YEAR 8

CambridgeMATHS NSW

STAGE 4

SECOND EDITION

STUART PALMER, KAREN McDAID
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JUSTIN ROBINSON, JENNY GOODMAN
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INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

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Additional resources for this publication at www.cambridge.edu.au/GO

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About the authors

Stuart Palmer was born and educated in NSW. He is a high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at The University of Sydney.

Karen McDaid has more that 17 years' experience teaching mathematics in several primary and secondary schools, and as a lecturer teaching mathematics to primary preservice teachers at Western Sydney University. Karen was the Professional Teachers Association representative on the NSW Board Curriculum Committee during the development and consultation phases of both the Australian Curriculum and the NSW Mathematics K-10 Syllabus for the Australian Curriculum. She has been active on the executive committee of the Mathematical Association of NSW since 2003 and was the NSW Councillor on the board of the Australian Association of Mathematics Teachers from 2014 to 2018. Karen co-authored the CambridgeMATHS NSW GOLD books Years 7 to 10. Karen is currently Head of Mathematics K to 12 at Cluey Learning.

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has over 20 years' experience teaching mathematics from Years 7 to 12. He has run workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, and is currently teaching both junior and senior mathematics in Victoria. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.

Justin Robinson is Head of Positive Education and a mathematics teacher at Geelong Grammar School. Prior to this, he spent 20 years teaching mathematics and was a key writer of in-house maths material. He has a keen interest in engaging all students through a wide variety of effective teaching methods and materials.

Jenny Goodman has worked for over 20 years in comprehensive State and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in NSW, WA, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

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Introduction and guide to this book

The **second edition** of this popular resource features a new interactive digital platform powered by Cambridge HOTmaths, together with improvements and updates to the textbook, and additional online resources such as video demonstrations of all the worked examples, Desmos-based interactives, carefully chosen HOTmaths resources including widgets and walkthroughs, and worked solutions for all exercises, with access controlled by the teacher. The Interactive Textbook also includes the ability for students to complete textbook work, including full working-out online, where they can self-assess their own work and alert teachers to particularly difficult questions. Teachers can see all student work, the questions that students have ‘red-flagged’, as well as a range of reports. As with the first edition, the complete resource is structured on detailed teaching programs for teaching the NSW Syllabus, now found in the Online Teaching Suite.

The chapter and section structure has been retained, and remains based on a logical teaching and learning sequence for the syllabus topic concerned, so that chapter sections can be used as ready-prepared lessons. Exercises have questions graded by level of difficulty and are grouped according to the **Working Mathematically components** of the NSW Syllabus, with enrichment questions at the end. Working programs for three ability levels (Building Progressing and Mastering) have been subtly embedded inside the exercises to facilitate the management of differentiated learning and reporting on students’ achievement (see page X for more information on the Working Programs). In the second edition, the *Understanding* and *Fluency* components have been combined, as have Problem-Solving and Reasoning. This has allowed us to better order questions according to difficulty and better reflect the interrelated nature of the Working Mathematically components, as described in the NSW Syllabus.

Topics are aligned exactly to the NSW Syllabus, as indicated at the start of each chapter and in the teaching program, except for topics marked as:
- REVISION — prerequisite knowledge
- EXTENSION — goes beyond the Syllabus
- FRINGE — topics treated in a way that lies at the edge of the Syllabus requirements, but which provide variety and stimulus.

See the Stage 5 books for their additional curriculum linkage.

The parallel ***CambridgeMATHS Gold*** series for Years 7–10 provides resources for students working at Stages 3, 4, and 5.1. The two series have a content structure designed to make the teaching of mixed ability classes smoother.

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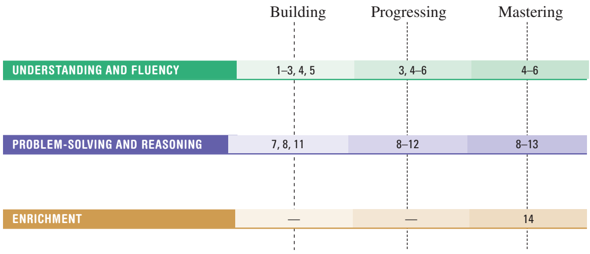
Guide to the working programs

It is not expected that any student would do every question in an exercise. The print and online versions contain working programs that are subtly embedded in every exercise. The suggested working programs provide three pathways through each book to allow differentiation for Building, Progressing and Mastering students.

Each exercise is structured in subsections that match the Working Mathematically strands, as well as Enrichment (Challenge).

The questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest-shaded colour) is the Building pathway
- The middle column (medium-shaded colour) is the Progressing pathway
- The right column (darkest-shaded colour) is the Mastering pathway.



	Building	Progressing	Mastering
UNDERSTANDING AND FLUENCY	1–3, 4, 5	3, 4–6	4–6
PROBLEM-SOLVING AND REASONING	7, 8, 11	8–12	8–13
ENRICHMENT	—	—	14

Gradients within exercises and question subgroups

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise, where there is an increasing level of mathematical sophistication required in the Problem-solving and Reasoning group of questions than in the Understanding and Fluency group, and within each group the first few questions are easier than the last.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway will likely need more practice at Understanding and Fluency but should also attempt the easier Problem-Solving and Reasoning questions.

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them if the chapter pre-tests can be used as a diagnostic tool.

For classes grouped according to ability, teachers may wish to set one of the Building, Progressing or Mastering pathways as the default setting for their entire class and then make individual alterations, depending on student need. For mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e … or b, d, f, …)
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- —: do not complete any of the questions in this section.

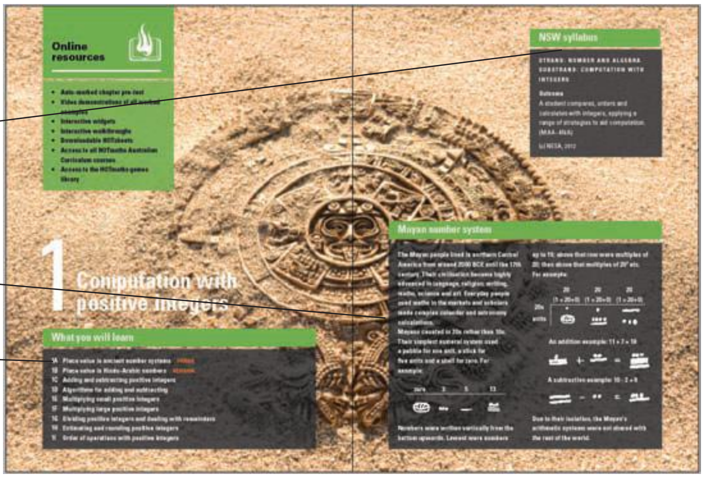
Guide to this book

Features:

NSW Syllabus: strands, substrands and content outcomes for chapter (see teaching program for more detail)

Chapter introduction: use to set a context for students

What you will learn: an overview of chapter contents



Online resources

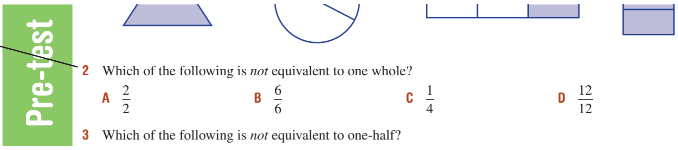
NSW syllabus

1 Computation with positive integers

What you will learn

Mayan number system

Pre-test: establishes prior knowledge (also available as an auto-marked quiz in the Interactive Textbook as well as a printable worksheet)



Pre-test

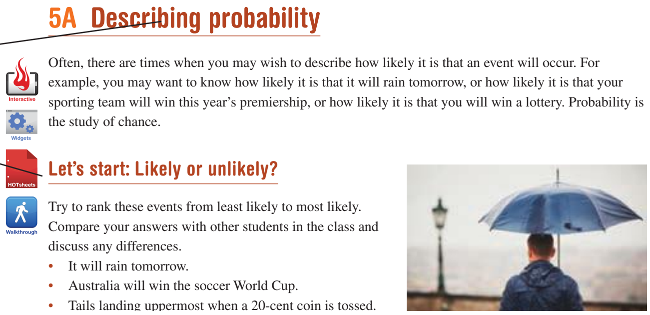
2 Which of the following is *not* equivalent to one whole?

A $\frac{2}{2}$ B $\frac{6}{6}$ C $\frac{1}{4}$ D $\frac{12}{12}$

3 Which of the following is *not* equivalent to one-half?

Topic introduction: use to relate the topic to mathematics in the wider world

Let's start: an activity (which can often be done in groups) to start the lesson



5A Describing probability

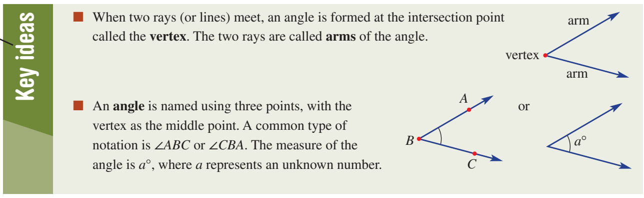
Often, there are times when you may wish to describe how likely it is that an event will occur. For example, you may want to know how likely it is that it will rain tomorrow, or how likely it is that your sporting team will win this year's premiership, or how likely it is that you will win a lottery. Probability is the study of chance.

Let's start: Likely or unlikely?

Try to rank these events from least likely to most likely. Compare your answers with other students in the class and discuss any differences.

- It will rain tomorrow.
- Australia will win the soccer World Cup.
- Tails landing uppermost when a 20-cent coin is tossed.

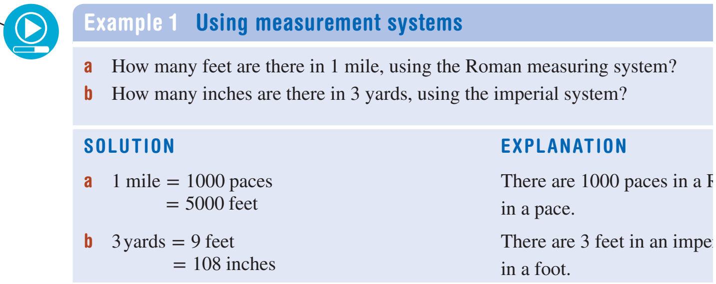
Key ideas: summarises the knowledge and skills for the lesson



Key ideas

- When two rays (or lines) meet, an angle is formed at the intersection point called the **vertex**. The two rays are called **arms** of the angle.
- An **angle** is named using three points, with the vertex as the middle point. A common type of notation is $\angle ABC$ or $\angle CBA$. The measure of the angle is $a°$, where a represents an unknown number.

Examples: solutions with explanations and descriptive titles to aid searches. Video demonstrations of every example are included in the Interactive Textbook.



Example 1 Using measurement systems

a How many feet are there in 1 mile, using the Roman measuring system?

b How many inches are there in 3 yards, using the imperial system?

SOLUTION	EXPLANATION
a 1 mile = 1000 paces = 5000 feet	There are 1000 paces in a F in a pace.
b 3 yards = 9 feet = 108 inches	There are 3 feet in an impe in a foot.

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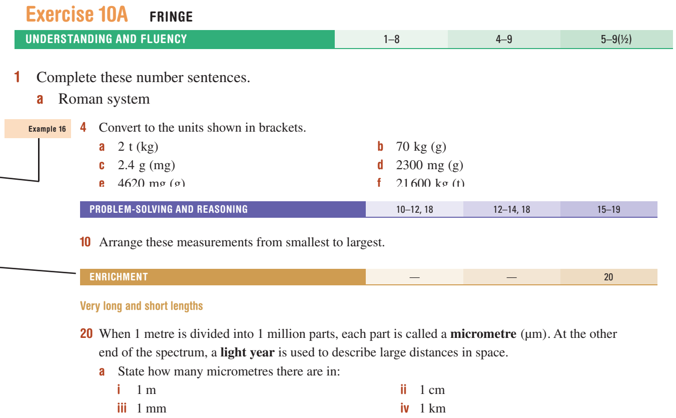
Exercise questions categorised by the **working mathematically components** and **enrichment**

Example references link exercise questions to worked examples.


Exercise 10A FRINGE

UNDERSTANDING AND FLUENCY	1–8	4–9	5–9(½)

1 Complete these number sentences.
a Roman system

Example 16

4 Convert to the units shown in brackets.
a 2 t (kg) b 70 kg (g)
c 2.4 g (mg) d 2300 mg (g)
e 4620 mg (g) f 21 600 kg (t)

PROBLEM-SOLVING AND REASONING	10–12, 18	12–14, 18	15–19

10 Arrange these measurements from smallest to largest.

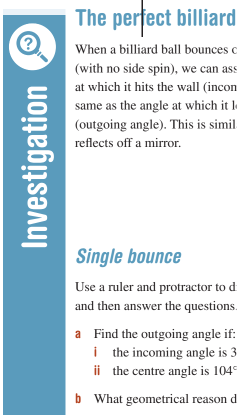
ENRICHMENT	—	—	20

Very long and short lengths

20 When 1 metre is divided into 1 million parts, each part is called a **micrometre** (μm). At the other end of the spectrum, a **light year** is used to describe large distances in space.
a State how many micrometres there are in:
i 1 m ii 1 cm
iii 1 mm iv 1 km

Investigations: inquiry-based activities

Puzzles and **challenges**


Investigation

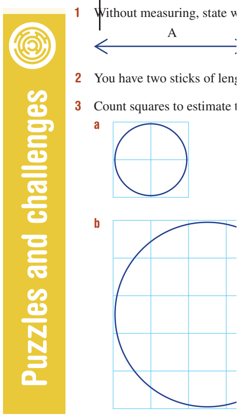
The perfect billiard

When a billiard ball bounces c
(with no side spin), we can ass
at which it hits the wall (incon
same as the angle at which it l
(outgoing angle). This is simil
reflects off a mirror.

Single bounce

Use a ruler and protractor to d
and then answer the questions.

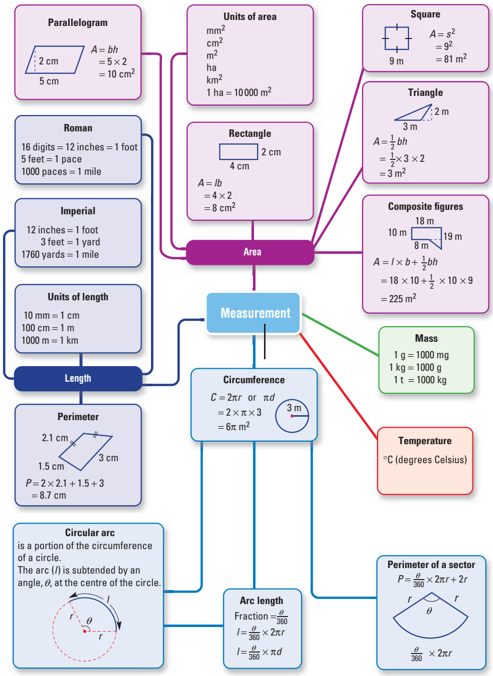
a Find the outgoing angle if:
i the incoming angle is 3
ii the centre angle is 104°
b What geometrical reason d


Puzzles and challenges

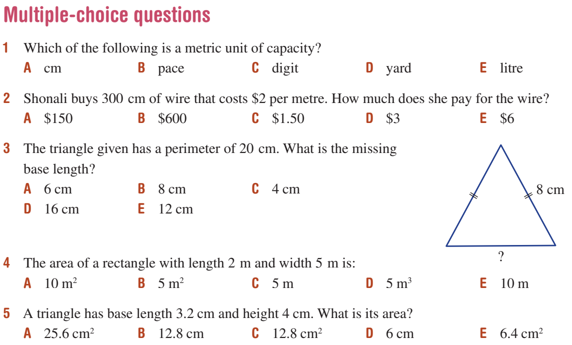
1 Without measuring, state w

2 You have two sticks of len
3 Count squares to estimate t
a
b

Chapter summary: mind map of key concepts & interconnections



Chapter reviews with **multiple-choice, short-answer** and **extended-response questions**


Multiple-choice questions

1 Which of the following is a metric unit of capacity?
A cm B pace C digit D yard E litre

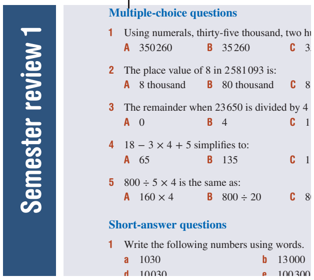
2 Shonali buys 300 cm of wire that costs $2 per metre. How much does she pay for the wire?
A $150 B $600 C $1.50 D $3 E $6

3 The triangle given has a perimeter of 20 cm. What is the missing base length?
A 6 cm B 8 cm C 4 cm
D 16 cm E 12 cm

4 The area of a rectangle with length 2 m and width 5 m is:
A 10 m² B 5 m² C 5 m D 5 m³ E 10 m

5 A triangle has base length 3.2 cm and height 4 cm. What is its area?
A 25.6 cm² B 12.8 cm C 12.8 cm² D 6 cm E 6.4 cm²

Two **Semester reviews** per book


Semester review 1

Multiple-choice questions

1 Using numerals, thirty-five thousand, two hu
A 350 260 B 35 260 C 3

2 The place value of 8 in 2 581 093 is:
A 8 thousand B 80 thousand C 8

3 The remainder when 23 650 is divided by 4
A 0 B 4 C 1

4 18 − 3 × 4 + 5 simplifies to:
A 65 B 135 C 1

5 800 ÷ 5 × 4 is the same as:
A 160 × 4 B 800 ÷ 20 C 8

Short-answer questions

1 Write the following numbers using words.
a 1030 b 13 000
d 10 030 e 100 300

Textbooks also include:

- Complete **answers**
- **Index**

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Overview of the digital resources

Interactive Textbook: general features

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available separately. (A **downloadable PDF textbook** is also included for offline use). These are its features, including those enabled when the students' ITB accounts are linked to the teacher's **Online Teaching Suite (OTS)** account.

