calculates the area of the loop")	Radius, $r = \frac{d}{2} = \frac{0.24}{2} = 0.12\,\text{m}$ Area, $A = \pi r^2$ $= \pi \times 0.12^2$ $= 0.045\,\text{m}^2$ (1 mark)
Step 4: Substitute the known values into the formula and solve for flux, ϕ. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides correct answer")	$\phi = BA$ $= 5.0 \times 10^{-3} \times 0.045$ $= 2.3 \times 10^{-4}\,\text{Wb}$ (2 s.f.) (1 mark)

Your turn

A circular loop placed at right angles in a field of strength 130 mT encloses 1.839 Wb of flux. **Calculate** the radius of the loop. Express your answer in cm and give your answer to an appropriate number of significant figures. (3 marks)

How is flux at an angle to a field calculated?

When the plane of the loop is not at 90° to the field, fewer flux lines will thread the same size loop. When the plane of the loop is parallel to the field, for instance, no lines will thread the loop, so the flux in the loop must be zero. A formula is needed that takes the angle into account.

The area of the loop is a vector quantity and points in the direction perpendicular to its surface (as shown in Figure 4). Because of that, the cosine function is used. The product of two vectors such as B and A (the "dot" product) is given by the product of the two magnitudes multiplied by the cosine of the angle between them. Therefore, the formula for magnetic flux becomes:

$$\phi = BA\cos\theta$$

where ϕ is the magnetic flux, B is magnetic field strength, A is the area of the loop and θ is the angle that the perpendicular to the plane of the loop makes with a field.

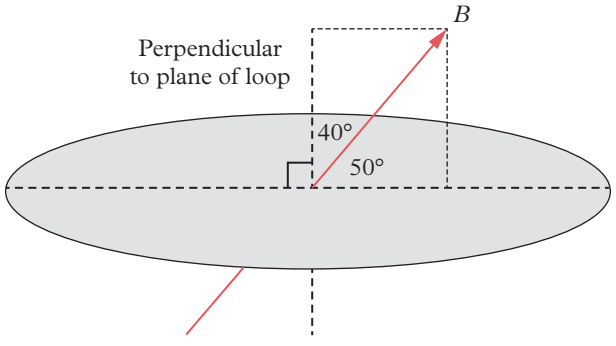


FIGURE 4 The field line, B, makes an angle of 50° to the plane of the loop, which is the same as 40° to the perpendicular to the loop. In this case $\theta = 40°$.

This is a tricky concept and might take a while to digest. The important part to remember is that the angle a loop makes with a field is the angle between the perpendicular to the loop and the field. That is, when the plane of the loop is at 90° to the field, the perpendicular is at an angle $\theta = 0°$ to the field.

In Figure 4, if the loop has an area of $1.5\,m^2$, and the field strength (flux density) is 2.0 T, then the amount of flux threading the loop will be:

$$\begin{aligned}\phi &= BA\cos\theta \\ &= 2.0 \times 1.5 \times \cos 40° \\ &= 2.3\text{Wb}\end{aligned}$$

Study tip

When completing questions on loops in magnetic fields, you should check the wording to see if it says the angle is to the "plane of the loop" or "perpendicular to the loop". You should be able to convert from one to the other. If θ is in the equation, use perpendicular.

Consider a loop of $1.0\,m^2$ with four lines (4 Wb) of flux threading it at right angles (Figure 5). This is a magnetic field strength of 4.0 T. The angle θ is the angle perpendicular to the angle the loop (black arrow) makes with the direction of the field (red arrows). In Figure 5A, the loop's perpendicular line is parallel to the field, so the angle $\theta = 0°$. In Figure 5B, the loop's perpendicular line is at 50° to the field, so $\theta = 50°$. In Figure 5C, the loop's perpendicular line is at right angles to the field, so the angle $\theta = 90°$. The calculations for Figure 5B and 5C are shown in Worked example 8.1B.

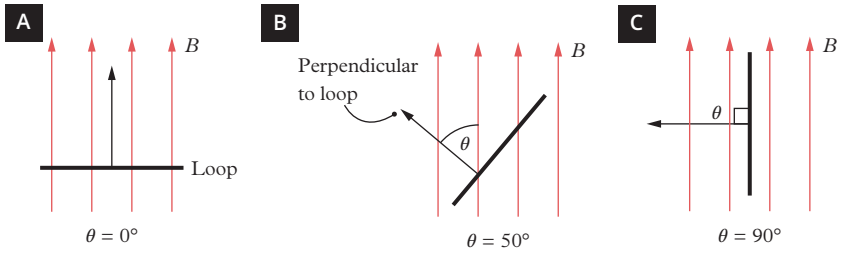


FIGURE 5 A loop rotating in a magnetic field: the loop is (A) perpendicular to the field, (B) at an angle to the field, and (C) parallel to the field. Note that θ is the angle between the perpendicular to the loop and the direction of the field.

Worked example 8.1B

Calculating flux at an angle to the loop

A loop of area $1.0\,m^2$ is placed in a magnetic field of strength 4.0 T, as shown in Figure 5. **Calculate** the magnetic flux threading the loop when its perpendicular makes an angle to the field of

a 50° (2 marks)

b 90°. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$A = 1.0\,m^2$, $B = 4.0$ T, angle = 50° and 90° $\phi = BA\cos\theta$
Step 3: Check that the angle given for part **a** is between the field and the perpendicular to the loop. Substitute known values into the equation and solve for flux, ϕ. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**a** $\theta = 50°$ between field and perpendicular to the loop $\phi = BA\cos\theta$ $= 4.0 \times 1.0 \times \cos 50°$ (1 mark) $= 2.6$ Wb (2 s.f.) (1 mark)

Think	Do
Step 4: Check that the angle given for part **b** is between the field and the perpendicular to the loop. The angle is 90°, so no flux threads the loop. Check this with a calculation. Substitute known values into the equation and solve for flux, ϕ. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	**b** $\theta = 90°$ between the field and the perpendicular to the loop. No flux threads the loop. $\phi = BA\cos\theta$ $= 4.0 \times 1.0 \times \cos 90°$ (1 mark) $= 0.0\,\text{Wb}$ (2 s.f.) (1 mark)

Your turn

The Earth's magnetic field has a strength of 53,158 nT at the Gabba cricket ground and is at an angle of 57.75° to the horizontal ground (or 32.25° between the field line and the perpendicular to the field). If the pitch is 20.12 m × 3.05 m, **calculate** the magnetic flux threading the pitch. (2 marks)

How is flux inside a solenoid determined?

A solenoid that is carrying an electric current will have a magnetic field inside the core that runs from one end to the other, so there will be lines of magnetic flux from one end to the other. To determine the amount of flux, you need to know the magnetic field strength, B, inside the solenoid, and the cross-sectional area, A. Recall from Module 7, the formula for the magnitude of the field strength in the solenoid's core is $B = \mu_0 nI$, where B is the magnitude of the field strength in the solenoid's core, μ_0 is the permeability constant of $1.26 \times 10^{-6}\,\text{T m A}^{-1}$, n is the number of turns per metre of length of the solenoid, and I is the current in ampere.

Worked example 8.1C

Calculating flux inside a solenoid

A 25 cm long solenoid of diameter 5.4 cm with 1,200 turns of wire carries a current of 1.5 A. **Determine** the flux inside the solenoid. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question. Convert to SI units as necessary.	$d = 5.4\,\text{cm} = 0.054\,\text{m}$, $L = 25\,\text{cm} = 0.25\,\text{m}$, $N = 1{,}200$, $I = 1.5\,\text{A}$
Step 3: We know the solenoid formula from Module 7, so we need to calculate the number of turns per metre, n. (1 mark for "Calculates the turns per metre")	$n = \frac{N}{L}$ $= \frac{1{,}200}{0.25}$ $= 4{,}800$ turns per metre (1 mark)
Step 4: Calculate the cross-sectional area of the solenoid. Firstly, calculate the radius, r, and then the area, A. (1 mark for "Calculates the cross-sectional area")	$r = \frac{0.24}{2} = 0.027$ $A_\perp = \pi r^2$ $= \pi \times (0.027)^2$ $= 0.0023\,\text{m}^2$ (1 mark)
Step 5: Calculate the field strength. (1 mark for "Demonstrates correct substitution")	$B = \mu_0 nI$ $= 1.26 \times 10^{-6} \times 4{,}800 \times 1.5$ (1 mark) $= 0.0091\,\text{T}$

Think	Do
Step 6: Calculate the amount of flux. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$\phi = BA_{\perp}$ $= 0.0091\text{T} \times 0.0023\ \text{m}^2$ $= 2.1 \times 10^{-5}\text{Wb}$ (2 s.f.) (1 mark)

Your turn

A 12.0 mm long solenoid in an electric guitar pickup of average diameter 2.0 cm with 6,500 turns of wire carries a current of 1.5 μA. **Determine** the flux inside the solenoid. Give your answer to an appropriate number of significant figures. (4 marks)

Challenge

Solenoids versus loops

A solenoid is made up of many loops placed together, so you should also be able to calculate the flux inside a single loop. The field strength, B, at the centre of a loop of radius R and carrying current I is given by the "field in a loop" formula: $B_{\text{loop}} = \frac{\mu_0 I}{2R}$.

Capital R is used for devices such as loops and solenoids, and lower-case r is used for the radial distance to a point in a magnetic or electric field. Your challenge is to show that the flux inside a single loop of radius 0.025 m when a current of 1.5 A flows is 7.4×10^{-8} Wb. (2 marks)

Real-world physics

Michael Faraday and his law

This module largely concerns Faraday's law of electromagnetic induction. These ideas are based on principles developed in 1830s that have remained unchanged for almost 200 years.

Michael Faraday (1791–1867) (Figure 6) received little formal education but was one of the most influential scientists in history. His ideas are the basis of electric motors and generators, and it was largely due to his insights into the underlying physics that electricity became such a fundamental part of life.

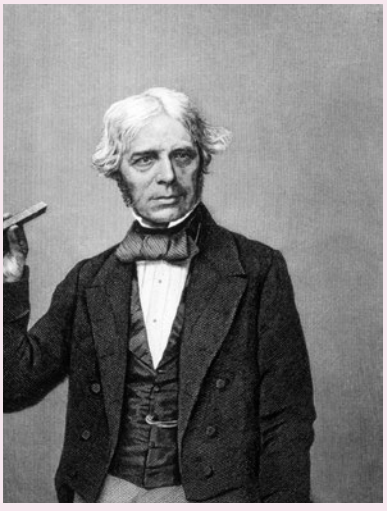


FIGURE 6 Michael Faraday depicted holding a bar magnet

In 1820, the Danish scientist Hans Christian Ørsted found that a magnetic compass was deflected by a current in a wire, an experiment for which he has been credited with the discovery of electromagnetism. Within two weeks of Ørsted publishing his results, André-Marie Ampère in France began investigating how two current-carrying wires would repel or attract each other. He was not convinced this was magnetic in origin. Like other scientists at the time, Faraday was keen to find out more.

In 1831 at the Royal Institution in London, Faraday wrapped two insulated coils of wire around an iron ring (as shown in Figure 7) and found that when he passed a current through one coil, a current was generated in the other coil. This established the idea that an electric current can induce a current in another wire, even if completely insulated from it. The connection was assumed to be via a magnetic effect.

In later experiments, Faraday moved a magnet through a loop of wire and found that an electric current was produced, thus confirming the connection between electricity and magnetism. It was found that a steady magnet would not induce a current, so the key had to be a change in the magnetic field.

Ampère was on to this idea too, but he had his magnet and coil in one room and the meter in another to keep it away from stray magnetism, and this spoiled his results. He moved the magnet in his coil, but by the time he got to the meter it had stopped flickering. Ampère couldn't afford an assistant, otherwise he would have been the first to discover this. Faraday did his experiments in a single room and was able to see the meter change. This was one of his greatest contributions to physics: a changing magnetic field produces electricity, and the voltage produced was proportional to the rate of change. Faraday took a theoretical leap and proposed that electromagnetic forces extended into the empty space around the conductor, which he named a magnetic "field". This idea was rejected by his fellow scientists at the time, and he did not live long enough to see the eventual acceptance of his idea by the scientific community.

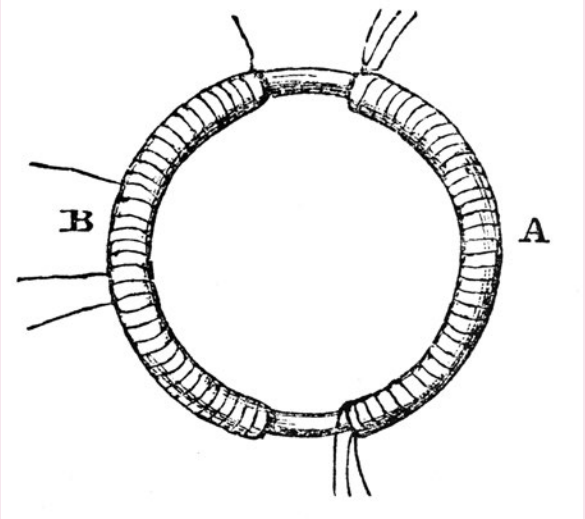


FIGURE 7 Faraday's induction coil consisted of two insulated coils of wire wrapped around an iron ring

Apply your understanding

1 In Faraday's time, a "field" was two-dimensional agricultural space. **Explain** how this is different to a magnetic field. (2 marks)

2 Faraday said that the voltage was proportional to the rate of change of a magnetic field. **Explain** whether this means the voltage was proportional to the time interval or inversely proportional to it. (2 marks)

3 **Examine** whether a voltage is generated when a magnet moves past a coil, or when a coil moves past a magnet, or both. (2 marks)

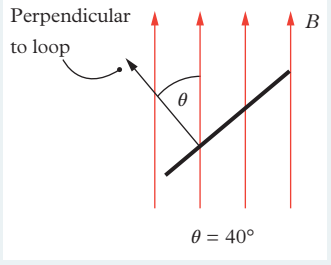
Check your learning 8.1

Check your learning 8.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** the difference between the terms "magnetic flux" and "magnetic flux density". (2 marks)

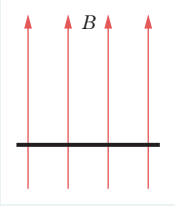
2 **Calculate** the flux threading a rectangular loop measuring 10 cm × 20 cm that is placed in a field of strength 4 mT, as shown in the diagram. (2 marks)



Analytical processes

3 **Determine** the flux inside a solenoid of diameter 4 cm, given that the solenoid has 800 turns in its length of 15 cm and carries a current of 1.5 A. (3 marks)

4 A loop is placed in a magnetic field, as shown in the diagram. **Deduce** the angle between the perpendicular to the loop and the field. (1 mark)



Knowledge utilisation

5 **Propose** the maximum flux in Wb you could achieve through a loop of wire if you vary the shape of the loop (e.g. square, rectangle, circle or triangle). The loop has a perimeter of 100 cm in a 5 mT magnetic field. **Justify** your proposal with a reasoned explanation. (4 marks)

6 A loop of area 0.10 m^2 has its perpendicular parallel to the magnetic field. A student claims that the change in flux will be the same when the plane is rotated from 0° to 30° as when it is rotated from 30° to 60°. **Evaluate** this claim. (3 marks)

Lesson 8.2
Electromagnetic induction and Faraday's law

Key ideas

- Electromagnetic induction involves the concepts of electromotive force (EMF) and Faraday's law.
- By moving a conductor in a magnetic field, an EMF can be induced.
- Problems involving electromagnetic induction can be solved using Faraday's law equations $emf = -\frac{N\Delta(BA_\perp)}{\Delta t}$ and $emf = -N\frac{\Delta\phi}{\Delta t}$.

Learning intentions and success criteria

What is electromotive force (EMF)?

Electromagnetic induction occurs when there is relative motion between a magnetic field and a conductor. In Faraday's experiment, a magnet moved towards a solenoid, but there would also be electromagnetic induction if the solenoid moved towards the magnet. In both cases, an **electromotive force (EMF)** or voltage is induced in the coil of the solenoid. If the coil is part of a circuit, then the induced EMF produces a current.

Figure 1 shows the process of induction. As the magnet approaches, the needle deflects to the left (Figure 1A). When the magnet is stationary (Figure 1B), the needle reads zero. But, when the magnet is pulled away from the coil (Figure 1C), the needle deflects to the right. Faraday published these results, and he is now considered to be the discoverer of electromagnetic induction.

electromagnetic induction
the production of an electromotive force (EMF) or voltage as a conductor moves within a magnetic field

electromotive force (EMF)
a difference in potential that can give rise to an electric current. It has the symbol *emf*, and is measured in volts (V).

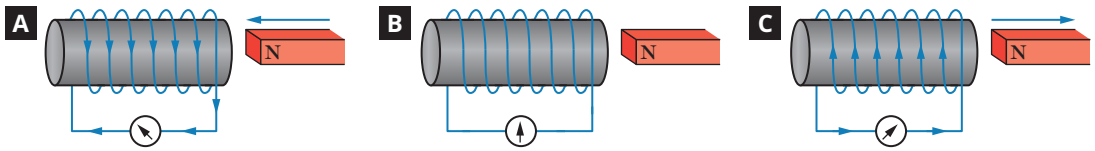


FIGURE 1 Faraday's experiment from 1831 is easily repeated in the classroom. (A) The magnet moves towards the solenoid. (B) The magnet is stationary. (C) The magnet moves away from the solenoid.

What factors affect the induced EMF?

When you move a magnet in and out of a coil, there are five variables that you can change to increase the EMF:

- **Magnetic field strength (*B*)** – using a stronger magnet or two magnets held together with a rubber band (with their like poles side by side) will increase the EMF.
- **Time interval (Δ*t*)** – reducing the time taken for one back-and-forth oscillation of the magnet (i.e. moving the magnet faster) will increase the EMF.
- **Area (*A*)** – using a coil with a bigger cross-sectional area means more lines of flux thread it and will increase the EMF.
- **Number of turns (*N*)** – increasing the number of turns of the coil will increase the EMF. For a coil wound so that the turns are close to each other, the change in flux through each turn is the same, and an EMF appears across each turn, so for N turns, the total EMF is n times the EMF of one turn.
- **Angle (*θ*)** – reducing the angle between the perpendicular to the plane of the coil and the direction of the incoming magnetic field will increase the EMF.

These factors all come together in **Faraday's law** thus:

$$emf \propto \Delta B$$
$$emf \propto \frac{1}{\Delta t}$$
$$emf \propto \Delta A_{\perp}$$
$$emf \propto N$$
$$emf = -\frac{N\Delta(BA_{\perp})}{\Delta t} \text{ (Faraday's law, first version)}$$

Faraday's law
a law stating that when the magnetic flux linking a circuit changes, an electromotive force (EMF) is induced in the circuit proportional to the rate of change of the flux linkage

We can now make a more general case for any angle, and not just perpendicular, by changing the area from $A_{\perp}$ to $A\cos\theta$. This gives the effective (reduced) area through which the flux passes. Faraday's law can also be written:

$$emf = -\frac{N\Delta(BA\cos\theta)}{\Delta t} \text{ (for any angle } \theta)$$

We also know from an earlier lesson that the magnetic flux ϕ is given by the equation $\phi = BA\cos\theta$, so Faraday's law can now be written as:

$$emf = -N\frac{\Delta\phi}{\Delta t} \text{ (Faraday's law, second version)}$$

That is, the EMF equals the rate of change of magnetic flux. We can also use the term "magnetic flux linkage" where the word "linkage" shows that the wire loop and the magnetic flux are linked together when producing an EMF.

The negative sign indicates that the induced EMF acts to oppose any change in magnetic flux. This is described later in the lesson on Lenz's law.

Study tip

Which is correct: EMF or *emf*? The abbreviation for electromagnetic force is EMF (in capitals, no italics) whereas the symbol *emf* (lower case, italics) is used in formulas and equations. That's why we say, "Calculate the EMF" and the answer might be "*emf* = 12 V". There is no penalty for getting it the wrong way around.

How do you calculate the magnitude of the EMF?

As you have seen, there are a number of scenarios that will generate an EMF. You can look at these in turn by changing one of the factors B, A, θ, N or Δt and keeping the others constant.

This leads to these situations. The subscripts f and i represent final and initial, respectively.

1 Changing the field strength, but keeping the area of the coil and angle constant

$$emf = \frac{-N\Delta(BA\cos\theta)}{\Delta t}$$
$$= \frac{-NA\cos\theta(B_f - B_i)}{\Delta t}$$

2 Changing the area of the coil, but keeping the field strength and angle constant

$$emf = \frac{-N\Delta(BA\cos\theta)}{\Delta t}$$
$$= \frac{-NB\cos\theta(A_f - A_i)}{\Delta t}$$

3 Changing the angle, but keeping the field strength and area of the coil constant

$$emf = \frac{-N\Delta(BA\cos\theta)}{\Delta t}$$
$$= \frac{-NBA(\cos\theta_f - \cos\theta_i)}{\Delta t}$$

For changes to time (Δt) the value can just be substituted in. Usually, we start timing from $t = 0$ so Δt is just the time on the measuring device. Also, for the number of turns (N), the value can just be substituted in as, if it is a fixed coil, there is no change during the measurement.

One mistake students make in exams is to only write the formula when they are asked to explain Faraday's law. That isn't enough; you also need to explain what it means. You also must careful with your wording – if you say EMF is proportional to magnetic flux, rather than proportional to the change in magnetic flux, your answer won't be marked correct.

Study tip

The negative sign in Faraday's law can be omitted if the EMF is stated as the magnitude (or absolute value):

$|emf| = N\frac{\Delta\phi}{\Delta t}$

Study tip

You are not expected to be able to use Ohm's law $\left(R = \frac{V}{I}\right)$ in Units 3 & 4 external assessment so you won't be asked to calculate a current from an EMF value.

Calculating the EMF for a change in field strength

The first way to generate an EMF is to vary the magnetic field strength, B. Either increasing or decreasing the field strength will lead to an EMF being generated, although the EMFs will have opposite signs (+/–). At the moment, we are just concerned with the magnitude of the EMF and not the sign. Note also that we can use the terms magnetic field strength B and magnetic flux density B as the same quantity with the same unit, tesla, T.

Calculating the EMF for a change in the time interval

A second way to generate an EMF is to vary the time taken for changes to the field strength, area or angle. According to Faraday's law, the EMF is inversely proportional to the time taken for the change in flux (i.e. $emf \propto \frac{1}{\Delta t}$). So, halving the time taken will double the EMF.

Calculating the EMF for a change in area

A third way to generate an EMF is to change the area of the loop through which the field passes. If the field strength and angle are held constant but the area of the loop is changed, the amount of magnetic flux that passes through the loop will change. For example, if the area is doubled, the amount of flux threading the loop will also double. This will lead to the production of an EMF. Similarly, if the area is halved, then the flux will also be halved and an EMF will be generated. Be careful not to assume that doubling the diameter of the loop will double the area – it won't, of course.

Calculating EMF for a change of angle

Another way to generate an EMF is to turn the loop by a certain angle to the direction of the field. The important point to remember here is the definition of the angle θ. It is the angle between the plane of the loop and the perpendicular to the field. Rotating a loop exposes less of the loop's area to the field, which means that less flux can thread the loop. As a result, the EMF is reduced by a factor of $\cos\theta$.

Worked example 8.2A

Calculating the EMF from change in field strength

A magnetic field of strength 4.0 mT threads (passes through) an open loop of area 1.5 m^2 at right angles to the plane of the loop (Figure 2). The field strength is suddenly reduced to 1.0 mT in 0.50 s.

a **Calculate** the magnitude of the change in flux. (2 marks)

b **Calculate** the magnitude of the EMF produced in millivolts (mV). (2 marks)

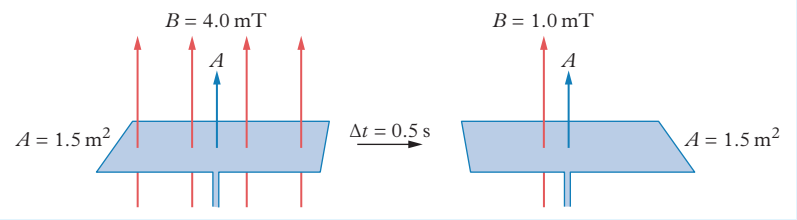


FIGURE 2 A magnetic field threading an open loop

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formulas.	$B_i = 4 \times 10^{-3}$ T, $B_f = 1 \times 10^{-3}$ T, $A = 1.5\,m^2$, $\Delta t = 0.50$ s $\phi = BA\cos\theta$ $emf = -N\frac{\Delta\phi}{\Delta t}$
Step 3: In this case, the value of magnetic field strength (B) is changing, while the angle (θ) and area (A) are held constant. Calculate the change in flux, $\Delta\phi$, using the initial and final magnetic field strength. (1 mark for "Demonstrates correct subsitution"; 1 mark for "Provides correct answer")	**a** $\Delta\phi = \Delta(BA\cos\theta)$ $= A \times \Delta B$ (as the angle $\theta = 0°$, so $\cos\theta = 1$) $= A(B_f - B_i)$ $= 1.5 \times (1 \times 10^{-3} - 4.0 \times 10^{-3})$ (1 mark) $= 4.5 \times 10^{-3}$ Wb (2 s.f.) (1 mark) (Ignore the negative sign.)
Step 4: Calculate the EMF. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer" – mark is still awarded if the answer is in volts)	**b** $emf = -N\frac{\Delta\phi}{\Delta t}$ $= -1 \times \frac{-4.5 \times 10^{-3}}{0.50}$ ($N = 1$ for a single loop) (1 mark) $= +9.0 \times 10^{-3}$ V $\lvert emf \rvert = 9.0$ mV (2 s.f.) (1 mark)

Your turn

An open loop of area 21.5 m^2 is placed at right angles to a magnetic field of strength 125.0 mT. The field strength is suddenly reduced to 10.0 mT (in the same direction) in 0.10 s.

a **Calculate** the magnitude of the change in flux. (2 marks)

b **Calculate** the magnitude of the EMF produced. (2 marks)

Worked example 8.2B

Calculating EMF for a change in time interval

A magnetic field of strength 4.0 mT threads (passes through) an open loop of area 1.5 m^2 at right angles to the plane of the loop (Figure 2 in Worked example 8.2A). The field strength is suddenly reduced to 1.0 mT in 0.50 s and generates an EMF of 9.0 mV.

a **Calculate** the magnitude of the EMF if it changes field strength in half that time (2 s.f.). (2 marks)

b **Determine** whether halving the time doubles the EMF. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer. "Determine" means to establish after a calculation.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$B_i = 4 \times 10^{-3}$ T, $B_f = 1 \times 10^{-3}$ T, $A = 1.5$ m^2, $\Delta t = 0.50$ s, EMF = 9.0 mV
Step 3: Calculate the EMF. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer" – mark is still awarded if the answer is in volts)	**a** $emf = \frac{-N\Delta(BA\cos\theta)}{\Delta t}$ $= \frac{-N(\Delta B)A\cos\theta}{\Delta t}$ $= \frac{-1 \times (1.0 - 4.0)\times10^{-3} \times 1.5 \times \cos 0°}{0.25}$ (1 mark) $= 1.8 \times 10^{-2}$ V = 18 mV (2 s.f.) (1 mark)
Step 4: Determine whether the new EMF is double the previous EMF and draw a conclusion. (1 mark for "Concludes that halving the time doubles the EMF")	**b** The new EMF of 18 mV is double the previous EMF of 9 mV, so this justifies the conclusion that halving the time doubles the EMF. (1 mark)

Your turn

An open loop of area 21.5 m^2 is at right angles to a magnetic field of strength 125.0 mT. The field strength is suddenly reduced to 10.0 mT (in the same direction) in 0.10 s, which generates an EMF across the ends of the open loop.

a **Calculate** the magnitude of the EMF generated if the time taken was doubled to 0.2 s. (2 marks)

b **Determine** whether doubling the time halves the EMF. (1 mark)

Worked example 8.2C

Calculating EMF for change in area

Consider the same setup as in Worked example 8.2A, in which a magnetic field of strength 4.0 mT threads (passes through) an open loop of area 1.50 m^2 at right angles to the field. The loop instead shrinks in area to 0.60 m^2 in 0.10 s, as shown in Figure 3. **Calculate** the magnitude of the EMF induced in the loop. Give your answer to an appropriate number of significant figures. (4 marks)

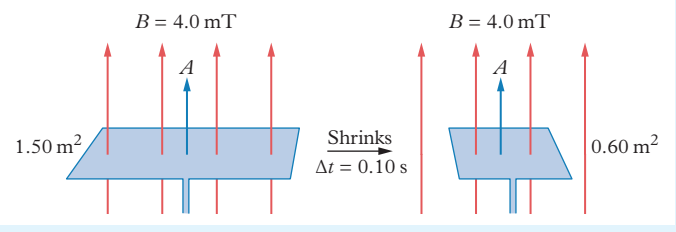


FIGURE 3 A magnetic field threading an open loop

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula. (1 mark for "Recognises the scenario relates to Faraday's law of electromagnetic induction")	$B = 4.0\,\text{mT}$, $A_i = 1.50\,\text{m}^2$, $A_f = 0.60\,\text{m}^2$, $\Delta t = 0.10\,\text{s}$ $emf = -N\frac{\Delta(BA)\cos\theta}{\Delta t}$ (1 mark)
Step 3: In this case, the area (A) is changing, while the angle (θ) and magnetic field strength (B) are being held constant. Show the modified relationship when only the area is changing. (1 mark for "Provides appropriate mathematical reasoning") Perform calculation. (1 mark for "Demonstrates correct substitution")	$emf = -\frac{NB\cos\theta \times \Delta A}{\Delta t}$ (B and θ are not changing.) $= -\frac{NB\cos\theta \times (A_f - A_i)}{0.10}$ (1 mark) $= -\frac{1 \times 4.0 \times 10^{-3} \times (0.60 - 1.50)}{0.10}$ (1 mark)
Step 4: State the answer using the correct units. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$\lvert emf \rvert = 0.036\,\text{V}$ (2 s.f.) (1 mark)

Your turn

An open circular loop of area $21.5\,\text{m}^2$ is placed at right angles to a magnetic field of strength 125.0 mT. The area shrinks to half its original diameter in 0.10 s. **Calculate** the magnitude of the EMF generated in the loop. Give your answer to an appropriate number of significant figures. (4 marks)

Worked example 8.2D

Calculating EMF for a change of angle

Consider the same setup as in Worked example 8.2A, in which a magnetic field of strength 4.0 mT threads an open loop of area $1.5\,\text{m}^2$ at right angles to the field. The loop is now rotated through 20° in 0.10 s, as shown in Figure 4. **Determine** the magnitude of the EMF generated in the loop. Give your answer to an appropriate number of significant figures. (3 marks)

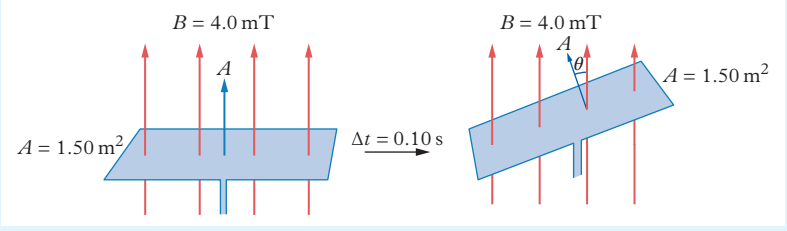


FIGURE 4 Rotating a loop in a magnetic field induces an EMF across its ends.

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formulas.	$B = 4\,\text{mT}$, $A = 1.5\,\text{m}^2$, angle of rotation = 20°

Think	Do
Step 3: In this question, the angle of the loop, θ, is changing, and the magnetic field strength, B, and the area, A, are being held constant. When the loop is at right angles (90°) to the field, the perpendicular to the loop is parallel (at 0°) to the field.	Therefore, $\theta_i = 0°$, $\theta_f = 20°$
Step 4: Select the appropriate formula and show how the change in angle is determined. (1 mark for "Provides appropriate mathematical reasoning") Substitute the known values into the formula. (1 mark for "Demonstrates correct substitution") Note that when there is a change in angle, you need to find the change in $\cos\theta$ – that is, $\Delta(\cos\theta)$, rather than the cosine of the change in θ (i.e. $\cos(\Delta\theta)$). Remember: it is $\Delta(\cos\theta)$, not $\cos(\Delta\theta)$.	$emf = -\frac{N\Delta(BA_\perp)}{\Delta t}$ $= -\frac{N\Delta(BA\cos\theta)}{\Delta t}$ $= -\frac{NBA \times \Delta\cos\theta}{\Delta t}$ (B and A are not changing) $= -\frac{NBA \times (\cos\theta_f - \cos\theta_i)}{0.10}$ (1 mark) $= -\frac{1 \times 4.0 \times 10^{-3} \times 1.5 \times (\cos 20° - \cos 0°)}{0.1}$ $= -\frac{6.0 \times 10^{-3} \times (0.94 - 1.0)}{0.10}$ $= -\frac{-0.36 \times 10^{-4}\,\text{Wb}}{0.10}$ (1 mark)
Step 5: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. Ignore the negative sign as only the magnitude is required. You can express the answer in V or mV. (1 mark for "Provides correct answer") Note that an EMF is only being generated during the movement of the coil. It will be zero before and after the motion.	$\lvert emf \rvert$ = 3.6 mV (2 s.f.) or 0.036 V (1 mark)

Your turn

An open loop of area 21.5 m^2 is placed at right angles to a magnetic field of strength 125.0 mT. The loop is now rotated through 30° in 0.25 s. **Determine** the magnitude of the EMF generated in the loop. Give your answer to an appropriate number of significant figures. (3 marks)

Challenge

Uneven rotation in a single-coil motor

A single-coil motor would have a very uneven rotation, as the force is a maximum when the plane of the loop is cutting the magnetic field fastest, which is when the loop is parallel to the field. How would a heavy loop even this out? Would a heavy loop slow it down more? (2 marks)

Challenge

Inducing an EMF with a copper skipping rope

If you used a skipping rope made of copper wire, would you feel the biggest EMF between your hands when you faced north or faced west? Why is this? (2 marks)

How is the direction of the current or EMF determined?

You have seen that when you move a magnet in and out of a solenoid, an EMF is created across the ends. Recall that the current flows in one direction as the magnet moves into the solenoid and flows in the opposite direction as the magnet moves out. The needle on a voltmeter will flicker from side to side; that is, from positive to negative and back. The needle of an ammeter will also flicker in the same way. This is because the electric charge oscillates back and forth in the wire.

So how do we work out the direction of the EMF and current at any moment? In this lesson we were able to work out the magnitude of the EMF or current but not which end of a coil became positive or negative, or which way the current circulated. This is where Lenz's law comes in, and that is dealt with in Lesson 8.3.

Check your learning 8.2

Check your learning 8.2: Complete these questions online or in your workbook.

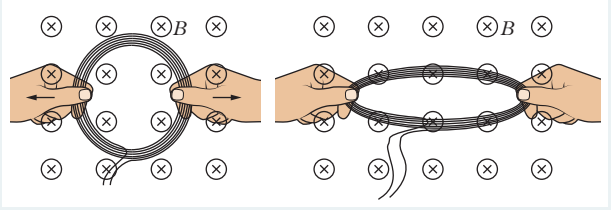
Retrieval and comprehension

1 **Describe** the concept of "electromagnetic induction". (1 mark)

2 **Identify** the four terms and their symbols contained in Faraday's law. (2 marks)

3 **Explain** how magnetic flux can be zero even though the magnetic field is not zero. (1 mark)

4 **Explain** why a voltage is induced in the coil in the diagram when it is pulled by the sides. (1 mark)



5 A square open loop of side 25 cm is threaded by a magnetic field of strength 2.2 mT. **Calculate** the magnitude of the EMF produced when the field strength falls to 1.0 T in 0.20 s. (3 marks)

6 A single proton is fired down a long tube that has a coil of wire wrapped around it to detect the passing of individual particles.

- **a** **Sketch** a graph of the voltage output of the coil as the proton passes through it. (2 marks)
- **b** **Explain** your reasoning. (3 marks)

Analytical processes

7 **Determine** the order from smallest to largest: 0.050 Wb, 1.6×10^{-3} Wb, 15 mWb. (1 mark)

8 A 10-turn coil lies in the plane of this page (or screen) and a uniform magnetic field of strength 0.90 T is directed into the page. The coil has an area of 0.150 m^2 and the sides pulled apart to have zero area in a time of 0.20 s. **Determine** the magnitude of the EMF produced. (3 marks)

9 **Determine** the number of turns in a coil that is pulled between the poles of a magnet so that the magnetic flux decreases from 3.1×10^{-4} Wb to 1.0×10^{-5} Wb in 0.020 s. The induced EMF is measured as 0.75 V. (3 marks)

Knowledge utilisation

10 A permanent magnet is tied to the end of a piece of string and set up like a pendulum. A loop of wire is placed on the floor directly under the pivot point. The magnet is drawn to one side and allowed to make one oscillation (out and back). **Propose** the number of times the loop experiences an increase in flux and the number of times the loop experiences a decrease in flux in that one oscillation. (2 marks)

Practical

Lesson 8.3
Induction of an electric current using a magnet and coil

Learning intentions and success criteria

This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.

Lesson 8.4
Lenz's law

Key ideas

- → Lenz's law can be used to determine the direction of an induced current or EMF during electromagnetic induction.
- → Lenz's law is consistent with the principle of conservation of energy.

Learning intentions and success criteria

What is Lenz's law?

The physicist Heinrich Lenz (1804–1864) first explained the direction of the induced current in a solenoid as being the result of a changing magnetic field. It is easy to push the magnet in and out when it is an ordinary school bar magnet and a small coil. However, if a very powerful magnet and a very large solenoid coil with thousands of turns are used, you would notice that a huge force would be needed to push the magnet into the solenoid. It appears that nature is trying to prevent, or oppose, the induced current in the coil. Lenz used the idea of nature trying to oppose any applied force. **Lenz's law**, as it is referred to today, states that nature does not provide something for nothing. In the field of electromagnetics, Lenz's law states:

> "The direction of an induced electric current always opposes the change in the circuit or the magnetic field that produces it."

Lenz's law
the direction of an induced electric current is such that it produces a magnetic field that opposes the change in the circuit or the magnetic field that produces it

How does conservation of energy apply to Lenz's law?

When an induced current flows through a solenoid, the magnetic field produced by the solenoid has a polarity that repels the incoming permanent magnet pole. It repels the magnet and so forces you to do work. If you didn't have to do work to produce a current, you would get electricity for no effort. Lenz's law is one expression of a fundamental law of nature – the **law of conservation of energy**:

> "Energy can neither be created nor destroyed; rather, it transforms from one form to another."

law of conservation of energy
the total energy of a system remains constant; energy can neither be created nor destroyed, but only changed from one form to another or transferred from one object to another

In Figure 1, you see that in the first case the north pole of a magnet is approaching so the coil produces a north pole at its top end to repel the incoming pole, as like poles repel. To generate a north pole at that end, a current has to flow anticlockwise as shown (as though you are looking from above). To determine the north pole, you can use Ampere's right-hand grip rule (discussed in Module 7). Point your fingers in the direction of the (positive) current in the coil and your thumb will point to the north pole.

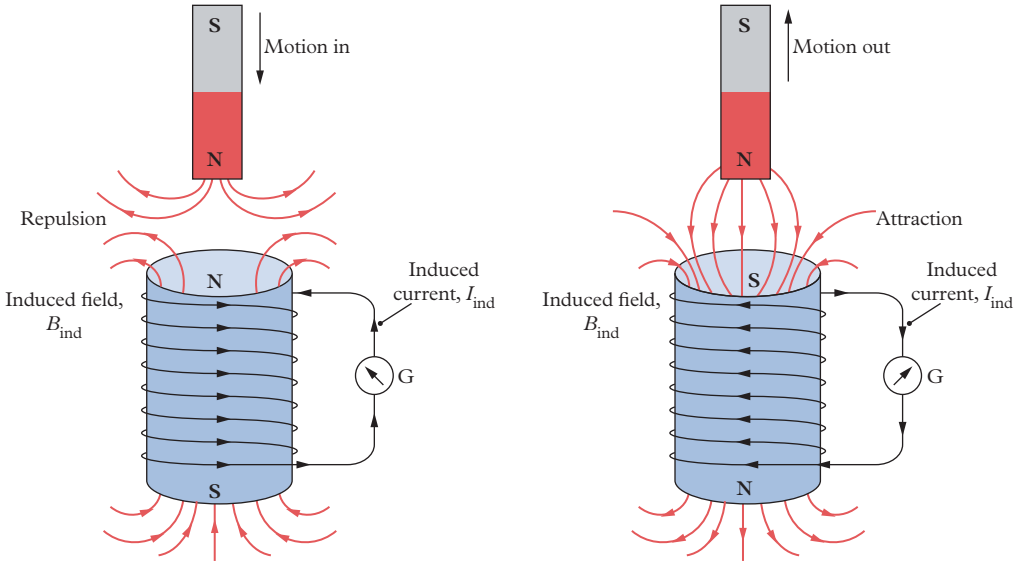


FIGURE 1 Direction of the induced field inside a coil as a magnet towards it and away from it

As the magnet moves away, the coil produces a south pole to try to keep the magnet there, as unlike poles attract. To get a south pole on the top end of the coil, you need an induced clockwise current (as viewed from above). Using Ampere's right-hand grip rule, your thumb should point to the bottom of the coil, indicating the location of the north pole.

In both situations, a force needs to be exerted and work needs to be done to continue moving the magnet. This work done is the origin of the induced electrical energy: mechanical energy is transformed into electrical energy. This principle is the basis of all electric generators. The big problem is how to produce a continuous flow of electrical energy from a generator, and not just single current pulses.

Challenge

Rare-earth magnets

Drop a rare-earth magnet through an aluminium tube and it will fall slowly. Drop a similar but unmagnetised object such as a steel nut, and it will fall fast. Why is this? (2 marks)

How is the direction of current in a loop determined?

As you saw earlier, there are many ways of changing the amount of flux in a loop: changing the field strength, moving the loop out of the field, changing the area and changing the angle.

1 Changing the field strength

Figure 2 shows a rectangular closed loop threaded by four lines of flux. The field is then reduced in strength over a time Δt, and now only one line of flux threads the loop. There has been a change in the flux, even though the area and angle have remained the same. According to Lenz's law, the loop reacts to this change by doing something to oppose it. In other words, the loop tries to restore the flux back to the original condition. It does this by inducing an anticlockwise current in the loop. That generates flux in the upwards direction to partially oppose the reduction in flux.

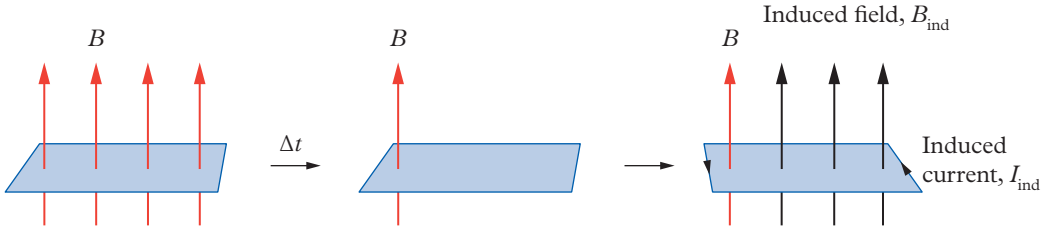


FIGURE 2 The flux threading the loop decreases and an induced current is generated.

2 Changing the amount of loop in the field

In Figure 3, initially 16 flux lines thread the loop, but when the loop is moved to the right and is partly out of the field, only 12 flux lines thread it. There has been a decrease in the flux inside the loop. The loop opposes this by inducing an anticlockwise current to produce more flux lines (a field) in the same direction as the original field. This is only a momentary current as it is an open loop and the charge can't move across the gap. The negative charge accumulates at terminal Y to give it a net negative (–) charge, which means terminal X has a deficiency of electrons and therefore a net positive charge (+).

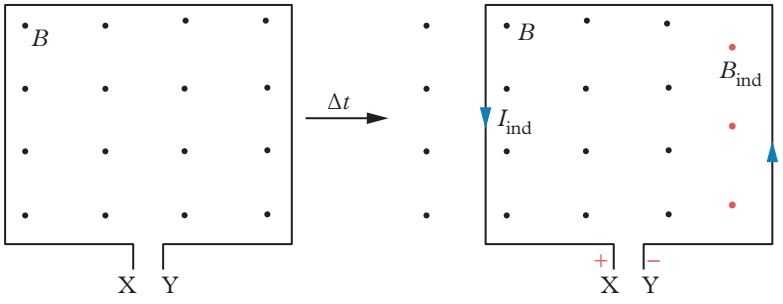


FIGURE 3 An open loop is moved to the side, partly out of the field. An anticlockwise current is induced to increase the field in the original direction.

3 Changing the area

In Figure 4, the field strength remains constant but the size of the loop changes. This could happen, for example, if the loop is made of a spring that shrinks to a smaller size when released. In this case, 25 flux lines (going into the page) thread a closed loop. After a time Δt, the loop shrinks and now there are just 9 flux lines threading the loop. There has been a decrease in flux inside the loop.

To oppose this change, it induces a current that generates flux lines (a magnetic field) into the page to restore the flux as before. For simplicity, only three of the induced flux lines are shown in the diagram.

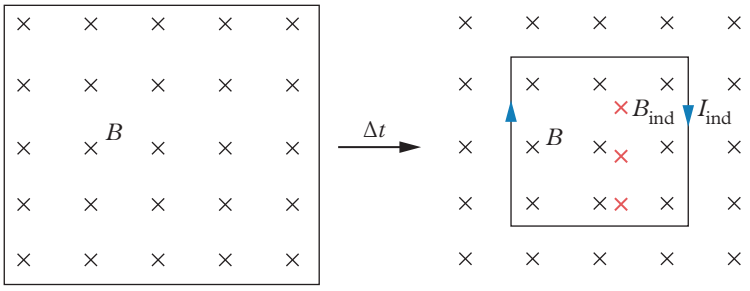


FIGURE 4 The area of a closed loop shrinks and a clockwise current I_{ind} is induced.

These induced flux lines are the result of a clockwise current in the loop. This can be verified by using Ampere's right-hand rule for loops and solenoids.

4 Changing the angle

Figure 5 shows a rectangular open loop at right angles to the field. The small arrow in the middle of the loop is the perpendicular to the plane of the loop. The angle between it and the field is 0°. This gives the maximum number of flux lines through the loop, because $\phi = BA\cos\theta$ and $\cos 0° = 1$.

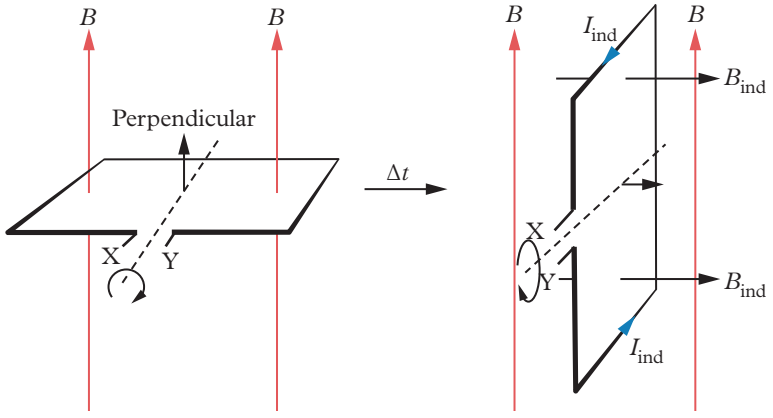


FIGURE 5 A rectangular open loop is rotated in a magnetic field that induces an EMF

The loop is then rotated clockwise. After time Δt, the plane of the loop is now parallel to the field, so the angle $\theta = 90°$ and $\cos 90° = 0$. This means the flux in the loop is at a minimum – in fact it is 0 Wb.

During the process of rotation, the loop cuts through the field lines and thus there is a change of flux, and an EMF is produced between the terminals X and Y.

According to Lenz's law, the loop opposes the change in flux, so a current is induced that produces flux from left to right inside the loop, labelled B_{ind}. Using Ampere's right-hand rule for loops and solenoids, an anticlockwise current develops in the loop; that is, up the back of the loop and down towards terminal X. This is labelled I_{ind} on the diagram.

An anticlockwise conventional current means a clockwise electron flow. Thus, X becomes positive as electrons leave that end, and Y becomes negative as electrons flow into that end.

Worked example 8.4A

Determining the direction of an induced current

A magnet approaches a closed loop from the right at velocity v, as shown in Figure 6.

a **Determine** the direction of the induced current in the loop. **Explain** your reasoning. (3 marks)

b **Justify** your answer. (1 mark)

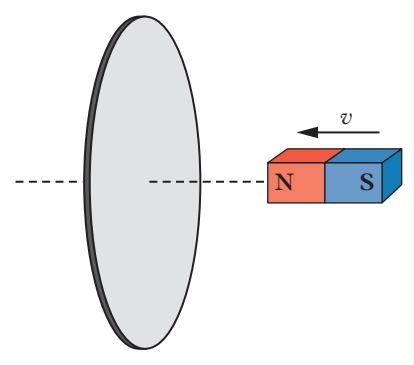
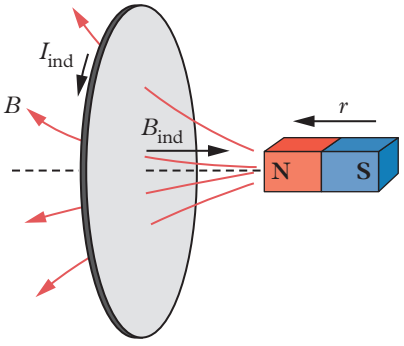


FIGURE 6 A magnet approaching a closed loop

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to ascertain after consideration. We must identify the appropriate information and use it to find an answer. "Explain" means to describe in more detail. "Justify" means to give reasons or evidence to support an answer. We must identify the theory that supports the answer and show how it applies.
Step 2: Gather the relevant information from the question and describe what the diagram shows.	**a** The north pole of the magnet is approaching the loop, so the flux lines thread the loop from right to left.
Step 3: State how the flux in the loop is changing. (1 mark for "Identifies the change in flux in the loop")	The flux into the loop is increasing as the magnet gets closer (even though the field strength of the magnet itself is not changing). (1 mark)
Step 4: Determine how the loop will respond. (1 mark for "Identifies the relationship between change in flux and response to the change")	The loop will oppose the increase in flux by generating flux in the opposite direction (back towards the magnet). (1 mark)
Step 5: Determine what induced current will achieve this using Ampere's right-hand grip rule. It can be useful to draw a diagram to make applying the right-hand rule easier. (1 mark for "Identifies the direction of the induced current")	An anticlockwise current will generate flux inside the loop from left to right. (1 mark) 
Step 6: Justify your answer by referring to the supporting theory. (1 mark for "Justifies the conclusion using Lenz's law")	**b** The direction of the induced current will oppose any change in flux, as described by Lenz's law. (1 mark)

Your turn

A magnet falls vertically through a closed loop, as shown in Figure 7.

a **Determine** the direction of the current in the loop as viewed from above, when the magnet is under the loop and moving downwards. **Explain** your reasoning. (3 marks)

b **Justify** your answer. (1 mark)

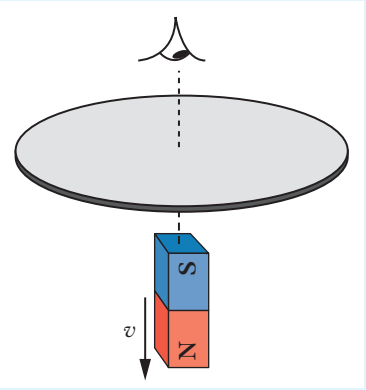


FIGURE 7 A magnet falling through a closed loop

How is the direction of current in a solenoid determined?

You were introduced to solenoids in Module 7 and how flux and EMF are determined earlier in this module. We will now apply Lenz's law to help work out the direction of an induced current and the polarity of terminals for solenoids.

Worked example 8.4B

Determining polarity for an induced EMF in a solenoid

Two identical solenoids, 1 and 2, are placed side by side as shown in Figure 8. When the switch is closed on solenoid 1, magnetic flux is generated that passes through (links) solenoid 2.

a **Determine** which terminal, X or Y, becomes positive while the switch is being closed. (4 marks)

b **Explain** whether terminals X and Y will continue to have net negative and positive charge while the switch remains closed. (2 marks)

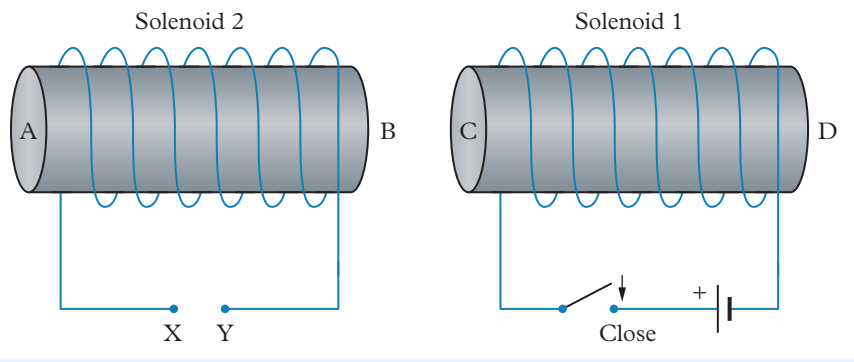


FIGURE 8 Two identical solenoids placed next to each other

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to ascertain after consideration. We must identify the appropriate information and use it to find an answer. "Explain" means to describe in more detail. We must identify the theory that supports the answer and show how it applies.
Step 2: Decide what happens in solenoid 1 when the switch is closed.	**a** When the switch is closed, current will travel from the positive terminal of the cell through the turns of wire and back to the other terminal. The current travels down the front of the solenoid.
Step 3: Use the right-hand rule to determine the polarity of solenoid 1. (1 mark for "Correctly determines the magnetic poles on solenoid 1")	Using Ampere's right-hand rule for solenoids, end C becomes a south pole, and end D becomes a north pole. Flux lines at the end C will point towards C. (1 mark)
Step 4: Decide how solenoid 2 will respond. (1 mark for "Identifies the relationship between the two solenoids is based on Lenz's law")	Solenoid 2 will develop a like pole, a south pole, at end B according to Lenz's law, as like poles repel each other. Flux lines at B will point towards B. (1 mark)
Step 5: Determine what current will be induced in the solenoid to generate a south pole at B. (1 mark for "Determines the direction of current in solenoid 1")	Using Ampere's right-hand rule for loops and solenoids, the current will travel up the front of solenoid 2 to generate a south pole at B. (1 mark)
Step 6: Determine how the direction of the current will affect the charge on terminals X and Y. (1 mark for "Describes how the two terminals become charged")	The current travels from end B of the solenoid to end A. This gives a net positive charge at X, so X becomes a positive terminal. Terminal Y has lost positive charge, so it has a net negative charge and becomes a negative terminal. The polarity of X and Y exists momentarily as the switch is being closed. (1 mark)
Step 7: Explain that an EMF is only induced while the flux is changing (as the switch is being closed). (1 mark for "Concludes that there will not be any lasting polarity"; 1 mark for "Explains that the reason for no lasting polarity is based on Faraday's law that says an EMF will develop while there is a change of flux")	**b** Terminals X and Y will not continue to have polarity. (1 mark) The polarity of X and Y exists momentarily as the switch is being closed, as this only happens while the flux is changing. Once the flux stops changing, terminals X and Y will have no net charge. This is consistent with Faraday's law. (1 mark)

Your turn

Two identical solenoids, 1 and 2, are placed side by side as shown in Figure 9. When the switch is closed on solenoid 1, magnetic flux links to solenoid 2.

a **Determine** which terminal, P or Q, becomes positive while the switch is being closed. (4 marks)

b **Explain** whether terminals P and Q will continue to have net negative and positive charge while the switch remains closed. (2 marks)

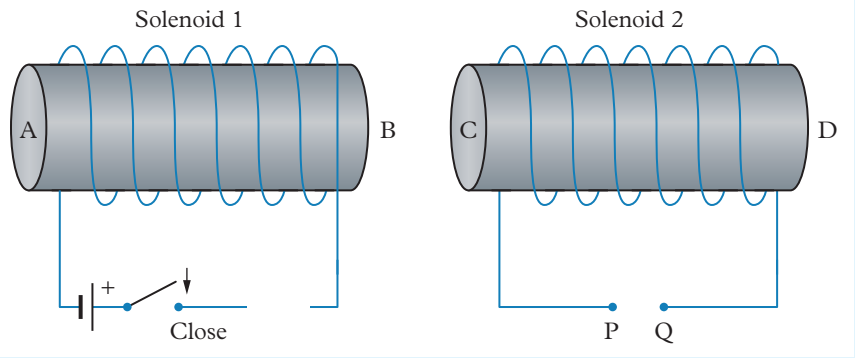


FIGURE 9 Two identical solenoids placed next to each other

How is the direction of a current in a straight wire determined?

You can also generate a current in a single wire – after all, a loop of wire is just a lot of short, straight segments joined end to end. Let's look at a simple case. If you have a wire between the poles of a horseshoe magnet, as shown in Figure 10, and pull the wire towards you, there is opposition to this motion and positive charge flows to the end on the right, leaving the end on the left with an excess of negative charge. (It is really the negative electrons moving, but everything is done in terms of positive charge moving.) Recall that the term "current" describes the flow of positive charge, and "electron flow" is used for the flow of negative charge (electrons).

Lenz developed his law for induced currents in loops, not in straight wires, but it can also be applied to wires. However, we will look at a more fundamental approach first, as this may be easier to understand to begin with.

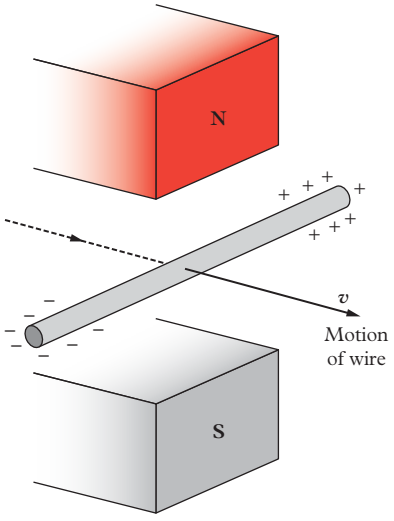


FIGURE 10 Pulling a wire between the poles of a magnet induces charge separation.

What forces act on charges in a wire moving in a magnetic field?

In Module 7 you saw how a charge moving in a magnetic field was deflected. This applied to streams of charges or charges contained in a wire. To work out the direction of the force on the charge, you used either Fleming's left-hand rule or the right-hand palm (slap) rule. They gave the same answers.

We can now apply similar thinking to determine the direction of an induced current in a wire that cuts a magnetic field. They are just opposites. These are summarised in Table 1.

TABLE 1 Currents and magnetic fields – summary of rules and formulas

Module	Inputs and outputs	Rule	Formula
7 Magnetic fields	magnetic field + current → force on charge	Motor rule (Fleming's left-hand rule, or right-hand palm rule)	$F = BIL \sin\theta$ $F = qvB \sin\theta$
8 Electromagnetic induction and radiation	magnetic field + force on charge → current	Generator rule (Fleming's right-hand rule)	$\text{EMF} = -N\frac{\Delta(BA_\perp)}{\Delta t}$

Method 1: The Demon Observer method

This method treats the wire as a collection of positive and negative charges (which it is) but considers the incoming wire in terms of a single positive particle in (or in this model – on) the wire. It goes like this:

Imagine yourself as a demon observer (microscopic in size) sitting on the wire holding a positive charge (Figure 11). Since the wire is moving to the right, that positive charge you're holding is moving to the right with the wire, making a current to the right, sitting in the magnetic field directed down the page. Therefore, that current (positive charge) would experience a force towards Y (using Fleming's left-hand rule or the right-hand palm rule). In the wire the positive charges are fixed. However, the electrons in the wire that are free to move will experience a force along the length of the wire towards X. As these electrons shift to end X of the wire, there is a net excess of negative charge at X (X is negative) and a net excess of positive charge at Y (Y is positive). This leads to a potential difference or EMF across the ends of the wire, with X having the lower positive potential and Y having the higher positive potential.

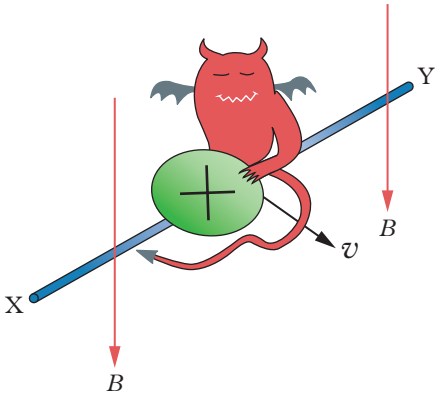


FIGURE 11 The demon on the wire

But doesn't the current go from high to low potential?

Students often say: if the current goes from X to Y, then X must be at a higher potential than Y because current goes from high positive potential to low positive potential. This seems to be the opposite of what we've just said about the demon observer. The answer is that work is done on the wire to push it through the field. This work causes the separation of charge, with the electrons being forced towards the X end. It is only while the wire is being moved in the field that this separation of charge occurs. When the wire stops moving in the field, no more work is being done so the electrons will leave X and shuffle back to their normal positions. This electron flow is the same as a (positive) current flowing from Y to X. So, a current does flow from higher potential (Y) to lower potential (X), but only when work is being done and the wire is moving in the field.

Method 2: Fleming's right-hand rule

Fleming's right-hand rule can be used to determine the direction of the induced current. Note that it is Fleming's "right-hand rule" – a new rule – and not the "left-hand rule" you met in the previous module. As we said earlier, electromagnetic induction can be viewed as the opposite of the electromagnetic force introduced in Module 7, so instead of Fleming's left-hand rule we use Fleming's right-hand rule to determine the direction of the induced current. Hold your thumb, index finger and middle finger at right angles to each other, as shown in Figure 12. Your index finger should point in the direction of the magnetic field (B) and your thumb should point in the direction of the force on the wire (F), i.e. the direction the wire is moving. Your middle finger will then point in the direction of the induced current (I). Note again: this is Fleming's right-hand rule for generating a current by electromagnetic induction. In Module 7 you learnt Fleming's left-hand rule for forces on a current-carrying wire.

Fleming's left-hand and right-hand rules are summarised in Table 2.

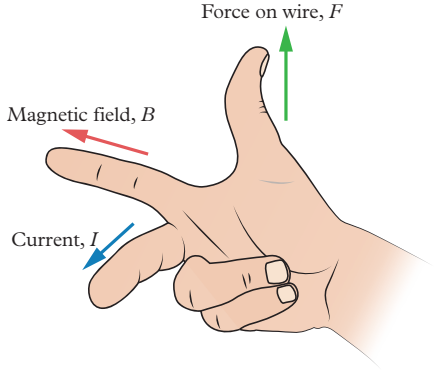


FIGURE 12 Fleming's right-hand rule (for generators)

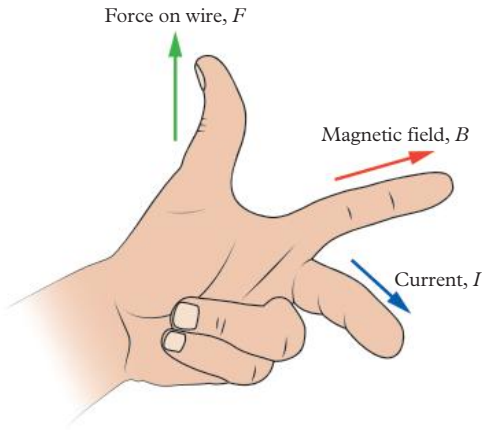
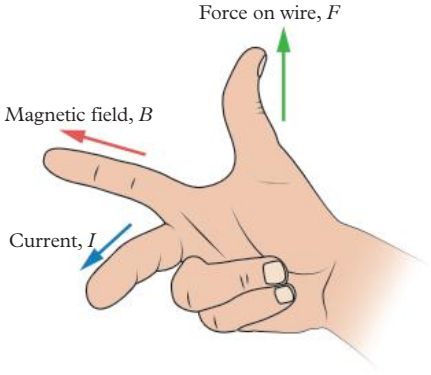
Study tip

You can remember the order for Fleming's right-hand rule starting with the thumb as FBI, like the US Federal Bureau of Investigation. This also works for Fleming's left-hand rule, but it's important to use the correct hand rule for what you're trying to work out.

Method 3: Right-hand palm (slap) rule

To use the right-hand palm rule for induced currents, we say the charge moves in the same direction as v (Figure 10), so the thumb (I, current) points that way. The fingers point in the direction of B, and the palm then shows the direction the positive charge would be pushed. For the arrangement in Figure 10, the fingers point down the page, the thumb points to the right in the direction of v, and the palm pushes the positive charge into the page (towards Y).

TABLE 2 Summary of Fleming's hand rules

Rule	Used in	Used to determine	Alternative name	Memory aid
Fleming's left-hand rule	Module 7 	Direction of force on a current-carrying wire in a magnetic field	Fleming's left-hand motor rule	LEFT F for force
Fleming's right-hand rule	Module 8 	Direction of current generated by electromagnetic induction	Fleming's right-hand generator rule	RIGHT G for generator

Method 4: Lenz's law for a moving wire in a magnetic field

Lenz's law deals with currents in loops of wire. It does not deal with currents in a segment of wire. However, we can consider the wire as part of an imaginary loop as shown in Figure 13.

The flux threading the loop is increasing as the wire moves into the field, so by Lenz's law a current is induced in the loop that opposes the change in the flux. Flux is generated inside the loop up the page and thus the current is anticlockwise according to Ampere's right-hand rule for loops and solenoids. For the "real" segment of the loop XY, the current is towards Y, so end Y will be positive and end X will be negative.

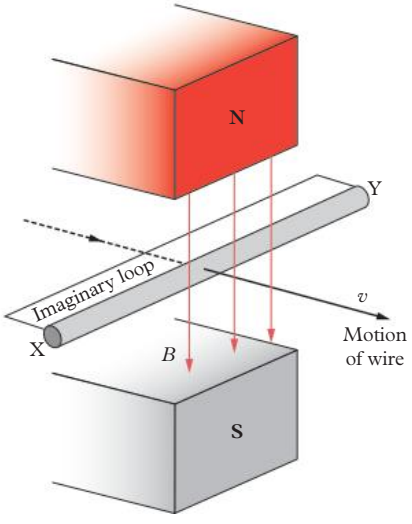


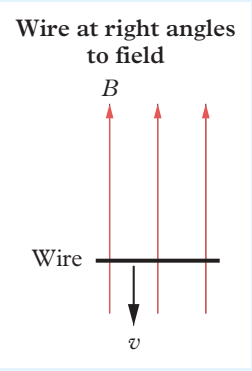
FIGURE 13 Using Lenz's law to determine direction of an EMF

Worked example 8.4C

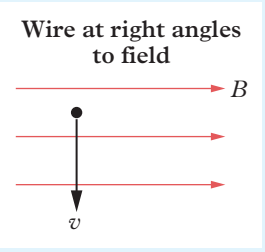
Determining the current for a moving wire in a magnetic field

A wire is placed in a magnetic field in four different orientations, as shown in Figure 14. The wire is then moved in the direction shown by the velocity, v. For each case, **determine** which end, if any, becomes positive. No reasons are necessary. (1 mark each)

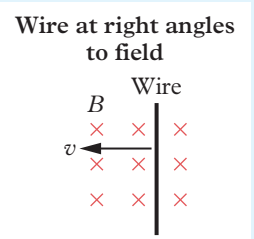
a



b



c



d

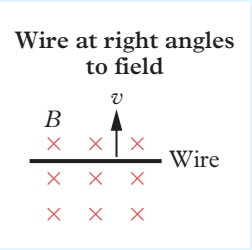


FIGURE 14 A wire in a magnetic field in four different orientations

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to ascertain after consideration. For each part we must apply the appropriate hand rule and use it to find an answer.
Step 2: For each scenario, identify whether the wire cuts any flux lines to work out whether an EMF is produced. Then use the correct hand rule to determine in which direction the charge moves. (1 mark for "States that neither end becomes positive")	**a** The wire does not cut any flux lines, so there is no induced current. No EMF is produced, so neither end is positive. (1 mark)
Step 3: Identify whether the wire cuts any flux lines to work out whether an EMF is produced. Then use the correct hand rule to determine in which direction the charge moves. (1 mark for "States that the end out of the page is positive") Use Fleming's right-hand rule or the right-hand palm rule.	**b** The wire moves at right angles to the field and cuts lines of flux, so an EMF is generated. Fleming's right-hand rule: Force, F (thumb), is down the page Magnetic field, B (index finger), is to the right Current, I (middle finger), is out of the page. The end of the wire out of the page is positive. (1 mark)
Step 4: Identify whether the wire cuts any flux lines to work out whether an EMF is produced. Then use the correct hand rule to determine in which direction the charge moves. (1 mark for "States that the bottom or lower end of the wire becomes positive") Use Fleming's right-hand rule or the right-hand palm rule.	**c** The wire moves at right angles to the field and cuts lines of flux, so an EMF is generated. Fleming's right-hand rule: Force, F (thumb), is to the left Magnetic field, B (index finger), is into the page Current, I (middle finger), is down the page. The bottom end of the wire is positive. (1 mark)
Step 5: Identify whether the wire cuts any flux lines to work out whether an EMF is produced. Then use the correct hand rule (Fleming's right-hand rule or the right-hand palm rule) to determine in which direction the charge moves. (1 mark for "States that the left end becomes positive")	**d** The wire moves at right angles to the field and cuts lines of flux, so an EMF is generated. Fleming's right-hand rule: Force, F (thumb), is up the page Magnetic field, B (index finger), is into the page Current, I (middle finger), is to the left. The left end of the wire is positive. (1 mark)

Your turn

A wire is placed in a magnetic field in four different orientations, as shown in Figure 15. The wire is then moved in the direction shown by the velocity v. For each case, **determine** which end, if any, becomes positive. No reasons are necessary. (1 mark each)

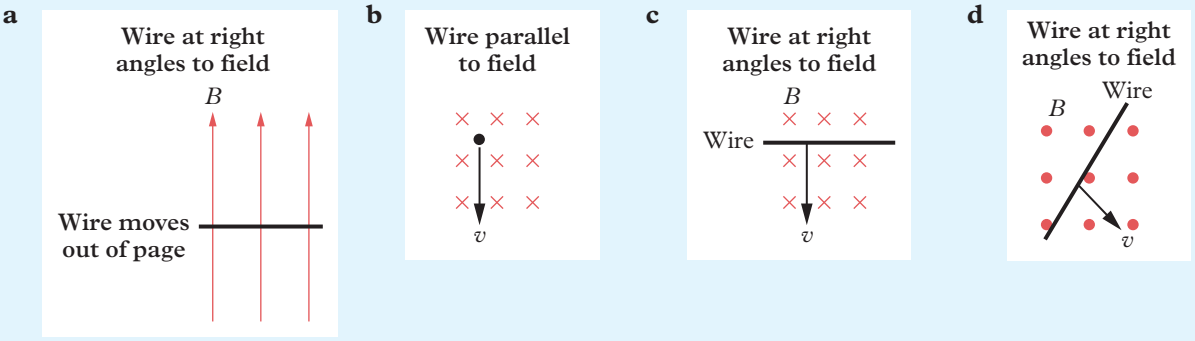


FIGURE 15 A wire in a magnetic field in four different orientations

How is Lenz's law used on a straight conductor in a rectangular loop?

Consider the apparatus depicted simply in Figure 16. It shows a metal rod AB resting on conducting rails connected to a galvanometer, G, between C and D. The apparatus is sitting in a magnetic field, directed into the page as shown. The conductor ABCD forms a loop through which the magnetic field passes.

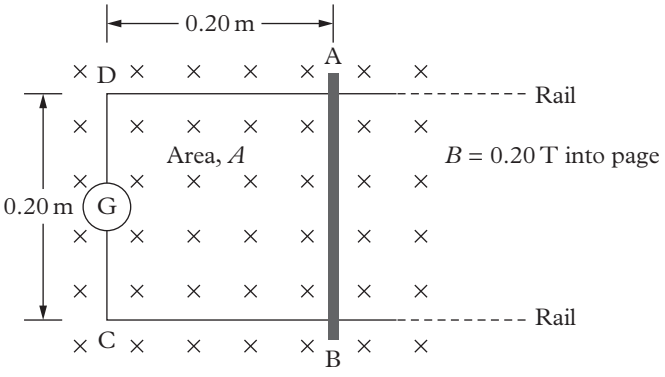


FIGURE 16 A metal rod on conducting rails in a magnetic field forms a loop.

Inducing an EMF by changing the field strength

Imagine the magnetic field strength in Figure 16 changing from 0.20 T to zero in 5.0 ms.

Magnitude of the EMF

The value of the EMF produced is given by Faraday's law:

$$\begin{aligned} emf &= -\frac{N\Delta(BA_\perp)}{\Delta t} \\ &= -\frac{NA_\perp \Delta B}{\Delta t} \quad (A \text{ is not changing}) \\ &= -\frac{NA_\perp \times (B_f - B_i)}{\Delta t} \\ &= -\frac{1 \times 0.20 \times 0.20 \times (0 - 0.20)}{5.0 \times 10^{-3}} \\ |emf| &= 1.6 \text{ V} \end{aligned}$$

Direction of the induced EMF and current

The direction of the induced current around the loop ABCD can be determined using Lenz's law. The field strength (B) is decreasing, so the loop opposes this change and attempts to keep the field strength the same. It does this by generating a field in the same direction as the original field to make up for the losses.

Ampere's right-hand rule for loops and solenoids (Lesson 7.5, Figure 4B) shows that the current must flow clockwise from D to A to B to C. This conclusion is justified because "the induced current will oppose any change, as described by Lenz's law". (You should say this if asked for justification.)

Inducing an EMF by moving the rod

Imagine now that the rod is moved to the right at a velocity of, say, 5.0 cm s^{-1}, with the magnetic field remaining constant at its initial value (Figure 17). The induced EMF from D to C can be calculated. This type of question is popular in every physics course including electromagnetic induction. You would be well advised to follow the logic.

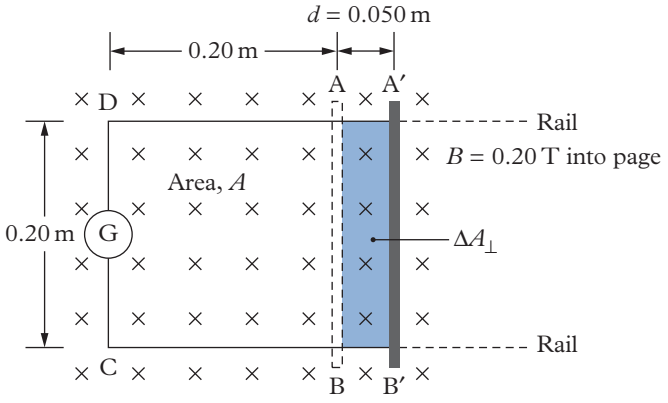


FIGURE 17 The shaded area contains the flux cut by the moving rod.

Magnitude of the induced EMF

In one second of time, the rod cuts through 5.0 cm (0.050 m) of the magnetic field (shown as blue shading) as the rod AB moves to the new position A′B′. This is an increase in area of $0.050 \times 0.20 = 0.010\,\text{m}^2$, so the change in area ($\Delta A_\perp$) is 0.010 m^2.

$$\begin{aligned} emf &= -\frac{N\Delta(BA_\perp)}{\Delta t} \\ &= -\frac{NB\Delta A_\perp}{\Delta t} \ (B \text{ is not changing}) \\ &= -\frac{1\,\text{turn} \times 0.20\,\text{T} \times 0.010\,\text{m}^2}{1.0\,\text{s}} \\ |emf| &= 2.0 \times 10^{-3}\,\text{V (2 s.f.)} \end{aligned}$$

Direction of the induced EMF and current

To determine the direction, use Lenz's law. When the rod slides to the right, the amount of flux enclosed by the loop increases. The loop opposes this change and generates a field that is in the opposite direction to the existing field to get it back to the original value. Using Ampere's right-hand rule for loops and solenoids (Lesson 7.5, Figure 4B), an anticlockwise current (from B′ to A′) is induced to achieve this.

Alternatively, you can use Fleming's right-hand (generator) rule. With your index finger (magnetic field, B) pointing into the page, your thumb (force, F) pushes the wire (AB) to the right, and your middle finger (current, I) points up the page (from B′ to A′). So the result is the same. However, Fleming's right-hand rule is just a way to determine the direction and is not "justification" for the direction. The principle behind Lenz's law is what justifies the direction of currents or flux: "the induced current will oppose any change in flux", or, more fully: "the induced current will produce flux in a direction that opposes any change in flux". Be aware that this must be stated fully, and that the terms "Fleming's right-hand rule", the "right-hand palm rule", and "Lenz's law" only name techniques and are not justifications.

Worked example 8.4D

Determining induced current for a rod on rails in a magnetic field

A copper rod RS is placed on conducting rails 45 cm apart that make an incomplete loop sitting in a magnetic field, as shown in Figure 18. The rod is moved with a constant velocity of 25 m s^{-1} to the left, and this creates an induced EMF across the rod of 165 mV.

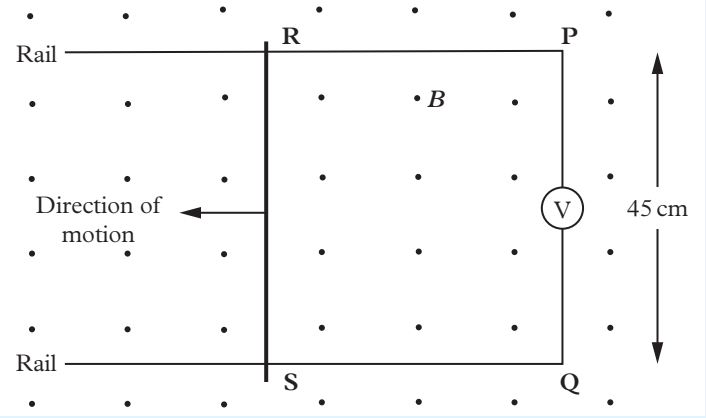
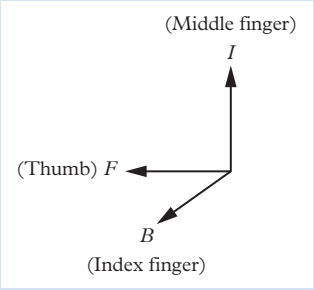


FIGURE 18 Moving a rod on a rail in a magnetic field

a **Determine** the magnitude (strength) of the magnetic field, B, experienced by the loop. Show your working. Give your answer to an appropriate number of significant figures. (3 marks)

b **Determine** the direction of the induced current in the loop. **Justify** your answer. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after a calculation. For part (a) we must identify the appropriate formula and use it to find an answer. "Justify" means to give reasons or evidence to support an answer.
Step 2: Gather the relevant data from the question and convert values to SI units where necessary. Select the appropriate formula.	**a** $v = 25\,\text{m s}^{-1}$, $V = 165\,\text{mV} = 0.165\,\text{V}$, width, $w = 45\,\text{cm} = 0.45\,\text{m}$, $n = 1$ turn $\lvert emf\rvert = -\frac{N\Delta(BA_\perp)}{\Delta t}$
Step 3: Rearrange the formula to show that the change in length divided by time equals the change in area. (1 mark for "Identifies relationship for the change in area over time")	$\lvert emf\rvert = -\frac{NB\Delta A_\perp}{\Delta t}$ (B is not changing) $= -\frac{NB\Delta(L \times W)}{\Delta t}$ (1 mark) $= -\frac{NBW \times \Delta L}{\Delta t}$ $= -NBW \times v$
Step 4: Substitute in the known values and solve for the magnetic field, B. (1 mark for "Provides appropriate mathematical reasoning")	$B = \frac{\lvert emf\rvert}{N \times W \times v}$ $= \frac{0.165}{1 \times 0.45 \times 25}$ (1 mark) $= 0.0147\,\text{T}$
Step 5: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. This is the magnitude, so no sign (+/–) is required. (1 mark for "Determines the magnitude of the magnetic field")	$B = 0.015\,\text{T}$ (2 s.f.) (1 mark)

Think	Do
Step 6: To determine the direction of the current in the loop, we need to apply Fleming's right-hand rule or use Lenz's law. This part is just about determining the direction of the current, not justifying your conclusion. **Fleming's right-hand rule**  **Lenz's law:** The amount of flux threading the loop is increasing. By Lenz's law the system opposes that change by inducing an anticlockwise current which generates flux into the page, which opposes the existing flux out of the page.	**b Using Fleming's right-hand rule:** Motion, v (thumb), is to the left Magnetic field, B (index finger), is out of the page Current, I (middle finger), is up the page, which is the direction of the current in the rod. The current then circulates in a clockwise direction. **Using Lenz's law:** The amount of flux threading the loop is increasing, and according to Lenz's law the system opposes that change by inducing an anticlockwise current. This generates flux into the page, which opposes the existing flux out of the page.
Step 7: State the direction of the current from Step 6. (1 mark for "Correctly concludes the direction of the induced current in the loop as clockwise")	Clockwise (1 mark), or Upwards in the rod S → R, and then R → P → Q → S (1 mark)
Step 8: To justify your answer in Step 7, you will need to show the underlying principle behind Fleming's right-hand rule and Lenz's law. It is not enough to say "Fleming's right-hand rule" or "Lenz's law"; they are names or descriptions for techniques, not justifications. (1 mark for "Provides correct justification")	The amount of flux threading the loop is increasing, and according to Lenz's law the system opposes that change by inducing an anticlockwise field. This generates flux into the page, which opposes the existing flux out of the page. (1 mark)

Your turn

A copper rod AB is placed on conducting rails 15 cm apart that make an incomplete loop sitting in a magnetic field, as shown in Figure 19. The rod is moved with a constant velocity of 8.6 m s^{-1} to the right, and this creates an induced EMF across the rod of 550 mV.

a **Determine** the magnitude (strength) of the magnetic field B experienced by the loop. Show your working. Give your answer to an appropriate number of significant figures. (3 marks)

b **Determine** the direction of the induced current in the loop. **Justify** your answer. (2 marks)

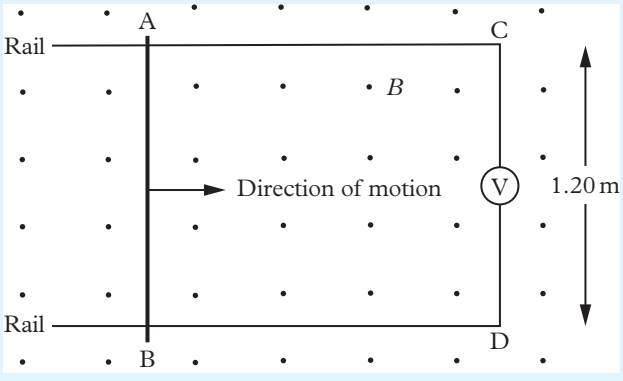


FIGURE 19 Inducing an EMF on a moving rod in a field

Challenge

The weight of a falling magnet

When you drop a rare-earth magnet through an aluminium tube, it falls slowly and doesn't touch the sides of the tube. If you hang the tube from a scale balance, **predict** whether the scale reading will increase or stay the same as the magnet falls through it. **Explain** your reasoning. (2 marks)

Study tip

Most Physics teachers adapt Faraday's law to incorporate the speed of a moving wire or rod:
Area swept by wire, $\Delta A_\perp$ = length, l × distance travelled, d

$$emf = \frac{B\,\Delta A_\perp}{\Delta t}$$
$$= \frac{B \times l \times d}{\Delta t}$$
$$= Bl \times \frac{d}{\Delta t}$$
$$= Blv$$

This formula is not mandatory in Senior Physics, so you are not expected to be able to use it. However, it is useful for checking the answers produced by the earlier method.

Check your learning 8.4

Check your learning 8.4: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Clarify** whether an induced voltage always produces a current. (1 mark)

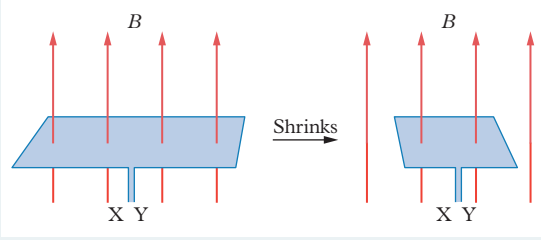
2 **Summarise** how Lenz's law applies to electromagnetic induction. (2 marks)

Analytical processes

3 The area of a loop halves as shown in the diagram.

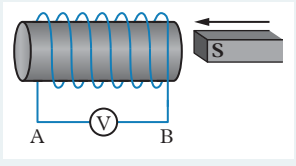
a **Deduce** whether an induced current is produced. (1 mark)

b **Determine** whether X or Y becomes positive if a current is produced. (1 mark)

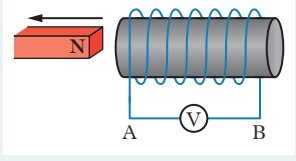


4 A magnet is brought up to a solenoid as shown in the diagram. **Determine** the direction of the induced current through the voltmeter (from A to B, or from B to A) in each case. (1 mark each)

a



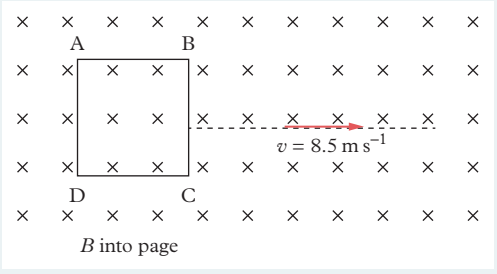
b



5 The diagram illustrates a conductive metal square coil positioned within a magnetic field of strength 150 mT. The coil has side AB = 5.0 cm and is moved sideways at 8.5 m s^{-1}. **Determine**

a the voltages induced across each of the sides of the coil AB, BC, CD, DA (4 marks)

b whether an induced current will flow around this coil as it is moved. **Explain** your answer. (1 mark)

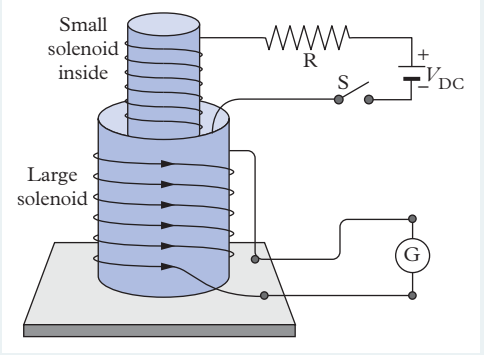


Knowledge utilisation

6 Consider the apparatus in the diagram.

a **Predict** which way the current will flow through the galvanometer (if at at all) when switch S is closed. (1 mark)

b **Propose**, with justification, what would occur if switch S is opened and closed repeatedly. (2 marks)



Lesson 8.5
Transformers

Key ideas

→ Transformers work on the principles of Faraday's law and electromagnetic induction.

→ Problems involving transformers can be solved using $I_pV_p = I_sV_s$ and $\frac{V_p}{V_s} = \frac{N_p}{N_s}$.

How do transformers work?

Learning intentions and success criteria

transformer
a device that transfers an alternating current from one circuit to another, usually with an increase (step-up transformer), or decrease (step-down transformer) in voltage

mutual induction
the production of an EMF in a circuit by a change in the current in an adjacent circuit that is linked to the first by the flux lines of a magnetic field

A basic **transformer** is just two solenoid coils wound onto a common soft iron core, as shown in Figure 1. When the switch is closed in the left-hand circuit, an expanding magnetic flux is produced that cuts the right-hand circuit. This induces an EMF pulse across the ends of the right-hand coil in such a way as to oppose this expanding flux, according to Lenz's law. No EMF is induced when the current in the left-hand coil is constant. However, if the switch is opened again, the magnetic flux cutting the right-hand coil collapses and again an EMF is induced, this time in the opposite direction to the original pulse. This process is called **mutual induction**. You may have seen an induction coil (a "spark coil") in your laboratory at school that works on this principle.

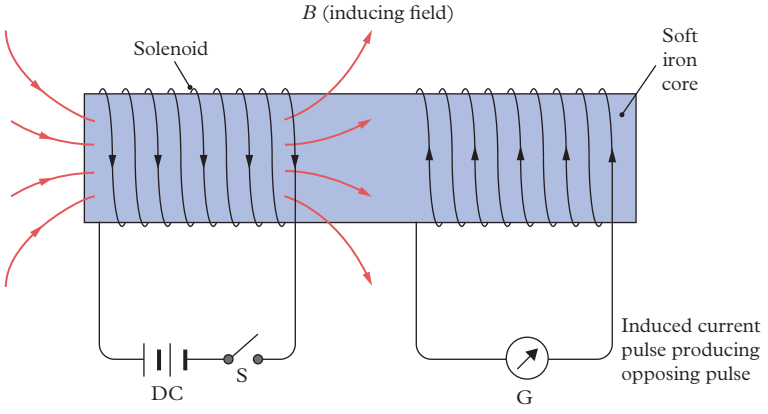


FIGURE 1 A schematic of a simple induction coil

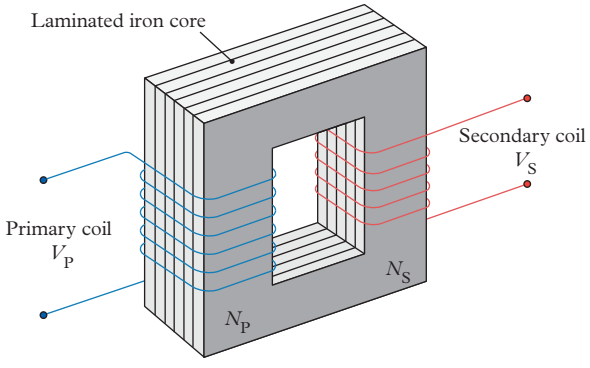


FIGURE 2 Soft iron core of a transformer

Another method of producing continuously expanding and collapsing magnetic fields is to use a primary circuit driven by an alternating current (AC).

Two coils – called the primary and secondary coils – are wound onto a common soft iron core (Figure 2). The soft iron core concentrates and strengthens the magnetic flux lines threading both coils. If the primary coil is fed with alternating (AC) voltages at a particular frequency, an induced alternating voltage of equal frequency will occur across the secondary coil. It wouldn't work with a battery as the input because the voltage would not change. Although the two coils are wrapped around the same core, there is no physical electrical connection between them.

Worked example 8.5A

Explaining how transformers work

Explain how transformers work in terms of Faraday's law and electromagnetic induction. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do. Note that this is not a "describe" question, so you are not describing the construction of a transformer.	"Explain" means to describe in more detail. We must define any pertinent terms, describe how they apply, and add extra relevant information about the process.
Step 2: Gather the relevant information from the question.	Consider three terms from the question: transformers, Faraday's law and electromagnetic induction.
Step 3: Define or describe the basis of a transformer. This is just to orient the marker, although no marks are allocated for this.	A transformer is a device that changes an alternating voltage to a higher or lower value by the use of two flux-linked coils.
Step 4: The first stage is to describe what is happening at the primary coil. The key term here is "alternating current", because if it is not alternating there is no change in flux. The second key term is "changing", referring to the flux. Without these two words there will be no mark. (1 mark for "Recognises that an alternating current creates a changing magnetic flux") Note: magnetic field or electromagnetic field would be accepted instead of magnetic flux.	An alternating current passing through the primary coils creates a changing magnetic flux. (1 mark)
Step 5: The next stage in the process is to describe what effect the changing magnetic flux in the first coil has on the secondary coil. The key words here are "changing", "inducing" and "EMF". (1 mark for "Recognises that an EMF is produced in the secondary coil by the changing magnetic flux in the primary coil")	An EMF is induced in the secondary coil by the changing magnetic flux in the primary coil. This is called "electromagnetic induction". (1 mark)
Step 6: The third stage is to relate Faraday's law to the process. Describe the relationship between EMF and the relative number of coils on the primary and secondary coils, and also to the rate of change of flux. (1 mark for "Identifies that the induced EMF is proportional to the number of coils and the rate of change of flux") Do not mention "opposing the change in flux", because that is Lenz's law.	The EMF is proportional to the ratio of coils on the primary and secondary coils, and also to the rate of change of flux. This is a statement of Faraday's law. (1 mark)

Your turn

Explain, in terms of Faraday's law and electromagnetic induction, why a transformer won't work if there is no changing flux in the primary coil. (3 marks)

What different types of transformers are there?

There are two main types of transformers. If the ratio of turns in the windings is varied, either a step-up or a step-down transformer is produced. For any transformer operating under ideal conditions (that is, no loss of energy), you can use the following versions of Faraday's law. The symbols V_p or V_s are used for the EMF (voltage) of the primary and secondary coil, respectively.

$$V_p = -N_p\left(\frac{\Delta\phi}{\Delta t}\right) \text{ and } V_s = -N_s\left(\frac{\Delta\phi}{\Delta t}\right)$$

"Lossless" transformers

A lossless transformer is one where there is no leakage of flux or energy from one side to the other. It is an ideal situation that is not delivered in practice. However, our discussion here will be in terms of lossless transformers.

What is the relationship between the primary and secondary coils?

The amount of flux threading each coil is the same if no energy is lost, therefore:

$$\left(\frac{\Delta\phi}{\Delta t}\right) = \frac{V_p}{N_p}$$
$$= \frac{V_s}{N_s}$$

which can be rearranged to:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

where V_p is the alternating voltage across the primary coil; V_s is the alternating voltage across the secondary coil; N_p is the number of turns on the primary coil; and N_s is the number of turns on the secondary coil.

- If $N_p > N_s$, then the transformer is a step-down transformer, which reduces the alternating voltage.
- If $N_p < N_s$, then the transformer is a step-up transformer, which increases the alternating voltage.

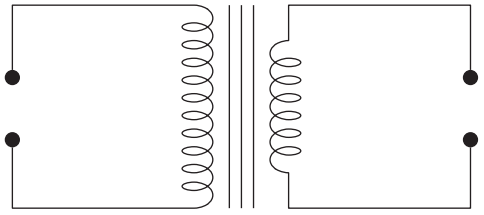


FIGURE 3 The circuit symbol for a transformer

Where are transformers used?

Electricity for Australian homes is normally generated at an AC voltage of 23 kV, which is stepped up to 330 kV for long-range transmission. It is then gradually stepped down at power-pole transformers in your street to 415 V, which then provides the 230 V used in household appliances.

Many household appliances contain miniature transformers. Appliances are designed to operate on a specific voltage, AC or DC. Not operating at the correct voltage can cause overheating and endanger the user. For example, your computer is likely to have an adapter that uses a step-down transformer for power, and further step-down and step-up transformers inside it to power particular bits of circuitry.

Without transformers, we couldn't have many of the electrical devices as we have today, and we couldn't use as much energy. The Real-world physics feature later in this lesson investigates the efficiency of transformer devices.

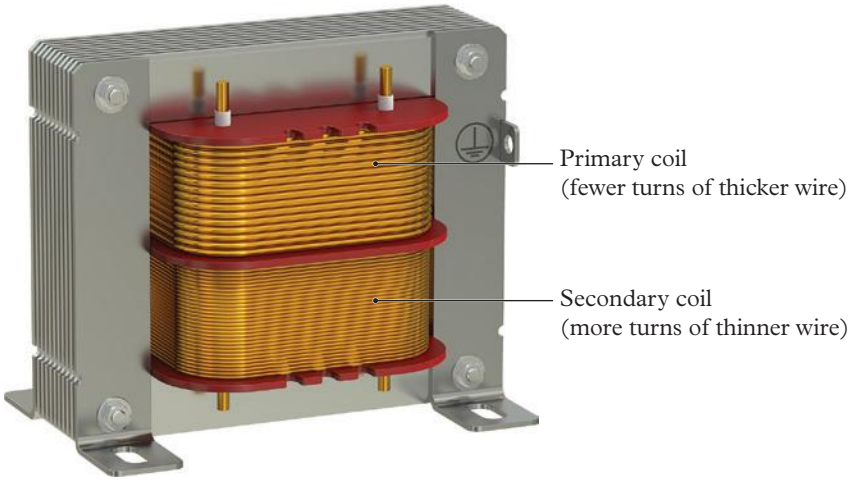


FIGURE 4 A transformer showing the two coils. The top coil has fewer turns and thicker wire than the lower coil. To act as a step-up transformer, the top coil is made the primary and the bottom coil is the secondary. To act as a step-down transformer, the reverse is true.

Challenge

Voltage change in a step-up transformer

In a step-up transformer, the voltage in the secondary coil is higher than in the primary coil. Where does the extra voltage come from? (1 mark)

Challenge

How safe is it to rub a pen on your jumper?

When you rub a pen on your jumper, the work you do induces an EMF in the pen and it can pick up bits of paper. The EMF can be high as 10,000 V. Why doesn't this harm you? People have been electrocuted by accident with household voltages of just 230 V. (1 mark)

Worked example 8.5B

Determining transformer characteristics

A transformer purchased from an electronics store is labelled as 230 V AC input, 50 V AC output. It has 400 turns of wire on its primary coil.

a **Calculate** the number of turns on the secondary coil. Express your answer to the nearest whole number. (2 marks)

b **Determine** whether this is a step-up or step-down transformer and state your reasoning. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer. "Determine" means to establish after a calculation.

Think	Do
Step 2: Gather the relevant data from the question and identify the appropriate formula.	$V_p = 230\,V$, $V_s = 50\,V$, $N_p = 400$ $\frac{V_p}{V_s} = \frac{N_p}{N_s}$
Step 3: Substitute the known values into the formula and calculate the number of turns on the secondary coil, N_s. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	**a** $\frac{V_p}{V_s} = \frac{N_p}{N_s}$ $N_s = \frac{N_p V_s}{V_p}$ $= \frac{400 \times 50}{240}$ (1 mark) = 83 turns (to the nearest whole number) (1 mark)
Step 4: Look at the relative number of turns on the coils. If $V_s < V_p$, it is a step-down transformer; if $V_s > V_p$, it is a step-up transformer. (1 mark for "Identifies the relationship between voltage ratio and the related terms")	**b** As $V_s < V_p$, it is a step-down transformer. (1 mark)

Your turn

A "power" transformer installed on a street pole is labelled as 11 kV AC input, 415 V AC output. It has 500 turns of wire on its primary coil.

a **Calculate** the number of turns on the secondary coil. Express your answer to the nearest whole number. (2 marks)

b **Determine** whether this is a step-up or step-down transformer and state your reasoning. (1 mark)

How does a generator work?

Our homes and offices are wired for an alternating current (AC) at 230 V. This current varies in a sinusoidal (sine wave) pattern that changes direction 50 times a second. The electrons vibrate back and forth in the wire but don't really go anywhere. It is not like direct current where the electrons move along the wire in one direction, from high potential to low potential.

A simple AC generator is shown in Figure 5. It consists of two permanent magnets with a rectangular open loop placed in the field between the poles of the magnets. As the loop rotates, it cuts the flux lines between the magnets and generates an EMF.

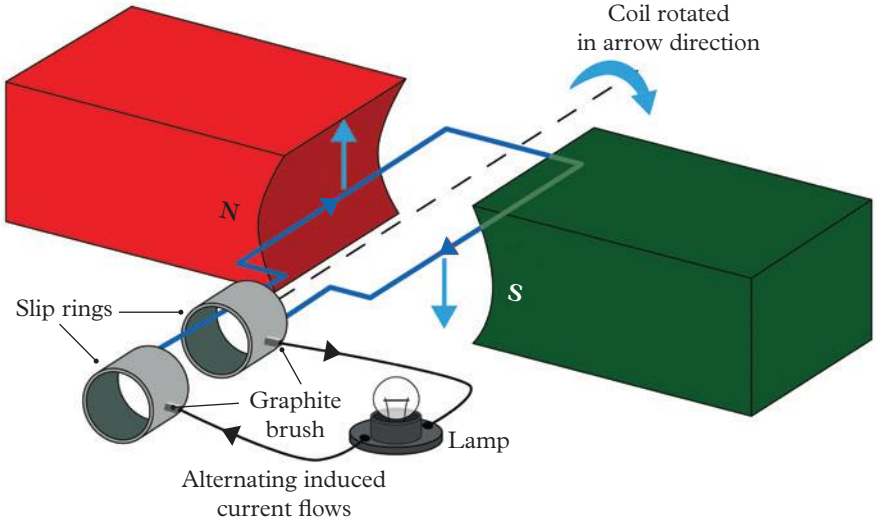


FIGURE 5 The basic parts of an AC generator

Generating alternating current

Alternating current is generated by rotating a loop or coil in a magnetic field. The axis of rotation of the loop needs to be perpendicular to the field. Figure 6 shows the changes in flux inside the loop as it rotates through four 90° stages.

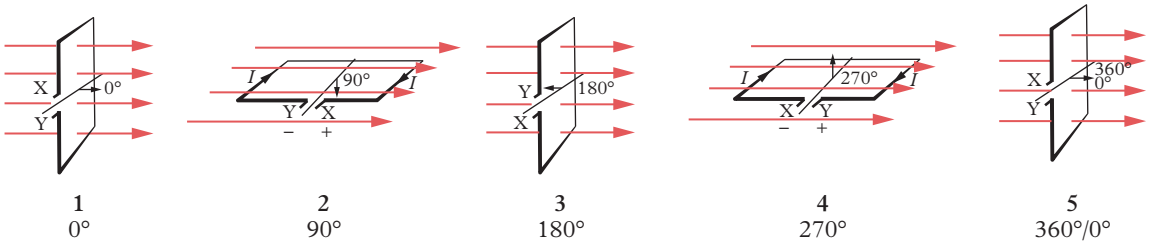


FIGURE 6 Change in flux as a loop rotates in a magnetic field

1. Angle = 0°. In position 1 the plane of the loop is at right angles to the field. This means the perpendicular to the loop is parallel to the field so the angle $\theta = 0°$. The amount of flux enclosed by the loop ($\phi = BA\cos\theta$) is a maximum (as $\cos 0° = 1$). However, in this position a slight rotation won't cause the wire to cut through any flux lines, so no EMF is generated. This is the position for zero EMF. Terminals X and Y have no net charge.
2. Angle = 90°. Here the plane of the loop is parallel to the field, so $\theta = 90°$. No flux is enclosed by the loop (as $\cos 90° = 0$). However, in this position a slight rotation will cause the wire to cut through the flux lines, so an EMF is generated. This will be the position for maximum EMF. Using Fleming's right-hand rule for generators, we can see that the current is clockwise, and thus terminal X becomes positive, and Y becomes negative. Note: as shown earlier there are other ways to work out the direction of the current. There is no one right way.
3. Angle = 180°. The loop is again perpendicular to the field but has now rotated 180° from its starting point. Zero EMF is developed, as it was for an angle of 0°. Terminal X and Y have no net charge.
4. Angle = 270°. The loop is again parallel to the field, similar to position 2 except that it is now upside down. Maximum EMF is developed, and the current is again clockwise. However, Y has become positive and X has become negative.
5. Angle = 360°. We are back to the starting position with X and Y uncharged.

AC waveforms

The waveform (Figure 7) shows the voltage on terminal X as a function of the angle of rotation. It is sinusoidal, and one full cycle occurs every 360°. In one-quarter of a cycle (90°), the EMF rises from zero volts to a maximum. The maximum is called the "peak EMF" and has the symbol V_{peak}. Its magnitude depends on the area of the loop, the number of turns, the field strength and the rate of rotation.

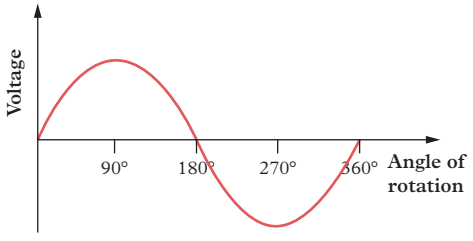


FIGURE 7 An AC waveform

What is average EMF?

This is a tricky term as it means different things to different people. Electrical engineers may say that for an alternating current the average EMF for one rotation of a coil is zero as it starts at 0 V and ends at 0 V, and in between this the positives and negatives cancel out. They prefer to talk about the "RMS voltage, V_{RMS}" or root-mean-square voltage, which gives a weighted average of EMFs over the quarter-cycle, but that is not a part of the syllabus.

In Unit 3 Physics the average EMF (as used in the external exam) is the value proportional to the rate of change of flux, as determined by Faraday's law, $emf = -N\frac{\Delta\phi}{\Delta t}$. That makes it simpler to calculate, although some would think it misleading. An example is given in Worked example 8.5C, so just follow that.

Worked example 8.5C

Calculating the EMF of a rotating loop

A rectangular open loop measuring 25 cm by 40 cm is placed in a uniform magnetic field of strength 0.15 T. It is oriented so that the plane of the loop is parallel to the field lines, as shown in Figure 8.

The loop is rotated clockwise at a frequency of 50 revolutions per second.

Calculate the average voltage produced. Give your answer to an appropriate number of significant figures. (4 marks)

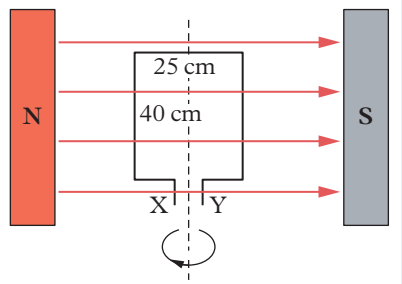


FIGURE 8 A loop with its plane parallel to the magnetic field lines and its axis of rotation perpendicular to the field

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formulas and use them to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	Length = 40 cm = 0.40 m, width = 25 cm = 0.25 m, frequency, $f = 50\ s^{-1}$, magnetic field, $B = 0.15$ T $\phi = BA\cos\theta$, $emf = -N\frac{\Delta\phi}{\Delta t}$
Step 3: We need to calculate the change in flux, $\Delta\phi$, to use in Faraday's law. To do so we need to calculate the minimum flux, ϕ_{min}, and the maximum flux, ϕ_{max}. The flux is at a minimum when the plane of the loop is parallel to the field ($\theta = 90°$), and at a maximum when the plane of the loop is at right angles to the field ($\theta = 0°$). This is one-quarter of a rotation. (1 mark for "Determines the initial and final flux threading the loop during one-quarter of a turn")	$\phi_{max} = BA\cos 0°$ $= 0.15 \times 0.25 \times 0.40 \times 1$ $= 0.015$ Wb $\phi_{min} = BA\cos 90°$ $= 0.15 \times 0.25 \times 0.40 \times 0$ $= 0$ Wb (1 mark)
Step 4: We now need to calculate the change in flux, $\Delta\phi$, for one-quarter of a rotation. (1 mark for "Determines the change in flux for a one-quarter turn")	$\Delta\phi = 0.015 - 0$ $= 0.015$ Wb (1 mark)
Step 5: We need to calculate the period of the rotation and hence the time for one-quarter of a rotation.	Time, T, for 1 rotation: $T = \frac{1}{f} = \frac{1}{50} = 0.020$ s Time, Δt, for $\frac{1}{4}$ rotation = 0.0050 s

Think	Do
Step 6: We can now calculate the average EMF generated over the one-quarter turn. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Use the correct units. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer") Note that the question asks for "average" EMF. This is not an appropriate quantity to calculate for an AC generator. However, just use the formula to the right and you will have an acceptable answer.	$emf = -N\frac{\Delta\phi}{\Delta t}$ $\lvert emf \rvert = \frac{0.015}{0.0050}$ (1 mark) $= 3.0\,\text{V}$ (1 mark)

Your turn

A rectangular coil of 100 loops has dimensions 0.50 m by 0.20 m and turns at a speed of 120 revolutions per second along an axis perpendicular to a uniform magnetic field. An average EMF of 1.2 V is generated. **Calculate** the magnetic field strength. Give your answer to an appropriate number of significant figures. (3 marks)

What variables affect the EMF in practice?

In the power supply industry, the voltage must be within a certain range; otherwise, it becomes difficult for users to control their own equipment. For example, when the voltage is too high, it becomes impossible for people with solar panels on their roof to feed electricity back into the grid. A high voltage also reduces the life of equipment, particularly if it results in higher temperatures. Likewise, too low a voltage can lead to "brown-outs", where equipment flickers off and on. Some devices will be severely affected, while others may not be affected at all. In Queensland the range is 230 V + 10% − 6%, which gives a range of 216–253 V. Generators must also keep their frequency within the range of 50 ± 0.15 Hz, as many timing devices depend on an accurate power-supply frequency. They can cope with short periods of 50 ± 0.5 Hz but may have to switch off some circuits to bring it back up to speed. For some renewable sources of power such as wind and water (hydro), the change in input speed may be difficult to control, but scientists and engineers have developed devices that can manage that.

This leads us to the question: how can we work out how fast a loop of a certain size needs to rotate within a given field to achieve a given voltage?

