MATHEMATICS

# OXFORD MATHS 9

SERIES CONSULTANT:
THOMAS CHRISTIANSEN

ALEXANDER BLANKSBY
MORGAN LEVICK
EUGENE ROIZMAN
JENNIFER NOLAN
MELANIE KOETSVELD
JOE MARSIGLIO

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Indexed by Max McMaster
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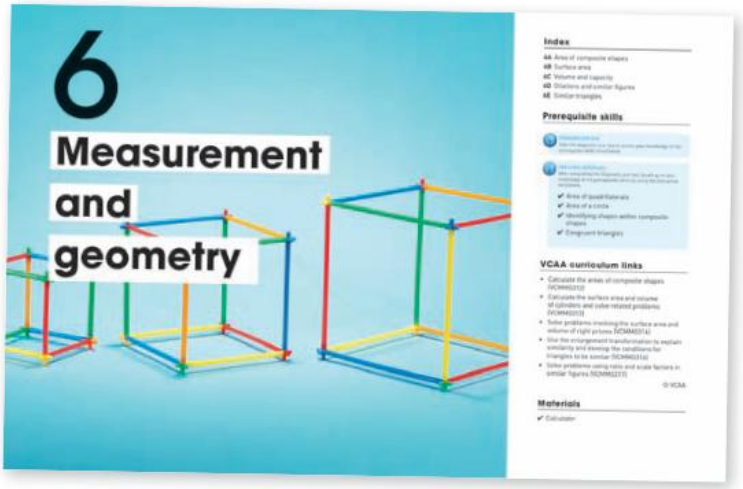
# Introducing Oxford Maths 7–10

*Oxford Maths 7-10 Victorian Curriculum* utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

## Key features of Student Books

- Complete access to all digital resources available on Student obook pro.
- Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- STEAM projects encourage inter-disciplinary thinking.
- Semester reviews provide an opportunity to revise key concepts from each semester.
- NAPLAN practice allows students to revise numeracy skills for the National Assessment Program.




**Each chapter opens with:**

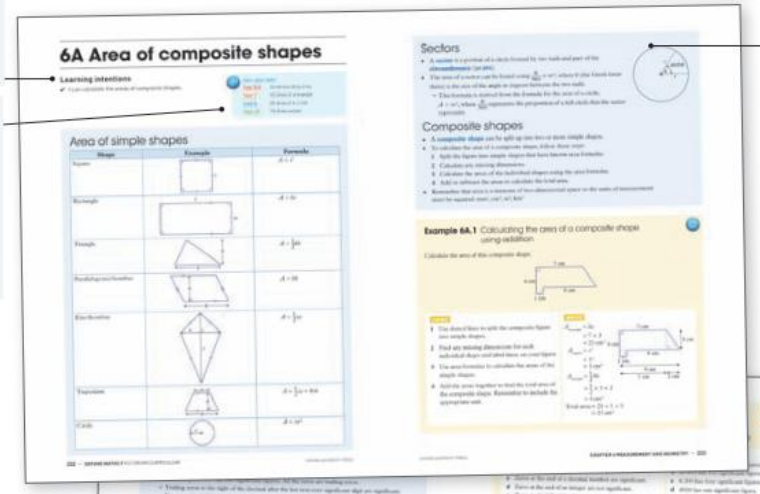
- **Prerequisite skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the VCAA mathematics syllabus.
- **Materials** used to complete the exercises.

**Learning intentions**

- Signpost the foundational skills being developed in each section.

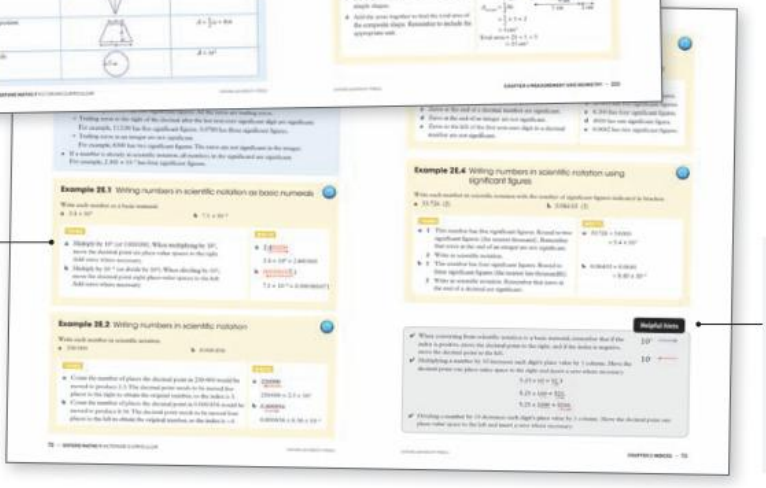
**Inter-year links**

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.




**New theory**

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life, and aid understanding.




**Worked examples**

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

**Helpful hints**

- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

### Understanding and fluency

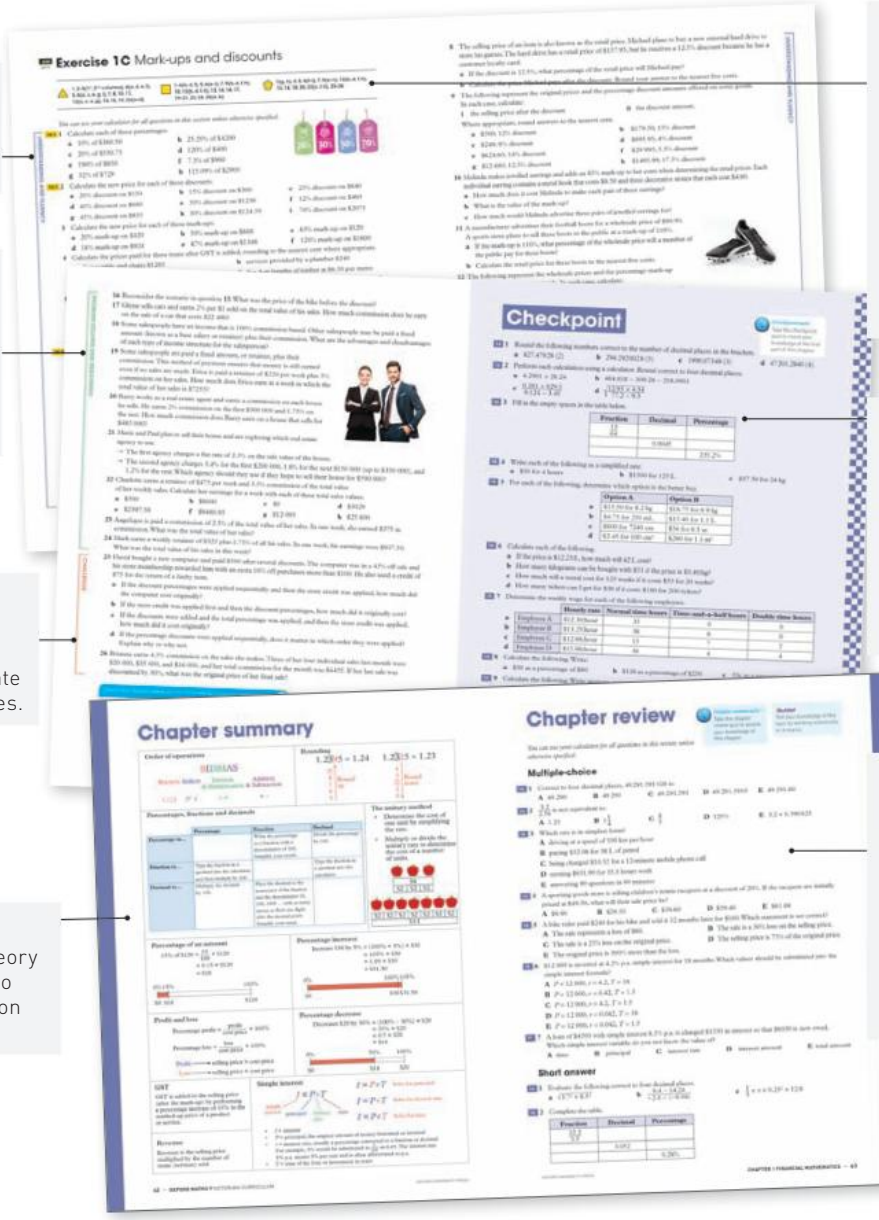
- Basic exercises dedicated to practising key concepts.

### Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

### Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.




### Differentiated learning pathways

- Each exercise is separated into three pathways, tailoring for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every chapter.

### Checkpoint

- A section in the middle of each chapter dedicated to summarising key skills and encouraging memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

### Chapter review

- Additional practice questions to further consolidate understanding at the end of each chapter.
- Reference to an online chapter review quiz to track results.
- Reference to Quizlet test to revise new terminology.

### Chapter summary

- Condenses all the theory from each section into one accessible revision page.

### Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.

### Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem-solve using design thinking.

### Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

## Key features of Student obook pro

obook pro

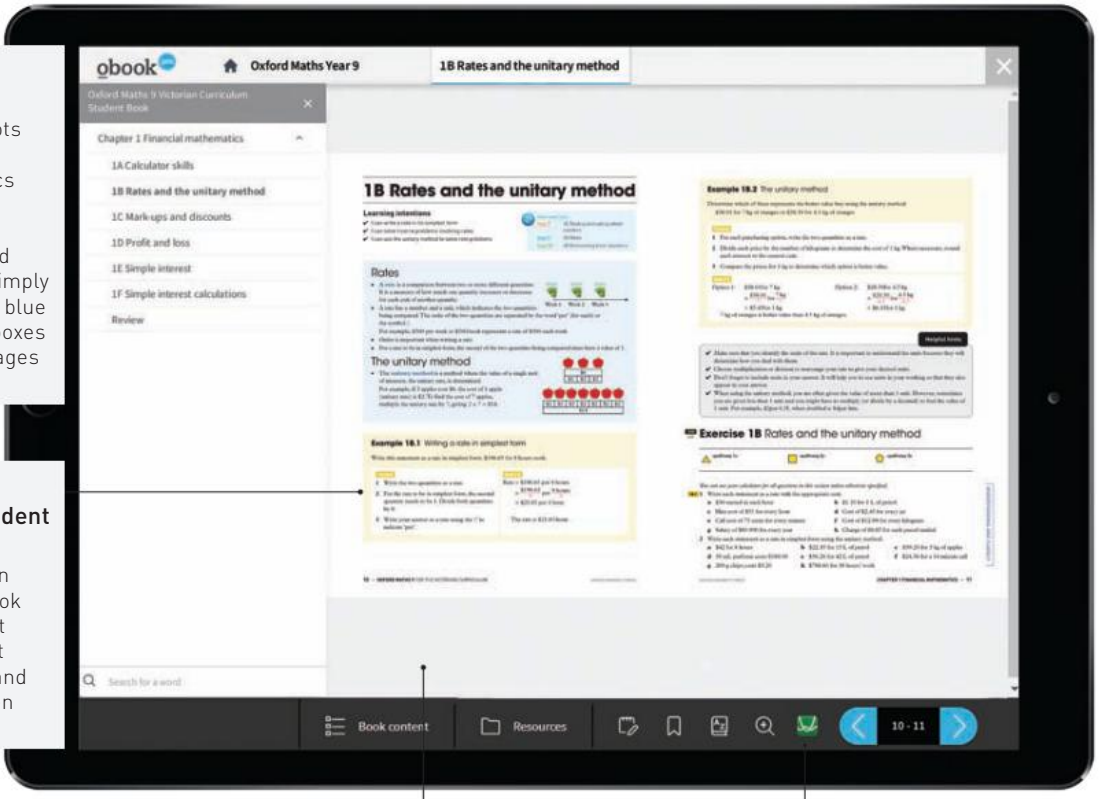
- Student obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- It offers a complete digital version of the Student Book with interactive note-taking, highlighting and bookmarking functionality, allowing students to revisit points of learning.
- A complete ePDF of the Student Book is also available for download for offline use and read-aloud functionality.

**Integrated digital resources**

- Integrated hotspots allow students to access diagnostics tests, quizzes, interactive skill sheets, videos and inter-year links simply by clicking on the blue digital resource boxes throughout the pages of the book.

**Complete digital version of the Student Book**

- The digital version of the Student Book is true to the print version, making it easy to navigate and transition between print and digital.




**Toolbar features**

- Notes can be added and saved to the text by simply selecting and highlighting.
- Bookmarks can be saved to any page.
- *Australian Concise Oxford Dictionary* can provide immediate definitions to any word within the text.

**Desmos integration**

- Our partership with Desmos allows students to access a suite of calculator tools as they read through the text, providing convenient graphical support as well as the opportunity to investigate plane geometry and Cartesian coordinates.

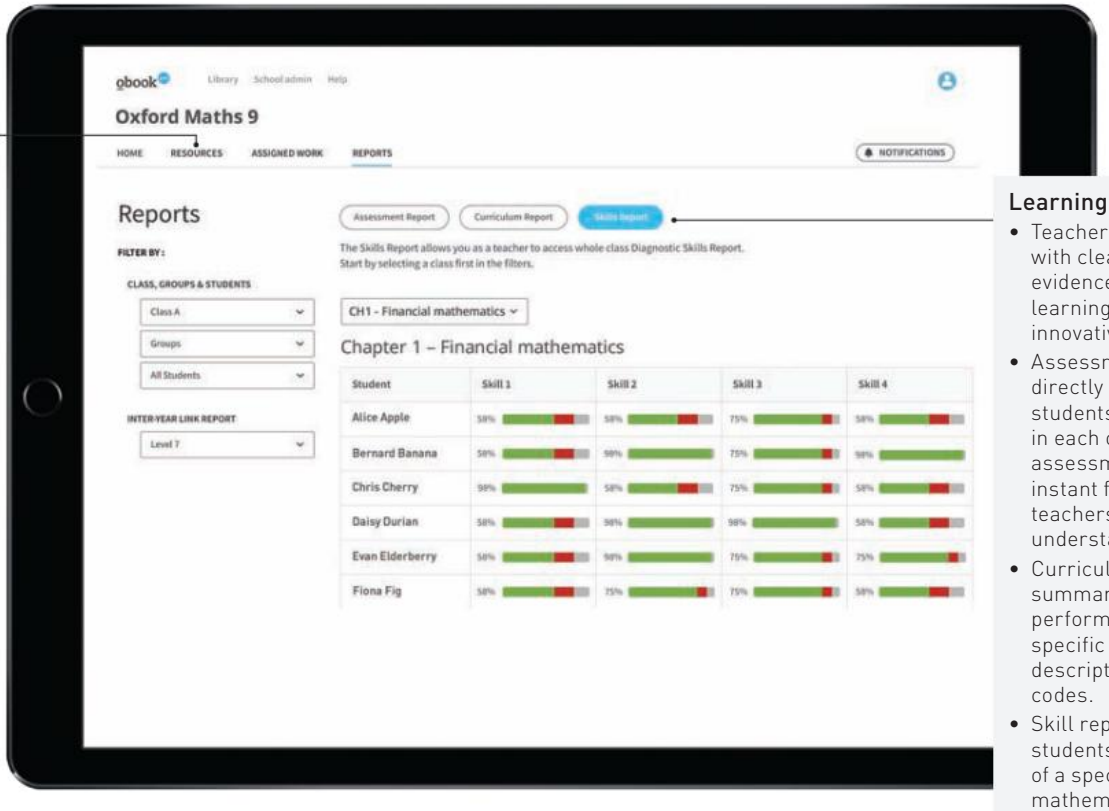
## Benefits for students

- Integrated *Australian Concise Oxford Dictionary* look-up feature
- Targeted instructional videos for every worked example question
- Groundwork resources to support assumed knowledge
- Interactive assessments to consolidate understanding
- Auto-marked practice exam question sets
- Integrated Quizlet sets, including real-time online quizzes with live leaderboards
- Access to online assessment results to track progress.

## Key features of Teacher obook pro

obook pro

- Teacher obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- Each chapter and topic of the Student Book is accompanied by full teaching support, including assessment reporting, worked solutions, chapter tests, detailed teacher notes and lesson plans.
- Teachers can use their Teacher obook pro to share notes and easily assign resources or assessments to students, including due dates and email notifications.




### Learning pathway reports

- Teachers are provided with clear and tangible evidence of student learning progress through innovative reports.
- Assessment reports directly show how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding.
- Curriculum reports summarise student performance against specific curriculum content descriptors and curriculum codes.
- Skill reports indicate the students' understanding of a specific skill in mathematics.

### Additional resources

- Each chapter of the Student Book is accompanied by additional interactive skillsheets, worksheets, investigations and topic quizzes to help students progress.

## Benefits for teachers

- Diagnostic pre-tests and chapter tests that track students' progress against Study Design key knowledge, providing detailed learning pathway reports that differentiate each student's ability in each skill
- Assign reading and assessments to students either individually, or in groups – administration is taken care of!
- Ability to set-up classes, monitor student progress and graph results
- Worked solutions for every Student Book question
- Detailed teacher notes, teaching programs and lesson plans.

# 1

# Financial mathematics

## Index


**1A** Calculator skills
**1B** Rates and the unitary method
**1C** Mark-ups and discounts
**1D** Profit and loss
**1E** Simple interest
**1F** Simple interest calculations


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Equivalent fractions
- ✔ Multiplying and dividing decimals
- ✔ Rounding decimals
- ✔ Calculating percentages

## Curriculum links

- Solve problems involving simple interest (VCMNA304)

© VCAA

## Materials

✔ Calculator

# 1A Calculator skills

## Learning intentions

- ✔ I can round decimals from calculator solutions to the appropriate decimal place.
- ✔ I can add, subtract, multiply and divide decimals using a calculator.
- ✔ I can convert between fractions, decimals and percentages using a calculator.

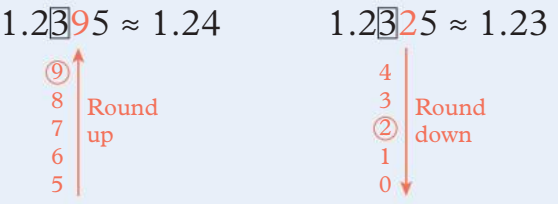
**Inter-year links**

| | |
|---|---|
| Year 5/6 | Place value |
| Year 7 | 1A Place value |
| Year 8 | 1A Rounding and estimating |
| Year 10 | 1A Calculating percentages |

## Rounding decimals

- A decimal number can be rounded to a given number of decimal places by considering the digit to the right of the specified place value.
  - → If this digit is 5 or more, then round up.
  - → If this digit is less than 5, then round down.

  For example, both numbers below have been rounded to two decimal places (hundredths).




- Amounts of money in dollars and cents are rounded to two decimal places.
- The value of each digit depends on the place or position of the digit in the number.
  For example, the decimal number 12 345.6789 is shown in the place value table.

| Ten thousands 10 000 | Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | . | 6 | 7 | 8 | 9 |

## Calculator skills

- Fractions can be typed into a calculator using the division key, ÷.
  For example, $\frac{1}{2}$ can be typed into a calculator as 1 ÷ 2.

BIDMAS

Brackets Indices Division & Multiplication Addition & Subtraction

- If a calculator does not have a button for indices, remember that indices are just repeated multiplication.
  For example, $2.4^3 = 2.4 \times 2.4 \times 2.4$.
- Most calculators are limited in their ability to maintain the correct order of operations when calculations are entered. Sometimes, parts of a calculation need to be entered separately before combining to find the result. In other cases, brackets can be typed into the calculator to ensure the correct order of operations if a calculator does not do this automatically. Remember **BIDMAS**.
- The rules for operations with decimals are also applied to calculations involving money.

# Converting between percentages, fractions and decimals using a calculator

| | Percentage | Fraction | Decimal |
|---|---|---|---|
| **Percentage to…** | | Write the percentage as a fraction with a denominator of 100. Simplify your result. | Divide the percentage by 100. |
| **Fraction to…** | Type the fraction as a quotient into the calculator, and then multiply by 100. | | Type the fraction as a quotient into the calculator. |
| **Decimal to…** | Multiply the decimal by 100. | Place the decimal as the numerator of the fraction and the denominator 10, 100, 1000 … with as many zeroes as there are digits after the decimal point. Simplify your result. | |

## **Example 1A.1** Rounding decimals

Round each number to two decimal places (the nearest hundredth).

**a** 5.7323 **b** −12.099 76

**THINK**

**a** **1** Draw a box around the second decimal place.

**2** Look at the digit to the right of the box. The digit to the right is a 2, which is less than 5. Do not change the digit in the box.

**3** Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.

**b** **1** Draw a box around the second decimal place.

**2** Look at the digit to the right of the box. The digit to the right is a 9, which is greater than 5. Add 1 to the digit in the box.

**3** As the boxed digit changes from 9 to 10, write zero in the box and then add one to the place to the left.

**4** Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same, except for 0, which changes to 1.

**WRITE**

**a** $5.7\boxed{3}23$

$5.7\boxed{3}23$

$5.7323 \approx 5.73$

**b** $-12.0\boxed{9}976$

$-12.0\boxed{9}976$

$-12.1\boxed{0}$

$-12.099\,76 \approx -12.10$

## Example 1A.2 Decimal calculator skills

Using a calculator, determine the result of each of the following. Round to the nearest thousandth.

**a** $\dfrac{142.56 - 23.34}{75.6 \times 4.59}$

**b** $23.2 \times 11.8 - (12.77^2 \times 3.9)$

**THINK**

**a** 1 Recall your knowledge of BIDMAS. Calculate the numerator and the denominator of the fraction separately using the calculator.

2 Divide the numerator by the denominator using the ÷ key. If your calculator has a memory function, select the results to use in your division calculation.

3 Round your answer to the nearest thousandth (three decimal places).

**b** 1 Recall your knowledge of BIDMAS. Determine the value inside the brackets first. If your calculator doesn't have a button for indices, remember that $12.77^2 = 12.77 \times 12.77$.

2 Now calculate the multiplication to the left of the subtraction sign.

3 Subtract your two results. If your calculator has a memory function, select the results to use in your subtraction calculation.

4 Round your answer to the nearest thousandth (three decimal places).

**WRITE**

**a** $142.56 - 23.34 = 119.22$
$75.6 \times 4.59 = 347.004$

$119.22 \div 347.004 = 0.3435695...$

$\approx 0.344$

**b** $12.77 \times 12.77 \times 3.9 = 635.98431...$

$23.2 \times 11.8 = 273.76$

$273.76 - 635.98431... = -362.22431...$

$\approx -362.224$

## Example 1A.3 Converting between fractions, decimals and percentages

Using a calculator, fill in the blanks of the following table.

| | Percentage | Fraction | Decimal |
|---|---|---|---|
| **a** | 40% | | |
| **b** | | $\frac{43}{50}$ | |
| **c** | | | 2.14 |

**THINK**

**a** 1 To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100. Simplify the fraction. Use your calculator to help you divide by common factors.

2 To convert a percentage to a decimal, divide the percentage by 100.

**b** **1** To convert a fraction to a percentage, type the fraction as a quotient into the calculator, and then multiply by 100.

**2** To convert a fraction to a decimal, type the fraction as a quotient into the calculator.

**c** **1** To convert a decimal to a percentage, multiply the decimal by 100.

**2** To convert a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000 … with as many zeroes as there are digits after the decimal point in the decimal fraction part of the number. Simplify the fraction.

**WRITE**

| | Percentage | Fraction | Decimal |
|---|---|---|---|
| **a** | 40% | $\frac{\cancel{40}^{2}}{\cancel{100}^{5}} = \frac{2}{5}$ | $40 \div 100 = 0.4$ |
| **b** | $(43 \div 50) \times 100 = 86\%$ | $\frac{43}{50}$ | $43 \div 50 = 0.86$ |
| **c** | $2.14 \times 100 = 214\%$ | $2\frac{\cancel{14}^{7}}{\cancel{100}^{50}} = 2\frac{7}{50}$ | 2.14 |

**Helpful hints**

- ✔ Make sure that you round to the appropriate number of decimal places, based on the real-life context. For example, if a chip packet costs $1 and you have $1.50, you can't buy 1.5 chip packets; you can only buy 1 packet! Also, don't forget that prices are usually rounded to the nearest 5 cents.
- ✔ Zeros between non-zero digits in a decimal (called placeholder zeros) must never be left out, or the value of the number will be changed.
- ✔ Zeros at the end of a decimal (called trailing zeros) do not change the value of the number.
- ✔ Don't forget to simplify your fractions.

ANS p474

# Exercise 1A Calculator skills

△ 1–2(1st, 2nd columns), 3(1st column), 4–12, 14 ■ 1–2(2nd, 3rd columns), 3–4(1st column), 5–7(d–f), 8, 9, 11, 13, 15–17, 18(a) ⬟ 1(g–i), 2(j–l), 4(g–i), 5–7(e, f), 8(c, d), 9, 12, 13, 15–19

*You can use your calculator for all questions in this section unless otherwise specified.*

1A.1 **1** Round each number correct to two decimal places (the nearest cent).

**a** $5.2134 **b** $127.529 **c** −$6.008

**d** $0.7649 **e** −$19.999 **f** $8.004

**g** −$5000.0005 **h** $39999.9999 **i** $624.7503

**2** Round each amount correct to the nearest five cents.

**a** $24.39 **b** $36.11 **c** $28.03

**d** $44.88 **e** $22.32 **f** $55.60

**g** $35.74 **h** $99.98 **i** $0.36

**j** $4.82 **k** $105.27 **l** $33.33

**3** Perform each calculation using a calculator.

**a** $37.84 + $156.32 **b** $352.36 – $87.84

**c** $523.68 + $364.62 + $92.65 **d** $17.80 × 8

**e** $110.40 ÷ 6 **f** $28.55 × 24

**g** 35 × $126.85 **h** $28.75 × 37.5

**i** $987.55 × 142.5 **j** $51.52 ÷ 11.2

1A.2 **4** Using a calculator, determine the result of each of the following. Round correct to the nearest thousandth.

**a** $12.7 \times 20.4 + \frac{1}{2} \times 9.8 \times 20.4^2$ **b** $432.7 \times (1 + 0.032)^8$

**c** $\pi \times 5.28^2 \times 10.1$ **d** $\dfrac{12.9 - (-13.8)}{-2.87 - 5.4}$

**e** $35 \times 0.64^4 \times (1 - 0.64)^3$ **f** $12.48 - 1.96 \times \dfrac{3.01}{\sqrt{16}}$

**g** $\sqrt{(7.3 - 9.4)^2 + (3.4 - (-5.2))^2}$ **h** $0.4 + 1.96 \times \sqrt{\dfrac{0.4 \times (1 - 0.4)}{25}}$

**i** $\sqrt{8.48(8.48 - 4.47)(8.48 - 6.08)(8.48 - 6.4)}$

1A.3 **5** Using a calculator, write each percentage as a fraction in its simplest form and as a decimal.

**a** 48% **b** 100% **c** 2.5%

**d** 1258% **e** 487.95% **f** 0.052%

**6** Using a calculator, write each fraction as a decimal, rounded to the nearest thousandth, and a percentage.

**a** $\frac{2}{7}$ **b** $\frac{13}{8}$ **c** $\frac{4}{9}$

**d** $\frac{50}{11}$ **e** $\frac{101}{99}$ **f** $\frac{82}{125}$

**7** Using a calculator, write each decimal as a fraction in its simplest form and as a percentage.

**a** 0.98 **b** 42.85 **c** 1.005

**d** 0.013 75 **e** 6.082 **f** 0.515 15

**8** Using a calculator, evaluate the following averages correct to two decimal places.

**a** $\dfrac{\$92.18 + \$20.28}{2}$ **b** $\dfrac{\$0.48 + \$2.29 + \$1.02}{3}$

**c** $\dfrac{\$0.31 + \$1.40 + \$91 + \$4.30}{4}$ **d** $\dfrac{\$101 + \$98 + \$240 + \$176 + \$64}{5}$

**9** Using a calculator, evaluate the following weighted averages correct to two decimal places.

**a** $\dfrac{4 \times \$2.30 + 9 \times \$5.25}{4 + 9}$

**b** $\dfrac{3 \times \$8.42 + 2 \times \$2.09 + 4 \times \$6.38}{3 + 2 + 4}$

**c** $\dfrac{10 \times \$0.30 + 12 \times \$0.87 + 15 \times \$0.05 + 11 \times \$0.49}{10 + 12 + 15 + 11}$

**d** $\dfrac{2 \times \$123.40 + 7 \times \$65.04 + 3 \times \$99.10 + 8 \times \$48.20 + 2 \times \$175.32}{2 + 7 + 3 + 8 + 2}$



**10** Consider the following purchase.

| | | | |
|---|---|---|---|
| chicken fillets | $5.50 | naan dippers | $2.95 |
| coconut curry | $2.75 | vegetable oil | $2.50 |
| beans | $2.90 | family pack of traditional lemonade | $4.50 |

**a** What is the least-valued note ($5, $10, $20, $50, $100) that can be used to make the purchase?

**b** How much change will be received when this note is used?

**11** Calculate the total of this purchase.

| | | | |
|---|---|---|---|
| 30 candles | $2.50 each | 2 vases | $18.50 each |
| 4 glass coasters | $7.99 each | 3 decorative pillows | $8.50 each |
| 12 artificial flowers | $3.50 each | 7 bags of lollies | $2 each |

**12** Five friends go to a cafe for lunch and decide to split their bill evenly between them. They ordered the following.

| | | | |
|---|---|---|---|
| Big Breakfast | $16.90 | Hot Chocolate | $3.40 |
| Eggs on Toast | $8.50 | Iced Coffee | $5.00 |
| Omelette | $11.90 | Caramel Thickshake | $4.50 |
| Triple Stack of Pancakes | $10.95 | Raspberry Fanta Spider | $4.80 |
| Burger and Chips | $11.95 | Fresh Juice | $5.50 |

**a** Determine the total of the bill.

**b** Determine how much each friend will pay.

**13** Zach buys a large meat-lovers pizza that costs $13.95, once a week, every week for a year. How much did Zach spend on pizza for the year?

**14** Janet hired a plumber to fix her dishwasher. The plumber charged her as follows:

| | |
|---|---|
| Call-out fee | $70 |
| Labour | $55 |
| Parts | $79.95 |

How much has the plumber charged Janet in total?

**15** When Cindy's dad makes a purchase, he will pay by card if the hundredths place is a 3, 4, 8, or 9, but will pay with cash if the hundredths place is any other digit. Explain why Cindy's dad pays in this way.

**16** Xander earns the current minimum wage of $753.80 per week. The current average weekly wage is $1304.70.

**a** Write Xander's weekly wage as a simplified fraction of the average weekly wage.

**b** Write the fraction in part **a** as a decimal and a percentage, correct to two decimal places.

**c** Yvette earns $2089.34 per week. Write Yvette's wage as a simplified fraction of the average weekly wage.

**d** Write the fraction in part **c** as a decimal and a percentage, correct to two decimal places.

**17** Share $25 624 in the ratio 8 : 3 : 11, correct to the nearest cent.

**18** Write the following as simplified fractions. Recall that the dot or dash over a number indicates that the decimal number is recurring.

For example: $0.\dot{2} = 0.222\ldots$ and $0.\overline{123} = 0.123\,123\ldots$

**a** $0.\dot{1}$ **b** $0.\overline{123}$ **c** $49.\overline{49}\%$

**d** $6.2\dot{9}\%$ **e** $14.\overline{321}$ **f** $0.00\overline{25}$



**19 a** You have the same number of each type of Australian coin and note. What is the maximum number of each you can have if your total is no more than $1000?

**b** Compared to your number of $2 coins, you have twice as many $1 coins, three times as many 50c coins, four times as many 20c coins, five times as many 10c coins, and six times as many 5c coins. What is the total maximum number of coins you have if your total is no more than $1000?

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1A

# 1B Rates and the unitary method

## Learning intentions

- ✔ I can write a rate in its simplest form.
- ✔ I can solve income problems involving rates.
- ✔ I can use the unitary method to solve rate problems.

**Inter-year links**

| | |
|---|---|
| Year 7 | 4F Dividing a decimal by a decimal |
| Year 8 | 3G Rates |

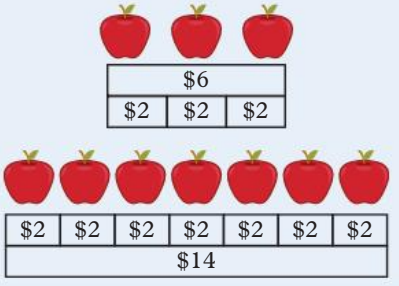
## Rates

- A **rate** is a comparison between two or more different quantities. It is a measure of how much one quantity increases or decreases for each unit of another quantity.
- A rate has a number and a unit, which indicates the two quantities being compared. The units of the two quantities are separated by the word 'per' (for each) or the symbol /.
  For example, $500 per week or $500/week represents a rate of $500 each week.

- Order is important when writing a rate.
- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.

## The unitary method

- The **unitary method** is a method where the value of a single unit of measure, the unitary rate, is determined. The unitary method can be used to determine the **best buy** for an item.
  For example, if 3 apples cost $6, the cost of 1 apple (unitary rate) is $2. To find the cost of 7 apples, multiply the unitary rate by 7, giving $2 \times 7 = \$14$.




### Example 1B.1 Writing a rate in simplest form

Write this statement as a rate in simplest form: $196.65 for 9 hours work.

**THINK**

1. Write the two quantities as a rate.
2. For the rate to be in simplest form, the second quantity needs to be 1. Divide both quantities by 9.
3. Write your answer as a rate using the '/' to indicate 'per'.

**WRITE**

Rate = $196.65 per 9 hours

$= \frac{\$196.65}{9}$ per $\frac{9 \text{ hours}}{9}$

= $21.85 per 1 hour

The rate is $21.85/hour.

## Example 1B.2 The unitary method

Determine which of these represents the best buy using the unitary method.
$38.01 for 7 kg of oranges or $28.50 for 4.5 kg of oranges

**THINK**

1 For each purchasing option, write the two quantities as a rate.

2 Divide each price by the number of kilograms to determine the cost of 1 kg. Where necessary, round each amount to the nearest cent.

3 Compare the prices for 1 kg to determine which option is better value.

**WRITE**

Option 1: $38.01 for 7 kg
$= \frac{\$38.01}{7}$ for $\frac{7 \text{ kg}}{7}$
= $5.43 for 1 kg

Option 2: $28.50 for 4.5 kg
$= \frac{\$28.50}{4.5}$ for $\frac{4.5 \text{ kg}}{4.5}$
= $6.33 for 1 kg

$38.01 for 7 kg of oranges is better value than $28.50 for 4.5 kg of oranges.

**Helpful hints**

- ✔ Make sure that you identify the units of the rate. It is important to understand the units because they will determine how you deal with them.
- ✔ Choose multiplication or division to rearrange your rate to give your desired units.
- ✔ Don't forget to include units in your answer. It will help you to use units in your working so that they also appear in your answer.
- ✔ When using the unitary method, you are often given the value of more than 1 unit. However, sometimes you are given less than 1 unit and you might have to multiply (or divide by a decimal) to find the value of 1 unit. For example, $2 per 0.5 litre when doubled is $4 per litre.

ANS p474

# Exercise 1B Rates and the unitary method

△ 1(1st column), 2–7, 8–9(a, c, e, g), 10, 12(a, b), 13, 14, 15(a–c), 18(a, b)
□ 1–2(e–h), 3–5, 6–9(b, d, f, h), 11, 12(c), 13, 14, 16(b, c, e, f), 18(d–f), 19a(ii, iv, vi), 20a(i, iii, v), 21(a, b)
⬠ 3, 6–9(b, d, f, h), 12, 15(c–e), 16(d–f), 17, 18(d–f), 19a(iv, vi, x, xii), 20a(3rd column), 20(b), 21–23

*You can use your calculator for all questions in this section unless otherwise specified.*

1B.1 **1** Write each statement as a rate with the appropriate unit.

**a** $30 earned in each hour
**b** $1.35 for 1 L of petrol
**c** Hire cost of $55 for every hour
**d** Cost of $2.45 for every jar
**e** Call cost of 75 cents for every minute
**f** Cost of $12.99 for every kilogram
**g** Salary of $60 000 for every year
**h** Charge of $6.85 for each parcel mailed

**2** Write each statement as a rate in simplest form using the unitary method.

**a** $42 for 8 hours
**b** $22.35 for 15 L of petrol
**c** $39.20 for 5 kg of apples
**d** 50 mL perfume costs $180.00
**e** $56.28 for 42 L of petrol
**f** $24.36 for a 14 minute call
**g** 200 g chips costs $3.20
**h** $768.60 for 36 hours work

**3** Write each statement as a rate in simplest form. Where necessary, round each amount to the nearest cent.

**a** $38.45 for 7 kg of oranges

**b** $156.00 for 6.5 hours work

**c** $22 collected in 60 minutes

**d** 5.5 m length of timber costs $45.50

**4** Calculate the cost of the items listed in parts **a–d**. Write your answers correct to the nearest:

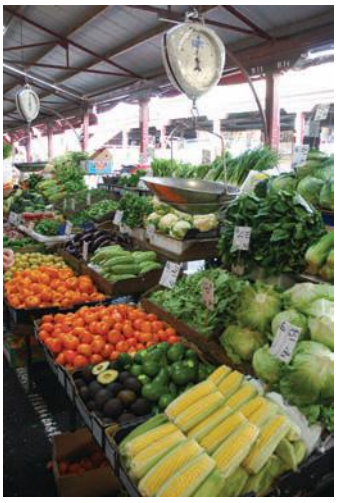
**i** cent **ii** five cents

**a** 5 kg of potatoes at $7.85 per kg

**b** 3.55 kg of apples at $4.90 per kg

**c** 0.825 kg of salad leaves at $7.20 per kg

**d** 2.6 kg of premium mince at $12.99 per kg



**5** For these calculations, round your answer to the nearest five cents.

**a** A bag of six cheese and bacon rolls is $5.94. What is the cost of one roll?

**b** 2.5 kg of pumpkin costs $11.90. What is the cost of 1 kg of pumpkin?

**c** A 24-can carton of soft drink sells for $14.88. What is the cost of one can?

**d** 300 g of shaved ham costs $7.98. What does it cost for 100 g?

1B.2 **6** Determine which of these represents the best buy using the unitary method.

| | Option A | Option B |
|---|---|---|
| **a** | $28.40 for 4 kg | $34 for 5 kg |
| **b** | $411.55 for 5 L | $812.30 for 10 L |
| **c** | $31.92 for 3.5 m | $19.10 for 2.5 m |
| **d** | $312.50 for 25 $m^2$ | $273.60 for 18 $m^2$ |
| **e** | $8.22 for 1.2 kg | $5.76 for 900 g |
| **f** | $9.99 for 1.25 L | $4.95 for 600 mL |
| **g** | $11.73 for 1235 cm | $8.65 for 9.4 m |
| **h** | $78 for 8000 $cm^2$ | $765.60 for 8 $m^2$ |

**7** The best buy can also be determined by finding the amount you can buy per dollar.

For example:

Option A: 48 pencils for $12

$= \frac{48 \text{ pencils}}{12} \text{ for } \frac{\$12}{12}$

= 4 pencils for $1

Option B: 50 pencils for $10

$= \frac{50 \text{ pencils}}{10} \text{ for } \frac{\$50}{10}$

= 5 pencils for $1

$10 for 50 pencils is better value than $12 for 48 pencils as you get more pencils for each $1 you spend.

Determine which of these represents the best buy using the unitary method per $1.

| | Option A | Option B |
|---|---|---|
| **a** | 4 kg for $2.50 | 3 kg for $1.80 |
| **b** | 50 g for $0.30 | 75 g for $0.50 |
| **c** | 250 mL for $3.20 | 750 mL for $10.20 |
| **d** | 2.5 L for $8.50 | 10 L for $35.50 |
| **e** | 6.8 m for $120 | 5.6 m for $100 |
| **f** | 4520 mm for $52 | 3780 mm for $42 |
| **g** | 49 $m^2$ for $200 | 62 $m^2$ for $250 |
| **h** | 1.98 ha for $365 000 | 12.14 ha for $3 900 000 |

**8** The cost of a plumber's work is $200 for 8 hours. Knowing this, we can determine the cost of the plumber for 1 hour of work by dividing the cost by 8. Then, we can multiply this amount to determine the cost of the plumber for any number of hours.

$$\begin{aligned}&\$200 \text{ per } 8 \text{ hours}\\ &= \frac{\$200}{8} \text{ per } \frac{8 \text{ hours}}{8}\\ &= \$25 \text{ per } 1 \text{ hour}\\ &= \$25 \times 5 \text{ per } 1 \times 5 \text{ hours}\\ &= \$125 \text{ per } 5 \text{ hours}\end{aligned}$$

**a** It costs $30 for 6 m. How much will it cost for 9 m?

**b** It costs $12 for 2 kg. How much will it cost for 93 kg?

**c** It costs $20 for 4 L. How much will it cost for 50 L?

**d** It costs $99 for 36 pieces. How much will it cost for 82 pieces?

**e** It costs $53 for 4 kg. How much will it cost for 85 kg?

**f** It costs $22 for 100 g. How much will it cost for 71 g?

**g** It costs $91 for 26 $m^2$. How much will it cost for 62 $m^2$?

**h** It costs $73 for 25 $m^3$. How much will it cost for 10 $m^3$?

**9** Sometimes it is important to stick to a budget and determine what you can afford with a fixed amount. Using the plumber from the example in question **8**, you can determine the amount of time you can afford with $120 by finding the time for $1.

$$\begin{aligned}&\$200 \text{ per } 8 \text{ hours}\\ &= \frac{\$200}{200} \text{ per } \frac{8 \text{ hours}}{200}\\ &= \$1 \text{ per } 0.04 \text{ hours}\\ &= \$1 \times 120 \text{ per } 0.04 \times 120 \text{ hours}\\ &= \$120 \text{ per } 4.8 \text{ hours}\end{aligned}$$

**a** It costs $82 for 41 m. How much can be purchased for $92?

**b** It costs $4 for 26 pieces. How much can be purchased for $36?

**c** It costs $84 for 14 L. How much can be purchased for $75?

**d** It costs $12 for 3 kg. How much can be purchased for $3?

**e** It costs $5 for 3 kg. How much can be purchased for $51?

**f** It costs $8 for 15 g. How much can be purchased for $70?

**g** It costs $16 for 29 $m^2$. How much can be purchased for $88?

**h** It costs $80 for 81 $m^3$. How much can be purchased for $44?



**10** Rafael works as a courier delivering parcels around the city. He earns $18.50 per hour.

**a** Write the information as a rate with the appropriate units.

**b** A wage is a payment made to workers based on a fixed hourly rate. Calculate Rafael's wage for a week in which he works 20 hours.

**c** In one particular week, Rafael's wage totalled $684.50. How many hours did Rafael work in this week?

**11** Lina works in research and earns a salary of $66 548. A salary is an annual amount of money that can be paid on a fortnightly or monthly basis.

**a** Write the information as a rate with the appropriate units.

**b** If Lina is paid monthly, write her monthly payment as a rate in simplest form.

**c** If Lina is paid fortnightly, write her fortnightly payment as a rate in simplest form.

**12** A person's pay before any deductions are subtracted is referred to as gross income. Examples of deductions include income tax, superannuation, union fees and payments to health benefits. The amount of pay after deductions have been subtracted is referred to as net income. Calculate the net income for each of these.

**a** Gross income of $498.95; income tax $56.80; union fees $9.45

**b** Weekly wage: 36 hours at $25.70 per hour; income tax $187.50; health fund $38.90

**c** Annual salary: $91 200 (paid monthly); monthly deductions: income tax $1807.80, superannuation $380 and health fund $61.25

**d** Weekly wage: 37.5 hours at $18.50 per hour; income tax $86.80; superannuation $20.45

**13** Workers on a wage who work beyond the normal hours may be eligible for overtime, which means they receive a higher rate of pay for the extra hours worked. Common overtime rates used are time-and-a-half (the worker is paid $1\frac{1}{2}$ times the normal hourly rate of pay) and double time (the worker is paid twice the normal hourly rate of pay).

For each of these normal hourly rates, calculate:

**i** the time-and-a-half rate **ii** the double time rate.

**a** $18 **b** $24 **c** $18.80

**d** $25.90 **e** $32.60 **f** $29.90

**14** Ryan is a bricklayer and is paid a wage of $28.90 per hour for a standard 36.5-hour week. The first 8 hours of overtime are paid at time-and-a-half and any additional hours are paid as double time.

**a** Calculate Ryan's gross income in a week in which he works 48.5 hours.

**b** Ryan's deductions for this week include income tax at $372.40, union fees $18.90 and superannuation $82.60. Calculate his net income for the week.

**15** Given the information in this table, calculate the net weekly income in each case.

| | Normal rate of pay $ | Hours worked | | | Deductions | | |
|---|---|---|---|---|---|---|---|
| | | Normal rate | Time-and-a-half | Double time | Income tax $ | Superannuation $ | Union fees $ |
| **a** | 12.40 | 20 | 0 | 0 | 19.90 | 0.00 | 0.00 |
| **b** | 25.00 | 35 | 6 | 8 | 326.00 | 35.00 | 24.50 |
| **c** | 35.60 | 28 | 5 | 0 | 255.10 | 30.00 | 0.00 |
| **d** | 19.90 | 10 | 1 | 3 | 34.90 | 0.00 | 8.75 |
| **e** | 26.80 | 36.5 | 3 | 4 | 269.90 | 78.25 | 21.80 |

**16** The price of petrol is given in cents per litre (c/L) correct to one decimal place. (Hint: prices are usually rounded to the nearest 5 cents when paying by cash.)

**a** The price of petrol today is 135.2 c/L. How much will it cost for 20 L if I pay by card?

**b** The price of petrol today is 119.9 c/L. How much will it cost for 15 L if I pay by card?

**c** The price of petrol today is 142.8 c/L. How much will it cost for 1250 mL if I pay by cash?

**d** I paid $35.28 by card for 25 L. What is the price of petrol today?

**e** The price of petrol today is 103.5 c/L. What is the maximum number of litres I can buy with $50 in cash correct to the nearest millilitre?

**f** I paid $30 by cash for 22 L. Between which two values was the price of petrol today?

**17** A friend tells you that the best buy is always the option with the lowest advertised price.

**a** Comment on the accuracy of this statement.

**b** When is the best buy the option with the lowest advertised price?

**18** A trip to the supermarket offers many opportunities to investigate purchases that represent the best buy. Determine which of these represents the better buy.

**a** a 45 g bag of crisps for $1.40 or a 175 g bag of crisps for $3.24

**b** an 800 g box of cereal for $3.00 or a 500 g box for $1.90

**c** a pre-packed 750 g bag of salted peanuts for $16.90 or peanuts sold loose for $23.95 per kg

**d** a 425 g jar of pasta sauce for $2.80 or a 680 g jar for $4.00

**e** 1.7 kg of sausages costing $8.00 or 560 g of sausages costing $3.50

**f** a 2 L bottle of fruit juice for $6.94 or a 500 mL bottle for $3.57

**19** There is a multiplicative relationship between any two non-zero values. That is, you can multiply any non-zero number by another, called the multiplier, to get any other number.

**a** Fill in the missing numbers in the boxes.

**i** $3 \times \frac{2}{3} = \square$ **ii** $3 \times \frac{4}{3} = \square$ **iii** $3 \times \frac{17}{3} = \square$ **iv** $3 \times \frac{\square}{3} = 14$

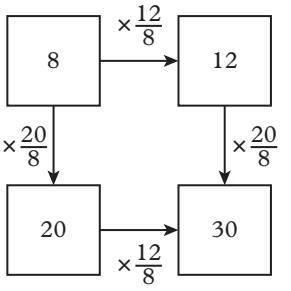
**v** $3 \times \frac{23}{\square} = 23$ **vi** $3 \times \frac{\square}{\square} = 31$ **vii** $7 \times \frac{\square}{\square} = 25$ **viii** $34 \times \frac{\square}{\square} = 5$

**ix** $8 \times \frac{\square}{\square} = 0$ **x** $0.8 \times \frac{\square}{\square} = 1.4$ **xi** $-4 \times \frac{\square}{\square} = 6$ **xii** $a \times \frac{\square}{\square} = b$

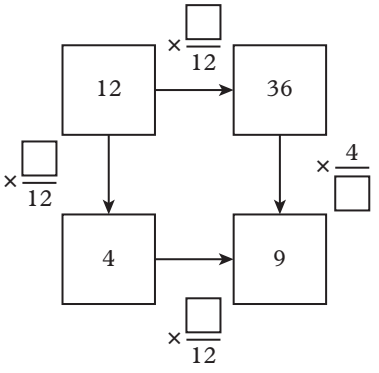
**b** Is it possible to fill in the boxes such that $0 \times \frac{\square}{\square} = 5$? If not, why not?

**20** We can show the multiplicative relationship between values using a proportion grid like the one shown. There is a common multiplier between two columns and another common multiplier between two rows.

**a** Fill in the missing numbers.




**i**

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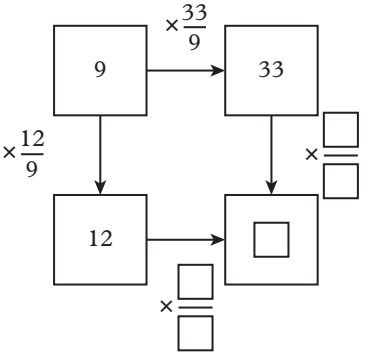


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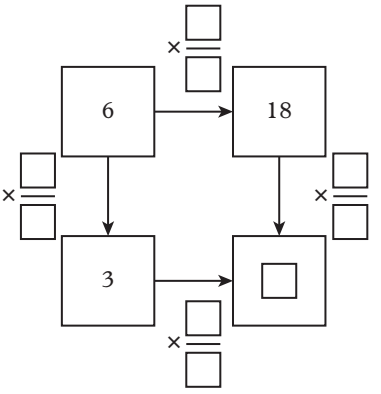


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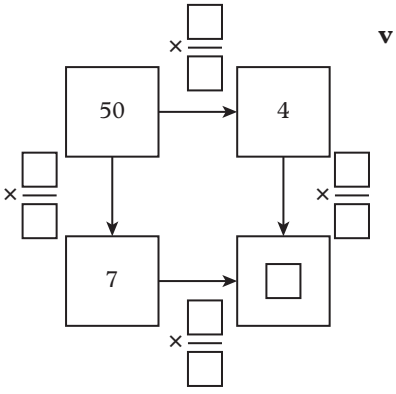


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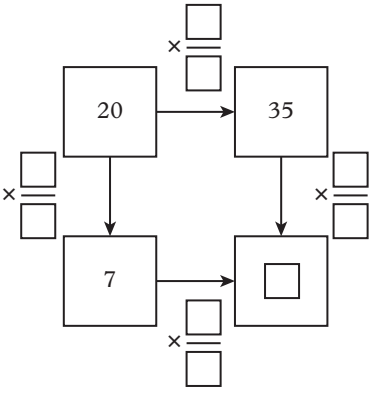


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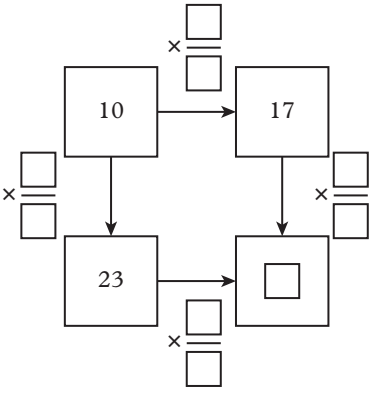


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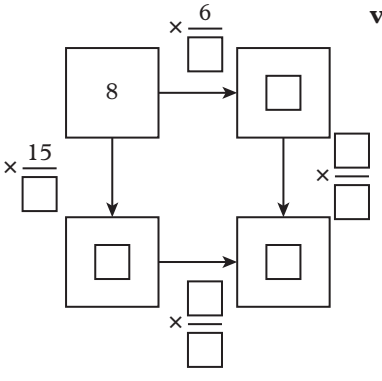
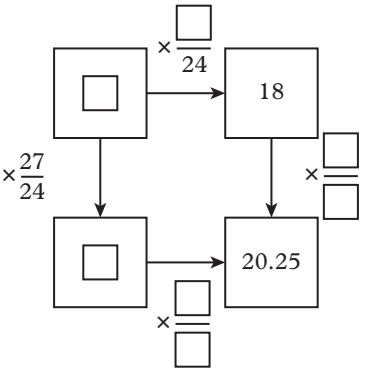
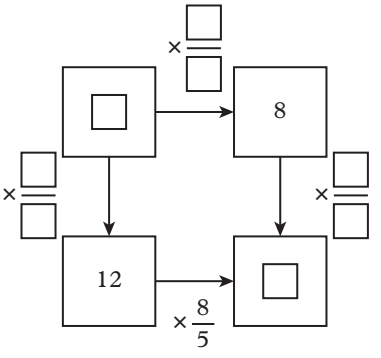



**viii**




**ix**

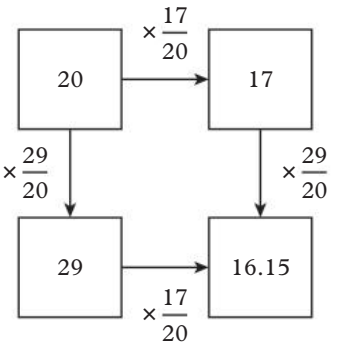


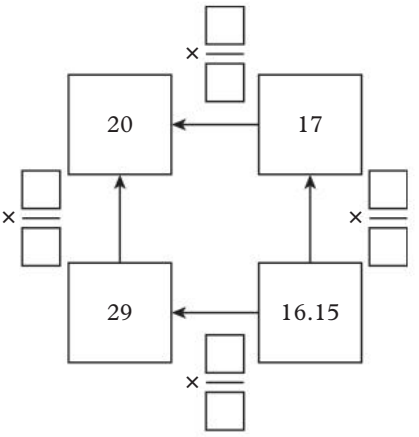
PROBLEM SOLVING AND REASONING

**b** If the directions of the arrows were reversed in this proportion grid like shown:

**i** what would the two multipliers be?

**ii** what is the relationship between the multipliers in both directions?



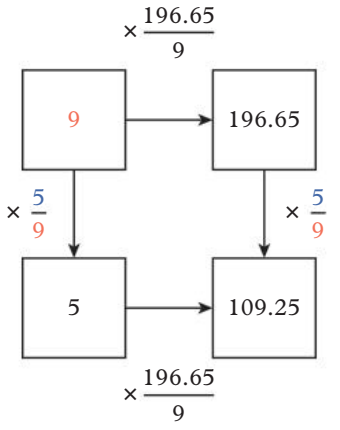

CHALLENGE

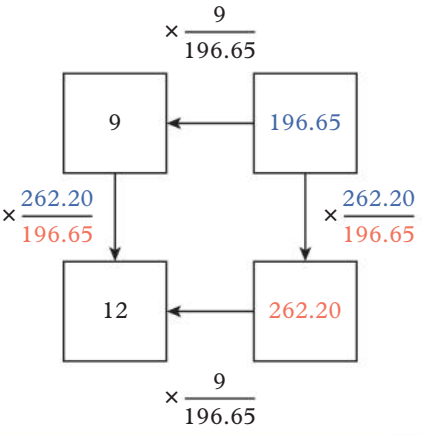
**21** The unitary method requires you to always find the unit rate before finding the value of the number you want. However, we can use a multiplier to skip finding the unit rate.

For example, if it costs \$196.65 for 9 hours work:

- then 5 hours of work would cost $\$196.65 \times \frac{5 \text{ hours}}{9 \text{ hours}} = \$109.25$
- then \$262.20 would pay for 9 hours $\times \frac{\$262.20}{\$196.65} = 12$ hours of work.

These can also be shown using proportion diagrams shown on the right.




Solve the following problems using multipliers.

**a** It costs \$30 for 22 m. How much can be purchased for \$102?

**b** It costs \$40 for 50 m. How much will it cost for 100 m?

**c** It costs \$100 for 50 m. How much will it cost for 40 m?

**d** It costs \$12 for 79 m. How much can be purchased for \$51?

**e** It costs \$8 for 109 m. How much can be purchased for \$166?

**f** It costs \$189 for 160 m. How much will it cost for 11 m?

**22** Gary's net pay for a week was \$1185.60. He had a deduction of \$341.70 for income tax and \$22.50 for union fees. He worked 30 hours at the normal rate, 8 hours at time-and-a-half and 6 hours at the double time rate. Calculate his normal hourly rate of pay.

**23** A plant grows 3.125% of its original height of 10 cm per day. After 45 days, the plant slows its growth to 1.5% of its original height per day. How tall will the plant be after 100 days?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet — Rates

Interactive skillsheet — The unitary method

Investigation — Dealing with dosages

Topic quiz — 1B

# 1C Mark-ups and discounts

## Learning intentions

- ✔ I can calculate mark-ups and discounts including GST.
- ✔ I can calculate the original amount after a mark-up or discount has been applied.
- ✔ I can solve problems involving commission.

Inter-year links

| | |
|---|---|
| Year 5/6 | Calculating percentages |
| Year 7 | 4I Calculating percentages |
| Year 8 | 3C Financial calculations |
| Year 10 | 1B Financial calculations |

## Percentage of a quantity

- To calculate a percentage of a quantity, convert the percentage to a decimal and then multiply by the quantity.

For example, $15\%$ of $\$120 = \frac{15}{100} \times \$120$
$= 0.15 \times \$120$
$= \$18$

- A **commission**, an amount earned by a salesperson, is a percentage of the total sales made by a salesperson during a period of time. Some salespeople earn a fixed amount or **retainer** plus commission, instead of a wage or a salary.
- Finding the commission value is the same as finding a percentage of the total sales.

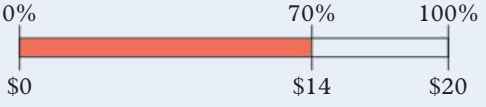
## Mark-ups, discounts and GST

- The **cost price** (wholesale price) is the cost paid to create or obtain a good or service before it is sold to the consumer.
- The **selling price** is the price that a product or service is sold at by the seller.
- A **mark-up** is the amount the cost price is increased by to give a profit and is usually expressed as a percentage.
- Finding the price after a mark-up is the same as increasing a quantity by a given percentage.
  1. Add the percentage to 100% and convert this new percentage to a decimal number.
  2. Multiply the amount to be increased by the decimal number.

Increase $\$30$ by $5\% = (100\% + 5\%) \times \$30$
$= 105\% \times \$30$
$= 1.05 \times \$30$
$= \$31.50$

- A **discount** is the amount the selling price is decreased by to sell at a lower price and is usually expressed as a percentage.
- Finding the price after a discount is the same as decreasing a quantity by a given percentage.
  1. Subtract the percentage from 100% and convert this new percentage to a decimal number.
  2. Multiply the amount to be decreased by the decimal number.

Decrease $\$20$ by $30\% = (100\% - 30\%) \times \$20$
$= 70\% \times \$20$
$= 0.7 \times \$20$
$= \$14$



- **GST** (Goods and Services Tax) was introduced by the Australian Government in 1999. Currently, the rate is 10% of the selling price of the product or service.
- GST is added to the price of a good or service by performing a percentage increase of 10% to the selling price of a product or service.

## Example 1C.1 Calculating a percentage of a quantity

Calculate 7% of $220.

**THINK**

1 Convert the percentage to a decimal.

2 Multiply by the quantity.

3 Round to the nearest cent.

**WRITE**

$$7\% \text{ of } \$220 = \frac{7}{100} \times \$220$$
$$= 0.07 \times \$220$$
$$= \$15.4$$

7% of $220 is $15.40.

## Example 1C.2 Calculating mark-ups and discounts

Calculate the selling price of:

**a** 25% mark-up on $350 **b** 30% discount on $299 **c** $418 plus GST

**THINK**

**a** 1 Mark-ups are a percentage increase. Add the percentage to 100% and convert this new percentage to a decimal number.

2 Multiply the amount to be increased by the decimal number. Round to the nearest cent.

**b** 1 This discount is a percentage decrease. Subtract the percentage from 100% and covert this new percentage to a decimal number.

2 Multiply the amount to be decreased by the decimal number. Round to the nearest cent.

**c** 1 GST is a tax applied to goods increasing the selling price by 10%. Add the percentage to 100% and convert this new percentage to a decimal number.

2 Multiply the amount to be increased by the decimal number. Round to the nearest cent.

**WRITE**

**a** Selling price $= (100\% + 25\%) \times \$350$
$= 125\% \times \$350$
$= 1.25 \times \$350$
$= \$437.50$

**b** Selling price $= (100\% - 30\%) \times \$299$
$= 70\% \times \$299$
$= 0.7 \times \$299$
$= \$209.30$

**c** Selling price $= (100\% + 10\%) \times \$418$
$= 110\% \times \$418$
$= 1.1 \times \$418$
$= \$459.80$

## Example 1C.3 Finding the original amount after a percentage increase or decrease

Find the original amount before a mark-up or a discount is applied.

**a** A sum of money increased by 60% is now $800.

**b** A sum of money decreased by 25% is now $640.

| THINK | WRITE |
|---|---|
| **a 1** To increase a sum of money by 60%, we multiply by 160%. To reverse the process, divide by 160%. | **a** Original amount = new amount ÷ (100% + percentage increase)<br>= $800 ÷ (100% + 60%)<br>= $800 ÷ 160% |
| **2** Convert 160% to a decimal number. | = $800 ÷ 1.6 |
| **3** Divide the new amount by the decimal number to calculate the original amount. | = $500 |
| **b 1** To decrease a number by 25%, we multiply by 75%. To reverse the process, divide by 75%. | **b** Original amount = new amount ÷ (100% − percentage increase)<br>= $640 ÷ (100% − 25%)<br>= $640 ÷ 75% |
| **2** Convert 75% to a decimal number. | = $640 ÷ 0.75 |
| **3** Divide the new amount by the decimal number to calculate the original amount. Round to the nearest cent. | = $853.33 |

### Helpful hints

✔ You can use fractions or decimals to find the percentage of a quantity. Both methods will give you the same answer.

For example (from Example 1C.1),

$$7\% \text{ of } \$220 = \frac{7}{\cancel{100}^{5}} \times \frac{\$\cancel{220}^{11}}{1}$$
$$= \frac{7 \times \$11}{5 \times 1}$$
$$= \frac{\$77}{5}$$
$$= \$15.40$$

✔ Can you see that writing a quantity as a percentage of a total and finding a percentage of a total are the opposite of each other?

| 20 as a percentage of 40 | 50% of 40 |
|---|---|
| $\frac{20}{40} = 50\%$ | $0.5 \times 40 = 20$ |

ANS p475

# Exercise 1C Mark-ups and discounts

▲ 1, 2–3(1st, 2nd columns), 4(a, d, e, f), 5, 6(a, c, e, g, i), 7, 8, 10, 11, 12(a, c, e, g), 14, 15, 19, 22(a–d)

■ 1–4(b, d, f), 5, 6(e–i), 7, 9(b, d, f, h), 10, 12(b, d, f, h), 13, 14, 16, 17, 19–21, 23, 24, 25(a, b)

⬟ 1(g, h), 4, 5, 6(f–i), 7, 9(e–h), 12(b, d, f, h), 13, 14, 18, 20, 22(c, f, h), 23–26

*You can use your calculator for all questions in this section unless otherwise specified.*

UNDERSTANDING AND FLUENCY

1C.1 **1** Calculate each of these percentages.

**a** 10% of $360.50
**b** 25.25% of $4200
**c** 20% of $550.75
**d** 120% of $400
**e** 190% of $850
**f** 7.3% of $960
**g** 32% of $729
**h** 115.09% of $2900

1C.1 **2** Calculate the new price for each of these discounts.

**a** 20% discount on $150
**b** 15% discount on $300
**c** 25% discount on $840
**d** 40% discount on $680
**e** 50% discount on $1238
**f** 12% discount on $460
**g** 45% discount on $855
**h** 30% discount on $124.50
**i** 70% discount on $2075

**3** Calculate the new price for each of these mark-ups.

**a** 20% mark-up on $420
**b** 50% mark-up on $668
**c** 65% mark-up on $120
**d** 18% mark-up on $924
**e** 87% mark-up on $1348
**f** 120% mark-up on $1600

**4** Calculate the prices paid for these items after GST is added, rounding to the nearest cent where appropriate.

**a** dining table and chairs $1285
**b** services provided by a plumber $240
**c** insurance purchased for a car $601.45
**d** five 3-m lengths of timber at $6.50 per metre
**e** electricity service and supply charge $314.65
**f** membership at a gymnasium at $72.95 per month

**5** For each of these, determine:

**i** the selling price
**ii** the mark-up or discount amount.

**a** A camera is purchased for $120 and sold later at a mark-up of 62%.
**b** A laptop originally marked at $1198 is offered for sale at a discount of 35%.
**c** Work tools each marked at $49.90 are offered for sale with a 15% discount.

1C.3 **6** Calculate the original price in each of these sales. Where necessary, round your answer to the nearest five cents.

**a** A mobile phone sells for $450 after a mark-up of 50%.
**b** A pair of sports shorts sells for $25 after a discount of 20%.
**c** Eyeliner sells for $11.85 following a 15% discount.
**d** A hardware store sells an electric chainsaw for $169 after it is marked up by 95%.
**e** A furniture store offers a leather lounge suite for sale for $9995 after a discount of 12.5%.
**f** Fitness equipment retails for $1499 following a 140% mark-up.
**g** The digital copy of a video game is on sale for $4.99 after a 90% discount.
**h** A jumper is on sale for $31.25 after a 37.5% discount.
**i** A painting is being sold for $1 200 000 after a 250% mark-up.

**7** The following items each include the GST charge in the price. Calculate the pre-GST price, rounding to the nearest cent where appropriate.

**a** telephone and internet services $155.65
**b** computer accessories purchased for $235.95
**c** garden maintenance provided for $182
**d** a necklace bought at a jewellery store for $120.50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**8** The selling price of an item is also known as the retail price. Michael plans to buy a new external hard drive to store his games. The hard drive has a retail price of $157.95, but he receives a 12.5% discount because he has a customer loyalty card.

**a** If the discount is 12.5%, what percentage of the retail price will Michael pay?

**b** Calculate the price Michael pays after the discount. Round your answer to the nearest five cents.

**9** The following represent the original prices and the percentage discount amounts offered on some goods. In each case, calculate:

**i** the selling price after the discount **ii** the discount amount.

Where appropriate, round answers to the nearest cent.

**a** $500; 12% discount **b** $179.50; 15% discount

**c** $249; 8% discount **d** $895.95; 4% discount

**e** $624.60; 14% discount **f** $29 995; 5.5% discount

**g** $12 680; 12.5% discount **h** $1495.99; 17.5% discount

**10** Melinda makes jewelled earrings and adds an 85% mark-up to her costs when determining the retail prices. Each individual earring contains a metal hook that costs $8.50 and three decorative stones that each cost $4.60.

**a** How much does it cost Melinda to make each pair of these earrings?

**b** What is the value of the mark-up?

**c** How much would Melinda advertise these pairs of jewelled earrings for?

**11** A manufacturer advertises their football boots for a wholesale price of $89.90. A sports store plans to sell these boots to the public at a mark-up of 110%.

**a** If the mark-up is 110%, what percentage of the wholesale price will a member of the public pay for these boots?

**b** Calculate the retail price for these boots to the nearest five cents.

**12** The following represent the wholesale prices and the percentage mark-up amounts offered on some goods. In each case, calculate:

**i** the retail price after the mark-up **ii** the mark-up amount.

Where appropriate, round answers to the nearest cent.

**a** $620; 24% mark-up **b** $89.95; 45% mark-up

**c** $1269; 80% mark-up **d** $450.50; 85.5% mark-up

**e** $6250; 140% mark-up **f** $350.99; 125% mark-up

**g** $14 625; 112.5% mark-up **h** $2295; 137.5% mark-up

**13** A girl's bicycle is reduced to $198 after a discount of 20%. To determine the original price, Jane thinks that she needs to calculate 20% of $198 and add the result to $198. Tim thinks that Jane has it wrong and that the calculation is more complex. Which person do you think is correct? Show working to support your answer.

**14** The unitary method can be used to solve percentage increase or decrease questions when the original amount is not known. Consider a television that has a retail price of $765 after a discount of 15%.

**a** A discount of 15% means you pay 85% of the original price. The unitary method requires you to find how much 1% represents (one unit). Calculate 1% of the original price.

**b** The original price of the television represents the full amount, or 100%. Use your answer to part **a** to calculate 100% of the original price. (Hint: Multiply the amount for 1% by 100.)

**c** What is the original price of the television?

**15** The method outlined in question **14** can also be applied to calculate the original amount after a mark-up has been applied. Consider a different television that retails for $1800 after an 80% mark-up. Calculate the wholesale price of the television. (Hint: A mark-up of 80% means you pay 180% of the original price.)

**16** Reconsider the scenario in question **13**. What was the price of the bike before the discount?

PROBLEM SOLVING AND REASONING

**17** Glenn sells cars and earns 2% per $1 sold on the total value of his sales. How much commission does he earn on the sale of a car that costs $22 490?

**18** Some salespeople have an income that is 100% commission based. Other salespeople may be paid a fixed amount (known as a base salary or retainer) plus their commission. What are the advantages and disadvantages of each type of income structure for the salesperson?

**19** Some salespeople are paid a fixed amount, or retainer, plus their commission. This method of payment ensures that money is still earned even if no sales are made. Erica is paid a retainer of $220 per week plus 5% commission on her sales. How much does Erica earn in a week in which the total value of her sales is $7255?

**20** Barry works as a real estate agent and earns a commission on each house he sells. He earns 2% commission on the first $300 000 and 1.75% on the rest. How much commission does Barry earn on a house that sells for $485 000?

**21** Maria and Paul plan to sell their house and are exploring which real estate agency to use.

→ The first agency charges a flat rate of 2.3% on the sale value of the house.

→ The second agency charges 3.4% for the first $200 000, 1.8% for the next $150 000 (up to $350 000), and 1.2% for the rest. Which agency should they use if they hope to sell their house for $590 000?

**22** Charlotte earns a retainer of $475 per week and 3.5% commission of the total value of her weekly sales. Calculate her earnings for a week with each of these total sales values.

**a** $500 **b** $8000 **c** $0 **d** $3029

**e** $2397.50 **f** $9480.95 **g** $12 095 **h** $25 800

**23** Angelique is paid a commission of 2.5% of the total value of her sales. In one week, she earned $375 in commission. What was the total value of her sales?

**24** Mark earns a weekly retainer of $325 plus 1.75% of all his sales. In one week, his earnings were $937.50. What was the total value of his sales in this week?

CHALLENGE

**25** David bought a new computer and paid $500 after several discounts. The computer was in a 45% off sale and his store membership rewarded him with an extra 10% off purchases more than $100. He also used a credit of $75 for the return of a faulty item.

**a** If the discount percentages were applied sequentially and then the store credit was applied, how much did the computer cost originally?

**b** If the store credit was applied first and then the discount percentages, how much did it originally cost?

**c** If the discounts were added and the total percentage was applied, and then the store credit was applied, how much did it cost originally?

**d** If the percentage discounts were applied sequentially, does it matter in which order they were applied? Explain why or why not.

**26** Brianna earns 4.5% commission on the sales she makes. Three of her four individual sales last month were $20 000, $35 000, and $16 000, and her total commission for the month was $4455. If her last sale was discounted by 30%, what was the original price of her final sale?

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Mark-ups, discounts and GST

Topic quiz
1C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

**1A** **1** Round the following numbers correct to the number of decimal places in the brackets.

**a** 827.47928 (2) **b** 294.2920028 (5) **c** 1990.07349 (3) **d** 47201.2840 (4)

**1A** **2** Perform each calculation using a calculator. Round correct to four decimal places.

**a** $4.2901 \times 28.24$ **b** $484.818 - 309.28 - 218.0901$

**c** $\dfrac{0.281 + 829.1}{9.124 - 3.45}$ **d** $\sqrt{\dfrac{12.95 \times 4.34}{77.2 \div 9.3}}$

**1A** **3** Fill in the empty spaces in the table below.

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{13}{64}$ | | |
| | 0.0045 | |
| | | 235.2% |

**1B** **4** Write each of the following as a simplified rate.

**a** $50 for 4 hours **b** $1500 for 125 L **c** $37.50 for 24 kg

**1B** **5** For each of the following, determine which option is the better buy.

| | Option A | Option B |
|---|---|---|
| **a** | $15.50 for 8.2 kg | $16.75 for 8.9 kg |
| **b** | $4.75 for 250 mL | $15.40 for 1.1 L |
| **c** | $800 for 7240 cm | $56 for 8.3 m |
| **d** | $2.45 for 100 $cm^2$ | $280 for 1.1 $m^2$ |

**1B** **6** Calculate each of the following.

**a** If the price is $12.25/L, how much will 42 L cost?

**b** How many kilograms can be bought with $51 if the price is $3.40/kg?

**c** How much will a rental cost for 125 weeks if it costs $55 for 20 weeks?

**d** How many tickets can I get for $30 if it costs $180 for 200 tickets?

**1B** **7** Determine the weekly wage for each of the following employees.

| | | Hourly rate | Normal time hours | Time-and-a-half hours | Double time hours |
|---|---|---|---|---|---|
| **a** | Employee A | $12.30/hour | 35 | 0 | 0 |
| **b** | Employee B | $13.25/hour | 38 | 6 | 0 |
| **c** | Employee C | $12.86/hour | 15 | 7 | 5 |
| **d** | Employee D | $15.98/hour | 38 | 4 | 4 |

**1C** **8** Calculate the following. Write:

**a** $30 as a percentage of $80 **b** $128 as a percentage of $250 **c** 35c as a percentage of $50

**1C** **9** Calculate the following. Write answers correct to the nearest cent.

**a** Increase $24 by 8%. **b** Increase $32.63 by 9.2%. **c** Decrease $158 by 45%.

**1C** **10** Calculate the following.

**a** $70 is 20% of what amount? **b** $184 is 15% more than what amount? **c** $132 is 34% less than what amount?

**1C** **11** **a** A car service cost $230 pre-GST. How much will the GST cost?

**b** A set of chairs cost $375 pre-GST. How much will it cost including GST?

**c** A new game console cost $599 including GST. Determine the pre-GST price.

**d** A taxi ride cost $18.87 including GST. How much GST was paid?

# 1D Profit and loss

## Learning intentions

- ✔ I can calculate the percentage profit or loss on the original price.
- ✔ I can calculate the percentage profit or loss on the selling price.
- ✔ I can determine the difference between profit and revenue.

Inter-year links

| | |
|---|---|
| Year 5/6 | Calculating percentages |
| Year 7 | 4I Calculating percentages |
| Year 8 | 3C Financial calculations |
| Year 10 | 1B Financial calculations |

## Express a quantity as a percentage of another

- To express a quantity as a percentage of another quantity:
  1. Make sure that both quantities are expressed in the same unit.
  2. Form a fraction with the numerator as the quantity you want to express as a percentage.
  3. Convert your fractions to a percentage.

## Profit and loss

- **Profit** is the difference between the selling price and the cost price. A profit occurs when the selling price is greater than the cost price.

  Profit = selling price – cost price
- The **percentage profit** is the profit as a percentage of the cost price.

  $$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

  For example, Sam made a profit of $20 when he sold a bike he bought for $100.

  $$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}}$$
  $$= \frac{20}{100}$$
  $$= 20\%$$
- A mark-up and percentage profit are both calculated as the percentage amount that the cost price is increased by to give the selling price.
- **Loss** is the difference between the cost price and the selling price. A loss occurs when the selling price is less than the cost price. Loss = cost price – selling price
- The **percentage loss** is the loss as a percentage of the cost price. Percentage loss $= \frac{\text{loss}}{\text{cost price}} \times 100\%$

  For example, Greg made a loss of $120 when he sold the phone he bought for $499.

  $$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}}$$
  $$= \frac{120}{499}$$
  $$= 24.05\%$$
- A discount is the amount the selling price is decreased by to sell at a lower price, whereas a percentage loss is calculated as the percentage amount that the cost price is decreased by to give the selling price.
- Percentage profit and loss calculations are generally written in relation to the cost price unless directly specified that it compares to the selling price.

  Profit ⟶ selling price > cost price

  Loss ⟶ selling price < cost price
- **Revenue** is the selling price multiplied by the number of items (or services) sold.

## Example 1D.1 Expressing a quantity as a percentage of another

**a** Write $60 as a percentage of $150.

**b** Write 90 cents as a percentage of $22, correct to the nearest whole number.

**THINK**

**a** 1 Express the amount as a fraction of the total and simplify.

2 To convert a fraction to a percentage, write the fraction with a denominator of 100.

3 Write your answer.

**b** 1 Express both quantities in the same unit.

2 Express the amount as a fraction of the total and simplify.

3 To convert a fraction to a percentage, multiply the fraction by 100%.

4 Cancel common factors to the numerators and denominators and simplify.

5 Divide 45 by 11 and write the answer, rounding appropriately. Write your answer.

**WRITE**

**a** $\frac{60}{150} \overset{\div 30}{=} \frac{2}{5}$

$= \frac{40}{100}$

$= 40\%$

$60 is 40% of $150.

**b** $22 = 2200 cents

$\frac{90}{2200} \overset{\div 10}{=} \frac{9}{220}$

$= \frac{9}{220} \times 100\%$

$= \frac{9}{\cancel{220}^{11}} \times \frac{\cancel{100}^{5}}{1}\%$

$= \frac{45}{11}\%$

$= 4.0909...\%$

$= 4\%$ (nearest whole number)

90 cents is 4% of $22.

## Example 1D.2 Calculating profit and loss

A television initially bought for $800 is later sold for $950.

**a** State if a profit or loss has been made and determine the amount.

**b** Write the profit or loss amount as a percentage of the cost price.

**THINK**

**a** The selling price is more than the cost price, so a profit has been made. Find the difference.

**b** Write the profit amount as a percentage of the cost price. Write the comparison as a fraction and convert it to a percentage.

**WRITE**

**a** A profit as been made.

Profit = selling price − cost price

$= \$950 - \$800$

$= \$150$

**b** Percentage profit $= \frac{\cancel{150}^{3}}{\cancel{800}^{16}}$

$= \frac{3}{16} \times 100\%$

$= \frac{3}{\cancel{16}^{4}} \times \frac{\cancel{100}^{25}}{1}\%$

$= 18.75\%$

The television sold for an 18.75% profit.

**Helpful hints**

- ✔ Remember that you can only generate a profit when: selling price > cost price and a loss when selling price < cost price. Think about the difference between the selling price and the cost price.
  - → When the result (selling price minus the cost price) is positive, there is a profit.
  - → When the result (selling price minus the cost price) is negative, there is a loss.
- ✔ When writing a percentage, make sure to carefully check what amount you are writing the percentage of.

ANS p476

# Exercise 1D Profit and loss

| | | |
|---|---|---|
| 1–6, 8, 9, 11, 13, 15, 16, 18, 21 | 1(b, d, f, h, j), 3, 4, 7, 8, 10–12, 14, 17, 19, 20, 23 | 1(g–j), 3, 4, 8, 11, 12, 17, 19, 20, 22–24 |

*You can use your calculator for all questions in this section unless otherwise specified.*

UNDERSTANDING AND FLUENCY

1D.1 **1** Write the first number as a percentage of the second number. Round answers to two decimal places where appropriate.

**a** \$45, \$225 **b** \$60, \$80 **c** \$36, \$144
**d** \$120, \$80 **e** \$99, \$600 **f** \$123, \$400
**g** \$67, \$90 **h** \$468, \$96 **i** \$2460, \$480
**j** \$123 456 543.21, \$111 111

**2** Determine the amount of profit or loss (in dollars) for each of the following.

**a** cost price \$35, selling price \$45 **b** cost price \$82, selling price \$68
**c** cost price \$92.50, selling price \$87.95 **d** cost price \$299.98, selling price \$145.50

1D.2 **3** For each of the following scenarios:

**i** state whether a profit or loss has been made and determine the amount
**ii** write the profit or loss amount as a percentage of the cost price, correct to two decimal places where appropriate.

**a** Shoes are bought for \$240 and later sold for \$180.
**b** A greengrocer buys a box of cherries for \$2.50 and sells them for \$9.80.
**c** An investor buys shares for \$5.20 and sells them for \$4.80.
**d** A car is purchased brand new for \$24 640 and sold for \$19 250.
**e** Coins are purchased in a set for \$120 and sold for \$350.
**f** A novel is purchased for \$29.95 and sold for \$8.

**4** **a** Calculate the percentages in question **3** if they are based on the selling price.
**b** How do the percentage amounts change if the percentages are based on the selling price?

**5** Calculate the percentage profit or loss on the cost price for each part in question **2**. Round your answers to two decimal places.

**6** Xiao pays \$198 for her wireless headphones and sells them to a friend for \$150 when a new model comes out.

**a** Did Xiao make a profit or a loss? State the amount of profit or loss.
**b** Write the amount in part **a** as a percentage of the original price.

**7** As Benjamin became more successful at his BMX racing, he chose to sell his bike to buy a better model. The bike, which had cost him \$240, was sold to a fellow competitor at a percentage profit of 5%.

**a** How much did Benjamin sell the bike for?
**b** The new bike Benjamin plans to buy will cost him \$900. Write this as percentage of the cost of his original bike.

**8** For each of these:

**i** state the value of the profit or loss

**ii** write the profit or loss as a percentage of the original price (rounded to the nearest 1%).

**a** cost price $48, selling price $34

**b** cost price $112.50, selling price $240

**c** cost price $35.90, selling price $85.95

**d** cost price $1649, selling price $1238

**e** cost price $29 895, selling price $17 500

**f** cost price $156 985, selling price $425 850

**9** John buys pears at the orchards for $2.95 per kilogram to sell at his market stall.

**a** How much does John mark up the cost of the pears per kilogram (see photo)?

**b** Write the mark-up amount as a percentage of the price John pays for the pears. State the mark-up as a percentage to the nearest 1%.

**10** A wireless printer is initially priced at $249.95 and is offered for sale at a discounted price of $222.50.

**a** State the amount of the discount.

**b** Write the discount as a percentage of the initial price to the nearest 1%.

**11** The revenue a company earns is the total amount of money received from sales.

Revenue = selling price × sales

Profit per sale is the difference in the selling price and cost price, which can be multiplied by the number of sales to determine the total profit.

For example, if the original price is $5, the selling price is $8, and 20 are sold, then:

Revenue = $8 × 20 = $160

Profit = ($8 − $5) × 20 = $3 × 20 = $60

For each of the following, calculate:

**i** revenue

**ii** total profit each company would make.

**a** cost price $2, selling price $5, 50 sold

**b** cost price $8, selling price $20, 100 sold

**c** cost price $10, selling price $40, 25 sold

**d** cost price $1.50, selling price $4.75, 45 sold

**e** cost price $7.99, selling price $13.99, 16 sold

**f** cost price 24c, selling price $1.99, 247 sold



**12** Joseph sells remote-controlled cars in his store. He knows that identical cars are being sold by a competitor for $65. Joseph can purchase these cars from a wholesaler for $28 per car.

**a** Joseph aims to make a 150% profit on the sale of each car and must add 10% for GST. Do you think this profit margin is a suitable pricing strategy? Briefly explain.

**b** Using whole number values, what is the maximum percentage increase Joseph should apply to the wholesale price? Remember to add the GST charge.

**c** Why is it necessary to consider a maximum percentage increase rather than any percentage increase Joseph wishes to apply?

**13** A small share portfolio was purchased at a price of $1200 and sold 12 months later for $1500.

**a** Write the increase in price as a percentage of the original price.

**b** Write the final selling price as a percentage of the original purchase price.

**c** Compare the percentage increase in part **a** with the answer in part **b**. Briefly explain how they are related.

**14** A car is bought for $20 000 and sold 6 months later for $16 000.

**a** Write the decrease in price as a percentage of the original price paid for the car.

**b** Write the final selling price as a percentage of the original purchase price.

**c** Compare the percentage decrease in part **a** with the answer in part **b**. Briefly explain how they are related.

**15** What percentage increases or decreases match the following profit/loss amounts?

**a** doubling your money **b** tripling your money **c** breaking even

**d** halving your money **e** quadrupling your money **f** losing all your money

**16** The finance report on the nightly news displays the daily movement in the cost of some common commodities. If the values given in this table represent the end-of-day trading figures, what were the values at the start of the day's trading?

| Commodity | Final price $ | Movement % |
|---|---|---|
| Gold | 1732.95 | ↓ 0.5 |
| Silver | 33.56 | ↓ 1.4 |
| Oil | 102.46 | ↑ 0.3 |
| Copper | 3.71 | ↑ 1.2 |

↓ represents a decrease in price

↑ represents an increase in price

**17** Olivia purchases a large block of land on which she plans to build four townhouses. The land costs $645 000 and the cost for each house is $230 000 (including plans, permits and other related charges). The project takes 2 years to complete and Olivia is charged $2300 in council rates each year. The real estate agency earns a commission of 1.75% for the sale of each townhouse. The four house sales are $485 000, $490 000, $472 000 and $461 000.

**a** What were the total expenses accrued by Olivia before the sale of the townhouses?

**b** From the total sales, how much of the money:

**i** goes to Olivia?

**ii** is earned as commission by the real estate agency?

**c** Does Olivia make a profit or a loss?

**d** Write your answer from part **c** as a percentage of Olivia's total expenses. Do not include the real estate agency's commission.

**18** Mario runs a hairdressing business from his home and sells shampoos, hair treatment and styling products to his customers. On all product sales, he plans to make a profit of 80% of the wholesale price he pays for the goods. On top of this, he knows he must add an additional 10% for GST. Mario believes he can determine the selling price by simply adding 90% to the wholesale price.

**a** A jar of styling gel has a wholesale price of $8.50. What will the price be after Mario's profit mark-up?

**b** What will the selling price be after GST is added?

**c** Increase $8.50 by 90% and compare your answer with the answer from part **b**. Is Mario's method of calculating the selling price correct? Why or why not?

**19** For each of the following, write, correct to two decimal places where appropriate.

**i** the profit as a percentage of the selling price

**ii** the selling price as a percentage of the profit

**iii** the selling price as a percentage of the cost price

**iv** the percentage mark-up from the cost price to the selling price

**a** cost price $2, selling price $5

**b** cost price $8, selling price $20

**c** cost price $10, selling price $40

**d** cost price $1.50, selling price $4.75

**e** cost price $7.99, selling price $13.99

**f** cost price 24c, selling price $1.99

**20** Determine the missing values in the table.

| Cost price | Selling price | Cost price as a percentage of selling price | Number of sales | Revenue | Total profit | Profit as a percentage of the revenue |
|---|---|---|---|---|---|---|
| $3.60 | $12 | | 35 | | | |
| | $15 | | 20 | | $200 | |
| | $21 | | | $777 | $296 | |
| | | | 150 | $375 | $210 | |
| | | | 25 | $875 | | 26% |
| | | | 125 | | $320 | 32% |
| | $99 | 51% | 8 | | | |
| $0.28 | | 25% | 271 | | | |
| | $32 | | | | $145.92 | 24% |
| | $6 | | 15 | | | 40% |

**21 a** Explain why finding 70% of a value is equal to decreasing the value by 30%.

**b** Explain why finding 130% of a value is equal to increasing the value by 30%.

**22** A small business said they earned $10 000 this week. They spent $12 000 to make the sales.

**a** If the $10 000 is the profit the business made, how much revenue did they earn?

**b** If the $10 000 is the revenue the business earned, how much profit or loss did they make?



**23 a** In question **18**, Mario likes to make a profit of 80% on his wholesale prices and then adds 10% for GST. What single calculation can Mario perform to determine his selling price for a jar of styling gel?

**b** A motorbike sells for $1200 after a mark-up of 60% and then GST is added. What single calculation can be performed to determine the wholesale price of the motorbike?

**c** GST is added to a price and then the item is discounted by 25% to sell for $400. What single calculation will determine the original pre-GST price?

**24** A company marks up their product's cost by 128% to its selling price. What percentage will the profit be of the revenue, correct to two decimal places?

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# 1E Simple interest

## Learning intentions

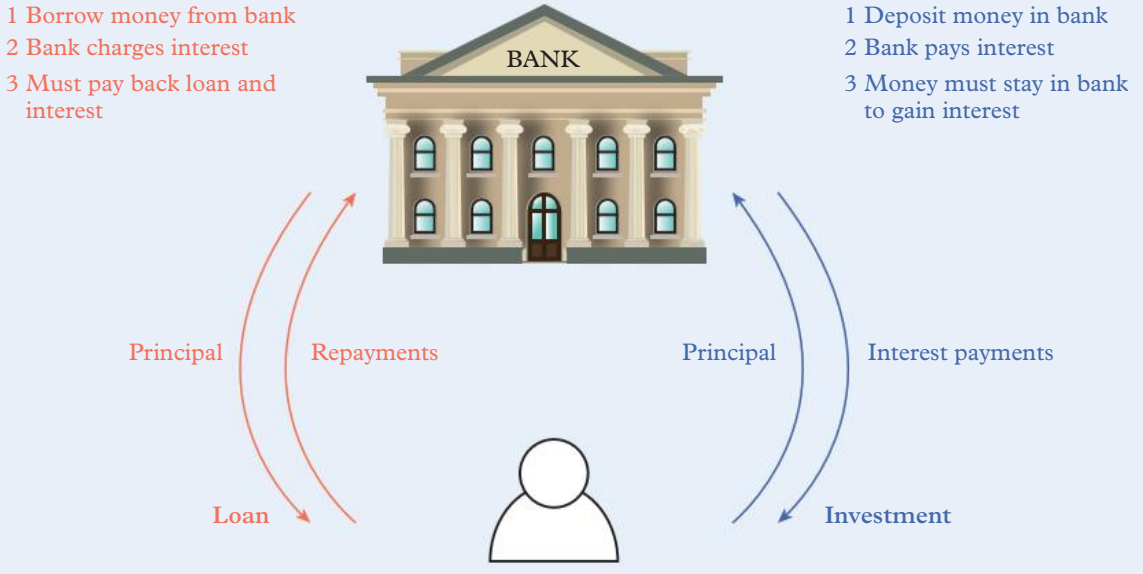
- ✔ I can calculate the amount of interest using a simple interest formula.
- ✔ I can calculate the simple interest on an investment.
- ✔ I can calculate the simple interest on a loan.

**Inter-year links**

| | |
|---|---|
| Year 7 | 6C Terms, expressions and equations |
| Year 8 | 6A Equations |
| Year 10 | 1C Simple interest |

## Loans and investments

- A **loan** is when you borrow money and pay **interest**. If you take a loan from a bank, the total of your repayments is more than the amount borrowed. Banks charge you interest for allowing you to have access to their money.
- An **investment** is when you deposit your money and earn interest. If you invest money with a bank rather than borrow it, the bank pays you interest for allowing them to have access to your money.




## Simple interest

- The amount of interest you pay on a loan (or earn on an investment) depends on the original amount you borrow, the interest rate charged and the time it takes to repay.
- **Simple interest** can be calculated using the formula:

$$I = PrT$$

simple interest: $I$ — principal: $P$ — interest rate: $r$ — time: $T$

- $I$ = interest
- $P$ = **principal,** the original amount of money borrowed or invested
- $r$ = **interest rate**, usually a percentage converted to a fraction or decimal
  For example, 5% would be substituted as $\frac{5}{100}$ or 0.05. The interest rate 5% p.a. means 5% per year and is often abbreviated to p.a.
- $T$ = time of the loan or investment in years
- The total you repay (or have invested) includes the original amount plus the interest.
  Total amount (loan/investment) = $P + I$

## Example 1E.1 Calculating interest rates

Write 8% interest rate as:

**a** a fraction in simplest form

**b** a decimal.

**THINK**

**a** To convert a percentage to a fraction, the percentage becomes the numerator and the denominator is 100. Simplify the fraction if required.

**b** To convert a percentage to a decimal, divide the percentage by 100.

**WRITE**

**a** $8\% = \frac{8^{2}}{\cancel{100}^{25}}$

$= \frac{2}{25}$

**b** $8\% = 8 \div 100$

$= 0.08$

## Example 1E.2 Calculating simple interest

For an investment of $5200 at an interest rate of 6% p.a. for 4 years, calculate:

**a** the amount of simple interest

**b** the value of the investment after 4 years.

**THINK**

**a** 1 Write the simple interest formula and identify the key terms: principal, interest rate, and time. The rate must be written as a fraction or a decimal.

2 Substitute the values into the formula and calculate the result.

3 Write the answer.

**b** 1 The value of the investment after 4 years is the interest amount added to the principal.

2 Write your final answer.

**WRITE**

**a** $I = PrT$

$P = \$5200$

$r = 6\% = 0.06$ per year

$T = 4$ years

$I = \$5200 \times 0.06 \times 4$

$= \$1248$

The simple interest earned in 4 years is $1248.

**b** Total amount $= P + I$

$= \$5200 + \$1248$

$= \$6448$

The value of the investment after 4 years is $6448.

### Helpful hints

- ✔ Be careful when converting your interest rate to a decimal or fraction. You can use the table from 1A to help you convert between them on your calculator.
- ✔ Remember to round your answers to the nearest cent. When using cash, round to the nearest 5 cents. For all other transactions, round to the nearest 1 cent.
- ✔ If you are finding the simple interest formula difficult, write it out in words to help you.
  Simple interest = principal × interest rate × time
- ✔ Remember that the principal is the initial amount invested or borrowed.
- ✔ Make sure that the interest rate $r$ and time $T$ have the same unit. If the interest rate is per annum, then the time must also be in years. If the interest rate is per month, then the time must be in months.

ANS p477

# Exercise 1E Simple interest

▲ 1–5, 7, 8(2nd column), 9, 10, 11, 13, 15 ■ 1–3, 4(b, d, f), 6, 7, 8(c, f, g, h), 10(a, c, e, g, i), 12, 13, 16, 18, 19(a) ⬟ 2, 3, 7, 8(c, e, g, i), 10(f–i), 13, 14, 16–20

*You can use your calculator for all questions in this section unless otherwise specified.*

UNDERSTANDING AND FLUENCY

1E.1 **1** Write these interest rates as:

**i** a fraction in simplest form **ii** a decimal.

**a** 7% **b** 11% **c** 24% **d** 6% **e** 10% **f** 12%

1E.2 **2** For each investment, calculate:

**i** the amount of simple interest **ii** the value of the investment after the time period listed.

**a** a simple interest investment of $5000 at an interest rate of 5% p.a. for 2 years

**b** a simple interest investment of $4800 at an interest rate of 4% p.a. for 3 years

**c** a simple interest investment of $12 500 at an interest rate of 8% p.a. for 5 years

**3** For each loan, calculate:

**i** the amount of simple interest **ii** the total amount to be repaid.

**a** a loan of $7500 at a simple interest rate of 5% p.a. over 3 years

**b** a loan of $10 800 at a simple interest rate of 12% p.a. over 5 years

**c** a loan of $25 000 at a simple interest rate of 7% p.a. over 8 years

**4** Calculate the simple interest given each of these. All interest rates are per annum.

**a** $P = \$4000$, $r = 6\%$, $T = 5$ years **b** $P = \$8650$, $r = 7\%$, $T = 4$ years

**c** $P = \$15\,000$, $r = 8\%$, $T = 10$ years **d** $P = \$9200$, $r = 4\%$, $T = 3$ years

**e** $P = \$19\,999$, $r = 15\%$, $T = 6$ years **f** $P = \$20\,000$, $r = 20\%$, $T = 5$ years

**5** Christian invests $3500 in a bank that offers the simple interest rate of 4.8% per annum. He plans to leave the money invested for 2 years.

**a** Identify the values of $P$, $r$ and $T$.

**b** How much simple interest does Christian earn?

**c** What is the total value of Christian's investment after 2 years?

**6** Jenna plans to start her business in massage therapy and needs to borrow $44 000 to assist with her set-up costs. She obtains an agreement with her lender to repay the money over 5 years with simple interest charged at 9.5% p.a.

**a** Identify the values of $P$, $r$ and $T$.

**b** How much simple interest is Jenna charged?

**c** What is the total amount that Jenna repays?

**7** **a** Calculate the amount of simple interest in each of the following situations.

**i** $5000 is invested at a simple interest rate of 4.75% p.a. for 3.5 years.

**ii** $5000 is borrowed at a simple interest rate of 4.75% for 3.5 years.

**b** Compare each of the answers in parts **ai** and **aii**. Briefly explain how the simple interest formula is used for investment and loan situations.

**c** Given that the simple interest calculations involving loans and investments are identical, how are the calculations different when they are interpreted?

**8** Convert each time to years. Where appropriate, write the fraction in simplest form.

| | | | | | |
|---|---|---|---|---|---|
| **a** | 11 months | **b** | 7 weeks | **c** | 26 weeks |
| **d** | 3 months | **e** | 271 days | **f** | 155 days |
| **g** | 15 months | **h** | 48 months | **i** | 84 weeks |
| **j** | 1241 days | **k** | 30 months | **l** | 286 weeks |

**9** A simple interest investment is made for 4 years and 3 months. Matthew thinks this is equivalent to 4.3 years but Lizzy is certain that Matthew is wrong. How is 4 years and 3 months written as a decimal in years?

**10** For the values given in the table below, calculate:

**i** the amount of simple interest

**ii** the total amount at the end of the term.

| | Principal $ | Simple interest rate % p.a. | Time |
|---|---|---|---|
| **a** | 9000 | 6 | 3 years |
| **b** | 10 500 | 4.5 | 6 months |
| **c** | 7500 | 9.8 | 130 weeks |
| **d** | 29 000 | 3.2 | 90 days |
| **e** | 8600 | 6.4 | 35 days |
| **f** | 155 570 | 12.5 | 11 months |
| **g** | 19 999 | 19.9 | 25 weeks |
| **h** | 45 950 | 14.05 | 2 years and 5 months |
| **i** | 208 654 | 8.75 | 5 weeks and 4 days |



**11** Sade is investigating which is the best way to calculate her simple interest for a short-term investment. She invests $2400 for the month of June at a simple interest rate of 4.6% p.a.

**a** Calculate the simple interest amount after writing the time as a fraction of the total number of months in the year.

**b** Now calculate the simple interest amount after writing the number of days in June as a fraction of the total number of days in the year.

**c** Which method of calculation would Sade be hoping would be used? Briefly explain why.

**d** If the values given represented a short-term loan instead of an investment, which method of calculation would Sade prefer? Briefly explain why.

**12** A bank is offering the simple interest rates shown in the table below for its customers to invest in a term deposit. The interest is calculated at the end of the investment. Jasmine has $20 000 to invest and plans to invest it for 12 months.

**a** What simple interest rate will Jasmine receive for her investment?

**b** How much interest does she earn?

**c** Jasmine's brother informed her that she would have earned more interest if she invested the money for one day less than 12 months. Investigate whether this statement is true and show working to support your finding.

| Term (months) | Interest on investment amount (% p.a.) | | |
|---|---|---|---|
| | $5000–<$10 000 | $10 000–<$50 000 | $50 000–<$100 000 |
| 1–2 | 2.5 | 2.5 | 2.8 |
| 3–6 | 3.25 | 3.25 | 3.25 |
| 7–11 | 5.5 | 5.55 | 5.5 |
| 12–24 | 5.3 | 5.25 | 5.2 |

**13** Simple interest assumes the same amount of interest is added every period (year, month, day, etc.). Therefore, if we know the interest amount per period, we can multiply that by the number of periods to determine the total interest using the unitary method.

For example: $P = \$4000$, $r = 4\%$ p.a., $T = 5$ years, $I = \$?$
4% p.a. × \$4000 = \$160 per 1 year
So for 5 years:
\$160 × 5 per 1 × 5 years = \$800 per 5 years

For each of the following calculate:

**i** the amount interest per period **ii** the total amount of interest

**iii** the total amount owing for a loan or in an account for an investment.

**a** $P = \$200$, $r = 5\%$ p.a., $T = 10$ years, $I = \$?$

**b** $P = \$1400$, $r = 6.2\%$ p.a., $T = 8$ years, $I = \$?$

**c** $P = \$500\,000$, $r = 8\%$ p.a., $T = 12.5$ years, $I = \$?$

**d** $P = \$750$, $r = 8\%$ p.a., $T = 20$ months, $I = \$?$

**e** $P = \$26\,000$, $r = 5\%$ p.a., $T = 72$ weeks, $I = \$?$

**f** $P = \$4380$, $r = 3.5\%$ p.a., $T = 153$ days, $I = \$?$

**14** The total amount owing for a loan or in an account for an investment can be determined by then adding the interest to the principal. Alternatively, we can find the percentage increase for the given time period, then apply the percentage increase to the principal.

For example: $P = \$4000$, $r = 4\%$ p.a., $T = 5$ years, $I = \$?$
4% per 1 year
So for 5 years:
4% × 5 per 1 × 5 year = 20% per 5 years
(100% + 20%) × \$4000 = \$4800

For each part in question **13**:

**i** determine the multiplier to determine the total amount using a percentage increase correct to four decimal places where appropriate

**ii** recalculate the total amount using the percentage increase in part **i**.

**15** Banks vary in the ways in which they calculate simple interest on savings and transaction accounts. Some accounts earn no interest while others attract bonus interest rates if certain conditions are met. If an account provides interest, it is most likely to be calculated on the daily account balance. Consider the account balances for the month of February shown.

**a** The opening balance of \$640.90 applies for the first 7 days of the month and each new balance applies from the date the transaction is made. How many days does each balance on this account apply for?

| Date | Transaction | Credits \$ | Debits \$ | Balance \$ |
|---|---|---|---|---|
| 01/02 | Opening balance | | | 640.90 |
| 08/02 | Withdrawal at Handybank | | −100.00 | 540.90 |
| 15/02 | Deposit | +240.00 | | 780.90 |
| 24/02 | EFTPOS purchase | | −125.40 | 655.50 |
| 28/02 | Interest | | | |

**b** The account attracts simple interest at a rate of 2.1% p.a. For each new balance in the account, calculate the simple interest based on the number of days each balance applies.

**c** Add all the amounts from part **b** to calculate the total interest for the month.

**d** What is the account balance at the end of February, if the total interest is added at the end of each period?

**16** This bank statement shows the transactions made during the month of August. Interest is calculated daily at a rate of 1.8% p.a.

| Date | Transaction | Credits \$ | Debits \$ | Balance \$ |
|---|---|---|---|---|
| 01/08 | Opening balance | | | 345.50 |
| 09/08 | ATM Withdrawal | | −50.00 | 295.50 |
| 14/08 | Deposit – Pay | +370.00 | | 665.50 |
| 16/08 | ATM Withdrawal | | −120.00 | 545.50 |
| 19/08 | EFTPOS Purchase | | −85.95 | 459.55 |
| 28/08 | Deposit – Pay | +370.00 | | 829.55 |
| 31/08 | Interest | | | |

**a** How much simple interest is earned during the month? **b** What is the final account balance?

**17** A bank offers a simple interest rate of 1.5% p.a. on its savings accounts, calculated daily. An extra 3.2% p.a. bonus rate is offered if no more than one withdrawal is made in the month and the account balance has increased by at least $200 for the month. Consider each of the account statements shown.

**A**

| Date | Transaction | Credits $ | Debits $ | Balance $ |
|---|---|---|---|---|
| 01/09 | Opening balance | | | 1200.85 |
| 15/09 | Deposit – Pay | +450.75 | | |
| 24/09 | Deposit – at branch | +820.00 | | |
| 29/09 | EFTPOS Purchase | | −245.85 | |
| 30/09 | Interest | | | |

**B**

| Date | Transaction | Credits $ | Debits $ | Balance $ |
|---|---|---|---|---|
| 01/01 | Opening balance | | | 1548.90 |
| 08/01 | EFTPOS Purchase | | −246.20 | |
| 15/01 | Deposit – Pay | +1920.00 | | |
| 29/01 | EFTPOS Purchase | | −85.94 | |
| 31/01 | Interest | | | |

**a** Will any of these accounts receive the bonus simple interest rate? Provide a reason to support your answer.

**b** Calculate the total interest earned on each account. You will need to determine the account balances following each transaction first.

**c** State the final account balance for each statement at the end of the month.

**18** Joel plans to buy a second-hand car for $12 500. He has saved $2500 and plans to borrow the remaining money from his bank at a simple interest rate of 8.5% p.a. for 3 years.

**a** The car-seller asks for a deposit of 15% of the selling price. Is Joel's savings enough to cover the deposit? (Note that a deposit is the first part of a payment often used as a promise to pay.)

**b** How much does Joel borrow to buy the car?

**c** Calculate the total amount, including simple interest, that Joel pays for the car.



**19** You have $2000 and wish to double this amount over 3 years. You plan to explore some different options to earn the most amount of simple interest possible.

**a** What is the annual simple interest rate that will enable this investment to double in 3 years?

**b** Explore how this rate changes if the time of the investment increases to:

**i** 4 years

**ii** 5 years

**iii** 6 years.

**c** Explore how this rate changes if the time of the investment decreases to:

**i** 2 years

**ii** 1 year.

**20** Provide three different annual interest rates and their corresponding times that would result in an investment of $5000 earning $1250 in simple interest.

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# 1F Simple interest calculations

## Learning intentions

- ✔ I can calculate the time period for a loan or investment using the simple interest formula.
- ✔ I can calculate the principal using the simple interest formula.
- ✔ I can calculate the interest rate using the simple interest formula.

**Inter-year links**

| | |
|---|---|
| Year 7 | 6H Solving equations using inverse operations |
| Year 8 | 6B Solving equations using inverse operations |
| Year 10 | 1C Simple interest |

## Calculating the principal, interest rate and time

- The simple interest formula is $I = PrT$.

$$I = \mathbf{P}rT \quad \text{Solve for principal}$$

$$I = P\mathbf{r}T \quad \text{Solve for interest rate}$$

$$I = PrT \quad \text{Solve for time}$$

- To solve the simple interest equation for $P$, $r$, or $T$:
  1. Write the simple interest formula and identify the known variables.
  2. Substitute the values into the formula and simplify the calculation.
  3. Solve the equation for the unknown value using inverse operations.
  4. Write the answer.

### Example 1F.1 Calculating the time period for an investment

How long will it take for an investment of \$4000 at an interest rate of 4% p.a. to earn \$800 in simple interest?

**THINK**

1. Write the simple interest formula and identify the variables. Write $r$ as a fraction or a decimal.
2. Substitute the values into the formula and simplify the calculation.
3. Solve the equation for $T$.
4. Write the answer and include the unit 'years' because $r$ is 'per annum'.

**WRITE**

$I = PrT$
$P = \$4000$
$r = 4\% = 0.04$ p.a.
$I = \$800$
$\$800 = \$4000 \times 0.04 \times T$
$\$800 = \$160 \times T$
$\frac{\$800}{\$160} = \frac{\$160 \times T}{\$160}$
$T = 5$
It will take 5 years for an investment of \$4000 to earn \$800 in simple interest.

## Example 1F.2 Calculating the principal value

How much needs to be invested at an interest rate of 6% p.a. for 3 years to earn $1440 in simple interest?

| THINK | WRITE |
|---|---|
| 1 Write the simple interest formula and identify the variables. $r$ should be written as a fraction or a decimal. | $I = PrT$<br>$r = 6\% = 0.06$ p.a.<br>$T = 3$ years<br>$I = \$1440$ |
| 2 Substitute the values into the formula and simplify the calculation. | $\$1440 = P \times 0.06 \times 3$<br>$\$1440 = P \times 0.18$ |
| 3 Solve the equation for $P$. | $\frac{\$1440}{0.18} = \frac{P \times 0.18}{0.18}$<br>$P = \$8000$ |
| 4 Write the answer. | $8000 needs to be invested to earn $1440 in simple interest over 3 years. |

## Example 1F.3 Calculating the interest rate

At what rate should $6000 be borrowed at over 6 years to be charged $864 in simple interest?

| THINK | WRITE |
|---|---|
| 1 Write the simple interest formula and identify the variables. $r$ should be written as a fraction or a decimal. | $I = PrT$<br>$P = \$6000$<br>$T = 6$ years<br>$I = \$864$ |
| 2 Substitute the values into the formula and simplify the calculation. | $\$864 = \$6000 \times r \times 6$<br>$\$864 = \$36\,000 \times r$<br>$\frac{\$864}{\$36\,000} = \frac{\$36\,000 \times r}{\$36\,000}$ |
| 3 Solve the equation for $r$ using inverse operations. | $r = 0.024$ |
| 4 Convert $r$ to a percentage by multiplying the decimal by 100. | $= 2.4\%$ |
| 5 Write the answer and include the unit p.a. because $T$ is in years. | The $6000 will need to be borrowed at 2.4% p.a. to be charged $864 in simple interest. |

**Helpful hints**

- ✔ Recall the calculator BIDMAS skills from 1A. You will need them as you solve the equations for the unknown variable with a calculator.
- ✔ Remember to round your answers to the nearest cent.
- ✔ To find the solution, the pronumeral does not have to appear on the left-hand side of the equation – if the pronumeral is by itself on one side of the equation, you have found the solution!

ANS p478

# Exercise 1F Simple interest calculations

1–9, 11, 12, 13(a–c) | 1(d–f), 2–4, 7, 8, 10, 12, 14–16 | 4, 7(b, d, f), 8, 12, 14–18

*You can use your calculator for all questions in this section unless otherwise specified.*

UNDERSTANDING AND FLUENCY

**1** Calculate the simple interest in each case.

**a** $P = \$2000$, $r = 7\%$, $T = 3$ years

**b** $P = \$250$, $r = 11\%$, $T = 1$ year

**c** $P = \$8500$, $r = 5\%$, $T = 4$ years

**d** $P = \$25\,000$, $r = 4\%$, $T = 5$ years

**e** $P = \$100\,000$, $r = 9.5\%$, $T = 4$ years

**f** $P = \$16\,000$, $r = 6\%$, $T = 2.5$ years

1F.1 **2** Find the value for $T$ in each of these.

**a** How long will it take for an investment of $8000 at a simple interest rate of 3% p.a. to earn $1200 in simple interest?

**b** How long will it take for an investment of $1250 at a simple interest rate of 4% p.a. to earn $350 in simple interest?

**c** How long does a loan of $15 000 at a simple interest rate of 9% p.a. take to earn $5400 in simple interest?

**d** How long will it take for an investment of $5600 at a simple interest rate of 5% p.a. to earn $1120 in simple interest?

1F.2 **3** Find the value for $P$ in each of these.

**a** How much needs to be invested at a simple interest rate of 8% p.a. for 5 years to earn $2000 in simple interest?

**b** How much is borrowed at a simple interest rate of 10% p.a. over 4 years to earn $6000 in simple interest?

**c** How much is borrowed at a simple interest rate of 9% p.a. over 5 years to earn $1800 in simple interest?

**d** How much needs to be invested at a simple interest rate of 6% p.a. for 2 years to earn $576 in simple interest?

**4** Find the unknown value in each of these.

**a** $I = \$600$, $P = \$3000$, $r = 4\%$, $T = ?$

**b** $I = \$1200$, $P = ?$, $r = 5\%$, $T = 4$ years

**c** $I = \$450$, $P = ?$, $r = 9\%$, $T = 2$ years

**d** $I = \$850$, $P = \$8500$, $r = 5\%$, $T = ?$

**e** $I = \$1000$, $P = ?$, $r = 8\%$, $T = 4$ years

**f** $I = \$5060$, $P = \$9200$, $r = 11\%$, $T = ?$

**5** Jessica has invested $4500 in a bank that offers simple interest of 5.0% p.a. She plans to earn $675 in interest.

**a** From the simple interest formula, which variable do you not know the value of?

**b** What variable does each of the given values represent?

**c** How long does the money need to be invested to earn $675 in simple interest?

**d** At a higher interest rate of 7.5% p.a., how much sooner can Jessica earn $675 in simple interest?

**6** Throughout the course of a simple interest investment, Stefan's money increased in value from $8400 to $8862. The interest was earned at a simple interest rate of 2.75% p.a.

**a** What is the total amount of interest earned on this investment?

**b** How many months was the initial amount of money invested for?

1F.3 **7** For the values given in the table, calculate the simple interest rate per annum that applies.

| | Simple interest $ | Principal $ | Time |
|---|---|---|---|
| **a** | 1640 | 8200 | 4 years |
| **b** | 420 | 3500 | 2 years |
| **c** | 985 | 9850 | 48 months |
| **d** | 1680 | 12 000 | 30 months |
| **e** | 3936 | 18 000 | 3 years and 5 months |
| **f** | 680 | 6400 | 4 years and 3 months |

**8** Use the simple interest formula to determine the values for the missing amounts in the table.

| | Simple interest $ | Principal $ | Rate % p.a. | Time |
|---|---|---|---|---|
| **a** | | 3700 | 5.6 | 4.5 years |
| **b** | 234 | | 4.8 | 13 months |
| **c** | 42 532 | 70 000 | | 6.2 years |
| **d** | 3549 | 19 500 | 5.2 | |
| **e** | 1711 | | 14.5 | 48 months |
| **f** | 2631.60 | 15 480 | | 130 weeks |
| **g** | 56.88 | 948 | | 1.2 years |
| **h** | 1534.40 | 13 700 | 6.4 | |



**9** Priyansha borrowed a sum of money from her parents to help her buy her first laptop. They agreed to charge interest at a rate of 4% p.a. over a period of 3 years. The total interest charge for the term of the loan is $144.

**a** From the simple interest formula, which variable is unknown?

**b** Which variable does each of the given values represent?

**c** How much money does Priyansha borrow from her parents?

**d** Priyansha plans to pay her parents $35 each month for 3 years and believes this will cover the agreed terms of their loan. Determine whether Priyansha's plans are correct and show working to support your finding.

**e** What are the exact monthly payments Priyansha needs to make to repay her loan?

**10** The cost of the latest tablet is $873. Although Gabriella has the savings to purchase the new tablet, she would rather let the interest earned from her investment cover the cost of the purchase.

**a** One bank offers her a simple interest rate of 7.2% p.a. for her investment of $10 000. How long does this money need to be invested to earn enough interest to pay for the tablet?

**b** Gabriella decides on 12 months to reach her goal. At the same rate of interest, how much does she need to invest in order to fully pay for the tablet with the interest she earns?

**11** Daniel has decided to learn the alto saxophone through his school music program. To encourage his development, his parents bought the saxophone shown through a purchase program arranged by his school valued at $1200. The repayment conditions involve quarterly payments over 3 years. The simple interest charged on the saxophone's cost is $162.

**a** What is the annual interest rate charged?

**b** What is the amount of each quarterly payment required?

**12 a** Alex is charged \$63.25 on a simple interest loan with an annual interest rate of 6.5% after 92 weeks. How much interest will he be charged after 100 weeks?

**b** Charlotte borrows \$1000 at a simple interest rate of 3.65% p.a. and is charged \$500 interest after a number of days. After how many days will Charlotte be charged \$1000 interest?

**c** Helen has a simple interest investment of 20c. After 1000 months, she earns \$1.05 in interest. Correct to the nearest year, how long will it take Helen to earn \$1000 in interest?

**13** The simple interest formula can be used to find the values of the principal, rate, or time if you know the interest amount and two other values, but you can also use your knowledge of rates and percentages. You can use the unitary method or a multiplier with the interest amount per annum to find the number of years for the desired interest amount.

$P = \$4000$, $r = 4\%$ p.a., $T =$ ? years, $I = \$800$

**Unitary method**

$4\%\text{p.a.} \times \$4000$

$= \$160$ per 1 year

$= \$160 \div 160$ per $1 \div 160$ years

$= \$1$ per $\frac{1}{160}$ years

$= \$1 \times 800$ per $\frac{1}{160} \times 800$ years

$= \$800$ per 5 years

**Multiplier method**

$4\%\text{p.a.} \times \$4000$

$= \$160$ per 1 year

$= \$160 \times \frac{800}{160}$ per $1 \times \frac{800}{160}$ years

$= \$800$ per 5 years

When the interest rate per annum or principal amount are not known, you can start by finding the interest per year then either writing it as a percentage of the principal or determining what percentage it is of the interest rate. For example:

$P = \$?$, $r = 6\%$ p.a., $T = 3$ years, $I = \$1440$

\$1440 per 3 years

$= \frac{\$1440}{3}$ per $\frac{3}{3}$ years

$= \$480$ per year

$P = \frac{\$480 \text{ per year}}{6\% \text{ per year}}$

$= \frac{\$480 \text{ per year}}{0.06 \text{ per year}}$

$= \$8000$

$P = \$6000$, $r = ?\%$ p.a., $T = 6$ years, $I = \$864$

\$864 per 6 years

$= \frac{\$864}{6}$ per $\frac{6}{6}$ years

$= \$144$ per year

$r = \frac{\$144 \text{ per year}}{\$6000} \times 100$

$= 2.4\%\text{p.a.}$

Determine the values for the missing amounts in the table using rates and percentages. Remember, per annum, p.a., means per year.

| | Simple interest \$ | Principal \$ | Rate % p.a. | Time |
|---|---|---|---|---|
| **a** | 297.50 | 850 | 5 | ? years |
| **b** | 786.24 | ? | 4.2 | 12 years |
| **c** | 569.43 | 999 | ? | 10 years |
| **d** | 125 | ? | 8 | 15 months |
| **e** | 120 | 5875 | ? | 120 weeks |
| **f** | 2 947 000 | 7 300 000 | 5 | ? days |

**14** An amount of \$4000 is invested at simple interest rate of 5.2% p.a. for a period of 3 years.

**a** Calculate the amount of simple interest that is earned on this investment.

**b** What is the value of the investment at the end of the 3-year term?

Investments involving simple interest result in the interest being passed on to the investor at maturity (at the end of the investment). Reconsider the simple interest investment of \$4000 at 5.2% p.a. for 3 years, but now calculate the simple interest during the investment period at yearly intervals and add these amounts to the principal.

**c** How much simple interest is earned in the first year of the investment?

**d** Add the interest amount from part **c** to the principal amount. This new amount is the principal for the second year of the investment.

**e** Use the new principal value to calculate the interest earned in the second year of the investment.

**f** Add the interest amount from part **e** to the principal amount for the second year. This new amount is the principal for the third year of the investment.

**g** Use the new principal value to calculate the interest earned in the third (final) year of the investment.

**h** Add the interest amount from part **g** to the principal amount for the third year. This new amount is the final value of the investment.

**i** Compare your answer from part **h** with the answer you obtained in part **b**. Which method of calculation resulted in a higher value at the end of 3 years? Why do you think this is so?

**15** The method of interest calculation you performed in question **14c–h** is known as compound interest, where interest is calculated on interest. You will study it in further detail next year. Calculate the final value of each of these investments by performing the interest calculations annually.

**a** $10 000 at 8% p.a. for 3 years

**b** $15 000 at 6.8% p.a. for 2 years

**c** $18 000 at 7.5% p.a. for 4 years

**d** $50 000 at 10% p.a. for 3 years

**16** For each investment in question **15**:

**i** determine the amount of interest earned over the investment term

**ii** calculate how much more was earned by using compound interest rather than simple interest.

**17** This bank statement is linked to a savings account and shows the transactions made during the month of April.

| Date | Transaction | Amount $ | Balance $ |
|---|---|---|---|
| 01/04 | Opening balance | | 2905.60 |
| 03/04 | Deposit – Pay | 1230.75 | 4136.35 |
| 08/04 | ATM Withdrawal | 250.00 | 3886.35 |
| 15/04 | EFTPOS Purchase | 499.95 | 3386.40 |
| 17/04 | Deposit – Pay | 1230.75 | 4617.15 |
| | Monthly interest | 7.54 | 4624.69 |

What is the annual interest rate (% p.a.) that applies to this account? (Remember that each new balance applies from the day of the transaction.) Round your answer to one decimal place.

**18** The statement shown is linked to a credit card where interest is charged from the day of purchase. To avoid additional charges, the total amount spent, plus interest, is to be paid each month.

| Date | Description | Amount $ |
|---|---|---|
| 06/07 | BPAY to electricity provider | 290.00 |
| 08/07 | Gym membership | 72.00 |
| 11/07 | Petrol | 45.00 |
| 20/07 | AFL tickets | 85.00 |
| 21/07 | Clothing store | 189.95 |
| 24/07 | Petrol | 52.87 |
| | Interest charge for the month of July | 6.31 |

**a** How much needs to be paid at the end of the month to avoid any additional charges?

**b** What is the annual interest rate (% p.a.) that is charged to this credit account?



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Interactive skillsheet
Calculating simple interest rate

Interactive skillsheet
Calculating the time period for an investment

Interactive skillsheet
Calculating the principle by simple interest formula

Investigation
What percentage interest are you really paying?

Topic quiz
1F

# Chapter summary

## Order of operations

BIDMAS

| Brackets | Indices | Division & Multiplication | Addition & Subtraction |
|---|---|---|---|
| ( ) [ ] | $2^2$ √ | ÷ × | + – |

## Rounding

$1.2395 \approx 1.24$ $1.2325 \approx 1.23$

9, 8, 7, 6, 5 → Round up

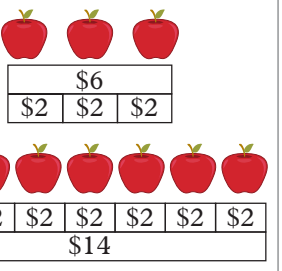
4, 3, 2, 1, 0 → Round down

## Percentages, fractions and decimals

| | Percentage | Fraction | Decimal |
|---|---|---|---|
| **Percentage to…** | | Write the percentage as a fraction with a denominator of 100. Simplify your result. | Divide the percentage by 100. |
| **Fraction to…** | Type the fraction as a quotient into the calculator, and then multiply by 100. | | Type the fraction as a quotient into the calculator. |
| **Decimal to…** | Multiply the decimal by 100. | Place the decimal as the numerator of the fraction and the denominator 10, 100, 1000 … with as many zeroes as there are digits after the decimal point. Simplify your result. | |

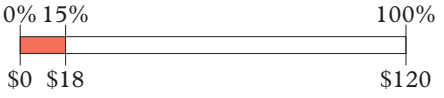
## The unitary method

- Determine the cost of one unit by simplifying the rate.
- Multiply or divide the unitary rate to determine the cost of a number of units.



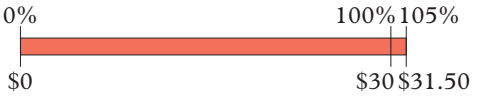

## Percentage of an amount

$$15\% \text{ of } \$120 = \frac{15}{100} \times \$120$$
$$= 0.15 \times \$120$$
$$= \$18$$




## Percentage increase

$$\text{Increase \$30 by 5\%} = (100\% + 5\\%) \times \$30$$
$$= 105\% \times \$30$$
$$= 1.05 \times \$30$$
$$= \$31.50$$




## Profit and loss

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

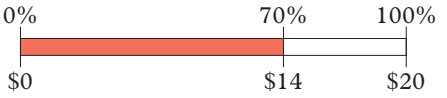
$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

Profit ⟶ selling price > cost price

Loss ⟶ selling price < cost price

## Percentage decrease

$$\text{Decreases \$20 by 30\%} = (100\% - 30\\%) \times \$20$$
$$= 70\% \times \$20$$
$$= 0.7 \times \$20$$
$$= \$14$$




## GST

GST is added to the selling price (after the mark-up) by performing a percentage increase of 10% to the marked-up price of a product or service.

## Revenue

Revenue is the selling price multiplied by the number of items (services) sold

## Simple interest

$I = PrT$ (simple interest: $I$ = interest, $P$ = principal, $r$ = interest rate, $T$ = time)

$I = PrT$ Solve for principal

$I = PrT$ Solve for interest rate

$I = PrT$ Solve for time

- $I$ = interest
- $P$ = principal, the original amount of money borrowed or invested
- $r$ = interest rate, usually a percentage converted to a fraction or decimal For example, 5% would be substituted as $\frac{5}{100}$ or 0.05. The interest rate 5% p.a. means 5% per year and is often abbreviated to p.a.
- $T$ = time of the loan or investment in years

# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

*You can use your calculator for all questions in this review unless otherwise specified.*

## Multiple-choice

1A **1** Correct to four decimal places, 49 291.591 028 is:

**A** 49 290 **B** 49 291 **C** 49 291.591 **D** 49 291.5910 **E** 49 291.60

1A **2** $\frac{3.2}{2.56}$ is not equivalent to:

**A** 1.25 **B** $1\frac{1}{4}$ **C** $\frac{4}{5}$ **D** 125% **E** $3.2 \times 0.390\,625$

1B **3** Which rate is in simplest form?

**A** driving at a speed of 100 km per hour

**B** paying \$52.06 for 38 L of petrol

**C** being charged \$10.32 for a 12-minute mobile phone call

**D** earning \$631.90 for 35.5 hours work

**E** answering 80 questions in 60 minutes

1C **4** A sporting goods store is selling children's tennis racquets at a discount of 20%. If the racquets are initially priced at \$49.50, what will their sale price be?

**A** \$9.90 **B** \$29.50 **C** \$39.60 **D** \$59.40 **E** \$61.88

1D **5** A bike rider paid \$240 for his bike and sold it 12 months later for \$180. Which statement is *not* correct?

**A** The sale represents a loss of \$60.

**B** The sale is a 30% loss on the selling price.

**C** The sale is a 25% loss on the original price.

**D** The selling price is 75% of the original price.

**E** The original price is 300% more than the loss.

1E **6** \$12 000 is invested at 4.2% p.a. simple interest for 18 months. Which values should be substituted into the simple interest formula?

**A** $P = 12\,000, r = 4.2, T = 18$

**B** $P = 12\,000, r = 0.42, T = 1.5$

**C** $P = 12\,000, r = 4.2, T = 1.5$

**D** $P = 12\,000, r = 0.042, T = 18$

**E** $P = 12\,000, r = 0.042, T = 1.5$

1F **7** A loan of \$4500 with simple interest 8.5% p.a. is charged \$1530 in interest so that \$6030 is now owed. Which simple interest variable do you not know the value of?

**A** time **B** principal **C** interest rate **D** interest amount **E** total amount

## Short answer

1A **1** Evaluate the following correct to four decimal places.

**a** $\sqrt{3.7^2 + 8.5^2}$ **b** $\frac{9.4 - 14.24}{-2.4 - (-6.04)}$ **c** $\frac{1}{3} \times \pi \times 0.25^2 \times 12.8$

1A **2** Complete the table.

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{32.2}{3.5}$ | | |
| | 3.052 | |
| | | 0.28% |

1B **3** Write each statement as a rate in simplest form.

**a** driving 185 km in 2 hours

**b** earning $193.80 for 8.5 hours work

**c** a 275 mL can of drink costing $2.50

1B **4** The hours worked by four employees are displayed in the table. The normal hourly rate of pay is $22.50. Use the information to determine each employee's gross income.

| | Total hours worked | | |
|---|---|---|---|
| **Employees** | **Normal rate** | **Time-and-a-half** | **Double time** |
| Employee 1 | 24 | 5 | 1 |
| Employee 2 | 30 | 0 | 6 |
| Employee 3 | 14 | 6 | 10 |
| Employee 4 | 0 | 15 | 10 |

1C **5** Calculate the price to be paid after:

**a** a 15% discount on $758

**b** a 22.5% discount on $84

**c** a 85% mark-up on $140

**d** a 155% mark-up on $68.

1C **6** Calculate the original price for:

**a** a mobile phone sold for $225 after a discount of 20%

**b** paint sold at $49.95 per can after a mark-up of 80%.

1D **7** For each of these:

**i** state if a profit or loss has been made and determine the amount

**ii** write the profit or loss amount as a percentage of the original price, correct to two decimal places.

**a** cost price $35, selling price $50

**b** cost price $104.50, selling price $85.85

**c** cost price $199.95, selling price $245.65

1D **8** Write these amounts as percentages.

**a** $55 as a percentage of $275

**b** $80 as a percentage of $120

**c** $150 as a percentage of $60

**d** $145 as a percentage of $25

1D **9** For each of the following, calculate:

**i** the revenue

**ii** the total profit

**iii** the percentage the revenue is of the profit.

**a** cost price $5, selling price $20, 10 sold

**b** cost price $0.99, selling price $2.50, 120 sold

1E **10** Calculate the simple interest in each case.

**a** $P = \$3000$, $r = 5\%$ p.a., $T = 4$ years

**b** $P = \$6400$, $r = 2.5\%$ p.a., $T = 3$ years

**c** $P = \$35\,000$, $r = 4.4\%$ p.a., $T = 5$ months

1F **11** Find the unknown value $P$, $T$ or $r$ when:

**a** $I = \$240$, $P = \$2000$, $r = 4\%$ p.a.

**b** $I = \$854.40$, $r = 8.9\%$ p.a., $T = 2$ years

**c** $I = \$1400$, $P = \$16\,000$, $r = 3.5\%$ p.a.

**d** $I = \$630$, $P = \$3500$, $T = 2$ years

**e** $I = \$1011.50$, $P = \$8500$, $T = 3.5$ years

## Analysis

**1** Kwame is planning to drive from Melbourne to Sydney. Kwame looks up directions on his map app and it says the journey is roughly 878 km and will take 9 hours and 12 mins.

**a** What would Kwame's average speed be in km/h for the journey, correct to two decimal places?

Kwame's car has an average fuel economy of 5.9 L per 100 km and its fuel tank capacity is 51 L.

**b** Can Kwame make it from Melbourne to Sydney with one tank of petrol?

Kwame decides to stop at Wagga Wagga for a break from driving and to refuel his car, which involves a slight detour. Kwame's map app says it is roughly 452 km from Melbourne to Wagga Wagga and 459 km from Wagga Wagga to Sydney.

**c** If Kwame's average speed is the same as in part **a**, correct to the nearest minute, how much longer will Kwame be driving than his map app's original prediction?

**d** How much petrol is Kwame expected to use from Melbourne to Wagga Wagga?

**e** The petrol at Wagga Wagga costs 145.7 c/L. Correct to the nearest cent, how much will refilling his tank cost?

**f** How much petrol is expected to be left in the tank when Kwame reaches Sydney?

**2** Julia manages a bookstore and earns an annual salary of $77 432. The normal hourly rate of $21.50 applies to her casual staff, although the opportunity for overtime is available. The store's rent is $1800 per week and Julia allows an extra $200 per week to cover other costs.

**a** Each week, Julia's deductions include $120.80 in income tax, $24.50 in superannuation and $8.50 in union fees. What is her weekly net income?

**b** One week, Julia has three staff working. Simon works 24 hours at the normal hourly rate. Melanie works 15 hours at the normal rate, 3 hours at time-and-a-half and 5 hours at double time. Tahlia works 30 hours at the normal rate and 4 hours at time-and-a-half. Calculate the gross weekly income for each employee.

**c** What is the minimum amount of money that Julia's store must make in sales each week to cover the cost of staff pay and store costs?

**d** The store rental is to increase by 40%. How much extra money does Julia need to make to cover the increase?

Julia buys books for $12 each and plans to sell them for $45 each.

**e** What is the percentage mark-up that Julia plans to make on the sale of each book?

**f** Julia notices that a rival bookstore sells identical books for $34. She changes her pricing so that she beats her rival's price by 10%. What is the retail price of the books now?

**g** What is the current selling price as a percentage of the initial price paid?

**h** What is the new percentage mark-up and how does it compare with the original percentage mark-up in part **e**?

The owners receive a quote for $48 000 to re-fit the store. They have half of this amount in savings and plan to borrow the remaining amount.

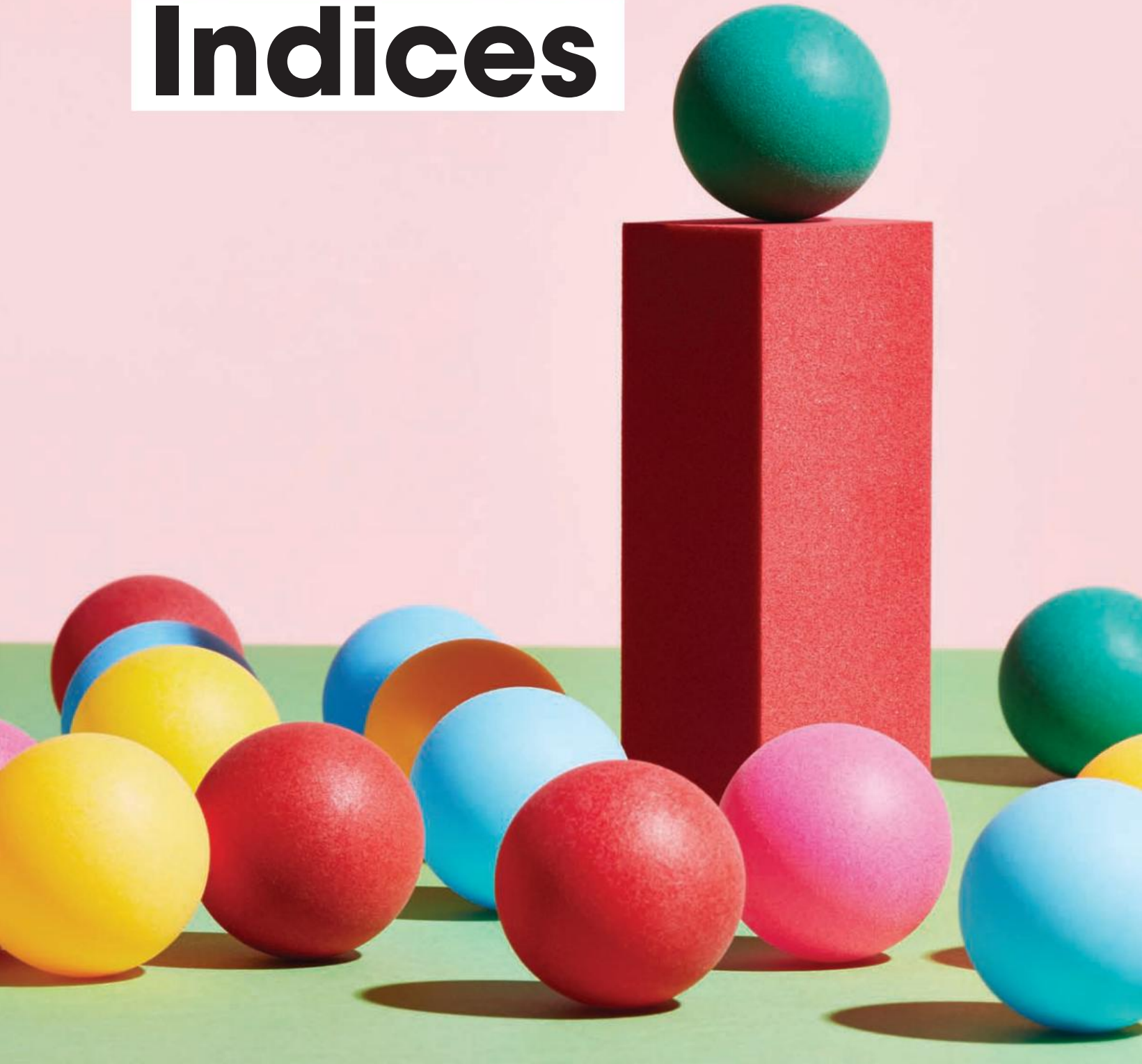
**i** The bank lends the money at a simple interest rate of 8.2% p.a. over 3 years. What is the total amount of money that must be repaid?

**j** If the money is repaid in equal monthly instalments, what is the amount?

**k** In total, how much did the store makeover cost?

# 2

# Indices



## Index


**2A** Indices
**2B** Index laws 1 and 2
**2C** Index law 3 and the zero index
**2D** Negative indices
**2E** Scientific notation
**2F** Surds


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Times tables
- ✔ Order of operations
- ✔ Prime factorisation
- ✔ Multiplying and dividing negative numbers
- ✔ Equivalent fractions

## Curriculum links

- Apply index laws to numerical expressions with integer indices (VCMNA302)
- Express numbers in scientific notation (VCMNA303)
- Extend and apply the index laws to variables, using positive integer indices and the zero index (VCMNA305)
- Investigate very small and very large time scales and intervals (VCMMG315)

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## Materials

- ✔ Calculator

# 2A Indices

## Learning intentions

- ✔ I can convert integers and terms between index and expanded form.
- ✔ I can calculate the value of numbers in index form.
- ✔ I can express integers as a product of prime factors.

**Inter-year links**

| | |
|---|---|
| Year 7 | 1G Indices and square numbers |
| Year 8 | 4A Indices |
| Year 10 | 2A Indices |

## Index notation

- **Index notation** or **index form** is used to represent repeated multiplication.
  - → $3^4$ is read as '3 to the power of 4'.
  - → $a^3$ is read as '$a$ to the power of 3'.
- The **base** is the number or variable that is multiplied repeatedly.
- The **index** or **exponent** indicates the number of times the base is multiplied.
- If no index is shown, then the base has an index of 1.

$$2 = 2^1$$
$$x = x^1$$

index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

index form expanded form basic numeral

index/exponent

base → $a^3 = a \times a \times a$

index form expanded form

- Index notation is also used to represent powers of negative numbers.

index/exponent

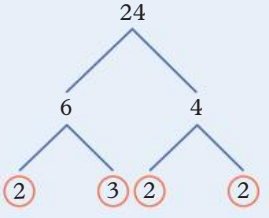
base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

index form expanded form basic numeral

  - → If the base is negative and the index is an even number, then the basic numeral will be positive.
  - → If the base is negative and the index is an odd number, then the basic numeral will be negative.

## Prime factorisation

- The **prime factorisation** of a positive integer is the product of all prime factors of that integer.
  - → Prime factorisation is often expressed in index form with the bases listed in ascending order.
    For example, the prime factorisation of 24 is: $24 = 2 \times 2 \times 2 \times 3$
    $= 2^3 \times 3$
- Prime factorisation can be performed using **factor trees**. In factor trees, composite numbers are broken down into pairs of factors until all factors are prime numbers.



## Example 2A.1 Calculating the value of a number in index form

Write the following in expanded form and then evaluate.

**a** $2^5$ **b** $(-4)^3$ **c** $\left(\frac{2}{5}\right)^4$

| THINK | WRITE |
|---|---|
| **a 1** Identify the base and the index. The base is 2 and the index is 5, so 2 is multiplied by itself 5 times. | **a** $2^5 = 2 \times 2 \times 2 \times 2 \times 2$ |
| **2** Perform the multiplication. | $= 4 \times 2 \times 2 \times 2$<br>$= 8 \times 2 \times 2$<br>$= 16 \times 2$<br>$= 32$ |
| **b 1** Identify the base and the index. The base is $-4$ and the index is 3, so $-4$ is multiplied by itself 3 times. | **b** $(-4)^3 = -4 \times -4 \times -4$ |
| **2** Perform the multiplication. Recall that if the base is negative and the index is an odd number, then the basic numeral will be negative. | $= 16 \times -4$<br>$= -64$ |
| **c 1** Identify the base and the index. The base is $\frac{2}{5}$ and the index is 4, so $\frac{2}{5}$ is multiplied by itself 4 times. | **c** $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ |
| **2** Perform the multiplication. Recall that to multiply fractions, you multiply the numerators together and the denominators together. | $= \frac{2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5}$<br>$= \frac{16}{625}$ |

## Example 2A.2 Writing variables in expanded form

Write the following in expanded form.

**a** $x^4$ **b** $(-ab)^3$ **c** $2xy^2z$

| THINK | WRITE |
|---|---|
| **a** Identify the base and the index. The base is $x$ and the index is 4, so $x$ is multiplied by itself 4 times. | **a** $x^4 = x \times x \times x \times x$ |
| **b** Identify the base and the index. The base is $-ab$ and the index is 3, so $-ab$ is multiplied by itself 3 times. | **b** $(-ab)^3 = -ab \times -ab \times -ab$ |
| **c** There are four bases in this term. Identify the bases and the matching index. Recall that if a base doesn't appear to have an index, then it has an index of 1. Therefore, 2, $x$ and $z$ each have an index of 1, and $y$ has an index of 2. | **c** $2xy^2z = 2 \times x \times y \times y \times z$ |

## **Example 2A.3** Prime factorisation using factor trees

Use a factor tree to express each number as the product of its prime factors. Write your answer in index form.

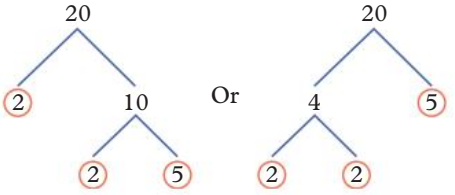
**a** 20 **b** 315

**THINK**

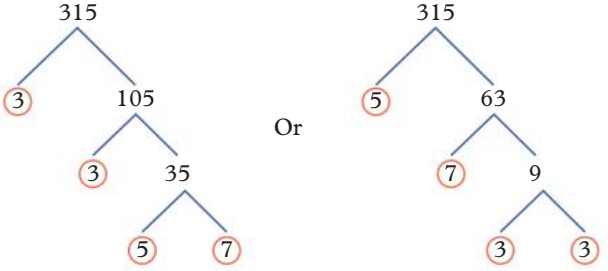
1. Identify a factor pair by dividing the composite number by its lowest prime factor. The lowest prime factor of an even number is always 2. Remember that if the sum of all the digits in a number is divisible by 3, then that number is also divisible by 3, and any number ending in 0 or 5 is divisible by 5.
2. Continue to split factors into further factor pairs until all factors are prime.
3. Write the composite number as a product of its prime factors. Write the answer in index form and list the bases in ascending order.

**WRITE**

**a** $20 = 2 \times 2 \times 5$
$= 2^2 \times 5$




**b** $315 = 3 \times 3 \times 5 \times 7$
$= 3^2 \times 5 \times 7$




**Helpful hints**

- ✔ Remember that raising a number to an index and multiplying are two different operations. For example: $2^4 \neq 2 \times 4$, $2^4 = 2 \times 2 \times 2 \times 2$
- ✔ Take care when writing indices – they should be a smaller size than the base and sit high up on the shoulder of the base to avoid confusion between $3^4$ and 34.
- ✔ When creating factor trees, remember that if a branch doesn't end on a prime number, then you must keep dividing the composite number until the branch ends on a prime.
- ✔ Recall that the first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

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# Exercise 2A Indices

| | | |
|---|---|---|
| 1–8, 9(a, c, e, g), 10, 11, 13, 14, 17(a, b) | 1–3(b, d, f, h), 4, 6–8, 9(b, d, f, h), 12, 13, 15, 17, 19 | 1–3(g, h), 4, 6(e–h), 8(e, f), 12–14, 16, 17(c, d), 18–20 |



**2A.1** **1** Write the following in expanded form and then evaluate.

**a** $6^4$ **b** $8^3$ **c** $(-2)^5$ **d** $(-3)^6$

**e** $\left(\frac{5}{4}\right)^3$ **f** $\left(\frac{1}{2}\right)^7$ **g** $\left(-\frac{2}{3}\right)^4$ **h** $\left(-\frac{3}{5}\right)^5$

**2A.2** **2** Write the following in expanded form.

**a** $b^6$ **b** $(-n)^5$ **c** $(-cd)^2$ **d** $(2pq)^4$

**e** $2pq^4$ **f** $-4a^2b^3c$ **g** $(3m^2)^5$ **h** $3(m^2)^5$

**3** Write the following in index form.

**a** $5 \times 5 \times 5 \times 5$

**b** $a \times a \times a \times a$

**c** $v \times k \times k \times v \times k \times v \times v \times 7$

**d** $qu \times qu \times qu \times qu \times qu$

**e** $-h \times -h \times -h$

**f** $-(h \times h \times h)$

**g** $n^3 \times n^3 \times n^3 \times n^3 \times n^3 \times n^3$

**h** $5b^3d^4 \times 5b^3d^4$

**2A.3** **4** Express each number as the product of its prime factors. Write your answer in index form.

**a** 50 **b** 72 **c** 135 **d** 378

**e** 152 **f** 812 **g** 550 **h** 1665

**5** Evaluate the following.

**a** $(0.2)^2$ **b** $(-0.2)^2$ **c** $(0.02)^2$

**d** $(0.2)^3$ **e** $(-0.2)^3$ **f** $(0.02)^3$

**g** $(0.2)^4$ **h** $(-0.2)^4$ **i** $(0.02)^4$

**6** Write the following in index form without brackets.

**a** $(-5)^4$ **b** $(-5)^3$ **c** $(ab)^4$ **d** $(5xy)^8$

**e** $5(xy)^8$ **f** $\left(\frac{11}{2}\right)^6$ **g** $(-3abc)^5$ **h** $(-3abc)^8$

**7** Substitute the given values and evaluate the expression.

**a** $x^3$, where $x = 7$

**b** $6a^4b^2$, where $a = -2$ and $b = \frac{1}{4}$

**c** $\frac{p^4}{qr^3}$, where $p = 3$, $q = 5$ and $r = -4$

**d** $2x^3 + 8x^2 + x + 7$, where $x = 10$

**8** Write the following in index form.

**a** $2 \times 2 \times 2 \times 3 \times 3$

**b** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 6$

**c** $13 \times 13 \times 13 \times 13 \times 17 \times 17 \times 17 \times 17 \times 17$

**d** $101 \times 101 \times 103 \times 103 \times 103 \times 103 \times 103$

**e** $4 \times 4 \times 4 \times x \times x \times x \times x$

**f** $7 \times 7 \times xy \times xy \times xy \times xy \times xy \times xy \times xy$

**9** Use the fact that $300 = 2^2 \times 3 \times 5^2$ to help find the prime factors of each of the following numbers and then write in index form.

**a** 600 **b** 150 **c** 900 **d** 1500

**e** 3000 **f** 1800 **g** 2100 **h** 2400

**10** Express each of the following in index form.

**a** $xxxxxxx$ **b** $aaabb$ **c** $3rssttt$ **d** $4eeeeeeeff$

**11** Explain the mistake in each of the following. Change the right-hand side so that the equation is correct.

**a** $tk^5 = t \times k \times t \times k \times t \times k \times t \times k \times t \times k$

**b** $(2rw)^4 = 2 \times r \times w \times w \times w \times w$

**c** $-3 \times (-2)^4 = 6^4$

**12** Substitute the given values and evaluate each expression.

**a** $(2x + 3)^8$, where $x = -2$

**b** $\left(\frac{y}{3}\right)^3 + 4\sqrt{y}$, where $y = 9$

**c** $ab^3 - ba^2$, where $a = 5$ and $b = -3$

**d** $2r^3 + 8r^2 - 3r$, where $r = -\frac{3}{2}$

**13 a** Evaluate each of the following.

**i** $(-1)^2$ **ii** $(-1)^3$ **iii** $(-1)^4$ **iv** $(-1)^5$ **v** $(-1)^6$

**vi** $(-1)^7$ **vii** $(-1)^8$ **viii** $(-1)^9$ **ix** $(-1)^{10}$ **x** $(-1)^{11}$

**b** Copy and complete the following sentences.

**i** When the index $n$ is odd, the basic numeral of $(-1)^n$ is _____.

**ii** When the index $n$ is even, the basic numeral of $(-1)^n$ is _____.

**c** Decide for each of the following whether the basic numeral will be positive or negative. Do not evaluate.

**i** $(-2)^{15}$ **ii** $(-4)^{27}$

**iii** $(-24)^{30}$ **iv** $(-17)^{198}$

**v** $(-16)^7 \times (-34)^{11}$ **vi** $(-8)^{14} \times (-5)^{27}$

**vii** $(-78)^{99} \times (-81)^{45} \times (-21)^{68}$ **viii** $\left(-\frac{77}{101}\right)^{108} \times \left(-\frac{301}{22}\right)^{404}$

**14** Consider each pair of numbers in index form.

**i** Using a calculator, evaluate each pair.

**ii** Describe how the two numbers are similar and how they are different in their index form and as a basic numeral.

**a** $(0.7)^3$ and $(0.07)^3$

**b** $(-0.4)^3$ and $-(0.4)^3$

**c** $(-1.2)^3$ and $(-1.2)^4$

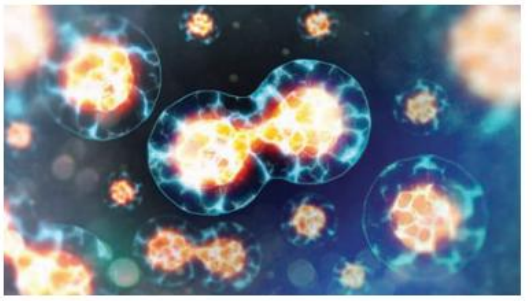
**d** $(2.1)^3$ and $(2.01)^3$

**15** A farmer's herd of cattle grows by approximately 20% each year. In 2020, the farmer had 20 cows.

**a** By what number can the number of cows be multiplied by to increase it by 20%?

**b** Predict the size of the farmer's herd in 2021, 2022 and 2025. Round to the nearest whole number.

**16** Three different groups of bacteria, Bacteria A, Bacteria B and Bacteria C, reproduce at different rates.



**a** Bacteria A splits into two bacteria every day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index form.

**b** Bacteria B splits into two bacteria twice each day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index form.

**c** Bacteria C splits into two bacteria once every two days.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index form.

**d** Populations of Bacteria A, B, and C each have 3 bacteria initially. Determine the size of each bacteria population after 3 days.

**17** The lowest common multiple is the product of the largest index of each prime factor or pronumeral in each term. The highest common factor is the product of the smallest index of each prime factor or pronumeral in each term. Find the lowest common multiple and highest common factor of each pair of terms. Write your answers in index form.

**a** $2^8 \times 3^5 \times 5^2 \times 7$ and $2^4 \times 3^{15} \times 5^2 \times 7^4$

**b** $a^8 b^5 c^2 d$ and $a^4 b^{15} c^2 d^4$

**c** $pq^5 r^7 s^2$ and $pq^3 r^{10} s^4$

**d** $8x^3 y^9 z^4$ and $12xy^3 z^4$

**18** For each of the following, state how many different sequences of answers there are. Write your answers in index notation.

**a** A quiz that has 10 true or false questions.

**b** A quiz that has 10 multiple choice questions each with options A, B, C, D, E.

**c** A quiz that has 12 true or false questions and 8 multiple choice questions with options A, B, C, D, E.

**19** Using positive and negative whole numbers (integers), see how many different index expressions that equal 64 you can find.

$\square^{\square} = 64$

**20** Evaluate each of the following.

**a** $\frac{ab^2}{c^3}$ where $a = 6$, $b = \frac{1}{3}$, and $c = -2$

**b** $\frac{p^3}{q^2}$ where $p = \frac{3}{2}$ and $q = \frac{2}{3}$

**c** $\frac{r^4}{(mn)^3}$ where $m = -0.5$, $n = 0.2$, and $r = -0.7$

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Indices
- Investigation: Using indices to determine how card tricks work
- Topic quiz: 2A

PROBLEM SOLVING AND REASONING

CHALLENGE

# 2B Index laws 1 and 2

## Learning intentions

- ✔ I can simplify products of numbers and variables with the same base.
- ✔ I can simplify quotients of numbers and variables with the same base.

**Inter-year link**

| | |
|---|---|
| Years 5/6 | Adding and subtracting whole numbers |
| Year 7 | 1B Adding whole numbers |
| Year 8 | 4B Multiplying and dividing numbers with the same base |
| Year 10 | 2A Indices |

## Index law 1: Multiplying terms with the same base

- When multiplying numbers in index form with the same base, add the indices and write the result in the same base. Writing the numbers or variables in expanded form and then simplifying achieves the same result, only at a slower rate.

$2^3 \times 2^5 = 2^{(3+5)} = 2^8$ $\quad a^3 \times a^5 = a^{(3+5)} = a^8$

For example, $2^3 \times 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$ is the same as $2^3 \times 2^5 = 2^{3+5} = 2^8$

- To multiply terms where variables have indices and coefficients:
  1. Multiply the coefficients of each term.
  2. Apply index law 1 and add the indices of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

## Index law 2: Dividing terms with the same base

- When dividing numbers in index form with the same base, subtract the second index from the first index and write the result in the same base. Writing the numbers or variables in expanded form and then simplifying achieves the same result, only at a slower rate.

$2^5 \div 2^3 = 2^{(5-3)} = 2^2$ $\quad a^5 \div a^3 = a^{(5-3)} = a^2$

- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the index of the number or variable in the denominator from the index of the number or variable in the numerator.

$\frac{2^5}{2^3} = 2^{(5-3)} = 2^2$ $\quad \frac{a^5}{a^3} = a^{(5-3)} = a^2$

- To divide numbers and variables with indices:
  1. Divide the coefficients by their highest common factor.
  2. Apply index law 2 and subtract the indices of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

### Example 2B.1 Simplifying numerical expressions using an index law

Use the appropriate index law to simplify each expression. Leave each answer in index form.

**a** $3^4 \times 3^2$ **b** $7^8 \div 7^5$ **c** $\frac{8^5}{8^2}$

| THINK | WRITE |
|---|---|
| **a** Index law 1: Add the indices of the common base, 3. | **a** $3^4 \times 3^2 = 3^{4+2}$<br>$= 3^6$ |
| **b** Index law 2: Subtract the indices of the common base, 7. | **b** $7^8 \div 7^5 = 7^{(8-5)}$<br>$= 7^3$ |
| **c** Remember that fractions can be written as division problems.<br>Index law 2: Subtract the indices of the common base, 8. | **c** $\frac{8^5}{8^2} = 8^{(5-2)}$<br>$= 8^3$ |

## Example 2B.2 Simplifying algebraic expressions using index law 1

Using index law 1, simplify each expression.

**a** $x^6 \times x^3$ **b** $2x^7 \times 3x^4$ **c** $a^3b^2 \times ab^{10}$

| THINK | WRITE |
|---|---|
| **a** Index law 1: Add the indices of the common base, $x$. | **a** $x^6 \times x^3 = x^{6+3}$<br>$= x^9$ |
| **b** 1 Multiply the coefficients of each term.<br>2 Index law 1: Add the indices of the common base, $x$.<br>3 Write the coefficient first, followed by the variable. | **b** $2x^7 \times 3x^4 = (2 \times 3) \times x^7 \times x^4$<br>$= 6 \times x^{(7+4)}$<br>$= 6 \times x^{11}$<br>$= 6x^{11}$ |
| **c** 1 Group the common bases. Remember that $a = a^1$.<br>2 Index law 1: Add the indices of the common bases, $a$ and $b$.<br>3 Write the variables in alphabetical order. | **c** $a^3b^2 \times ab^{10} = a^3 \times b^2 \times a^1 \times b^{10}$<br>$= a^{(3+1)} \times b^{(2+10)}$<br>$= a^4b^{12}$ |

## Example 2B.3 Simplifying algebraic expressions using index law 2

Using index law 2, simplify each expression.

**a** $x^5 \div x^2$ **b** $\frac{8x^9}{12x^5}$ **c** $a^5b^3 \div a^2b$

| THINK | WRITE |
|---|---|
| **a** Index law 2: Subtract the indices of the common base, $x$. | **a** $x^5 \div x^2 = x^{5-2}$<br>$= x^3$ |
| **b** 1 Divide the coefficients by the highest common factor.<br>2 Index law 2: Subtract the indices of any common bases.<br>3 Write the coefficient first, followed by the variable. | **b** $\frac{8x^9}{12x^5} = \frac{8^2 \times x^9}{12^3 \times x^5}$<br>$= \frac{2}{3} \times \frac{x^9}{x^5}$<br>$= \frac{2}{3} \times x^{(9-5)}$<br>$= \frac{2}{3}x^4$ or $\frac{2x^4}{3}$ |
| **c** 1 Index law 2: Subtract the indices of the common bases, $a$ and $b$. Remember that $b = b^1$.<br>2 Write the variables in alphabetical order. | **c** $a^5b^3 \div a^2b = a^{(5-2)}b^{(3-1)}$<br>$= a^3b^2$ |

**Helpful hints**

- ✔ 'Simplify' and 'evaluate' are two different command terms:
  - → To **simplify** in this chapter, use index laws to combine the terms and hence reduce the complexity of the calculation or numerical expression.
  - → To evaluate or 'find the value' of a calculation or numerical expression, convert the expression from index form into a basic numeral.
- ✔ Indices only apply to the number or pronumeral immediately to the left of the index. For example, in the term $4gh^3$, the index of 3 only applies to the variable $h$, so $4gh^3 = 4 \times g \times h \times h \times h$.
- ✔ Recall the rules for multiplying positive and negative numbers. $+ \times - = -$ $- \times - = +$

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# Exercise 2B Index laws 1 and 2

△ 1–5, 6–8(1st, 3rd columns), 9, 11(a, b), 14 ■ 1(f, h, j, l), 3–9(2nd, 4th columns), 10, 11(b, c), 12(a, b), 14, 15(a–d) ⬠ 5–9(2nd, 4th columns), 10, 11(c, d), 12(b, c), 13, 15, 16

UNDERSTANDING AND FLUENCY

**2B.1** **1** Use the appropriate index law to simplify each expression. Leave each answer in index form.

**a** $3^5 \times 3^4$ **b** $7^8 \div 7^2$ **c** $(-2)^7 \times (-2)^5$ **d** $6 \times 6^2$

**e** $(-8)^{13} \div (-8)^6$ **f** $10^2 \times 10^9$ **g** $3^6 \div 3^5$ **h** $5^3 \div 5$

**i** $\frac{4^7}{4^5}$ **j** $\frac{(-9)^{18}}{(-9)^9}$ **k** $\frac{13^9}{13^6}$ **l** $2^5 \times 2^2 \times 2^3$

**2** Using a calculator, calculate the basic numeral for question **1** parts **a** to **d**.

**2B.2** **3** Using index law 1, simplify each expression.

**a** $3y^3 \times y^6$ **b** $g^2 \times 7g^5$ **c** $2b^8 \times 3b^3$ **d** $-6k^5 \times 2k^8$

**e** $-2b^8 \times -3b^3$ **f** $-5g^5 \times -2g \times -8g^5$ **g** $3c \times 3c^7 \times 3c^6$ **h** $p^6 \times -3p^2 \times -5p^2$

**2B.3** **4** Using index law 2, simplify each expression.

**a** $a^6 \div a^4$ **b** $d^7 \div d^6$ **c** $g^{11} \div g$ **d** $p^{10} \div p^7$

**e** $a^8 \div a^3$ **f** $n^{14} \div n^{11}$ **g** $r^9 \div r$ **h** $8x^{17} \div x^6$

**5** Use the index laws to simplify each expression.

**a** $3x^5 \times 4x^6$ **b** $5x^4 \times 2x^3$ **c** $-8x^2 \times 3x^7$ **d** $-6x^{10} \times -9x$

**e** $6x^7 \div (2x^3)$ **f** $-20x^6 \div (-5x^2)$ **g** $4x^8 \div (10x^7)$ **h** $15x^{12} \div (9x^4)$

**i** $\frac{-24t^{18}}{3t^6}$ **j** $\frac{-20r^8}{-32r^2}$ **k** $\frac{10c^7}{2c^3}$ **l** $\frac{15y^{12}}{6y^5}$

**6** Use the index laws to simplify:

**a** $\frac{ax^{13}}{ax^4}$ **b** $\frac{a^2b}{a^3}$ **c** $\frac{m^5n}{m^5}$ **d** $\frac{b^{20}d}{b^{12}d}$

**e** $\frac{x^4 \times x^3}{x^2}$ **f** $\frac{m^7 \times m^6}{m^9}$ **g** $\frac{6a^2 \times a^8}{a^4}$ **h** $\frac{n^5 \times n^7}{n^3 \times n^4}$

**i** $\frac{5d^6 \times d^3}{d^9}$ **j** $\frac{8t^2 \times t^3}{-2t^3}$ **k** $\frac{-4k \times 3k^9}{-6k^{10}}$ **l** $\frac{15e^{13}}{3e^8 \times 5e^5}$

**7** Simplify each expression.

**a** $a^3b^4 \times a^6b^2$ **b** $6m^5n^2 \times -3m^6n$ **c** $\frac{c^2d^9}{d^7}$ **d** $\frac{k^3m^8}{km^5}$

**e** $x^2 \times y^5 \times x^6 \times y^2$ **f** $3g^4 \times 5h^3 \times 2g^6$ **g** $a^5b^4 \times a^3b^2$ **h** $5x^6y^5 \times 3x^2y^5$

**i** $9w^4x^8 \times 6x^5y^4$ **j** $\frac{1}{tu^3} \times \frac{tu^7}{5}$ **k** $\frac{-6e^5f^{11}}{8e^4f}$ **l** $-4v^9 \times -9y^3 \times -3v^8y^7$

**8** Use the index laws to simplify each expression.

**a** $\frac{x^7 \times x^3}{x^4}$ **b** $\frac{2k^4 \times k^5}{k^6}$ **c** $\frac{4a^2 \times 3a^6}{2a^7}$ **d** $\frac{5m^2 \times 2x^4}{10x^6}$

**e** $\frac{a^5b^7 \times a^3b^6}{a^8b^{10}}$ **f** $\frac{n^{17}p^{13}}{n^3p^2 \times np^8}$ **g** $\frac{-6jq^5 \times 5j^7q^2}{15j^3q}$ **h** $\frac{6w^9x^6 \times 3w^4x^5}{9w^5x^4 \times w^6x^3}$

**9** Use the index laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

**a** $x^3 \times x^4 = x^{12}$ **b** $k^3 + k^3 = k^6$ **c** $y^7 \div y = y^6$ **d** $a^5 \times a \times a^5 = a^{10}$

**e** $m^3n^5 \times m^2n^4 = m^{14}n^{14}$ **f** $100^8 \div 100^2 = 100^4$ **g** $\frac{m^3 \times m^7}{m^{11}} = \frac{1}{m}$ **h** $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2$

**10** If the index of the denominator is greater than the index of the numerator, we can instead subtract the numerator's index from the denominator's index, leaving the base on the denominator. For example: $\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2}$

**a** Simplify the following. Write your answers in index form.

**i** $\frac{3^4}{3^{10}}$ **ii** $\frac{5^2}{5^8}$ **iii** $\frac{2^5 \times 3^2}{2^9 \times 3^7}$ **iv** $\frac{2^5 \times 3^7}{2^9 \times 3^2}$ **v** $\frac{2^9 \times 3^2}{2^5 \times 3^7}$

**b** Copy and complete the following.

**i** $\frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{\square \times \square} = \frac{1}{2^{\square}} = \frac{1}{2^{(\square - \square)}}$

**ii** $\frac{2^4}{2^8} = \frac{\square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square \times \square} = \frac{\square}{\square \times \square \times \square \times \square} = \frac{\square}{2^{\square}} = \frac{\square}{2^{(\square - \square)}}$

**iii** $\frac{5^6}{5^7} = \frac{\square \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square} = \frac{\square}{\square} = \frac{\square}{\square^{\square}} = \frac{\square}{\square^{(\square - \square)}}$

**c** Explain why subtracting the indices on the numerator or denominator when the base is the same gives the index of the number on the numerator or denominator when simplified.

**11** Write the following products in index form with prime number bases.

**a** $2 \times 4 \times 8 \times 16 \times 32$ **b** $3 \times 9 \times 27 \times 81 \times 243$ **c** $6 \times 36 \times 216$ **d** $4 \times 16 \times 64 \times 256 \times 1024$

**12** Determine the values of the unknowns.

**a** $2^5 \times 3^{4x} \times 5^{12} \times 7^{z+3} = 2^w \times 3^{12} \times 5^{6y} \times 7^{11}$

**b** $(5^x \times 7^{4y} \times 11^z) \times (3^9 \times 5^6 \times 11) = 3^9 \times 5^{15} \times 7^{24} \times 11^5$

**c** $\frac{11^a \times 13^b \times 17^{2c} \times 19^8}{11^5 \times 13^6 \times 17^3 \times 19^d} = 11^{11} \times 13^3 \times 17^5 \times 19^2$

**13** Do the index laws for multiplying and dividing terms in index form work when the terms have different bases? Explain, using $2^4 \times 3^2$ and $y^8 \div x^5$ as examples.

**14** Fill in the box to make the statement true.

**a** $2^{\square} = 8$ **b** $3^{\square} = 27$ **c** $5^{\square} = 25$ **d** $10^{\square} = 10\,000$

**15** Fill in the box to make the statement true. Start by writing the base on the right as a power of the base on the left. For example: $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{12}$

**a** $2^{\square} = 8^4$ **b** $3^{\square} = 27^5$ **c** $5^{\square} = 25^9$ **d** $10^{\square} = 10\,000^3$

**e** $4^{\square} = 16^7$ **f** $2^{\square} = 32^6$ **g** $6^{\square} = 216^2$ **h** $3^{\square} = 243^6$

**16** Simplify each of the following expressions.

**a** $a^mb^x \times a^nb^y$ **b** $a^mb^x \div (a^nb^y)$

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Index laws 1 and 2

Topic quiz
2B

# 2C Index law 3 and the zero index

## Learning intentions

- ✔ I can raise a number or a variable to two indices using an index law.
- ✔ I can evaluate calculations involving the zero index.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Multiplying and dividing whole numbers |
| Year 7 | 1D Multiplying whole numbers |
| Year 8 | 4C Raising indices and the zero index |
| Year 10 | 2A Indices |

## Index law 3: Raising a number or a variable to two indices

- When raising a number or a variable to two indices, multiply the indices. Writing the numbers or variables in expanded form and applying index law 1 achieves the same result, only at a slower rate.
  For example, $(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$ is the same as $(2^3)^5 = 2^{3\times5}$
  $= 2^{15}$ $= 2^{15}$

$$(2^3)^5 = 2^{(3\times5)} \qquad (a^2)^3 = a^{(2\times3)}$$
$$= 2^{15} \qquad = a^6$$

- To raise an index by another index:
  1. Multiply the index of every base inside the brackets by the index outside the brackets. If there is no index applied to a number or a variable, the index is 1 and still must be multiplied.
  2. Write the coefficient first, followed by the variables listed in alphabetical order.
- Every base number or variable inside brackets should have its index multiplied by the index outside the brackets.

$$(2 \times 3)^5 = 2^5 \times 3^5 \qquad (ab)^3 = a^3 \times b^3$$
$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} \qquad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

## The zero index

- Excluding 0, any number or variable with an index of 0 is equal to 1. This is because for any non-zero base: an index indicates the number of times we multiply 1 by the base. If we multiply 1 by the base zero times, we haven't multiplied so we are left with 1.
  For example, $1 = \frac{a^m}{a^m}$
  $= a^{(m-m)}$
  $= a^0$
  Therefore, $a^0 = 1$
  $2^0 = 1$ $(-k)^0 = 1$
- The order of operations also applies to simplification. Calculations in grouping symbols should be simplified first. Remember BIDMAS: Brackets, Indices, Division and Multiplication, Addition and Subtraction.

## Example 2C.1 Simplifying numerical expressions using index law 3

Use index law 3 to simplify the following products. Give your answer in index form.

**a** $(3^4)^5$ **b** $(4^3 \times 7)^2$ **c** $\left(\frac{5}{2}\right)^4$ **d** $3(2^2)^4$

**THINK**

**a** Multiply the index of 3 by the index outside the brackets.

**b** Multiply the index of every base inside the brackets by the index outside the brackets. Remember that base numbers that do not appear to have an index have an index of 1, so $7 = 7^1$.

**c** Multiply the index of every base inside the brackets by the index outside the brackets. Remember that $5 = 5^1$.

**d** Multiply the index of 2 by the index outside the brackets. The index outside the brackets only applies to the term inside the brackets.

**WRITE**

**a** $(3^4)^5 = 3^{(4\times5)}$
$= 3^{20}$

**b** $(4^3 \times 7)^2 = 4^{3\times2} \times 7^{1\times2}$
$= 4^6 \times 7^2$

**c** $\left(\frac{5}{2}\right)^4 = \frac{5^{1\times4}}{2^{1\times4}}$
$= \frac{5^4}{2^4}$

**d** $3(2^2)^4 = 3 \times 2^{2\times4}$
$= 3 \times 2^8$

## Example 2C.2 Simplifying algebraic expressions using index law 3

Using index law 3, simplify each expression.

**a** $(2x^4)^3$ **b** $(3a^2b^3)^2$ **c** $\left(\frac{-x}{y^2}\right)^3$ **d** $4(a^2b)^4$

**THINK**

**a** 1 Multiply the index of every base inside the brackets by the index outside the brackets. Remember that $2 = 2^1$.
2 Write the coefficient first, followed by the variables listed in alphabetical order.

**b** 1 Multiply the index of every base inside the brackets by the index outside the brackets.
2 Write the coefficient first, followed by the variables listed in alphabetical order. Simplify where possible.

**c** Multiply the index of every base inside the brackets by the index outside the brackets. Recall that if the base is negative and the index is an odd number, then the basic numeral will be negative.

**d** 1 Multiply the index of every base inside the brackets by the index outside the brackets. The index only applies to the terms inside the brackets.
2 Write the coefficient first, followed by the variables listed in alphabetical order.

**WRITE**

**a** $(2x^4)^3 = 2^{1\times3}x^{4\times3}$
$= 2^3x^{12}$ or $8x^{12}$

**b** $(3a^2b^3)^2 = 3^{1\times2}a^{2\times2}b^{3\times2}$
$= 3^2a^4b^6$
$= 9a^4b^6$

**c** $\left(\frac{-x}{y^2}\right)^3 = \frac{(-x)^{1\times3}}{y^{2\times3}}$
$= \frac{-x^3}{y^6}$
$= -\frac{x^3}{y^6}$

**d** $4(a^2b)^4 = 4 \times a^{2\times4}b^{1\times4}$
$= 4a^8b^4$

## Example 2C.3 Simplifying expressions using the zero index

Use the property $a^0 = 1$ to simplify each expression.

**a** $23^0$ **b** $3x^0$ **c** $(x^2)^0$

**THINK**

**a** Any number, excluding 0, to the index of 0 is equal to 1.

**b** Any variable to the index of 0 is equal to 1. Recall that an index only applies to the number or pronumeral immediately to its left.

**c** 1 Multiply the index of every base inside the brackets by the index outside the brackets.

2 Any variable to the index of 0 is equal to 1.

**WRITE**

**a** $23^0 = 1$

**b** $3x^0 = 3 \times x^0$
$= 3 \times 1$
$= 3$

**c** $(x^2)^0 = x^{2\times 0}$
$= x^0$
$= 1$

## Example 2C.4 Simplifying expressions using multiple index laws

Use the appropriate index law to simplify each expression.

**a** $(x^3)^5 \times x^2$ **b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3}$

**THINK**

**a** 1 Use index law 3 to simplify the first term. Multiply the index of every base inside the brackets by the index outside the brackets.

2 Apply index law 1 and add the indices of the common base, $x$.

**b** 1 Simplify the brackets using index law 3. Remember BIDMAS.

2 Simplify the numerator and simplify the denominator.

3 Divide the numerator by the denominator. Divide the coefficients. Keep the base and subtract the indices.

4 Use the property $a^0 = 1$ to simplify further.

**WRITE**

**a** $(x^3)^5 \times x^2 = x^{3\times 5} \times x^2$
$= x^{15} \times x^2$
$= x^{(15+2)}$
$= x^{17}$

**b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3} = \dfrac{4x^8 \times 3x^5}{2x^4 \times x^9}$
$= \dfrac{12x^{13}}{2x^4 \times x^9}$
$= \dfrac{12x^{13}}{2x^{13}}$
$= 6x^0$
$= 6 \times 1$
$= 6$

**Helpful hints**

✔ Take care not to mix up the index laws.
→ across a multiplication sign, add indices
→ across a division sign, subtract indices
→ across brackets, multiply indices

✔ Remember that $2^0 = 1$, not 0.

| Index law | Example |
|---|---|
| 1 | $a^5 \times a^3 = a^{5+3}$ |
| 2 | $a^5 \div a^3 = a^{5-3}$ |
| 3 | $(a^5)^3 = a^{5\times 3}$ $(ab)^3 = a^3 b^3$ $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ |
| zero index | $a^0 = 1$ |

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# Exercise 2C Index law 3 and the zero index

1–10(1st, 3rd columns), 12, 13(a–e), 15(a, b)

3–8(2nd, 4th columns), 9, 10(c, h, j, l), 11, 13–15, 16(b), 17

6–8(2nd, 4th columns), 9, 10, 13, 15, 16(c, d), 18–20



2C.1 **1** Use index law 3 to simplify the following. Give your answer in index form.

**a** $(6^4)^3$ **b** $(3^2)^2$ **c** $(3^3 \times 4)^2$ **d** $(2^6)^4$

**e** $(5 \times 2^7)^4$ **f** $\left(\frac{3}{4}\right)^2$ **g** $\left(\frac{5^3}{2^2}\right)^4$ **h** $\left(\frac{1}{8^5}\right)^5$

**i** $(-3^4)^7$ **j** $(-3^5)^4$ **k** $(-7^4 \times -11^3)^7$ **l** $\left(\frac{13^8}{-17^4}\right)^6$

2C.2 **2** Using index law 3, simplify each expression.

**a** $(b^5)^2$ **b** $(m^4)^2$ **c** $(j^5)^2$ **d** $(j^2)^5$ **e** $(n^{10})^8$ **f** $(p^{11})^9$

**3** Using index law 3, simplify each expression. Give your answer in index form.

**a** $(xy)^6$ **b** $(2d)^3$ **c** $(-5k)^7$ **d** $(9p)^{10}$

**e** $(-3m)^4$ **f** $\left(\frac{8}{p}\right)^2$ **g** $\left(\frac{x}{y}\right)^6$ **h** $(gh)^2$

**i** $(ab)^5$ **j** $\left(\frac{k}{m}\right)^3$ **k** $(-2x)^8$ **l** $\left(-\frac{d}{3}\right)^5$

**4** Using index law 3, simplify each expression. Give your answer in index form.

**a** $(3x^6)^4$ **b** $5(a^4b)^7$ **c** $\left(\frac{2m}{n}\right)^3$ **d** $\left(\frac{a^2}{b^5}\right)^4$

**e** $-2(u^3)^4$ **f** $\frac{4}{3}(v^7w^3)^{10}$ **g** $\frac{7}{9}\left(\frac{p}{q^6}\right)^7$ **h** $\frac{1}{2^3}(3^2r^9)^5$

**i** $8\left(\frac{1}{5^6t^{11}}\right)^4$ **j** $7(3i^{17})^2$ **k** $2^3(2^4c^5)^8$ **l** $\frac{5}{7}\left(\frac{5x^{25}}{7y^{30}}\right)^3$

2C.3 **5** Use the property $a^0 = 1$ to simplify each expression.

**a** $34^0$ **b** $(18)^0$ **c** $y^0$ **d** $(7a)^0$

**6** Use the property $a^0 = 1$ to simplify each expression.

**a** $2x^0$ **b** $(2x)^0$ **c** $-7y^0$ **d** $(-7y)^0$

**e** $-(-3c)^0$ **f** $8^0 + 4^0$ **g** $2 \times 5^0 - 3^0$ **h** $m^0 + m^0$

**i** $n^0 + p^0$ **j** $a^0 + b^0 + c^0$ **k** $(x + y)^0$ **l** $(-a^0)^4$

**m** $(5^3)^0$ **n** $(-8)^0$ **o** $-8^0$ **p** $-(-3)^0$

2C.4 **7** Use the index laws to simplify each expression.

**a** $(x^2)^4 \times x^5$ **b** $(x^5)^3 \times x^7$ **c** $x^3 \times (x^4)^6$ **d** $(x^3)^2 \times (x^7)^3$

**e** $\frac{x^4 \times (x^3)^5}{x^9}$ **f** $\frac{(w^2)^4 \times (w^5)^2}{(w^4)^3}$ **g** $\frac{6(b^4)^4 \times (b^3)^2}{18b^{21}}$ **h** $\frac{e^5 \times e^8}{e^3 \times e^4}$

**i** $\frac{(x^6)^2 \times x^3}{x^5 \times (x^2)^5}$ **j** $\frac{4a^6 \times 6(a^3)^4}{2a^4 \times 3a^5}$ **k** $\frac{t^8}{(t^2)^5} \times \frac{(t^6)^7}{t^{15}}$ **l** $(f^6)^9 \times \left(\frac{f^7}{f^2}\right)^{11}$

**8** Use the index laws to simplify each expression.

**a** $a^3 \div a^3$ **b** $-7x^9 \div x^9$ **c** $(m^2)^3 \div m^6$ **d** $-18(b^4)^5 \div [-6(b^5)^4]$

**e** $y^7 \times y \div y^8$ **f** $(k^6)^0 \times k^2$ **g** $5g^4 \times 2(-g^7)^0$ **h** $3(w^5)^2 \div (w^2)^5$

**i** $x^8 \times (x^2)^5 \div x^3$ **j** $4p^7 \times 3p^2 \div (6p^9)$ **k** $16(b^3)^3 \div [-2(b^2)^4]$ **l** $4m^5 \times m \div [10(m^3)^2]$

**9** Use the index laws to simplify each expression.

**a** $\frac{5(n^7)^2 \times -6(n^2)^3}{15n^2 \times (n^3)^6}$ **b** $\frac{(k^8)^2 \times k \times k^9}{k^3 \times (k^4)^2 \times k^5}$ **c** $(x^4)^2y^7 \times x^3y^2$

**d** $\frac{4(m^3)^4n^2 \times (m^2)^3}{8m^5n^6 \times mn}$ **e** $\frac{-3h^7k^5 \times 2h^6(k^3)^2}{(h^2)^6k^3 \times -6h(k^4)^2}$

**10** Use the index laws to simplify each expression.

**a** $(xy)^3 \times x^6y^4$ **b** $(2k)^5 \times (7k)^2$ **c** $(-3x^6)^4$ **d** $-5(a^4b)^7$

**e** $\frac{x^4}{y^5} \times \left(\frac{x}{y}\right)^6$ **f** $\left(\frac{2m}{n}\right)^3$ **g** $\left(\frac{a^2}{b^5}\right)^4$ **h** $\left(\frac{w^5x^3}{y^4}\right)^2$

**i** $\left(\frac{k^3m}{n^2}\right)^5 \times \left(\frac{n^3}{k^2m}\right)^4$ **j** $\left(\frac{t^4}{r^2p^3}\right)^5 \times \frac{(4r^5)^2}{p^6t^7}$ **k** $\frac{(a^3b^2)^5 \times (ab^4)^6}{(a^5b)^4}$ **l** $\frac{(3e^4)^2(2h^6)^3}{(e^2h^3)^4}$

**11 a** Simplify $a^3 \div a^3$ by first writing as a fraction with each term in expanded form.

**b** Simplify $a^3 \div a^3$ using an index law. Leave your answer in index form.

**c** Use your answers to parts **a** and **b** to explain why $a^0 = 1$.

**12** Use the index laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

**a** $(3g)^4 = 3^4 \times g^4$ **b** $-8^0 = -1$ **c** $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y}$ **d** $\frac{(k^3)^2 \times k^4}{k^2} = k^5$

**e** $6 + k^0 = 7$ **f** $100^9 \div 100^9 = 0$ **g** $\frac{m^3 \times m^8}{m^{11}} = 1$

**13** Find the value of $x$ that will make each statement true.

**a** $2^x = 2^7$ **b** $5^x \times 5^2 = 5^6$ **c** $4^x = 1$ **d** $7^x \div 7^3 = 7^5$

**e** $(9^x)^2 = 9^6$ **f** $\left(\frac{2}{3}\right)^x = \frac{32}{243}$ **g** $\frac{6^x \times 6^3}{6^5} = 6^5$ **h** $(3a^x)^4 = 81a^{20}$

**14** Eden simplified $3^4 \times (3^5)^3$ as $(3^9)^3 = 3^{27}$. Explain and correct her mistake.

**15** Index law 3 can be explained in terms of index law 1.
Complete the following.

**a** $(2^3)^5 = (2^3) \times (2^3) \times ____ \times ____ \times ____$
$= 2^{3+3+\square+\square+\square}$
$= 2^{3\times\square}$

**b** $(x^7)^4 = (x^7) \times ____ \times ____ \times ____$
$= x^{7+\square+\square+\square}$
$= x^{7\times\square}$

**c** $(2 \times 3)^4 = (2 \times 3) \times ____ \times ____ \times ____$
$= 2 \times __ \times __ \times __ \times 3 \times __ \times __ \times __$
$= 2^{\square} \times 3^{\square}$

**d** $\left(\frac{2}{3}\right)^6 = \frac{2}{3} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square}$
$= \frac{2^{\square}}{3^{\square}}$

**16** We can describe multiplication in terms of repeated addition, $2 \times 3 = 2 + 2 + 2$, and raising to the power of an index in terms of repeated multiplication, $2^3 = 2 \times 2 \times 2$. However, repeatedly raising a number to the same index does not require a new operation as it can be simplified to index form.
For example: $(((2^5)^5)^5) = 2^{5\times5\times5} = 2^{(5^3)} = 2^{125}$
Write the following in index form with a single index.

**a** $(((((3^2)^2)^2)^2)^2)^2$ **b** $(((5^3)^3)^3)^3$ **c** $((((7^4)^4)^4)^4)^4$ **d** $((((10^5)^5)^5)^5)^5$

**17** A cube has side lengths of 85 cm. What is the volume of the cube in $cm^3$?
Write your answer in index form.

**18** A rubber band is stretched to $\frac{4}{3}$ of its current length, and this is repeated another four times until it snaps. How many times longer was the rubber band when it snapped than it was originally?

**19** Use index law 3 to show that $(a^m)^n = (a^n)^m$.

**20** Solve the following equation for $x$.

$$\frac{12x^6y^7}{7x^2y^{10}z^4} \times \frac{35y^3z^4}{3x^3} = 3$$

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Interactive skillsheet
Index law 3 and the zero index

Topic quiz
2C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

2A **1** Write the following in expanded form then evaluate.

**a** $2^6$ **b** $(-3)^4$ **c** $(-4)^3$ **d** $\left(\frac{5}{6}\right)^3$

2A **2** Write the following in expanded form.

**a** $a^6$ **b** $(-b)^4$ **c** $(3y)^5$ **d** $3(xy)^5$

2A **3** Write the following in index form.

**a** $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $u \times u \times u \times u$

**c** $4b \times 4b \times 4b \times 4b \times 4b$ **d** $-7 \times k \times k \times k \times h \times h \times h \times h \times h$

2A **4** Write the following numbers as a product of their prime factors. Express your answer in index form.

**a** 28 **b** 72 **c** 484 **d** 270

2B **5** Use the index laws to simplify each expression. Express your answers in index form.

**a** $8^5 \times 8^6$

**b** $5^7 \times 7^4 \times 5^3 \times 7^8$

**c** $\frac{6^8}{6^3}$

**d** $\frac{3^{14} \times 10^{12}}{3^6 \times 10^5}$

2B **6** Use the index laws to simplify each expression.

**a** $a^3 \times a^9$

**b** $4b^{11}c^8 \times -3b^7c^{13}$

**c** $\frac{u^{14}}{u^9}$

**d** $\frac{-15p^{17}q^{21}}{-21p^3q^4}$

2B **7** Use the index laws to simplify each expression. Write your answer in index form.

**a** $\frac{3^7 \times 3^{12}}{3^9}$

**b** $\frac{k^{23}}{k^7 \times k^8}$

**c** $\frac{c^3t^8}{c^{14}t^7}$

**d** $\frac{9d^7w^4}{10d^{12}w^7} \times -\frac{25d^{17}w^{12}}{6dw^2}$

2C **8** Use the index laws to simplify each expression.

**a** $87^0$ **b** $t^5 \div t^5$ **c** $-(4g)^0$ **d** $7a^0 + (8b)^0$

2C **9** Use the index laws to simplify each expression. Write your answers in index form.

**a** $(3^4)^6$ **b** $(j^5)^9$ **c** $(-5a^3b^7)^6$ **d** $-\left(\frac{3p^5}{2q^7}\right)^8$

2C **10** Use the index laws to simplify each expression. Write your answers in index form.

**a** $\frac{t^4 \times (t^2)^3}{t^{10}}$

**b** $\frac{3(g^2)^8 \times (3g^5)^3}{(3g)^{11}}$

**c** $\frac{(5m^{11}n^{10})^8 \times (5mn^6)^6}{(5m^9n^7)^2 \times (5m^2n^3)^5}$

**d** $\frac{(8j^5p)^0 \times 6(j^0p^4)^3}{(j^7p^2)^6}$

# 2D Negative indices

## Learning intentions

- ✔ I can write a term with a negative index as a term with a positive index.
- ✔ I can write a term with a positive index as a term with a negative index.
- ✔ I can apply index laws to numerical and algebraic expressions with negative indices.
- ✔ I can simplify and evaluate numerical and algebraic expressions with negative indices.

**Inter-year links**

| | |
|---|---|
| Year 7 | 3F Multiplying fractions |
| Year 8 | 2C Multiplying and dividing fractions |
| Year 10 | 2B Negative indices |

## Negative indices

- A **negative index** is the reciprocal of the base with the positive index. $2^{-1} = \left(\frac{2}{1}\right)^{-1} = \frac{1}{2}$ $a^{-1} = \left(\frac{a}{1}\right)^{-1} = \frac{1}{a}$
- The reciprocal of a number or a variable $n$ is $\frac{1}{n}$ so the reciprocal of $\frac{m}{n}$ is $\frac{n}{m}$. The reciprocal of a fraction is found by swapping the numerator and denominator.
- A negative index can be used to write a fraction in index form.
- For numbers and variables with a negative index, any non-zero number $a$, and positive integer $m$: $2^{-3} = \left(\frac{2}{1}\right)^{-3} = \frac{1}{2^3}$ $a^{-m} = \left(\frac{a}{1}\right)^{-m} = \frac{1}{a^m}$
- For numbers and variables in the denominator of a fraction with a negative index, any non-zero number $a$, and positive integer $m$: $\frac{1}{2^{-3}} = \left(\frac{1}{2}\right)^{-3} = 2^3$ $\frac{1}{a^{-m}} = \left(\frac{1}{a}\right)^{-m} = a^m$
- The index laws apply to expressions containing terms with negative indices.
- The above rules can be simplified to the following.

  For any non-zero $a$, and positive integer $m$: $a^{-1} = \frac{1}{a}$, $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$

### Example 2D.1 Determining the reciprocal

Determine the reciprocal of each of the following.

**a** $\frac{3}{2}$ **b** $\frac{1}{4}$ **c** 3 **d** $2x$ **e** $\frac{1}{2x}$

**THINK**

1. Write the base with a negative index as a fraction if it is not already.
2. Find the reciprocal of the fraction. Swap the numerator and denominator.
3. Simplify the result.

**WRITE**

**a** $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$

**b** $\left(\frac{1}{4}\right)^{-1} = \frac{4}{1}$
$= 4$

**c** $3^{-1} = \left(\frac{3}{1}\right)^{-1}$
$= \frac{1}{3}$

**d** $(2x)^{-1} = \left(\frac{2x}{1}\right)^{-1}$
$= \frac{1}{2x}$

**e** $\left(\frac{1}{2x}\right)^{-1} = \frac{2x}{1}$
$= 2x$

## Example 2D.2 Writing a term with a positive index

Write each term with a positive index.

**a** $3^{-3}$ **b** $7^{-4}$ **c** $x^{-3}$ **d** $(2x)^{-2}$

**THINK**

1. Write the base with a negative index as a fraction if it is not already.
2. Find the reciprocal of the fraction and write the index as a positive number.
3. Use index rule 3 to remove the brackets.
4. Simplify the result.

**WRITE**

**a** $3^{-3} = \left(\frac{3}{1}\right)^{-3}$
$= \left(\frac{1}{3}\right)^{3}$
$= \frac{1}{3^3}$

**b** $7^{-4} = \left(\frac{7}{1}\right)^{-4}$
$= \left(\frac{1}{7}\right)^{4}$
$= \frac{1}{7^4}$

**c** $x^{-3} = \left(\frac{x}{1}\right)^{-3}$
$= \left(\frac{1}{x}\right)^{3}$
$= \frac{1}{x^3}$

**d** $(2x)^{-2} = \left(\frac{2x}{1}\right)^{-2}$
$= \left(\frac{1}{2x}\right)^{2}$
$= \frac{1}{(2x)^2}$
$= \frac{1}{4x^2}$

## Example 2D.3 Writing fractions with positive indices

Write each fraction in index form with a positive index.

**a** $\frac{1}{3^{-2}}$ **b** $\frac{1}{(2x)^{-1}}$ **c** $\left(\frac{2}{5}\right)^{-3}$ **d** $\left(\frac{3x}{y}\right)^{-2}$

**THINK**

1. Write the base with a negative index as a fraction if it is not already.
2. Find the reciprocal of the fraction and write the index as a positive number.
3. Use index rule 3 to remove the brackets.
4. Simplify the calculation. Dividing by a fraction is the same as multiplying by the reciprocal.
5. Simplify the result.

**WRITE**

**a** $\frac{1}{3^{-2}} = \frac{1}{\left(\frac{3}{1}\right)^{-2}}$
$= \frac{1}{\left(\frac{1}{3}\right)^{2}}$
$= 1 \times \frac{3^2}{1}$
$= 3^2$

**b** $\frac{1}{(2x)^{-1}} = \frac{1}{\left(\frac{2x}{1}\right)^{-1}}$
$= \frac{1}{\left(\frac{1}{2x}\right)^{1}}$
$= 1 \times \frac{2x}{1}$
$= 2x$

**c** $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$
$= \frac{5^3}{2^3}$

**d** $\left(\frac{3x}{y}\right)^{-2} = \left(\frac{y}{3x}\right)^{2}$
$= \frac{y^2}{3^2x^2}$

## Example 2D.4 Writing terms with positive indices

Write each term with positive indices.

**a** $x^{-5}y^3$ **b** $\frac{5x^{-4}}{y^{-7}}$ **c** $\frac{6a^7b^{-2}}{3c^{-4}}$

**THINK**

**a** 1 Write the term as a product of two factors and write the base with a negative index as a fraction.
2 Find the reciprocal of the fraction and write the index as a positive number.
3 Simplify the result.

**b** 1 Write the term as a product of three factors and write the base with a negative index as a fraction.
2 Find the reciprocal of the fraction and write the index as a positive number.
3 Simplify the result.

**c** 1 Cancel 6 and 3 by a common factor of 3.
2 Write the term as a product of four factors and write the base with a negative index as a fraction.
3 Find the reciprocal of the fraction and write the index as a positive number.
4 Simplify the result.

**WRITE**

**a** $x^{-5}y^3 = \left(\frac{x}{1}\right)^{-5} \times y^3$
$= \frac{1}{x^5} \times y^3$
$= \frac{y^3}{x^5}$

**b** $\frac{5x^{-4}}{y^{-7}} = 5 \times \left(\frac{x}{1}\right)^{-4} \times \frac{1}{\left(\frac{y}{1}\right)^{-7}}$
$= 5 \times \frac{1}{x^4} \times 1 \times \frac{y^7}{1}$
$= \frac{5y^7}{x^4}$

**c** $\frac{6a^7b^{-2}}{3c^{-4}} = \frac{6^2a^7b^{-2}}{3^1c^{-4}}$
$= \frac{2a^7b^{-2}}{c^{-4}}$
$= 2 \times a^7 \times \left(\frac{b}{1}\right)^{-2} \times \frac{1}{\left(\frac{c}{1}\right)^{-4}}$
$= 2 \times a^7 \times \frac{1}{b^2} \times c^4$
$= \frac{2a^7c^4}{b^2}$

## Example 2D.5 Simplifying expressions with negative indices using index laws

Use an appropriate index law to simplify each expression. Write your answers using positive indices.

**a** $3^5 \times 3^{-7}$ **b** $2^4 \div 2^{-3}$ **c** $(5^{-6})^2 \times 5^3$

**THINK**

**a** 1 Apply index law 1 to multiply the terms. Write the base and add the indices.
2 Find the reciprocal of the fraction and write the index as a positive number.

**b** Apply index law 2 to divide the terms. Write the base and subtract the indices.

**c** 1 Apply index law 3 to simplify the first term. Multiply the index of every base inside the brackets by the index outside the brackets.
2 Apply index law 1 to multiply the terms. Write the base and add the indices.
3 Find the reciprocal of the fraction and write the index as a positive number.

**WRITE**

**a** $3^5 \times 3^{-7} = 3^{(5+(-7))}$
$= 3^{-2}$
$= \frac{1}{3^2}$

**b** $2^4 \div 2^{-3} = 2^{(4-(-3))}$
$= 2^7$

**c** $(5^{-6})^2 \times 5^3 = 5^{-6\times2} \times 5^3$
$= 5^{-12} \times 5^3$
$= 5^{(-12+3)}$
$= 5^{-9}$
$= \frac{1}{5^9}$

**Helpful hints**

✔ Keep the index laws and the rules for negative indices close by until they become second nature.

$$a^{-1} = \frac{1}{a} \qquad a^{-m} = \frac{1}{a^m} \qquad \frac{1}{a^{-m}} = a^m$$

✔ If you want to move a number or a variable from the numerator to the denominator, remember that 1 will be left in its place, not zero.

For example: $\frac{a}{b} = \frac{a}{1} \times \frac{1}{b}$

$= \frac{1}{a^{-1}} \times \frac{1}{b}$

$= \frac{1}{a^{-1}b}$

✔ Don't confuse negative indices with negative numbers. For example: $2^{-3} = \frac{1}{2^3}$ and $2^{-3} \neq -(2^3)$

ANS p482

# Exercise 2D Negative indices

△ 1–8(1st column), 9–12(1st, 3rd columns), 14, 15(a–d), 16, 19, 22a–b(i, ii, iii), 26(a, b)

□ 1(f–h), 2(a, c, e), 3–10(4th column), 11–12(2nd, 4th columns), 14, 15(e–h), 17, 20, 23, 25(a, b), 26(c, g, h)

⬠ 1(c, f, h), 6–12(4th column), 13, 14, 15(e–h), 18, 21, 22, 24, 26(2nd, 4th columns), 28

**2D.1** **1** Determine the reciprocal of each of the following.

**a** $\frac{8}{7}$ **b** $\frac{1}{2}$ **c** $-3$ **d** $3w$

**e** $\frac{1}{d}$ **f** $\frac{r}{m}$ **g** $\frac{1}{5y}$ **h** $u^2$

**2D.2** **2** Write each term with a positive index.

**a** $5^{-1}$ **b** $8^{-1}$ **c** $(-2)^{-1}$ **d** $x^{-1}$

**e** $p^{-1}$ **f** $(3w)^{-1}$

**3** Write each of the following in index form with a negative index.

**a** $\frac{1}{5}$ **b** $-\frac{1}{13}$ **c** $5$ **d** $-8$

**e** $\frac{1}{m}$ **f** $x$ **g** $\frac{1}{5y}$ **h** $3w$

**i** $\frac{4}{3}$ **j** $\frac{5}{6}$ **k** $\frac{w}{a}$ **l** $v^3$

**4** Write each term with a positive index.

**a** $4^{-2}$ **b** $2^{-6}$ **c** $(-9)^{-3}$ **d** $(-5)^{-4}$

**e** $-7^{-8}$ **f** $10^{-5}$ **g** $a^{-4}$ **h** $x^{-7}$

**i** $k^{-10}$ **j** $m^{-2}$ **k** $u^{-9}$ **l** $g^{-11}$

**5** Write each fraction in index form with a negative index.

**a** $\frac{1}{3^4}$ **b** $\frac{1}{4^7}$ **c** $\frac{1}{6^5}$ **d** $\frac{1}{(-5)^3}$

**e** $\frac{1}{(-9)^2}$ **f** $-\frac{1}{11^6}$ **g** $\frac{1}{n^2}$ **h** $\frac{1}{g^{11}}$

**i** $\frac{1}{x^8}$ **j** $\frac{1}{a^9}$ **k** $\frac{1}{p^4}$ **l** $\frac{1}{w^7}$

2D.3 **6** Write each fraction in index form with positive indices.

**a** $\frac{1}{2^{-3}}$ **b** $\frac{1}{5^{-6}}$ **c** $\frac{1}{(-8)^{-4}}$ **d** $\frac{1}{3^{-9}}$

**e** $\frac{1}{(-7)^{-5}}$ **f** $-\frac{1}{4^{-2}}$ **g** $\frac{1}{x^{-7}}$ **h** $\frac{1}{y^{-3}}$

**i** $\frac{1}{c^{-4}}$ **j** $\frac{1}{z^{-35}}$ **k** $\frac{1}{(5t)^{-4}}$ **l** $\frac{1}{(uv)^{-15}}$

**7** Write each fraction in index form with positive indices.

**a** $\left(\frac{4}{5}\right)^{-2}$ **b** $\left(\frac{7}{3}\right)^{-1}$ **c** $\left(-\frac{3}{4}\right)^{-3}$ **d** $\left(-\frac{a}{2}\right)^{-4}$

**e** $\left(\frac{6}{m}\right)^{-1}$ **f** $\left(\frac{144}{u}\right)^{-1}$ **g** $\left(\frac{x}{y}\right)^{-6}$ **h** $\left(-\frac{9}{7}\right)^{11}$

**i** $\left(-\frac{13}{17}\right)^{-14}$ **j** $\left(\frac{500}{43}\right)^{-11}$ **k** $\left(\frac{u}{97}\right)^{-198}$ **l** $\left(\frac{123}{g}\right)^{-654}$

2D.4 **8** Write each term with positive indices.

**a** $x^{-2}y^{3}$ **b** $m^{6}n^{-4}$ **c** $a^{-1}c^{7}$ **d** $2k^{5}p^{-3}$

**e** $5a^{-8}b^{2}$ **f** $-4x^{-6}w^{-2}$ **g** $a^{4}b^{-5}c^{7}$ **h** $k^{-3}m^{5}n^{-8}$

**i** $7b^{9}c^{-6}d$ **j** $3x^{-2}y^{-7}z^{-4}$ **k** $-12p^{17}q^{-18}u^{-21}$ **l** $-34j^{-65}b^{-78}$

**9** Write each term with positive indices.

**a** $\frac{a^{-5}}{b^{-3}}$ **b** $\frac{k^{-6}}{p^{-2}}$ **c** $\frac{h^{-4}}{m^{-7}}$ **d** $\frac{e^{8}}{d^{-5}}$

**e** $\frac{u^{3}}{w^{-8}}$ **f** $\frac{3x^{2}}{y^{-4}}$ **g** $\frac{8h^{5}}{g^{-6}}$ **h** $\frac{-6d^{-5}}{3c^{-9}}$

**i** $\frac{-4m^{-7}}{-10n^{-3}}$ **j** $\frac{y^{-8}}{6e^{5}}$ **k** $\frac{6g^{-99}}{-12h^{11}}$ **l** $\frac{14r^{-72}}{35c^{-101}}$

**10** Write each term with positive indices.

**a** $\frac{m^{-3}n^{4}}{p^{-6}}$ **b** $\frac{8c^{2}d^{-5}}{2e^{-6}}$ **c** $\frac{4k^{-1}n^{-3}}{6p^{-4}}$ **d** $\frac{3^{-1}a^{2}c^{-5}}{b^{-3}d^{4}}$

**e** $\frac{m^{-6}n}{7^{-1}k^{-2}}$ **f** $\frac{5^{-1}k^{-2}x^{3}}{2u^{-5}w^{8}}$ **g** $\frac{-5q^{4}r^{-4}}{p^{-1}s^{3}}$ **h** $\frac{7b^{14}u^{10}}{3^{-2}g^{-15}k^{-17}}$

**i** $\frac{t^{-4}u^{-3}}{5v^{7}w^{12}}$ **j** $\frac{-4^{-3}l^{-88}n^{41}}{7^{-2}m^{-59}v}$ **k** $\frac{a^{97}b^{-105}c^{153}}{d^{-167}e^{111}f^{-89}}$ **l** $\frac{k^{402}l^{812}}{m^{-999}n^{-571}}$

2D.5 **11** Use an appropriate index law to simplify each expression. Write your answers in index form with positive indices.

**a** $4^{-5} \times 4^{2}$ **b** $7^{3} \times 7^{-4}$ **c** $2^{-6} \times 2^{8}$ **d** $(-3)^{-1} \times (-3)^{-5}$

**e** $5^{7} \times 5^{-3}$ **f** $(-2)^{-4} \div (-2)^{3}$ **g** $9^{5} \div 9^{7}$ **h** $3^{6} \div 3^{-2}$

**i** $4^{-1} \div 4^{8}$ **j** $10^{-7} \div 10^{-4}$ **k** $211^{-9} \times 211^{-5}$ **l** $13^{-87} \div 13^{13}$

**12** Use an appropriate index law to simplify each expression. Write your answers in index form with positive indices.

**a** $(5^{-3})^{2}$ **b** $(3^{-2})^{4}$ **c** $(-2^{-4})^{-1}$ **d** $(3^{-1})^{4} \times 3^{2}$

**e** $(6^{-5})^{3} \times 6^{11}$ **f** $(4^{-2})^{3} \times (4^{-5})^{-1}$ **g** $9^{3} \times 9^{-6} \times 9^{2}$ **h** $\frac{5^{4} \times 5^{-2}}{5^{-6}}$

**i** $\frac{7^{-5} \times 7^{-3}}{7^{-4} \times 7^{-7}}$ **j** $\frac{2^{8} \times (2^{-2})^{3}}{2^{5}}$ **k** $\frac{(99^{-12})^{-6} \times 99^{15}}{(99^{8})^{-5}}$ **l** $\frac{(15^{-9})^{8} \times (15^{7})^{6}}{(15^{11})^{12}}$

**13** Using a calculator, calculate the basic numeral for parts **a** to **f** in question **11**. Write your answer as a whole number or fraction.

**14** Use an appropriate index law to write each expression without brackets using positive indices only.

**a** $(a \times b)^{-7}$ **b** $(3 \times x)^{-5}$ **c** $(4 \times y)^{-1}$

**d** $(7k)^{-2}$ **e** $(2p)^{-3}$ **f** $(mk)^{-11}$

**15** Find the value of $x$ that will make each statement true.

**a** $2^{x} = \frac{1}{2^{3}}$ **b** $5^{x} = \frac{1}{5^{7}}$ **c** $3^{x} = \frac{1}{3}$ **d** $6^{x} = \frac{1}{6^{-2}}$

**e** $4^{x} = \frac{1}{16}$ **f** $3^{x} = \frac{1}{27}$ **g** $5^{x} = \frac{1}{25}$ **h** $10^{x} = \frac{1}{10\,000}$

**16** A microscopic worm is $4^{-3}$ mm in length. Using a calculator, write this length in millimetres:

**a** as a fraction

**b** as a decimal.

**17** The time for light to travel 3 m is about $10^{-8}$ s. Using a calculator, write this time in seconds:

**a** as a fraction

**b** as a decimal.

**18** The diameter of a strand of human hair is about $5^{-6}$ m. Using a calculator, write this measurement in metres:

**a** as a fraction

**b** as a decimal.

**19 a** Complete this table.

| **Index form** | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ | $2^{-1}$ | $2^{-2}$ | $2^{-3}$ | $2^{-4}$ | $2^{-5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Basic numeral** | 32 | 16 | 8 | | | 1 | | $\frac{1}{4}$ | | | |

**b** Describe the pattern you can see in the table.

**c** Following the pattern, write $2^{-6}$ as a fraction.

**d** If $2^{10}$ is 1024, write the value of $2^{-10}$ as a fraction.

**e** If $2^{-7}$ is $\frac{1}{128}$, write the value of $2^7$.

**20 a** Complete this table.

| **Index form** | $3^5$ | $3^4$ | $3^3$ | $3^2$ | $3^1$ | $3^0$ | $3^{-1}$ | $3^{-2}$ | $3^{-3}$ | $3^{-4}$ | $3^{-5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Basic numeral** | | 81 | 27 | | | 1 | | $\frac{1}{9}$ | | | |

**b** Describe the pattern you can see in the table.

**c** Following the pattern, write $3^{-6}$ as a fraction.

**d** If $3^8$ is 6561, write the value of $3^{-8}$ as a fraction.

**e** If $3^{-7}$ is $\frac{1}{2187}$, write the value of $3^7$.

**21 a** Complete this table.

| **Index form** | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ |
|---|---|---|---|---|---|---|---|---|---|
| **Basic numeral** | | | 100 | | | $\frac{1}{10}$ | $\frac{1}{100}$ | | |

**b** Describe the pattern you can see in the table.

**c** Write the value of each term as a whole number.

**i** $10^5$ **ii** $10^6$ **iii** $10^7$ **iv** $10^8$ **v** $10^9$

**d** Write the value of each term as a fraction.

**i** $10^{-5}$ **ii** $10^{-6}$ **iii** $10^{-7}$ **iv** $10^{-8}$ **v** $10^{-9}$

**e** Write $10^{-1}$ as:

**i** a fraction **ii** a decimal.

**f** Write each term as a decimal. (Hint: Use the matching fractions from your table.)

**i** $10^{-2}$ **ii** $10^{-3}$ **iii** $10^{-4}$

**g** Write each fraction in part **d** as a decimal.

**h** Explain any shortcuts you have used to obtain your answers to parts **c** to **g**.

PROBLEM SOLVING AND REASONING

**22 a** Without using a calculator, find the whole number value of each of the following. (Hint: What shortcut can you use when multiplying by a positive power of 10?)

**i** $2 \times 10^4$ **ii** $7 \times 10^3$ **iii** $3 \times 10^5$

**iv** $4 \times 10^{11}$ **v** $9 \times 10^7$

**b** Write each expression as a fraction involving positive indices.

**i** $5 \times 10^{-2}$ **ii** $8 \times 10^{-5}$ **iii** $2 \times 10^{-3}$

**iv** $7 \times 10^{-4}$ **v** $6 \times 10^{-9}$

**c** Without using a calculator, find the decimal value of each result in part **b**. (Hint: What shortcut can you use when dividing by a positive power of 10?)

**d** Use your results from part **c** to describe a shortcut that can be used when multiplying by a negative power of 10.

**23 a** Complete the following by writing the missing numerals and operation.

**i** $2 \times 3^{-1} = 2 \times \frac{\square}{\square} = \frac{2}{\square} = 2 \square 3$

**ii** $2 \div 3^{-1} = \frac{2}{\square} = 2 \times \frac{\square}{\square} = 2 \square 3$

**b** Explain the connection between multiplication, division, and reciprocals.

**24 a** Evaluate the following.

**i** $(3^{-1})^{-1}$ **ii** $(5^{-1})^{-1}$ **iii** $\left(\left(\frac{5}{4}\right)^{-1}\right)^{-1}$

**b** Use index laws to explain why $(a^{-1})^{-1} = a$.

**c** Explain what $(a^{-1})^{-1} = a$ means in terms of reciprocals.

**25** Write the following as products without fractions by using negative indices.

For example: $\frac{9x^3}{y^2} = 9x^3y^{-2}$

**a** $\frac{2}{a^2 b}$ **b** $-\frac{3t^2}{v^3}$ **c** $\frac{x^4}{5y^4}$ **d** $\frac{1}{pq^5r^{-2}}$

CHALLENGE

**26** Use an appropriate index law to simplify each expression. Write your answer using positive indices only.

**a** $x^4 \times x^{-6}$ **b** $x^{-3} \times x^{-1}$ **c** $4x^{-2} \times 2x^5$ **d** $5x^{-8} \times 6x^3$

**e** $3x^7 \times x^{-7}$ **f** $x^5 \div x^{-4}$ **g** $x^{-10} \div x^{-7}$ **h** $4x^3 \div (2x^{-2})$

**i** $6x^{-6} \div (18x^4)$ **j** $8x^7 \div (14x^{11})$ **k** $(x^{-2})^3 \times x^4$ **l** $(x^4)^5 \times x^{-9}$

**m** $(x^{-5})^3 \times 4x^2$ **n** $2x^{-3} \times (x^{-1})^5$ **o** $(x^{-4})^2 \times (x^{-3})^{-1}$ **p** $(xy)^{-7}$

**27** Write all answers from question **26** that have a positive index using a negative index.

**28** Write $a^m \div a^n$ in each of the following different forms.

**a** $a^{\square} \times a^{\square} = a^{(\square+\square)}$

**b** $\frac{1}{a^{\square} \times a^{\square}} = \frac{1}{a^{(\square+\square)}}$

**c** two variations of $a^{\square} \div a^{\square} = a^{(\square-\square)}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Negative indices

Topic quiz
2D

# 2E Scientific notation

## Learning intentions

- ✔ I can convert between scientific notation and basic numerals.
- ✔ I can identify significant figures in a number and express it in scientific notation.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Place value |
| Year 7 | 1A Place value |
| Year 8 | 1A Rounding and estimating |

## Scientific notation

- **Scientific notation** (or **standard form**) is a simple way of writing very large and very small numbers.
- A number is written in scientific notation if it is the product of a number, the significand, $a$, between 1 (inclusive) and 10 (exclusive) or $-1$ and $-10$, and a power of 10, written in index form.
  - → That is, $a \times 10^m$ where $1 \leq a < 10$ or $-10 < a \leq -1$ and $m$ is an integer.
    - If $m$ is a positive integer, the number is larger than or equal to 10.
    - If $m$ is a negative integer, the number is between 0 and 1.
    - If $m$ is zero, the number is either between 1 and 10 or $-1$ and $-10$.

significand

$1230 = 1.23 \times 10^3$

basic numeral — scientific notation

significand

$0.00\,123 = 1.23 \times 10^{-3}$

basic numeral — scientific notation

- Approximate value is a term referring to a value obtained by a calculation that uses rounded values. Numbers in scientific notation are usually approximate values.
- The index laws can be used to perform operations on numbers in scientific notation.
- To convert a number in scientific notation to a basic numeral, the index indicates the number of places the decimal point is moved.
  - → If the index is positive, move the decimal point to the right.
  - → If the index is negative, move the decimal point to the left.

$10^+$ →

$10^-$ ←

- To write a number in scientific notation, place the decimal point after the first non-zero digit and multiply by the appropriate power of 10.

$31500 = 3.15 \times 10^4$ ← index of 4

move 4 spaces to the left

$0.042 = 4.2 \times 10^{-2}$ ← index of –2

move 2 spaces to the right

- Scientific notation takes advantage of our base 10 number system as each place value represents a power of 10. Where we can write the expanded form of a numeral using powers of 10, scientific notation uses only the highest power of 10 from the expansion.

| Place value | Ten thousands | Thousands | Hundreds | Tens | Ones | . | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|---|---|
| Index form | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ | . | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
| Basic numeral | 10 000 | 1000 | 100 | 10 | 1 | . | 0.1 | 0.01 | 0.001 |

# Significant figures

- **Significant figures** are the number of digits in a number that contribute to the level of **accuracy**.
- When counting significant figures, count the first non-zero digit from left to right.
  - → All non-zero digits are significant.
    For example, 7.789 has four significant figures as all are non-zero.
  - → Zeros between two non-zero digits are significant.
    For example, 4056 has four significant figures including the zero between 4 and 5.
  - → Leading zeros are not significant.
    For example, 0.051 has two significant figures. All the zeros are leading zeros.
  - → Trailing zeros to the right of the decimal after the last non-zero significant digit are significant.
    For example, 112.00 has five significant figures. 0.0780 has three significant figures.
  - → Trailing zeros in an integer are not significant.
    For example, 8300 has two significant figures. The zeros are not significant in the integer.
- If a number is already in scientific notation, all numbers in the significand are significant.
  For example, $2.301 \times 10^{-2}$ has four significant figures.

## Example 2E.1 Writing numbers in scientific notation as basic numerals

Write each number as a basic numeral.

**a** $2.4 \times 10^6$ **b** $7.1 \times 10^{-8}$

**THINK**

**a** Multiply by $10^6$ (or 1 000 000). When multiplying by $10^6$, move the decimal point six place-value spaces to the right. Add zeros where necessary.

**b** Multiply by $10^{-8}$ (or divide by $10^8$). When dividing by $10^8$, move the decimal point eight place-value spaces to the left. Add zeros where necessary.

**WRITE**

**a** 2.400000

$2.4 \times 10^6 = 2\,400\,000$

**b** 000000007.1

$7.1 \times 10^{-8} = 0.000\,000\,071$

## Example 2E.2 Writing numbers in scientific notation

Write each number in scientific notation.

**a** 230 000 **b** 0.000 856

**THINK**

**a** Count the number of places the decimal point in 230 000 would be moved to produce 2.3. The decimal point needs to be moved five places to the right to obtain the original number, so the index is 5.

**b** Count the number of places the decimal point in 0.000 856 would be moved to produce 8.56. The decimal point needs to be moved four places to the left to obtain the original number, so the index is −4.

**WRITE**

**a** 230000

$230\,000 = 2.3 \times 10^5$

**b** 0.000856

$0.000\,856 = 8.56 \times 10^{-4}$

## Example 2E.3 Identifying significant figures

How many significant figures are shown in each number?

**a** 5.42 **b** 20 803 **c** 6.200 **d** 4000 **e** 0.0082

**THINK**

**a** All non-zero digits are significant.
**b** Zeros between non-zero digits are significant.
**c** Zeros at the end of a decimal number are significant.
**d** Zeros at the end of an integer are not significant.
**e** Zeros to the left of the first non-zero digit in a decimal number are not significant.

**WRITE**

**a** 5.42 has three significant figures.
**b** 20 803 has five significant figures.
**c** 6.200 has four significant figures.
**d** 4000 has one significant figure.
**e** 0.0082 has two significant figures.

## Example 2E.4 Writing numbers in scientific notation using significant figures

Write each number in scientific notation with the number of significant figures indicated in brackets.

**a** 53 726 (2) **b** 0.084 03 (3)

**THINK**

**a** **1** This number has five significant figures. Round to two significant figures (the nearest thousand). Remember that zeros at the end of an integer are not significant.
**2** Write in scientific notation.
**b** **1** This number has four significant figures. Round to three significant figures (the nearest ten-thousandth).
**2** Write in scientific notation. Remember that zeros at the end of a decimal are significant.

**WRITE**

**a** $53\,726 \approx 54\,000$
$= 5.4 \times 10^4$

**b** $0.084\,03 \approx 0.0840$
$= 8.40 \times 10^{-2}$

**Helpful hints**

✔ When converting from scientific notation to a basic numeral, remember that if the index is positive, move the decimal point to the right, and if the index is negative, move the decimal point to the left.

$10^+ \rightarrow$

$10^- \leftarrow$

✔ Multiplying a number by 10 increases each digit's place value by 1 column. Move the decimal point one place-value space to the right and insert a zero where necessary.

$5.23 \times 10 = 52.3$

$5.23 \times 100 = 523.$

$5.23 \times 1000 = 5230.$

✔ Dividing a number by 10 decreases each digit's place value by 1 column. Move the decimal point one place-value space to the left and insert a zero where necessary.

ANS p484

# Exercise 2E Scientific notation

▲ 1, 2, 3(1st column), 4, 5–9(1st, 2nd columns), 10–12, 14(a–c), 17, 20, 22 ■ 3(d, g, i, k), 4, 5–9(1st, 2nd columns), 13, 14, 16, 21, 23, 26 ⬟ 4, 5–9(2nd, 3rd columns), 14, 15, 18, 19, 24, 25, 27

UNDERSTANDING AND FLUENCY

**1** Calculate each of these. (Hint: Move the decimal point an appropriate number of places.)

**a** $5.4 \times 100$ **b** $7.36 \times 10\,000$ **c** $-1.8 \times 1000$

**d** $4.05 \times 100\,000$ **e** $2.753 \times 1\,000\,000$ **f** $\frac{6.1}{10}$

**g** $\frac{8.22}{1\,000\,000}$ **h** $\frac{-9.76}{10\,000}$ **i** $\frac{7.003}{100\,000}$

**2** Write each number as a power of 10.

**a** 100 **b** 1000 **c** 10 000 **d** 100 000 **e** 1 000 000

**f** 0.1 **g** 0.01 **h** 0.001 **i** 0.0001 **j** 0.000 01

2E.1 **3** Write each number as a basic numeral.

**a** $3.2 \times 10^5$ **b** $8.14 \times 10^9$ **c** $-5.0 \times 10^2$

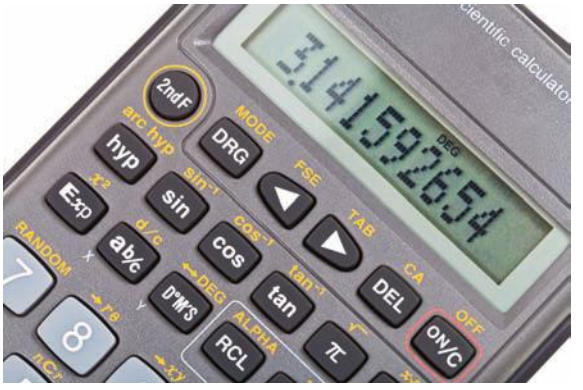
**d** $-2.345 \times 10^7$ **e** $1.1 \times 10^4$ **f** $6.4 \times 10^{-3}$

**g** $7.28 \times 10^{-6}$ **h** $9 \times 10^{-7}$ **i** $-3.02 \times 10^{-5}$

**j** $-5.41 \times 10^{-2}$ **k** $4.5 \times 10^{11}$ **l** $6.12 \times 10^{-9}$

**m** $5.7 \times 10^{-1}$ **n** $1.3068 \times 10^3$ **o** $2.7316 \times 10^{-4}$

**4** Calculators have different methods for displaying scientific notation. Most scientific calculators have a button for entering numbers in scientific notation quickly. It is usually labelled with a bold **E**, Exp, $\times 10^x$ or $\times 10^n$.
Check with your teacher if you cannot find this button.
To use the button, type the significand, press the scientific notation button, and then type the index of 10.




**a** Use a calculator to verify each number in question **3**.

**b** Were there any numbers that you could not easily obtain on your calculator? Explain.

2E.2 **5** Write each number in scientific notation.

**a** 4500 **b** 7 320 000 **c** 200 000 **d** −190

**e** 3216 **f** 0.0063 **g** 0.000 000 18 **h** 0.05

**i** −0.000 0702 **j** 0.427 **k** 11 220 **l** 0.000 004

**m** −568.2 **n** 0.000 249 **o** 679 300 **p** −0.0102

2E.3 **6** How many significant figures are shown in each number?

**a** 345 **b** 25 000 **c** 5072

**d** 400 **e** −809 **f** 0.59

**g** −0.003 **h** 1.472 **i** 48.062

**j** −7.300 **k** 36 020 **l** 0.009 04

**7** How many significant figures are shown in each number?

**a** $2.4 \times 10^3$ **b** $5.06 \times 10^{-4}$ **c** $1.900 \times 10^7$

**d** $8.0 \times 10^5$ **e** $-3.206 \times 10^{-9}$ **f** $7.00 \times 10^5$

**8** Round each number to the number of significant figures indicated in brackets.

**a** $2.58 \times 10^5$ (2) **b** $-5.037 \times 10^4$ (3) **c** $9.1042 \times 10^6$ (4) **d** $-6.00 \times 10^3$ (2)

**e** 458 (2) **f** 73 051 (4) **g** 1279 (1) **h** 40 008 (1)

**i** −5.1437 (3) **j** 0.0349 (2) **k** −42.0607 (4) **l** 0.852 (1)

2E.4 **9** Write each number in scientific notation with the number of significant figures indicated in brackets.

**a** 327 (2) **b** 48 654 (3) **c** −190 760 (4) **d** 2621 (1)

**e** 0.4031 (3) **f** −0.0544 (2) **g** 0.000 207 193 (4) **h** −0.008 327 (1)

**i** 758.4 (2) **j** −20 703.02 (4) **k** 40.155 (3) **l** 54 007.63 (5)

**10** Consider the following numbers **A–H**.

**A** $3.4 \times 10^4$ **B** $2.03 \times 10^{-3}$ **C** $-0.58 \times 10^6$ **D** $60.34 \times 10^2$

**E** 0.009 **F** $-4.19 \times 10^3$ **G** $700 \times 10^5$ **H** $9 \times 10^{-4}$

**a** Which numbers are written in scientific notation?

**b** Which numbers are written with three significant figures?

**c** Which numbers are larger than 10?

**d** Which numbers are less than 1?

**11** Write each approximate measurement in scientific notation.

**a** A medium-sized grain of sand has a length of 0.0005 m.

**b** The Maroondah Reservoir has a capacity of 22 000 ML.

**c** The thickness of the epidermal layer of skin on your eyelid is 0.048 mm.

**d** An estimate for the world's population in 2050 is 9 300 000 000.

**12** Write each approximate measurement as a basic numeral.

**a** The number of times the wings of a hummingbird flap in a minute is $6.4 \times 10^3$.

**b** The diameter of a virus is $8 \times 10^{-5}$ mm.

**c** The distance from the Sun to Earth is $1.496 \times 10^8$ km.

**d** The radius of an electron is $2.8 \times 10^{-13}$ cm.

**13** Complete the table below by writing the numbers as a product with each of the powers of 10. Underline the answers that are in scientific notation. The first row has been completed for you.

| | **1234.56** | **4.0191** | **0.0492** | **0.007 40** |
|---|---|---|---|---|
| $\times 10^3$ | $\underline{1.234\ 56 \times 10^3}$ | $0.004\ 0191 \times 10^3$ | $0.000\ 0492 \times 10^3$ | $0.000\ 007\ 40 \times 10^3$ |
| $\times 10^2$ | | | | |
| $\times 10^1$ | | | | |
| $\times 10^0$ | | | | |
| $\times 10^{-1}$ | | | | |
| $\times 10^{-2}$ | | | | |
| $\times 10^{-3}$ | | | | |

**14** We can perform arithmetic operations in scientific notation. Multiplication and division can be performed by multiplying or dividing the significands and then using index law 1 or 2 to multiply or divide the powers of 10. For example:

$(2.1 \times 10^7) \times (8.4 \times 10^3)$
$= (2.1 \times 8.4) \times (10^7 \times 10^3)$
$= 17.64 \times 10^{10}$
$= (17.64 \div 10) \times 10^{(10+1)}$
$= 1.764 \times 10^{11}$

$(2.1 \times 10^7) \div (8.4 \times 10^3)$
$= (2.1 \div 8.4) \times (10^7 \div 10^3)$
$= 0.25 \times 10^4$
$= (0.25 \times 10) \times 10^{(4-1)}$
$= 2.5 \times 10^3$

Evaluate the following products and quotients. Write your answer in scientific notation.

**a** $(1.7 \times 10^5) \times (4 \times 10^2)$ **b** $(8 \times 10^7) \div (4 \times 10^5)$ **c** $(-5 \times 10^{-5}) \times (-9 \times 10^8)$

**d** $(-6 \times 10^9) \div (1.5 \times 10^5)$ **e** $(4.1 \times 10^{-6}) \times (-3 \times 10^4)$ **f** $(7.2 \times 10^{-2}) \div (2.4 \times 10^{-7})$

**15** Addition and subtraction require digits with the same place value to be added together. Therefore, in scientific notation, both numbers must be written using the same power of 10 so that the digits in the significands have the same place value relative to the decimal point. For example:

$(2.1 \times 10^5) + (8.4 \times 10^3)$
$= (2.1 \times 10^5) + (0.084 \times 10^5)$
$= (2.1 + 0.084) \times 10^5$
$= 2.184 \times 10^5$

$(2.1 \times 10^{-2}) - (8.4 \times 10^{-3})$
$= (2.1 \times 10^{-2}) - (0.84 \times 10^{-2})$
$= (2.1 - 0.84) \times 10^{-2}$
$= 1.26 \times 10^{-2}$

Evaluate the following sums and differences. Write your answer in scientific notation.

**a** $(3.4 \times 10^2) + (7.3 \times 10^5)$ **b** $(8.52 \times 10^4) - (1.6 \times 10^3)$ **c** $(6.03 \times 10^{-3}) + (2.7 \times 10^{-4})$

**d** $(8.2 \times 10^{-3}) - (3.5 \times 10^{-2})$ **e** $(-9.8 \times 10^3) + (-7.7 \times 10^2)$ **f** $(1.01 \times 10^5) - (7.5 \times 10^3)$

**16** Light travels at a speed of approximately $3.00 \times 10^{10}$ cm/s. How many kilometres does it travel in 1 hour? Give your answer in scientific notation.

**17** Earth revolves around the Sun at an average speed of $10^5$ km/h.

**a** What distance does Earth travel in 1 day?

**b** How many days would it take Earth to travel $9.6 \times 10^8$ km?

**18** The Australian $1 coin has a mass of 9 g and a thickness of $3 \times 10^{-1}$ cm.

**a** Sarah has a pile of these coins on her desk. She stacks as many of them as she can on top of each other between two shelves in a bookcase. The distance separating the shelves is 26 cm.

**i** How many coins are in the stack?

**ii** What would be the mass of these coins?

**b** Ben takes Sarah's stack of coins and places them end-to-end in a line. The line stretches to a length of 2.15 m. What is the diameter of a $1 coin?

**19** The Sun is $1.52 \times 10^8$ km from Earth. Light from the Sun travels towards Earth at a speed of $3 \times 10^8$ m/s. How long does it take this light to reach Earth? Give your answer to the nearest minute.

**20 a** Round each of the following to one, two and three significant figures.

**i** 1.901 **ii** 1.994 **iii** 1.997

**iv** 2.003 **v** 2.006 **vi** 2.098

**b** Explain how the significant trailing zeros are important in determining to how many significant figures a number is rounded.

**21** Consider 0.41, 0.000 000 000 0012 and −0.000 034.

**a** Round each number to one significant figure.

Assume, only for this question, that we included leading zeros as significant.

**b** Round each number to one significant figure.

**c** Explain why not including leading zeros as significant is more useful than including them. Consider how including them is similar to rounding to a place value or number of decimal places.

**22** Explain the mistake each student made.

**a** Jane rounded 4.1025 to three significant figures as 4.103.

**b** Kaleb rounded 0.0432 to three significant figures as 0.04.

**c** Lisa rounded 102 948.3618 to three significant figures as 102 900.

**d** Marius rounded 102 948.3618 to three significant figures as 103.

**23 a** Write the following in seconds in scientific notation.

**i** 47 minutes **ii** 14 days

**iii** 40 weeks **iv** 1 year (not a leap year)

**b** Write the following in the unit in brackets correct to three significant figures.

**i** 10 000 seconds (hours) **ii** 1 000 000 seconds (days)

**iii** 1 000 000 000 seconds (years) **iv** 1 000 000 000 000 seconds (millennia)

**24** We use metric prefixes for very large and very small measurements. These are closely related to engineering notation or engineering form, which are similar to scientific notation.

A number written in engineering notation is the product of a number, the significand, $a$, between positive or negative 1 (inclusive) and positive or negative 1000 (exclusive) and a power of 1000 written as a power of 10 in index form. That is, $a \times 10^{3m}$ where $1 \leq a < 1000$ or $-1000 < a \leq -1$ and $m$ is an integer.

For example, $3.456 \times 10^9$, $34.56 \times 10^6$, $345.6 \times 10^{-6}$ are in engineering form but $0.3456 \times 10^9$, $3456 \times 10^6$, $345.6 \times 10^{-5}$ are not in engineering form.

| Prefix | Abbreviation | Meaning | Power of 10 |
|---|---|---|---|
| tera | T | one trillion of the unit | $10^{12}$ |
| giga | G | one billion of the unit | $10^9$ |
| mega | M | one million of the unit | $10^6$ |
| kilo | k | one thousand of the unit | $10^3$ |
| milli | m | one-thousandth of the unit | $10^{-3}$ |
| micro | μ | one-millionth of the unit | $10^{-6}$ |
| nano | n | one-billionth of the unit | $10^{-9}$ |
| pico | p | one-trillionth of the unit | $10^{-12}$ |

**a** Write the following in seconds in engineering form.

**i** 7.3 kiloseconds (7.3 ks)
**ii** 9.1 microseconds (9.1 μs)
**iii** 54 nanoseconds (54 ns)
**iv** 82 teraseconds (82 Ts)
**v** 129 megaseconds (129 Ms)
**vi** 974 picoseconds (974 ps)

**b** Write the following times in engineering form using the appropriate prefix.

**i** $5.601 \times 10^7$ seconds
**ii** $9.2 \times 10^5$ seconds
**iii** $4.31 \times 10^{-5}$ seconds
**iv** $7.88 \times 10^{-7}$ seconds
**v** $8 \times 10^{-2}$ seconds
**vi** $1.0356 \times 10^{14}$ seconds



**25** Sound travels at 330 m/s, whereas light travels at $3 \times 10^5$ km/s.

**a** Compare the speed of light and the speed of sound.

A timekeeper stands at the end of a 100 m straight running track. The starting gun at the beginning of the track goes off.

**b** How long does it take:

**i** for the sight of the smoke to reach the timekeeper?
**ii** for the sound of the gun to reach the timekeeper?

**c** What advice should you give the timekeeper in order to have an accurate recording of the time of the race?

**26** The circumference of a hydrogen atom is $7.98 \times 10^{-9}$ cm. How far would a line of 1 million hydrogen atoms stretch if placed next to each other?

**27** Consider the multiplication problem $2^{350} \times 3^2 \times 4^3 \times 5^{355}$. Write the exact answer in scientific notation.

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Scientific notation

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Measuring large units in our solar system

CAS instructions
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Topic quiz
2E

# 2F Surds

## Learning intentions

- ✔ I can simplify roots to surds.
- ✔ I can multiply surds.
- ✔ I can divide surds.

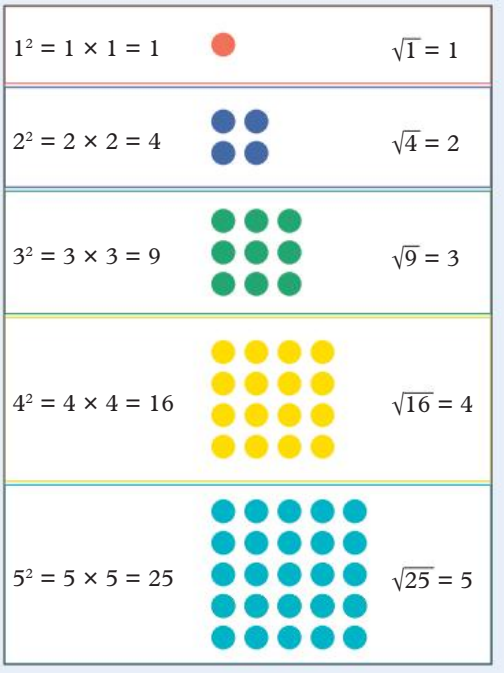
**Inter-year links**

| | |
|---|---|
| Year 7 | 1G Indices and square roots |
| Year 8 | 4E Roots |
| Year 10 | 3A Rational and irrational numbers |

## Surds

- A **perfect square** is the square of an integer.
  For example, 1, 4, 9, 16, 25… are perfect squares.
- A **surd** is an irrational number that cannot be simplified.
  - → $\sqrt{2}$ cannot be written as a fraction, integer or recurring or terminating decimal, so it is a surd.
  - → $\sqrt{4}$ can be simplified to 2, so it is not a surd.

  $\sqrt{1} = 1$
  $\sqrt{2} \approx 1.4142135\ldots$
  $\sqrt{3} \approx 1.7320508\ldots$
  $\sqrt{4} = 2$
- Surds can be other types of roots such as the square root, $\sqrt{2}$, the cube root, $\sqrt[3]{2}$, or fourth root, $\sqrt[4]{2}$.
- Exact value refers to a value given precisely as measured or recorded, not rounded off. Surds are a way to present numbers as exact values that have not been rounded.




## Basic rules for square roots

- The inverse operation of taking a square root is an index of 2.
- The inverse operation of an index of 2 is taking a square root.
- A square root can be written as the product of the square roots of its positive factors.
- A square root divided by a square root can be written as the square root of the quotient.

$(\sqrt{2})^2 = 2$ $\quad$ $(\sqrt{a})^2 = a$

$\sqrt{2^2} = 2$ $\quad$ $\sqrt{a^2} = a$

$\sqrt{6} = \sqrt{2} \times \sqrt{3}$ $\quad$ $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$\dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$ $\quad$ $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

## Simplifying square roots to surds

- To simplify a square root with a perfect square factor:
  1. Rewrite the number as a product of its factors, including the square number.
  2. Write the square root as a product of the square root of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
  3. Simplify the perfect square roots.
  4. Multiply the remaining square roots and simplify the expression.

$\sqrt{12} = \sqrt{4 \times 3}$
$= \sqrt{4} \times \sqrt{3}$
$= 2 \times \sqrt{3}$
$= 2\sqrt{3}$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# Multiplying and dividing surds

- To multiply square roots, use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ in reverse, then simplify.

  For example, $\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8}$
  $= \sqrt{16}$
  $= 4$
- To divide square roots, use the rule $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ to first simplify the fraction under the root, and then simplify the square root.

  For example, $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}}$
  $= \sqrt{4}$
  $= 2$

## Example 2F.1 Simplifying surds

Simplify the following surds.

**a** $\sqrt{8}$ **b** $\sqrt{22}$ **c** $\sqrt{108}$

**THINK**

**a** 1 Rewrite the number as a product of its factors, including the square number 4.
2 Write the square root as a product of the square root of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
3 Simplify the perfect square roots.
4 Multiply the remaining square roots and simplify the expression.

**b** Rewrite the number as a product of its factors. There are no square number factors so $\sqrt{22}$ is already in its simplest form.

**c** 1 Rewrite the number as a product of its factors, including the square numbers 4 and 9.
2 Write the square root as a product of the square root of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
3 Simplify the perfect square roots and multiply together.
4 Multiply the remaining square roots and simplify the expression.

**WRITE**

**a** $\sqrt{8} = \sqrt{4 \times 2}$
$= \sqrt{4} \times \sqrt{2}$
$= 2 \times \sqrt{2}$
$= 2\sqrt{2}$

**b** $\sqrt{22} = \sqrt{2 \times 11}$
$= \sqrt{2} \times \sqrt{11}$
Therefore, $\sqrt{22}$ is already in its simplest form.

**c** $\sqrt{108} = \sqrt{4 \times 9 \times 3}$
$= \sqrt{4} \times \sqrt{9} \times \sqrt{3}$
$= 2 \times 3 \times \sqrt{3}$
$= 6 \times \sqrt{3}$
$= 6\sqrt{3}$

## Example 2F.2 Writing simplified surds as square roots

Write the simplified surd, $4\sqrt{3}$, as the square root of a single number.

**THINK**

1 Write the simplified surd as a product of the integer and square root.
2 Rewrite the integer as a square root using $a = \sqrt{a^2}$.
3 Multiply the square roots together using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

**WRITE**

$4\sqrt{3} = 4 \times \sqrt{3}$
$= \sqrt{16} \times \sqrt{3}$
$= \sqrt{16 \times 3}$
$= \sqrt{48}$

## Example 2F.3 Multiplying surds

Simplify the following expressions containing surds.

**a** $\sqrt{15} \times \sqrt{6}$ **b** $\sqrt{8} \times \sqrt{75}$

**THINK**

**a** **1** Multiply the square roots together using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

**2** Identify a perfect square factor and write the number as a product of the factors.

**3** Simplify the square root.

**b** **1** Multiply the square roots together using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

**2** Identify a perfect square factor and write the number as a product of the factors.

**3** Simplify the square root.

**WRITE**

**a** $\sqrt{15} \times \sqrt{6} = \sqrt{15 \times 6}$
$= \sqrt{90}$
$= \sqrt{9 \times 10}$
$= \sqrt{9} \times \sqrt{10}$
$= 3 \times \sqrt{10}$
$= 3\sqrt{10}$

**b** $\sqrt{8} \times \sqrt{75} = \sqrt{8 \times 75}$
$= \sqrt{600}$
$= \sqrt{100 \times 6}$
$= \sqrt{100} \times \sqrt{6}$
$= 10 \times \sqrt{6}$
$= 10\sqrt{6}$

## Example 2F.4 Dividing surds

Simplify the following expressions containing square roots.

**a** $\frac{\sqrt{27}}{\sqrt{3}}$ **b** $\frac{\sqrt{35}}{\sqrt{15}}$

**THINK**

**a** **1** Divide the square roots using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

**2** Simplify the fraction.

**3** Simplify the square root.

**b** **1** Divide the square roots using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

**2** Simplify the fraction by cancelling a common factor of 5.

**3** Simplify the square roots. $\sqrt{7}$ and $\sqrt{3}$ are surds. Write in the form $\frac{\sqrt{a}}{\sqrt{b}}$.

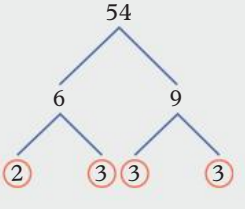
**WRITE**

**a** $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}}$
$= \sqrt{9}$
$= 3$

**b** $\frac{\sqrt{35}}{\sqrt{15}} = \sqrt{\frac{35}{15}}$
$= \sqrt{\frac{7}{3}}$
$= \frac{\sqrt{7}}{\sqrt{3}}$

**Helpful hints**

- ✔ Remember that factor trees can help to determine square number factors as a number that is multiplied by itself to give a square number. For example, $3 \times 3 = 3^2 = 9$ is a perfect square factor of 54.
- ✔ If there are no repeated prime factors in a surd, then the surd cannot be simplified.
- ✔ Simplifying surds is an important skill for Year 11 and 12 Maths.