The features described below are illustrated in the screenshot below.

1 Every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the 'flipped classroom'

2 Seamlessly blend with Cambridge HOTmaths, including hundreds of interactive widgets, walkthroughs and games and access to Scorcher

3 **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher

4 **Desmos interactives** based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics

5 The **Desmos scientific calculator** is also available for students to use (as well as the graphics calculator and geometry tools)

6 Auto-marked practice quizzes in each section with saved scores

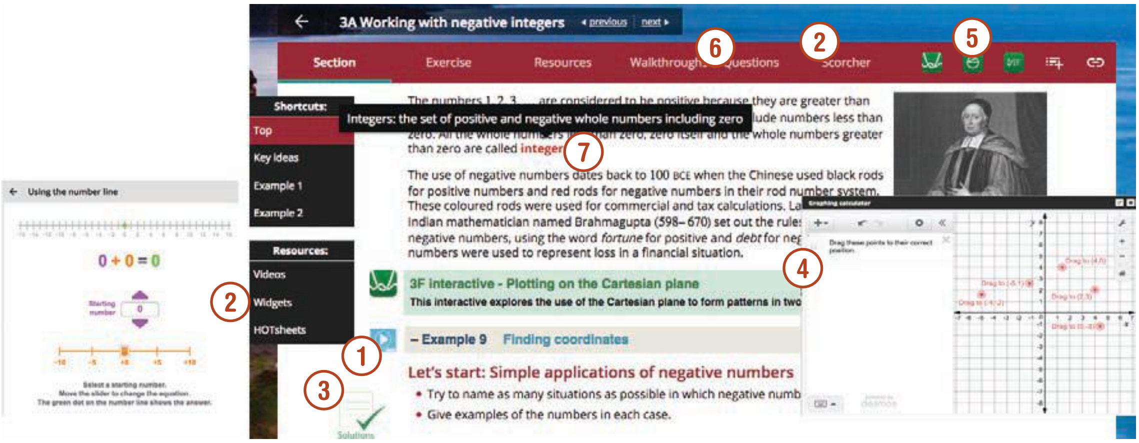
7 **Definitions** pop up for key terms in the text, and access to the Hotmaths **dictionary**

Not shown but also included:

- Access to alternative HOTmaths lessons is included, including content from previous year levels.
- Auto-marked pre-tests and multiple-choice review questions in each chapter.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



The numbers 1, 2, 3, … are considered to be positive because they are greater than … lude numbers less than zero. All the whole numbers … than zero, zero itself and the whole numbers greater than zero are called integer

Integers: the set of positive and negative whole numbers including zero

The use of negative numbers dates back to 100 BCE when the Chinese used black rods for positive numbers and red rods for negative numbers in their rod number system. These coloured rods were used for commercial and tax calculations. La… Indian mathematician named Brahmagupta (598–670) set out the rules … negative numbers, using the word *fortune* for positive and *debt* for neg… numbers were used to represent loss in a financial situation.

3F interactive - Plotting on the Cartesian plane

This interactive explores the use of the Cartesian plane to form patterns in two

– Example 9 Finding coordinates

Let's start: Simple applications of negative numbers

- Try to name as many situations as possible in which negative numb
- Give examples of the numbers in each case.

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Interactive Textbook: Workspaces and self-assessment tools

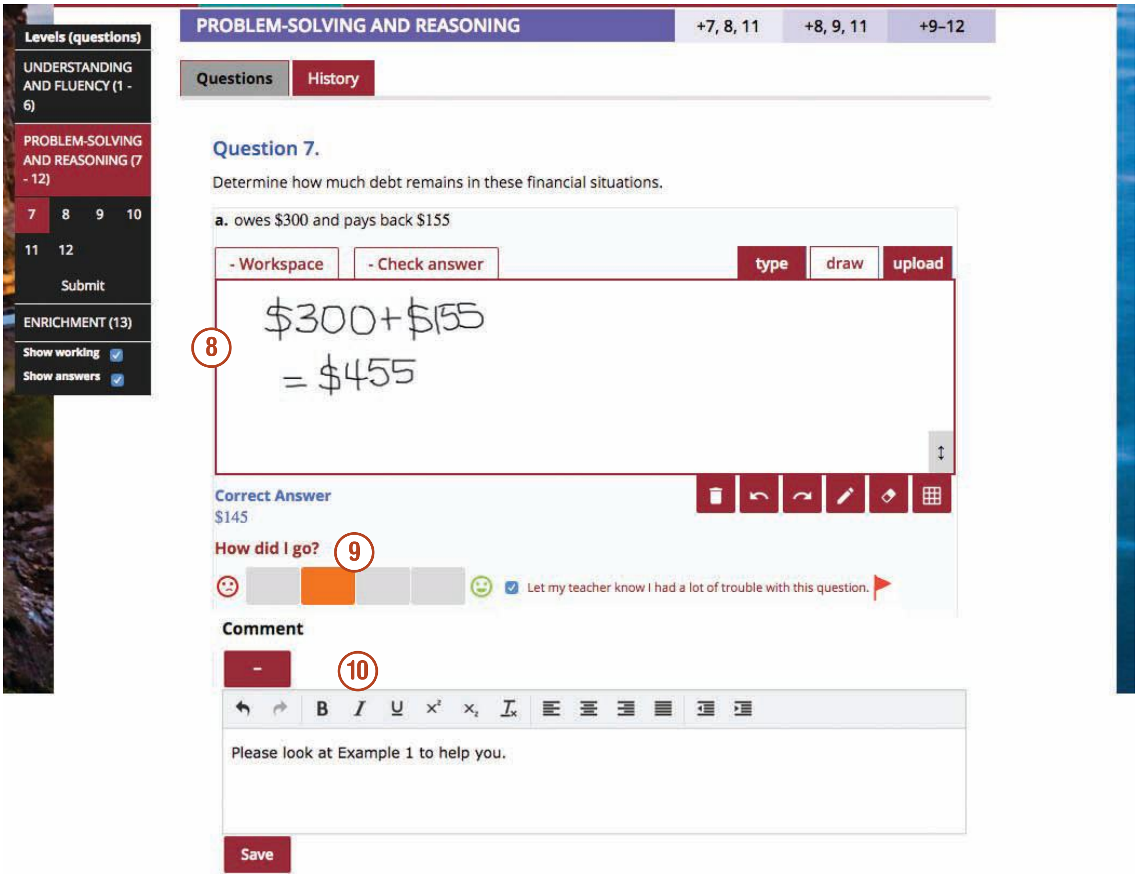
Almost every question in ***CambridgeMATHS NSW Second Edition*** can be completed and saved by students, including showing full working-out and students critically assessing their own work. This is done via the workspaces and self-assessment tools that are found below every question in the Interactive Textbook.

8 The new **Workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploding images of writing or drawing.

9 The new **self-assessment tools** enable students to check answers including questions that have been red-flagged, and can rate their confidence level in their work, and alert teachers to questions the student has had particular trouble with. This self-assessment helps develop responsibility for learning and communicates progress and performance to the teacher.

10 Teachers can view the students' self-assessment individually or provide feedback. They can also view results by class.

WORKSPACES AND SELF-ASSESSMENT



Question 7.

Determine how much debt remains in these financial situations.

a. owes $300 and pays back $155

$300+$155

= $455

Correct Answer

$145

How did I go?

☑ Let my teacher know I had a lot of trouble with this question.

Comment

Please look at Example 1 to help you.

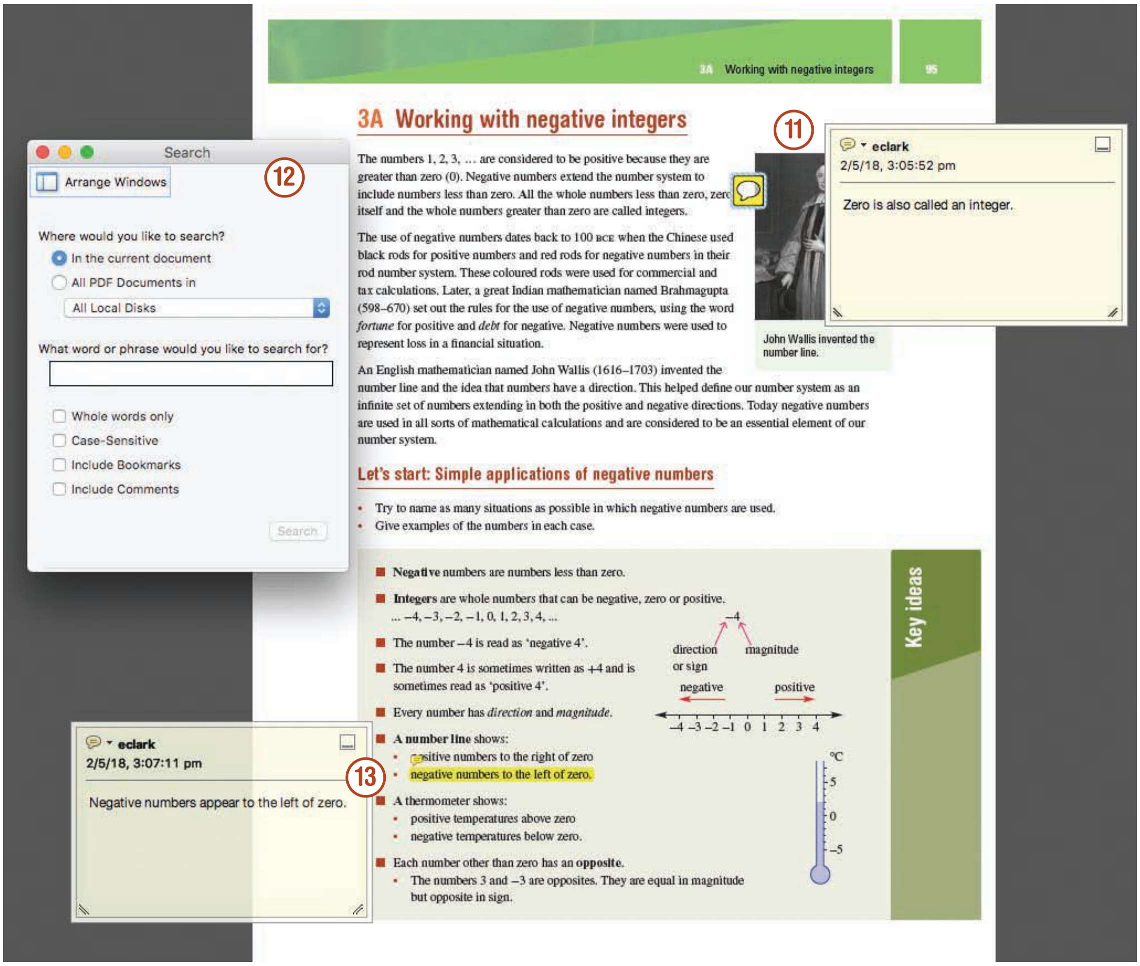
Downloadable PDF Textbook

The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

The features include:

11 PDF note-taking
12 PDF search features are enabled
13 highlighting functionality.

PDF TEXTBOOK



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3A Working with negative integers

The numbers 1, 2, 3, … are considered to be positive because they are greater than zero (0). Negative numbers extend the number system to include numbers less than zero. All the whole numbers less than zero, zerc itself and the whole numbers greater than zero are called integers.

The use of negative numbers dates back to 100 BCE when the Chinese used black rods for positive numbers and red rods for negative numbers in their rod number system. These coloured rods were used for commercial and tax calculations. Later, a great Indian mathematician named Brahmagupta (598–670) set out the rules for the use of negative numbers, using the word *fortune* for positive and *debt* for negative. Negative numbers were used to represent loss in a financial situation.

John Wallis invented the number line.

An English mathematician named John Wallis (1616–1703) invented the number line and the idea that numbers have a direction. This helped define our number system as an infinite set of numbers extending in both the positive and negative directions. Today negative numbers are used in all sorts of mathematical calculations and are considered to be an essential element of our number system.

eclark
2/5/18, 3:05:52 pm
Zero is also called an integer.

Let's start: Simple applications of negative numbers

- Try to name as many situations as possible in which negative numbers are used.
- Give examples of the numbers in each case.

- **Negative** numbers are numbers less than zero.
- **Integers** are whole numbers that can be negative, zero or positive.
  … −4, −3, −2, −1, 0, 1, 2, 3, 4, …
- The number −4 is read as 'negative 4'.
- The number 4 is sometimes written as +4 and is sometimes read as 'positive 4'.
- Every number has *direction* and *magnitude*.
- **A number line** shows:
  - sitive numbers to the right of zero
  - negative numbers to the left of zero.
- A thermometer shows:
  - positive temperatures above zero
  - negative temperatures below zero.
- Each number other than zero has an **opposite**.
  - The numbers 3 and −3 are opposites. They are equal in magnitude but opposite in sign.

eclark
2/5/18, 3:07:11 pm
Negative numbers appear to the left of zero.