Worked example 8.5D

Calculating the speed of a rotating loop

A rectangular coil is made up of 1,530 loops, each measuring 50 cm by 20 cm. It is placed in a uniform magnetic field of strength 0.15 T and is oriented so that the plane of the loop is parallel to the field line, similar to the setup in Figure 8 in Worked example 8.5C.

Calculate the number of revolutions per second (rps) needed to generate an average EMF of 11.0 kV. Express your answer to the nearest whole number. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find the number or answer by using mathematical processes and showing working. We must identify the appropriate formulas and use them to find an answer.
Step 2: Gather the relevant data from the question and identify the appropriate formula.	Length = 50 cm = 0.50 m, width = 20 cm = 0.20 m, $B = 0.15\,\text{T}$, $emf = 11.0\,\text{kV} = 11{,}000\,\text{V}$ $\phi = BA\cos\theta$, $emf = -N\frac{\Delta\phi}{\Delta t}$
Step 3: To calculate the rotational speed in revolutions per second (rps), we need to calculate the time taken to make one-quarter of a revolution so that we can apply Faraday's law. This is because the average EMF will be the rate of change of flux from minimum flux to maximum flux. The flux is at a minimum when the plane of the loop is parallel to the field ($\theta = 90°$), and at a maximum when it is at right angles to the field ($\theta = 0°$). This is one-quarter of a rotation. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Determines the change in flux for one-quarter turn")	$emf = -N\frac{\Delta\phi}{\Delta t}$ $\lvert emf\rvert = -N\frac{B(A\cos 0° - A\cos 90°)}{\Delta t}$ (1 mark) $\Delta t = -N\frac{BA(\cos 0° - \cos 90°)}{emf}$ $= \frac{1{,}528 \times 0.15 \times (0.50 \times 0.20) \times (1 - 0)}{11{,}000}$ $= 0.0021\,\text{s}$ (for $\frac{1}{4}$ turn) (1 mark)
Step 4: We now need to calculate the time for one full rotation – its period, T. (1 mark for "Calculates period correctly") From that we can calculate its frequency, f, which is the number of rotations per second. Give your answer to the nearest whole number and use the correct units. (1 mark for "Provides correct answer")	$T = 4 \times 0.0021$ $= 0.0083\,\text{s}$ (1 mark) $f = \frac{1}{T}$ $= \frac{1}{0.0083}$ $= 120$ rps (to the nearest whole number) (1 mark)

Your turn

A square coil with a side of 20 cm contains 6,500 turns. It is placed in a uniform magnetic field of 0.80 T with the plane of the loop perpendicular to the field.

Calculate the frequency in revolutions per second (rps) that will generate a desired average EMF of 167.5 V. Express your answer to the nearest whole number. (4 marks)

How is electromagnetic induction used with renewable energy?

Electric generators rely on the concepts of electromagnetic induction developed by Faraday in 1831. You may think that something more modern has been developed in today's world or quantum physics and nanotechnology. Actually, a lot has been done in the past two centuries to improve the efficiency of generators, but they still rely on the same principle of a coil being rotated in a magnetic field, or vice versa. Renewable energy generators have just replaced the non-renewable fuels such as coal, oil and nuclear that turn the coil with renewable ones such as wind, hydro and geothermal.

FIGURE 9 Inserting a metal fluoride layer in multilayered perovskite-silicon solar cells can enhance performance.

However, progress is being made in other forms of renewable energy such as batteries and solar photovoltaic cells. Thousands of research teams around the world are developing new methods for renewable energy production. Projects are underway in Queensland to investigate new materials (Figure 9), including organic semiconductor materials, to increase the performance of solar cells. In other areas researchers are making great progress with sustainable, long-lasting and low-cost energy storage devices, including better battery technology and supercapacitors.

The teams behind this research will move on to new areas, but using scientific knowledge to develop useful technology will continue as always.

Skill drill

Identification of relevant trends

Science inquiry skills: Processing and analysing data (Lesson 1.7); Planning investigations (Lesson 1.4)

Students investigated the research question: does the voltage ratio for a simple transformer equal the turns ratio? They used two solenoids and linked them with a single 10 mm diameter length of steel rod as shown in Figure 10.

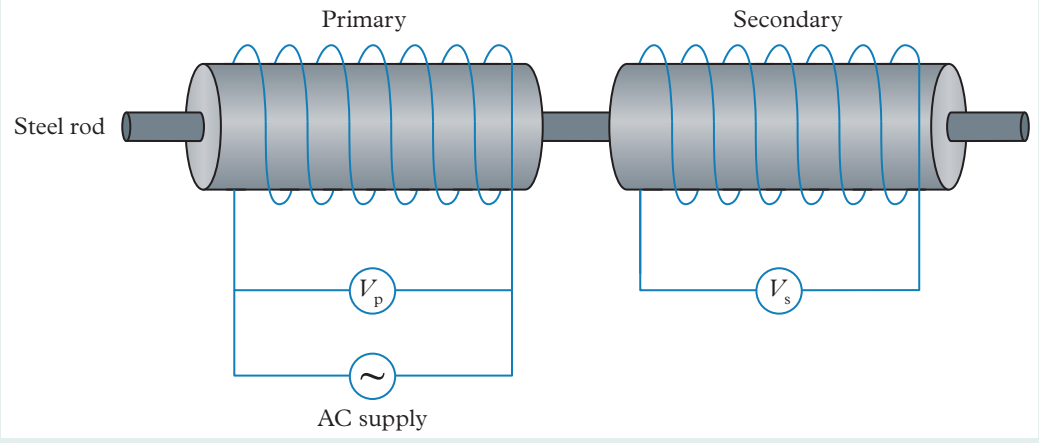


FIGURE 10 A simple transformer as used in a student experiment

The voltage across the primary coil was supplied by a variable AC power supply with a constant frequency of 50 Hz. As they varied the primary voltage they measured the secondary voltage. The ratio $\frac{V_p}{V_s}$ is called the voltage ratio and in theory should equal the turns ratio $\frac{N_p}{N_s}$. The students only had time to take one set of readings. Their results are given in Table 1.

TABLE 1 Student results for a practical on transformers

V_p (V)	1.6	3.7	5.6	7.9	10.1	12.3
V_s (V)	0.1	0.5	1.1	1.7	2.4	3.0

Practise your skills

1. Plot a graph with V_p on the x-axis and determine the equation for the line. **Determine** whether the graph shows direct proportion between the variables, as proposed in the rationale. (3 marks)
2. **Determine** whether the turns ratio is equal to the gradient or $\frac{1}{\text{gradient}}$. (1 mark)
3. **Evaluate** whether the results collected "sufficient and relevant" data to draw a conclusion. (2 marks)

Study tip

One of the suggested experiments is to investigate the induced EMF in an AC transformer. An interesting modification would be to research the effect of changing frequency on the efficiency, with particular emphasis on efficiency peaks. This could lead to a discussion of mutual induction (back EMF) between conductors in the transformer winding.

Real-world physics

Transformers in practice

Transformers sound like marvellous devices for varying AC voltages, and they are. However, energy is not created in these devices – the electrical power available at the output is never greater than the electrical power supplied to the input of the transformer. An ideal or "lossless" transformer has no energy loss, so:

input power in the primary coil = output power in the secondary coil

$$P_p = P_s$$

$$I_p V_p = I_s V_s$$

In essence, when the voltage in the secondary coil is greater than in the primary coil (step up), the current in the secondary coil must be less than in the primary coil (step down).

In practice, although transformers are very efficient devices, there is always some energy loss. Where does the energy go? The answer is in the core. When an alternating current passes through the primary coil, tiny circulating currents called eddy currents are set up in the soft iron core. This causes heating and represents lost energy.

This is obviously not a good thing, and to minimise it, laminated cores are used (Figure 11). Insulated laminations are used to separate the core into smaller sections, significantly reducing the size of eddy currents and thus reducing the energy lost as heat.

The wires of the coil also lose energy through ohmic heating due to coil resistances.

The efficiency (η) of a transformer is measured by the ratio of power produced in the secondary coil to the power input from the primary coil:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{V_s I_s}{V_p I_p} \times 100\%$$

It is also known that the frequency of the alternating current affects the efficiency of a transformer. Some frequencies are favoured, but in general efficiency is reduced at higher frequencies. Typical electromagnetic transformers are about 90–95% efficient in operation. The point at which efficiency is a maximum is called "resonance" and is a point at which there is a maximum transfer of energy.

FIGURE 11 Transformer core being assembled with sheets of silicon steel laminations

Apply your understanding

1 **Calculate** the efficiency of a power transformer used in a computer adapter that supplies a current of 0.9 mA at 230 V AC to produce 20 mA at 12 V AC. (2 marks)
2 **Explain** which coil has the greater current in a step-down transformer: the primary coil or the secondary coil. (2 marks)
3 **Evaluate** this statement: "Transformers can never be more than 100% efficient." (2 marks)

Challenge

Oil, heat and transformers

Small transformers are air cooled, but the big transformers on power poles along the street often have a jacket of oil around them. Why do they get hot, and what is the purpose of the oil? (1 mark)

Check your learning 8.5

Check your learning 8.5: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** whether a transformer will operate if a 1.5 V battery is used as the primary input. (2 marks)

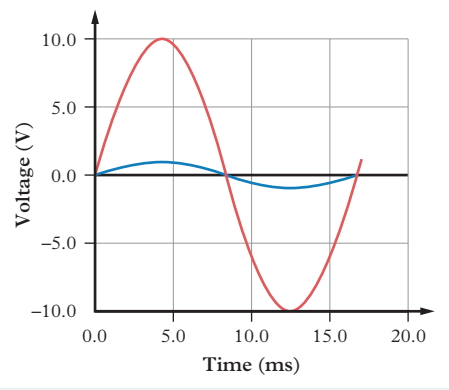
2 An iron core is used in most transformers. **Explain** whether it true to say that without it, the transformer output produces a lower power than with it. (2 marks)

3 **Explain** whether it is true to say that a step-up transformer steps up the voltage but it also steps down the number of turns. (2 marks)

4 The diagram shows the primary and secondary waveforms of a step-up transformer. **Calculate**

a the approximate turns ratio (2 marks)

b the frequency of the AC signal. (2 marks)



Analytical processes

5 A transformer with a turns ratio ($N_p : N_s$) of 26 : 1 is used to charge a portable drill from a domestic supply of 230 V. The charger draws 0.30 A from the household outlet.

a **Determine** the voltage output of the charger. (2 marks)

b **Determine** whether the transformer is step-up or step-down. (1 mark)

Knowledge utilisation

6 A street transformer converts 12 kV in the overhead wires to 415 V for household use.

a **Determine** the turns ratio required of the transformer used. (2 marks)

b **Propose** whether it would be a step-up or a step-down transformer. (1 mark)

7 Neon lights require at least 12 kV for their operation, and big advertising signs operate from a 230 V line.

a **Calculate** the turns ratio for the transformer that would be needed. (2 marks)

b **Predict** whether it would be a step-up or a step-down transformer. (1 mark)

Practical

Lesson 8.6
Transformers: Induced EMF from an AC generator

Learning intentions and success criteria

oxforddigital

This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.

Lesson 8.7
Electromagnetic radiation

Key ideas

- → Electromagnetic radiation is radiant energy consisting of synchronised oscillations of electric and magnetic fields – electromagnetic waves – propagated at the speed of light in a vacuum.
- → Examples of electromagnetic radiation range from low frequency waves up to high frequency ionising gamma rays, with visible light in-between.

Learning intentions and success criteria

What is electromagnetic radiation?

In the previous sections, you learnt that electric fields can produce magnetic fields, and that magnetic fields can produce electric fields. If there was no loss of energy during this process, we could have a self-perpetuating (self-maintaining) system of fields in sync with each other. We do have this – it is called electromagnetic radiation.

In the 1860s, James Clerk Maxwell (1831–79), a Scottish mathematician and theoretical physicist, first proposed that light consists of **electromagnetic waves**. These propagate through space as changing electric and magnetic fields that are in phase and at right angles to each other and the direction of propagation (Figure 1). This model is still accepted today. We refer to it as **electromagnetic radiation**.

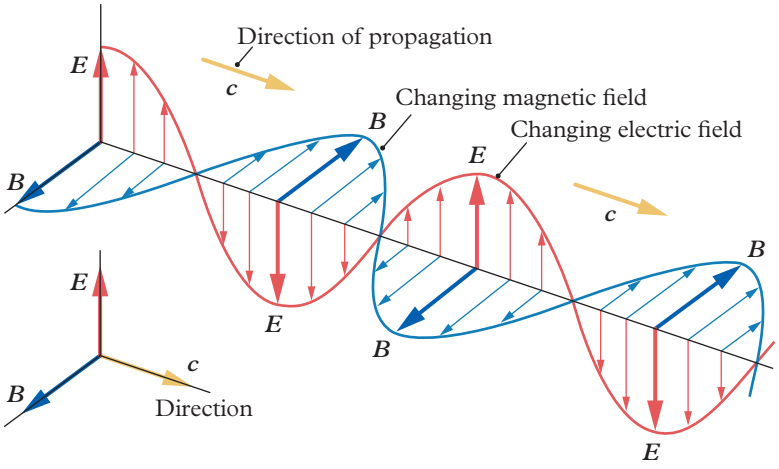


FIGURE 1 An electromagnetic wave has the changing magnetic fields, *B*, and electric fields, *E*, at right angles to each other and to the direction of propagation. The waves are only shown propagating in one direction here. In reality, they would be in all directions.

How are electromagnetic waves produced?

A simple way to understand the production of electromagnetic waves is to imagine a short conducting rod that can be connected to a battery. As the charge oscillates up and down the rod, an electric field is produced in the plane of the rod, but an oscillating magnetic field is produced perpendicular to the rod. What is interesting is that although they oscillate at right angles to each other, they oscillate in phase. This combination of mutually linked fields is called an electromagnetic field or wave.

electromagnetic wave
waves produced by an oscillating electric charge that radiate out at the speed of light as mutually perpendicular electric and magnetic fields

electromagnetic radiation
radiant energy consisting of synchronised oscillations of electric and magnetic fields – electromagnetic waves – propagated at the speed of light in a vacuum

transverse wave
a wave where the direction of oscillation of particles is perpendicular to the direction of energy transfer

synchronised
waves that have the same frequency and a constant phase difference. For electromagnetic radiation there is no phase difference; that is, when one reaches a maximum so does the other.

How the electric and magnetic fields are linked

A steadily changing magnetic field can induce a steadily changing electric field and vice versa. Their properties can be summarised as follows.

- An oscillating electric field, E, generates an oscillating magnetic field, B.
- An oscillating magnetic field, B, generates an oscillating electric field, E.
- The planes of the two fields are perpendicular (at 90°) to each other.
- The fields are also perpendicular to the direction of travel of the wave, meaning that an electromagnetic wave is a **transverse wave**.
- The oscillations are **synchronised** and are in phase.
- The energy of the wave is stored in the linked electric and magnetic fields.
- Electromagnetic waves travel away from the source at the speed of light, c.
- They do not need a medium for their propagation (they can travel in a vacuum).
- When the source is turned off, the waves keep travelling.

Worked example 8.7A

Describing features of electromagnetic radiation

Describe how electromagnetic radiation is propagated by the interaction between electric and magnetic fields. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Describe" means to give an account of the characteristics or features of something. This question is worth 2 marks, so you need to present two cognitions (points). It mentions the interaction between two things (electric and magnetic fields), so we should describe them (1 point) and then describe the relationship between them (1 point) and what this results in.
Step 2: Gather the relevant information from the question. We need to describe what electromagnetic radiation is.	Electromagnetic radiation is the propagation of energy through space by mutually linked electric and magnetic fields.
Step 3: We need to say what an electric field is and what oscillating means.	An electric field is the region of space near an electric charge. An oscillating charge is one that moves back and forth and therefore has an associated oscillating electric field.
Step 4: We need to say what an oscillating electric field produces.	This oscillating electric field induces a magnetic field that oscillates at the same rate.
Step 5: We need to describe how one "propagates" the other. (1 mark for "Describes the production of a magnetic field")	This oscillating magnetic field then induces an oscillating electric field. (1 mark)
Step 6: We need to describe how the two fields are related to each other. (1 mark for "Describes the formation of a magnetic field perpendicular to the electric field")	This electric field is perpendicular to the magnetic field. (1 mark)
Step 7: We need to conclude by linking the mutually oscillating fields to the final term "electromagnetic radiation".	These linked fields propagate outwards, in a direction perpendicular to both fields, as electromagnetic radiation.

Your turn

Describe the concept of electromagnetic radiation in terms of electromagnetic induction. (2 marks)

Challenge

Speed of light – theoretical value

When Maxwell developed his equations for electromagnetic waves, he found that the velocity was dependent on two constants that you used in previous modules.

For electric field strength he used the permittivity of free space:

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$$

For magnetic field strength he used the permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{TmA^{-1}}$$

Maxwell showed that the speed of light could be given by:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Compare this value to the accepted value of 299,792,458 $\mathrm{m\,s^{-1}}$. (2 marks)

What is the electromagnetic spectrum?

The wave equation

The wavelength and frequency of a wave are connected by the speed of light, c:

$$v = f\lambda \text{ or } c = f\lambda$$

where v is the speed of propagation of the wave (for light, $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$ in air or a vacuum), f is the frequency and λ is the wavelength.

Spectral regions

Electromagnetic waves are split into different categories or "spectral regions" based on their frequency or wavelength, as shown in Figure 2.

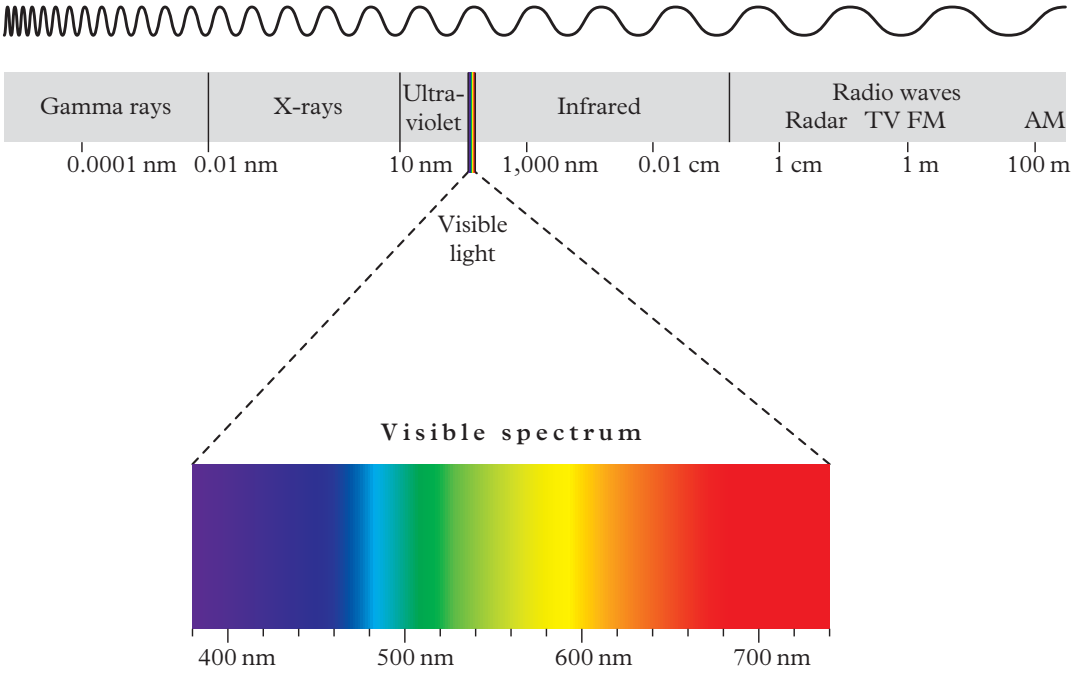


FIGURE 2 The spectral regions of electromagnetic waves

Visible light, for example, ranges from violet to red. Violet light has a wavelength of 400 nm (400×10^{-9} m), corresponding to a frequency of 7.5×10^{14} Hz. Red light has a wavelength of 700 nm, and a frequency of 4.3×10^{14} Hz. Any electromagnetic wave with a frequency (or wavelength) between those values can be seen by humans.

Visible light makes up a very small part of the full electromagnetic spectrum. Electromagnetic waves with higher energy (higher frequency, shorter wavelength) than visible light include ultraviolet light, X-rays and gamma rays. Lower energy waves (lower frequency, longer wavelength) include infrared, microwaves, and radio and television waves.

Real-world physics

How dangerous is mobile-phone radiation?

Two stories that keep doing the rounds are that mobile phones can cook eggs and that overhead power lines will give you cancer. Some people have claimed that the radiation from a mobile phone cooks the proteins in an egg, and they then ask "imagine what it can do to the proteins in our brains…" However, the average power from a mobile phone is about 0.25 W – and that is when the phone is a long distance from the base station. When near the base station, the power drops to about 2 mW. Is that enough to cook and egg – or do anything for that matter? It turns out that it is not possible, but now many people are convinced it is true. The same goes for cancer from wi-fi or under power lines. Is there enough transmitted radiation to do harm?

FIGURE 3 Sarah Loughran

You could ask whether there is any scientific evidence about the risks. The problem is that the effect is likely to be small if at all because of the low power, and any damage might not show up for a long time. It is sometimes difficult to get a definite answer as the threshold for damage is a matter of definition and what is "damage" to one researcher is acceptable to another. The best way to tackle this for high school Physics is to consider reviews of the evidence by experts in the field and see if there is any consensus.

For example, a 2024 review by the World Health Organization looked more closely at possible health effects associated with exposure to radio waves. Associate Professor Sarah Loughran said: "This systematic review provides the strongest evidence to date that radio waves from wireless technologies are not a hazard to human health. No association between mobile phone use and brain cancer, or any other head or neck cancer, was found. There was also no association with cancer if a person used a mobile phone for ten or more years (prolonged use)."

Dr Loughran is currently Director of Radiation Research and Advice, and the Principal Researcher and Director of the Electromagnetic Energy Program at the Australian Radiation Protection and Nuclear Safety Agency, Chief Investigator for the Australian Centre for Electromagnetic Bioeffects Research and an adjunct researcher at the University of Wollongong.

Source: Loughran, S., & Karapidis, K. (2024, September 4). "Mobile phones are not linked to brain cancer, according to a major review of 28 years of research". *The Conversation*. https://theconversation.com/mobile-phones-are-not-linked-to-brain-cancer-according-to-a-major-review-of-28-years-of-research-237882

Apply your understanding

Two graphs presented in the article (Figure 4 and Figure 5) show mobile phone use in Australia and brain-cancer rates.

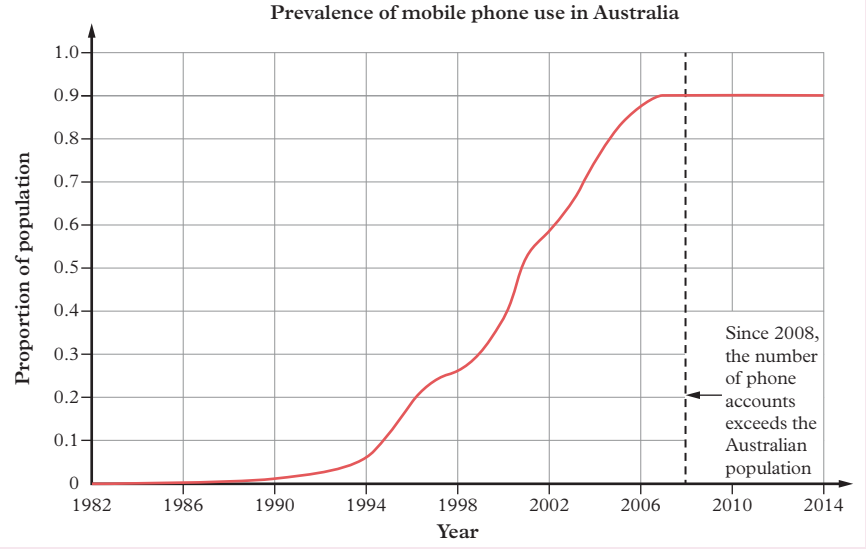


FIGURE 4 The rise in mobile phone usage in Australia

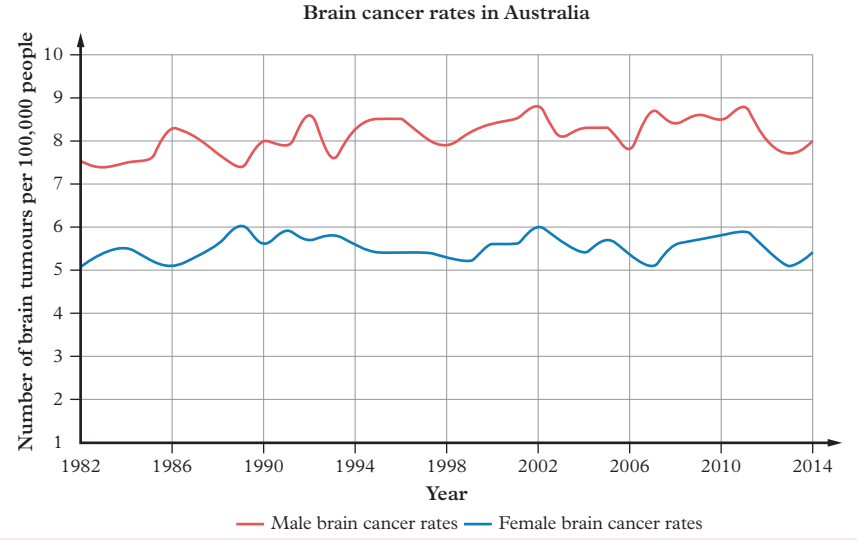


FIGURE 5 Variations in brain cancer rates in Australia

1 Based on the graphs, **comment** on the evidence for brain cancer from mobile phone use. (2 marks)

2 **Infer** why the graph of mobile phone use levels off after the dotted line. (2 marks)

3 The evidence came from 63 studies on mobile phones and cancer published between 1994 and 2022. **Evaluate** whether this information would constitute a selection of "sufficient and relevant" sources on which to draw a conclusion. (2 marks)

4 Mobile phones operate in the 2,000 MHz region and beyond. **Determine** whether this is higher or lower than the frequencies of visible light. Use Figure 5 to help you decide. (2 marks)

5 **Determine** which has higher energy: mobile phone radiation or ultraviolet radiation. (1 mark)

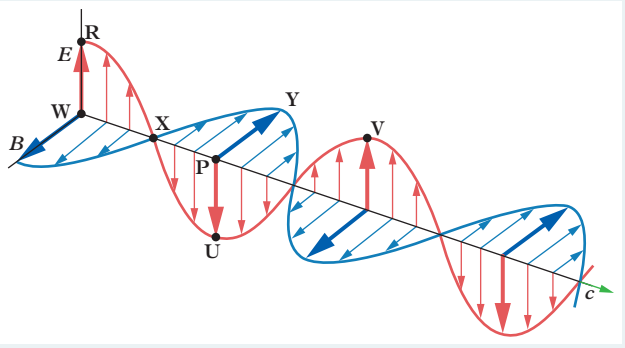
Note: if you were thinking of using "Mobile phones cause cancer" as a claim for your Research investigation (IA3) be aware that this topic is from Unit 3 (not Unit 4 as required) so is out of sequence. It may also veer too far into the biology of cancer cells and be outside the scope of physics.

Check your learning 8.7

Check your learning 8.7: Complete these questions online or in your workbook.

Retrieval and comprehension

Use the diagram to answer the following questions.



1 An electromagnetic wave is shown in the diagram.
   - **a** **Identify** the colour that represents the electric field and the colour that represents the magnetic field. Give reasons for your answer. (1 mark)
   - **b** **Identify** the direction in which the wave is travelling and explain your reasoning. (1 mark)
   - **c** **Explain** how you can tell that the two waves are in phase. (1 mark)
   - **d** **Explain** whether the wave is transverse or longitudinal. (2 marks)
   - **e** **Identify** two points that are one wavelength apart. (1 mark)

2 **Identify** the direction of the electric field and of the magnetic field at the following points in the diagram. (1 mark each)
   - **a** P **b** Q **c** S

3 **Identify** which pair of points on the wave in the diagram are in phase: P and U, or T and U. **Explain** your reasoning. (2 marks)

4 **Calculate** the wavelength of a microwave of frequency 5×10^{10} Hz. (1 mark)

Analytical processes

5 **Compare** the magnitude and direction of the field strengths at points R and U in the diagram. (2 marks)

6 Consider the wave in the diagram. Imagine the distance between points X and Z is 1.0 m.
   - **a** **Determine** the frequency of the wave. (1 mark)
   - **b** **Identify** the spectral region to which it belongs (visible, radio etc.). (1 mark)

Knowledge utilisation

7 The wave in the diagram represents green light. **Evaluate** the approximate distance between X and Z. (3 marks)

8 **Decide** which wave crest came first in time: V or R. **Justify** your answer. (2 marks)

MODULE 8

Lesson 8.8
Review: Electromagnetic induction and radiation

Summary

8.1
- The characteristics of a magnetic field can be described in terms of magnetic flux and magnetic flux density.
- Problems involving the magnetic flux in an electric current-carrying loop can be solved using $\phi = BA\cos\theta$.

8.2
- Electromagnetic induction involves the concepts of electromotive force (EMF) and Faraday's law.
- By moving a conductor in a magnetic field, an EMF can be induced.
- Problems involving electromagnetic induction can be solved using Faraday's law equations $emf = -\frac{N\Delta(BA_{\perp})}{\Delta t}$ and $emf = -N\frac{\Delta\phi}{\Delta t}$.

8.3
- Practical: Induction of an electric current using a magnet and coil

8.4
- Lenz's law can be used to determine the direction of an induced current or EMF during electromagnetic induction.
- Lenz's law is consistent with the principle of conservation of energy.

8.5
- Transformers work on the principles of Faraday's law and electromagnetic induction.
- Problems involving transformers can be solved using $I_p V_p = I_s V_s$ and $\frac{V_p}{V_s} = \frac{N_p}{N_s}$.

8.6
- Practical: Transformers: induced EMF from an AC generator

8.7
- Electromagnetic radiation is radiant energy consisting of synchronised oscillations of electric and magnetic fields – electromagnetic waves – propagated at the speed of light in a vacuum.
- Examples of electromagnetic radiation range from low frequency waves up to high frequency ionising gamma rays, with visible light in-between.

Key formula

Flux in a loop	$\phi = BA\cos\theta$
Faraday's law of induction (1st version)	$emf = -\frac{N\Delta(BA_{\perp})}{\Delta t}$
Faraday's law of induction (2nd version)	$emf = -N\frac{\Delta\phi}{\Delta t}$
Transformer – energy conservation relationship	$I_p V_p = I_s V_s$
Transformer formula	$\frac{V_p}{V_s} = \frac{N_p}{N_s}$

Review questions 8.8A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 Magnetic flux density has the units

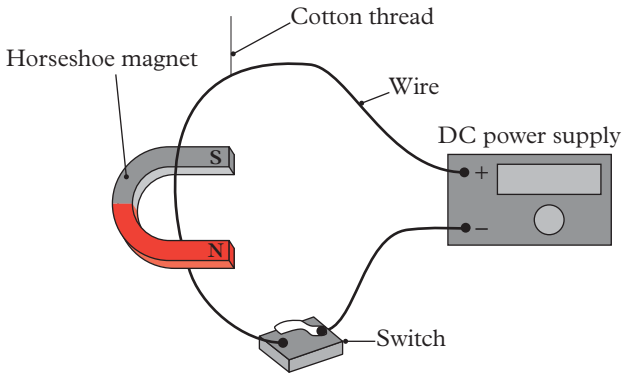
A $\mathrm{Wb\,m^{-2}}$

B $\mathrm{Wb\,A^{-1}\,m^{-1}}$

C $\mathrm{A\,m^{-1}}$

D $\mathrm{T\,m}$

2 A student sets up the apparatus shown in the diagram.



When the student closes the switch, a 3.0 A current moves through the wire, and the wire experiences a force and moves. With the switch closed, the student moves the wire so that it experiences a force of zero.

Determine the new position of the wire.

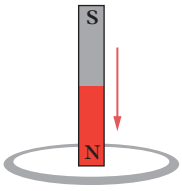
A The wire is parallel to the direction of the magnetic field.

B The wire is at 45° to the direction of the magnetic field.

C The wire is at 60° to the direction of the magnetic field.

D The wire is perpendicular to the direction of the magnetic field.

3 A bar magnet with the north pole facing downwards is held above a horizontal circular coil and allowed to fall through the loop, as shown in the diagram. Which of the following statements about the induced current is true, if the apparatus is viewed from above?



A It flows in a clockwise direction.

B It flows in an anticlockwise direction.

C It flows first in a clockwise and then in an anticlockwise direction.

D It flows first in an anticlockwise and then in a clockwise direction.

4 Determine which scenario would increase the voltage produced when a permanent magnet is brought towards the end of a solenoid.

A Having two magnets with unlike poles together

B Increasing the angle between the direction of the magnet and the axis of the coil

C Increasing the number of turns

D Reducing the number of lines of flux by using a weaker magnet

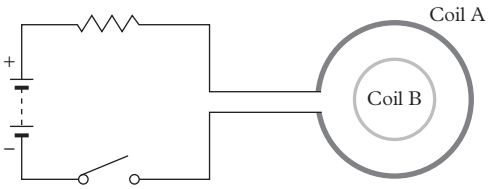
5 Coil A is connected to a circuit that includes a battery, a switch and a resistor. Coil B lies in the same plane as coil A, as shown in the diagram. Determine the direction of the induced current in coil B at the moment when the switch is closed.

A Clockwise

B Anticlockwise

C Clockwise to anticlockwise

D Anticlockwise to clockwise



6 The electric and magnetic fields in an electromagnetic wave are oriented

A parallel to the direction of travel of the wave, as well as to each other.

B parallel to the direction of travel of the wave, and perpendicular to each other.

C perpendicular to the direction of travel of the wave, and parallel to each other.

D perpendicular to the direction of travel of the wave and also to each other.

7 Select which one of the following statements is correct for a step-up transformer.

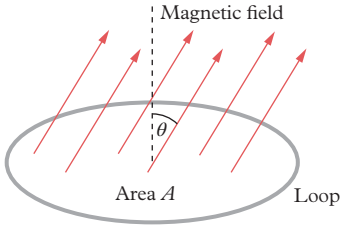
A Primary current < secondary current

B Primary voltage < secondary voltage

C Primary frequency < secondary frequency

D Primary power < secondary power (for a non-ideal transformer)

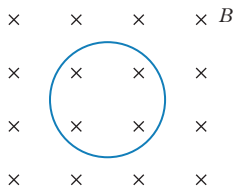
8 The diagram shows a uniform magnetic field of strength B threading a circular loop of area A. The field makes an angle θ with a perpendicular to the plane of the loop. The strength of B is increasing at a uniform rate of R. Which one of the following best describes the EMF induced in the loop?



A $RA\cos\theta$ **B** $RA\sin\theta$

C $\frac{RA}{\cos\theta}$ **D** $\frac{RA}{\sin\theta}$

9 Determine the direction of the induced current in the circular loop of wire in the diagram if the magnetic field, B, increases.



A Clockwise

B Anticlockwise

C Into the page

D Out of the page

10 To reduce power losses in the transmission lines between a power station and a factory, two transformers are used. The first is located at the power station and the second is at the factory. Which one of the following gives the correct types of transformer?

A Step-down, step-up

B Step-down, step-down

C Step-up, step-up

D Step-up, step-down

11 Which one or more of the following statements must be true about an ideal transformer?

I The power output exceeds the power input.

II The magnetic flux produced by the primary coil entirely links the secondary coil.

III There are more turns on the secondary coil than on the primary coil.

A I and II only

B I and III only

C II only

D III only

12 An ideal transformer has N_p primary turns and N_s secondary turns. The electrical power input into the primary coil is P_p. Determine the power output of the secondary coil, P_s.

A $\frac{N_p}{N_s}P_p$ **B** $\frac{1}{P_p}$

C $\frac{N_s}{N_p}P_p$ **D** P_p

13 One coil in a transformer has 500 turns and experiences a voltage of 20 kV. Meanwhile, the other experiences a voltage of 300 kV. Determine the number of turns on the other coil.

A 3,000

B 5,500

C 7,500

D 9,000

14 The diagram shows a transformer. If the upper coil is the primary, identify what type of transformer it is.

A Step-up

B Step-down

C Switch mode

D Grounding

15 All electromagnetic waves travel through a vacuum at

A the same speed.

B speeds that are proportional to their frequency.

C speeds that are inversely proportional to their frequency.

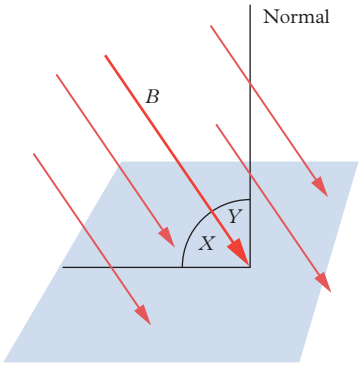
D none of the above.

Review questions 8.8B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 The diagram shows lines of flux (B) cutting (threading) a horizontal plane. The formula $\phi = BA\cos\theta$ is used to calculate the flux. **Explain** whether angle X or angle Y represents θ. (2 marks)



17 A magnet is brought up to the end of a solenoid and an EMF is induced. **Describe** how the EMF would change if

- **a** a similar coil with fewer turns was used (1 mark)
- **b** the coil was moved faster (1 mark)
- **c** a stronger magnetic field was used. (1 mark)

18 **Explain** why the operation of a transformer depends on AC rather than DC voltage. (2 marks)

19 **Identify** the direction in which a wire would have to move in a magnetic field for there to be no induced current. (1 mark)

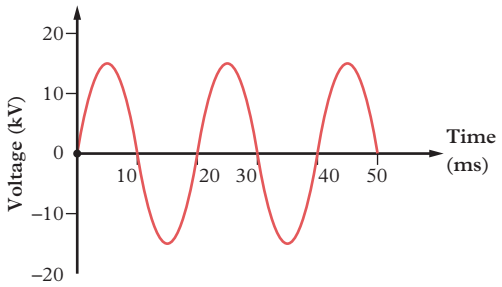
20 **Explain** why the solenoids used in electromagnets and transformers need to have soft iron cores, and state why the cores are laminated. (2 marks)

21 **Describe** the factors affecting the output EMF of any practical transformer. If these transformers are not ideal, state where the energy is lost between input and output. (2 marks)

22 A transformer is designed to step down the 230 V mains voltage used in Australia to a voltage of 6.3 V AC. **Calculate** the number of turns on the secondary coil given that the primary coil has 2,000 turns. (2 marks)

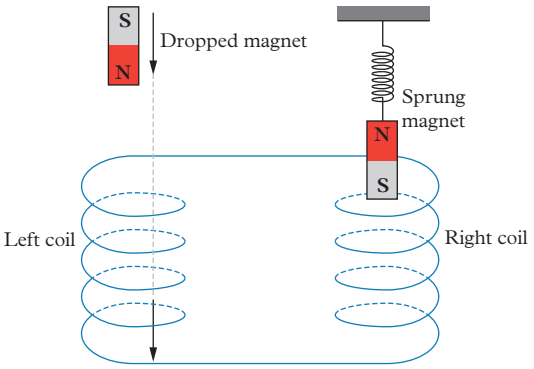
23 The graph shows the output of a generator that is rotating at 50 Hz in a magnetic field of 0.40 T. **Describe** the changes to the output waveform when

- **a** the magnetic field changes to 0.80 T (3 marks)
- **b** the field remains the same, but the rate of rotation increases to 100 Hz. (2 marks)

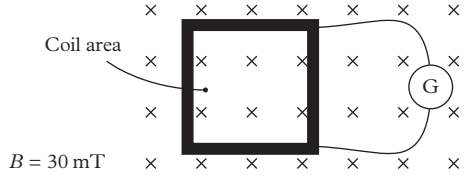


24 The diagram illustrates two solenoid coils connected in series.

- **a** **Describe** the behaviour of the suspended magnet in the right-hand coil after a bar magnet is dropped through the left-hand coil as shown. (1 mark)
- **b** **Explain** your reasoning. (3 marks)



25 A galvanometer is connected across a 200-turn coil of area 50 cm^2, as shown in the diagram. This assembly is perpendicular to the field and its intensity varies from 30 mT to 10 mT in a time of 0.02 s. **Calculate** the voltage reading on the galvanometer. (3 marks)



Analytical processes

26 **Distinguish** between magnetic flux and magnetic field. (1 mark)

27 **Determine** which is greater: 15 mT or 0.016 T. (1 mark)

28 **Categorise** the following field strengths from smallest to largest. (1 mark)
0.011 T, 1.8×10^{-3} T, 15 mT, 16,500 nT

29 **Determine** the magnitude of magnetic flux in the following situations.

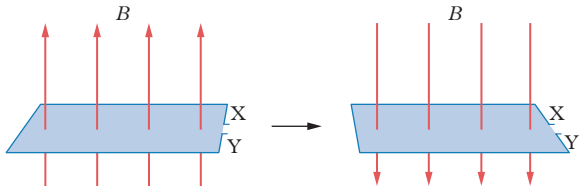
a A 1 cm^2 loop is placed perpendicular to a magnetic field of strength 4.5×10^{-10} T. (2 marks)

b A 0.5 cm^2 loop is placed perpendicular to the magnetic field 5 cm from a wire carrying a steady current of 5 A. (2 marks)

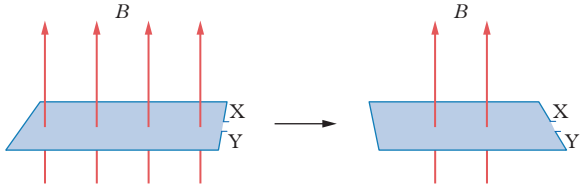
c A 60 mm^2 loop is placed in a magnetic field of strength 6.0×10^{-9} T such that the plane of the loop makes an angle of 60° to the field lines. (2 marks)

d A loop of diameter 20 cm is placed in a magnetic field of strength 5 mT at an angle of 35° between the plane of the loop and the field direction. (2 marks)

30 **Determine** the direction of the current (in terms of X and Y) when the change shown in the diagram is made to the field threading a loop. (3 marks)

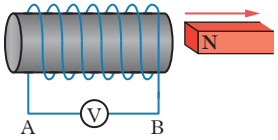


31 **Determine** which terminal, X or Y, becomes positive when the change shown in the diagram is made to the magnetic field strength in a loop of constant area. **Explain** your answer. (2 marks)

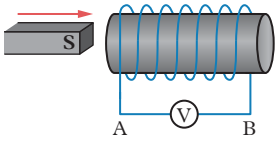


32 For each of the scenarios shown in **a** and **b**, **determine** the direction of the current (if any) when a magnet is moved relative to a solenoid. **Explain** your answers.

a (2 marks)

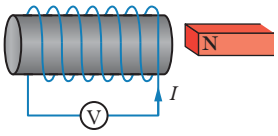


b (2 marks)

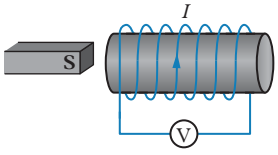


33 For each of the scenarios shown in the following diagrams, **determine** the direction of motion of the magnet (towards the solenoid, away from the solenoid, or no movement). **Explain** your answers.

a (2 marks)



b (2 marks)



34 **Determine** the EMF induced in a coil that has 1,200 evenly spaced turns and a radius of 2.5 cm, when a magnet is brought near its end so that the magnetic field inside it is increased from 0 to 0.060 T in 20 ms. (3 marks)

35 A copper bar 50 cm long is moved at right angles to a magnetic field of 0.80 T at a speed of 130 $cm\,s^{-1}$.

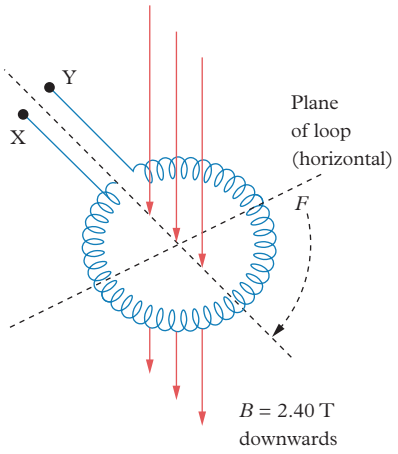
a **Determine** the EMF induced in the bar. (3 marks)

b **Determine** the order of magnitude of the EMF induced in the bar. (1 mark)

36 A train is moving due north at a constant speed of 100 $km\,h^{-1}$, through an area where the vertical component of Earth's magnetic field is 5.4×10^{-5} T. **Determine** the EMF induced in a carriage axle 1.25 m long. (3 marks)

37 An aircraft is flying through Earth's magnetic field at a steady horizontal speed of 800 $km\,h^{-1}$. The vertical component of Earth's magnetic field is 5.0×10^{-5} T. **Determine** the EMF generated between the wing tips if they are 25 m apart. (3 marks)

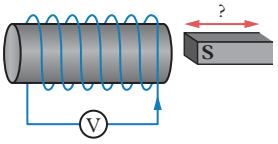
38 The diagram shows a circular spring loop perpendicular to a magnetic field. The spring is released and its area changes from 0.55 m^2 to 0.15 m^2 in 0.40 s. **Determine** the EMF induced between the points X and Y shown on the loop. (3 marks)



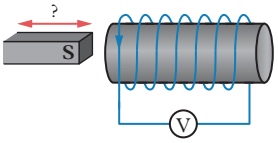
Knowledge utilisation

39 For each of the scenarios shown in the following diagrams, **propose** the direction in which the magnet must have been moved (towards or away from the solenoid). **Explain** your answers. The current is shown on the diagram for each case.

a (2 marks)

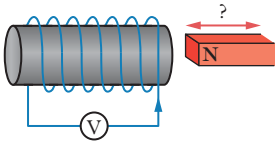


b (2 marks)

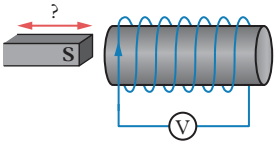


40 For each of the scenarios shown in the following diagrams, **predict** the direction in which the magnet must have been moved (to the left or the right) to give rise to the induced current shown. **Explain** your answers.

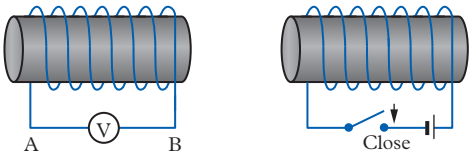
a (2 marks)



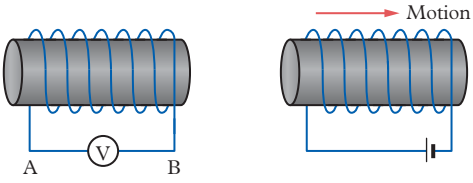
b (2 marks)



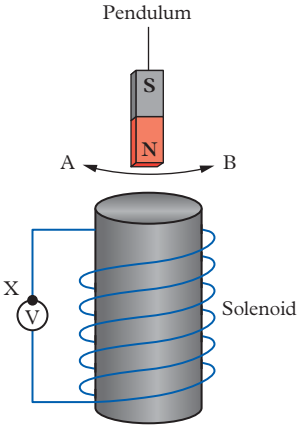
41 **Propose** what effect (if any) there would be on the solenoid to the left of the diagram if the switch for the solenoid to the right is closed. **Explain** your answer. (2 marks)



42 In the diagram, the solenoid to the right is moved as shown. **Predict** the effect (if any) on the solenoid to the left. **Explain** your answer. (2 marks)



43 A permanent magnet is tied to a length of string and allowed to swing freely (like a pendulum) over the top of a solenoid, as shown in the diagram. The magnet is released from position A and allowed to swing to position B and back to A.



a **Sketch** the shape of the resultant voltage waveform. (1 mark)

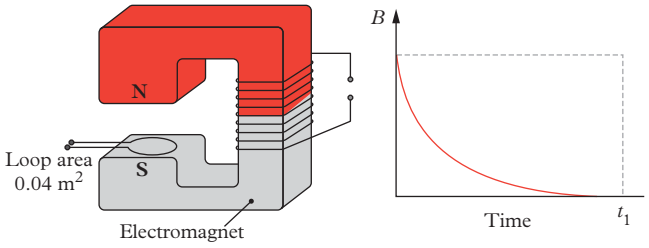
b **Justify** your proposed shape by referring to physics principles. (3 marks)

c **Evaluate** the claim: "the frequency of the voltage waveform is equal to the frequency of the pendulum's displacement". (2 marks)

44 A flexible loop of wire is held between the poles of an electromagnet that provides a uniform field B in the region of the loop. At time $t = 0$, the current through the electromagnet is turned off and the field B falls to zero at time t_1, as shown in the graph.

a **Sketch** a corresponding graph of the induced EMF across the ends of the loop as a function of time. (2 marks)

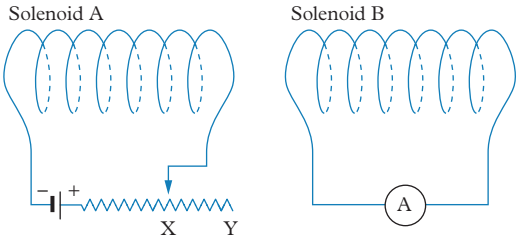
b **Determine** the average EMF induced if time $t_1 = 2.0\,\text{s}$, given that the loop has an area of $0.04\,\text{m}^2$, and $B = 0.6\,\text{T}$ at time $t = 0$ s. (2 marks)



45 A variable resistor is connected into a circuit with solenoid A and a battery, as shown in the diagram. The resistor is varied from position X to Y, which reduces the size of the current.

a **Predict** the direction of induced current in solenoid B. (1 mark)

b **Explain** your analysis. (2 marks)



Data drill

Linearising experimental transformer data

Students proposed a research question "How does the amount of flux linked in a transformer vary with the amount of iron core linking them?" They conducted an experiment on a simple transformer made from two identical solenoids with an iron bar linking both of them, as shown in Figure 1. A constant AC voltage of 2.00 V was connected across the primary coil, and the voltage on the secondary coil was read. This was repeated two more times. The number of iron bars was increased and the reading taken again. The ratio of secondary voltage to primary voltage, $\frac{V_s}{V_p}$, was calculated. The results are in Table 1. A graph of the voltage ratio was plotted against the number of rods (Figure 2).

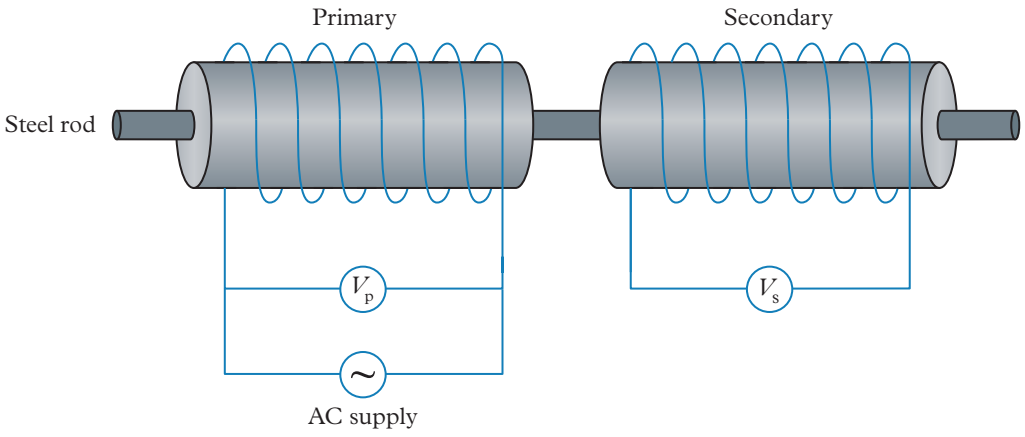


FIGURE 1 Experimental setup: two solenoids linked by an iron rod

TABLE 1 Results for experiment to find the effect of varying the iron core in a simple transformer

Number of rods	Primary voltage	Secondary voltage, V_s (V)					$\frac{V_s}{V_p}$
N	V_p (V)	Trial 1	Trial 2	Trial 3	Avg (V)	$\delta\bar{x}$ (V)	
0	2.00	0.08	0.08	0.08	0.08	0.0005	0.04
1	2.00	0.52	0.52	0.52	0.52	0.0019	0.26
2	2.00	0.68	0.67	0.67	0.68	0.0075	0.34
3	2.00	0.77	0.76	0.75	0.76	0.0073	0.38
4	2.00	0.84	0.83	0.81	**Q1**	**Q2**	0.41
5	2.00	0.83	0.87	0.87	0.86	0.0191	0.43

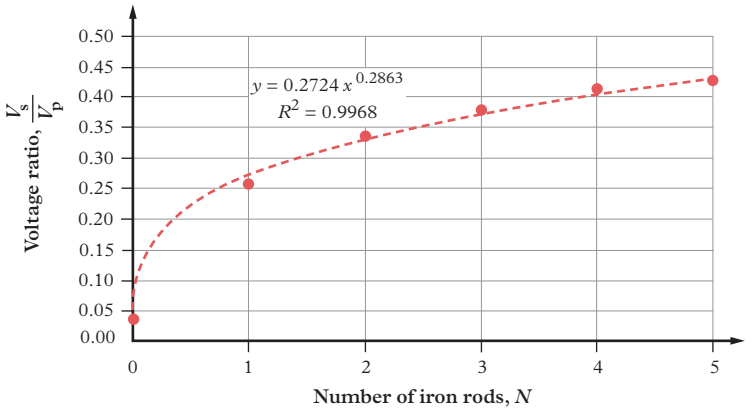


FIGURE 2 Graph of experimental results

Apply understanding

1 **Calculate** the average secondary voltage for the setup with four iron rods. (1 mark)

2 **Calculate** the absolute uncertainty for the secondary voltage reading for four iron rods. (1 mark)

Analyse data

3 Using the equation shown on the graph, **identify** an equation that quantifies the voltage ratio in terms of the number of iron rods. (1 mark)

4 **Identify** how you might linearise these data. (2 marks)

Interpret evidence

5 For an ideal "lossless" transformer: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$.

Draw a conclusion about whether this transformer is ideal. Justify your answer by referring to the data. (2 marks)

6 **Predict** whether the transformer becomes more ideal as the number of rods increases. (2 marks)

Module 8 checklist: Electromagnetic induction and radiation

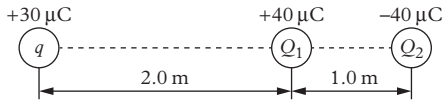
UNIT 3

Topic 2 review

Multiple choice

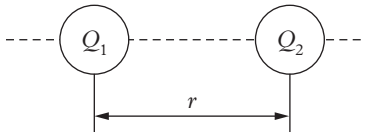
(1 mark each)

1 The force between Q_1 and Q_2 is shown as F in the diagram. Determine the magnitude of the force between Q_1 and q.



A $\frac{F}{2}$ **B** $\frac{3F}{16}$ **C** $\frac{3F}{8}$ **D** $\frac{12F}{16}$

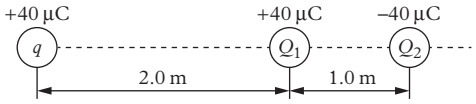
2 Two equally charged spheres Q_1 and Q_2 are a distance r apart. The force exerted on each sphere by the other has magnitude F.



If the charge on Q_1 doubled, and the charge on Q_2 trebled (×3), the force on each sphere now has a magnitude

A $5F$ **B** $6F$ **C** $\frac{F}{5}$ **D** $\frac{F}{6}$

3 Three charged objects are placed in a line as shown in the diagram.

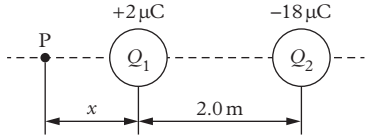


What is the force in newton on Q_1 due to q?

A 7.2×10^{12} N **B** 3.6×10^{12} N

C 7.2 N **D** 3.6 N

4 Q_1 and Q_2 are two oppositely charged objects placed 2.0 m apart, as shown in the diagram. There is a point P to the left of Q_1 where the electric field strength is zero; that is, the electric fields from both objects cancel out.



What is the value of x?

A 0.5 m **B** 1.0 m **C** 9.0 m **D** 18.0 m

5 Calculate the electric field strength at a point P between a charge A (–5 μC) and charge B (–1 μC) that are separated by a distance of 15 cm. Point P is 10 cm to right of A and is 5 cm to left of B.

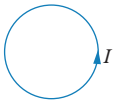
A $0.9 \times 10^6\,\text{N C}^{-1}$ to the right

B $8.1 \times 10^6\,\text{N C}^{-1}$ to the right

C $0.9 \times 10^6\,\text{N C}^{-1}$ to the left

D $8.1 \times 10^6\,\text{N C}^{-1}$ to the left

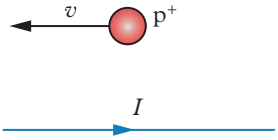
6 A circular loop of wire has an anticlockwise current running through it, as shown in the diagram. Determine the direction of the magnetic field inside of the loop.



A North **B** South

C Into the page **D** Out of the page

7 A horizontal wire is carrying current I to the east, and a proton is moving with a velocity v to the west, as shown in the diagram, looking from above. They are both in the same horizontal plane.



What is the direction of the magnetic force acting on the proton due to the current-carrying wire?

A North **B** South

C West **D** East

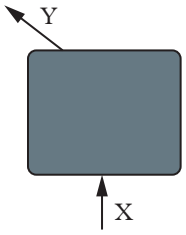
8 Three parallel wires carry equal currents in the directions shown in the diagram.

The magnetic fields due to X and Y pass through wire Z. In what direction does the magnetic force act on Z?

A To the left **B** To the right

C Up the page **D** Down the page

9 An electron enters a magnetic field (shown in the diagram inside the grey box) in the direction X and is deflected and leaves in direction Y.



The direction of the magnetic field inside the box is

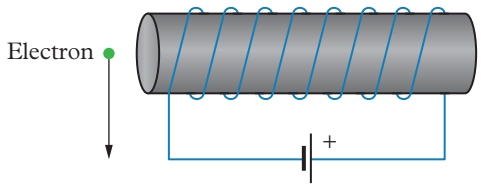
A into the page.

B out of the page.

C in the direction of X.

D in the direction opposite to X.

10 An electron passes the pole of an electromagnet as shown in the diagram.



What is the direction of the magnetic force on the electron?

A Out of the page **B** Into the page

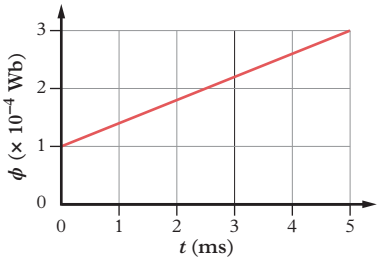
C To the right **D** To the left

11 A rectangular loop of wire with dimensions 0.2 m × 0.5 m is placed in a uniform magnetic field of magnitude 2 T. The magnetic field is perpendicular to the plane of the loop. Determine the magnetic flux threading the loop.

A 0.1 Wb **B** 0.2 Wb

C 0.3 Wb **D** 0.4 Wb

12 The magnetic flux threading a solenoid with 1,000 turns changes as shown in the diagram.



Determine the EMF produced.

A 0.04 V **B** 0.06 V

C 40 V **D** 60 V

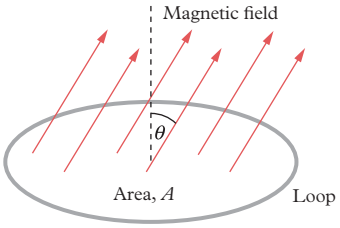
13 A uniform magnetic field of field strength B threads a circular loop of area A, as shown in the diagram. The field makes an angle θ with a line perpendicular to the plane of the loop. The strength of the magnetic field is increasing at a uniform rate. Determine the direction of the induced current (as viewed from above).

A Clockwise in the loop

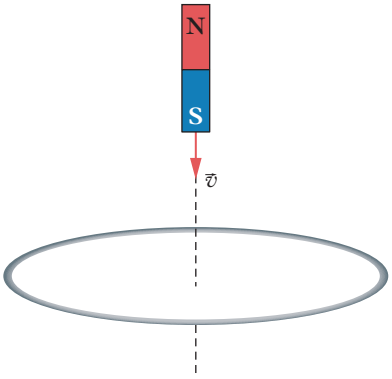
B Anticlockwise in the loop

C Clockwise in a circle perpendicular to the field line

D Anticlockwise in the loop perpendicular to the field line



14 A magnet is allowed to fall through a loop of wire as shown in the diagram. When viewed from above, what is the direction of the current as it enters and then leaves the loop?



A Clockwise, clockwise

B Clockwise, anticlockwise

C Anticlockwise, clockwise

D Anticlockwise, anticlockwise

15 In a step-up transformer, how does the rate of change of flux and the voltage in the secondary coil compare to the rate of change of flux and voltage in the primary coil? In the secondary coil

A the rate is the same but voltage is less.

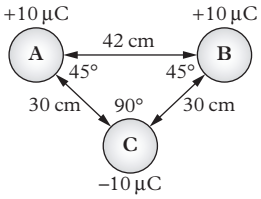
B the rate is the same but voltage is greater.

C the rate is greater but the voltage is greater.

D the rate is greater but the voltage is the same.

Short response

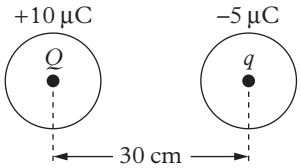
16 Three point-charges are arranged in a right-angled triangle, as shown in the diagram. The size of the charge and the distances between the charges are also shown.



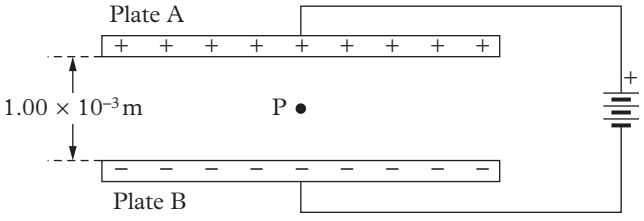
a **Determine** the magnitude and direction of the force on charge A due to charge B. (3 marks)

b **Determine** the magnitude and direction of the net force acting on charge C. (4 marks)

17 **Determine** the position, P, where the field strength is zero (besides infinity) for the system of charges shown in the diagram. State the location of P in relation to a distance from the negative charge. Show your working. (4 marks)



18 Two parallel plates, A and B, separated by a distance of 1.00×10^{-3} m are charged to a potential difference by a battery. The electric field strength between the plates is a uniform $1.2 \times 10^{3}\,\text{N C}^{-1}$. An alpha particle with a charge of $+2e$ is located at point P, halfway between the two plates.

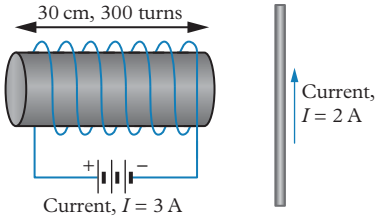


a **Calculate** the force experienced by the alpha particle. (2 marks)

b **Determine** what time will be taken for the alpha particle to strike one of the plates. (2 marks)

19 An electron with a velocity of $3.0 \times 10^{7}\,\text{m s}^{-1}$ passes through Earth's magnetic field of 25×10^{-6} T at an angle of 40° to the direction of the field. **Calculate** the force experienced by the electron. (2 marks)

20 A 30 cm long solenoid has 300 turns of wire and carries a current of 3 A, as shown in the diagram. A copper wire carrying a current of 2 A up the page is placed near the end of the solenoid.



a **Determine** the magnitude and direction of the magnetic field strength of the coil. (2 marks)

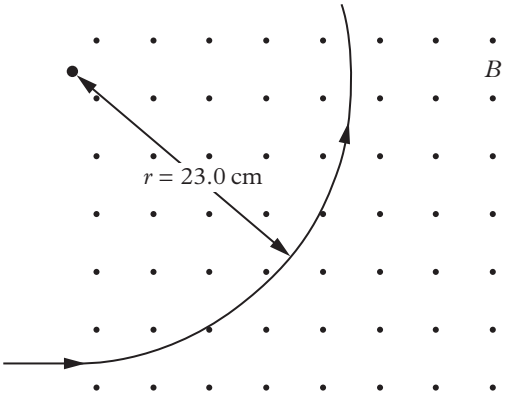
b **Determine** the direction of the force on the wire due to the solenoid's magnetic field. (1 mark)

c A 10 cm segment of the wire is in the solenoid's field. **Determine** the magnitude of the force acting on the wire. (2 marks)

21 A particle with a net elementary charge of $1e$ enters a magnetic field of 86.6 μT at right angles as shown in the diagram. The particle has a velocity of $3.5 \times 10^{6}\,\text{m s}^{-1}$ and moves in a circular path of radius 23.0 cm.

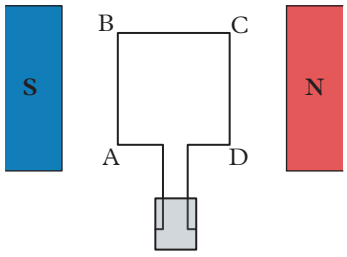
a **Determine** whether the charge on the particle is positive or negative. **Explain** your answer. (2 marks)

b **Determine** whether the particle is an electron. Show your working. (3 marks)



22 A student sets up a simple DC motor, consisting of a square coil of wire sitting in the region between the north and south poles of two magnets.

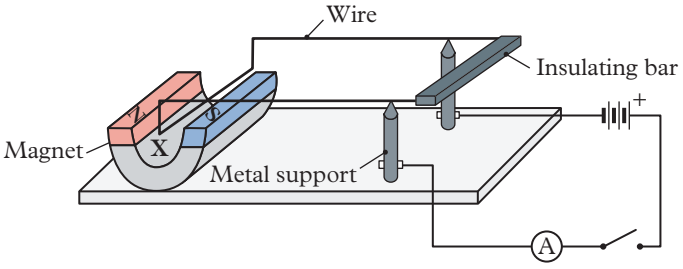
The coil has 15 loops of wire, side lengths of 10.0 cm and a current flowing of 1.50 A. The strength of the magnetic field in the region of the coil is 0.550 T. At the instant shown, the coil spins clockwise as viewed from above. That is, side AB is moving away from you and CD is moving towards you.



a **Determine** whether the current in wire AB is flowing upwards, flowing downwards, or is zero. (1 mark)

b **Determine** the force acting on the wire segment AB at this moment. (3 marks)

23 The diagram shows a piece of apparatus called a current balance. A current can be passed through a 15.0 cm long wire that sits between the poles of a magnet. When a current of 3.8 A is passed through the wire, it experiences a force of 0.020 N.

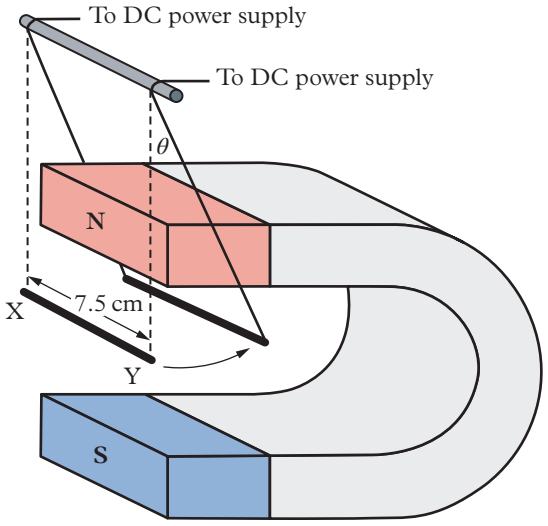


When the switch is closed, the part of the wire labelled X experiences a force.

a **Determine** the direction the wire moves (up or down). **Explain** your reasoning. (2 marks)

b **Determine** the strength of the magnetic field between the poles of the magnet. (3 marks)

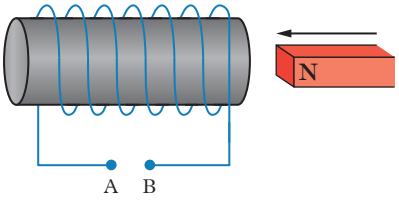
24 A device was set up to demonstrate the forces on a current-carrying wire in a magnetic field. It consists of a metal rod XY of mass 3.0 g suspended in a magnetic field of 45 mT. The dotted lines show the position of the rod when no current is present. When a current, I, of 5.6 A passes through the rod, it experiences a force and moves through an angle θ as shown.



a **Determine** the direction of the current through the rod either from X to Y or Y to X. **Explain** your reasoning. (2 marks)

b **Determine** the angle θ in degrees. (4 marks)

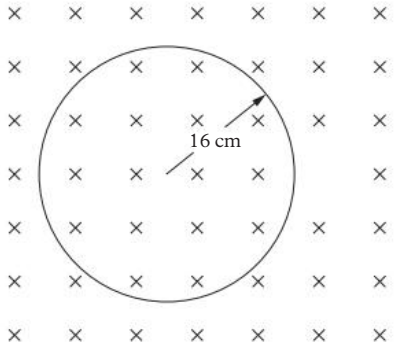
25 A bar magnet is brought up to a short coil, as shown in the diagram.



a **Determine** which of the terminals, A or B, will become positive. (1 mark)

b **Explain** your answer in terms of Lenz's law. (2 marks)

26 A circular loop of wire has a radius of 16 cm. It is placed in a 1.5 T magnetic field into the page, as shown in the diagram. The magnetic field is reduced uniformly to 1.0 T over a period of 0.20 s.



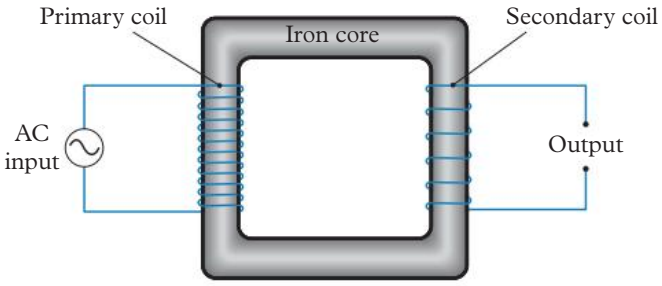
a **Determine** the initial magnetic flux in the loop. (2 marks)

b **Determine** the EMF generated by the collapsing field. (2 marks)

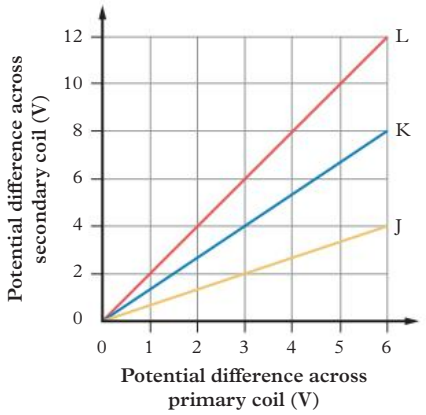
c **Determine** the direction of the induced current in the loop as the field collapses. (1 mark)

d **Explain** how you worked out the direction of the current in the loop. (2 marks)

27 The construction of a simple transformer is shown in the diagram.



A student makes three simple transformers, J, K and L. The graph shows how the potential difference across the secondary coil of each transformer varies as the potential difference across the primary coil of each transformer is changed.

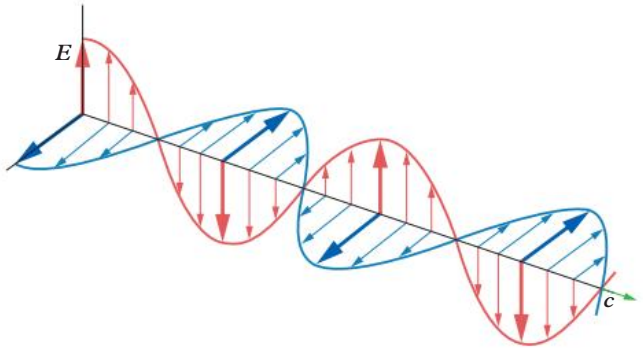


a **Describe** the purpose of the iron core in the transformer. (1 mark)

b **Determine** whether transformer J is a step-up or step-down transformer. **Explain** your reasoning. (2 marks)

c Each of the transformers has 50 turns on the primary coil. **Calculate** the number of turns on the secondary coil of transformer L. (2 marks)

28 The diagram represents an electromagnetic wave travelling to the right.



The blue and red arrows represent synchronised electric and magnetic fields. **Explain** what this means. (2 marks)

TOTAL MARKS

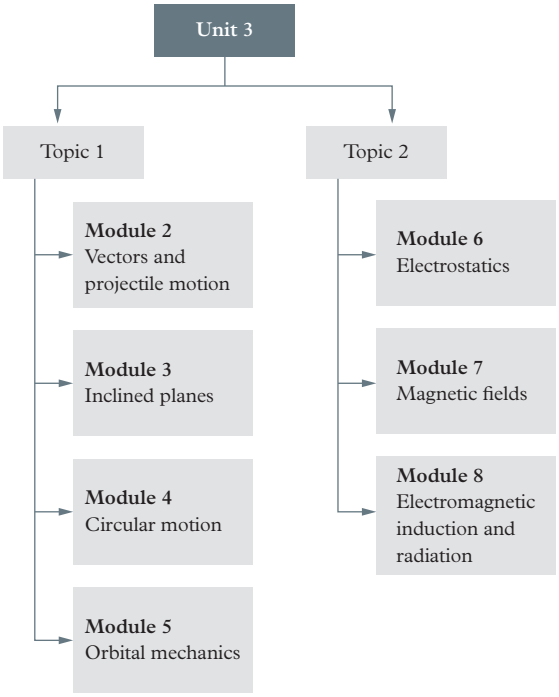
/74

UNIT 3

Review

Part A – Revisit and revise

Part A of the Unit review asks you to reflect on your learning and identify areas in which you need more work.



Part B – Exam essentials

Now that you've completed your revision for Unit 3, it's time to learn and practise some of the skills you'll need to answer exam questions like a pro! Our expert authors have created the following advice and tips to help you maximise your results on the end-of-year examination.

Exam tip 1: Check the scales on graphs

A big mistake students make with graphs is to read the scale divisions incorrectly. There may be four or five or more minor gridlines on a graph between the major gridlines. You need to be able to work out what each minor division is worth. If you get it wrong, your answer may still sound plausible but will be out by enough to lose you marks.

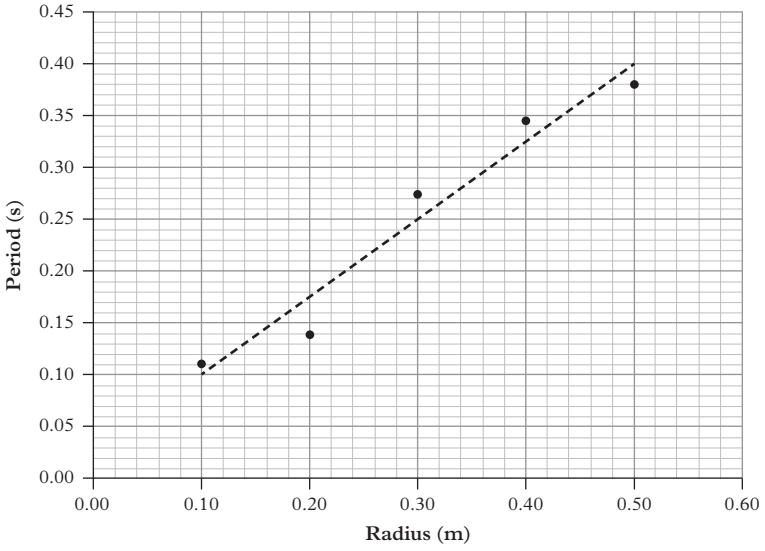
See it in action

Read the real exam question that follows and see how the tip has made the difference between a response that has scored full marks and a response where marks have been lost.

QUESTION 23 (4 marks)

An experiment was conducted to understand an object undergoing circular motion, with the radius of motion acting as the independent variable and the speed kept constant.

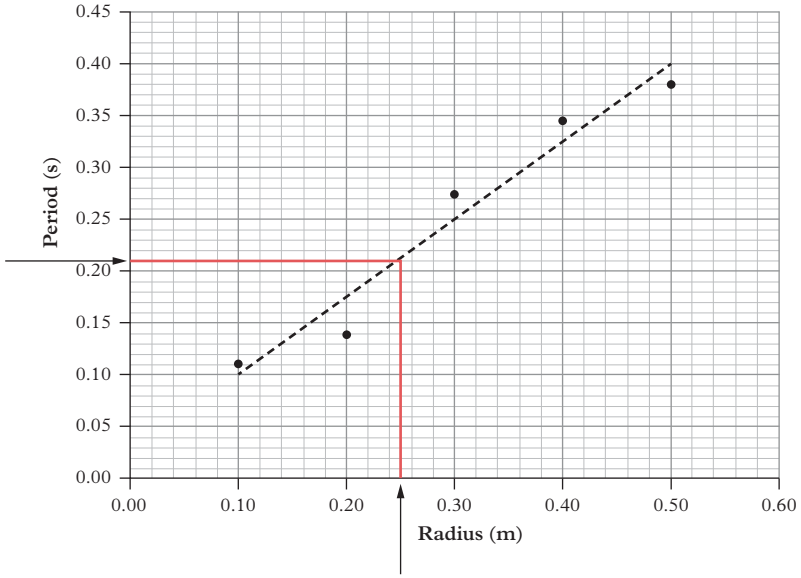
The data comparing the period and radius of motion is shown.



a) Identify the period expected for a 25 cm radius of motion. [*1 mark*]

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

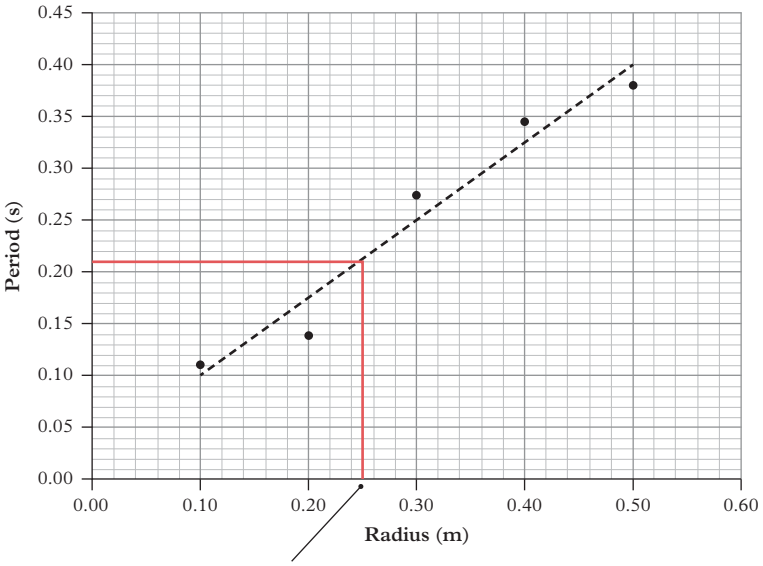
Complete response



Period = 0.21 s

By interpreting the scale correctly, the student has been able to correctly identify that 25 cm (0.25 m on the *x*-axis) corresponds to a period of 0.21 s on the *y*-axis. [1 mark]

Incomplete response



This student has misread the x-value and used 0.26 m, which gives a y-value of 0.22 s.

Period = 0.22 s

By interpreting the x-axis scale incorrectly, the student has not been able to determine the correct period on the y-axis. [0 marks]

Think like an assessor

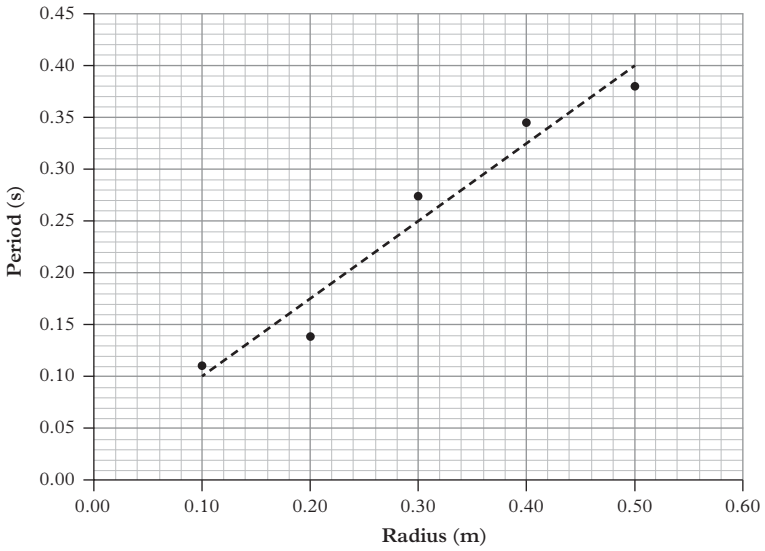
To maximise your marks on an exam, it can help to think like a QCAA assessor. Consider how many marks each question is worth and what information the assessor is looking for.

A student has given the following response in a practice exam. Imagine you are a QCAA assessor and use the marking guide to mark the response.

QUESTION 23 (3 marks)

An experiment was conducted to understand an object undergoing circular motion, with the radius of motion acting as the independent variable and the speed kept constant.

The data comparing the period and radius of motion is shown.



b) Determine the constant speed of the object. Show your working. [*3 marks*]

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{0.40}{0.50} = 0.80\,\text{m s}^{-1}$$

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Marking guide

Question 23b	Calculates the gradient [1 mark]. Accept values between 0.71 and 0.79 inclusive. Question says "show your working", so some working must be shown. Allow a follow-through mark for an incorrect period and/or coordinates from part (a). Do not accept co-ordinates going through the origin, i.e. (0, 0). The data point must be on the trendline.
	Recognises the relationship between gradient and average speed, i.e. gradient = $\frac{2\pi}{v}$ [1 mark]
	Determines the constant speed of the object. [1 mark] Allow a follow-through mark if the average speed is correctly calculated from an incorrect gradient. Do not penalise for incorrect decimal places/significant figures, units or rounding.

Source: Adapted from *QCAA 2024 Physics marking guide and response* © State of Queensland (QCAA)

Fix the response

Consider where you did and did not award marks for the student's response. How could the response be improved? Write your own response to the same question to receive full marks from a QCAA assessor.

Exam tip 2: Don't get misled by irrelevant information

Exam writers will often try to give you some context for the question being asked. It may be additional data, a practical application or a historical note. Students often think that all data has to be relevant – but it may not be.

See it in action

Read the real exam question that follows and see how this tip has made a difference between a response that scored full marks and a response where marks were lost.

QUESTION 25 (4 marks)

A transformer with a turns ratio of 48:1 is set up to reduce a 240 V input.

a) Explain how a transformer works in terms of Faraday's law and electromagnetic induction. [*3 marks*]

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Complete response

Describes Faraday's law [1 mark]

According to Faraday's law, when the magnetic flux linking a circuit changes, an EMF is induced in the circuit proportional to the rate of change of flux. According to Faraday's law, when the magnetic flux linking a circuit changes, an EMF is induced in the circuit proportional to the rate of change of flux. Although the circuits aren't connected, a current will be induced in the secondary circuit.

Explains how changing voltage in the primary coil affects rate of change of magnetic flux [1 mark]

Explains that an AC voltage is induced in the secondary coil by electromagnetic induction [1 mark]

Incomplete response

A formula does not "describe" Faraday's law. "Describe" in this case means to give an account of the characteristics or features of Faraday's law. [0 marks]

Faraday's law is emf = $\frac{\Delta\phi}{\Delta t}$ (n is proportional to EMF ∴ more turns = higher EMF).

Transformers transfer energy from one side of a core to another attached to a battery to another. These sides have coils/turns; a step-down transformer will have more turns on the side attached to a battery and transfer less volts to the other side, and a step-up has less coils on the side attached to a battery and will transfer more volts. The turns/coils never directly touch each other and transfer their energy through electromagnetic induction.

The mention of the turns ratio and voltage at the start of the question has misled the student. Their response is a description of the structure of the two types of transformers. It is not about Faraday's law, which is about the rate of change of flux in the primary being linked to the same rate of change in the secondary. [0 marks]

The student has correctly identified that the coils are not connected and that induction is taking place. [1 mark]

Think like an assessor

To maximise your marks on an exam, it can help to think like a QCAA assessor. Consider how many marks each question is worth and what information the assessor is looking for.

A student has given the following response in a practice exam. Imagine you are a QCAA assessor and use the marking guide to mark the response.

QUESTION 25 (2 marks)

Describe how electromagnetic radiation is propagated by the interaction between electric and magnetic fields.

The electric field creates a magnetic field, which creates an electric field, and so on. $c = f\lambda$

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Marking guide

Question 25

- Describes the production of an induced magnetic field. [1 mark]
- Describes the formation of an induced electric field, perpendicular to the magnetic field. [1 mark]

Source: Adapted from *QCAA 2024 Physics marking guide and response* © State of Queensland (QCAA)

Fix the response

Consider where you did and did not award marks for the student's response. How could the response be improved? Write your own response to the same question to receive full marks from a QCAA assessor.

Exam tip 3: Get the algebra right

Physics questions often involve using algebra in some form or another. We have many variables in physics that are expressed as formulas and equations. Students sometimes think that variables work differently to numbers. But they don't – they work the same way. One of the most common mistakes is to say $Q \times Q = 2Q$ instead of Q^2.

See it in action

Read the real exam question that follows and see how this tip has made a difference between a response that has scored full marks and a response where marks have been lost.

QUESTION 26 (3 marks)

The centres of two small equally positively charged metallic spheres are separated by a distance of 0.30 m and experience a force of 0.025 N between them.

Calculate the charge on each of the metallic spheres. (Show your working.)

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Complete response

$F = \frac{kQq}{r^2}$

$Q = q$, therefore

$F = \frac{kQQ}{r^2}$ — The student recognises that the scenario relates to Coulomb's law. [1 mark]

$= \frac{kQ^2}{r^2}$

$0.025 = \frac{9 \times 10^9 \times Q^2}{0.30^2}$ — The student has used correct mathematical reasoning ($Q \times Q = Q^2$). [1 mark]

$Q = \sqrt{\frac{0.025 \times 0.30^2}{9 \times 10^9}}$

$= 5.0 \times 10^{-3}$ C — The student has calculated the charge. [1 mark]

Incomplete response

$F = \frac{kQq}{r^2}$

$Q = q$, therefore

$F = \frac{kQQ}{r^2}$ — The student recognises that the scenario relates to Coulomb's law. [1 mark]

$0.025 = \frac{9 \times 10^9 \times 2 \times Q}{0.30^2}$ — The student has not used correct mathematical reasoning and has written that $Q \times Q = 2Q$. [0 marks]

$Q = \sqrt{\frac{0.025 \times 0.30^2}{2 \times 9 \times 10^9}}$

$= 2.5 \times 10^{-13}$ C — The student has consequentially calculated the wrong charge. [0 marks]

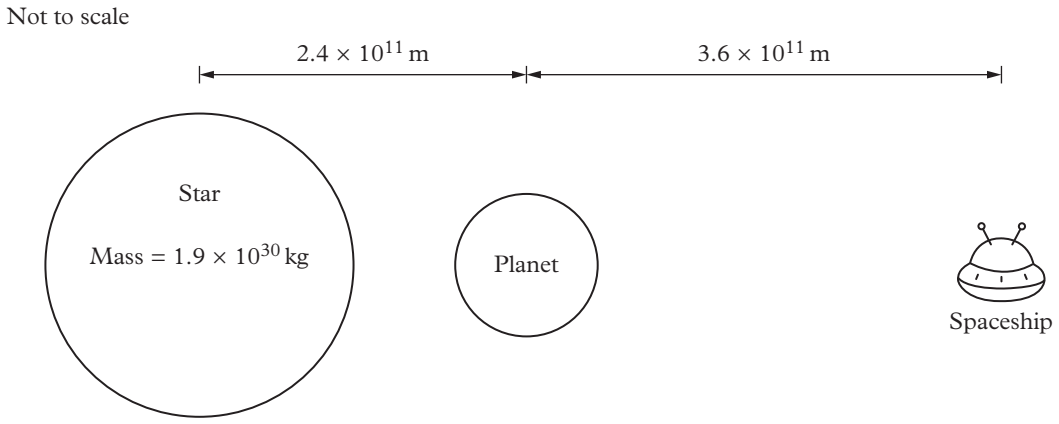
Think like an assessor

To maximise your marks on an exam, it can help to think like a QCAA assessor. Consider how many marks each question is worth and what information the assessor is looking for.

A student has given the following response in a practice exam. Imagine you are a QCAA assessor and use the marking guide to mark the response.

QUESTION 2 (6 marks)

In a distant solar system, a star, planet and spaceship system are aligned as shown.



a) Calculate the strength of the star's gravitational field experienced by the spaceship. Show your working.

[*3 marks*]

$g_{star} = \frac{GM}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{30}}{(3.6 \times 10^{11})^2}$

$= 9.8 \times 10^{-4}\ m\ s^{-2}$

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Marking guide

Question 2a

- Recognises the scenario relates to the gravitational field about a central body. [1 mark]
- Recognises the total radial distance is the sum of the star–planet distance and the planet–spaceship distance. [1 mark]
- Calculates the gravitational field strength to be $3.5 \times 10^{-4}\,m\,s^{-2}$. [1 mark]

Source: Adapted from *QCAA 2024 Physics marking guide and response* © State of Queensland (QCAA)

Fix the response

Consider where you did and did not award marks for the student's response. How could the response be improved? Write your own response to the same question to receive full marks from a QCAA assessor.

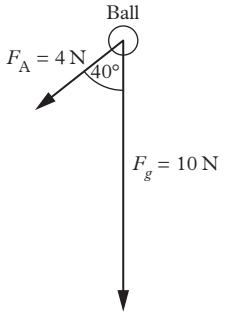
Part C – Exam practice

Now it's time to put the tips and advice you've learned into practice while you complete these exam-style questions!

Multiple choice

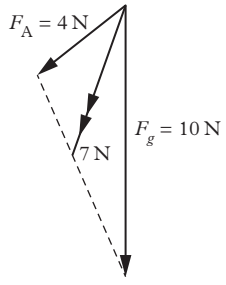
(1 mark each)

1 A ball with a weight F_g of 10 N is falling to the ground and experiences an applied sideways force F_A of 4 N due to the wind. This is shown in the vector diagram.

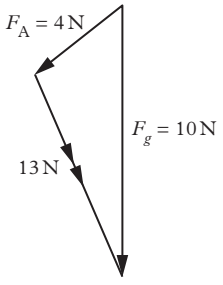


Select which of the following shows the resultant vector when the two vectors F_g and F_A are added.

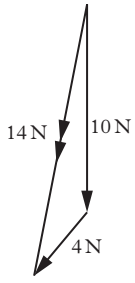
A



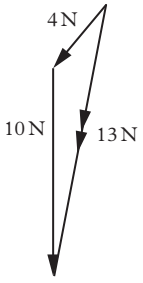
B



C



D



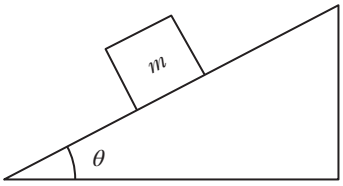
2 A missile is fired at 80 m s^{-1} at an angle of 30° with the horizontal and lands far away at the same height. Determine approximately how many seconds the missile was airborne for.

A 2 s **B** 4 s **C** 7 s **D** 8 s

3 A ball is launched horizontally from a cliff with an initial velocity of u m s^{-1}. After a time of 3.0 s, the ball's velocity is in the direction 45° from the horizontal. Determine the magnitude of the velocity in m s^{-1} at 3.0 s.

A 1.0 **B** 1.4 **C** 29.4 **D** 41.6

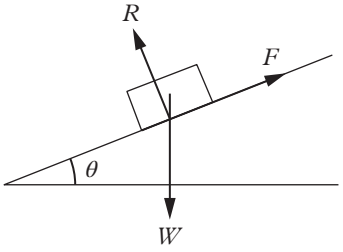
4 An object of mass m is placed on a frictionless inclined plane as indicated in the diagram. Select one of the following expressions that represent the magnitude of the normal force on the mass due to the inclined plane.



A mg **B** $mg\tan\theta$

C $mg\cos\theta$ **D** $mg\sin\theta$

5 A block of weight W slides down an inclined plane at a constant speed, as shown in the diagram. The normal reaction acting between the block and the plane is R and the frictional force between the block and the plane is F. The incline is at an angle θ to the horizontal.



What is the magnitude of F?

A $R\sin\theta$ **B** $W\sin\theta$

C $R\cos\theta$ **D** $W\cos\theta$

6 A body moves with uniform speed around a circle of radius r. The period of the motion is T. Select which one of the following gives the speed of the body.

A zero **B** $\frac{\pi r^2}{T}$

C $\frac{2\pi r}{T}$ **D** $\frac{2\pi T}{r}$

7 A car of mass 2,000 kg moves on a circular road with a speed of 10 m s^{-1}. Its direction has changed by 90° after travelling 314 m on the road.

Determine the centripetal force acting on the car.

A 500 N

B 1,000 N

C 1,500 N

D 2,000 N

8 The acceleration due to gravity on the Earth's surface is represented by g. If a planet had a mass that was four times the mass of the Earth, how many times larger must the planet's radius be than Earth's to experience the same acceleration due to gravity, g, on its surface?

A 0.25 **B** 0.5
C 1.5 **D** 2.0

9 Two masses have a gravitational force of 200 N between them. The distance between the masses is then doubled. Determine the new gravitational force between them.

A 50 N **B** 100 N
C 200 N **D** 400 N

10 Select one of the following as a correct statement of Kepler's second law of planetary motion.

A A line drawn from a planet to the Sun always sweeps over equal areas in equal intervals of time.
B A planet moves slowly when it is nearest the Sun and more rapidly when it is furthest from the Sun.
C The square of the orbital period of a planet is proportional to the cube of its semimajor axis.
D The orbit of a planet around the Sun is an ellipse, with the Sun at one focus.

11 The mean distance of Mars from Sun is 1.524 times the distance of Earth from the Sun.

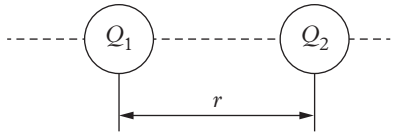
Determine the period of revolution of Mars around Sun.

A 1.23 y **B** 1.32 y
C 1.88 y **D** 3.53 y

12 The value of Coulomb's constant is: $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$. Calculate the magnitude of the permittivity of free space, ε_0.

A 1.1×10^{11} **B** 3.6×10^{10}
C 2.8×10^{-11} **D** 8.8×10^{-12}

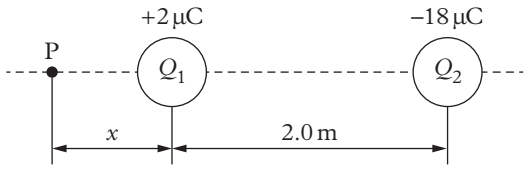
13 Two equally charged spheres Q_1 and Q_2 are a distance r apart, as shown in the diagram. The force exerted on each sphere by the other has magnitude F.



The charge on Q_1 is doubled, and the charge on Q_2 trebled (×3). Determine the magnitude of the force on each sphere now.

A $\frac{F}{6}$ **B** $\frac{F}{5}$ **C** $5F$ **D** $6F$

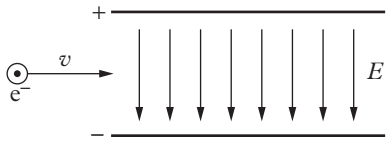
14 Q_1 and Q_2 are two oppositely charged objects placed 2.0 m apart, as shown in the diagram.



There is a point P to the left of Q_1 where the electric field strength is zero. Calculate the value of x.

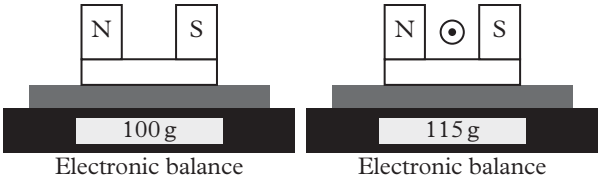
A 0.5 m **B** 1.0 m
C 9.0 m **D** 18.0 m

15 An electron with a velocity v and charge e^- is fired into a region between two charged plates in which there is an electric field of magnitude E. A magnetic field is introduced that produces a magnetic force that opposes the force on the particle due to the electric field, as shown in the diagram. Determine the direction of the magnetic field.



A Directed into the page
B Directed out of the page
C Toward the top of the page
D Toward the bottom of the page

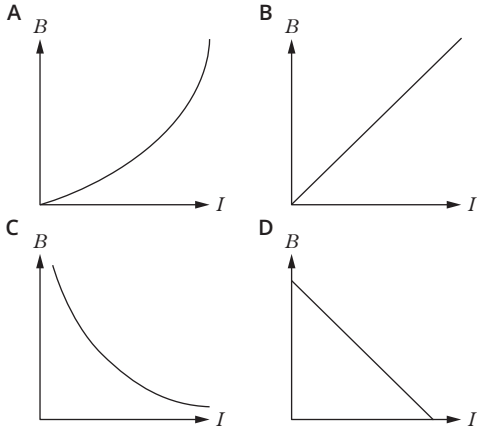
16 The diagram shows a magnet placed onto an electronic balance that gives a reading of 100 g. The same magnet is placed on the electronic balance but a wire carrying a current out of the page is placed directly in between the north and south ends of the magnet. The balance now reads 115 g.



Predict what would happen if the current in the wire were increased.

A The balance reading would increase.
B The balance reading would decrease.
C The balance reading would stay the same.
D The balance reading would increase for a few seconds and then go back to its original reading.

17 A current-carrying wire is placed perpendicular to a magnetic field and experiences a force. The current is gradually increased and the magnetic field strength changed so that the force on the wire remains constant.

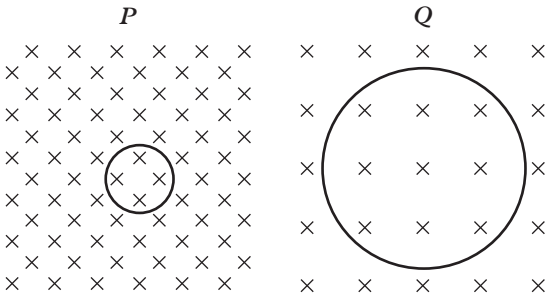


Select which of the graphs shown correctly shows the relationship between magnetic field strength, B, and current, I.

A Graph A **B** Graph B

C Graph C **D** Graph D

18 Different magnetic fields pass through two loops of wire, P and Q, as shown in the diagram.



Select the option that correctly identifies the loop with the greater magnetic flux density, and the loop with the greater magnetic flux, respectively.

A P, P **B** P, Q

C Q, P **D** Q, Q

19 A coil of wire has a large number of turns. It is moved relative to a fixed magnetic field. Determine what the EMF generated will be equal to.

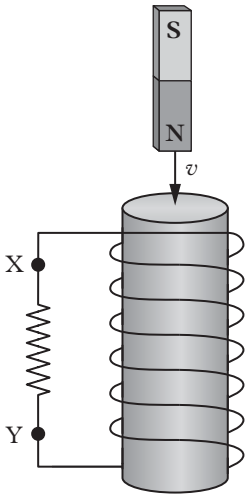
A Its speed relative to the magnetic field

B The amount of magnetic flux threading the coil

C The change of the magnetic flux through the coil

D The rate of change of the magnetic flux through the coil

20 A magnet is held vertically above a solenoid as shown in the diagram. The magnet is allowed to fall. Determine the flow of charge through the resistor between X and Y as the magnet enters the solenoid and then exits at the bottom of the solenoid.



A X to Y, then Y to X

B Y to X, then X to Y

C X to Y both on entry and exit

D Y to X both on entry and exit

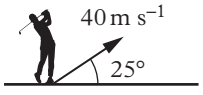
Short response

21 In 1971, during the *Apollo 15* mission, an astronaut famously dropped a hammer and a feather at the same time on the Moon's surface. Both objects landed on the Moon's surface at the same time.

a If the astronaut threw the hammer horizontally at the same time as he dropped the feather, **explain** whether or not they would still hit the ground at the same time. (3 marks)

b The gravitational field strength on the surface of the Moon is about $\frac{1}{6}$ of that on Earth. **Determine** the horizontal distance covered by the hammer if it were thrown horizontally at $5.0\,\text{m}\,\text{s}^{-1}$ from a chest height of 1.3 m. (2 marks)

22 A golfer hits a ball at a speed of $40\,\text{m}\,\text{s}^{-1}$ at an elevation of 25° to the horizontal, as shown in the diagram.



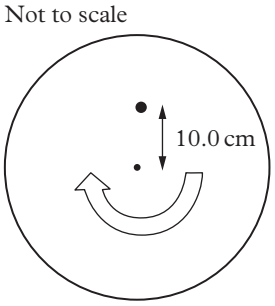
The golfer later tells friends that the ball was stopped in flight when it struck a tree 6.0 m up from its base. Determine how far the tree must have been from the launch point. (5 marks)

23 A 25 kg bag of cement slides down a 30° incline. There are frictional forces of 100.0 N acting.

a **Calculate** the net force down the incline. Show your working. (2 marks)

b **Determine** the acceleration of the bag of cement. Show your working. (1 mark)

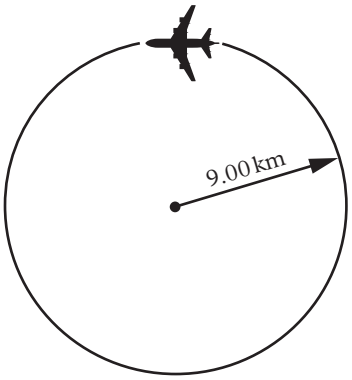
24 The diagram shows a 50.0 g brass mass resting 10.0 cm from the centre of a turntable of radius 40.0 cm. The turntable is rotated at 1.50 revolutions per second (rps).



a **Calculate** the centripetal acceleration of the brass mass at the instant shown in the diagram. (4 marks)

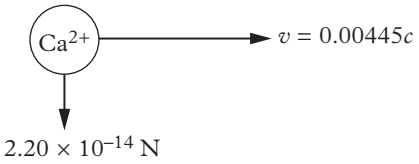
b **Calculate** the centripetal force acting. (1 mark)

25 A 75.0 kg pilot is flying an aircraft around an airport while waiting to land. The pilot is flying the plane in a horizontal circular path of radius 9.00 km, as shown in the diagram. Each revolution around the airport takes 525 s to complete.

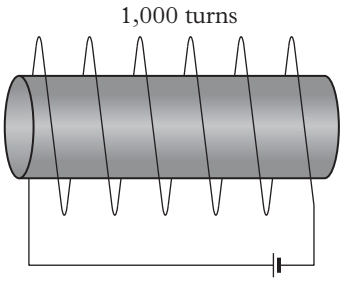


Determine the magnitude of both the centripetal acceleration of the pilot and the centripetal force acting on them during this horizontal flight. (4 marks)

26 A calcium ion (Ca^{2+}) travelling in an easterly direction at 0.00445 times the speed of light in a magnetic field experiences a force of 2.20×10^{-14} N to the south, as shown in the diagram. **Determine** the magnitude and direction of the magnetic field influencing the calcium ion. Ignore any relativistic effects. (4 marks)



27 A current of 3.0 A flows through a solenoid of diameter 5.0 cm that has 1,000 turns in its 10 cm length, as shown in the diagram.



a **Determine** the magnetic field strength in the solenoid. Show your working. (3 marks)

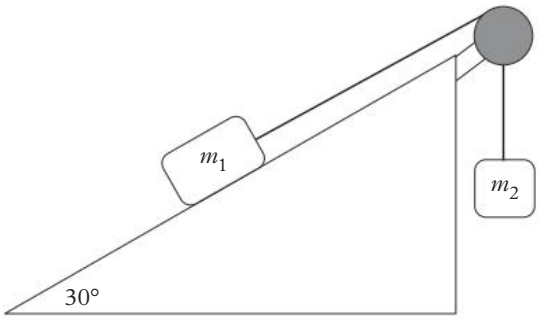
b **Determine** the amount of flux threading the solenoid. Show your working. (2 marks)

c **Determine** which end of the solenoid is the north pole – the left or the right end. (1 mark)

28 A golfer has developed a trick shot. She hits a golf ball at 30 m s^{-1} at an angle of 60°, and while it is in the air, she hits another ball from the same place at the complementary angle of 30° at the same speed. Both balls land in a bucket some distance away at the same time.

Determine how many seconds after she hits the first ball she must hit the second ball for the shot to work. Show your working. (5 marks)

29 A 410.0 kg coal truck (m_1) is on a 30° incline attached to cable from the surface. A hanging mass m_2 of 300.0 kg is used to provide the applied force in the cable, as shown in the diagram.

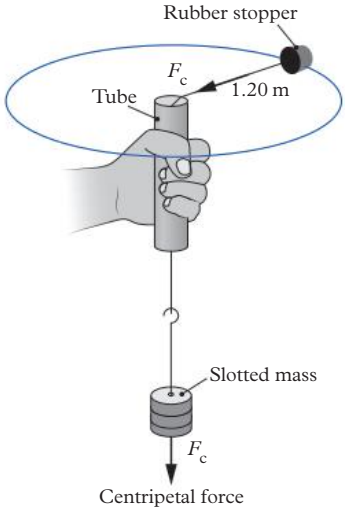


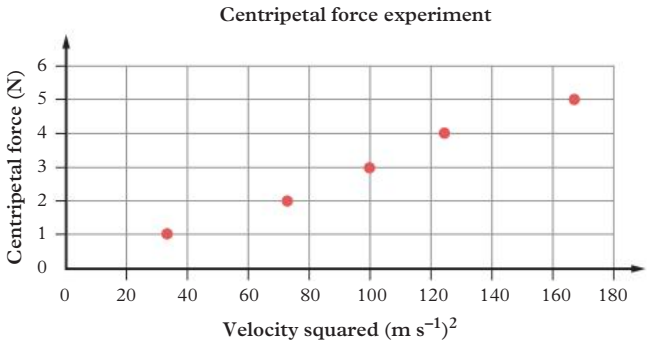
a **Determine** the acceleration (magnitude and direction) of mass m_1, assuming there is no friction between m_1 and the inclined plane. Show your working. (3 marks)

b Now assume there is frictional force of 400 N between m_1 and the inclined plane. **Determine** the acceleration (magnitude and direction) of m_1. Show your working. (3 marks)

30 A centripetal force experiment was conducted in which a rubber stopper attached to a string was whirled in a horizontal circle of radius 1.20 m at constant speed, as shown in the diagram.

The results for the experiment are shown in the graph.





a Use the graph to **calculate** the mass of the rubber stopper. Show your working. Express your answer in kilograms. (3 marks)

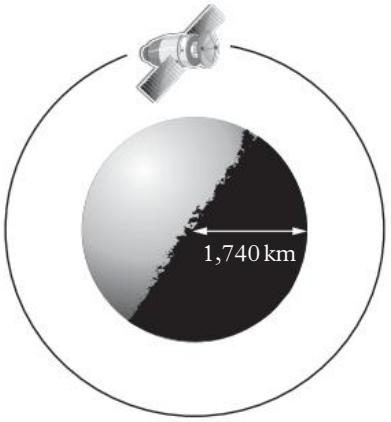
b The thread has a breaking strain of 6.0 N. **Calculate** the maximum speed at which the rubber stopper can be whirled without breaking the line. Show your working. (2 marks)

31 A satellite is placed in geostationary orbit around the Earth – that is, revolving at the same rate as Earth rotates. The average radius of the Earth is 6,378 km.

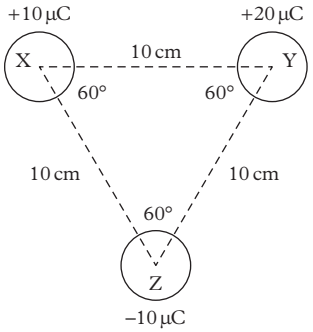
a **Calculate** the satellite's orbital distance from the Earth. Show your working. Express your answer in kilometres. (3 marks)

b **Determine** what fraction of the gravitational field strength at Earth's surface the satellite would experience. Show your working. Express your answer as a decimal fraction. (2 marks)

32 The lunar module *Eagle* orbited the Moon after completing the first successful lunar landing in July 1969. *Eagle* orbited 111 km above the surface of the Moon, as shown in the diagram. The mass of the Moon is 7.35×10^{22} kg and its radius is 1,740 km. **Calculate** the period of *Eagle's* orbit. (4 marks)

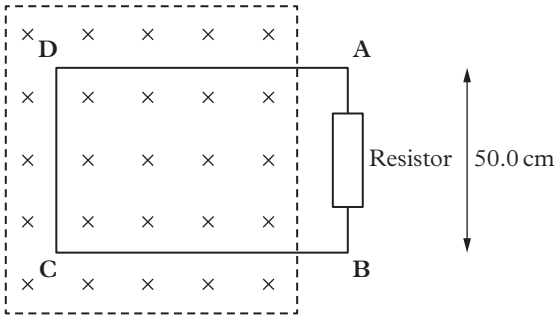


33 Three charged objects are arranged in a triangle as shown in the diagram.



Determine the net force (magnitude and direction) acting on object Z. Show your working. (5 marks)

34 A rectangular loop of wire ABCD of width 50.0 cm is moving with a constant velocity of 0.025 m s^{-1} to the right in a uniform magnetic field. The magnetic field has a strength of 0.855 T. The plane of the loop at right angles to the magnetic field, as shown in the diagram. A resistor is attached between A and B.