ANS p485

# Exercise 2F Surds

▲ 1(a, c, e), 2–7(1st, 3rd columns), 8–10, 11(a, d, f, g), 12, 15(a–d)

■ 2–7(2nd, 4th columns), 8–10, 11(c, g, h), 12, 13, 15–17, 18(a, c, e, g)

⬟ 3(e–h), 5–7(f–h), 8–10, 11(f–h), 14–16, 18(b, d, f, h), 19–21

**1** Evaluate the following products.

| | | | | | |
|---|---|---|---|---|---|
| **a** | **i** $4 \times 2$ | **ii** $4 \times 3$ | **iii** $4 \times 5$ | **iv** $4 \times 6$ | **v** $4 \times 7$ |
| **b** | **i** $9 \times 2$ | **ii** $9 \times 3$ | **iii** $9 \times 5$ | **iv** $9 \times 6$ | **v** $9 \times 7$ |
| **c** | **i** $25 \times 2$ | **ii** $25 \times 3$ | **iii** $25 \times 5$ | **iv** $25 \times 6$ | **v** $25 \times 7$ |
| **d** | **i** $4 \times 4 \times 2$ | **ii** $4 \times 4 \times 3$ | **iii** $4 \times 4 \times 5$ | **iv** $4 \times 4 \times 6$ | **v** $4 \times 4 \times 7$ |
| **e** | **i** $4 \times 9 \times 2$ | **ii** $4 \times 9 \times 3$ | **iii** $4 \times 9 \times 5$ | **iv** $4 \times 9 \times 6$ | **v** $4 \times 9 \times 7$ |
| **f** | **i** $4 \times 4 \times 4 \times 2$ | **ii** $4 \times 4 \times 4 \times 3$ | **iii** $4 \times 4 \times 4 \times 5$ | **iv** $4 \times 4 \times 4 \times 6$ | **v** $4 \times 4 \times 4 \times 7$ |

2F.1 **2** Simplify the following surds.

| | | | |
|---|---|---|---|
| **a** $\sqrt{20}$ | **b** $\sqrt{27}$ | **c** $\sqrt{28}$ | **d** $\sqrt{50}$ |
| **e** $\sqrt{72}$ | **f** $\sqrt{800}$ | **g** $\sqrt{147}$ | **h** $\sqrt{384}$ |
| **i** $\sqrt{432}$ | **j** $\sqrt{847}$ | **k** $\sqrt{486}$ | **l** $\sqrt{891}$ |

2F.2 **3** Write each of the following simplified surds as the square root of a single number.

| | | | |
|---|---|---|---|
| **a** $3\sqrt{5}$ | **b** $4\sqrt{6}$ | **c** $2\sqrt{3}$ | **d** $2\sqrt{11}$ |
| **e** $5\sqrt{10}$ | **f** $8\sqrt{7}$ | **g** $13\sqrt{3}$ | **h** $12\sqrt{5}$ |

2F.3 **4** Write each of the following products of square roots as the square root of a single number.

| | | | |
|---|---|---|---|
| **a** $\sqrt{5} \times \sqrt{6}$ | **b** $\sqrt{8} \times \sqrt{12}$ | **c** $\sqrt{9} \times \sqrt{15}$ | **d** $\sqrt{24} \times \sqrt{14}$ |
| **e** $\sqrt{13} \times \sqrt{27}$ | **f** $\sqrt{50} \times \sqrt{12}$ | **g** $\sqrt{101} \times \sqrt{7}$ | **h** $\sqrt{123} \times \sqrt{45}$ |

2F.4 **5** Simplify each of the following quotients of square roots.

| | | | |
|---|---|---|---|
| **a** $\frac{\sqrt{24}}{\sqrt{8}}$ | **b** $\frac{\sqrt{65}}{\sqrt{5}}$ | **c** $\frac{\sqrt{126}}{\sqrt{6}}$ | **d** $\frac{\sqrt{144}}{\sqrt{16}}$ |
| **e** $\frac{\sqrt{132}}{\sqrt{12}}$ | **f** $\frac{\sqrt{136}}{\sqrt{8}}$ | **g** $\frac{\sqrt{360}}{\sqrt{15}}$ | **h** $\frac{\sqrt{420}}{\sqrt{14}}$ |

**6** Simplify the following products of square roots.

| | | | |
|---|---|---|---|
| **a** $\sqrt{6} \times \sqrt{8}$ | **b** $\sqrt{20} \times \sqrt{30}$ | **c** $\sqrt{18} \times \sqrt{6}$ | **d** $\sqrt{35} \times \sqrt{15}$ |
| **e** $\sqrt{32} \times \sqrt{20}$ | **f** $\sqrt{63} \times \sqrt{54}$ | **g** $\sqrt{60} \times \sqrt{96}$ | **h** $\sqrt{75} \times \sqrt{105}$ |

**7** Simplify the following quotients of square roots.

| | | | |
|---|---|---|---|
| **a** $\frac{\sqrt{960}}{\sqrt{3}}$ | **b** $\frac{\sqrt{160}}{\sqrt{20}}$ | **c** $\frac{\sqrt{882}}{\sqrt{7}}$ | **d** $\frac{\sqrt{4752}}{\sqrt{12}}$ |
| **e** $\frac{\sqrt{3456}}{\sqrt{8}}$ | **f** $\frac{\sqrt{2520}}{\sqrt{10}}$ | **g** $\frac{\sqrt{2016}}{\sqrt{24}}$ | **h** $\frac{\sqrt{3600}}{\sqrt{75}}$ |

**8** Consider the following numbers.

$\sqrt{9}, \sqrt{6}, 6^2, \sqrt[3]{6}, \sqrt{\frac{4}{9}}, \sqrt{8}, \sqrt{27}, 4^3, \sqrt{0.16}, \sqrt[4]{16}, \sqrt{\frac{7}{5}}, \sqrt{1.6}$

**a** List the roots.

**b** Of the square roots, list the surds.

**9** Evaluate the following.

| | | | |
|---|---|---|---|
| **a** $(\sqrt{3})^2$ | **b** $\sqrt{5^2}$ | **c** $\sqrt{8 \times 8}$ | **d** $\sqrt{7} \times \sqrt{7}$ |
| **e** $\frac{\sqrt{2}}{\sqrt{2}}$ | **f** $5\sqrt{2} \times \sqrt{50}$ | **g** $\frac{3\sqrt{6}}{\sqrt{54}}$ | **h** $\frac{\sqrt{24}}{2\sqrt{6}}$ |

UNDERSTANDING AND FLUENCY

**10** The coefficient of a surd represents the number of surds that have been added together. This is similar to the coefficient of a variable. For example:

$\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 5 \times \sqrt{2} = 5\sqrt{2}$

**a** Write each of the following sums in the form $a\sqrt{b}$.

**i** $\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$ **ii** $\sqrt{5} + \sqrt{5} + \sqrt{5}$ **iii** $\sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6}$ **iv** $\sqrt{10} + \sqrt{10}$

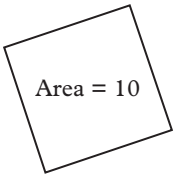
**b** Write the following as repeated sums.

**i** $5\sqrt{3}$ **ii** $2\sqrt{6}$ **iii** $8\sqrt{2}$ **iv** $3\sqrt{11}$

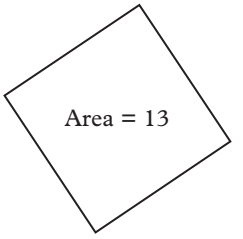
**11** Determine the side lengths of the following squares. Write your answers as simplified surds. All measurements are in metres.

**a**

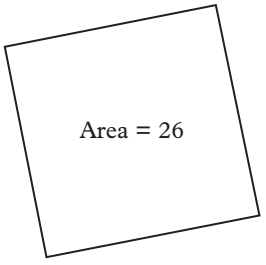
**b**



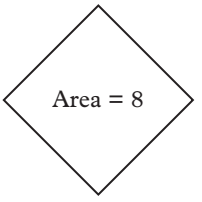

**c**



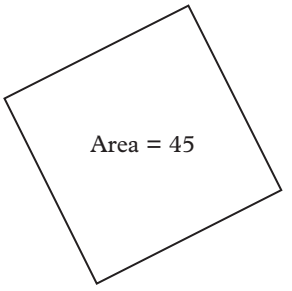

**d**



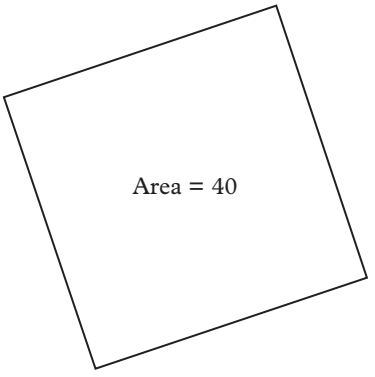

**e**



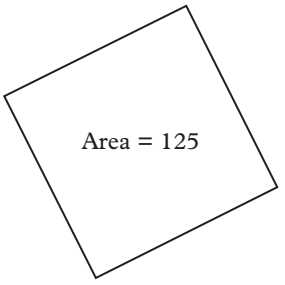

**f**




**g**




**h**




**12** When simplifying a surd, write the number as a product of its prime factors to help you find the square factors. You can do this by pairing repeated prime factors. For example:

$\sqrt{3 \times 7 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11}$
$= \sqrt{3 \times (7 \times 7) \times (7 \times 7) \times (11 \times 11) \times 11}$
$= \sqrt{3} \times \sqrt{7 \times 7} \times \sqrt{7 \times 7} \times \sqrt{11 \times 11} \times \sqrt{11}$
$= 7 \times 7 \times 11 \times \sqrt{3 \times 11}$
$= 539\sqrt{33}$

Simplify the following by pairing repeated prime factors.

**a** $\sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$ **b** $\sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 7 \times 11 \times 11}$

**c** $\sqrt{2^4 \times 3^2 \times 3 \times 5 \times 7}$ **d** $\sqrt{2^3 \times 3 \times 5^5}$

**13 a** Factorise the following numbers by writing them as a product of their prime factors.

**i** 288 **ii** 270 **iii** 980 **iv** 1125

**b** Hence, simplify the following surds.

**i** $\sqrt{288}$ **ii** $\sqrt{270}$ **iii** $\sqrt{980}$ **iv** $\sqrt{1125}$

PROBLEM SOLVING AND REASONING

**14** When simplifying products of surds, writing the numbers as products of their prime factors before multiplying can help you to find the square factors. Pair repeated factors from both sets of products. For example:

$$\begin{aligned}\sqrt{15} \times \sqrt{6} &= \sqrt{3 \times 5} \times \sqrt{2 \times 3} \\ &= \sqrt{2 \times 3 \times 3 \times 5} \\ &= \sqrt{9 \times 2 \times 5} \\ &= \sqrt{9} \times \sqrt{10} \\ &= 3\sqrt{10}\end{aligned}$$

Simplify the following by first writing the numbers as a product of their prime factors.

**a** $\sqrt{21} \times \sqrt{14}$ **b** $\sqrt{27} \times \sqrt{33}$ **c** $\sqrt{30} \times \sqrt{18}$ **d** $\sqrt{42} \times \sqrt{231}$

**15** To multiply or divide surds with coefficients, multiply or divide the coefficients and then multiply or divide the numbers in the surd.

For example: $2\sqrt{3} \times 4\sqrt{5} = 2 \times 4 \times \sqrt{3 \times 5} = 8\sqrt{15}$ and $\frac{8\sqrt{6}}{2\sqrt{3}} = \frac{8}{2}\sqrt{\frac{6}{3}} = 4\sqrt{2}$

Simplify the following products and quotients.

**a** $4\sqrt{3} \times 7\sqrt{2}$ **b** $5\sqrt{6} \times 8\sqrt{15}$ **c** $\frac{12\sqrt{56}}{3\sqrt{8}}$ **d** $\frac{7\sqrt{72}}{14\sqrt{12}}$

**e** $12\sqrt{70} \times 7\sqrt{50}$ **f** $\frac{24\sqrt{120}}{88\sqrt{24}}$ **g** $9\sqrt{91} \times 16\sqrt{78}$ **h** $\frac{196\sqrt{252}}{16\sqrt{42}}$

**16** Evaluate the following products and quotients.

**a** $\sqrt{120} \times \sqrt{270}$ **b** $\frac{\sqrt{96}}{\sqrt{24}}$ **c** $\sqrt{35} \times \sqrt{28} \times \sqrt{45}$ **d** $\frac{\sqrt{2700}}{\sqrt{12}}$

**17 a** Evaluate each of the following.

**i** $2^3$ **ii** $3^3$ **iii** $5^3$ **iv** $6^3$ **v** $7^3$ **vi** $8^3$

**b** Simplify the following surds.

**i** $\sqrt{8}$ **ii** $\sqrt{27}$ **iii** $\sqrt{125}$ **iv** $\sqrt{216}$ **v** $\sqrt{343}$ **vi** $\sqrt{512}$

**c** Explain how to simplify the square root of a cube number $\sqrt{a^3}$.

**18** Surds can be added or subtracted in the same way we add like terms. You can add or subtract the coefficients of surds that have the same surd factor after simplifying.

For example: $7\sqrt{2} + 3\sqrt{2} = 10\sqrt{2}$ and $7\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$

Simplify the following sums and differences.

**a** $5\sqrt{3} + 2\sqrt{3}$ **b** $5\sqrt{3} - 2\sqrt{3}$

**c** $12\sqrt{5} + \sqrt{5}$ **d** $9\sqrt{11} - 8\sqrt{11}$

**e** $4\sqrt{7} + 2\sqrt{7} - 6\sqrt{3} + 2\sqrt{7} - 5\sqrt{3}$ **f** $12\sqrt{3} - 4\sqrt{5} - 7\sqrt{3} + 9\sqrt{2} + 8\sqrt{2} + 7\sqrt{5} - 4\sqrt{2} + 2\sqrt{3}$

**g** $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32}$ **h** $\sqrt{12} + \sqrt{20} + \sqrt{45} + \sqrt{48}$

**19** Evaluate the following.

**a** $\frac{\sqrt{2} \times \sqrt{10}}{\sqrt{5}}$ **b** $\frac{\sqrt{24}}{\sqrt{5}} \times \frac{\sqrt{15}}{\sqrt{6}}$ **c** $\frac{\sqrt{8+13}}{\sqrt{4+3}}$

**20** Evaluate the following.

**a** $(5\sqrt{3})^2$ **b** $(3\sqrt{5})^2$ **c** $(4\sqrt{7})^2$ **d** $(8\sqrt{6})^2$

**21** Determine the smallest surd you would need to multiply (or divide) these surds by so that the product (or quotient) is not a surd.

**a** $\sqrt{8}$ **b** $\sqrt{45}$ **c** $\sqrt{12}$ **d** $\sqrt{50}$ **e** $\sqrt{75}$ **f** $\sqrt{54}$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Simplifying surds

Interactive skillsheet
Multiplying and dividing surds

CAS instructions
Simplifying surds

Topic quiz
2F

# Chapter summary

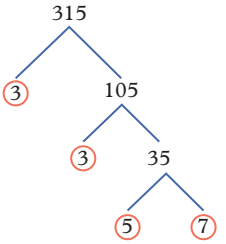
## Indices

index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

index form | expanded form | basic numeral

## Prime factorisation




$315 = 3 \times 3 \times 5 \times 7$
$= 3^2 \times 5 \times 7$

## Index law 1: Multiplying numbers with the same base

$2^3 \times 2^5 = 2^{(3+5)} = 2^8$

$a^3 \times a^5 = a^{(3+5)} = a^8$

## Index law 2: Dividing numbers with the same base

$2^5 \div 2^3 = 2^{(5-3)} = 2^2$

$a^5 \div a^3 = a^{(5-3)} = a^2$

## Index law 3: Raising a number or variable to two indices

$(2^3)^5 = 2^{(3\times5)} = 2^{15}$

$(a^2)^3 = a^{(2\times3)} = a^6$

$(2\times3)^5 = 2^5 \times 3^5$

$(ab)^3 = a^3 \times b^3$

$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$

$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

## Negative indices

$2^{-1} = \left(\frac{2}{1}\right)^{-1} = \frac{1}{2}$

$a^{-1} = \left(\frac{a}{1}\right)^{-1} = \frac{1}{a}$

$\frac{1}{2^{-3}} = \frac{1}{\left(\frac{1}{2}\right)^{-3}}$

$\frac{1}{a^{-m}} = \frac{1}{\left(\frac{a}{1}\right)^{-m}}$

$= \frac{1}{\left(\frac{1}{2}\right)^{-3}}$

$= \frac{1}{\left(\frac{1}{a}\right)^{m}}$

$= 1 \times \frac{2^3}{1}$

$= 1 \times \frac{a^m}{1}$

$= 2^3$

$= 2^3$

## The zero index

$2^0 = 1$ $a^0 = 1$

## Scientific notation

significand

$1230 = 1.23 \times 10^3$

basic numeral | scientific notation

index of 4

$3.1500 \times 10^4 = 31\,500$

move 4 spaces to the right

index of 4

$31500 = 3.15 \times 10^4$

move 4 spaces to the left

significand

$0.00123 = 1.23 \times 10^{-3}$

basic numeral | scientific notation

index of –2

$004.2 \times 10^{-2} = 0.042$

move 2 spaces to the left

index of –2

$0.042 = 4.2 \times 10^{-2}$

move 2 spaces to the right

## Surds

$(\sqrt{2})^2 = 2$ $(\sqrt{a})^2 = a$

$\sqrt{2^2} = 2$ $\sqrt{a^2} = a$

## Multiplying surds

$\sqrt{6} = \sqrt{2} \times \sqrt{3}$

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

## Dividing surds

$\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$



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# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

2A **1** Which of the following is **not** equivalent to $9(xy)^4$?

**A** $9 \times xy \times xy \times xy \times xy$ **B** $-3^2x^4y^4$ **C** $9x^4y^4$

**D** $9xxxxyyyy$ **E** $(-3)^2x^4y^4$

2A **2** Which of the following is the prime factorisation of 360?

**A** $6^2 \times 10$ **B** $4 \times 9 \times 10$ **C** $2^3 \times 3^2 \times 5$ **D** $2^2 \times 3^2 \times 10$ **E** $3 \times 10 \times 12$

2B **3** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

**A** $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{3a}$ **D** $\frac{6ab^2}{18a^2}$ **E** $3ab^2$

2B **4** Which statement does not correctly represent one of the index laws?

**A** $m^5 \times m^2 = m^{5+2}$ **B** $(p \times q)^8 = p^8 \times q^8$ **C** $w^7 \div w^5 = w^{7-5}$

**D** $a^5 \times a = a^6$ **E** $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y}$

2C **5** Using the index laws, $\frac{5x^{13} \times 2x^4}{4x^8 \times x^0}$ simplifies to:

**A** $\frac{10x^9}{4x^8}$ **B** $\frac{5x^9}{2}$ **C** $\frac{5x^{17}}{2x^8}$ **D** $\frac{5}{2x^9}$ **E** $10x^9$

2D **6** Which of the following is *not* the reciprocal of $\frac{4}{3}$?

**A** $\frac{3}{4}$ **B** $3 \times 4^{-1}$ **C** $\left(\frac{4}{3}\right)^{-1}$ **D** $\frac{1}{3 \times 4}$ **E** $1 \div \frac{4}{3}$

2D **7** Which statement is false?

**A** $\frac{1}{7} = 7^1$ **B** $4^{-2} = \frac{1}{16}$ **C** $\frac{1}{3^6} = 3^{-6}$ **D** $7^3 \times 7^{-5} = \frac{1}{49}$ **E** $\frac{5^{-3}}{5^4} = 5^{-7}$

2E **8** Which number is equivalent to $6.4724 \times 10^2$?

**A** 0.647 24 **B** 64.724 **C** 0.064 724 **D** 64 724 **E** 647.24

2F **9** Which of the following is a surd?

**A** $\sqrt{25}$ **B** $\sqrt{\frac{49}{36}}$ **C** $\sqrt{196}$ **D** $\sqrt{91}$ **E** $\sqrt{0.16}$

2F **10** Which of the following is not fully simplified?

**A** $\sqrt{10}$ **B** $5\sqrt{24}$ **C** $7\sqrt{14}$ **D** $2\sqrt{22}$ **E** $16\sqrt{7}$

## Short answer

2A **1** Evaluate the following.

**a** $3^4$ **b** $(-5)^3$ **c** $-4^3$ **d** $\left(\frac{3}{2}\right)^5$ **e** $(0.6)^3$ **f** $(1.2)^4$

2A **2** Write the following in index form.

**a** $17 \times 17 \times 17 \times 17 \times 17 \times 17$

**b** $-5b^2 \times -5b^2 \times -5b^2 \times -5b^2 \times -5b^2$

**c** $-10 \times f \times f \times f \times v \times v \times v \times v \times v \times v \times v$

**d** $\frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3}$

2C **3** Simplify each expression using the index laws.

**a** $a^{11} \times a^5$ **b** $b^9 \div b^8$ **c** $(c^8)^2$

**d** $18d^7 \div (54d^4)$ **e** $(e^5)^5 \times (e^{11})^2$ **f** $5a^0 + 3b^0 + 1c^0$

2C **4** Simplify each expression.

**a** $\frac{m^3n^4 \times m^9n^{11}}{m^7n^7}$

**b** $\frac{(3k^5l^2)^3 \times (2k^3l^3)^4}{(2k^3l^2)^3}$

2D **5** Write each term with a positive index.

**a** $4^{-1}$ **b** $b^{-1}$ **c** $m^{-7}$ **d** $\frac{6x^3}{2y^{-3}}$

**e** $\frac{f^{-4}}{g^{-5}}$ **f** $\frac{a^{-3}b}{5^{-2}c^{-4}}$ **g** $(p^{-2})^5 \times p^{-5}$ **h** $\left(\frac{k}{l}\right)^{-7}$

2D **6** **a** If $4^8 = 65\,536$, write the value of $4^{-8}$ as a fraction.

**b** If $7^{-3} = \frac{1}{343}$, write the value of $7^3$.

2E **7** Write each number as a basic numeral.

**a** $5.876 \times 10^4$ **b** $9.02 \times 10^{-6}$

2E **8** State the number of significant figures in each part of question **7**.

2E **9** Write each number in scientific notation.

**a** 540 000 **b** 0.000 76

2E **10** Round each of the following to the number of significant figures indicated.

**a** 879 (2) **b** $2.58 \times 10^5$ (1)

2E **11** A scientist estimates that there are $3.40 \times 10^4$ bacteria in one sample and $4.6 \times 10^3$ in the second. Write the total number of bacteria:

**a** as a basic numeral

**b** rounded to two significant figures

**c** in scientific notation.

2F **12** Simplify the following square roots.

**a** $\sqrt{72}$ **b** $\sqrt{49}$ **c** $\sqrt{243}$

**d** $\sqrt{256}$ **e** $\sqrt{1000}$ **f** $\sqrt{350}$

2F **13** Simplify the following products and quotients.

**a** $\sqrt{5} \times \sqrt{7}$ **b** $\sqrt{60} \times \sqrt{70}$ **c** $\frac{\sqrt{180}}{\sqrt{36}}$ **d** $\frac{\sqrt{1625}}{\sqrt{13}}$

2F **14** Determine a simplified surd expressing the side length of a square with an area of:

**a** 30 m$^2$ **b** 71 m$^2$ **c** 44 m$^2$ **d** 189 m$^2$

2F **15** Simplify the following surds.

**a** $\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$ **b** $\sqrt{2^4 \times 2 \times 3^6 \times 3 \times 5}$

**c** $\sqrt{2^3 \times 5^4 \times 7}$ **d** $\sqrt{2^2 \times 3^2 \times 5^3 \times 7}$

## Analysis

**1** In September 2020, the population of each Australian state was recorded. The figure for each state is shown in the table.

| State | Population at 30 June 2020 ('000) |
|---|---|
| NSW | 8166.4 |
| Vic | 6680.6 |
| Qld | 5184.8 |
| SA | 1770.6 |
| WA | 2667.1 |
| Tas | 541.4 |
| NT | 246.5 |
| ACT | 431.2 |

Source: http://www.abs.gov.au/ausstats/abs@.nsf/mf/3101.0/

**a** Which states and territories have a population listed to:

**i** four significant figures?

**ii** five significant figures?

**b** Copy the table and add three additional columns.

**c** In the first new column, write the population of each state and territory in full.

**d** In the second new column, round each population to its leading digit.

**e** In the third new column, write each population in scientific notation to one significant figure.

**f** Use your answers from part **e** to determine the following. Write the values in scientific notation.

**i** Which state or territory has the highest population?

**ii** Which state or territory has the lowest population?

**iii** Calculate the difference between the highest and the lowest population.

**iv** Calculate the total population of SA, Tas, ACT and NT.

**v** Calculate the total population of Australia.

**g** The actual total population value recorded at the end of September 2020 was 25 693 100. Calculate the difference between your answer to **f** part **v** and the actual value. Why is there a difference?

**2** Thy and Asha are playing a game. They are using die rolls and a coin flip to generate the product of three numbers in index form per round. They each roll the dice to determine the value of the bases and indices and flip the coin to determine if each index is positive or negative.

The products generated after each round are multiplied together with the goal to end up with the least number of remaining factors after three rounds.

The table below shows the numbers Thy and Asha got in their three rounds.

| | Thy | Asha |
|---|---|---|
| **Round 1** | $2^4 \times 4^3 \times 5^{-3}$ | $1^3 \times 3^6 \times 5^{-5}$ |
| **Round 2** | $3^2 \times 5^6 \times 6^{-3}$ | $2^{-3} \times 3^1 \times 6^4$ |
| **Round 3** | $2^{-6} \times 2^3 \times 3^4$ | $3^{-5} \times 4^2 \times 5^3$ |

**a** Use the facts that $4 = 2^2$ and $6 = 2 \times 3$ to write Thy and Asha's round 1, 2 and 3 numbers in index form with positive indices using only the bases 2, 3 and 5.

**b** Determine Thy and Asha's final number for their game by multiplying their round 1, 2 and 3 numbers together and simplifying the products in index form with positive indices.

**c** Who won the game with the least number of factors (find the sum of the positive indices)?

**d** Did the winner have the smaller value? Explain.

Thy and Asha decide to play one more round of the game.

**e** What products do Thy and Asha need to generate to end up with a total product of 1?

Thy and Asha decide to change the rules so that they can choose which base gets which index. Their products from the first two rounds are given in the table below.

| | Thy | Asha |
|---|---|---|
| **Round 1** | $4^{-3} \times 5^4 \times 6^5$ | $2^{-3} \times 5^{-2} \times 6^4$ |
| **Round 2** | $1^5 \times 2^3 \times 3^{-4}$ | $3^{-3} \times 4^4 \times 5^2$ |

**f** Determine the product of round 1 and 2 for Thy and Asha. Write the products in index form.

For round 3:

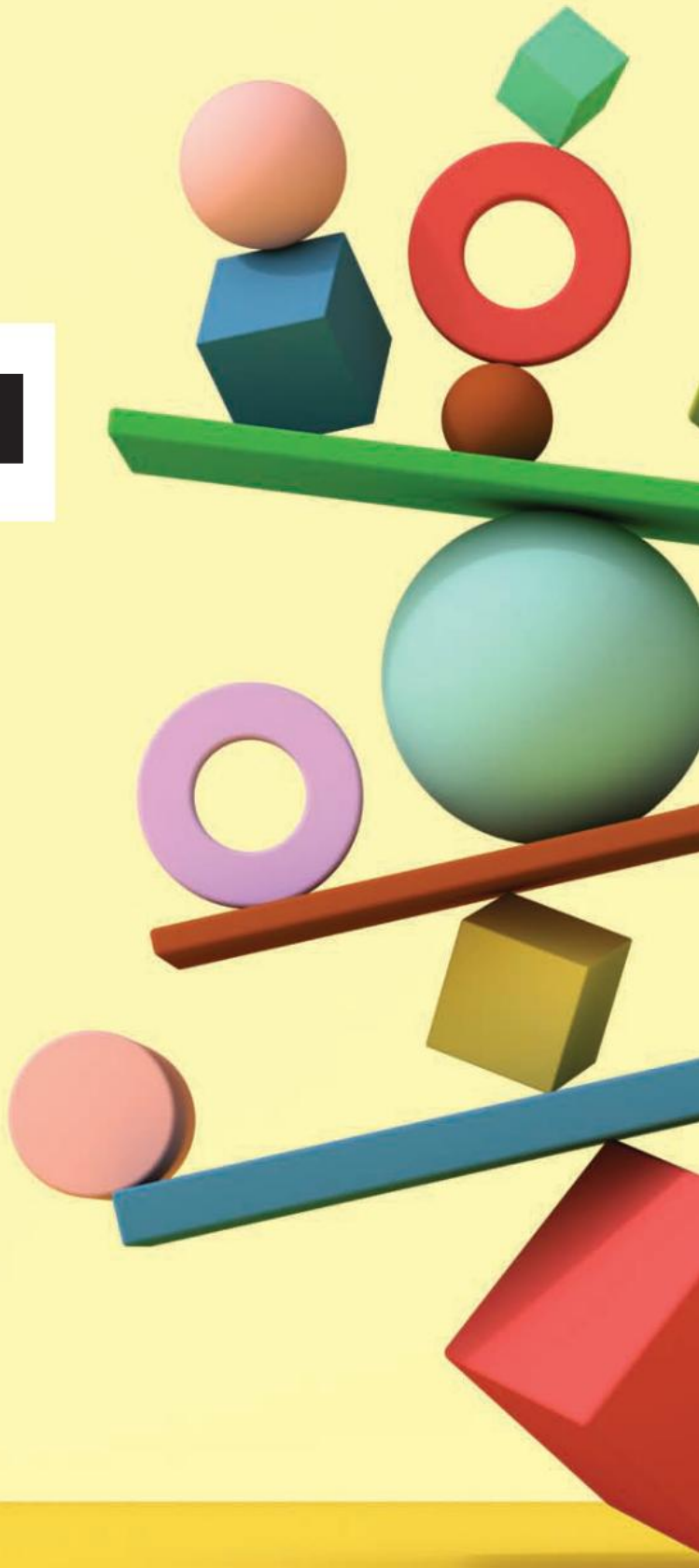
- Thy gets the bases 2, 4 and 6 and the indices −6, −4 and 1.
- Asha gets the bases 1, 5 and 5 and the indices −4, 3 and 6.

**g** Determine which index should go with which base so that Thy and Asha get the minimum number of factors remaining for the game.

**h** Who wins this game and by how many factors?

# 3

# Algebra



## Index


**3A** Simplifying
**3B** Expanding
**3C** Factorising using the HCF
**3D** Factorising the difference of two squares
**3E** Factorising quadratic expressions


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Adding and subtracting with negative numbers
- ✔ Grouping symbols
- ✔ Index laws
- ✔ Highest common factor
- ✔ Substitution

## Curriculum links

- Extend and apply the index laws to variables, using positive integer indices and the zero index (VCMNA305)
- Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (VCMNA306)

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# 3A Simplifying

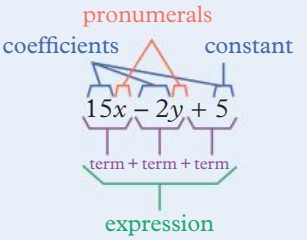
## Learning intentions

- ✔ I can simplify algebraic terms involving addition and subtraction.
- ✔ I can simplify algebraic terms involving multiplication and division.

**Inter-year links**

| | |
|---|---|
| Year 7 | 6F Simplifying |
| Year 8 | 5C Adding and subtracting algebraic terms |
| Year 10 | 2C Simplifying |

## Terms and expressions




- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an **unknown** or a **variable**.
- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- A **term** is part of an expression that is separated from the other parts by a plus or minus sign (as long as the plus or minus sign is not inside any brackets).

  Note: If the term is separated on the left by a minus sign, then the term is negative.
- A **coefficient** is the number acting as a multiplier in an algebraic term, usually written before the pronumeral. A pronumeral without a number preceding it has a coefficient of 1.
- A **constant** is a term without any pronumerals. It also counts as a coefficient.

## Adding and subtracting algebraic terms

- **Like terms** contain the same pronumerals with the same indices.
  - → The order of pronumerals can be different in two like terms.
    For example, $xyz$, $4yxz$, and $7zxy$ are all like terms.
  - → You can write terms containing indices in **expanded form** to determine whether they are like terms.
    For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$, so $a^2b$ and $ab^2$ are not like terms.

$3a^2b = 3 \times a \times a \times b$

index form | expanded form

- Like terms can be added or subtracted by adding or subtracting the coefficients of the terms.
  For example, $a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$

## Multiplying algebraic terms

- To multiply algebraic terms:
  1. Write the coefficients and pronumerals for each term in expanded form, without expanding index form.
  2. Multiply the coefficients together.
  3. Apply index law 1 to multiply the numbers in index form. Keep the base and add the indices.
  4. Simplify by leaving out the multiplication signs. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$3a^3b \times -2a^2b^2 = 3 \times -2 \times a^{(3+2)} \times b^{(1+2)}$$
$$= -6a^5b^3$$

# Dividing algebraic terms

- Remember that quotients can be expressed as fractions.
  For example, $x \div 7 = \frac{x}{7}$
- To divide algebraic terms:
  1. Write the coefficients and pronumerals for each term in expanded form, without expanding index form.
  2. Divide the coefficients by the HCF.
  3. Apply index law 2 for the division of numbers in index form. Keep the base and subtract indices.
  4. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$\frac{6a^5b^3}{3a^3b} = \frac{6 \times a^5 \times b^3}{3 \times a^3 \times b}$$
$$= 2 \times a^{(5-3)} \times b^{(3-1)}$$
$$= 2a^2b^2$$

## Example 3A.1 Adding and subtracting algebraic terms

Simplify each expression where possible.

**a** $10a - 6a + a$

**b** $4xy + 2x - 5xy$

**c** $-5bc + 2b - 3cb - \frac{b}{2}$

**d** $6x^2y - 3y^3 - 2y^2x + y^3$

**THINK**

**a** Identify like terms and simplify by adding and subtracting the coefficients.

**b** 1 Rearrange the expression so that like terms are grouped together.
2 Simplify by adding and subtracting the coefficients.

**c** 1 Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term.
2 Simplify by adding and subtracting the coefficients. Remember that fractional coefficients can be written in two ways, so $\frac{b}{2} = \frac{1}{2}b$.

**d** 1 Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term. Note that $xy^2$ and $x^2y$ are not like terms.
2 Simplify by adding and subtracting the coefficients.

**WRITE**

**a** $10a - 6a + a = (10 - 6 + 1)a$
$= 5a$

**b** $4xy + 2x - 5xy = 2x + 4xy - 5xy$
$= 2x + (4 - 5)xy$
$= 2x - xy$

**c** $-5bc + 2b - 3cb - \frac{b}{2}$
$= -5bc - 3cb + 2b - \frac{b}{2}$
$= (-5 - 3)bc + \left(2 - \frac{1}{2}\right)b$
$= -8bc + \frac{3}{2}b$

**d** $6x^2y - 3y^3 - 2y^2x + y^3$
$= 6x^2y - 3y^3 + y^3 - 2y^2x$
$= 6x^2y + (-3 + 1)y^3 - 2y^2x$
$= 6x^2y - 2y^3 - 2y^2x$

## Example 3A.2 Multiplying algebraic terms

Simplify the following products.

**a** $4de \times 7ab$ **b** $5x^2y^5 \times -2kwxy^3$

**THINK**

**a** 1 Write in expanded form.
2 Multiply the coefficients together.
3 Simplify by leaving out the multiplication signs. Write the pronumerals in alphabetical order.

**b** 1 Write the coefficients and pronumerals for each term in expanded form, without expanding index form.
2 Multiply the coefficients together.
3 Apply index law 1 for the multiplication of indices, so $x^2 \times x = x^{(2+1)}$ and $y^5 \times y^3 = y^{(5+3)}$.
4 Simplify by leaving out the multiplication signs.

**WRITE**

**a** $4de \times 7ab = 4 \times d \times e \times 7 \times a \times b$
$= 28 \times d \times e \times a \times b$
$= 28abde$

**b** $5x^2y^5 \times -2kwxy^3$
$= 5 \times x^2 \times y^5 \times -2 \times k \times w \times x \times y^3$
$= -10 \times x^{(2+1)} \times y^{(5+3)} \times k \times w$
$= -10 \times x^3 \times y^8 \times k \times w$
$= -10kwx^3y^8$

## Example 3A.3 Dividing algebraic terms

Simplify the following quotients.

**a** $\frac{-15xy}{10x}$ **b** $\frac{8a^3b}{2a^2b^2}$

**THINK**

**a** 1 Write in expanded form.
2 Cancel the coefficients, and divide –15 and 10 by the HCF of 5. Cancel any common pronumerals from the numerator and denominator.
3 Simplify the numerator and the denominator.

**b** 1 Write the coefficients and pronumerals for each term in expanded form, without expanding index form.
2 Divide each coefficient by the HCF.
3 Apply index law 2 to cancel out repeated pronumerals that appear in both the numerator and denominator. Keep the base and subtract indices.
4 Simplify the numerator and the denominator. Use $a^{-m} = \frac{1}{a^m}$ to write the pronumeral as a fraction with a positive index.

**WRITE**

**a** $\frac{-15xy}{10x} = \frac{-15 \times x \times y}{10 \times x}$
$= \frac{-15^{3} \times x^{1} \times y}{10^{2} \times x^{1}}$
$= -\frac{3y}{2}$

**b** $\frac{8a^3b}{2a^2b^2} = \frac{8 \times a^3 \times b}{2 \times a^2 \times b^2}$
$= \frac{8^{4}}{2^{1}} \times \frac{a^3}{a^2} \times \frac{b}{b^2}$
$= 4 \times a^{(3-2)} \times b^{(1-2)}$
$= 4ab^{-1}$
$= \frac{4a}{b}$

**Helpful hints**

✔ Recall the rules for writing algebraic notation:
- → Products are simplified by leaving out the multiplication sign and placing the coefficient first.
  For example: $7 \times x = 7x$ and $7 \times (x + 2) = 7(x + 2)$
- → When a pronumeral is multiplied by 1, the 1 is not shown.
  For example: $1 \times x = x$
- → Quotients are represented by fractions.
  For example: $x \div 7 = \frac{x}{7}$ and $(x + 2) \div 7 = \frac{x + 2}{7}$
- → Terms with fractional coefficients can be written in two ways.
  For example: $\frac{x}{7} = \frac{1}{7}x$

✔ Recall the rules for multiplying and dividing positive and negative numbers:

$+ \times - = -$ $\quad + \times + = +$

$- \times + = -$ $\quad - \times - = +$

✔ Take care not to mix up the index laws and definitions.
- → across a multiplication sign, add indices
- → across a division sign, subtract indices
- → across brackets, multiply indices
- → zero index: $a^0 = 1$
- → negative index: $a^{-1} = \frac{1}{a}$

| Index law | Example |
|---|---|
| 1 | $a^5 \times a^3 = a^{5+3}$ |
| 2 | $a^5 \div a^3 = a^{5-3}$ |
| 3 | $(a^5)^3 = a^{5\times3}$<br>$(ab)^3 = a^3b^3$<br>$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ |

ANS p487

# Exercise 3A Simplifying

▲ 1, 2, 3–4(1st column), 5, 6–8(1st column), 9–12, 13(a, d, e, f), 14(a, c, f), 15, 16, 21, 23(a, b)

■ 4, 5, 6–8(2nd column), 9, 10, 14(a, c, f), 17–20, 22, 23

⬟ 5, 7, 8(f, j, m), 9, 10, 14(a, c, f), 17–20, 22–24

**1** Consider these terms: $3x$, $7xy$, $-x$, $2x^2$, $xw$, $20x$

**a** Which are like terms?

**b** Explain how you can tell.

**c** What is the coefficient of each term?

**2** List each group of like terms from these terms: $2ba^2$, $3a$, $2b^2a$, $6a^3$, $aaa$, $6a$, $3aab$, $6ab^2$, $6a^2a$, $6a^2b$, $3abb$

3A.1 **3** Simplify each expression where possible.

**a** $6a - 4a + 8a$

**b** $4k - 5k - 7k$

**c** $x^2 + 3x^2 + 2x^2$

**d** $3cd + cd - 9cde$

**e** $3x + 4y + 9x + 2y$

**f** $7a + 5b - 3a + b$

**g** $m - 2p + 4p + 8m$

**h** $3 + 5k - 2 - 6k$

**i** $4xy + 3x^2 - xy + 2x^2$

**j** $d + de^2 + d - 5de^2$

**k** $5m^3 + 7 - m^3 - 5$

**l** $abc + ab + ac + 3ab$

**4** Simplify each expression.

**a** $6x + 3y - x + 2y + 5x - 4y$

**b** $8ab - 4b - b + b^2 + a - 3ab$

**c** $2k + 3km - 6k + 4 + 4k - km$

**d** $4x^2 - 7x^2 - 3x + 5 + 6x - 9$

**e** $9a - 4a^2 + a^3 + 5a^2 - 3 - 7a$

**f** $m^2n + 3m^2 + 5nm^2 - 2n^2 + 4mn^2 - 3m^2$

**5** Simplify each expression by first using index laws.

**a** $7a^2 + 5(3a)^2 + (-5a)^2 - 9aa$

**b** $-5bbb + 40b^2b - (2b)^3 + 2(-3b)^3$

**c** $9c^2d^2 - (6cd)^2 + 15(cd)^2 + 9d^2c^2$

**d** $r(pq)^2 + \frac{q^2}{(pr)^{-1}} + ppqqr + p^2q^2r + pq^2r$

**e** $\frac{2}{f^{-1}} + \frac{3}{e^{-1}} + \frac{4}{(ef)^0} + \frac{5}{e^{-1}} + \frac{6}{f^{-1}}$

**f** $\frac{4y}{x^{-2}} + \frac{1}{(3xy)^{-1}} - \frac{2x}{y^{-1}} + \frac{1}{2x^{-1}y^{-1}} + \frac{3x^2}{y^{-1}}$

3A.2 **6** Simplify each expression.

**a** $2ab \times 3cd$

**b** $-5xy \times 4mp$

**c** $9gh \times g$

**d** $4km \times -6kn$

**e** $7jp \times 8bpt$

**f** $-x^2y \times -ay$

**g** $6a^2b \times 3acd$

**h** $-10hk \times 2hkp$

**i** $3b \times -2b \times b$

**j** $m^2n \times 4n \times kn$

**k** $-5xy \times x^2 \times -3xy$

**l** $8abc \times 7a^3c \times b^2$

**7** Simplify each expression by first using index laws.

**a** $(5a^3z^8)^2 \times 3(a^3z^5)^4$

**b** $(-3by^4)^3 \times (-2yb^7)^4$

**c** $5(cmx)^5 \times c(-2mx)^3 \times mx(-2c)^2$

**d** $4(d^3n^{-2}w)^5 \times 11(d^{-2}n^5w^{-3})^4$

**e** $(5e^3p^{12}t^5)^{-1} \times (e^{10}p^4t^2)^0 \times (-3e^4p^6t^7)^{-2}$

**f** $-6(g^{-2}q^5u^3)^5 \times 4(-g^4q^{-4}u^5)^{-3} \times -7(g^{-2}qu^7)^{-4}$

3A.3 **8** Simplify each expression.

**a** $\frac{abc}{ac}$

**b** $\frac{kmn}{kp}$

**c** $\frac{12cd}{3d}$

**d** $\frac{-16w}{8xw}$

**e** $\frac{7ef}{-14ef}$

**f** $\frac{-5xy}{-20x}$

**g** $\frac{18ab}{15ac}$

**h** $\frac{4mn}{22mn}$

**i** $\frac{18a^2}{3a}$

**j** $\frac{6x^2y}{-2x}$

**k** $\frac{15mn^2}{9m}$

**l** $\frac{-3a^2bc}{12ab}$

**9** Simplify each expression using index laws.

**a** $\left(\frac{12d^8}{48d^3}\right)^{-1}$

**b** $\left(\frac{c^3d^{-4}}{c^{-5}d^{-9}}\right)^{-3}$

**c** $\frac{(-7u^6r^{-5})^{-2}}{(-5u^5r^{-2})^{-3}}$

**d** $\frac{(2k^{-3}p^4)^5}{18(k^{-3}p^6)^{-3}}$

**e** $\frac{1.6xy^{-4}z^0}{0.8(x^2z^3)^{-4}(y^{-2}z^3)^5}$

**f** $\frac{4.4f^{-8}}{1.1(f^5)^2} \times \frac{0.35(f^{-4})^{-3}}{0.07f^{-7}}$

**10** Simplify each expression. (Hint: First write each as an algebraic fraction.)

**a** $m^4p^3q \div m^4$

**b** $-2a^8de^7 \div (a^8e^7)$

**c** $6ax \div (-2ac)$

**d** $-5km \div (-10mp)$

**e** $7a^2bc \div (abd)^2$

**f** $3mn^2w^2 \div (9n^3w)^2$

**g** $-12x^2y \div [8(xyz)^3]$

**h** $-ab^3cd \div (-2abc^2)^{-4}$

**i** $8km^2n \div [-12(k^2mn)^{-2}]$

**11** Answer true or false to each statement.

**a** Two like terms can be added to form one new term.

**b** Any term can be subtracted from another term to form one new term.

**c** Two terms can be multiplied to form one new term only if they are like terms.

**d** Any term can be divided by another term to form one new term.

**12** Students in a class were asked to simplify three algebraic expressions. Three sets of working for each expression are shown below. One set is correct and the other two sets contain errors.

For each expression, choose the correct set and then identify the errors in the other two sets of working.

**a** **Expression 1:** $4a - 3b + 2 + 2a + 8b - 7$

| **Set A** | **Set B** | **Set C** |
|---|---|---|
| $4a - 3b + 2 + 2a + 8b - 7$ | $4a - 3b + 2 + 2a + 8b - 7$ | $4a - 3b + 2 + 2a + 8b - 7$ |
| $= 4a + 2a + 2 + 7 + 3b + 8b$ | $= 4a + 2a + 2 - 7 - 3b + 8b$ | $= 4a - 2a + 2 - 7 - 3b + 8b$ |
| $= 6a + 9 + 11b$ | $= 6a - 5 + 5b$ | $= 2a + 9 + 5b$ |

**b** **Expression 2:** $-3ab \times 4bc$

| Set A | Set B | Set C |
|---|---|---|
| $-3ab \times 4bc$<br>$= -3 \times 4 \times a \times b^{(1+1)} \times c$<br>$= -12ab^2c$ | $-3ab \times 4bc$<br>$= -3 \times 4 \times a \times b^{(1+1)} \times c$<br>$= 12 \times a \times b^2 \times c$<br>$= 12abc$ | $-3ab \times 4bc$<br>$= -3 \times 4 \times a \times b^{(1+1)} \times c$<br>$= -12 \times a \times b \times 2 \times c$<br>$= -24abc$ |

**c** **Expression 3:** $4a^2bc \div (8abd)$

| Set A | Set B | Set C |
|---|---|---|
| $4a^2bc \div (8abd)$<br>$= \frac{4a^2bc}{8abd}$<br>$= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$<br>$= \frac{\cancel{4}^{1}}{\cancel{8}^{2}} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$<br>$= \frac{1}{2} \times a^{(2-1)} \times 1 \times c \times \frac{1}{d}$<br>$= 2acd$ | $4a^2bc \div (8abd)$<br>$= \frac{4a^2bc}{8abd}$<br>$= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$<br>$= \frac{\cancel{4}^{1}}{\cancel{8}^{2}} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$<br>$= \frac{1}{2} \times 1 \times 1 \times c \times \frac{1}{d}$<br>$= \frac{c}{2d}$ | $4a^2bc \div (8abd)$<br>$= \frac{4a^2bc}{8abd}$<br>$= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$<br>$= \frac{\cancel{4}^{1}}{\cancel{8}^{2}} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$<br>$= \frac{1}{2} \times a^{(2-1)} \times 1 \times c \times \frac{1}{d}$<br>$= \frac{ac}{2d}$ |

**13** If $x = 3$ and $y = -2$, evaluate each expression. To make it easier, simplify each expression first.

**a** $5x - 6y + 7y + 3x$ **b** $3xy - 8xy - xy$ **c** $5x \times 3y$

**d** $\frac{10x}{xy}$ **e** $xy \times xy^2$ **f** $6x^2y \div (2xy)$

**g** $x + y - 2xy + 5y - x$ **h** $x \times 3x - 2x^2 + 4x - y$ **i** $xy^{12} \div (x^4y^8)$

**14** Evaluate each expression if $a = 2$, $b = -1$ and $c = 5$. Remember to simplify each expression first.

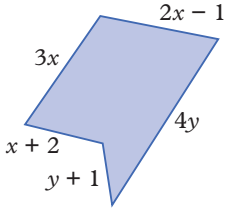
**a** $3a + 2b + 7c - a - 5c + b$ **b** $7ab + 4a - 5a + ab$

**c** $a^2b + ab^2 + ac - 3a^2b + 2ac$ **d** $2abc \times bc \times 5a$

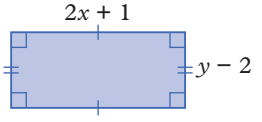
**e** $18ab^2c \div (6bc)$ **f** $3ac^2 \times 4ab \div (9abc)$

**15** Write the perimeter of each shape as an algebraic expression in simplest form.

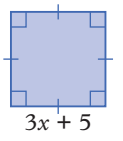
**a**




**b**



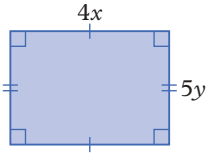

**c**



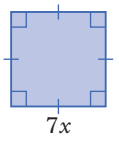

**16** Calculate the perimeter of each shape in question **15** for $x = 4$ cm and $y = 5$ cm.

**17** Write the shaded area in each shape as an algebraic expression in simplest form.

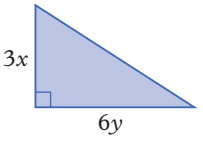
**a**



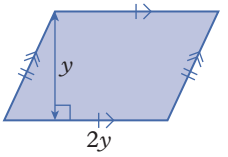

**b**




**c**




**d**




**18** Calculate the area shaded in each shape in question **17** for $x = 3$ m and $y = 2$ m.

PROBLEM SOLVING AND REASONING

**19** The area of a rectangle is $16xy$.

**a** If the length is $8x$, write an expression for the width of the rectangle.

**b** If the length of the rectangle is 16 m and $y$ is 5 m, calculate the width and the area of the rectangle.

**20** The area of a right-angled triangle is $6x^2$.

**a** If the base length is $4x$, write an expression for the height of the triangle.

**b** If the height of the triangle is 12 cm, calculate the area and the base length of the triangle.

**21** A rectangle has a width of $k$.

**a** If the length of the rectangle is twice the width, write an expression for:

**i** the perimeter of the rectangle

**ii** the area of the rectangle.

**b** Calculate the perimeter and the area of the rectangle when $k = 5$ cm.

**22** Lana plants a 1-metre-wide flowerbed around a square section of lawn.

**a** If the lawn has a length of $x$ metres, write an expression for:

**i** the perimeter of the lawn

**ii** the area of the lawn

**iii** the perimeter around the outer edge of the flowerbed

**iv** the area of the flowerbed, given that the total area of the lawn and flowerbed is $(x^2 + 4x + 4)$ m$^2$.

**b** When $x = 8$, calculate:

**i** the area of the flowerbed

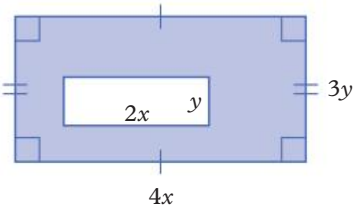
**ii** the length of edging needed around the inner edge of the flowerbed

**iii** the length of edging needed around the outer edge of the flowerbed

**iv** the area to be mown.

CHALLENGE

**23** Consider the shaded region of this shape.




**a** Write an algebraic expression for the area of the shaded region.

**b** If $x = 4$ cm and $y = 5$ cm, calculate the area of the shaded region.

**c** Write an expression for the total length of the outer and inner edges of the shape.

**d** Use your sets of values from part **b** to calculate the total length of the outer and inner edges of the shape.

**e** If $x = 12$ cm and the area of the shaded region is 210 cm$^2$, determine the total length of the outer and inner edges of the shape.

**24** Write $((2x)^7 \times (4x)^{-3} \times (8x)^3 \div (16x)^4)^8$ in the form $2^{\square}x^{\square}$.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Like terms
- Interactive skillsheet: Multiplying terms
- Interactive skillsheet: Dividing terms
- Investigation: Like terms, Snap
- Investigation: How many times?
- Topic quiz: 3A

# 3B Expanding

## Learning intentions

- ✔ I can expand algebraic expressions of the form $a(b + c)$.
- ✔ I can expand algebraic expression of the form $(a + b)(c + d)$.

Inter-year links

| | |
|---|---|
| Year 7 | 6F Simplifying |
| Year 8 | 5F Expanding |
| Year 10 | 2D Expanding |

## The distributive law

- A **binomial** is an expression containing two terms that are either added or subtracted.
  For example, $x - 7$, $2x + y$, and $x^2 + y^2$ are all binomials.
- The **distributive law** is used to distribute or **expand** products over addition and subtraction.
  Note: The distributive law only applies if the expression inside the brackets is not raised to the power of any index (other than 1).
- The distributive law for expanding over one binomial expression is: $a(b + c) = ab + ac$

$$a(b + c) = ab + ac$$

- To expand algebraic expressions with one pair of brackets:
  1 Multiply each term inside the brackets by the term outside the brackets.
  2 Simplify the results by performing any multiplication, addition and subtraction.
- The distributive law for expanding over a **binomial product** is: $(a + b)(c + d) = ac + ad + bc + bd$

$$(a + b)(c + d) = ac + ad + bc + bd$$

- To expand binomial products:
  1 Multiply each term inside the second pair of brackets, $c$ and $d$, by each term in the first pair of brackets, $a$ and $b$.
  2 Simplify the results by performing any multiplication, addition and subtraction.
- Where possible, expanded expressions should be simplified.
- The distributive law for expanding a binomial product can be derived by treating the first binomial as a single term and then applying the distributive law for expanding over one pair of brackets.

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= c(a + b) + d(a + b) \\ &= ca + cb + da + db \\ &= ac + ad + bc + bd\end{aligned}$$

- The **difference of two squares** is a specific form of expansion with a simplified rule:

$$(a + b)(a - b) = a^2 - b^2$$

- The expansion of a perfect square is a specific form of expansion with a simplified rule:

$$(a + b)^2 = a^2 + 2ab + b^2$$

## Example 3B.1 Expanding over one binomial expression

Expand and simplify each expression using the distributive law.

**a** $3(k + 2)$ **b** $-5(3a + 8)$ **c** $x(x - 7)$

**THINK**

**a** 1 Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications.

**b** 1 Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications. Take care with + and − signs when simplifying.

**c** 1 Multiply each term inside the brackets by the term outside the brackets. Note that the second term inside the brackets is negative.

2 Simplify the results by performing the multiplications. Remember that $x \times x = x^2$.

**WRITE**

**a** $3(k + 2) = 3 \times k + 3 \times 2$

$= 3k + 6$

**b** $-5(3a + 8) = -5 \times 3a + (-5) \times 8$

$= -15a - 40$

**c** $x(x - 7) = x \times x + x \times (-7)$

$= x^2 - 7x$

## Example 3B.2 Expanding and simplifying over one binomial expression

Expand and simplify each expression.

**a** $8x(6y - 9)$ **b** $-3a(5a - 4b)$

**THINK**

**a** 1 Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications.

**b** 1 Multiply each term inside the brackets by the term outside the brackets. Enclose the second term in brackets to separate the plus sign and the negative coefficient of the $b$ term.

2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.

**WRITE**

**a** $8x(6y - 9) = 8x \times 6y + 8x \times (-9)$

$= 48xy - 72x$

**b** $-3a(5a - 4b) = -3a \times 5a + (-3a \times -4b)$

$= -15a^2 + 12ab$

## Example 3B.3 Expanding binomial products

Expand each algebraic expression to remove the brackets.

**a** $(a + 9)(b + 2)$ **b** $(x + 4)(x - 6)$ **c** $(3 - a)(t + 6)$

**THINK**

**a** **1** Multiply each term inside the second pair of brackets, $b$ and 2, by the first term in the first pair of brackets, $a$, and then the second term in the first pair of brackets, 9.

**2** Simplify each term.

**b** **1** Multiply each term inside the second pair of brackets, $x$ and $-6$, by the first term in the first pair of brackets, $x$, and then the second term in the first pair of brackets, 4.

**2** Simplify each term.

**3** Simplify any like terms.

**c** **1** Multiply each term inside the second pair of brackets, $t$ and 6, by the first term in the first pair of brackets, 3, and then the second term in the first pair of brackets, $-a$.

**2** Simplify each term.

**WRITE**

**a** $(a + 9)(b + 2) = a \times b + a \times 2 + 9 \times b + 9 \times 2$

$= ab + 2a + 9b + 18$

**b** $(x + 4)(x - 6) = x \times x + x \times (-6) + 4 \times x + 4 \times (-6)$

$= x^2 - 6x + 4x - 24$

$= x^2 - 2x - 24$

**c** $(3 - a)(t + 6) = 3 \times t + 3 \times 6 + (-a) \times t + (-a) \times 6$

$= 3t + 18 - at - 6a$

### Helpful hints

- ✔ When multiplying by a negative term, use brackets for clarity.
  For example: $-7p(3q - 4) = -7p \times 3q + (-7p \times -4)$
- ✔ Show all your working to avoid arithmetical errors and to ensure all signs are correct.
- ✔ After expanding expressions containing brackets, always simplify your results by looking for like terms. Simplify, simplify, simplify!
- ✔ A good way of remembering which terms to multiply when expanding a **binomial product** is to memorise 'FOIL' (First terms, Outer terms, Inner terms, Last terms).

First, Outer: F. O. I. L.

$(a + b)(c + d) = ac + ad + bc + bd$

Inner, Last

Note: This rule only works for expressions of the form $(a + b)(c + d)$.

ANS p488

# Exercise 3B Expanding

▲ 1(1st column), 2, 3–4(2nd column), 5–8(1st, 2nd columns), 9–13, 21, 22 ■ 2–8(2nd column), 11–17, 19(a, c, d), 20, 24 ⬟ 4(b, d, f), 5–8(2nd column), 11–18, 23, 24(d, f), 25–27

UNDERSTANDING AND FLUENCY

**3B.1** **1** Expand each algebraic expression to remove the brackets.

**a** $4(a + 3)$ **b** $7(b + 5)$ **c** $3(c - 2)$

**d** $5(d - 1)$ **e** $6(4 + e)$ **f** $-2(f + 8)$

**g** $-3(g + 4)$ **h** $-8(h - 5)$ **i** $-4(x - 9)$

**j** $-5(2 - j)$ **k** $k(p + 6)$ **l** $a(b - 4)$

**m** $6(3m + k)$ **n** $n(2p + 4q)$ **o** $x(x - 7y)$

**2** Expand and simplify each algebraic expression. Remember, $\frac{x}{4} = \frac{1}{4}x$ and $\frac{x+3}{4} = \frac{1}{4}(x + 3)$.

**a** $\frac{1}{2}(8x + 12)$ **b** $\frac{8x + 12}{4}$

**c** $\frac{18 - 6n}{6}$ **d** $\frac{7xy + 5xz}{x}$

**e** $\frac{35ab - 50ac}{-5a}$ **f** $\frac{-21t^2 - 49t}{-7t}$

**3** Expand and simplify each algebraic expression.

**a** $3(x + 2) + 8x$ **b** $11 + 5(p - 1)$

**c** $a(b + 4) - 2a$ **d** $-7\ (1 - y) + 4y + 3$

**e** $5k + 2 + 4(h - k)$ **f** $m(m - 6) - m^2$

**4** Expand and simplify each algebraic expression.

**a** $2(x + 5) + 3(x - 6)$ **b** $8(k - 3) + 5(k + 4)$

**c** $3(p + 7) - 4(5 - p)$ **d** $x(x + 1) + 3(x + 4)$

**e** $m(m + 2) + 3(m + 2)$ **f** $y(y - 5) - 2(y - 5)$

**3B.2** **5** Expand and simplify each expression.

**a** $3a(4z + 5)$ **b** $8b(7 + 5y)$ **c** $2c(7 - 3x)$

**d** $-5d(4w + 5)$ **e** $-6e(8v - 9)$ **f** $-10e(4t + 3u)$

**g** $4g(2g - 7t)$ **h** $-6h(5r - 7h)$ **i** $3ij(4ik + 5jk)$

**j** $3m^3(4m^2 + 5m^5)$ **k** $\frac{20n^6 - 12n^5}{4n^2}$ **l** $\frac{p^5(pq^4 - p^4q^8)}{q^4}$

**3B.3** **6** Expand each algebraic expression to remove the brackets.

**a** $(a + 3)(b + 4)$ **b** $(c + 2)(d + 7)$ **c** $(m + 5)(n + 1)$

**d** $(x + 9)(y + 3)$ **e** $(k + 6)(p - 2)$ **f** $(f + 4)(g - 1)$

**g** $(a - 5)(c + 3)$ **h** $(w - 7)(f + 2)$ **i** $(x - 4)(y - 8)$

**j** $(j - 9)(k - 5)$ **k** $(2a + 7)(b + 3)$ **l** $(5c + 2)(3d - 4)$

**7** Expand each algebraic expression to remove the brackets.

**a** $(a + 2)(a + 3)$ **b** $(x + 5)(x + 10)$ **c** $(d + 4)(d - 6)$

**d** $(y + 3)(y - 8)$ **e** $(k - 7)(k + 9)$ **f** $(m - 6)(m + 3)$

**g** $(5e - 2)(e - 4)$ **h** $(7a - 8)(3a - 1)$ **i** $(3y - 5)(2y - 1)$

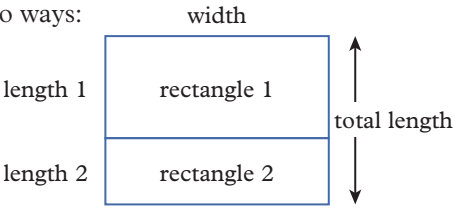
**8** Expand each algebraic expression to remove the brackets.

**a** $(5 - a)(b + 4)$ **b** $(x + 6)(3 - x)$ **c** $(2c + 3)(5 - 4c)$

**d** $(11d - 9e)(4d - 6e)$ **e** $(6p + 5q)(8q - 7p)$ **f** $(x + 2)(x - 3) + 3x^2 - 8x + 5$

**g** $(x^3 + x^2)(x^5 + x)$ **h** $(4x^2 + 7)(12 - 5x^2)$ **i** $(x + 2)(x + 3) + (x + 1)(x + 5)$

**9** The area of the large rectangle in this diagram can be determined in two ways:




Area of large rectangle = width × total length

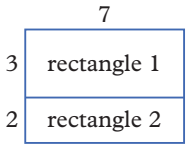
Area of large rectangle = area of rectangle 1 + area of rectangle 2
= width × length 1 + width × length 2

For each diagram below:

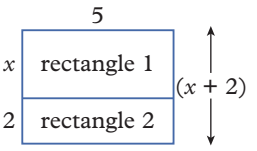
**i** write an expression for the area of the large rectangle by multiplying the total length by the width

**ii** write an expression for the area of the large rectangle by adding the area of rectangle 1 and rectangle 2.

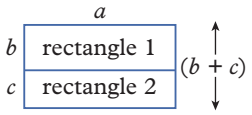
**a**




**b**



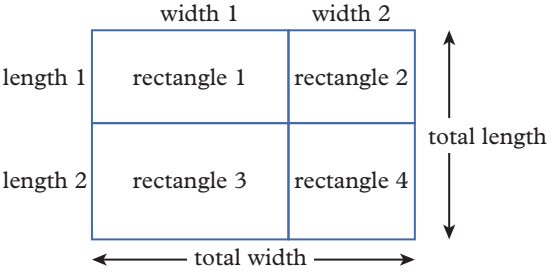

**c** Use the rectangle below to explain how $a(b + c)$ can be expanded to obtain $ab + ac$.




**10** The area of the largest rectangle in this diagram can be determined in two ways:

Area of largest rectangle = total width × total length

Area of largest rectangle = area of rectangle 1 + area of rectangle 2 + area of rectangle 3 + area of rectangle 4
= width 1 × length 1 + width 2 × length 1 + width 1 × length 2 + width 2 × length 2

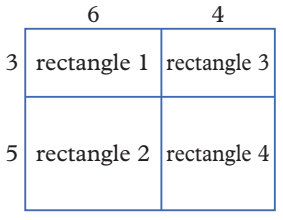



For each diagram below:

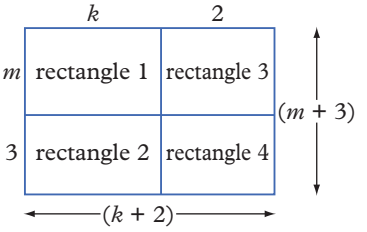
**i** write an expression for the area of the large rectangle by multiplying the total length by the total width

**ii** write an expression for the area of the large rectangle by adding the area of rectangle 1, rectangle 2, rectangle 3, and rectangle 4.

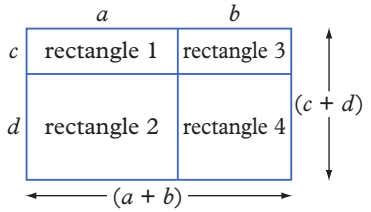
**a**




**b**




**c** Use this rectangle to explain how $(a + b)(c + d)$ can be expanded to obtain $ac + ad + bc + bd$.




**11** Expand each algebraic expression to remove the brackets.

**a** $(a + 3)(a - 3)$ **b** $(b + 2)(b - 2)$ **c** $(c + 5)(c - 5)$

**d** $(3p - 8)(3p + 8)$ **e** $(h - 1)(h + 1)$ **f** $(k + m)(k - m)$

**12** Describe the pattern or shortcut you can see in question **11**. What is special about the two binomial factors that are multiplied together?

**13** The pattern you observed in question **11** is known as the difference of two squares. This rule can be written as: $(a + b)(a - b) = a^2 - b^2$

**a** Why do you think the rule is called the difference of two squares?

**b** Does it matter whether the product is $(a + b)(a - b)$ or $(a - b)(a + b)$? Explain.

**c** Use the rule (or shortcut) to expand each algebraic expression.

**i** $(x - 2)(x + 2)$ **ii** $(y + 9)(y - 9)$ **iii** $(m + 6)(m - 6)$

**iv** $(3d - 10)(3d + 10)$ **v** $(m + n)(m - n)$ **vi** $(3 + x)(3 - x)$

**14 a** Use the difference of two squares expansion rule to expand $(100 + 3)(100 - 3)$.

**b** Use your answer to work out the result for $103 \times 97$ without using a calculator.

**c** Work out the result for each product without using a calculator.

**i** $102 \times 98$ **ii** $95 \times 105$

**iii** $1001 \times 999$ **iv** $994 \times 1006$

**15** Expand each algebraic expression to remove the brackets and simplify.

**a** $(a + 2)(a + 2)$ **b** $(b + 7)(b + 7)$ **c** $(c + 4)(c + 4)$

**d** $(3p - 5)(3p - 5)$ **e** $(h - 1)(h - 1)$ **f** $(m + n)(m + n)$

**16** Describe the pattern or shortcut you can see in question **15**. What is special about the two binomial factors that are multiplied together?

**17** The pattern you observed in question **15** is known as the expansion of a perfect square. This rule can be written as: $(a + b)^2 = a^2 + 2ab + b^2$.

**a** Why do you think the rule is called the expansion of a perfect square?

**b** Use the rule (or shortcut) to expand each algebraic expression.

**i** $(x + 3)^2$ **ii** $(y + 6)^2$ **iii** $(m + 2)^2$

**iv** $(4b + 11)^2$ **v** $(m + n)^2$ **vi** $(5 + x)^2$

**c** Is $(a - b)^2 = a^2 - 2ab + b^2$ also an expansion of a perfect square? Explain.

**d** Expand each algebraic expression.

**i** $(a - 2)^2$ **ii** $(b - 4)^2$ **iii** $(c - 7)^2$

**iv** $(5w - 6)^2$ **v** $(k - p)^2$ **vi** $(3 - x)^2$

**18** A binomial is an algebraic expression with two terms.

**a** Write three examples of a binomial.

**b** $(x + 1)(x - 2)$ is an example of a binomial product. Explain what the term 'binomial product' means.

**c** How many terms do you think a trinomial has?

**d** Write three examples of a trinomial.

**e** Find out what a quadratic trinomial is and provide three examples.

**19** We can also expand products of sums and differences with more than two terms; it just involves adding more products together. For example:

$2(3x + 5y - 7) = 2 \times (3x) + 2 \times (5y) + 2 \times (-7)$
$= 6x + 10y - 14$

and

$(a + b + c)(x + y) = x(a + b + c) + y(a + b + c)$
$= ax + bx + cx + ay + by + cy$

It is recommended to expand in two stages as shown above or use the grid method to ensure that no products are missed.

Expand each algebraic expression.

**a** $4(3a - 5b + 10)$ **b** $2x(3w - 5x + 7y - 4z)$ **c** $5[(x + 3)(x + 4)]$

**d** $(3a + 5b - 6)(2x + 3y)$ **e** $(x + 3y + 6)(2x + y + 3)$ **f** $(x + 2)(x^2 + 6x + 7)$

**20** Which of the following is not equivalent to $3(x + 2)(x + 8)$?

**A** $(3x + 6)(x + 8)$ **B** $(x + 2)(3x + 24)$ **C** $3(x^2 + 10x + 16)$

**D** $3x^2 + 30x + 48$ **E** $(3x + 6)(3x + 24)$

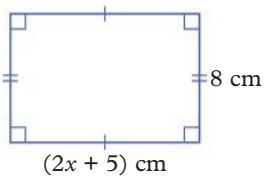
**21** **a** For the rectangle on the right, write the area as a product of two factors using brackets.

**b** Expand the expression to remove brackets.

**c** When $x = 3$, calculate the area using your answer to:

**i** part **a** **ii** part **b**.

**d** Compare your answers in part **c**. How does this show that you have expanded the area expression correctly?



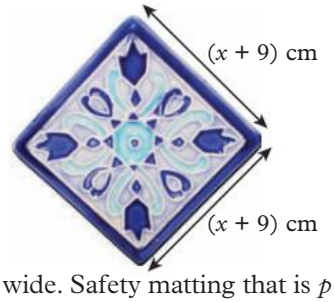

**22** For each of the following rectangles:

**i** write the area as a binomial product **ii** expand the binomial product to remove brackets

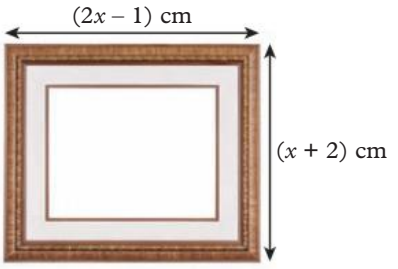
**iii** calculate the area when $x = 5$.

**a**

**b**




**c**




**23** A rectangular trampoline is 6 m long and 4 m wide. Safety matting that is $p$ metres wide is to be placed around the perimeter of the trampoline.

**a** Draw a labelled diagram of the top of the trampoline and the matting around it.

**b** Write an expression for:

**i** the total length of the trampoline and matting

**ii** the total width of the trampoline and matting.

**c** Write an expression for the total area taken up by the trampoline and matting. Simplify the expression by expanding to remove any brackets.

**d** Write an expression for the area of the matting only.

**e** If $p = 2$, calculate the area taken up by:

**i** the trampoline **ii** the trampoline and matting **iii** the matting.

**24** Expand and simplify each expression.

**a** $(10y - 7)^2$ **b** $x^4(x^3 - x^2)$ **c** $(x^2 - 5)(x^2 + 5)$

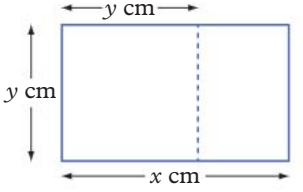
**d** $(y^6 + 3y)^2$ **e** $a^3(a^2 + 4) - a^2(a^3 + 9a)$ **f** $x^5y^2(xy^4 + 2w)$

**25** Show that the expression $(a - b)^2 + (c - d)^2$ is equivalent to $(b - a)^2 + (d - c)^2$.

**26** A rectangular piece of paper is $x$ cm long and $y$ cm wide. The paper is torn along a line parallel to its width, forming a square of side length $y$ cm, and another rectangle.

**a** Write the dimensions of the newly formed square and rectangle.

**b** Prove that the area of the original rectangle is the same as the total area of the two new shapes.




**27** Paul has invented a new form of expansion as: $a \dashv b = b - a + ab$.

If, using this expansion, $3 \dashv 8$ has the same value as $6 \dashv x$, what is the value of $x$?

CHALLENGE

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Expanding over one pair of brackets
- Interactive skillsheet: Expanding binomial products
- Investigation: It's a mystery
- CAS instructions: Expanding
- Topic quiz: 3B

# 3C Factorising using the HCF

## Learning intentions

- ✔ I can find the HCF of two or more algebraic terms.
- ✔ I can factorise algebraic expressions by taking out the HCF.
- ✔ I can factorise algebraic expressions with binomial factors.
- ✔ I can factorise algebraic expressions by grouping terms.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Factors and multiples |
| Year 7 | 2D Factors and the highest common factor |
| Year 8 | 5G Factorising |
| Year 10 | 2F Factorising |

## Factorising terms

- **Factorising** an algebraic expression is the opposite to expanding an algebraic expression.
- An expression is in simplest factorised form if all the terms inside the brackets have no common factors.
- An expression is in expanded form if it has no brackets and is simplified.

expanding →
$7(a + 2) = 7a + 14$
← factorising

$2x^2 - 6x = 2x(x - 3)$
expanded form  factorised form

## The highest common factor (HCF) of an expression

- The **highest common factor (HCF)** of two or more algebraic terms is the 'highest' or 'largest' combination of coefficients, pronumerals and algebraic expressions that are factors of every term.
- The HCF of an expression can have numerical and pronumeral terms.
- The HCF can be a binomial factor.
- To identify the HCF of an expression:
  1. Find the HCF of the coefficients.
  2. Find the HCF of the pronumerals.
  3. Multiply the factors from steps 1 and 2, and then simplify.

| Algebraic terms | HCF |
|---|---|
| $x + 7x$ | $x$ |
| $6m + 12mn + 8mp$ | $2m$ |
| $15a^3bc + 9a^2c$ | $3a^2c$ |
| $x(x - 1) + 2(x - 1)$ | $(x - 1)$ |

## Factorising algebraic expressions

- To factorise an expression:
  1. Identify the HCF of the expression.
  2. Write the HCF in front of the brackets. Inside the brackets, divide each term in the original expression by the HCF.
  3. Simplify the bracketed expression.

$7a + 14 = 7\left(\frac{7^1a}{7^1} + \frac{14^2}{7^1}\right)$
$= 7(a + 2)$

## Factorising by grouping terms

- To factorise an expression with four terms by grouping:
  1. Identify pairs of terms with common factors. Group the pairs of terms, remembering to move the positive or negative sign with the term.
  2. Factorise each pair of terms by dividing out the HCF.
  3. Factorise using the binomial factor and simplify the expression.

$x^2 + 2x - 3x - 6 = x(x + 2) - 3(x + 2)$
$= (x + 2)(x - 3)$

## Example 3C.1 Identifying the HCF

Find the highest common factor (HCF) of each pair of terms.

**a** 24 and 84 **b** $a^2b$ and $b^2ac$ **c** $12x^2y^3$ and $-18x^3y$

**THINK**

**a** **1** Write the prime factorisation for each number. Use factor trees to help factorise.

**2** Multiply the common factors to find the HCF.

**b** **1** Write each term in expanded form.

**2** Multiply the common factors to find the HCF.

**c** **1** Find the HCF of the coefficients. The negative sign can be represented using the factor '−1'.

**2** Find the HCF of the pronumerals.

**3** Multiply the common factors to find HCF. Simplify the product.

**WRITE**

**a** $24 = 2 \times 2 \times 2 \times 3$
$84 = 2 \times 2 \times 3 \times 7$
$\text{HCF} = 2 \times 2 \times 3$
$= 12$

**b** $a^2b = a \times a \times b$
$b^2ac = b \times b \times a \times c$
$\text{HCF} = a \times b$
$= ab$

**c** The HCF of 12 and −18 is 6.

The HCF of $x^2y^3$ and $x^3y$ is $x^2y$.

The HCF of $12x^2y^3$ and $-18x^3y$ is $6x^2y$.

## Example 3C.2 Factorising algebraic expressions

Factorise each expression.

**a** $6a + 18$ **b** $x^2 - 7x$ **c** $10k^3m^2 + 15k^2m$

**THINK**

**a** **1** Identify the HCF.

**2** Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.

**3** Simplify the bracketed expression.

**b** **1** Identify the HCF.

**2** Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.

**3** Simplify the bracketed expression. Use the index laws to help you.

**c** **1** Identify the HCF.

**2** Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.

**3** Simplify the bracketed expression. Use the index laws to help you.

**WRITE**

**a** HCF = 6

$6a + 18 = 6\left(\frac{\cancel{6}^1a}{\cancel{6}^1} + \frac{\cancel{18}^3}{\cancel{6}^1}\right)$

$= 6(a + 3)$

**b** HCF = $x$

$x^2 - 7x = x\left(\frac{x^2}{x} - \frac{7x}{x}\right)$

$= x(x^{(2-1)} - 7x^{(1-1)})$
$= x(x - 7)$

**c** HCF = $5k^2m$

$10k^3m^2 + 15k^2m$

$= 5k^2m\left(\frac{\cancel{10}^2k^3m^2}{\cancel{5}^1k^2m} + \frac{\cancel{15}^3k^2m}{\cancel{5}^1k^2m}\right)$

$= 5k^2m(2k^{(3-2)}m^{(2-1)} + 3k^{(2-2)}m^{(1-1)})$
$= 5k^2m(2km + 3)$

## Example 3C.3 Factorising using a HCF that is a binomial factor

Factorise each expression.

**a** $y(x+3)+7(x+3)$ **b** $4k(2-m)-5(2-m)$

**THINK**

**a**
1. Identify the HCF. It is the binomial factor of $(x+3)$.
2. Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.
3. Simplify the bracketed expression.

**b**
1. Identify the HCF. It is the binomial factor of $(2-m)$.
2. Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.
3. Simplify the bracketed expression.

**WRITE**

**a** HCF $= (x+3)$

$$y(x+3)+7(x+3) = (x+3)\left(\frac{y\cancel{(x+3)^1}}{\cancel{(x+3)^1}} + \frac{7\cancel{(x+3)^1}}{\cancel{(x+3)^1}}\right)$$

$$= (x+3)(y+7)$$

**b** HCF $= (2-m)$

$$4k(2-m)-5(2-m) = (2-m)\left(\frac{4k\cancel{(2-m)^1}}{\cancel{(2-m)^1}} - \frac{5\cancel{(2-m)^1}}{\cancel{(2-m)^1}}\right)$$

$$= (2-m)(4k-5)$$

## Example 3C.4 Factorising by grouping terms

Factorise $xy + 2x + 3y + 6$ by grouping terms.

**THINK**

1. Check for a HCF of all four terms. Group the terms in pairs and identify the HCF for each pair.
2. Factorise each pair of terms by dividing out the HCF.
3. Factorise using the binomial factor of $(y+2)$. Simplify the expression.

**WRITE**

$xy + 2x + 3y + 6 = (xy + 2x) + (3y + 6)$
HCF of $xy$ and $2x$ is $x$.
HCF of $3y$ and $6$ is $3$.

$xy + 2x + 3y + 6 = x(y+2) + 3(y+2)$

$$= (y+2)(x+3)$$

**Helpful hints**

- ✔ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.
- ✔ The divisibility rules can help you to find the HCF. Recall that a number is:
  - → divisible by 2, if the number is even
  - → divisible by 3, if the number's digits add to a multiple of 3
  - → divisible by 5, if the number ends in 0 or 5.
- ✔ The great thing about factorising is that you can always check your answer by expanding the brackets!

$$2x - 16 = 2(x-8) = 2x - 16$$

ANS p489

# Exercise 3C Factorising using the HCF

| | | |
|---|---|---|
| 1–4(1st, 2nd columns), 5, 6, 7–10(1st, 2nd columns), 11, 12, 15, 16(a, b), 17 | 2–4(3rd column), 5–6(e–h), 7–10(2nd, 3rd columns), 13–16, 18, 20, 24 | 4(g–i), 7–10(2nd, 3rd columns), 15, 16, 19–25 |

3C.1 **1** Find the highest common factor (HCF) of each pair of terms.

**a** 4 and 28 **b** 6 and 10 **c** 15 and 35

**d** $d^2$ and $3d$ **e** $2e$ and $2k$ **f** 3 and $-6$

**g** 12 and $-8$ **h** $9h$ and $-15h^2$ **i** $24x^2$ and $36x$

**2** Find the HCF of each pair of terms.

**a** $bc$ and $cd$ **b** $2xy$ and $2y$ **c** $18mn$ and $-9m$

**d** $abcd$ and $bdf$ **e** $8xy$ and $28y$ **f** $6k^2$ and $-10k$

**g** $p$ and $11p^2$ **h** $45ab$ and $-40cd$ **i** $3pq$ and $6p$

**3** Factorise each expression using the HCFs from question **2**.

**a** $bc + cd$ **b** $2xy + 2y$ **c** $18mn - 9m$

**d** $abcd + bdf$ **e** $8xy + 28y$ **f** $6k^2 - 10k$

**g** $p + 11p^2$ **h** $45ab - 40cd$ **i** $3pq + 6p$

**4** Find the HCF of each pair of terms.

**a** $a^4$ and $a^7$ **b** $5b^6$ and $3b^2$ **c** $16c^5$ and $-2c^3$

**d** $6d^4z^7$ and $6d^3z^5$ **e** $7e^6y^5$ and $14e^7y^8$ **f** $-20fx^{12}$ and $6f^3x^9$

**g** $-2g^2$ and $-2gh^5$ **h** $12i^6k^5$ and $-18j^3k^8$ **i** $x^9y^{11}z^{13}$ and $-x^{11}y^7z^{16}$

3C.2 **5** Factorise each expression.

**a** $10x + 5$ **b** $3y - 21$ **c** $8k + 12$ **d** $15 - 6d$

**e** $28x + 21y$ **f** $20n - 50$ **g** $x^2y + 3xy^4$ **h** $n^3m^2 - 9n^2m$

**6** Factorise each expression.

**a** $20ab - 5b$ **b** $8d + 8cde$ **c** $15x^2 + 10x$ **d** $4k^2 - 22k$

**e** $30n - 18n^2$ **f** $16a^2 + a$ **g** $2h^2 - 14h$ **h** $6p + 6p^2$

**7** Factorise each expression. Remember to use the index laws to help you simplify.

**a** $24m^6 + 16m^4$ **b** $21q^5r^2 + 35q^3r^6$ **c** $15t^4u^2 - 5t^8u^7$

**d** $12c^9d^5 + 6c^4d^4$ **e** $7e^{11}f^3 - 7e^5f^3$ **f** $3i^5j^2k^6 + 27ij^7k$

**g** $a^7b^{12}c^7 + b^3c^9\mathrm{d}^4$ **h** $5mpq - 3np^{10}q^5$ **i** $u^4w^5y^2z^3 + v^4w^5x^2z^3$

**8** Factorise each expression by factoring out a negative HCF. Remember that $-5mn = -1 \times 5 \times m \times n$.

**a** $-5mn - 10n$ **b** $-14xy - 7x$ **c** $-6c + 6cd$

**d** $-a^2 - 3a$ **e** $-4k^2 - 2k$ **f** $-8x^2 + 8x$

**g** $-12 - 3xy$ **h** $-16m - 10m^2$ **i** $-9x^2y + 18xy$

3C.3 **9** Factorise each expression.

**a** $x(w + 4) + 2(w + 4)$ **b** $y(x - 1) + 7(x - 1)$ **c** $a(a + 6) - 3(a + 6)$

**d** $p(5 - n) + 8(5 - n)$ **e** $3k(4 - k) - 5(4 - k)$ **f** $2x(3x - 4) + 9(3x - 4)$

**g** $4g(2g + 1) + (2g + 1)$ **h** $2h(8n - d) - (8n - d)$ **i** $3a(7x + 6y) + 2b(7x + 6y)$

3C.4 **10** Factorise each expression by grouping terms.

**a** $ab + 5b + 4a + 20$ **b** $xy - 6x + 7y - 42$ **c** $mn + 4m - 2n - 8$

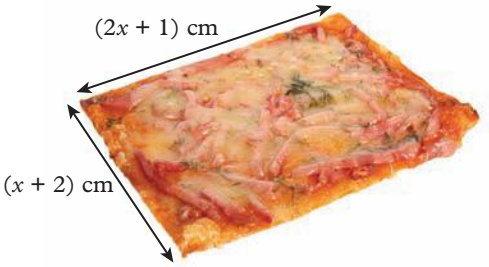
**d** $y^2 + 3y + 5y + 15$ **e** $k^2 - 7k + 2k - 14$ **f** $6x + 18 + x^2 + 3x$

**g** $a^2 - 7a - 2a + 14$ **h** $p^2 + 5p - 2p - 10$ **i** $6c^2 - 2c + 9c - 3$

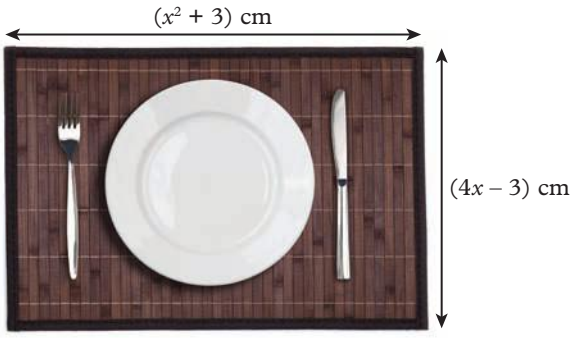
**11** Explain the difference between expanding an expression and factorising an expression.

**12 a** Write an expression in factorised form for the perimeter of each rectangular shape.

**i**




**ii**




**b** Calculate the perimeter of each rectangle when $x = 5$.

**13** Write an expression for the missing side length of each rectangular object, given the area shown.

**a** Area of rug is $(8x + 20)$ m$^2$.

**b** Area of stained glass panel is $(m^2 + 15m)$ cm$^2$.

**14** Write an expression for the perimeter of each item in question **13** in factorised form.

**15** Expressions with more than two terms can be factorised in the same way as those with two terms. For example, to factorise $8a + 12b - 6c$ write the HCF, 2, in front of the brackets and divide each term inside the brackets by the HCF. So, $2\left(\frac{8a}{2} + \frac{12b}{2} - \frac{6c}{2}\right) = 2(4a + 6b - 3c)$. Factorise each expression.

**a** $27x - 9y + 15z$

**b** $45p - 50q - 5$

**c** $4 - 20i + 40j - 60k$

**d** $a^5 + a^3 + a^2$

**e** $18b^3c^5 - 36b^4c^4 + 24b^8c^5$

**f** $84r^{12}t^8 + 7r^5t^7 + 49r^8t^{14} + 14r^9t^8$

**16** We can take fractions out as a factor so that all terms in the brackets no longer involve fractions. For example:

$$\begin{aligned}\frac{5}{3}a + \frac{5}{6}b &= \frac{10}{6}a + \frac{5}{6}b \\ &= \frac{1}{6}\left(\frac{10}{6}a \times 6 + \frac{5}{6}b \times 6\right) \\ &= \frac{1}{6}(10a + 5b) \\ &= \frac{5}{6}\left(\frac{\cancel{10}^{2}a}{5} + \frac{\cancel{5}^{1}b}{5}\right) \\ &= \frac{5}{6}(2a + b)\end{aligned}$$

Factorise each expression.

**a** $\frac{5}{2}x + \frac{3}{2}y$

**b** $\frac{9}{5}g + \frac{3}{5}h$

**c** $\frac{5}{2}p + \frac{15}{4}q$

**d** $\frac{7}{3}m - \frac{5}{4}n$

**17** A rectangle has an area of $(10x - x^2)$ mm$^2$.

**a** Write the area in factorised form.

**b** List possible expressions for the length and width of the rectangle.

**c** Suggest values for the length and width of three different rectangles that have the given area.

**18** A triangle has an area of $(21x - 3x^2)$ cm$^2$.

**a** List possible expressions for the height and base length of the triangle.

**b** Suggest numerical values for the height and base length of three different triangles that have the given area.

**19** A square-based prism has a volume of $(9x^3 + 45x^2)$ cm$^3$ and a height of $(x + 5)$ cm. Determine the length of the square base of the prism in terms of $x$.

**20** Consider the expression $x^2 + 3x - 4x - 12$.

**a** Factorise the expression by grouping the first two terms together, and the last two terms together.

**b** Perform the factorisation again by grouping the first and third terms together, and the second and fourth terms together.

**c** Compare your answers to parts **a** and **b**.

**21** A number is represented by the pronumeral $n$.

**a** Write down the next two consecutive numbers in terms of $n$.

**b** Write the sum of these three consecutive numbers.

**c** Factorise your expression from part **b**. Explain the answer.

**d** Investigate to see whether the same outcome results from the sum of three consecutive even numbers.

PROBLEM SOLVING AND REASONING

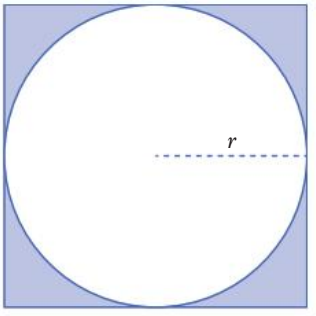
CHALLENGE

**22** In question **21**, an odd number (three) of consecutive numbers were added together. Investigate to see the outcome when an even number of consecutive numbers are added together. Write a factorised expression for the sum of 10 consecutive numbers, where the first number is $n$.

**23** Completely factorise each expression.

**a** $x(y + 5) - (3y + 15)$ **b** $p(2q - 3) - 2q + 3$ **c** $2(a^2 - 3a) + (9 - 3a)$

**24** A circle of radius $r$ cm fits within a square as shown in the diagram. Write a factorised expression for the area of the square not covered by the circle. Leave your answer in terms of $r$ and $\pi$.




**25** It is possible to take any term or expression as a factor of another expression by dividing it out. For example, to take 7 out as a factor of $3x + 5$, write the 7 in front of the brackets and inside the brackets divide each term by 7. So, $3x + 5 = 7\left(\frac{3x}{7} + \frac{5}{7}\right)$.

**a** Take 5 out as a factor of $9x + 17$.

**b** Take 12 out as a factor of $12x^2 + 3x + 6$.

**c** Take $x$ out as a factor of $3x^2 + 5xy + y^2$.

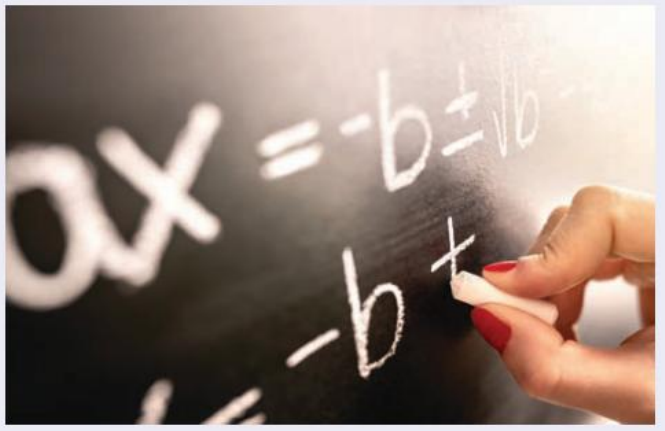
**d** Take $x + 2$ out as a factor of $7(x + 2)^2 + 5(x + 2) + 9$.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Factorising using the HCF
- Interactive skillsheet: Factorising by grouping terms
- Investigation: Surface area of a soft drink can
- Topic quiz: 3C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.



**3A** **1** Simplify the expressions.

**a** $-3a + 6b - 5a - 8b + 12a - 5$

**b** $9t - t^2 + 4t - 6 + 5t^2 + 2 - t$

**c** $7x^3 - 4x^2 - 3x + 2 + 8x - 3x^2 - 5x^3$

**d** $7cd^2 - 7d^2 + 2c - 8d^2 - 3d^2c + 5dd$

**3A** **2** Simplify the expressions.

**a** $5a \times -7b$

**b** $-8ac \times 3a \times -2bc$

**c** $\dfrac{45gh}{20g}$

**d** $\dfrac{18t^2u}{54tu^2}$

**3A** **3** Simplify the expressions.

**a** $(-5a^3b^6)^2 \times (-2a^2b^4)^5$

**b** $\left(-\dfrac{72k^4p^3}{16k^5p}\right)^3$

**c** $(8r^{-3}t^4)^{-1} \times \left(\dfrac{1}{3}r^5t^{-4}\right)^{-2}$

**d** $-42(x^3y^{-5}z^4)^{-4} \div [-63(x^{-5}y^3z^{-2})^8]$

**3A** **4** Expand the following.

**a** $8(a + 3)$

**b** $-4(2b - 5)$

**c** $3c(7 - y)$

**d** $9d(2d - 1)$

**3B** **5** Expand and simplify the following.

**a** $5(w + 7) - 2w + 6$

**b** $8 - 3(x - 9) + 2x$

**c** $10y - (12 - 5y) + 3$

**d** $z(z + 2) + 3(z - 5)$

**3B** **6** Expand and simplify the following.

**a** $(p + 2)(v + 5)$

**b** $(4 - q)(u + 3)$

**c** $(r + 6)(r - 2)$

**d** $(2t - 5)(3t - 7)$

**3B** **7** Expand and simplify the following.

**a** $(a^4 + 2a^2)(a^3 + 5)$

**b** $(b^2 - 3b)(b + 8) + 2b^2 - 4b + 1$

**c** $(c + 2)(c - 9) - 4(c - 2)$

**d** $(5 - d)(4 - d) + (2d + 1)(3d - 1)$

**3B** **8** Factorise the following.

**a** $6a + 9$

**b** $12b - 36$

**c** $-28c - 7$

**d** $8d^2 + 36d$

**3C** **9** Factorise the following.

**a** $2m(3g + 2) + 5(3g + 2)$

**b** $8(h - 3) - n(h - 3)$

**c** $p(q + 7r) - (q + 7r)$

**d** $2x(x + 3) - 5(x + 3)$

**3C** **10** Factorise the following by grouping.

**a** $ab + 5a + 4b + 20$

**b** $12cd - 8c - 21d + 14$

**c** $15ef - 5e + 3f - 1$

**d** $x^2 + 6x + 5x + 30$

# 3D Factorising the difference of two squares

## Learning intentions

✔ I can factorise expressions in the form $a^2 - b^2$.

**Inter-year links**

| | |
|---|---|
| Year 7 | 1G Indices and square roots |
| Year 8 | 5G Factorising |
| Year 10 | 2F Factorising |

## The difference of two squares

- Binomials that have the same terms but one different sign, such as $a + b$ and $a - b$, are **conjugates** of each other.
- If a pair of conjugates are multiplied together, $(a + b)(a - b)$, then the middle terms add to zero and the solution is the difference of two squares:

$$\begin{aligned}(a + b)(a - b) &= a^2 + ab - ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

The difference of two squares rule can be used to factorise expressions of the form $a^2 - b^2$:

$$a^2 - b^2 = (a - b)(a + b)$$

## Factorising the difference of two squares

1 Check for common factors. Write the factor in front of the brackets. Inside the brackets, divide each term by the factor.
2 Write the expressions in the brackets as the difference of two squares.
3 Factorise using the difference of two squares rule.

$$\begin{aligned}3x^2 - 12 &= 3(x^2 - 4) \\ &= 3(x^2 - 2^2) \\ &= 3(x - 2)(x + 2)\end{aligned}$$

Remember the first 10 square numbers so that they can be easily identified.

| | $1^2$ | $2^2$ | $3^2$ | $4^2$ | $5^2$ | $6^2$ | $7^2$ | $8^2$ | $9^2$ | $10^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **Perfect square** | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |

### Example 3D.1 Factorising simple expressions using the difference of two squares rule

Factorise $x^2 - 25$.

**THINK**

1 There are no common factors. Write the expressions as a difference of two squares.
2 Factorise using the difference of two squares rule with $a = x$ and $b = 5$.

**WRITE**

$$x^2 - 25 = x^2 - 5^2$$

$$= (x - 5)(x + 5)$$

## Example 3D.2 Factorising expressions using the difference of two squares rule

Factorise each quadratic expression.

**a** $9y^2 - 16$ **b** $4k^2 - 9m^2$

**THINK**

**a** **1** There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the square of the term with a coefficient and a pronumeral.

**2** Factorise using the difference of two squares rule with $a = 3y$ and $b = 4$.

**b** **1** There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the squares of the terms with a coefficient and a pronumeral.

**2** Factorise using the difference of two squares rule with $a = 2k$ and $b = 3m$.

**WRITE**

**a** $9y^2 - 16 = (3y)^2 - 4^2$

$= (3y - 4)(3y + 4)$

**b** $4k^2 - 9m^2 = (2k)^2 - (3m)^2$

$= (2k - 3m)(2k + 3m)$

## Example 3D.3 Factorising complex expressions using the difference of two squares rule

Factorise each quadratic expression.

**a** $2x^2 - 8y^2$ **b** $(m + 4)^2 - 1$

**THINK**

**a** **1** Check for common factors: 2. Write the factor in front of the brackets. Inside the brackets, divide each term by the factor.

**2** Write the expressions as a difference of two squares.

**3** Factorise using the difference of two squares rule with $a = x$ and $b = 2y$. Write the factor of 2 before the brackets.

**b** **1** There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the square of the term with a coefficient and a pronumeral.

**2** Factorise using the difference of two squares rule with $a = m + 4$ and $b = 1$.

**3** Simplify the expression in each pair of brackets.

**WRITE**

**a** $2x^2 - 8y^2 = 2(x^2 - 4y^2)$

$= 2(x^2 - (2y)^2)$

$= 2(x - 2y)(x + 2y)$

**b** $(m + 4)^2 - 1 = (m + 4)^2 - 1^2$

$= (m + 4 - 1)(m + 4 + 1)$

$= (m + 3)(m + 5)$

**Helpful hints**

- ✔ Memorise the first 15 square numbers to help you with factorising and mental arithmetic.
- ✔ You can check your factorisation by expanding the brackets!
- ✔ Simplify the expression inside the brackets before you give your final answer.
- ✔ The difference of two squares rule only applies to two square terms that are subtracted, not added together. For example, $x^2 - 4$ can be factorised using the rule but $x^2 + 4$ cannot.

$$a^2 - 9^2 = (a - 3)(a + 3)$$
$$= a^2 - 3a + 3a - 9$$
$$= a^2 - 9$$

ANS p490

# Exercise 3D Factorising the difference of two squares

△ 1, 2, 3(1st, 2nd columns), 4–5(1st column), 6(1st, 2nd columns), 7-9(1st column), 10, 13–15, 18(a, b)

□ 3(2nd, 3rd columns), 4–5(2nd column), 6(2nd, 3rd columns), 7-9(2nd column), 12, 15, 16, 18(a, c), 20(a, b, e), 21

⬠ 4–5(2nd column), 6(3rd column), 7-9(2nd column), 11, 15–17, 18(c, d), 19, 20(b, d, f, h), 21–23

**1** **a** Expand $(3x + 7)(3x - 7)$.

**b** Expand $(3x + 7)(3x - 7)$ using the difference of two squares rule.

**c** Explain how you factorise $9x^2 - 49$.

3D.1 **2** Factorise each expression.

**a** $x^2 - 36$ **b** $a^2 - 100$ **c** $9 - y^2$

3D.2 **3** Factorise each expression using the difference of two squares rule.

**a** $25m^2 - 4$ **b** $49 - 64c^2$ **c** $(ab)^2 - 9$

**d** $x^2y^2 - w^2$ **e** $p^2 - q^2r^2$ **f** $b^2n^2 - d^2m^2$

**g** $(5jk)^2 - (6mn)^2$ **h** $81a^2b^2 - 49c^2d^2$ **i** $144p^2q^2 - 121r^2t^2$

3D.3 **4** Factorise each expression.

**a** $3x^2 - 12$ **b** $18p^2 - 50$

**c** $500 - 245g^2$ **d** $7 - 28u^2$

**e** $36r^2 - 100$ **f** $6400 - 12100z^2$

**g** $6x^2 - 6y^2$ **h** $pq^2 - pr^2$

**i** $u^2t^2 - d^2u^2$ **j** $36dk^2 - 16dj^2$

**k** $162a^2b^2 - 242a^2d^2$**1** **l** $9x - 81x^3$

**5** Factorise each expression.

**a** $(k - 3)^2 - 16$ **b** $(7 - m)^2 - 1$

**c** $9 - (y + 2)^2$ **d** $4 - (a + 2)^2$

**e** $(y + 5)^2 - x^2$ **f** $u^2 - (t - 9)^2$

**g** $(2p + 4)^2 - p^2$ **h** $(9 - 4m)^2 - m^2$

**i** $(x + 3)^2 - x^2$ **j** $(q + 6)^2 - (r - 5)^2$

**k** $(7x - 4)^2 - (3x - 5)^2$ **l** $(z + 1)^2 - (z - 1)^2$

**6** Factorise each expression.

**a** $-x^2 + 36$ **b** $-16 + b^2$ **c** $-25 + 9h^2$

**d** $-49d^2 + 64n^2$ **e** $-(rp)^2 + 1$ **f** $-t^2u^2 + w^2x^2$

**7** Factorise each expression.

**a** $t^2 - \frac{4}{9}$ **b** $r^2 - \frac{25}{49}$

**c** $\frac{81}{16} - u^2$ **d** $\frac{9}{100}d^2 - \frac{121}{144}$

**e** $\frac{1}{64}k^2 - \frac{1}{25}$ **f** $\frac{w^2}{169} - \frac{x^2}{196}$

**8** Factorise each expression. (Hint: It might help to convert each decimal to a fraction first.)

**a** $z^2 - 0.16$ **b** $y^2 - 1.21$

**c** $0.04 - x^2$ **d** $c^2 - 0.0025$

**e** $0.49g^2 - 0.01h^2$ **f** $0.0144 - 0.0121k^2$

**9** Factorise each expression as the difference of two squares. (Hint: Use the index laws.)

**a** $x^6 - 9$ **b** $36 - y^{10}$

**c** $z^{22} - 49$ **d** $4p^{14} - 25$

**e** $\frac{q^{18}}{16} - \frac{81}{121}$ **f** $-r^{30}t^{50} + 1$

**10 a** Write the following differences as a product.

**i** $14^2 - 4^2$ **ii** $10^2 - 15^2$

**iii** $99^2 - 98^2$ **iv** $16 - 64$

**v** $169 - 9$ **vi** $225 - 196$

**b** Evaluate each difference in part **a** using the product.

**c** Write the following products as a difference of two square numbers.

**i** $(8 + 5)(8 - 5)$ **ii** $(12 + 18)(12 - 18)$

**iii** $(30 + 1)(30 - 1)$ **iv** $12 \times 6$

**v** $40 \times -20$ **vi** $13 \times 15$

**d** Evaluate each product in part **c** using the difference.

**11** Without using a calculator, work out the value of each problem.

**a** $632 - 372$

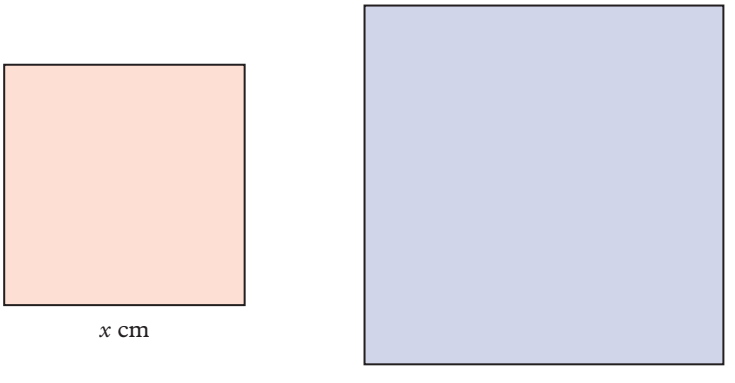
**b** $15.192 - 14.812$

**c** $\left(\frac{13}{25}\right)^2 - \left(\frac{12}{25}\right)^2$

**12 a** Write a simplified expression to represent the difference between the squares of two consecutive numbers $n$ and $(n + 1)$.

**b** If $n$ is always an integer, what type of number is the difference of the squares of two consecutive numbers?

**13** A square has side lengths of $x$ cm. A second square has side lengths 2 cm longer than the first square. Write a factorised expression to represent the difference between the areas of the two perfect squares.



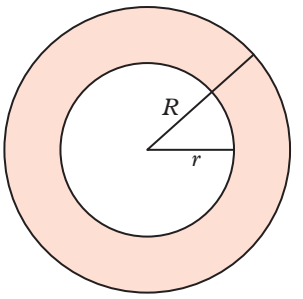

**14** Use the difference of two squares rule to fully factorise each expression.

**a** $x^4 - 16$

**b** $x^8 - y^8$

PROBLEM SOLVING AND REASONING

**15** An annulus (plural annuli) is a 2D shape that looks like a donut, with a circle cut out of the centre of another circle. The radius of the larger circle is labelled as $R$ and the radius of the smaller circle is labelled as $r$.

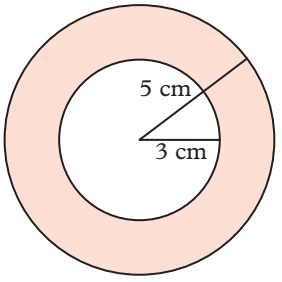



**a** Write an expression, in terms of $r$ and $R$, for the area of the annulus. Remember, the area of a circle with radius of $r$ units is $\pi r^2$.

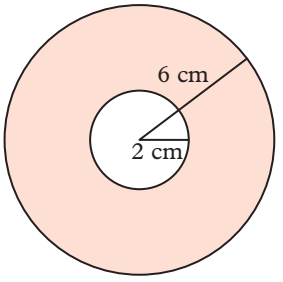
**b** Factorise the expression from part **a**.

**c** Calculate the area of the following annuli. Write your answers correct to two decimal places.

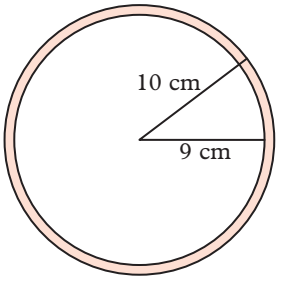
**i**




**ii**

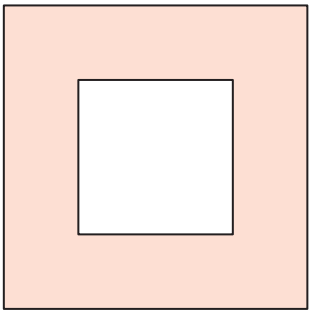



**iii**




**16** A square annulus is similar to an annulus but uses squares instead of circles.

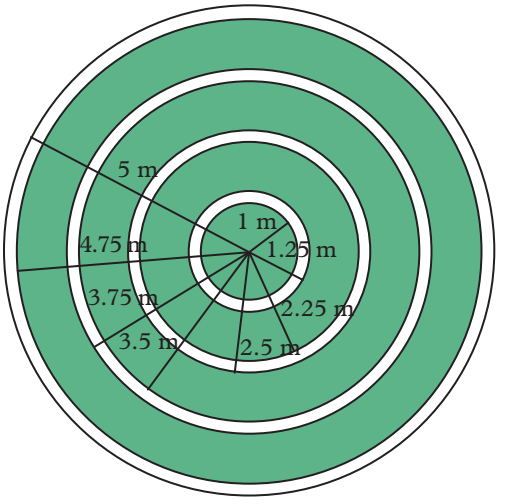
The square annulus shown has integer side lengths where the larger square has a side length 2 cm greater than the smaller square. If the shaded area of the square annulus is 24 cm$^2$, determine the side lengths of the two squares.



**17** A landscaper has designed a concentric circle pattern for the lawn at a university. The innermost circle of lawn has a diameter of 1 m and is surrounded by a concrete ring that is 0.25 m wide. The concrete ring is surrounded by a 1-m-wide ring of lawn. Three more rings of concrete and two more rings of lawn of the same width surround the original circle.

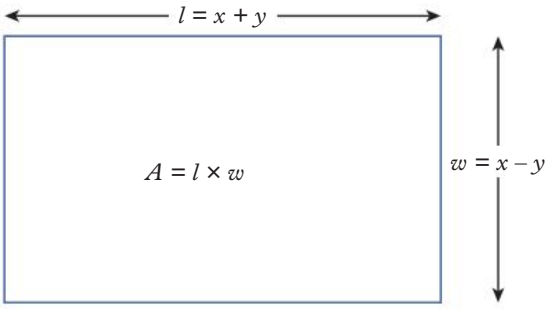
**a** Write a calculation using the radii lengths for the area of concrete needed.

**b** By factorising, show that the area of concrete needed is given by $0.25\pi(5 + 4.75 + 3.75 + 3.5 + 2.5 + 2.25 + 1.25 + 1)$ m$^2$

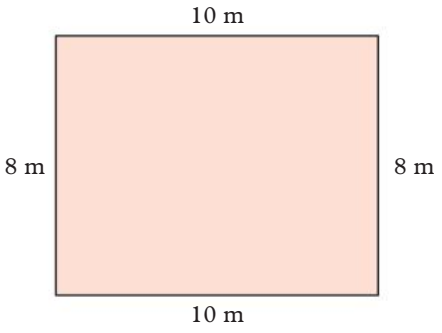


**18** If the length of a rectangle is given by $l = x + y$ and the width is given by $w = x - y$, then the area of any rectangle, $A = l \times w$, can be expressed as the difference of two squares $A = x^2 - y^2$.

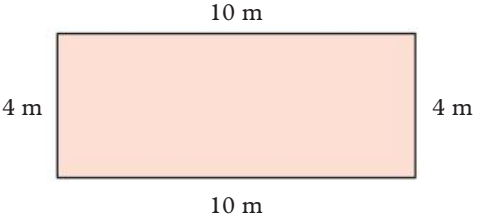
For each of the following rectangles, state the side lengths ($x$ and $y$) of two squares, where the difference in the areas of the squares is the same as the area of the rectangle.



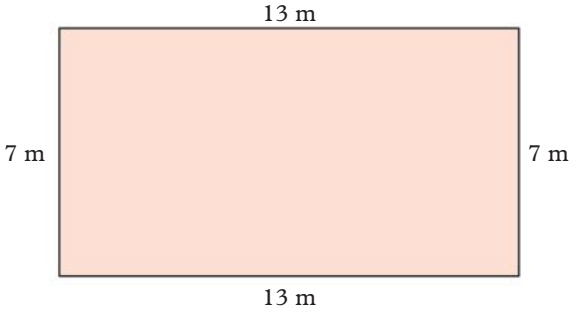

**a**



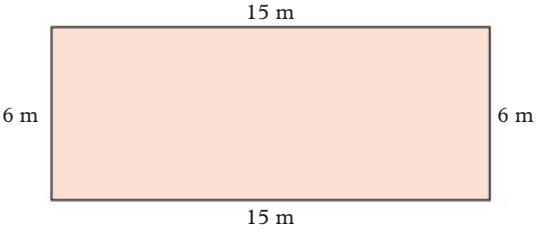

**b**




**c**




**d**




**19** A rectangle has a length 6 cm longer than its width. The rectangle's area is a square number, $y^2$, less than another square number, $x^2$. What is the value of $y^2$ in square centimetres?

**20** We can factorise the difference of any two terms by using square roots. For example, $x - y$ can be factorised to $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$. Factorise the following to the product of conjugates. Remember to evaluate square roots and simplify surds where possible.

**a** $a - 5$ **b** $b^2 - 7$

**c** $c - 8$ **d** $3d - 16$

**e** $4e - 45$ **f** $2f^2 - 75$

**g** $x^3 - y^5$ **h** $8x^2 - 12y^2$



**21** Show how $(x + 1)(x + y)^2 - (x + 1)(x - y)^2$ can be written in its simplest factorised form of $4xy(x + 1)$.

**22** **a** Expand $(x + y)(x^2 - xy + y^2)$.

**b** Use your result in part **a** to factorise $x^3 + 8$.

**c** Predict the factorised form of $x^3 - 8$.

**23** These are the known facts about two numbers: The difference between two numbers is 5 and the difference between the squares of the two numbers is 155. What is the sum of the two numbers?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Factorising the difference of two squares

CAS instructions
Factorising

Topic quiz
3D

# 3E Factorising quadratic expressions

## Learning intentions

- ✔ I can factorise simple quadratic trinomials.
- ✔ I can factorise quadratic trinomials by first taking out a common factor.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Factors and multiples |
| Year 7 | 2D Factors and the highest common factor |
| Year 8 | 5G Factorising |
| Year 10 | 2G Factorising quadratic expressions |

## Quadratic trinomials

- A **quadratic** is an algebraic expression that contains a squared pronumeral, with no indices greater than 2 in the expression.
  For example, $6t^2$, $x^2 + 5$, $k^2 + 14k + 30$, and $4b^2 - a^2$ are all quadratic expressions.
- An expression with three terms is called a **trinomial**.
- A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where $a$, $b$ and $c$ are constants.

$$ax^2 + bx + c$$

  → $a$ is the **leading coefficient**
  → $b$ is the coefficient of the **linear term**
  → $c$ is the **constant term**

- Many quadratic trinomials can be factorised to produce a binomial product.

expanding ←

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

quadratic trinomial expanded form — binomial product factorised form

→ factorising

## Factorising quadratics of the form $x^2 + bx + c$

- Expanding a binomial product of the form $(x + m)(x + n)$, where $m$ and $n$ are constants, gives a quadratic trinomial of the form $x^2 + bx + c$.

$$\begin{aligned}(x + m)(x + n) &= x(x + m) + n(x + m)\\ &= x^2 + mx + nx + mn\\ &= x^2 + (m + n)x + mn\\ &= x^2 + bx + c\end{aligned}$$

where $b = m + n$ and $c = m \times n$

- The process can be reversed to factorise quadratics of the form $x^2 + bx + c$, by finding two numbers ($m$ and $n$) that add to give $b$ and multiply to give $c$.

  For example, $x^2 + 4x + 3 = x^2 + (3 + 1)x + (3 \times 1)$
  $= (x + 3)(x + 1)$

## Example 3E.1 Finding the numbers for a given product and sum

Identify which two numbers add to give the first number and multiply to give the second number.

**a** Sum: 5, Product: 6

**b** Sum: −2, Product: −8

**THINK**

**a** **1** List the factor pairs of 6. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.

**2** Add the factor pairs together and determine which pair adds to 5.

**b** **1** List the factor pairs of −8. Remember that a negative multiplied by a positive is a negative, so there are two combinations of factor pairs with the same numerals.

**2** Add the factor pairs together and determine which pair adds to −2.

**WRITE**

**a** $1 \times 6 = 6 \rightarrow 1 + 6 = 7$
$-1 \times -6 = 6 \rightarrow -1 + (-6) = -7$
$2 \times 3 = 6 \rightarrow 2 + 3 = 5$
$-2 \times -3 = 6 \rightarrow -2 + (-3) = -5$

The numbers are 2 and 3.

**b** $-1 \times 8 = -8 \rightarrow -1 + 8 = 7$
$1 \times -8 = -8 \rightarrow 1 + (-8) = -7$
$-2 \times 4 = -8 \rightarrow -2 + 4 = 2$
$2 \times -4 = -8 \rightarrow 2 + (-4) = -2$

The numbers are 2 and −4.

## Example 3E.2 Factorising simple quadratic trinomials

Factorise the quadratic trinomial $x^2 + 7x + 10$.

**THINK**

**1** List the factor pairs of the constant term, 10. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.

**2** Add the factor pairs together and identify which pair adds to the linear coefficient, 7.

**3** Write the expression in factorised form.

**4** Check your result by expanding.

**WRITE**

$1 \times 10 = 10 \rightarrow 1 + 10 = 11$
$-1 \times -10 = 10 \rightarrow -1 + (-10) = -11$
$2 \times 5 = 10 \rightarrow 2 + 5 = 7$
$-2 \times -5 = 10 \rightarrow -2 + (-5) = -7$

The numbers are 2 and 5.

$x^2 + 10x + 7 = (x + 2)(x + 5)$

Check: $(x + 2)(x + 5) = x^2 + 2x + 5x + 10$
$= x^2 + 7x + 10$



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## Example 3E.3 Factorising more complex quadratic trinomials (+ and –)

Factorise each quadratic trinomial.

**a** $m^2 + 2m - 3$ **b** $x^2 - 7x - 8$

**THINK**

1. List the factor pairs of the constant term. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.
2. Add the factor pairs together and identify which pair adds to the linear term.
3. Write the expression in factorised form.
4. Check your result by expanding.

**WRITE**

**a** $1 \times -3 = -3 \rightarrow 1 + (-3) = -2$
$-1 \times 3 = -3 \rightarrow -1 + 3 = 2$
$m^2 + 2m - 3 = (m + 3)(m - 1)$
Check: $(m + 3)(m - 1) = m^2 - m + 3m - 3$
$= m^2 + 2m - 3$

**b** $1 \times -8 = -8 \rightarrow 1 + (-8) = -7$
$-1 \times 8 = -8 \rightarrow -1 + 8 = 7$
$2 \times -4 = -8 \rightarrow 2 + (-4) = -2$
$-2 \times 4 = -8 \rightarrow -2 + (4) = 2$
$x^2 - 7x - 8 = (x - 8)(x + 1)$
Check: $(x - 8)(x + 1) = x^2 + x - 8x - 8$
$= x^2 - 7x - 8$

## Example 3E.4 Factorising quadratic trinomials by first taking out a common factor

Factorise by first taking out a common factor.

**a** $2x^2 - 14x + 12$ **b** $-x^2 + 4x - 3$

**THINK**

1. If the coefficient of $x^2$ is not 1, check for a common factor of the three terms. Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF. A common factor can be –1.
2. Factorise the expression inside the brackets. List the factor pairs of the constant term.
3. Add the factor pairs together and identify which pair adds to the linear term.
4. Write the expression in factorised form.
5. Check your result by expanding.

**WRITE**

**a** $2x^2 - 14x + 12 = 2(x^2 - 7x + 6)$
$1 \times 6 = 6 \rightarrow 1 + 6 = 7$
$-1 \times -6 = 6 \rightarrow -1 + (-6) = -7$
$2 \times 3 = 6 \rightarrow 2 + 3 = 5$
$-2 \times -3 = 6 \rightarrow -2 + (-3) = -5$
$2(x^2 - 7x + 6) = 2(x - 6)(x - 1)$
Check: $2(x - 6)(x - 1) = 2(x^2 - x - 6x + 6)$
$= 2(x^2 - 7x + 6)$
$= 2x^2 - 14x + 12$

**b** $-x^2 + 4x - 3 = -(x^2 - 4x^2 + 3)$
$1 \times 3 = 3 \rightarrow 1 + 3 = 4$
$-1 \times -3 = 3 \rightarrow -1 + (-3) = -4$
$-(x^2 - 4x + 3) = -(x - 3)(x - 1)$
Check: $-(x - 3)(x - 1) = -(x^2 - x - 3x + 3)$
$= -(x^2 - 4x + 3)$
$= -x^2 + 4x - 3$

**Helpful hints**

- ✔ The order of the binomial products doesn't matter. This is because of the commutative law for multiplication. For example: $2 \times 3 = 3 \times 2$ and $(x - 2)(x + 3) = (x + 3)(x - 2)$
- ✔ The order of the terms in a quadratic trinomial does not change the value of the expression. This is because of the commutative law for addition. For example: $x^2 + 6x + 8 = 6x + 8 + x^2 = 8 + x^2 + 6x$
- ✔ Always look for the HCF of the three terms before factorising. You might get lucky and find that it makes factorising easier.
- ✔ You can check your factorisation of the binomial products by expanding the brackets!

$$x^2 + 4x + 3 = (x+1)(x+3)$$
$$= x^2 + 3x + x + 3$$
$$= x^2 + 4x + 3$$

- ✔ You don't always need to consider all the factor pairs of the constant term, $c$, to factorise a quadratic of the form $x^2 + bx + c$.
- ✔ If the constant term, $c$, is positive, then both factors will have the same sign. So:
  - → if $b$ is positive, then both factors will be positive
  - → if $b$ is negative, then both factors will be negative.

ANS p491

# Exercise 3E Factorising quadratic expressions

| | | |
|---|---|---|
| 1–3(1st, 2nd columns), 4, 5–6(1st, 2nd columns), 7–13, 16(a–d), 18 | 4, 6–11, 13, 14, 16(b, e, g, h, i), 18, 19, 21(a, c, d, f) | 4(c, f, i, l), 6(c, f, i, l), 7–11, 14, 15, 16(d, g, h, i), 17, 20, 21(b, c, d, f, i), 22, 23 |

UNDERSTANDING AND FLUENCY

3E.1 **1** Identify which two numbers add to give the first number and multiply to give the second number.

**a** Sum: 5, Product: 4 **b** Sum: 6, Product: 8 **c** Sum: 13, Product: 22

**d** Sum: 9, Product: 20 **e** Sum: 10, Product: 24 **f** Sum: 7, Product: 12

**g** Sum: 13, Product: 42 **h** Sum: 12, Product: 35 **i** Sum: 8, Product: 16

**2** Identify which two numbers add to give the first number and multiply to give the second number.

**a** Sum: 2, Product: −8 **b** Sum: −1, Product: −6 **c** Sum: −8, Product: 12

**d** Sum: 3, Product: −10 **e** Sum: −8, Product: −9 **f** Sum: −5, Product: 6

**g** Sum: 5, Product: −6 **h** Sum: −6, Product: −27, **i** Sum: −12, Product: 11

**3** Use your results from question **1** to factorise each of these quadratic trinomials.

**a** $x^2 + 5x + 4$ **b** $x^2 + 6x + 8$ **c** $x^2 + 13x + 22$

**d** $x^2 + 9x + 20$ **e** $x^2 + 10x + 24$ **f** $x^2 + 7x + 12$

**g** $x^2 + 13x + 42$ **h** $x^2 + 12x + 35$ **i** $x^2 + 8x + 16$

3E.2 **4** Factorise each quadratic trinomial.

**a** $a^2 + 4a + 3$ **b** $b^2 + 9b + 14$

**c** $c^2 + 7c + 6$ **d** $d^2 + 10d + 21$

**e** $e^2 + 8e + 7$ **f** $f^2 + 8f + 15$

**g** $g^2 + 11g + 28$ **h** $h^2 + 13h + 36$

**i** $x^2 + 9x + 18$ **j** $j^2 + 14j + 45$

**k** $k^2 + 11k + 30$ **l** $y^2 + 13y + 40$

**5** Use your results from question **2** to factorise each of these quadratic trinomials.

**a** $x^2 + 2x - 8$ **b** $x^2 - x - 6$ **c** $x^2 - 8x + 12$

**d** $x^2 + 3x - 10$ **e** $x^2 - 8x - 9$ **f** $x^2 - 5x + 6$

**g** $x^2 + 5x - 6$ **h** $x^2 - 6x - 27$ **i** $x^2 - 12x + 11$

3E.3 **6** Factorise each quadratic trinomial.

**a** $a^2 + 2a - 3$ **b** $b^2 - 2b - 15$ **c** $c^2 - 5c + 4$

**d** $d^2 + 5d - 14$ **e** $e^2 - 10e + 24$ **f** $f^2 - 3f - 10$

**g** $g^2 + g - 12$ **h** $h^2 - 8h + 15$ **i** $x^2 - 5x - 24$

**j** $j^2 - 10j + 16$ **k** $k^2 + 3k - 18$ **l** $y^2 - y - 2$

3E.4 **7** Factorise each quadratic trinomial by first taking out a common factor.

**a** $3x^2 + 9x + 6$ **b** $2x^2 + 16x + 30$ **c** $5x^2 + 15x - 20$

**d** $-4x^2 - 20x - 24$ **e** $-6x^2 + 36x - 30$ **f** $-x^2 - 2x + 35$

**8** Factorise each quadratic trinomial by first taking out a common factor.

**a** $ax^2 + 8ax + 12a$ **b** $bx^2 - 11bx + 28b$ **c** $10cx^2 - 10cx - 300c$

**d** $-2dx^2 + 34dx - 120d$ **e** $p^2x^2 + 13p^2x + 30p^2$ **f** $q^2x^2 + 15qx^2 + 44x^2$

**9** Factorise each quadratic.

**a** $x^2 - 6x + 9$ **b** $x^2 + 0x - 9$ **c** $x^2 - 6x + 0$

**10** Factorise each quadratic.

**a** $x^2 - 25$ **b** $x^2 - 10x + 25$ **c** $x^2 - 25x$

**d** $x^2 + 14x + 49$ **e** $x^2 - 49x$ **f** $x^2 - 49$

**11** Factorise each quadratic by first reordering and simplifying the terms of each expression.

**a** $21 + 10a + a^2$ **b** $2b - 35 + b^2$ **c** $3c + c^2 - 18$

**d** $48 + 2d - d^2$ **e** $-24 - 10e - e^2$ **f** $10f - f^2 - 16$

**g** $10g + 70 + g^2 + 7g + 2$ **h** $-2h + 5 + h^2 - 19 - 3h$ **i** $50 - 20x - x^2 + 7x - 86$

**12 a** Determine all the positive integers (including zero) that can be substituted into the constant term of $x^2 + 6x + \square$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

**b** Determine all the positive integers up to 40 that can be substituted into the constant term of $x^2 + 6x - \square$ so that it can be factorised. Write both the expanded and factorised form for each possible value.

**c** When the coefficient of $x$ is positive, how can you tell if the factors are both positive or if one is positive and one negative?

**13 a** Determine all the positive integers that can be substituted into the constant term of $x^2 - 6x + \square$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

**b** Determine all the positive integers up to 40 that can be substituted into the constant term of $x^2 - 6x - \square$ (including zero) so that it can be factorised. Write both the expanded and factorised form for each possible value.

**c** When the coefficient of $x$ is negative, how can you tell if the factors are both negative or if one is positive and one negative?

**14 a** Determine all the positive integers that can be substituted into the constant term of $x^2 + \square x + 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

**b** Determine all the positive integers that can be substituted into the constant term of $x^2 - \square x + 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

**c** When the constant term is positive:

**i** how can you tell if the factors need to be positive or negative?

**ii** is the coefficient of $x$ the sum or difference of the positive values of the factors?

**15 a** Determine all the positive integers (including zero) that can be substituted into the constant term of $x^2 - \square x - 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

**b** Determine all the positive integers (including zero) that can be substituted into the constant term of $x^2 + \square x - 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

**c** When the constant term is negative:

**i** how can you tell which factor must be positive and which must be negative?

**ii** is the coefficient of $x$ the sum or difference of the positive values of the factors?

**16** Use the expansion of a perfect square rule, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$, to completely factorise each quadratic trinomial as a perfect square.

**a** $x^2 + 8x + 16$ **b** $y^2 - 10y + 25$ **c** $v^2 + 2v + 1$

**d** $(2z)^2 - 12z + 9$ **e** $9w^2 + 42x + 49$ **f** $3h^2 - 24h + 24$

**g** $-k^2 - 18k - 81$ **h** $f^2 + 2fg + g^2$ **i** $(5p)^2 - 40pq + (4q)^2$

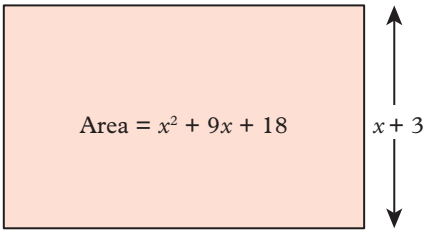
**17** The following quadratic trinomials cannot be factorised as a perfect square.

**i** $x^2 + 6x - 9$ **ii** $-x^2 + 6x + 9$ **iii** $x^2 + 3x + 9$ **iv** $x^2 + 6x + 3$

**a** Describe how you can check if a quadratic trinomial is a perfect square.

**b** State why each of the above quadratic trinomials are not perfect squares.

**18** A rectangle has an area of $x^2 + 9x + 18$ and a width of $x + 3$.




**a** Determine the expression of the length of the rectangle.

**b** Write the area of the rectangle as a product of its length and width.

**c** If $x = 2$ m, calculate the area using:

**i** the product of the length and width

**ii** the expression $x^2 + 9x + 18$.

**19 a** Write an expression for the missing side length for each rectangular object.

**i** area within frame is $(x^2 + 7x - 18)$ cm$^2$

**ii** area of billboard is $(9y^2 - 16)$ m$^2$

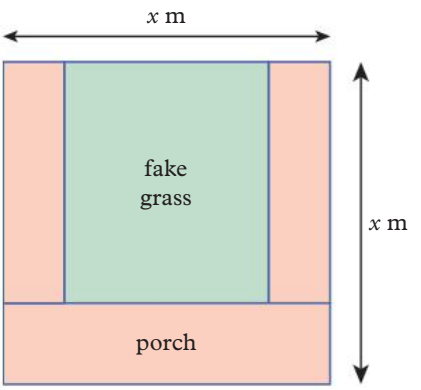
**b** Find the value of $x$ that gives an area of 152 cm$^2$ for part **i**.

**c** Find the value of $y$ that gives an area of 425 m$^2$ for part **ii**.

**d** Determine the positive value of $x$ and $y$ such that the area of the rectangular objects is zero.

**e** Substitute $x = 1$ and $y = 1$ into the length, width, and area expression and state why they are not appropriate for the items.

**20** Two friends, Melissa and Lena, want to renovate their square-shaped backyards by adding a full-length rectangular porch and installing a rectangular section of fake grass with garden beds on either side. Both friends want to have the same width for their porches and their garden beds but have different-sized backyards, so they decide to use algebra to determine the area of fake grass that they need for both. They've already decided on the width of their porches, and have narrowed down the area of fake grass to four options:




**i** $(x^2 - 5x + 6)$ m$^2$

**ii** $(x^2 - 7x + 10)$ m$^2$

**iii** $(x^2 - 4x + 4)$ m$^2$

**iv** $(x^2 - 6x + 8)$ m$^2$

**a** Factorise each quadratic expression.

**b** Consider the factors of each equation and relate them to the possible side lengths of the fake grass. Judging from Melissa and Lena's four options, what is the width of Melissa and Lena's porches?

**c** Lena also suggested the fake grass areas $(x^2 - x - 6)$ m$^2$ and $(x^2 + x - 6)$ m$^2$. Explain why Melissa said these areas wouldn't be possible.

**d** Melissa and Lena decide to have 2 m of space either side of the lawn for their garden beds. Which quadratic expression did they decide on?

**e** If Melissa's backyard is 6 m by 6 m, determine what area of fake grass she will install.

**f** If Lena's backyard is 7.5 m by 7.5 m, determine what area of fake grass she will install.

**g** Determine how much area Melissa and Lena will each have for planting flowers.



**21** Factorise each quadratic.

**a** $(3x)^2 + 5(3x) - 14$ **b** $(11x)^2 + 2(11x) - 80$ **c** $(5x)^2 - 12(5x) + 32$

**d** $(x + 2)^2 + 3(x + 2) - 4$ **e** $(x - 4)^2 - 11(x - 4) + 30$ **f** $(x - 9)^2 + 14(x - 9) + 45$

**g** $(3x)^2 - (3x) - 42$ **h** $(2x)^2 + 7(2x) - 30$ **i** $(8x)^2 + 32(2x) - 48$

**22** Fully factorise each expression.

**a** $(x^2)^2 - 13(x^2) + 36$ **b** $(x^2)^2 - 12(x^2) - 64$ **c** $(x^2)^2 + 24(x^2) - 25$

**23** We can factorise quadratic trinomials by splitting the linear term once we have determined the two factors and then use grouping. For example, two numbers that add to 7 and multiply to 12 are 3 and 4 so:

$$\begin{aligned} x^2 + 7x + 12 &= x^2 + 3x + 4x + 12 \\ &= x(x + 3) + 4(x + 3) \\ &= (x + 3)(x + 4) \end{aligned}$$

Factorise each of the following by splitting the linear term and using grouping.

**a** $x^2 + 5x + 4$ **b** $x^2 + 6x + 8$ **c** $x^2 + 13x + 22$

**d** $x^2 + 3x - 10$ **e** $x^2 - 8x - 9$ **f** $x^2 - 5x + 6$

Check your Student obook pro for these digital resources and more:

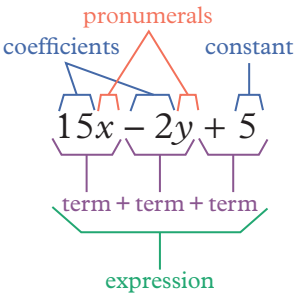
Interactive skillsheet
Factorising quadratic expressions

Worksheet
Factorising quadratic expressions

Topic quiz
3E

# Chapter summary

## Simplifying terms and expressions




## Adding and subtracting by terms

- Like terms contain the exact same pronumerals.

$$3a^2b = 3 \times a \times a \times b$$

index form — expanded form

- Like terms can be added and subtracted by adding and subtracting their coefficients.

$$a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$$

## Multiplying algebraic terms

$$3a^3b \times -2a^2b^2 = 3 \times -2 \times a^{(3+2)} \times b^{(1+2)}$$
$$= -6a^5b^3$$

## Dividing algebraic terms

$$\frac{6a^5b^3}{3a^3b} = \frac{\cancel{6}^{2} \times a^5 \times b^3}{\cancel{3}^{1} \times a^3 \times b}$$
$$= 2 \times a^{(5-3)} \times b^{(3-1)}$$
$$= 2a^2b^2$$

## Expanding

- Use the distributive law to expand brackets

$$a(b + c) = ab + ac$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

## Factorising by grouping terms

$$x^2 + 2x - 3x - 6 = x(x + 2) - 3(x + 2)$$
$$= (x + 2)(x - 3)$$

## Factorising quadratic trinomials

$$ax^2 + bx + c$$

To factorise a quadratic trinomial of the form $x^2 + bx + c$:

- find two numbers that add to give $b$ and multiply to give $c$
- substitute those two numbers into the binomial product factorised form.

expanding ←

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

quadratic trinomial expanded form — binomial product factorised form

→ factorising

## Factorising using the HCF

expanding ←

$$2x^2 - 6x = 2x(x - 3)$$

expanded form — factorised form

## Difference of two squares

expanding ←

$$a^2 - b^2 = (a + b)(a - b)$$

difference of two squares expanded form — binomial product factorised form

→ factorising

## Expansion of a perfect square

expanding ←

$$a^2 + 2ab + b^2 = (a + b)^2$$

quadratic trinomial expanded form — perfect square factorised form

→ factorising

# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams

## Multiple-choice

3A **1** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

**A** $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{12a}$ **D** $\frac{6ab^2}{18a^2}$ **E** $\frac{b^2}{3a}$

**2** Which is not a like term to $4ab^3$?

**A** $3abbb$ **B** $4ba^3$ **C** $-4b^3a$ **D** $2bab^2$ **E** $\frac{ab^3}{7}$

**3** If $x = -3$ and $y = 2$ then $-5y^3x^2$ is equal to:

**A** $-360$ **B** $360$ **C** $-540$ **D** $540$ **E** $-9000$

3B **4** Which expression is equivalent to $8 - 2(3 - 5g)$?

**A** $18 - 30g$ **B** $10g + 2$ **C** $10g + 10$ **D** $2 - 10g$ **E** $10 - 10g$

**5** Which expression is **not** equivalent to $(3x + 4)(2y - 5)$?

**A** $2y(3x + 4) - 5(3x + 4)$ **B** $6xy - 20$

**C** $6xy - 15x + 8y - 20$ **D** $3x \times 2y + 3x \times (-5) + 4 \times 2y + 4 \times (-5)$

**E** $3x(2y - 5) + 4(2y - 5)$

**6** Which expression cannot be expanded using the difference of two squares rule?

**A** $(x + 6)(x - 6)$ **B** $(7 - p)(7 + p)$ **C** $(2x - 7)(2x + 7)$

**D** $(q + 9)(-q + 9)$ **E** $(d + 5)(d - 2)$

3B **7** Which statement is *incorrect*?

**A** $(d + 3)(d - 7) = d^2 + 4d - 21$ **B** $-3(b + 5) = -3b - 15$

**C** $(m - 4)(m + 4) = m^2 - 16$ **D** $(2b + 5)(3b - 2) = 6b^2 + 11b - 10$

**E** $(e - 6)^2 = e^2 - 12e + 36$

3C **8** When fully factorised, the expression $9x^2 - 3x^3$ factorises to:

**A** $3(3x^2 - x^3)$ **B** $3x^2(3 - x)$ **C** $x^2(9 - 3x)$

**D** $3x(3x - x^2)$ **E** $-3x(x^2 - 3x)$

**9** Which of the following is **not** a factor of $12x^2y - 24xy^2$?

**A** $x^2$ **B** $12xy$ **C** $12$ **D** $6y$ **E** $3x$

**10** The expression $3x - 2$ is **not** a factor of which expression?

**A** $24xy + 15x - 18y - 10$ **B** $6(3x - 2) - y(3x - 2)$

**C** $3x(5y + 9) - 2(5y + 9)$ **D** $21x - 14$ **E** $3x - 2$

3D **11** The expression $w^2 - 49$ factorises to:

**A** $(w - 7)^2$ **B** $(w + 7)^2$ **C** $(w + 7)(w - 7)$

**D** $(7 + w)(7 - w)$ **E** $(w - 7)(7 - w)$

**12** When fully factorised, the expression $36x^2z - 100y^2z$ factorises to:

**A** $4z(5y + 3x)(5y - 3x)$ **B** $z(6x - 10y)(6x - 10y)$ **C** $4z(6x - 10y)(6x - 10y)$

**D** $4z(3x + 5y)(3x - 5y)$ **E** $z(6x + 10y)(6x - 10y)$

**13** The expression $(a - 3)^2 - 9$ factorises to:

**A** $(a + 6)(a - 12)$ **B** $(a - 12)(a + 6)$ **C** $a^2 - 6a$

**D** $a(a + 6)$ **E** $a(a - 6)$

3E **14** The expression $t^2 + 3t - 18$ factorises to:

**A** $(t + 9)(t - 2)$ **B** $(t - 6)(t + 3)$ **C** $(t + 6)(t - 3)$

**D** $(t - 9)(t + 2)$ **E** $(t + 5)(t - 2)$

3E **15** When fully factorised, the expression $3y^2 - 33y + 84$ factorises to:

**A** $3(y - 7)(y - 4)$ **B** $(3y - 7)(3y - 4)$ **C** $(3y - 21)(y - 4)$

**D** $3(y + 7)(y + 4)$ **E** $(y - 7)(3y - 12)$

**16** Which of the following is not equivalent to $b^2 - 8b + 16$?

**A** $(b - 4)^2$ **B** $(4 - b)^2$ **C** $(b - 4)(4 - b)$

**D** $(4 - b)(4 - b)$ **E** $(b - 4)(b - 4)$

## Short answer

3A **1** Simplify each expression.

**a** $15t - 7t + 8t$ **b** $a - 7p - 11p + 12a$

**c** $3k + 5km - 7k - 15 + 2k - 4km$ **d** $6m^2n - 2m^2 + 7nm^2 + 11n^2 - 4mn^2 - 3m^2$

3A **2** Simplify each expression.

**a** $4xy \times 11xyz$ **b** $9mnp \times 2m^3p \times 4n^2$

**c** $15de \div (18df)$ **d** $11klmn \div (-22klm^2)$

3B **3** Expand each product.

**a** $4(z - 7)$ **b** $-8(5 - 3y)$ **c** $5x(7 - 6w)$

**d** $(u + 3)(t - 4)$ **e** $(5r + 6)(8 - 3v)$ **f** $(9p + 11q)(7m - 3n)$

**4** Expand and simplify each expression to remove the brackets.

**a** $5(a + 2) - 3(7 - a)$ **b** $(b - 11)(b + 2)$ **c** $(3c - 2)(4c - 5)$

**d** $(d + w)(d - w)$ **e** $(6 + e)^2$ **f** $(f - 9)^2$

**5** Expand and simplify each product.

**a** $x^4(x^3 - x^2)$ **b** $y^3z^4(y^9 - z^2)$ **c** $(x^5 - y^3)(x^6 + y^4)$

3C **6** Factorise each expression.

**a** $4a - 24$ **b** $36pq^2 + 144pq$

**c** $7d(8 - d) - 4(8 - d)$ **d** $5e + 15ef + 2 + 6f$

**7** Factorise each expression.

**a** $6z^5 - 5z^4$ **b** $9d^2y + 15dx^2$

**c** $-36r^{10}t^8u^6 - 96r^7t^8u^3$ **d** $a^5b^2c^7d^8 + a^6b^5c^9d^5 + a^4b^2c^8d^7$

**8** Factorise each expression.

**a** $24x^2(x^2 + 2) - 8x(x^2 + 2)$ **b** $(r + q)(t + 5) + (r + q)(p - 3)$ **c** $4x^2 + 16x - 6x - 24$

3D **9** Factorise each expression using the difference of two squares rule.

**a** $a^2 - 64$ **b** $121 - b^2$ **c** $36m^2 - 49n^2$

**d** $(p + 1)^2 - 4$ **e** $2e^2 - 32$ **f** $(f + 4)^2 - (f - 5)^2$

**10** Factorise each expression.

**a** $y^{10} - z^{10}$ **b** $q^2 - (q + 4)^2$ **c** $-r^2 + 100$ **d** $-64 + 81j^2$

**11** Factorise each expression.

**a** $\frac{196}{25}x^2 - \frac{169}{4}y^2$ **b** $\frac{144}{c^2} - \frac{225}{d^2}$

**c** $-\frac{u^2}{16} + \frac{36}{49}$ **d** $0.04v^2 - 1.21$

3E **12** Factorise each quadratic trinomial.

**a** $a^2 + 6a + 5$ **b** $b^2 - 7b + 12$ **c** $c^2 + 4c - 21$ **d** $d^2 - 16d - 36$

**13** Factorise each quadratic trinomial.

**a** $15 + 8x + x^2$ **b** $60 + 20x - 5x^2$

**c** $32x - 2x^2 - 120$ **d** $240 - 6x - 3x^2$

**14** Factorise each quadratic.

**a** $98 - 2x^2$ **b** $16y^2 + 64x^2$ **c** $x^2 - 10x + 25$ **d** $9x^2 - 36x$

## Analysis

**1** Expressions in the form $(x + a)^2 - b^2$ can be factorised using the difference of two squares rule.

**a** Factorise each of the following using the difference of two squares rule.

**i** $(x + 8)^2 - 9$ **ii** $(x - 10)^2 - 16$ **iii** $(x + 11)^2 - 25$

**b** Describe the connection between the numbers in the brackets in factor form and the values of $a$ and $b$ in $(x + a)^2 - b^2$ form.

**c** Expand each of the following.

**i** $(x + 8)^2 - 9$ **ii** $(x - 10)^2 - 16$ **iii** $(x + 11)^2 - 25$

**d** Describe the connection between the coefficient of $x$ in the expanded form and the value of $a$ in $(x + a)^2 - b^2$ form.

**e** Describe the connection between the constant in the expanded form and the values of $a$ and $b$ in $(x + a)^2 - b^2$ form.

**f** Fill in the spaces so that the expressions are equivalent.

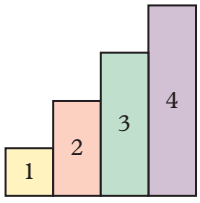
**i** $(x + \square)^2 - 4 = x^2 + 6x + 5 = (x + \square)(x + \square)$

**ii** $(x - 5)^2 - \square = x^2 - \square x + \square = (x - 9)(x - 1)$

**iii** $(x + \square)^2 - \square = x^2 + 14x - 51 = (x - \square)(x + \square)$

**2** Malak and Samara are investigating how to quickly add the positive integers from 1 to $n$: 1, 2, 3, 4, 5, ..., $n$

Malak finds the expression $\frac{1}{2}n(n + 1)$ and Samara finds the expression $\frac{(n + 1)^2 - (n + 1)}{2}$, which will give the sum of the integers from 1 to 4, if $n = 4$. Visually, the first four positive integers can be represented using bars.




**a** Fully expand both of Malak's and Samara's expressions to show that they are equivalent.

**b** Calculate the sum of the first five positive integers:

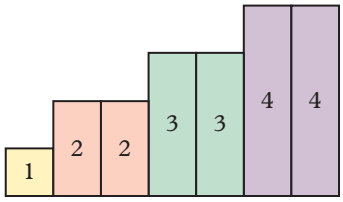
**i** by manually computing the sum 1 + 2 + 3 + 4 + 5

**ii** by using one of the expressions.

**c** Calculate the sum of the first 100 positive integers.

Malak decides to investigate the sum of the first $n$ positive even numbers: 2, 4, 6, 8, 10,..., $n$

Malak finds the expression $n^2 + n$, which will give the sum of the positive even numbers from 2 to 8, if $n = 4$. Visually, the first four even numbers can be represented using double bars.



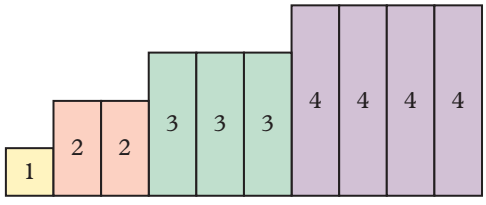

**d** Factorise $n^2 + n$.

**e** Calculate the sum of the first 100 positive even numbers.

**f** Describe the connection between the sum of the first $n$ positive even numbers and the sum of the first $n$ positive integers.

Samara decides to investigate the sum of the first $n$ square numbers: 1, 4, 9, 16, 25,..., $n$

Samara finds the expression $\frac{1}{6}(n^2 + n)(2n + 1)$, which will give the sum of the square numbers from 1 to 16, if $n = 4$. Visually, the first four square numbers can be represented using squares of bars.




**g** Fully expand $\frac{1}{6}(n^2 + n)(2n + 1)$.

**h** Calculate the sum of the first five square numbers:

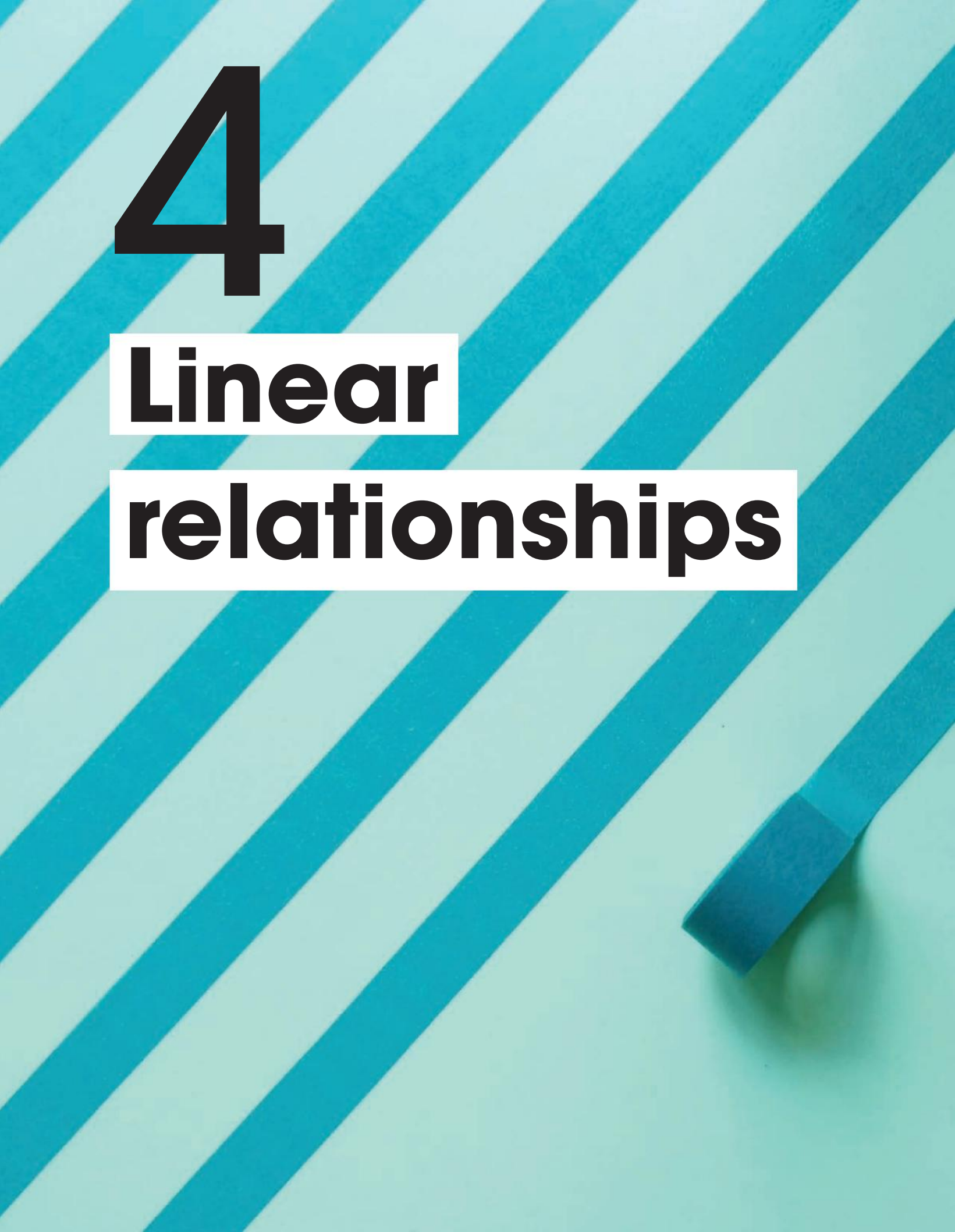
**i** by manually computing the sum 1 + 4 + 9 + 16 + 25

**ii** by using one of the expressions.

**i** Calculate the sum of the first 100 square numbers.

# 4

# Linear relationships



## Index

**4A** Solving linear equations
**4B** Plotting linear relationships
**4C** Gradient and intercepts
**4D** Sketching linear graphs using intercepts
**4E** Determining linear equations
**4F** Midpoint and length of a line segment
**4G** Direct proportion

## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Equivalent fractions
- ✔ Order of operations
- ✔ Expanding over one pair of brackets
- ✔ The Cartesian plane
- ✔ Plotting graphs

## Curriculum links

- Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (VCMNA301)
- Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (VCMNA308)
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (VCMNA309)
- Sketch linear graphs using the coordinates of two points and solve linear equations (VCMNA310)

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## Materials

- ✔ Calculator
- ✔ Graph paper
- ✔ Ruler

# 4A Solving linear equations

## Learning intentions

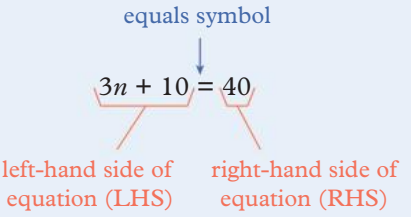
- ✔ I can solve linear equations using inverse operations.
- ✔ I can solve linear equations with the unknown on both sides using inverse operations.

**Inter-year links**

- Year 7 6H Solving equations using inverse operations
- Year 8 6C Solving equations with the unknown on both sides
- Year 10 4A Solving linear equations

## Linear equations

- An **equation** is a mathematical statement that shows equivalence between the expression on the left-hand side (LHS) and the right-hand side (RHS) of the equation.
- An algebraic equation contains one or more pronumerals (such as $x$, $y$, $a$ or $b$) that represent values, sometimes referred to as unknowns. If the pronumeral represents an unknown that can have multiple values, then it is called a variable.




- A **linear equation** is an equation containing only pronumerals that are raised to a power of 1.

| Linear equations | Non-linear equations |
|---|---|
| $y = x$ | $y = x^2$ |
| $\frac{n}{4} = 2$ | $\frac{n^4}{4} = 2$ |
| $2a - \frac{3}{5}b = 10$ | $a^3 + 6b = 0$ |

## Solving linear equations using inverse operations

- A **solution** is a value for an unknown that makes the equation a true statement.
- To check whether a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.
- To solve equations using **inverse operations**, identify and apply the inverse operation(s) required to reverse the operation(s) and isolate the unknown on the LHS of the equation ($x = \ldots$).
  - → For equations involving more than one operation, inverse operations must be performed in the reverse order to BIDMAS.
  - → A useful shorthand is to put the inverse operation in brackets to the left of the equation for each line of working out.
  - → To solve an equation in which the unknown appears on both sides of the equation, use inverse operations to eliminate the pronumeral term from one side of the equation, then solve the equation using inverse operations.

| Operation | Inverse operation |
|---|---|
| +3 | −3 |
| −3 | +3 |
| ×3 | ÷3 |
| ÷3 | ×3 |

For example, $4x - 2 = 3x + 1$ $(-3x)$
$x - 2 = 1$ $(+2)$
$x = 3$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## Example 4A.1 Solving two-step equations using inverse operations

Solve the following equations using inverse operations.

**a** $\frac{x}{3} - 5 = 7$ **b** $10 = -2(x + 6)$

**THINK**

**a** Identify and apply the inverse operation to both sides of the equation in the reverse order of BIDMAS, and then write the solution to the equation.

**b** Divide both sides by −2, remembering that the sign changes when you multiply or divide by a negative number. Subtract 6 from both sides. Note that $-11 = x$ is the same as $x = -11$.

**WRITE**

**a** $\frac{x}{3} - 5 = 7 \quad (+5)$

$\frac{x}{3} = 12 \quad (\times 3)$

$x = 36$

**b** $10 = -2(x + 6) \quad (\div -2)$

$-5 = x + 6 \quad (-6)$

$x = -11$

## Example 4A.2 Solving three-step equations using inverse operations

Solve:

**a** $12 = \frac{7x}{2} - 9$ **b** $\frac{-x - 4}{3} = 1$

**THINK**

**a** Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, and then write the solution to the equation.

**b** Identify and apply the inverse operation to both sides of the equation in the reverse order to BIDMAS, then write the solution to the equation. Note that $-x = 7$ is not the solution, as the $-x$ has a coefficient of −1.

**WRITE**

**a** $12 = \frac{7x}{2} - 9 \quad (+9)$

$21 = \frac{7x}{2} \quad (\times 2)$

$42 = 7x \quad (\div 7)$

$x = 6$

**b** $\frac{-x - 4}{3} = 1 \quad (\times 3)$

$-x - 4 = 3 \quad (+4)$

$-x = 7 \quad (\div -1)$

$x = -7$

## Example 4A.3 Solving equations with the unknown on both sides

Solve each equation for $x$.

**a** $4x + 7 = 2x - 3$ **b** $3(2x + 1) = -17 - 4x$

**THINK**

**a** 1 Eliminate the pronumeral term from one side of the equation by subtracting the pronumeral with the smaller coefficient, $2x$, from both sides of the equation.

2 Solve the equation using inverse operations.

**WRITE**

**a** $4x + 7 = 2x - 3 \quad (-2x)$

$2x + 7 = -3 \quad (-7)$

$2x = -10 \quad (\div 2)$

$x = -5$

**b** **1** Remove the brackets by expanding the expression on the left-hand side of the equation.

**2** Eliminate the negative pronumeral term by adding $4x$ to both sides of the equation.

**3** Solve the equation using inverse operations.

**b** $3(2x + 1) = -17 - 4x$

$6x + 3 = -17 - 4x \quad (+4x)$

$10x + 3 = -17 \quad (-3)$

$10x = -20 \quad (\div 10)$

$x = -2$

**Helpful hints**

- ✔ Writing the inverse operation beside the appropriate line of working out is a great way to keep track of your calculations!

$\frac{x}{3} - 5 = 7 \quad (+5)$

- ✔ Remember that you have to apply the inverse operation to both sides of the equation.
- ✔ Remember to define your pronumerals before writing equations to represent the variables in worded problems. For example, let $n$ = number of eggs in a carton or let $w$ = weight of eggs (grams).
  Note: '$n$ = eggs in a carton' would not be correct, as a pronumeral must always represent a quantity. Similarly, '$w$ = weight of eggs' would not be correct because it doesn't specify a unit of measurement.

ANS p493

# Exercise 4A Solving linear equations

▲ 1(a, b, e, g, l), 2–6, 7(a, c, f, g), 8–14, 16(a, b, c), 17, 21 ■ 2–13, 15, 16, 18, 19, 23(a, b) ⬟ 2–5, 7(e–h), 9, 11–13, 16(c, d), 20–23

UNDERSTANDING AND FLUENCY

**4A.1** **1** Solve the following equations using inverse operations.

**a** $4x + 5 = 29$ **b** $\frac{x + 3}{2} = 4$ **c** $\frac{x}{4} - 2 = 7$ **d** $\frac{2x}{5} = 4$

**e** $-2(x + 6) = 28$ **f** $-17 = -3x - 5$ **g** $9 = \frac{x}{5} + 6$ **h** $\frac{x - 4}{5} = -1$

**i** $-20 = 4(x - 2)$ **j** $-5x + 1 = 16$ **k** $\frac{-3x}{2} = -18$ **l** $\frac{x}{2} + 7 = 4$

**4A.2** **2** Solve the following equations using inverse operations.

**a** $\frac{3x + 4}{5} = 2$ **b** $\frac{-3x}{4} + 1 = 7$ **c** $\frac{5(x - 1)}{2} = 20$ **d** $4(x + 3) - 2 = 30$

**e** $\frac{x - 2}{6} - 3 = 0$ **f** $1 = \frac{11x}{4} + 23$ **g** $3 = \frac{-7x + 6}{2}$ **h** $-5(x + 2) - 7 = 3$

**i** $2 = \frac{2(x + 8)}{3}$ **j** $-16 = 4(5 - x) + 4$ **k** $6 = \frac{-x}{2} - 4$ **l** $\frac{2 - 3x}{5} = -2$

**3** Solve:

**a** $\frac{x - 2.9}{4} = 1$ **b** $\frac{x - 4}{3} + 11.2 = 9$ **c** $\frac{3x}{4} - 1.9 = 6.2$ **d** $\frac{5x + 2}{3} = -4.6$

**4** **a** Solve $5(x - 2) = 20$ by first dividing both sides by 5.

**b** Another way to solve this equation is first to expand the expression on the left-hand side. Try this method. Do you obtain the same solution?

**c** Solve $5(x - 2) = 18$ by first dividing both sides by 5.

**d** Solve $5(x - 2) = 18$ by first expanding the expression on the left side.

**e** Which method did you find easier to use when solving $5(x - 2) = 18$? Explain.

**5** Solve the following equations using the appropriate method from question **4**. Where relevant, write the solution as an improper fraction.

**a** $4(x - 1) = 8$ **b** $3(x + 7) = -6$ **c** $2(x - 3) = 5$

**d** $5(x + 4) = 8$ **e** $-4(x + 2) = -24$ **f** $-6(x - 2) = 1$

**6** **a** What value of $x$ would make $\frac{12}{x} = 3$ a true statement?

**b** What is the first step to solving this equation using inverse operations?

**c** Solve $\frac{12}{x} = 3$ using inverse operations. Use substitution to check that your solution is correct.

**7** Solve the following equations using inverse operations. Use substitution to check that your solution is correct.

**a** $\frac{10}{x} = 2$ **b** $\frac{21}{x} = -7$ **c** $\frac{-18}{x} = 3$ **d** $\frac{-24}{x} = -6$

**e** $\frac{11}{x} = 2$ **f** $\frac{8}{x} = -5$ **g** $\frac{9.6}{x} = 2$ **h** $-\frac{1}{x} = 3$

4A.3 **8** Solve each equation for $x$.

**a** $6x + 5 = 4x + 9$ **b** $3x - 11 = x + 3$ **c** $3x - 8 = 6x - 5$ **d** $-15 - 2x = 4x + 3$

**e** $3x + 7 = -3 - 2x$ **f** $9x - 4 = 10x - 11$ **g** $10 - x = 7x - 22$ **h** $-5x - 5 = 11 - x$

**9** Solve each equation for $x$. Use substitution to check that your solution is correct.

**a** $3(x - 2) = 8x - 1$ **b** $2x - 1 = 5(x - 2)$ **c** $2(3x - 4) = 5x - 1$ **d** $-3(-2x - 1) = -18 - x$

**e** $4(x + 3) = 5(x + 1)$ **f** $5(x + 9) = -3(x - 7)$ **g** $-6(1 - x) = 3(x - 8)$ **h** $-6(x + 1) = -10(x - 3)$

**10** Consider the equation $\frac{2x + 5}{3} = \frac{x - 4}{3}$.

**a** The first step to solving this equation using inverse operations is to multiply both sides by 3. Multiply both sides of the equation by 3 to obtain an equivalent equation.

**b** Solve the equation obtained in part **a**. Use substitution to check that your solution is correct.

**11** Solve:

**a** $\frac{3x - 4}{7} = \frac{x + 6}{7}$ **b** $\frac{5x + 2}{4} = \frac{2x - 7}{4}$ **c** $\frac{2x + 3}{11} = \frac{15 - 4x}{11}$ **d** $\frac{4(2x + 1)}{9} = \frac{x - 17}{9}$

**12** Consider the equation $\frac{5x - 1}{2} = \frac{x + 5}{3}$.

**a** The first step to solving this equation using inverse operations is to find a common denominator. Write an equivalent equation where the fractions have a common denominator.

$\frac{__(5x - 1)}{__} = \frac{__(x + 5)}{__}$

**b** Solve the equation obtained in part **a**, using the method from question **10**. Use substitution to check that your solution is correct.

**13** Use the method in question **12** to solve the following equations. Use substitution to check that your solution is correct.

**a** $\frac{4x + 1}{5} = \frac{3x + 2}{4}$ **b** $\frac{13x - 8}{10} = \frac{4x - 2}{3}$ **c** $\frac{2x - 1}{3} = \frac{4x + 11}{9}$ **d** $\frac{x + 15}{4} = \frac{x + 9}{6}$

**14** Trent is sharing a bag of jellybeans equally with three of his friends and finds that there are two left over. Consider the number of jellybeans that each person receives, including Trent, if there were 34 jellybeans in the bag.

**a** Define a pronumeral to represent the unknown quantity in this problem.

**b** Use this pronumeral to write an equation to represent the problem.

**c** Solve the equation using inverse operations.

**d** How many jellybeans did each person receive?

**15** Lily is saving to buy a pair of sneakers that cost $395. She is able to save $70 per month. If she currently has $115, consider the number of months it will take for Lily to buy the shoes.

**a** Define a pronumeral to represent the unknown quantity in the problem.

**b** Use this pronumeral to write an equation to represent the problem.

**c** Solve the equation using inverse operations.

**d** In how many months can Lily buy the shoes?

PROBLEM SOLVING AND REASONING

**16** For each problem, set up an equation and solve it using inverse operations.

**a** Jack buys three model planes online for a total cost of $590, which includes the delivery charge of $35. What is the cost of each model plane?

**b** Emma and Maggie score a total of 35 goals in a basketball match. Maggie scores seven more goals than Emma. How many goals did Emma score?

**c** The perimeter of a rectangular playing field is 100 m. If the length is 12 m longer than the width, what are the dimensions of the playing field?

**d** The cost of hiring a party venue is $500. There is a $26 per person charge for food. If Kasey has a budget of $2700 for the party, what is the maximum number of people that can attend?

**17** Sarah and Josh have the same amount of money. Sarah buys seven sushi rolls and has $1.50 left over. Josh buys four sushi rolls and has $12 left over.

**a** If $x$ represents the cost of one sushi roll ($), which equation fits this situation?

**A** $7x + 150 = 4x + 12$ **B** $7x + 1.5 = 4x + 12$ **C** $7x - 1.5 = 4x - 12$

**D** $7x - 150 = 4x - 12$ **E** $7x + 12 = 4x + 1.5$

**b** Solve the equation to find the cost of one sushi roll.

**18** At the cinema, the cost of five boxes of popcorn and two choc-tops is the same as the cost of three boxes of popcorn and seven choc-tops. The cost of a choc-top is $4.50 but you don't know the cost of the popcorn.

**a** Write an equation to represent this situation.

**b** What was the cost of a box of popcorn?

**19** Violetta cooked sausages for the school sausage sizzle. Each sausage was placed in bread with tomato sauce. Twenty of these were sold with mustard. Half of those left were sold with fried onions. If there were 18 sausages sold with fried onions, how many sausages did Violetta cook?

**20** One angle in a triangle is 30°. The second angle is twice the size of the third angle in the triangle. Find the size of the largest angle.

**21** The sum of three consecutive integers is 13 more than the smallest of the three numbers. Identify the three numbers using algebra.

CHALLENGE

**22** The linear equations you have dealt with have been sometimes equal. That is, for a particular value of the pronumeral, the left- and right-hand sides are equal. The linear equations $2x + 5 = 2x + 5$, $3x + 5 = 3x + 5$ and $3x + 6 = 3x + 6$ are equal for all values of $x$. That is, regardless of the value of $x$, the equations are always equal. The linear equations $2x + 5 = 2x + 6$, $3x + 5 = 3x + 6$ and $3x + 4 = 3x + 6$ are equal for no value of $x$. That is, regardless of the value of $x$, the equations are never equal.

**a** For each of the following, determine whether the equation will always, sometimes or never be equal.

**i** $2x + 5 + 3x = 1 + 5x + 4$ **ii** $7x - 2 + 4 = 3x + 4x - 6$ **iii** $2x + 3x - 5 = 6x - 9$

**iv** $15 - 12x = -3(4x + 5)$ **v** $4(6x + 10) = 8(3x + 5)$ **vi** $4(3 - 4x) + x = 4 - 3(5x - 3)$

**b** When will a linear equation with one pronumeral always be equal?

**c** When will a linear equation with one pronumeral never be equal?

**23** Solve the following equations for $x$. Use substitution to check your solution.

**a** $\frac{3x+2}{2} + \frac{5-x}{3} = \frac{5x+1}{4} + \frac{7x-2}{6}$ **b** $\frac{5x-1}{2} = \frac{x+5}{3} + 1$ **c** $\frac{5x-1}{2} = \frac{x+5}{3} + x$

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Solving equations using inverse operations
- Interactive skillsheet: Solving equations with the unknown on both sides
- Worksheet: Solving linear equations
- CAS instructions: Solving equations
- Topic quiz: 4A

# 4B Plotting linear relationships

## Learning intentions

- ✔ I can plot linear relationships from tables of values and equations.
- ✔ I can identify independent and dependent variables.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 5D The Cartesian plane |
| Year 8 | 6D Plotting linear and non-linear relationships |
| Year 10 | 4C Sketching linear graphs |

## Linear relationships

- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of **coordinate points** or a graph.
- A **linear relationship** is a relationship between two variables, the independent variable and the dependent variable, which produces a linear graph.
- A **linear graph** is a straight line on a **Cartesian plane**.
- The **independent variable** is the quantity that does not depend on the other variable.
- The value of the **dependent variable** depends on the value of the independent variable.

  For example, if a car is moving at a constant speed, then *time* is an independent variable, the *distance travelled by the car* is a dependent variable, and the relationship between them is linear.
- It is conventional for the independent variable to be:
  - → listed in the top row of a table of values

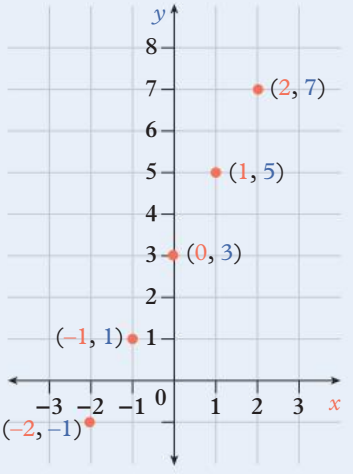
| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −1 | 1 | 3 | 5 | 7 |

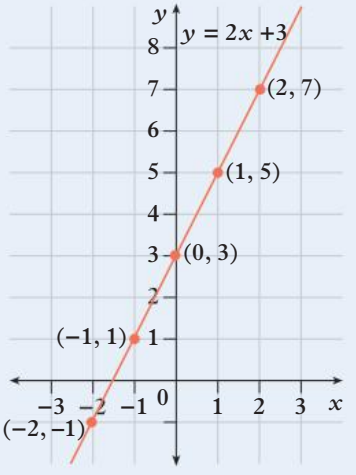
  - → listed first in a pair of coordinates

    (−2, −1), (−1, 1), (0, 3), (1, 5), (2, 7)
  - → shown on the horizontal axis.

    Independent variable: $x$

    Dependent variable: $y$




## Plotting linear relationships

- A **plot** is composed of individual coordinate points.

  A linear graph is a continuous line made up of an infinite number of coordinate points.
- To sketch a graph from an equation follow these steps:
  1. Construct a table of values by selecting values for $x$, then substituting each value of $x$ into the equation to find the corresponding value of $y$.
  2. Write out the coordinate points listed in the table.
  3. Plot the coordinate points on the Cartesian plane.
  4. Join the points using a straight line.

## Example 4B.1 Plotting linear relationships from a table of values

Use the table of values to construct a plot of the relationship between $x$ and $y$. Is the relationship linear or non-linear?

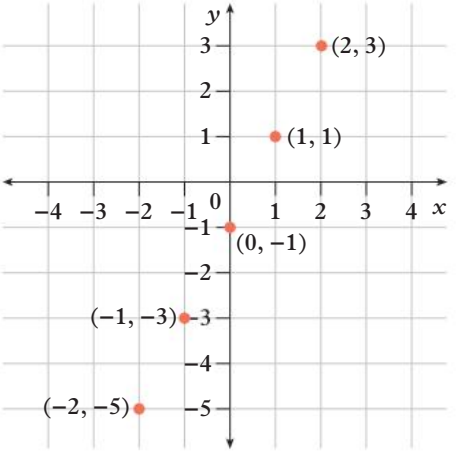
| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −5 | −3 | −1 | 1 | 3 |

**THINK**

1. Write out the coordinate points listed in the table.
2. Plot the points on the Cartesian plane.
3. Consider whether the points form a straight line. As the points form a straight line, the relationship is linear.

**WRITE**

(−2, −5), (−1, −3), (0, −1), (1, 1), (2, 3)




The relationship is linear.

## Example 4B.2 Graphing linear relationships from an equation

Plot a graph of $y = -x - 3$ by first completing a table of values for $x$ from −3 to 2.

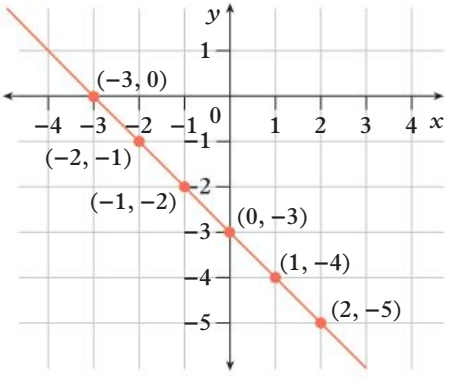
**THINK**

1. Construct a table of values for $x$ from −3 to 3. Substitute each value of $x$ into the equation to find the corresponding value of $y$.
2. Write out the coordinate points listed in the table.
3. Plot the points on the Cartesian plane.
4. Join the points with a straight line.

**WRITE**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $y$ | 0 | −1 | −2 | −3 | −4 | −5 |

(−3, 0), (−2, −1), (−1, −2), (0, −3), (1, −4), (2, −5)



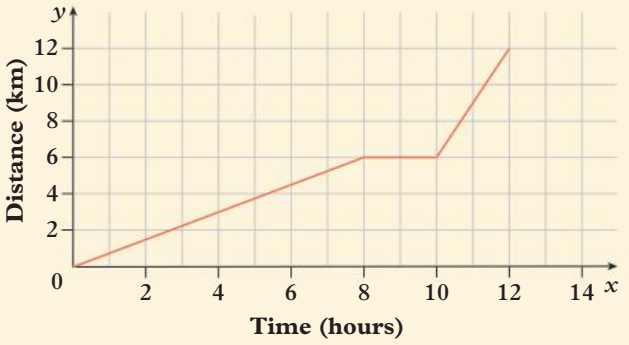
## Example 4B.3 Identifying independent and dependent variables

Identify the independent and dependent variables in the following relationships. Explain why for each case.

**a**

| Age (years) | 10 | 11 | 12 | 13 |
|---|---|---|---|---|
| Height (cm) | 140 | 148 | 154 | 160 |

**b**




**c** the number of oranges you put in a juicer and the volume of orange juice produced by the juicer

**THINK**

**a** In a table of values, the values of the independent variable are listed in the first row and the values of the dependent variable are listed in the second row. To explain why height depends on age, think about which variable cannot change without the other.

**b** In a graph, the independent variable is shown on the horizontal axis and the dependent variable is shown on the vertical axis. To explain why distance depends on time, think about which variable cannot change without the other.

**c** The volume of orange juice *depends* on the number of oranges you put in a juicer. To explain why, think about which one is required to make the other.

**WRITE**

**a** Independent variable: **Age** because a person ages constantly, but their height might not change.
Dependent variable: **Height** because as a person ages their height changes.

**b** Independent variable: **Time** because time passes regardless of an object's movement.
Dependent variable: **Distance** because something cannot move without time passing.

**c** Independent variable: **Number of oranges** because oranges cannot be made from orange juice.
Dependent variable: **Volume of orange juice** because orange juice can be made from oranges.

**Helpful hints**

- ✔ Remember that in Cartesian coordinates, the $x$-coordinate is always listed first, followed by the $y$-coordinate.
- ✔ When constructing plots and sketches, always label your $x$- and $y$-axis and label your graph with the equation of the graph.

$(-4, 1)$

$x$-coordinate – horizontal distance from the origin

$y$-coordinate – vertical distance from the origin

ANS p494

# Exercise 4B Plotting linear relationships

1, 2(a, b, d, e, g, i), 3–8, 9(a–d) 13 | 2(f, g, h, i), 3–8, 9(f, h, i, j), 10(a), 12, 13, 16(a, b) | 2(h, i), 3, 5–8, 9(i, j), 10(b), 11, 13–16

UNDERSTANDING AND FLUENCY

4B.1 **1 a** Use the following tables of values to construct a plot of each relationship between $x$ and $y$.

**i**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |

**ii**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 11 | 6 | 3 | 2 | 3 | 6 | 11 |

**iii**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 10 | 9 | 8 | 7 | 6 | 5 | 4 |

**iv**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −27 | −8 | −1 | 0 | 1 | 8 | 27 |

**b** Classify each relationship as linear or non-linear.

4B.2 **2** For each of the following linear relationships construct a table of values for $x$ from −3 to 3 and then write out the coordinate points listed in the table.

**a** $y = x + 2$ **b** $y = x - 4$ **c** $y = 3 - x$

**d** $y = 2 - x$ **e** $y = -x - 3$ **f** $y = 4x$

**g** $y = 2x + 1$ **h** $y = 3x - 2$ **i** $y = 4 - 2x$

4B.3 **3** Identify the independent and dependent variables in the following relationships. Explain why for each case.

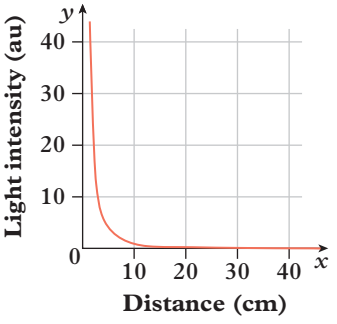
**a**

| Radius (cm) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Area (cm²) | 3.14 | 12.57 | 28.27 | 50.27 |

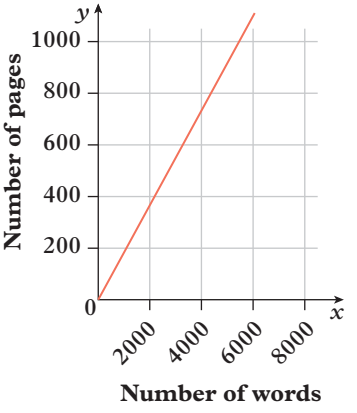
**b**

| Number sold | 20 | 40 | 60 | 80 |
|---|---|---|---|---|
| Revenue ($) | 100 | 200 | 300 | 400 |

**c**




**d**




**e** the *amount of breath* used to play a note on a flute and the *volume* of the note produced

**f** the *chance* of drawing the ace of spades from a standard deck after removing cards that are not the ace of spades and the *number of cards* remaining in the deck

**4 a** Complete the following tables of values using the equations provided.

**i** $x + y = 24$

| $x$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**ii** $xy = 24$

| $x$ | −12 | −6 | −2 | 2 | 6 | 12 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**iii** $y = x(x + 2)$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**iv** $x = 4y - 2$

| $x$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $y$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |

**b** Plot the coordinates from the tables of values and classify each relationship as linear or non-linear. Do not attempt to join the points with curves or lines.

**5 a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

**i** $y = x$ **ii** $y = 2x$ **iii** $y = 3x$ **iv** $y = 4x$

**b** Describe the similarities and differences between each of the four graphs.

**c** Describe the relationship between the coefficient of $x$ and the steepness of the corresponding graph.

**d** Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

**i** $y = 5x$ **ii** $y = 3.5x$

**6 a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

**i** $y = -x$ **ii** $y = -2x$ **iii** $y = -3x$ **iv** $y = -4x$

**b** How do these graphs differ from those sketched in question **5**?

**c** Describe the relationship between the coefficient of $x$ and the feature of the graph identified in part **b**.

**d** Without using a table of values, sketch the graphs of the following relationships on the same set of axes.

**i** $y = -5x$ **ii** $y = -1.5x$

**7 a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

**i** $y = x$ **ii** $y = x + 1$ **iii** $y = x + 2$ **iv** $y = x + 3$

**b** Describe the similarities and differences between each of the four graphs.

**c** Describe the relationship between the equations and corresponding graphs.

**d** Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

**i** $y = x + 4$ **ii** $y = x + 1.5$

**8** Many of the linear graphs you have sketched so far have equations of the form and then write out the coordinate points listed in the table. $y = \ldots$ This makes it easy to produce a table of values by substituting values for $x$. However, linear equations can take different forms. Consider the equation $4x + y = 6$.

**a** Rearrange the equation so that $y$ is isolated on the LHS ($y = \ldots$).

**b** Complete a table for $x$ from $-2$ to 2.

**c** Sketch the graph of $4x + y = 6$ and label the graph with its equation.

**9** Sketch a graph of each linear relationship by rearranging the equation so that $y$ is isolated on the LHS ($y = \ldots$) and then completing a table of values from $x = -2$ to $x = 2$.

**a** $x + y = 5$ **b** $x + y = -1$ **c** $2x + y = 3$ **d** $y - x = 4$ **e** $y - 3x = 1$

**f** $x + y + 2 = 0$ **g** $x - y = 3$ **h** $4x - y = -1$ **i** $3x - y = 0$ **j** $6x + 2y = 8$

**10 a** To make a table of values for an equation such as $2x + 3y = 12$, start with the values for $2x$ and $3y$ that add to 12, then determine the corresponding values for $x$ and $y$ using division.

| **2x** | −3 | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|---|
| **3y** | 15 | 12 | 9 | 6 | 3 | 0 | −3 |
| ***x*** | −1.5 | | | | | | |
| ***y*** | 5 | | | | | | |

**b** Complete the table of values below for $4x - 3y = 36$.

| **4x** | 0 | 12 | | | 42 | | 54 |
|---|---|---|---|---|---|---|---|
| **3y** | | | −18 | 0 | | 12 | |
| ***x*** | | | | | | | |
| ***y*** | | | | | | | |

**11** Steve is training for the cross-country race. He runs laps of the oval every other day and records the number of laps he ran.

| **Day** | 1 | 3 | 5 | 7 | 9 | 11 | 13 |
|---|---|---|---|---|---|---|---|
| **Number of laps** | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 |

**a** Identify the independent variable and dependent variable.

PROBLEM SOLVING AND REASONING

**b** **i** What feature(s) of the table of values indicates that the graph is likely to be linear?

**ii** Hence, what is the minimum number of points you need to plot to sketch the line?

**c** Sketch the relationship on a Cartesian plane and extend the line to day 25.

**d** Use the graph to determine:

**i** the number of laps Steve should run on day 17 **ii** the day Steve should run 8 laps.

**12** An equation that gives the conversion from Euros to Australian dollars on a particular day is AUD = 1.6 × EUR.

**a** Identify the independent variable and dependent variable.

**b** **i** What feature(s) of the equation indicates that the graph is likely to be linear?

**ii** Hence, what is the minimum number of points you need to plot to sketch the line?

**c** Sketch the relationship on a Cartesian plane and extend the line to 40 EUR.

**d** Use the graph to determine:

**i** the amount in AUD for 30 EUR **ii** the amount in EUR for 24 AUD.

**13** A bus is hired for a school trip to the snow. The school pays $250 towards the bus hire and each student is charged $40.

**a** If $m$ is the total amount of money collected for bus hire and $n$ students go on the trip, write an equation for the relationship between $m$ and $n$.

**b** Identify the independent and dependent variables in the relationship.

**c** Calculate how much money would be collected for bus hire if:

**i** 0 students go on the trip

**ii** 20 students go on the trip.

**d** Use your answer to part **c** to sketch a graph of this relationship using a scale from 0 to 30 along the horizontal axis.

**e** Is the relationship linear? Explain.

**f** Use the graph to find the total amount of money collected if 30 students go on the trip.

**g** Use the graph to determine how many students need to go on the trip to collect a total of $850 for bus hire.

**h** The hire cost of the bus is $1300. Use the graph to determine the minimum number of students who need to go on the trip to cover the hire cost.

**14** The formula $F = 1.8C + 32$ describes the relationship between temperatures in degrees Celsius, $C$, and temperatures in degrees Fahrenheit, $F$.

**a** Plot the graph of this relationship. Show a scale from −50 to 50 along the horizontal axis.

**b** Use the graph to find the temperature in °F for 30°C.

**c** Use the graph to find the temperature in °C for −22°F.

CHALLENGE

**15** Use the graph in question **14** to find where the temperature in °C has the same numerical value as the matching temperature in °F.

**16** For each of the following pairs of linear relationships, plot two graphs on the same Cartesian plane to determine the coordinates where the two lines intersect.

**a** $y = x + 1$ and $y = 2x$ **b** $y = 5x$ and $y = \frac{1}{3}x$

**c** $y = \frac{1}{2}x - 1$ and $y = -x + 2$ **d** $y + 2x = 12$ and $3y + x = 16$

Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** Plotting linear relationships
- **Interactive skillsheet** Classifying variables
- **Investigation** How accurate is a 'rule of thumb' temperature formula?
- **CAS instructions** Graphing functions
- **Topic quiz** 4B

# 4C Gradient and intercepts

## Learning intentions

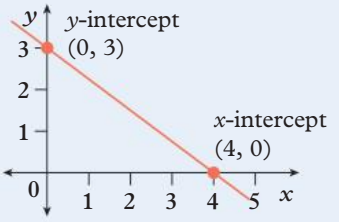
- ✔ I can identify the $x$- and $y$-intercepts of a linear graph.
- ✔ I can determine the gradient of a line segment and a graph.

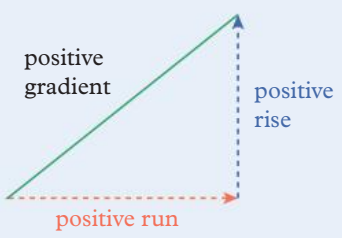
**Inter-year links**

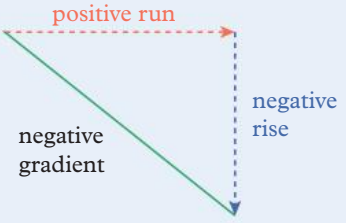
| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 6D Plotting linear and non-linear relationships |
| Year 8 | 6F Finding linear equations |
| Year 10 | 4C Sketching linear graphs |

## Features of linear graphs

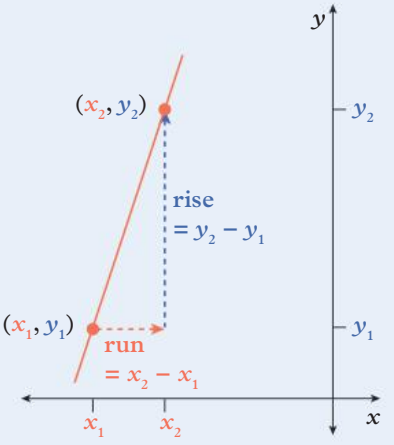
- The features of a linear graph include its $x$-intercept, $y$-intercept and gradient.
- The **$x$-intercept** is the point where the graph crosses the $x$-axis.
- The **$y$-intercept** is the point where the graph crosses the $y$-axis.
- The **gradient** is a numerical measure of the slope of the graph. It is also referred to as the rate of change between the two variables.



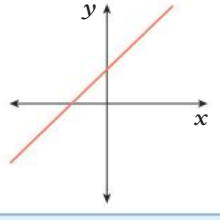
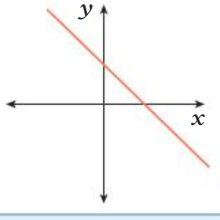
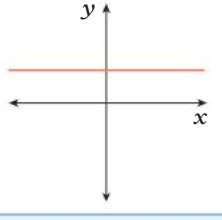
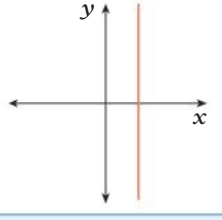




- The gradient of a linear graph is a constant because the gradient between any two points on the line is the same. This means that the rate of change between two variables in a linear relationship does not change.



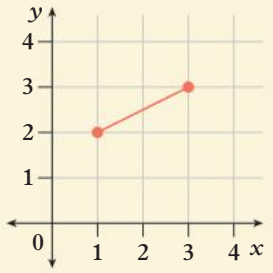

- The value of the gradient is the number of units that the graph increases in the vertical direction for every 1 unit that it increases in the horizontal direction. So, if the gradient is 3, then the linear graph increases 3 units up in the vertical direction for every 1 unit that it increases to the right in the horizontal direction.
- The formula for the gradient, $m$, between any two points, $(x_1, y_1)$ and $(x_2, y_2)$, is: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Gradients can be positive, negative, zero or undefined.

| positive | negative | zero | undefined |
|---|---|---|---|
|  |  |  |  |

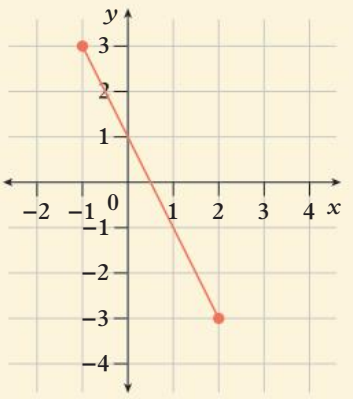
## Example 4C.1 Determining the gradient of a line segment by identifying rise and run

Find the gradient of each line segment.

**a**




**b**




**THINK**

**a** **1** Determine the run, the horizontal distance between the endpoints of the line segment. The run is 2 units.

**2** Determine the rise, the vertical distance between the endpoints of the line segment. The rise is 1 unit.

**3** Calculate the gradient by dividing the rise by the run. Simplify the gradient where possible.

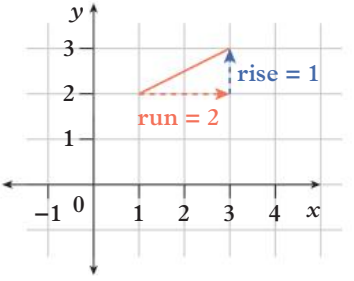
**b** **1** Determine the run, the horizontal distance between the endpoints of the line segment. The run is 3 units.

**2** Determine the rise, the vertical distance between the endpoints of the line segment. The rise is −6 units.

**3** Calculate the gradient by dividing the rise by the run. Simplify the gradient where possible.

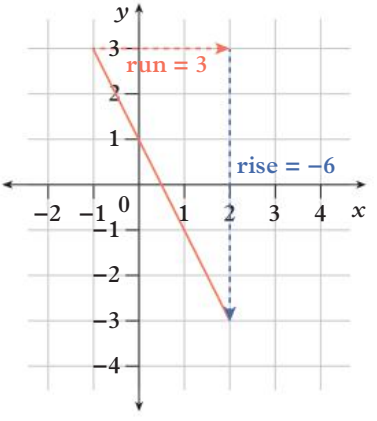
**WRITE**

**a**




$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{1}{2}$$

**b**




$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-6}{3}$$
$$= -2$$



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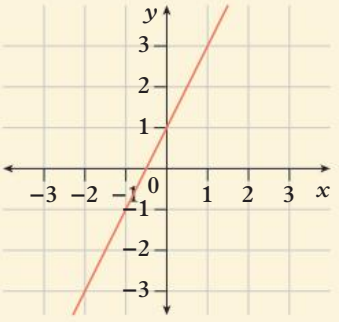
## **Example 4C.2** Determining the gradient, *x*-intercept and *y*-intercept

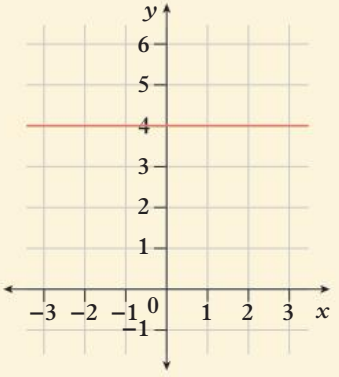
For each of the linear graphs shown, determine the:

**i** gradient **ii** *x*-intercept **iii** *y*-intercept

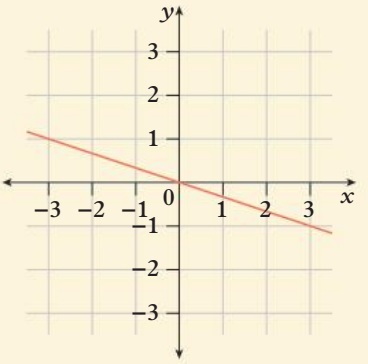
**a**




**b**




**c**




**THINK**

**a i** Select any two points that have integer coordinates. Determine the rise and run, and then calculate the gradient.

**ii** State the coordinates of the point at which the graph crosses the *x*-axis.

**iii** State the coordinates of the point at which the graph crosses the *y*-axis.

**b i** The graph is a horizontal line, so it has a zero gradient.

**ii** The graph does not cross the *x*-axis, so it does not have an *x*-intercept.

**iii** State the coordinates of the point at which the graph crosses the *y*-axis.

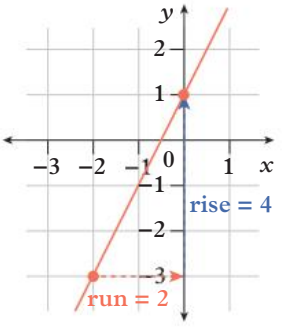
**c i** Select any two points that have integer coordinates. Determine the rise and run, and then calculate the gradient. Remember that if the graph is sloping down to the right then the gradient will be negative.

**ii** State the coordinates of the point at which the graph crosses the *x*-axis.

**iii** State the coordinates of the point at which the graph crosses the *y*-axis.

**WRITE**

**a i**




$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{2}$$
$$= 2$$

**ii** *x*-intercept: $\left(-\frac{1}{2}, 0\right)$

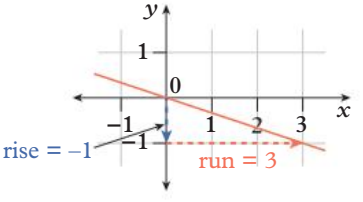
**iii** *y*-intercept: $(0, 1)$

**b i** Gradient = 0

**ii** no *x*-intercept

**iii** *y*-intercept: $(4, 0)$

**c i**




**ii** *x*-intercept: $(0, 0)$

**iii** *y*-intercept: $(0, 0)$

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-1}{3}$$
$$= -\frac{1}{3}$$

## Example 4C.3 Determining gradient using two coordinate points

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining the points (3,2) and (9,5).

**THINK**

1. Define the points $(x_1, y_1)$ and $(x_2, y_2)$. The order of the coordinates will not affect the value of the gradient.
2. Substitute the $x$- and $y$-coordinates into the gradient formula.
3. Calculate and simplify the gradient.

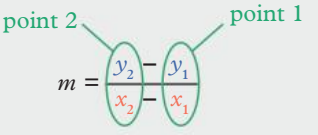
**WRITE**

Let $(x_1, y_1) = (3,2)$ and $(x_2, y_2) = (9,5)$

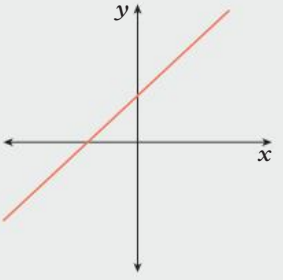
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 2}{9 - 3}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

### Helpful hints

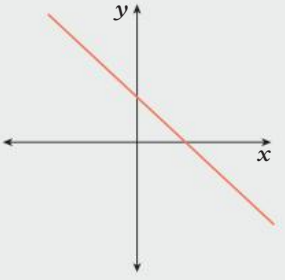
- ✔ The order in which you substitute points into the formula for the gradient of a line won't affect your final value – you just need to make sure the $x$- and $y$-coordinates of a given point match up vertically!
- ✔ Positive gradients can be described as increasing gradients as the value of $y$ increases from left to right.
- ✔ Negative gradients can be described as decreasing gradients as the value of $y$ decreases from left to right.




Positive gradient: Increasing




Negative gradient: Decreasing



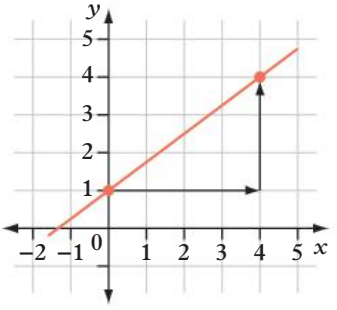

ANS p498

# Exercise 4C Gradient and intercepts

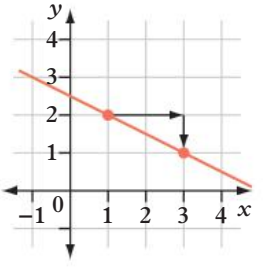
△ 1–5, 6(1st column), 7–10, 12, 16 ■ 3–5, 6(2nd column), 8(b, c, f), 9–11, 13(a, c, f), 16, 17(a–c) ⬠ 3, 4(c, e, h), 6(2nd column), 7, 9, 13–16, 17(e, f), 18

**1** Use the rise and run marked on the graphs below to determine the gradient of each line segment.

**a**

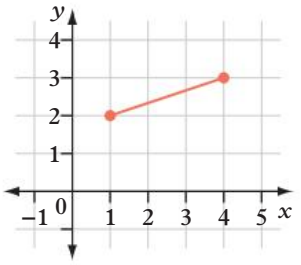



**b**

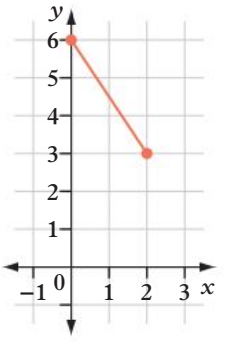


4C.1 **2** Find the gradient of each line segment.

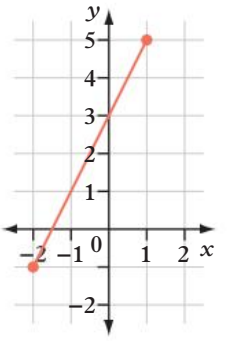
**a**




**b**

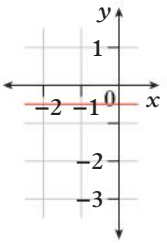



**c**

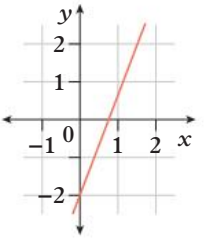



**3** Classify the gradients of the following lines as positive, negative, zero or undefined.

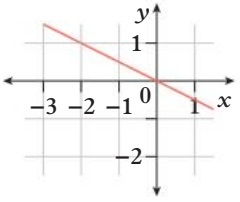
**a**



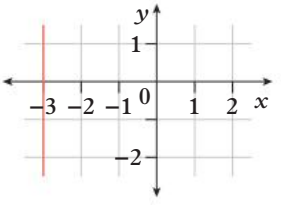

**b**



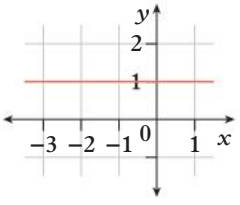

**c**



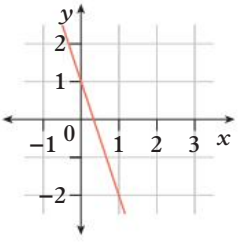

**d**




**e**



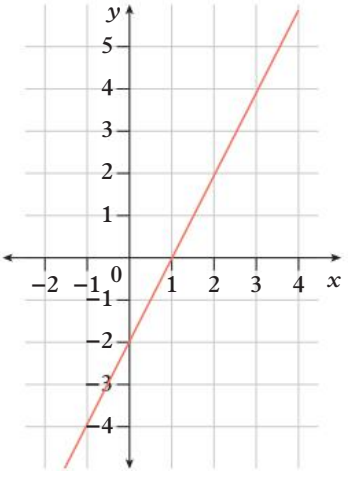

**f**



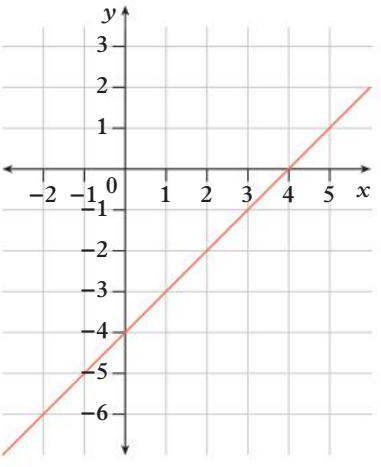

4C.2 **4** For the linear graph shown, determine the:

**i** gradient **ii** $x$-intercept **iii** $y$-intercept

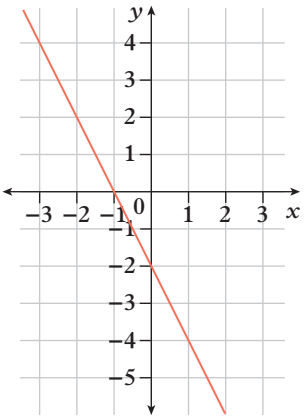
**a**



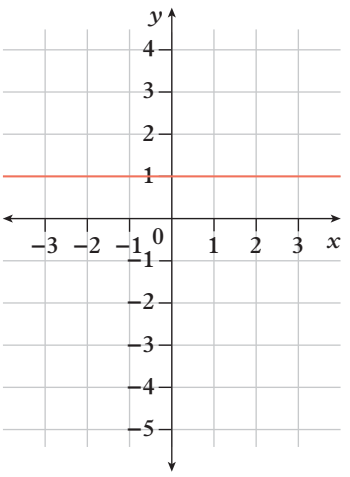

**b**



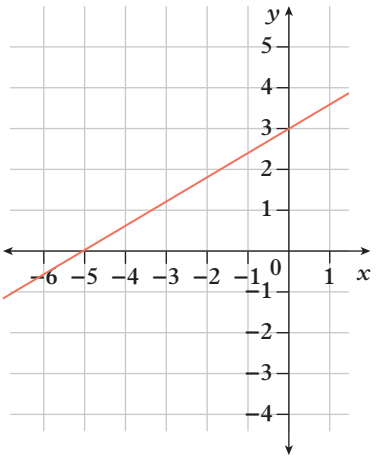
**c**



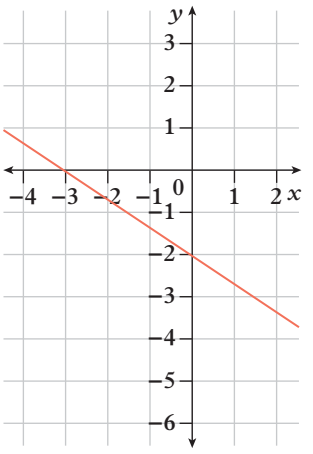

**d**



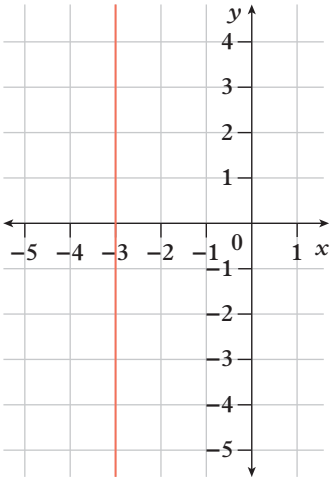

**e**



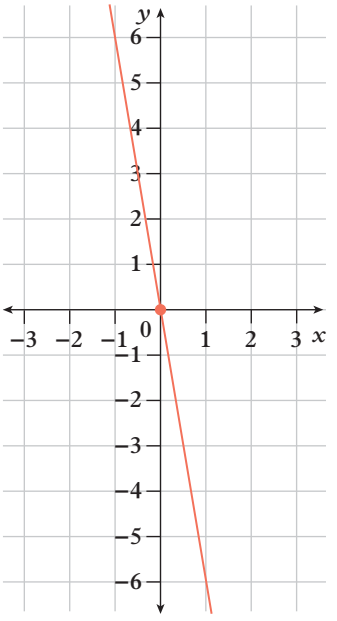

**f**




**g**




**h**




**5** **i** Plot each pair of points on the Cartesian plane and join them with a straight line to form a line segment.

**ii** Determine the gradient of the line segment.

**a** (2, 3) and (6, 8)

**b** (1, 2) and (3, 6)

**c** (3, 7) and (4, 4)

**d** (−4, 5) and (2, −3)

4C.3 **6** Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining each pair of points.

**a** (2, 4) and (5, 6)
**b** (3, 1) and (7, 4)
**c** (1, 2) and (4, 8)
**d** (4, 6) and (7, 5)
**e** (−2, 5) and (3, 7)
**f** (0, 8) and (1, 5)
**g** (−1, −6) and (−1, −1)
**h** (0, 5) and (1, 0)
**i** (−4, −3) and (−1, 4)
**j** (−3, 3) and (−3, −5)
**k** (−5, −6) and (−4, −8)
**l** (−9, 3) and (−6, 3)

**7** Consider the following pairs of points.

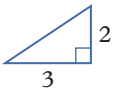
**i** Plot each pair on the Cartesian plane and then join them with a straight line. Ensure your lines cross both axes.

**ii** Determine the $x$- and $y$-intercept of the line, if they exist. Write the intercepts as coordinates.

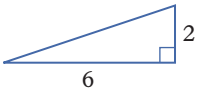
**a** (2, 2) and (5, 8)
**b** (1, 2) and (3, 6)
**c** (−6, 6) and (−4, 3)
**d** (−2, 1) and (−2, −3)

**8** Calculate the gradient of the hypotenuse of these right-angled triangles.

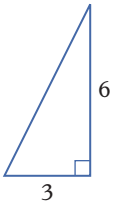
**a**



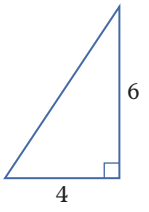

**b**



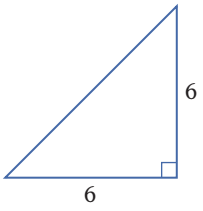

**c**



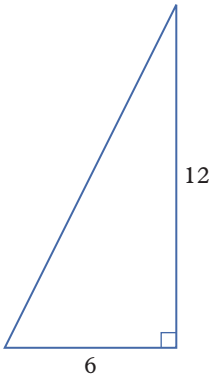

**d**



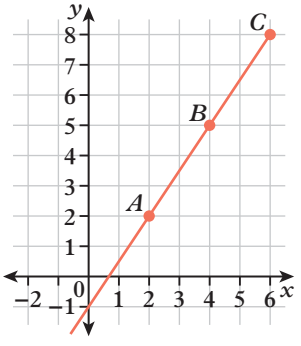

**e**




**f**




**9** Consider the graph below.

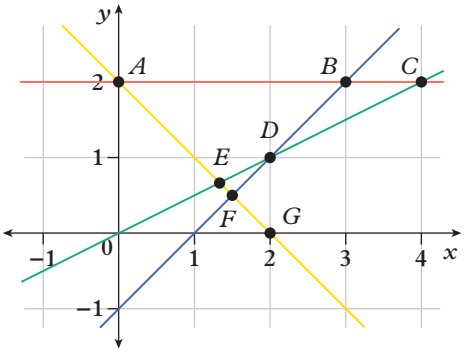



**a** Determine the gradient of the following line segments:

**i** $AB$
**ii** $AC$

**b** Compare your answers to part **a** and describe the relationship between the gradient of a line and the gradient of the line segments on that line.

**10** Consider the four linear graphs on the Cartesian plane below.




**a** Calculate the gradient for the following line segments:

**i** $AB$ **ii** $BD$ **iii** $CD$ **iv** $AH$

**b** Use your answers from part **a** and your observations in question **9** to determine the gradient for the following line segments.

**i** $DF$ **ii** $BC$ **iii** $DG$ **iv** $FG$

**11** Kane calculates the gradient of the line segment joining $(2, 8)$ and $(6, 5)$, as shown below. Identify Kane's error and calculate the correct value for the gradient of the line segment.

Kane

$$m = \frac{8-5}{6-2}$$
$$= \frac{3}{4}$$

**12** Todd and Bridget calculated the gradient of the line segment joining $(-1, 9)$ and $(3, -3)$, as shown below. Explain why both students have produced correct calculations.

Todd

$$m = \frac{-3-9}{3-(-1)}$$
$$= \frac{-12}{4}$$
$$= -3$$

Bridget

$$m = \frac{9-(-3)}{-1-3}$$
$$= \frac{12}{-4}$$
$$= -3$$

**13** The vertical rise and the horizontal run can be determined from the gradient if it is written as a fraction. For example, a gradient of $3 = \frac{3}{1}$, so a gradient with a graph of 3 has a rise of 3 for a run of 1. For negative gradients, negative fractions can be expressed as a negative numerator and a positive denominator. For example, a gradient of $-\frac{2}{3} = \frac{-2}{3}$, so a graph with a gradient of $-\frac{2}{3}$ has a rise of $-2$ for a run of 3.

For each gradient:

**i** write the gradient as a fraction with a positive denominator

**ii** identify the rise

**iii** identify the run.

**a** 2 **b** $-3$ **c** $-4$ **d** $-1$

**e** $\frac{5}{4}$ **f** $\frac{-3}{8}$ **g** $-\frac{2}{7}$ **h** $-\frac{1}{5}$

**14** Ash calculated the gradient of several line segments between points along a straight line. The gradients he calculated are listed below.

$$\frac{6}{10}, \frac{-3}{-5}, \frac{3}{5}, \frac{0.6}{1}, \frac{1.8}{3}, \frac{-1.5}{-2.5}$$

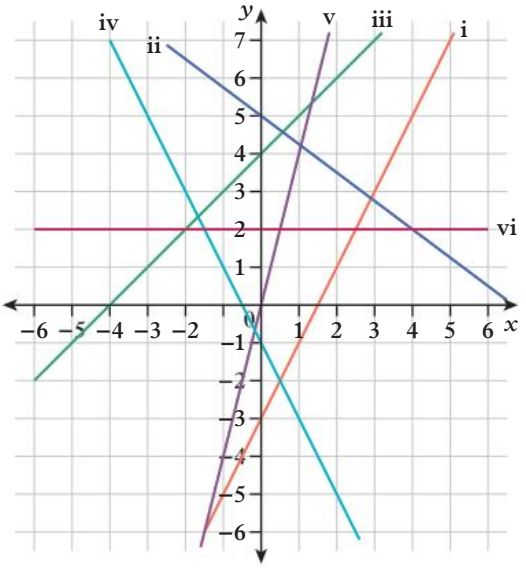
Ash believes that since the rise and run are not equal for all of the fractions, the gradient of the line is not constant. Explain why Ash is wrong and state the constant gradient of the line.

**15** Complete the table below.

| **Run** | 8 | −20 | | | 4 | |
|---|---|---|---|---|---|---|
| **Rise** | | | −4 | −16 | | 30 |
| **Gradient** | $\frac{3}{2}$ | $-\frac{4}{5}$ | $\frac{1}{7}$ | $-\frac{2}{5}$ | 7 | −5 |

**16** Consider these linear graphs.

**a** Complete the following table by calculating the gradient and identifying the $y$-intercept of each graph.




| | **Equation** | **Gradient** | ***y*-intercept** |
|---|---|---|---|
| **i** | $y = 2x - 3$ | | |
| **ii** | $y = -\frac{3}{4}x + 5$ | | |
| **iii** | $y = x + 4$ | | |
| **iv** | $y = -2x - 1$ | | |
| **v** | $y = 4x$ | | |
| **vi** | $y = 2$ | | |

**b** What pattern can you see in the table that allows you to identify the gradient and the $y$-intercept from the equation for the graph?

**c** Predict the gradient and $y$-intercept of the linear graphs with the following rules:

**i** $y = 6x + 4$ **ii** $y = x - 5$ **iii** $y = -3x$

**17** Determine the value of the unknowns in each of the following.

**a** A line with a gradient of 3 passes through the points $(1, 4)$ and $(x, 10)$.

**b** A line with a gradient of −2 passes through the points $(1, 4)$ and $(3, y)$.

**c** A line with a gradient of 2 passes through the points $(1, 4)$ and $(3, y)$.

**d** A line with a gradient of 2 passes through the points $(0, 4)$ and $(3, y)$.

**e** A line with a gradient of $-\frac{1}{2}$ passes through the points $(1, 4)$ and $(3, y)$.

**f** A line with a gradient of $\frac{3}{2}$ passes through the points $(-1, -4)$ and $(x, 8)$.

**18** The gradient of a line is 2 and it passes through the points $(0, 4)$ and $(x, y)$.

**a** Write an equation for the gradient of this line.

**b** Rearrange the equation in part **a**, to solve for $y$ in terms of $x$.

The gradient of another line is 2 and it passes through the points $(1, 4)$ and $(x, y)$.

**c** Write an equation for the gradient of this line.

**d** Rearrange the equation in part **a**, to solve for $y$ in terms of $x$.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Gradients

Interactive skillsheet
Intercepts

Worksheet
Identifying features of a linear graph

Topic quiz
4C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

4A **1** Solve the following equations for $x$.

**a** $\frac{x}{3} + 2 = -7$ **b** $4(x + 2) = 28$

**c** $\frac{4 - x}{9} = -2$ **d** $5(2x - 3) - 8x = -1$

4A **2** Solve the following equations for $x$.

**a** $\frac{24}{x} = 4$ **b** $\frac{3}{x} = 18$ **c** $\frac{5}{x} = 0.8$ **d** $\frac{10}{x} = 0.25$

4A **3** Solve the following equations for $x$ (variables on both sides).

**a** $5x + 7 = 2x - 5$ **b** $7 - 4x = 6x - 13$ **c** $7(x - 3) = -56$ **d** $8(2x + 7) = 4(3x + 11)$

4A **4** Jamie orders five equally priced video games online for a total cost of $409.70, which includes the delivery charge of $9.95.

**a** Define a pronumeral to represent the unknown quantity in this problem.

**b** Use this pronumeral to write an equation to represent the problem.

**c** Solve the equation using inverse operations.

**d** What is the cost of each game?

4B **5** Consider the following tables of coordinate points.

**i** Plot the points from the following tables provided.

**ii** State whether the graph is linear or non-linear.

**a**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −10 | −8 | −6 | −4 | −2 | 0 | 2 |

**b**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 4.5 | 4 | 3.5 | 3 | 2.5 | 2 | 1.5 |

**c**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 5 | 2.25 | 0 | −1.75 | −3 | −3.75 | −4 |

**d**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −2 | 0 | 2 | 4 | 6 | 8 | 12 |

4B **6** Sketch a graph of each of the following linear relationships by first completing a table of values for $x$ from −3 to 3.

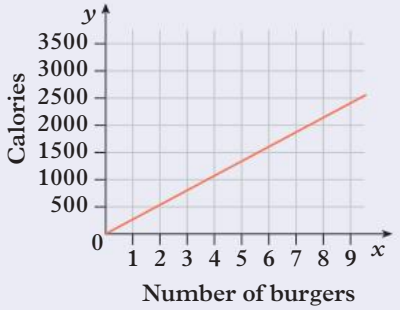
**a** $y = 2x - 5$ **b** $y = 5 - x$ **c** $y = -3x + 1$ **d** $y = -2x - 3$

4B **7** Identify the independent and dependent variables in the following relationships.

**a**

| **Length (cm)** | 5 | 10 | 15 | 20 |
|---|---|---|---|---|
| **Volume (cm³)** | 125 | 1000 | 3375 | 8000 |

**b**




**c** time spent playing a video game and the number of achievements unlocked

4B **8** Complete the table of values below for $2x + 5y = 20$.

| $2x$ | −5 | | 5 | 10 | | 20 | 25 |
|---|---|---|---|---|---|---|---|
| $5y$ | 25 | 20 | 15 | | 5 | 0 | |
| $x$ | | | | | | | |
| $y$ | | | | | | | |



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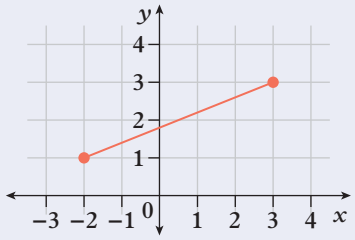
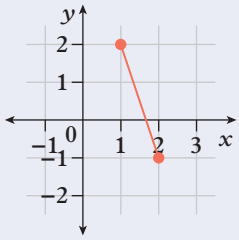
4C **9** Determine the gradients of the following line segments.

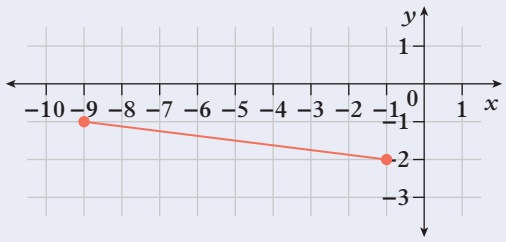
**a**



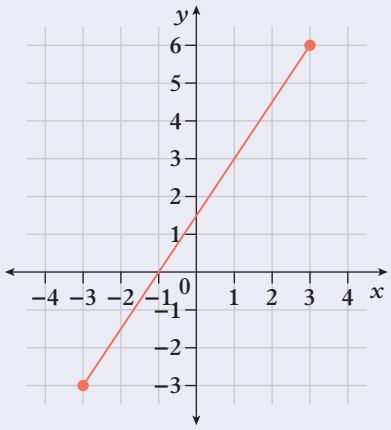

**b**




**c**

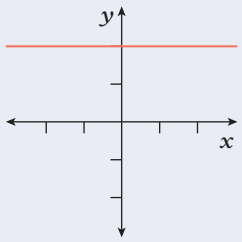



**d**

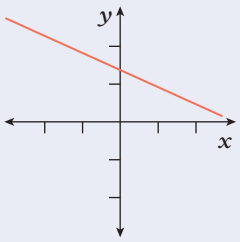



4C **10** State whether the following lines have a gradient that is positive, negative, zero or undefined.

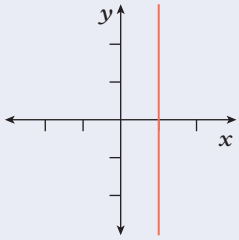
**a**



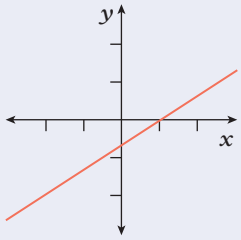

**b**




**c**




**d**

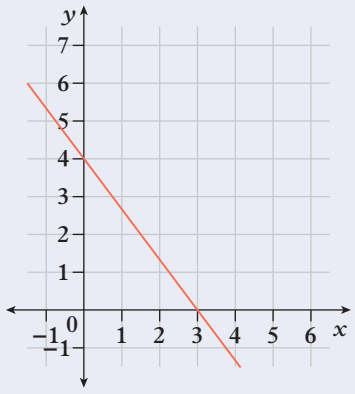



4C **11** Determine the gradients of the lines that pass through the following pairs of points.

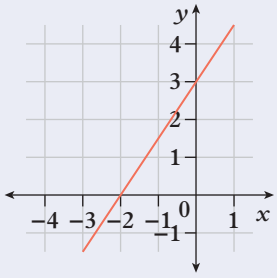
**a** (1,5) and (3,11) **b** (−1,3) and (2,−6) **c** (0,5) and (3,0) **d** (−6,−2) and (−1,−8)

4C **12** State the $x$- and $y$-intercepts of the following lines as coordinates.

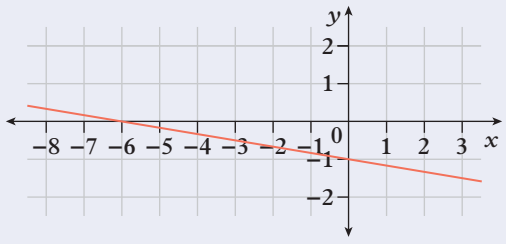
**a**




**b**




**c**



# 4D Sketching linear graphs using intercepts

## Learning intentions

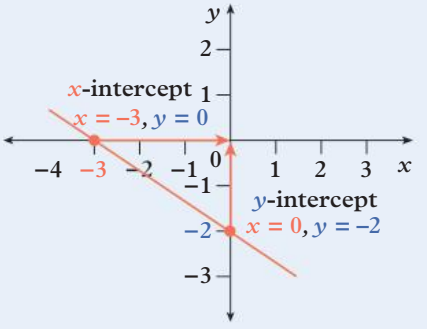
- ✔ I can calculate the $x$- and $y$-intercepts of a linear graph from its equation.
- ✔ I can sketch linear graphs with two intercepts using the $x$- and $y$-intercepts.
- ✔ I can sketch linear graphs with one intercept.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 5D The Cartesian plane |
| Year 8 | 6D Plotting linear and non-linear relationships |
| Year 10 | 4C Sketching linear graphs |

## Sketching linear graphs with two intercepts

- A minimum of two coordinate points are required to sketch a linear graph.
- The $x$-intercept is the point where the linear graph crosses the $x$-axis and $y = 0$.
- The $y$-intercept is the point where the linear graph crosses the $y$-axis and $x = 0$.
- Sketch a linear graph with two intercepts by first finding the $x$- and $y$-intercepts:
  1. Determine the $x$-intercept by substituting $y = 0$ into the equation of the line and solving for $x$.
  2. Determine the $y$-intercept by substituting $x = 0$ into the equation of the line and solving for $y$.
  3. Plot and label the $x$- and $y$-intercepts on the Cartesian plane.
  4. Draw a straight line through the two points.




For example, to sketch the graph of $y = 2x + 3$:

For $x$-intercept, let $y = 0$:

$0 = 2x + 3$

$-3 = 2x$

$x = -\frac{3}{2}$

$x$-intercept: $\left(-\frac{3}{2}, 0\right)$

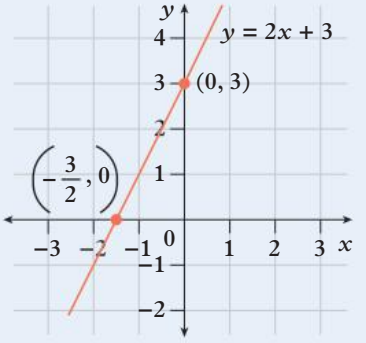
For $y$-intercept, let $x = 0$:

$y = 2 \times 0 + 3$

$y = 0 + 3$

$y = 3$

$y$-intercept: $(0, 3)$



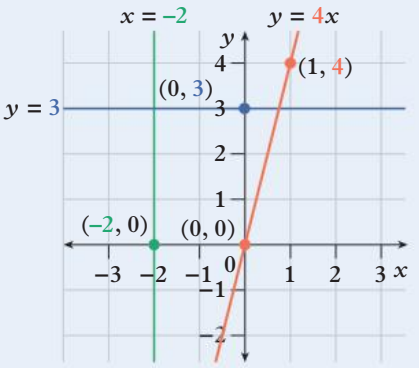

## Sketching linear graphs with one intercept

- There are three cases in which a linear graph has only one intercept:

| Description | General equation | Intercept | Gradient |
|---|---|---|---|
| Vertical lines | $x = a$ | $x$-intercept: $(a, 0)$ | undefined |
| Horizontal lines | $y = b$ | $y$-intercept: $(0, b)$ | 0 |
| Lines that pass through the origin | $y = mx$ | origin: $(0, 0)$ | $m$ |

where $a$, $b$ and $m$ are constants.

For example, the graphs of $x = -2$, $y = 3$ and $y = 4x$ are shown on the Cartesian plane below.




- To sketch a linear graph that passes through the origin (0,0), determine a second point on the graph by substituting any value of $x$ into the equation and then solve for $y$.
- For example, in the relationship $y = 4x$, $y = 4 \times 1 = 4$ for $x = 1$. Therefore (1, 4) is another point on $y = 4x$.

## Example 4D.1 Calculating *x*- and *y*-intercepts

Determine the coordinates of the $x$- and $y$-intercepts of the graphs of the linear relationships:

**a** $x + 5y = 10$ **b** $y = 3x - 4$

**THINK**

**a** **1** To determine the $x$-coordinate of the $x$-intercept, substitute $y = 0$ into the equation and solve for $x$. The coordinates of the $x$-intercept have the form $(x, 0)$.

**2** To determine the $y$-coordinate of the $y$-intercept, substitute $x = 0$ into the equation and solve for $y$. The coordinates of the $y$-intercept have the form $(0, y)$.

**b** **1** To determine the $x$-coordinate of the $x$-intercept, substitute $y = 0$ into the equation and solve for $x$. The coordinates of the $x$-intercept have the form $(x, 0)$.

**2** To determine the $y$-coordinate of the $y$-intercept, substitute $x = 0$ into the equation and solve for $y$. The coordinates of the $y$-intercept have the form $(0, y)$.

**WRITE**

**a** For $x$-intercept, let $y = 0$:

$x + 5(0) = 10$

$x + 0 = 10$

$x = 10$

$x$-intercept: (10, 0)

For $y$-intercept, let $x = 0$:

$0 + 5y = 10$

$5y = 10 \quad (\div 5)$

$y = 2$

$y$-intercept: (0, 2)

**b** For $x$-intercept, let $y = 0$:

$0 = 3x - 4 \quad (+4)$

$4 = 3x \quad (\div 3)$

$x = \frac{4}{3}$

$x$-intercept: $\left(\frac{4}{3}, 0\right)$

For $y$-intercept, let $x = 0$:

$y = 3(0) - 4$

$= 0 - 4$

$= -4$

$y$-intercept: (0, –4)

## Example 4D.2 Sketching linear graphs with two intercepts

Sketch a graph of $4x - y = 8$ by first finding the $x$- and $y$-intercepts.

**THINK**

1 Determine the $x$-intercept by substituting $y = 0$ into the equation and solving for $x$. Determine the $y$-intercept by substituting $x = 0$ into the equation and solving for $y$.

2 Plot and label the $x$- and $y$-intercepts on the Cartesian plane.

3 Rule a straight line through the points. Label the graph with its equation.

**WRITE**

For $x$-intercept, let $y = 0$:

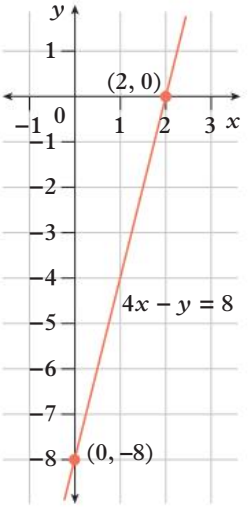
$4x - 0 = 8$
$4x = 8$ $(\div 4)$
$x = 2$

$x$-intercept: $(2, 0)$

For $y$-intercept, let $x = 0$:

$4(0) - y = 8$
$0 - y = 8$
$-y = 8$ $(\div -1)$
$y = -8$

$y$-intercept: $(0, -8)$




## Example 4D.3 Sketching vertical and horizontal lines

Sketch a graph of the following linear relationships.

**a** $x = 4$ **b** $y = -2$

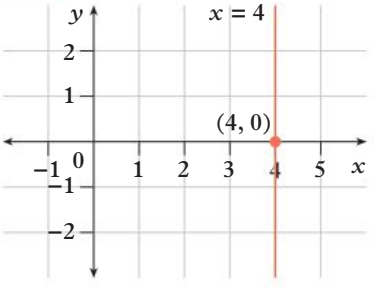
**THINK**

**a** The graph of $x = 4$ is a vertical line that passes through the point $(4, 0)$. Rule a vertical line through $(4, 0)$ and label the $x$-intercept. Label the graph with its equation.

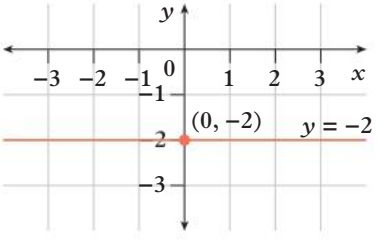
**b** The graph of $y = -2$ is a horizontal line that passes through the point $(0, -2)$. Rule a horizontal line through $(0, -2)$ and label the $y$-intercept. Label the graph with its equation.

**WRITE**

**a**




**b**



## Example 4D.4 Sketching linear graphs that pass through the origin

Sketch the graph of $y = -3x$.

**THINK**

1 The $x$- and $y$-intercepts are both $(0, 0)$ so the graph passes through the origin $(0,0)$. Determine a second point on the graph by substituting any value of $x$ into the equation and solving for $y$.

2 Plot and label the two points on the Cartesian plane.

3 Rule a straight line through the points and label the graph with its equation.

**WRITE**

For $x$-intercept, let $y = 0$:

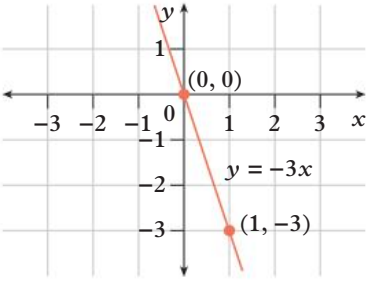
$0 = -3x$ $(\div -3)$
$x = 0$

$x$- and $y$-intercept: $(0, 0)$

For a second point, let $x = 1$:

$y = -3(1)$
$= -3$

Second point: $(1, -3)$




**Helpful hints**

- ✔ Find the $x$- and $y$-intercepts before you start sketching so you can plan the scale on the axes of your Cartesian plane.
- ✔ Always label your graph with the following information:
  - → the equation of the graph
  - → any $x$- and $y$-intercepts

ANS p502

# Exercise 4D Sketching linear graphs using intercepts

1–7, 9, 11, 13, 14 | 3–8, 10, 12–14, 16(a, b) | 3, 6, 7, 12–17

4D.1

**1** Determine the coordinates of the $x$- and $y$-intercepts of the graphs of the following linear relationships.

**a** $x + 4y = 12$ **b** $y = x + 4$ **c** $2x + y = 6$
**d** $y = x - 5$ **e** $y = -2x + 8$ **f** $y = 3x - 6$
**g** $5x + y = -10$ **h** $y = -x + 7$ **i** $y = 4 - x$

**2** Sketch the linear graphs that have the following $x$- and $y$-intercepts.

**a** $x$-intercept: $(2,0)$, $y$-intercept: $(0,1)$ **b** $x$-intercept: $(-3,0)$, $y$-intercept: $(0,5)$
**c** $x$-intercept: $(-1,0)$, $y$-intercept: $(0,-1)$ **d** $x$-intercept: $\left(\frac{1}{2},0\right)$, $y$-intercept: $\left(0,-\frac{5}{2}\right)$

UNDERSTANDING AND FLUENCY

4D.2 **3** Sketch a graph of each of the following linear relationships by first finding the $x$- and $y$-intercepts.

**a** $4x + y = 4$ **b** $y = x - 2$ **c** $-2x + 3y = 6$

**d** $y = -3x + 3$ **e** $5x + y = -5$ **f** $y = 2 - x$

**g** $2y = 2 - 4x$ **h** $3y - 9x + 12 = 0$ **i** $4x - 3y - 8 = 0$

4D.3 **4** Sketch a graph of the following linear relationships.

**a** $x = 1$ **b** $y = 4$ **c** $x = -3$ **d** $y = 0$

4D.4 **5** Sketch a graph of the following linear relationships.

**a** $y = 3x$ **b** $y = -2x$ **c** $y = 6x$ **d** $y = -x$

**6** Use the most appropriate method to sketch the graph of each linear relationship.

**a** $2x - 5y = 10$ **b** $y = 4x + 2$ **c** $x + y = 6$

**d** $y = -3x$ **e** $y = 7$ **f** $y = 6 - 3x$

**g** $x = 1$ **h** $y = 4(x - 1)$ **i** $x + 3y - 9 = 0$

**j** $y = x$ **k** $y = -3x + 5$ **l** $y = -7$

PROBLEM SOLVING AND REASONING

**7** Without sketching, determine how many intercepts the graph of each relationship has.

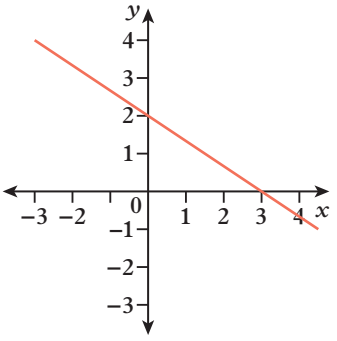
**a** $y = 10$ **b** $y = 2x + 1$ **c** $x = -12$

**d** $y = 15x$ **e** $5x - y = 25$ **f** $x + y = 1$

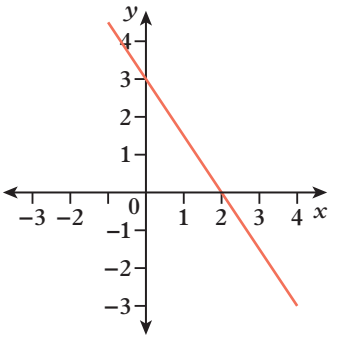
**g** $y = x$ **h** $x = 1$ **i** $-2y = -7x$

**8** Which of these three graphs is a correct sketch of $3x + 2y = 6$? Identify the errors in the other two options.

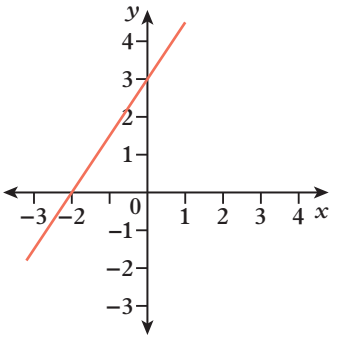
**A**




**B**




**C**




**9** Decide whether each statement about the graph of $y = 3x + 6$ is true or false. Correct each false statement.

**a** The $y$-intercept is 6. **b** The $x$-intercept is 2.

**c** The relationship is linear. **d** The point (1, 9) lies on the line.

**e** The line passes through the origin. **f** The gradient of the line is 3.

**10** Decide whether each statement about the graph of $y = -4x$ is true or false. Correct each false statement.

**a** The $x$-intercept is $-4$. **b** The $y$-intercept is 0.

**c** The line passes through the origin. **d** The point (1, 4) lies on the line.

**e** The gradient is negative. **f** The gradient of the line is $-\frac{1}{4}$.

**11 a** Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x + 3y = 18$, $2x + 3y = 0$ and $2x + 3y = -12$

**b** State the similarities between the equations and their graphs.

**c** Describe the impact of the differences between the equations and their graphs.

**12 a** Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x - 3y = 12$, $-2x + 3y = 12$ and $-2x - 3y = 12$

**b** State the similarities between the equations and their graphs.

**c** Describe the impact of the differences between the equations and their graphs.

**13** Determine the gradient of the following lines by finding and using the $x$- and $y$-intercepts.

**a** $3x - 5y = 30$ **b** $7x + 6y + 9 = 0$ **c** $10y = 3 - 5x$ **d** $8x = 4y - 3$

**14** Tony is buying a skateboard on a purchase plan where he makes a regular payment each week. He creates the formula $y = 300 - 25x$ to describe the relationship between the amount still owed in dollars ($y$) after a number of weeks ($x$).

- **a** Explain why Tony has chosen to solve for $y$ (amount still owed in dollars) in terms of $x$ (number of weeks), rather than the other way around. (Hint: Think about the relationship between the variables.)
- **b** Sketch the graph of this relationship by first determining the coordinates of the $x$- and $y$-intercepts.
- **c** What does the $y$-intercept represent on this graph?
- **d** What does the $x$-intercept represent on this graph?
- **e** Describe the purchase plan Tony is using. When will he be able to bring his skateboard home?

**15** A rainwater tank has a capacity of 1500 L and feeds a drip system to water the garden. At the beginning of April, the tank is full, but it is empty at the end of the last day of the month. Let $x$ represent the number of days from the start of April and $y$ represent the number of litres of water in the tank. Assume a constant rate of water use and no further rain during April.

- **a** Identify the independent variable and dependent variable in this situation. Explain your reasoning.
- **b** Explain why this relationship can be represented by a linear graph.
- **c** Determine the coordinates of the $x$-intercept for this relationship.
- **d** Determine the coordinates of the $y$-intercept for this relationship.
- **e** Use these intercepts to sketch a graph of the relationship.
- **f** Use the graph to estimate the number of litres of water in the tank at the end of the day on 10 April.
- **g** Use the graph to estimate when there is 600 L of water left in the tank.

**16** **a** Explain why the reciprocals of $a$ and $b$ are respectively the $x$- and $y$-intercepts of $ax + by = 1$.

- **b** Explain how you could find the $x$- and $y$-intercepts of linear equations in the form $ax + by = d$.
- **c** For the equation $ax + by = d$, write an expression for $d$ in terms of $a$ and $b$ such that the $x$-intercept is equal to $b$ and the $y$-intercept is equal to $a$.

**17** Determine the axis intercepts of the following lines.

**a** $3x - 5y = -7$ **b** $\frac{2}{3}x + \frac{5}{2}y = \frac{4}{5}$ **c** $\sqrt{2}x - \sqrt{3}y = \sqrt{6}$ **d** $\sqrt{8}x + \sqrt{18}y = \sqrt{2}$

Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** Sketching linear graphs using intercepts
- **Investigation** Making a stained glass window
- **Topic quiz** 4D

PROBLEM SOLVING AND REASONING

CHALLENGE

# 4E Determining linear equations

## Learning intentions

✔ I can determine the linear equation of a graph.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 5D The Cartesian plane |
| Year 8 | 6F Finding linear equations |
| Year 10 | 4D Determining linear equations |

## Gradient-intercept form

- The equation of a linear graph can be expressed in **gradient-intercept form**:

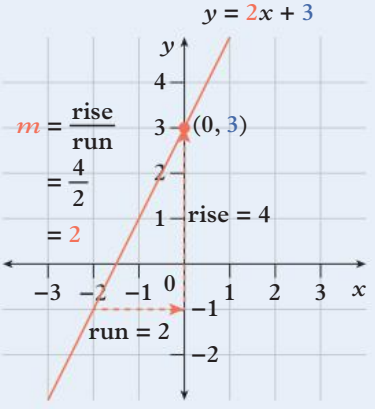
$$y = mx + c$$

($m$: gradient; $c$: $y$-coordinate of $y$-intercept)

→ $m$ is the gradient of the line

→ $c$ is the $y$-coordinate of the $y$-intercept

For example, the equation of the linear graph below is $y = 2x + 3$. It has a gradient of 2 and $y$-intercept at $(0, 3)$ so $m = 2$ and $c = 3$.




## Determining the equation of a linear graph

- To find the equation of a linear graph in the form of $y = mx + c$, determine the value of the:

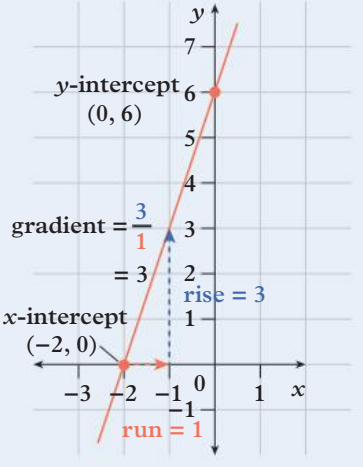
→ gradient, for any two points $(x_1, y_1)$ and $(x_2, y_2)$: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

→ $y$-coordinate of the $y$-intercept: $(0, c)$

For example, this linear graph has a gradient of 3 and a $y$-intercept at $(0, 6)$.

$m = 3$ and $c = 6$ so the equation of the graph is $y = 3x + 6$.

Note: The $x$-intercept at $(-2, 0)$ is not used in this calculation.



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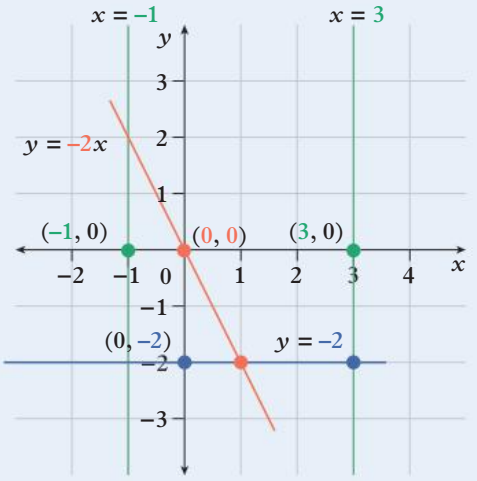
- For lines with only one intercept, the equations can be determined using the following general equations where $a$, $b$ and $m$ are constants.
  - → A vertical line with an $x$-intercept at $(a, 0)$ is given by $x = a$.
    For example, the graph of the equation $x = -1$ is a vertical line with an $x$-intercept at $(-1, 0)$.
  - → A horizontal line with an $y$-intercept at $(0, b)$ is given by $y = b$.
    For example, the graph of the equation $y = -2$ is a horizontal line with a $y$-intercept at $(0, -2)$.
  - → A line that passes through the origin $(0, 0)$ with gradient $m$ is given by $y = mx$.
    For example, the graph of the equation $y = -2x$ passes through the origin and has a gradient of $-2$.




## Example 4E.1 Determining the gradient and $y$-intercept of a linear equation

Identify the gradient, $m$, and the coordinates of the $y$-intercept of the linear graphs with the following equations.

**a** $y = 3x - 8$ **b** $y = 4 - \frac{3x}{5}$

**THINK**

**a** Compare the equation to the gradient-intercept form $y = mx + c$ to identify the values of $m$ and $c$.

**b** Rearrange the equation so that it is in gradient-intercept form $y = mx + c$. Note that the coefficient of $x$ will be the gradient and the constant term will be the $y$-intercept.

**WRITE**

**a** $y = 3x - 8$
$m = 3, c = -8$

**b** $y = 4 - \frac{3x}{5}$
$y = -\frac{3}{5}x + 4$
$m = -\frac{3}{5}$
$c = 4$

## Example 4E.2 Determining the equation of a linear graph given the gradient and the $y$-intercept

Determine the equation of a linear graph with a gradient of $-3$ and a $y$-intercept of $(0, 7)$.

**THINK**

Identify the values of gradient $m$ and the $y$-coordinate of the $y$-intercept $c$, and then substitute into the gradient-intercept form, $y = mx + c$.

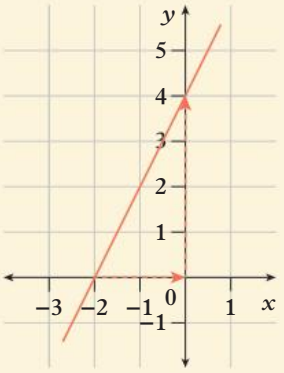
**WRITE**

$m = -3$
$c = 7$
$y = -3x + 7$

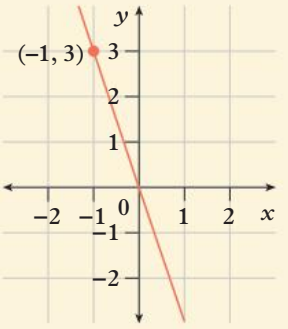
## Example 4E.3 Determining the equation of a linear graph given the *y*-intercept and a second point

Determine the equation of each of the following linear graphs.

**a**




**b**




**THINK**

**a** **1** Determine the value of the gradient $m$ by identifying the rise and run between any two integer coordinates on the graph.

**2** Determine the value of the constant $c$ by identifying the $y$-coordinate of the $y$-intercept.

**3** Substitute the values of $m$ and $c$ into the general equation for a straight line.

**b** **1** Determine the value of the gradient $m$ by identifying the rise and run between any two integer coordinates on the graph.