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Online Teaching Suite

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the assets and resources are in one place for easy access. The features include:

14 The HOTmaths learning management system with class and student analytics and reports, and communication tools

15 Teacher's view of a student's working and self-assessment, including multiple progress and completion reports viewable at both student and class level, as well as seeing the questions that a class has flagged as being difficult

16 A HOTmaths-style test generator

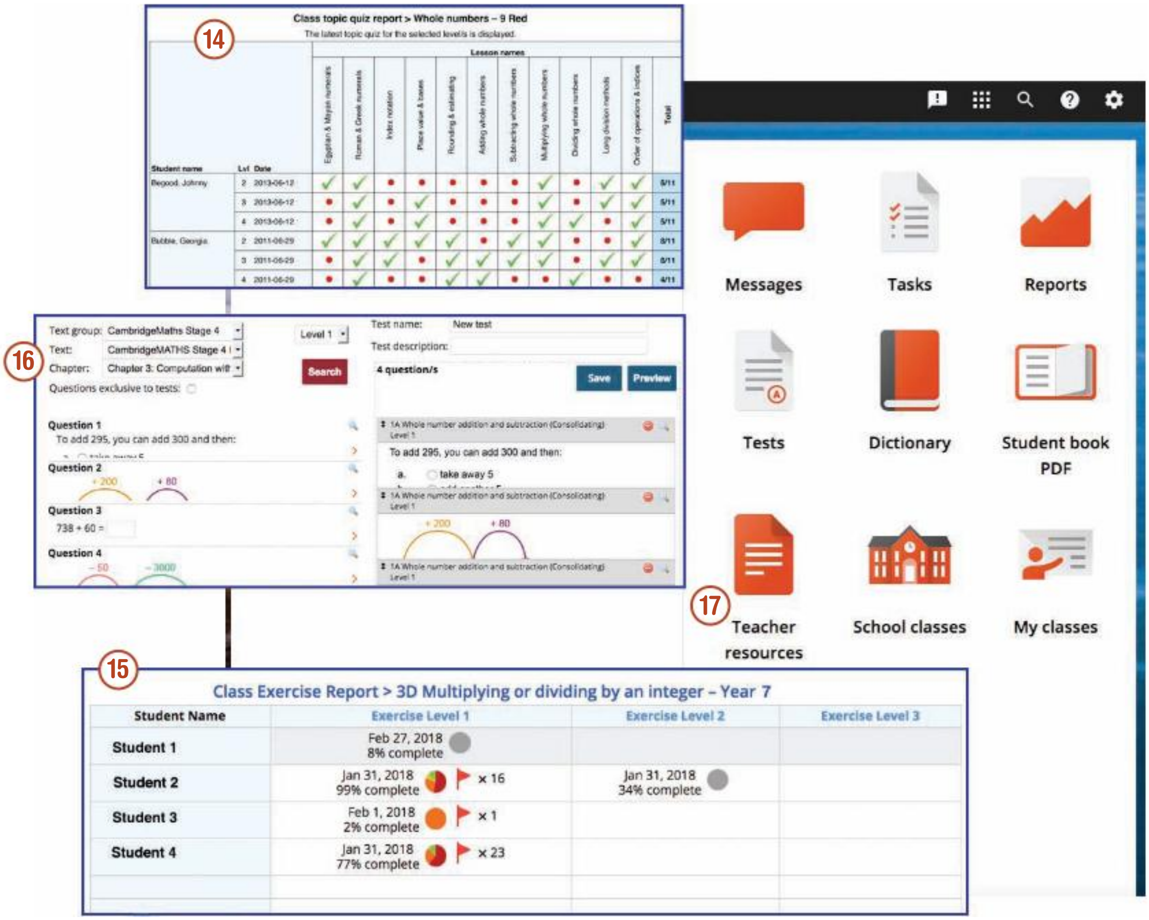
17 Chapter tests and worksheets

Not shown but also available:

- Editable teaching programs and curriculum grids.

ONLINE TEACHING SUITE POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



Class topic quiz report > Whole numbers – 9 Red

The latest topic quiz for the selected level/s is displayed.

Student name	Lvl	Date	Egyptian & Mayan numerals	Roman & Greek numerals	Index notation	Place value & bases	Rounding & estimating	Adding whole numbers	Subtracting whole numbers	Multiplying whole numbers	Dividing whole numbers	Long division methods	Order of operations & indices	Total
Begood, Johnny	2	2013-06-12	✓	✓	●	●	●	●	●	✓	●	✓	✓	5/11
	3	2013-06-12	●	✓	●	✓	●	●	●	✓	●	✓	✓	5/11
	4	2013-06-12	●	✓	●	✓	●	●	●	✓	✓	●	✓	5/11
Bubble, Georgia	2	2011-06-29	✓	✓	✓	✓	✓	●	✓	✓	●	●	✓	8/11
	3	2011-06-29	●	✓	✓	●	✓	✓	✓	✓	●	✓	✓	8/11
	4	2011-06-29	●	✓	●	●	✓	✓	●	●	✓	●	●	4/11

Text group: CambridgeMaths Stage 4

Text: CambridgeMATHS Stage 4

Chapter: Chapter 3: Computation wit

Questions exclusive to tests: ○

Level 1

Test name: New test

Test description:

4 question/s

Question 1

To add 295, you can add 300 and then:

Question 2

+ 200 + 80

Question 3

738 + 60 =

Question 4

− 50 − 3000

1A Whole number addition and subtraction (Consolidating) Level 1

To add 295, you can add 300 and then:

a. ○ take away 5

1A Whole number addition and subtraction (Consolidating) Level 1

+ 200 + 80

1A Whole number addition and subtraction (Consolidating) Level 1

Class Exercise Report > 3D Multiplying or dividing by an integer – Year 7

Student Name	Exercise Level 1	Exercise Level 2	Exercise Level 3
Student 1	Feb 27, 2018 8% complete		
Student 2	Jan 31, 2018 99% complete ⚑ × 16	Jan 31, 2018 34% complete	
Student 3	Feb 1, 2018 2% complete ⚑ × 1		
Student 4	Jan 31, 2018 77% complete ⚑ × 23		

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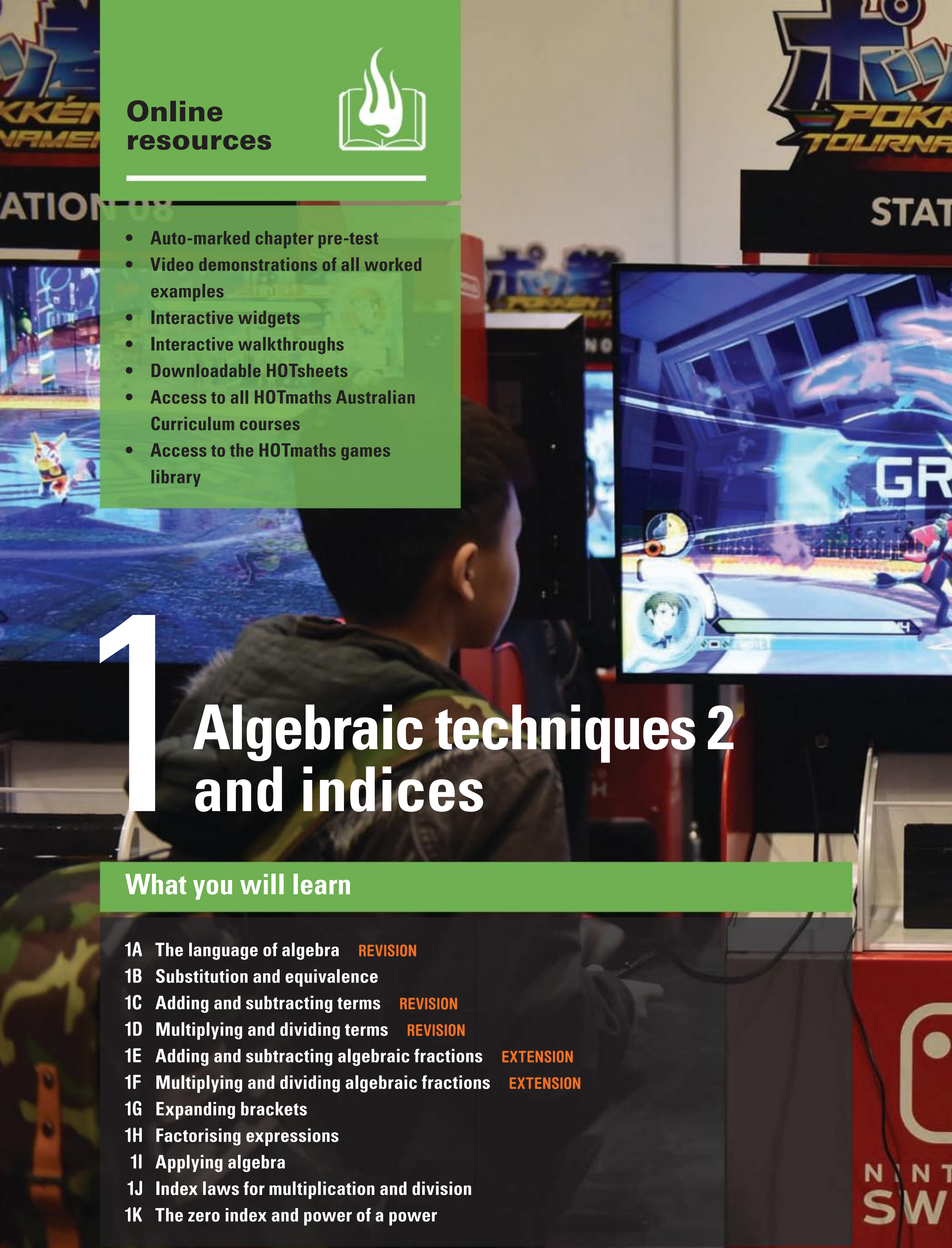


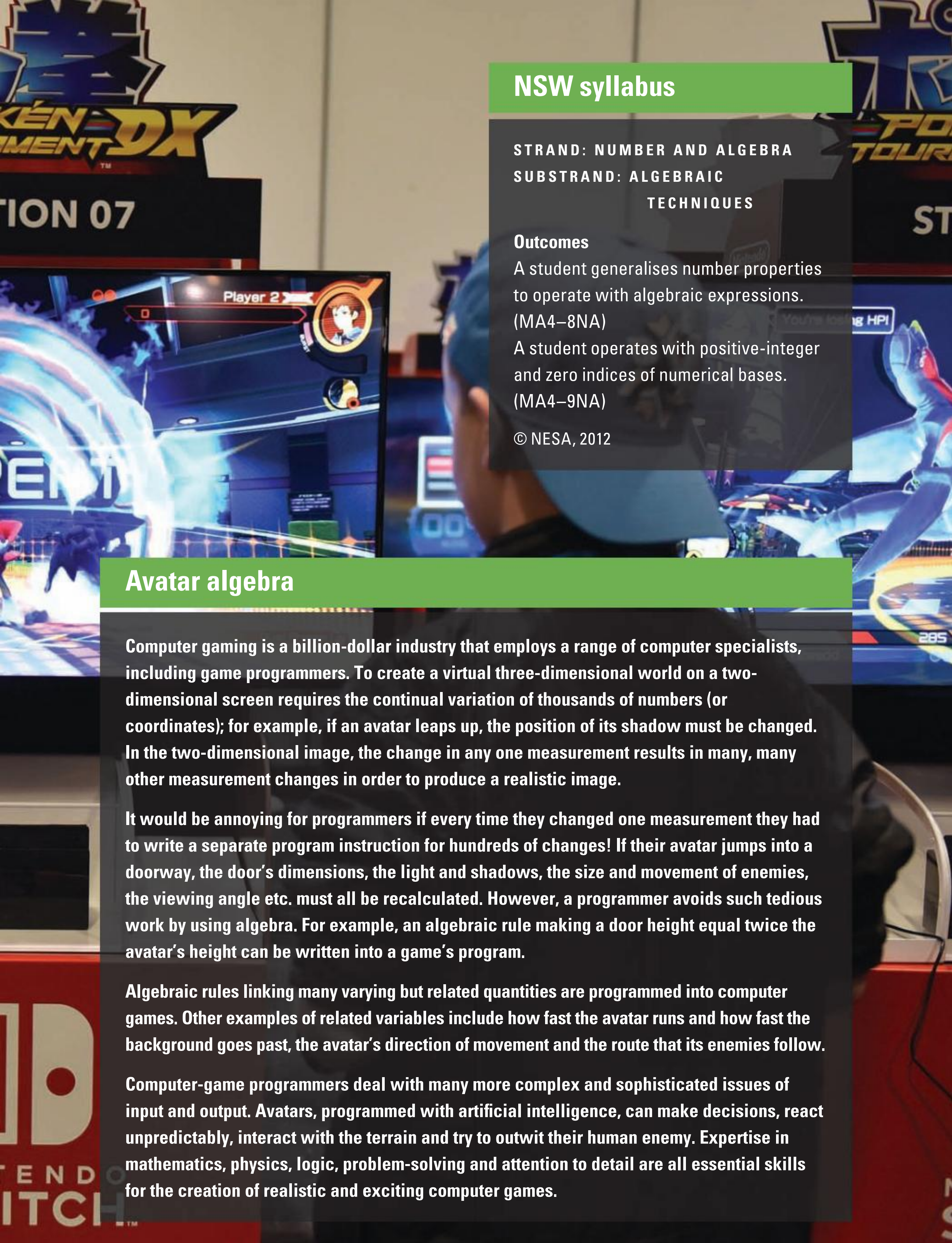
Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

1

Algebraic techniques 2 and indices

What you will learn



NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: ALGEBRAIC TECHNIQUES

Outcomes

A student generalises number properties to operate with algebraic expressions. (MA4–8NA)

A student operates with positive-integer and zero indices of numerical bases. (MA4–9NA)

Avatar algebra

Computer gaming is a billion-dollar industry that employs a range of computer specialists, including game programmers. To create a virtual three-dimensional world on a two-dimensional screen requires the continual variation of thousands of numbers (or coordinates); for example, if an avatar leaps up, the position of its shadow must be changed. In the two-dimensional image, the change in any one measurement results in many, many other measurement changes in order to produce a realistic image.

It would be annoying for programmers if every time they changed one measurement they had to write a separate program instruction for hundreds of changes! If their avatar jumps into a doorway, the door's dimensions, the light and shadows, the size and movement of enemies, the viewing angle etc. must all be recalculated. However, a programmer avoids such tedious work by using algebra. For example, an algebraic rule making a door height equal twice the avatar's height can be written into a game's program.

Algebraic rules linking many varying but related quantities are programmed into computer games. Other examples of related variables include how fast the avatar runs and how fast the background goes past, the avatar's direction of movement and the route that its enemies follow.

Computer-game programmers deal with many more complex and sophisticated issues of input and output. Avatars, programmed with artificial intelligence, can make decisions, react unpredictably, interact with the terrain and try to outwit their human enemy. Expertise in mathematics, physics, logic, problem-solving and attention to detail are all essential skills for the creation of realistic and exciting computer games.

Pre-test

1 Evaluate:

a $8 + 4 \times 6$ **b** $4 \times 5 - 2 \times 3$

c $12 - (6 + 2) + 8$ **d** $3(6 + 4)$

2 Evaluate:

a $10 + 6 - 12$ **b** $4 - 7$

c $-3 - 8$ **d** $-1 - 1 - 1$

3 Write an expression for:

a 5 more than x

b 7 less than m

c the product of x and y

d half of w

e double the sum of p and q

f half of the sum of p and q

4 If $y = 2x + 5$, find the value of y when $x = 1.2$.

5 Complete the tables using the given equations.

a $M = 2A + 3$

A	0	3	7	10
M				

b $y = \frac{1}{2}(x + 1)$

x	1	3	11	0
y				

6 Substitute $x = 6$ and $y = -2$ into each expression and then evaluate.

a $x + y$ **b** xy **c** $3x - y$ **d** $2x + 3y$

7 Write these numbers in expanded form.

a 5^2 **b** 2^4 **c** 3^3 **d** $(-8)^3$

8 Evaluate:

a 5^3 **b** $4^2 + 5^2$ **c** $(-4)^2$ **d** $3^3 - 2^2$

9 Complete:

a $5 + 5 + 5 + 5 = 4 \times \square$

b $6 + 6 + 6 + 6 + 6 + 6 = \square \times 6$

c $10 \div 5 = \frac{\square}{5}$

d $9 \times 9 \times 9 \times 8 \times 8 = 9^{\square} \times 8^{\square}$

e $117 \times 21 = 117 \times 20 + 117 \times \square$

10 Write down the HCF (highest common factor) of:

a 24 and 36

b 15 and 36

c 48 and 96

1A The language of algebra REVISION

A **pronumeral** is a letter that can represent one or more numbers. For instance, x could represent the number of goals a particular soccer player scored last year. Or p could represent the price (in dollars) of a book. The word **variable** is also used to describe a letter that represents an unknown value or quantity.

x could represent the number of goals this soccer player scored last year.

Let's start: Algebra sort

Consider the four expressions $x + 2$, $x \times 2$, $x - 2$ and $x \div 2$.