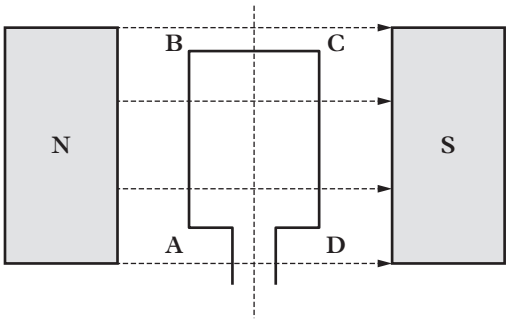


a **Determine** the magnitude of the induced EMF generated in the loop across the resistor. (3 marks)

b **Identify** the direction of the induced current through the resistor (section AB) and **explain** your reasoning. (2 marks)

c With reference to Faraday's law and the magnetic flux within the loop ABCD, **explain** why the current in the loop is constant. (2 marks)

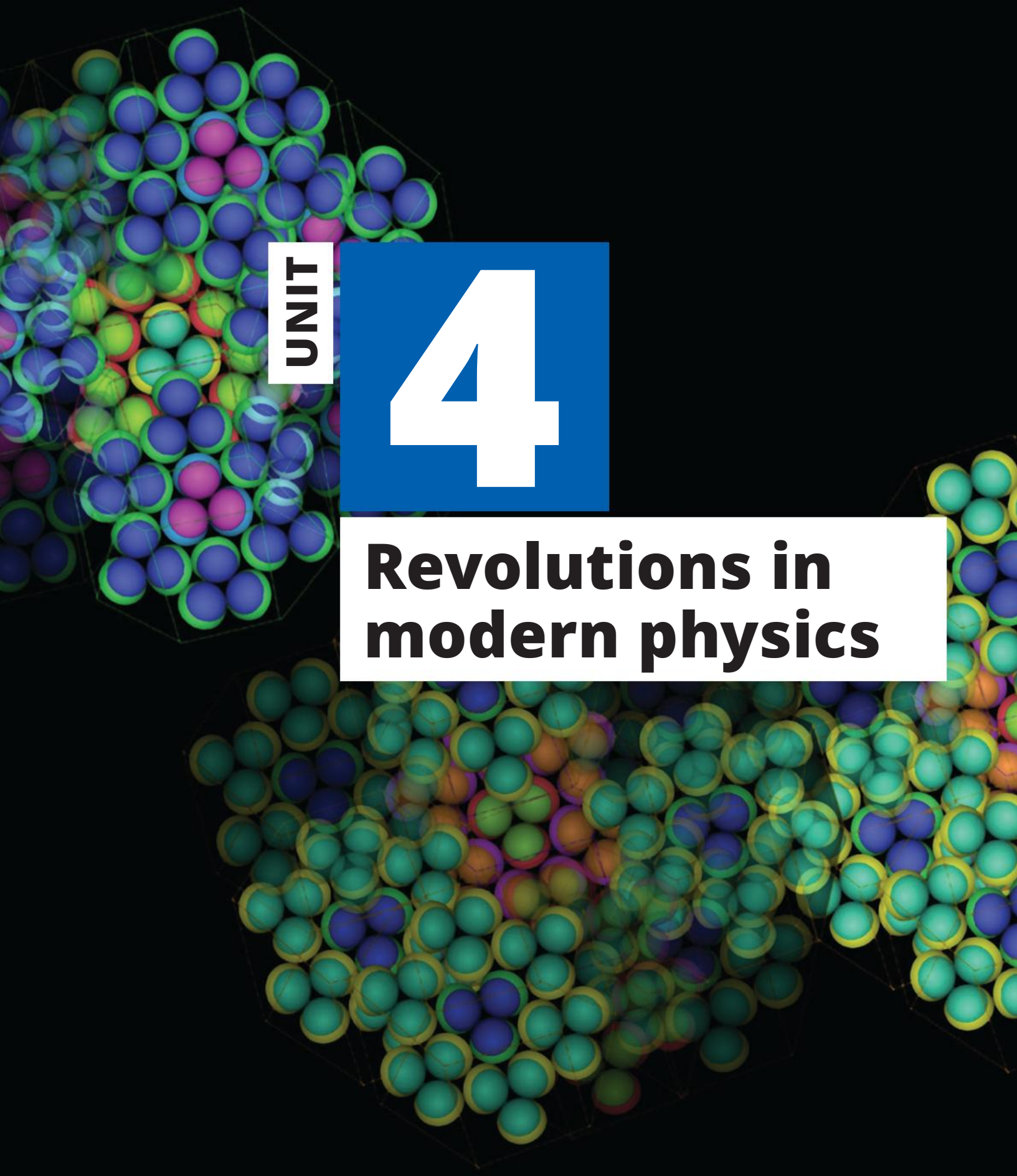
35 The diagram shows the structure of a simple AC generator.



The coil ABCD consists of 30 turns, rotates around its central axis, and has dimensions AB = CD = 20.0 cm and AD = BC = 10.0 cm. It lies in a uniform magnetic field of strength 0.400 T. The coil rotates at a rate of 600 revolutions per minute.

Calculate the average EMF generated by the coil over one-quarter of a turn from when the plane of the loop is parallel to the field to when the plane of the loop is at right angles to the field. (4 marks)

TOTAL MARKS
/93



UNIT

4

Revolutions in modern physics

Unit 4 overview

In Unit 4, students examine observations of relative motion, light and matter that could not be explained by classical physics theories, and investigate how the shortcomings of existing theories led to the development of the special theory of relativity and the quantum theory of light and matter. The development of quantum theory and the theory of relativity fundamentally changed our understanding of how nature operates and led to the development of a wide range of new technologies, including those that revolutionised the storage, processing and communication of information. Students evaluate the contribution of the quantum theory of light to the development of the quantum theory of the atom, and examine the Standard Model of particle physics and how it relates to the Big Bang theory.

Participation in a range of experiments and investigations will allow students to apply their understanding of relativity, black-body radiation, wave–particle duality and the quantum theory of the atom to make and/or explain observations of a range of phenomena such as atomic emission and absorption spectra, the photoelectric effect, lasers and Earth's energy balance.

Throughout the unit, students develop skills in planning and conducting investigations, interpreting results, synthesising evidence to support conclusions, recognising and defining the realm of validity of physical theories and models, and communicating these conclusions to others in a range of formats.

Unit objectives

1. Describe ideas and findings about special relativity, quantum theory and the Standard Model.
2. Apply understanding of special relativity, quantum theory and the Standard Model.
3. Analyse data about special relativity, quantum theory and the Standard Model.
4. Interpret evidence about special relativity, quantum theory and the Standard Model.
5. Evaluate processes, claims and conclusions about special relativity, quantum theory and the Standard Model.
6. Investigate phenomena associated with special relativity, quantum theory and the Standard Model.

Unit 4 Topics

Topic	Module	
Topic 1 Special relativity	**Module 9**	Special relativity: Time and motion
	Module 10	Special relativity: Length, momentum and energy
Topic 2 Quantum theory	**Module 11**	Quantum theory and light
	Module 12	Quantum theory and atoms
Topic 3 The Standard Model	**Module 13**	The Standard Model
	Module 14	Particle interactions

MODULE 9

Special relativity: Time and motion

Einstein's name is always attached to the theory of relativity, yet he acknowledges many famous scientists before him, such as Maxwell, Michelson, Lorentz and Poincaré, whose work underpins his theory. Einstein questioned the accepted theories of time and motion of earlier nineteenth-century physics. In 1905, he came up with a theory of his own which, in German, he called "das Prinzip der Relativität", which is translated into English as "the Principle of Relativity". It wasn't until 1916 that Einstein expanded this original work with his general theory of relativity, which explained the law of gravitation and its relation to other forces of nature. The original theory then became known as "special relativity". It applies to all matter, from subatomic particles to outer space, and describes all their physical phenomena except gravity.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to special relativity: time and motion before you start.

Subject matter

Science understanding

- → Describe observations of natural phenomena that cannot be explained by classical physics, e.g. the presence of muons in the atmosphere and the momentum of high speed particles in particle accelerators.
- → Describe the concepts of frame of reference and inertial frame of reference.
- → State the two postulates of special relativity.
- → Explain how motion can only be measured relative to an observer.
- → Explain the concept of simultaneity.
- → Describe the consequences of the constant speed of light in a vacuum, e.g. time dilation and length contraction.
- → Describe the concepts of time dilation, proper time interval, relativistic time interval, length contraction, proper length, relativistic length, rest mass and relativistic momentum.
- → Describe the phenomena of time dilation and length contraction, including examples of experimental evidence of the phenomena.

→ Solve problems involving time dilations, length contraction and relativistic momentum using
$t = \frac{t_0}{\sqrt{(1 - \frac{v^2}{c^2})}}$, $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ and $\Delta E = \Delta m c^2$.

→ Explain paradoxical scenarios that may arise as a result of special relativity including the twins' paradox, flashlights on a train, and the ladder in the barn paradox.

Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Albert Einstein and Amalie 'Emmy' Noether who furthered our understanding of relativity.

→ Explore how special relativity built upon the work of previous scientists and led to the development of relativistic theories of gravitation, mass–energy equivalence and quantum field theory.

Science inquiry skills

→ Consider the experimental evidence that supports the phenomena of time dilation and its real-world applications.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Lesson 9.1
Special relativity

Key ideas

- → A reference frame that moves with constant velocity with respect to an inertial frame is itself also an inertial frame.
- → There is no absolute frame of reference. All inertial reference frames are equivalent for the description of mechanical phenomena, and no one inertial reference frame is any better than another.
- → There are two postulates of the special theory of relativity:
  1. The laws of physics have the same form in all inertial reference frames.
  2. Light propagates through empty space with a definite speed, *c*, independent of the speed of the source or the observer.

Where did Newtonian theory fall short?

Learning intentions and success criteria

For more than two centuries after its inception, the Newtonian view of the world dominated, to the point that scientists developed an almost blind faith in this theory. Almost all problems could be explained using the Newtonian approach. Nevertheless, by the end of the nineteenth century new experimental data began to accumulate that was difficult to explain using Newtonian theory. New theories soon replaced the old ones. In 1900, Lord Kelvin said that there were "19th century clouds" hanging over the physics of the time, referring to problems that had resisted explanation using the Newtonian approach:

- Light appeared to be a wave, but the medium for its propagation (the "aether") was undetectable.
- The equations describing electricity and magnetism were inconsistent with Newton's descriptions of space and time.

From the 1940s, the problem of accounting for the lifetimes of muons (radioactive particles) at different altitudes stimulated interest in how the theory of relativity could be incorporated to explain the results. This provides a good case study on how the theory of relativity was confirmed.

What was the problem with muon decay?

A muon is a subatomic particle that is created in particle collisions. When cosmic ray protons from the Sun collide with oxygen and nitrogen atoms in the upper atmosphere, subatomic particles called **pions** are created. These decay within a few metres into **muons** and **muon neutrinos**. The muons generally continue in the same direction as the original proton, at a velocity near the speed of light (>0.99*c*). But the muons are short-lived, with an average lifespan of 2.2 μs (microseconds), and quickly decay into an electron and two neutrinos. The generated muons rain down at high speed, but in the few microseconds it takes them to reach the ground, some of them will have decayed.

In 1962, the physicists David Frisch and James Smith at the Massachusetts Institute of Technology were experimenting to prove to their students that relativity is real and that it is needed to solve the riddle of muons. They were testing one of the consequences of high-speed travel – that time slows and distances contract.

pion
a subatomic particle produced in the atmosphere as a result of the collision of cosmic ray protons with nitrogen and oxygen atoms; can be charged, π^+ or π^-, or uncharged, π^0

muon
an elementary particle similar to the electron but with a greater mass; it is the product of the decay of pions

muon neutrino
an almost massless and neutral elementary particle produced in radioactive decay of pions

special theory of relativity (special relativity)
a theory that all motion must be defined relative to a frame of reference; it consists of two principal postulates. It explains how space and time are linked for objects moving at constant speed in a straight line, and forms part of the basis of modern physics.

The scientists set up equipment on the top of the nearby Mount Washington and measured how many muons were present in the atmosphere by counting the number that passed through their counter in one hour at that altitude (1,910 m) (Figure 2). Their equipment counted 563 per hour. They then travelled back down to sea level and repeated the count. It should have taken the muons 6.4 μs to travel the intervening distance, which meant that most of them should have decayed. Using Newtonian (non-relativistic) calculations, only 27 per hour should have been detected at sea level. However, they counted 409 per hour, as predicted by the **special theory of relativity (special relativity)**.

This was a major confirmation of Einstein's theory. To the observers, the distance from mountaintop to sea level was 1,910 m. That is, the distance was 1,910 m in the scientists' "frame of reference", but to the muons the distance was much shorter – in the muons' frame of reference the distance was 183 m. This is a consequence of the relativity postulate for very high speeds and is called "length contraction" (that is, the path length contracted from 1,910 m to 183 m as measured in the muon's frame of reference).

The other measurement that changed was the muons' lifetime. The mean lifetime in the muons' frame of reference is 2.2 μs but in the scientists' frame of reference it became 23 μs, so to the scientists the muons took longer to decay. This too is a consequence of relativity and is known as "time dilation" ("dilation" means enlargement).

FIGURE 1 Cosmic ray protons from the Sun strike atoms in the atmosphere to produce high-energy pions that decay to muons.

Other experimental evidence supporting the phenomena of time dilation

The muon experiment is supported by similar results elsewhere. In 1978 this was confirmed by the European Organisation for Nuclear Research (CERN) in Geneva in a laboratory using a 600 m evacuated tube and charged pions, π^+, with a speed of $0.99936c$. Their rest lifetime of 9.31×10^{-10} s had been dilated to 2.603×10^{-8} s. Without accounting for special relativity all of the pions would have decayed in that distance, but the researchers found that just 3.5 per cent had decayed.

Study tip
If you could ride along with a muon or any other particle you would measure its rest lifetime.

What the muon experiment says about a particle's lifetime

We can sum up the observations by saying that the average lifetime of an unstable subatomic particle is longer than measured by a stationary detector because the particle experiences time differently to the detector. However, this is difficult to demonstrate because the times involved may be incredibly short (a few nanoseconds), and it is very hard to measure lifetimes accurately at relativistic speeds.

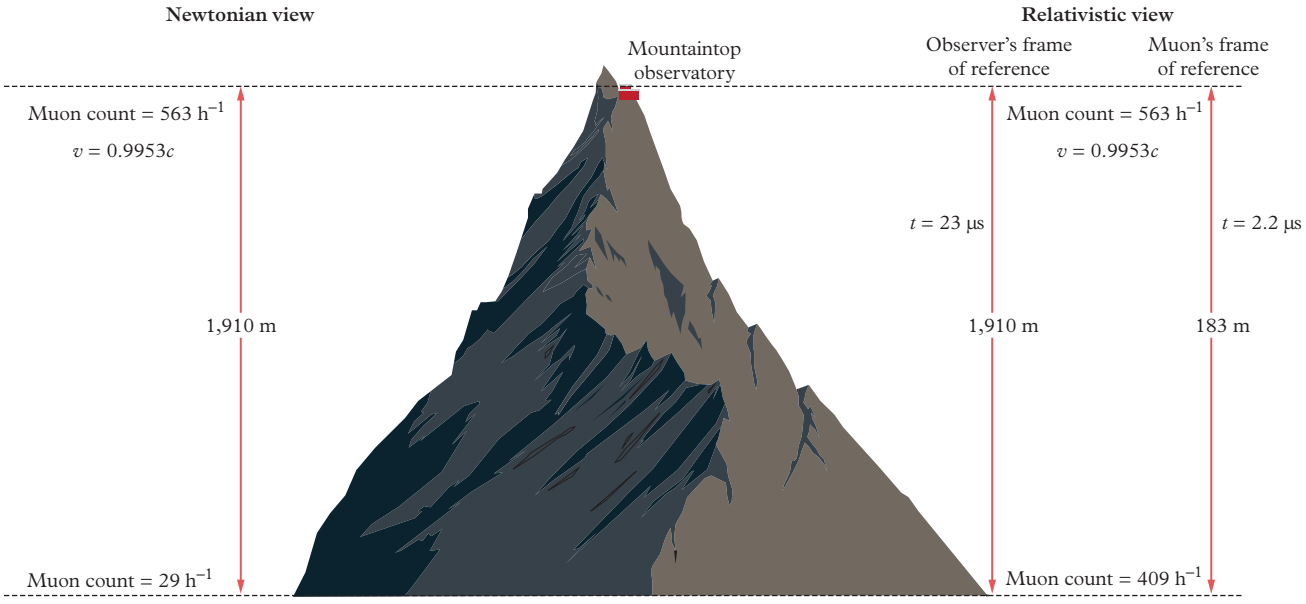


FIGURE 2 The muon experiment confirms aspects of special relativity.

What are frames of reference?

The term **frame of reference** used in the description of the muon experiment is one of the cornerstones of any study of modern physics. When you describe the motion of an object, you need something to compare it against. This is the central idea of **relative motion**. When a car drives past at 60 km h^{-1}, this speed is relative to the ground or Earth. That is your frame of reference. To the person inside the car, it could be they who are stationary and the Earth going by at 60 km h^{-1}. The car becomes their frame of reference.

A frame of reference is an arbitrary or abstract coordinate system that defines the location of the observer. It is also called a reference frame.

frame of reference
an arbitrary set of axes with reference to which the position or motion of something is described or physical laws are formulated

relative motion
the motion of an object with regard to some other moving object; the motion is not necessarily calculated with reference to Earth, but is the velocity of the object in reference to the other moving object as if it were in a static state

Inertial and non-inertial frames of reference

Let us consider how the laws of physics hold for the different frames of reference. Consider three simple experiments you may have done:

- Timing the swing of a pendulum to confirm g, the acceleration due to gravity
- Pulling a block along the bench with hanging weights to confirm Newton's second law $F = ma$
- Swinging a rubber stopper in a circle above your head to confirm the centripetal motion formula $F_c = \frac{mv^2}{r}$

Your results will not be different from those of other students who used a different coordinate system. For example, students at other schools in Queensland will achieve the same results. This is because they are all at rest in your frame of reference and, because the laws of inertia hold, you can say you are all in an inertial frame of reference.

But what if you were travelling aboard a ship and doing Physics classes while on holidays? Imagine the ship is moving at constant speed. Will you get the same results as you would on land? The answer to this is yes, of course you will. If the laws of inertia hold, then the ship is said to be an **inertial frame of reference**.

In general, a frame of reference that is stationary or moving at constant velocity is an inertial frame. However, in a frame of reference that is accelerating, things are different.

inertial frame of reference
a non-accelerating frame of reference in which Newton's laws of motion hold

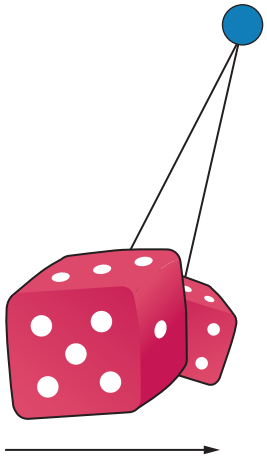


FIGURE 3 As the car accelerates forwards the fluffy dice seem to be pushed back. But what is pushing them? By the way, it's illegal to hang them on the rearview mirror in Queensland.

Imagine doing a pendulum experiment while accelerating. The pendulum will make an angle to the vertical even when it is not swinging, in the same way that something hanging from the rearview mirror of a car moves when the car accelerates away at the lights (Figure 3). An inertial frame is also one in which there is no gravitational field present. On the surface of the Earth the gravitational field is weak enough to be zero for this definition. Einstein worried about gravity in his later general theory of relativity. We ignore it in this unit.

Table 1 provides examples of inertial and non-inertial frames of reference. In special relativity the terms are usually shortened to "inertial frame" and "non-inertial frame"; the term "of reference" is often left out.

TABLE 1 Frames of reference

Inertial reference frames	Non-inertial reference frames
At rest on Earth	Accelerating upwards in an elevator
In a car at constant velocity	Going around a corner
In a spaceship at constant velocity	In free-fall

Note: the surface of Earth is considered an inertial reference frame even though it is undergoing centripetal acceleration. The acceleration is considered small enough to be disregarded.

Challenge

The Gravitron

The Gravitron is an amusement park ride that spins riders around in a giant wheel. They feel "pressed" against the wall because of their inertia and the normal force from the wall they stand against.

Decide if riding in a Gravitron that spins at constant speed is an example of an inertial reference frame. **Explain** your answer. (2 marks)

What are the postulates of special relativity?

In 1905, Albert Einstein put forward a revolutionary theory that we now use to explain phenomena such as muon decay. His special theory of relativity changed the way we understand nature.

Special relativity is a deceptively simple theory and has only two assumptions or "postulates" – the **two postulates of special relativity**.

two postulates of special relativity the two assumptions of the special theory of relativity: 1, that the laws of physics are the same in all inertial frames of reference; 2, that the speed of light in a vacuum has the same value, c, in all inertial frames of reference

- The laws of physics are the same in all inertial (uniformly moving) frames of reference.
- The speed of light in a vacuum has the same value, c, in all inertial frames of reference.

Most physicists agree that the second postulate is redundant as it is a logical consequence of the first.

When you are asked about the two correct postulates of special relativity, be careful about the wording, because these statements have tripped up people for the past century. Some examples of incorrect or irrelevant statements are as follows.

- Statement: "The speed of light has the same value c in all inertial frames of reference."
  Correction: This is only true for the speed of light in a vacuum. In water or glass, for instance, the speed of light is less.
- Statement: "The laws of physics are not the same in all inertial frames of reference."
  Correction: They are the same in all inertial frames.
- Statement: "Events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial."
  Correction: This is true, but it is not one of the two postulates of special relativity. (It is covered in a later lesson.)
- Statement: $\Delta E = \Delta mc^2$. Correction: This is also true, but it is not one of the postulates of special relativity.

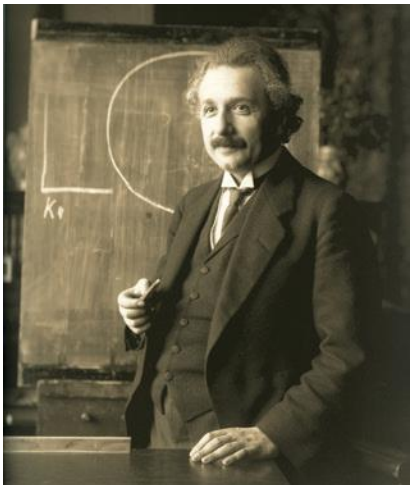


FIGURE 4 Albert Einstein, 1921

Check your learning 9.1

Check your learning 9.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** briefly how the muon experiment confirmed the theory of relativity. (2 marks)
2 **Identify** the two postulates of special relativity. (2 marks)
3 **Clarify** whether the surface of Earth is an inertial reference frame. (1 mark)
4 **Identify** if this equation is one of Einstein's postulates of special relativity: $\Delta E = \Delta mc^2$. (1 mark)

Analytical processes

5 **Distinguish** between inertial and non-inertial frames of reference. (1 mark)
6 **Deduce** which of the following is an inertial reference frame (or a very good approximation).
   a Your bedroom (1 mark)
   b A car rolling down a steep hill (1 mark)
   c A train coasting along a level track (1 mark)
   d A rocket being launched (1 mark)
   e A roller-coaster going over the top of a hill (1 mark)
   f A skydiver falling at terminal speed (1 mark)

Knowledge utilisation

7 Pi-mesons are subatomic particles with extremely short lifetimes. A collection of pi-mesons were observed by a detector to exist for 9.28×10^{-8} s when travelling at 0.96 times the speed of light. However, it is known that the average lifetime of these particles is 2.6×10^{-8} s. **Propose** an explanation for the difference between the observed and expected average lifetimes. (2 marks)

Lesson 9.2
Relative motion

Key ideas

- → All observers in inertial frames of reference agree on their relative speed.
- → Relativistic speeds are arbitrarily chosen as those greater than 0.1*c*.

Learning intentions and success criteria

What is relative motion?

For Newton, there was a "master" or absolute inertial frame: a frame that was stationary relative to absolute space. However, subsequent investigations showed that there is no master frame and that all motion is relative.

What this means is that the idea of being stationary or of moving is dependent on the observer. For example, you are travelling in a train at 60 km h^{-1} along a railway. You see houses go past, and you imagine that you are stationary and the houses (Earth) are moving at 60 km h^{-1}. But to people in the houses looking at the train, it is you who is moving.

FIGURE 1 From our stationary perspective, the rain appears to fall at an angle. To the cyclist, the rain may appear to fall vertically.

"Who is really moving?" The answer is that you are moving relative to the stationary observer outside the train, but the stationary observer is moving relative to you. So, you are each moving relative to the other. Both your inertial frame and the observer's inertial frame are equally valid. There is no way of working out who is really moving because any experiments in the train would give the same results as on the platform. The platform and the train are both inertial frames. There is one frame in which the train is moving, another in which the observer is moving, and another in which both frames are moving. However, the one thing all observers agree on is the relative speed.

A good example is of a cyclist riding through rain (Figure 1). To an observer on the ground the rain may appear to fall at an angle, but to the cyclist, the rain falls vertically. They disagree about the direction of the rain.

How is relative motion shown with the cannonball and the boat?

An old favourite example to illustrate this further is a cannonball dropped from the mast of a yacht sailing past an observer on the shore (Figure 3). For a sailor on the yacht, the cannonball appears to fall straight down (Figure 3A). From the point of view of an observer on shore, the cannonball falls with a uniform acceleration downwards while moving with constant speed in the horizontal direction; that is, the cannonball follows a parabolic path relative to the shore, just like a rock thrown horizontally off a cliff (Figure 3B). For both observers the cannonball lands at the base of the mast, and the laws of inertia are the same in both reference frames even though the paths are different.

However, in frames moving relative to each other, the velocity of an object will appear different.

Not all things change when viewed in different reference frames. For example, the number of atoms in an object doesn't change. If you time your pulse rate at home as 60 beats per minute, then you will also time it as 60 beats per minute aboard a moving train. But if you are sitting down on the train as it travels along the railway line at $5\,m\,s^{-1}$, you could say your speed is zero relative to the train ($v_{person-train} = 0\,m\,s^{-1}$), and the speed of the train relative to the Earth ($v_{train-Earth}$) = $5\,m\,s^{-1}$. Your speed relative to the Earth ($v_{person-Earth}$) would then also be $5\,m\,s^{-1}$.

FIGURE 2 Who is really moving: the people on the platform or the people on the train, or both?

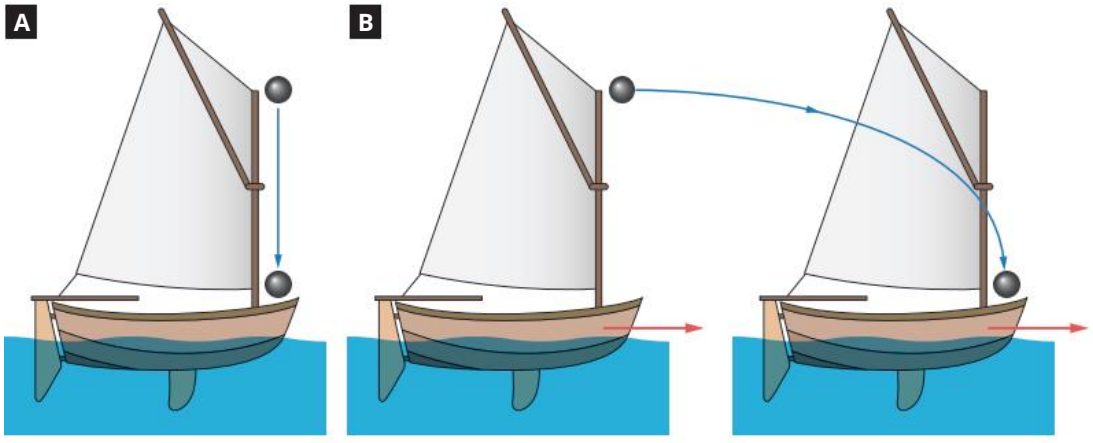


FIGURE 3 A falling cannonball travels different paths depending on the frame of reference: (A) from aboard the yacht; (B) from the shore as the yacht sails past you.

Challenge

Where does the pen land?

If you were running along and dropped a pen would it land at your feet, in front of you, or behind you? **Explain** your answer. (2 marks)

Is the speed of light constant?

As you saw in earlier modules, changing magnetic fields produce electricity; conversely, changing electric fields produce magnetism. In the mid-19th century, the great Scottish physicist James Clerk Maxwell deduced that, as each field could create the other, a "wondrous new phenomenon" would result. Once a changing field of one type appears, self-perpetuating systems of electric and magnetic fields take on an independent existence that is no longer associated with what started them, and which would propagate through space as an electromagnetic wave. Maxwell explored the properties of these waves theoretically and calculated their speed as $3.00 \times 10^8\,\text{m s}^{-1}$ (equal to the speed of light). The symbol "c" was chosen to represent the speed of light. It was the initial letter in the Latin word "celeritas" meaning "swiftness" (as in accelerate). Today, the accepted value is $299{,}792{,}458\,\text{m s}^{-1}$, which is rounded to 2 decimal places as $3.00 \times 10^8\,\text{m s}^{-1}$. It does have 3 significant figures, but we just write it as $3 \times 10^8\,\text{m s}^{-1}$ for convenience. So, there was no doubt that the speed of Maxwell's electromagnetic waves was the speed of light, and his brilliant conclusion was inescapable: light is an electromagnetic wave.

Study tip

In this unit, the speed of light of $299{,}792{,}458\,\text{m s}^{-1}$ is rounded to one significant figure as $3 \times 10^8\,\text{m s}^{-1}$.

Nineteenth-century physicists were familiar with the properties of water waves, sound waves and waves on springs. These waves all needed a medium for their propagation, be it water, air or steel. There was no reason to think that Maxwell's electromagnetic waves should be any different. They called this transparent medium "aether" and assumed it permeated all space.

It was therefore presumed that the velocity of light given by Maxwell's equations must be with respect to this "aether". Ultimately, scientists found that there was no aether; no material medium was needed for the propagation of light, and the velocity of light had a constant value in free space irrespective of the motion of the source or the motion of the observer. Einstein used these findings to develop his theory of relativity.

Study tip

The speed of light in a vacuum is $3 \times 10^8\,\text{m s}^{-1}$. Other high speeds can be stated as a fraction of this. For example, $0.8c$ is equal to $0.8 \times 3 \times 10^8 = 2.4 \times 10^8\,\text{m s}^{-1}$. Likewise, speeds in metres per second can be converted to units of c by dividing by 3×10^8. For example, a speed of $2 \times 10^8\,\text{m s}^{-1}$, in units of c would be:

$$v = 2 \times 10^8\,\text{m s}^{-1} = \frac{2 \times 10^8}{3 \times 10^8} c = 0.67c$$

Adding relativistic velocities

Einstein made his second postulate that "the speed of light in a vacuum has the same value, c, in all inertial frames of reference" and this leads to an interesting conclusion: all observers regardless of whether they are moving or not will measure the same speed of light, c, regardless of whether the source of light is moving or not.

Non-relativistic example: Car shooting tennis balls

Imagine being an observer who is watching a car that is travelling along a road at a leisurely $0.9\,\text{m s}^{-1}$. Imagine that the car has a cannon on the roof that shoots a tennis ball forwards at $1.0\,\text{m s}^{-1}$ relative to the car. The observer sees the ball coming towards them at $1.9\,\text{m s}^{-1}$ (Figure 4).

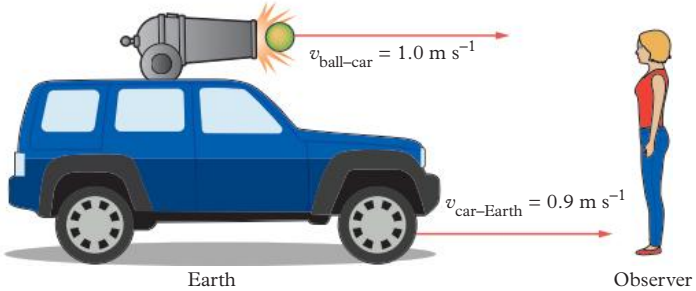


FIGURE 4 Newtonian addition of velocities

$$v_{\text{ball–Earth}} = v_{\text{ball–car}} + v_{\text{car–Earth}}$$
$$1.9\,\text{m s}^{-1} = 1.0\,\text{m s}^{-1} + 0.9\,\text{m s}^{-1}$$

where $v_{\text{ball–Earth}}$ is the velocity of the ball relative to Earth, $v_{\text{ball–car}}$ is the velocity of the ball relative to the car and $v_{\text{car–Earth}}$ is the velocity the car relative to Earth.

Relativistic example: Car with lamp

Contrast this to a headlamp on a very fast car. It sends out a light wave at $1.0c$ (Figure 5). The observer sees the car coming towards them at $0.9c$, and the light leaving the car in a forwards direction at $1.0c$ relative to the car.

In Newtonian terms the observer should measure the speed of light as $1.9c$; however, they measure it at $1.0c$ instead. The velocity of the source makes no difference.

$$v_{\text{light–Earth}} = v_{\text{light–car}} + v_{\text{car–Earth}}$$

$$1.9c \neq 1.0c + 0.9c$$

$$1.0c = 1.0c + 0.9c \text{ (using relativistic addition of velocities)}$$

where $v_{\text{light–Earth}}$ is the velocity of the light relative to Earth, $v_{\text{light–car}}$ is the velocity of the light relative to the car and $v_{\text{car–Earth}}$ is the velocity of the car relative to Earth. There is a special formula for the relativistic addition of velocities but it is not a part of the syllabus.

Study tip

"Relativistic speed" is a term used by physicists to denote that relativistic formulas (not Newtonian or classical formulas) should be used. It is an arbitrary cut-off, but you can regard anything greater than $0.1c$ as being relativistic.

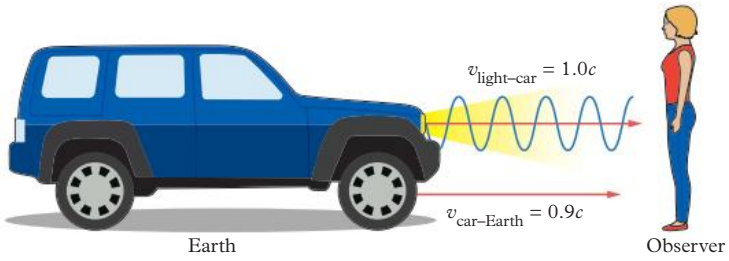


FIGURE 5 Failure of Newtonian kinematics at high speeds

The following Real-world physics confirms the unchanging speed of light.

Real-world physics

Pulsars confirm special relativity

There is a pair of burnt-out neutron stars in the constellation Aquilia 1,500 light-years (1.4×10^{16} km) from us. The stars, called PSR 1913+16, are a binary pulsar; that is, they orbit around a common centre and give off pulses of light waves as they spiral to their death. Because we see them side on (like looking at a saucer from the side), sometimes they are approaching and sometimes receding (moving away) – at a speed of 300 km s^{-1} (about 0.1 per cent the speed of light) (Figure 6).

In 1974, the US astronomers Joe Taylor and Russel Hulse measured the characteristics of light emanating from the pulsars and found the speed of the light to be the same irrespective of whether they were approaching us or receding from us. For this, and other confirmations of relativity, they were awarded the Nobel Prize in Physics in 1993.

These burnt-out stars provide an example of the confirmation of the unchanging speed of light.

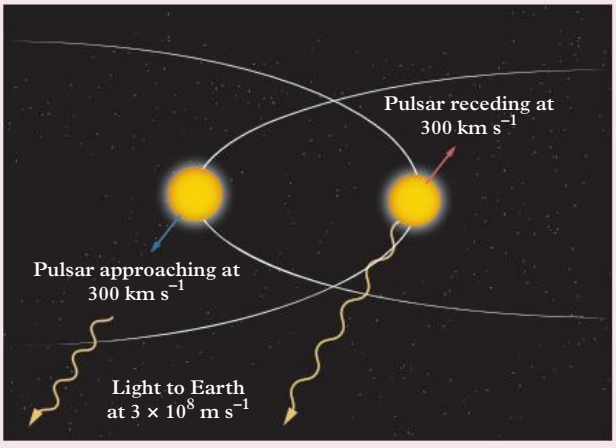


FIGURE 6 The light from binary pulsars arrives on Earth at c, irrespective of whether the stars are approaching or receding.

Apply your understanding

1 If special relativity were refuted by the evidence from these observations, **propose** what the astronomers would have noticed. (1 mark)

2 **Explain** why it is necessary to be looking at the pulsars side on. That is, why not observe them at right angle to their plane of revolution? (1 mark)

3 **Prove** that 1,500 light-years is equal to 1.4×10^{16} km. (3 marks)

Challenge

Velocity of a rocket relative to Earth

Einstein derived a formula to allow for the relativistic addition of velocities.

$$v_{AB} = \frac{v_A - v_B}{1 - \frac{v_A v_B}{c^2}}$$

This means that v cannot be $>1.0c$, to avoid violation of the second postulate of relativity.

Rocket B was travelling away from Earth at a velocity of $v_{BE} = +0.60c$ when it launched rocket A forwards in the same direction at a velocity of $+0.60c$ relative to itself ($v_{AB} = +0.60c$) (Figure 7).

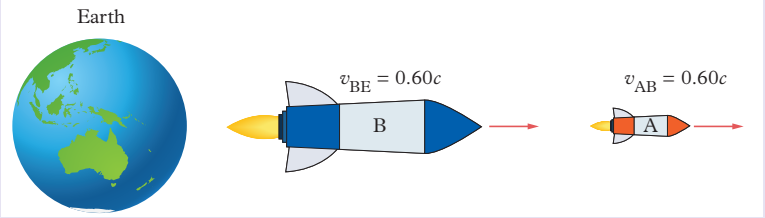


FIGURE 7 Rocket B ($v_{BE} = +0.60c$) launches rocket A ($v_{AB} = +0.60c$). Rocket A is moving at $0.60c$ relative to rocket B.

Prove that the velocity of rocket A relative to Earth (v_A) is not $+1.2c$ (as this would contradict Einstein's second postulate), but that v_A is only $+0.88c$. (3 marks)

Challenge

Can you add velocities?

Evaluate whether the statement "you can't just simply add velocities together" is incorrect. (2 marks)

Check your learning 9.2

Check your learning 9.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Describe** what is meant by "relativistic speed". (1 mark)

2 **Calculate** what a speed of $0.5c$ is in $m\,s^{-1}$. (1 mark)

3 **Identify** whether this sentence is true or false: "An object's velocity can only be measured in a frame of reference in which it is moving." (1 mark)

Analytical processes

4 A very fast car travels at $0.6c$ relative to Earth and fires a particle forwards at $+0.5c$ relative to the car. **Determine** which of the following is the speed of the particle relative to Earth: $<1.0c$, $1.0c$, $>1.0c$. (1 mark)

5 A mirror moves towards you at speed v. You shine a light towards it and the light beam bounces back at you. **Determine** the speed of the reflected beam. (1 mark)

Knowledge utilisation

6 The speed of light in glass is only $2 \times 10^8 \text{ m s}^{-1}$. Write a short paragraph to **evaluate** the proposition that "Einstein made a blunder in saying that the speed of light is constant". (2 marks)

7 **Propose** a method for measuring the speed of light in a laboratory. (1 mark)

8 Part A of the diagram shows a cannon ball dropped from the top of the mast of a boat at rest to observer standing on the shore. Part B shows the same cannonball dropped from the top of the mast according to an observer standing on the shore who sees the boat as moving.

a **Predict** what the path in the diagram would look like if gravity were

i less than 9.8 m s^{-1} (1 mark)

ii more than 9.8 m s^{-1} (1 mark)

iii zero. (1 mark)

b **Predict** how the path in the diagram would differ if the cannonball was half the original mass. (1 mark)

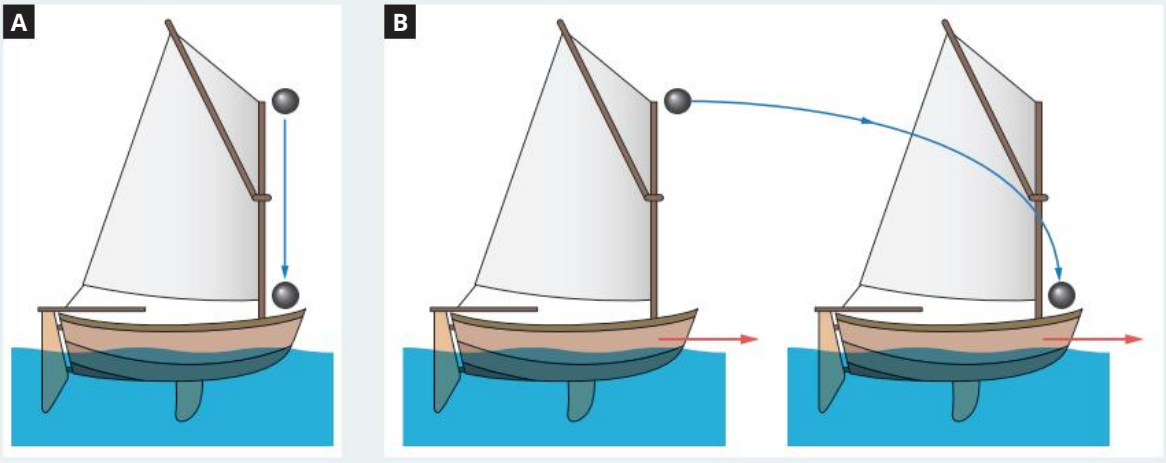
c The mast was 20 m high, and the boat sailed at a speed of 20 m s^{-1} relative to the shoreline. **Determine** how many seconds the cannonball would take to hit the deck in both diagrams. (2 marks)

d **Calculate** how far to the right the boat would have travelled in this time. (1 mark)

e **Calculate** average velocity of the cannonball relative to the shoreline in its journey shown in diagram B. (3 marks)

f **Determine** how far the cannonball would have travelled in the diagram relative to the shoreline. You will need to work out the "arc length" of the parabola. Caution – this is a very difficult question, and you will need to use calculus! A calculation like this would definitely not be on the external exam. (5 marks)

9 You are in a spaceship travelling in outer space. **Evaluate** the claim that there is no way you can measure your speed or acceleration without looking outside. (3 marks)



Lesson 9.3
Simultaneity

Key ideas

- → Two events in a reference frame are simultaneous if light signals from the events reach an observer halfway between the events at the same time.
- → Two events that are simultaneous to one observer are not necessarily simultaneous to a second observer moving with respect to the first.

What is simultaneity?

Learning intentions and success criteria

Imagine that at your school there are two bells, one at each end of the school. You hear both bells sound at the same time. Could there be a situation in which another observer hears one bell before the other? In other words, can an event (the sounding of the bells) be simultaneous to one observer but not to another?

event
an act or action with a distinct beginning and end, e.g. the swing of a pendulum, ticking of a watch, motion of an object (including a light pulse) from one place to another, decay of a nucleus, propagation of an energy wave (light, sound, water)

simultaneity
the relationship between two events assumed to be happening at the same time in a particular frame of reference

What does "simultaneous" mean? Two **events** are simultaneous if they occur at the same time. But how can you tell if the two bells rang at the same time? If the bells were side by side, there would be no problem; but when events are separated in space it gets difficult. If you were midway between the two bells and you heard them ring at the same time, you could say they were simultaneous. But what if you were closer to one bell than the other? If you still heard them at the same time, the more distant one must have sounded first because the sound had to travel further to your ears.

Does this apply to light as well? If you were looking out of your window at dusk and two streetlights came on at the same time, you could say the events were simultaneous if you were midway between them (Figure 1). If you were not midway, you would have to calculate the time it took to get from each event to your position so that you could work out when the events actually occurred. If both lights appeared to turn on at the same time but one was closer to you than the other, the closer light must have turned on after the more distant light. They were not simultaneous. **Simultaneity** can be described thus: two events are simultaneous in a given reference frame if light signals from the events reach an observer who is midway between them at the same time. A broader definition is that simultaneity is the relationship between two events that are assumed to be happening at the same time in a particular frame of reference.

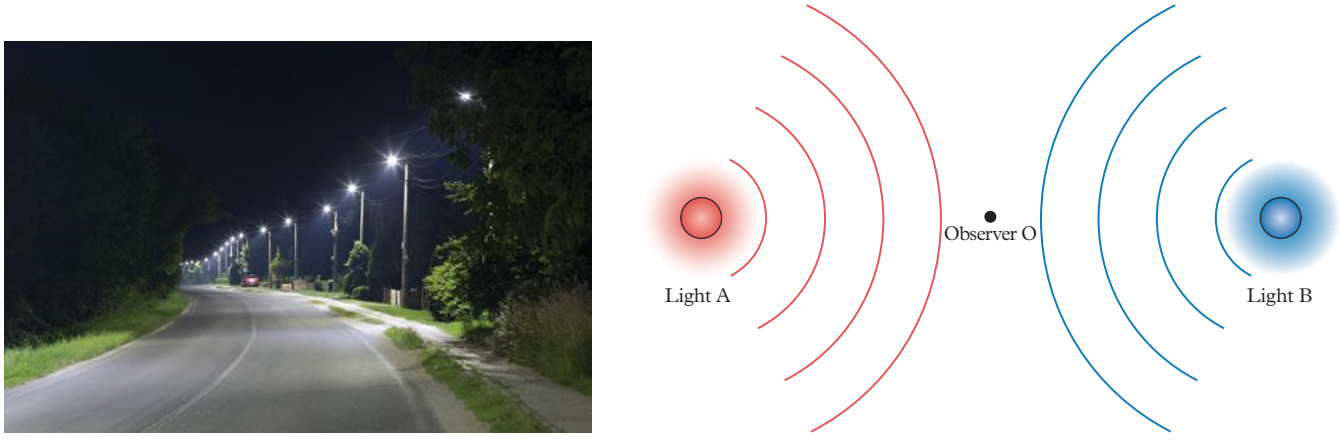


FIGURE 1 A moment after streetlights A and B turn on, light waves travel outwards. If they arrive at stationary observer O at the same time, she can say the events are simultaneous because she is midway between the two lights.

Events that are not simultaneous

It is easy to get confused with all the requirements for declaring something to be simultaneous. For example, the following all sound reasonable but are not correct descriptions of "simultaneous" in physics.

- Description: Two events occur at the same time.
  Correction: They also have to happen in the same frame of reference.
- Description: An observer sees two events occurring at the same time.
  Correction: The observer also must be at rest with respect to the events.
- Description: Two events happen at the same time when observed from any frame of reference.
  Correction: This must be the same frame of reference.

In what way is simultaneity relative?

relativity of simultaneity events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial

Two events that are simultaneous in one frame are not necessarily simultaneous in another frame that is moving relative to the first. This is known as the **relativity of simultaneity**. It is a paradox introduced by Einstein. He called such paradoxes "Gedanken" ("thinking") experiments. They were true premises (ideas or statements) that seemed to result in self-contradictory conclusions, which is the definition of a paradox. There are three that you should be familiar with. This is the first.

FIGURE 2 The "flashlights on a train" paradox is one of Einstein's "thinking" experiments.

The "flashlights on a train" paradox

Imagine an observer sitting on an embankment at the side of a train track. A train is the moving frame of reference, and the embankment is the stationary frame.

Imagine that in the centre of the train carriage there is a person holding a flashlight that can send out a pulse of light in the forwards direction and in the backwards direction at the same time, as shown in Figure 3. Also, imagine that the front and rear doors of the carriage are opened automatically when the light pulses arrive.

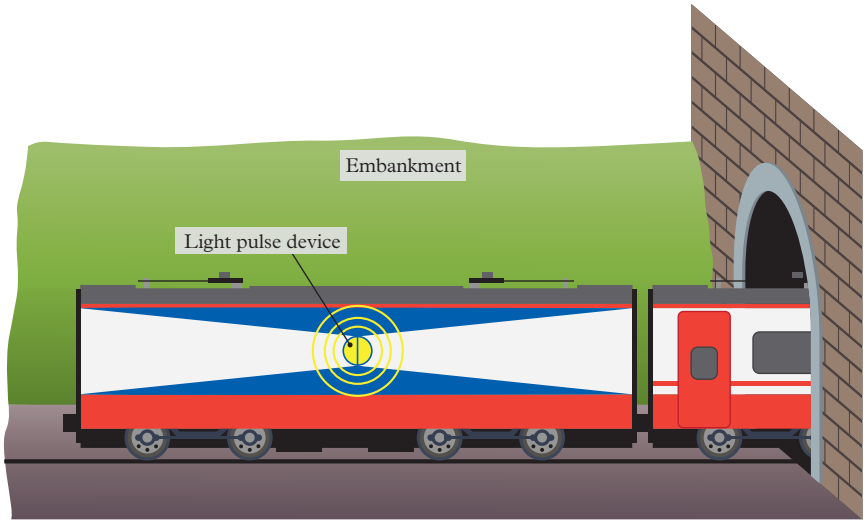


FIGURE 3 The train with the light pulse device

To the person holding the flashlight, the doors of the carriage will open simultaneously (Figure 4A). But to a person on the embankment, the rear door will open before the front door (Figure 4B). This is because the stationary observer sees the back door move forwards to meet the light pulse while the front door moves away from the light pulse, so the light gets to the rear door before it can get to the front door.

Therefore, the opening of the doors may be simultaneous to one observer (on the train) but not simultaneous to another (on the embankment). Students often ask, "Who is right? Do the doors really open together or not?" The answer is that both are right. It depends on your frame of reference. Remember, there is no absolute or best frame of reference; some are just more useful than others. You, as an observer, will decide which is the more useful frame.

What we have just witnessed is called the "loss of simultaneity". That describes how, when observers are moving relative to each other (are in different frames of reference), events that were simultaneous when the observers were stationary with respect to each other (in the same frame of reference) are now not simultaneous.

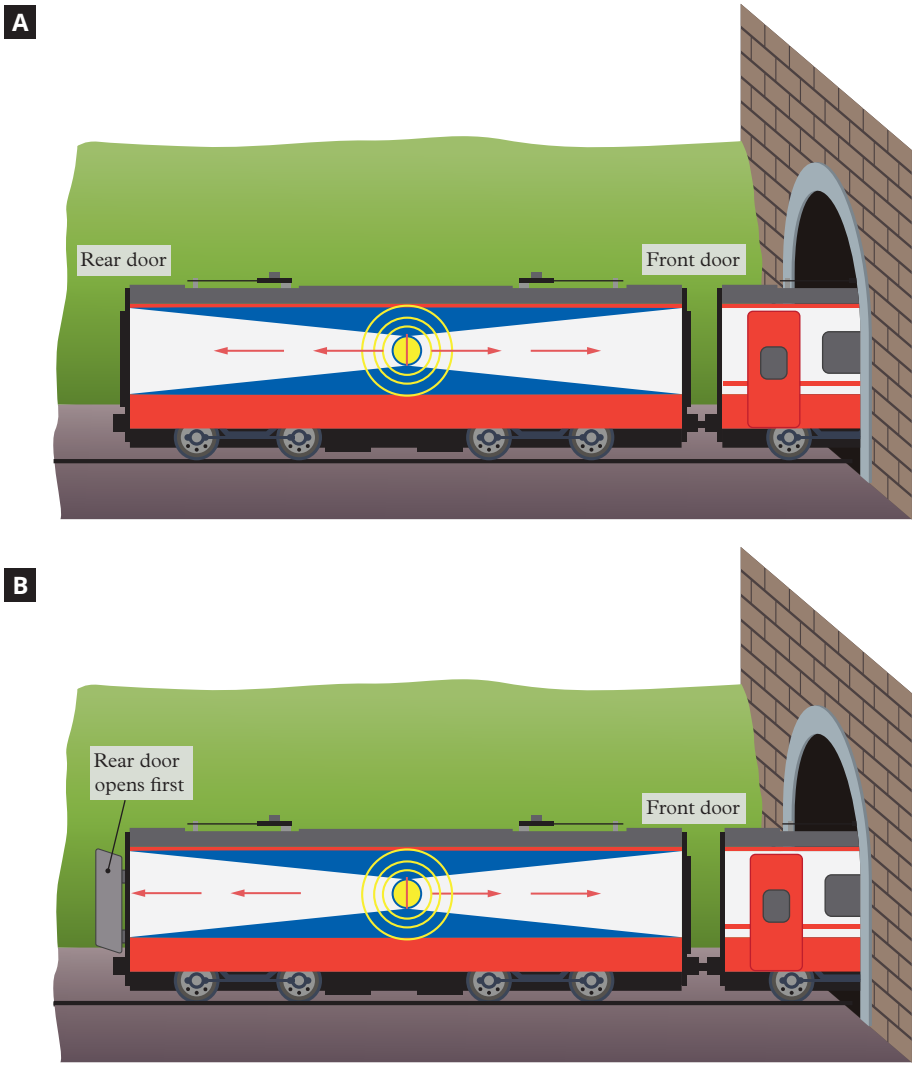


FIGURE 4 (A) Motion of the light pulse as seen by an observer inside the train. (B) Motion of the light pulse as seen by the observer on the embankment – it reaches the rear door before it reaches the front door.

Challenge

Image in a moving mirror

When Einstein was a boy, he wondered about the following question. A runner holds a mirror at arm's length in front of his face. Can he see himself in the mirror if he runs at the speed of light?

When you look at yourself in a mirror, light travels from your face to the mirror and then is reflected back to your eyes, as shown in Figure 5A.

Einstein wondered how light could ever get from your face to the mirror if the mirror is travelling away from the light beam at the speed of light (Figure 5B). The light would never catch up to the mirror! He soon realised the flaw in the logic. Can you? **Propose** your reasons. (2 marks)

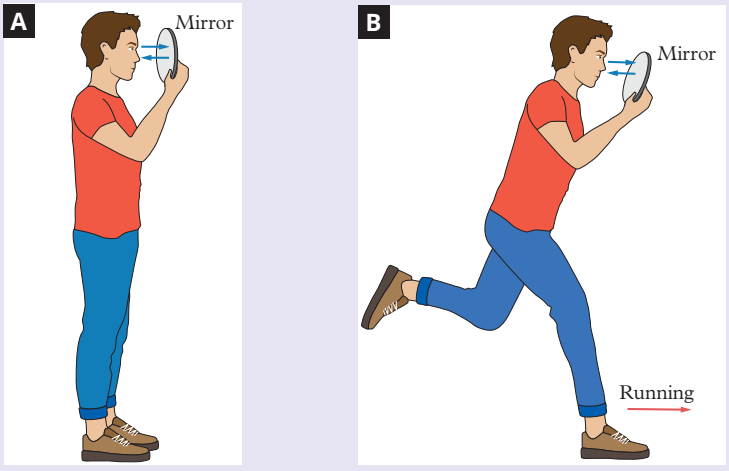


FIGURE 5 (A) The motion of light when you are at rest. (B) Your face and mirror are moving at speed *c*. Can you see yourself now?

Check your learning 9.3

Check your learning 9.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** the meaning of "simultaneity". (2 marks)

2 **Describe** how the "flashlight on a train" paradox is resolved. (2 marks)

3 **Identify** the term that states that "events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial". (1 mark)

Analytical processes

4 A light in the middle of a train carriage is used to open doors at each end of the carriage. One door is red and the other is black. To an observer in the carriage the doors open at the same time. To an observer on the platform the red door opens first. **Determine** the direction in which the train is travelling and state this clearly. (1 mark)

5 Two bells in a school are rung and a person midway between them hears them at the same time. However, a person standing closer to one bell hears it ring first. **Determine** whether the bells rang simultaneously or not and explain your reasoning. (2 marks)

Knowledge utilisation

6 **Evaluate** this claim: "Events that are simultaneous in one frame will be simultaneous in another frame travelling at constant speed with respect to it." (2 marks)

7 It's raining. A cyclist says the rain appears to be falling at an angle, but a bystander says it is falling vertically. **Evaluate** whether they are both correct and it just depends on the frame of reference of each, or if the bystander is really correct. (2 marks)

Lesson 9.4
Relativity of time

Key ideas

- → The time interval for an event is dependent on the motion of the observer with respect to the event.
- → An observer moving relative to an event will measure the time interval as longer than would an observer at rest to the event. This is called time dilation and is summed up as "Moving clocks are measured to run slowly".
- → Problems involving time dilation can be solved using: $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ where t_0 is proper time and t is dilated or relativistic time interval. The relativistic (dilated) time interval is longer than the proper time interval.

Learning intentions and success criteria

How is time variable?

What is time? When Einstein was asked, he said "what a clock measures" and left it at that. You usually think of time marching on, oblivious to anything you may be doing. Although time seems to drag when you are doing something boring and goes fast when you are having fun, this is not time in a physical sense, only your perception of time. However, physical, measurable time can also vary.

The following example may convince you that the time interval between two events cannot be the same for two observers in motion with respect to each other. The first observer is sitting on the bus. This bus has a light source on the floor and a mirror directly above it on the ceiling. A brief flash of light leaves the source and travels upwards to hit the mirror, reflect and return to the source, as shown in Figure 1A.

To a second observer who is sitting at a bus stop watching the bus pass by, the flash occurs when the bus is located to the left, strikes the ceiling mirror when the bus is directly in front of the second observer, and then returns to the source when the bus is to the right of the observer, as shown in Figure 1B.

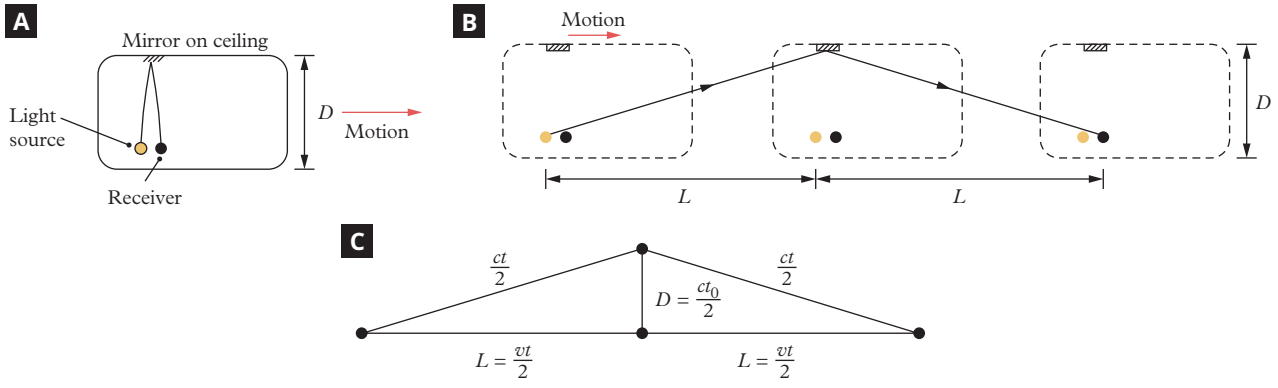


FIGURE 1 (A) Path of light as viewed by the observer aboard the bus, and (B) path of light as viewed by the observer at the bus stop – the path looks much longer. (C) Part of the derivation of Einstein's formula

This is labelled in Figure 1 is as follows.

- The distance from the source to the ceiling mirror is D.
- To the second observer (at the bus stop) the bus is travelling to the right at velocity v and moves a distance L in the time between the flash and the light striking the ceiling.
- The bus moves another distance L by the time the light goes from the mirror back to the source. This makes the total distance moved by the bus equal to $2L$.
- Remember, light travels at a speed c for both observers (Einstein's second postulate).

To the first observer aboard the bus, the light travels a distance of $2D$ (up to the mirror and back to the source) in a time t_0.

Proper time interval

In the example in Figure 1, the observer on the bus measures what is called the **proper time interval**. It is the time interval for an event where the start and finish occur in the same place in space. The start and finish have not moved relative to the observer. In other words, they were at rest relative to the observer. "Proper time" and "proper time interval" are two different things. "Proper time" is the time shown by a clock at rest relative to the observer. The proper time interval, t, is the difference in proper time for some event.

proper time interval
the time between two events measured by an observer at rest relative to the events

Using the velocity formula $v = \frac{s}{t}$:

$$t = \frac{s}{v}$$

As the distance travelled by the light according to the observer on the bus is $2D$, and the speed of light is c, then:

$$t_0 = \frac{2D}{c}.$$

When rearranged, this can give the distance as $D = \frac{ct_0}{2}$ (Figure 1C), where D is the distance from the source to the ceiling mirror, c is the speed of light and t_0 is the proper time as measured by the observer on the bus. To the second observer (at the bus stop), the light has travelled a triangular path in time t (t with no subscript, as distinct from proper time, t_0). As the speed of light is the same for both observers and the light travelled a longer path according to the observer at the bus stop, it must have taken longer than the proper time, t_0.

Relativistic time interval

In our example, the observer at the bus stop has measured what is called the **relativistic time interval** or "dilated" time. Relativistic time interval is the time between two events that occur in two different places in space. In this case the two events (the flash leaving the source and the flash arriving back at the source) are separated by a distance of $2L$. They have moved relative to the outside observer.

relativistic time interval
the time between two events measured by an observer moving with respect to the events; also known as dilated time interval

Looking at Figure 1C, as seen by the observer at the bus stop, the total distance s travelled by the light is calculated using the velocity formula $v = \frac{s}{t}$, or $s = vt$, where v is the velocity of light and t is the time taken by the flash to strike the mirror and return to the source. In this case, the distance travelled by the light is ct. In the diagram, the length of the hypotenuse equals $\frac{ct}{2}$. The time taken, t, for the bus to go from start to finish is given by $t = \frac{2L}{v}$, which can be rearranged as $L = \frac{vt}{2}$, the base of each triangle.

Using Pythagoras' theorem on one of the right-angled triangles:

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{ct_0}{2}\right)^2 + \left(\frac{vt}{2}\right)^2$$
$$(ct)^2 = (ct_0)^2 + (vt)^2$$
$$t^2 = t_0^2 + \frac{v^2}{c^2}t^2$$
$$t_0^2 = t^2 - \frac{v^2}{c^2}t^2$$
$$t_0^2 = t^2\left(1 - \frac{v^2}{c^2}\right)$$
$$t^2 = \frac{t_0}{1 - \frac{v^2}{c^2}}$$

Take square roots of both sides:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where t is the time measured by an observer at the bus stop (the relativistic time interval), t_0 is the time measured by the observer on the bus at rest to the event (the proper time), v is the velocity of the bus relative to the observer at the bus stop and c is the speed of light.

Because v is always less than c, the value of $\sqrt{1 - \frac{v^2}{c^2}}$ must always be less than 1, so $t > t_0$.

What is time dilation?

In the bus scenario, we can see that the time between the two events (the flash of light leaving the source and arriving back at the source) is greater for the second observer (at the bus stop) than for the first observer (on the bus).

time dilation
the difference in the time interval between two events as measured by observers that are moving with respect to each other

This is a general result of the theory of relativity and is known as **time dilation**. Sometimes this is stated simply as "moving clocks are measured to run slow", but you need to be careful with the interpretation.

Challenge

Can you use your pulse to measure time?

Galileo measured the period of swinging lamps using his pulse.

a **Propose** why did not he use a watch. (1 mark)

b **Propose** two advantages and disadvantages of using your pulse as a timekeeper. (2 marks)

FIGURE 2 The clock in Bern, Switzerland, that inspired Albert Einstein in 1905

Do moving clocks run slow?

One evening in May 1905, Einstein was on his way home from his work at the Swiss Patent Office in Bern, Switzerland. He was riding in a tram along Kramgasse Street in the old city centre when he passed the big medieval clock tower (Figure 2). He gazed up at the tower and suddenly imagined what would happen if the tram raced away from the tower at the speed of light.

Einstein reasoned that while he was stopped beside the clock, he would be able to see the hands tick over. The light from the clock's hand would travel to his eyes and he would see movement.

He also reasoned that if he were to travel at the speed of light, the light from the hand of the clock would never catch him and he would not see the hands tick over. It would seem as though the clock – and time – had stopped. Einstein also realised that his wristwatch would continue to show that time was really passing. Within six weeks of this moment Einstein had completed a paper outlining the principles of relativity.

You can think of the principles of relativity as "moving clocks run slow". However, this only refers to clocks moving with respect to you. The clocks will be stationary to someone moving along with the clock – that is, to someone in the same frame of reference as the clock. Moving is relative! Time really does pass more slowly in a frame of reference moving with respect to you (in Einstein's case, the moving frame of the clock).

Imagine using a clock to time some event. To someone stationary with respect to the clock, the time for the event may be 1 minute. For example, you could measure the time between the start and finish of the events in a frame moving relative to you as 2 minutes (the time between hand movements on the clock). To someone stationary with respect to the clock, the time between the events may be 1 minute. If they say 1 minute and you say 2 minutes, it seems longer to you. This is the inevitable outcome of the two postulates of the theory of relativity! Equally, you see your wristwatch tick over every minute as you are stationary with respect to that watch, but to an observer on the ground as you travel past, your watch would be moving and would take 2 minutes to tick over. They are the ones moving relative to your watch.

You might ask whether this is really true or whether it just appears to be true. All you can say is that it does not violate the laws of physics and that it has been confirmed by many experiments, so it can be called a "fact". However, like all "facts" in science, it can be replaced if better theories and experiments come along. For now, special relativity works beautifully well.

The concept of time dilation is hard to accept, as it violates common sense, but you can see from the earlier equation for relativistic time, $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, that the time dilation effect is negligible unless v is reasonably close to c. Table 1 shows the ratio of $\frac{v}{c}$ (called the speed factor, β) for different speeds, and the ratio of $\frac{t}{t_0}$ (called the Lorentz factor, γ, after the Dutch physicist H.A. Lorentz, who developed the formula before Einstein, but didn't realise its significance). The Lorentz factor can also be written as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1 - \beta^2}}$$

Note, though, that the symbol beta (β) is not a part of the syllabus and does not appear in the Formula and data book. It is just for your convenience.

TABLE 1 Time dilation factors at various speeds

Speed	$\beta = \frac{v}{c}$	$\gamma = \frac{t}{t_0}$
Car at 60 km h^{-1}	0.0000	1.000000
Jet at 8,000 km h^{-1}	0.0000068	1.000000
Parker space probe at 343,100 km h^{-1}	0.0003177	1.000000
Electron in a cathode ray tube	0.1	1.005
Lightning strike	0.3	1.05
Speed of light in optical fibre	0.7	1.40
1 GeV electron	0.99999988	2,000
20 GeV electron	0.99999999967	40,000
Light pulse	1.000	infinite

Study tip

The speed factor, β, equal to $\frac{v}{c}$, is not required for the external exam but is a shorthand way of problem-solving. An object travelling at $0.8c$ has a speed factor $\beta = 0.8$.

Table 1 shows that at ordinary speeds (e.g. a car) relativistic effects are negligible, but at speeds approaching the speed of light, the effects are dramatic. An experimenter working with 1 MeV electrons travelling at 0.94 times the speed of light ($0.94c$) would know that 1 s of time in the electron's frame of reference is the same as 2.9 s of time in the laboratory frame of reference. Imagine that the electron orbited a nucleus every second when viewed from a frame of reference stationary to the electron (that is, if you rode along with the electron). To an observer in the laboratory, the electron would take 2.9 s to orbit once. The electron's "clock" runs slow according to the other observer.

Thousands of experiments have confirmed the theory of relativity. For example, in 1971 extremely accurate clocks were flown around the world on commercial aircraft, and when they were compared to the clocks left back in the laboratory, time dilation was confirmed.

How do we know which clock is moving?

The thing that confuses students most of all in this work is defining which is the moving clock and which is the stationary clock. Einstein said that motion is relative, so you could say either is stationary or either is moving. So, which is "t" and which is "t_0" in the formula?

Synchronising clocks

Before you can compare clocks, you need to ensure they are synchronised. This means they agree on the time of day. If clocks are side by side, it is easy to check if they are synchronised; however, if they are on different planets, it is harder. You could:

- synchronise them when they are together and then move them apart
- send out a signal to both clocks simultaneously from a point midway between them
- send a signal from Earth to a distant clock and work out the time delay based on how far away it is, then adjust the distant clock by that amount.

Study tip

The Lorentz factor $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ can be used as shorthand in writing equations. For example, $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ becomes $t = t_0\gamma$. This is not a part of the syllabus and is not in the Formula and data book, but it can be handy to speed up calculations.

Identifying proper and dilated time intervals

A good demonstration that may help you understand the difference between proper and dilated time intervals is to consider a toy car running across the front bench of the laboratory (Figure 3).

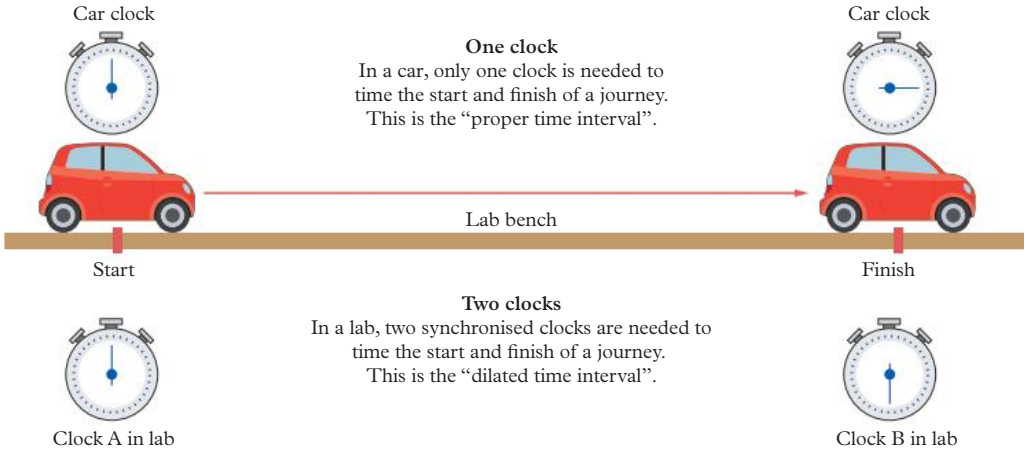


FIGURE 3 Measuring the time taken for a toy car to travel across the bench – which is the proper time interval?

Study tip

If you could ride along with any short-lived subatomic particle, you would measure its rest lifetime.

The simplest way to work out who is measuring proper time is to ask how many clocks are needed by each observer to measure the time interval – in this case, for the journey across the bench. An imaginary observer driving the toy car needs just one clock (fixed to the car).

The start and finish line will appear in the same place (under the clock on the car). So the observer in the car measures proper time. The events (start and finish) occur in one place to the car driver. However, the observer sitting in the laboratory will need two clocks – one at the start and one at the finish. This observer is measuring dilated (relativistic) time, as the events are in two different places to this observer.

Timing space travel

Many analogies for time dilation used by Einstein and his colleagues involved space travel. Imagine that you are in fact timing a rocket flight to the Moon. There is a clock on board the rocket for the space travellers and there are synchronised clocks on Earth and on the Moon.

The two events – take-off and landing – are measured by the space travellers with a single clock on the spacecraft, but the observers on Earth need two clocks for their timing – one on Earth to register the time of take-off and a synchronised one on the Moon to register the time of landing. Thus, the space travellers have measured the proper time interval, t_0, because the two events were measured in the one place by one clock at rest with respect to both events. The people on Earth measured the dilated (relativistic) time interval, t. The relativistic time interval is longer than the proper time interval: $t > t_0$.

Study tip

In the external exam, the number of significant figures (s.f.) will not be stated. It is up to you to choose an appropriate number. Using 2 s.f. is generally appropriate, but 3 s.f. would be okay. You will not be penalised if you use too many. In all cases, do not use 1 s.f. as it will probably be unacceptable.

Worked example 9.4A

Calculating dilated time

A very fast train travelling at $0.80c$ passes through a very long railway station. An observer on board at the front of the train measures the time interval to go from one end of the station to the other as 1.0 s. **Calculate** the length of time in seconds a person on the platform would measure for this time interval. Give your answer to an appropriate number of significant figures. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find an answer or number by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$v = 0.80c$, t (train) = 1.0 s, t (platform) = ? $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
Step 3: Identify the proper and dilated time intervals. The observer on the train measures the proper time, t_0, because she stays in the same place with respect to the train and would only need one clock to measure the time interval from one end of the station to the other. It would be as if the station was rushing past her. (1 mark for "Identifies t_0 as 1.0 s")	The proper time, t_0, would be measured by the observer on the train and would equal 1.0 s. (1 mark)
Step 4: The observer on the platform measures the dilated (relativistic) time interval, t, because to them the start and end of the time interval occur in two places – one at the start of the platform and one at the other end of the platform. They see the train as moving and would therefore need two clocks (one at each end) to measure the time interval. (1 mark for "Identifies t as the time measured by the platform observer")	The dilated (relativistic) time, t, would be measured by the observer on the platform. (1 mark)

Think	Do
Step 5: Substitute in the known values and complete the calculation. Give your answer to an appropriate number of significant figures – in this case, 2 s.f. Check you have the correct units. (1 mark for "Correctly substitutes values"; 1 mark for "Provides correct answer of 1.7 s")	The velocity of the train, v, is $0.8c$. $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $= \frac{1.0}{\sqrt{1 - \frac{(0.80c)^2}{c^2}}}$ (cancel out the c^2 terms) $= \frac{1.0}{\sqrt{1 - (0.80)^2}}$ (1 mark) $= \frac{1.0}{\sqrt{0.36}}$ = 1.7 s (2 s.f.) (1 mark)

Your turn

A very fast train travelling at $0.90c$ passes through a very long railway station. An observer on the platform measures the time interval for the front of the train to go from one end of the station to the other end as 3.0 s. **Calculate** the length of time an observer located in the front of the train would measure for the same event. Give your answer to an appropriate number of significant figures. (4 marks)

What is mean lifetime?

mean lifetime
the average time before decay of an elementary particle as measured by an observer at rest to the particle; also known as rest life

A term commonly used when describing extremely high-speed subatomic particles is "rest life" or **mean lifetime**. Many of these elementary particles decay very quickly. Recall that muons decay into an electron and two neutrinos in an average time of 2.2 μs (2.2 microseconds or 2.2×10^{-6} s) as measured by an observer at rest relative to the muon. This is its mean lifetime or rest life, which is the average value for all muons. Some muons live shorter and some live longer. As this is measured at rest relative to the muon, it is the proper time, t_0. It is different to the half-life term you encountered in Unit 1. They are related $\left(\text{mean lifetime} = \frac{\text{half life}}{0.693}\right)$, but we use "mean lifetime" in special relativity.

Worked example 9.4B

Calculating the lifetime of a muon in motion

Determine the mean lifetime of a pion (an elementary particle) as measured in the laboratory if it is travelling at $0.669c$ with respect to the laboratory. Its mean lifetime at rest is 3.5×10^{-8} s. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	mean lifetime at rest = 3.5×10^{-8} s, $v = 0.669c$ $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
Step 3: Determine whether the mean lifetime at rest is proper time interval or dilated time interval. (1 mark for "Determines the time given as the proper time interval")	The mean lifetime of a pion at rest, 3.5×10^{-8} s, must be proper time interval because the observer is at rest relative to the pion. (1 mark)

Think	Do
Step 4: The question asks for the mean lifetime of a pion according to an observer in the laboratory. As the pion is travelling at $0.669c$ relative to the laboratory, it is not in the same frame of reference as the laboratory. Therefore, that observer measures the dilated (relativistic) time, t. You would expect the pion to live longer than the mean lifetime, 3.5×10^{-8} s, because of time dilation. Give your answer to an appropriate number of significant figures, in this case, 3 s.f. (1 mark for "Correctly substitutes values"; 1 mark for "Calculates the dilated time correctly")	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $= \frac{3.5 \times 10^{-8}}{\sqrt{1 - \frac{(0.669c)^2}{c^2}}}$ (cancel out the c^2 terms) $= \frac{3.5 \times 10^{-8}}{\sqrt{1 - (0.669)^2}}$ (1 mark) $= \frac{3.5 \times 10^{-8}}{0.743}$ $= 4.71 \times 10^{-8}$ s (3 s.f.) (1 mark)

Your turn

Determine the mean lifetime of a neutral pion (π^0) as measured in the laboratory if it is travelling at $0.98c$ with respect to the laboratory. Its mean lifetime at rest is 80 attoseconds (80×10^{-18} s or 8.0×10^{-17} s). Give your answer to an appropriate number of significant figures. (3 marks)

Worked example 9.4C

Calculating velocity from proper and dilated time intervals

An electron takes 0.25 μs to move between the plates of an electron gun as measured by an observer holding the gun. The gun is aboard a spaceship moving past Earth. On Earth, observers measure the time for the electron to move between the plates as 0.80 μs. **Calculate** the speed of the spaceship. Give your answer to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data and select the appropriate formulas.	Time for observer holding electron gun = 0.25 μs Time for observer moving relative to electron gun = 0.80 μs $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
Step 3: Determine which time value is the proper time interval and which is the dilated time interval. (1 mark for "Determines the time intervals correctly") Note: you could identify which is t and which is t_0 by recalling that $t > t_0$.	The observer holding the electron gun is stationary relative to the gun, so the time they measure, 0.25 μs, is proper time interval, t_0. The observer on Earth is moving relative to the electron gun, so the time they measure, 0.80 μs, is the dilated (relativistic) time interval, t. (1 mark)
Step 4: Rearrange the equation to make the v term the subject. Substitute the known values into the equation. Our answer for v will be in units of c, so we can use this to simplify the equation in line 3. Solve for velocity, and make sure you have used the correct units. Give your answer to an appropriate number of significant figures – in this case, 2 s.f. (1 mark for "Correctly substitutes values"; 1 mark for "Provides correct answer")	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $0.80 \times 10^{-6} = \frac{0.25 \times 10^{-6}}{\sqrt{1 - v^2}}$ (v is in units of c) $\sqrt{1 - v^2} = \frac{0.25 \times 10^{-6}}{0.80 \times 10^{-6}}$ (1 mark) $1 - v^2 = \left(\frac{0.25}{0.80}\right)$ $v = \sqrt{0.9023}$ $= 0.95c$ (1 mark)

Your turn

A muon is approaching Earth and decays 2.2 ns after it is formed in its own frame of reference. In Earth's frame of reference, it decays 23 ns after it is formed. **Calculate** the speed of the muon in units of c according to Earth observers. Give your answer to an appropriate number of significant figures. (3 marks)

Skill drill

Assessing and modifying research questions

Science inquiry skills: Understanding the scientific method (Lesson 1.3); Conducting your research investigation (Lesson 1.12)

Writing a research question

Just like in the student experiment, you will be expected to write a research question to direct your research investigation. There is no such thing as a "correct" or "incorrect" research question. For every claim, there are many possible research questions. However, your research question needs to follow a specific structure to meet the assessment (ISMG) criteria.

- Relevant: is the research question appropriate to the claim and/or topics covered in Unit 4?
- Specific: is the research question clearly defined?
- Question format: is it written in question format?

Your research question needs to be specific to get you top marks. Just like for a student experiment, where your research question should be able to guide someone to complete the same experiment, your research question for the research investigation should guide someone to conduct the same research.

Practise your skills

1 Copy Table 2 and **assess** the research questions. Tick or cross the criteria boxes to determine if they satisfy the three assessment criteria. **Modify** those that do not get three ticks so that they satisfy all three criteria. (3 marks)

TABLE 2 Suggested research questions

Research question	Relevant?	Specific?	Question format?	Modified research question
Humans may be able to reach nearby stars in less than a lifetime as a consequence of special relativity.				
Is nuclear energy too costly for Australia?				
Is special relativity real or just theoretical?				

Check your learning 9.4

Check your learning 9.4: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** what is meant by "moving clocks run slow". (1 mark)

2 **Recall** whether the need for two clocks to time an event means you are measuring the proper or the relativistic (dilated) time interval. (1 mark)

3 **Calculate** the speed of a pion if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 3.1×10^{-8} s. (3 marks)

4 **Clarify** whether the proper time interval is longer or shorter than the relativistic time interval for an event. (1 mark)

Analytical processes

5 **Distinguish** proper time interval and relativistic time interval. (1 mark)

6 **Determine** the rest lifetime of an elementary particle travelling at a speed of $2.85 \times 10^8\,\text{m}\,\text{s}^{-1}$ if its average lifetime at this speed is measured to be $2.50 \times 10^{-8}\,\text{s}$. (3 marks)

7 Sarah boards a spaceship and flies past Earth at 0.800 times the speed of light. Her twin sister, Emma, stays on Earth. At the instant Sarah's ship passes Earth, they both start timers. Sarah stops her timer after 60.0 s have passed on her timer. **Determine** how much time has passed on Emma's timer for the same event. (3 marks)

8 **Determine** whether the ratio $t : t_0$ doubles when comparing speeds of $0.4c$ and $0.8c$. (3 marks)

Knowledge utilisation

9 **Predict** how t and t_0 compare at a speed of $1.0c$. (2 marks)

10 **Evaluate** the claim that a moving clock that runs at half the rate of a clock at rest must be moving at $0.5c$. (3 marks)

11 A person holds a clock as they move along at speed v. A stationary observer notices that the moving clock runs at half the rate of their own clock. Assuming the clocks are accurate, **determine** the speed v. (3 marks)

MODULE 9

Lesson 9.5

Review: Special relativity: Time and motion

Summary

9.1

- A reference frame that moves with constant velocity with respect to an inertial frame is itself also an inertial frame.
- There is no absolute frame of reference. All inertial reference frames are equivalent for the description of mechanical phenomena, and no one inertial reference frame is any better than another.
- There are two postulates of the special theory of relativity:
  1. The laws of physics have the same form in all inertial reference frames.
  2. Light propagates through empty space with a definite speed, c, independent of the speed of the source or the observer.

9.2

- All observers in inertial frames of reference agree on their relative speed.
- Relativistic speeds are arbitrarily chosen as those greater than $0.1c$.

9.3

- Two events in a reference frame are simultaneous if light signals from the events reach an observer halfway between the events at the same time.
- Two events that are simultaneous to one observer are not necessarily simultaneous to a second observer moving with respect to the first.

9.4

- The time interval for an event is dependent on the motion of the observer with respect to the event.
- An observer moving relative to an event will measure the time interval as longer than would an observer at rest to the event. This is called time dilation and is summed up as "Moving clocks are measured to run slowly".
- Problems involving time dilation can be solved using: $t = \dfrac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ where t_0 is proper time and t is dilated or relativistic time interval. The relativistic (dilated) time interval is longer than the proper time interval.

Key formulas

Time dilation	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

Review questions 9.5A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 A spaceship moves towards you at a speed of $\frac{1}{3}c$, where c is the speed of light. The spaceship emits a beam of light in your direction. As measured in your frame of reference, the speed of the light emitted by the spaceship is

A $\frac{4}{3}c$ **B** $1c$ **C** $\frac{2}{3}c$ **D** $\frac{1}{3}c$

2 Select the statement that describes one of the postulates of Einstein's special theory of relativity.

A Nothing can travel faster than the speed of light.
B The laws of physics are the same in all inertial (non-accelerated) frames of reference.
C All inertial observers obtain the same result when measuring the time and position of an event.
D Time dilates for a moving observer.

3 Select the option about which observers in different inertial frames of reference will agree.

A The simultaneity of events at separate locations
B The rate at which their clocks run
C The number of seconds elapsed for an event
D None of the above

4 The special theory of relativity describes problems involving

A inertial frames of reference.
B non-inertial frames of reference.
C fast-moving frames of reference.
D accelerated frames of reference.

5 The muon detection experiment helped confirm one aspect of special relativity, namely that

A to a muon distance is contracted.
B muons are created in the upper atmosphere.
C muons are relatively short-lived.
D fewer muons reach the ground than expected.

6 If you approach a lamp while travelling at seven-tenths the speed of light ($0.7c$), you will measure the speed of light from the lamp to be

A $0.3c$ **B** $0.70c$
C $1.0c$ **D** $1.7c$

7 A train travelling at $0.9c$ along a straight, horizontal track at a constant speed approaches a rail crossing that flashes a light once per second. An observer on the train will measure the time between flashes to be

A somewhat greater than one second.
B equal to one second.
C somewhat less than one second.
D much greater than one second.

8 The proper time interval is the elapsed time between two events measured in a frame of reference in which the two events

A are simultaneous.
B occur at the same time.
C are measured with synchronised clocks.
D occur at the same point in space.

9 A meteor is heading for a planet at high speed. Select the statement that best describes the time of the meteor's descent to the planet's surface as measured aboard the meteor, and the time as measured by the observers on the surface of the planet.

A Only observers on the meteor measure the proper time.
B Both measure proper time in their own reference frames.
C Neither measure proper time.
D Only the observers on the planet measure proper time.

10 Select one of the following natural phenomena that cannot be explained by Newtonian physics.

A The absence of muons anywhere in the atmosphere
B The presence of muons in the atmosphere
C The relatively high number of muons close to Earth
D The relatively low number of muons close to Earth

11 Identify the statement that best describes the meaning of "frame of reference".
- **A** The concept that a person's location is linked to their perception of time
- **B** The location of an observer
- **C** Where the laws of physics are obeyed
- **D** The difference between the location of one observer and another

12 Identify the correct relationship between t and t_0 at relativistic speeds.

A $t < t_0$ **B** $t > t_0$ **C** $t \geq t_0$ **D** $t \leq t_0$

13 You are sitting on a park bench, and you see a cyclist riding towards you at a speed of $\frac{1}{8}c$. Determine the speed of the cyclist relative to your frame of reference.

A $1.0c$ **B** $\frac{1}{8}c$ **C** $\frac{7}{8}c$ **D** $\frac{1}{2}c$

14 Select the statement that best summarises the special theory relativity.
- **A** A theory that states that the laws of physics are all the same in all inertial (uniformly moving) frames of reference
- **B** A theory that explains how space and time are linked for objects moving in a straight line
- **C** A theory that describes how space and time are intertwined
- **D** A theory that describes time dilation with respect to a reference frame

15 If you walk towards a lighthouse beacon while you are travelling at five-tenths the speed of light ($0.5c$), determine what you will perceive the speed of the light from the lighthouse beacon to be.

A $0.25c$ **B** $0.5c$ **C** $1.0c$ **D** $1.5c$

Review questions 9.5B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 **Explain** whether, if you ran at the speed of light, you could see your own reflection in a mirror in your hand. (1 mark)

17 **Clarify** whether can you tell which rocket is really moving when two rockets are moving relative to each other. (1 mark)

18 **Explain** whether light from a moving torch would be faster than light from a torch at rest. (1 mark)

19 **Calculate** the speed of a pion if its rest life is 2.6×10^{-8} s, but to a laboratory observer it appears to live for 5.2×10^{-8} s. (3 marks)

20 Sitting in the middle of a movie cinema, you notice that the two lights on either side of the cinema, which are controlled by a single switch, appear to turn off at the same time. However, when you sit on the right-hand side of the cinema, you perceive the light on the right-hand side to go off before that on the left-hand side. **Explain** why you perceive this. **Construct** a diagram to assist your response. (2 marks)

Analytical processes

21 **Contrast** inertial and non-inertial frames of reference. (1 mark)

22 **Deduce** whether anything travels faster than light. (2 marks)

23 **Determine** the speed at which sunlight would pass you if you were on a spacecraft travelling at $0.6c$ away from the Sun. (1 mark)

24 An aeroplane speeds along a runway at a velocity of $0.7c$. A passenger on board observes that it takes 8 seconds for the plane to take off. **Compare** the take-off time measured by the passenger to the take-off time measured by an individual in the airport terminal. (2 marks)

25 The mean lifetime of a subatomic particle is measured in a laboratory to be 4.2×10^{-7} s when the particle travels at $0.55c$ with respect to the laboratory. **Determine** the proper time interval of the subatomic particle's lifetime. (3 marks)

26 A particular elementary particle travels at a speed of $2.6 \times 10^{8}\,\text{m s}^{-1}$. At this speed, the average lifetime is measured to be 2.20×10^{-7} s.
- **a** **Derive** its speed in units of c. (1 mark)
- **b** **Determine** the particle's lifetime at rest. (3 marks)

27 A train travelling at relativistic speed passes through a station. An observer on the platform measures the time taken for the train to pass. The driver of the train has synchronised clocks at the front and back of the train, and she also measures the time for the same event, but she disagrees with the value measured by the observer on the platform. One time is twice that of the other. **Determine** the speed of the train and which observer has the higher figure. (3 marks)

28 You are standing on top of a moving train and throw a rock straight upwards (as it appears to you).

a **Deduce** how the motion of the rock would appear to your mother, who is standing beside the train. (1 mark)

b **Deduce** whether the rock would land behind the carriage or on top of it. (1 mark)

Knowledge utilisation

29 One of the earliest attempts to measure the speed of light was by the Italian scientist Galileo in 1638. He attempted to measure the speed of light by measuring the time lag between one observer turning on a lamp and another observer noting this and turning on a second lamp on a distant hill 1.7 km away. His results were inconclusive. **Propose** a reason for this. (2 marks)

30 You are aboard a train moving through a station and there is a clock on the wall of the carriage. **Decide** who sees the clock ticking over faster: you or someone on the station platform. (2 marks)

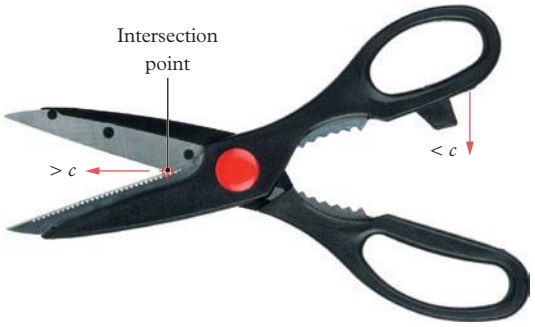
31 A rocket ship is travelling at relativistic speed past an observer R on a runway on a distant planet. Observer R is located midway between two landing lights, X and Y. The astronaut S inside the rocket ship is directly opposite observer R, when R notes that lights X and Y give out a flash of light simultaneously. **Propose** whether the lamps flash simultaneously according to observer S. (2 marks)

32 You have a powerful laser and shine it on the left-hand side of the moon, then move it to the right-hand side of the moon so that it draws a track on the surface. The moon is 3,474 km wide, and it takes one-hundredth of a second for the laser spot to travel this distance. **Evaluate** this claim: the spot is travelling faster than the speed of light. (2 marks)

33 Because of the rotational motion of Earth about its axis, a point on the equator moves with a speed $460\,\text{km}\,\text{h}^{-1}$ relative to a point on the North Pole. **Propose** whether this means that a clock placed on the equator runs more slowly than a similar clock placed at the North Pole. (2 marks)

34 Pions have a half-life of 1.8×10^{-8} s, which means that every 1.8×10^{-8} s, the number of pions present halves. A target is struck by extremely high-energy protons, which results in a beam of pions exploding out of the target at a speed of $0.98c$. The number of pions present is half the original number after they have travelled just 26.59 m. However, you would think that particles travelling at $0.98c$ would travel just 5.29 m in the 1.8×10^{-8} s. **Propose** a reason for this anomaly. (3 marks)

35 The Superluminal Scissors: You have a very large pair of scissors and as you close them, you notice that the intersection point (i.e. the point where the blades meet) travels along the blades, as shown in the diagram. You find that the quicker you close them, the faster the intersection point moves. When you close them really quickly you find that the intersection point travels along the blades faster than the speed of light (i.e. at "superluminal" speed). It seems to have disproven Einstein's claim that nothing can travel faster than light. **Assess** the evidence to explore whether your experiment does disprove his claim. (2 marks)



36 The driver in a relativistic high-speed train passes two synchronised clocks on a station platform placed 200.0 m apart. The clocks register a time interval of 7.326×10^{-7} s for the front of the train to pass by. The driver and an observer on the platform agree that the speed is $0.910c$ but disagree about the time interval.

a **Evaluate** the claim that the speed is $0.910c$. (2 marks)

b **Propose** what time the train driver would record for the time interval. (2 marks)

Data drill

The lifetime of a subatomic particle was measured at various relativistic speeds. The results are shown in Table 1. The column labelled Lorentz factor is derived from the velocity using the formula $\gamma = \frac{1}{\sqrt{1 - v^2}}$ (where the velocity is in units of c).

TABLE 1 The lifetime of a subatomic particle measured at various relativistic speeds

v (c)	Lorentz factor, γ	Dilated time, t (μs)
0.2	1.02	78.6
0.4	1.09	84.0
0.6	Question 2	96.3
0.8	1.67	128.3

A graph of the dilated time, t, vs the Lorentz factor is shown in Figure 1.

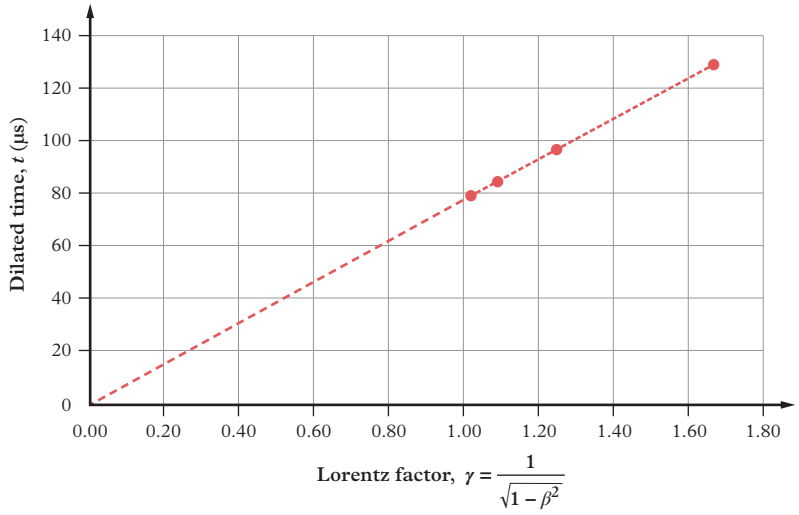


FIGURE 1 A graph of the dilated time, t, versus the Lorentz factor, γ. Note that the velocity is given in units of c.

Apply understanding

1 **Calculate** the speed of the particle in metres per second when $v = 0.6c$. (1 mark)

2 Using the formula, **calculate** the Lorentz factor for a velocity of $0.6c$. Show your working. (2 marks)

Analyse data

3 **Identify** the relationship between the dilated time and the Lorentz factor. (3 marks)

Interpret evidence

4 In terms of the formulas for time dilation, **determine** what the gradient represents. (1 mark)

5 Use the gradient of the graph to **draw a conclusion** about the rest lifetime of the subatomic particle. (3 marks)

Module 9 checklist: Special relativity: Time and motion

MODULE 10

Special relativity: Length, momentum and energy

Introduction

Two observers moving relative to each other will agree on their relative velocities. However, velocity is a relationship between distance and time and, as Module 9 showed, the observers will disagree about the time interval between events. They will distinguish these intervals as proper time and relativistic time. The consequence is that they will also disagree about the distance travelled during those times, which leads to the ideas of proper length and relativistic length.

In this module you will be introduced to the idea of length contraction. You will see that whether length really contracts depends on the relative motion of the observers. When length contraction is put together with time dilation, you end up with many paradoxes about momentum, time travel and high-speed interactions.

This module will also introduce you to one of the most profound consequences of Einstein's theories – that mass and energy are equivalent properties of matter. The proof of this is in the destructive power of the atomic bomb and in its constructive use in nuclear power generation.

Prior knowledge

Prior knowledge quiz

Check your understanding of concepts related to special relativity: length, momentum and energy before you start.

Subject matter

Science understanding

- → Describe observations of natural phenomena that cannot be explained by classical physics, e.g. the presence of muons in the atmosphere and the momentum of high speed particles in particle accelerators.
- → Describe the consequences of the constant speed of light in a vacuum, e.g. time dilation and length contraction.
- → Describe the concepts of time dilation, proper time interval, relativistic time interval, length contraction, proper length, relativistic length, rest mass and relativistic momentum.

- → Describe the phenomena of time dilation and length contraction, including examples of experimental evidence of the phenomena.
- → State the mass–energy equivalence relationship.
- → Solve problems involving time dilations, length contraction and relativistic momentum using $t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$, $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ and $\Delta E = \Delta m c^2$.
- → Explain the implications of relativistic momentum of objects increasing as they approach the speed of light.
- → Explain paradoxical scenarios that may arise as a result of special relativity including the twins' paradox, flashlights on a train, and the ladder in the barn paradox.

Science as a human endeavour

- → Appreciate the significant contributions of scientists such as Albert Einstein and Amalie "Emmy" Noether who furthered our understanding of relativity.
- → Explore how special relativity built upon the work of previous scientists and led to the development of relativistic theories of gravitation, mass–energy equivalence and quantum field theory.
- → Explore how special relativity leads to the idea of mass–energy equivalence, which has subsequently been applied in nuclear fission reactors.

Science inquiry skills

- → Consider whether information could be transmitted at speeds faster than the speed of light.
- → Consider the experimental evidence that supports the phenomena of time dilation and its real-world applications.
- → Explore why the speed of light is the maximum possible speed in our universe.

Lesson 10.1
Length contraction

Key ideas

- → Moving objects contract in their direction of motion: $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, where L_0 is the proper (rest) length and L is the relativistic (contracted) length.
- → The following equation can be used to solve problems about space travel: $v = \frac{L_0}{t} = \frac{L}{t_0}$

Learning intentions and success criteria

What is length contraction?

length contraction
the shorter measurement made by an observer moving relative to the object in the direction of the length being measured

One of the strange consequences of special relativity is **length contraction**. An object moving at relativistic speeds ($> 0.1c$) will contract along its direction of travel. For example, an observer watching a 1 m ruler travelling past lengthwise at $0.8c$ would measure it as only 60 cm long. Its width and height would remain the same.

Students often ask whether the ruler really contracts or whether it just appears to contract. The answer is that it really does contract – as measured by someone who sees the ruler moving. If you rode along with the ruler, you would still measure it to be 1 m long.

The amount of contraction of the object depends on the speed of the object relative to the observer. Good confirmation of this has been obtained at the Stanford Linear Accelerator (SLAC) in California, USA (Figure 1). The SLAC is a 3.2 km long tube in which particles are accelerated to over $0.999999c$. To the particles, the tube is whizzing past at $0.999999c$ and appears to be only about a metre long.

FIGURE 1 The Stanford Linear Accelerator (SLAC) at Stanford University

Challenge

"Mr Tompkins in Wonderland"

Physicist George Gamow wrote a science fantasy book called *Mr Tompkins in Wonderland* (1940). In the book, the speed of light was walking pace. Mr Tompkins had studied relativity, and when he began speeding on a bicycle, he thought he should be immediately contracted in length. To his great surprise, nothing happened to him or to his bicycle. However, the streets got shorter, and the windows of the shops looked like narrow slits. A policeman walking past looked really thin. The policeman said it was he, Tompkins, and his bicycle that got thin. Mr Tompkins exclaimed "By Jove! This is where the word relativity comes in!"
Explain Mr Tompkins's observations. (2 marks)

Does length contract in all directions at relativistic speeds?

Length contraction applies to everything from distances between stars to distances between atoms. But this contraction occurs only along the direction of motion. For example, if a car travelled forwards at a relativistic speed, it would appear to shrink in length (from, say, 4 m to 2 m), but its height would remain the same at 1.5 m and its width would remain the same at 2 m. If you could run as fast, your height and width would also remain the same, but you would get thinner in your direction of motion (Figure 2).

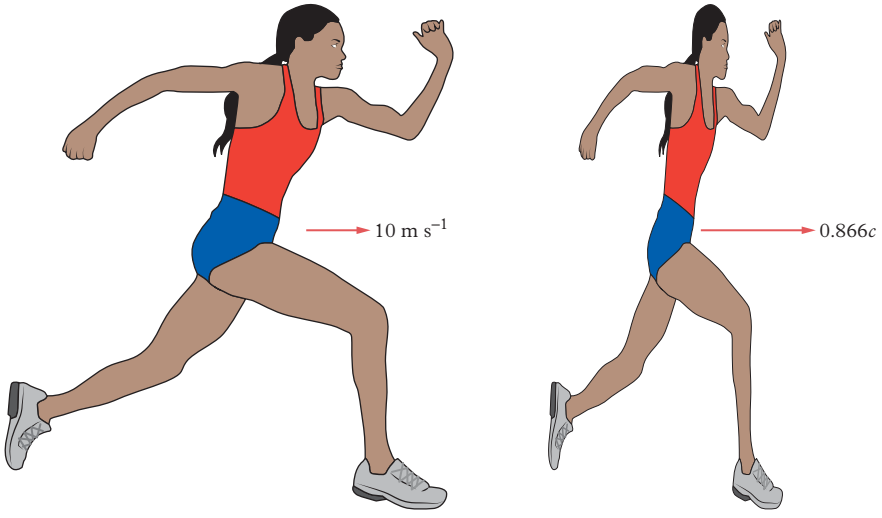


FIGURE 2 Running at 0.866c will halve your length in the direction of motion but leave your height and width unchanged.

Other properties affected by relativity

Other properties are affected by relativistic speeds. For example, if a book were flung past you at a relativistic speed, the following would occur:

- its dimensions would contract in the direction of motion
- because of this, its volume would get smaller
- there would be no change to the number of atoms or pages, so its rest mass would not change (dealt with later)
- its momentum would increase (dealt with later)
- its density would increase, because its volume had decreased and its rest mass had not changed.

How is length contraction calculated?

We can now look at another practical example to see how length changes depending on the frame of reference of the observer. The nearby star Rigil is 800 light-years away, which means it takes light 800 years to reach Earth from Rigil. If you could travel at the speed of light, it would take 800 years to get there. We can't travel at that speed, but let's imagine that we can travel at $0.999c$. Using our relativistic time formula, $t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$, from Module 9, the 800-year journey (represented by t) becomes 35.8 years to someone aboard the spaceship, which is more reasonable.

You might wonder how it can do this in such a short time if nothing can go faster than light, and even light takes 800 years to get here from Rigil. The answer is that, although time is stretched (dilated) in different reference frames, length is squeezed (contracted).

To understand this, we will look at the journey from Earth to Rigil from both frames of reference.

The Earth–Rigil frame of reference

Imagine that an observer on Earth watches a spaceship take off for the star Rigil at a speed of v. Both the astronauts and the Earth observer will agree on the speed of the spaceship. Earth and Rigil are at rest relative to one another, so they form a single frame of reference (Figure 3). The distance between Earth and Rigil is 800 light-years and has the symbol L_0. This is called the rest length or **proper length** because it is the length measured by an observer at rest ($v = 0$, hence the subscript 0) relative to both Earth and Rigil.

proper length
the length as measured by an observer at rest with respect to the object

The time taken for the journey according to observers on Earth or Rigil is the dilated time, t, because the departure and arrival events are separated in space and require two separate clocks for their measurement.

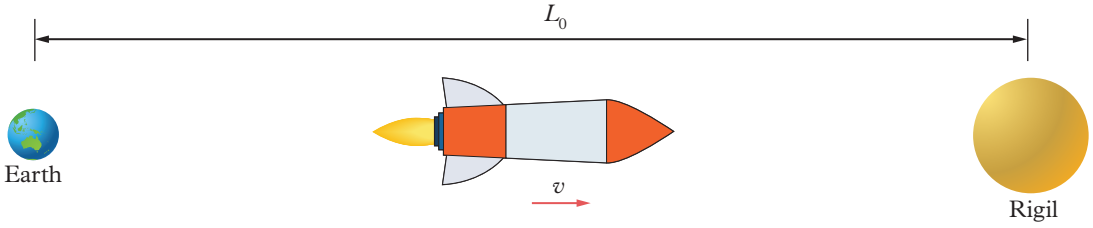


FIGURE 3 As viewed from Earth, the rocket is the moving frame.

The spaceship frame of reference

Figure 4 shows the journey from Earth to Rigil from the astronauts' perspective. They can picture themselves as being stationary and therefore assume that Earth is rushing away from them and Rigil is approaching them. The Earth–Rigil frame is moving with respect to the astronauts, so they measure the distance between Earth and Rigil as the contracted or **relativistic length**, L. The time taken for the journey as measured by the astronauts is t_0 (the proper time), because they are measuring the departure and arrival events in the same place in space (inside their spaceship) and can use one clock to do this.

relativistic length
the length as measured by an observer moving with respect to the object in the direction of motion

The astronauts and the Earth observers do, however, agree on the relative velocity, v, between the two frames of reference. The quantities measured by the astronauts and Earth observers are summarised in Table 1.

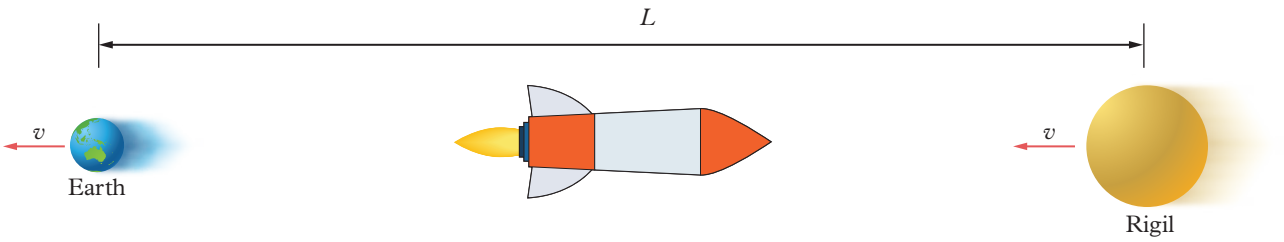


FIGURE 4 As viewed from the spaceship, Earth–Rigil is the moving frame of reference.

TABLE 1 Quantities measured in each frame of reference

Frame of reference	Earth–Rigil	Spaceship
Time for journey	t	t_0
Distance travelled	L_0	L
Velocity	v	v

Relationships between the frames of reference

The time for the journey is t for the Earth observers and t_0 for the astronauts. The distance travelled is L_0 for the Earth observers and L for the astronauts. They both agree that the velocity of the spaceship is v. As the relationship between t and t_0 is given by $t_0 = t\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, the time measured by the astronauts, t_0, is less than that measured by Earth observers, t (i.e. $t_0 < t$). But as they agree on the velocity of the spaceship, v, and because distance = $v \times t$, the distance travelled by the astronauts must also be less than that measured by Earth observers. In other words, $L < L_0$.

We now have two relationships for velocity: for the astronauts, $v = \frac{L_0}{t}$, and for the Earth observers, $v = \frac{L}{t_0}$. Because the velocity is the same for both, we can equate these:

$$v = \frac{L_0}{t} = \frac{L}{t_0}$$

If we rearrange the second part, we get $L = vt_0$, and if we replace t_0 with the equation for time dilation, we get:

$$\begin{aligned} L &= v \times t_0 \\ &= v \times t\sqrt{\left(1 - \frac{v^2}{c^2}\right)} \\ &= L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)} \end{aligned}$$

Study tip

The Lorentz factor, $\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$, can be used as shorthand in writing equations. For example, $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$ then becomes $L = \frac{L_0}{\gamma}$. Alternatively, using the speed factor, β, where $\beta = \frac{v}{c}$, the equation for contracted length can be written as $L = L_0\sqrt{1 - \beta^2}$.

Study tip

The zero in L_0 can be thought of as the velocity of the observer with respect to what is being measured: zero means at rest.

Worked example 10.1A

Calculating the length contraction of a spaceship

A spaceship passes you at a speed of $0.80c$. You measure its length to be 90 m. **Calculate** the length it would be to observers on board the spaceship. Give your answer to an appropriate number of significant figures. (2 marks)

FIGURE 5 A representation of the speed of light in space

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$v = 0.80c$, length = 90 m $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$
Step 3: Identify whether the length is the proper length or the relativistic (contracted) length. Identify whether or not you are at rest relative to the spaceship. (1 mark for "Identifies 90 m as the relativistic length")	You are not at rest relative to the spaceship, so you measure the relativistic length, L, as 90 m. We need to calculate the length measured by the observers on board the spaceship. They are at rest with respect to the spaceship, so they will measure the proper length, L_0. (1 mark)
Step 4: Substitute the known values into the formula and solve for L_0. Give your answer to an appropriate number of significant figures – in this case 2 s.f. (1 mark for "Correctly calculates the length")	$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$ $L_0 = \frac{L}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ $= \frac{90}{\sqrt{1 - 0.8^2}}$ $= \frac{90}{0.6}$ = 150 m (1 mark)

Your turn

You are on Earth and watch a spaceship pass by at a speed of $0.95c$. Observers aboard the spaceship say they have measured its length to be 55 m. **Calculate** the length you would measure it to be. Give your answer to an appropriate number of significant figures. (2 marks)

Challenge

Running a red light

Colour also changes at relativistic speeds. Light moving towards you looks more blue as its frequency is increased, and light moving away from you appears more red as its frequency is decreased.