**2** Determine the value of the constant $c$ by identifying the $y$-coordinate of the $y$-intercept.

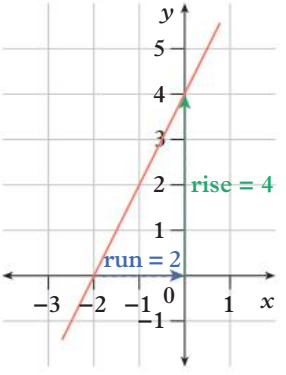
**3** Substitute the values of $m$ and $c$ into the general equation for a straight line.

**WRITE**

**a** $m = \frac{\text{rise}}{\text{run}}$

$= \frac{4}{2}$

$= 2$




$y$-intercept: (0,4)

$c = 4$

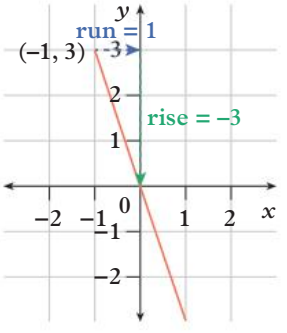
$y = mx + c$

$y = 2x + 4$

**b** $m = \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{1}$

$= -3$




$y$-intercept: (0,0)

$c = 0$

$y = mx + c$

$y = -3x + 0$

$y = -3x$



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## Example 4E.4 Determining the equation of a linear graph given two points

Find the equation of the linear graph that passes through the points (6, 5) and (2, −3).

| THINK | WRITE |
|---|---|
| 1 Calculate the value of the gradient $m$. | $m = \frac{y_2 - y_1}{x_2 - x_1}$<br>$= \frac{5 - (-3)}{6 - 2}$<br>$= \frac{8}{4}$<br>$= 2$ |
| 2 Substitute the value of the gradient $m$ into the general equation for a straight line, $y = mx + c$. | $y = 2x + c$ |
| 3 Select one of the two coordinates, substitute the values of $x$ and $y$ into the partially completed equation, and solve for $c$. | From the point (6, 5):<br>Let $x = 6$ and $y = 5$:<br>$5 = 2(6) + c$<br>$5 = 12 + c \quad (-12)$<br>$c = -7$ |
| 4 State the equation of the line. | $y = 2x - 7$ |

**Helpful hints**

✔ The different methods for finding the equation of a line can be overwhelming, so remember that to fully define a line you just need to find the gradient $m$ and the $y$-intercept $(0, c)$, which you can then substitute into the formula for a linear graph, $y = mx + c$.

ANS p504

# Exercise 4E Determining linear equations

△ 1(1st, 2nd columns), 2(a, c, e, g), 3, 4(a, c, e, g), 5, 6(a, c, e, g), 7(a, c, e), 8, 11(a, b), 13

□ 1(f, g, j, k, l), 3(b, e, g, h), 4(b, d, f, h), 5, 6(e–h), 7(b, d, f), 9, 10, 11(a, b, d), 13, 14(a, b)

⬠ 1(j–l), 3(f, h), 4(b, d, f, h), 5, 6(e–h), 7(b, d, f), 10, 11(c, d), 12, 13, 14(c), 15

4E.1 **1** Identify the gradient, $m$, and the coordinates of the $y$-intercept of the linear graphs with the following equations.

**a** $y = 2x + 5$ **b** $y = 4x + 1$ **c** $y = -3x + 7$ **d** $y = -5x - 3$

**e** $y = x - 6$ **f** $y = 1 - x$ **g** $y = \frac{4}{3}x + 2$ **h** $y = \frac{x}{2} - 8$

**i** $y = -\frac{4x}{3} + \frac{1}{4}$ **j** $y = 9$ **k** $y = -7x$ **l** $y = 5 - \frac{2}{5}x$

4E.2 **2** Determine the equations of the lines in the following graphs in the form $y = mx + c$.

**a** gradient: 3, $y$-intercept: (0,4) **b** gradient: −2, $y$-intercept: (0,10)

**c** gradient: 1, $y$-intercept: (0,−7) **d** gradient: −12, $y$-intercept: (0, −1)

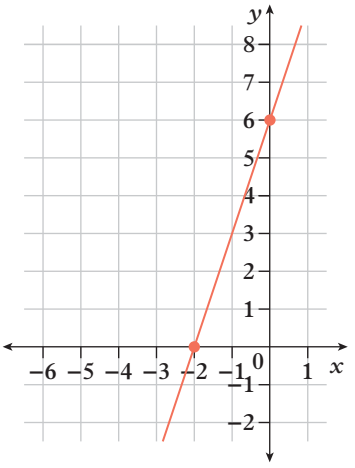
**e** gradient: −1, $y$-intercept: (0,20) **f** gradient: 5, $y$-intercept: (0,0)

**g** gradient: $\frac{1}{3}$, $y$-intercept: $\left(0, \frac{4}{5}\right)$ **h** gradient: $-\frac{3}{4}$, $y$-intercept: (0,0)

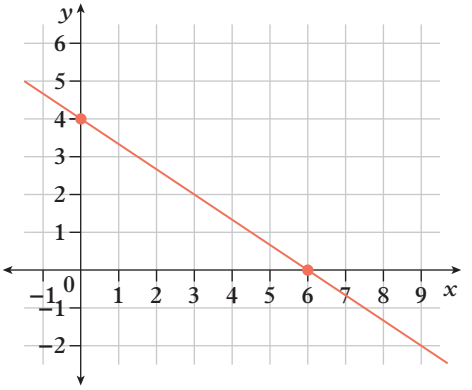
UNDERSTANDING AND FLUENCY

4E.3 **3** Determine the equation of each of the following linear graphs.

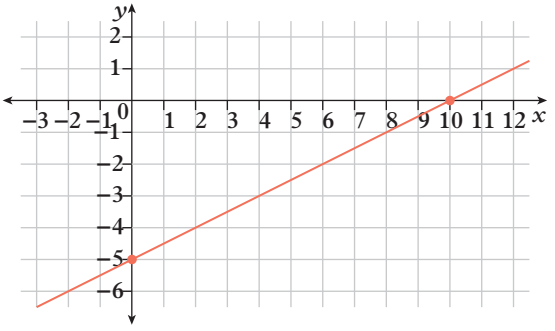
**a**



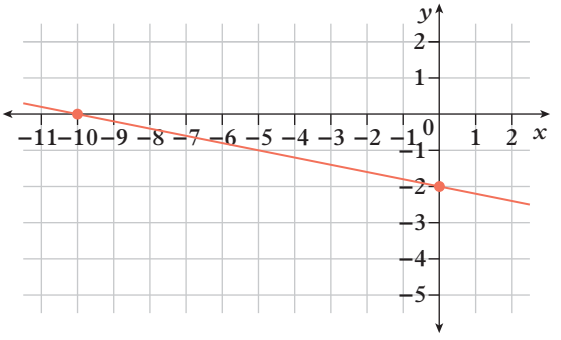
**b**



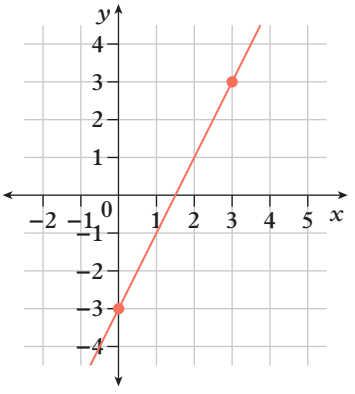
**c**



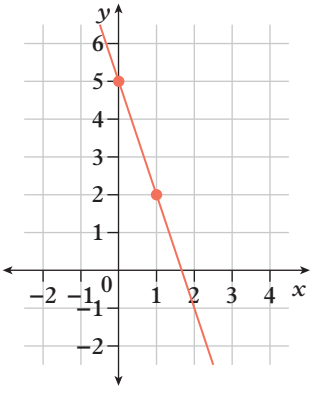
**d**



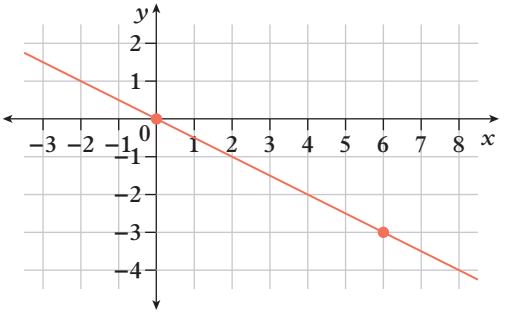
**e**



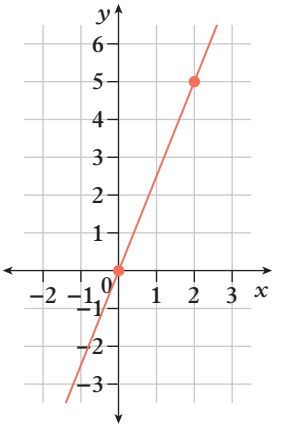
**f**



**g**



**h**





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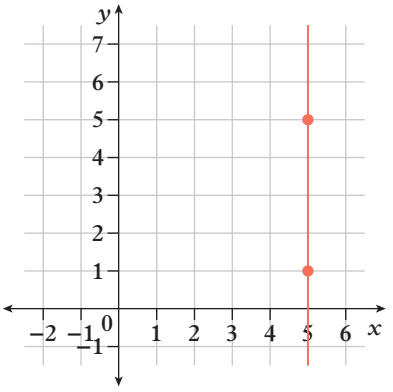
4E.4 **4** Find the equations of the linear graphs that pass through each pair of points.

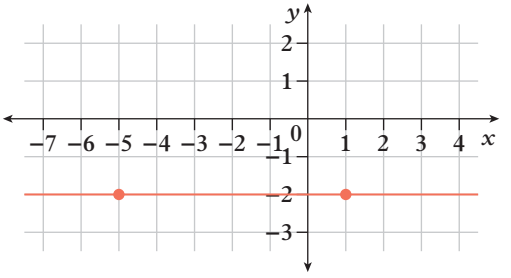
**a** (1,−20) and (11,0)
**b** (2,4) and (3,2)
**c** (−1,6) and (3,2)
**d** (−4,−12) and (−2,−6)
**e** (2,5) and (1,3)
**f** (9,−4) and (11,6)
**g** (−1,−2) and (5,−5)
**h** (6,−3) and (9,1)

**5** Find the equation of the linear graphs below. Recall that the general equation for a vertical line is $x = a$ and the general equation for a horizontal line is $y = b$, where $a$ and $b$ are constants.

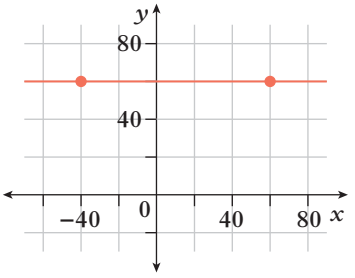
**a**



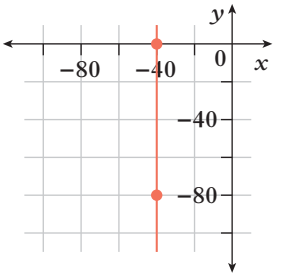

**b**



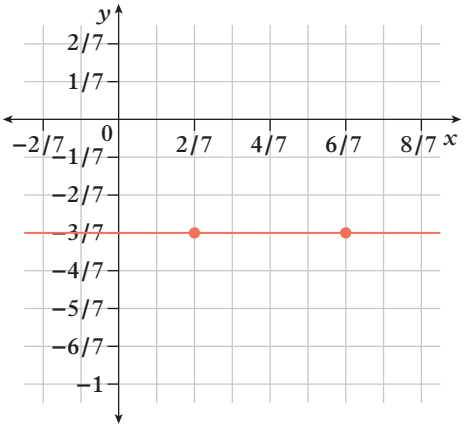

**c**



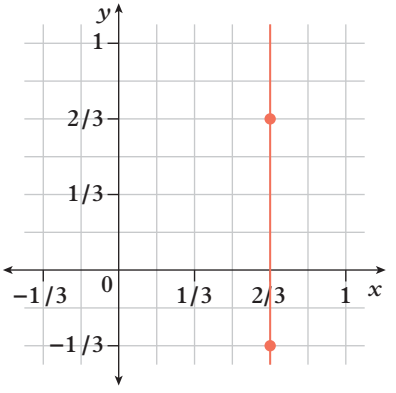

**d**




**e**




**f**

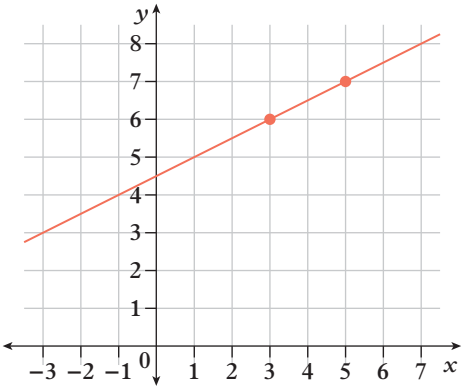



**6** Determine the equations of the linear graphs that have the following intercepts.

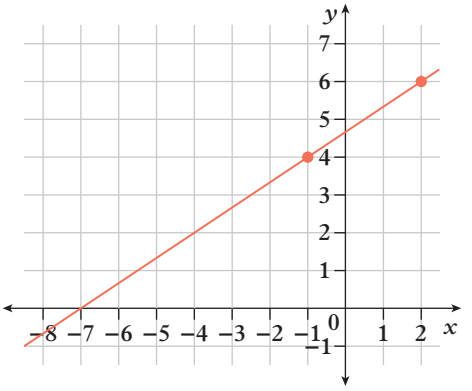
**a** $x$-intercept: (4,0), $y$-intercept: (0,12)
**b** $x$-intercept: (−5,0), $y$-intercept: (0,10)
**c** $x$-intercept: (3,0), $y$-intercept: (0,−21)
**d** $x$-intercept: (−6,0), $y$-intercept: (0,−24)
**e** $x$-intercept: (−1,0), $y$-intercept: (0,10)
**f** $x$-intercept: (18,0), $y$-intercept: (0,6)
**g** $x$-intercept: (5,0), $y$-intercept: (0,−2)
**h** $x$-intercept: (−6,0), $y$-intercept: (0,9)

**7** Identify two integer coordinates on each linear graph and use these to determine their equations.

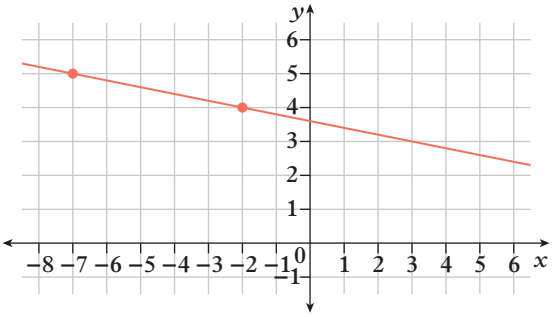
**a**



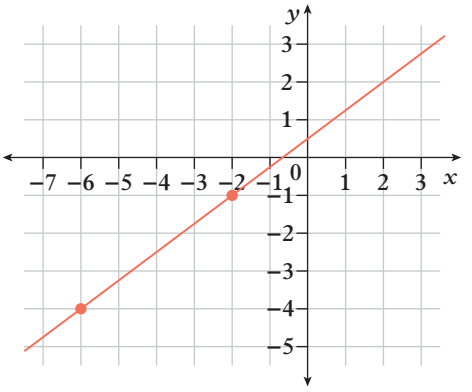

**b**



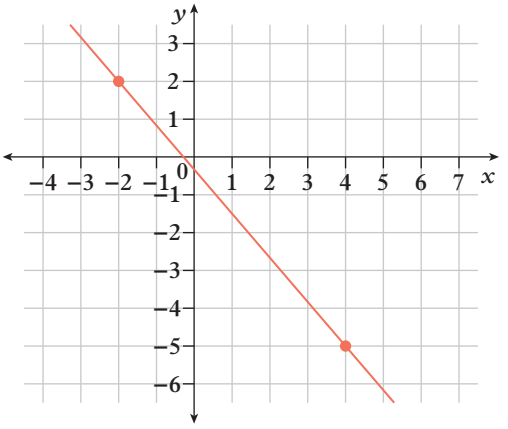

**c**



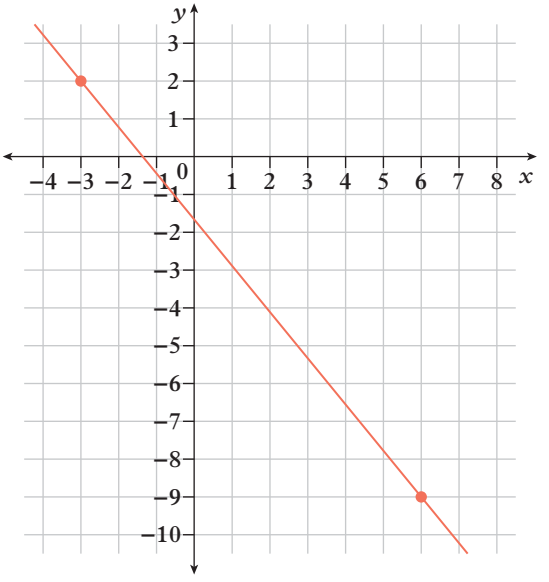

**d**




**e**




**f**




**8** Azami invests some money into a simple interest account. After 2 years the account has $2500 and after 5 years the account has $3400.

**a** Identify the independent variable and dependent variable.

**b** Determine the equation that will determine the amount in Azami's account, $\$A$, after $n$ years.

**9** Roland started a business and his earnings are increasing at a constant rate. After opening for 7 days, he earned $450 in profit and 14 days after opening he earned $870 in profit.

**a** Write a linear equation that gives the amount of profit, $\$P$, that Roland receives after $n$ days.

Being overly optimistic about his launch, Roland uses his model to predict the profit he will earn 1 year (365 days) after opening.

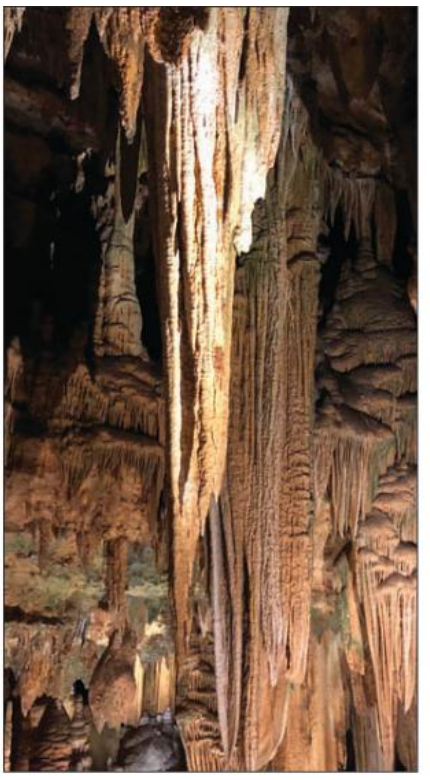
**b** What is the predicted profit Roland will earn exactly one year (365 days) after opening?

**10** As water drips from the ceiling of a cave it creates a long rock formation that hangs from the ceiling called a stalactite. A specific stalactite is measured to be 85 centimetres long in 2010 and then 10 years later it is measured to be 87 centimetres long.

**a** Identify the independent variable and dependent variable.

**b** Determine a linear equation for the growth of the stalactite, $g$, in centimeters $n$ years after 2010.

**c** Use your equation to estimate the age of the stalactite in 2010. What year did it begin to form?



**11** Determine the equation of the lines with the following tables of values.

**a**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −32 | −23 | −14 | −5 | 4 | 13 | 22 |

**b**

| $x$ | −3 | −1 | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|---|---|
| $y$ | −2 | 1 | 4 | 7 | 10 | 13 | 16 |

**c**

| $x$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|
| $y$ | 48 | 51 | 54 | 57 | 60 | 63 | 66 |

**d**

| $x$ | 1 | 4 | 9 | 16 | 25 | 36 | 49 |
|---|---|---|---|---|---|---|---|
| $y$ | 28 | 22 | 12 | −2 | −20 | −42 | −68 |

**12** A carpenter sells his wares based on the cost of the materials and the time spent crafting. A particular collection all cost the same in materials but take different amounts of time to craft. A piece that takes 16 hours to craft costs \$845 and a piece that takes 28 hours to craft costs \$1385. The carpenter then marks up the total cost by 120% to sell.

**a** Write the equation that gives the total cost, \$$C$, for a piece in this collection that takes $n$ hours to craft.

**b** Write the equation that gives the revenue, \$$R$, for a piece in this collection that takes $n$ hours to craft.

**c** Write the equation that gives the profit, \$$P$, for a piece in this collection that takes $n$ hours to craft.

**13 a** Factorise the right-hand side of each of the following equations.

**b** Find the $x$-intercept of each of the following equations.

**c** Describe the connection between the factor form in part **a** and the $x$-intercept in part **b**.

**i** $y = 3x + 12$ **ii** $y = -5x - 15$

**iii** $y = 2x - 10$ **iv** $y = 6x - 8$

**v** $y = 21 - 7x$ **vi** $y = 16x - 40$

**14** Find the equations of the linear graphs that pass through each pair of points.

**a** $\left(\frac{2}{3}, 0\right)$ and $\left(0, \frac{7}{5}\right)$

**b** $\left(-\frac{2}{5}, \frac{3}{4}\right)$ and $\left(\frac{1}{5}, -\frac{1}{4}\right)$

**c** $\left(-\frac{9}{2}, -\frac{7}{4}\right)$ and $\left(-\frac{9}{8}, -\frac{5}{6}\right)$



**15** The product of the gradient, $m$, and $y$-intercept, $c$, for a particular line is 99. The sum of the gradient and $y$-intercept is 20.

**a** Write the two possible equations for the line.

**b** Write the equation of the line that is always halfway between the two lines from part **a**.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Determining linear equations

Investigation
The human body

Topic quiz
4E

# 4F Midpoint and length of a line segment

## Learning intentions

- ✔ I can determine the midpoint of a line segment on the Cartesian plane.
- ✔ I can calculate the length of a line segment on the Cartesian plane.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 1G Indices and square roots |
| Year 8 | 4E Roots |

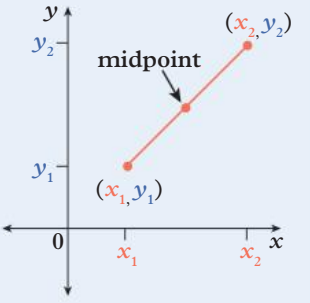
## Midpoint of a line segment

- The **midpoint** of a line segment is the point located exactly halfway between the endpoints of the line segment. If the coordinates of two endpoints of a line segment are $(x_1, y_1)$ and $(x_2, y_2)$, then:

  midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

  → The $x$-coordinate of the midpoint is the average of the $x$-coordinates of the endpoints.

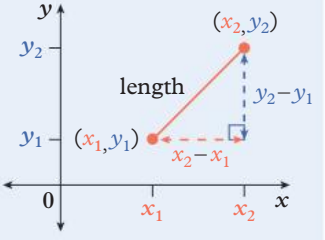
  → The $y$-coordinate of the midpoint is the average of the $y$-coordinates of the endpoints.




## Length of a line segment

- The formula for the length of a line segment can be determined by using Pythagoras' Theorem, which is covered in Chapter 7. If the coordinates of two endpoints are $(x_1, y_1)$ and $(x_2, y_2)$, then the length of the line segment (in units) is:

  length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$




### Example 4F.1 Determining the midpoint of a line segment

Find the coordinates of the midpoint of the line segment joining $(-1, 4)$ and $(7, 9)$.

**THINK**

1. Calculate the $x$-coordinate of the midpoint by averaging the $x$-coordinates of the endpoints.
2. Calculate the $y$-coordinate of the midpoint by averaging the $y$-coordinates of the endpoints.
3. Write the coordinates of the midpoint.

**WRITE**

$x$-coordinate of the midpoint $= \frac{-1 + 7}{2} = 3$

$y$-coordinate of the midpoint $= \frac{4 + 9}{2} = \frac{13}{2}$

Coordinates of the midpoint: $\left(3, \frac{13}{2}\right)$

## Example 4F.2 Calculating the length of a line segment

Calculate the length of the line segment joining $(-1, 4)$ and $(7, 9)$, correct to one decimal place.

**THINK**

1 Define the points $(x_1, y_1)$ and $(x_2, y_2)$. The order of the coordinates will not affect the result.

2 Substitute the $x$- and $y$-coordinates into the formula for the length of a line segment.

3 Use a calculator to evaluate the root and round the answer to one decimal place as specified by the question.

**WRITE**

Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (7, 9)$

length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7 - (-1))^2 + (9 - 4)^2}$

$= \sqrt{8^2 + 5^2}$

$= \sqrt{64 + 25}$

$= \sqrt{89}$

$\approx 9.4$ units

**Helpful hints**

- ✔ Take care when substituting negative values into the formula for the length of a line segment – watch carefully for any changes in sign!
- ✔ If you are already familiar with Pythagoras' Theorem, it may help to draw the line segment as the longest side of a right-angled triangle and identify the lengths of the two shorter sides.

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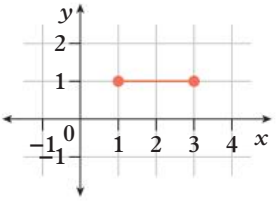
# Exercise 4F Midpoint and length of a line segment

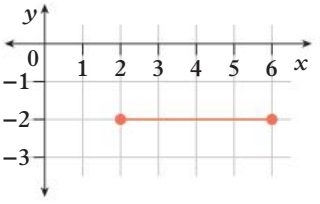
1, 2–3(1st, 2nd columns), 4, 6, 7, 9, 11 | 2–3(2nd, 3rd columns), 4, 8–10, 12, 13(a), 15 | 2–3(3rd column), 4, 8, 10, 12, 13(b), 14, 16–18

**1** State the midpoint of the following line segments.

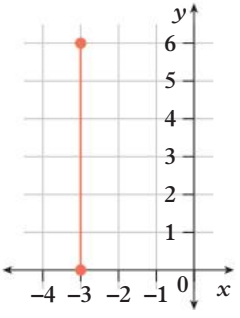
**a**



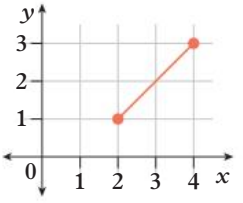

**b**




**c**

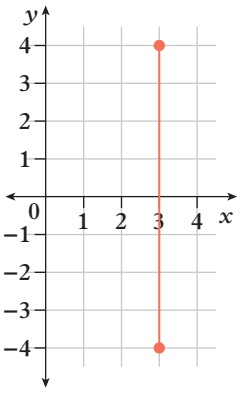



**d**

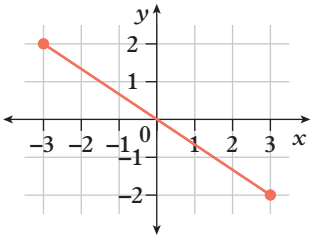



UNDERSTANDING AND FLUENCY

**e**




**f**




4F.1 **2** Find the coordinates of the midpoint of the line segment joining each pair of points.

**a** (1, 4) and (3, 10)
**b** (2, 5) and (8, 3)
**c** (1, 0) and (5, 2)
**d** (2 , 6) and (2 , 10)
**e** (0, 5) and (8, 9)
**f** (3, −4) and (7, 6)
**g** (2, −1) and (6, 7)
**h** (−3, −4) and (5, −4)
**i** (−4, −2) and (−2, 2)
**j** (3, 9) and (4, 8)
**k** (5, 0) and (8, 11)
**l** (−5, 7) and (5, −7)

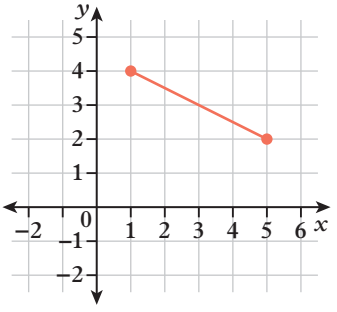
4F.2 **3** Calculate the length of the line segment joining each pair of points correct to one decimal place.

**a** (2, 5) and (3, 7)
**b** (3, 4) and (5, 8)
**c** (6, 2) and (9, 3)
**d** (−4, 5) and (−4, 9)
**e** (2, −4) and (4, 2)
**f** (5, 0) and (8, −4)
**g** (0, −1) and (1, −2)
**h** (7 , 8) and (−7 , 8)
**i** (4, −3) and (6, 0)
**j** (−3, 6) and (−2, 2)
**k** (−5, −4) and (−1, −2)
**l** (6, −5) and (−6 , 5)

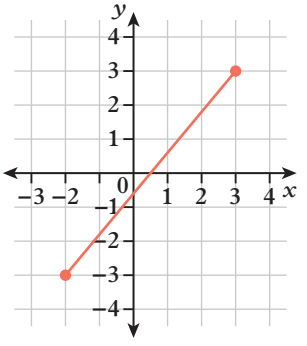
**4** For each line segment, find:

**i** the midpoint
**ii** the length (1 d.p.)
**iii** the gradient.

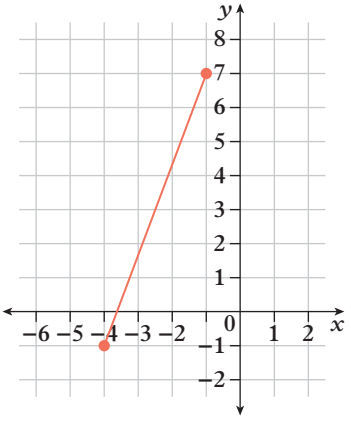
**a**



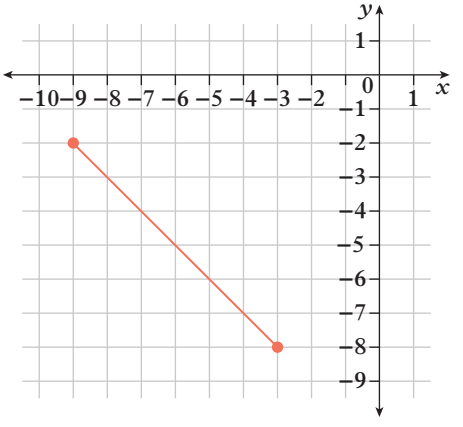

**b**




**c**




**d**




**5** Calculate the length between the pairs of points in question **3** and the midpoint of each line segment, correct to 1 decimal place.

**6** The midpoint of a line segment $AB$ has the coordinates (6, 4). If point $A$ has the coordinates (2, 3), find the coordinates of point $B$.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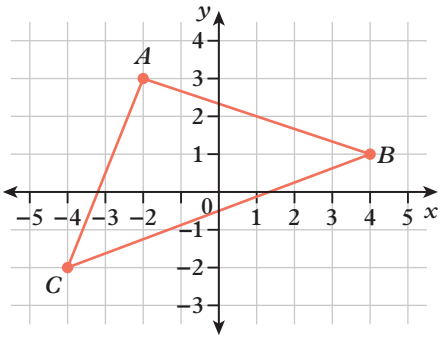
**7** The midpoint of a line segment $CD$ has the coordinates $(-5, 1)$. If point $D$ has the coordinates $(4, -7)$, find the coordinates of point $C$.

**8** A point, $A$, on a circle has the coordinates $(-1, -1)$.

**a** If the centre of the circle is at $(2, 3)$, calculate the radius of the circle.

**b** Identify the coordinates of another point on the circle that forms a diameter with point $A$.

**9** Calculate the perimeter of $\Delta ABC$.




**10** Calculate the perimeter of each shape correct to one decimal place.

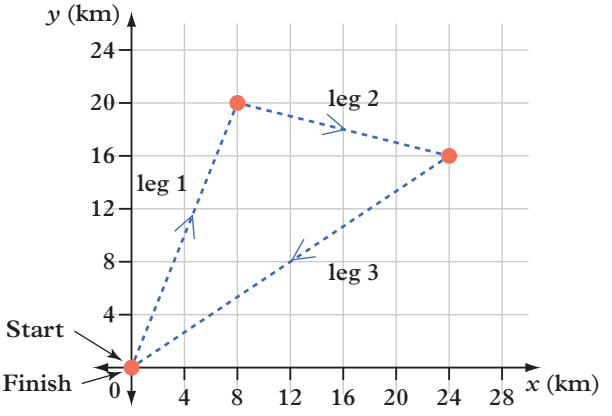
**a** triangle with vertices at $(-3, -2)$, $(-2, 4)$ and $(4, 2)$

**b** square with vertices at $(-1, 2)$, $(2, 5)$, $(5, 2)$ and $(2, -1)$

**c** rectangle with vertices at $(-4, -2)$, $(2, 4)$, $(4, 2)$ and $(-2, -4)$

**d** trapezium with vertices at $(-3, 3)$, $(1, 5)$, $(3, 3)$ and $(2, -2)$

**11** A yacht race follows a triangular course that has been mapped on to the Cartesian plane. The scales on the axes represent distances in kilometres. The race begins and ends at the origin.




**a** Calculate the length of each leg of the race correct to one decimal place.

**b** Calculate the total distance covered during the race correct to one decimal place.

**c** An observer's boat is located close to the midpoint of the second leg of the race. Determine the distance between the observer's boat and the finishing point, correct to one decimal place.

**12** Consider this parallelogram drawn on the Cartesian plane.

**a** List the coordinates of the vertices of the parallelogram.

**b** Find the coordinates of the midpoint of:

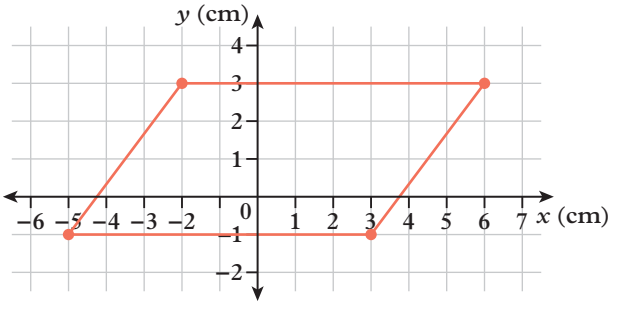
**i** the longer diagonal

**ii** the shorter diagonal.

**c** What do you notice about your answers to part **b**?

**d** Calculate the perimeter of the parallelogram.

**e** Find the difference in length of the two diagonals correct to one decimal place.



PROBLEM SOLVING AND REASONING

**13** A shape (in blue) is drawn on the Cartesian plane. A smaller shape (in orange) is then formed by joining the midpoints of the vertices of the original shape.

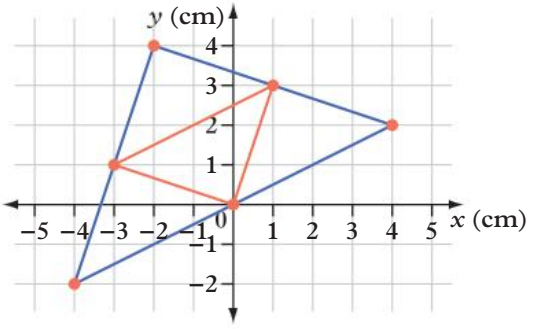
In each case, find the perimeter (correct to one decimal place) of:

**i** the blue shape

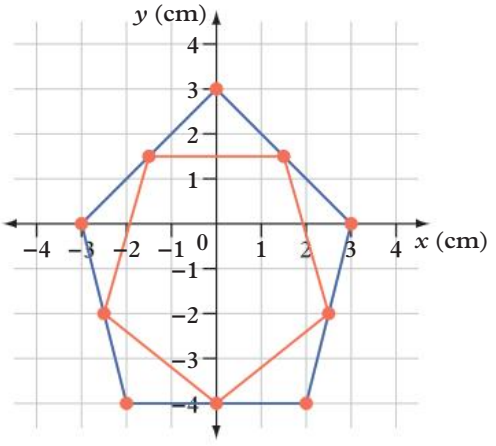
**ii** the orange shape.

**iii** Compare the perimeter of the orange shape to the perimeter of the blue shape.

**a**




**b**




**14** A quadrilateral $ABCD$ has vertices at $A(-4, 2)$, $B(-1, 4)$, $C(3, -2)$ and $D(0, -4)$. The scales on the axes represent distances in metres.

**a** Determine the length of each side of the quadrilateral correct to one decimal place.

**b** Determine the coordinates of the midpoint of each diagonal.

**c** Determine the distance between each vertex and the midpoint correct to one decimal place.

**d** Use your answers for parts **a–c** to identify the shape of the quadrilateral $ABCD$. Explain your reasoning.

**e** Hence use the correct formula to determine the area of the quadrilateral, correct to one decimal place.

CHALLENGE

**15 a** Prove that triangle $ABC$ with vertices at $A(3, 6)$, $B(-1, -2)$ and $C(-5, 2)$ is an isosceles triangle. State any dimensions in simplified surd form.

**b** Calculate the area of the triangle using your knowledge of the midpoint and length of a line segment.

**16** Prove that quadrilateral $ABCD$ with vertices at $A(-5, 3)$, $B(-1, 5)$, $C(3, -2)$ and $D(-1, -4)$ is a parallelogram. State any dimensions in simplified surd form.

**17** A line segment has endpoints at $A(-1, -1)$ and $B(2, 5)$. If point $C$ lies between $A$ and $B$ such that $AC$ is $\frac{1}{3}$ the length of $AB$, determine the exact coordinates of $C$.

**18** A line segment has a midpoint at the origin, a length of 10 units and a gradient of 2. Determine the coordinates of the two endpoints. Write your answer in simplified surd form. (Hint: Sketch a diagram and then consider only one half of the line segment.)

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Midpoint of a line segment

Interactive skillsheet
Length of a line segment

Investigation
From square to triangle

Topic quiz
4F

# 4G Direct proportion

## Learning intentions

- ✔ I can identify when two variables are directly proportional to each other.
- ✔ I can solve simple rate problems.

**Inter-year links**

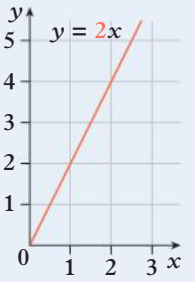
| | |
|---|---|
| Years 5/6 | Multiplying and dividing whole numbers |
| Year 7 | 5E Interpreting graphs |
| Year 8 | 3G Rates |

## Direct proportion

$y = kx$

constant of proportionality
= rate of change
= gradient

- $y$ is said to be **directly proportional** to $x$ if:
  - → when $x = 0$, $y = 0$
  - → the **rate of change** of $y$ with respect to $x$ is constant.
- Direct proportionality is denoted using the symbol $\propto$.
- If $y \propto x$, then the equation for the relationship between $x$ and $y$ is $y = kx$, where $k$ is the rate of change of $y$ with respect to $x$ and is called the **constant of proportionality**.
- The graph of $y = kx$ is a straight line that passes through the origin $(0, 0)$ and has a gradient of $k$.
  For example, in the relationship represented by the graph to the right, the constant of proportionality is 2.



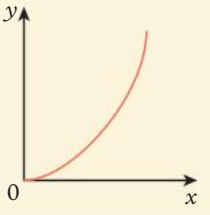

### Example 4G.1 Identifying direct proportion

Determine whether $x$ is directly proportional to $y$ in each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

**a**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | −7 | −14 | −21 | −28 |

**b**




**THINK**

**a** 1 Check that $x = 0$ when $y = 0$.

2 Check whether the rate of change is constant. To determine the rate of change from a table of values, calculate $\frac{y}{x}$ for each pair of coordinates excluding $(0, 0)$.

3 State whether or not $y \propto x$.

**b** 1 Check that $x = 0$ when $y = 0$.

2 Identify whether the gradient of the graph is constant.

3 State whether or not $y \propto x$.

**WRITE**

**a** When $x = 0$, $y = 0$.

$\frac{y}{x} = \frac{-7}{1} = -7$ $\quad$ $\frac{y}{x} = \frac{-21}{3} = -7$

$\frac{y}{x} = \frac{-14}{2} = -7$ $\quad$ $\frac{y}{x} = \frac{-28}{4} = -7$

$y$ is directly proportional to $x$.

**b** The graph passes through the origin $(0, 0)$.

The graph of $y$ versus $x$ is non-linear and, therefore, does not have a constant gradient.

$y$ is not directly proportional to $x$.

## **Example 4G.2** Finding the constant of proportionality

Find the constant of proportionality if $y \propto x$ using the given information, then write the equation for each directly proportional relationship.

**a** $y = 18$ when $x = 3$

**b**

| $x$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | 0 | 24 | 48 | 72 | 96 |

**c**

**THINK**

**a** **1** If $y \propto x$, $y = kx$. Substitute the given values into $y = kx$ and solve for $k$.

**2** Substitute into the general equation $y = kx$.

**b** **1** Select any pair of values in the table and substitute the corresponding values of $x$ and $y$ into $y = kx$, then solve for $k$.

**2** Substitute into the general equation $y = kx$.

**c** **1** As the graph passes through the origin, (0, 0), to find the gradient, $k$, select any point on the graph, substitute the corresponding values of $x$ and $y$ into $y = kx$, then solve for $k$.

**2** Substitute into the general equation $y = kx$.

**WRITE**

**a** $y = kx$

Let $x = 3$ and $y = 18$:

$18 = k \times 3$
$18 = 3k \quad (\div 3)$
$k = 6$

$y = 6x$

**b**

| $x$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | 0 | 24 | 48 | 72 | 96 |

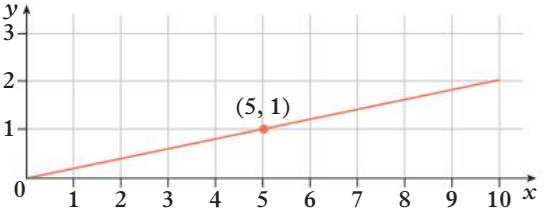
$y = kx$

Let $x = 2$ and $y = 24$:

$24 = k \times 2$
$24 = 2k \quad (\div 2)$
$k = 12$

$y = 12x$

**c**




$y = kx$

Let $x = 5$ and $y = 1$:

$1 = k \times 5$
$1 = 5k \quad (\div 5)$
$k = \frac{1}{5}$

$y = \frac{1}{5}x$



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## **Example 4G.3** Solving simple rate problems

Jamie completes a 200 metre sprint in 40 seconds.

**a** Assuming Jamie runs at a constant speed, sketch a graph of Jamie's distance, $d$, from the starting line against the time, $t$, for the duration of the sprint.

**b** **i** Determine the gradient of the graph.

**ii** Express Jamie's average speed as a rate.

**c** Determine the equation for the relationship between $d$ and $t$.

**d** How far had Jamie run after 10 seconds?

**THINK**

**a** Plot the endpoints:
Start $t = 0$ s, Jamie is $d = 0$ m from the starting line: (0, 0).
End $t = 40$ s, Jamie is $d = 200$ m from the starting line: (40, 200).

**b** **i** To find the gradient, select the points (0,0) and (40, 200), and calculate $m = \frac{\text{rise}}{\text{run}}$.

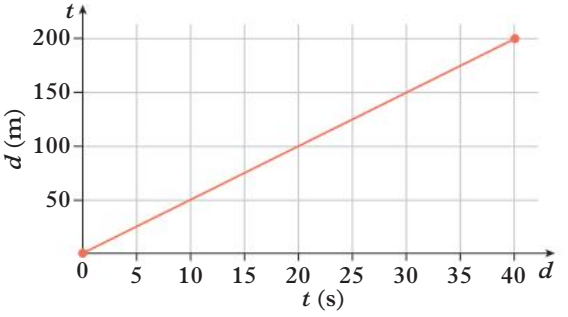
**ii** The rate of change of $d$ with respect to $t$ is the same as the gradient of the graph of $d$ vs. $t$. Jamie runs 5 metres per second.

**c** The equation for direct proportion is $y = kx$, where $k$ is the constant of proportionality.

**d** Substitute time, $t = 10$ seconds into the equation and solve for the distance, $d$.

**WRITE**

**a**




**b** **i** $m = \frac{\text{rise}}{\text{run}}$
$= \frac{200}{40}$
$= 5$

**ii** Speed = 5 m/s

**c** $d = 5t$

**d** Let $t = 10$
$d = 5(10)$
$= 50$ m

**Helpful hints**

- ✔ Direct proportion is an example of linear algebra and linear graphs. Remember that for the relationship $y = kx$, $k$ = constant of proportionality = rate of change = gradient.
- ✔ For a relationship to be directly proportional, the constant of proportionality $k$ must be a constant, but it can still be fractional, irrational or negative!

ANS p506

# Exercise 4G Direct proportion

1–4, 6, 7 | 1–3, 5, 6, 8, 9(a–e) | 1, 3, 5, 6, 8–10

UNDERSTANDING AND FLUENCY

4G.1 **1** Determine whether $x$ is proportional to $y$ for each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

**a**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 4 | 8 | 12 | 16 |

**b**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 1 | 8 | 27 | 64 |

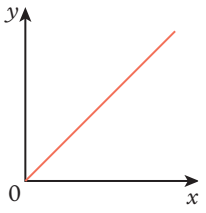
**c**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | −9 | −18 | −27 | −36 |

**d**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 1 | 2 | 4 | 6 | 8 |

**e**

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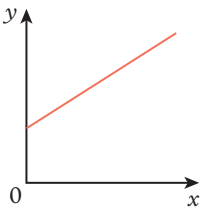
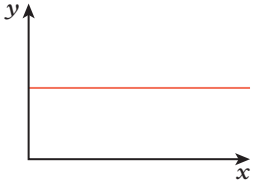
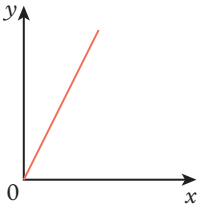


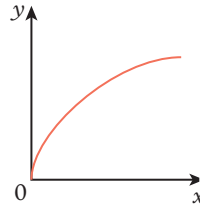

**g**



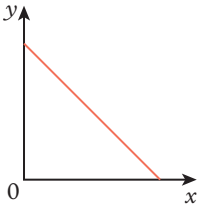

**h**




**i**




**j**




**2** Identify and state the constant of proportionality for each relationship.

**a** $y = 4x$

**b** $m = -10n$

**c** $h = 3.5t$

**d** $y = \frac{x}{3}$

4G.2 **3** Find the constant of proportionality using the given information, then write the equation for each directly proportional relationship.

**a** $y = 50$ for $x = 5$

**b** $y = 63$ for $x = -7$

**c** $y = 16$ for $x = 64$

**d** $y = -12$ for $x = 18$

**e**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | −6 | −12 | −18 | −24 |

**f**

| $x$ | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| $y$ | 0 | 25 | 50 | 75 | 100 |

**g**

| $x$ | 0 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| $y$ | 0 | −5 | −10 | −15 | −20 |

**h**

| $x$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | 0 | 10 | 20 | 30 | 40 |



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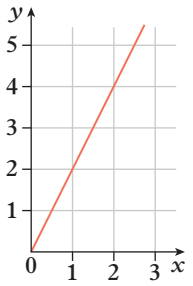
**i**

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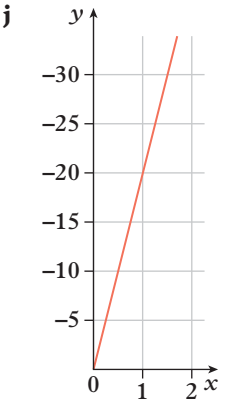
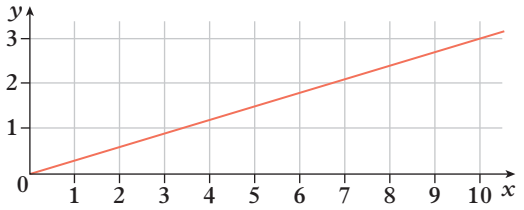
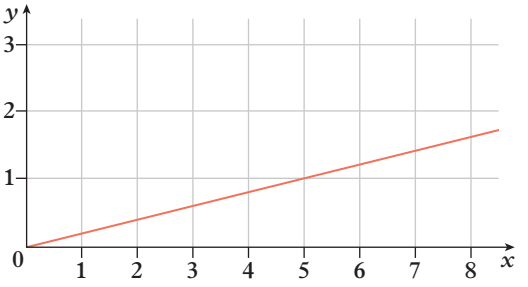



**k**



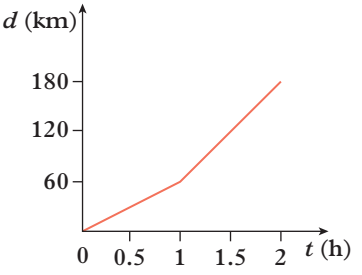

**l**




**4G.3** **4** Julian earns $24 an hour working at the local deli.

- **a** **i** Sketch a graph of Julian's earnings, $c$, against $t$, the number of hours he works during an 8-hour shift.
  - **ii** Determine the gradient of the graph.
- **b** Express Julian's hourly wage as a rate.
- **c** Determine the equation for the relationship between $c$ and $t$.
- **d** If Julian has to leave halfway through the 8-hour shift, how much will he earn in total?

**5** The graph on the right shows the distance travelled by a car over time.




- **a** Describe the car's journey, including the speed at which the car is travelling at different times in the journey.
- **b** Write an equation for the relationship between $d$ and $t$ during the first hour of the journey.
- **c** Write an equation for the relationship between $d$ and $t$ during the second hour of the journey.
- **d** Is $d \propto t$? Why or why not?

**6** The cost, $C$ dollars, of building a house is directly proportional to the area, $A$, of the floor space in square metres. It costs $90 000 to build a house with a floor space of 150 m$^2$.

- **a** Given $C \propto A$, determine the constant of proportionality.
- **b** Write an equation for the relationship between $C$ and $A$.
- **c** Use your answer from part **a** to calculate the floor space of a house that you could build with a budget of $126 000.

PROBLEM SOLVING AND REASONING

CHALLENGE

**7** A physiotherapist sees 160 patients every week.

**a** If the physiotherapist works 40 hours each week, on average, how many patients does the physiotherapist see per hour?

**b** On average, how many minutes does the physiotherapist spend with each patient?

**c** If the physiotherapist wishes to earn at least $10 000 every week, what is the minimum he must charge each patient?

**8** Jasmine lives 1200 m from school. She walks to school each day at a speed of 1 m/s. Nelson lives 4.5 km from school. He rides his bike to school each day at a speed of 15 m/s.

**a** If both Jasmine and Nelson arrive at school at 8.30 am, what time did each student leave home?

**b** Jasmine and Nelson live 4.8 km apart. They plan to meet each other one morning and both leave their homes at 9.00 am. At what time will they meet?

**9** Tom is riding in a cycling event. His distance from the start line at given times is recorded. The table shows values for $t$ (number of hours) and $d$ (distance from the start line in km).

| $t$ | 0 | 1 | 4 | 9 | 16 |
|---|---|---|---|---|---|
| $d$ | 0 | 20 | 40 | 60 | 80 |

**a** Plot the points on a Cartesian plane and then join them with a smooth line.

**b** Is the relationship between $t$ and $d$ an example of direct proportion? Explain.

**c** Complete this table.

| $\sqrt{t}$ | | | | | |
|---|---|---|---|---|---|
| $d$ | 0 | 20 | 40 | 60 | 80 |

**d** Plot the points on a Cartesian plane and join them with a smooth line.

**e** Is the relationship between $\sqrt{t}$ and $d$ an example of direct proportion? Explain.

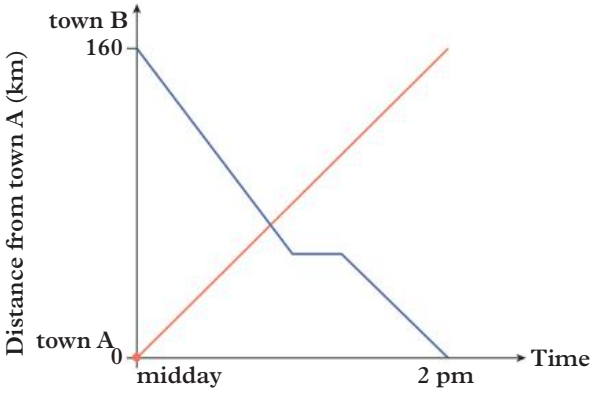
**f** Determine the constant of proportionality for the relationship between $\sqrt{t}$ and $d$ and hence write an equation for the relationship.

**g** What distance is Tom from the start line after 36 hours?

**10** The graph below shows the journeys made by two friends between towns A and B. Both friends leave their homes at midday and arrive at their destinations 2 hours later.

Helena travels at a constant speed from town A to town B. Julia travels at a constant speed for the first hour, then rests for 15 minutes before continuing on to town A at the same speed travelled as Helena.

Determine how far each of them was from town A after travelling for half an hour.




Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Direct proportion

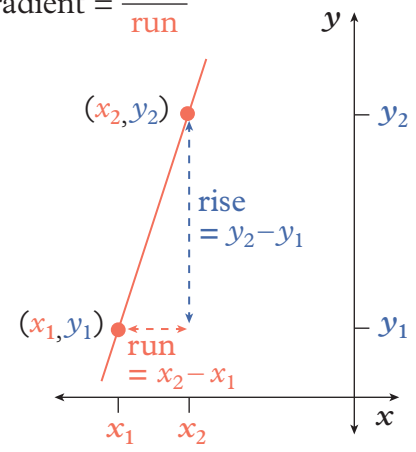
Investigation
How much gold is actually in a piece of gold jewellery?

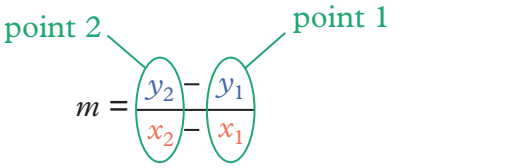
Topic quiz
4G

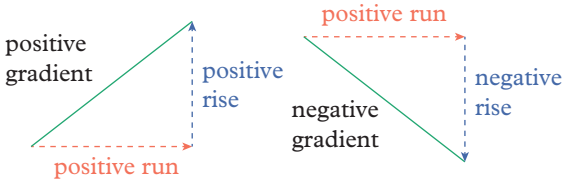
# Chapter summary

## Gradient

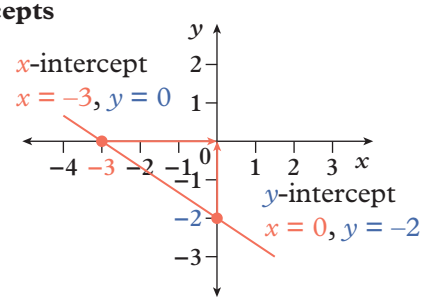
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$



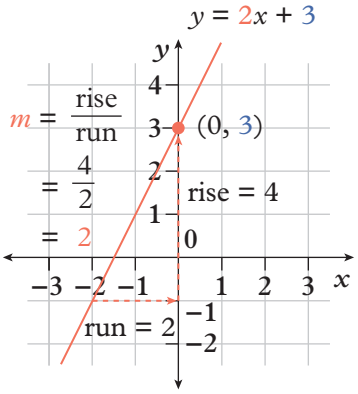

## Intercepts




## Gradient-intercept form

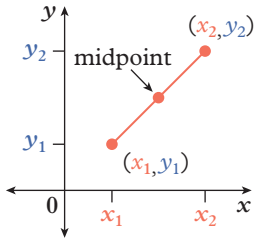
$$y = mx + c$$

$m$: gradient; $c$: $y$-coordinate of $y$-intercept



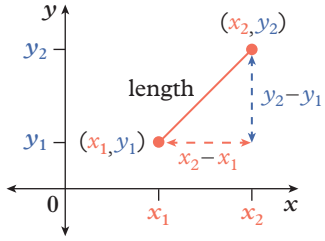

## Midpoint

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$




## Length

$$\text{Length} = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$




## Direct proportion

→ when $x = 0$, $y = 0$

→ the rate of change of $y$ with respect to $x$ is constant

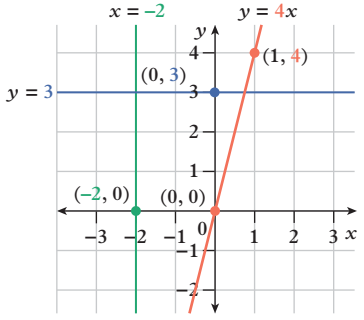
$y \propto x \longrightarrow y = kx$

$k$: constant of proportionality = rate of change = gradient

## Linear graphs with one intercept

| Description | General equation | Intercept | Gradient |
|---|---|---|---|
| Vertical lines | $x = a$ | $x$-intercept: $(a, 0)$ | undefined |
| Horizontal lines | $y = b$ | $x$-intercept: $(0, b)$ | 0 |
| Lines that pass through the origin | $y = mx$ | origin: $(0, 0)$ | $m$ |

where $a$, $b$ and $m$ are constants.



# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

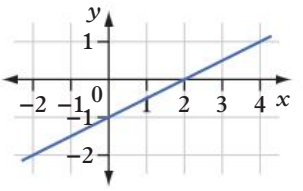
4A **1** The solution to the equation $3x - 9 = 12$ is:
**A** $x = 1$ **B** $x = 7$ **C** $x = 12$ **D** $x = 13$ **E** $x = 63$

4A **2** Which of the following equations is not equivalent to $3(2x - 5) = 21$?
**A** $2x - 5 = 7$ **B** $x = 6$ **C** $6x - 5 = 21$ **D** $2x = 12$ **E** $6x = 36$

4A **3** The value of $x$ in the equation $5x - 2 = 3x + 7$ is:
**A** $\frac{5}{8}$ **B** $1\frac{1}{8}$ **C** $2\frac{1}{2}$ **D** $4\frac{1}{2}$ **E** $5\frac{1}{2}$

4B **4** Which point does *not* lie on the graph of $y = 2x - 4$?
**A** $(0, -4)$ **B** $(1, -2)$ **C** $(4, 4)$ **D** $(-1, -2)$ **E** $(-2, -8)$

4C **5** The gradient of the linear graph shown to the right is:
**A** $-2$ **B** $-\frac{1}{2}$ **C** $\frac{1}{2}$
**D** 1 **E** 2




4C **6** The gradient of the line segment joining the points $(-2, 4)$ and $(7, -4)$ is:
**A** $\frac{8}{9}$ **B** $-\frac{8}{9}$ **C** $\frac{9}{8}$ **D** $-\frac{9}{8}$ **E** 0

4C **7** A line with a gradient of 0:
**A** is vertical **B** is horizontal **C** is undefined
**D** increases from left to right **E** decreases from left to right

4D **8** At the point where a line crosses the $x$-axis:
**A** $x = y$ **B** $y = -x$ **C** $y = 1$ **D** $x = 0$ **E** $y = 0$

4D **9** The $y$-intercept for the graph of $2y = 3x - 4$ is:
**A** $(0, -4)$ **B** $(0, -2)$ **C** $(0, 0)$ **D** $(0, 2)$ **E** $(0, 4)$

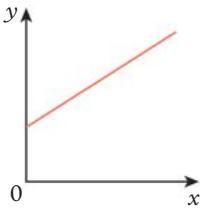
4E **10** In the general equation of a linear graph, $y = mx + c$, the pronumeral $c$ represents the:
**A** $y$-intercept **B** $x$-intercept **C** rise **D** run **E** gradient

4E **11** A linear graph has a gradient of 3 and a $y$-intercept of $(0, -2)$. The equation of the graph is:
**A** $y = -2x + 3$ **B** $y = \frac{1}{3}x - 2$ **C** $y = 3x - 2$ **D** $y = -3x + 2$ **E** $3x - 2y = 1$

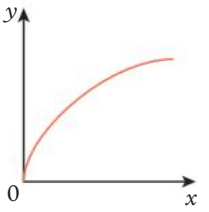
4F **12** The coordinates of the midpoint of the line segment joining $(3, 5)$ and $(7, 9)$ are:
**A** $(5, 7)$ **B** $(7, 5)$ **C** $(2, 2)$ **D** $(4, 8)$ **E** $(1,2)$

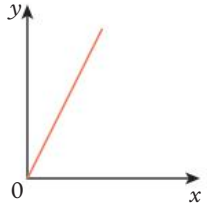
4F **13** The length of the line segment joining $(3, 5)$ and $(7, 9)$ is closest to:
**A** 4 **B** 5 **C** 6 **D** 7 **E** 8

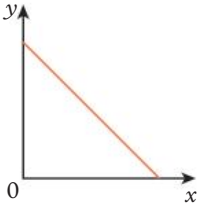
4G **14** For which graph is $y \propto x$?

**A**



**B**

**C**



**D**



**E**




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## Short answer

**4A** **1** Solve each equation using inverse operations.

**a** $3x - 4 = 2$ **b** $2 = 4 - 3x$ **c** $\frac{x+2}{3} = 4$ **d** $\frac{x}{3} - 2 = -1$

**e** $\frac{x-2}{3} + 5 = 4$ **f** $\frac{4}{x} + 3 = -2$ **g** $4(x - 3) = 3$ **h** $2(3 - x) - 7 = -2$

**4A** **2** Solve each equation for $x$.

**a** $7x - 3 = 4x + 9$ **b** $x + 8 = 1 - 6x$

**4A** **3** One more than three-fifths of the class is 16 people.

**a** Define a pronumeral to represent the unknown quantity in this problem.

**b** Use this pronumeral to write an equation to represent the problem.

**c** Solve the equation.

**d** How many people are in the class?

**4A** **4** Jacob's brother is 9 years older than Jacob now and will be twice as old as Jacob next year.

**a** Define a pronumeral to represent the unknown quantity in this problem.

**b** Use this pronumeral to write an equation to represent the problem.

**c** Solve the equation.

**d** How old is Jacob this year?

**4B** **5** Complete a table of values from $x$ is equal to $-2$ to 2 for each linear relationship, then construct a plot of the relationship between $x$ and $y$.

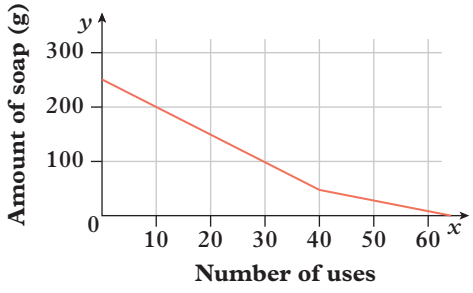
**a** $y = x + 5$ **b** $y = x - 5$ **c** $y = -x + 5$ **d** $y = -x - 5$

**4B** **6** Identify the independent and dependent variables in the following relationships.

**a**

| **Accuracy (%)** | 75 | 80 | 90 | 98 |
|---|---|---|---|---|
| **Number of attempts** | 3 | 8 | 12 | 15 |

**b**

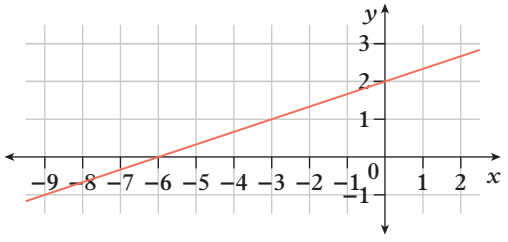



**c** the number of victories and the number of chess matches played.

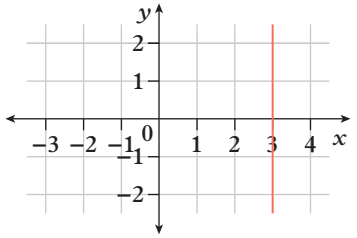
**4C** **7** For each linear graph shown, determine the:

**i** gradient **ii** $x$-intercept **iii** $y$-intercept

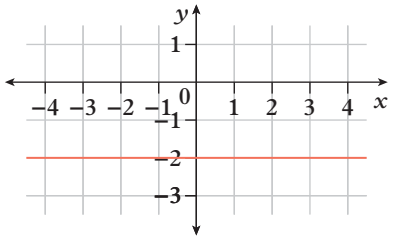
**a**



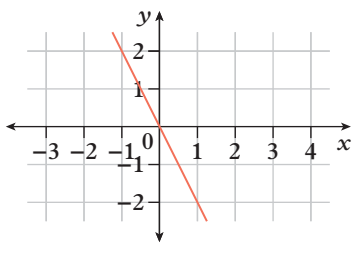

**b**




**c**




**d**



4C **8** Find the gradient of the line segment joining each pair of points.

**a** (2, 3) and (−2, −3) **b** (−3, −2) and (−2, −3)

4D **9** For of the graph of each linear relationship below, determine the coordinates of the:

**i** $x$-intercept **ii** $y$-intercept

**a** $2x + 3y = 18$ **b** $3x - y = 6$ **c** $y = 4x - 2$

**d** $2y = 5x - 3$ **e** $x - 2y = 4$ **f** $-y = 4 - x$

4D **10** Use your answers from question **9** to sketch each linear graph.

4D **11** Use the most appropriate method to sketch the graph of each linear relationship.

**a** $3x + 2y = 4$ **b** $4 - 2x = 3y$ **c** $y = \frac{1}{3}x$ **d** $y = 5 - \frac{2}{3}x$

**e** $y = 5$ **f** $2x + 3y + 6 = 0$ **g** $y = \frac{3}{4}x - 2$ **h** $x = -7$

4E **12** Write the equations of the linear graphs with the following properties.

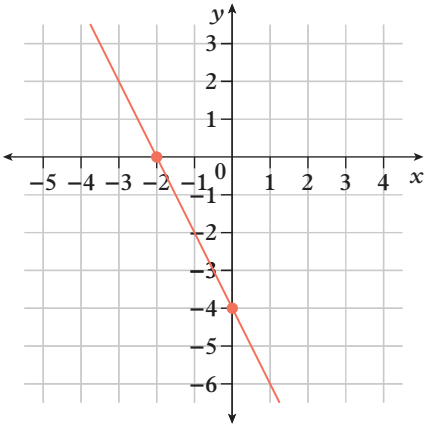
**a** gradient: 4, $y$-intercept: (0, −2)

**b** gradient: $\frac{1}{4}$, $y$-intercept: (0, 0)

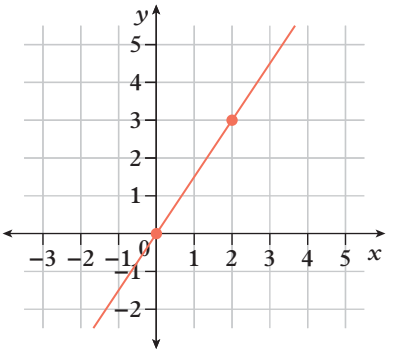
**c** gradient: 0, $y$-intercept: $\left(0, -\frac{1}{2}\right)$

4E **13** Find the equation of each of the following linear graphs.

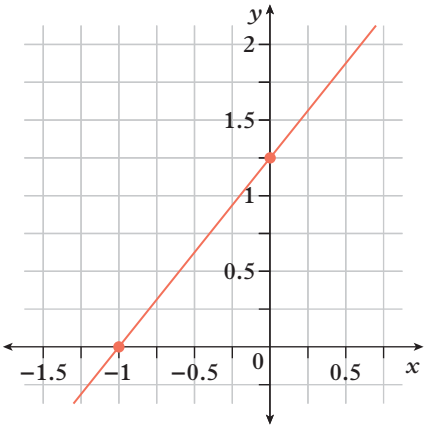
**a**



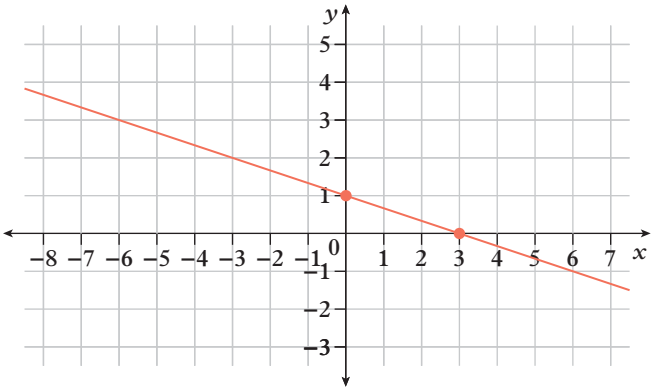

**b**




**c**




**d**




**14** Determine the equations of the linear graphs that pass through each pair of points.

**a** (−5,2) and (1,−4) **b** (100,39) and (−47,39)

**c** (−3,4) and (6,−8) **d** (7,12) and (−3,4)

4F **15** For the line segment that joins each of the following pairs of points, find the:

**i** midpoint **ii** length (correct to one decimal place).

**a** (2, 3) and (8, 7) **b** (−4, 6) and (−3, 5)

**c** (2, −4) and (−2, 4) **d** (−3, −1) and (2, −9)

4G **16** Determine whether $x$ is directly proportional to $y$ in each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

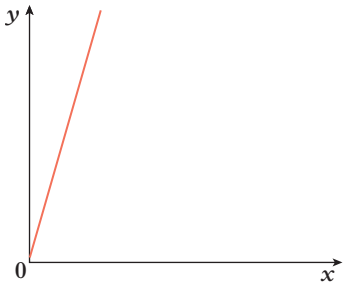
**a**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 1 | 4 | 9 | 16 |

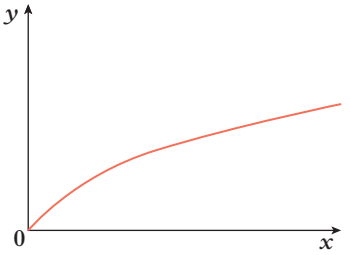
**b**

| $x$ | 0 | 3 | 6 | 9 | 12 |
|---|---|---|---|---|---|
| $y$ | 0 | −1.5 | −3 | −4.5 | −6 |

**c**




**d**




4G **17** Water flows into an empty tank at a constant rate. After 4 hours, the tank contains 2000 litres of water.

**a** **i** Sketch a graph of the volume of water in the tank, $V$, against time, $t$ from $t = 0$ to 4 hours.

**ii** Determine the gradient of the graph.

**b** Determine the flow rate of the water.

**c** Determine the equation for the relationship between $V$ and $t$.

**d** If the tank has a total capacity of 10 000 litres, how long will it take to fill the tank to full capacity?

## Analysis

**1** The cross-section of a building is drawn on a Cartesian plane with the scale on the axes showing length in metres. The $x$-axis represents ground level.

**a** On the same Cartesian plane, sketch the graph of:

**i** $4y - x = 12$ **ii** $y = 5 - \frac{1}{4}x$

**b** To represent the cross-section of the building, shade the area between the graph of $4y - x = 12$ and the $x$-axis from $x = 0$ to $x = 4$, as well as the area between the graph of $y = 5 - \frac{1}{4}x$ and the $x$-axis between $x = 4$ and $x = 8$.

**c** How tall is the building at its highest point?

**d** What is the distance from the top of the roof to the lower edge of the roof, correct to one decimal place?

**e** What is the positive gradient of the roof?

**f** If a chimney is to be placed halfway along the slope of the roof on the side with the positive gradient, describe its position on the Cartesian plane.

**2** A rectangle $DEFG$ has vertices at $D(-2, -1)$, $E(0, 1)$, $F(3, -2)$ and $G(1, -4)$.

**a** Draw the rectangle on the Cartesian plane.

**b** Calculate the lengths of all the sides of the rectangle, correct to one decimal place.

**c** Using your sketch, identify the $y$-intercepts of the line segments:

**i** $\overline{DE}$ **ii** $\overline{EF}$ **iii** $\overline{DG}$ **iv** $\overline{FG}$

**d** Find the gradients of the lines through:

**i** $\overline{DE}$ **ii** $\overline{EF}$ **iii** $\overline{DG}$ **iv** $\overline{FG}$

**e** If point $P$ is the midpoint of $\overline{DE}$, point $Q$ is the midpoint of $\overline{EF}$, $R$ is the midpoint of $\overline{FG}$ and S is the midpoint of $\overline{DG}$. Find the coordinates of:

**i** $P$ **ii** $Q$ **iii** $R$ **iv** $S$

**f** Describe the shape of the figure $PQRS$. Justify the statements you make.

**g** If the original figure $DEFG$ had been a square instead of a rectangle, explain how this would affect the shape of $PQRS$. Support your answer with mathematical evidence.

# 5

# Non-linear relationships

## Index


**5A** Solving quadratic equations
**5B** Plotting quadratic relationships
**5C** Sketching parabolas using intercepts
**5D** Sketching parabolas using transformations
**5E** Circles and other non-linear relationships


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Solving equations using inverse operations
- ✔ Factorising quadratic expressions
- ✔ Factorising the difference of two squares
- ✔ The Cartesian plane
- ✔ Plotting graphs

## Curriculum links

- Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (VCMNA311)

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## Materials

- ✔ Calculator
- ✔ Graph paper

# 5A Solving quadratic equations

## Learning intentions

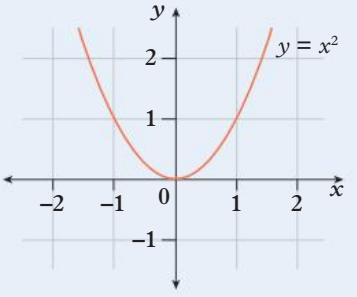
✔ I can solve simple quadratic equations.

**Inter-year links**

| | |
|---|---|
| Year 7 | 6C Terms, expressions and equations |
| Year 9 | 6A Equations |
| Year 10 | 5A Solving quadratic relationships |

## Quadratic equations

- A **quadratic equation** is a single-variable equation in which the highest index is 2.
- The general form of a quadratic equation is: $ax^2 + bx + c = 0$ where $a$, $b$ and $c$ are constants.
- Similarly, the general form of a quadratic relationship is: $y = ax^2 + bx + c$
- A quadratic relationship produces a parabola-shaped graph, as shown on the right.




## The Null Factor Law

- The **Null Factor Law** states that if the product of two factors is 0, then one or both of the factors must be 0.
  If $a \times b = 0$, then $a = 0$ or $b = 0$.
- Applying the Null Factor Law to quadratic equations in **factor form**, $(x - p)(x - q) = 0$:
  If $(x - p)(x - q) = 0$, then $x - p = 0$ or $x - q = 0$.

## Solving quadratic equations

- To solve a quadratic equation in general form:
  1 Factorise the quadratic expression.
  2 Apply the Null Factor Law by equating each factor to 0.
  3 Solve each linear equation.

  For example, $x^2 + 2x - 3 = 0$
  $(x + 3)(x - 1) = 0$
  $x + 3 = 0$ or $x - 1 = 0$
  $x = -3$ or $x = 1$
- Quadratic equations can have 0, 1 or 2 solutions.

| Type of quadratic equation | Example | Number of solutions |
|---|---|---|
| $ax^2 + bx + c = 0$ cannot be factorised | $x^2 + x + 1 = 0$ | 0 |
| Factorised form is a perfect square: $(x + a)^2 = 0$ | $x^2 + 2x + 1 = 0$<br>$(x + 1)^2 = 0$<br>$x + 1 = 0$<br>$x = -1$ | 1 |
| Factorised form is $(x + a)(x + b) = 0$, where $a \neq b$ | $x^2 + 3x + 2 = 0$<br>$(x + 1)(x + 2) = 0$<br>$x + 1 = 0$ or $x + 2 = 0$<br>$x = -1$ or $x = -2$ | 2 |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## Example 5A.1 Solving factorised quadratic equations

Solve each quadratic equation.

**a** $(x - 6)(x + 2) = 0$ **b** $x(x - 4) = 0$ **c** $(2x + 1)(4x - 3) = 0$

**THINK**

1 Check that the quadratic expression is in factor form and is equal to 0.

2 Apply the Null Factor Law by equating each factor to 0.

3 Solve each linear equation.

**WRITE**

**a** $(x - 6)(x + 2) = 0$
$x - 6 = 0$ or $x + 2 = 0$
$x = 6$ or $x = -2$

**b** $x(x - 4) = 0$
$x = 0$ or $x - 4 = 0$
$x = 0$ or $x = 4$

**c** $(2x + 1)(4x - 3) = 0$
$2x + 1 = 0$ or $4x - 3 = 0$
$2x = -1$ or $4x = 3$
$x = -\frac{1}{2}$ or $x = \frac{3}{4}$

## Example 5A.2 Factorising and solving quadratic equations

Solve:

**a** $x^2 - 3x - 10 = 0$ **b** $x^2 - 9 = 0$

**THINK**

1 Factorise the quadratic expression.

2 Apply the Null Factor Law by equating each factor to 0.

3 Solve each linear equation.

**WRITE**

**a** $x^2 - 3x - 10 = 0$
$(x + 2)(x - 5) = 0$
$x + 2 = 0$ or $x - 5 = 0$
$x = -2$ or $x = 5$

**b** Using the difference of two squares rule:
$x^2 - 9 = 0$
$(x + 3)(x - 3) = 0$
$x + 3 = 0$ or $x - 3 = 0$
$x = -3$ or $x = 3$

## Example 5A.3 Rearranging quadratic equations before solving

Solve $x^2 + 12 = 8x$

**THINK**

1 Subtract the expression on the RHS of the equation from both sides, so that the RHS of the equation is equal to 0.

2 Factorise the quadratic expression.

3 Apply the Null Factor Law by equating each factor to 0.

4 Solve each linear equation.

**WRITE**

$x^2 + 12 = 8x$
$x^2 - 8x + 12 = 0$ $(-8x)$

$(x - 2)\ (x - 6) = 0.$
$x - 2 = 0$ or $x - 6 = 0.$

$x = 2$ or $x = 6.$

## **Example 5A.4** Solving quadratic equations with common factors

Solve:

**a** $2x^2 + 22x + 36 = 0$ **b** $-x^2 + 10x = 25$

| THINK | WRITE |
|---|---|
| **a 1** Take the highest common factor (HCF) out of the quadratic expression. | **a** $2x^2 + 22x + 36 = 0$<br>$2(x^2 + 11x + 18) = 0$ |
| **2** Divide both sides by the HCF. Note that you can never divide both sides by the variable, $x$, in case $x = 0$. | $\frac{2(x^2 + 11x + 18)}{2} = \frac{0}{2}$ |
| **3** Factorise the quadratic expression. | $x^2 + 11x + 18 = 0$<br>$(x + 2)(x + 9) = 0$ |
| **4** Apply the Null Factor Law by equating the factors to 0. | $x + 2 = 0$ or $x + 9 = 0$ |
| **5** Solve each linear equation. | $x = -2$ or $x = -9$ |
| **b 1** Rearrange so that the right-hand side is 0. | **b** $-x^2 + 10x = 25$<br>$-x^2 + 10x - 25 = 0$ |
| **2** Take out the HCF. Remember, HCF can be $-1$. | $-1(x^2 - 10x + 25) = 0$ |
| **3** Factorise. | $\frac{-1(x^2 - 10x + 25)}{-1} = \frac{0}{-1}$ |
| **4** Apply the Null Factor Law. Remember, there is one solution when the factorised form is a perfect square. | $x^2 - 10x + 25 = 0$<br>$(x - 5)^2 = 0$ |
| **5** Solve the linear equation. | $x - 5 = 0$<br>$x = 5$ |

### Helpful hints

- ✔ Watch for changes in sign when solving quadratic equations for $x$.

$$(x + 6)(x - 2) = 0$$
$$x + 6 = 0 \text{ or } x - 2 = 0$$
$$x = -6 \text{ or } x = +2$$

- ✔ Although it is possible to add and subtract $x$, you **must not** divide both sides of an equation by $x$ in case $x = 0$, since dividing anything by 0 is **undefined**.
  For example, $x^2 = 5x$ should be solved by first subtracting $5x$ from both sides and then factorising. If, instead, you divided both sides by $x$, then you would only find the solution $x = 5$ and miss the solution $x = 0$.
- ✔ Always check the coefficients for a highest common factor (HCF). You can factor out the HCF from any quadratic equation, $ax^2 + bx + c = 0$, by dividing on both sides. This is because $\frac{0}{\text{HCF}} = 0$ for any HCF.

$$3x^2 - 18x + 27 = 0$$
$$3(x^2 - 6x + 9) = 0$$
$$\frac{3(x^2 - 6x + 9)}{3} = \frac{0}{3}$$
$$x^2 - 6x + 9 = 0$$

ANS p508

# Exercise 5A Solving quadratic equations

1, 2(a, c, e, g), 3–11, 13, 16 1, 2(f, g, h), 4–10, 12, 13, 15, 18 1, 5–7, 9, 10, 12, 15–18

**1** Which of the following are quadratic equations?

**a** $x^2 - 2 = 0$ **b** $3(x + 1) = 0$ **c** $x^2 + x = 5$ **d** $x^2 + 5x + 6$

**e** $2x^2 + x - 4 = 0$ **f** $x^3 + 8 = 0$ **g** $6x + 1 = 2x - 5$ **h** $x^2 + 7x = x - 3$

**2** Use substitution to check whether the value in brackets is a solution to the given quadratic equation.

**a** $(x - 4)(x - 5) = 0$ $(x = 5)$ **b** $(x + 2)(x - 8) = 0$ $(x = 2)$

**c** $x(x - 6) = 0$ $(x = 3)$ **d** $x^2 + 8x + 7 = 0$ $(x = -1)$

**e** $x^2 - 4x + 4 = 0$ $(x = -2)$ **f** $x^2 - 49 = 0$ $(x = 7)$

**g** $x^2 - 2x - 15 = 0$ $(x = -3)$ **h** $x^2 - 8x + 12 = 0$ $(x = -4)$

**3** Solve each of the following linear equations.

**a** $x + 3 = 0$ **b** $x - 6 = 0$ **c** $4x - 8 = 0$ **d** $3x + 1 = 0$

5A.1 **4** Solve each of the following quadratic equations.

**a** $(x + 2)(x - 3) = 0$ **b** $(x - 7)(x - 1) = 0$ **c** $(x + 4)(x - 4) = 0$

**d** $x(x - 6) = 0$ **e** $(x + 5)(x + 1) = 0$ **f** $x(x + 2) = 0$

**g** $(x - 8)(x + 8) = 0$ **h** $(2x + 2)(x - 7) = 0$ **i** $x(x - 11) = 0$

**j** $(2x + 3)(x - 5) = 0$ **k** $(4x - 2)(4x - 2) = 0$ **l** $(3x + 5)(2x + 3) = 0$

5A.2 **5** Factorise and then solve each quadratic equation.

**a** $x^2 + 7x + 10 = 0$ **b** $x^2 - 3x + 2 = 0$ **c** $x^2 + 5x = 0$

**d** $x^2 - 3x = 0$ **e** $x^2 - 36 = 0$ **f** $x^2 + 10x + 21 = 0$

5A.3 **6** Rearrange, factorise and then solve each quadratic equation.

**a** $x^2 - 2x = 8$ **b** $x^2 = 1$ **c** $x^2 = -8x$

**d** $x^2 + 3 = 4x$ **e** $6x = -x^2 - 9$ **f** $1 = 2x - x^2$

**7** Use substitution to check that your solutions for questions **5** and **6** are correct.

**8** Solve each quadratic equation by first dividing both sides by the highest common factor (HCF).

**a** $2(x + 8)(x - 2) = 0$ **b** $-3(x - 1)(x - 4) = 0$

**c** $-7(x + 6)(x - 6) = 0$ **d** $-5x(x + 9) = 0$

5A.4 **9** Solve each quadratic equation by first dividing both sides by the highest common factor (HCF).

**a** $3x^2 + 9x + 6 = 0$ **b** $2x^2 + 8x = 0$ **c** $2x^2 - 4x - 16 = 0$

**d** $3x^2 - 12x + 12 = 0$ **e** $-5x^2 - 5x + 10 = 0$ **f** $-4x^2 + 8x = 0$

**g** $-x^2 - 10x - 21 = 0$ **h** $-3x^2 - 24x - 48 = 0$ **i** $-2x^2 + 32 = 0$

**10** Solve each equation.

**a** $x^2 + 14x + 48 = 0$ **b** $3x^2 - 12x = 0$ **c** $x - x^2 = 0$

**d** $x^2 - 64 = 0$ **e** $-x^2 - 2x + 3 = 0$ **f** $-x^2 + 8x - 16 = 0$

**g** $-x^2 + 5x = 6$ **h** $2x^2 + 50 = -20x$ **i** $20 = 5x^2$

**11 a** Determine how many solutions each of the following quadratic equations has.

**i** $(x - 4)(x - 7) = 0$ **ii** $(x - 4)(x - 4) = 0$ **iii** $x^2 + 4 = 0$

**b** Identify and describe the feature(s) of each equation in part **a** that result in the number of solutions that the equation has.

PROBLEM SOLVING AND REASONING

**12** State how many solutions each equation has.

**a** $x^2 + 3x - 10 = 0$ **b** $x^2 - 4 = 0$ **c** $x^2 - 6x + 9 = 0$ **d** $x^2 + 1 = 0$

**e** $x^2 + 7x = 0$ **f** $x^2 + 12x + 32 = 0$ **g** $x^2 - x - 72 = 0$ **h** $x^2 + 2x + 5 = 0$

**13** The length of a rectangular mouse pad is 8 cm longer than its width.

**a** Write an expression for the area of the mouse pad.

**b** Expand the expression.

**c** The area of the mouse pad is estimated to be 560 cm$^2$. Write an equation for the area of the mouse pad.

**d** Factorise and solve the equation. Which value of $x$ is a feasible solution in this scenario? Explain.

**e** State the dimensions of the mouse pad.

**14** The area of a rectangular sand pit is 35 m$^2$. The length is 2 m longer than the width.

**a** Write a quadratic equation to represent this scenario.

**b** Solve the quadratic equation.

**c** State the dimensions of the sand pit.

**15** The width of a laptop screen is 12 cm less than its length. If the area of the screen is 640 cm$^2$, use algebra to determine the dimensions of the screen.

**16** Alec throws a tennis ball back on to the court from the spectator stand. The height of the ball above the surface of the tennis court can be represented by the quadratic relationship $h = -4(t + 1)(t - 2)$, where $h$ is the height in metres after $t$ seconds in the air.

**a** What is the height of the ball after:

**i** 1 second? **ii** 2 seconds?

**b** What is the height of the ball when Alec releases it from his hand?

**c** How long does it take for the ball to hit the tennis court after it is thrown?

**d** Explain why there is only one time value for your answer to part **c** even though you have solved a quadratic equation that has two solutions.

CHALLENGE

**17** The height above the ground, $h$, of a firework rocket $x$ seconds after it is launched is given by the equation $h = -5x^2 + 40x$.

**a** When is the rocket at a height of 60 m above the ground?

**b** Why are there two solutions? Explain the significance of each solution.

**18** An equation containing a pronumeral that is taken to the power of three (and no higher) is called a cubic equation. Use the Null Factor Law to solve the following cubic equations.

**a** $(x - 1)(x - 3)(x - 5) = 0$

**b** $x(x + 6)(x - 6) = 0$

**c** $(x + 11)(x - 4)^2 = 0$

**d** $(2x - 4)(3x + 1)(4x - 6) = 0$

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Solving quadratic equations

CAS instructions
Solving equations

Topic quiz
5A

# 5B Plotting quadratic relationships

## Learning intentions

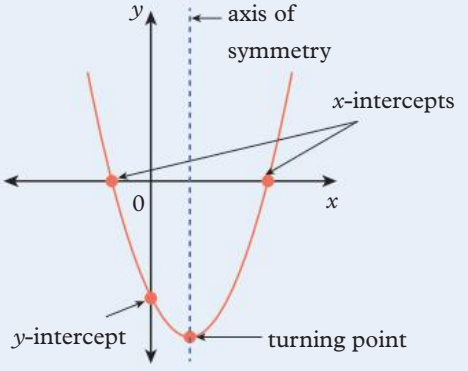
- ✔ I can identify the key features of a quadratic graph.
- ✔ I can plot quadratic relationships from tables of values and equations.

**Inter-year links**

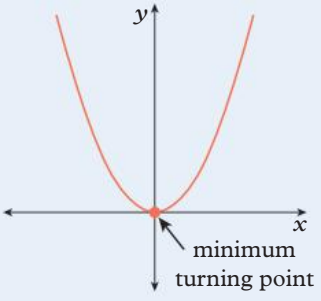
| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 5D The Cartesian plane |
| Year 8 | 6D Plotting linear and non-linear relationships |

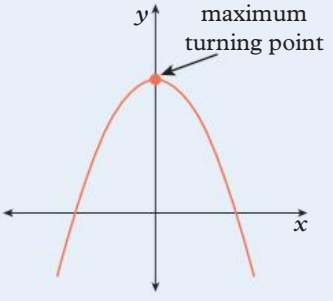
## Quadratic graphs

- The graph of a quadratic relationship is called a **parabola**.
- The key features of a parabola include:
  - → $x$- and $y$-intercepts
    - – all parabolas have one $y$-intercept
    - – parabolas can have no, one or two $x$-intercept(s)
  - → a **turning point** or vertex
  - → an **axis of symmetry**, $x = a$, where $a$ is the $x$-coordinate of the turning point.
- If there are two $x$-intercepts, then the axis of symmetry will be at the midpoint between them.



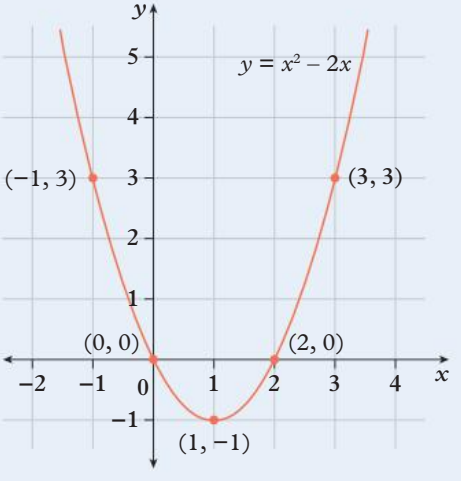

- The turning point can be a **minimum turning point** or **maximum turning point**.




## Plotting quadratic graphs

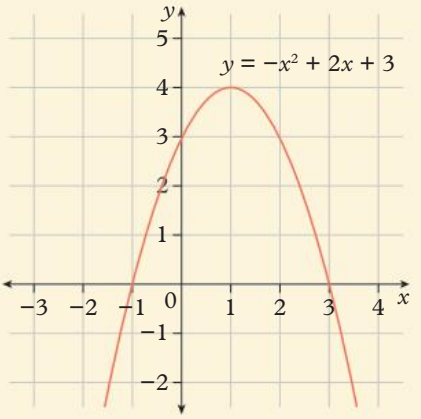
- To plot the graph of a quadratic equation, create a table of coordinate points and join the points on a Cartesian plane. In section 5C you will **sketch** parabolas using the key features of the graphs.
  1. Construct a table of $x$- and $y$-values by selecting values for $x$, then substituting each value of $x$ into the equation to find the corresponding value of $y$.
  2. Write the coordinate points in the table.
  3. Plot the coordinate points on a Cartesian plane.
  4. Join the points using a smooth curve to form a parabola and label the graph with its equation.



## Example 5B.1 Identifying the features of a quadratic graph

For the quadratic graph shown, identify:

**a** the coordinates of the $x$-intercepts
**b** the coordinates of the $y$-intercept
**c** the coordinates of the turning point
**d** whether the turning point is a maximum or minimum
**e** the equation of the axis of symmetry.




**THINK**

**a** State the coordinates of the points where the graph crosses the $x$-axis.
**b** State the coordinates of the point where the graph crosses the $y$-axis.
**c** State the coordinates of the point where the graph changes direction.
**d** The turning point is the highest point on the parabola, so it is a maximum turning point.
**e** State the equation of the axis of symmetry.

**WRITE**

**a** $x$-intercepts: $(-1,0)$ and $(3,0)$
**b** $y$-intercept: $(0,3)$
**c** turning point: $(1,4)$
**d** maximum turning point
**e** axis of symmetry: $x = 1$

## Example 5B.2 Graphing quadratic relationships from an equation

Sketch a graph of $y = x^2 - 2x - 1$ by first completing a table of values for $x$ from $-1$ to 3.

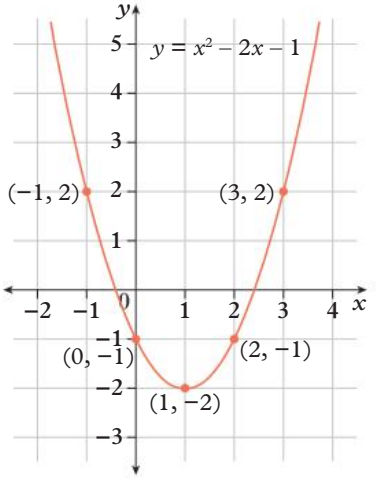
**THINK**

1 Construct a table of values for $x$ from $-1$ to 3. Substitute each value of $x$ into the equation to find the corresponding value of $y$.
2 List the coordinate points in a table.
3 Plot the points on a Cartesian plane.
4 Join the points with a smooth curve. Label the graph with its equation.

**WRITE**

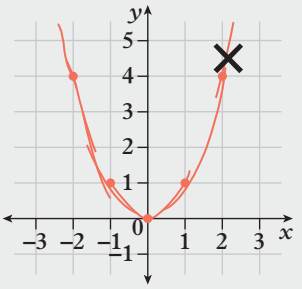
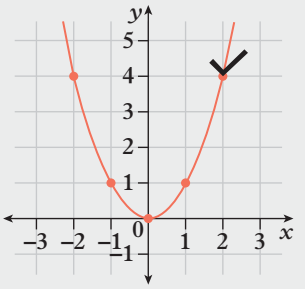
| $x$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | 2 | $-1$ | $-2$ | $-1$ | 2 |

$(-1, 2)$, $(0, -1)$, $(1, -2)$, $(2, -1)$, $(3, 2)$

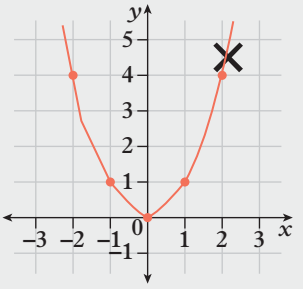
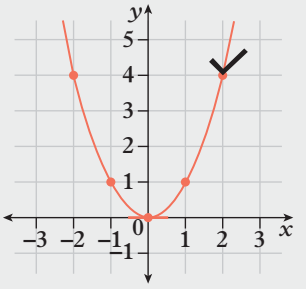


**Helpful hints**

✔ Rather than drawing 'hairy' parabolas, try starting at the turning point and then drawing two smooth, continuous and **symmetrical** curves through the other plotted points.

✔ Draw a short horizontal dash where the turning point is to ensure the parabola does not end up with a pointy turning point.

✔ Take care when reading a question. If you are asked to find the $x$- and $y$-intercepts, then you must give your answer as coordinates. So if the coordinates of the intercepts are $(5,0)$ and $(0,-1)$, **do not** write $x = 5$ and $y = -1$.

ANS p509

# Exercise 5B Plotting quadratic relationships

△ 1–4, 6, 8, 9, 12 ▭ 1(b, d, f, h), 2, 3, 5, 7, 8, 10, 13 ⬠ 1(d, f, h), 2, 3, 7, 8, 10, 11, 13, 14

5B.1 **1** For each quadratic graph, identify:

**i** the coordinates of any $x$-intercepts

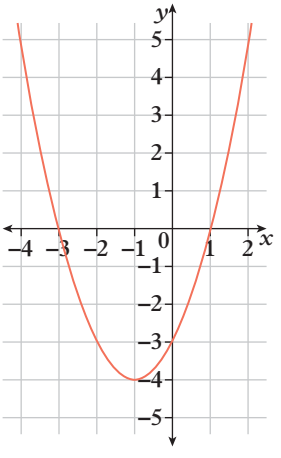
**ii** the coordinates of the $y$-intercept

**iii** the coordinates of the turning point

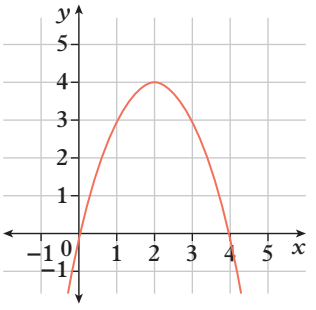
**iv** whether the turning point is a maximum or minimum

**v** the equation of the axis of symmetry.

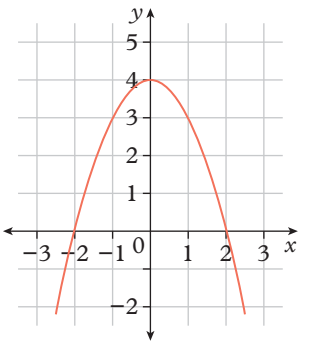
**a**



**b**

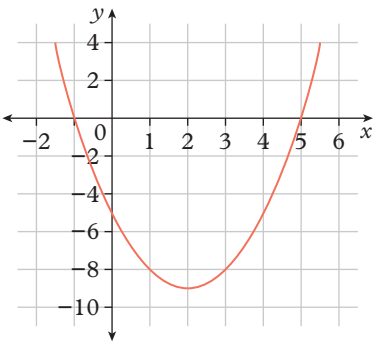


**c**

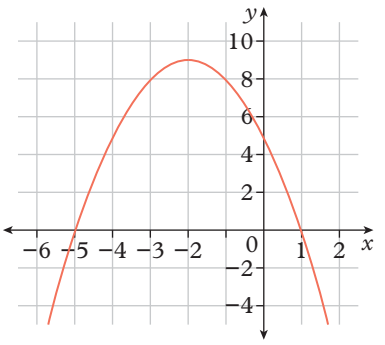


UNDERSTANDING AND FLUENCY

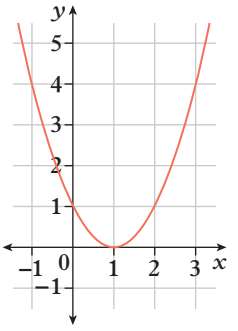
**d**



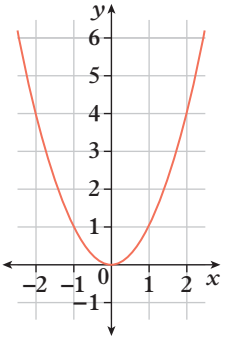

**e**



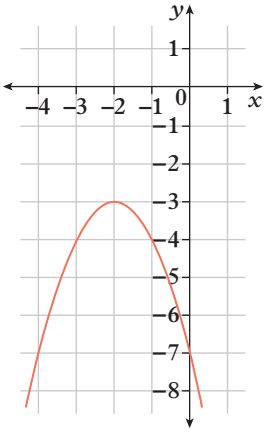

**f**



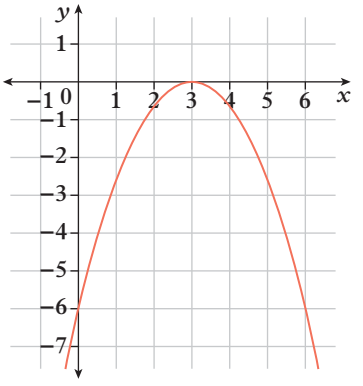

**g**




**h**




**i**




**2** For each quadratic relationship:

**i** complete the table of values

**ii** plot the graph

**iii** identify the axis intercepts and the turning point, and then state the equation of the axis of symmetry

**iv** state whether the turning point is a maximum or minimum.

**a** $y = x^2 + 2x - 8$

| $x$ | −5 | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | |

**b** $y = 9 - x^2$

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | |

5B.2 **3** For each quadratic relationship:

**i** Plot its graph by first completing a table of coordinate points, using the $x$-values given in brackets.

**ii** Identify the coordinates of the $x$- and $y$-intercepts, and the turning point, and state the equation of the axis of symmetry.

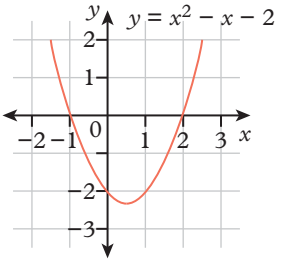
**a** $y = -x^2 - 6x - 5$ $(-6 \text{ to } 0)$ **b** $y = x^2 - 4$ $(-3 \text{ to } 3)$ **c** $y = x^2 - 2x - 15$ $(-4 \text{ to } 6)$

**d** $y = x^2 + 4x$ $(-5 \text{ to } 1)$ **e** $y = -x^2 + 2x$ $(-1 \text{ to } 3)$ **f** $y = x^2 - 6x + 9$ $(0 \text{ to } 6)$

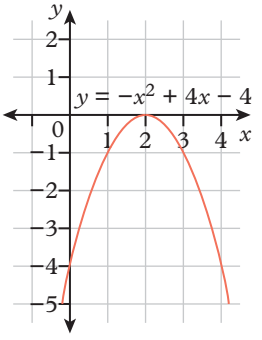


**4 a** How many $x$-intercepts do each of the following parabolas have?

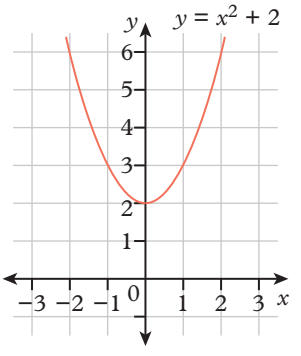
**i**




**ii**




**iii**




**b** Can a parabola have more than two $x$-intercepts? Explain.

**c** How many $y$-intercepts does each parabola in part **a** have?

**d** Describe the link between the $y$-intercept of a parabola and the general equation, $y = ax^2 + bx + c$.

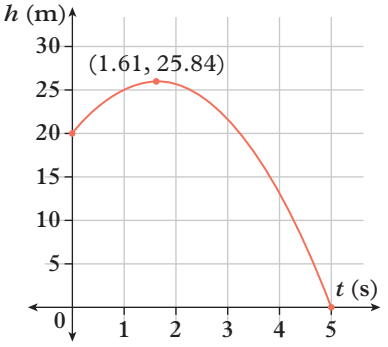
**5** For each of the following parabolas state the number of:

**i** $x$-intercepts **ii** $y$-intercepts

**a** parabola with a minimum turning point at $(-3, 1)$

**b** parabola with a maximum turning point at $(10, 0)$

**6** The graph below shows the height, $h$ metres, of a firework rocket $t$ seconds after it is launched.




**a** From what height is the rocket launched?

**b** What is the maximum height of the rocket to the nearest metre?

**c** For how long is the rocket in the air?

**7** Alani's hair clip falls to the ground while she is on a roller coaster. The position of the clip as it falls can be described by the relationship $h = 100 - 4t^2$, where $h$ is the height of the clip above the ground in metres after $t$ seconds.

**a** Plot the graph of the relationship for $t$-values from 0 to 5.

**b** Why didn't you draw the parabola for values less than 0 or greater than 5?

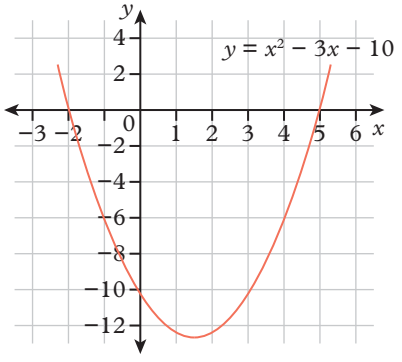
**c** What is the height of the clip above the ground after:

**i** 2 s? **ii** 3 s?

**d** From what height above the ground did the hair clip start to fall?

**e** How long did it take for the hair clip to hit the ground?

**8** Consider the graph of $y = x^2 - 3x - 10$ shown below.




**a** Identify the $x$-coordinates of the $x$-intercepts from the graph.

**b** Solve the quadratic equation $x^2 - 3x - 10 = 0$ by first factorising the quadratic expression, and then applying the Null Factor Law.

**c** Compare your answers for parts **a** and **b**. What do you notice?

**d** Explain how you can use the graph of $y = x^2 - 3x - 10$ to solve $x^2 - 3x - 10 = 0$.

**9** Determine the equation for the axis of symmetry of a parabola that has:

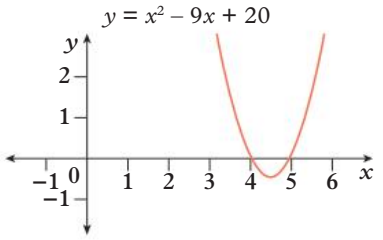
**a** $x$-intercepts at $(2, 0)$ and $(8, 0)$

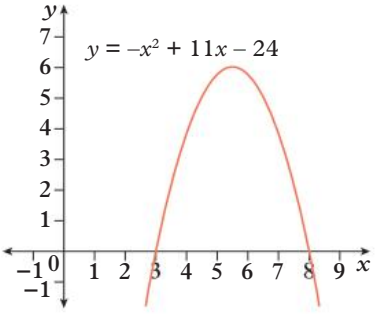
**b** $x$-intercepts at $(-3, 0)$ and $(3, 0)$

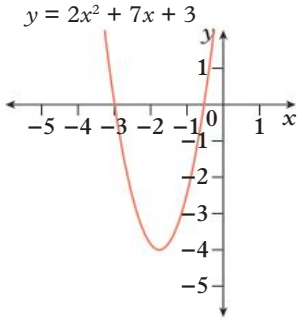
**c** one $x$-intercept at $(-4, 0)$ (Hint: Draw a rough sketch of the parabola and consider the midpoint.)

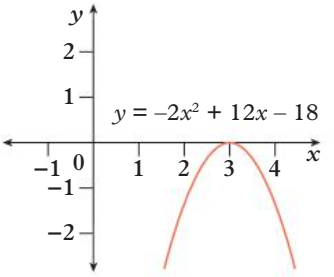
PROBLEM SOLVING AND REASONING

**10** Use your observations from question **8** to solve each quadratic equation using the following graphs.




**a** $x^2 - 9x + 20 = 0$ **b** $-x^2 + 11x - 24 = 0$ **c** $2x^2 + 7x + 3 = 0$ **d** $-2x^2 + 12x - 18 = 0$

**11** The $x$-intercepts of a quadratic graph are $(-3, 0)$ and $(5, 0)$. If the graph has a maximum turning point and the maximum value of $y$ is 10:

**a** find the equation of the axis of symmetry **b** determine the coordinates of the turning point.

**12** The axis of symmetry of a quadratic graph is $x = -4$. If the graph passes through the $x$-axis at $(-8, 0)$, determine the coordinates of the second $x$-intercept.

CHALLENGE

**13** The equation for a graph with $x$-intercepts at $(-7,0)$ and $(5,0)$ is written as $y = (x - p)(x - q)$, where $p$ and $q$ are constants.

**a** Determine the $x$-intercepts of $y = (x - p)(x - q)$ in terms of $p$ and $q$.

**b** Hence identify appropriate values for $p$ and $q$ and write the equation for the graph.

**c** Determine the minimum value of $y$.

**d** Determine the equation for a graph with $x$-intercepts at $(-7,0)$ and $(5,0)$ and a maximum value of 36.

**14** Kim throws a javelin. The position of the tip of the javelin can be represented by the quadratic relationship $y = -0.05x^2 + 1.5x + 1.55$, where $y$ is the height above the ground and $x$ is the horizontal distance from where the javelin was thrown. Both $x$ and $y$ are in metres.

**a** Plot the graph of this relationship. Use 0, 5, 10, 15, …, 30 as the $x$-values in the table.

**b** At what height off the ground was the javelin thrown?

**c** Let $y = 0$ and solve the quadratic equation for $x$. (Hint: Start by multiplying both sides of the equation by $-20$.)

**d** What horizontal distance did the javelin travel before hitting the ground?

**e** What was the maximum height reached by the javelin?

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Plotting quadratic relationships
- Investigation: Quadratics and types of numbers
- CAS instructions: Graphing functions
- Topic quiz: 5B

# 5C Sketching parabolas using intercepts

## Learning intentions

- ✔ I can find the $x$- and $y$-intercepts of a quadratic graph from its equation.
- ✔ I can find the coordinates of the turning point of a quadratic graph with two $x$-intercepts.
- ✔ I can sketch quadratic graphs with two $x$-intercepts using the $x$- and $y$-intercepts and the turning point.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 6E Substitution |
| Year 8 | 5B Substitution |
| Year 10 | 5C Sketching parabolas using intercepts |

## Sketching quadratic graphs

- Before sketching quadratic graphs from equations of the form $y = ax^2 + bx + c$, determine the coordinates of any $x$-intercepts, the $y$-intercept and the turning point.
- To find the $x$-intercepts from an equation of the form $y = ax^2 + bx + c$:

**1** Substitute $y = 0$ into the quadratic equation.

**2** Factorise the quadratic expression.

**3** Solve the equation by applying the Null Factor Law. (Note: A parabola can have 0, 1, or 2 $x$-intercepts)

$y = x^2 + 2x - 3$
Let $y = 0$:
$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
$x + 3 = 0$ or $x - 1 = 0$
$x = -3$ or $x = 1$
$x$-intercepts: $(-3, 0)$ and $(1, 0)$

- To find the $y$-intercept, substitute $x = 0$ into the quadratic equation.

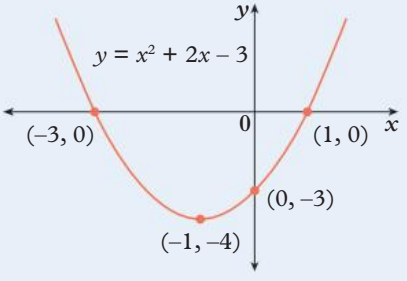
$y = x^2 + 2x - 3$
Let $x = 0$:
$y = (0)^2 + 2(0) - 3$
$y = -3$
$y$-intercept: $(0, -3)$

- The turning point is located on the axis of symmetry, which is halfway between the $x$-intercepts. So to find the coordinates of the turning point for a parabola with two $x$-intercepts:

**1** Calculate the $x$-coordinate of the turning point by finding the midpoint of the $x$-intercepts.

**2** Calculate the $y$-coordinate of the turning point by substituting the $x$-coordinate into the quadratic equation.

$$x = \frac{-3 + 1}{2}$$
$$= \frac{-2}{2}$$
$$= -1$$
$y = x^2 + 2x - 3$
Let $x = -1$:
$y = (-1)^2 + 2(-1) - 3$
$= 1 - 2 - 3$
$= -4$
Turning point: $(-1, -4)$



## Example 5C.1 Finding the coordinates of the *x*- and *y*-intercepts

For each quadratic relationship, find the coordinates of the:

**i** *x*-intercepts **ii** *y*-intercept

**a** $y = x^2 - 6x$ **b** $y = x^2 + 7x + 10$

**THINK**

**i** **1** To find the *x*-coordinate of the *x*-intercept, substitute $y = 0$ into the quadratic equation.

**2** Factorise the quadratic expression.

**3** Apply the Null Factor Law to solve the equation.

**ii** To find the *y*-coordinate of the *y*-intercept, substitute $x = 0$ into the quadratic equation and solve for *y*.

**WRITE**

**a** **i** $y = x^2 - 6x$

For *x*-intercepts, let $y = 0$:

$x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0$ or $x - 6 = 0$

$x = 0$ or $x = 6$

*x*-intercepts:

(0, 0) and (6, 0)

**ii** $y = x^2 - 6x$

For *y*-intercept, let $x = 0$:

$y = (0)^2 - 6(0)$

$= 0$

*y*-intercept: (0, 0)

**b** **i** $y = x^2 + 7x + 10$

For *x*-intercepts, let $y = 0$:

$x^2 + 7x + 10 = 0$

$(x + 2)(x + 5) = 0$

$x + 2 = 0$ or $x + 5 = 0$

$x = -2$ or $x = -5$

*x*-intercepts:

(−2, 0) and (−5, 0)

**i** $y = x^2 + 7x + 10$

For *y*-intercept, let $x = 0$:

$y = (0)^2 + 7(0) + 10$

$= 10$

*y*-intercept: (0, 10)

## Example 5C.2 Finding the coordinates of the turning point using *x*-intercepts

Find the coordinates of the turning point of $y = (x + 3)(x - 9)$.

**THINK**

**1** Identify the coordinates for the *x*-intercepts.

**2** Calculate the *x*-coordinate of the turning point by finding the midpoint of the *x*-intercepts.

**3** Calculate the *y*-coordinate of the turning point by substituting the *x*-coordinate into the quadratic equation.

**WRITE**

*x*-intercepts: (−3,0) and (9,0)

$x = \frac{-3 + 9}{2}$

$= \frac{6}{2}$

$= 3$

Let $x = 3$:

$y = (x + 3)(x - 9)$

$= (3 + 3)(3 - 9)$

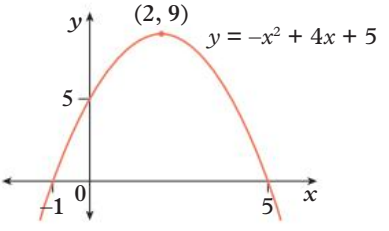
$= 6 \times -6$

$= -36$

Turning point: (3, −36)

## **Example 5C.3** Sketching parabolas using the intercepts

Sketch the graph of $y = -x^2 + 4x - 5$ by first finding the $x$- and $y$-intercepts. Label the turning point with its coordinates.

| THINK | WRITE |
|---|---|
| **1** Find the $x$-intercepts by substituting $y = 0$ into the equation. | $y = -x^2 + 4x + 5$<br>For the $x$-intercepts, let $y = 0$:<br>$-x^2 + 4x + 5 = 0$ |
| **2** Divide by the coefficient of $x^2$, so that the equation is of the form $x^2 + bx + c = 0$. | $-(x^2 - 4x - 5) = 0$<br>$x^2 - 4x - 5 = 0$ |
| **3** Factorise the quadratic. | $(x + 1)(x - 5) = 0$ |
| **4** Apply the Null Factor Law to solve each linear equation for $x$. | $x + 1 = 0$ or $x - 5 = 0$<br>$x = -1$ or $x = 5$<br>$x$-intercepts: $(-1, 0)$ and $(5, 0)$ |
| **5** Find the $y$-intercept by substituting $x = 0$ into the equation and solving for $y$. | For the $y$-intercept, let $x = 0$:<br>$y = -(0)^2 + 4(0) + 5$<br>$= 5$<br>$y$-intercept: $(0, 5)$ |
| **6** Find the coordinates of the turning point. | For the turning point:<br>$x = \frac{-1 + 5}{2}$<br>$= \frac{4}{2}$<br>$= 2$<br>Let $x = 2$:<br>$y = -(2)^2 + 4(2) + 5$<br>$= -4 + 8 + 5$<br>$= 9$<br>Turning point: $(2, 9)$ |
| **7** Mark the $x$- and $y$-intercepts on the axes, plot the turning point, then draw the parabola. Label the equation of the graph and the coordinates of the turning point. |  |

### Helpful hints

- ✔ Label your working out so that it is clear that you are calculating the $x$-intercepts, the $y$-intercept and the turning point. This can make it easier to check your calculations on a test.
- ✔ Always label the key features of a graph such as the $x$-intercepts, the $y$-intercept and the turning point when sketching parabolas.
- ✔ For any given parabola, $y = ax^2 + bx + c$:
  - if $a > 0$, then the parabola has a minimum turning point
  - if $a < 0$, then the parabola has a maximum turning point.

ANS p511

# Exercise 5C Sketching parabolas using intercepts

1, 2, 3(a, b, d, i), 4–8, 12, 14 | 2–6, 8–11, 13, 16(a) | 4–6, 8, 10, 11, 13, 15, 16, 17

UNDERSTANDING AND FLUENCY

**1** For each quadratic relationship, find the coordinates of the:

**i** $x$-intercepts **ii** $y$-intercept

**a** $y = (x + 7)(x - 5)$ **b** $y = 3(x - 2)(x - 6)$ **c** $y = (3x + 12)(4x + 8)$

5C.1 **2** For each quadratic relationship, find the coordinates of the:

**i** $x$-intercepts **ii** $y$-intercept

**a** $y = x^2 - 2x$ **b** $y = x^2 + 8x$ **c** $y = x^2 + 6x + 8$

**d** $y = x^2 - 8x + 12$ **e** $y = x^2 - 4x - 5$ **f** $y = x^2 - 9$

5C.2 **3** Find the coordinates of the turning point for each quadratic relationship.

**a** $y = x(x - 4)$ **b** $y = (x - 3)(x - 5)$ **c** $y = (x + 4)(x - 6)$

**d** $y = -(x + 1)(x + 3)$ **e** $y = (x - 2)(x + 2)$ **f** $y = -(x + 2)(x - 4)$

**g** $y = x(x + 3)$ **h** $y = -x(x + 7)$ **i** $y = 2(x - 2)(x - 8)$

**4** Sketch the graph of each quadratic relationship by first finding the $x$- and $y$-intercepts. Label the turning point with its coordinates.

**a** $y = (x + 2)(x + 4)$ **b** $y = (x + 3)(x - 3)$ **c** $y = x(x - 2)$

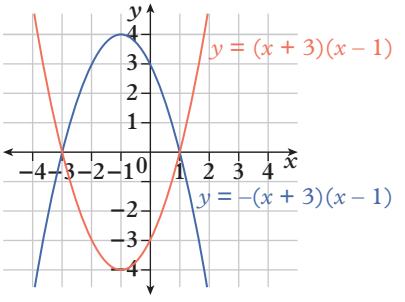
**d** $y = -(x - 2)(x - 6)$ **e** $y = (x - 4)^2$ **f** $y = (2x - 4)(x - 6)$

**5** Consider the graphs of $y = (x + 3)(x - 1)$ and $y = -(x + 3)(x - 1)$.

**a** Identify the $x$-intercepts for each graph.

**b** Find the turning point of each graph.

**c** Describe the similarities and differences between the two graphs.




5C.3 **6** Sketch the graph of each quadratic relationship by first finding the $x$- and $y$-intercepts. Label the turning point with its coordinates.

**a** $y = x^2 - 6x + 5$ **b** $y = x^2 + 4x - 12$ **c** $y = -x^2 + 6x + 7$

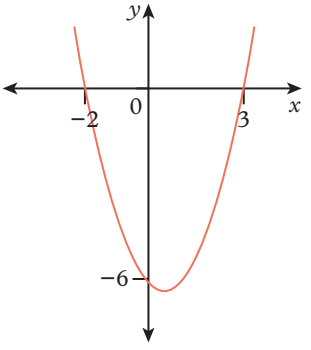
**d** $y = x^2 + 4x$ **e** $y = -x^2 - 8x - 12$ **f** $y = x^2 - 4$

**g** $y = x^2 + 2x - 15$ **h** $y = 10x - x^2$ **i** $y = x^2 - 6x - 7$

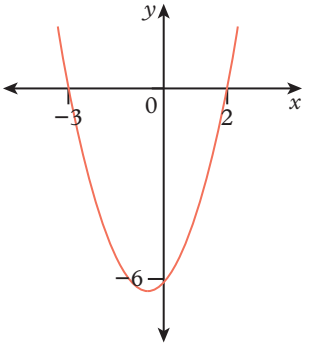
**j** $y = x^2 - 5x$ **k** $y = 16 - x^2$ **l** $y = -x^2 + x$

**7** Match each graph with its rule from the list below.

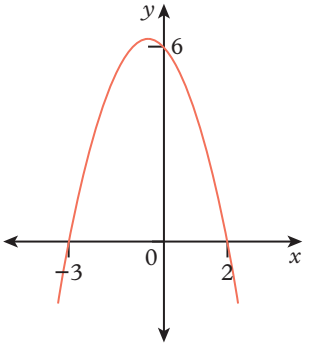
**a**




**b**




**c**




**A** $y = x^2 + x - 6$ **B** $y = -x^2 - x + 6$ **C** $y = x^2 - x - 6$

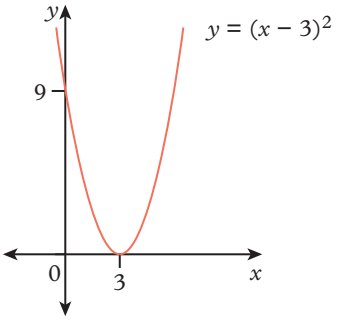
**8** A parabola with a maximum turning point is called an inverted parabola. Explain how you can tell whether a parabola will be upright or inverted from its equation. Refer to your answers to question **6** as examples in your explanation.

**9** Without drawing a sketch, decide whether these equations will produce upright or inverted parabolas.

**a** $y = x^2 - 2x - 3$ **b** $y = x^2 - 100$ **c** $y = -x^2 - 8x + 15$

**d** $y = -x^2 + 6x$ **e** $y = 9x^2 - 10x$ **f** $y = 81 - 4x^2$

**10** Consider the graph shown to the right.




**a** Is the parabola upright or inverted?

**b** How many $y$-intercepts does the parabola have? List the coordinates of the $y$-intercept(s).

**c** How many $x$-intercepts does the parabola have? List the coordinates of the $x$-intercept(s).

**d** What are the coordinates of the turning point?

**e** Describe the relationship between the $x$-intercept(s) and the turning point.

**f** Describe the relationship between equations of the form $y = a(x - p)^2$ (where $a$ and $p$ are constants), and the coordinates of the turning point.

**11** Sketch the graph of each quadratic relationship. Label the turning point with its coordinates.

**a** $y = (x - 1)^2$

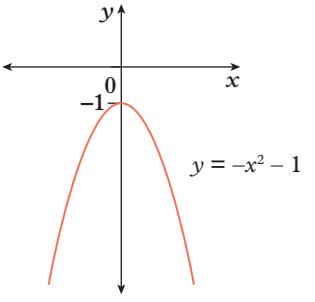
**b** $y = -(x + 2)^2$

**c** $y = x^2 + 8x + 16$

**d** $y = x^2 - 4x + 4$

**e** $y = -x^2 - 6x - 9$

**12** Consider the graph shown to the right.




**a** Is the parabola upright or inverted?

**b** How many $y$-intercepts does the parabola have? List the coordinates of the $y$-intercept(s).

**c** How many $x$-intercepts does the parabola have? List the coordinates of the $x$-intercept(s).

**d** What are the coordinates of the turning point?

**e** Describe the relationship between the equations of the form $y = ax^2 + c$ (where $a$ and $c$ are constants), and the coordinates of the turning point.

**13** Rhys fires an arrow from a bow. The path of the arrow can be represented by the quadratic relationship $h = -0.1(d + 1)(d - 15)$ where $h$ is the height above the ground and $d$ is the horizontal distance from where the arrow was fired. Both $h$ and $d$ are in metres.

**a** Sketch the graph of this relationship by first finding the intercepts and turning point.

**b** Between what two values of $d$ does the graph represent the path of the arrow?

**c** How high does the arrow reach?

**d** At what height above the ground was the arrow fired?

**e** What horizontal distance did the arrow fly before hitting the ground?

PROBLEM SOLVING AND REASONING

**14** A soccer ball is kicked off the ground. The path of the ball can be represented by the quadratic relationship $y = -0.2x^2 + 2.4x$, where $x$ is the horizontal and $y$ is the vertical distance. Both $x$ and $y$ are in metres.

**a** Sketch the graph of this relationship by first finding the intercepts and turning point.

**b** What was the maximum height of the soccer ball?

**c** What horizontal distance had the soccer ball travelled when it was at its maximum height?

**d** What horizontal distance did the soccer ball travel before hitting the ground?

**15** During a game of netball, Ayca throws the ball to her teammate Siegrid. The ball's position is described by the equation $h = -4t^2 + 2t + 2$, where $h$ is the height of the ball above the ground in metres and $t$ is the time the ball spends in the air in seconds.

**a** If Siegrid catches the ball 0.5 seconds after Ayca throws it to her, from what height does Siegrid catch the ball?

**b** List the coordinates that represent the point at which the ball is thrown and the point at which it is caught. What do you notice about the height of the ball in each case?

**c** Use your observations from part **b** and the symmetry of the parabola to determine the coordinates for the turning point.

**d** Hence sketch the graph from $t = 0$ seconds to $t = 0.5$ seconds.

**e** What is the maximum height of the netball?

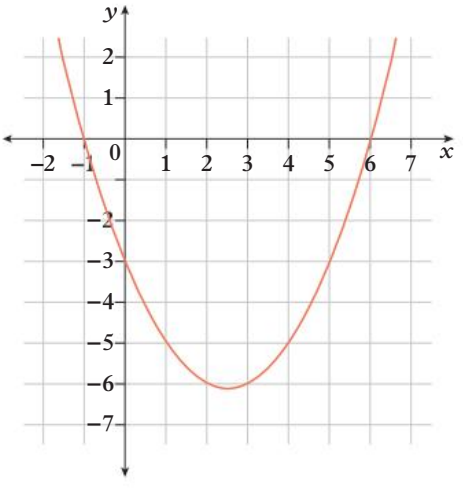
CHALLENGE

**16** The equation for a quadratic graph with at least one $x$-intercept can be expressed in factor form:

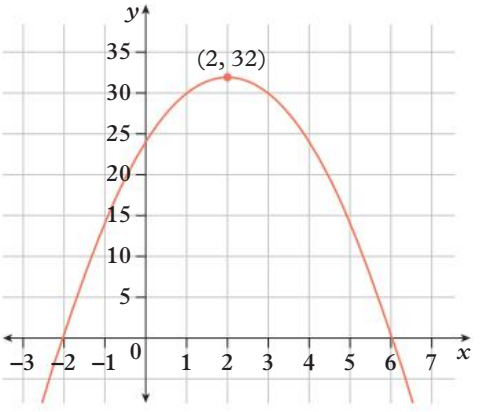
$$y = a(x - p)(x - q)$$

The values of $p$ and $q$ are equal to the $x$-coordinates of the $x$-intercepts. The value of $a$ can be determined by substituting the coordinates of any other points on the graph into the values for $x$ and $y$. Use the factor form to determine the equation of each of the following quadratic graphs.

**a**




**b**




**17** Find the equation of the parabola that has only one $x$-intercept at $(3, 0)$ and passes through the point $(-1, 6)$.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Sketching parabolas using intercepts
- Investigation: Fencing the chicken run
- Topic quiz: 5C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

5A **1** Solve each quadratic equation using the Null Factor Law.

**a** $(x - 4)(x - 3) = 0$

**b** $(x - 2)(x + 2) = 0$

**c** $(x + 9)(x + 9) = 0$

**d** $x(x + 4) = 0$

5A **2** Solve:

**a** $x^2 - 12x + 35 = 0$

**b** $x^2 + 3x - 28 = 0$

**c** $x^2 - 12x + 36 = 0$

**d** $x^2 - 81 = 0$

5A **3** Solve the following quadratic equations by first identifying a HCF.

**a** $2x^2 - 32 = 0$

**b** $8x^2 - 16x - 64 = 0$

**c** $-2x^2 - 12x - 10 = 0$

**d** $45x - 3x^2 = 0$

5A **4** The length of a rectangular swimming pool is 4 m more than its width.

**a** Write an expression for the area of the swimming pool.

**b** If the area of the swimming pool is 60 $m^2$, write an equation for the area.

**c** Solve the equation. Which value of the variable is a feasible solution in this scenario? Explain.

**d** State the dimensions of the pool.

5B **5** For the quadratic graph shown identify:

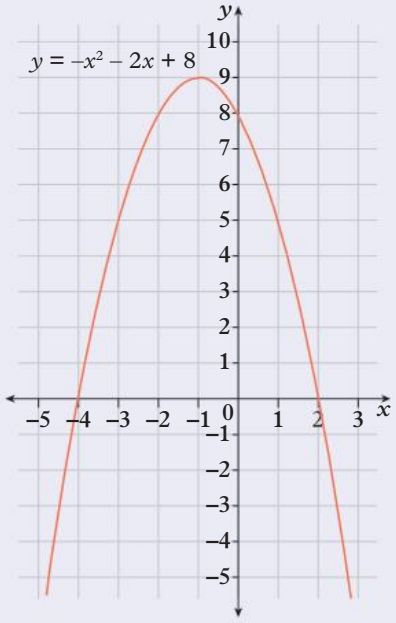
**a** the coordinates of the $x$-intercepts

**b** the coordinates of the $y$-intercept

**c** the coordinates of the turning point

**d** whether it is a maximum or minimum turning point

**e** the equation of the axis of symmetry.




5B **6** Sketch a graph of $y = 2x^2 - 8$ by first completing a table of values for $x$ from $-3$ to 3.

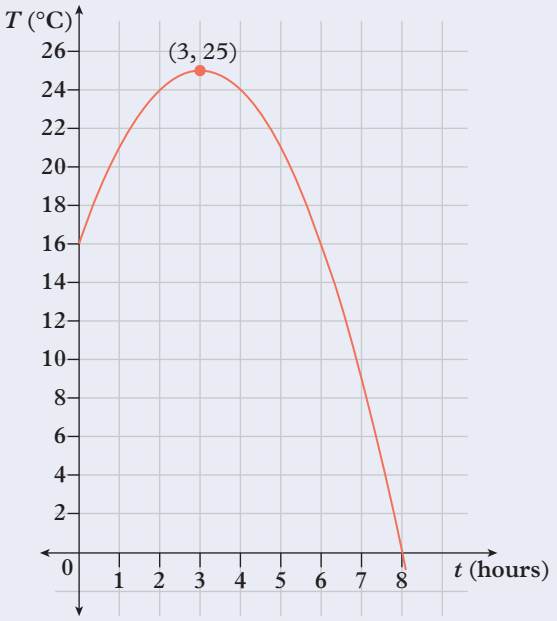
5B **7** The heater inside a greenhouse stops working on a freezing winter day. This graph shows the temperature, $T$, in degrees Celsius inside the greenhouse, $t$ hours after midday.

**a** What is the temperature in the greenhouse at midday?

**b** What is the maximum temperature in the greenhouse?

**c** At what time does the heater break?

**d** After what time does the temperature of the greenhouse drop below 0°C?




5B **8** A quadratic graph has a minimum turning point at (5, 0).

**a** What is the equation of the axis of symmetry?

**b** What are the coordinates of the $x$-intercept(s)?

5C **9** For each quadratic relationship, find the coordinates of the:

**i** $x$-intercept(s)

**ii** $y$-intercept.

**a** $y = x^2 + 3x$

**b** $y = 2x^2 - 50$

**c** $y = x^2 + 2x - 8$

**d** $y = x^2 - 16x + 64$

5C **10** Find the coordinates of the turning point for each quadratic relationship.

**a** $y = (x - 2)(x + 4)$

**b** $y = x(x - 4)$

**c** $y = (x - 6)(x + 6)$

**d** $y = -(x - 8)(x - 12)$

5C **11** Sketch the graph of $y = x^2 - 4x + 3$ by first finding the $x$- and $y$-intercepts. Label the turning point with its coordinates.

5C **12** A cannon is fired up into the air as an experiment. The height, $h$, in metres of the cannonball above the ground is given by $h = 15d - d^2$, where $d$ is the horizontal distance in metres from where the cannonball is fired.

**a** Sketch the graph of this relationship by first finding the intercepts. Label the turning point with its coordinates.

**b** How far from the cannon does the cannonball land?

**c** What is the maximum height reached by the cannonball?

# 5D Sketching parabolas using transformations

## Learning intentions

✔ I can sketch quadratic graphs of the form $y = a(x - h)^2 + k$ using reflections, vertical translations and horizontal translations.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 8E Rotations and reflections |
| Year 8 | 7C Transformations |
| Year 10 | 5D Sketching parabolas using transformations |

## Turning point form

- The graph of $y = x^2$ is an upright parabola with a minimum turning point at the origin, $(0, 0)$.
- Quadratic relationships can be written in **turning point form**:

$$y = a(x - h)^2 + k$$

vertical translation of $k$ units
$k > 0$ up
$k > 0$ down

dilation
$a > 0$ upright
$a < 0$ inverted

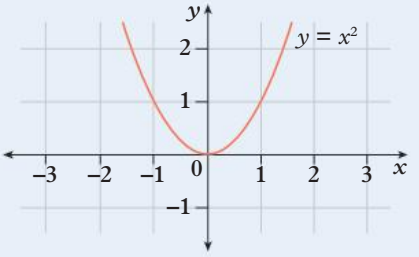
horizontal translation of $h$ units
$h > 0$ right
$h < 0$ left

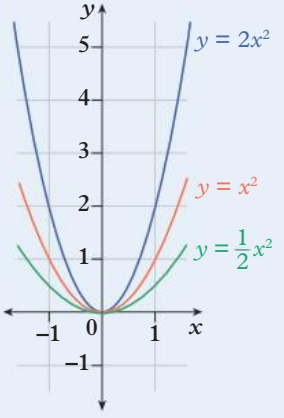
where:

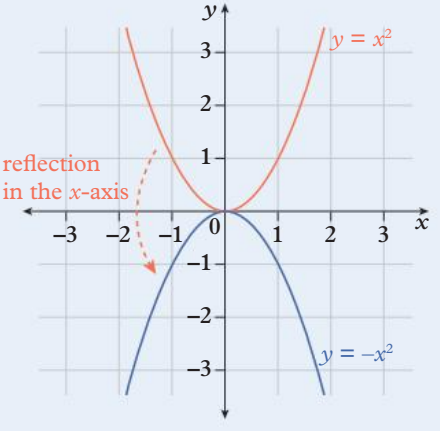
→ the stretch in the $y$-direction is by a factor of $a$

→ the coordinates of the turning point are $(h, k)$.

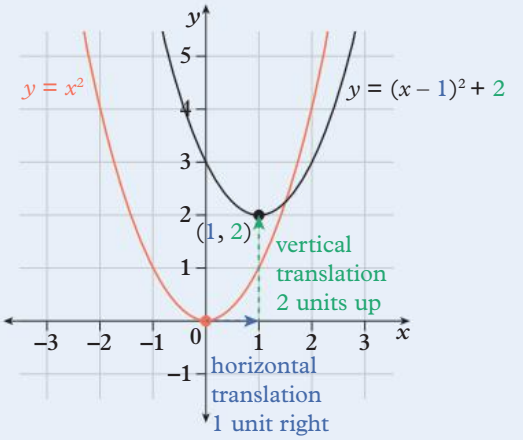
- **Transformations** can be performed on the graph of $y = x^2$ to sketch a graph of $y = a(x - h)^2 + k$.
- A **stretch** by a factor of $a$ in the $y$-direction will:
  → *extend* the parabola in the $y$-direction if $a > 1$
  → *compress* the parabola in the $y$-direction if $0 < a < 1$.
- A **reflection** in the $x$-axis will take place if $a = -1$.
- A reflection and a stretch will take place if $a < 0$ and $a \neq -1$.
- Note: Reflections and stretches must be applied before translations.



- A **horizontal translation** will move the parabola $h$ units:
  - → *right* if $h > 0$
  - → *left* if $h < 0$
- A **vertical translation** will move the parabola $k$ units:
  - → *up* if $k > 0$
  - → *down* if $k < 0$




## Example 5D.1 Sketching a parabola using translations

Sketch the graph of $y = x^2$ on a Cartesian plane, and then perform a translation to sketch the graph of each quadratic relationship. Label the $y$-axis at the intercept and label the turning point with its coordinates.

**a** $y = x^2 - 3$ **b** $y = (x - 4)^2$

**THINK**

**a** 1 Identify the transformation. The equation has the form $x^2 + k$ where $k = -3$, so the graph of $y = x^2$ undergoes a vertical translation of 3 units down. So the $y$-intercept is at the turning point.

$y = x^2 - 3$

vertical translation of 3 units down

2 Sketch the graph of $y = x^2$ and label its turning point, $(0, 0)$. Translate the turning point 3 units down to find the turning point of $y = x^2 - 3$.

**b** 1 Find the $y$-coordinate of the $y$-intercept and mark it on the $y$-axis.

2 Identify the transformation. The equation has the form $(x - h)^2$ where $h = 4$, so the graph of $y = x^2$ undergoes a horizontal translation of 4 units to the right.

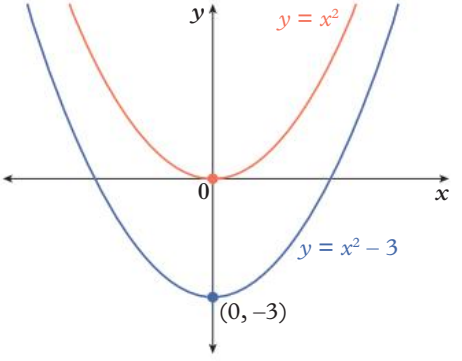
$y = (x - 4)^2$

horizontal translation of 4 units to the right

3 Sketch the graph of $y = x^2$ and label its turning point, $(0, 0)$. Translate the turning point 4 units to the right to find the turning point of $y = (x - 4)^2$.

**WRITE**

**a**

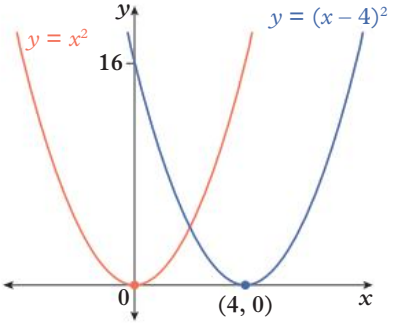



**b** $y = (x - 4)^2$

Let $x = 0$:

$y = (0 - 4)^2$

$= 16$



## **Example 5D.2** Sketching a parabola using multiple transformations

Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the $y$-axis at the intercept and label the turning point with its coordinates.

**a** $y = -(x + 3)^2$ **b** $y = (x - 2)^2 - 1$

**THINK**

**a** **1** Find the $y$-coordinate of the $y$-intercept and mark it on the $y$-axis.

**2** Identify the transformations. The equation form is $a(x - h)^2$ so the graph of $y = x^2$:

- is inverted as $a = -1$
- is translated 3 units to the left as $h = -3$.

The turning point $(h, k)$ is $(-3, 0)$.

$y = -(x + 3)^2$

inverted; horizontal translation of 3 units to the left

**3** Sketch the graph.

**b** **1** Find the $y$-coordinate of the $y$-intercept and mark it on the $y$-axis.

**2** Identify the transformations. The equation form is $(x - h)^2 + k$ so the graph of $y = x^2$:

- is translated 2 units to the right as $h = 2$
- is translated 1 unit down as $k = -1$.

The turning point $(h, k)$ is $(2, -1)$.

$y = (x - 2)^2 - 1$

horizontal translation of 2 units to the right; vertical translation of 1 unit down

**3** Sketch the graph.

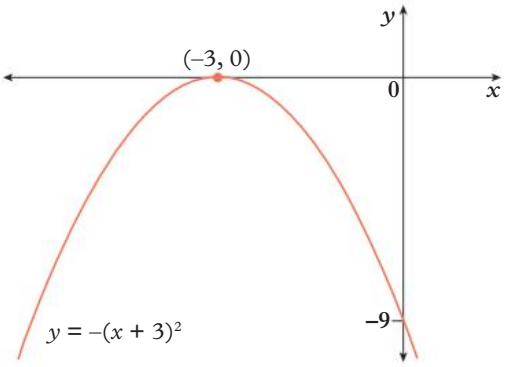
**WRITE**

**a** $y = -(x + 3)^2$

Let $x = 0$:

$y = -(0 + 3)^2$

$= -9$



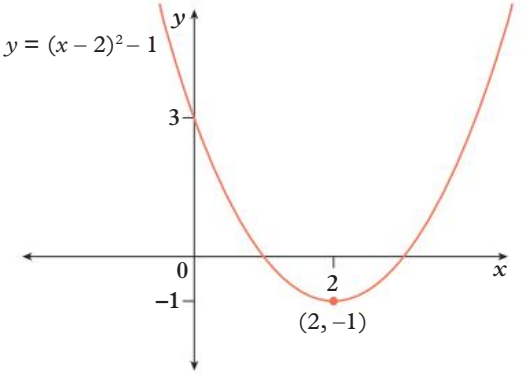

**b** $y = (x - 2)^2 - 1$

Let $x = 0$:

$y = (0 - 2)^2 - 1$

$= 4 - 1$

$= 3$




**Helpful hints**

- ✔ In turning point form:
  - → vertical translations are determined by the constant *outside* the brackets
  - → horizontal translations are determined by the constant *inside* the brackets with $x$.

$y = a(x - h)^2 + k$

horizontal translation of $h$ units; vertical translation of $k$ units

- ✔ It is always helpful to label the key features of your graph before sketching. The turning point, $y$-intercept, and a dashed line for the axis of symmetry will help you sketch an accurate parabola.
- ✔ Remember that if $a$ is negative, you must reflect the graph in the $x$-axis before translating.

ANS p514

# Exercise 5D Sketching parabolas using transformations

1–7, 8(a–d), 9(a, c, e, f), 10, 14, 17(a)

2–6, 8, 9(b, d, f, h), 10, 13–15, 16(a, e), 17(a–c)

5, 6, 8(f–i), 9(b, d, f, h), 11, 13, 14, 16(b, d, f, h, j), 17, 18

UNDERSTANDING AND FLUENCY

**1** Complete the descriptions of transformations below.

**a** To sketch the graph of $y = x^2 - 10$, the graph of $y = x^2$ is translated _____ units _____.

**b** To sketch the graph of $y = (x - 5)^2$, the graph of $y = x^2$ is translated _____ units _____.

**c** To sketch the graph of $y = (x + 12)^2$, the graph of $y = x^2$ is translated _____ units _____.

**d** To sketch the graph of $y = x^2 + 8$, the graph of $y = x^2$ is translated _____ units _____.

5D.1 **2** Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical translation to sketch the graph of each quadratic relationship on the same set of axes. Label the $y$-axis at the intercept and label the turning point on each parabola with its coordinates.

**a** $y = x^2 + 3$ **b** $y = x^2 + 1$ **c** $y = x^2 - 2$

**d** $y = x^2 + 6$ **e** $y = x^2 - 4$ **f** $y = x^2 + 9$

**3** Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a horizontal translation to sketch the graph of each quadratic relationship on the same set of axes. Label the $y$-axis at the intercept and label the turning point on each parabola with its coordinates.

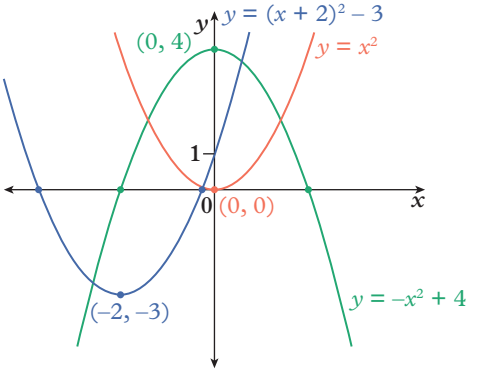
**a** $y = (x - 3)^2$ **b** $y = (x - 1)^2$ **c** $y = (x + 2)^2$

**d** $y = (x + 4)^2$ **e** $y = (x - 5)^2$ **f** $y = (x + 7)^2$

**4** Complete the following descriptions of transformations.

**a** To sketch the graph of $y = (x + 2)^2 - 3$, the graph of $y = x^2$ is translated _____ units _____ and _____ units _____.

**b** To sketch the graph of $y = -x^2 + 4$, the graph of $y = x^2$ is _____ across the $x$-axis and translated _____ units _____.




5D.2 **5** Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the $y$-axis at the intercept and label the turning point on each parabola with its coordinates.

**a** $y = -(x - 2)^2$ **b** $y = (x - 2)^2 + 3$

**c** $y = -(x + 6)^2$ **d** $y = -x^2 + 4$

**e** $y = (x - 1)^2 - 2$ **f** $y = -x^2 - 3$

**g** $y = -(x + 4)^2 + 6$ **h** $y = -(x - 7)^2 - 5$

**6** For each quadratic relationship:

**i** identify whether its graph will be an upright or inverted parabola

**ii** find the coordinates of the $x$- and $y$-intercepts

**iii** write the coordinates of the turning point

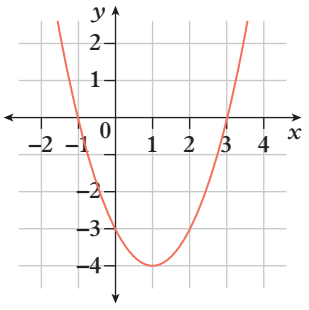
**iv** sketch the graph of the relationship.

**a** $y = (x - 2)^2 - 1$ **b** $y = -(x + 1)^2 + 9$

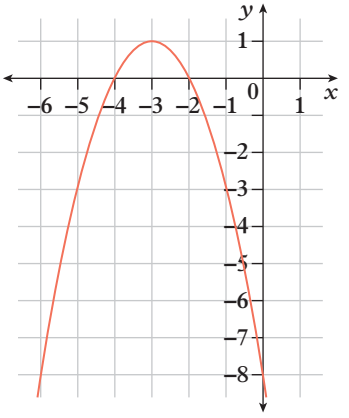
**c** $y = (x - 3)^2 - 4$ **d** $y = -(x + 4)^2 + 1$

**7** Consider the quadratic graphs below.

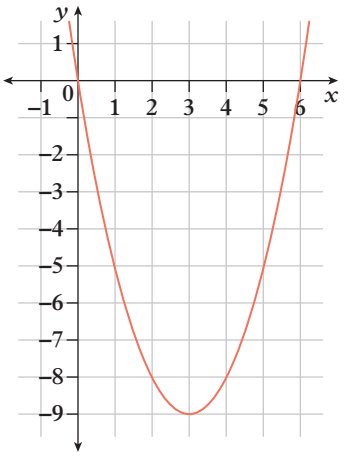
**a**



**b**



**c**



**i** State whether the parabola is upright or inverted.

**ii** Identify the coordinates of the turning point.

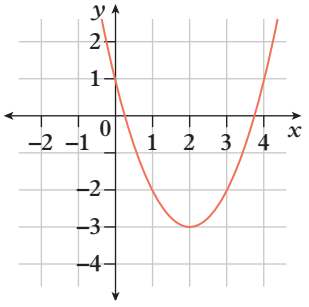
If each graph has an equation of the form $y = a(x - h)^2 + k$:

**iii** decide whether $a$ is positive or negative

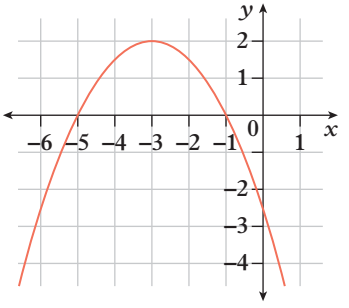
**iv** determine the values of $h$ and $k$.

**8** Match each graph with its equation from the list provided below.

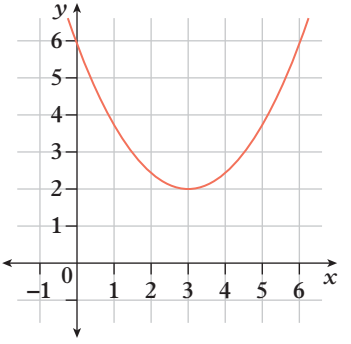
**a**



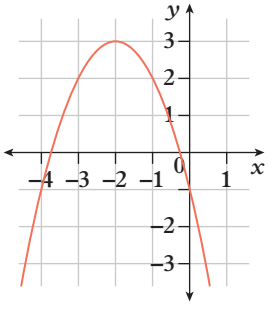
**b**



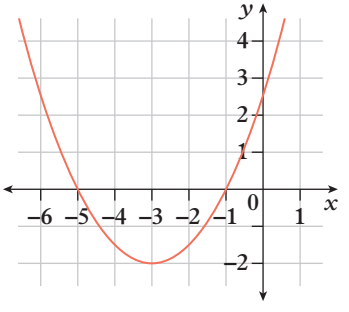
**c**



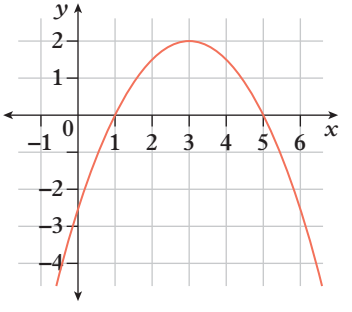
**d**



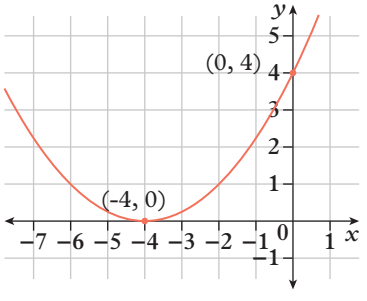
**e**



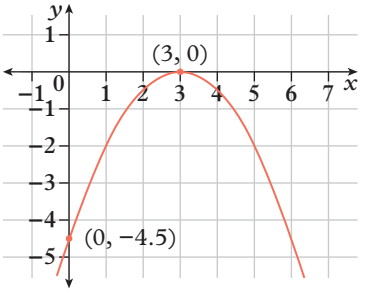
**f**



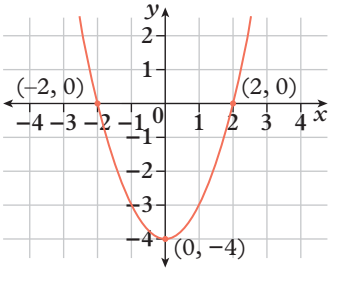
**g**




**h**




**i**




**A** $y = (x - 3)^2 + 2$

**B** $y = -(x - 3)^2 + 2$

**C** $y = (x + 3)^2 - 2$

**D** $y = -(x + 3)^2 + 2$

**E** $y = (x - 2)^2 - 3$

**F** $y = -(x + 2)^2 + 3$

**G** $y = x^2 - 4$

**H** $y = \frac{1}{4}(x + 4)^2$

**I** $y = -\frac{1}{2}(x - 3)^2$

**9** Write the equation for each of the quadratic graphs described below. Assume each parabola has the same shape as $y = x^2$; that is, the stretch factor is +1 or −1.

**a** upright, turning point at (3, 7)

**b** upright, turning point at (−2, 5)

**c** inverted, turning point at (2, 4)

**d** inverted, turning point at (6, −1)

**e** upright, turning point at (9, 0)

**f** inverted, turning point at (0, 4)

**g** inverted, turning point at (−1, −2)

**h** upright, turning point at (0, −5)

**10** Consider the quadratic relationship $y = (x - 4)^2 + 5$.

**a** What are the coordinates of the turning point?

**b** What is the smallest $y$-value that this relationship can have?

**c** Is there a maximum value for $y$? Explain your answer.

**11** Consider the quadratic relationship $y = -(x + 1)^2 + 2$. What is the maximum $y$-value in this relationship? Explain.

**12** Jenna throws a netball to a teammate. The height of the ball can be represented by the relationship $h = -(d - 2)^2 + 6$, where $h$ is the height in metres and $d$ is the horizontal distance from where Jenna throws the ball.

**a** What are the coordinates of the turning point of this relationship?

**b** What is the value of $h$ when:

**i** $d = 0$?

**ii** $d = 4$?

**c** Sketch the graph of this relationship from $d = 0$ to $d = 4$.

**d** Use the graph to find:

**i** the height at which the ball left Jenna's hands

**ii** the maximum height of the ball during the pass to her teammate.

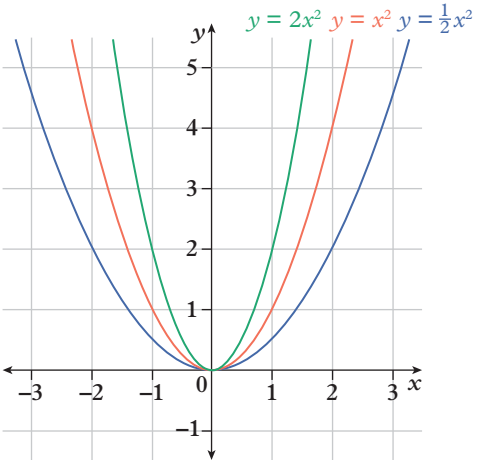
**13** The path of a firework follows the relationship $y = -(x - 10)^2 + 100$, where $y$ is the height of the firework for a horizontal distance $x$ from where it was launched. Both $x$ and $y$ are in metres.

**a** Sketch the graph of this relationship from when the firework was launched to when it landed on the ground after exploding. (Hint: Use the fact that a parabola is symmetrical.)

**b** What was the maximum height of the firework?

**c** How far from the launch pad did the firework land?

**14** Consider the graphs of the form $y = ax^2$.




**a** What is the value of the stretch factor, $a$, for $y = x^2$?

**b** Which graph is narrower than the graph of $y = x^2$? State the value of the graph's stretch factor.

**c** Which graph is wider than the graph of $y = x^2$?

**d** Describe how the value of the stretch factor affects the width of the corresponding parabola compared to the parabola $y = x^2$.



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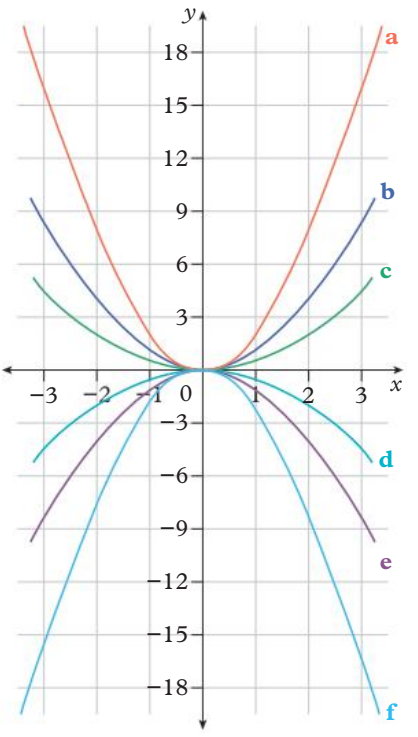
**15** Match each graph on the Cartesian plane to the right with its equation from the list provided below.




**A** $y = x^2$ **B** $y = -2x^2$ **C** $y = -\frac{1}{2}x^2$

**D** $y = \frac{1}{2}x^2$ **E** $y = 2x^2$ **F** $y = -x^2$

**16** For each quadratic relationship, identify:

**i** whether the graph will have greater, lesser or equal width to the graph of $y = x^2$

**ii** whether the parabola will be upright or inverted

**iii** the coordinates of the turning point.

**a** $y = 2(x - 4)^2 - 3$ **b** $y = \frac{1}{2}(x + 2)^2 + 6$

**c** $y = 4(x + 2)^2$ **d** $y = -5x^2 - 4$

**e** $y = -\frac{1}{2}(x - 7)^2$ **f** $y = -3(x + 1)^2 + 5$

**g** $y = -\frac{1}{3}(x - 3)^2 - 4$ **h** $y = \frac{1}{4}x^2 + 3$

**i** $y = -(x - 5)^2 + 4$ **j** $y = 4(x + 3) - 7$

**17** Write an equation for the parabola produced after performing each set of transformations on the graph of $y = x^2$.

**a** stretch by a factor of 3, then a translation of 2 units right

**b** reflection in the $x$-axis, then a translation of 4 units down and 5 units left

**c** stretch by a factor of $\frac{1}{2}$, then a reflection in the $x$-axis

**d** stretch by a factor of 4, reflection in the $x$-axis, then a translation of 2 units left

**18** A ball is kicked during a competition to see who can kick it the highest. The height of the ball, $h$, in metres over a horizontal distance, $d$, in metres is given by $h = -(d - 3)^2 + 9$.

**a** Sketch a graph showing the path of the ball. Label the turning point with its coordinates.

**b** What is the maximum height reached by the ball?

**c** When the ball lands, haw far away is it from where it was kicked?

A second ball is kicked from the same point. It travels 4 metres further from where it was kicked to where it lands.

**d** Assuming the second ball also follows a parabolic pathway with a stretch factor of 1, determine the equation for the path of the second basketball.

**e** On the same set of axes used in part **a**, sketch the path of the second ball. Label the turning point with its coordinates.

**f** How much higher does the second ball reach than the first ball?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Sketching parabolas using transformations

Worksheet
Quadratic relationships

Topic quiz
5D

# 5E Circles and other non-linear relationships

## Learning intentions

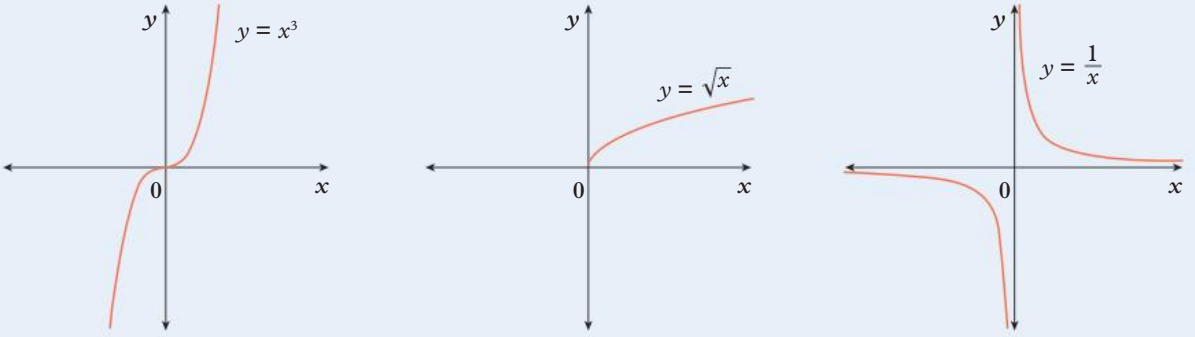
- ✔ I can sketch circles of the form $(x - h)^2 + (y - k)^2 = r^2$ using vertical translations and horizontal translations.
- ✔ I can plot non-linear relationships from a table of values and equations.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 7 | 8D Translations |
| Year 8 | 7C Transformations |
| Year 10 | 5E Circles |

## Non-linear relationships

- Linear relationships produce straight-line graphs. **Non-linear relationships** produce curved-line graphs.
- A **non-linear equation** is an equation containing at least one pronumeral that is raised to an index other than 1.



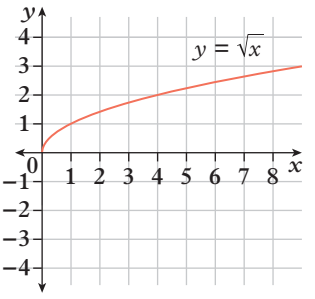
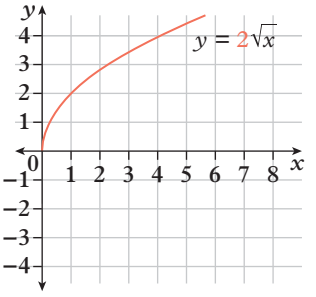
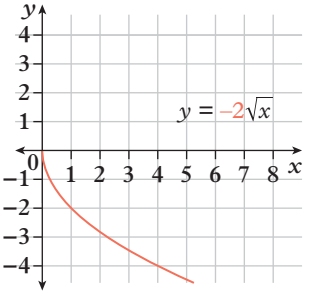
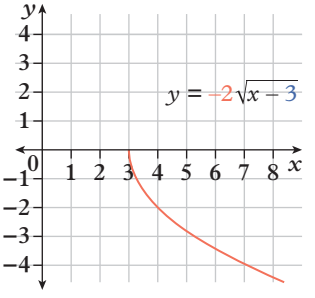
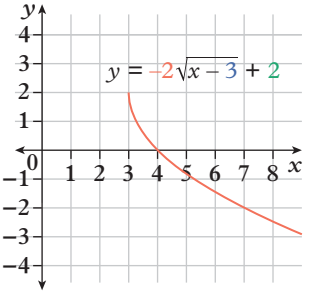

- For all linear and non-linear relationships:
  - → The $x$-intercept(s) are the point(s) where the graph crosses the $x$-axis and $y = 0$.
  - → The $y$-intercept(s) are the point(s) where the graph crosses the $y$-axis and $x = 0$.

## Square root of a number

- Many non-linear equations are solved by taking the square root of a number. In these cases, it is important to understand that there are two possible solutions for the square root of a positive number.
- The square root of a number has a positive and negative solution.
  For example, $2 \times 2 = 4$ and $-2 \times -2 = 4$, so the equation $x^2 = 4$ has two solutions:
  $x = 2$ and $x = -2$
- The **plus-minus symbol**, $\pm$, is used as shorthand to write both results in an efficient way.
  For example, $x^2 = 4$
  $x = \pm\sqrt{4}$
  $= \pm 2$
- The graph of $y^2 = x$ is a sideways parabola, where $y = \sqrt{x}$ represents its upper half with positive $y$ values and $y = -\sqrt{x}$ represents the lower half with negative $y$ values.
- The graph of $y = \sqrt{x}$ does not have a plus-minus symbol and so it implicitly takes the positive root of $x$.

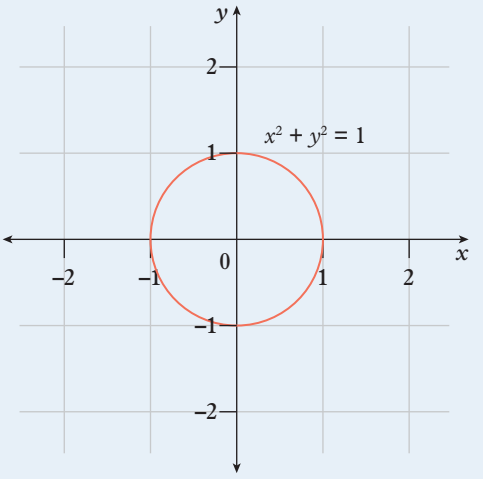
# General transformations

- Transformations can be performed on any non-linear relationship of the form $y = \ldots$, much like the transformations applied to the turning point form of a quadratic graph.

| Transformation on equation | Transformation on graph | Example |
|---|---|---|
| No transformation. | No transformation. |   |
| Multiply the RHS by a constant, $a$. | Stretch by a factor of $a$ in the $y$-direction. |   |
| Multiply the RHS by $-1$. | Reflect in the $x$-axis. |   |
| Substitute $x$ for $x - h$. | Translate $h$ units in the $x$-direction. |   |
| Substitute $y$ for $y - k$.<br>Note: $y - k = \ldots$ is equivalent to $y = \ldots + k$, by applying inverse operations. | Translate $k$ units in the $y$-direction. |   |

# Circles

- The graph of $x^2 + y^2 = 1$ is a circle with a centre at (0, 0) and a radius $r = 1$ unit.




- The **general equation of a circle** is:

$$(x - h)^2 + (y - k)^2 = r^2$$

radius of $r$ units

horizontal translation of $h$ units

$h > 0$ right
$h < 0$ left

vertical translation of $k$ units

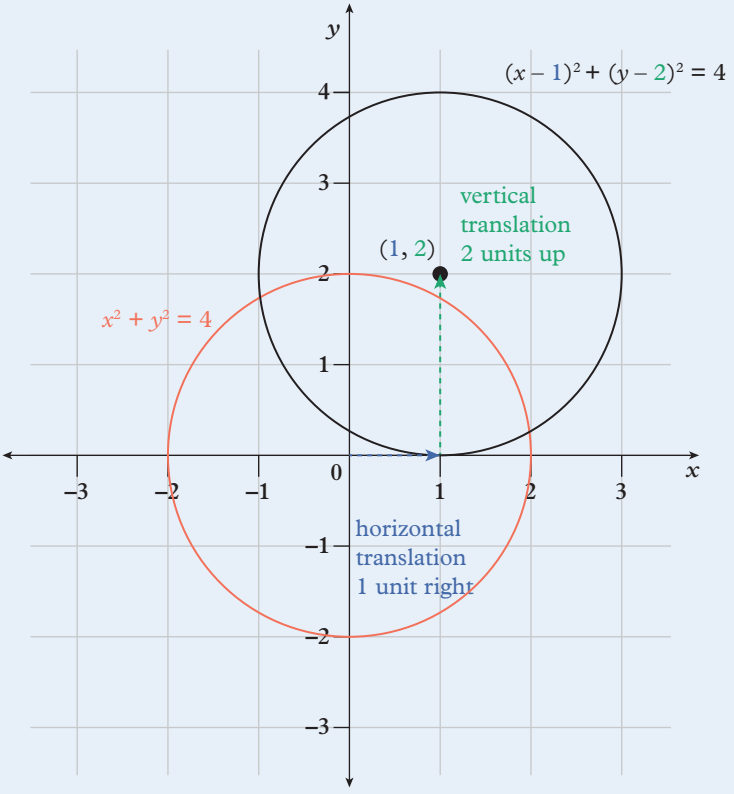
$k > 0$ up
$k < 0$ down

where:

→ the circle has a radius of $r$ units

→ the coordinates of the centre of the circle are $(h, k)$.

- Transformations can be performed on the graph of $x^2 + y^2 = r^2$ to sketch a graph of $(x - h)^2 + (y - k)^2 = r^2$.
  - → A horizontal translation will move the circle $h$ units:
    - *right* if $h > 0$
    - *left* if $h < 0$
  - → A vertical translation will move the circle $k$ units:
    - *up* if $k > 0$
    - *down* if $k < 0$
- The horizontal and vertical extremes of a circle are the furthest points left, right, up and down on the graph. These are useful coordinates for sketching the circle.



## **Example 5E.1** Sketching simple circles

Sketch the graph of $x^2 + y^2 = 36$ by first finding the horizontal and vertical extremes of the circle.

**THINK**

1. Identify the centre of the circle. The equation has the form $x^2 + y^2 = r^2$, so $h = 0$ and $k = 0$, and the centre of the circle is at the origin $(0, 0)$.
2. Identify the radius. The equation has the form $x^2 + y^2 = r^2$ so $r^2 = 36$. A radius cannot be negative, so ignore the negative value.
3. Identify the horizontal extremes. These will be the $x$-intercepts 6 units left $(-)$ and right $(+)$ of the origin.
4. Identify the vertical extremes. These will be the $y$-intercepts 6 units below $(-)$ and above $(+)$ the origin.
5. Sketch the graph. Label the graph with its equation.

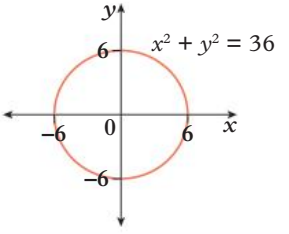
**WRITE**

Centre: $(0, 0)$

$r^2 = 36$
$r = \pm\sqrt{36}$
$= \pm 6$
The radius is 6 units.

Horizontal extremes: $(-6, 0)$ and $(6, 0)$

Vertical extremes: $(0, -6)$ and $(0, 6)$




## **Example 5E.2** Sketching circles using translations

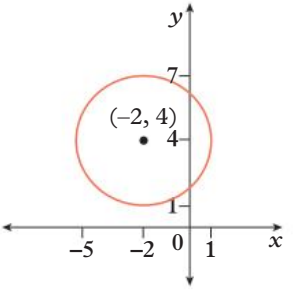
Sketch the graph of $(x + 2)^2 + (y - 4)^2 = 9$ using transformations.

**THINK**

1. Identify the translations. The equation has the form $y = (x - h)^2 + (y - k)^2 = r^2$ where $h = -2$ and $k = 4$, so the coordinates of the centre $(h, k) = (-2, 4)$.
2. Identify the radius.
3. Sketch the graph by marking the centre of the circle at $(-2, 4)$, then identifying the horizontal and vertical extremes that are 3 units left, right, below and above the centre: $(-5, 4)$, $(1, 4)$, $(-2, 1)$ and $(-2, 7)$.

**WRITE**

Centre: $(-2, 4)$
$r^2 = 9$
$r = \pm\sqrt{9}$
$= \pm 3$
The radius is 3 units.



## Example 5E.3 Determining the equation of a circle

Determine the equation of a circle with a radius of 6 units and a centre at $(2, -5)$.

**THINK**

The general equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$. Identify the values of $h$, $k$ and $r$, then substitute them into $(x - h)^2 + (y - k)^2 = r^2$ and simplify.

**WRITE**

Centre: $(2, -5) = (h, k)$

$h = 2$

$k = -5$

Radius: $r = 6$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - (-5))^2 = 6^2$

$(x - 2)^2 + (y + 5)^2 = 36$

**Helpful hints**

- ✔ Remember that the values of $h$ and $k$ are the opposite sign to how they appear in the equation.
  For example, in the equation $(x + 2)^2 + (y - 4)^2 = 9$, $h = -2$ and $k = +4$.
- ✔ For circles centred at the origin, $x^2 + y^2 = r^2$, you can find horizontal and vertical extremes by finding the $x$- and $y$-intercepts.
  For example, to find the $x$-intercepts of $x^2 + y^2 = 4$, let $y = 0$:
  $$x^2 + 0^2 = 4$$
  $$x^2 = 4$$
  $$x = \pm 2$$
  So, the horizontal extremes are $(2, 0)$ and $(-2, 0)$.
  Note: Finding the $x$- and $y$-intercepts of a circle becomes more complicated and less useful for sketching circles that are not centred on the origin.

ANS p516

# Exercise 5E Circles and other non-linear relationships

1–6, 8, 9, 11(a, b) | 2–5, 8–11, 13(a), 14 | 3–5, 7, 9, 10, 11(b, c), 12(a, b, d), 15, 16(b)

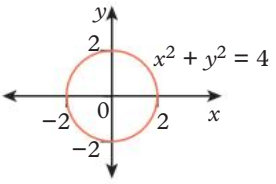
UNDERSTANDING AND FLUENCY

**1** For each circle below, identify:

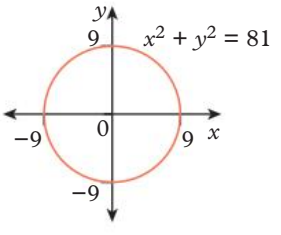
**i** the coordinates of the centre

**ii** the radius.

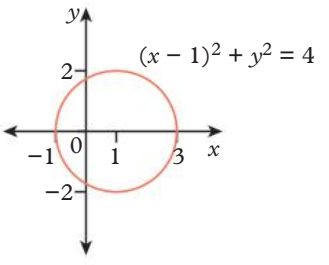
**a**



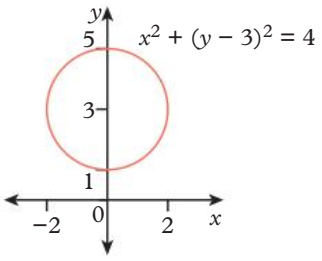

**b**



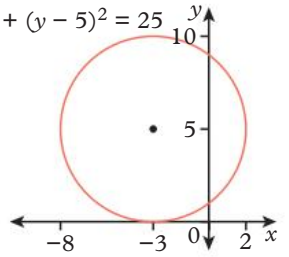

**c**



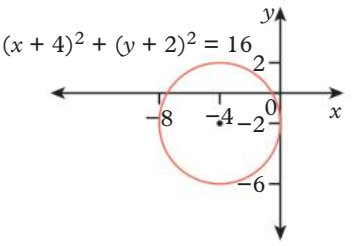
**d**




**e**




**f**




5E.1 **2** Sketch the following graphs by first finding the horizontal and vertical extremes of the circle.

**a** $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 25$ **c** $x^2 + y^2 = 1$

**d** $x^2 + y^2 = 100$ **e** $x^2 + y^2 = 49$ **f** $x^2 + y^2 = 144$

5E.2 **3** Sketch the following graphs using transformations.

**a** $(x - 2)^2 + (y - 3)^2 = 4$ **b** $(x - 1)^2 + (y - 5)^2 = 9$

**c** $(x + 3)^2 + (y - 2)^2 = 36$ **d** $(x - 4)^2 + (y + 3)^2 = 25$

**e** $x^2 + y^2 = 1$ **f** $(x - 6)^2 + y^2 = 4$

**g** $x^2 + (y + 4)^2 = 49$ **h** $(x + 5)^2 + (y + 1)^2 = 16$

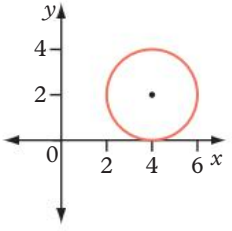
5E.3 **4** Determine the equation of each of the following circles.

**a** circle with radius of 4 units and centre at (3, 5) **b** circle with radius of 5 units and centre at (−2, 4)

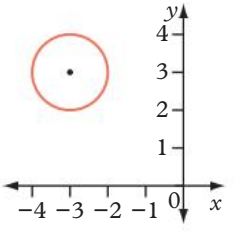
**c** circle with radius of 9 units and centre at (−7, −6) **d** circle with radius of 11 units and centre at (4, −8)

**5** Identify the centre and the radius of each circle and hence write its equation.

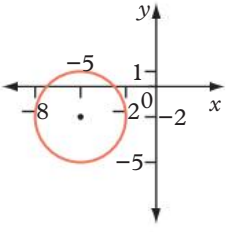
**a**




**b**




**c**




**6** An unusual circular running track is mapped on to a Cartesian plane using the relationship $(x - 30)^2 + (y - 40)^2 = 2500$.
All measurements are in metres.

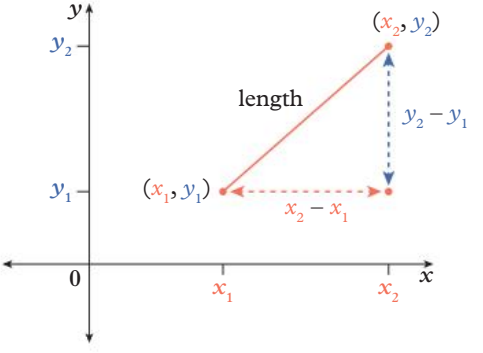
**a** Sketch the graph of this relationship.

**b** Calculate the length of the running track to the nearest metre.

**c** The surface of the ground inside the running track is to be sown with grass seed. To the nearest square metre, what area is to be sown?

**7** From Chapter 4 we know that the length between any two points, $(x_1, y_1)$ and $(x_2, y_2)$, on a Cartesian plane is given by the formula:

$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$




**a** How can you use the length formula to find a point on the Cartesian plane that forms a line segment with the origin, (0,0), of length = 1 unit? Give your answer in the form of an equation.

**b** How can you use the length formula to find a point on the Cartesian plane that forms a line segment with the point $(h, k)$, of length = $r$ units? Do you recognise this equation?

**c** What property of circles connects them with the equation for the length between two points?

**8** Consider the basic cubic relationship, $y = x^3$.

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −27 | | | | | | |

**a** Complete the table of values for the relationship $y = x^3$.

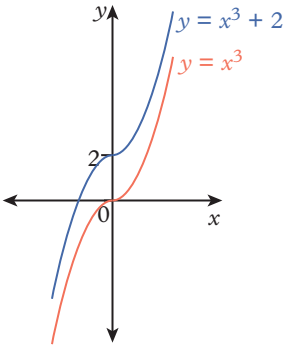
**b** An important feature of the graph of $y = x^3$ is the stationary point at (0,0), where the slope of the graph is flat (gradient = 0) at a single point. Use the table of values to construct a plot of $y = x^3$. Label the point of inflection with its coordinates on your graph and join the plotted points with a smooth curve.

**c** Describe the shape of the graphs on the right.

**d** The graphs of $y = x^3$ and $y = x^3 + 2$ are shown to the right.

**i** Identify the coordinates of the point of inflection for $y = x^3 + 2$.

**ii** Describe the translation required to produce the graph of $y = x^3 + 2$ from the graph of $y = x^3$.




**e** Use your understanding of translations to describe how the graphs of these cubic relationships can be produced from the graph of $y = x^3$.

**i** $y = x^3 + 1$ **ii** $y = x^3 - 3$ **iii** $y = (x - 2)^3$ **iv** $y = (x + 4)^3$ **v** $y = (x - 1)^3 + 2$

**f** Use your answers to part **e** to sketch each relationship. Label the point of inflection with its coordinates on each graph.

**9** Consider the non-linear relationship $y = \sqrt{x}$, where only the positive roots of $x$ are taken.

| $x$ | 0 | 1 | 4 | 9 | 16 | 25 |
|---|---|---|---|---|---|---|
| $y$ | 0 | | | | | |

**a** Complete the table of values for the relationship $y = \sqrt{x}$.

**b** Use the table of values to construct a plot of $y = \sqrt{x}$. Join the plotted points with a smooth curve.

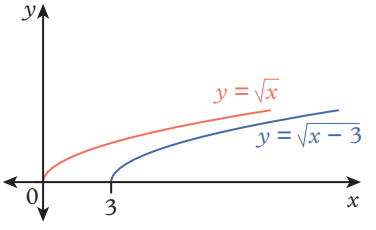
**c** Describe the shape of the graph.

**d** An important feature of the graph of $y = \sqrt{x}$ is the point on the graph where $y$ is a minimum. Label this point with its coordinates on your graph.

**e** The graphs of $y = \sqrt{x}$ and $y = \sqrt{x - 3}$ are shown to the right.

**i** Identify the coordinates of the point on the graph of $y = \sqrt{x - 3}$ where $y$ is a minimum.

**ii** Describe the translation required to produce the graph of $y = \sqrt{x - 3}$ from the graph of $y = \sqrt{x}$.




**f** Use your understanding of translations to describe how the graphs of these relationships can be produced from the graph of $y = \sqrt{x}$.

**i** $y = \sqrt{x} + 2$ **ii** $y = \sqrt{x} - 1$ **iii** $y = \sqrt{x - 1}$ **iv** $y = \sqrt{x + 4}$ **v** $y = \sqrt{x - 2} + 3$

**g** Use your answers to part **f** to sketch each relationship. Label the point on each graph where $y$ is a minimum with its coordinates on each graph.

**10** Consider the relationship $y = \frac{1}{x}$.

| $x$ | −3 | −2 | −1 | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-\frac{1}{3}$ | | | −2 | | | | | 2 | | | |

**a** Complete the table of values for the relationship $y = \frac{1}{x}$.

**b** Use the table of values to construct a plot of $y = \frac{1}{x}$. This graph is called a hyperbola.

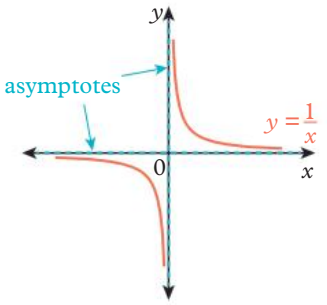
**c** Explain why the graph does not join as the values for $x$ get closer to 0 on either side? (Hint: What is the $y$-value when $x = 0$?)

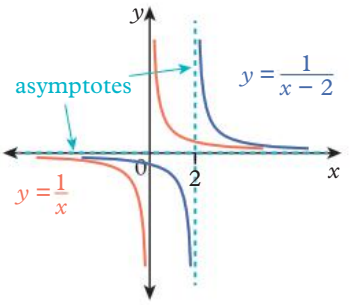
**d** Use similar reasoning to explain why the graph does not join as the values for $y$ get closer to 0 on either side.

**e** An important feature of the graph of $y = \frac{1}{x}$ are the asymptotes, the lines that form boundaries between each part of the hyperbola. For $y = \frac{1}{x}$, the asymptotes are on the $x$- and $y$-axes.

Write the equation of the asymptote for $y = \frac{1}{x}$ that is on the:

**i** $x$-axis **ii** $y$-axis.




**f** The graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x-2}$ are shown above.

**i** Identify the equation of each asymptote for the graph of $y = \frac{1}{x-2}$.

**ii** Describe the translation required to produce the graph of $y = \frac{1}{x-2}$ from the graph of $y = \frac{1}{x}$.

**g** Use translations to describe how each relationship below can be produced from the graph of $y = \frac{1}{x}$.

**i** $y = \frac{1}{x} + 2$ **ii** $y = \frac{1}{x} - 3$ **iii** $y = \frac{1}{x-4}$ **iv** $y = \frac{1}{x+1}$ **v** $y = \frac{1}{x-3} + 2$

**h** Use your answers to part **g** to sketch each relationship. Clearly show the asymptotes for each graph.

**11** Consider the relationships in questions **8–10**.

**a** Sketch the graphs of $y = -x^3$, $y = -x^3 + 4$ and $y = -(x-3)^3$ on the same Cartesian plane.

**b** Sketch the graphs of $y = -\sqrt{x}$, $y = -\sqrt{x} + 3$ and $y = -\sqrt{x-4}$ on the same Cartesian plane.

**c** Sketch the graphs of $y = -\frac{1}{x}$, $y = -\frac{1}{x} + 4$ and $y = -\frac{1}{x-5}$ on the same Cartesian plane.

**12** Use your knowledge of transformations to sketch the graph of each of the following relationships by first finding any $x$- and $y$-intercepts. Label all intercepts with their coordinates.

**a** $x^2 + y^2 = 64$ **b** $y = 3\sqrt{x+1} - 6$ **c** $y = (x+2)^3 - 1$ **d** $y = \frac{1}{x+1} + 1$

**13** Sketch the following circles. Label the $x$- and $y$-intercepts with their exact coordinates in simplified surd form.

**a** $(x-1)^2 + (y-2)^2 = 4$ **b** $(x+5)^2 + (y-2)^2 = 16$

**14** Find the equation of the circle that has a centre at (2, 1) and passes through the point (6, 1).

**15 a** Sketch a graph of $y = \sqrt{x}$ and $y = -\sqrt{x}$ on the same Cartesian plane.

**b** Write one equation to describe all the points on both graphs. What is the shape of the graph for this equation?

**c** Sketch a graph of $y = x^2$ on the same Cartesian plane and label the coordinates where the two graphs intersect each other.

**d** Draw a dotted line that passes through the points where the two graphs intersect each other. Write an equation for this line.

**e** Describe the transformation required to turn the graph of $y = x^2$ to the graph in part **b**.

**16** Use your observations from question **15** to sketch graphs for each equation.

**a** $x = y^3$ **b** $x = \frac{1}{y}$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Circles
- Interactive skillsheet: Non-linear relationships
- Worksheet: Non-linear relationships
- CAS instructions: Graphing circles
- Topic quiz 5E

# Chapter summary

## Quadratics

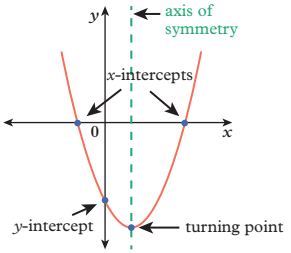
Quadratic equation: $ax^2 + bx + c = 0$

Quadratic relationship: $y = ax^2 + bx + c$

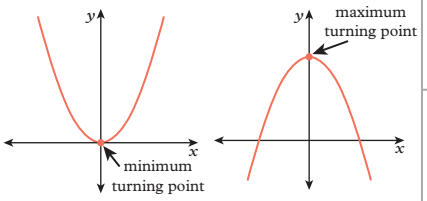
where $a$, $b$ and $c$ are constants.

- Produces a parabola shaped.

## Parabolas




## Turning points

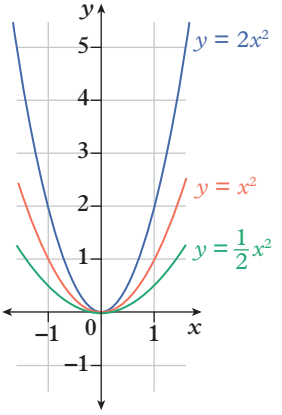



## Turning point form

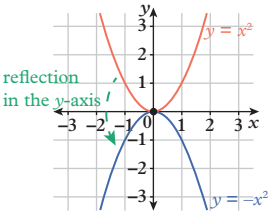
$y = a(x - h)^2 + k$

- $k$: vertical translation of $k$ units; If $k > 0$, up; If $k < 0$, down
- $a$: stretch in the $y$-direction; If $a > 0$, extend in the $y$-direction; If $0 < a < 1$, compress in the $y$-direction; But if $a < 0$, invert
- $h$: horizontal translation of $h$ units; If $h > 0$, right; If $h < 0$, left

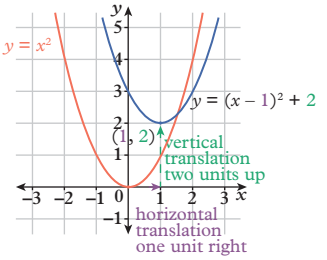
## Stretches in the $y$-direction




## Reflection




## Translation

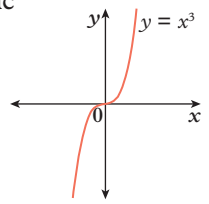



## The Null Factor Law

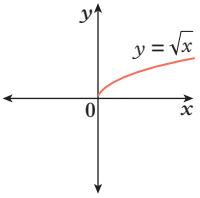
- If $a \times b = 0$, then $a = 0$ or $b = 0$.
- If $(x - p)(x - q) = 0$, then $x - p = 0$ or $x - q = 0$.

## Other non-linear relationships

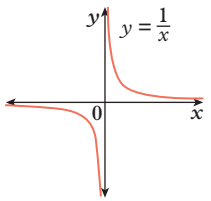
cubic




square root

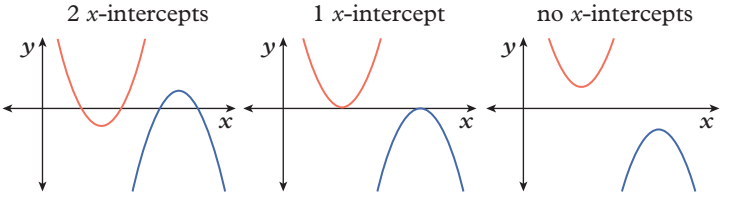



hyperbola




## $x$-intercepts

Let $y = 0$ and factorise the quadratic to solve for $x$.




## Circles

vertical translation of $k$ units

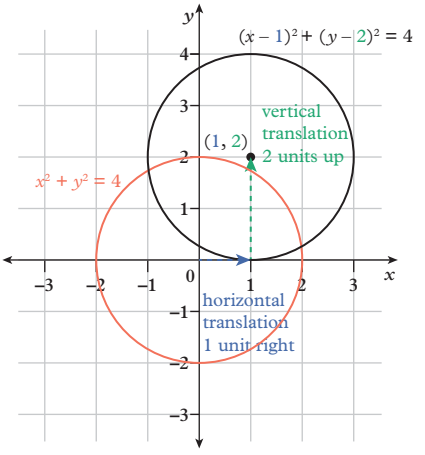
$k > 0$ up
$k < 0$ down

radius of $r$ units

$(x - h)^2 + (y - k)^2 = r^2$

horizontal translation of $h$ units

$h > 0$ right
$h < 0$ left



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# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams

## Multiple-choice

5A **1** The solution(s) to the quadratic equation $x(x + 1) = 0$ are:

**A** $x = -1$ **B** $x = 1$ **C** $x = -1$ or $x = 1$ **D** $x = 0$ or $x = -1$ **E** $x = 0$ or $x = 1$

5A **2** In the quadratic equation $x^2 - 13x - 48 = 0$ , $x =$

**A** $-3$ or $16$ **B** $-16$ or $3$ **C** $-12$ or $4$ **D** $-6$ or $8$ **E** $-8$ or $6$

5B **3** If a parabola intersects the $x$-axis when $x = -1$ and $-5$, the turning point could be:

**A** $(-3, 7)$ **B** $(-2, 2)$ **C** $(0, -3)$ **D** $(2, -6)$ **E** $(3, 4)$

5C **4** The coordinates of the $x$-intercepts of the graph of $y = x^2 - 4x - 12$ are:

**A** $(0, -6), (0, 2)$ **B** $(0, 6), (0, -2)$ **C** $(-6, 0), (2, 0)$ **D** $(6, 0), (-2, 0)$ **E** $(6, 0), (2, 0)$

5C **5** The $y$-coordinate of the $y$-intercept of the graph of $y = (x - 5)(x + 2)$ is:

**A** $0$ **B** $-2$ **C** $-10$ **D** $5$ **E** $10$

5D **6** The graph of $y = x^2$ is translated 5 units down. The equation of the translated graph is:

**A** $y = x^2 + 5$ **B** $y = 5x^2$ **C** $y = x^2 - 5$ **D** $y = (x + 5)^2$ **E** $y = \frac{1}{5}x^2$

5D **7** Which of the following transformations is **not** required to sketch the graph of $y = -4(x + 2)^2 + 1$?

**A** $y = x^2$ must be dilated.

**B** $y = x^2$ must be reflected in the $x$-axis.

**C** $y = x^2$ must be translated up.

**D** $y = x^2$ must be translated to the left.

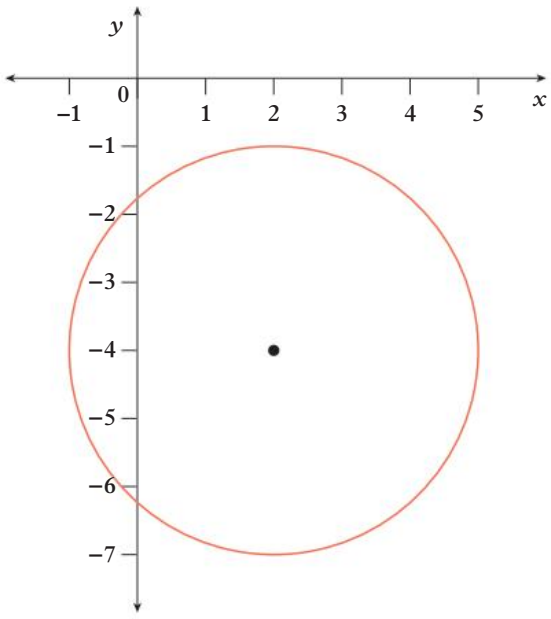
**E** $y = x^2$ must be rotated 90° in the clockwise direction.

5D **8** The coordinates of the turning point of the graph of $y = -(x - 4)^2$ are:

**A** $(0, 0)$ **B** $(4, 0)$ **C** $(-4, 0)$ **D** $(0, 4)$ **E** $(0, -4)$

Questions **9** and **10** refer to the graph of $(x - 2)^2 + (y + 4)^2 = 9$.

5E **9** The radius of the circle is:




**A** 1 unit **B** 2 units **C** 3 units **D** 4 units **E** 9 units

5E **10** The centre of the circle in question **9** is located at:

**A** $(-2, -4)$ **B** $(2, -4)$ **C** $(-2, 4)$ **D** $(4, -2)$ **E** $(-4, 2)$

## Short answer

5A **1** Solve:

**a** $x^2 - 5x + 6 = 0$ **b** $x^2 + x - 30 = 0$ **c** $x^2 + 9 = 0$ **d** $3x^2 - 36x = 0$

5B **2** Sketch a graph of the following relationships by first completing a table of values for $x$ from $-3$ to 3. Use your graph to identify:

**i** the coordinates of the $x$- and $y$-intercepts

**ii** the coordinates of the turning point

**iii** whether there is a maximum or a minimum turning point

**iv** the equation of the axis of symmetry.

**a** $y = 4x^2 - 4$ **b** $y = -4x^2 - 4x$ **c** $y = x^2 - 4x + 4$

5C **3** Complete the following for each quadratic relationship below.

**i** Find the coordinates of the $x$- and $y$-intercepts.

**ii** Find the coordinates of the turning point.

**iii** Sketch the graph.

**a** $y = -x^2 - 4x$ **b** $y = x^2 - x - 12$

**c** $y = (x + 5)(x - 4)$ **d** $y = -(x + 2)(x + 1)$

5D **4** Sketch a graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the $y$-axis at the intercept and label the turning point with its coordinates.

**a** $y = x^2 + 3$ **b** $y = (x - 1)^2$ **c** $y = x^2 - 5$ **d** $y = (x + 4)^2$

5D **5** Sketch the graph by performing transformations on the graph of $y = x^2$. Label the turning point on each parabola with its coordinates.

**a** $y = (x + 1)^2 - 1$ **b** $y = -(x + 3)^2 - 3$ **c** $y = (x - 4)^2 + 4$ **d** $y = -(x - 2)^2 - 2$

5E **6** Sketch graphs of the following relationships by first finding the $x$- and $y$-intercepts.

**a** $x^2 + y^2 = 9$ **b** $x^2 + y^2 = 81$

5E **7** Sketch each circle using transformations.

**a** $(x + 4)^2 + y^2 = 1$ **b** $x^2 + (y + 3)^2 = 9$

**c** $(x - 4)^2 + (y - 3)^2 = 16$ **d** $(x + 3)^2 + (y + 5)^2 = 49$

**8** Match the table of values with the correct graph.

**a**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | undefined | undefined | 0 | 1 | $\sqrt{2}$ |

**b**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $-8$ | $-1$ | 0 | 1 | 8 |

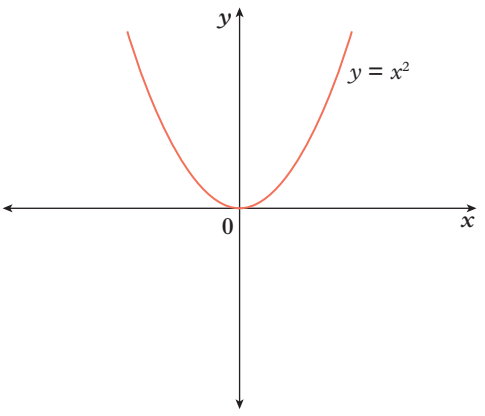
**c**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 4 | 1 | 0 | 1 | 4 |

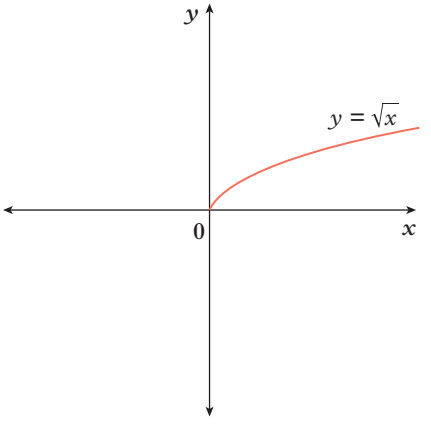
**d**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $-0.5$ | $-1$ | undefined | 1 | 0.5 |

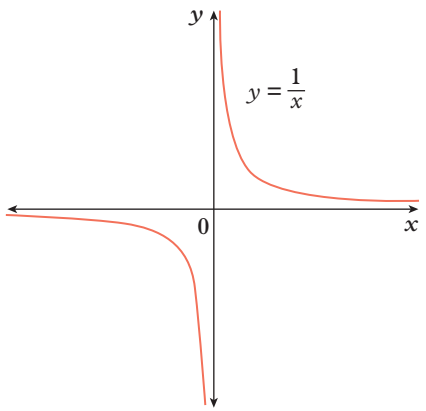
**A**



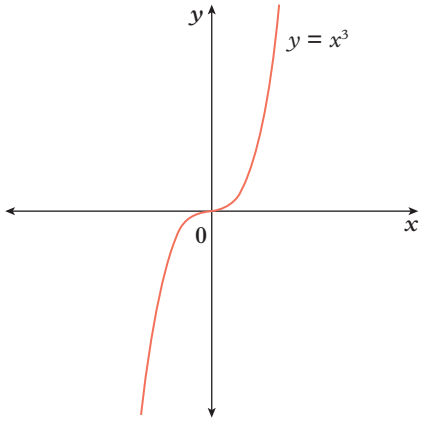

**B**



**C**




**D**




## Analysis

A parabolic through-arch bridge has an upper arch and a lower arch that are closely modelled by the following quadratic relationships.

$$\text{Lower arch: } h = -\frac{1}{120}d^2 + d$$

$$\text{Upper arch: } h = -\frac{1}{180}(d - 60)^2 + 40$$

where $h$ is the height of the arch at a horizontal distance of $d$ from the left side of the bridge. Both $h$ and $d$ are in metres.

**a** Sketch the two arches on the same Cartesian plane from $d = 0$ m to $d = 120$ m.

**b** For each parabola, determine:

- **i** the coordinates of the $y$-intercepts
- **ii** the coordinates of the turning points
- **iii** the equation of the axis of symmetry.

**c** What is the height of the upper arch above the lower arch at each end of the bridge?

**d** What is the span of the bridge? Show this:

- **i** graphically, by referring to your graphs in part **a**
- **ii** algebraically, by solving a quadratic equation.

The distance between the arches does not remain constant over the span of the bridge.

**e** Find the height of the highest point of the:

**i** lower arch **ii** upper arch.

**f** How far apart are the two arches at their highest point?

**g** How far apart are the two arches at a horizontal distance of 30 m from the left side of the bridge?

**h** Describe the distance between the two arches over the span of the bridge.

# Semester 1 review

## Short answer

**1** Round the following as specified.

**a** 20.040 194 (3 decimal places)

**b** 20.040 194 (3 significant figures)

**c** 0.000 782 718 (4 decimal places)

**d** 0.000 782 718 (4 significant figures)

**2** Determine the following.

**a** The cost for 50 m of timber if 12 m costs $138.

**b** The time taken to travel 3.2 km at 5 km/h.

**c** The gross income if someone who earns $18/hour worked 42 hours, of which 3 hours were at time-and-a-half and 2.5 hours were at double-time.

**d** The value of $y$ for $x = 36$ if $y$ is directly proportional to $x$ and $y = 90$ when $x = 12$.

**3** Determine the following.

**a** The price per kilogram of chicken fillets if 2.5 kg costs $22.50.

**b** The fortnightly wage of someone whose yearly salary is $85 800.

**c** The average speed of a car (km/h) if it travels 44 km in 33 minutes.

**d** The constant of proportionality, $k$, if $y$ is directly proportional to $x$ and $y = 84$ for $x = 16$.

**4** Determine the following amounts, correct to the nearest cent.

**a** The GST on a pre-GST sale price of $234.50.

**b** The selling price, when the cost price of $62 is marked up 80%.

**c** The sale price when the selling price of $89.99 is discounted by 20%.

**d** The value of sales made when 12% commission of the sales is $1800.

**5** Determine the following percentages.

**a** 35 as a percentage of 140

**b** 500 as a percentage of 80

**c** The percentage profit of the cost price when the cost price is $20 and the selling price is $45.

**d** The percentage profit of the selling price when the cost price is $240 and the selling price is $500.

**6** **a** $4500 is invested at a simple interest rate of 8% p.a. How much interest is earned after 9 years?

**b** $150 is borrowed and $35 interest is charged after 2 years. At what annual simple interest rate was the $150 borrowed, correct to two decimal places?

**c** $5 interest is earned on an investment after 2 weeks with a simple interest rate of 6% p.a. How much was invested, correct to the nearest cent?

**d** $100 000 is borrowed at a simple interest rate of 7.2% p.a. After how many months is $1200 of interest charged?

**7** Write the following numbers in index form (as the product of powers of prime factors).

**a** 81 **b** 500 **c** 216 **d** 2700

**8** Write the following as a basic numeral.

**a** $7^3$ **b** $3 \times 5^3$

**c** $8.213 \times 10^4$ **d** $1.0530 \times 10^{-2}$

**e** $2^{-3} \times 3^3$ **f** $\left(\frac{5}{7}\right)^{-2}$

**9** Write the following in index form with positive indices.

**a** $(2^3 \times 7^2)^4 \times (2^9 \times 7^5)^2$

**b** $\dfrac{11^{12} \times 13^9 \times 17^6}{11^4 \times 13^9 \times 17^{15}}$

**c** $101^{-5} \times 97^5 \times 101^{-7} \times 97^{-9}$

**d** $\left(\dfrac{41^{-3}}{57^5}\right)^{-8} \times \left(\dfrac{57^{-4}}{41^{-7}}\right)^6$

**10** Simplify the following. Write your answer using positive indices.

**a** $(ab^3)^4 \times (ba^6)^2$

**b** $\dfrac{c^2de^8}{c^{12}de^7}$

**c** $f^{-4}g^6 \times f^3g^{-2}$

**d** $\dfrac{(p^8q^{-4})^3 \times (p^{-5}q^3)^8}{(p^{-3}q^{-6})^2 \times (p^{-2}q^{-9})^3}$

**11** State the number of significant figures.

**a** 946 025 **b** 120 000 000 **c** 0.002 520 20 **d** 630.00

**12** Write the following numbers in scientific notation.

**a** 751 425 963, correct to four significant figures

**b** 25.360 141, correct to five significant figures

**c** 0.000 085 945 6, correct to two significant figures

**d** 75 000, correct to three significant figures

**13** Expand the following products.

**a** $8(7 - 2x)$ **b** $-2x^3(3x^2 - 8)$ **c** $(x + 2)(x - 7)$

**d** $(5x - 2)(2y + 3)$ **e** $5 - 4(3x + 7)$ **f** $2x(x + 9) + 3(x^2 - 4x + 2)$

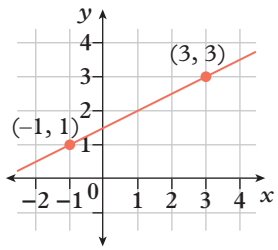
**14** Factorise the following expressions.

**a** $12a + 18b^2 - 6cd$ **b** $21g^2 - 7g$ **c** $a^2 - 25$

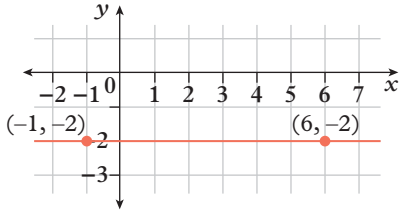
**d** $8m^2 - 18n^2$ **e** $(t + 2)^2 - 49$ **f** $r^2 + 5r - 36$

**15** Calculate the gradient of the following.

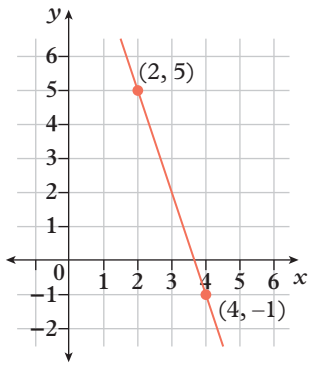
**a**



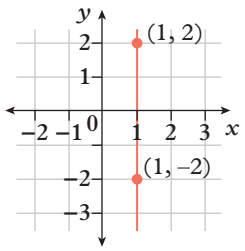

**b**



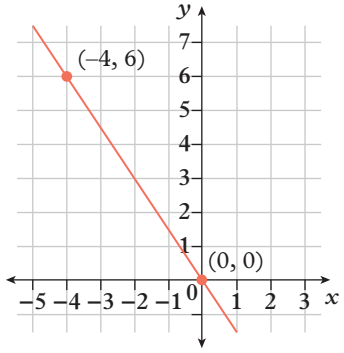

**c**




**d**




**e**



**16** Solve the following equations for $x$.

**a** $12x - 7 = 4$ **b** $9 - 2x = 15$ **c** $4x + 9 = 6x - 7$

**d** $\frac{21}{x} = 3$ **e** $\frac{4}{x} = 5$ **f** $(x + 3)(x - 7) = 0$

**g** $(9x - 1)(5x + 3) = 0$ **h** $x^2 + 12x + 35 = 0$

**17** Plot the graphs of the following equations by completing each table of values and plotting the coordinates.

**a** $y = 3x - 4$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** $y = -\frac{5}{2}x$

| $x$ | $-4$ | $-2$ | 0 | 2 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**c** $y = (x + 1)^2 - 5$

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**d** $y = -x^2 + 6x - 2$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**18** Determine the $x$- and $y$-intercepts for the graphs of each of the following equations.

**a** $y = 3x - 6$ **b** $5x + 8y = -60$

**c** $y = (x - 5)(x + 9)$ **d** $y = x^2 - 6x - 40$

**19** Sketch the graphs of the following equations.

**a** $2x + 3y = 12$ **b** $y = 2$

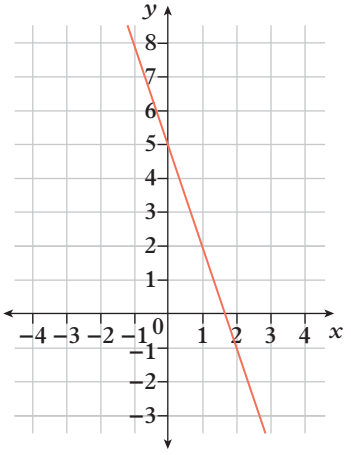
**c** $y = 2x$ **d** $y = x^2$

**e** $y = (x - 2)(x + 3)$ **f** $y = x^2 - 5x - 6$

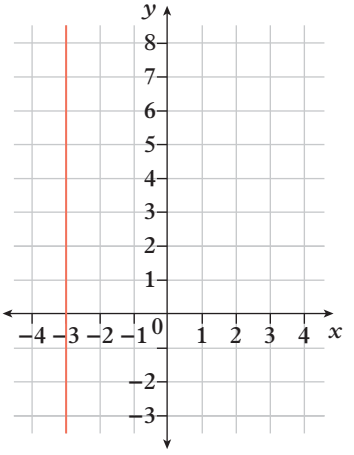
**g** $y = (x + 4)^2 - 4$ **h** $(x + 1)^2 + (y - 2)^2 = 16$

**20** Determine the equations of the following graphs.

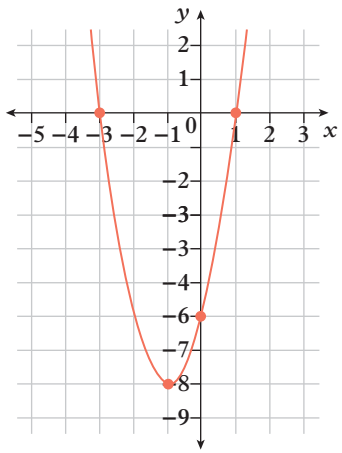
**a**



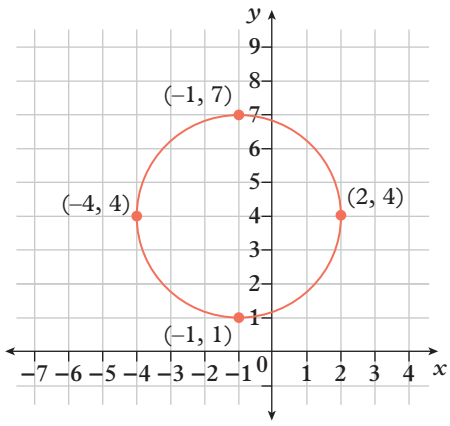

**b**



**c**

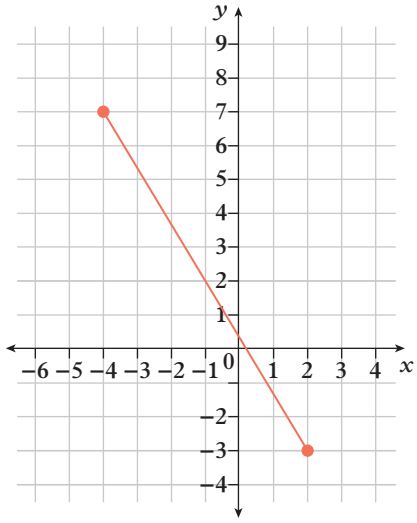



**d**

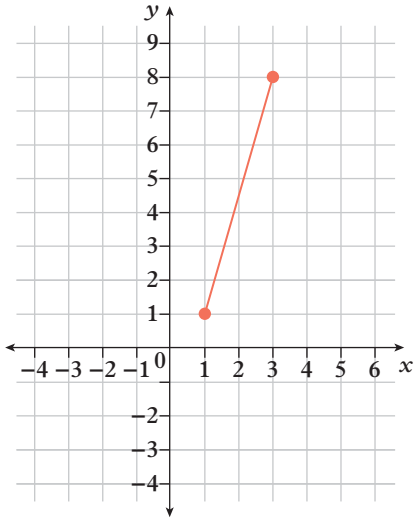



**21** Find the midpoint of the following straight-line segments.

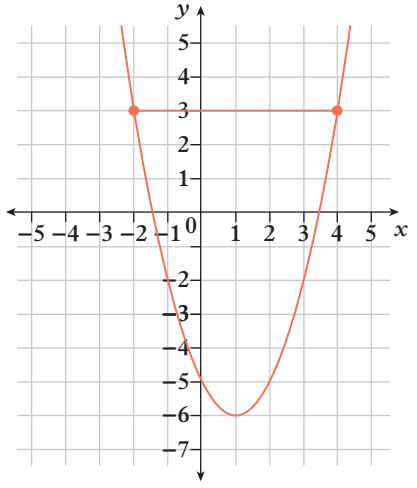
**a**



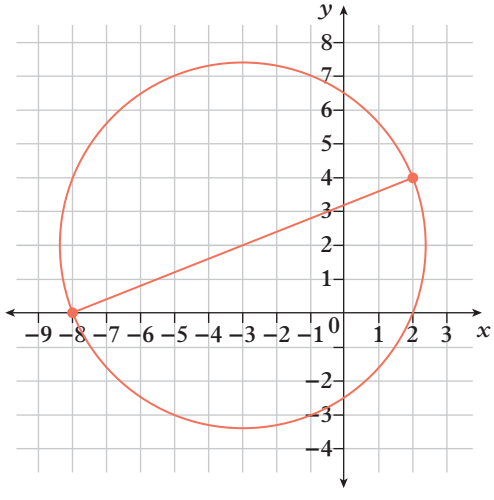

**b**




**c**

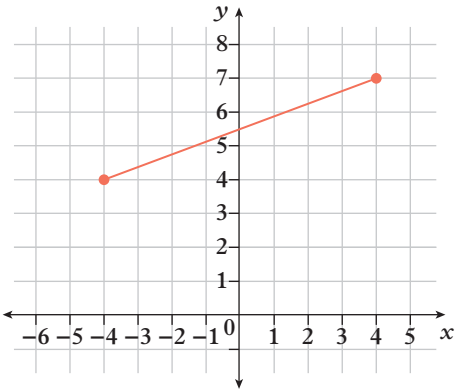



**d**

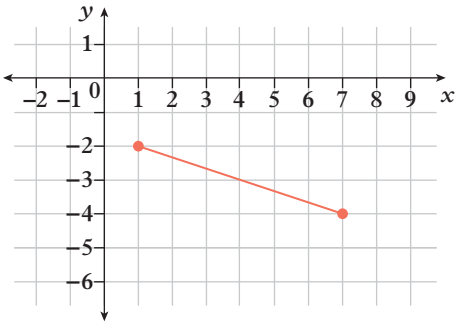


**22** Determine the length of the following line segments, correct to two decimal places.

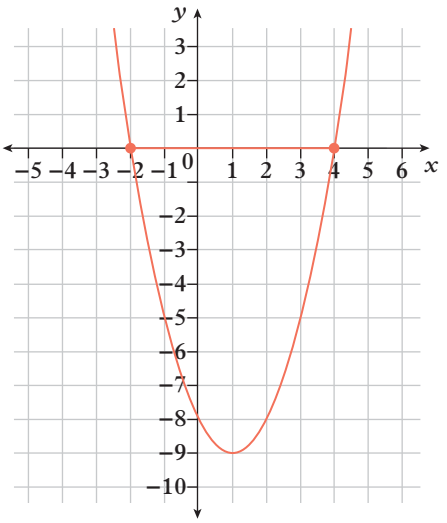
**a**



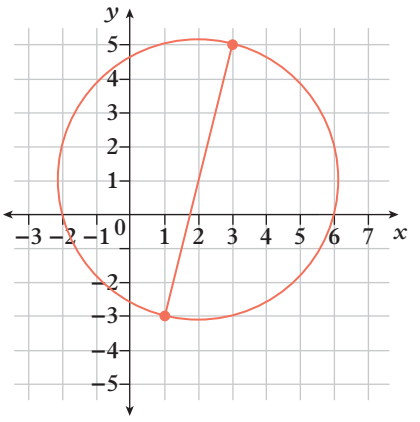

**b**




**c**




**d**




**23** Simplify the following.

**a** $\sqrt{32}$

**b** $\sqrt{180}$

**c** $\sqrt{84}$

**d** $\sqrt{1008}$

**24** Simplify each of the following. (Write as an expression with a single surd.)

**a** $\sqrt{3} \times \sqrt{11}$

**b** $\sqrt{45} \div \sqrt{15}$

**c** $7\sqrt{2} \times 4\sqrt{6}$

**d** $\dfrac{24\sqrt{200}}{8\sqrt{20}}$

## Analysis

**1** Janessa is driving at an average speed of 80 km/h from her workplace to her home, a distance of 25 km.

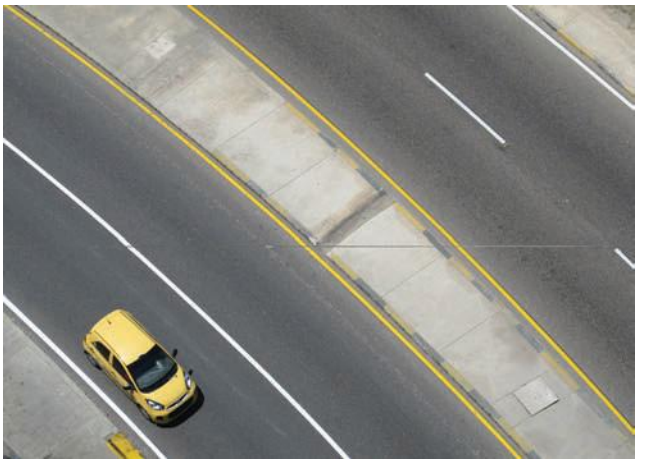
**a** Write an equation in terms of the distance Janessa has travelled, $d$ kilometres, and the time elapsed since leaving work, $t$ minutes.

**b** State the dependent and independent variables.

**c** Determine the expected time it will take for Janessa to drive home in minutes and seconds.

**d** After 10 minutes, Janessa has to drive at 40 km/h for 2 km of roadwork before returning to her average speed of 80 km/h. How much longer will it take for her journey home?

**e** Draw a distance-time graph of Janessa's journey home.



**2** Ben has a simple interest bank account earning 6.2% per annum calculated and paid monthly on the minimum monthly balance. Ben's bank statement for October is shown below.

| Date | Transaction | Credits $ | Debits $ | Balance $ |
|---|---|---|---|---|
| 01/10 | Opening balance | | | 10 000.00 |
| 05/10 | Grocery shopping | | −213.50 | 9786.50 |
| 14/10 | Wage | +1880.00 | | 11 666.50 |
| 19/10 | EFTPOS purchase | | −350.00 | 11 316.50 |
| 28/10 | Wage | +1880.00 | | 13 196.50 |
| 31/10 | Interest | | | |

**a** Calculate the interest that Ben's account earned in October.

**b** If Ben worked four 40-hour weeks, what is his hourly net wage?

When grocery shopping on the 5/10, Ben had to decide between two options:

- 12 rolls of toilet paper for $12.96
- 24 rolls of toilet paper for $25.68

**c** Which option is the best buy?

**d** What percentage of the grocery shopping was the best buy option, correct to one decimal place?

**e** The EFTPOS purchase on the 19/10 was after a 30% discount. Determine the price before the sale.

**3** Lisa has modelled the profit she will likely earn in her candle business, $\$P$, when selling $n$ candles (units) with the equation $P = -n^2 + 410n - 18\,000$.

**a** Calculate the profit earned or loss made when she sells these units:

**i** 100 **ii** 200 **iii** 400 **iv** 50 **v** 360

**b** **i** Determine the number of units that would make the company the most profit. Hint: Find the midpoint between 50 and 360.

**ii** State the amount of profit Lisa would make.

**c** Use your answer for part **b** to write the equation for the profit in the form $P = a(n + b)^2 + c$.

**d** If the selling price per unit is $500 and the number of units in part **b** is sold, determine:

**i** the total cost

**ii** the percentage the profit is of the total revenue earned, correct to one decimal place.

**e** If the selling price per unit is $500, write an equation for:

**i** the total revenue, $\$R$

**ii** the total cost, $\$C$

# EXPLORATIONS 1

## 1 Overtake

A car, a van, a truck and a bicycle are all travelling in the same direction on the same road, each at its own constant speed. At 10 am, the car overtakes the van; at noon, it overtakes the truck; at 2 pm it overtakes the bicycle. At 4 pm, the truck overtakes the bicycle. At 6 pm, the van overtakes the truck.

**a** Suppose the speed of the car is 120 km/h and the speed of the truck is 80 km/h.

**i** Find the speeds of the van and the bicycle. Then find the time at which the van overtakes the bicycle.

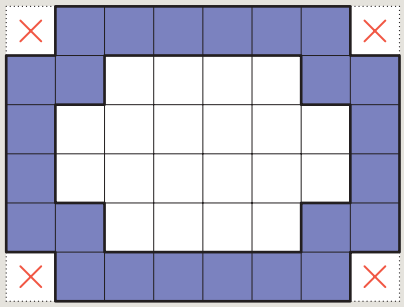
**ii** A motorbike overtakes the van at 10 am and the bicycle at 3 pm. What is its speed?

**iii** A semi-trailer travelling at 80 km/h in the opposite direction passes the van at 6 pm. At what time does it pass the bicycle?

**b** Let $c$ and $t$ represent the speeds, in km/h, of the car and truck, respectively. Find the speeds of the van and the bicycle in terms of $c$ and $t$. Then show that the time at which the van overtakes the bicycle is always the same, regardless of the speeds of the car and the truck.

## 2 Borders

The design shown is formed by removing one square from each corner of a rectangular grid and shading a border of thickness 1 unit. In this $6 \times 8$ example, there are 24 shaded squares in the border and 20 unshaded squares in the interior.



**a** Ignoring rotations, we want to find the number of such designs that have the same numbers of border and interior squares. Follow these steps:

**i** Let the dimensions of the original rectangular grid be $x \times y$. Find expressions, in terms of $x$ and $y$, for the number of border squares and the number of interior squares.

**ii** Setting the expressions equal, rearrange the equation to get 0 on the right-hand side.

**iii** The left-hand side can almost be fully factorised as a binomial product of the form $(x - ___)(y - ___)$. What number do you need to add to both sides of the equation to make this work?

**iv** Remember that $x$ and $y$ are positive integers, so the factorised expression on the left-hand side of your equation should match up with a factorisation of the number on the right-hand side. Deduce that there are six possible pairs of positive integer solutions for $x$ and $y$.

**v** Why does this mean that there are only three possible designs?

**b** Adapt the above methods to find, ignoring rotations, the number of such designs in which there are:

**i** twice as many interior squares as border squares

**ii** twice as many border squares as interior squares.

## 3 Absurd surds

What does the infinite surd expression $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$ equal? Does it even equal anything? Maybe as you add more and more sixes, the result gets larger and larger without ever settling down.

**a** One way to get an idea of what's going on is to use the 'answer' button on your scientific calculator, as follows. The instructions for your model might be slightly different.

- Enter √ 6 = – this calculates the first approximation of $\sqrt{6}$ and stores it as the most recent answer.
- Enter √ 6 + Ans = – this calculates the next approximation $\sqrt{6 + \text{Ans}}$ based on the previous answer.
- Now repeatedly enter = – this repeats the previous calculation as often as you like.
- What number do the approximations appear to get closer to?

**b** Now let's prove that your observation in part **a** is correct. Let $x$ be the final answer, then notice that $x = \sqrt{6 + x}$. Square both sides and solve the quadratic equation. Which solution can you ignore and why?

**c** **i** Adapt the method in part **b** to calculate the exact values of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}$ and $\sqrt{2 - \sqrt{2 - \sqrt{2 - \ldots}}}$

**ii** Find two different infinite surd expressions that are equal to 5.

## 4 Snowflakes

There is a famous conjecture in mathematics about so-called *hailstone* numbers. Starting with any positive integer, we halve the number if it is even, otherwise we triple it and add 1. Now repeat this process to form a sequence of numbers called hailstones, because of the way they rise and fall like real hailstones forming in a cloud. The conjecture states that every number will eventually 'fall to the ground' by reaching the number 1. Even though this idea dates back to 1937, when it was first introduced by Lothar Collatz, mathematicians still do not know whether this will always happen. You might like to check what happens when you start with 27; it takes a surprisingly long time for the hailstones to fall!

Let's look at a gentler version of this problem: *snowflake* numbers. Again, if a number is even we halve it, otherwise we triple it and *subtract* 1. For example, the snowflakes of 11 are 32, 16, 8, 4, 2, 1, 2, 1 . . . and so on. The sequence starting with 11 has a cycle of length 2 that repeats indefinitely, which we can represent as follows:

$$11 \longrightarrow 32 \longrightarrow 16 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$$

We call them snowflakes because, being lighter than hailstones, they might never fall to the ground!

**a** Show that the snowflakes of 12 also result in a cycle of length 2.

**b** Show that the snowflakes of 13 result in a cycle of length 5.

**c** Find a cycle of snowflakes with length greater than 5.

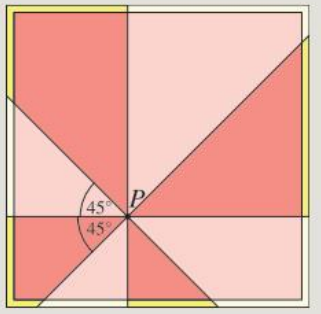
**d** Explain why no cycle of snowflakes can contain a multiple of 3.

**e** Find all cycles of length 5.

**f** Find an odd number whose snowflakes are alternately even and odd, which do not form a cycle for at least 1000 terms.

## 5 Square pizzas

Angelo likes square pizzas because they fit inside square boxes more efficiently than round pizzas do. However, not having as many symmetries, he worries that it might not be possible to share square pizzas as fairly as round ones. By 'sharing fairly', Angelo means that everyone gets the same amount of topping (total area of their slices) and the same amount of crust (total length of boundary shared with the original square).




Angelo makes a cut parallel to one pair of sides, and then makes a second cut parallel to the other pair of sides. Through the point $P$ where these two cuts meet, he makes two more cuts at 45° to the others, as shown.

**a** A pizza has side length 20 cm and the point $P$ is 8 cm from the left edge and 6 cm from the bottom edge. Find the length of each piece of crust (ignoring its thickness) and the area of each slice (including the crust).

**b** Verify that the pizza in part **a** will be shared fairly among two people if they take alternate slices, shaded lighter and darker in the diagram.

**c** Show that, regardless of the size of the pizza and the location of the point $P$, the method above will always share a pizza fairly among two people. (Hint: Use coordinates $(0, 0)$, $(c, 0)$ and $(a, b)$ for the bottom-left corner, bottom-right corner and intersection point $P$, respectively. What if $P$ is on a diagonal? What if it isn't?)

**d** Next, Angelo is interested in sharing a square pizza fairly among three people. He wants to do this by making three straight cuts from a common point somewhere inside the square. Is this possible?

**e** For what values of $n$ is it possible to share a square pizza fairly among $n$ people by making $n$ straight cuts from a common point?

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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Problemo
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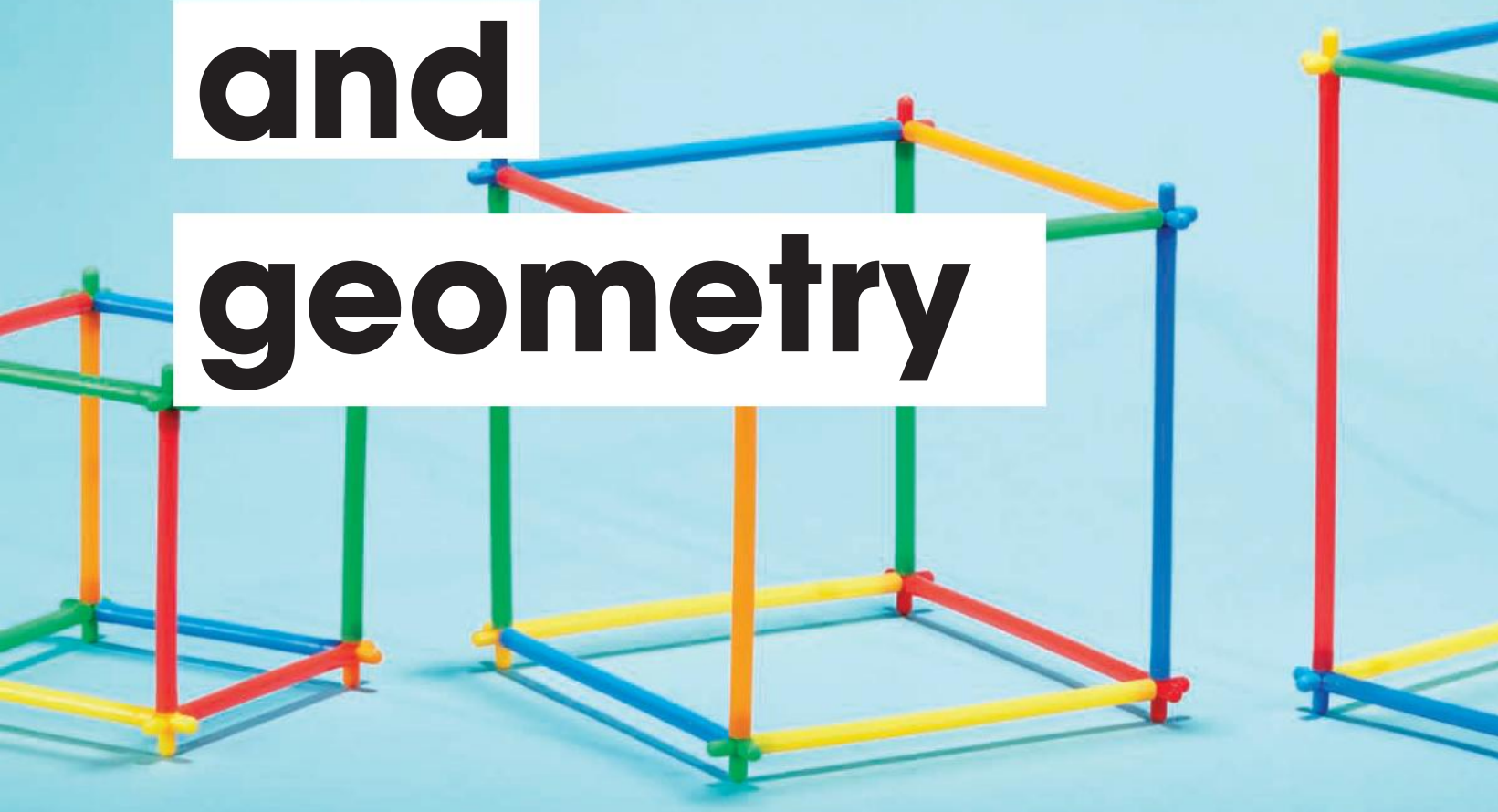
# 6

# Measurement and geometry



## Index


**6A** Area of composite shapes
**6B** Surface area
**6C** Volume and capacity
**6D** Dilations and similar figures
**6E** Similar triangles


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Area of quadrilaterals
- ✔ Area of a circle
- ✔ Identifying shapes within composite shapes
- ✔ Congruent triangles

## VCAA curriculum links

- Calculate the areas of composite shapes (VCMMG312)
- Calculate the surface area and volume of cylinders and solve related problems (VCMMG313)
- Solve problems involving the surface area and volume of right prisms (VCMMG314)
- Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (VCMMG316)
- Solve problems using ratio and scale factors in similar figures (VCMMG317)

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## Materials

- ✔ Calculator

# 6A Area of composite shapes

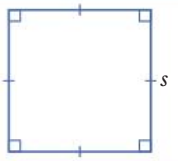
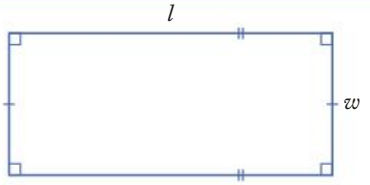
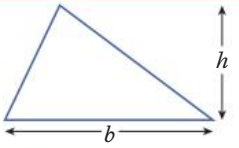
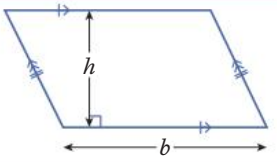
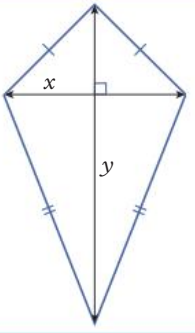
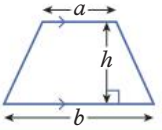
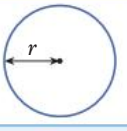
## Learning intentions

✔ I can calculate the areas of composite shapes.

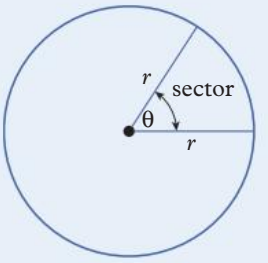
**Inter-year links**

| | |
|---|---|
| Year 5/6 | Understanding area |
| Year 7 | 9D Area of a triangle |
| Year 8 | 8E Area of a circle |
| Year 10 | 9A Area review |

## Area of simple shapes

| Shape | Example | Formula |
|---|---|---|
| Square |  | $A = s^2$ |
| Rectangle |  | $A = lw$ |
| Triangle |  | $A = \frac{1}{2}bh$ |
| Parallelogram/rhombus |  | $A = bh$ |
| Kite/rhombus |  | $A = \frac{1}{2}xy$ |
| Trapezium |  | $A = \frac{1}{2}(a + b)h$ |
| Circle |  | $A = \pi r^2$ |

# Sectors



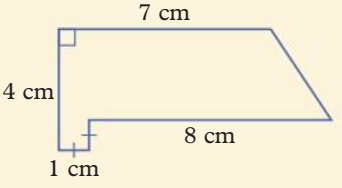

- A **sector** is a portion of a circle formed by two radii and part of the **circumference** (an **arc**).
- The area of a sector can be found using $\frac{\theta}{360} \times \pi r^2$, where θ (the Greek letter theta) is the size of the angle in degrees between the two radii.
  - → This formula is derived from the formula for the area of a circle, $A = \pi r^2$, where $\frac{\theta}{360}$ represents the proportion of a full circle that the sector represents.

# Composite shapes

- A **composite shape** can be split up into two or more simple shapes.
- To calculate the area of a composite shape, follow these steps:
  1. Split the figure into simple shapes that have known area formulas.
  2. Calculate any missing dimensions.
  3. Calculate the areas of the individual shapes using the area formulas.
  4. Add or subtract the areas to calculate the total area.
- Remember that area is a measure of two-dimensional space so the units of measurement must be squared: $mm^2$, $cm^2$, $m^2$, $km^2$

## Example 6A.1 Calculating the area of a composite shape using addition

Calculate the area of this composite shape.




**THINK**

1. Use dotted lines to split the composite figure into simple shapes.
2. Find any missing dimensions for each individual shape and label these on your figure.
3. Use area formulas to calculate the areas of the simple shapes.
4. Add the areas together to find the total area of the composite shape. Remember to include the appropriate unit.

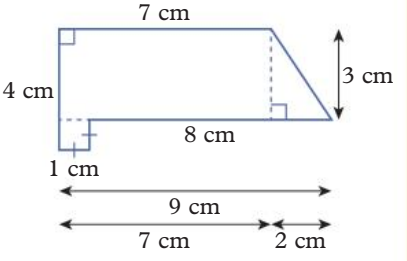
**WRITE**

$A_{rectangle} = lw$
$= 7 \times 3$
$= 21\ cm^2$

$A_{square} = s^2$
$= 1^2$
$= 1\ cm^2$

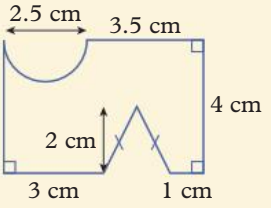
$A_{triangle} = \frac{1}{2}bh$
$= \frac{1}{2} \times 3 \times 2$
$= 3\ cm^2$

Total area $= 21 + 1 + 3$
$= 25\ cm^2$



## Example 6A.2 Calculating the area of a composite shape using subtraction

Calculate the area of this composite shape, correct to one decimal place.




**THINK**

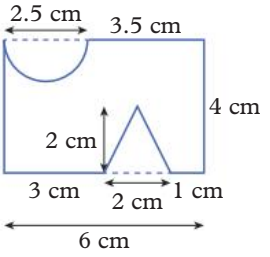
1 Use dotted lines to split the composite figure into simple shapes.

2 Find any missing dimensions for each individual shape and label these on your figure.

3 Use area formulas to calculate the areas of the simple shapes.

4 Subtract the smaller areas from the main area to find the total area of the composite shape. Remember to round your answer and include the appropriate unit.

**WRITE**




$A_{\text{rectangle}} = lw$

$= 6 \times 4$

$= 24\,\text{cm}^2$

$A_{\text{triangle}} = \frac{1}{2}bh$

$= \frac{1}{2} \times 2 \times 2$

$= 2\,\text{cm}^2$

$A_{\text{semicircle}} = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 1.25^2$

$\approx 2.45\ldots\,\text{cm}^2$

Total area $= 24 - 2 - 2.45\ldots$

$= 19.54\ldots$

$\approx 19.5\,\text{cm}^2$

**Helpful hints**

- ✔ When calculating the areas of composite shapes, a useful technique can be to number the different parts, e.g. $A_1$ = Area of shape 1. This is useful when there is more than one of a particular type of simple shape in the composite figure.
- ✔ Many composite shapes can be split up in more than one way. There is not a correct way to split up a composite shape, but you should look for the easiest method. However the shape is split up, the total area should be the same!

ANS p527

# Exercise 6A Area of composite shapes

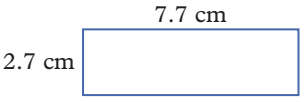
1–3, 4(b, c, e), 5–7, 8(c, d, f), 9, 12, 13(a, b)

1, 2, 4, 5, 7, 8(b, d, e, f), 10, 12, 13(c, d, f), 15, 18(a, b)

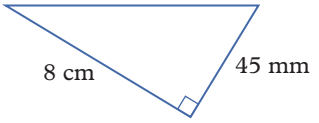
2, 4, 5, 8(b, e, f), 10, 11, 13(d, e, f), 14, 16, 17, 18(b, c)

**1** Calculate the area of these simple shapes. Give your answers to two decimal places where necessary.

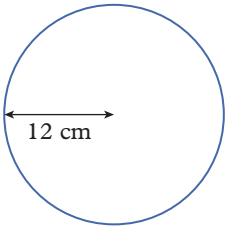
**a**



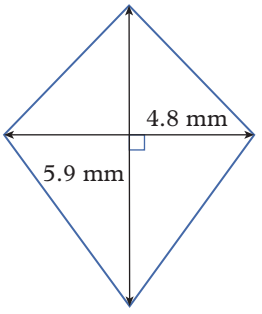

**b**



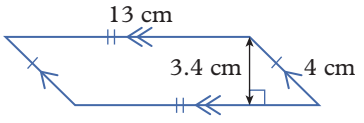

**c**



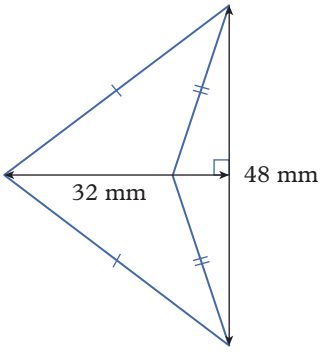

**d**




**e**

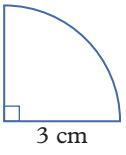



**f**

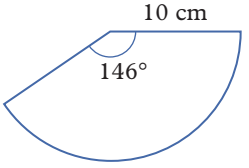



**2** Calculate the area of each sector correct to two decimal places.

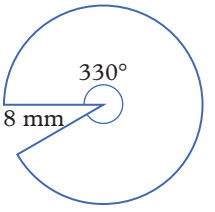
**a**




**b**

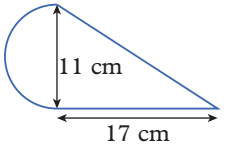



**c**




**3** Identify the basic shapes within each composite shape.

**a**

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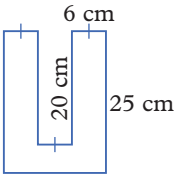
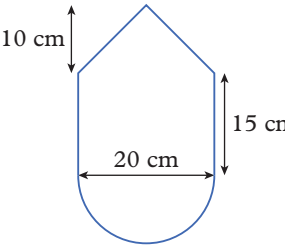



**c**

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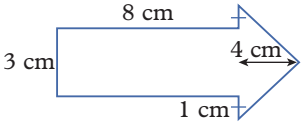


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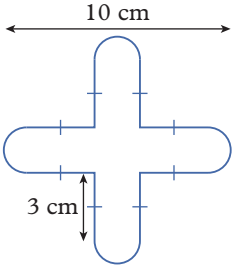


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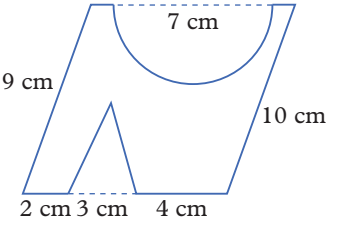
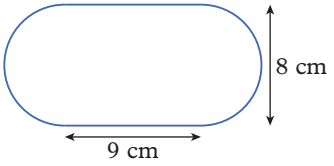



UNDERSTANDING AND FLUENCY

**6A.1** **4** Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

**a**

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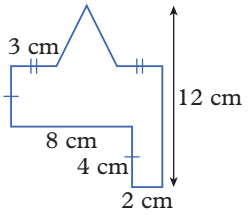
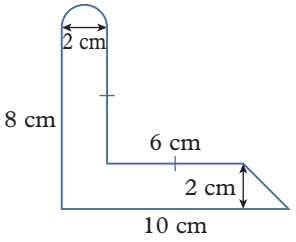



**c**

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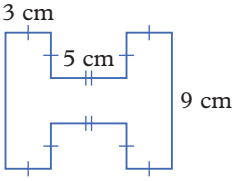


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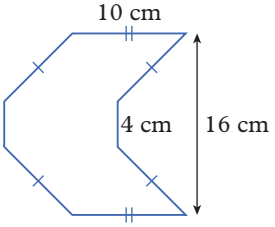


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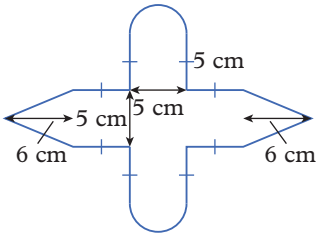
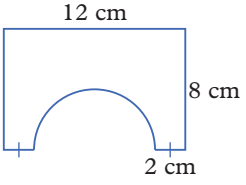



**6A.2** **5** Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

**a**

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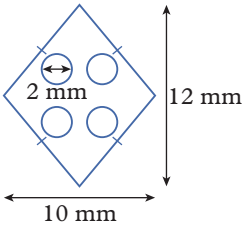


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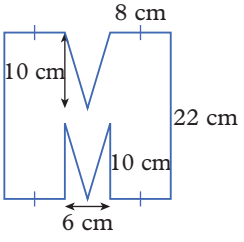


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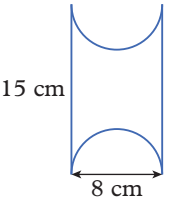
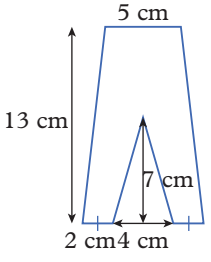
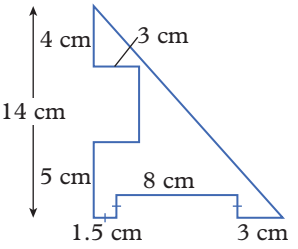



**e**

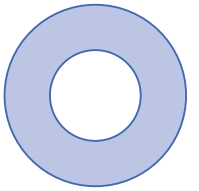



**f**




**6** Calculate the area of each composite shape in question **3**. Give your answers to two decimal places where necessary.

**7** This figure is known as an annulus (plural annuli).



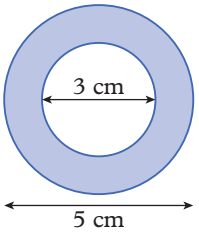
**a** What is an annulus?

**b** Explain how you can find the area of this annulus using a formula.

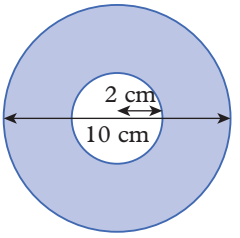
**c** Find the area of this annulus if it has an outer diameter of 16 cm and an inner diameter of 9 cm. Give your answer to two decimal places.