- If you know that x is 10, sort the four values from lowest to highest.
- Give an example of a value of x that would make $x \times 2$ less than $x + 2$.
- Try different values of x to see if you can:
  - make $x \div 2$ less than $x - 2$
  - make $x + 2$ less than $x - 2$
  - make $x \times 2$ less than $x \div 2$

Key ideas

- In algebra, a letter can be used to represent one or more numbers. These letters are called **pronumerals** or **variables**.
- $a \times b$ is written ab and $a \div b$ is written $\frac{a}{b}$.
- $a \times a$ is written a^2.
- An **expression** is a combination of numbers and pronumerals combined with mathematical operations. For example, $3x + 2yz$ and $8 \div (3a - 2b) + 41$ are expressions.
- A **term** is a part of an expression with only pronumerals, numbers, multiplication and division. For example, $9a$, $10cd$, and $\frac{3x}{5}$ are all terms.
- A **coefficient** is the number in front of a pronumeral. If the term is being subtracted, the coefficient is a negative number, and if there is no number in front, the coefficient is 1. For the expression $3x + y - 7z$, the coefficient of x is 3, the coefficient of y is 1 and the coefficient of z is -7.
- A term that does not contain any variables is called a **constant term**.
- The **sum** of a and b is $a + b$.
- The word **difference** usually implies subtraction. However, the order with subtraction is important. 'Subtract a from b' means $b - a$, whereas 'subtract b from a' means $a - b$.
- The **product** of a and b is $a \times b$, which is written as ab.
- The **quotient** of a and b is $a \div b$ or $\frac{a}{b}$.
- The **square** of a is $a \times a$, which is written as a^2.
- The **square** of ab can be written as $(ab)^2$ or a^2b^2.

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Example 1 Using the language of algebra

a List the individual terms in the expression $4a + b - 12c + 5$.
b In the expression $4a + b - 12c + 5$ state the coefficients of a, b, c and d.
c What is the constant term in $4a + b - 12c + 5$?
d State the coefficient of b in the expression $3a + 4ab + 5b^2 + 7b$.

SOLUTION	EXPLANATION
a There are four terms: $4a$, b, $12c$ and 5.	Each part of an expression is a term. Terms get added (or subtracted) to make an expression.
b The coefficient of a is 4. The coefficient of b is 1. The coefficient of c is -12. The coefficient of d is 0.	The coefficient is the number in front of a pronumeral. For b the coefficient is 1 because b is the same as $1 \times b$. For c the coefficient is -12 because this term is being subtracted. For d the coefficient is 0 because there are no terms with d.
c 5	A constant term is any term that does not contain a pronumeral.
d 7	Although there is a 4 in front of ab and a 5 in front of b^2, neither of these is a term containing just b, so they should be ignored.

Example 2 Creating expressions from a description

Write an expression for each of the following.

a the sum of 3 and k
b the product of m and 7
c 5 is added to one half of k
d the sum of a and b is doubled

SOLUTION	EXPLANATION
a $3 + k$	The word 'sum' means +.
b $m \times 7$ or $7m$	The word 'product' means ×.
c $\frac{1}{2}k + 5$ or $\frac{k}{2} + 5$	One half of k can be written $\frac{1}{2} \times k$ (because 'of' means ×), or $\frac{k}{2}$ because k is being divided by 2.
d $(a + b) \times 2$ or $2(a + b)$	The values of a and b are being added and the result is multiplied by 2. Grouping symbols (the brackets) are required to multiply the whole result by 2 and not just the value of b.

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Exercise 1A REVISION

UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	3, 4–6(½), 7	4–7(½)

Example 1a,b **1** The expression $3a + 2b + 5c$ has three terms.
- **a** List the terms.
- **b** State the coefficient of:
  - **i** a
  - **ii** b
  - **iii** c
- **c** Write another expression with three terms.

Example 1c,d **2** The expression $5a + 7b + c - 3ab + 6$ has five terms.
- **a** State the constant term.
- **b** State the coefficient of:
  - **i** a
  - **ii** b
  - **iii** c
- **c** Write another expression that has five terms.

Example 2a,b **3** Match each of the following worded statements with the correct mathematical expression.
- **a** the sum of x and 7
- **b** 3 less than x
- **c** x is divided by 2
- **d** x is tripled
- **e** x is subtracted from 3
- **f** x is divided by 3

- **A** $3 - x$
- **B** $\frac{x}{3}$
- **C** $x - 3$
- **D** $3x$
- **E** $\frac{x}{2}$
- **F** $x + 7$

4 For each expression:
- **i** State how many terms there are.
- **ii** List the terms.

- **a** $7a + 2b + c$
- **b** $19y - 52x + 32$
- **c** $a + 2b$
- **d** $7u - 3v + 2a + 123c$
- **e** $10f + 2be$
- **f** $9 - 2b + 4c + d + e$
- **g** $5 - x^2y + 4abc - 2nk$
- **h** $ab + 2bc + 3cd + 4de$

5 For each of the following expressions, state the coefficient of b.
- **a** $3a + 2b + c$
- **b** $3a + b + 2c$
- **c** $4a + 9b + 2c + d$
- **d** $3a - 2b + f$
- **e** $b + 2a + 4$
- **f** $2a + 5c$
- **g** $7 - 54c + d$
- **h** $5a - 6b + c$
- **i** $4a - b + c + d$
- **j** $2a + 4b^2 - 12b$
- **k** $7a - b + c$
- **l** $8a + c - 3b + d$

Example 2c,d

6 Write an expression for each of the following.

a 7 more than y
b 3 less than x
c the sum of a and b
d the product of 4 and p
e half of q is subtracted from 4
f one-third of r is added to 10
g the sum of b and c multiplied by 2
h the sum of b and twice the value of c
i the product of a, b and c divided by 7
j a quarter of a added to half of b
k the quotient of x and $2y$
l the difference between a and half of b
m the product of k and itself
n the square of w
o subtract p from q
p subtract 3 lots of x from the square of y

7 Describe each of the following expressions in words.

a $3 + x$
b $a + b$
c $4 \times b \times c$
d $2a + b$
e $(4 - b) \times 2$
f $4 - 2b$

PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13, 14	10–15

8 Marcela buys seven plants from the local nursery.

a If the cost is $\$x$ for each plant, write an expression for the total cost in dollars.
b If the cost of each plant is decreased by \$3 during a sale, write an expression for:
  i the new cost per plant in dollars
  ii the new total cost in dollars of the seven plants

9 Francine earns $\$p$ per week for her job. She works 48 weeks each year. Write an expression for the amount she earns:

a in a fortnight
b in 1 year
c in 1 year if her wage is increased by \$20 per week after she has already worked 30 weeks in the year

10 Jon likes to purchase DVDs of some TV shows. One show, *Numbers*, costs $\$a$ per season, and another show, *Proof by Induction*, costs $\$b$ per season. Write an expression for the cost of:

a four seasons of *Numbers*
b seven seasons of *Proof by Induction*
c five seasons of both shows
d all seven seasons of each show, if the final price is halved when purchased in a sale

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11 A plumber charges a \$70 call-out fee and then \$90 per hour. Write an expression for the total cost of calling a plumber out for x hours.

12 A mobile phone call costs 20 cents connection fee and then 50 cents per minute.
- **a** Write an expression for the total cost (in cents) of a call lasting t minutes.
- **b** Write an expression for the total cost (in dollars) of a call lasting t minutes.
- **c** Write an expression for the total cost (in dollars) of a call lasting t hours.

13 If x is a positive number, classify the following statements as true or false.
- **a** x is always smaller than $2 \times x$.
- **b** x is always smaller than $x + 2$.
- **c** x is always smaller than x^2.
- **d** $1 - x$ is always less than $4 - x$.
- **e** $x - 3$ is always a positive number.
- **f** $x + x - 1$ is always a positive number.

14 If b is a negative number, classify the following statements as true or false. Give a brief reason.
- **a** $b - 4$ *must* be negative.
- **b** $b + 2$ *could* be negative.
- **c** $b \times 2$ *could* be positive.
- **d** $b + b$ *must* be negative.

15 What is the difference between $2a + 5$ and $2(a + 5)$? Give an expression in words to describe each of them and describe how the grouping symbols change the meaning.

ENRICHMENT	–	–	16

Algebraic alphabet

16 An expression contains 26 terms, one for each letter of the alphabet. It starts $a + 4b + 9c + 16d + 25e + \dots$.
- **a** What is the coefficient of f?
- **b** What is the coefficient of z?
- **c** Which pronumeral has a coefficient of 400?
- **d** One term is removed and now the coefficient of k is zero. What was the term?
- **e** Another expression containing 26 terms starts $a + 2b + 4c + 8d + 16e + \dots$. What is the sum of all the coefficients?

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1B Substitution and equivalence

The pronumerals in an algebraic expression may be replaced by numbers. This is referred to as **substitution**, or **evaluation**. In the expression $4 + x$ we can substitute $x = 3$ to get the result 7. Two expressions are called **equivalent** if they always give the same result when a number is substituted. For example, $4 + x$ and $x + 4$ are equivalent because $4 + x$ and $x + 4$ will give equal numbers no matter what value of x is substituted.

Let's start: AFL algebra

In Australian Rules football, the final team score is given by $6x + y$, where x is the number of goals and y is the number of behinds scored.

- State the score if $x = 3$ and $y = 4$.
- If the score is 29, what are the values of x and y? Try to list all the possibilities.
- If $y = 9$ and the score is a two-digit number, what are the possible values of x?

Key ideas

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value.

  For example, if $a = 3$ and $b = 4$, then we can evaluate the expression $7a + 2b + 5$:

  $$\begin{aligned} 7a + 2b + 5 &= 7(3) + 2(4) + 5 \\ &= 21 + 8 + 5 \\ &= 34 \end{aligned}$$

- Two expressions are **equivalent** if they have equal values regardless of the number that is substituted for each pronumeral. The laws of arithmetic help to determine equivalence.
  - The **commutative laws** of arithmetic tell us that $a + b = b + a$ and $a \times b = b \times a$ for any values of a and b.
  - The **associative laws** of arithmetic tell us that $a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$ for any values of a and b.

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Example 3 Substituting values

Substitute $x = 3$ and $y = 6$ to evaluate the following expressions.

a $5x$

b $5x^2 + 2y + x$

SOLUTION

a $5x = 5(3)$
$= 15$

b $5x^2 + 2y + x = 5(3)^2 + 2(6) + (3)$
$= 5(9) + 12 + 3$
$= 45 + 12 + 3$
$= 60$

EXPLANATION

Remember that $5(3)$ is another way of writing 5×3.

Replace all the pronumerals by their values and remember the order in which to evaluate (multiplication before addition).

Example 4 Deciding if expressions are equivalent

a Are $x - 3$ and $3 - x$ equivalent expressions?

b Are $a + b$ and $b + 2a - a$ equivalent expressions?

SOLUTION

a No

b Yes

EXPLANATION

a The two expressions are equal if $x = 3$ (both equal zero).
But if $x = 7$ then $x - 3 = 4$ and $3 - x = -4$.
Because they are not equal for every single value of x, they are not equivalent.

b Regardless of the values of a and b substituted, the two expressions are equal. It is not possible to check every single number but we can check a few to be reasonably sure they seem equivalent.
For instance, if $a = 3$ and $b = 5$, then $a + b = 8$ and $b + 2a - a = 8$.
If $a = 17$ and $b = -2$, then $a + b = 15$ and $b + 2a - a = 15$.

Exercise 1B

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 9(½), 11	4, 5–9(½), 10	6–9(½), 10

1 What number is obtained when $x = 5$ is substituted into the expression $3 \times x$?

2 What is the result of evaluating $20 - b$ if b is equal to 12?

3 What is the value of $a + 2b$ if a and b both equal 10?

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4 **a** State the value of $4 + 2x$ if $x = 5$.
b State the value of $40 - 2x$ if $x = 5$.
c Are $4 + 2x$ and $40 - 2x$ equivalent expressions?

Example 3a

5 Substitute the following values of x into the expression $7x + 2$.
a 4 **b** 5 **c** 2 **d** 8
e 0 **f** −6 **g** −9 **h** −3

Example 3b

6 Substitute $a = 4$ and $b = -3$ into each of the following.
a $5a + 4$ **b** $3b$ **c** $a + b$
d $ab - 4 + b$ **e** $2 \times (3a + 2b)$ **f** $100 - (10a + 10b)$
g $\frac{12}{a} + \frac{6}{b}$ **h** $\frac{ab}{3} + b$ **i** $\frac{100}{a + b}$
j $a^2 + b$ **k** $5 \times (b + 6)^2$ **l** $a - 4b$

7 Evaluate the expression $2x - 3y$ when:
a $x = 10$ and $y = 4$
b $x = 1$ and $y = -10$
c $x = 0$ and $y = -2$
d $x = -10$ and $y = -6$
e $x = -7$ and $y = -9$
f $x = -2$ and $y = 9$

8 Evaluate the expression $4ab - 2b + 6c$ when:
a $a = 4$ and $b = 3$ and $c = 9$
b $a = -8$ and $b = -2$ and $c = 9$
c $a = -1$ and $b = -8$ and $c = -4$
d $a = 9$ and $b = -2$ and $c = 5$
e $a = -8$ and $b = -3$ and $c = 5$
f $a = -1$ and $b = -3$ and $c = 6$

Example 4

9 For the following, state whether they are equivalent (E) or not (N).
a $x + y$ and $y + x$ **b** $3 \times x$ and $x + 2x$
c $4a + b$ and $4b + a$ **d** $7 - x$ and $4 - x + 3$
e $4(a + b)$ and $4a + b$ **f** $4 + 2x$ and $2 + 4x$
g $\frac{1}{2} \times a$ and $\frac{a}{2}$ **h** $3 + 6y$ and $3(2y + 1)$

10 For each of the following, two of the three expressions are equivalent. State the odd one out.
a $4x$, $3 + x$ and $3x + x$
b $2 - a$, $a - 2$ and $a + 1 - 3$
c $5t - 2t$, $2t + t$ and $4t - 2t$
d $8u - 3$, $3u - 8$ and $3u - 3 + 5u$

11 Copy and complete the following table.

x	1	2	3	4	5	6	10	100
$x + 3$	4	5	6					

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PROBLEM-SOLVING AND REASONING	12, 15	12–16	12, 15–18

12 Give three expressions that are equivalent to $2x + 4y + 5$.

13 Copy and complete the following table.

x	3		0.25		−2	
$4x + 2$	14	6				
$4 - 3x$	−5					−2
$2x - 4$				8		

14 **a** Evaluate x^2 for the following values of x. Recall that $(-3)^2 = -3 \times -3$.
- **i** 3
- **ii** −3
- **iii** 7
- **iv** −7

b Given that $25^2 = 625$, what is the value of $(-25)^2$?

c Explain why the square of a number is the square of its negative.

d Why is $(-3)^2 = 9$ but $-3^2 = -9$? What effect do the brackets have?

15 **a** Explain with an example why $a \div (b \times c)$ is not equivalent to $(a \div b) \times c$.

b Does this contradict the associative law (see Key ideas)? Justify your answer.

c Is $a \div (b \div c)$ equivalent to $(a \div b) \div c$? Why or why not?

16 Are the following statements true for all values of p?
(*Hint*: try substituting $p = 0$, $p = 1$, $p = 2$ and $p = 3$ into each statement.)
- **a** $p + p = 2p$
- **b** $p \times p = 2p$
- **c** $6p - p = 6$
- **d** $p \times p = p^2$
- **e** $p - p = 0$
- **f** $p \div p = 0$

17 **a** By substituting a range of numbers for a and b, determine whether $(ab)^2$ is equivalent to a^2b^2.

b Is $(a + b)^2$ equivalent to $a^2 + b^2$? Why or why not?

c Is $\sqrt{ab}$ equivalent to $\sqrt{a} \times \sqrt{b}$? Why or why not?

d Is $\sqrt{a + b}$ equivalent to $\sqrt{a} + \sqrt{b}$? Why or why not?

e For pairs of expressions in **a**–**d** that are not equivalent, find a few values for a and b that make them equal.