A physicist driving a very fast sports car is booked for travelling through a red traffic light. The physicist argues that because he was travelling fast with respect to the light, the light had its wavelength altered so it appeared green to him. The judge said that she would let the physicist off the charge of running a red light but would fine him 1 cent for every metre per second he was travelling over 100 km h^{-1}.

Calculate how much the physicist was fined. (4 marks)

Note: the frequency of red light is 4.5×10^{14} Hz and that of green light is 6.0×10^{14} Hz. The formula for the change in frequency is:

$$f_0 = f\sqrt{\frac{\left(1 - \frac{v}{c}\right)}{\left(1 + \frac{v}{c}\right)}}$$

where f_0 is the frequency of the light with respect to an observer in the frame of reference of the light source (i.e. the police).

Is a light-year a measure of distance?

A **light-year** (ly) is the distance travelled by light in a vacuum in one year. This distance is $3 \times 10^8 \times 60 \times 60 \times 24 \times 365 = 9.5 \times 10^{15}$ m. The distance to the red star Betelgeuse in the constellation Orion is 650 ly, while our Sun is only 500 light-seconds away from us.

light-year (ly)
the distance travelled by light in one year (9.5×10^{15} m)

Calculating travel time using light-years

When a distance in light-years (ly) is divided by a speed in units of c, the answer is time in years (y). For example, to calculate the time it would take to travel a distance of 10 ly at a speed of $0.5c$:

$$t = \frac{\text{distance in ly}}{\text{speed in } c}$$
$$= \frac{10\,\text{ly}}{0.5\,c}$$
$$= 20\ \text{y}$$

The logic behind this is that light travels at a speed of c, but it also travels at 1 ly per year. So, c has the units ly y^{-1}. Thus, the above equation can be written as:

$$t = \frac{\text{distance in ly}}{\text{speed in } c}$$
$$= \frac{10\,\text{ly}}{0.5\ \text{ly y}^{-1}}$$
$$= 20\ \text{y}$$

A common trick in using light-years in equations is to give them the symbol "cy", so 10 light-years would be written as 10 cy. For example: How many years does it take an object to travel a distance of 10 light-years at a speed of $0.5c$? Answer: $t = \frac{s}{v} = \frac{10\,\text{cy}}{0.5c} = 20\,\text{y}$. It is not really the correct use of symbols, but it can make the calculations easier to understand.

Study tip

Proper length is the measurement you make when you are at rest with the object being measured – for example, an observer on Earth measuring the distance to another planet.

The distance between planets is always the proper length, L_0, unless specifically stated otherwise.

Worked example 10.1B

Calculations involving space travel

A spaceship leaves Earth and travels at $0.98c$ to reach the star Ajax, 36 light-years (3.4×10^{17} m) away.

a **Determine** how many years it would take the spaceship to reach Ajax as measured by an observer on Earth. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** how many years it would take the spaceship to reach Ajax as measured by an observer on the spaceship. Give your answer to an appropriate number of significant figures. (2 marks)

c **Determine** the distance travelled in light-years according to an observer on the spaceship. Give your answer to an appropriate number of significant figures. (2 marks)

d **Identify** whether the spaceship's velocity measured by both observers would be the same. (1 mark)

Note: the distance is provided in both light-years and metres. Either can be used.

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after a calculation. "Identify" means to recognise and state. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$v = 0.98c$, distance = 36 ly $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, $v = \frac{L_0}{t} = \frac{L}{t_0}$
Step 3: Identify whether the distance of 36 ly is the proper length or the relativistic (contracted) length. (1 mark for "Provides correct reasoning")	**a** Distances between planets are measured in the frame of reference of the Earth and stars. This means 36 ly is the proper length, L_0. (1 mark)

Think	Do
Step 4: Identify whether the time measured by an Earth observer is the proper time interval, t_0, or the relativistic (dilated) time interval, t. To do this, we must consider whether the take-off and landing occur in two separate places and thus would require two separate clocks. If so, it is the relativistic time interval.	The "event" of the spaceship taking off and landing occurs in two separate places, so the Earth observers must be measuring the relativistic (dilated) time interval, t.
Step 5: Select the appropriate formula. For our purposes it needs to have L_0 and t.	$v = \frac{L_0}{t}$
Step 6: Rearrange and substitute in the known values. Note that 36 light-years is written as 36 cy to help with the calculation. Complete the calculation. Give your answer to an appropriate number of significant figures – in this case 2 s.f. (1 mark for "Correctly calculates the Earth time interval")	$t = \frac{L_0}{v}$ $= \frac{36\,\text{cy}}{0.98c}$ (writing 36 ly as 36 cy) $= 36.73\,\text{y}$ $= 37\,\text{y}$ (2 s.f.) (1 mark)
Step 7: To calculate time elapsed aboard the spaceship, we need to identify whether the time measured aboard the spaceship is the proper time interval, t_0, or the relativistic (dilated) time interval, t. To do this we must consider whether the take-off and landing occur in one place according to the spaceship observers and thus would require just one clock. If so, it is the proper time interval.	**b** The "event" of the spaceship taking off and landing occurs in one place according to the spaceship observers. They see the Earth disappear and then the star Ajax appear and only need the one clock. Therefore, the spaceship observers must be measuring the proper time interval, t_0.
Step 8: Select the appropriate formula. This needs to have t_0 and t. Rearrange and substitute in the known values. Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Correctly calculates the spaceship time interval")	$t_0 = t\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$ $= 36.73 \times \sqrt{1 - 0.98^2}$ (1 mark) $= 7.3\,\text{y}$ (2 s.f.) (1 mark)
Step 9: To calculate the distance as measured by observers on the spaceship, we need to identify whether the distance (length) they measure is the proper length, L_0, or the relativistic length, L. To do this, we must consider whether the spaceship observers are in the same frame of reference as Earth and Ajax or are moving relative to them. If the observers are moving relative to Earth and Ajax, they are measuring the relativistic length, L.	**c** The Earth observers are at rest relative to the Earth–Ajax frame of reference so they measure proper length, L_0, as 36 ly. The spaceship observers are moving relative to the Earth–Ajax frame of reference, so the spaceship observers are measuring the relativistic (contracted) length, L.
Step 10: Select the appropriate formula. It needs to have L_0 and L. Rearrange and substitute in the known values. Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Correctly calculates the spaceship distance measurement")	$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$ $= 36\,\text{ly} \times \sqrt{1 - 0.98^2}$ (1 mark) $= 7.2\,\text{ly}$ (2 s.f.) (1 mark)
Step 10 (continued): An alternative method can be used to calculate L.	$L = vt_0$ $= 0.98c \times 7.3\,\text{y}$ $= 7.2\,\text{ly}$
Step 11: The velocity is relative, so each will see the other moving at that speed. (1 mark for "Provides correct reasoning")	**d** They will both agree that the velocity is $0.98c$. (1 mark)

Your turn

The star Trappist-1 is a dwarf star about the size of Jupiter and is 40 ly (3.78×10^{17} m) from Earth (Figure 6). Twins George and Astrid on Earth are 25 years old when Astrid takes off on a spaceship journey to Trappist-1 at a speed of $0.60c$.

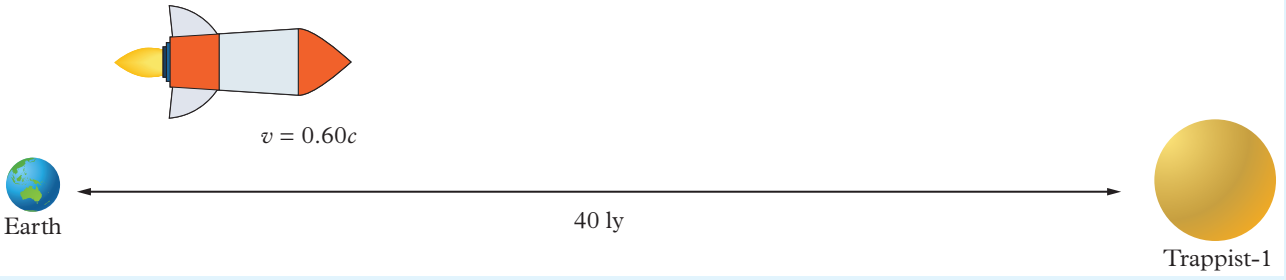


FIGURE 6 Spaceship journey to Trappist-1 at a speed of $0.60c$

a **Determine** how many years will it take Astrid's spaceship to reach Trappist-1 as measured by George on Earth. (3 marks)

b **Determine** how many years will it take Astrid's spaceship to reach Trappist-1 as measured by the clocks aboard Astrid's spaceship. (2 marks)

c **Determine** the distance travelled in light-years according to Astrid on the spaceship (2 s.f.). (2 marks)

d **Identify** whether the spaceship's velocity measured by both observers would be the same. (1 mark)

Note: the distance is provided in both light-years and metres. Either can be used.

Check your learning 10.1

Check your learning 10.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 A 1 cm cube of copper is seen passing at a speed of $0.8c$. **Describe** changes in its dimensions (in all three directions) and its density. (2 marks)

2 Two people, A and B, have metre rulers with the dimensions 100 cm × 2 cm × 0.5 cm. Person A measures B's ruler as it passes at high speed to be 100 cm × 1 cm × 0.5 cm. **Explain** how this is possible. (2 marks)

3 **Calculate**

a 1.8×10^7 m s^{-1} as units of c (1 mark)

b $0.95c$ as m s^{-1} (1 mark)

c 30 ly as km. (1 mark)

4 An aeroplane that has a rest length of 40.0 m is moving at a uniform velocity with respect to Earth at a speed of 630 m s^{-1}. **Calculate** the length of the aeroplane as measured from Earth. (3 marks)

Analytical processes

5 You decide to travel to a star that is 85 light-years away. **Determine** how fast you would have to travel so the distance would only be 20 light-years to you. (3 marks)

6 A friend travels past you in her fast sports car at a speed of $0.760c$. The car appears to be 5.80 m long and 1.45 m high.

a **Calculate** its length and height at rest. (4 marks)

b **Determine** how many seconds you saw elapse on your friend's watch when 20.0 s passed in Earth's frame of reference. (3 marks)

c **Deduce** how fast you appeared to be travelling to your friend. (1 mark)

7 After the Sun, the nearest star visible to the naked eye is Rigil Kentaurus, which is 4.35 light-years away. A spacecraft is sent there from Earth at a speed of $0.80c$.

a **Determine** how many years it would take to reach Rigil Kentaurus from Earth as measured by observers

i on Earth (1 mark)

ii on the spacecraft. (3 marks)

b **Calculate** the distance travelled according to observers on the spacecraft. (1 mark)

Knowledge utilisation

8 An astronaut leaves Earth and travels at $0.95c$ to another galaxy 5 light-years away as measured by the astronaut. When she arrives, she immediately sends a radio signal back to Earth to confirm her arrival. **Determine** how long after her departure observers on Earth would receive her radio signal. (4 marks)

Lesson 10.2
Rest mass and relativistic momentum

Key ideas

- → Newtonian momentum and relativistic momentum have similar values at low speeds but diverge at relativistic speeds (>0.1c).
- → Problems involving relativistic momentum can be solved using the formula:

$$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

- → Relativistic momentum is conserved in all interactions, providing no external force acts.
- → Einstein's mass–energy equivalence relationship shows the relationship between energy and mass.
- → Problems involving Einstein's mass–energy equivalence can be solved using $\Delta E = \Delta m c^2$.

Learning intentions and success criteria

What is rest mass?

rest mass the mass of an object when measured in the same frame of reference as the observer

Einstein defined **rest mass** as the mass of an object when measured at rest; that is, in the same reference frame as the observer. He used the symbol m, but others have added the subscript zero (m_0) to keep it consistent with other "rest" quantities such as length (L_0) and time (t_0).

Is momentum affected by relativity?

In the Newtonian world, momentum is defined as the product of mass and velocity, $p = m_0 v$. You have learnt that time and distance change at relativistic speeds, so what about momentum?

Physicists have found that the momentum of an object is affected by relativity. At relativistic speeds, its momentum becomes much greater than its Newtonian momentum, $p = m_0 v$. The momentum becomes:

$$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where p_v is the **relativistic momentum**, m_0 is the rest mass, v is the velocity of the object, and c is the speed of light. The subscript v in p_v indicates that it is the relativistic version of momentum.

We already know that velocity, v, is invariant (i.e. does not change according to the observer). So does that mean the mass must increase for an object moving at relativistic speeds if its momentum increases? Sometimes relativistic momentum used to be explained by saying that the mass of an object increases with velocity and that this mass should be defined as its relativistic mass. However, in 1948 Einstein forcefully warned against the concept of mass increasing with velocity. Unfortunately, this warning was ignored. The concept of relativistic mass is now considered outdated and somewhat of a misconception. It is not used in the syllabus or in this work here. It is best to think of the term "mass" as meaning "rest mass" with either symbol m or m_0 and that it doesn't change (is invariant).

Note that momentum and velocity are vector quantities. The v^2 term in $\frac{v^2}{c^2}$ is not a vector quantity, though, because the product of two vectors is not a vector.

You have been able to use v in units of c when calculating relativistic time and length, because the c^2 terms cancelled out. However, for this equation, the extra v term on the numerator (top line) means c wouldn't cancel out. Therefore, you need to use units of $\mathrm{m\,s^{-1}}$ for momentum questions.

FIGURE 1 Would the space shuttle's mass have changed as its velocity increasesd?

Study tip

The equation for momentum $p = mv$ can also be written as $p = m_0 v$ if you choose to use m_0 instead of m for rest mass. It makes no difference.

relativistic momentum
the momentum of an object as measured by an observer moving relative to the object

Study tip

If you could ride along with a particle, you would measure its rest mass. Mass doesn't change with speed, either to stationary or moving observers. We don't talk of "relativistic mass".

Challenge

Do we measure rest mass in the lab?

Explain whether an electronic balance in the lab measures rest mass. (2 marks)

Worked example 10.2A

Comparing relativistic and Newtonian momentum

The Stanford Linear Accelerator (SLAC) can accelerate electrons to over $0.999c$.

a **Calculate** the Newtonian momentum and the relativistic momentum of the electron travelling at a speed of exactly $0.999c$. The rest mass of an electron is 9.109×10^{-31} kg. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** how many times greater the relativistic value is than the Newtonian value. Give your answer to an appropriate number of significant figures. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer. "Determine" means to establish after a calculation.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$v = 0.999c$, $m_0 = 9.109 \times 10^{-31}$ kg $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$
Step 3: Convert the velocity from units of c to m s^{-1}. Calculate the Newtonian momentum using the Newtonian formula from classical mechanics. (1 mark for "Calculates the classical momentum correctly")	**a** $p = mv$ $= 9.109 \times 10^{-31} \times 0.999c$ $= 9.109 \times 10^{-31} \times 0.999 \times 3 \times 10^{8}$ $= 2.73 \times 10^{-22}$ $= 2.7 \times 10^{-22}$ kg m s^{-1} (2 s.f.) (1 mark)
Step 4: The relativistic momentum is calculated using the special relativity version of the momentum formula. Note, though, the speed v above the line has to be in units of m s^{-1}. Below the line, the c terms cancel out and the factor becomes 0.999 without any units. Give your answer to an appropriate number of significant figures – in this case 2 s.f. (1 mark for "Calculates the relativistic momentum correctly")	$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ $= \frac{9.109 \times 10^{-31} \times 0.999 \times 3 \times 10^{8}}{\sqrt{(1 - 0.999^2)}}$ $= \frac{2.73 \times 10^{-22}}{0.0447}$ $= 6.1 \times 10^{-21}$ kg m s^{-1} (2 s.f.) (1 mark)
Step 5: Calculate the ratio of the two values for momentum. Give your answer to an appropriate number of significant figures – in this case 2 s.f. (1 mark for "Calculates the ratio correctly")	**b** $\frac{\text{Relativistic momentum}}{\text{Newtonian momentum}} = \frac{6.11 \times 10^{-21}}{2.73 \times 10^{-22}}$ $= 22$ $p_{\text{relativistic}} = 22 \times p_{\text{Newtonian}}$ (2 s.f.) (1 mark)

Your turn

At full power the Large Hadron Collider (LHC) has trillions of protons racing around its accelerator ring 11,245 times a second, at $0.999999991c$.

a **Calculate** the Newtonian momentum and the relativistic momentum of a proton travelling at that speed. The rest mass of proton is $1.6726219 \times 10^{-27}$ kg. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** how many times greater the relativistic value is than the Newtonian value. Give your answer to an appropriate number of significant figures. (1 mark)

Worked example 10.2B

Calculating speed from momentum

Calculate the speed of a 1.00 g copper target travelling at relativistic speed with a momentum of 500,000 kg m s^{-1} (5.0×10^5 kg m s^{-1}). Give your answer as a speed in units of c to an appropriate number of significant figures. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formula.	$m_0 = 1.00\,\text{g} = 1.00 \times 10^{-3}\,\text{kg}$, $p_v = 5.0 \times 10^5\,\text{kg m s}^{-1}$ $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$
Step 3: Substitute in the known values. Remember to convert the mass in grams to kg, if you have not done so already. Recall that v must be in units of m s^{-1} and not in units of c. (1 mark for "Demonstrates correct substitution")	$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ $500{,}000 = \frac{1.00 \times 10^{-3} v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ (1 mark)
Step 4: Rearrange and simplify. Recall that squaring both sides is the easiest way to deal with square roots in these sorts of problems.	$5.0 \times 10^5 \times \sqrt{\left(1 - \frac{v^2}{c^2}\right)} = 1.00 \times 10^{-3} v$ (square both sides) $25 \times 10^{10} \times \left(1 - \frac{v^2}{c^2}\right) = 1.00 \times 10^{-6} v^2$ $25 \times 10^{16} \times \left(1 - \frac{v^2}{c^2}\right) = v^2$
Step 5: Continue rearranging and calculate v. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	$25 \times 10^{16} = 25 \times 10^{16} \times \frac{v^2}{c^2} + v^2$ $25 \times 10^{16} = \frac{25 \times 10^{16} v^2}{(3 \times 10^8)^2} + v^2$ (1 mark) $25 \times 10^{16} = 2.78\,v^2 + v^2$ $v^2 = 6.618 \times 10^{16}$ $v = 2.57 \times 10^8$ $= 2.6 \times 10^8\,\text{m s}^{-1}$ $= 0.86c$ (2 s.f.) (1 mark)

Your turn

Calculate the speed of an electron that has a momentum of 1.56×10^{-21} kg m s^{-1}. Give your answer in units of c to an appropriate number of significant figures. (3 marks)

Is relativistic momentum conserved?

As with Newtonian (or "classical") momentum, relativistic momentum is conserved in all interactions. This comes with the caution of "as long as no external force is applied to the objects", as for Newtonian momentum. The principles of **conservation of momentum** are used to analyse collisions in atom-smashing colliders such as the Large Hadron Collider (LHC) and the SLAC.

conservation of momentum
for a collision occurring between two objects in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision

How are mass and energy related?

The famous equation $\Delta E = \Delta m c^2$ is called "Einstein's **mass–energy equivalence relationship**". According to this equation, 1 kg mass of any matter is equivalent to 9×10^{16} J of energy. This is a huge amount of energy and is the same whether the matter is uranium, hydrogen, wood, oil or anything else. It does not depend on atomic number, mass number, electronic configuration and so on.

mass–energy equivalence relationship
relates change in mass to change in energy, given by $\Delta E = \Delta m c^2$

It means that "mass" is the connecting link between energy and matter. However, it seems that Einstein was somewhat loose in his use of symbols in his popular writings. In formal writings he used $E_0 = mc^2$ with the zero subscript to show that he was referring to rest energy (E_0). This was to distinguish it from any kinetic energy a particle could have due to its motion.

Challenge

Speed

An unstable particle is at rest and suddenly decays into two fragments. One of the fragments has a speed of $0.60c$ and a mass of 2×10^{-27} kg; the other has a mass of 8×10^{-27} kg. **Determine** the speed of the other fragment. (3 marks)

What does the equivalence of mass and energy mean?

To avoid the complications introduced by having to consider what is rest energy and what is total energy, you will look at the change in energy, ΔE. As stated earlier, the formula is quite simple:

$$\Delta E = \Delta m c^2$$

where ΔE is change in energy, Δm is the change in mass and c is the speed of light.

This formula works both ways. Energy and mass are equivalent: mass can appear as energy, and energy can appear as mass. It is common to say mass is "converted to" energy, but Einstein made it clear that mass and energy were just two different expressions of the same thing. Since the value of c is very high, a very small change in mass is equivalent to a large amount of energy. Conversely, even a large amount of energy appears as a very small change in mass. This was discussed in detail in the study of nuclear physics in Unit 1.

How to apply mass–energy equivalence

Recall from Unit 1 that binding energy released is related to mass defect. When you use energy to pull a nucleus apart, the constituent particles have a total mass greater than that of the nucleus. The energy you used to overcome the binding energy appears as mass (Δm). But the energy calculated using Einstein's equation doesn't have to involve nuclear particles. When you change the energy of any system, that energy appears as mass. Consider the charging of your phone. It takes 2 hours at a current of 2.4 A to charge a 3.6 V phone battery. That equates to energy given by:

$$\begin{aligned} W &= VIt \\ &= 3.6 \times 2.4 \times 2.0 \times 60 \times 60 \\ &= 62{,}208 \text{ J} \end{aligned}$$

This is a change in energy (ΔE) equivalent to a change in mass (Δm):

$$\begin{aligned} \Delta E &= \Delta m c^2 \\ \Delta m &= \frac{\Delta E}{c^2} \\ &= \frac{62{,}208}{(3 \times 10^8)^2} \\ &= 6.9 \times 10^{-13} \text{ kg (2 s.f.)} \end{aligned}$$

That's one-billionth of a gram, or a nanogram, and is undetectable on an electronic balance, no matter how expensive.

In the following examples, we see how mass–energy calculations can be applied. Firstly, in Worked example 10.2C, we look at mass being used to provide binding energy for the nucleus. In Worked example 10.2D, we look at mass providing energy to be released in nuclear decay. Lastly, in Worked example 10.2E, we compare the energy released by nuclear fission with the energy from burning fossil fuels.

Worked example 10.2C

Calculating the energy equivalent of mass defect

Calculate the binding energy in joules for a carbon-12 nuclide equivalent to a mass defect of 0.098940 amu. Give your answer to an appropriate number of significant figures. Note: 1 amu = 1.66×10^{-27} kg. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formulas.	$\Delta E = \Delta m c^2$, $\Delta m = 0.098940$ amu
Step 3: Convert amu to kg by multiplying by 1.66×10^{-27}. Note: you could also combine this with the next step, but it is shown here for clarity.	$\Delta m = 0.098940\,\text{amu} \times 1.66 \times 10^{-27}\,\text{kg amu}^{-1}$ $= 1.643 \times 10^{-28}\,\text{kg}$
Step 4: Substitute in the known values and complete the calculation. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check you have used the correct units. (1 mark for "Correctly substitutes values"; 1 mark for "Provides correct answer")	$E = \Delta m c^2$ $= 1.643 \times 10^{-28} \times (3 \times 10^8)^2$ (1 mark) $= 1.48 \times 10^{-11}$ J (3 s.f.) (1 mark)

Your turn

Calculate the mass of a substance (in amu) that is equivalent to 2.0×10^{-10} J. Give your answer to an appropriate number of significant figures. (2 marks)

Worked example 10.2D

Calculating the energy released in nuclear decay

Calculate the amount of energy that would be released if a pi-meson (π^0) of rest mass of 2.4×10^{-28} kg decayed and its entire rest mass were transformed completely into electromagnetic radiation. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formulas.	$\Delta E = \Delta m c^2$, $\Delta m = m_0 = 2.4 \times 10^{-28}$ kg
Step 3: Substitute in the known values and complete the calculation. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Check you have used the correct units. (1 mark for "Demonstrates correct substitution"; 1 mark for "Provides correct answer")	$\Delta E = \Delta m c^2$ $= 2.4 \times 10^{-28} \times (3 \times 10^8)^2$ (1 mark) $= 2.2 \times 10^{-11}$ J (2 s.f.) (1 mark)

Your turn

You have a 500 g lump of coal to be used as fuel.

a **Calculate** the amount of energy that would be released if its entire mass were transformed into energy. Give your answer to an appropriate number of significant figures. (1 mark)

b **Explain** why coal doesn't provide this amount of energy when it is used as fuel. (1 mark)

Worked example 10.2E

Comparing nuclear energy to fossil fuels

When 1 kg of petrol is burnt, it releases 30 MJ (3.0×10^7 J) of energy. When 1 kg of U-235 undergoes fission, the mass converted to energy is 0.047 g. **Determine** which fuel produces more energy per kilogram reacted. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formulas.	$\Delta E = \Delta m c^2$ For petrol: 1 kg releases 30×10^6 J. For U-235: 1 kg releases the energy equivalent of 0.047 g.
Step 3: Calculate the amount of energy equivalent to 0.047 g. You will need to convert grams to kilograms first. (1 mark for "Provides appropriate mathematical reasoning")	$\Delta E = \Delta m c^2$ $= 0.047 \times 10^{-3} \times (3 \times 10^8)^2$ $= 4.23 \times 10^{12}$ J (1 mark)
Step 4: Compare your value for Step 3 with the energy released by 1 kg petrol. (1 mark for "Identifies the relationship between petrol and uranium energy outputs")	U-235 releases 4.23×10^{12} J kg^{-1}, whereas petrol releases 3.0×10^7 J kg^{-1}. U-235 releases about 100,000 times more energy per kg. (1 mark)

Your turn

Determine which type of nuclear reaction produces more energy per kilogram of reactants: a fusion reaction in which reactants with a mass of 6.64×10^{-27} kg lose 4.765×10^{-29} kg of it as energy, or a fission reaction in which 1 kg of reactants produce 7.38×10^{13} J. (2 marks)

Does relativistic momentum suggest a universal speed limit?

According to Newtonian mechanics, if a rocket engine was used to propel a spacecraft through space, then its speed would continue to increase forever (or until its fuel was used up). Newton's second law of motion implies that as long as there is an unbalanced force, acceleration will continue ($F = ma$). For example, if a 10,000 N force was applied to a 100 kg satellite, then its acceleration would be 100 m s^{-2}. The time taken to go from rest to the speed of light could be calculated thus:

$$a = \frac{v - u}{t}$$

$$t = \frac{v - u}{a}$$

$$= \frac{3 \times 10^8 - 0}{100}$$

$$= 3 \times 10^6 \text{ s} \quad \text{(about 1 month)}$$

If the force was continued for another month, then its speed would be twice that of light, and so on. Clearly, this is wrong, because we know that nothing travels faster than the speed of light in a vacuum. It turns out that relativistic momentum explains this. As speed approaches c, the momentum approaches infinity (Figure 2).

The amount of effort needed for each increment of velocity becomes larger and larger until the added effort makes no difference. The spacecraft just wouldn't get noticably faster. This implies a cosmic speed limit.

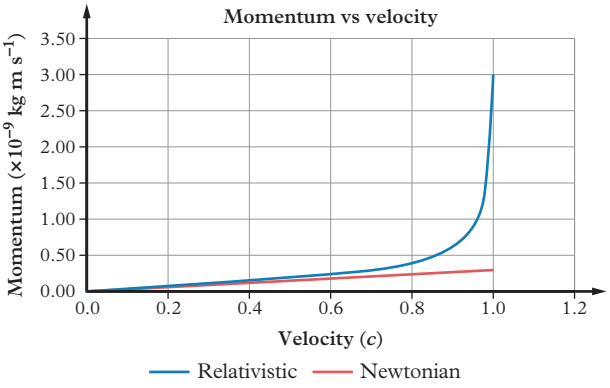


FIGURE 2 Comparison of relativistic and Newtonian momentum as velocity increases. Velocity is in units of c. Note that there is little difference until about $0.5c$.

Expansion of the universe faster than the speed of light

It is established that no material object can travel at the speed of light as it would require infinite energy to get to that speed. Some things do travel faster than light – but they are not material objects.

For example, since the Big Bang, space has expanded much faster than the speed of light. However, it is not matter that is travelling faster than the speed of light, but the space between the matter. This Big Bang paradox doesn't violate the special theory of relativity as no matter is breaking the light barrier.

Challenge

Nuclear reactors

You may have seen pictures of nuclear reactors that show fuel rods in pools of water with an eerie blue glow. This blue light is Cherenkov radiation. It has been said that Cherenkov radiation inside a nuclear reactor will travel faster than the speed of light in the watery medium. **Explain** whether this would break one of Einstein's postulates. (2 marks)

Real-world physics

Conversion of mass in a nuclear reactor

Special relativity leads to the idea of mass–energy equivalence, which has been applied in nuclear reactors. Nuclear energy from the neutron bombardment of uranium is a good example of mass appearing as energy. The reactants, a U-235 nucleus and a slow neutron, turn into fission fragments that have a lower total mass than the reactants. This mass defect appears as a large amount of energy. For example, when 1 kg of U-235 undergoes fission, there is a mass defect of 0.8 g, which is equivalent to 10^{14} J – a staggering amount.

A typical reactor produces energy at the rate of 3,000 MW (3×10^9 J s^{-1}) but, because of energy losses, the electrical output is only 1,000 MW. To produce this, about 1 kg of U-235 is "converted" to energy in a year. The most common form of nuclear fission power reactors is the thermal reactor. It consists of a containment vessel in which U-235 fuel rods are housed. These fuel rods are immersed in liquid (usually water or heavy water).

This absorbs the thermal energy produced by the fission reaction. The heated water runs through steam turbines, which turn the electrical generators to produce electrical energy that gets transmitted to an electrical distribution network for industry and our homes. Safety concerns, high costs, and a lack of suitable ways to store the waste for the millions of years it will remain radioactive have so far been enough to stop nuclear reactors from being built in Australia.

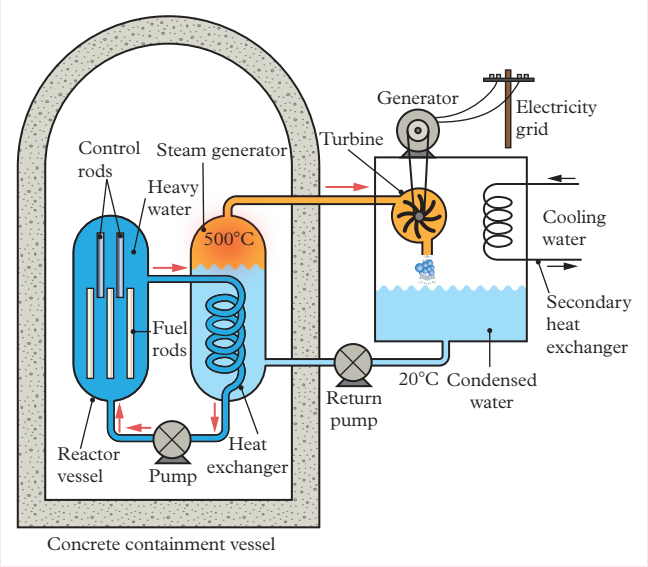


FIGURE 3 Schematic diagram of a typical pressurised water reactor in a thermal nuclear power station

Apply your understanding

1 **Interpret** the statement that "nuclear energy is based on Einstein's mass–energy equivalence principle". (1 mark)

2 The energy efficiency of a coal-fired power plant in converting heat into electricity is about 37%. **Propose** an argument supporting and an argument opposing the claim that a nuclear-fission power plant is better than a coal-fired power plant. (2 marks)

Check your learning 10.2

Check your learning 10.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Recall** whether relativistic momentum is conserved in collisions of subatomic particles when it is known that Newtonian momentum is conserved. (1 mark)

2 **Explain** whether relativistic momentum is conserved when an external force acts during an interaction. (1 mark)

3 A fission reactor might use around 27 tonnes of uranium per year. **Explain** whether all 27 tonnes of U-235 fuel in a nuclear reactor is converted to an equivalent amount of energy. (1 mark)

Analytical processes

4 **Determine** the momentum of a neutral pi-meson, π^0 ($m_0 = 2.4 \times 10^{-28}$ kg) travelling at a velocity of $0.992c$ using the

a relativistic formula (2 marks)

b Newtonian formula. (1 mark)

5 **Deduce** the velocity (in units of c) for a kaon, K^0 ($m_0 = 8.87077 \times 10^{-28}$ kg) that has a relativistic momentum of 5.33957×10^{-19} kg m s^{-1}. (3 marks)

6 **Determine** the increase in mass when 1.0 kg of water is placed on an electronic balance and heated from 20.0°C to 100.0°C, and identify whether the scale reading on the balance would increase as energy is transferred and converted to an equivalent mass. Recall that the energy to heat water is given by $Q = mc\Delta T$, where m is the rest mass of water, and c is the specific heat capacity of water = 4,180 J kg^{-1} K^{-1}. (3 marks)

Knowledge utilisation

7 Imagine the ocean increased in temperature by 1.0°C this century. It is claimed that the mass of the ocean would increase by a measurable amount if this happened. **Evaluate** this claim using Einstein's mass–energy equivalence postulate, $\Delta E = \Delta mc^2$. Assume the ocean has a mass of 1.4×10^{21} kg and a specific heat capacity of 4,000 J kg^{-1} K^{-1}. (3 marks)

8 Australia's OPAL nuclear reactor is for experimental and industrial purposes and not for power generation. Because of this, it has a small energy output of 20 MW (20 MJ s^{-1}). **Evaluate** the claim that the mass of the reactor would change by an insignificant amount if it ran every second for a full year (365 days). (3 marks)

9 **Evaluate** the claim that "special relativity leads to the idea of mass–energy equivalence, which has been applied in nuclear fission reactors". (2 marks)

Lesson 10.3
Paradoxical scenarios

Key ideas

→ Apparent paradoxes about relativistic time and length can be resolved when the principles of relativity are applied correctly.

What is a paradox?

Learning intentions and success criteria

The consequences of Einstein's special theory of relativity do seem rather weird and unbelievable. You have to accept clocks running slow, things getting shorter, disagreements about who is moving, and unbelievable amounts of energy coming from a tiny nucleus. These ideas give rise to some strange **paradoxes** – self-contradictory conclusions from true premises. They are not really paradoxes because they can be resolved when proper time and rest length are no longer wrongly applied. Let's look at some examples.

paradox
a self-contradictory conclusion that comes from true premises (ideas or statements)

What is the twins' paradox?

Not long after Einstein proposed his theory, an apparent paradox was pointed out – the twins' paradox. Suppose that there is a pair of 25-year-old twins, and one of them takes off in a spaceship travelling at very high speeds to a distant star and back again, while the other twin remains on Earth. When the travelling twin returns, they have aged differently according to the concept of time dilation (Figure 1). The question is: which twin has aged more? In the examples that follow, assume the travelling twin travelled at an average speed of 0.80c for the journey and had aged 30 years aboard the spaceship before returning.

Study tip

To identify the proper time and proper length and remember which is contracted and which is dilated, try the following: "Moving clocks run slow; they must measure t_0 (t-oh)." "Moving objects appear short; they must measure L_0 (L-nought)."

1 The travelling twin leaves the Earth twin for space.

2 The travelling twin travels at higher speeds than the Earth twin.

3 The travelling twin returns to Earth.

4 The Earth twin finds that his travelling brother has not aged as much.

FIGURE 1 The twins' paradox: one twin leaves Earth to travel through space. When he returns, he seems to be younger than his Earth twin.

Earth's reference frame

According to the twin who remained on Earth, the travelling twin will have aged less.

- Proper time (aboard spaceship): $t_0 = 30$ years
- Velocity: $v = 0.8c$
- Dilated time:

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$
$$= \frac{30 \text{ years}}{\sqrt{(1 - 0.8^2)}}$$
$$= 50 \text{ years}$$

Hence, the time elapsed on Earth is 50 years, so this is how much the Earth twin will have aged. The Earth twin will be 75 years old (25 + 50). The travelling twin will be 55 years old (25 + 30 years).

The spaceship's reference frame

Since "everything is relative", all inertial reference frames are equally as good as each other. The travelling twin could make all the same claims as the Earth twin, only in reverse. The travelling twin could claim that, since Earth is moving away at high speed, time passes more slowly on Earth and so the Earth twin will have aged less. In this case, proper time will be time measured aboard the "moving" Earth ($t_0 = 30$ years) and dilated time will be measured by the "stationary" spaceship ($t = 50$ years). This is the opposite of what the Earth twin would say. They both can't be right, can they? When the spaceship returns to Earth, they can stand beside one another and compare ages and clocks. Only one of the scenarios will be correct – but which one?

Resolving the twins' paradox

uniform motion
motion of an object that is not undergoing acceleration

The problem can be resolved by deciding who is travelling with **uniform motion** – that is, in an inertial frame of reference and not accelerating. The travelling twin must change velocity at the beginning and end of the trip and also when turning around in space. Therefore, he is not travelling with uniform motion and is not in an inertial frame of reference while he is accelerating, even if these acceleration periods occupy only a tiny portion of the total time. So, the Earth twin measures proper length, and the travelling twin measures the relativistic (contracted) length. But as both twins agree on the relative velocity, the travelling twin must measure a shorter time (to cover the shorter length) and thus returns to Earth having aged less than the Earth twin. Even when the acceleration periods are considered, Einstein's theory of general relativity, which deals with accelerating reference frames, confirms this result.

The ultimate judge, of course, is experiment. In 1971, precise clocks were sent around the world in jet planes. They confirmed that less time had passed for the travelling clocks.

Challenge

Infinite speed

Imagine that the speed of light was infinite. **Explain** how this would affect the ideas about relativistic length contraction and time dilation. (2 marks)

What is the pole (ladder) in the barn paradox?

Imagine a barn, 40 m long, with a door on each end. A runner has a pole (or, in some examples, a ladder) that is exactly as long as the barn (at rest), as shown in Figure 2A. The plan is for the runner to run into the barn at $0.80c$, and a farmer inside will close both doors for a brief time to demonstrate that the pole fits. The question is: will both the runner and the farmer agree that the pole fits inside the barn and the doors can be shut while it is in there?

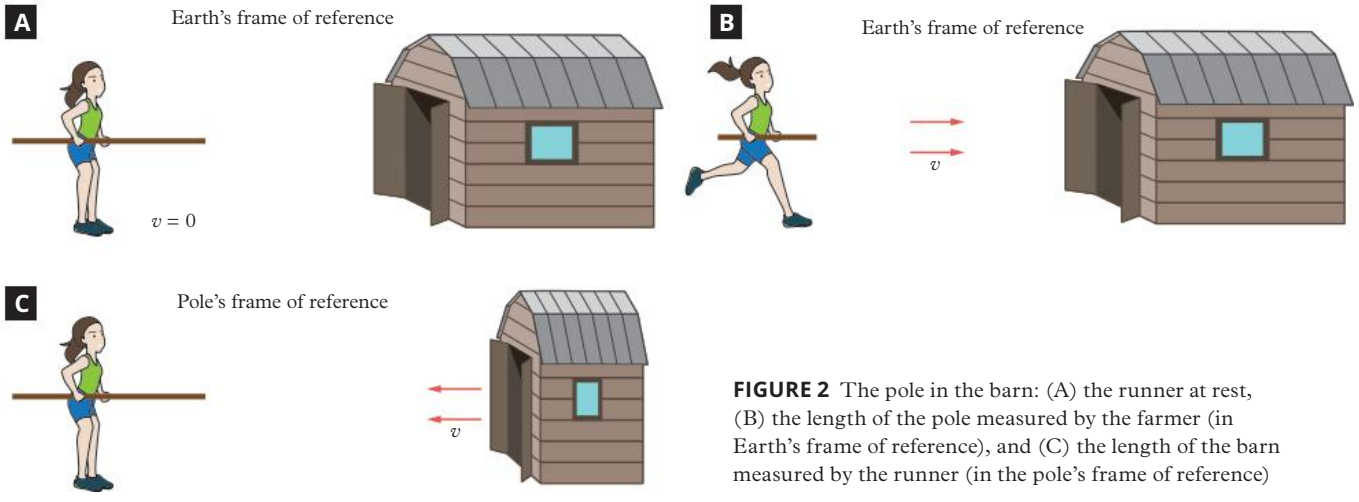


FIGURE 2 The pole in the barn: (A) the runner at rest, (B) the length of the pole measured by the farmer (in Earth's frame of reference), and (C) the length of the barn measured by the runner (in the pole's frame of reference)

In the reference frame of the barn, the runner enters the barn with the pole at $0.80c$ (Figure 2B). According to relativity, in the reference frame of the barn (and Earth), the pole is moving and experiences length contraction, and is now 24 m long. Being shorter, it fits easily inside the 40 m long barn, and the farmer can shut both doors momentarily.

In the reference frame of the pole (Figure 2C), the barn is rushing towards it at $0.80c$, so the barn experiences length contraction and is smaller – in fact, it is now just 24 m long. The pole, at 40 m, is too long for a barn that measures 24 m. There is no way that the farmer can close both doors with the pole inside.

In one reference frame, the pole fits in the barn; in the other it does not. Does the pole get caught in the doors or does it remain unscathed?

Resolving the pole in the barn paradox

This apparent paradox results from the mistaken assumption of absolute simultaneity (Module 9). The pole fits into the barn only if both of its ends are simultaneously inside the barn. The paradox is resolved when it is considered that, in relativity, simultaneity is relative to each observer, making the answer of whether the pole fits inside the barn also relative to each observer.

The runner's idea of simultaneity is when she sees both doors close at the same time while inside the barn.

The farmer's idea of simultaneity is also when he sees both doors close at the same time while the pole is inside the barn. He finds they do, so he finds that the pole fits. They both agree that the pole remains unscathed.

Real-world physics

Relativity, satellites and the Global Positioning System

There's an array of satellites orbiting Earth that allow you to work out your exact position. These form the Global Positioning System (GPS). Using light signals from extremely accurate on-board clocks, users can pinpoint a location on Earth to within a few metres. This is the basis of a car's navigation system and finding your location on phone map apps, although they also use phone masts and wi-fi signals for location.

The satellites are moving at 14,000 km h^{-1} with an orbital time of 12 hours. This means they and their clocks are travelling much faster than clocks on Earth. As you already know, "moving clocks run slow" – at this speed by about 7 μs per day. The GPS unit in your car adjusts for this loss of time and works out the real time at which the signal was received. There are 24 of these satellites active at any one time, so any place on Earth can get a signal from at least four satellites (and usually several more).

By knowing the transmitted time and the speed of light, it is possible for your GPS device to calculate the distance between it and each of the four satellites. Using simple geometry, it can work out where on Earth you are. The GPS system has an accuracy of about 50 nanoseconds (50 ns = 50×10^{-9} s), and in that time light can travel about 15 m, so that is the best navigational accuracy you could hope for. However, when other, land-based location systems are added, accuracy can be increased to less than a metre.

How is relativity taken into account?

Special relativity accounts for a correction of −7.0 μs, but a bigger correction is needed because of general relativity effects. The effect of gravity on time forms a part of the theory of general relativity, but that is beyond the scope of this course. However, it means a correction of +45.9 μs is needed to the ground-based time. Together the two effects of relativity account for a correction of +38.7 μs per day.

Our global navigation system would not work without an understanding of relativity. That sounds like a good claim for a research question.

FIGURE 3 Thirty-one satellites provide complete GPS coverage of Earth. Twenty four are operational at any time; the rest mean coverage is not affected if a satellite needs maintenance.

Apply your understanding

1 **Describe** how a difference in time signals can be used to calculate a position on Earth. (2 marks)

2 **Explain** which theory of relativity, special or general, has the bigger impact on the calculation of position. (1 mark)

3 A GPS satellite makes two orbits in 24 hours.

- **a** **Determine** its orbital distance above the surface of the Earth. Hint: you will need to use formulas and data from the QCAA Formula and data book. Assume the radius of the Earth, R_E = 6,378 km. (2 marks)
- **b** **Calculate** the time it takes a light signal to go from the satellite to Earth. (2 marks)
- **c** **Determine** the order of magnitude of the answer to part **b**. (1 mark)

Skill drill

Relativity and aging

Science inquiry skills: Evaluating evidence (Lesson 1.8)

A student was given the claim for a research investigation that "space travel is the answer to the aging process". They compiled a table of the fastest probes and spaceships launched from Earth to date (Guo, 2024).

TABLE 1 Mission data for some of our fastest spacecraft

Year launched	Craft	Max speed (km h^{-1})	Max speed (c)	Humans on board
1969	*Apollo 10* command module "Charlie Brown"	39,705	3.679×10^{-5}	Yes
1999	*Stardust*	46,440	4.303×10^{-5}	Yes
1977	*Voyager 1*	61,500	5.698×10^{-5}	No
1974	Helios satellites	252,792	2.342×10^{-4}	No
2018	Parker Solar Probe	586,800	5.44×10^{-4}	No

They drew a conclusion that "There is currently no spaceship that can travel at speeds close enough to the speed of light to observe signs of anti-aging. This prospect is not expected to be achieved within the foreseeable future."

Practise your skills

1 **Propose** how the student may have drawn such a conclusion from these results. (1 mark)
2 **Judge** whether there is sufficient data to draw a justified conclusion. (1 mark)
3 **Assess** any limitations on the conclusion. (1 mark)

Source: Guo, Y. (2024). Flying Parker Solar Probe to touch the Sun. *Acta Astronautica*, *214*, 110–124.

Check your learning 10.3

Check your learning 10.3: Complete these questions online or in your workbook.

Retrieval and comprehension

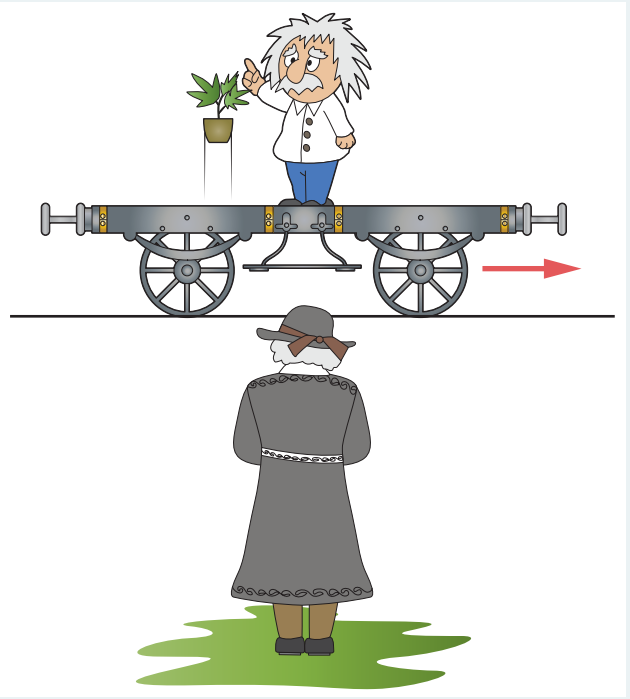
1 **Describe** the twins' paradox in terms of inertial reference frames. (2 marks)
2 **Explain** why you can't have faster-than-light movement of physical objects. (1 mark)
3 A ladder and a barn are measured as 15 m and 10 m by observers stationary to them. A person takes the ladder and runs at the barn at 0.90*c*.
   a Determine the length they measure the ladder to be and the barn to be. (2 marks)
   b Determine the length a farmer, stationary to the barn, measures the ladder to be and the barn to be. (2 marks)
4 A young-looking astronaut has just arrived home from a long space trip. She rushes up to a grey-haired old man and in the ensuing conversation refers to him as her child. **Explain** how this is possible. (3 marks)

Analytical processes

5 As you are travelling away from Earth at a speed of 0.6*c*, your pulse rate is being measured by a monitor on your wrist. The results are automatically radioed to Earth. **Determine** if observers on Earth would find the pulse rate increased when compared to normal. (2 marks)

Knowledge utilisation

6 Consider the following scenario as shown in the image. Albert Einstein is on an ultrafast railway carriage and his wife Mileva Einstein is standing in a nearby field watching her high-speed husband go past, travelling at close-to-light speed relative to her. As he passes, he drops Mileva's favourite potted plant, and to him the pot appears to shatter as it hits the floor of his ultrafast railway carriage.



Propose what Mrs Einstein observes with respect to the pot's
   a motion (momentum and velocity in the vertical plane) (2 marks)
   b physical characteristics (size and shape etc.) (2 marks)
   c fate on impact. (2 marks)

Justify your answer by identifying and explaining the relevant principles of special relativity.
Note: experts have problems answering this. Your task is to present an argument and draw a conclusion. You would be marked on the quality of your argument, not on getting a correct answer.

MODULE 10

Lesson 10.4
Review: Special relativity: Length, momentum and energy

Summary

10.1
- Moving objects contract in their direction of motion: $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, where L_0 is the proper (rest) length and L is the relativistic (contracted) length.
- The following equations can be used to solve problems about space travel: $v = \frac{L_0}{t} = \frac{L}{t_0}$

10.2
- Newtonian momentum and relativistic momentum have similar values at low speeds but diverge at relativistic speeds (>0.1c).
- Problems involving relativistic momentum can be solved using the formula: $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$
- Relativistic momentum is conserved in all interactions, providing no external force acts.
- Einstein's mass–energy equivalence relationship shows the relationship between energy and mass.
- Problems involving Einstein's mass–energy equivalence can be solved using $\Delta E = \Delta m c^2$.

10.3
- Apparent paradoxes about relativistic time and length can be resolved when the principles of relativity are applied correctly.

Key formulas

Length contraction	$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$
Space-travel relationships	$v = \frac{L_0}{t} = \frac{L}{t_0}$
Relativistic momentum	$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$
Mass–energy equivalence	$\Delta E = \Delta m c^2$

Revision questions 10.4A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 Why don't we notice relativistic effects in the normal motion of our everyday lives?
- **A** The Earth orbits the Sun so it undergoes centripetal acceleration.
- **B** The Earth rotates on its own axis so is not an inertial frame of reference.
- **C** They only apply at the speed of light.
- **D** They are too small to notice at normal speeds.

2 Einstein's two postulates of the special theory of relativity
- **A** state explicitly that nothing can travel faster than the speed of light.
- **B** provide for a rest mass that increases to infinity as the speed approaches c.
- **C** apply only to tiny atomic particles.
- **D** require that all of the laws of physics are the same in every inertial frame.

3 All observers in different inertial systems will agree about
- **A** the simultaneity of events at separate locations.
- **B** the rate at which their clocks run.
- **C** the lengths they measure along the direction of their relative travel.
- **D** none of the above.

4 A meteor is heading directly towards the surface of a planet at a constant speed of $0.8c$. Observers on the surface of the planet observe it at a time when it is a distance h above the surface in their reference frame. The observers calculate the time that the meteor will take to reach the surface of the planet as 784 μs.
The value that is closest to the distance h is

A 105 km. **B** 200 km.
C 235 km. **D** 380 km.

5 Identify the statement that best defines relativistic length.

A The length measured by an observer moving in the opposite direction to the object
B The difference between the length measured by an observer and the known length
C The length as measured by an observer moving with respect to the object in the direction of motion
D The length measured by an observer at rest in relation to the object

6 A motorcycle speeds past you at $0.60c$ and appears to be 1.2 m in length. Determine the length of the motorcycle as seen by the rider.

A 1.88 m **B** 1.50 m
C 1.92 m **D** 1.75 m

7 A spacecraft travels at a speed of $0.75c$ to a star 20 light-years from Earth, as measured by an astronaut on board. Determine the duration measured by an astronaut on board.

A 45.71 years
B 30.24 years
C 26.67 years
D 13.28 years

8 A spaceship passes Earth at $0.60c$ and appears to have a width of 5.0 m to Earth observers. Which one of the following would be the closest value of its width if it landed on Earth?

A 4 m **B** 5 m **C** 6 m **D** 8 m

9 A spaceship approaches the planet Mars, travelling at $0.50c$ with respect to Mars. Its crew shines a laser at Mars. The beam strikes a Martian mirror and is reflected back to the ship. The crew on the ship will measure the speed of the reflected beam to be

A $0.50c$. **B** $0.75c$. **C** $1.0c$. **D** $1.5c$.

10 The resolution to the twins' paradox lies in the observation that

A no two people can be exactly the same age.
B the twin who stays on Earth ages differently because of Earth's gravitational field.
C the predictions of special relativity only apply to subatomic particles.
D one of the twins must accelerate in order to leave Earth and come back.

11 The relativistic momentum of a subatomic particle of mass 1.62×10^{-27} kg moving at $0.92c$ is

A 3.80×10^{-27} kg m s^{-1}.
B 9.70×10^{-27} kg m s^{-1}.
C 1.14×10^{-18} kg m s^{-1}.
D 2.91×10^{-18} kg m s^{-1}.

12 The rest energy, in joules, of an electron of mass 9.11×10^{-31} kg is

A 3.0×10^{-39} J. **B** 2.7×10^{-22} J.
C 8.2×10^{-14} J. **D** 1.1×10^{-11} J.

13 A 1-metre-long ruler, at an angle of 30 degrees with the x-axis, is traveling at $0.6c$ in the direction of the positive y-axis. To a stationary observer, how long does the metre ruler appear to be?

A 1.0 m **B** $\frac{1}{5}\sqrt{\frac{29}{2}}$ m
C $\frac{\sqrt{91}}{10}$ m **D** $\frac{4}{5}$ m

14 What is the consequence of special relativity on time?

A It makes synchronisation of clocks impossible as every observer can claim their clock is correct.
B Time passes more quickly in a non-inertial frame of reference.
C Time passes more slowly in a stationary frame of reference.
D The rate at which time passes depends on the speed of the frame of reference.

15 A stationary observer views the approach of a spaceship moving at a relativistic speed. Which one of the following quantities would she measure as less than if the spaceship were stationary with respect to her?

A Momentum
B Height
C Length
D Density

Revision questions 10.4B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

16 A proton travels at 0.6c around a circular tunnel in the ground that is 23 km long. **Identify** whether the proton would see this as greater than or less than 23 km in its own frame of reference. (1 mark)

17 **Explain** why you can't just keep applying a force to an object to increase its speed to be faster than light. (1 mark)

18 **Calculate** the speed of a pion if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 5.2×10^{-8} s (2 s.f.). (2 marks)

19 **Clarify** whether anything can travel faster than light. (1 mark)

Analytical processes

20 You are travelling away from Earth at a speed of 0.5c. **Deduce** whether your mass, height, or waist measurement would change. **Infer** what observers on Earth using telescopes would say about these things. (2 marks)

21 Consider the piece of paper on which one page of a book is printed. **Deduce** which of the following properties of the piece of paper will not change – that is, which are independent of whether the paper is at rest or in motion relative to you. (1 mark each)

- **a** Thickness of the paper
- **b** Mass of the paper
- **c** Volume of the paper
- **d** Number of atoms in the paper
- **e** Chemical composition of the paper
- **f** Speed of the light reflected by the paper

22 **Determine** the rest length of a spaceship given that it passes you at a speed of 0.75c and you measure its length to be 120 m (2 s.f.). (2 marks)

23 **Determine** how much time it would take to travel to a star 24 light-years from Earth at a speed of 2.4×10^{8} m s^{-1}, according to

- **a** the spaceship travellers (2 s.f.) (2 marks)
- **b** an observer on Earth (2 s.f.). (2 marks)

24 Electrons travel through a linear accelerator 1.5 km long at a speed of 0.999999218c. **Determine** the length of this tube in the electrons' reference frame (2 s.f.). (2 marks)

25 In a reactor, 1.0 mg of mass is converted to energy.

- **a** **Determine** the theoretical amount of energy in joules that could be obtained from this conversion (2 s.f.). (1 mark)
- **b** **Determine** the order of magnitude of the energy produced in joules. (1 mark)

26 A 100 MeV electron travelling at 0.999987c moves along the axis of an evacuated tube. The tube is at rest relative to a laboratory observer, S, who measures its length as 3.00 m. Another observer, S′, moving with the electron, sees the tube moving past her. **Determine** the length that the tube would appear to the observer S′ (2 s.f.). (2 marks)

27 The star system Alpha Centauri is 4.0 light-years away. **Determine** at what constant velocity a spacecraft would have to travel from Earth if it is to reach the star in 3.0 years, as measured by travellers on the spacecraft (2 s.f.). (3 marks)

28 After the Sun, our next nearest star is Proxima Centauri, which is 4.225 light-years away. A spacecraft travels there from Earth at a speed of 0.80c.

- **a** **Determine** how many years it would take to reach Proxima Centauri from Earth, as measured by an observer on Earth (2 s.f.). (2 marks)
- **b** **Determine** how many years the trip would take as measured by an observer on the spacecraft (2 s.f.). (2 marks)
- **c** **Calculate** the distance travelled according to an observer on the spacecraft (2 s.f.). (2 marks)

29 Imagine a rocket takes off for a distant planet and can travel at many times the speed of light. (You know that this is not possible, but just imagine it is for this question.) Observers on the planet are viewing the incoming spaceship through a powerful telescope. **Deduce** what they would see from the moment the rocket leaves Earth until it lands on their planet (2 s.f.). (2 marks)

Knowledge utilisation

30 **Propose** the speed at which sunlight would pass you if you were on a rocket travelling at $0.6c$ away from the Sun. **Explain** your answer. (2 marks)

31 You want to get to a star 100.0 light-years away and have calculated you could do it in 4.5 years if you travelled at $0.999c$. The problem is that the human body can't withstand acceleration greater than $5g$. **Predict** the time, as measured by an Earth observer, it would it take you to reach $0.999c$ from rest at an acceleration of $5g$ (2 s.f.). (3 marks)

32 Imagine you have slid into a parallel universe in which the year is 1840. A US marshal (sheriff) is travelling by in a train at $0.75c$ and approaches two gunslingers about to have a duel. The gunslingers are both the same distance from the railway line. Gunslinger A is closer to the marshal. The marshal sees both men draw their guns at the same time. **Decide** and **explain** who actually drew first in

- **a** the marshal's frame of reference (2 marks)
- **b** the gunslingers' frame of reference. (2 marks)

33 The Andromeda galaxy is 100 (1.00×10^2) light-years away, and astronauts plan to travel there at $0.98c$ to get there in a reasonable time.

- **a** **Predict** how much time the journey would take in their frame of reference (2 s.f.). (2 marks)
- **b** Once the astronauts arrive, they send a radio signal back to Earth. **Determine** how long after the astronauts left Earth would observers on Earth have to wait to receive the radio signal (2 s.f.). (2 marks)

34 The Large Hadron Collider has a circular track exactly 26,659 m long. Protons have been made to whizz around the track at speeds very close to the speed of light. Most calculators can't work with enough decimal places to use the exact number, so we can use a slower proton as an example. Imagine a proton moving at $0.99999c$ around the track. Give your answers to 2 significant figures for the following questions.

- **a** **Determine** the length of the track as seen by this proton. (2 marks)
- **b** **Propose** how many seconds the scientists would measure for the proton to complete one revolution of the track. (2 marks)
- **c** **Deduce** how long it would take in the reference frame of the proton. (2 marks)
- **d** **Determine** the relativistic momentum of the proton, given that its rest mass is 1.67×10^{-27} kg. (2 marks)

35 Suppose that a special breed of cat (*Felix schrödingerus*) lives for exactly 7.00 years according to its own body clock. When such a cat is born, it is put aboard a spaceship and sent off at a speed of $0.80c$ toward the star system Alpha Centauri.

- **a** **Determine** how far from Earth (reckoned in the reference frame of Earth) the cat would be when it dies (2 s.f.). (3 marks)
- **b** As soon as the cat dies, a radio signal announcing the death of the cat will be sent from the spaceship. **Decide** in how many years after the departure of the spaceship the signal would reach Earth (2 s.f.). (Recall that radio signals travel at the speed of light.) (2 marks)

36 A spacecraft of rest mass 20,000 kg (2.0×10^4 kg) has been accelerated from rest to $0.25c$.

- **a** **Determine** its momentum (2 s.f.). (2 marks)
- **b** **Determine** by what percentage you would be in error if you used the classical formula for momentum (2 s.f.). (2 marks)

37 You are sitting in your car when a very fast sports car passes you at a speed of $0.18c$. The driver of the sports car says his car is 6.00 m long and yours is 6.15 m long. **Determine** what you would measure for the length of

- **a** the sports car (2 s.f.) (2 marks)
- **b** your car (2 s.f.). (2 marks)

38 Train carriages A and B each have a rest length of 10.0 m and are directly approaching each other at high speed. Ann is in carriage A, and as she passes the front of carriage B she starts her clock. She stops her clock when she passes the rear end of carriage B. Ann's clock reading shows it took carriage B a time of 2.0×10^{-7} s to pass her. **Propose**, with evidence, what the speed of the carriage B is (2 s.f.). (2 marks)

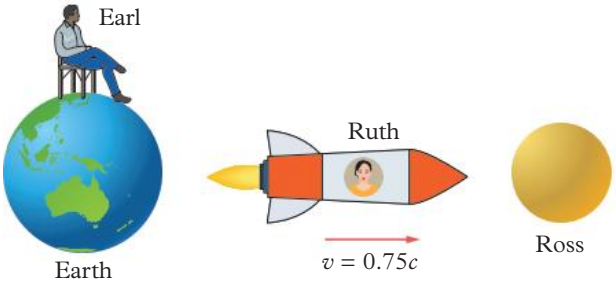
39 Trains A and B are about to pass each other at extremely high speeds from opposite directions. Train A has a rest length of 100.0 m and train B measures 50.0 m at rest. Albert is in train A, and as he passes the front of train B he starts his clock. He stops his clock when he passes the rear end of train B. Albert's clock reading shows it took train B exactly 1.00 μs to pass him. **Determine** the speed of train B (2 s.f.). (2 marks)

40 A 20-year-old astronaut leaves Earth in a rocket for a distant planet 9.0 light-years away (as measured by Earth observers). When he lands on the planet, he sends a radio signal back to Earth, which arrives 19 years after he left Earth.

a **Deduce** the speed in units of c at which he travelled (2 s.f.). (4 marks)

b A radio signal was sent back to the planet confirming that Earth had received the signal from him. **Predict** how old he would be when the return signal was received on the planet (2 s.f.). (3 marks)

41 Twins Earl and Ruth are 20 years old when Ruth is sent off into space for a mission to the distant star Ross. Her spacecraft travels at $0.80c$.



She returns to Earth, and Earl is now 35 years old by his reckoning.

Determine how old Ruth would be, to the nearest whole number, by her own reckoning. (3 marks)

Data drill

Subatomic particles

A subatomic particle was travelling in a circular path at various relativistic speeds around the Large Hadron Collider. Its position in the frame of reference of the particle at certain times was calculated. The column labelled speed factor, β, is derived from the velocity using the formula $\beta = \sqrt{1 - v^2}$ (where the velocity is in units of c). A graph of the relativistic length, L, of the circular path versus the speed factor is shown in Figure 1.

TABLE 1 The velocity, speed factor, β, and relativistic length of a subatomic particle travelling in a circular path at various relativistic speeds

Velocity, v (c)	Speed factor, β	Relativistic length, L (m)
0.60	0.80	18,400
0.70	0.71	16,425
0.80	**Q2**	13,800
0.90	0.44	10,025
0.99	0.14	3,245

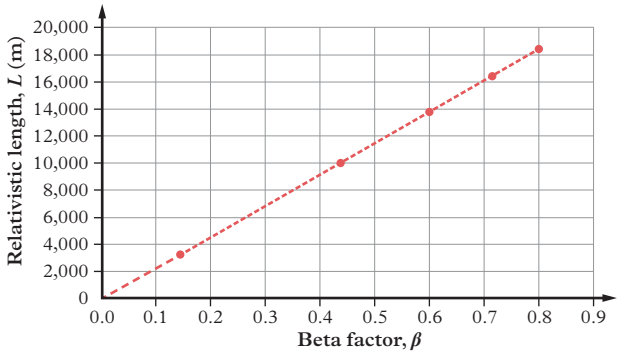


FIGURE 1 Graph of the relativistic length, L, of the circular path versus the speed factor, β

Apply understanding

1 **Calculate** the speed of the particle in metres per second when $v = 0.80c$. (1 mark)

2 Using the formula, **calculate** the speed factor for a velocity of $0.80c$. Show your working. (2 marks)

Analyse evidence

3 **Identify** the relationship between relativistic length and the speed factor. (2 marks)

Interpret evidence

4 In terms of the formulas for length contraction, **determine** what the gradient represents. (1 mark)

5 By calculating the gradient of the graph, **draw a conclusion** about the proper length of the path taken by the subatomic particle. (3 marks)

Module 10 checklist: Special relativity: Length, momentum and energy

UNIT 4

Topic 1 review

Multiple choice

(1 mark each)

1 A very fast spaceship is travelling at a relativistic speed near a planet. Deduce which one of the following observers is measuring its rest length.

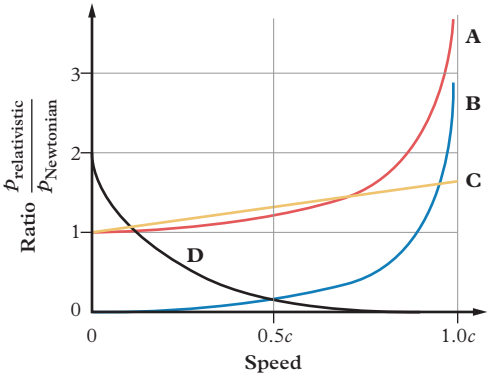
A The observer on the spaceship measuring the time taken for light to travel between two points on the planet

B The observer on the planet measuring the time taken for light to travel from the front to the back of the spaceship

C The observer on the spaceship measuring the time taken for light to travel from the front to the back of the spaceship

D The observer on the planet measuring the difference in the arrival time of light from the front and the back of the spaceship

2 Select the graph in the diagram that represents the ratio of relativistic momentum to Newtonian momentum as a function of speed.



3 Calculate the energy equivalent to 1 g of uranium.

A 3×10^5 J

B 3×10^{16} J

C 9×10^{13} J

D 9×10^{16} J

4 Which one of the following is a key postulate of special relativity?

A The speed of light in any medium has the same value, c, in all inertial frames of reference.

B The laws of physics are the same in all inertial (uniformly moving) frames of reference.

C Relative motion occurs when one object moves relative to a stationary object.

D A frame of reference that is stationary is an inertial frame.

5 An unstable particle is moving at a speed of 2.4×10^8 m s^{-1} relative to a laboratory. Its lifetime is measured by a stationary observer in the laboratory to be 4.7×10^{-6} s. What is the lifetime of the particle, measured in the rest frame of the particle?

A 1.69×10^{-6} s

B 2.82×10^{-6} s

C 5.87×10^{-6} s

D 7.80×10^{-6} s

6 Pions are particles created in the upper atmosphere by cosmic rays from the Sun. Pions decay with an average rest lifetime of 26 nanoseconds in

A the cosmic rays' frame of reference.

B each pion's own frame of reference.

C any inertial frame of reference.

D Earth's frame of reference.

7 A rocket with a velocity of $0.80c$ journeys to the star Procyon. A stationary observer on Earth measures the journey taking 19.0 years. Calculate the time taken for the journey for an observer on the rocket.

A 6.9 years

B 11 years

C 32 years

D 53 years

8 Identify the correct consequence of the speed of light in a vacuum being the same for all observers.

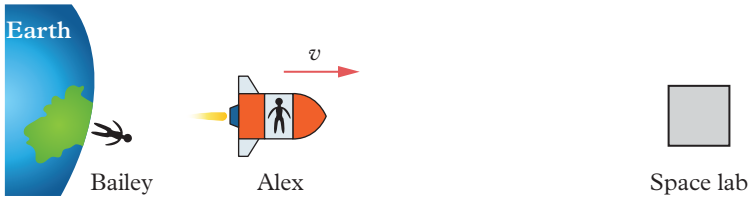
A A clock that is moving relative to an observer will tick faster than a clock that is at rest in the observer's own frame of reference.

B The length of a moving object is smaller than the length of the object when it is measured in its own frame of reference.

C Observations of events in distant galaxies are observations of events that occurred in the past.

D The frame of reference of the observer is always the real frame of reference.

9 Alex and Bailey have identical quartz clocks that use the precise period of vibration of quartz crystals to determine time. Bailey and his clock are on Earth. Alex accompanies her clock on a rocket travelling at constant high velocity, v, past Earth and towards a space lab (which is stationary relative to Earth), as shown in the diagram.



Which one of the following statements correctly describes the behaviour of these two clocks?

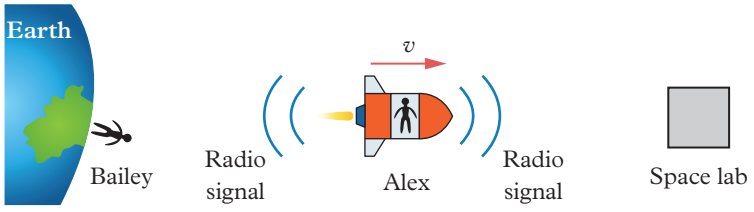
A The period of vibration in Alex's clock (as observed by Alex) will be shorter than the period of vibration in Bailey's clock (as observed by Bailey).

B The period of vibration in Alex's clock (as observed by Alex) will be longer than the period of vibration in Bailey's clock (as observed by Bailey).

C The period of vibration in Alex's clock (as observed by Alex) will be the same as the period of vibration in Bailey's clock (as observed by Bailey).

D Only the time on Bailey's clock is reliable because it is in a frame that is not moving.

10 When Alex (from question **9**) is halfway between Earth and the space lab, she sends a radio pulse towards Earth and towards the space lab, as shown in the diagram.



As observed by Alex, which one of the following statements correctly gives the order in which this signal is received by Bailey and by the space lab?

A Bailey receives the signal first.

B The space lab receives the signal first.

C The signal is received by Bailey and the space lab at the same time.

D It is not possible to predict since special relativity applies to light but not to radio signals.

11 A space lab travelling at $u = 0.80c$ away from Earth can record high-energy charged particles passing through its detectors. One particle is travelling towards Earth at $v = 0.91c$ relative to the space lab.

Two detectors, numbered 1 and 2 in the diagram, are 2.0 m apart in the space lab's frame.

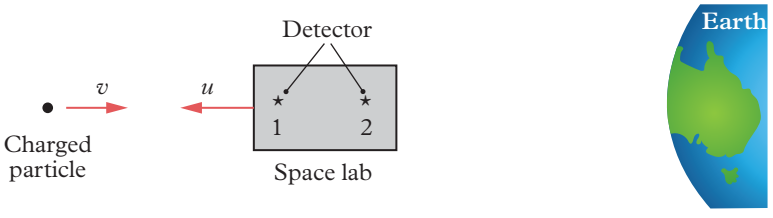
What distance apart are the two detectors in this particular particle's frame?

A 0.83 m

B 1.2 m

C 3.3 m

D 4.8 m



12 A very fast spaceship X with a rest length of 80.0 m is travelling at $0.80c$ past another spaceship Y. Which one of the following statements by the observer on spaceship Y is true?

A The observer on X is aging at a different rate to observers on Y.

B The length of X is shorter than 80.0 m.

C The length of X is longer than 80.0 m.

D The clock on X is running fast.

13 Which one of the following is true about momentum?

A Relativistic momentum doubles when speed is doubled.

B Newtonian momentum remains constant as speed is increased.

C An object travelling at $1.00c$ would have an infinite amount of momentum.

D At low speeds Newtonian momentum is just slightly greater than relativistic momentum.

14 Which one of the following shows the amount of energy equivalent to 2.0 kg of mass?

A $\frac{(3 \times 10^8)^2}{2.0}$ J

B $\frac{2.0}{(3 \times 10^8)^2}$ J

C $2.0 \times (3 \times 10^8)^2$ J

D $(2.0 \times 3 \times 10^8)^2$ J

15 Chris travels at high speed from the Earth to the star Proxima Centauri, four light years away as measured by Earth observers. In Chris's frame of reference

A the distance is longer than four light years.

B the distance is shorter than four light years.

C the trip takes more time than it does in the Earth's frame of reference.

D the trip takes the same time as it does in the Earth's frame of reference.

Short response

16 A muon moves at a speed of $0.9c$ down a tube that is 30 m long as measured in the reference frame of the laboratory.

a **Determine** how the length of the tube appears to an observer in the reference frame of the muon. (2 marks)

b **Determine** how the width of the tube as measured in the muon's frame of reference would compare to the width as measured by a laboratory observer. (1 mark)

17 A muon is created in our upper atmosphere and travels toward the ground at $0.998c$. Its lifetime is 2.2×10^{-6} s.

a **Determine** the lifetime of the muon as observed in the Earth's frame of reference. (3 marks)

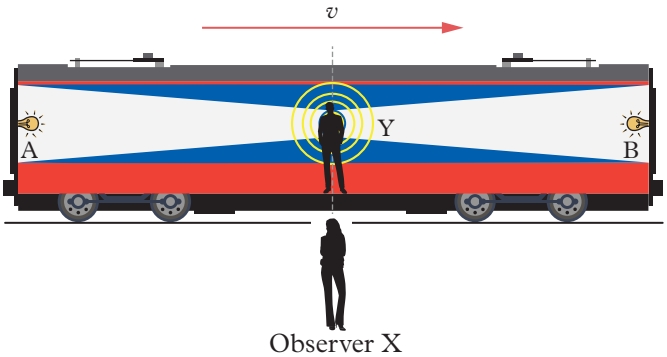
b **Determine** the distance that would be covered by the Earth as it sped toward the muon according to its own reference frame. (3 marks)

c **Determine** what distance the muon would cover in our reference frame. (1 mark)

18 **Identify** the two postulates of Einstein's special relativity. (2 marks)

19 Pions are particles that are present in cosmic rays striking Earth. Pions decay with an average life of 26 ns. A pion is approaching Earth at a speed of $0.98c$. **Determine** the life of the pion as measured by Earth observers. (3 marks)

20 A train with a proper length of 500 m is moving at a speed $v = 0.76c$ with respect to an external stationary observer X, as shown in the diagram.



When exactly half the train has passed next to observer X, two light bulbs, A and B, placed at opposite ends of the train are turned on simultaneously, according to observer Y on the train midway between the lights.

a **Identify** which light bulb is turned on first according to observer X. (1 mark)

b **Identify** how this is different to what observer Y sees and **explain** who is correct. (3 marks)

21 A spaceship is observed to be 45 m long while at rest on Earth. As it passes Earth, it appears to be 8 m long. **Determine** how fast the ship is travelling. (3 marks)

22 Alpha Centauri, our nearest star system, lies 4.37 light-years from the Sun. **Determine** the fraction of the speed of light at which a ship would have to travel for the crew to experience only 1 year in transit. (4 marks)

23 A spaceship travels to Sirius, which is about 8.7 light years away from Earth.

a **Determine** how far this distance appears to you if you are on a spaceship travelling at $0.99c$. (3 marks)

b **Determine** how many years it would take according to your watch to travel to Sirius at $0.99c$. (1 mark)

24 Two identical spaceships are 30.0 m long when measured at rest. Later, they are flying towards each other at speeds of $0.42c$, relative to each other.

a **Determine** what each of the pilots observes the other ship's length to be. (3 marks)

b **Explain**, with reference to special relativity, your answer to part **a**. (1 mark)

25 The nearest galaxy is Ursa Major III, about 33 thousand light years from Earth.

a Without relativistic effects, **determine** how long it would take us to reach Ursa Major III if we could travel at half the speed of light. (3 marks)

b Without relativistic effects, **determine** how long it would take us to reach Ursa Major III if we could travel at the speed of light. (1 mark)

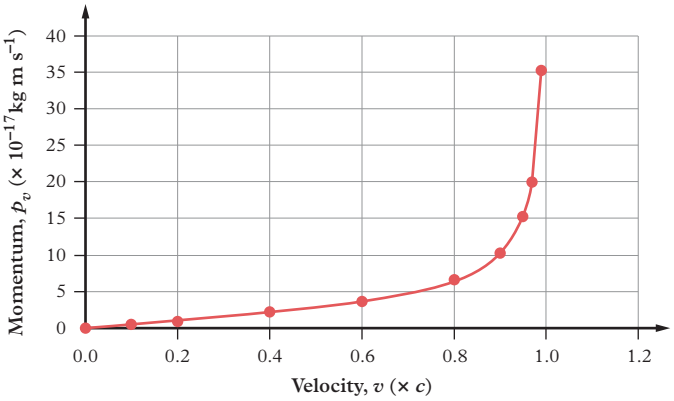
c Considering relativistic effects, **determine** how long it would take us to reach Ursa Major III if we travelled at $0.99c$. (2 marks)

d **Determine** how fast we would need to travel if we wanted to reach Ursa Major III in 50 years. (2 marks)

26 A proton (mass 1.673×10^{-27} kg) ejected by a supernova attains a velocity of $0.9996c$. **Calculate** the magnitude of the momentum of the proton. (2 marks)

27 A spacecraft of rest mass 100 kg is travelling through space at $0.5c$. **Determine** the ratio of the relativistic to the Newtonian calculations of its momentum. (4 marks)

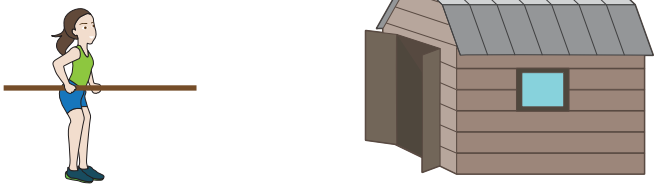
28 The Large Hadron Collider accelerates protons to very high speeds, reaching speeds of $0.999999991c$, before two proton streams collide. As the protons increase in speed, the momentum of the protons as measured in the laboratory frame changes according to the rules of special relativity. The graph shows the observed momentum of the protons as a function of velocity.



a **Calculate** the momentum of a proton at $0.3c$ using the relativistic momentum formula. (2 marks)

b **Explain** how the graph shows that there is a maximum speed limit for all particles with mass. (2 marks)

29 An athlete is carrying a pole measured at 8.0 m when stationary. It is carried horizontally by the athlete, as shown in the diagram, who runs through a barn at a speed of $0.92c$. The barn is 6.0 m long and has open doors at either end.



There is a farmer, B, sitting inside the barn (not shown) who presses a button when they see that the pole is completely inside the barn. The farmer sees both the front and rear door close simultaneously and then immediately open again. This is a paradox because the farmer knows that the rest length of the pole is longer than the barn, but sees the pole fit inside the barn with both doors shut. But the athlete says the barn is too small to fit the pole. **Explain** how this paradox is resolved. Support your explanation and conclusion with calculations. (8 marks)

TOTAL MARKS

/75 marks

MODULE 11

Quantum theory and light

Introduction

Until the end of the nineteenth century, classical physics, based on Newton's laws, successfully explained all most of our natural surroundings of matter, space and time. At the turn of the century, experimental observations by physicists and subsequent theoretical explorations began to raise questions about the validity of the Newtonian laws, especially at very small distances, at very high speeds and within the emerging world of the atom. Observations such as the dark lines in the spectra of light emitted by heated gases or gas discharges could not be explained by classical physics. Light itself was difficult to explain, as it seemed to have both a particle nature and a wave nature. Research in the field of thermodynamics, which described the changes in a system in terms of large-scale measurable properties, began to seek a microscopic interpretation of interactions between individual particles. New explanations and models were needed, and this ultimately led to the beautiful but strange world of quantum theory.

Prior knowledge

Prior knowledge quiz

Check your understanding of concepts related to quantum theory and light before you start.

Subject matter

Science understanding

- → Explain how the double slit experiment provides evidence for the wave model of light.
- → Describe light as an electromagnetic wave.
- → Explain the concept of black-body radiation and the significance of the evidence it provides.
- → Describe the photoelectric effect in terms of the photon.
- → Describe the concepts of threshold frequency and work function.
- → Solve problems involving black-body radiation and the photoelectric effect using

$$\lambda_{max} = \frac{b}{T},\ E = hf = \frac{hc}{\lambda},\ E_k = hf - W,\ W = hf_0.$$

- → Describe wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light.
- → Interpret data related to the photoelectric effect.

Science as a human endeavour

- → Consider how theories are contested, refined or replaced when new evidence challenges them, or when a new model or theory has greater explanatory power.

Science inquiry skills

- → Investigate variables related to the photoelectric effect such as
  - → photoelectron energy or velocity
  - → electrical potential difference across the anode and cathode
  - → wavelength or frequency of incident light
  - → work functions of surfaces.

Practicals

This lesson is available on Oxford Digital.

Lesson 11.5 Investigating the photoelectric effect

Lesson 11.1
The wave model of light

Key ideas

- → Young's double-slit experiment demonstrates the wave nature of light by allowing two coherent beams of light to overlap on a screen to form an interference pattern.
- → Young observed interference patterns in light that were similar to patterns he saw in water waves and heard in sound waves.
- → Light is an electromagnetic wave produced by an oscillating electric charge resulting in mutually perpendicular electric and magnetic fields.
- → Electromagnetic radiation is energy radiating out as electromagnetic waves, propagated at the speed of light in a vacuum.
- → The wave model of light uses wave characteristics such as wavelength, frequency and speed to describe behaviour of light such as polarisation, interference and diffraction.
- → The wave equation is $v = f\lambda$, where v is speed (velocity), f is frequency and λ is wavelength. For electromagnetic radiation in a vacuum, $v = c$.

Learning intentions and success criteria

Is light a particle or a wave?

By the end of the nineteenth century, the dispute over competing models of light was just about over. The wave model had won the battle, and the particle model was put on hold. Even Newton's support for the particle model couldn't withstand the amazing success of the wave model. Waves and particles were just as successful for explaining reflection, rectilinear (straight-line) propagation, speed and shadows. But the particle model had little success in explaining polarisation, or the diffraction (bending) of light as it passes around objects and through thin slits. The particle model was completely unable to deal with interference of light, which produces bright and dark fringes when light passes through thin slits and is the source of the beautiful colours in beetles' wings (Figure 1).

FIGURE 1 The beautiful iridescent colours in this jewel bug's wings are produced by the interference of light and can only be explained by the wave model of light.

What did Young's double-slit experiment uncover?

One of the most resounding demonstrations in support of the wave model was provided in 1801 by English scientist Thomas Young (Figure 2), some 100 years after Newton published his particle model. His was the first serious challenge to Newton's particle theory, which proposed the existence of "corpuscles" of light. These "corpuscles" could be likened to the modern idea of photons. Newton was aware of diffraction but failed to recognise its significance.

The origins of Young's theory

Young wrote a thesis on the physical and mathematical properties of sound; then, in 1800, he presented a paper to the Royal Society in London in which he argued that light was also a wave. His idea was greeted with some scepticism because it contradicted Newton's corpuscular theory. Young knew that sound was a wave phenomenon and that if two sound waves of equal intensity were 180° out of phase, they would experience destructive interference and cancel out. Young argued that if light was a wave phenomenon, then similar interference effects should occur. This proposal inspired Young to perform an experiment that is now called **Young's double-slit experiment**.

FIGURE 2 Thomas Young

Young's method

Young split a beam of sunlight to produce two **coherent** (in phase) beams that he let fall onto a screen, where they overlapped to produce a diffraction pattern of light and dark bands (Figure 3).

Coherent beams of light are needed for this experiment, but Young knew that the usual light sources at the time (candles and lanterns) could not produce coherent light. He made a pinhole in his window shutter and directed the beam of sunlight across the room to strike a piece of card edge-on. Some light went to the left side of the card and some went to the right. Because the two beams were both from the same source (the pinhole), Young considered them as coming from two coherent sources.

Young's double-slit experiment demonstrates the wave nature of light by allowing two coherent beams of light to overlap on a screen to form an interference pattern

coherent waves with the same frequency and amplitude, and a constant phase relationship. Conventional light sources such as candles and incandescent bulbs are incoherent sources; laser beams are coherent.

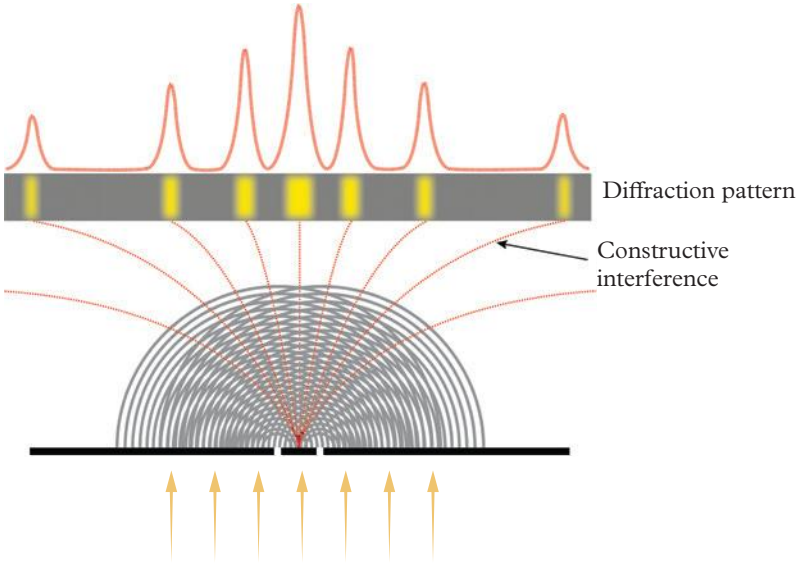


FIGURE 3 A stylised version of Young's double-slit experiment showing the waves of light and the bright and dark bands on the screen. The peaks at the top show the relative intensity (brightness) of each band.

Study tip

To help you visualise double-slit interference, you can view the pattern produced by a laser beam passed through a diffraction grating. Your school may have them both.

The implications of Young's double-slit experiment

The explanation for the pattern Young observed was that when crests of light waves met other crests of light waves, or troughs met troughs, there would be constructive interference, and bright bands would be produced. In positions where a crest and a trough met there would be destructive interference, and dark bands would be produced. This interference is a behaviour seen in mechanical waves. It provides evidence, but not necessarily proof, for the wave nature of light. However, the particle model had no explanation for these results, so the wave model was confirmed. After that, the particle theory of light wasn't heard of until the twentieth century, some 100 years later.

What does the wave model of light suggest?

wave model of light
uses mechanical wave characteristics such as wavelength, frequency and speed to describe the behaviour of light such as polarisation, interference and diffraction

Some 200 years after Young proposed his idea on the wave nature of light, we now have a model that builds on all that has gone before. Today, our **wave model of light** conceptualises light as an electromagnetic (EM) wave – it is an oscillation of the electromagnetic field. Examples include gamma rays, ultraviolet light, infrared light, X-rays and radio waves. They all share the same characteristics: they do not need a medium in which to travel, so electromagnetic waves can get to us from distant stars by passing through the vacuum of space.

An electromagnetic wave is produced by oscillating an electric charge, which produces oscillating magnetic fields and electric fields that are at right angles to each other and to the direction the wave is travelling (Figure 4). Both fields oscillate in phase, and they travel at a speed of $3 \times 10^8\,\text{m s}^{-1}$ in a vacuum as electromagnetic radiation. We use the symbol c for the speed of light in a vacuum. In material media such as glass, plastic or water, their speed is lower.

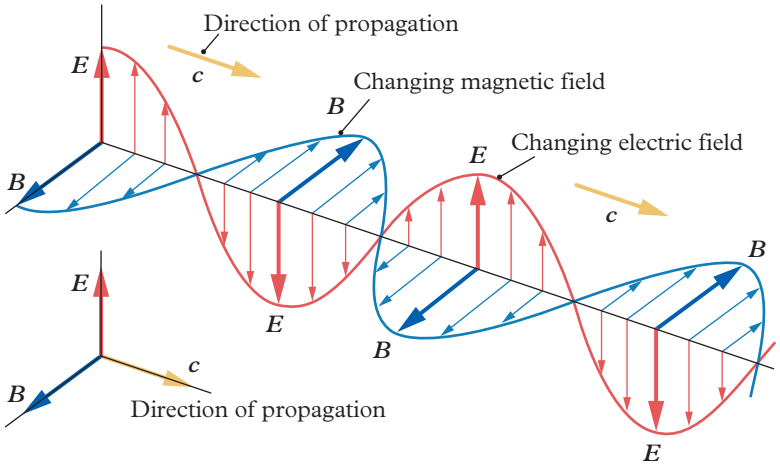


FIGURE 4 In an electromagnetic wave, the changing magnetic, B, and electric, E, fields are at right angles to each other and to the direction of propagation. The waves are shown propagating in only one direction but, in reality, they would be in all directions.

Spectral regions

Recall from Module 8 that electromagnetic waves can be divided into "spectral regions" based on their frequency or wavelength. This is shown in Figure 5. Visible light, for example, ranges from violet to red. Violet light has a wavelength of 400 nm and a frequency of 7.5×10^{14} Hz. Red light has a wavelength of 700 nm and a frequency of 4.3×10^{14} Hz. Any electromagnetic wave with a frequency (or wavelength) between those values can be seen by humans, but visible light makes up a very small part of the full electromagnetic spectrum. Electromagnetic waves that are of higher energy than visible light (higher frequency, shorter wavelength) include ultraviolet light, X-rays and gamma rays. Lower energy waves (lower frequency, longer wavelength) include infrared light, microwaves, and radio and television waves.

The wave equation

You will have used the wave equation in earlier years:

$$v = f\lambda$$

where v is the speed (velocity) of the wave, f is the frequency and λ is the wavelength.

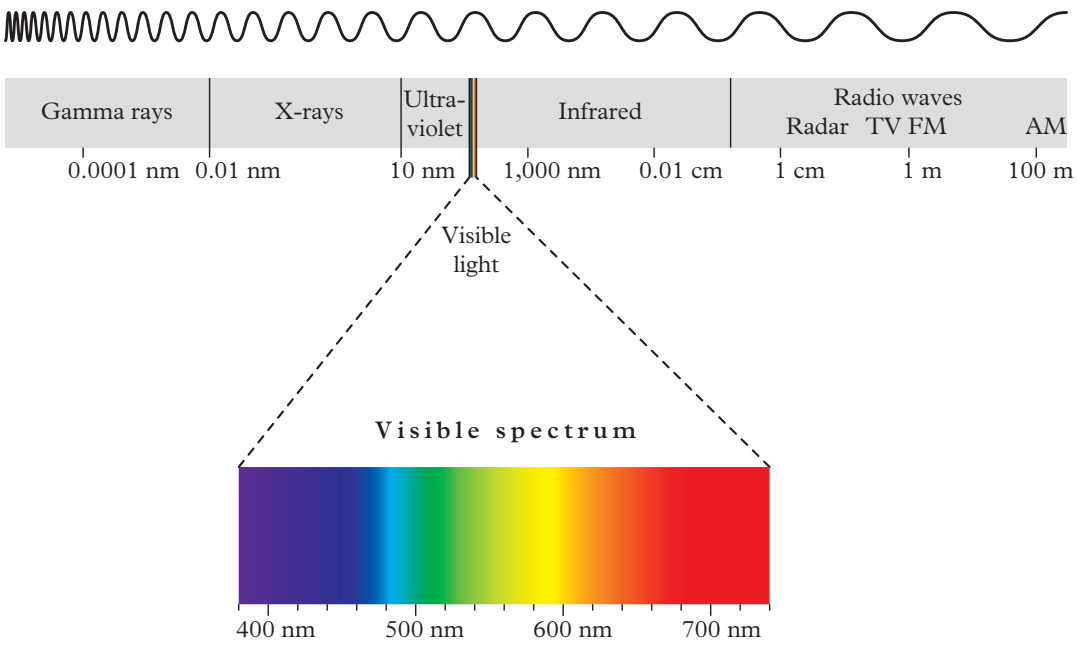


FIGURE 5 The electromagnetic spectrum with wavelengths for each spectral region

Study tip

Wavelengths of light are commonly given in nanometres (1 nm = 10^{-9} m). For example, the wavelength of mid-green is about 550 nm or 550 × 10^{-9} m. To convert nm to m, just multiply the value in nm by 10^{-9}. To convert from m to nm, divide your value in m by 10^{-9}. Frequencies can be given in hertz (Hz) or terahertz (1 THz = 10^{12} Hz). Unless otherwise stated, assume the speed of light, *c*, is 3 × 10^{8} m s^{-1}.

Worked example 11.1A

Calculating light wavelength and frequency and identifying spectral region and colour

A beam of light has a wavelength of 7.00 × 10^{-7} m.

a **Calculate** its wavelength in nanometres. (1 mark)

b **Calculate** its frequency. (1 mark)

c **Identify** its spectral region and colour using Figure 5. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. "Identify" means to recognise and state. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formula.	$v = f\lambda$, $\lambda = 7.00 \times 10^{-7}$ m
Step 3: Convert metres to nm by dividing by 10^{-9}. (1 mark for "Converts m to nm")	**a** $\lambda = 7.00 \times 10^{-7}$ m $= \frac{7.00 \times 10^{-7}\text{ m}}{10^{-9}\text{ m nm}^{-1}}$ = 700 nm (1 mark)
Step 4: Rearrange the wave equation and substitute in the known values. Complete the calculation. Give your answer to an appropriate number of significant figures, in this case 3 s.f. Check you have used the correct units. (1 mark for "Correctly calculates frequency")	**b** $f = \frac{v}{\lambda}$ $= \frac{3 \times 10^{8}\text{ m s}^{-1}}{7 \times 10^{-7}\text{ m}}$ (1 mark) $= 4.28 \times 10^{14}$ Hz
Step 5: Identify the spectral region and colour from the diagram. (1 mark for "Correctly identifies region as visible and colour as red")	**c** It is in the visible region, red. (1 mark)

Your turn

A beam of light has a wavelength of 4.10×10^{-7} m.

a **Calculate** its wavelength in nanometres. (1 mark)

b **Calculate** its frequency. (1 mark)

c **Identify** its spectral region. (1 mark)

Challenge

Micro

Explain whether the prefix "micro" in the word "microwave" means that microwaves have wavelengths smaller than visible light. (1 mark)

Challenge

Underwater double-slit experiment

Propose how the distance between bright fringes would change if Young's double-slit experiment were performed underwater. (2 marks)

Check your learning 11.1

Check your learning 11.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how Young's double-slit experiment provides evidence for the wave model of light. (2 marks)

2 **Describe** the relationship between the electric and magnetic fields in an electromagnetic wave. (1 mark)

3 **Identify** the colour of visible light that has the longest wavelength. (1 mark)

4 **Identify** what is oscillating to produce an electromagnetic wave. (1 mark)

5 **Construct** a diagram to show two waves undergoing constructive and destructive interference. (2 marks)

6 **Construct** a diagram to represent two coherent electromagnetic waves of the same frequency, wavelength and amplitude. (1 mark)

Analytical processes

7 **Determine** the colour of light of frequency 400 THz (tera, T = 10^{12}). (2 marks)

8 **Determine** the frequency range of visible light, given the range of wavelengths is from 390 nm to 720 nm (nano, n = 10^{-9}). (2 marks)

9 **Determine** the wavelength in picometres for a gamma wave of frequency 5×10^{17} Hz (pico, p = 10^{-12}). (2 marks)

Lesson 11.2
Black-body radiation

Key ideas

- → A black body is an object that absorbs all radiation falling on it, of all wavelengths. It is a perfect absorber or emitter of radiation.
- → Black-body radiation is the radiation emitted by a black body from the conversion of thermal energy. It has a characteristic frequency distribution that depends on the black body's temperature.
- → Wien's displacement law states that the black-body radiation curve for different temperatures peaks at a maximum wavelength (λ_{max}), which is inversely proportional to the temperature T (in kelvin, K). The formula for this is: $\lambda_{max} = \frac{b}{T}$, where b is a constant of proportionality called Wien's displacement constant and is equal to 2.898×10^{-3} m K.
- → Earth's energy balance is explained by the greenhouse effect and black-body radiation.

What is black-body radiation?

The electromagnetic wave model of light was very successful in explaining many phenomena such as diffraction, interference and polarisation. However, it couldn't explain other phenomena being investigated in the late 1800s. This included the light produced from heated objects. The object being heated, commonly a big block of hollowed-out charcoal, was called a **black body**. This is an object that absorbs all radiation; that is, it doesn't reflect any light shining onto it. So, as the black body is heated, any electromagnetic radiation that it emits is solely coming from within itself and has been converted from thermal energy. This is called **black-body radiation**.

What is Wien's displacement law?

German physicist Wilhelm Wien (pronounced "ve-en"; 1864–1928) worked on the problem of black-body radiation through the late 1880s and early 1890s. He made one of the most significant contributions to our understanding of the inner workings of an atom, and he was awarded a Nobel Prize in Physics in 1911 for his work on black-body radiation.

Wien's study of the emission of light from metals for his PhD led him to investigate the distribution of wavelengths of emitted light as an object is heated. He published his law in 1893 to much acclaim. **Wien's displacement law** states that black-body radiation curves for an object at different temperatures have peaks at different wavelengths. Each peak wavelength is inversely proportional to the temperature. It is called "displacement" because it shows how much the peak is displaced as the temperature changes. It is shown graphically in Figure 1. The peak wavelength has the symbol λ_{max}, although some sources use λ_{peak}. The syllabus uses λ_{max}.

Learning intentions and success criteria

black body
an object that absorbs all radiation falling on it, at all wavelengths; it is a perfect absorber or emitter of radiation

black-body radiation
the radiation emitted by a black body from the conversion of thermal energy, and which has a characteristic frequency distribution that depends on the temperature

Wien's displacement law
states that the black-body radiation curve for different temperatures peaks at a maximum wavelength (λ_{max}) that is inversely proportional to the temperature T (in kelvin, K)

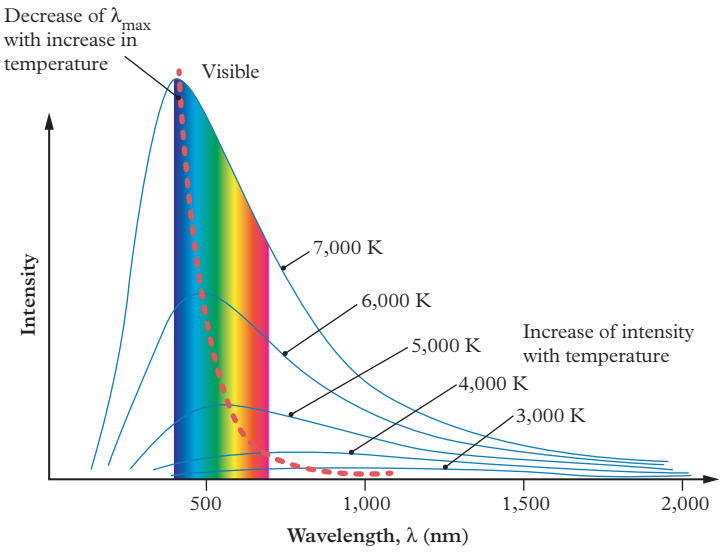


FIGURE 1 Black-body radiation graph, sometimes called Wien's distribution curve, showing the intensity of radiation at different wavelengths for six different temperatures. The dotted line through the middle shows how the maximum wavelength, λ_{max}, changes with temperature according to Wien's displacement law.

Study tip

A good way to visualise the changes in the spectrum of light with temperature is to use the bulb from a raybox kit at voltages from 2 V to 12 V and notice how it goes from yellow to white.

Features of Wien's displacement law

Wien's displacement law has four important features:

1. All black bodies at the same temperature produce the same spectrum; that is, the spectrum depends on the temperature and not on the substance from which the object is made.
2. Making an object hotter produces more radiation (of all wavelengths) across the whole spectrum.
3. The higher the temperature, the shorter the peak wavelength of the spectrum. The peak wavelength corresponds to the wavelength at which the maximum intensity of light is emitted.
4. Visible light is only a small region of the whole emission spectrum, which also includes infrared (IR), and some ultraviolet (UV) if the object is very hot.

Intensity of radiation

Figure 1 shows wavelength (*x*-axis) against intensity (*y*-axis) for black bodies at different temperatures. You learnt previously that "intensity" is defined as "the average rate of flow of energy per unit area". This is just power per square metre ($W\,m^{-2}$). For our discussion, the unit used is quite arbitrary as it is the relative amount of energy we are considering. Physicists often use the related term "spectral radiance", and in the external exam you may see intensity called "luminosity" or "the energy of the photon". They just differ in the way the area enclosing the radiation is measured. For us it all comes under the general name of "intensity".

Unfortunately, the symbol for both intensity and current is *I*, so we need to be clear which one we are using. Current, *I*, will be in ampere (A), milliampere (mA), or microampere (μA). Intensity, *I*, will be in watts per square metre ($W\,m^{-2}$), but generally the unit is not shown.

Formula for Wien's displacement law

As already stated, the peak wavelength is inversely proportional to the temperature of the black body, i.e. $\lambda_{max} \propto \frac{1}{T}$. The equation Wien developed for the displacement law is:

$$\lambda_{max} = \frac{b}{T}$$

where λ_{max} is the peak wavelength in metres (m), T is the temperature in kelvin, and b is a constant of proportionality called Wien's displacement constant, equal to 2.898×10^{-3} m K. Note that the units m K are metre-kelvin, not millikelvin and that the temperature in kelvin, $T_K = T_C + 273$.

Worked example 11.2A

Calculating peak wavelengths using Wien's law

Calculate the peak wavelength in nanometres and its region in the visible spectrum for radiation from the following bodies given their effective surface temperature. Give your answer to an appropriate number of significant figures.

a Sun at 5,800 K (3 marks)

b Earth at 16.0°C (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Identify this as a question about Wien's displacement law. Gather the relevant data from the question and state the appropriate equation.	**a** $\lambda_{max} = \frac{b}{T}$, $T = 5{,}800$ K, $b = 2.898 \times 10^{-3}$ m K
Step 3: Substitute values into the equation. Solve for lambda-max (1 mark for "Provides correct answer in m")	$\lambda_{max(Sun)} = \frac{2.898 \times 10^{-3}\ \text{m K}}{5{,}800\ \text{K}}$ $= 5.00 \times 10^{-7}$ m (1 mark)
Step 4: Convert the answer to nanometres. (1 mark "Converts to nm correctly") Give your answer to an appropriate number of significant figures, in this case 3 s.f.	500 nm (or 5.00×10^{2} nm) (1 mark)
Step 5: Identify the spectral region for 500 nm. (1 mark for "Identifies correct region of the spectrum")	This is in the blue-green region. (1 mark)
Step 6: Convert Earth's temperature in °C to K. (1 mark for "Converts temperature to K correctly")	**b** $T_K = T_C + 273$ $= 16.0 + 273$ $= 289$ K (1 mark)
Step 7: Substitute the known values into the equation. Solve for lambda-max. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides correct answer in nm")	$\lambda_{max(Earth)} = \frac{2.898 \times 10^{-3}\ \text{m K}}{289\ \text{K}}$ $= 1.00 \times 10^{-5}$ m $= 10{,}000$ nm $= 1.00 \times 10^{4}$ nm (3 s.f) (1 mark)
Step 8: Identify the spectral region for 10,000 nm. (1 mark for "Identifies correct region of the spectrum")	This is in the infrared region. (1 mark)

Your turn

a **Calculate** the peak wavelength and spectral region for radiation emitted by a source with a surface temperature of 5,727°C. Give your answer to an appropriate number of significant figures. (3 marks)

b **Determine** the surface temperature in kelvin of an object that emits a peak surface radiation of 414 nm. Give your answer to an appropriate number of significant figures. (3 marks)

How does black-body radiation relate to the colour we see?

Colour change when heating an iron bar

An iron bar at room temperature is emitting light – but not in the visible region of the spectrum. It is mostly in the infrared. When you heat it, it will start with a deep red colour, and as you heat it further it will change to orange, then yellow and so on. We can explain this in terms of black-body radiation, where the iron bar's thermal energy is converted to electromagnetic radiation, and the peak wavelength of radiation is related to the temperature of the iron bar. This is outlined by Wien's law, $\lambda_{max} = \frac{b}{T}$. As the temperature of the iron bar increases, the peak wavelength of its radiation will get shorter, which is seen as a colour change from red to yellow.

Challenge

The colour of the Sun

The Sun has a surface temperature of 5,700 K. This puts its peak wavelength in the blue-green area of the spectrum. So, you may say "well it doesn't look green". This is only the peak wavelength colour. All of the other wavelengths are added in but in lower amounts, and this blends to give it a white colour. Sunlight gets scattered as it comes through the atmosphere. Light from one end of the visible spectrum comes through mostly unscattered, while light from the other end of the visible spectrum gets scattered and causes the sky to look blue. The amount of scattering is proportional to the fourth power of the wavelength.

Propose why the Sun looks yellow from the Earth. (3 marks)

How does black-body radiation relate to the greenhouse effect?

The consequences of the results from Worked example 11.2A are important to our survival. Visible light passes through our atmosphere easily and is not absorbed. It strikes the surface of Earth and is absorbed by the ground and buildings, which causes them to warm. However, the wavelengths re-radiated by Earth's surface and objects on it are in the infrared (IR) part of the spectrum, so are longer than those from the Sun. These infrared waves are strongly absorbed by the atmospheric gases that cause a blanket effect to keep Earth warm – the **greenhouse effect**. An increased concentration of these "greenhouse gases" (mainly CO_2 and methane) causes the atmosphere to retain more heat than it would without human activity, warming the planet and causing climate change.

greenhouse effect the process by which certain gases in the atmosphere, known as greenhouse gases, warm Earth by absorbing the long-wavelength radiation from Earth's surface

Scientists have produced many models to explain the greenhouse effect and how it is intensified by human activity. For example, they look at how Earth's climate has changed in the past to help predict how the climate could change for different increases in global temperature. Their models are tested against the evidence, reviewed and analysed, and modified to find a better fit. The models are based largely on several laws you are now familiar with: Wien's law, and the laws of conservation of mass and conservation of energy.

Incoming radiation

The temperature of the surface of the Sun is about 5,614°C (5,887 K), so it radiates light mostly in the visible spectral region with some in the ultraviolet and infrared. Earth is cooler, so it radiates at longer wavelengths – mainly in the infrared. The amount of energy absorbed by Earth depends on the reflectivity of the clouds and Earth's surface. Snow and sea ice, being white, are good reflectors and contribute to a high reflectivity.

However, if this polar ice melts to form sea water or to expose the underlying rocks, the reflectivity drops and the darker surfaces absorb more radiation, leading to greater heating.

Outgoing radiation

For Earth to maintain a constant temperature, the outgoing radiant energy must be the same as the incoming radiant energy. This doesn't have to be at the same wavelength, as long as the total energies are equal. Very little of the radiation from Earth's surface goes directly back to outer space (Figure 2). Most of it is absorbed by gases in the atmosphere such as carbon dioxide (CO_2), methane (CH_4), water (H_2O), and nitrogen oxides (NO_x). These are the most commonly known and most abundant greenhouse gases. These gases occupy a layer about 5 km from Earth's surface, but at this altitude they are cooler than the surface and don't radiate as much. You can assume that the atmosphere is now like a black-body emitter that radiates some of the absorbed radiation out into space. However, the gases emit in all directions and much of the emitted radiation finds its way back to Earth's surface, which is warmed to an average of about 15°C (288 K). Earth's average temperature of +15°C is because of the greenhouse gas effect; without it the temperature would be −18°C.

FIGURE 2 Radiation pathways on Earth. Sunlight (yellow) is partially reflected by clouds and Earth's surface (mainly ice and snow). The rest heats the surface. Infrared radiation (red) is emitted from Earth. Some goes directly into space, but the rest is absorbed by the atmosphere and re-emitted either out to space or back to Earth to further warm the surface.

How are greenhouse gas levels changing?

The levels of greenhouse gases in the atmosphere are affected by human activity, mostly from burning fossil fuels. Human production of greenhouse gases since the Industrial Revolution has increased the amount of energy retained by the atmosphere, which makes Earth hotter than it otherwise would have been. This is called the enhanced greenhouse effect, and it leads to changes in Earth's climate called anthropogenic climate change.

Scientists make climate models and test them against historical data such as temperature and rainfall. If the models are successful, climate scientists can then use them to make predictions about the future climate. Tens of thousands of scientists work in collaboration to examine how well models fit past and current data, and look for any anomalies the model can't explain. There are always disputes, but the fundamental principles of the enhanced greenhouse model are overwhelmingly accepted by the scientific community.

Skill drill

Developing a research question

Science inquiry skills: Conducting your research investigation: (Lesson 1.12)

Just like in the student experiment, you will be expected to write a research question to direct your research investigation. There is no such thing as a "correct" or "incorrect" research question. For every claim, there are many possible research questions. However, your research question needs to follow a specific structure to meet the assessment criteria (ISMG).

- Relevant: is the research question appropriate to the claim and/or topics covered in the unit?
- Specific: is the research question clearly defined?
- Question format: is it written in question format?