**8** Calculate the area of each annulus correct to two decimal places.

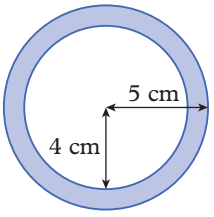
**a**



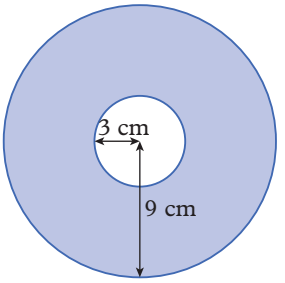

**b**



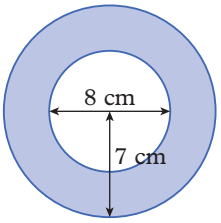

**c**



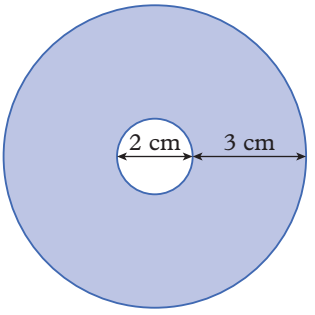

**d**



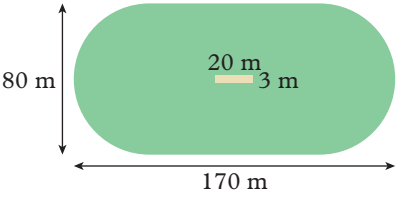

**e**



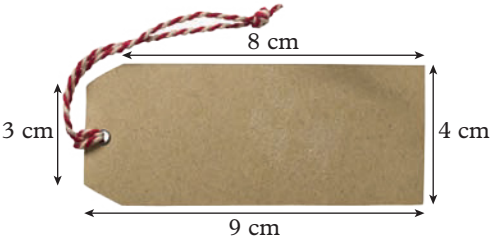

**f**




**9** What area of grass needs to be mowed on this oval, assuming that the cricket pitch is artificial and does not require mowing? Give your answer to two decimal places.




**10** Emily makes and sells custom tags through the website *Etsy*.




**a** What is the area of cardboard in a tag with the dimensions shown?

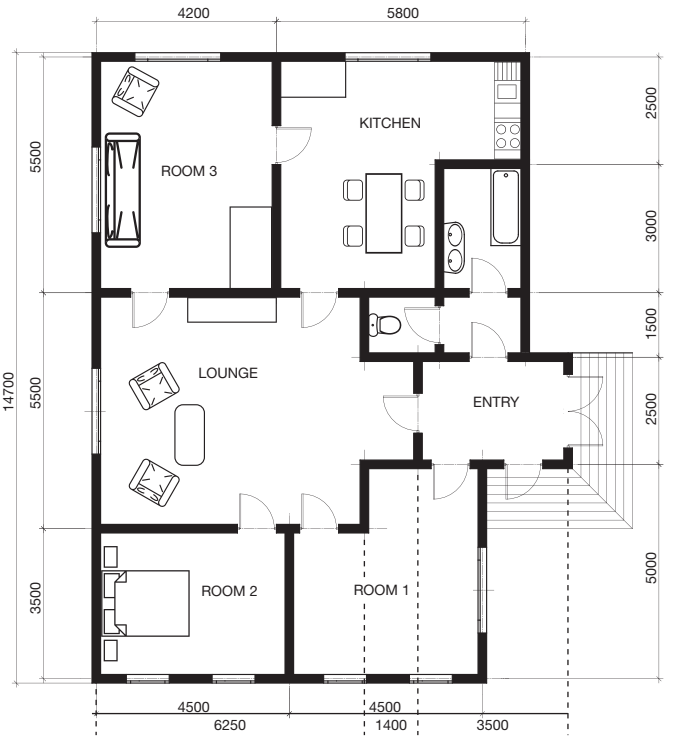
**b** What is the total area of cardboard required to make 550 tags with the dimensions shown? Explain why Emily might need to buy more than this amount of cardboard in order to make the tags?

**11** Shannon wants to lay new carpet in his house. Consider the floor plan with measurements in millimetres.

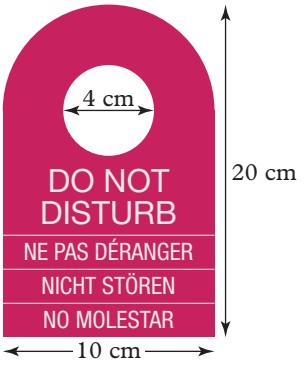
**a** What area of the house would he need to cover if everything except the bathroom, kitchen and toilet was to be carpeted? Write this in square metres. (Hint: You may find it easier to convert the measurements to metres.)

**b** How much would it cost if the carpet is priced at \$45/m²?

**c** How much money would he save if he chose a cheaper carpet at \$30/m²?

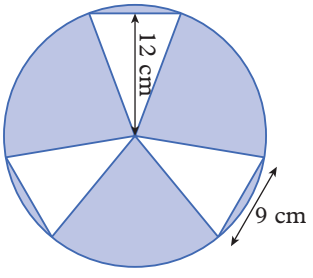



**12** Find the amount of cardboard in this DO NOT DISTURB sign correct to two decimal places.

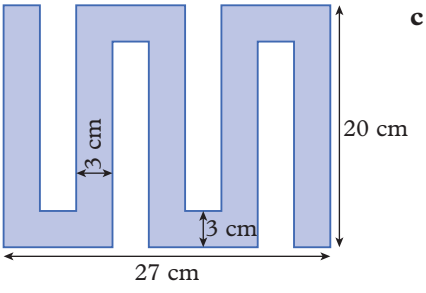



**13** Calculate the shaded area of each shape. Give your answers to two decimal places where necessary.

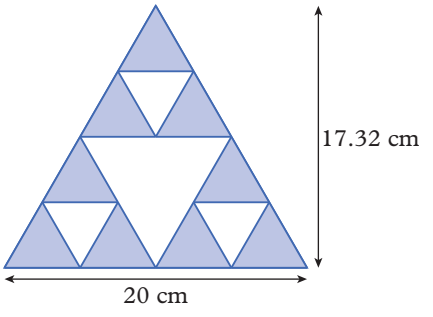
**a**



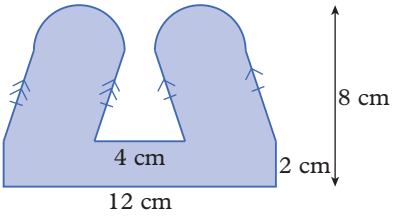

**b**



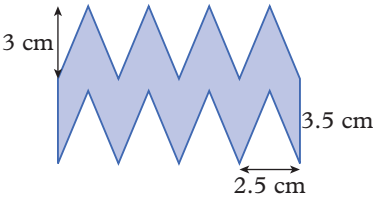

**c**




**d**




**e**

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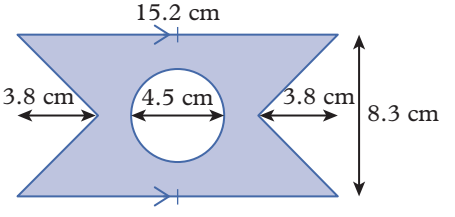
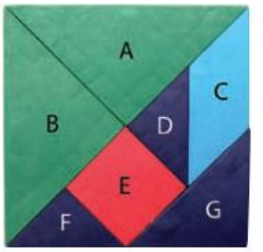


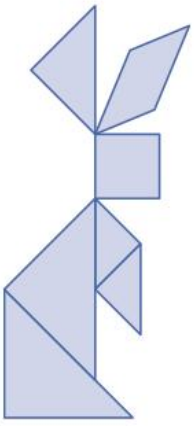
**14** The tangram is an ancient Chinese puzzle that consists of seven pieces (usually placed in a square as shown) that can be rearranged to form a variety of shapes.



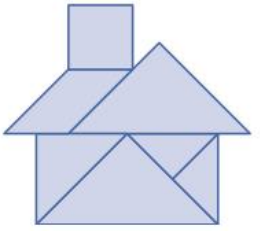

**a** What is the area of each individual piece in this tangram square if the entire square has side lengths of 10 cm?

**b** What is the total area of each of these shapes that can be made from the seven pieces of the tangram?

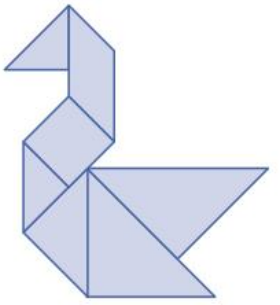
**i**



**ii**



**iii**

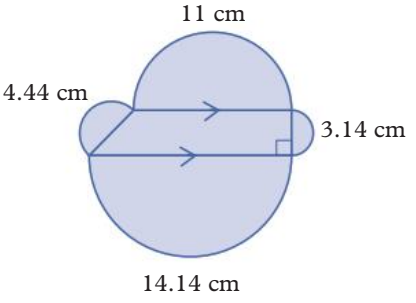


**15** Find the area of an annulus with an outer circumference of 35 cm and an inner circumference of 25 cm. Give your answer to two decimal places.

**16** A circular skirt, without hems, is cut from a circle of material with a hole for the waist. If a particular circular skirt had an inner circumference of 70 cm and was 60 cm in length when worn, what was the area of material used to create the skirt? Give your answer to two decimal places.

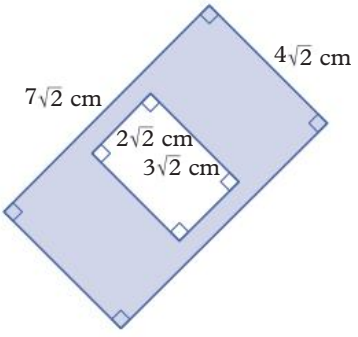


**17** Calculate the shaded area correct to two decimal places.

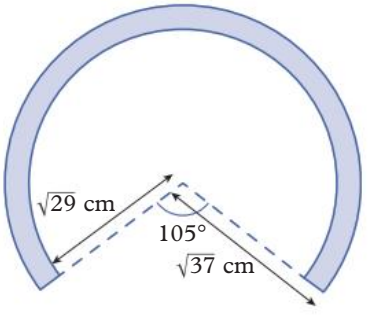



**18** Calculate the shaded area of each shape. Give your answers correct to two decimal places where necessary.

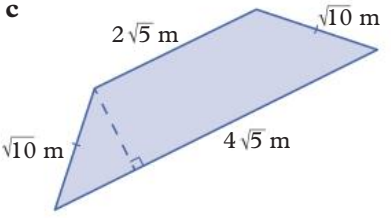
**a**




**b**




**c**




Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** — Area of composite shapes
- **Worksheet** — Identifying simple shapes within composite shapes
- **Topic quiz** — 6A

# 6B Surface area

## Learning intentions

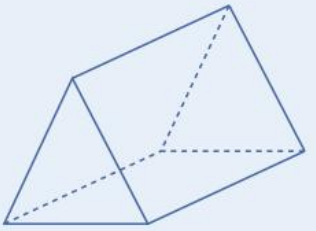
- ✔ I can calculate the surface area of right prisms.
- ✔ I can calculate the surface area of cylinders.

**Inter-year links**

| | |
|---|---|
| Year 7 | 9E Surface area |
| Year 8 | 8D Area of quadrilaterals |
| Year 10 | 9B Surface area of prisms and cylinders |

## Surface area

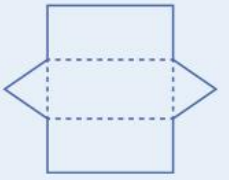
- The **total surface area** (TSA) of a three-dimensional (3D) object is the total area of the outer surface of that object.
- To calculate the total surface area of a 3D object:
  1. Determine the number of **faces**.
  2. Calculate the area of each face.
  3. Add the areas of the faces together.



Triangular prism

## Nets

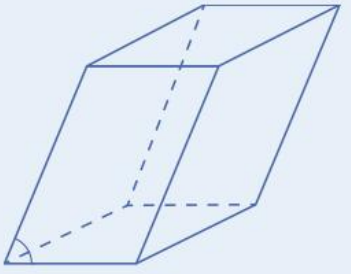
- A **net** is a 2D plan that can be folded to form a 3D object. For example, the net shown on the right is the 2D plan of the triangular prism above.
- Nets can be used to show all the faces of 3D objects and hence to help calculate the surface areas of those objects.



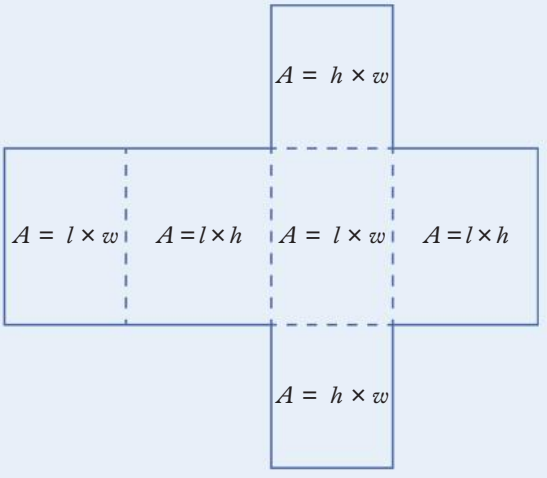
Net

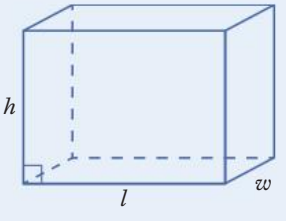
## Right prisms

- A **prism** is a 3D object with straight sides, two identical bases and a constant cross-section.
- A **right prism** has right angles between the base and the sides.
  - → All the sides (non-bases) of a right prism are rectangles.
  - → The surface area of a rectangular prism (or cuboid) can be calculated using the formula:
    TSA = $2lw + 2lh + 2hw$, where:
    $l$ is the length of the base rectangle
    $w$ is the width of the base rectangle
    $h$ is the height of the prism.



Non-right prism

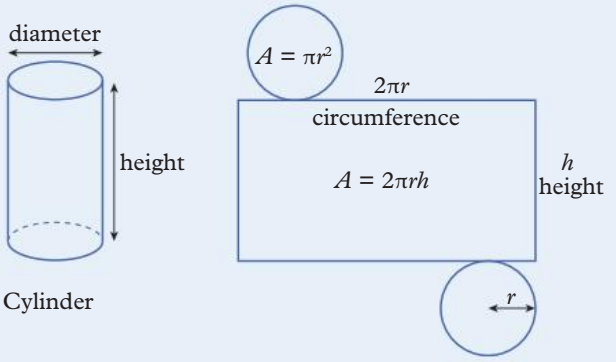



Right prism

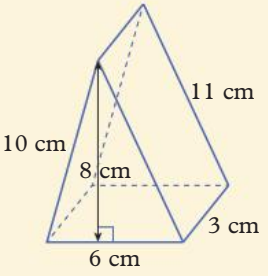
# Cylinders

- A **cylinder** is a 3D object with a circular base and a constant circular cross-section.
  - → The net of a cylinder is formed by a rectangle and two identical circular bases.
  - → The surface area of a cylinder can be calculated using the formula TSA = $2\pi rh + 2\pi r^2$, where $2\pi rh$ is the area of the rectangle, $2\pi r^2$ is the area of the two base circles and the radius if half of the diameter: $r = D \div 2$.




## Example 6B.1 Calculating the surface area of right prisms

Calculate the surface area of this triangular right prism.




**THINK**

1 Determine the number of faces. A triangular right prism has two identical triangular faces and three rectangular faces.

2 Calculate the area of each face.

3 Add the areas of the faces together. Include the appropriate unit.

**WRITE**

Area of triangular face $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 6 \times 8$
$= 24\,\text{cm}^2$

Area of bottom rectangular face $= l \times w$
$= 6 \times 3$
$= 18\,\text{cm}^2$

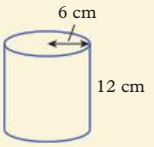
Area of left-hand rectangular face $= 10 \times 3$
$= 30\,\text{cm}^2$

Area of right-hand rectangular face $= 11 \times 3$
$= 33\,\text{cm}^2$

TSA $= 2 \times 24 + 18 + 30 + 33$
$= 129\,\text{cm}^2$

## **Example 6B.2** Drawing the net of a cylinder

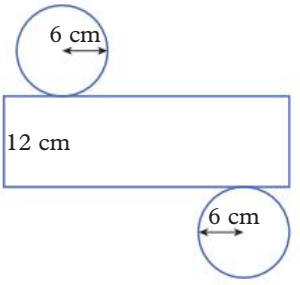
Draw the net of this cylinder, labelling dimensions accurately.



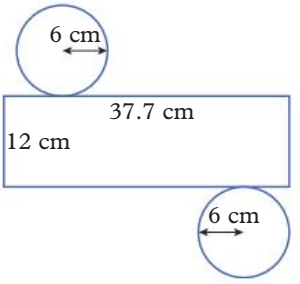

**THINK**

1. Draw a net of a cylinder, remembering that it consists of a rectangle and two circles.
2. Label the radius of the circles and the height of the rectangle. Include appropriate units.
3. The length of the rectangle is the circumference of the circular ends. Use the formula $C = 2\pi r$ to calculate this length, correct to one decimal place.
4. Label the length of the rectangle to complete your net. Include appropriate units.

**WRITE**

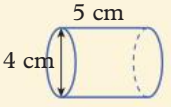



$C = 2\pi r$
$= 2\pi \times 6$
$\approx 37.7$ cm




## **Example 6B.3** Calculating the surface area of a cylinder

Calculate the surface area of this cylinder correct to two decimal places.




**THINK**

1. Identify the measurements for the radius ($r$) and the height ($h$) and substitute these into the formula.
2. Write the formula for surface area of a cylinder.
3. Substitute the values for $r$ and $h$ into the formula and calculate the result correct to two decimal places. Include the appropriate unit.

**WRITE**

$r = D \div 2$
$= 4 \div 2$
$= 2$ cm
$h = 5$ cm

$\text{TSA} = 2\pi rh + 2\pi r^2$
$= 2 \times \pi \times 2 \times 5 + 2 \times \pi \times 2^2$
$= 20\pi + 8\pi$
$= 28\pi$
$= 87.964...$
$\approx 87.96\,\text{cm}^2$

**Helpful hints**

- ✔ The number of faces in a prism is equal to the number of sides in the matching ends plus two.
- ✔ All of the non-matching end faces in a right prism are rectangles.
- ✔ A prism can be orientated in any way. Identify the matching ends to determine the shape of the base of a prism.

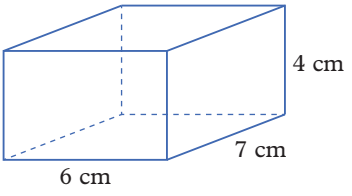
ANS p528

# Exercise 6B Surface area

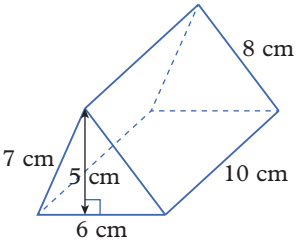
1(a, b, e, f, h), 2–4, 5(a, e, f), 6, 7, 10, 11, 15 | 1(b, c, g, h), 2, 5, 8, 9, 14, 15, 19 | 2, 5, 9, 11–13, 16–21

6B.1 **1** Calculate the surface area of each right prism.

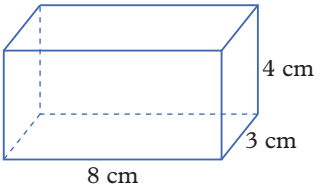
**a**



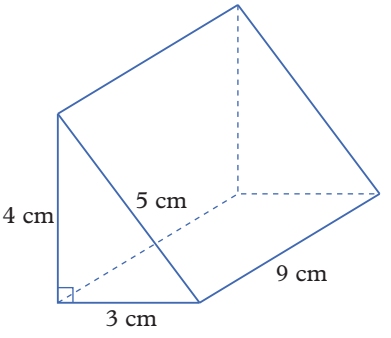

**b**




**c**




**d**

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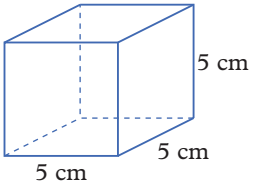


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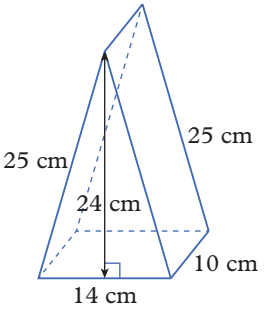


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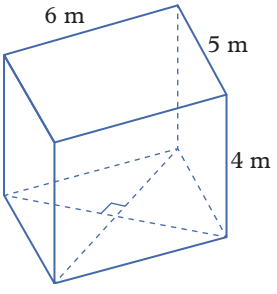
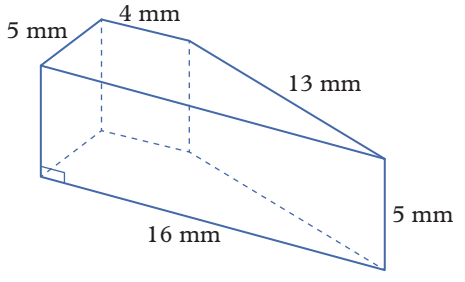


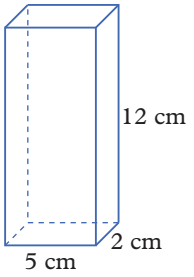

**h**



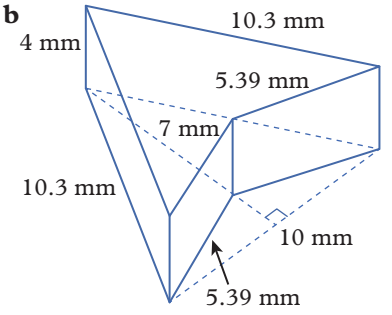

UNDERSTANDING AND FLUENCY

**2** Calculate the surface area of each right prism.

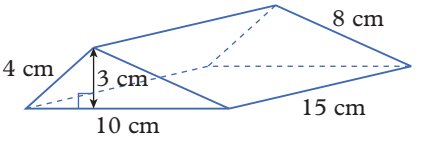
**a**




**b**




**c**

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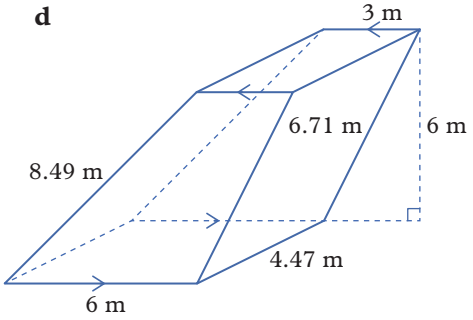


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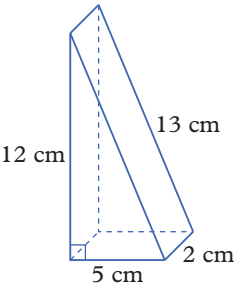


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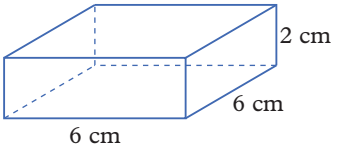
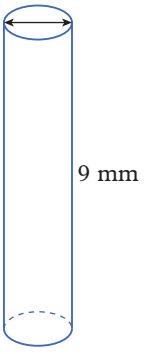
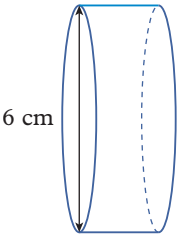



6B.3 **3** Draw a net for each cylinder, labelling the dimensions correct to two decimal places where appropriate.

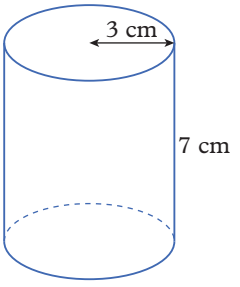
**a**




**b**




**c**

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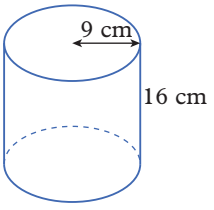
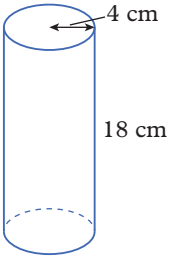



**e**

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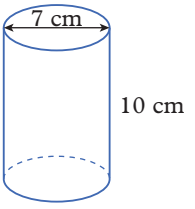
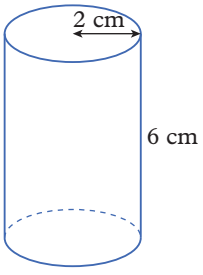


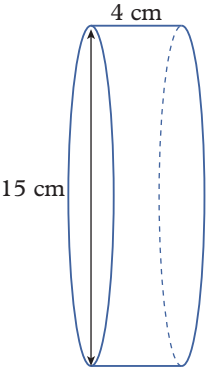

**4** Calculate the surface area of each cylinder in question **3**. Give your answers to two decimal places.

6B.3 **5** Calculate the total surface area for each cylinder correct to two decimal places.

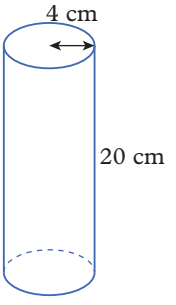
**a**



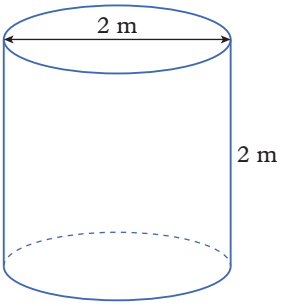

**b**



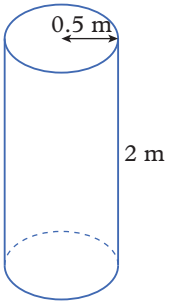

**c**



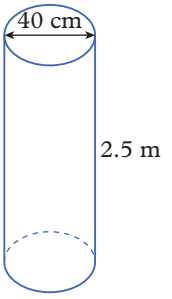
**d**




**e**

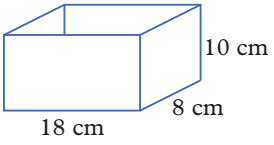



**f**

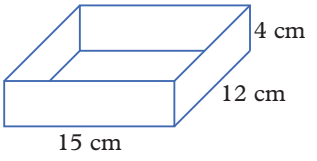



**6** Find the surface area of these boxes if you were to consider:

**a**




**b**

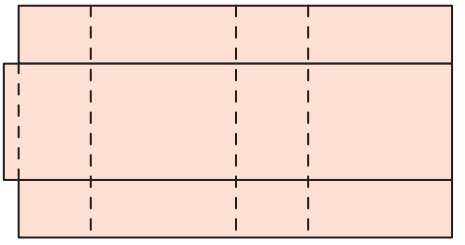



**i** the amount of material required to build the boxes

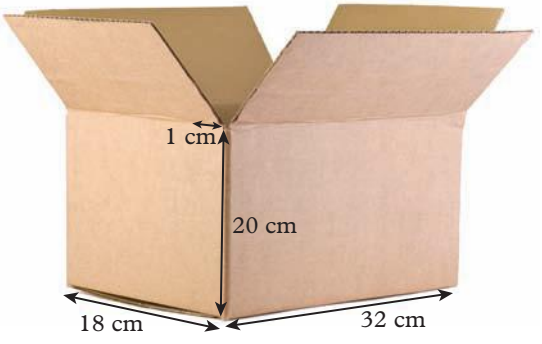
**ii** the amount of paint (in $cm^2$) required to paint all surfaces of the boxes (inside and outside).

**7** Calculate the outer surface area of this box and its lid, if the box is 22 cm tall and has a radius of 15 cm. The lid is 4 cm tall and has a diameter of 31 cm. Give your answer to two decimal places.

**8** The net of a basic cardboard box is shown. The dashed lines represent folds, and the solid lines represent cuts. The tab on the left is used to join the central section of the box, and the flaps at the top and bottom are all half of the width of the box. Calculate the amount of cardboard required to make these boxes.



**a**




**b**

**9** A cylindrical pool is 2 m deep and has a radius of 6.5 m. How much would it cost, to the nearest dollar, to paint its interior if it needs two coats of paint that costs $50 per litre? Assume that 1 L covers 15 $m^2$.

**10** Maria has the choice of two paint rollers. One roller is 25 cm long and has a radius of 4 cm. The other roller is 30 cm long and has a diameter of 6 cm. Assuming they have the same absorbency, which roller would need to be re-dipped in paint the least often?

PROBLEM SOLVING AND REASONING

**11** A block of butter is in the shape of a rectangular prism.

**a** If the block was 7 cm wide, 16 cm long and 6 cm high, calculate its total surface area.

**b** If the block was cut halfway along the length (that is, to give two pieces 8 cm long), what is the surface area:

**i** of each piece? **ii** in total?

**c** How is your answer to part **b ii** different from your answer to part **a**?

**d** Imagine that you cut the block of butter into 1-cm cubes. What would be the total surface area of the butter now?

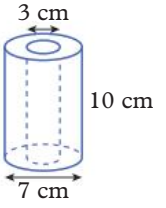
**e** If somebody wanted to melt butter quickly, what would you recommend to them? Why?

**12** Billy is having his room painted. His room is 6 m long, 5 m wide and 2.5 m high and has a large window on one wall that measures 150 cm × 95 cm. The walls are to be painted blue and the ceiling is to be painted cream.

**a** What area is to be painted blue and what area is to be painted cream?

**b** How many litres of each colour paint is needed if 1 L of paint covers about 15 $m^2$ and the room will take two coats?

**13 a** What is the surface area of the tube on the right, including the internal exposed surface? Give your answer correct to two decimal places.




**b** Lucian answered part **a** as 296.88 $cm^2$ and Curtis said it was 282.74 $cm^2$. Explain where they went wrong.

**14** A cylinder has a radius of 4 cm and a surface area of 300 $cm^2$. What is its height to the nearest centimetre?

**15** A cube has a surface area of 150 $cm^2$.

**a** What is the surface area of each face?

**b** What is the side length of the cube?

**c** Write a formula that will help you determine the side length of any cube if you know its surface area.

**16** The surface area for any prism can be calculated using the formula:

SA = 2 × area of base + perimeter of base × height

Explain why this formula works.

**17** Explain why knowing the surface area of a cube is enough to determine its side lengths but knowing the surface area of a rectangular prism is not.

**18** If you double the height of a cylinder, does its surface area also double? Explain, using an example.

**19** The surface area of a sphere can be calculated using the formula $TSA = 4\pi r^2$. Calculate the surface area of a sphere, correct to two decimal places, that has:

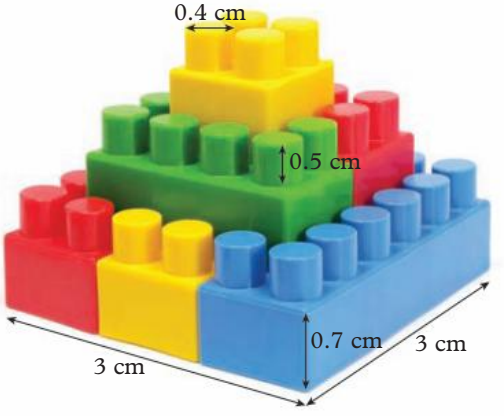
**a** radius 4 cm **b** radius 7 mm

**c** diameter 10 cm **d** circumference 20 cm.

CHALLENGE

**20** Write a formula that will calculate the height of a cylinder, given its surface area and radius.

**21** Calculate the surface area, correct to two decimal places, of the structure shown in the image on the right.




Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Surface area of prisms
- Interactive skillsheet — Surface area of cylinders
- Investigation — Dissecting cubes
- Topic quiz — 6B

# 6C Volume and capacity

## Learning intentions

- ✔ I can calculate the volume of right prisms and cylinders.
- ✔ I can calculate the capacity of cylindrical objects.

**Inter-year links**

| | |
|---|---|
| Year 5/6 | Volume and capacity |
| Year 7 | 9F Volume and capacity |
| Year 8 | 8F Volume of prisms |
| Year 10 | 9C Volume of prisms and cylinders |

## Volume

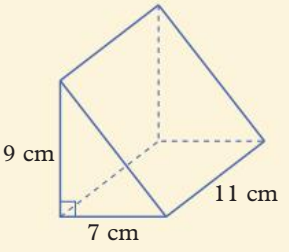
- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.
- The volume of a prism can be determined by multiplying the base area by the height of the prism.
  $V = Ah$, where $A$ is the area of the base and $h$ is the height of the prism.
- The volume of a cylinder can also be found by multiplying the base area by the height. As the base area is $A = \pi r^2$, this gives the following formula:
  $V = \pi r^2 h$, where $r$ is the radius of the base and $h$ is the height of the cylinder.
- Remember that volume is a measure of three-dimensional space so the units of linear measures must be cubed: $mm^3$, $cm^3$, $m^3$.

## Capacity

- The **capacity** of a 3D object is a measure of how much the object can hold.
  A container with an inside volume of:
  - → 1 $cm^3$ holds 1 mL of liquid
  - → 1000 $cm^3$ holds 1 L of liquid
  - → 1 $m^3$ holds 1 kL of liquid.

### Example 6C.1 Calculating the volume of a right prism

Calculate the volume of this triangular prism.




**THINK**

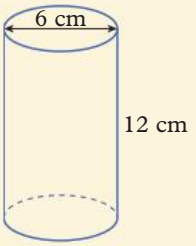
1. Identify the shape of the base of the right prism (triangle with base length $b$ and height $h_1$) and write the appropriate formula for the area: $A_{base} = \frac{1}{2}bh_1$
2. Substitute the values for the base area ($A$) and the height of the prism ($h_2$) into the formula for the volume of a prism: $V = Ah_2$
3. Calculate the result. Remember to include the appropriate unit.

**WRITE**

$$A_{base} = \frac{1}{2}bh_1$$
$$= \frac{1}{2} \times 7 \times 9$$
$$= 31.5$$
$$V = Ah_2$$
$$= 31.5 \times 11$$
$$= 346.5 \text{ cm}^3$$

## **Example 6C.2** Calculating the volume of a cylinder

Calculate the volume of this cylinder correct to two decimal places.




**THINK**

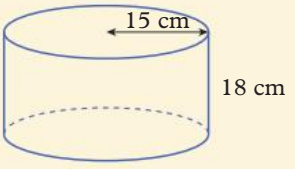
1. Identify the values for the radius of the base ($r$) and the height of the cylinder ($h$).
2. Substitute the values for $r$ and $h$ into the formula for the volume of a cylinder.
3. Calculate the result using the π button on your calculator and round your answer to two decimal places. Remember to include the appropriate unit.

**WRITE**

$r = D \div 2$
$= 6 \div 2$
$= 3$ cm
$h = 12$ cm
$V = \pi r^2 h$
$= \pi \times 3^2 \times 12$
$= 108\pi$
$= 333.292...$
$\approx 333.29\,\text{cm}^3$

## **Example 6C.3** Calculating the capacity of a cylinder

Calculate the capacity of this cylinder correct to the nearest litre.




**THINK**

1. Identify the values for the radius of the base ($r$) and the height of the cylinder ($h$).
2. Calculate the volume of the cylinder, using the π button on your calculator.
3. Convert the volume into capacity by using an appropriate conversion factor.

**WRITE**

$r = 15$ cm
$h = 18$ cm
$V = \pi r^2 h$
$= \pi \times 15^2 \times 18$
$= 4050\pi$
$= 12\,723.45...\ \text{cm}^3$
$= (12\,723.45... \div 1000)$ L
$\approx 13$ L

**Helpful hints**

✔ Remember that volume and capacity are not the same thing! Capacity is a measure of how much an object can hold (mL, L, kL), whereas volume is a measure of how much space an object occupies ($\text{mm}^3$, $\text{cm}^3$, $\text{m}^3$).

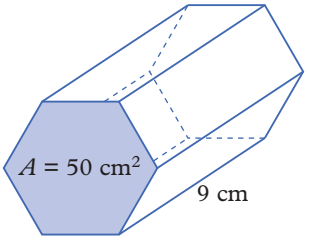
ANS p529

# Exercise 6C Volume and capacity

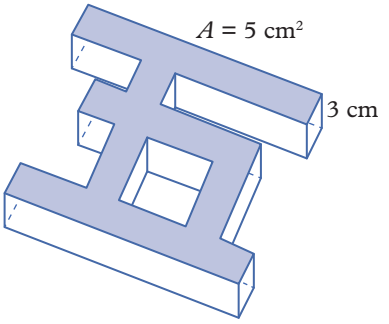
1–6, 9, 10, 13, 16(a, b), 18(a) | 1–4(b, d, f), 5–7, 11, 13, 16, 18, 20 | 2–4(c, f), 5, 6, 8, 12, 14, 15, 16(c), 17–21

**1** Calculate the volume of each prism by using the formula $V = Ah$.

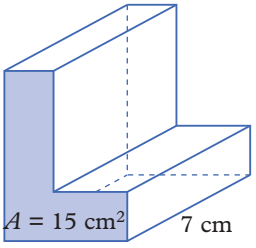
**a**



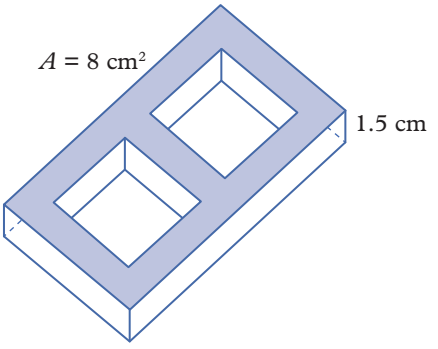

**b**



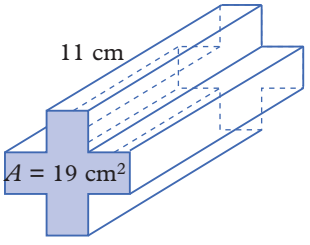

**c**



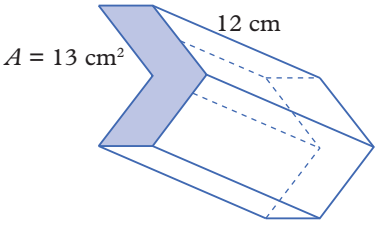

**d**




**e**

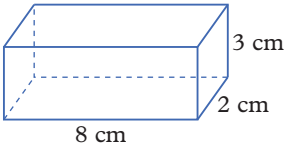



**f**

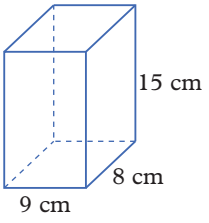



**2** The volume of a rectangular prism can be calculated using the formula $V = lwh$. Calculate the volume of each rectangular prism.

**a**




**b**

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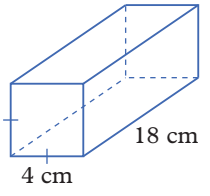


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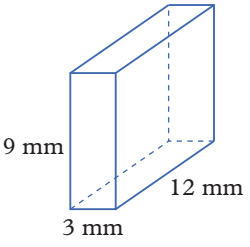


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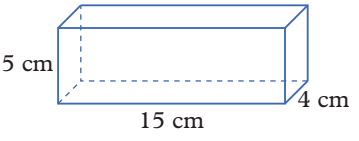
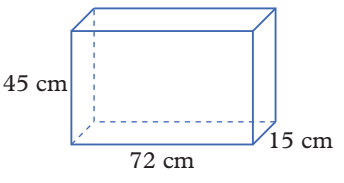
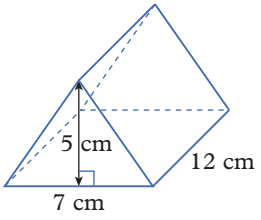



**f**

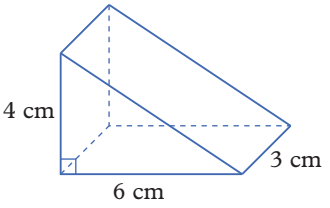


**6C.1** **3** Calculate the volume of each triangular prism.

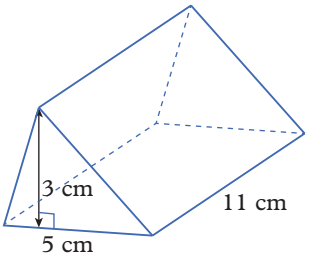
**a**



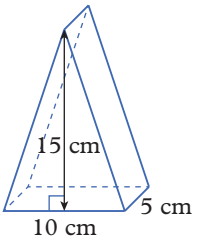

**b**




**c**




**d**

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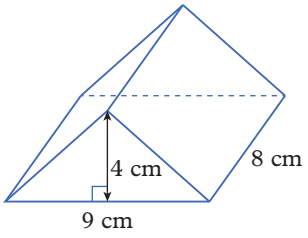


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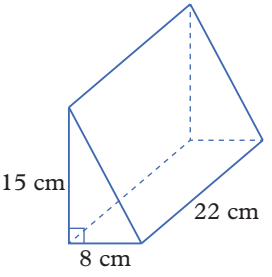
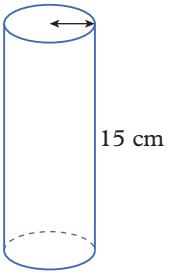
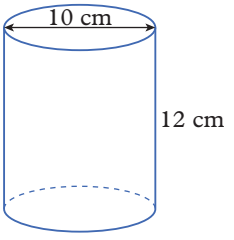



**6C.2** **4** Calculate the volume of each cylinder. Give your answers to two decimal places.

**a**




**b**

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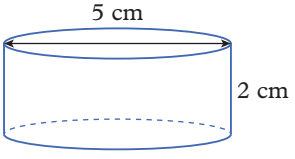
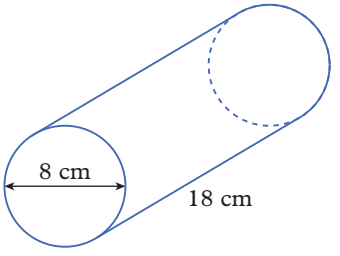
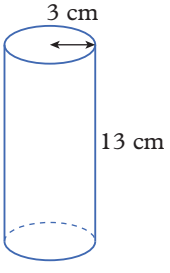


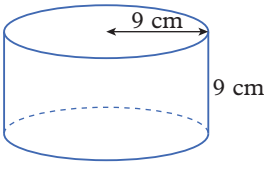

**d**




**e**




**f**



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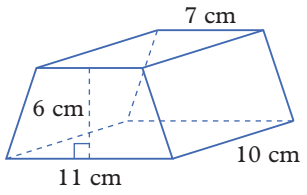
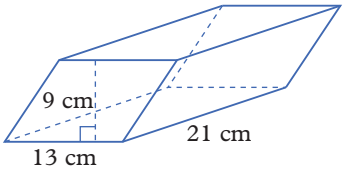
**5** Calculate the volume of each prism.

**a**




**b**

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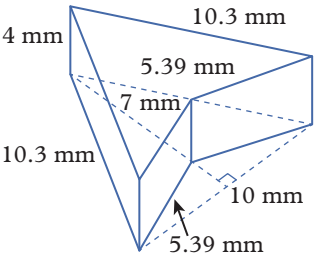
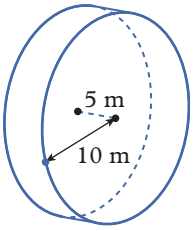
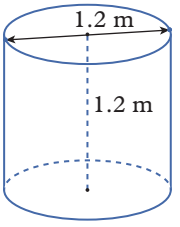



6C.2 **6** Calculate the capacity of each cylinder, correct to the nearest litre.

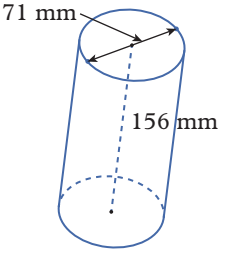
**a**




**b**




**c**




**7** A prism has a volume of 240 cm$^3$.

**a** If the prism has a height of 8 cm, what is the area of its base?

**b** If the prism is rectangular and has a base area of 60 cm$^2$, find its height.

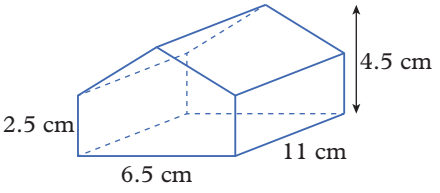
**8** A cylinder has a volume of 150 cm$^3$.

**a** If it has a radius of 6 cm, find its height.

**b** If it has a height of 6 cm, find its radius.

Give your answers to two decimal places.

**9** Calculate the volume of this prism.




**10** A can of condensed milk has an internal diameter of 7 cm and a height of 12 cm.

**a** What is the volume of condensed milk in the can? Give your answer to the nearest cm$^3$.

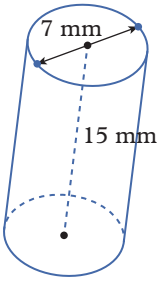
**b** If condensed milk has a density of 1.3 g/cm$^3$, what is the weight of the milk if the can is filled? Give your answer to the nearest gram.
(Weight = volume × density)

**11** A $1 coin has a diameter of 2.5 cm and a thickness of 3 mm.

**a** How much metal is in a stack of 30 $1 coins? Write your answer in both cubic millimetres and cubic centimetres, correct to two decimal places.

**b** How many $1 coins could be made from 1000 cm$^3$ of metal?

**12** The capacity of the cylinder shown on the right correct to the nearest litre is 0 L. Explain why rounding to this degree of accuracy does not make sense.




**13** Oliver has two cylindrical glasses. The first glass has a diameter of 6.5 cm and a height of 22 cm. The second glass has a radius of 5 cm and a height of 10 cm. Which glass has the greatest capacity and by how much? Give your answer in millilitres correct to two decimal places.

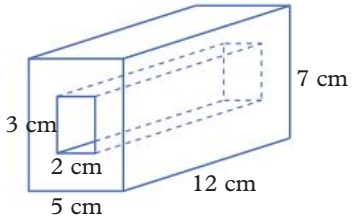
PROBLEM SOLVING AND REASONING

**14** What is the capacity of a cylindrical bottle cap if it has an internal diameter of 2.8 cm and a height of 1.1 cm? Give your answer in mL correct to two decimal places.

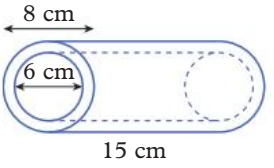
**15** Explain why if the radius of a cylinder is doubled it will have twice the volume of the same cylinder with its height doubled.

**16** Find the volume of these hollow objects. Where necessary, give your answers to two decimal places.

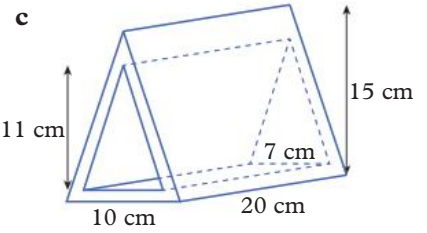
**a**




**b**




**c**

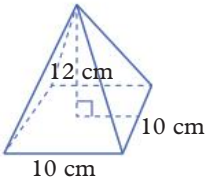



**17** A candle has a diameter of 6 cm and a height of 16 cm. If the wick can be thought of as a cylinder with a diameter of 3 mm, what is the amount of wax contained within a single candle? Give your answer in both cubic centimetres and in millilitres correct to two decimal places.

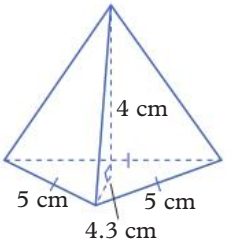
CHALLENGE

**18** The volume of any pyramid is equal to $\frac{1}{3}$ the volume of the corresponding prism with the same base and height, $V = \frac{1}{3}Ah$. Calculate the volume for each pyramid shown below.

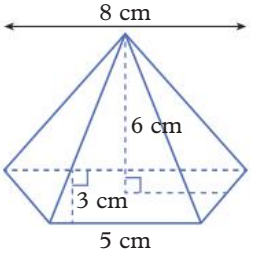
**a**




**b**




**c**

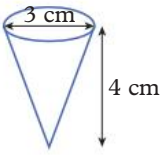



**19** A cone can be thought of as a 'pyramid' that relates to the cylinder.

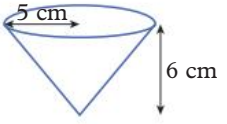
**a** Use the formula for the volume of a cylinder to write a formula that will calculate the volume of a cone in terms of $r$ and $h$.

**b** Calculate the volume of each cone correct to two decimal places.

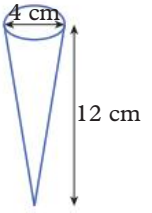
**i**




**ii**




**iii**




**20** The volume of a sphere can be calculated using the formula $V = \frac{4}{3}\pi r^3$. Calculate the volume of a sphere, correct to two decimal places, that has:

**a** radius 3 cm **b** radius 6 cm **c** diameter 8 cm **d** circumference 21 cm.

**21** An ice-cream cone has a diameter of 5 cm and height of 12 cm.

**a** How much ice cream can the cone hold? Assume that the ice-cream fills the entire cone and has a perfect half-sphere on its top. Give your answer to the nearest millilitre.

**b** How many cones can be completely filled using a 1.5 L tub of ice-cream?

Check your Student obook pro for these digital resources and more:

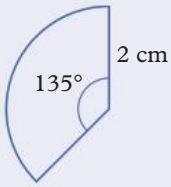
- Interactive skillsheet: Volume of prisms
- Interactive skillsheet: Volume of cylinders
- Interactive skillsheet: Capacity
- Investigation: Volume in cooking and medicine
- Topic quiz: 6C

# Checkpoint

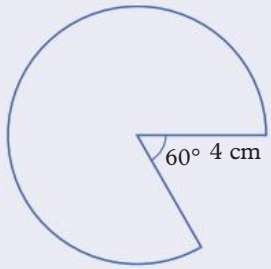
**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

**6A** **1** Calculate the area of these sectors correct to two decimal places.

**a**

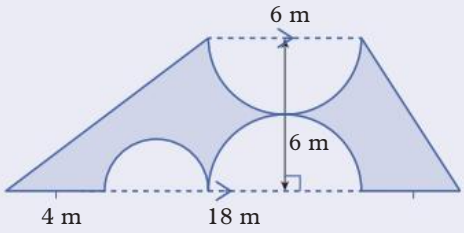



**b**

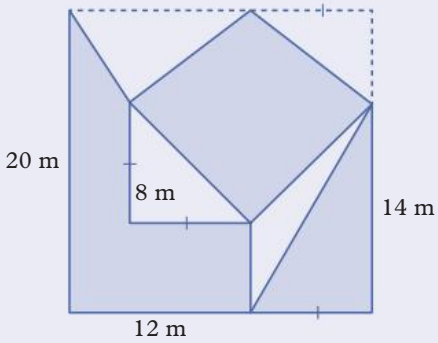



**6A** **2** Calculate the shaded area of these composite shapes correct to two decimal places.

**a**

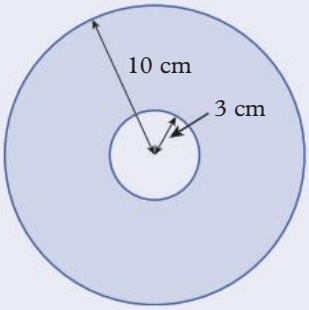



**b**

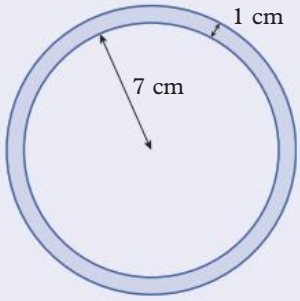



**6A** **3** Calculate the area of these annuli correct to two decimal places.

**a**

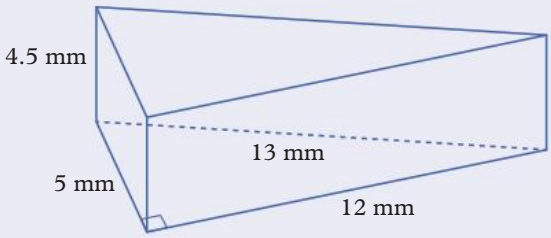



**b**

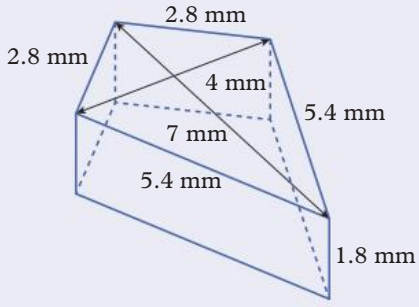



**6B** **4** Calculate the surface area of these prisms.

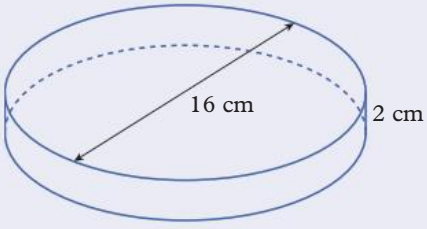
**a**




**b**



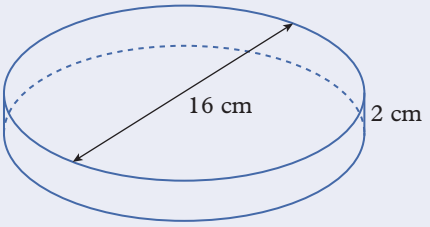

**6B** **5** Draw the net of this cylinder, labelling dimensions correct to two decimal places where appropriate.



6B **6** Calculate the surface area of these cylinders, correct to two decimal places.

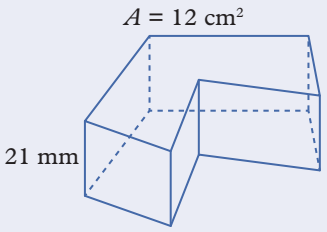
**a**

**b**

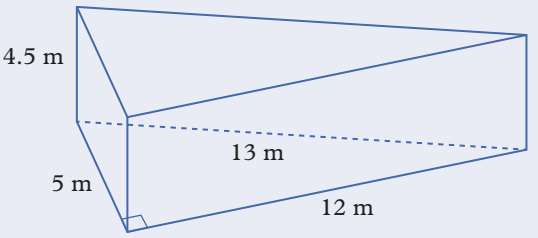



6C **7** Calculate the volume of these prisms.

**a**



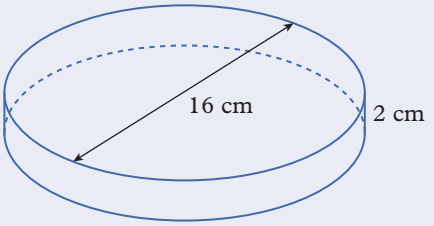

**b**




6C **8** Calculate the volume of these cylinders, correct to two decimal places.

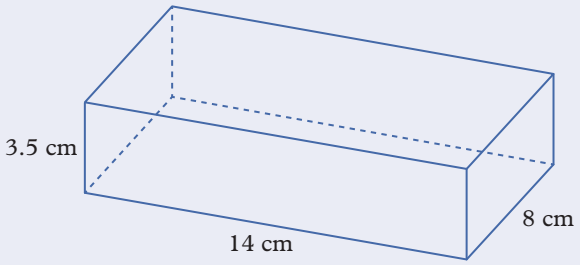
**a**

**b**

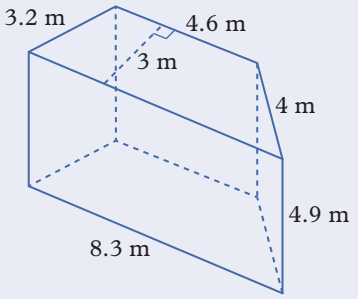



6C **9** Calculate the capacity of these prisms in litres.

**a**

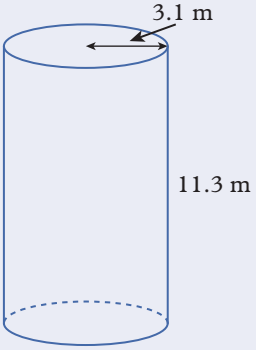



**b**

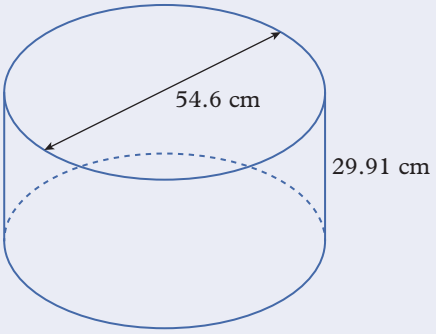



6C **10** Calculate the capacity of these cylinders in litres, correct to two decimal places.

**a**




**b**



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# 6D Dilations and similar figures

## Learning intentions

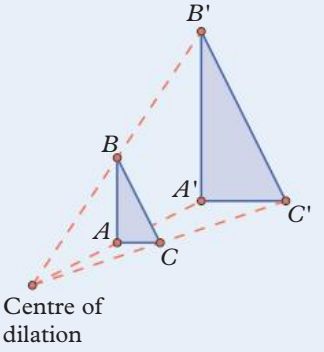
- ✔ I can determine scale factors.
- ✔ I can find unknown lengths in similar figures.
- ✔ I can perform dilations.

**Inter-year links**

| | |
|---|---|
| Year 7 | 3B Equivalent fractions |
| Year 10 | 7C Congruence and similarity |

## Dilations

- A **dilation** is a transformation that changes the size of an object. For a dilation, the shape and orientation of the image remains the same as the original shape.
- A dilation can be either an **enlargement** or a **reduction**.
  - → An enlargement creates an image that is larger than the original shape.
  - → A reduction creates an image that is smaller than the original shape.
- A **centre of dilation** and **scale factor** need to be specified to perform a dilation.



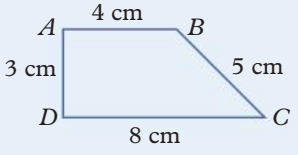

- The scale factor can be found using the formula:

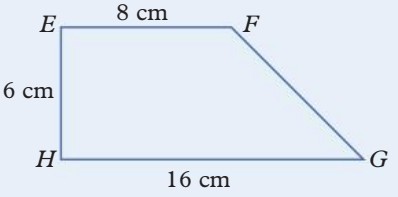
  $$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

  - → If the scale factor is between 0 and 1, the image will be a reduction.
  - → If the scale factor is greater than 1, the image will be an enlargement.
- A scale factor can also be written as a ratio. For example, a scale factor of $\frac{1}{10}$ is the same as a ratio of 1 : 10.
- The side length of a dilation can be found by multiplying the original side length by the scale factor. For example, when dilating a shape with a side $AB$:
  Length of $A'B'$ = length of $AB$ × scale factor.

## Similar figures

- Two figures are said to be **similar** if they are the same shape. Similar figures do not need to be the same size.
  - → An image produced by dilation will be a similar figure with the original shape. For example, $ABCD$ and $EFGH$ are similar figures.




- If two figures are similar, all corresponding angles are equal in size and all corresponding sides will be in the same ratio.
- If the scale factor between two similar figures is 1, then the two figures are **congruent**.

- The symbol ~ can be used to indicate similarity. When making a similarity statement, corresponding vertices must be put in the same order.
  - → For example, $ABCD \sim EFGH$.
  - → The symbol ≅ can be used to indicate congruence.
- If two figures are known to be similar, an unknown side length or angle can be found by using the scale factor.

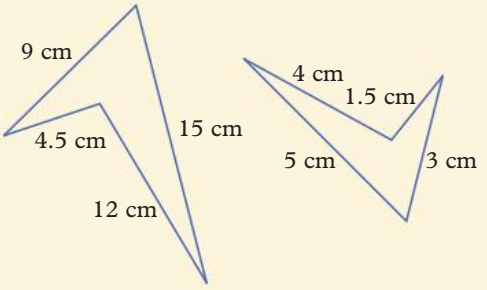
# Areas of similar figures

- The **area scale factor** can be found using the formula:

  $$\text{Area scale factor} = \frac{\text{image area}}{\text{original area}}$$
- The area scale factor between two similar figures is equal to the square of the scale factor between the figures.
  - → For example, if the scale factor between two figures is 2, the area scale factor will be $2^2 = 4$.

## Example 6D.1 Finding the scale factor

Find the scale factor for this pair of similar figures if the figure on the left is the original shape.




**THINK**

1 Identify two corresponding sides. The longest side of the original figure is 15 cm and the longest side of the image is 5 cm.

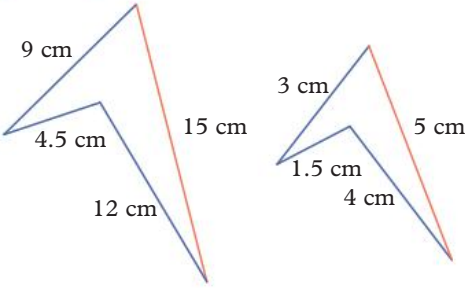
2 Substitute these lengths into the formula:

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

3 Match the remaining corresponding sides and check that each pair of sides is in the same ratio.

4 Check that the scale factor seems reasonable (between 0 and 1 means a reduction).

**WRITE**




$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{5 \text{ cm}}{15 \text{ cm}}$$
$$= \frac{1}{3}$$

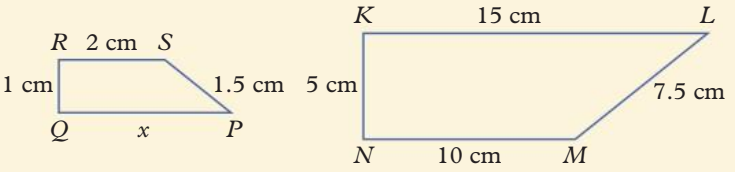
Check each corresponding pair of sides:

$$\text{Scale factor} = \frac{3}{9} = \frac{1.5}{4.5} = \frac{4}{12} = \frac{1}{3}$$

The scale factor of $\frac{1}{3}$ is a reduction, as required.

## **Example 6D.2** Finding an unknown side length in similar figures

Find the value of $x$, given that these two figures are similar.




**THINK**

1. Identify two corresponding sides and calculate the scale factor. Use corresponding vertices to write the side names in the same order. As $R$ corresponds to $N$ and $S$ corresponds to $M$, then side $RS$ corresponds to side $NM$ (not $MN$).
2. Find a pair of corresponding sides that involve $x$.
3. Substitute in the known values and solve for $x$.

**WRITE**

Sides $RS$ and $NM$ are corresponding.

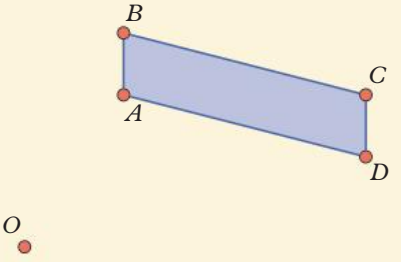
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{10 \text{ cm}}{2 \text{ cm}}$$
$$= 5$$

Side $QP$ has length $x$. Sides $QP$ and $KL$ are corresponding.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$5 = \frac{15}{x}$$
$$5 \times \frac{x}{5} = \frac{15}{x} \times \frac{x}{5}$$
$$x = \frac{15}{5}$$
$$x = 3 \text{ cm}$$

## **Example 6D.3** Performing a dilation

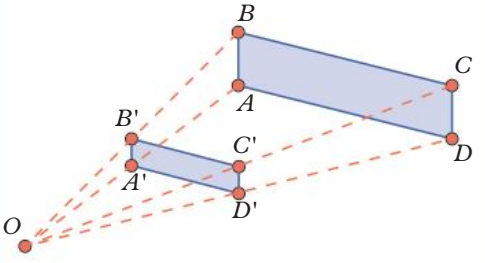
Dilate this figure by a scale factor of $\frac{1}{2}$ using the centre of dilation $O$.




**THINK**

1. Use dotted lines to connect the centre of dilation with each of the vertices on the original shape.
2. The scale factor is $\frac{1}{2}$ so measure half the distance from the centre of dilation to each of the vertices on the original shape and mark these points. These are the corresponding vertices of the image.
3. Label the vertices of the image and join them together to complete the dilation.

**WRITE**



## Example 6D.4 Finding the area scale factor

A figure is dilated by a scale factor of 3. What is its area scale factor?

**THINK**

Take the square of the scale factor to calculate the area scale factor.

**WRITE**

Area scale factor $= 3^2$
$= 9$

**Helpful hints**

✔ The image from a dilation will always be in the same orientation as the original shape; however, similar shapes do not need to be in the same orientation.

ANS p530

# Exercise 6D Dilations and similar figures

△ 1–5, 6(a–c), 7–9, 12, 13, 15, 17, 21(a, b)

□ 2–5, 6(d–f), 7, 8, 13, 14, 16–18, 22, 25

⬠ 3–5, 6(e, f), 8, 10, 11, 16, 17, 19, 20, 22–24

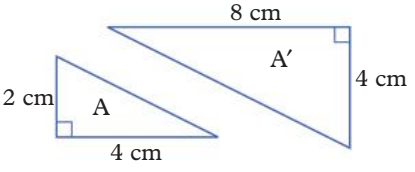
UNDERSTANDING AND FLUENCY

**1** Decide whether each scale factor produces an enlargement or a reduction.

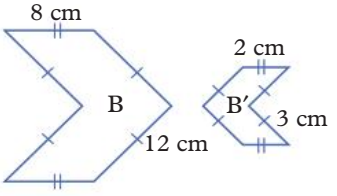
**a** 4 **b** $\frac{1}{3}$ **c** 2 **d** 6 **e** $\frac{1}{10}$ **f** $\frac{1}{5}$

6D.1 **2** Find the scale factor for each pair of similar figures.

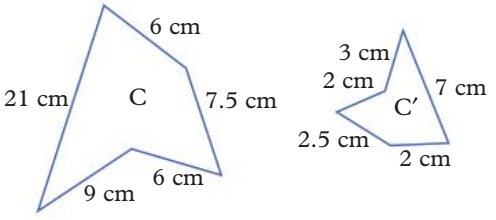
**a**



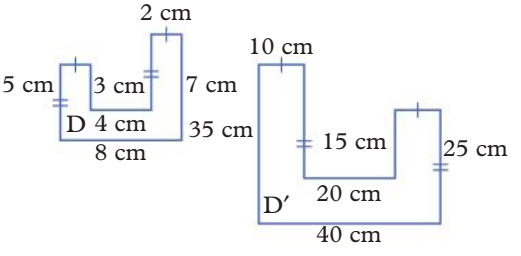

**b**



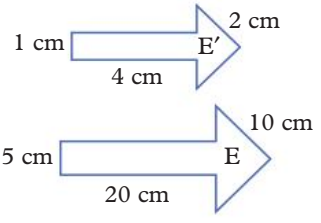

**c**



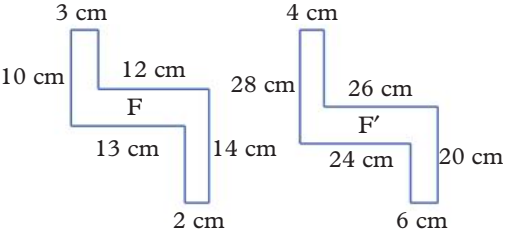

**d**




**e**




**f**



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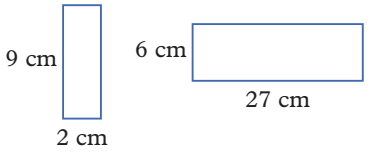
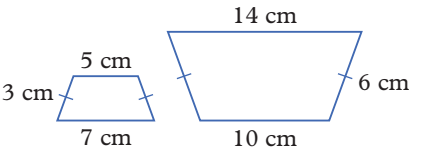
**3** Find the scale factor for the similar figures if the figure on the left of each pair is the original shape.

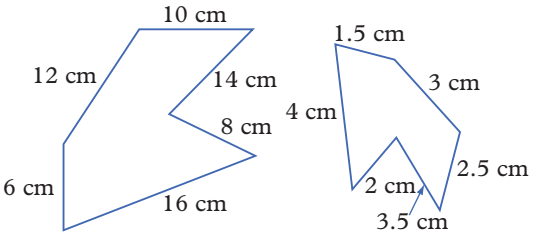
**a**



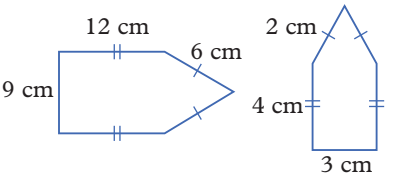

**b**




**c**

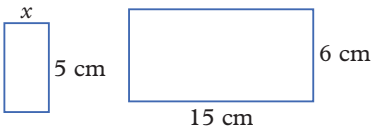



**d**

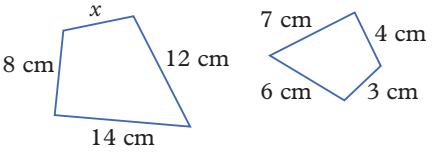



6D.2 **4** Find the value of $x$, given that these pairs of shapes are similar.

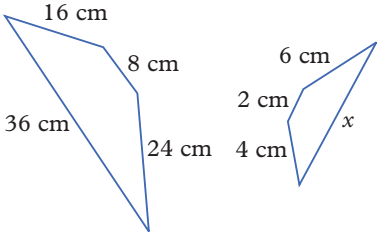
**a**



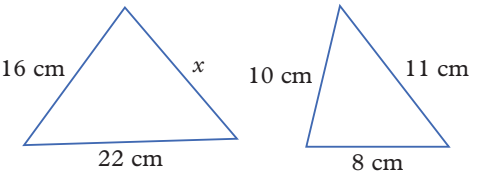

**b**



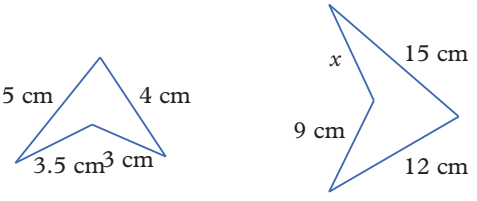

**c**



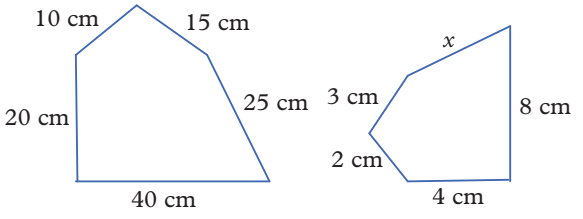

**d**




**e**



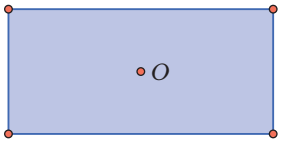

**f**



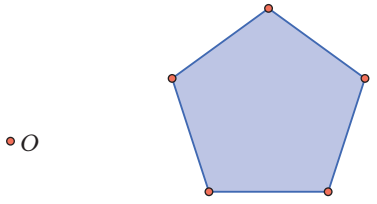

6D.3 **5** Use the worksheet available on your obook pro to dilate each figure using the centre of dilation $O$ and a scale factor of:

**i** 3 **ii** $\frac{1}{3}$

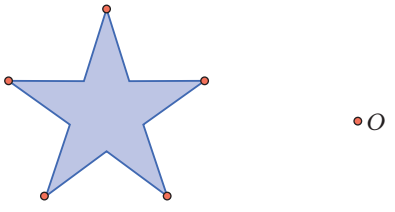
**a**




**b**



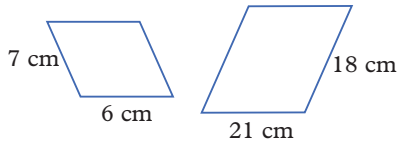

**c**



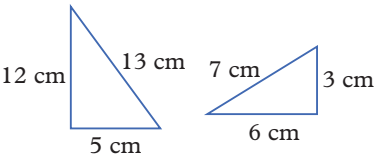
UNDERSTANDING AND FLUENCY

**6** Decide whether each pair of shapes is similar by calculating and comparing the scale factor for each pair of corresponding sides.

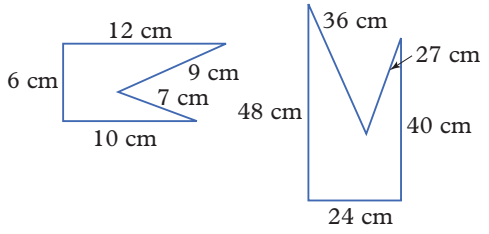
**a**



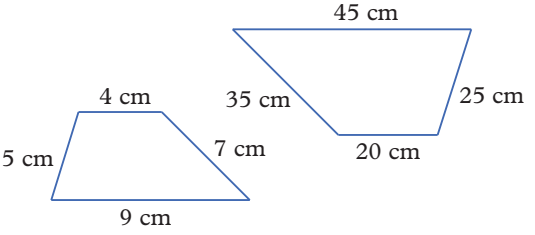

**b**



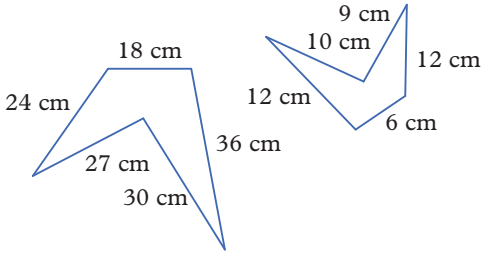

**c**



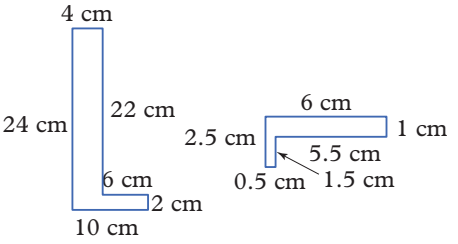

**d**




**e**




**f**




6D.4 **7** For each scale factor, what is the area scale factor?

**a** 2 **b** 3 **c** 4 **d** $\frac{1}{2}$ **e** $\frac{1}{3}$ **f** $\frac{1}{4}$

**8** Copy and complete this table.

| | Original area | Length scale factor | New area |
|---|---|---|---|
| **a** | 12 cm$^2$ | 2 | |
| **b** | 5 mm$^2$ | 3 | |
| **c** | 10 cm$^2$ | 5 | |
| **d** | 40 cm$^2$ | $\frac{1}{2}$ | |
| **e** | 27 cm$^2$ | $\frac{1}{3}$ | |
| **f** | 100 mm$^2$ | $\frac{1}{5}$ | |

PROBLEM SOLVING AND REASONING

**9** A microscope with a scale factor of 20 000 was used to view a bacterial cell, which appeared 5.2 cm long. What was the actual length of the bacterial cell?

**10** Explain why a circle is similar to all other circles, why a square is similar to all other squares, but a rectangle is not similar to all other rectangles.

**11** Finn was explaining a dilation to a classmate and said that a dilation of $\frac{2}{3}$ was the same as dilating by a factor of 2 and then dilating by a factor of $\frac{1}{3}$. Can you explain why he is correct?

**12** Explain the difference between congruent figures and similar figures.

**13** One standard print size for photographs is 10 cm × 15 cm. Alice wanted to enlarge a photograph to place it onto a canvas measuring 40 cm × 50 cm. Are the photograph and its corresponding canvas art similar figures?

**14** Ethan needed to increase the size of the area of his working space to nine times its previous size in order to accommodate a new project. Explain why he does not need to increase the length measurements of his working space by nine.

**15** A farmer bought new land that was next to a paddock he already owned. If the area that he owned increased by a factor of 4, by what scale factor did the length of the land increase by?

**16** A radar has a scanning radius of roughly 5 km. If the owner wanted to cover an area of 100 km$^2$, by how much would the radius of the radar need to increase?

**17** A ratio can be used instead of a fraction to describe a dilation. For example, a figure that is 10 times as small as the original has a scale factor of $\frac{1}{10}$, which is the ratio 1 : 10.

**a** How would a scale factor of $\frac{1}{2}$ be written as a ratio?

**b** Explain the relationship between the fraction and the ratio. (Hint: What parts of the ratio relate to the numerator and denominator?)

**c** Consider an enlargement with a scale factor of 2. What is 2 written as a fraction?

**d** Use your answer to part **c** to write a scale factor of 2 as a ratio.

**e** Write these ratios as scale factors.

**i** 1 : 4 **ii** 1 : 100 **iii** 1 : 2500

**iv** 3 : 4 **v** 2 : 1 **vi** 7 : 2

**f** Explain how you can tell if a similar figure is an enlargement or reduction by the order of the ratio.

**18** Timothy got a model plane for his birthday, which had a scale factor of 1 : 100.

**a** What does this scale factor mean?

**b** If the real-life plane has a length of 34 m, what is the length of the model plane?

**c** If the model plane has a wingspan of 27 cm, what is the wingspan of the real-life plane?

**19** A model car is constructed using a ratio of 1 : 40.

**a** If the model car has a length of 10 cm, what is the length of the real car?

**b** If the real car has a height of 1.5 m, what is the height of the model car?

**c** Do you think that a model car can be a true similar figure to a real car? Explain.

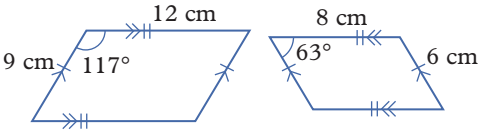
**20** Marc has a hiking map with a ratio scale of 1 : 75 000.

**a** Marc planned to walk the length of a 15 km track. What distance would this be on the map?

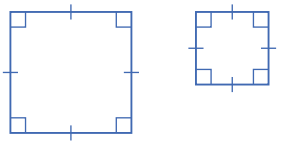
**b** At lunchtime, he had walked a distance shown as 7 cm on the map. How far did he have left to walk on the track?

**21** Use your understanding of quadrilateral properties to decide whether each pair of shapes is similar.

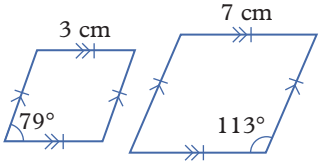
**a**



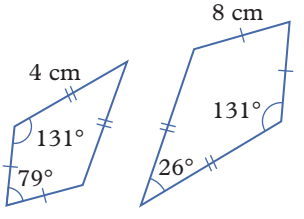

**b**



**c**

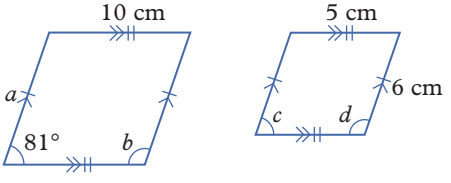



**d**

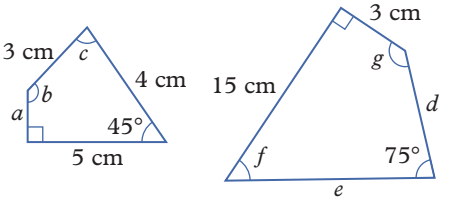



**22** Use your understanding of quadrilateral properties to find the value of the pronumerals in each pair of similar figures.

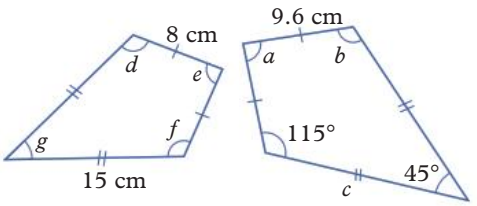
**a**



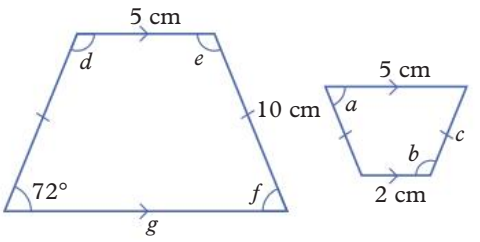

**b**



**c**




**d**




**23** Elliot drew a regular hexagon with side lengths of 3 cm.

**a** What is the difference between a regular and an irregular hexagon?

**b** If Elliot drew another regular hexagon with different side lengths, explain how you know without measuring anything that the two hexagons must be similar.

CHALLENGE

**24** Emily bought Timothy a model plane to go into a special glass case that allows for a maximum height of 15 cm.

**a** If the plane was 30 m tall in real life, what maximum scale factor could be applied to the model plane?

When the plane arrives, Emily finds that the plane actually measures 16 cm in height.

**b** What scale factor has actually been applied to the model plane?

Assuming that all dimensions must stay in the same ratio, it is important that the plane fits into the glass case.

**c** What scale factor would the model need to be dilated by to fit into the space? (Hint: Which measurement is the original and which is the image?)

**d** What scale factor would the space need to be dilated by to fit the current model?

**e** How are the answers to parts **c** and **d** different and how are they similar?

**25** This table shows the dimensions of standard paper sizes, rounded to the nearest millimetre. Use the table to decide whether any or all of the following sizes are similar figures. Compare:

**a** sizes within the A group (e.g. A2, A4 and A7)

**b** sizes within the B group (e.g. B0, B3 and B6)

**c** sizes within the C group (e.g. C1, C5 and C8)

**d** sizes between different groups (e.g. A0, B0 and C0).

| | A sizes (mm) | B sizes (mm) | C sizes (mm) |
|---|---|---|---|
| **0** | 841 × 1189 | 1000 × 1414 | 917 × 1297 |
| **1** | 594 × 841 | 707 × 1000 | 648 × 917 |
| **2** | 420 × 594 | 500 × 707 | 458 × 648 |
| **3** | 297 × 420 | 353 × 500 | 324 × 458 |
| **4** | 210 × 297 | 250 × 353 | 229 × 324 |
| **5** | 148 × 210 | 176 × 250 | 162 × 229 |
| **6** | 105 × 148 | 125 × 176 | 114 × 162 |
| **7** | 74 × 105 | 88 × 125 | 81 × 114 |
| **8** | 52 × 74 | 62 × 88 | 57 × 81 |
| **9** | 37 × 52 | 44 × 62 | 40 × 57 |
| **10** | 26 × 37 | 31 × 44 | 28 × 40 |

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Similar figures

Worksheet
6D Dilations

Topic quiz
6D

# 6E Similar triangles

## Learning intentions

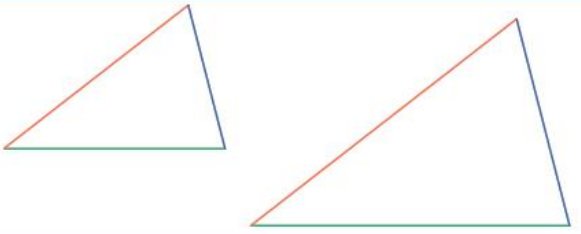
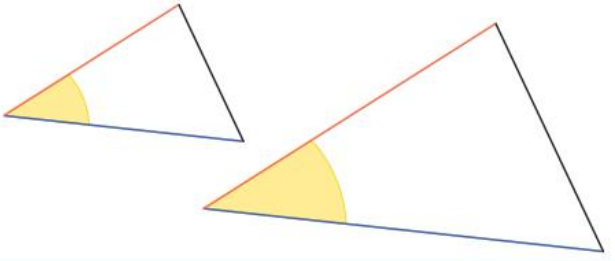
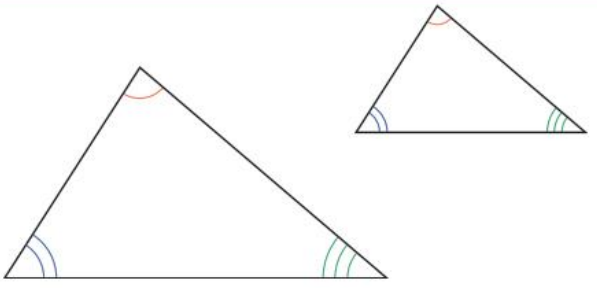
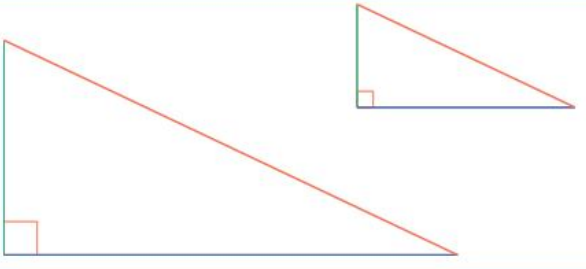
- ✔ I can find unknown angles and lengths in similar triangles.
- ✔ I can determine whether two triangles are similar.

**Inter-year links**

| | |
|---|---|
| Year 7 | 7D Classifying triangles |
| Year 8 | 7E Congruent triangles |
| Year 10 | 7C Congruence and similarity |

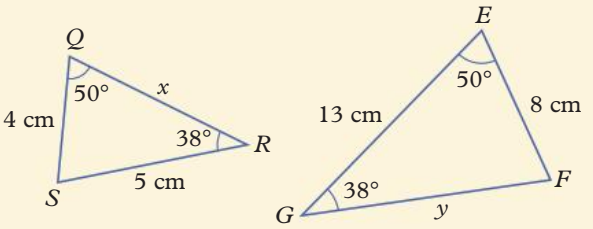
## Similar triangles

- Two triangles are similar if:
  - → corresponding pairs of angles are equal, and
  - → corresponding pairs of side lengths are in the same ratio.
- If a pair of triangles meet any of the following conditions, they are similar:

| Condition | Meaning | Example |
|---|---|---|
| **SSS** | All three pairs of corresponding side lengths are in the same ratio. |  |
| **SAS** | Two corresponding pairs of side lengths are in the same ratio and the included angles are equal. |  |
| **AAA** | All three pairs of corresponding angles are equal. |  |
| **RHS** | The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle. |  |

## Example 6E.1 Finding unknown side lengths in similar triangles

Find the value of the unknown side lengths in this pair of similar triangles.




**THINK**

1. Identify the equal corresponding angles and then match the corresponding sides opposite these angles. Sides $QS$ and $EF$ are both opposite the 38° angle. Sides $RS$ and $GF$ are both opposite the 50° angle.
2. Using the pair of corresponding sides that have known measurements ($QS$ and $EF$), find the scale factor between $\Delta QSR$ and $\Delta EFG$. Consider $\Delta QSR$ as the original triangle and $\Delta EFG$ as the image.
3. Use the scale factor formula to find $y$ in $\Delta EFG$.
4. Use the scale factor formula to find $x$ in $\Delta QSR$.

**WRITE**

Corresponding pairs: $\overline{QS} = 4$ cm and $\overline{EF} = 8$ cm

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{8}{4}$$
$$= 2$$

Corresponding pairs: $\overline{RS} = 5$ cm and $\overline{GF} = y$

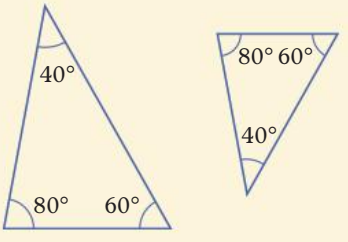
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$2 = \frac{y}{5}$$
$$y = 2 \times 5$$
$$y = 10 \text{ cm}$$

Corresponding pairs: $\overline{QR} = x$ and $\overline{EG} = 13$ cm

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$2 = \frac{13}{x}$$
$$x = \frac{13}{2}$$
$$x = 6.5 \text{ cm}$$

## Example 6E.2 Identifying similarity conditions

Which condition would you use to check that these triangles are similar?




**THINK**

Look at the information provided in each triangle. All angles are given in both triangles.

**WRITE**

The similarity condition AAA can be used to check whether these triangles are similar.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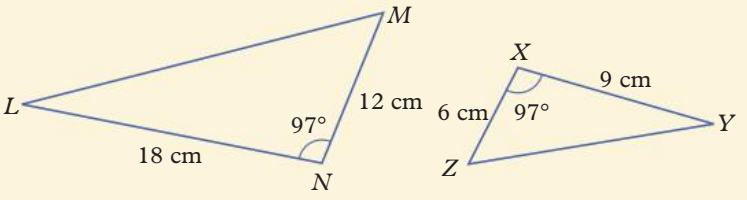
## Example 6E.3 Determining if two triangles are similar

Determine whether these two triangles are similar.




**THINK**

1 Both triangles show two sides and an included angle, so we can use the similarity condition SAS to determine whether the two triangles are similar.

2 Calculate the scale factor between the two shorter sides ($MN$ and $XZ$) and the two longer sides ($LN$ and $XY$).

3 Compare the scale factors to determine whether the similarity condition is met.

4 Write your final answer.

**WRITE**

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{\overline{XZ}}{\overline{MN}}$$
$$= \frac{6 \text{ cm}}{12 \text{ cm}}$$
$$= \frac{1}{2}$$

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{\overline{XY}}{\overline{LN}}$$
$$= \frac{9 \text{ cm}}{18 \text{ cm}}$$
$$= \frac{1}{2}$$

The two triangles are similar as they meet the similarity condition SAS.

**Helpful hints**

✔ Make sure that you identify the corresponding sides and angles correctly. This is a key skill. Remember that the given triangles may not be in the same orientation.

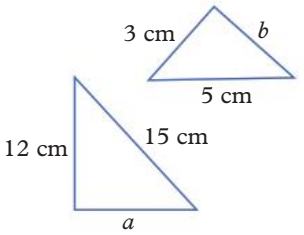
ANS p531

# Exercise 6E Similar triangles

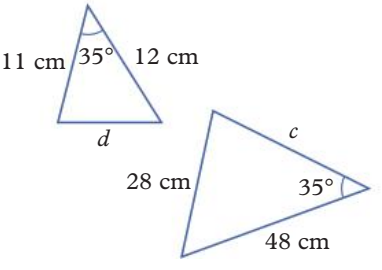
1, 2, 4–6, 9, 11, 12, 14 | 1, 3–5, 7, 8, 10, 13, 15, 19(a, b) | 1, 4, 5, 7, 11, 13, 14, 16–20

6E.1 **1** Find the values of the unknown side lengths in each pair of similar triangles.

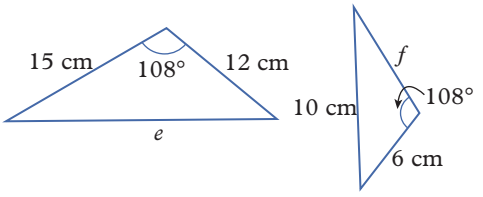
**a**



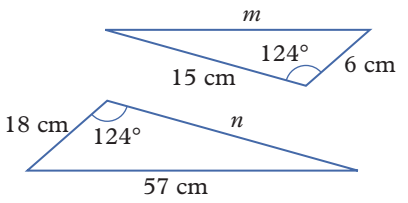

**b**



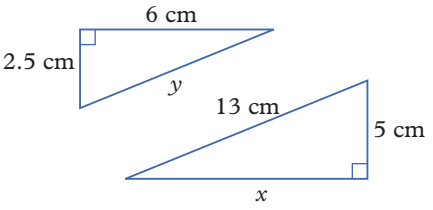
**c**



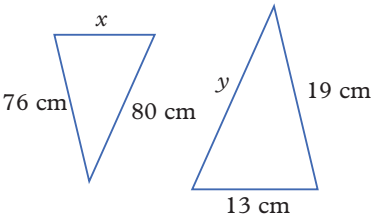

**d**




**e**

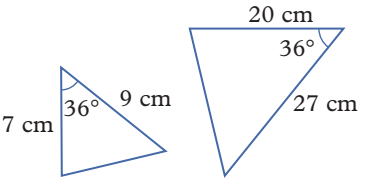



**f**

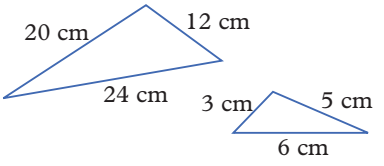



**6E.2** **2** State the condition you would use to decide whether each pair of triangles is similar.

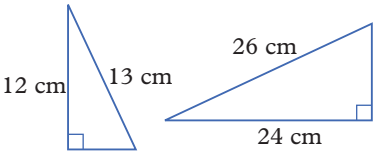
**a**



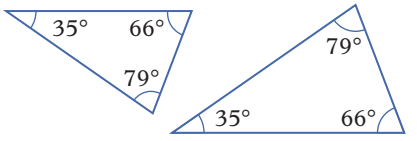

**b**



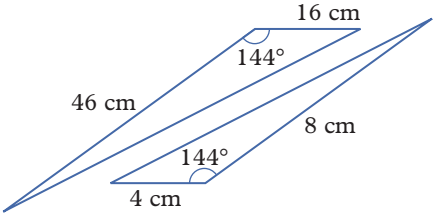

**c**



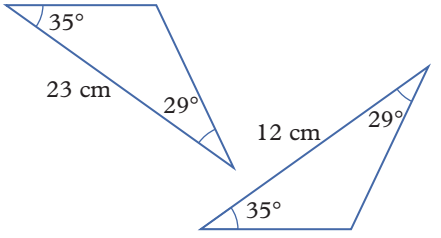

**d**




**e**

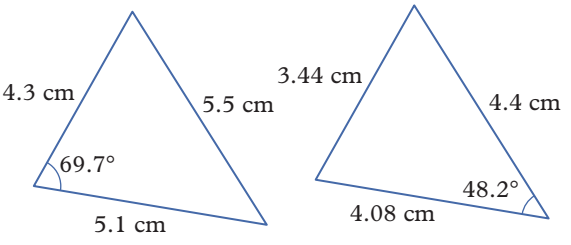



**f**

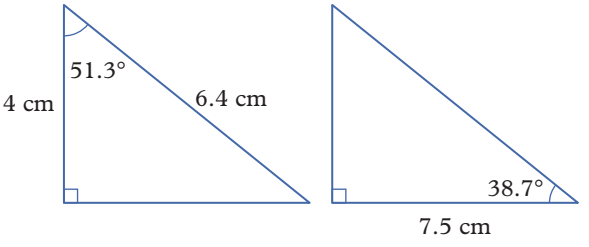



**3** Which condition(s) could you use to check that these triangles are similar?

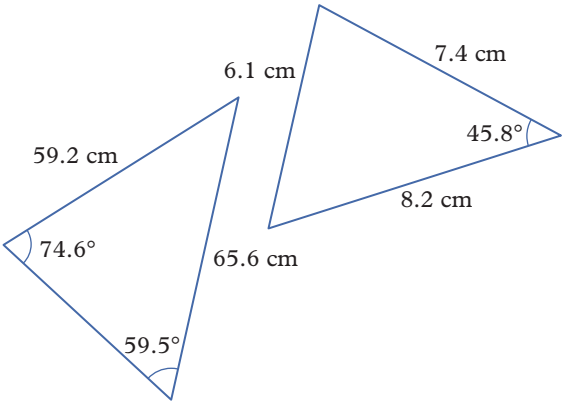
**a**



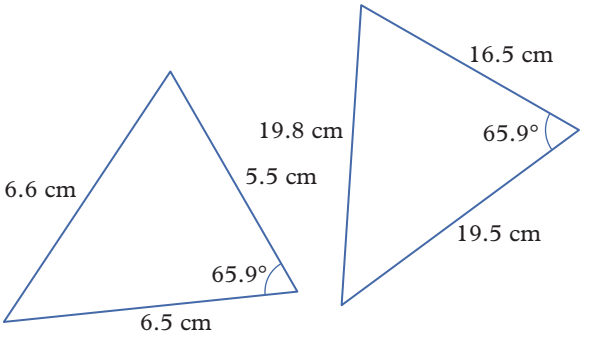

**b**




**c**

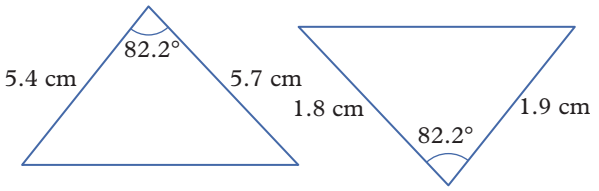



**d**

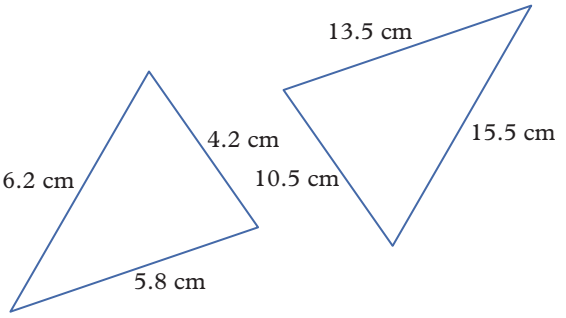


6E.3 **4** Determine if these pairs of triangles are similar. State the similarity condition used or explain why the triangles are not similar.

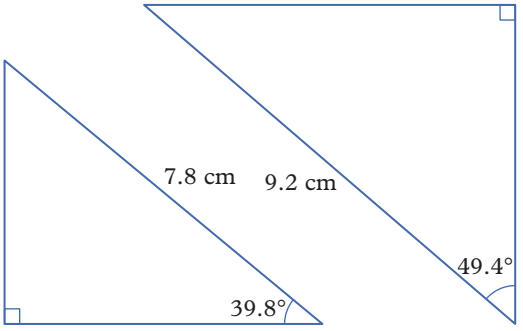
**a**



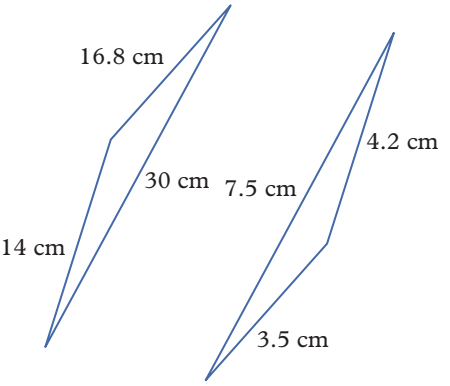

**b**



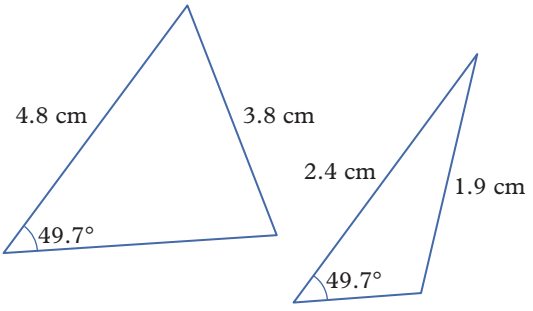

**c**



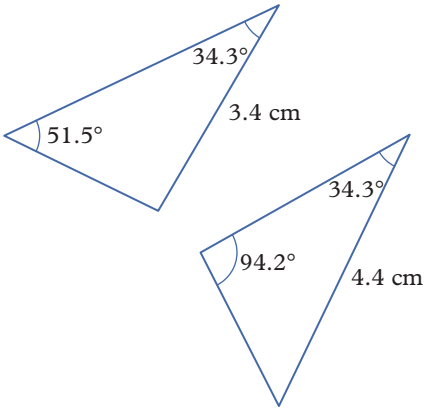

**d**




**e**

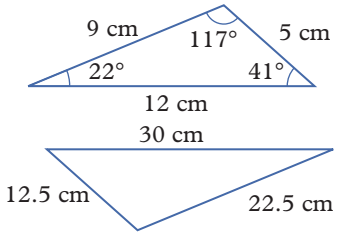



**f**

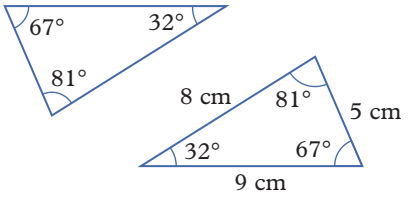



**5** Determine whether these pairs of triangles are similar.

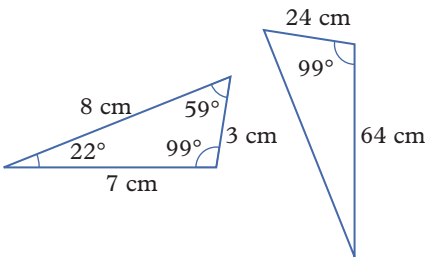
**a**



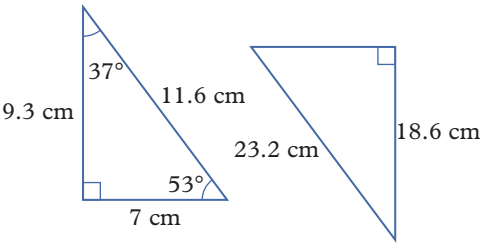

**b**



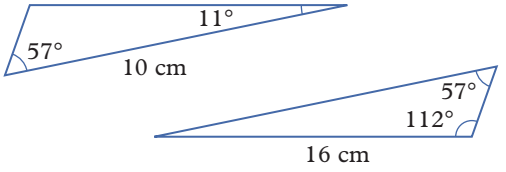

**c**



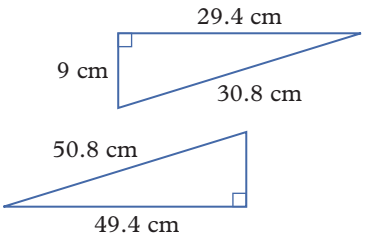

**d**



**e**




**f**




**6** Decide if the pairs of triangles from question **2** are similar.

**7** Consider the four similar-triangle conditions. What conditions need to be added to each of the conditions for the similar triangles to be congruent?

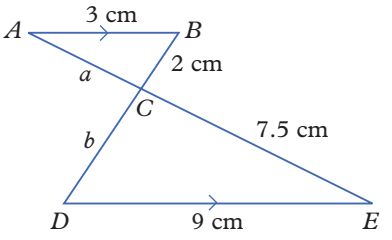
**8** Why are all equilateral triangles similar to each other?

**9** Explain why only two angles need to be checked when using AAA as a test for similarity.

**10 a** Using an understanding of angles and parallel lines, explain why these pair of triangles are similar.

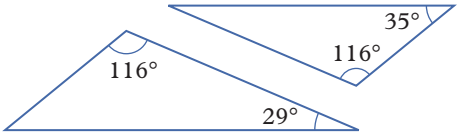
**b** If triangle *ABC* is the original figure, find the scale factor between the two triangles.

**c** Find the length of the unknown sides.

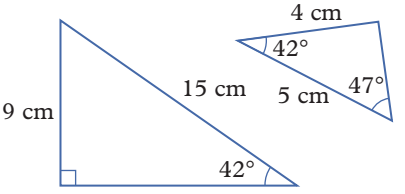



**11** Use your understanding of triangle properties to determine whether these pairs of triangles are similar.

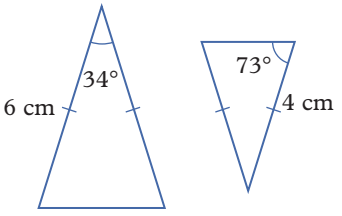
**a**



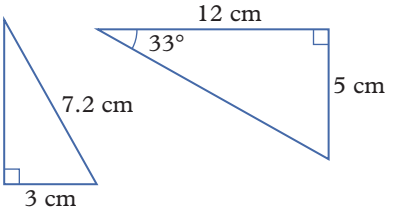

**b**



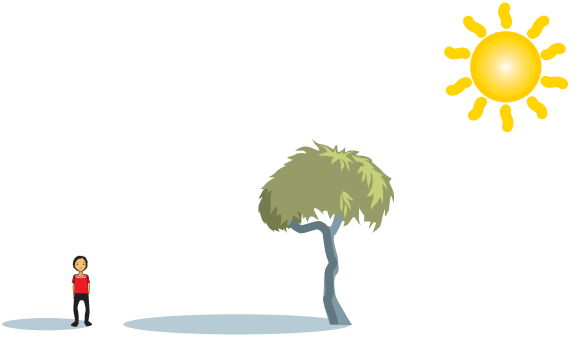

**c**




**d**



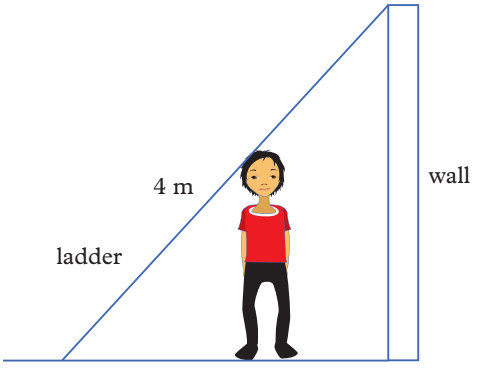

**12** Similar triangles can be used to find the heights or lengths of objects that are difficult to measure. Consider this diagram showing the shadows cast by a person and a tree. The person is 1.76 m tall and casts a shadow of 2.1 m. If the tree casts a shadow of 7.8 m, how tall is the tree?



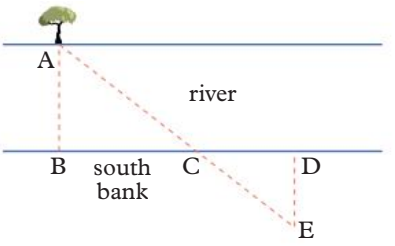
**13** Luke wants to measure the heights of the walls in his house and then order some wall decorations. He places his 4 m ladder on an angle so that it hits the top of the wall, as shown in the diagram. The ladder has rungs every 50 cm: at 50 cm, 1 m, etc. If Luke, at 170 cm tall, stood directly under the ladder and his head came into contact with the fifth rung, find:

**a** the length of the ladder where it touched Luke's head

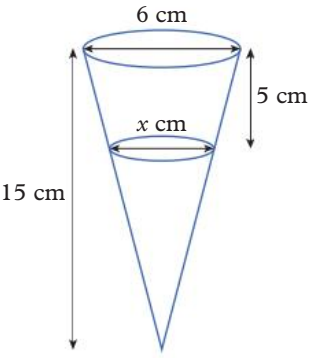
**b** the height of Luke's walls.



**14** In order to find the width of a river, Ahmed places three markers along its southern bank. Marker $B$ is directly opposite a tree at position $A$. Marker $C$ is 50 m east of marker $B$ and 40 m west of marker $D$. Ahmed then stands at point $E$, 30 m south of marker $D$ and he can see marker $C$ in a direct line with the tree at point $A$. Mark in these measurements on the diagram and then use similar triangles to find the width of the river.




**15** An ice-cream cone is 15 cm high with an opening of diameter 6 cm. If chocolate coats the inside of the cone from the bottom to 5 cm below its top, find the largest diameter of the chocolate coating inside the cone.




**16** Draw diagrams to help you solve these similar triangle problems.

**a** Scott has a red triangular flag that is 10 cm high and 15 cm long. He wants to create a larger version from 2 m of red material. If he uses its full length, what height would this larger flag be?

**b** Christy bought a new netball ring but she wasn't sure that it was the official height (3.05 m). She measured her shadow as 117 cm and the netball ring's shadow as 195 cm. If Christy is 1.8 m tall, is the netball ring the official height?

**c** Oscar wants to know the height of a flagpole. A guide wire positioned 3.5 m from its base reached to exactly half its height. Oscar stood exactly halfway between the flagpole and where the guide wire attached to the ground and the guide wire touched the top of his head. If Oscar is 1.75 m tall, how tall is the flagpole?

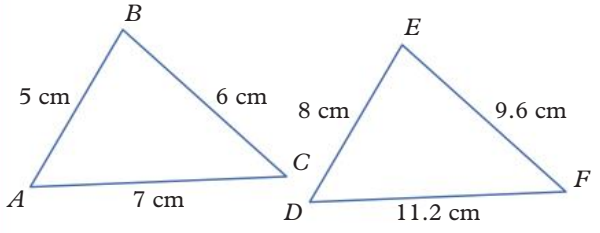
**17** Sean and Tania want to know whose house is taller. One morning, Sean measures his shadow to be 1.9 m and the shadow of his house is 3.42 m. Later that day, Tania measures her shadow to be 2.3 m and the shadow of her house is 3.8 m. If Sean is 172 cm tall and Tania is 165 cm tall, whose house is taller?

**18** Explain why you cannot just compare the shadow lengths of the houses in question **17** to decide which house is taller.

**19** The process of showing that two triangles are similar can be formalised and generalised further. We want to prove beyond a shadow of a doubt that a given fact is true by writing up bulletproof reasoning behind every step we take. We call these geometric proofs.

Follow the worked examples and then complete the proofs.

Example: Prove that $\triangle ABC \sim \triangle DEF$.




$\frac{DE}{AB} = \frac{8\text{ cm}}{5\text{ cm}} = 1.6$

$\frac{EF}{BC} = \frac{9.6\text{ cm}}{6\text{ cm}} = 1.6$

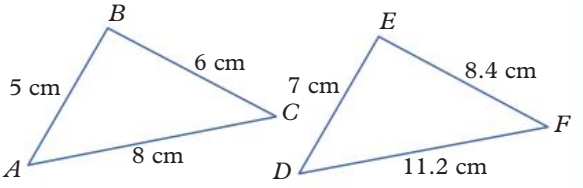
$\frac{FD}{CA} = \frac{11.2\text{ cm}}{7\text{ cm}} = 1.6$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

Corresponding sides are in the same ratio.

$\therefore \triangle ABC \sim \triangle DEF$ by SSS

**a** Prove that $\triangle ABC \sim \triangle DEF$.




$\frac{DE}{AB} = \frac{\square}{\square} =$

$\frac{EF}{BC} = \frac{\square}{\square} =$

$\frac{FD}{CA} = \frac{\square}{\square} =$

$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

Corresponding sides are in the same ratio.

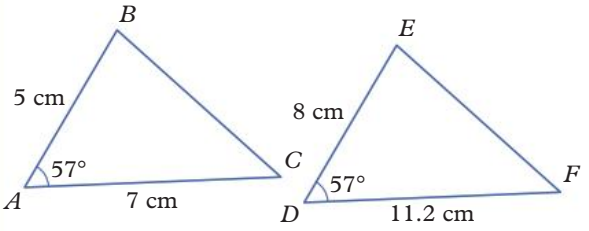
$\therefore \triangle$____ $\sim \triangle$____ by ____

PROBLEM SOLVING AND REASONING

CHALLENGE

CHALLENGE

Example: Prove that $\triangle ABC \sim \triangle DEF$.




$\frac{DE}{AB} = \frac{8 \text{ cm}}{5 \text{ cm}} = 1.6$

$\frac{FD}{CA} = \frac{11.2 \text{ cm}}{7 \text{ cm}} = 1.6$

$\angle EDF = 57°$

$\angle BAC = 57°$

$\frac{DE}{AB} = \frac{FD}{CA}$

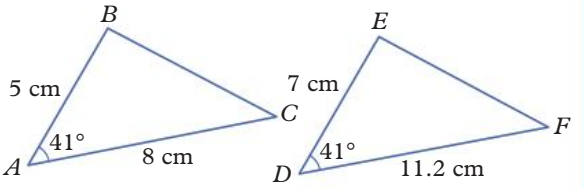
Corresponding sides are in the same ratio.

$\angle EDF = \angle BAC$

Corresponding included angles are equal.

$\therefore \triangle ABC \sim \triangle DEF$ by SAS

**b** Prove that $\triangle ABC \sim \triangle DEF$.




$\frac{DE}{AB} = \frac{\square}{\square} =$

$\frac{FD}{CA} = \frac{\square}{\square} =$

$\angle EDF =$ ____°

$\angle BAC =$ ____°

$\frac{\square}{\square} = \frac{\square}{\square}$

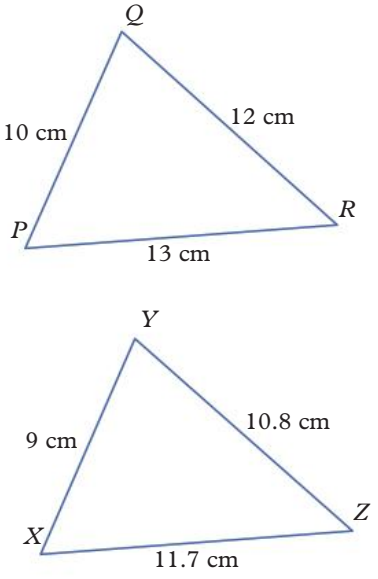
Corresponding sides are in the same ratio.

$\angle$____ = $\angle$____

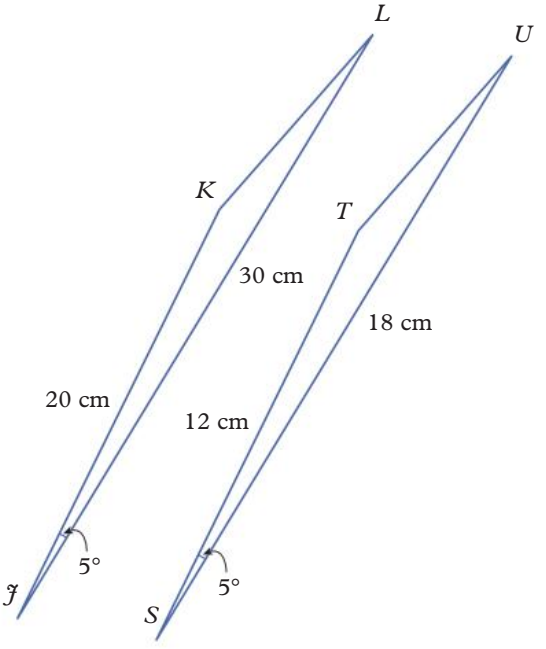
Corresponding included angles are equal.

$\therefore \triangle$____ ~ $\triangle$____ by ____

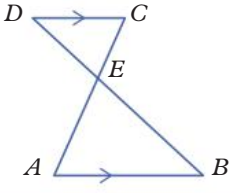
**c i** Prove that $\triangle PQR \sim \triangle XYZ$.




**ii** Prove that $\triangle JKL \sim \triangle STU$.




**20** Prove that $\triangle ABE \sim \triangle CDE$.




Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Similar triangles

Investigation
Using mirrors to calculate length

Topic quiz
6E



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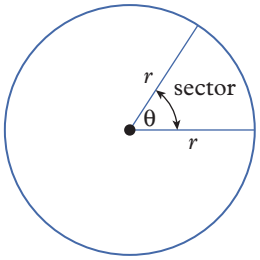
# Chapter summary

## Area of a sector




$$A = \frac{\theta}{360} \times \pi r^2$$

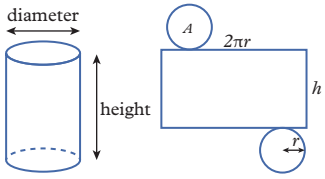
## Area of composite shapes

1. Split the figure into simple shapes that have known area formulas.
2. Calculate any missing dimensions.
3. Calculate the areas of the individual shapes using the area formulas.
4. Add or subtract the areas to calculate the total area.

## Surface area

1. Determine the number of faces.
2. Calculate the area of each face.
3. Add the areas of the faces together.

- Right prism: TSA = $2lw + 2lh + 2hw$
- Cylinder: TSA = $2\pi rh + 2\pi r^2$

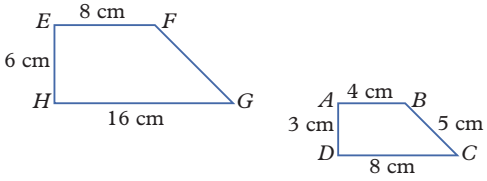



## Dilations

- An enlargement creates an image that is larger than the original shape.
- A reduction creates an image that is smaller than the original shape.
- A centre of dilation and scale factor are needed to perform a dilation.

## Similar figures

- Same shape but do not need to be the same size.
- All corresponding angles are equal in size.
- All corresponding sides are in the same ratio.




## Scale factor

$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

- Scale factor between 0 and 1: reduction.
- Scale factor is greater than 1: enlargement.
- Length of $A'B'$ = length of $AB$ × scale factor
- A scale factor of $\frac{1}{10}$ is the same as a ratio of 1 : 10.

## Area scale factor

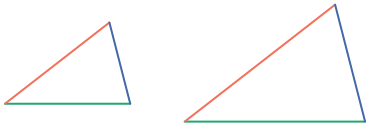
$$\text{Area scale factor} = \frac{\text{image area}}{\text{original area}}$$

$$\text{Area scale factor} = \text{scale factor}^2$$

## Similar triangles

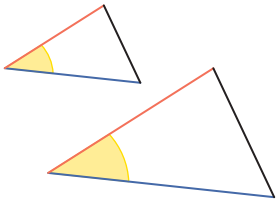
### SSS

All three pairs of corresponding side lengths are in the same ratio.



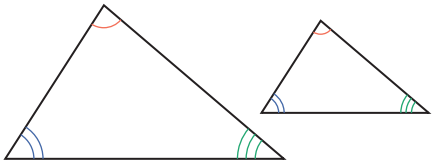
### SAS

Two corresponding pair of side lengths are in the same ratio and the included angles are equal.



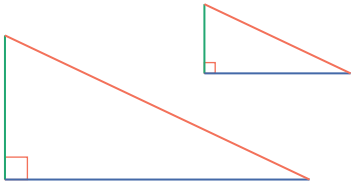
### AAA

All the three pairs of corresponding angles are equal.



### RHS

The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle.



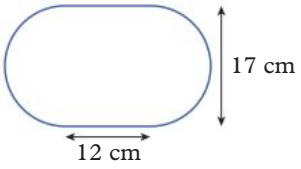
# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams

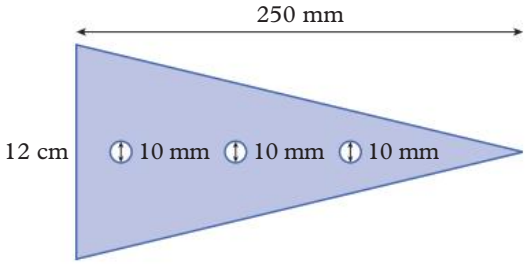
## Multiple-choice

6A **1** The area of this shape is closest to:



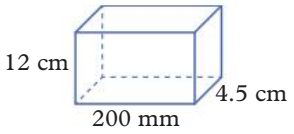

**A** 77 cm$^2$ **B** 204 cm$^2$ **C** 431 cm$^2$ **D** 1112 cm$^2$ **E** 641 cm$^2$

6A **2** This figure has an area closest to:



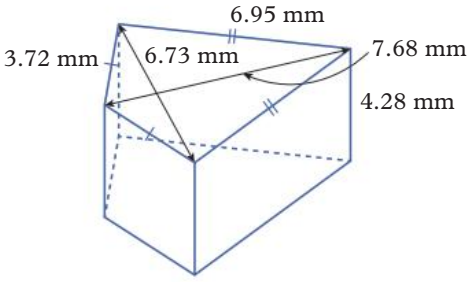

**A** 140.58 cm$^2$ **B** 147.64 cm$^2$ **C** 149.21 cm$^2$ **D** 150 cm$^2$ **E** 152.36 cm$^2$

6B **3** The surface area of this rectangular prism is:




**A** 6708 cm$^2$ **B** 1080 cm$^2$ **C** 768 cm$^2$ **D** 678 cm$^2$ **E** 384 cm$^2$

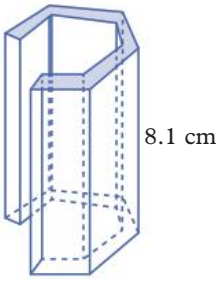
6B **4** The surface area of this prism is:




**A** 150.7444 mm$^2$ **B** 143.0216 mm$^2$ **C** 110.655 12 mm$^2$ **D** 221.217 792 mm$^2$ **E** 194.708 mm$^2$

6C **5** The volume of this prism is:

$P$ = 19.02 cm, $A$ = 4.22 cm$^2$




**A** 154.062 cm$^3$ **B** 80.2644 cm$^3$ **C** 114.4464 cm$^3$ **D** 650.141 64 cm$^3$ **E** 34.182 cm$^3$

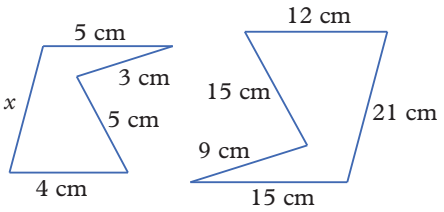
6C **6** The volume of the prism in question **4** is:

**A** 150.7444 cm$^3$ **B** 143.0216 cm$^3$ **C** 110.608 896 cm$^3$ **D** 221.217 792 cm$^3$ **E** 194.708 cm$^3$

6D **7** Which scale factor will not result in an enlargement?

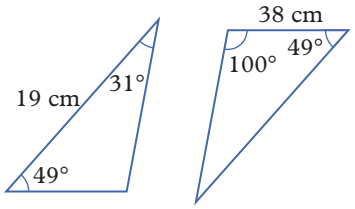
**A** $\frac{1}{3}$ **B** $\frac{5}{4}$ **C** 2 **D** $\frac{10}{3}$ **E** 4

6D **8** Find the value of $x$ for these similar figures.



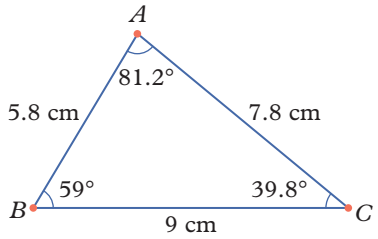

**A** 21 cm **B** 63 cm **C** 7 cm **D** 8.75 cm **E** 12 cm

6E **9** Which test would you use to decide whether these two triangles are similar?

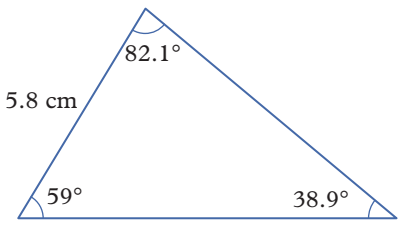



**A** AAA **B** SSS **C** SAS **D** RHS **E** AAS

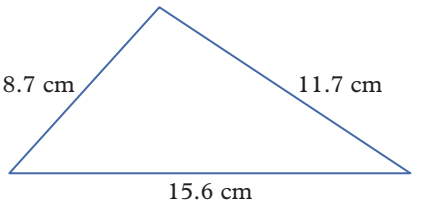
6E **10** Which triangle is similar to $\triangle ABC$?



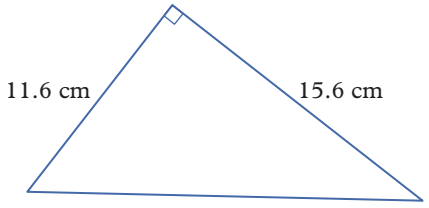

**A**



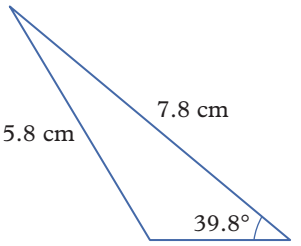

**B**



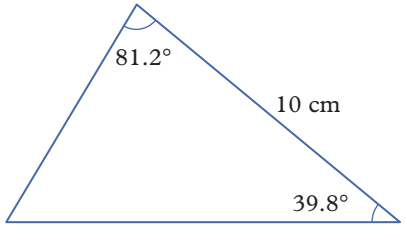

**C**




**D**

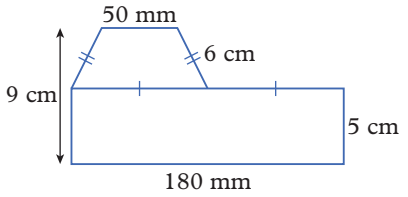



**E**

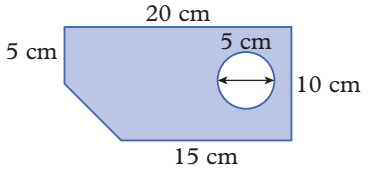


## Short answer

6A **1** Calculate the area of this composite shape in square centimetres.

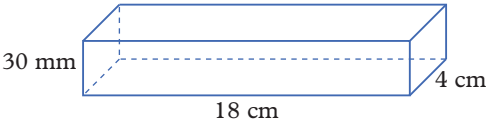



6A **2** Calculate the shaded area correct to two decimal places.

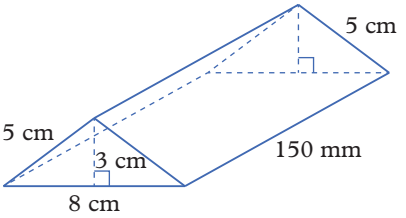



6B **3** Calculate the surface area of each prism.

**a**

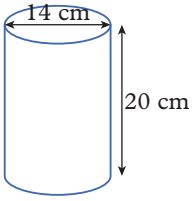



**b**

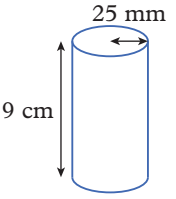



6B **4** Calculate the surface area of each cylinder correct to two decimal places.

**a**




**b**




6C **5** Calculate the volume of each prism in question **3**.

6C **6** Calculate the volume of each cylinder in question **4**. Give your answers to two decimal places.

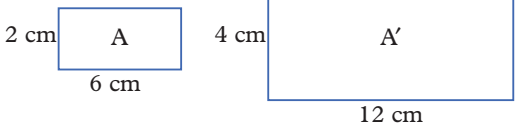
6C **7** The volume of a cylindrical water tank with radius 10 m is 4000 $m^3$.

**a** What is the capacity of the water tank?

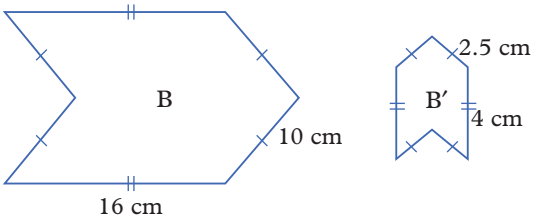
**b** Calculate the height of the tank to the nearest metre.

6D **8** Describe each transformation as a reduction or an enlargement and include the scale factor.

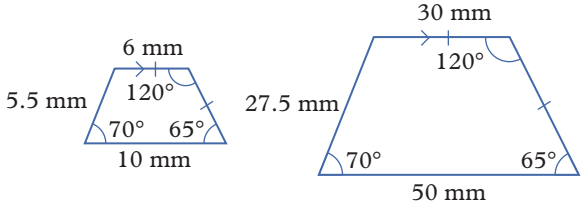
**a**




**b**




6D **9** Decide if this pair of shapes is similar, giving a reason for your answer.



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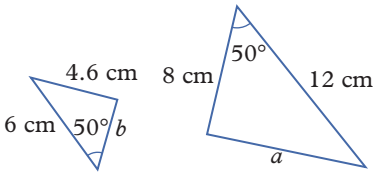
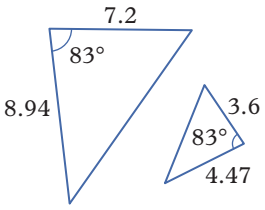
6E **10** Given that these two triangles are similar, find the value of the unknown side lengths.



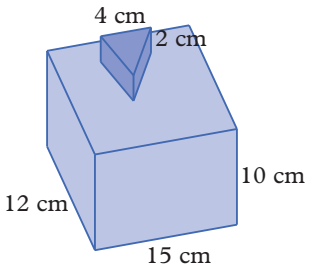

6E **11** Determine whether these two triangles are similar. If similar, state the similarity condition you used, or explain why they are not similar.

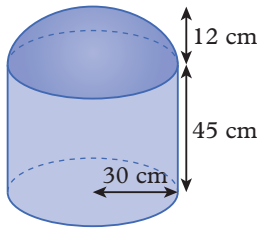



## Analysis

**1** Angelique is making two sculptures. Her designs are shown.

In the first sculpture, the rectangular parts will be painted purple and the triangular part will be painted pink. The triangular prism has a height of 2 cm and the cross-section is a right-angled triangle with a base length of 5 cm and a perpendicular height of 4 cm.




**a** Calculate the area to be painted each colour ($cm^2$), assuming all sides, including the bottom, will be painted before assembly.

**b** Angelique has a tin of purple paint that will cover 1 $m^2$. Does she have enough purple paint for two coats?

**c** After painting two purple coats, what area would the remaining purple paint cover?

**d** Pink paint comes in smaller tins which contain enough paint to cover 90 $cm^2$. What is the maximum number of coats possible from one tin?

**e** After painting the maximum number of pink coats, what area would the remaining pink paint cover?

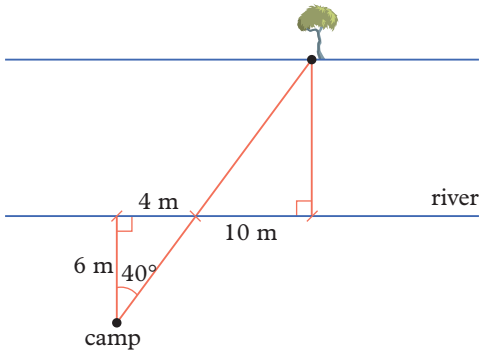
In the second sculpture, the cylinder will be painted, including its top and bottom circular faces. The dome on top of the cylinder will be made of clear acrylic and will not be painted.

**f** Calculate the total surface area to be painted in $cm^2$, correct to two decimal places.

**g** Angelique decides to paint the cylinder green. This paint comes in tins that cover 1 $m^2$. How many tins will she require?

**h** Is there any green paint remaining? If so, what area would it cover in $cm^2$ correct to two decimal places?

**2** While preparing to study a species of gum tree on a field trip, Stephanie set up a camp 6 m from a river's edge. From the camp, she measures the angle to a gum tree she wishes to get to and records various measurements, as shown in the diagram.



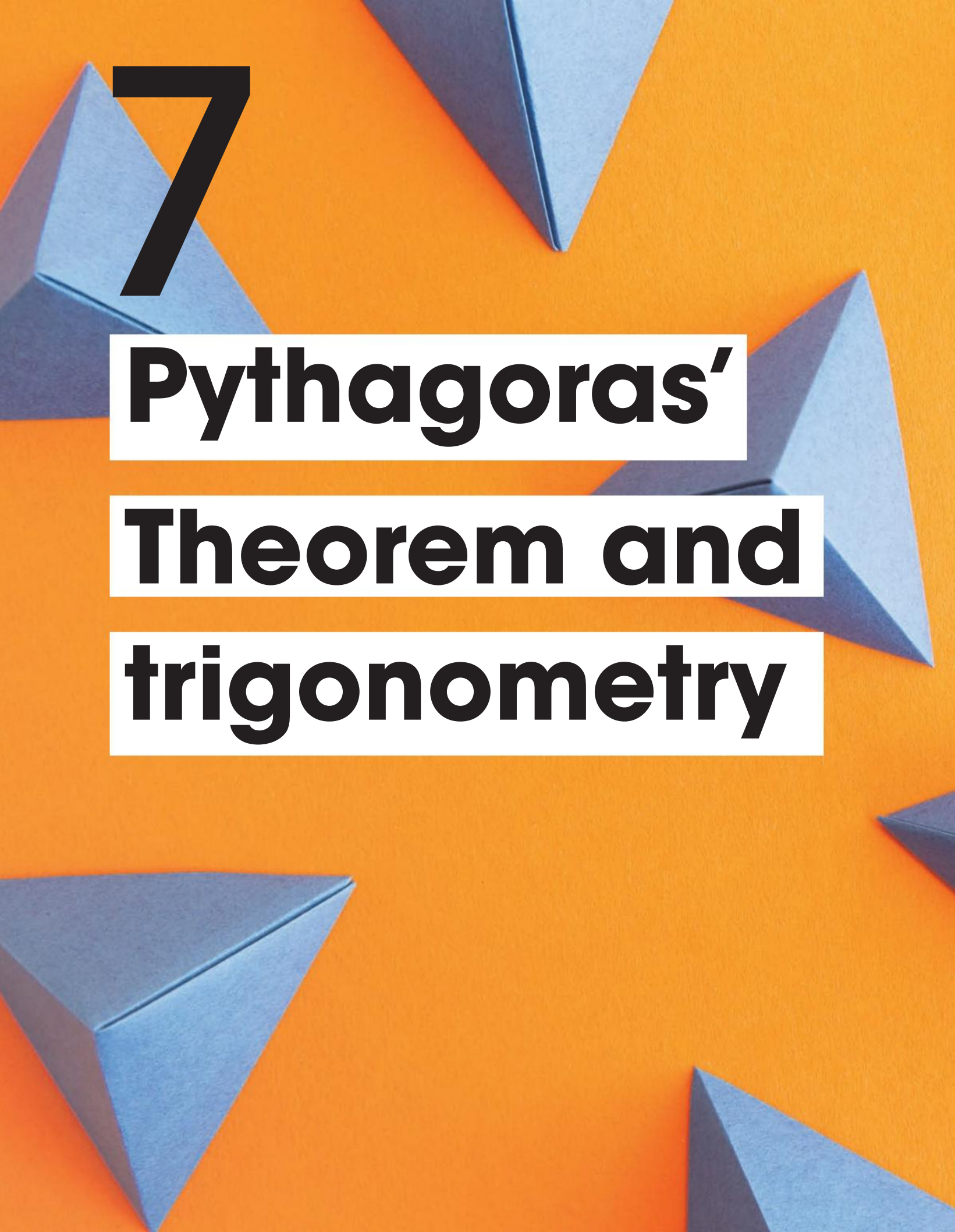

**a** Considering both the triangles shown in Stephanie's diagram, what are the angle sizes in each triangle?

**b** Are the two triangles congruent or only similar? Provide a reason to support your answer.

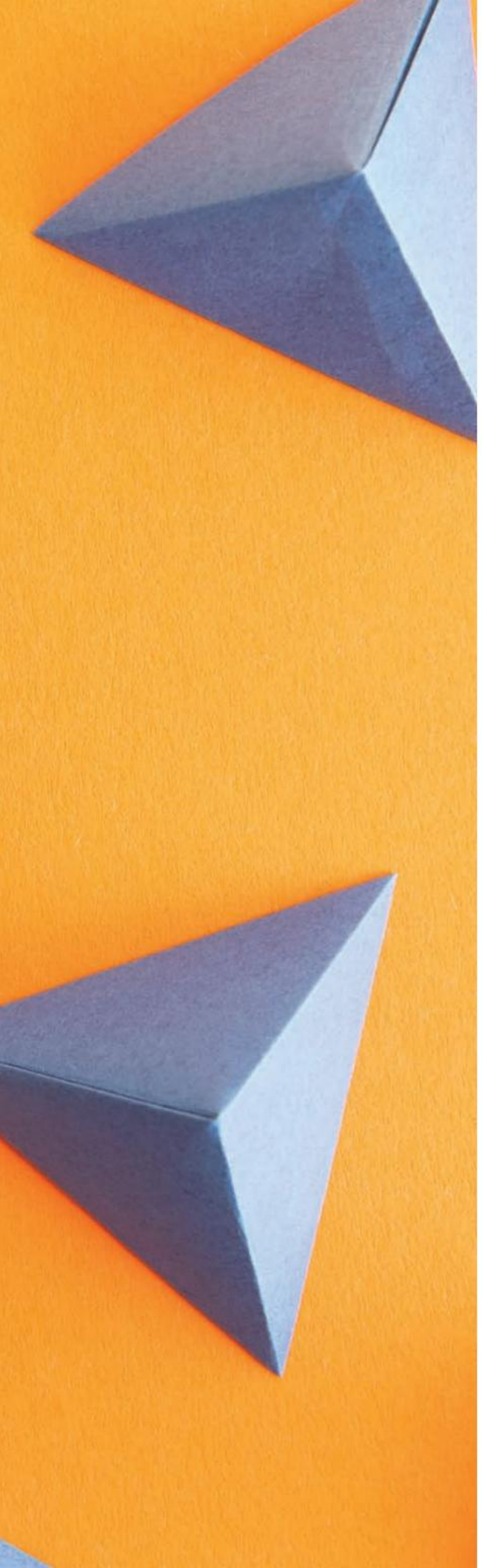
**c** What is the scale factor linking the triangle attached to the camp site to the triangle attached to the gum tree?

**d** Briefly explain how the triangles can be used to determine the width of the river.

**e** Calculate the width of the river.



# 7 Pythagoras' Theorem and trigonometry



## Index


**7A** Angles and lines

**7B** Pythagoras' Theorem

**7C** Using Pythagoras' Theorem to find the length of a shorter side

**7D** Trigonometric ratios

**7E** Using trigonometry to find lengths

**7F** Using trigonometry to find angles


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Triangle properties
- ✔ Squares and square roots
- ✔ Substitution
- ✔ Solving equations with inverse operations

## Curriculum links

- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles (VCMMG318)
- Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (VCMMG319)
- Apply trigonometry to solve right-angled triangle problems (VCMMG320)

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## Materials

- ✔ Calculator

# 7A Angles and lines

## Learning intentions

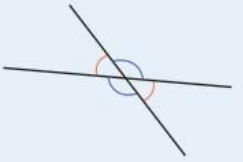
- ✔ I can calculate and use complementary and supplementary angles.
- ✔ I can calculate and use angles at a point.
- ✔ I can calculate and use related angles.
- ✔ I can calculate unknown interior and exterior angles of triangles.

Inter-year links

| | |
|---|---|
| Years 5/6 | Understanding angles |
| Year 7 | 7B Angles at a point |
| Year 8 | 7A Angles |
| Year 10 | 7A Geometry review |

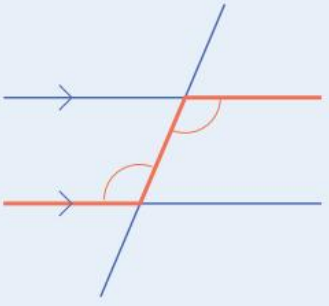
## Angle properties

- Angles at a point that form a straight line add to 180°.
- Angles at a point that form a revolution add to 360°.
- Two angles that add to 90° are called **complementary angles**.
- Two angles that add to 180° are called **supplementary angles**.
- **Vertically opposite angles** are equal in size.

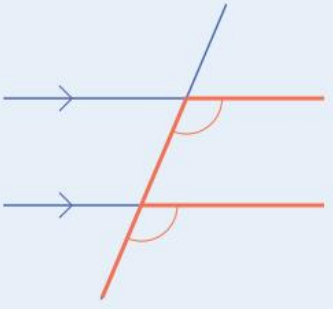


## Angles and parallel lines

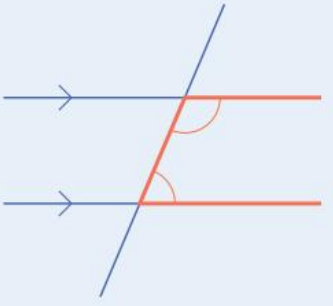
- When two **parallel lines** are intersected by another line (a **transversal**) some related angles are formed.
  - → **Alternate angles** are equal.
  - → **Corresponding angles** are equal.
  - → **Co-interior angles** are supplementary.



alternate angles



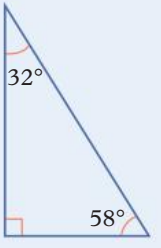
corresponding angles



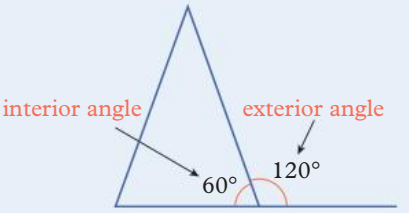
co-interior angles

## Interior and exterior angles

- An **interior angle** is an angle formed inside a polygon.
  The internal angle sum of a triangle is 180°.
  90° + 32°+ 58° = 180°

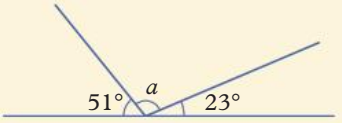


- An **exterior angle** of a triangle can be formed by extending one of its sides. The sum of the interior and exterior angle is 180° and a straight line is formed. An exterior angle is equal to the sum of the opposite interior angles.




## Example 7A.1 Finding angle size using complementary and supplementary angles

Find the size of the unknown angle.




**THINK**

1 Identify the relationship between the unknown and known angles. Angles at a point that form a straight line add to 180°.

2 Subtract the known angles from 180° to find the value of $a$.

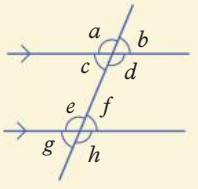
**WRITE**

$51° + a + 23° = 180°$

$a = 180° - 51° - 23°$
$= 106°$

## Example 7A.2 Finding angles related to parallel lines

If angle $c$ is equal to 41°, name its alternate angle and state its value.



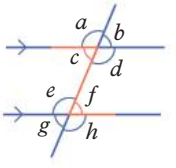

**THINK**

1 Angles $c$ and $f$ are inside the two angles of the letter 'Z', so they are alternate angles.

2 Alternate angles formed by parallel lines are equal in size, so $f = c$.

**WRITE**

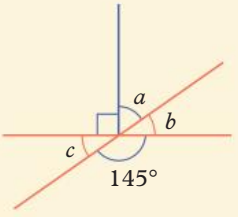
$c$ and $f$ are alternate angles.




$f = 41°$

## **Example 7A.3** Finding angles around a point

Find the size of the unknown angle.




**THINK**

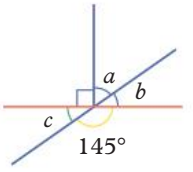
1 Identify any relationships between unknown and known angles. Angles $c$ and 145° are supplementary, so $c = 180° - 145°$.

2 Angle $b$ is vertically opposite to $c$ and vertically opposite angles are equal in size.

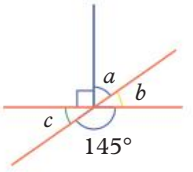
3 Angles $a$ and $b$ are complementary angles, so $a = 90° - b$.

**WRITE**

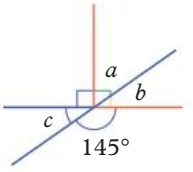
$c = 180° - 145°$
$= 35°$




$b = c$
$= 35°$

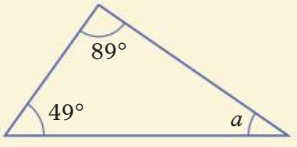



$a = 90° - 35°$
$= 55°$




## **Example 7A.4** Finding the size of an angle in a triangle

Find the size of angle $a$.




**THINK**

The internal angles in a triangle add to 180°. Subtract each of the known angles from 180° to calculate the third angle, $a$.

**WRITE**

$a = 180° - 89° - 49°$
$= 42°$

**Helpful hints**

- ✔ Unless stated, you will not need to measure any of the angles in a diagram. All of the information required to answer questions is already provided.
- ✔ Don't assume that two lines are parallel just because they appear to be parallel in the diagram! Parallel lines will be marked with arrowheads or this information will be given in the question.
- ✔ Transversal lines can form acute, obtuse or right angles with parallel lines.
- ✔ Sides lengths that are opposite to equal angles in a triangle will be equal in size. Likewise, angles that are opposite to equal side lengths will be equal in size.

ANS p532

# Exercise 7A Angles and lines

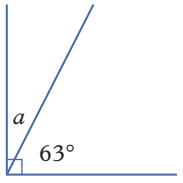
1, 2, 4–7, 8(a, c, e), 9, 12, 15, 17 | 2, 4–7, 8(e, f), 10–12, 17, 18, 20 | 3, 6, 7, 8(e, f), 11, 13, 14, 16, 17(a–d), 19, 21, 22

**1** For each angle, find the following. (If it is not possible, write N/A.)

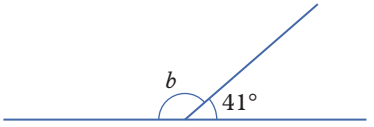
**a** 23° **b** 47° **c** 176° **d** 9°
**e** 97° **f** 115° **g** 17° **h** 186°
**i** complementary angle
**ii** supplementary angle

7A.1 **2** Find the size of each unknown angle.

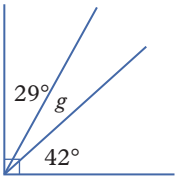
**a**



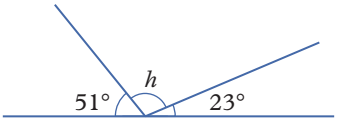

**b**



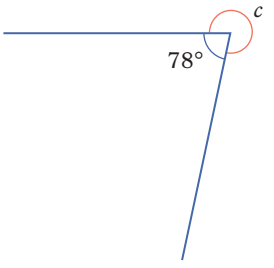

**c**



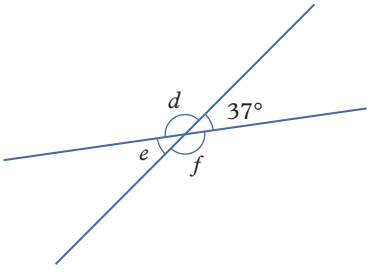

**d**



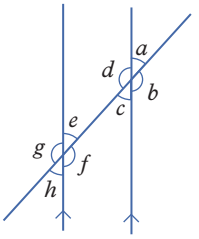

**e**




**f**




7A.2 **3** Use this figure to name each angle below and state its value from the given information.




**a** $\angle d = 123°$; alternate angle
**b** $\angle a = 46°$; corresponding angle
**c** $\angle h = 37°$; vertically opposite angle
**d** $\angle f = 98°$; co-interior angle
**e** $\angle b = 138°$; corresponding angle
**f** $\angle e = 53°$; alternate angle

UNDERSTANDING AND FLUENCY

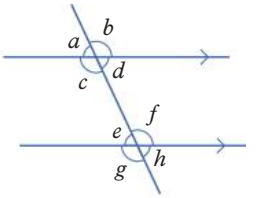
**4** For the figure shown, find the angle that is:

**a** alternate to $c$ **b** corresponding to $g$

**c** co-interior to $f$ **d** vertically opposite to $d$

**e** alternate to $e$ **f** co-interior to $c$

**g** vertically opposite to $a$ **h** corresponding to $b$.

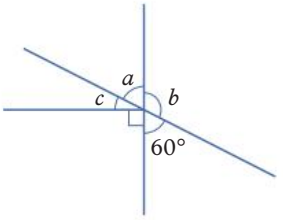



**5** Using the figure in question **4**, if $a$ is equal to 78°, find the value of:

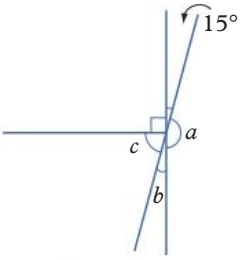
**a** $b$ **b** $c$ **c** $d$ **d** $e$ **e** $f$ **f** $g$.

7A.3 **6** Find the size of the labelled unknown angles.

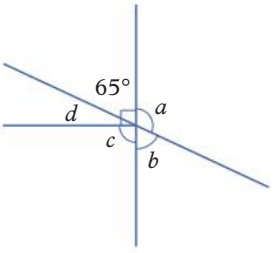
**a**



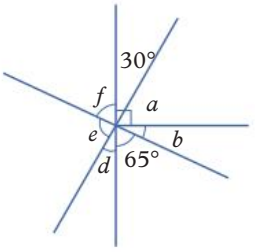

**b**



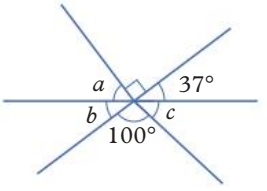

**c**



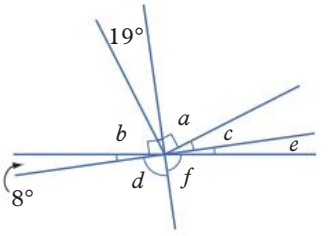

**d**




**e**

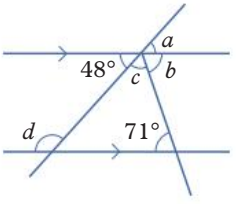



**f**

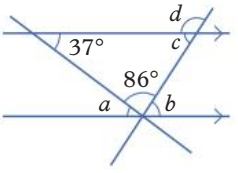



**7** Find the size of the labelled unknown angles in each diagram.

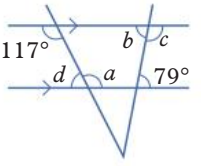
**a**



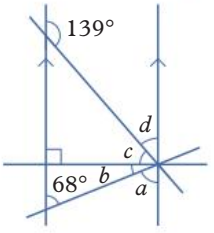

**b**



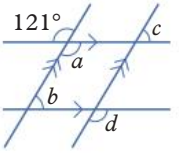

**c**



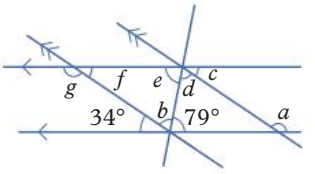

**d**




**e**

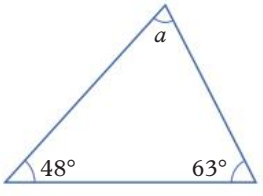



**f**

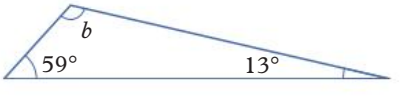



7A.4 **8** Find the size of each labelled unknown angle.

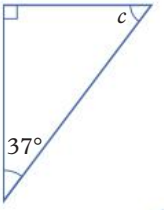
**a**



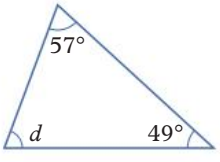

**b**



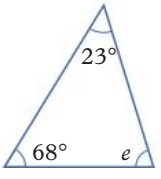

**c**



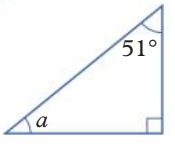

**d**




**e**

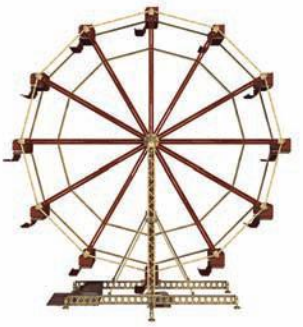



**f**



**9** **a** Explain how you would find the angle between each carriage of the Ferris wheel.

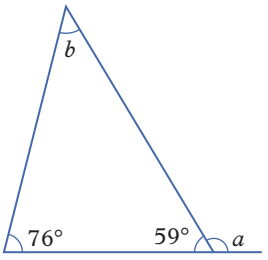
**b** Find the angle between each carriage.



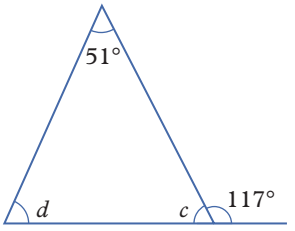
**10** In Example 7A.3, one method for finding the three unknown angles was shown. State another two ways you could find the size of the unknown angles.

**11** **a** Use your understanding of angles within a triangle and supplementary angles to find the size of each labelled unknown angle.

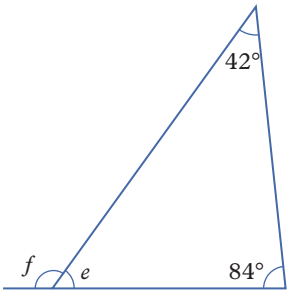
**i**




**ii**




**iii**




**b** Using your understanding of supplementary angles, explain why any exterior angle of a triangle is equal to the sum of the two opposite interior angles.

**c** What can you say about the sum of the exterior angles of a triangle?

**12** A ladder leans up against a wall.

**a** If the ladder makes an angle of 32° with the wall, what angle does it make with the ground?

A stepladder stands with two legs of equal length on the ground.

**b** If the stepladder is opened up to make an angle of 43° at its top, what angle do its legs make with the ground?

**c** What is the obtuse angle that both sides of the ladder make with the ground – the angle on which you would climb it?

**13** Consider any pair of parallel lines cut by a transversal. Explain how, if you know the size of one angle, you are able to find the size of all remaining angles without using a protractor.

**14** Describe a process you could use to determine whether two lines are parallel.

**15** **a** Complete this table. The first column has been started for you.

| | **56°** | **78°** | **19°** | **146°** | **27°** | **103°** |
|---|---|---|---|---|---|---|
| **Complementary** | 34° | | | | | |
| **Supplementary** | | | | | | |
| **Vertically opposite** | 56° | | | | | |
| **Co-interior on parallel lines** | | | | | | |
| **Alternate on parallel lines** | | | | | | |
| **Corresponding on parallel lines** | | | | | | |
| **At a point** | 304° | | | | | |

**b** Why have two cells in the table been shaded out?

**c** Write a rule for each row so that somebody who doesn't know what the terms mean could complete the table.

**16** If you shine light onto a mirror, it reflects off at the same angle, as shown in figure A.

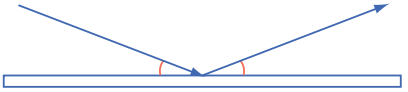


**Figure A**

**a** If you shine a light onto a mirror at 32°:

**i** what is the angle between the reflected ray of light and the mirror?

**ii** what is the angle between the two rays of light?

**iii** what is the angle between the ray of light and a line perpendicular to the mirror?

In physics, an imaginary line that is perpendicular to the mirror is known as the 'normal'. The law of reflection says that angle $a$ (the angle of incidence) is equal to angle $b$ (the angle of reflection), as shown in figure B.

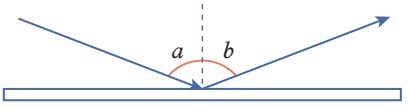



**Figure B**

**b** Explain how this is the same rule as you were using in part **a**.

**c** If $a$ was equal to 54°, find the size of:

**i** angle $b$

**ii** the angle the light ray makes with the mirror.

**17** Rather than dividing angles into decimals (such as 15.5°), they are usually divided into minutes and seconds. This convention is known as degrees-minutes-seconds (DMS).

**a** How many seconds are in a minute?

**b** How many minutes do you think might be in a degree?

**c** How many minutes are there in:

**i** $\frac{1}{2}°$

**ii** $\frac{1}{4}°$

**iii** $\frac{1}{3}°$?

Minutes are represented by a prime (′) and seconds are represented by a double prime (″). For example, 14°35′22″ represents 14 degrees, 35 minutes and 22 seconds.

**d** Is 14°35′22″ greater or less than 14.5°?

**e** Write each angle using the words: degrees, minutes and seconds.

**i** 83°16′55″

**ii** 27°43′04″

**iii** 154°09′37″

**f** Write each angle using DMS conventions.

**i** 230 degrees, 29 minutes and 13 seconds

**ii** 67 degrees, 18 minutes and 2 seconds

**iii** 192 degrees, 56 minutes and 42 seconds

**18 a** Convert each angle into DMS conventions. (Hint: 1° = 60′)

**i** 86.75° **ii** 113.8° **iii** 217.1° **iv** 9.65°

**b** Convert each DMS angle into degrees as a decimal number.

**i** 196°42′ **ii** 98°51′ **iii** 23°33′ **iv** 107°03′

**19** Consider this analogue clock, which shows 6 pm.

**a** What is the angle between the two hands of the clock?

**b** What angle would the minute hand of a clock turn to display these times?

**i** 3 am **ii** 9 pm **iii** 5 pm **iv** 11 am **v** 8 pm

**c** What is the angle between any two adjacent hour-marks?

**d** How many degrees are in the following fractions of the angle in part **c**?

**i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$

**e** The small hand moves between each number at a rate of one number per hour. At the half-hour mark, the small hand is halfway between two numbers. Use your answers to part **d** to find the smallest angle between the two hands of a clock displaying:

**i** 4.30 pm **ii** 7.15 am **iii** 9.45 pm **iv** 1.15 pm **v** 5.45 pm

**20** Remembering that there are also 60 seconds in a minute, perform these conversions. Round your decimal answers to two decimal places.

**a** 72°30′30′ into degrees (decimal)

**b** 46.3975° into DMS

**c** 89°36′18′ into degrees (decimal)

**d** 34.1575° into DMS

**21** Find the size of the smallest angle between the two hands of a clock displaying these times.

**a** 11.25 am

**b** 2.05 pm

**c** 10.17 am

**d** 6.32 am

**e** 12.48 pm

**22** Two mirrors are joined together at 90°. Use your understanding of the law of reflection, complementary angles and angles in parallel lines to draw a diagram to show what would happen if you shone a light at an angle of 50° with respect to the mirror into the perpendicular mirrors. Label all angles. (Hint: Use figure B in question **16** as a starting point and place a vertical mirror at the right edge of the horizontal mirror.)

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Angles at a point

Interactive skillsheet
Angles and parallel lines

Topic quiz
7A

# 7B Pythagoras' Theorem

## Learning intentions

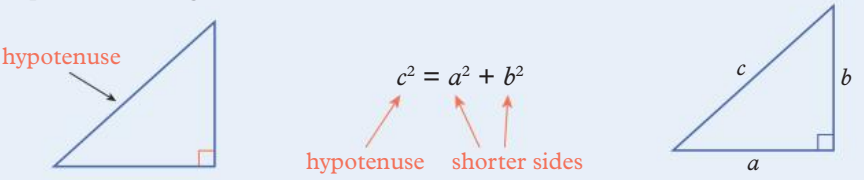
- ✔ I can correctly use the plus-minus symbol when determining the roots of a number.
- ✔ I can use Pythagoras' Theorem to determine whether a triangle is right angled.
- ✔ I can use Pythagoras' Theorem to determine the hypotenuse of a right-angled triangle.

**Inter-year links**

| | |
|---|---|
| Year 7 | 1G Indices and square roots |
| Year 8 | 4E Roots |
| Year 10 | 8A Pythagoras' Theorem |

## Pythagoras' Theorem in right-angled triangles

- The **hypotenuse** is the longest side of a right-angled triangle opposite a right angle.
- **Pythagoras' Theorem** states the relationship between the three side lengths of a right-angled triangle. That is, the sum of the squares of the shorter sides is equal to the square of the hypotenuse.
  The relationship for the triangle shown is:




- A triangle is right-angled if it has side lengths that satisfy Pythagoras' Theorem.
- Any set of three whole numbers that satisfy Pythagoras' Theorem is called a **Pythagorean triad** (or **Pythagorean triple**).
  For example, 3, 4, 5 is a Pythagorean triad as $5^2 = 3^2 + 4^2$.

## Finding the length of the hypotenuse

- To find the length of the hypotenuse using Pythagoras' Theorem:
  1. State Pythagoras' Theorem and define $a$, $b$ and $c$.
  2. Substitute the values for $a$, $b$ and $c$ and simplify the equation.
  3. Solve for $c$ by finding the square root of both sides of the equation using inverse operations.
- The result for the length of the hypotenuse can be written as an exact value (whole number, terminating decimal or in surd form) or approximated to a specific number of decimal places.

## Square root of a number

- A quadratic equation has two solutions, a positive solution and a negative solution. The positive solution is the square root of the number.
  For example, $6 \times 6 = 36$ and $-6 \times -6 = 36$ so the equation $x^2 = 36$ has two solutions:
  $x = 6$ and $x = -6$
- The plus-minus symbol, $\pm$, can be used to write both results in an efficient way.
  For example, $x^2 = 36$
  $x = \pm\sqrt{36}$
  $x = \pm 6$

## Example 7B.1 Determining the square root of a number

Solve the following equations for $x$, rounding to two decimal places as needed.

**a** $x^2 = 49$ **b** $x^2 = 24$

**THINK**

1 Recall that the inverse operation for squaring a number is taking the square root. Use the times tables or square numbers to determine which number multiplied by itself is equal to the number under the square root.

2 Use the calculator to find the approximate square root. Round to two decimal places.

**WRITE**

**a** $7 \times 7 = 49$ and $-7 \times -7 = 49$

$x^2 = 49$

$x = \pm\sqrt{49}$

$= \pm 7$

**b** $x^2 = 24$

$x = \pm\sqrt{24}$

$\sqrt{24} \approx 4.8989...$

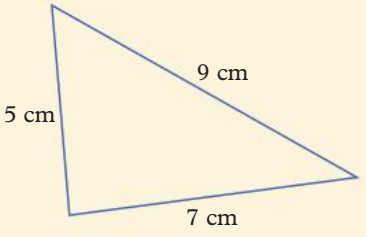
$\approx 4.8\boxed{9}8$

$\approx 4.90$

So, $x \approx \pm 4.90$

## Example 7B.2 Using Pythagoras' Theorem to determine whether a triangle is right-angled

Use Pythagoras' Theorem to decide whether this is a right-angled triangle.




**THINK**

1 Identify the length of the longest side and define as $c$.

2 Calculate the value of $c^2$.

3 Identify the lengths of the two shorter sides and define as $a$ and $b$.

4 Calculate the value of $a^2 + b^2$.

5 Compare $c^2$ to $a^2 + b^2$.

If the two values are equal, then the triangle is right-angled.

If the two values are not equal, then it is not a right-angled triangle.

**WRITE**

Let $c = 9$ cm.

$c^2 = 9^2$

$= 81$

Let $a = 5$ cm and $b = 7$ cm.

$a^2 + b^2 = 5^2 + 7^2$

$= 25 + 49$

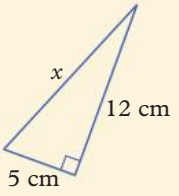
$= 74$

$c^2 \neq a^2 + b^2$

The triangle is not a right-angled triangle.

## Example 7B.3 Calculating the length of the hypotenuse (whole number)

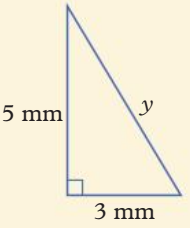
Use Pythagoras' Theorem to calculate the length of the hypotenuse of this triangle.




| THINK | WRITE |
|---|---|
| **1** The triangle is right-angled, so state Pythagoras' Theorem and define $a$, $b$ and $c$. | $c^2 = a^2 + b^2$ where $a = 5$ cm, $b = 12$ cm and $c = x$. |
| **2** Substitute the values for $a$, $b$ and $c$, and simplify the equation. | $x^2 = 5^2 + 12^2$<br>$= 25 + 144$<br>$= 169$ |
| **3** Solve for $x$ by finding the square root of both sides of the equation using inverse operations. Write the positive solution only with the correct unit, as $x$ represents a length. | $x = \sqrt{169}$<br>$= 13$ cm |

## Example 7B.4 Calculating the length of the hypotenuse (decimal value)

Calculate the length of the hypotenuse of this triangle, rounded to two decimal places.




| THINK | WRITE |
|---|---|
| **1** The triangle is right-angled, so state Pythagoras' Theorem and define $a$, $b$ and $c$. | $c^2 = a^2 + b^2$ where $a = 3$ mm, $b = 5$ mm and $c = y$. |
| **2** Substitute the values for $a$, $b$ and $c$, and simplify the equation. | $y^2 = 3^2 + 5^2$<br>$= 9 + 25$<br>$= 34$ |
| **3** Solve for $y$ by finding the square root of both sides of the equation. Write the positive solution only, as $y$ represents a length. | $y = \sqrt{3}$ |
| **4** Use a calculator to find the approximate length. Round to two decimal places. (Note that $\sqrt{34}$ mm is the exact length.) | $\approx 5.83$ mm |

**Helpful hints**

- ✔ Remember that the plus-minus sign is used to present two numbers in a short-hand way. It is also correct to write the two answers separately.
  For example, as $6 \times 6 = 36$ and $-6 \times -6 = 36$, then if $x^2 = 36$, $x = \pm 6$.
- ✔ Make sure you correctly identify the hypotenuse. It is the longest side that is opposite the right angle. It may not be obvious, so always look at the numbers carefully.
- ✔ Pythagoras' Theorem is an excellent formula to memorise. You will use it many times in your VCE Maths courses. Writing the formula from memory before substituting the values in will help you to memorise it quicker.
- ✔ It doesn't matter which shorter sides you assign to be $a$ or $b$, but the hypotenuse must always be $c$!

ANS p533

# Exercise 7B Pythagoras' Theorem

△ 1(a, c, f, g), 2–9, 12, 14, 15, 16(a, b, f)

□ 1(b, d, h), 3, 4, 6–8, 10, 13, 16(c, e, f), 17, 20(a, b), 23

⬠ 3, 7, 8, 10, 11, 13, 18, 19, 20(c, d), 21, 22

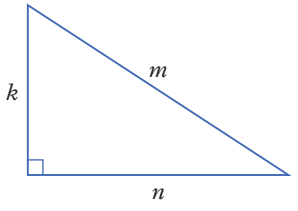
7B.1 **1** Solve the following equations for $x$, rounding to two decimal places as needed.

**a** $x^2 = 25$ **b** $x^2 = 81$ **c** $x^2 = 80$ **d** $x^2 = 31$

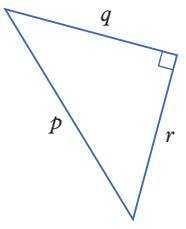
**e** $x^2 = 2.25$ **f** $x^2 = 5.23$ **g** $x^2 = \frac{36}{121}$ **h** $x^2 = \frac{43}{12}$

**2** Identify the pronumeral shown on the hypotenuse in each triangle.

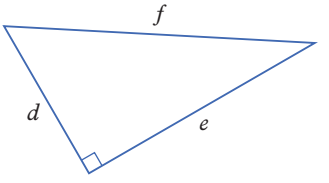
**a**




**b**

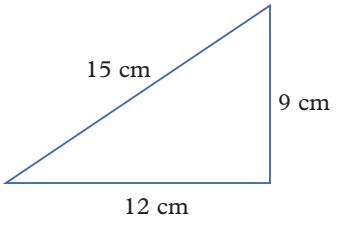



**c**

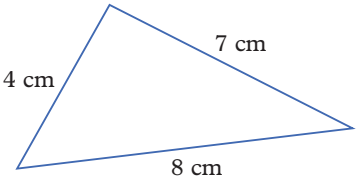



7B.2 **3** Use Pythagoras' Theorem to decide whether these are right-angled triangles.

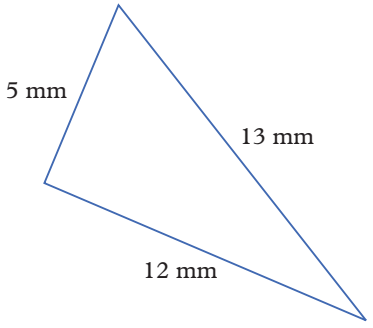
**a**



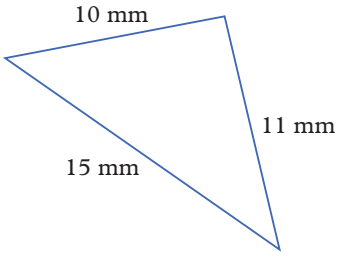

**b**




**c**




**d**




UNDERSTANDING AND FLUENCY

**4** Decide whether each triangle with the given side lengths is right-angled. (Hint: Which side length is the hypotenuse?)

**a** 6 mm, 8 mm, 10 mm

**b** 7 cm, 24 cm, 25 cm

**c** 8 cm, 10 cm, 14 cm

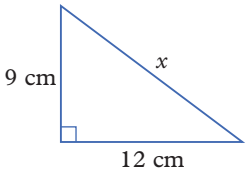
**d** 11 mm, 15 mm, 19 mm

**e** 20 cm, 21 cm, 29 cm

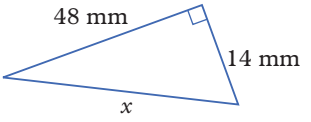
**f** 10 mm, 18 mm, 20 mm

7B.3 **5** Use Pythagoras' Theorem to calculate the length of the hypotenuse in each triangle.

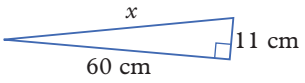
**a**



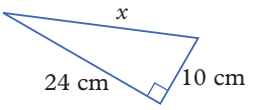

**b**




**c**

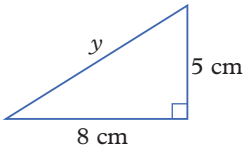



**d**

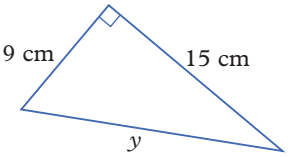



7B.4 **6** Calculate the length of the hypotenuse in each triangle, correct to two decimal places.

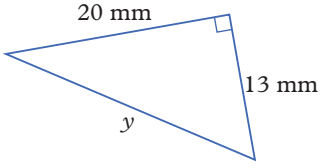
**a**



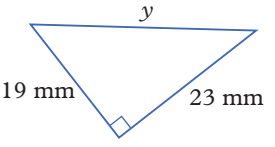

**b**




**c**

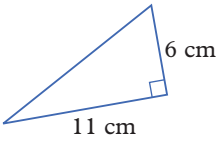



**d**

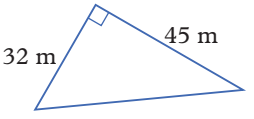



**7** Find the unknown length in each right-angled triangle, correct to two decimal places.

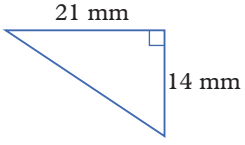
**a**



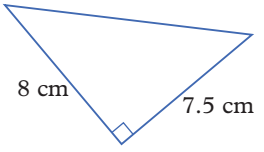

**b**




**c**

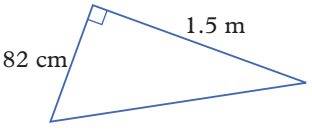



**d**

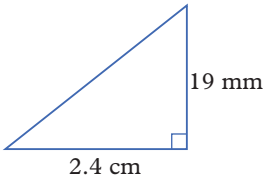



**8** Find the length of the hypotenuse in each right-angled triangle, correct to the nearest centimetre. (Hint: Convert all length measurements to centimetres first.)

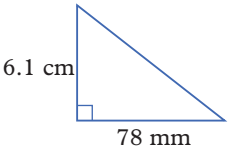
**a**



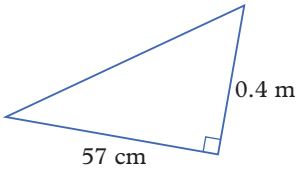

**b**



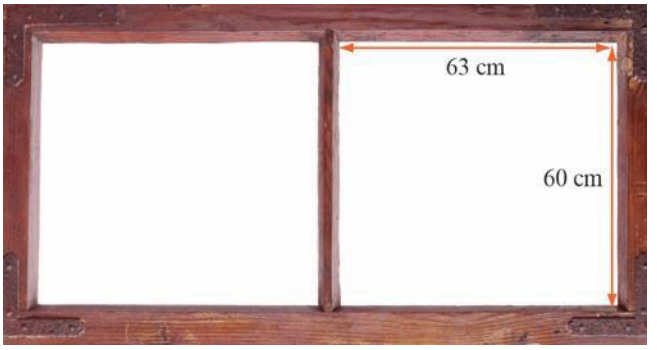

**c**




**d**



**9** The timber frames in a window should meet at right angles. The diagonal length of the glass in one section of the window below is measured to be 87 cm. Use Pythagoras' Theorem to check whether this window frame has been made correctly.




**10** A rectangular picture frame measures 70 cm by 24 cm. The picture framer checks that the frame is 'square' (right angled) by measuring the distance from the top left corner to the bottom right corner of the frame. The measured distance is 73 cm.

**a** Draw a diagram of the frame and label it with all the known information.

**b** Use Pythagoras' Theorem to check whether the frame is 'square'. If it is not, should the diagonal distance be longer or shorter?

**c** Suggest a length for the diagonal distance.

**11** Both multiplication and addition are commutative. That is, when numbers or variables are multiplied or added, the order doesn't matter, as the result will be the same.
For example: $2 \times 3 = 3 \times 2$ and $2 + 3 = 3 + 2$
In Pythagoras' Theorem, $a$ and $b$ refer to the lengths of the two shorter sides of a right-angled triangle. Does it matter which of the shorter sides is labelled $a$ and which is labelled $b$? Explain by referring to commutative properties.

**12** Pythagorean triples can be generated using formulas. One method for finding the Pythagorean triples uses three formulas and two known values, $m$ and $n$ where $m > n$, to generate shorter side lengths $a$ and $b$, and the hypotenuse $c$. The formulas are listed below.

→ $a = m^2 - n^2$

→ $b = 2mn$

→ $c = m^2 + n^2$

Use the formulas to complete the table below to generate Pythagorean triples. The first row has been completed for you.

| **Values** | | **Pythagorean triple** | | |
|---|---|---|---|---|
| ***m*** | ***n*** | $a = m^2 - n^2$ | $b = 2mn$ | $c = m^2 + n^2$ |
| 2 | 1 | $2^2 - 1^2 = 3$ | $2 \times 2 \times 1 = 4$ | $2^2 + 1^2 = 5$ |
| 3 | 1 | | | |
| 3 | 2 | | | |
| 4 | 1 | | | |
| 4 | 2 | | | |
| 4 | 3 | | | |
| 5 | 1 | | | |
| 5 | 2 | | | |
| 5 | 3 | | | |
| 5 | 4 | | | |
| 6 | 1 | | | |

**13** Pythagorean triples form the lengths of right-angled triangles. Some triples are multiples of another triple and they form similar triangles.

For example, (6, 8, 10) and (9, 12, 15) are multiples of (3, 4, 5).

The Pythagorean triple that does not have a common factor between the three sides is called a primitive Pythagorean triple.

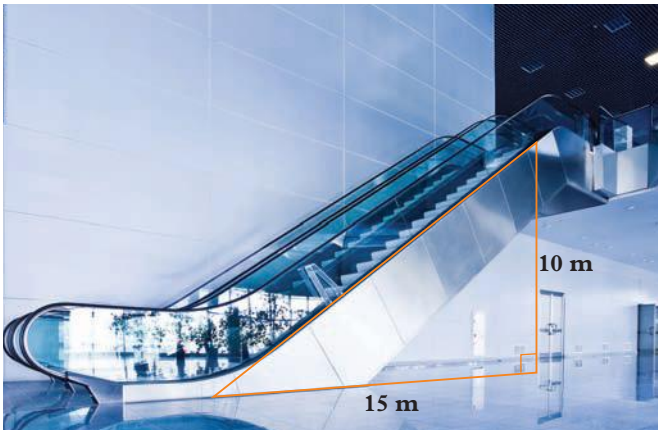
**a** List the primitive Pythagorean triples from question **12**.

**b** Determine the primitive Pythagorean triple for each of the Pythagoreans triples below.

**i** (50, 120, 130) **ii** (24, 45, 51) **iii** (27, 36, 45)

**iv** (36, 160, 164) **v** (390, 432, 582) **vi** (231, 2420, 2431)

**14** An escalator rises a vertical distance of 10 m for a horizontal distance of 15 m. What distance would you travel from the bottom to the top of the escalator? Give your answer correct to two decimal places.




**15** Consider triangle A shown.

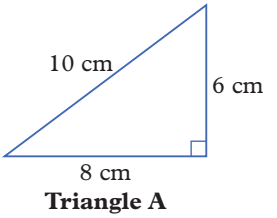
**a** Use Pythagoras' Theorem to confirm that triangle A is right-angled.

**b** A new triangle, Triangle B, is formed by doubling the side lengths of triangle A. Does Pythagoras' Theorem still work for triangle B?

**c** Triangle C is formed by tripling the side lengths of triangle A. Does Pythagoras' Theorem still work for triangle C?

**d** Triangle D is formed by halving the side lengths of triangle A. Does Pythagoras' Theorem still work for triangle D?

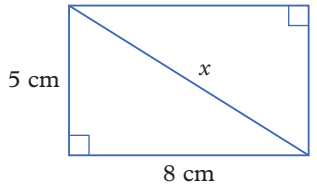
**e** Will Pythagoras' Theorem still work if the side lengths of triangle A are multiplied by 10?



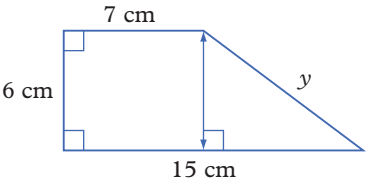
**Triangle A**

**16** Use Pythagoras' Theorem to calculate the unknown length indicated by the pronumeral in each diagram, correct to two decimal places.

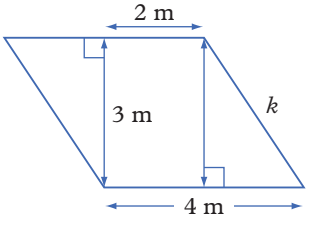
**a**



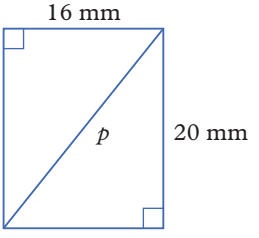

**b**



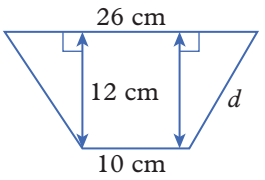

**c**



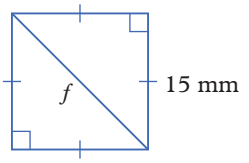

**d**



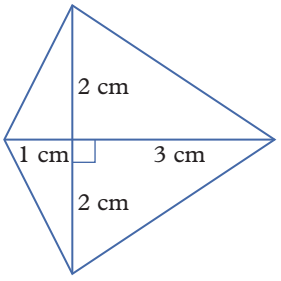
**e**




**f**




**17** Calculate the perimeter of the kite shown correct to two decimal places.



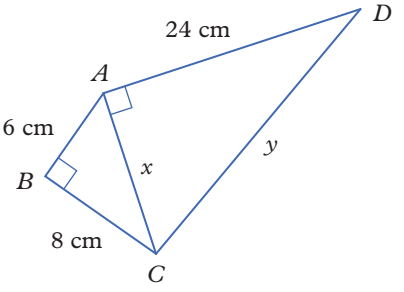

**18** Consider the shape *ABCD* shown on the right.

**a** How many right-angled triangles can you see? Name them.

**b** Calculate the length of *AC*. Which side lengths did you use in your calculation?

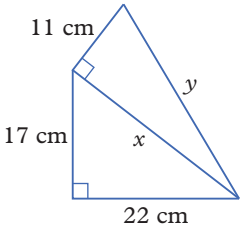
**c** Calculate the length of *CD*. Explain how you were able to do this.

**d** Find the perimeter of the shape *ABCD*.

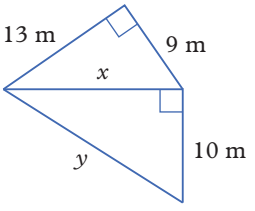



**19** Find the lengths indicated by $x$ and $y$ in each diagram, correct to two decimal places.

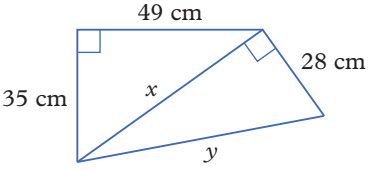
**a**




**b**

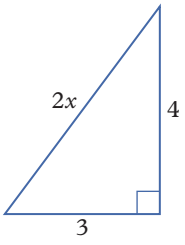



**c**

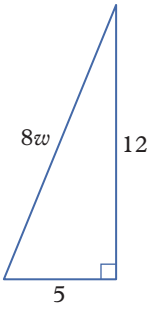



**20** Determine the value of the pronumeral in each of the following. Write your answer as a fraction where appropriate.

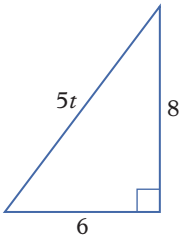
**a**



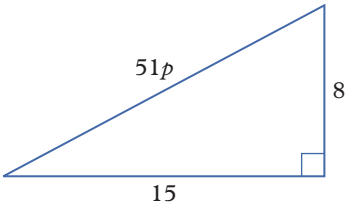

**b**




**c**




**d**



CHALLENGE

**21** Builders in ancient Egypt used a piece of rope with 12 equally spaced knots as a tool. How did this show that a corner was 'square'? Explain using a diagram.

**22** Pythagoras' Theorem is one of the most thoroughly proved theorems with hundreds of unique proofs (mathematical explanations). One proof, which we will look at, is known as 'Proof by rearrangement'.

Consider Diagram 1 and Diagram 2. Both diagrams have the same four right-angled triangles with shorter sides of $a$ and $b$, and a hypotenuse with length $c$.

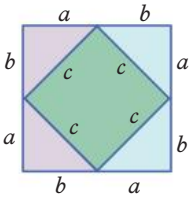



**Diagram 1**

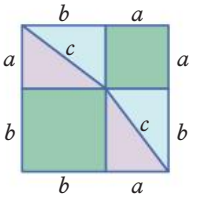



**Diagram 2**

**a** Explain why the two squares have the same area.

**b** Using your ideas from part **a**, explain how this pair of diagrams shows that $c^2 = a^2 + b^2$.

We can proof this algebraically by writing an equation equating the areas of each square.

**c** Write an equation for the total area in each diagram by adding the area of their smaller sections.

**d** The area in both diagrams is the same, so we can equate the equations from part **c.** Show that this equation is equivalent to $c^2 = a^2 + b^2$.

Another proof for Pythagoras' Theorem can be shown using Diagram 3.

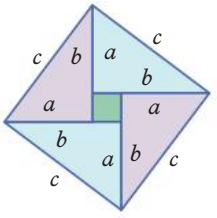


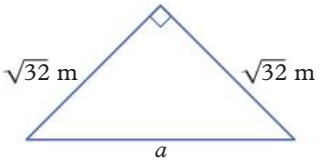

**Diagram 3**

**e** Using this diagram, explain why $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$ by considering the area of each part.

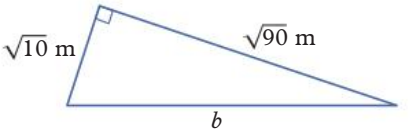
**f** Show that the equation from part **e** is equivalent to $c^2 = a^2 + b^2$.

**23** Determine the value of the pronumerals. Give your answer as a simplified surd where required.

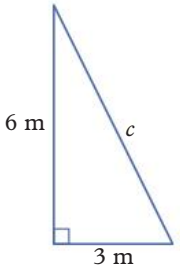
**a**




**b**




**c**

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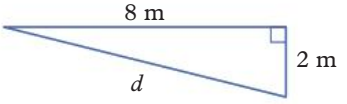



Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Pythagoras' Theorem
- Interactive skillsheet — Finding the length of the hypotenuse
- Investigation — Pythagoras' Theorem with shapes other than squares
- Topic quiz — 7B

# 7C Using Pythagoras' Theorem to find the length of a shorter side

## Learning intentions

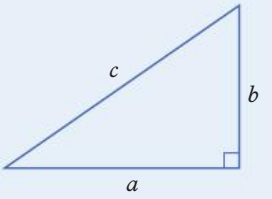
✔ I can use Pythagoras' Theorem to determine the shorter side of a right-angled triangle.

**Inter-year links**

| | |
|---|---|
| Year 7 | 6H Solving equations using inverse operations |
| Year 8 | 6B Solving equations using inverse operations |
| Year 10 | 8A Pythagoras' Theorem |

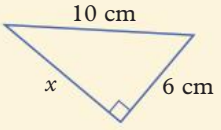
## Finding the length of a shorter side

- Pythagoras' Theorem for a right-angled triangle with side lengths $a$, $b$ and $c$, where $c$ is the length of the hypotenuse, is $c^2 = a^2 + b^2$.
- To find the length of one of the shorter sides using Pythagoras' Theorem:
  1. State Pythagoras' Theorem and define $a$, $b$ and $c$.
  2. Substitute the values for $a$, $b$ and $c$, and simplify the equation.
  3. Solve for $a$ or $b$ by finding the square root of both sides of the equation using inverse operations. Write the positive solution only and include the correct unit for the side length.




### Example 7C.1 Calculating the length of a shorter side (whole number)

Use Pythagoras' Theorem to calculate the unknown side length in this triangle.




**THINK**

1. The triangle is right-angled, so state Pythagoras' Theorem and define $a$, $b$ and $c$.
2. Substitute the values for $a$, $b$ and $c$, and simplify the equation.
3. Solve for $x$ by first subtracting 36 from both sides of the equation and then finding the square root. Write the positive solution only with its unit, as $x$ represents a length.

**WRITE**

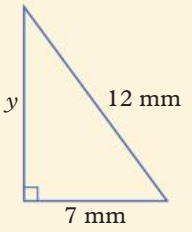
$c^2 = a^2 + b^2$ where $a = x$, $b = 6$ cm and $c = 10$ cm.

$10^2 = x^2 + 6^2$

$100 = x^2 + 36$
$64 = x^2$
$\sqrt{64} = x$
$8 = x$
$x = 8$ cm

## Example 7C.2 Calculating the length of a shorter side (decimal value)

Calculate the unknown side length in this triangle, correct to two decimal places.




**THINK**

1. The triangle is right-angled, so state Pythagoras' Theorem and define $a$, $b$ and $c$.
2. Substitute the values for $a$, $b$ and $c$, and simplify the equation.
3. Solve for $y$ by first subtracting 49 from both sides of the equation and then finding the square root. Write the positive solution only, as $y$ represents a length.
4. Use a calculator to find the approximate length correct to two decimal places. (Note that $\sqrt{95}$ mm is the exact length.)

**WRITE**

$c^2 = a^2 + b^2$ where $a = 7$ mm, $b = y$ and $c = 12$ mm.

$12^2 = 7^2 + y^2$

$144 = 49 + y^2$

$95 = y^2$

$\sqrt{95} = y$

$y \approx 9.75$ mm

**Helpful hints**

- ✔ A common mistake in solving the Pythagoras' Theorem often occurs in the second-last step. When looking at $x^2 = 16$, instead of taking the square root of both sides, some students will halve the result. The 2 is an index, not a factor!
- ✔ Remember that $c$ represents the length of the hypotenuse in $c^2 = a^2 + b^2$, not the unknown side. So when finding the length of the shorter side, substitute the pronumeral into $a$ or $b$.
- ✔ The solution of an equation $x^2 = 64$ has two solutions, $x = \pm 8$, but you only write the positive solution if $x$ represents a length.

ANS p534

# Exercise 7C Using Pythagoras' Theorem to find the length of a shorter side

△ 1, 2–4(1st column), 5–9, 10(a, b) □ 2–4(2nd column), 5, 6–7(2nd column), 9, 11, 12, 15, 18(a, b) ⬠ 4, 5, 6–7(2nd column), 11–14, 15(b), 16, 17, 18(c, d)

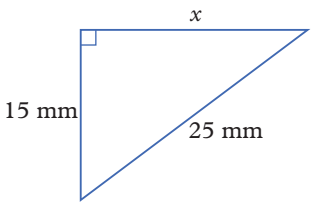
**1** Solve the following equations for $x$. Write your answers correct to two decimal places.

**a** $x^2 + 25 = 36$ **b** $x^2 + 7^2 = 10^2$ **c** $16 + x^2 = 64$

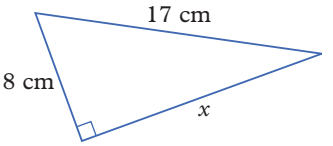
**d** $81 = x^2 + 9$ **e** $12^2 = 5^2 + x^2$ **f** $36 = x^2 + x^2$

7C.1 **2** Use Pythagoras' Theorem to calculate the unknown side length in each triangle.

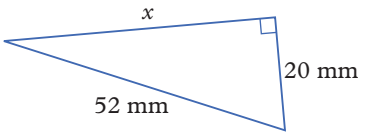
**a**



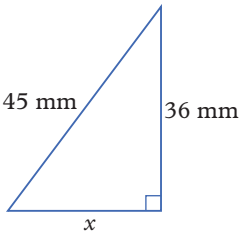

**b**



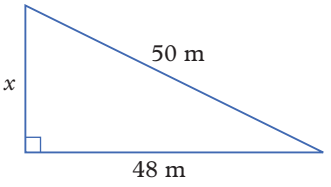

**c**



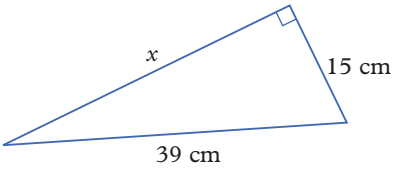

**d**




**e**

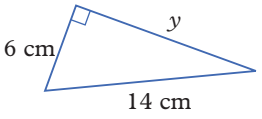



**f**

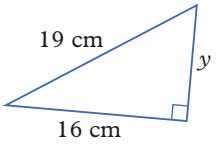



7C.2 **3** Calculate the unknown side length in each triangle, correct to two decimal places.

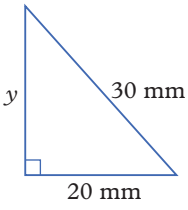
**a**



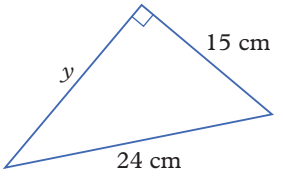

**b**



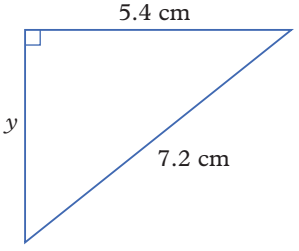

**c**



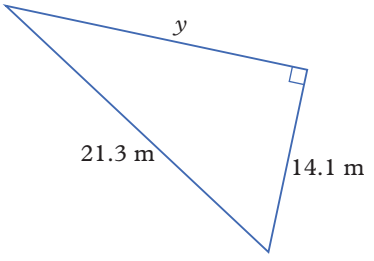

**d**




**e**

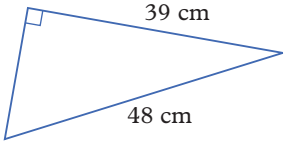



**f**

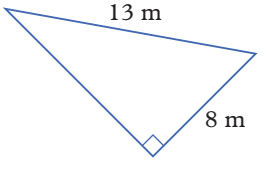



**4** Find the unknown length in each right-angled triangle, correct to two decimal places.

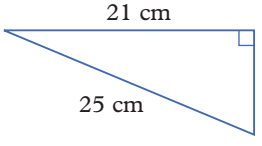
**a**



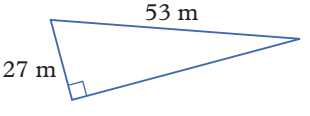

**b**




**c**

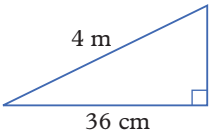



**d**

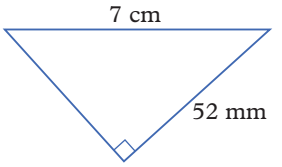


**5** Find the unknown length in each right-angled triangle, correct to the nearest centimetre. (Hint: Convert all length measurements to centimetres first.)

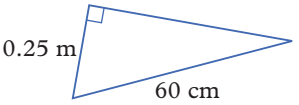
**a**



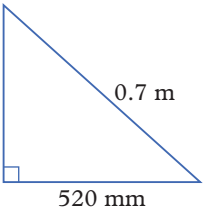

**b**




**c**

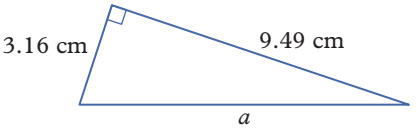



**d**

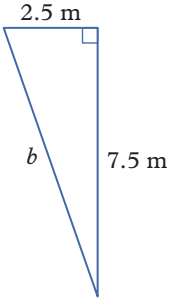



**6** Calculate the length of the unknown side in each right-angled triangle, correct to two decimal places.

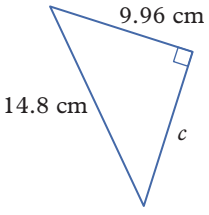
**a**



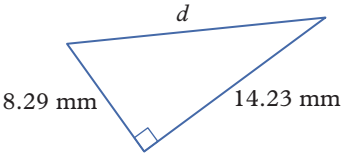

**b**



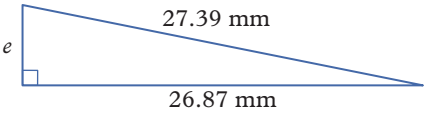

**c**



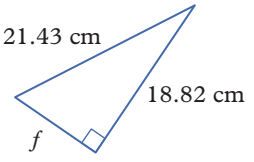

**d**




**e**

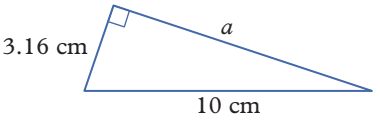



**f**

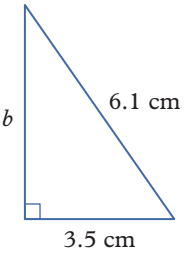



**7** Calculate the area of the following right-angled triangles correct to two decimal places.

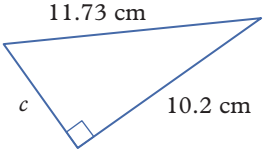
**a**



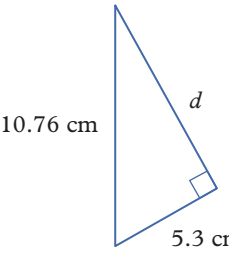

**b**



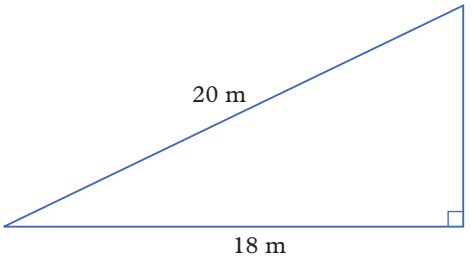

**c**



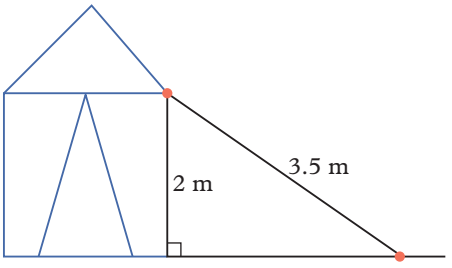

**d**



**8** A BMX rider uses the ramp shown in this diagram to perform stunts. How high is the ramp? Give your answer correct to two decimal places.

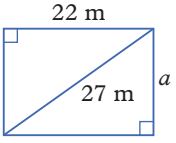



**9** A guy rope on a tent is 3.5 m long and is attached to the tent at a height of 2 m. How far from the tent does the guy rope reach the ground? Give your answer correct to two decimal places.

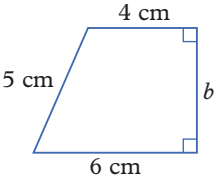



**10** Use Pythagoras' Theorem to calculate the unknown length indicated by the unknown in each diagram, correct to two decimal places.

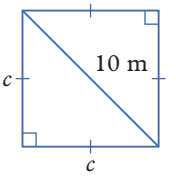
**a**




**b**

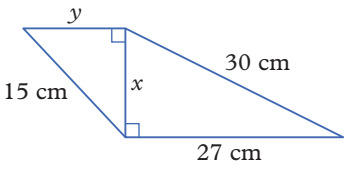



**c**

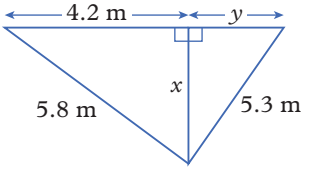



**11** Find the lengths labelled $x$ and $y$ in each diagram, correct to two decimal places.

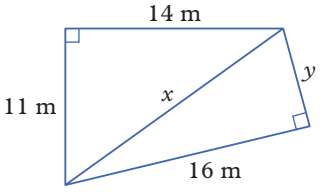
**a**



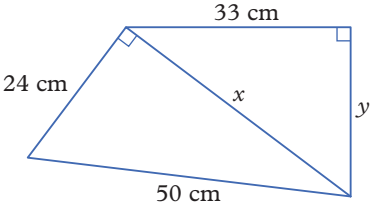

**b**




**c**



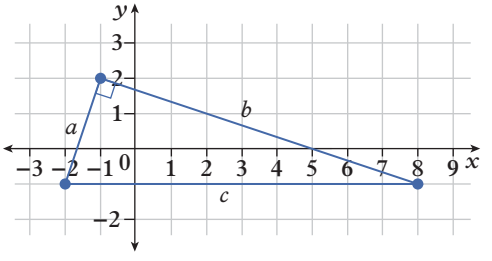

**d**




**12** For each shape in question **11**, find correct to two decimal places:

**i** the perimeter **ii** the area.

**13** Consider the triangle shown on the Cartesian plane.




**a** State the length of $c$.

**b** Determine the square of the lengths of $a$ and $b$.

**c** Show that the triangle is right-angled.

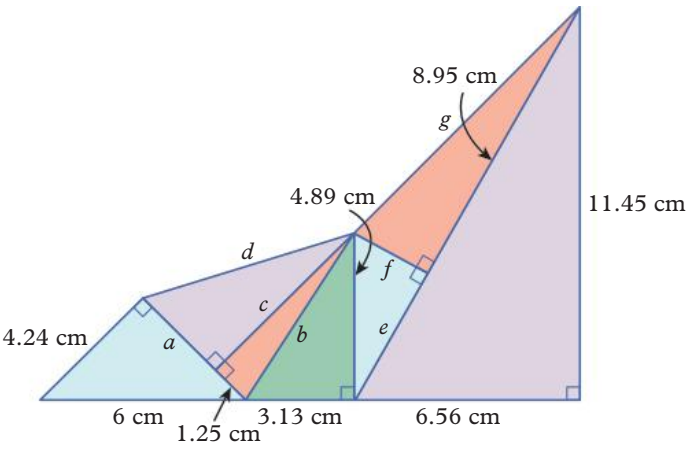
**d** Determine the midpoint of each side length. Write your answer using decimals.

**e** Determine the squares of the lengths between each pair of midpoints.

**f** Determine whether the triangle formed by the midpoints is right-angled.

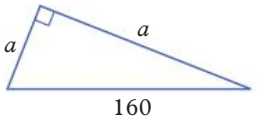
PROBLEM SOLVING AND REASONING

**14** Determine the value of all pronumerals, $a$–$g$, correct to two decimal places.

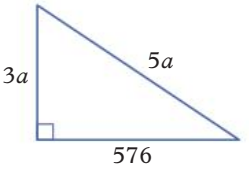



**15** Determine the value of the unknown, correct to two decimal places where appropriate.

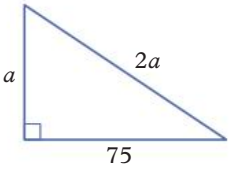
**a**




**b**




**c**




**16** Joey is determining the unknown side length of a right-angled triangle with two side lengths: 7 cm and 15 cm. Joey writes down all possible calculations for the third length.

**i** $\sqrt{7^2 + 15^2}$ **ii** $\sqrt{15^2 + 7^2}$ **iii** $\sqrt{7^2 - 15^2}$ **iv** $\sqrt{15^2 - 7^2}$

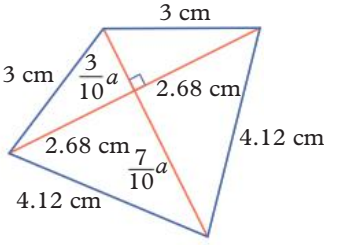
**a** If the missing side length is the hypotenuse, which calculation(s) could be used?

**b** If the missing side length is not the hypotenuse, which calculation(s) could be used?

**c** Is there a calculation listed that does not result in the side length of a right-angled triangle? Explain why or why not.

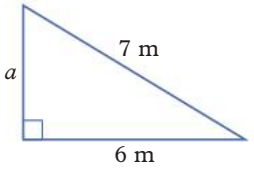
CHALLENGE

**17** Calculate the area of the kite shown. Give your answer correct to the nearest integer.

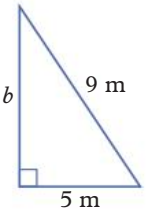



**18** Determine the value of the pronumerals writing your answer as a simplified surd where required.

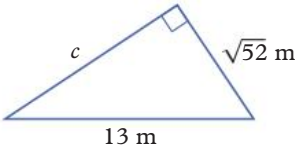
**a**



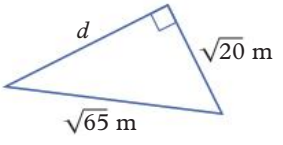

**b**




**c**




**d**




Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Finding the length of a shorter side

Topic quiz
7C



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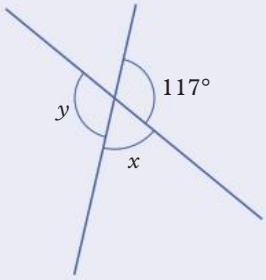
# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

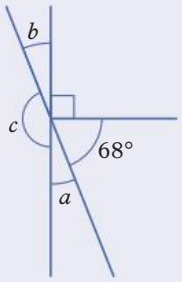
**7A** **1** **a** Calculate the complement of 37°.
**b** Calculate the supplement of 23°.

**7A** **2** Determine the value of the pronumerals.

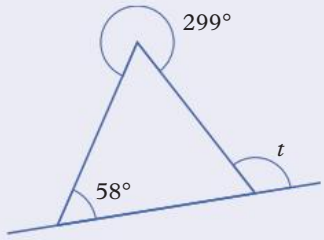
**a**




**b**

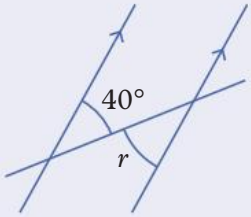



**c**

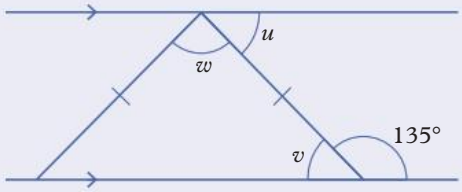



**7A** **3** Determine the value of the pronumerals.

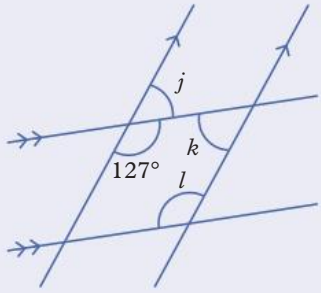
**a**




**b**




**c**

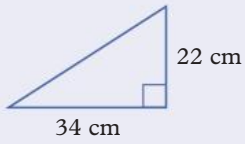



**7B** **4** Solve the following equations for $x$.
**a** $x^2 = 64$ **b** $x^2 = 7^2 + 24^2$

**7B** **5** Determine whether the following triples could be side lengths of a right-angled triangle.
**a** (21, 28, 35) **b** (24, 121, 123)

**7B** **6** Determine the length of the missing side, correct to two decimal places.

**a**

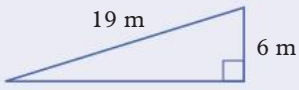



**b**

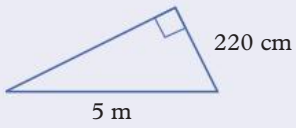
**7C** **7** Solve the following equations for $x$.
**a** $25 = x^2 + 16$ **b** $13^2 = x^2 + 12^2$

**7C** **8** Determine the length of the missing side, correct to two decimal places.

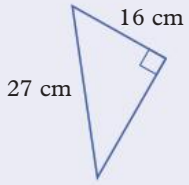
**a**




**b**

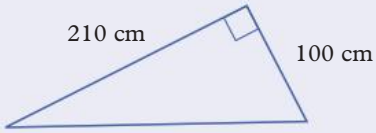



**c**

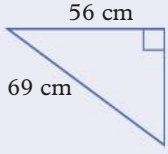



**7C** **9** Determine the length of the missing side, correct to two decimal places.

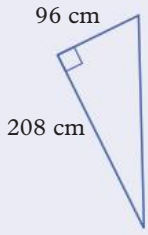
**a**




**b**




**c**



# 7D Trigonometric ratios

## Learning intentions

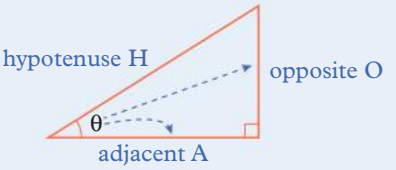
- ✔ I can identify the hypotenuse, opposite and adjacent sides of a right-angled triangle containing a reference angle.
- ✔ I can calculate sine, cosine and tangent ratios of an angle using side length relationships.

**Inter-year links**

| | |
|---|---|
| Year 7 | 3H Ratios |
| Year 8 | 3D Ratios |
| Year 10 | 8B Trigonometry |

## Trigonometry

- **Trigonometry** involves relationships between angles and side lengths in a triangle.




- In a right-angled triangle with a reference angle:
  - → the **opposite side** is opposite the reference angle
  - → the **adjacent side** is next to or adjacent the reference angle that is not the hypotenuse
  - → the hypotenuse is always the longest side and is opposite the right angle.
- The symbol θ (*theta*, a Greek letter) is often used to represent the reference angle.

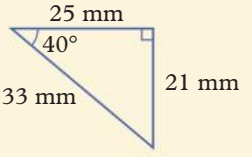
## Trigonometric ratios

- For any right-angled triangle with a reference angle of θ, the:
  - → **sine** of angle θ is the ratio of the lengths of the opposite side to the hypotenuse
  - → **cosine** of angle θ is the ratio of the lengths of the adjacent side to the hypotenuse
  - → **tangent** of angle θ is the ratio of the lengths of the opposite side to the adjacent side.
- The ratios can be written as fractions:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

### Example 7D.1 Labelling the sides of a right-angled triangle with respect to a reference angle

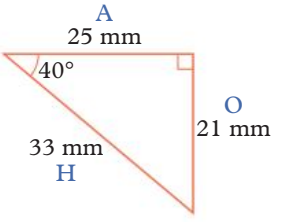
Label the sides of this triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the angle of 40°.




**THINK**

1. Label the hypotenuse with H. It is the longest side and is opposite the right angle.
2. Identify the opposite side and the adjacent side with respect to the reference angle of 40°. Label the side opposite 40° as O and the side next to 40° (adjacent) as A.

**WRITE**





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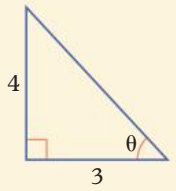
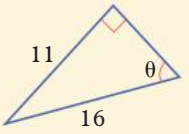
## **Example 7D.2** Determining the trigonometric ratios

For each triangle, identify which trigonometric ratio can be used and then determine its value, based on the side lengths provided.

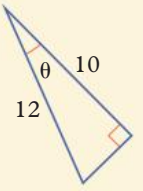
**a**




**b**




**b**




**THINK**

**a** **1** Identify the given sides with respect to the reference angle. The side length opposite the angle is 4. The side length adjacent to the angle is 3. Label the sides accordingly.

**2** Decide which trigonometric ratio to use. As O and A are involved, use tangent.

**3** Substitute the values into the equation to determine the ratio.

**b** **1** Identify the given sides with respect to the angle. The side length opposite the angle is 11. The hypotenuse is 16. Label the sides accordingly.

**2** Decide which trigonometric ratio to use. As O and H are involved, use sine.

**3** Substitute the values into the equation to determine the ratio.

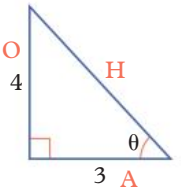
**c** **1** Identify the given sides with respect to the angle. The side length adjacent to the angle is 10. The hypotenuse is 12. Label the sides accordingly.

**2** Decide which trigonometric ratio to use. As A and H are involved, use cosine.

**3** Substitute the values into the equation to determine the ratio.

**WRITE**

**a**

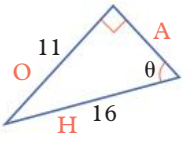



$O = 4, A = 3$

$\tan(\theta) = \frac{O}{A}$

$= \frac{4}{3}$

**b**

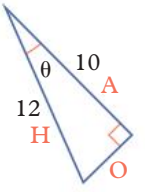



$O = 11, H = 16$

$\sin(\theta) = \frac{O}{H}$

$= \frac{11}{16}$

**c**




$A = 10, H = 12$

$\cos(\theta) = \frac{A}{H}$

$= \frac{10}{12}$

$= \frac{5}{6}$

**Helpful hints**

- ✔ Trigonometry can involve a lot of calculator work. You will need to know how to enter the trigonometric function in your scientific calculator.
- ✔ To remember the ratio of the two different sides according to the reference angle, you can use SOH CAH TOA. Like with Pythagoras' Theorem, writing the equation from memory before substituting the values in will help you to memorise it quicker.

$$\sin(\theta) = \frac{O}{H} \quad \cos(\theta) = \frac{A}{H} \quad \tan(\theta) = \frac{O}{A} \longrightarrow \text{SOH CAH TOA}$$

- ✔ The sine, cosine or tangent values do not have units. The units cancel out in the ratio.
- ✔ Do not write sin =, cos = or tan = without the angle. Sine, cosine and tangent do not have a value without an angle.
- ✔ The tangent of the angle is the gradient of the hypotenuse of a triangle in the first quadrant with the reference angle placed at the origin.
- ✔ The prefix 'co-' in cosine is short for complementary, as cosine is the sine of the complementary angle, the other non-right-angle in a right-angle triangle, which is opposite the side adjacent to the reference angle.

ANS p535

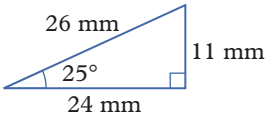
# Exercise 7D Trigonometric ratios

△ 1–5, 6(a, c, e), 7(b, d, f), 8–10, 13, 14, 18 ■ 2–5, 7–9, 11, 15, 16, 18, 19 ⬟ 3–5, 8, 9, 11, 12, 15, 17, 19, 20

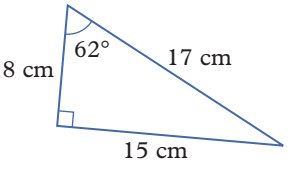
UNDERSTANDING AND FLUENCY

**7D.1** **1** Label the sides of each triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the given reference angle.

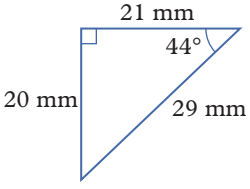
**a**



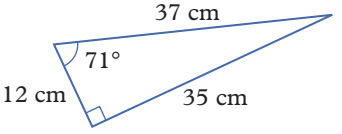

**b**




**c**

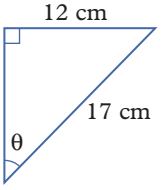



**d**

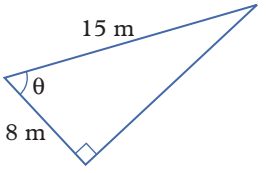



**7D.2** **2** For each triangle, determine which trigonometric ratio can be used based on the side lengths provided.

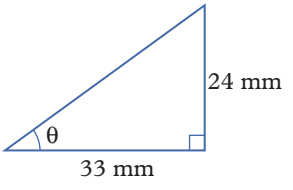
**a**



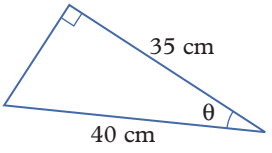

**b**




**c**




**d**

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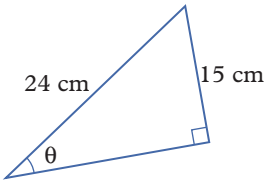


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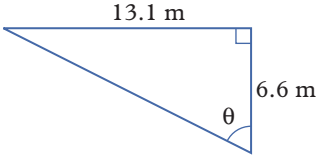
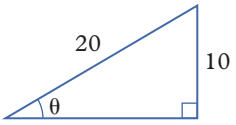
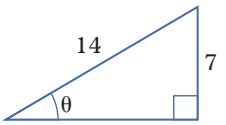


**3** For each of the triangles, determine the ratio equal to the sine of angle θ. Simplify your fractions where possible.

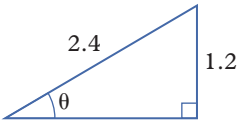
**a**



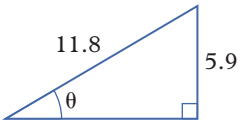

**b**




**c**

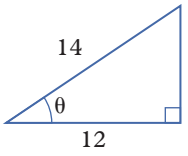



**d**

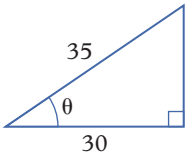



**4** For each of the triangles, determine the ratio equal to the cosine of angle θ. Simplify your fractions where possible.

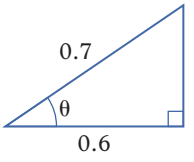
**a**



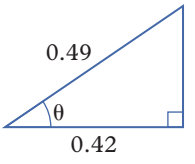

**b**




**c**

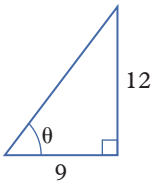



**d**

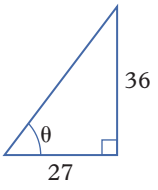



**5** For each of the triangles, determine the ratio equal to the tangent of angle θ. Simplify your fractions where possible.

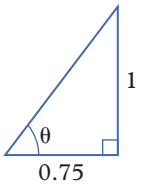
**a**



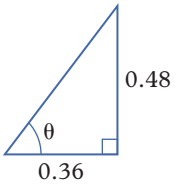

**b**




**c**




**d**




**6** Evaluate the following using a calculator correct to four decimal places. Ensure your calculator is in degree mode.

**a** $\sin(20°)$ **b** $\sin(36°)$ **c** $\cos(36°)$

**d** $\tan(36°)$ **e** $\tan(63°)$ **f** $\cos(63°)$

**7** Evaluate the following using a calculator correct to four significant figures. Ensure your calculator is in degree mode.

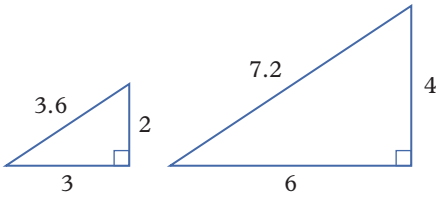
**a** $6\sin(20°)$ **b** $\dfrac{6}{\sin(20°)}$ **c** $\dfrac{\sin(20°)}{6}$

**d** $(\cos(20°))^2 + (\sin(20°))^2$ **e** $(\cos(20°))^2 - (\sin(20°))^2$ **f** $\dfrac{\sin(20°)}{\cos(20°)}$

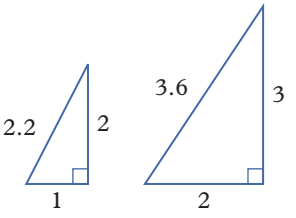
**8** **a** Describe the relationship between the triangles in questions **3**, **4** and **5**.

**b** Determine which pairs of triangles will have the same corresponding sine, cosine and tangent values for their non-right angles.

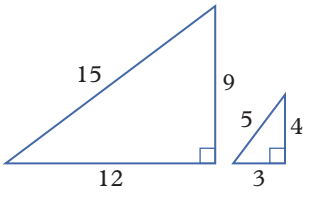
**i**



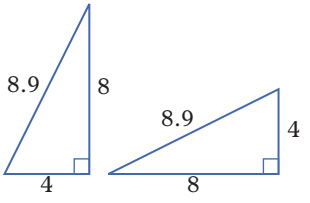

**ii**



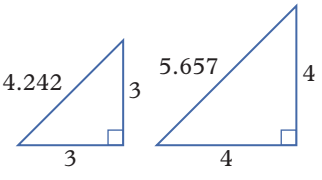

**iii**



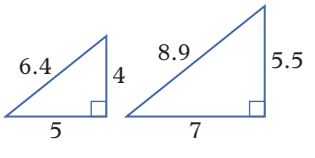

**iv**




**v**

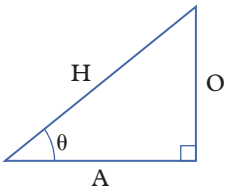



**vi**

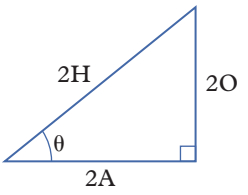


**c** For each of the triangles, determine the ratio equal to the sine, cosine and tangent of angle θ. Simplify your fractions where possible.

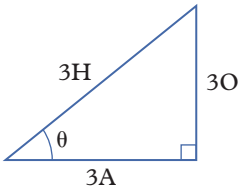
**i**



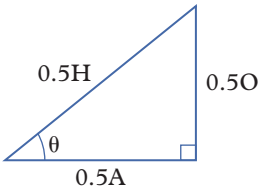

**ii**



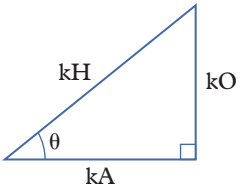

**iii**



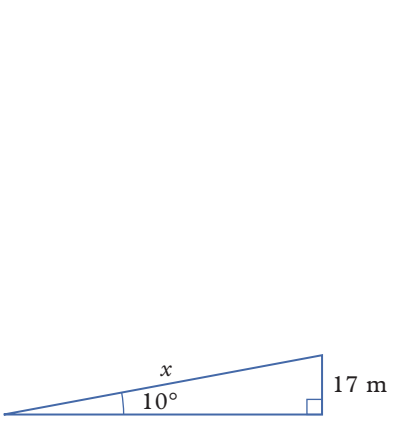

**iv**

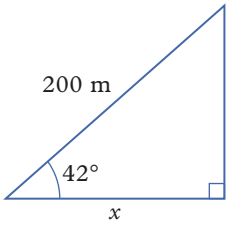


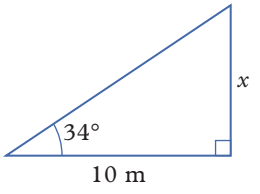

**v**

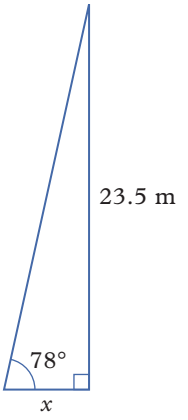



**9** For each of the following, use the given approximate trigonometric ratio to determine the value of $x$.




**a** $\sin(10°) = 0.17$ **b** $\cos(42°) = 0.74$ **c** $\tan(34°) = 0.67$ **d** $\tan(78°) = 4.7$



**10 a** Using your calculator, complete the table below. Write your answers correct to four decimal places.

| θ | $\sin(\theta)$ | $\cos(\theta)$ | $\tan(\theta)$ | $\frac{1}{\tan(\theta)}$ |
|---|---|---|---|---|
| 1° | | | | |
| 5° | | | | |
| 15° | | | | |
| 30° | | | | |
| 45° | | | | |
| 60° | | | | |
| 75° | | | | |
| 85° | | | | |
| 89° | | | | |

**b** Which column(s) increase as the angle, θ, increases? Which column(s) decrease as the angle, θ, increases?

**c** Fill in the blanks of the following equations.

**i** $\sin(27°) = \cos(___°)$ **ii** $\sin(___°) = \cos(56°)$

**iii** $\tan(12°) = \frac{1}{\tan(___°)}$ **iv** $\tan(___°) = \frac{1}{\tan(47°)}$

**d** Describe the relationship between sine and cosine. Use the word *complementary* in your description.

**e** Describe the relationship between tangent and the reciprocal of tangent. Use the word *complementary* in your description.

**11** There are two special triangles that can help us determine the exact values of trigonometric ratios for common angles.

Diagram 1 shows a square with side lengths of 1 unit. If we cut the square in half along a diagonal, we obtain a right-angled isosceles triangle. This is the first special triangle.

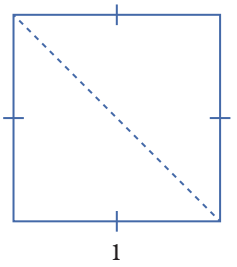



**Diagram 1**

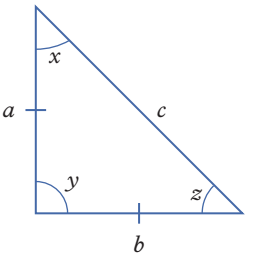



**Diagram 2**

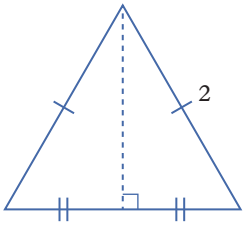



**Diagram 3**

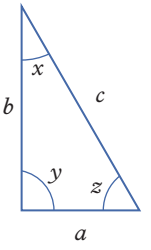



**Diagram 4**

**a** Determine the value of all side lengths and angles in Diagram 2 using geometry and Pythagoras' Theorem. Write answers exactly, as a square root, where required.

**b** Using the values determined in part **a**, write down the exact values of the sine, cosine and tangent ratios of 45°.

The second special triangle is formed by cutting an equilateral triangle of side length 2 units in half down the middle, as shown in Diagram 3.

**c** Determine the value of all side lengths and angles in Diagram 4 using geometry and Pythagoras' Theorem. Write answers exactly, as a fraction or a square root, where required.

**d** Using the values determined in part **c**, write down the exact values of the sine, cosine and tangent ratios of 30° and 60°.

**e** Evaluate the following on a calculator.

**i** $(\sin(45°))^2$ **ii** $(\cos(30°))^2$ **iii** $(\tan(30°))^2$ **iv** $(\sin(60°))^2$

**v** $(\sin(30°))^2$ **vi** $(\cos(60°))^2$ **vii** $(\tan(45°))^2$ **viii** $(\cos(45°))^2$

**f** Describe how you could use the calculations in part **e** to obtain the exact values of the sine, cosine and tangent ratios of 30°, 45° and 60° without the special triangles.

**12 a** Explain, by referring to the hypotenuse as the longest side of a right-angled triangle, why the values of sine and cosine of θ are never greater than 1 for $0° < \theta < 90°$.

**b** Explain why the values of tangent, unlike sine and cosine, can be greater than 1 for $0° < \theta < 90°$

**c** Under what condition(s) is the value of tangent greater than 1?

**13 a** Add another column to the table in question **10** with the heading $\frac{\sin(\theta)}{\cos(\theta)}$. Complete the column by performing the division $\sin(\theta) \div \cos(\theta)$ for each angle.

**b** Fill in the blanks of the following equations.

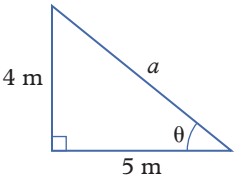
**i** $\frac{\sin(27°)}{\cos(27°)} = \tan(___°)$ **ii** $\frac{\sin(___°)}{\cos(56°)} = \tan(56°)$

**iii** $\frac{\sin(12°)}{\cos(___°)} = \tan(12°)$ **iv** $\frac{\sin(___°)}{\cos(___°)} = \tan(47°)$

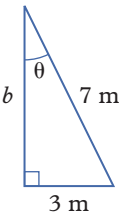
**c** Describe the relationship between $\frac{\sin(\theta)}{\cos(\theta)}$ and $\tan(\theta)$.

**14 a** For each of the triangles below, use Pythagoras' Theorem to determine the missing length correct to two decimal places.

**i**



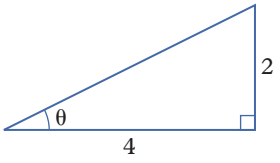

**ii**



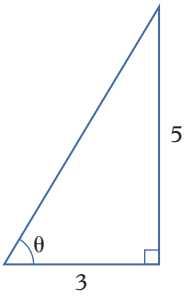

**b** Write the sine, cosine and tangent ratios of θ as a decimal correct to four decimal places where required.

**15 a** Calculate the gradient of the hypotenuse in each right-angled triangle.

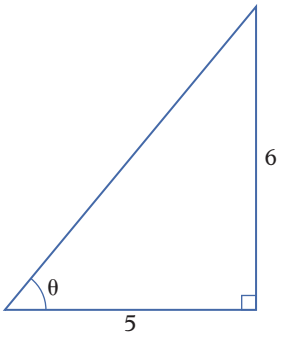
**i**



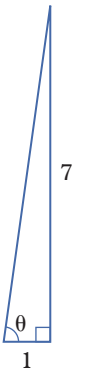

**ii**




**iii**



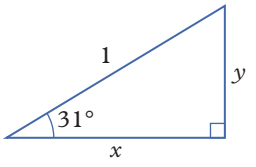

**iv**




**b** Write the value of the tangent of θ for each right-angled triangle in part **a**.

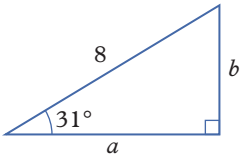
**c** Describe the relationship between the gradient of a line segment and the tangent of the angle from the horizontal.

**16 a** Write an equation using the sine ratio and cosine ratio for this triangle. Do not evaluate the sine and cosine values.

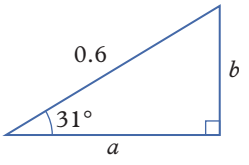



**b** Determine the lengths of $a$ and $b$ in these similar triangles correct to four significant figures.

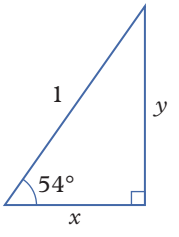
**i**




**ii**

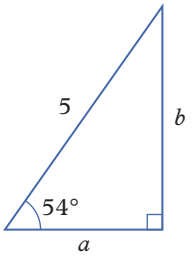



**c** Write an equation using the sine ratio and cosine ratio for this triangle. Do not evaluate the sine and cosine values.

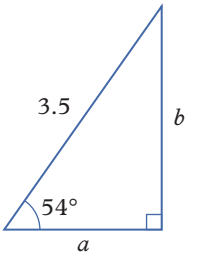



**d** Determine the lengths of $a$ and $b$ in these similar triangles correct to four significant figures.

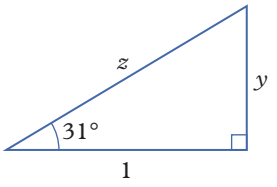
**i**




**ii**

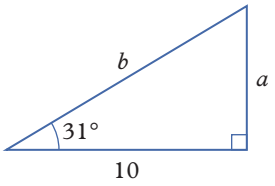


**17 a** Write an equation using the cosine ratio and tangent ratio for this triangle. Do not evaluate the cosine and tangent values.

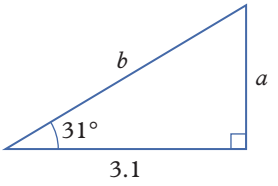



**b** Determine the lengths of $a$ and $b$ in these similar triangles correct to four significant figures.

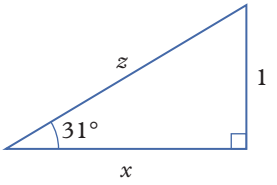
**i**




**ii**

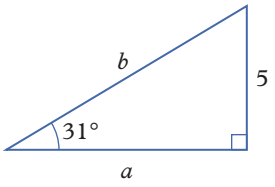



**c** Write an equation using the sine ratio and tangent ratio for this triangle. Do not evaluate the sine and tangent values.

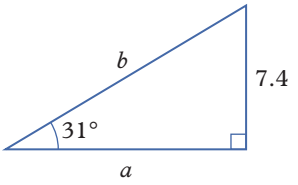



**d** Determine the lengths of $a$ and $b$ in these similar triangles correct to four significant figures.

**i**

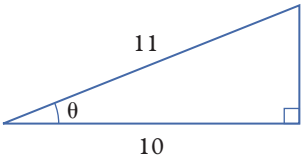



**ii**



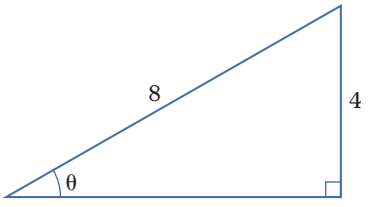

**18** Explain and correct the error each person has made when writing the trigonometric ratio.

**a**



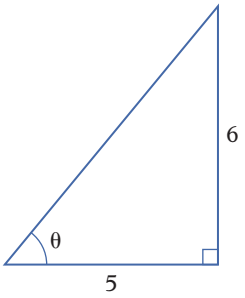

$$\cos = \frac{10}{11}$$

**b**




$$\sin(\theta) = \frac{8}{4}$$
$$\sin(\theta) = 2$$

**c**



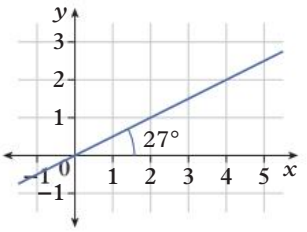

$$\cos(\theta) = \frac{6}{5}$$

CHALLENGE

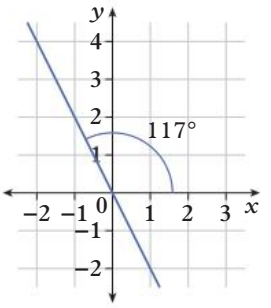
**19** In question **15**, we saw that the tangent of the angle from the horizontal is equal to the gradient of the hypotenuse. This is also true for lines inclined at any angle, not just $0° < \theta < 90°$.

**a** State the tangent of the angle for each by determining the gradient of the line.

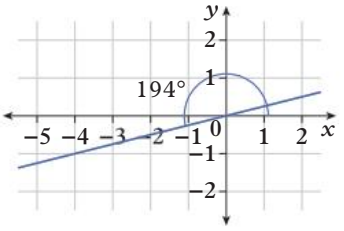
**i**



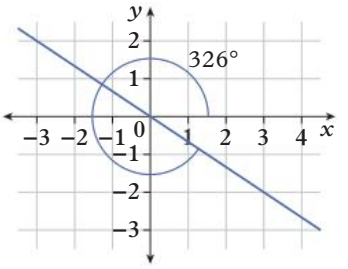

**ii**




**iii**




**iv**




**b** Evaluate the following with a calculator.

**i** tan(45°) **ii** tan(135°) **iii** tan(225°) **iv** tan(315°)

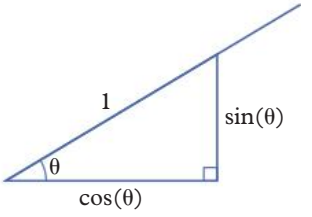
**c** Explain the similarity of the values in part **b**. Drawing a to-scale diagram may help.

**d** Complete the following sentences:

**i** The gradient of a horizontal line is _____. Therefore, the tangent of 0° is _____.

**ii** The gradient of a vertical line is _____. Therefore, the tangent of 90° is _____.

**20** We saw in question **13** that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. The tangent of the angle from the horizontal is equal to the gradient, so $\frac{\sin(\theta)}{\cos(\theta)} = \frac{\text{rise}}{\text{run}}$. Considering the ray:




- the sine of the angle is the amount of rise when the hypotenuse is 1
- the cosine of the angle is the amount of run when the hypotenuse is 1.

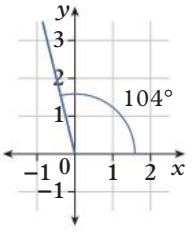
**a** Evaluate the following with a calculator.

**i** sin(30°) **ii** sin(150°) **iii** sin(210°) **iv** sin(330°)

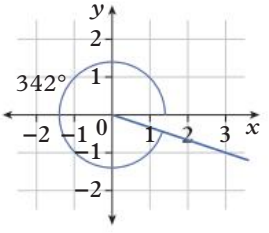
**v** cos(60°) **vi** cos(120°) **vii** cos(240°) **viii** cos(300°)

**b** State whether the sine and cosine values will be positive or negative for the angles shown.

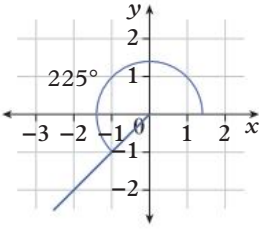
**i**




**ii**




**iii**

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Trigonometry

CAS instructions
Trigonometry

Topic quiz
7D

# 7E Using trigonometry to find lengths

## Learning intentions

✔ I can find the unknown lengths in a right-angled triangle.

**Inter-year links**

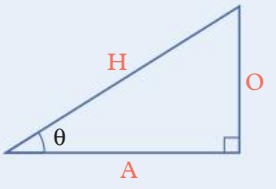
| | |
|---|---|
| Year 7 | 6H Solving equations using inverse operations |
| Year 8 | 6B Solving equations using inverse operations |
| Year 10 | 8C Applications of trigonometry |

## Using trigonometry to find a side length

- A trigonometric ratio can be used to find an unknown side length in a right-angled triangle if an angle (other than the right angle) and one side length are known.
- To find a side length:
  1. Identify the given sides with respect to the angle θ. Label the sides accordingly.
  2. Decide which trigonometric ratio to use.
  4. Substitute in the known side length and angle.
  5. Solve the equation for the unknown.
- Use a calculator to perform the final calculation and, where appropriate, round your answer.

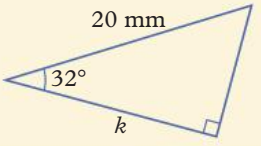
SOH CAH TOA

$\sin(\theta) = \frac{O}{H}$ $\cos(\theta) = \frac{A}{H}$ $\tan(\theta) = \frac{O}{A}$




### Example 7E.1 Identifying which trigonometric ratio to use to write an equation

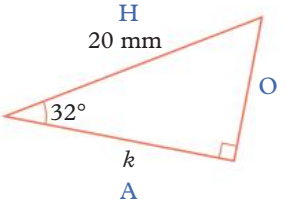
Determine which trigonometric ratio should be used, based on the side lengths provided, and hence write an equation that involves $k$.




**THINK**

1. Identify the given sides with respect to the angle 32°. The side length adjacent to the angle is $k$. The hypotenuse is 20 mm. Label the sides accordingly.
2. Decide which trigonometric ratio to use. As A and H are involved, use cosine.
3. Substitute for θ, A and H to write an equation involving $k$.

**WRITE**




A = $k$, H = 20 mm

$\cos(\theta) = \frac{A}{H}$

$\cos(32°) = \frac{k}{20}$

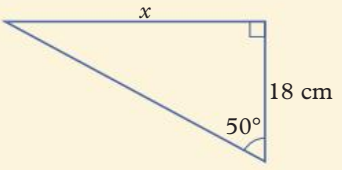
## **Example 7E.2** Solving equations involving trigonometric ratios

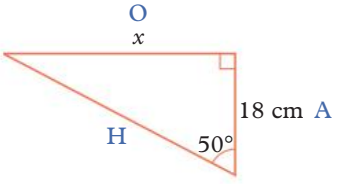
Solve the equation $\sin(23°) = \frac{m}{4}$. Round your answer to one decimal place.

| THINK | WRITE |
|---|---|
| **1** Rearrange the equation to make $m$ the subject of the formula. | $m = 4 \times \sin(23°)$ |
| **2** Use your calculator to evaluate the trigonometric ratio $\sin(23°)$ and perform the multiplication operation $(\times 4)$. Round the answer to one decimal place. | $m \approx 6.3$ |

## **Example 7E.3** Using trigonometry to find an unknown side length in the numerator

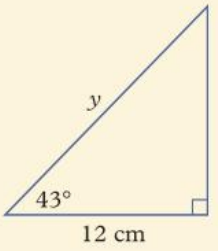
Use trigonometry to find the side length $x$ in this triangle, correct to two decimal places.




| THINK | WRITE |
|---|---|
| **1** Identify the given sides with respect to the angle 50°. The side length adjacent to the angle is 18 cm. The side length opposite the angle is $x$. Label the sides accordingly. |   |
| | $O = x$, $A = 18$ cm |
| **2** Decide which trigonometric ratio to use. As O and A are involved, use tangent. | $\tan(\theta) = \frac{O}{A}$ <br> $\tan(50°) = \frac{x}{18}$ |
| **3** Substitute for θ, O and A. | $18 \times \tan(50°) = x$ |
| **4** Solve the equation for $x$. Multiply both sides of the equation by 18. | $21.4515... \approx x$ |
| **5** Use a calculator to multiply 18 by $\tan(50°)$. Round the value of $x$ to two decimal places. | $x \approx 21.45$ cm |

## **Example 7E.4** Using trigonometry to find an unknown side length in the denominator

Use trigonometry to find the side length $y$ in this triangle, correct to two decimal places.




**THINK**

1 Identify the given sides with respect to the angle 43°. The side length adjacent to the angle is 12 cm. The hypotenuse is $y$. Label the sides accordingly.

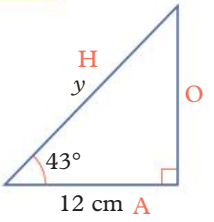
2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.

3 Substitute for θ, A and H.

4 Solve the equation for $y$. Multiply both sides of the equation by $y$, and then divide both sides of the equation by $\cos(43°)$.

5 Use a calculator to divide 12 by $\cos(43°)$. Round the value of $y$ to two decimal places.

**WRITE**




$A = 12$ cm, $H = y$

$\cos(\theta) = \frac{A}{H}$

$\cos(43°) = \frac{12}{y}$

$y \times \cos(43°) = 12$

$y = \frac{12}{\cos(43°)}$

$y \approx 16.4079.....$

$y \approx 16.41$ cm

**Helpful hints**

- ✔ Make sure your calculator is in degree mode, not radian mode. Otherwise, all your answers will be wrong! Check the DEG or D symbol is on screen, not RAD or R or GRAD or G.
- ✔ Remember that when solving an equation for an unknown, you can check your answer by substituting it back into the original equation.
- ✔ Drawing your diagram close to scale (making all angles roughly the correct size) will let you check that your answer is reasonable by referring to the relative length of the other sides. Always ensure that at the end of the question the hypotenuse is still the longest side of the triangle.
- ✔ SOH-CAH-TOA is still a useful pneumonic device for this section. If helpful for you, write the following at the top of your page. Writing the equation from memory before substituting the values in will help you memorise it quicker.

$$S\frac{O}{H}\,C\frac{A}{H}\,T\frac{O}{A}$$

ANS p537

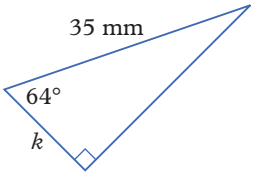
# Exercise 7E Using trigonometry to find lengths

1–6, 8, 9, 12, 13(a–d) | 2, 3, 5–7, 10, 12, 13, 17, 18 | 3, 5–7, 12, 14–17, 19

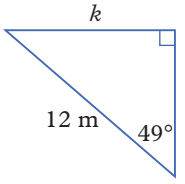
UNDERSTANDING AND FLUENCY

7E.1 **1** Determine which trigonometric ratio can be used, based on the side lengths provided, and hence write an equation that involves $k$.

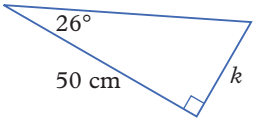
**a**



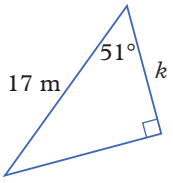

**b**



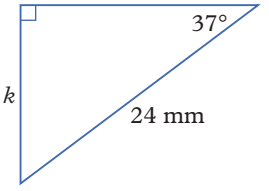

**c**



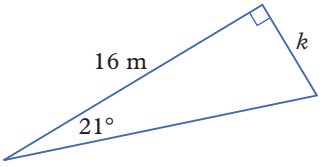

**d**




**e**




**f**



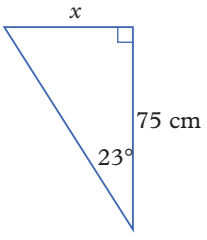

7E.2 **2** Solve the following equations for the unknown, correct to two decimal places.

**a** $0.4226 = \frac{x}{5}$ **b** $0.4226 = \frac{5}{x}$ **c** $\sin(25°) = \frac{x}{5}$ **d** $\sin(25°) = \frac{5}{x}$

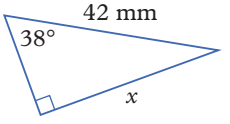
**e** $\cos(53°) = \frac{x}{11}$ **f** $\cos(53°) = \frac{11}{x}$ **g** $\tan(11°) = \frac{x}{20}$ **h** $\tan(79°) = \frac{20}{x}$

7E.3 **3** Use trigonometry to find the side length $x$ in each triangle, correct to two decimal places.

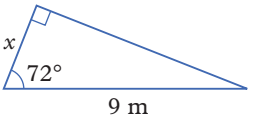
**a**



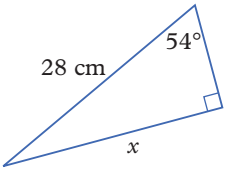

**b**



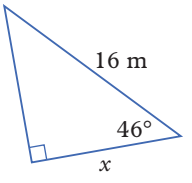

**c**



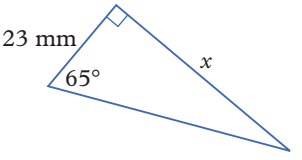

**d**



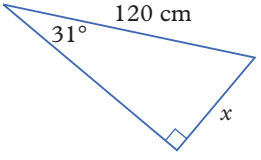

**e**



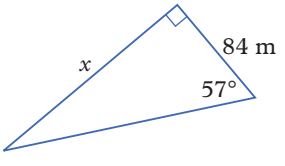

**f**



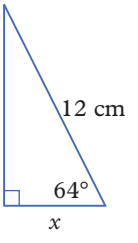

**g**




**h**




**i**




**4** Solve each equation formed in question **1** to calculate the value of $k$, correct to two decimal places.



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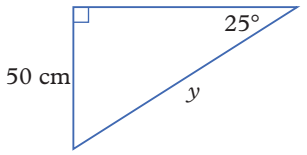
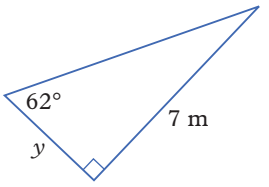
7E.4 **5** Find the side length $y$ in each triangle, correct to two decimal places.