18 Sometimes when two expressions are equivalent you can explain why they are equivalent. For example, $x + y$ is equivalent to $y + x$ because 'the order in which you add numbers does not matter', or 'because addition is commutative'. For each of the following pairs of expressions, try to describe why they are equivalent.
- **a** $x \times y$ and $y \times x$
- **b** $x + x$ and $2x$
- **c** $y - y$ and 0
- **d** $\frac{1}{2} \times x$ and $x \div 2$
- **e** $a \times 3a$ and $3a^2$
- **f** k^2 and $(-k)^2$

ENRICHMENT	–	–	19

Missing values

19 Find the missing values in the table below.

a	5	8			−20			
b	2		1					
$a + b$		10		10		7		−19
$a + 2b$				17	0		11	
$a - b$			1			13		
$a \quad 2b$							29	20

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1C Adding and subtracting terms REVISION

An expression such as $3x + 5x$ can be simplified to $8x$, but an expression such as $3x + 5y$ cannot be simplified. The reason is that $3x$ and $5x$ are **like terms**—they have exactly the same pronumerals. The terms $3x$ and $5y$ are not like terms. Also, $4ab$ and $7ba$ are like terms because ab and ba are equivalent, as multiplication is commutative. However, a^2b, ab and ab^2 are all unlike terms, since a^2b means $a \times a \times b$, which is different from $a \times b$ and $a \times b \times b$.

Let's start: Like terms

- Put these terms into groups of like terms.
  $4a$ $5b$ $2ab$ $3ba$ $2a$ $7b^2$ $5a^2b$ $9aba$
- What is the sum of each group?
- Ephraim groups $5a^2b$ and $2ab$ as like terms, so he simplifies $5a^2b + 2ab$ to $7ab$. How could you demonstrate to him that $5a^2b + 2ab$ is not equivalent to $7ab$?

Key ideas

- **Like terms** contain exactly the same pronumerals with the same powers; the pronumerals do not need to be in the same order. For example, $4ab$ and $7ba$ are like terms.
- When like terms are added or subtracted the expression can be simplified.
  For example, $3xy + 5xy = 8xy$.
- The sign in front of the term stays with the term when the expression is rearranged.

$$\begin{aligned} &3x \boxed{+\,7y}\boxed{-\,2x} + 3y + x - 4y \\ = {}&3x \boxed{-\,2x} + x \boxed{+\,7y} + 3y - 4y \end{aligned}$$

Example 5 Identifying like terms

a Are $5abc$ and $-8abc$ like terms?
b Are $12xy^2$ and $4y^2x$ like terms?
c Are $3ab^2$ and $7a^2b$ like terms?

SOLUTION	EXPLANATION
a Yes	Both terms have exactly the same pronumerals: a, b and c.
b Yes	When written out in full $12xy^2$ is $12 \times x \times y \times y$ and $4y^2x$ is $4 \times y \times y \times x$. Both terms include one x and two occurrences of y being multiplied, and the order of multiplication does not matter.
c No	$3ab^2 = 3 \times a \times b \times b$ and $7a^2b = 7 \times a \times a \times b$. They are not like terms because the first includes only one a and the second includes two.

Example 6 Simplifying by combining like terms

Simplify the following by combining like terms.

a $7t + 2t - 3t$ **b** $4x + 3y + 2x + 7y$ **c** $7ac + 3b - 2ca + 4b - 5b$

SOLUTION	EXPLANATION
a $7t + 2t - 3t = 6t$	These are like terms, so they can be combined: $7 + 2 - 3 = 6$
b $4x + 3y + 2x + 7y$ $= 4x + 2x + 3y + 7y$	Move the like terms next to each other.
$= 6x + 10y$	Combine the pairs of like terms.
c $7ac + 3b - 2ca + 4b - 5b$ $= 7ac - 2ca + 3b + 4b - 5b$ $= 5ac + 2b$	Move the like terms together. Recall that the subtraction sign stays in front of $2ca$ even when it is moved. $7 - 2 = 5$ and $3 + 4 - 5 = 2$.

Exercise 1C REVISION

UNDERSTANDING AND FLUENCY	1–4, 6–7(½)	5, 6–7(½), 8	6–7(½), 8

Example 5a **1** Classify the following pairs as like terms (L) or not like terms (N).

a $3a$ and $5a$ **b** $7x$ and $-12x$
c $2y$ and $7y$ **d** $4a$ and $-3b$
e $7xy$ and $3y$ **f** $12ab$ and $4ba$
g $3cd$ and $-8c$ **h** $2x$ and $4xy$

Example 5b,c **2** Classify the following pairs as like terms (L) or not like terms (N).

a $-3x^2y$ and $5x^2y$ **b** $12ab^2$ and $10b^2a$
c $2ab^2$ and $10ba^2$ **d** $7qrs$ and $-10rqs$
e $11q^2r$ and $10rq$ **f** $-15ab^2c$ and $-10cba^2$

3 **a** If $x = 3$, evaluate $5x + 2x$.
b If $x = 3$, evaluate $7x$.
c $5x + 2x$ is equivalent to $7x$. True or false?

4 **a** If $x = 3$ and $y = 4$, evaluate $5x + 2y$.
b If $x = 3$ and $y = 4$, evaluate $7xy$.
c $5x + 2y$ is equivalent to $7xy$. True or false?

5 **a** Substitute $x = 4$ into the expression $10x - 5x + 2x$.
b Substitute $x = 4$ into:
i $3x$ **ii** $5x$ **iii** $7x$
c Which one of the expressions in part **b** is equivalent to $10x - 5x + 2x$?

Example 6a **6** Simplify the following by combining like terms.

a $3x + 2x$ **b** $7a + 12a$ **c** $15x - 6x$
d $4xy + 3xy$ **e** $16uv - 3uv$ **f** $10ab + 4ba$
g $11ab - 5ba + ab$ **h** $3k + 15k - 2k$ **i** $15k - 2k - 3k$

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Example 6b,c

7 Simplify the following by combining like terms.

- **a** $7f + 2f + 8 + 4$
- **b** $10x + 3x + 5y + 3y$
- **c** $2a + 5a + 13b - 2b$
- **d** $10a + 5b + 3a + 4b$
- **e** $10 + 5x + 2 + 7x$
- **f** $10a + 3 + 4b - 2a - b$
- **g** $10x + 31y - y + 4x$
- **h** $11a + 4 - 2a + 12a$
- **i** $7x^2y + 5x + 10yx^2$
- **j** $12xy - 3yx + 5xy - yx$
- **k** $-4x^2 + 3x^2$
- **l** $-2a + 4b - 7ab + 4a$
- **m** $10 + 7q - 3r + 2q - r$
- **n** $11b - 3b^2 + 5b^2 - 2b$

8 For each expression choose an equivalent expression from the options **A**–**E** listed.

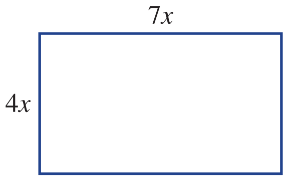
- **a** $7x + 2x$
- **b** $12y + 3x - 2y$
- **c** $3x + 3y$
- **d** $8y - 2x + 6y - x$
- **e** $4xy + 5yx$

- **A** $10y + 3x$
- **B** $9xy$
- **C** $9x$
- **D** $3y + 3x$
- **E** $14y - 3x$

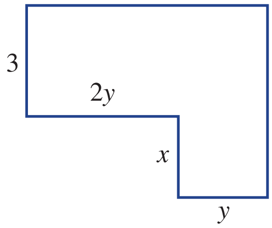
PROBLEM-SOLVING AND REASONING	9, 10, 15	10–12, 15, 16	12–14, 16, 17

9 Write expressions for the perimeters of the following shapes in simplest form.

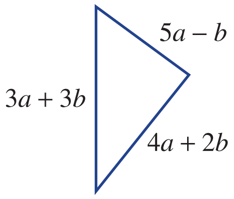
a



b



c



10 Towels cost \$$c$ each at a shop.

- **a** John buys three towels, Mary buys six towels and Naomi buys four towels. Write a fully simplified expression for the total amount spent on towels.
- **b** On another occasion, Chris buys n towels, David buys twice as many as Chris and Edward buys three times as many as David. Write a simplified expression for the total amount they spent on towels.

11 State the missing numbers to make the following equivalences true.

- **a** $10x + 6y - \square x + \square y = 3x + 8y$
- **b** $5a - 7b + \square a + \square b = 11a$
- **c** $\square c + \square d + \square = 4c + 2d + 1 + 3c + 7d + 4$
- **d** $\square a^2b + \square b^2a + 2a^2b + b^2a = 7b^2a + 10a^2b$

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12 Add the missing expressions to the puzzle to make all six equations true.

$$\begin{array}{ccccc}
\boxed{5x} & + & \boxed{} & = & \boxed{7x}\\
+ & & + & & +\\
\boxed{y} & + & \boxed{} & = & \boxed{}\\
= & & = & & =\\
\boxed{} & + & \boxed{} & = & \boxed{7x + 3y}
\end{array}$$

13 In how many ways could the blank boxes be filled if all the coefficients must be positive integers?
$\square a + \square b + \square a = 10a + 7b$

14 Simplify $a - 2a + 3a - 4a + 5a - 6a + \ldots + 99a - 100a$.

15 Prove that $10x + 5y + 7x + 2y$ is equivalent to $20x - 3x + 10y - 3y$. (*Hint*: simplify both expressions.)

16 **a** Make a substitution to prove that $4a + 3b$ is not equivalent to $7ab$.
b Is $4a + 3b$ ever equal to $7ab$? Try to find some values of a and b to make $4a + 3b = 7ab$ a true equation.
c Is $4a + 3a$ ever not equal to $7a$? Explain your answer.

17 **a** Decide whether $7x - 3x$ is equivalent to $7x + (-3x)$. Explain why or why not.
b Fill in the missing numbers to make the following equivalence true.
$14a + 3b + 2ab + \square a + \square b + \square ba = a$

ENRICHMENT	–	–	18

Missing expressions

18 **a** Fill in the missing expressions to make all eight equations true.

$$\begin{array}{ccccccc}
\boxed{5a} & + & \boxed{} & + & \boxed{4a} & = & \boxed{10a + 3}\\
+ & & + & & + & & +\\
\boxed{2b} & + & \boxed{4 + a} & + & \boxed{} & = & \boxed{}\\
+ & & + & & + & & +\\
\boxed{} & + & \boxed{} & + & \boxed{-3a} & = & \boxed{1}\\
= & & = & & = & & =\\
\boxed{} & + & \boxed{3a + 8} & + & \boxed{} & = & \boxed{10a + 2b + 8}
\end{array}$$

b Design your own 'missing values' puzzle like the one in part **a**. It should have only one possible solution.

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1D Multiplying and dividing terms REVISION

Interactive
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A term written as $4ab$ is shorthand for $4 \times a \times b$. Observing this helps to see how we can multiply terms.

$$\begin{aligned} 4ab \times 3c &= 4 \times a \times b \times 3 \times c \\ &= 4 \times 3 \times a \times b \times c \\ &= 12abc \end{aligned}$$

Division is written as a fraction so $\frac{12ab}{9ad}$ means $(12ab) \div (9ad)$. To simplify a division we look for common factors:

$$\frac{{}^{4}\cancel{12} \times \cancel{a} \times b}{{}^{3}\cancel{9} \times \cancel{a} \times d} = \frac{4b}{3d}$$

$a \div a = 1$ for any value of a except 0, so $\frac{a}{a}$ cancels to 1.

Let's start: Multiple ways

Multiplying $4a \times 6b \times c$ gives you $24abc$.

- In how many ways can positive integers fill the blank boxes in $\square a \times \square b \times \square c = 24abc$?
- In how many other ways can you multiply three terms to get $24abc$? For example, $12ab \times 2 \times c$. You should assume the coefficients are all integers.

Key ideas

- $12abc$ means $12 \times a \times b \times c$.
- When multiplying, the order is not important: $2 \times a \times 4 \times b = 2 \times 4 \times a \times b$.
- x^2 means $x \times x$ and x^3 means $x \times x \times x$.
- When dividing, cancel any common factors.

  For example: $\frac{{}^{3}\cancel{15}x\cancel{y}}{{}^{4}\cancel{20}\cancel{y}z} = \frac{3x}{4z}$

Example 7 Multiplying and dividing terms

a Simplify $7a \times 2bc \times 3d$.

b Simplify $3xy \times 5xz$.

c Simplify $\frac{10ab}{15bc}$.

d Simplify $\frac{18x^2y}{8xz}$.

SOLUTION	EXPLANATION
a $7a \times 2bc \times 3d$	
$= 7 \times a \times 2 \times b \times c \times 3 \times d$ $= 7 \times 2 \times 3 \times a \times b \times c \times d$	Write the expression with multiplication signs and bring the numbers to the front.
$= 42abcd$	Simplify: $7 \times 2 \times 3 = 42$ and $a \times b \times c \times d = abcd$.

b $3xy \times 5xz$

$= 3 \times x \times y \times 5 \times x \times z$

$= 3 \times 5 \times x \times x \times y \times z$

Write the expression with multiplication signs and bring the numbers to the front.

$= 15x^2yz$

Simplify, remembering that $x \times x = x^2$.

c $\frac{10ab}{15bc} = \frac{{}^{2}\cancel{10} \times a \times \cancel{b}}{{}^{3}\cancel{15} \times \cancel{b} \times c}$

$= \frac{2a}{3c}$

Write the numerator and denominator in full, with multiplication signs. Cancel any common factors and remove the multiplication signs.

d $\frac{18x^2y}{8xz} = \frac{{}^{9}\cancel{18} \times \cancel{x} \times x \times y}{{}^{4}\cancel{8} \times \cancel{x} \times z}$

$= \frac{9xy}{4z}$

Write the numerator and denominator in full, remembering that x^2 is $x \times x$. Cancel any common factors and remove the multiplication signs.

Exercise 1D REVISION

UNDERSTANDING AND FLUENCY	1–5, 6(½), 7, 8	4, 5–6(½), 7, 8(½)	6(½), 7, 8(½)

1 Which is the correct way to write $3 \times a \times b \times b$?

A $3ab$ **B** $3ab^2$ **C** ab^3 **D** $3a^2b$

2 Simplify these fractions.

a $\frac{12}{20}$ **b** $\frac{5}{15}$ **c** $\frac{12}{8}$ **d** $\frac{15}{25}$

3 Which one of these is equivalent to $a \times b \times a \times b \times b$?

A $5ab$ **B** a^2b^3 **C** a^3b^2 **D** $(ab)^5$

4 Write these without multiplication signs.

a $3 \times x \times y$

b $5 \times a \times b \times c$

c $12 \times a \times b \times b$

d $4 \times a \times c \times c \times c$

Example 7a

5 Simplify the following.

a $7d \times 9$ **b** $5a \times 2b$

c $3 \times 12x$ **d** $4a \times 2b \times cd$

e $3a \times 10bc \times 2d$ **f** $4a \times 6de \times 2b$

Example 7b

6 Simplify the following.

a $8ab \times 3c$ **b** $a \times a$

c $3d \times d$ **d** $5d \times 2d \times e$

e $7x \times 2y \times x$ **f** $5xy \times 2x$

g $4xy \times 2xz$ **h** $4abc \times 2abd$

i $9ab \times 2a^2$ **j** $-3xz \times (-2z)$

k $-5xy \times 2yz$ **l** $4xy^2 \times 4y$

7 Write the following terms as a product of prime numbers and pronumerals.

a $18xy$ **b** $12abc$ **c** $15x^2$ **d** $51ab^2$

Example 7c,d

8 Simplify the following divisions by cancelling any common factors.

a $\frac{5a}{10a}$ **b** $\frac{7x}{14y}$

c $\frac{10xy}{12y}$ **d** $\frac{ab}{4b}$

e $\frac{7xyz}{21yz}$ **f** $\frac{2}{12x}$

g $\frac{-5x}{10yz^2}$ **h** $\frac{12y^2}{-18y}$

i $\frac{-4a^2}{8ab}$ **j** $\frac{21p}{-3q}$

k $\frac{-21p}{-3p}$ **l** $\frac{-15z}{-20z^2}$

PROBLEM-SOLVING AND REASONING	9–10(½), 12	9, 10, 12, 13	10(½), 11–14

9 Express each of the following divisions as fractions, then simplify where possible.

a $x \div 3$ **b** $m \div n$

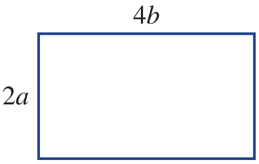
c $10a \div 12b$ **d** $x^2 \div 9$

e $(a + b) \div 3$ **f** $12mn \div 16mn$

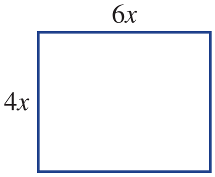
g $16a^2 \div 4a$ **h** $4ab \div 12b^2$

10 Write a simplified expression for the area of the following shapes. Recall that rectangle area = length × breadth.

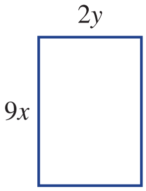
a



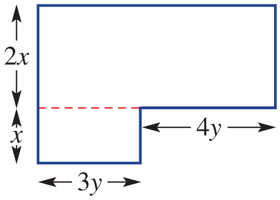
b



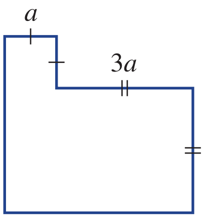
c



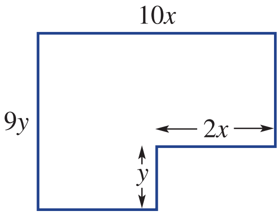
d



e



f



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11 Fill in the missing terms to make the following equivalences true.

a $3x \times \square \times z = 6xyz$

b $4a \times \square = 12ab^2$

c $-2q \times \square \times 4s = 16qs$

d $\dfrac{\square}{4r} = 7s$

e $\dfrac{\square}{2ab} = 4b$

f $\dfrac{14xy}{\square} = -2y$

12 A box has a height of x cm. It is three times as wide as it is high, and two times as long as it is wide. Find an expression for the volume of the box, given that volume = length × breadth × height.