Practise your skills

1 A claim has been made that "using the concept of black-body radiation to study Earth's energy balance enables scientists to monitor changes in global temperature". **Propose** whether a research question would need to mention water vapour and carbon dioxide given the evidence for changes in climate could be due to the enhanced greenhouse effect and anthropogenic climate change. (2 marks)

2 A claim stated above is "climate change can be modelled using black-body radiation". A teacher has given their students a published example of a research question developed from the claim: "Is there a relationship between the atmospheric concentration of water vapour and carbon dioxide greenhouse gases and the rate of global warming due to an increased absorption of black-body radiation?" **Evaluate** (make an appraisal by weighing up or assessing strengths, implications and limitations) whether the question is appropriate to the claim. (2 marks)

3 Another student has taken the same claim and proposed another research question that is supposed to be unique: "Is there a relationship between the atmospheric concentration of carbon dioxide greenhouse gas and the rate of global warming due to an increased absorption of black-body radiation?" **Evaluate** whether the question is appropriate to the claim and is sufficiently different to the published research question. (2 marks)

Study tip

Black-body radiation and the greenhouse effect are good concepts to explore in the research investigation as there are many claims made in relation to these concepts and a great deal of secondary evidence in the literature available to help students evaluate their claim.

Challenge

Maximum intensity

The temperature of a radiating object is increased. **Identify** how the maximum intensity and peak wavelength changes. (2 marks)

Challenge

Black light

Ultraviolet lights used at parties are called "black lights". **Explain** why they are given this name. (1 mark)

FIGURE 3 Person wearing ultraviolet paint illuminated by ultraviolet lights at a party

Check your learning 11.2

Check your learning 11.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how the frequency of light is related to temperature. (2 marks)

2 **Describe** the way in which Wien's law shows an inverse relationship. (1 mark)

3 **Explain** what is meant by the claim that a black body is a perfect absorber and emitter. (2 marks)

Analytical processes

4 The Moon has an average temperature 100°C during its daytime and –173°C during its night. **Determine** the peak wavelength and spectral regions of the radiation in the daytime and at night. (3 marks)

5 **Determine** the peak wavelength for a steel horseshoe that has been heated to 350°C. (2 marks)

6 **Determine** the temperature in degrees Celsius that would produce radiation with a peak wavelength of 1.0 μm (micro, $\mu = 10^{-6}$). (2 marks)

Knowledge utilisation

7 Earth radiates in the infrared, which is invisible to our eyes. **Propose** how it is that we can see Earth from space. (1 mark)

8 Radiation produced 180,000 years after the Big Bang has been detected by University of Arizona physicists on a spectrometer in outback Western Australia. It indicated the temperature of space at that time was just 1.5 K.

a **Determine** the peak wavelength of the light, expressed in cm. (2 marks)

b **Propose** how this energy gets to Earth when it is so far away. (2 marks)

Lesson 11.3
The particle model of light

Key ideas

- → Max Planck postulated the quantum nature of energy by suggesting radiation is absorbed and released by atoms in discrete packets or quanta.
- → Electromagnetic energy quanta are called photons. Their energy is directly related to their frequency: $E = hf = \frac{hc}{\lambda}$.

Learning intentions and success criteria

What is the ultraviolet catastrophe?

Wien's equation and the shapes of his graphs were accurate for a wide range of temperatures but ran into two major problems. One was that classical physics could not explain the shape of the graphs. Another was that it didn't work well for low temperatures, where the radiation had long wavelengths and low frequencies.

Other physicists, namely Rayleigh and Jeans from England, developed equations based on Maxwell's electromagnetic theories. Their formula $I \sim \frac{1}{\lambda^4}$ predicted that at very high temperatures (and hence very small wavelengths), a black body should emit enormous amounts of ultraviolet light, X-rays and gamma rays – but in practice it did not. According to their model, the total energy of these high-energy waves would be greater than the energy used to make them. This is clearly impossible, and the failure of classical physics to explain the lack of ultraviolet light, X-rays and gamma rays at high temperatures was described as the "ultraviolet catastrophe".

Wien's equation didn't work well for long wavelengths (low frequencies) and the Rayleigh–Jeans equations didn't work for short wavelengths (high frequencies). The equations were not valid at the extremes, and the wave model was no longer a full description of how light behaves. A new model was needed.

What are Planck's quanta?

The German physicist Max Planck (1858–1947) combined half of each equation to cobble together an equation that worked for the whole range of wavelengths and could describe black-body distribution. In doing so, Planck devised a revolutionary theory of subatomic matter. He proposed that the energy released by a black body was, in fact, emitted by atoms, and that these atoms could only vibrate at certain frequencies that were multiples of a smallest value. He had to assume that the energy released by the atoms was not given off continuously, but in small energy packets that he called **quanta** (singular quantum), from the Latin "quantus", meaning "how much".

quanta
the smallest discrete packets of energy of electromagnetic waves, also later known as photons; singular "quantum"

Planck found that each frequency, f, of light emitted by the atoms is proportional to the change in energy of the atom. For example, as violet light is twice the frequency of red light, the energy quanta of violet light are twice the size of those of red light. He didn't know why this was, but his predictions were in perfect agreement with experimental results. He didn't believe that radiant energy was transferred in these quantum packets and held on to the belief that light consisted of waves in the classical sense.

The difference in the Planck, Wien and Rayleigh–Jeans models is shown in Figure 1. Experimental results are shown as black markers, and these are in perfect agreement with Planck's law (solid line). Wien's law (dotted line) is consistent with results at low wavelengths but fails at high wavelengths. The classical (wave) model by Rayleigh and Jeans is good at long wavelengths (low frequencies) but fails in the UV region, where it predicts the emission of huge amounts of energy.

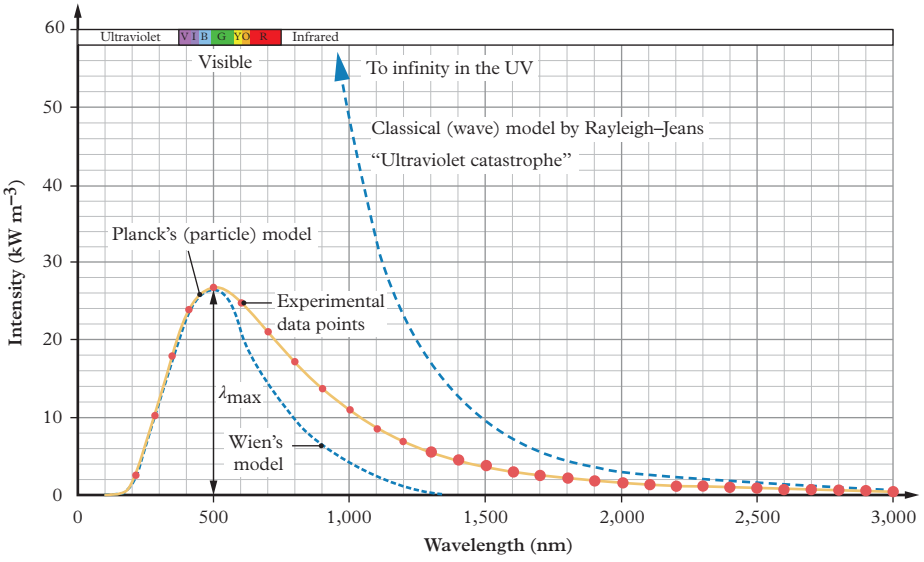


FIGURE 1 Black-body radiation curve showing the intensity of light radiation emitted over the spectrum from wavelengths of 0 nm up to 3,000 nm

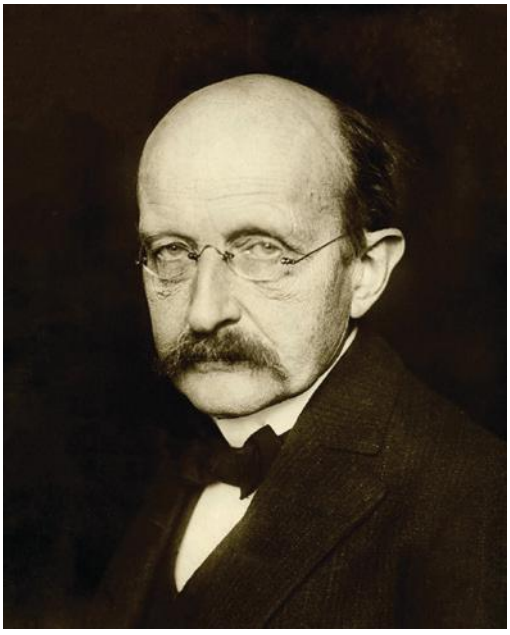


FIGURE 2 Max Planck

What is Planck's constant?

Planck devised an equation that related the energy of light quanta with their frequency. This is now called **Planck's equation**, $E = hf$, where E is the energy of the light quanta (J), h is a constant called **Planck's constant** (6.626×10^{-34} Js) and f is the frequency of the light (Hz).

Planck's equation
a relationship that relates frequency to energy: $E = hf$. It accounts for the quantised nature of light and plays a key role in understanding phenomena such as the photoelectric effect and Planck's law of black-body radiation.

Planck's constant
a fundamental constant used in quantum mechanics that relates frequency to energy (symbol: h; SI unit: joule second; unit symbol: Js), equal to 6.626×10^{-34} Js. Also known as the Planck constant.

photon
a quantum of all forms of electromagnetic radiation

These quanta were later called **photons**. To help you understand this we will also refer to them as photons even though, historically, they weren't named that until 25 years later.

We can rearrange the wave equation for light, $c = f\lambda$, to make f the subject: $f = \frac{c}{\lambda}$, where c is the velocity of light ($3 \times 10^8\,\text{m s}^{-1}$), f is frequency (Hz), and λ is wavelength (m). Substituting this into Planck's equation means energy can be calculated using either frequency or wavelength:

$$E = hf = \frac{hc}{\lambda}$$

This equation is in your QCAA Formula and data book.

quantised
formed into quanta with certain discrete energies

discrete
individually separate and distinct

Planck's interpretation of black-body radiation provided evidence that electromagnetic radiation is **quantised** into **discrete** values.

As Planck's constant is extremely small, energy quanta (photons) are not noticeable in most everyday circumstances. A typical light source such as a ceiling light releases billions of billions (10^{18}) of photons per second.

With this idea in place, Planck was able to describe the reason for the absence of high-energy emissions from black bodies. The vibrating atoms were simply not large enough to provide the necessary energy changes. He found that certain states of vibration of the atoms were more likely, and this accounted for the peak in the frequency distribution curves.

As you will see later, Planck's idea that the whole atom vibrates is, in fact, not quite correct. Energy emissions are due to electron movements (transitions) within the atom. Quantum theory today shows that electrons in atoms can only move between defined energy levels within the atom. Planck himself did not have any evidence for energy quanta, but it was an excellent idea that perfectly described solutions to several problems in physics at the time. Not many physicists accepted Planck's interpretation as it didn't relate to classical physics. However, Albert Einstein did accept it and used it as the basis for a revolution in physics. Planck's work is regarded as the beginning of modern quantum physics and earned him the 1918 Nobel Prize in Physics.

Study tip

This will give you a feeling for the energy of a photon. The number of photons streaming from a fluorescent lamp in your lab is about 10^{18} per second. However, the dark-adapted eye (stand in complete darkness for a few minutes) can see a single photon.

Challenge

Energy per photon

Determine which laser light contains more energy per photon: a red laser ($\lambda = 635\,\text{nm}$) or a blue laser ($\lambda = 442\,\text{nm}$). (2 marks)

What is the electron volt?

The joule as a unit of energy is useful in thermodynamics, where thousands of joules are needed to heat a beaker of water, or megajoules are needed to boil water. But for binding energy in nuclear physics and black-body radiation in quantum mechanics, the values are very small, such as $10^{-19}\,\text{J}$, and it becomes tedious to have to talk in those amounts. Physicists introduced a new unit for energy – the **electron volt (eV)** – which is the energy needed to accelerate an electron through an electrical potential difference of 1 volt. The formula for the work done, W, in moving a charge q across a potential difference V, $W = qV$, tells you how much energy that is. The charge on an electron is $1.60 \times 10^{-19}\,\text{C}$, so:

$$W = 1.60 \times 10^{-19} \times 1 = 1.60 \times 10^{-19}\,\text{J}$$

$$1 \text{ electron volt (eV)} = 1.60 \times 10^{-19}\,\text{J}$$

electron volt (eV)
a unit of energy equal to the work done in accelerating an electron through an electrical potential difference of 1 V.
$1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$.
(This value is in your QCAA Formula and data book)

The prefixes kilo, k ($1\,\text{keV} = 10^3\,\text{eV}$), and mega, M ($1\,\text{MeV} = 10^6\,\text{eV}$), are commonly used.

Worked example 11.3A

Calculating the energy of a photon

Calculate the energy, in J and eV, of a quantum (photon) of violet electromagnetic radiation of wavelength 450 nm. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and state the appropriate formula.	$E = \frac{hc}{\lambda}$, $\lambda = 450\,\text{nm} = 4.50 \times 10^{-7}\,\text{m}$
Step 3: Substitute the known values into the equation and solve for E. (1 mark for "Correctly calculates energy in J")	$E = \frac{hc}{\lambda}$ $= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{450 \times 10^{-9}}$ $= 4.417 \times 10^{-19}\,\text{J}$ (1 mark)
Step 4: Convert J to eV by dividing by the magnitude of the charge on an electron, $1.6 \times 10^{-19}\,\text{J eV}^{-1}$. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "correctly calculates value in eV")	$E = \frac{4.417 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-19}\,\text{J eV}^{-1}}$ $= 2.8\,\text{eV}$ (2 s.f.) (1 mark)

Your turn

Calculate the energy, in J and eV, of a quantum (photon) of violet electromagnetic radiation of wavelength 750 nm. Give your answer to an appropriate number of significant figures. (2 marks)

Check your learning 11.3

Check your learning 11.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Describe** the problems with Wien's theory of black-body radiation and **explain** how Planck's model overcame these. (2 marks)

2 **Clarify** what experimental evidence Planck had for the existence of quanta (photons). (1 mark)

Analytical processes

3 **Determine** the energy in joules of a 2.49 eV quantum (photon) of green light. (1 mark)

4 **Determine** the energy in both joules and eV for a single quantum (photon) of green light of wavelength 500 nm. (2 marks)

5 A particular light wave has a frequency of $4.3 \times 1{,}014$ Hz.

- **a** **Determine** its wavelength. (1 mark)
- **b** **Determine** the energy of one quantum (photon) of this light. (1 mark)
- **c** **Deduce** what colour it would appear to our eyes. (1 mark)

6 Using Planck's equation, **determine**

- **a** the energy in eV of a quantum (photon) of light of wavelength 750 nm (2 marks)
- **b** the wavelength of X-ray quanta of energy 3 keV. (2 marks)

7 **Deduce** whether red or blue light has more energetic quanta (photons). (2 marks)

Knowledge utilisation

8 **Predict** the spectral region (e.g. ultraviolet, gamma) to which the following quanta belong.

- **a** 0.50 eV (1 mark)
- **b** 100 eV (1 mark)
- **c** 20 keV (1 mark)

9 **Decide** whether the "ultraviolet catastrophe" was evidence in favour of the wave model or the quantum model of light. (2 marks)

Lesson 11.4
The photoelectric effect

Key ideas

- The photoelectric effect provides evidence for the particle model of light.
- Light shone on a metal surface will cause electrons to be emitted as a photocurrent if the light is above the threshold frequency. Below that frequency, no electrons are emitted for any intensity of light. Above the threshold frequency, the photocurrent is proportional to the intensity of the light and electrons are emitted almost immediately.
- A graph of the maximum kinetic energy of the emitted electrons, $E_{k(max)}$ (in J), versus light frequency, f, can be interpreted as follows:
  - the gradient is equal to Planck's constant, h
  - the y-intercept is equal in magnitude to the work function, W (the minimum energy needed to remove an electron from the metal)
  - the x-intercept is the threshold frequency, f_0.
- Light exhibits properties of both waves and particles, called wave–particle duality:
  - diffraction, Young's double-slit experiment and polarisation support the wave model
  - the photoelectric effect, photon momentum and Compton scattering support the particle model.

Learning intentions and success criteria

What is the photoelectric effect?

In the 1880s, the German physicist Heinrich Hertz (1857–94) attempted to verify an aspect of Maxwell's proposals on electromagnetism. He was trying to produce electromagnetic waves that would travel through space to the other side of the lab and induce a spark in an induction coil. Not only did Hertz get sparks, but he also noticed that, when he used ultraviolet light, the sparks in the induction coil got bigger. He called this phenomenon the **photoelectric effect**. However, as the structure of the atom was not known at the time, the explanation was just that the energy originates inside the atom.

photoelectric effect
the emission of electrons (or other free carriers) when light shines on a material

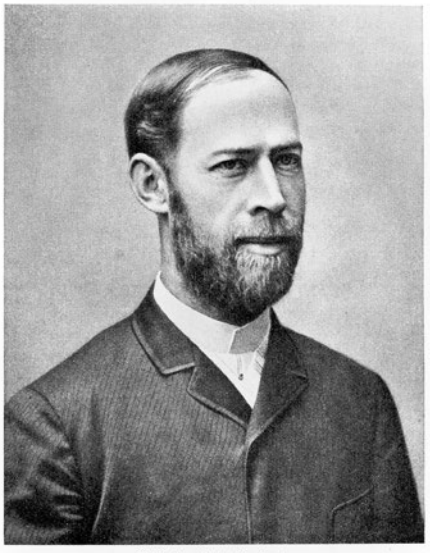


FIGURE 1 Heinrich Hertz 1894

What do photoelectric experiments show?

A simple demonstration to physics students at the time of Hertz's experiments was to shine different lights on a negatively charged electroscope that had a square of zinc metal sitting on the top (Figure 2). With ordinary incandescent light from a light bulb there was no change, but when ultraviolet light was shone on the electroscope the leaves collapsed, indicating that the zinc was losing electrons and these were being pulled from the electroscope.

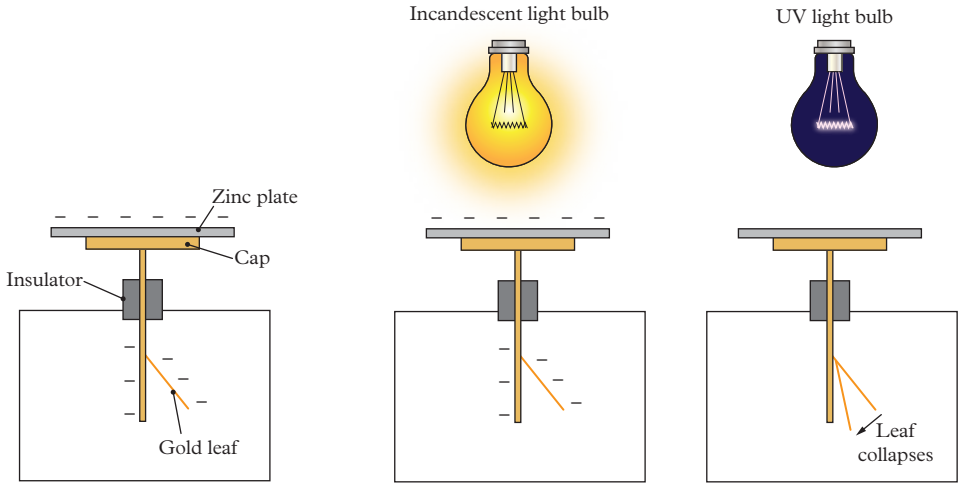


FIGURE 2 Typical demonstration of the photoelectric effect. (A) The electroscope is negatively charged with the gold leaf extended. (B) In the presence of an incandescent bulb there is no change. (C) When a UV light is introduced, the gold leaf collapses, indicating loss of the negative charge.

Study tip

Try the demonstration in Figure 2 to help you appreciate the meaning of the photoelectric effect. UV bulbs are sold as UV aquarium lights or UV LED fishing black torches. Make sure you use a clean piece of zinc on top.

In 1902, one of Hertz's students, Philipp Lenard (1862–1947), constructed a glass tube with a metal plate at each end (Figure 3). He removed the air to create an evacuated chamber so there was nothing, not even air, between the two metal plates, and connected a battery. The metal plate on the left was called the emitter or **cathode**, and the rays that were emitted from it were called "cathode rays" – now known to be electrons. The metal plate on the right was called the collector or **anode**. The tube itself was called a cathode-ray tube. No electron current was registered on the ammeter, as would be expected, since a vacuum is a poor conductor.

cathode
the electrode from which electrons are emitted

anode
the electrode that attracts electrons

The three variables that Lenard could change were:

- the frequency of the incident light
- the intensity of the incident light
- the voltage between the cathode (emitter) and anode (collector).

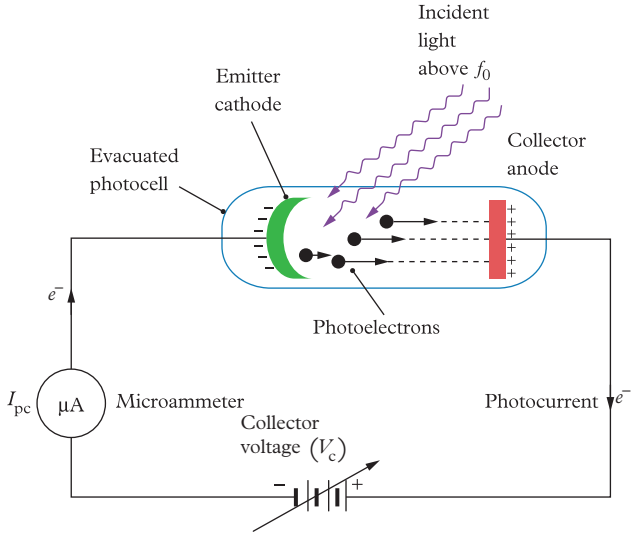


FIGURE 3 Lenard's photoelectric apparatus in forward bias (anode is positive)

Experiment 1: Forward bias

In his first set of experiments, Lenard connected the tube to the battery in what we call "forward bias". That is, the anode is connected to the positive terminal of the battery and the cathode is connected to the negative side. We say that the anode is positive with respect to the cathode. Lenard shone light on the metal cathode in the tube and slowly increased the frequency. Nothing happened until the frequency reached the ultraviolet region, then instantly a current started to flow and registered on the microammeter. This current, I_{pc}, was called a **photocurrent**. The ultraviolet light had caused cathode rays (electrons) to be ejected from the metal into the tube. These electrons became known as **photoelectrons**. As the anode was positive, the electrons were attracted to it and thus completed the circuit. Lenard called this frequency the **threshold frequency**, f_0. This is shown in Figure 4A.

photocurrent
an electric current produced by the photoelectric effect

photoelectron
electron emitted from an atom by interaction with a photon, especially from a solid surface by the action of light

threshold frequency
the minimum frequency of a photon that can eject an electron from a surface

Lenard then repeated the experiment to determine the effect of intensity on the current. He kept the tube in forward bias and held the frequency of incident light constant. A photocurrent, I_{pc}, flowed. He then increased the intensity (brightness) of the incident light. He found that the more intense the light, the bigger the current. In fact, the current was proportional to the brightness. This is shown in Figure 4B.

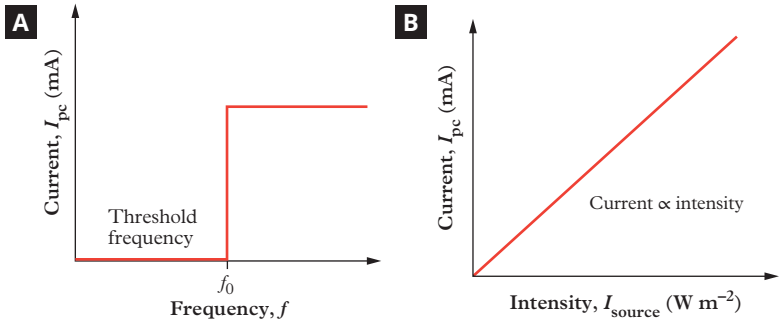


FIGURE 4 The results of Lenard's first set of experiments. Lenard found that (A) no current flowed until the frequency of the light reached a certain "threshold" value, f_0, and (B) when current flowed it was proportional to intensity.

Study tip

Photoelectrons and photons are different particles. They are often confused in students' answers, so be careful, especially when you're under exam pressure. Sometimes we also see them referred to as protons or positrons in exam answers.

Study tip

If you study chemistry as well as physics, it's easy to get the terms anode and cathode mixed up. In chemistry, in a voltaic (galvanic) cell the anode is negative, and the cathode is positive. However, in an electrolytic cell, the anode is positive and the cathode is negative. Keep the chemistry and physics definitions separate. Remember – in physics the cathode and anode can be made either positive or negative.

Summary of findings from Experiment 1

1. **Incident light needed to be above the threshold frequency.** Electrons were ejected from the metal only if the frequency of the incident light exceeded a minimum value called the threshold frequency, f_0.
2. **Electrons were ejected instantly.** There was almost no time delay between the light striking the surface and electrons being ejected.
3. **Intensity had no effect if the frequency was too low.** If the frequency of light was below the threshold frequency, electrons were not ejected and there was no flow of photocurrent, even when very intense light was used.
4. **Intensity and current are proportional.** Once a photocurrent was registered, increasing the intensity of the incident light increased the photocurrent proportionally.

Experiment 2: Reverse bias

Lenard then experimented with the voltage. He wanted to measure the kinetic energy of the emitted electrons and did this by reversing the battery connections of the cathode (emitter) and anode (collector). Previously he had the anode positive to attract any emitted photoelectrons.

With the battery reversed (Figure 5) the anode was negative with respect to the cathode. When light struck the cathode, electrons were emitted into the tube as usual. But with the anode now negative, these electrons were repelled. Only the most energetic electrons could make it across the gap from the cathode to the anode. Lenard made the anode more and more negative and noticed that the current decreased. At a certain reverse voltage, the current reached zero. This reverse voltage was now large enough to stop even the most energetic electrons from crossing the gap, stopping the photocurrent completely. This voltage was a direct measure of the maximum kinetic energy, $E_{k(max)}$, of the electrons. He called this the **stopping potential**, V_s. It is also called the "cut-off" potential. It didn't matter if he used high-intensity (bright) light or low-intensity (dim) light – the stopping potential was the same for the same frequency of incident light. This behaviour is shown in Figure 6A.

stopping potential the negative potential on the collector at which the photoelectric current becomes zero. Symbol V_s, but sometimes shown as V_0, or V_{stop}.

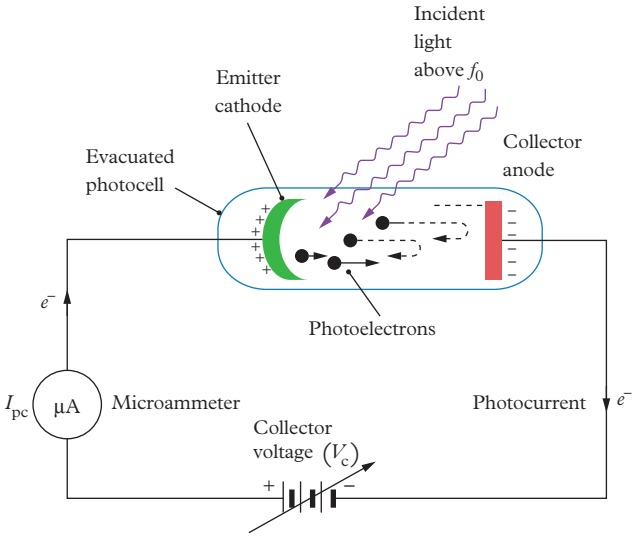


FIGURE 5 Lenard's photoelectric apparatus in reverse bias with the anode (collector) made negative by the battery

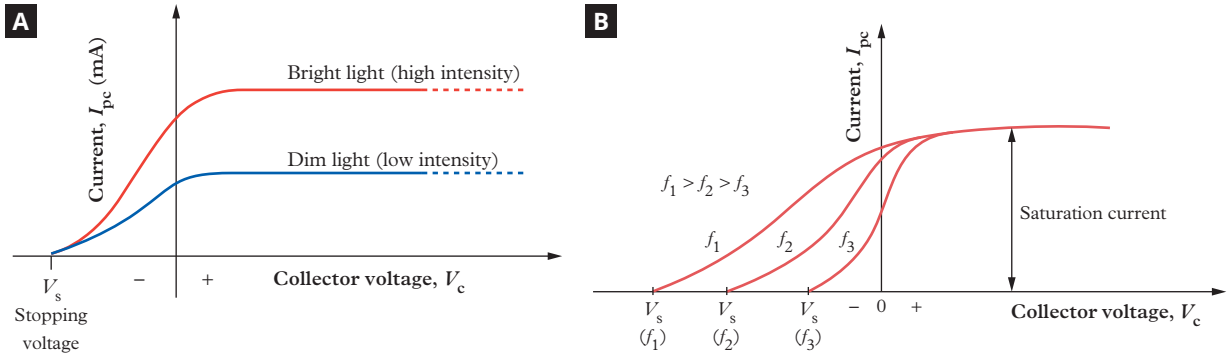


FIGURE 6 Results of Lenard's photoelectric experiment. (A) The stopping potential for a metal depends on the frequency, not the intensity. It is a negative value, but usually just the magnitude is quoted. (B) For a given metal, the higher the frequency, the greater the stopping potential.

Lastly, Lenard measured the stopping potential for light of different frequencies. He found that the higher the frequency of light, the greater the stopping potential required. This meant that electrons emitted using high frequency light had greater maximum kinetic energy than electrons emitted using lower frequency light. In fact, the maximum kinetic energy of the emitted electrons was proportional to the frequency of the incident light. This is shown in Figure 6B.

Summary of findings from Experiment 2

For a given metal:

1 stopping potential is independent of intensity

2 stopping potential is proportional to frequency.

Lenard's experimental observations were impossible to explain using conventional wave theories of light. A new model based on the idea of quanta (photons) was developed by Planck and given a sound theoretical basis by Einstein. Table 1 shows how experiments on the photoelectric effect provide evidence of the quantised nature of photons.

TABLE 1 Summary of evidence provided by experiments on the photoelectric effect for the quantised nature of photons.

Observations of the photoelectric effect	Does evidence support classical (wave) theory?	Does evidence support Planck's/Einstein's quantisation of photons?
1 Electrons are ejected instantly.	No. Wave theory wrongly predicts a delay of hours at low intensity as energy builds up.	Yes. If the photon has enough energy, it transfers it to the electron instantly.
2 The number of electrons (current) is proportional to intensity of the incident light above threshold frequency.	No. Wave theory wrongly says greater intensity light has greater energy (amplitude), and that more electrons should be emitted at any frequency.	Yes. Greater intensity means a greater number of photons, so more electrons should be emitted, as long as the frequency is above f_0.
3 Light with a frequency less than the threshold value will not eject electrons, no matter how great its intensity.	No. Wave theory wrongly says frequency is irrelevant because if you shine light of any frequency on the metal for long enough, enough energy will build up to eject the electron. Under the wave model, it would just take longer for low-intensity light to eject electrons.	Yes. The energy of the photon equals hf, and if it is below this value, no electrons will be emitted, no matter how great the intensity of light.
4 The kinetic energy of ejected electrons is proportional to the frequency of light and not the intensity.	No. Wave theory wrongly says the electrons' maximum kinetic energy depends only on intensity, not on frequency.	Yes. Intensity has no effect on the kinetic energy of the electrons. Doubling the intensity just doubles the number of photons, not their kinetic energy.

Stopping potential and kinetic energy

You have seen how the stopping potential is considered a measure of the kinetic energy of the electrons. The relationship between the two quantities is based on the formula for potential (voltage) $V = \frac{W}{q}$, where W is the work done on the electron by the electric field and q is the charge on the particle (the electron). An electron has one elementary charge, so $q_e = 1.60 \times 10^{-19}$ J. The work done causes the electron to acquire an equal amount of kinetic energy according to the law of conservation of energy. Thus:

$$W = q_e V$$

$$W = E_{k(max)} = \frac{1}{2} m v^2$$

Worked example 11.4A

Calculating kinetic energy from stopping potential

The stopping potential required to prevent current through a photocell is 3.2 V. For the photoelectrons being emitted, **calculate** (to an appropriate number of significant figures) their maximum

a kinetic energy in joules (2 marks)

b kinetic energy in electron volts (eV) (2 marks)

c velocity in m s^{-1}. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to find a numerical answer using mathematical processes and showing working. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and state the appropriate formulas.	$W = q_e V$ $W = E_{k(max)} = \frac{1}{2}mv^2$ $q_e = 1.6 \times 10^{-19}\,\text{C}$ $m_e = 9.109 \times 10^{-31}\,\text{kg}$ $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$
Step 3: To find the maximum kinetic energy in joules, multiply the stopping potential, V_0, by the elementary charge, q_e. Make sure you use the stopping potential in volts and the elementary charge in coulombs. Rearrange the relevant formulas and substitute in the known values to solve for kinetic energy. (1 mark for "Substitutes correctly into the equation")	**a** $E_k = q_e V$ $= 1.60 \times 10^{-19} \times 3.2$ (1 mark) $= 5.12 \times 10^{-19}$
Step 4: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Check you have used the correct units. (1 mark for "Provides correct answer")	$E_k = 5.1 \times 10^{-19}\,\text{J}$ (1 mark)
Step 5: Convert joules to electron volts by dividing J by $1.6 \times 10^{-19}\,\text{J eV}^{-1}$. (1 mark for "Substitutes correctly into the equation")	**b** $E_{k(max)} = 5.12 \times 10^{-19}\,\text{J}$ $= \frac{5.1 \times 10^{-19}\,\text{J}}{1.60 \times 10^{-19}\,\text{J eV}^{-1}}$ $= 3.2\,\text{eV}$ (1 mark)
Step 6: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Check you have used the correct units. Note that this is the stopping potential value stated in eV. (1 mark for "Provides correct answer")	$E_k = 3.2\,\text{eV}$ (1 mark)
Step 7: To find the velocity, use the kinetic energy formula with the mass of the electron in kg. (1 mark for "Substitutes correctly into the equation")	**c** $E_{k(max)} = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2E_{k(max)}}{m_e}}$ (1 mark) $= \sqrt{\frac{2 \times 5.12 \times 10^{-19}}{9.109 \times 10^{-31}}}$ $= 1.06 \times 10^6\,\text{m s}^{-1}$
Step 8: Give your answer to an appropriate number of significant figures, in this case 2 s.f. Check you have used the correct units. (1 mark for "Provides answer")	$v = 1.1 \times 10^6\,\text{m s}^{-1}$ (1 mark)

Your turn

The stopping potential required to prevent current through a photocell is 2.85 V. For the photoelectrons being emitted, calculate, to an appropriate number of significant figures, their maximum

a kinetic energy in J (2 marks)

b kinetic energy in electron volts (eV) (2 marks)

c velocity in m s^{-1}. (2 marks)

How did Einstein explain the photoelectric effect?

A youthful Albert Einstein wondered how it could be that light, which was considered a wave, could interact with an atom that exists at only a point. He took the quantum idea of Planck seriously, although other physicists continued to defend the classical wave model. His thoughts along these lines culminated in his famous paper of 1905. In it, Einstein made one of the most revolutionary statements in the history of physics:

> "…the energy…consists of a finite number of energy quanta which…can only be produced and absorbed as complete units".

As mentioned earlier, energy quanta eventually became known as photons. Einstein predicted that his light quanta each had energy that was a product of the frequency, f, and Planck's constant, h.

How are the energies of the photon and the electron related?

Einstein assumed that the light quanta, now called photons, interacted with the surface electrons in the metal so that a single photon ejects a single electron. The photon will give either all of its energy to the electron or none of it. Each electron can only absorb the energy of one photon. The collision interactions between photons and electrons in the metal are totally elastic and obey the law of conservation of energy (Figure 7). Einstein defined three forms of energy in the system:

- Photon energy, $E = hf$, which is frequency dependent
- **Work function** or energy required to unbind the electron from the metal, $W = hf_0$, where f_0 is the threshold frequency. This is the energy of a photon at the threshold frequency.
- Maximum kinetic energy of the electron, $E_{k(max)}$, of the ejected electrons from the metal surface. Electrons emitted from the surface of the metal will have the maximum kinetic energy. Electrons in the metal that are not at the surface will need a little more energy to bring them to the surface. In this case they will have lower kinetic energies when emitted. You can see that there is a range of kinetic energies, with the maximum for electrons close to the surface. Recall that these ejected electrons are called photoelectrons.

work function
the minimum energy required to remove an electron from a solid (symbol: W; SI unit: joule; unit symbol: J). Formula: $W = hf_0$

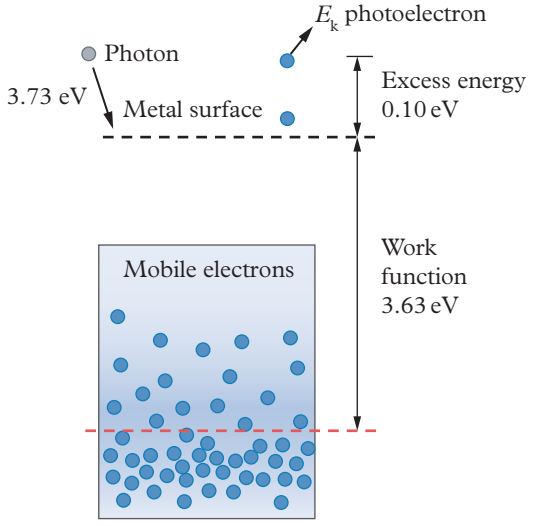


FIGURE 7 Work function and excess energy. An incoming photon with 3.73 eV of energy transfers it to the metal surface. The work function is 3.63 eV, so 3.63 eV is needed to free the electron from the metal. The excess 0.10 eV is transformed into kinetic energy of the emitted photoelectron.

TABLE 2 Work functions for some common metals

Metal	W (eV)
Rubidium	2.05
Sodium	2.36
Calcium	2.87
Zinc	3.63
Aluminium	4.28
Copper	4.64
Gold	5.10
Platinum	5.63

Challenge

Origin of the name photon

The photon was named by US physicist Gilbert Lewis in 1926 using the Greek "phos", meaning "light". **Identify** whether the following statements are true.

a All words beginning with the prefix "phos" are related to the concept of light. (1 mark)

b The planet Venus used to be called Phosphor when appearing as the morning star. (1 mark)

How do photon energy and work function determine if an electron is released?

Whether an electron is released or not depends on whether the incoming photon has more energy than the work function. Let's consider the case of zinc, which has a work function of 3.63 eV. That means light with a photon energy less than 3.63 eV will not eject an electron. But a photon energy greater than 3.63 eV will eject an electron. The calculations for this follow.

Case 1: No photoelectron released

If the incident photon has less energy than the work function, then no photoelectron is released. For example, if zinc is illuminated by violet light of frequency 7×10^{14} Hz, the energy of the violet photon is given by:

$$\begin{aligned} E &= hf \\ &= 6.626 \times 10^{-34} \times 7 \times 10^{14} \\ &= 4.64 \times 10^{-19}\ \text{J} \end{aligned}$$

Convert this to electron volts (eV):

$$\begin{aligned} E(\text{eV}) &= \frac{E(\text{J})}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} \\ &= \frac{4.64 \times 10^{-19}}{1.6 \times 10^{-19}} \\ &= 2.90\ \text{eV} \end{aligned}$$

This is less than the work function of zinc, 3.63 eV, so no photoelectron will be ejected.

Case 2: Photoelectron is released

If we illuminate zinc with ultraviolet light, for example of frequency 9×10^{14} Hz, the energy of the incident photon is:

$$\begin{aligned} E &= hf \\ &= 6.626 \times 10^{-34} \times 9 \times 10^{14} \\ &= 5.96 \times 10^{-19}\ \text{J} \\ &= 3.73\ \text{eV} \end{aligned}$$

This is greater than the work function so a photoelectron will be ejected. The excess energy (3.73 eV – 3.63 eV = 0.10 eV) will be transformed into the kinetic energy of the photoelectron.

Worked example 11.4B

Determining if frequency is high enough for ejection

Ultraviolet light with a frequency of 1.5×10^{15} Hz is shone on the surface of aluminium metal, which has a work function of 4.28 eV. **Determine** whether a photoelectron will be ejected. Show your working. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you.	"Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formula.	$E = hf$, $f = 1.5 \times 10^{15}$ Hz, W(Al) = 4.28 eV
Step 3: To determine whether the photon has enough energy, we first need to calculate the photon energy. (1 mark for "Calculates the energy in J")	$E = hf$ $= 6.626 \times 10^{-34} \times 1.5 \times 10^{15}$ $= 9.94 \times 10^{-19}$ J (1 mark)
Step 4: To compare this with the work function in eV, we need to convert the photon energy to eV. (1 mark for "Correctly converts J to eV")	$E = \frac{9.94 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 6.21$ eV (1 mark)
Step 5: If the calculated value is higher than the work function of aluminium, a photoelectron will be ejected. (1 mark for "Justifies the conclusion by comparing eV values")	The energy of the photon, 6.21 eV, is greater than the work function of aluminium, 4.28 eV, so a photoelectron will be ejected. (1 mark)
Step 6: An elaboration is not required but is added here for completeness.	The photoelectron will have an excess energy of 6.21 eV – 4.28 eV = 1.93 eV, which it will carry away as kinetic energy.

Your turn

Ultraviolet light with a frequency of 12×10^{14} Hz is shone on the surface of copper metal, which has a work function of 4.64 eV. **Determine** whether a photoelectron will be ejected. Show your working. (3 marks)

Does each metal have its own threshold frequency?

The threshold frequency for metals is in the order of 1,014 Hz to 1,015 Hz. This corresponds to wavelengths in the range of 10^{-6} m to 10^{-7} m or 100 nm to 1,000 nm. It's handy to recall to check your answers are the right magnitude.

Worked example 11.4C

Determining work function from threshold frequency

The threshold frequency for a metal illuminated by violet light is 6.93×10^{14} Hz.

a **Determine** the work function, W. (2 marks)

b **Identify** the likely metal from the values in Table 2. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Determine" means to establish after a calculation. "Identify" means to recognise and state. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formula.	$f_0 = 6.93 \times 1014$ Hz, $E = hf_0$

Think	Do
Step 3: Substitute the known values into the formula and make the calculation. (1 mark for "Correctly substitutes values"; 1 mark for "Calculates the work function in J or eV")	**a** $W = hf_0$ $= 6.626 \times 10^{-34} \times 6.93 \times 10^{14}$ (1 mark) $= 4.59 \times 10^{-19}\,\text{J}$ $= \frac{4.59 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-19}\,\text{J eV}^{-1}}$ $= 2.87\,\text{eV}$ (1 mark)
Step 4: If the work function is still in J, convert it to eV. Compare the value to those in Table 2 and identify the metal. (1 mark for "Concludes that the metal is calcium") If you get a value somewhere between two given values, check your working and, if it's correct, explain that the value is between two values, giving details.	**b** The metal is mostly likely to be calcium ($W = 2.87\,\text{eV}$). (1 mark)

Your turn

The threshold frequency for a metal illuminated by ultraviolet light is $1.03 \times 10^{15}\,\text{Hz}$.

a **Determine** the work function, W. (2 marks)

b **Identify** the likely metal from the values in Table 2. (1 mark)

How is kinetic energy calculated?

We have described the relationship between photon energy and work function, and now we can look at the kinetic energy of the emitted electron. If we know the excess energy after a photon causes an electron to be ejected, we say this transformed into the kinetic energy of the electron. That also enables us to calculate the velocity of the electron, as we know its mass. The kinetic energy is:

$$E_{k(max)} = \text{energy of photon} - \text{work function}$$
$$E_{k(max)} = hf - W$$

where $E_{k(max)}$ is the maximum kinetic energy of an emitted photoelectron (J or eV), and the rest of the symbols have been described earlier. Here is an example.

Worked example 11.4D

Determining kinetic energy

A barium metal cathode is illuminated with 420 nm violet light. The metal's work function is 2.52 eV. The mass of an electron, $m_e = 9.1 \times 10^{-31}\,\text{kg}$. **Determine** (to an appropriate number of significant figures)

a the maximum kinetic energy (2 marks)

b the velocity of a photoelectron with this energy. (1 mark)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas. This question can be done in joules or electron-volts. We will start with joules.	$E_k = hf - W = \frac{hc}{\lambda} - W$, $\lambda = 420\,\text{nm}$, $W = 2.52\,\text{eV}$

Think	Do
Calculation in joules Step 3: Calculate the work function, W, in J. (1 mark for "Provides appropriate mathematical reasoning")	**a** $W = 2.52\,\text{eV}$ $= 2.52 \times 1.60 \times 10^{-19}$ (1 mark) $= 4.03 \times 10^{-19}\,\text{J}$
Step 4: Use the photoelectric equation to calculate the kinetic energy. Note that wavelength is in nm, so it first has to be converted to m ($\times 10^{-9}$) and then to a frequency using $f = \frac{c}{\lambda}$. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Provides correct answer")	$E_k = hf - W$ $= \frac{hc}{\lambda} - W$ $= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{420 \times 10^{-9}}\,\text{J} - 4.03 \times 10^{-19}\,\text{J}$ $= 4.73 \times 10^{-19}\,\text{J} - 4.03 \times 10^{-19}\,\text{J}$ $= 0.70 \times 10^{-19}\,\text{J}$ $= 7.0 \times 10^{-20}\,\text{J}$ (1 mark)
Step 5: Now calculate the velocity from the kinetic energy. Give your answer to an appropriate number of significant figures, in this case 2 s.f. Check you have used the correct units. (1 mark for "Provides correct answer")	**b** $E_k = \frac{1}{2}mv^2$ $7.0 \times 10^{-20} = \frac{1}{2} \times 9.1 \times 10^{-31} v^2$ $v^2 = \frac{2 \times 7.0 \times 10^{-20}}{9.1 \times 10^{-31}}$ $= 1.54 \times 10^{11}$ $v = 3.9 \times 10^5\,\text{m s}^{-1}$ (1 mark)
Calculation in eV Step 6: Calculate the kinetic energy in eV. The energy term $\frac{hc}{\lambda}$ will be in J, so divide by 1.6×10^{-19} to convert to eV. (1 mark for "Provides appropriate mathematical reasoning"; 1 mark for "Provides correct answer")	**a** $E_k = hf - W$ $= \frac{hc}{\lambda} - W$ (divide by $1.6 \times 10^{-19}\,\text{J eV}^{-1}$) $= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{420 \times 10^{-9} \times 1.60 \times 10^{-19}} - 2.52$ (1 mark) $= 2.96 - 2.52$ $= 0.44\,\text{eV}$ (1 mark)
Step 7: Convert 0.44 eV to J and calculate the velocity from the kinetic energy. (1 mark for "Provides correct answer")	**b** $E_k = \frac{1}{2}mv^2$ $0.44 \times 1.60 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31} v^2$ $0.704 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31} v^2$ $v^2 = \frac{2 \times 0.704 \times 10^{-19}}{9.1 \times 10^{-31}}$ $= 1.547 \times 10^{11}$ $v = 3.9 \times 10^5\,\text{m s}^{-1}$ (1 mark)

Your turn

A rubidium metal cathode is illuminated with 400 nm violet light. The metal's work function is 2.05 eV. The mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg. **Determine** (to an appropriate number of significant figures)

a the maximum kinetic energy (2 marks)

b the velocity of a photoelectron with this energy. (1 mark)

How do we interpret graphs of photon kinetic energy?

Einstein's photoelectric equation relates the incoming photon energy with the ejected photoelectron energy and the metal's work function so that conservation of energy is maintained. The logic is that the energy of the incoming photon (hf) is used partly to release the electron from the surface of the metal (the work function, W) and the left-over energy gives the electron its kinetic energy (E_k). Thus:

$$hf = W + E_k$$
$$E_k = hf - W$$

We can compare this with the general equation for a straight-line graph:

$$y = mx + c$$

You can see that the equation presents as a linear graph of kinetic energy against frequency, with Planck's constant h as the gradient (m). If you extrapolate the graph back to the vertical axis ($x = 0$), the y-intercept (c) will be the negative value of the work function, W. You may come across slightly different versions of these graphs.

- Graph type 1: the y-axis shows the stopping potential in V and the maximum kinetic energy in eV.
- Graph type 2: the y-axis shows the maximum kinetic energy in joules.

The dependent variable on the y-axis can be given in volts, electron volts, or joules. These graphs look the same, and students often get them mixed up. We'll clear up the differences now.

Graph type 1: Kinetic energy in eV

In the experimental determination of work functions and kinetic energy, a metal is subjected to a light of varying frequency. The voltage on the photoelectric tube is adjusted to just stop the fastest, most energetic electrons from crossing the tube between the plates. This voltage stops the current in the circuit and is called the "stopping potential" or "stopping voltage", V_s. At this voltage, the maximum kinetic energy of the photoelectrons is that value in eV. In summary, the magnitude of the stopping potential in volts is an exact measure of the maximum kinetic energy, $E_{k(max)}$, of the emitted photoelectrons in electron volts in eV at that frequency.

Interpreting an eV vs *f* graph

A graph of stopping potential versus frequency can be interpreted as a graph of maximum kinetic energy versus frequency. Hence, the y-intercept of the graph represents the work function W, in electron volts. The threshold frequency, f_0, is the x-intercept, and the gradient is Planck's constant, h. It won't be in the units J s as the energy is in eV, not J. The gradient will be in units of eV s, but you can convert it using the relationship 1 eV = 1.6×10^{-19} J (which is in the QCAA Physics Formula and data book).

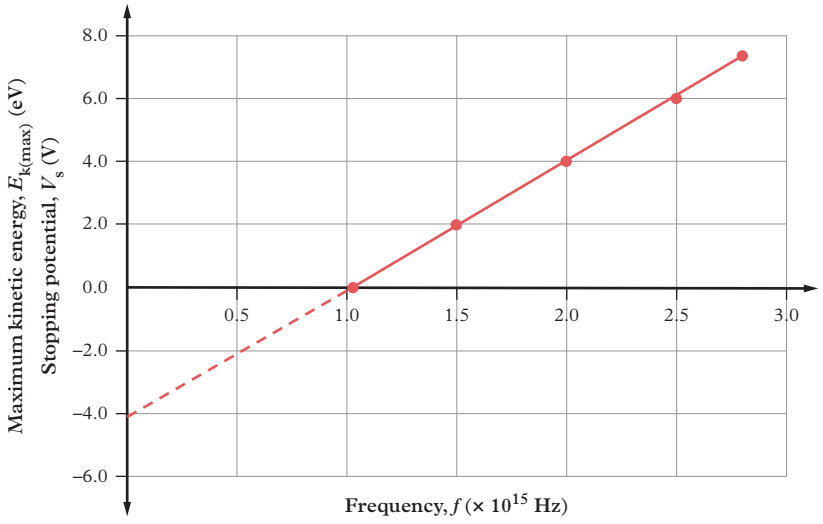


FIGURE 8 Graph of maximum kinetic energy and the magnitude of the stopping potential vs frequency. The dashed portion of the line is an extrapolation of the line joining the data points.

The stopping potential label

The example in Figure 8 shows results for aluminium. The y-axis has two labels using the same scale. One is the stopping potential, V_s, in volts (V). If light with a frequency less than the threshold frequency, f_0, of 1.03×10^{15} Hz (where the line crosses the x-axis) is incident on the metal, no photoelectrons will be emitted. Therefore, the stopping potential needed to stop them travelling across the phototube between the plates is also zero.

Above the threshold frequency, photoelectrons are emitted by the metal surface and a voltage is needed to repel them from making their way across the phototube to the other plate. This voltage is the stopping potential and has the units of volts. For example, at a frequency of 2.0×10^{15} Hz a stopping potential of 4.0 V is needed to prevent the electrons from reaching the opposite plate, and at a frequency of 2.5×10^{15} Hz a stopping potential of 6.0 V is needed. Note that this is just the magnitude of the stopping potential.

The maximum kinetic energy label

The second label on the *y*-axis is maximum kinetic energy, $E_{k(max)}$, and has the units of electron volts, eV. So, for light with a frequency of 2.0×10^{15} Hz, and where the stopping potential is 4.0 V, the maximum kinetic energy is 4.0 eV. Note: for a stopping potential $V_s = 4.0$ V, $E_{k(max)} = 4.0$ eV. Remember not to get these numbers and units back to front!

Interpreting a photoelectric graph

The following questions and answers provide a summary of how the graph in Figure 8 can be interpreted.

Question: What is the stopping potential in volts for light of frequency 2.8×10^{15} Hz?

Answer: 7.4 V

Question: What frequency gives the photoelectron a maximum kinetic energy of 3.0 eV?

Answer: 1.75×10^{15} Hz

Question: What is the work function?

Answer: 4.2 eV (not −4.2 eV)

Question: What is the energy in eV of the incident photoelectron for light of frequency 2.8×10^{15} Hz?

Answer: Add the work function to the kinetic energy of the photoelectrons at that frequency:

$$E = 4.2\,\text{eV} + 7.4\,\text{eV} = 11.6\,\text{eV}.$$

Note you can check this by using $E = hf$, which gives 1.85×10^{-19} J or 11.6 eV.

Question: What is the gradient in eV s?

Answer: 4.1×10^{-15} eV s

Question: What is the gradient in J s and how does it relate to Planck's constant, *h*?

Answer: Gradient $= 4.1 \times 10^{-15}$ eV s $\times$ 1.6×10^{-19} J eV^{-1}

$= 6.6 \times 10^{-34}$ J s

This is the same as Planck's constant.

Graph type 2: Kinetic energy in joules

The graph in Figure 8 showing energy in eV can replotted using energy in joules. This is the more common way of representing the graph, but you need to be aware of both forms.

Figure 9 represents a typical set of graphs obtained from photoelectric experiments carried out on various metals. In these graphs the *y*-axis has the units of energy in joules. Again, the gradient of each graph is equal to Planck's constant. The intercept on the *x*-axis is the threshold frequency, f_0, and the intercept on the *y*-axis is the work function, W (stated as a positive value). Work functions are commonly expressed in terms of electron volts (eV) but can also be expressed in joules.

The graph in Figure 9 shows the kinetic energy versus frequency for two metals – rubidium and calcium. They both have the same gradient, which you would expect as it represents *h*.

For rubidium:

- the threshold frequency, f_0, is where the line cuts the x-axis, so $f_0 = 4.8 \times 10^{14}$ Hz
- the work function, W, is the negative of the y-intercept, so $W = 3.2 \times 10^{-19}$ J (= 2.0 eV). Check this using $E = hf$:

$$f = \frac{E}{h} = \frac{3.2 \times 10^{-19}}{6.626 \times 10^{-34}} = 4.8 \times 10^{14} \text{ Hz}$$

- Planck's constant, h, is the gradient, so $h = 6.626 \times 10^{-34}$ J s.

For calcium:

- the threshold frequency, f_0, is where the line cuts the x-axis, so $f_0 = 6.89 \times 10^{14}$ Hz
- the work function, W, is the negative of the y-intercept, so $W = 4.6 \times 10^{-19}$ J = 2.87 eV). Check this using $E = hf$:

$$f = \frac{E}{h} = \frac{4.6 \times 10^{-19}}{6.626 \times 10^{-34}} = 6.9 \times 10^{14} \text{ Hz}$$

- Planck's constant, h, is the gradient, so $h = 6.626 \times 10^{-34}$ J s.

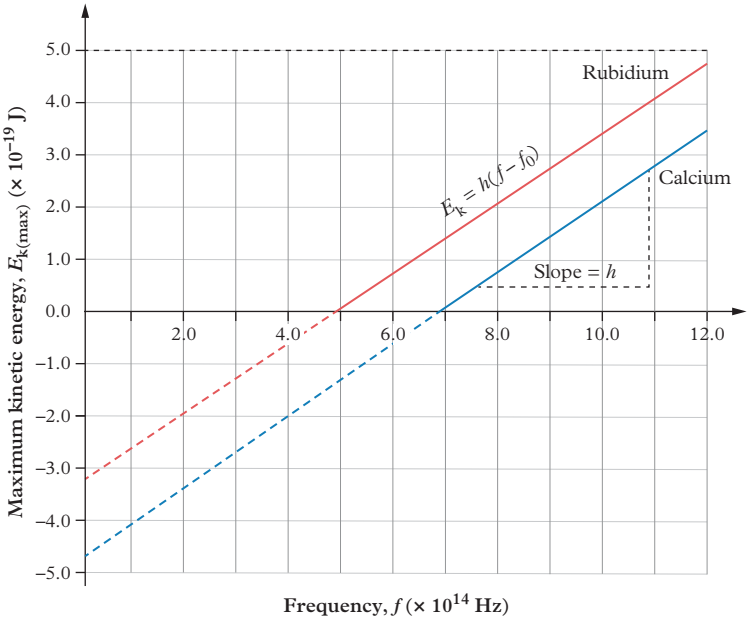


FIGURE 9 Results from photoelectric experiments: $E_{k(max)}$ vs frequency

This type of graph can also reveal other information about the elements. If we look at the graph for rubidium, we can find the kinetic energy, E_k, of the photoelectrons for any frequency light. For example, at a frequency of 10×10^{14} Hz, $E_k = 3.8 \times 10^{-19}$ J.

Study tip

The type of graph in Figure 9 almost always appears in external exams, often in multiple-choice questions. Try to familiarise yourself with it. Learn that gradient = h, x-intercept = f_0, and y-intercept (magnitude) = W.

Worked example 11.4E

Interpreting graphs with energy in eV

A photoelectric experiment was conducted by shining light of different frequencies on to the surface of a metal. The maximum kinetic energy of the ejected photoelectrons was measured. The data is shown in Figure 10.

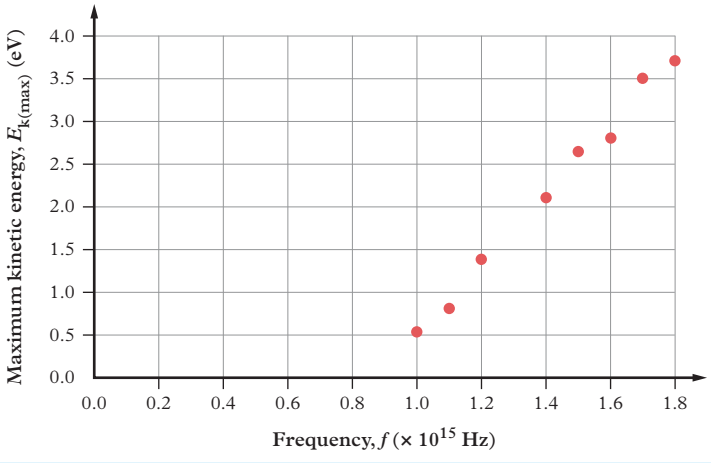
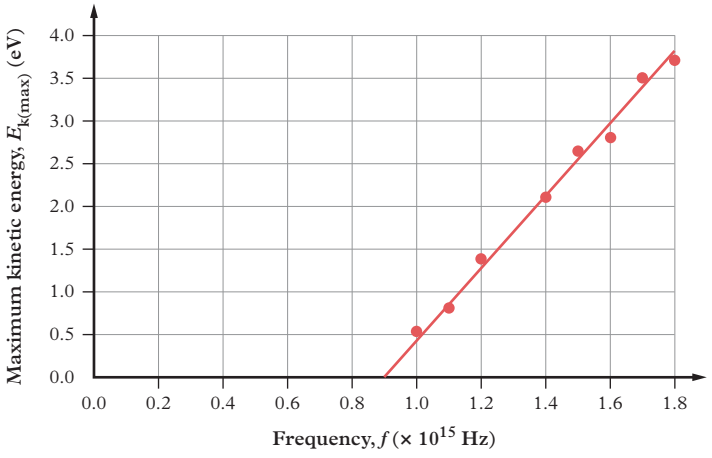


FIGURE 10 Graph of data for a photoelectric effect experiment

a Using the graph, **identify** the relationship between the kinetic energy, $E_{k(max)}$, and the incident frequency, f. (5 marks)

b **Determine** the work function of the metal. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Identify" means to recognise and state. "Determine" means to establish after a calculation". For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formulas.	**a** $E_k = hf - W$
Step 3: Draw a line of best fit. (1 mark for "Draws accurate line of best fit") You may annotate your graph, but this is not necessary for marking.	 (1 mark)
Step 4: Identify the x-intercept as 0.90×10^{15} Hz. (1 mark for "Provides a correct value for the x-intercept") Accept between 0.85×10^{15} and 0.95×10^{15} Hz.	x-intercept = 0.90×10^{15} Hz (1 mark)
Step 5: Identify the gradient as 4.28×10^{-15} eV s^{-1}. (1 mark for "Provides a correct value for the gradient") Accept any other value if it is the product of acceptable variations to rounding or differences in values read off the graph. In this case accept values between 3.9×10^{-15} and 4.6×10^{-15}, inclusive. No marks awarded if the gradient is assumed to be Planck's constant (it must be determined from the graph). No penalty for missing or wrong units.	Gradient $= \frac{3.85 - 0}{(1.80 - 0.90) \times 10^{15}}$ $= 4.28 \times 10^{-15}$ eV s^{-1} (1 mark)
Step 6: Identify the y-intercept. (1 mark for "Provides a correct value for the y-intercept") Accept any other value if it is the product of acceptable variations to rounding or differences in values read off the graph. In this case accept values of magnitude between 3.7 to 4.0 eV, inclusive.	When $y = 0$, $x = 0.90 \times 10^{-15}$ $y = mx + c$ $0 = 4.28 \times 10^{-15} \times 0.90 \times 10^{15} + c$ $c = -3.9$ $= -3.9$ eV (1 mark)
Step 7: Identify the relationship between the kinetic energy and the frequency. (1 mark for "Correctly provides the equation") Accept answer in the form of $y = (4.3 \times 10^{15}x - 3.9)$ eV, including units. Accept values in the equation in the ranges stated above.	Linear relationship: $y = mx + c$ $E_{k(max)} = (4.3 \times 10^{15} f - 3.9)$ eV (1 mark)

Think	Do
Step 8: Determine the work function as the negative of the y-intercept (i.e. 3.9 eV). (1 mark for "Provided pertinent mathematical operation(s) correctly performed or sound mathematical reasoning") Accept answers between 3.7 eV and 4.0 eV. You may deduce the work function by approximating the y-intercept by extrapolation and using it to estimate the work function. You could also determine it using $W = hf_0$, as shown in the "Do" column. Full marks should be awarded for either approach. No mark to be awarded if the work function is given as a negative value. The work function may also be stated in joules (accept values between 5.9×10^{-19} J and 6.4×10^{-19} J. In all cases units must be given.	**b** $W = 3.9$ eV or $W = hf_0$ $= 6.626 \times 10^{-34} \times 0.90 \times 10^{15}$ $= 5.96 \times 10^{-19}$ J $= \frac{5.96 \times 10^{-19}}{1.6 \times 10^{-19}}$ [convert J to eV] $= 3.7$ eV (1 mark)

Your turn

A photoelectric experiment was conducted by shining light of different frequencies on to the surface of a metal. The maximum kinetic energy of the ejected photoelectrons was measured. The data is shown in Figure 11.

a **Identify** the relationship between the kinetic energy, $E_{k(max)}$, and the incident frequency, f. (5 marks)

b **Determine** the work function of the metal in eV. Give your answer to an appropriate number of significant figures. (1 mark)

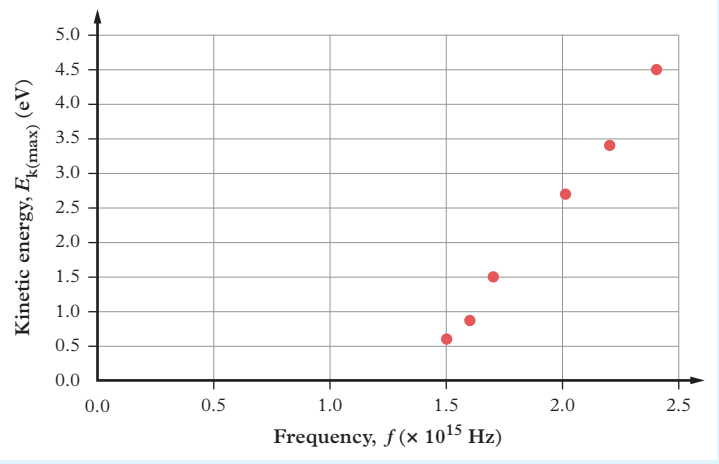


FIGURE 11 Graph of data for a photoelectric effect experiment

Worked example 11.4F

Interpreting graphs with energy in joules

A photoelectric experiment was carried out by shining light of different frequencies on to a metal surface. The maximum kinetic energy of the electron was plotted against the frequency as shown in Figure 12.

a **Identify** the mathematical relationship between E_k and f using the information from the graph. Show your working. (5 marks)

b Using the table of work function values (Table 2), **determine** which metal was being used in the experiment. (3 marks)

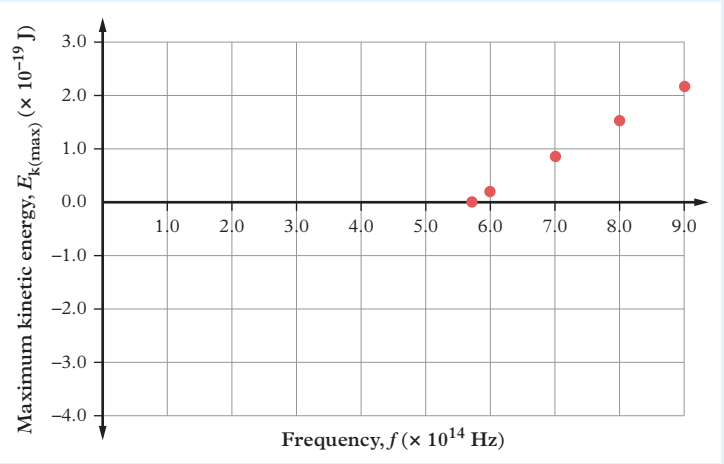
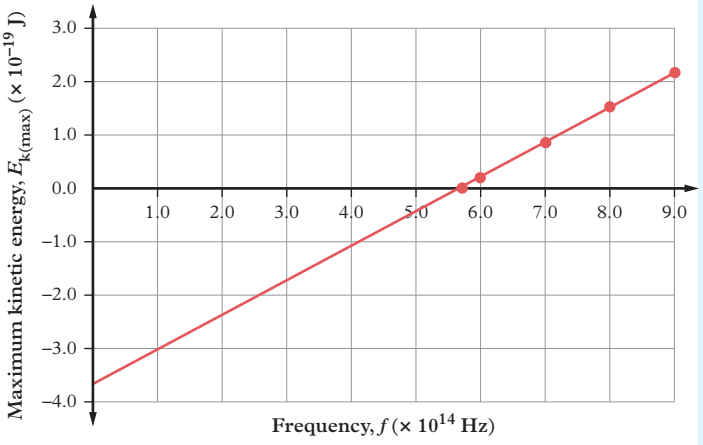


FIGURE 12 Graph of data for a photoelectric effect experiment

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Identify" means to recognise and state. "Determine" means to establish after a calculation. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant information from the question and select the appropriate formulas. (1 mark for "Recognises the scenario relates to $E_k = hf - W$")	**a** $E_k = hf - W$ (1 mark)
Step 3: Draw a line of best fit. (1 mark for "Draws accurate line of best fit") The line may not necessarily be extrapolated to the y-axis. You can annotate the graph, but this is not necessary for marking.	 (1 mark)
Step 4: Identify the gradient as 6.67×10^{-34} eV s^{-1}. (1 mark for "Provides a correct value for the gradient") Accept any other value if it is the product of acceptable variations to rounding or differences in values read off the graph. In this case accept values between 6.4×10^{-34} and 6.8×10^{-34}, inclusive. No marks awarded if the gradient is assumed to be Planck's constant (it must be determined from the graph). No penalty for missing or wrong units.	Gradient $= \frac{2.2 \times 10^{-19} - (-3.8 \times 10^{-19})}{(9.0 - 0.0) \times 10^{15}}$ $= 6.67 \times 10^{-34}$ J s^{-1} (1 mark)

Think	Do
Step 5: Identify the y-intercept. (1 mark for "Provides a correct value for the y-intercept") Accept any other value if it is the product of acceptable variations to rounding or differences in values read off the graph. In this case accept values of magnitude between 3.6×10^{-19} J and 3.8×10^{-19} J, inclusive.	y-intercept = -3.7×10^{-19} J (1 mark)
Step 6: Identify the relationship between the kinetic energy and the frequency. (1 mark for "Correctly provides the equation") Accept answers in the form of $y = (6.7 \times 10^{-34}x - 3.7 \times 10^{-19})$. Accept values in the equation in the ranges stated above.	$E_{k(max)} = (6.7 \times 10^{-34} f - 3.7 \times 10^{-19})$ J (1 mark)
Step 7: Determine the work function as the negative of the y-intercept; that is, 3.7×10^{-19} J. (1 mark for "Provides a correct value for the work function in J") Accept answers between 3.6×10^{-19} J and 3.8×10^{-19} J. Convert your answer to a value in eV. (1 mark for "Provides a correct value for the work function in eV") Accept values between 2.2 eV and 2.4 eV. No mark awarded if the work function is given as a negative value. You can also deduce the work function using $W = hf_0$. Full marks should be awarded. Identify the metal. (1 mark for "Correctly identifies the metal as sodium"). Allow follow-through error if metal is correctly identified based on incorrect value).	**b** $W = 3.7 \times 10^{-19}$ J (1 mark) $= \frac{3.7 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 2.3$ eV (1 mark) or $W = hf_0 = 6.626 \times 10^{-34} \times 5.7 \times 10^{14}$ $= 3.78 \times 10^{-19}$ J $= \frac{3.78 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 2.36$ eV The metal is sodium. (1 mark)

Your turn

a **Identify** the mathematical relationship between E_k and f using the information from Figure 13. Show your working. (5 marks)

b Using the table of work function values (Table 2), **determine** which metal was being used in the experiment. (3 marks)

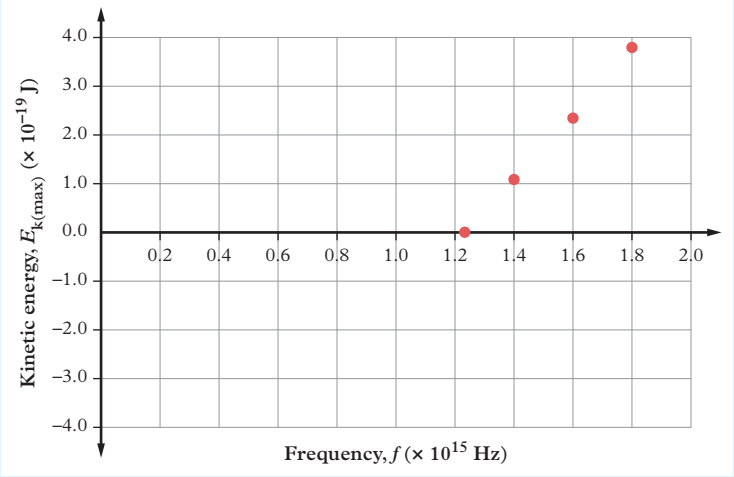


FIGURE 13 Graph of data for a photoelectric effect experiment

How do we interpret other photoelectric-effect graphs?

There are three variables that are typically displayed on a photoelectric-effect graph: frequency, kinetic energy and work function. So far, you have seen the kinetic energy of emitted photons plotted as a function of frequency. But you could also plot kinetic energy as a function of work function for a variety of different metals while keeping frequency constant.

The photoelectric equation is: $E_k = hf - W$. We can rearrange this to:

$$E_{k(max)} = -W + hf$$

Again, we can compare this with the general equation for a straight-line graph:

$$y = mx + c$$

If we plot $E_{k(max)}$ on the y-axis and work function W on the x-axis, we can see that the gradient will be –1, and the y-intercept will be hf, which is the energy of the photons.

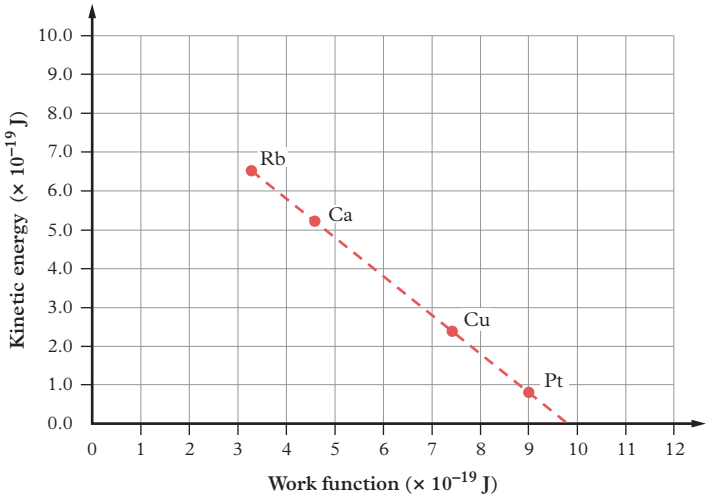


FIGURE 14 Kinetic energy, $E_{k(max)}$, plotted as a function of work function for the metals Rb, Ca, Cu and Pt.

Example of a kinetic energy vs work function graph

An example of this is shown in Figure 14, where the names of the four metals are shown beside the data points. Their work functions (which are independent of frequency) can be read off the x-axis. Typically, a question might be "What are the wavelength of the incident light?" To solve this, we need to find the frequency of the light and from that determine the wavelength. It would be worth 4 marks.

Gradient

As already stated, the gradient should be –1. We can check this to make sure our thinking is correct:

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{(0.00 - 6.5)\times 10^{-19}}{(9.8 - 3.3)\times 10^{-19}} = -1.0$$

Photon energy

Secondly, we said that the y-intercept is the photon energy, hf. If you extrapolate (using a ruler or a straight line) the y-intercept is 9.8 ($\times 10^{-19}$J). Alternatively, you could substitute some values into the equation and solve for the y-intercept, c:

$$\begin{aligned} y &= mx + c \\ 6.5 \times 10^{-19} &= -1.0 \times 3.3 \times 10^{-19} + hf \\ hf &= 6.5 \times 10^{-19} + 3.3 \times 10^{-19} \\ &= 9.8 \times 10^{-19}\,\text{J} \end{aligned}$$

Photon frequency and wavelength

Now that we have the photon energy in joules, we can calculate the frequency using $E = hf$:

$$E = hf = 9.8 \times 10^{-19}\,\text{J}$$

$$f = \frac{9.8 \times 10^{-19}}{h} = \frac{9.8 \times 10^{-19}}{6.626 \times 10^{-34}} = 1.48 \times 10^{15}\,\text{Hz}$$

Finally, we can use the frequency to calculate wavelength:

$$\begin{aligned} \lambda &= \frac{v}{f} \\ &= \frac{3 \times 10^8}{1.48 \times 10^{15}} \\ &= 2.03 \times 10^{-7}\,\text{m} \\ &= 203\,\text{nm} \end{aligned}$$

How do photoelectric experiments provide evidence for the quantised nature of photons?

The syllabus expects you to be able to "describe the photoelectric effect in terms of the photon". "Describe" means to present the pertinent facts and provide some helpful elaboration. This could be worth up to 3 marks in a test. The main points to consider are evidence and conclusions.

1 **Evidence** – remember that the photoelectric effect experiment demonstrates that:
   - the number of electrons ejected from a metal plate is proportional to the intensity of incident light
   - the kinetic energy of the ejected electrons is proportional to the frequency of incident light.

2 **Conclusions** – from observations we can draw a conclusion that energy is quantised as packets of energy known as quanta or photons. This comes from the fact that:
   - frequencies lower than a threshold value will not eject electrons
   - as the frequency increases, the kinetic energy of the ejected electrons increases
   - a greater intensity of light causes more electrons to be ejected (only above the threshold frequency), which suggests that the greater the intensity, the more photons (or packets of energy) are incident on the metal plate.

11.4 Real-world physics

Ultraviolet photoelectron spectroscopy

When electrons are emitted from a metal surface by UV light they have a range of kinetic energies, from a maximum value ($E_{k(max)}$), right through to the slowest ones that were buried several nanometres below the surface. This is called their photoelectron spectra, and the process of analysing this is called ultraviolet photoelectron spectroscopy (UPS). Not only can it measure the work function of metals, but it is now possible to measure the chemical structure of both organic and inorganic substances to a depth of 9 nm. The emitted photoelectrons are filtered by kinetic energy by the analyser and counted by the detector. The lower the kinetic energy of the photoelectron, the more tightly it was bound to the nucleus, so the higher is its binding energy.

FIGURE 15 The ultraviolet photoelectron spectroscope at QUT

The UPS at Queensland University of Technology is used for many types of research such as superconductors, thin film biomedical coatings, organic electronics and photocatalysts.

Apply your understanding

1 A UPS uses UV light of wavelength 58.5 nm. **Determine** the energy of the photons in eV (3 s.f.). (1 mark)

2 An X-ray version of the UPS is called XPS. It has photon energies of 1,486.7 eV. **Calculate** the wavelength this corresponds to (3 s.f.). (1 mark)

3 The XPS photons can penetrate down to more than 10 nm, which is much further than the 1–9 nm for UPS. **Explain** why X-rays penetrate further. (1 mark)

Does a photon have momentum?

Even though Einstein received the Nobel Prize in Physics in 1921, physicists still did not accept his 1905 photon concept. However, further evidence for the particle nature of electromagnetic radiation came in 1923, with the discovery of X-ray scattering by Arthur Holly Compton (1892–1962). Compton showed that X-ray photons behaved like particles with definite momentum characteristics. Compton was a joint winner of the 1927 Nobel Prize in Physics, mostly for his work on photon momentum, and the behaviour he observed is now called Compton scattering.

Is light a particle or a wave?

Einstein acknowledged that there was a conflict between the two models of light – the particle model and the wave model. That is, if light is made of particle-like photons, how can it show wave-like properties in Young's double-slit experiment, in which dark and light bands were explained in terms of wave properties of light such as interference and diffraction?

A classical (Newtonian) particle, when faced with Young's double slit, would have to go through one slit or the other. If light consisted of classical particles, you would see two bright spots on the screen – but instead you see an interference pattern. Einstein proposed that a photon must somehow go through both slits and interfere with itself. Photons thus have both wave-like and particle-like characteristics at the same time. When light's properties are tested, it behaves either as a wave or as a particle, depending on the apparatus used for the test. It is the testing that forces light to adopt either of its two personalities as wave or particle. This is called the **wave–particle duality** of light. You can see from Table 3 that both models are needed to explain the physical phenomena we can observe.

wave–particle duality
light can exhibit either wave or particle behaviour, depending on the situation. More fully, a complete description of light behaviour at the atomic level requires the application of two different but complementary models – the wave model and the particle model – but the two are never applied at the same time to the same physical experiment.

TABLE 3 Summary of phenomena explained by the wave and/or particle model

Phenomenon	Explained by wave model?	Explained by particle model?
Reflection	yes	yes
Refraction	yes	no
Diffraction (Young's experiment)	yes	no
Polarisation	yes	no
Black-body radiation	no	yes
Photoelectric effect	no	yes
Photon momentum (Compton scattering)	no	yes

Study tip
You can think of light this way: it is both a wave and a particle until you observe it; then it becomes one or the other.

Challenge

Power per photon

In a dark room let your eyes become dark-adapted over a couple of minutes. Give your eyelid a sharp tap with your finger and you should see a flash. A single rod (light receptor in your eye) will detect a single photon, but your visual system only responds when between two and ten photons are absorbed by your rods within 0.1 s; you will then see the flash. **Predict** how much power two visible photons will give to your eyes in 0.1 s. (2 marks)

Challenge

Can you shut a door with photons?

It is said you can shut a door by firing photons at it. If it takes 5 J to close a door, **determine** how many photons of wavelength 635 nm are needed. (2 marks)

Challenge

Crookes radiometer

A Crookes radiometer consists of four paddles suspended on a needle point in a low-pressure glass container (Figure 16). It was invented by English scientist William Crookes in 1873. One side of the paddle is painted black and the other side white. You may have one in the classroom. When placed in sunlight, the radiometer turns around. **Explain** whether the black side moves away from the Sun or towards it, and why this happens. It will turn in the opposite direction when placed near a block of dry ice (–44°C). Most people (even scientists) give the wrong explanation. (2 marks)

FIGURE 16 A Crookes radiometer when light is on it

Skill drill

Avoiding traps with exponents

Science inquiry skills: Processing and analysing data (Lesson 1.7)

Now that you're nearing the end of the coursework, it's time to reflect on simple errors that lose you marks in the external exam. In the rush to save time, students often make mistakes that they wouldn't normally make, and a loss of 5 marks or more is not uncommon. Here we want you to practise simple, exam-style algebra and calculations, particularly division, involving exponents.

Practise your skills

1 In your head, **calculate** the answer to: $(0.3)^2$, $1 \div \frac{1}{2}$, $(2 \times 3)^2$. (1 mark)

2 Using a calculator, **calculate** the value given from $\frac{3 \times 10^{-3}}{9 \times 10^{9} \times (10 \times 10^{-6})^2}$ and state the order of magnitude of the answer. (1 mark)

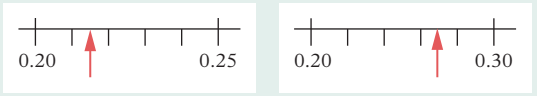
3 **Solve** the following expressions for r:

a $9.8 \times 10^{-14} = \frac{(27.3 \times 24 \times 60 \times 60)^2}{r^3}$ (1 mark)

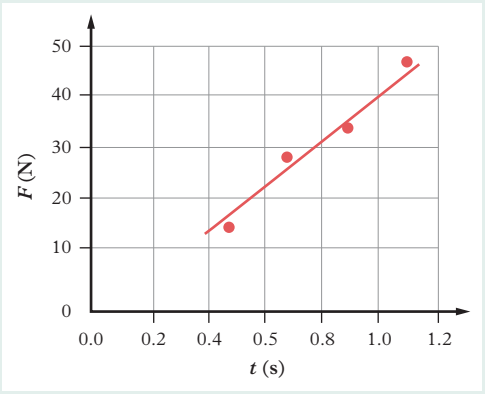
b $r \times r = 4 \times 10^{-8}$ (1 mark)

4 **Explain** the following: if charge Q equals charge q, does $\frac{kQq}{r^2}$ equal $\frac{kQ^2}{r^2}$ or $\frac{k2Q}{r^2}$? (1 mark)

5 **Determine** the scale reading on the two scales in the diagram. (1 mark each)



6 **Determine** the gradient of the line on the graph. (1 mark)



7 **Determine** the order of magnitude of the answer in joules for the following calculation:

$W = E_{k(max)} - hf$

$= 7.7 \times 10^{-18} - 6.626 \times 10^{-34} \times 9.4 \times 10^{15}$

(2 marks)

Check your learning 11.4

Check your learning 11.4: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** how collector current depends on threshold frequency. (2 marks)

2 **Describe** the terms meaning of "Planck's constant" and "work function". (2 marks)

3 **Describe** a photon and **describe** two similarities and two differences between a photon of violet light and a photon of ultraviolet light. (3 marks)

4 **Describe** the wave–particle duality of light and identify evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light. (3 marks)

Analytical processes

5 **Determine** the work function of a particular metal if photoelectrons are only produced when the metal is illuminated with light that has a wavelength of less than 400 nm. (3 marks)

6 **Determine** the wavelength of light that produces electrons with a maximum kinetic energy of 1.32 eV from a copper surface. The work function of copper is 4.64 eV. (3 marks)

7 Rubidium metal, which has a threshold frequency of 5.0×10^{14} Hz, is illuminated by light photons of frequency 8.1×10^{14} Hz.
   a **Determine** the value of the work function. (2 marks)
   b **Determine** the maximum velocity of the photoelectrons ($m_e = 9.109 \times 10^{-31}$ kg). (3 marks)
   c **Determine** the order of magnitude of the velocity of the photoelectrons calculated in part **b**. (1 mark)

8 **Deduce** how the graph in Figure 6A would appear if classical physics was a good model for the photoelectric effect. Explain your reasoning. (2 marks)

9 **Determine** whether light of frequency 5.25×10^{14} Hz would allow for the photoelectric effect to be demonstrated in rubidium and sodium metals. (3 marks)

Knowledge utilisation

10 **Predict** the order of the stopping potentials for metals A, B and C, which have work functions of 2.0 eV, 2.5 eV and 3.0 eV, respectively, and all of which produce photoelectrons when irradiated with UV light. (2 marks)

11 A photoelectric-effect experiment was performed in which a metal surface was illuminated by light of varying wavelengths and the photoelectric current was measured. For each metal, the threshold wavelength was noted. The results are shown in the table.

Metal target	Threshold wavelength (nm)
Calcium	428
Copper	264
Platinum	197

   a **Determine**, for each metal
      i the threshold frequency in Hz (1 mark)
      ii the work function in J (1 mark)
      iii the work function in eV. (1 mark)
   b **Assess** the percentage error in the work function for each metal by comparing it to the accepted values, namely calcium, 2.87 eV; copper, 4.64 eV; and platinum, 6.35 eV. (3 marks)

12 **Propose** an answer to the following question. "How can light be both a wave and a particle?" (2 marks)

Practical

Lesson 11.5
Investigating the photoelectric effect

oxforddigital **This practical lesson is available on Oxford Digital. It is also provided as part of a printable resource that can be used in class.**

Learning intentions and success criteria

MODULE 11

Lesson 11.6
Review: Quantum theory and light

Summary

11.1

- Young's double-slit experiment demonstrates the wave nature of light by allowing two coherent beams of light to overlap on a screen to form an interference pattern.
- Young observed interference patterns in light that were similar to patterns he saw in water waves and heard in sound waves.
- Light is an electromagnetic wave produced by an oscillating electric charge resulting in mutually perpendicular electric and magnetic fields.
- Electromagnetic radiation is energy radiating out as electromagnetic waves, propagated at the speed of light in a vacuum.
- The wave model of light uses wave characteristics such as wavelength, frequency and speed to describe behaviour of light such as polarisation, interference and diffraction.
- The wave equation is $v = f\lambda$, where v is speed (velocity), f is frequency and λ is wavelength. For electromagnetic radiation in a vacuum, $v = c$.

11.2

- A black body is an object that absorbs all radiation falling on it, of all wavelengths. It is a perfect absorber or emitter of radiation.
- Black-body radiation is the radiation emitted by a black body from the conversion of thermal energy. It has a characteristic frequency distribution that depends on the black body's temperature.
- Wien's displacement law states that the black-body radiation curve for different temperatures peaks at a maximum wavelength (λ_{max}), which is inversely proportional to the temperature T (in kelvin, K). The formula for this is: $\lambda_{max} = \frac{b}{T}$, where b is a constant of proportionality called Wien's displacement constant and is equal to $2.898 \times 10^{-3}\,\text{m K}$.
- Earth's energy balance is explained by the greenhouse effect and black-body radiation.

11.3

- Max Planck postulated the quantum nature of energy by suggesting radiation is absorbed and released by atoms in discrete packets or quanta.
- Electromagnetic energy quanta are called photons. Their energy is directly related to their frequency:

$$E = hf = \frac{hc}{\lambda}.$$

11.4

- The photoelectric effect provides evidence for the particle model of light.
- Light shone on a metal surface will cause electrons to be emitted as a photocurrent if the light is above the threshold frequency. Below that frequency, no electrons are emitted for any intensity of light. Above the threshold frequency, the photocurrent is proportional to the intensity of the light and electrons are emitted almost immediately.
- A graph of the maximum kinetic energy of the emitted electrons, $E_{k(max)}$ (in J), versus light frequency, f, can be interpreted as follows:
  - the gradient is equal to Planck's constant, h
  - the y-intercept is equal in magnitude to the work function, W (the minimum energy needed to remove an electron from the metal)
  - the x-intercept is the threshold frequency, f_0.

- Light exhibits properties of both waves and particles, called wave–particle duality:
  - diffraction, Young's double-slit experiment and polarisation support the wave model
  - the photoelectric effect, photon momentum and Compton scattering support the particle model.

11.5
- Practical: Investigating the photoelectric effect

Key formulas

Wien's displacement law	$\lambda_{max} = \frac{b}{T}$
Black-body radiation	$E = hf = \frac{hc}{\lambda}$
Kinetic energy and work function	$E_k = hf - W$
Work function and threshold frequency	$W = hf_0$

Key constants

Electron-volt	$1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$
Elementary charge	$q_e = 1.60 \times 10^{-19}\,\text{C}$
Planck's constant	$h = 6.626 \times 10^{-34}\,\text{Js}$
Speed of light in a vacuum	$c = 3 \times 10^{8}\,\text{ms}^{-1}$
Wien's displacement constant	$b = 2.898 \times 10^{-3}\,\text{mK}$

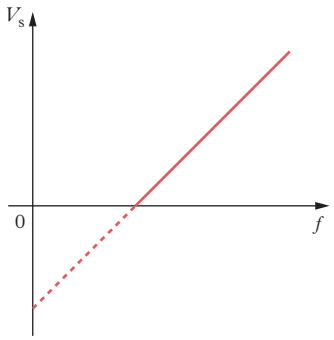
Review questions 11.6A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 An aluminium cathode is illuminated with ultraviolet light in a photoelectric experiment. A graph of the stopping potential versus frequency is shown. Planck's constant can be determined by multiplying the charge on an electron by

A the gradient.
B the x-intercept.
C the y-intercept.
D the inverse of the gradient.



2 A metal is illuminated with light of increasing frequency, and at a particular minimum value electrons start being ejected. If the work function is given the symbol W, which one of the following best describes the maximum kinetic energy of the electrons?

A $hf - W$
B $hf + W$
C $\frac{W}{hf}$
D $h(f - W)$

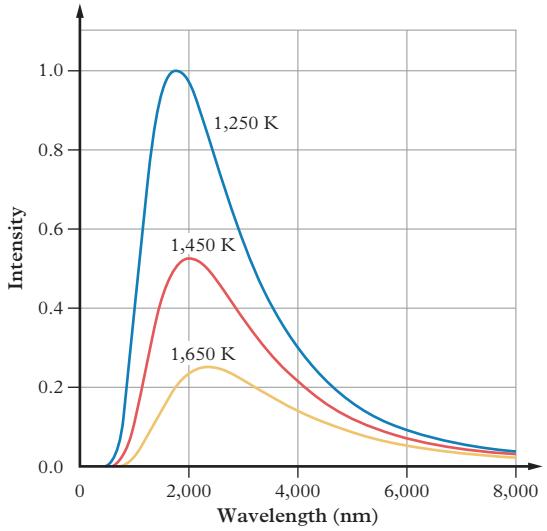
3 When a metal is illuminated by light, electrons can only be photo-emitted if the light

A is in the visible region of the spectrum.
B has a frequency in the UV or X-ray range.
C has a certain minimum wavelength.
D has a certain minimum frequency.

4 Light is described as an electromagnetic wave produced by an oscillating charge that generates
A photons that travel at the speed of light.
B mutually perpendicular oscillating magnetic and electric fields.
C electric and magnetic fields 90° out of phase to each other.
D fields that oscillate parallel to the direction of propagation.

5 Select the diagram that best represents the distribution of energies of a black-body radiator.

A



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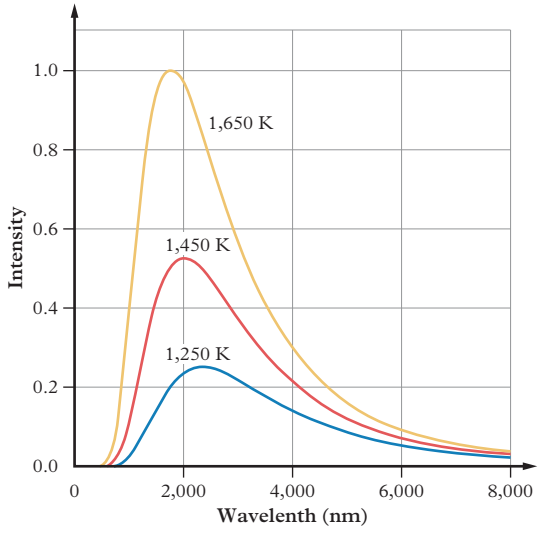
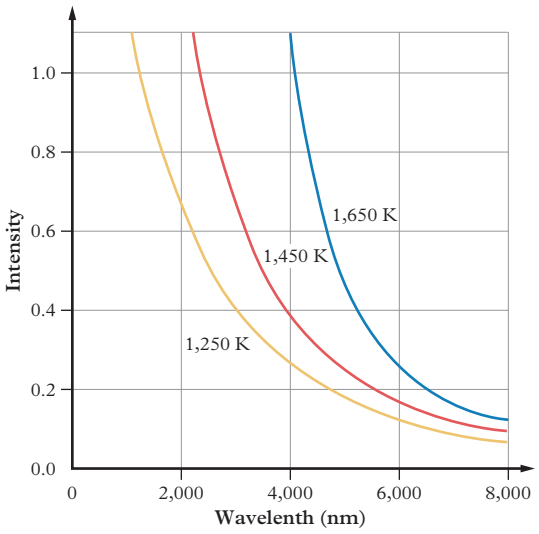
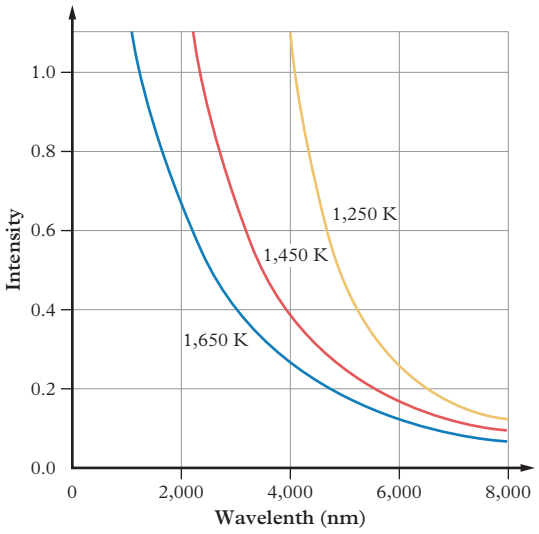


C



D

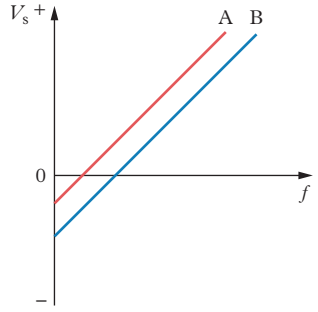


6 Wien's law predicts that, for a black-body radiator, the frequency at which maximum radiation of energy occurs is proportional to
A T^{-4}
B T^{-1}
C T
D T^{4}

7 The frequency of a photon with a wavelength of 550 nm is
A 5.45×10^{-4} Hz
B 5.45×10^{5} Hz
C 5.45×10^{12} Hz
D 5.45×10^{14} Hz

8 The energy of a photon with a wavelength of 550 nm is
A 0.1 eV
B 1.0 eV
C 2.3 eV
D 3.6 eV

9 The graph shows the photoelectric results for metals A and B. Select the statement that is true about the graph.



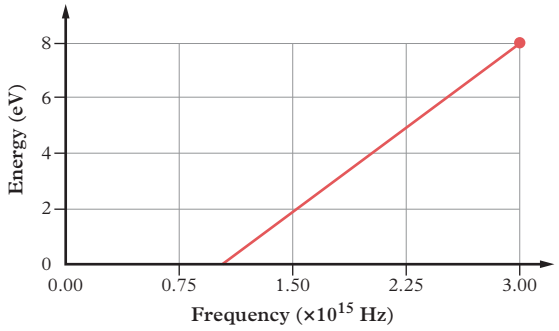
A There is significant systematic error as the lines don't pass through the origin.

B Metals A and B have the same gradient so are likely to be the same metal.

C Metal A intercepts the x-axis as a lower value than metal B, so metal A has the smaller work function.

D The outer electron of metal A is less strongly held than the outer electron of metal B.

10 The graph shows the results from a photoelectric experiment. Analyse the graph and determine the threshold frequency.



A 1.5×10^{15} Hz

B 0.75×10^{15} Hz

C 2.25×10^{15} Hz

D 1.125×10^{15} Hz

11 A photon with an energy of 9.08×10^{-19} J hits a piece of metal that has a work function of 4.64×10^{-19} J. Determine the maximum kinetic energy of the ejected photoelectron.

A 3.26×10^{-19} J

B 4.44×10^{-19} J

C 1.58×10^{-19} J

D 1.0×10^{-19} J

12 Light with a frequency of 3.0×10^{16} Hz strikes a surface. Its frequency is then increased to 6.0×10^{16} Hz. Photoelectrons start to be emitted by the surface once the frequency reaches 5.0×10^{16} Hz. If the frequency is kept constant at 6.0×10^{16} Hz but the intensity is increased, deduce how this would affect the maximum kinetic energy of the emitted photoelectrons.

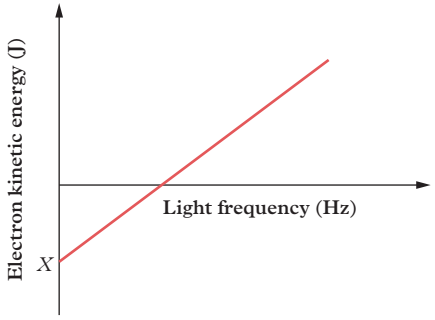
A It would stay the same.

B It would double (×2).

C It would halve ($\times\frac{1}{2}$).

D It would quadruple (×4).

13 The graph shows the relationship between the maximum kinetic energy of emitted photoelectrons and the frequency of the incident light. Identify what point X on the graph represents.



A Work function energy

B Threshold frequency

C Incident photon intensity

D Photoelectron frequency

14 Identify how the energy of a photon can be described.

A Directly proportional to wavelength

B Inversely proportional to work

C Directly proportional to frequency

D Inversely proportional to speed

Review questions 11.6B Short response

Retrieval and comprehension

15 **Explain** why it is necessary to use a coherent light source for Young's double-slit experiment. (2 marks)

16 **Explain** whether it is minimum energy or minimum frequency that determines whether an electron is photoejected from a metal. (2 marks)

17 **Clarify** whether the statement that "hot objects glow red, very hot glow blue" is true for Wien's law. (2 marks)

18 **Clarify** whether Planck's equation, $E = hf$, applies to all transverse waves. (2 marks)

Analytical processes

19 **Determine** the temperature, in °C, of an object emitting light with a peak wavelength of 15 cm. (3 marks)

20 **Determine** the energy, in both joules and electron volts, of a photon with wavelength 5.5×10^{-7} m. (3 marks)

21 **Determine** the wavelength of a 6 keV photon, and **identify** what spectral region it would be in. (3 marks)

22 Red light from a source has a frequency of 460 THz (tera, T = 10^{12}). **Determine** the speed of a photon of red light. (2 marks)

23 **Determine** the colour and wavelength (in nm) of light with a frequency of 500 THz (T = 10^{12}). (3 marks)

24 A beam of blue light has a wavelength of 450 nm.

- **a** **Determine** the energy of a quantum of this light in J. (2 marks)
- **b** **Determine** the order of magnitude of a quantum of this light in eV. (1 mark)

25 The threshold frequency for a particular metal is 2.5×10^{14} Hz. Light of frequency 6.0×10^{14} Hz falls onto the surface.

- **a** **Identify** the colour of the incident light. (1 mark)
- **b** **Determine** the energy of the incident photon. (2 marks)
- **c** **Apply** the work function equation to determine the work function of the metal. (2 marks)
- **d** **Determine** the maximum kinetic energy of the photoelectrons. (2 marks)
- **e** **Deduce** the maximum velocity of the photoelectrons. (2 marks)

Knowledge utilisation

26 It is found that a neutron travelling at 1.98×10^4 m s^{-1} has the same energy as a photon of frequency 5×10^{14} Hz.

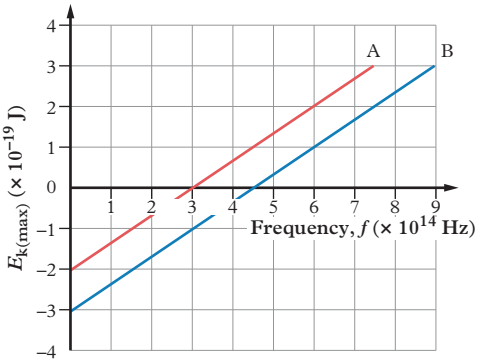
- **a** **Determine** the mass of the neutron in kg. (4 marks)
- **b** **Determine** the order of magnitude of this mass in kg. (1 mark)

27 The table contains data obtained from a photoelectric experiment.

f (× 10^{14}) (Hz)	3.75	4.5	5.5	7.0	8.0	8.9
$E_{k(max)}$ (eV)	0.50	0.80	1.02	1.75	2.3	2.5

- **a** **Construct** a graph of $E_{k(max)}$ (vertical axis) versus frequency. (3 marks)
- **b** **Propose** values for
  - **i** Planck's constant (in J s) (2 marks)
  - **ii** the threshold frequency for the metal (in Hz) (2 marks)
  - **iii** the work function of the metal (in eV). (2 marks)

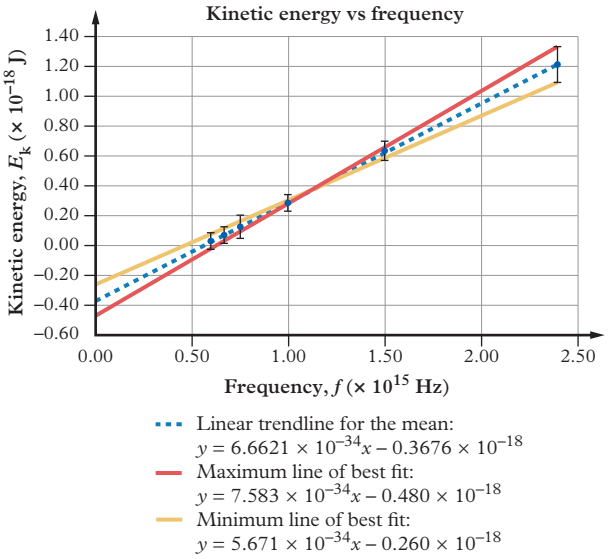
28 The graph in the diagram is used to measure the work function of two unknown metals.



- **a** **Deduce** the work function of each metal. (2 marks)
- **b** **Calculate** the value of Planck's constant. (3 marks)
- **c** **Predict** the shape of the curve for metal A if a less intense light was used. (2 marks)
- **d** **Propose** a reasoned interpretation of the data if the gradient for metal B was steeper than the gradient for metal A. (2 marks)

29 Students conducted a photoelectric experiment to determine the kinetic energy being given to electrons in a sodium target by the incident photons of light. A set of parallel plates was set up in a vacuum. One plate consisted of sodium metal that would be exposed to light of different frequencies. The plates were connected to a battery to provide a potential difference across them. If the light was able to eject electrons from the sodium target, it would cause a current to flow in the circuit, as shown by an ammeter. The source of potential difference could be adjusted to "stop" the current and cause the current to fall to zero.

A graph of kinetic energy versus frequency was plotted, as shown. A linear trendline was found and the formula determined. Error bars were added, and maximum and minimum linear lines of best fit were added. Formulas for these lines were also determined. Note that all of the values on the y-axis are in joules.



a **Determine** the experimental value for Planck's constant, and its absolute and percentage uncertainty. (3 marks)

b **Assess** the experimental value for Planck's constant for percentage error by comparing it to the accepted value. (2 marks)

c **Determine** the experimental work function for sodium, and its absolute uncertainty. (3 marks)

d **Assess** the results for percentage error in the work function by comparing the result to the accepted value of 2.36 eV. (3 marks)

30 A photoelectric experiment like the one outlined in question **29** provided the results listed in the table. A metal target was illuminated with light of wavelength 150–250 nm, and the voltage needed to stop any photocurrent was recorded for each trial wavelength.

Wavelength, λ (nm)	Magnitude of stopping potential, V_s (V)
150	4.00
175	2.85
200	1.90
225	1.20
250	0.65

a **Determine** the maximum kinetic energy of the electrons for a wavelength of 150 nm. (1 mark)

b **Construct** a graph of kinetic energy (vertical axis) versus frequency (horizontal axis). (3 marks)

c **Develop** a formula for the linear trendline. (3 marks)

d **Describe** the meaning of the gradient, the x-intercept and the y-intercept. (3 marks)

e **Determine** the experimental value for Planck's constant. (3 marks)

f **Assess** the accuracy of the experiment for determining Planck's constant. (3 marks)

g **Determine** the experimental value for the work function of the metal (in eV). (3 marks)

h **Predict** which metal target may have been used by referring to the data in Table 2, Lesson 11.4. (1 mark)

Data drill

Students investigated the photoelectric experiment using an online simulation. Light was shone onto a platinum surface, which caused a photoelectron to be released. As the wavelength of the light was changed, the maximum kinetic energy of the electrons was measured. The data is shown as a plot of kinetic energy versus wavenumber. Wavenumber is the reciprocal of wavelength and is used in spectroscopy experiments instead of frequency.

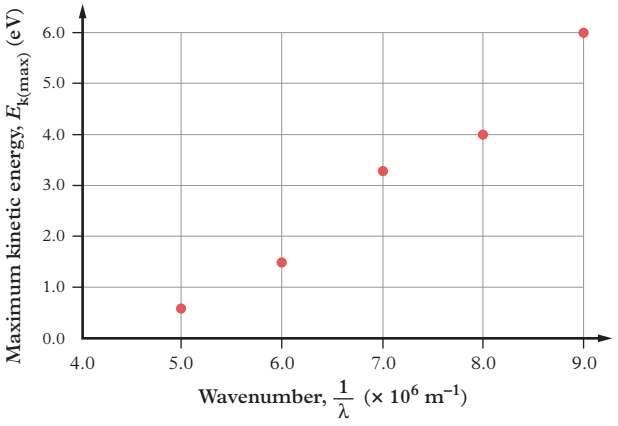


FIGURE 1 Results from photoelectric experiment

Apply understanding

1 **Identify** the wavelength of the light in nanometres, to the nearest whole number, for a wavenumber of 6.0×10^6 m^{-1}. (2 marks)

2 **Calculate** the frequency of light with a wavenumber of 8.0×10^6 m^{-1}. Give your answer in scientific notation to an appropriate number of significant figures. (1 mark)

Analyse data

3 **Identify** a mathematical relationship between the maximum kinetic energy, $E_{k(max)}$, of photoelectrons ejected from the platinum surface and the wavenumber, $\frac{1}{\lambda}$, of the incident light. Use evidence from the graph to support your answer. Hint: draw a line of best fit and determine this gradient. (1 mark)

Interpret evidence

4 **Determine** the work function in J (2 s.f.). (4 marks)

Module 11 checklist: Quantum theory and light

MODULE 12

Quantum theory and atoms

Introduction

For much of nineteenth century, the atom was thought to be nothing more than a tiny indivisible sphere. But towards the end of the century and into the first half of twentieth century, various models were proposed and discarded as more evidence was obtained. By the mid-1900s, the model we have today was conceptualised by scientists who built on each other's work.

Although the idea of an atom goes back to the ancient Greeks, only a handful of basic atomic models have been proposed. Each model has contributed to how we think of the structure of the atom today. The first model was Dalton's billiard-ball model in 1808, then J.J. Thomson's "plum pudding" model in 1897, followed by Rutherford's planetary model in 1898, Bohr's atomic model in 1913 and, finally, the quantum mechanical model in 1926.

This module describes how discoveries in experimental physics and chemistry led to the development of new theories about the atom, which were eventually replaced as new evidence became available or when a new model or theory could better explain the experimental results.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to quantum theory and atoms before you start.

Subject matter

Science understanding

→ Compare the different models of the atom proposed by Rutherford and Bohr.

→ Explain how Bohr's model of the hydrogen atom integrates light quanta and atomic energy states to explain the specific wavelengths in the hydrogen line spectrum.

→ Solve problems involving the line spectra of simple atoms using atomic energy states or atomic energy level diagrams using $n\lambda = 2\pi r$, $mvr = \frac{nh}{2\pi}$, $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ and $\lambda = \frac{h}{p}$.

→ Describe wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light.

Science as a human endeavour

→ Explore the historical development of the model of the atom in terms of traditional models.

→ Consider how theories are contested, refined or replaced when new evidence challenges them, or when a new model or theory has greater explanatory power.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Lesson 12.1
The Rutherford model of the atom

Key ideas

- → The spectrum of hydrogen hinted at the structure of an atom but was inconclusive.
- → Rutherford proposed a model of the atom that consisted of a dense, positive central core surrounded by negatively charged, orbiting electrons.
- → Limitations of the Rutherford model included the failure to explain the stability of the atom and to account for spectral lines in hydrogen.

What does the spectrum of hydrogen tell us?

Learning intentions and success criteria

The work of Planck and Einstein has given us an understanding of particles and radiation, but it doesn't tell us much about the structure of atoms and molecules. Let's begin our investigation with the simplest atom – hydrogen.

If hydrogen gas is ionised by subjecting it to 20,000 V in a discharge tube, it emits a characteristic purple glow. When this glowing light is passed through a prism, four very distinct lines can be seen, as shown in Figure 1. This is called an emission spectrum.