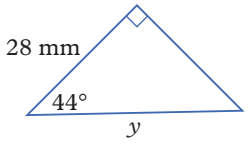
**a**



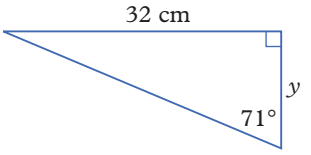

**b**



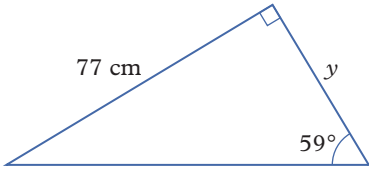

**c**



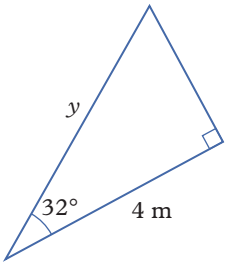

**d**




**e**

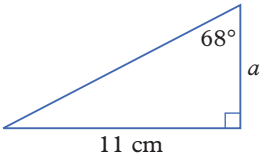



**f**

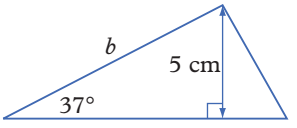



**6** Find the value of the pronumeral in each triangle, correct to two decimal places.

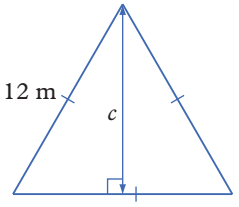
**a**



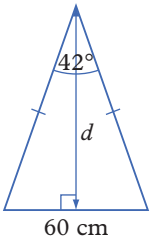

**b**



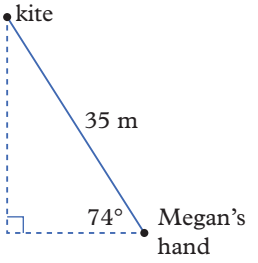

**c**




**d**



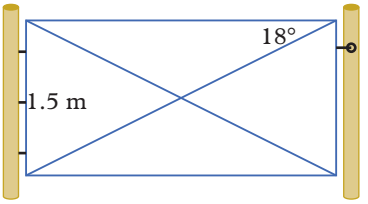

**7** Megan holds the string attached to her kite at a height of 1 m above the ground. The 35 m long string makes an angle of 74° with the horizontal. Find the following correct to two decimal places.




**a** Use trigonometry to calculate the vertical distance from one end of the string to the other.

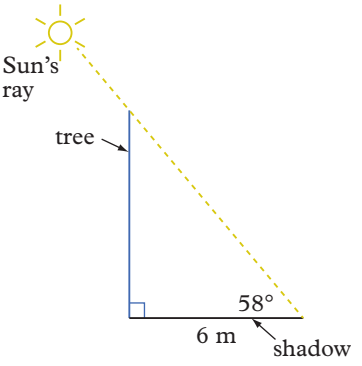
**b** What is the height of the kite above the ground?

**8** A farm gate, 1.5 m high, has diagonal supporting braces that make an angle of 18° with the horizontal. How wide is the gate, correct to two decimal places?




**9** A shadow is formed when the Sun's rays are blocked by an object. The angle of the Sun's rays determine how long a shadow will be.

At a certain time of day, the Sun's rays make an angle of 58° with the ground and a tree forms a shadow that is 6 m long. Find the following correct to two decimal places.




**a** Use the diagram to calculate the height of the tree.

**b** At the same time, another tree forms a shadow that is 10 m long. How high is this tree?

**10** The two identical sides of this ladder meet at an angle of 50° and are 1.8 m apart where they touch the ground. Find the following correct to two decimal places.

**a** How high is the top of the ladder above the ground?

**b** How long is each side of the ladder?

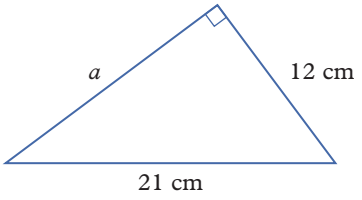
**11** A ramp for wheelchair access to a building is to be built at an angle of 3° to the horizontal. The front door of the building is 45 cm above ground level. How long should the ramp be? Give your answer in metres, correct to two decimal places.

**12 a** What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using trigonometry?

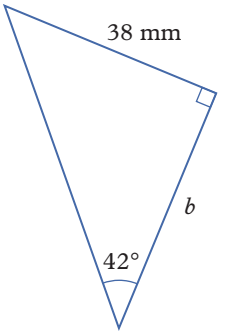
**b** What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using Pythagoras' Theorem?

**13** Find the length of the unknown side correct to one decimal place.

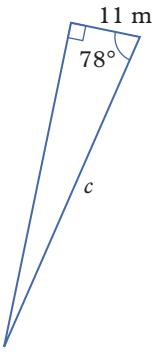
**a**



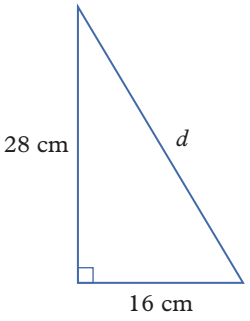

**b**



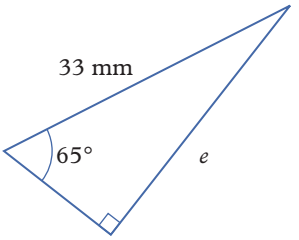

**c**



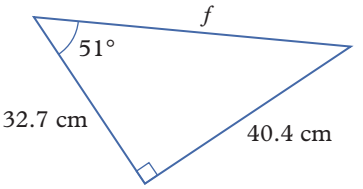

**d**




**e**

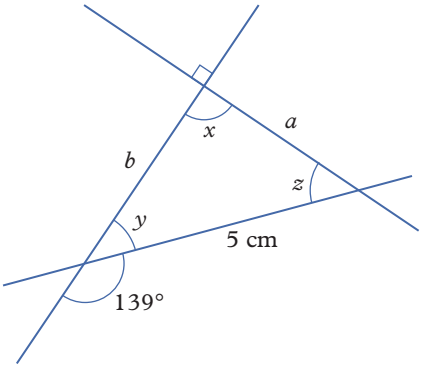



**f**

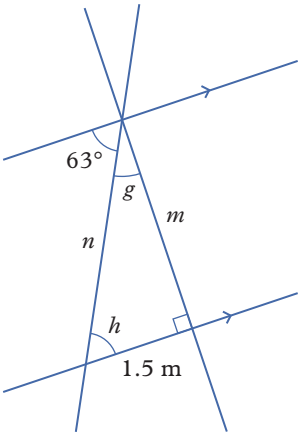



**14** Determine the values of the pronumerals correct to two decimal places.

**a**

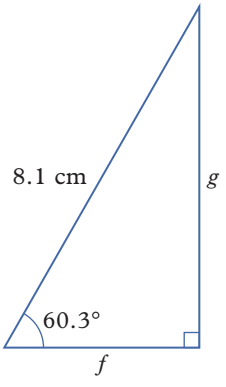



**b**

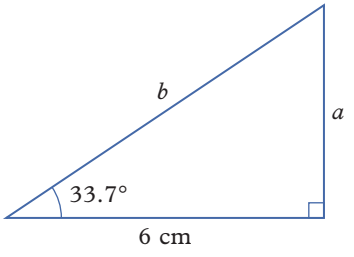



**15** Use the specified ratio, geometry and Pythagoras' Theorem to determine the value of the pronumerals correct to two decimal places.

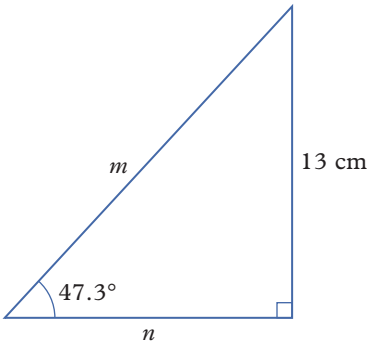
**a** sine



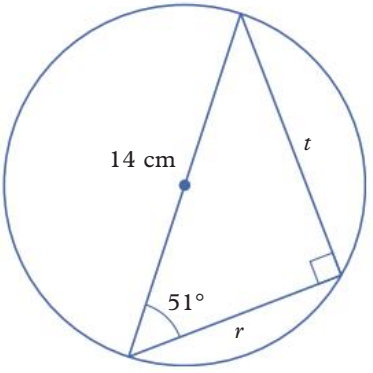

**b** tangent




**c** sine

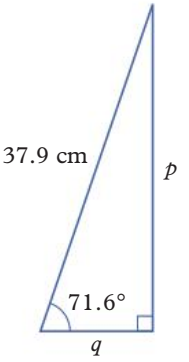


**16** Determine the lengths $r$ and $t$ correct to two decimal places.

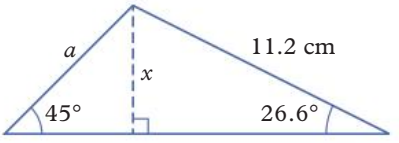



CHALLENGE

**17** Use the tangent ratio and Pythagoras' Theorem **only** to determine the value of the pronumerals correct to two decimal places. (Hint: Start by writing $p$ in terms of $q$.)




**18** We can find lengths in non-right-angled triangles by constructing right-angled triangles made up of known lengths. Consider the non-right-angled triangle shown with a perpendicular height added as a dotted line.




**a** Calculate the length of $x$.

**b** Using part **a**, calculate the length of $a$.

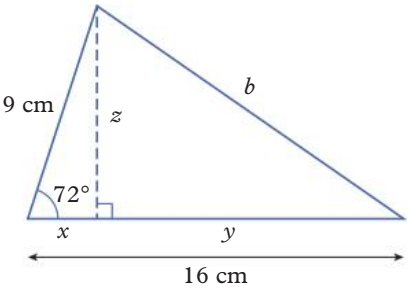
**19** Consider the non-right-angled triangle shown below.

**a** Calculate the length of $x$.

**b** Calculate the length of $y$.

**c** Calculate the length of $z$.

**d** Hence, calculate the length of $b$.




Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** — Finding lengths by trigonometry
- **Investigation** — Using trigonometry to find an approximate value for π
- **Topic quiz** — 7E

# 7F Using trigonometry to find angles

## Learning intentions

✔ I can find an unknown angle in a right-angled triangle.

**Inter-year links**

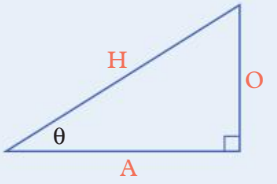
| | |
|---|---|
| Year 7 | 6H Solving equations using inverse operations |
| Year 8 | 6B Solving equations using inverse operations |
| Year 10 | 8C Applications of trigonometry |

## Using trigonometry to find an angle

- A trigonometric ratio can be used to find an unknown angle in a right-angled triangle if two side lengths are known.
- A calculator can be used to obtain the angle from its sine, cosine or tangent value. This is known as finding the **inverse sine** ($\sin^{-1}$), **inverse cosine** ($\cos^{-1}$) or **inverse tangent** ($\tan^{-1}$) of a value.
- To find an angle:
  1. Identify the given sides with respect to the angle. Label the sides accordingly.
  2. Decide which trigonometric ratio to use.
  4. Substitute in the known side lengths.
  5. Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
  6. Use the calculator to find the result.

SOH CAH TOA

$\sin(\theta) = \frac{O}{H}$ $\cos(\theta) = \frac{A}{H}$ $\tan(\theta) = \frac{O}{A}$




### Example 7F.1 Solving trigonometric equations to find the value of an angle

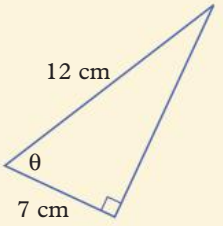
Solve each equation to find the value of θ, correct to the nearest degree.

**a** $\tan(\theta) = 2.4$ **b** $\sin(\theta) = \frac{22}{35}$

| THINK | WRITE |
|---|---|
| **a 1** Rearrange to make θ the subject of the equation using the inverse tangent. | $\tan(\theta) = 2.4$ |
| **2** Use the $\tan^{-1}$ key on the calculator to find the inverse tangent of 2.4. Round the value of θ to the nearest degree. | $\theta = \tan^{-1}(2.4)$<br>$\approx 67.3801...$<br>$\approx 67°$ |
| **b 1** Rearrange to make θ the subject of the equation using the inverse sine. | $\sin(\theta) = \frac{22}{35}$ |
| **2** Use the $\sin^{-1}$ key on the calculator to find the inverse sine of $\frac{22}{35}$. Round the value of θ to the nearest degree. | $\theta = \sin^{-1}\left(\frac{22}{35}\right)$<br>$= 38.9448...$<br>$\approx 39°$ |

## **Example 7F.2** Using trigonometry to find an unknown angle

Use trigonometry to find the angle θ in this triangle, correct to the nearest degree.




**THINK**

**1** Identify the given sides with respect to the angle. The side length adjacent to the angle is 7 cm. The hypotenuse is 12 cm. Label the sides accordingly.

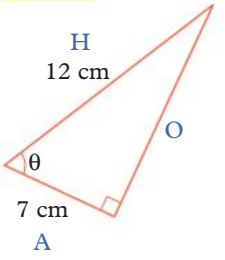
**2** Decide which trigonometric ratio to use. As A and H are involved, use cosine.

**3** Substitute for A and H.

**4** Rearrange to make θ the subject of the equation using the inverse cosine.

**5** Use the $\cos^{-1}$ key on the calculator to find the inverse cosine of $\frac{7}{12}$. Round the value of θ to the nearest degree.

**WRITE**




A = 7 cm, H = 12 cm

$\cos(\theta) = \frac{A}{H}$

$\cos(\theta) = \frac{7}{12}$

$\theta = \cos^{-1}\left(\frac{7}{12}\right)$

$= 54.3146...$

$\approx 54°$

**Helpful hints**

- ✔ While a superscript of **−1**, like $2^{-1}$, usually indicates 2 to the power of **−1** (the reciprocal), $\sin^{-1}$ does *not* mean sine to the power of −1 (the reciprocal of the value of sine): it is the inverse of sine. The notation of $^{-1}$ as the inverse of a function is explored in VCE Mathematical Methods.
- ✔ It can be easy to choose the wrong trigonometric function when determining the angle. Make sure you take your time and label the sides of the triangles. This can help to minimise errors.
- ✔ If you get the ratio upside-down for sine and cosine when determining the angle, your calculator will display an error as no real angle has a sine or cosine ratio greater than 1. However, if you get the ratio upside-down for tangent, your calculator will not show an error.

ANS p537

# Exercise 7F Using trigonometry to find angles

1–3(a, c, e, f), 4–6, 8, 10, 12 | 2–3(b, d, f, h), 4–7, 11–13, 17 | 2–3(c, e, f), 4(b, d, f), 5, 7, 9, 13–17

7F.1 **1** Calculate each of these, correct to the nearest degree.

**a** $\sin^{-1}(0.23)$ **b** $\cos^{-1}(0.72)$ **c** $\tan^{-1}(1.46)$

**d** $\sin^{-1}\left(\frac{2}{3}\right)$ **e** $\cos^{-1}\left(\frac{24}{25}\right)$ **f** $\tan^{-1}\left(\frac{39}{11}\right)$

**2** Find the unknown angle θ, correct to the nearest degree.

**a** $\sin(\theta) = 0.34$ **b** $\cos(\theta) = 0.81$ **c** $\tan(\theta) = 0.65$ **d** $\tan(\theta) = 0.47$

**e** $\cos(\theta) = 0.99$ **f** $\sin(\theta) = 0.01$ **g** $\tan(\theta) = 3.14$ **h** $\sin(\theta) = 0.71$

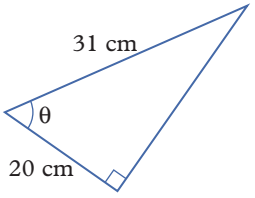
**3** Solve each equation to find the value of θ, correct to the nearest degree.

**a** $\sin(\theta) = \frac{4}{5}$ **b** $\cos(\theta) = \frac{15}{19}$ **c** $\tan(\theta) = \frac{8}{21}$ **d** $\sin(\theta) = \frac{23}{26}$

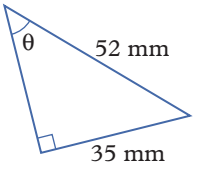
**e** $\cos(\theta) = \frac{11}{31}$ **f** $\sin(\theta) = \frac{9}{23}$ **g** $\cos(\theta) = \frac{17}{31}$ **h** $\tan(\theta) = \frac{26}{3}$

7F.2 **4** Use trigonometry to find the angle θ in each triangle, correct to the nearest degree.

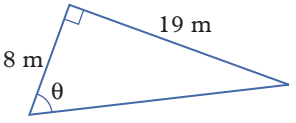
**a**



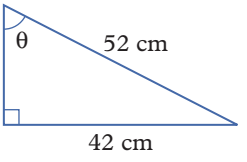

**b**



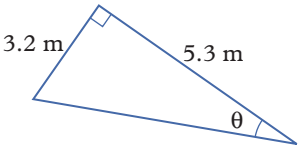

**c**



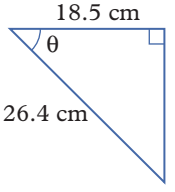

**d**




**e**

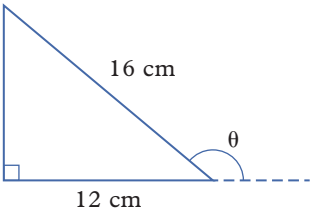



**f**

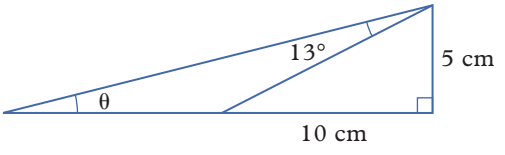



**5** Determine the value of θ in each of the following correct to one decimal place.

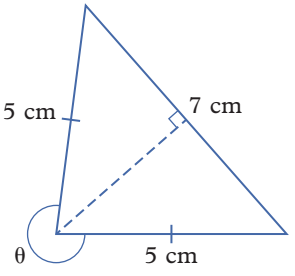
**a**



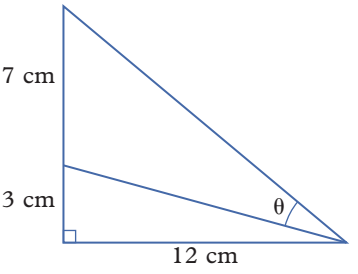

**b**




**c**




**d**

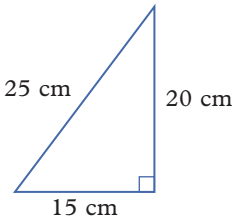



UNDERSTANDING AND FLUENCY

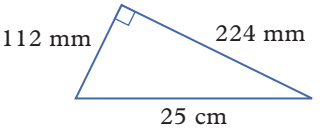
UNDERSTANDING AND FLUENCY

**6** Calculate the size of the non-right angles in the following right-angled triangles. Give your answers in degrees correct to one decimal place.

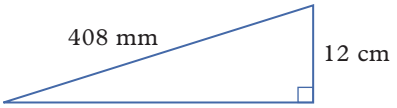
**a**



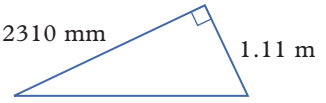

**b**




**c**

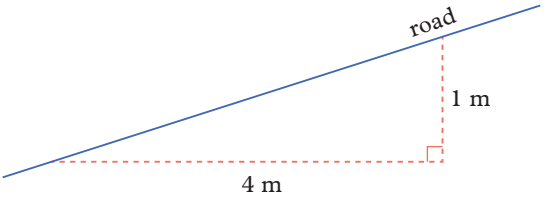



**d**




PROBLEM SOLVING AND REASONING

**7** A road has a gradient of 1 in 4; that is, it rises 1 m vertically for every 4 m horizontally.




**a** What angle does the road surface make with the horizontal? Give your answer correct to the nearest degree.

**b** How far have you travelled along the road if you are now 1.7 m higher than when you started? Give your answer correct to two decimal places.

**8** The roof of a holiday house has the dimensions shown. Calculate, correct to the nearest degree:

**a** the angle the roof makes with the horizontal

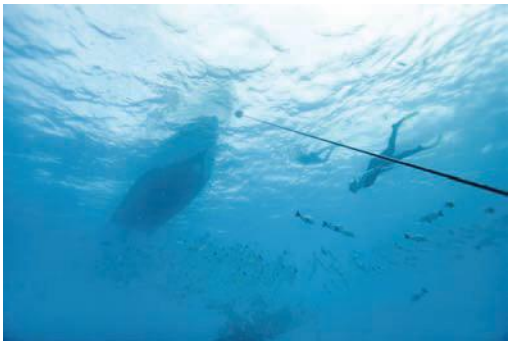
**b** the angle formed where the two sections of roof meet.

**9** An anchor holding a boat in position lies on the seabed at a depth of 7.5 m. It is attached to the boat by a chain that is 8.4 m long.

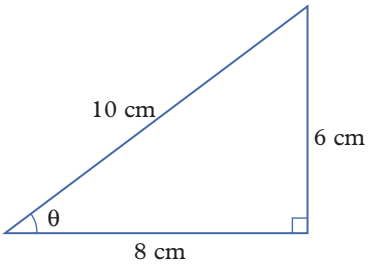
**a** Draw a diagram of this scenario.

**b** What angle does the chain make with the vertical? Give your answer correct to the nearest degree.

**c** If the chain was longer, would this angle be larger or smaller? Explain.



**10** Consider the right-angled triangle shown.




**a** Calculate the value of θ to the nearest degree using a calculator and:

**i** the sine ratio

**ii** the cosine ratio

**iii** the tangent ratio.

**b** Draw your own scale diagram of the triangle and measure the angle θ with a protractor.

**c** Compare your answers to parts **a** and **b**. Comment on the advantages and disadvantages of using:

**i** trigonometry

**ii** measurements from a scale diagram.

**11** Nadia and Alex are set a problem-solving task by their teacher. Their challenge is to work out the height of a tree in their school yard without climbing it or using a ladder. The only allowable equipment is a 1 m ruler, a tape measure and a calculator.

Nadia and Alex discuss some possible ideas, take some measurements simultaneously and draw these diagrams.

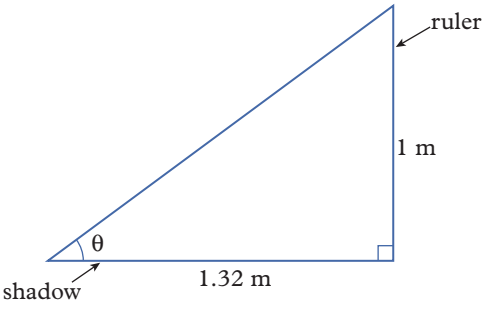



**Diagram 1**

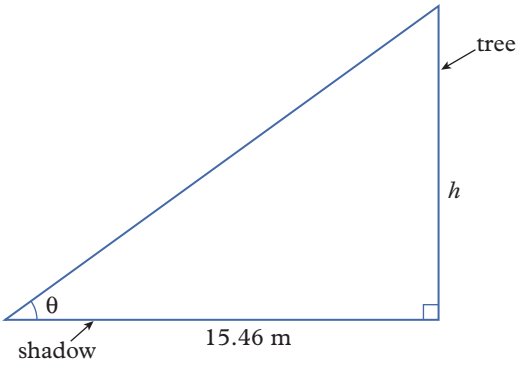



**Diagram 2**

**a** What you think their strategy is?

**b** Use trigonometry to calculate the value of θ to the nearest degree in Diagram 1.

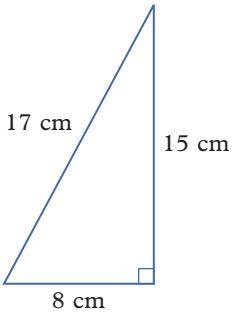
**c** Explain why this value of θ can be used in Diagram 2.

**d** Use trigonometry to calculate the height of the tree correct to two decimal places.

**e** Instead of using trigonometry, Nadia and Alex could have used their knowledge of similar triangles. Explain how this strategy could be used to calculate the height of the tree.

**f** Can you think of any other strategies that could have been used? Try them.

**12** Consider the right-angled triangle shown.




**a** Correct to the nearest degree, calculate the size of the non-right angles using a calculator and:

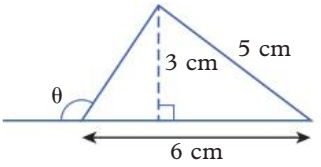
**i** the sine ratio **ii** the cosine ratio **iii** the tangent ratio.

**b** After the size of one non-right angle has been determined, how else could the size of the other non-right angle be determined?

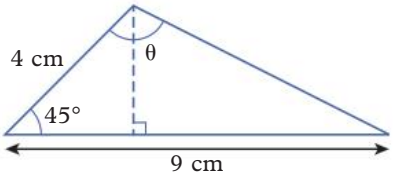
PROBLEM SOLVING AND REASONING

**13** Determine the value of θ in each of the following correct to one decimal place.

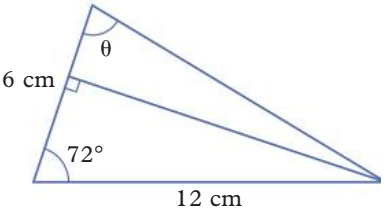
**a**




**b**

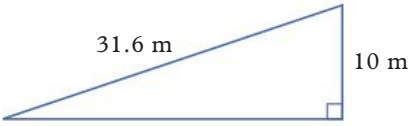



**c**

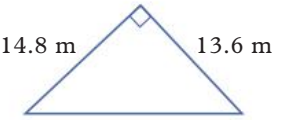



**14** Calculate the size of the non-right angles and the missing side length in the following right-angled triangles using the sine ratio and Pythagoras' Theorem correct to one decimal place.

**a**

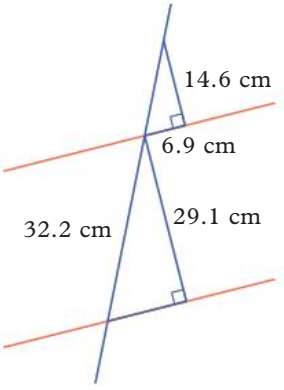



**b**

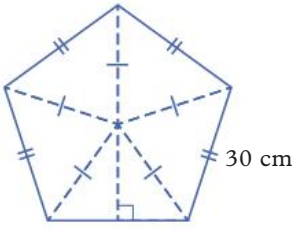



CHALLENGE

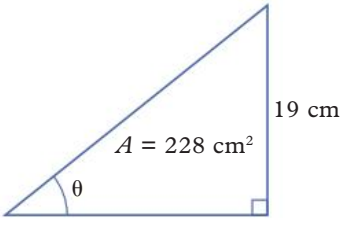
**15** Determine if the two red lines are parallel. Explain why or why not.




**16** Calculate the area of this regular pentagon correct to one decimal place.




**17** The area of the triangle shown is 228 cm$^2$. Determine the size of the angle θ correct to one decimal place.




Check your Student obook pro for these digital resources and more:

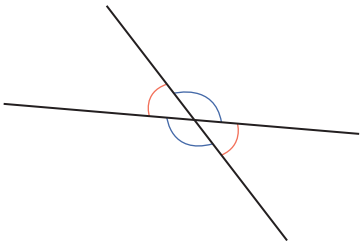
Interactive skillsheet
Finding angles by trigonometry

Topic quiz
7F

# Chapter summary

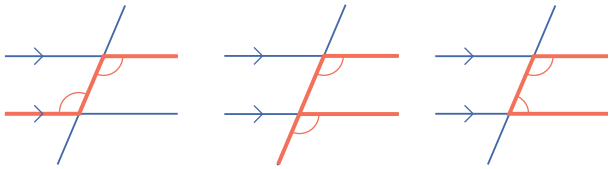
## Angles

- Angles at a point that form a straight line add to 180°.
- Angles at a point that form a revolution add to 360°.
- Two angles that add to 90° are called complementary angles.
- Two angles that add to 180° are called supplementary angles.
- Vertically opposite angles are equal in size.

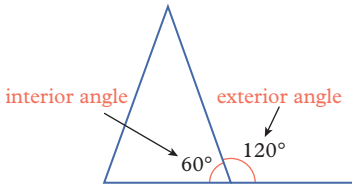


## Angles and parallel lines

- Alternate angles are equal.
- Corresponding angles are equal.
- Co-interior angles are supplementary.



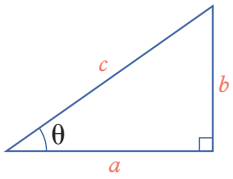
## Interior and exterior angles




## Pythagoras' Theorem

$c^2 = a^2 + b^2$

hypotenuse, shorter sides

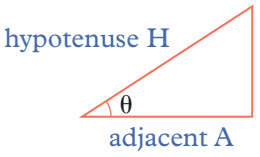



- A triangle is right-angled if it has side lengths that satisfy Pythagoras' Theorem.
- Any set of three whole numbers that satisfy Pythagoras' Theorem is called a Pythagorean triad (or Pythagorean triple).
  For example, 3, 4, 5 is a Pythagorean triad as $3^2 + 4^2 = 5^2$.

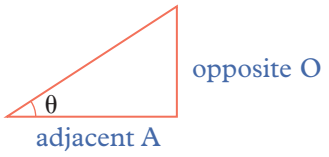
## Trigonometry

**SOH** $\sin(\theta) = \frac{O}{H}$

**CAH** $\cos(\theta) = \frac{A}{H}$




**TOA** $\tan(\theta) = \frac{O}{A}$




To find a side length:

1. Identify the given sides with respect to the angle θ. Label the sides accordingly.
2. Decide which trigonometric ratio to use.
3. Substitute in the known side length and angle.
4. Solve the equation for the unknown.
5. Use a calculator to perform the final calculation.

To find an angle:

1. Identify the given sides with respect to the angle. Label the sides accordingly.
2. Decide which trigonometric ratio to use.
3. Substitute in the known side lengths.
4. Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
5. Use a calculator to find the result.

# Chapter review

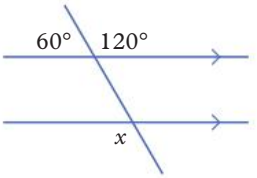
**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

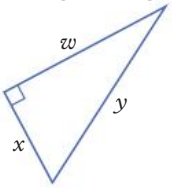
## Multiple-choice

7A **1** What is the size of angle $x$?

**A** $x = 30°$
**B** $x = 60°$
**C** $x = 120°$
**D** $x = 180°$
**E** not enough information

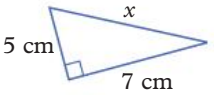



7B **2** Using Pythagoras' Theorem, which statement shows the relationship between the side lengths of the triangle?



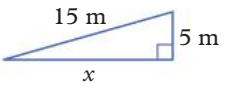

**A** $w^2 = x^2 + y^2$ **B** $x^2 = w^2 + y^2$ **C** $y^2 = x^2 + w^2$ **D** $y^2 = x^2 - w^2$ **E** $y^2 = w^2 - x^2$

7B **3** Which equation can be used to find the length of the hypotenuse in this triangle?




**A** $x = 7 + 5$ **B** $7 = x + 5$ **C** $7^2 = x^2 + 5^2$ **D** $5^2 = x^2 + 7^2$ **E** $x^2 = 5^2 + 7^2$

7C **4** The unknown side length $x$ in this triangle, correct to the nearest m, is:

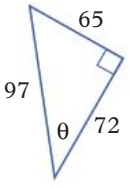



**A** 3 m **B** 10 m **C** 14 m **D** 16 m **E** 20 m

7D **5** Sine is the trigonometric ratio of which side to which side of a right-angled triangle?

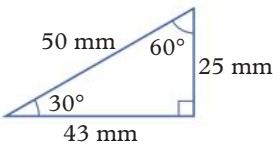
**A** hypotenuse to opposite side
**B** opposite side to adjacent side
**C** adjacent side to hypotenuse
**D** opposite side to hypotenuse
**E** hypotenuse to adjacent side

7D **6** Which of the following equations about this triangle is not correct?



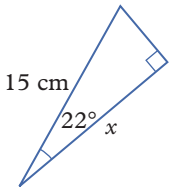

**A** $A = \frac{1}{2} \times 72 \times 65$ **B** $97^2 = 65^2 + 72^2$ **C** $\sin(\theta) = \frac{65}{97}$ **D** $\cos(\theta) = \frac{72}{97}$ **E** $\tan(\theta) = \frac{72}{65}$

7D **7** Using the lengths on the triangle shown, the cosine of 30° is:




**A** $\frac{43}{50}$ **B** $\frac{25}{50}$ **C** $\frac{25}{43}$ **D** $\frac{50}{43}$ **E** $\frac{50}{25}$

7E **8** Which equation can be used to find $x$ in this triangle?

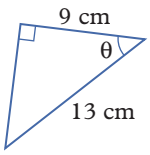



**A** $\cos(22°) = \frac{x}{15}$ **B** $\tan(22°) = \frac{x}{15}$ **C** $\sin(22°) = \frac{x}{15}$ **D** $\cos(22°) = \frac{15}{x}$ **E** $\sin(22°) = \frac{15}{x}$

7F **9** If $\sin(\theta) = \frac{12}{15}$, then $\theta$ is equal to:

**A** $\sin\left(\frac{15}{12}\right)$ **B** $\sin^{-1}\left(\frac{12}{15}\right)$ **C** $\sin^{-1}\left(\frac{15}{12}\right)$ **D** $\sin\left(\frac{12}{15}\right)^{-1}$ **E** $\frac{\sin^{-1}(12)}{15}$

7F **10** The angle $\theta$ in this triangle is closest to:

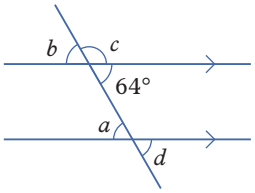



**A** 35° **B** 44° **C** 46° **D** 55° **E** 0.81°

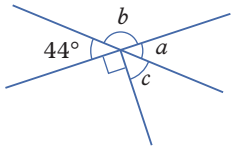
## Short answer

7A **1** Find the value of each pronumeral.

**a**

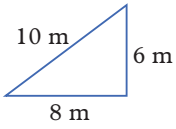



**b**

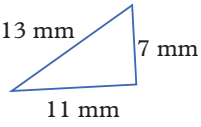



7B **2** Use Pythagoras' Theorem to decide whether these are right-angled triangles.

**a**




**b**



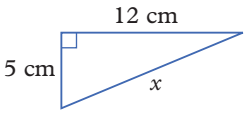

7B **3** A triangle has side lengths 30 mm, 40 mm and 50 mm.

**a** Show that the triangle is right-angled.

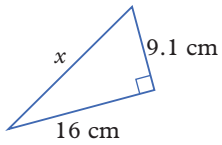
**b** Are the side lengths of the triangle an example of a Pythagorean triad? Briefly explain why or why not.

7B **4** Calculate the length of the hypotenuse in each triangle, correct to two decimal places.

**a**

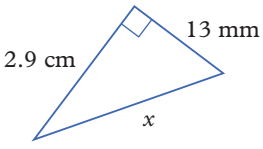



**b**

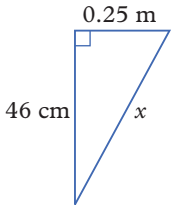



7B **5** Calculate the length of the hypotenuse in each triangle, correct to the nearest centimetre.

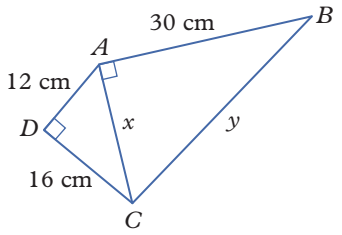
**a**




**b**

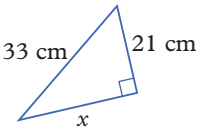


7B **6** Find the length of the sides represented by $x$ and $y$ in this diagram, correct to two decimal places.

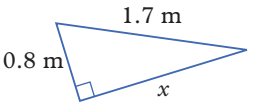



7C **7** Calculate the side length $x$ in each triangle, correct to two decimal places.

**a**

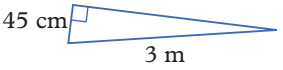



**b**

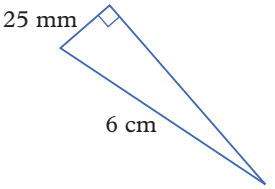



7C **8** Calculate the unknown length in each triangle, correct to the nearest centimetre.

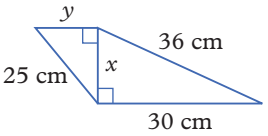
**a**



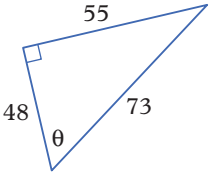

**b**




7C **9** Find the length of the sides represented by $x$ and $y$ in this diagram, correct to two decimal places.

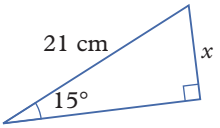



7D **10** Write equations for the sine, cosine and tangent of θ for this triangle.

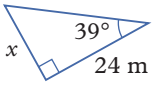



7E **11** Find the side length $x$ in each triangle, correct to two decimal places.

**a**

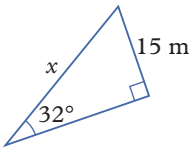



**b**

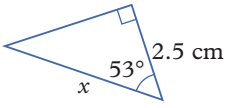



7E **12** Find the side length $x$ in each triangle, correct to two decimal places.

**a**

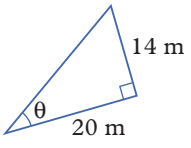



**b**

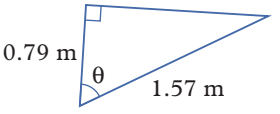



7F **13** Find the angle θ in each triangle, correct to the nearest degree.

**a**

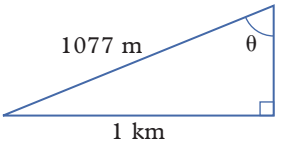



**b**

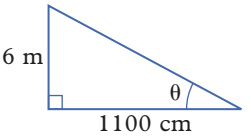



7F **14** Find the angle θ in each triangle, correct to the nearest degree.

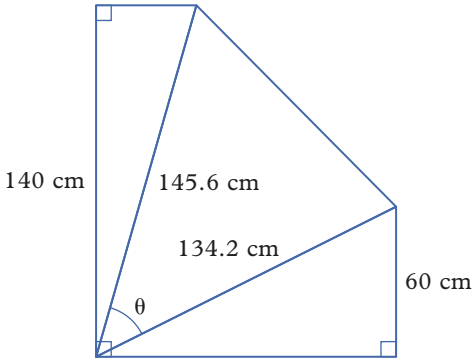
**a**




**b**

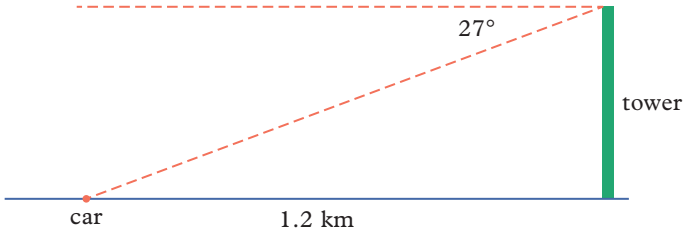


7F **15** Calculate the size of the angle θ, correct to one decimal place.




## Analysis

**1** From the top of a tower, Kim looks down to her family's car in a nearby car park at an angle of 27° from the horizontal as shown. She knows that the car is 1.2 km from the base of the tower.




**a** Consider the information provided.

**i** Is there enough information to use Pythagoras' Theorem to calculate the height of the tower? Explain why or why not.

**ii** Is there enough information to use trigonometry to calculate the height of the tower? Explain why or why not.

**b** Using your answer from part **a**, draw a triangle and label the sides with the appropriate information.

**c** Calculate the height of the tower to the nearest metre.

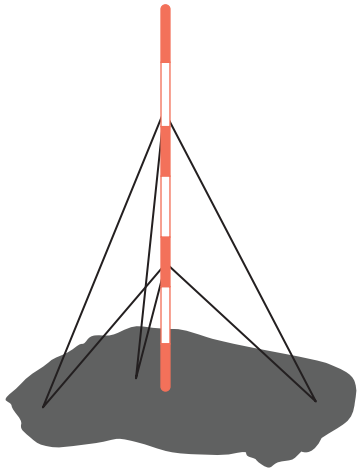
**d** Show another two different triangles that also could have been used to calculate the height of the tower. Label each triangle with its relevant information and show that each triangle produces the same tower height as you calculated in part **c**.

Kim wonders whether she can use Pythagoras' Theorem to calculate the direct distance from her eye level to the car in the car park.

**e** Show that there is enough information for Kim to calculate the direct distance to the car using Pythagoras' Theorem. Write your answer to the nearest metre.

**f** Check your answer in part **e** using trigonometry. Show that both methods produce the same value.

**2** A 150 m tall AM radio tower is supported by six guy wires equally spaced around the tower connecting to the ground at three points. Three of the wires are connected one-third of the way up the tower and the other three are connected one-third of the way down from the top of the tower.



**a** If the shorter wires are each 90 m long, how far from the base of the tower are they connected, correct to one decimal place?

**b** Determine the total length of wire being used to support the tower, correct to the nearest metre.

**c** Determine the straight-line distance between two of the connecting points on the ground, correct to one decimal place.

**d** Determine the angles the wires make with the ground, correct to two decimal places.

**e** Determine the angle between the two wires where they meet at the ground, correct to two decimal places.

# 8

# Statistics

## Index


**8A** Classifying and displaying data
**8B** Grouped data and histograms
**8C** Summary statistics from tables and displays
**8D** Describing data
**8E** Comparing data


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Interpreting graphs
- ✔ Column graphs and dot plots
- ✔ Stem-and-leaf plots
- ✔ Calculating percentages

## VCAA curriculum links

- Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (VCMSP323)
- Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (VCMSP324)
- Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bimodal' (VCMSP325)
- Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (VCMSP326)

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# 8A Classifying and displaying data

## Learning intentions

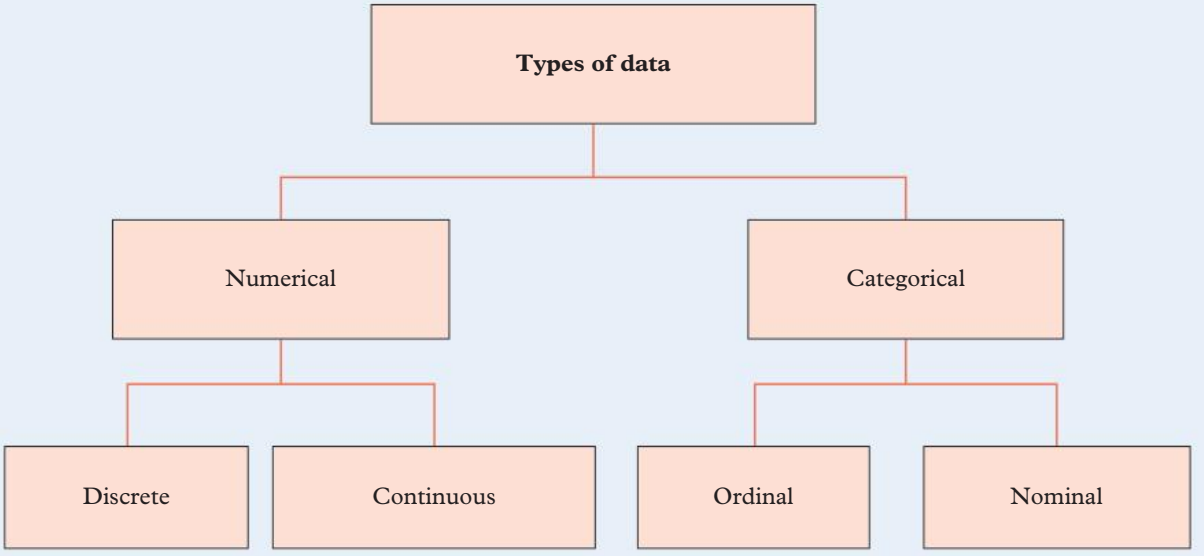
- ✔ I can classify data involving more than one variable.
- ✔ I can display data involving more than one variable.

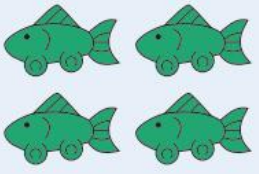
Inter-year links

| | |
|---|---|
| Year 5/6 | Interpreting data |
| Year 7 | 5E Interpreting graphs |
| Year 8 | 9C Presenting data |
| Year 10 | 10E Scatterplots and bivariate data |

## Classifying data

- **Numerical data** can be classified as either **discrete** (countable) or **continuous** (measured).
- **Categorical data** can be classified as either **ordinal** (can be ordered) or **nominal** (unordered categories).



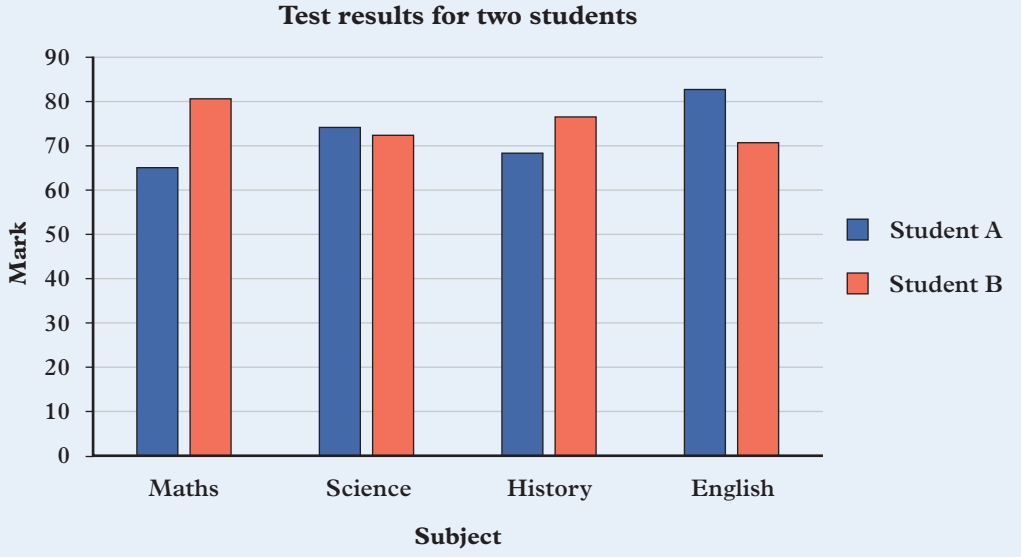



| | |
|---|---|
| A | Epping |
| B | Northcote |
| C | Ivanhoe |
| D | Hawthorn |

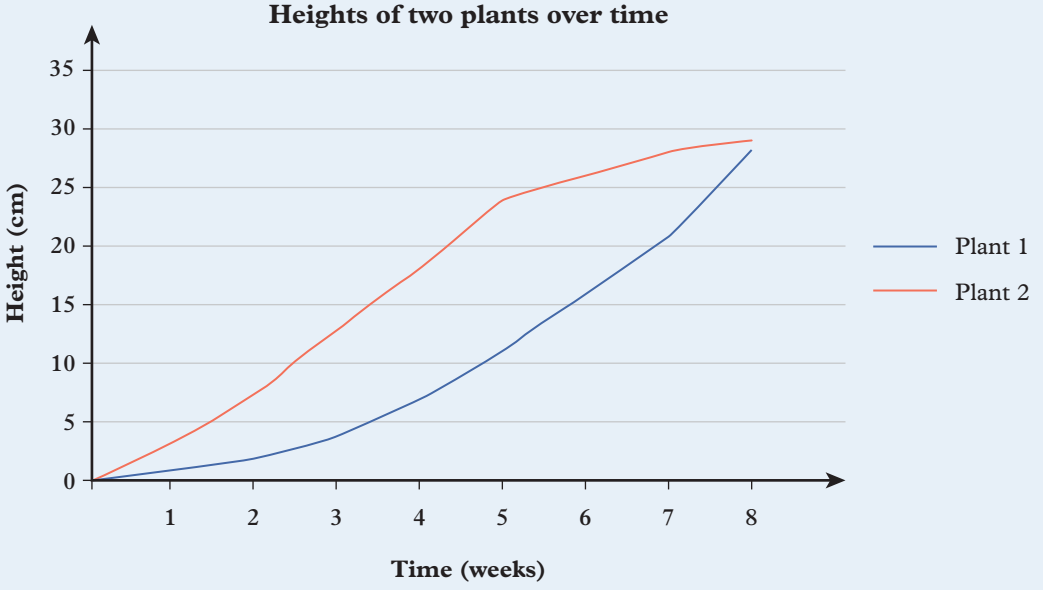
- **Univariate data** is data that only concerns a single variable.
- **Bivariate data** is data that shows the relationship between two variables. Bivariate data often falls within more than one classification. For example, comparing the amount of rainfall in different cities could involve a numerical variable, the amount of rainfall, and a categorical variable, the city.

# Displaying data

- **Column graphs** can be used to display data involving one numerical variable and one categorical variable.
- The numerical variable is displayed on the vertical axis and the categorical data is displayed on the horizontal axis.
- **Side-by-side column graphs** can be used to display data involving one numerical variable and two categorical variables. The second categorical variable is detailed in the legend.




- **Line graphs** can be used to display data involving two numerical variables. The horizontal axis must be a continuous variable and usually displays the independent variable. The vertical axis usually displays the dependent variable. For example, the height of a plant depends on the length of time it has been growing. Distance–time graphs are an example of line graphs.
- **Multiple line graphs** can be used to display data involving two numerical variables, and one categorical variable. The categorical variable is detailed in the legend.




- All graphs should include clearly labelled axes with evenly spaced scales, and a title and legend where necessary.

## Example 8A.1 Classifying univariate data

Match each of the following variables with the data classification.

**a** Weights (in kg) of African elephants
**b** Quality of sleep (Excellent, Good, Fair, Poor, Extremely Poor)
**c** Favourite school subject of Year 9 students
**d** Number of pets owned by different celebrities

**A** Numerical – discrete
**B** Numerical – continuous
**C** Categorical – ordinal
**D** Categorical – nominal

**THINK**

1 Consider which variables are numerical and decide which must be measured and which must be counted.

2 Weights of African elephants and number of pets are numerical, since each data point must be collected as numbers. Weights of elephants must be measured, so this variable is continuous. The number of pets must be counted, so this variable is discrete.

3 Consider which variables are categorical and decide which has an inherent order and which does not.

4 Quality of sleep and favourite school subject are categorical, because respondents must select one of several categories, rather than responding with a number. The categories given for quality of sleep have an inherent order (in order of increasing or decreasing quality), so this variable is ordinal. The list of different school subjects does not have an inherent order, so this variable is nominal.

**WRITE**

**a** B **b** C **c** D **d** A

## Example 8A.2 Classifying bivariate data

Identify the variables when researching the following issues and classify each variable as either numerical or categorical.

**a** Time spent commuting to work from different suburbs
**b** Number of customers in a store at various times of the day

**THINK**

**a** 1 Identify the two variables: time duration and name of suburb.
2 Classify each of these variables from the previous step. Time is a numerical variable and the suburb is a categorical variable.

**b** 1 Identify the two variables: number of customers and time of day.
2 Classify each of these variables from the previous step. Number of customers and time of day are both numerical variables.

**WRITE**

**a** Time: numerical; suburbs: categorical

**b** Number of customers: numerical; time of day: numerical

## Example 8A.3 Interpreting line graphs

Consider the line graph on the right.

**a** What information does the graph show?

**b** Classify the variables in the graph.

**c** What time period does it cover?

**d** The maximum number of customers appeared in which week?

**e** Between which weeks does the number of customers increase the fastest?

**THINK**

**a** Look at the title of the graph and the title of the axes.

**b** There are two variables in a line graph; a numerical variable on the vertical axis, and a different numerical variable on the horizontal axis.

**c** Look at the range on the horizontal axis.

**d** Identify the week that has the greatest value on the vertical axis.

**e** Identify between which two points was the greatest increase on the vertical axis.

**WRITE**

**a** The number of customers over a period of weeks

**b** Time (weeks): continuous, Number of customers: discrete

**c** 10 weeks

**d** Week 6

**e** Weeks 2 and 3

## Example 8A.4 Drawing a line graph

The data in the following table was recorded over 10 days.

| **Day** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Weight of puppy (g)** | 92 | 100 | 110 | 125 | 123 | 131 | 142 | 150 | 156 | 164 |

Draw a line graph to represent this data.

**THINK**

1 Determine which variable should be placed on which axis. The independent variable (days) should be placed on the horizontal axis and the dependent variable (weight) should be placed on the vertical axis.

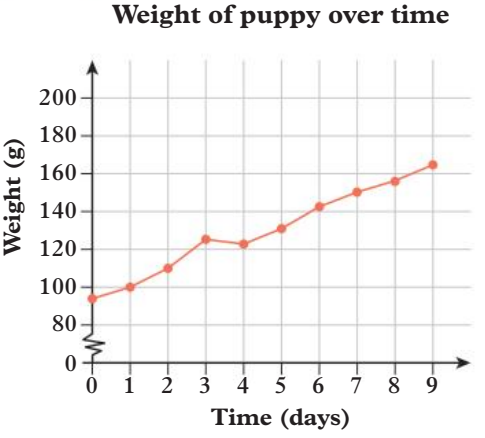
2 Draw the vertical axis. The top of the vertical scale should be a round number that is equal to or greater than the largest value.

3 Draw the horizontal scale, providing enough space to fit in all the independent values.

4 Plot the points from the table and join subsequent points with straight lines.

5 Include a title and axis labels.

**WRITE**



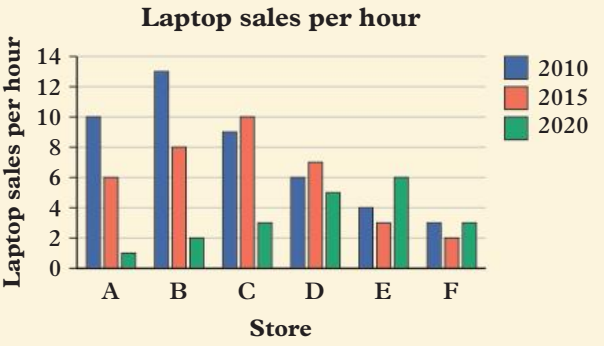
## Example 8A.5 Interpreting side-by-side column graphs

Consider the side-by-side column graph on the right.

**a** Which variables are displayed in the graph?

**b** In which year and at which store were the most sales per hour achieved?

**c** In which year and at which store were the least sales per hour achieved?




**THINK**

**a** Look at the titles of the axes and the legend.

**b** Identify the tallest column. Use the horizontal axis and the legend to interpret which store and year this bar represents.

**c** Identify the shortest column. Use the horizontal axis and the legend to interpret which store and year this bar represents.

**WRITE**

**a** laptop sales per hour, store and year

**b** Store B in 2010

**c** Store A in 2020

**Helpful hints**

✔ Univariate (single variable) data is usually displayed in a column graph with the frequency of the categories on the vertical axis. Bivariate data consists of two distinct variables.

ANS p538

# Exercise 8A Classifying and displaying data

△ 1–6, 8, 10–12, 14 □ 2–4, 7, 9, 11, 13, 14, 17 ⬠ 3, 4, 7, 11, 15–17

UNDERSTANDING AND FLUENCY

8A.1 **1** Match each of the following variables with the data classification.

**a** Eye colour of students

**b** Number of students in each class

**c** Time (in minutes) spent travelling to school

**d** Final grade on an end-of-year exam (A+, A, B+, B, etc.)

**A** Numerical – discrete

**B** Numerical – continuous

**C** Categorical – ordinal

**D** Categorical – nominal

**2** Consider the following four variables.

**1** Height (in cm) of AFL players

**2** Method of transport for commuting to school (car, bus, train, etc.)

**3** Mark (out of 40) on a Mathematics test

**4** Belt level in karate (white, orange, blue, etc.)

**a** Classify each variable as numerical or categorical.

**b** Classify each numerical variable as either discrete or continuous, or each categorical variable as either ordinal or nominal.

8A.2 **3** In each of the following situations, identify the two variables and classify each as either numerical or categorical.

**a** A car company hires an independent researcher to find the fuel consumption (kilometres per litre of petrol) for each of their car models.

**b** A business owner investigates the relationship between the number of ice cream sales per day in her shop and the maximum daily temperature (in °C).

**c** On each student's report, a school lists the student's exam score, as a percentage, for each subject.

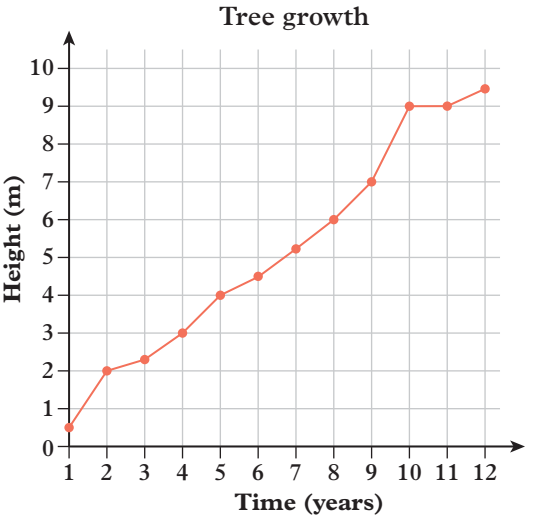
8A.3 **4** Consider the line graph on the right.

**a** What does the graph show?

**b** Classify the variables in the graph.

**c** What time period does it cover?

**d** Between which years did the tree grow the fastest?




**5** Consider the line graph on the right.

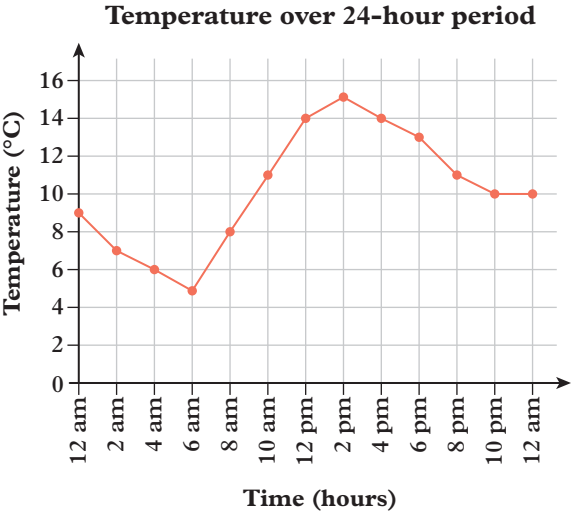
**a** What does the graph show?

**b** Classify the variables in the graph.

**c** What is the temperature at 10 am?

**d** When is the temperature 15°C?

**e** What is the coolest temperature recorded? At what time does it occur?




8A.4 **6** The data in the following table shows the estimated market value of a gaming console for 10 months after its purchase. Draw a line graph to represent this data.

| **Time since purchase (months)** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Market value ($)** | 600 | 400 | 350 | 320 | 300 | 270 | 250 | 250 | 220 | 220 | 220 |

**7** The following data shows the number of kilometres driven by Antoine over a period of 9 days.

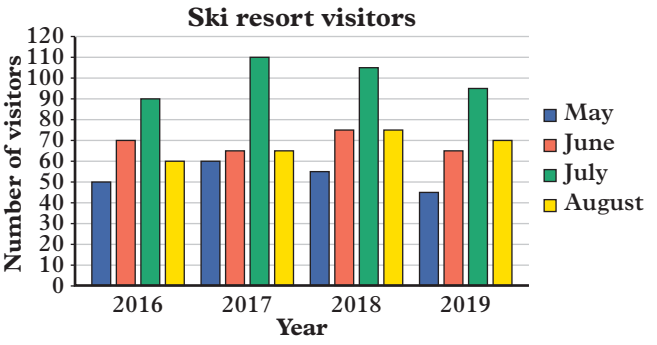
| **Day** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **Number of km driven** | 18 | 11 | 0 | 18 | 15 | 8 | 10 | 6 | 13 |

**a** Draw a line graph to represent this data.

**b** On which day(s) did Antoine drive the most kilometres?

**c** On how many days did Antoine drive more than 10 km?

**8A.5** **8** Consider the side-by-side column graph on the right, which shows the number of people who visited a ski resort during the months it was open from 2016 to 2019.

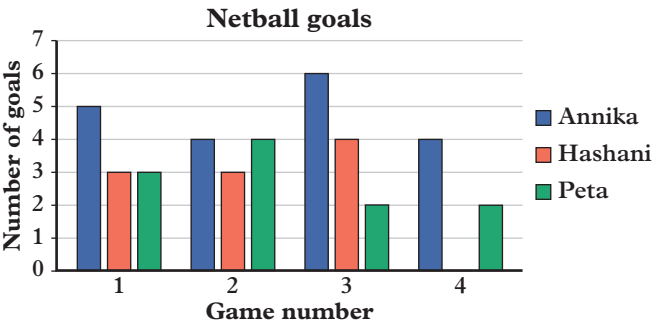



**a** What are the different variables in the graph?

**b** In which month of which year did the resort have the most visitors?

**c** In which month of which year did the resort have the least visitors?

**9** Consider the side-by-side column graph on the right.




**a** What are the different variables in the graph?

**b** In which game did one of the three girls not score a goal?

**c** In which game did the three players score the most goals combined?

**d** Which of the three players scored the least number of goals across the four games?



**10** The following data is the monthly profit (rounded to the nearest $100) of a small business over the course of a year.

| **Month number** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Profit ($)** | 4000 | 5300 | 3800 | 4100 | 4300 | 4000 | 3800 | 3400 | 5100 | 5400 | 4900 | 4600 |

**a** Draw a line graph to represent this data, using an appropriate scale.

**b** In which month was the greatest profit?

**c** In which month was the least profit?

**d** In which month was the greatest increase in profit over the previous month? How much was this increase?

**e** In which month was the greatest decrease in profit over the previous month? How much was this decrease?

**11** Students in years 8, 9 and 10 were asked to choose their preferred ice-cream flavour from three choices given. The data is shown below.

| | **Year 8** | **Year 9** | **Year 10** |
|---|---|---|---|
| **Chocolate** | 140 | 135 | 130 |
| **Strawberry** | 100 | 90 | 70 |
| **Vanilla** | 60 | 75 | 60 |

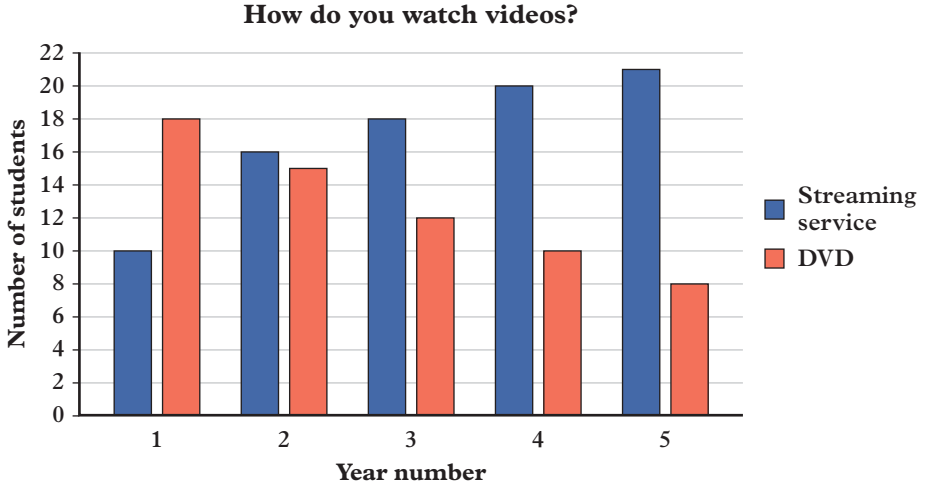
**a** Create a side-by-side column graph for the set of data if the category on the horizontal axis is year level.

**b** Create a side-by-side column graph for the set of data if the category on the horizontal axis is ice-cream flavour.

**c** Which of the two graphs is better for showing which ice-cream flavour is the most popular with Year 9 students?

**d** Is it accurate to say that, in general, Year 9 students prefer chocolate ice-cream more than Year 10 students? Why or why not?

**12** Each year a Mathematics teacher has a Year 9 class with exactly 25 students. He asks each student "Do you have access to an online streaming service (like Netflix)?" and "Is there a DVD player in your house?" The following side-by-side column graph shows the data collected over a 5-year period.




**a** How many students had access to a streaming service in year 4?

**b** In which year was the number of students in each category closest to being equal?

**c** Explain why the two column values for each year can add to more than the number of students in the class.

**d** What would you expect the two columns to look like in 10 years, assuming the teacher continues collecting data? Explain your answer.

**13** A specialty store sells Christmas items all year round. The table below shows the average number of customers per day, rounded to the nearest 10 customers, in each month over the course of 12 months.

| **Month** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Average daily customers** | 10 | 20 | 20 | 30 | 60 | 170 | 300 | 40 | 20 | 10 | 0 | 10 |

**a** Draw a line graph to represent this data.

**b** It is safe to assume that, in this data set, month 12 does not represent December? Which month number do you think represents December? Explain your answer.

**c** Is it accurate to say that the store had no customers at all in month 11? Explain your answer.

**14** The following side-by-side column graph shows the total rainfall (in mm) in different cities in Australia, Pakistan and Malaysia, over the years 2010, 2011 and 2012.

**a** Based on this data, which of the five cities is the wettest?

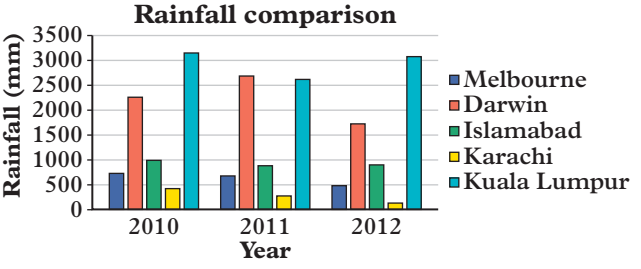
**b** Based on this data, which of the five cities is the driest?

**c** In which city and which year was the rainfall the highest?

**d** In which city and which year was the rainfall the lowest?

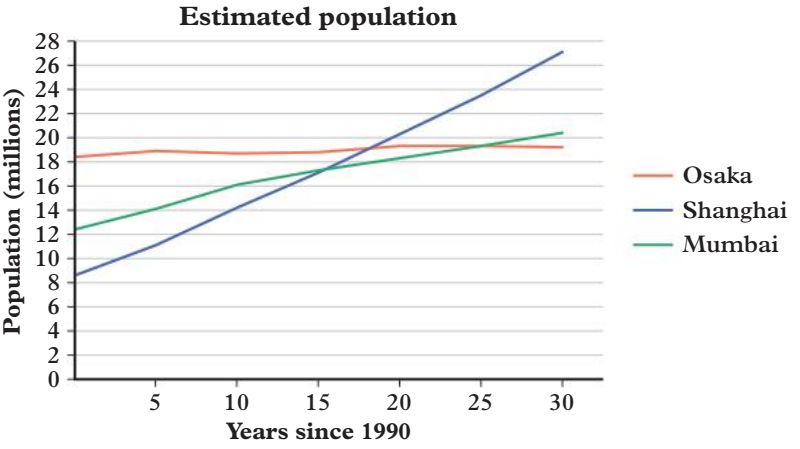
**e** Which city had an increase in rainfall between 2010 and 2011?

**f** Which city had the most consistent rainfall across the three years?



PROBLEM SOLVING AND REASONING

**15** The following multiple line graph shows the estimated metro population of three Asian cities since 1990.




**a** Which city had the highest population in 1990?

**b** Which city had the highest population in 2010?

**c** Approximately how many people were living in Mumbai in 2000?

**d** Which city had the highest population in 2020?

**e** In which year, approximately, did the population of Shanghai surpass the population of Osaka?

**f** Describe the change in each city's population between 1990 and 2020.

CHALLENGE

**16** The table below shows the monthly profit, rounded to the nearest $1000, of a small ice-cream store in Geelong. In this case, January is represented by month 1.

| **Month** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Profit ($)** | 9000 | 8000 | 6000 | 3000 | 1000 | −1000 | −2000 | 0 | 3000 | 4000 | 7000 | 9000 |

**a** Draw a line graph to represent this data. Remember to make space below the horizontal axis for the negative values.

**b** Explain what the negative values in month 6 and month 7 mean.

**c** Explain what the 0 in month 8 means.

**d** What was the total profit of the ice-cream store over the course of the year?

**e** From month 8 to month 12 the profit is increasing each month. Do you expect this trend to continue forever? Explain your answer.

**f** How would you expect the monthly profit of an ice-cream store to be different for a similarly sized store in the northern hemisphere (for example, in a small town in Spain)? Explain your answer.

**17** The following table shows the height (in cm) of a person over the first 15 years of their life.

| **Age (years)** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Height (cm)** | 52 | 75 | 85 | 92 | 100 | 108 | 112 | 120 | 128 | 135 | 140 | 147 | 154 | 160 | 165 | 172 |

**a** Draw a line graph to represent this data.

**b** The person had a height of 52 cm when they were born and 172 cm 15 years later. Using these two points, find the average height they gained per year.

**c** Assuming the person continues growing at this average rate per year, how tall would they be at 25 years old?

**d** Instead, suggest a reasonable height that the person might be at 25 years old.

**e** Explain what you would expect the rest of the graph to look like if it continued to when the person was 25 years old.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Classifying data
- Interactive skillsheet: Distance–time graphs
- Interactive skillsheet: Side-by-side column graphs
- CAS instructions: Line graphs
- Topic quiz: 8A

# 8B Grouped data and histograms

## Learning intentions

- ✔ I can group numerical data into class intervals.
- ✔ I can represent grouped numerical data in a histogram.

Inter-year links

| | |
|---|---|
| Year 5/6 | Data displays |
| Year 7 | 10C Constructing and interpreting graphs |
| Year 8 | 9C Presenting data |
| Year 10 | 10C Distributions of data |

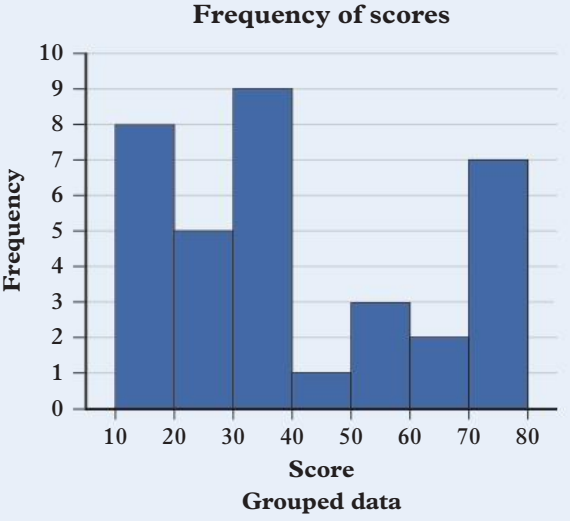
## Grouping data into class intervals

- **Grouped data** is numerical data that has been sorted into groups or **class intervals**. **Ungrouped data** is raw data that has not been placed into class intervals.
- Class intervals should be chosen so that a frequency table contains 5 to 10 classes.
  - → Class intervals should start at a round number, for example 0 or 10.
- Before constructing a frequency table with class intervals, identify whether the numerical data is discrete or continuous.
  - → Discrete data uses class intervals such as 0–9, which means any value from 0 to 9, including both 0 and 9.
  - → Continuous data uses class intervals such as 0–<10, which means any value from 0 to 10, including 0 but not including 10.

| Discrete interval | Continuous interval |
|---|---|
| 0–9 | 0–<10 |
| 10–19 | 10–<20 |

## Histograms

- A **histogram** is a special kind of column graph that can be used to represent grouped continuous or discrete numerical data.
  - → Ungrouped discrete numerical data is usually represented by using a standard column graph.
- Histograms have no gap between columns, but there is a small gap between the vertical axis and the first column when the intervals do not start at 0.
- When displaying grouped data in a histogram, the **lower bound** of the class interval is shown on the left of each column and the **upper bound** of the class interval is shown on the right of each column.
  - → For discrete data, where the upper and lower bounds of subsequent classes are not equal, the lower bound of the higher class interval should be shown. For example, for the discrete intervals 0–9, 10–19, 20–29, the intervals on the horizontal axis are labelled 0, 10, 20.



## Example 8B.1 Understanding histograms

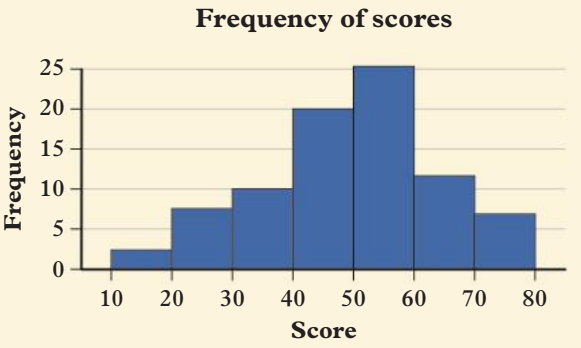
Consider this histogram representing continuous data.

**a** What is the width of each class interval?

**b** What is the frequency of the class 30–<40?

**c** What is the most common class? What is its frequency?

**d** Which class has a frequency of 20?




**THINK**

**a** Look at the horizontal axis. The width of the class intervals is the width of each column.

**b** Look at the column covering the marks 30 to 40 and read off the frequency from the vertical axis.

**c** Identify the tallest column. Use the horizontal axis to identify the class interval and read off the frequency from the vertical axis.

**d** Identify the column with a frequency of 20 on the vertical axis, and then use the horizontal axis to identify the class interval.

**WRITE**

**a** The width of each class interval is 10.

**b** The class 30–<40 has a frequency of 10.

**c** The most common class is 50–<60 and it has a frequency of 25.

**d** The class interval with a frequency of 20 is 40–<50.

## Example 8B.2 Drawing a frequency table to represent grouped data

Use the frequency table below with class intervals of width 10 to represent this continuous data.

4.5, 11.6, 67.3, 33.7, 28.1, 36.4, 22.6, 54.8, 1.4, 66.8, 36.4, 29.3, 37.8, 42.3, 52.1, 38.3

| **Class** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Frequency** | | | | | | | |

**THINK**

**1** Identify the minimum and maximum scores in the data set. The minimum score is 1.4 and the maximum score is 67.3.

**2** The data is continuous so the class intervals must be in the form of 0–<10, etc.

**3** Draw the frequency table and group the raw data into each class interval.

**WRITE**

| **Class** | 0–<10 | 10–<20 | 20–<30 | 30–<40 | 40–<50 | 50–<60 | 60–<70 |
|---|---|---|---|---|---|---|---|
| **Frequency** | 2 | 1 | 3 | 5 | 1 | 2 | 2 |

## Example 8B.3 Drawing a histogram

Draw a histogram to represent the data in this frequency table.

| Class | 10–<15 | 15–<20 | 20–<25 | 25–<30 | 30–<35 | 35–<40 |
|---|---|---|---|---|---|---|
| Frequency | 7 | 5 | 3 | 4 | 3 | 2 |

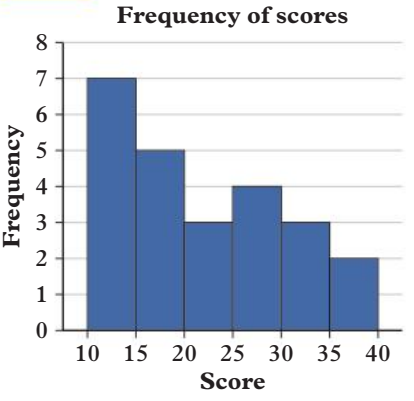
**THINK**

1 Draw the axes with an even scale that allows the minimum and maximum values to be shown. Ensure that there is a half space between the vertical axis and first class interval.

2 Use the frequency table to draw the columns of the histogram.

3 Label both axes and provide a title.

**WRITE**




**Helpful hints**

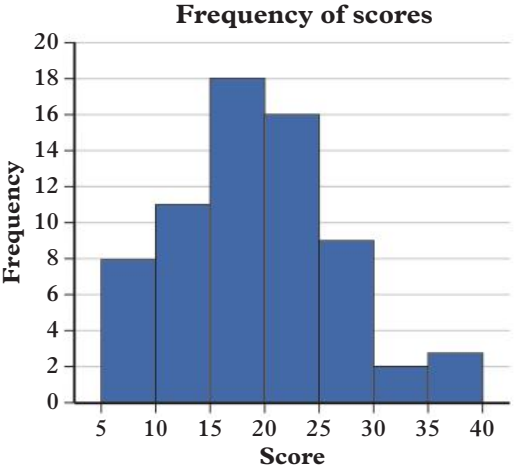
- ✔ The class interval 0–9 contains 10 values. If this doesn't make sense to you, try counting the values using your hands: 0, 1, 2 … 8, 9.
- ✔ When grouping raw data, a tally column can help to ensure you don't miss any scores.
- ✔ You don't need to order the values in a data set before grouping data. Avoiding this unnecessary step can save you time when answering questions.

ANS p540

# Exercise 8B Grouped data and histograms

1–3, 4(a, c), 5(a, b), 6(a, d), 8, 10, 13, 16 | 2, 4(b, c), 5(a, c), 6(d, e), 7, 11, 14, 16, 19 | 4(d), 5(a, c), 6(e, f), 9, 12, 15, 17–20

8B.1 **1** Consider this histogram representing continuous data.




**a** What is the size of each class interval?

**b** What is the frequency of class 20–<25?

**c** What is the most common class? What frequency does it have?

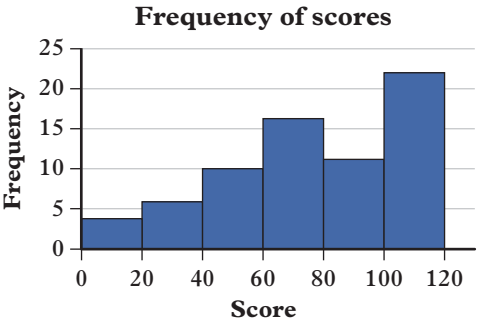
**d** Which class has a frequency of 9?

**e** What is the total frequency of scores that are larger than 25?

**f** What is the total frequency of scores that are less than or equal to 15?

**g** What percentage of the scores are more than 35?

**2** Consider the histogram on the right representing continuous data.




**a** How many class intervals are there?

**b** What is the least common class? What frequency does it have?

**c** How many people were surveyed?

**3** Joseph collected the following data on the heights of the people in his class.

145, 183, 167, 172, 161, 158, 153, 168, 165, 174, 157, 152, 173, 166, 158, 159, 160, 171, 171, 161, 165, 172, 165, 158, 154

**a** Identify the minimum value in the data set.

**b** Identify the maximum value in the data set.

**c** Calculate the range of the data.

**d** Group the data into class intervals of 5 (e.g. 150–<155) in a frequency table.

8B.2 **4** Draw a frequency table with the given class interval widths to represent each of these continuous data sets.

**a** 5, 16, 28, 24, 31, 39, 3, 18, 13, 11, 25, 33, 8, 12, 19, 21, 31, 28 [class interval width = 5]

**b** 14.5, 73.2, 22.1, 43.9, 42.0, 58.4, 19.8, 37.6, 62.1, 29.4, 34.5, 72.1, 59.1, 52.3, 63.1, 26.3, 34.0, 41.9, 48.5, 16.4, 31.2, 52.9 [class interval width = 10]

**c** 1.2, 5.4, 9.3, 11.4, 3.3, 4.7, 3.3, 3.9, 4.8, 6.6, 2.9, 1.9, 10.6, 9.7, 10.8, 3.6, 4.8, 2.7, 2.1, 1.7, 1.9, 11.9, 6.7, 5.4, 5.1, 1.6, 1.8 [class interval width = 2]

**d** 42, 79, 56, 49, 77, 50, 51, 46, 48, 72, 61, 78, 63, 45, 58, 53, 73, 58, 49, 61, 68, 67, 43, 49, 75, 77, 58, 54, 67, 72, 51, 56, 53, 48, 76, 78, 72, 42, 48, 53 [class interval width = 5]

8B.3 **5** Use each frequency table to draw a histogram.

**a**

| **Class** | 5–<10 | 10–<15 | 15–<20 | 20–<25 | 25–<30 | 30–<35 | 35–<40 | 40–<45 | 45–<50 |
|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | 8 | 6 | 7 | 2 | 3 | 1 | 5 | 6 | 9 |

**b**

| **Class** | 0–<20 | 20–<40 | 40–<60 | 60–<80 | 80–<100 | 100–<120 |
|---|---|---|---|---|---|---|
| **Frequency** | 14 | 21 | 18 | 13 | 8 | 2 |

**c**

| **Class** | 0–<10 | 10–<20 | 20–<30 | 30–<40 | 40–<50 | 50–<60 | 60–<70 |
|---|---|---|---|---|---|---|---|
| **Frequency** | 5 | 8 | 12 | 3 | 11 | 9 | 6 |

**6** Use each data set to draw an appropriate histogram with the given class interval widths.

**a** 13, 46, 13, 17, 35, 9, 22, 15, 8, 2, 35, 42, 42, 17, 16, 22, 29, 31, 47, 29, 13, 20, 36, 47, 28, 23, 30, 38 [class interval width = 10]

**b** 18.1, 24.5, 32.1, 15.6, 22.5, 29.1, 34.6, 16.7, 19.4, 17.5, 21.8, 27.5, 29.2, 30.1, 20.0, 33.1, 32.8, 31.9, 33.8, 14.3 [class interval width = 5]

**c** 64, 18, 120, 7, 29, 40, 145, 38, 72, 38, 18, 29, 2, 56, 49, 87, 99, 104, 59, 5, 29, 112, 118, 34, 59, 29, 19, 13 [class interval width = 20]

**d** 125, 726, 632, 465, 428, 257, 283, 399, 619, 402, 132, 196, 183, 743, 120, 703, 336, 652, 349, 402, 560, 144, 759, 717, 588, 185, 464, 685, 268, 352, 310, 408, 114, 782, 189 [class interval width = 100]

**e** 1.25, 1.89, 1.09, 1.76, 1.15, 1.36, 1.55, 1.67, 1.99, 1.32, 1.08, 1.14, 1.17, 1.62, 1.88, 4.9, 1.68, 1.49, 1.08, 1.16, 1.24, 1.19, 1.26, 1.83, 1.52, 1.18, 1.07, 1.42, 1.01, 1.19 [class interval width = 0.2]

**f** 25, 58, 48, 33, 26, 53, 42, 49, 58, 53, 46, 24, 58, 53, 46, 41, 38, 47, 44, 58, 53, 57, 39, 21, 48, 46, 42, 58, 52, 43, 42, 37, 36, 27, 46, 42, 49, 53, 57, 59 [class interval width = 5]

**7** Data was collected on the time spent listening to music in hours per week, as shown below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 10 | 2 | 4 | 24 | 3 | 7 | 9 | 5 | 15 |
| 17 | 16 | 19 | 20 | 5 | 3.5 | 5 | 10 | 14 | 7 |
| 7 | 5 | 9 | 10 | 17 | 7 | 4 | 10 | 11 | 16 |
| 4 | 5.5 | 12 | 14 | 6 | 7 | 12 | 14 | 16 | 3 |

**a** Create a frequency table with class intervals of width 5 to collate the data.

**b** Draw a histogram to represent this data.

**8** Data was collected on the weights (in kg) of various dogs, as shown below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11.2 | 14.8 | 6.4 | 19.7 | 22.1 | 4.8 | 5.4 | 12.6 | 18.2 | 9.2 |
| 13.1 | 20.1 | 8.6 | 4.9 | 8.4 | 17 | 6.2 | 14.3 | 26.3 | 16.6 |
| 18.6 | 19.2 | 14.5 | 28.3 | 16.2 | 9 | 11.4 | 14.9 | 11.3 | 13.2 |

**a** Fill in the following frequency table.

| Class | Frequency |
|---|---|
| 0–<5 | |
| 5–<10 | |
| 10–<15 | |
| 15–<20 | |
| 20–<25 | |
| 25–<30 | |

**b** Based on your frequency table, draw a histogram to represent this data.

**9** Data was collected on the ages of people in a train carriage one day.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 16 | 22 | 19 | 28 | 25 | 14 | 18 | 19 | 21 |
| 32 | 27 | 22 | 14 | 15 | 15 | 23 | 22 | 17 |
| 14 | 18 | 26 | 24 | 39 | 31 | 20 | 15 | 13 |
| 16 | 16 | 24 | 29 | 30 | 19 | 20 | 21 | 21 |

**a** Arrange this data in a frequency table with class intervals of width 5.

**b** Draw a histogram to represent this data.

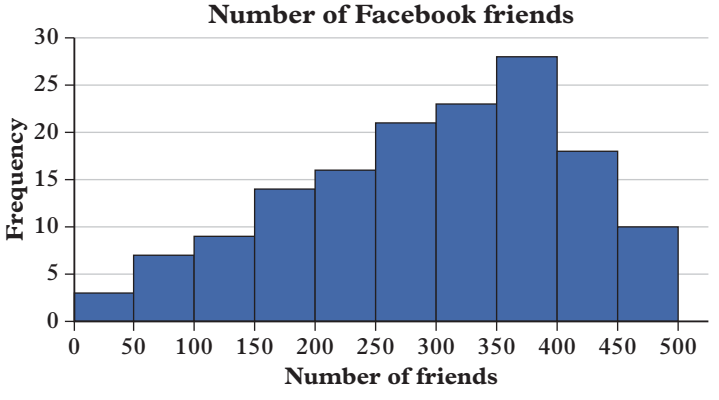
**10** Consider the following histogram.

**a** What does it show?

**b** What is the size of its class intervals?

**c** State the most common class and its frequency.

**d** If you were to add a piece of data with a value of 140 to the histogram, to which class interval would you add it?



UNDERSTANDING AND FLUENCY

**11** Data collected on the ages of customers in a clothes store in a day is shown in the table below.

**a** Why is this table difficult to read?

**b** Redraw the table with class intervals of width 10 so that it is easier to read.

| Class | Frequency |
|---|---|
| <10 | 2 |
| 10–<12 | 6 |
| 12–<14 | 12 |
| 14–<16 | 16 |
| 16–<18 | 14 |
| 18–<20 | 13 |
| 20–<22 | 11 |
| 22–<24 | 9 |
| 24–<26 | 10 |
| 26–<28 | 8 |
| 28–<30 | 6 |

| Class | Frequency |
|---|---|
| 30–<32 | 4 |
| 32–<34 | 6 |
| 34–<36 | 5 |
| 36–<38 | 3 |
| 38–<40 | 2 |
| 40–<42 | 3 |
| 42–<44 | 1 |
| 44–<46 | 1 |
| 46–<48 | 0 |
| 48–<50 | 1 |
| ≥50 | 6 |

PROBLEM SOLVING AND REASONING

**12** Explain why you can't accurately decrease the size of the class intervals in this table.

| Class | Frequency |
|---|---|
| 0–<20 | 16 |
| 20–<40 | 48 |
| 40–<60 | 42 |

**13** The following ordered data shows the number of kilometres travelled by different people to attend a conference.

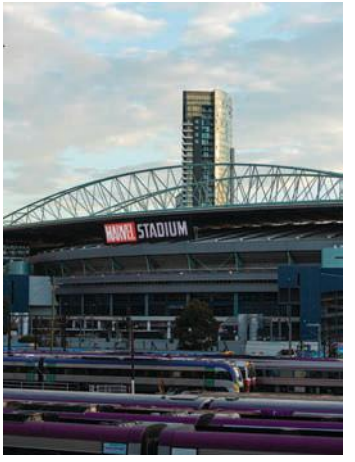
4, 4, 11, 12, 12, 13, 19, 19, 20, 21, 21, 25, 28, 31, 33, 34, 42, 44, 52, 55, 57, 58

**a** Create a histogram with class intervals of width 20 to represent this data.

**b** Create a histogram with class intervals of width 10 to represent this data.

**c** Create a histogram with class intervals of width 5 to represent this data.

**d** Which of the three histograms do you think displays the data most appropriately? Explain your answer.




**14** This data was collected on how many thousands of people were present at AFL matches at Marvel Stadium, Melbourne.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 51 | 35 | 22 | 25 | 46 | 34 | 21 | 8 | 28 | 11 | 19 |
| 22 | 31 | 38 | 42 | 22 | 22 | 48 | 30 | 25 | 26 | 15 | 21 |
| 18 | 18 | 41 | 39 | 33 | 33 | 38 | 25 | 28 | 10 | 17 | 13 |

**a** Create a histogram with class intervals of width 5 to represent this data.

**b** Create a histogram with class intervals of width 10 to represent this data.

**c** What is similar and what is different about your graphs for parts **a** and **b**?

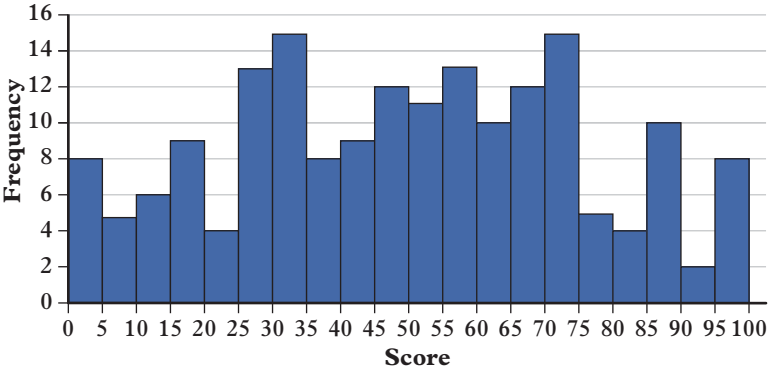
**15** Consider the histogram on the right.

**a** What is the width of the class intervals?

**b** What can you say about the number of classes? Is it easy to read? Can you see a pattern?

**c** Redraw the histogram with class intervals of width 10.

**d** What patterns can you see now?



**16** Stem-and-leaf plots can be used to display discrete data, where each piece of data is split into two parts (a stem and a leaf). The key indicates the value of the data. Consider this stem-and-leaf plot.

| Stem | Leaf |
|---|---|
| 1 | 2 6 7 8 8 9 |
| 2 | 0 1 5 6 7 |
| 3 | 3 6 8 |
| 4 | 0 2 |
| 5 | 1 |

Key: 1 | 2 = 12

**a** Which part of the plot represents the class intervals?

**b** What is the size of the class intervals?

**c** How many people were surveyed?

**d** What advantage does a stem-and-leaf plot have over a histogram?

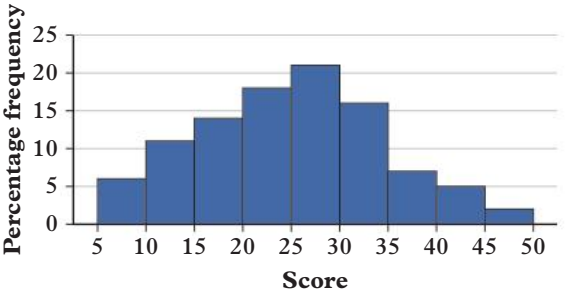
**17** Use the stem-and-leaf plot shown to draw a histogram.

| Stem | Leaf |
|---|---|
| 1 | 0 0 1 1 3 4 |
| 1* | 5 5 5 6 6 7 9 |
| 2 | 0 1 2 2 3 3 4 4 6 |
| 2* | 5 6 6 7 8 8 9 |
| 3 | 0 1 1 2 2 3 4 |
| 3* | 5 6 7 8 |

Key: 1 | 3 = 13

**18** Explain why you can't accurately draw a stem-and-leaf plot from a histogram.

**19** Consider this percentage frequency histogram.




**a** How is it different from a normal histogram?

**b** What percentage of scores are between 30 and 35?

**c** What percentage of scores are greater than 35?

**d** Without performing a calculation, state the sum of the percentage frequency columns. Explain how you know.

**e** If there were 400 scores in total, calculate:

**i** the number of scores between 15 and 20

**ii** the number of scores less than 25

**iii** the number of scores between 20 and 40.

**20** Create a percentage frequency histogram with class intervals of width 0.2 to represent this data set.

Weights of newborn babies at a particular hospital in one week (in kg):

3.25, 4.15, 2.75, 3.60, 3.95, 3.05, 2.85, 4.20, 1.95, 3.50, 3.65, 3.15, 3.70, 3.95, 4.10, 4.85, 2.90, 3.10, 3.30, 3.25, 3.50, 4.05, 3.45, 3.85, 3.75, 3.15, 3.45, 3.20, 3.25, 4.25, 2.55, 2.95, 3.40, 3.85, 3.80, 3.55, 3.20, 3.00, 3.20, 3.75, 4.00, 4.15, 3.80, 3.75, 3.40, 3.25, 3.15, 3.05, 3.85, 2.95

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Histograms

Investigation Pocket money

CAS instructions Histograms

Topic quiz 8B

# 8C Summary statistics from tables and displays

## Learning intentions

- ✔ I can calculate summary statistics from frequency tables.
- ✔ I can calculate summary statistics from displays.
- ✔ I can calculate summary statistics of grouped data.

**Inter-year links**

| | |
|---|---|
| Year 5/6 | Understanding data |
| Year 7 | 10B Summary statistics |
| Year 8 | 9B Summary statistics |
| Year 10 | 10A Five-number summary and interquartile range |

## Measures of centre and spread

- The **mean**, **median** and **mode** are **measures of centre**, as they each represent a central point in a data set.

  $$\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$

  Median = middle value when data is listed in order (divides a data set in half)

  Mode = most common value (there can be more than one mode)
- The **range** is a **measure of spread**, as it gives an indication of the spread of the data.

  Range = maximum value – minimum value

  For example: The numbers of humpback whales passing the Portland coast each week during a period of the migration season were counted. The results are shown below.

  2, 13, 16, 28, 35, 35, 46, 29, 16, 24

  $$\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$
  $$= \frac{2 + 13 + 16 + 28 + 35 + 35 + 46 + 29 + 16 + 24}{10}$$
  $$= \frac{244}{10}$$
  $$= 24.4$$

  List data in order: 2, 13, 16, 16, 24, 28, 29, 35, 35, 46

  The number of data values is even, so the median is the mean of the two middle values.

  $$\text{Median} = \frac{24 + 28}{2}$$
  $$= 26$$

  The highest frequency is 2 for both values 16 and 35.

  Modes = 16 and 35

  Range = 46 – 2
  = 44

## Calculating summary statistics from data displays

- To calculate the mean from a frequency table, insert a column multiplying each individual score by its frequency, and divide the total of this column by the total of the frequencies.
- To calculate the median from a frequency table, insert a column for the **cumulative frequency** (a running total of the frequencies) to help identify where the middle value lies.
- To calculate summary statistics from a display that details individual scores (e.g. stem-and-leaf plots, dot plots or column graphs) the data can be treated as either a raw list or as a frequency table.

## Example 8C.1 Summary statistics from a frequency table

For the data shown in the following frequency table, find the:

**a** mean **b** median **c** mode **d** range

| Score ($x$) | Frequency ($f$) |
|---|---|
| 1 | 4 |
| 2 | 7 |
| 3 | 8 |
| 4 | 8 |
| 5 | 2 |
| 6 | 1 |

**THINK**

**a** **1** Insert a 'Score × frequency' column and a 'Total' row and then complete the table. The total of the 'Frequency' column is the number of data values in the table.

**2** To find the mean, divide 'Score × frequency' total by the 'Frequency' total.

**b** **1** Insert a 'Cumulative frequency' column in the table. Check that the uppermost number matches the number of data values.

**2** Divide the number of data values by 2 to find out how many values lie below the median. Then identify the row that contains the median score by using the cumulative frequency column.

**c** Identify the score with the highest frequency.

**d** Identify the minimum and maximum values in the score column of the data set. Subtract the minimum value from the maximum value.

**WRITE**

| Score ($x$) | Frequency ($f$) | Score × frequency ($x \times f$) | Cumulative frequency |
|---|---|---|---|
| 1 | 4 | $1 \times 4 = 4$ | 4 |
| 2 | 7 | $2 \times 7 = 14$ | $4 + 7 = 11$ |
| 3 | 8 | $3 \times 8 = 24$ | $11 + 8 = 19$ |
| 4 | 8 | $4 \times 8 = 32$ | $19 + 8 = 27$ |
| 5 | 2 | $5 \times 2 = 10$ | $27 + 2 = 29$ |
| 6 | 1 | $6 \times 1 = 6$ | $29 + 1 = 30$ |
| **Total** | **30** | **90** | |

**a** $\text{Mean} = \frac{90}{30}$

$= 3$

**b** There are 30 values in the data set. $\frac{30}{2} = 15$ so 15 data values lie below the median.

The number of data values is even so the median is between the 15th and 16th value.

The 12th to 19th scores are 3, so both the 15th and 16th score are 3.

Median = 3

**c** The highest frequency is 8 for both scores 3 and 4.

Modes = 3, 4

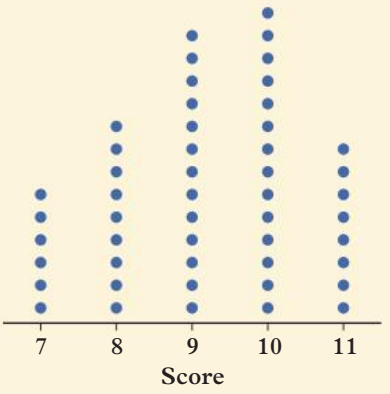
**d** Range = 6 − 1

= 5

## **Example 8C.2** Finding summary statistics from a dot plot

For the data shown in the following dot plot, find the:

**a** mean **b** median **c** mode **d** range




**THINK**

**a** **1** Create a frequency table by counting the number of dots in each column.

**2** Insert a 'Total' row, a 'Score × frequency' column and a 'Cumulative frequency' column in the table.

**3** Find the mean by dividing the 'Score × frequency' total by the 'Frequency' total.

**b** Divide the number of values by 2 to find out how many values lie below the median.

**c** Find the mode by identifying the score with the highest frequency.

**d** Find the range by subtracting the minimum score from the maximum score.

**WRITE**

| Score ($x$) | Frequency ($f$) | Score × frequency ($x \times f$) | Cumulative frequency |
|---|---|---|---|
| 7 | 6 | 42 | 6 |
| 8 | 9 | 72 | 15 |
| 9 | 13 | 117 | 28 |
| 10 | 14 | 140 | 42 |
| 11 | 8 | 88 | 50 |
| **Total** | **50** | **459** | |

**a** Mean $= \frac{459}{50}$

$= 9.18$

**b** $\frac{50}{2} = 25$ so 25 data values lie below the mean.

The number of data values is even so the median is between the 25th and 26th score.

The 16th to 28th scores are 9, so both the 25th and 26th score are 9.

Median = 9

**c** The highest frequency is 14 for the score 10.

Mode = 10

**d** Range = 11 − 7

$= 4$

## Example 8C.3 Finding summary statistics from a stem-and-leaf plot

For the data shown in the following stem-and-leaf plot, find the:

**a** mean **b** median **c** mode **d** range

| Stem | Leaf |
|---|---|
| 1 | 5 9 |
| 2 | 3 7 8 8 9 |
| 3 | 0 1 2 2 6 9 9 |
| 4 | 0 2 3 3 6 6 6 7 8 |
| 5 | 1 5 |

Key 2|3 = 23

**THINK**

1. Find the mean by dividing the sum of scores by the number of scores.
2. Divide the number of values by 2 to find out how many values lie below the median.
3. Find the mode by identifying the most common scores.
4. Find the range by calculating the difference between the minimum score and the maximum score.

**WRITE**

**a** Mean $= \frac{915}{25}$
$= 36.6$

**b** $\frac{25}{2} = 12.5$ so 12 data values lie below the mean.
The number of data values is odd so the median is the 13th score, which is 39.
Median = 39

**c** The highest frequency is 3 for the score 46.
Mode = 46

**d** Range = 55 − 15
= 40

**Helpful hints**

- ✔ When calculating summary statistics from a stem-and-leaf plot make sure that you use the full data values by combining the stem and leaf and using the key.
- ✔ Remember that if the median falls between two different values, the median will be the average (mean) of these values.
- ✔ A data set can have more than one mode.

ANS p543

# Exercise 8C Summary statistics from tables and displays

1(a, c, e), 2, 4, 5(a, b), 6(a, b), 9, 11(a, c, d, e), 13(a–c)

1(b, d, f), 3, 5, 6(b, c), 7, 10, 13–15

1(d, f), 3, 5–6(b, c), 10, 12–14, 16

UNDERSTANDING AND FLUENCY

8C.1 **1** For the data shown in each frequency table, find the **i** mean, **ii** median, **iii** mode and **iv** range. Give your answers to two decimal places where necessary.

**a**

| Score ($x$) | Frequency ($f$) |
|---|---|
| 1 | 6 |
| 2 | 7 |
| 3 | 5 |
| 4 | 3 |
| 5 | 1 |

**b**

| Score ($x$) | Frequency ($f$) |
|---|---|
| 10 | 8 |
| 20 | 6 |
| 30 | 8 |
| 40 | 2 |

**c**

| Score ($x$) | Frequency ($f$) |
|---|---|
| 13 | 3 |
| 14 | 4 |
| 15 | 8 |
| 16 | 11 |
| 17 | 12 |
| 18 | 4 |

**d**

| Score ($x$) | Frequency ($f$) |
|---|---|
| 0 | 11 |
| 1 | 13 |
| 2 | 6 |
| 3 | 3 |
| 4 | 1 |

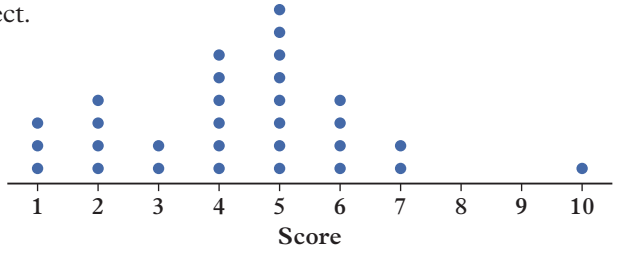
**e**

| Score ($x$) | Frequency ($f$) |
|---|---|
| 15 | 29 |
| 20 | 41 |
| 25 | 58 |
| 30 | 72 |

**f**

| Score ($x$) | Frequency ($f$) |
|---|---|
| 1 | 6 |
| 2 | 11 |
| 3 | 9 |
| 4 | 4 |
| 5 | 3 |
| 19 | 1 |

8C.2 **2** This dot plot shows the time in hours to complete a project.



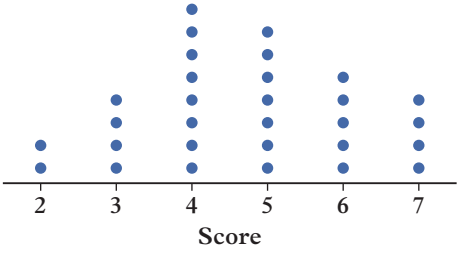

**a** How many people were surveyed?

**b** State the mode and range.

**c** Create a frequency table.

**d** Use the frequency table to calculate the median and mean of project completion time.

**3** Find the mean, median, mode and range for the data displayed in this dot plot.



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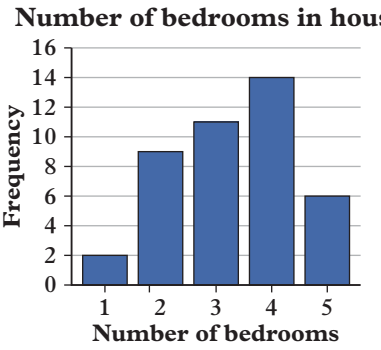
**4** This column graph shows the number of bedrooms in a survey of houses.



**a** What is the most common number of bedrooms in the houses surveyed?

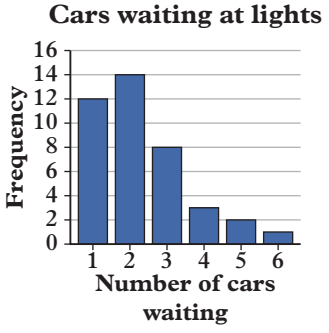
**b** What is the range of the number of bedrooms?

**c** Create a frequency table to represent the data shown in the column graph.

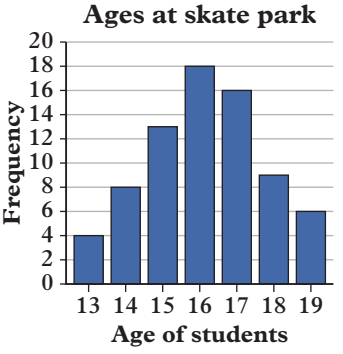
**d** Use the frequency table to calculate the median and the mean number of bedrooms. Give your answers to two decimal places where necessary.

**5** Find the mean, median, mode and range for the data shown in each column graph. Give your answers to two decimal places where necessary.

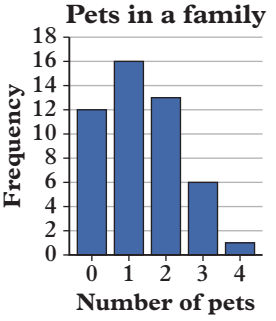
**a**




**b**




**c**




8C.3 **6** Find the mean, median, mode and range for the data displayed in each stem-and-leaf plot. Give your answers to two decimal places where necessary.

**a**

| Stem | Leaf |
|---|---|
| 3 | 2 4 7 |
| 4 | 2 2 2 6 9 |
| 5 | 3 6 8 9 9 |
| 6 | 1 1 4 6 7 7 8 8 |
| 7 | 0 4 |

Key 3 | 2 = 32

**b**

| Stem | Leaf |
|---|---|
| 1 | 2 3 8 8 8 9 |
| 2 | 0 0 1 2 3 6 7 8 |
| 3 | 2 3 7 9 |
| 4 | 4 6 |
| 5 | 2 |

Key 1 | 3 = 1.3

**c**

| Stem | Leaf |
|---|---|
| 1 | 0 1 2 4 5 6 7 7 9 |
| 2 | 2 3 4 4 6 8 8 |
| 3 | 0 1 1 1 3 |
| 4 | 3 7 9 |
| 5 | 1 |
| 6 | |
| 7 | 7 |

Key 1 | 2 = 120

**7** A number of people were surveyed on how many pairs of shoes they bought in a year. Use the results shown in the table to find the mean, median, mode and range. Give your answers to two decimal places where necessary.

| Score ($x$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency ($f$) | 9 | 13 | 21 | 18 | 13 | 11 | 7 | 4 | 1 | 3 |

**8** Explain why the only summary statistic that you can find for categorical data is the mode.

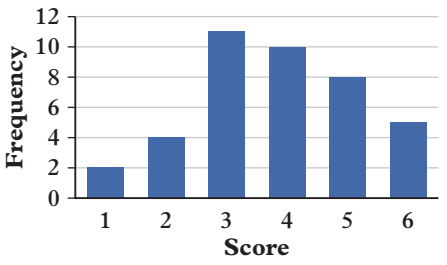
**9** Match these summary statistics with these graphs.

**a** mean = 3.3, median = 3, mode = 3, range = 5

**b** mean = 3.825, median = 4, mode = 3, range = 5

**c** mean = 40.74, median = 36.5, mode = 74, range = 97

**A**




**B**

**C**

| Stem | Leaf |
|---|---|
| 0 | 1 2 6 |
| 1 | 2 6 7 7 8 9 |
| 2 | 0 1 1 2 4 6 7 7 8 9 |
| 3 | 0 3 4 4 5 6 7 9 |
| 4 | 0 1 2 2 5 9 |
| 5 | 0 1 2 3 5 |
| 6 | 0 1 4 8 9 |
| 7 | 4 4 4 4 |
| 8 | 1 9 |
| 9 | 8 |

Key 2 | 1 = 21

**10** Draw a dot plot on a number line as shown below using a different data set consisting of exactly 9 numbers for each of the following scenarios. In each case, write a sentence explaining your reasoning for placing the dots in the positions you chose.

**a** The minimum value is 2 and the range is 7.

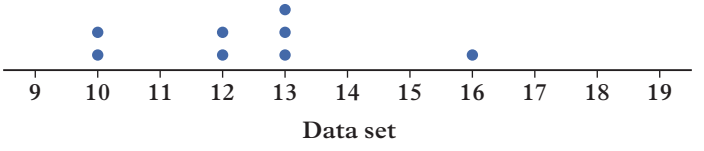
**b** The median and the maximum value are both 6.

**c** The mode includes four different numbers.

**11** The following dot plot is missing one dot.

Find a possible value of the missing dot for each of the following scenarios.

In each case, state whether your answer is unique or if it is one answer out of several that are possible.




**a** The maximum value is 17.

**b** The mean is 12.

**c** The median is 13.

**d** The modes are 10 and 13.

**e** The range is 7.

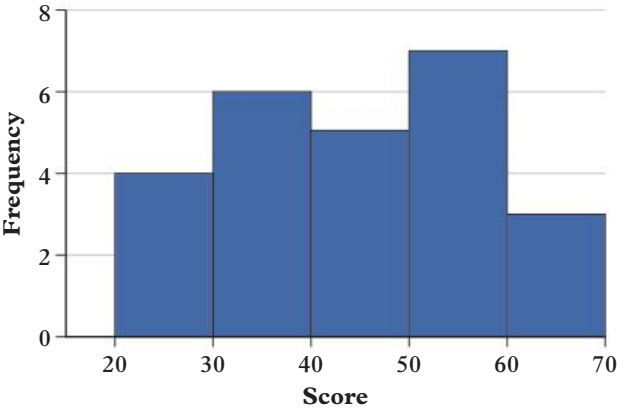
**f** The mean is a whole number.

**12** A large data set contains 457 data points that are listed in a spreadsheet. The numbers are ordered from smallest to largest and are listed in rows of 30. This means the number in column 1 of row 1 is the smallest number in the set, the number in column 1 of row 2 is the 31st smallest number in the set, and so on.

**a** Find the row number and column number of the greatest number in the data set.

**b** Find the row number and column number of the median of the data set.

**13** A researcher has collected discrete data and arranged it into the following histogram. The data is discrete, so the first interval is 20–29, the second interval is 30–39, etc. The raw data (i.e. the list of numbers that make up the data set) is not available.




- **a** What is the maximum possible value of the range? Explain your reasoning.
- **b** What is the minimum possible value of the range? Explain your reasoning.
- **c** Explain why it is not possible to find the median.
- **d** In which interval is the median contained?
- **e** Explain why it is not possible to calculate the mean or even to solve for the interval in which it is contained.

**14** For the histogram given in question **13**, it is not possible to calculate the exact value of the mean. However, it is possible to find the minimum and maximum possible values of the mean.

- **a** Find the minimum possible value of the mean by assuming every data point occurs at the lowest possible value of its interval.
- **b** Find the maximum possible value of the mean by assuming every data point occurs at the highest possible value of its interval.
- **c** Find the average value of the minimum possible mean and the maximum possible mean. Is this a good way to estimate the mean for data presented in a histogram? Explain your answer.

**15** Elana has completed four Maths tests this year with an average score of 78%.

- **a** What is the maximum value her average could increase to after completing the next test?
- **b** What is the minimum value her average could drop to after completing the next test?
- **c** What percentage score must she achieve on her next test to increase her average to 80%?

**16** Another measure of spread is the standard deviation. It measures the average spread of each data value from the mean. A small standard deviation means that most values are close to the mean and a large standard deviation means that the values are spread far from the mean. Standard deviation is best found using a calculator with the appropriate function. It can also be found (for a sample) using the formula:

$$s = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n - 1}}$$

where $s$ represents standard deviation, $\Sigma$ means 'the sum of', $\bar{x}$ represents the mean, $x$ represents an individual score and $n$ represents the number of scores.

- **a** Use your calculator or the standard deviation formula to find the standard deviation for each data set, correct to two decimal places.
  - **i** 4, 8, 2, 6, 4, 9, 7, 7, 4, 1, 2, 4, 6, 9, 7, 4, 6, 7, 8, 1, 2, 3, 1, 6, 2, 6
  - **ii** 22, 27, 35, 64, 12, 74, 37, 93, 27, 33, 11, 71, 64, 42, 81, 37, 13, 19, 88, 50
  - **iii** 103, 118, 109, 111, 117, 116, 117, 117, 105, 107, 116, 113, 108, 109, 112, 113
- **b** Use your results to state whether each data set from part **a** has a large or small spread from the mean.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Summary statistics from frequency tables
- Interactive skillsheet: Summary statistics from displays
- Investigation: Estimating population means and medians
- CAS Instructions: Measures of centre
- Topic quiz: 8C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

8A **1** Classify the following variables as numerical or categorical.

**a** Time spent watching television

**b** Most used smartphone app

**c** Programming skill level (Beginner, Intermediate, Advanced)

**d** Number of members in a chess club

8A **2** Consider the following table of data. Draw a line graph to represent this data.

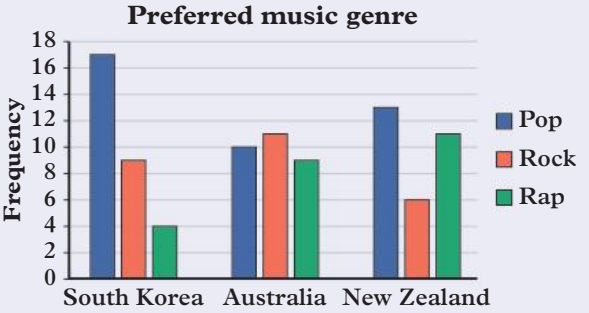
| Hours since midnight | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature (°C) | 8 | 9 | 9 | 11 | 10 | 12 | 13 | 15 | 17 | 18 | 17 | 20 | 22 |

8A **3** Thirty teenagers from three different countries were asked to choose their preferred music genre. The results are shown in the side-by-side column graph on the right.

**a** Which music genre was most preferred by South Koreans?

**b** The teenagers of which country preferred rap to rock?

**c** What was the most popular music genre, overall?

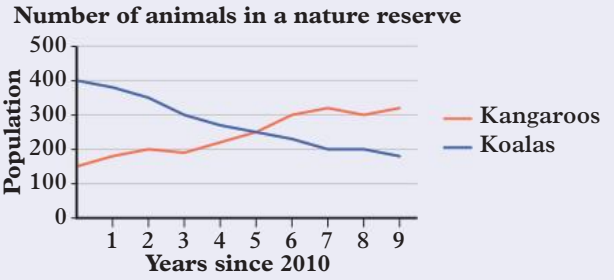



8A **4** Consider the multiple line graph on the right, which shows the number of two different animals in a nature reserve since 2010.

**a** The population of which animal in the reserve is increasing?

**b** During which year was the population of each animal the same?

**c** Between which two years did the population of koalas remain steady?

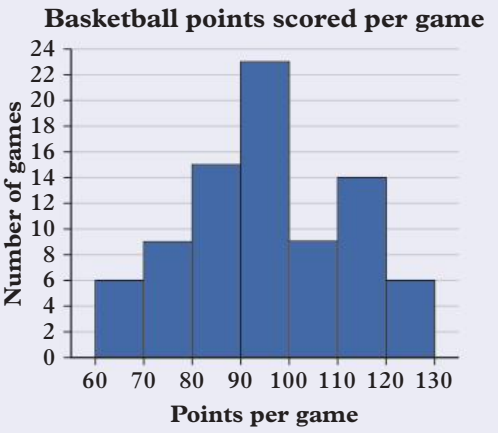



8B **5** Teams in the NBA play 82 games of basketball in a regular season. The number of points scored per game for a certain team is displayed in the histogram on the right.

**a** Which interval contains the most values?

**b** In how many games did the team score at least 120 points?

**c** In how many games did the team score less than 80 points?




8B **6** A researcher asked 24 people how many minutes they spend in the shower on average. The results are shown below.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 12 | 14 | 8 | 6 | 4 | 4 | 9 | 8 | 7 | 6 | 11 |
| 16 | 18 | 21 | 3 | 4 | 9 | 8 | 10 | 12 | 13 | 14 | 20 |

**a** Draw a frequency table with class intervals of 5 to represent this data.

**b** Use the frequency table to draw a histogram for this data.



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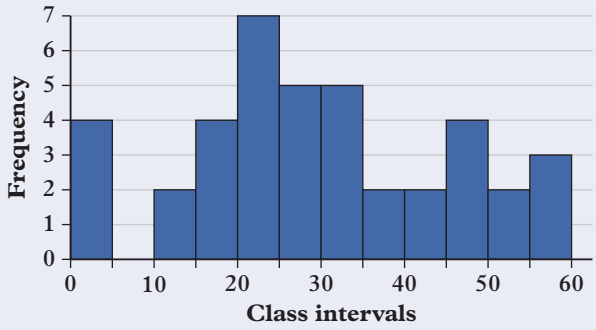
**8B** **7** Redraw the following histogram with class intervals of 10, starting at 0.




**8C** **8** A researcher asked 40 people how many hours they slept last night, rounded to the nearest hour.
The results are shown in the frequency table on the right.

**a** Find the median number of hours slept.

**b** Find the mean number of hours slept.

**c** How many people slept for at least 8 hours?

| Score ($x$) | Frequency ($f$) |
|---|---|
| 5 | 3 |
| 6 | 7 |
| 7 | 14 |
| 8 | 11 |
| 9 | 4 |
| 11 | 1 |

**8C** **9** Consider the stem-and-leaf plot on the right.

**a** Find the median of the data set.

**b** Find the mode of the data set.

**c** Find the range of the data set.

| Stem | Leaf |
|---|---|
| 7 | 5 7 |
| 8 | 2 2 2 8 9 |
| 9 | 3 4 5 |
| 10 | 1 2 3 6 7 7 8 |
| 11 | 0 1 3 6 |

Key 8|9 = 89

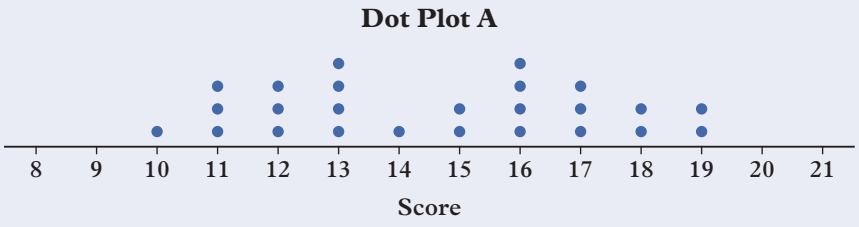
**8C** **10** Consider the following two dot plots.
Which dot plot has:

**a** a range of 7?

**b** two different modes?

**c** a median of 15?

**d** the greater mean?



# 8D Describing data

## Learning intentions

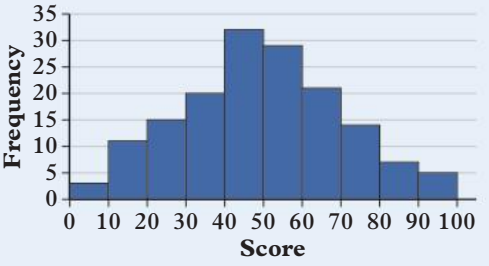
- ✔ I can identify whether a data set is symmetric, skewed or bimodal.
- ✔ I can determine which measure of centre best describes a data set.

**Inter-year links**

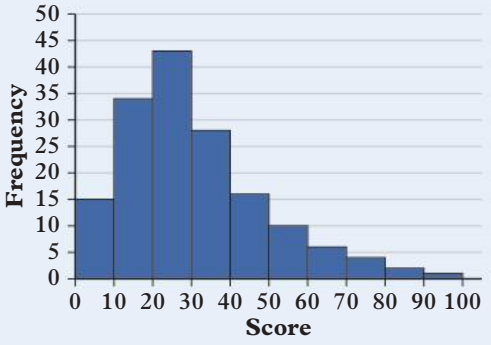
| | |
|---|---|
| Year 5/6 | Understanding data |
| Year 7 | 10B Summary statistics |
| Year 8 | 9D Analysing data |
| Year 10 | 10C Distributions of data |

## Describing data

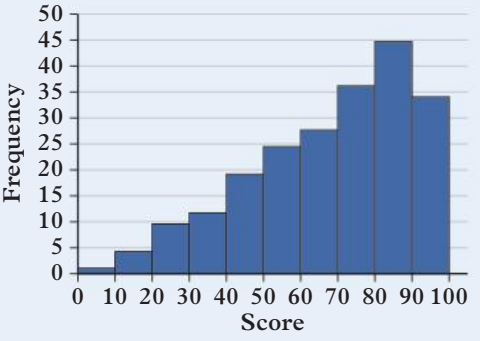
- By displaying numerical data, you can identify the shape of the distribution of the data and use this to describe data sets. **Symmetric** distributions have a peak in the middle and are roughly evenly spread on either side.



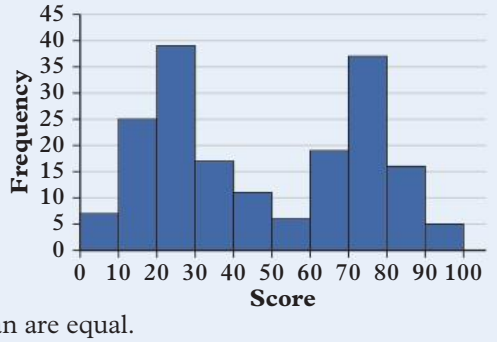

- A distribution is **skewed** when the data points are clustered on one side of the distribution. There is a tail on the other side of the distribution.
  - → **Positively skewed** distributions are skewed towards the vertical axis with a tail on the right of the distribution.




  - → **Negatively skewed** distributions are skewed away from the vertical axis with a tail on the left of the distribution.




- A distribution is **bimodal** if there are two clear peaks in the data.




- The mean, median and mode can all be used to represent a central point of a data set, but the shape of the distribution should be considered before deciding which measure of centre to use for this purpose.
- When describing symmetric distributions, either the mean or the median are appropriate representations of the centre. In perfect symmetric distributions, the mean, mode and median are equal.
- When describing skewed distributions, the median is a better representation of the centre than the mean, as the mean is skewed by the values in the tail.
- Bimodal distributions are best described using the mode of each peak.
- An **outlier** is a data point that is significantly higher or lower than the other values in a data set.
  - → The presence of outliers can have a significant impact on the mean, so distributions with outliers are best described by using the median.
  - → Outliers can be determined using several different methods; in Year 10 you will be given a formula for determining whether a data point is an outlier. At this stage, only clear outliers will be considered.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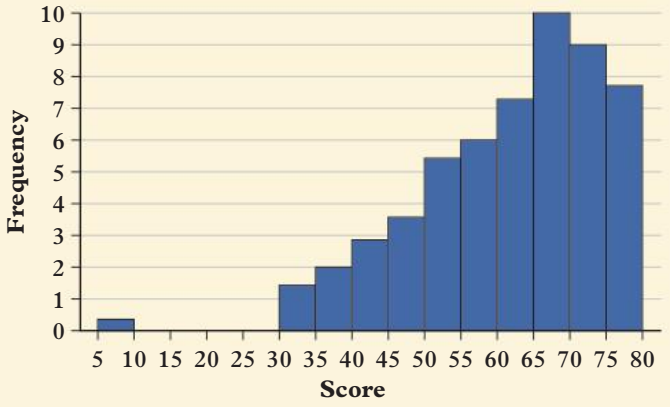
## **Example 8D.1** Describing distributions

Describe the distribution of this histogram.




**THINK**

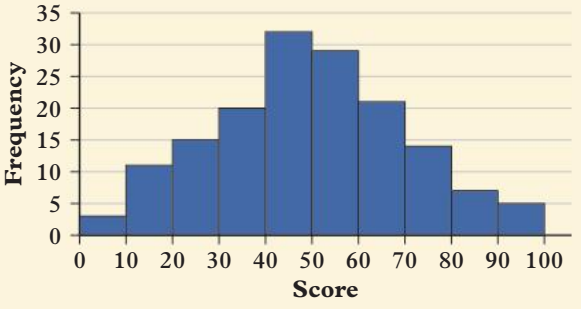
1. Identify whether the distribution is symmetric, skewed or bimodal.
2. Identify any outliers.
3. Write your answer.

**WRITE**

The distribution is negatively skewed with an outlier between 5 and 10.

## **Example 8D.2** Deciding which measure of centre best describes a distribution

Decide which measure(s) of centre would best describe this distribution.




**THINK**

1. Identify whether the distribution is symmetric, skewed or bimodal.
2. Identify how the shape of the distribution determines which measure of centre should be used to describe the distribution.
3. Write your answer.

**WRITE**

The distribution is symmetric so the centre of the distribution could be described by the mean or the median.

## Example 8D.3 Describing a histogram

Use this data to:

**a** draw a histogram with class intervals of width 10

**b** describe its distribution

**c** state the best measure of centre to describe the data, providing a reason.

49, 3, 16, 12, 20, 49, 22, 37, 32, 18, 34, 13, 4, 7, 17, 9, 13, 59, 25, 1, 15, 30, 23, 27, 4, 17, 26, 3, 10, 5, 8, 2, 31, 11, 8, 20, 27, 13, 16, 11

**THINK**

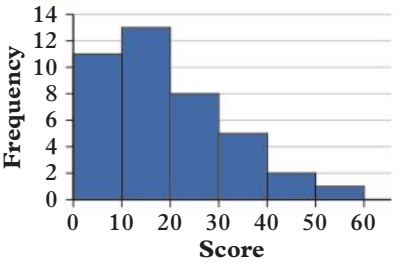
**a** Use a frequency table to draw a histogram with an even scale and labels on both axes.

**b** Decide whether the data is skewed.

**c** Identify the best measure of centre to describe the data. The data is positively skewed, not symmetric or bimodal, so the median should be used.

**WRITE**

**a**




**b** The distribution is positively skewed.

**c** The median should be used to describe the centre, because the distribution is skewed, which affects the value of the mean.

**Helpful hints**

✔ The peaks in a bimodal distribution do not need to be of equal height.

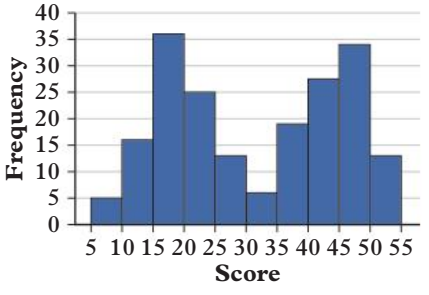
ANS p544

# Exercise 8D Describing data

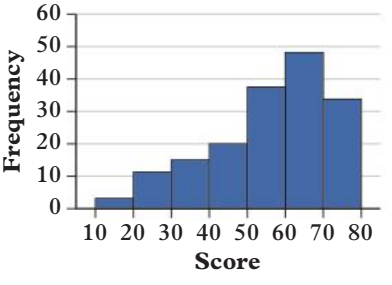
△ 1–3, 5(a, b), 6, 8, 9 □ 1, 2, 4, 5(b, c), 6, 9–11, 12(a, b) ⬠ 2, 3, 5(b, c), 7, 9, 11–13

8D.1 **1** Describe the distribution of each histogram.

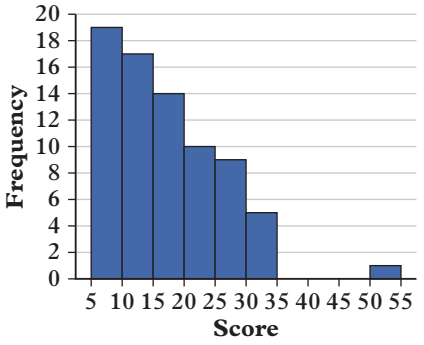
**a**



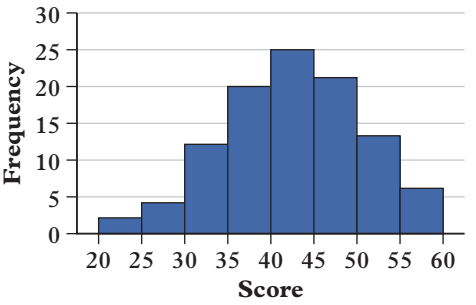

**b**



**c**




**d**




**2** Describe the distribution of each stem-and-leaf plot.

**a**

| Stem | Leaf |
|---|---|
| 1 | 0 2 |
| 2 | 1 1 4 |
| 3 | 0 4 5 6 |
| 4 | 4 5 |
| 5 | 3 |

Key 1 | 2 = 12

**b**

| Stem | Leaf |
|---|---|
| 0 | 1 4 6 8 |
| 1 | 3 4 5 5 6 |
| 2 | 1 2 4 4 |
| 3 | 4 8 9 |
| 4 | 3 |
| 5 | 1 |

Key 1 | 3 = 13

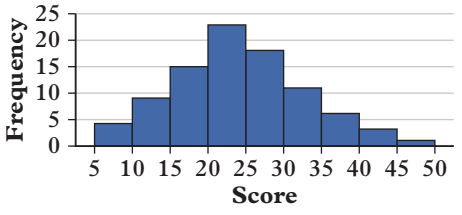
**c**

| Stem | Leaf |
|---|---|
| 0 | 9 |
| 1 | |
| 2 | |
| 3 | 1 9 |
| 4 | 4 5 6 |
| 5 | 4 5 5 6 |
| 6 | 0 1 1 2 4 |

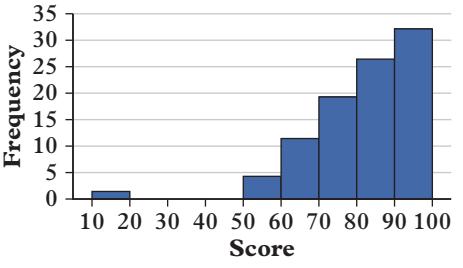
Key 3 | 9 = 39

8D.2 **3** Decide which measure(s) of centre would best describe each distribution.

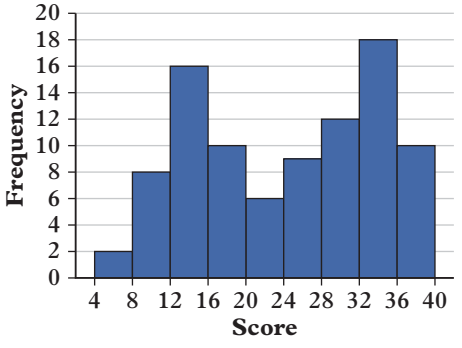
**a**



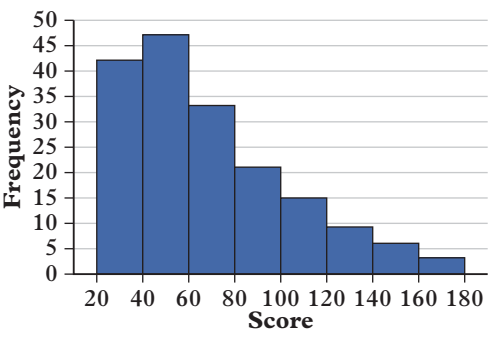

**b**




**c**




**d**




**4** Decide which measure(s) of centre would best describe each distribution in question **1**.

8D.3 **5** For each data set:

**i** draw a histogram with the given class interval width

**ii** describe its distribution

**iii** state the best measure of centre to describe the data, providing a reason.

**a** 35, 33, 42, 99, 54, 68, 4, 91, 97, 55, 99, 86, 40, 58, 41, 95, 38, 62, 35, 88, 82, 98, 77, 69, 78, 78, 82, 81, 98, 57, 88, 41, 60, 85, 82, 85, 91, 90, 80, 49, 58, 66, 97, 95, 82, 84, 78, 91, 62, 42 [width = 10]

**b** 9, 2, 3, 9, 43, 8, 2, 15, 12, 17, 10, 10, 14, 9, 34, 7, 12, 18, 18, 47, 2, 12, 24, 34, 19, 1, 12, 18, 35, 47, 6, 14, 8, 35, 7, 9, 4, 17, 2, 20, 8, 12, 21, 24, 48, 6, 7, 8, 17, 41 [width = 5]

**c** 49, 23, 45, 23, 31, 77, 62, 21, 52, 51, 60, 54, 46, 69, 27, 60, 142, 41, 32, 80, 52, 21, 80, 65, 37, 33, 74, 45, 48, 78, 70, 21, 55, 64, 33, 42, 59, 67, 32, 79, 30 [width = 10]

**6** This stem-and-leaf plot does not seem to have a noticeable pattern. However, it is possible to redraw this stem-and-leaf plot with more stems, just like histograms can be redrawn with different class sizes. For example, the number 1 can be the stem for two leaves, with the numbers 10–<15 listed in the first stem labelled 1 and the numbers 15–<20 listed in the second stem labelled 1⋆.

| Stem | Leaf |
|---|---|
| 1 | 0 0 0 1 2 2 2 2 2 3 3 5 5 5 5 5 6 6 7 8 9 |
| 2 | 0 0 1 2 2 2 3 3 3 9 |

Key 1 | 3 = 13

**a** Redraw it as a split stem-and-leaf plot with:

**i** four stems **ii** ten stems.

**b** Describe the distributions that you see from the split stem-and-leaf plots.

**7** The following tables show the data that Ava collected on the weight of dogs (in kilograms) in her community.

| Class | 0–<2 | 2–<4 | 4–<6 | 6–<8 | 8–<10 | 10–<12 | 12–<14 | 14–<16 | 16–<18 | 18–<20 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 1 | 6 | 3 | 5 | 6 | 5 | 8 | 6 | 7 | 3 |

| Class | 20–<22 | 22–<24 | 24–<26 | 26–<28 | 28–<30 | 30–<32 | 32–<34 | 34–<36 | 36–<38 | 38–<40 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 7 | 2 | 5 | 3 | 4 | 5 | 4 | 2 | 1 | 4 |

**a** Draw a histogram to represent the data.

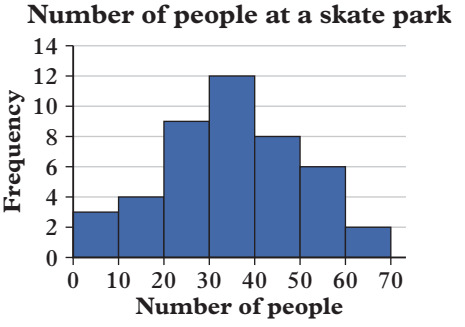
**b** What pattern can you see?

**c** Reorder the data into class intervals of width 4 and redraw your histogram.

**d** What pattern can you now see?

**e** Write a sentence describing the weight distribution of dogs as shown in your second histogram.

**8** The histogram on the right shows the number of people at a skate park recorded at various times.




**a** Describe the shape of the distribution.

**b** What measure(s) of centre would be the most appropriate to use?

**c** Create a frequency table that represents the histogram.

**d** Use the frequency table to determine in which class interval the median lies.

**e** Write a sentence that describes the distribution in terms of the number of people at the skate park.

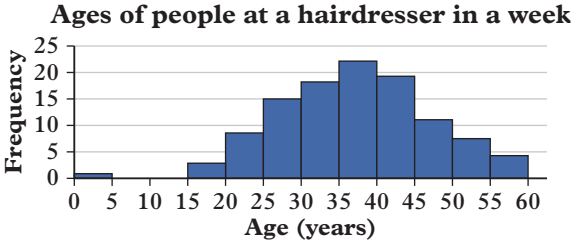
**9** For each data distribution:

**i** describe its shape

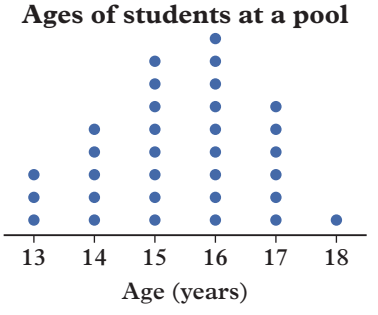
**ii** calculate the appropriate measure(s) of centre

**iii** write a sentence that summarises the graph in its context.

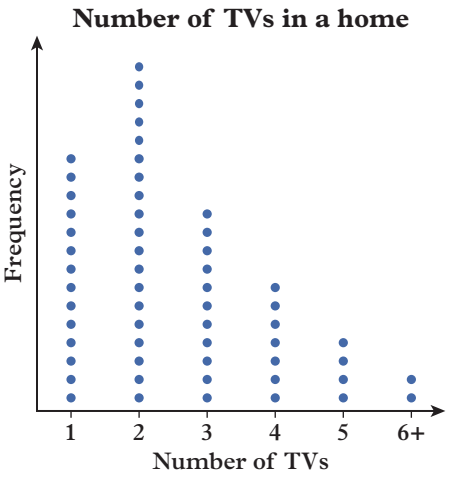
**a**




**b**



**c**



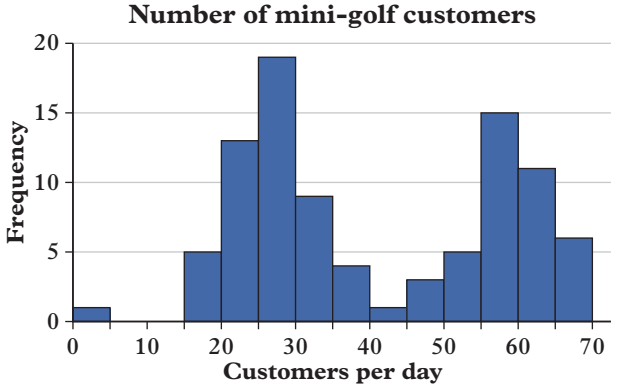

**d** **Length of hair in Year 9 students (cm)**

| Stem | Leaf |
|---|---|
| 0 | 1 2 2 2 3 3 3 4 4 6 6 6 6 7 8 8 8 9 |
| 1 | 0 2 4 6 8 |
| 2 | 0 1 5 8 8 9 9 |
| 3 | 0 1 2 2 2 2 2 4 5 5 5 5 6 7 8 9 9 |
| 4 | 1 2 2 4 4 5 6 6 8 9 |
| 5 | 0 4 5 6 8 8 8 |
| 6 | 2 9 |

Key 1 | 2 = 12

**10** A mini-golf course is open every day of the year except Christmas day. The number of customers who visited the course each day in the months of October, November and December has been placed in the frequency table on the right. The histogram for this data is also shown.

| Class | Frequency |
|---|---|
| 0–<5 | 1 |
| 5–<10 | 0 |
| 10–<15 | 0 |
| 15–<20 | 5 |
| 20–<25 | 13 |
| 25–<30 | 19 |
| 30–<35 | 9 |
| 35–<40 | 4 |
| 40–<45 | 1 |
| 45–<50 | 3 |
| 50–<55 | 5 |
| 55–<60 | 15 |
| 60–<65 | 11 |
| 65–<70 | 6 |




**a** Suggest a reason for why the class interval 0–<5 has a frequency of 1.

**b** Suggest a reason for why the number of daily customers at the mini-golf course has a bimodal distribution.

**c** Give examples of other businesses that will most likely have a bimodal distribution for the number of daily customers.

**11** A boat tour company offers ocean cruises where customers have a chance to see different wildlife, including orcas (killer whales). The following data set is the number of orcas that were spotted during tours for each day of February.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 10 | 0 | 0 | 8 | 0 | 0 | 0 | 26 | 0 | 0 |
| 0 | 13 | 0 | 41 | 0 | 0 | 7 | 7 | 0 | 0 | 0 | 0 | 0 | 0 |

The whale watching company decides to run an advertisement during March on social media with the claim "Last month we saw an average of 4 killer whales per tour!"

**a** Explain why this claim, while technically true, is misleading.

**b** Why would the company not want to advertise the median number of whales they saw per tour?

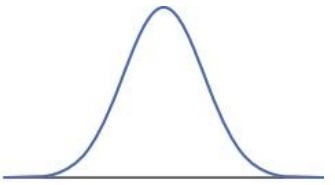
**c** Create a histogram with class intervals of size 10 to display the data.

**d** Explain why the class interval 0–<10 is misleading in the histogram.

**e** The company decides to change the advertisement to be more honest. The new advertisement will instead highlight the percentage of tours on which killer whales were spotted. Complete the sentence below to help the company write their new advertisement.

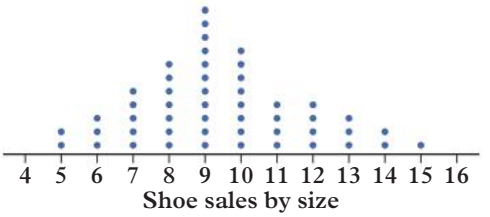
"Last month we saw killer whales on ___% of tours!"

**12** A common distribution that arises from many sets of real-life data is called the normal distribution or bell curve. A perfect normal distribution has many nice features, including that it is symmetric and that its mean, median and mode are all equal. When a real-life data set follows this pattern closely, the data is described as "approximately normally distributed".



An example of this phenomenon is the number of sales of shoes by size, with the most common sales for adult males being of size 9 and the number of sales decreasing for shoe sizes on either side of this centre.

The dot plot below shows one day's worth of sales by size in a shoe store that sells adult male shoes.



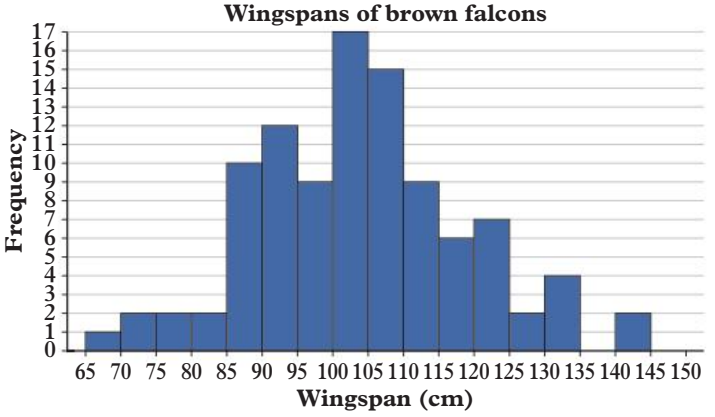

**a** Find the mean, median and mode of this data set. Round the value of the mean to one decimal place.

**b** Explain why, of the three values you found in part **a**, the mode is the most relevant statistic for the owner of the shoe store.

**c** What percentage of shoe sales were of sizes between 7 and 11, including 7 and 11?

**d** What percentage of shoe sales were of sizes between 5 and 13, including 5 and 13?

**13** The bell-curve pattern can also be seen in histograms of continuous data. Approximately normally distributed data appears often in nature, from the heights and weights of most species of animals to the maximum daily temperatures in a city.




The histogram above was created from data collected on the wingspans of 100 adult brown falcons and placed in class intervals of size 5. The brown falcon is a bird species native to Australia and New Guinea, which is known to have a mean wingspan of approximately 105 cm.

**a** Redraw this histogram with class intervals of size 10, starting at 60.

**b** Redraw this histogram with class intervals of size 15, starting at 60.

**c** Which class interval size produces the histogram that most clearly shows the bell shape? Explain your answer.

**d** What percentage of the studied brown falcons had a wingspan between 90 cm and 120 cm?

**e** What percentage of the studied brown falcons had a wingspan between 75 cm and 135 cm?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Describing data

Topic quiz
8D

# 8E Comparing data

## Learning intentions

- ✔ I can compare data sets in different graphical displays.
- ✔ I can compare data sets using summary statistics.
- ✔ I can construct back-to-back stem-and-leaf plots.

Inter-year links

Year 7 10D Stem-and-leaf plots

Year 8 9D Analysing data

## Comparing data sets

- Data sets are easier to compare when displayed in the same format; for example, when two or more data sets are displayed in histograms.
- By placing the data in graphical displays, the range and shape of the distributions can be compared.
- To make a more thorough comparison of two or more data sets, summary statistics should be calculated so the centre and spread of each data set can be compared.

## Back-to-back stem-and-leaf plots

- **Back-to-back stem-and-leaf plots** share the same stem with one data set on the right of the stem and the other data set on the left. For example, these two ordered stem-and-leaf plots show the ages of people swimming in two different pools.

| Stem | Leaf<br>Pool A |
|---|---|
| 0 | 2 7 9 |
| 1 | 1 4 5 6 7 9 |
| 2 | 0 1 1 5 6 8 8 |
| 3 | 1 2 2 4 9 9 9 |
| 4 | 2 4 5 7 |
| 5 | 2 3 |
| 6 | |
| 7 | 9 |

Key: 1 | 4 = 14

| Stem | Leaf<br>Pool B |
|---|---|
| 0 | |
| 1 | 7 7 7 7 8 9 |
| 2 | 2 3 4 4 6 7 8 9 |
| 3 | 2 2 4 5 5 7 9 |
| 4 | 2 2 6 7 |
| 5 | 1 9 |
| 6 | 2 |
| 7 | |

Key: 1 | 4 = 14

- The two stem and leaf plots can be combined into this back-to-back stem-and-leaf plot.

| Leaf<br>Pool B | Stem | Leaf<br>Pool A |
|---|---|---|
| | 0 | 2 7 9 |
| 9 8 7 7 7 7 | 1 | 1 4 5 6 7 9 |
| 9 8 7 6 4 4 3 2 | 2 | 0 1 1 5 6 8 8 |
| 9 7 5 5 4 2 2 | 3 | 1 2 2 4 9 9 9 |
| 7 6 2 2 | 4 | 2 4 5 7 |
| 9 1 | 5 | 2 3 |
| 2 | 6 | |
| | 7 | 9 |

Key: 1 | 4 = 14

## Example 8E.1 Drawing a back-to-back stem-and-leaf plot to compare data sets

Draw an ordered back-to-back stem-and-leaf plot and make a brief comparison of the two data sets detailing the ages of people at a pool.

**Winter:** 45, 23, 15, 36, 57, 31, 9, 38, 44, 56, 52, 13, 36, 27, 48, 44, 48, 14, 27, 45

**Summer:** 31, 16, 14, 15, 23, 56, 24, 18, 17, 8, 11, 13, 16, 21, 17, 36, 20, 17, 14, 15

**THINK**

1. Identify the minimum and maximum numbers for both data sets.
2. The youngest is 8 and the oldest is 57, so use six stems: 0, 1, 2, 3, 4, 5
3. Draw the back-to-back stem-and-leaf plot with these stems, and place the winter leaves on the left and the summer leaves on the right. Write the leaves in order and include a key.
4. Look at the centre and spread of the two data sets. Where does the centre appear to be for each set? What does this indicate about the ages at different times of year?

**WRITE**

| Leaf Winter | Stem | Leaf Summer |
|---|---|---|
| 9 | 0 | 8 |
| 5 4 3 | 1 | 1 3 4 4 5 5 6 6 7 7 7 8 |
| 7 7 3 | 2 | 0 1 3 4 |
| 8 6 6 1 | 3 | 1 6 |
| 8 8 5 5 4 4 | 4 | |
| 7 6 2 | 5 | 6 |

Key: 1 | 4 = 14

The two data sets have a similar range of age brackets, but they are skewed in different directions and have different measures of centre. The data suggests that the pool attracts younger people in summer and older people in winter.

## Example 8E.2 Comparing data sets using summary statistics

Rachel investigated the average age of customers in two different cafes in her local area. Use her results to compare the ages of the customers in cafes.

| Cafe | Mean | Median | Range |
|---|---|---|---|
| **Gumtree** | 21 | 22 | 17 |
| **Jinx** | 27 | 21 | 45 |

**THINK**

1. Compare the given measure of spread (range).
2. Compare the given measures of centre (mean and median). The medians are similar, but Jinx has a higher mean than Gumtree. This difference combined with Jinx's large range suggests that Jinx's distribution may be skewed or affected by an outlier.
3. Summarise your findings.

**WRITE**

Jinx (45) covers a much wider range of ages than Gumtree (17).

The average ages are similar but Jinx (27) has a slightly higher mean than Gumtree (21). Jinx may have a positively skewed distribution or a data value for an older customer that is an outlier.

The mean customer age at the cafes is similar but Jinx has a larger range of ages.

**Helpful hints**

- ✔ When dealing with information from samples, the conclusions you draw will not be certain, so use words to show this, such as, 'The data indicates that …' or 'It appears that …'
- ✔ If you want to compare data sets but the data you're given is not in the same format, put the data sets in the same format before you begin the comparison.

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# Exercise 8E Comparing data

1, 2(a, b), 3(a, b), 4(a–c), 5, 6, 8, 9, 13

2(c, d), 3(b, c), 4(c, e), 6, 10, 11, 14, 16

3(b, c), 4(c, e), 7, 11, 12, 14, 15, 17

**1** Use this back-to-back stem-and-leaf plot to answer these questions.

**a** What is the maximum score:

**i** in group A? **ii** in group B? **iii** overall?

| Leaf Group A | Stem | Leaf Group B |
|---|---|---|
| 9 7 4 3 1 | 0 | 7 9 |
| 9 8 8 6 4 3 2 1 0 | 1 | 3 4 6 8 |
| 7 5 3 0 | 2 | 0 1 1 4 4 4 5 8 9 |
| 8 3 | 3 | 2 2 3 4 7 9 9 |
| 1 | 4 | 0 3 5 |

Key: 1 | 3 = 13

**b** What is the most common score:

**i** in group A? **ii** in group B? **iii** overall?

**c** How would you describe the distribution of:

**i** group A? **ii** group B?

**2** Make a brief comparison of the two data sets in each back-to-back stem-and-leaf plot.

**a**

| Leaf Group A | Stem | Leaf Group B |
|---|---|---|
| 1 | 1 | 0 1 3 5 8 8 |
| 8 7 2 | 2 | 2 4 5 6 7 8 9 9 |
| 9 7 6 3 1 | 3 | 4 6 7 8 |
| 9 8 7 7 6 5 3 1 1 | 4 | 0 1 2 |
| 9 7 6 4 3 1 0 | 5 | 1 3 |

Key: 1 | 9 = 19

**b**

| Leaf Group A | Stem | Leaf Group B |
|---|---|---|
| | 3 | 0 0 1 2 4 5 6 |
| 9 8 7 6 5 4 4 4 | 4 | 1 2 5 5 6 7 8 9 9 |
| 8 7 6 6 5 4 3 1 1 0 | 5 | 0 3 4 5 5 6 8 |
| 8 7 6 5 4 3 3 0 | 6 | |
| | 7 | |
| | 8 | 1 |

Key: 3 | 2 = 3.2

**c**

| Leaf Group A | Stem | Leaf Group B |
|---|---|---|
| | 0 | 2 3 4 |
| 9 7 6 5 5 5 | 1 | 1 7 8 9 9 |
| 9 8 7 7 6 5 4 4 3 | 2 | 0 2 3 5 5 7 9 |
| 9 7 6 5 4 4 4 3 | 3 | 0 4 6 7 8 8 |
| 3 2 2 1 | 4 | 0 1 5 6 |
| | 5 | 0 6 |

Key: 1 | 7 = 170

**d**

| Leaf Group A | Stem | Leaf Group B |
|---|---|---|
| | 5 | 1 |
| 9 8 7 1 | 6 | |
| 7 6 5 4 3 3 3 2 1 | 7 | 8 9 9 |
| 9 8 7 5 4 2 2 1 0 0 | 8 | 1 2 3 4 6 7 8 8 |
| | 9 | 0 0 5 6 7 8 8 8 9 |

Key: 5 | 1 = 51

8E.1 **3** For each of the following draw a back-to-back stem-and-leaf plot and use it to make a brief comparison of the two data sets.

**a** Number of people at a cinema over 6 months of weekends:

**Friday:** 65, 48, 67, 55, 32, 92, 64, 51, 49, 57, 76, 61, 29, 46, 61, 59, 53, 67, 72, 88, 71, 58, 54, 57, 56, 42, 30

**Saturday:** 67, 78, 84, 26, 37, 42, 99, 84, 75, 68, 64, 55, 75, 85, 59, 66, 77, 78, 83, 81, 92, 94, 77, 76, 79, 89

**b** Daily maximum temperature in February:

**Darwin:** 32.3, 32.1, 32.9, 33.2, 33.5, 33.8, 33.4, 32.8, 32.5, 33.3, 32.5, 29.5, 32.8, 33.4, 33.1, 32.7, 33.4, 31.4, 33.0, 31.3, 29.9, 30.8, 30.0, 32.2, 32.1, 32.6, 31.8, 30.4

**Adelaide:** 20.6, 23.7, 25.4, 29.4, 34.2, 38.4, 27.0, 28.3, 28.0, 26.3, 28.3, 31.2, 33.8, 34.3, 36.0, 37.1, 39.2, 40.5, 26.6, 28.4, 32.1, 33.8, 35.6, 38.2, 28.7, 33.4, 21.5, 22.7

**c** Mass of dogs of two breeds (to nearest hundred grams):

**Boston Terriers:** 5100, 6800, 7800, 10500, 9200, 5500, 4900, 11200, 8600, 9200, 10500, 4900, 5500, 7600, 8400, 6800, 9200, 8600, 7500, 4600, 8300, 10 500, 11 000, 8000, 7600, 5100, 6700, 8300, 9200, 10 000, 4900, 7500, 7900, 6200, 8200, 4600, 7800, 8100, 6400, 9900

**French Bulldogs:** 9800, 12900, 8600, 9900, 9800, 12 500, 11 200, 9100, 12 900, 11 200, 9500, 9200, 10 500, 11 900, 12 500, 11 800, 9400, 9800, 10 800, 10 600, 11 500, 9600, 12 300, 13 000, 11 000, 12 000, 10 600, 11 100, 12 700, 10 900, 9500, 11 500, 12 900, 10 100, 11 100, 9200, 11 500, 10 900, 9800, 11 300

8E.2 **4** For each table, use the given statistics to make a comparison of the two data sets.

**a**

| Heights in class | Mean | Median | Range |
|---|---|---|---|
| 9A | 172 cm | 168 cm | 46 cm |
| 9B | 166 cm | 167 cm | 22 cm |

**b**

| Jeans price | Mean | Median | Range |
|---|---|---|---|
| Store A | $120 | $122 | $35 |
| Store B | $124 | $122 | $30 |

**c**

| Internet usage per week | Mean | Median | Range |
|---|---|---|---|
| Primary school students | 4.5 hours | 4 hours | 8 hours |
| Secondary school students | 11.2 hours | 12 hours | 17 hours |

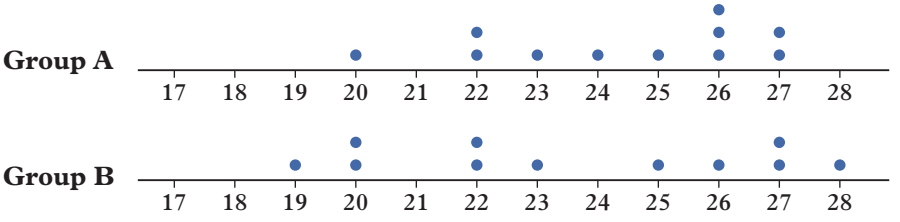
**d**

| SD card capacity | Mean | Median | Range |
|---|---|---|---|
| Store A | 20 GB | 16 GB | 60 GB |
| Store B | 16 GB | 16 GB | 60 GB |

**e**

| Goals per game | Mean | Median | Range |
|---|---|---|---|
| Player A | 4.2 | 4 | 9 |
| Player B | 4.8 | 4 | 7 |

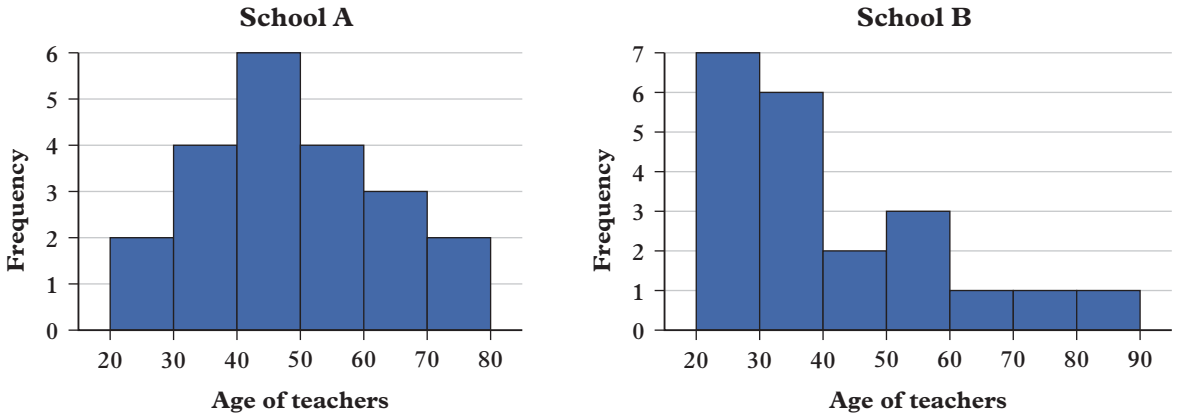
**5** Consider the following two dot plots.




**a** Which group has the greater range? Justify your answer with calculations.

**b** Which group has the greater median? Justify your answer.

**6** Consider the following two histograms that show the ages of teachers from two different schools.




**a** Which school has the oldest teacher?

**b** Which school has the most teachers below the age of 40?

**c** Make a brief comparison of the two data sets.

**7** Consider this back-to-back stem-and-leaf plot detailing the heights of Year 7 and Year 9 students (cm). Note the order of the data in the leaves of this stem-and-leaf plot.

| Leaf Year 7 | Stem | Leaf Year 9 |
|---|---|---|
| 8 | 14 | |
| 6 7 9 9 | 15 | 9 |
| 0 0 1 1 2 2 2 3 4 6 7 8 | 16 | 9 9 8 7 5 4 3 1 |
| 0 1 2 3 4 | 17 | 9 8 8 7 6 5 5 4 3 3 3 1 |
| 2 | 18 | 7 3 2 1 |
| | 19 | 2 |

Key: 14|8 = 148

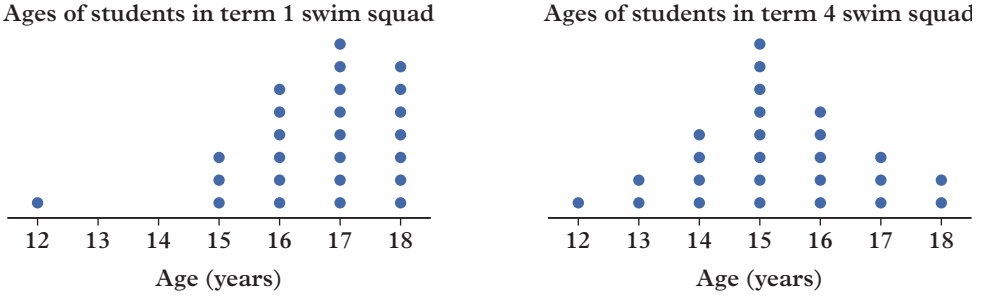
**a** Make a brief comparison of the two data sets, writing your answer in context.

**b** Calculate the mean (to two decimal places), median and range for both data sets displayed in the stem-and-leaf plot.

**c** Use these statistics to make a more detailed comparison of the two data sets.

**d** Does your answer to part **c** support your answer to part **a**? Explain.

**8** **a** Compare the shape of these two dot plots.




**b** Find the mean, median and range of each dot plot.

**c** Write a couple of sentences comparing the two data sets with reference to their summary statistics. Be sure to explain the meaning of the range and centre in terms of the context of the data sets.

**9** A class of Year 8 students and a class of Year 9 students were each asked to calculate the average number of minutes they spend studying per school night. The data is shown in the stem-and-leaf plots on the right.

**a** Redraw these separate stem-and-leaf plots as one back-to-back stem-and-leaf plot with the Year 8 group on the left and the Year 9 group on the right.

**b** Calculate the range of each group.

**c** Calculate the median of each group.

**d** Is it accurate to say that, in general, the Year 9 group studies more than the Year 8 group? Justify your answer.

**Year 8**

| Stem | Leaf |
|---|---|
| 0 | 0 5 |
| 1 | 4 |
| 2 | 3 5 7 7 |
| 3 | 0 0 2 3 8 |
| 4 | 1 4 4 9 |
| 5 | 5 8 9 |
| 6 | 0 5 |
| 7 | 1 4 |
| 8 | 3 6 |

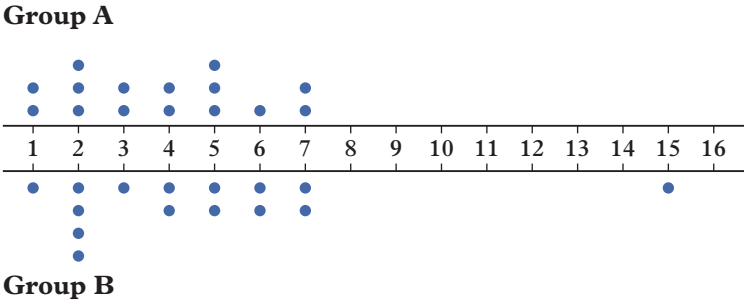
Key: 4 | 1 = 41

**Year 9**

| Stem | Leaf |
|---|---|
| 1 | 8 9 |
| 2 | 0 6 |
| 3 | 5 7 |
| 4 | 5 8 9 |
| 5 | 1 2 4 5 |
| 6 | 0 5 |
| 7 | 1 4 6 7 7 |
| 8 | 3 6 |
| 9 | 0 2 7 |

Key: 5 | 2 = 52

**10** Two groups of 15 people were asked how many times they go to the hairdressers each year. Their responses are recorded in the back-to-back dot plot below. The data for Group A is above the number line and the data for Group B is below the number line.




**a** Find the median of each group.

**b** Complete the table below. Round the mean of each data set to 1 decimal place.

| | Mean | Range |
|---|---|---|
| **A** | | |
| **B** | | |

**c** Explain why the summary statistics in part **b** may be misleading to a person who has not seen the back-to-back dot plot.

**11** Consider the following stem-and-leaf plot and histogram.

**a** Explain why you cannot redraw the histogram as a stem-and-leaf plot.

**b** Redraw the Group A data in the stem-and-leaf plot as a histogram with class intervals of 10 starting at 0.

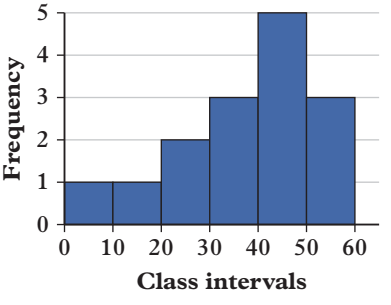
**c** Use the two histograms to make a brief comparison of the two data sets.

**Group A**

| Stem | Leaf |
|---|---|
| 0 | 3 8 9 |
| 1 | 2 4 6 7 |
| 2 | 3 7 7 |
| 3 | 1 5 |
| 4 | 6 7 |
| 5 | 1 |

Key: 3 | 5 = 35

**Group B**

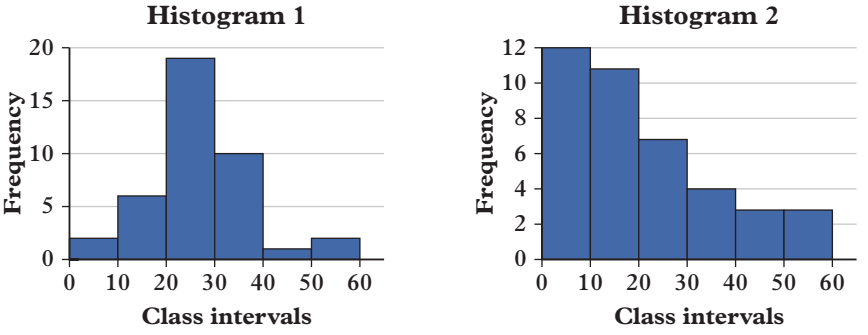


**12** Consider the following summary statistics for two different data sets.

| | Mean | Median | Range |
|---|---|---|---|
| **A** | 21 | 17 | 57 |
| **B** | 27 | 25 | 56 |

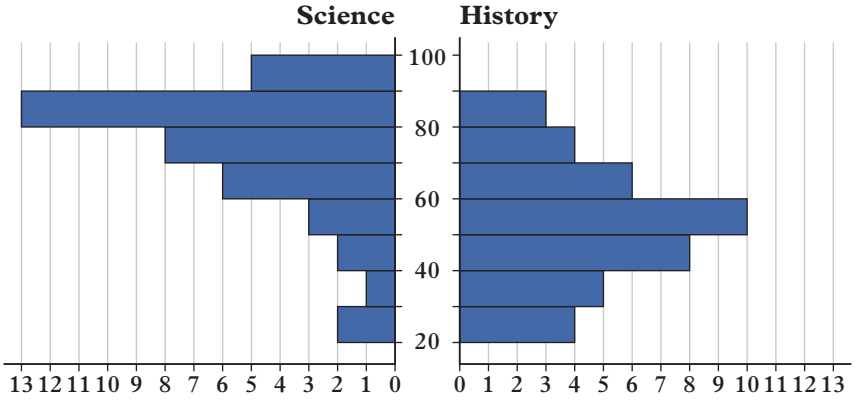
These data sets have been drawn as histograms, but they were mistakenly drawn without labels.

Which data set matches with which histogram? Explain your answer.




**13** Two sets of grouped data can be displayed in one back-to-back histogram. The following back-to-back histogram shows the exam scores (out of 100) for the same 40 students in two different exams: Science and History.

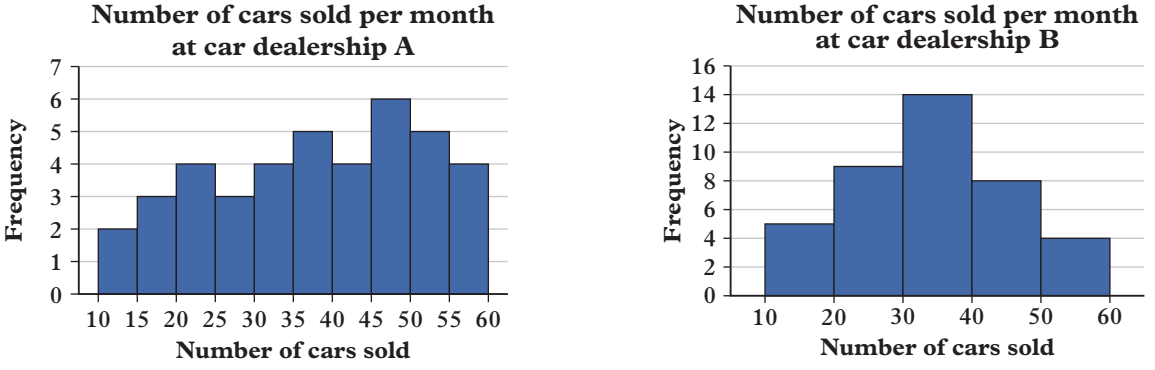
**a** Describe the distribution of the Science exam scores.




**b** Describe the distribution of the History exam scores.

**c** How many exam scores overall were in the interval 80–<90?

**14** Consider these two histograms.




**a** Why is it difficult to make a quick comparison of the two histograms as presented?

**b** Redraw the first histogram so that its class intervals match those of the second.

**c** Use your answer to part **b** to make a comparison of the two data sets.

**d** Why can't you redraw the second histogram to match the first histogram?

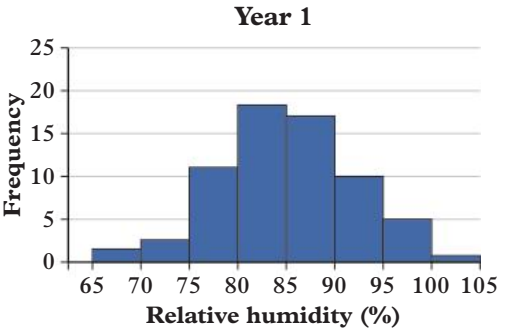
PROBLEM SOLVING AND REASONING

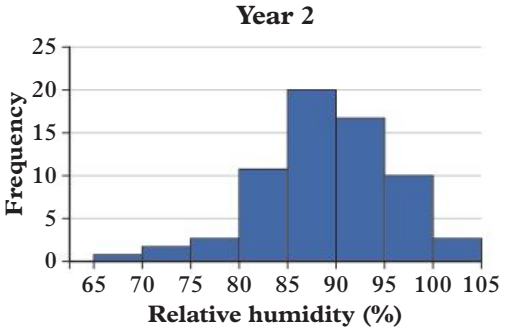
**15** Relative humidity is an indicator of how much water vapor is in the air. As part of his research on climate change, Patrick recorded the relative humidity of this hometown every day for 3 consecutive years. The data he collected has been drawn as histograms shown on the right and below.

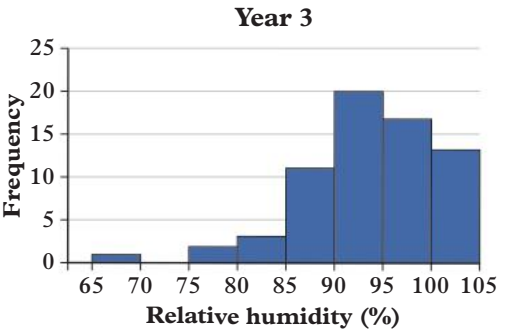
**a** What is the most common class interval in Year 1, Year 2, and Year 3?

**b** Which year has the greater median?

**c** What would you expect the histogram to look like in Year 4, assuming Patrick continues his research.




CHALLENGE

**16** A study on the gaming habits of year 9 students focused on three separate classes, each with exactly 25 students. Each student was asked how many hours they spent per week playing video games. The study found the following results.

| | Mean | Median | Range |
|---|---|---|---|
| **Class A** | 5.5 | 5 | 12.5 |
| **Class B** | 7 | 6 | 28 |
| **Class C** | 6.5 | 8.5 | 14.5 |

**a** One class had an outlier. Which class had the outlier and how can you tell?

**b** The median for class C is much higher than the mean. Does this mean the data for class C is positively skewed or negatively skewed? Explain your answer.

**c** In general, which class plays video games the most? Justify your answer.

**17** In a school there are 250 students in Year 8 and 200 students in Year 9. The following statistics have been calculated about the students' heights (in cm).

| | Mean | Median | Minimum | Maximum |
|---|---|---|---|---|
| **Year 8** | 160 | 161 | 142 | 173 |
| **Year 9** | 166 | 164 | 144 | 179 |

**a** Find the range for the heights of each year level.

**b** Find the range for the heights of all students in two year levels combined.

**c** Find the average height of all students in the two year levels combined, rounded to 1 decimal place. Hint: The answer is not 163 cm.

**d** Explain why it is not possible to find the median height of all students in the two year levels combined.

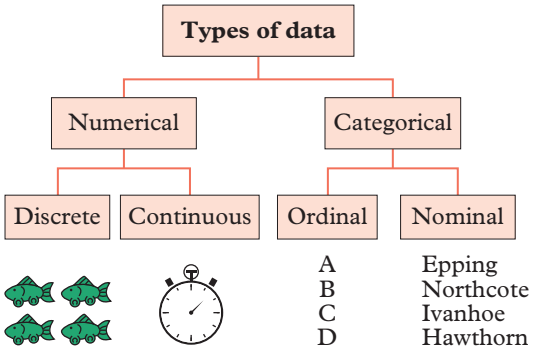
Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Comparing data

Topic quiz
8E

# Chapter summary

## Classifying data



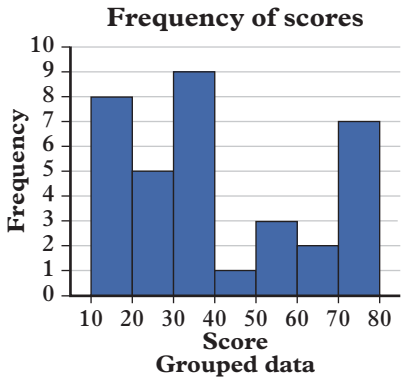

- Bivariate data is data that shows the relationship between two variables.

## Grouping data

- Frequency tables: 5–10 class intervals
- Discrete data: 0–9
- Continuous data: 0–<10

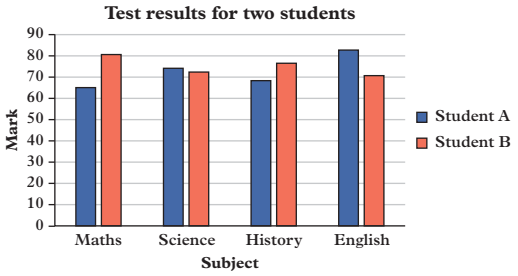
## Histograms

- Lower bound on left of each column
- For continuous data: upper bound on right of each column
- For discrete data: lower bound of next class interval on right of each column

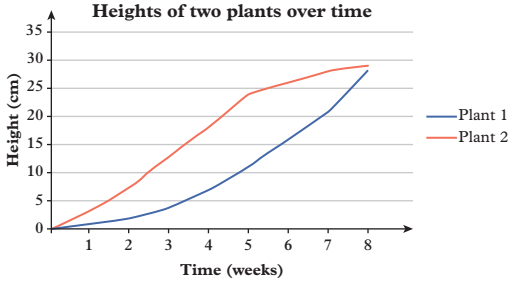



## Displaying data

- Column graphs: numerical variable (vertical axis), categorical data (horizontal axis)
- Side-by-side column graph: second categorical variable in legend




- Line graphs: two numerical variables, horizontal axis must be continuous
- Multiple line graphs: catagorical variable in legend

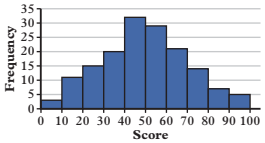



## Measures of centre and spread

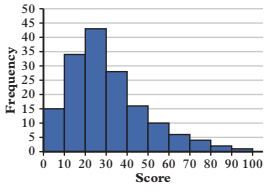
- **Mean** = $\frac{\text{sum of all data points}}{\text{total number of data points}}$
- **Median** = middle value when data is listed in order
- **Mode** = most common value
- **Range** = maximum value – minimum value
- To calculate the mean and median from a frequency table, add 'score × frequency' and 'cumulative frequency' columns to the table.

## Data distributions

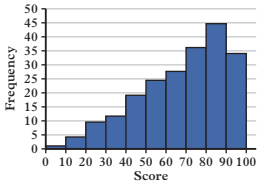
Symmetric: peak in middle, even either side



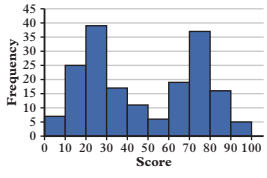

Positive skew: skew to vertical axis, tail on right




Negative skew: skew from vertical axis, tail on left




Bimodal: two clear peaks




## Describing distributions

- Symmetric: use mean or median
- Skewed: use median
- Bimodal distribution: use mode of peaks
- Outliers: use median

## Comparing data sets

- Put data sets in same format
- Compare centre and spread

| Leaf Pool B | Stem | Leaf Pool A |
|---|---|---|
| | 0 | 2 7 9 |
| 9 8 7 7 7 7 | 1 | 1 4 5 6 7 9 |
| 9 8 7 6 4 4 3 2 | 2 | 0 1 1 5 6 8 8 |
| 9 7 5 5 4 2 2 | 3 | 1 2 2 4 9 9 9 |
| 7 6 2 2 | 4 | 2 4 5 7 |

# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

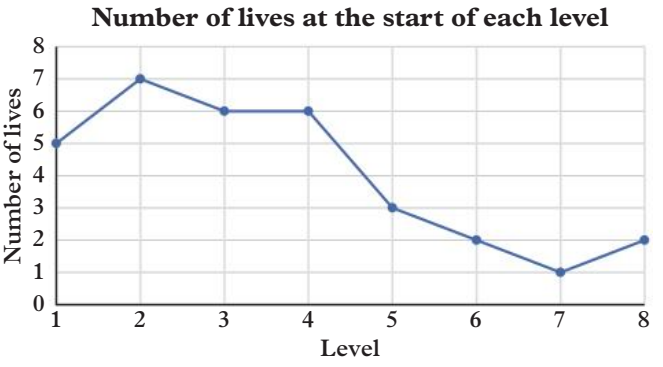
## Multiple-choice

8A **1** Which of the following variables is categorical?

**A** Number of pets owned
**B** Percentage score on a test
**C** Preferred music genre
**D** Time spent waiting in line at the canteen
**E** Number of books sold

8A **2** The line graph on the right tracks the number of lives a gamer has when they begin each level of a video game. The gamer gains two lives during each level. During which level did they lose the most lives?

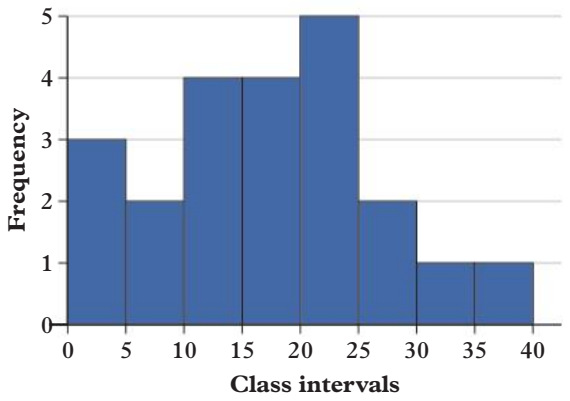
**A** 1
**B** 2
**C** 3
**D** 4
**E** 5



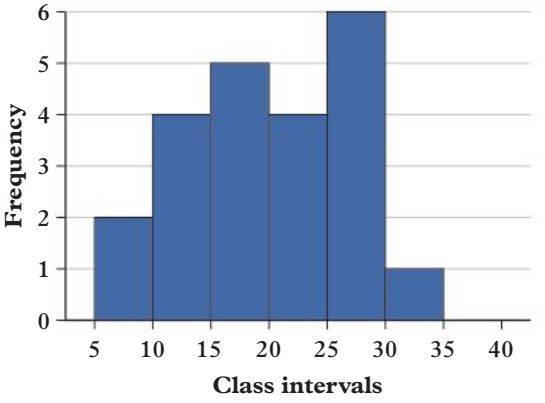

8B **3** The table of data below is represented in which histogram?

| Class | 0–<5 | 5–<10 | 10–<15 | 15–<20 | 20–<25 | 25–<30 | 30–<35 | 35–<40 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 3 | 2 | 4 | 4 | 5 | 2 | 1 | 1 |

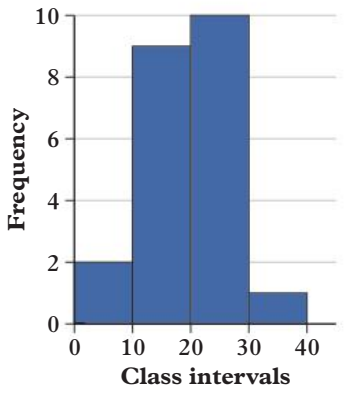
**A**



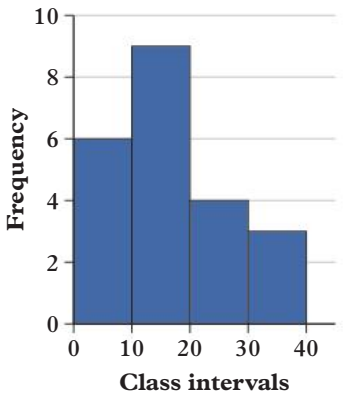

**B**



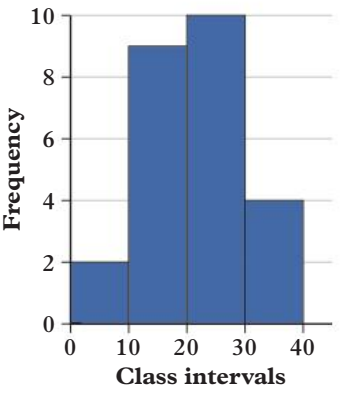

**C**




**D**




**E**





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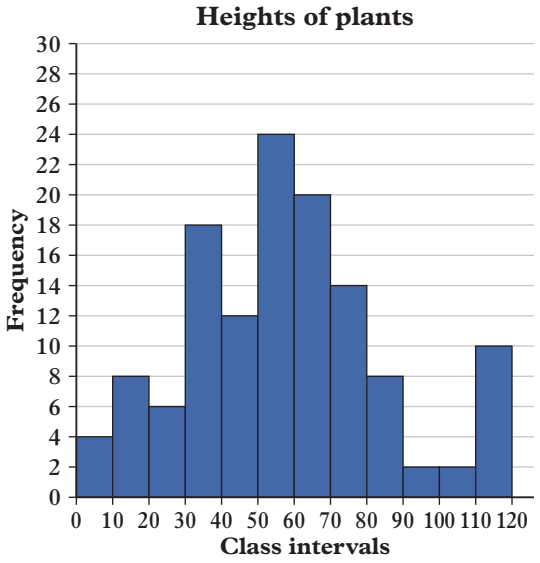
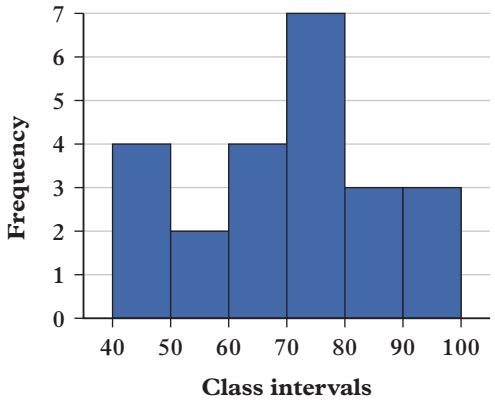
8B **4** The histogram displays the results of research where the heights of plants (in cm) were measured.




How many plants are less than 70 cm tall?

**A** 14 **B** 20 **C** 36 **D** 44 **E** 92

8C **5** In the following histogram, which class interval contains the median?




**A** 40–<50 **B** 50–<60 **C** 60–<70 **D** 70–<80 **E** 80–<90

8C **6** What is the range of the data shown in the stem-and-leaf plot below?

| Stem | Leaf |
|---|---|
| 1 | 4 7 8 |
| 2 | 3 8 |
| 3 | 8 9 |
| 4 | 1 |
| 5 | 6 7 |
| 6 | 0 2 4 5 |
| 7 | 6 6 |
| 8 | 9 |
| 9 | |
| 10 | 1 4 |
| 11 | 2 6 |

Key: 5 \| 6 = 5.6

**A** 60 **B** 7.5 **C** 10.2 **D** 6 **E** 102

8D **7** Which statement is true if describing distribution of data?

**A** Symmetric distributions have a centre closer to the right of the distribution, away from the $y$-axis.

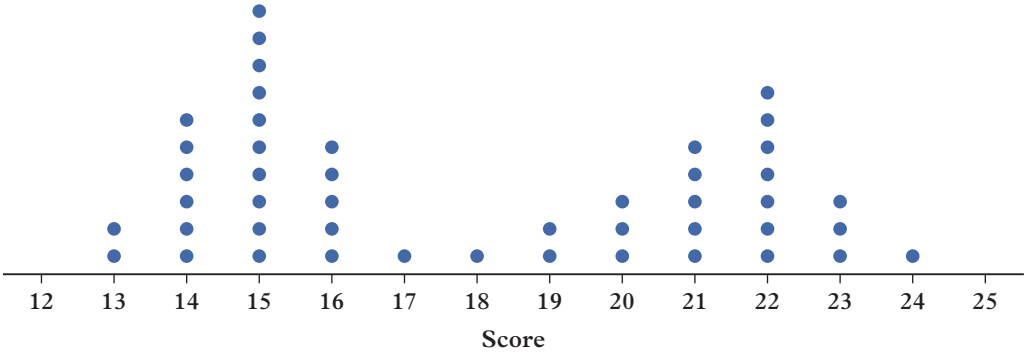
**B** Any distribution that is skewed or has an outlier should have its centre described using the median rather than the mean.

**C** Positively skewed distributions have a middle peak and a roughly even spread on either side of this peak.

**D** Negatively skewed distributions have a centre closer to the left of the distribution, towards the vertical axis.

**E** The peaks in a bimodal distribution must be exactly the same height.

8D **8** Which of the following is an appropriate measure of centre for the dot plot shown?




**A** mean **B** range **C** median **D** mode **E** no centre

8E **9** How is each data set distributed in the back-to-back stem-and-leaf plot on the right?

**A** Both groups are symmetric.

**B** Group A is negatively skewed and Group B is positively skewed.

**C** Group A is positively skewed and Group B is negatively skewed.

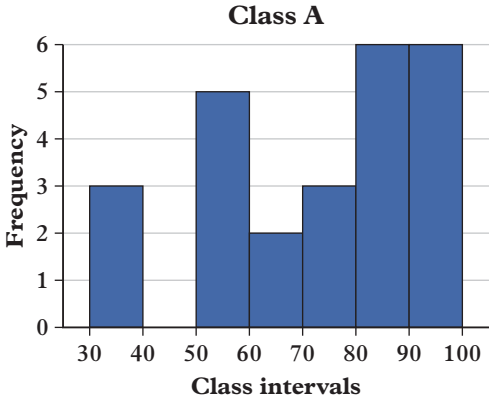
**D** Both groups are negatively skewed.

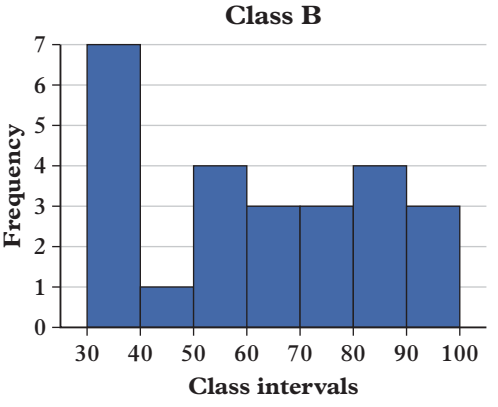
**E** Both groups are positively skewed.

| Leaf Group A | Stem | Leaf Group B |
|---|---|---|
| 9 8 6 6 4 4 3 2 | 0 | |
| 8 7 5 4 3 1 | 1 | 9 |
| 7 7 1 | 2 | 1 |
| 5 | 3 | 3 4 |
| 4 | 4 | 2 4 7 9 |
| 1 | 5 | 2 3 3 4 6 8 9 9 |
| | 6 | 1 5 6 6 8 |

Key: 1 | 9 = 19

8E **10** The two histograms below show the Geography exam scores of two different Year 9 classes. Which of the following statements is definitely true?




**A** The range of each class is at least 60.

**B** There are more students in Class A than in Class B.

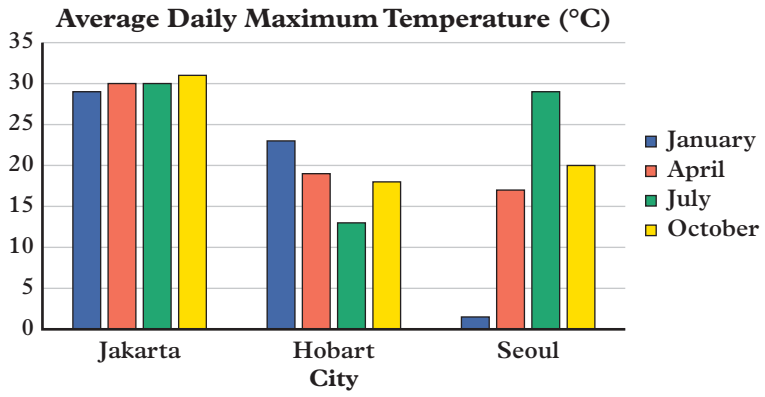
**C** The range of Class A is greater than the range of Class B.

**D** The median of Class A is greater than the median of Class B.

**E** Class B performed better on the exam than Class A.

## Short answer

8A **1** The following column graph shows the average daily maximum temperature in three cities across four different months.




**a** Identify the three variables shown in the graph and classify each as either numerical or categorical.

**b** Which city has the most consistent temperature over the year?

**c** Of the months shown, when is Hobart hotter than Seoul?

**d** Suggest a reason that the coldest months in Hobart and Seoul do not occur at the same time.

8A **2** Maria has started a new job selling cars at a car dealership. The number of cars she sold during each of her first 9 months is shown in the table below.

| **Month** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **Number of cars sold** | 2 | 4 | 8 | 14 | 12 | 15 | 19 | 21 | 22 |

**a** Draw a line graph to represent this data.

**b** In which month(s) did Maria sell less cars than in the previous month?

**c** At the car dealership, Maria is paid a fixed salary of $4000 per month plus an extra $200 for each car she sells. What was her total income from this job during month 6?

8B **3** Data was collected on the number of hours Year 9 students spent using various forms of social media over the course of a week.

45 21 37 21 20 17 31 32 11 15 17 18 20 31 48 32 21 5 11 7 19 18 27 42

40 21 32 23 24 19 38 37 14 15 19 28 22 35 41 33 27 2 10 5 18 18 25 43

**a** Construct a frequency table with class intervals of width 10.

**b** How many people were surveyed?

**c** What is the most common class interval?

**d** Draw a histogram to represent this data.

**e** Which interval is the most common?

8C **4** Find the mean, median, mode and range for this data set, correct to two decimal places where appropriate.

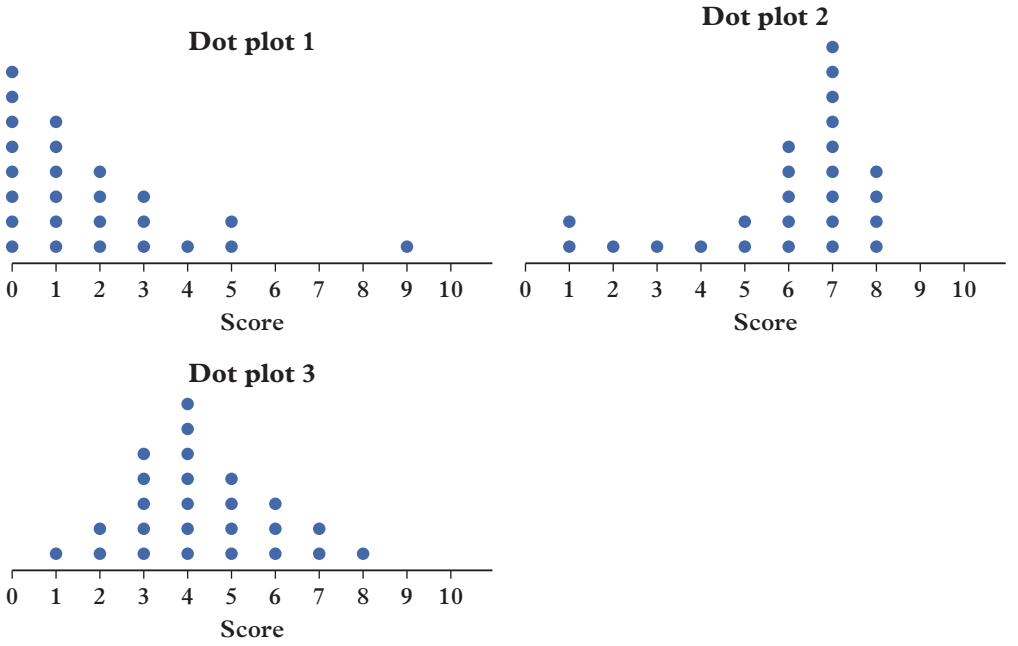
| **Score** | **Frequency** |
|---|---|
| 3 | 10 |
| 4 | 12 |
| 5 | 23 |
| 6 | 5 |

8C **5** Calculate the mean, mode, median and range for the data represented in this stem-and-leaf plot, correct to two decimal places where appropriate.

| Stem | Leaf |
|---|---|
| 0 | 2 4 5 7 7 7 7 |
| 1 | 5 8 8 8 9 |
| 2 | 1 1 3 4 7 9 |
| 3 | 7 8 8 |
| 4 | 1 2 5 |
| 5 | 8 |

Key 2 | 4 = 24

8D **6** Consider the following three dot plots.




**a** Which dot plot has the greatest range?

**b** Which dot plot is positively skewed?

**c** Which dot plot has a roughly symmetric distribution?

**d** Is the mean or median the most appropriate measure of centre for dot plot 2? Explain your reasoning.

**e** One of the dot plots represents the number of pets owned by each student in a Year 9 class. Which dot plot do you think represents this data? Explain your reasoning.

8D **7** The ages of 53 people are shown in the stem-and-leaf plot below.

| Stem | Leaf |
|---|---|
| 0 | 4 4 5 5 7 8 |
| 1 | 0 1 1 1 3 4 4 5 6 6 6 6 7 9 9 9 |
| 2 | 0 0 1 3 4 4 4 4 5 6 6 6 7 7 7 8 9 9 |
| 3 | 0 0 0 0 1 1 1 2 2 3 3 3 3 |

Key: 1 | 3 = 13

**a** Find the median age of the group.

**b** Find the range of the data set.

**c** Draw a histogram to represent this data. Use class intervals of 5 with the first interval starting at 0.

**d** Describe the distribution of ages in the data set.

8E **8** The results achieved in Maths tests by two different classes are recorded in a back-to-back stem-and-leaf plot.

| Leaf Class 9A | Stem | Leaf Class 9B |
|---|---|---|
| 8 5 5 3 | 4 | 6 7 8 8 9 |
| 9 7 3 | 5 | 1 3 6 7 |
| 7 6 5 4 3 | 6 | 3 4 7 |
| 0 | 7 | 2 4 8 |
| 9 7 5 3 1 | 8 | 2 5 6 7 |
| 0 | 9 | 8 9 |

Key: 4|6 = 46

**a** Find the mean, median, mode and range for each class.

**b** Compare the Maths results of the two classes.

8E **9** The following data comparing the number of hours spent at football training per fortnight were recorded for Year 9 boys from two different secondary schools.

**School 1:** 35 11 27 11 10 7 21 22 1 5 7 8 10 21 38 22 11 5 1 7 9 8 17 32

**School 2:** 12 14 16 17 20 11 22 11 11 16 19 14 2 4 10 19 17 6 4 6 2 7 16 31

**a** Collate the data in a back-to-back stem-and-leaf plot using class intervals of 5.

**b** Calculate the mean, median and range for each school, correct to one decimal place.

**c** Which measure of centre would you use to discuss the results for each school?

## Analysis

This stem-and-leaf plot shows the measurements of shrubs taken at different nurseries.

| Leaf Brisbane | Stem | Leaf Sydney |
|---|---|---|
| 7 4 4 4 | 11 | 4 4 4 5 7 8 9 |
| 7 6 4 3 2 2 | 12 | 1 3 6 7 |
| 8 7 6 2 1 | 13 | 2 3 9 |
| 1 0 | 14 | 1 4 6 7 8 9 |
| 9 8 5 3 2 | 15 | 1 2 5 5 |

Key: 11|4 = 11.4 cm

**a** Compare the number of shrubs measured at each location.

**b** Calculate the mean, median, mode and range of shrub heights at each location, correct to two decimal places where necessary.

**c** Write a short comparison of the height of shrubs at the two locations.

The manager of the nursery company wanted to collate the data for all of his businesses.

**d** Generate lists of raw data for Brisbane and Sydney. Using this data and the lists below, create a frequency table using suitable class intervals and collate all of the data about shrub height.

**Victoria:** 12.1, 11.7, 18.3, 11.4, 11.4, 14.5, 15.6, 17.1, 16.5, 18.6, 13.0, 12.6, 12.4, 10.9, 11.4, 14.0, 16.9, 17.1, 16.5, 18.6

**Western Australia:** 10.2, 10.6, 19.3, 11.4, 11.4, 15.9, 14.7, 18.3, 17.7, 13.4, 10.0, 12.5

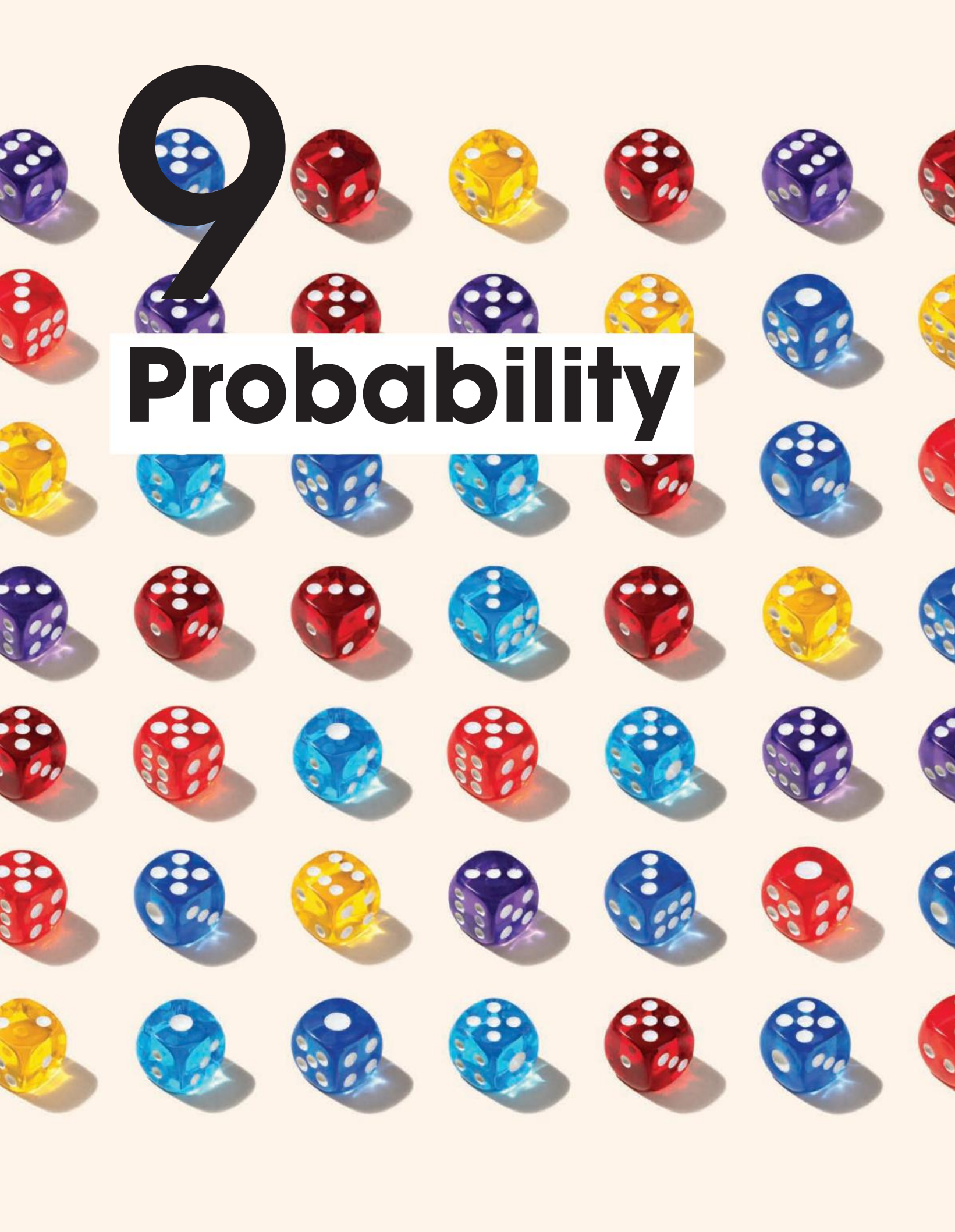
**e** How many plants were measured in total?

**f** What is the modal class?

**g** Represent this data as a histogram and comment on the shape of the distribution.

**h** Calculate the mean shrub height for Victoria and Western Australia correct to two decimal places.

**i** Write a statement that could be used in a marketing campaign, which includes information about the smallest and largest shrubs and the average shrub height.



# 9 Probability

## Index


**9A** Two-step chance experiments
**9B** Experiments with replacement
**9C** Experiments without replacement
**9D** Relative frequency
**9E** Two-way tables
**9F** Venn diagrams


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Calculating theoretical probability
- ✔ Fractions, decimals and percentages
- ✔ Adding and subtracting fractions
- ✔ Multiplying and dividing fractions

## VCAA curriculum links

- List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (VCMSP321)
- Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (VCMSP322)

## Materials

- ✔ Calculator

# 9A Two-step chance experiments

## Learning intentions

- ✔ I can create tree diagrams and arrays to represent sample spaces.
- ✔ I can use tree diagrams and arrays to calculate probabilities.

Inter-year links

| | |
|---|---|
| Year 5/6 | Understanding probability |
| Year 7 | 10F Theoretical probability |
| Year 8 | 9E Theoretical probability |
| Year 10 | 11A Theoretical probability |

## Calculating theoretical probability

- The **sample space** of an experiment is a list of all possible outcomes in the **experiment**.
- An **event** is a collection of one or more outcomes from the sample space of an experiment.
- The probability of an event occurring can be written as Pr(A), where A represents all possible outcomes in the event.
- If all the outcomes in an experiment are equally likely to occur, the **theoretical probability** of an event occurring is the ratio of the number of outcomes in the event to the number of outcomes in the sample space. It can be expressed as a fraction, decimal or percentage and calculated using the formula:
  - → $\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$
- The complement of an event A is the event that A does not occur. Events A and 'not A' are **complementary events**.
  - → Complementary events are denoted using the symbol prime (′).
    A and A′ are complementary events.
  - → All outcomes in the sample space that are not in A must be in A′.
    $\Pr(A) + \Pr(A') = 1$
- When calculating the probability of an event involving multiple outcomes, the probabilities of each outcome are added together to give the probability of the event.
  - → Make sure you do not include any individual outcome more than once. For example, when a standard six-sided die is rolled, the probability of the event 'rolling an even or a prime number' must only count the outcome '2' once.

## Two-step chance experiments

- Two-step chance experiments involve two experiments, which may be completed at the same time, or one after the other. For example, rolling a standard six-sided die and tossing a coin is a two-step chance experiment.
- The outcomes for two-step experiments can be recorded in an **array** or in a **tree diagram**.

## Arrays

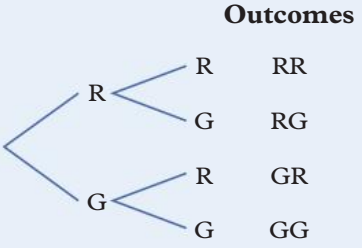
- An array is a list of items arranged in rows and columns.
- The sample space of a two-step chance experiment shown in an array have the outcomes of one event in columns and the outcomes of the other event in rows. For example, this array shows the outcomes from rolling a standard six-sided die and tossing a coin.
  - → There are two outcomes when a coin is tossed: heads (H) or tails (T).
  - → There are six outcomes when a die is rolled: 1, 2, 3, 4, 5, 6.

| | | Rolling a standard six-sided die | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Tossing a coin | H | (1, H) | (2, H) | (3, H) | (4, H) | (5, H) | (6, H) |
| | T | (1, T) | (2, T) | (3, T) | (4, T) | (5, T) | (6, T) |

- When calculating probability using an array, all the outcomes will be equally likely, and the theoretical probability formula can be used.
- Arrays are best used to display the sample space when there are a large number of outcomes in either step of a two-step experiment.

# Tree diagrams

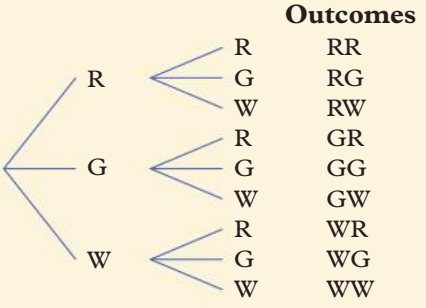
- In a tree diagram, the outcomes of each step of a chance experiment are listed vertically, with the events joined with branches. For example, this tree diagram shows the outcomes for selecting two marbles at random from a bag of red and green marbles.




- If the outcomes in a tree diagram are equally likely, the theoretical probability formula can be used.

## **Example 9A.1** Understanding tree diagrams

This tree diagram displays the outcomes of selecting two lollies at random from a jar containing red (R), green (G) and white (W) lollies.




Use this tree diagram to find:

**a** the total number of outcomes

**b** the number of outcomes containing at least 1 white lolly.

**THINK**

**a** Count the number of final outcomes at the right end of the tree diagram.

**b** Trace the branches carefully and count the number that contains at least one white lolly.

**WRITE**

**a** There are 9 possible outcomes: RR, RG, RW, GR, GG, GW, WR, WG, WW

**b** 5 outcomes contain a white lolly: RW, GW, WR, WG, WW

## **Example 9A.2** Calculating probability using an array

Use an array to calculate the probability of a total of 10 or above when rolling two standard six-sided dice.

**THINK**

1. Draw an outline for the array, with enough columns to list all the outcomes for the first step, and enough rows to list all the outcomes for the second step.
2. Complete the array by showing the result for the first step before the result for the second step in brackets, separated by a comma.
3. Count the outcomes that have a total 10 or more (these cells have been highlighted in the answer).
4. Count the total number of outcomes in the array.
5. Use the theoretical probability formula and simplify if possible.

**WRITE**

| | | 1st dice roll | | | | | |
|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** |
| **2nd dice roll** | **1** | (1, 1) | (2, 1) | (3, 1) | (4, 1) | (5, 1) | (6, 1) |
| | **2** | (1, 2) | (2, 2) | (3, 2) | (4, 2) | (5, 2) | (6, 2) |
| | **3** | (1, 3) | (2, 3) | (3, 3) | (4, 3) | (5, 3) | (6, 3) |
| | **4** | (1, 4) | (2, 4) | (3, 4) | (4, 4) | (5, 4) | (6, 4) |
| | **5** | (1, 5) | (2, 5) | (3, 5) | (4, 5) | (5, 5) | (6, 5) |
| | **6** | (1, 6) | (2, 6) | (3, 6) | (4, 6) | (5, 6) | (6, 6) |

$$\Pr(10 \text{ or above}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$
$$= \frac{6}{36}$$
$$= \frac{1}{6}$$

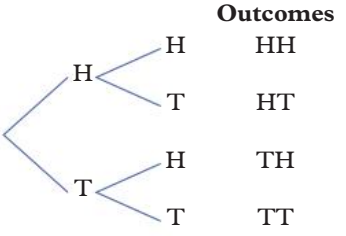
## **Example 9A.3** Calculating probability using a tree diagram

Use a tree diagram to calculate the probability of exactly one tail when tossing a coin twice.

**THINK**

1. Draw the first two branches to represent the first coin toss. Label the ends of these branches with H and T to represent the two different outcomes, heads and tails.
2. From each branch, draw another two branches to represent the next coin toss and label them appropriately.
3. Write the final outcome on each of the four branches to complete the tree diagram.
4. There are four possible outcomes and two outcomes contain exactly one tail. As each of the outcomes are equally likely, the probability formula can be used.

**WRITE**




$$\Pr(\text{exactly one tail}) = \frac{2}{4}$$
$$= \frac{1}{2}$$

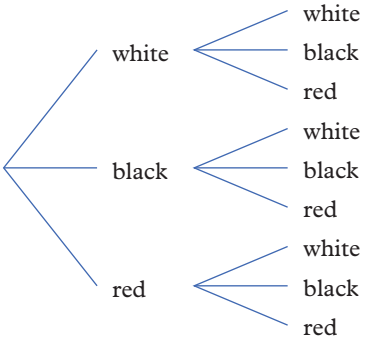
**Helpful hints**

- ✔ In other contexts, an event means an occurrence or an occasion, but in probability it means a collection of outcomes from the sample space. Don't get confused!
- ✔ When creating a tree diagram, make sure you leave plenty of space to draw in all the branches!

# ANS p550 Exercise 9A Two-step chance experiments

△ 1–8, 10, 11 □ 2–4, 6, 8, 9, 11–13 ⬠ 2–4, 6, 9–13

**9A.1** **1** Consider this tree diagram, which shows the results of choosing coloured balloons at random.



**a** How many possible outcomes are there?

**b** How many of these outcomes contain at least one red?

**9A.2** **2** Consider this array, which shows the results of two people randomly picking a number between 1 and 3.

**a** How many possible outcomes are there?

**b** State the probability of obtaining a sum of:

**i** 3

**ii** an odd number

**iii** a prime number

**iv** a square number

**v** a number greater than 6.

| | | 1st number | | |
|---|---|---|---|---|
| | | **1** | **2** | **3** |
| **2nd number** | **1** | (1, 1) | (2, 1) | (3, 1) |
| | **2** | (1, 2) | (2, 2) | (3, 2) |
| | **3** | (1, 3) | (2, 3) | (3, 3) |

**3** Two four-sided dice (numbered 1–4) are rolled and the numbers showing are added together to give the final outcome.

**a** Create an array that lists all the outcomes.

**b** How many equally likely outcomes are detailed in the array?

**c** What is the most likely final outcome? What is the probability of this occurring?

**d** What are the least likely final outcomes? What is the probability of these occurring?

**e** State the probability of rolling two four-sided dice (numbered 1–4) and obtaining a sum:

**i** of 3 **ii** greater than 6

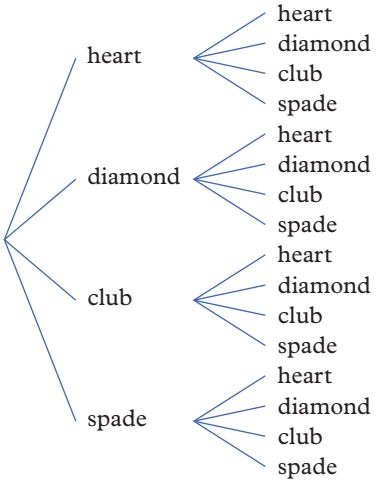
**iii** of an odd number **iv** less than 5.

**4** Two cards are drawn out of a standard deck of cards. Assuming that all outcomes are equally likely, use the tree diagram to find the probability of selecting:

**a** at least one diamond **b** a club and a heart

**c** two spades **d** not a heart

**e** a diamond or a spade **f** a spade then a club.




UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

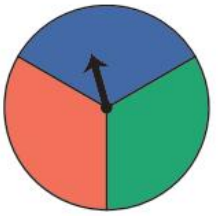
**9A.3** **5** A coin was tossed twice. Use a tree diagram to find the probability of:

**a** two heads

**b** one head and one tail

**c** no heads

**6** This spinner was spun twice. Use a tree diagram to find the probability of:



**a** two different results

**b** spinning blue at least once

**c** spinning the same colour both times.

**7** This array represents a single round of a game of rock (r), paper (p), scissors (s).

| | | **Your choice** | | |
|---|---|---|---|---|
| | | **r** | **p** | **s** |
| **Opponent's choice** | **r** | (r, r) | (p, r) | (s, r) |
| | **p** | (r, p) | (p, p) | (s, p) |
| | **s** | (r, s) | (p, s) | (s, s) |

rock beats scissors

paper beats rocks

scissors beats paper

**a** List the outcomes that result in you:

**i** winning

**ii** losing

**iii** drawing the game.

**b** Hence calculate the probability that you win a game of rock, paper, scissors.

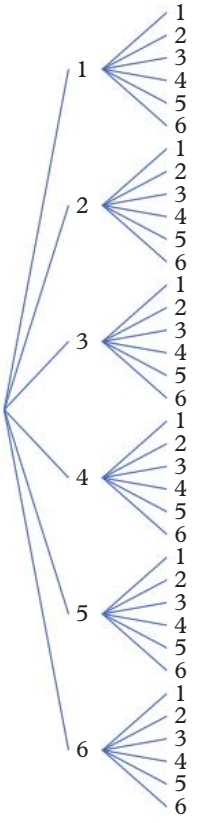
**8** Consider sitting a quiz consisting of multiple-choice questions, with answers A–D.

**a** Draw a tree diagram to represent the possible different options for the first two questions.

**b** Draw an array to represent the possible different options for the first two questions.

**c** Explain why you could use a tree diagram but not an array to represent guessing the answers to the first three questions.

**9** The tree diagram on the right shows the result of rolling two standard six-sided dice.




**a** How many possible outcomes are there?

**b** Use the tree diagram to find the probability of rolling:

**i** two odd numbers **ii** one even number

**iii** a double **iv** a total of 6.

**c** Why would it be more appropriate to use an array to display the sample space?

**10** Tree diagrams aren't limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you toss a coin and then roll a standard six-sided die.

**a** Draw a tree diagram to represent this two-step experiment.

**b** How many outcomes are there?

**c** What is the probability that you toss a tail:

**i** and roll a 6? **ii** and roll a number less than 4?

**iii** or roll a 6? **iv** or roll a number less than 4?

**d** Explain the mistake made by someone who answers $\frac{8}{12}$ to part **c iii**.

**11** Similar to tree diagrams, arrays are not limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you select a vowel (A, E, I, O, U) at random and roll a standard four-sided die.

**a** Draw an array to represent this two-step experiment.

**b** How many outcomes are there?

**c** What is the probability that:

**i** you select E and roll a 3? **ii** you select E or roll a 3?

**iii** you select a vowel with only straight lines and roll a number less than 4?

**iv** you select a vowel with only straight lines or roll a number less than 4?

**12** Determine the missing numbers and list all combinations of outcomes in each step such that:

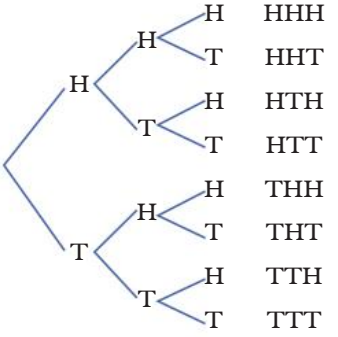
**a** the number of outcomes in each step is greater than 1

**b** the first step has either less than or equal to the number of outcomes in the second step.

| Experiment | Number of outcomes in first step | Number of outcomes in second step | Total number of outcomes | Number of favourable outcomes | Probability of event |
|---|---|---|---|---|---|
| 1 | 3 | 5 | | | $\frac{7}{15}$ |
| 2 | 3 | 5 | | | $\frac{2}{5}$ |
| 3 | 8 | 12 | | | $\frac{1}{4}$ |
| 4 | 4 | | 36 | | $\frac{3}{4}$ |
| 5 | | | 10 | | $\frac{9}{10}$ |
| 6 | | | 12 | | $\frac{5}{6}$ |
| 7 | | | | 11 | $\frac{11}{14}$ |
| 8 | | | | 12 | $\frac{2}{3}$ |
| 9 | | | | 35 | $\frac{5}{7}$ |
| 10 | | | | 66 | $\frac{2}{11}$ |



**13** Experiments can have three or more steps. When using a tree diagram to represent this, we add more sets of branches to the end of each branch. For example, if a coin is tossed three times (or three coins are tossed), then the tree diagram that represents the sample space would look like this.




**a** Use the tree diagram shown to determine the probability of tossing a fair coin three times and getting:

**i** three heads **ii** two heads and one tail in any order

**iii** two tails and one head in any order

**iv** an alternating sequence of heads and tails in any order.

**b** Explain why you couldn't use an array to show the sample space for an experiment with three or more steps where each set of outcomes at each step is shown. That is, without using the outcomes of a group of steps against the outcomes of another group.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Arrays
- Interactive skillsheet: Tree diagrams
- Investigation: Dominoes and tree diagrams
- Topic quiz: 9A

# 9B Experiments with replacement

## Learning intentions

✔ I can calculate probabilities for two-step chance experiments with replacement.

**Inter-year links**

| | |
|---|---|
| Year 7 | 10F Theoretical probability |
| Year 8 | 9E Theoretical probability |
| Year 10 | 11B Experiments with and without replacement |

## Combining like outcomes

- When dealing with an experiment with like outcomes, these outcomes can be grouped together.
- When grouping outcomes, the probability of each grouped outcome occurring can be calculated by summing the probabilities of the outcomes within the group.
- When outcomes are not equally likely, a tree diagram with probabilities written on the branches can be used to determine the probability of each outcome.
- The probabilities for each final outcome can be determined by multiplying the probabilities of each branch leading to that final outcome.
  → The probabilities of all the final outcomes will add to 1.

## Experiments with replacement

- Experiments with replacement involve selecting an item at random, recording the result, and then replacing the item before making another selection.
- When replacing the item before making another selection, the probabilities for each step of the experiment will remain the same.

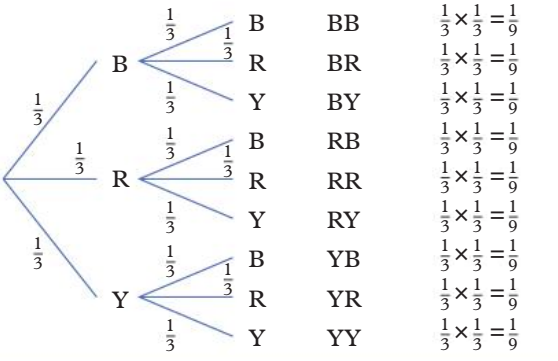
### Example 9B.1 Representing experiments with equally likely outcomes

A box contains equal numbers of blue, red and yellow activity cards. A card is selected at random, its colour is recorded and then it is replaced. Another card is then selected at random. Draw a tree diagram to represent this situation, complete with probabilities on each branch and for each final outcome.

**THINK**

1. Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.
2. Write the probability of each branch on your tree diagram. Each colour is equally likely so there is a $\frac{1}{3}$ chance of selecting a blue card, a $\frac{1}{3}$ chance of selecting a red card and a $\frac{1}{3}$ chance of selecting a yellow card.
3. Multiply the probabilities along each branch to determine the probability of each final outcome.

**WRITE**




| Outcomes | Probability |
|---|---|
| BB | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| BR | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| BY | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| RB | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| RR | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| RY | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| YB | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| YR | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |
| YY | $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ |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## Example 9B.2 Calculating probabilities for experiments with equally likely outcomes

Use the tree diagram from Example 9B.1 to find the probability that:

**a** two blue cards are selected

**b** a yellow card is selected first.

**THINK**

**a** Identify the outcome in which both selections are blue cards.

**b** 1 Identify the outcomes in which a yellow activity card is selected first.

2 Find the sum of the probabilities and simplify.

**WRITE**

**a** $\Pr(\text{two blue}) = \Pr(\text{BB})$

$= \frac{1}{9}$

**b** $\Pr(\text{yellow first}) = \Pr(\text{YB}, \text{YR}, \text{YY})$

$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$

$= \frac{3}{9}$

$= \frac{1}{3}$

## Example 9B.3 Representing experiments with outcomes that are not equally likely

A bag contains 15 red balls and 5 green balls. A ball is selected at random, its colour is recorded and then it is replaced. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment.

**THINK**

1 Draw a tree diagram to represent this experiment, listing all the final outcomes.

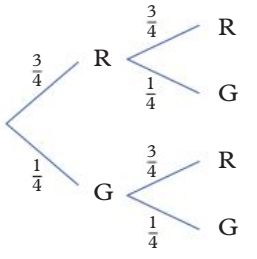
2 Find the probability of a green ball being selected: $\Pr(\text{G}) = \frac{5}{20} = \frac{1}{4}$

Find the probability of a red ball being selected: $\Pr(\text{R}) = \frac{15}{20} = \frac{3}{4}$

3 Write the probability on each branch.

4 Multiply the probabilities along each branch to determine the probability of each final outcome.

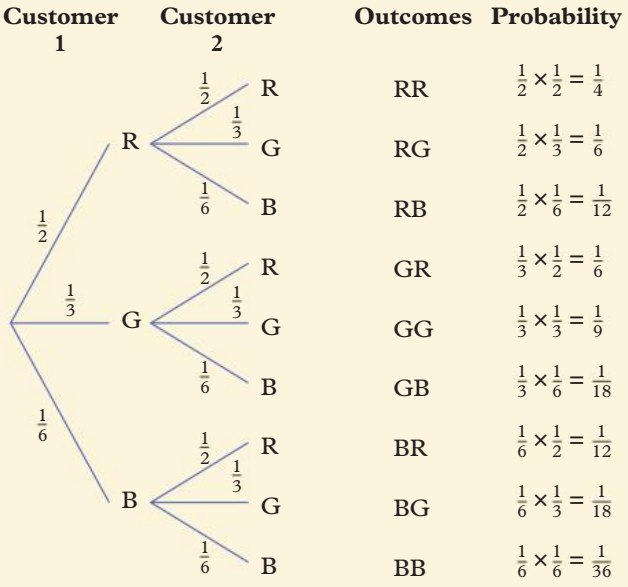
**WRITE**




| Outcomes | Probability |
|---|---|
| RR | $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ |
| RG | $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ |
| GR | $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$ |
| GG | $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ |

## Example 9B.4 Calculating probabilities for experiments with outcomes that are not equally likely

This tree diagram shows the possible outcomes when two customers select a ball at random from a box containing 3 red balls, 2 green balls and 1 blue ball. The balls are returned after the first customer's selection.




Calculate the probability of selecting:

**a** exactly one blue ball

**b** a blue ball and a green ball.

**THINK**

**a** Identify the outcomes in which exactly one blue ball is selected. Find the sum of the probabilities of the outcomes and simplify.

**b** Identify the outcomes in which a blue ball and a green ball are selected (in any order). Find the sum of the probabilities of the outcomes and simplify.

**WRITE**

**a** $\Pr(\text{exactly one blue}) = \Pr(\text{RB}) + \Pr(\text{GB}) + \Pr(\text{BR}) + \Pr(\text{BG})$

$$= \frac{1}{12} + \frac{1}{18} + \frac{1}{12} + \frac{1}{18}$$
$$= \frac{10}{36}$$
$$= \frac{5}{18}$$

**b** $\Pr(\text{blue and green}) = \Pr(\text{GB}) + \Pr(\text{BG})$

$$= \frac{1}{18} + \frac{1}{18}$$
$$= \frac{2}{18}$$
$$= \frac{1}{9}$$

**Helpful hints**

- ✔ The sum of the probabilities at each step of a two-step experiment must add to 1.
- ✔ After calculating the probabilities of the final outcomes, check that the sum of these probabilities is equal to 1.

ANS p551

# Exercise 9B Experiments with replacement

△ 1–9, 13 □ 2, 4–7, 10, 11, 13, 14, 16 ⬠ 2, 4, 6, 7, 11, 12, 15, 16



**1** For each chance experiment, state the theoretical probability of the outcome in brackets in any given trial.

**a** selecting a marble and recording its colour from a bag with 10 blue, 5 red, 10 yellow and 5 green marbles (selecting a green marble)

**b** rolling a standard six-sided die and recording the number on top (rolling a 5)

**c** drawing a card from a standard deck of 52 playing cards (drawing a red picture card – jack, queen or king)

**d** selecting a gift voucher from a lucky dip containing twelve $5 vouchers, eight $10 vouchers, and four $20 vouchers (selecting a $20 voucher)

9B.1 **2** Draw a tree diagram with probabilities on the branches to represent each of these chance experiments.

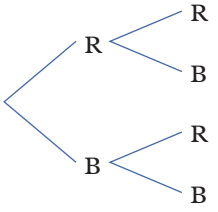
**a** A pencil case contains equal numbers of red and blue pens. A pen is selected, its colour is recorded and then it is replaced. This is repeated one more time.

**b** A box contains equal numbers of $5, $20 and $75 vouchers. A voucher is selected, its value is recorded and then it is replaced. This is repeated one more time.

**c** A box contains 11 cards, numbered 1 to 11. A card is selected, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated one more time.

**d** A ball-pit contains equal numbers of blue, red, yellow and green balls. A ball is selected, its colour is recorded and then it is replaced. This is repeated one more time.

9B.2 **3** A bag contains six red counters and six black counters. A counter is drawn twice, being replaced after the first draw. This tree diagram shows the probabilities of the two selections.




**a** In any given trial, what is the probability of selecting a black counter?

**b** How many final outcomes are there?

**c** What is the probability of selecting:

**i** two black counters? **ii** exactly one black counter? **iii** two red counters?

**4** A box contains milk, dark and white chocolates in equal numbers. A chocolate is selected at random from the box, its flavour recorded and then it is replaced. This is repeated once more. What is the probability that:

**a** both chocolates are white?

**b** at least one chocolate is dark?

**c** the first chocolate is white and the second chocolate is milk?

**d** one chocolate is white and one chocolate is milk?

**5** There are 52 cards in a standard pack of playing cards with 13 cards of each suit (clubs, diamonds, hearts and spades). A card is chosen at random from a pack, its suit is recorded and then it is replaced. This is repeated once more. What is the probability of selecting:

**a** two hearts? **b** at least one diamond? **c** exactly one club?

**d** no spades? **e** at least one diamond or spade? **f** one heart and one club?

UNDERSTANDING AND FLUENCY

9B.3 **6** Draw a tree diagram with probabilities on the branches for each chance experiment.

**a** A pencil case contains five blue pens and two red pens. A pen is chosen, its ink colour is recorded and then it is replaced. This is repeated one more time.

**b** A box contains fifteen $5 vouchers, ten $20 vouchers and five $75 vouchers. A voucher is chosen, its value is recorded and then it is replaced. This is repeated one more time.

**c** A box contains 15 cards, numbered 1 to 15. A card is chosen, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated a total of two times.

**d** A ball-pit contains five blue, four red, three green and two yellow balls. A ball is chosen, its colour recorded, and then it is replaced. This is repeated one more time.

9B.4 **7** A lucky dip contains 10 pink gift vouchers for $100 and 40 green gift vouchers for $10. A voucher is chosen, its value is recorded and then it is replaced. If this is repeated, find the probability that:

**a** a $100 voucher was selected twice

**b** a $100 voucher was not selected at all

**c** a $100 voucher was selected first, and a $10 voucher selected second.

**8** The contents of a pencil case are shown here. The owner of the pencil case takes out a pen at random for each lesson.

**a** Use the pens shown to draw a tree diagram to represent the colours of the pens chosen for the first two lessons of the day. Remember to include the probabilities along each branch and for the final outcomes.

**b** Find the probability that the owner chooses:

**i** a blue pen each time

**ii** a red pen each time

**iii** a black pen each time

**iv** a blue pen, then a black pen

**v** a blue pen, then a red pen

**vi** a red pen, then a black pen.

PROBLEM SOLVING AND REASONING

**9** The probability of selecting a picture card from a pack is $\frac{3}{13}$.
A card is chosen from a standard pack of cards and it is recorded whether it was a picture card or not, before the card is replaced. This is repeated once more.

**a** Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome as a decimal number rounded to four decimal places.

**b** Find the probability of selecting:

**i** exactly one picture card

**ii** at least one picture card

**iii** two picture cards

**iv** no picture cards.

**10** In Example 9B.4 part **a** there are 4 favourable outcomes out of 9 possible outcomes. Explain why the theoretical probability of selecting exactly one blue ball is $\frac{5}{18}$ instead of $\frac{4}{9}$.

**11** A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you choose from a box at random. If you take a red ball you get 10% off, if you take a green ball you get 25% off and if you take a blue ball you get 50% off. There are 10 balls of each colour in the box.
You can have a second chance if you don't take a blue ball out of the box first up, as long as you put the first ball back into the box.
Use a tree diagram or other means to show that you have a $\frac{5}{9}$ chance of selecting a blue ball from the box.

**12** An array can be used to list the outcomes of a two-step experiment with replacement.

**a** Explain why an array may be a better choice to show the outcomes of a two-step experiment with replacement than a tree diagram when there is a large number of equally likely outcomes for each step (even when like outcomes are combined).

**b** Explain how you could represent outcomes that are not equally likely in an array.

**13** A sock drawer contains 10 socks; some are black and some are white. You need to find out how many of each colour are in the drawer, but you can only select one sock at a time and place it back.

**a** If you selected with replacement 10 times and selected 3 black socks and 7 white socks, does this mean that there are 3 black and 7 white socks in the drawer? Explain.

**b** If you selected with replacement 50 times, selecting 21 black socks and 29 white socks, how many socks of each colour would you estimate are in the drawer?

**c** If you selected with replacement 80 times, selecting 34 black socks and 46 white socks, does this support your previous estimate?

**14** A bag contains a number of aqua marbles and a different number of purple marbles. A marble is taken out of the bag at random, the colour is observed, and then the marble is replaced into the bag before a second marble is taken out of the bag at random. If the probability of selecting an aqua marble twice in a row is $\frac{4}{25}$, determine:

**a** the probability of selecting an aqua marble on any given selection

**b** the probability of selecting a purple marble on any given selection

**c** the probability of selecting an aqua and a purple marble in any order

**d** the number of aqua and purple marbles in the bag if there are 35 marbles.

**15** A bag contains different numbers of white, black, and red tiles. A tile is taken out of the bag at random, the colour is observed, then the tile is replaced into the bag before a second tile is taken out of the bag at random. If the probability of selecting a white tile twice in a row is $\frac{25}{144}$ and the probability of selecting a white and a black tile in either order is $\frac{5}{24}$, determine:

**a** the probability of selecting a white tile on any given selection

**b** the probability of selecting a black tile on any given selection

**c** the probability of selecting a red tile on any given selection

**d** the probability of selecting two different coloured tiles in either order.

**16** A bag contains different numbers of tiles labelled A, B, and C. A tile is taken out of the bag at random, the letter is observed, then the tile is replaced into the bag before a second tile is taken out of the bag at random.

Let the probability of selecting a tile labelled A on any given selection be $\Pr(\text{A}) = a$, and the probability of selecting a tile labelled B on any given selection be $\Pr(\text{B}) = b$.

Write an expression for each of the following in terms of $a$ and $b$.

**a** the probability of selecting two tiles labelled A

**b** the probability of selecting a tile labelled A and a tile labelled B in any order

**c** the probability of selecting a tile labelled C on any given selection

**d** the probability of selecting a tile labelled A and a tile labelled C in any order in:

**i** factorised form **ii** expanded form.

**e** the probability of selecting two tiles labelled C in:

**i** factorised form **ii** expanded form.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Experiments with replacement

Topic quiz
9B

PROBLEM SOLVING AND REASONING

CHALLENGE

# 9C Experiments without replacement

## Learning intentions

✔ I can calculate probabilities for two-step chance experiments without replacement.

**Inter-year links**

| | |
|---|---|
| Year 7 | 10F Theoretical probability |
| Year 8 | 9E Theoretical probability |
| Year 10 | 11B Experiments with and without replacement |

## Experiments without replacement

- Experiments without replacement involve selecting an item at random, recording the result, and then not replacing the item before making another selection.
- The selected items are not replaced, so the probability of the remaining items being selected is changed.
- A tree diagram or list of outcomes can help to find the probabilities of individual outcomes or events involving more than one outcome.

  For example, the following tree diagram shows the results of a customer choosing two socks at random from 10 red, 10 green and 10 blue socks. The first sock is not replaced so the probabilities change for subsequent selections.

  If a pair of red socks is selected, the first selection is from 10 red socks out of 30, and the second selection is from 9 red socks out of 29.

  Selection 1: $\Pr(\text{R}) = \frac{10}{30} = \frac{1}{3}$ Selection 2: $\Pr(\text{R}) = \frac{9}{29}$

  If a red then a green sock is selected, the first selection is from 10 red socks out of 30 and the second selection is from 10 green socks out of 29.

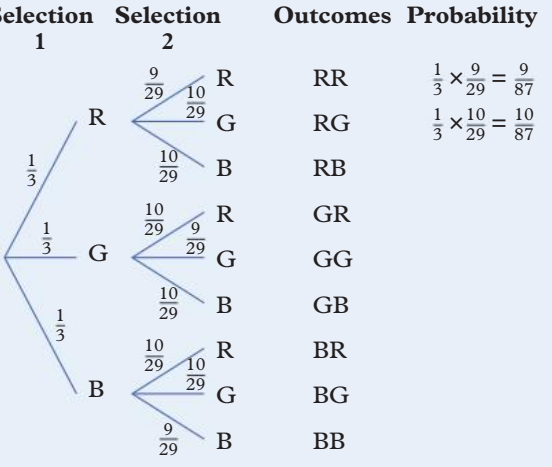
  Selection 1: $\Pr(\text{R}) = \frac{10}{30} = \frac{1}{3}$ Selection 2: $\Pr(\text{G}) = \frac{10}{29}$

  The probability of selecting a pair of socks that are the same colour can be calculated:

  $\Pr(\text{RR}) = \Pr(\text{GG}) = \Pr(\text{BB}) = \frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$

  The probability of selecting a pair of socks that are different colours can be calculated:

  $\Pr(\text{RG}) = \Pr(\text{RB}) = \Pr(\text{GR}) = \Pr(\text{GB}) = \Pr(\text{BR}) = \Pr(\text{BG}) = \frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$




### Example 9C.1 Representing experiments without replacement

A lucky dip contains five red gift vouchers for $50 and five green gift vouchers for $5. A gift voucher is selected and the customer keeps it. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment.

**THINK**

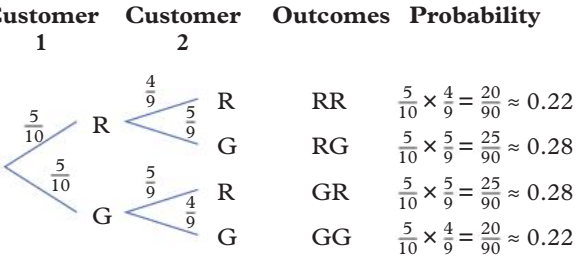
1 Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.

2 Write the probability at the first branches of your tree diagram. Initially there is a $\frac{5}{10}$ chance of selecting a red gift voucher, and a $\frac{5}{10}$ chance of selecting a green gift voucher.

3 Determine the probabilities at the second branches and write on your tree diagram.
- If you select a red gift voucher, 4 red and 5 green vouchers remain.
- If you select a green gift voucher, 5 red and 4 green gift vouchers remain.

4 Multiply the probabilities along each branch to determine the probability of each final outcome.

**WRITE**




## Example 9C.2 Calculating probabilities for experiments without replacement

Use the tree diagram from Example 9C.1 to find the probability that:

**a** both customers select a $50 voucher

**b** the first customer selects a $5 voucher and the second customer selects a $50 voucher.

**THINK**

**a** Identify the outcome where both customers select a $50 voucher.

**b** Identify the outcome where the first customer selects a $5 voucher and the second customer selects a $50 voucher.

**WRITE**

**a** $\Pr(\text{both \$50}) = \Pr(\text{RR})$

$= \frac{20}{90}$

$= \frac{2}{9}$

**b** $\Pr(\text{first \$5 and second \$50}) = \Pr(\text{GR})$

$= \frac{25}{90}$

$= \frac{5}{18}$

**Helpful hints**

- ✔ The number of total outcomes will reduce by 1 after the first step of a two-step chance experiment without replacement.
- ✔ The number of favourable outcomes will reduce by 1 only for the outcome occurring in the first step.
- ✔ When calculating probabilities involving multiple final outcomes, it is often easier to not simplify fractions until the end of your working, as it is easier to add fractions when they have a common denominator.

ANS p553

# Exercise 9C Experiments without replacement

△ 1(a, b, d), 2–4, 6, 7, 9, 10, 14 □ 1(b, c, e), 2, 4–6, 8, 11, 12, 13(a), 15 ⬠ 1(c, f), 4–6, 11–13, 15–17

UNDERSTANDING AND FLUENCY

9C.1 **1** Draw a tree diagram with probabilities on the branches for each of these chance experiments without replacement.

**a** A drawer contains five black socks and five white socks. A sock is selected at random and its colour recorded. This is repeated one more time.

**b** An esky contains six cans of Coke and six cans of Pepsi. A can is selected at random and its type recorded. This is repeated one more time.

**c** A box contains five 16 GB SD cards, five 32 GB SD cards and five 64 GB SD cards. A card is selected at random and its capacity recorded. This is repeated once more.

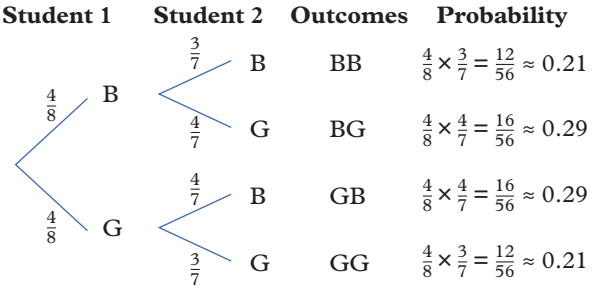
**d** A bowl contains 10 Smarties and 10 M&M's. A chocolate is selected at random and its type recorded. This is repeated once more.

**e** A small ball-pit contains 10 blue, 10 red, 10 yellow and 10 green balls. A ball is selected at random and its colour recorded. This is repeated one more time.

**f** A box contains 10 names from 9D, 10 names from 9E and 10 names from 9F. A name is selected at random and the class is recorded. This is repeated a total of once more.

**2** The following tree diagram shows how two students can be selected at random from a group of four boys and four girls. Find the probability of selecting:

**a** two boys **b** no boys **c** a boy and a girl.




**3** A lucky dip contains four purple gift vouchers for $100 and four yellow gift vouchers for $10. A gift voucher is chosen at random and the customer keeps it. If this was repeated for a second customer, find the probability that:

**a** both customers select a $100 voucher

**b** the first customer selects a $100 voucher and the second customer selects a $10 voucher.

**4** Each bonbon in a pack of 12 contains one toy, and there are three different kinds of toys: a whistle, a yo-yo and a bouncy ball. In total, the pack contains four of each kind of toy. You and a friend each select a bonbon at random from the pack.

**a** Draw a tree diagram to represent this situation. Be sure to include probabilities on each branch and the final probabilities for each outcome.

**b** Find the probability that:

**i** you both select a bonbon with a whistle

**ii** the first bonbon has a yo-yo and the second has a bouncy ball.

**5** Consider the bonbons from question **4**. Find the probability that:

**a** you both select a bonbon with a different toy

**b** at least one of you selects a bonbon with a bouncy ball

**c** a bonbon with a whistle is not chosen.

**6** Experiments without replacement can also start with unequally likely outcomes. A lucky dip contains 15 red, 10 green and 5 blue balls. Customers select one ball at random and they do not replace the ball before the next customer makes a selection.

**a** Complete the tree diagram.

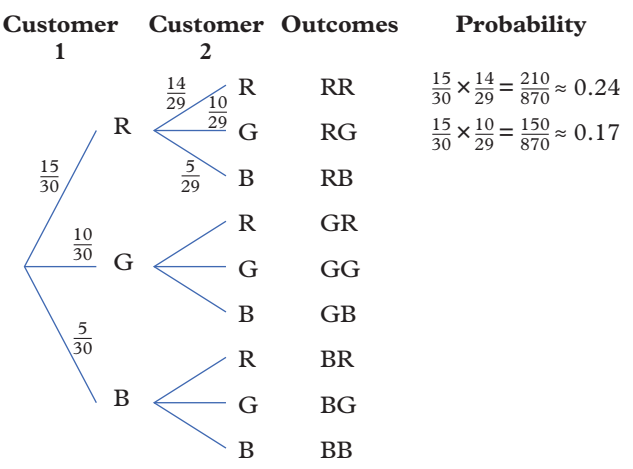
**b** Use the tree diagram to calculate the probability that:

**i** both customers select a blue ball

**ii** both customers select a green ball

**iii** both customers select a red ball

**iv** the first customer selects a red ball and the second selects a green ball.




**7** Use the tree diagram from question **6** to calculate the probability of selecting:

**a** at least one green ball **b** at least one red ball **c** at least one blue ball

**d** exactly one red ball **e** exactly one green ball **f** a blue ball and a green ball.

**8** There are 52 cards in a standard pack of playing cards, with 13 cards of each suit (clubs, diamonds, hearts and spades).

A card is selected and its suit is recorded. The card is not replaced and another card is selected.

What is the probability of selecting:

**a** two hearts? **b** at least one diamond?

**c** no spades? **d** one heart and one club?

**9** A drawer contains two pink socks, two purple socks and two green socks. Use a tree diagram or other means to alculate the probability that you take out a pair when you select two socks from the drawer at random.

**10** Imagine instead that the drawer from question **9** contains six socks of each colour. How does this change the probability of selecting a pair when you select two socks from the drawer at random?

**11** If a drawer contained five red socks, four black socks and three white socks, find the probability that the first two socks selected at random from the drawer form a pair.

**12** A sports team needs to select a captain and a vice-captain. Five people have put their names forward: Adrian, Chantelle, Katie, Ben and Sam.

**a** Draw a tree diagram to represent the selection (start with 'captain' branches).

**b** How does this tree diagram differ from the other ones you have done beforehand? (Hint: Does the second set of branches contain the same number as the first set of branches?)