13 A square has a side length of x cm.

a State its area in terms of x.

b State its perimeter in terms of x.

c Prove that its area divided by its perimeter is equal to a quarter of its side length.

14 Joanne claims that the following three expressions are equivalent: $\dfrac{2a}{5}, \dfrac{2}{5} \times a, \dfrac{2}{5a}$.

a Is she right? Try different values of a.

b Which two expressions are equivalent?

c There are two values of a that make all three expressions equal. What are they?

ENRICHMENT	–	–	15

Multiple operations

15 Simplify the following expressions, remembering that you can combine like terms when adding or subtracting.

a $\dfrac{2ab \times 3bc \times 4cd}{4a \times 3bc \times 2d}$

b $\dfrac{12a^2b + 4a^2b}{4b + 2b}$

c $\dfrac{7x^2y - 5yx^2}{12xy}$

d $\dfrac{8a^2b + (4a \times 2ba)}{3ba - 2ba}$

e $\dfrac{10abc + 5cba + 5a \times bc}{4c \times 10ab}$

f $\dfrac{10x^2y - (4x \times 6xy)}{7xy^2}$

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1E Adding and subtracting algebraic fractions EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

An algebraic fraction is a fraction that could include any algebraic expression in the numerator or the denominator.

$$\underbrace{\frac{2}{3} \quad \frac{5}{7} \quad \frac{20}{3}}_{\text{Fractions}} \qquad\qquad \underbrace{\frac{2x}{5} \quad \frac{3}{7a+4} \quad \frac{2x-4y}{7a+9b}}_{\text{Algebraic fractions}}$$

The rules for working with algebraic fractions are the same as the rules for normal fractions. For example, two fractions with the same denominator can be added or subtracted easily.

Normal fractions	Algebraic fractions
$\frac{2}{13}+\frac{7}{13}=\frac{9}{13}$	$\frac{5x}{13}+\frac{3x}{13}=\frac{8x}{13}$
$\frac{8}{11}-\frac{2}{11}=\frac{6}{11}$	$\frac{5x}{11}-\frac{2y}{11}=\frac{5x-2y}{11}$

If two fractions do not have the same denominator, they must be converted to have the lowest common denominator (LCD) before adding or subtracting.

Normal fractions	Algebraic fractions
$\frac{2}{3}+\frac{1}{5}=\frac{10}{15}+\frac{3}{15}$	$\frac{2a}{3}+\frac{b}{5}=\frac{10a}{15}+\frac{3b}{15}$
$=\frac{13}{15}$	$=\frac{10a+3b}{15}$

Let's start: Adding thirds and halves

Dallas and Casey attempt to simplify $\frac{x}{3}+\frac{x}{2}$. Dallas gets $\frac{x}{5}$ and Casey gets $\frac{5x}{6}$.

- Which of the two students has the correct answer? You could try substituting different numbers for x.
- How can you prove that the other student is incorrect?
- What do you think $\frac{x}{3}+\frac{x}{4}$ is equivalent to? Compare your answers with those of others in the class.

Key ideas

- An **algebraic fraction** is a fraction with an algebraic expression as the numerator or the denominator.
- The **lowest common denominator** (or LCD) of two algebraic fractions is the smallest number that is a multiple of both denominators.
- Adding and subtracting algebraic fractions requires that they both have the same denominator.
  For example, $\frac{2x}{5}+\frac{4y}{5}=\frac{2x+4y}{5}$.

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Example 8 Working with denominators

a Find the lowest common denominator of $\frac{3x}{10}$ and $\frac{2y}{15}$.

b Convert $\frac{2x}{7}$ to an equivalent algebraic fraction with the denominator 21.

SOLUTION	EXPLANATION
a 30	The multiples of 10 are 10, 20, 30, 40, 50, 60 etc. The multiples of 15 are 15, 30, 45, 60, 75, 90 etc. The smallest number in both lists is 30.
b $\frac{2x}{7} = \frac{3 \times 2x}{3 \times 7}$	Multiply the numerator and denominator by 3, so that the denominator is 21.
$= \frac{6x}{21}$	Simplify the numerator: $3 \times 2x$ is $6x$.

Example 9 Adding and subtracting algebraic fractions

Simplify the following expressions.

a $\frac{3x}{11} + \frac{5x}{11}$ **b** $\frac{4a}{3} + \frac{2a}{5}$ **c** $\frac{6k}{5} - \frac{3k}{10}$ **d** $\frac{a}{6} - \frac{b}{9}$

SOLUTION	EXPLANATION
a $\frac{3x}{11} + \frac{5x}{11} = \frac{3x + 5x}{11}$	The two fractions have the same denominator, so the two numerators are added.
$= \frac{8x}{11}$	$3x$ and $5x$ are like terms, so they are combined to $8x$.
b $\frac{4a}{3} + \frac{2a}{5} = \frac{5 \times 4a}{15} + \frac{3 \times 2a}{15}$	The LCD = 15, so both fractions are converted to have 15 as the denominator.
$= \frac{20a}{15} + \frac{6a}{15}$	Simplify the numerators.
$= \frac{26a}{15}$	Combine: $20a + 6a$ is $26a$.
c $\frac{6k}{5} - \frac{3k}{10} = \frac{12k}{10} - \frac{3k}{10}$	LCD = 10, so convert the first fraction (multiplying numerator and denominator by 2).
$= \frac{12k - 3k}{10}$	Combine the numerators.
$= \frac{9k}{10}$	Simplify: $12k - 3k = 9k$.
d $\frac{a}{6} - \frac{b}{9} = \frac{3a}{18} - \frac{2b}{18}$	LCD = 18, so convert both fractions to have 18 as a denominator.
$= \frac{3a - 2b}{18}$	Combine the numerators. Note this cannot be further simplified since $3a$ and $2b$ are not like terms.

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Exercise 1E EXTENSION

UNDERSTANDING AND FLUENCY	1–6, 7–8(½)	3–6, 7–9(½)	6, 7–9(½)

1 Complete these sentences.

a For the fraction $\frac{2}{7}$ the numerator is 2 and the denominator is _____.

b For $\frac{4}{9}$ the numerator is _____ and the denominator is _____.

c The expression $\frac{12x}{5}$ is an example of an __________ fraction.

d The denominator of $\frac{5x+3}{7}$ is _____.

2 Find the lowest common denominator (LCD) of the following pairs of fractions.

a $\frac{1}{3}$ and $\frac{2}{5}$ **b** $\frac{1}{4}$ and $\frac{1}{5}$ **c** $\frac{3}{7}$ and $\frac{5}{6}$ **d** $\frac{2}{3}$ and $\frac{1}{6}$

3 Find the missing numerator to make the following equations true.

a $\frac{2}{3} = \frac{\square}{6}$ **b** $\frac{4}{7} = \frac{\square}{21}$ **c** $\frac{1}{3} = \frac{\square}{12}$ **d** $\frac{6}{11} = \frac{\square}{55}$

4 Evaluate the following, by first converting to the lowest common denominator.

a $\frac{1}{4} + \frac{1}{3}$ **b** $\frac{2}{7} + \frac{1}{5}$ **c** $\frac{1}{10} + \frac{1}{5}$ **d** $\frac{2}{5} - \frac{1}{4}$

Example 8a

5 Find the LCD of the following pairs of algebraic fractions.

a $\frac{x}{3}$ and $\frac{2y}{5}$ **b** $\frac{3x}{10}$ and $\frac{21y}{20}$ **c** $\frac{x}{4}$ and $\frac{y}{5}$ **d** $\frac{x}{12}$ and $\frac{y}{6}$

Example 8b

6 Copy and complete the following, to make each equation true.

a $\frac{x}{5} = \frac{\square}{10}$ **b** $\frac{2a}{7} = \frac{\square}{21}$ **c** $\frac{4z}{5} = \frac{\square}{20}$ **d** $\frac{3k}{10} = \frac{\square}{50}$

Example 9a,b

7 Simplify the following sums.

a $\frac{x}{4} + \frac{2x}{4}$ **b** $\frac{5a}{3} + \frac{2a}{3}$ **c** $\frac{2b}{5} + \frac{b}{5}$ **d** $\frac{4k}{3} + \frac{k}{3}$

e $\frac{a}{2} + \frac{a}{3}$ **f** $\frac{a}{4} + \frac{a}{5}$ **g** $\frac{p}{2} + \frac{p}{5}$ **h** $\frac{q}{4} + \frac{q}{2}$

i $\frac{2k}{5} + \frac{3k}{7}$ **j** $\frac{2m}{5} + \frac{2m}{3}$ **k** $\frac{7p}{6} + \frac{2p}{5}$ **l** $\frac{x}{4} + \frac{3x}{8}$

Example 9c

8 Simplify the following differences.

a $\frac{3y}{5} - \frac{y}{5}$ **b** $\frac{7p}{13} - \frac{2p}{13}$ **c** $\frac{10r}{7} - \frac{2r}{7}$ **d** $\frac{8q}{5} - \frac{2q}{5}$

e $\frac{p}{2} - \frac{p}{3}$ **f** $\frac{2t}{5} - \frac{t}{3}$ **g** $\frac{9u}{11} - \frac{u}{2}$ **h** $\frac{8y}{3} - \frac{5y}{6}$

i $\frac{r}{3} - \frac{r}{2}$ **j** $\frac{6u}{7} - \frac{7u}{6}$ **k** $\frac{9u}{1} - \frac{3u}{4}$ **l** $\frac{5p}{12} - \frac{7p}{11}$

Example 9d

9 Simplify the following expressions, giving your final answer as an algebraic fraction. (*Hint*: $4x$ is the same as $\frac{4x}{1}$.)

a $4x + \frac{x}{3}$

b $3x + \frac{x}{2}$

c $\frac{a}{5} + 2a$

d $\frac{8p}{3} - 2p$

e $\frac{10u}{3} + \frac{3v}{10}$

f $\frac{7y}{10} - \frac{2x}{5}$

g $2t + \frac{7p}{2}$

h $\frac{x}{3} - y$

i $5 - \frac{2x}{7}$

PROBLEM-SOLVING AND REASONING	10, 13a,b	10, 11, 13	11–14

10 Cedric earns an unknown amount \$$x$ every week. He spends $\frac{1}{3}$ of his income on rent and $\frac{1}{4}$ on groceries.

a Write an algebraic fraction for the amount of money he spends on rent.

b Write an algebraic fraction for the amount of money he spends on groceries.

c Write a simplified algebraic fraction for the total amount of money he spends on rent and groceries.

11 Egan fills the bathtub so it is a quarter full and then adds half a bucket of water. A full bathtub can contain T litres and a bucket contains B litres.

a Write the total amount of water in the bathtub as the sum of two algebraic fractions.

b Simplify the expression in part **a** to get a single algebraic fraction.

c If a full bathtub contains 1000 litres and the bucket contains 2 litres, how many litres of water are in the bathtub?

12 Afshin's bank account is halved in value and then \$20 is removed. If it initially had \$$A$ in it, write an algebraic fraction for the amount left.

13 a Demonstrate that $\frac{x}{2} + \frac{x}{3}$ is equivalent to $\frac{5x}{6}$ by substituting at least three different values for x.

b Demonstrate that $\frac{x}{4} + \frac{x}{5}$ is not equivalent to $\frac{2x}{9}$.

c Is $\frac{x}{2} + \frac{x}{5}$ equivalent to $x - \frac{x}{3}$? Explain why or why not.

14 a Simplify:

i $\frac{x}{2} - \frac{x}{3}$

ii $\frac{x}{3} - \frac{x}{4}$

iii $\frac{x}{4} - \frac{x}{5}$

iv $\frac{x}{5} - \frac{x}{6}$

b What patterns did you notice in the results above?

c Write a difference of two algebraic fractions that simplifies to $\frac{x}{110}$.

ENRICHMENT	–	–	15

Equivalent sums and differences

15 For each of the following expressions, find a single equivalent algebraic fraction.

a $\frac{z}{4} + \frac{z}{3} + \frac{z}{12}$

b $\frac{2x}{5} + \frac{x}{2} - \frac{x}{5}$

c $\frac{7u}{2} + \frac{3u}{4} - \frac{5u}{8}$

d $\frac{8k}{3} + \frac{k}{6} - \frac{5k}{12}$

e $\frac{p}{4} + \frac{p}{2} - 3$

f $\frac{u}{3} + \frac{u}{4} + \frac{u}{5}$

g $\frac{5j}{12} - \frac{j}{3} + 2$

h $\frac{7t}{5} - \frac{t}{3} + \frac{2r}{15}$

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1F Multiplying and dividing algebraic fractions EXTENSION

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As with fractions, it is generally easier to multiply and divide algebraic fractions than it is to add or subtract them.

$$\frac{3}{5}\times\frac{2}{7}=\frac{6}{35}\quad\begin{array}{l}\longleftarrow 3\times 2\\ \longleftarrow 5\times 7\end{array}$$

Fractions

$$\frac{4x}{7}\times\frac{2y}{11}=\frac{8xy}{77}$$

Algebraic fractions

Dividing is done by multiplying by the reciprocal of the second fraction.

$$\begin{aligned}\frac{4}{5}\div\frac{1}{3}&=\frac{4}{5}\times\frac{3}{1}\\&=\frac{12}{5}\end{aligned}$$

Fractions

$$\begin{aligned}\frac{2x}{5}\div\frac{3y}{7}&=\frac{2x}{5}\times\frac{7}{3y}\\&=\frac{14x}{15y}\end{aligned}$$

Algebraic fractions

Let's start: Always the same

One of these four expressions always gives the same answer, no matter what the value of x is.

$$\frac{x}{2}+\frac{x}{3}\qquad\frac{x}{2}-\frac{x}{3}\qquad\frac{x}{2}\times\frac{x}{3}\qquad\frac{x}{2}\div\frac{x}{3}$$

- Which of the four expressions always has the same value?
- Can you explain why this is the case?
- Try to find an expression involving two algebraic fractions that is equivalent to $\frac{3}{8}$.

Key ideas

- To multiply two algebraic fractions, multiply the **numerators** and the **denominators** separately. Then cancel any common factors in the numerator and the denominator. For example:

$$\begin{aligned}\frac{2x}{5}\times\frac{10y}{3}&=\frac{{}^{4}\cancel{20}xy}{\cancel{15}^{3}}\\&=\frac{4xy}{3}\end{aligned}$$

- The **reciprocal** of an algebraic fraction is formed by swapping the numerator and denominator.