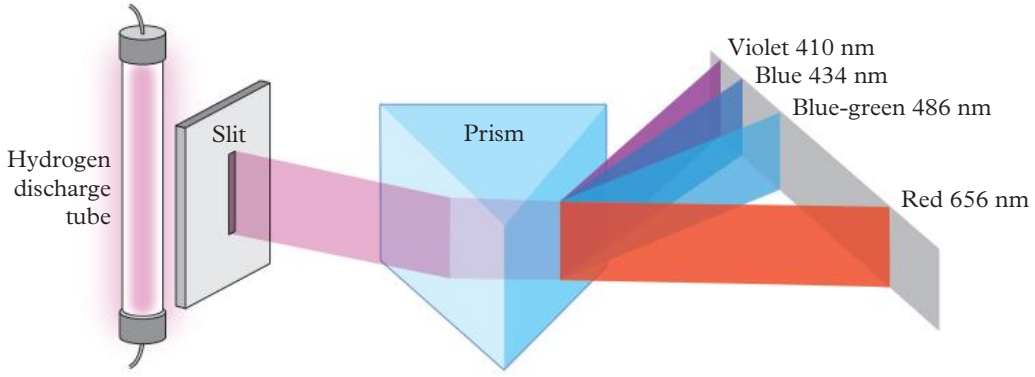


FIGURE 1 A prism is used to split up the different colours that make up the hydrogen emission spectrum. The wavelengths of the four lines are shown.

A second type of spectrum for hydrogen can also be produced. It is an absorption spectrum and is observed when white light is passed through cold atomic hydrogen gas. The gas removes some of the wavelengths from the white light and leaves black lines in their place in the spectrum. The black lines have the same wavelengths as the coloured lines of the emission spectrum.

This spectrum was not new to scientists. Spectroscopy was being used at the time to identify individual atoms or molecules. The word "spectrum" comes from the Latin "specere", meaning "to look at", and scientists were looking at the light given off by heated or ionised samples of all sorts of materials. The first prism spectroscope (as shown in Figure 1) was designed by Gustav Kirchhoff and Robert Bunsen in 1859 while working on chemical analysis.

In 1885, Johann Balmer, a Swiss physicist, worked out a mathematical expression that gave the values for the four lines seen in the hydrogen spectrum.

Table 1 lists the wavelengths and colours of the four visible emission lines for hydrogen.

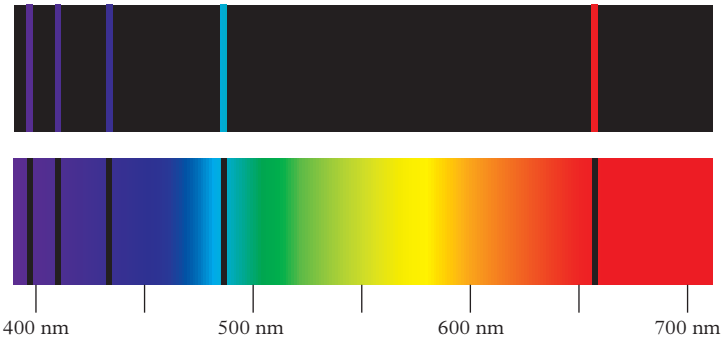


FIGURE 2 The hydrogen emission spectrum (top) and the absorption spectrum (bottom) show the four lines in the visible region, with the wavelength scale in nanometres along the bottom. The leftmost violet line is in the ultraviolet region and cannot be seen by eye.

TABLE 1 Wavelength and colour of the four emission lines of the Balmer series

	$n = 3$	$n = 4$	$n = 5$	$n = 6$
Wavelength	656.3 nm	486.1 nm	434.1 nm	410.2 nm
Colour	Red	Aqua	Blue	Violet

In 1888, the Swedish physicist Johannes Rydberg expanded Balmer's result in the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \text{ for } n = 3, 4, 5...$$

where λ is the wavelength of the absorbed or emitted light and R is the Rydberg constant ($1.097 \times 10^7\,\text{m}^{-1}$).

No-one was able to suggest what these numbers meant in terms of the structure of hydrogen, and they remained a puzzle. Rydberg found that the formula also worked for other atoms that were hydrogen-like – that is, atoms with just one electron, such as He^+, Li^{2+} and Be^{3+}. When numbers for n greater than 6 were used, the formula gave wavelengths in the ultraviolet region that couldn't be seen with the naked eye. Later observations confirmed that the formula held for numbers greater than 6.

Worked example 12.1A

Calculating wavelengths using the Rydberg formula

The emission spectrum of hydrogen has a line at 397 nm that was first observed in the 1860s. Using the Rydberg formula, **assess** whether this is the $n = 7$ line. (4 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Assess" means to make a judgment about the outcomes. Firstly, we need to calculate the answer and then compare it to the accepted value.
Step 2: Gather the relevant information from the question and select the appropriate formula.	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, $\lambda = 397\,\text{nm}$

Think	Do
Step 3: Substitute the known values into the equation. Use $n = 7$ to test whether it gives the correct answer. You could also use 397 nm (converted to m) for λ and solve for n, but that approach is more complex and more prone to errors. (1 mark for "Correctly substitutes values"). You may like to rearrange this equation before substitution to minimise transcription errors.) Note that the value opposite is for $\frac{1}{\lambda}$ and you will find λ in the next step.	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{7^2}\right)$ $= 1.097 \times 10^7 \times 0.2296$ (1 mark) $= 2.519 \times 10^6$
Step 4: Solve for λ by taking the reciprocal of $\frac{1}{\lambda}$. (1 mark for "Calculates the wavelength in metres")	$\lambda = 3.97 \times 10^{-7}$ m (1 mark)
Step 5: Convert to nm by dividing by 10^{-9} or by multiplying by 10^9. (1 mark for "Demonstrates correct conversion from m to nm")	$\lambda = 3.97 \times 10^{-7}\,\text{m} \times \frac{1\,\text{m}}{10^9\,\text{nm}}$ (1 mark) $= 397\,\text{nm}$
Step 6: Complete your answer by stating that the values for the wavelength are the same. (1 mark for "Assesses that the wavelength is the $n = 7$ line")	The value of 397 nm is the same as the wavelength for the $n = 7$ line. (1 mark)

Your turn

As spectrographic techniques developed, scientists uncovered more lines in the spectrum of hydrogen. In the 1980s they found a very faint line at 389 nm. Using the Rydberg formula, **assess** the n value of this line. (4 marks)

What was the Rutherford model?

In 1911, the New Zealand–born, British physicist Ernest Rutherford (1871–1937) carried out one of the most important physics experiments into the structure of the atom of the twentieth century.

He was aware of the so-called "plum pudding model" developed by J.J. Thomson, which modelled the atom as tiny, negatively charged electrons embedded in a positively charged "dough", giving a neutral atom overall. Rutherford expected to find results consistent with Thomson's atomic model when he began a series of tests from 1908 onwards. He began by testing the passage of alpha rays through gold foil (Figure 3).

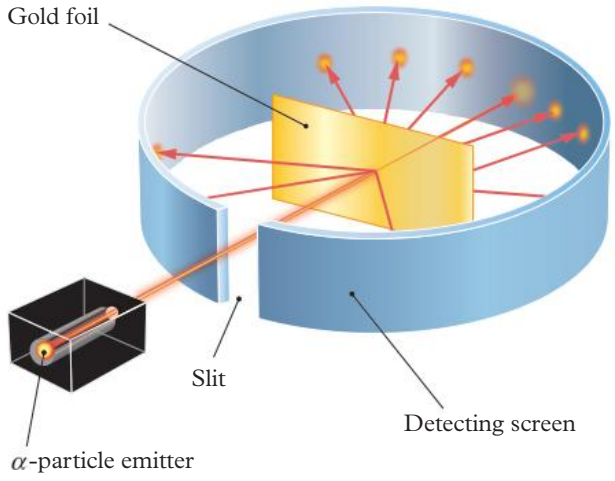


FIGURE 3 Schematic of Rutherford's gold foil apparatus. Most alpha particles passed straight through, but a small percentage (roughly 1 in 8,000) were deflected at large angles, and some even bounced back.

Rutherford was looking for alpha particles with very high deflection angles. He had his assistants Hans Geiger and Ernest Marsden perform the work under his guidance. Rutherford wasn't really expecting anything major, as he pictured the alpha particles as bullets passing right through the "plum pudding". But huge deflections were found for some alpha particles. Rutherford interpreted this as meaning most of the mass of the atom was concentrated in a very small, positively charged region. This led him to formulate a new model of the atom – the **Rutherford model** – that a very small, charged nucleus containing much of the atom's mass was orbited by low-mass electrons (Figure 4).

Rutherford model
a small, central, positively charged nucleus with negatively charged electrons orbiting it

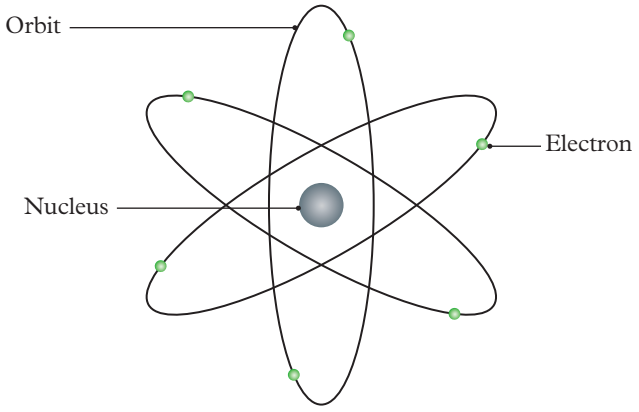


FIGURE 4 Rutherford's model of the atom.

Rutherford proposed the simplest atom of hydrogen as a single positive charge with a single negative electron circling like a planet (Figure 4). He saw the dense positive core as a single entity and his model did not have separate positive particles, later called protons, in the core of bigger atoms. So, he had no need to explain why the positive particles in the core didn't repel and fly apart.

Challenge

Electrons in space

Explain what keeps the electrons from flying off into space in Rutherford's planetary model of the atom. (2 marks)

What were the limitations of the Rutherford model?

The Rutherford model had two serious flaws. The syllabus doesn't require you to describe the limitations of the Rutherford model, so you won't be questioned on them in an external exam. However, it is useful to see that models are replaced when new evidence challenges them, or a different model has greater explanatory power.

First, an electron orbiting a nucleus would be undergoing centripetal acceleration. According to the electromagnetic equations of Maxwell, it would therefore continuously radiate electromagnetic energy. The electron would therefore continuously lose energy, which would cause it to spiral in towards the nucleus. Maxwell's equations predicted that the Rutherford atom would be highly unstable and could not exist for more than around 10^{-8} seconds. But this is not what happens: atoms are stable.

Second, the model couldn't explain the lines in the hydrogen emission spectrum. In the Rutherford model, electrons were allowed to have any energy, and any light produced from them would give a continuous spectrum like a rainbow, and not the clear lines other scientists like Balmer and Rydberg were seeing. This indicated there was something incomplete with the Rutherford nuclear model of the atom. Obviously, a revised model was needed.

A further problem with Rutherford's model came when, in 1917, Rutherford discovered the proton. His model then had to explain why these positive nuclear particles did not repel each other and fly apart. Any new model had to keep them together, as well as explain the two limitations from his existing model. In 1921, Rutherford theorised about the existence of neutrons, which would somehow compensate for the repelling effect of the positive charges of protons by causing an attractive nuclear force. Even so, he was unable to explain what held the protons together in the nucleus. The neutron was discovered by his co-worker James Chadwick in 1932.

From about 1911 onwards the Danish physicist Niels Bohr had been working on his own model of the atom. He was aware that the Rutherford model could not explain the spectrum of hydrogen, so he took a different approach and made a momentous leap with a new model of the atom – one that is still much the same today.

Check your learning 12.1

Check your learning 12.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Describe** the two major features of the Rutherford model of the atom. (2 marks)

Analytical processes

2 **Determine** the wavelength of the violet spectral line of hydrogen ($n = 6$) using the Rydberg formula and compare it to the experimental value of 410.2 nm. (3 marks)

3 **Deduce** whether refraction of light as it passes through a prism increases as its wavelength increases or as its frequency increases. Analyse the diagram in Figure 1 to draw your conclusion. (2 marks)

4 **Infer** how scientists know there is helium on the Sun if they have not been there to collect samples. (1 mark)

Knowledge utilisation

5 **Prove** that using $n = 4$ in the Rydberg formula gives a frequency of 6.17×10^{14} Hz, consistent with spectroscopic evidence. (3 marks)

Lesson 12.2
The Bohr model of the atom

Key ideas

- → Postulates of the Bohr model are the existence of specific quantised energy levels for electron orbits, that transitions between energy levels produced absorption or emission of radiation, and that angular momentum is quantised.
- → Atomic emission spectra of simple atoms can be described mathematically using the Bohr model of the atom.
- → The excitation energy states of any atom can be shown in a quantum energy-level diagram.
- → Emitted or absorbed energy can be calculated using $\Delta E = E_f - E_i$.
- → The difference in energy between states can be used to calculate the frequency or wavelength of emitted or absorbed light using Planck's equation: $\Delta E = hf = \frac{hc}{\lambda}$.

Learning intentions and success criteria

What are the postulates for the Bohr model?

In 1911, Bohr was invited by Rutherford to undertake his postdoctoral research with him at the University of Manchester in England. After returning to his home country of Denmark, Bohr applied Planck's quantum theory to Rutherford's nuclear model and abandoned efforts to describe the atom in classical terms. In 1913, he created the **Bohr model** of the atom (Figure 1), which proposed a revolutionary hypothesis with three major postulates.

Bohr model
model of the atom in which electrons orbit the nucleus in particular circular orbits called "orbitals" or energy levels, with fixed angular momentum and energy. Their distance from the nucleus (their radius or orbit) is proportional to their energy. When an electron moves between these orbitals, it is accompanied by the emission or absorption of a photon.

orbital
region of space around the nucleus of an atom where an electron is likely to be found

1 **Electrons in an atom exist in specific quantised energy levels or orbitals**
Bohr stated that normal atoms exist with their electrons in specific energy levels. At first he called them "stationary states". He did not mean that the electrons were stationary (not moving) but rather that the electrons orbit at fixed distances from the nucleus without emitting energy. He later used the term **orbital** to distinguish them from the planetary "orbits" of the Rutherford model.

2 **Transition between energy levels (orbitals) absorbs or emits electromagnetic radiation**
- Any permanent change in an electron's motion must be accompanied by a complete transition from one energy level to another.
- When an electron moves from one energy level to another, it is accompanied by the emission or absorption of a photon.
- This photon's energy is given by Planck's equation $\Delta E = hf = \frac{hc}{\lambda}$.
- If energy is added to any atom, such as by particle bombardment or sufficient heating, then the electrons are forced into higher energy or "excited" states temporarily by absorbing discrete light quanta.
- As the atom restabilises, the electron transitions back down to an energy level in one jump or a series of allowed steps. Each energy level (orbital) jump results in the emission of a light quantum of discrete, predictable value.
- If an atom absorbs too much energy, then the outermost electron will be promoted completely away from the attraction of the nucleus and will be removed from the atom; the atom is said to be ionised. The energy required is called the "ionisation energy", and for an electron in the $n = 1$ level of the hydrogen atom it is equal to 2.18×10^{-18}J.

3 Angular momentum of an electron in an energy level is quantised

In the Rutherford model of the atom, the electrons could orbit in circular orbits at any distance from the nucleus (i.e. at any radius). This would mean that every element could emit a full spectrum of light as any electron transition would be possible. But this is not the case, so electrons must orbit at certain fixed orbits or radii.

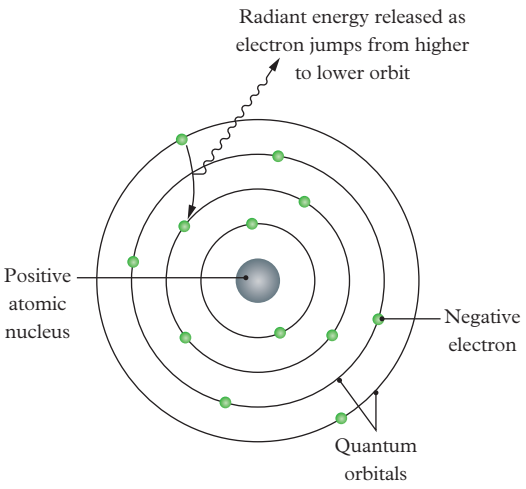


FIGURE 1 Bohr's atomic model. The centre is the nucleus, and the dots are the electrons on different levels orbiting the nucleus.

Bohr made his third postulate not about the radius of an electron's orbit but about the related quantity, **angular momentum**: $L = mvr$.

$$mvr = \frac{nh}{2\pi}$$

where m is the mass of the electron (9.109×10^{-31} kg), v is velocity of the electron, r is the radius of the electron's orbit, n is the energy level (1, 2, 3...), and h is Planck's constant (6.626×10^{-34} Js). This formula was developed by treating the motion of electrons in an atom as consisting of standing waves in circular orbits about the nucleus. This is mentioned later but is mostly beyond the syllabus.

The formula equates angular momentum (mvr) to integer multiples (n) of the constant $\frac{h}{2\pi}$, which therefore means angular momentum is quantised. For example, the first energy level ($n = 1$) has an angular momentum of $\frac{1h}{2\pi}$.

angular momentum for circular motion, the momentum of a particle equal to mvr (symbol: L; unit: kg m^2 s^{-1})

Quantised angular momentum

Firstly, the question of what "radius" means in terms of an electron's orbit intrigues physicists to this day. In terms of a planetary model, which isn't far off from what Bohr postulated, the radius is simple: it is equivalent to a radius such as Earth's (almost circular) orbit around the Sun. However, it is now known that the motion of an electron is not a distinct orbit but more like a fuzzy cloud around the nucleus. It is more like trying to measure the radius of the orbit of a fruit fly as it moves around a honey pot. The probability of finding the electron close to the nucleus is high, and that probability gets lower further away from the nucleus.

To calculate a radius, you need a velocity, and you do not have that. You will not be required to calculate radius or angular momentum in the external exam, but you do need to know that angular momentum is quantised. That is, it increases in integer multiples of $\frac{h}{2\pi}$. This is different to what we see in classical physics. A satellite orbiting the Earth can have any value of angular momentum, so long as gravity can provide the centripetal force. Not so with electrons. They can only have discrete values of angular momentum. This, we will see later, is a consequence of the wave nature of the atom.

Study tip

A handy way of recalling Bohr's three postulates is STAB: specific energy levels, transitions, angular momentum, Bohr.

To show how quantisation works, here is the angular momentum, $L = mvr = \frac{nh}{2\pi}$, for the first three energy levels of a hydrogen atom. You can see that the angular momentum for $n = 2$ is double the angular momentum for $n = 1$, and so on.

For $n = 1$:

$$L = \frac{1 \times 6.626 \times 10^{-34}}{2\pi} = 1.055 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$$

For $n = 2$:

$$L = \frac{2 \times 6.626 \times 10^{-34}}{2\pi} = 2.109 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$$

For $n = 3$:

$$L = \frac{3 \times 6.626 \times 10^{-34}}{2\pi} = 3.164 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$$

Challenge

Multiple lines in hydrogen spectra

When you look at the spectrum of hydrogen in the lab, you only see light from four transitions.

1 **Explain** why you do not observe light from other transitions. (2 marks)

2 **Explain** why the spectrum of hydrogen contains so many lines when hydrogen contains only one electron. (2 marks)

What were the limitations of the Bohr model?

The Bohr model replaced the Rutherford model because it could explain observations that Rutherford's could not. However, the Bohr model also could not explain all observations. You will not need to describe these in the external exam, but it can be useful to understand how the model of the atom developed.

The Bohr model could not explain the:

- **spectra of large atoms.** The Bohr model could only successfully explain the spectra of hydrogen and hydrogen-like ions with one electron.
- **relative spectra intensity.** The Bohr model could not explain why the intensity of spectral lines was not equal. This suggests that some transitions are favoured more than others.
- **hyperfine spectral lines.** With better equipment, previously undiscovered spectral lines were observed that accompanied the other more visible lines. These were called "hyperfine lines". These were not explained by the Bohr model.
- **Zeeman effect.** When hydrogen gas is excited in a magnetic field, the emission spectrum shows a splitting of lines. This is now known to be due to the magnetic field of the electron, but it was not explained by the Bohr model.
- **specific energy levels.** Bohr proposed that the electrons were in specific quantised energy levels, but he could not explain why.

Nevertheless, his application of the quantum theory to atomic structure was very important, and for his work, Bohr was awarded the Nobel Prize in Physics in 1922. Today, the artificial radioactive element of atomic number 107 is called bohrium (Bh) in his honour.

How are atomic spectra analysed using the Bohr model?

principal quantum number a discrete variable assigned to each electron in an atom to describe the energy level of the electron, with higher numbers representing higher potential energy for electrons further from the nucleus (symbol: n).

The Bohr model could explain the absorption and emission spectra of elements. Now it was possible to relate the electron transitions and the wavelengths of light quantitatively.

Bohr could choose orbitals for the hydrogen electron that would give exactly the wavelengths for the emitted spectral lines of the hydrogen spectrum, according to the Rydberg equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{ for } n = 3,4,5...$$

The terms n_i and n_f are called the initial and final **principal quantum numbers**.

What's the difference between emission and absorption transitions?

An emission spectrum is produced when atoms are heated or subjected to an electrical discharge. The electrons end up at high energy levels and fall to lower levels, thus emitting photons. Some may be visible, such as when you heat copper salts in a flame and get an apple-green colour. Other photons may be in the ultraviolet or infrared regions. It all depends on the starting and final energy levels that the electron transitions between.

In essence, an emission spectrum is when electrons transition from high energy to low energy. An absorption spectrum is when electrons transition from low to high energy. We said earlier that atomic hydrogen has an emission spectrum with a purple glow. That is because its photons in the visible region are mostly blues and violets. There are red photons, although fewer, and together this mix appears purple to our eyes.

Figure 2 shows the energy-level diagram for hydrogen. This shows the Bohr orbitals and their corresponding energies in joules and eV, with the series of spectral lines present in the hydrogen spectrum. The spectral series, showing the possible transitions between energy levels, are named after their discoverers.

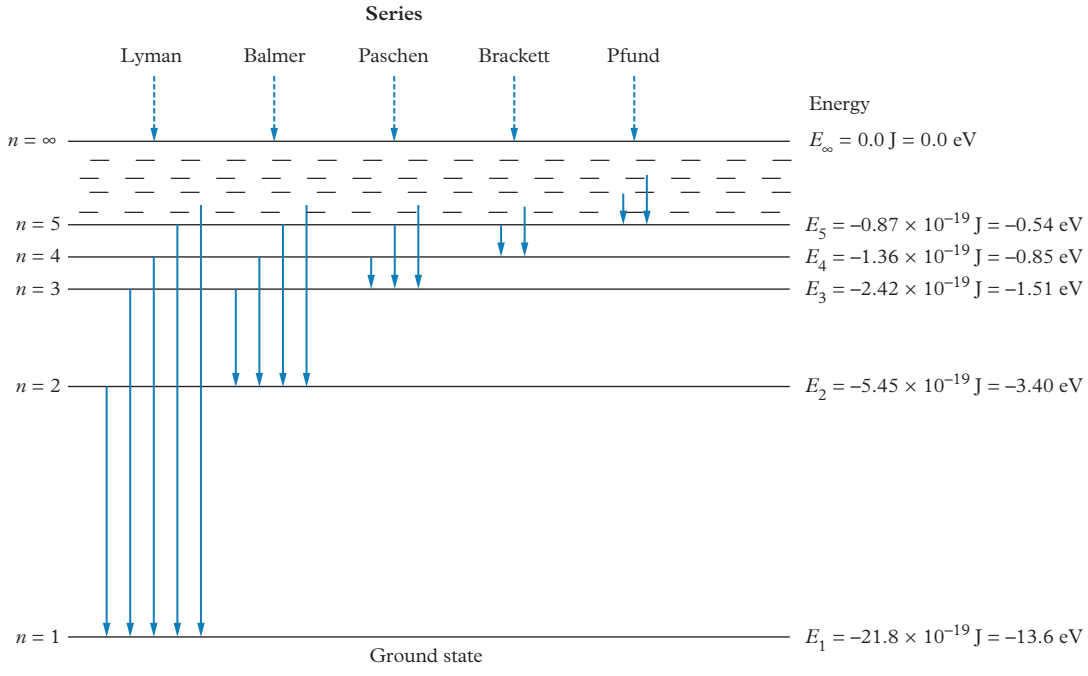


FIGURE 2 Energy-level diagram for hydrogen. The arrows pointing down indicate that light is being emitted.

Notice that in the diagram the energy associated with each level is negative. For example, the ground state energy ($n = 1$) has a value of -21.8×10^{-19}J. This represents the amount of energy needed to remove an electron from the ground state, $n = 1$, to infinity, ∞. It is called the ionisation energy for hydrogen. Because we can say that the electron at infinity has zero energy, by definition all possible energy states of the hydrogen atom can be regarded as negative.

The four lines of the Balmer series are in the visible region and correspond to the transitions in Table 1. To calculate the frequency of light based on the energy of the photon, we use Planck's equation, $E = hf$. We assume that the law of conservation of energy applies, so the photon energy exactly matches the change in energy of the electron involved in the transition.

Study tip

In the external exam, you will see energy-level diagrams like Figure 2, with energies as negative values. However, you may come across energy-level diagrams that show the ground state ($n = 1$) as zero energy and the other energy levels as positive values. These diagrams show the potential energy of the electron in an excited state rather than the energy needed to move the electron to infinity. You can work out electron transitions in the same way – just pay attention to the labels on the diagram.

TABLE 1 Data from the measured spectrum of hydrogen

Colour	Wavelength λ (nm)	Frequency f (Hz)	Photon energy E (J)	Transition $n_i \rightarrow n_f$
Observed	Measured	$f = \frac{c}{\lambda}$	$\Delta E = hf$	$\Delta E = E_f - E_i$
Red	656	4.57×10^{14}	3.02×10^{-19}	$3 \rightarrow 2$
Aqua	486	6.17×10^{14}	4.08×10^{-19}	$4 \rightarrow 2$
Blue	434	6.91×10^{14}	4.57×10^{-19}	$5 \rightarrow 2$
Violet	410	7.32×10^{14}	4.84×10^{-19}	$6 \rightarrow 2$

Worked example 12.2A

Calculating emission spectrum energy transitions in joules

a Using the energy-level diagram for hydrogen (Figure 2) **identify** the energy of an electron, in joules, in both the fifth and second energy levels of the hydrogen atom. (1 mark)

b **Calculate** the frequency of the light emitted when an electron transitions from the fifth to the second energy level. Give your answer to an appropriate number of significant figures. (2 marks)

c **Calculate** the wavelength of this emitted light in nanometres. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Identify" means to recognise and state. "Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Recognise that this is a question about light emitted during an electron transition. Gather the relevant data from the question and select the appropriate formula. In this case, we need to use Planck's equation.	The transition is from $n = 5$ to $n = 1$. $\Delta E = hf$
Step 3: From Figure 2, identify the energy of the electron associated with each level. (1 mark for "Determines the energy of the two levels")	**a** $n = 5$: $E_5 = -0.87 \times 10^{-19}$ J $n = 2$: $E_2 = -5.45 \times 10^{-19}$ J (1 mark)
Step 4: The energy change happens in the transition from the fifth ($n = 5$) to the second ($n = 2$) energy level. The negative value means the electron is losing energy and hence a photon is emitted so that energy is conserved. Calculate the energy difference. (1 mark for "Calculates the change in energy")	**b** $\Delta E = E_2 - E_5$ $= -5.45 \times 10^{-19} - (-0.87 \times 10^{-19})$ $= -4.58 \times 10^{-19}$ J (1 mark)
Step 5: Rearrange the equation to make f the subject, and substitute in the known values. Calculate the frequency, f. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Determines the frequency")	$E = hf$ $f = \frac{E}{h}$ $= \frac{4.58 \times 10^{-19}}{6.626 \times 10^{-34}}$ $= 6.91 \times 10^{14}$ Hz (1 mark)
Step 6: Use the wave equation to calculate the wavelength in metres. (1 mark for "Provides appropriate mathematical reasoning")	**c** $v = f\lambda$ $\lambda = \frac{c}{f}$ $= \frac{3 \times 10^8}{6.91 \times 10^{14}}$ $= 4.34 \times 10^{-7}$ m (1 mark)

Think	Do
Step 7: Convert to nm by dividing by 10^{-9}. The approach shown here is a simple "×1" method to convert units. You multiply by 1 nm divided by 10^{-9} m (which are equal), which means you are just multiplying by 1 and the answer is in nm. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Provides correct answer"). Allow follow-through errors.	$\lambda = \frac{4.34 \times 10^{-7}\,\text{m}}{10^{-9}\,\text{m nm}^{-1}}$ $= 434\,\text{nm}$ (1 mark)

Your turn

a Using the energy-level diagram for hydrogen (Figure 2), **identify** the energy of an electron, in joules, in both the third and first energy levels of the hydrogen atom. (1 mark)

b **Calculate** the frequency of the light emitted when an electron transitions from the third to the first energy level. Give your answer to an appropriate number of significant figures. (2 marks)

c **Calculate** the wavelength of this emitted light in nanometres. Give your answer to an appropriate number of significant figures. (2 marks)

How do we use energy-level diagrams in electron volts?

You may have noticed that energy-level diagrams show the energy of each level in electron volts, eV, as well as (or instead of) joules, J (Figure 3). You will often see energy-level diagrams in eV in the external exams. You can determine the wavelength of light produced when an electron transitions between levels given in eV in the same way you would for energy levels given in J.

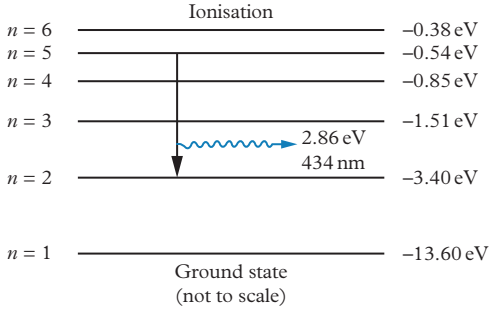


FIGURE 3 Energy-level diagram for hydrogen in eV. The *y*-axis is not to scale.

In this case, the vertical axes are labelled with the energy in eV, and an electron is moving from the $n = 5$ to the $n = 2$ energy level. We can calculate its energy in eV, convert that to J, then calculate its wavelength in nm as follows:

$$\begin{aligned}\Delta E &= E_\text{f} - E_\text{i}\\ &= -3.40 - (-0.54)\\ &= -2.86\ \text{eV}\\ &= 2.86 \times 1.60 \times 10^{-19}\,\text{J}\\ &= 4.58 \times 10^{-19}\,\text{J}\\ \lambda &= \frac{hc}{\Delta E}\\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{4.58 \times 10^{-19}}\\ &= 4.34 \times 10^{-7}\,\text{m}\\ &= 434\,\text{nm (blue)}\end{aligned}$$

Worked example 12.2B

Determining emission transitions in eV

An energy-level diagram for aluminium is shown in Figure 4. Consider an aluminium atom that emits a photon with a wavelength of 1,108.2 nm during a particular electron transition. **Determine** the principal quantum numbers, n, of the initial and final energy levels associated with this transition. (4 marks)

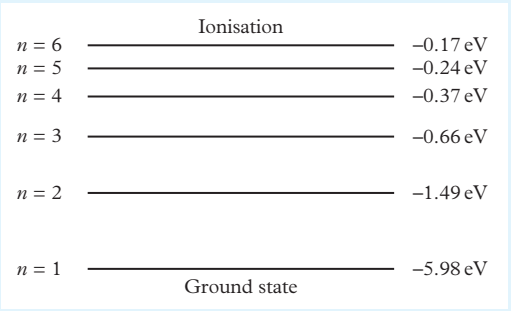


FIGURE 4 Energy-level diagram for aluminium

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$\lambda = 1{,}108.2\,\text{nm}$ $\Delta E = hf = \frac{hc}{\lambda}$
Step 3: Calculate the energy of the photon associated with the transition using the version of Planck's equation in terms of wavelength. (1 mark for "Correctly substitutes values"; 1 mark for "Provides correct value for energy")	$\Delta E = \frac{hc}{\lambda}$ $= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1{,}108.2 \times 10^{-9}}$ (1 mark) $= 1.79 \times 10^{-19}\,\text{J}$ (1 mark)
Step 4: Convert J to eV by dividing by 1.6×10^{-19}. (1 mark for "Converts J to eV")	$\Delta E = 1.79 \times 10^{-19}\,\text{J}$ $= \frac{1.79 \times 10^{-19}}{1.60 \times 10^{-19}}$ $= 1.12\,\text{eV}$ (1 mark)
Step 5: The question asked you to identify the principal quantum numbers (energy levels). You need to identify these values from the energy-level diagram (Figure 4) in the question. It is a matter of trial-and-error to identify the pair of levels that have the correct difference (1.12 eV). Don't try every possible combination or it will take forever. Write down a few possible pairs that seem to have the right difference and find one that fits. (1 mark for "Identifies the n value of the initial and final energy levels")	$n = 4, E_4 = -0.37\,\text{eV}$ $n = 2, E_2 = -1.49$ $\Delta E = -1.49 - (-0.37)$ $= 1.12\,\text{eV}$ (1 mark)

Your turn

An energy-level diagram for aluminium is shown in Figure 4. An aluminium atom emits a photon with a wavelength of 233.5 nm during a particular electron transition. **Determine** the principal quantum numbers, n, of the initial and final energy levels associated with this transition. (4 marks)

How is an absorption spectrum analysed?

When white light is passed through a heated cloud of gas, the electrons in atoms will jump to higher energy levels if the energy of an incoming photon exactly matches the difference in energy between any two levels in the gas. For example, the energy difference between the second energy level ($n = 2$) and the fourth energy level ($n = 4$) in hydrogen is 2.55 eV (4.09×10^{-19} J). So, a photon with that energy ($\lambda = 486$ nm) will cause a $2 \rightarrow 4$ transition. If photons of different energies (e.g. 3.00 eV (4.80×10^{-19} J), corresponding to a wavelength of 414 nm) bombard hydrogen, it won't cause a $2 \rightarrow 4$ transition because the photon energy is not an equal match. It may cause a different transition where the energies do match.

Be warned though: even though the photon has more than enough energy, don't think that some of the energy of the photon can be used, leaving the photon with the leftover energy. This does *not* happen. It's all or nothing.

An example of two successful transitions is shown in Figure 5. Consider a mixture of photons, some red with an energy of 1.89 eV, and some aqua with an energy of 2.55 eV. If they are allowed to strike atomic hydrogen, they will cause two transitions, because the energy of each colour exactly matches the transition energy required. The red photon matches the 2 → 3 transition, and the aqua photon matches the 2 → 4 transition already discussed. Therefore, both are absorbed.

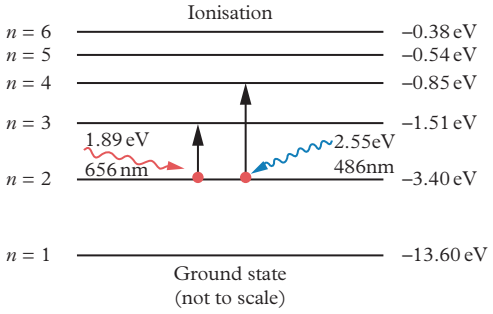


FIGURE 5 Energy-level diagram for hydrogen showing transitions from $n = 2$ to $n = 3$ and $n = 4$.

Absorption spectrum calculations in joules

While the following worked example shows you how to determine possible absorption transitions in eV, you could do the same if the energy diagram were in joules, or the incoming photons were described in terms of wavelength, frequency or energy in joules. It is just a matter of converting from one unit to whatever the energy diagram is presented in (J or eV).

Worked example 12.2C

Determining absorption transitions in eV

The first three energy levels in magnesium are occupied by electrons in the ground state, and the energy of each level is shown in Figure 6. Ultraviolet light with photon energies of 0.85 eV, 1.91 eV, 2.76 eV, 5.75 eV and 6.80 eV is shone through magnesium vapour. **Determine** absorption transitions that may occur. (3 marks)

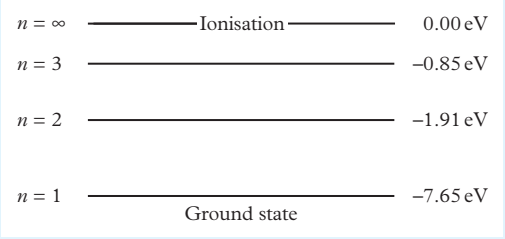
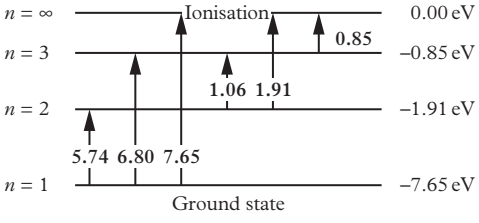


FIGURE 6 Energy-level diagram for magnesium

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formula. In this case we need to know the energy difference between pairs of energy levels. This includes the ionisation level, $n = \infty$, for which the energy is 0 eV. Identify the transitions using a diagram, then make the calculations for each transition. (1 mark for "Identifies all possible transitions"; 1 mark for "Determines the correct magnitude of energy values in eV")	$\Delta E = E_f - E_i$  (1 mark) $n = 1 \rightarrow 2$, $\Delta E = -1.91 - -7.65 = 5.74$ eV $n = 1 \rightarrow 3$, $\Delta E = -0.85 - -7.65 = 6.80$ eV $n = 1 \rightarrow \infty$, $\Delta E = 0 - -7.65 = 7.65$ eV $n = 2 \rightarrow 3$, $\Delta E = -0.85 - -1.91 = 1.06$ eV $n = 2 \rightarrow \infty$, $\Delta E = 0 - -1.91 = 1.91$ eV $n = 3 \rightarrow \infty$, $\Delta E = 0 - -0.85 = 0.85$ eV (1 mark)

Think	Do
Step 3: Identify which photons match the energy-level transitions you calculated in Step 2, and which levels the electrons transition between. (1 mark for "Provides appropriate mathematical reasoning to identify the 1 unsuccessful and 4 successful transitions")	0.85 eV, 3 → ∞ (ionisation) – successful 1.91 eV, 2 → ∞ (ionisation) – successful 2.76 eV, nil – unsuccessful 5.75 eV, 1 → 2 – successful 6.80 eV, 1 → 3 – successful (1 mark)

Your turn

The first three energy levels in sodium are occupied by electrons in the ground state, and the energy of each level is shown in Figure 7. Ultraviolet light with photon energies of 0.27 eV, 0.71 eV, 2.80 eV, 4.56 eV and 5.13 eV is shone through sodium vapour. **Determine** the absorption transitions that may occur. (3 marks)

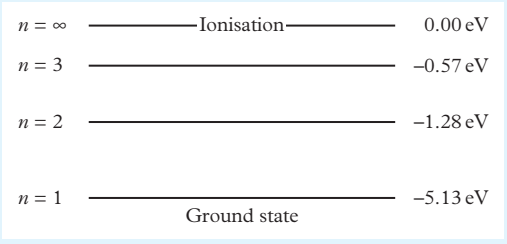


FIGURE 7 Energy-level diagram for sodium vapour

Check your learning 12.2

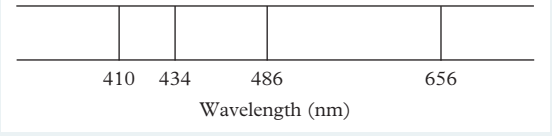
Check your learning 12.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** these terms as applied to quantum atomic theory.
   - **a** Quantum orbital (1 mark)
   - **b** Ionisation energy (1 mark)
   - **c** Principal quantum number (1 mark)

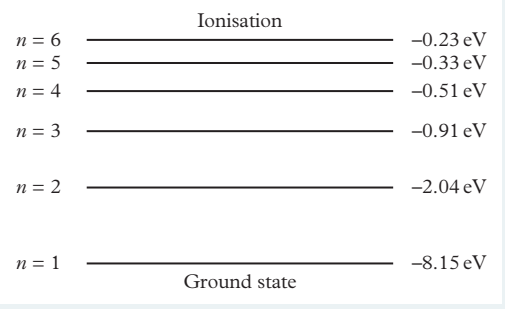
2 **Explain** why the spectrum of hydrogen contains several very bright lines, but the atom itself contains only one electron and one proton. (2 marks)

3 **Explain** why hydrogen has only four emission-spectrum lines in the visible (400–700 nm) region of the spectrum, as shown in the diagram. (2 marks)



4 **Explain** how the Bohr model of the hydrogen atom uses the concepts of light quanta and atomic energy states to explain the specific wavelengths in the hydrogen line spectrum. (2 marks)

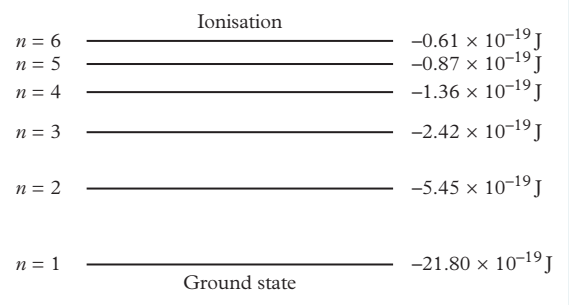
5 The diagram shows the energy levels, in eV, for silicon. **Demonstrate** that when an electron drops from its second energy level to the ground state, an ultraviolet photon is emitted. (3 marks)



Analytical processes

6 **Compare** the different models of the atom proposed by Rutherford and Bohr. (2 marks)

7 The energy-level diagram for hydrogen is shown.



a **Calculate** the energy in joules that must be supplied to raise the atom from quantum state $n = 2$ to $n = 5$. (1 mark)

b **Determine** the frequency of the photon emitted in an electron transition from $n = 5$ to $n = 2$. (1 mark)

c An electron transitions in two stages: from $n = 3$ to $n = 2$, and then from $n = 2$ to $n = 1$. **Determine** if this would produce the light of the same colour as a single transition from $n = 3$ to $n = 1$. (2 marks)

d **Determine** the wavelengths in nm of the first two lines of the Paschen series ($4 \rightarrow 3$, $5 \rightarrow 3$) in the spectrum of hydrogen. (2 marks)

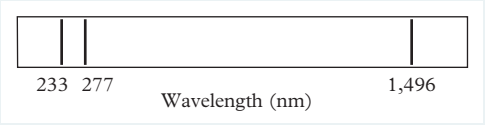
e Light of three different wavelengths, 121.6 nm, 102.6 nm and 150.5 nm, is shone through atomic hydrogen vapour. **Determine** if any of them will be absorbed by the hydrogen gas. Show your working. (2 marks)

Knowledge utilisation

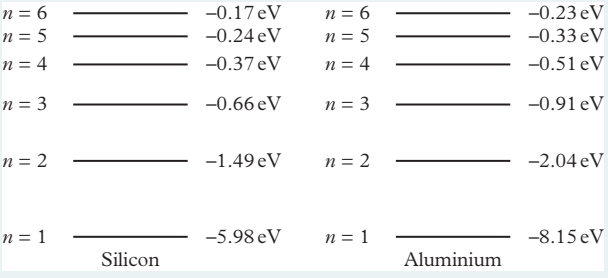
8 **Evaluate** the proposition that the attraction between the nucleus and the outer electron in mercury is lower than the attraction of the nucleus of hydrogen for its outer ($n = 1$) electron. (2 marks)

9 **Assess** whether in the Bohr model of the hydrogen atom, an electron transition from $n = 2$ to $n = 3$ involves the same energy as a transition from $n = 3$ to $n = 2$. (2 marks)

10 The following atomic spectrum is of a substance believed to contain either silicon or aluminium.



The first six energy levels of silicon and aluminium are shown in the following diagram. **Determine** if the spectrum belongs to aluminium or to silicon. Show your working. (5 marks)



Lesson 12.3
Wave–particle duality

Key ideas

- At the atomic level, matter can exhibit wave or particle characteristics.
- Bohr's model was modified by de Broglie to explain that the stability of atoms was due to wave-like electron orbitals, in which each energy level is a standing wave.
- All particles have an associated wavelength and momentum given by the de Broglie equation: $\lambda = \frac{h}{p} = \frac{h}{mv}$.
- De Broglie's condition for a standing wave was given by the equation $n\lambda = 2\pi r$.

Learning intentions and success criteria

What is wave–particle duality?

As you learnt in Module 11, Einstein led the revolution in blurring the distinction between a wave and a particle by introducing the idea of wave–particle duality. The wave–particle duality concept for light and other forms of electromagnetic energy is our current explanation for the behaviour of light in different circumstances. In general, if light energy is interacting with other forms of light energy (for example, in optical effects such as interference and diffraction), then the wave model is the best explanation. If light is interacting with matter (for example, in Compton's work verifying that photons of light have momentum), then the particle model is the best explanation.

Does wave–particle duality apply to matter as well as light?

In 1924, the French physicist Louis-Victor de Broglie (pronounced "de-broy") (1892–1987) took the relationship for the momentum of a photon a little further. He thought that if light were a particle and particles could have a wavelength, then why shouldn't other particles such as electrons and protons have a wavelength? That is, if light can be either a wave or a particle, why couldn't matter also be a wave or a particle? De Broglie hypothesised that matter could demonstrate wave-like properties.

How did de Broglie calculate wavelength for matter?

de Broglie wavelength
a concept in quantum mechanics that describes the wavelength, λ, of a particle, such as an electron, as it behaves like a wave. It is given by the equation $\lambda = \frac{h}{p}$, where h is Planck's constant and p is its momentum, given by $p = mv$.

To derive a formula for wavelength, de Broglie equated Einstein's mass–energy equivalence formula, $\Delta E = mc^2$, and Planck's equation, $\Delta E = hf = \frac{hc}{\lambda}$, as follows:

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc} \text{ (for a photon)}$$

$$\lambda = \frac{h}{mv} \text{ (for a particle)}$$

$$\lambda = \frac{h}{p} \text{ (as } p = mv)$$

where λ is the wavelength of the photon or particle, h is Planck's constant, and p is the momentum of the photon or particle. The wavelength for particles became known as the **de Broglie wavelength**.

Worked example 12.3A

Determining the de Broglie wavelength

An electron ($m = 9.109 \times 10^{-31}$ kg) has a non-relativistic velocity of $6.00 \times 10^{5}\,\text{m s}^{-1}$.

a **Calculate** its momentum in kg m s^{-1}. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** the de Broglie wavelength of the electron in nanometres. Give your answer to an appropriate number of significant figures. (2 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For the first part we must identify the appropriate formula and use it to find an answer. "Determine" means to establish after a calculation
Step 2: Gather the relevant data from the question and select the appropriate formula.	**a** $m = 9.109 \times 10^{-31}$ kg, $v = 6.00 \times 10^{5}\,\text{m s}^{-1}$ $\lambda = \frac{h}{p}$
Step 3: Calculate the classical momentum using the formula $p = mv$. Give your answer to an appropriate number of significant figures – in this case 3 s.f. (1 mark for "Substitutes correctly"; 1 mark for "Calculates the classical momentum correctly")	$p = mv$ $= 9.109 \times 10^{-31} \times 6.00 \times 10^{5}$ (1 mark) $= 5.47 \times 10^{-25}\,\text{kg m s}^{-1}$ (1 mark)
Step 4: Determine the de Broglie wavelength by substituting your value for the momentum from Step 3 into the de Broglie formula. (1 mark for "Determines the wavelength in metres". Allow follow through errors. Award the full 2 marks if the two calculations in steps 3 and 4 are combined.)	**b** $\lambda = \frac{h}{p}$ $= \frac{6.626 \times 10^{-34}}{5.47 \times 10^{-25}}$ $= 1.21 \times 10^{-9}$ m (1 mark)
Step 5: Convert the wavelength to nm by dividing by 10^{-9}. Give your answer to an appropriate number of significant figures, in this case 3 s.f. (1 mark for "Converts m to nm correctly" Allow follow through errors.)	$\lambda = 1.21 \times 10^{-9} \times \frac{1\,\text{nm}}{10^{-9}\,\text{m}}$ $= 1.21$ nm (1 mark)

Your turn

A positron ($m = 9.109 \times 10^{-31}$ kg) is created with a non-relativistic velocity of $7.00 \times 10^{6}\,\text{m s}^{-1}$.

a **Calculate** its momentum in kg m s^{-1}. Give your answer to an appropriate number of significant figures. (2 marks)

b **Determine** its wavelength in nanometres. Give your answer to an appropriate number of significant figures. (3 marks)

Study tip

For calculations involving de Broglie wavelength, you need to check whether the particle is travelling at non-relativistic speeds (< 0.1c) or relativistic speeds (> 0.1c). If the speed is non-relativistic, use $p = mv$, but if the speed is relativistic, use $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$. It is rather unlikely that you would find a relativistic de Broglie wavelength question on the external exam. If so, you would be told it is relativistic. QCAA does not set out to trap you, so the question would be quite clear.

How to deal with relativistic momentum

For high-energy electrons travelling at relativistic speeds (> 0.1c), the effects of relativity on the de Broglie wavelength must be considered. Momentum increases with velocity according to the Newtonian momentum formula $p = mv$. However, once speeds become clearly relativistic (say, when $v > 0.1c$), the relativistic momentum must be considered. So, you should always consider whether the velocity is relativistic. If $v > 0.1c$, then you should calculate the relativistic momentum before substituting it into the de Broglie equation to calculate the wavelength. Thus:

For $v < 0.1c$, use $p = mv$.

For $v > 0.1c$, use $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$.

Why is 0.1c considered the cut-off for using Newtonian formulas? As you saw in the special relativity modules, we consider 0.1c as the cut-off. At this speed, the difference between Newtonian and relativistic calculations is noticable for values to 2 significant figures. Actually, for 2 significant figures the cut-off is 0.24c, and for values to 3 significant figures the cut-off is 0.05c, so we use a simple arbitrary value in-between.

Challenge

Calculating relativistic de Broglie wavelength

For non-relativistic speeds less than 0.1c, the de Broglie wavelength is given by $p = mv$. For relativistic speeds, we need to include the relativistic momentum formula, so the de Broglie wavelength becomes: $p_v = \gamma m_0 v$, where $\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$.

a **Apply** the relativistic formula for momentum to show that an electron travelling at 0.78c has a de Broglie wavelength of 1.95×10^{-12} m. (2 marks)

b **Calculate** the de Broglie wavelength for an electron travelling at 0.5c. (2 marks)

Can we calculate the de Broglie wavelength of macroscopic objects?

The de Broglie wavelengths of anything other than subatomic particles are very short. It makes little sense, for instance, to think of the de Broglie wavelength of a tennis ball or car driving down a highway, even though their wavelength can be calculated. For example, a 1 kg rock travelling at 1 m s^{-1} would have a de Broglie wavelength of 6.626×10^{-34} m.

Although the wavelength of macroscopic objects is so small it's pretty meaningless, you may still need to calculate it in an exam, so we will work through an example here.

Worked example 12.3B

Calculating de Broglie wavelength for a macroscopic object

The world's fastest serve of a tennis ball (m = 56 g) is a speed of 263 km h^{-1} by Australian Sam Groth in South Korea in 2012. **Calculate** the de Broglie wavelength of the ball in metres. Give your answer to an appropriate number of significant figures using scientific notation. (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. We must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formula.	$m = 56\,\text{g} = 0.056\,\text{kg}$, $v = 263\,\text{km h}^{-1}$ $\lambda = \frac{h}{p}$
Step 3: Convert km h^{-1} to m s^{-1}. (1 mark for "Converts speed to m s^{-1}")	$v = 263\,\text{km h}^{-1}$ $= \frac{263 \times 1{,}000}{60 \times 60}$ $= 73.1\,\text{m s}^{-1}$ (1 mark)
Step 4: Note that this speed is non-relativistic ($<0.1c$) so we just use the formula $p = mv$. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Demonstrates correct substitution"; 1 mark for "Calculates the wavelength as 1.6×10^{-34} m". Allow follow through errors for wrong conversions.)	$\lambda = \frac{h}{mv}$ $= \frac{6.626 \times 10^{-34}}{56 \times 10^{-3} \times 73.1}$ (1 mark) $= 1.6 \times 10^{-34}\,\text{m}$ (1 mark)

Your turn

The world's fastest delivery of a cricket ball ($m = 160\,\text{g}$) was $161.3\,\text{km h}^{-1}$ bowled by Shoaib Akhtar (Pakistan) against England in 2003. **Calculate** the wavelength of the ball in metres to an appropriate number of significant figures using scientific notation. (3 marks)

How can the de Broglie wavelength be measured and applied?

The de Broglie wavelength can be measured using various experimental techniques, such as electron diffraction. Measurements have confirmed the predictions of non-relativistic and relativistic calculations, which have provided evidence for the wave–particle duality of matter.

De Broglie wavelength calculations have practical applications in fields such as electron microscopy, where knowing the wavelength of an electron beam allows the size of an object to be accurately calculated at a nanoscale level. They are also used in particle accelerators, where the precise measurement and manipulation of particle wavelengths is crucial for producing high-energy collisions.

Real-world physics

Electron microscopy

Scientists have found that good resolution in microscopy is limited to about half of the wavelength of the lamp used to illuminate the object. As shown by Young's double-slit experiment, the longer the wavelength of light, the greater the amount of diffraction (bending), and hence the greater the interference (blurring of the image). Because our eyes can only detect light of wavelength greater than 400 nm, the best resolution that can be achieved by light microscopes is about 200 nm. A gold atom, with a size of 0.288 nm, is much too small to be seen, so you need a much smaller wavelength.

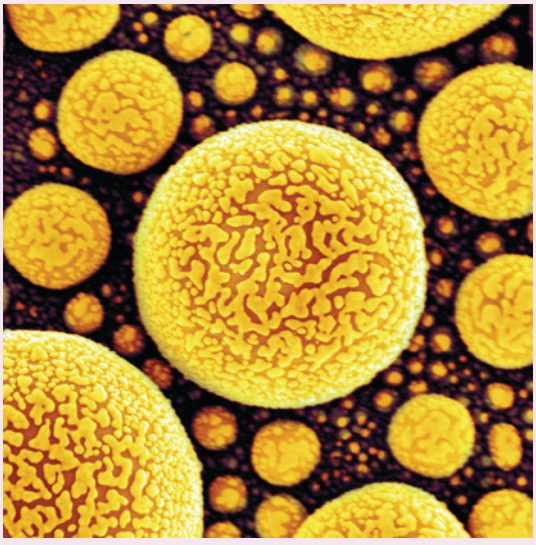


FIGURE 1 Individual gold atoms as seen by a scanning electron microscope. The low diffraction of high-frequency electron waves is clear confirmation of the wave nature of matter.

When it was discovered that electrons had a wave-like nature and that their wavelength was extremely small, new possibilities opened up. At high energies, electrons could have wavelengths as small as 1 pm (picometre, pm = 10^{-12} m, or 0.001 nm). It was then proposed that electron waves could be used in microscopy. The practical limit for electrons used in microscopes is a wavelength of about 1 nm, which means clear viewing (resolution) of objects of less than 0.5 nm is possible. A gold atom (Figure 1), with a diameter of 0.288 nm can be seen, although with some fuzziness. This clearly supports the wave nature of matter.

Apply your understanding

1 Potassium has an atomic diameter of 0.454 nm. **Assess** whether that would make it easier or harder to see under a microscope compared to gold atoms. (2 marks)

2 The diameter of a sodium atom is 372 pm. **Identify** what the diameter would be if it was converted to nm. (2 marks)

3 The average width of a bacterium is 0.2 μm. **Assess** whether this is bigger or smaller than the width a gold atom, and by how many times. (3 marks)

How did de Broglie apply wave theory to electrons in atoms?

The idea that particles such as electrons could have wave-like properties led to a better understanding of the behaviour of electrons in atoms. The problem of how an electron could exist in quantum orbitals without losing energy could be explained by de Broglie's ideas. If electrons had wave-like properties, then, it was argued, they should form standing waves as a result of constructive and destructive interference and therefore not lose energy.

Implications of wave–particle duality for matter

This idea allowed Bohr's quantum orbits (now called orbitals to distinguish them from planetary-type orbits) to be considered as electron wave orbitals whose circumference contained an integral number (whole number) of wavelengths, as shown in Figure 2A.

The standing waves of electrons in orbitals would not lose energy, and the angular momentum of the electrons in their orbitals would be quantised. This de Broglie prediction was experimentally verified by the American team of Clinton Davisson and Lester Germer, as well as the British physicist George Thomson. They showed that a beam of electrons scattered by crystals does, in fact, produce a characteristic wave-diffraction pattern.

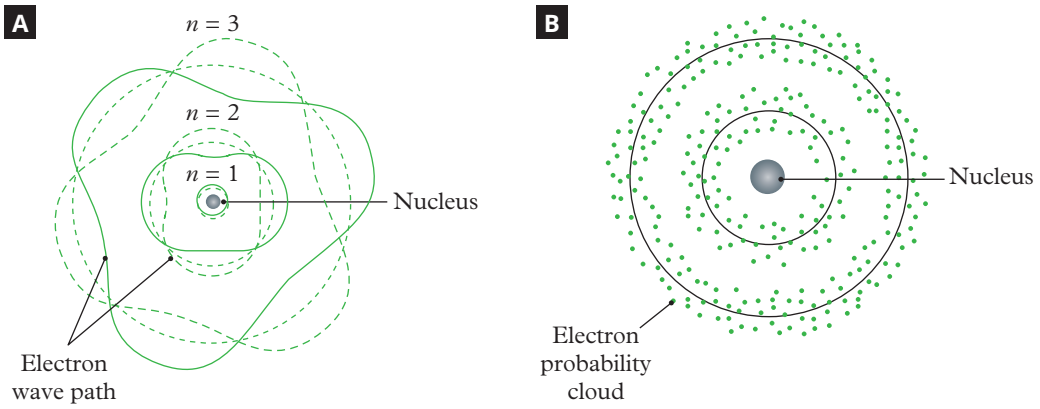


FIGURE 2 (A) Orbital wavelengths and (B) electron clouds. Note the similarity.

Challenge

Photon or electron?

Distinguish between a photon and a photoelectron. (1 mark)

Challenge

Same speed, different wavelength

For an electron and a proton travelling at the same speed, **explain** which has the shorter wavelength. (2 marks)

How did de Broglie derive the angular momentum formula?

Bohr said that each orbit had to be a whole number of wavelengths – that is, $n\lambda = 2\pi r$, where λ is the wavelength, n is a whole number representing the energy level, r is the radius of the orbit, and v is the velocity of the electron. Let's start with that premise and replace λ with $\frac{h}{mv}$ from the earlier equation for the de Broglie wavelength:

$$n\lambda = 2\pi r$$
$$\frac{nh}{mv} = 2\pi r$$
$$mvr = \frac{nh}{2\pi}$$

It shows that angular momentum mvr is quantised. This form of the equation became known as Bohr's condition, and orbital electrons must satisfy this condition. It also allows us to calculate the radius of the orbital if we know the velocity, and vice versa. De Broglie was awarded the Nobel Prize in Physics in 1929 for this work. Today, modern atomic theory does not allow subatomic particles to be treated in the same way as large objects, so some of Bohr's theory has been replaced by other models. The idea of quantised angular momentum, though, has been retained.

Worked example 12.3C

Calculating the radius of the first energy level in helium

Calculate the radius of the $n = 1$ energy level for a single-electron helium ion if the electron's kinetic energy is 8.70×10^{-18} J. Give your answer to an appropriate number of significant figures. (5 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Calculate" means to determine or find a number or answer by using mathematical processes. For each part we must identify the appropriate formula and use it to find an answer.
Step 2: Gather the relevant data from the question and select the appropriate formulas.	$E_k = \frac{1}{2}mv^2$, $E_k = 8.70 \times 10^{-18}$ J $mvr = \frac{nh}{2\pi}$, $m_e = 1.909 \times 10^{-31}$ kg
Step 3: Rearrange the kinetic energy equation to make v the subject. Substitute values and solve for velocity. (1 mark for "Substitutes values correctly"; 1 mark for "Determines correct value for velocity")	$E_k = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2E_k}{m_e}}$ $= \sqrt{\frac{2 \times 8.70 \times 10^{-18}}{9.109 \times 10^{-31}}}$ (1 mark) $= 4.37 \times 10^6$ m s^{-1} (1 mark)
Step 4: Rearrange the Bohr angular momentum formula to make r the subject. Substitute values and calculate the radius. (1 mark for "Demonstrates correct subsitution"; 1 mark for "Calculates the radius in m correctly")	$mvr = \frac{nh}{2\pi}$ $r = \frac{nh}{2\pi mv}$ $= \frac{1 \times 6.626 \times 10^{-34}}{2\pi \times 9.109 \times 10^{-31} \times 4.37 \times 10^6}$ (1 mark) $= 2.65 \times 10^{-11}$ m (1 mark)
Step 5: Convert the radius to nm. Give your answer to an appropriate number of significant figures, in this case 2 s.f. (1 mark for "Calculates the radius in nm" Allow follow through errors. Accept 0.025–0.026 nm)	$r = 0.026$ nm (1 mark)

Your turn

Calculate the radius of the $n = 1$ energy level for a hydrogen atom if the electron's kinetic energy is 2.18×10^{-18} J. Give your answer to an appropriate number of significant figures. (5 marks)

Real-world physics

Heisenberg and uncertainty

The wave–particle concept has led to very complex mathematical models of the nature of atomic structure, called wave mechanics. In 1927, Werner Heisenberg proposed that for an orbital electron in a standing wave. The big question raised by Schrödinger's wave equation was: "What exactly does it describe?" The German physicist Max Born proposed that the square of the wave amplitude at any point is a measure of the probability that an electron can be found at that point at any given time. This gives us a picture of an electron cloud around the nucleus. This probability is as close as scientists can get to defining the position of any electron and is a result of the uncertainty principle. Heisenberg pointed out that any measurement made on a physical system will, in fact, change the system itself and introduce a fundamental uncertainty into measurements of all other properties of that system.

FIGURE 3 Werner Heisenberg, who developed the uncertainty principle.

The principle states: "It is impossible to measure the position and the corresponding momentum of a particle simultaneously with complete accuracy."

Again, it might be obvious that this effect is only important in the subatomic domain. For example, if an electron is measured with a velocity of $4.4 \times 10^6\,\text{m s}^{-1}$ with an uncertainty, Δv, of 0.1%, then the uncertainty in the position, Δx, of the electron can be calculated as $1.3 \times 10^{-8}\,\text{m}$. This uncertainty is, in fact, about 100 times the diameter of the hydrogen atom, so according to the uncertainty principle, we cannot even determine if the electron is within the atom. The uncertainty principle places large limits on the measurement of atomic properties. As Heisenberg said: "Particles do not oscillate because they are a wave. Their wave nature means they are spread out in a fuzzy ball as they travel." He was awarded the 1932 Nobel Prize in Physics for his contribution to quantum mechanics.

Quantum mechanics has solved a lot of the great scientific problems that previously troubled physicists. It is interesting to note, however, that even Albert Einstein had difficulties with the ideas of quantum mechanics and had many famous arguments with Niels Bohr on the subject.

Apply your understanding

1 A formula for uncertainty is $\Delta x = \frac{h}{4\pi m \Delta v}$, where Δv is the uncertainty in the velocity. **Prove** that the positional uncertainty, Δx, of an electron travelling at $2.186 \times 10^6\,\text{m s}^{-1}$ with a velocity uncertainty, Δv, of 0.1% is $2.4 \times 10^{-7}\,\text{m}$. (2 marks)

2 **Propose** whether the uncertainty in position gets bigger or smaller as the uncertainty in velocity increases. (1 mark)

Skill drill

Identifying the correct formula

Science inquiry skills: Preparing for your exams (Lesson 1.13)

Identifying the correct formula from the QCAA Formula and data book is a skill that most students don't practice. Many of the formulas look similar but with different combinations of m, r, v, n, f, λ, p, E and so on. You should be able to identify every formula in the booklet and state what it is used for. Identifying a formula is common in multiple-choice questions on the external exam. Work with a partner to try the following questions.

Practise your skills

1 **Identify** the formula linking the momentum of an electron to the energy level in an atom. (1 mark)

2 The value of Planck's constant, h, is 6.626×10^{-34}. **Identify** the units for this constant. (1 mark)

Check your learning 12.3

Check your learning 12.3: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Describe** wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light. (2 marks)

2 An electron is travelling at a non-relativistic $0.01c$. **Calculate** its

a de Broglie wavelength (1 mark)

b momentum (1 mark)

c kinetic energy. (1 mark)

Analytical processes

3 A proton travelling at $1.0149 \times 10^{4}\,\text{m s}^{-1}$ is found to have the same kinetic energy as a photon of light of frequency $1.300 \times 10^{14}\,\text{Hz}$. **Determine** the mass of the proton. (3 marks)

4 **Deduce** the wavelength of an electron in the ground state ($n = 1$) of a uranium atom, given that the electron is at a radius of 225.9 pm ($1\,\text{pm} = 10^{-12}\,\text{m}$). (2 marks)

5 **Determine** the radius of the $n = 2$ orbital of a caesium atom, given that the electron's wavelength is 1.77 nm. (2 marks)

6 **Determine** the de Broglie wavelength of a neutron ($m = 1.6749 \times 10^{-27}\,\text{kg}$) travelling at the following speeds.

a $0.02c$ (2 marks)

b $20{,}000\,\text{m s}^{-1}$ (2 marks)

7 A coccus bacterium is 0.5 µm in length. **Deduce** with reasoning whether the wavelength of an electron microscope should be half this or twice this to achieve a clear image. (2 marks)

MODULE 12

Lesson 12.4
Review: Quantum theory and atoms

Summary

12.1
- The spectrum of hydrogen hinted at the structure of an atom but was inconclusive.
- Rutherford proposed a model of the atom that consisted of a dense, positive central core surrounded by negatively charged, orbiting electrons.
- Limitations of the Rutherford model included the failure to explain the stability of the atom and to account for spectral lines in hydrogen.

12.2
- Postulates of the Bohr model are the existence of specific quantised energy levels for electron orbits, that transitions between energy levels produced absorption or emission of radiation, and that angular momentum is quantised.
- Atomic emission spectra of simple atoms can be described mathematically using the Bohr model of the atom.
- The excitation energy states of any atom can be shown in a quantum energy-level diagram.
- Emitted or absorbed energy can be calculated using $\Delta E = E_f - E_i$.
- The difference in energy between states can be used to calculate the frequency or wavelength of emitted or absorbed light using Planck's equation: $\Delta E = hf = \frac{hc}{\lambda}$.

12.3
- At the atomic level, matter can exhibit wave or particle characteristics.
- Bohr's model was modified by de Broglie to explain that the stability of atoms was due to wave-like electron orbitals, in which each energy level is a standing wave.
- All particles have an associated wavelength and momentum given by the de Broglie equation: $\lambda = \frac{h}{p} = \frac{h}{mv}$.
- De Broglie's condition for a standing wave was given by the equation $n\lambda = 2\pi r$.

Key formulas

Electron wavelength and radius	$n\lambda = 2\pi r$
Angular momentum (*L*)	$mvr = \frac{nh}{2\pi}$
Rydberg formula	$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$
de Broglie wavelength	$\lambda = \frac{h}{p}$

Review questions 12.4A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 According to the Rutherford model of the atom, the atomic nucleus contains
- **A** all of the atom's positive charge and most of the atom's mass.
- **B** all of the atom's negative charge and most of the atom's mass.
- **C** all of the atom's electric charge and none of the atom's mass.
- **D** all of the atom's mass and none of the atom's electric charge.

2 Following his gold foil experiment, Rutherford concluded that most of the mass of the atom was concentrated in a small area. The observation that led Rutherford to this conclusion was that
- **A** some alpha particles were absorbed.
- **B** most alpha particles were absorbed.
- **C** most alpha particles were deflected.
- **D** some alpha particles were deflected.

3 In Rutherford's experiment, a thin gold foil was bombarded with alpha particles. According to the Thomson "plum-pudding" model of the atom
- **A** all the alpha particles should have been deflected by the foil.
- **B** all the alpha particles should have bounced straight back from the foil.
- **C** alpha particles should have passed through the foil with little or no deflection.
- **D** alpha particles should have become embedded in the foil.

4 Rutherford's alpha-particle scattering experiment established that
- **A** protons are not evenly distributed throughout an atom.
- **B** electrons have a negative charge.
- **C** protons are 1,840 times heavier than electrons.
- **D** gold foil deflects all the alpha particles.

5 The Bohr model of the atom was able to explain the Balmer series because
- **A** larger orbits required electrons to have more negative energy to match the angular momentum.
- **B** differences between the energy levels of the orbits matched the differences between energy levels of the line spectra.
- **C** electrons could exist only in allowed orbits and nowhere else.
- **D** electrons with different energies passed through the prism to produce different spectral lines.

6 An electron will jump to a higher energy level when the energy of the incoming photon
- **A** is greater than the energy of the destination energy level.
- **B** is greater than the energy difference between levels.
- **C** is equal to the energy difference between levels.
- **D** is equal to the energy of the initial energy level.

7 The bright-line emission spectrum of an element is due to
- **A** electrons giving off radiation as they move in circular orbits.
- **B** electrons undergoing constructive and destructive interference.
- **C** absorption of photons from incident radiation.
- **D** electrons transitioning between energy levels in an atom.

8 An electron and a proton both moving at non-relativistic speeds have the same de Broglie wavelength. Which of the following is also the same for the two particles?
- **A** Speed
- **B** Kinetic energy
- **C** Momentum
- **D** Frequency

9 A rock of mass 0.100 kg is thrown with a speed of 50.0 m s^{-1}. Determine its de Broglie wavelength.
- **A** 6.63×10^{-34} m
- **B** 5.30×10^{-34} m
- **C** 1.33×10^{-34} m
- **D** 1.66×10^{-31} m

10 The de Broglie wavelength applies to
- **A** photons only.
- **B** photons and electrons.
- **C** electrons, protons and neutrons.
- **D** all particles.

11 In the equation $mvr = \frac{nh}{2\pi}$, which physical quantity is quantised?

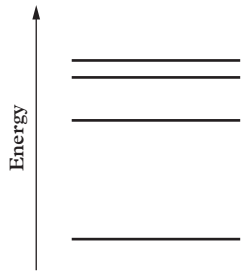
A Linear momentum

B Angular momentum

C Centripetal acceleration

D Atomic radius

12 The diagram shows four energy levels in an atom.



The number of transitions possible within these four lines is

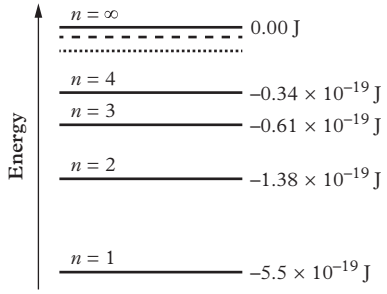
A 3

B 4

C 6

D 7

13 The diagram shows energy levels for a sodium atom.



Select the statement that best indicates the transitions that would give the highest wavelength radiation and the highest energy photons, respectively.

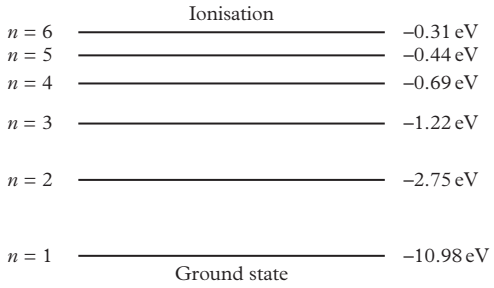
A $3 \rightarrow 2$, $3 \rightarrow 1$

B $3 \rightarrow 1$, $3 \rightarrow 1$

C $4 \rightarrow 3$, $\infty \rightarrow 1$

D $\infty \rightarrow 2$, $3 \rightarrow \infty$

14 The diagram shows the energy levels for a phosphorus atom. The values are in eV.
The expression that best states the energy of the photon in joules for the $3 \rightarrow 1$ transition is



A $\frac{9.76}{1.6 \times 10^{-19}}$ J

B $9.76 \times 1.6 \times 10^{-19}$ J

C $9.76 \times 6.626 \times 10^{-34}$ J

D $\frac{1.6 \times 10^{-19}}{9.76}$ J

Review questions 12.4B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

15 **Explain** the importance of the large deflection angle in Rutherford's gold-foil experiment. (2 marks)

16 **Identify** the three postulates of the Bohr model. (3 marks)

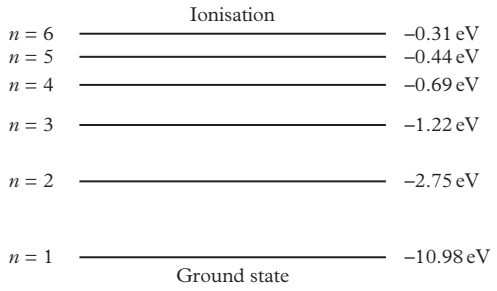
17 **Describe** the wave–particle duality of light. (1 mark)

18 **Identify** evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light. (2 marks)

19 **Explain**, for the Rydberg formula, whether it is true to say that the bigger the difference between the n_i and n_f values, the longer the wavelength. (2 marks)

20 **Clarify** whether the angular momentum of an electron refers to the electron spinning on its own axis or orbiting the nucleus. (1 mark)

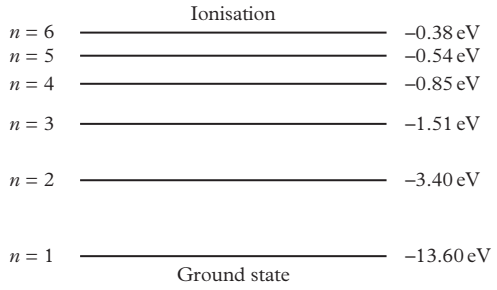
21 Consider the energy-level diagram for phosphorus shown in the diagram.



Identify the number of the initial and final energy levels for a transition in which an electron absorbs a photon of 1.96 eV. (2 marks)

Analytical processes

22 The diagram shows the atomic energy levels for hydrogen.



A hydrogen atom is bombarded with an incoming photon of wavelength 365 nm.
Determine if this photon will be absorbed by an electron in the hydrogen atom. **Explain** your answer. (4 marks)

23 An electron ($m_e = 9.109 \times 10^{-31}$ kg) is travelling at 0.05c. **Determine** its

a de Broglie wavelength (2 marks)

b momentum (2 marks)

c kinetic energy. (2 marks)

24 It is found that a neutron travelling at 1.99×10^4 m s^{-1} has the same kinetic energy as a photon of light with a frequency 5×10^{14} Hz. **Determine** the mass of the neutron. (3 marks)

25 **Determine** the radius of the $n = 1$ energy level in a francium atom given that the de Broglie wavelength of the electron is 0.469 nm ($m_e = 9.109 \times 10^{-31}$ kg). (3 marks)

26 **Determine** the de Broglie wavelength of a neutron ($m = 1.6749 \times 10^{-27}$ kg) travelling at 0.08c. (2 marks)

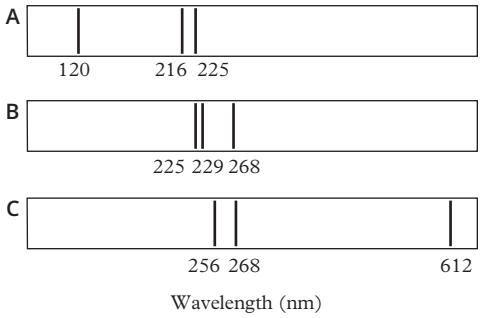
27 A proton ($m_p = 1.672 \times 10^{-27}$ kg) has the same momentum as an electron travelling at 0.09c.

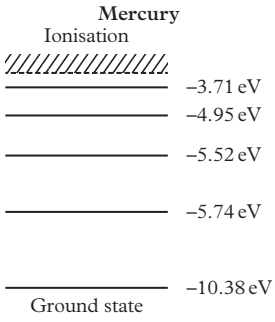
a **Compare** their speeds. (2 marks)

b **Compare** their wavelengths. (2 marks)

Knowledge utilisation

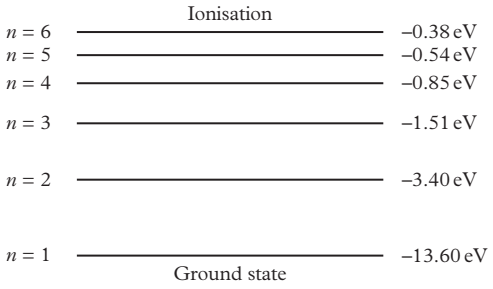
28 The partial spectrum of three elements A, B and C was prepared. One of the elements was mercury, whose atomic energy diagram is shown.





Determine which substance, A, B or C, is mercury. **Explain** your answer. (5 marks)

29 The energy-level diagram for hydrogen is shown in the diagram.

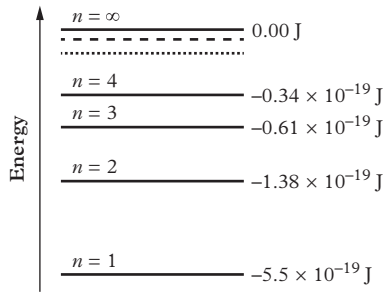


a **Deduce** the amount of energy that must be supplied to raise the atom from quantum state $n = 1$ to $n = 4$. (2 marks)

b **Determine** the energy needed to ionise the atom from its ground state. (2 marks)

c **Predict** the frequency of the photon emitted in an electron transition from $n = 5$ to $n = 1$. (3 marks)

30 Consider the energy-level diagram for the sodium atom shown in the diagram.



a **Determine** the energy of the photon emitted in a transition from $n = 4$ to $n = 2$. (2 marks)

b **Predict** the transition that will have to take place for an electron to completely absorb a 1.04 eV photon. (2 marks)

31 The energies of possible quantum states for a gas are listed below. Assume that the ground-state energy has been included in this list.

-8.64×10^{-19} J

-5.76×10^{-19} J

-16.6×10^{-19} J

-11.5×10^{-19} J

-6.72×10^{-19} J

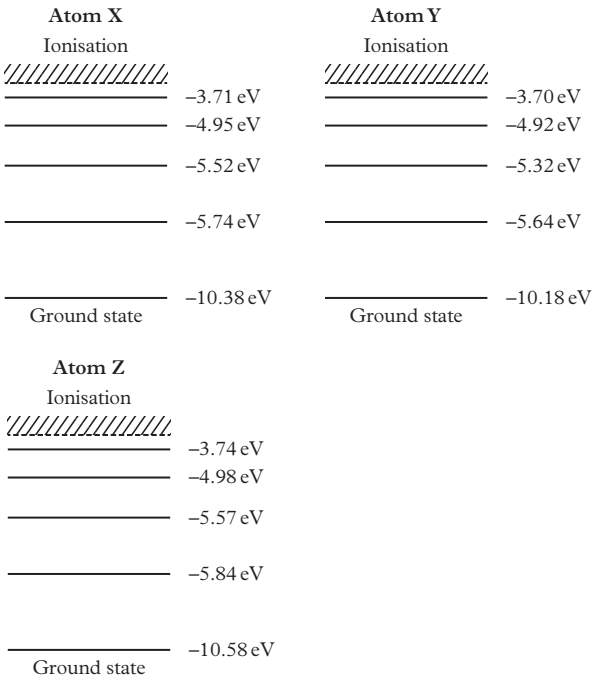
a **Construct** an appropriate energy-level diagram by reorganising and assessing these values. (2 marks)

b **Propose** the shortest and longest wavelength expected in the emission spectra of this gas under excitation. (4 marks)

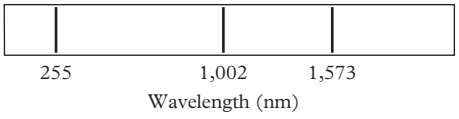
c **Determine** how much energy is required (in J) to cause the gas atoms to transition from energy level $n = 3$ to energy level $n = 4$. (2 marks)

32 An electron at rest is accelerated through a potential difference of 100.0 V, where it gains substantial kinetic energy. A student claims that because its energy is increasing, its wavelength must also be increasing. **Evaluate** this claim. Your answer should include a calculation of its wavelength after being accelerated. (3 marks)

33 The atomic energy levels of three atoms are shown in the diagram (not to scale).



The emission spectrum of an unknown element is also shown.



Determine which atom is most likely to be the unknown atom. (5 marks)

Data drill

Atomic energy level graphs: *E* vs *n*

Students conducted a simulation to investigate the trend in electron energy at various levels in a hydrogen atom. They plotted the energy in electron volts of the electron at each level against the principal quantum number, n, of the level. It appeared to be an inverse squared relationship, so they linearised the data by plotting energy, E, versus $\frac{1}{n^2}$. Their table and the graph are shown.

TABLE 1 Results from simulation

n	$\frac{1}{n^2}$	E (eV)
1	1.00	13.63
2	0.25	3.41
3	0.11	1.51
4	0.06	0.85
5	**Q1**	0.55

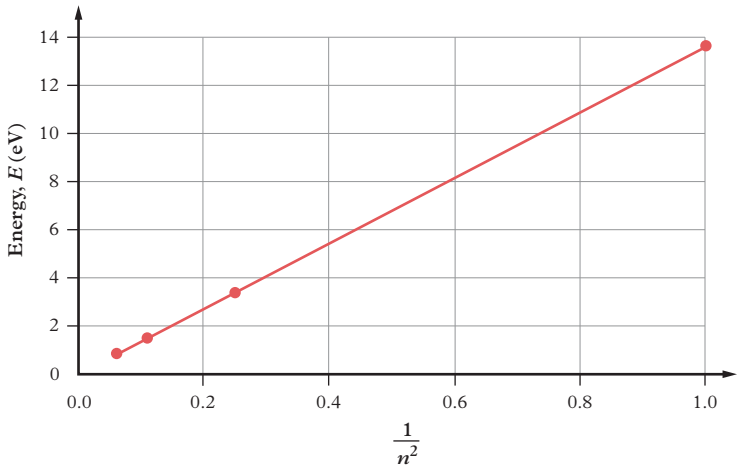


FIGURE 1 Results from simulation.

Apply understanding

1 **Calculate** the $\frac{1}{n^2}$ value in the table for $n = 5$ (labelled Q1). (1 mark)

2 **Calculate** the energy for $n = 1$ in joules. (2 marks)

Analyse data

3 **Identify** a mathematical equation for the trendline for energy, E_n, of an electron and its principal quantum number, n. Use evidence from the graph or data table to support your answer. (3 marks)

Interpret evidence

4 Using the trendline equation, **determine** the energy in electronvolts for $n = 6$. (2 marks)

5 **Deduce** why there were no data on the x-axis between 0.3 and 1.0. (2 marks)

Module 12 checklist: Quantum theory and atoms

UNIT 4

Topic 2 review

Multiple choice

(1 mark each)

1 Young's double-slit experiment demonstrated that light
- **A** undergoes diffraction and interference like sound and water waves.
- **B** behaves like a particle in some experiments.
- **C** produces a rainbow from a prism.
- **D** is a longitudinal wave-like sound.

2 Select the option that gives the relationships in Wien's displacement law.
- **A** The maximum temperature produced by light of a given wavelength
- **B** The wavelength of radiation emitted by an object at a given temperature
- **C** The temperature of an object when heated by light of a particular wavelength
- **D** The wavelength of radiation at which total emissive power is maximum at a given temperature

3 The surface of the Sun has a temperature of 5,527°C and can be modelled as a black body that produces photons across a range of wavelengths. Calculate the wavelength at which the maximum number of photons are produced from the surface of the Sun.
- **A** 477 nm
- **B** 500 nm
- **C** 524 nm
- **D** 2,000 nm

4 The law that states the relationship $\lambda_{max} T$ = constant is
- **A** de Broglie's law.
- **B** Kirchhoff's law.
- **C** Planck's law.
- **D** Wien's law.

5 As a black body is heated, what happens to the intensity and wavelength of the emitted light?
- **A** Intensity is decreased; wavelength is decreased.
- **B** Intensity is decreased; wavelength is increased.
- **C** Intensity is increased; wavelength is decreased.
- **D** Intensity is increased; wavelength is increased.

6 A photon has
- **A** non-zero mass but finite value of momentum.
- **B** zero mass and a finite value of momentum.
- **C** non-zero mass but zero momentum.
- **D** zero mass and zero momentum.

7 Ultraviolet photons of wavelength 200 nm are incident on platinum, which has a work function of 5.6 eV. Determine the maximum kinetic energy of the emitted photoelectrons.
- **A** 0.6 eV
- **B** 1.2 eV
- **C** 5.6 eV
- **D** 6.2 eV

8 Light of frequency 1.85×10^{15} Hz is incident on a metal surface with a work function of 5.38×10^{-19} J. Determine the kinetic energy of a photoelectron ejected from the metal surface.
- **A** 3.4 eV
- **B** 4.3 eV
- **C** 7.7 eV
- **D** 11.0 eV

9 Ultraviolet photons of energy 6.2 eV are incident on aluminium, which has a work function of 4.3 eV. Determine the option that gives the maximum kinetic energy of the emitted photoelectrons.
- **A** $1.9 \times 1.6 \times 10^{-19}$ J
- **B** $6.2 \times 1.6 \times 10^{-19}$ J
- **C** $\frac{1.9}{1.6 \times 10^{-19}}$ J
- **D** $\frac{10.5}{1.6 \times 10^{-19}}$ J

10 Recall the name of the frequency below which no electrons are emitted from metal surface.
- **A** Maximum frequency
- **B** Minimum frequency
- **C** Threshold frequency
- **D** Ionisation frequency

11 In the photoelectric effect, the energies of electrons liberated by light depend on the

A source of the light.

B intensity of the light.

C voltage of the source.

D frequency of the light.

12 The stopping potential in a photoelectric chamber is 3.00 V. The work function, W, for the metal is 2.20 eV. Calculate the incident photon energy and frequency.

A 1.12×10^{-19} J, 1.69×10^{14} Hz

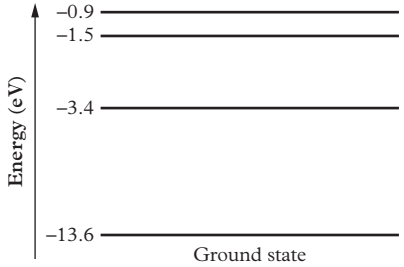
B 1.12×10^{-19} J, 1.26×10^{15} Hz

C 8.32×10^{-19} J, 1.26×10^{15} Hz

D 8.32×10^{-19} J, 1.69×10^{14} Hz

13 The four lowest energy levels of a hydrogen atom are shown in the diagram.

Four different photons, W, X, Y and Z, are incident on a hydrogen atom in its ground state. Their energies are: $E_W = 8.6$ eV, $E_X = 10.2$ eV, $E_Y = 12.1$ eV, $E_Z = 12.6$ eV. Determine which photon(s) could be absorbed by the atom.



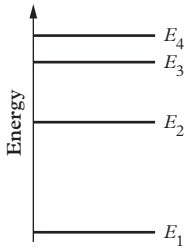
A W only

B X and Y only

C Y and Z only

D X, Y and Z only

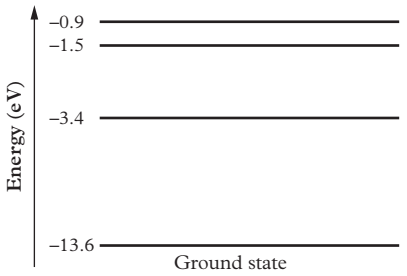
14 The energy levels in an atom are shown in the diagram. The transition E_3 to E_1 produces a red emission line in a spectrum. Select one of the following that could produce the higher frequency blue line.



A E_2 to E_1 **B** E_4 to E_1

C E_4 to E_3 **D** E_4 to E_2

15 The four lowest energy levels of a hydrogen atom are shown in the diagram.



A photon with an energy of 1.9 eV strikes a hydrogen atom in its ground state. Which one of the following is most likely to happen?

A The photon is completely absorbed and causes a transition of an electron from level 2 to level 3.

B The photon is absorbed and causes a $3 \rightarrow 4$ transition, which emits a photon with 1.3 eV.

C The photon is absorbed and raises the ground state electron to −11.5 eV.

D Nothing happens and the photon retains its 1.9 eV.

Short response

16 The wave model of light was supported by phenomena such as diffraction and interference. However, the wave model could not explain some phenomena that were being investigated in the late 1800s.

a **Explain** the role of Young's double-slit experiment in providing a model for light. (1 mark)

b **Identify** one piece of evidence for the particle model and explain why it refutes the wave model of light. (1 mark)

17 A demonstration of the photoelectric effect is set up in which light with a wavelength of 310 nm shines on a metal. As a result, electrons are ejected from the metal at a non-relativistic speed of $0.0024c$.

a **Determine** the work function of the metal. (3 marks)

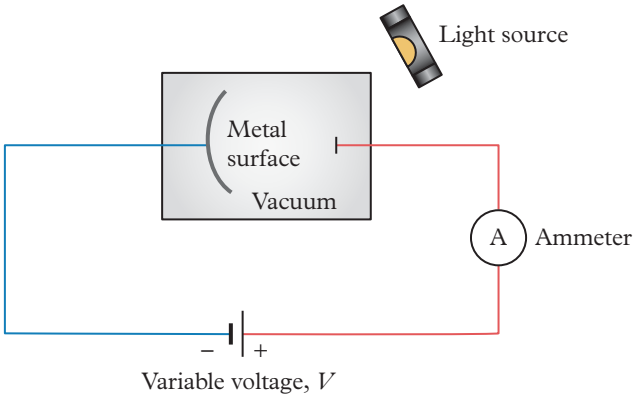
b The wavelength of the light is doubled to 620 nm. **Predict**, with justification, whether any electrons are produced from the metal and, if so, **calculate** their speed it has changed. (3 marks)

18 The photoelectric effect is demonstrated by shining light of a single wavelength on a metal surface. As a result, electrons are ejected from the metal. If the wavelength of light is 290 nm, electrons are ejected with speeds up to $0.0019c$.

a **Determine** the work function of the metal. (3 marks)

b **Propose** changes, if any, that would be observed if the intensity of the light doubled while the initial wavelength of 290 nm remained constant. (2 marks)

19 A photoelectric effect apparatus is set up as shown in the diagram, so that when monochromatic light shines upon a metal surface, all the electrons emitted from the metal surface can be detected by a simple circuit.



A light source of power 3.00 W emits light with a wavelength of 4.60×10^{-7} m. All of the light is incident on a metal surface with a threshold frequency of 5.45×10^{14} Hz and causes electrons to be emitted at a rate of 6.69×10^{18} electrons per second.

a **Calculate** the work function of the metal. (2 marks)

b **Calculate** the maximum kinetic energy of a photoelectron ejected from the metal when it reaches the collector plate. (3 marks)

A new light source is now shone upon the metal plate, which has the same power output, but a longer wavelength of 6.90×10^{-7} m.

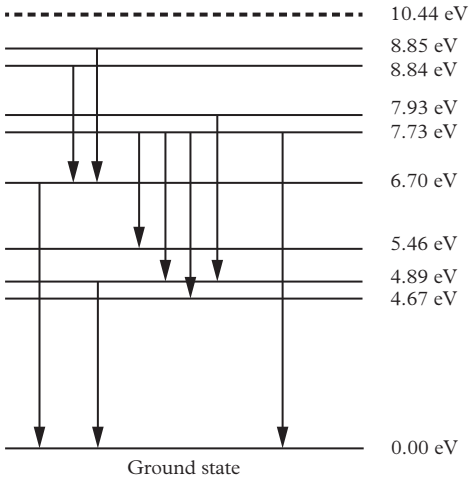
c **Describe** the impact on the photoelectron current of the change of wavelength and explain why it has changed. (4 marks)

20 A hydrogen atom absorbs a photon of ultraviolet light and its electron jumps from level 1 to level 4.

a **Determine** the change in energy of the atom in joules. (3 marks)

b **Determine** the wavelength of the photon in nanometres. (1 mark)

21 The diagram shows the allowed electron transitions for mercury. If there is no allowable transition downwards between the energy levels shown, you can also assume this transition is not allowable upwards.



A ground state electron in a mercury atom is bombarded with electrons of kinetic energy 3.06 eV.

a **Determine** all possible electron transitions. (2 marks)

b **Determine** which two energy levels are involved in the production of the violet line in mercury's emission spectrum, with a wavelength of 409 nm. (3 marks)

22 A simplified diagram of the energy levels for a mercury atom is shown (not to scale).

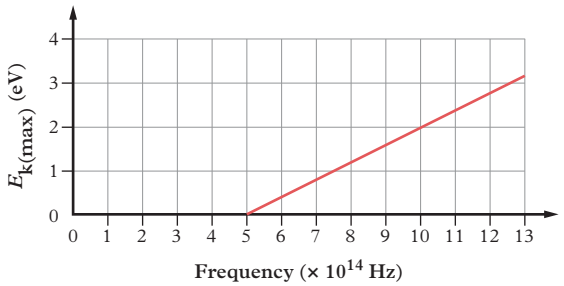
a **Explain** why a mercury atom, while in the first excited state, can absorb a 1.8 eV photon but cannot emit a photon of this energy. (2 marks)

b In a sample of excited mercury atoms, all of the energy levels shown in the diagram are occupied. One of the energy levels is labelled x eV. The emission spectrum of mercury shows lines at approximately 0.9 eV, 1.5 eV and 2.2 eV. Use this information and the diagram to **calculate** x. Give your reasoning. (3 marks)

23 Students conducted three different experiments to investigate the photoelectric effect.

For experiment 1, light of various frequencies was shone on the surface of metal A.

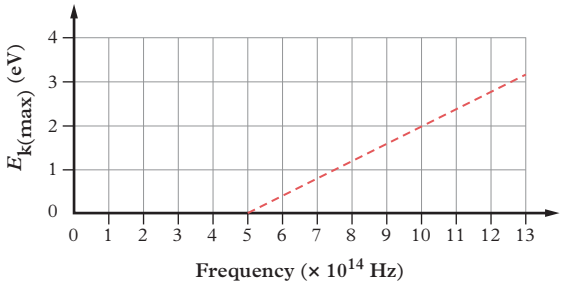
The diagram shows the graph of maximum kinetic energy, $E_{k(max)}$, of the photoelectrons against frequency from experiment 1.



a **Determine** the threshold frequency for metal A. (1 mark)

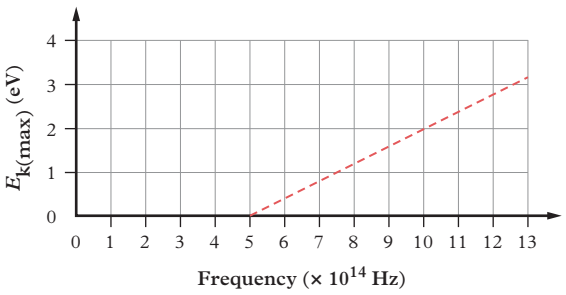
b Use the graph to **calculate** the value of Planck's constant for experiment 1. (3 marks)

c In experiment 2, the intensity of light of each frequency is doubled and again shone on metal A. Copy the graph and draw a solid line to **construct** the graph of maximum kinetic energy against frequency for experiment 2. The dotted line represents the graph from experiment 1. (2 marks)



d In experiment 3, metal A is replaced with metal B that has a work function 50% larger than that of metal A. The original light intensity is used.

Copy the graph and draw a solid line to **construct** the graph of maximum kinetic energy versus frequency for experiment 3. A dotted line shows the results of experiment 1. (2 marks)



24 Classical physics considers electromagnetic radiation as a wave only, and this leads to the conclusion that even relatively low temperature objects should emit high energy radiation.

Referring to the statement above, **explain** how experimental findings have altered our understanding and supported the particle model of light. **Construct** a diagram to support your explanation. (4 marks)

25 According to one model of atoms, electrons in atoms move in stable circular orbits around the nucleus. In an atom modelled in this way, an electron is moving at $2.0 \times 10^{6}\,m\,s^{-1}$. Take the mass of an electron as 9.1×10^{-31} kg. **Calculate** the de Broglie wavelength of this electron. Give your answer in nanometres. (2 marks)

26 **Describe** how the wave nature of electrons can be used to explain the quantised energy levels in atoms. (3 marks)

27 Physicists use the expression "wave–particle duality" because light sometimes behaves like a particle and electrons sometimes behave like waves.

a What evidence do we have that light can behave like a particle? **Explain** how this evidence supports a particle model of light. (2 marks)

b What evidence do we have that electrons can behave like waves? **Explain** how this evidence supports a wave model of electrons. (2 marks)

28 **Explain** why Einstein's explanation of the photoelectric effect required light to be composed of particles rather than waves. (3 marks)

29 **Identify** the three main postulates of Bohr's model of the atom. (3 marks)

TOTAL MARKS

/78 marks

MODULE 13

The Standard Model

Introduction

Up until the end of the nineteenth century, the atom was thought to be a single hard indivisible particle, like a billiard ball. This was Dalton's model introduced in 1808. Little changed until Thomson discovered the electron in 1897, and Rutherford identified the nucleus in 1898. But, to probe deeper inside an atom, the atom had to be smashed apart. This idea was proposed by Rutherford during his presidential address to the Royal Society of London in 1938. He said that to find out the secrets of an atom, he needed a beam of charged particles more energetic than those produced by natural radioactivity. Physicists at the Cavendish Laboratory at the University of Cambridge in England took up the challenge and invented the first particle accelerator.

Since then, physicists have discovered strange antimatter particles such as positrons and antiprotons, as well as muons and well over 100 other new particles. Almost a century after the first accelerator was developed, a new generation of high-energy accelerators is shared by research groups from numerous laboratories and funds from all over the world. Trying to classify the many particles that have been found has sent physics on a wild goose chase, but there is now a model that ties them all together. It is not perfect, but the Standard Model is the best physicists have come up with (for the time being).

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to the Standard Model before you start.

Subject matter

Science understanding

→ Describe the concepts of elementary particles and antiparticles.
→ Identify the six types of quarks.
→ Describe baryons and mesons.
→ Identify the six types of leptons.
→ Identify the four gauge bosons.
→ Compare the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons.
→ Contrast the fundamental forces experienced by quarks and leptons.

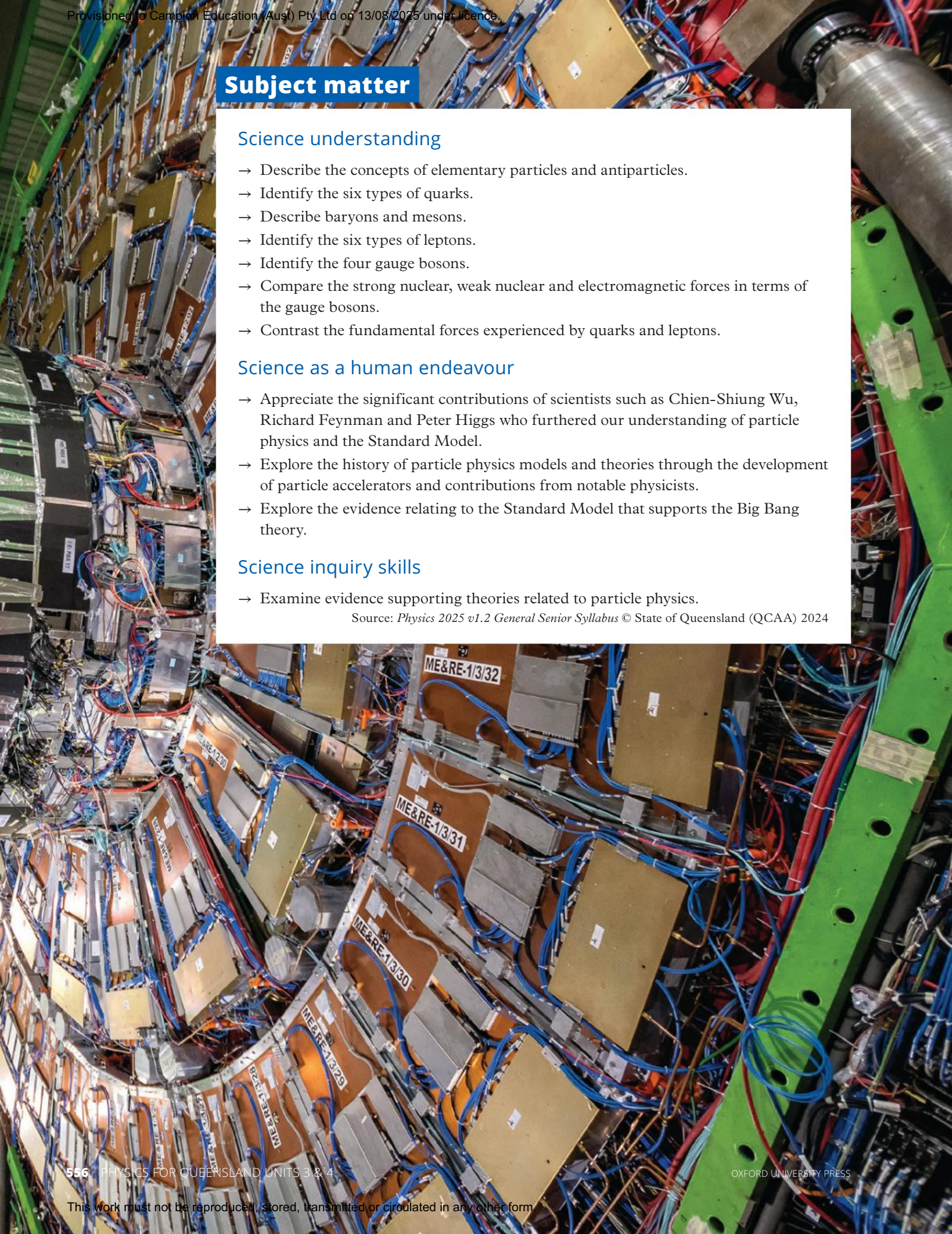
Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Chien-Shiung Wu, Richard Feynman and Peter Higgs who furthered our understanding of particle physics and the Standard Model.
→ Explore the history of particle physics models and theories through the development of particle accelerators and contributions from notable physicists.
→ Explore the evidence relating to the Standard Model that supports the Big Bang theory.

Science inquiry skills

→ Examine evidence supporting theories related to particle physics.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024



Lesson 13.1
Matter and antimatter

Key ideas

- → There are two types of elementary matter particles: leptons and quarks. These elementary particles have no internal structure.
- → Particles and their antiparticles have identical masses but opposite values of charge.
- → There are six quarks, called up (u), down (d), strange (s), charm (c), top (t) and bottom (b).
- → Hadrons are composite particles that are made up of quarks and/or antiquarks. There are two types of hadrons: baryons and mesons.
- → Baryons include the neutron and the proton and consist of three quarks or three antiquarks, whereas mesons include pions and kaons and consist of a quark and antiquark.
- → There are six leptons in the Standard Model: the electron, the muon, the tau particles and their associated neutrinos – namely, the electron neutrino, the muon neutrino and the tau neutrino. There are also three corresponding antileptons and three corresponding antineutrinos.

Learning intentions and success criteria

Study tip

Do not get elementary and fundamental mixed up. We have elementary particles and fundamental forces. We do not use the older term "fundamental particles" anymore.

What is the Standard Model?

Matter is made up of hundreds of different particles that move around and interact with each other in a particular manner. Physicists have developed a model to describe the nature and properties of these particles called the **Standard Model**. The Standard Model states that all the known matter particles are made of elementary particles called quarks and leptons, and they interact by exchanging force-carrier particles called gauge bosons.

The Standard Model is a good theory, but it doesn't explain everything. For example, gravity has not been successfully included in the model. Nevertheless, experiments have verified its predictions to incredible precision, and all the particles predicted by this theory have been found.

Standard Model
a theory describing three of the four known fundamental forces in the universe (the strong nuclear, weak nuclear and electromagnetic forces), as well as classifying all known elementary particles

What are elementary particles?

In physics, an **elementary particle** is a particle with no underlying structure, which means it is not composed of other particles. The current model views the elementary particles as leptons, quarks, gauge bosons (photons, W and Z bosons, and gluons), and the Higgs boson.

You have learnt that matter is composed of atoms, which used to be thought of as elementary particles. Then the electron and proton were discovered, followed by the photon, neutron, neutrino, strong and weak nuclear force bosons and a whole avalanche of other particles. Once the elementary quarks and leptons were identified, physicists needed to find ways to understand their properties and how they combine to help us understand matter.

elementary particle
a particle with no substructure, and thus not composed of other particles

Matter and antimatter

As well as having particles of matter, physicists said there should also be **antiparticles of matter** – that is, particles with the same mass as their matter equivalent, but with the opposite charge. In the 1920s, an antiparticle to the electron was proposed.

antiparticle of matter
a particle that has the same mass but opposite charge to its corresponding matter particle; for example, a positron and an electron

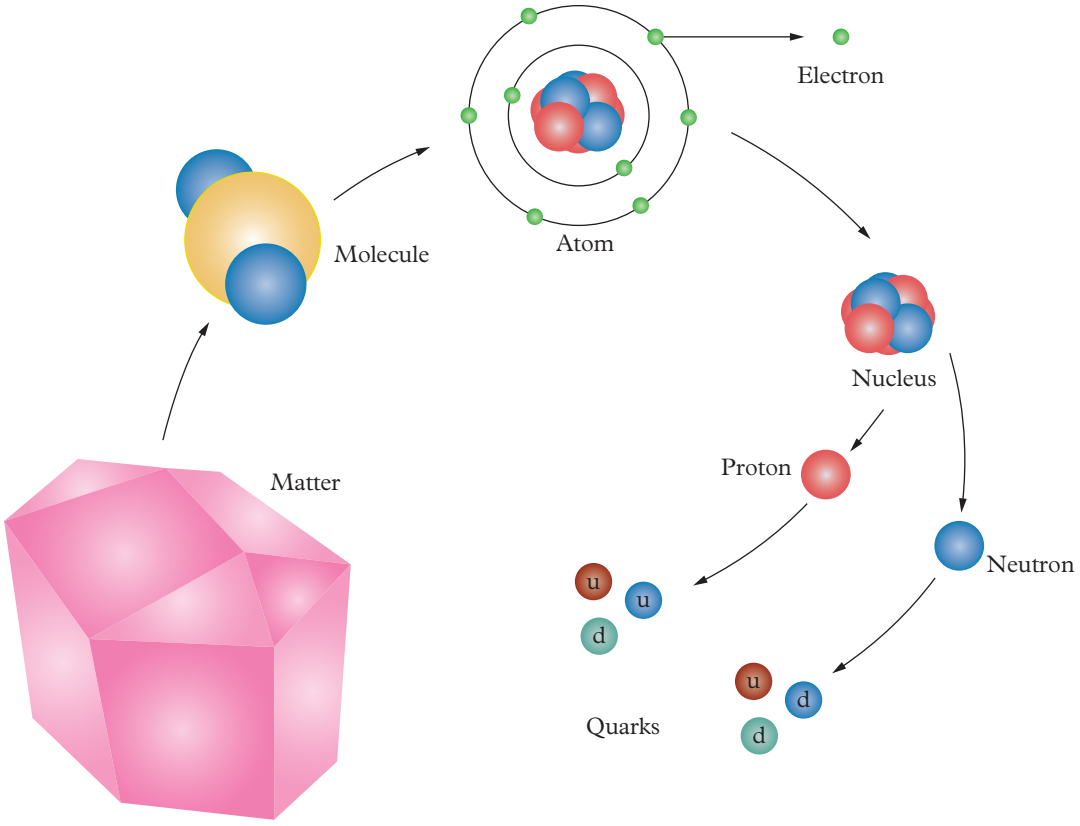


FIGURE 1 The structure of matter shown to subatomic levels.

antiproton
the antiparticle of the proton, with an electric charge of $-1e$. It is relatively stable, but it is typically short-lived because any collision with a proton causes both particles to be annihilated in a burst of energy.

matter
a physical substance that has mass and takes up space by having volume, especially as distinct from energy

antimatter
matter that is composed of the antiparticles of ordinary matter particles

It was called a positron – that is, a positive electron – and was discovered in 1932. The theory was that all particles should have a corresponding antiparticle, and to support this, the **antiproton** (a negatively charged proton) was finally discovered in 1955.

Antiparticles do not live very long in the presence of matter. For example, a positron is stable when it is by itself, but it doesn't survive long. When it meets an electron, the two annihilate each other and give off two gamma rays with their combined energy. This annihilation also occurs for all other particle–antiparticle pairs. During the first second of the Big Bang, the hot and dense universe was filled with particle–antiparticle pairs coming in and out of existence. For each particle formed, an antiparticle was also formed (according to a property called "symmetry"). These annihilated each other in a burst of gamma rays. If **matter** and **antimatter** are created and destroyed together, it seems the universe should contain nothing else but leftover energy.