**c** How many different combinations of captains and vice-captains are there? Remember that order is important!

**d** If each person has an equal chance, then find the probability that:

**i** Katie is selected captain **ii** Sam is selected either captain or vice-captain

**iii** Adrian is captain and Chantelle is vice-captain **iv** Ben does not get a position

**v** Katie and Sam both get a position.

**13** An array can be used to show the outcomes for a two-step experiment without replacement. It is important to show the outcomes that cannot occur as they are the repeated outcomes that have been removed after the first step. We show this by placing a cross in that cell. For example, when selecting two marbles from a bag that contains five red marbles and four blue marbles without replacement, the array can be drawn as shown.

**a** Determine the probability of selecting the following combinations of marbles from the bag with five red marbles and four blue marbles.

**i** two red marbles

**ii** two blue marbles

**iii** a red marble then a blue marble

**iv** a red and a blue marble in either order

| Second \ First | R | R | R | R | R | B | B | B | B |
|---|---|---|---|---|---|---|---|---|---|
| R | × | RR | RR | RR | RR | BR | BR | BR | BR |
| R | RR | × | RR | RR | RR | BR | BR | BR | BR |
| R | RR | RR | × | RR | RR | BR | BR | BR | BR |
| R | RR | RR | RR | × | RR | BR | BR | BR | BR |
| R | RR | RR | RR | RR | × | BR | BR | BR | BR |
| B | RB | RB | RB | RB | RB | × | BB | BB | BB |
| B | RB | RB | RB | RB | RB | BB | × | BB | BB |
| B | RB | RB | RB | RB | RB | BB | BB | × | BB |
| B | RB | RB | RB | RB | RB | BB | BB | BB | × |

**b** Another bag contains three green marbles and one purple marble. Two marbles are selected from the bag at random without replacement.

**i** Draw an array to show the outcomes for this experiment.

**ii** Determine the probability of selecting one purple marble out of the two marbles.

**iii** Determine the probability of selecting two purple marbles.

**14** A box of chocolates contains four milk chocolates, three white chocolates and two dark chocolates. Two chocolates are selected from the box, without looking. Find the probability of selecting:

**a** two white chocolates

**b** no dark chocolates

**c** two different types of chocolate

**d** at least one white chocolate.

**15** A shuffle play removes any song that has been played in the current play - through of the playlist, then randomly selects from the remaining tracks. A paticular playlist contains 12 songs from album A, 7 songs from album B, and 9 songs from album C. What is the probability that:

**a** two songs in a row are from album B

**b** two songs in a row are from the same album

**c** two songs in a row are from different albums.

**16** A lottery consists of 45 numbered balls, of which six winning balls are chosen at random, with none of the six balls drawn replaced. To win the first division prize, you must pick all six winning numbers. What is the probability of winning the first division prize in this lottery? Express your answer as both a fraction and a decimal correct to three significant figures.

**17** An album contains nine songs and the lengths of the songs are shown in the table in minutes and seconds. Find the probability that the first two different songs played on shuffle are above the mean length of a track on the album.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4:17 | 1:08 | 4:22 | 1:38 | 4:16 | 0:53 | 4:20 | 1:44 | 4:46 |

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Interactive skillsheet
Experiments without replacement

Investigation
Draws for tennis tournaments

Topic quiz
9C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

**9A** **1** A fair coin is tossed twice in a row. Draw a tree diagram that shows all outcomes for this experiment.

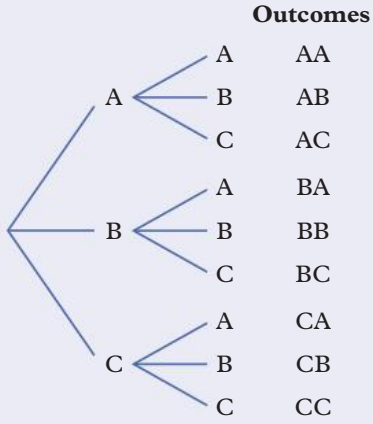
**9A** **2** A fair, four-sided die numbered 1 to 4 is rolled twice in a row. Draw an array that shows all outcomes for this experiment.

**9A** **3** Use the tree diagram shown on the right to determine the probability of rolling:

**a** two consonants

**b** a vowel and a consonant

**c** at least one consonant.




**9B** **4** Use the array shown to determine the probability of:

**a** B and Y

**b** B or Y or both

**c** B or Y but not both.

| | A | B | C | D |
|---|---|---|---|---|
| **X** | (A, X) | (B, X) | (C, X) | (D, X) |
| **Y** | (A, Y) | (B, Y) | (C, Y) | (D, Y) |
| **Z** | (A, Z) | (B, Z) | (C, Z) | (D, Z) |

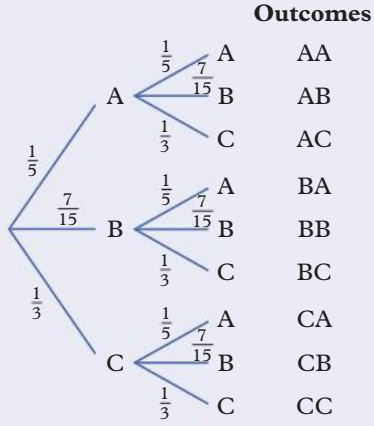
**9B** **5** A bag contains three pieces of marble and four pieces of quartz. A piece is selected from the bag at random, returned to the bag, and then another piece is selected. Draw a tree diagram showing all outcomes with probabilities on the branches for this experiment.

**9B** **6** Use the tree diagram shown on the right to determine the probability of:

**a** the same letter twice in a row

**b** at least one A

**c** less than two Cs.




**9B** **7** Use your tree diagram from question **6** to determine the probability of:

**a** selecting a piece of marble and quartz

**b** selecting a piece of marble second

**c** selecting at least one piece of quartz.

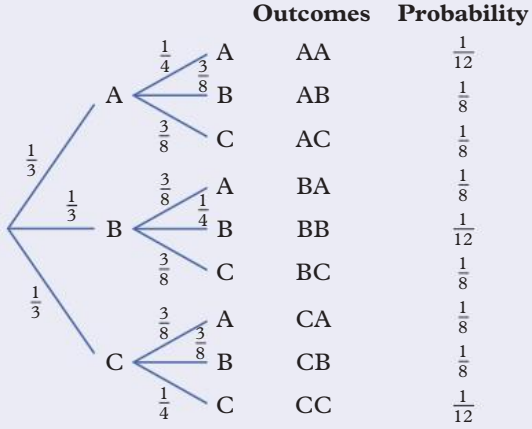
**9C** **8** A bag contains nine tiles, three tiles labelled A and six labelled B. A tile is drawn at random, the letter observed, then another tile is drawn without replacing the first. Draw a tree diagram with probabilities on the branches for this experiment.

**9C** **9** Use the tree diagram shown on the right to determine the probability of:

**a** the same letter twice in a row

**b** at least one A

**c** less than two Cs.




**9C** **10** Use your tree diagram from question **9** to determine the probability of:

**a** the same letter twice in a row

**b** a tile labelled A second

**c** at least one B.

# 9D Relative frequency

## Learning intentions

- ✔ I can calculate relative frequencies.
- ✔ I can calculate the expected number of times an outcome will occur in an experiment.

Inter-year links
Year 5/6 Chance experiments
Year 7 10G Experimental probability

## Relative frequency

- **Relative frequency** is calculated using the results of experiments or surveys rather than using theoretical probability.
- The relative frequency of a random event is the number of times a favourable outcome occurs divided by the total number of trials.
  - → Relative frequency = $\frac{\text{number of occurrences}}{\text{total number of trials}}$
- The **expected number** of each outcome in a chance experiment can be used to investigate whether there is any bias in an experiment. To calculate an expected number, multiply the theoretical probability by the number of trials in the experiment.
  - → Expected number = theoretical probability × number of trials
- If there is bias in a chance experiment, the experiment will favour one outcome over another outcome even when they are supposed to have an equal likelihood of occurring.
  - → When considering if a chance experiment is biased, consider both the expected number of occurrences and the actual number of occurrences. If there is a sizable difference, consider the total number of trials.
  - → With a small number of trials, the results will often vary significantly from the expected numbers, but in the long term you can expect the relative frequency of a random event to approach the theoretical probability due to a powerful concept in probability called the law of large numbers.
  - → Start from the assumption that bias does not exist, and if you suspect bias may exist, carry out more trials to confirm your assumption.
- The theoretical probabilities of the outcomes from surveys and statistical investigations are not usually known. So, the relative frequency can be used to estimate the theoretical probability of outcomes in real-world events.

### Example 9D.1 Calculating relative frequencies

In Melbourne it rained for 11 days in March. Find the relative frequency of rainy days in March correct to two decimal places.

**THINK**

1. Write the relative frequency formula.
2. Identify the number of occurrences (11) and the total number of outcomes (31). Substitute into the formula and solve, simplifying if possible.
3. Convert to a decimal number and write your answer.

**WRITE**

Relative frequency = $\frac{\text{number of occurrences}}{\text{total number of outcomes}}$

$= \frac{11}{31}$

$\approx 0.35$

The relative frequency of rainy days in Melbourne in March is 0.35.

## Example 9D.2 Calculating an expected number

Find the expected number of 5s or 6s if a standard six-sided die is rolled 90 times.

**THINK**

1 Write the formula for an expected number.

2 Identify the theoretical probability of rolling a 5 or a 6 $\left(\frac{2}{6}\right)$ and the number of trials (90) and substitute into the formula.

**WRITE**

Expected number = theoretical probability × number of trials

$$= \frac{2}{6} \times 90$$
$$= 30$$

## Example 9D.3 Comparing the relative frequency and expected number

A standard six-sided die is rolled 80 times and a 5 is obtained 17 times.

**a** Find the expected number of 5s if a standard six-sided die is rolled 80 times.

**b** Comment on whether you believe this die may be biased.

**THINK**

**a** 1 Write the expected number formula.

2 Substitute the theoretical probability $\left(\frac{1}{6}\right)$ and the number of trials (80) into the formula and evaluate.

**b** Look to see if there is a significant difference between the expected number and the actual number. Consider the number of trials and make a reasonable assessment with the given information.

**WRITE**

**a** Expected number = theoretical probability × number of trials

$$= \frac{1}{6} \times 80$$
$$\approx 13$$

**b** Although there is a difference between the expected number and the actual number, the difference is not significant. To determine whether the die is biased, more trials should be conducted.

**Helpful hints**

- ✔ The minimum value of a relative frequency is 0. For example, the number of rainy days cannot be −2 days out of 31 days. The minimum number of rainy days is 0 days out of 31 days.
- ✔ The maximum value of a relative frequency is 1. For example, the number of rainy days cannot be 32 days out of 31 days. The maximum number of rainy days is 31 days out of 31 days.
- ✔ The results from an experiment or survey are more reliable when the sample size is larger. This should be considered when drawing conclusions from data.

ANS p556

# Exercise 9D Relative frequency

1–3, 5, 6, 7(a, b), 8, 10 | 1, 2, 4–6, 8, 10, 11, 13(a, b) | 1–2(d–f), 5, 6, 9, 10, 12, 13

UNDERSTANDING AND FLUENCY

9D.1 **1** Find the relative frequency of the outcomes in the following situations. Give your answers as fractions and decimals, rounded to two decimal places where necessary.

**a** Of 50 families surveyed about pets, 18 had a pet dog.

**b** Of 300 people surveyed about being right-handed, left-handed or ambidextrous, 3 people were ambidextrous.

**c** Of 56 people surveyed about playing sport, 21 people did not play a team sport.

**d** Of 450 people surveyed, 300 people had less than two video game consoles.

**e** Of 348 people surveyed about pizza or pasta, 87 people preferred pasta.

**f** Of 1200 cats that had their tails measured, 655 had tails longer than 30 cm.

9D.2 **2** Find the expected number of:

**a** heads if a coin is tossed 250 times

**b** 1s or 2s if a standard six-sided die is rolled 120 times

**c** consonants if a letter is selected randomly from the alphabet 130 times

**d** 6s if a standard six-sided die is rolled 30 times

**e** two-digit integers if an integer from 1 to 999 is selected 444 times

**f** double heads if a pair of coins is tossed 856 times.

**3** A standard six-sided die is rolled 150 times and the results are shown in this table.

| **Outcome** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Frequency** | 38 | 32 | 19 | 24 | 21 | 16 |

**a** Find the relative frequency of rolling a number greater than 2, based on this data. Give your answer as a fraction and a decimal rounded to two decimal places.

**b** Assuming the die is not biased, describe how you would expect the relative frequency of rolling a number greater than 2 to change in the long term.

**4** A standard pack of 52 cards has four suits (diamonds, clubs, hearts and spades), with each suit consisting of 10 number cards (Ace to 10) and three picture cards (Jack, Queen and King).

A card is chosen at random from a pack and replaced. This is repeated 129 times for a total of 130 selections.

**a** What is the expected number of picture cards?

**b** If seven picture cards were obtained, find the relative frequency of selecting a picture card. Give your answer as a fraction and a decimal correct to three decimal places.

**c** Assuming the deck is not biased, describe how you would expect the relative frequency of selecting a picture card to change in the long term.

**5** A number of experiments were performed and their results recorded below. For each experiment, find the number of times each outcome occurred.

**a** Total number of trials = 40

| **Outcome** | Heads | Tails |
|---|---|---|
| **Relative frequency** | 0.625 | 0.375 |

**b** Total number of trials = 120

| **Outcome** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Relative frequency** | 0.15 | 0.2 | 0.175 | 0.1 | 0.125 | 0.25 |

**c** Total number of trials = 60

| **Outcome** | Hearts | Diamonds | Clubs | Spades |
|---|---|---|---|---|
| **Relative frequency** | 0.2 | 0.3 | 0.35 | 0.15 |

9D.3 **6** Consider each of these situations:

**a** rolling a 4, if a standard six-sided die is rolled 180 times and a 4 is obtained 27 times

**b** tossing a tail, if a coin is tossed 36 times and a head is obtained 16 times

**c** rolling an odd number, if a standard six-sided die is rolled 200 times and an odd number is obtained 53 times

**i** Find the expected number of outcomes. Where necessary, give your answer to the nearest whole number.

**ii** Comment on whether you believe the experiment may be biased.

**7** A magician uses a number of props in his show to demonstrate magic tricks. He tosses a coin, rolls a six-sided die and selects a card from a pack. To test whether the props are fair (standard) or biased, the results over several shows were recorded and are shown in the tables below.

**a**

| **Outcome** | Heads | Tails |
|---|---|---|
| **Frequency** | 8 | 2 |

**b**

| **Outcome** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Frequency** | 966 | 971 | 1036 | 994 | 1031 | 1002 |

**c**

| **Outcome** | Hearts | Diamonds | Clubs | Spades |
|---|---|---|---|---|
| **Frequency** | 38 | 31 | 19 | 24 |

For each experiment:

**i** find the total number of trials

**ii** state the theoretical probability of each outcome

**iii** calculate the expected number of each outcome

**iv** calculate the relative frequency of each outcome based on the data

**v** state whether you think the prop used is fair, biased or if there are not enough trials to make a firm decision

**vi** give a reason to support your answer to part **v**.

**8** Three coins (5c, 10c and 20c) are tossed one at a time.

**a** There are 8 possible outcomes; for example, HTH represents a head, tail and head outcome. List all the possible outcomes.

**b** What is the theoretical probability of tossing a 'triple'?

**c** How many 'triples' would you expect to get if you performed 40 trials?

The experiment is performed 40 times and 5 triples are recorded.

**e** Does the actual occurrence of a 'triple' match the expected number?

**f** Describe how you would expect the relative frequency to change if you performed 4000 trials.

PROBLEM SOLVING AND REASONING

**9** A four-sided die (numbered 1–4) is rolled 20 times and the number 2 appears uppermost on 13 occasions. Explain why, considering the law of large numbers, this does not mean that the die is biased, and what should be done to check for any bias.

**10** Tulio reads a headline that said '51% of adults worldwide don't get enough sleep'. Concerned for his parents' health, he asks if they both get the recommended 7–8 hours of sleep, which they do. Tulio is now sceptical of the headline.

**a** Explain why Tulio's sample would not verify or refute the statistics in the headline regardless of his parents' responses.

Later, Tulio finds another article that says 'one in three Australians not getting enough sleep'. He believes this is more accurate because he only asked two people, so the third person he asks won't be getting enough sleep.

**b** Assuming the statistic is accurate, explain why Tulio would not be guaranteed to have a person that is not getting enough sleep next.

**c** Give at least two reasons why the statistics from the two headlines are different.

**11** Miguel read two similar headlines '39% of cricket fans are female' and '20% of cricket fans are female'. After reading the articles more closely, Miguel found out that the 39% group comprised over 19 000 interviews, whereas the 20% group was from a sample of 2000. Which percentage is more reliable? Why?

**12** Julia read an article that said, 'about 10% of the world population is left-handed.' The article also mentions that it was only about 4% in 1920 when left-handedness was stigmatised. Explain why we do not know the exact percentage of the world population that is left-handed.

CHALLENGE

**13** Sometimes people fake data, because it can be either time-consuming or costly to collect primary data. Consider the instance of faking data on coin tosses, where people who create fake lists of results often avoid putting in long streaks of heads or tails because they think it would not happen in reality.

**a** If you toss a coin 7 times, what is the probability you get 7 heads?

**b** Explain why you would expect to get at least one streak of 7 or more heads if you toss a coin 1000 times.

**c** If you toss a coin 1000 times, how many streaks of (at least) 5 heads in a row do you expect?

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Relative frequency

Topic quiz
9D

# 9E Two-way tables

## Learning intentions

- ✔ I can create two-way relative frequency tables.
- ✔ I can use two-way tables to solve problems.

Inter-year links

| | |
|---|---|
| Year 5/6 | Understanding probability |
| Year 7 | 10F Theoretical probability |
| Year 8 | 9F Two-way tables |
| Year 10 | 11C Two-way tables and Venn diagrams |

## Two-way tables

- **Two-way tables** display the results of a survey or experiment with two different categories. The results can be displayed as raw numbers or as relative frequencies. For example, both of these two-way tables display the same data.

| | Dark hair | Light hair | Total |
|---|---|---|---|
| **Dark eyes** | 16 | 4 | 20 |
| **Pale eyes** | 8 | 12 | 20 |
| **Total** | 24 | 16 | 40 |

| | Dark hair | Light hair | Total |
|---|---|---|---|
| **Dark eyes** | 0.4 | 0.1 | 0.5 |
| **Pale eyes** | 0.2 | 0.3 | 0.5 |
| **Total** | 0.6 | 0.4 | 1 |

## Identifying the required outcome

- The language used to describe an outcome can help you to identify which cells of the two-way table are being referred to.
  - → 'and' means both outcomes must be met. For example, 16 people have dark eyes and dark hair.
  - → 'or' can be inclusive, meaning if both outcomes are met the outcome is favourable. For example, 4 + 8 + 12 = 24 have pale eyes or light hair.
  - → 'or' can also be exclusive, meaning if both outcomes are met but the outcome is not favourable. For example, 4 + 8 = 12 people have pale eyes or light hair but not both.

### Example 9E.1 Understanding a two-way table

Consider this two-way table, which shows whether Year 8 and Year 9 students prefer jam or Vegemite.

**a** How many students were surveyed?

**b** How many students are in Year 9 and prefer Vegemite?

| | Year 8 | Year 9 | Total |
|---|---|---|---|
| **Jam** | 28 | 38 | 66 |
| **Vegemite** | 25 | 34 | 59 |
| **Total** | 53 | 72 | 125 |

**THINK**

**a** The total number of people surveyed is displayed in the bottom right-hand corner.

**b** Identify the cell that is in the 'Year 9' column and the 'Vegemite' row.

**WRITE**

**a** 125 students were surveyed.

**b** 34 students are in Year 9 and prefer Vegemite.

## Example 9E.2 Creating a two-way relative frequency table

This two-way table shows whether a group of 50 people from Melbourne and Perth own a pet or not. Convert it into a two-way relative frequency table.

| | Melbourne | Perth | Total |
|---|---|---|---|
| **Owns pet** | 13 | 19 | 32 |
| **Does not own pet** | 11 | 7 | 18 |
| **Total** | 24 | 26 | 50 |

**THINK**

Divide the values in each cell by the value in the total cell in the bottom right-hand corner.
The totals should add to 1 in the bottom right-hand corner.

**WRITE**

| | Melbourne | Perth | Total |
|---|---|---|---|
| **Owns pet** | $\frac{13}{50} = 0.26$ | $\frac{19}{50} = 0.38$ | $\frac{32}{50} = 0.64$ |
| **Does not own pet** | $\frac{11}{50} = 0.22$ | $\frac{7}{50} = 0.14$ | $\frac{18}{50} = 0.36$ |
| **Total** | $\frac{24}{50} = 0.48$ | $\frac{26}{50} = 0.52$ | 1 |

## Example 9E.3 Creating and completing a relative frequency two-way table

A survey collected information on whether motorists from the country and city drive automatic or manual cars. 45% of the respondents are from the city and drive automatic cars, 43% of the respondents are from the country and 34% of the respondents drive manual cars.
Use this information to complete a relative frequency two-way table.

**THINK**

1. Draw the table with one of the categories (car type) placed in the rows and the other category (location) placed in the columns.
2. Convert the percentages to relative frequencies by dividing by 100, e.g. 43% = 0.43.
3. Insert the given information in the relevant cells. Remember that the total in the bottom right-hand corner is always 1.
4. The numbers in each row and column add to give the value in the total cells, so wherever there are two out of three cells in a row or column, the unknown value can be found by using addition or subtraction.

**WRITE**

| | City | Country | Total |
|---|---|---|---|
| **Automatic** | 0.45 | 0.66 – 0.45 = 0.21 | 1 – 0.34 = 0.66 |
| **Manual** | 0.57 – 0.45 = 0.12 | 0.43 – 0.21 = 0.22 | 0.34 |
| **Total** | 1 – 0.43 = 0.57 | 0.43 | 1 |

## Example 9E.4 Calculating probability using a two-way table

Consider the two-way table in example 9E.1, showing whether Year 8 and 9 students prefer jam or Vegemite. What is the probability of a student chosen randomly from the group being in Year 8 or preferring jam (or both)?

| | Year 8 | Year 9 | Total |
|---|---|---|---|
| **Jam** | 28 | 38 | 66 |
| **Vegemite** | 25 | 34 | 59 |
| **Total** | 53 | 72 | 125 |

**THINK**

1 Identify the relevant cells in the table that represent this outcome (any cell in the Year 8 column or Jam row).

2 Divide the total of these cells by the total in the table. Simplify if possible.

**WRITE**

$$\frac{28 + 38 + 25}{125} = \frac{91}{125}$$

**Helpful hints**

- ✔ When adding the totals in rows and columns, be careful not to double the cells where the rows and columns intersect.
- ✔ In some problems it may appear that there is insufficient information to answer the question. Start by writing down what you know and then use this information to find the unknown values.

ANS p557

# Exercise 9E Two-way tables

△ 1, 3–10, 12 □ 2, 4–9, 11, 14, 15(a), 16(a–d) ⬠ 4, 6–9, 11, 13, 15(c), 16, 17

9E.1 9E.4

**1** Consider this two-way table, which shows whether students in primary school (PS) and high school (HS) prefer to watch or play sport.

| | PS | HS | Total |
|---|---|---|---|
| **Watch sport** | 8 | 7 | 15 |
| **Play sport** | 22 | 18 | 40 |
| **Total** | 30 | 25 | 55 |

**a** How many students were surveyed in total?

**b** How many students in high school prefer to watch sport?

**c** What is the probability of a student chosen randomly from the group being in high school and preferring to watch sport?

**2** Consider this two-way table, which shows the hair colours and eye colours of a group of people.

| | Fair | Dark | Total |
|---|---|---|---|
| **Blue/green** | 23 | 11 | 34 |
| **Brown** | 6 | 35 | 41 |
| **Total** | 29 | 46 | 75 |

**a** How many people were surveyed in total?

**b** How many people with dark hair have blue/green eyes?

**c** What is the probability of a person chosen randomly from the group having dark hair or blue/green eyes but not both?

**3** This two-way table shows whether Year 8 and Year 9 students prefer alternative or mainstream music. Use this two-way table to find the probability of a person chosen randomly from the group:

| | Year 8 | Year 9 | Total |
|---|---|---|---|
| **Alternative** | 14 | 23 | 37 |
| **Mainstream** | 29 | 19 | 48 |
| **Total** | 43 | 42 | 85 |

**a** being a Year 8 student who prefers mainstream music

**b** preferring alternative music

**c** being a Year 9 student

**d** being a Year 9 student who prefers alternative music.

**4** This two-way table shows whether Year 7, 8 and 9 students prefer tennis, basketball or hockey. Use this table to find the probability that a person chosen randomly from this group:

| | Year 7 | Year 8 | Year 9 | Total |
|---|---|---|---|---|
| **Tennis** | 23 | 26 | 37 | 86 |
| **Basketball** | 19 | 42 | 34 | 95 |
| **Hockey** | 31 | 13 | 25 | 69 |
| **Total** | 73 | 81 | 96 | 250 |

**a** is in Year 9

**b** plays basketball

**c** is in Year 8 and plays tennis

**d** plays hockey or tennis

**e** does not play hockey

**f** is in Year 7 and doesn't play basketball.

9E.2 **5** This two-way table shows whether students from two classes prefer sweet or savoury food. Convert the two-way table into a two-way relative frequency table.

| | Class A | Class B | Total |
|---|---|---|---|
| **Prefer sweet food** | 23 | 32 | 55 |
| **Prefer savoury food** | 29 | 16 | 45 |
| **Total** | 52 | 48 | 100 |

**6** This two-way table shows whether a group of introverts and extroverts prefer to read books, play games or play sports. Convert the two-way table into a two-way relative frequency table.

| | Reads books | Plays games | Plays sports | Total |
|---|---|---|---|---|
| **Introvert** | 56 | 44 | 32 | 132 |
| **Extrovert** | 12 | 44 | 68 | 124 |
| **Total** | 68 | 88 | 100 | 256 |

9E.3 **7** A survey collected information on whether people with light or dark hair preferred pop music or rock music. 22% of the respondents had light hair and preferred pop music, 65% of the respondents had dark hair and 40% of the respondents preferred rock music.

Use this information to complete a relative frequency two-way table.

**8** A survey collected information on whether people who subscribe to Spotify also subscribe to Netflix. $\frac{1}{6}$ of the respondents had a subscription to both Spotify and Netflix, $\frac{1}{4}$ of the respondents had a subscription to Spotify, and $\frac{1}{3}$ did not have a subscription to Netflix. Use this information to complete a relative frequency two-way table.

**9** **a** Copy and complete this two-way table, which shows whether people prefer to watch films at the cinema or at home, and whether they prefer action or comedy films.

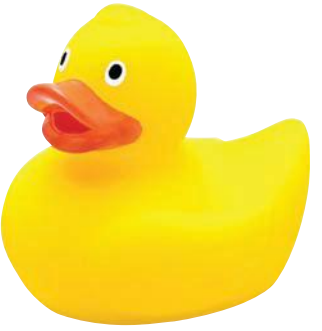
| | Cinema | Home | Total |
|---|---|---|---|
| **Action** | 22 | | |
| **Comedy** | | 14 | 33 |
| **Total** | | | 85 |

**b** Use it to calculate the probability that a person chosen randomly from the group is someone who prefers to:

**i** watch films at the cinema **ii** watch comedy films at home **iii** watch action films.

**10** A group of 200 people with a single pet were surveyed on their pets. Of the 113 who owned cats, 29 had a purebred. This gives a total of 104 pet owners who owned a purebred. Create a two-way table showing this information and use it to calculate the probability that a person chosen at random is an owner of a purebred that is not a cat.

**11** A group of people were surveyed on their bathing habits. 60% of women surveyed preferred a bath to a shower, whereas 80% of men said they preferred a shower rather than a bath. 55% of the group was female.



**a** Create a relative frequency two-way table to represent this information. Hint: The statement '60% of women prefer a bath' refers to 60% of the proportion of women, not the total.

**b** If a person was randomly selected from a group of 500 people, find the probability that they are:

**i** a male who prefers to shower

**ii** someone who prefers a bath

**iii** a female who prefers to shower.

**c** Of a group of 500 people, find the number of people who:

**i** prefer a bath to a shower **ii** are female **iii** are male and prefer a bath.

**12** Elsa recorded the make and colour of cars that went past her house over a week and recorded her results as shown. Add totals to her results and then find the probability that a car going past her house is:

**a** white **b** a Holden **c** a white Holden

**d** blue or red **e** not a Toyota **f** not black

**g** neither red nor a Ford **h** blue but not a Hyundai **i** a Mitsubishi but not silver.

| | Ford | Hyundai | Holden | Mitsubishi | Toyota | Other |
|---|---|---|---|---|---|---|
| **Silver** | 11 | 9 | 17 | 8 | 6 | 12 |
| **White** | 15 | 12 | 16 | 11 | 8 | 10 |
| **Red** | 12 | 10 | 8 | 6 | 13 | 14 |
| **Blue** | 8 | 15 | 12 | 7 | 9 | 11 |
| **Black** | 13 | 8 | 10 | 10 | 7 | 8 |
| **Other** | 4 | 3 | 4 | 2 | 1 | 2 |

**13** $n(\text{A})$ means the number of elements in category A. For the relative frequency two-way table shown, determine the population size if:

**a** $n(\text{A}) = 30$ **b** $n(\text{B}) = 72$ **c** $n(\text{A and B}) = 98$ **d** $n(\text{A or B}) = 102$

| | A | A' | Total |
|---|---|---|---|
| **B** | 0.1 | 0.35 | 0.45 |
| **B'** | 0.4 | 0.15 | 0.55 |
| **Total** | 0.5 | 0.5 | 1 |

**14** Consider the relative frequency two-way table below. If the total frequency is 120, then draw the frequency two-way table.

| | A | A' | Total |
|---|---|---|---|
| **B** | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{7}{12}$ |
| **B'** | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{5}{12}$ |
| **Total** | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 |

PROBLEM SOLVING AND REASONING

**15** Complete each pair of frequency and relative frequency two-way tables.

**a**

| | A | A' | Total |
|---|---|---|---|
| **B** | 40 | | 75 |
| **B'** | | | |
| **Total** | | | |

| | A | A' | Total |
|---|---|---|---|
| **B** | 0.2 | | |
| **B'** | | | |
| **Total** | 0.4 | | 1 |

**b**

| | C | C' | Total |
|---|---|---|---|
| **D** | | | 9 |
| **D'** | | 9 | |
| **Total** | | | |

| | C | C' | Total |
|---|---|---|---|
| **D** | | 0.25 | |
| **D'** | | | |
| **Total** | 0.3 | | 1 |

**c**

| | E | E' | Total |
|---|---|---|---|
| **F** | 18 | | |
| **F'** | | 12 | |
| **Total** | | | |

| | E | E' | Total |
|---|---|---|---|
| **F** | | $\frac{1}{9}$ | $\frac{11}{18}$ |
| **F'** | | | |
| **Total** | | | 1 |

CHALLENGE

**16** Consider this two-way table, which shows whether a group of Year 8 and Year 9 students prefer Maths or English.

| | Year 8 | Year 9 | Total |
|---|---|---|---|
| **Maths** | | 11 | |
| **English** | 10 | | |
| **Total** | 22 | | 50 |

**a** Copy and complete the table.

**b** Use it to find the probability that a student selected randomly from the group:

**i** prefers Maths **ii** is in Year 9 **iii** is in Year 9 and prefers Maths.

**c** A student is selected randomly from the group. You know that they are in Year 9.

**i** How many students are in Year 9? **ii** How many students in Year 9 prefer Maths?

**iii** What is the probability that a student in Year 9 prefers Maths?

**d** What is the difference between parts **b iii** and **c iii**?

The problem represented in part **c iii** is an example of conditional probability. It looks at the probability of an outcome given certain conditions. In part **c iii**, you are looking for the probability that a student likes Maths, given that they are in Year 9. This means that you consider only the limited group of the condition rather than the entire population.

**e** Use these steps to calculate the probability of a student being in Year 8, given that they prefer English.

**i** What is the condition? How many students are in this group?

**ii** What is the specific group you are after? (Hint: It is not just somebody in Year 8.)

**iii** How many students are in this specific group?

**iv** Use your answers to parts **i** and **iii** to calculate the probability of selecting somebody in Year 8, given that they prefer English.

**17** Use the two-way table in question **4** to calculate the probability of randomly selecting a person from the group that:

**a** plays hockey, given that they are in Year 8

**b** plays tennis, given that they are in Year 9

**c** is in Year 7, given that they play basketball

**d** is in Year 8, given that they play tennis

**e** is in Year 9, given that they *don't* play hockey

**f** is not in Year 7, given that they play basketball.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Two-way tables

Topic quiz
9E

# 9F Venn diagrams

## Learning intentions

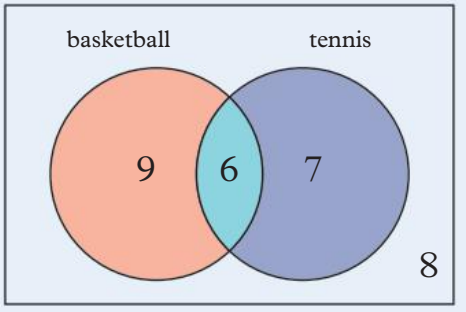
- ✔ I can use set notation to represent sets of data.
- ✔ I can create Venn diagrams to represent two or more sets of data.
- ✔ I can calculate the probabilities of outcomes using data displayed in Venn diagrams.

**Inter-year links**

| | |
|---|---|
| Year 5/6 | Understanding probability |
| Year 7 | 10F Theoretical probability |
| Year 8 | 9G Venn diagrams |
| Year 10 | 11C Two-way tables and Venn diagrams |

## Venn diagrams

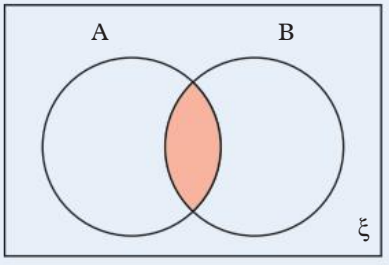
- In mathematics, a **set** is a collection of distinct objects.
- A **Venn diagram** is used to display the relationship between different sets of data.
- In a Venn diagram, each circle represents a different set and the rectangle containing the circles represents the entire data set.
- Frequencies are placed within Venn diagrams to represent the number of data points for each region of the diagram.
  For example, the Venn diagram on the right shows whether Year 9 students play either basketball or tennis. 6 students play both basketball and tennis (the region where these sets overlap).



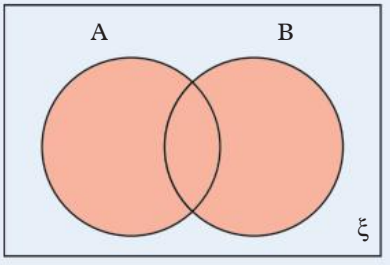

## Set notation

- All elements contained in the rectangle of a Venn diagram belong to the **universal set**, which is represented by the Greek letter xi (ξ).
- $A \cap B$ means the **intersection** of sets A and B, which includes only the elements that are in A and are also in B. This can be read as 'A and B'.
- $A \cup B$ means the **union** of sets A and B, which includes all elements in either A or B or both. This can be read as 'A or B'.
  → The intersection ($A \cap B$) and union ($A \cup B$) of sets A and B are also sets themselves.
- $A'$ means the **complement** of A, which includes all elements not in A.

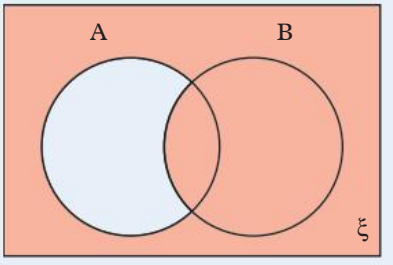
$A \cap B$



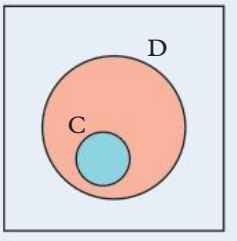

$A \cup B$




$A'$




- $C \subset D$ means C is a **subset** of D. This means that set C is contained within set D and every element in set C is also in set D.



## Example 9F.1 Understanding Venn diagrams

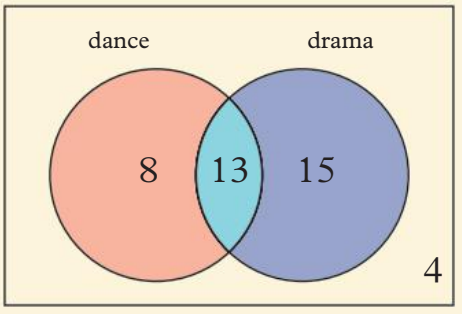
This Venn diagram shows whether people attend optional dance and drama classes.

**a** How many people attend both the dance and drama classes?

**b** How many people attend the drama class?

**c** How many people do not attend the dance class?

**d** How many people were surveyed in total?




**THINK**

**a** Identify the required region, which is the intersection of both sets. This is the middle section where the two circles overlap.

**b** Identify the required region, which is the 'drama' circle. Add the numbers in this region.

**c** Identify the required region, which is everything outside the 'dance' circle. Add the numbers in this region.

**d** Find the total of all numbers in the Venn diagram.

**WRITE**

**a** 13 people attend both classes.

**b** 13 + 15 = 28 attend the drama class.

**c** 15 + 4 = 19 people do not attend the dance class.

**d** 8 + 13 + 15 + 4 = 40 people were surveyed in total.

## Example 9F.2 Using Venn diagrams to calculate relative frequencies

Use the Venn diagram from Example 9F.1 to find the relative frequency that a person chosen randomly from the group attends the dance class.

**THINK**

**1** Identify the required region, which is the 'dance' circle. Add the numbers in this region.

**2** Find the total number of people who were surveyed by adding all the numbers in the Venn diagram (don't forget the number outside the circles).

**3** To determine the relative frequency, divide the number in the required region by the total number.

**WRITE**

8 + 13 = 21 people attended the dance class.

8 + 13 + 15 + 4 = 40 were surveyed.

$$\text{Relative frequency} = \frac{21}{40}$$
$$= 0.525$$

## Example 9F.3 Creating a Venn diagram

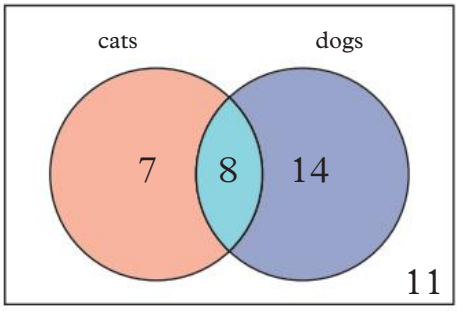
In a group of 40 people, 15 have a cat and 22 have a dog. There are 11 people who do not have either a cat or a dog. Draw a Venn diagram to represent this situation.

**THINK**

1. Draw two circles, one representing cats and the other representing dogs. Draw a rectangle around the two diagrams.
2. Place the number of people who do not have either a cat or a dog (11) in the rectangle outside the circles.
3. Determine the number of people who have a cat only by subtracting the number of people who have a dog (22) and the number of people who do not have either a cat or a dog (11) from the total (40).
4. Repeat step 3 to determine the number of people who have a dog only.
5. Determine the number of people who have a cat and a dog by subtracting the number of people who have a dog only (14) from the number of people who have a dog (22).

**WRITE**

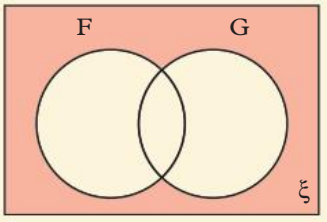
Number of people who have a cat only
$= 40 - 22 - 11$
$= 7$
Number of people with a dog only
$= 40 - 15 - 11$
$= 14$
Number of people with a cat and a dog
$= 22 - 14$
$= 8$



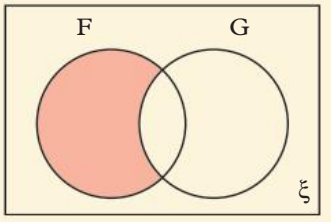

## Example 9F.4 Using set notation

Use set notation to describe the following regions in this Venn diagram.

**a**




**b**




**THINK**

**a** 1. Look at the shaded region. This represents the elements not in F and not in G.
2. Use set notation to represent the region. Use the complement symbol G′ for 'not in G' and the symbol ∩ for 'and'.

**b** 1. Look at the shaded region. This represent the elements in F and not in G.
2. Use set notation to represent the region.

**WRITE**

**a** $F' \cap G'$

**b** $F \cap G'$

Note: There is sometimes more than one way to describe a region. For example, the region in part **a** could also be described as $(F \cup G)'$.

**Helpful hints**

✔ When creating Venn diagrams, check that the total in all the individual regions equals the number in the entire data set.

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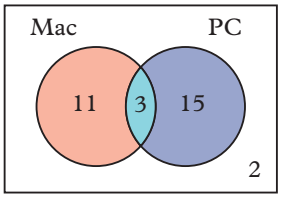
# Exercise 9F Venn diagrams

△ 1, 3, 4, 6–8, 11, 12 ▢ 2, 4–9, 12, 13, 16 ⬠ 5, 7–10, 12–15, 17

UNDERSTANDING AND FLUENCY

9F.1 **1** Consider the Venn diagram on the right, which shows whether people own a Mac or a PC.



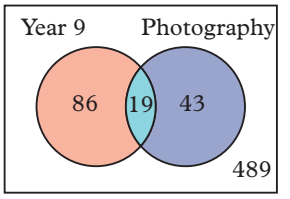

**a** How many people own a Mac?

**b** How many people own a PC but not a Mac?

**c** How many people don't own either a Mac or a PC?

**d** How many people were surveyed in total?

**2** Consider the Venn diagram on the right, which shows whether people are in Year 9 and whether they take a Photography class.



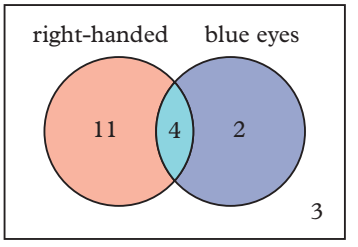

**a** How many people take Photography?

**b** How many people are in Year 9 and take Photography?

**c** How many people are not in Year 9?

**d** How many people were surveyed in total?

9F.2 **3** This Venn diagram shows whether people are right-handed and whether they have blue eyes. Use this diagram to find the relative frequency of a person:




**a** being right-handed with blue eyes

**b** having blue eyes

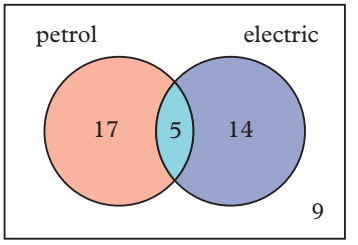
**c** being right-handed with eyes not blue

**d** not having blue eyes

**e** being left-handed with blue eyes

**f** being left-handed.

**4** This Venn diagram shows whether people drive petrol or electric cars. Use this diagram to find the relative frequency of a person who:




**a** drives petrol cars only

**b** drives both petrol and electric cars

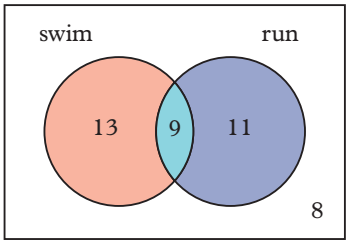
**c** drives electric cars

**d** drives neither

**e** does not drive petrol cars

**f** drives petrol or electric cars (but not both).

**5** This Venn diagram shows whether people like to swim or run for exercise. Use this diagram to find the relative frequency of a person who:




**a** swims and runs

**b** does not run

**c** runs but does not swim

**d** does not swim or run

**e** swims

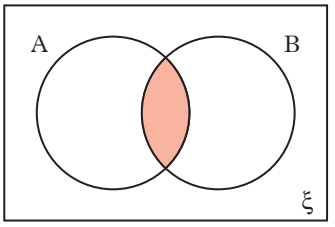
**f** swims or runs.

9F.3 **6** In a group of 50 students, 24 are in Year 9, 19 walk to school, and 16 are not in Year 9 and do not walk to school. Draw a Venn diagram to represent this situation.

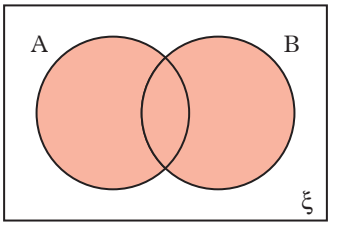
**7** In a group of 160 students, 78 take music, 95 take drama, and 46 take music and drama. Draw a Venn diagram to represent this situation.

9F.4 **8** Use set notation to describe the following regions in the Venn diagrams.

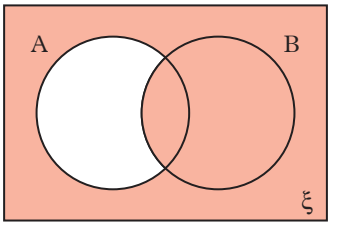
**a**



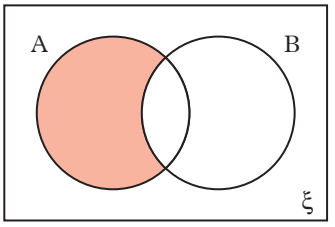

**b**



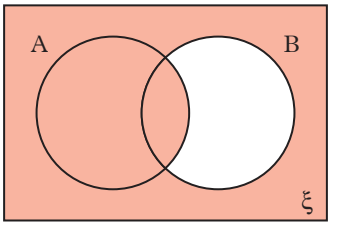

**c**



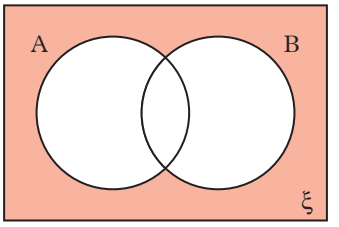

**d**



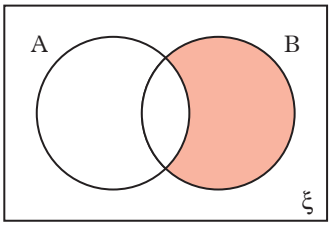

**e**



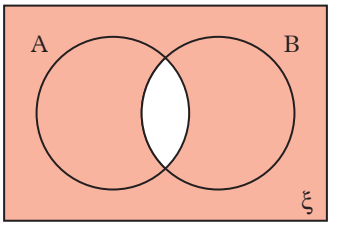

**f**



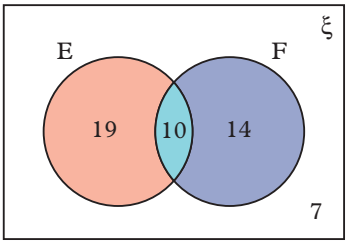

**g**




**h**




**9** Consider this Venn diagram.




**a** Given that $n(F)$ means the number of elements in set F, find:

**i** $n(E)$ **ii** $n(E \cup F)$ **iii** $n(F')$

**iv** $n(E \cap F)$ **v** $n(\xi)$ **vi** $n(E' \cap F)$.

**b** Given that Pr(E) means the probability of selecting an element from set E, find:

**i** Pr(F) **ii** Pr(E′) **iii** $\Pr(E \cap F)$

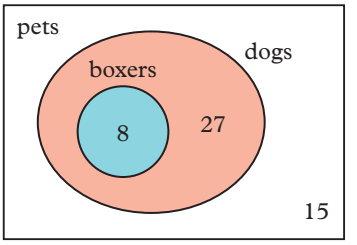
**iv** $\Pr(E \cup F)'$ **v** $\Pr(E' \cap F')$ **vi** $\Pr(F \cup E')$.

**10** Consider the Venn diagrams in question **8**.

**a** State which pairs are complementary by writing both set equalities using the symbol for the complementary event, ′. For example, $(A \cap B)' = A' \cup B'$ and $(A' \cup B')' = A \cap B$.

**b** Explain the relationship between the complements of intersections and unions (Note: These relationships are known as De Morgan's laws).

**11** Venn diagrams can be used to show the relationship between three or more sets and also include subsets. Consider these two Venn diagrams, the first of which details the pets of pet owners, and the second of which shows whether people take three subjects in school.

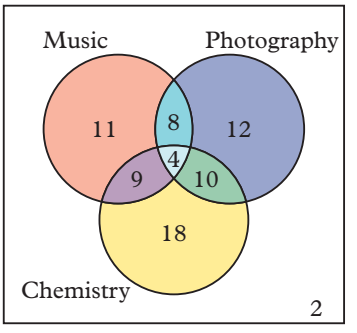



**a** Which Venn diagram shows subsets of data? Explain the relationship shown in this Venn diagram.

**b** Copy the first Venn diagram three times and on separate copies shade the section that represents:

**i** pets that are not dogs **ii** dogs that are not boxers

**iii** all dogs.




**c** The other Venn diagram shows the relationship between three different sets. Copy this Venn diagram five times and shade the section that represents students:

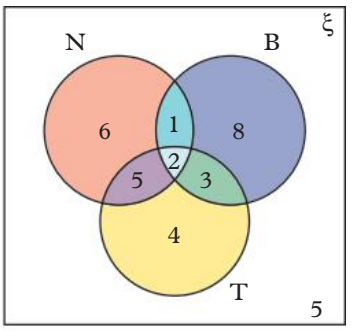
**i** taking all three elective subjects **ii** taking only Photography

**iii** taking Music or Chemistry **iv** taking Music

**v** not taking any of these three electives.

PROBLEM SOLVING AND REASONING

**12** This Venn diagram shows the number of students who play netball, basketball and tennis. Find the probability that a student chosen randomly:

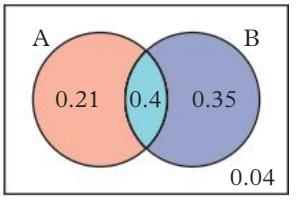



**a** plays netball but not tennis
**b** plays basketball only
**c** plays all three sports
**d** doesn't play any of these sports
**e** plays basketball or tennis
**f** plays tennis and netball.

**13** Use the following information to draw a Venn diagram that shows the relationship between three different sets of data. In a group of 100 people surveyed:

- 35 liked action films
- 46 liked horror films
- 5 people liked all three genres
- 16 only liked horror
- 45 liked romance
- 55 did not like romance films
- 15 people liked both action and romance
- 18 liked both horror and romance
- 19 people did not like films in any of these three genres.

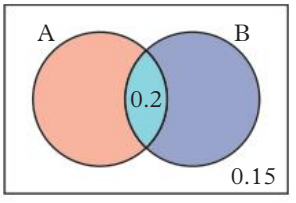
**14** The Venn diagram on the right shows the relative frequency of each intersection between the events A, B and their complements. If the sample size was 400, redraw the Venn diagram using frequencies.

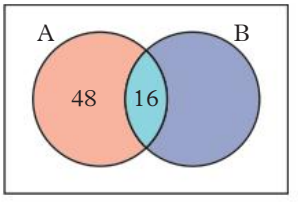



**15** The following Venn diagrams represent the same sample of people for the events A and B.

**a** Complete the Venn diagrams.

**b** Calculate: **i** Pr(A) **ii** Pr(B)

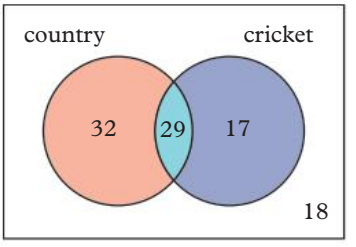



CHALLENGE

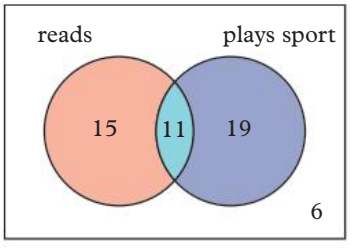
**16** The probability of an event can change when new information is obtained. For example, the probability of selecting a diamond from a deck of cards is $\frac{1}{4}$. However, if you are told the card is red then the probability it is a diamond is now $\frac{1}{2}$, based on this new information. This would usually be asked with wording such as "What is the probability that a randomly selected card from a deck is a diamond, given that it is red?"

This Venn diagram shows whether people are from the country and if they like cricket or not. Use this diagram to calculate the probability that a person chosen randomly:




**a** likes cricket, given that they are from the country
**b** is from the country, given that they like cricket
**c** does not like cricket, given that they are from the country
**d** is not from the country, given that they do not like cricket.

**17** This Venn diagram shows whether people like to read and/or play sport. Use this diagram to calculate the probability that a person chosen randomly:




**a** reads given that they play sport
**b** plays sport given that they read
**c** does not play sport given that they read
**d** does not read given that they do not play sport.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Venn diagrams
- Interactive skillsheet: Identifying the required outcome
- Interactive skillsheet: Set notation
- Investigation: Venn diagram applications
- Topic quiz: 9F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# Chapter summary

## Theoretical probability

- Sample space: all possible outcomes
- Event: collection of one or more outcomes from the sample space
- If outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

- A' means not A. A and A' are complementary events.

$\Pr(A) + \Pr(A') = 1$

## Arrays

- List of items arranged in rows and columns.
- Use when there is a large number of outcomes in either step of a two-step experiment.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **H** | (1, H) | (2, H) | (3, H) | (4, H) | (5, H) | (6, H) |
| **T** | (1, T) | (2, T) | (3, T) | (4, T) | (5, T) | (6, T) |

## Experiments with/without replacement

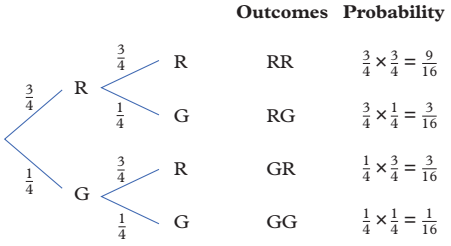
- With replacement: probabilities for each step remain the same.
- Without replacement: probabilities for all remaining steps are changed.

## Relative frequency

- Relative frequency = $\frac{\text{number of occurrences}}{\text{total number of trials}}$
- Expected number = theoretical probability × number of trials
- With a small number of trials, results may vary significantly from expected.
- In the long term, the relative frequency will approach the theoretical probability.

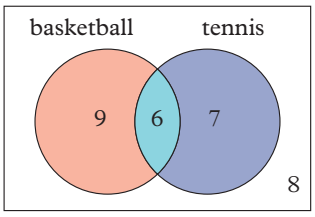
## Combining like outcomes

- When grouping like outcomes, the probability of the grouped outcome is the sum of the probabilities in the group.
- The probabilities of final outcomes can be found by multiplying the probabilities leading to that outcome.




## Venn diagrams

- Each region represents a different set.
- Rectangle represents the entire data set.
- Frequencies represent number of data points for each region of the diagram.




## Two-way tables

- Raw numbers or relative frequencies.

| | Dark hair | Light hair | Total |
|---|---|---|---|
| **Dark eyes** | 0.4 | 0.1 | 0.5 |
| **Pale eyes** | 0.2 | 0.3 | 0.5 |
| **Total** | 0.6 | 0.4 | 1 |

- 'and': both outcomes must be met
- Inclusive 'or': if both outcomes are met the outcome is favourable.
- Exclusive 'or': if both outcomes are met the outcome is not favourable.

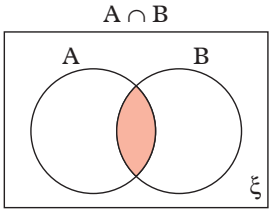
## Set notation

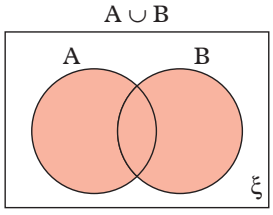
- Everything within the rectangle of a Venn diagram belongs to the universal set, $\xi$.

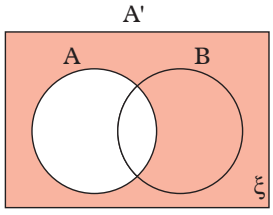
$A \cap B$ is the intersection of sets A and B (A and B)

$A \cup B$ is the union of sets A and B (A or B)

A' is the complement A (not A)



# Chapter review

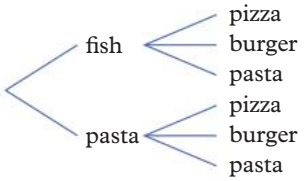
**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

9A **1** If there is an equal probability of choosing every option on the following menu, the probability that a person orders pasta for both entrée and main is:

**Meal choices at a local cafe**



**A** $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{1}{6}$ **D** $\frac{1}{2}$ **E** $\frac{1}{9}$

9A **2** Two numbers are chosen randomly and their product is recorded. One number is chosen from the set $\{1, 2, 3, 4, 5\}$ and the other is chosen from the set $\{2, 3, 4\}$. The probability of which pair of events is not equal?

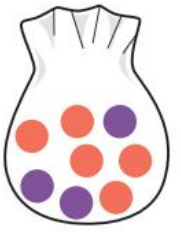
**A** Pr(4), Pr(8)
**B** Pr(odd), Pr(multiple of 5)
**C** Pr(multiple of 3), Pr(multiple of 2)
**D** Pr(square number), Pr(multiple of 10)
**E** Pr(prime), Pr(cube number)

9B **3** There is a $\frac{1}{13}$ chance of selecting an Ace from a standard pack of playing cards. A card is selected at random and an Ace is chosen. The card is replaced into the pack. What is the probability of selecting an Ace on the second selection?

**A** $\frac{1}{13}$ **B** $\frac{1}{169}$ **C** $\frac{12}{13}$ **D** $\frac{144}{169}$ **E** $\frac{3}{676}$

Questions **4** and **5** refer to the diagram on the right.

A marble is selected and not replaced and then a second selection is made.



9C **4** The first marble selected is orange. What is the probability the second marble selected will also be orange?

**A** $\frac{4}{7}$ **B** $\frac{3}{7}$ **C** $\frac{1}{7}$ **D** $\frac{5}{7}$ **E** $\frac{2}{7}$

9C **5** The probability of selecting two purple marbles is:

**A** $\frac{3}{8}$ **B** $\frac{2}{7}$ **C** $\frac{5}{14}$ **D** $\frac{3}{28}$ **E** $\frac{3}{4}$

9D **6** A six sided die is rolled many times. Consider this table showing the frequencies of the outcomes of this experiment.

| Number on die | Frequency |
|---|---|
| 1 | 45 |
| 2 | 35 |
| 3 | 27 |
| 4 | 30 |
| 5 | 32 |
| 6 | 31 |

Which of these statements is true?

**A** The number of trials performed was 200 rolls.
**B** The relative frequency of rolling an even number is $\frac{13}{25}$.
**C** The number rolled the least frequently was 1.
**D** The relative frequency of rolling a 6 is $\frac{1}{6}$.
**E** The expected number for rolling a 4 is 30.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9E **7** The relative frequency of a person preferring coffee is:

| | Under 30s | 30 and over | Total |
|---|---|---|---|
| **Prefer coffee** | 176 | 98 | |
| **Prefer tea** | 101 | 25 | 126 |
| **Total** | 277 | | 400 |

**A** $\frac{137}{200}$ **B** $\frac{49}{200}$ **C** $\frac{11}{25}$ **D** $\frac{101}{400}$ **E** $\frac{176}{277}$

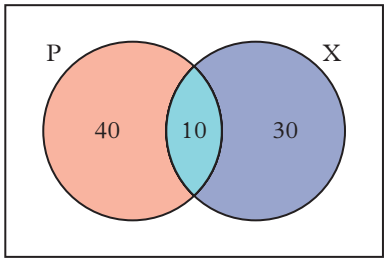
9E **8** This two-way table is to be converted to a relative frequency two-way table. Which relative frequency will not appear?

| | A | A' | Total |
|---|---|---|---|
| **B** | 5 | 5 | 10 |
| **B'** | 7 | 8 | 15 |
| **Total** | 12 | 13 | 25 |

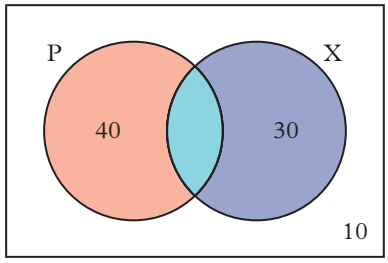
**A** 0.2 **B** 0.28 **C** 0.52 **D** 0.5 **E** 0.4

9F **9** A group of 80 students were asked if they owned a PlayStation (P) or an Xbox (X). 40 students owned a PlayStation, 30 students owned an Xbox, and 10 said they owned both. Which of the following Venn diagrams correctly displays this information?

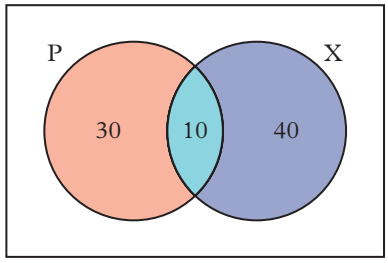
**A**



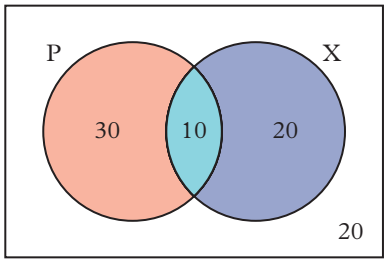

**B**



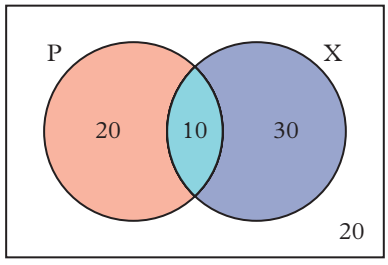

**C**



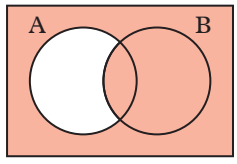

**D**




**E**




9F **10** Which of the following is represented by the shaded section in the Venn diagram?




**A** $A \cap B$ **B** $A \cup B$ **C** $A' \cup B$
**D** $A' \cap B$ **E** $A \cap B'$

## Short answer

9A **1** A company runs a competition where one out of every four purchases contains the winning bar code for an ebook.

**a** What is the probability of not winning an ebook?

**b** Complete a tree diagram showing all probabilities on the branches for two trials.

**c** Calculate the probability of:

**i** winning twice in a row

**ii** not winning twice in a row

**iii** winning on the first try and then not winning on the second try.

9A **2** A coin is tossed and a fair, standard six-sided die is rolled.

**a** Show all possible outcomes on an array.

**b** Calculate the probability that:

**i** the coin lands on heads and the die lands on 2

**ii** the coin lands on heads or the die lands on 2 or both

**iii** the coin lands on heads or the die lands on an even number, but not both.

**c** Calculate the probability that the coin lands on heads and the die lands on a multiple of 3.

**d** Calculate the probability that the coin lands on heads or the die lands on a factor of 6 or both.

9B **3** A bag contains six chocolates. Two have orange wrappers, one has a green wrapper and three have pink wrappers. A chocolate is chosen at random and the colour is recorded, the chocolate is replaced, and then another selected.

**a** Show all possible outcomes on a tree diagram.

**b** Calculate the probability that:

**i** both wrappers are orange **ii** both wrappers are green **iii** both wrappers are pink.

**c** Calculate the probability that the first wrapper is green.

**d** Calculate the probability that the second wrapper is pink.

9C **4** There are four suits (clubs, diamonds, hearts and spades) in a standard pack of 52 cards. Two cards are selected from a pack and the suits are noted. Assuming selection without replacement, calculate:

**a** Pr(two spades) **b** Pr(heart, then spade).

9D **5** Calculate the relative frequency of:

**a** a 3 being rolled , given that 45 out of 120 rolls of the die landed on 3

**b** the number 387 being generated when a random integer was generated 1000 times and 387 appeared 5 times

**c** selecting someone in a shoe store at random wearing boots when 300 people visited the shoe store in a day and 33 were wearing boots

**d** selecting a blue puzzle piece at random when a 1000-piece puzzle has 650 pieces with similar shades of blue on them.

9D **6** Calculate the expected number of the following:

**a** rolling a 1 when a 12-sided die is rolled 420 times

**b** generating 676 when a random integer from 1 to 1000 is generated 10 000 times

**c** *Hungry Like the Wolf* playing from a workout playlist with 73 tracks, when 365 tracks are played on random shuffle

**d** rolling a double if a pair of dice is thrown 150 times while playing a board game

9E **7** Refer to this two-way table for Years 8, 9 and 10 students and their preferred brands of make-up.

| | Year 8 | Year 9 | Year 10 | Total |
|---|---|---|---|---|
| **Napoleon Perdis** | 32 | 25 | 45 | |
| **Rimmel** | 22 | | 22 | 54 |
| **Covergirl** | 17 | 31 | 13 | |
| **Maybelline** | | 29 | | |
| **Total** | 120 | | 95 | 310 |

**a** Copy and complete the table.

**b** Calculate the probability that a student chosen at random from the group:

**i** prefers Maybelline **ii** is in Year 8 and prefers Rimmel

**iii** is in Year 9 and prefers Napoleon Perdis.

**c** Calculate the probability that a student chosen from the Year 9 students:

**i** prefers Napoleon Perdis **ii** prefers Covergirl or Maybelline.

9F **8** 120 secondary school students were surveyed about whether they like to play video games and board games. Of the 20 Year 9 students surveyed, 8 play video games, 10 play board games, and this includes 3 who play both. Of the students in other year levels who were surveyed, 55 play video games, 60 play board games and this includes 35 who play both.

Let A be the set of Year 9 students and let V be the set of students who play video games.

**a** Use a Venn diagram with circles for the sets A and V to display this information.

**b** Calculate:

**i** $\Pr(A \cap V)$ **ii** $\Pr(A \cup V)$ **iii** $\Pr(A' \cap V')$ **iv** $\Pr(V)$

**c** Use a Venn diagram with circles for the sets B and V to display the information.

Let B be the set of students who play board games.

**d** Calculate:

**i** $\Pr(B \cap V)$ **ii** $\Pr(B' \cup V)$ **iii** $\Pr(B \cap V')$ **iv** $\Pr(B)$

## Analysis

When moving from Year 9 into Year 10, students undertake core studies such as English and Mathematics, and can select from a range of electives to complete their study program.

A student must select two from these four electives: Health and Human Development (HHD), Art, Italian and Food Technology. The student writes each subject on a card and then chooses two at random from a hat.

**a** Would this selection be with or without replacement? Explain your reasoning.

**b** **i** Draw a tree diagram to represent the possible subject combinations.

**ii** How many combinations are possible?

**c** Find the probability of selecting:

**i** Art and Food Technology **ii** Italian and HHD.

**d** The senior students at the school were surveyed about the Food Technology and Art electives offered. 12% of the respondents were Year 12 students who preferred Art to Food Technology; 28% of the respondents were Year 11 students who preferred Food Technology to Art. Overall, 52% of the respondents were in Year 11.

**i** Construct a two-way table showing these percentages as relative frequencies.

Based on these survey results, if a person was randomly selected from the 200 students who participated in the survey, find the probability that they are:

**ii** a Year 12 student who prefers Food Technology

**iii** a Year 11 student who prefers Art

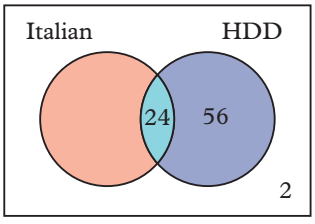
**iv** a student who prefers Art.

Of these 200 students, find the number of people who:

**v** are in year 12 and prefer Food Technology

**vi** prefer Food Technology.

**e** An analysis of numbers of students studying Italian and/or HHD was also undertaken and the results collated into this Venn diagram.




**i** How many of the 150 students study both Italian and HHD?

**ii** How many students study neither subject?

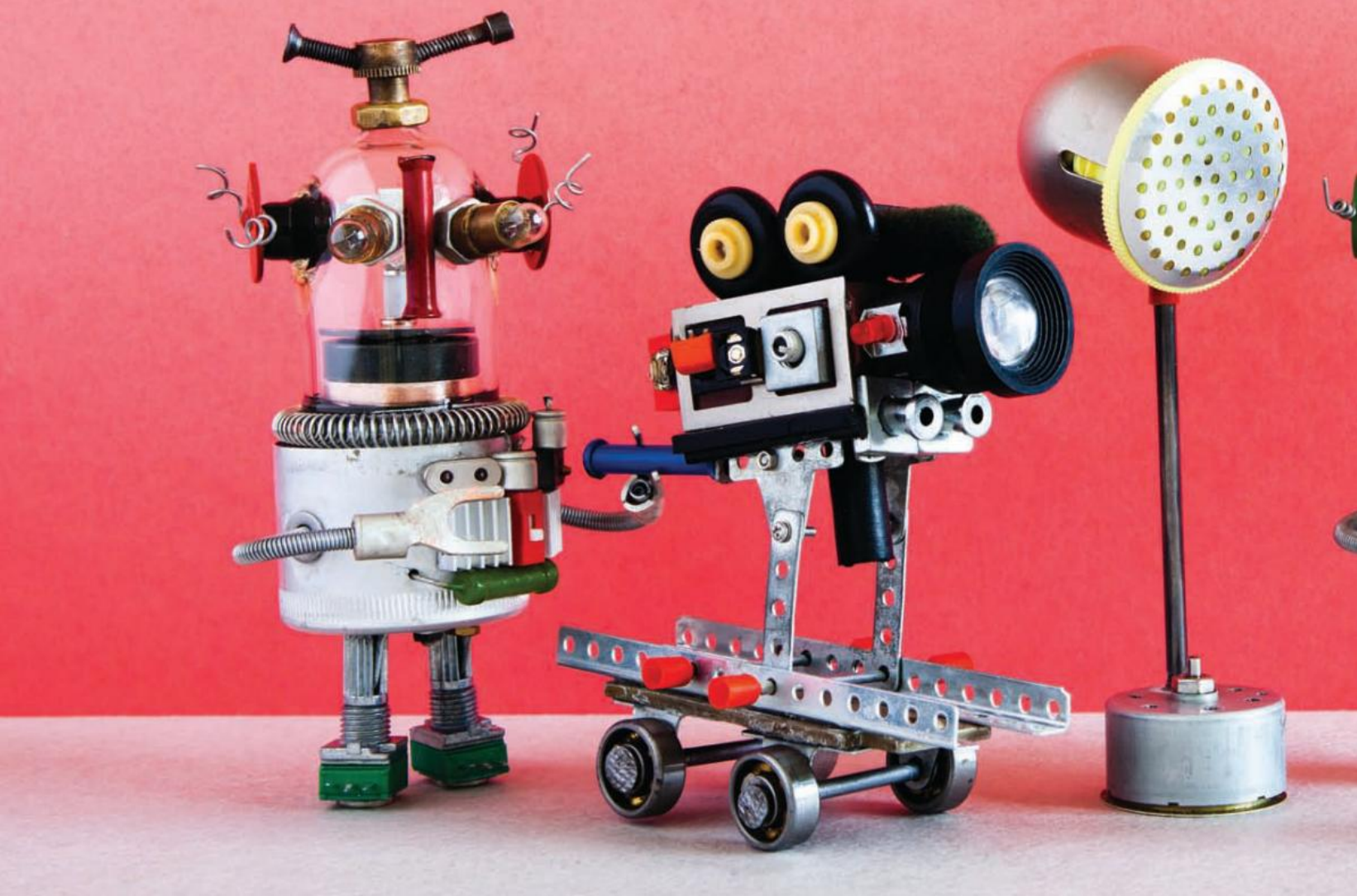
**iii** How many students study Italian only?

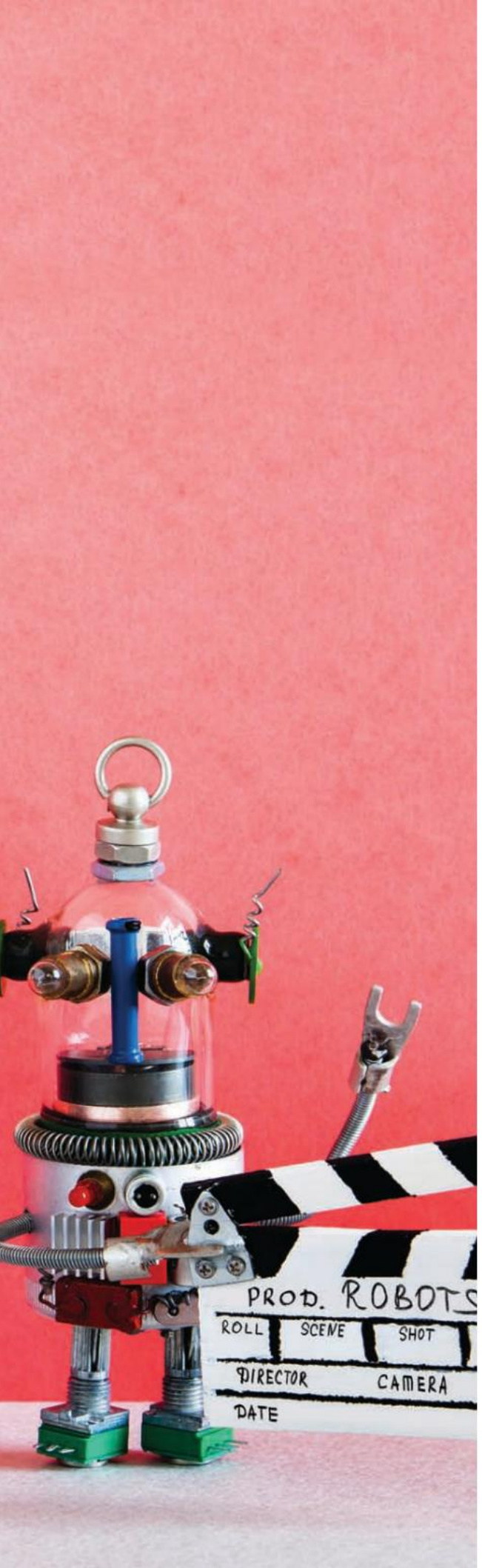
**iv** If selecting a student at random from the cohort, what is the probability that they study Italian only?

**v** If selecting a student at random from the cohort, what is the probability that they study HHD only?

# 10

# Computational thinking






## Index

**10A** Nested loops
**10B** Sorting a list of numbers
**10C** Functions

## Prerequisite skills

Before starting this chapter, please ensure you have completed the Computational Thinking chapters from Oxford Maths 7 and Oxford Maths 8.

Inter-year links

| | |
|---|---|
| Year 7 | 11 Computational thinking |
| Year 8 | 10 Computational thinking |
| Year 10 | 12 Computational thinking |

## Curriculum links

Apply set structures to solve real-world problems (VCMNA307)

## Materials

✔ Computer
✔ Python (online or downloaded)

Note: Python is used as the coding language in this chapter. Instructions will vary for other coding languages.

# 10A Nested loops

## Learning intentions

- ✔ I can understand what is meant by a nested loop.
- ✔ I can identify the inner and outer loops in a nested loop.
- ✔ I can predict the output of a nested loop by drawing a tree diagram.

**Inter-year links**

| | |
|---|---|
| Year 7 | 11A Printing information to the computer screen |
| Year 8 | 10A Lists |
| Year 8 | 10B Introduction to loops |

## Nested loops

- A **nested loop** is a structure that contains at least two ordinary **loops**, where one loop is carried out within the other loop.
  - → The **inner loop** repeats a particular action or set of actions a given number of times. The inner loop is always **indented** four spaces to the right from the outer loop above it.
  - → The **outer loop** repeats the inner loop a given number of times. For a 10-week term, the outer loop would repeat the inner loop 10 times.

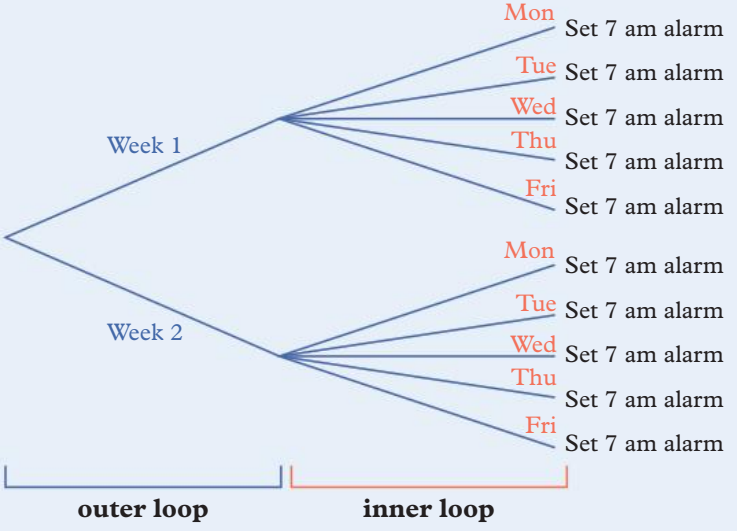
  For example, below is a nested loop used to set a morning alarm for each school day throughout a term.

```
for each week of the term:
    for each weekday of the week:
        set an alarm for 7 am
```

outer loop

inner loop

- Tree diagrams can be used to represent the inner and outer loops visually. The following tree diagram shows the example loop above for the first two weeks of the term.



- A nested loop can be thought of as a quick way to generate all the outcomes in a two-step experiment. For example, the following code generates all possible outcomes when a four-sided die numbered 1 to 4 is rolled twice and the two numbers are added.

```
For number1 in [1, 2, 3, 4]:
    For number2 in [1, 2, 3, 4]:
        print (number1+number2)
```

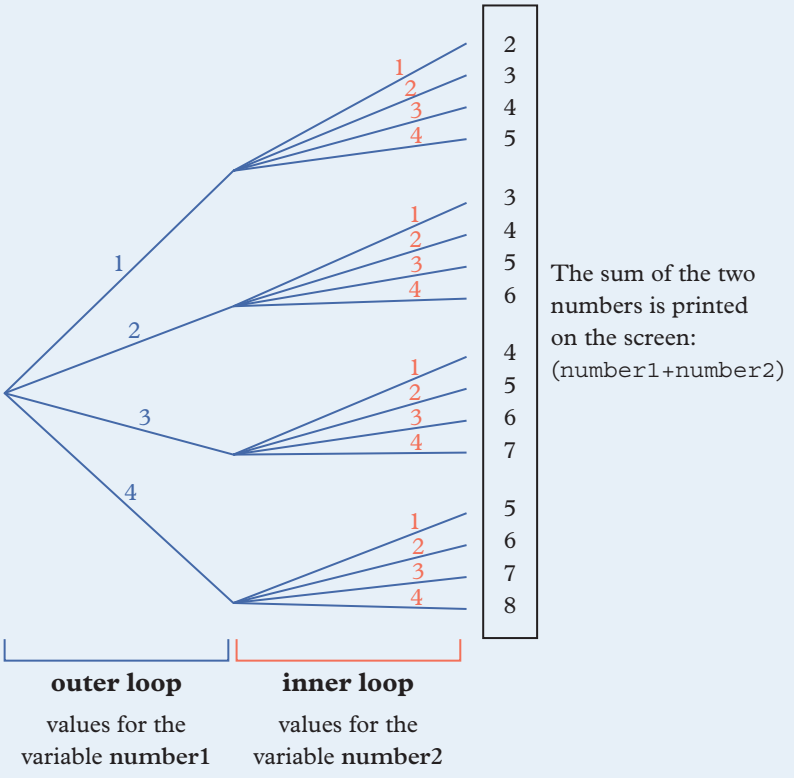
outer loop

inner loop

→ The outer loop is looping through 1, 2, 3, 4 using a variable `number1`.

→ The inner loop is looping through 1, 2, 3, 4 using a variable `number2`.

→ The tree diagram that represents all the possible outcomes is:




The sum of the two numbers is printed on the screen: `(number1+number2)`

- It is important to note the order in which nested loops perform their steps. For each item or value in the outer loop, the entire inner loop is repeated before moving on to the next item or value in the outer loop. For example, initially `number1 = 1`. It doesn't change until `number2` has looped through all its options from 1 to 4. Only then does `number1` change its value to 2 and the inner loop starts again.

# Python commands

- When coding in Python, the command **print( )** is used to print information to the screen. Any information you want to see on the screen should be placed inside the brackets.
- The position of each item in the list is called its **index**. The index of a list starts at 0 rather than 1.

```
L = [5, 2, 7, 4, 9]
```

Index: 0 1 2 3 4

## Example 10A.1 Drawing a tree diagram for a nested loop

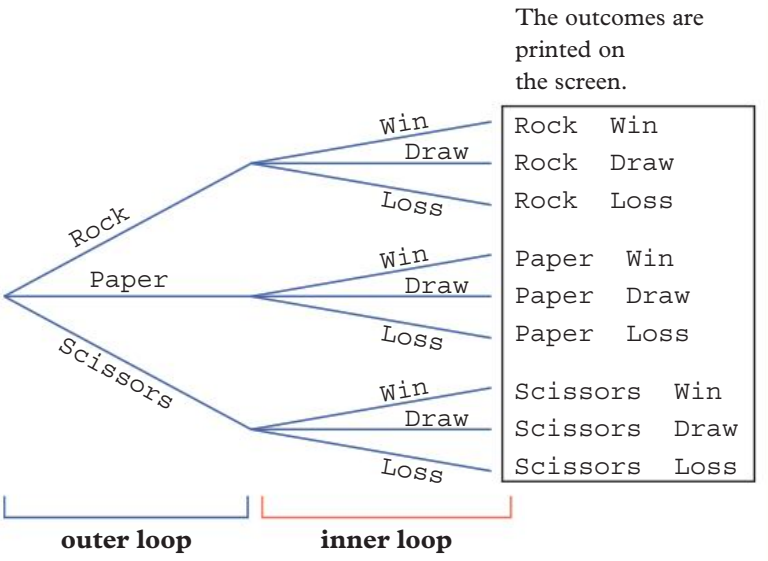
Consider the code given below. Draw a tree diagram to represent the nested loop below.

```
for choice in ['Rock','Paper','Scissors']:
    for outcome in ['Win','Draw','Loss']:
        print(choice, outcome)
```

**THINK**

1. Determine the output of the outer loop. The outer loop is looping through 'Rock', 'Paper' and 'Scissors' using a variable called `choice`.
2. Determine the output of the inner loop. The inner loop is looping through 'Win', 'Draw' and 'Loss' using a variable called `outcome`.
3. Draw the branches for the outer loop, ensuring to leave space between each branch.
4. Draw the branches of the inner loop at the end of each branch of the outer loop.

**WRITE**




## Example 10A.2 Nested loops with a condition

For the code below, write down what will be printed on screen in the correct order.

```
for x in [1,2,3,4,5]:
    for y in [2,3,5,7,11]:
        if x > y:
            print(x,">",y)
```

**THINK**

1. Determine the output of the outer loop. The outer loop is looping through the integers `[1,2,3,4,5]` using a variable called `x`.
2. Determine the output of the inner loop. The inner loop is looping through the integers `[2,3,5,7,11]` using a variable called `y`.
3. Consider the condition of the inner loop. An if statement is indented four spaces to the right of the inner loop, so the code will check whether `x > y` at each point in the nested loop.
4. Consider the print conditions of the code. When `x > y`, the code will print `x > y`. Determine the values where this condition is met.

**WRITE**

```
3 > 2
4 > 2
4 > 3
5 > 2
5 > 3
```

## **Example 10A.3** Nested loops with lists

For the code below, determine what is printed to the screen, in the correct order.

```
biscuit = ['Anzac','Choc chip','Tim Tam','Wagon Wheel']
tea = ['Black','Earl Grey','Green']
    for i in [0,1,2,3]:
        for j in [2,1,0]:
            print(biscuit[i],"with",tea[j],"tea please.")
```

**THINK**

1. The first two lines define two variables, *biscuit* and *tea*.
2. Determine the output of the outer loop. The outer loop is looping through the integers `[0,1,2,3]` using a variable called `i`.
3. Determine the output of the inner loop. The inner loop is looping through the integers `[2,1,0]` using a variable called `j`. Note that the `j` variable is looping through the numbers in descending order – don't let this trick you!
4. Consider the print conditions of the code. The last line prints a request for a particular biscuit and tea but refers to its index. For example, `biscuit[2]` would refer to the third biscuit (Tim Tam) because we start counting at 0. Recall that the start of a list is referred to as index 0, not index 1.
5. Determine the outputs where the print condition is met. Remember that the order is important.

**WRITE**

```
Anzac with Green tea please.
Anzac with Earl Grey tea please.
Anzac with Black tea please.
Choc chip with Green tea please.
Choc chip with Earl Grey tea
  please.
Choc chip with Black tea please.
Tim Tam with Green tea please.
Tim Tam with Earl Grey tea please.
Tim Tam with Black tea please.
Wagon Wheel with Green tea please.
Wagon Wheel with Earl Grey tea
  please.
Wagon Wheel with Black tea please.
```

**Helpful hints**

- ✔ Sometimes, rather than referring to the items in a list directly, it is useful to refer to their position in the list, or their index. Recall that the start of a list is referred to as index 0, not index 1.
- ✔ It is very easy to ignore indentation because we tend to ignore white space. However, for Python, it's crucial. Ensure that the inner loop is always indented four spaces to the right of the outer loop.
- ✔ It is common to forget the colon (:) at the end of a for loop, particularly when dealing with nested loops. If you do forget, Python will helpfully point out that there's something missing.

```
For j in [2,1,0]
              ^
SyntaxError: invalid syntax
```

- ✔ It can be tempting to try to skip steps such as drawing the tree diagram when working with nested loops. This is likely to result in errors that are hard to find. Taking the time to draw the tree diagram will save time later.
- ✔ If you have forgotten some of the coding language, use your inter-year links at the beginning of this chapter to remind yourself of the Year 7 and 8 content.

# ANS p560 Exercise 10A Nested loops

**10A.1** **1** Draw a tree diagram to represent each of the nested loops below.

**a**
```
for food in ['burger','pizza','sushi']:
    for drink in ['cola','juice','water']:
        print(food, drink)
```

**b**
```
for console in ['Xbox','PlayStation','Switch']:
    for game in ['Fortnite','Call of Duty','FIFA']:
        print(console, game)
```

**c**
```
for hour in [6,7,8,9,10]:
    for period in ['AM', 'PM']:
        print(hour, period)
```

**d**
```
for spread in ['peanut butter','honey','jam']:
    for bread in ['multigrain','wholegrain']:
        print (spread, bread)
```

**e**
```
for team1 in ['Collingwood','Richmond','Carlton']:
    for team2 in ['Fremantle','Port Adelaide','Brisbane']:
        print (team1, team2)
```

**10A.2** **2** For each of the codes below, determine what is printed to the screen, in the correct order. (Hint: Draw a tree diagram to help you).

**a**
```
for x in [2,4,6,8]:
    for y in [9, 6, 3]:
        if x + y > 10:
            print(x+y, "is greater than 10!")
```

**b**
```
for word in ['chair','house','field']:
    for vowel in ['a', 'e', 'i', 'o', 'u']:
        if vowel in word:
            print(word, "has an", vowel)
```

**c**
```
for n1 in [-1,2,-3]:
    for n2 in [3,-2,1]:
        if n1 * n2 > 0:
            print(n1 * n2,"is a positive number")
        elif n1 * n2 < 0:
            print(n1 * n2,"is a negative number")
```

**d**
```
for hour in [10,11]:
    for minute in [0,10,20,30,40,50]:
        if minute == 0:
            print(hour,'oclock')
        elif minute < 30:
            print(minute,'past',hour)
        elif minute == 30:
            print('half past',hour)
        else:
            print(60-minute,'to',hour+1)
```

10A.3 **3** Each of the codes below loop through the indices (positions) of items in lists. For each, write down what will be displayed on the screen.

**a**
```
consonants = ['B','C','D','F','G','H','J','K','L','M']
vowels = ['A','E','I','O','U']
for i in [0,5,8]:
    for j in [0,1,2]:
        print(consonants[i],vowels[j])
```

**b**
```
adjectives = ['green','shiny','sweet']
fruits = ['apple','pear','grapes']
for i in [0,2,1]:
    for j in [1,2,0]:
        print(adjectives[i],fruits[j])
```

**c**
```
primes = [2,3,5,7,11]
composites = [4,6,8,9,10]
for i in [0,2,4]:
    for j in [2,3,4]:
        if primes[i] > composites[j]:
            print("prime is bigger")
        else:
            print("composite is bigger")
```

**d**
```
length = [11,5,14,6]
width = [6,4,15,3]
height = [3,5,8,20,10]
for i in [1,3]:
    for j in [0,2,4]:
        print('Volume =',length[i]*width[i]*height[j])
```

**e**
```
homescore = [63,101,49,89]
awayscore = [19,99,57,74]
    for i in [0,2,1,3]:
        if homescore[i] > awayscore[i]:
            print('Home Team Win:',homescore[i],'-',awayscore[i])
        else:
            print('Away Team Win:',homescore[i],'-',awayscore[i])
```

# 10B Sorting a list of numbers

### Learning intentions

- ✔ I can sort a list of numbers using a nested loop.
- ✔ I can find the median of a list of sorted numbers.

## Algorithms for sorting a list of numbers

- Sorting a list of numbers in ascending order is a very common task in coding and many **algorithms** have been designed for this purpose. One such algorithm is the **bubble sort algorithm** that uses a nested loop and a series of swaps to sort a list.

## Bubble sort

- The bubble sort algorithm considers an unsorted list of numbers, compares each pair of neighbouring numbers in the list and then swaps them if they are not in ascending order. This process is then repeated with one less pair being compared each time.
- Consider an unsorted list of five numbers as an example (7 2 1 8 6).

**1** Compare all four neighbouring pairs of numbers.

| | |
|---|---|
| [7 2] 1 8 6 | 7 and 2 are swapped as 7 > 2 |
| 2 [7 1] 8 6 | 7 and 1 are swapped as 7 > 1 |
| 2 1 [7 8] 6 | 7 and 8 are not swapped as 7 < 8 |
| 2 1 7 [8 6] | 8 and 6 are swapped as 8 > 6 |

**2** Then, compare the first three neighbouring pairs of numbers.

| | |
|---|---|
| [2 1] 7 6 8 | 2 and 1 are swapped as 2 > 1 |
| 1 [2 7] 6 8 | 2 and 7 are not swapped as 2 < 7 |
| 1 2 [7 6] 8 | 7 and 6 are swapped as 7 > 6 |

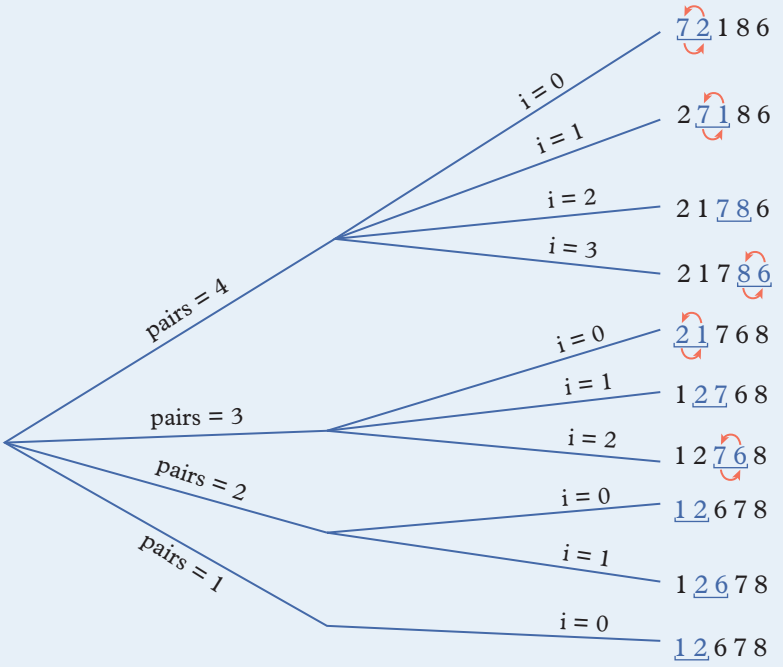
**3** Next, compare the first two neighbouring pairs of numbers.

| | |
|---|---|
| [1 2] 6 7 8 | 1 and 2 are not swapped as 1 < 2 |
| 1 [2 6] 7 8 | 2 and 6 are not swapped as 2 < 6 |

**4** Finally, compare the first (one) neighbouring pair of numbers.

| | |
|---|---|
| [1 2] 6 7 8 | 1 and 2 are not swapped as 1 < 2 |

- The process can be represented with a tree diagram as shown on the next page.
  - → The outer loop in the tree diagram is the number of pairs being compared. Note that this goes down by 1 each time.
  - → The inner loop tracks where each comparison starts.




# Writing the bubble sort code

**1** Create a variable called length to store the length of the list of the numbers.

```
length = len(numbers)
```

**2** To write the outer loop code, loop through 4, 3, 2 and 1 variable pairs.
For example, `for pairs in [4,3,2,1]`
However, this would only work for a list with 5 numbers. More generally, we want the variable pairs to loop backwards, starting at one less than the length of the list (in this case, 5 – 1 = 4) and going down to 1. We can do this in the general case with:

```
length = len(numbers)
for pairs in range (length-1,0,-1):
```

The loop starts at 1 less than the length of the list.

The range() function doesn't include the second number, so this loop will finish at 1 as required.

The –1 tells the loop to count in steps of –1. It will count backwards.

**3** To write the code for the inner loop, note that the inner loop always starts at 0 and finishes one number lower than the value of pairs.
For example, `for i in range(0,pairs,1):`

**4** An if statement checks whether to swap the current pair of numbers. The last line is the code for swapping the order of two items in a list using Python.

```
if numbers[i] > numbers[i+1]:
        numbers[i], numbers[i+1] = numbers[i+1], numbers[i]
```

**5** The complete bubble sort code is as follows:

```
length = len(numbers)
for pairs in range(length-1,0,-1):
    for i in range(0,pairs,1):
        if numbers[i] > numbers[i+1]:
            numbers[i], numbers[i+1] = numbers[i+1], numbers[i]
```

## **Example 10B.1** Understanding bubble sort

Show each of the steps of the bubble sort algorithm for this list of numbers: 9 1 8 4

| THINK | WRITE |
|---|---|
| **1** Compare all three neighbouring pairs of numbers and swap their positions when the left number is greater than the right number. | 9 8 1 4<br>8 9 1 4<br>8 1 9 4 |
| **2** Compare the first two neighbouring pairs of numbers and swap their positions when the left number is greater than the right number. | 8 1 4 9<br>1 8 4 9 |
| **3** Compare the first pair of numbers and swap their positions if the left number is greater than the right number. | 1 4 8 9 |

## **Example 10B.2** Drawing a tree diagram for bubble sort

Consider the code given below. Draw a tree diagram to represent what is happening in this nested loop.

```
numbers = [1,9,2,5]
length = len(numbers)
for pairs in range(length-1,0,-1):
     for i in range(0,pairs,1):
          if numbers[i] > numbers[i+1]:
               numbers[i], numbers[i+1] = numbers[i+1], numbers[i]
               print(numbers[i], numbers[i+1], "SWAP")
          else:
               print(numbers[i], numbers[i+1], "NO SWAP")
```

**THINK**

**1** Consider the code. The bubble sort algorithm has an extra first line to define the `numbers` list as 1, 9, 2 and 5. The last lines have been changed to print the two numbers being compared after sorting each pair, as well as a label indicating whether or not the numbers were swapped.

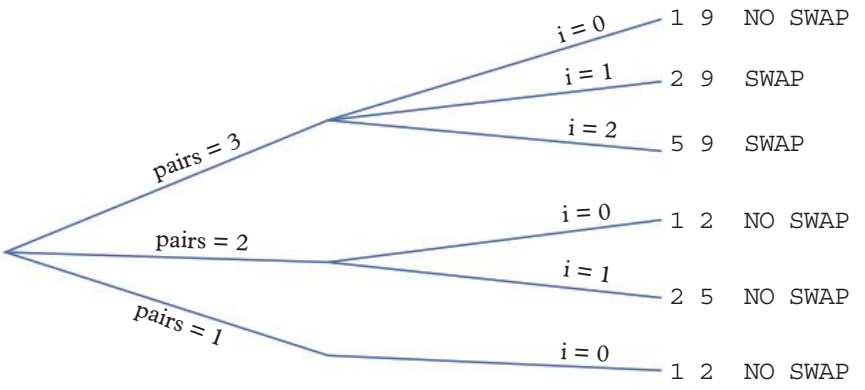
**2** Consider the outer loop: `for pairs in range(length-1,0,-1):`
There are four numbers so the outer loop will loop through 3, 2 and 1 pairs, and there will be three branches in the tree diagram.

**3** Consider the inner loop: `for i in range(0,pairs,1):`
The number of branches is always 1 less than the number of pairs in the outer loop.

**4** Consider the if statement: `if numbers[i] > numbers[i+1]:`
This statement checks whether to swap the pair of numbers.

**5** Write the numbers that will be printed in each loop and the accompanying "SWAP" or "NO SWAP" label.

**WRITE**




## **Example 10B.3** Finding the highest number in a list

For a given list of integers called `numbers`, write code to find the highest number in the list.

**THINK**

1. The bubble sort algorithm sorts numbers into ascending order, so we can sort the list and then simply look at the rightmost (last) number. Write bubble sort code for the list of integers called `numbers`.
2. Write a line of code printing the rightmost number in the list. Recall that to access the last item in a list, we will use `numbers[-1]`. The negative sign on the -1 indicates we want to count from the back of the list.

**WRITE**

```
length = len(numbers)
for pairs in range(length-1,0,-1):
     for i in range(0,pairs,1):
          if numbers[i] > numbers[i+1]:
               numbers[i], numbers[i+1] = numbers[i+1], numbers[i]
                    print(numbers[-1])
```

## **Example 10B.4** Finding the median of a list of numbers

For a given list of integers called `numbers`, write code to find the median of the list.

**THINK**

1 Consider the process of finding the median. Placing the numbers in ascending order can be achieved using the bubble sort algorithm.

2 Consider the two conditions for the median. Count the number of integers in the list.

→ For an odd number, the median is the middle integer.

→ For an even number, the median is the mean of the middle two integers.

→ To check whether the length of the list is even, use the following line of code:
`if length % 2 == 0:`

3 Determine the code for an even-numbered list.

→ To find the two numbers in the middle of a list of an even number of integers, the two numbers required will be in the positions:
$\frac{\text{length of list}}{2} - 1$ and $\frac{\text{length of list}}{2}$.

→ Code:
```
middle_left = int(len(numbers) / 2 - 1)
middle_right = int(len(numbers) / 2)
```

→ To find the median of the two middle numbers, use the following line of code:
`median = (numbers[middle_left] + numbers[middle_right])/2`

4 Determine the code for an odd-numbered list.

→ The formula for the index of the integer in the middle: $\frac{\text{length of list} - 1}{2}$

Note: This formula gives the position of the integer in the list, not the value of the integer. For example, if the formula gives an index of 10, then the middle integer is the 10th number in an ordered list.

→ Code: `middle = (len(numbers) - 1) / 2`

5 Print the result.

**WRITE**

```
length = len(numbers)
for pairs in range(length-1,0,-1):
    for i in range(0,pairs,1):
        if numbers[i] > numbers[i+1]:
            numbers[i], numbers[i+1] = numbers[i+1], numbers[i]
    if length % 2 == 0:
        middle_left = int(len(numbers) / 2 - 1)
        middle_right = int(len(numbers) / 2)
        median = (numbers[middle_left] + numbers[middle_right])/2
    else:
        middle = int((len(numbers) - 1) / 2)
        median = numbers[middle]
            print(median)
```

**Helpful hints**

- ✔ Division in Python will convert integers into a type of number called a *float*, which will show up as a decimal. As a result, whole numbers after division will often look like 5.0 instead of 5. If you require the integer only, use `int()`.
- ✔ One of the trickier aspects of the bubble sort algorithm is understanding how the `range()` function works. It is useful to experiment with simple for loops to see how changing each of the three inputs impacts the output. For example, run something like:

```
for i in range(2, 10, 3):
    print(i)
```

then change each of 2, 10 and 3 to see what happens.

# ANS p561 Exercise 10B Sorting a list of numbers

10B.1 **1** Show each of the steps of the bubble sort algorithm for the following lists. (Hint: Refer to the example in the theory section above to see how to annotate the steps.)

**a** 1 8 2 5 **b** 2 7 0 3 **c** 8 6 2 9 5 **d** 6 5 4 3 2 1

10B.2 **2** Consider the code given below. Draw a tree diagram to represent what is happening in this nested loop and what the print output will be, if any, after each loop.

```
numbers = [3,1,7,2,8]
length = len(numbers)
for pairs in range(length-1,0,-1):
    for i in range(0,pairs,1):
        if numbers[i] > numbers[i+1]:
            numbers[i], numbers[i+1] = numbers[i+1], numbers[i]
            print(numbers[i], numbers[i+1])
```

10B.3 **3** Write a bubble sort code to find the lowest number in a list called `numbers`.

**4** Write a bubble sort code to find the second-highest number in a list called `numbers`.

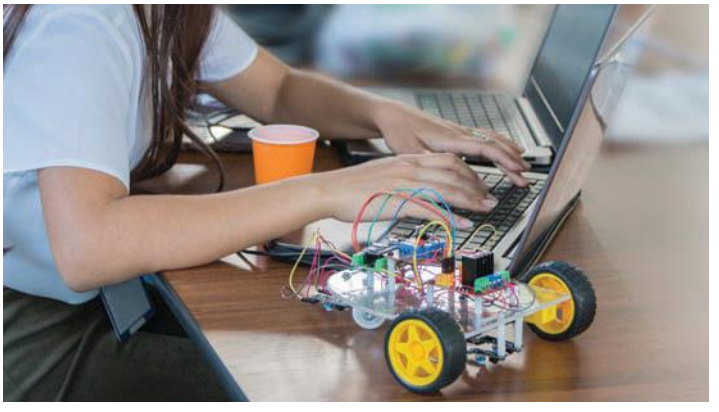
10B.4 **5** Write code that finds the median of an already sorted list of numbers where there are an *odd* number of items in the list called `numbers`.

**6** Write code that finds the median of an already sorted list of numbers where there are an *even* number of items in the list called `numbers`.

**7** Does the bubble sort algorithm help in calculating the mean (average) of a list? Explain why or why not.

**8** Is it possible to find the highest number in a list without using a nested loop? (Hint: Consider your solutions to question **1** of this exercise. What do you notice?)

**9** Rewrite the bubble sort algorithm so that it sorts a list called `numbers` in descending order instead.



# 10C Functions

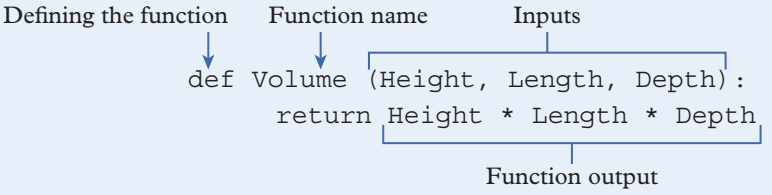
## Learning intentions

- ✔ I understand that a function is a machine with inputs and a return.
- ✔ I can read and write functions.

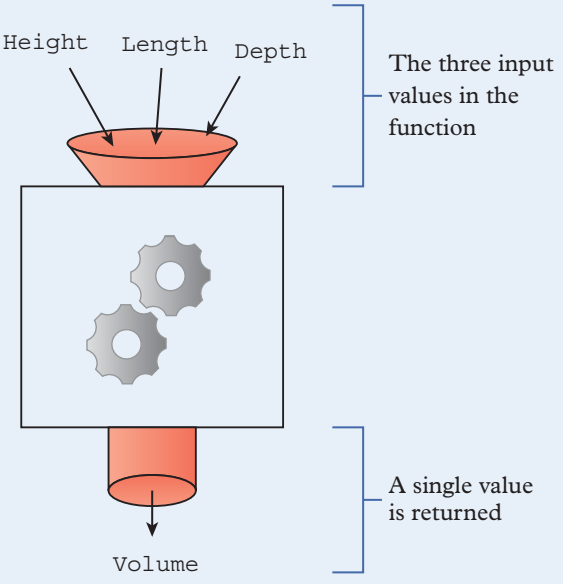
## Defining a function

- In coding, there is often a need to re-use the same piece of code repeatedly. You can **define** a function and then re-use the same piece of code many times.
- To define or create a function:
  1. Begin the code line with `def`, which is short for define.
  2. Create a name for the function and include **inputs** instead of the function brackets.
  3. Indent the code and write `return` followed by the output of the function.

  For example, a function is defined that will input the Height, Length and Depth of a rectangular prism and **return** its volume:




- It is useful to visualise a function as a physical machine, where the inputs are what goes in, and the return is what comes out.




- Defining a function is like building a machine. To get it to work, you have to **call** the function and give it some inputs. Just writing `return` will not print the required code on the screen.

  For example, `print(Volume(10,5,3))`

  will compute $10 \times 5 \times 3$ (as that is what we defined the function to do) and will print 150 on the screen.

## **Example 10C.1** Reading and calling functions

Consider the code given below which defines a function, followed by a call of the function. Determine what the call will return.

```
def function1(x):
    if x > 10:
        return x - 5
    else:
        return x + 3
```

**a** `print(function1(5))`

**b** `print(function1(15))`

**THINK**

**a** **1** Determine the properties of the function. The code defines a function called `function1` with an input of `x`. If the input value is greater than 10, the return is the result of `x - 5`. If the input is less than or equal to 10, the return is the result of `x + 3`.

**2** The call is asking to print the return of the `function1` with an input of 5. As 5 is less than 10, the return is `5 + 3`.

**b** The call is asking to print the return of the `function1` with an input of 15. As 15 is greater than 10, the return is `15 - 5`.

**WRITE**

**a** `8`

**b** `10`

## **Example 10C.2** Writing functions

Write a function that inputs a list and returns the third number in the list.

**THINK**

**1** Define the function. It doesn't matter what the function or the list are called if the names are consistent in the code. Here, the function has been called `third` and the input list has been called `L`.

**2** Return the element in position 2. Recall that the third number in a list is in position (index) 2.

**WRITE**

```
def third(L):
    return L[2]
```

## Example 10C.3 Writing a function to find the median of a sorted list

Write a function that will input a sorted list and will return the median of that list.

**THINK**

1. Define the function.
2. The code for finding a median has already been written in Example 10B.4. Note, however, that we are only using the second part of the example as in this case we are assuming the list is already sorted.
3. Return the median.

**WRITE**

```
def find_median(numbers):
    length = len(numbers)
    if length % 2 == 0:
        middle_left = int(len(numbers) / 2 - 1)
        middle_right = int(len(numbers) / 2)
        median = (numbers[middle_left] + numbers[middle_right])/2
    else:
        middle = int((len(numbers) - 1) / 2)
        median = numbers[middle]
    return median
```

**Helpful hints**

- ✔ Remember that defining a function will not print anything to the screen. It is just creating a machine that you can use later. Only when you call the function will you see what it returns.
- ✔ A function *can* print a value as part of its function, but it often returns (or outputs) something as well.

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# Exercise 10C Functions

10C.1 **1** For each of the functions below, consider what the call of the function with the specific given value/s will return:

**a**
```
def triangle(b, h):
    return 0.5 * b * h
```
**i** `print(triangle(2, 8))`

**ii** `print(triangle(8, 5))`

**b**
```
def function2(L):
    for item in L:
        return(item)
```
**i** `print(function2([2,7,4]))`

**ii** `print(function2(['hat','coat','boots','umbrella']))`

**c**
```
def find(x, L):
    if x in L:
        return True
    else:
        return False
```
**i** `print(find(3, [2,4,5]))`
**ii** `print(find(7, [2,3,5,7]))`

**d**
```
def function3(x,y,z):
    if x > y + z :
        return x
    elif y > x + z :
        return y
    else:
        return z
```
**i** `print(function3(2,7,4))`
**ii** `print(function3(5,2,4))`
**iii** `print(function3(10,-15,12))`

**e**
```
def digits(number):
    if number < 10:
        return "1 digit"
    elif number < 100:
        return "2 digits"
    else:
        return "3 or more digits"
```
**i** `print(digits(7))`
**ii** `print(digits(216))`
**iii** `print(digits(55))`
**iv** `print(digits(-15))`

10C.2 **2** Write a function that returns the second element in an input list.

**3** Write a function that returns the length of an input list.

**4** Write a function that returns the length of an input list without using the `len()` command. (Hint: You will need to use a for loop and a variable that keeps count).

10C.3 **5** Write bubble sort code as a function, taking in an unsorted list and returning a sorted list.

**6** Write a function that finds the lowest number in an unsorted list.

**7** Write a function that determines whether an input list has an odd or even number of items.

**8** Write a function that finds the median of an unsorted list. (Hint: Combine the idea from Example 10C.3 and Question **7** above).

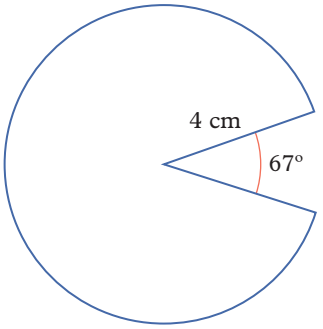
**9** Write a function that uses an altered version of bubble sort code to sort an input list of integers in descending order.

# Semester 2 review

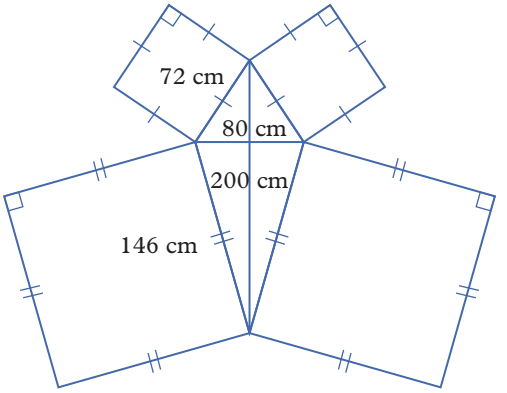
## Short answer

**1** Calculate the areas of these composite shapes, correct to two decimal places.

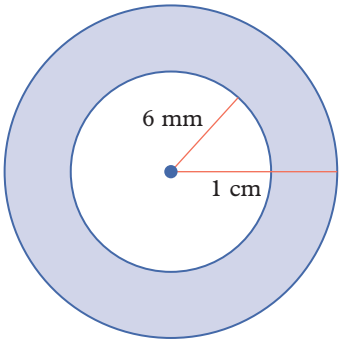
**a**



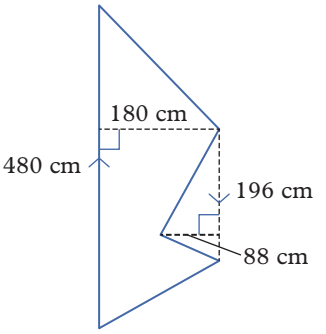

**b**




**c**

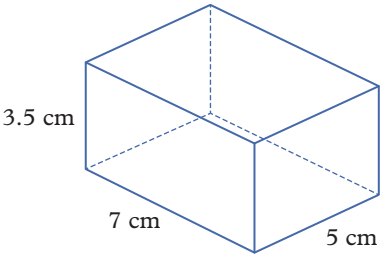



**d**

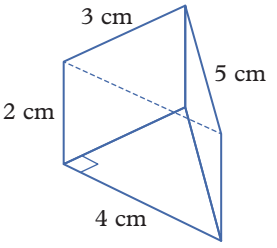



**2** Calculate the surface area of the following objects, correct to one decimal place where required.

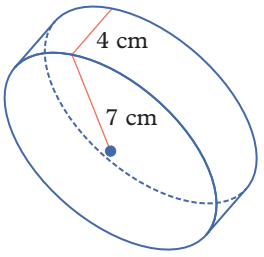
**a**



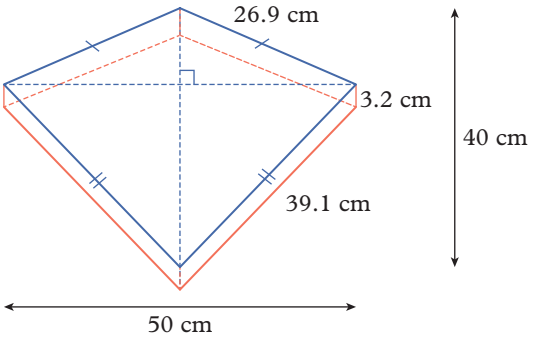

**b**




**c**



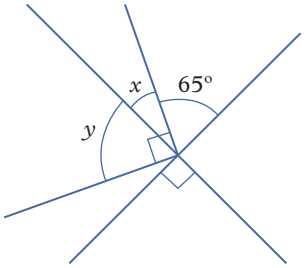

**d**



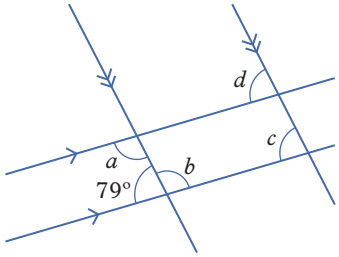

**3** Calculate the volume of the objects in question **2**, correct to one decimal place where required.

**4** Determine the values of the pronumerals in each diagram.

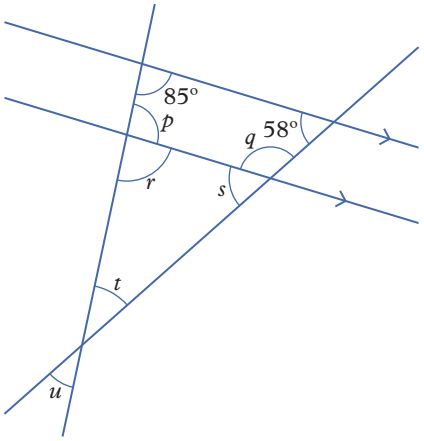
**a**



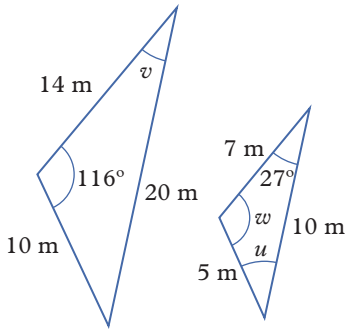

**b**



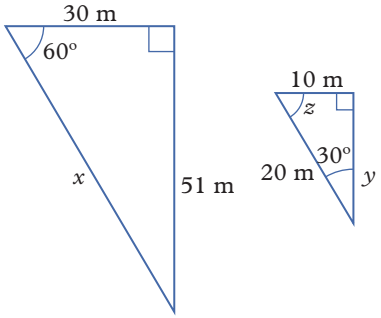

**c**



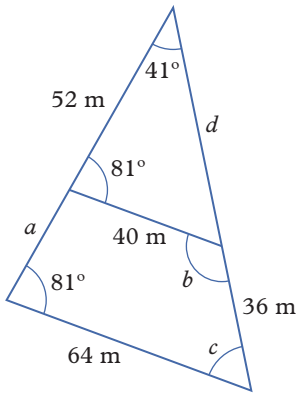

**d**




**e**

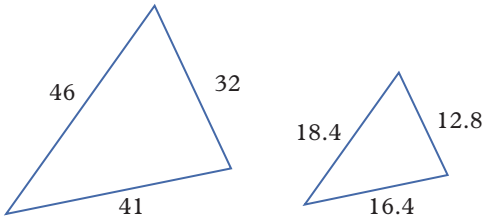



**f**

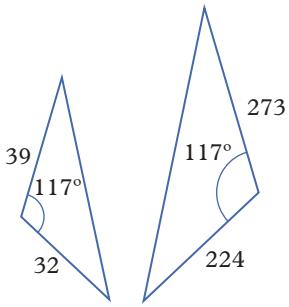



**5** Determine if the following pairs of triangles are similar. If they are similar, state the condition that explains why.

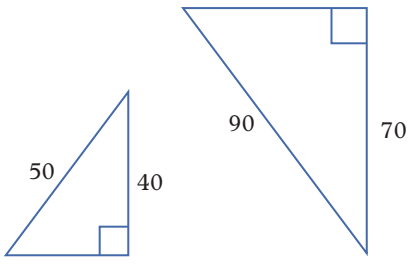
**a**



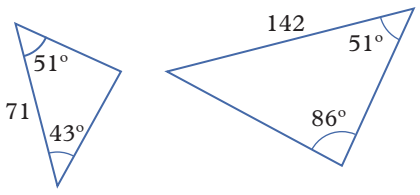

**b**



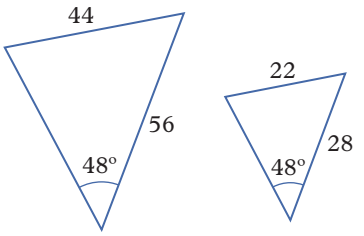

**c**



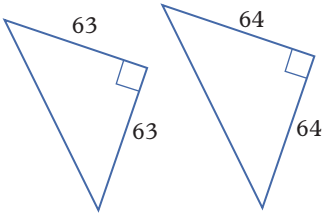

**d**



**e**




**f**




**6** For each of the following pairs of similar triangles state the following values. Where appropriate, round to four decimals places.

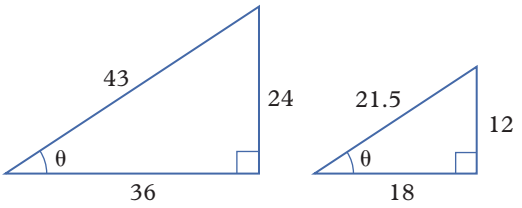
**i** the scale factor (original on left)

**ii** $\sin(\theta)$

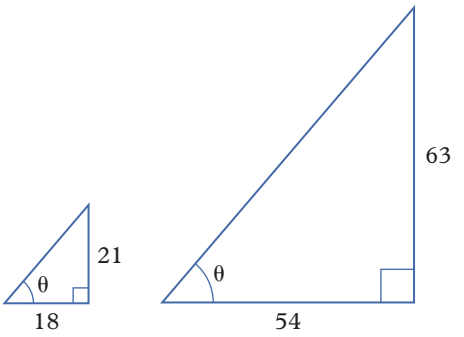
**iii** $\cos(\theta)$

**iv** $\tan(\theta)$

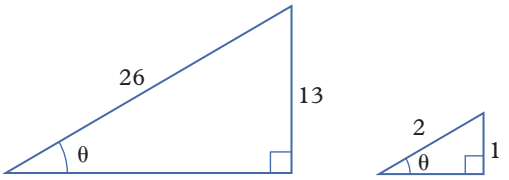
**a**



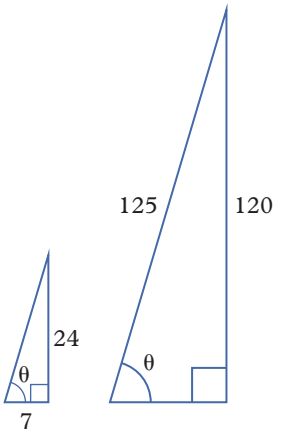

**b**




**c**

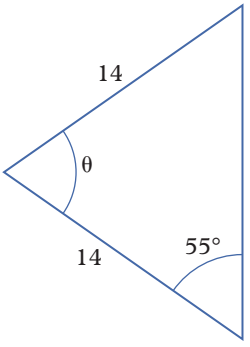



**d**

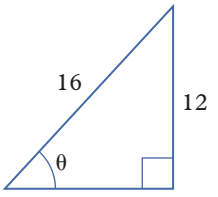



**7** Determine the value of θ in each of the following, correct to the nearest degree.

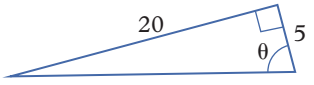
**a**



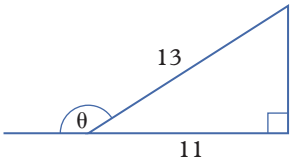

**b**




**c**

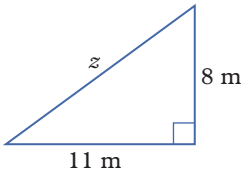



**d**

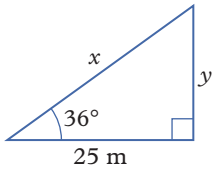


**8** Determine the length of the unknown side lengths, correct to three significant figures.

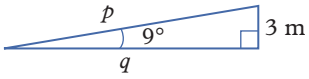
**a**



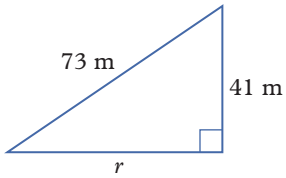

**b**




**c**




**d**




**9** Determine whether these side lengths can form right-angled triangles.

**a** 7.7 cm, 41.7 cm, 42.7 cm

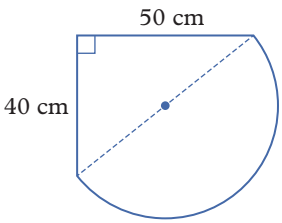
**b** 29 mm, 128 mm, 131 mm

**c** 20 cm, 48 cm, 52 cm

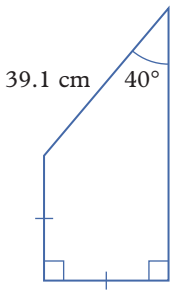
**d** 4 m, 4.2 m, 5.8 m

**10** Calculate the area of these shapes, correct to one decimal place.

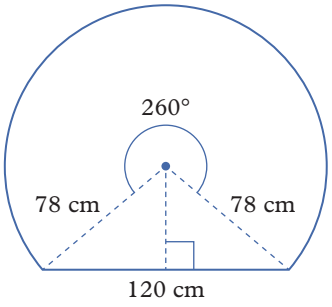
**a**



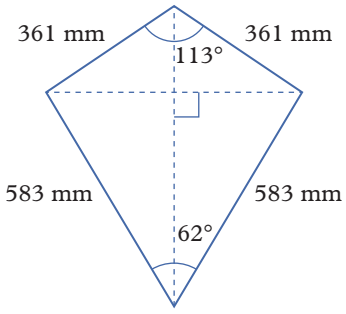

**b**




**c**



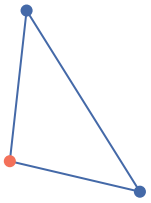

**d**




**11** Dilate each figure using the orange centre of dilation and the scale factor provided.

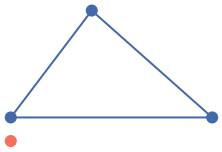
**a**

Scale factor: $\frac{1}{2}$



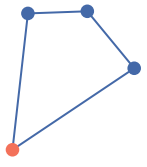
**b**

Scale factor: 2



**c**

Scale factor: 3



**12** Classify each of the following variables as:

**i** categorical or numerical

**ii** ordinal, nominal, discrete or continuous

**a** name of train line

**b** distance to Southern Cross Station

**c** number of passengers on the train

**13** Draw an appropriate frequency table to represent each of these numerical data sets.

**a** 24.9, 31.1, 26.8, 19.2, 19.1, 29.8, 26, 25.5, 16, 23.7, 18.1, 25.8, 23, 18.3, 7.7, 15.3, 26.7, 27.6, 6.6, 23.8

**b** 45, 94, 94, 75, 113, 75, 98, 87, 73, 51, 4, 90, 94, 48, 99, 62, 63, 64, 32, 101, 81, 119, 85, 98, 103, 92, 90, 86, 84, 56, 99, 81, 86, 82, 75, 38, 89, 89, 71, 46

**14** Construct the specified graph for each of the following data sets.

**a** column graph

| Favourite game | Frequency |
|---|---|
| Board games | 10 |
| Card games | 45 |
| Video games | 50 |
| Physical games | 25 |

**b** line graph

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Number | 12 | 20 | 16 | 18 | 24 | 22 | 24 | 24 | 26 |

**c** column graph

| Value | Frequency |
|---|---|
| 8 | 10 |
| 9 | 26 |
| 10 | 34 |
| 11 | 38 |
| 12 | 30 |

**d** histogram

| Class | Frequency |
|---|---|
| 0–<10 | 30 |
| 10–<20 | 60 |
| 20–<30 | 10 |
| 30–<40 | 90 |
| 40–<50 | 30 |

**15** Determine each of the following values. Round to two decimal places if required.

**i** mean **ii** median **iii** mode **iv** range

**a** 21, 19, 19, 19, 17, 16, 18, 26, 24, 19

**b**

| Score ($x$) | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| Frequency ($f$) | 6 | 12 | 18 | 20 | 14 |

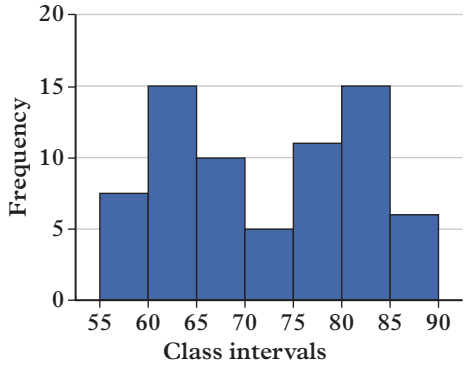
**c**

| Stem | Leaf |
|---|---|
| 2 | 0 3 4 6 7 |
| 3 | 1 2 3 3 4 4 4 5 7 |
| 4 | 1 2 5 |
| 5 | 1 3 3 6 8 |
| 6 | 0 1 1 2 4 6 7 |

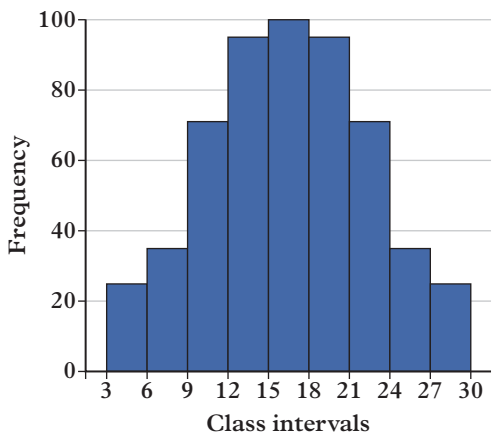
Key 2 | 1 = 2.1

**16** Describe the shape of the distribution in each of the following graphs.

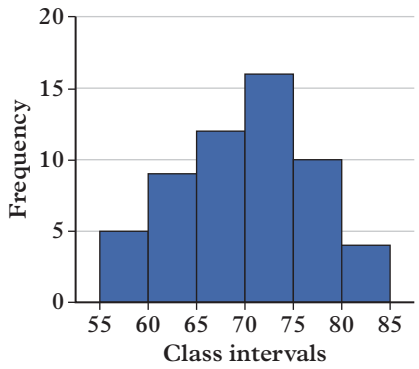
**a**



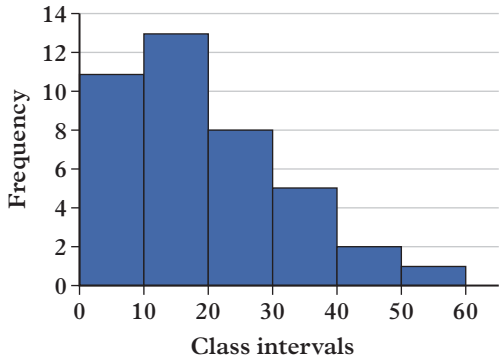

**b**



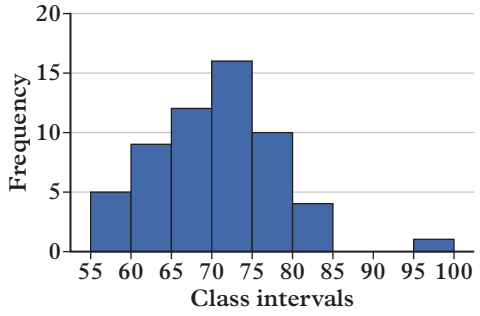

**c**



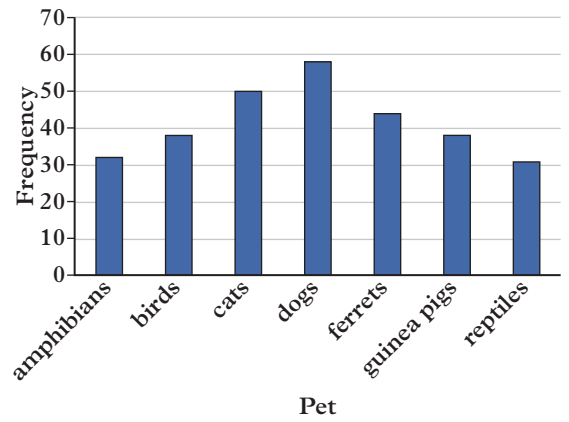

**d**




**e**




**f**




**17** State which measures of centre are most appropriate to use to describe each of the distributions in question **16**.

**18** Brea plans to play video games on two different consoles this afternoon. She has an Xbox, a PlayStation and a Nintendo Switch to choose from.

**a** Construct a tree diagram to show all outcomes in the sample space.

**b** Assuming each outcome is equally likely, calculate the probability that Brea plays:

**i** Xbox then PlayStation

**ii** Xbox second

**iii** Xbox either first or second

**iv** Xbox and PlayStation in either order.

**19** A fair six-sided die is rolled and the result is noted. Following that a card is drawn from a standard 52-card deck and the suit is noted.

**a** Construct an array to show all outcomes in the sample space.

**b** Calculate the probability of:

**i** rolling a 6 and drawing a spade

**ii** rolling an even number and drawing a red card (heart or diamond)

**iii** rolling an even number or drawing a red card or both

**iv** rolling an even number or drawing a red card but not both.

**20** A bag contains 21 coloured tokens: 4 red, 8 blue and 9 yellow. Two tokens are selected at random from the bag simultaneously. Calculate the probability that:

**a** a yellow and blue token are selected, if the first token is replaced

**b** a yellow and blue token are selected, if the first token is not replaced

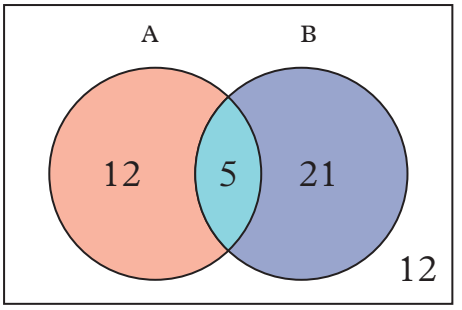
**c** two red tokens are selected, if the first token is not replaced

**d** two of the same-coloured tokens are selected, if the first token is not replaced.

**21 a** Display the information in this two-way table in a Venn diagram.

| | **A** | **A'** | **Total** |
|---|---|---|---|
| **B** | 11 | 57 | 68 |
| **B'** | 47 | 25 | 72 |
| **Total** | 58 | 82 | 140 |

**b** Display the information in this Venn diagram in a two-way table.




**22** A school has 212 Year 9 students. Of the 98 students with a smartphone, 56 have a gaming console. 80 students do not have a gaming console.

**a** Show this information in a two-way table.

**b** Calculate the relative frequency of students that:

**i** do not have a smartphone

**ii** have a gaming console

**iii** have neither a smartphone nor a gaming console

**iv** have a smartphone or a gaming console but not both.

**23 a** Convert this two-way table to a relative frequency two-way table.

| | A | A' | Total |
|---|---|---|---|
| **B** | 75 | 5 | 80 |
| **B'** | 35 | 10 | 45 |
| **Total** | 110 | 15 | 125 |

**b** Convert this relative frequency two-way table to a two-way table given that the sample size is 120.

| | A | A' | Total |
|---|---|---|---|
| **B** | $\frac{17}{40}$ | $\frac{29}{120}$ | $\frac{2}{3}$ |
| **B'** | $\frac{1}{8}$ | $\frac{5}{24}$ | $\frac{1}{3}$ |
| **Total** | $\frac{11}{20}$ | $\frac{9}{20}$ | 1 |

**24** A large box contains some light globes from two companies: Phillips and Osram. A sample of 90 light globes is taken and 46 are Phillips light globes, 39 are fluorescent and 21 are Osram LED light globes.

**a** Show this information on a Venn diagram.

**b** From the sample of 90 light globes, calculate the relative frequency of:

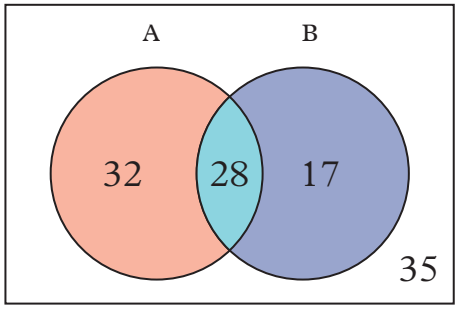
**i** Phillips fluorescent light globes

**ii** LED light globes

**iii** Osram light globes

**iv** fluorescent or not Phillips or both.

**25** Consider this Venn diagram.




**a** Given that $n(A)$ means the number of elements in set A, find:

**i** $n(A)$ **ii** $n(A \cup B)$ **iii** $n(A')$

**iv** $n(A \cap B)$ **v** $n(\xi)$ **vi** $n(A' \cap B)$

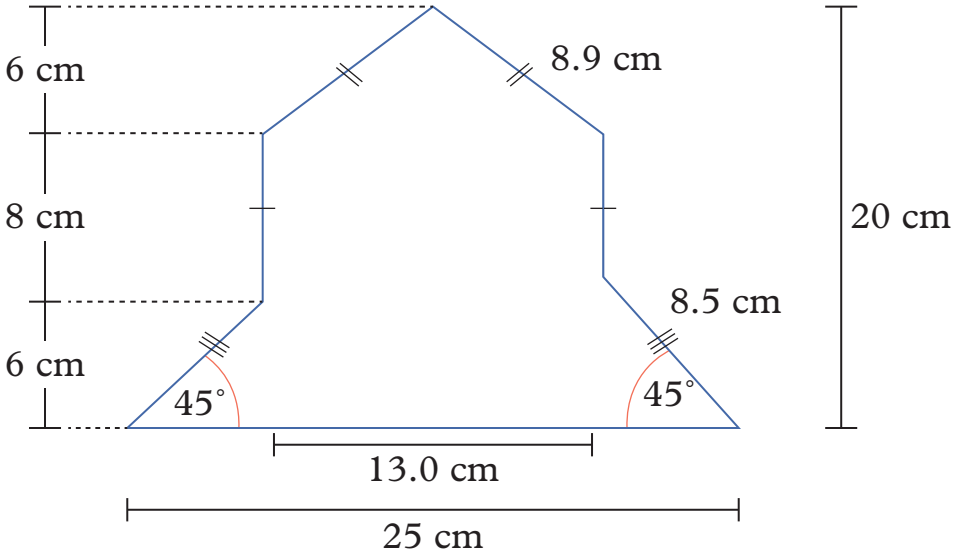
**b** Given that Pr(A) means the probability of selecting an element from set A, find:

**i** $\Pr(A)$ **ii** $\Pr(A')$ **iii** $\Pr(A \cap B)$

**iv** $\Pr((A \cup B)')$ **v** $\Pr(A' \cap B')$ **vi** $\Pr(A \cup B')$

## Analysis

**1** Xav is planning to create a flat plywood spaceship to mount on his wall. He draws a rough diagram of the spaceship using a scale factor of 1 : 5. The desired lengths and angles in the spaceship are shown.




**a** The angles and integer lengths in the diagram are correct, but Xav measured the other lengths. Use trigonometry or Pythagoras' Theorem to check whether the lengths 8.9 cm, 8.5 cm and 13.0 cm are correct. If not, calculate the correct lengths correct to one decimal place.

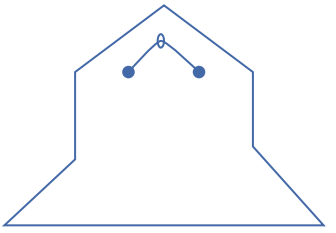
**b** Draw a labelled diagram of the spaceship showing the correct lengths and angles.

**c** Calculate the area of:

**i** the diagram of the spaceship

**ii** the actual plywood spaceship.

To mount the plywood spaceship on the wall, Xav wants to attach an 80 cm string to two points on the back to hang on a hook. The hook will pull the string 20 cm up a vertical distance from where the string is connected, as shown.



**d** Determine the horizontal distance between the two points where Xav will attach the string, correct to one decimal place.

**e** Determine the angle the string will make with the line between the two attachment points.

**f** Determine whether the string is parallel to the edge of the plywood. Explain why or why not.

**2** Arielle and Brenna have started to track their sleeping habits. They recorded how many minutes each slept in February, correct to the nearest 10 minutes.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Arielle:** | 550, | 530, | 480, | 480, | 470, | 410, | 480, |
| | 510, | 520, | 490, | 430, | 480, | 480, | 370, |
| | 460, | 440, | 490, | 480, | 500, | 520, | 470, |
| | 470, | 410, | 480, | 490, | 500, | 430, | 470 |
| **Brenna:** | 570, | 420, | 560, | 460, | 530, | 480, | 510, |
| | 410, | 450, | 510, | 430, | 540, | 600, | 570, |
| | 600, | 440, | 530, | 460, | 630, | 370, | 580, |
| | 500, | 500, | 550, | 510, | 400, | 510, | 460 |

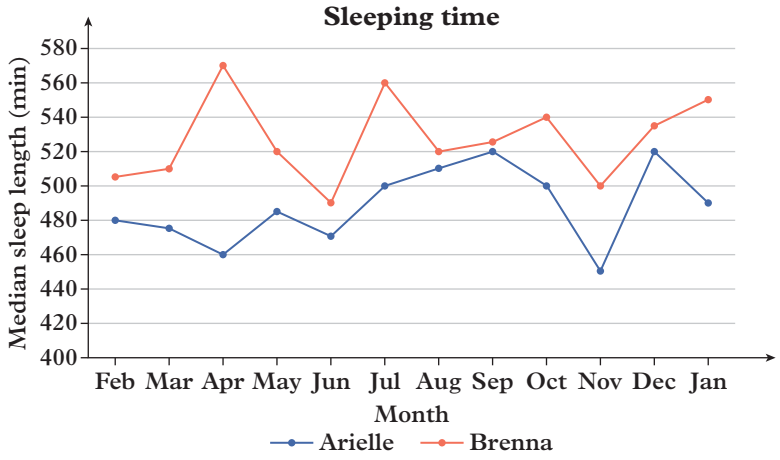
**a** Construct a back-to-back stem-and-leaf plot using the data above. Use intervals of 50 and the key 1 | 2 = 120 minutes.

**b** Compare the shape of the two data sets.

**c** Determine the median and range for Arielle's and Brenna's minutes of sleeping.

**d** Write two sentences comparing the two data sets with reference to their summary statistics.

The median values of Arielle's and Brenna's sleep times are calculated each month for a year and are plotted on the line graph below.




**e** State the independent and dependent variables.

**f** Compare the trends in Arielle's and Brenna's median sleep length over the year.

**3** An avid puzzle collector has puzzles with 1000, 5000 and 10 000 pieces. Some of the puzzles have irregular-shaped designs, as shown in the table below.

| | 1000 pieces | 5000 pieces | 10 000 pieces |
|---|---|---|---|
| **Number of puzzles in collection** | 20 | 9 | 1 |
| **Number with irregular-shaped designs** | 5 | 2 | 0 |

**a** Determine the relative frequency of:

**i** 1000-piece puzzles out of all puzzles

**ii** puzzles with irregular designs out of all puzzles

**iii** 1000-piece puzzles with irregular designs out of all 1000-piece puzzles

**iv** 1000-piece puzzles with regular designs out of all puzzles with regular designs

The puzzle collector selects two puzzles at random with replacement.

**b** Determine the probability of choosing:

**i** an irregular-shaped 1000-piece puzzle and then a regular-shaped puzzle that is not 1000 pieces

**ii** a 1000-piece puzzle and then an irregular shaped design.

The puzzle collector starts with a rectangular 1000-piece puzzle. The puzzle actually consists of 1026 pieces in a 38-piece by 27-piece grid. The collector separates the corner, edge, and centrepieces.

**c** Calculate the probability of selecting at random:

**i** an edge piece connecting to any corner piece

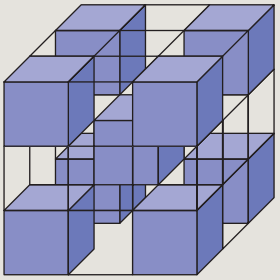
**ii** a centre piece connecting to any edge piece.

# EXPLORATIONS 2

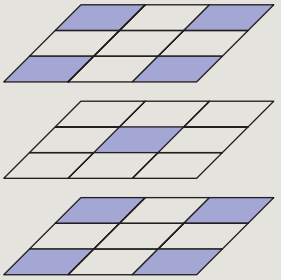
## 1 Cube stacks

Jali has a large supply of blue and clear cubes of identical size. She arranges 9 blue cubes and 18 clear cubes into a 3 × 3 × 3 stack as shown below. The simpler diagram in the middle shows the position of the blue cubes in each 3 × 3 horizontal layer. Jali notices that this arrangement results in the same pattern, shown on the right, when she views the stack from all six directions: top, bottom, left, right, front and back. In fact, she realises that if two cubes are removed from the top and bottom layers, it is still possible to see the same final pattern from all directions.

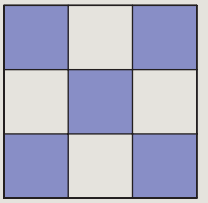
Jali wonders what other symmetrical patterns can be seen from all directions and how many blue cubes are needed to make them.



3D view



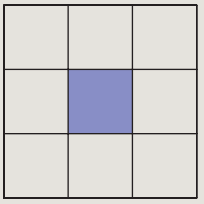
layer view



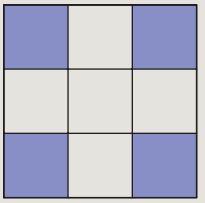
directional view

**a** For each of the following, determine both the maximum and minimum number of blue cubes that Jali can use in a 3 × 3 × 3 stack so that the resulting pattern can be seen from all six directions. Draw a layer view for each.

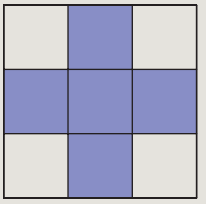
**i**



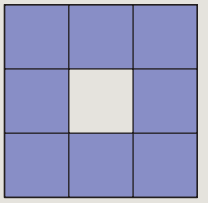
**ii**



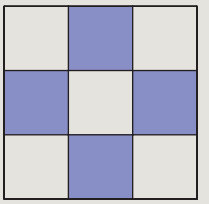
**iii**



**iv**



**b** Jali wants to make the symmetrical pattern shown on the right.



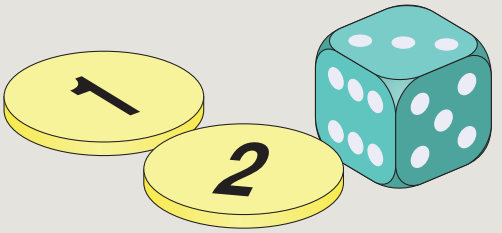
**i** Describe a stack in which this pattern can be seen from exactly two directions.

**ii** Describe a stack in which this pattern can be seen from exactly four directions.

**iii** Explain why it is not possible to see this pattern from all six directions.

## 2 Counter attack

Connie has a number of counters, each labelled 1 on one side and 2 on the other. She flips the counters and adds up the numbers showing on top. At the same time, Dieter selects a die and rolls it. The game ends in a draw if Connie and Dieter get the same score, otherwise the winner is the player with the higher score.




**a** Connie flips two counters and Dieter rolls a standard 6-sided die.

**i** Calculate the probability that both players get a score of 3.

**ii** Explain why Dieter is more likely to win than Connie.

**b** Connie flips three counters and Dieter rolls a 6-sided die. Is this version of the game fair? That is, do Connie and Dieter have the same chance of winning?

**c** Dieter selects an octahedral die numbered 1 to 8. How many counters must Connie flip for the game to be fair?

**d** Connie flips five counters. Assuming they are numbered 1, 2, 3 … how many sides must there be on Dieter's die for the game to be fair?

**e** Generalise part **d** as follows: Connie flips $n$ counters and Dieter spins a spinner with $k$ wedges of equal size labelled 1, 2, 3 …, k. Find the relationship between n and k if the game is fair.

**f** Is there a fair version of the game if Dieter uses a dodecahedral die numbered 1 to 12? What about an icosahedral die numbered 1 to 20?

## 3 Differagons

A differagon is a polygon in which each side length is a different positive integer.

**a** For each of the following types of differagon, find the least possible perimeter:

**i** 3-sided
**ii** 3-sided with a right angle
**iii** 4-sided
**iv** 4-sided with one right angle
**v** 4-sided with two adjacent right angles
**vi** 4-sided with two opposite right angles
**vii** 5-sided with three consecutive right angles
**viii** 5-sided with three right angles, not all consecutive.

**b** Which types of differagon listed in part **a** can have an odd perimeter? Explain.

**c** Diago claims he has found a 6-sided differagon ABCDEF with the following properties:

- angles ABC, ACD, ADE and AFE are all right angles
- BC = 2, CD = 6, DE = 3 and EF = 7.

Show that Diago must have made a mistake.

## 4 All sorts

Imagine you have a row of cards in front of you, each with a single letter written on it. The goal is to sort them into alphabetical order. Here are three different approaches to this problem. As you work through them, see what shortcuts you can come up with to answer the questions as efficiently as possible.

**a** The SelectSort algorithm works as follows:

- Find the letter that appears earliest in the alphabet (in the case of repeats, choose the first occurrence).
- Swap it with the letter in the first position.
- Repeat the above steps with the remainder of the list.

For example, consider the cards S O R T in that order. The first step swaps O (the earliest letter) with S (the first letter), producing O S R T. Now repeat this procedure with the remaining list S R T, swapping R and S. The final step applied to S T has no effect, since S is earliest and in the first position. So only two steps are required to arrive at the final order, O R S T.

For each of the following, find the number of steps that SelectSort will take to sort the cards into alphabetical order.

**i** O R D E R S **ii** S O R T E D **iii** A R R A N G I N G **iv** D I S O R G A N I S E D

**b** In the SwapThree algorithm, a move consists of reversing the order of three consecutive cards. For example, S O R T can be converted to R O S T by reversing the first three cards, or to S T R O by reversing the last three. For each of the following, decide whether it is possible to sort the cards into alphabetical order using SwapThree and, if so, determine the minimum number of moves required.

**i** O R D E R S **ii** S O R T E D **iii** A R R A N G I N G **iv** D I S O R G A N I S E D

**c** Each pass of the iSort algorithm works as follows:

- Compare the first two letters in the list. Whichever is earlier in the alphabet is moved to a new list. The other stays in the original list.
- Repeat the previous step with the remaining letters, moving one letter to the end of the new list at each stage.
- When there is one letter remaining, move it to the end of the new list.

For example, comparing the first two letters of S O R T, the letter O starts the new list and S R T remain in the original list. Comparing the first two letters again, R is moved to the new list, leaving S T. Finally, S and then T are moved to the new list, giving O R S T. This is already in alphabetical order, so the original list was sorted in one pass of the iSort algorithm. But in general, more passes may be needed.

For each of the following, find the number of passes of iSort needed to sort the cards into alphabetical order.

**i** O R D E R S **ii** S O R T E D **iii** A R R A N G I N G **iv** D I S O R G A N I S E D

# Naplan practice

## CHAPTER 1 Financial mathematics

### Calculator

**1** John's annual pay is $55 827. Which statement is an estimate for his fortnightly pay?

- ☐ $55 827 ÷ 26
- ☐ $60 000 ÷ 26
- ☐ $60 000 ÷ 30
- ☐ $50 000 ÷ 30

**2** You can buy a 5 kg bag of apples for $14.50. Which rate statement is *not* true?

- ☐ $14.50 per 5 kg
- ☐ $2.90 per kg
- ☐ $9.50 per kg
- ☐ $7.25 per 2.5 kg

Questions **3** and **4** refer to this information.

Ainslee earns $15.80 per hour. In one particular week, she worked 14 hours at the standard rate of pay, 5 hours at time-and-a-half and 3 hours at double time.

**3** Which calculation would determine her gross income?

- ☐ 14 × $15.80 + 5 × $15.80 + 3 × $15.80
- ☐ 14 × $15.80 + 5 × 1.5 × $15.80 + 3 × 2 × $15.80
- ☐ 14 × $15.80 + 5 × 1.5 × $15.80 + 3 × $15.80
- ☐ 14 × $15.80 + 5 × $23.07 + 3 × $31.06

**4** Ainslee has these items deducted from her pay in this week:

income tax: $54.20, union fees: $8.50, superannuation: $15.75.

What is her net income?

**5** Shorts originally priced at $79.00 are offered for sale at a discount of 15%. Which represents the calculation for the discount amount?

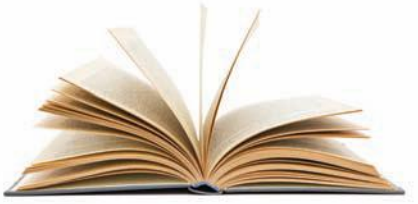
- ☐ 85% of $79.00
- ☐ 15% of $79.00
- ☐ 115% of $79.00
- ☐ $79.00 – 15% of $79.00

**6** The wholesale price on a television is $820. After a mark-up of 85%, what is the selling price of this television?

**7** A refrigerator originally marked at $1245 is discounted by 17%. What is the selling price?

**8** A novel sells for $15 after a discount of 20%. Which of these represents the original price before the discount was applied?

- ☐ $15 ÷ 80 × 100
- ☐ $15 + 20% of $15
- ☐ 80% of $15
- ☐ $15 × 80 ÷ 100



**9** A salesman receives 2.4% of the total sales made during a week. What is his pay in a week where his total sales are $14 580?

**10** What is $450 written as a percentage of $550? Round your answer to two decimal places.

Questions **11** and **12** refer to this information.

The original price for a coin set was $40. The selling price, some time later, was $150.

**11** Which statement is correct?

- ☐ The coins were sold at a loss of $110.
- ☐ The coins were sold at a profit of $110.
- ☐ The coins were sold at a profit of $150.
- ☐ The percentage profit is 375%.

**12** What is the profit or loss amount as a percentage of the original price?

Questions **13** and **14** refer to this information.

A loan of $12 500 is charged a simple interest rate of 8% p.a. for a period of 3 years.

**13** How much interest is charged to the loan?

**14** What is the total amount to be repaid?

Questions **15** and **16** relate to this savings account bank statement for the month of April.

| Date | Transaction | Credits $ | Debits $ | Balance $ |
|---|---|---|---|---|
| 01/04 | Opening balance | | | 825.00 |
| 05/04 | Withdrawal at Handybank | +100.00 | | 925.00 |
| 13/04 | Deposit – Pay | +740.00 | | 1665.00 |
| 25/04 | EFTPOS purchase | | −225.00 | 1440.00 |
| 30/04 | Interest | | | |

**15** How many days does the highest monthly balance apply to this account?

**16** Interest for this account is calculated daily at a rate of 3.0% p.a. How much interest is earned during the month?

**17** How much needs to be invested at an interest rate of 5% p.a. for 4 years to earn $1000 in simple interest?

**18** How long does a loan of $25 000 at an interest rate of 8% p.a. take to earn $7000 simple interest?

# CHAPTER 2 Indices

## Non-calculator

**1** What is $2^3 + 3^2$?

12 ☐ 17 ☐ 25 ☐ 36 ☐

**2** Simplify $x^6 \times x^4 \times x^{-11}$ using positive indices.

**3** Simplify $\frac{w^{11}}{w^6}$.

**4** Write $2.97 \times 10^4$ as a basic numeral.

**5** What is 0.000 000 000 000 000 000 000 000 029 in scientific notation?

**6** Which symbol makes the following statement true?

$4.98 \times 10^3$ ___ $0.0498 \times 10^6$

> ☐ < ☐ = ☐ ≤ ☐

**7** What does $(5 \times 10^2) \times (4 \times 10^3)$ simplify to in scientific notation?

$9 \times 10^5$ ☐ $2 \times 10^6$ ☐ $20 \times 10^5$ ☐ $9 \times 10^1$ ☐

## Calculator

**8** Which expression is equivalent to $5a^2 \times 3a^4$?

$8a^6$ ☐ $8a^8$ ☐ $15a^6$ ☐ $15a^8$ ☐

**9** Which expression shows $\frac{8r^{-2}p^{-6}}{12p^{-2}}$ in simplified form with positive indices only?

$\frac{2r^2}{3p^4}$ ☐ $\frac{8r^2}{12p^4}$ ☐ $\frac{2}{3r^2p^4}$ ☐ $\frac{2}{3r^2p^{-8}}$ ☐

**10** Which expression is equivalent to $\frac{x^5yz^2}{x^3y^4}$?

$\frac{x^2z^2}{y^3}$ ☐ $x^2y^3z^2$ ☐ $x^8y^5z^2$ ☐ $\frac{z^2}{x^2y^3}$ ☐

**11** Which expression is equivalent to $\left(\frac{2m^6}{n^4}\right)^3$?

$\frac{2m^{18}}{n^4}$ ☐ $\frac{6m^9}{n^7}$ ☐ $\frac{6m^{18}}{n^{12}}$ ☐ $\frac{8m^{18}}{n^{12}}$ ☐

**12** Which of the following is equivalent to $(2x^2y)^0 \times 3x^0$?

$1$ ☐ $6x^2$ ☐ $3$ ☐ $3x^2y$ ☐

**13** Which is equivalent to $4^{-6}$?

$-\frac{1}{4^{-6}}$ ☐ $-4^6$ ☐ $\frac{1}{4^6}$ ☐ $-(-4)^6$ ☐

# CHAPTER 3 Algebra

## Non-calculator

**1** Which expression is equivalent to $7(2b + 4)$?

$14b + 4$ ☐ $14b + 28$ ☐ $9b + 4$ ☐ $18b$ ☐

**2** What is the value of $4x^2$ when $x = -5$?

$100$ ☐ $-100$ ☐ $400$ ☐ $220$ ☐

**3** If $a = -3$ and $b = 3$, what is the value of $a^2 + b^2$?

**4** What term makes this statement true for all values of $x$?

$7(x + 6) +$ ____$(x - 2) = 10x + 36$

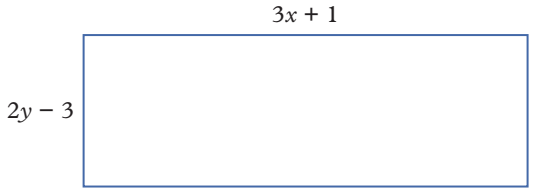
**5** If $m = -7$, what is the value of $2m - 11$?

**6** Which expression is equivalent to $5a + 6 + 7a - 11$?

☐ $12a - 5$ ☐ $7a$

☐ $12a + 17$ ☐ $11a - 4$

Questions **7–10** refer to this rectangle.




**7** Write the perimeter of this shape as an algebraic expression in simplest form.

**8** Calculate the perimeter when $x = 5$ cm and $y = 2$ cm.

**9** Write an algebraic expression for the area of the shape.

**10** Calculate the area when $x = 12$ cm and $y = 8$ cm.

## Calculator

**11** In a set of three consecutive whole numbers, the value of the lowest number is $w$. What is the value of the highest number in the set?

☐ $3w$ ☐ $w + 3$

☐ $3 - w$ ☐ $w + 2$

**12** What is the highest common factor of $18x^2yz$ and $12xy^2z$?

☐ $6xyz$ ☐ $9x^2yz$

☐ $6xy^2z$ ☐ $6xy$

**13** Expand $(a - 3)(a + 5)$.

**14** Factorise $b^2 - 13b + 40$.

**15** Factorise $m^2 + m - 12$.

**16** Factorise $7d^2 - 14d + 9d - 18$.

**17** What is the expanded form of $(x + 2y)^2$?

☐ $x^2 + 4y^2$

☐ $x^2 + 2y^2$

☐ $x^2 + 2xy + 2y^2$

☐ $x^2 + 4xy + 4y^2$

**18** What is the factorised form of $p^2 - 8p + 16$?

☐ $(p + 4)^2$

☐ $(p - 2)^2$

☐ $(p - 4)^2$

☐ $(p + 2)^2$

Questions **19** and **20** refer to a rectangle with an area of $(50x - 5x^2)$ cm$^2$.

**19** Write the area in factor form.

**20** List the expressions for the length and width of the rectangle.

# CHAPTER 4 Linear relationships

## Non-calculator

**1** The solution to $\frac{2x-3}{4} = 3$ is:

- ☐ $x = 4\frac{1}{2}$
- ☐ $x = 7\frac{1}{2}$
- ☐ $x = 12$
- ☐ $x = 34$

**2** Which equation does *not* have the solution $x = 3$?

- ☐ $\frac{4x-2}{5} = 2$
- ☐ $\frac{9-x}{6} = 1$
- ☐ $5 - 3x = 4$
- ☐ $4 - 2x = -2$

**3** Which point does *not* lie on the graph of $y = 2x - 4$?

- ☐ $(1, -2)$
- ☐ $(2, 1)$
- ☐ $(3, 2)$
- ☐ $(4, 4)$

**4** Which point does *not* lie on the graph of $-x + 2y = 4$?

- ☐ $(-1, 1.5)$
- ☐ $(2, 3)$
- ☐ $(-2, 1)$
- ☐ $(-3, 3.5)$

**5** What is the gradient of the line with equation $-4x + 2y = 9$?

Questions **6–7** refer to this information. Consider the linear relationship with the rule $2x - 3y = -6$.

**6** What is the $x$-intercept of the graph?

**7** What is the $y$-intercept of the graph?

Questions **8–10** refer to this information.
Let $A$ be the point $(3, 5)$ and $B$ be the point $(-1, 2)$.

**8** What is the gradient of the line connecting $A$ and $B$?

- ☐ $-\frac{3}{4}$
- ☐ $-\frac{4}{3}$
- ☐ $\frac{3}{4}$
- ☐ $\frac{4}{3}$

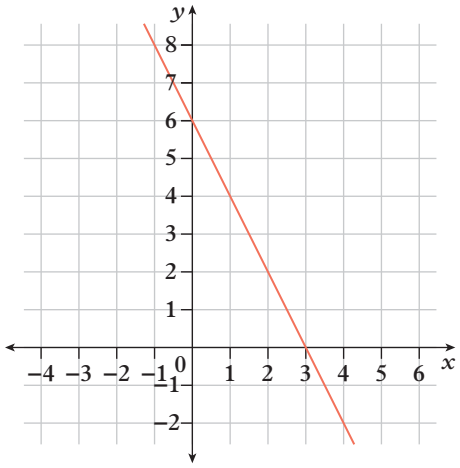
**9** What is the midpoint of the line segment $AB$?

- ☐ $(2, 3.5)$
- ☐ $(1, 3.5)$
- ☐ $(4, 7)$
- ☐ $(2, 3)$

**10** What is the distance between the two points $A$ and $B$?

- ☐ 2
- ☐ 5
- ☐ 7
- ☐ $\sqrt{7}$

**11** What is the equation of the graph shown below?




- ☐ $y = 3x + 6$
- ☐ $y = -2x + 3$
- ☐ $y = -3x + 6$
- ☐ $y = -2x + 6$

**12** If $y \propto x$ and $y = 2$ when $x = 10$, what is the value of $x$ when $y = 10$?

- ☐ 50
- ☐ 10
- ☐ 5
- ☐ 2

## Calculator

**13** Answer true or false to this statement: The equation $4x - 3 = 2$ is a linear equation.

**14** If 5 is added to 3 times a number $n$ and the answer is divided by 4, then the result is 6. The equation representing this process is:

- ☐ $5 + \frac{3n}{4} = 6$
- ☐ $\frac{3(5+n)}{4} = 6$
- ☐ $\frac{5(3 \times n)}{4} = 6$
- ☐ $\frac{3n+5}{4} = 6$

**15** A number $n$ is divided into 4, and then 5 is added. The result is 10. Write this as an equation.

**16** Consider an odd integer represented by $n$. The next odd integer would be represented by:

- ☐ $n + 1$
- ☐ $n + 2$
- ☐ $n - 1$
- ☐ $n - 2$

**17** The sum of three consecutive odd numbers is 33. What is the smallest of the three numbers?

**18** Which rule would *not* produce a linear graph?

- ☐ $y = \frac{x}{2} + 3$
- ☐ $y = 3x - \frac{5}{2}$
- ☐ $y = \frac{4}{x} + 5$
- ☐ $y = \frac{1}{2}x - 2$

**19** The gradient of the graph of $y = x - 6$ is:

- ☐ positive
- ☐ negative
- ☐ zero
- ☐ undefined

**20** The gradient of the graph of $y = -4$ is:

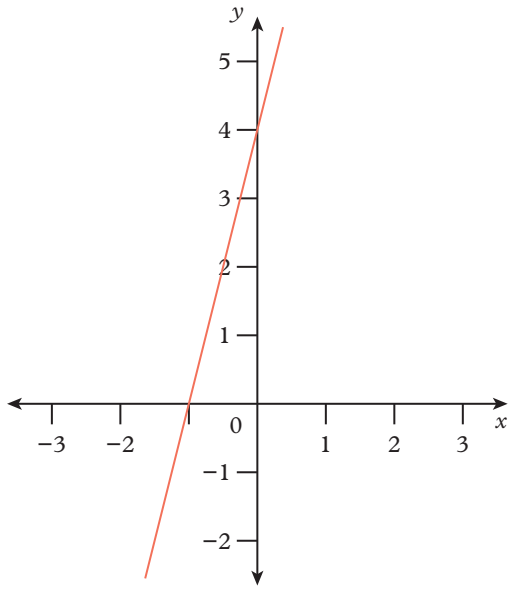
- ☐ positive
- ☐ negative
- ☐ zero
- ☐ undefined

**21** The gradient of the graph of $x = 10$ is:

- ☐ positive
- ☐ negative
- ☐ zero
- ☐ undefined

**22** Compare the gradients of the graphs of $y = -x$ and $y = -4x$. Which is the steeper line?

Questions **23–25** refer to the linear graph drawn on this Cartesian plane.




**23** What is the $x$-coordinate of the $x$-intercept?

- ☐ $-1$
- ☐ 0
- ☐ 1
- ☐ 4

**24** What is the $y$-coordinate of the $y$-intercept?

- ☐ $-1$
- ☐ 0
- ☐ 1
- ☐ 4

**25** What is the gradient?

- ☐ $-4$
- ☐ $-\frac{1}{4}$
- ☐ $\frac{1}{4}$
- ☐ 4

Questions **26** and **27** refer to this information.
A line segment $\overline{AB}$ has a midpoint $M(2, 4)$.
The coordinates of the point $B$ are $(5, 6)$.

**26** What are the coordinates of the point $A$?

**27** What is the length of the line segment $\overline{AM}$, correct to one decimal place?

Questions **28–30** refer to a triangle, $\Delta PQR$ with vertices at $P(-1, 3)$, $Q(-1, -2)$ and $R(4, -2)$.

**28** Which line segment represents the longest side of the triangle?

- ☐ $\overline{PQ}$
- ☐ $\overline{QR}$
- ☐ $\overline{PR}$
- ☐ $\overline{QP}$

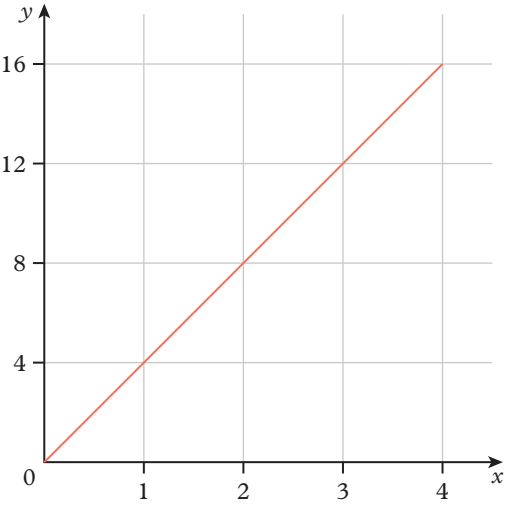
**29** Calculate the length of the longest side of the triangle, correct to one decimal place.

**30** Find the coordinates of the line segment $\overline{QR}$.

**31** Which of these statements is *not* correct? The graph of a direct proportion relationship:

- ☐ passes through the origin
- ☐ is linear
- ☐ has a constant gradient
- ☐ shows decreasing $y$-values from left to right

**32** If $y \propto x$, use this graph to write the rule for the relationship between $x$ and $y$.




# CHAPTER 5 Non-linear relationships

## Non-calculator

**1** If $(x - 6)(x + 1) = 0$, the two solutions for $x$ are:

- ☐ $x = -6$ or $x = 1$
- ☐ $x = 6$ or $x = -1$
- ☐ $x = -6$ or $x = -1$
- ☐ $x = 6$ or $x = 1$

**2** The expanded form of the equation $(x - 8)(x + 2) = 0$ is:

- ☐ $x^2 + 6x - 16 = 0$
- ☐ $x^2 - 6x - 16 = 0$
- ☐ $x^2 - 10x - 16 = 0$
- ☐ $x^2 + 10x - 16 = 0$

**3** What is the solution or solutions to $x^2 - 11x + 10 = 0$?

**4** For $y = x^2 + 3x - 2$, what is the value of $y$ when $x = -1$?

**5** Write the coordinates of the $x$-intercepts for the graph of $y = x^2 + x - 20$.

**6** The axis of symmetry for the graph of $y = x^2 + 2x - 15$ has the rule:

- ☐ $x = 1$
- ☐ $x = -1$
- ☐ $y = 1$
- ☐ $y = -1$

**7** Write the coordinates of the turning point for the graph of $y = x^2 - 8x + 15$.

## Calculator

**8** What transformation has been performed on the graph of $y = x^2$ to produce the graph of $y = x^2 - 3$?

- ☐ stretched by factor of 3 in the $y$-direction
- ☐ horizontal translation of 3 units
- ☐ reflection in the $x$-axis
- ☐ vertical translation of 3 units

**9** Which relationship would produce the most stretched parabola when graphed on the same Cartesian plane?

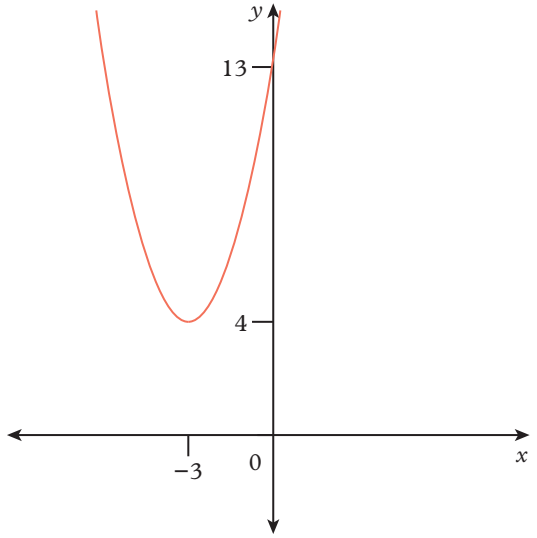
- ☐ $y = x^2 + 4$
- ☐ $y = -4x^2 - 5$
- ☐ $y = \frac{1}{2}x^2 + 1$
- ☐ $y = 2x^2 + 3$

**10** What are the coordinates of the $y$-intercept for the graph of $y = -3(x - 2)^2 - 4$?

**11** What are the coordinates of the turning point for the graph of $y = -3(x - 2)^2 - 4$?

**12** Which rule best matches the graph shown?

- ☐ $y = (x - 3)^2 + 4$
- ☐ $y = (x - 3)^2 - 4$
- ☐ $y = (x + 3)^2 + 4$
- ☐ $y = (x + 3)^2 - 4$

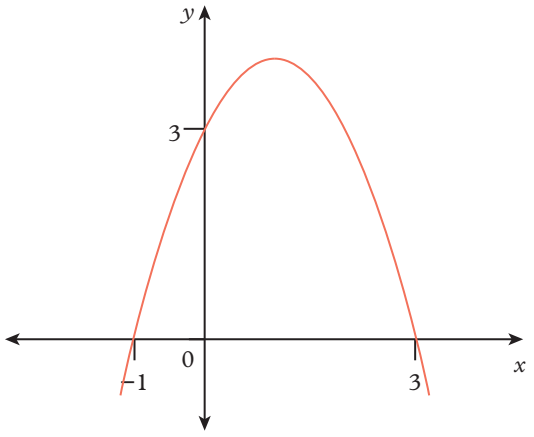



**13** How many $x$-intercepts does the graph of $y = (x - 3)^2$ have?

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ |

**14** Which rule best matches the graph shown?

- ☐ $y = x^2 + 2x + 3$
- ☐ $y = -x^2 + 2x + 3$
- ☐ $y = x^2 - 2x - 3$
- ☐ $y = -x^2 + 2x - 3$




**15** A circle has its centre at $(2, -3)$ and a radius of 5 units. What is its rule?

- ☐ $(x - 2)^2 + (y + 3)^2 = 25$
- ☐ $(x + 2)^2 + (y - 3)^2 = 25$
- ☐ $(x - 2)^2 + (y - 3)^2 = 5$
- ☐ $(x + 2)^2 + (y + 3)^2 = 5$

**16** What are the coordinates of the centre and the radius of the circle with the rule $x^2 + (y - 8)^2 = 4$?

- ☐ $(0, -8)$, 2 units
- ☐ $(0, 8)$, 2 units
- ☐ $(0, 8)$, 4 units
- ☐ $(8, 0)$, 4 units

**17** What is the area of the circle with the rule $(x - 2)^2 + (y + 5)^5 = 36$? Write your answer to the nearest square unit.

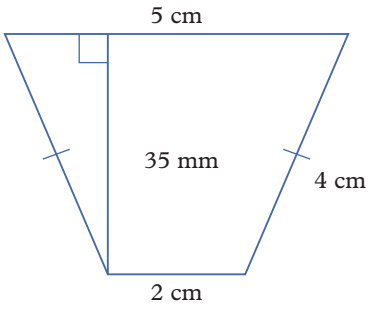
# CHAPTER 6 Measurement and geometry

## Calculator

**1** The area of a rectangle is 96 mm². If the rectangle has a width of 8 mm, calculate the length.

- ☐ 12 mm
- ☐ 24 mm
- ☐ 48 mm
- ☐ 72 mm

Questions **2** and **3** refer to this shape.

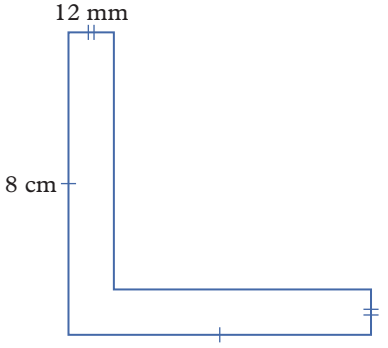



**2** The correct formula to use when calculating the area of this shape is:

- ☐ $A = xy$
- ☐ $A = \frac{1}{2}bh$
- ☐ $A = \frac{1}{2}(a + b)h$
- ☐ $A = lw$

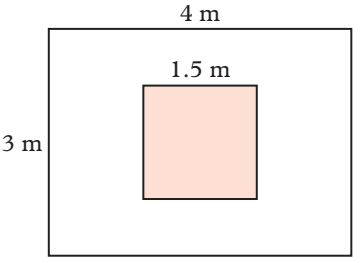
**3** The area of the shape is:

**4** The area of this composite shape is:




- ☐ 19.2 cm²
- ☐ 96 cm²
- ☐ 46.24 cm²
- ☐ 17.76 cm²

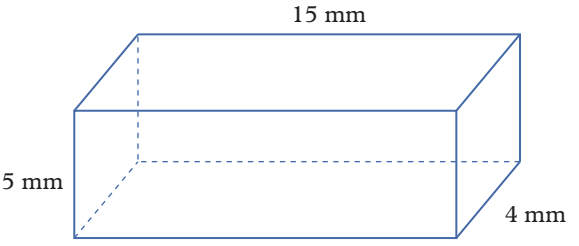
**5** The shaded square represents a garden and the unshaded part represents the paving around the garden. What is the area of paving?




- ☐ 12 m²
- ☐ 9.75 m²
- ☐ 2.25 m²
- ☐ 14.25 m²

**6** The surface area of a cube is 726 cm². How long is an edge of the cube?

cm




Questions **7** and **8** refer to this prism.

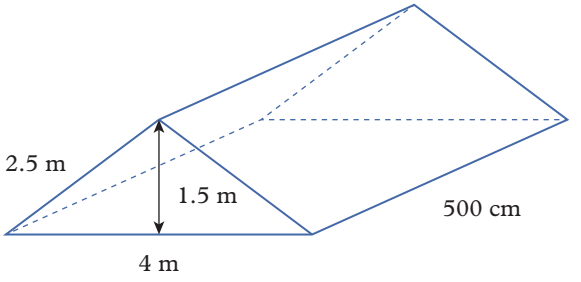
**7** The surface area of this rectangular prism in square centimetres is:

cm²

**8** The volume of the rectangular prism, correct to one decimal place, is:

cm³

Questions **9** and **10** refer to this prism.



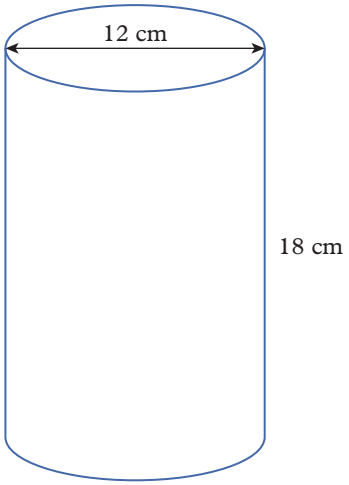

**9** This solid triangular prism is to have all surfaces painted. If two coats of paint are required, what is the total area to be painted?

- ☐ 51 m²
- ☐ 102 m²
- ☐ 43.5 m²
- ☐ 48 m²

**10** The volume of the triangular prism, correct to one decimal place, is:

m³

Questions **11** and **12** refer to this cylinder.



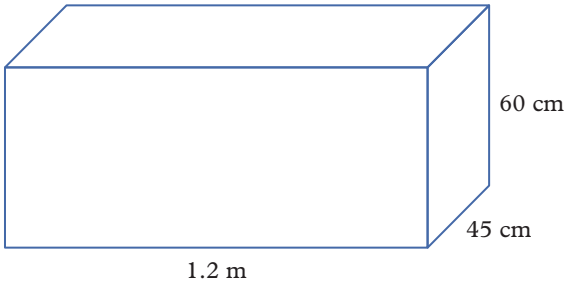

**11** The surface area of this cylinder to the nearest centimetre is:

- ☐ 905 cm²
- ☐ 2262 cm²
- ☐ 904 cm²
- ☐ 2261 cm²

**12** The volume of the cylinder, correct to two decimal places, is:

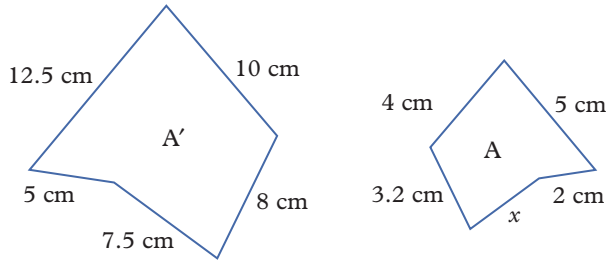
_____ cm³

**13** Given that 1000 cm³ is 1 L, what is the capacity of the following prism?




- ☐ 3.24 L
- ☐ 32.4 L
- ☐ 324 L
- ☐ 3240 L

Questions **14** and **15** refer to these shapes.




**14** Describe the dilation from A to A′.

_____

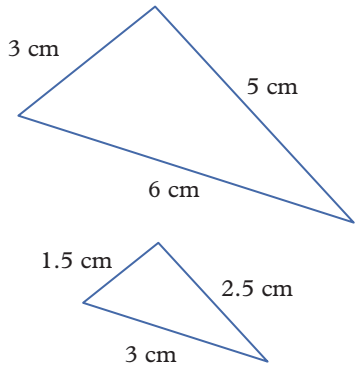
**15** What is the value of $x$?

_____

**16** Which statement cannot be used to describe two triangles as similar?

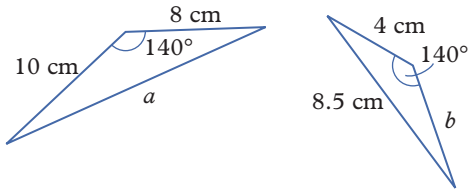
- ☐ All corresponding angles are equal and all corresponding sides are in the same ratio.
- ☐ All corresponding sides of the two triangles are in the same ratio.
- ☐ The two triangles contain a right angle and the hypotenuse lengths are in the same ratio.
- ☐ All corresponding angles are equal in size.

**17** Which condition would you use to check that these triangles are similar?




- ☐ AAA
- ☐ SSS
- ☐ SAS
- ☐ RHS

**18** What is the value of each unknown side length?



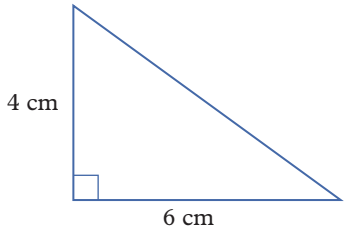

- ☐ $a$ = 17 cm and $b$ = 5 cm
- ☐ $a$ = 4.25 cm and $b$ = 5 cm
- ☐ $a$ = 8.5 cm and $b$ = 10 cm
- ☐ $a$ = 17 cm and $b$ = 4 cm

**19** If the sides of a rectangle were dilated by a scale factor of 4, then the area scale factor is:

| 2 | 4 | 8 | 16 |
|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ |

**20** The triangle shown is dilated by a length scale factor of 2.

What will the area of the dilated image be?

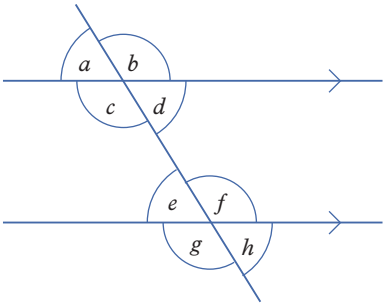



_____

# CHAPTER 7 Pythagoras' Theorem and trigonometry

## Non-calculator

Questions **1–3** refer to this diagram.




**1** The angle that is alternate to $c$ is:

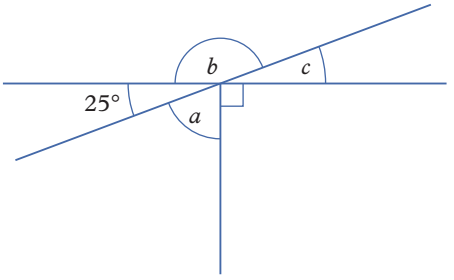
☐ $b$ ☐ $f$ ☐ $e$ ☐ $g$

**2** If $b$ is equal to 125°, what is the value of $d$?

**3** Which statement is *not* true?

☐ Angles $a$ and $d$ are vertically opposite.
☐ Angles $d$ and $h$ are corresponding.
☐ Angle $g$ is alternate to $d$.
☐ Angles $d$ and $f$ are co-interior.

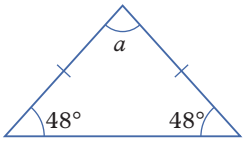
**4** What are the values for the unknown angles?



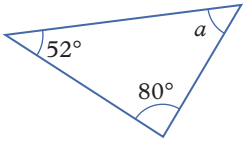

☐ $a = 65°, b = 155°, c = 25$
☐ $a = 65°, b = 155°, c = 65°$
☐ $a = 65°, b = 65°, c = 25°$
☐ $a = 75°, b = 165°, c = 25°$

**5** In which triangle is the value for $a$ equal to 48°?

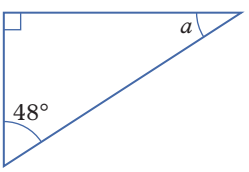
☐



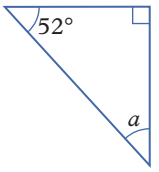

☐




☐



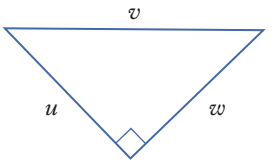

☐




**6** The two smaller sides of a right-angled triangle are 3 m and 4 m. What is the length of the third side?

## Calculator

Questions **7** and **8** refer to this triangle.



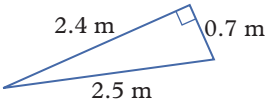

**7** Which pronumeral represents the hypotenuse in the triangle?

**8** Using Pythagoras' Theorem, which of these statements shows the relationship between the side lengths of the triangle?

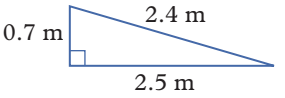
☐ $w^2 = u^2 + v^2$ ☐ $v^2 = u^2 + w^2$
☐ $u^2 = v^2 + w^2$ ☐ $v = u + w$

**9** A right-angled triangle contains the side lengths 0.7 m, 2.4 m and 2.5 m. Which of these triangles best matches this description?

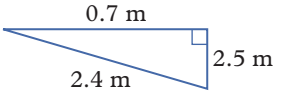
☐ Triangle A



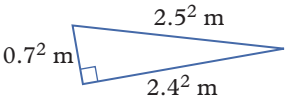

☐ Triangle B



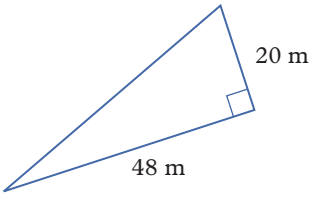

☐ Triangle C




☐ Triangle D




**10** What is the length of the hypotenuse in this triangle?




☐ 68 m ☐ 8.24 m
☐ 52 m ☐ 43.63 m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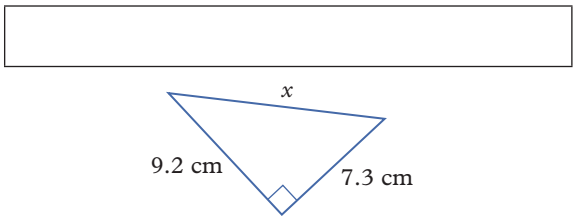
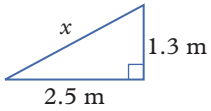
**11** What is the value of $x$ in this triangle, correct to two decimal places?

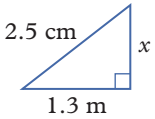



**12** A ladder leans against a vertical wall of a building so that the top of the ladder reaches 2.5 m up the wall and the foot is 1.3 m from the base of the wall. Which of these triangles correctly displays this information?

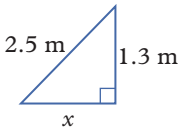
☐ Triangle A



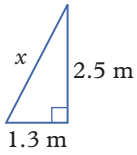

☐ Triangle B



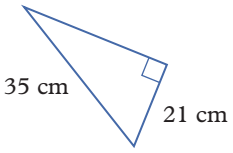

☐ Triangle C




☐ Triangle D

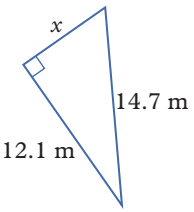



**13** What is the length of the unknown side in this triangle, correct to the nearest centimetre?

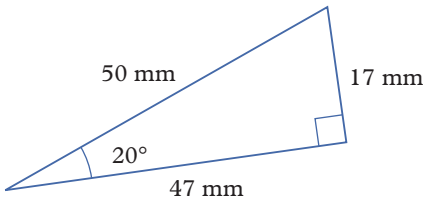



☐ 14 cm ☐ 28 cm
☐ 41 cm ☐ 784 cm

**14** What is the value of $x$ in this triangle, correct to two decimal places?




Questions **15** and **16** refer to this triangle.



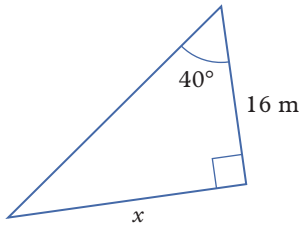

**15** What is the value of $\cos(20°)$ as a fraction?

☐ $\frac{47}{50}$ ☐ $\frac{17}{50}$ ☐ $\frac{17}{47}$ ☐ $\frac{47}{17}$

**16** What is the value of $\tan(20°)$, correct to two decimal places?

☐ 0.36 ☐ 2.76
☐ 0.94 ☐ 0.34

**17** Which equation can be used to find the side length $x$ in the triangle?



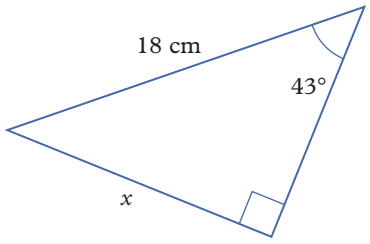

☐ $\tan(40°) = \frac{16}{x}$

☐ $\tan(40°) = \frac{x}{16}$

☐ $\cos(40°) = \frac{x}{16}$

☐ $\sin(40°) = \frac{x}{16}$

**18** What is the side length $x$ in this triangle, correct to two decimal places?




**19** If $\tan(\theta) = \frac{22}{25}$, then $\theta$ is equal to:

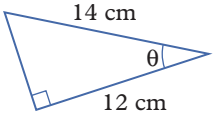
☐ $\tan^{-1}\left(\frac{22}{25}\right)$

☐ $\tan^{-1}\left(\frac{25}{22}\right)$

☐ $\tan\left(\frac{22}{25}\right)$

☐ $\tan\left(\frac{22}{25}\right)^{-1}$

**20** What is the angle θ in this triangle?



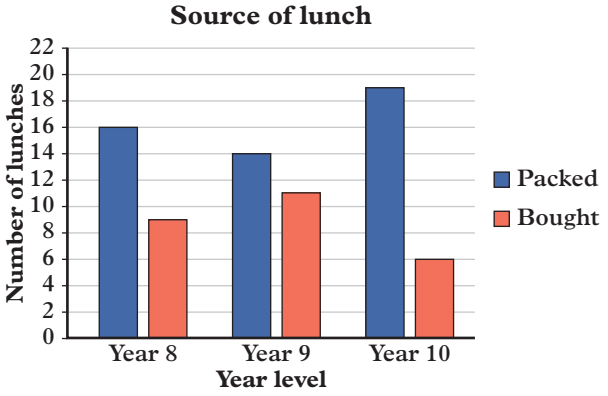

## CHAPTER 8 Statistics

### Calculator

**1** Which of these is classified as ordinal data?

- ☐ eye colour (hazel, blue, green)
- ☐ pizza sizes (small, medium, family)
- ☐ the number of SMS messages sent in a month
- ☐ the time taken to walk from home to school

**2** The students from three different classes were asked after lunch if they brought a packed lunch to school or if they bought their lunch at the canteen. The results are shown in the following side-by-side column graph.




How many Year 9 students brought a packed lunch?

Questions **3** and **4** refer to the following data, which is listed below and also displayed in the stem-and-leaf plot.

25, 27, 29, 41, 32, 35, 20, 27, 49

| Stem | Leaf |
|---|---|
| 2 | 5 7 7 9 |
| 3 | 2 5 |
| 4 | 1 9 |

**3** Which score is missing in the stem-and-leaf plot?

**4** What is an appropriate key for this stem-and-leaf plot?

Questions **5** and **6** refer to this frequency table.

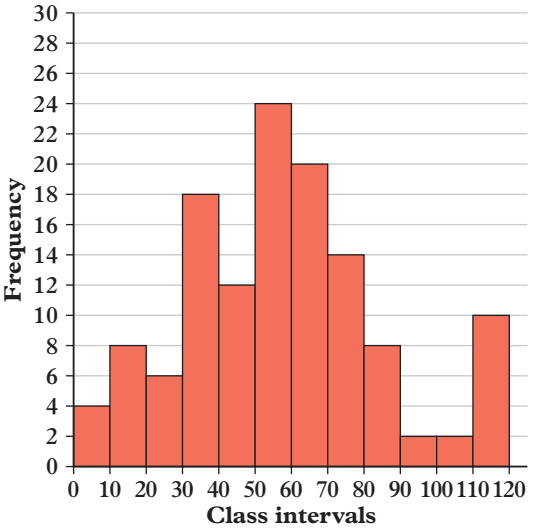
| Score | Frequency |
|---|---|
| 24 | 97 |
| 25 | 111 |
| 26 | 378 |
| 27 | 246 |
| 28 | |
| 29 | 301 |
| Total | 1325 |

**5** What is the missing value in the frequency table?

**6** Find the percentage of scores that are 25 or less, correct to one decimal place.

Questions **7** and **8** refer to the histogram below.

The histogram displays the results of research where the heights of plants (in cm) were measured.




**7** How many plants are less than 70 cm tall?

92 ☐ 36 ☐ 20 ☐ 14 ☐

**8** What percentage of plants are 100 cm or taller, correct to two decimal places?

- ☐ 1.56%
- ☐ 12%
- ☐ 9.38%
- ☐ 90.62%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Questions **9** and **10** refer to the data set below.

15 17 18 45 13 15 15 16 15

**9** Which value is closest to the mean of this data set?

- ☐ 18
- ☐ 15
- ☐ 32
- ☐ 19

**10** A score was recorded incorrectly in the list of data. If the score of 45 should be 15, which statement is true?

- ☐ The range will be unchanged.
- ☐ The mean will be unchanged.
- ☐ The mode will be unchanged.
- ☐ The median will change.

**11** A friend purchased 12 concert tickets for a total of $1188. What is the average price per ticket?

**12** The heights of a group of Year 9 students were recorded, as follows:

145 cm, 152 cm, 147 cm, 1.35 m, 165 cm, 170 cm.

Which statement is false?

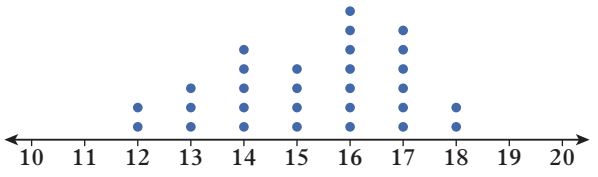
- ☐ The mean height is 1.52 m.
- ☐ The standard deviation is large, indicating a significant spread from the mean.
- ☐ The range is 35 cm.
- ☐ The median height is 152 cm.

**13** Twenty teachers were asked how many cups of coffee they have each day. The results are displayed in the frequency table below.

What is the mean number of cups of coffee consumed per day?

| Score | Frequency |
|---|---|
| 0 | 3 |
| 1 | 8 |
| 2 | 6 |
| 3 | 2 |
| 6 | 1 |

**14** The ages of students in a chess club at a particular high school are shown on the dot plot below. What is the median age of students in the chess club?




15 ☐ 15.3 ☐ 16 ☐ 17 ☐

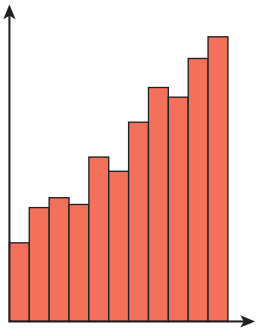
**15** What is the range of the data in the following stem-and-leaf plot?

| Stem | Leaf |
|---|---|
| 2 | 3 4 7 9 |
| 3 | 5 6 |
| 4 | 1 3 8 |
| 5 | 2 4 6 6 |
| 6 | 5 7 |

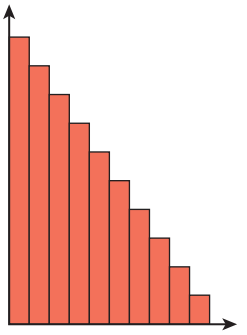
Key: 3|6 = 3.6

**16** Which graph shows a positively skewed distribution?

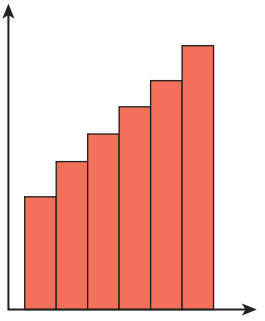
☐ Graph A



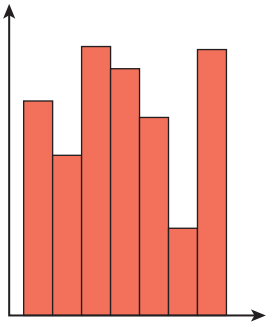
☐ Graph B



☐ Graph C



☐ Graph D



# CHAPTER 9 Probability

## Non-calculator

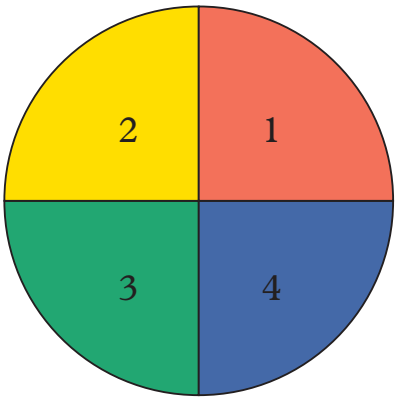
**1** The theoretical probability of rolling a number greater than 4 on a standard die is:

$\frac{1}{6}$ ☐ $\frac{1}{3}$ ☐ $\frac{1}{2}$ ☐ 0 ☐

**2** Two spinners are spun at the same time. The first spinner has four equally likely outcomes and the second has five equally likely outcomes. The total number of possible outcomes to be represented on a tree diagram is:

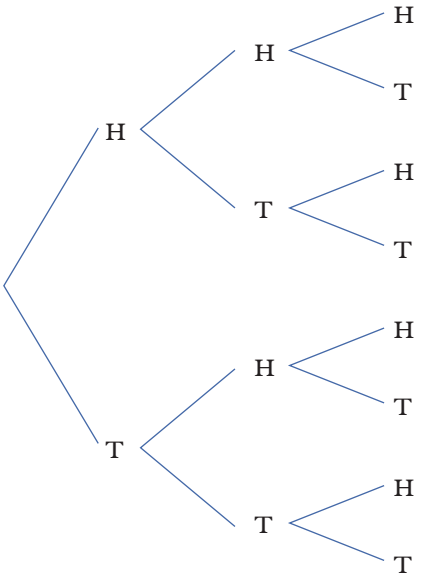
4 ☐ 5 ☐ 10 ☐ 20 ☐

**3** The following spinner is spun twice. What is the probability that the sum of the two numbers is 6?




☐

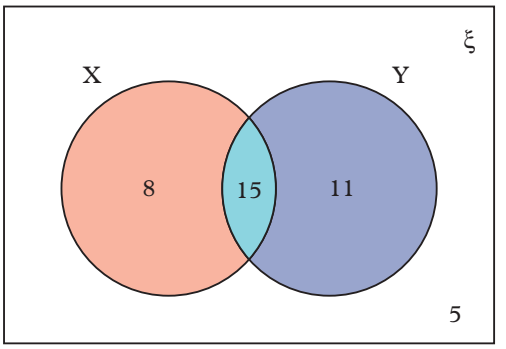
**4** The following tree diagram represents the possible outcomes when a coin is tossed three times.




What is the probability that the coin lands on tails at least twice?

$\frac{1}{8}$ ☐ $\frac{1}{4}$ ☐ $\frac{3}{8}$ ☐ $\frac{1}{2}$ ☐

Questions **5–7** refer to the following Venn diagram. The number of elements in sets X and Y is shown in the Venn diagram.




**5** $n(Y)$ is:

8 ☐ 11 ☐ 23 ☐ 26 ☐

**6** $n(X \cup Y)$ is:

23 ☐ 26 ☐ 34 ☐ 39 ☐

**7** $n(X \cap Y)$ is:

8 ☐ 15 ☐ 11 ☐ 5 ☐

## Calculator

**8** The results of an experiment are recorded in this table.

| Option | Frequency |
|---|---|
| A | 400 |
| B | 250 |
| C | 360 |
| D | 375 |
| E | 198 |
| F | 220 |
| G | 315 |
| H | |

Which of these statements is *not* correct?

☐ If 2500 trials were performed, the frequency for outcome H was 382.

☐ If 2500 trials were performed, the experimental probability for option A is $\frac{4}{25}$.

☐ A spinner with six equal segments may have been used in this simulation.

☐ If an additional 2500 trials were performed and the options were equally likely, the experimental probability for each option would theoretically get closer to $\frac{1}{8}$.

**9** The following result was recorded when a fair coin was tossed a number of times.

| **Outcome** | Heads | Tails |
|---|---|---|
| **Relative frequency** | | 0.465 |

Which of these statements is false?

- ☐ The relative frequency of heads for this experiment is 0.535.
- ☐ If 3000 trials were performed, 1605 heads were recorded.
- ☐ If 5000 trials were performed, 2500 tails would be expected in theory.
- ☐ The relative frequency of heads for this experiment is 0.465.

Questions **10** and **11** refer to the information and table below.

A survey of 700 students was conducted relating to student enjoyment in different subjects at primary, secondary and tertiary levels.

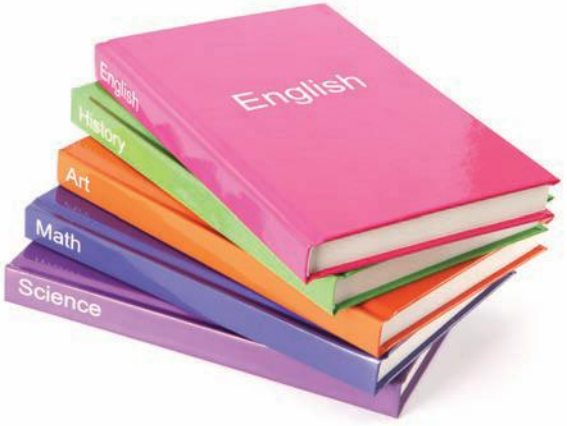
| | **Primary** | **Secondary** | **Tertiary** | **Total** |
|---|---|---|---|---|
| **Maths** | 100 | | 50 | |
| **English** | | 69 | 122 | |
| **Sport** | 75 | 96 | | |
| **Total** | 250 | 250 | | 700 |

**10** The probability that a student chosen at random from the group was a secondary student who enjoyed Maths is:

$\frac{1}{7}$ ☐ $\frac{1}{14}$ ☐ $\frac{17}{140}$ ☐ $\frac{47}{140}$ ☐

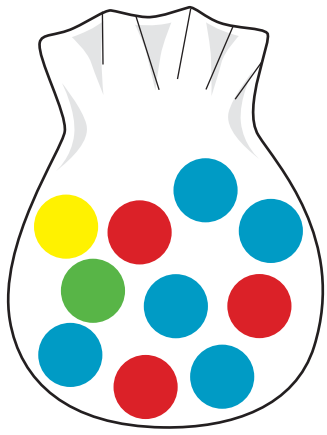
**11** The probability that a student chosen from the primary group of the survey group enjoyed English is:

$\frac{2}{5}$ ☐ $\frac{3}{10}$ ☐ $\frac{69}{250}$ ☐ $\frac{3}{28}$ ☐




Questions **12–15** refer to the following diagram.

A bag contains a number of coloured marbles: 1 yellow, 1 green, 3 red and 5 blue. A marble is drawn, the colour recorded, the marble is replaced and then another is selected.



**12** The probability of selecting two blue marbles is:

$\frac{1}{2}$ ☐ $\frac{1}{4}$ ☐ $\frac{9}{100}$ ☐ $\frac{1}{100}$ ☐

**13** The probability of selecting a red marble first is:

$\frac{3}{20}$ ☐ $\frac{3}{100}$ ☐ $\frac{9}{100}$ ☐ $\frac{3}{10}$ ☐

Now assume that marbles are selected without replacement.

**14** The probability of selecting a blue marble on the second trial, given that the first marble was not blue, is:

$\frac{5}{9}$ ☐ $\frac{1}{2}$ ☐ $\frac{3}{9}$ ☐ $\frac{1}{9}$ ☐

**15** The probability of selecting three red marbles, if a third trial is performed, is:

$\frac{2}{9}$ ☐ $\frac{1}{120}$ ☐ $\frac{2}{72}$ ☐ 0 ☐

[STEAM project 1]

# How can we use sustainable farming practices so that no one goes hungry in the future?

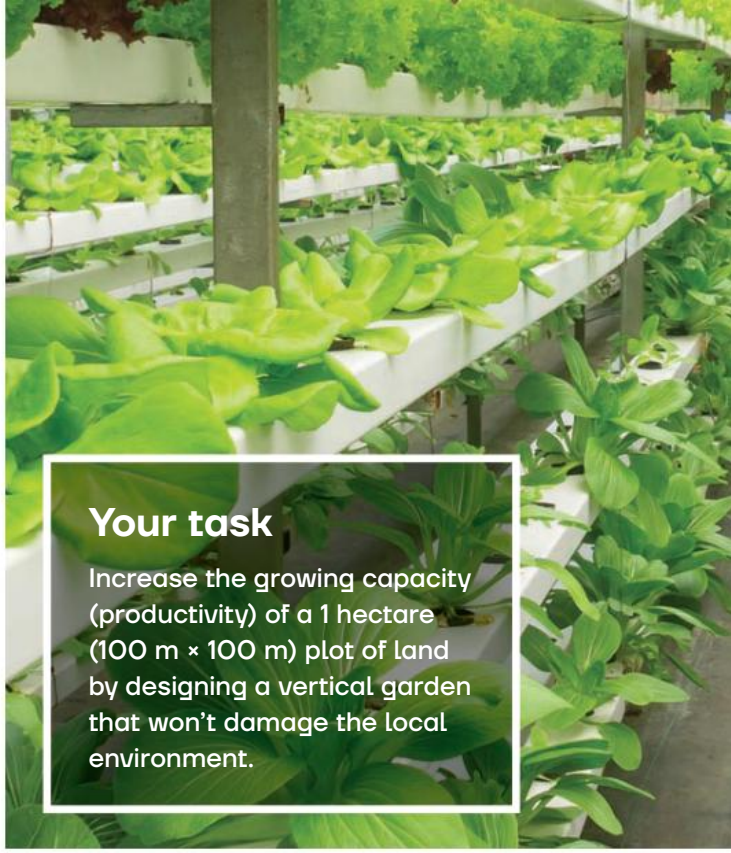
The United Nations ranks food shortages and hunger among the most serious issues affecting humankind. It predicts that more than 840 million people will be hungry by 2030. Even in a high-income country such as Australia, 5 per cent of the population are unable to access enough nutritious food. The experience of having inadequate access to food, or having an inadequate supply of food, is known as food insecurity. Food insecurity is linked to poor general health, higher rates of some cancers and higher mortality.

Rapid climate change is increasing threats to Australia's and the world's food security. Changes in the amount of rainfall, longer droughts and an increase in the number of extreme weather events are expected to disrupt the amount and quality of food that Australia can produce. A hotter climate is expected to cause stress in livestock animals such as chickens, sheep and cattle, and to increase the amount of water needed for crop irrigation.

## Sustainable farming

Sustainable farming practices use methods that balance the needs of all members of the community. This means that new and old technologies are used to make sure that food production is:

- economically viable – if farmers cannot make enough money to survive, then the farming practice is not sustainable
- socially supportive – if the lifestyle of the farming community is not supported, then people will not want to live in the area



**Your task**

Increase the growing capacity (productivity) of a 1 hectare (100 m × 100 m) plot of land by designing a vertical garden that won't damage the local environment.

**Figure 1** Vertical farming allows people to grow more food in a smaller space.

**Figure 2** Drought impacts Australia's production of important crops, such as wheat.

- ecologically sound – if the local environment is not supported, then the land will be unable to support food production. Sustainable farming also works to maintain the diversity of the local wildlife.

Sustainable farming uses technology to increase the production of fresh, nutritious food while minimising the impact on the local environment.



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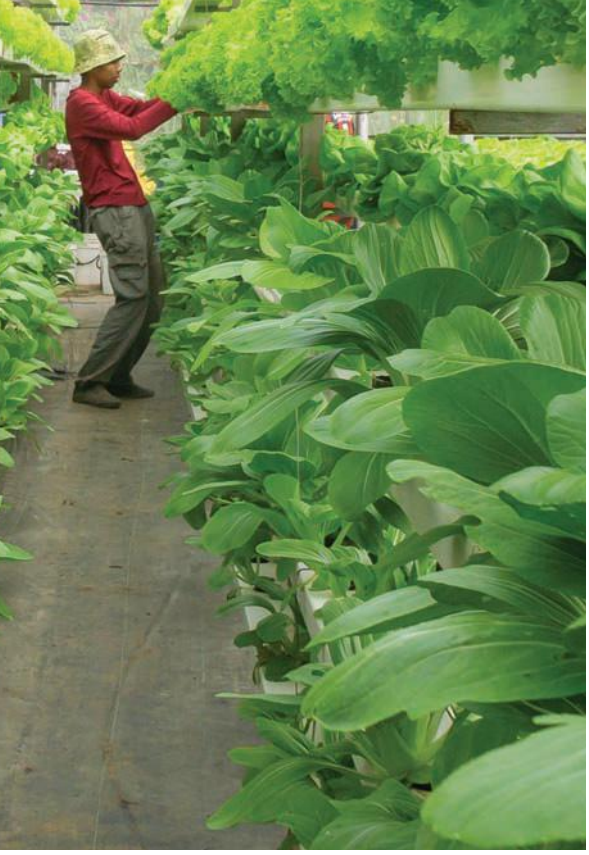

## HUMANITIES

In Geography this year, you will learn about food security around the world and food production in Australia. You will investigate the factors that influence crop yield (such as soil moisture) and how food production can alter a biome. In Economics and Business, you will study the agricultural resources (such as wheat) that form a large part of Australia's trade economy.

To complete this task successfully, you will need to investigate the environmental constraints on agricultural production in Australia, such as climate and distribution of water resources. You will also need to understand the extent to which agricultural innovations have overcome these constraints.

You will find more information on this in Chapter 3 'Food security', Chapter 2 'Biomes' and Chapter 17 'Understanding the global economy' of *Oxford Humanities 9 Victorian Curriculum*.

## MATHS

In Maths this year, you will build on your knowledge of measurement and geometry to determine areas and volumes of more complicated shapes. You will be introduced to Pythagoras' Theorem and trigonometry. You will also extend your skills in collecting, representing and investigating data.

To complete this task successfully, you will need to perform calculations involving angles, lengths and areas of two-dimensional and three-dimensional shapes. You will need to apply your understanding of scale factors to build a prototype of your designed product. To consider the situation at local, national and international scales, you will need skills in dealing with ratios and proportions. You may also find it helpful to use scientific notation for very large or very small numbers.

You will find help for applying these maths skills in Chapter 6 'Measurement and geometry', Chapter 7 'Pythagoras' Theorem and trigonometry', and section 2E 'Scientific notation' of *Oxford Maths 9 Victorian Curriculum*.

## SCIENCE

In Science this year, you will learn about the biotic and abiotic factors that support and maintain ecosystems. You will consider the role of different nutrients and sunlight on plants and the effects each of these will have on the surrounding ecosystem.

To complete this task successfully, you will need to understand the factors required to keep a system, such as a vertical garden, alive. You may need to consider how these factors can be monitored and controlled automatically. You will also need to be familiar with the scientific method, and understand how to conduct a fair test.

You will find more information on this in Chapter 2 'Ecosystems' of *Oxford Science 9 Victorian Curriculum*.

[STEAM project 1]

# The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

## Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, those who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

## Define

Before you start to design your vertical garden, you need to define the criteria that you will use to test the success of your vertical garden in achieving your goal.

### Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide:

'How can we help (the group) to solve (the problem) so that (the reason)?'

### Determine the criteria

1 What is the total area of the 100 m × 100 m plot of land? (Remember to use the correct units.)

2 If the plants are planted 25 cm apart in a 100 m row, and the rows are placed 1 m apart, how many plants could be planted in the plot of land? HINT: Draw the plot of land to make sure you reach maximum capacity.

3 What criteria will you use to measure the success of your solution or design? How will you measure how much the end-users have been helped?

## Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

As a group, brainstorm ways the problem can be solved. Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Once you have many possible solutions, it is time to sort them by possibility. Select three to five ideas that are possible. Research whether these ideas have already been produced by someone else. If they are already on the market, can you make a better version?

## Build

Draw your top two vertical garden designs. Label each part of the designs. Include the materials that will be used for their construction.

### Include in the designs:

**a** the total surface area available for plant growth

**b** a description of how food production will be increased



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c a description of how the design (inputs and waste) will impact the local ecosystem

d a description of how the workers will access all areas of the design to tend the plants

e at least one advantage and disadvantage of each design.

Select one of the designs to take to the building and testing stage.

## Build the prototype

You will need to build at least three versions of your vertical garden design prototype. The first prototype garden will be tested for effectiveness. The second prototype will be used to survey the group you are helping. The third prototype will be used for the presentation. The prototype may be full size, or it may be a scale model (10 cm represents 1 m). Use the following questions as a guideline for your prototype:

- What materials will you use?
- What material will you use to represent the plants?
- How will you represent the height, width and angle of the finished prototype?

# Test

## Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first vertical garden prototype. You will test the prototype more than once, to compare results, so you will need to control your variables between tests. What criteria will you use to determine the success of your prototype? Conduct your tests and record your results.

## Prototype 2

If your prototype will be used to help market gardeners, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would they consider buying it?)

If your prototype will be used to help another group, or native plants and animals, you will need to consider how you could test the impact it will have. (Will the prototype affect normal behaviours? How will the prototype affect the soil or waterways?)

## Prototype 3

Use the information you have obtained from testing the first two versions to adapt your last prototype to be more effective and usable for the group you are helping. You may want to use the first two prototypes to demonstrate how the design has been improved.

# Communicate

Present your vertical garden design to the class as though you are trying to get your peers to invest in it. Describe the criteria and testing used to measure the effectiveness of your vertical garden design.

In your presentation, you will need to:

- explain why we need to be more efficient with food production
- describe the key features of your design and how they improve or solve the problem of food shortages
- show a labelled, to-scale diagram of your prototype
- describe how the ecosystem will be affected by the installation of the prototype
- explain the relevant scientific principles that support your designed solution (e.g. water cycle, photosynthesis, nitrogen/carbon cycle)
- quantify the increase in food production that your design allows; present calculations to justify your claim
- present a calculation for the estimated cost of producing a full-size model of your design
- explain the implications of your design at a state or national level, by comparing the benefits and costs.

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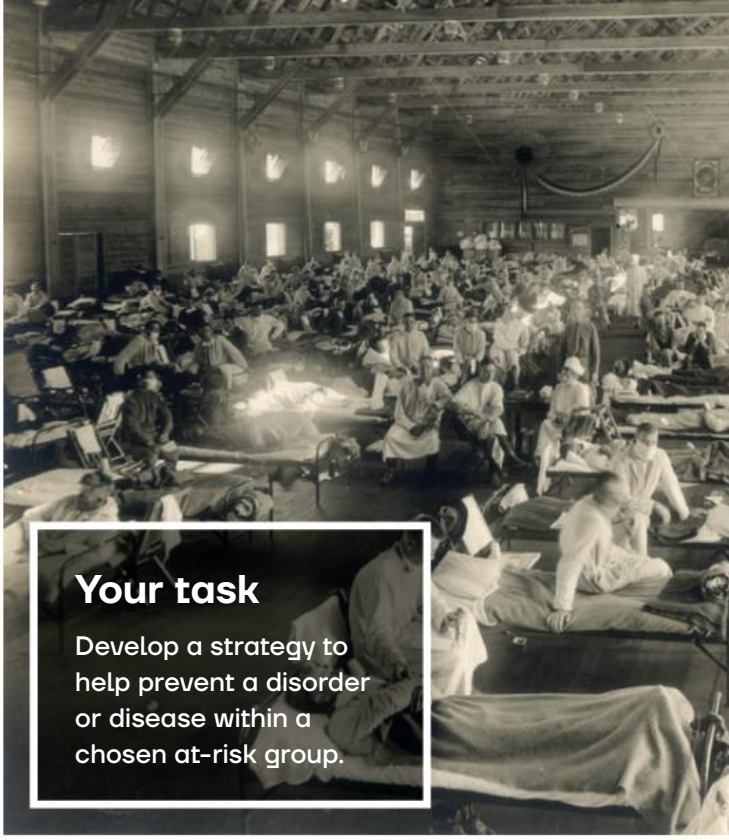
**Assessment resources**
Find information about assessment for this project.

[STEAM project 2]

# How can we harness technology so that we can live healthier lives?



**Your task**

Develop a strategy to help prevent a disorder or disease within a chosen at-risk group.

**Figure 1** During the 1918 flu pandemic (sometimes called the Spanish flu), an estimated 500 million people, or a third of the world's population, were infected with the virus.

A disorder or disease is a condition that affects the normal functioning of the body. Different disorders and diseases can affect many parts of the body. They can be caused by infectious agents such as bacteria or viruses that spread from person to person. Some disorders or diseases are inherited. Environmental factors (such as pollution or diet) can also have an impact on the development of disorders or diseases.

Heart disease, a non-infectious disease, is the leading cause of death globally. Mental health disorders, such as depression, bipolar disorder and dementia, also affect many people around the world.

Disorders and disease affect both high-income and low-income countries, but there are large differences in the ability of different healthcare systems to provide adequate care for people. The need for low-cost health care has led many researchers to investigate how technology can be used to help people live healthier lives.

## Prevention of disorders and disease

There are many disorders and diseases that can be prevented through simple, low-cost interventions. Below are a few examples.

- Wearing a helmet or a seat belt has been shown to decrease the risk of brain injury from a road accident. In Vietnam, when wearing a helmet was made mandatory for motorcycle riders, it resulted in a 16 per cent decrease in head injuries.
- The use of mosquito nets can help to prevent malaria, a disease that can lead to life-long neurological impairment, such as epilepsy in children if they have a severe infection.

**Figure 2** Healthcare workers wear personal protective equipment (PPE) to prevent the spread of infectious disease.

- Providing vaccinations for viruses such as polio and meningitis can also prevent neurological conditions.
- Promoting a healthy lifestyle and educating the population about the importance of diet can reduce the prevalence of stroke. In Japan, campaigns and treatment for high blood



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pressure have reduced the rate of strokes by 70 per cent.

- Personal protective equipment (PPE) is used to protect people from catching infectious diseases, such as Covid-19.

## HUMANITIES

In Economics and Business this year, you will learn about different economic systems, and how health services are provided in Australia. In Geography, you will study how people are interconnected through travel, technology and trade. These connections affect where and how people access the services they need. In History, you will examine the experiences of different groups during the Industrial Revolution, and the reforms made to improve living standards.

To complete this task successfully, you will need to research the demographics of your local area, and the location and accessibility of health services. You should also consider the economic performance of your area to determine what type of preventative strategy would be most successful for your at-risk group.

You will find more information on this in Chapter 17 'Understanding the global economy', Chapter 4 'An interconnected world' and Chapter 8 'The Industrial Revolution' of *Oxford Humanities 9 Victorian Curriculum*.

## MATHS

In Maths this year, you will extend your skills in representing and interpreting data. You will consider media reports that use statistics and collect secondary data to investigate social issues. You will relate real-world data to probabilities of events, and compare data sets using summary statistics and different graphical displays. You will evaluate and represent data, both with and without digital technology.

To complete this task successfully, you will need to find data to quantify the problem, work out how much your strategy will cost, and calculate a quantitative, evidence-based estimate of the possible benefits of your strategy. You will need skills in dealing with ratios, proportions and percentages to consider the situation at local, national and international scales.

You will find help for applying these maths skills in sections 1A 'Calculator skills' and Chapter 8 'Statistics' of *Oxford Maths 9 Victorian Curriculum*.

## SCIENCE

In Science this year, you will learn about how the body coordinates and regulates its internal systems so that it can respond to changes. When things change in the environment (such as the emergence of a disease-causing agent), or a part of the body fails, the normal functioning of the body is interrupted. The body needs to respond and attempt to return to a normal homeostatic state before permanent damage is caused.

To complete this task successfully, you will need to identify how the body's systems work together to maintain a functioning body. You should consider the type of disorder or disease that you will be fighting, and how it may cause changes in the body's normal function and response mechanisms.

You will find more information on this in Chapter 3 'Control and regulation' of *Oxford Science 9 Victorian Curriculum*.

[STEAM project 2]

# The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

## Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for? Is it the people directly affected by the disorder or disease, or do their families and carers need support too?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

## Define

Before you start to design your solution to the problem, you need to define the parameters you are working towards.

### Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

### Determine the criteria

1 Describe the type of life that the people you are helping lived before their lives were affected by the disorder or disease.

2 Describe how the people affected by the disease have needed to change their lives to cope with the effects of the disorder or disease.

3 Describe how you will know that you have made their lives better as a result of your prototype strategy.

## Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

Outline the criteria or requirements your design must fulfil (i.e. usability, accessibility, cost).

Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

## Build

Each group member should draw an individually designed solution. Label each part of the design. Include the material that will be used for its construction.

Include in the individual designs:

**a** a detailed diagram of the design

**b** a description of why it is needed by the individual or group



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c a description of any similar designs that are already available to buy

d an outline of why your idea or design is better than others that can be purchased.

Present your design to your group.

## Build the prototype

Choose one design and build two or three prototypes.

Use the following questions as a guideline for your prototype.

- What materials or technology will you need to build or represent your prototype design?
- What skills will you need to construct your prototype design?
- How will you make sure your prototype design is able to be used by the people who need it?
- How will you describe the way the prototype design will work?

# Test

## Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first prototype. You will test the prototype more than once, to compare results, so you will need to control your variables between tests.

What criteria will you use to determine the success of your prototype?

Conduct your tests and record your results in an appropriate table.

## Prototype 2

If your prototype will be used to help individuals with the disorder or disease, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would it make their life easier or harder? Would they consider buying it? How much would they be willing to pay to access the design?)

## Prototype 3

Your last prototype should be adapted using the information gathered from testing the first two versions to make it more effective and usable for the group you are helping. You may want to use the first two prototypes as a demonstration of how the design has been improved over time.

# Communicate

Present your design to the class as though you are trying to get your peers to invest in your design.

In your presentation, you will need to:

- outline the relevant disorder or disease and how it affects individuals, as well as society as a whole
- create a working model, or a detailed series of diagrams, with a description of how it will be used by an individual and group
- explain how you changed your design as a result of testing or feedback
- describe how the design will improve the life of an individual or group
- describe how many people need or will use the design
- describe how individuals will be able to access the design (will they need to purchase it or will it be publicly funded?)
- describe how the design will improve an individual's ability to contribute to society as a whole.

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# Answers

## CHAPTER 1 Financial mathematics

### EX p7 1A Calculator skills

**1** **a** \$5.21 **b** \$127.53 **c** −\$6.01
**d** \$0.76 **e** −\$20.00 **f** \$8.00
**g** −\$5000.00 **h** \$40 000.00 **i** \$624.75

**2** **a** \$24.40 **b** \$36.10 **c** \$28.05
**d** \$44.90 **e** \$22.30 **f** \$55.60
**g** \$35.75 **h** \$100.00 **i** \$0.35
**j** \$4.80 **k** \$105.25 **l** \$33.35

**3** **a** \$194.16 **b** \$264.52 **c** \$980.95
**d** \$142.40 **e** \$18.40 **f** \$685.20
**g** \$4439.75 **h** \$1078.13 **i** \$140 725.88
**j** \$184.30

**4** **a** 2298.264 **b** 556.704 **c** 884.584
**d** −3.229 **e** 0.274 **f** 11.005
**g** 8.853 **h** 0.592 **i** 13.029

**5** **a** $\frac{21}{50} = 0.42$ **b** 1 **c** $\frac{1}{40} = 0.025$
**d** $\frac{629}{50} = 12\frac{29}{50} = 12.58$
**e** $\frac{9759}{2000} = 4\frac{1759}{2000} = 4.8795$
**f** $\frac{13}{25\,000} = 0.000\,52$

**6** **a** 0.286, 28.571% **b** 1.625, 162.5%
**c** 0.444, 44.444% **d** 4.545, 454.545%
**e** 1.020, 102.020% **f** 0.656, 65.6%

**7** **a** $\frac{49}{50} = 98\%$ **b** $\frac{857}{20} = 42\frac{17}{20} = 4285\%$
**c** $\frac{201}{200} = 1\frac{1}{200} = 100.5\%$ **d** $\frac{11}{800} = 1.375\%$
**e** $\frac{3041}{500} = 6\frac{41}{500} = 608.2\%$ **f** $\frac{10\,303}{20\,000} = 51.515\%$

**8** **a** \$56.23 **b** \$1.26 **c** \$24.25 **d** \$135.80

**9** **a** \$4.34 **b** \$6.11 **c** \$0.41 **d** \$78.89

**10** **a** \$50 **b** \$28.90

**11** \$225.46

**12** **a** \$83.40 **b** \$16.68

**13** \$725.40

**14** \$204.95

**15** Cindy's dad is minimising the cost of his purchase as purchasing with cash when the hundredths place is a 3, 4, 8, or 9, will round up and cost 1 or 2 cents more than his purchase, but will round down when the hundredths place is a 1, 2, 6, or 7 and cost 1 or 2 cents less than his purchase. When the hundredths place is a 0 or 5 it will not change when using cash, so it does not matter if he pays with cash or by card.

**16** **a** $\frac{7538}{13\,047}$ **b** 0.58, 57.78%
**c** $\frac{10\,4467}{65\,235}$ **d** 1.60, 160.14%

**17** \$9317.82, \$3494.18, \$12 812.00

**18** **a** $\frac{1}{9}$ **b** $\frac{41}{333}$ **c** $\frac{49}{99}$
**d** $6\frac{3}{10} = \frac{63}{10}$ **e** $14\frac{107}{333} = \frac{4769}{333}$ **f** $\frac{1}{3996}$

**19** **a** 5 **b** 294

### EX p11 1B Rates and the unitary method

**1** **a** \$30/h **b** \$1.35/L **c** \$55/h
**d** \$2.45/jar **e** \$0.75/min **f** \$12.99/kg
**g** \$60 000/year **h** \$6.85/parcel mailed

**2** **a** \$5.25/h **b** \$1.49/L **c** \$7.84/kg
**d** \$3.60/mL **e** \$1.34/L **f** \$1.74/min
**g** \$0.02/g or \$1.60/100 g **h** \$21.35/h

**3** **a** \$5.49/kg **b** \$24/h **c** \$0.37/min **d** \$8.27/m

**4** **a** **i** \$39.25 **ii** \$39.25
**b** **i** \$17.40 **ii** \$17.40
**c** **i** \$5.94 **ii** \$5.95
**d** **i** \$33.77 **ii** \$33.75

**5** **a** \$1.00 **b** \$4.75 **c** \$0.60 **d** \$2.65

**6** **a** B **b** B **c** B **d** A
**e** B **f** A **g** B **h** B

**7** **a** B **b** A **c** A **d** A
**e** A **f** B **g** B **h** A

**8** **a** \$45 **b** \$558 **c** \$250
**d** \$225.50 **e** \$1126.25 **f** \$15.62
**g** \$217 **h** \$29.20

**9** **a** 46 m **b** 234 pieces **c** 12.5 L
**d** 0.75 kg **e** 30.6 kg **f** 131.25 g
**g** 159.5 $m^2$ **h** 44.55 $m^3$

**10** **a** \$18.50/h **b** \$370 **c** 37 h

**11** **a** \$66 548/year **b** \$5545.67/month
**c** \$2559.54/fortnight

**12** **a** \$432.70 **b** \$698.80 **c** \$5350.95 **d** \$586.50

**13** **a** **i** \$27 **ii** \$36
**b** **i** \$36 **ii** \$48
**c** **i** \$28.20 **ii** \$37.60
**d** **i** \$38.85 **ii** \$51.80
**e** **i** \$48.90 **ii** \$65.20
**f** **i** \$44.85 **ii** \$59.80

**14** **a** \$1632.85 **b** \$1158.95

**15** **a** \$228.10 **b** \$1114.50 **c** \$978.70
**d** \$304.60 **e** \$943.25

**16** **a** \$27.04 **b** \$17.99 **c** \$1.80 **d** 141.1 c/L
**e** 48.329 L **f** 136.2 c/L and 136.5 c/L

**17 a** The statement is not accurate. The best buy is the option that has the lowest price per unit or lowest price when written as rates with the same units.

**b** It is the best buy when the amount of the cheaper option is the same or more than the other option.

**18 a** 175-g bag for $3.24 **b** 800-g box for $3.00

**c** 750-g bag for $16.90 **d** 680-g jar for $4.00

**e** 1.7 kg for $8.00 **f** 2-L bottle for $6.94

**19 a** **i** $3 \times \frac{2}{3} = 2$ **ii** $3 \times \frac{4}{3} = 4$

**iii** $3 \times \frac{17}{3} = 17$ **iv** $3 \times \frac{14}{3} = 14$

**v** $3 \times \frac{23}{3} = 23$ **vi** $3 \times \frac{31}{3} = 31$

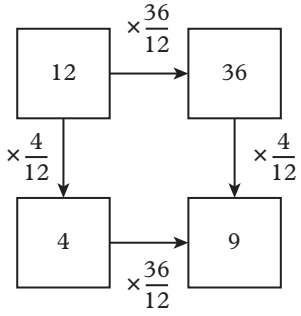
**vii** $7 \times \frac{25}{7} = 25$ **viii** $34 \times \frac{5}{34} = 5$

**ix** $8 \times \frac{0}{8} = 0$ **x** $0.8 \times \frac{1.4}{0.8} = 1.4$

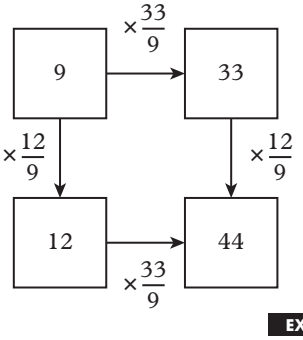
**xi** $-4 \times \frac{6}{-4} = 6$ **xii** $a \times \frac{b}{a} = b$

**b** No, because multiplying by zero is always zero. Dividing by zero is not defined, so we cannot use $0 \times \frac{5}{0} = 5$.

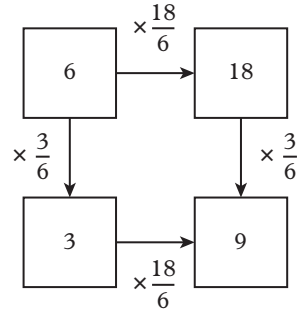
**20 a** **i**



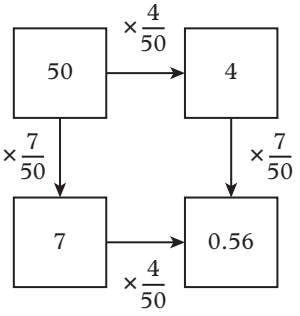

**ii**



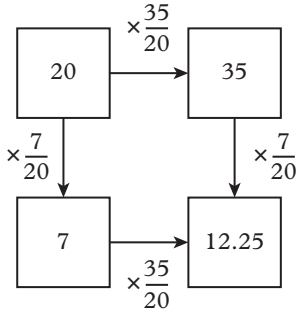

**iii**



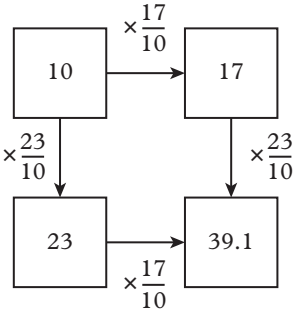

**iv**



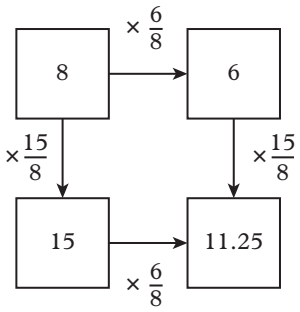

**v**



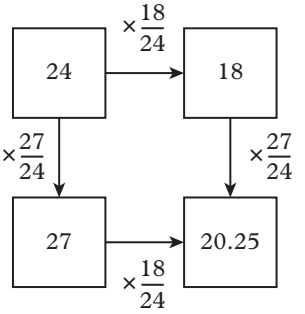

**vi**



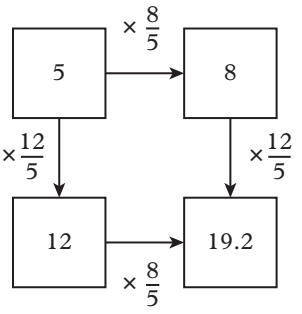

**vii**




**viii**




**ix**




**b** **i** $\frac{20}{17}$ and $\frac{20}{29}$

**ii** They are reciprocals of each other. They multiply to 1.

**21 a** 74.8 m **b** $80

**c** $80 **d** 335.75 m

**e** 2261.75 m **f** $12.99

**22** $28.69/h

**23** 32.3125 cm

## 1C Mark-ups and discounts

EX p20

**1 a** $36.05 **b** $1060.50 **c** $110.15

**d** $480 **e** $1615 **f** $70.08

**g** $233.28 **h** $3337.61

**2 a** $120 **b** $255 **c** $630

**d** $408 **e** $619 **f** $404.80

**g** $470.25 **h** $87.15 **i** $622.50

**3 a** $504 **b** $1002 **c** $198

**d** $1090.32 **e** $2520.76 **f** $3520

**4 a** $1413.50 **b** $264 **c** $661.60

**d** $107.25 **e** $346.12 **f** $80.25

**5 a** **i** $194.40 **ii** $74.40

**b** **i** $778.70 **ii** $419.30

**c** **i** $42.42 **ii** $7.48

**6 a** $300 **b** $31.25 **c** $13.95

**d** $86.65 **e** $11 422.85 **f** $624.60

**g** $49.90 **h** $50 **i** $342 857.15

**7 a** $141.50 **b** $214.50 **c** $165.45

**d** $109.55

**8 a** 87.5% **b** $138.20

**9** **a** **i** \$440 **ii** \$60
**b** **i** \$152.58 **ii** \$26.92
**c** **i** \$229.08 **ii** \$19.92
**d** **i** \$860.11 **ii** \$35.84
**e** **i** \$537.16 **ii** \$87.44
**f** **i** \$28 345.28 **ii** \$1649.72
**g** **i** \$11 095 **ii** \$1585
**h** **i** \$1234.19 **ii** \$261.80

**10** **a** \$44.60 **b** \$37.91 **c** \$82.51

**11** **a** 210% **b** \$188.80

**12** **a** **i** \$768.80 **ii** \$148.80
**b** **i** \$130.43 **ii** \$40.48
**c** **i** \$2284.20 **ii** \$1015.20
**d** **i** \$835.68 **ii** \$385.18
**e** **i** \$15 000 **ii** \$8750
**f** **i** \$789.73 **ii** \$438.74
**g** **i** \$31 078.13 **ii** \$16 453.13
**h** **i** \$5450.63 **ii** \$3155.63

**13** Jane is finding 20% of the discount price, not the original price. Her calculations will give: $0.2 \times \$198 = \$39.60$
Original price = \$198 + \$39.60 = \$237.60
Find 20% of \$237.60 and see if it gives a reduced price of \$198.
A 20% discount means you pay 80% or 0.8 of the original price of the bike:
$\$198 \div 0.8 = \$247.50$
20% of \$247.50 gives a reduced price of \$198.
Hence, Tim is correct.

**14** **a** \$9 **b** \$900 **c** \$900

**15** \$1000

**16** \$247.50

**17** \$449.80

**18** An advantage is the more you sell the more you earn; a disadvantage is you will not earn much money if your sales are low. You will feel the pressure of needing to make sales at all times.

**19** \$582.75

**20** \$9237.50

**21** They should choose the second agency, which charges \$11 180 commission, as compared to the \$13 570 commission charged by the first agency.

**22** **a** \$492.50 **b** \$755 **c** \$475 **d** \$581.02
**e** \$558.91 **f** 806.83 **g** \$898.33 **h** \$1378

**23** \$15 000

**24** \$35 000

**25** **a** \$1161.62 **b** \$1085.10 **c** \$1277.78
**d** It does not matter as multiplication is commutative. That is, a 45% discount then a 10% is the same as a 10% discount then a 45% discount:
$0.55 \times 0.90 = 0.90 \times 0.55$

**26** \$40 000

## 1 Checkpoint

**1** **a** 827.48 **b** 294.292 00
**c** 1990.073 **d** 47 201.2840

**2** **a** 121.1524 **b** −42.5521
**c** 146.1722 **d** $\sqrt{\frac{12.95 \times 4.34}{77.2 \div 9.3}}$

**3**

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{13}{64}$ | 0.203 125 | 20.3125 |
| $\frac{9}{2000}$ | 0.0045 | 0.45% |
| $\frac{294}{125} = 2\frac{44}{125}$ | 2.352 | 235.2% |

**4** **a** \$12.50/hour **b** \$12/L **c** \$1.5625/kg

**5** **a** B **b** B **c** B **d** A

**6** **a** \$514.50 **b** 15 kg
**c** \$343.75 **d** 33 tickets

**7** **a** \$430.50 **b** \$622.75 **c** \$456.53 **d** \$830.96

**8** **a** 37.5% **b** 51.2% **c** 0.7%

**9** **a** \$25.82 **b** \$35.63 **c** \$86.90

**10** **a** \$350 **b** \$160 **c** \$200

**11** **a** \$23 **b** \$412.50 **c** \$544.55 **d** \$1.72

## 1D Profit and loss

EX p26

**1** **a** 20% **b** 75% **c** 25% **d** 150%
**e** 16.5% **f** 30.75% **g** 74.44% **h** 487.5%
**i** 512.5% **j** 111 111%

**2** **a** profit of \$10 **b** loss of \$14
**c** loss of \$4.55 **d** loss of \$154.48

**3** **a** **i** loss of \$60 **ii** 25%
**b** **i** profit of \$7.30 **ii** 292%
**c** **i** loss of \$0.40 **ii** 7.69%
**d** **i** loss of \$5390 **ii** 21.88%
**e** **i** profit of \$230 **ii** 191.67%
**f** **i** loss of \$21.95 **ii** 73.29%

**4** **a** **i** 33.33% **ii** 74.49%
**iii** 8.33% **iv** 28%
**v** 65.71% **vi** 274.38%
**b** If a loss occurred the percentages will be greater, but if a profit occurred the percentages will be less.

**5** **a** 28.57% **b** 17.07% **c** 4.92% **d** 51.50%

**6** **a** loss of \$48 **b** 24.24%

**7** **a** \$252 **b** 375%

**8** **a** **i** loss of \$14 **ii** 29%
**b** **i** profit of \$127.50 **ii** 113%
**c** **i** profit of \$50.05 **ii** 139%
**d** **i** loss of \$411 **ii** 25%
**e** **i** loss of \$12 395 **ii** 41%
**f** **i** profit of \$268 865 **ii** 171%

**9** **a** \$2.55 per kg **b** 86%

**10** **a** \$27.45 **b** 11%

**11 a** **i** \$250 **ii** \$150

**b** **i** \$2000 **ii** \$1200

**c** **i** \$1000 **ii** \$750

**d** **i** \$213.75 **ii** \$146.25

**e** **i** \$223.84 **ii** \$96

**f** **i** \$491.53 **ii** \$432.25

**12 a** No, as it makes Joseph's cars more expensive than his competitor's.

**b** 111% profit

**c** A maximum percentage increase enables Joseph to keep his selling price below the selling price of his competitor's cars.

**13 a** 25% **b** 125%

**c** 125% = 100% (the purchase price)
+ 25% (the price increase)

**14 a** 20% **b** 80%

**c** 20% + 80% = 100%
If percentage decrease is subtracted from 100%, the remaining percentage value gives the percentage of the original price of car.

**15 a** 100% increase **b** 200% increase

**c** 0% increase or decrease **d** 50% decrease

**e** 300% increase **f** 100% decrease

**16**

| Commodity | Starting price \$ | Final price \$ | Movement % |
|---|---|---|---|
| Gold | 1741.66 | 1732.95 | ↓ 0.5 |
| Silver | 34.04 | 33.56 | ↓ 1.4 |
| Oil | 102.15 | 102.46 | ↓ 0.3 |
| Copper | 3.67 | 3.71 | ↓ 1.2 |

**17 a** \$1 569 600

**b** **i** \$1 874 610 **ii** \$33 390

**c** profit of \$305 010 **d** 19.43%

**18 a** \$15.30 **b** \$16.83

**c** \$16.15; less than price in part b, because it doesn't take into account that GST is 10% of the marked-up price, not the wholesale price.

**19 a** **i** 60% **ii** 166.67%

**iii** 250% **iv** 150%

**b** **i** 60% **ii** 166.67%

**iii** 250% **iv** 150%

**c** **i** 75% **ii** 133.33%

**iii** 400% **iv** 300%

**d** **i** 68.42% **ii** 146.15%

**iii** 316.67% **iv** 216.67%

**e** **i** 75.09% **ii** 133.17%

**iii** 175.09% **iv** 75.09%

**f** **i** 87.94% **ii** 113.71%

**iii** 829.17% **iv** 729.17%

**20**

| Cost price | Selling price | Cost price as a percentage of selling price | Number of sales |
|---|---|---|---|
| \$3.60 | \$12 | 30% | 35 |
| \$5 | \$15 | 33.33% | 20 |
| \$13 | \$21 | 61.90% | 37 |
| \$1.10 | \$2.50 | 44% | 150 |
| \$9.10 | \$35 | 74% | 25 |
| \$5.44 | \$8 | 68% | 125 |
| \$50.49 | \$99 | 51% | 8 |
| \$0.28 | \$1.12 | 25% | 271 |
| \$24.32 | \$32 | 76% | 19 |
| \$3.60 | \$6 | 60% | 15 |

| Revenue | Total profit | Profit as a percentage of the revenue |
|---|---|---|
| \$420 | \$294 | 70% |
| \$300 | \$200 | 66.67% |
| \$777 | \$296 | 38.10% |
| \$375 | \$210 | 56% |
| \$875 | \$227.50 | 26% |
| \$1000 | \$320 | 32% |
| \$792 | \$388.08 | 49% |
| \$303.52 | \$227.64 | 75% |
| \$608 | \$145.92 | 24% |
| \$90 | \$36 | 40% |

**21 a** To decrease by 30%, we can multiply by (100% − 30%) = 70%, which is the same percentage we would multiply by to find 70% of its value.

**b** To increase by 30%, we can multiply by (100% + 30%) = 130%, which is the same percentage we would multiply by to find 130% of its value.

**22 a** \$22 000 **b** \$2000 loss

**23 a** selling price = wholesale price × 1.8 × 1.1
= wholesale price × 1.98

He can add 98% to the wholesale price or multiply the wholesale price by 1.98.

**b** \$1200 = wholesale price × 1.6 × 1.1
= wholesale price × 1.76

So wholesale price = \$1200 ÷ 1.76 = \$681.82
The single calculation is dividing by 1.76.

**c** \$400 = original price × 1.1 × 0.75
= original price × 0.825

So original price = \$400 ÷ 0.825 = \$484.85
The single calculation is dividing by 0.825.

**24** 56.14%

## EX p32 1E Simple interest

**1 a** **i** $\frac{7}{100}$ **ii** 0.07

**b** **i** $\frac{11}{100}$ **ii** 0.11

**c** **i** $\frac{6}{25}$ **ii** 0.24

**d** **i** $\frac{3}{50}$ **ii** 0.06

**e** **i** $\frac{1}{10}$ **ii** 0.1

**f** **i** $\frac{3}{25}$ **ii** 0.12

**2** **a** **i** \$500 **ii** \$5500

**b** **i** \$576 **ii** \$5376

**c** **i** \$5000 **ii** \$17 500

**3** **a** **i** \$1125 **ii** \$8625

**b** **i** \$6480 **ii** \$17 280

**c** **i** \$14 000 **ii** \$39 000

**4** **a** \$1200 **b** \$2422 **c** \$12 000

**d** \$1104 **e** \$17 999.10 **f** \$20 000

**5** **a** $P = \$3500$, $r = 4.8\%$, $T = 2$ years

**b** \$336 **c** \$3836

**6** **a** $P = \$44\,000$, $r = 9.5\%$, $T = 5$ years

**b** \$20 900 **c** \$64 900

**7** **a** **i** \$831.25 **ii** \$831.25

**b** Answers to parts **a i** and **ii** are identical. Simple interest is calculated in the same manner for investments and loans. In each case, simple interest is found by multiplying principal, rate (as a fraction or decimal) and time.