  For example, the reciprocal of $\frac{3b}{4}$ is $\frac{4}{3b}$.

- To divide one fraction by another, multiply the first fraction by the reciprocal of the second fraction. For example:

$$\begin{aligned}\frac{2a}{5}\div\frac{3b}{4}&=\frac{2a}{5}\times\frac{4}{3b}\\&=\frac{8a}{15b}\end{aligned}$$

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Example 10 Multiplying algebraic fractions

Simplify the following products.

a $\frac{2a}{5} \times \frac{3b}{7}$ **b** $\frac{4x}{15} \times \frac{3y}{2}$

SOLUTION	EXPLANATION
a $\frac{2a}{5} \times \frac{3b}{7} = \frac{6ab}{35}$	$2a \times 3b = 6ab$ and $5 \times 7 = 35$.
b Method 1:	
$\frac{4x}{15} \times \frac{3y}{2} = \frac{12xy}{30}$	$4x \times 3y = 12xy$ $15 \times 2 = 30$
$= \frac{2xy}{5}$	Divide by a common factor of 6 to simplify.
Method 2: $\frac{{}^{2}\cancel{4}x}{{}^{5}\cancel{15}} \times \frac{{}^{1}\cancel{3}y}{\cancel{2}^{1}} = \frac{2xy}{5}$	First divide by any common factors in the numerators and denominators: $4x$ and 2 have a common factor of 2. Also $3y$ and 15 have a common factor of 3.

Example 11 Dividing algebraic fractions

Simplify the following divisions.

a $\frac{3a}{8} \div \frac{b}{5}$ **b** $\frac{u}{4} \div \frac{15p}{2}$

SOLUTION	EXPLANATION
a $\frac{3a}{8} \div \frac{b}{5} = \frac{3a}{8} \times \frac{5}{b}$	Take the reciprocal of $\frac{b}{5}$, which is $\frac{5}{b}$.
$= \frac{15a}{8b}$	Multiply as before: $3a \times 5 = 15a$, $8 \times b = 8b$.
b $\frac{u}{4} \div \frac{15p}{2} = \frac{u}{4} \times \frac{2}{15p}$	Take the reciprocal of $\frac{15p}{2}$, which is $\frac{2}{15p}$.
$= \frac{2u}{60p}$	Multiply as before: $u \times 2 = 2u$ and $4 \times 15p = 60p$.
$= \frac{u}{30p}$	Cancel the common factor of 2.

Exercise 1F EXTENSION

UNDERSTANDING AND FLUENCY	1–6, 7(½)	3(½), 4, 5–8(½)	5–8(½)

1 Write the missing number.

a $\frac{2}{3} \times \frac{4}{5} = \frac{\square}{15}$ **b** $\frac{1}{2} \times \frac{3}{7} = \frac{\square}{14}$ **c** $\frac{4}{5} \times \frac{7}{11} = \frac{28}{\square}$ **d** $\frac{3}{4} \times \frac{5}{8} = \frac{15}{\square}$

2 Which one of the following shows the correct first step in calculating $\frac{2}{3} \div \frac{4}{5}$?

A $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} \times \frac{5}{4}$ **B** $\frac{2}{3} \div \frac{4}{5} = \frac{2}{4} \times \frac{3}{5}$ **C** $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$ **D** $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} \times \frac{4}{5}$

3 Calculate the following by hand, remembering to simplify your result.

a $\frac{2}{3} \times \frac{7}{10}$ **b** $\frac{1}{5} \times \frac{10}{11}$ **c** $\frac{1}{4} \times \frac{6}{11}$ **d** $\frac{5}{12} \times \frac{6}{10}$

e $\frac{2}{3} \div \frac{4}{5}$ **f** $\frac{1}{4} \div \frac{2}{3}$ **g** $\frac{4}{17} \div \frac{1}{3}$ **h** $\frac{2}{9} \div \frac{1}{2}$

4 Simplify the following algebraic fractions.

a $\frac{2x}{6}$ **b** $\frac{5xy}{2y}$ **c** $\frac{7ab}{14b}$ **d** $\frac{2bc}{5bc}$

Example 10a

5 Simplify the following products.

a $\frac{x}{3} \times \frac{2}{5}$ **b** $\frac{1}{7} \times \frac{a}{9}$

c $\frac{2}{3} \times \frac{4a}{5}$ **d** $\frac{4c}{5} \times \frac{1}{5}$

e $\frac{4a}{3} \times \frac{2b}{5}$ **f** $\frac{3a}{2} \times \frac{7a}{5}$

Example 10b

6 Simplify the following products, remembering to cancel any common factors.

a $\frac{6x}{5} \times \frac{7y}{6}$ **b** $\frac{2b}{5} \times \frac{7d}{6}$

c $\frac{8a}{5} \times \frac{3b}{4c}$ **d** $\frac{9d}{2} \times \frac{4e}{7}$

e $\frac{3x}{2} \times \frac{1}{6x}$ **f** $\frac{4}{9k} \times \frac{3k}{2}$

Example 11

7 Simplify the following divisions, cancelling any common factors.

a $\frac{3a}{4} \div \frac{1}{5}$ **b** $\frac{2x}{5} \div \frac{3}{7}$

c $\frac{9a}{10} \div \frac{1}{4}$ **d** $\frac{2}{3} \div \frac{4x}{7}$

e $\frac{4}{5} \div \frac{2y}{3}$ **f** $\frac{1}{7} \div \frac{2}{x}$

g $\frac{4a}{7} \div \frac{2}{5}$ **h** $\frac{4b}{7} \div \frac{2c}{5}$

i $\frac{2x}{5} \div \frac{4y}{3}$ **j** $\frac{2y}{x} \div \frac{3}{y}$

k $\frac{5}{12x} \div \frac{7x}{2}$ **l** $\frac{4a}{5} \div \frac{2b}{7a}$

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8 Simplify the following. $\left(\text{Recall that } 3 = \frac{3}{1}.\right)$

a $\frac{4x}{5} \times 3$

b $\frac{4x}{5} \div 3$

c $2 \div \frac{x}{5}$

d $4 \times \frac{a}{3}$

e $5 \times \frac{7}{10x}$

f $1 \div \frac{x}{2}$

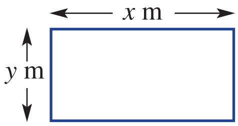
PROBLEM-SOLVING AND REASONING	9, 10, 12	9, 10, 12, 13	10, 11, 13–14

9 Helen's family goes to dinner with Tess' family. The bill comes to a total of $\$x$ and each family pays half.

a Write an algebraic fraction for the amount Helen's family pays.

b Helen says that she will pay for one-third of her family's bill. Write an algebraic fraction for the amount she pays.

10 The rectangular field below has length x metres and breadth y metres.



a Write an expression for the area of the field.

b A smaller section is fenced off. It is $\frac{1}{2}$ the length and $\frac{3}{4}$ the breadth.

i Write an expression for the length of the smaller section.

ii Write an expression for the breadth of the smaller section.

iii Hence, write an expression for the area of the smaller section.

c To find the proportion of the field that is fenced off, you can divide the fenced area by the total area. Use this to find the proportion of the field that has been fenced.

11 Write an algebraic fraction for the result of the following operations.

a A number q is halved and then the result is tripled.

b A number x is multiplied by $\frac{2}{3}$ and the result is divided by $1\frac{1}{3}$.

c The fraction $\frac{a}{b}$ is multiplied by its reciprocal $\frac{b}{a}$.

d The number x is reduced by 25% and then halved.

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12 Recall that any value x can be thought of as the fraction $\frac{x}{1}$.

- **a** Simplify $x \times \frac{1}{x}$.
- **b** Simplify $x \div \frac{1}{x}$.
- **c** Show that $x \div 3$ is equivalent to $\frac{1}{3} \times x$ by writing them both as algebraic fractions.
- **d** Simplify $\frac{a}{b} \div c$.
- **e** Simplify $a \div \frac{b}{c}$.

13 **a** Simplify each of the following expressions.

- **i** $\frac{x}{5} + \frac{x}{6}$
- **ii** $\frac{x}{5} - \frac{x}{6}$
- **iii** $\frac{x}{5} \times \frac{x}{6}$
- **iv** $\frac{x}{5} \div \frac{x}{6}$

b Which one of the expressions above will always have the same value regardless of x?

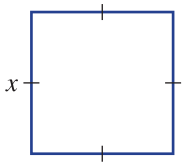
14 Assume that a and b are any two whole numbers.

- **a** Prove that $1 \div \frac{a}{b}$ is the same as the reciprocal of the fraction $\frac{a}{b}$.
- **b** Find the reciprocal of the reciprocal of $\frac{a}{b}$ by evaluating $1 \div \left(1 \div \frac{a}{b}\right)$.

ENRICHMENT	–	–	15

Irrational squares

15 Consider a square with side length x.



- **a** Write an expression for the area of the square.
- **b** The length of each side is now halved. Give an expression for the area of the new square.
- **c** If each side of the original square is multiplied by $\frac{3}{5}$, show that the resulting area is less than half the original area.
- **d** If each side of the original square is multiplied by 0.7, find an expression for the area of the square. Recall that $0.7 = \frac{7}{10}$.
- **e** Each side of the square is multiplied by some amount, which results in the square's area being halved. Try to find the amount by which they were multiplied, correct to three decimal places.

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1G Expanding brackets

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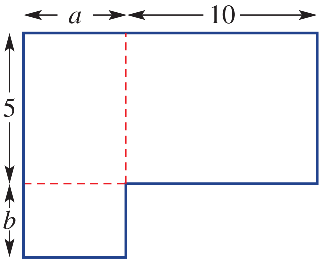
Two expressions might look quite different when in fact they are equivalent. For example, $2(3 - 7b)$ is equivalent to $4b + 6(1 - 3b)$ even though they look quite different. One use for expanding brackets is that it allows us quite easily to convert between equivalent expressions.

Let's start: Room plans

An architect has prepared floor plans for a house but some numbers are missing. Four students have attempted to describe the total area of the plans shown.



Alice says it is $5a + 50 + ab$.
Brendan says it is $5(a + 10) + ab$.
Charles says it is $a(5 + b) + 50$.
David says it is $(5 + b)(a + 10) - 10b$.

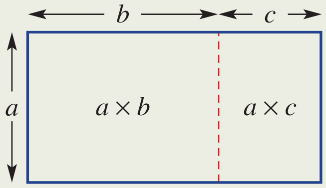
- Discuss which of the students is correct.
- How do you think each student worked out their answer?
- The architect later told them that $a = 4$ and $b = 2$. What value would each of the four students get for the area?

Key ideas

- The **distributive law** is used to rewrite an expression without the brackets.
  - $a(b + c) = a \times b + a \times c$
    $= ab + ac$
  - $a(b - c) = a \times b - a \times c$
    $= ab - ac$

  For example: $4(2x + 5) = 8x + 20$ and $3(5 - 2y) = 15 - 6y$.

- The distributive law can be illustrated by considering rectangle areas.



Area $= a(b + c)$
Area $= ab + ac$

- The distributive law is used in arithmetic. For example:

$5 \times 31 = 5 \times (30 + 1)$
$= 5 \times 30 + 5 \times 1$
$= 150 + 5$
$= 155$

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Example 12 Expanding using the distributive law

Expand the following.

a $3(2x + 5)$ **b** $-8(7 + 2y)$ **c** $4x(2x - y)$

SOLUTION

a $3(2x + 5) = 3(2x) + 3(5)$

$= 6x + 15$

b $-8(7 + 2y) = -8(7) + (-8)(2y)$

$= -56 + (-16y)$

$= -56 - 16y$

c $4x(2x - y) = 4x(2x) - 4x(y)$

$= 8x^2 - 4xy$

EXPLANATION

Distributive law: $3(2x + 5) = 3(2x) + 3(5)$

Simplify the result.

Distributive law: $-8(7 + 2y) = -8(7) + (-8(2y))$

Simplify the result.

Adding $-16y$ is the same as subtracting positive $16y$.

Distributive law: $4x(2x - y) = 4x(2x) - 4x(y)$

Simplify the result, remembering $4x \times 2x = 8x^2$.

Example 13 Expanding and combining like terms

Expand the brackets in each expression and then combine like terms.

a $3(2b + 5) + 3b$ **b** $12xy + 7x(2 - y)$

SOLUTION

a $3(2b + 5) + 3b = 3(2b) + 3(5) + 3b$

$= 6b + 15 + 3b$

$= 9b + 15$

b $12xy + 7x(2 - y)$

$= 12xy + 7x(2) - 7x(y)$

$= 12xy + 14x - 7xy$

$= 5xy + 14x$

EXPLANATION

Use the distributive law.

Simplify the result.

Combine the like terms.

Use the distributive law.

Simplify the result.

Combine the like terms.

Exercise 1G

UNDERSTANDING AND FLUENCY	1–6, 7–8(½)	3, 5, 6–8(½), 10(½)	6–10(½)

1 The area of this combined rectangle is $5(2 + x)$.

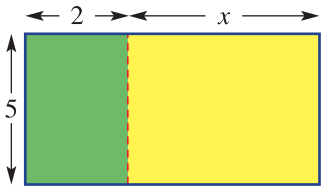
a What is the area of the green rectangle?

b What is the area of the yellow rectangle?

c Write an expression for the sum of these two areas.

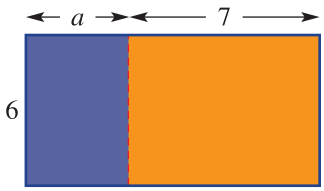
d Hence, complete the following.

The expanded form of $5(2 + x)$ is ____________.



2 The expression $3(2 + 7x)$ is equivalent to $2 + 7x + 2 + 7x + 2 + 7x$. Simplify this expression by combining like terms.

3 Consider this combined rectangle, which has an area of $6(a + 7)$.



- **a** What is the area of the purple rectangle?
- **b** What is the area of the orange rectangle?
- **c** Write an expression for the sum of these two areas.
- **d** Write the expanded form for $6(a + 7)$.

4 Which of the following is the correct expansion of $4(3x + 7)$?

A $12x + 7$ **B** $4x + 28$ **C** $3x + 28$ **D** $12x + 28$

5 Complete the following expansions.

- **a** $2(b + 5) = 2b + \square$
- **b** $3(2x + 4y) = 6x + \square y$
- **c** $5(3a - 2) = 15a - \square$
- **d** $7(4x - 2y) = 28x - \square y$

Example 12a,b

6 Use the distributive law to expand these expressions.

a $9(a + 7)$ **b** $2(2 + t)$ **c** $8(m - 10)$ **d** $3(8 - v)$
e $-5(9 + g)$ **f** $-7(5b + 4)$ **g** $-9(u + 9)$ **h** $-8(5 + h)$
i $5(6 - j)$ **j** $6(2 - m)$ **k** $3(10 - b)$ **l** $2(c - 8)$

Example 12c

7 Use the distributive law to remove the grouping symbols.

a $8z(k - h)$ **b** $6j(k + a)$ **c** $4u(r - 2q)$ **d** $2p(3c - v)$
e $m(10a + v)$ **f** $-2s(s + 5g)$ **g** $-3g(8q + g)$ **h** $-f(n + 4f)$
i $-8u(u + 10t)$ **j** $-s(t + 5s)$ **k** $m(2h - 9m)$ **l** $4a(5w - 10a)$

Example 13

8 Simplify the following by expanding and then collecting like terms.

- **a** $7(9f + 10) + 2f$
- **b** $8(2 + 5x) + 4x$
- **c** $4(2a + 8) + 7a$
- **d** $6(3v + 10) + 6v$
- **e** $7(10a + 10) + 6a$
- **f** $6(3q - 5) + 2q$
- **g** $6(4m - 5) + 8m$
- **h** $4(8 + 7m) - 6m$

9 The distributive law also allows expansion with more than two terms in the brackets, for instance $3(2x - 4y + 5) = 6x - 12y + 15$. Use this fact to simplify the following.

a $2(3x + 2y + 4z)$ **b** $7a(2 - 3b + 4y)$
c $2q(4z + 2a + 5)$ **d** $-3(2 + 4k + 2p)$
e $-5(1 + 5q - 2r)$ **f** $-7k(r + m + s)$

10 Simplify the following by expanding and then collecting like terms.

- **a** $3(3 + 5d) + 4(10d + 7)$
- **b** $10(4 + 8f) + 7(5f + 2)$
- **c** $2(9 + 10j) + 4(3j + 3)$
- **d** $2(9 + 6d) + 7(2 + 9d)$
- **e** $6(10 - 6j) + 4(10j - 5)$
- **f** $8(5 + 10g) + 3(4 - 4g)$

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PROBLEM-SOLVING AND REASONING	11, 15a,b	12, 14–16	12–15, 17, 18

11 Write an expression for each of the following and then expand it.

- **a** A number t has 4 added to it and the result is multiplied by 3.
- **b** A number u has 3 subtracted from it and the result is doubled.
- **c** A number v is doubled, and then 5 is added. The result is tripled.
- **d** A number w is tripled, and then 2 is subtracted. The result is doubled.