However, everything around us – from bacteria to galaxies – is made almost entirely of matter. There is not much antimatter around, which means that something must have happened to tip the balance towards matter. In that first second of the Big Bang, for every billion antimatter particles produced, there were one billion and one matter particles produced. That extra one matter particle per billion was enough to build the universe as it is today. There are still antimatter particles around, but they are difficult to find.

What are quarks and antiquarks?

quark
subatomic particles that make up hadrons and are governed by the strong nuclear force; there are six quarks in the Standard Model

Quarks are elementary particles. We have found no evidence that they can be broken up, nor is there anything to suggest that they have an internal structure (unlike an atom which has internal structure), so really do seem to be elementary particles. However, quarks don't exist alone and are only found in groups of two or three to make up particles called hadrons (such as the proton, neutron and meson).

There are six types (or "flavours") of quarks. These are grouped according to their mass and charge and when they were discovered. They were discovered two at a time, so the first two were called the first **generation** (or family) and are the up (u) and down (d) quarks. Then the next two were the second generation and are the charm (c) and strange (s) quarks. Finally, the third generation are the top (t) and bottom (b) quarks. Each quark has a corresponding **antiquark**, which is shown by a bar above the symbol. An antiquark is an example of an antiparticle: a particle that has the same mass but opposite charge to its matter equivalent. For example, the up quark, u, has an antiquark called antiup ($\bar{u}$). This makes 12 quarks and antiquarks in total. They have whimsical names, but don't get misled – this is serious business.

generation
a division of the elementary particles according to the Standard Model. There are three generations, or families, of elementary particles grouped according to mass and charge and when they were discovered

antiquark
a particle with the same mass and opposite charge to a corresponding quark; for example, a strange quark and an antistrange quark are said to be antiparticles

Quarks and electric charge

Quarks have fractional electric charge (charge that is a fraction of the elementary charge, e) – either $+\frac{2}{3}e$ or $-\frac{1}{3}e$. Three quarks – the up, charm and top quarks (the first members of each of the three generations) – have a charge $+\frac{2}{3}e$, and down, strange and bottom quarks (the second member of each generation) have a charge of $-\frac{1}{3}e$. The six antiquarks have the opposite charge to their corresponding quarks. This is shown in Table 1.

Study tip

Charm quarks are sometimes referred to as charmed quarks. Both names are correct.

Quarks and colour charge

Before we look at this further, you should know that quark colours are not a part of the syllabus. It is included here as extension material and is a useful way to understand how quarks combine.

Physicists found they needed an additional quantity to fully describe a quark. This had to have three different values, so they devised the "colour charge" (Figure 2). A quark can take one of three colour charges: red (r), green (g), or blue (b). An antiquark can take one of three anticolours: antired ($\bar{r}$), antigreen ($\bar{g}$), and antiblue ($\bar{b}$), which are usually represented as cyan, magenta and yellow respectively. They have nothing to do with optical colour such as the red of a rose – it is just a self-consistent system that relies on the analogy of colour mixing.

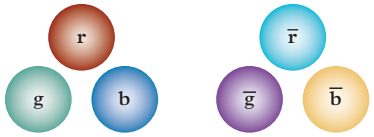


FIGURE 2 Example of the colours. The quark colours are red, green and blue, and the antiquark colours are antired (cyan), antigreen (magenta) and antiblue (yellow).

The purpose of colour charge will be seen in the next section. Again, it is not required for external assessment.

Table 1 shows the 12 quarks and their electric charges and colour charges.

TABLE 1 Generations of quarks and antiquarks and their properties

Generations	Quarks				Antiquarks			
	Flavour	Symbol	Charge	Colours	Flavour	Symbol	Charge	Colours
First	up	u	$+\frac{2}{3}e$	r, g, b	antiup	$\bar{u}$	$-\frac{2}{3}e$	$\bar{r}, \bar{g}, \bar{b}$
	down	d	$-\frac{1}{3}e$	r, g, b	antidown	$\bar{d}$	$+\frac{1}{3}e$	$\bar{r}, \bar{g}, \bar{b}$
Second	charm	c	$+\frac{2}{3}e$	r, g, b	anticharm	$\bar{c}$	$-\frac{2}{3}e$	$\bar{r}, \bar{g}, \bar{b}$
	strange	s	$-\frac{1}{3}e$	r, g, b	antistrange	$\bar{s}$	$+\frac{1}{3}e$	$\bar{r}, \bar{g}, \bar{b}$
Third	top	t	$+\frac{2}{3}e$	r, g, b	antitop	$\bar{t}$	$-\frac{2}{3}e$	$\bar{r}, \bar{g}, \bar{b}$
	bottom	b	$-\frac{1}{3}e$	r, g, b	antibottom	$\bar{b}$	$+\frac{1}{3}e$	$\bar{r}, \bar{g}, \bar{b}$

What are hadrons?

hadron
particle composed of quarks and held together by gluons; for example, mesons and baryons

gluon
fundamental exchange particle that operates between quarks and hence underlies the strong nuclear force between nucleons (protons and neutrons) in a nucleus

meson
composite subatomic particle composed of one quark and one antiquark, held together by the strong nuclear force

baryon
composite subatomic particle that contains three quarks held together by the strong nuclear force

Particles composed of quarks are called **hadrons** (from the Greek "hadros", meaning "bulky", which is what they are). As well as the quarks, hadrons need to have force-carrying particles called **gluons** to hold the quarks together. You will not need to know the term "hadron" for the external exam.

Quark composites (hadrons) are formed in two main ways: in pairs to form **mesons**, or in threes to form **baryons**.

Quarks are always found bound up in hadrons, either in mesons or baryons. So far, they have not been found alone.

Mesons

Mesons are named from the Greek "mesos", meaning "middle", because they are between protons and electrons in mass. Mesons are part of the hadron particle family and are defined simply as particles composed of a quark and an antiquark bound together by the strong nuclear force. All mesons are unstable, with the longest lived lasting for just under a second. They can carry an electric charge of –1, 0, or +1. Some examples of mesons are shown in Figure 3. The first meson is called a pion (π^+) and is shown in Figure 3A. It is made up of an up quark and an antidown quark ($u\bar{d}$) and has a net electric charge of $+1e$. In this example the colours are red and antired (cyan), which add to white when combined. This is called colour neutral (white); all composite particles must be colour neutral.

The second meson, shown in Figure 3B, is a charmed eta-c (η_c) meson made up of a charm and an anticharm quark ($c\bar{c}$), and has a net electric charge of zero. The colours used are green and antigreen (magenta), which again add to white. The third example (Figure 3C) is a pion (π^-) made up of a down quark and an antiup quark, and has a net electric charge of $-1e$. The π^- is the antiparticle to the π^+.

The top quark can't form a meson because it has an extremely short lifetime of 5×10^{-25} s.

Study tip

Some mesons are their own antiparticle. For example, the charmed eta-c meson ($c\bar{c}$) has an antiparticle ($\bar{c}c$) that is identical. However, the pion ($u\bar{d}$) has an antiparticle ($\bar{u}d$) that is different, because their charges are different ($+1e$ and $-1e$ respectively).

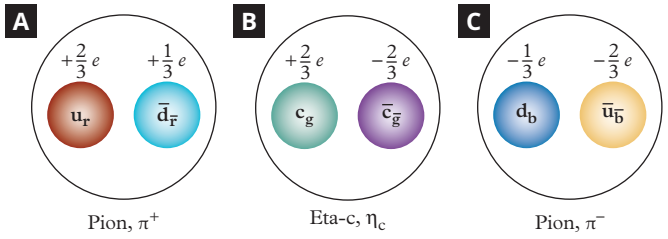


FIGURE 3 Mesons are composed of a quark and an antiquark bound together by the strong nuclear force.

Baryons

Baryons are named from the Greek "barys", meaning "heavy", because when they were named, they were the heaviest of the known nuclear particles. They are composites of three quarks (qqq), or three antiquarks ($\bar{q}\bar{q}\bar{q}$). A baryon with three antiquarks is called an **antibaryon**. The quarks or antiquarks are held together by the strong nuclear force.

antibaryon
composite subatomic particle that contains three antiquarks held together by the strong nuclear force

The most familiar baryons are the proton and the neutron. The proton was discovered in 1917 and the neutron in 1932, before the term "baryon" came into use. The antiproton wasn't found until 1955 and the antineutron a year later. Neutral particles are much harder to detect as they don't interact with matter. Antiparticles are also hard to detect as they annihilate themselves when they meet ordinary matter. More about that in the next module.

The quarks or antiquarks that make up baryons can be of any flavour (u, d, c, s, t, b), although top quarks are not likely to form baryons (or mesons, as mentioned earlier) because of their short lifetime.

The electric charge of baryons can be −1, 0, or +1. Some examples of baryons and antibaryons are shown in Table 2 and Figure 4.

TABLE 2 Properties of some baryons and their quark composition

Baryon	Symbol	Quarks	Electric charge
Neutron	n^0 or n	udd	$+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} = 0$
Antineutron	$\overline{n}$	$\overline{udd}$	$-\frac{2}{3}, +\frac{1}{3}, +\frac{1}{3} = 0$
Proton	p^+	uud	$+\frac{2}{3}, +\frac{2}{3}, -\frac{1}{3} = +1$
Antiproton	$\overline{p}$	$\overline{uud}$	$-\frac{2}{3}, -\frac{2}{3}, +\frac{1}{3} = -1$
Omega minus	Ω^-	sss	$-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3} = -1$
Lambda	Λ	uds	$+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} = 0$

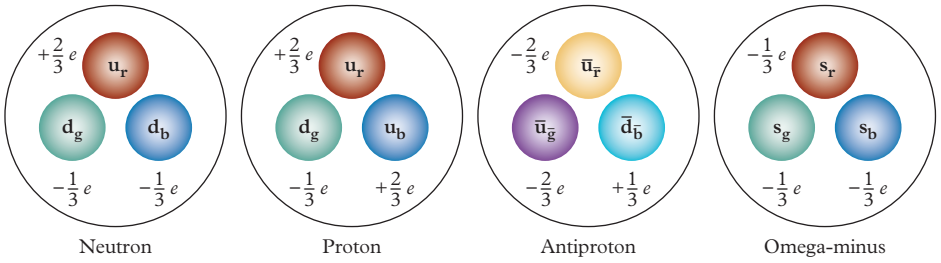


FIGURE 4 Examples of baryons. The colours have been randomly chosen, but they all add to white (colour neutral).

Study tip

Quarks have a property called "colour" or "colour charge". It is a very useful property, but it has nothing to do with physical colours that we see. While it is hugely important to particle physicists, it will not be examined in any external assessment.

How can quarks be summed up?

The properties of quarks can be summarised into five main points. Each of these points is usually necessary to get full marks in a question asking you to describe quarks, so they are important to learn. Quarks:

- are elementary particles that come in six flavours (types), each with its own antiparticle
- are always found in composite particles and are not known to exist alone
- have a fractional positive or negative charge and are bound together by the strong nuclear force to form baryons (three quarks or three antiquarks) or mesons (a quark and an antiquark), known collectively as hadrons
- can be arranged into three families or generations (u, d), (c, s), (t, b) of increasing mass and shortened lifetime
- are affected by the four fundamental forces (which will be described in the next lesson).

Challenge

Mesons or baryons?

Identify which of the following combinations of quarks exist as particles. For those that do, **identify** whether they are mesons or baryons.

a $u\overline{u}d$ (1 mark)

b ss (1 mark)

c $u\overline{d}$ (1 mark)

Challenge

Colour-charge rules

Using colour-charge rules, **decide** what the colour of the last quark in each of the following combinations must be to make each particle colour neutral. Note: you won't need to do this in the external exam, but understanding colour-charge rules can help you understand how quarks can combine.

a Sigma baryon (Σ^0): $u_r d_b s$ (1 mark)
b Phi meson (ϕ): $s_r \bar{s}$ (1 mark)
c Strange D-meson (D_s^+): $c_b \bar{s}$ massless (1 mark)
d Double charmed Xi baryon (Ξ_{cc}^{++}): $d_b c_g c$ (1 mark)
e Antiproton ($\bar{p}$): $\bar{u}_r \bar{u}_b \bar{d}$ (1 mark)

What are leptons?

lepton
a class of elementary particles that respond only to the weak force and the gravitational force. They can carry one unit of electric charge or are neutral, and those that are charged experience the electromagnetic force. There are six leptons in the Standard Model.

The other type of elementary particle is called is the **lepton**. Unlike quarks, leptons do not form stable composite particles (hadrons) such as baryons and mesons; they can exist on their own. The first lepton to be discovered was the electron, which was discovered by J.J. Thomson in 1897. It is also the smallest. The largest, the tau lepton, is about as twice as heavy as a proton.

There are six leptons in the Standard Model: the electron, muon and tau particles and their associated neutrinos (the electron neutrino, the muon neutrino and the tau neutrino). As well as these six, there are the corresponding six antileptons. Like quarks, leptons are divided into generations according to increasing mass and decreasing lifetime. Table 3 sets out their properties.

Study tip

The antiparticle of an electron neutrino is more commonly referred to as an "electron antineutrino", not an "antielectron neutrino". It's the neutrino that the "anti-" applies to.

Unlike the quarks, which interact via the strong nuclear force, leptons interact via the weak nuclear force. Leptons that have an electric charge also interact via the electromagnetic force. Electrons and electron neutrinos are stable particles, but muons and tau particles have mean lifetimes of only 2.2×10^{-6} s and 2.9×10^{-13} s respectively. Muons and tau particles have too much energy and tend to break down into the stable first-generation particles such as the electron and electron neutrino. Leptons can be divided up into charged and uncharged particles: the charged leptons (also known as the electron-like leptons), and the neutral leptons, which are better known as neutrinos. Because neutrinos are uncharged, you can't distinguish a neutrino from its antineutrino by changing the electric charge from negative to positive. Like quarks, we use a bar over the top of the symbol to indicate the antiparticle of a neutrino, so an electron antineutrino is $\bar{\nu}_e$. Antineutrinos are different to their corresponding neutrinos, even though they have identical characteristics except one. The symbol for the neutrino is Greek and is pronounced "nu".

TABLE 3 Properties of leptons. The charges are given in units of e, the fundamental charge (i.e. the charge on an electron).

Generation	Lepton			Antilepton		
	Flavour	Symbol	Charge	Flavour	Symbol	Charge
First	electron	e^-	–1	positron	e^+	+1
	electron neutrino	ν_e	0	electron antineutrino	$\bar{\nu}_e$	0
Second	muon	μ^-	–1	antimuon	μ^+	+1
	muon neutrino	ν_μ	0	muon antineutrino	$\bar{\nu}_\mu$	0
Third	tau	τ^-	–1	antitau	τ^+	+1
	tau neutrino	ν_τ	0	tau antineutrino	$\bar{\nu}_\tau$	0

How can leptons be summed up?

To summarise, leptons:

- are elementary particles that don't form stable composite particles and can exist on their own
- come in six different types, each with its own antiparticle, which are grouped into three generations
- can have an electric charge (the electron-type) or are neutral (the neutrino type)
- interact via the weak nuclear force but not the strong nuclear force
- are also affected by the electromagnetic force if they are charged.

As a side note, although leptons do not form stable composites, they can and do form short-lived composites during interactions. For example, when a high-energy electron and positron collide with each other, they form a short-lived positronium atom that orbits a centre of mass and lasts about 1.1 ms before the electron and positron annihilate each other and turn into a burst of gamma rays (usually two). This was discovered in 1951. It is described in the next module, and you will need to understand it.

Study tip

Quarks and leptons are usually grouped together by particle physicists as the convenient term "fermions" (pronounced fermi-ons) in honour of the Italian physicist Enrico Fermi. The term is not included in the syllabus. Another overarching term is "hadron", which is a particle made up of quarks, such as protons, neutrons, mesons; hence the naming of the Large Hadron Collider, which makes protons (examples of hadrons) collide at extremely high speeds.

Worked example 13.1A

Contrasting the electric charge on quarks and leptons

Contrast the electrical charge on an antiup quark and an antitau particle. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Contrast" means to identify differences. There are two marks allocated, so we need to identify two aspects of the electrical charge that are different. To be sure, we are not just describing two aspects of the same property; we should choose one property that is about the sign of the charge (+/–) and one that is about the magnitude. The key points are: state **differences** about sign and magnitude.
Step 2: Identify one difference (the sign of the charge) to show how it applies to each particle. (1 mark for "Identifies the difference between the sign of charge of the particles")	The antiup quark has a negative charge and the antitau has a positive charge. (1 mark)

Think	Do
Step 3: Identify one difference (magnitude of charge) and elaborate on how it applies to each particle. (1 mark for "Identifies the difference in magnitude of the charges") You don't have to say the value of the charge, but you can if you like (no extra marks): antiup $\frac{2}{3}e$ and antitau $1e$, where e is the elementary charge. There is no point saying they are elementary particles or even that antineutrinos are leptons. This will not be awarded marks, as it is not a difference so is not relevant.	The antiup quark has fractional charge whereas the antitau has integer (whole number) charge. (1 mark)

Your turn

Contrast the electrical change of an anticharm particle and a muon. (2 marks)

Check your learning 13.1

Check your learning 13.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Recall** the names of the six quarks. (1 mark)

2 **Describe** "baryon" and "meson". (2 marks)

3 **Describe** how you can tell whether a particle is a meson or a baryon by its quark content. (1 mark)

4 **Describe** the concepts of elementary particles and antiparticles. (2 marks)

5 Copy and complete the table and **identify** with a tick or cross whether each term in the first row applies to the composite particles i–iv, whose quarks are given. (4 marks)

Analytical processes

6 **Deduce** whether there are any quark–antiquark combinations that result in a non-integer (fractional) electric charge. (2 marks)

Knowledge utilisation

7 **Decide** what conditions are necessary for a particle and its antiparticle to be identical. (1 mark)

8 A hadron has a charge of $+1e$. **Propose** the possible composition of the hadron and describe it as a baryon or a meson. (2 marks)

9 A particle is identified with a charge of $+\frac{2}{3}e$. **Evaluate** the possibility that it could be either an up quark or a meson made up of an antistrange quark and an antibottom quark. (2 marks)

Composite particle	Hadron	Baryon	Muon	Antimuon	Meson	Lepton	Antilepton
i $u\bar{d}$							
ii dds							
iii ccc							
iv $\bar{d}\bar{u}c$							

Lesson 13.2
Gauge bosons – the force carriers

Key ideas

→ In addition to the six elementary leptons and six elementary quarks, the Standard Model includes four mediating particles, called gauge bosons, that are associated with three of the four fundamental forces.

→ The four fundamental forces are strong nuclear, electromagnetic, weak nuclear and gravitational force.

Learning intentions and success criteria

What are the fundamental forces?

There are four **fundamental forces** in the universe. They act between bodies of matter and are mediated by one or more particles. The four fundamental forces from strongest to weakest, are strong nuclear force, electromagnetic force, weak nuclear force and gravitational force. These forces act over different ranges. We think of force as being a measure of a push or a pull; however, physicists have a different measure but agree on the relative strength. This section describes the gauge bosons for each force and contrasts the four forces as experienced by quarks and leptons.

The three forces of the Standard Model operate by exchanging particles between the objects they operate on. These force-carrier particles are called **gauge bosons**. The word "gauge" means "measurement" and refers to the idea that you can't measure the properties of these bosons directly but only indirectly by other measurements.

The exchange of gauge bosons produces the force or – in the language of physics – mediates the force. "Mediate" comes from the Latin "medius", meaning "in the middle", which is where these bosons are – in the middle, between the particles. Each of the three fundamental forces described by the Standard Model has its own corresponding boson:

- The electromagnetic force is mediated by the photon.
- The strong nuclear force is mediated by the gluon.
- The weak nuclear force is mediated by the W (W^+ and W^-) and Z^0 gauge bosons.

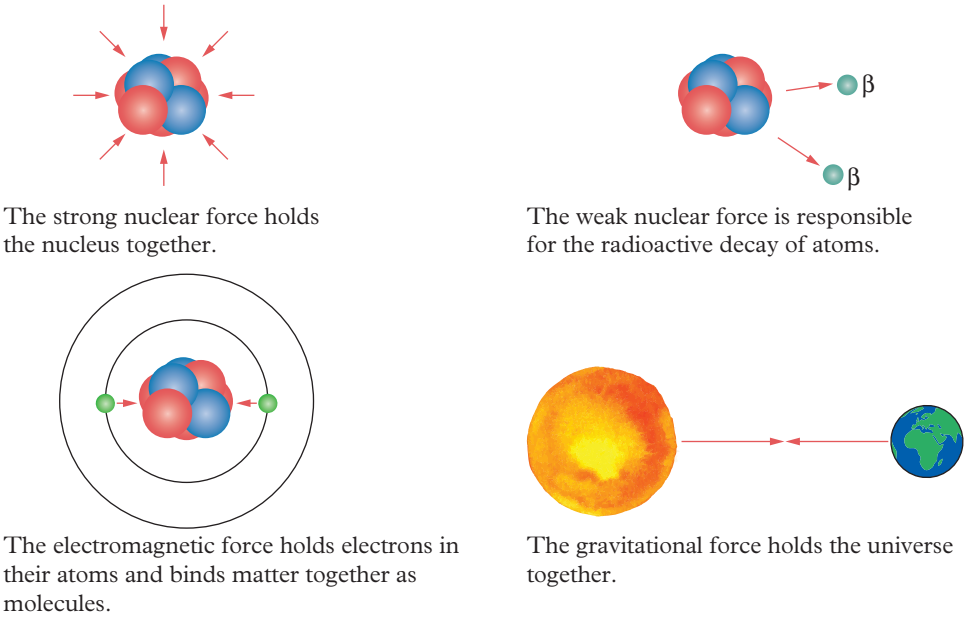
These four gauge bosons are summarised in Table 1. All gauge bosons have antiparticles. The photon, Z^0 boson and gluons are their own antiparticles. The W^+ and W^- are antiparticles of each other.

fundamental force
force that acts between bodies of matter and is mediated by one or more particles. The fundamental forces are, from strongest to weakest, the strong nuclear force, electromagnetic force, weak nuclear force and gravitational force

gauge boson
mediating particle that governs particle interaction and the mediation of three of the four fundamental forces. There are four gauge bosons in the Standard Model.

TABLE 1 A comparison of the Standard Model forces in terms of the gauge bosons. The table does not include gravity, which is not a part of the Standard Model.

Standard Model force	Strong nuclear force	Weak nuclear force	Electromagnetic force
Particles experiencing the force	Quarks	Quarks, leptons	Quarks and electrically charged leptons
Gauge boson	Gluon, g	W^+, W^-, Z^0	Photon, γ



The strong nuclear force holds the nucleus together.

The weak nuclear force is responsible for the radioactive decay of atoms.

The electromagnetic force holds electrons in their atoms and binds matter together as molecules.

The gravitational force holds the universe together.

FIGURE 1 The four fundamental forces of the universe are the strong nuclear force, the weak nuclear force, the electromagnetic force and the gravitational force.

What is the strong nuclear force?

strong nuclear force
the strongest of the four fundamental forces; binds quarks together to make subatomic particles such as protons and neutrons and underlies interactions between all particles containing quarks and/or antiquarks; sometimes called the strong force. The strong nuclear force between quarks is mediated by gluons.

mediating particle
a descriptive name for the gauge bosons, which govern the interaction of the four fundamental forces. Also known as carrier particles or exchange particles; expressed, for example, as "gluons are the mediating particle for the force between quarks".

The **strong nuclear force** (also called the "strong force") acts between quarks to bind them together into protons and neutrons (nucleons). It only acts on quarks within nucleons and over extremely small distances: about 10^{-15} m (a femtometre, fm), or about the size of a nucleon, for these quark–quark interactions. The strong nuclear force is highly repulsive at separations below 0.5 fm but is very attractive up to a nuclear separation of 3.0 fm. The maximum attractive value occurs at around 1.0 fm, which is a typical value for nucleon separation. The **mediating particle** for the strong nuclear force is the gluon.

Gluons

A gluon is an elementary particle that acts as the exchange particle for the strong nuclear force between quarks. It is similar to the exchange of photons in the electromagnetic force between two electrically charged particles. So, when quarks are close to each other there is an exchange of gluons that creates a very strong force that binds the quarks together. You could say the gluons "glue" quarks together, forming baryons such as protons and neutrons. Isolated gluons have never been observed experimentally, only their effects. Gluons have neutral charge and are their own antiparticle.

Challenge

Femtometre

The range of the strong nuclear force is one femtometre (10^{-15} m). **Determine** the number of femtometres (fm) in 1 metre. (1 mark)

Real-world physics

Naming of the strong nuclear force

FIGURE 2 Hideki Yukawa

There is a lot of confusion about the naming of the strong nuclear force. In 1937, Japanese physicist Hideki Yukawa proposed that the attractive force between nucleons (protons and neutrons) be called the "strong nuclear force" and that it was due to the exchange of particles. These particles were discovered in 1947 and were named pi-mesons (π) or just pions. With the development of quark theory in 1963, it was realised that protons and neutrons (nucleons) were not elementary particles because within them were quarks, and quarks were the elementary particles. Similarly, mesons (such as pions) were not elementary particles because they were made of a quark and antiquark. Therefore the "strong nuclear force" between nucleons was no longer considered a fundamental force. The new fundamental force was the force between quarks and was to be called the "strong force", mediated by gluons (which you will meet later in this lesson). Note that the word "nuclear" has been removed. However, this "strong force" between quarks also extends out from any nucleon and is responsible for the force between nucleons that stops protons flying apart – so we end up with a confusion of terms.

Today, in the Standard Model, we can call the fundamental force the "strong nuclear force" (as in the syllabus), but it is sometime shortened to the "strong force" even in exam questions. The fact that this force has a residual component that acts between nucleons is confusing and this aspect is sometimes called the "residual strong nuclear force".

Apply your understanding

1 **Identify** the carrier of the force between a proton and a neutron. (1 mark)

2 **Propose** the identities of two quarks that have the strong nuclear force pulling them together but the electromagnetic force pushing them apart. There can be many answers. (2 marks)

What is the electromagnetic force?

Electrically charged objects such as all quarks, and leptons with charge, experience the **electromagnetic force**. As a reminder, the charged leptons are the electrons, muons, tau particles (also called "tauons"), and their antiparticles. Neutrinos are not charged and so are not affected. The electromagnetic force holds electrons in their atoms and binds matter together as molecules. But it also causes protons in the nucleus to repel each other. Like gravity, it operates to the edges of the universe, but because it can cause both attraction and repulsion, its effect over vast distances is not as pronounced as that of gravity. Nevertheless, it is the second strongest force, being just one-hundredth that of the strong nuclear force. You learnt about this force as Coulomb's law. The mediating particle for the electromagnetic force is the photon.

electromagnetic force
the second strongest of the four fundamental forces. The electromagnetic force is mediated by photons.

Study tip

Don't get your forces confused. The fundamental force is the "electromagnetic force". This is different to the "electromotive force" (EMF) you met in Module 8 Electromagnetic induction and radiation. In multiple-choice questions you may even see "electromagnetic field" or the nonsense word "electromotive field". They are common traps.

Photons

The photon is a gauge boson that mediates the electromagnetic force (and its composites, the electrostatic force and the magnetic force). You learnt about the photon as a particle in Module 11. It is a massless, electrically neutral particle that travels at the speed of light, c, in free space. Photons are considered to be their own antiparticle (which becomes important in Module 14).

What is the weak nuclear force?

weak nuclear force
the third strongest force of the four fundamental forces. It is responsible for radioactive decay and is mediated by W (W^+ or W^-) and Z^0 bosons.

All quarks and all leptons experience this force. It is independent of electric charge. The **weak nuclear force** is primarily responsible for slow nuclear processes such as the radioactive decay of atoms, and it seems to control energy-producing fusion reactions in stars. It is the third strongest force, at about 10^{-5} times that of the strong nuclear force. The mediating particles for the weak nuclear force are the W (W^+ or W^-) and Z^0 bosons.

W and Z bosons

The W and Z bosons act between quarks and leptons and mediate the weak nuclear force. The W bosons are weak, hence the symbol W, and they can have either a positive charge (W^+) or a negative charge (W^-). The Z boson is neutral and has the symbol Z^0. The W and Z bosons are almost 100 times as massive as the proton – heavier, even, than an entire iron atom. W^+ and W^- are each other's antiparticle and the neutral Z_0 boson is its own antiparticle.

What is the gravitational force?

Gravity is one of the four fundamental forces, but it is yet to be incorporated into the Standard Model. It is included here for completeness, but we will stress that it is not a part of the Standard Model.

All objects with mass experience the gravitational force. Hence, quarks and leptons, and any composite of these, will feel the gravitational force. When you learnt about Newton's law of gravitation, you will have learnt that this force is the product of the masses and the inverse square of the distance between them. The gravitational force keeps planets in orbit, controls the expansion of the universe and stops us from falling off Earth. It is the weakest of all forces, at about 10^{-39} of the strong nuclear force, but acts over an infinite distance.

What makes gravity so different from the other forces is that it is only attractive. No-one has found negative mass, so every particle in the universe experiences an attractive force. Gravity can act from one side of the universe to the other with no opposition. However, it can be ignored inside the atom as it is weak compared to the other forces.

The mediating particle for gravity is proposed to be the "graviton", and if it were found it would possibly see gravity added to the Standard Model. However, despite decades of work to find it and to include it in what is called "the grand unification", so far it remains undiscovered. The best current model we have for gravity is in terms of gravitational waves, and waves are not a part of the Standard Model. Gravity, gravitational waves, and gravitons are dealt with separately in the Real-world physics feature "Search for the elusive graviton".

Challenge

Why do you notice gravity so much?

Gravity is the weakest fundamental force, so why do you notice it so much? **Decide** whether the following are true or false and **explain** your reasoning.

1. Gravity acts only on charged particles. (1 mark)
2. Gravity can act only on elementary particles, not on composites such as atoms. (1 mark)
3. Gravitational forces have a short range. (1 mark)

Real-world physics

Search for the elusive graviton

Physicists have two superb theories that explain the universe. One is general relativity, which explains how gravity acts, and the other is quantum physics, which uses the Standard Model to explain things at the atomic scale. Both have been confirmed millions of times, but they don't fit together well. To develop an overarching theory that combines gravity into the Standard Model, physicists came up with a hypothetical particle, the graviton.

According to physicists, it will be a long time before gravitons are considered part of the established subatomic "zoo" of particles. The graviton is the hypothetical elementary particle that mediates the force of gravity. It has yet to be detected, but it is predicted to be massless because the gravitational force is very long range and appears to propagate at the speed of light. You know from the theory of special relativity that particles with mass can't travel at the speed of light as their momentum would become infinite.

Although gravitational waves have been detected, the graviton has not. It has been calculated that to capture a graviton, a detector the size of Jupiter would have to be placed in close orbit around a neutron star and, even then, gravitons would only be detected every 10 years, even under the most favourable conditions.

The detection of gravitational waves, however, provides information about certain properties of the graviton. For example, if gravitational waves were observed to propagate slower than c, that would imply that the graviton has mass. The LIGO observations of gravitational waves in 2015 put an upper bound of 2.1×10^{-58} kg on the graviton's mass – very close to zero! Astronomical observations of the rotation of galaxies might give conclusive evidence that gravitons have non-zero mass.

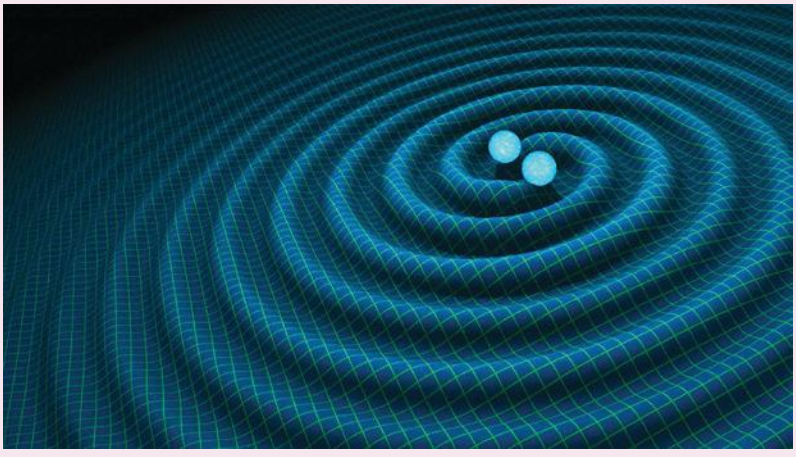


FIGURE 3 Impression of gravitational waves radiating out from the collision of two neutron stars. Gravitons would be represented by dots radiating outwards on the crests and in the troughs of the waves.

Apply your understanding

1 The electromagnetic force and the gravitational force act across the whole universe but there is one main difference between them. **Explain** the difference in terms of attraction and repulsion. (2 marks)

2 If the upper bound for the mass of a graviton is 2.1×10^{-58} kg, **calculate** the equivalent amount of energy in joules this would be equal to. (2 marks)

3 The gravitational waves detected in 2016 were from the collision of two black holes 1.3 billion light-years away from Earth. **Calculate** how long it would have taken a graviton, if it existed, to get from there to Earth. (2 marks)

How do the four fundamental forces differ?

The four fundamental forces and their mediating particles are summarised in Table 2. As discussed earlier, the Standard Model includes only three of the four fundamental forces.

TABLE 2 A contrast of the fundamental forces experienced by quarks and leptons. A femtometre (fm) = 10^{-15} m.

Fundamental force	Strong nuclear force (Standard Model)	Weak nuclear force (Standard Model)	Electromagnetic force (Standard Model)	Gravitational force
Described by the Standard Model?	Yes	Yes	Yes	No
Relative strength	1	10^{-2}	10^{-5}	10^{-39}
Range	10^{-15} m (1 fm)	10^{-18} m (10^{-3} fm)	Infinite	Infinite
Acts on	Quarks	Quarks Leptons	Quarks Charged leptons	Quarks Leptons

Study tip

It would be useful to be able to reproduce Tables 1 and 2 from memory. A good task would be to combine them into one big table.

Challenge

The "Oh-My-God!" particle

What is the "Oh-My-God" particle? **Identify** whether it is a particle that produces a field that interacts with particles and gives them mass, or a particle that is so fast it almost travels at the speed of light. (1 mark)

Worked example 13.2A

Contrasting the nature of quarks and leptons and the interaction of forces

Contrast the properties of a charm quark and a muon. (2 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Contrast" means to identify differences. There are two marks allocated, so we need to identify two properties that are different. To be sure we are not just describing two aspects of the same property, we should choose one property that is about the nature of the particles (from Lesson 13.1) and one that is about the interaction of forces (from this lesson). We then need to state the **differences** about their nature (1 mark) and forces (1 mark).
Step 2: In terms of "nature", they are both elementary particles. You will not get a mark for stating this – it is a similarity rather than a difference. Identify one difference about their nature and elaborate to show how it applies to each particle. In this case, we have chosen whether they can form composite particles or not. (1 mark for "Identifies the difference between the nature of the particles")	The charm quark can combine with other quarks to form composite particles such as a meson (quark–antiquark pair) or a baryon (3 quarks or 3 antiquarks). The muon particle is a lepton and thus does not form composite particles. (1 mark)

Think	Do
Step 3: Alternative answer to Step 2, choosing electric charge as the property. Recall that all quarks have fractional charge whereas leptons have unit charge (0 or 1). However, you must relate this to the charm quark and muon to get the mark.	The charm quark has fractional charge of $+\frac{2}{3}e$. The muon has a unit charge of $-1e$. (alternative 1 mark)
Step 4: Identify one difference about the interaction of forces and elaborate to show how it applies to each particle. (1 mark for "Identifies the difference between the interaction of forces")	The charm quark experiences the strong nuclear force. The muon does not experience the strong nuclear force. (1 mark)

Your turn

Contrast the properties of a tau particle and a strange quark. (2 marks)

What is the Higgs boson?

There is another type of boson called a "scalar boson" of which the **Higgs boson** is the only present example. So according to the Standard Model there are five elementary bosons: the four gauge bosons that act as force carriers between quarks and leptons, and the Higgs (scalar) boson, which is not a force carrier but gives particles quarks and leptons their mass. The Higgs boson is beyond the scope of the syllabus.

Higgs boson
an elementary particle in the Standard Model that represents the Higgs field – a field that pervades the universe and is responsible for giving certain elementary particles mass.

Study tip

The Higgs boson and the graviton are not a part of the Senior Physics curriculum. You will need to know only the four gauge bosons: photons, the W and Z^0 bosons, and gluons.

Real-world physics

Peter Higgs and the Higgs boson

The Standard Model proposes that the forces between quarks and between leptons are transmitted by particles known as gauge bosons. What puzzled physicists was that some particles, such as W and Z bosons, had mass but others, such as gluons and photons, didn't.

To account for this, Professor Peter Higgs (1929–2024) and others proposed a particle now known as the Higgs boson, and its associated field, the Higgs field, that causes other particles to have mass. Higgs bosons are manifestations of the Higgs field, which uses the bosons to transmit its effects. This is similar to the way in which an electromagnetic field uses photons to transmit its effects. The Higgs boson is difficult to detect directly because it has a tiny lifetime – about 10^{-23} seconds – but nevertheless it was discovered in 2013 at the Large Hadron Collider in Switzerland. Since then, scientists have uncovered various decay modes of the particle and confirmed that the Higgs boson does give mass to the tau lepton and the top and bottom quarks, particles from the heaviest of the three families of matter particles. They are seeking proof that the Higgs boson also gives mass to the top quark, but because the top quark is heavier than the Higgs boson, the boson cannot decay into a pair of top quarks. This leaves a bit of a conundrum that needs a bit more thought.

Currently, it seems that the Higgs boson does not have an antiparticle, but the current model for cosmogenesis (creation of the cosmos) allows for there to have been both Higgs bosons and anti-Higgs bosons in the very early universe.

FIGURE 4 Two famous Scottish physicists: Peter Higgs in 2018 and, on the poster in the background, William Thomson, Lord Kelvin, in 1906. Kelvin suggested that atoms were vortices in the aether surrounding us. This was rejected at the time but, in retrospect, he may have been on to something.

Apply your understanding

1 The Higgs field is associated with Higgs bosons. **Identify** which bosons are associated with the electromagnetic field. (1 mark)
2 **Compare** a photon and a Higgs boson for one similarity and one difference. (2 marks)
3 **Deduce** one similarity and one difference between a graviton and a Higgs boson. (2 marks)
4 **Propose** whether quarks are affected by the Higgs field. (1 mark)

Real-world physics

Particles created in the Big Bang

The Big Bang theory of the birth of the universe relies on evidence from the Standard Model of particle physics. Until the 1920s, the general opinion was that the universe was in an unchanging state – the steady-state model – made of fixed stars that had always shone and would continue to shine forever. This view was well entrenched – even Einstein believed it!

Gradually, scientists began to make predictions that there were galaxies outside our own. The astronomer Edwin Hubble was the first to find evidence for this. By observing the light from these galaxies, he found that they were moving away from our own galaxy, and the further away the galaxy, the faster it was moving away. The spectral lines of elements such as hydrogen and helium appeared at longer wavelengths than normal. This shift in wavelength was towards the red end of the spectrum and was given the name "redshift". It suggested that the light source – the galaxies – must be moving away from the observer.

This was a monumental breakthrough. If galaxies are moving away from each other, it means there must have been a time when they were all together. That time was nearly 14 billion years ago – the time of the Big Bang – when the whole universe was a tiny fraction of the size of the full stop at the end of this sentence.

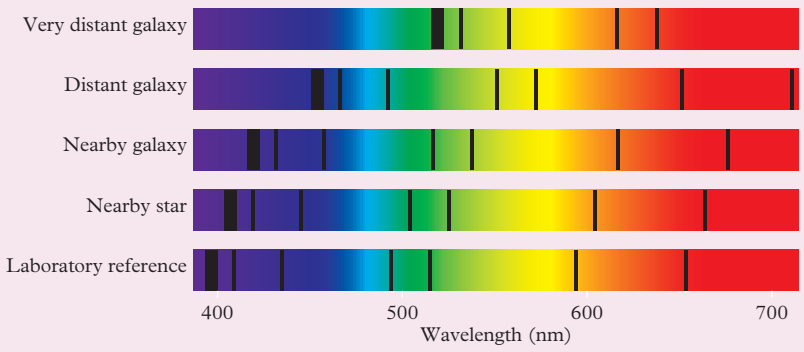


FIGURE 5 A comparison of redshifts for the spectra of starlight. The further the galaxies are away from us, the bigger the redshift. This implies there was a beginning with zero size.

The Big Bang

Physicists can use the Standard Model and its rules of particle interactions to make predictions about the early universe. They can then compare these predictions to conditions in our universe today. For example, the amounts of the light elements such as hydrogen, helium and lithium are consistent with models of the early universe after the Big Bang.

The Primordial Era: t = 0 to 3 minutes

About 13.8 billion years ago, the universe was just a tiny speck in space, a billionth the size of a proton. It had almost infinite density and temperature. This was the beginning of time ($t = 0$) as we know it.

An enormous explosion (the Big Bang) occurred at $t = 0$. At 10^{-43} s after the Big Bang, the universe began to expand and cool at a fantastic rate. In this very short time, the temperature fell from 10^{32} K to 10^{20} K.

Physicists don't know what particles were present at the Big Bang, but they do know that whatever they were, these particles soon decayed into lighter particles such as quarks, electrons, gluons, photons, neutrinos and dark matter. For every one billion antimatter particles produced, one billion and one matter particles were produced. As the universe cooled, the matter and antimatter quarks annihilated each other, leaving the excess fraction of matter quarks to survive.

Because there were more quarks than antiquarks, more protons and neutrons formed than antiprotons and antineutrons. These particles and antiparticles continued to annihilate each other, but as the temperature dropped further, fewer and fewer particles and their antiparticles were created, which just left an excess of protons and neutrons.

These protons and neutrons still exist today – you are made up of quarks from the Big Bang.

Nucleosynthesis

About one second after the Big Bang, when the temperature had dropped to 10^{10} K, the universe was cool enough to allow nucleosynthesis, in which protons and neutrons fused to make light atomic nuclei, mainly helium ($^4_2\mathrm{He}$), with some traces of deuterium ($^2_1\mathrm{H}$) and lithium ($^7_3\mathrm{Li}$). This continued for about three minutes, until the temperature of the ever-expanding universe dropped to a mere 10^9 K and nucleosynthesis stopped.

The Stelliferous Era: t = 3 minutes to 10^{14} years

After the explosive first three minutes, the universe settled into a much calmer phase. For the next 300,000 years, the universe consisted of a sea of hydrogen and helium nuclei, photons, free electrons and the mysterious dark matter.

Gravity started pulling the hydrogen and helium together to produce the vast collections of gas and other matter we now call galaxies. Our Sun ignited some 4.6 billion years (4.6×10^9 years) ago and has enough hydrogen to last another 6 billion years.

The future

There seem to be two possible scenarios for the future of the universe: the Big Freeze, in which the universe continues to expand until it dies a cold death, or the Big Rip, in which the expansion proceeds at an increasing rate. At present, the evidence points to the Big Rip, but astronomical knowledge develops quickly, and in your lifetime our model of the universe will continue to be developed.

Apply your understanding

1 **Explain** what is meant by "matter beat antimatter" in the creation of the universe. (1 mark)

2 Figure 6 shows the spectra of three Type 1a supernovae, labelled star A, star B and star C. The black lines represent the presence of hydrogen in the stars. The spectrum of a hydrogen sample in a laboratory on Earth is shown as the reference diagram. Use your understanding of astronomy to answer the following questions.

a **Propose** what the lines represent. (1 mark)

b **Deduce** which supernova would be the brightest. (1 mark)

c **Justify** your choice by evaluating the evidence for and against this statement in terms of speed (towards us or away from us) and distance from us. (2 marks)

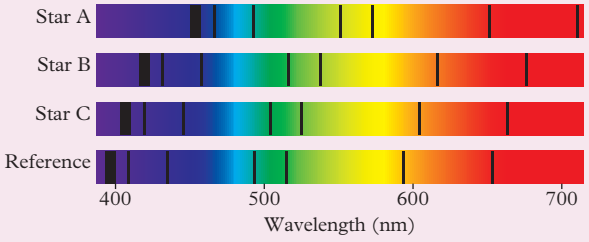


FIGURE 6 Spectra of supernovae.

Skill drill

Recognising limitations of the evidence

Science inquiry skills: Evaluating evidence (Lesson 1.8); Conducting your research investigation (Lesson 1.12)

When you are completing the research investigation for internal assessment, the syllabus requires you to analyse and interpret research evidence about special relativity, quantum theory or the Standard Model. After recognising and analysing the limitations in your evidence, you are required to suggest logical improvements and extensions to the investigation.

Any suggested improvements that you propose should be directly linked to the limitations you identify and should also be relevant to the claim. For every limitation you identify, you need to offer a way that this could be fixed. Similarly, your suggested extensions need to complement the current findings, while also providing new (or more) information that could be used to further evaluate the claim and draw a valid conclusion.

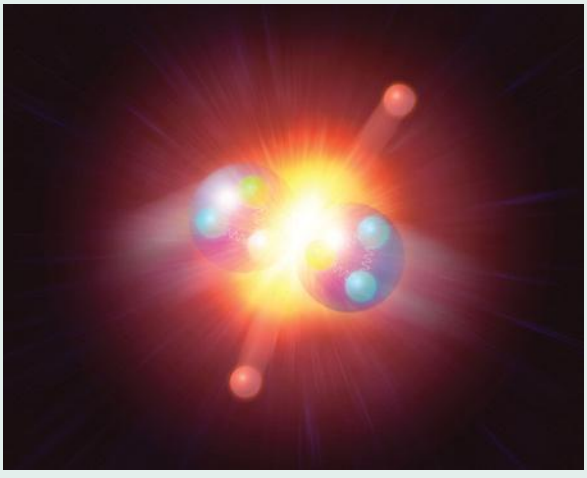


FIGURE 7 An illustration of Higgs bosons (in pink) being produced by two colliding protons, with quarks visible.

Consider the following claim:

"Particle accelerators are allowing scientists to decode the universe."

The following research question has been provided:

"Have there been significant advancements in physics from the use of the Large Hadron Collider?"

The following evidence has been provided:

Since the Large Hadron Collider (LHC) began functioning in September 2008, many scientific breakthroughs have been made. The discovery of the Higgs boson in 2012 is one. Originally theorised in 1964 by Francois Englert and Peter Higgs, the Higgs boson supports the theory of the Standard Model and provides evidence to support the idea of the Higgs field and the Big Bang theory as the origin of the universe (CERN, 2019). Both Englert and Higgs received Nobel Prizes in 2013 for their theoretical prediction of the existence of this elementary particle. The LHC enabled its actual discovery.

The LHC created particle interactions above 6.5 TeV, eclipsing the energy levels of its predecessor, the Large Electron–Positron Collider (LEP). Moreover, the LHC also allows for the interaction of more particles than the LEP could. This has allowed the LHC to provide data to four experimental collaborations around the globe, contributing to more than 2,000 scientific papers being published by 2018 (Lincoln, 2018). The LHC also has the potential to provide evidence to support (or reject) the Supersymmetry Model, a theoretical model that would account for gravity, unlike the Standard Model (CERN, 2019). CERN announced that when the LHC was made operational again in 2021 after scheduled upgrades, it was able to produce 13 TeV collisions (6.5 TeV per beam) (CERN, 2019; Lincoln, 2018). This is an increase from the 8 TeV collisions recorded in 2012 (CERN, 2019).

Before its most recent scheduled shutdown in late 2018, the LHC had only recorded 3% of its expected data (Lincoln, 2018). This has left scientists pondering, if such discoveries can be made from only 3% of maximum data, how many more phenomena the LHC might reveal. CERN (2019) intends to use the upgrades to the LHC to enable physicists to keep hunting for evidence that might shed light on the theories of the universe.

Practise your skills

1 **Identify** two limitations of the evidence presented. (2 marks)

2 **Propose** possible improvements and extensions to the investigation. **Explain** how these improvements and extensions will allow the claim to be evaluated further and valid conclusions to be made. (4 marks)

CERN. (2019). *The Higgs boson*. https://home.cern/science/physics/higgs-boson

Lincoln, D. (2018). *In 10 years, the Large Hadron Collider was a smash - with more discoveries to come*. https://www.livescience.com/64392-large-hadron-collider-discoveries.html

Check your learning 13.2

Check your learning 13.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Identify** the particle involved in the strong nuclear force between nucleons. (1 mark)

2 **Recall** the names of the four gauge bosons. (1 mark)

3 **Describe** the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons. (3 marks)

Analytical processes

4 **Contrast** the fundamental forces experienced by quarks and leptons. (2 marks)

5 **Contrast** the forces experienced by a positron with that of a bottom quark. (2 marks)

Knowledge utilisation

6 A gauge boson mediates between two elementary particles. **Propose** what you would need to know about the particles to decide which gauge boson it was. (2 marks)

7 The table lists elementary particles and possible forces between them. **Modify** the table so the two columns match correctly. (3 marks)

Particles	Feel the
a charged leptons	**i** weak force
b all quarks	**ii** weak and electromagnetic forces
c all leptons and all quarks	**iii** weak, electromagnetic and strong forces

MODULE 13

Lesson 13.3
Review: The Standard Model

Summary

13.1
- There are two types of elementary matter particles: leptons and quarks. These elementary particles have no internal structure.
- Particles and their antiparticles have identical masses but opposite values of charge.
- There are six quarks, called up (u), down (d), strange (s), charm (c), top (t) and bottom (b).
- Hadrons are composite particles that are made up of quarks and/or antiquarks. There are two types of hadrons: baryons and mesons.
- Baryons include the neutron and the proton and consist of three quarks or three antiquarks, whereas mesons include pions and kaons and consist of a quark and antiquark.
- There are six leptons in the Standard Model: the electron, the muon, the tau particles and their associated neutrinos – namely, the electron neutrino, the muon neutrino and the tau neutrino. There are also three corresponding antileptons and three corresponding antineutrinos.

13.2
- In addition to the six elementary leptons and six elementary quarks, the Standard Model includes four mediating particles, called gauge bosons, that are associated with three of the four fundamental forces.
- The four fundamental forces are strong nuclear, electromagnetic, weak nuclear and gravitational force.

Review questions 13.3A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 The elementary particles of the Standard Model are
- **A** baryons and leptons.
- **B** baryons and quarks.
- **C** leptons and quarks.
- **D** photons and baryons.

2 The muon and the electron are both
- **A** baryons.
- **B** hadrons.
- **C** leptons.
- **D** mesons.

3 Determine which combination of quarks would produce a neutrally charged baryon.
- **A** $\overline{udd}$
- **B** $\bar{u}dd$
- **C** uud
- **D** udd

4 Which one of the following is composed of quarks?
- **A** Neutron
- **B** Muon
- **C** Gluon
- **D** Boson

5 The elementary particle that carries no charge is the
- **A** neutron.
- **B** meson.
- **C** neutrino.
- **D** quark.

6 The components of a meson are
- **A** a quark and an antiquark.
- **B** two quarks and one antiquark.
- **C** a muon and an antimuon.
- **D** a gluon and a quark.

7 The gluon is the force carrier for the
- **A** electromagnetic force.
- **B** gravity.
- **C** strong nuclear force.
- **D** weak force.

8 Which of the following statements is correct?
- **A** The interactions between subatomic particles are mediated by the exchange of particles called gauge bosons.
- **B** The strong nuclear force is mediated by the exchange of baryons.
- **C** The weak nuclear force is mediated by Z bosons that have either a positive or negative electric charge.
- **D** The electromagnetic force is mediated by massless bosons.

9 The carrier particles that are transmitted between quarks are
- **A** gluons.
- **B** pions.
- **C** photons.
- **D** W and Z bosons.

10 The strong nuclear force between protons and neutrons is due to an exchange of
- **A** muons.
- **B** gluons.
- **C** photons.
- **D** leptons.

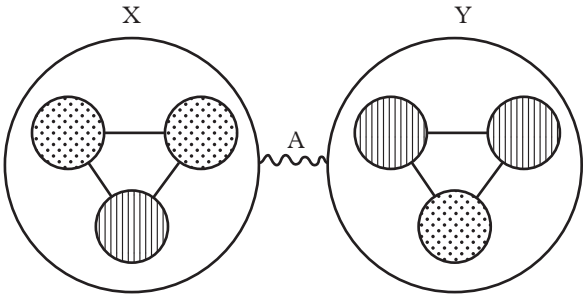
11 The force that is not described by the Standard Model is
- **A** electromagnetism.
- **B** gravitation.
- **C** strong force.
- **D** weak force.

12 Identify the statement that best describes a lepton particle.
- **A** An elementary particle that responds to strong forces and weak forces, but not gravitational forces
- **B** An elementary particle that responds to weak forces and gravitational forces, but not strong forces
- **C** An elementary particle that responds to strong forces and gravitational forces, but not weak forces
- **D** An elementary particle that responds to gravitational forces and electromagnetic forces, but not weak forces

13 Identify the correct order, from weakest force to strongest force.
- **A** Weak, electromagnetic, gravitation, strong nuclear
- **B** Electromagnetic, gravitational, weak, strong nuclear
- **C** Gravitational, weak, electromagnetic, strong nuclear
- **D** Weak, gravitational, electromagnetic, strong nuclear

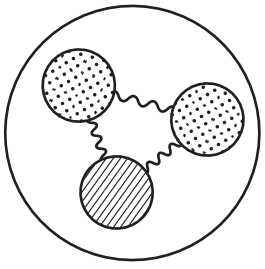
14 Identify the force that holds particles together in a nucleus.
- **A** Electromagnetic force
- **B** Gravitational force
- **C** Strong nuclear force
- **D** Weak force

15 The diagrams represent a proton and a neutron. The circles with dots represent a positive charge and the striped circles represent a negative charge. The wavy line, A, represents the main force between X and Y.



Which one of the following options states the names of X, Y and A in order?
- **A** Neutron, proton, gauge boson
- **B** Proton, neutron, gauge boson
- **C** Neutron, proton, meson
- **D** Proton, neutron, meson

16 The diagram represents an antiproton.



The circles with the dots represent
- **A** an antiup quark.
- **B** an antidown quark.
- **C** an up quark.
- **D** a down quark.

Review questions 13.3B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

17 **Recall** the names and charges of the six types of leptons and their antiparticles. (2 marks)

18 **Identify** which of the following are elementary particles: proton, neutron, quark, muon, electron, gluon, Higgs boson, atom, photon. (3 marks)

19 **Clarify** whether a positron and an antielectron are identical particles. (1 mark)

20 Baryons and mesons are similar in many ways but have two major differences in their composition. **Identify** these two differences. (2 marks)

21 **Describe** two properties of the particles made up of the following quarks. (1 mark each)

a $u\bar{d}$

b $\bar{d}u$

c ddc

d uss

Analytical processes

22 **Compare** the properties of the first generation of quarks and the first generation of leptons. (2 marks)

23 **Contrast** the force between leptons with the force between antileptons. (2 marks)

24 **Categorise** the six quarks into their three generations (families) and distinguish between each generation. (3 marks)

25 **Classify** the four types of gauge bosons by

a the force they mediate (4 marks)

b the particles they mediate between. (4 marks)

26 **Compare** the properties of particles with antiparticles. (2 marks)

27 **Determine** the charge on these baryons. (4 marks)

	Name	Symbol	Quarks
a	Charmed lambda	Λ_c^+	udc
b	Sigma	Σ^+	uus
c	Charmed sigma	Σ_c^{++}	uuc
d	Xi	Ξ^-	dss

28 **Determine** the charge on these mesons. (3 marks)

	Name	Symbol	Quarks
a	Charmed eta meson	η_c	$c\bar{c}$
b	Strange B meson	B_s^0	$s\bar{b}$
c	Strange D meson	D_s^+	$c\bar{s}$

29 The symbols for a D meson and its antiparticle are D^0 and $\overline{D}^0$, but not necessarily in that order. **Infer** which is the particle and which is the antiparticle, with reasons. (2 marks)

Knowledge utilisation

30 **Create** three lists that divide the elementary particles of the Standard Model (including the gauge bosons) into groups based on their charge (+, −, neutral). (3 marks)

31 Physicists have defined a category of particles known as hadrons. Any composite particle containing quarks (or antiquarks) is a hadron. **Assess** which of the following are hadrons: proton, neutron, electron, muon, meson, gluon, graviton, Higgs boson, alpha particle, beta particle. (2 marks)

32 **Propose** the total number of elementary particles in the Standard Model. You should not include quark colours in your answer, but you will need to provide the names of all particles. (5 marks)

Data drill

Modelling the electromagnetic force between protons in the nucleus

The Coulomb (electromagnetic) force between two protons in an atom was being measured as a function of their separation distance. This is shown in Table 1. The separation distance, d, is in units of femtometre, fm, where $1\,\text{fm} = 10^{-15}\,\text{m}$.

TABLE 1 The Coulomb force between two protons in an atom measured as a function of their separation distance

Distance, d (fm)	$\frac{1}{d^2}$ ($\times 10^{30}\,\text{m}^{-2}$)	Coulomb force (N)
0.50	4.00	921.6
0.75	1.78	409.6
1.00	1.00	230.4
1.50	**Q1**	102.4
2.00	0.25	57.6
3.00	0.11	25.6

A graph of the force, F, versus the inverse of the square of the distance is shown in Figure 1.

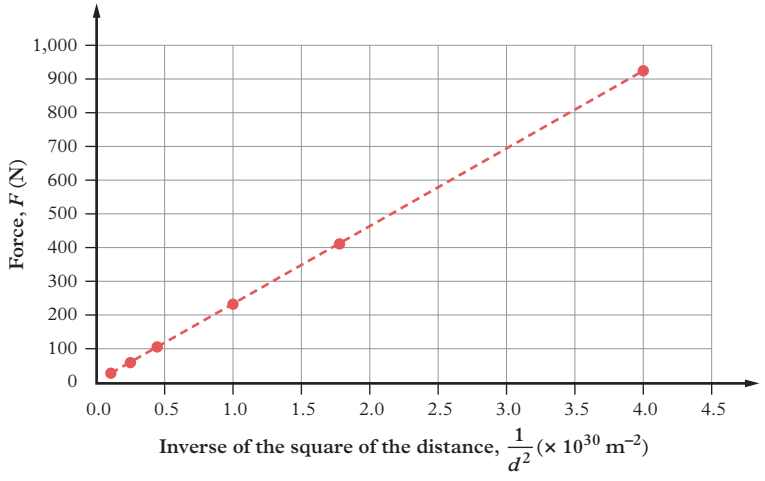


FIGURE 1 Force vs the inverse of the square of the distance for two protons.

Apply understanding

1 **Calculate** the missing value from the table for the inverse of the square of the distance when the distance is 1.5 fm. State the units you have used. (2 marks)

2 **Identify** the sign and magnitude of the charge on a proton in coulomb. Use your QCAA Formula and data book to find this. (1 mark)

Analyse data

3 **Identify** the relationship between the Coulomb force between protons and the separation distance. (3 marks)

Interpret evidence

4 In terms of Coulomb's formula for the force between electrically charged particles, **determine** what the gradient represents. (2 marks)

5 Use the gradient of the graph to **draw a conclusion** about the experimental value for the charge on a proton. (2 marks)

Module 13 checklist: The Standard Model

MODULE 14

Particle interactions

Introduction

There is an old saying in physics that "everything that can happen without violating a conservation law does happen". It doesn't say "might happen"; it says "does happen". So, if a physicist knows a particular event is possible and therefore must be happening somewhere in the universe, they set themselves the task of observing it and quantifying it. That's just one of the things that drive science along.

Some interactions can be ruled out because they don't follow various conservation laws – for example, conservation of energy, momentum and charge – so there is no point looking for them. This module looks at a variety of particle interactions and shows you how to represent them in diagrams and use properties such as lepton number, baryon number and symmetry to work out what interactions are allowed and what interactions are not allowed.

Prior knowledge

Prior knowledge quiz Check your understanding of concepts related to particle interactions before you start.

Subject matter

Science understanding

- → Describe the concepts of lepton number and baryon number.
- → Solve problems relating to the conservation of lepton number and baryon number in particle interactions using $B = n_b - n_{\bar{b}}$, $B = \frac{1}{3}(n_q - n_{\bar{q}})$ and $L = n_l - n_{\bar{l}}$.
- → Describe electron/electron, electron/positron and neutron decay interactions using particle interaction diagrams.
- → Describe how symmetry in particle interactions occurs to maintain the principles of conservation.

Science as a human endeavour

→ Appreciate the significant contributions of scientists such as Chien-Shiung Wu, Richard Feynman and Peter Higgs who furthered our understanding of particle physics and The Standard Model.

→ Explore the history of particle physics models and theories through the development of particle accelerators and contributions from notable physicists.

→ Appreciate that particle accelerators like the Australian Synchrotron and Large Hadron Collider are developed through multinational collaborations between science organisations and governments.

Science inquiry

→ Examine evidence supporting theories related to particle physics.

Source: *Physics 2025 v1.2 General Senior Syllabus* © State of Queensland (QCAA) 2024

Lesson 14.1
Conservation in interactions

Key ideas

- → Baryon and lepton numbers help us understand which reactions can happen and which cannot happen.
- → Baryons have a baryon number, B, of +1, and antibaryons have a baryon number of –1. Leptons and antileptons have a baryon number of 0.
- → Quarks have a baryon number of $+\frac{1}{3}$, and antiquarks have a baryon number of $-\frac{1}{3}$.
- → Leptons have a lepton number, L, of +1, and antileptons have a lepton number of –1. Baryons and antibaryons have a lepton number of 0.
- → Baryon number and lepton number are always conserved in a reaction.

What is baryon number?

Learning intentions and success criteria

baryon number
a quantum number of a system defined by $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks; it is strictly conserved and additive

One of the great principles of physics is conservation. A conservation law states that a particular measurable property of an isolated physical system does not change as the system changes over time. You have already learnt about conservation of energy, mass, momentum and electric charge. These all still hold for particle interactions. Next, we will introduce some more particle properties and their corresponding conservation laws that help explain why some reactions occur and others do not.

These apply to such quantities as lepton number, baryon number and strangeness. These quantities are conserved in certain classes of physics processes, but not in all. This section will look at two of the most fundamental predictions of conservation laws in particle physics – the laws of conservation of **baryon number** and lepton number.

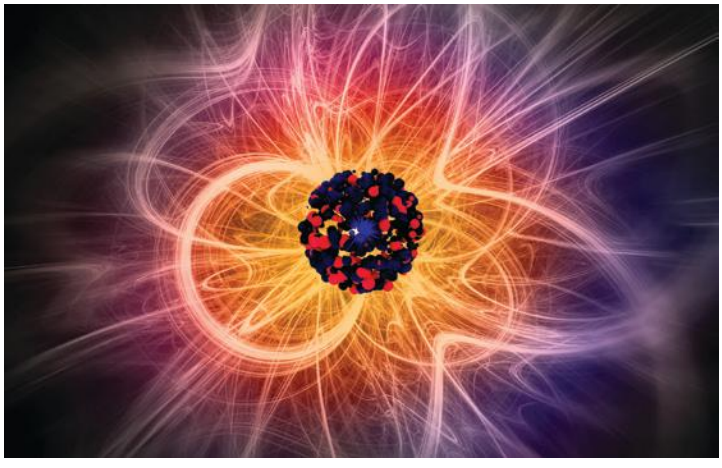


FIGURE 1 An abstract illustration of particles in collision

In Module 13, you learnt that baryons are composite subatomic particles made up of three quarks (or in the case of antibaryons, three antiquarks) held together by the strong nuclear force. Put simply, if a particle is a baryon, its baryon number, B, is +1. If the particle is an antibaryon, its baryon number is −1. Mesons, which are made up of a quark and antiquark pair and are not baryons, must have a baryon number of 0. That sums it up well, but let's dig a little deeper. For instance, what is the baryon number of a quark? Well, as there are three quarks in a baryon, each quark has a baryon number of $+\frac{1}{3}$, and as there are three antiquarks in an antibaryon, each antiquark has a baryon number of $-\frac{1}{3}$.

For example, a baryon is made of three quarks, so its baryon number is $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. A meson is a quark and antiquark pair, so its baryon number is $+\frac{1}{3} + -\frac{1}{3} = 0$, as you'd expect, because it is not a baryon.

Baryon number is defined by $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks. Some examples of baryon numbers for different particles are shown in Table 1.

TABLE 1 Baryon numbers of some common particles

Particle	Quark composition	$B = \frac{1}{3}\left(n_q - n_{\bar{q}}\right)$	Baryon number, B
quark	q	$B = \frac{1}{3}(1 - 0)$	$+\frac{1}{3}$
antiquark	$\bar{q}$	$B = \frac{1}{3}(0 - 1)$	$-\frac{1}{3}$
proton, p+	uud	$B = \frac{1}{3}(3 - 0)$	+1
neutron, n^0	udd	$B = \frac{1}{3}(3 - 0)$	+1
antiproton, $\bar{p}$	$\bar{u}\bar{u}\bar{d}$	$B = \frac{1}{3}(0 - 3)$	–1
antineutron, $\bar{n}$	$\bar{u}\bar{d}\bar{d}$	$B = \frac{1}{3}(0 - 3)$	–1
meson (not a baryon)	$q\bar{q}$	$B = \frac{1}{3}(1 - 1)$	0
lepton (not a baryon)	e^-	$B = \frac{1}{3}(0 - 0)$	0

Study tip

You are expected to be able to determine baryon number quantitatively (i.e. by calculation) so don't let strange names for baryons and mesons confuse you. Just look at how many quarks and antiquarks are in the particle and apply the equation.

How is baryon number conserved?

Now you know what baryon number is, we can look at how it is conserved.

You will recall that when you balanced a nuclear equation in Unit 1, the mass numbers of the reactants had to equal the mass number of the products. For example, consider the alpha bombardment of beryllium:

$$^4_2\text{He} + ^9_4\text{Be} \rightarrow ^{12}_6\text{C} + ^1_0\text{n}$$

The mass numbers (top numbers) need to be equal on both sides. In this example $4 + 9 = 12 + 1$, so the mass number is balanced (or conserved).

Similarly, baryon numbers must be the same before and after an interaction (i.e. baryon number is conserved), and you can add and subtract the number of baryons (i.e. it is additive). More formally, baryon number is a strictly conserved additive quantum number of a system. Mathematically, the baryon number is defined as:

$$B = n_b - n_{\bar{b}}$$

where n_b is the number of baryons, and $n_{\bar{b}}$ is the number of antibaryons.

As protons and neutrons are baryons (composites of three quarks), the equation for alpha bombardment of beryllium could also be referred to as baryon conservation. However, you have seen that there are more baryons than just protons and neutrons, so the idea of **conservation of baryon number** means something broader in this module.

conservation of baryon number in any interaction, baryon number is conserved

In Worked example 14.1A, the baryon numbers of all species in an interaction are given, and you have to check whether the baryon number of the reactants, B(reactants), equals the baryon number of the products, B(products).

Study tip

The symbol for a proton is p^+, and the symbol for a neutron is n^0 or just n.

Worked example 14.1A

Conservation of baryon number where *B* is given

The particle interaction $\Omega^- \rightarrow \Xi^0 + K^-$ shows the possible decay of an omega particle.

For each particle in the interaction, the following information is given, in order: the symbol for the particle, its name, its quark constituents, and its baryon number.

Ω^-, omega (sss), $B = +1$

Ξ^0, Xi (uss), $B = +1$

K^-, kaon ($s\bar{u}$), $B = 0$

Determine if the interaction violates the law of conservation of baryon number and hence **assess** if it is "allowable". (3 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. "Assess" means to make a judgment about the outcomes. There are 3 marks: 1 mark for determining the values of B, 1 mark for determining if B has been conserved, and 1 mark for assessing whether this makes the interaction forbidden (not allowed).
Step 2: Determine the baryon number, B, for each particle in the interaction. In this case, the B values are given. (1 mark for "Determines correct B values")	Ω^-, $B = +1$ Ξ^0, $B = +1$ (1 mark) K^-, $B = 0$
Step 3: Set out the values so that the B values for the reactants and products are clear.	**Particle** | Ω^- | $\rightarrow$ | $\Xi^0 + K^-$ ***B*** | +1 | = | +1 + 0
Step 4: Calculate the sum of baryon numbers for the reactants and the products. If they are equal, B is conserved. (1 mark for "Determines that the baryon number is conserved") Allow follow through errors (FTE) if the baryon number is incorrectly determined.	B(reactants) = +1, B(products) = (+1 + 0) = +1 B is conserved. (1 mark)
Step 5: State whether the reaction is allowed based on whether it meets the law of conservation of baryon number. (1 mark for "Concludes with justification that the interaction is allowed (permitted)") Allow FTE from the previous step. Note: it is highly unlikely that exotic quark composites such as the first two would appear on the external exam. It is used here to develop understanding.	The interaction is allowed in terms of baryon number as it does not violate the law of conservation of baryon number. (1 mark)

Your turn

The particle interaction $\Sigma_b^- \rightarrow \Lambda_b^0 + \pi^-$ shows the possible decay of a bottom sigma particle.

For each particle in the interaction, the following information is given, in order: the symbol for the particle, its name, its quark constituents, and its baryon number. (2 marks)

Σ_b^-, bottom sigma (ddb), $B = +1$

Λ_b^0, bottom lambda (udb), $B = +1$

π^-, pion ($d\bar{u}$), $B = 0$

Determine if the interaction violates the law of conservation of baryon number and hence **assess** whether it is "allowable". (3 marks)

How is baryon number determined?

A more complex situation arises when only the quark composition in an interaction is given. In this case, you first need to use the formula for baryon number, $B = \frac{1}{3}(n_q - n_{\bar{q}})$, to determine B, before determining whether B is conserved. This is shown in Worked example 14.1B. Note that there are leptons in this reaction.

Worked example 14.1B

Conservation of baryon number where *B* is calculated

The particle interaction $\Sigma^+ \rightarrow p^+ + \pi^0$ shows a possible decay pathway for the sigma-plus baryon.

For each particle in the interaction, the following information is given, in order: the symbol for the particle, its name, and its quark constituents.

Σ^+, sigma plus (uus)

p^+, proton (uud)

π^0, neutral pion ($u\bar{u}$)

Determine if the interaction violates the law of conservation of baryon number and hence **assess** whether it is "allowable". Show your working. (3 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. "Assess" means to make a judgement about the outcomes. There are 3 marks: they are likely to be 1 mark for determining the values of *B*, 1 mark for determining if *B* has been conserved, and 1 mark for assessing whether this makes the interaction forbidden (not allowed).
Step 2: Determine the baryon number, *B*, for each particle in the interaction. (1 mark for "Determines correct *B* values")	$B(\Sigma^+) = \frac{1}{3}(n_q - n_{\bar{q}}) = \frac{1}{3}(3 - 0) = 1$ $B(p^+) = \frac{1}{3}(n_q - n_{\bar{q}}) = \frac{1}{3}(3 - 0) = 1$ (1 mark) $B(\pi^0) = \frac{1}{3}(n_q - n_{\bar{q}}) = \frac{1}{3}(1 - 1) = 0$
Step 3: Set out the values so that the baryon number for each reactant and product is clear.	$B(\Sigma^+) = +1$ $B(p^+) = 1$ $B(\pi^0) = 0$
Step 4: Calculate the sum of baryon numbers for the reactant and for the products. If they are equal, baryon number is conserved. (1 mark for "Determines that the baryon number is conserved") Allow follow-through errors (FTE) if baryon number is incorrectly determined.	B(reactants) = +1, B(products) = (+1 + 0) = +1 *B* is conserved. (1 mark)
Step 5: State whether the reaction is "allowed" based on whether it meets the law of conservation of baryon number. (1 mark for "Concludes with justification that the interaction is allowed (permitted)"). Allow FTE from the previous step.	The interaction is allowed in terms of baryon number as it does not violate the law of conservation of baryon number. (1 mark)

Your turn

The particle interaction $\pi^- + p^+ \rightarrow n^0 + \pi^0 + \pi^- + \pi^+$ shows the possible interaction of a proton with a negative pion (a meson) particle.

For each particle in the interaction, the following information is given, in order: the symbol for the particle, its name, and its quark constituents.

p^+, proton (uud)

n^0, neutron (udd)

π^0, neutral pion ($u\bar{u}$)

π^-, negative pion ($d\bar{u}$)

π^+, positive pion ($u\bar{d}$)

Determine if the interaction violates the law of conservation of baryon number and hence **assess** whether it is "allowable". Show your working. (3 marks)

What is lepton number?

conservation of lepton number
in any interaction, lepton number is conserved

lepton number
a conserved quantum number representing the difference between the number of leptons and the number of antileptons in an elementary particle reaction: $L = n_l - n_{\bar{l}}$

Another useful particle conservation law is the law of **conservation of lepton number**. In particle physics, **lepton number** is a conserved quantum number representing the difference between the number of leptons and the number of antileptons in an elementary particle reaction. This means that, like baryon number, lepton number is the same before and after a reaction. It is an additive quantum number, so its sum is preserved in interactions. Mathematically, the lepton number is defined as:

$$L = n_l - n_{\bar{l}}$$

where n_l is the number of leptons, and $n_{\bar{l}}$ is the number of antileptons.

However, a lepton number is commonly given to individual particles: leptons have a lepton number of +1, antileptons have a lepton number of −1, and non-leptons have a lepton number of 0.

FIGURE 2 Neutrino detectors at the IceCube neutrino observatory, waiting to be installed in the ice under the Amundsen–Scott South Pole Station. Conservation of lepton number would be violated in beta emission interactions without the elusive neutrino.

Study tip

Lepton number is not the same as electric charge. For example, an electron, e^-, has an electric charge of −1 because it has a negative charge, but it has a lepton number of +1 because it is a lepton.

Study tip

Neutrinos are uncharged, but they still have a lepton number of +1. Antineutrinos have a lepton number of −1.

TABLE 2 Lepton numbers of some leptons and antileptons

Generation	Leptons, $L = +1$		Antileptons, $L = -1$	
	Flavour	Symbol	Flavour	Symbol
First	electron	e^-	positron	e^+
	electron neutrino	ν_e	electron antineutrino	$\bar{\nu}_e$
Second	muon	μ^-	antimuon	μ^+
	muon neutrino	ν_μ	muon antineutrino	$\bar{\nu}_\mu$
Third	tau	τ^-	antitau	τ^+
	tau neutrino	ν_τ	tau antineutrino	$\bar{\nu}_\tau$

Non-leptons ($L = 0$) are baryons (qqq), such as protons and neutrons, or mesons ($q\bar{q}$), such as kappa-zero (K^0) and pions (π^+, π^0, π^-).

The symbol for an antilepton is written in two ways. If it is "electron-like", such as an electron, muon or tau, it is written with the opposite sign (+ or –), but no bar over the top. If it is "neutrino-like", such as the antineutrinos of the electron, muon and tau, it is written with a bar over the top.

Worked example 14.1C

Determining lepton number in a multiple-choice question

Determine the lepton number for, in order, a tau particle, tau neutrino, tau antineutrino and a proton. (1 mark)

A +1, +1, 0, 0
B +1, +1, −1, 0
C −1, +1, −1, +1
D +1, +1, +1, +1

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We first must calculate the lepton number of each particle.
Step 2: Gather the relevant information from the question and select the appropriate formula.	$L = n_l - n_{\bar{l}}$
Step 3: A tau is a lepton; it is not an antilepton. Because it is a lepton you could just give its lepton number, $L = +1$, and not show the calculation.	A tau is a lepton: $L = n_l - n_{\bar{l}} = 1 - 0 = +1$
Step 4: A tau neutrino is a lepton; it is not an antilepton. Because it is a lepton you can give its lepton number, $L = +1$, and not show the calculation.	A tau neutrino is a lepton: $L = n_l - n_{\bar{l}} = 1 - 0 = +1$
Step 5: A tau antineutrino is an antilepton. Because it is an antilepton you can give its lepton number, $L = -1$, and not show the calculation.	A tau antineutrino is an antilepton: $L = n_l - n_{\bar{l}} = 0 - 1 = -1$
Step 6: A proton is a baryon, so it contains no leptons or antileptons. Because it is not a lepton you could just give its lepton number, $L = 0$, and not show the calculation.	A proton contains no leptons or antileptons (it is a baryon): $L = n_l - n_{\bar{l}} = 0 - 0 = 0$
Step 7: List the lepton numbers in order and choose the option that matches.	The answer is +1, +1, −1, 0, which is option B. (1 mark)

Your turn

Determine the lepton number for, in order, a neutron, muon, meson and muon antineutrino. (1 mark)

A 0, +1, 0, −1 **B** 0, −1, +1, −1 **C** −1, +1, 0, −1 **D** −1, −1, −1, +1

How is lepton number conserved?

Study tip

You are expected to calculate lepton number, and you should be able to determine if it is conserved. The calculation is required for the external exam.

Like baryon number, lepton number is also conserved in particle interactions. The method of testing for conservation is much like the process for baryons. This is shown in Worked example 14.1D.

Worked example 14.1D

Conservation of lepton number where *L* is calculated

Determine whether lepton number is conserved, and draw a conclusion about whether the reaction is allowable, for the following particle interactions. Show your working.

a The decay of a neutron: $n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$ (3 marks)

b The decay of a muon: $\mu^- \rightarrow e^- + \bar{\nu}_e$ (3 marks)

Think	Do
Step 1: Look at the cognitive verb and mark allocation to determine what the question is asking you to do.	"Determine" means to establish after a calculation. We must calculate the lepton number for all particles, compare the values for the reactants and products to see if lepton number is conserved, and draw a conclusion about whether the reaction is allowable.
Step 2: Gather the relevant information from the question and select the appropriate formula.	$L = n_l - n_{\bar{l}}$

Think	Do
Step 3: State which particles are leptons and non-leptons (baryons or mesons) and use that to identify their lepton numbers.	**a** The neutron and proton are baryons, so $L = 0$. The electron is a lepton, so $L = +1$, and the electron antineutrino has the opposite sign, $L = -1$.
Step 4: Set out the values so the lepton number for each reactant and product is clear. (1 mark for "Correctly determines the lepton number for each reactant and product")	<table><tr><th></th><th>n^0</th><th>$\rightarrow$</th><th>p^+</th><th>e^-</th><th>$\bar{\nu}_e$</th></tr><tr><td>L</td><td>0</td><td>=</td><td>0</td><td>+1</td><td>−1</td></tr></table> (1 mark)
Step 5: Calculate the sum of lepton numbers for the reactants and the products. If they are equal, lepton number is conserved. (1 mark for "Shows correct working to determine that lepton number is conserved")	L(reactants) = 0, L(products) = 0, so they are equal. Lepton number is conserved. (1 mark)
Step 6: State if the reaction is allowable under the law of conservation of lepton number. (1 mark for "Draws a correct justified conclusion that it is allowable")	The reaction is allowable as the lepton number is conserved. (1 mark)
Step 7: State which particles are leptons and non-leptons (baryons or mesons) and use that to identify their lepton numbers. Then set out the values so the lepton number for each reactant and product is clear. (1 mark for "Correctly determines the lepton number for each reactant and product")	**b** The muon and electron are leptons ($L = +1$). The antineutrino is an antilepton ($L = -1$). <table><tr><th></th><th>μ^-</th><th>$\rightarrow$</th><th>e^-</th><th>$\bar{\nu}_e$</th></tr><tr><td>L</td><td>+1</td><td>≠</td><td>+1</td><td>−1</td></tr></table> L(reactants) = +1, L(products) = 0 (1 mark)
Step 8: Make a statement about conservation of lepton number, and about whether the reaction is allowable. (1 mark for "Concludes with justification that lepton number is not conserved"; 1 mark for "Assesses that the reaction is not allowable")	L(reactants) ≠ L(products). Lepton number is not conserved (1 mark) The law of conservation of lepton number is violated, so the reaction is not allowable. (1 mark)

Your turn

Determine whether lepton number is conserved, and draw a conclusion whether the reaction is allowable, for the following particle interactions. Show your working.

a the decay of a neutron: $\bar{\nu}_e + p^+ \rightarrow n^0 + e^+$ (3 marks)

b the decay of a muon: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ (3 marks)

How does conservation of baryon and lepton number apply together?

For a particle reaction to occur, it has to obey all of the various conservation laws. As well as conservation of electrical charge and momentum, you have seen there must be conservation of baryon number and lepton number.

Physicists have divided lepton conservation up into three subcategories – electron lepton (L_e), muon lepton (L_μ) and tau lepton (L_τ) numbers – all of which have to be conserved. However, our rule for leptons in general will suffice for this level of work. Physicists have discovered that there are other conservation rules that must be applied, such as conservation of charm and strangeness (for baryons and mesons), but they are beyond this course.

Worked example 14.1E shows you how to apply conservation of baryon number and lepton number to the same reaction.

Worked example 14.1E

Determining baryon and lepton conservation together

In a hypothesised particle interaction, a neutron decays into a proton, a positron and an electron antineutrino.

a **Construct** a balanced equation for this interaction. (1 mark)

b **Assess** whether this interaction violates the laws of conservation of baryon and lepton number. (4 marks)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Construct" means to display information in a diagrammatic or logical form. "Assess" means to make a judgement about the outcomes.
Step 2: Construct an equation using the correct symbols. (1 mark for "Constructs correct symbol equation") Note: you can also write the neutron and proton as just n and p without the charges.	**a** $n^0 \rightarrow p^+ + e^+ + \bar{\nu}_e$ (1 mark)
Step 3: State which particles are baryons and which are leptons, and use that to identify their baryon and lepton numbers. Set out the values so the baryon number and lepton numbers for the reactant and products are clear. Prepare a table and fill in the columns with the particle and its baryon and lepton numbers. (2 marks for "Correctly determines the baryon and lepton numbers for each reactant and product")	**b** Neutrons and protons are baryons so are not leptons ($B = 1$, $L = 0$). The positron and electron antineutrino are not baryons ($B = 0$) but are antileptons ($L = -1$). B(reactants) = +1, B(products) = +1 L(reactants) = 0, L(products) = –2 (2 marks)
Step 4: Make a statement about conservation of baryon number and lepton number, and about whether the reaction is allowable. (1 mark for "Concludes with justification that lepton number is not conserved"; 1 mark for "Assesses that the reaction is not allowable")	B(reactants) = B(products), so baryon number is conserved. L(reactants) ≠ L(products), so lepton number is not conserved. (1 mark) The law of conservation of lepton number is violated, so the reaction is not allowable. (1 mark)

	n^0	→	p^+	e^+	$\bar{\nu}_e$
B	+1	=	+1	0	0
L	0	≠	0	−1	−1

Your turn

In a hypothesised particle interaction, a lambda zero particle, Λ^0 (uds), decays into a proton, an electron and an electron antineutrino.

a **Construct** an equation for this interaction. (1 mark)

b **Assess** whether this interaction violates the laws of conservation of baryon and lepton numbers. (4 marks)

Skill drill

Making use of summary tables

Science inquiry skills: Processing and analysing data (Lesson 1.7)

A great way of comparing the characteristics of particles is to use summary tables. That way you can group together similar properties such as electric charge or baryon number and see the commonalities. It helps you sort out the "particle zoo" that is the Standard Model.

Practise your skills

1 The table lists three particles of the Standard Model and their characteristics A, B, C and D. **Determine** what properties the four letters stand for. (4 marks)

	A	B	C	D
$\bar{d}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	no
μ^+	+1	0	–1	no
Z^0	0	0	0	yes

2 The properties of four particles are listed across the top of the following summary table. **Determine** what particles are represented by the letters A, B, C and D. (4 marks)

	Baryon number	Lepton number	Electric charge	Baryon?
A	+1	0	+1	yes
B	+1	0	0	yes
C	0	−1	0	no
D	$-\frac{1}{3}$	0	$-\frac{2}{3}$	no

3 This summary table shows four fundamental forces labelled A, B, C and D. The particles affected by these forces are in the first column. For each force, the particles that they affect are shown by ticks. **Determine** what A, B, C and D represent. (4 marks)

	A	B	C	D
u	✓	✓	✓	✓
e^-	✓	✓	×	✓
ν_e	×	✓	×	✓
γ	×	×	×	×
Standard Model	✓	✓	✓	×

Challenge

Quick decay of a lepton

Explain whether it is possible for the short-lived tau lepton (whose mass is almost twice that of a proton) to decay into only baryons or mesons. (2 marks)

Check your learning 14.1

Check Your Learning 14.1: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** what the conservation of baryon and lepton numbers means for the likelihood of a given reaction occurring. (1 mark)

2 **Recall** the baryon number for these particles: muon, proton, baryon, meson. (2 marks)

3 **Recall** the lepton number for these particles: muon, proton, baryon, meson. (2 marks)

Analytical processes

4 **Determine** the baryon number for these particles: an uus quark composite, an $\overline{\text{sss}}$ quark composite and an $\text{u}\overline{\text{d}}$ quark composite. (3 marks)

5 **Determine** the lepton number for these particles: an uud quark composite, a $\overline{\text{ccs}}$ quark composite and a $\text{d}\overline{\text{s}}$ quark composite. (3 marks)

6 The pi-negative meson, π^-, has the quark composition $\text{u}\overline{\text{d}}$. It reacts with a proton (uud) to produce a kappa-zero meson, K^0 ($\text{d}\overline{\text{s}}$) and a lambda-zero baryon, Λ^0 (uds), according to the equation $\pi^- + p \rightarrow K^0 + \Lambda^0$. **Determine** whether the reaction would violate the laws of conservation of baryon and lepton numbers. (3 marks) Note: reactions involving exotic quark composites are unlikely to appear on the external exam. They are used here to develop understanding.

Knowledge utilisation

7 Lepton number was introduced in 1953 to explain the Cowan–Reines neutrino experiment, which investigated these two possible reactions:

$$\overline{\nu} + n^0 \rightarrow p^+ + e^- \text{ and } \overline{\nu} + p^+ \rightarrow n^0 + e^+$$

Predict which reaction would have violated conservation of lepton number and therefore did not occur. (3 marks)

Lesson 14.2
Feynman diagrams

Key ideas

- → Feynman diagrams are a graphical representation of particle interactions showing time along the horizontal axis and space along the vertical axis.
- → Examples of particle interactions that can be represented with Feynman diagrams include electron–electron scattering, electron–positron (Bhabha) scattering and annihilation, and neutron decay.
- → Feynman diagrams can show baryon decay in terms of baryons or quarks.

What are Feynman diagrams?

Learning intentions and success criteria

Over this course of study, you have learnt about objects interacting with each other. You have seen how energy and momentum are conserved in collisions and explosions, and more recently how charge, baryon number and lepton number are conserved. You have also seen how leptons and baryons interact via mediating particles such as gluons, photons, and W and Z bosons. Trying to illustrate all of this in simple diagrams is not easy.

For example, how do you show two electrons approaching and colliding with each other, exchanging a photon as the mediating particle and then rebounding away from each other? The interaction of subatomic particles can be complex and difficult to understand intuitively. In 1948, the US physicist Richard Feynman devised a type of diagram to show this that proved so useful and popular that it is still in use today. These are called **Feynman diagrams**. You will see them called "particle interaction diagrams" in the syllabus and in external exams, but that term isn't commonly used by particle physicists. We call them Feynman diagrams here. When you are asked to draw a particle interaction diagram in an external exam, make sure you draw a Feynman diagram.

Feynman diagram
a graphical representation of particle interactions, with time along the horizontal axis and space along the vertical axis

What are the conventions of Feynman diagrams?

Feynman diagrams follow several conventions to represent space–time, and different types of particles and antiparticles and their motion.

Convention 1: Representing space–time

Space is shown on the vertical axis and time is on the horizontal axis (Figure 1). Feynman diagrams can be thought of as space and time diagrams that show the sequence of interactions of particles. They are read from left to right. However, they are not "trajectory" diagrams showing the direction and speed of the particles, like a vector diagram showing projectile motion.

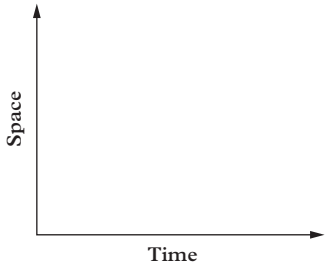


FIGURE 1 Feynman diagrams are drawn with space on the vertical axis and time on the horizontal axis. This is how axes will be labelled on the external exam.

Convention 2: Representing particle motion

Particles are shown as straight-line arrows in the direction of time, with a letter indicating the type of particle. For example, an electron and an electron neutrino are shown in Figure 3.

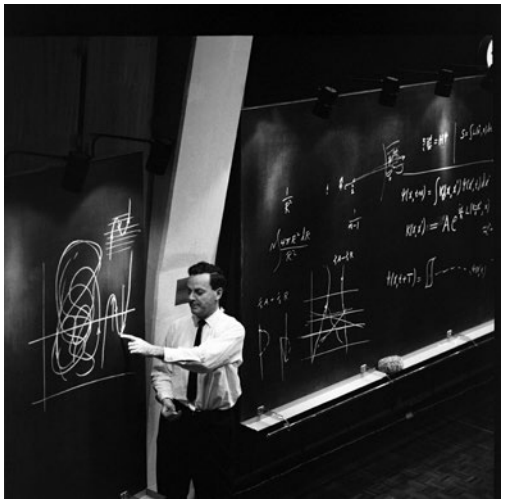


FIGURE 2 Feynman diagrams were named after their inventor Richard Feynman, physicist and 1965 Nobel laureate.

FIGURE 3 Particles are shown as if travelling to the right, in the direction of time.

Convention 3: Representing antiparticle motion

Antiparticles are shown as straight-line arrows pointing in the opposite direction to time. For example, an electron antineutrino and a positron are shown in Figure 4.

FIGURE 4 Antiparticles are shown as if travelling backwards in time. Remember that the arrow for time points to the right.

The arrows make it seem as if antiparticles travel backwards in time. They don't – it is just a useful way to draw them. Originally, it was to show the flow of negative charge, but now it is used to show the difference between particles (→) and antiparticles (←). Physicists liken it to the way we can think of conventional (electric) current as being electrons travelling backwards in a circuit, instead of the flow of positive charges.

Convention 4: Representing gauge bosons

Gauge bosons are shown as wiggly lines, with a letter indicating the type of gauge boson. Examples are shown in Figure 5. Gauge bosons are the force carriers, and the lines representing them are usually placed between the particles. You will only need to know how to represent the four gauge bosons in the Standard Model (not any other type of boson).

You will need to understand and draw Feynman diagrams with "virtual" gauge bosons. These show the transfer of energy during particle interactions, but they are only "internal" – that is, they are not part of the initial and final conditions. In this case, they are not force carriers but a way of indicating a transfer of energy without needing to give specific details.

FIGURE 5 Gauge bosons (photon γ, W^+, W^- and Z^0) are represented by wavy lines.

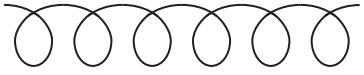


FIGURE 6 A gluon is represented by a string of loops.

The gluon, which is the gauge boson for the strong nuclear force between quarks and nucleons, is commonly represented by a string of loops (Figure 6). You will not be using Feynman diagrams involving gluons on the external exam.

Summary of Feynman diagram conventions for particles

Use the following conventions when you are constructing Feynman diagrams.

1. Represent space on the vertical axis and time on the horizontal axis.
2. Represent particles as straight-line arrows in the direction of time, with a letter indicating the type of particle.
3. Represent antiparticles as straight-line arrows in the opposite direction to time, with a letter indicating the type of antiparticle.
4. Represent gauge bosons as wiggly lines, with a letter indicating the type of gauge boson. Recall that these will be virtual bosons if they are internal (i.e. not present at the start of finish of the interaction).

Study tip

Time will always be on the horizontal axis in this module and in any external exam. Some sources choose to have the axes reversed, with time on the vertical axis. Be careful if you use these sources.

How are particle interactions shown?

Feynman diagrams show what particles are present before and after an interaction, but they also need to show how the particles interact.

To do this, several terms and conventions are used that you will need to understand.

Study tip

You will need to know how to represent virtual W and Z bosons and real and virtual photons. You will not be expected to use Feynman diagrams containing gluons.

Convention 1: Vertex representing the point of interaction

The point at which particles interact is called a **vertex** (plural vertices). At the vertex they will emit or absorb new particles, deflecting one another or changing type. For example, in Figure 7, an electron (e^-) emits a photon (γ) at the vertex and continues with lower energy. This is the case when an electron in an atom drops from a high energy level to a low level and emits a photon. Some diagrams will show a vertex as a dot; others will just show the three lines coming together.

vertex
(plural vertices) the point where particles interact. At the vertex they will emit or absorb new particles, deflecting one another, or changing type.

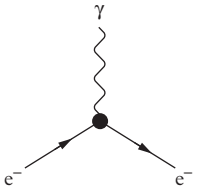


FIGURE 7 A vertex can be drawn as a dot (as it is here) or as a point where lines representing particles join. Here it shows where an electron (e^-) emits a photon (γ).

Convention 2: Vertex locations for external lines (real particles)

For external (real) particles, if the particle is incoming (before the interaction), the vertex is on the right. If the particle is outgoing (after the interaction) the vertex is on the left. This is what you would expect, as time runs from left to right. The lines are drawn horizontally here to save space. Antiparticles are shown in the opposite direction.

TABLE 1 Representing incoming and outgoing external particles

External particle	Incoming	Outgoing
Lepton, quark		
Antilepton, antiquark		
Gauge boson		

Convention 3: Vertex locations for internal lines (virtual particles)

For internal lines (representing virtual particles), the vertices are at both ends.

Virtual gauge boson (internal line with two vertices)	

FIGURE 8 Representing virtual particles

There are other types of internal lines, but they are not dealt with in this course.

Convention 4: Vertex joining a fixed number of lines

In Feynman diagrams, a vertex always has three lines attached to it: one gauge boson line, and two lines for leptons, quarks or quark composites: one with the arrow pointing towards the vertex and one pointing away from it (Figure 9).

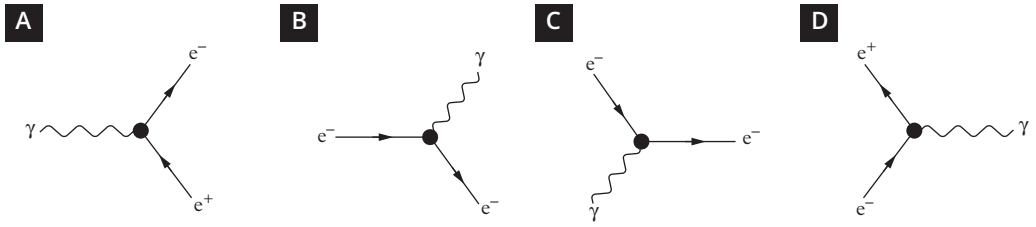


FIGURE 9 Four examples showing how each vertex has three lines attached. (A) a photon "pair producing" an electron and positron, (B) an electron emitting a photon, (C) an electron absorbing a photon, and (D) an electron and positron annihilating each other to form a photon.

What types of particle interactions exist?

It's important that you understand three types of particle interactions that show how fundamental particles behave, and that you can draw Feynman diagrams for them. They are:

- electron and electron
- electron and positron
- neutron decay.

We will go through each in turn.

What occurs during electron–electron interactions?

One of the things electrons do when they collide is to scatter off in various directions, unchanged other than in their velocity. The original particle colliders were designed specifically for electron–electron collisions, but we also see this occurring naturally in the repulsion of electrons in the helium atom, for example. The process, known as Møller scattering, can be represented in equation form as follows:

$$e^- + e^- \rightarrow e^- + e^-$$

This is shown in the Feynman diagram in Figure 10. An electron approaches another electron, and they interact. This is not a vector diagram, so the lengths of the arrows and angles do not necessarily represent the magnitude and direction of velocity.

The electrostatic repulsion between the two electrons gets stronger and stronger as they invade each other's space, so they begin to slow down. At a certain distance, they exchange a photon, γ – a virtual photon, that is.

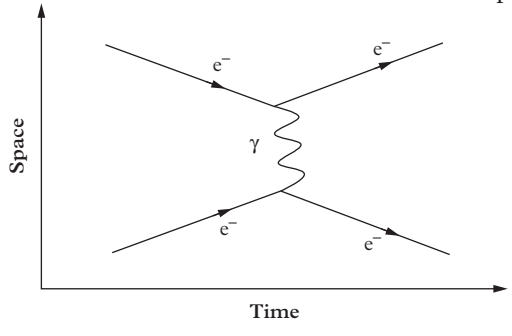


FIGURE 10 An electron–electron interaction in Møller scattering. Can you identify the vertices and state how many lines are attached to each?

This virtual photon only exists as long as the interaction takes place, and it doesn't have an independent existence of its own. It is considered a quantum of energy like a real photon, but unlike a real photon, a virtual photon doesn't have a well-defined energy and momentum, which means it doesn't have a well-defined wavelength or frequency either.

After the interaction the two electrons move away from each other (scatter) due to electrostatic repulsion. This is shown as the two lines of the Feynman diagram moving apart.

Summary of electron–electron interactions

Electron–electron interactions can be summarised into three main points. Each of these points are usually worth 1 mark in a question asking you to describe this interaction, so they are important to learn.

- Two electrons approach each other and feel a repulsive force due to their similar negative electric charge.
- They interact by exchanging a virtual photon.
- They scatter (move away from each other) because of their similar negative charge.

Study tip

Learn Figure 10 as an electron–electron interaction. You do not have to use the term "Møller scattering". Note that the photon line (the wiggly line) is almost vertical, showing that the electrons interacted ("scattered") but didn't change into something else. You don't need to comment on the change of velocities.

What occurs during electron–positron interactions?

In particle physics, the interaction between an electron and a positron can be represented in equation form as:

$$e^- + e^+ \rightarrow e^- + e^+$$

This hardly tells us anything about what is happening, and the interaction can actually take two forms.

Bhabha scattering or electron–positron interaction

The first form of this interaction is called Bhabha scattering after the Indian physicist Homi J. Bhabha (1909–66). You need to know this as an electron–positron interaction.

In Figure 11 you can see an electron and a positron moving towards each other. The positron, e^+, at the bottom left, has a left-facing arrow to show it is an antiparticle (not that it is travelling backwards in time). As the electron and positron move closer together, they interact, exchanging a virtual photon, γ, and scatter off each other. Bhabha gained international prominence after deriving a correct expression for the probability of this type of scattering.

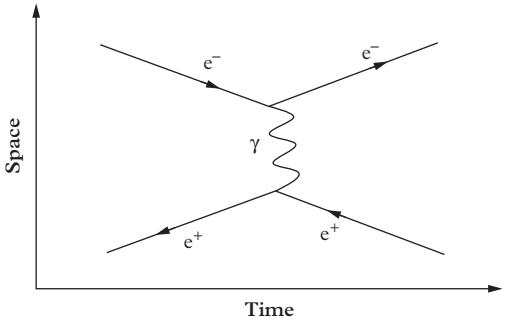


FIGURE 11 Bhabha scattering in an electron–positron interaction. Can you identify the two vertices? Does this diagram obey the convention of three lines at each vertex?

Summary of Bhabha scattering or electron–positron interaction

Bhabha scattering or electron–positron interaction can be summarised into three main points. Each point is usually worth one mark in a question that asks you to describe this type of particle interaction, so make sure you remember each of them.

- An electron, e^-, and a positron, e^+, approach each other and feel an attractive electromagnetic force due to their opposing electric charges.
- They interact by exchanging a virtual photon.
- They scatter (move away from each other).

Study tip

Learn Figure 11 as an electron–positron interaction. It is also known by the term "Bhabha scattering". Note again that the photon wiggly line is almost vertical, showing that the electrons interacted ("scattered") but didn't change into something else. Show you know which is the electron and which is the positron by stating the symbols in any answer.

Things to look out for when describing electron–positron interactions

When describing this scattering there are many things you should say. The main one is that they "exchange" a virtual photon. Don't say that they create a virtual photon. You should also avoid saying that it is gamma radiation – in fact, don't talk about its properties at all, because it is virtual and you just don't know. You can say that it is "short-lived". There are more tips at the end of this section.

Electron–positron annihilation and pair production

The second process that can happen during an electron–positron interaction is shown in Figure 12. It is called electron–positron annihilation and pair production.

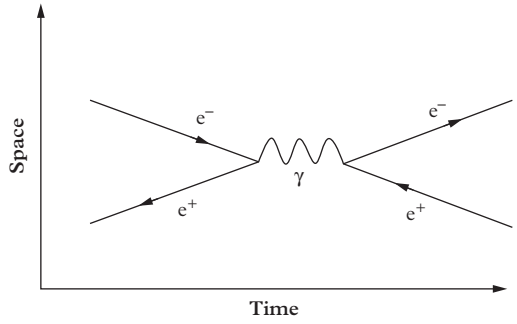


FIGURE 12 Electron–positron annihilation, again with two vertices. Each vertex has three lines.

Again, the electron and positron approach each other on the left of the diagram. The left-facing arrow indicates the positron, e^+. As they get closer, they do something rather strange. Rather than scatter off each other, they annihilate (destroy) each other. They form a virtual photon, γ, which is short-lived (perhaps as short as 10^{-20} s) and doesn't really exist in its own right. But then out of the virtual photon's energy, a new electron–positron pair is created. Their direction is unknown, because a Feynman diagram does not show particle trajectories, only possible interactions. This particular interaction is well-known and has a high probability of occurring. It is a great example of photon energy (γ) being expressed as matter (e^-, e^+). This is the same process as the creation of matter in the Big Bang: energy was converted into particle–antiparticle pairs.

Summary of electron–positron annihilation

annihilation
the process that occurs when a subatomic particle collides with its respective antiparticle to produce other particles such as photons

Electron–positron **annihilation** can be summarised into three main points. You should learn this. It could be worth 3 marks in an exam.

- An electron, e^-, and a positron, e^+, approach each other and interact by colliding.
- The electron and positron annihilate each other and create a virtual photon.
- The virtual photon "pair produces" a new electron and positron.

Things to look out for when describing electron–positron annihilation

A few things you should be mindful of when describing electron–positron annihilation and pair production are listed below.

- The key point about describing an interaction is to ensure you get the sequence of events in the right order. Start at the earliest part of the timeline and tell the interaction story as time progresses. There are three stages you need to describe in this sequence. "Describe" is to state with some extra information or elaboration. The cognitive verb is not "explain", so you don't need to give reasons for each stages.
- You can use dot points to describe the interaction.
- Use the symbols for electrons and positrons when describing this interaction to show you understand which is which. Make sure you do not call them both "electrons". You could call the positron an "antielectron". Also make sure you don't call the positron a "proton" in the rush of an exam – it happens, and this is wrong, and will prevent you getting full marks.
- There is no need to mention the directions or velocities of the incoming particles. The diagram shows a sequence of possible interactions. It does not show trajectories.
- Ensure you call the wiggly line with a vertex at either end a "virtual photon". Don't just say a "γ" is produced; spell it out. Don't call it a mediating particle, a force carrier, an exchange particle, a gamma photon, gamma radiation or electromagnetic radiation. It is none of these. It is just a "virtual photon" whose properties are not well-defined.
- Don't use too many descriptive words that might make your answer unclear. "Annihilation", "pair producing" and "virtual photon" are essential, but don't say "a mediating particle is formed by annihilation". It's a mixed message, with one part being wrong.
- Make it clear that the electron and positron produced at the end are called "new" or "the incoming ones have been converted to new ones". Don't say the virtual photon undergoes "decay". It is not entirely wrong, but as it is not a particle in its own right it is not decaying but changing out of its transient state. It is best to only describe matter particles as undergoing decay.

Study tip

Learn Figure 12 as an electron–positron annihilation. Note this time that the wiggly line representing the photon is **horizontal**, showing that the electron and positron actually combined (annihilated) in this interaction to form the virtual photon. The electron and positron effectively disappeared by combining to form this virtual photon, which then turned back into a new electron and positron.

What occurs during neutron decay?

For the third type of interaction, consider the beta decay of carbon-14:

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + \overline{\nu}_e$$

Carbon-14 is unstable and has too many neutrons for the number of protons. It can shed some of its excess energy by undergoing beta decay, in which one of the neutrons turns into a proton. It uses some of its excess energy to form a temporary (virtual) particle called the W^- boson, which disappears by forming an electron and an electron antineutrino:

$$n^0 \rightarrow p^+ + e^- + \overline{\nu}_e$$

This is a process called quark flavour changing. The neutron is a baryon composed of udd quarks, and the proton is a baryon with uud quarks. All that is happening is that a down quark in the neutron turns into an up quark and produces a virtual W^- boson: $d \rightarrow u + W^-$. The W^- boson immediately transforms its energy into mass as an electron and an electron antineutrino: $W^- \rightarrow e^- + \overline{\nu}_e$. This can be represented in a simple diagram (Figure 13).

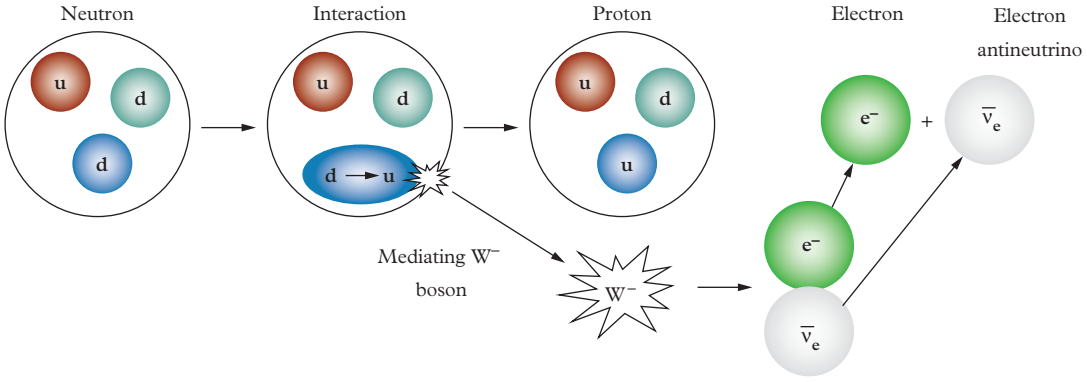


FIGURE 13 A neutron decays into a proton, and an electron and an electron antineutrino are emitted.

This sort of diagram does not convey as much information as a Feynman diagram. The two Feynman diagrams in Figure 14 and Figure 15 should be understood. Let's take a closer look at what is occurring in each.

Beta-negative decay of a neutron

The neutron decay shown in Figure 14 can be summarised into three main points.

- A neutron, n^0, decays.
- It forms a proton, p^+, and a virtual W^- boson.
- The virtual W^- boson transforms its energy into mass in the form of an electron, e^-, and an electron antineutrino, $\overline{\nu}_e$.

When you are describing what is happening in Feynman diagrams like Figure 13, be mindful of the following things.

- The proton has positive charge, so the W boson must be negative to conserve the neutral charge of the neutron.
- The W^- boson is virtual so little can be said about it other than that it carries away some of the mass of the neutron as energy.
- The W^- boson then converts energy back to mass in the form of two leptons.
- One lepton is the electron, e^-, which carries a negative electric charge, and the other is an electron antineutrino, which carries no charge, so charge is conserved ($-1 \rightarrow -1$).

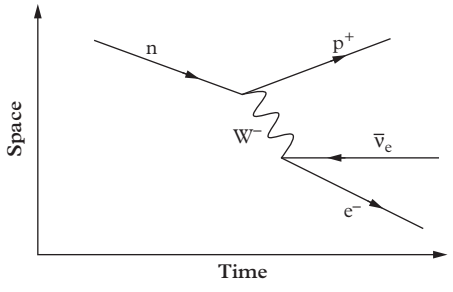


FIGURE 14 Beta-negative decay of a neutron as depicted by a Feynman diagram

- The emission of an electron in beta decay is always accompanied by the production of an antineutrino. It is an electron antineutrino as it is associated with the emitted electron. Don't call it an "antielectron neutrino" as it is not associated with an antielectron (i.e. a positron). One of its roles is to help conserve momentum.

How is beta-negative decay of a neutron shown using quarks?

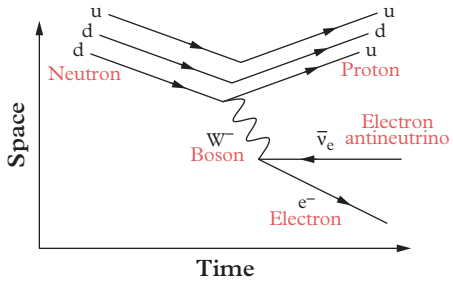


FIGURE 15 Beta-negative decay of a neutron showing flavour change of a down quark

Feynman diagrams can also represent quarks. Figure 15 shows the beta-negative decay of a neutron, this time showing the flavour change of a down quark.

A summary of what is occurring in Figure 15 is given below. Each of the points are usually worth one mark in a question that asks you to describe what is occurring. You should learn this!