**c** Interest on an investment is a payment to the investor and on a loan is an charge.

**8** **a** $\frac{11}{12}$ **b** $\frac{7}{52}$ **c** $\frac{1}{2}$

**d** $\frac{1}{4}$ **e** $\frac{271}{365}$ **f** $\frac{31}{73}$

**g** $1\frac{1}{4}$ **h** 4 **i** $1\frac{8}{13}$

**j** $3\frac{2}{5}$ **k** $2\frac{1}{2}$ **l** $5\frac{1}{2}$

**9** 4.25 years

**10** **a** **i** \$1620 **ii** \$10 620

**b** **i** \$236.25 **ii** \$10 736.25

**c** **i** \$1837.50 **ii** \$9337.50

**d** **i** \$228.82 **ii** \$29 228.82

**e** **i** \$52.78 **ii** \$8652.78

**f** **i** \$17 825.73 **ii** \$173 395.73

**g** **i** \$1913.37 **ii** \$21 912.37

**h** **i** \$15 601.94 **ii** \$61 551.94

**i** **i** \$1950.77 **ii** \$210 604.77

**11** **a** \$9.20 **b** \$9.07

**c** Sade would hope the method that writes time as a fraction of total number of months in year was used. She earns more interest with this method.

**d** Sade would prefer the method that uses the number of days in June as a fraction of total number of days in year. She would pay less interest on her loan using this method.

**12** **a** 5.25% **b** \$1050

**c** This statement is true. If Jasmine invests her money for 364 days, the calculations are:

$$I = \frac{\$20\,000 \times 0.0555 \times 364}{365} = \$1106.96$$

She receives slightly more interest.

**13** **a** **i** \$10 **ii** \$100 **iii** \$300

**b** **i** \$86.80 **ii** \$694.40 **iii** \$2094.40

**c** **i** \$40 000 **ii** \$500 000 **iii** \$1 000 000

**d** **i** \$5 **ii** \$100 **iii** \$850

**e** **i** \$25 **ii** \$1800 **iii** \$27 800

**f** **i** \$4.20 **ii** \$642.50 **iii** \$5022.60

**14** **a** **i** 1.5 **ii** \$300

**b** **i** 1.496 **ii** \$2094.40

**c** **i** 2 **ii** \$1 000 000

**d** **i** $\frac{34}{30} \approx 1.1333$ **ii** \$850

**e** **i** $\frac{139}{130} \approx 1.0692$ **ii** \$27 800

**f** **i** $\frac{9188}{9125} \approx 1.0069$ **ii** \$5022.60

**15 a, b**

| Balance \$ | Days applied for | Interest earned \$ |
|---|---|---|
| 640.90 | 7 days | 0.26 |
| 540.90 | 7 days | 0.22 |
| 780.90 | 9 days | 0.40 |
| 655.50 | 5 days | 0.19 |

**c** \$1.07 **d** \$656.57

**16** **a** \$0.72 **b** \$830.27

**17** **a** Yes, account A will receive the bonus interest rate. They have made only one withdrawal for the month and the account balance has increased by more than \$200. Account B will not receive the bonus because there is more than one withdrawal.

**b** Account A: \$6.23, Account B: \$3.06

**c** Account A: \$2231.98, Account B: \$3139.82

**18** **a** The minimum deposit is \$1875, so Joel's savings are enough to cover it.

**b** \$10 000 **c** \$15 050

**19** **a** $33\frac{1}{3}\%$ p.a.

**b** **i** 25% p.a. **ii** 20% p.a. **iii** $16\frac{2}{3}\%$ p.a.

**c** **i** 50% p.a. **ii** 100% p.a.

**20** Sample answer: 5% p.a. for 5 years, 2.5% for 10 years, 10% for 2.5 years

EX p38

## 1F Simple interest calculations

**1** **a** \$420 **b** \$27.50 **c** \$1700

**d** \$5000 **e** \$38 000 **f** \$2400

**2** **a** $T = 5$ years **b** $T = 7$ years **c** $T = 4$ years

**d** $T = 4$ years

**3** **a** $P = \$5000$ **b** $P = \$15\,000$ **c** $P = \$4000$

**d** $P = \$4800$

**4** **a** $T = 5$ years **b** $P = \$6000$ **c** $P = \$2500$

**d** $T = 2$ years **e** $P = \$3125$ **f** $T = 5$ years

**5** **a** $T$, number of years

**b** $P$ = \$4500, $r$ = 5.0% p.a., $I$ = \$675

**c** 3 years

**d** One year sooner ($T$ = 2 years compared to $T$ = 3 years)

**6** **a** \$462 **b** 24 months

**7** **a** 5.0% p.a. **b** 6.0% p.a. **c** 2.5% p.a.

**d** 5.6% p.a. **e** 6.4% p.a. **f** 2.5% p.a.

**8** **a** \$932.40 **b** \$4500 **c** 9.8%

**d** 3.5 years or 3 years and 6 months

**e** \$2950 **f** 6.8% **g** 5.0%

**h** 1.75 years or 1 year and 9 months

**9** **a** $P$, amount borrowed

**b** $r$ = 4.0% p.a., $T$ = 3 years, $I$ = \$144

**c** \$1200

**d** Total amount to be repaid = \$1200 + \$144 = \$1344
Repayments = \$35 × 36 = \$1260
(3 years = 36 months)
Repayments of \$35 per month will not be enough to repay the loan over 3 years.

**e** \$37.22 (will be 8 cents short at the end, but can make this adjustment later)

**10** **a** 1.2125 years = a year and 78 days or about a year and two and a half months

**b** \$12 125

**11** **a** 4.5% **b** \$113.50

**12** **a** \$68.75 **b** 10 000 days **c** 79 365 years

**13** **a** 7 years **b** \$1560 **c** 5.7%

**d** \$1250 **e** 9.8% **f** 2947 days

**14** **a** \$624 **b** \$4624 **c** \$208

**d** \$4208 **e** \$218.82 **f** \$4426.82

**g** \$230.19 **h** \$4657.01

**i** Calculating interest at yearly intervals gives a higher value at the end of three years. \$4657.01 is more than \$4624. Interest is being added at the end of each year, so because the principal is greater each time interest is calculated, the interest payments will be higher.

**15** **a** \$12 597.12 **b** \$17 109.36 **c** \$24 038.44

**d** \$66 550

**16** **a** **i** \$2597.12 **ii** \$197.12

**b** **i** \$2109.36 **ii** \$69.36

**c** **i** \$6038.44 **ii** \$638.44

**d** **i** \$16 550 **ii** \$1550

**17** 2.2%

**18** **a** \$741.13

**b** 16.8% (rounded to one decimal place)

### Chapter review

#### Multiple-choice

**1** D **2** C **3** A **4** C

**5** B **6** E **7** A

#### Short answer

**1** **a** 9.2704 **b** −1.3297 **c** 0.8378

**2**

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{32.2}{3.5}$ | 9.2 | 920% |
| $\frac{763}{250} = 3\frac{13}{250}$ | 3.052 | 305.2% |
| $\frac{7}{2500}$ | 0.0029 | 0.29% |

**3** **a** 92.5 km/h **b** \$22.80/h

**c** \$0.01/mL or \$0.91/100 mL

**4**

| | |
|---|---|
| Employee 1 | \$753.75 |
| Employee 2 | \$945.00 |
| Employee 3 | \$967.50 |
| Employee 4 | \$956.25 |

**5** **a** \$644.30 **b** \$65.10 **c** \$259 **d** \$173.40

**6** **a** \$281.25 **b** \$27.75

**7** **a** **i** profit of \$15 **ii** 42.86%

**b** **i** loss of \$18.65 **ii** 17.85%

**c** **i** profit of \$45.70 **ii** 22.86%

**8** **a** 20% **b** 66.67% **c** 250% **d** 580%

**9** **a** **i** \$200 **ii** \$150 **iii** 75%

**b** **i** \$300 **ii** \$181.20 **iii** 60.4%

**10** **a** \$600 **b** \$480 **c** \$641.67

**11** **a** $T$ = 3 years **b** $P$ = \$4800 **c** $T$ = 2.5 years

**d** $r$ = 9% p.a. **e** $r$ = 3.4% p.a.

#### Analysis

**1** **a** 95.43 km/h **b** no **c** 21 minutes

**d** 26.668 L **e** \$38.86 **f** 23.919 L

**2** **a** \$1341.26

**b** Simon: \$516; Melanie: \$634.25; Tahlia: \$806.25

**c** \$5297.76 per week **d** \$720

**e** 275% **f** \$30.60 **g** 255%

**h** The new percentage mark-up is 155%, which is 120% less than the percentage mark-up in part **e**.

**i** \$29 904 **j** \$830.67 **k** \$53 904

## CHAPTER 2 Indices

EX p51

### 2A Indices

**1** **a** $6 \times 6 \times 6 \times 6 = 1296$

**b** $8 \times 8 \times 8 = 512$

**c** $-2 \times -2 \times -2 \times -2 \times -2 = -32$

**d** $-3 \times -3 \times -3 \times -3 \times -3 \times -3 = 729$

**e** $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{125}{64}$

**f** $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{128}$

**g** $-\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = \frac{16}{81}$

**h** $-\frac{3}{5} \times -\frac{3}{5} \times -\frac{3}{5} \times -\frac{3}{5} \times -\frac{3}{5} = -\frac{243}{3125}$

**2** **a** $b \times b \times b \times b \times b \times b$

**b** $-n \times -n \times -n \times -n \times -n$

**c** $-cd \times -cd$

**d** $2pq \times 2pq \times 2pq \times 2pq$

**e** $2 \times p \times q \times q \times q \times q$

**f** $-4 \times a \times a \times b \times b \times b \times c$

**g** $3m^2 \times 3m^2 \times 3m^2 \times 3m^2 \times 3m^2$

**h** $3 \times m^2 \times m^2 \times m^2 \times m^2 \times m^2$

**3** **a** $5^4$ **b** $a^4$ **c** $7k^3v^4$

**d** $(qu)^5$ or $q^5u^5$ **e** $(-h)^3$ **f** $-h^3$

**g** $(n^3)^6$ **h** $(5b^3d^4)^2$

**4** **a** $2 \times 5^2$ **b** $2^3 \times 3^2$ **c** $3^3 \times 5$

**d** $2 \times 3^3 \times 7$ **e** $2^3 \times 19$ **f** $2^2 \times 7 \times 29$

**g** $2 \times 5^2 \times 11$ **h** $3^2 \times 5 \times 37$

**5** **a** 0.04 **b** 0.04 **c** 0.0004

**d** 0.008 **e** −0.008 **f** 0.000 008

**g** 0.0016 **h** 0.0016 **i** 0.000 000 16

**6** **a** $5^4$ **b** $-5^3$ **c** $a^4b^4$

**d** $5^8x^8y^8$ **e** $5x^8y^8$ **f** $\frac{11^6}{2^6}$

**g** $-3^5a^5b^5c^5$ **h** $3^8a^8b^8c^8$

**7** **a** 343 **b** 6 **c** $-\frac{81}{320}$ **d** 2817

**8** **a** $2^3 \times 3^2$ **b** $5^6 \times 6$ **c** $13^4 \times 17^5$

**d** $101^2 \times 103^5$ **e** $4^3 \times x^4$ **f** $7^2(xy)^7$ or $7^2x^7y^7$

**9** **a** $2^3 \times 3 \times 5^2$ **b** $2 \times 3 \times 5^2$

**c** $2^2 \times 3^2 \times 5^2$ **d** $2^2 \times 3 \times 5^3$

**e** $2^3 \times 3^2 \times 5^3$ **f** $2^3 \times 3^2 \times 5^2$

**g** $2^2 \times 3 \times 5^2 \times 7$ **h** $2^5 \times 3 \times 5^2$

**10** **a** $x^7$ **b** $a^3b^2$ **c** $3rs^2t^3$ **d** $4e^7f^2$

**11** **a** Only $k$ has the index 5, so the $t$ should not be repeatedly multiplied. $tk^5 = t \times k \times k \times k \times k \times k$

**b** All of $2rw$ is raised to the power of 4, so $2rw$ should be repeatedly multiplied. $(2rw)^4 = 2rw \times 2rw \times 2rw \times 2rw$

**c** The product was evaluated before the power. $-3 \times (-2)^4 = -3 \times 16 = -48$

**12** **a** 1 **b** 39 **c** −60

**d** $\frac{63}{4} = 15\frac{3}{4}$

**13** **a** **i** 1 **ii** −1 **iii** 1 **iv** −1 **v** 1

**vi** −1 **vii** 1 **viii** −1 **ix** 1 **x** −1

**b** **i** When the index, $n$, is odd, the basic numeral of $(-1)^n$ is −1.

**ii** When the index, $n$, is even, the basic numeral of $(-1)^n$ is 1.

**c** **i** negative **ii** negative

**iii** positive **iv** positive

**v** positive **vi** negative

**vii** positive **viii** positive

**14** **a** **i** 0.343 and 0.000 343

**ii** In index form, both are the cube of a decimal with the digit 7 in different place values: one is 7 tenths, the other is 7 hundredths. The basic numerals use the same digits 343 also in different place values; 0.000 343 is $10^3 = 1000$ times smaller than 0.343.

**b** **i** −0.064 and −0.064

**ii** In index form, both are the cube of 0.4 but one has a negative inside the brackets, the other outside. The basic numerals for both are exactly the same.

**c** **i** −1.728 and 2.0736

**ii** In index form, both raise −1.2 to a power: one is to the power of 3; the other to the power of 4. The basic numeral for the power of 3 is negative while the power of 4 is positive. The two numerals have different values.

**d** **i** 9.261 and 8.120 601

**ii** In index form, both are cubes of a number that uses the digits 2 and 1. The basic numerals share similar digits where 8.120 601 has its digits spaced out one place more than 9.261 does.

**15** **a** 1.2

**b** 2021: 24, 2022: 29, 2025: 50

**16** **a** 3 days: $2^3$, 8 days: $2^8$, 12 days: $2^{12}$

**b** 3 days: $2^6$, 8 days: $2^{16}$, 12 days: $2^{24}$

**c** 3 days: 2, 8 days: $2^4$, 12 days: $2^6$

**d** A: 24, B: 192, C: 6

**17** **a** LCM: $2^8 \times 3^{15} \times 5^2 \times 7^4$ HCF: $2^4 \times 3^5 \times 5^2 \times 7$

**b** LCM: $a^8b^{15}c^2d^4$ HCF: $a^4b^5c^2d$

**c** LCM: $pq^5r^{10}s^4$ HCF: $pq^3r^7s^2$

**d** LCM: $24x^3y^9z^4$ HCF: $4xy^3z^4$

**18** **a** $2^{10}$ **b** $5^{10}$ **c** $2^{12} \times 5^8$

**19** $64^1 = 64$, $8^2 = 64$, $(-8)^2 = 64$, $4^3 = 64$, $2^6 = 64$, $(-2)^6 = 64$

**20** **a** $-\frac{1}{12}$ **b** $\frac{243}{32}$ **c** $-\frac{2401}{10} = -240.1$

## EX p56 2B Index laws 1 and 2

**1** **a** $3^9$ **b** $7^6$ **c** $(-2)^{12}$ **d** $6^3$

**e** $(-8)^7$ **f** $10^{11}$ **g** $3^1$ or 3 **h** $5^2$

**i** $4^2$ **j** $(-9)^9$ **k** $13^3$ **l** $2^{10}$

**2** **a** 19 683 **b** 117 649 **c** 4096 **d** 216

**3** **a** $3y^9$ **b** $7g^7$ **c** $6b^{11}$

**d** $-12k^{13}$ **e** $6b^{11}$ **f** $-80g^{11}$

**g** $27c^{14}$ **h** $15p^{10}$

**4** **a** $a^2$ **b** $d$ **c** $g^{10}$

**d** $p^3$ **e** $a^5$ **f** $n^3$

**g** $r^8$ **h** $8x^{11}$

**5** **a** $12x^{11}$ **b** $10x^7$ **c** $-24x^9$ **d** $54x^{11}$

**e** $3x^4$ **f** $4x^4$ **g** $\frac{2x}{5}$ **h** $\frac{5x^8}{3}$ or $\frac{5}{3}x^8$

**i** $-8t^{12}$ **j** $\frac{5r^6}{8}$ or $\frac{5}{8}r^6$ **k** $5c^4$ **l** $\frac{5y^7}{2}$ or $\frac{5}{2}y^7$

**6** **a** $x^9$ **b** $\frac{b}{a}$ **c** $n$ **d** $b^8$

**e** $x^5$ **f** $m^4$ **g** $6a^6$ **h** $n^5$

**i** 5 **j** $-4t^2$ **k** 2 **l** 1

**7** **a** $a^9b^6$ **b** $-18m^{11}n^3$ **c** $c^2d^2$
**d** $k^2m^3$ **e** $x^8y^7$ **f** $30g^{10}h^3$
**g** $a^8b^6$ **h** $15x^8y^{10}$ **i** $54w^4x^{13}y^4$
**j** $\frac{u^4}{5}$ or $\frac{1}{5}u^4$ **k** $-\frac{3}{4}ef^{10}$ **l** $-108v^{17}y^{10}$

**8** **a** $x^6$ **b** $2k^3$ **c** $6a$
**d** $\frac{m^2}{x^2}$ **e** $b^3$ **f** $n^{13}p^3$
**g** $-2j^5q^6$ **h** $2w^2x^4$

**9** **a** False: When multiplying you add, not multiply indices. $x^3 \times x^4 = x^7$
**b** False: There is no index law for adding terms, but you can add these as they are like terms. $k^3 + k^3 = 2k^3$
**c** true
**d** False: There is a power of 1 in the middle term, which must be added. $a^5 \times a \times a^5 = a^{11}$
**e** False: Add the powers of $m$ and $n$ separately. $m^3n^5 \times m^2n^4 = m^5n^9$
**f** False: When dividing you subtract, not divide indices. $100^8 \div 100^2 = 100^6$
**g** true
**h** False: The first $b^6$ has been missed. $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^5b^6$

**10** **a** **i** $\frac{1}{3^6}$ **ii** $\frac{1}{5^6}$ **iii** $\frac{1}{2^4 \times 3^5}$
**iv** $\frac{3^5}{2^4}$ **v** $\frac{2^4}{3^5}$
**b** Copy and complete the following.
**i** $\frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2^2} = \frac{1}{2^{(5-3)}}$
**ii** $\frac{2^4}{2^8} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{2^4} = \frac{1}{2^{(8-4)}}$
**iii** $\frac{5^6}{5^7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5} = \frac{1}{5^1} = \frac{1}{5^{(7-6)}}$
**c** Subtracting the indices on the numerator (larger index on the numerator) or denominator (larger index on the denominator) gives the index on the numerator or denominator when simplified because the common bases are cancelled.

**11** **a** $2^{15}$ **b** $3^{15}$ **c** $2^6 \times 3^6$ **d** $2^{30}$

**12** **a** $w = 5, x = 3, y = 2, z = 8$
**b** $x = 9, y = 6, z = 4$
**c** $a = 16, b = 9, c = 4, d = 6$

**13** No. An index represents the number of times the base is repeatedly multiplied. If you write $2^4 \times 3^2$ as $(2 \times 3)^{(4+2)}$, you are saying that $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is equal to $(2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$, which is not true. Similarly, if you write $y^8 \div x^5$ as $\left(\frac{y}{x}\right)^{(8-5)}$, you are saying that $\frac{y \times y \times y \times y \times y \times y \times y \times y}{x \times x \times x \times x \times x}$ is equal to $\left(\frac{y}{x}\right) \times \left(\frac{y}{x}\right) \times \left(\frac{y}{x}\right)$, which is not true.

**14** **a** $2^3 = 8$ **b** $3^3 = 27$
**c** $5^2 = 25$ **d** $10^4 = 10\,000$

**15** **a** $2^{12} = 8^4$ **b** $3^{15} = 27^5$ **c** $5^{18} = 25^9$
**d** $10^{12} = 10\,000^3$ **e** $4^{14} = 16^7$ **f** $2^{30} = 32^6$
**g** $6^6 = 216^2$ **h** $3^{30} = 243^6$

**16** **a** $a^{m+n}b^{x+y}$ **b** $a^{m-n}b^{x-y}$

## EX p61 2C Index law 3 and the zero index

**1** **a** $6^{12}$ **b** $3^4$ **c** $3^6 \times 4^2$ **d** $2^{24}$
**e** $5^4 \times 2^{28}$ **f** $\frac{3^2}{4^2}$ **g** $\frac{5^{12}}{2^8}$ **h** $\frac{1}{8^{25}}$
**i** $-3^{28}$ **j** $3^{20}$ **k** $7^{28} \times 11^{21}$ **l** $\frac{13^{48}}{17^{24}}$

**2** **a** $b^{10}$ **b** $m^8$ **c** $j^{10}$ **d** $j^{10}$ **e** $n^{80}$ **f** $p^{99}$

**3** **a** $x^6y^6$ **b** $2^3d^3$ **c** $(-5)^7k^7 = -5^7k^7$
**d** $9^{10}p^{10}$ **e** $3^4m^4$ **f** $\frac{8^2}{p^2}$
**g** $\frac{x^6}{y^6}$ **h** $g^2h^2$ **i** $a^5b^5$
**j** $\frac{k^3}{m^3}$ **k** $2^8x^8$ **l** $-\frac{d^5}{3^5}$

**4** **a** $3^4a^{24}$ **b** $5a^{28}b^7$
**c** $\frac{2^3m^3}{n^3}$ **d** $\frac{a^8}{b^{20}}$
**e** $-2u^{12}$ **f** $\frac{4}{3}v^{70}w^{30} = \frac{4v^{70}w^{30}}{3}$
**g** $\frac{7p^7}{9q^{42}}$ **h** $\frac{3^{10}}{2^3}r^{45} = \frac{3^{10}r^{45}}{2^3}$
**i** $\frac{8}{5^{24}t^{44}}$ **j** $3^2 \times 7 \times i^{34} = 63i^{34}$
**k** $2^{35}c^{40}$ **l** $\frac{5^4x^{75}}{7^4y^{90}}$

**5** **a** 1 **b** 1 **c** 1 **d** 1

**6** **a** 2 **b** 1 **c** $-7$ **d** 1
**e** $-1$ **f** 2 **g** 2 **h** 2
**i** 2 **j** 3 **k** 1 **l** 1
**m** 1 **n** 1 **o** $-1$ **p** $-1$

**7** **a** $x^{13}$ **b** $x^{22}$ **c** $x^{27}$ **d** $x^{27}$
**e** $x^{10}$ **f** $w^6$ **g** $\frac{b}{3}$ **h** $e^6$
**i** 1 **j** $4a^9$ **k** $t^{25}$ **l** $f^{109}$

**8** **a** 1 **b** $-7$ **c** 1 **d** 3
**e** 1 **f** $k^2$ **g** $10g^4$ **h** 3
**i** $x^{15}$ **j** 2 **k** $-8b$ **l** $\frac{2}{5}$

**9** **a** $-2$ **b** $k^{10}$ **c** $x^{11}y^9$ **d** $\frac{m^{12}}{2n^5}$ **e** 1

**10** **a** $x^9y^7$ **b** $1568k^7$ **c** $81x^{24}$ **d** $-5a^{28}b^7$
**e** $\frac{x^{10}}{y^{11}}$ **f** $\frac{8m^3}{n^3}$ **g** $\frac{a^8}{b^{20}}$ **h** $\frac{w^{10}x^6}{y^8}$
**i** $k^7mn^2$ **j** $\frac{16t^{13}}{p^{21}}$ **k** $ab^{30}$ **l** $72h^6$

**11 a** $a^3 \div a^3 = \frac{a^3}{a^3} = \frac{a \times a \times a}{a \times a \times a} = 1$

**b** $a^3 \div a^3 = a^3 - {}^3 = a^0$

**c** The answers to parts **a** and **b** should be the same so $a^0 = 1$.

**12 a** True **b** True

**c** False: Not every power in the bracket has been raised to the sixth power. $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$

**d** False: The bracket has been expanded incorrectly. $\frac{(k^3)^2 \times k^4}{k^2} = \frac{k^6 \times k^4}{k^2} = k^8$

**e** True **f** False, $100^9 \div 100^9 = 1$

**g** True

**13 a** 7 **b** 4 **c** 0 **d** 8

**e** 3 **f** 5 **g** 7 **h** 5

**14** Eden added the 4 and 5 indices before multiplying the 5 by the 3 index. $3^4 \times (3^5)^3 = 3^4 \times 3^{15} = 3^{19}$

**15 a** $(2^3)^5 = (2^3) \times (2^3) \times (2^3) \times (2^3) \times (2^3)$
$= 2^{3+3+3+3+3}$
$= 2^{3\times5}$

**b** $(x^7)^4 = (x^7) \times (x^7) \times (x^7) \times (x^7)$
$= x^{7+7+7+7}$
$= x^{7\times4}$

**c** $(2 \times 3)^4 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= 2^4 \times 3^4$

**d** $\left(\frac{2}{3}\right)^6 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$
$= \frac{2^6}{3^6}$

**16 a** $3^{64}$ **b** $5^{81}$ **c** $7^{1024}$ **d** $10^{3125}$

**17** $8^{15}$ cm$^3$

**18** $\frac{4^5}{3^5}$

**19** $(a^m)^n = a^{m\times n} = a^{n\times m} = (a^n)^m$

**20** $x = \frac{3}{20}$

## Checkpoint

**1 a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

**b** $-3 \times -3 \times -3 \times -3 = 81$

**c** $-4 \times -4 \times -4 = -64$

**d** $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

**2 a** $a \times a \times a \times a \times a \times a$

**b** $-b \times -b \times -b \times -b$

**c** $3y \times 3y \times 3y \times 3y \times 3y$

**d** $3 \times xy \times xy \times xy \times xy \times xy$

**3 a** $8^7$ **b** $u^4$ **c** $(4b)^5$ **d** $-7k^3h^5$

**4 a** $2^2 \times 7$ **b** $2^3 \times 3^2$

**c** $2^2 \times 11^2$ **d** $2 \times 3^3 \times 5$

**5 a** $8^{11}$ **b** $5^{10} \times 7^{12}$ **c** $6^5$ **d** $3^8 \times 10^7$

**6 a** $a^{12}$ **b** $-12b^{18}c^{21}$ **c** $u^5$ **d** $\frac{5}{7}p^{14}q^{17}$

**7 a** $3^{10}$ **b** $k^8$

**c** $\frac{t}{c^{11}}$ **d** $-\frac{15}{4}d^{11}w^7$

**8 a** 1 **b** 1 **c** $-1$ **d** 8

**9 a** $3^{24}$ **b** $j^{45}$

**c** $(-5)^6a^{18}b^{42} = 5^6a^{18}b^{42}$ **d** $-\frac{3^8p^{40}}{2^8q^{56}}$

**10 a** 1 **b** $\frac{g^{20}}{3^7}$ **c** $5^7m^{122}n^{87}$ **d** $\frac{6}{j^{42}}$

## 2D Negative indices

EX p67

**1 a** $\frac{7}{8}$ **b** 2 **c** $-\frac{1}{3}$ **d** $\frac{1}{3w}$

**e** $d$ **f** $\frac{m}{r}$ **g** $5y$ **h** $\frac{1}{u^2}$

**2 a** $\frac{1}{5}$ **b** $\frac{1}{8}$ **c** $-\frac{1}{2}$

**d** $\frac{1}{x}$ **e** $\frac{1}{p}$ **f** $\frac{1}{3w}$

**3 a** $5^{-1}$ **b** $(-13)^{-1}$ **c** $\frac{1}{5^{-1}}$

**d** $-\frac{1}{8^{-1}}$ **e** $m^{-1}$ **f** $\frac{1}{x^{-1}}$

**g** $(5y)-^{-1}$ **h** $\frac{1}{(3w)^{-1}}$ **i** $\frac{3^{-1}}{4^{-1}}$

**j** $\frac{6^{-1}}{5^{-1}}$ **k** $\frac{a^{-1}}{w^{-1}}$ **l** $\frac{1}{v^{-3}}$

**4 a** $\frac{1}{4^2}$ **b** $\frac{1}{2^6}$ **c** $\frac{1}{(-9)^3} = -\frac{1}{9^3}$

**d** $\frac{1}{(-5)^4} = \frac{1}{5^4}$ **e** $-\frac{1}{7^8}$ **f** $\frac{1}{10^5}$

**g** $\frac{1}{a^4}$ **h** $\frac{1}{x^7}$ **i** $\frac{1}{k^{10}}$

**j** $\frac{1}{m^2}$ **k** $\frac{1}{u^9}$ **l** $\frac{1}{g^{11}}$

**5 a** $3^{-4}$ **b** $4^{-7}$ **c** $6^{-5}$

**d** $(-5)^{-3} = -5^{-3}$ **e** $(-9)^{-2} = 9^{-2}$ **f** $-11^{-6}$

**g** $n^{-2}$ **h** $g^{-11}$ **i** $x^{-8}$

**j** $a^{-9}$ **k** $p^{-4}$ **l** $w^{-7}$

**6 a** $2^3$ **b** $5^6$ **c** $(-8)^4 = 8^4$

**d** $3^9$ **e** $(-7)^5 = -7^5$ **f** $-4^2$

**g** $x^7$ **h** $y^3$ **i** $c^4$

**j** $z^{35}$ **k** $(5t)^4 = 5^4t^4 = 625t^4$

**l** $(uv)^{15} = u^{15}v^{15}$

**7 a** $\frac{5^2}{4^2}$ **b** $\frac{3}{7}$ **c** $-\frac{4^3}{3^3}$

**d** $\frac{2^4}{a^4}$ **e** $\frac{m}{6}$ **f** $\frac{u}{144}$

**g** $\frac{y^6}{x^6}$ **h** $-\frac{7^{11}}{9^{11}}$ **i** $\frac{17^{14}}{13^{14}}$

**j** $\frac{43^{11}}{500^{11}}$ **k** $\frac{97^{198}}{u^{198}}$ **l** $\frac{g^{654}}{123^{654}}$

**8 a** $\frac{y^3}{x^2}$ **b** $\frac{m^6}{n^4}$ **c** $\frac{c^7}{a}$

**d** $\frac{2k^5}{p^3}$ **e** $\frac{5b^2}{a^8}$ **f** $-\frac{4}{w^2x^6}$

**g** $\frac{a^4c^7}{b^5}$ **h** $\frac{m^5}{k^3n^8}$ **i** $\frac{7b^9d}{c^6}$

**j** $\frac{3}{x^2y^7z^4}$ **k** $\frac{-12p^{17}}{q^{18}u^{21}}$ **l** $\frac{-34}{j^{65}b^{78}}$

**9** **a** $\frac{b^3}{a^5}$ **b** $\frac{p^2}{k^6}$ **c** $\frac{m^7}{h^4}$

**d** $d^5e^8$ **e** $u^3w^8$ **f** $3x^2y^4$

**g** $8g^6h^5$ **h** $-\frac{2c^9}{d^5}$ **i** $\frac{2n^3}{5m^7}$

**j** $\frac{1}{6e^5y^8}$ **k** $-\frac{1}{2g^{99}h^{11}}$ **l** $\frac{2c^{101}}{5r^{72}}$

**10** **a** $\frac{n^4p^6}{m^3}$ **b** $\frac{4c^2e^6}{d^5}$ **c** $\frac{2p^4}{3kn^3}$

**d** $\frac{a^2b^3}{3c^5d^4}$ **e** $\frac{7k^2n}{m^6}$ **f** $\frac{u^5x^3}{10k^2w^8}$

**g** $-\frac{5pq^4}{r^4s^3}$ **h** $63b^{14}g^{15}k^{17}u^{10}$ **i** $\frac{1}{5t^4u^3v^7w^{12}}$

**j** $-\frac{49m^{59}n^{41}}{64l^{88}v}$ **k** $\frac{a^{97}c^{153}d^{167}f^{89}}{b^{105}e^{111}}$ **l** $k^{402}l^{812}m^{999}n^{571}$

**11** **a** $\frac{1}{4^3}$ **b** $\frac{1}{7}$ **c** $2^2$

**d** $\frac{1}{(-3)^6} = \frac{1}{3^6}$ **e** $5^4$ **f** $\frac{1}{(-2)^7} = -\frac{1}{2^7}$

**g** $\frac{1}{9^2}$ **h** $3^8$ **i** $\frac{1}{4^9}$

**j** $\frac{1}{10^3}$ **k** $\frac{1}{211^{14}}$ **l** $\frac{1}{13^{100}}$

**12** **a** $\frac{1}{5^6}$ **b** $\frac{1}{3^8}$ **c** $-2^4$

**d** $\frac{1}{3^2}$ **e** $\frac{1}{6^4}$ **f** $\frac{1}{4}$

**g** $\frac{1}{9}$ **h** $5^8$ **i** $7^{13}$

**j** $\frac{1}{2^3}$ **k** $99^{127}$ **l** $\frac{1}{15^{162}}$

**13** **a** $\frac{1}{64}$ **b** $\frac{1}{7}$ **c** 4

**d** $\frac{1}{729}$ **e** 625 **f** $-\frac{1}{128}$

**14** **a** $\frac{1}{a^7b^7}$ **b** $\frac{1}{3^5x^5} = \frac{1}{243x^5}$ **c** $\frac{1}{4y}$

**d** $\frac{1}{7^2k^2} = \frac{1}{49k^2}$ **e** $\frac{1}{2^3p^3} = \frac{1}{8p^3}$ **f** $\frac{1}{m^{11}k^{11}}$

**15** **a** $-3$ **b** $-7$ **c** $-1$ **d** 2

**e** $-2$ **f** $-3$ **g** $-2$ **h** $-4$

**16** **a** $\frac{1}{64}$ mm **b** 0.015 625 mm

**17** **a** $\frac{1}{100\,000\,000}$ **b** 0.000 000 01

**18** **a** $\frac{1}{15\,625}$ **b** 0.000 064

**19** **a**

| Index form | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ | $2^{-1}$ | $2^{-2}$ | $2^{-3}$ | $2^{-4}$ | $2^{-5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Basic numeral | 32 | 16 | 8 | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{32}$ |

**b** Each term is half of the term before it. The basic numeral of a term with base 2 and a negative index is the reciprocal of the value of the term with the matching positive index.

**c** $\frac{1}{64}$ **d** $\frac{1}{1024}$ **e** 128

**20** **a**

| Index form | $3^5$ | $3^4$ | $3^3$ | $3^2$ | $3^1$ | $3^0$ | $3^{-1}$ | $3^{-2}$ | $3^{-3}$ | $3^{-4}$ | $3^{-5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Basic numeral | 243 | 81 | 27 | 9 | 3 | 1 | $\frac{1}{3}$ | $\frac{1}{9}$ | $\frac{1}{27}$ | $\frac{1}{81}$ | $\frac{1}{243}$ |

**b** Each term is one third of the term before it. The basic numeral of a term with base 3 and a negative index is the reciprocal of the value of the term with the matching positive index.

**c** $\frac{1}{729}$ **d** $\frac{1}{6561}$ **e** 2187

**21** **a**

| Index form | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ |
|---|---|---|---|---|---|---|---|---|---|
| Basic numeral | 10 000 | 1000 | 100 | 10 | 1 | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ | $\frac{1}{10\,000}$ |

**b** Each term is one tenth of the term before it. The basic numeral of a term with base 10 and a negative index is the reciprocal of the value of the term with the matching positive index.

**c** **i** 100 000 **ii** 1 000 000

**iii** 10 000 000 **iv** 100 000 000

**v** 1 000 000 000

**d** **i** $\frac{1}{100\,000}$ **ii** $\frac{1}{1\,000\,000}$

**iii** $\frac{1}{10\,000\,000}$ **iv** $\frac{1}{100\,000\,000}$

**v** $\frac{1}{1\,000\,000\,000}$

**e** **i** $\frac{1}{10}$ **ii** 0.1

**f** **i** $\frac{1}{100} = 0.01$ **ii** $\frac{1}{1000} = 0.001$

**iii** $\frac{1}{10\,000} = 0.0001$

**g** **i** $\frac{1}{100\,000} = 0.000\,01$

**ii** $\frac{1}{1\,000\,000} = 0.000\,001$

**iii** $\frac{1}{10\,000\,000} = 0.000\,000\,1$

**iv** $\frac{1}{100\,000\,000} = 0.000\,000\,01$

**v** $\frac{1}{1\,000\,000\,000} = 0.000\,000\,001$

**h** The number of decimal places in the decimal matches the number of zeros in the denominator of the fraction, which matches the power of 10.

**22 a** **i** 20 000 **ii** 7000
**iii** 300 000 **iv** 400 000 000 000
**v** 90 000 000

**b** **i** $\frac{5}{100}$ **ii** $\frac{8}{100\,000}$ **iii** $\frac{2}{1000}$
**iv** $\frac{7}{10\,000}$ **v** $\frac{6}{1000\,000\,000}$

**c** **i** 0.05 **ii** 0.000 08 **iii** 0.002
**iv** 0.0007 **v** 0.000 000 006

**d** When multiplying a whole number by a negative power of ten, move the decimal point one place to the left for each zero you see.

**23 a** **i** $2 \times 3^{-1} = 2 \times \frac{1}{3} = \frac{2}{3} = 2 \div 3$

**ii** $2 \div 3^{-1} = \frac{2}{3^{-1}} = 2 \times \frac{1}{3^{-1}} = 2 \times 3$

**b** Multiplication by the reciprocal of a number is equivalent to division by that number. Division by the reciprocal of a number is equivalent to multiplication by that number.

**24 a** **i** 3 **ii** 5 **iii** $\frac{5}{4}$

**b** Using index law 3, $(a^{-1})^{-1} = a^{(-1 \times -1)} = a^1 = a$.

**c** The reciprocal of the reciprocal of a number is the original number.

**25 a** $2a^{-2}b^{-1}$ **b** $-3t^2v^{-3}$ **c** $5^{-1}x^4y^{-4}$ **d** $p^{-1}q^{-5}r^2$

**26 a** $\frac{1}{x^2}$ **b** $\frac{1}{x^4}$ **c** $8x^3$ **d** $\frac{30}{x^5}$
**e** 3 **f** $x^9$ **g** $\frac{1}{x^3}$ **h** $2x^5$
**i** $\frac{1}{3x^{10}}$ **j** $\frac{4}{7x^4}$ **k** $\frac{1}{x^2}$ **l** $x^{11}$
**m** $\frac{4}{x^{13}}$ **n** $\frac{2}{x^8}$ **o** $\frac{1}{x^5}$ **p** $\frac{1}{x^7y^7}$

**27 c** $\frac{8}{x^{-3}} = \frac{1}{8^{-1}x^{-3}}$ **e** $\frac{1}{3^{-1}}$ **f** $\frac{1}{x^{-9}}$
**h** $\frac{2}{x^{-5}} = \frac{1}{2^{-1}x^{-5}}$ **l** $\frac{1}{x^{-11}}$

**28 a** $a^m \times a^{-n} = a^{(m+(-n))}$

**b** $\frac{1}{a^{-m} \times a^n} = \frac{1}{a^{(-m+n)}}$

**c** $a^m \div a^n = a^{(m-n)}$ and $a^{-n} \div a^{-m} = a^{(-n-(-m))}$

EX p74

## 2E Scientific notation

**1 a** 540 **b** 73 600
**c** −1800 **d** 405 000
**e** 2 753 000 **f** 0.61
**g** 0.000 008 22 **h** −0.000 976
**i** 0.000 070 03

**2 a** $10^2$ **b** $10^3$ **c** $10^4$ **d** $10^5$ **e** $10^6$
**f** $10^{-1}$ **g** $10^{-2}$ **h** $10^{-3}$ **i** $10^{-4}$ **j** $10^{-5}$

**3 a** 320 000 **b** 8 140 000 000
**c** −500 **d** −23 450 000
**e** 11 000 **f** 0.0064
**g** 0.000 007 28 **h** 0.000 000 9
**i** −0.000 030 2 **j** −0.0541
**k** 450 000 000 000 **l** 0.000 000 006 12
**m** 0.57 **n** 1306.8
**o** 0.000 273 16

**4 a** Check the answers on your calculator with the answers in question **3**.

**b** Some of the very large and very small numbers are difficult to see on the calculator due to limited screen size.

**5 a** $4.5 \times 10^3$ **b** $7.32 \times 10^6$
**c** $2 \times 10^5$ **d** $-1.9 \times 10^2$
**e** $3.216 \times 10^3$ **f** $6.3 \times 10^{-3}$
**g** $1.8 \times 10^{-7}$ **h** $5 \times 10^{-2}$
**i** $-7.02 \times 10^{-5}$ **j** $4.27 \times 10^{-1}$
**k** $1.122 \times 10^4$ **l** $4 \times 10^{-6}$
**m** $-5.682 \times 10^2$ **n** $2.49 \times 10^{-4}$
**o** $6.793 \times 10^5$
**p** $-1.02 \times 10^{-2}$

**6 a** 3 **b** 2 **c** 4 **d** 1
**e** 3 **f** 2 **g** 1 **h** 4
**i** 5 **j** 4 **k** 4 **l** 3

**7 a** 2 **b** 3 **c** 4 **d** 2 **e** 4 **f** 3

**8 a** $2.6 \times 10^5$ **b** $-5.04 \times 10^4$
**c** $9.104 \times 10^6$ **d** $-6.0 \times 10^3$
**e** 460 **f** 73 050
**g** 1000 **h** 40 000
**i** −5.14 **j** 0.035
**k** −42.06 **l** 0.9

**9 a** $3.3 \times 10^2$ **b** $4.87 \times 10^4$
**c** $-1.908 \times 10^5$ **d** $3 \times 10^3$
**e** $4.03 \times 10^{-1}$ **f** $-5.4 \times 10^{-2}$
**g** $2.072 \times 10^{-4}$ **h** $-8 \times 10^{-3}$
**i** $7.6 \times 10^2$ **j** $-2.070 \times 10^4$
**k** $4.02 \times 10^1$ **l** $5.4008 \times 10^4$

**10 a** A, B, F, H **b** B, F
**c** A, D, G **d** B, E, H

**11 a** $5 \times 10^{-4}$ m **b** $2.2 \times 10^4$ ML
**c** $4.8 \times 10^{-2}$ mm **d** $9.3 \times 10^9$ people

**12 a** 6400 times in a minute
**b** 0.000 08 mm
**c** 149 600 000 km
**d** 0.000 000 000 000 28 cm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**13**

| | 1234.56 | 4.0191 |
|---|---|---|
| $\times 10^3$ | $1.23456 \times 10^3$ | $0.004\,019\,1 \times 10^3$ |
| $\times 10^2$ | $12.345\,6 \times 10^2$ | $0.040\,191 \times 10^2$ |
| $\times 10^1$ | $123.456 \times 10^1$ | $0.401\,91 \times 10^1$ |
| $\times 10^0$ | $1234.56 \times 10^0$ | $4.0191 \times 10^0$ |
| $\times 10^{-1}$ | $12\,345.6 \times 10^{-1}$ | $40.191 \times 10^{-1}$ |
| $\times 10^{-2}$ | $123\,456 \times 10^{-2}$ | $401.91 \times 10^{-2}$ |
| $\times 10^{-3}$ | $1\,234\,560 \times 10^{-3}$ | $4019.1 \times 10^{-3}$ |

| | 0.0492 | 0.007 40 |
|---|---|---|
| $\times 10^3$ | $0.000\,049\,2 \times 10^3$ | $0.000\,007\,40 \times 10^3$ |
| $\times 10^2$ | $0.000\,492 \times 10^2$ | $0.000\,074\,0 \times 10^2$ |
| $\times 10^1$ | $0.004\,92 \times 10^1$ | $0.000\,740 \times 10^1$ |
| $\times 10^0$ | $0.0492 \times 10^0$ | $0.007\,40 \times 10^0$ |
| $\times 10^{-1}$ | $0.492 \times 10^{-1}$ | $0.0740 \times 10^{-1}$ |
| $\times 10^{-2}$ | $4.92 \times 10^{-2}$ | $0.740 \times 10^{-2}$ |
| $\times 10^{-3}$ | $49.2 \times 10^{-3}$ | $7.40 \times 10^{-3}$ |

**14 a** $6.8 \times 10^7$ **b** $2.0 \times 10^2$ **c** $4.5 \times 10^4$
**d** $-4.0 \times 10^4$ **e** $-1.23 \times 10^{-1}$ **f** $3.0 \times 10^5$

**15 a** $7.3034 \times 10^5$ **b** $8.36 \times 10^4$ **c** $6.3 \times 10^{-3}$
**d** $-2.68 \times 10^{-2}$ **e** $-1.057 \times 10^4$ **f** $9.35 \times 10^4$

**16** $1.08 \times 10^9$ km/h

**17 a** $2.4 \times 10^6$ km **b** 400 days

**18 a i** 86 coins **ii** 774 g
**b** 0.025 m = 2.5 cm

**19** Approximately 507 seconds or about 8 minutes

**20 a i** 2 (1 s.f.), 1.9 (2 s.f.), 1.90 (3 s.f.)
**ii** 2 (1 s.f.), 2.0 (2 s.f.), 1.99 (3 s.f.)
**iii** 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)
**iv** 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)
**v** 2 (1 s.f.), 2.0 (2 s.f.), 2.01 (3 s.f.)
**vi** 2 (1 s.f.), 2.1 (2 s.f.), 2.10 (3 s.f.)
**b** If the trailing zeros were not written as significant, then many of these numbers would round to the same number for different significant figures, which means we would be unable to determine the number of significant figures the number was rounded to consistently.

**21 a** 0.4 (1 s.f.), 0.000 000 000 001 (1 s.f.), −0.000 03 (1 s.f.)
**b** 0, 0, and 0
**c** If leading zeros are significant, then all values just more than −0.5 and just less than 0.5 would round to 0 for 1 significant figure, which does not provide useful information about very small numbers and is no different to rounding to the nearest integer (or decimal place for more significant figures).

**22 a** Jane rounded to 3 decimal places, not 3 significant figures.
**b** Kaleb included the leading zeros as significant.
**c** Lisa did not include the 0 between the 1 and 2 as significant.
**d** Marius did not keep the place value of the significant figures.

**23 a i** $2.82 \times 10^3$ seconds
**ii** $1.209\,6 \times 10^6$ seconds
**iii** $2.419\,2 \times 10^7$ seconds
**iv** $3.153\,6 \times 10^7$ seconds
**b i** 2.78 hours **ii** 11.6 days
**iii** 3.17 years **iv** 3.17 millenia

**24 a i** $7.3 \times 10^3$ seconds
**ii** $9.1 \times 10^{-6}$ seconds
**iii** $54 \times 10^{-9}$ seconds
**iv** $82 \times 10^{12}$ seconds
**v** $129 \times 10^6$ seconds
**vi** $974 \times 10^{-12}$ seconds
**b i** $56.01 \times 10^6$ seconds, 56.01 megaseconds (56.01 Ms)
**ii** $920 \times 10^3$ seconds, 920 kiloseconds (920 ks)
**iii** $43.1 \times 10^{-6}$ seconds, 43.1 microseconds (43.1 μs)
**iv** $788 \times 10^{-9}$ seconds, 788 nanoseconds (788 ns)
**v** $80 \times 10^{-3}$ seconds, 80 milliseconds (80 ms)
**vi** $103.56 \times 10^{12}$ seconds, 103.56 terraseconds (103.56 Ts)

**25 a** Light is much faster than sound.
**b i** $3.3 \times 10^{-7}$ seconds
**ii** about 0.30 seconds
**c** Watch for the smoke rather than listen for the shot (or listen for the shot and add 0.3 s onto the time).

**26** Diameter of one atom = $2.54 \times 10^{-9}$ cm
The line will be $2.54 \times 10^{-3}$ cm long.

**27** $1.8 \times 10^{356}$

## EX p81 2F Surds

**1 a i** 8 **ii** 12 **iii** 20 **iv** 24 **v** 28
**b i** 18 **ii** 27 **iii** 45 **iv** 54 **v** 63
**c i** 50 **ii** 75 **iii** 125 **iv** 150 **v** 175
**d i** 32 **ii** 48 **iii** 80 **iv** 96 **v** 112
**e i** 72 **ii** 108 **iii** 180 **iv** 216 **v** 252
**f i** 128 **ii** 192 **iii** 320 **iv** 384 **v** 448

**2 a** $2\sqrt{5}$ **b** $3\sqrt{3}$ **c** $2\sqrt{7}$ **d** $5\sqrt{2}$
**e** $6\sqrt{2}$ **f** $20\sqrt{2}$ **g** $7\sqrt{3}$ **h** $8\sqrt{6}$
**i** $12\sqrt{3}$ **j** $11\sqrt{7}$ **k** $9\sqrt{6}$ **l** $9\sqrt{11}$

**3 a** $\sqrt{45}$ **b** $\sqrt{96}$ **c** $\sqrt{12}$ **d** $\sqrt{44}$
**e** $\sqrt{250}$ **f** $\sqrt{448}$ **g** $\sqrt{507}$ **h** $\sqrt{720}$

**4 a** $\sqrt{30}$ **b** $\sqrt{96}$ **c** $\sqrt{135}$ **d** $\sqrt{336}$
**e** $\sqrt{351}$ **f** $\sqrt{600}$ **g** $\sqrt{707}$ **h** $\sqrt{5535}$

**5 a** $\sqrt{3}$ **b** $\sqrt{13}$ **c** $\sqrt{21}$ **d** $\sqrt{9}$
**e** $\sqrt{11}$ **f** $\sqrt{17}$ **g** $\sqrt{24}$ **h** $\sqrt{30}$

**6 a** $4\sqrt{3}$ **b** $10\sqrt{6}$ **c** $6\sqrt{3}$ **d** $5\sqrt{21}$
**e** $8\sqrt{10}$ **f** $9\sqrt{42}$ **g** $24\sqrt{10}$ **h** $15\sqrt{35}$

**7** **a** $8\sqrt{5}$ **b** $2\sqrt{2}$ **c** $3\sqrt{14}$ **d** $6\sqrt{11}$
**e** $12\sqrt{3}$ **f** $6\sqrt{7}$ **g** $2\sqrt{21}$ **h** $4\sqrt{3}$

**8** **a** $\sqrt{9}, \sqrt{6}, \sqrt[3]{6}, \sqrt{\frac{4}{9}}, \sqrt{8}, \sqrt{27}, \sqrt{0.16}, \sqrt[4]{16}, \sqrt{\frac{7}{5}}, \sqrt{1.6}$

**b** $\sqrt{6}\ \sqrt{8}, \sqrt{27}, \sqrt{\frac{7}{5}}, \sqrt{1.6}$

**9** **a** 3 **b** 5 **c** 8 **d** 7
**e** 1 **f** 50 **g** 1 **h** 1

**10** **a** **i** $4\sqrt{3}$ **ii** $3\sqrt{5}$
**iii** $6\sqrt{6}$ **iv** $2\sqrt{10}$

**b** **i** $\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$
**ii** $\sqrt{6} + \sqrt{6}$
**iii** $\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$
**iv** $\sqrt{11} + \sqrt{11} + \sqrt{11}$

**11** **a** $\sqrt{5}$ m **b** $\sqrt{10}$ m **c** $\sqrt{13}$ m **d** $\sqrt{26}$ m
**e** $2\sqrt{2}$ m **f** $3\sqrt{5}$ m **g** $2\sqrt{10}$ m **h** $5\sqrt{5}$ m

**12** **a** $6\sqrt{10}$ **b** $99\sqrt{35}$ **c** $12\sqrt{105}$ **d** $50\sqrt{30}$

**13** **a** **i** $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
**ii** $2 \times 3 \times 3 \times 3 \times 5$
**iii** $2 \times 2 \times 5 \times 7 \times 7$
**iv** $3 \times 3 \times 5 \times 5 \times 5$

**b** **i** $12\sqrt{2}$ **ii** $3\sqrt{30}$
**iii** $14\sqrt{5}$ **iv** $15\sqrt{15}$

**14** **a** $\sqrt{3 \times 7} \times \sqrt{2 \times 7} = 7\sqrt{6}$
**b** $\sqrt{3^3} \times \sqrt{3 \times 11} = 9\sqrt{11}$
**c** $\sqrt{2 \times 3 \times 5} \times \sqrt{2 \times 3^2} = 6\sqrt{15}$
**d** $\sqrt{2 \times 3 \times 7} \times \sqrt{3 \times 7 \times 11} = 21\sqrt{22}$

**15** **a** $28\sqrt{6}$ **b** $120\sqrt{10}$
**c** $4\sqrt{7}$ **d** $\frac{\sqrt{6}}{2}$
**e** $840\sqrt{35}$ **f** $\frac{3\sqrt{5}}{11} = \frac{3}{11}\sqrt{5}$
**g** $1872\sqrt{42}$ **h** $\frac{49\sqrt{6}}{4} = \frac{49}{4}\sqrt{6}$

**16** **a** 180 **b** 2 **c** 210 **d** 15

**17** **a** **i** 8 **ii** 27 **iii** 125
**iv** 216 **v** 343 **vi** 512

**b** **i** $2\sqrt{2}$ **ii** $3\sqrt{3}$ **iii** $5\sqrt{5}$
**iv** $6\sqrt{6}$ **v** $7\sqrt{7}$ **vi** $8\sqrt{8}$ or $16\sqrt{2}$

**c** The square root of a cube number, $\sqrt{a^3}$, simplifies to $a\sqrt{a}$ as there is always a square factor of $a$. This is on the condition that $a$ does not have any square numbers as a factor, as then $a\sqrt{a}$ could be simplified further.

**18** **a** $7\sqrt{3}$ **b** $3\sqrt{3}$
**c** $13\sqrt{5}$ **d** $\sqrt{11}$
**e** $8\sqrt{7} - 11\sqrt{3}$ **f** $13\sqrt{2} + 7\sqrt{3} + 3\sqrt{5}$
**g** $10\sqrt{2}$ **h** $6\sqrt{3} + 5\sqrt{5}$

**19** **a** 2 **b** $2\sqrt{3}$ **c** $\sqrt{3}$

**20** **a** 75 **b** 45 **c** 112 **d** 384

**21** **a** $\sqrt{2}$ **b** $\sqrt{5}$ **c** $\sqrt{3}$
**d** $\sqrt{2}$ **e** $\sqrt{3}$ **f** $\sqrt{6}$

## Chapter review

### Multiple-choice

**1** B **2** C **3** C **4** E **5** B
**6** D **7** A **8** E **9** D **10** B

### Short answer

**1** **a** 81 **b** −125 **c** −64
**d** $\frac{243}{32}$ **e** 0.216 **f** 2.0736

**2** **a** $17^6$ **b** $(-5b^2)^5$ **c** $-10f^3v^7$ **d** $\left(\frac{b^4d^5}{6n^3}\right)^4$

**3** **a** $a^{16}$ **b** $b$ **c** $c^{16}$
**d** $\frac{d^3}{3}$ **e** $e^{47}$ **f** 9

**4** **a** $m^5n^8$ **b** $54k^{18}l^{12}$

**5** **a** $\frac{1}{4}$ **b** $\frac{1}{b}$ **c** $\frac{1}{m^7}$
**d** $3x^3y^3$ **e** $\frac{g^5}{f^4}$ **f** $\frac{5^2bc^4}{a^3} = \frac{25bc^4}{a^3}$
**g** $\frac{1}{p^{15}}$ **h** $\frac{l^7}{k^7}$

**6** **a** $\frac{1}{65\,536}$ **b** 343

**7** **a** 58 760 **b** 0.000 009 02

**8** **a** 4 **b** 3

**9** **a** $5.4 \times 10^5$ **b** $7.6 \times 10^{-4}$

**10** **a** 880 **b** 300 000

**11** **a** 38 600 **b** 39 000
**c** $3.86 \times 10^4$ or $3.9 \times 10^4$ (rounded)

**12** **a** $6\sqrt{2}$ **b** 7 **c** $9\sqrt{3}$
**d** 16 **e** $10\sqrt{10}$ **f** $5\sqrt{14}$

**13** **a** $\sqrt{5} \times \sqrt{7}$ **b** $\sqrt{60} \times \sqrt{70}$
**c** $\frac{\sqrt{180}}{\sqrt{36}}$ **d** $\frac{\sqrt{1625}}{\sqrt{13}}$

**14** **a** $\sqrt{30}$ m **b** $\sqrt{71}$ m
**c** $2\sqrt{11}$ m **d** $3\sqrt{21}$ m

**15** **a** $12\sqrt{3}$ **b** $108\sqrt{30}$
**c** $100\sqrt{14}$ **d** $30\sqrt{35}$

### Analysis

**1** **a** **i** Tas, NT, ACT
**ii** NSW, Vic, QLD, SA, WA

**b–e**

| State | Population | Population (rounded to leading digit) | Population (scientific notation) |
|---|---|---|---|
| NSW | 8 166 400 | 8 000 000 | $8 \times 10^6$ |
| Vic | 6 680 600 | 7 000 000 | $7 \times 10^6$ |
| QLD | 5 184 800 | 5 000 000 | $5 \times 10^6$ |
| SA | 1 770 600 | 2 000 000 | $2 \times 10^6$ |
| WA | 2 667 100 | 3 000 000 | $3 \times 10^6$ |
| Tas | 541 400 | 500 000 | $5 \times 10^5$ |
| NT | 246 500 | 200 000 | $2 \times 10^5$ |
| ACT | 431 300 | 400 000 | $4 \times 10^5$ |

**f** **i** NSW **ii** NT **iii** $8 \times 10^6$
**iv** $3 \times 10^6$ **v** $2.6 \times 10^7$

**g** $4.069 \times 10^5$ or 406 900. The difference is due to rounding.

**2 a**

| | Thy | Asha |
|---|---|---|
| **Round 1** | $\frac{2^{10}}{5^3}$ | $\frac{3^6}{5^5}$ |
| **Round 2** | $\frac{5^6}{2^3 \times 3}$ | $2 \times 3^5$ |
| **Round 3** | $\frac{3^4}{2^3}$ | $\frac{2^4 \times 5^3}{3^5}$ |

**b** Thy: $2^4 \times 3^3 \times 5^3$, Asha: $\frac{2^5 \times 3^6}{5^2}$

**c** Thy

**d** No. Asha had more factors, but the factors of 5 are divided (not multiplied) by the other factors, reducing the value. Thy's value was 54 000, whereas Asha's value was 933.12.

**e** Thy: $2^{-4} \times 3^{-3} \times 5^{-3} = \frac{1}{2^4 \times 3^3 \times 5^3}$,
Asha: $2^{-5} \times 3^{-6} \times 5^2 = \frac{5^2}{2^5 \times 3^6}$

**f** Thy: $2^2 \times 3 \times 5^4$, Asha: $2^9 \times 3$

**g** Thy: $2^{-6} \times 4^1 \times 6^{-4}$, Asha: $1^6 \times 5^3 \times 6^{-4}$

**h** Asha by two factors

# CHAPTER 3 Algebra

## EX p93 3A Simplifying

**1 a** $3x, -x, 20x$

**b** Like terms have exactly the same pronumerals raised to exactly the same powers. The coefficient can vary.

**c** 3, 7, −1, 2, 1, 20

**2** $\{2ba^2, 3aab, 6a^2b\}$, $\{3a, 6a\}$, $\{2b^2a, 6ab^2, 3abb\}$, $\{6a^3, aaa, 6a^2a\}$

**3 a** $10a$ **b** $-8k$ **c** $6x^2$
**d** $4cd - 9cde$ **e** $12x + 6y$ **f** $4a + 6b$
**g** $9m + 2p$ **h** $1 - k$ **i** $3xy + 5x^2$
**j** $2d - 4de^2$ **k** $4m^3 + 2$ **l** $abc + 4ab + ac$

**4 a** $10x + y$ **b** $5ab - 5b + b^2 + a$
**c** $2km + 4$ **d** $-3x^2 + 3x - 4$
**e** $2a + a^2 + a^3 - 3$ **f** $6m^2n + 4mn^2 - 2n^2$

**5 a** $70a^2$ **b** $-27b^3$ **c** $-3c^2d^2$
**d** $3p^2q^2r + 2pq^2r$ **e** $8e + 8f + 4$
**f** $7x^2y + \frac{3}{2}xy = 7x^2y + \frac{3xy}{2}$

**6 a** $6abcd$ **b** $-20mpxy$ **c** $9g^2h$
**d** $-24k^2mn$ **e** $56bjp^2t$ **f** $ax^2y^2$
**g** $18a^3bcd$ **h** $-20h^2k^2p$ **i** $-6b^3$
**j** $4km^2n^3$ **k** $15x^4y^2$ **l** $56a^4b^3c^2$

**7 a** $75a^{18}z^{36}$ **b** $-432b^{31}y^{16}$
**c** $-160c^8m^9x^9$ **d** $\frac{44d^7n^{10}}{w^7} = 44d^7n^{10}w^{-7}$
**e** $\frac{1}{45e^{11}p^{24}t^{19}}$ **f** $-\frac{168q^{33}}{g^{14}u^{22}} = 168g^{-14}q^{33}u^{-22}$

**8 a** $b$ **b** $\frac{mn}{p}$ **c** $4c$ **d** $-\frac{2}{x}$
**e** $-\frac{1}{2}$ **f** $\frac{y}{4}$ **g** $\frac{6b}{5c}$ **h** $\frac{2}{11}$
**i** $6a$ **j** $-3xy$ **l** $\frac{5n^2}{3}$ **m** $-\frac{ac}{4}$

**9 a** $\frac{4}{d^5}$ **b** $\frac{1}{c^{24}d^{15}}$ **c** $-\frac{125u^3r^4}{49}$
**d** $\frac{16p^{38}}{9k^{24}}$ **e** $2x^9y^6z^{27}$ **f** $20f$

**10 a** $p^3q$ **b** $-2d$ **c** $-\frac{3x}{c}$
**d** $\frac{k}{2p}$ **e** $\frac{7c}{bd^2}$ **f** $\frac{m}{27n}$
**g** $-\frac{3}{2xy^2z^3}$ **h** $-16a^5b^7c^9d$ **i** $-\frac{2k^5m^4n^3}{3}$

**11 a** true **b** false **c** false **d** true

**12 a** Set B is correct. Set A: all signs changed to + when expression rearranged. Set C: The sign in front of the $2a$ term has changed from + to − and 2 − 7 should be −5, not 9.

**b** Set A is correct. Set B: $-3 \times 4$ should be −12, not 12, and $b$ should be squared in the last step. Set C: $b \times b$ (or $b^2$) has been written as $b \times 2$.

**c** Set C is correct. Set A: The terms are not in their appropriate locations in the fraction in the last step. Set B: error in cancelling the '$a$' terms.

**13 a** $8x + y = 22$ **b** $-6xy = 36$ **c** $15xy = -90$
**d** $\frac{10}{y} = -5$ **e** $x^2y^3 = -72$ **f** $3x = 9$
**g** $6y - 2xy = 0$ **h** $x^2 + 4x - y = 23$
**i** $\frac{y^4}{x^3} = \frac{16}{27}$

**14 a** $2a + 3b + 2c = 11$ **b** $8ab - a = -18$
**c** $-2a^2b + ab^2 + 3ac = 40$ **d** $10a^2b^2c^2 = 1000$
**e** $3ab = -6$ **f** $\frac{4ac}{3} = \frac{40}{3} = 13\frac{1}{3}$

**15 a** $6x + 5y + 2$ **b** $4x + 2y - 2$ **c** $12x + 20$

**16 a** 51 cm **b** 24 cm **c** 68 cm

**17** **a** $20xy$ **b** $49x^2$ **c** $9xy$ **d** $2y^2$

**18** **a** 120 m$^2$ **b** 441 m$^2$

**c** 54 m$^2$ **d** 8 m$^2$

**19** **a** $2y$ **b** 10 m, 160 m$^2$

**20** **a** $3x$ **b** 96 cm$^2$, 16 cm

**21** **a** **i** $6k$ **ii** $2k^2$

**b** **i** 30 cm **ii** 50 cm$^2$

**22** **a** **i** $4x$ m **ii** $x^2$ m$^2$

**iii** $(4x + 8)$ m **iv** $(4x + 4)$ m$^2$

**b** **i** 36 m$^2$ **ii** 32 m

**iii** 40 m **iv** 64 m$^2$

**23** **a** $10xy$ **b** 200 cm$^2$ **c** $12x + 8y$

**d** 88 cm, 104 cm, 172 cm **e** 110 cm

**24** $((2x)^7 \times (4x)^{-3} \times (8x)^3 \div (16x)^4)^8 = 2^{128}x^{88}$

## EX 3B Expanding

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**1** **a** $4a + 12$ **b** $7b + 35$ **c** $3c - 6$

**d** $5d - 5$ **e** $24 + 6e$ **f** $-2f - 16$

**g** $-3g - 12$ **h** $-8h + 40$ **i** $-4x + 36$

**j** $-10 + 5j$ **k** $kp + 6k$ **l** $ab - 4a$

**m** $18m + 6k$ **n** $2np + 4nq$ **o** $x^2 - 7xy$

**2** **a** $4x + 6$ **b** $2x + 3$ **c** $3 - n$

**d** $7y + 5z$ **e** $-7b + 10c = 10c - 7b$

**f** $3t + 7$

**3** **a** $11x + 6$ **b** $5p + 6$ **c** $ab + 2a$

**d** $11y - 4$ **e** $4h + k + 2$ **f** $-6m$

**4** **a** $5x - 8$ **b** $13k - 4$ **c** $7p + 1$

**d** $x^2 + 4x + 12$ **e** $m^2 + 5m + 6$ **f** $y^2 - 7y + 10$

**5** **a** $12az + 15$ **b** $56b + 40by$ **c** $14c - 6cx$

**d** $-20dw - 25d$ **e** $-48ev + 54e$ **f** $-40et - 30eu$

**g** $8g^2 - 28gt$ **h** $-30hr + 42h^2$ **i** $12i^2jk + 15ij^2k$

**j** $12m^5 + 15m^8$ **k** $5n^4 - 3n^3$ **l** $p^6 - p^9q^4$

**6** **a** $ab + 4a + 3b + 1$ **b** $cd + 7c + 2d + 14$

**c** $mn + m + 5n + 5$ **d** $xy + 3x + 9y + 27$

**e** $kp - 2k + 6p - 12$ **f** $fg - f + 4g - 4$

**g** $ac + 3a - 5c - 15$ **h** $wf + 2w - 7f - 14$

**i** $xy - 8x - 4y + 32$ **j** $jk - 5j - 9k + 45$

**k** $2ab + 6a + 7b + 21$ **l** $15cd - 20c + 6d - 8$

**7** **a** $a^2 + 5a + 6$ **b** $x^2 + 15x + 50$

**c** $d^2 - 2d - 24$ **d** $y^2 - 5y - 24$

**e** $k^2 + 2k - 63$ **f** $m^2 - 3m - 18$

**g** $5e^2 - 22e + 8$ **h** $21a^2 - 31a + 8$

**i** $6y^2 - 13y + 5$

**8** **a** $5b + 20 - ab - 4a$ **b** $-x^2 - 3x + 18$

**c** $-8c^2 - 2c + 15$ **d** $44d^2 - 102de + 54e^2$

**e** $-42p^2 + 13pq + 40p^2$ **f** $4x^2 - 9x - 1$

**g** $x^8 + x^7 + x^4 + x^3$ **h** $-20x^4 + 13x^2 + 84$

**i** $2x^2 + 11x + 11$

**9** **a** **i** Area of large rectangle $= 7(3 + 2)$
$= 7 \times 5$
$= 35$

**ii** Area of large rectangle $= 7 \times 3 + 7 \times 2$
$= 21 + 14$
$= 35$

**b** **i** Area of large rectangle $= 5(x + 2)$

**ii** Area of large rectangle $= 5x + 5 \times 2$
$= 5x + 10$

**c** The area of the large rectangle is length × width or $a \times (b + c)$.
The total area of the two smaller rectangles is $a \times b + a \times c$.
As the two areas are equal, $a(b + c) = ab + ac$.

**10** **a** **i** Area of large rectangle $= (6 + 4)(3 + 5)$
$= 10 \times 8$
$= 80$

**ii** Area of large rectangle $= 6 \times 3 + 6 \times 5 + 4 \times 3 + 4 \times 5$
$= 18 + 30 + 12 + 20$
$= 80$

**b** **i** Area of large rectangle $= (k + 2)(m + 3)$

**ii** Area of large rectangle $= km + 3k + 2m + 2 \times 3$
$= km + 3k + 2m + 6$

**c** The area of the large rectangle is length × width or $(a + b) \times (c + d)$.
The total area of the four smaller rectangles is $a \times c + a \times d + b \times c + b \times d$.
As the two areas are equal,
$(a + b)(c + d) = ac + ad + bc + bd$.

**11** **a** $a^2 - 9$ **b** $b^2 - 4$ **c** $c^2 - 25$

**d** $9p^2 - 64$ **e** $h^2 - 1$ **f** $k^2 - m^2$

**12** The middle two terms always combine to give zero, so you are left with the first term squared minus the second term squared. The two factors are almost the same with one set of brackets containing a sum of two terms and the other containing a difference of the same two terms.

**13** **a** The result is the first term squared minus the second term squared.

**b** No, as you can multiply two factors in any order.

**c** **i** $x^2 - 4$ **ii** $y^2 - 81$ **iii** $m^2 - 36$

**iv** $12d^2 - 100$ **v** $m^2 - n^2$ **vi** $9 - x^2$

**14** **a** $(100 + 3)(100 - 3) = 100^2 - 3^2$

**b** $103 \times 97 = (100 + 3)(100 - 3) = 10\,000 - 9 = 9991$

**c** **i** 9996 **ii** 9975 **iii** 999 999

**iv** 999 964

**15** **a** $a^2 + 4a + 4$ **b** $b^2 + 14b + 49$

**c** $c^2 + 8c + 16$ **d** $9p^2 - 30p + 25$

**e** $h^2 - 2h + 1$ **f** $m^2 + 2mn + n^2$

**16** The result is always the first term squared then 2 times the product of both terms + second term squared. The two factors are exactly the same.

**17** **a** $(a + b)^2$ is a perfect square.

**b** **i** $x^2 + 6x + 9$ **ii** $y^2 + 12y + 36$

**iii** $m^2 + 4m + 4$ **iv** $16b^2 + 88b + 121$

**v** $m^2 + 2mn + n^2$ **vi** $25 + 10x + x^{22}$



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**c** $(a - b)^2$ is also a perfect square. The rule or pattern holds true. The negative sign in the factor $(a - b)$ means that the middle term is $-2ab$ rather than $2ab$.

**d** **i** $a^2 - 4a + 4$ **ii** $b^2 - 8b + 16$
**iii** $c^2 - 14c + 49$ **iv** $25w^2 - 60w + 36$
**vi** $k^2 - 2kp + p^2$ **vi** $9 - 6x + x^2$

**18 a** Some possible answers are: $2x + 5$, $9p - 7$, $6c^2 - 14$, and $ab + 4c$.

**b** A binomial product is where two terms are multiplied by two terms.

**c** three terms

**d** Some possible answers are: $x + 6y + 9$, $a - 2d + 4$ and $81 - 18z + y$.

**e** A quadratic trinomial is an expression with three terms, where the highest power of a variable is 2. For example: $p^2 - 10p + 25$, $b^2 + 22b + 121$ and $9 - 6xy + x^2$.

**19 a** $12a - 20b + 40$

**b** $6xw - 10x^2 + 14xy - 8xz)$

**c** $5x^2 + 35x + 60$

**d** $6ax + 10bx - 12x + 9ay + 15by - 18y$

**e** $2x^2 + 7xy + 15x + 3y^2 + 15y + 18$

**f** $x^3 + 8x^2 + 19x + 14$

**20** E

**21 a** $(8)(2x + 5)$ **b** $16x + 40$

**c** **i** 88 cm$^2$ **ii** 88 cm$^2$

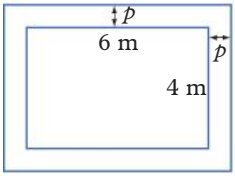
**d** The answers should be the same if you have expanded correctly.

**22 a** **i** $(x + 7)(x + 3)$ **ii** $x^2 + 10x + 21$
**iii** 96 m$^2$

**b** **i** $(x + 9)(x + 9)$ **ii** $x^2 + 18x + 81$
**iii** 196 mm$^2$

**c** **i** $(2x - 1)(x + 2)$ **ii** $2x^2 + 3x - 2$
**iii** 63 cm$^2$

**23 a**




**b** **i** $(2p + 6)$ m **ii** $(2p + 4)$ m

**c** $(2p + 6)(2p + 4) = 4p^2 + 20p + 24$

**d** $(4p^2 + 20p + 24) - (6 \times 4) = 4p^2 + 20p$

**e** **i** 24 m$^2$ **ii** 80 m$^2$ **iii** 56 m$^2$

**24 a** $100y^2 - 140y + 49$ **b** $x^7 - x^6$

**c** $x^4 - 25$ **d** $y^{12} + 6y^7 + 9y^2$

**e** $-5a^3$ **f** $x^6y^6 + 2wx^5y^2$

**25** $(a - b)^2 + (c - d)^2$
$= (a^2 - 2ab + b^2) + (c^2 - 2cd + d^2)$
$= (b^2 - 2ab + a^2) + (d^2 - 2cd + d^2)$
$= (b - a)^2 + (d - c)^2$

**26 a** Square: length = $y$, width = $y$
Rectangle: length = $y$, width = $x - y$

**b** Area of the original rectangle = $xy$
Total area of the two shapes = $(y)(x - y) + y^2$
$= xy - y^2 + y^2$
$= xy$

**27** $x = 5$

EX p107

## 3C Factorising using the HCF

**1 a** 4 **b** 2 **c** 5
**d** $d$ **e** 2 **f** 3
**g** 4 **h** $3h$ **i** $12x$

**2 a** $c$ **b** $2y$ **c** $9m$
**d** $bd$ **e** $4y$ **f** $2k$
**g** $p$ **h** 5 **i** $3p$

**3 a** $c(b + d)$ **b** $2y(x + 1)$ **c** $9m(2n - 1)$
**d** $bd(ac + f)$ **e** $4y(2x + 7)$ **f** $2k(3k - 5)$
**g** $p(1 + 11p)$ **h** $5(9ab - 8cd)$ **i** $3p(q + 2)$

**4 a** $a^4$ **b** $b^2$ **c** $2c^3$
**d** $6d^3z^5$ **e** $7e^6y^5$ **f** $2fx^9$
**g** $-2g$ **h** $6k^5$ **i** $-x^9y^7z^{13}$

**5 a** $5(2x + 1)$ **b** $3(y - 7)$ **c** $4(2k + 3)$
**d** $3(5 - 2d)$ **e** $7(4x + 3y)$ **f** $10(2n - 5)$
**g** $xy(x + 3y^3)$ **h** $mn^2(mn - 9)$

**6 a** $5b(4a - 1)$ **b** $8d(1 + ce)$ **c** $5x(3x + 2)$
**d** $2k(2k - 11)$ **e** $6n(5 - 3n)$ **f** $a(16a + 1)$
**g** $2h(h - 7)$ **h** $6p(1 + p)$

**7 a** $8m^4(3m^2 + 2)$ **b** $7q^3r^2(3q^2 + 5r^4)$
**c** $5t^4u^2(3 - t^4u^5)$ **d** $6c^4d^4(2c^5d + 1)$
**e** $7e^5f^3(e^6 - 1)$ **f** $3ij^2k(i^4k^6 + 9j^7)$
**g** $b^3c^7(a^7b^9 + c^2\mathrm{d}^4)$ **h** $pq(5m - 3np^9q^4)$
**i** $w^5z^3(u^4y^2 + v^4x^2)$

**8 a** $-5n(m + 2)$ **b** $-7x(2y + 1)$ **c** $-6c(1 - d)$
**d** $-a(a + 3)$ **e** $-2k(2k + 1)$ **f** $-8x(x - 1)$
**g** $-3(4 + xy)$ **h** $-2m(8 + 5m)$ **i** $-9xy(x - 2)$

**9 a** $(w + 4)(x + 2)$ **b** $(x - 1)(y + 7)$
**c** $(a + 6)(a - 3)$ **d** $(5 - n)(p + 8)$
**e** $(4 - k)(3k - 5)$ **f** $(3x - 4)(2x + 9)$
**g** $(2\mathrm{g} + 1)(4\mathrm{g} + 1)$ **h** $(8n - d)(2h - 1)$
**i** $(7x + 6y)(3\mathrm{a} + 2\mathrm{b})$

**10 a** $(a + 5)(b + 4)$ **b** $(y - 6)(x + 7)$
**c** $(n + 4)(m - 2)$ **d** $(y + 3)(y + 5)$
**e** $(k - 7)(k + 2)$ **f** $(x + 3)(6 + x)$
**g** $(a - 7)(a - 2)$ **h** $(p + 5)(p - 2)$
**i** $(3c - 1)(2c + 3)$

**11** Expanding means to write an expression without brackets. Factorising means to write an expression in factor form and generally involves using brackets. Factorising and expanding are opposite processes.

**12 a** **i** $6(x + 1)$ cm **ii** $2x(x + 4)$ cm
**b** **i** 36 cm **ii** 90 cm

**13 a** $(2x + 5)$ m **b** $(m + 15)$ cm

**14 a** $2(2x + 9)$ m **b** $2(2m + 15)$ cm

**15** **a** $3(9x - 3y + 5z)$ **b** $5(9p - 10q - 1)$

**c** $4(1 - 5i + 10j - 15k)$ **d** $a^2(a^3 + a + 1)$

**e** $6b^3c^4(3c - 6b + 4b^5c)$

**f** $7r^5t^7(12r^7t + 1 + 7r^3t^7 + 2r^4t)$

**16** **a** $\frac{1}{2}(5x + 3y)$ **b** $\frac{3}{5}(3g + h)$

**c** $\frac{5}{4}(2p + 3q)$ **d** $\frac{1}{12}(28m - 15n)$

**17** **a** $x(10 - x)$ mm$^2$

**b** Length and width are $x$ mm and $(10 - x)$ mm.

**c** Sample answers: 6 mm by 4 mm, 7 mm by 3 mm, 9 mm by 1 mm.

**18** **a** Some possible answers are:

Height = $(7 - x)$ cm, base length = $6x$ cm

Height = $2(7 - x)$ cm, base length = $3x$ cm

Height = 6 cm, base length = $x(7 - x)$ cm

**b** Some possible answers, using the first sample answer in **a**:

Height = 3 cm, base length = 24 cm ($x = 4$)

Height = 6 cm, base length = 6 cm ($x = 1$)

Height = 2 cm, base length = 30 cm ($x = 5$)

**19** Length = width = $3x$ cm, Height = $(x + 5)$ cm

**20** **a** $(x + 3)(x - 4)$ **b** $(x - 4)(x + 3)$

**c** They are equivalent.

**21** **a** $n + 1, n + 2$ **b** $3n + 3$

**c** $3(n + 1)$; three times middle number

**d** $n + (n + 2) + (n + 4) = 3n + 6 = 3(n + 2)$; also three times middle number

**22** For four consecutive numbers, the sum is four times the average of the two middle numbers.

For six consecutive numbers, the sum is six times the average of the two middle numbers.

For 10 consecutive numbers, the sum is 10 times the average of the two middle numbers.

$$10 \times \left[\frac{(n + 4) + (n + 5)}{2}\right] = 10 \times \left(\frac{2n + 9}{2}\right) = 5(2n + 9)$$

**23** **a** $(y + 5)(x - 3)$ **b** $(2q - 3)(p - 1)$

**c** $(a - 3)(2a - 3)$

**24** Area of square = $l \times w = 2r \times 2r = 4r^2$

Area of circle = $\pi r^2$

Area of shaded section = $4r^2 - \pi r^2 = r^2(4 - \pi)$

**25** **a** $5\left(\frac{9x}{5} + \frac{17}{5}\right)$

**b** $12\left(x^2 + \frac{x}{4} + \frac{1}{2}\right)$

**c** $x\left(3x + 5y + \frac{y^2}{x}\right)$

**d** $(x + 2)\left(7(x + 2) + 5 + \frac{9}{x + 2}\right)$

## Checkpoint

**1** **a** $4a - 2b - 5$ **b** $4t^2 + 12t - 4$

**c** $2x^3 - 7x^2 + 5x + 2$ **d** $4cd^2 - 2d^2 + 2c - 8d^2$

**2** **a** $-35ab$ **b** $48a^2bc^2$

**c** $\frac{9h}{4}$ **d** $\frac{t}{3u}$

**3** **a** $-800a^{16}b^{32}$ **b** $-\frac{729p^6}{8k^3}$

**c** $\frac{9t^4}{8r^7}$ **d** $\frac{2x^{28}}{3y^4}$

**4** **a** $8a + 24$ **b** $-8b + 20$

**c** $21c - 3cy$ **d** $18d^2 - 9d$

**5** **a** $3w + 41$ **b** $-x + 35$

**c** $15y - 9$ **d** $z^2 + 5x - 15$

**6** **a** $pv + 2v + 5p + 10$ **b** $12 - 3q + 4u + 12$

**c** $r^2 + 4r - 12$ **d** $6t^2 - 29t + 35$

**7** **a** $a^7 + 2a^5 + 5a^4 + 10a^2$ **b** $b^3 + 7b^2 - 28b + 1$

**c** $c^2 - c - 10$ **d** $7d^2 - 8d + 19$

**8** **a** $3(2a + 3)$ **b** $12(b - 3)$

**c** $-7(4c + 1)$ **d** $4d(2d + 9)$

**9** **a** $(3g + 2)(2m + 5)$ **b** $(h - 3)(8 - n)$

**c** $(q + 7r)(p - 1)$ **d** $(x + 3)(2x - 5)$

**10** **a** $(b + 5)(a + 4)$ **b** $(3d - 2)(4c - 7)$

**c** $(3f - 1)(5e + 1)$ **d** $(x + 6)(x + 5)$

EX p113

## 3D Factorising the difference of two squares

**1** **a** $9x^2 - 49$ **b** $9x^2 - 49$

**c** Show each term as a square ($(3x)^2$ and $7^2$) and then write the base of each term in two pairs of brackets. One pair of brackets will contain the sum of the two bases and the other the difference.

**2** **a** $(x + 6)(x - 6)$ **b** $(a + 10)(a - 10)$

**c** $(3 + y)(3 - y)$

**3** **a** $(5m + 2)(5m - 2)$ **b** $(7 + 8c)(7 - 8c)$

**c** $(ab + 3)(ab - 3)$ **d** $(xy + w)(xy - w)$

**e** $(p + qr)(p - qr)$ **f** $(bn + dm)(bn - dm)$

**g** $(5jk + 6mn)(5jk - 6mn)$

**h** $(9ab + 7cd)(9ab - 7cd)$

**i** $(12pq + 11rt)\ (12pq - 11rt)$

**4** **a** $3(x + 2)(x - 2)$ **b** $2(3p + 5)(3p - 5)$

**c** $5(10 + 7g)(10 - 7g)$ **d** $7(1 + 2u)(1 - 2u)$

**e** $4(6r + 5)(6r - 5)$ **f** $100(8 + 11z)(8 - 11z)$

**g** $6(x + y)(x - y)$ **h** $p(q + r)(q - r)$

**i** $u^2(t + d)(t - d)$ **j** $4d(3k + 2j)(3k - 2j)$

**k** $2a^2(9b + 11d)(9b - 11d)$ **l** $9x(1 + x)(1 - x)$

**5** **a** $(k + 1)(k - 7)$ **b** $(8 - m)(6 - m)$

**c** $(y + 5)(1 - y)$ **d** $-a(a + 4)$

**e** $(y + 5 + x)(y + 5 - x)$ **f** $(u + t - 9)(u - t + 9)$

**g** $(3p + 4)(p + 4)$ **h** $(9 - 3m)(9 - 5m)$

**i** $3(2x + 3)$ **j** $(p + r + 1)(p - r + 11)$

**k** $(10x - 9)(4x - 1)$ **l** $4z$

**6** **a** $(6 + x)(6 - x)$ **b** $(b + 4)(b - 4)$

**c** $(3h + 5)(3h - 5)$ **d** $(8n + 7d)(8n - 7d)$

**e** $(1 + rp)(1 - rp)$ **f** $(tu + wx)(tu - wx)$

**7** **a** $\left(t+\frac{2}{3}\right)\left(t-\frac{2}{3}\right)$ **b** $\left(r+\frac{5}{7}\right)\left(r-\frac{5}{7}\right)$

**c** $\left(\frac{9}{4}+u\right)\left(\frac{9}{4}-u\right)$

**d** $\left(\frac{3}{10}d+\frac{11}{12}\right)\left(\frac{3}{10}d-\frac{11}{12}\right)$

**e** $\left(\frac{1}{8}k+\frac{1}{5}\right)\left(\frac{1}{8}k-\frac{1}{5}\right)$ **f** $\left(\frac{w}{13}+\frac{x}{14}\right)\left(\frac{w}{13}-\frac{x}{14}\right)$

**8** **a** $(z+0.4)(z-0.4)$ **b** $(y+1.1)(y-1.1)$

**c** $(0.2+x)(0.2-x)$ **d** $(c+0.05)(c-0.05)$

**e** $(0.7g+0.1h)(0.7g-0.1h)$

**f** $(0.12+0.11k)(0.12-0.11k)$

**9** **a** $(x^3+3)(x^3-3)$ **b** $(6+y^5)(6-y^5)$

**c** $(z^{11}+7)(z^{11}-7)$ **d** $(2p^7-5)(2p^7-5)$ EX p120

**e** $\left(\frac{q^9}{4}+\frac{9}{11}\right)\left(\frac{q^9}{4}-\frac{9}{11}\right)$ **f** $(1+r^{15}t^{25})(1-r^{15}t^{25})$

**10** **a** **i** $18\times10$ **ii** $25\times-5$ **iii** $197\times1$

**iv** $12\times-4$ **v** $16\times10$ **vi** $29\times1$

**b** **i** 180 **ii** −125 **iii** 197

**iv** −48 **v** 160 **vi** 29

**c** **i** $64-9$ **ii** $144-324$ **iii** $900-1$

**iv** $81-9$ **v** $900-100$ **vi** $196-1$

**d** **i** 55 **ii** −180 **iii** 899

**iv** 72 **v** −800 **vi** 195

**11** **a** 2600 **b** 11.4 **c** $\frac{1}{25}$

**12** **a** $2n+1$ **b** odd number

**13** $4(x+1)$

**14** **a** $(x^2+4)(x+2)(x-2)$

**b** $(x^4+y^4)\ (x^2+y^2)\ (x+y)(x-y)$

**15** **a** $\pi R^2-\pi r^2$ **b** $\pi(R+r)(R-r)$

**c** **i** 50.27 cm² **ii** 100.53 cm² **iii** 59.69 cm²

**16** 5 cm and 7 cm

**17** **a** $(5^2\pi-4.75^2\pi+3.75^2\pi-3.5^2\pi+2.5^2\pi-2.25^2\pi+1.25^2\pi-1^2\pi)$ m²

**b** $(5^2\pi-4.75^2\pi+3.75^2\pi-3.5^2\pi+2.5^2\pi-2.25^2\pi+1.25^2\pi-1^2\pi)$

$=\pi((5^2-4.75^2)+(3.75^2-3.5^2)+(2.5^2-2.25^2)+(1.25^2-1^2))$

$=\pi((5+4.75)(5-4.75)+(3.75+3.5)(3.75-3.5)+(2.5+2.25)(2.5-2.25)+(1.25+1)(1.25-1))$

$=\pi(0.25(5+4.75)+0.25(3.75+3.5)+0.25(2.5+2.25)+0.25(1.25+1))$

$=0.25\pi(5+4.75+3.75+3.5+2.5+2.25+1.25+1)$m²

**18** **a** 9 m and 1 m **b** 7 m and 3 m

**c** 10 m and 3 m **d** 10.5 m and 4.5 m

**19** 9 cm²

**20** **a** $(\sqrt{a}+\sqrt{5})(\sqrt{a}-\sqrt{5})$ **b** $(b+\sqrt{7})(b-\sqrt{7})$

**c** $(\sqrt{c}+2\sqrt{2})(\sqrt{c}-2\sqrt{2})$ **d** $(\sqrt{3d}+4)(\sqrt{3d}-4)$

**e** $(2\sqrt{e}+3\sqrt{5})(2\sqrt{e}-3\sqrt{5})$

**f** $(f\sqrt{2}+5\sqrt{3})(f\sqrt{2}-5\sqrt{3})$

**g** $(x\sqrt{x}+y^2\sqrt{y})(x\sqrt{x}-y^2\sqrt{y})$

**h** $4(x\sqrt{2}+\sqrt{3})(x\sqrt{2}-\sqrt{3})$

**21** $(x+1)(x+y)^2-(x+1)(x-y)^2$

$=(x+1)[(x+y)^2-(x-y)^2]$

$=(x+1)[(x+y)+(x-y)][(x+y)-(x-y)]$

$=(x+1)(x+y+x-y)(x+y-x+y)$

$=(x+1)(2x)(2y)$

$=4xy(x+1)$

**22** **a** $x^3+y^3$ **b** $(x+2)(x^2-2x+4)$

**c** $(x-2)(x^2+2x+4)$

You can use the previous result, replacing 2 with −2.

**23** 31

## 3E Factorising quadratic expressions

**1** **a** 1 and 4 **b** 2 and 4 **c** 2 and 11

**d** 4 and 5 **e** 4 and 6 **f** 3 and 4

**g** 6 and 7 **h** 5 and 7 **i** 4 and 4

**2** **a** −2 and 4 **b** −3 and 2 **c** −6 and −2

**d** −2 and 5 **e** −9 and 1 **f** −3 and −2

**g** −1 and 6 **h** −9 and 3 **i** −11 and −1

**3** **a** $(x+1)(x+4)$ **b** $(x+2)(x+4)$

**c** $(x+2)(x+11)$ **d** $(x+4)(x+5)$

**e** $(x+4)(x+6)$ **f** $(x+3)(x+4)$

**g** $(x+6)(x+7)$ **h** $(x+5)(x+7)$

**i** $(x+4)^2$

**4** **a** $(a+3)(a+1)$ **b** $(b+2)(b+7)$

**c** $(c+1)(c+6)$ **d** $(d+3)(d+7)$

**e** $(e+1)(e+7)$ **f** $(f+3)(f+5)$

**g** $(g+4)(g+7)$ **h** $(h+4)(h+9)$

**i** $(x+3)(x+6)$ **j** $(j+5)(j+9)$

**k** $(k+5)(k+6)$ **l** $(y+5)(y+8)$

**5** **a** $(x-2)(x+4)$ **b** $(x-3)(x+2)$

**c** $(x-6)(x-2)$ **d** $(x-2)(x+5)$

**e** $(x-9)(x+1)$ **f** $(x-3)(x-2)$

**g** $(x-1)(x+6)$ **h** $(x-9)(x+3)$

**i** $(x-11)(x-1)$

**6** **a** $(a-1)(a+3)$ **b** $(b-5)(b+3)$

**c** $(c-4)(c-1)$ **d** $(d-2)(d+7)$

**e** $(e-6)(e-4)$ **f** $(f-5)(f+2)$

**g** $(g-3)(g+4)$ **h** $(h-5)(h-3)$

**i** $(x-8)(x+3)$ **j** $(j-8)(j-2)$

**k** $(k-3)(k+6)$ **l** $(y-2)(y+1)$

**7** **a** $3(x+1)(x+2)$ **b** $2(x+3)(x+5)$

**c** $5(x-1)(x+4)$ **d** $-4(x+2)(x+3)$

**e** $-6(x-5)(x-1)$ **f** $-(x-5)(x+7)$

**8** **a** $a(x+2)(x+6)$ **b** $b(x-7)(x-4)$

**c** $10c(x-6)(x+5)$ **d** $-2d(x-5)(x-12)$

**e** $p^2(x+3)(x+10)$ **f** $x^2(q+11)(q+4)$

**9** **a** $(x-3)^2$ **b** $(x-3)(x+3)$

**c** $(x+0)(x-6)=x(x-6)$

**10 a** $(x-5)(x+5)$ **b** $(x-5)^2$

**c** $x(x-25)$ **d** $(x+7)^2$

**e** $x(x-49)$ **f** $(x-7)(x+7)$

**11 a** $(x+3)(x+7)$ **b** $(x-5)(x+7)$

**c** $(x-6)(x+3)$ **d** $-(x+6)(x-8)$

**e** $-(x+4)(x+6)$ **f** $-(x-2)(x-8)$

**g** $(x+8)(x+9)$ **h** $(x-7)(x+2)$

**i** $-(x-4)(x-9)$

**12 a** $x^2 + 6x + 0 = (x+0)(x+6)$

$x^2 + 6x + 5 = (x+1)(x+5)$

$x^2 + 6x + 8 = (x+2)(x+4)$

$x^2 + 6x + 9 = (x+3)(x+3)$

**b** $x^2 + 6x - 7 = (x-1)(x+7)$

$x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 + 6x - 27 = (x-3)(x+9)$

$x^2 + 6x - 40 = (x-4)(x+10)$

**c** When the coefficient of $x$ is positive, both factors are positive when the constant is positive. When the coefficient of $x$ is positive, one factor is positive and one is negative when the constant is negative.

**13 a** $x^2 - 6x - 7 = (x-1)(x-7)$

$x^2 - 6x - 16 = (x-2)(x-8)$

$x^2 - 6x - 27 = (x-3)(x-9)$

$x^2 - 6x - 40 = (x-4)(x-10)$

**b** $x^2 - 6x + 0 = (x-0)(x-6)$

$x^2 - 6x + 5 = (x-1)(x-5)$

$x^2 - 6x + 8 = (x-2)(x-4)$

$x^2 - 6x + 9 = (x-3)(x-3)$

**c** When the coefficient of $x$ is negative, both factors are negative when the constant is positive. When the coefficient of $x$ is negative, one factor is positive and one is negative when the constant is negative.

**14 a** $x^2 + 65x + 64 = (x+1)(x+64)$

$x^2 + 34x + 64 = (x+2)(x+32)$

$x^2 + 20x + 64 = (x+4)(x+16)$

$x^2 + 16x + 64 = (x+8)(x+8)$

**b** $x^2 - 65x + 64 = (x-1)(x-64)$

$x^2 - 34x + 64 = (x-2)(x-32)$

$x^2 - 20x + 64 = (x-4)(x-16)$

$x^2 - 16x + 64 = (x-8)(x-8)$

**c i** If the coefficient of $x$ is positive, use positive factors when the constant is positive. If the coefficient of $x$ is negative, use negative factors when the constant is positive.

**ii** sum

**15 a** $x^2 - 63x - 64 = (x+1)(x-64)$

$x^2 - 30x - 64 = (x+2)(x-32)$

$x^2 - 12x - 64 = (x+4)(x-16)$

$x^2 - 0x - 64 = (x+8)(x-8)$

**b** $x^2 + 63x - 64 = (x-1)(x+64)$

$x^2 + 30x - 64 = (x-2)(x+32)$

$x^2 + 12x - 64 = (x-4)(x+16)$

$x^2 + 0x - 64 = (x-8)(x+8)$

**c i** If the coefficient of $x$ is positive, the larger factor is positive and the smaller factor is negative when the constant is negative. If the coefficient of $x$ is negative, the larger factor is negative and the smaller factor is positive when the constant is negative.

**ii** difference

**16 a** $(x+4)(x+4) = (x+4)^2$

**b** $(y-5)(y-5) = (y-5)^2$

**c** $(v+1)(v+1) = (v+1)^2$

**d** $(2z-3)(2z-3) = (2z-3)^2$

**e** $(3w+7)(3w+7) = (3w+7)^2$

**f** $3(h-2)(h-2) = 3(h-2)^2$

**g** $-(k+9)(k+9) = -(k+9)^2$

**h** $(f+g)(f+g) = (f+g)^2$

**i** $(5p-4q)(5p-4q) = (5p-4q)^2$

**17 a** Check for and factorise out any common factors. Halve the coefficient of the linear term and check if it is equal to the product of the square root of the squared terms (or squared term and constant). Both squared terms must be positive.

**b i** both squared terms are not positive

**ii** both squared terms are not positive

**iii** half of the linear term is not the product of the square root of the squared terms

**iv** half of the linear term is not the product of the square root of the squared terms

**18 a** $(x+6)$ **b** $(x+3)(x+6)$

**c i** Length = 8 m, width = 5 m, area = $8 \times 5 = 40$ m$^2$

**ii** Area = $2^2 + 9 \times 2 + 18 = 40$ m$^2$

**19 a i** $(x+9)$ cm **ii** $(3y+4)$ m

**b** 10 **c** 7

**d i** $x = 2$ **ii** $y = \frac{4}{3}$

**e** While one of the lengths is positive, the other length and area are both negative, which does not make sense for a physical rectangle. That is, the values for the length, width, and area for both items must all be positive.

**20 a i** $(x-2)(x-3)$ **ii** $(x-2)(x-5)$

**iii** $(x-2)(x-2)$ **iv** $(x-2)(x-4)$

**b** 2 m

**c** $x^2 - x - 6 = (x-2)(x+1)$ and $x^2 + x - 6 = (x-2)(x+3)$ both are wider than their backyards' width, $x$ m.

**d** $x^2 - 6x + 8$

**e** 8 m$^2$

**f** 19.25 m$^2$

**g** Melissa: 8 m$^2$, Lena: 14 m$^2$

**21** **a** $(3x-2)(3x+7)$ **b** $(11x+10)(11x-8)$
**c** $(5x-4)(5x-8)$ **d** $(x+6)(x+1)$
**e** $(x-10)(x-11)$ **f** $x(x-4)$
**g** $3(x+2)(3x-7)$ **h** $2(x+5)(2x-3)$
**i** $16(2x-1)(2x+3)$

**22** **a** $(x-2)(x+2)(x-3)(x+3)$
**b** $(x^2+4)(x-4)(x+4)$
**c** $(x^2+25)(x-1)(x+1)$

**23** **a** $(x+4)(x+1)$ **b** $(x+4)(x+2)$
**c** $(x+11)(x+2)$ **d** $(x+5)(x-2)$
**e** $(x-9)(x+1)$ **f** $(x-3)(x-2)$

## Chapter review

### Multiple-choice

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1** | E | **2** | B | **3** | A | **4** | B |
| **5** | B | **6** | E | **7** | A | **8** | B |
| **9** | A | **10** | A | **11** | C | **12** | D |
| **13** | E | **14** | C | **15** | A | **16** | C |

### Short answer

**1** **a** $16t$ **b** $13a-18p$ **c** $-2k+km-15$
**d** $13m^2n-5m^2+11n^2-4mn^2$

**2** **a** $44x^2y^2z$ **b** $72m^4n^3p^2$ **c** $\frac{5e}{6f}$ **d** $-\frac{n}{2m}$

**3** **a** $4x-28$
**b** $-40+24y$
**c** $35x-30wx$
**d** $ut-4u+3t-12$
**e** $40r-15rv+48-18v$
**f** $63mp-27np+77mq-33nq$

**4** **a** $8a-11$ **b** $b^2-9b-22$
**c** $12c^2-23c+10$ **d** $d^2-w^2$
**e** $36+12e+e^2$ **f** $f^2-18f+81$

**5** **a** $x^7-x^6$ **b** $y^{12}z^4-y^3z^6$
**c** $x^{11}+x^5y^4-x^6y^3+y^7$

**6** **a** $4(a-6)$ **b** $36pq(q+4)$
**c** $(8-d)(7d-4)$ **d** $(1+3f)(5e+2)$

**7** **a** $z^4(6z-5)$ **b** $3d(3dy+5x^2)$
**c** $-12r^7t^8u^3(3r^3u^3+8)$
**d** $a^4b^2c^7d^5(ad^3+a^2b^3c^2+cd^2)$

**8** **a** $8x(x^2+2)(3x-1)$ **b** $(r+q)(t+p+2)$
**c** $2(x+4)(2x-3)$

**9** **a** $(a+8)(a-8)$ **b** $(11+b)(11-b)$
**c** $(6m+7n)(6m-7n)$ **d** $(p+3)(p-1)$
**e** $2(e+4)(e-4)$ **f** $9(2f-1)$

**10** **a** $(y^5-z^5)(y^5+z^5)$ **b** $-8(q+2)$
**c** $(10+r)(10-r)$ **d** $(9j-8)(9j+8)$

**11** **a** $\left(\frac{14}{5}x+\frac{13}{2}y\right)\left(\frac{14}{5}x-\frac{13}{2}y\right)$ **b** $\left(\frac{12}{c}+\frac{15}{d}\right)\left(\frac{12}{c}-\frac{15}{d}\right)$
**c** $\left(\frac{6}{7}+\frac{u}{4}\right)\left(\frac{6}{7}-\frac{u}{4}\right)$
**d** $(0.2v-1.1)(0.2v+1.1)$

**12** **a** $(a+5)(a+1)$ **b** $(b-4)(b-3)$
**c** $(c-3)(c+7)$ **d** $(d-18)(d+2)$

**13** **a** $(x+3)(x+5)$ **b** $-5(x-6)(x+2)$
**c** $-2(x-10)(x-6)$ **d** $-3(x+10)(x-8)$

**14** **a** $2(7-x)(7+x)$ **b** $16(y^2+4x^2)$
**c** $(x-5)^2$ **d** $9x(x-4)$

### Analysis

**1** **a** **i** $(x+5)(x+11)$ **ii** $(x-6)(x-14)$
**iii** $(x+6)(x+16)$
**b** The numbers in the brackets in factor form are $a+b$ and $a-b$.
**c** **i** $x^2+16x+55$ **ii** $x^2-20x+84$
**iii** $x^2+22x+96$
**d** The coefficient of $x$ in the expanded form is $2a$.
**e** The constant in the expanded form is $a^2-b^2$.
**f** **i** $(x+3)^2-4=x^2+6x+5=(x+1)(x+5)$
**ii** $(x-5)^2-16=x^2-10x+9=(x-9)(x-1)$
**iii** $(x+7)^2-100=x^2+14x-51=(x-3)(x+17)$

**2** **a** $\frac{1}{2}n(n+1)=\frac{1}{2}n^2+\frac{1}{2}n$, $\frac{(n+1)^2-(n+1)}{2}=\frac{1}{2}n^2+\frac{1}{2}n$
**b** **i** 15 **ii** 15
**c** 5050 **d** $n(n+1)$ **e** 10 100
**f** The sum of the first $n$ even numbers is double the sum of the first $n$ counting numbers.
**g** $\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n=\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$
**h** **i** 55 **ii** 55
**i** 338 350

# CHAPTER 4 Linear relationships

## EX p132 4A Solving linear equations

**1** **a** $x=6$ **b** $x=5$ **c** $x=36$ **d** $x=10$
**e** $x=-20$ **f** $x=4$ **g** $x=15$ **h** $x=-1$
**i** $x=-3$ **j** $x=-3$ **k** $x=12$ **l** $x=-6$

**2** **a** $x=2$ **b** $x=-8$ **c** $x=9$ **d** $x=5$
**e** $x=20$ **f** $x=-8$ **g** $x=0$ **h** $x=-4$
**i** $x=-5$ **j** $x=10$ **k** $x=-20$ **l** $x=4$

**3** **a** $x=6.9$ **b** $x=-2.6$ **c** $x=10.8$ **d** $x=-3.16$

**4** **a** $x=6$ **b** Yes, $x=6$ **c** $x=5.6$ **d** $x=5.6$
**e** Expanding brackets first means you do not have to work with fractions or decimals until the last step.

**5** **a** $x=3$ **b** $x=-9$ **c** $x=\frac{11}{2}$
**d** $x=-\frac{12}{5}$ **e** $x=4$ **f** $x=\frac{11}{6}$

**6** **a** $x=4$ **b** multiply both sides by $x$
**c** $x=4$

**7** **a** $x=5$ **b** $x=-3$ **c** $x=-6$ **d** $x=4$
**e** $x=\frac{11}{2}$ **f** $x=-\frac{8}{5}$ **g** $x=4.8$ **h** $x=-\frac{1}{3}$

**8** **a** $x=2$ **b** $x=7$ **c** $x=-1$ **d** $x=-3$
**e** $x=-2$ **f** $x=7$ **g** $x=4$ **h** $x=-4$

**9** **a** $x = -1$ **b** $x = 3$ **c** $x = 7$ **d** $x = -3$
**e** $x = 7$ **f** $x = -3$ **g** $x = -6$ **h** $x = 9$

**10** **a** $2x + 5 = x - 4$ **b** $x = -9$

**11** **a** $x = 5$ **b** $x = -3$ **c** $x = 2$ **d** $x = -3$

**12** **a** $\frac{3(5x-1)}{6} = \frac{2(x+5)}{6}$ **b** $x = 1$

**13** **a** $x = 6$ **b** $x = -4$ **c** $x = 7$ **d** $x = -27$

**14** **a** Let $n$ = number of jellybeans
**b** $4n + 2 = 34$
**c** $n = 8$
**d** Each person receives 8 jellybeans.

**15** **a** Let $x$ = number of months
**b** $70x + 115 = 395$
**c** $x = 4$ **d** 4 months

**16** **a** \$185 **b** 14 goals
**c** 31 m × 19 m **d** 84 people

**17** **a** B **b** \$3.50

**18** **a** Let $p$ be the cost of a box of popcorn.
$5p + 9 = 3p + 31.5$
**b** \$11.25

**19** 56 sausages

**20** 100°

**21** Numbers are 5, 6 and 7.

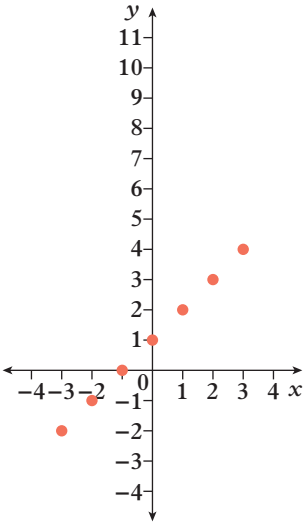
**22** **a** **i** always **ii** never **iii** sometimes
**iv** never **v** always **vi** never
**b** After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are equal. Alternatively, the equation reduces to 0 = 0.
**c** After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are not equal. Alternatively, the equation reduces to an equation that is not equal, such as 0 = 2.

**23** **a** $x = \frac{11}{5} = 2.2$ **b** $x = \frac{19}{13}$ **c** $x = \frac{13}{7}$

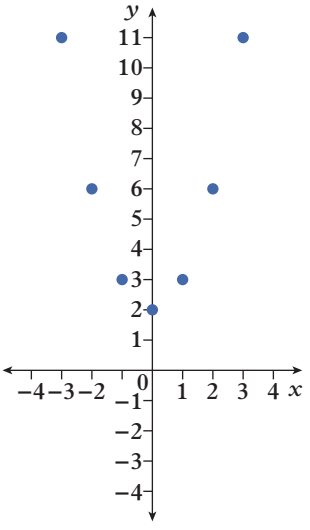
EX p138

## 4B Plotting linear relationships

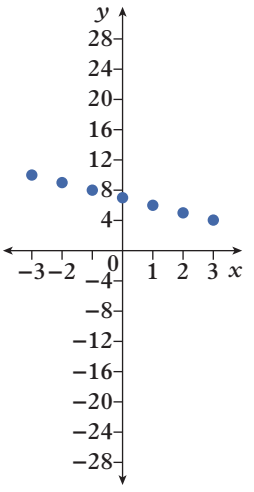
**1** **a** **i**



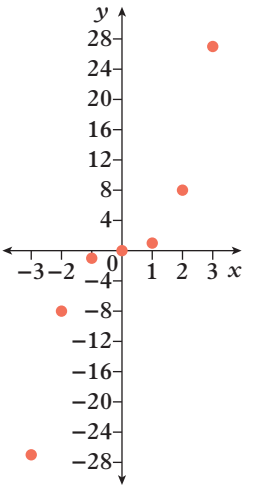

**ii**




**b** **i**




**ii**




**b** **i** linear **ii** non-linear **iii** linear
**iv** non-linear

**2** **a** $y = x + 2$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |

(−3, −1), (−2, 0), (−1, −1), (0, 2), (1, 3), (2, 4), (3, 5)

**b** $y = x - 4$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −7 | −6 | −5 | −4 | −3 | −2 | −1 |

(−3, −7), (−2, −6), (−1, −5), (0, −4), (1, −3), (2, −2), (3, −1)

**c** $y = 3 - x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

(−3, 6), (−2, 5), (−1, 4), (0, 3), (1, 2), (2, 1), (3, 0)

**d** $y = 2 - x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 5 | 4 | 3 | 2 | 1 | 0 | −1 |

(−3, 5), (−2, 4), (−1, 3), (0, 2), (1, 1), (2, 0), (3, −1)

**e** $y = -x - 3$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | −1 | −2 | −3 | −4 | −5 | −6 |

(−3, 0), (−2, −1), (−1, −2), (0, −3), (1, −4), (2, −5), (3, −6)

**f** $y = 4x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −12 | −8 | −4 | 0 | 4 | 8 | 12 |

(−3, -12), (−2, −8), (−1, −4), (0, 0), (1, 4), (2, 8), (3, 12)

**g** $y = 2x + 1$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −5 | −3 | −1 | 1 | 3 | 5 | 7 |

(−3, −5), (−2, −3), (−1, −1), (0, 1), (1, 3), (2, 5), (3, 7)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**h** $y = 3x - 2$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −11 | −8 | −5 | −2 | 1 | 4 | 7 |

(−3, −11), (−2, −8), (−1, −5), (0, −2), (1, 1), (2, 4), (3, 7)

**i** $y = 4 - 2x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 10 | 8 | 6 | 4 | 2 | 0 | −2 |

(−3, 10), (−2, 8), (−1, 6), (0, 4), (1, 2), (2, 0), (3, −2)

**3 a** IV: radius, DV: area – because the formula for the area of a circle depends on the radius.

**b** IV: Number sold
DV: Revenue because it depends on the number of items sold.

**c** IV: Distance
DV: Light intensity because it depends on the distance from the source.

**d** IV: Number of words
DV: Number of pages because it depends on the number of words.

**e** IV: Amount of breath
DV: Volume because the volume of the note depends on the amount of breath used to play a note.

**f** IV: Number of cards
DV: Chance because the chance of drawing an ace of spades depends on the number of cards remaining.

**4 a i**

| $x$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
|---|---|---|---|---|---|---|---|
| $y$ | 24 | 21 | 18 | 15 | 12 | 9 | 6 |

**ii**

| $x$ | −12 | −6 | −2 | 2 | 6 | 12 |
|---|---|---|---|---|---|---|
| $y$ | −2 | −4 | −12 | 12 | 4 | 2 |

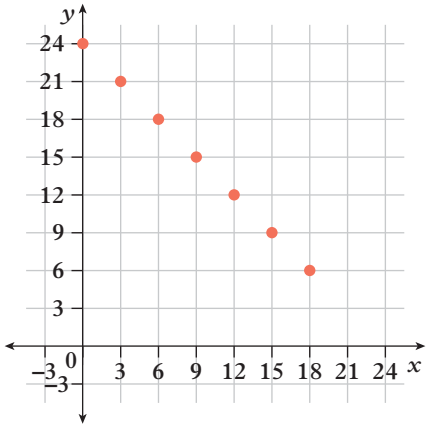
**iii**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 3 | 0 | −1 | 0 | 3 | 8 | 15 |

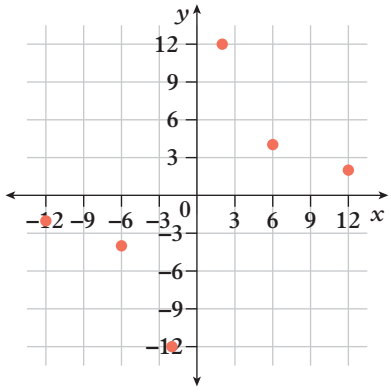
**iv**

| $x$ | −14 | −10 | −6 | −2 | 2 | 6 | 10 |
|---|---|---|---|---|---|---|---|
| $y$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |

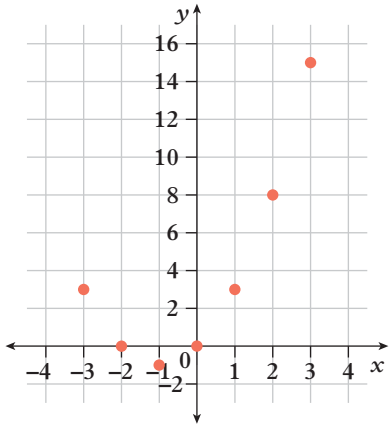
**b i** linear



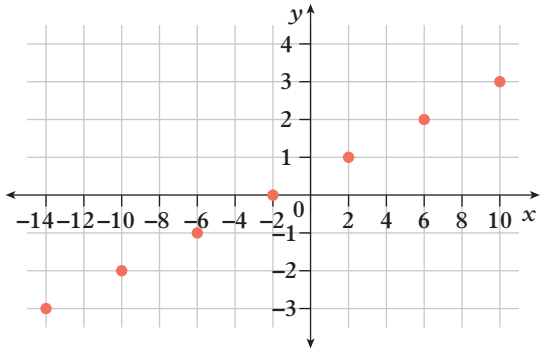

**ii** non-linear




**iii** non- linear




**iv** linear




**5 a i** $y = x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |

**ii** $y = 2x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −6 | −4 | −2 | 0 | 2 | 4 | 6 |

**iii** $y = 3x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −9 | −6 | −3 | 0 | 3 | 6 | 9 |

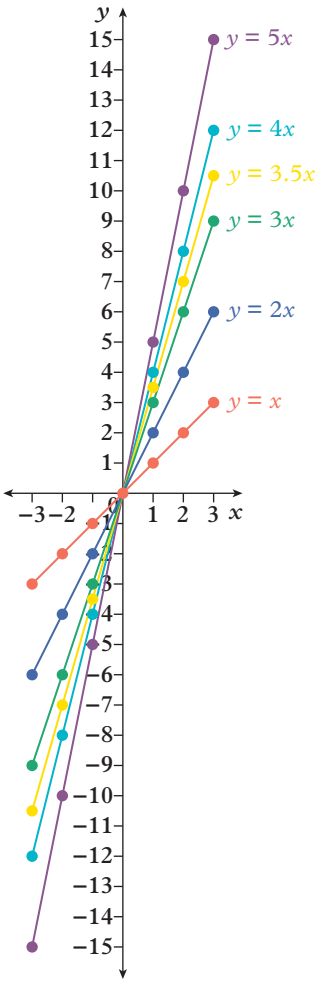
**iv** $y = 4x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −12 | −8 | −4 | 0 | 4 | 8 | 12 |

**b** All four graphs pass through the origin. However, the steepness of each line is different.

**c** As the coefficient of $x$ increases, the steepness of the corresponding graph increases.

**a, d**




**6 a i** $y = -x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 3 | 2 | 1 | 0 | −1 | −2 | −3 |

**ii** $y = -2x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 6 | 4 | 2 | 0 | −2 | −4 | −6 |

**iii** $y = -3x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 9 | 6 | 3 | 0 | −3 | −6 | −9 |

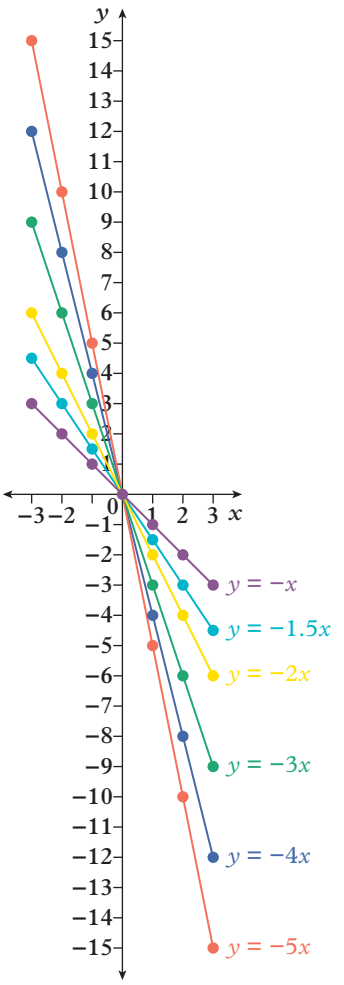
**iv** $y = -4x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 12 | 8 | 4 | 0 | −4 | −8 | −12 |

**b** The values of $y$ decrease from left to right.

**c** As the coefficient of $x$ decreases, the steepness of the corresponding graph increases.

**a, d**




**7 a i** $y = x$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |

**ii** $y = x + 1$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |

**iii** $y = x + 2$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |

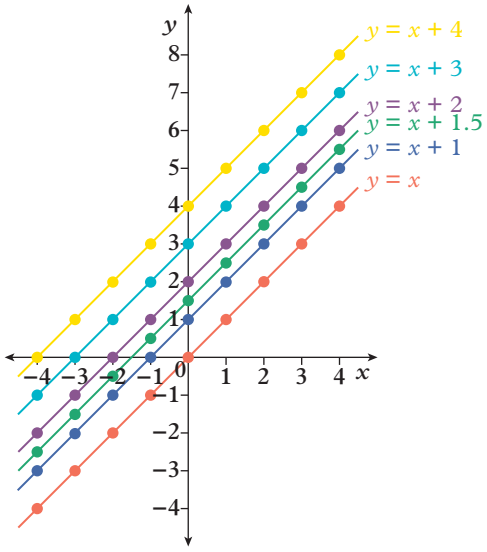
**iv** $y = x + 3$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

**b** All four graphs have the same steepness, but pass through different points on the $x$- and $y$-axes.

**c** As the value added to $x$ increases, the graph moves upwards.

**a, d**

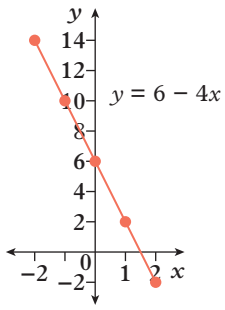



**8** **a** $y = 6 - 4x$

**b**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 14 | 10 | 6 | 2 | −2 |

**c**




**9** **a** $y = 5 - x$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 7 | 6 | 5 | 4 | 3 |

**b** $y = -1 - x$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 1 | 0 | −1 | −2 | −3 |

**c** $y = 3 - 2x$

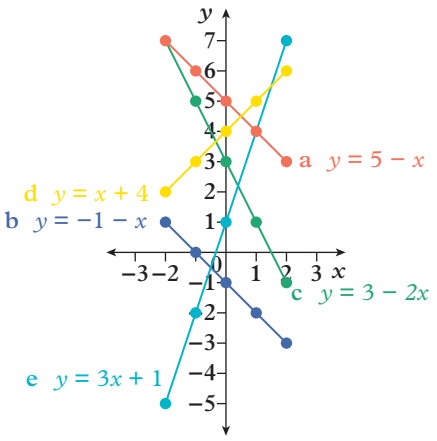
| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 7 | 5 | 3 | 1 | −1 |

**d** $y = x + 4$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 2 | 3 | 4 | 5 | 6 |

**e** $y = 3x + 1$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −5 | −2 | 1 | 4 | 7 |




**f** $y = -2 - x$

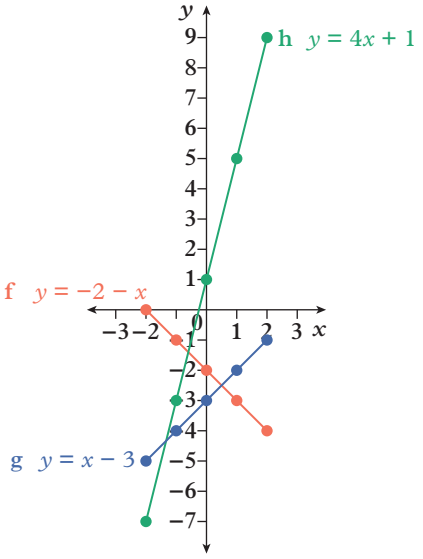
| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0 | −1 | −2 | −3 | −4 |

**g** $y = x - 3$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −5 | −4 | −3 | −2 | −1 |

**h** $y = 4x + 1$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −7 | −3 | 1 | 5 | 9 |

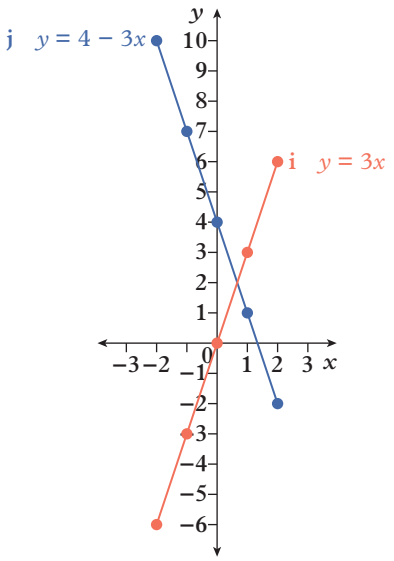



**i** $y = 3x$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −6 | −3 | 0 | 3 | 6 |

**j** $y = 4 - 3x$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 10 | 7 | 4 | 1 | −2 |




**10 a**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $2x$ | −3 | 0 | 3 | 6 | 9 | 12 | 15 |
| $3y$ | 15 | 12 | 9 | 6 | 3 | 0 | −3 |
| $x$ | −1.5 | 0 | 1.5 | 3 | 4.5 | 6 | 7.5 |
| $y$ | 5 | 4 | 3 | 2 | 1 | 0 | −1 |

**b**

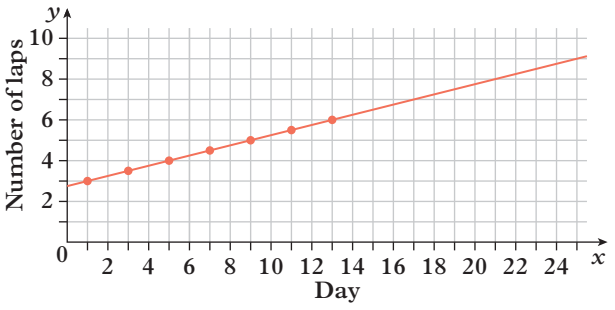
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $4x$ | 0 | 12 | 18 | 36 | 42 | 48 | 54 |
| $3y$ | −36 | −24 | −18 | 0 | 6 | 12 | 18 |
| $x$ | 0 | 3 | 4.5 | 9 | 10.5 | 12 | 13.5 |
| $y$ | −12 | −8 | −6 | 0 | 2 | 4 | 6 |

**11 a** IV: day number, DV: number of laps

**b i** the number of laps increases by the same amount every 2 days

**ii** 2 points

**c**



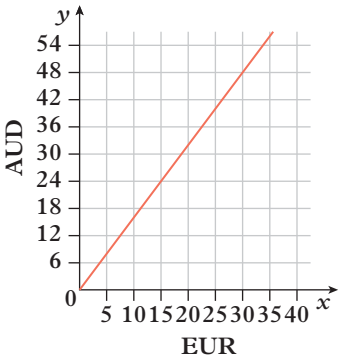

**d i** 7 laps **ii** day 21

**12 a** IV: EUR (Euro), DV: AUD (Australian dollar)

**b i** both variables are linear and not multiplied together

**ii** 2 points

**c**



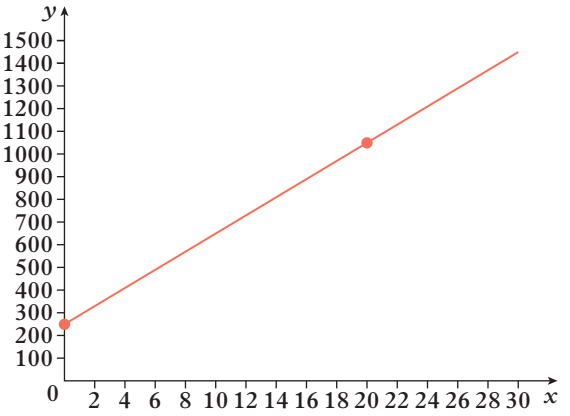

**d i** 48 AUD **ii** 15 EUR

**13 a** $m = 40n + 250$

**b** Independent variable: number of students, $n$
Dependent variable: total amount of money collected for bus hire, $m$

**c i** \$250 **ii** \$1050

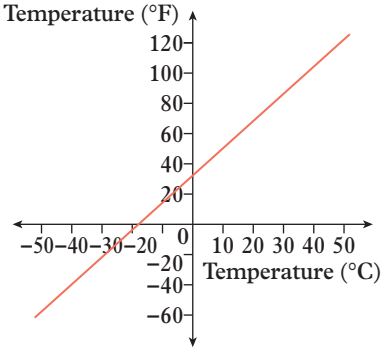
**d**




**e** Relationship is linear because it forms a straight line.

**f** \$1450 **g** 15 students **h** 27 students

**14 a**




**b** 86°F **c** −30°C

**15** −40°F = −40°C

**16 a** (1,2) **b** (0,0) **c** (2,0) **d** (4,4)

## EX p144 4C Gradient and intercepts

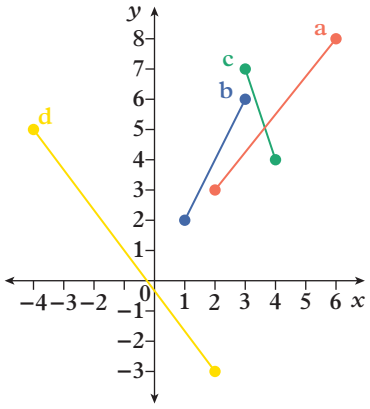
**1 a** $\frac{3}{4}$ **b** $-\frac{1}{2}$

**2 a** $\frac{1}{3}$ **b** $-\frac{3}{2}$ **c** 2

**3** **a** zero **b** positive **c** negative
**d** undefined **e** zero **f** negative

**4** **a** **i** 2 **ii** 1 **iii** $-2$
**b** **i** 1 **ii** 4 **iii** $-4$
**c** **i** $-2$ **ii** $-1$ **iii** $-2$
**d** **i** 0 **ii** none **iii** 1
**e** **i** $\frac{3}{5}$ **ii** $-5$ **iii** 3
**f** **i** $-\frac{2}{3}$ **ii** $-3$ **iii** $-2$
**g** **i** undefined **ii** $-3$ **iii** none
**h** **i** $-6$ **ii** 0 **iii** 0

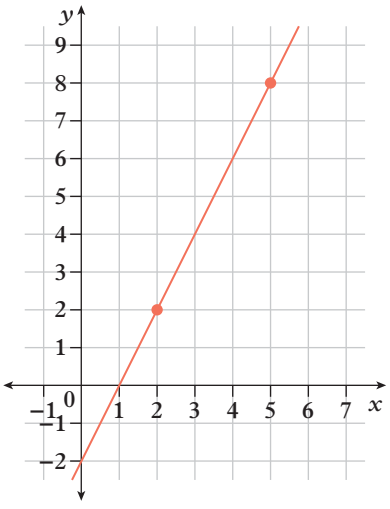
**5** **i**




**a** **ii** $\frac{5}{4}$
**b** **ii** 2
**c** **ii** $-3$
**d** **ii** $-\frac{4}{3}$

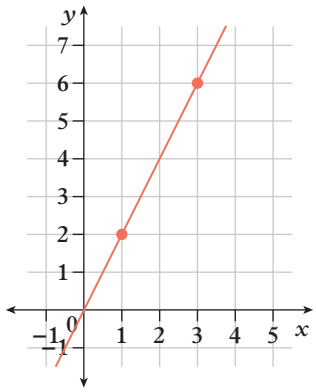
**6** **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** 2 **d** $-\frac{1}{3}$
**e** $\frac{2}{5}$ **f** $-3$ **g** undefined **h** $-5$
**i** $\frac{7}{3}$ **j** undefined **k** $-2$ **l** 0

**7** **a** **i**



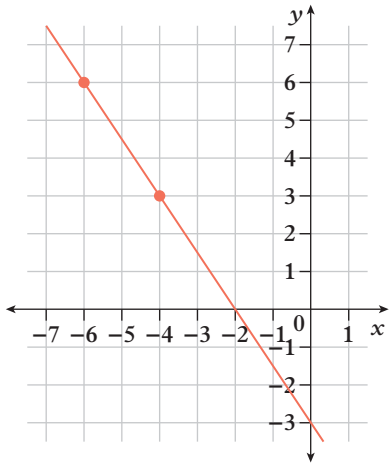

**ii** (1,0),(0,−2)

**b** **i**



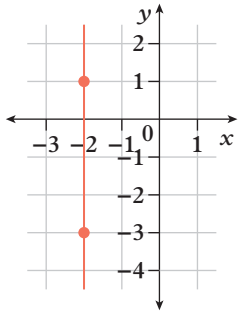

**ii** (0,0)

**c** **i**




**ii** (−2,0),(0,−3)

**d** **i**




**ii** (−2, 0)

**8** **a** $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** 2 **d** $\frac{3}{2}$ **e** 1 **f** 2

**9** **a** **i** $\frac{3}{2}$ **ii** $\frac{3}{2}$

**b** The answers are the same. The gradient of a line is the same as the gradient of any line segment on that line.

**10** **a** **i** 0 **ii** 1 **iii** $\frac{1}{2}$ **iv** $-1$
**b** **i** $\frac{1}{2}$ **ii** 0 **iii** 1 **iv** $-1$

**11** Kane subtracted the second $y$-coordinate from the first, and the first $x$-coordinate from the second; $m = -\frac{3}{4}$.

**12** The order in which the coordinates are substituted into the gradient formula does not affect the value of the gradient.

**13** **a** **i** $\frac{2}{1}$ **ii** 2 **iii** 1

**b** **i** $\frac{-3}{1}$ **ii** $-3$ **iii** 1

**c** **i** $\frac{-4}{1}$ **ii** $-4$ **iii** 1

**d** **i** $\frac{-1}{1}$ **ii** $-1$ **iii** 1

**e** **i** $\frac{5}{4}$ **ii** 5 **iii** 4

**f** **i** $\frac{-3}{8}$ **ii** $-3$ **iii** 8

**g** **i** $\frac{-2}{7}$ **ii** $-2$ **iii** 7

**h** **i** $\frac{-1}{5}$ **ii** $-1$ **iii** 5

**14** All of the gradients Ash calculated are equivalent to $\frac{3}{5} = 0.6$ after simplifying. It does not matter that the rise and run are not equal, only that the value of the fractions (gradients) are. The gradient of the line is $\frac{3}{5} = 0.6$.

**15**

| **Run** | 8 | $-20$ | $-28$ | 40 | 4 | $-6$ |
|---|---|---|---|---|---|---|
| **Rise** | 12 | 16 | $-4$ | $-16$ | 28 | 30 |
| **Gradient** | $\frac{3}{2}$ | $-\frac{4}{5}$ | $\frac{1}{7}$ | $-\frac{2}{5}$ | 7 | $-5$ |

**16** **a**

| | **Equation** | **Gradient** | ***y*-intercept** |
|---|---|---|---|
| **i** | $y = 2x - 3$ | 2 | $(0, -3)$ |
| **ii** | $y = -\frac{3}{4}x + 5$ | $-\frac{3}{4}$ | $(0, 5)$ |
| **iii** | $y = x + 4$ | 1 | $(0, 4)$ |
| **iv** | $y = -2x - 1$ | $-2$ | $(0, -1)$ |
| **v** | $y = 4x$ | 4 | $(0, 0)$ |
| **vi** | $y = 2$ | 0 | $(0, 2)$ |

**b** gradient is coefficient of $x$; $y$-intercept is the constant

**c** **i** gradient = 6, $y$-intercept: $(0, 4)$

**ii** gradient = 1, $y$-intercept: $(0, -5)$

**iii** gradient = $-3$, $y$-intercept: $(0, 0)$

**17** **a** $x = 3$ **b** $y = 0$ **c** $y = 8$

**d** $y = 10$ **e** $y = 3$ **f** $x = 7$

**18** **a** $\frac{y-4}{x} = 2$ **b** $y = 2x + 4$

**c** $\frac{y-4}{x-1} = 2$ **d** $y = 2x + 2$

## Checkpoint

**1** **a** $x = -27$ **b** $x = 5$ **c** $x = 22$ **d** $x = 7$

**2** **a** $x = 6$ **b** $x = \frac{1}{6}$ **c** $x = 6.25$ **d** $x = 40$

**3** **a** $x = -4$ **b** $x = 2$ **c** $x = -5$ **d** $x = -3$

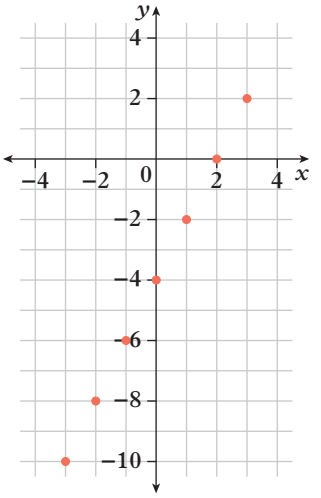
**4** **a** Let $n$ be the cost of one video game.

**b** $5n + 9.95 = 409.7$

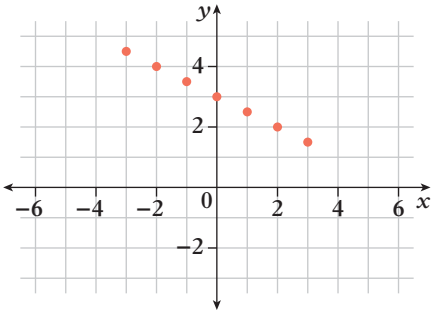
**c** $n = 79.95$

**d** \$79.95

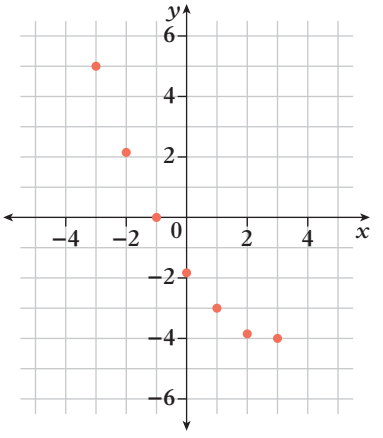
**5** **a** **i** **ii** linear




**b** **i** **ii** linear




**c** **i** **ii** non-linear



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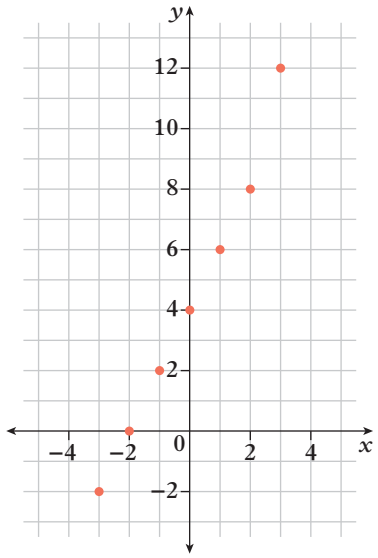
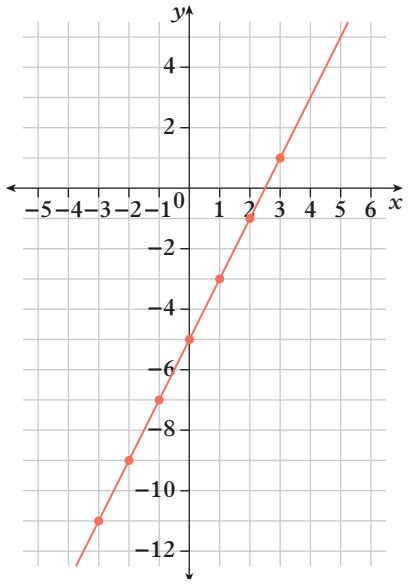
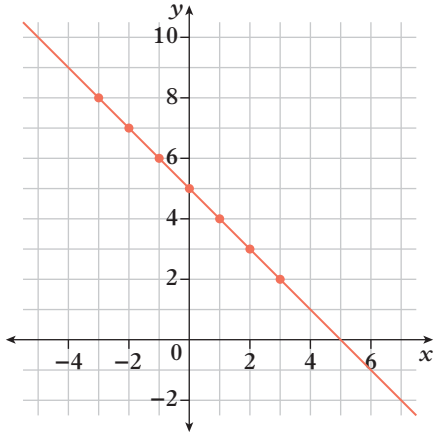
**d i ii** non-linear



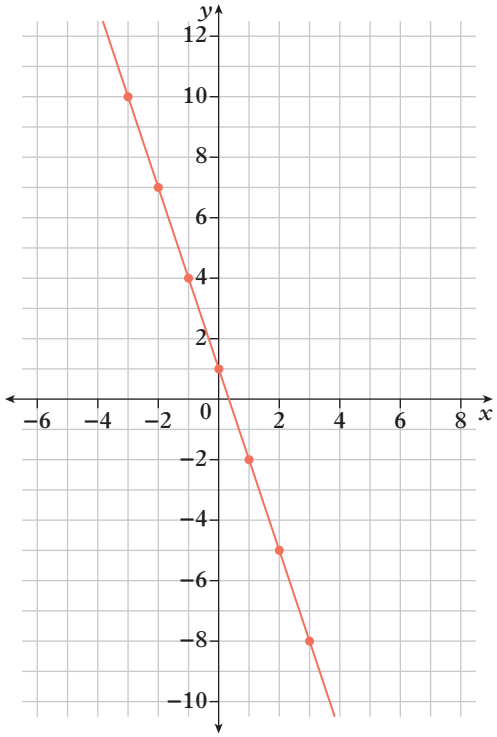

**6 a**



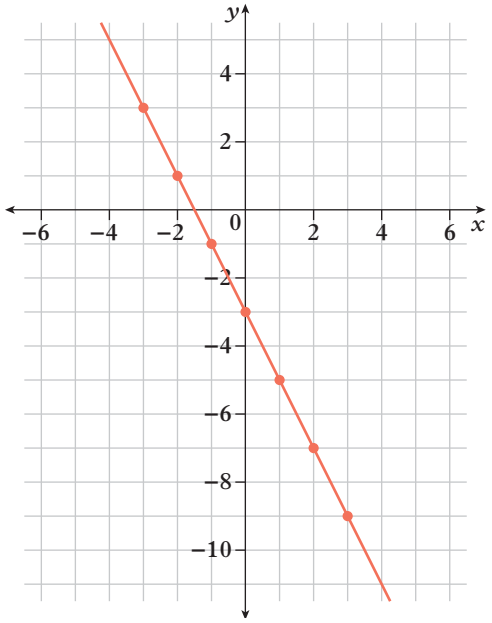

**b**




**c**




**d**




**7 a** IV: length, DV: volume

**b** IV: number of burgers, DV: calories

**c** IV: time, DV: number of achievements

**8**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $2x$ | −5 | 0 | 5 | 10 | 15 | 20 | 25 |
| $5y$ | 25 | 20 | 15 | 10 | 5 | 0 | −5 |
| $x$ | −2.5 | 0 | 2.5 | 5 | 7.5 | 10 | 12.5 |
| $y$ | 5 | 4 | 3 | 2 | 1 | 0 | −1 |

**9** **a** $\frac{2}{5}$ **b** $-3$ **c** $-\frac{1}{8}$ **d** $\frac{3}{2}$

**10** **a** zero **b** negative **c** undefined **d** positive

**11** **a** 3 **b** $-3$ **c** $-\frac{5}{3}$ **d** $-\frac{6}{5}$

**12** **a** $(3,0),(0,4)$ **b** $(-2,0),(0,3)$ **c** $(-6,0),(0,-1)$

EX p155

## 4D Sketching linear graphs using intercepts

**1** **a** (12, 0); (0, 3)

**b** (−4, 0); (0, 4)

**c** (3, 0); (0, 6)

**d** (5, 0); (0, −5)

**e** (4, 0); (0, 8)

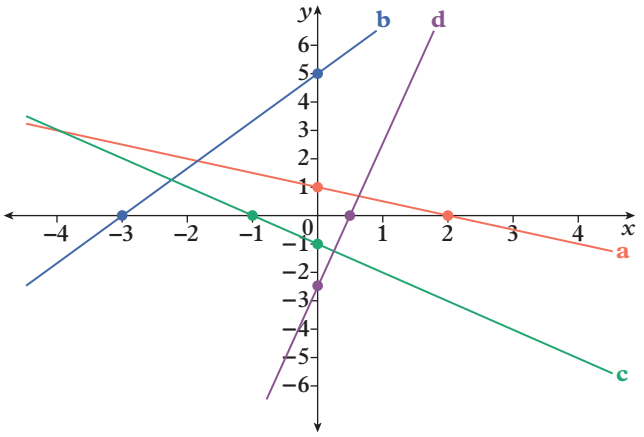
**f** (2, 0); (0, −6)

**g** (−2, 0); (0, −10)

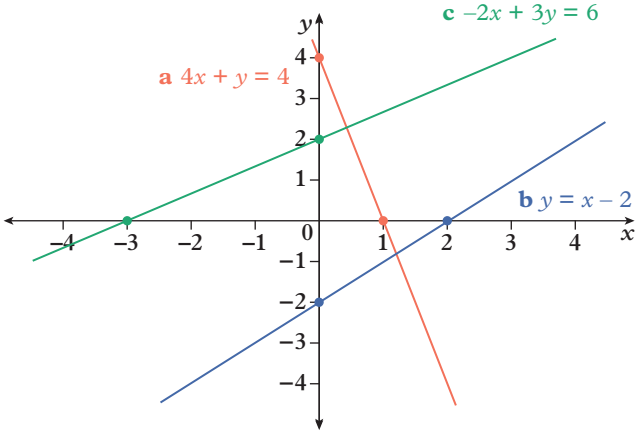
**h** (7, 0); (0, 7)

**i** (4, 0); (0, 4)

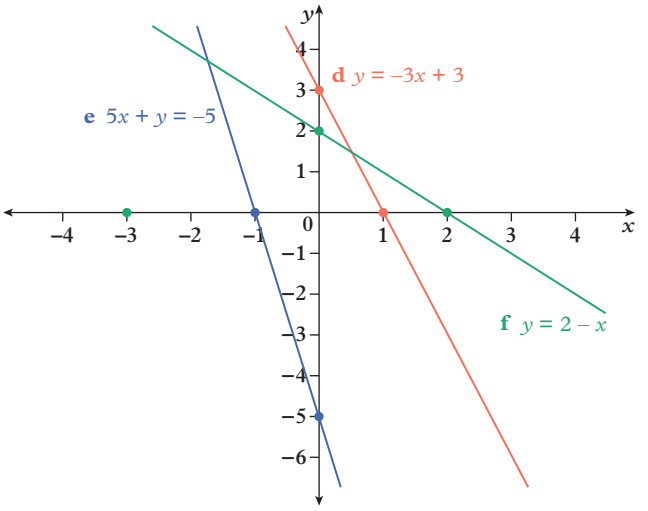
**2** **a–d**



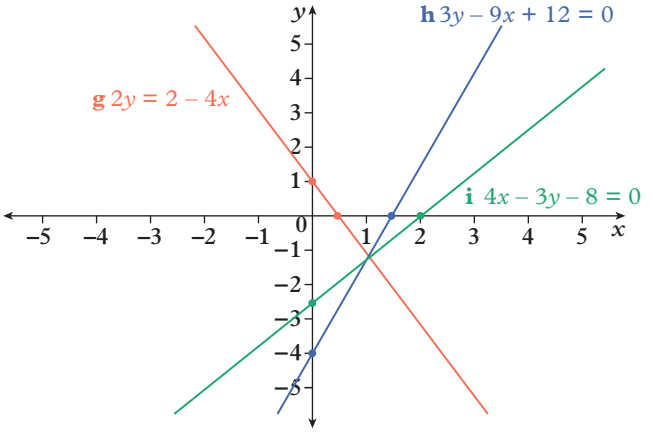

**3** **a–c**



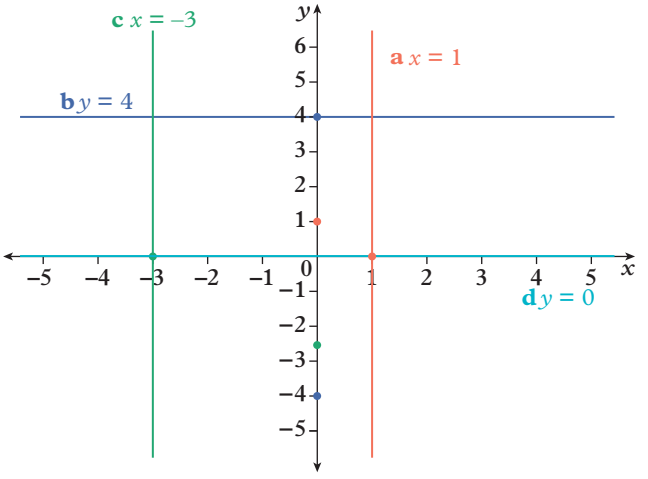

**d–f**



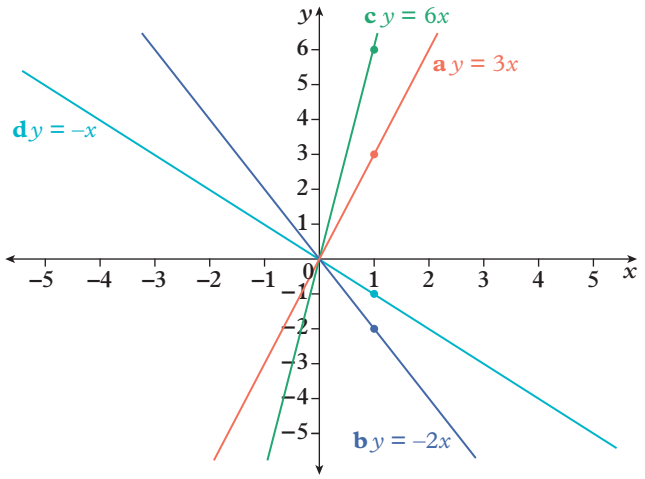

**g–i**



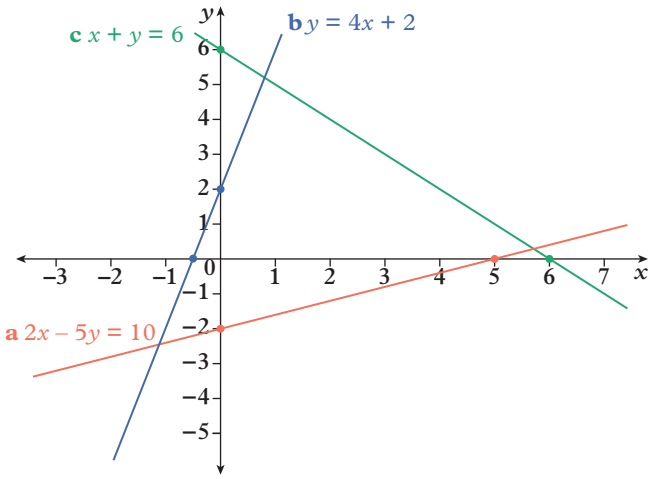

**4**



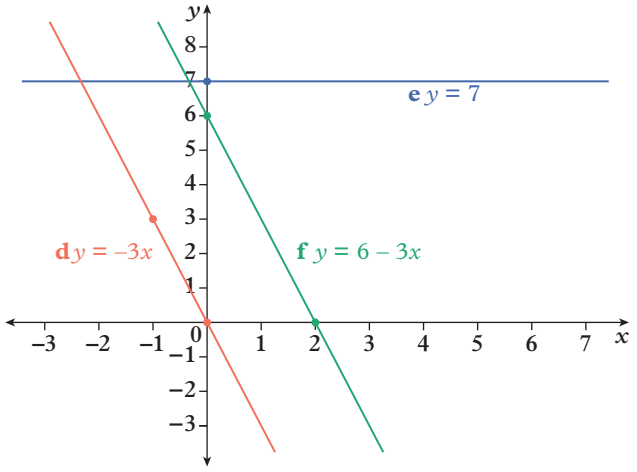
**5**



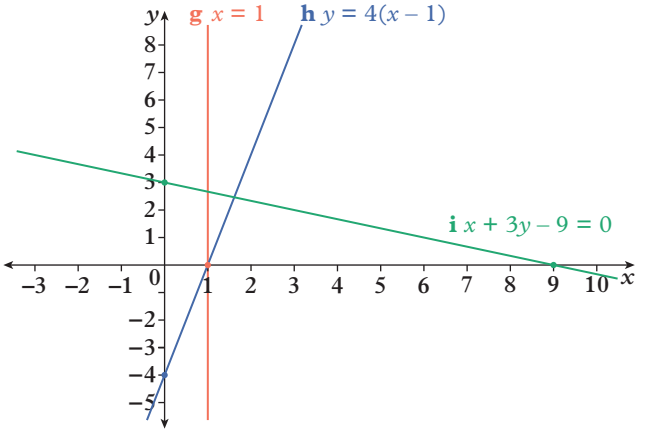

**6 a–c**



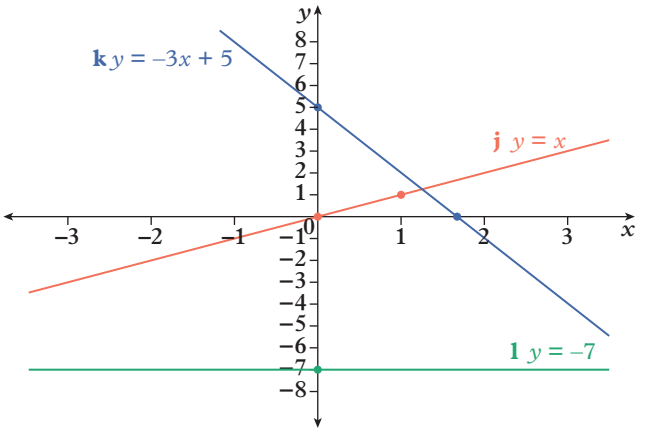

**d–f**




**g–i**




**j–l**



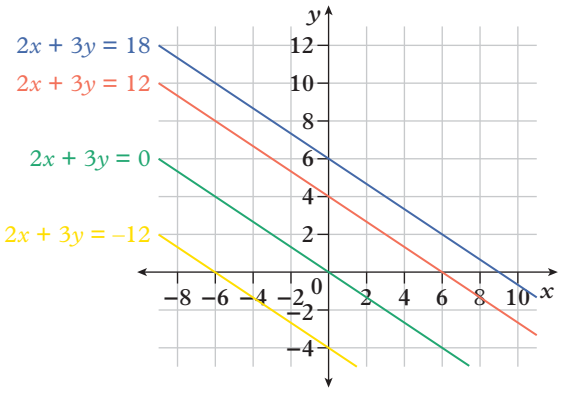

**7** **a** 1 **b** 2 **c** 1
**d** 1 **e** 2 **f** 2
**g** 1 **h** 1 **i** 1

**8** Graph B is correct. In graph A the coefficients of variables have been used as intercepts. Graph B is a correct sketch of $3x + 2y = 6$. Graph C has an $x$-intercept at $-2$ instead of $+2$.

**9** **a** true
**b** false; the $x$-intercept is $-2$
**c** true
**d** true
**e** false; the line passes through $(-2, 0)$ and $(0, 6)$
**f** true

**10** **a** false; the graph passes through the origin
**b** true
**c** true
**d** false; the point $(1, -4)$ lies on the line
**e** true
**f** false; the gradient of the line is $-4$

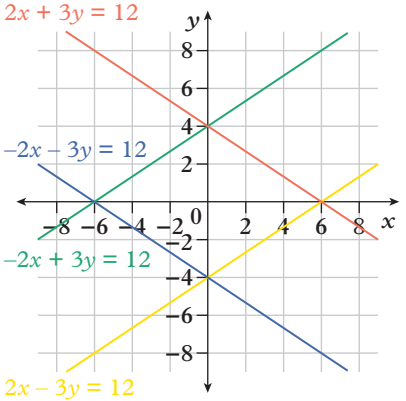
**11 a**




**b** The equations have the same variable sum, $2x + 3y$, and are parallel (have the same gradient).

**c** Increasing the constant translates the line up and to the right. Decreasing the constant translates the line down and to the left. If the constant is 0, the line passes through the origin. If the constant is positive, the line has positive intercepts. If the constant is negative, the line has negative intercepts.

**12 a**



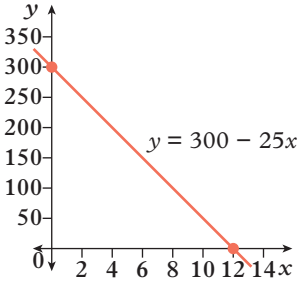

**b** All equations are equal to the same positive constant and have coefficients with the same unsigned values (but not necessarily the same sign).

**c** The term(s) with a positive coefficient have positive intercepts. The term(s) with a negative coefficient have negative intercepts. Compared to $2x + 3y = 12$, the line is reflected in the $x$-axis when the $y$-term is negative and reflected in the $y$-axis when the $x$-term is negative.

**13 a** $\frac{3}{5}$ **b** $-\frac{7}{6}$ **c** $-\frac{1}{2}$ **d** 2

**14 a** The independent variable is $x$ (number of weeks) so Tony should calculate $y$ (amount still owed in dollars), which depends on the number of payments over $x$ weeks.

**b**




**c** original cost of skateboard

**d** number of weeks needed to pay off skateboard

**e** Tony is paying \$25 a week towards the skateboard. He will pay it off in 12 weeks.

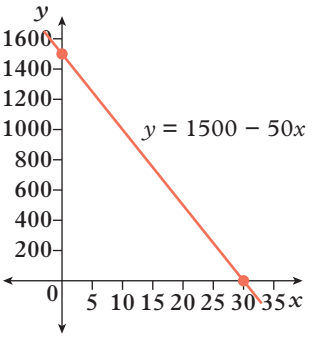
**15 a** IV: number of days, DV: litres of water – because time passing does not depend on the volume of water in the tank, but the volume of water in the tank depends on the time.

**b** Yes, the constant rate of water use and no additions to the tank suggest a constant gradient and hence a linear relationship between time, $x$, and amount of water, $y$.

**c** (30, 0)

**d** (0, 1500)

**e**




**f** 1000 L

**g** end of day 18

**16 a** When $x = 0$, the equation becomes $by = 1$, which has the solution $y = \frac{1}{b}$ (the $y$-intercept), the reciprocal of $b$. Likewise, when $y = 0$, the equation becomes $ax = 1$, which has the solution $x = \frac{1}{a}$ (the $x$-intercept), the reciprocal of $a$.

**b** Divide both sides of the equation by $d$, then the reciprocals of the coefficients of $x$ and $y$ will be the $x$- and $y$-intercepts, respectively.

**c** $d = ab$

**17 a** $\left(-\frac{7}{3}, 0\right)$ and $\left(0, \frac{7}{5}\right)$ **b** $\left(\frac{6}{5}, 0\right)$ and $\left(0, \frac{8}{25}\right)$

**c** $(\sqrt{3}, 0)$ and $(0, -\sqrt{2})$ **d** $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{3}\right)$

## EX p161 4E Determining linear equations

**1 a** $m = 2, c = 5$ **b** $m = 4, c = 1$

**c** $m = -3, c = 7$ **d** $m = -5, c = -3$

**e** $m = 1, c = -6$ **f** $m = -1, c = 1$

**g** $m = \frac{4}{3}, c = 2$ **h** $m = \frac{1}{2}, c = -8$

**i** $m = -\frac{4}{3}, c = \frac{1}{4}$ **j** $m = 0, c = 9$

**k** $m = -7, c = 0$ **l** $m = -\frac{2}{5}, c = 5$

**2 a** $y = 3x + 4$ **b** $y = -2x + 10$

**c** $y = x - 7$ **d** $y = -12x - 1$

**e** $y = -x + 20$ **f** $y = 5x$

**g** $y = \frac{1}{3}x + \frac{4}{5}$ **h** $y = -\frac{3}{4}x$

**3** **a** $y = 3x + 6$ **b** $y = -\frac{2}{3}x + 4$

**c** $y = \frac{1}{2}x - 5$ **d** $y = -\frac{1}{5}x - 2$

**e** $y = 2x - 3$ **f** $y = -3x + 5$

**g** $y = -\frac{1}{2}x$ **h** $y = \frac{5}{2}x$

**4** **a** $y = 2x - 22$ **b** $y = -2x + 8$

**c** $y = -x + 5$ **d** $y = 3x$

**e** $y = 2x + 1$ **f** $y = 5x - 49$

**g** $y = -\frac{1}{2}x - \frac{5}{2}$ **h** $y = \frac{4}{3}x - 11$

**5** **a** $x = 5$ **b** $y = -2$

**c** $y = 60$ **d** $x = -40$

**e** $y = -\frac{3}{7}$ **f** $x = \frac{2}{3}$

**6** **a** $y = -3x + 12$ **b** $y = 2x + 10$

**c** $y = 7x - 21$ **d** $y = -4x - 24$

**e** $y = 10x + 10$ **f** $y = -\frac{x}{3} + 6$

**g** $y = \frac{2}{5}x - 2$ **h** $y = \frac{3}{2}x + 9$

**7** **a** $y = \frac{1}{2}x + \frac{9}{2}$ **b** $y = \frac{2}{3}x + \frac{14}{3}$

**c** $y = -\frac{1}{5}x + \frac{18}{5}$ **d** $y = \frac{3}{4}x + \frac{1}{2}$

**e** $y = -\frac{7}{6}x - \frac{1}{3}$ **f** $y = -\frac{11}{9}x - \frac{5}{3}$

**8** **a** IV: number of years, DV: amount in Azami's account

**b** $A = 300n + 1900$

**9** **a** $P = 60n + 30$ **b** \$21 930

**10** **a** IV: number of years from 2010, DV: length of stalactite in centimetres

**b** $g = \frac{1}{5}n + 85$

**c** The stalactite is 425 years old in 2010, which means it began to form in 1585.

**11** **a** $y = 9x - 5$ **b** $y = \frac{3}{2}x + \frac{5}{2}$

**c** $y = \frac{3}{10}x + 45$ **d** $y = -2x + 30$

**12** **a** $C = 45n + 125$ **b** $R = 99n + 275$

**c** $P = 54n + 150$

**13** **a** **i** $y = 3(x + 4)$ **ii** $y = -5(x + 3)$

**iii** $y = 2(x - 5)$

**iv** $y = 2(3x - 4)$ **v** $y = 7(3 - x)$ or $y = -7(x - 3)$

**vi** $y = 8(2x - 5)$

**b** **i** $x = -4$ **ii** $x = -3$ **iii** $x = 5$

**iv** $x = \frac{4}{3}$ **v** $x = 3$ **vi** $x = \frac{5}{2}$

**c** The $x$-intercept is the value of $x$ that makes the bracketed factor equal to zero. It is the opposite sign of the constant divided by the coefficient of $x$.

**14** **a** $y = -\frac{21}{10}x + \frac{7}{5}$

**b** $y = -\frac{5}{3}x + \frac{1}{12}$

**c** $y = \frac{22}{81}x - \frac{19}{36}$

**15** **a** $y = 9x + 11$ and $y = 11x + 9$ **b** $y = 10x + 10$

EX p167

## 4F Midpoint and length of a line segment

**1** **a** (2, 1) **b** (4, −2) **c** (3, 0)

**d** (−3, 3) **e** (3, 0) **f** (0, 0)

**2** **a** (2, 7) **b** (5, 4) **c** (3, 1) **d** (2,8)

**e** (4, 7) **f** (5, 1) **g** (4, 3) **h** (1,−4)

**i** (−3, 0) **j** (3.5, 8.5) **k** (6.5, 5.5) **l** (0,0)

**3** **a** 2.2 units **b** 4.5 units **c** 3.2 units

**d** 4.0 units **e** 6.3 units **f** 5.0 units

**g** 1.4 units **h** 14.0 units **i** 3.6 units

**j** 4.1 units **k** 4.5 units **l** 15.6 units

**4** **a** **i** (3, 3) **ii** 4.5 units **iii** $-\frac{1}{2}$

**b** **i** (0.5, 0) **ii** 7.8 units **iii** $1\frac{6}{5}$

**c** **i** (−2.5, 3) **ii** 8.5 units **iii** $\frac{8}{3}$

**d** **i** (−6, −5) **ii** 8.5 units **iii** −1

**5** **a** 1.1 units **b** 2.2 units **c** 1.6 units **d** 2.0 units

**e** 3.2 units **f** 2.5 units **g** 0.7 units **h** 7.0 units

**i** 1.8 units **j** 2.1 units **k** 2.2 units **l** 7.8 units

**6** (10, 5)

**7** (−14, 9)

**8** **a** 5 units **b** (5, 7)

**9** 20.3 units

**10** **a** 20.5 units **b** 17.0 units **c** 22.6 units **d** 19.5 units

**11** **a** leg 1: 21.5 km; leg 2: 16.5 km; leg 3: 28.8 km

**b** 66.8 km **c** 24.1 km

**12** **a** (−5, −1), (−2, 3), (6, 3) and (3, −1)

**b** **i** (0.5, 1) **ii** (0.5, 1)

**c** midpoints of both diagonals have same coordinates

**d** 26 cm **e** 5.3 cm

**13** **a** **i** 21.6 cm **ii** 10.8 cm

**iii** Perimeter of red shape = $\frac{1}{2}$, perimeter of blue shape

**b** **i** 20.7 cm **ii** 16.7 cm

**iii** Perimeter of red shape ≈ $\frac{4}{5}$, perimeter of blue shape

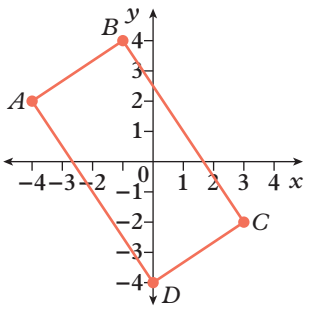
**14** **a** $AB$: 3.6 m, $BC$: 7.2 m, $CD$: 3.6 m and $AD$: 7.2 m

**b** Midpoint for both diagonals (−0.5, 0)

**c** $A$: 4.0 m, $B$: 4.0 m, $C$: 4.0 m, $D$: 4.0 m

**d** The parallelogram is a rectangle because it has two opposite pairs of sides of equal length, diagonals equal in length and bisect each other, with each vertex the same distance from the midpoint.

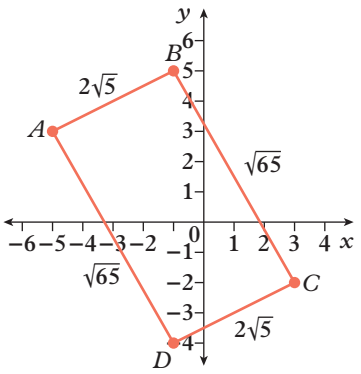
**e** 26 m$^2$




**15 a** Two sides are equal in length, $AB = AC = 4\sqrt{5}$ units and $BC = 4\sqrt{2}$ units

**b** 24 units$^2$

**16** Two pairs of equal-length sides, $AB = CD = 2\sqrt{5}$ units and $BC = AD = \sqrt{65}$ units; two pairs of parallel sides with gradients $\frac{1}{2}$ and $-\frac{7}{4}$.




**17** $(0, 1)$

**18** $(\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, -2\sqrt{5})$

## EX 4G Direct proportion

p174

**1 a** $y \propto x$

**b** $y$ is not directly proportional to $x$ because the rate of change is not constant.

**c** $y \propto x$

**d** $y$ is not directly proportional to $x$ because when $x = 0$, $y = 1$.

**e** $y \propto x$

**f** $y$ is not directly proportional to $x$ as the graph does not pass through $(0, 0)$.

**g** $y$ is not directly proportional to $x$ as the graph does not pass through $(0, 0)$ and $y$ does not increase as $x$ increases.

**h** $y \propto x$

**i** $y$ is not directly proportional to $x$ as the gradient is not constant.

**j** $y$ is not directly proportional to $x$ as the graph does not pass through $(0, 0)$.

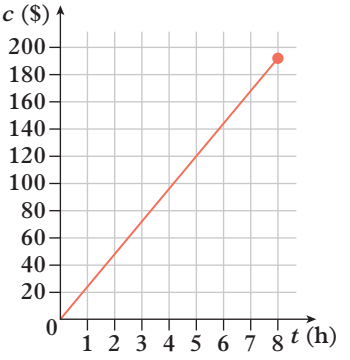
**2 a** $k = 4$ **b** $k = -10$ **c** $k = 3.5$ **d** $k = \frac{1}{3}$

**3 a** $y = 10x$ **b** $y = -9x$ **c** $y = \frac{1}{4}x$ **d** $y = -\frac{2}{3}x$

**e** $y = -6x$ **f** $y = 5x$ **g** $y = -\frac{1}{2}x$ **h** $y = 5x$

**i** $y = 2x$ **j** $y = -20x$ **k** $y = \frac{3}{10}x$ **l** $y = \frac{3}{4}x$

**4 a i**




**ii** 24

**b** \$24/hour

**c** $c = 24t$

**d** \$96

**5 a** The car travelled at a speed of 60 km/h for 1 hour, then at a speed of 120 km/h for second hour.

**b** $d = 60t$ **c** $d = 120t - 60t$

**d** No. The rate of change of $d$ with respect to $t$ is not constant over the duration of the journey.

**6 a** $k = 600$ **b** $C = 600A$ **c** 210 m$^2$

**7 a** 4 patients per hour

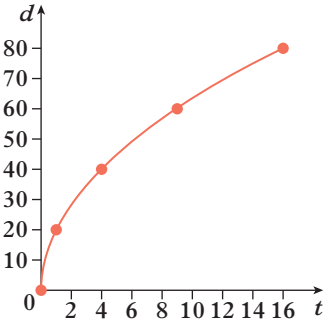
**b** 15 min

**c** \$62.50 per patient

**8 a** Jasmine 8.10 am, Nelson 8.25 am

**b** 9.05 am

**9 a**

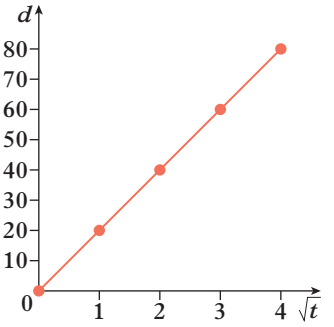



**b** No; as the graph is not a straight line passing through $(0, 0)$

**c**

| $\sqrt{t}$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $d$ | 0 | 20 | 40 | 60 | 80 |

**d**



**e** Yes; it forms a straight-line graph that passes through (0, 0).

**f** $k = 20$, $d = 20\sqrt{t}$ **g** 120 km

**10** Helena is 40 km from town A and Julia is 110 km from town A.

## Chapter review

### Multiple-choice

**1** B **2** C **3** D **4** D **5** C **6** B **7** B

**8** E **9** B **10** A **11** C **12** A **13** C **14** D

### Short answer

**1** **a** $x = 2$ **b** $x = \frac{2}{3}$ **c** $x = 10$ **d** $x = 3$

**e** $x = -1$ **f** $x = -\frac{4}{5}$ **g** $x = 3\frac{3}{4}$ **h** $x = \frac{1}{2}$

**2** **a** $x = 4$ **b** $x = -1$

**3** **a** Let $n$ be the number of people in the class.

**b** $\frac{3}{5}n + 1 = 16$ **c** $n = 25$ **d** 25 people

**4** **a** Let $j$ be Jacob's age this year.

**b** $(j + 9) + 1 = 2(j + 1)$

**c** $j = 8$

**d** 8 years old

**5** **a**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 | 7 |

**b**

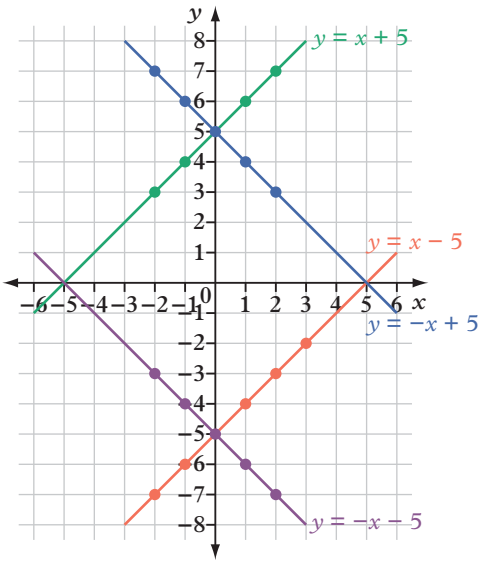
| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −7 | −6 | −5 | −4 | −3 |

**c**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 7 | 6 | 5 | 4 | 3 |

**d**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −3 | −4 | −5 | −6 | −7 |




**6** **a** IV: number of attempts, DV: accuracy

**b** IV: number of uses, DV: amount of soap

**c** IV: number of chess matches played, DV: the number of victories

**7** **a** **i** $\frac{1}{3}$ **ii** (−6,0) **iii** (0,2)

**b** **i** undefined **ii** (3,0)

**iii** no $y$-intercept

**c** **i** 0 **ii** no $x$-intercept **iii** (0,−2)

**d** **i** −2 **ii** (0,0) **iii** (0,0)

**8** **a** $\frac{3}{2}$ **b** −1

**9** **a** **i** (9,0) **ii** (0,6)

**b** **i** (2,0) **ii** (0,−6)

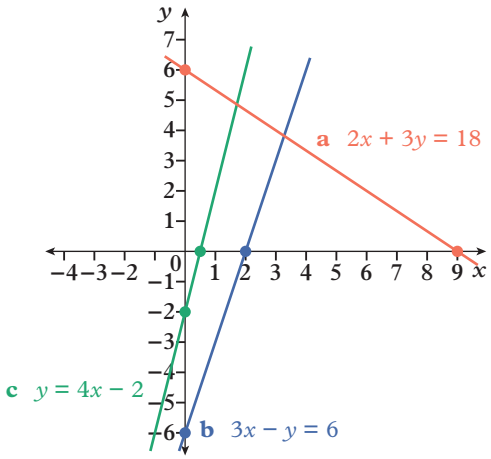
**c** **i** $\left(\frac{1}{2},0\right)$ **ii** (0,−2)

**d** **i** $\left(\frac{3}{5},0\right)$ **ii** $\left(0,-\frac{3}{2}\right)$

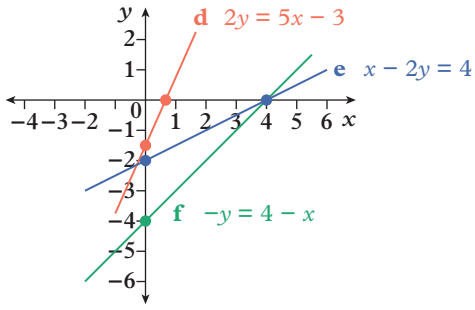
**e** **i** (4,0) **ii** (0,−2)

**f** **i** (4,0) **ii** (0,−4)

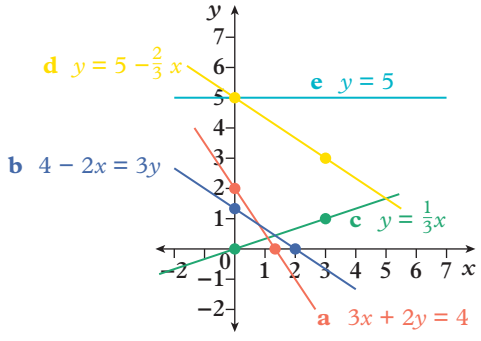
**10** **a–c**



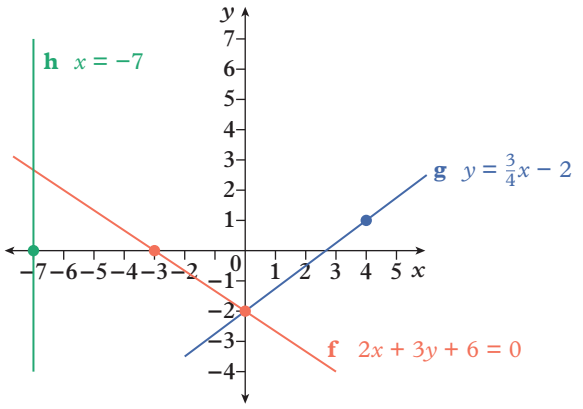

**d–f**




**11** **a–e**



**f–h**




**12** **a** $y = 4x - 2$ **b** $y = \frac{1}{4}x$ **c** $y = -\frac{1}{2}$

**13** **a** $y = -2x - 4$ **b** $y = \frac{3}{2}x$

**c** $y = 1.25x + 1.25$ **d** $y = -\frac{1}{3}x + 1$

**14** **a** $y = -x - 3$ **b** $y = 39$

**c** $y = -\frac{4}{3}x$ **d** $y = \frac{4}{5}x + \frac{32}{5}$

**15** **a** **i** (5, 5) **ii** 7.2 units

**b** **i** (−3.5, 5.5) **ii** 1.4 units

**c** **i** (0, 0) **ii** 8.9 units

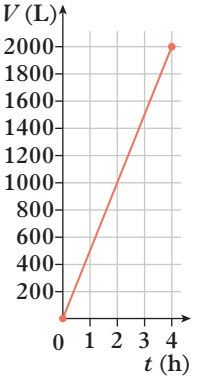
**d** **i** (−0.5, −5) **ii** 9.4 units

**16** **a** $y$ is not directly proportional to $x$; the rate of change is not constant.

**b** $y \propto x$ **c** $y \propto x$

**d** $y$ is not directly proportional to $x$ as the gradient is not constant.

**17** **a** **i**




**ii** 500

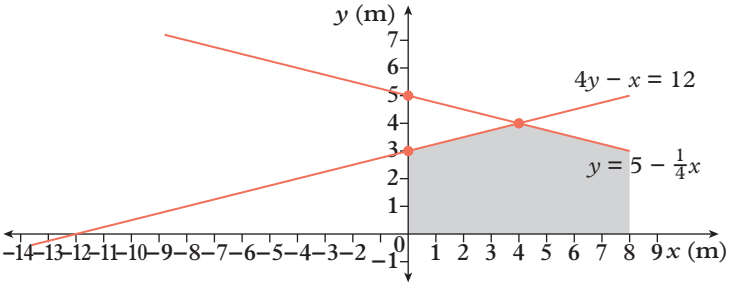
**b** 500 L/hr

**c** $V = 500t$

**d** 20 hours

## Analysis

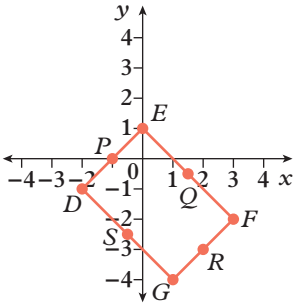
**1** **a**, **b**




**c** 4 m **d** 4.1 m **e** $\frac{1}{4}$

**f** Chimney will be at (2, 3.5).

**2** **a**




**b** $DE$ and $FG$ are 2.8 units long, $EF$ and $DG$ 4.2 units long.

**c** **i** (0, 1) **ii** (0, 1) **iii** (0, −3) **iv** none

**d** **i** 1 **ii** −1 **iii** −1 **iv** 1

**e** **i** (−1, 0) **ii** (1.5, −0.5)

**iii** (2, −3) **iv** (−0.5, −2.5)

**f** $PQRS$ is a rhombus with all sides equal in length ($PQ$, $RS$, $SP$ and $QR$).

**g** If $DEFG$ were square, then $PQRS$ would also have been square because the midpoints of each side would have been the same distance from midpoints of diagonals, which cross at 90°.

# CHAPTER 5 Non-linear relationships

## EX 5A Solving quadratic equations

p187

**1** **a**, **c**, **e** and **h**

**2** **a** yes **b** no **c** no **d** yes

**e** no **f** yes **g** yes **h** no

**3** **a** $x = -3$ **b** $x = 6$ **c** $x = 2$ **d** $x = -\frac{1}{3}$

**4** **a** $x = -2$ or $x = 3$ **b** $x = 1$ or $x = 7$

**c** $x = -4$ or $x = 4$ **d** $x = 0$ or $x = 6$

**e** $x = -5$ or $x = -1$ **f** $x = -2$ or $x = 0$

**g** $x = -8$ or $x = 8$ **h** $x = -1$ or $x = 7$

**i** $x = 0$ or $x = 11$ **j** $x = -\frac{3}{2}$ or $x = 5$

**k** $x = \frac{1}{2}$ **l** $x = -\frac{5}{3}$ or $-\frac{3}{2}$

**5** **a** $x = -2$ or $x = -5$ **b** $x = 1$ or $x = 2$

**c** $x = 0$ or $x = -5$ **d** $x = 0$ or $x = 3$

**e** $x = -6$ or $x = 6$ **f** $x = -7$ or $x = -3$

**6** **a** $x = -2$ or $x = 4$ **b** $x = -1$ or $x = 1$
**c** $x = 0$ or $x = -8$ **d** $x = 1$ or $x = 3$
**e** $x = -3$ **f** $x = 1$

**7** Check answers in questions 5 and 6.

**8** **a** $x = -8$ or $x = 2$ **b** $x = 1$ or $x = 4$
**c** $x = -6$ or $x = 6$ **d** $x = -9$ or $x = 0$

**9** **a** $x = -2$ or $x = -1$ **b** $x = 0$ or $x = -4$
**c** $x = -2$ or $x = 4$ **d** $x = 2$
**e** $x = -2$ or $x = 1$ **f** $x = 0$ or $x = 2$
**g** $x = -7$ or $x = -3$ **h** $x = -4$
**i** $x = -4$ or $x = 4$

**10** **a** $x = -8$ or $x = -6$ **b** $x = 0$ or $x = 4$
**c** $x = 0$ or $x = 1$ **d** $x = -8$ or $x = 8$
**e** $x = -3$ or $x = 1$ **f** $x = 4$
**g** $x = 2$ or $x = 3$ **h** $x = -5$
**i** $x = -2$ or $x = 2$

**11** **a** **i** 2 **ii** 1 **iii** 0
**b** **i** The equation is factorised into 2 different linear expressions.
**ii** The equation is factorised into a perfect square.
**iii** The equation cannot be factorised.

**12** **a** 2 **b** 2
**c** 1 **d** 0
**e** 2 **f** 2
**g** 2 **h** 0

**13** **a** $x(x + 8)$ cm$^2$ **b** $x^2 + 8x$ cm$^2$
**c** $x^2 + 8x = 560$
**d** $x = 20$ or $x = -28$. $x = 20$ is the feasible solution and $x = -28$ is not, as it's not possible to have a negative length.
**e** width = 20 cm and length = 28 cm

**14** **a** $x^2 + 2x = 35$ **b** $x = -7$ or $x = 5$
**c** length = 7 m, width = 5 m

**15** $x(x - 12) = 640$, length = 32 cm, width = 20 cm

**16** **a** **i** 8 m **ii** 0 m
**b** 8 m **c** 2 s
**d** It's not possible to have negative time values.

**17** **a** 2 seconds and 6 seconds
**b** The first time is when the rocket is ascending, the second time is when the rocket is descending.

**18** **a** $x = 1$, $x = 3$ or $x = 5$
**b** $x = 0$, $x = -6$ or $x = 6$
**c** $x = -11$ or $x = 4$
**d** $x = 2$, $x = -\frac{1}{3}$ or $x = \frac{3}{2}$

## EX p191 5B Plotting quadratic relationships

**1** **a** **i** $(-3, 0)$, $(1, 0)$
**ii** $(0, -3)$
**iii** $(-1, -4)$
**iv** minimum
**v** $x = -1$

**b** **i** $(0, 0)$, $(4, 0)$
**ii** $(0, 0)$
**iii** $(2, 4)$
**iv** maximum
**v** $x = 2$

**c** **i** $(-2, 0)$, $(2, 0)$
**ii** $(0, 4)$
**iii** $(0, 4)$
**iv** maximum
**v** $x = 0$

**d** **i** $(-1, 0)$, $(5, 0)$
**ii** $(-5, 0)$
**iii** $(2, -9)$
**iv** minimum
**v** $x = 2$

**e** **i** $(-5, 0)$, $(1, 0)$
**ii** $(0, 5)$
**iii** $(-2, 9)$
**iv** maximum
**v** $x = -2$

**f** **i** $(1, 0)$
**ii** $(0, 1)$
**iii** $(1, 0)$
**iv** minimum
**v** $x = 1$

**g** **i** $(0, 0)$
**ii** $(0, 0)$
**iii** $(0, 0)$
**iv** minimum
**v** $x = 0$

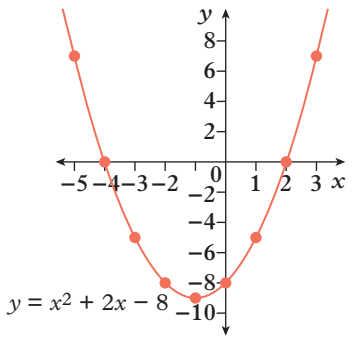
**h** **i** none **ii** $(0, -7)$
**iii** $(-2, -3)$ **iv** maximum
**v** $x = -2$

**i** **i** $(3, 0)$ **ii** $(0, -6)$
**iii** $(3, 0)$ **iv** maximum
**v** $x = 3$

**2** **a** **i**

| $x$ | −5 | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 7 | 0 | −5 | −8 | −9 | −8 | −5 | 0 | 7 |

**ii**



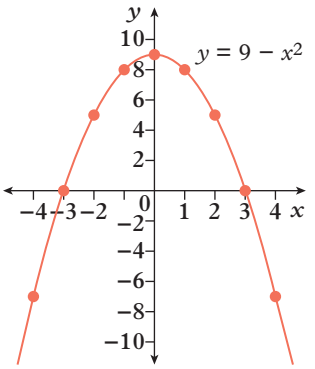
**iii** $x$-intercepts: $(-4, 0)$ and $(2, 0)$
$y$-intercept: $(0, -8)$
turning point: $(-1, -9)$
axis of symmetry: $x = -1$

**iv** minimum

**b i**

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | −7 | 0 | 5 | 8 | 9 | 8 | 5 | 0 | −7 |

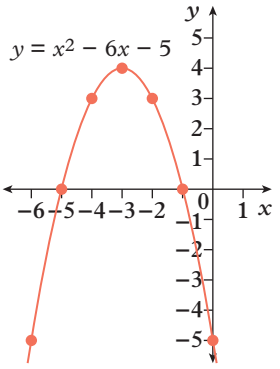
**ii**




**iii** $x$-intercepts: $(-3, 0)$ and $(3, 0)$
$y$-intercept: $(0, 9)$
turning point: $(0, 9)$
axis of symmetry: $x = 0$

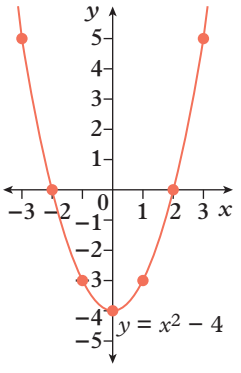
**iv** maximum

**3 a i**



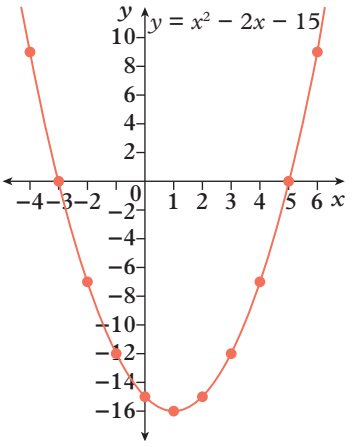

**ii** $x$-intercepts: $(-5, 0)$ and $(-1, 0)$
$y$-intercept: $(0, -5)$
turning point: $(-3, 4)$
axis of symmetry: $x = -3$

**b i**



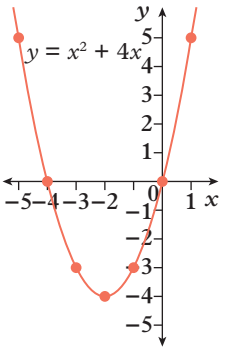

**ii** $x$-intercepts: $(-2, 0)$ and $(2, 0)$
$y$-intercept: $(0, -4)$
turning point: $(0, -4)$
axis of symmetry: $x = 0$

**c i**



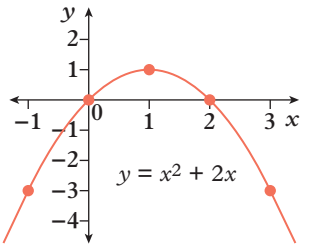

**ii** $x$-intercepts: $(-3, 0)$ and $(5, 0)$
$y$-intercept: $(0, -15)$
turning point: $(1, -16)$
axis of symmetry: $x = 1$

**d i**



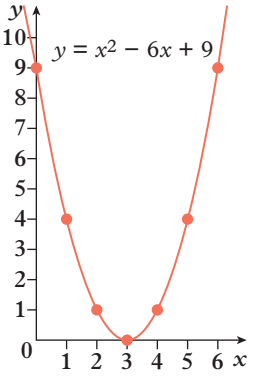

**ii** $x$-intercepts: $(-4, 0)$ and $(0, 0)$
$y$-intercept: $(0, 0)$
turning point: $(-2, -4)$
axis of symmetry: $x = -2$

**e i**




**ii** $x$-intercepts: $(0, 0)$ and $(2, 0)$
$y$-intercept: $(0, 0)$
turning point: $(1, 1)$
axis of symmetry: $x = 1$

**f i**



**ii** $x$-intercept: (3, 0)
$y$-intercept: (0, 9)
turning point: (3, 0)
axis of symmetry: $x = 3$

**4 a i** 2 **ii** 1 **iii** 0

**b** A parabola changes direction only once, so there can only be a maximum of two $x$-intercepts.

**c** one

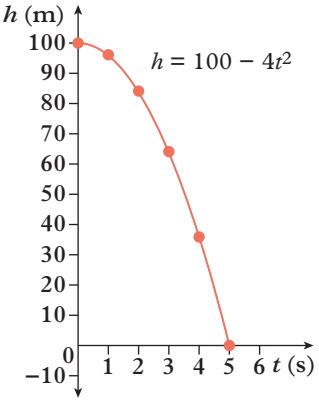
**d** The $y$-coordinate of the $y$-intercept is given by the constant term, $c$, in the equation.

**5 a i** no $x$-intercepts **ii** 1 $y$-intercept

**b i** 1 $x$-intercept **ii** 1 $y$-intercept

**6 a** 20 m **b** 26 m **c** 5 s

**7 a**




**b** It's not possible to have negative time values and it is not possible for the hairclip to fall below ground level.

**c i** 84 m **ii** 64 m

**d** 100 m **e** 5 s

**8 a** (–2, 0) and (5, 0) **b** $x = -2$ or $x = 5$

**c** They are the same.

**d** The $x$-intercepts of a parabola represent the solutions to a quadratic equation.

**9 a** $x = 5$ **b** $x = 0$ **c** $x = -4$

**10 a** $x = 4$ and $x = 5$

**b** $x = 3$ and $x = 8$

**c** $x = -3$ and $x = -0.5$

**d** $x = 3$

**11 a** $x = 1$ **b** (1, 10)

**12** (0, 0)

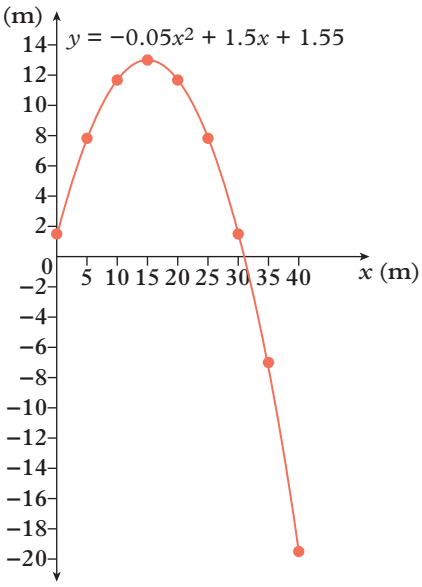
**13 a** $x$-intercepts: $(p, 0)$ and $(q, 0)$

**b** $y = (x + 7)(x - 5)$

**c** $y = -36$

**d** $y = -(x + 7)(x - 5)$

**14 a**




**b** 1.55 m

**c** $x = -1$ and $x = 31$

**d** 31 m

**e** 12.8 m

EX p198

## 5C Sketching parabolas using intercepts

**1 a i** (–7, 0) and (5, 0) **ii** (0, –35)

**b i** (2, 0) and (6, 0) **ii** (0, 36)

**c i** (–4, 0) and (–2, 0) **ii** (0, 96)

**2 a i** (0, 0) and (2, 0) **ii** (0, 0)

**b i** (–8, 0) and (0, 0) **ii** (0, 0)

**c i** (–4, 0) and (–2, 0) **ii** (0, 8)

**d i** (2, 0) and (6, 0) **ii** (0, 12)

**e i** (–1, 0) and (5, 0) **ii** (0, –5)

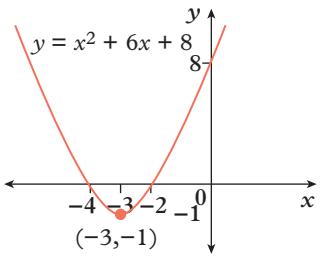
**f i** (–3, 0) and (3, 0) **ii** (0, –9)

**3 a** (2, –4) **b** (4, –1) **c** (1, –25) **d** (–2, 1)

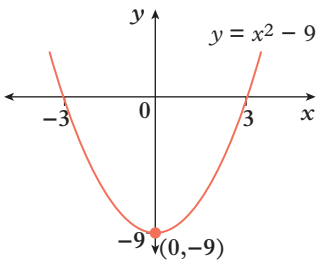
**e** (0, –4) **f** (1, 9) **g** $\left(-\frac{3}{2}, -\frac{9}{4}\right)$ **h** $\left(-\frac{7}{2}, \frac{49}{4}\right)$

**i** (5, –18)

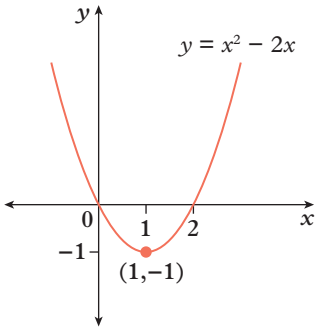
**4 a**



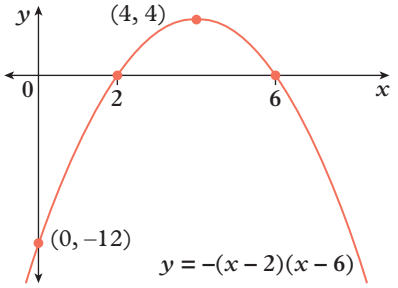

**b**



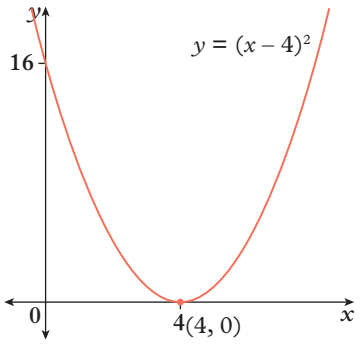
**c**



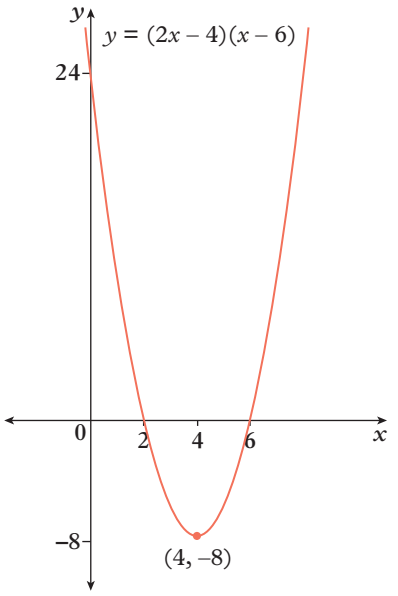

**d**




**e**




**f**

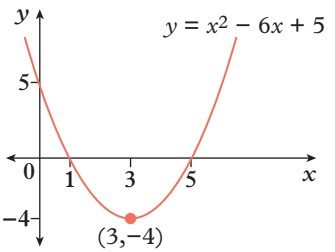



**5** **a** Both graphs have $x$-intercepts at $(-3, 0)$ and $(1, 0)$.

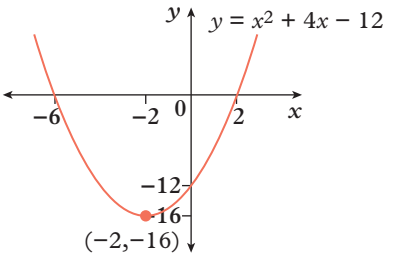
**b** $y = (x + 3)(x - 1)$ has turning point at $(-4, 0)$ and $y = -(x + 3)(x - 1)$ has turning point at $(4, 0)$.

**c** Both graphs have the same $x$-intercepts and the same shape, but they are reflections of each other in the $x$-axis. So $y = (x + 3)(x - 1)$ has a minimum turning point and $y = -(x + 3)(x - 1)$ has a maximum turning point.

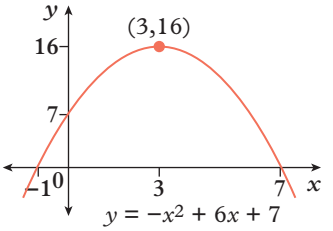
**6** **a**



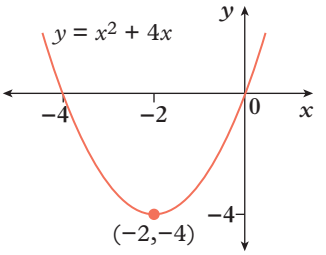

**b**



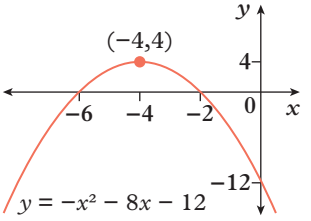

**c**



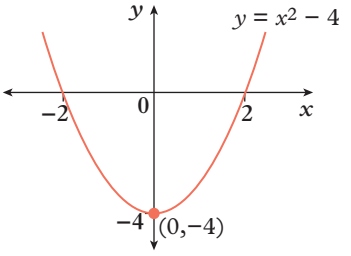

**d**



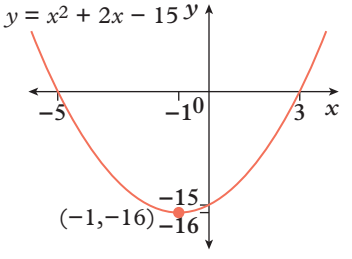

**e**




**f**




**g**



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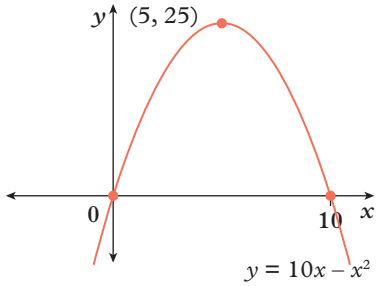
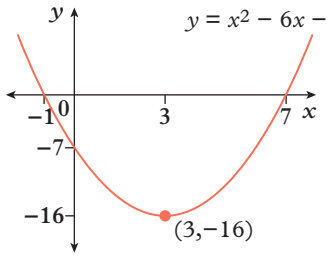
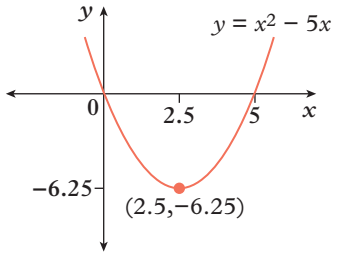
**h**



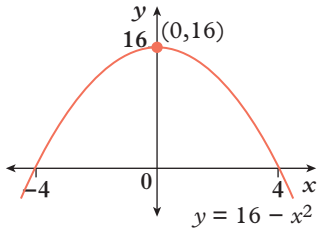

**i**



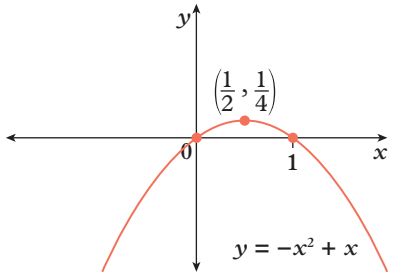

**j**




**k**




**l**



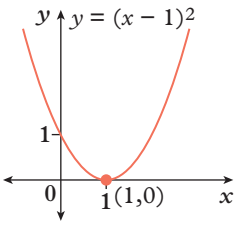

**7** **a** C **b** A **c** B

**8** Look at coefficient of $x^2$ term ($a$). If $a > 0$, parabola is upright, e.g. $y = x^2 + x + 6$ produces an upright parabola. If $a < 0$, parabola is inverted, e.g. $y = -x^2 - x + 6$ produces an inverted parabola.

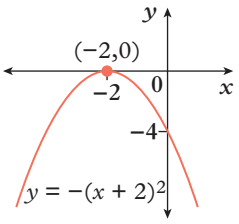
**9** **a** upright **b** upright **c** inverted
**d** inverted **e** upright **f** inverted

**10** **a** upright
**b** one (0, 9)
**c** one (3, 0)
**d** (3, 0)
**e** The turning point is also the $x$-intercept.
**f** The negative value of $p$ is the $x$-coordinate of the turning point. The turning point is also the $x$-intercept, so the $y$-coordinate of the turning point will be 0.

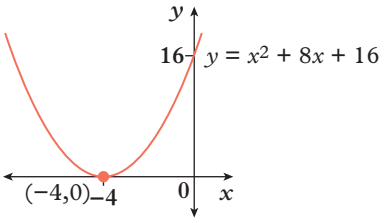
**11** **a**



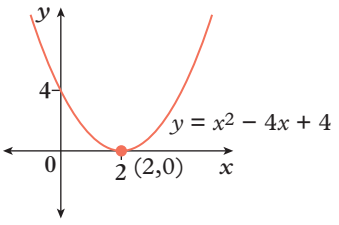

**b**



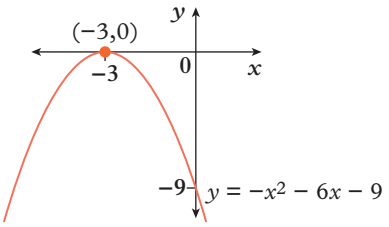

**c**




**d**

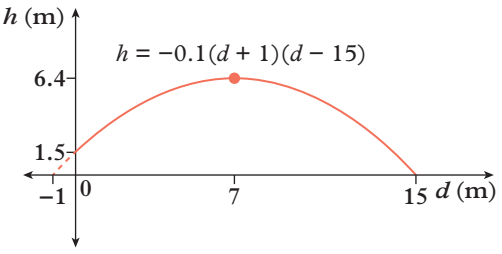



**e**




**12** **a** inverted **b** one (0, −1)
**c** none **d** (0, −1)
**e** The value of $c$ is the $y$-coordinate of the turning point. The turning point is also the $y$-intercept, so the $x$-coordinate of the turning point will be 0.

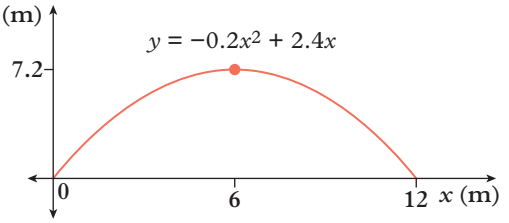
**13** **a**



**b** The graph represents the path of the arrow from $d = 0$ m to $d = 15$ m.

**c** 6.4 m **d** 1.5 m **e** 15 m

**14 a**



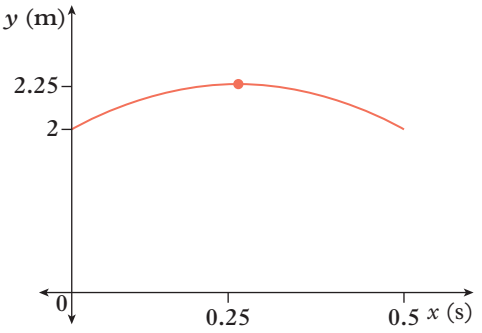

**b** 7.2 m **c** 6 m **d** 12 m

**15 a** 2 m

**b** (0, 2) and (0.5, 2), The ball is thrown and caught from the same height at 2 m.

**c** (0.25, 2.25)

**d**




**e** 2.25 m

**16 a** $y = \frac{1}{2}(x - 6)(x + 1)$

**b** $y = -2(x - 6)(x + 2)$

**17** $y = \frac{3}{8}(x - 3)^2$

## Checkpoint

**1 a** $x = 3$ or $x = 4$ **b** $x = -2$ or $x = 2$

**c** $x = -9$ **d** $x = 0$ or $x = -4$

**2 a** $x = 5$ or $x = 7$ **b** $x = -7$ or $x = 4$

**c** $x = 6$ **d** $x = -9$ or $x = 9$

**3 a** $x = -4$ or $x = 4$ **b** $x = -2$ or $x = 4$

**c** $x = -5$ or $x = -1$ **d** $x = 0$ or $x = 15$

**4 a** $x(x + 4)$ m² **b** $x(x + 4) = 60$

**c** $x = -10$ or $x = 6$. $x = 6$ is the feasible solution as it is not possible to have a negative length.

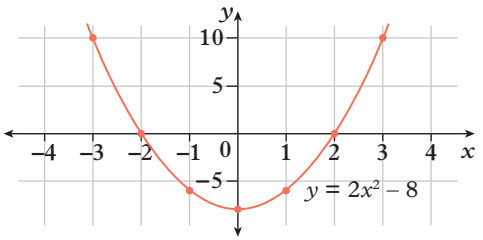
**d** width = 6 m, length = 10 m

**5 a** (2, 0) and (–4, 0) **b** (0, 8)

**c** (−1, 9) **d** maximum

**e** $x = -1$

**6**




**7 a** 16°C **b** 25°C **c** 3 pm **d** 8 pm

**8 a** $x = 5$ **b** (5, 0)

**9 a i** (0, 0) and (−3, 0) **ii** (0, 0)

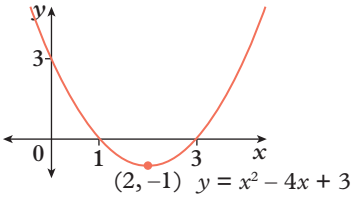
**b i** (−5, 0) and (5, 0) **ii** (0, −50)

**c i** (−4, 0) and (2, 0) **ii** (0, −8)

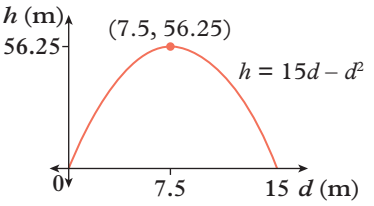
**d i** (8, 0) **ii** (0, 64)

**10 a** (−1, −9) **b** (2, −4) **c** (0, −36) **d** (10, 4)

**11**




**12 a**




**b** 15 m **c** 56.25 m

## EX 5D Sketching parabolas using transformations

p206

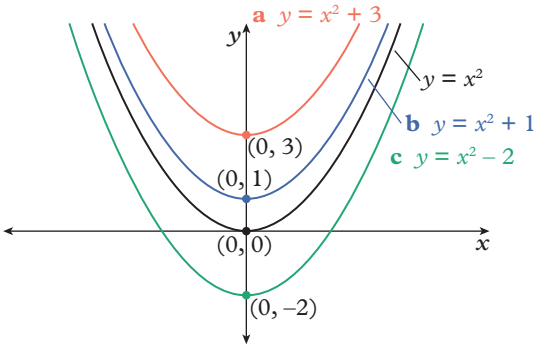
**1 a** 10, down

**b** 5, right

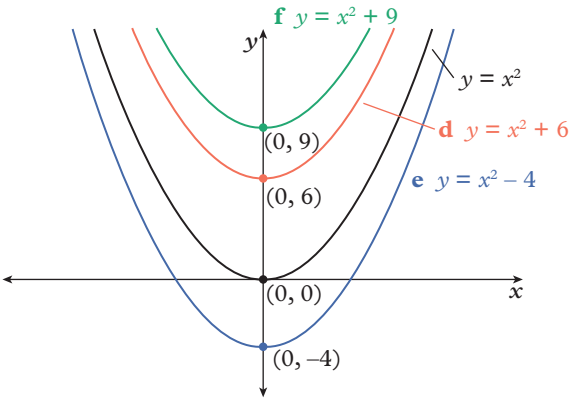
**c** 12, left

**d** 8, up

**2 a–c**




**d–f**



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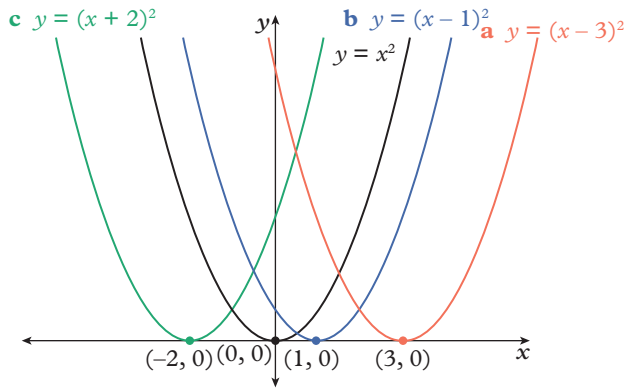
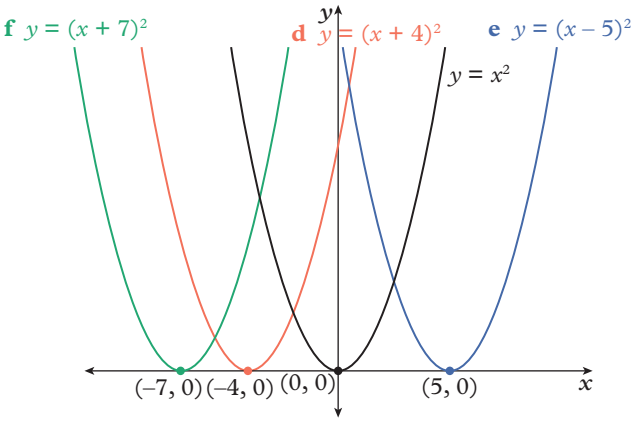
**3** **a–c**

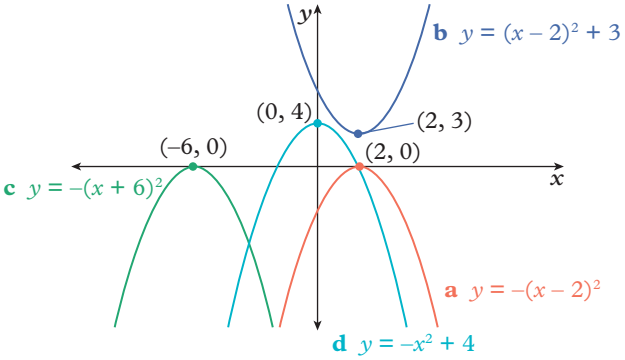



**d–f**

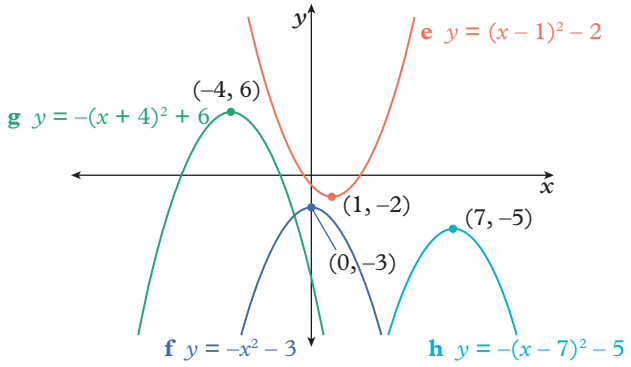



**4** **a** 2, left, 3, down **b** reflected, 4, up

**5** **a–d**




**e–h**

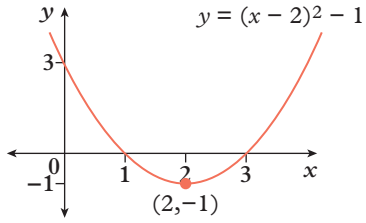



**6** **a** **i** upright

**ii** $x$-intercepts: (1, 0) and (3, 0); $y$-intercept: (0, 3)

**iii** (2, −1)

**iv**

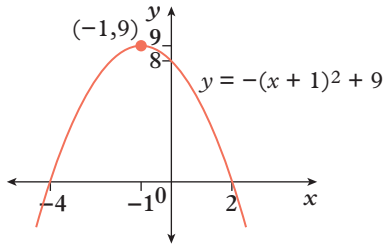



**b** **i** inverted

**ii** $x$-intercepts: (−4, 0) and (2, 0); $y$-intercept: (0, 8)

**iii** (−1, 9)

**iv**

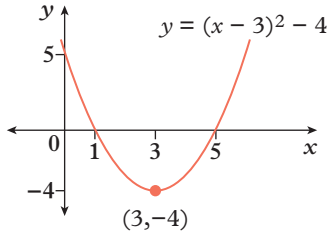



**c** **i** upright

**ii** $x$-intercepts: (1, 0) and (5, 0); $y$-intercept: (0, 5)

**iii** (3, −4)

**iv**

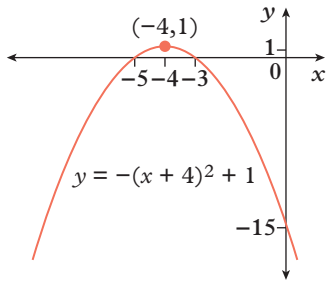



**d** **i** inverted

**ii** $x$-intercepts: (−5, 0) and (−3, 0); $y$-intercept: (0, −15)

**iii** (−4, 1)

**iv**




**7** **a** **i** upright **ii** (1, −4)
**iii** $a$ is positive **iv** $h = 1, k = -4$
**b** **i** inverted **ii** (−3, 1)
**iii** $a$ is n--egative **iv** $h = -3, k = 1$
**c** **i** upright **ii** (3, −9)
**iii** $a$ is positive **iv** $h = 3, k = -9$

**8** **a** E **b** D **c** A **d** F **e** C
**f** B **g** H **h** I **i** G

**9** **a** $y = (x - 3)^2 + 7$ **b** $y = (x + 2)^2 + 5$

**c** $y = -(x - 2)^2 + 4$ **d** $y = -(x - 6)^2 - 1$

**e** $y = (x - 9)^2$ **f** $y = -x^2 + 4$

**g** $y = -(x + 1)^2 - 2$ **h** $y = x^2 - 5$

**10** **a** (4, 5) **b** 5

**c** There is no maximum value for $y$ because the parabola is upright (it has a minimum turning point).

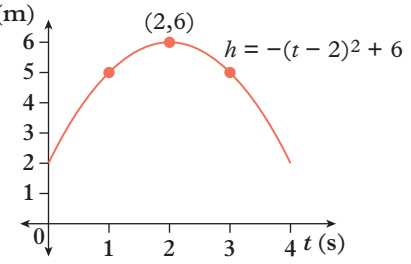
**11** Graph is inverted so it has a maximum turning point. The $y$-coordinate of the turning point is 2, so the maximum $y$-value is 2.

**12** **a** (2, 6)

**b** **i** 2 m

**ii** 2 m

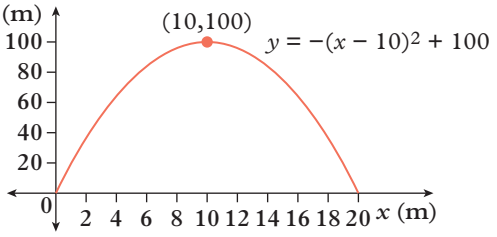
**c**




**d** **i** 2 m

**ii** 6 m

**13** **a**




**b** 100 m **c** 20 m

**14** **a** $a = 1$

**b** $y = 2x^2$, $a = 2$

**c** $y = \frac{1}{2}x^2$, $a = \frac{1}{2}$

**d** If $a > 1$, the graph will be narrower than $y = x^2$. If $0 < a < 1$, the graph will be wider than $y = x^2$.

**15** **a** E **b** A **c** D **d** C **e** F **f** B

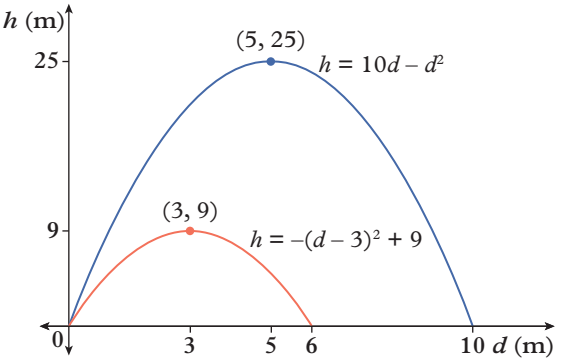
**16**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **a** | **i** | narrower | **ii** | upright | **iii** | (4, −3) |
| **b** | **i** | wider | **ii** | upright | **iii** | (−2, 6) |
| **c** | **i** | narrower | **ii** | upright | **iii** | (−2, 0) |
| **d** | **i** | narrower | **ii** | inverted | **iii** | (0, −4) |
| **e** | **i** | wider | **ii** | inverted | **iii** | (7, 0) |
| **f** | **i** | narrower | **ii** | inverted | **iii** | (−1, 5) |
| **g** | **i** | wider | **ii** | inverted | **iii** | (3, −4) |
| **h** | **i** | wider | **ii** | upright | **iii** | (0, 3) |
| **i** | **i** | the same | **ii** | inverted | **iii** | (5, 4) |
| **j** | **i** | wider | **ii** | upright | **iii** | (−3, −7) |

**17** **a** $y = 3(x - 2)^2$ **b** $y = -(x + 5)^2 - 4$

**c** $y = -\frac{1}{2}x^2$ **d** $y = -4(x + 2)^2$

**18** **a, e**




**b** 9 m

**c** 6 m

**d** $h = -(d - 5)^2 + 25$

**f** 16 m

## EX p214 5E Circles and other non-linear relationships

**1** **a** **i** (0, 0) **ii** 2

**b** **i** (0, 0) **ii** 9

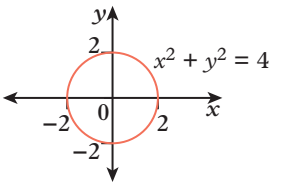
**c** **i** (1, 0) **ii** 2

**d** **i** (0, 3) **ii** 2

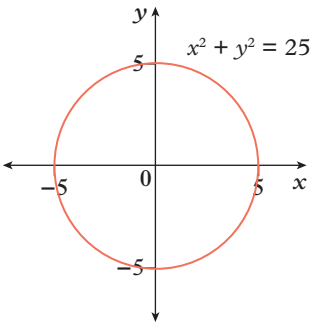
**e** **i** (−3, 5) **ii** 5

**f** **i** (−4, −2) **ii** 4

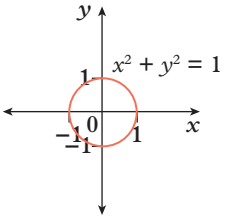
**2** **a**



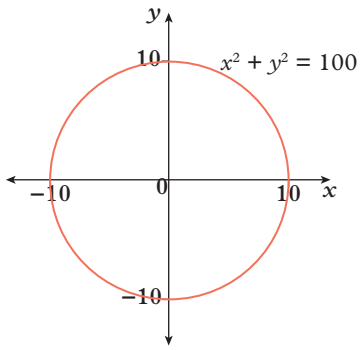

**b**



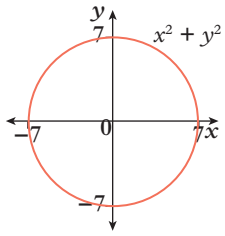

**c**



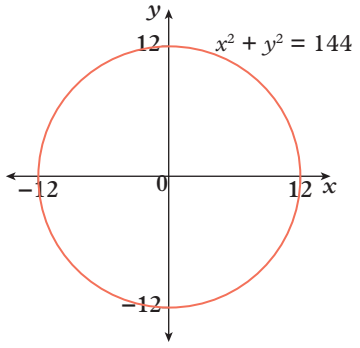
**d**



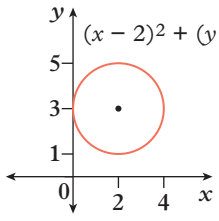

**e**



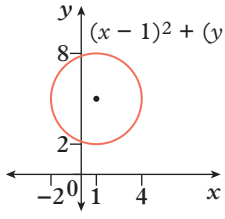

**f**



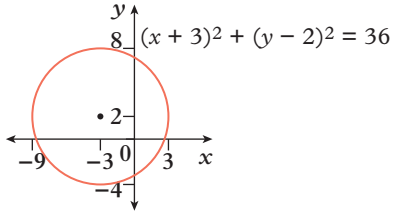

**3 a**



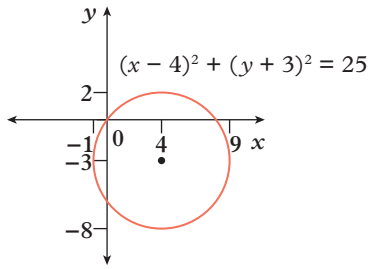

**b**



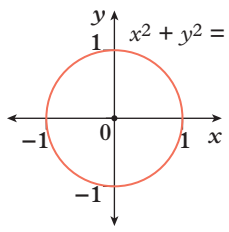

**c**



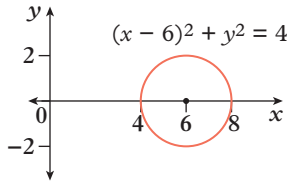

**d**



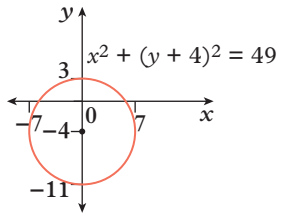

**e**



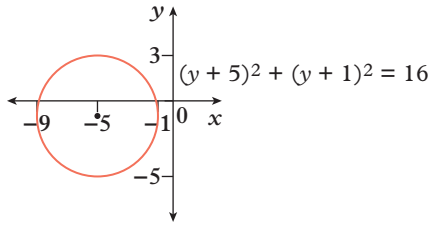

**f**




**g**




**h**




**4 a** $(x - 3)^2 + (y - 5)^2 = 16$

**b** $(x + 2)^2 + (y - 4)^2 = 25$

**c** $(x + 7)^2 + (y + 6)^2 = 81$

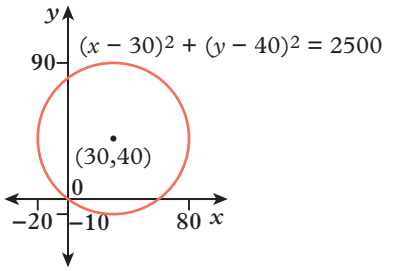
**d** $(x - 4)^2 + (y + 8)^2 = 121$

**5 a** centre (4, 2); radius = 2 units
$(x - 4)^2 + (y - 2)^2 = 4$

**b** centre (−3, 3); radius = 1 unit
$(x + 3)^2 + (y - 3)^2 = 1$

**c** centre (−5, −2); radius = 3 units
$(x + 5)^2 + (y + 2)^2 = 9$

**6 a**




**b** 314 m **c** 7854 m²

**7 a** Let Length = 1:
$(x_1,y_1) = (0,0)$, and $(x_2,y_2) = (x,y)$
then square both sides: $x^2 + y^2 = 1$

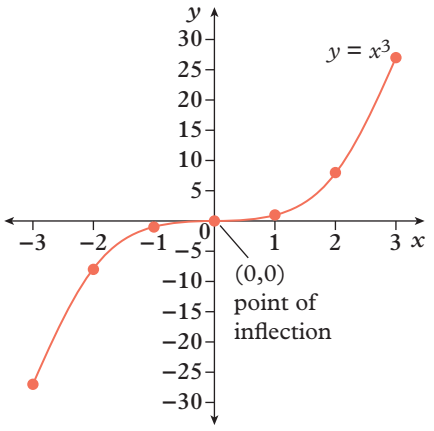
**b** The length formula becomes the general equation for a circle on a Cartesian plane: $(x - h)^2 + (y - k)^2 = r^2$

**c** A circle is a set of points that are an equal distance (or length) from a central point. So if the length of a line segment remains constant while the coordinates for one endpoint are variable, then the formula for length of a line segment will become the equation for a circle.

**8 a**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −27 | −8 | −1 | 0 | 1 | 8 | 27 |

**b**




**c** As $x$ increases, $y$ increases. Graph appears to level out at $x = 0$ before rising more steeply again.

**d i** (0, 2) **ii** translate graph 2 units up

**e i** translate 1 unit up

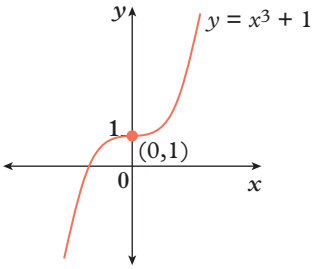
**ii** translate 3 units down

**iii** translate 2 units right

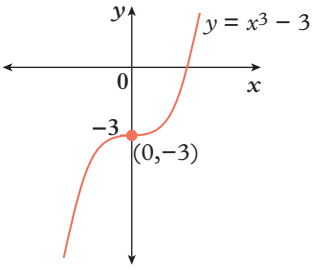
**iv** translate 4 units left

**v** translate 1 unit right and 2 units up

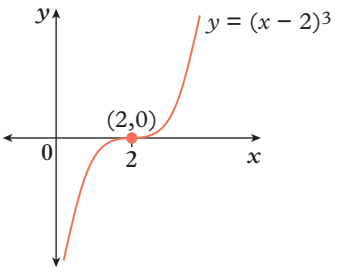
**f i**



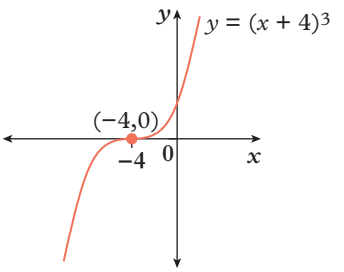

**ii**



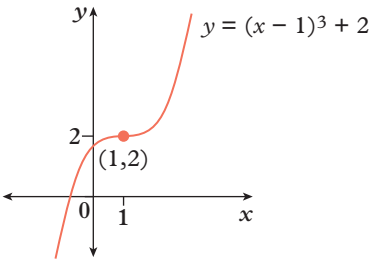

**iii**




**iv**



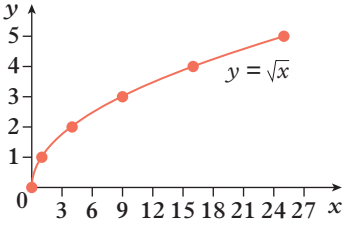

**v**




**9 a**

| $x$ | 0 | 1 | 4 | 9 | 16 | 25 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 | 4 | 5 |

**b, d**




No; not possible to take the square root of a negative number.

**c** As $x$ increases, $y$ increases, starting off steep and becoming less steep. The graph starts at the origin and extends into the first quadrant.

**e i** (3, 0) **ii** translate 3 units to right

**f** **i** translate 2 units up

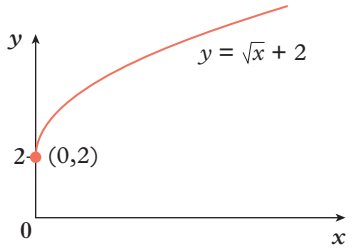
**ii** translate 1 unit down

**iii** translate 1 unit right

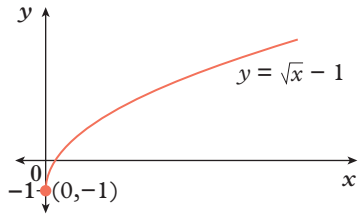
**iv** translate 4 units left

**v** translate 2 units right and 3 units up

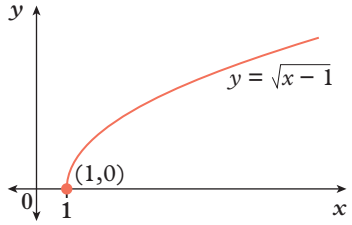
**g** **i**



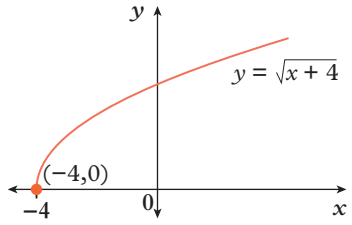

**ii**



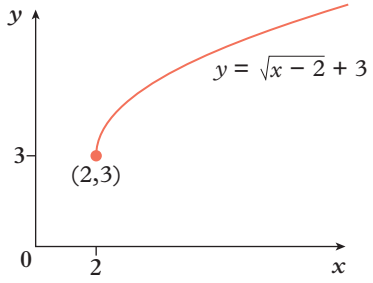

**iii**




**iv**



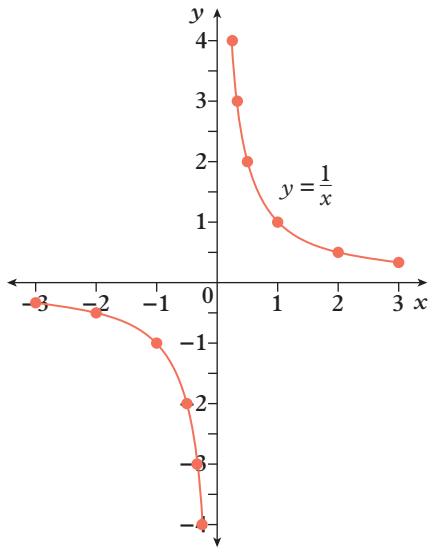

**v**




**10 a**

| $x$ | −3 | −2 | −1 | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-\frac{1}{3}$ | $-\frac{1}{2}$ | −1 | −2 | −3 | −4 | 4 | 3 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ |

**b**




**c** $y$ is undefined when $x = 0$. As $x$ approaches 0 from the left, the graph approaches $-\infty$ (very small $y$-values) and as $x$ approaches 0 from the right, the graph approaches $+\infty$ (very large $y$-values).

**d** $x$ is undefined when $y = 0$ because the equation is $x = \frac{1}{y}$. As $y$ approaches 0 from below, the graph approaches $-\infty$ (very small negative $y$-values) and as $y$ approaches 0 from above, the graph approaches $+\infty$ (very small positive $y$-values).

**e** **i** $y = 0$ **ii** $x = 0$

**f** **i** $y = 0$ (horizontal) and $x = 2$ (vertical)

**ii** translate 2 units right

**g** **i** translate 2 units up

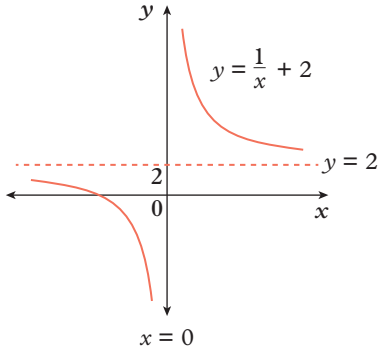
**ii** translate 3 units down

**iii** translate 4 units right

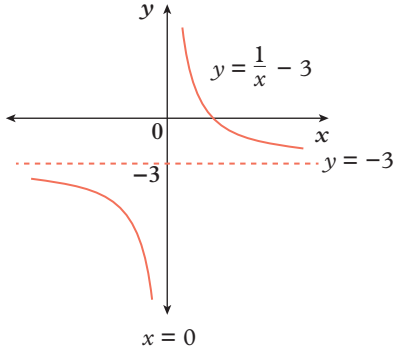
**iv** translate 1 unit left

**v** translate 3 units right and 2 units up

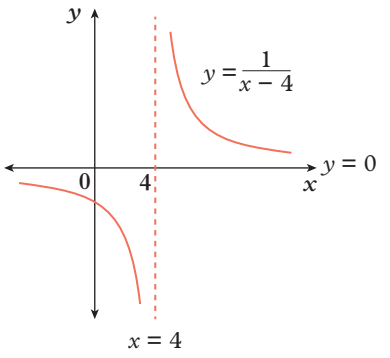
**h** **i**



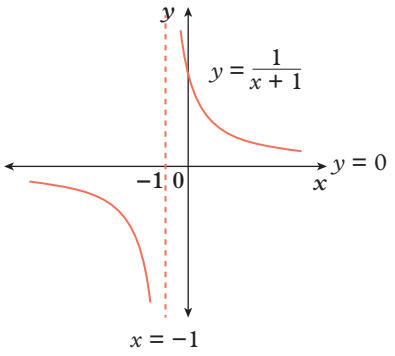

**ii**



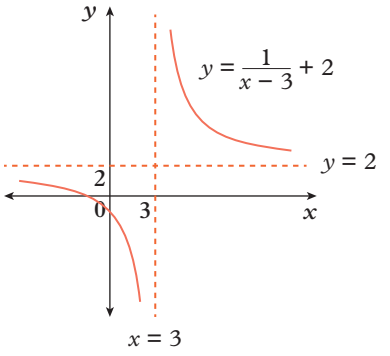
**iii**



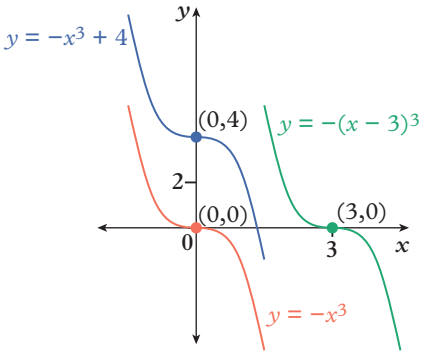

**iv**



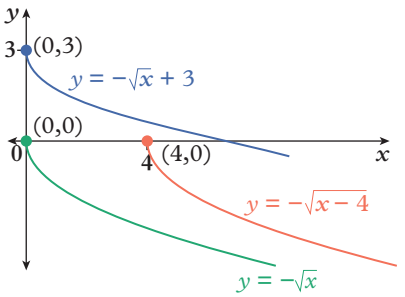

**v**



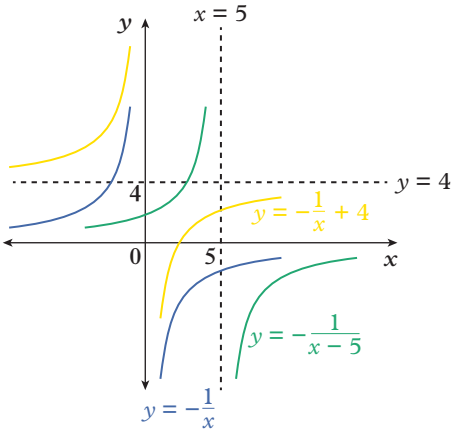

**11 a**



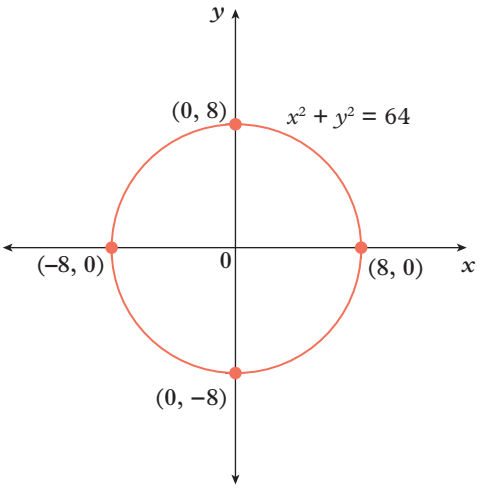

**b**



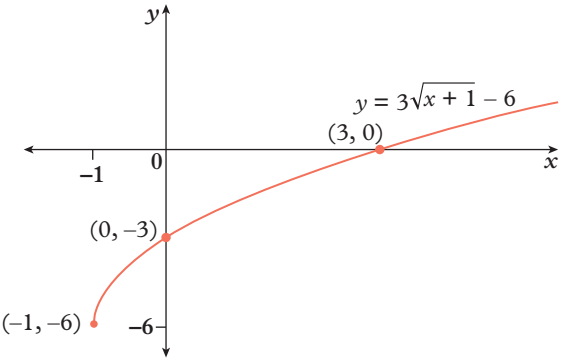

**c**



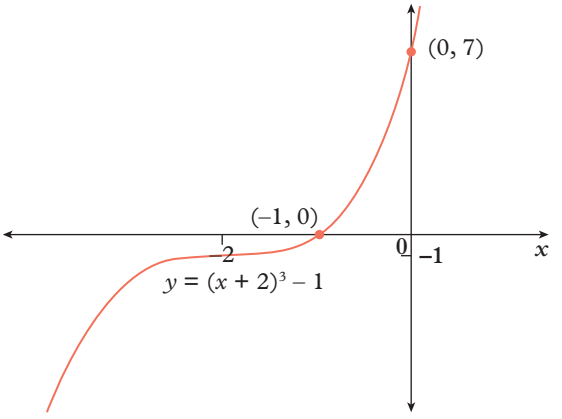

**12 a**



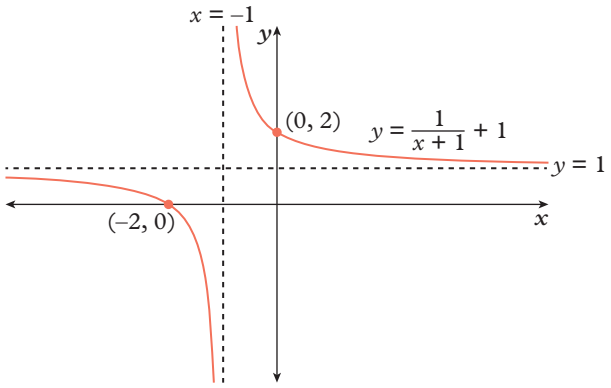

**b**



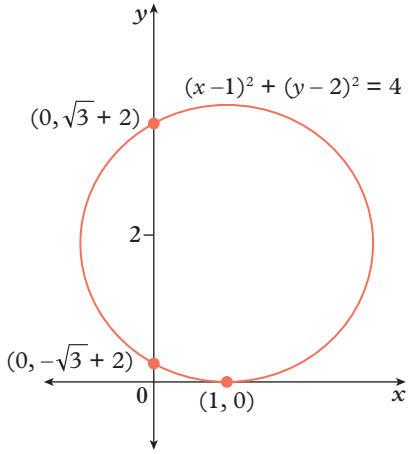

**c**



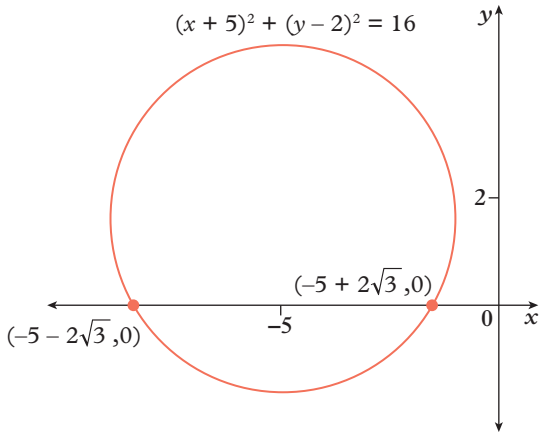
**d**




**13 a**

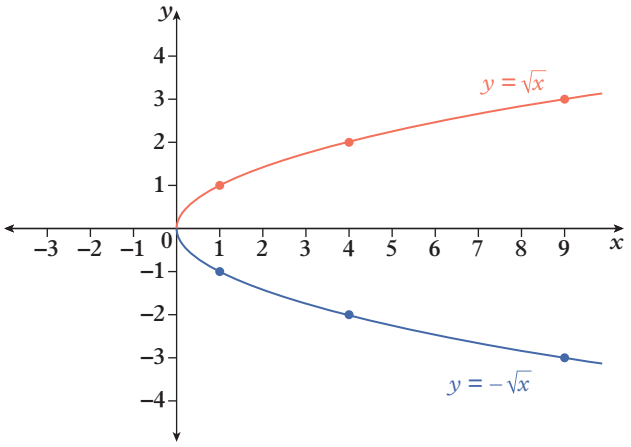



**b**



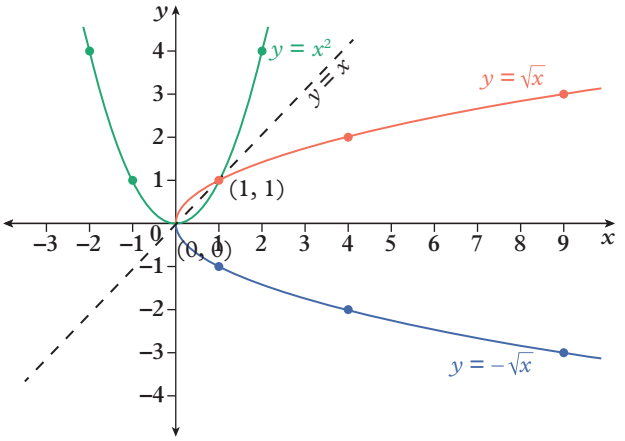

**14** $(x-2)^2 + (y-1)^2 = 16$

**15 a**



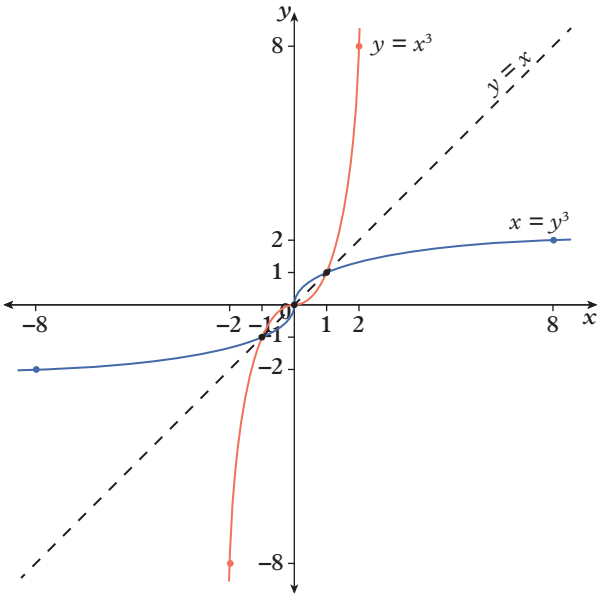

**b** $y = \pm\sqrt{x}$ or $y^2 = x$

**c, d**

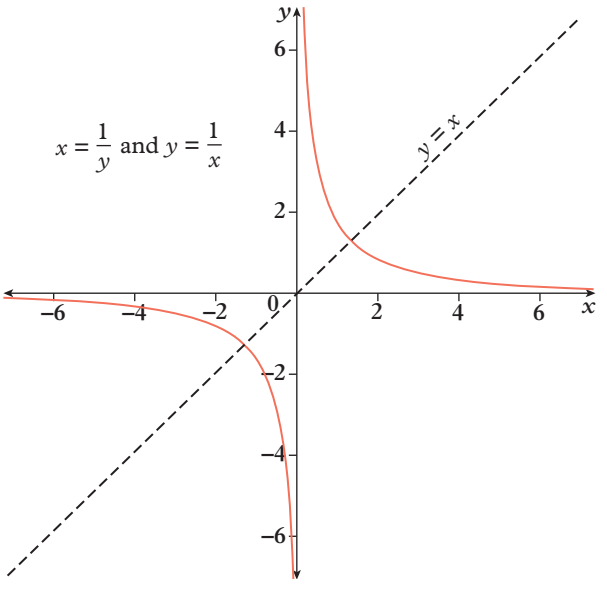



**e** The graph of $y^2 = x$ and $y = x^2$ are reflected in the line $y = x$, or the graph of $y = x^2$ is rotated 90 degrees clockwise to make $y^2 = x^2$.