- A neutron (composed of up, down, down (udd) quarks) decays.
- A down quark undergoes a flavour change to form an up quark and a virtual W^- boson.
- The new up quark together with the existing up and down quarks from the neutron become a proton, p^+.
- The virtual W^- boson transforms its energy into mass in the form of an electron, e^-, and an electron antineutrino, $\bar{\nu}_e$.

When you are describing what is happening in Feynman diagrams like Figure 15, be mindful of the following things.

- You do not have to say the incoming particle is a neutron or a baryon, but it is safer to mention that it is a 3-quark composite. Do not just say "composite", which could mean a baryon or a meson.
- You must identify the components of the incoming particle as quarks, and you should name them or use their symbols. If you use both, make sure they match.
- You do not need to say the down quark "flavour changes". Just "changes" is appropriate.
- There is a net loss of a down quark and a net gain of an up quark to form the proton.
- The neutron has neutral charge and the proton after the interaction has positive charge. Therefore, the W boson must be negative to conserve the neutral charge, so it is a W^- boson.
- The W^- boson is virtual because it is represented by an internal line, so little can be said about it other than that it carries away some of the mass of the original baryon (neutron) as energy.
- The W^- boson then converts its energy back to mass in the form of the two leptons. One is the electron, e^-, which carries a negative charge, and the other is an electron antineutrino, $\bar{\nu}_e$, which carries no charge. Therefore, charge is again conserved.
- The emission of an electron in beta decay is always accompanied by the production of an antineutrino. In this case, it is an electron antineutrino as it is associated with the emitted electron. One of its roles is to help conserve momentum.

Study tip

Ensure you can understand, construct and interpret Feynman diagrams of a neutron decaying into a proton. They are complex and highly likely to appear in the external exam, possibly in multiple-choice questions.

How is beta-positive decay of a proton shown using quarks?

Study tip

You will not be asked about a proton decaying into a neutron in any external exam. It is included here as another example. It is useful to understand it, but there is no need to learn it off by heart.

As a comparison, consider an example of beta-positive decay in which a positron and an electron neutrino are formed. In this case, it is a proton that changes into a neutron by quark flavour changing. An up quark changes into a down quark. The virtual W^+ boson carries away some of the energy of the decay. This energy is converted into mass in the form of a positron and an electron neutrino. It is hypothetical and has yet to be observed.

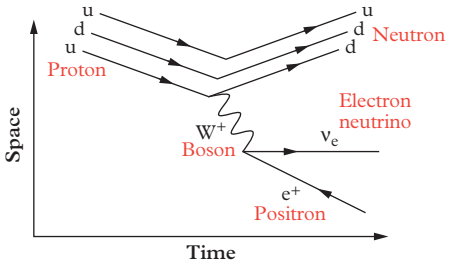


FIGURE 16 Beta-positive decay of a proton showing flavour change of an up quark

Challenge

Feynman diagrams with time on the vertical axis

In some books the axes for Feynman diagrams are reversed so that time appears on the vertical axis, and position along the horizontal axis. **Construct** a new diagram of Figure 17 so that it follows the usual convention with time along the bottom. (3 marks)

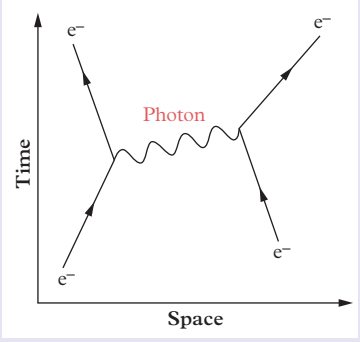


FIGURE 17 Feynman diagram with axes reversed

Real-world physics

Particle accelerators – the synchrotron

In the study of atomic particles, or high-energy physics, scientists require atom-smashing machines. These large accelerators, such as the 13.5 TeV (tera, T, = 10^{12}) Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) in Geneva, are used to study the collisions of highly energetic particles to learn about the structure of matter. Many of these large accelerators use powerful magnetic fields to deflect the charged particles into circular paths. It was at the LHC that the Higgs boson was finally confirmed in 2012.

One of the earliest devices, developed in 1930, was called a cyclotron. More modern versions of the original cyclotron are called synchrocyclotrons, synchrotrons, tevatrons and supercolliders.

Australian Synchrotron

Australia has a synchrotron in Melbourne that opened in 2007. Its construction involved collaboration between Australian and New Zealand science organisations, as well as state and federal governments.

The synchrotron provides researchers access to the specialised tools and techniques needed for research in areas such as advanced manufacturing, food, health and medical research, materials and textile science.

The electrons start in the electron gun, in which a hot metal cathode produces free electrons. These are fired into the linear accelerator in packets of about 100 million, spaced two nanoseconds apart, where they are accelerated to 99.9997% of the speed of light. The energy of the electrons is increased further in the booster ring, before they are transferred to the storage ring.

In the storage ring, electromagnets create a magnetic field that exerts a force on the electrons to keep them travelling in a circular path. Because the electrons are moving charges, they also generate a magnetic field. As they are constantly accelerating to keep them travelling in their circular path, their magnetic field is constantly changing, which in turn creates a changing electric field. These changes in the electric field, in turn, generate an associated magnetic field. The changing electric and magnetic fields form electromagnetic waves that propagate outwards from the path of the beam. This electromagnetic radiation is called synchrotron radiation or synchrotron light.

Because the electrons are moving at velocities very close to the speed of light, relativistic effects apply, so the path length appears shortened to the electrons, meaning that the frequency of the radiation is increased. Therefore, most of the light is X-ray, with some ultraviolet, visible and infrared light.

Synchrotron radiation gives scientists a mighty toolkit of advanced analytical and imaging techniques. It is very intense, and it covers a large part of the spectrum, including infrared, visible light, ultraviolet and X-rays, giving a consistent and reliable supply of X-ray and infrared photons to the thousands of national and international researchers who use the synchrotron's unique capabilities each year via its beamlines (workstations).

One example of research using the synchrotron beamlines is the international efforts to beat the malaria parasite, which infects about half a billion people around the world every year and kills more than one million, many of them children. Researchers have been able to obtain high-resolution crystal structures of a key malarial enzyme in the hope of inhibiting its action.

Apply your understanding

1 **Recall** the purpose of having charged particles travel at such high speeds. (1 mark)

2 To use the electrons, a magnetic field is applied to the clockwise beam of electrons to make them move to the outside of the ring and into openings called beamlines. **Determine** the direction a magnetic field would have to be applied to make the electron path bend to the outside. (3 marks)

3 In the Australian Synchrotron, the storage ring is 216 m in circumference and electrons travel around it at a speed of $0.9999998c$ (where c is the speed of light). To an electron travelling at such speeds, the ring appears to be only a few metres long. **Evaluate** this statement by using the special relativity formula for length contraction. (3 marks)

Check your learning 14.2

Check Your Learning 14.2: Complete these questions online or in your workbook.

Retrieval and comprehension

1 **Explain** whether the purpose of a Feynman diagram is to show the trajectory of the particles or the nature of the interaction. (1 mark)

2 **Recall** the convention about time and space used in constructing Feynman diagrams. (1 mark)

3 **Sketch** the Feynman symbol for a gauge boson. (1 mark)

4 Consider the beta decay of a neutron.

- **a** **Sketch** a Feynman diagram that indicates what happens at the quark level. (3 marks)
- **b** **Identify** whether baryon number and lepton number are conserved. (2 marks)

5 **Explain** what the arrow for a positron represents in an electron–positron interaction (Bhabha scattering). (1 mark)

Analytical processes

6 **Interpret** the interaction in which an electron and positron annihilate to form a photon, to suggest where the new electron and positron came from. (1 mark)

7 **Analyse** the following interaction and draw a Feynman diagram for it.

An up quark interacts with an antidown quark to form a W^+ boson. The W^+ boson immediately decays into an antimuon and a muon neutrino. (3 marks) Note: in the external exam you will not be required to draw Feynman diagrams other than those mentioned in the syllabus.

Knowledge utilisation

8 **Evaluate** this claim by referring to the appropriate Feynman diagram: "When a neutron decays into a proton, a positive charge is created out of nothing." (2 marks)

Lesson 14.3
Symmetry in particle interactions

Key ideas

- → Symmetry describes whether an interaction is the same before and after a particular operation.
- → Symmetry reversal includes charge reversal and time reversal.
- → Symmetry is important as it allows physicists to predict which interactions may or may not be successful.
- → Some properties are always conserved in any symmetry operation. These include baryon and lepton numbers, energy, momentum and electric charge.
- → Other properties are not always conserved in symmetry operations, and this opens up new areas of physics to explore.

Learning intentions and success criteria

What is symmetry?

symmetry
the concept that laws of physics do not vary under transformations in an inertial (constant velocity) frame of reference, such as in a different place or at a different time, or when symmetry operations are performed on particle interactions and the conservation laws are not violated

One important feature of modern physics is the use of **symmetry**. For example, an experiment performed in one location should have the same result as an identical experiment performed in a different place or at a different time (although Einstein would add "providing it's in an inertial frame of reference").

During the twentieth century, particle physicists believed that the laws of physics governing elementary particles would not change when tested under various transformations such as the reversal of direction in space or time. Physicists hoped to answer the question of why the Big Bang produced more particles than antiparticles, when theory suggested they should be exactly equal. They found in practice that for every 10 billion antimatter particles produced, there were 10 billion and one matter particles. Why this "symmetry violation" happened is one of the big unanswered questions of modern physics.

There are many forms of symmetry that physicists need to consider, but there are two that have been tested and found to hold: charge-reversal symmetry and time-reversal symmetry. There are others such as crossing symmetry and parity (mirror image) symmetry, but two will be enough to give you the idea. We will consider examples of each of these to help understand what symmetry means and to determine when it has been violated.

What is charge-reversal symmetry?

charge-reversal symmetry
the idea that interactions are not affected if all charges are swapped (i.e. positive for negative and vice versa)

Charge-reversal symmetry, C, says that interactions still occur if the signs of all charges are reversed. Reversing the charges means all negative charges are made positive, and all positive charges are made negative. It is based on the idea that there is nothing special about what we call positive charge, and nature treats it as equal and opposite to negative charge.

This idea also includes switching from particle to antiparticle and vice versa, such as neutrinos to antineutrinos, and neutrons to antineutrons. In Module 13, you learnt that an antiparticle is just a particle with opposite electric charge. A good example is the decay of a pion:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

In its charge-reversed equivalent, its particles are swapped for their antiparticles. The positive charges are swapped for negative ones, so π^+ becomes π^- and μ^+ becomes μ^-, and the neutrino, ν_μ, becomes an antineutrino, $\overline{\nu}_\mu$. Therefore:

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$

Both versions of this decay have been seen experimentally, so we can say charge reversal is symmetrical.

Note that uncharged gauge bosons such as the photon are unchanged during charge reversal symmetry operations.

Worked example 14.3A

Predicting the outcome of a charge-reversal symmetry operation

Muons decay according to this equation: $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$. A charge-reversal symmetry operation is performed on this decay.

a **Identify** what is meant by "charge-reversal symmetry operation". (1 mark)

b **Determine** the decay equation after the operation has been applied. (1 mark)

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Identify" means to recognise and state. "Determine" means to ascertain after consideration. We need to consider the meaning of the symmetry operation and then perform it.
Step 2: Define "charge-reversal symmetry operation". (1 mark for "Provides suitable definition")	**a** An operation where all electric charges are reversed (1 mark)
Step 3: Swap all charges in the decay equation. (1 mark for "Provides correct equation")	**b** $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$ (1 mark)

Your turn

A positive pion decays according to this equation: $\pi^+ \rightarrow \mu^+ + \nu_\mu$. A charge-reversal symmetry operation is performed on this decay.

a **Identify** what is meant by "charge-reversal symmetry operation". (1 mark)

b **Determine** the decay equation after the operation has been applied. (1 mark)

What is time-reversal symmetry?

Richard Feynman said that "the fundamental physical laws, on a microscopic and fundamental level, are completely reversible in time". Yet our everyday experience tells us that there are many situations in which a system seems to go in one direction and not the reverse, suggesting that there actually is a preferred direction in time. For example, a glass falling to the ground and smashing into tiny pieces is a process that always seems to occur in the same way and never in the reverse; the pieces of broken glass never reform into an unbroken glass. However, Feynman used the words "on a microscopic and fundamental level", and therein lies the difference. At an atomic scale time is reversible. In other words, products become reactants, and reactants become products.

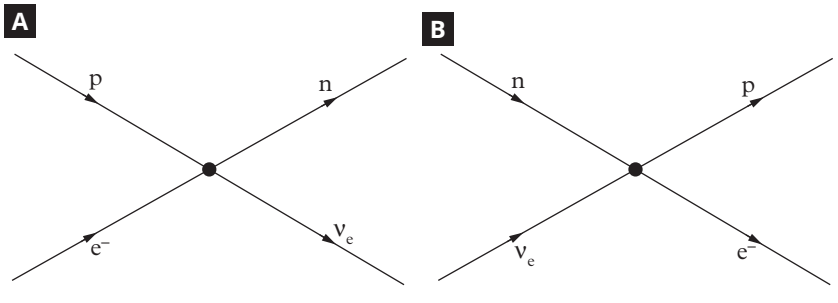


FIGURE 1 Feynman diagrams showing the time-reversal symmetry of (A) an inverse beta decay to (B) a beta-negative decay

time-reversal symmetry
an interaction can still occur if the flow of time is reversed (products become reactants, and reactants become products)

Consider a particle interaction called inverse beta decay, in which a proton in a nucleus captures an orbital electron to form a neutron and an electron neutrino (Figure 1A). This process is also known as electron capture and is a well-known way in which radioisotopes gain stability by getting rid of protons.

Time is shown along the horizontal axis going from past (on the left) to the future (on the right). To run this backwards in time, just flip the Feynman diagram about a vertical axis, as in Figure 1B. Now you can see a neutron grab an electron neutrino and form a proton and an electron. This is known as beta decay and is a well-known way in which a nucleus gets rid of a neutron. So time is reversible, as shown by this example of **time-reversal symmetry**, T.

Worked example 14.3B

Predicting the outcome of a time-reversal symmetry operation

Figure 2 shows an inverse beta decay. It undergoes a time-reversal symmetry operation.

a **Describe** the particle interactions for the original and time-reversed versions. (2 marks)

b **Construct** a Feynman diagram to show how it appears after the time-reversal symmetry operation has been applied. (1 mark)

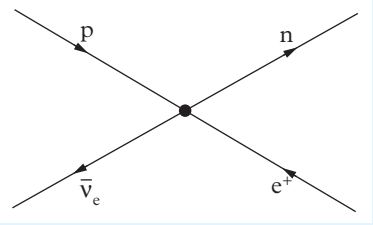
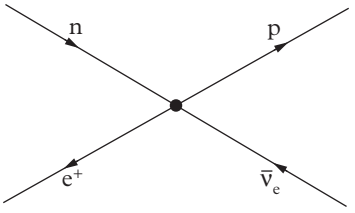


FIGURE 2 Inverse beta decay

Think	Do
Step 1: Look at the cognitive verbs and mark allocation to determine what the questions are asking you to do.	"Describe" means to give an account of a process, or of the characteristics or features of something. We must decide what the operation requires. "Construct" means to display information in a diagrammatic or logical form. We then need to draw a diagram to represent the new reaction.
Step 2: State what the original diagram represents. (1 mark for "Correctly describes the original interaction")	**a** A proton interacts with an electron antineutrino to form a neutron and a positron. (1 mark)
Step 3: Determine what will happen after a time-reversal operation. (1 mark for "Correctly describes the interaction after a time-reversal symmetry operation is applied")	After the time-reversal symmetry operation, a neutron interacts with a positron to form a proton and an electron antineutrino. (1 mark)
Step 4: Draw a diagram to represent the new interaction. Ensure the arrows point to the left for antiparticles. (1 mark for "Constructs a correct diagram of the new interaction")	**b**  (1 mark)

Your turn

Figure 3 shows a common decay process. It undergoes a time-reversal symmetry operation.

a **Describe** the particle interactions for the original and time-reversed versions. (2 marks)

b **Construct** a Feynman diagram to show how it appears after the time-reversal symmetry operation has been applied. (1 mark)

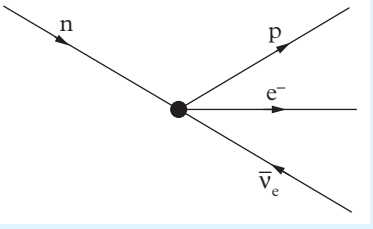


FIGURE 3 A common decay process

How can we determine the probability of a reaction occurring?

The most powerful use of Feynman diagrams is to determine the probability of a reaction occurring. This is too complex to cover in detail here, but, roughly, every line in a Feynman diagram can be assigned a measure of probability (by looking up a table), which can then be summed for the whole diagram. Physicists refer to it as measuring the "scattering amplitude" and regard it as the diagram's most important job. For example, there are nine different ways two electrons can interact, but some are highly unlikely (i.e. have low amplitudes).

But you can try to predict which of two interactions may be more likely by considering what the interaction involves. For example, consider the reactions in Figure 4. You may have done this operation in the "Your turn" question at the end of Worked example 14.1B. They are time-reversed versions of the same interaction. In the first case, a neutron decays into a proton, an electron and an electron antineutrino. In the second case, a proton must collide with an electron and an electron antineutrino at the same time. Although protons and electrons attract each other by an electromagnetic force, protons and neutrinos do not – they would probably just scatter. The probability of these three particles coming together at the same time is very low, so this reaction is less likely to occur than the first.

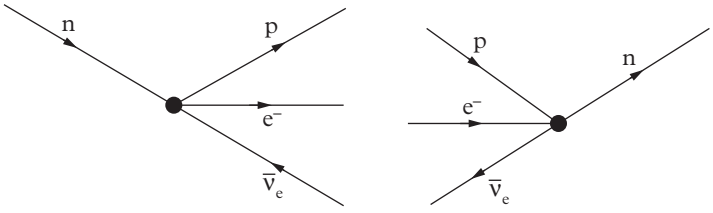


FIGURE 4 Time-reversal symmetry applied to neutron decay has a lower probability of occurring.

What is symmetry violation?

Symmetry reversal holds for most interactions, but there are cases in which violations occur (i.e. the interaction is only possible in one direction). Whatever the type of symmetry, energy, momentum, electric charge, baryon number and lepton number must also be conserved for a reaction to be possible. The violation of symmetries (symmetry breaking) is not a bad thing. True, it stops a particular symmetry from being called a general law of nature, but it gives even more clues to the inner working of the atom. And that surely is a good thing!

How can we summarise symmetry?

A key objective of this section is for you to be able to "Describe how symmetry in particle interactions occurs to maintain the principles of conservation". Here is a suitable response:

1. **Symmetry operations.** Symmetry refers to the concept that the laws of nature still apply when certain changes or operations are applied. Two important symmetries in particle interactions are charge-reversal symmetry and time-reversal symmetry.
2. **Symmetry requires conservation.** Symmetry operations must allow for the principles of conservation to be maintained. This includes conservation of lepton number, baryon number, electric charge, energy and momentum.
3. **Symmetry is upheld by nature.** Symmetry operations are generally upheld by nature, and this enables physicists to predict new reactions, although the probability of the new interaction may be unknown.
4. **Symmetry can be violated.** Sometimes symmetry is violated in nature, which means that particular form of symmetry cannot be a universal law of nature.
5. **Symmetry breaking.** Symmetry violation provides physicists with information about interactions that can help them investigate further.

Study tip

The syllabus doesn't mention any particular type of symmetry operation, so the best thing to do is learn the five points in the summary and be able to explain one operation as an example.

Real-world physics

Parity – another form of symmetry

Consider a "mirror image" of our universe, one in which all objects have their positions reflected by an arbitrary plane. This is said to be parity inversion.

For a long time, left and right were believed to be just conventions, and it was thought nature had no preference for one over the other. Physicists believed that the laws of physics would work the same way in a mirror universe. They said that there was no experiment you could do that would tell you whether you were in your own world or a mirror world. This was verified for particle interactions involving the strong nuclear force and the electromagnetic force. The world was said to obey parity conservation, and this became a well-established principle in the field.

However, during the 1950s, physicists puzzled over the decay of the kaon, a meson made up of a quark and an antiquark. There seemed to be two types of kaons: one that decayed into two pions, and another that decayed into three pions. One possible explanation was that it was the same particle undergoing two different decay modes. If this were the case, for reasons outside the scope of this module, parity could not be conserved.

In 1956, while visiting scientists at Brookhaven National Laboratory in New York, Tsung-Dao (T.D.) Lee and Chen Ning (Frank) Yang predicted the non-conservation of parity in the weak interaction. This was very controversial, so they examined experimental data from the Brookhaven particle accelerator as well as the literature from almost 40 years of research. They found that although there was a great deal of support for parity conservation in the strong and electromagnetic interactions, there was no evidence for it in the weak interaction.

Their prediction was confirmed experimentally when C.S. Wu and collaborators examined the beta-negative decay of cobalt-60 in 1957. They oriented the nuclei in cobalt to all spin one way (e.g. anticlockwise when viewed from above). They counted the number of electrons emitted upwards and downwards and found them equal. However, when the spin of the nuclei was reversed (that is, the mirror image), they found that more electrons were emitted downwards than upwards. This meant that the decay process varied between the original and mirror image – parity was not conserved.

Lee and Yang went on to win the 1957 Nobel Prize in Physics for their work on this revolutionary discovery.

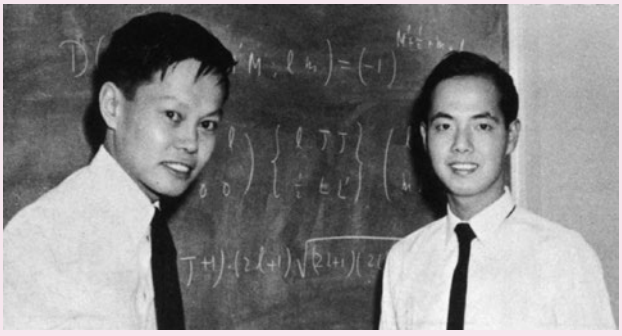


FIGURE 5 Chen Ning Yang (born 1922) of Princeton University and Tsung-Dao Lee (1926–2024) of Columbia University, whose work in disproving the conservation of parity principle won them the 1957 Nobel Prize in Physics

Apply your understanding

1 **Clarify** whether Yang and Lee found that parity was conserved or not conserved in kaon decay. (1 mark)

2 One of the reactions tested was for a kaon (meson; $u\bar{s}$), which decays into two positive pions, π^+ (mesons), and a negative pion, π^-, according to this reaction: $K^+ \rightarrow 2\,\pi^+ + \pi^-$. The positive pions are each made of the quarks $u\bar{d}$, and the negative pion is the positive pion's antiparticle. **Prove** that the net reaction is one of a flavour change of a strange antiquark to a down antiquark. (3 marks)

3 **Determine** whether lepton number and baryon number are both conserved in the kaon decay. (3 marks)

Challenge

Time reversal and the Big Bang

In the Big Bang model of the universe time only goes forwards, and therefore time-reversal, T, may not be a valid symmetry of the universe. It is also believed that matter and antimatter were originally created in equal amounts in the Big Bang. However, we observe that we live in a matter-dominated universe, so baryons outnumber antibaryons.

Discuss whether this means that charge, and/or time symmetry must have been violated to explain the difference between matter and antimatter. (2 marks)

Check your learning 14.3

Check Your Learning 14.3: Complete these questions online or in your workbook.

Retrieval and comprehension

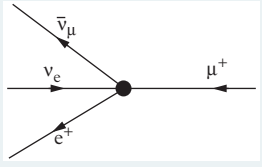
1 **Recall** two types of symmetry operations. (1 mark)

2 **Explain** the changes that occur for time-reversal symmetry. (1 mark)

3 **Recall** the name given to the operation in which particles are replaced with their antiparticles. (1 mark)

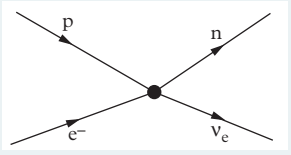
Analytical processes

4 **Derive** a time-reversed symmetry Feynman diagram based on the diagram shown. (2 marks)



Knowledge utilisation

5 **Modify** the Feynman diagram of electron capture by a proton shown in the diagram to represent the process after a charge-reversal symmetry operation is applied. (2 marks)



6 The following reaction shows a possible hypothetical decay of a muon:

$$\mu^+ \rightarrow \bar{\nu}_\mu + e^+ + \nu_e$$

a **Construct** a Feynman diagram to represent this interaction. (2 marks)

b **Modify** the diagram from part **a** to represent the process after a charge-reversal symmetry operation has been applied. (1 mark)

7 **Evaluate** this statement about particle interactions: "Symmetry-breaking is not a good thing to do." (1 mark)

8 **Assess** this claim about symmetry: "Our understanding of the atomic world would have progressed faster if violations of symmetry had not been found." (1 mark)

MODULE 14

Lesson 14.4
Review: Particle interactions

Summary

14.1
- Baryon and lepton numbers help us understand which reactions can happen and which cannot happen.
- Baryons have a baryon number, B, of +1, and antibaryons have a baryon number of –1. Leptons and antileptons have a baryon number of 0.
- Quarks have a baryon number of $+\frac{1}{3}$, and antiquarks have a baryon number of $-\frac{1}{3}$.
- Leptons have a lepton number, L, of +1, and antileptons have a lepton number of –1. Baryons and antibaryons have a lepton number of 0.
- Baryon number and lepton number are always conserved in a reaction.

14.2
- Feynman diagrams are a graphical representation of particle interactions showing time along the horizontal axis and space along the vertical axis.
- Examples of particle interactions that can be represented with Feynman diagrams include electron–electron scattering, electron–positron (Bhabha) scattering and annihilation, and neutron decay.
- Feynman diagrams can show baryon decay in terms of baryons or quarks.

14.3
- Symmetry describes whether an interaction is the same before and after a particular operation.
- Symmetry reversal includes charge reversal and time reversal.
- Symmetry is important as it allows physicists to predict which interactions may or may not be successful.
- Some properties are always conserved in any symmetry operation. These include baryon and lepton numbers, energy, momentum and electric charge.
- Other properties are not always conserved in symmetry operations, and this opens up new areas of physics to explore.

Key formulas

Baryon number (from number of baryons and antibaryons)	$B = n_{\mathrm{b}} - n_{\bar{\mathrm{b}}}$
Baryon number (from number of quarks and antiquarks)	$B = \frac{1}{3}(n_{\mathrm{q}} - n_{\bar{\mathrm{q}}})$
Lepton number	$L = (n_{\mathrm{l}} - n_{\bar{\mathrm{l}}})$

Review questions 14.4A Multiple choice

Review questions: Complete these questions online or in your workbook.

(1 mark each)

1 Select which of the following would have a baryon number of 0.

A Proton
B Neutron
C Antibaryon
D Neutrino

2 A particle is made up of the following combination: $\overline{sss}$. Which one of the following contains terms that all apply to the particle?

A Meson, antiquarks, gluons
B Baryon, antiquarks, gluons
C Antibaryon, antiquarks, antigluons
D Antibaryon, antiquarks, gluons

3 Which one of the following correctly lists the baryon number and lepton number respectively for the charmed bottom xi-prime particle Ξ'^{0}_{cb}, given that it is a quark composite (dcb)?

A –1, –1
B –1, 0
C +1, 0
D +1, +1

4 In a particular reaction, $\Xi^0 \rightarrow p^+ + \pi^-$, a baryon (usc) decays into a proton (uud) and a pion ($d\bar{u}$). The quark composition of each particle is given in brackets.

Which one of the following correctly states whether the reaction can happen and, if not, why?

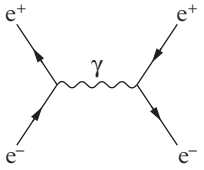
A It is allowed.
B Forbidden; conservation of baryon number is violated.
C Forbidden; conservation of lepton number is violated.
D Forbidden; conservation of both baryon and lepton number is violated.

5 Which one of the following correctly matches a mediating particle and the fundamental force it mediates, respectively?

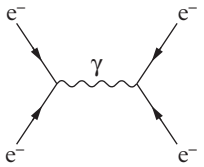
A Boson, weak nuclear force
B Gluon, strong nuclear force
C Meson, weak nuclear force
D Quark, strong nuclear force

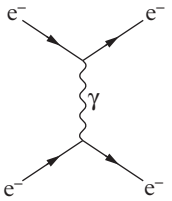
6 Determine what process this Feynman diagram shows.

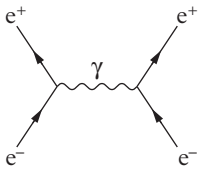
A An electron and a positron annihilate to form a photon.
B An electron turns into a photon and a positron.
C An electron and positron bounce off a photon.
D A photon forms a pair of electrons.

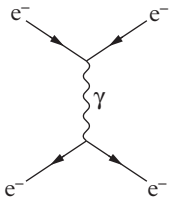


7 Identify which of the following Feynman diagrams shows an electron–electron interaction.

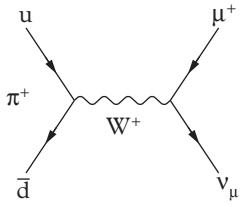
A


B


C


D


8 A pion (π^+) is a quark composite ($u\bar{d}$). Its decay can be represented in a Feynman diagram as shown.



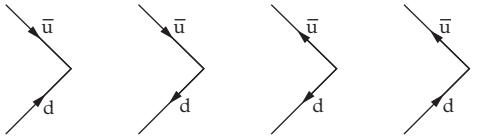
Which one of the following best states the baryon number before and after the interaction?

A 0, 0
B 1, 1
C 2, 2
D $+\frac{1}{3}$, 0

9 Symmetry in particle interactions occurs to maintain
- **A** the principles of conservation.
- **B** the transfer of energy by virtual bosons.
- **C** equal numbers of annihilations and pair productions.
- **D** conservation of particle and antiparticle numbers.

10 If the meson in question **8** was made up of $d\bar{u}$ quark combination instead of a $u\bar{d}$ combination, which one of the diagrams would best represent its initial interaction?

A B C D



11 Which one of the following correctly lists the baryon number and lepton number respectively for the double-charmed anti-omega particle $\overline{\Omega}_{c\bar{c}}$, which is a quark composite $(\overline{scc})$?
- **A** –1, –1
- **B** –1, 0
- **C** +1, 0
- **D** +1, +1

12 A particle has a baryon number of 0, and a lepton number of −1. Which one of the following could it be?
- **A** Strange-D meson, D_s^-, $(s\bar{c})$
- **B** Double-charmed bottom antiomega, $\overline{\Omega}_{bc\bar{c}}$, $(\overline{cc}\overline{b})$
- **C** Tau, τ
- **D** Muon antineutrino, $\overline{\nu}_\mu$

13 For the decay reaction $\Sigma^- \rightarrow \Lambda^0 + e^+ + \overline{\nu}_e$, the identity of the unfamiliar particles and quark composition are Σ^-, sigma (dds) and Λ^0, lambda (uds).

Which one of the following is a true characterisation in terms of baryon and lepton conservation of the reaction?
- **A** It is allowed.
- **B** Forbidden; conservation of baryon number is violated.
- **C** Forbidden; conservation of lepton number is violated.
- **D** Forbidden; conservation of both baryon and lepton number is violated.

14 For the decay reaction $\pi^+ \rightarrow \mu^+ + \gamma$, the identity of the particles and quark composition are π^+, pion $(u\bar{d})$; μ^+, antimuon (lepton) and γ, gamma (electromagnetic radiation).

Which one of the following is a true characterisation in terms of baryon and lepton conservation of the reaction?
- **A** It is allowed.
- **B** Forbidden; conservation of baryon number is violated.
- **C** Forbidden; conservation of lepton number is violated.
- **D** Forbidden; conservation of both baryon and lepton number is violated.

15 Consider the neutrino decay: $\overline{\nu}_e + p^+ \rightarrow e^- + n^0$. Use charge-reversal symmetry to identify another possible interaction.
- **A** $\overline{\nu}_\mu + \overline{p} \rightarrow e^+ + \overline{n^0}$
- **B** $\nu_e + \overline{p} \rightarrow e^+ + \overline{n^0}$
- **C** $\overline{\nu}_e + p^- \rightarrow e^+ + n^-$
- **D** $p^+ \rightarrow e^+ + n^0 + \nu_e$

16 Consider the following muon decay after a charge-reversal symmetry operation has been applied:

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

Determine the original decay.
- **A** $\mu^- \rightarrow e^- + \nu_e + \overline{\nu}_\mu$
- **B** $\mu \rightarrow e^- + \nu_e + \nu_\mu$
- **C** $\overline{\mu} \rightarrow p + \overline{\nu}_e + \nu_\mu$
- **D** $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$

Review questions 14.4B Short response

Review questions: Complete these questions online or in your workbook.

Retrieval and comprehension

17 **Describe** the concept of lepton number and baryon number and explain their difference. (3 marks)

18 **Identify** the forces that can be experienced by leptons. (1 mark)

19 **Describe** the significance of the electron antineutrino produced in the decay of a neutron to form a proton. (2 marks)

20 **Sketch** the Feynman symbol for a Z^0 boson. (1 mark)

21 **Describe** how symmetry in particle interactions occurs to maintain the principles of conservation. (5 marks)

22 **Explain** the following interactions using Feynman diagrams.

- **a** An electron and positron annihilating into a virtual photon, only to then "pair produce" another electron and positron. (3 marks)
- **b** A neutron decaying into a proton in terms of the particles at a nucleon level and showing the mediating boson. (3 marks)
- **c** A neutron decaying into a proton in terms of the quarks, showing the role of the W^- boson. (3 marks)

23 **Construct** a Feynman diagram for the following interaction. A tau lepton decays into a tau neutrino mediated by a W^- boson. The W^- boson immediately decays into an electron and an electron antineutrino. (3 marks)

Analytical processes

24 **Determine** the baryon number for these particles. (1 mark each)

- **a** uuc
- **b** $\bar{s}s\bar{b}$
- **c** $\bar{u}b$
- **d** tau particle

25 Consider this decay reaction: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$.

- **a** **Determine** the lepton number for the reactants and the products. (2 marks)
- **b** **Compare** the L values to determine whether lepton number is conserved. (2 marks)

26 A suspected reaction is between a meson, π^-, and a proton, p, to form a kappa zero particle, K^0, made of the quarks $d\bar{s}$.

$$\pi^- + p^+ \rightarrow K^0$$

Determine whether the reaction violates the lepton or baryon conservation laws and hence whether it is likely to be observed to any extent. (2 marks)

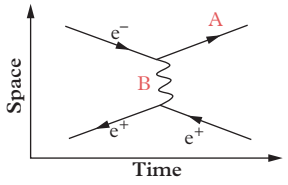
27 The following reactions depict the decay of a muon before and after a symmetry reversal operation is applied.

i $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

ii $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

- **a** **Deduce**, with reasons, the type of symmetry reversal that is occurring. (2 marks)
- **b** **Determine** whether lepton number has been conserved in each case. (1 mark)

28 The interaction of some fundamental particles is shown in the diagram.



- **a** **Deduce** the charge on particle A and explain your reasoning. (2 marks)
- **b** **Identify**, with reasons, whether this is a scattering or an annihilation interaction. (2 marks)
- **c** **Deduce** the identity of boson B and explain how you arrived at this conclusion. (2 marks)

Knowledge utilisation

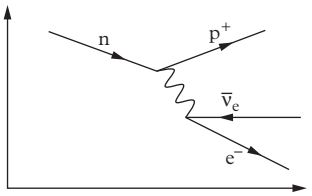
29 **Determine** whether the following decay of a tau particle would violate the conservation of lepton number. (1 mark)

$$\tau^- \rightarrow \mu^- + \nu_\mu$$

30 Consider the beta decay of a neutron.

- **a** **Construct** a Feynman diagram that indicates what happens at the quark level. (3 marks)
- **b** **Assess** whether baryon number and lepton number are conserved. Show your working. (2 marks)

31 The Feynman diagram shown represents the interaction of some fundamental particles.



a **Recall** the labels given to the vertical and horizontal axes respectively. (1 mark)

b **Describe** the importance of the electron antineutrino in this interaction. (1 mark)

c **Construct** a Feynman diagram after charge-reversal symmetry is applied. (2 marks)

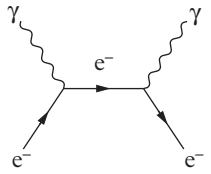
d **Construct** a Feynman diagram after time-reversal symmetry is applied. (2 marks)

e **Assess**, with reasons, which reaction has the greater probability of occurring: the original interaction or the time-reversed interaction. (2 marks)

32 **Assess** the following beta-negative decay of a neutron for conservation of baryon number. (2 marks)

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

33 The diagram shows an interaction called Compton scattering.



a **Describe** the three stages of the interaction. (3 marks)

b **Justify** why the diagram represents scattering, not annihilation. (1 mark)

Data drill

Translating information in symmetry operations using symbols and notation

Students were testing another form of symmetry operations called "crossing symmetry". They took a particular particle interaction and anticipated other possible interactions by replacing a particle with its antiparticle on the other side of the interaction. They found that the reaction $\pi^+ \rightarrow e^+ + \nu_e$ could be "crossed" to give $\pi^+ + \bar{\nu}_e \rightarrow e^+$.

They considered the hypothetical interaction:

$$A + B \rightarrow C + D$$

They then proposed the following interactions:

$$A + B + \overline{C} \rightarrow D \quad (1)$$

$$A + \overline{C} \rightarrow \overline{B} + D \quad (2)$$

$$\overline{C} + D \rightarrow \overline{A} + \overline{B} \quad (3)$$

$$B \rightarrow \overline{A} + C + D \quad (4)$$

Apply understanding

1 **Identify** which symbol would be used for the antiparticle of D. (1 mark)

2 **Identify** how many crossing symmetry operations were performed in equation 1. (1 mark)

Analyse data

3 **Classify** whether each of the proposed crossings are valid or not. (3 marks)

Interpret evidence

4 **Determine** an example of a "crossing" interaction not already shown here. (2 marks)

5 **Deduce** why the probability of equation 1 might be low. (2 marks)

Module 14 checklist: Particle interactions

UNIT 4

Topic 3 review

Multiple choice

(1 mark each)

1 Select the statement that best describes an antiproton.

A The mass of a proton and the charge of an electron

B The mass of an electron and the charge of a proton

C The mass of a neutron and the charge of a proton

D The mass of a proton and the charge of a neutron

2 Select the sequence that correctly lists the four fundamental forces of nature in order of decreasing strength.

A Strong nuclear, electromagnetic, weak nuclear, gravitational

B Electromagnetic, strong nuclear, weak nuclear, gravitational

C Strong nuclear, gravitational, weak nuclear, electromagnetic

D Strong nuclear, electromagnetic, gravitational, weak nuclear

3 Select the option that correctly represents a meson.

A udc

B $\overline{ud}$

C cc

D $c\overline{c}$

4 Select the option that correctly lists the baryon number and lepton number, respectively, for the charmed-lambda particle, Λ^+_c, given that it is a quark composite (udc).

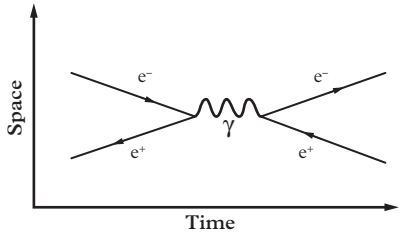
A −1, −1

B −1, 0

C +1, 0

D +1, +1

5 Select the statement that is a correct description of the Feynman diagram shown.



A An electron turns into a positron and a gamma particle.

B An electron and a positron bounce off a gamma particle.

C An electron and a positron bounce off a gamma particle and turn into their opposites.

D An electron and a positron collide and form a virtual photon, which forms an electron and a positron.

6 The standard model proposes that all matter is composed of

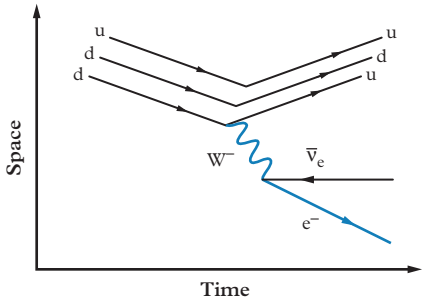
A three quarks and three leptons.

B six leptons and four bosons.

C six quarks and four leptons.

D six quarks and six leptons.

7 What interaction is represented in the diagram?



A A neutron decaying into a proton

B Three quarks colliding with a boson

C A neutron and a proton annihilating to form a W^- boson

D The collision of a down quark and an up quark with a W^- boson to undergo a flavour change

8 For the decay reaction $\Omega^- \rightarrow \Xi^0 + e^+ + \nu_e$, the name, quark composition and baryon numbers of the Ω^- and Ξ^0 particles are:

Ω^-, omega (sss), $B = +1$

Ξ^0, Xi (uss), $B = +1$

The reaction is described as

A allowed.

B forbidden; conservation of lepton number is violated.

C forbidden; conservation of baryon number is violated.

D forbidden; conservation of both baryon and lepton number is violated.

9 Which of the following particles is governed by the weak nuclear force and is also influenced by the electromagnetic force?

A Uncharged leptons

B Neutrinos

C Quarks

D Mesons

10 Symmetry occurs in particle interactions to

A maintain the principle of conservation of lepton number, baryon number and momentum.

B allow reactions to proceed in the forward and reverse directions.

C ensure the mass–energy equivalence principle is obeyed.

D allow virtual particles to come in and out of existence.

11 Which one of the following options correctly names a charged lepton and the Standard Model force(s) that act on it?

A Muon neutrino; weak nuclear and electromagnetic forces

B Muon; weak nuclear and strong nuclear forces

C Muon neutrino; weak nuclear force only

D Muon; weak nuclear force and electromagnetic force only

12 Which one of the following statements is true about a quark and a lepton?

A All quarks have a fractional charge, but all leptons have a charge of either +1e or −1e.

B All quarks have an electric charge, but some leptons are neutral.

C All quarks and all leptons experience the electromagnetic force.

D Quarks have antiparticles, leptons do not.

13 Which one of the following options lists possible composite particles of the four quarks c, c, d, $\overline{c}$?

A cc, cd, $c\overline{c}$, $d\overline{c}$

B ccd, $c\overline{c}$, $d\overline{c}$

C ccc, ccd, cdc, dcc

D ccc, ccd, $cc\overline{c}$, $cd\overline{c}$

14 Which of the following pairs of particles have the same magnitude and sign of charge?

A $\overline{sss}$, e^-

B $\overline{\nu}_e$, $\overline{\nu}_\mu$

C uud, udd

D $c\overline{s}$, $\overline{\nu}_e$

15 Where are gluons and leptons likely to be found in an atom?

A Both between quarks

B Both in an orbital cloud surrounding the nucleus

C The gluon between quarks; the leptons inside the nucleus

D The gluon between quarks; the lepton in an orbital cloud surrounding the nucleus

Short response

16 **Identify** the symbols or names of the six charged leptons. (3 marks)

17 Mesons can be positive, negative or neutral. Using first generation quarks and antiquarks, **demonstrate** how these three types of mesons can be constructed. The names of the mesons are not required. (3 marks)

18 Leptons can be charged or uncharged.

a **Identify** one force that charged and uncharged leptons have in common. (1 mark)

b **Identify** one force that charged and uncharged leptons do not have in common. (1 mark)

19 **Determine** whether all leptons are affected by the force that has photons as the mediating gauge boson. **Explain** your answer. (2 marks)

20 **Determine** whether baryon number is conserved for the following interactions.

a $e^- + p^+ \rightarrow \nu_e + n^0$ (1 mark)

b $\nu_\mu + \tau^- \rightarrow \mu^- + \nu_\tau$ (1 mark)

21 **Determine** whether lepton number is conserved in the following interactions.

a $\nu_e + n^0 \rightarrow e^- + p^+$ (1 mark)

b $\tau^- \rightarrow e^- + \nu_e + \nu_\tau$ (1 mark)

22 The decay reaction $\Sigma^- \rightarrow \Lambda^0 + e^+ + \overline{\nu}_e$ involves two unfamiliar particles with quark compositions as follows: Σ^-, sigma-minus, (dds) and Λ^0, lambda-zero, (uds). They have the same baryon number of +1 and the same lepton number of 0. **Determine** whether the reaction is allowable or forbidden, based on conservation of baryon and lepton numbers. Note: this type of question involving exotic particles is unlikely to appear on the external exam. It is here because it is a good challenge that will brighten up your day. (3 marks)

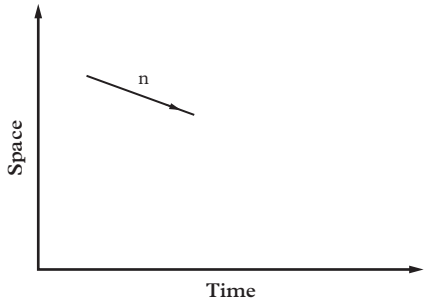
23 Carbon-14 is a radioactive nuclide that decays according to the following reaction.

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \overline{\nu}_e$$

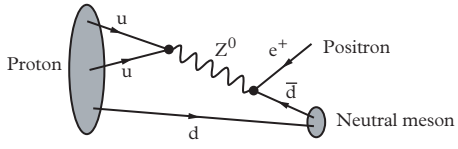
It sheds some of its excess energy by turning one of its neutrons into a proton. It uses some of its excess energy to form a temporary (virtual) intermediate particle called the W^- boson, which disappears by forming an electron and an electron antineutrino, as shown.

$$n \rightarrow p + e^- + \overline{\nu}_e$$

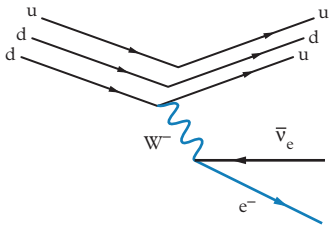
The interaction can be represented by a Feynman diagram, the start of which is shown in the diagram. **Construct** a labelled Feynman diagram to represent this interaction. (3 marks)



24 **Explain** one reason why the reaction represented by the Feynman diagram shown cannot occur. (2 marks)



25 The particle interaction diagram shown represents beta decay of a nuclide.



a **Determine** whether this is a proton decaying into a neutron or a neutron decaying into a proton. (1 mark)

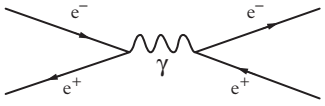
b **Determine** whether this is an example of beta-negative or beta-positive decay. (1 mark)

c **Explain** the role of the antineutrino in this interaction. (2 marks)

d **Explain** in terms of conservation principles why the gauge boson is negative. (2 marks)

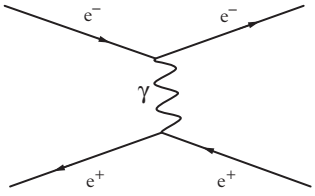
e **Explain** how baryon number and lepton number are conserved in this interaction. (4 marks)

26 A lepton interaction is shown in the Feynman diagram.



There are three stages in this interaction. **Describe** what is happening in each stage. (3 marks)

27 An interaction between charged leptons is shown in the Feynman diagram.

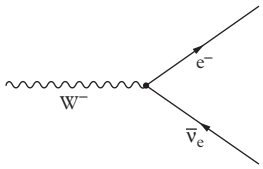


a **Determine** whether this is an example of scattering or annihilation. Justify your answer. (2 marks)

b **Explain** the role of the gauge boson in this interaction. (1 mark)

c **Explain** how the conservation of lepton number is obeyed in this interaction. (2 marks)

28 A particular charged gauge boson can transition into two particles according to the Feynman diagram shown.

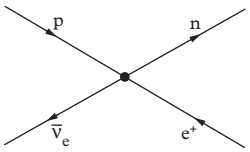


One symmetry operation is "charge-reversal symmetry", which states that interactions are not affected if all charges are swapped (positive for negative and vice versa).

a **Draw** a particle interaction diagram (Feynman diagram) after a charge-reversal symmetry operation has been performed on this particle interaction. (2 marks)

b **Determine** whether lepton number has been conserved in both interactions. (2 marks)

29 "Time-reversal symmetry" is an operation in which an interaction looks the same if time is reversed, so products become reactants, and reactants become products. The diagram shows an interaction called inverse-beta decay.



a **Sketch** the interaction after performing a time-reversal symmetry operation. (2 marks)

b **Determine** whether conservation of baryon number and conservation of lepton number have been obeyed in this symmetry operation. (4 marks)

c **Determine** whether the particle interaction after the time-reversal symmetry operation has been performed is "allowed" (permitted). Explain your reasoning. (2 marks)

d **Identify** one other conservation law that must be obeyed in particle interactions. (1 mark)

TOTAL MARKS

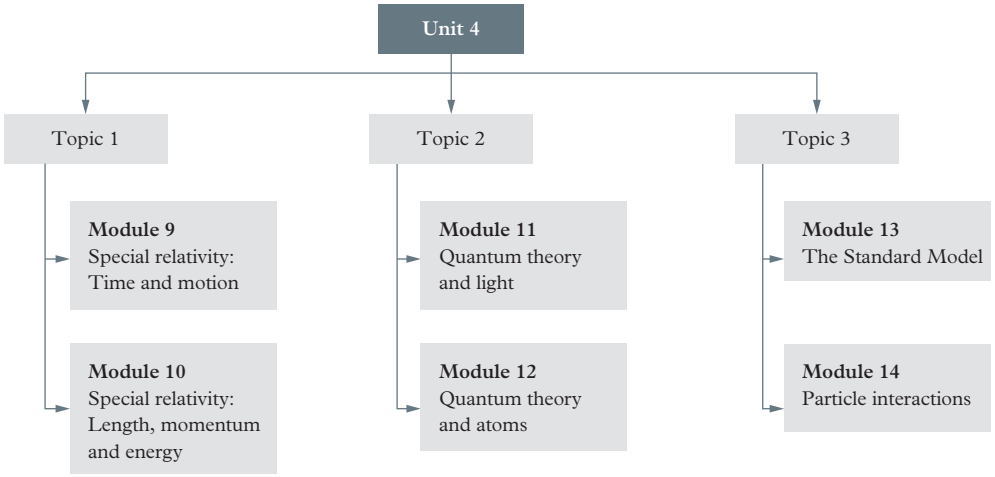
/68 marks

UNIT 4

Review

Part A – Revisit and revise

Part A of the Unit review asks you to reflect on your learning and identify areas in which you need more work.



Part B – Exam essentials

Now that you've completed your revision for Unit 4, it's time to learn and practise some of the skills you will need to answer exam questions like a pro! To help you, our expert authors have created the following advice and tips to help you maximise your results on the end-of-year examination.

Exam tip 1: Describe processes like you are telling a story.

There are 33 instances in Units 3 and 4 of the syllabus where you need to "describe" something – for example, an idea, concept, observation or law. You're bound to get this type of question in the external exam.

"Describe" means give an account of a situation, event, pattern or process, or of the characteristics or features of something. If it is a process, such as how a transformer works or how particles interact, then a good tip is to tell it like a story, with a beginning, middle and end.

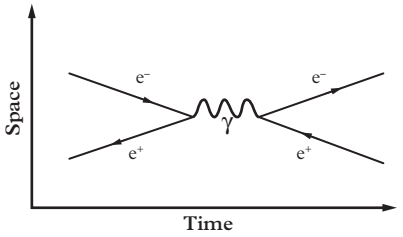
This is just a tip. You can still get full marks if you make a suitable response in another format, as long as you show reasonable understanding.

See it in action

Read the real exam question below and see how this tip has made the difference between a response that has scored full marks and a response where marks have been lost.

QUESTION 21 (3 marks)

Describe the particle interaction shown.



Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Complete response

The start of the response talks about how two separate particles approach each other. Words like "colliding" or "meeting in space" could also be used. This sentence identifies by name that the interaction involves an electron and a positron and doesn't confuse positron with proton. [1 mark]

The Feynman diagram represents the interaction between an electron and a positron. During this interaction, the two particles are annihilated into a photon. The photon will then "pair produce" another electron and positron.

The middle sentence identifies that a photon is created through annihilation. [1 mark] Rather than using the symbol γ (which would not be enough), the response uses the term "photon". The photon is virtual so "virtual photon" would also be accepted but is not essential.

The final sentence describes the creation of another electron and positron by "pair production". [1 mark] Be careful not to say the original electron and positron are back again. These are new particles, hence the word "another".

Incomplete response

This is a Feynman diagram, but the question hasn't asked for a definition of one. [0 marks]

This is a Feynman diagram. It is a graphical representation of particle interactions showing time along the horizontal axis and space along the vertical axis. The e^- and e^+ come together to produce a γ particle. This γ particle is a force carrier that mediates the interaction. The γ particle decays to form the e^- and e^+ which were there at the start.

The response needs to name the three particles and not just restate the symbols. [0 marks]

The photon, γ, may be a force carrier and thus a mediator but, in this interaction, it is a particle produced by annihilation. [0 marks]

Again, the response needs to name the electron and positron, not just restate the symbols. The original electron and positron have not returned; new particles are formed by "pair production". [0 marks]

Think like an assessor

To maximise your marks on an exam, it can help to think like a QCAA assessor. Consider how many marks each question is worth and what information the assessor is looking for.

A student has given the following response in a practice exam. Imagine you are a QCAA assessor and use the marking guide below to mark the response.

QUESTION 7 (4 marks)

a) Contrast baryons and mesons in terms of composition. [2 marks]

Similarities: Baryons and mesons are composed of quarks. The quarks can be the same type or different types.
Differences: Baryons are composed of three quarks but can't be all the same type (e.g. a proton is uud). Mesons are composed of two quarks which must be different (e.g. ud).

b) Contrast quarks and leptons in terms of the possible fundamental forces they experience. [2 marks]

	Strong nuclear force	Weak nuclear force	Electromagnetic force	Gravitational force
Quark	yes	yes	yes	yes
Charged leptons	yes	yes	yes	yes
Uncharged leptons	no	yes	no	yes

Source: *QCAA 2024 External Examination Paper 2* © State of Queensland (QCAA)

Marking guide

Question 7a	• Identifies baryons are composed of three quarks [1 mark] • Identifies mesons are composed of a quark and an antiquark [1 mark]
Question 7b	• Recognises that quarks experience all four forces (SNF, WNF, EM, gravitational) whereas leptons do not experience the SNF [1 mark] Accept lists or tables of fundamental forces that quarks and leptons experience individually. • Identifies that only charged leptons would experience the EM force [1 mark]

Source: Adapted from *QCAA 2024 Physics marking guide and response* © State of Queensland (QCAA)

Fix the response

Consider where you did and did not award marks in the above response. How could the response be improved? Write your own response to the same question to receive full marks from a QCAA assessor.

Exam tip 2: Explain rather than define.

The cognitive verb "explain" is used 14 times in the syllabus for Units 3 and 4, so it is tempting to think you could just learn those 14 explanations off by heart. However, in the external exam you will also see "explain" questions about concepts that are "describe" in the syllabus. This will necessarily have extra detail such as a particular context or have a diagram, and this adds more complexity than you may have expected.

"Describe" typically is to *define* the idea or concept mentioned in the question and then *elaborate* on it.

"Explain" means to "make an idea or situation plain or clear by *describing* it in more detail or revealing relevant facts". The simplest way to think of "explain" is that it is a more detailed description.

You may have no trouble providing a definition but will often find you are not sure how much extra detail is required. You also need to work out what you're trying to explain – for example, a term, a process or a concept. You need to identify key terms in the question to know what the examiner is after.

See it in action

Read the real exam question below and see how this tip has made a difference between a response that has scored full marks and a response where marks have been lost.

QUESTION 5 (3 marks)

Explain the significance of the threshold frequency when incident light with a range of frequencies shines on a metal.

Source: *QCAA 2024 External Examination Paper 2* © State of Queensland (QCAA)

Complete response

The opening sentence of the response provides the "definition" of threshold frequency. [1 mark] This definition can be put into your own words, as the syllabus doesn't have set definitions.

The question mentions "a range of frequencies", so the response needs to consider frequencies below and above the threshold. This first elaboration of the response is about frequencies "below". [1 mark] Students sometimes think that if nothing is happening there is no need to mention it. However, "nothing happening" is very important. You should say something, even if it's that photons will scatter off the metal's surface.

Threshold frequency is the minimum frequency at which a photon has sufficient energy to dislodge an electron from the surface of a metal. If the frequency is less than threshold there will be no emission (ejection) of an electron as the quanta of energy in the photon is less than the work function. If the frequency is equal to or greater than the threshold there will be emission (ejection) of electrons with greater energy as the frequency increases (up to a limit).

The response elaborates about frequencies above threshold. [1 mark]

Incomplete response

The opening sentence defines the particle model instead of the threshold frequency. [0 marks]

This sentence elaborates about the frequency being greater than the threshold frequency. [1 mark] However, the question is about frequency and not energy and work function, so the third sentence is irrelevant.

According to the particle model, light consists of particles known as photons. When light is shone on to a metallic plate, the plate is bombarded by photons. If the frequency of the light is greater than the threshold frequency of the metal, they will liberate electrons from the surface with kinetic energy equal to the energy of the photon (E = hf) less the work function of the metal. Each metal has a different threshold frequency and work function depending on its composition. Increasing the intensity of the light will lead to a greater number of electrons being emitted.

Intensity is not mentioned in the question and is not needed in the response. This statement is only true for light above the threshold frequency, so it is also incomplete and somewhat misleading. [0 marks]

Think like an assessor

To maximise your marks on an exam, it can help to think like a QCAA assessor. Consider how many marks each question is worth and what information the assessor is looking for.

A student has given the following response in a practice exam. Imagine you are a QCAA assessor and use the marking guide below to mark the response.

QUESTION 22 (5 marks)

Special relativity accounts for the observation that more muons from cosmic rays are detected near Earth's surface than expected. Explain this phenomenon by outlining why Newtonian physics cannot explain this scenario while special relativity can. Refer to the frames of reference of both the travelling muons and the observer near the Earth's surface.

Muons are short-lived particles that have a lifetime measured in microseconds. In Newtonian physics this means they shouldn't have enough time to reach the Earth below after being created in the atmosphere.
Muons travel at close to the speed of light – about 0.994c. To an observer travelling with the muon, that is, in the muon's frame of reference, the lifetime will be a few microseconds. However, an observer moving relative to the muon, that is, in the Earth's frame of reference, will see a dilated time for this event that is 10 times as long – maybe as much as 23 microseconds. So, to Earth observers the muons will have more time to reach the ground and will therefore travel further before they decay.
Special relativity also means length contraction. Scientists will see the distance between the atmosphere and the ground as being shorter than it really is so will see more muons making it to the ground.

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Marking guide

Question 22	• Identifies that muons have a very short half-life (or lifetime) [1 mark] • Explains that muons are unlikely to reach the Earth's surface using Newtonian physics [1 mark] • Identifies that muons travel at relativistic speeds [1 mark] • Explains that from the observer's frame of reference, the muon's half-life will be longer due to time dilation [1 mark] • Explains that from the muon's frame of reference, it experiences length contraction [1 mark]

Source: Adapted from *QCAA 2024 Physics marking guide and response* © State of Queensland (QCAA)

Fix the response

Consider where you did and did not award marks in the above response. How could the response be improved? Write your own response to the same question to receive full marks from a QCAA assessor.

Exam tip 3: Watch out for the familiarity trap.

When you see a special relativity question about "twins", it is tempting to think it must be a "twins paradox" question where one twin travels at high speed and then returns to Earth to greet the stay-at-home twin. However, it may be a completely different scenario. So, don't assume questions are like the ones you've done before. When you are under time pressure in exams, any way to save time is great, which leads you to answer questions in the same way you have used in practice. It is a shortcut we all use, and it's the trap of familiarity.

See it in action

Read the real exam question below and see how this tip has made a difference between a response that has scored full marks and a response where marks have been lost.

QUESTION 23 (2 marks)

Twin astronauts conduct an experiment where one travels with a velocity close to the speed of light to a distant planet, while the other stays on Earth. Each twin expects the other to be a different age by the time the first twin reaches and remains on the distant planet.

Use the theory of special relativity to explain why the twins will no longer be the same age, and draw a conclusion about which twin will be younger.

Source: *QCAA 2020 External Examination Paper 1* © State of Queensland (QCAA)

Complete response

The student has made it clear that the time for an event will be different if the observers are moving relative to each other. This provides an explanation of why the twins will no longer be the same age. [required for the 2 marks]

The special theory of relativity states that there is no absolute time. The measurement of time is relative to the frame of reference in which it is measured. The time measured by a clock at rest to an observer is greater than the time measured by an observer moving relative to the clock. From the perspective of either twin, the other twin will be younger.

The response supports the explanation using the special theory of relativity. Either twin can claim to be at rest (stationary) and say the other is moving. [required for the 2 marks]

The student has drawn a conclusion about the respective ages of the twins. This indicates an understanding of the twins paradox. [required for the 2 marks]

Incomplete response

This statement correctly says that the time for an event will be different if the observers are moving relative to each other. This correctly provides an explanation of why the twins will no longer be the same age. [required for the 2 marks]

The twin will age differently as they are in motion relative to each other. The measurement of time depends on who is moving and who is at rest. The time measured on a clock at rest to an observer is greater than the time measured by an observer moving relative to the clock. The twin that went to the distant planet had to speed up and slow down so must be the one who moved. Their clock moved slowly (dilated time). When this twin returns to Earth, the travelling twin will be younger than the Earth twin because the travelling twin's clocks slowed down.

This is a correctly stated consequence of special relativity and supports the explanation. Either twin can truthfully claim to be at rest (stationary) and say the other is moving. It is a correct interpretation of special relativity. [required for the 2 marks]

The twin that went to the distant planet did not return to Earth, so the twins never got back together to compare clocks. The response has made an assumption that is not in the question. [Unsuitable response so only 2 out of 3 points made, therefore 1 mark for whole question.]

Think like an assessor

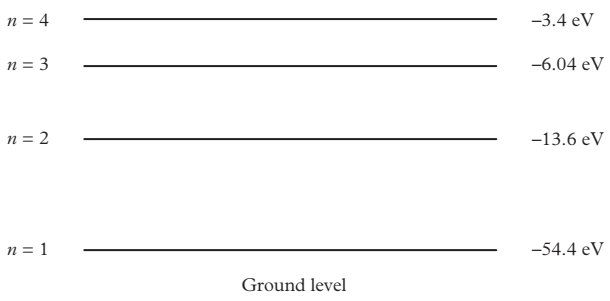
To maximise your marks on an exam, it can help to think like a QCAA assessor. Consider how many marks each question is worth and what information the assessor is looking for.

A student has given the following response in a practice exam. Imagine you are a QCAA assessor and use the marking guide below to mark the response.

QUESTION 24 (7 marks)

The atomic energy level diagram for an unknown multi-electron ion is shown.

Not to scale



a) Calculate the wavelength of light emitted as electrons move from $n = 4$ to $n = 2$. Show your working. [4 marks]

$\Delta E = E_f - E_i$

$= -13.6 - (-3.4)$

$= 10.2$ eV

$E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{E}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{10.2}$

$= 1.9 \times 10^{-26}$ m

A photon with 40.8 eV of energy is incident on the unknown ion and collides with an electron in the first energy level.

b) Explain what would happen within the ion in terms of the photon and electron. [3 marks]

The 40.8 eV corresponds to a jump from the first energy level (n = 1) to the second energy level (n = 2):

$\Delta E = E_f - E_i$

$= E_2 - E_1$

$= -13.6 - (-54.4) = 40.8$ eV

The photon has enough energy to cause this jump and transfers it to the electron. The electron would absorb the photon and jump to the second level. The photon would carry away any left-over energy which could cause a transition elsewhere in the atom.

$\Delta E = E_f - E_i$

$= E_2 - E_1$

$= -13.6 - (-54.4) = 40.8\,\text{eV}$

Source: *QCAA 2024 External Examination Paper 1* © State of Queensland (QCAA)

Marking guide

Question 24a	• Calculates the energy change from $n = 2$ and $n = 4$ to be 10.2 eV [1 mark] • Converts between eV and J [1 mark] • Shows mathematical reasoning [1 mark] • Calculates the wavelength [1 mark] Allow follow-through marks for correct *conversion* and *wavelength* based on incorrect change in energy between energy levels. For *mathematical reasoning*, accept formula or substitution of values. Do not penalise for incorrect decimal places/significant figures or rounding.
Question 24b	• Identifies photon is absorbed by electron [1 mark] • Explains photon energy as equivalent to gap between $n = 1$ and $n = 2$ [1 mark] • Explains that photon will be absorbed because electron in $n = 1$ will jump to $n = 2$ [1 mark]

Source: Adapted from *QCAA 2024 Physics marking guide and response* © State of Queensland (QCAA)

Fix the response

Consider where you did and did not award marks in the above response. How could the response be improved? Write your own response to the same question to receive full marks from a QCAA assessor.

Part C – Exam practice

Now it's time to put the tips and advice you've learned into practice while you complete these exam-style questions!

Multiple choice

(1 mark each)

1 Which one of the following is a postulate of special relativity?

A The laws of physics are the same for all observers in uniformly moving frames of reference.

B Physicists can make no measurements in a moving reference frame.

C The relative speed of two objects is the same for all observers.

D Time passes faster in a moving frame of reference.

2 Pions are particles created in the upper atmosphere by cosmic rays from the Sun. Pions decay with an average rest lifetime of 26 nanoseconds (ns).

Consider one particular pion approaching Earth at a speed of $0.98c$. Its lifetime is 26 ns in its own frame of reference after it is formed. For how much time did the pion exist as observed in Earth's frame of reference?

A 5.2 ns **B** 26 ns

C 130 ns **D** 650 ns

3 A stationary observer views the approach of a spaceship moving at a relativistic speed. Which one of the following quantities would be measured as less than if it were stationary with respect to them?

A Height

B Length

C Density

D Momentum

4 A metre ruler is 100 cm long, 2 cm wide and 0.5 cm thick. It is held vertically and made to move sideways at $0.6c$. What would its volume be to a stationary observer?

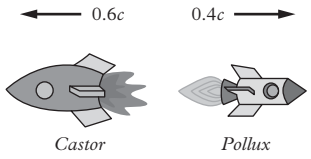
A $60\,cm^3$

B $80\,cm^3$

C $100\,cm^3$

D $125\,cm^3$

5 Two spacecraft travel in opposite directions, with spacecraft *Castor* travelling at a speed of $0.6c$ and spacecraft *Pollux* travelling at a speed of $0.4c$. Both are travelling relative to the fixed stars of the galaxy. The situation is shown in the diagram. A radio signal is emitted by *Castor* towards *Pollux*. The navigator of *Pollux* uses the classical physics understanding of radio waves travelling at a speed relative to a medium fixed with respect to the galaxy.



How can proper time be measured for the interval between the radio signal being emitted on *Castor* and the signal reaching *Pollux*?

A Using measurements made by the crew on *Castor*

B Using measurements made by the crew on *Pollux*

C No single observer can measure proper time for this case

D Using measurements made by an observer stationary at the point where the signal was emitted

6 Astronauts travel at a velocity of $0.9c$ to Alpha Centauri. Newtonian physics predicts that this journey would take 4.86 years. How many years will the journey take in the frame of reference of the astronauts?

A 0.923 **B** 1.54 **C** 2.12 **D** 11.1

7 A 1.00 kg mass of water is heated by the addition of 334 kJ of energy. What is its increase in mass, in kilograms, due to the conversion of energy to mass?

A $\frac{334 \times 10^3}{(3 \times 10^8)^2}$ **B** $1.00 + \frac{334 \times 10^3}{(3 \times 10^8)^2}$

C $\frac{(3 \times 10^8)^2}{334 \times 10^3}$ **D** $1.00 + \frac{(3 \times 10^8)^2}{334 \times 10^3}$

8 The sun emits its maximum emissive power radiation of wavelength 0.52 μm. Calculate its surface temperature, assuming it to be a black body.

A 2345 K **B** 5573 K **C** 9847 K **D** 6492 K

9 The threshold frequency for photoelectric emission in copper is 1.10×10^{15} Hz. Determine the maximum kinetic energy of the photoelectrons emitted when light of frequency 1.50×10^{15} Hz is directed on a copper surface.

A 1.32 eV **B** 1.66 eV **C** 2.40 eV **D** 3.22 eV

10 Photons are incident on a metal surface and electrons are emitted from the surface. Which one of the following changes may result in *no* electrons being emitted from the surface?

A Halving the wavelength of the photons

B Doubling the wavelength of the photons

C Halving the number of photons incident on the surface per second

D Doubling the number of photons incident on the surface per second

11 Aluminium has a work function of 3.3 eV. When ultraviolet photons with an energy of 5.3 eV are incident on it, what is the maximum kinetic energy of the emitted photoelectrons?

A $2.0 \times 1.6 \times 10^{-19}$ J **B** $5.3 \times 1.6 \times 10^{-19}$ J

C $\frac{2.0}{1.6 \times 10^{-19}}$ J **D** $\frac{8.6}{1.6 \times 10^{-19}}$ J

12 Which model of the atom would predict the continuous emission of electromagnetic energy, resulting in the instability of the atom?

A Bohr's model of the atom

B Dalton's model of the atom

C Rutherford's model of the atom

D Thomson's plum pudding model of the atom

13 Which one of the following is regarded as a major success of the Bohr model of the atom?

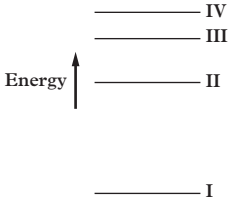
A Explaining the hydrogen spectrum

B Proposing the existence of the neutron

C That the electron could be a wave and a particle

D The further development of the Rydberg equation to predict undiscovered spectral lines

14 The diagram shows the first four energy levels in an atom.



Which transition will produce the highest frequency photon and the longest wavelength photon respectively?

A IV → I, IV → III **B** IV → I, IV → I

C IV → III, IV → III **D** IV → III, IV → I

15 The energy of a photon is 4.2 eV. Determine its momentum.

A 2.24×10^{-24} kg m s^{-1} **B** 2.24×10^{-27} kg m s^{-1}

C 3.32×10^{-27} kg m s^{-1} **D** 3.32×10^{-24} kg m s^{-1}

16 Select which one has the greater wavelength: a cricket ball travelling at $20\,m\,s^{-1}$ or an electron travelling at $2 \times 10^6\,m\,s^{-1}$.

A The electron has the greater wavelength.

B The cricket ball has the greater wavelength.

C There is no difference.

D The cricket ball does not have a wave nature.

17 Determine the frequency of a microwave photon whose momentum is $1.2 \times 10^{-31}\,kg\,m\,s^{-1}$.

A 1.8×10^{10} Hz **B** 2.4×10^{10} Hz

C 3.6×10^{10} Hz **D** 5.4×10^{10} Hz

18 Where are a gluon and a lepton, respectively, likely to be found in an atom?

A Both are between baryons.

B Both are in an orbital cloud surrounding the nucleus.

C The gluon is between baryons; the lepton is between electrons.

D The gluon is between quarks; the lepton is orbiting the nucleus.

19 Select the option that contains only quarks.

A Tau, bottom, up, down

B Up, down, strange, muon

C Muon, boson, meson, gluon

D Charm, strange, top, bottom

20 A common type of symmetry operation is "crossing symmetry" in which particles are replaced with their antiparticle on the opposite side of the equation. Which one of the following represents crossing symmetry?

A $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \xrightarrow{\text{symmetry operation}} \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

B $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \xrightarrow{\text{symmetry operation}} e^- + \bar{\nu}_e + \nu_\mu \rightarrow \mu^+$

C $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \xrightarrow{\text{symmetry operation}} e^+ + \nu_e + \bar{\nu}_\mu \rightarrow \mu^+$

D $\mu^- + e^- + \bar{\nu}_e + \nu_\mu \xrightarrow{\text{symmetry operation}} e^- + \nu_{e+} + \bar{\nu}_{\mu-} \rightarrow \bar{\mu}$

Short response

21 Subatomic particles called muons travel close to the speed of light. They have an average lifetime of $2.20\,\mu s$ as measured in their frame of reference, but scientists measured their lifetime to be $23.0\,\mu s$.

a **Explain** how this difference can be accounted for. Make calculations to support your answer. (2 marks)

b **Determine** how far the muons would travel in their lifetime in the frame of reference of scientists in the laboratory. (2 marks)

22 A rocket of length 5.0 m passes an observer on Earth. The observer measures the passing rocket to be 3.0 m long. **Determine** the velocity of the rocket in the reference frame of the Earth-based observer. (3 marks)

23 Twins A and B are exactly 20 years old on Earth. Twin A stays on Earth, but Twin B leaves Earth in a spaceship travelling at almost the speed of light, arrives at a distant planet and stays there. The twins agree that their ages will be different, but they disagree about who is older.

Determine which twin will be older. **Justify** your response. (2 marks)

24 An electron is travelling at $0.99c$. **Determine** how many times greater its relativistic momentum is than its Newtonian momentum. (3 marks)

25 **Explain** why an object with mass can never reach the speed of light in a vacuum. (1 mark)

26 **Describe** the models of the atom proposed by Rutherford and Bohr. (2 marks)

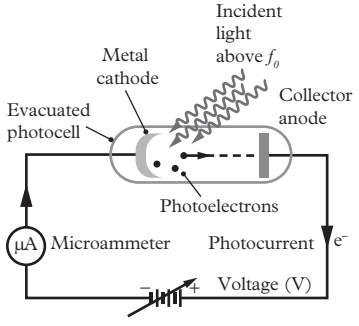
27 In the spectrum of hydrogen, the Lyman series describes transitions in which the electron falls from higher energy levels to the first energy level ($n = 1$).

a **Calculate** the wavelength in nanometres, nm, of the photon emitted in a $4 \rightarrow 1$ transition in hydrogen. (3 marks)

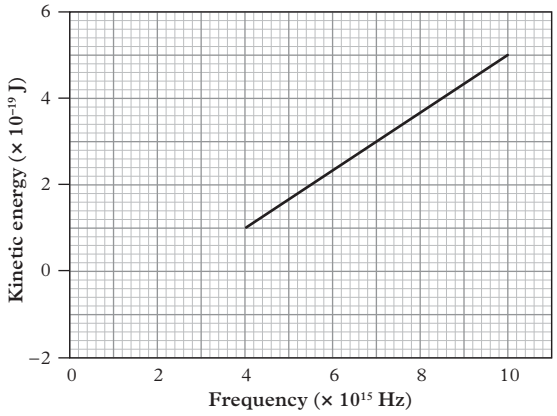
b **Determine** which spectral region the emitted light will belong to using the table of frequencies provided. (2 marks)

Spectral region	Frequency range
Infrared	1×10^{14} Hz to 4×10^{14} Hz
Visible	4×10^{14} Hz to 7.5×10^{14} Hz
Ultraviolet	7.5×10^{14} Hz to 1×10^{20} Hz

28 A photoelectric experiment was carried out in which light of different frequencies was used to illuminate a metal plate in an evacuated chamber, as shown in the diagram.



For each frequency, the maximum kinetic energy of the photoelectrons was measured. The results have been plotted on the graph.



a **Determine** the approximate maximum kinetic energy of an electron (in J) when light with a wavelength of 50.0 nm is shone on it. (2 marks)

b **Determine** the threshold frequency of the metal. (1 mark)

c **Determine** the work function of the metal (in eV). (2 marks)

d **Identify** what the value of the gradient represents. (1 mark)

29 Two overlapping beams of light from two flashlights (torches) fall on a screen, but no interference pattern is observed. **Explain** whether this supports or refutes the wave model of light. (2 marks)

30 **Identify** the names of the gauge bosons in the Standard Model and match them with the names of the forces they mediate. (3 marks)

31 Protons have an extremely long lifetime, and none have been observed to decay. However, a scientist evaluated the following interaction as a possible decay reaction.

$$p^+ \rightarrow n^0 + e^+ + \overline{\nu}_e$$

Explain whether it obeys the conservation laws for baryon number and lepton number. (3 marks)

32 An astronaut leaves Earth and travels at $0.994c$ to another galaxy 6.0 ly (light-years) away, as measured by the astronaut. One light-year is 9.46×10^{15} m. When they arrive at the distant galaxy, they send a light (radio) signal back to Earth to announce their arrival.

Calculate how many years after the astronaut left Earth the signal arrived back on Earth, according to Earth observers. (5 marks)

33 A beam of light from a laser is perpendicular to a plane mirror. **Determine** the speed of the reflected beam if the mirror is moving directly towards the laser with speed v. **Explain** your reasoning. (2 marks)

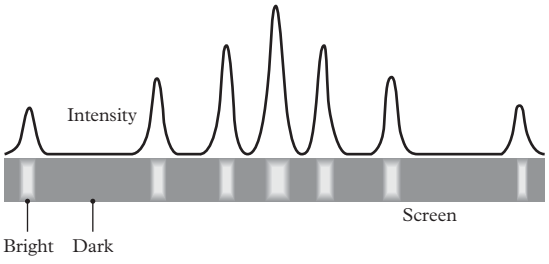
34 A spaceship passes you at a speed of $0.75c$. You measure its length to be 100.0 m.

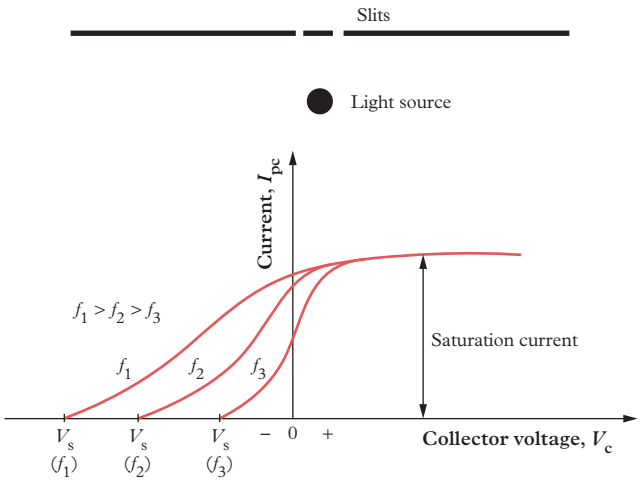
a **Propose** whether this is the rest length or the relativistic length. Justify your decision. (2 marks)

b **Determine** its length as measured by observers aboard the spaceship. Show your working. (2 marks)

35 An electron has a rest mass of 9.109×10^{-31} kg and is travelling at $0.1c$. It is accelerated to a higher speed, so that its relativistic momentum is triple its classical (Newtonian) momentum. **Determine** this speed in units of c. Show your working. (3 marks)

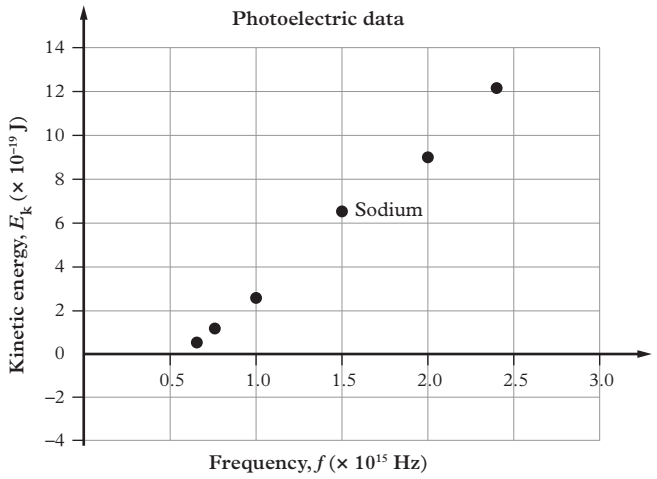
36 The following diagram represents Young's double-slit experiment. The graph shows the current in a photoelectric experiment for light of three different frequencies of the same intensity incident on some zinc metal.





Describe how these diagrams support the notion of wave–particle duality. (4 marks)

37 A photoelectric experiment was carried out on sodium metal, and the results were plotted as shown in the graph.



a **Identify** the mathematical relationship in the form $y = mx + c$ between kinetic energy and frequency for sodium. Show your working. (3 marks)

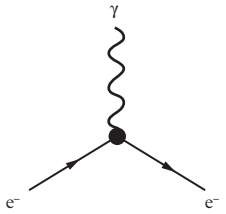
b **Determine** the experimental value of Planck's constant from the graph. (1 mark)

c **Determine** the value for the work function of sodium in electron volts. (2 marks)

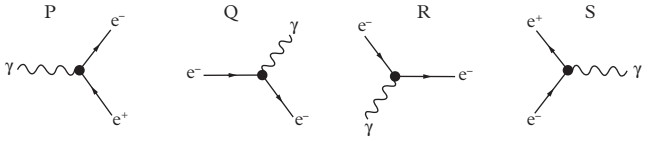
d Rubidium metal has a work function of 2.05 eV. **Explain** where on the graph the line for rubidium would go in terms of its y-intercept, x-intercept and gradient. (3 marks)

38 **Explain** the difference between "scattering" and "pair production" when an electron and positron interact. **Construct** Feynman diagrams to support your explanation. (3 marks)

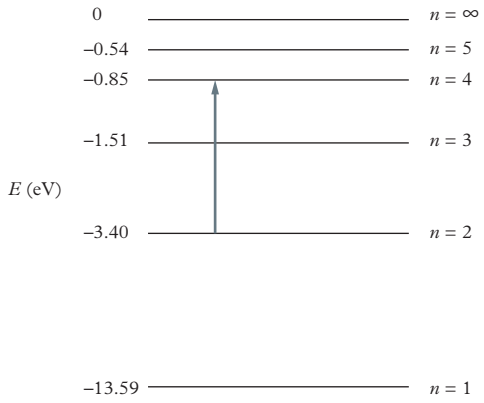
39 The image shows a section of a Feynman diagram. It is claimed to show an electron transition from an $n = 3$ orbital level to an $n = 2$ level. **Explain** whether this diagram is consistent with this interaction. (2 marks)



40 The following Feynman diagrams show four particle interactions.



Determine which diagram (P, Q, R or S) is consistent with the electron transition shown in the energy-level diagram below. **Explain** your answer. (3 marks)



41 **Determine** whether the following interaction is allowable. **Justify** your answer. (2 marks)

$$\bar{\nu} + n^0 \rightarrow p^+ + e^-$$

TOTAL MARKS
/91

MODULE 15

Practical manual

Introduction

The QCAA Physics General Senior Syllabus expects that students will engage in aspects of the work of a scientist by engaging in scientific inquiry.

The practicals in this module have been trialled, and safety instructions are provided; however, it is the legal obligation of the teacher to perform their own risk assessments before participating in any practical activity. These practicals are not prescriptive, and schools may adapt them to their own needs.

For this module, safety hazards will be highlighted at the top of each practical. When undertaking practical experiments, you should always wear a lab coat, safety glasses and enclosed footwear, and have long hair tied back. Below is a summary of general safety cautions:

- → Burns – hot water and hotplates can cause burns in the lab. These can be avoided by not moving boiling water and avoiding splashing, as well as leaving heated samples to cool before using them. It is also important to ensure that each member of your group is aware that a hotplate is on, and once you have finished with it, use a hazard sign to warn others that it may still be hot.
- → Electrical cords – always keep electrical cords away from water and hot metal surfaces.
- → Electric shock – electrical equipment can cause electric shocks and can cause serious or fatal injury. When using electrical equipment, make sure that there are no exposed wires.
- → Glass – glass can shatter and cut you. Be careful when you are using thermometers, conical flasks, beakers or other glassware in the lab. Make sure you handle each item with care. Do not use your hands to pick up broken glass.
- → In the event that an injury or accident does happen, make sure you tell your teacher immediately.

Please familiarise yourself with your school's safety procedures, including the location of first aid kits, safety equipment and chemical waste disposal, and the procedures for the setup and pack-down of practical stations. If you are unsure of any steps in a practical, check with your teacher for the best course of action.

Module 1 Physics toolkit is a good reference for how to keep safe in the laboratory. It also contains detailed guidance for completing data collection and analysis for any practical you undertake in your senior Physics course.

oxforddigital

Full lessons for this module are available on Oxford Digital. They are also provided as part of a printable resource that can be used in class.

Unit 3 Practicals

- **Lesson 2.4** Angled projection and distance
- **Lesson 3.4** The parallel component of the weight of an object on an inclined plane
- **Lesson 4.4** Centripetal forces on an object undergoing horizontal circular motion
- **Lesson 5.5** Investigating orbital radius and mass using a simulation
- **Lesson 6.3** The effects of electrostatic charge on various materials
- **Lesson 7.3** Strength of a magnet at various distances
- **Lesson 7.4** Strength of a magnetic field at various distances from a current-carrying wire
- **Lesson 7.8** Force acting on a conductor in a magnetic field
- **Lesson 8.3** Induction of an electric current using a magnet and coil
- **Lesson 8.6** Transformers: induced EMF from an AC generator

Unit 4 Practical

- **Lesson 11.5** Investigating the photoelectric effect

Glossary

A

absolute error
the difference between the observed (measured) value and the accepted (true) value

absolute uncertainty
the dispersion (spread) of values around the mean, $\delta\bar{x}$, reported by half the range of the measurements; an indicator of the precision of measurements

accuracy
the difference between the observed (measured) value and a currently accepted value of a quantity

angular momentum
for circular motion, the momentum of a particle equal to mvr (symbol: L; unit: $\mathrm{kg\,m^2\,s^{-1}}$)

annihilation
the process that occurs when a subatomic particle collides with its respective antiparticle to produce other particles such as photons

anode
the electrode that typically attracts electrons

anomaly
a false data point often the result of a faulty observation or wrong equipment

antibaryon
composite subatomic particle that contains three antiquarks held together by the strong nuclear force

antimatter
matter that is composed of the antiparticles of ordinary matter particles

antiparticle of matter
a particle that has the same mass but opposite charge to its corresponding matter particle; for example, a positron and an electron

antiproton
the antiparticle of the proton, with an electric charge of $-1e$. It is relatively stable, but it is typically short-lived because any collision with a proton causes both particles to be annihilated in a burst of energy.

antiquark
a particle with the same mass and opposite charge to a corresponding quark; for example, a strange quark and an antistrange quark are said to be antiparticles

applied force
force applied to an object by a person or another object (symbol: F_A; SI unit: newton; unit symbol: N)

average speed
the rate of change of distance calculated by the formula average speed $= \frac{\text{distance}}{\text{time}}$; a scalar quantity (symbol: v; SI unit: metres per second; unit symbol: $\mathrm{m\,s^{-1}}$)

B

baryon
composite subatomic particle that contains three quarks held together by the strong nuclear force

baryon number
a quantum number of a system defined by $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks; it is strictly conserved and additive

black body
an object that absorbs all radiation falling on it, at all wavelengths; it is a perfect absorber or emitter of radiation

black hole
a region of space where the gravitational field is so intense that no matter or almost no radiation, except a type called Hawking radiation, can escape

black-body radiation
the radiation emitted by a black body from the conversion of thermal energy, and which has a characteristic frequency distribution that depends on the temperature

Bohr model
model of the atom in which electrons orbit the nucleus in particular circular orbits called "orbitals" or energy levels, with fixed angular momentum and energy. Their distance from the nucleus (their radius or orbit) is proportional to their energy. When an electron moves between these orbitals, it is accompanied by the emission or absorption of a photon.

C

cathode
the electrode from which electrons are typically emitted

centripetal acceleration
the acceleration experienced by any object moving in a circular path directed towards the centre of motion (symbol: a_c; SI unit: metres per second squared; unit symbol: $\mathrm{m\,s^{-2}}$)

centripetal force
the force acting on an object travelling in a circle that constantly either pulls or pushes the object towards the centre of motion (symbol: F_c; SI unit: newton; unit symbol: N)

charge-reversal symmetry
the idea that interactions are not affected if all charges are swapped (i.e. positive for negative and vice versa)

coherent
waves with the same frequency and amplitude, and a constant phase relationship. Conventional light sources such as candles and incandescent bulbs are incoherent sources; laser beams are coherent

combine
(vectors) add two or more vectors to determine the resultant vector

complementary angles
angles that add to 90°; for example, 60° is the complementary angle (complement) of 30° and 75° is the complement of 15°

components
(of a vector) the part of a vector that acts in a given direction

conservation of baryon number
the total baryon number of particles before an interaction is equal to the total baryon number of particles after the interaction

conservation of lepton number
the total lepton number of particles before an interaction is equal to the total lepton number of particles after the interaction

conservation of momentum
for a collision occurring between two objects in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision

Coulomb's constant, *k*
a constant of proportionality relating the force between charged objects to the magnitude of their charge and separation distance; $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$

Coulomb's law
states that like electric charges repel and opposite electric charges attract, with a force proportional to the product of the electric charges and inversely proportional to the square of the distance between them, expressed by the formula $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$

D

de Broglie wavelength
a concept in quantum mechanics that describes the wavelength, λ, of a particle, such as an electron, as it behaves like a wave. It is given by the equation $\lambda = \frac{h}{p}$, where h is Planck's constant and p is its momentum, given by $p = mv$

directly proportional
relationship that is linear and the line goes through the origin (0, 0)

discrete
individually separate and distinct

E

electric field
the region of space near an electrically charged particle or object within which a force is exerted on other electrically charged particles or objects

electric field strength
the intensity of an electric field at a particular location (symbol: E; SI unit: newtons per coulomb; unit symbol: $\mathrm{N\,C^{-1}}$); often expressed as volts per metre, $\mathrm{V\,m^{-1}}$

electrical potential
the electrical potential energy stored per unit charge at any given point (symbol: V; SI unit: volt; unit symbol: V)

electrical potential difference
the work done in moving a unit charge between the final and the initial positions in an electric field (symbol: ΔV; SI unit: volt; unit symbol: V)

electrical potential energy
the capacity of electric charge carriers to do work due to their position in an electric field (symbol: U; SI unit: joule; unit symbol: J)

electromagnetic force
the second strongest of the four fundamental forces. The electromagnetic force is mediated by photons.

electromagnetic induction
the production of an electromotive force (EMF) or voltage as a conductor moves within a magnetic field

electromagnetic radiation
radiant energy consisting of synchronised oscillations of electric and magnetic fields – electromagnetic waves – propagated at the speed of light in a vacuum

electromagnetic wave
waves produced by an oscillating electric charge that radiate out at the speed of light as mutually perpendicular electric and magnetic fields

electromotive force (EMF)
a difference in potential that can give rise to an electric current. It is often abbreviated to EMF and is measured in volts (V)

electron volt (eV)
a unit of energy equal to the work done on in accelerating an electron through an electrical potential difference of 1 V. $1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$ (This value is in your QCAA Formula and data book)

elementary particle
a particle with no substructure, and thus not composed of other particles

elevation angle
the angle at which a projectile is launched with respect to the horizontal

event
an act or action with a distinct beginning and end, e.g. the swing of a pendulum, ticking of a watch, motion of an object (including a light pulse) from one place to another, decay of a nucleus, propagation of an energy wave (light, sound, water)

extrapolation
the prediction of values beyond the range of data points by extending the trendline

F

Faraday's law
a law stating that when the magnetic flux linking a circuit changes, an electromotive force (EMF) is induced in the circuit proportional to the rate of change of the flux linkage

Feynman diagram
a graphical representation of particle interactions, with time along the horizontal axis and space along the vertical axis; also called a particle interaction diagram

first law of planetary motion (law of orbits)
all planets move about the Sun in elliptical orbits, having the Sun as one of their foci

force
a push or pull between objects, which may cause one or all objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape

frame of reference
an arbitrary set of axes with reference to which the position or motion of something is described or physical laws are formulated

free-fall acceleration
the acceleration of a body falling freely in a vacuum near the surface of an astronomical body in the local gravitational field

friction
the resistance to motion of a surface moving relative to another

fundamental force
a force that acts between bodies of matter and is mediated by one or more particles. The fundamental forces are, from strongest to weakest, the strong nuclear force, electromagnetic force, weak nuclear force and gravitational force.

G

gauge boson
mediating particle that governs particle interaction and the mediation of three of the four fundamental forces. There are four gauge bosons in the Standard Model.

generation
a division of the elementary particles according to the Standard Model. There are three generations, or families, of elementary particles grouped according to mass and charge and when they were discovered.

gluon
fundamental exchange particle that operates between quarks and hence underlies the strong nuclear force between nucleons (protons and neutrons) in a nucleus

gradient
the slope of a graph

gravitational field direction
in the direction of the net gravitational force

gravitational field strength
the net force, F_g, per unit mass, m, at a particular point in the gravitational field (symbol, g)

gravitational field
the region of space surrounding a body in which another body experiences a force of gravitational attraction

gravity
the force of attraction between objects with mass

greenhouse effect
the process by which certain gases in the atmosphere, known as greenhouse gases, warm Earth by absorbing the long-wavelength radiation from Earth's surface

H

hadron
particle composed of quarks and held together by gluons; for example, mesons and baryons

Higgs boson
an elementary particle in the Standard Model that represents the Higgs field – a field that pervades the universe and is responsible for giving certain elementary particles mass

horizontal component of velocity
the resolution of a projectile's velocity in the horizontal direction

I

impact speed
the speed at which an object hits a surface; the final speed

impact velocity
the velocity of a projectile as it strikes the ground; the magnitude of impact velocity is impact speed

inclined plane
a flat surface raised at one end, used as an aid for raising or lowering a load

inertia
the tendency of an object to resist change in its motion

inertial frame of reference
a non-accelerating frame of reference in which Newton's laws of motion hold

interpolation
the prediction of values between data points using a trendline

L

launch velocity
the velocity at which a projectile is launched; a vector quantity that has both magnitude and direction, usually specified as an angle of elevation to the horizontal

law of conservation of energy
the total energy of a system remains constant; energy can neither be created nor destroyed, but only changed from one form to another or transferred from one object to another

length contraction
the shorter measurement made by an observer moving relative to the object in the direction of the length being measured

Lenz's law
the direction of an induced electric current is such that it produces a magnetic field that opposes the change in the circuit or the magnetic field that produces it

lepton
a class of elementary particles that respond only to the weak force and the gravitational force. They can carry one unit of electric charge or are neutral, and those that are charged experience the electromagnetic force. There are six leptons in the Standard Model.

lepton number
a conserved quantum number representing the difference between the number of leptons and the number of antileptons in an elementary particle reaction: $L = n_l - n_{\bar{l}}$

light-year (ly)
the distance travelled by light in one year (9.5×10^{15} m)

linearising
the process of transforming non-linear data by applying a mathematical function to one of the variables so that the relationship between the variables becomes closer to a straight line. Each type of relationship has a different method for linearising. If the linearised graph is indeed linear, then we can say that the relationship has been correctly identified.

loop
a circuit, often a conducting wire, where you start and end at the same point; can be "open" or "closed"

M

magnetic field
a region of space where a magnetic force is experienced by a moving charge, i.e. where magnetism is mediated

magnetic field line
the direction an isolated north pole would move in the field, or the direction a compass needle would point

magnetic flux
a measurement of the total magnetic field that passes through a given area; a measure of the number of magnetic field lines passing through the given area (symbol: ϕ; SI unit: weber; unit symbol: Wb)

magnetic flux density
the strength of a magnetic field or the number of magnetic field lines per unit area (symbol: B; SI unit: weber per square metre; unit symbol: Wb m^{-2} or T)

magnetic force
the force exerted by a magnetic field on other magnetic materials

mass
an object's resistance to a change in motion; also commonly stated as the amount of matter in an object (symbol: m; SI unit: kilogram; unit symbol: kg)

mass–energy equivalence relationship
relates change in mass to change in energy, given by $\Delta E = \Delta mc^2$

matter
a physical substance that has mass and takes up space by having volume, especially as distinct from energy

maximum trendline
a line of best fit of maximum gradient within the bounds of the error bars

mean
the average of multiple values

mean lifetime
the average time before decay of an elementary particle as measured by an observer at rest to the particle; also known as rest life

mediating particle
a descriptive name for the gauge bosons, which govern the interaction of the four fundamental forces. Also known as carrier particles or exchange particles; expressed, for example, as "gluons are the mediating particle for the force between quarks".

meson
composite subatomic particle composed of one quark and one antiquark, held together by the strong nuclear force

minimum trendline
a line of best fit of minimum gradient within the bounds of the error bars

muon
an elementary particle similar to the electron but with a greater mass; it is the product of the decay of pions

muon neutrino
an almost massless and neutral elementary particle produced in radioactive decay of pions

mutual induction
the production of an EMF in a circuit by a change in the current in an adjacent circuit that is linked to the first by the flux lines of a magnetic field

N

Newton's law of universal gravitation
the force of attraction between each pair of point particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

normal force
the force acting along an imaginary line drawn perpendicular to the surface

O

orbital
region of space around the nucleus of an atom where an electron is likely to be found

outlier
a value data point that is much smaller or larger than most of the other values in a set of data; often quantified as being greater than three standard deviations from the mean

P

parabola
a graph of a quadratic function for which the power a is 2, e.g. $y = x^2$

paradox
a self-contradictory conclusion that comes from true premises (ideas or statements)

percentage error
the absolute error expressed as a percentage of the accepted (true) value

percentage uncertainty
an indicator of uncertainty in which the range of values for a measurement result (the absolute uncertainty) is expressed as a percentage of the observed measurement

period
the time taken to complete one revolution calculated by the formula $\text{period} = \frac{\text{time}}{\text{no. of revolutions}}$ (symbol: T; SI unit: second; unit symbol: s)

photocurrent
an electric current produced by the photoelectric effect

photoelectric effect
the emission of electrons (or other free carriers) when light shines on a material

photoelectron
electron emitted from an atom by interaction with a photon, especially from a solid surface by the action of light

photon
a quantum of all forms of electromagnetic radiation

physics
the systematic study of energy and matter and how they interact with each other to explain observed phenomena

pion
a subatomic particle produced in the atmosphere as a result of the collision of cosmic ray protons with nitrogen and oxygen atoms; can be charged, π^+ or π^-, or uncharged, π^0

Planck's constant
a fundamental constant used in quantum mechanics that relates frequency to energy (symbol: h; SI unit: joule second; unit symbol: Js), equal to 6.626×10^{-34} Js. Also known as the Planck constant.

Planck's equation
a relationship that relates frequency to energy: $E = hf$. It accounts for the quantised nature of light and plays a key role in understanding phenomena such as the photoelectric effect and Planck's law of black-body radiation.

point charge
a representation of a charged object as a single point in which all charge is concentrated rather than having dimensions in space; an idealised situation where the size of the point is considered negligible

postulates of special relativity
the two assumptions of the special theory of relativity: 1, that the laws of physics are the same in all inertial frames of reference; 2, that the speed of light in a vacuum has the same value, c, in all inertial frames of reference

precision
the range of values found around the observed (measured) value; how close two or more measurements of the same object or phenomenon are to each other (i.e. the "uncertainty" of the measurement); the higher the uncertainty the lower the precision

principal quantum number
a discrete variable assigned to each electron in an atom to describe the energy level of the electron, with higher numbers representing higher potential energy for electrons further from the nucleus (symbol: n)

proper length
the length as measured by an observer at rest with respect to the object

proper time interval
the time between two events measured by an observer at rest relative to the events

Q

quanta
the smallest discrete packets of energy of electromagnetic waves, also later known as photons; singular "quantum"

quantised
formed into quanta with certain discrete energies

quark
subatomic particles that make up hadrons and are governed by the strong nuclear force; there are six quarks in the Standard Model

R

range
the horizontal displacement of a projectile from launch to impact

relative motion
the motion of an object with regard to some other moving object; the motion is not necessarily calculated with reference to Earth, but is the velocity of the object in reference to the other moving object as if it were in a static state

relativistic length
the length as measured by an observer moving with respect to the object in the direction of motion

relativistic momentum
the momentum of an object as measured by an observer moving relative to the object

relativistic time interval
the time between two events measured by an observer moving with respect to the events; also known as dilated time interval

relativity of simultaneity
events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial

resolving
determining the components of a vector, usually at right angles to each other. Also called "decomposition".

rest mass
the mass of an object when measured in the same frame of reference as the observer

resultant vector
a single vector that is a combination of two or more other vectors

rotational speed
the number of revolutions an object makes per second, as distinguished from the term "average speed", which is the linear speed

Rutherford model
a small, central, positively charged nucleus with negatively charged electrons orbiting it

S

Schwarzschild radius
a point near a black hole where the gravity is so powerful that nothing, not even light, can escape; given by the formula $r_s = \frac{GM}{c^2}$ where M is the mass of the black hole and c is the speed of light ($3 \times 10^8 \text{ m s}^{-1}$)

second law of planetary motion (law of areas)
a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time

sidereal period
the time it takes for a planet to complete one orbit of another body relative to the stars

simultaneity
the relationship between two events assumed to be happening at the same time in a particular frame of reference

solenoid
a long straight coil of wire used to generate a controlled and almost uniform magnetic field

special theory of relativity (special relativity)
a theory that all motion must be defined relative to a frame of reference; it consists of two principal postulates. It explains how space and time are linked for objects moving at constant speed in a straight line, and forms part of the basis of modern physics.

standard deviation
a measure of the amount of variation of a set of data values

Standard Model
a theory describing three of the four known fundamental forces in the universe (the strong nuclear, weak nuclear and electromagnetic forces), as well as classifying all known elementary particles

stopping potential
the negative potential on the collector at which the photoelectric current becomes zero. Symbol V_s, but sometimes shown as V_0, or V_{stop}.

strong nuclear force
the strongest of the four fundamental forces; binds quarks together to make subatomic particles such as protons and neutrons and underlies interactions between all particles containing quarks and/or antiquarks; sometimes called the strong force. The strong nuclear force between quarks is mediated by gluons.

symmetry
the concept that laws of physics do not vary under transformations in an inertial (constant velocity) frame of reference, such as in a different place or at a different time, or when symmetry operations are performed on particle interactions and the conservation laws are not violated

synchronised
waves that have the same frequency and a constant phase difference. For electromagnetic radiation there is no phase difference; that is, when one reaches a maximum so does the other.

T

tangential velocity
the linear velocity of an object undergoing circular motion, where the magnitude is the speed of the object, and the direction is a tangent to the circular path at that moment (directed towards the centre); for circular motion, the term is usually abbreviated to "velocity"

tension
the pulling force transmitted along a rope, string, cable or chain on an object

tesla
the SI unit of magnetic field strength; $1 \text{ T} = 1 \text{ N C}^{-1} \text{ m}^{-1} \text{ s}$

third law of planetary motion (law of periods)
the square of the sidereal period of a planet is directly proportional to the cube of its mean distance from the Sun: $T^2 \propto r^3$

threshold frequency
the minimum frequency of a photon that can eject an electron from a surface

time dilation
the difference in the time interval between two events as measured by observers that are moving with respect to each other

time-reversal symmetry
an interaction can still occur if the flow of time is reversed (products become reactants, and reactants become products)

trajectory
the path taken by a projectile in flight

transformer
a device that transfers an alternating current from one circuit to another, usually with an increase (step-up transformer), or decrease (step-down transformer) in voltage

transverse wave
a wave where the direction of oscillation of particles is perpendicular to the direction of energy transfer

trendline
a line of best fit which shows the general direction that a group of points seem to follow

U

uniform circular motion
the motion of an object travelling at a constant speed in a circle

uniform electric field
a field that has constant strength, as found between charged parallel plates

uniform motion
motion of an object that is not undergoing acceleration

V

vector
a variable quantity, such as velocity, force or gravitational field strength, that has magnitude and direction

vertex
(plural vertices) the point where particles interact. At the vertex they will emit or absorb new particles, deflecting one another, or changing type.

vertical component of velocity
the resolution of a projectile's velocity in the vertical direction

W

wave model of light
uses mechanical wave characteristics such as wavelength, frequency and speed to describe the behaviour of light such as polarisation, interference and diffraction

wave–particle duality
light can exhibit either wave or particle behaviour, depending on the situation. More fully, a complete description of light behaviour at the atomic level requires the application of two different but complementary models – the wave model and the particle model – but the two are never applied at the same time to the same physical experiment.

weak nuclear force
the third strongest force of the four fundamental forces. It is responsible for radioactive decay and is mediated by W (W^+ or W^-) and Z^0 bosons.

weight
a measure of the force of gravity acting on an object (symbol: F_g; SI unit: newton; unit symbol: N)

Wien's displacement law
states that the black-body radiation curve for different temperatures peaks at a maximum wavelength (λ_{max}) that is inversely proportional to the temperature, T (SI unit: kelvin; unit symbol: K)

work function
the minimum energy required to remove an electron from a solid (symbol: W; SI unit: joule; unit symbol: J). Formula: $W = hf_0$

Y

Young's double-slit experiment
demonstrates the wave nature of light by allowing two coherent beams of light to overlap on a screen to form an interference pattern

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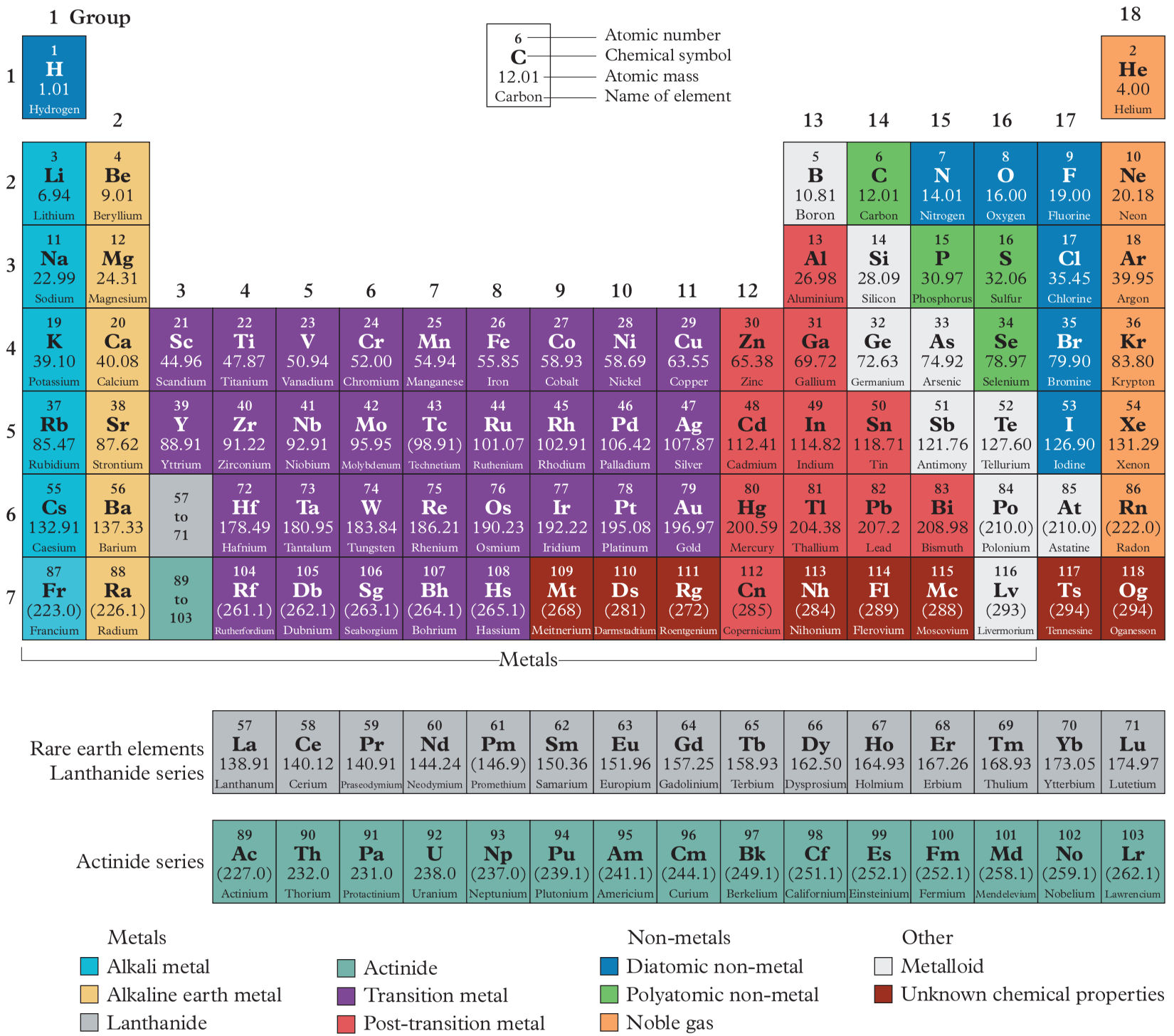

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Notes

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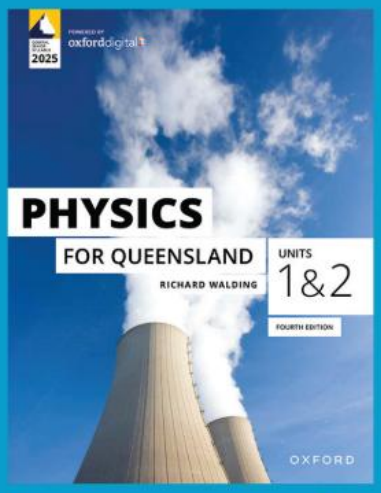

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