**16 a**




**b**



## Chapter review

### Multiple-choice

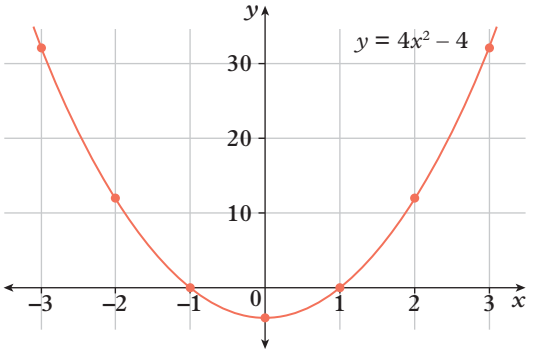
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | D | **2** | A | **3** | A | **4** | D | **5** | C |
| **6** | C | **7** | E | **8** | B | **9** | C | **10** | B |

### Short answer

**1** **a** $x = 2$ or $x = 3$ **b** $x = -6$ or $x = 5$

**c** no solutions **d** $x = 0$ or $x = 12$

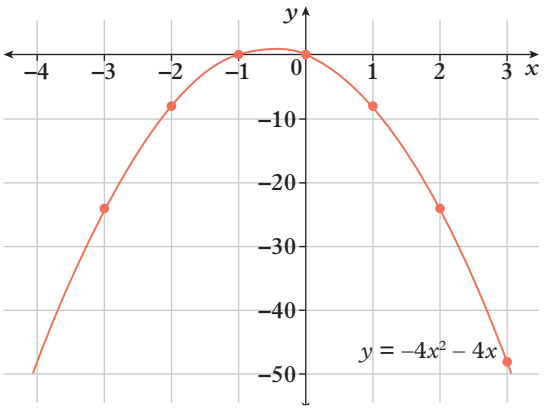
**2** **a**




**i** $x$-intercepts: $(-1, 0)$ and $(1, 0)$, $y$-intercept: $(0, -4)$

**ii** $(0, -4)$ **iii** minimum **iv** $x = 0$

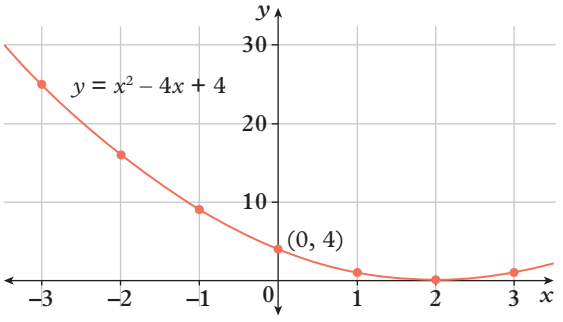
**b**




**i** $x$-intercepts: $(-1, 0)$ and $(0, 0)$, $y$-intercept: $(0, 0)$

**ii** $(-0.5, 1)$ **iii** maximum **iv** $x = -0.5$

**c**



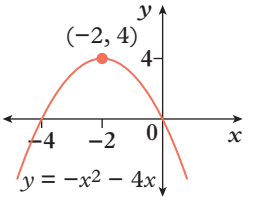

**i** $x$-intercept: $(2, 0)$, $y$-intercept: $(0, 4)$

**ii** $(2, 0)$ **iii** minimum **iv** $x = 2$

**3** **a** **i** $x$-intercepts: $(-4, 0)$ and $(0, 0)$, $y$-intercept: $(0, 0)$

**ii** $(-2, 4)$

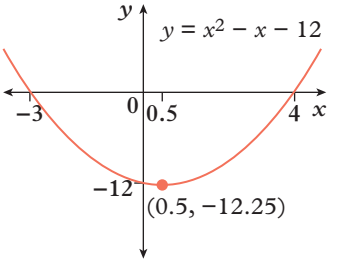
**iii**




**b** **i** $x$-intercepts: $(-3, 0)$ and $(4, 0)$, $y$-intercept: $(0, -12)$

**ii** $(0.5, -12.25)$

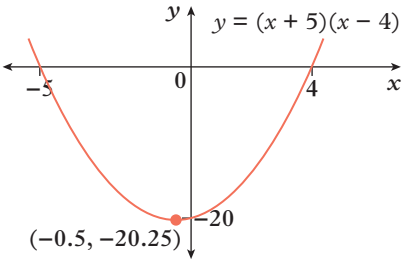
**iii**




**c** **i** $x$-intercepts: $(-5, 0)$ and $(4, 0)$, $y$-intercept: $(0, -20)$

**ii** $(-0.5, -20.25)$

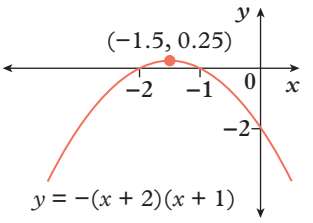
**iii**



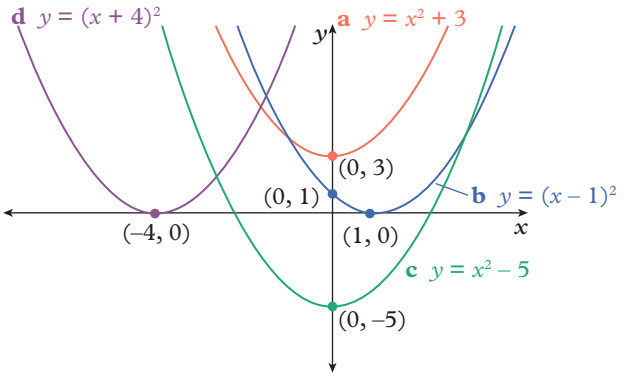

**d** **i** $x$-intercepts: $(-2, 0)$ and $(-1, 0)$, $y$-intercept: $(0, -2)$

**ii** $(-1.5, 0.25)$

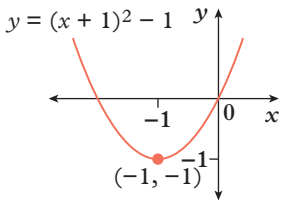
**iii**



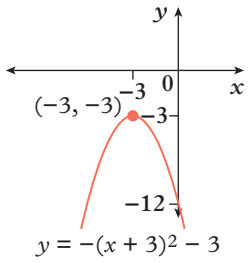

**4** **a–d**



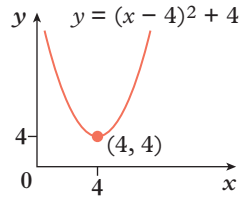
**5 a**



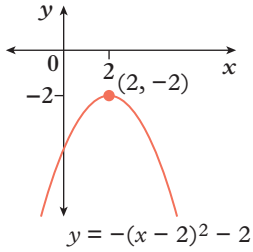

**b**



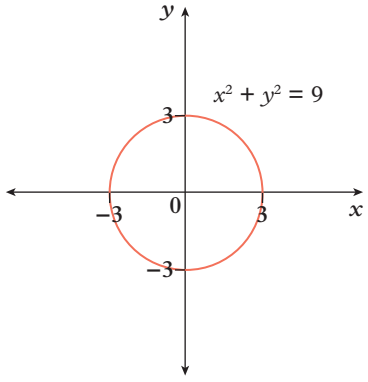

**c**



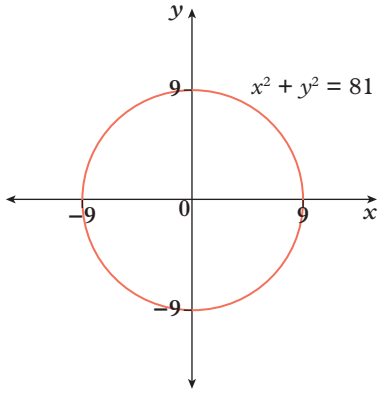

**d**



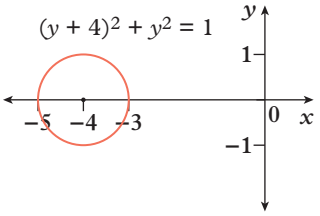

**6 a**



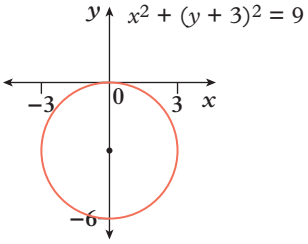

**b**



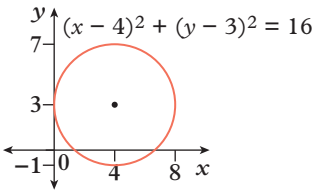

**7 a** centre (−4, 0); radius = 1 unit



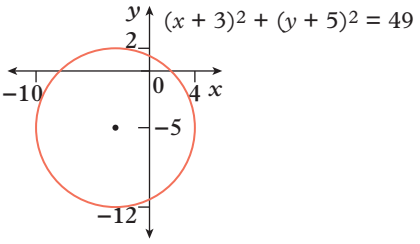

**b** centre (0, −3); radius = 3 units




**c** centre (4, 3); radius = 4 units



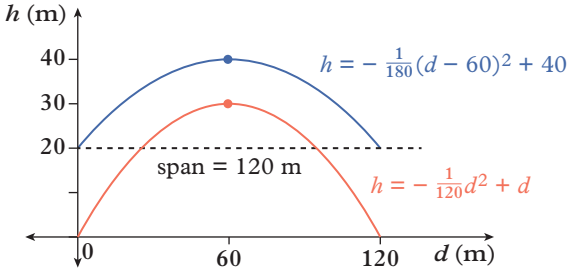

**d** centre (−3, −5); radius = 7 units




**8 a** B
**b** D
**c** A
**d** C

## Analysis

**a, d(i)**



**b** **i** upper arch: (0, 20), lower arch: (0, 0)

**ii** upper arch: (60, 40), lower arch: (60, 30)

**iii** Both have the same axis of symmetry, $d = 60$.

**c** 20 m

**d** **i** 120 m

**ii** Sample answer: Let $h = 0$

$$-\frac{1}{120}d^2 + d = 0$$

$$-d\left(\frac{1}{120}d - 1\right) = 0$$

$d = 0$ or $d = 120$

So, span = 120 m

**e** **i** 30 m **ii** 40 m

**f** 10 m

**g** 12.5 m

**h** The two arches are 20 m apart at the start. Approaching the highest points on both arches, the distance between the two arches decreases until it is a minimum of 10 m apart at the top of the arch. Approaching the far right side of the bridge, the distance between the two arches increases until they are a distance of 20 m apart at the end of the bridge.

EX p222

## SEMESTER 1 Review

### Short answer

**1** **a** 20.040 **b** 20.0

**c** 0.0008 **d** $7.827 \times 10^{-4}$

**2** **a** \$575

**b** 0.64 h or 38 min 24 sec

**c** \$828

**d** $y = 270$

**3** **a** \$9/kg **b** \$3300/fortnight

**c** 80 km/h **d** $k = 5.25$

**4** **a** \$23.45 **b** \$111.60 **c** \$71.99 **d** \$15 000

**5** **a** 25% **b** 625% **c** 125% **d** 108.3%

**6** **a** \$3240 **b** 11.67%

**c** \$2166.67 **d** 2 months

**7** **a** $3^4$ **b** $2^2 \times 5^3$

**c** $2^3 \times 3^3$ **d** $2^2 \times 3^3 \times 5^2$

**8** **a** 343 **b** 375

**c** 82 130 **d** 0.010 530

**e** $\frac{27}{8} = 3.375$ **f** $\frac{49}{25} = 1.96$

**9** **a** $2^{30} \times 7^{18}$ **b** $\frac{11^8}{17^9}$

**c** $\frac{1}{97^4 \times 101^{12}}$ **d** $41^{66} \times 57^{16}$

**10** **a** $a^{16}b^{14}$ **b** $\frac{e}{c^{10}}$ **c** $\frac{g^4}{f}$ **d** $\frac{q^{51}}{p^4}$

**11** **a** 6 **b** 2 **c** 6 **d** 5

**12** **a** $7.514 \times 10^8$

**b** $2.5360 \times 10$ or $2.5360 \times 10^1$

**c** $8.6 \times 10^{-5}$

**d** $7.50 \times 10^4$

**13** **a** $56 - 16x$ **b** $-6x^5 + 16x^3$

**c** $x^2 - 5x - 14$ **d** $10xy + 15x - 4y - 6$

**e** $-12x - 23$ **f** $5x^2 + 6x + 6$

**14** **a** $6(2a + 3b^2 - cd)$ **b** $7g(3g - 1)$

**c** $(a - 5)(a + 5)$ **d** $2(2m - 3n)(2m + 3n)$

**e** $(t - 5)(t + 9)$ **f** $(r - 4)(r + 9)$

**15** **a** $\frac{1}{2}$ **b** 0

**c** $-3$ **d** undefined

**e** $-\frac{3}{2}$

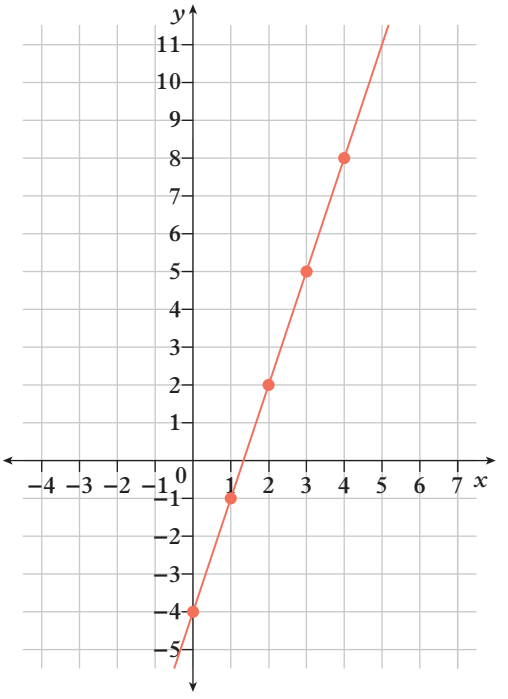
**16** **a** $x = \frac{11}{12}$ **b** $x = -3$

**c** $x = 8$ **d** $x = 7$

**e** $x = \frac{4}{5}$ **f** $x = -3, x = 7$

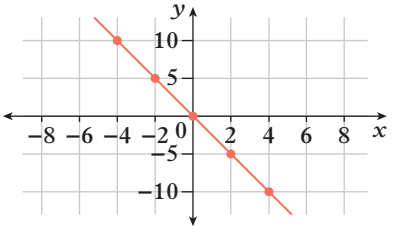
**g** $x = \frac{1}{9}, x = -\frac{3}{5}$ **h** $x = -5, x = -7$

**17** **a**



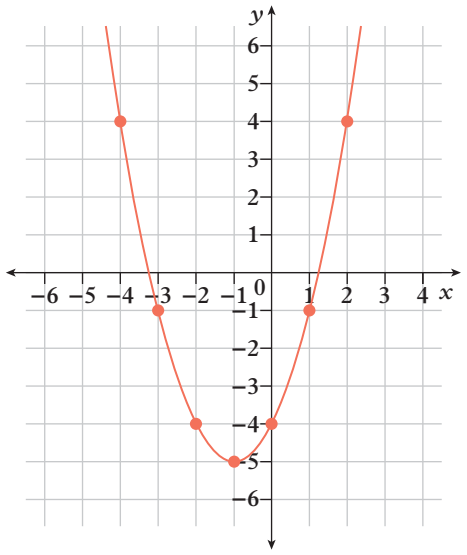

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | −4 | −1 | 2 | 5 | 8 |

**b**



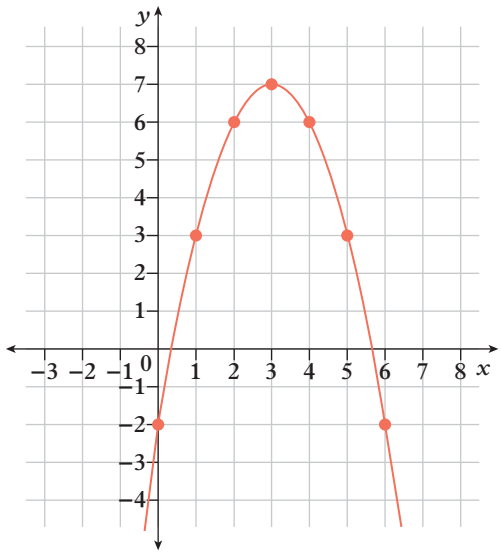

| $x$ | −4 | −2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| $y$ | 10 | 5 | 0 | −5 | −10 |

**c**



| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $y$ | 4 | −1 | −4 | −5 | −4 | −1 | 4 |

**d**



| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | −2 | 3 | 6 | 7 | 6 | 3 | −2 |

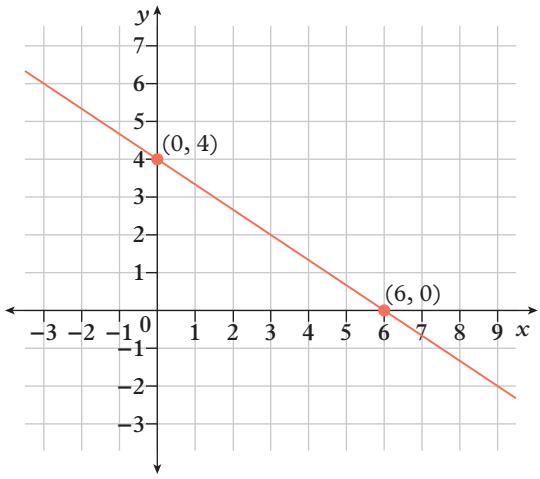
**18 a** $(2,0),(0,-6)$

**b** $(-12,0),\left(0,-\frac{15}{2}\right)$

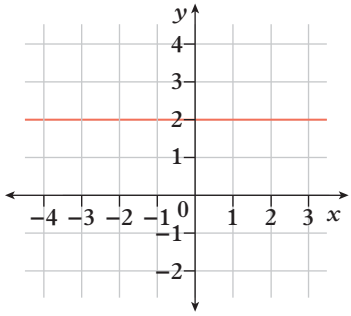
**c** $(5,0),(-9,0),(0,-45)$

**d** $(10,0),(-4,0),(0,-40)$

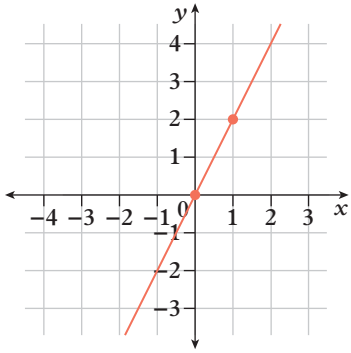
**19 a**



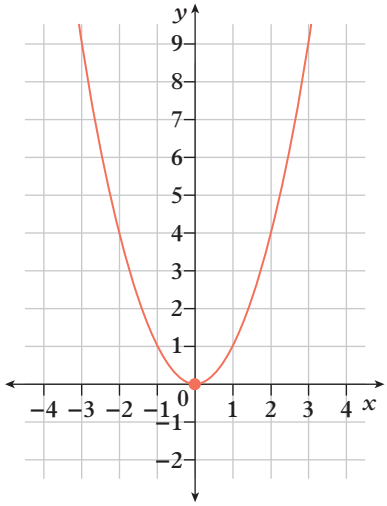

**b**



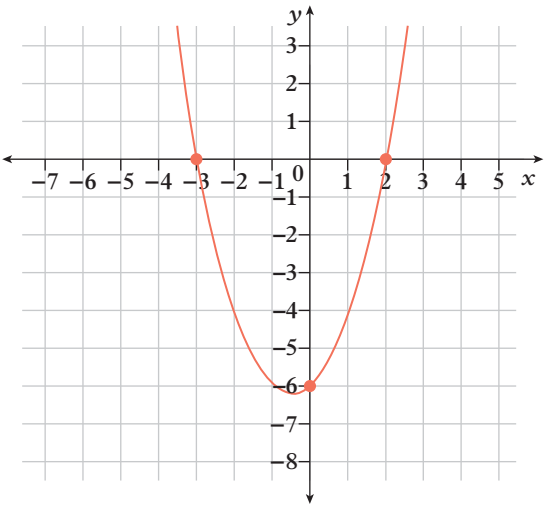
**c**



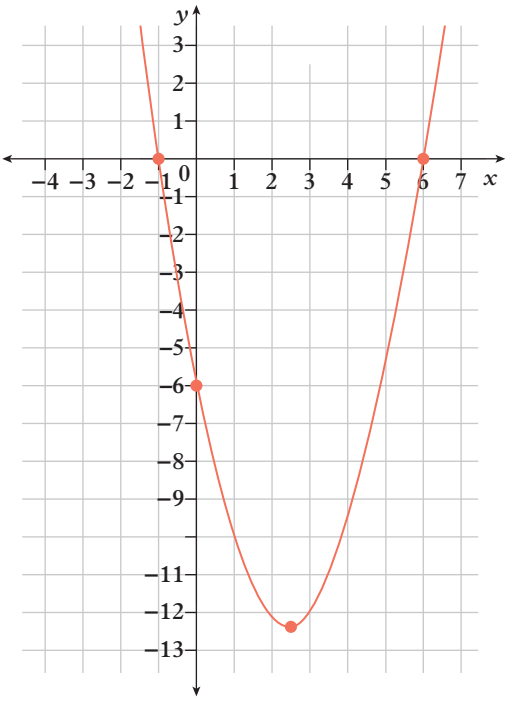
**d**



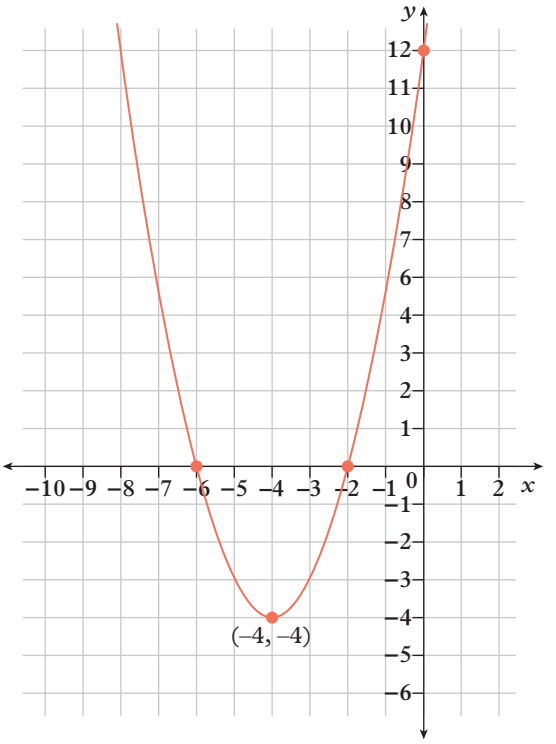
**e**



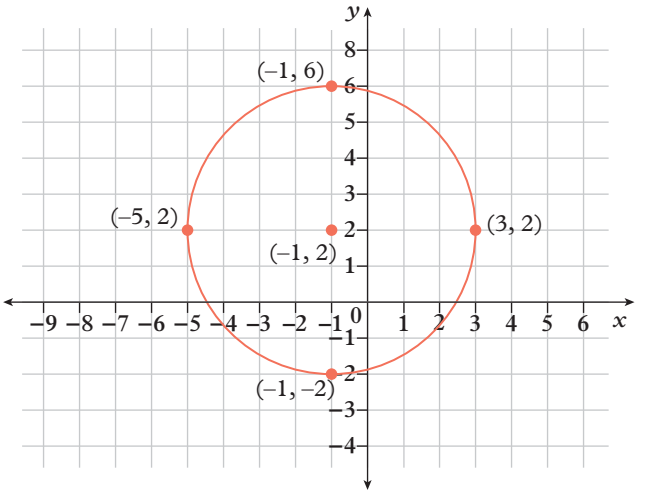
**f**



**g**




**h**




**20** **a** $y = -3x + 5$

**b** $x = -3$

**c** $y = 2(x + 1)^2 - 8$ or $y = 2(x + 3)(x - 1)$ or $y = 2x^2 + 4x - 6$

**d** $(x + 1)^2 + (y - 4)^2 = 9$

**21** **a** $(-1,2)$ **b** $(2,4.5)$ **c** $(1,3)$ **d** $(-3,2)$

**22** **a** 8.54 units **b** 6.32 units

**c** 6.00 units **d** 8.25 units

**23** **a** $4\sqrt{2}$ **b** $6\sqrt{5}$ **c** $2\sqrt{21}$ **d** $12\sqrt{7}$

**24** **a** $\sqrt{33}$ **b** $\sqrt{3}$ **c** $56\sqrt{3}$ **d** $3\sqrt{10}$

## Analysis

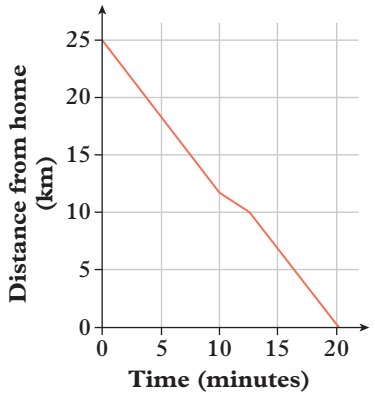
**1** **a** $d = \frac{4}{3}t$

**b** IV: Time, DV: Distance

**c** 18 min 45 sec

**d** 1.5 minutes or 1 min 30 sec or 90 seconds

**e**




**2** **a** \$50.56

**b** \$23.50/hour

**c** 24 rolls

**d** 12.0%

**e** \$500

**3** **a** **i** \$13 000 profit

**ii** \$24 000 profit

**iii** \$14 000 loss

**iv** \$0 profit (break-even)

**v** \$0 profit (break-even)

**b** **i** 205 units

**ii** \$24 025 profit

**c** $P = -(n - 205)^2 + 24025$

**d** **i** \$78 475

**ii** 23.4%

**e** **i** $R = 500n$

**ii** $C = n^2 + 90n + 18\,000$

## EX p228 EXPLORATIONS 1

**1** **a** **i** The van's speed is 90 km/h and the bicycle's is 40 km/h. The van overtakes the bicycle at 4:24 pm.

**ii** 104 km/h

**iii** 6:40 pm

**b** Speed of van $= \frac{1}{4}(c + 3t)$ km/h

Speed of bicycle $= (2t - c)$ km/h

The van overtakes the bicycle at 4:24 pm, which is independent of $c$ and $t$.

**2** **a** **i** The number of border squares is $2x + 2y - 4$ and the number of interior squares is $xy - 2x - 2y$.

**ii** $xy - 4x - 4y + 4 = 0$

**iii** 12, so the equation becomes $(x - 4)(y - 4) = 12$

**iv** $(x, y) = (5, 16), (6, 10), (7, 8), (8, 7), (10, 6)$ or $(16, 5)$

**v** Switching $x$ and $y$ rotates the design, so we can ignore the last three solutions.

**b** **i** Three: $(x, y) = (7, 34), (8, 20)$ or $(10, 13)$

**ii** One: $(x, y) = (4, 8)$

**3** **a** To four decimal places: 2.4495, 2.9068, 2.9844, 2.9974, 2.9996, …, which appears to approach 3.

**b** Solve $x^2 - x - 6 = 0$ to give $x = -2$ or 3 but ignore $x = -2$ as $x$ must be positive.

**c** **i** 4 and 1

**ii** $\sqrt{20 + \sqrt{20 + \ldots}}$ and $\sqrt{30 - \sqrt{30 - \ldots}}$

**4** **a** $12 \longrightarrow 6 \longrightarrow 3 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$

**b** $13 \longrightarrow \cdots \longrightarrow 14 \longrightarrow 7 \longrightarrow 20 \longrightarrow 10 \longrightarrow 5$

**c** This cycle has length 18: There are other solutions.

$17 \longrightarrow 50 \longrightarrow 25 \longrightarrow \cdots \longrightarrow 136 \longrightarrow 68 \longrightarrow 34$

**d** Use the fact that a multiple of 3 can only be the result of halving.

**e** There is only one: the cycle in part **b**

**f** One solution is $2^{500} + 1$. There are many others.

**5** **a** Clockwise from the top left, the lengths and areas are as follows:

| **Length (cm)** | 14 | 14 | 12 | 12 | 6 | 6 | 8 | 8 |
|---|---|---|---|---|---|---|---|---|
| **Area (cm²)** | 80 | 96 | 72 | 54 | 18 | 17 | 30 | 32 |

**b** Dark length $= 14 + 12 + 6 + 8 = 40$ cm

$= \frac{1}{2}$ of total

Dark area $= 80 + 72 + 18 + 30 = 200$ cm²

$= \frac{1}{2}$ of total

**c** Show that, for any $a$ and $b$, the total dark length is always $2c$ and the total dark area is always $\frac{1}{2}c^2$.

**d** Let $P$ be at the centre and choose any three points on the boundary that divide the perimeter into three equal lengths. Show that cutting from $P$ to the three boundary points creates three fair slices.

**e** It is possible for all positive integers by adapting the strategy in part **d**.

# CHAPTER 6 Measurement and geometry

## EX p235 6A Area of composite shapes

**1** **a** 20.79 cm² **b** 1800 mm² **c** 452.39 cm²

**d** 14.16 mm² **e** 44.2 cm² **f** 768 mm²

**2** **a** 7.07 cm² **b** 127.41 cm² **c** 184.31 mm²

**d** 30.39 cm²

**3** **a** triangle, semicircle

**b** three rectangles

**c** triangle, rectangle, semicircle

**d** triangle, rectangle

**e** rectangles, semicircles

**f** parallelogram, triangle, semicircle

**4** **a** 122.27 cm² **b** 56 cm² **c** 31.57 mm²

**d** 69 mm² **e** 160 cm² **f** 174.63 m²

**5** **a** 70.87 cm² **b** 47.43 mm² **c** 424 cm²

**d** 69.73 mm² **e** 70.5 cm² **f** 60.5 cm²

**6** **a** 141.02 mm² **b** 330 cm² **c** 557.08 cm²

**d** 34 cm² **e** 34.28 m² **f** 63.26 cm²

**7** **a** An annulus is the shape formed by two different-sized circles with a common centre. It is the area between the two circles.

**b** area of outer circle – area of inner circle

**c** 137.44 $cm^2$

**8** **a** 12.57 $cm^2$ **b** 65.97 $cm^2$ **c** 28.27 $m^2$

**d** 226.19 $cm^2$ **e** 103.67 $cm^2$ **f** 47.12 $cm^2$

**9** 12 166.55 $m^2$

**10** **a** 35.5 $cm^2$

**b** 19 525 $cm^2$ Sample answers: To cater for wastage between tags when they are cut out; in case any mistakes are made in production of tags.

**11** **a** 105.35 $m^2$ **b** $4740.75 **c** $1580.25

**12** 176.70 $cm^2$

**13** **a** 354.01 $cm^2$ **b** 336 $cm^2$ **c** 97.425 $cm^2$

**d** 68.57 $cm^2$ **e** 35 $m^2$ **f** 78.72 $cm^2$

**14** **a** A, B: 25 $cm^2$, C, E, G: 12.5 $cm^2$, D, F: 6.25 $cm^2$

**b** **i** 100 $cm^2$ **ii** 100 $cm^2$ **iii** 100 $cm^2$

**15** 47.75 $cm^2$

**16** 15 509.73 $cm^2$

**17** 31.81 $cm^2$

**18** **a** 44 $m^2$ **b** 7.33 $cm^2$ **c** 15 $cm^2$

## EX 6B Surface area

p243

**1** **a** 188 $cm^2$ **b** 240 $cm^2$ **c** 136 $m^2$

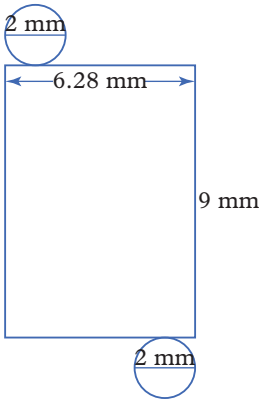
**d** 120 $cm^2$ **e** 150 $cm^2$ **f** 976 $cm^2$

**g** 148 $m^2$ **h** 290 $mm^2$

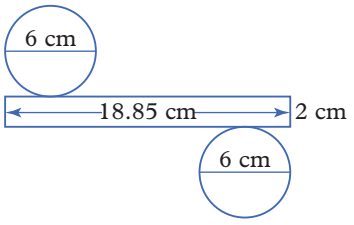
**2** **a** 188 $cm^2$ **b** 195.52 $mm^2$ **c** 360 $cm^2$

**d** 162.174 $m^2$ **e** 120 $cm^2$ **f** 120 $cm^2$

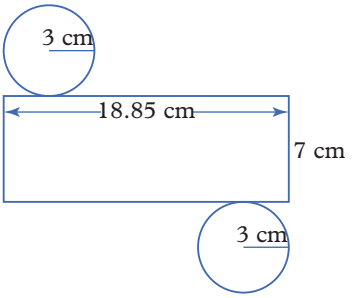
**3** **a**



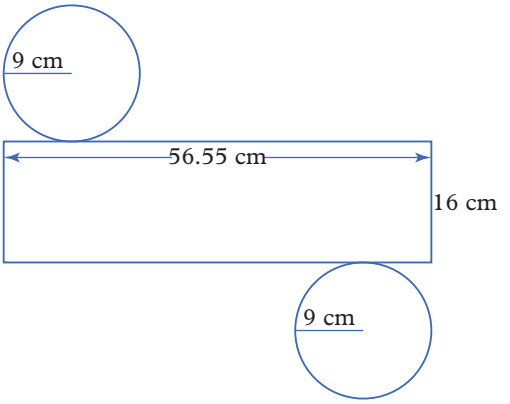

**b**



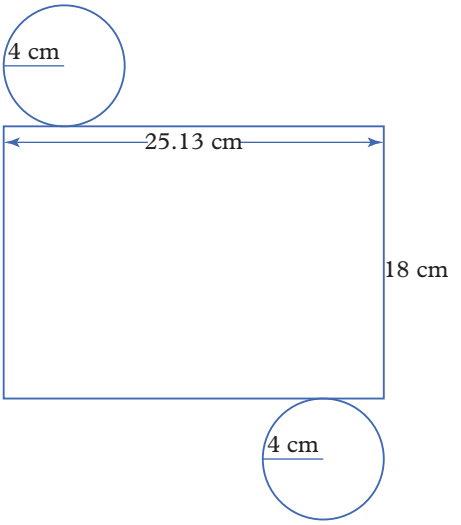

**c**



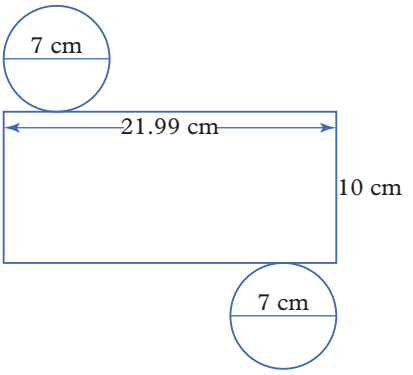

**d**




**e**




**f**




**4** **a** 62.83 $mm^2$ **b** 94.25 $cm^2$ **c** 188.50 $cm^2$

**d** 1413.72 $cm^2$ **e** 552.92 $cm^2$ **f** 296.88 $cm^2$

**5** **a** 100.53 $cm^2$ **b** 541.92 $cm^2$ **c** 603.19 $cm^2$

**d** 18.85 $m^2$ **e** 7.85 $m^2$ **f** 3.39 $m^2$

**6** **a** **i** 664 cm$^2$ **ii** 1328 cm$^2$

**b** **i** 396 cm$^2$ **ii** 792 cm$^2$

**7** 3924.63 cm$^2$

**8** **a** 3820 cm$^2$ **b** 2961.2844 cm$^2$

**9** $1429 ($1450 if rounded to needing 29 L of paint)

**10** first roller with radius 4 cm and length 25 cm

**11** **a** 500 cm$^2$

**b** **i** 292 cm$^2$ **ii** 584 cm$^2$

**c** Answer to **bii** is 84 cm$^2$ greater, because two more faces (7 cm by 6 cm) are created when the block of butter is cut in half.

**d** 4032 cm$^2$

**e** Sample answer: Cutting butter into smaller pieces increases the amount of surface exposed to the heat, so butter will melt quicker.

**12** **a** cream: 30 m$^2$, blue: 53.575 m$^2$

**b** cream: 4 L, blue: 7.14 L

**13** **a** 376.99 cm$^2$

**b** Lucian has calculated the surface area of the whole cylinder (ignoring the hollowed-out section) and Curtis forgot to include the area of the inside surface.

**14** 8 cm

**15** **a** 25 cm$^2$ **b** 5 cm **c** $l = \sqrt{\frac{\text{TSA}}{6}}$

**16** Sample answer: 2 × area of base calculates the surface area of the matching ends of a prism. The surface area of each of the rectangular faces joining the matching ends can be calculated by multiplying the section of the perimeter of base that face occupies by the height, so together the area of all of the rectangular faces joining the matching ends is perimeter of base × height.

**17** Sample answer: A cube has six identical square faces hence all sides are equal in length. Therefore, the length of one face will determine the length of all sides. A rectangular prism could have a different length, width and height.

**18** No, surface area will not double if you double the height of the cylinder. The surface area of the curved surface will double, but the area of the ends will not change.

**19** **a** 201.06 cm$^2$ **b** 615.75 mm$^2$

**c** 314.16 cm$^2$ **d** 127.32 cm$^2$

**20** $h = \frac{\text{SA} - 2\pi r^2}{2\pi r}$ or $h = \frac{\text{SA}}{2\pi r} - r$

**21** 57.42 cm$^2$

## EX p249 6C Volume and capacity

**1** **a** 450 cm$^3$ **b** 15 cm$^3$ **c** 105 cm$^3$

**d** 12 cm$^3$ **e** 209 cm$^3$ **f** 156 cm$^3$

**2** **a** 48 cm$^3$ **b** 1080 cm **c** 288 cm$^3$

**d** 324 mm$^3$ **e** 300 cm$^3$ **f** 48 600 m$^3$

**3** **a** 210 cm$^3$ **b** 36 cm$^3$ **c** 82.5 cm$^3$

**d** 375 cm$^3$ **e** 1320 cm$^3$ **f** 144 cm$^3$

**4** **a** 424.12 cm$^3$ **b** 942.48 mm$^3$ **c** 39.27 cm$^3$

**d** 904.78 mm$^3$ **e** 367.57 cm$^3$ **f** 2290.22 cm$^3$

**5** **a** 540 cm$^3$ **b** 2457 cm$^3$ **c** 99.74 mm$^3$

**6** **a** 1 570 796 L **b** 1357 L **c** 1 L (618 mL)

**7** **a** 30 cm$^2$ **b** 4 cm

**8** **a** 1.33 cm **b** 2.82 cm

**9** 250.25 cm$^3$

**10** **a** 462 cm$^3$ **b** 600 g

**11** **a** 44 178.65 mm$^3$ or 44.18 cm$^3$ **b** 679 coins

**12** The cylinder has size and capacity, it's just small and rounds to 0 when measured in litres. Saying that its capacity is 0 L is misleading.

**13** Second glass holds 55.37 mL more.

**14** 6.77 mL

**15** In the calculation for the volume of a cylinder the radius is squared, so if the radius is doubled, the volume will be multiplied by 4 ($2^2 = 4$). The height is only used once in the calculation for the volume, so if the height is doubled, the volume will also be doubled.

**16** **a** 348 cm$^3$ **b** 329.87 cm$^3$ **c** 730 cm$^3$

**17** 451.26 cm$^3$ or 451.26 mL

**18** **a** 400 cm$^3$ **b** 14.33 cm$^3$ **c** 39 cm$^3$

**19** **a** $V = \frac{1}{3}\pi r^2 h$

**b** **i** 9.42 cm$^3$ **ii** 157.08 cm$^3$ **iii** 50.27 cm$^3$

**20** **a** 113.10 cm$^3$ **b** 904.78 cm$^3$ **c** 268.08 cm$^3$

**d** 156.39 cm$^3$

**21** **a** 111 mL **b** 13

### Checkpoint

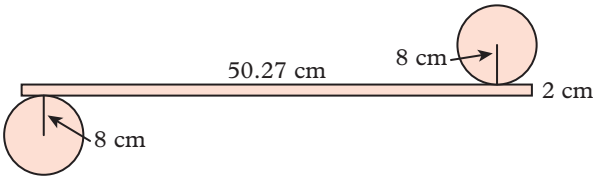
**1** **a** 4.71 cm$^2$ **b** 41.89 cm$^2$

**2** **a** 37.44 m$^2$ **b** 284 m$^2$

**3** **a** 285.88 cm$^2$ **b** 47.12 cm$^2$

**4** **a** 273 mm$^2$ **b** 57.52 mm$^2$

**5**




**6** **a** 710.50 cm$^2$ **b** 502.65 cm$^2$

**7** **a** 25.2 cm$^3$ **b** 135 m$^3$

**8** **a** 1219.94 cm$^3$ **b** 402.12 cm$^3$

**9** **a** 0.392 L or 392 mL **b** 94 815 L

**10** **a** 341 154.97 L **b** 70.03 L

## EX 6D Dilations and similar figures

p258

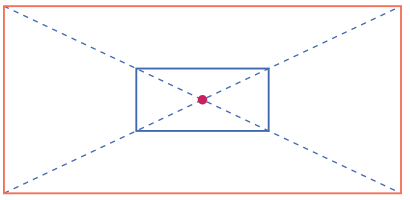
**1** **a** enlargement **b** reduction **c** enlargement
**d** enlargement **e** reduction **f** reduction

**2** **a** 2 **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
**d** 5 **e** $\frac{1}{5}$ **f** 2

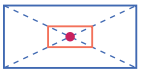
**3** **a** 3 **b** 2 **c** $\frac{1}{4}$ **d** $\frac{1}{3}$

**4** **a** $x = 2$ cm **b** $x = 6$ cm **c** $x = 9$ cm
**d** $x = 20$ cm **e** $x = 10.5$ cm **f** $x = 5$ cm

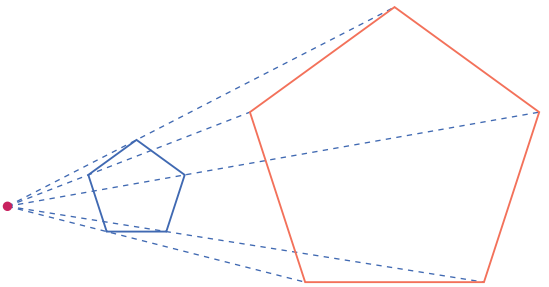
**5** **a** **i**



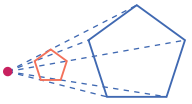
**ii**



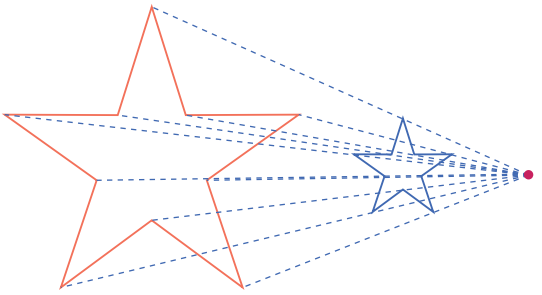
**b** **i**



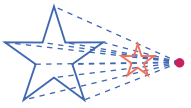
**ii**



**c** **i**



**ii**



**6** **a** similar **b** not similar **c** not similar
**d** similar **e** not similar **f** similar

**7** **a** 4 **b** 9 **c** 16
**d** $\frac{1}{4}$ **e** $\frac{1}{9}$ **f** $\frac{1}{16}$

**8** **a** 48 cm$^2$ **b** 45 mm$^2$ **c** 250 cm$^2$
**d** 10 cm$^2$ **e** 3 cm$^2$ **f** 4 mm$^2$

**9** 0.0026 mm

**10** All circles and squares are exactly the same shape, whereas rectangles can vary in shape.

**11** Finn is correct because $\frac{2}{3}$ is the same as $2 \times \frac{1}{3}$.

**12** Congruent figures are the same size and shape. Similar figures are the same shape, but not necessarily the same size.

**13** No

**14** To increase area by scale factor 9, increase length by scale factor $\sqrt{9} = 3$.

**15** 2

**16** 13%

**17** **a** 1 : 2

**b** The first part of the ratio is the numerator and the second part is the denominator.

**c** $\frac{2}{1}$ **d** 2 : 1

**e** **i** $\frac{1}{4}$ **ii** $\frac{1}{100}$ **iii** $\frac{1}{2500}$
**iv** $\frac{3}{4}$ **v** 2 **vi** $\frac{7}{2}$

**f** If the first part of the ratio is smaller than the second part, it is a reduction. If the first part of the ratio is larger than the second part, it is an enlargement.

**18** **a** Size of plane is reduced by a scale factor of 100.

**b** 34 cm **c** 27 m

**19** **a** 4 m **b** 3.75 cm

**c** If all proportions are in same ratio, yes; however, it would be very difficult to manufacture a model car with every aspect in same ratio.

**20** **a** 20 cm **b** 9750 m or 9.75 km

**21** **a** similar **b** similar **c** not similar
**d** not similar

**22** **a** $a = 12$ cm, $b = 99°$, $c = 81°$, $d = 99°$

**b** $a = 1$ cm, $b = 150°$, $c = 75°$, $d = 9$ cm, $e = 12$ cm, $f = 45°$, $g = 150°$

**c** $a = 85°$, $b = 115°$, $c = 18$ cm, $d = 115°$, $e = 85°$, $f = 115°$, $g = 45°$

**d** $a = 72°$, $b = 108°$, $c = 4$ cm, $d = 108°$, $e = 108°$, $f = 72°$, $g = 12.5$ cm

**23** **a** A regular hexagon has all side lengths equal and all angles equal (120°), an irregular hexagon does not.

**b** Because all angles will be same (they are for every regular hexagon) and all side lengths must be equal, any regular hexagon is similar to any other regular hexagon.

**24** **a** $\frac{1}{200}$ **b** $\frac{2}{375}$ **c** $\frac{15}{16}$ **d** $\frac{16}{15}$

**e** The numerators and denominators are swapped because in part **c** the plane is the original, but in part **d** the case is the original.



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**25 a** Looking at all paper sizes (from A0 to A8), the sizes are almost all similar (scale factor around 0.71), though rounding limits exact dilations to even numbers and odd numbers.

**b** similar relationship in B-sized paper

**c** similar relationship in C-sized paper

**d** B series covers sizes that are not found in the A series, so they are not similar. The C series are meant to be similar to A sizes, but there are some slight differences due to rounding errors.

## EX 6E Similar triangles

p265

**1 a** $a = 9$ cm, $b = 4$ cm

**b** $c = 44$ cm, $d = 7$ cm

**c** $e = 20$ cm, $f = 7.5$ cm

**d** $m = 19$ cm, $n = 45$ cm

**e** $x = 12$ cm, $y = 6.5$ cm

**f** $x = 52$ cm, $y = 20$ cm

**2 a** SAS **b** SSS **c** RHS

**d** AAA **e** SAS **f** AAA

**3 a** SSS **b** AAA **c** SAS

**d** SSS or SAS

**4 a** Yes: SAS

**b** No: all corresponding side pairs are not in the same ratio

**c** No: corresponding angles are not equal.

**d** Yes: SSS

**e** No: SSA is not a condition for similarity

**f** Yes: AAA

**5 a** similar **b** similar **c** not similar

**d** similar **e** similar **f** not similar

**6 a** not similar **b** similar **c** similar

**d** similar **e** not similar **f** similar

**7** With all the similarity conditions, if one pair of corresponding sides are equal then the triangles will be congruent.

**8** All equilateral triangles have three equal angles (60°), meeting similarity condition AAA.

**9** The third angle is equal to 180° minus the sum of the other two angles. Therefore, since the two triangles have the same two angles, their third angles will also be equal.

**10 a** Using knowledge of alternate angles within parallel lines ($\angle ABC = \angle EDC$ and $\angle BAC = \angle DEC$) and vertically opposite angles ($\angle ACB = \angle ECD$), these triangles have three corresponding equal pairs of angles and hence are similar.

**b** scale factor = 3

**c** $a = 2.5$ cm, $b = 6$ cm

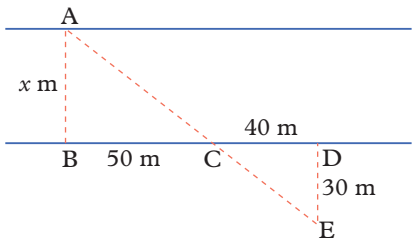
**11 a** similar **b** not similar **c** similar

**d** not similar

**12** $x = 6.54$ m

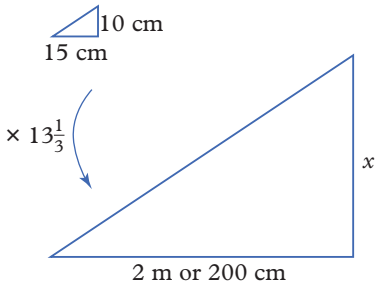
**13 a** 2.5 m **b** 2.72 m tall

**14** $x = 37.5$ m

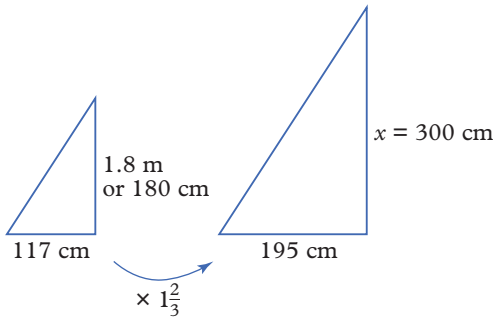



**15** 4 cm

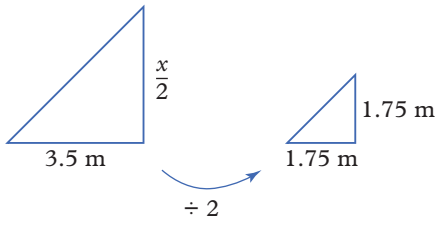
**16 a** 133.33 cm




**b** Ring is 5 cm too short, at 300 cm tall.




**c** 7 m




**17** Sean's house is 3.1 m tall; Tania's house is 2.7 m tall. Sean's house is taller.

**18** You need to have the second similar triangle to compare house heights and shadows because measurements were taken at different times of day so the length of the shadows would change.

**19 a** $\frac{DE}{AB} = \frac{7}{5} = 1.4$

$\frac{EF}{BC} = \frac{8.4}{6} = 1.4$

$\frac{FD}{CA} = \frac{11.2}{8} = 1.4$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

Corresponding sides are in the same ratio

$\therefore \triangle ABC \sim \triangle DEF$ by SSS

**b** $\frac{DE}{AB} = \frac{7}{5} = 1.4$
$\frac{FD}{CA} = \frac{11.2}{8} = 1.4$
$\angle EDF = 41°$
$\angle BAC = 41°$
$\frac{DE}{AB} = \frac{FD}{CA}$
Corresponding sides are in the same ratio.
$\angle EDF = \angle BAC$
Corresponding angles are equal
$\therefore \triangle ABC \sim \triangle DEF$ by SAS

**c i** $\frac{XY}{PQ} = \frac{9}{10} = 0.9$
$\frac{YZ}{QR} = \frac{10.8}{12} = 0.9$
$\frac{ZX}{RP} = \frac{11.7}{13} = 0.9$
$\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX}$
Corresponding sides are in the same ratio
$\therefore \triangle PQR \sim \triangle XYZ$ by SSS

**ii** $\frac{ST}{JK} = \frac{12}{20} = 0.6$
$\frac{SU}{JL} = \frac{18}{30} = 0.6$
$\angle KJL = 5°$
$\angle TSU = 5°$
$\frac{JK}{ST} = \frac{JL}{SU}$
Corresponding sides are in the same ratio
$\angle KJL = \angle TSU$
Corresponding included angles are equal
$\therefore \triangle JKL \sim \triangle STU$ by SAS

**20** $\angle AEB = \angle CED$
vertically opposite angles are equal
$AB$ is parallel to $CD$ ($AB \parallel CD$)
$\angle EAB \cong \angle ECD$
alternate angles are equal when parallel lines are cut by a transversal.
$\angle EBA \cong \angle EDC$
alternate angles are equal when parallel lines are cut by a transversal.
$\therefore \triangle ABE \sim \triangle CED$ by AAA.

## Chapter review

### Multiple-choice

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | C | **2** | D | **3** | C | **4** | B | **5** | E |
| **6** | C | **7** | A | **8** | C | **9** | A | **10** | E |

### Short answer

**1** 118 cm²

**2** 167.87 cm²

**3** **a** 276 cm² **b** 294 cm²

**4** **a** 1187.52 cm² **b** 180.64 cm²

**5** **a** 216 cm³ **b** 180 cm³

**6** **a** 3078.76 cm³ **b** 176.71 cm³

**7** **a** 4000 kL or 4 ML **b** 13 m

**8** **a** enlargement; scale factor = 2
**b** reduction; scale factor = $\frac{1}{4}$

**9** The shapes are similar as all sides are in the same ratio and the corresponding angles are the same.

**10** $a$ = 9.2 cm and $b$ = 4 cm

**11** Yes, by SAS

### Analysis

**1** **a** purple area = 920.8 cm², pink area = 10 cm²
**b** yes
**c** yes, 8158.4 cm² or about 0.82 m²
**d** 9 coats
**e** 10 cm²
**f** 14 137.17 cm²
**g** 2 tins
**h** yes, 5862.83 cm²

**2** **a** camp triangle: 40°, 90°, 50° gum-tree triangle: 40° (gum tree), 90°, 50°
**b** Corresponding angles are same but side lengths are not; triangles are similar (AAA) but not congruent.
**c** 2.5
**d** Side length of 6 m in 'camp triangle' corresponds to side in 'gum triangle' that represents width of river. To determine width of the river, multiply 6 m by scale factor (6 × 2.5).
**e** 15 m

# CHAPTER 7 Pythagoras' Theorem and trigonometry

## EX p281 7A Angles and lines

**1** **a** **i** 67° **ii** 157°
**b** **i** 43° **ii** 133°
**c** **i** N/A **ii** 4°
**d** **i** 81° **ii** 171°
**e** **i** N/A **ii** 83°
**f** **i** N/A **ii** 65°
**g** **i** 73° **ii** 163°
**h** **i** N/A **ii** N/A

**2** **a** $a = 27°$ **b** $b = 139°$ **c** $g = 19°$ **d** $h = 106°$
**e** $c = 282°$ **f** $d = 143°, e = 37°, f = 143°$

**3** **a** $f = 123°$ **b** $e = 46°$ **c** $e = 37°$ **d** $c = 82°$
**e** $f = 138°$ **f** $c = 53°$

**4** **a** $f$ **b** $c$ **c** $d$ **d** $a$ **e** $d$ **f** $e$
**g** $d$ **h** $f$

**5** **a** $b = 102°$ **b** $c = 102°$ **c** $d = 78°$
**d** $e = 78°$ **e** $f = 102°$ **f** $g = 102°$

**6** **a** $a = 60°, b = 120°, c = 30°$
**b** $a = 165°, b = 15°, c = 75°$
**c** $a = 115°, b = 65°, c = 90°, d = 25°$

**d** $a = 60°, b = 25°, d = 30°, e = 85°, f = 65°$

**e** $a = 43°, b = 37°, c = 43°$

**f** $a = 71°, b = 63°, c = 19°, d = 90°, e = 8°, f = 82°$

**7 a** $a = 48°, b = 71°, c = 61°, d = 132°$

**b** $a = 37°, b = 57°, c = 57°, d = 123°$

**c** $a = 117°, b = 79°, c = 101°, d = 63°$

**d** $a = 68°, b = 22°, c = 49°, d = 41°$

**e** $a = 121°, b = 59°, c = 59°, d = 121°$

**f** $a = 146°, b = 67°, c = 34°, d = 67°, e = 79°, f = 34°, g = 146°$

**8 a** 69° **b** 108° **c** 53° **d** 74°
**e** 89° **f** 39°

**9 a** Divide 360° by the number of carriages in the Ferris wheel.

**b** $360° \div 12 = 30°$

**10 Method 1**: Find angle $b$ first:

$180° - 145° = 35°$, $c$ is vertically opposite $b$, so $c = 35°$

$a = 180° - 90° - 35° = 55°$.

**Method 2**: $c = 180° - 145° = 35°$

$a$ is complementary to angle $c$:
$a = 90° - 35° = 55°$

$b$ is vertically opposite to $c$: $b = 35°$.

**11 a i** $a = 121°, b = 45°$ **ii** $c = 63°, d = 66°$
**iii** $e = 54°, f = 126°$

**b** Supplementary angles add to 180°, so an interior and its exterior angle add to 180°. The sum of that interior angle and the other two opposite interior angles is also 180°, so the exterior angle must be equal to the sum of the opposite interior angles.

**c** If the interior angles are $a$, $b$ and $c$, and you add the three exterior angles, you will get:

$(a + b) + (a + c) + (b + c)$

$= 2(a + b + c) = 2 \times 180° = 360°$

**12 a** 58° **b** 68.5° **c** 111.5°

**13** There are only two different-sized (and supplementary) angles formed in this scenario that can be found using angle facts.

**14** Check that when the two lines are cut by a transversal, corresponding or alternate angles are equal. Also check that co-interior angles add to 180°.

**15 a**

| | 56° | 78° | 19° | 146° | 27° | 103° |
|---|---|---|---|---|---|---|
| **Complementary** | 34° | 12° | 71° | | 63° | |
| **Supplementary** | 124° | 102° | 161° | 34° | 153° | 77° |
| **Vertically opposite** | 56° | 78° | 19° | 146° | 27° | 103° |
| **Co-interior** | 124° | 102° | 161° | 34° | 153° | 77° |
| **Alternate** | 56° | 78° | 19° | 146° | 27° | 103° |
| **Corresponding** | 56° | 78° | 19° | 146° | 27° | 103° |
| **Around a point** | 304° | 282° | 341° | 214° | 333° | 257° |

**b** It is not possible to make complementary angles with one angle greater than 90°.

**c** Vertically opposite, alternate and corresponding angles are equal.
Co-interior angles add to 180°. Complementary angles add to 90°.
Supplementary angles add to 180°.
Angles around a point add to 360°.
Let the angle be $A$.
Complementary = $90° - A$ if $A < 90°$,
Supplementary = $180° - A$, Vertically opposite = $A$,
Co-interior = $180° - A$, Alternate = $A$,
Corresponding = $A$, Around a point = $360° - A$.

**16 a i** 32° **ii** 116° **iii** 58°

**b** Angles $a$ and $b$ are equal, as are the angles between the light rays and the mirror. The angles $a$ and $b$ are complementary to the angles in part **a**, so if $a$ and $b$ are equal, so are the angles in part **a**.

**c i** 54° **ii** 36°

**17 a** 60 seconds **b** 60 minutes

**c i** 30 minutes **ii** 15 minutes **iii** 20 minutes

**d** greater than 14.5°

**e i** 83 degrees, 16 minutes and 55 seconds
**ii** 27 degrees, 43 minutes and 4 seconds
**iii** 154 degrees, 9 minutes and 37 seconds

**f i** 230°29′13″ **ii** 67°18′2″
**iii** 192°56′42″

**18 a i** 86°45′ **ii** 113°48′ **iii** 217°6′ **iv** 9°39′

**b i** 196.7° **ii** 98.85° **iii** 23.55° **iv** 107.05°

**19 a** 180°

**b i** 90° **ii** 270° **iii** 150° **iv** 330°
**v** 240°

**c** 30°

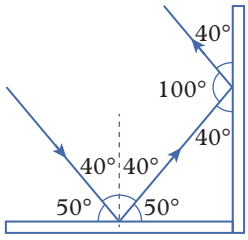
**d i** 15° **ii** 7.5° **iii** 22.5°

**e i** 45° **ii** 127.5° **iii** 22.5° **iv** 52.5°
**v** 97.5°

**20 a** 72.51° **b** 46°23′51″ **c** 89.61°
**d** 34°9′27″

**21 a** 167.5° **b** 32.5° **c** 153.5°
**d** 4° **e** 96°

**22**




## EX p289 7B Pythagoras' Theorem

**1 a** $x = \pm 5$ **b** $x = \pm 9$
**c** $x = \pm 8.94$ **d** $x = \pm 5.57$
**e** $x = \pm 1.5$ **f** $x = \pm 2.29$
**g** $x = \pm\frac{6}{11} \approx \pm 0.55$ **h** $x = \pm 1.89$

**2** **a** $m$ **b** $p$ **c** $f$

**3** **a** right-angled **b** not right-angled
**c** right-angled **d** not right-angled

**4** **a** right-angled **b** right-angled
**c** not right-angled **d** not right-angled
**e** right-angled **f** not right-angled

**5** **a** 15 cm **b** 50 mm **c** 61 cm **d** 26 cm

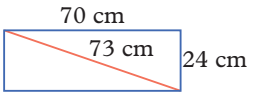
**6** **a** 9.43 cm **b** 17.49 cm **c** 23.85 mm
**d** 29.83 mm

**7** **a** 12.53 cm **b** 55.22 m **c** 25.24 mm
**d** 10.97 cm

**8** **a** 171 cm **b** 3 cm **c** 10 cm **d** 70 cm

**9** Yes, because $60^2 + 63^2 = 87^2$.

**10** **a**




**b** The frame is not square. The diagonal distance should be longer.

**c** 74 cm

**11** No, because $a^2 + b^2$ is the same as $b^2 + a^2$.

**12**

| Values | | Pythagorean triple | | |
|---|---|---|---|---|
| $m$ | $n$ | $a = m^2 - n^2$ | $b = 2mn$ | $c = m^2 + n^2$ |
| 2 | 1 | $2^2 - 1^2 = 3$ | $2 \times 2 \times 1 = 4$ | $2^2 + 1^2 = 5$ |
| 3 | 1 | 8 | 6 | 10 |
| 3 | 2 | 5 | 12 | 13 |
| 4 | 1 | 15 | 8 | 17 |
| 4 | 2 | 12 | 16 | 20 |
| 4 | 3 | 7 | 24 | 25 |
| 5 | 1 | 24 | 10 | 26 |
| 5 | 2 | 21 | 20 | 29 |
| 5 | 3 | 16 | 30 | 34 |
| 5 | 4 | 9 | 40 | 41 |
| 6 | 1 | 35 | 12 | 37 |

**13** **a** (3, 4, 5), (5, 12, 13), (15, 8, 17), (7, 24, 25), (21, 20, 29), (9, 40, 41), (35, 12, 37)

**b** **i** (5, 12, 13) **ii** (8, 15, 17) **iii** (3, 4, 5)
**iv** (9, 40, 41) **v** (65, 72, 97) **vi** (21, 220, 221)

**14** 18.03 m

**15** **a** $10^2 = 8^2 + 6^2$, $100 = 64 + 36$

**b** yes; $20^2 = 12^2 + 16^2$

**c** yes; $30^2 = 18^2 + 24^2$

**d** yes; $5^2 = 3^2 + 4^2$

**e** yes; $100^2 = 60^2 + 80^2$

**16** **a** 9.43 cm **b** 10 cm **c** 3.61 m
**d** 25.61 mm **e** 14.42 cm **f** 21.21 mm

**17** 11.68 cm

**18** **a** Triangle $ABC$ and Triangle $ACD$

**b** $AC = 10$ cm; $AB = 6$ cm and $BC = 8$ cm

**c** $CD = 26$ cm; the hypotenuse in triangle $ABC$ becomes one of the smaller sides in triangle $ACD$.

**d** 64 cm

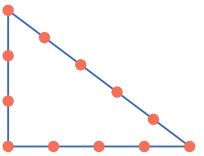
**19** **a** $x = 27.80$ cm, $y = 29.90$ cm

**b** $x = 15.81$ m, $y = 18.71$ m

**c** $x = 60.22$ cm, $y = 66.41$ cm

**20** **a** $x = \frac{5}{2}$ **b** $w = \frac{13}{8}$ **c** $t = 2$ **d** $p = \frac{1}{3}$

**21** The rope with 12 equally spaced knots forms a right-angled triangle with sides of 3 intervals, 4 intervals and 5 intervals. Pythagoras' Theorem applies ($3^2 + 4^2 = 5^2$).



**22** **a** Both squares have side lengths of $a + b$, so they both have an area of $(a + b)^2$.

**b** Both squares have the same area and include four identical triangles. So, the remaining space must be equal. The remaining area in Diagram 1 is a square with side lengths of $c$ so it has an area of $c^2$.

The remaining area of Diagram 2 is two squares, one with a side length of $a$ and the other with side length $b$. So their areas are $a^2$ and $b^2$ respectively. Therefore, equating the remaining areas, $c^2 = a^2 + b^2$.

**c** Diagram 1: $c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$

Diagram 2: $a^2 + b^2 + 4 \times \frac{1}{2}ab = a^2 + b^2 + 2ab$

**d** $c^2 + 2ab = a^2 + b^2 + 2ab$
$c^2 = a^2 + b^2$

**e** The total area of Diagram 3 with side lengths of $c$ is $c^2$. The square is made up of four right-angled triangles with an area of $\frac{1}{2}ab$ and a square with side lengths of $b - a$ and an area of $(b - a)^2$.

**f** $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$
$c^2 = b^2 - 2ab + a^2 + 2ab$
$c^2 = a^2 + b^2$

**23** **a** $a = 8$ m **b** $b = 10$ m
**c** $c = 3\sqrt{5}$ m **d** $d = 2\sqrt{17}$ m

EX p296

## 7C Using Pythagoras' Theorem to find the length of a shorter side

**1** **a** $x = \pm 3.32$ **b** $x = \pm 7.14$ **c** $x = \pm 6.93$
**d** $x = \pm 8.49$ **e** $x = \pm 10.91$ **f** $x = \pm 4.24$

**2** **a** 20 mm **b** 15 cm **c** 48 mm
**d** 27 mm **e** 14 m **f** 36 cm

**3** **a** 12.65 cm **b** 10.25 cm **c** 22.36 mm
**d** 18.73 cm **e** 4.76 cm **f** 15.96 m

**4** **a** 27.98 cm **b** 10.25 m **c** 13.56 cm
**d** 45.61 m

**5** **a** 398 cm **b** 5 cm **c** 55 cm **d** 47 cm

**6** **a** 10.00 cm **b** 7.91 m **c** 10.95 cm
**d** 16.47 mm **e** 5.31 mm **f** 10.25 cm

**7** **a** 14.99 cm$^2$ **b** 8.74 cm$^2$ **c** 29.54 cm$^2$
**d** 24.82 cm$^2$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**8** 8.72 m

**9** 2.87 m

**10** **a** 15.65 m **b** 4.58 cm **c** 7.07 m

**11** **a** $x = 13.08$ cm, $y = 7.35$ cm

**b** $x = 4$ m, $y = 3.48$ m

**c** $x = 17.80$ m, $y = 7.81$ m

**d** $x = 43.86$ cm, $y = 28.90$ cm

**12** **a** **i** 79.35 cm **ii** 224.65 cm$^2$

**b** **i** 18.78 m **ii** 15.36 m$^2$

**c** **i** 48.81 m **ii** 139.48 m$^2$

**d** **i** 135.90 cm **ii** 1003.17 cm$^2$

**13** **a** $c = 10$ **b** $a^2 = 10$ and $b^2 = 90$

**c** A triangle is right-angled if its side lengths satisfy Pythagoras' Theorem. $c^2 = 10^2 = 100$ and $a^2 + b^2 = 10 + 90 = 100$.
So, the triangle is right-angled.

**d** $(-1.5, 0.5), (3.5, 0.5), (3, -1)$

**e** 22.5, 2.5, 25

**f** $c^2 = 25$ and $a^2 + b^2 = 22.5 + 2.5 = 25$. So, the triangle is right-angled.

**14** $a = 3.00$ cm, $b = 5.81$ cm, $c = 5.67$ cm, $d = 6.41$ cm, $e = 4.25$ cm, $f = 2.43$ cm, $g = 9.27$ cm

**15** **a** $a = 8.94$ **b** $a = 144$ **c** $a = 43.30$

**16** **a** **i** and **ii** **b** **iv**

**c** **iii** The calculation subtracts the square of the shorter side from the square of the hypotenuse. This is the wrong way around. The calculation is the square root of a negative number which is not a real number, needed for the length of a right-angled triangle.

**17** 12 cm$^2$

**18** **a** $a = \sqrt{13}$ m

**b** $b = 2\sqrt{14}$ m

**c** $c = 3\sqrt{13}$ m

**d** $d = 3\sqrt{5}$ m

## Checkpoint

**1** **a** 53° **b** 157°

**2** **a** $x = 63°$, $y = 117°$

**b** $a = 22°$, $b = 22°$, $c = 158°$

**c** $t = 119°$

**3** **a** $r = 40°$

**b** $j = 53°$, $k = 53°$, $l = 127°$

**c** $u = 45°$, $v = 45°$, $w = 90°$

**4** **a** $x = \pm 8$ **b** $x = \pm 25$

**5** **a** yes **b** no

**6** **a** 40.50 cm **b** 112.36 mm or 11.24 cm

**7** **a** $x = \pm 3$ **b** $x = \pm 5$

**8** **a** 18.03 m **b** 449.00 cm or 4.49 m

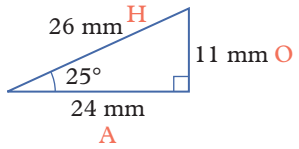
**c** 21.75 cm

**9** **a** 232.59 cm **b** 40.31 cm **c** 229.09 cm

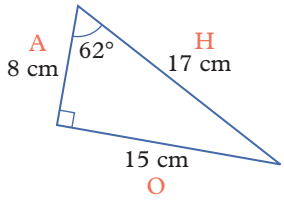
EX p304

## 7D Trigonometric ratios

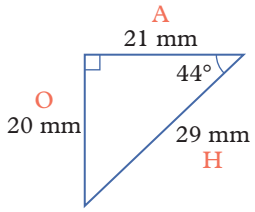
**1** **a**



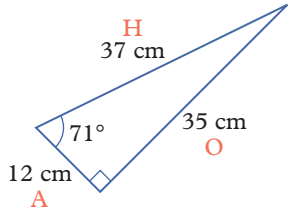

**b**




**c**




**d**




**2** **a** $\sin(\theta) = \frac{12}{17}$ **b** $\cos(\theta) = \frac{8}{15}$

**c** $\tan(\theta) = \frac{24}{33} = \frac{8}{11}$ **d** $\cos(\theta) = \frac{35}{40} = \frac{7}{8}$

**e** $\sin(\theta) = \frac{15}{24} = \frac{3}{8}$ **f** $\tan(\theta) = \frac{13.1}{6.6} = \frac{131}{6.6}$

**3** **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

**4** **a** $\frac{6}{7}$ **b** $\frac{6}{7}$ **c** $\frac{6}{7}$ **d** $\frac{6}{7}$

**5** **a** $\frac{4}{3}$ **b** $\frac{4}{3}$ **c** $\frac{4}{3}$ **d** $\frac{4}{3}$

**6** **a** 0.3420 **b** 0.5878 **c** 0.8090

**d** 0.7265 **e** 1.9626 **f** 0.4540

**7** **a** 2.052 **b** 17.54 **c** 0.057 00

**d** 1 **e** 0.7660 **f** 0.3640

**8** **a** They are similar triangles.

**b** **i**, **iii**, **iv**, **v**

**c** **i** $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

**ii** $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

**iii** $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

**iv** $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

**v** $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

**9** **a** 100 m **b** 148 m **c** 6.7 m **d** 5 m

**10 a**

| θ | $\sin(\theta)$ | $\cos(\theta)$ | $\tan(\theta)$ | $\frac{1}{\tan(\theta)}$ |
|---|---|---|---|---|
| 1 | 0.0175 | 0.9998 | 0.0175 | 57.2900 |
| 5 | 0.0872 | 0.9962 | 0.0875 | 11.4301 |
| 15 | 0.2588 | 0.9659 | 0.2679 | 3.7321 |
| 30 | 0.5000 | 0.8660 | 0.5774 | 1.7321 |
| 45 | 0.7071 | 0.7071 | 1.0000 | 1.0000 |
| 60 | 0.8660 | 0.5000 | 1.7321 | 0.5774 |
| 75 | 0.9659 | 0.2588 | 3.7321 | 0.2679 |
| 85 | 0.9962 | 0.0872 | 11.4301 | 0.0875 |
| 89 | 0.9998 | 0.0175 | 57.2900 | 0.0175 |

**b** Sine and tangent increase, and cosine and reciprocal tangent decrease.

**c** **i** $\sin(27°) = \cos(63°)$

**ii** $\sin(34°) = \cos(56°)$

**iii** $\tan(12°) = \frac{1}{\tan(78°)}$

**iv** $\tan(43°) = \frac{1}{\tan(47°)}$

**d** The cosine of an angle is the sine of the complementary angle. The sine of an angle is the cosine of the complementary angle.

**e** The reciprocal of the tangent of an angle is the tangent of the complementary angle.

**11 a** $a = 1, b = 1, c = \sqrt{2}, x = 45°, y = 90°, z = 45°$

**b** $\sin(45°) = \frac{1}{\sqrt{2}}, \cos(45°) = \frac{1}{\sqrt{2}}, \tan(45°) = 1$

**c** $a = 1, b = \sqrt{3}, c = 2, x = 30°, y = 90°, z = 60°$

**d** $\sin(30°) = \frac{1}{2}, \cos(30°) = \frac{\sqrt{3}}{2}, \tan(30°) = \frac{1}{\sqrt{3}}$

$\sin(60°) = \frac{\sqrt{3}}{2}, \cos(60°) = \frac{1}{2}, \tan(60°) = \sqrt{3}$

**e** **i** $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{3}$ **iv** $\frac{3}{4}$

**v** $\frac{1}{4}$ **vi** $\frac{1}{4}$ **vii** 1 **viii** $\frac{1}{2}$

**f** Determine the square of the value of sine/cosine/tangent of 30°/45°/60°, then write the square root of the squared value.

**12 a** The values of sine and cosine of θ for $0° < \theta < 90°$ are the ratio of a shorter side to the longer hypotenuse. This means all values of sine and cosine can be considered as smaller number divided by a larger number, which results in a value less than one.

**b** Tangent is the ratio of the two shorter sides. The two shorter sides have no restriction on which is longer or shorter.

**c** The tangent ratio is greater than one when the opposite side is longer than the adjacent side, which occurs when $45° < \theta < 90°$.

**13 a**

| θ | $\tan(\theta)$ | $\frac{\sin(\theta)}{\cos(\theta)}$ |
|---|---|---|
| 1 | 0.0175 | 0.0175 |
| 5 | 0.0875 | 0.0875 |
| 15 | 0.2679 | 0.2679 |
| 30 | 0.5774 | 0.5774 |
| 45 | 1 | 1 |
| 60 | 1.7321 | 1.7321 |
| 75 | 3.7321 | 3.7321 |
| 85 | 11.4301 | 11.4301 |
| 89 | 57.2900 | 57.2900 |

**b** **i** $\frac{\sin(27°)}{\cos(27°)} = \tan(27°)$

**ii** $\frac{\sin(56°)}{\cos(56°)} = \tan(56°)$

**iii** $\frac{\sin(12°)}{\cos(12°)} = \tan(12°)$

**iv** $\frac{\sin(47°)}{\cos(47°)} = \tan(47°)$

**c** $\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$

**14 a** **i** $a = 6.40$ m **ii** $b = 6.32$ m

**b** **i** $\sin(\theta) = 0.6247, \cos(\theta) = 0.7809, \tan(\theta) = 0.8$

**ii** $\sin(\theta) = 0.4286, \cos(\theta) = 0.9035, \tan(\theta) = 0.4743$

**15 a** **i** $\frac{1}{2}$ **ii** $\frac{5}{3}$ **iii** $\frac{6}{5}$ **iv** 7

**b** **i** $\frac{1}{2}$ **ii** $\frac{5}{3}$ **iii** $\frac{6}{5}$ **iv** 7

**c** The tangent of the angle from the horizontal is equal to the gradient of the hypotenuse.

**16 a** $x = \cos(31°), y = \sin(31°)$

**b** **i** $a = 6.857, b = 4.120$

**ii** $a = 0.5143, b = 0.3090$

**c** $x = \cos(54°), y = \sin(54°)$

**d** **i** $a = 2.939, b = 4.045$

**ii** $a = 2.057, b = 2.832$

**17 a** $y = \tan(31°), \frac{1}{z} = \cos(31°)$ or $z = \frac{1}{\cos(31°)}$

**b** **i** $a = 6.009, b = 11.67$

**ii** $a = 1.863, b = 3.617$

**c** $\frac{1}{x} = \tan(31°)$ or $x = \frac{1}{\tan(31°)}, \frac{1}{z} = \sin(31°)$ or $z = \frac{1}{\sin(31°)}$

**d** **i** $a = 8.321, b = 9.708$

**ii** $a = 12.32, b = 14.37$

**18 a** The person has not written the angle, θ. $\cos(\theta) = \frac{10}{11}$

**b** The person has written the reciprocal of the sine ratio. So, the fraction is upside down.

$\sin(\theta) = \frac{4}{8}$ $\frac{1}{\sin(\theta)} = \frac{8}{4}$

or

$\sin(\theta) = \frac{1}{2}$ $\frac{1}{\sin(\theta)} = 2$

**c** The person has written the cosine ratio instead of the tangent ratio. $\tan(\theta) = \frac{6}{5}$

**19 a** **i** $\tan(27°) = \frac{1}{2}$ **ii** $\tan(117°) = -2$
**iii** $\tan(194°) = \frac{1}{4}$ **iv** $\tan(326°) = -\frac{2}{3}$

**b** **i** 1 **ii** −1 **iii** 1 **iv** −1

**c** All four angles (45°, 135°, 225°, and 315°) are 45° rotations from the horizontal, so the gradient of the corresponding line will either be +1 or −1 for a line inclined at a 45° angle depending on which way it is rotated.

**d** **i** The gradient of a horizontal line is zero. Therefore, the tangent of 0° is zero.

**ii** The gradient of a vertical line is undefined. Therefore, the tangent of 90° is undefined.

**20 a** **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $-\frac{1}{2}$ **iv** $-\frac{1}{2}$
**v** $\frac{1}{2}$ **vi** $-\frac{1}{2}$ **vii** $-\frac{1}{2}$ **viii** $\frac{1}{2}$

**b** **i** sine: positive, cosine: negative

**ii** sine: negative, cosine: positive

**iii** sine: negative, cosine: negative

## EX p314 7E Using trigonometry to find lengths

**1 a** $\cos(\theta) = \frac{A}{H}$, $\cos(64°) = \frac{k}{35}$

**b** $\sin(\theta) = \frac{O}{H}$, $\sin(49°) = \frac{k}{12}$

**c** $\tan(\theta) = \frac{O}{A}$, $\tan(26°) = \frac{k}{50}$

**d** $\cos(\theta) = \frac{A}{H}$, $\cos(51°) = \frac{k}{17}$

**e** $\sin(\theta) = \frac{O}{H}$, $\sin(37°) = \frac{k}{24}$

**f** $\tan(\theta) = \frac{O}{A}$, $\tan(21°) = \frac{k}{16}$

**2 a** $x = 2.11$ **b** $x = 11.83$
**c** $x = 2.11$ **d** $x = 11.83$
**e** $x = 6.62$ **f** $x = 18.28$
**g** $x = 3.89$ **h** $x = 3.89$

**3 a** 31.84 cm **b** 25.86 mm **c** 2.78 m
**d** 22.65 cm **e** 11.11 m **f** 49.32 mm
**g** 61.80 cm **h** 129.35 m **i** 5.26 cm

**4 a** 15.34 mm **b** 9.06 m **c** 24.39 cm
**d** 10.70 m **e** 14.44 mm **f** 6.14 m

**5 a** 118.31 cm **b** 3.72 m **c** 38.92 mm
**d** 11.02 cm **e** 46.27 cm **f** 4.72 m

**6 a** 4.44 cm **b** 8.31 cm **c** 10.39 m
**d** 78.15 cm

**7 a** 33.64 m **b** 34.64 m

**8** 4.62 m

**9 a** 9.60 m **b** 16.00 m

**10 a** 1.93 m **b** 2.13 m

**11** 8.60 m

**12 a** One angle size (other than the right angle) and one side length

**b** Two side lengths

**13 a** $a = 17.2$ cm **b** $b = 42.2$ mm **c** $c = 52.9$ m
**d** $d = 32.2$ cm **e** $e = 29.9$ mm **f** $f = 52.0$ cm

**14 a** $a = 3.28$ cm, $b = 3.77$ cm, $x = 90°$, $y = 41°$, $z = 49°$

**b** $g = 27°$, $h = 63°$, $m = 2.94$ m, $n = 3.30$ m

**15 a** $f = 4.01$ cm, $g = 7.04$ cm

**b** $a = 4.00$ cm, $b = 7.21$ cm

**c** $m = 17.69$ cm, $n = 12.00$ cm

**16** $r = 8.81$ cm and $t = 10.88$ cm

**17** $p = 35.96$ cm, $q = 11.96$ cm

**18 a** $x = 5.01$ cm **b** $a = 7.09$ cm

**19 a** $x = 2.78$ cm **b** $y = 13.22$ cm

**c** $z = 8.56$ cm **d** $b = 15.75$ cm

## EX p321 7F Using trigonometry to find angles

**1 a** 13° **b** 44° **c** 56° **d** 42° **e** 16° **f** 74°

**2 a** 20° **b** 36° **c** 33° **d** 25°
**e** 8° **f** 1° **g** 72° **h** 45°

**3 a** 53° **b** 38° **c** 21° **d** 62°
**e** 69° **f** 23° **g** 57° **h** 83°

**4 a** 50° **b** 42° **c** 67° **d** 54° **e** 31° **f** 46°

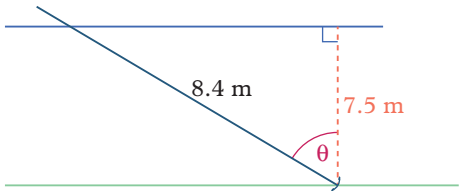
**5 a** θ = 138.6° **b** θ = 13.6°
**c** θ = 271.1° **d** θ = 25.8°

**6 a** 36.9°, 53.1° **b** 26.4°, 63.6°
**c** 72.9°, 17.1° **d** 25.7°, 64.3°

**7 a** 14° **b** 7.01 m

**8 a** 47° **b** 86°

**9 a**




**b** 27°

**c** Larger. If the chain is longer, the angle with the vertical would be greater. If the chain is shorter, the angle is closer to the vertical, hence making less angle size with the vertical.

**10 a** **i** 37° **ii** 37° **iii** 37°

**b** θ = 37°

**c** **i** Using trigonometry can be accurate to a required degree of accuracy but is difficult to perform without a calculator.

**ii** It is an advantage to use a scale diagram to check the results from trigonometric calculations, but it is difficult to record measurements with sufficient accuracy.

**11 a** First, calculate the angle of the Sun's rays at the time of measurement by using the length of a metre ruler's shadow. Then use the length of the tree's shadow as a measurement to calculate the height of the tree.

**b** 37°

**c** It refers to the angle of the Sun's rays at the time of measurement, both measurements being taken at the same time.

**d** 11.71 m

**e** By forming an equivalent ratio statement using the corresponding sides.

$x : 1\text{ m} = 15.46\text{ m} : 1.32\text{ m}$

$$x = \frac{1 \times 15.46}{1.32}$$

$$= 11.71\text{ m}$$

**f** You could draw the diagram to scale.

**12 a i** 28°, 62° **ii** 28°, 62° **iii** 28°, 62°

**b** The two non-right angles are complementary, so $90° - \theta$.

**13 a** $\theta = 123.7°$ **b** $\theta = 110.4°$ **c** $\theta = 78.6°$

**14 a** 30.0 m, 18.4°, 71.6°

**b** 20.1 m, 42.6°, 47.4°

**15** The corresponding angles in the bottom left of each triangle are approximately 64.707° and 64.652°, so the lines are very nearly parallel.

**16** 1548.4 $cm^2$

**17** $\theta = 38.4°$

## Chapter review

### Multiple-choice

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | C | **2** | C | **3** | E | **4** | C | **5** | D |
| **6** | E | **7** | A | **8** | A | **9** | B | **10** | C |

### Short answer

**1 a** $a = 64°$, $b = 64°$, $c = 116°$, $d = 64°$

**b** $a = 44°$, $b = 136°$, $c = 46°$

**2 a** right-angled **b** not right-angled

**3 a** $a^2 + b^2 = 30^2 + 40^2 = 2500$, $c^2 = 50^2 = 2500$

**b** Yes, as they are three whole numbers that satisfy Pythagoras' Theorem.

**4 a** 13 cm **b** 18.41 cm

**5 a** 3 cm **b** 52 cm

**6** $x = 20$ cm, $y = 36.06$ cm

**7 a** 25.46 cm **b** 1.50 m

**8 a** 297 cm **b** 5 cm

**9** $x = 19.90$ cm, $y = 15.13$ cm

**10** $\sin(\theta) = \frac{55}{73}$, $\cos(\theta) = \frac{48}{73}$, $\tan(\theta) = \frac{55}{48}$

**11 a** 5.44 cm **b** 19.43 m

**12 a** 28.31 m **b** 4.15 cm

**13 a** 35° **b** 60°

**14 a** $\theta = 68°$ **b** $\theta = 29°$

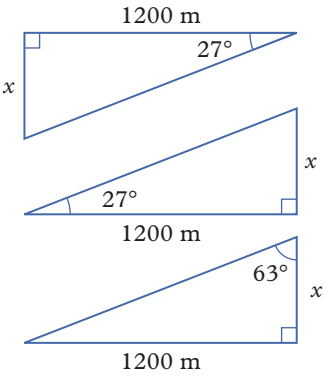
**15** $\theta = 47.5°$

### Analysis

**1 a i** No; need the value of two side lengths to find the value of the third side length.

**ii** yes; one angle and value of one side length has been given.

**b** In total, there are three figures that can be used to determine the height of the tower.




**c** 611 m

**d** See figures in part **b**. Note that all triangles result in the same length; that is, $x = 611$ m.

**e** 1347 m **f** $\frac{1200}{\cos(27°)} \approx 1347$ m

**2 a** 74.8 m **b** 599 m **c** 129.6 m

**d** 33.75°, 53.19° **e** 19.44°

# CHAPTER 8 Statistics

EX p336

## 8A Classifying and displaying data

**1 a** D **b** A **c** B **d** C

**2 a** 1 and 3 are numerical; 2 and 4 are categorical

**b** height: continuous, mark: discrete, method of transport: nominal, karate belt: ordinal

**3 a** fuel consumption: numerical, car model: categorical

**b** number of ice cream sales: numerical, maximum daily temperature: numerical

**c** exam score: numerical, subject: categorical

**4 a** The height of a tree, measured every year

**b** number of years: continuous, height: continuous

**c** 1 year old to 12 years old

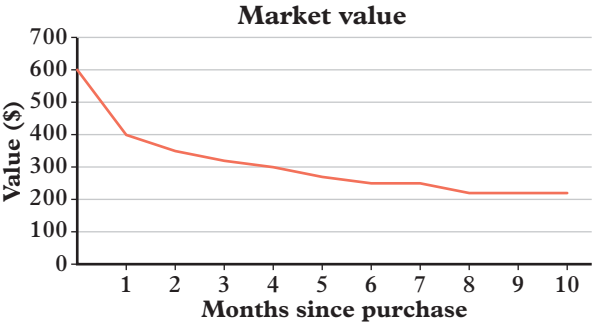
**d** between years 9 and 10

**5 a** Temperature over a 24-hour period, measured every 2 hours

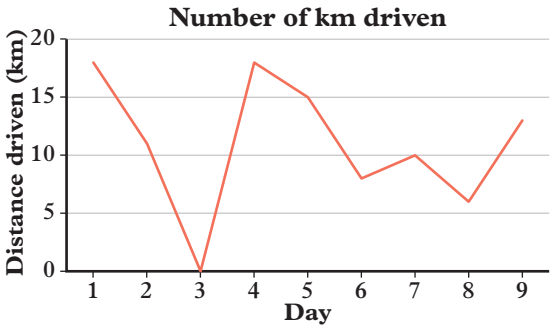
**b** time: continuous, temperature: continuous

**c** 11°C **d** 2 pm **e** 5°C, 6 am

**6**



7 **a**




**b** day 1 and day 4

**c** 5

8 **a** number of people, month, year

**b** July 2017

**c** May 2019

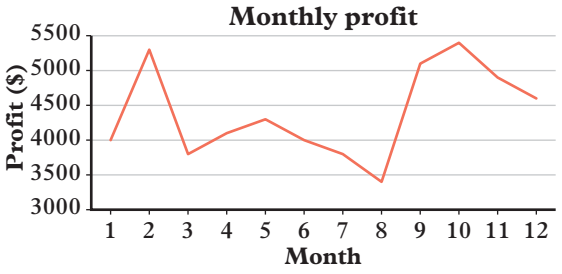
9 **a** number of goals, player, game number

**b** game 4

**c** game 3 (total of 12 goals)

**d** Hashani

10 **a**



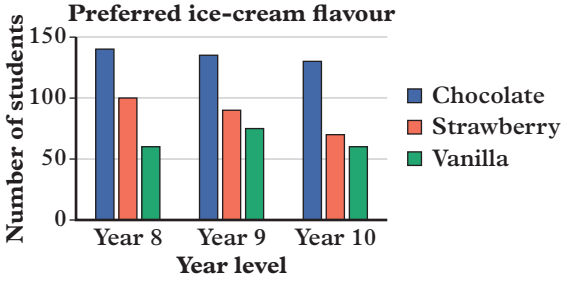

**b** month 10

**c** month 8 fit

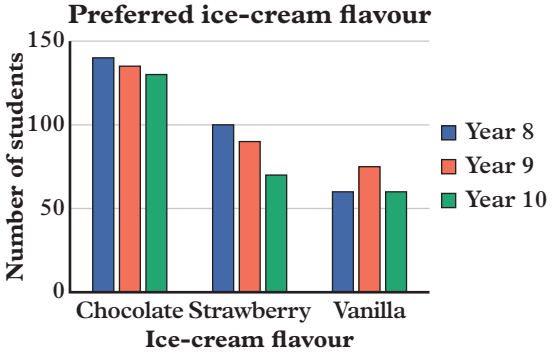
**d** month 9, $1700

**e** month 3, $1500

11 **a**




**b**




**c** The graph from part **a** (year levels on the horizontal axis)

**d** No. A lower percentage of Year 9s (45%) prefer chocolate compared to Year 10s (50%).

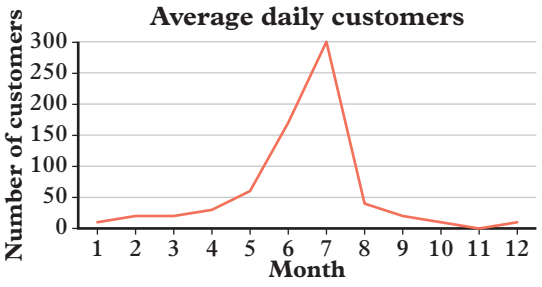
12 **a** 20

**b** Year 2

**c** Students can have access to a streaming service and also have a DVD player in their house.

**d** Almost all students to have access to a streaming service and few to have a DVD player.

13 **a**




**b** Month 7. It is the most popular month, and it is followed by a large drop off, which makes sense because January would not be busy.

**c** No. The data is rounded to the nearest 10 customers and it is the average number of daily customers for the month, not the total number of customers in the month. The average could have been just below 5 customers per day, which would be around 150 customers total for the month.

14 **a** Kuala Lumpur

**b** Karachi

**c** Kuala Lumpur, 2010

**d** Karachi, 2012

**e** Darwin

**f** Islamabad

15 **a** Osaka

**b** Shanghai

**c** 16 million

**d** Shanghai

**e** Approximately 2008

**f** Osaka has remained relatively stable. Shanghai has been increasing steadily. Mumbai has also been increasing steadily but not as fast as Shanghai.

16 **a**

**b** Negative profit means the store lost money.

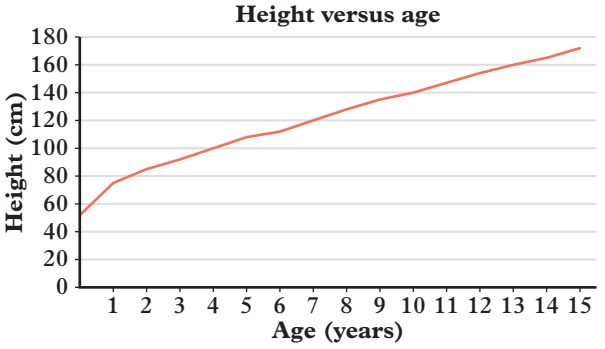
**c** $0 profit means the store broke even (did not make money or lose money)

**d** $47 000

**e** No. The profit is most likely associated with the time of year so it will be back down again in the middle of next year.

**f** Summer in the northern hemisphere occurs in the middle of the year so the months with the highest profit would be the months in the middle of the year, unlike the Geelong store, which has the highest profits at the start and end of the year.

**17**




**b** 8 cm

**c** 252 cm

**d** 185 cm (answers will vary)

**e** Continue increasing until the age of 18 or 19, then remain constant.

## EX p343 8B Grouped data and histograms

**1** **a** 5 **b** 16

**c** 15–<20, 18 **d** 25–<30

**e** 14 **f** 19

**g** 4.5%

**2** **a** 6 **b** 0–<20, 4 **c** 69

**3** **a** 145 **b** 183 **c** 38

**d**

| Class | Frequency |
|---|---|
| 145–<150 | 1 |
| 150–<155 | 3 |
| 155–<160 | 5 |
| 160–<165 | 3 |
| 165–<170 | 6 |
| 170–<175 | 6 |
| 175–<180 | 0 |
| 180–<185 | 1 |
| Total | 25 |

**4** **a**

| Class | Frequency |
|---|---|
| 0–<5 | 1 |
| 5–<10 | 2 |
| 10–<15 | 3 |
| 15–<20 | 3 |
| 20–<25 | 2 |
| 25–<30 | 3 |
| 30–<35 | 3 |
| 35–<40 | 1 |

**b**

| Class | Frequency |
|---|---|
| 10–<20 | 3 |
| 20–<30 | 3 |
| 30–<40 | 4 |
| 40–<50 | 4 |
| 50–<60 | 4 |
| 60–<70 | 2 |
| 70–<80 | 2 |

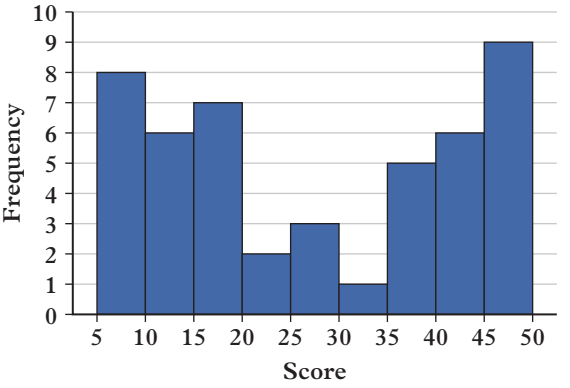
**c**

| Class | Frequency |
|---|---|
| 0–<2 | 6 |
| 2–<4 | 7 |
| 4–<6 | 6 |
| 6–<8 | 2 |
| 8–<10 | 2 |
| 10–<12 | 4 |

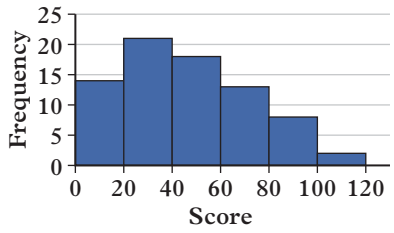
**d**

| Class | Frequency |
|---|---|
| 40–<45 | 3 |
| 45–<50 | 8 |
| 50–<55 | 7 |
| 55–<60 | 5 |
| 60–<65 | 3 |
| 65–<70 | 3 |
| 70–<75 | 4 |
| 75–<80 | 7 |

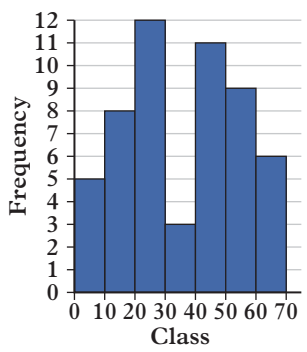
**5** **a**



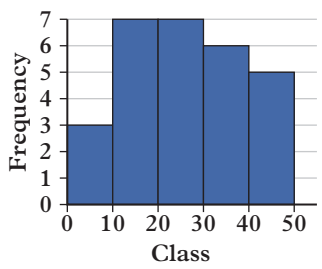
b



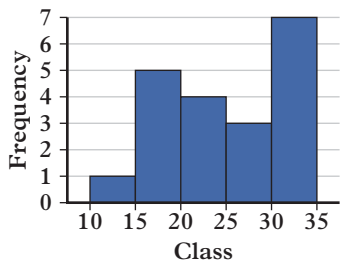

c



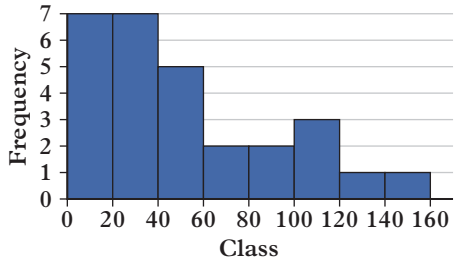

6 a



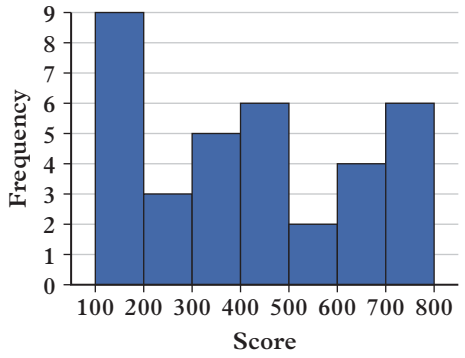

b



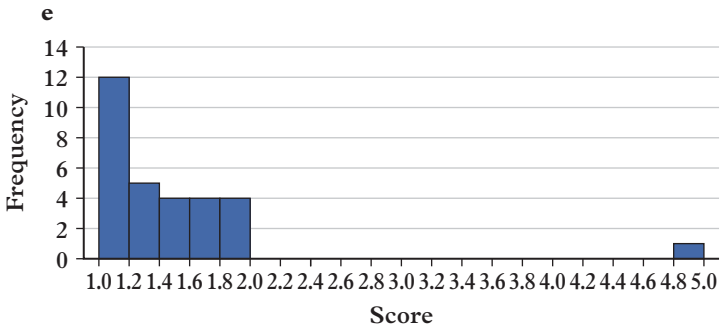

c



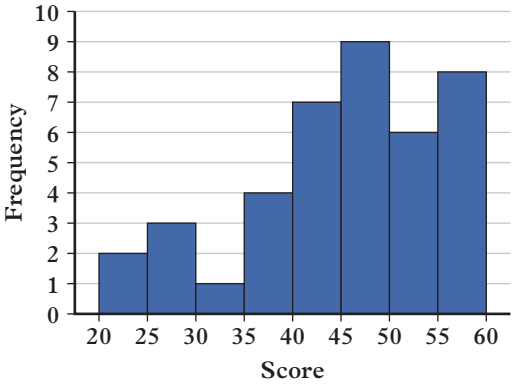

d




e



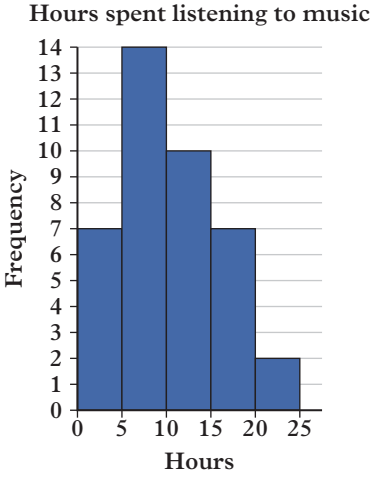

f




7 a

| Hours | Frequency |
|---|---|
| 0–<5 | 7 |
| 5–<10 | 14 |
| 10–<15 | 10 |
| 15–<20 | 7 |
| 20–<25 | 2 |

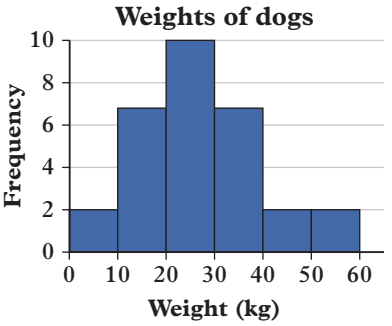
b




8 a

| Class | Frequency |
|---|---|
| 0–<5 | 2 |
| 5–<10 | 7 |
| 10–<15 | 10 |
| 15–<20 | 7 |
| 20–<25 | 2 |
| 25–<30 | 2 |

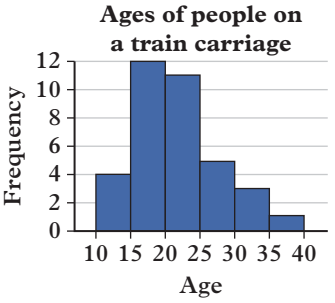
**b**




**9 a**

| Ages of people on a train | Frequency |
|---|---|
| 10–<15 | 4 |
| 15–<20 | 12 |
| 20–<25 | 11 |
| 25–<30 | 5 |
| 30–<35 | 3 |
| 35–<40 | 1 |

**b**




**10 a** number of Facebook friends

**b** 50 **c** 350–399; 28

**d** 100–149

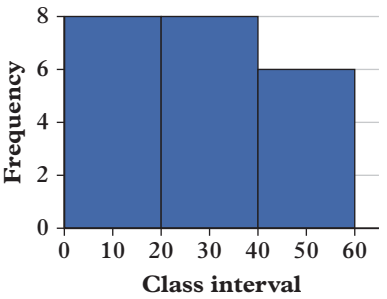
**11 a** There are too many class intervals, which makes the table long and difficult to read.

**b**

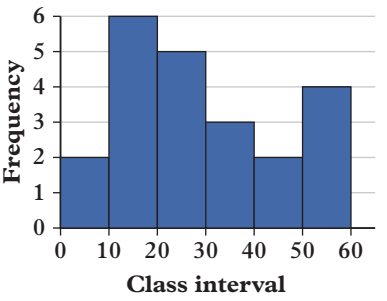
| Ages of customers | Frequency |
|---|---|
| 0–<10 | 2 |
| 10–<20 | 61 |
| 20–<30 | 44 |
| 30–<40 | 20 |
| 40–<50 | 6 |
| ≥50 | 6 |

**12** Without the raw data it is not possible to determine the number in each of smaller class intervals.

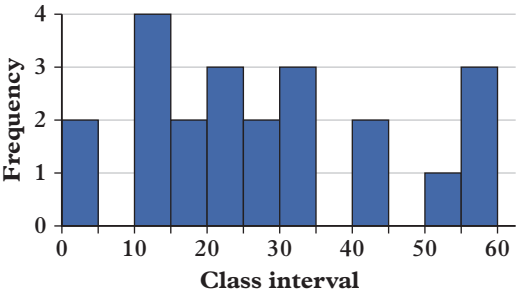
**13 a**




**b**

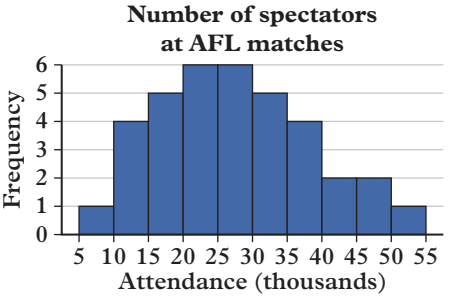



**c**

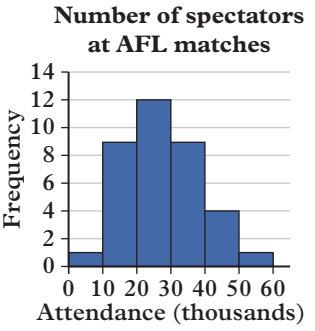



**d** Sample answer: The histogram in part **b** is most appropriate. The intervals in part **a** are too wide to give an accurate representation of the spread of the data. Both parts **b** and **c** display the data effectively, but the histogram in part **b** looks the neatest as it does not have intervals with frequency of 0 like part **c** does.

**14 a**




**b**

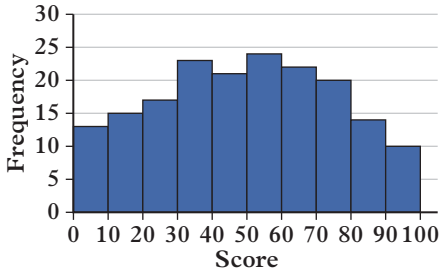



**c** Similar shape for each histogram (roughly symmetrical). Using smaller class intervals has resulted in smaller frequency values. That is, largest frequency is 6 compared with 12.

**15 a** 5

**b** There appears to be no pattern. Too many intervals make it harder to read.

**c**




**d** The distribution is symmetrical, with lower frequencies at either end and higher frequencies in the middle.

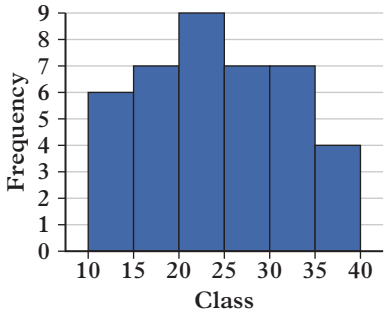
**16 a** The stems represent the class intervals.

**b** 10; e.g. 10–19, 20–29 etc.

**c** 17

**d** In a stem-and-leaf plot, each individual piece of data can be seen.

**17**




**18** Histograms do not provide specific data within each class interval; they only provide the frequency of data for each class interval.

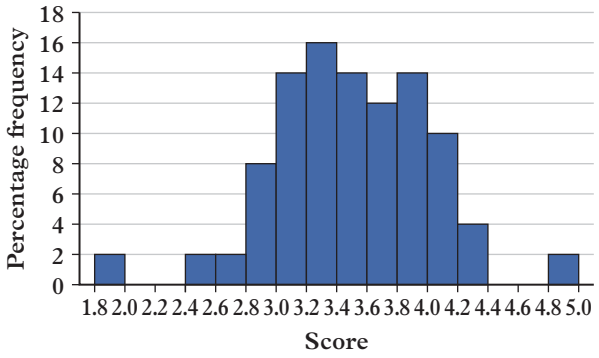
**19 a** The vertical axis represents percentage frequency rather than frequency.

**b** 16% **c** 14%

**d** 100%; because percentages of a whole add to 100%

**e i** 56 **ii** 196 **iii** 248

**20**




## EX 8C Summary statistics from tables and displays

p352

**1 a i** 2.36 **ii** 2 **iii** 2 **iv** 4

**b i** 21.67 **ii** 20 **iii** 10, 30 **iv** 30

**c i** 15.88 **ii** 16 **iii** 17 **iv** 5

**d i** 1.12 **ii** 1 **iii** 1 **iv** 4

**e i** 24.33 **ii** 25 **iii** 30 **iv** 15

**f i** 3.09 **ii** 2.5 **iii** 2 **iv** 18

**2 a** 30 people **b** mode = 5, range = 9

**c**

| Hours | Frequency |
|---|---|
| 1 | 3 |
| 2 | 4 |
| 3 | 2 |
| 4 | 6 |
| 5 | 8 |
| 6 | 4 |
| 7 | 2 |
| 8 | 0 |
| 9 | 0 |
| 10 | 1 |
| Total | 30 |

**d** mean = 4.3, median = 4.5

**3** mean = 4.7, median = 5, mode = 4, range = 5

**4 a** four bedrooms **b** 4

**c**

| Number of bedrooms | Frequency |
|---|---|
| 1 | 2 |
| 2 | 9 |
| 3 | 11 |
| 4 | 14 |
| 5 | 6 |
| Total | 42 |

**d** mean = 3.31, median = 3

**5 a** mean = 2.3, mode = 2, median = 2, range = 5

**b** mean = 16.15, mode = 16, median = 16, range = 6

**c** mean = 1.33, mode = 1, median = 1, range = 4

**6 a** mean = 55.43, median = 59, mode = 42, range = 42

**b** mean = 2.70, median = 2.3, mode = 1.8, range = 4

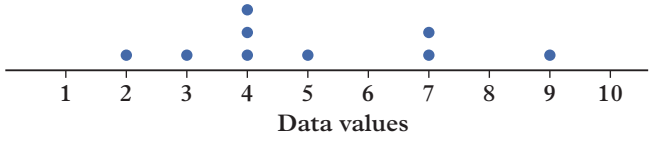
**c** mean = 280.38, median = 250, mode = 310, range = 670

**7** mean = 4.21, median = 4, mode = 3, range = 9

**8** Categorical data does not contain any numerical scores. You can only find the most popular/common category that is the mode.

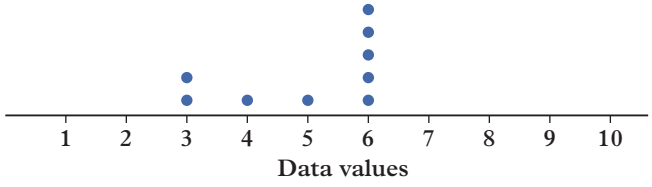
**9 a** B **b** A **c** C

**10 a** Answers will vary.



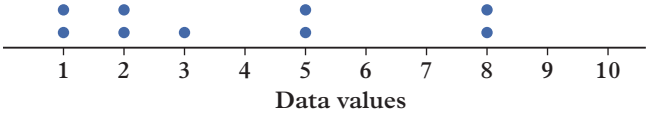

The lowest value must be 2, the highest value must be 9, and the other dots can be anywhere from 2 to 9.

**b** Answers will vary.



There must be at least five dots on 6 and all other dots must be below 6.

**c** Answers will vary.




There must be four values with exactly two dots each and one value with exactly one dot.

**11 a** 17 (unique)

**b** 9 (unique)

**c** Any number greater than or equal to 13 (not unique)

**d** 10 (unique)

**e** 9 or 17 (not unique)

**f** Any multiple of 9 (not unique)

**12 a** 16th row, 7th column

**b** 8th row, 19th column

**13 a** 49, because the highest possible value is 69 and the lowest possible value is 20

**b** 31, because the minimum value of the highest number is 60 and the maximum value of the lowest number is 29.

**c** The raw data is not available.

**d** 40–<49

**e** It is not possible to calculate the mean because the raw data is not available. It is not possible to know which interval contains the mean since, unlike the median, the mean is not found based on position.

**14 a** 39.6

**b** 48.6

**c** 44.1. Yes, it is reasonable to assume that the values in each interval are evenly spread out rather than all clustered at the top or all clustered at the bottom.

**15 a** 82.4% **b** 62.4% **c** 88%

**16 a** **i** 2.56 **ii** 26.37 **iii** 4.73

**b** The second set of data has a large spread of data from the mean (standard deviation = 26.37), but the other two data sets have a small spread of data from the mean (standard deviations = 2.56 and 4.73 respectively).

## Checkpoint

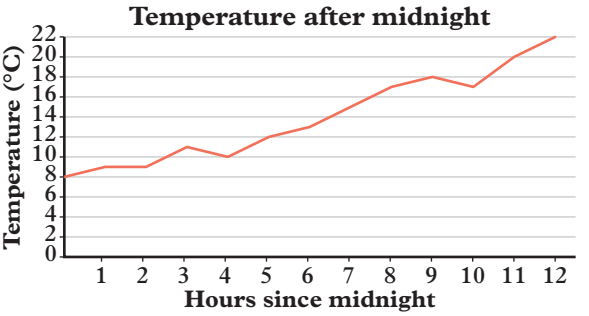
**1 a** numerical

**b** categorical

**c** categorical

**d** numerical

**2**




**3 a** pop

**b** New Zealand

**c** pop

**4 a** kangaroos

**b** 2015

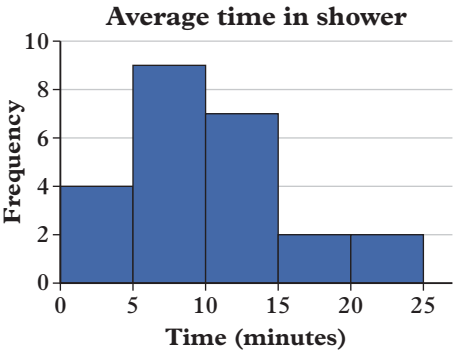
**c** between 2017 and 2018

**5 a** 90–99 **b** 6 **c** 15

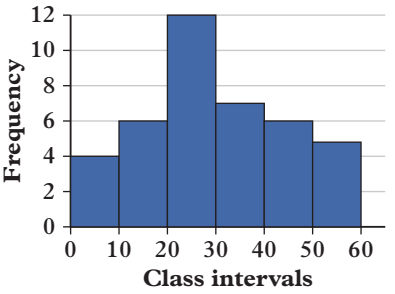
**6 a**

| Class | Frequency |
|---|---|
| 0–<5 | 4 |
| 5–<10 | 9 |
| 10–<15 | 7 |
| 15–<20 | 2 |
| 20–<25 | 2 |

**b**




**7**




**8 a** 7 **b** 7.25 **c** 16

**9 a** 101 **b** 82 **c** 41

**10 a** B **b** A **c** A **d** B

## EX 8D Describing data

p360

**1 a** bi-modal **b** negatively skewed

**c** positively skewed with an outlier

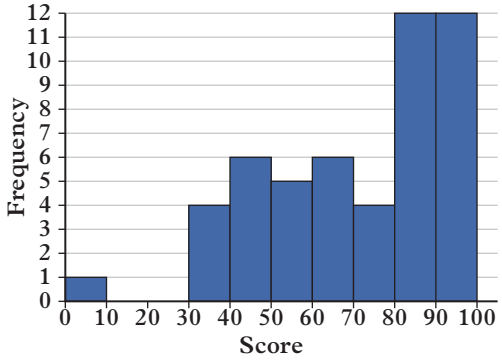
**d** symmetric

**2** **a** symmetric **b** positively skewed
**c** negatively skewed with an outlier

**3** **a** mean or median **b** median
**c** mode of each peak **d** median

**4** **a** mode of each peak **b** median
**c** median **d** mean or median

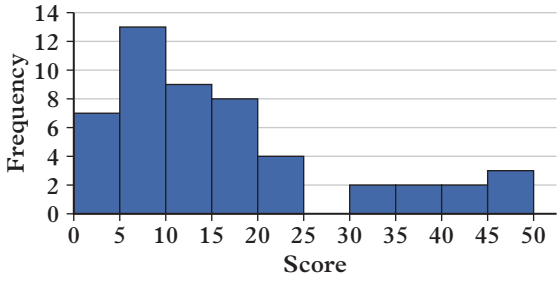
**5** **a** **i**




**ii** negatively skewed

**iii** Median: The graph is skewed and there is an outlier present, therefore the mean may not be an accurate measure of centre.

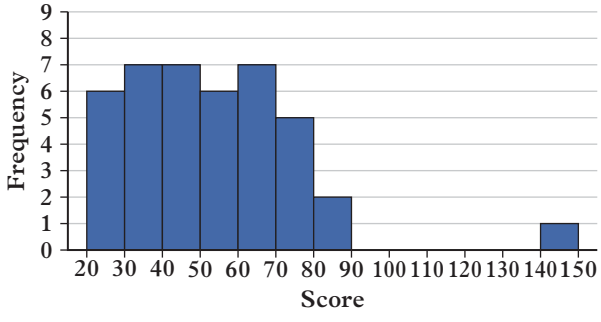
**b** **i**




**ii** positively skewed

**iii** Median: The graph is skewed, therefore the mean may not be an accurate measure of centre.

**c** **i**




**ii** symmetric or positively skewed with an outlier

**iii** median. An outlier is present, therefore the mean may not be an accurate measure of centre.

**6** **a**

| Stem | Leaf |
|---|---|
| 1 | 0 0 0 1 2 2 2 2 2 3 3 |
| 1* | 5 5 5 5 5 6 6 7 8 9 |
| 2 | 0 0 1 2 2 2 3 3 3 |
| 2* | 9 |

Key: 1 | 3 = 13

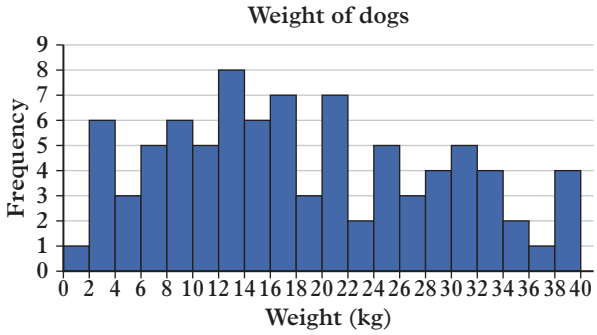
| Stem | Leaf |
|---|---|
| 1 | 0 0 0 1 |
| 1* | 2 2 2 2 2 3 3 |
| 1^ | 5 5 5 5 5 |
| 1# | 6 6 7 |
| 1+ | 8 9 |
| 2 | 0 0 1 |
| 2* | 2 2 2 3 3 3 |
| 2^ | |
| 2# | |
| 2+ | 9 |

Key: 1 | 3 = 13

**b** **i** positively skewed

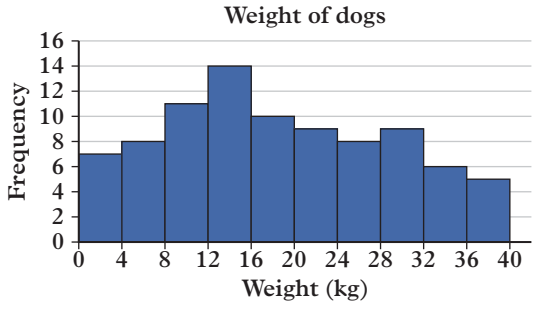
**ii** bi-modal with an outlier

**7** **a**




**b** no pattern visible

**c**




**d** very slightly positively skewed

**e** Weight of dogs ranges from below 2 kg up to 40 kg. The distribution is fairly even and roughly symmetric with a peak (modal class) around 12–16 kg.

**8** **a** symmetric **b** mean or median

**c**

| Number of people | Frequency |
|---|---|
| 0–<10 | 3 |
| 10–<20 | 4 |
| 20–<30 | 9 |
| 30–<40 | 12 |
| 40–<50 | 8 |
| 50–<60 | 6 |
| 60–<70 | 2 |
| Total | 44 |

**d** 30–<40

**e** Sample answer: The average number of people in the skate park is between 30 and 40.

**9** **a** **i** symmetrical with an outlier

**ii** median is 35–<40

**iii** Ages of people at a hairdresser's in a week usually ranges from 15 to 60 years, with the median age being between 35 and 40 years.

**b** **i** negatively skewed

**ii** mean = 15.41, median = 15.5

**iii** Ages of students at a pool ranges from 13 to 18 years, with median age 15.5 years

**c** **i** positively skewed **ii** median = 2

**iii** The majority of homes surveyed have one or two TVs, with the most common number of TVs in each home being two.

**d** **i** bi-modal

**ii** mode of each peak = 0–<10 cm and 30–<40 cm

**iii** Of the Year 9 students surveyed, most have either short hair ranging from 1 to 10 cm in length or longer hair ranging from 30 to 40 cm in length.

**10** **a** There were 0 customers on Christmas day because the course was closed.

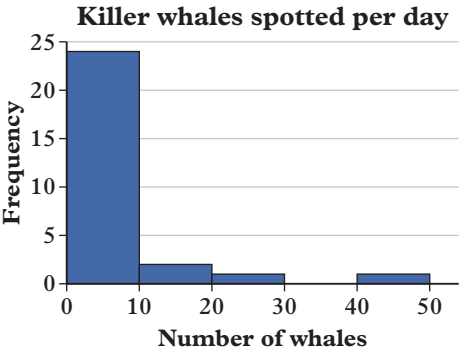
**b** A mini golf course is likely to have few customers during the week and do the majority of their business on Friday, Saturday, and Sunday.

**c** Any business that is much busier on weekends: cinemas, golf clubs, paintball, amusement parks, bars, hotel, etc.

**11** **a** It could lead people to believe that customers usually see about 4 whales per tour, which is not true.

**b** The median number of whales spotted is 0, which would not be a statistic worth advertising.

**c**




**d** The 24 numbers in this interval are not evenly spread out. 21 of them are 0.

**e** 25

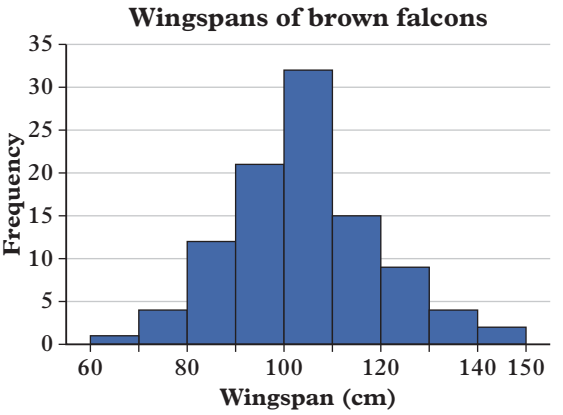
**12** **a** mean: 9.4, median: 9, mode: 9

**b** The mode tells the owner which shoe size is most popular. The median and mean do not tell the owner relevant data for stocktaking.

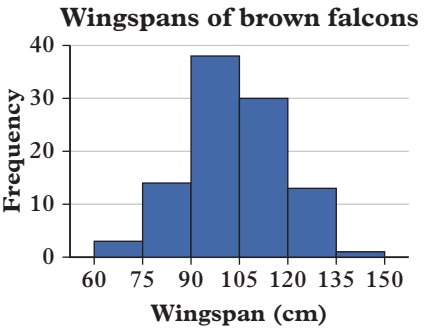
**c** 70%

**d** 94%

**13** **a**




**b**




**c** The histogram with class intervals of size 10, because the smaller size creates intervals that do not follow the pattern properly and the larger size intervals make the pattern less obvious.

**d** 68%

**e** 95%

## EX p367 8E Comparing data

**1** **a** **i** 41 **ii** 45 **iii** 45

**b** **i** 18 **ii** 24 **iii** 18 and 24

**c** **i** positively skewed **ii** symmetrical

**2 a** Group A and Group B have a similar spread of data but are skewed in different directions. Group A is negatively skewed, whereas Group B is positively skewed. Group A has a larger centre than group B.

**b** Both groups A and B are symmetric, but group B has a larger range due to the presence of an outlier. The median should be used as a measure of centre for group B, which has a smaller centre than group A.

**c** Both groups have a symmetric distribution and have a similar centre, but Group B is spread over a larger range.

**d** Both groups are negatively skewed, but group A has a smaller centre than Group B. Group B has an outlier present and a larger range.

**3 a**

| Leaf Friday | Stem | Leaf Saturday |
|---|---|---|
| 9 | 2 | 6 |
| 2 0 | 3 | 7 |
| 9 8 6 2 | 4 | 2 |
| 9 8 7 7 6 5 4 3 1 | 5 | 5 9 |
| 7 7 5 4 1 1 | 6 | 4 6 7 8 |
| 6 2 1 | 7 | 5 5 6 7 7 8 8 9 |
| 8 | 8 | 1 3 4 4 5 9 |
| 2 | 9 | 2 4 9 |

Key: 2|9 = 29

Data sets are spread roughly over same range but data set for Friday night is symmetrical, whereas the data set for Saturday night is negatively skewed. More people seem to attend cinema on Saturday night than on Friday night.

**b**

| Leaf Adelaide | Stem | Leaf Darwin |
|---|---|---|
| 6 | 20 | |
| 5 | 21 | |
| 7 | 22 | |
| 7 | 23 | |
| | 24 | |
| 4 | 25 | |
| 6 3 | 26 | |
| 0 | 27 | |
| 7 4 3 3 0 | 28 | |
| 4 | 29 | 5 9 |
| | 30 | 0 4 8 |
| 2 | 31 | 3 4 8 |
| 1 | 32 | 1 1 2 3 5 5 6 7 8 8 9 |
| 8 8 4 | 33 | 0 1 2 3 4 4 4 5 8 |
| 3 2 | 34 | |
| 6 | 35 | |
| 0 | 36 | |
| 1 | 37 | |
| 4 2 | 38 | |
| 2 | 39 | |
| 5 | 40 | |

Key: 32|9 = 32.9

Adelaide has a much larger range of temperatures than Darwin. Adelaide seems to have a roughly symmetric distribution with a lower average temperature than Darwin. Darwin's temperatures cover a small range and appear to be negatively skewed, with a higher average temperature than Adelaide.

**c**

| Leaf Boston Terriers | Stem | Leaf French Bulldogs |
|---|---|---|
| 9 9 9 6 6 | 4 | |
| 5 5 1 1 | 5 | |
| 8 8 7 4 2 | 6 | |
| 9 8 8 6 6 5 5 | 7 | |
| 6 6 4 3 3 2 1 0 | 8 | 6 |
| 9 2 2 2 2 | 9 | 1 2 2 4 5 5 6 8 8 8 8 9 |
| 5 5 5 0 | 10 | 1 5 6 6 8 9 9 |
| 2 0 | 11 | 0 1 1 2 2 3 5 5 5 8 9 |
| | 12 | 0 3 5 5 7 9 9 9 |
| | 13 | 0 |

Key: 10|9 = 10.9 kg

Both dog breeds appear to have roughly symmetric distributions, with Boston Terriers covering a larger range of masses. French Bulldogs appear to be heavier on average than Boston Terriers.

**4 a** Average heights of students in both classes are similar but 9A has a larger range of heights. The larger range suggests that 9A heights may be affected by presence of an outlier. Median should be used as measure of centre.

**b** Average price of jeans and range of prices in both stores are similar. Distributions seem to be symmetrical. Mean should be used as measure of centre.

**c** Average use of internet in secondary school is greater than average use in primary school. Range of hours is also greater in secondary school. Median should be used as measure of centre.

**d** Both stores have the same range and median of storage capacity in their cards, however store A has a larger mean than store B (and larger than its median), which suggests that store A has a skewed distribution and the median is a better representation of centre.

**e** Players A and B have the same median but player B has a larger mean. Combined with the smaller range, it appears that player B is a more consistent goal kicker with a positively skewed distribution, which would make them a better goal kicker.

**5 a** B. The range of B is 9, whereas the range of A is 7.

**b** A. The median of A is 25, whereas the median of B is 23.

**6 a** School B

**b** School B (13 at school B and 6 at school A)

**c** The two data sets are spread out over roughly the same age brackets, but distributed differently. School A has a roughly symmetric distribution, whereas school B is positively skewed.

**7 a** The range of heights is similar for Year 7 and Year 9 but Year 9 heights are slightly skewed in the negative direction. Year 9 students seem to be taller overall.

**b**

| Class heights | Mean (cm) | Median (cm) | Range (cm) |
|---|---|---|---|
| Year 7 | 164.22 | 162 | 34 |
| Year 9 | 173.54 | 173.5 | 33 |

**c** Range of heights for Year 7 and Year 9 students is almost identical. Measures of centre are close for each group, but the mean and median for Year 9 is higher, indicating the Year 9 students are taller than the Year 7 students.

**d** Yes; mean and median indicate that the Year 9 students are slightly taller than the Year 7 students and ranges of heights for Year 7 students and Year 9 students are almost identical.

**8 a** dot plot 1: negatively skewed; dot plot 2: symmetrical

**b** dot plot 1: mean = 16.6, median = 17, range = 6
dot plot 2: mean = 15.24, median = 15, range = 6

**c** Scores in both data sets are spread over same range, 6. The data set in first dot plot has a higher mean, median and mode than the data set in the second dot plot. Therefore, students participating in the school play in term 1 tended to be older than students who participated in the school play in term 4.

**9 a**

| Leaf Year 8 | Stem | Leaf Year 9 |
|---|---|---|
| 5 0 | 0 | |
| 4 | 1 | 8 9 |
| 7 7 5 3 | 2 | 0 6 |
| 8 3 2 0 0 | 3 | 5 7 |
| 9 4 4 1 | 4 | 5 8 9 |
| 9 8 5 | 5 | 1 2 4 5 |
| 5 0 | 6 | 0 5 |
| 4 1 | 7 | 1 4 6 7 7 |
| 6 3 | 8 | 3 6 |
| | 9 | 0 2 7 |

Key: 5|2 = 52

**b** Year 8: 86, Year 9: 79

**c** Year 8: 41, Year 9: 55

**d** Yes, it is true in general because the median for Year 9 students is much higher.

**10 a** Group A: 4, Group B: 4

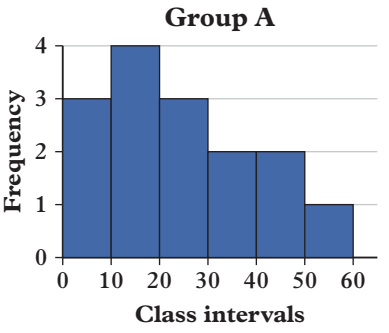
**b**

| | Mean | Range |
|---|---|---|
| **A** | 3.8 | 6 |
| **B** | 4.7 | 14 |

**c** The two data sets are very similar apart from the outlier in Group B.

**11 a** We do not have access to the raw data for the histogram. All that is known is how many numbers are in each interval, but not the actual values of those numbers.

**b**




**c** The two data sets are spread out over roughly the same values but distributed differently. Group A is positively skewed, whereas Group B is negatively skewed. Group B would have a higher centre than group A.

**12** A matches with histogram 2 and B matches with histogram 1.

Reasons may vary. Any of the following are acceptable:

The median for histogram 2 is in the interval 10–<20 which matches with data set A.

The median for histogram 1 is in the interval 20–<30, which matches with data set B. Histogram 2 will have a lower mean than histogram 1 because so many more values are stacked in the first two bins.

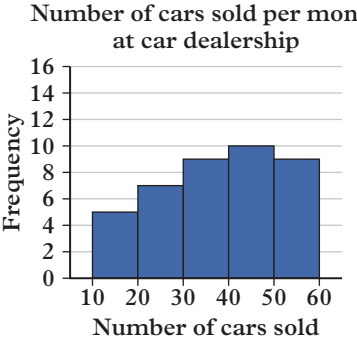
**13 a** negatively skewed

**b** roughly symmetric

**c** 16

**14 a** Class intervals used are not same size.

**b**




**c** Both histograms display data spread over same range. The first histogram displays data that is negatively skewed. Car dealership A sold an average of 30–40 cars a month, with the most frequent number sold in any month being 40–50 cars. The second histogram displays data that is symmetrical. Car dealership B sold an average of 30–40 cars, which is also the modal class for this data.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**d** We do not know how the data is spread over the class intervals to enable us to draw the histogram with smaller class intervals.

**15 a** Year: 1 80–85, Year: 2 85–90, Year: 3 90–95

**b** Year 3

**c** If the trend continues, in Year 4 the graph will be negatively skewed with a modal class of 95–100.

**16 a** Class B; it has a range of 28.

**b** Negatively skewed. A higher median than mean indicates that the centre of the distribution is a high value but there is a long tail of lower values to bring the mean down.

**c** Class C because its median is the highest by far. Class B has the highest mean but was affected by one outlier, which does not indicate that class B plays video games more in general.

**17 a** Year 8: 31, Year 9: 35

**b** 37

**c** 162.7 cm

**d** Summary statistics can not help us to find the middle value of all 450 student heights; the raw data is needed.

## Chapter review

### Multiple-choice

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | C | **2** | D | **3** | A | **4** | E | **5** | D |
| **6** | C | **7** | B | **8** | E | **9** | C | **10** | D |

### Short answer

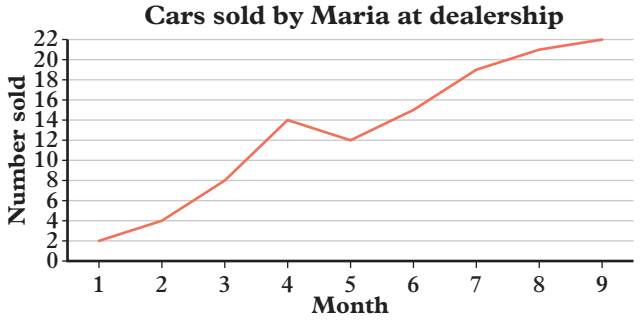
**1 a** average daily maximum temperature: numerical, city: categorical, month: categorical

**b** Jakarta

**c** January and April

**d** Hobart is in the southern hemisphere and Seoul is in the northern hemisphere, so they do not experience summer and winter at the same times.

**2 a**




**b** month 5

**c** $7000

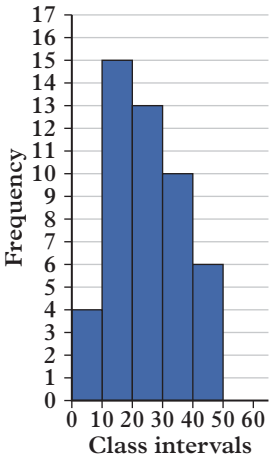
**3 a**

| Class interval | Frequency |
|---|---|
| 0–<10 | 4 |
| 10–<20 | 15 |
| 20–<30 | 13 |
| 30–<40 | 10 |
| 40–<50 | 6 |
| Total | 48 |

**b** 48

**c** 10–<20

**d**




**e** 10–<20

**4** mean = 4.46, median = 5, mode = 5, range = 3

**5** mean = 22.84, median = 21, mode = 7, range = 56

**6 a** dot plot 1

**b** dot plot 1

**c** dot plot 3

**d** Median. The mean is not an appropriate measure of centre because the data is skewed.

**e** Dot plot 1. Reasons may vary. Any of the following are appropriate reasons.

None of the other dot plots have zeros but we would expect some students to have no pets.

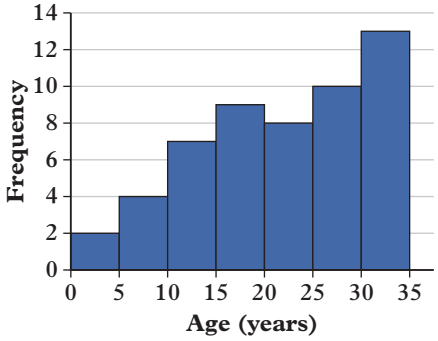
Most values are 0, 1, and 2, which fits the context.

The values in dot plots 2 and 3 are too large.

**7 a** 24

**b** 29

**c** Age




**d** negatively skewed

**8** **a** Class 9A: mean = 66.32, median = 65, mode = 45, range = 47

Class 9B: mean = 67.14, median = 64, mode = 48, range = 53

**b** The means and medians are very similar despite the stem-and-leaf plots looking quite different, but Class 9B has a slightly larger range.

**9** **a**

| Leaf School 1 | Stem | Leaf School 2 |
|---|---|---|
| 1 1 | 0 | 2 2 4 4 |
| 9 8 8 7 7 7 5 5 | 0* | 6 6 7 |
| 1 1 1 0 0 | 1 | 0 1 1 1 2 4 4 |
| 7 | 1* | 6 6 6 7 7 9 9 |
| 2 2 1 1 | 2 | 0 2 |
| 7 | 2* | |
| 2 | 3 | 1 |
| 8 5 | 3* | |

Key: 1 | 1 = 11

**b** School 1: mean = 14.4, median = 10.5, range = 37

School 2: mean = 12.8, median = 13, range = 29

**c** In discussing the results, the median will give the best measure of centre, since in school 1 the mean is affected by the positive skew.

### Analysis

**a** Brisbane: 22 shrubs were measured, Sydney: 24 shrubs were measured

**b** Brisbane: mean = 13.34, median = 13.15, mode = 11.4, range = 4.5

Sydney: mean = 13.33, median = 13.25, mode = 11.4, range = 4.1

**c** The data for the two locations is very similar: 22 shrubs were measured in Brisbane and 24 were shrubs measured in Sydney, and the mean and mode for both locations is almost identical. The median is slightly higher for Sydney. It could be said that the average shrub height was 13.3 cm.

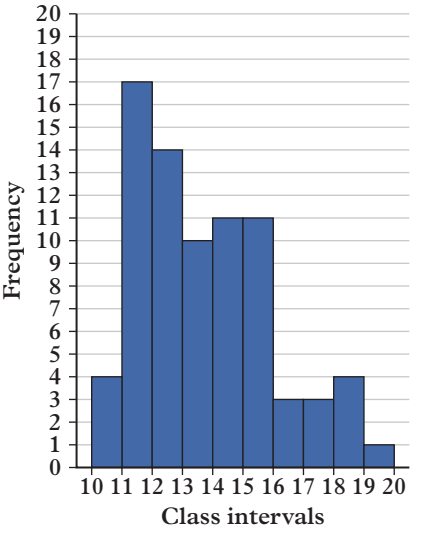
**d**

| Class interval | Frequency |
|---|---|
| 10–<11 | 4 |
| 11–<12 | 17 |
| 12–<13 | 14 |
| 13–<14 | 10 |
| 14–<15 | 11 |
| 15–<16 | 11 |
| 16–<17 | 3 |
| 17–<18 | 3 |
| 18–<19 | 4 |
| 19–<20 | 1 |
| Total | 78 |

**e** 78 plants

**f** 11–<12

**g**




The data displayed in the histogram is positively skewed.

**h** Victoria = 14.53, WA = 13.78

**i** If purchasing a shrub from this nursery chain, it will most likely be in height range of 10–20 cm, with the average shrub height being about 14 cm.

# CHAPTER 9 Probability

## EX 9A Two-step chance experiments

p385

**1** **a** 9 **b** 5

**2** **a** 9

**b** **i** $\frac{2}{9}$ **ii** $\frac{4}{9}$ **iii** $\frac{5}{9}$

**iv** $\frac{1}{3}$ **v** 0

**3** **a**

| | | 1st dice roll | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| 2nd dice roll | 1 | 2 | 3 | 4 | 5 |
| | 2 | 3 | 4 | 5 | 6 |
| | 3 | 4 | 5 | 6 | 7 |
| | 4 | 5 | 6 | 7 | 8 |

**b** 16 **c** 5, $\frac{1}{4}$ **d** 2 or 8, $\frac{1}{16}$

**e** **i** $\frac{1}{8}$ **ii** $\frac{3}{16}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{8}$

**4** **a** $\frac{7}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{16}$

**d** $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{16}$

**5** **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

**6** **a** $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{1}{3}$

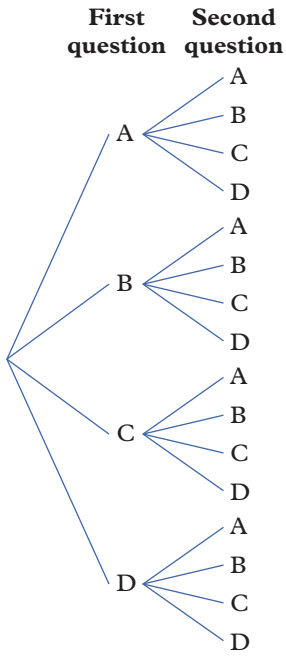
**7** **a** **i** rock, scissors; paper, rock; scissors, paper

**ii** rock, paper; paper, scissors; scissors, rock

**iii** rock, rock; paper, paper, scissors, scissors

**b** $\frac{1}{3}$

**8 a**




**b**

| | | Second question | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **First question** | **A** | A, A | A, B | A, C | A, D |
| | **B** | B, A | B, B | B, C | B, D |
| | **C** | C, A | C, B | C, C | C, D |
| | **D** | D, A | D, B | D, C | D, D |

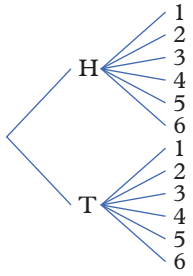
**c** With a tree diagram you can add a third step to the right of the second step, whereas it is not possible to add another dimension to an array to represent the third step.

**9 a** 36

**b** **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{6}$ **iv** $\frac{5}{36}$

**c** When there are a large number of outcomes in each step of a two-step chance experiment, the information is easier to interpret in an array than a tree diagram, and it can be easier to locate the favourable outcomes in an array.

**10 a**




**b** 12

**c** **i** $\frac{1}{12}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{12}$ **iv** $\frac{3}{4}$

**d** Tails on the coin toss and a 6 on the die is counted as one outcome, not two.

**11 a**

| | | Vowel | | | | |
|---|---|---|---|---|---|---|
| | | **A** | **E** | **I** | **O** | **U** |
| **Die roll** | **1** | (A, 1) | (E, 1) | (I, 1) | (O, 1) | (U, 1) |
| | **2** | (A, 2) | (E, 2) | (I, 2) | (O, 2) | (U, 2) |
| | **3** | (A, 3) | (E, 3) | (I, 3) | (O, 3) | (U, 3) |
| | **4** | (A, 4) | (E, 4) | (I, 4) | (O, 4) | (U, 4) |

**b** 20

**c** **i** $\frac{1}{20}$ **ii** $\frac{2}{5}$

**iii** $\frac{9}{20}$ **iv** $\frac{9}{10}$

**12**

| Experiment | Number of outcomes in first step | Number of outcomes in second step | Total number of outcomes | Number of favourable outcomes | Probability of event |
|---|---|---|---|---|---|
| **1** | 3 | 5 | 15 | 7 | $\frac{7}{15}$ |
| **2** | 5 | 3 | 15 | 6 | $\frac{2}{5}$ |
| **3** | 12 | 8 | 96 | 24 | $\frac{1}{4}$ |
| **4** | 4 | 9 | 36 | 27 | $\frac{3}{4}$ |
| **5** | 2 | 5 | 10 | 9 | $\frac{9}{10}$ |
| **6** | 2 or 3 | 6 or 4 | 12 | 10 | $\frac{5}{6}$ |
| **7** | 2 | 7 | 14 | 11 | $\frac{11}{14}$ |
| **8** | 2 or 3 | 9 or 6 | 18 | 12 | $\frac{2}{3}$ |
| **9** | 7 | 7 | 49 | 35 | $\frac{5}{7}$ |
| **10** | 3 or 11 | 121 or 33 | 363 | 66 | $\frac{2}{11}$ |

**13 a** **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{4}$

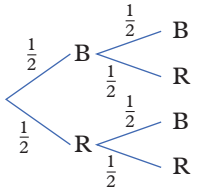
**b** It is not possible to add another dimension to an array to represent the third or more steps.

## 9B Experiments with replacement

EX p391

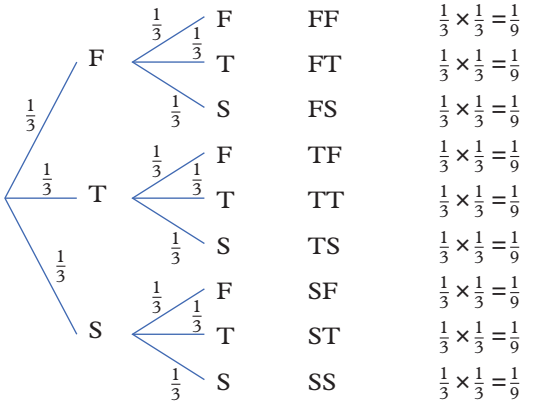
**1 a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{3}{26}$ **d** $\frac{1}{6}$

**2 a**

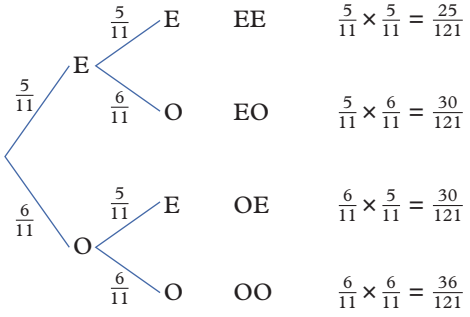



| Outcomes | Probability |
|---|---|
| BB | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
| BR | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
| RB | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
| RR | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |

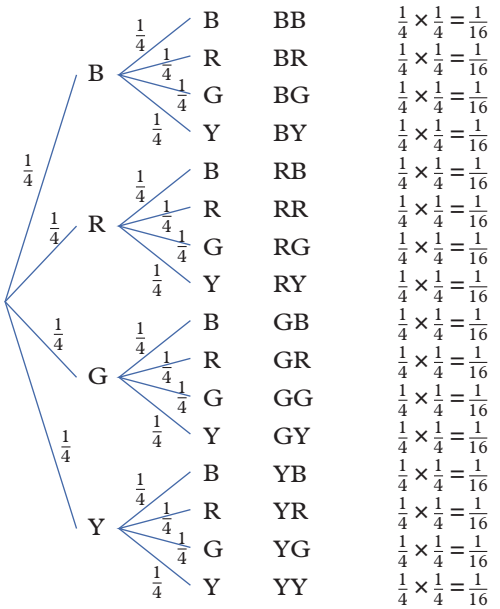
**b**




**c**




**d**



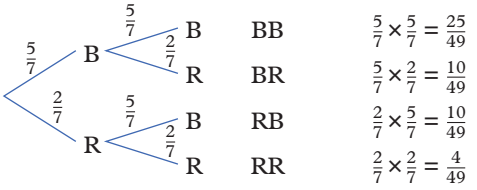

**3** **a** $\frac{1}{2}$ **b** 4

**c** **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$

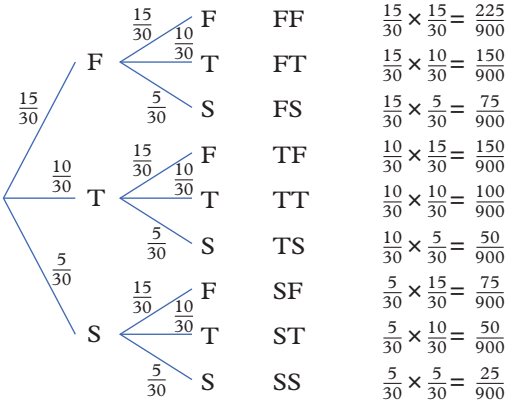
**4** **a** $\frac{1}{9}$ **b** $\frac{5}{9}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$

**5** **a** $\frac{1}{16}$ **b** $\frac{7}{16}$ **c** $\frac{3}{8}$ **d** $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{8}$

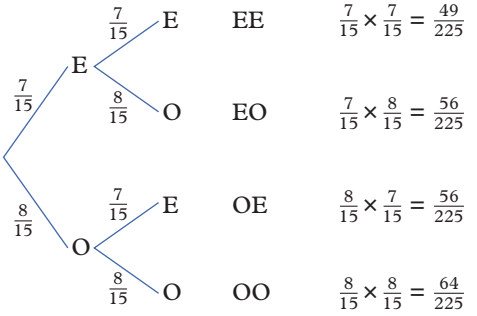
**6** **a**



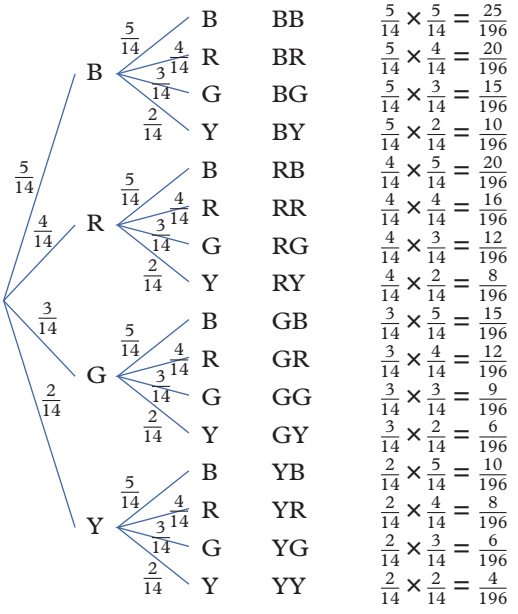

**b**




**c**

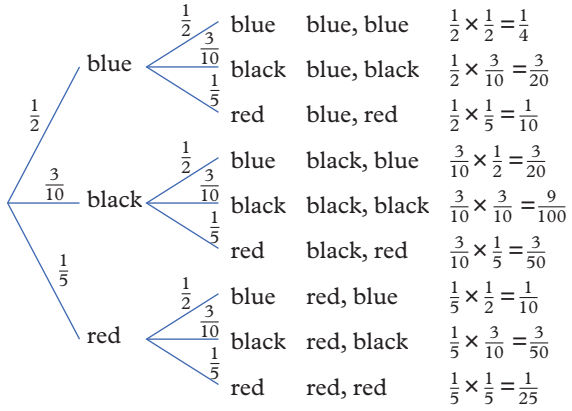



**d**




**7** **a** $\frac{1}{25}$ **b** $\frac{16}{25}$ **c** $\frac{4}{25}$

**8 a**

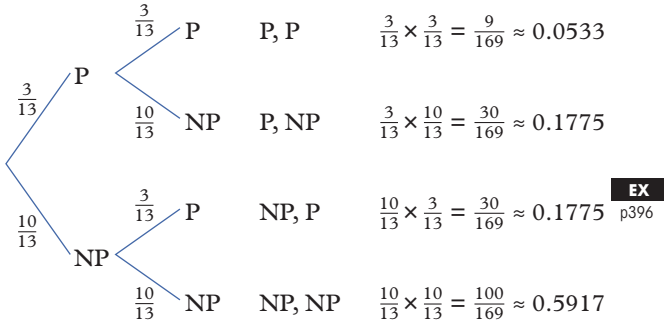



| Outcomes | Probability |
|---|---|
| blue, blue | $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ |
| blue, black | $\frac{1}{2}\times\frac{3}{10}=\frac{3}{20}$ |
| blue, red | $\frac{1}{2}\times\frac{1}{5}=\frac{1}{10}$ |
| black, blue | $\frac{3}{10}\times\frac{1}{2}=\frac{3}{20}$ |
| black, black | $\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$ |
| black, red | $\frac{3}{10}\times\frac{1}{5}=\frac{3}{50}$ |
| red, blue | $\frac{1}{5}\times\frac{1}{2}=\frac{1}{10}$ |
| red, black | $\frac{1}{5}\times\frac{3}{10}=\frac{3}{50}$ |
| red, red | $\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$ |

**b i** $\frac{1}{4}$ **ii** $\frac{1}{25}$ **iii** $\frac{9}{100}$

**iv** $\frac{3}{20}$ **v** $\frac{1}{10}$ **vi** $\frac{3}{50}$

**9 a**




| Outcomes | Probability |
|---|---|
| P, P | $\frac{3}{13}\times\frac{3}{13}=\frac{9}{169}\approx 0.0533$ |
| P, NP | $\frac{3}{13}\times\frac{10}{13}=\frac{30}{169}\approx 0.1775$ |
| NP, P | $\frac{10}{13}\times\frac{3}{13}=\frac{30}{169}\approx 0.1775$ |
| NP, NP | $\frac{10}{13}\times\frac{10}{13}=\frac{100}{169}\approx 0.5917$ |

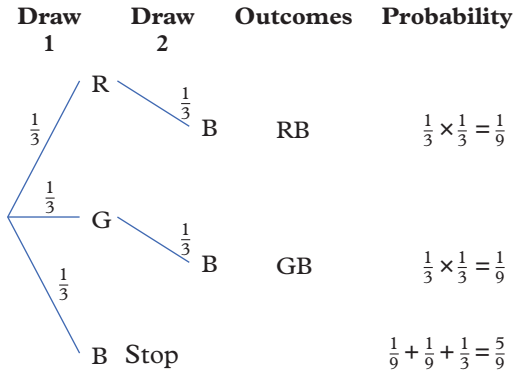
**b i** $\frac{60}{169}\approx 0.3550$ **ii** $\frac{69}{169}\approx 0.4083$

**iii** $\frac{9}{169}\approx 0.0533$ **iv** $\frac{100}{169}\approx 0.5917$

**10** The grouped outcomes of this chance experiment are not equally likely, so the theoretical probability is calculated by adding the probabilities of the successful outcomes instead of using the theoretical probability formula.

**11** Pr(blue ball) = Pr(blue ball on first draw) or Pr(not blue, then blue)

$$= \frac{1}{3} + \left(2 \times \frac{1}{3} \times \frac{1}{3}\right)$$
$$= \frac{5}{9}$$




**12 a** If the two-step chance experiment with replacement has lots of outcomes, they may be easier to see in an array than a tree diagram, and it can be easier to locate the favourable outcomes.

**b** If outcomes are not equally likely, then include the outcomes in the array in the same proportion as the likelihood. For example, if the first event is a ball is drawn at random from 1 blue and 2 red, then the array would have three columns: blue, red, red.

**13 a** Sample answer: No not necessarily. The socks are replaced each time, so you cannot say that this is the exact number of white and black socks. Some of the same socks may have been chosen multiple times.

**b** 4 black socks and 6 white socks

**c** Yes, this supports the previous estimate.

**14 a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{12}{25}$

**d** 14 aqua, 21 purple

**15 a** $\frac{5}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{47}{72}$

**16 a** $a^2$ **b** $2ab$ **c** $1 - a - b$

**d i** $a(1 - a - b)$ **ii** $a - a^2 - ab$

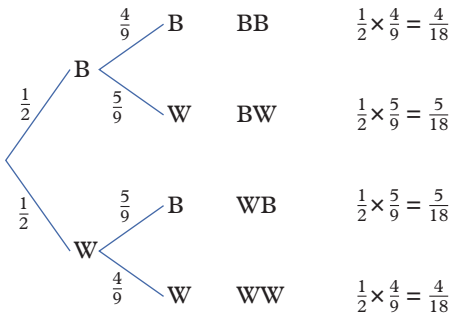
**e i** $(1 - a - b)^2$

**ii** $1 - 2a - 2b + 2ab + a^2 + b^2$

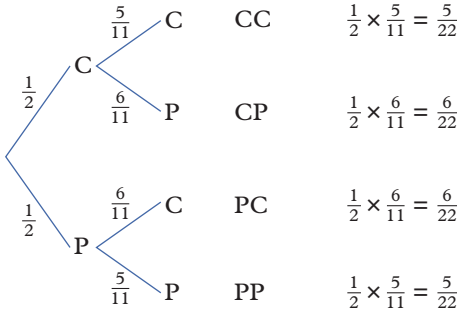
## 9C Experiments without replacement

EX p396

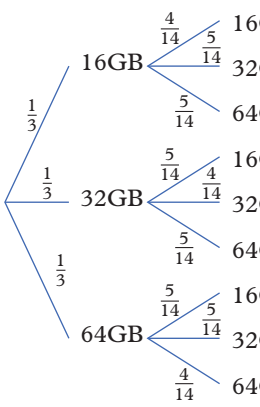
**1 a**




**b**

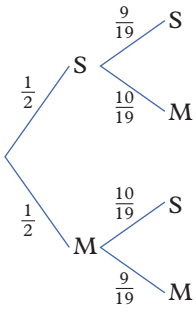


**c**



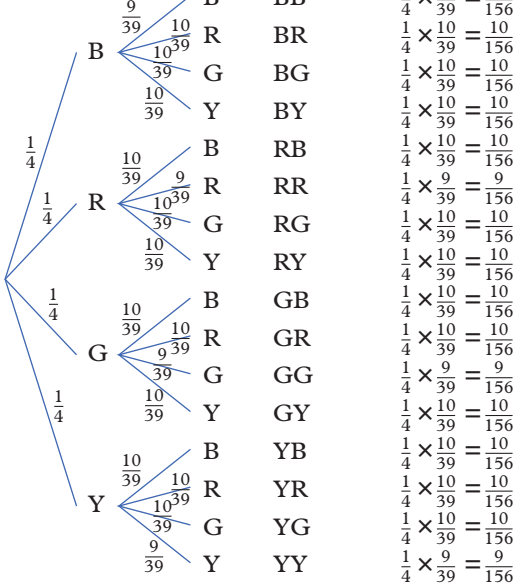

| Outcomes | | Probability |
|---|---|---|
| 16GB | 16, 16 | $\frac{1}{3} \times \frac{4}{14} = \frac{4}{42}$ |
| 32GB | 16, 32 | $\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$ |
| 64GB | 16, 64 | $\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$ |
| 16GB | 32, 16 | $\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$ |
| 32GB | 32, 32 | $\frac{1}{3} \times \frac{4}{14} = \frac{4}{42}$ |
| 64GB | 32, 64 | $\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$ |
| 16GB | 64, 16 | $\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$ |
| 32GB | 64, 32 | $\frac{1}{3} \times \frac{5}{14} = \frac{5}{42}$ |
| 64GB | 64, 64 | $\frac{1}{3} \times \frac{4}{14} = \frac{4}{42}$ |

**d**



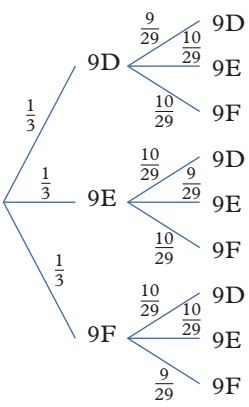

| Outcomes | | Probability |
|---|---|---|
| S | SS | $\frac{1}{2} \times \frac{9}{19} = \frac{9}{38}$ |
| M | SM | $\frac{1}{2} \times \frac{10}{19} = \frac{10}{38}$ |
| S | MS | $\frac{1}{2} \times \frac{10}{19} = \frac{10}{38}$ |
| M | MM | $\frac{1}{2} \times \frac{9}{19} = \frac{9}{38}$ |

**e**




| Outcomes | | Probability |
|---|---|---|
| B | BB | $\frac{1}{4} \times \frac{9}{39} = \frac{9}{156}$ |
| R | BR | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| G | BG | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| Y | BY | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| B | RB | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| R | RR | $\frac{1}{4} \times \frac{9}{39} = \frac{9}{156}$ |
| G | RG | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| Y | RY | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| B | GB | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| R | GR | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| G | GG | $\frac{1}{4} \times \frac{9}{39} = \frac{9}{156}$ |
| Y | GY | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| B | YB | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| R | YR | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| G | YG | $\frac{1}{4} \times \frac{10}{39} = \frac{10}{156}$ |
| Y | YY | $\frac{1}{4} \times \frac{9}{39} = \frac{9}{156}$ |

**f**

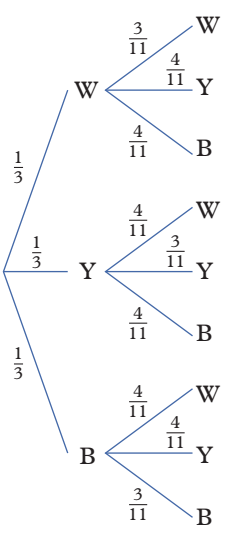



| Outcomes | | Probability |
|---|---|---|
| 9D | 9D, 9D | $\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$ |
| 9E | 9D, 9E | $\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$ |
| 9F | 9D, 9F | $\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$ |
| 9D | 9E, 9D | $\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$ |
| 9E | 9E, 9E | $\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$ |
| 9F | 9E, 9F | $\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$ |
| 9D | 9F, 9D | $\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$ |
| 9E | 9F, 9E | $\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$ |
| 9F | 9F, 9F | $\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$ |

**2** **a** 0.21 **b** 0.21 **c** 0.58

**3** **a** $\frac{3}{14}$ **b** $\frac{2}{7}$

**4** **a**

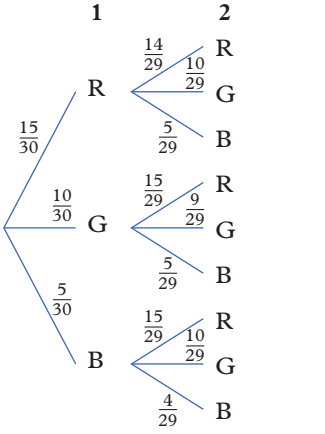



| Outcomes | | Probability |
|---|---|---|
| W | WW | $\frac{1}{3} \times \frac{3}{11} = \frac{3}{33}$ |
| Y | WY | $\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$ |
| B | WB | $\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$ |
| W | YW | $\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$ |
| Y | YY | $\frac{1}{3} \times \frac{3}{11} = \frac{3}{33}$ |
| B | YB | $\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$ |
| W | BW | $\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$ |
| Y | BY | $\frac{1}{3} \times \frac{4}{11} = \frac{4}{33}$ |
| B | BB | $\frac{1}{3} \times \frac{3}{11} = \frac{3}{33}$ |

**b** **i** $\frac{1}{11}$ **ii** $\frac{4}{33}$

**5** **a** $\frac{8}{11}$ **b** $\frac{19}{33}$ **iii** $\frac{14}{33}$

**6** **a**




| Customer 1 | Customer 2 | Outcomes | Probability |
|---|---|---|---|
| R | R | RR | $\frac{15}{30} \times \frac{14}{29} = \frac{210}{870} \approx 0.24$ |
| R | G | RG | $\frac{15}{30} \times \frac{10}{29} = \frac{150}{870} \approx 0.17$ |
| R | B | RB | $\frac{15}{30} \times \frac{5}{29} = \frac{75}{870} \approx 0.09$ |
| G | R | GR | $\frac{10}{30} \times \frac{15}{29} = \frac{150}{870} \approx 0.17$ |
| G | G | GG | $\frac{10}{30} \times \frac{9}{29} = \frac{90}{870} \approx 0.10$ |
| G | B | GB | $\frac{10}{30} \times \frac{5}{29} = \frac{50}{870} \approx 0.06$ |
| B | R | BR | $\frac{5}{30} \times \frac{15}{29} = \frac{75}{870} \approx 0.09$ |
| B | G | BG | $\frac{5}{30} \times \frac{10}{29} = \frac{50}{870} \approx 0.06$ |
| B | B | BB | $\frac{5}{30} \times \frac{4}{29} = \frac{20}{870} \approx 0.02$ |

**b** **i** $\frac{20}{870} = \frac{2}{87} \approx 0.0230$

**ii** $\frac{90}{870} = \frac{3}{29} \approx 0.1034$

**iii** $\frac{210}{870} = \frac{7}{29} \approx 0.2414$

**iv** $\frac{150}{870} = \frac{5}{29} \approx 0.1724$

**7** **a** $\frac{490}{870} = \frac{49}{87} \approx 0.5632$ **b** $\frac{660}{870} = \frac{22}{29} \approx 0.7586$

**c** $\frac{270}{870} = \frac{9}{29} \approx 0.3103$ **d** $\frac{450}{870} = \frac{15}{29} \approx 0.5172$

**e** $\frac{400}{870} = \frac{40}{87} \approx 0.4598$ **f** $\frac{100}{870} = \frac{10}{87} \approx 0.1149$

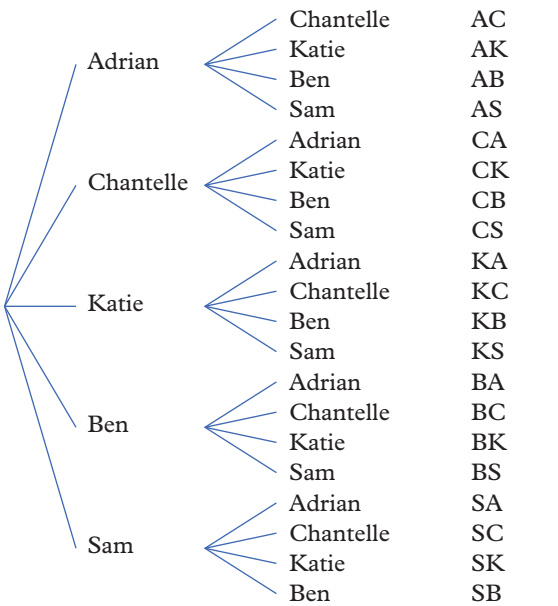
**8** **a** $\frac{1}{17}$ **b** $\frac{15}{34}$ **c** $\frac{19}{34}$ **d** $\frac{13}{102}$

**9** $\frac{1}{5} = 0.2$

**10** Pr(pair) $= \frac{5}{17} \approx 0.2941$; probability of a pair increases slightly.

**11** $\frac{38}{132} = \frac{19}{66} \approx 0.2879$

**12** **a**




**b** Second set of branches has one fewer branch. Once captain has been selected there is one fewer person to choose from for vice-captain's position.

**c** 20

**d** **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{20}$

**iv** $\frac{3}{5}$ **v** $\frac{1}{10}$

**13** **a** **i** $\frac{5}{18}$ **ii** $\frac{1}{6}$ **iii** $\frac{5}{18}$ **iv** $\frac{5}{9}$

**b** **i**

| | G | G | G | P |
|---|---|---|---|---|
| **G** | × | GG | GG | GP |
| **G** | GG | × | GG | GP |
| **G** | GG | GG | × | GP |
| **P** | PG | PG | PG | × |

**ii** $\frac{1}{2}$ **iii** 0

**14** **a** $\frac{1}{12}$ **b** $\frac{7}{12}$ **c** $\frac{13}{18}$ **d** $\frac{7}{12}$

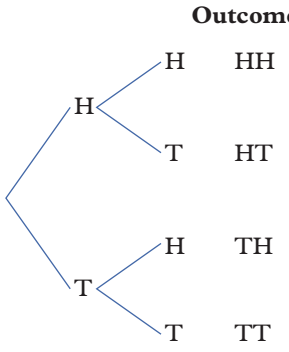
**15** **a** $\frac{1}{18}$ **b** $\frac{41}{126}$ **c** $\frac{85}{126}$

**16** $\frac{1}{8\,145\,060} \approx 0.000\,000\,123$

**17** $\frac{5}{18}$

## Checkpoint

**1**



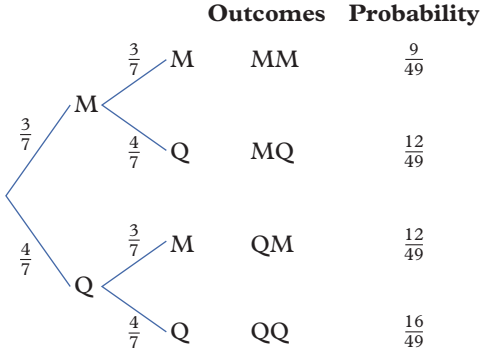

**2**

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **1** | (1, 1) | (2, 1) | (3, 1) | (4, 1) |
| **2** | (1, 2) | (2, 2) | (3, 2) | (4, 2) |
| **3** | (1, 3) | (2, 2) | (3, 3) | (3, 4) |
| **4** | (1, 4) | (2, 4) | (3, 4) | (4, 4) |

**3** **a** $\frac{4}{9}$ **b** $\frac{4}{9}$ **c** $\frac{8}{9}$

**4** **a** $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{5}{12}$

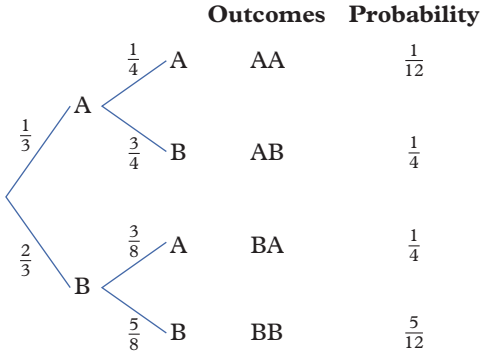
**5**




**6** **a** $\frac{83}{225}$ **b** $\frac{9}{25}$ **c** $\frac{8}{9}$

**7** **a** $\frac{24}{49}$ **b** $\frac{3}{7}$ **c** $\frac{40}{49}$

**8**



**9** **a** $\frac{1}{4}$ **b** $\frac{7}{12}$ **c** $\frac{11}{12}$

**10** **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{11}{12}$

## EX 9D Relative frequency

p402

**1** **a** $\frac{9}{25} \approx 0.36$ **b** $\frac{1}{100} = 0.01$

**c** $\frac{3}{8} \approx 0.63$ **d** $\frac{2}{3} \approx 0.67$

**e** $\frac{3}{4} = 0.75$ **f** $\frac{131}{240} \approx 0.55$

**2** **a** 125 **b** 40 **c** 105 **d** 5

**e** 40 **f** 214

**3** **a** $\frac{8}{15} \approx 0.53$

**b** In the long term, you would expect the experimental probability of rolling a die and obtaining a number greater than 2 to increase as it approaches the theoretical probability of $\frac{2}{3} \approx 0.67$.

**4** **a** 30

**b** $\frac{7}{130} \approx 0.054$

**c** In the long term, you would expect the experimental probability of drawing a picture card to increase as it approaches the theoretical probability of $\frac{3}{13} \approx 0.231$.

**5** **a** heads: 25; tails: 15

**b** one: 18; two: 24; three: 21; four: 12; five: 15; six: 30

**c** hearts: 12; diamonds: 18; clubs: 21; spades: 9

**6** **a** **i** 30

**ii** There is only a slight difference between the expected number and the actual number, so there is no evidence that the experiment is biased.

**b** **i** 18

**ii** There is only a slight difference between the expected number and the actual number, so there is no evidence that the experiment is biased.

**c** **i** 100

**ii** There is a significant difference between the expected number and the actual number and the number of trials is large. It appears that this experiment is biased.

**7** **a** **i** 10

**ii** $\Pr(\text{heads}) = \frac{1}{2}$, $\Pr(\text{tails}) = \frac{1}{2}$

**iii** heads = 5, tails = 5

**iv** $\Pr(\text{heads}) = \frac{4}{5} = 0.8$, $\Pr(\text{tails}) = \frac{1}{5} = 0.2$

**v** There are not enough trials to make a firm decision.

**vi** There are only 10 trials; this is too few to tell if it is biased.

**b** **i** 6000

**ii** $\Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \Pr(6)$

$= \frac{1}{6} \approx 0.17$

**iii** expected number = 1000 for all outcomes

**iv** $\Pr(1) \approx 0.16$, $\Pr(2) \approx 0.16$, $\Pr(3) \approx 0.17$, $\Pr(4) \approx 0.17$, $\Pr(5) \approx 0.17$, $\Pr(6) \approx 0.17$

**v** The prop is fair.

**vi** The frequency is very close to the expected number for each outcome. The relative frequency for each outcome is approximately equal to its theoretical probability after a large number of trials.

**c** **i** 112

**ii** Pr(hearts) = Pr(diamonds) = Pr(clubs) = Pr(spades) = $\frac{1}{4}$

**iii** expected number = 28 for all outcomes

**iv** Pr(hearts) ≈ 0.34, Pr(diamonds) ≈ 0.28, Pr(clubs) ≈ 0.17, Pr(spades) ≈ 0.21

**v** The prop is biased.

**vi** Not all frequencies are close to expected numbers of each outcome. Hearts and diamonds are greater than expected, and clubs and spades are less than expected. Relative frequencies for all outcomes are not approximately equal to their theoretical probabilities after a reasonable number of trials.

**8** **a** HHH, HHT, HTH, HTT, THH, TTH, THT, TTT

**b** $\frac{1}{4}$ **c** 10

**d** No, the actual occurrences are only half of the expected number.

**e** The relative frequency should get closer to the theoretical probability after 4000 trials.

**9** With a small number of rolls, the actual number (13) can be significantly different from the expected number (5) due to random chance. As the number of rolls increases, the relative frequency will approach the theoretical probability if no bias exists. Therefore, to check for any bias a higher number of trials should take place.

**10** **a** A sample size of two is not a broad enough sample to verify a claim about the worldwide adult population.

**b** While the expected number would be 1 for a group of 3, 'one in three' does not mean one in any group of three. It means that any given Australian has a one in three chance of being an Australian who does not get enough sleep.

**c** The first headline is worldwide while the second is Australian only. The samples may be biased. The sample sizes may be too small to draw wide-scale conclusions from.

**11** The 39% is likely to be more reliable as a larger sample has been taken.

**12** The number of people who are left-handed changes year to year as more people are born and die.

**13** **a** $\frac{1}{128}$

**b** A streak of 7 of more heads has a 1 in 128 chance of occurring, so it is very likely to occur if you toss a coin 1000 times.

**c** 31

## EX 9E Two-way tables

p407

**1** **a** 55 **b** 7 **c** $\frac{7}{55}$

**2** **a** 75 **b** 11 **c** $\frac{58}{75}$

**3** **a** $\frac{29}{85}$ **b** $\frac{37}{85}$ **c** $\frac{42}{85}$ **d** $\frac{23}{85}$

**4** **a** $\frac{48}{125}$ **b** $\frac{19}{50}$ **c** $\frac{13}{125}$ **d** $\frac{31}{50}$

**e** $\frac{181}{250}$ **f** $\frac{27}{125}$

**5**

| | Class A | Class B | Total |
|---|---|---|---|
| **Prefer sweet food** | 0.23 | 0.32 | 0.55 |
| **Prefer savoury food** | 0.29 | 0.16 | 0.45 |
| **Total** | 0.52 | 0.48 | 1 |

**6**

| | Reads books | Plays games | Plays sports | Total |
|---|---|---|---|---|
| **Introvert** | 0.218 75 | 0.171 875 | 0.125 | 0.515 625 |
| **Extrovert** | 0.046 875 | 0.171 875 | 0.265 625 | 0.484 375 |
| **Total** | 0.265 625 | 0.343 75 | 0.390 625 | 1 |

**7**

| | Light hair | Dark hair | Total |
|---|---|---|---|
| **Prefer pop music** | 0.22 | 0.38 | 0.6 |
| **Prefer rock music** | 0.13 | 0.27 | 0.4 |
| **Total** | 0.35 | 0.65 | 1 |

**8**

| | Spotify | No Spotify | Total |
|---|---|---|---|
| **Netflix** | $\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{2}{3}$ |
| **No Netflix** | $\frac{1}{12}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |
| **Total** | $\frac{1}{4}$ | $\frac{3}{4}$ | 1 |

**9** **a**

| | Cinema | Home | Total |
|---|---|---|---|
| **Action** | 22 | 30 | 52 |
| **Comedy** | 19 | 14 | 33 |
| **Total** | 41 | 44 | 85 |

**b** **i** $\frac{41}{85}$ **ii** $\frac{14}{85}$ **iii** $\frac{52}{85}$

**10**

| | Cat | No cat | Total |
|---|---|---|---|
| **Purebred** | 29 | 75 | 104 |
| **Not purebred** | 84 | 12 | 96 |
| **Total** | 113 | 87 | 200 |

$\Pr(\text{non-cat purebred}) = \frac{75}{200}$

$= \frac{3}{8}$

**11** **a**

| | Women | Men | Total |
|---|---|---|---|
| **Bath** | 0.33 | 0.09 | 0.42 |
| **Shower** | 0.22 | 0.36 | 0.58 |
| **Total** | 0.55 | 0.45 | 1 |

**b** **i** $\frac{9}{25}$ **ii** $\frac{21}{50}$ **iii** $\frac{11}{50}$

**c** **i** 210 **ii** 275 **iii** 45

**12** **a** $\frac{18}{83}$ **b** $\frac{67}{332}$ **c** $\frac{4}{83}$

**d** $\frac{125}{332}$ **e** $\frac{72}{83}$ **f** $\frac{69}{83}$

**g** $\frac{109}{166}$ **h** $\frac{47}{332}$ **i** $\frac{9}{83}$

**13** **a** 60 **b** 160 **c** 980 **d** 120

**14**

| | A | A' | Total |
|---|---|---|---|
| **B** | 40 | 30 | 70 |
| **B'** | 20 | 30 | 50 |
| **Total** | 60 | 60 | 120 |

**15** **a**

| | A | A' | Total |
|---|---|---|---|
| **B** | 40 | 35 | 75 |
| **B'** | 40 | 85 | 125 |
| **Total** | 80 | 120 | 200 |

| | A | A' | Total |
|---|---|---|---|
| **B** | 0.2 | 0.175 | 0.375 |
| **B'** | 0.2 | 0.425 | 0.625 |
| **Total** | 0.4 | 0.6 | 1 |

**b**

| | C | C' | Total |
|---|---|---|---|
| **D** | 4 | 5 | 9 |
| **D'** | 2 | 9 | 11 |
| **Total** | 6 | 14 | 20 |

| | C | C' | Total |
|---|---|---|---|
| **D** | 0.2 | 0.25 | 0.45 |
| **D'** | 0.1 | 0.45 | 0.55 |
| **Total** | 0.3 | 0.7 | 1 |

**c**

| | E | E' | Total |
|---|---|---|---|
| **F** | 18 | 4 | 22 |
| **F'** | 2 | 12 | 14 |
| **Total** | 20 | 16 | 36 |

| | E | E' | Total |
|---|---|---|---|
| **F** | $\frac{1}{2}$ | $\frac{1}{9}$ | $\frac{11}{18}$ |
| **F'** | $\frac{1}{18}$ | $\frac{1}{3}$ | $\frac{7}{18}$ |
| **Total** | $\frac{5}{9}$ | $\frac{4}{9}$ | 1 |

**16** **a**

| | Year 8 | Year 9 | Total |
|---|---|---|---|
| **Maths** | 12 | 11 | 23 |
| **English** | 10 | 17 | 27 |
| **Total** | 22 | 28 | 50 |

**b** **i** $\frac{23}{50}$ **ii** $\frac{14}{25}$ **iii** $\frac{11}{50}$

**c** **i** 28 **ii** 11 **iii** $\frac{11}{28}$

**d** Part **c iii** says the student is in Year 9, hence the total number of possible outcomes are restricted to the total number of Year 9 students (28). In part **b iii** you are selecting from the total number of students (50), and of those you need to select a student in Year 9 who likes Maths.

**e** **i** likes English; 27

**ii** Year 8 students who like English

**iii** 10

**iv** $\frac{10}{27}$

**17** **a** $\frac{13}{81}$ **b** $\frac{37}{96}$ **c** $\frac{1}{5}$

**d** $\frac{13}{43}$ **e** $\frac{71}{181}$ **f** $\frac{4}{5}$

## EX 9F Venn diagrams

p414

**1** **a** 14 **b** 15 **c** 2 **d** 31

**2** **a** 62 **b** 19 **c** 532 **d** 637

**3** **a** $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{11}{20}$

**d** $\frac{7}{10}$ **e** $\frac{1}{10}$ **f** $\frac{1}{4}$

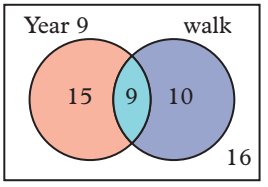
**4** **a** $\frac{17}{45}$ **b** $\frac{1}{9}$ **c** $\frac{19}{45}$

**d** $\frac{1}{5}$ **e** $\frac{23}{45}$ **f** $\frac{31}{45}$

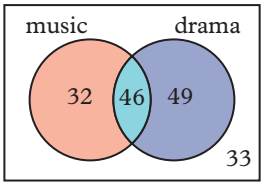
**5** **a** $\frac{9}{41}$ **b** $\frac{21}{41}$ **c** $\frac{11}{41}$

**d** $\frac{8}{41}$ **e** $\frac{22}{41}$ **f** $\frac{33}{41}$

**6**




**7**




**8** **a** $A \cap B$ **b** $A \cup B$ **c** $A' \cup B$

**d** $A \cap B'$ **e** $A \cup B'$ **f** $A' \cap B'$

**g** $A' \cap B$ **h** $A' \cup B'$

**9** **a** **i** 29 **ii** 43 **iii** 26

**iv** 10 **v** 50 **vi** 14

**b** **i** $\frac{12}{25}$ **ii** $\frac{21}{50}$ **iii** $\frac{1}{5}$

**iv** $\frac{7}{50}$ **v** $\frac{7}{50}$ **vi** $\frac{31}{50}$

**10** **a** $(A \cap B)' = A' \cup B'$ and $(A' \cup B')' = A \cap B$

$(A' \cap B)' = A \cup B'$ and $(A \cup B')' = A' \cap B$

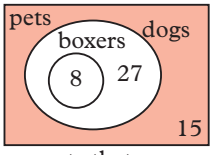
$(A \cap B')' = A' \cup B$ and $(A' \cup B)' = A \cap B'$

$(A' \cap B')' = A \cup B$ and $(A \cup B)' = A' \cap B'$

**b** The complement of an intersection is the union of the complements of the two events. The complement of a union is the intersection of the complements of the two events.

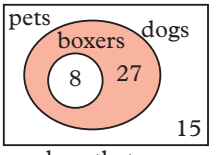
**11** **a** The first Venn diagram shows a subset of data. All the set of 'Boxers' belongs to the set of 'Dogs', hence 'Boxers' is a subset of 'Dogs'.

**b** **i**



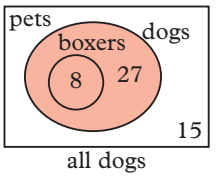

pets that are not dogs

**ii**



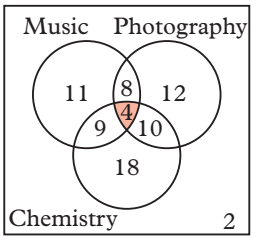

dogs that are not boxers

**iii**

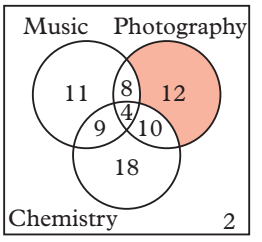



all dogs

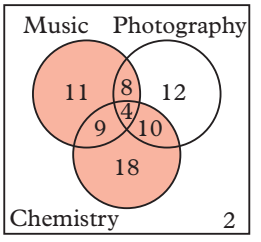
**c** **i**



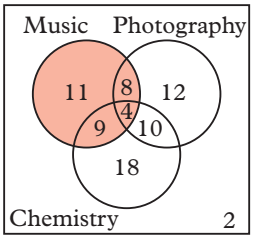

**ii**



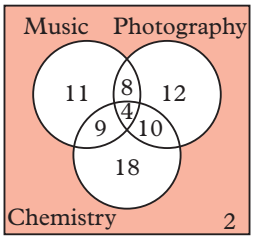

**iii**




**iv**



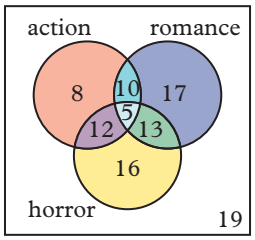

**v**



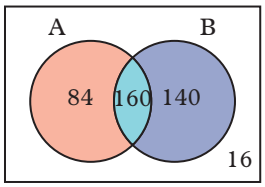

**12** **a** $\frac{7}{34}$ **b** $\frac{4}{17}$ **c** $\frac{1}{17}$

**d** $\frac{5}{34}$ **e** $\frac{23}{34}$ **f** $\frac{7}{34}$

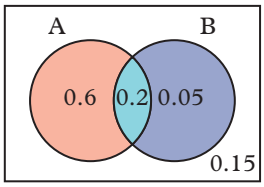
**13** Sample answer:

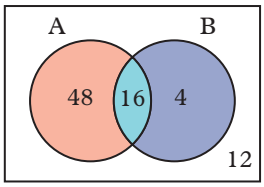



**14**




**15 a**




**b i** 0.8 **ii** 0.25

**16 a** $\frac{29}{61}$ **b** $\frac{29}{46}$ **c** $\frac{32}{61}$ **d** $\frac{9}{25}$

**17 a** $\frac{11}{30}$ **b** $\frac{11}{26}$ **c** $\frac{15}{26}$ **d** $\frac{2}{7}$

## Chapter review

### Multiple-choice

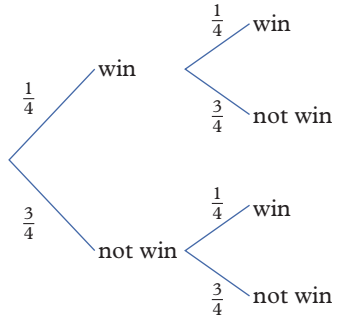
**1** C **2** C **3** A **4** A **5** D

**6** A **7** A **8** D **9** D **10** C

### Short answer

**1 a** $\frac{3}{4}$

**b** **Chance of winning an eBook**




**c i** $\frac{1}{16}$ **ii** $\frac{15}{16}$ **iii** $\frac{3}{16}$

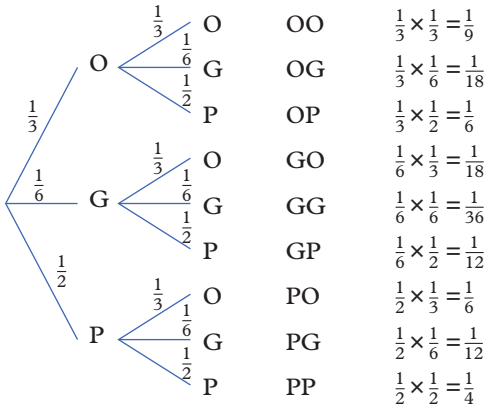
**2 a**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **H** | (H, 1) | (H, 2) | (H, 3) | (H, 4) | (H, 5) | (H, 6) |
| **T** | (T, 1) | (T, 2) | (T, 3) | (T, 4) | (T, 5) | (T, 6) |

**b i** $\frac{1}{12}$ **ii** $\frac{7}{12}$ **iii** $\frac{1}{2}$

**c** $\frac{1}{6}$ **d** $\frac{7}{12}$

**3 a**




**b i** $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{4}$

**c** $\frac{1}{6}$ **d** $\frac{1}{2}$

**4 a** $\frac{1}{17}$ **b** $\frac{13}{204}$

**5 a** $\frac{3}{8}$ **b** $\frac{1}{200}$ **c** $\frac{11}{100}$ **d** $\frac{13}{20}$

**6 a** 35 **b** 10 **c** 5 **d** 25

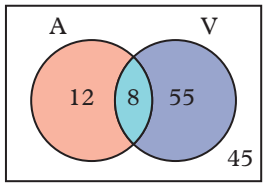
**7 a**

| | Year 8 | Year 9 | Year 10 | Total |
|---|---|---|---|---|
| **Napoleon Perdis** | 32 | 25 | 45 | 102 |
| **Rimmel** | 22 | 10 | 22 | 54 |
| **Covergirl** | 17 | 31 | 13 | 61 |
| **Maybelline** | 49 | 29 | 15 | 93 |
| **Total** | 120 | 95 | 95 | 310 |

**b i** $\frac{3}{10}$ **ii** $\frac{11}{155}$ **iii** $\frac{5}{62}$

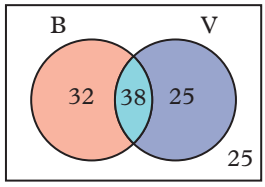
**c i** $\frac{5}{19}$ **ii** $\frac{12}{19}$

**8 a**




**b i** $\frac{1}{15}$ **ii** $\frac{5}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{21}{40}$

**c**

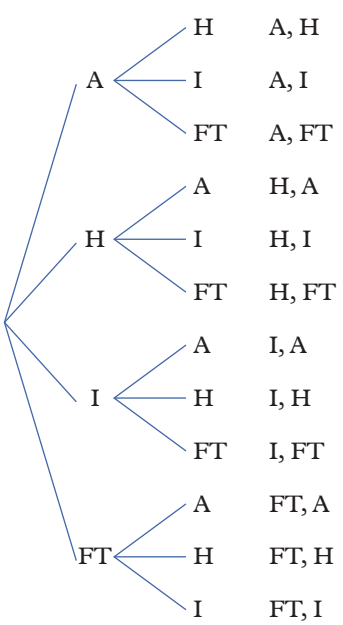



**d i** $\frac{19}{60}$ **ii** $\frac{11}{15}$ **iii** $\frac{4}{15}$ **iv** $\frac{7}{12}$

## Analysis

**a** Without replacement, as the same subject cannot be selected twice.

**b i**




**Key:**
A = Art
H = Health and Human Development
I = Italian
FT = Food Technology

**ii** 12

**c i** $\frac{1}{6}$ **ii** $\frac{1}{6}$

**d i**

| | Year 11 | Year 12 | Total |
|---|---|---|---|
| **Art** | 0.24 | 0.12 | 0.36 |
| **Food technology** | 0.28 | 0.36 | 0.64 |
| **Total** | 0.52 | 0.48 | 1.00 |

**ii** $\frac{9}{25}$ **iii** $\frac{6}{25}$ **iv** $\frac{9}{25}$

**v** 72 **vi** 128

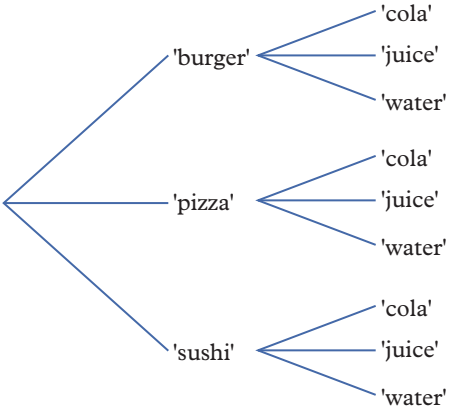
**e i** 24 **ii** 2 **iii** 68

**iv** $\frac{34}{75}$ **v** $\frac{28}{75}$

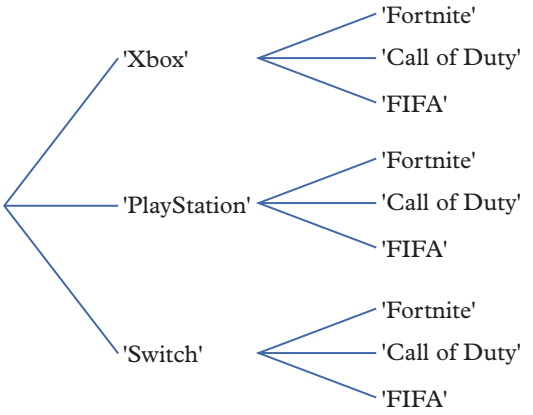
# CHAPTER 10 Computational thinking

## EX p428 10A Nested loops

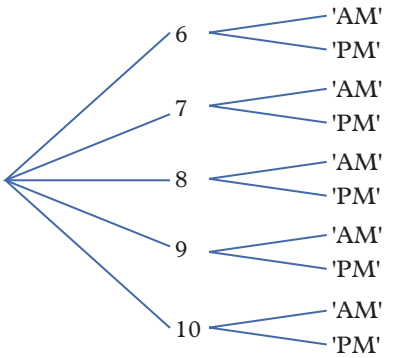
**1 a**



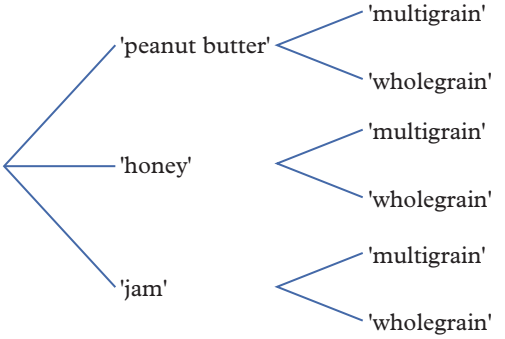

**b**



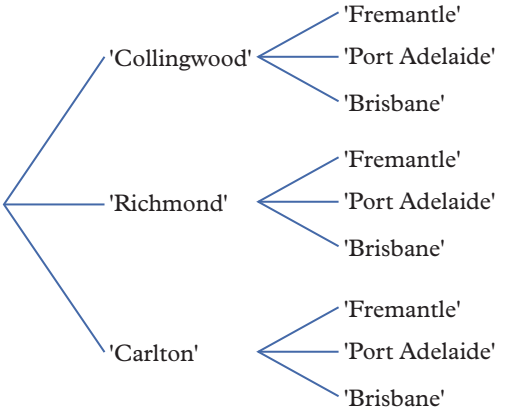

**c**




**d**




**e**




**2 a**

```
11 is greater than 10!
13 is greater than 10!
15 is greater than 10!
12 is greater than 10!
17 is greater than 10!
14 is greater than 10!
11 is greater than 10!
```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**b**
```
chair has an a
chair has an i
house has an e
house has an o
house has an u
field has an e
field has an i
```
**c**
```
-3 is a negative number
2 is a positive number
-1 is a negative number
6 is a positive number
-4 is a negative number
2 is a positive number
-9 is a negative number
6 is a positive number
-3 is a negative number
```
**d**
```
10 oclock
10 past 10
20 past 10
half past 10
20 to 11
10 to 11
11 oclock
10 past 11
20 past 11
half past 11
20 to 12
10 to 12
```

**3 a**
```
B A
B E
B I
H A
H E
H I
L A
L E
L I
```
**b**
```
green pear
green grapes
green apple
sweet pear
sweet grapes
sweet apple
shiny pear
shiny grapes
shiny apple
```
**c**
```
composite is bigger
composite is bigger
composite is bigger
composite is bigger
composite is bigger
composite is bigger
prime is bigger
prime is bigger
prime is bigger
```
**d**
```
Volume = 60
Volume = 160
Volume = 200
Volume = 54
Volume = 144
Volume = 180
```
**e**
```
Home Team Win: 63 - 19
Away Team Win: 49 - 57
Home Team Win: 101 - 99
Home Team Win: 89 - 74
```

## EX p435 10B Sorting a list of numbers

**1 a**

1 8 2 5
1 2 8 5
1 2 5 8
1 2 5 8
1 2 5 8
1 2 5 8

**b**

2 7 0 3
2 0 7 3
2 0 3 7
0 2 3 7
0 2 3 7
0 2 3 7

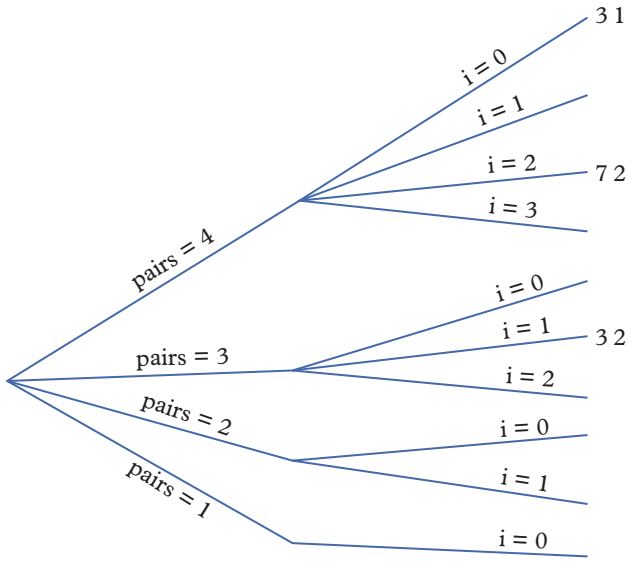
**c**

6 8 2 9 5
6 2 8 9 5
6 2 8 9 5
6 2 8 5 9
2 6 8 5 9
2 6 8 5 9
2 6 5 8 9
2 6 5 8 9
2 5 6 8 9
2 5 6 8 9

**d**

5 6 4 3 2 1
5 4 6 3 2 1
5 4 3 6 2 1
5 4 3 2 6 1
5 4 3 2 1 6
4 5 3 2 1 6
4 3 5 2 1 6
4 3 2 5 1 6
4 3 2 1 5 6
3 4 2 1 5 6
3 2 4 1 5 6
3 2 1 4 5 6
2 3 1 4 5 6
2 1 3 4 5 6
1 2 2 4 5 6

**2**



**3**
```
length = len(numbers)
for pairs in range(length-1,0,-1):
 for i in range(0,pairs,1):
  if numbers[i] > numbers[i+1]:
   numbers[i], numbers[i+1] =
   numbers[i+1], numbers[i]
print(numbers[0])
```

**4**
```
length = len(numbers)
for pairs in range(length-1,0,-1):
 for i in range(0,pairs,1):
  if numbers[i] > numbers[i+1]:
   numbers[i], numbers[i+1] = num
   numbers[i+1], numbers[i]
print(numbers[-2])
```

**5**
```
middle = int((len(numbers) - 1) / 2)
median = numbers[middle]
print(median)
```

**6**
```
middle_left = int(len(numbers) / 2 - 1)
middle_right = int(len(numbers) / 2)
median = (numbers[middle_left] +
numbers[middle_right])/2
print(median)
```

**7** The bubble sort algorithm sorts the numbers in a list. To calculate the mean of a list of numbers, all numbers have to be added up so there is no benefit to having the numbers sorted. Therefore, the bubble sort algorithm doesn't help.

**8** Comparing each pair of numbers in a list and swapping if they are out of order guarantees that the highest number will be at the end of the list (the other numbers will not necessarily be in the right order though). So to find the highest number, it is enough to just run the first "branch" of the bubble sort algorithm (that is, comparing each pair just once) and then looking at the last number in the list.

**9**
```
length = len(numbers)
for pairs in range(length-1,0,-1):
 for i in range(0,pairs,1):
  if numbers[i] < numbers[i+1]:
   numbers[i], numbers[i+1] =
   numbers[i+1], numbers[i]
print(numbers)
```

## EX 10C Functions

p438

**1** **a** **i** `8.0` **ii** `20.0`
**b** **i** `2` **ii** `hat`
**c** **i** `False` **ii** `True`
**d** **i** `7` **ii** `4` **iii** `10`
**e** **i** `1 digit`
**ii** `3 or more digits`
**iii** `2 digits`
**iv** `1 digit`

**2**
```
def second(L):
 return L[1]
```

**3**
```
def length(L):
 return len(L)
```

**4**
```
def length(L):
 count = 0
 for item in L:
 count = count + 1
 return count
```

**5**
```
def BubbleSort(numbers):
 length = len(numbers)
 for pairs in range(length-1,0,-1):
  for i in range(0,pairs,1):
   if numbers[i] > numbers[i+1]:
    numbers[i], numbers[i+1] =
    numbers[i+1], numbers[i]
 return numbers
```

**6**
```
def BubbleSort_lowest(numbers):
 length = len(numbers)
 for pairs in range(length-1,0,-1):
  for i in range(0,pairs,1):
   if numbers[i] > numbers[i+1]:
    numbers[i], numbers[i+1] =
    numbers[i+1], numbers[i]
 return numbers[0]
```

**7**
```
def odd_or_even(numbers):
 if len(numbers) % 2 == 0:
  return "even"
 else:
  return "odd"
```

**8**
```
def find_median(numbers):
 length = len(numbers)
 for pairs in range(length-1,0,-1):
  for i in range(0,pairs,1):
   if numbers[i] > numbers[i+1]:
    numbers[i], numbers[i+1] =
    numbers[i+1], numbers[i]
 if length % 2 == 0:
  middle_left = int(len(numbers) / 2 - 1)
  middle_right = int(len(numbers) / 2)
  median = (numbers[middle_left] +
  numbers[middle_right])/2
 else:
  middle = int((len(numbers) - 1) / 2)
  median = numbers[middle]
 return median
```

**9**
```
def BubbleSort_descend(numbers):
 length = len(numbers)
 for pairs in range(length-1,0,-1):
  for i in range(0,pairs,1):
   if numbers[i] < numbers[i+1]:
    numbers[i], numbers[i+1] =
    numbers[i+1], numbers[i]
 return numbers
```

## EX SEMESTER 2 Review

p440

### Short answer

**1** **a** 40.91 cm$^2$ **b** 61 000.00 cm$^2$
**c** 201.06 mm$^2$ or 2.01 cm$^2$ **d** 52 216.00 cm$^2$

**2** **a** 154 cm$^2$ or 15 400 mm$^2$ **b** 36 cm$^2$
**c** 483.8 cm$^2$ **d** 2211.2 cm$^2$

**3** **a** 122.5 cm$^3$ **b** 12 cm$^3$
**c** 615.8 cm$^2$ **d** 3200 cm$^3$

**4** **a** $x = 25°, y = 65°$
**b** $a = 101°, b = 101°, c = 79°, d = 79°$
**c** $p = 95°, q = 122°, r = 85°, s = 58°, t = 37°, u = 37°$
**d** $w = 116°, v = 27°, u = 37°$
**e** $z = 60°, x = 60$ m, $y = 17$ m
**f** $a = 31.2$ m, $b = 122°, c = 58°, d = 60$ m

**5** **a** yes, SSS **b** yes, SAS **c** no
**d** yes, AAA **e** no **f** yes, SAS

**6** **a** **i** $\frac{1}{2}$ **ii** 0.5581
**iii** 0.8372 **iv** 0.6667
**b** **i** 3 **ii** 0.7593
**iii** 0.6508 **iv** 1.1667
**c** **i** $\frac{1}{13}$ **ii** 0.500
**iii** 0.8660 **iv** 0.5774
**d** **i** 5 **ii** 0.96
**iii** 0.28 **iv** 3.4286

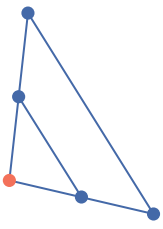
**7** **a** 70° **b** 49° **c** 76° **d** 148°

**8** **a** $z = 13.6$ m
**b** $x = 30.9$ m, $y = 18.2$ m
**c** $p = 19.2$ m, $q = 18.9$ m
**d** $r = 60.4$ m

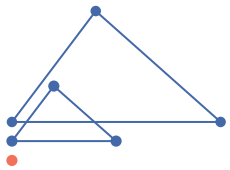
**9** **a** no **b** no
**c** yes **d** yes

**10** **a** 2610.1 cm$^2$ **b** 1008.1 cm$^2$
**c** 16 799.9 cm$^2$ **d** 210 276.5 mm$^2$

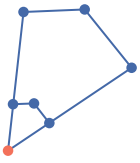
**11** **a**



**b**



**c**



**12** **a** **i** categorical **ii** nominal
**b** **i** numerical **ii** continuous
**c** **i** numerical **ii** discrete

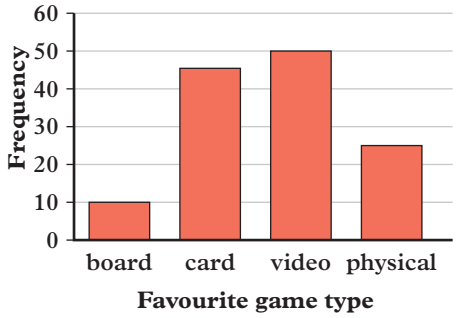
**13** **a** Sample answer:

| Class | Frequency |
|---|---|
| 5–<10 | 2 |
| 10–<15 | 0 |
| 15–<20 | 6 |
| 20–<25 | 4 |
| 25–<30 | 7 |
| 30–<35 | 1 |

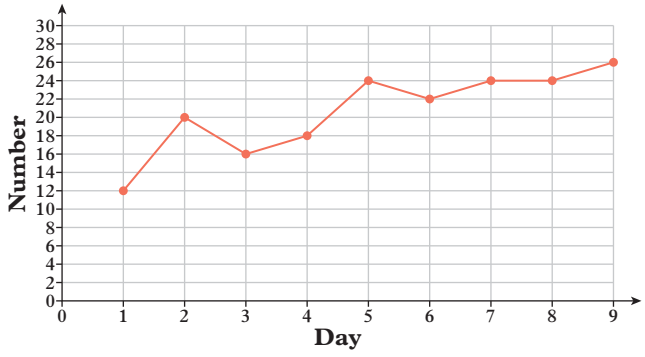
**b**

| Class | Frequency |
|---|---|
| 20–<40 | 2 |
| 40–<60 | 5 |
| 60–<80 | 8 |
| 80–<100 | 21 |
| 100–<120 | 4 |

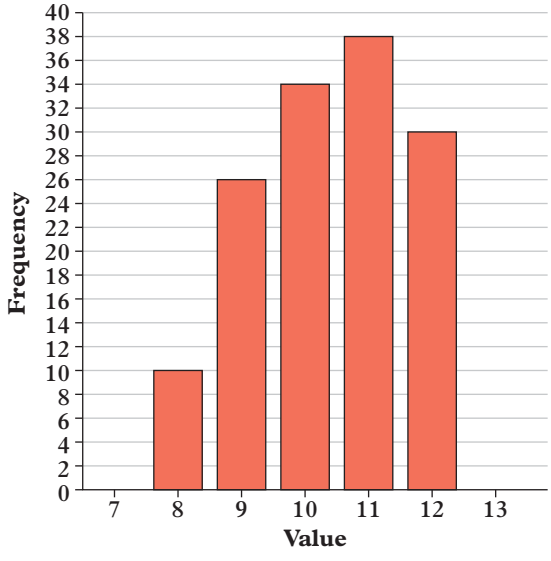
**14** **a**



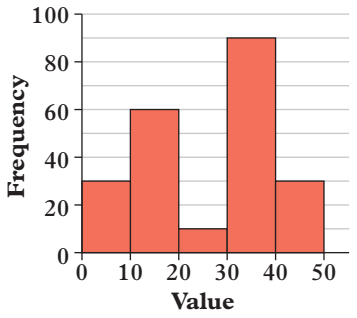

**b**




**c**



**d**

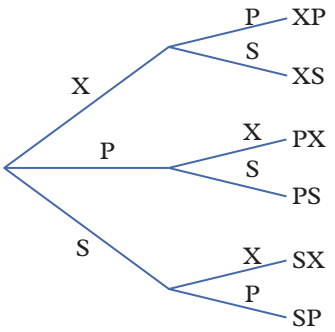



**15 a** **i** 19.8 **ii** 19
**iii** 19 **iv** 10
**b** **i** 12.34 **ii** 12
**iii** 13 **iv** 4
**c** **i** 4.36 **ii** 4.1
**iii** 3.4 **iv** 4.7

**16 a** bimodal symmetric **b** perfectly symmetric
**c** negatively skewed **d** positively skewed
**e** negatively skewed with outlier
**f** symmetric

**17 a** mode of each peak **b** mean, median or mode
**c** mean, median or mode **d** median
**e** median or mode **f** mode

**18 a**




**b** **i** $\frac{1}{6}$ **ii** $\frac{1}{3}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$

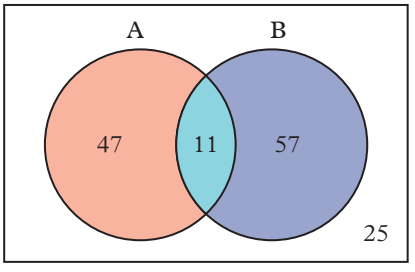
**19 a**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Spade** | (1, ♠) | (2, ♠) | (3, ♠) | (4, ♠) | (5, ♠) | (6, ♠) |
| **Club** | (1, ♣) | (2, ♣) | (3, ♣) | (4, ♣) | (5, ♣) | (6, ♣) |
| **Diamond** | (1, ♦) | (2, ♦) | (3, ♦) | (4, ♦) | (5, ♦) | (6, ♦) |
| **Heart** | (1, ♥) | (2, ♥) | (3, ♥) | (4, ♥) | (5, ♥) | (6, ♥) |

**b** **i** $\frac{1}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$ **iv** $\frac{1}{2}$

**20 a** $\frac{16}{49}$ **b** $\frac{12}{35}$ **c** $\frac{1}{35}$ **d** $\frac{1}{3}$

**21 a** Sample answer:




**b**

| | A | A′ | Total |
|---|---|---|---|
| **B** | 5 | 21 | 26 |
| **B′** | 12 | 12 | 24 |
| **Total** | 17 | 33 | 50 |

**22 a**

| | S | S′ | Total |
|---|---|---|---|
| **G** | 56 | 76 | 132 |
| **G′** | 42 | 38 | 80 |
| **Total** | 98 | 114 | 212 |

**b** **i** $\frac{57}{106}$ **ii** $\frac{33}{53}$ **iii** $\frac{19}{106}$ **iv** $\frac{59}{106}$

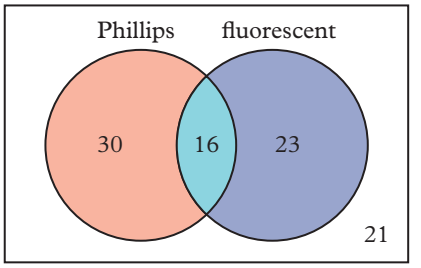
**23 a**

| | A | A′ | Total |
|---|---|---|---|
| **B** | $\frac{3}{5}$ | $\frac{1}{25}$ | $\frac{16}{25}$ |
| **B′** | $\frac{7}{25}$ | $\frac{2}{25}$ | $\frac{9}{25}$ |
| **Total** | $\frac{22}{25}$ | $\frac{3}{25}$ | 1 |

**b**

| | A | A′ | Total |
|---|---|---|---|
| **B** | 51 | 29 | 80 |
| **B′** | 15 | 25 | 40 |
| **Total** | 66 | 54 | 120 |

**24 a**



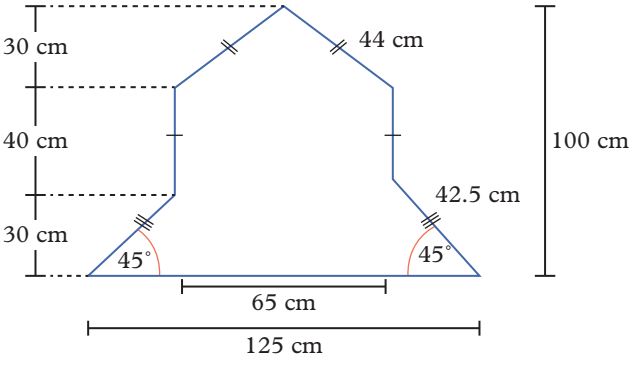

**b** **i** $\frac{23}{45}$ **ii** $\frac{17}{30}$ **iii** $\frac{22}{45}$ **iv** $\frac{2}{3}$

**25 a** **i** 60 **ii** 77 **iii** 52
**iv** 28 **v** 112 **vi** 17
**b** **i** $\frac{15}{28}$ **ii** $\frac{13}{28}$ **iii** $\frac{1}{4}$
**iv** $\frac{5}{16}$ **v** $\frac{5}{16}$ **vi** $\frac{95}{112}$

## Analysis

**1 a** 8.9 cm should be 8.8 cm, correct to one decimal place. 8.5 cm and 13.0 cm are correct.

**b**





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**c** **i** 257 cm$^2$ **ii** 6425 cm$^2$

**d** 69.3 cm **e** 30°

**f** They are not parallel as the angle the edge of the plywood makes with the horizontal (≈ 42.7°) is not the same as the angle the string makes with the horizontal (30°), and these are corresponding angles.

**2 a**

| Leaf Arielle | Stem | Leaf Brenna |
|---|---|---|
| 7 | 3* | 7 |
| 4 3 3 1 1 | 4 | 0 1 2 3 4 |
| 9 9 9 8 8 8 8 8 8 7 7 7 7 6 | 4* | 5 6 6 6 8 |
| 3 2 2 1 0 0 | 5 | 0 0 1 1 1 1 3 3 4 |
| 5 | 5* | 5 6 7 7 8 |
| | 6 | 0 0 3 |

Key: 1 | 2 = 120

**b** Arielle's minutes of sleeping is negatively skewed whereas Brenna's minutes of sleeping is approximately symmetric.

**c** Arielle: median = 480 minutes, range = 180 minutes
Brenna: median = 510 minutes, range = 260 minutes

**d** Arielle sleeps on average less than Brenna with a median of 480 minutes of sleep compared to 510 minutes of sleep for Brenna. Arielle has a more consistent length of sleep with a range of 180 minutes as compared to 260 minutes for Brenna.

**e** Independent variable: Month
Dependent variable: Median sleep length

**f** Arielle and Brenna have valleys in June and November, but their peaks don't coincide. Both Arielle and Brenna's sleep length is, on average, increasing.

**3 a** **i** $\frac{2}{3}$ **ii** $\frac{7}{30}$

**iii** $\frac{1}{4}$ **iv** $\frac{15}{23}$

**b** **i** $\frac{2}{23}$ **ii** $\frac{7}{45}$

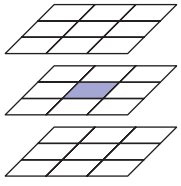
**c** **i** $\frac{4}{63}$ **ii** $\frac{9}{64}$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –1 | 0 | 1 | 2 | 3 | 4 | 5 |

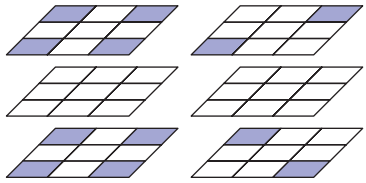
## EX p450 EXPLORATIONS 2

**1 a** Rotations and reflections of the minimum diagrams in **ii** and **iv** are also possible.

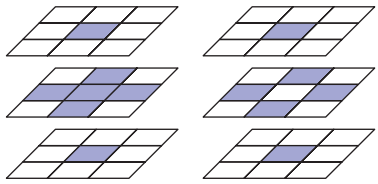
**i** maximum = minimum = 1



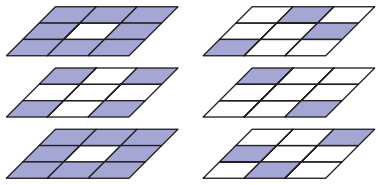
**ii** maximum = 8 minimum = 4



**iii** maximum = 7 minimum = 6

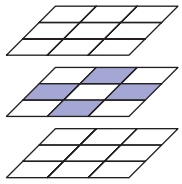


**iv** maximum = 20 minimum = 8

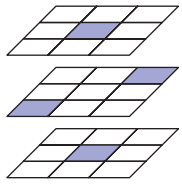


**b** One possible arrangement is given for **i** and **ii**. There are others.

**i** Top/bottom views:



**ii** Four side views:



**iii** If the pattern can be viewed from all four sides, deduce that the top and bottom layers must have a cube in the centre, as in **ii** above, so the pattern cannot be seen from above or below.

**2 a** **i** $\frac{1}{12}$

**ii** Dieter has probability $\frac{1}{2}$ of winning but draws are also possible.

**b** No, Connie has probability $\frac{7}{12}$ of winning.

**c** 3 coins

**d** 14 sides

**e** $k = 3n - 1$

**f** No for dodecahedral (cannot have $n = 13$ counters).
Yes for icosahedral (with $n = 7$ counters).

**3 a** **i** 9 **ii** 12 **iii** 10 **iv** 10

**v** 16 **vi** 20 **vii** 18 **viii** 22

**b** Types **i**, **iii** and **iv** only.

**c** $AB = AF$, so not all side lengths are different.

**4 a** **i** 2 steps **ii** 5 steps

**iii** 6 steps **iv** 9 steps

**b** **i** 2 moves **ii** impossible

**iii** 9 moves **iv** impossible

**c** **i** 2 passes **ii** 5 passes

**iii** 5 passes **iv** 9 passes

## EX NAPLAN practice

p452

### CHAPTER 1 Financial mathematics

**1** \$60 000 ÷ 30
**2** \$9.50 per kg
**3** 14 × \$15.80 + 5 × 1.5 × \$15.80 + 3 × 2 × \$15.80
**4** \$356.05
**5** 15% of \$79.00
**6** \$1517
**7** \$1033.35
**8** \$15 ÷ 80 × 100
**9** \$349.92
**10** 81.82%
**11** Coins sold at a profit of \$110.
**12** 275%
**13** \$3000
**14** \$15 500
**15** 12 days
**16** \$3.23
**17** \$5000
**18** 3.5 years

### CHAPTER 2 Indices

**1** 17
**2** $\frac{1}{x}$
**3** $w^5$
**4** 29 700
**5** $2.9 \times 10^{-26}$
**6** $<$
**7** $2 \times 10^6$
**8** $15a^6$
**9** $\frac{2}{3r^2p^4}$
**10** $\frac{x^2z^2}{y^3}$
**11** $\frac{8m^{18}}{n^{12}}$
**12** 3
**13** $\frac{1}{4^6}$

### CHAPTER 3 Algebra

**1** $14b + 28$
**2** 100
**3** 18
**4** 3
**5** $-25$
**6** $12a - 5$
**7** $6x + 4y - 4$
**8** 34 cm
**9** $6xy + 2y - 9x - 3$
**10** 481 cm$^2$
**11** $w + 2$
**12** $6xyz$
**13** $a^2 + 2a - 15$
**14** $(b - 5)(b - 8)$
**15** $(m - 3)(m + 4)$
**16** $(d - 2)(7d + 9)$
**17** $x^2 + 4xy + 4y^2$
**18** $(p - 4)^2$
**19** $5x(10 - x)$
**20** length = $5x$ cm and width = $(10 - x)$ cm

### CHAPTER 4 Linear relationships

**1** $7\frac{1}{2}$
**2** $5 - 3x = 4$
**3** $(2, 1)$
**4** $(-3, 3.5)$
**5** 2
**6** $-3$
**7** 2
**8** $\frac{3}{4}$
**9** $(1, 3.5)$
**10** 5
**11** $y = -2x + 6$
**12** 50
**13** true
**14** $\frac{3n + 5}{4} = 6$
**15** $\frac{4}{n} + 5 = 10$
**16** $n + 2$
**17** 9
**18** $y = \frac{4}{x} + 5$
**19** positive
**20** zero
**21** undefined
**22** $y = -4x$
**23** $-1$
**24** 4
**25** 4
**26** $(-1, 2)$
**27** 3.6 units
**28** $\overline{PR}$
**29** 7.1 units
**30** $(1.5, -2)$
**31** shows decreasing $y$-values from left to right
**32** $y = 4x$

### CHAPTER 5 Non-linear relationships

**1** $x = 6$ or $x = -1$
**2** $x^2 - 6x - 16 = 0$
**3** $x = 1$ or $x = 10$
**4** $-4$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**5** $(-5, 0)$ and $(4, 0)$

**6** $x = -1$

**7** $(4, -1)$

**8** vertical translation of 3 units

**9** $y = -4x^2 - 5$

**10** $(0, -16)$

**11** $(2, -4)$

**12** $y = (x + 3)^2 + 4$

**13** 1

**14** $y = -x^2 + 2x + 3$

**15** $(x - 2)^2 + (y + 3)^2 = 25$

**16** $(0, 8)$, 2 units

**17** 113 square units

## CHAPTER 6 Measurement and geometry

**1** 12 mm

**2** $A = \frac{1}{2}(a + b)h$

**3** 12.25 $cm^2$

**4** 17.76 $cm^2$

**5** 9.75 $m^2$

**6** 11 cm

**7** 3.1 $cm^2$

**8** 0.3 $cm^3$

**9** 102 $m^2$

**10** 15 $m^3$

**11** 905 $cm^2$

**12** 2035.75 $cm^3$

**13** 324 L

**14** enlargement with scale factor 2.5

**15** $x = 3$ cm

**16** The two triangles contain a right angle and hypotenuse lengths are in same ratio.

**17** SSS

**18** $a = 17$ cm, $b = 5$ cm

**19** 16

**20** 48 $cm^2$

## CHAPTER 7 Pythagoras' Theorem and trigonometry

**1** $f$

**2** 55°

**3** $g$ is alternate to $d$.

**4** $a = 65°$, $b = 155°$, $c = 25°$

**5** second triangle

**6** 5 m

**7** $v$

**8** $v^2 = u^2 + w^2$

**9** Triangle A

**10** 52 m

**11** 11.74 cm

**12** Triangle D

**13** 28 cm

**14** 8.35 m

**15** $\frac{47}{50}$

**16** 0.36

**17** $\tan(40°) = \frac{x}{16}$

**18** 12.28 cm

**19** $\tan^{-1}\left(\frac{22}{25}\right)$

**20** 31°

## CHAPTER 8 Statistics

**1** pizza sizes

**2** 14

**3** 20

**4** Sample answer: Key: $2|7 = 27$

**5** 192

**6** 15.7%

**7** 92

**8** 9.38%

**9** 19

**10** Mode will be unchanged.

**11** $99

**12** The median height is 152 cm.

**13** 1.6

**14** 16

**15** 4.4

**16** Graph B

## CHAPTER 9 Probability

**1** $\frac{1}{3}$

**2** 20

**3** $\frac{3}{16}$

**4** $\frac{1}{2}$

**5** 26

**6** 34

**7** 15

**8** A spinner with six equal segments may have been used in this simulation.

**9** Relative frequency of heads for this experiment is 0.465.

**10** $\frac{17}{140}$

**11** $\frac{3}{10}$

**12** $\frac{1}{4}$

**13** $\frac{3}{10}$

**14** $\frac{5}{9}$

**15** $\frac{1}{120}$

# Glossary

### AAA
abbreviation for angle–angle–angle. Triangles are similar if the three angles they contain are the same. The condition AAA does not necessarily mean the triangles are congruent (more information is needed)

### accuracy
the closeness of a measurement to its exact value

### adjacent side
(trigonometry) the side, in a right-angled triangle, next to the reference angle that is not the hypotenuse

### algorithm
clear set of instructions for solving a problem

### alternate angles
pair of angles not adjacent to (next to) each other on opposite sides of the transversal between two lines, sometimes called 'Z' angles

### arc
the curved part of a sector. Angles are marked using an arc

### area scale factor
the scale factor between the areas of two similar shapes

### array
items arranged in rows and columns

### axis of symmetry
imaginary line that divides a symmetrical shape or graph so that one side is a reflection of the other

### back-to-back stem-and-leaf plot
stem-and-leaf plot that displays two sets of data using the one set of stems

### base
**1** in a value expressed in index form, the base is the number that is repeatedly multiplied; for example, $2^4$ has a base of 2

**2** in a 2D shape, the base is a side that is perpendicular to its height (see height)

**3** in a right 3D object, the base is a face that is perpendicular to its height

### best buy
found by comparing a number of purchase options to find the one that is the best value for money

### BIDMAS
order of operations: brackets, indices, division and multiplication, addition and subtraction (*see* order of operations)

### bimodal
describes a distribution with two clear peaks

### binomial
an expression containing two terms

### binomial product
a factorised expression that is the product of two terms such as $(a + b)(c + d)$

### bivariate data
data that shows the relationship between two variables

### bubble sort algorithm
sorting algorithm that uses a nested loop and a series of swaps to sort a list

### call (a variable)
a line of code that refers to a variable defined earlier in the code

### capacity
amount of a substance that a 3D object (container) can hold (mL, L)

### Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described

### categorical data
non-numerical data that can be organised into categories

### centre of dilation
a point in a shape from which a dilation is made; often one of the vertices of the shape

### circumference
perimeter of a circle

### class intervals
groupings of data into equal-sized classes

### coefficient
the number multiplied by a pronumeral

### co-interior angles
pair of angles on the same side of the transversal between two lines, sometimes called 'C angles'

### column graph
graph where the frequency of categorical data is presented in columns

### commission
payment to an employee calculated by finding a percentage of sales made

### complement
everything in the universal set that is not in the given set; for example, if $\xi = \{1, 2, 3, 4, 5\}$ and set A contains even numbers, $A = \{2, 4\}$, then the complement of A contains odd numbers, $A' = \{1, 3, 5\}$

### complementary angles
angles that add to make a right angle (90°)

### complementary events
two events are complementary if their probabilities add to 1

### composite shape
figures made up of more than one type of simple shape

### congruent
figures that are identical in shape and size but can be in any position or orientation

### conjugates
binomial products with the same terms but one different sign such as $a + b$ and $a - b$

### constant
a term without any pronumeral part

### constant term
the term in an expression that has no variable or the variable is to power 0; for example, –1 is the constant term in $3x^2 + 4x - 1$

### constant of proportionality
if two quantities $x$ and $y$ are related so that $y = kx$, $k$ is the constant of proportionality. $k$ is equal to the rate of change of $y$ with respect to $x$ and the gradient of the graph of $y$ vs. $x$

### continuous data
numerical data that can be measured; for example, height of a plant

### coordinate points
pair of numbers that describe the position of a point on the Cartesian plane

### corresponding angles
pair of angles on the same side of the transversal and in corresponding positions on the two lines (both above a line or both below a line), sometimes called 'F angles'

### cosine (cos)
in trigonometry, the cosine of an angle in a right-angled triangle equals the ratio of the length of the adjacent side to the length of the hypotenuse

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

### cost price
cost paid to create or obtain a good or service before it is sold to consumer

### cumulative frequency
running total of frequencies in a frequency table

### cylinder
3D object with a uniform circular cross-section

### define (a variable)
command used when creating a function that is called later in the code

### dependent variable
variable that depends on another variable (*see* independent variable) in a relationship of two variables. It is listed second in a table of values and in pairs of coordinates, and shown on the vertical axis of a Cartesian plane

### difference of two squares
rule for factorising a difference of perfect squares, $a^2 - b^2 = (a + b)(a - b)$

### dilation
transformation that produces an enlargement or a reduction of the original figure

### directly proportional
two variables are said to be in direct proportion if $y = kx$, where $k$ is the constant of proportionality, shown using the symbol $\propto$

### discount
an amount off the selling price: New selling price = original selling price − discount

### discrete data
numerical data that can be counted; for example, number of siblings

### distributive law
states that the same result is obtained if you add the numbers inside a pair of brackets first before multiplying by the common factor or multiply each number inside the brackets by the number outside the brackets before adding: $a(b + c) = ab + ac$

### enlargement
larger image produced after a figure has been dilated

### equation
collection of two or more algebraic terms separated by mathematical operation symbols and containing an equals sign

### event
outcome or group of outcomes in a sample space; for example, a single outcome is rolling a 2; a group of outcomes is rolling a 2 or a 4

### expanded form
power written as a repeated multiplication; for example, $4^3$ written in expanded form is $4 \times 4 \times 4$ algebraic expression written without brackets, as opposed to factor form expand to convert an expression from factor to expanded form or to remove brackets by multiplying by a common factor; for example, $3(2x + 1)$ expands to $6x + 3$

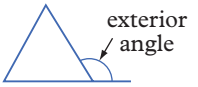
### expected number
number of favourable outcomes expected in a probability experiment: expected number = theoretical probability × number of trials

### experiment
repeatable procedure that has a clearly defined set of possible results

### exponent (index)
*see* index

### expression
either a single term or the sum or difference of two or more terms

### exterior angle
angle formed outside a triangle where a side has been extended




### faces
flat surfaces of a polyhedron, where each face is a polygon

### factor form
algebraic expression written as the product of its factors, as opposed to expanded form

### factor tree
a tree diagram where composite numbers are broken down into factor pairs until prime numbers are found

### factorise
to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$

### general equation of a circle
$(x - h)^2 + (y - k)^2 = r^2$, where the circle has a radius of $r$ units and the coordinates of the centre of the circle are $(h, k)$

### GST, Goods and Services Tax
10% tax on the selling price of products or services

### gradient *m*
numerical measure of the slope of a graph.

$$\text{gradient} = \frac{\text{rise}}{\text{run}},\ m = \frac{y_2 - y_1}{x_2 - x_1}$$

### gradient-intercept form
general form of a linear equation, $y = mx + c$, where $m$ is the gradient and $c$ is the $y$-coordinate of the $y$-intercept of the corresponding linear graph

### grouped data
numerical data that has been sorted into groups or class intervals

### highest common factor (HCF)
greatest factor that is common to two or more given numbers

### histogram
type of column graph that can be used to display grouped continuous or discrete numerical data where the bounds of each class interval are located at the edges of the columns

### horizontal translation
transformation where a shape or object is translated (moved) left or right in a straight line without turning or changing size.

### hypotenuse
the longest side of a right-angled triangle; it is the side opposite the right angle

### independent variable
quantity that changes independently in a relationship between two variables. It is listed first in a table of values and in pairs of coordinates, and shown along the horizontal axis on the Cartesian plane

### indent
a gap/space used in Python to indicate a logical structure

### index (coding)
refers to the position of an item in a list. The index of a list starts at 0 rather than 1; for example,

```
L = [5, 2, 7, 4, 9]
Index 0  1  2  3  4
```

### index (exponent) (plural indices)
for a value expressed in index form, the index indicates the number of times the base is written as a repeated multiplication

### index notation (index form)
shorter form of writing a repeated multiplication, where a number is written with a base and an index; for example, $2^4$ is written in index form

### inner loop
a loop within a loop that repeats a set of actions a given number of times in a nested loop

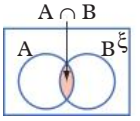
### inputs
values entered into a function

### interest
extra payment made for a loan or paid on an investment

### interest rate
rate, usually expressed as a percentage, at which interest is charged for a loan or paid for an investment

### interior angle
an angle formed inside the triangle. The internal angle sum of a triangle is 180°

### intersection
the intersection of two sets ($A \cap B$) is the set of elements that appear in both A and B




### inverse operation
operation that reverses the effect of a previous operation; for example, addition and subtraction are inverse operations

### inverse cosine ($\cos^{-1}$)
used to obtain the angle from its cosine value

### inverse sine ($\sin^{-1}$)
used to obtain the angle from its sine value

### inverse tangent ($\tan^{-1}$)
used to obtain the angle from its tangent value

### investment
money put into a bank, financial institution or other business for which interest is paid to the investor

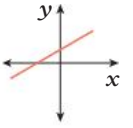
### leading coefficient
the number multiplied by the highest power of a variable in an expression; for example, 3 is the leading coefficient in $3x^2 + 4x - 1$

### like terms
terms that contain the same pronumerals

### line graph
graph that displays the relationship between two variables where points are plotted and joined by a straight line. A continuous variable must be displayed on the horizontal axis; for example, time

### linear equation
equation that contains a pronumeral term or terms where the highest power is 1; for example, $2x + 3 = 6$

### linear graph
straight line graph of a linear relationship




### linear relationship
relationship between two variables that produces a linear graph

### linear term
the term with a variable of power 1 in an expression; for example, $4x$ is the linear term in $3x^2 + 4x - 1$

### loan
the amount of money given by a bank or other institution to a person for a limited time and on which they must pay interest

### loop
the structure for repeating a procedure until a stopping condition is met

### loss
the amount lost when the selling price is less than the cost price

### lower bound
abbreviation for lower boundary; the lowest value in an interval

### mark-up
amount added to an original price; for example, a cost price has a 25% mark-up added to obtain the selling price

### maximum turning point
a point at which a parabola changes direction and has its maximum $y$-value

### mean
a measure of the centre of a data set (average) calculated by adding all of the data values and then dividing by the number of values

### measure of centre
number used to represent the central point in a data set. Measures of centre include the mean, median and mode

### measure of spread
number used to indicate how spread out a data set is. Measures of spread include the range and standard deviation

### median
a measure of the centre of a data set. For an ordered set of data, it is the middle number if there is an odd number of values, and the average of the two middle numbers if there is an even number of values

### midpoint
halfway point or middle point of an interval or line segment

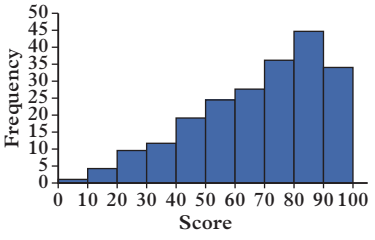
### minimum turning point
a point at which a parabola changes direction and has its minimum $y$-value

### mode
a measure of the centre of a data set determined by finding the value, or values, that occur most frequently in the data set

### multiple line graph
graph used to display two sets of bivariate numerical data. The categorical variables are detailed in the legend

### negative index (negative exponent)
index with a negative value indicating repeated division

### negatively skewed
describes a distribution that is skewed away from the vertical axis with a tail to the left




### nested loop
structure consisting of a loop within a loop, known as the outer and inner loop

### net
2D plan that can be folded to form a three-dimensional object

### nominal data
categorical data with categories that cannot be ordered; for example, eye colour

### non-linear equation
equation that contains at least one pronumeral term with a power other than 1

### non-linear relationship
relationship between two variables that produces a non-linear graph

### Null Factor Law
states that if the product of two factors is 0, then one or both of the factors must equal 0

### numerical data
data sets of numbers that can be counted or measured

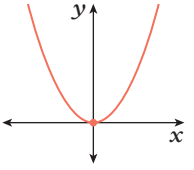
### opposite side
(trigonometry) the side opposite the reference angle in a right-angled triangle

### ordinal data
categorical data that can be placed in categories in a specific order; for example, a rating system of 1–5

### outer loop
a loop that repeats the actions of the inner loop a given number of times in a nested loop

### outlier
extreme piece of data that is much higher or lower than the rest of the data set

### parabola
the graph of a quadratic relationship




### parallel lines
lines (or rays or segments) that are always the same distance apart so that they never meet. Parallel lines are labelled with matching arrowheads

### percentage profit
amount of profit expressed as a percentage of the cost price

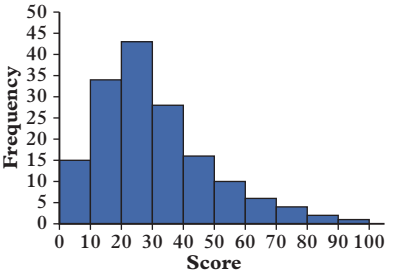
### percentage loss
amount of loss expressed as a percentage of the cost price

### perfect square
the square of an integer; for example 1, 4, 9 and 25 are perfect squares

### plot
graphical representation of a relationship consisting of individual coordinate points

### plus-minus symbol
± indicates both positive and negative values or both addition and subtraction

### positively skewed
describes a distribution that is skewed towards the vertical axis with a tail to the right




### prime factorisation
the product of all the prime factors of an integer

### principal
sum of money invested or amount borrowed for a loan

### print
to display text, variables or other information on a screen

### prism
3D object with two ends that are identical polygons and joined by straight edges

### profit
amount made when the selling price is greater than the cost price

### pronumeral
letter or symbol that takes the place of a number

### Pythagoras' Theorem
states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the two other sides, $c^2 = a^2 + b^2$, where $c$ is the length of the hypotenuse and $a$ and $b$ are the other side lengths

### Pythagorean triad (Pythagorean triple)
any set of three whole numbers that satisfy Pythagoras' Theorem

### quadratic
An algebraic expression that contains a pronumeral with a power of 2, and with no indices greater than 2

### quadratic trinomial
expression of the form $ax^2 + bx + c$, where $a$, $b$ and $c$ are constants

### quadratic equation
an equation that contains a pronumeral term or terms with the highest power of 2; for example, $x^2 + 3x - 2 = 10$. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where $a$, $b$ and $c$ are constants

### range
a measure of spread determined by finding the difference between the highest and lowest value in a data set

### rate
comparison between two or more different quantities. It is a measure of how a quantity changes for each unit of another quantity

### rate of change
the change in one variable compared to the change in another variable

### reduction
smaller image produced after a figure has been dilated

### reflection
transformation where a shape or object is reflected (flipped) in a mirror line to produce its exact mirror image

### relative frequency
the number of times a favourable outcome occurs divided by the total number of outcomes

$$\text{Relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

### retainer
a fixed amount paid to a person who also earns commissions on sales

### return
the value produced by a function

### revenue
income generated by the sale of goods or services

### RHS
abbreviation for right-angle–hypotenuse–side. Right-angled triangles are similar and congruent if their hypotenuses are the same length and one other corresponding pair of sides has the same length.

### right prism
prism with sides that make right angles with its base

### sample space
list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a six-sided die is {1, 2, 3, 4, 5, 6}

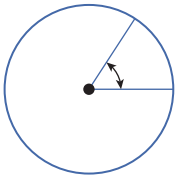
### SAS
abbreviation for side–angle–side. Triangles are similar and congruent if two corresponding sides are equal in length and the angle between them is the same

### scale factor
indicates how many times larger or smaller an image is after a figure has been dilated:

$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

### scientific notation (standard form)
a value written as a number from 1 up to, but not including, 10 (with any number of decimal places) multiplied by a power of 10

### sector
portion of a circle formed by two radii and part of the circumference



### selling price (retail price)
price for which an item is sold

### set
a collection of distinct objects or numbers

### side-by-side column graph
graph used to display frequency of two sets of categorical data. The categorical variables are detailed in the legend

### significant figures
the number of digits in a number that contribute to its accuracy

### similar
figures with the same shape but not the same size. Similar figures are produced by a dilation

### simple interest, *I*
an amount paid on a loan or investment. $I = PrT$, where $I$ is the amount of interest, $P$ is the principal, $r$ is the interest rate and $T$ is time

### sine (sin)
the sine of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the hypotenuse

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

### sketch
a simple graph in which main features such as intercepts and turning points are labelled

### skewed
describes a data set where data points are clustered to the left or right of the distribution

### solution
value of the pronumeral that makes an equation a true statement

### SSS
abbreviation for side–side–side. Triangles are similar and congruent if all three corresponding sides are equal in length

### standard form
*see* scientific notation

### stretch
a type of transformation that can extend or compress the graph of a function in the $x$- or $y$-direction (*see* dilation)

### subset
a set contained within another set

### supplementary angles
angles that add to make a straight angle (180°)

### surd
irrational root of an integer that cannot be simplified to remove the root; for example $\sqrt{2}$

### symmetrical (symmetric)
**1** property of a shape or object where it can be divided into two halves that are mirror reflections of each other
**2** of graphs, having a similar distribution of frequencies either side of a central peak

### tangent (tan)
in trigonometry, the tangent of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the adjacent side

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

### term
a single pronumeral or number, or the product of a number and one or more pronumerals

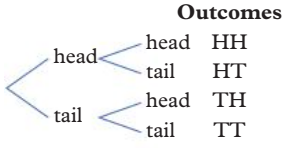
### theoretical probability
the probability determined by mathematical reasoning, not experiments

### total surface area (TSA)
total area of the outer surface of an object. The total surface area of a prism is the sum of the areas of each face of the prism

### transformations
general name for translations, reflections, rotations and dilations

### transversal
line that crosses (intersects) two or more lines

### tree diagram
a branched display of outcomes for a multi-step experiment; for example, the outcomes when two coins are tossed




### trigonometry
the study of relationships between angles and side lengths of a triangle

### trinomial
an expression containing three terms

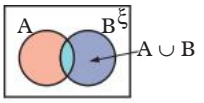
### turning point
the point where a parabola changes direction

### turning point form
general for of a quadratic equation $y = a(x - h)^2 + k$, where $a$ is the dilation factor and the coordinates of the turning point are $(h, k)$

### two-way table
display of outcomes for a two-step probability experiment in a table

### ungrouped data
raw data that has not been placed into class intervals

### union
the union of two sets ($A \cup B$) is the set of elements in A or B or both




### unitary method
problem solving technique that involves first finding the value of a single unit; for example if 3 apples cost $5, to determine the cost of 7 apples, calculate the cost of 1 apple, then multiply by 7

### univariate data
a data set for one variable

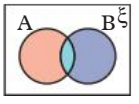
### universal set
all possible outcomes in a chance experiment or the set of all elements, ε, inside the rectangle of a Venn diagram

### unknown
a quantity or measurement required in a problem that has one possible value only and is usually represented by a pronumeral

### upper bound
abbreviation for upper boundary; the largest value in an interval

### variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

### Venn diagram
a display of the relationship between different sets of data. The universal set, ξ, is represented by a rectangle and each set of data is represented by a circle

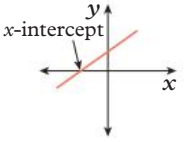



### vertical translation
transformation in which a shape or object is translated (moved) up or down in a straight line without turning or changing size

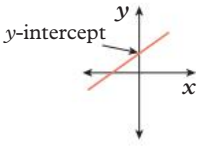
### vertically opposite angles
pair of equal angles not adjacent to each other, where two lines intersect; sometimes called 'X angles'

### volume
amount of space that a three-dimensional (3D) object occupies ($cm^3$, $m^3$)

### *x*-intercept
point where a line crosses the *x*-axis of a Cartesian plane




### *y*-intercept
point where a line crosses the *y*-axis of a Cartesian plane



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**Pilots need to apply trigonometry when they are making course corrections due to wind. There is a lot of mental maths to be done in the air!**

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