12 Match each operation on the left with an equivalent one on the right. (*Hint*: first convert the descriptions into algebraic expressions.)

- **a** The number x is doubled and 6 is added.
- **b** The number x is reduced by 5 and the result is doubled.
- **c** The number x is added to double the value of x.
- **d** The number x is halved, then 3 is added and the result is doubled.
- **e** 2 is subtracted from one-third of x and the result is tripled.

- **A** x is doubled and reduced by 10.
- **B** The number x is tripled.
- **C** x is decreased by 6.
- **D** x is increased by 3 and the result is doubled.
- **E** x is increased by 6.

13 The number of boys in a classroom is b and the number of girls is g. Each boy has five pencils and each girl has three pencils.

- **a** Write an expression for the total number of pencils in the class.
- **b** If the pencils cost $2 each, write and expand an expression for the total cost of all the pencils in the room.
- **c** Each boy and girl also has one pencil case, costing $4 each. Write a simplified and expanded expression for the total cost of all pencils and cases in the room.
- **d** If there are 10 boys and eight girls in the room, what is the total cost for all the pencils and cases in the room?

14
- **a** When expanded, $4(2a + 6b)$ gives $8a + 24b$. Find two other expressions that expand to give $8a + 24b$.
- **b** Give an expression that expands to $4x + 8y$.
- **c** Give an expression that expands to $12a - 8b$.
- **d** Give an expression that expands to $18ab + 12ac$.

15 The distributive law is often used in multiplication of whole numbers. For example:
$17 \times 102 = 17 \times (100 + 2) = 17(100) + 17(2) = 1734$

- **a** Use the distributive law to find the value of 9×204. Start with $9 \times 204 = 9 \times (200 + 4)$.
- **b** Use the distributive law to find the value of 204×9. Start with $204 \times 9 = 204 \times (10 - 1)$.
- **c** Given that $a \times 11 = a \times (10 + 1) = 10a + a$, find the value of these multiplications.
  i 14×11 **ii** 32×11 **iii** 57×11 **iv** 79×11
- **d** It is known that $(x + 1)(x - 1)$ expands to $x^2 - 1$. For example, if $x = 7$ this tells you that $8 \times 6 = 49 - 1 = 48$. Use this fact to find the value of:
  i 7×5 **ii** 21×19 **iii** 13×11 **iv** 201×199
- **e** Using a calculator, or otherwise, evaluate 15^2, 25^2 and 35^2. Describe how these relate to the fact that $(10n + 5)(10n + 5)$ is equivalent to $100n(n + 1) + 25$.

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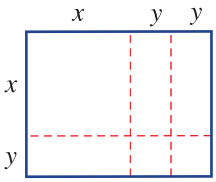
16 Prove that $4a(2 + b) + 2ab$ is equivalent to $a(6b + 4) + 4a$ by expanding both expressions.

17 Find an expanded expression for $(x + y)(x + 2y)$ by considering the diagram below. Ensure your answer is simplified by combining any like terms.



18 Prove that the following sequence of operations has the same effect as doubling a number.

1 Take a number, add 2.
2 Multiply by 6.
3 Subtract 6.
4 Multiply this result by $\frac{1}{3}$.
5 Subtract 2.

ENRICHMENT	–	–	19

Expanding algebraic fractions

19 To simplify $\frac{x+5}{3} + \frac{x}{2}$ change both fractions to have a common denominator of 6, giving $\frac{2(x+5)}{6} + \frac{3x}{6}$. Then expand to finish off the simplification: $\frac{2x+10}{6} + \frac{3x}{6} = \frac{5x+10}{6}$.

Use this method to simplify the following sums.

a $\frac{x+1}{3} + \frac{x}{2}$

b $\frac{x+5}{5} + \frac{x}{3}$

c $\frac{3x}{8} + \frac{x-1}{4}$

d $\frac{x+2}{4} + \frac{x+1}{3}$

e $\frac{2x+1}{5} + \frac{3x+1}{10}$

f $\frac{2x-1}{7} + \frac{3x+2}{5}$

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1H Factorising expressions

Factorising is the opposite procedure to expanding. It allows us to simplify expressions and solve harder mathematical problems. Because $3(2x + 5)$ expands to $6x + 15$, this means that the factorised form of $6x + 15$ is $3(2x + 5)$. The aim when factorising is to write expressions as the product of two or more factors.

$$\begin{tikzcd} \boxed{3(2x - 5)} \arrow[rr, bend left, "\text{Expanding}"] & = & \boxed{6x + 15} \arrow[ll, bend left, "\text{Factorising}"] \end{tikzcd}$$

Let's start: Expanding gaps

- Try to fill in the gaps to make the following equivalence true: ☐(☐ + ☐) = $12x + 18xy$.
- In how many ways can this be done? Try to find as many ways as possible.
- If the aim is to make the term outside the brackets as large as possible, what is the best possible solution to the puzzle?

Key ideas

- The **highest common factor** (HCF) of a set of terms is the largest factor that divides into each term. For example:
  HCF of $15x$ and $21y$ is 3.
  HCF of $10a$ and $20c$ is 10.
  HCF of $12x$ and $18xy$ is $6x$.
- To **factorise** an expression, first take the HCF of the terms outside the brackets and divide each term by it, leaving the result in brackets.
  For example:
  $10x + 15y$
  HCF = 5
  Result $5(2x + 3y)$
  HCF ↗ $10x \div 5$ ↗ $15y \div 5$ ↖
- To check your answer, expand the factorised form.
  For example: $5(2x + 3y) = 10x + 15y$ ✓

Example 14 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of:

a 20 and 35 **b** $18a$ and $24ab$ **c** $12x$ and $15x^2$

SOLUTION	EXPLANATION
a 5	5 is the largest number that divides into 20 and 35.
b $6a$	6 is the largest number that divides into 18 and 24, and a divides into both terms.
c $3x$	3 divides into both 12 and 15, and x divides into both terms.

Example 15 Factorising expressions

Factorise the following expressions.

a $6x + 15$ **b** $12a + 18ab$ **c** $21x - 14y$

d $-10a - 20$ **e** $x^2 + 4x$

SOLUTION	EXPLANATION
a $6x + 15 = 3(2x + 5)$	HCF of $6x$ and 15 is 3. $6x \div 3 = 2x$ and $15 \div 3 = 5$.
b $12a + 18ab = 6a(2 + 3b)$	HCF of $12a$ and $18ab$ is $6a$. $12a \div 6a = 2$ and $18ab \div 6a = 3b$.
c $21x - 14y = 7(3x - 2y)$	HCF of $21x$ and $14y$ is 7. $21x \div 7 = 3x$ and $14y \div 7 = 2y$. The subtraction sign is included, as in the original expression.
d $-10a - 20 = -10(a + 2)$	HCF of $-10a$ and -20 is 10 but because both terms are negative we bring -10 out the front. $-10a \div -10 = a$ and $-20 \div -10 = 2$.
e $x^2 + 4x = x(x + 4)$	HCF of x^2 and $4x$ is x. $x^2 \div x = x$ and $4x \div x = 4$.

Exercise 1H

UNDERSTANDING AND FLUENCY	1–5, 6–7(½)	4(½), 5, 6–9(½)	7, 8–9(½)

1 List the factors of:

a 20 **b** 12 **c** 15 **d** 27

2 The factors of 14 are 1, 2, 7 and 14. The factors of 26 are 1, 2, 13 and 26. What is the highest factor that 14 and 26 have in common?

3 Find the highest common factor of the following pairs of numbers.

a 12 and 18 **b** 15 and 25 **c** 40 and 60 **d** 24 and 10

4 Fill in the blanks to make these expansions correct.

a $3(4x + 1) = \square x + 3$ **b** $5(7 - 2x) = \square - 10x$

c $6(2 + 5y) = \square + \square y$ **d** $7(2a - 3b) = \square - \square$

e $3(2a + \square) = 6a + 21$ **f** $4(\square - 2y) = 12 - 8y$

g $7(\square + \square) = 14 + 7q$ **h** $\square(2x + 3y) = 8x + 12y$

5 Verify that $5x + 15$ and $5(x + 3)$ are equivalent by copying and completing the table below.

x	2	7	4	0	-3	-6
$5x + 15$						
$5(x + 3)$						

Example 14

6 Find the highest common factor (HCF) of the following pairs of terms.

a 15 and $10x$ **b** $20a$ and $12b$ **c** $27a$ and $9b$

d $7xy$ and $14x$ **e** $-2yz$ and $4xy$ **f** $11xy$ and $-33xy$

g $8qr$ and $-4r$ **h** $-3a$ and $6a^2$ **i** $14p$ and $25pq$

Example 15a

7 Factorise the following by first finding the highest common factor. Check your answers by expanding them.

a $3x + 6$ **b** $8v + 40$ **c** $15x + 35$
d $10z + 25$ **e** $40 + 4w$ **f** $5j - 20$
g $9b - 15$ **h** $12 - 16f$ **i** $5d - 30$

Example 15b,c

8 Factorise the following.

a $10cn + 12n$ **b** $24y + 8ry$
c $14jn + 10n$ **d** $24g + 20gj$
e $10h + 4z$ **f** $30u - 20n$
g $40y + 56ay$ **h** $12d + 9dz$
i $21hm - 9mx$ **j** $49u - 21bu$
k $28u - 42bu$ **l** $21p - 6c$

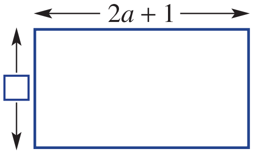
Example 15d,e

9 Find the factorisation of the following expressions.

a $-3x - 6$ **b** $-5x - 20$
c $-10a - 15$ **d** $-30p - 40$
e $x^2 + 6x$ **f** $k^2 + 4k$
g $2m^2 + 5m$ **h** $6p^2 + 3p$

PROBLEM-SOLVING AND REASONING	10, 12	10, 11, 13	11, 13, 14

10 The area of the rectangle shown is $10a + 5$. One side's measurement is unknown.



a What is the value of the unknown measurement?
b Write an expression for the perimeter of the rectangle.

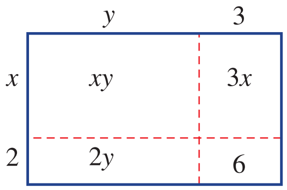
11 A group of students lines up for a photo.

They are in six rows with x students in each row. Another 18 students join the photo.

a Write an expression for the total number of students in the photo.
b Factorise the expression above.
c How many students would be in each of the six rows now? Write an expression.
d If the photographer wanted just three rows, how many students would be in each row? Write an expression.
e If the photographer wanted just two rows, how many students would be in each row? Write an expression.

12 **a** Expand $2(x + 1) + 5(x + 1)$ and simplify.
b Factorise your result.
c Make a prediction about the equivalent of $3(x + 1) + 25(x + 1)$ if it is expanded, simplified and then factorised.
d Check your prediction by expanding and factorising $3(x + 1) + 25(x + 1)$.

13 Consider the diagram shown below. What is the factorised form of $xy + 3x + 2y + 6$?



14 In English, people often convert between ‘factorised’ and ‘expanded’ sentences. For instance, ‘I like John and Mary’ is equivalent in meaning to ‘I like John and I like Mary’. The first form is factorised with the common factor that I like them. The second form is expanded.

- **a** Expand the following sentences.
  - **i** I eat fruit and vegetables.
  - **ii** Rohan likes Maths and English.
  - **iii** Petra has a computer and a television.
  - **iv** Hayden and Anthony play tennis and chess.
- **b** Factorise the following sentences.
  - **i** I like sewing and I like cooking.
  - **ii** Olivia likes ice-cream and Mary likes ice-cream.
  - **iii** Brodrick eats chocolate and Brodrick eats fruit.
  - **iv** Adrien likes chocolate and Adrien likes soft drinks, and Ben likes chocolate and Ben likes soft drinks.

ENRICHMENT	–	–	15

Factorising fractions

15 Factorising can be used to simplify algebraic fractions. For example, $\frac{5x + 10}{7x + 14}$ can be simplified by first factorising the numerator and the denominator $\frac{5\cancel{(x + 2)}}{7\cancel{(x + 2)}} = \frac{5}{7}$. Factorise and then simplify the following fractions as much as possible.

a $\frac{2x + 4}{5x + 10}$ **b** $\frac{7x - 7}{2x - 2}$ **c** $\frac{3ac + 5a}{a + 2ab}$

d $\frac{4a + 2b}{8c + 10d}$ **e** $\frac{5q - 15}{3q - 9}$ **f** $\frac{7p + 14pq}{9p + 18pq}$

g $\frac{7a - 21}{2a - 6}$ **h** $\frac{12p}{8p + 2pq}$ **i** $\frac{100 - 10x}{20 - 2x}$

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1I Applying algebra

Interactive
Widgets
HOTsheets
Walkthrough

The skills of algebra can be applied to many situations within other parts of mathematics, as well as to other fields such as engineering, sciences and economics.

Algebra can be applied to both the engineering of the ride and the price of the tickets.

Let's start: Carnival conundrum

Alwin, Bryson and Calvin have each been offered special deals for the local carnival.

- Alwin can pay $50 to go on all the rides all day.
- Bryson can pay $20 to enter the carnival and then pay $2 per ride.
- Calvin can enter the carnival at no cost and then pay $5 per ride.

- Which of them has the best deal?
- In the end, each of them decides that they were happiest with the deal they had and would not have swapped. How many rides did they each go on? Compare your different answers.

Key ideas

- Different situations can be **modelled** with algebraic expressions.
- To apply a rule, first the variables should be clearly defined. Then known values are substituted for the variables.

e.g. Total cost is

$$2 \times \overset{\nearrow}{n} + 3 \times \overset{\nwarrow}{d}$$

n = number of minutes
d = distance, in km

Example 16 Writing expressions from descriptions

Write an expression for the following situations.

a The total cost of k bottles if each bottle costs $4
b The area of a rectangle if its breadth is 2 cm more than its length and its length is x cm
c The total cost of hiring a plumber for n hours if they charge a $40 call-out fee and $70 per hour

SOLUTION	EXPLANATION
a $4 \times k = 4k$	Each bottle costs $4, so the total cost is $4 multiplied by the number of bottles purchased.
b $x \times (x + 2) = x(x + 2)$	Length = x so breadth = $x + 2$. The area is length × breadth.
c $40 + 70n$	$70 per hour means that the cost to hire the plumber would be $70 \times n$. Additionally $40 is added for the call-out fee, which is charged regardless of how long the plumber stays.

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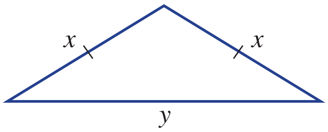
Exercise 1I

UNDERSTANDING AND FLUENCY	1–6	3–7	5–7

1 Evaluate the expression $3d + 5$ when:
a $d = 10$ **b** $d = 12$ **c** $d = 0$

2 Find the value of $30 + 10x$ when:
a $x = 2$ **b** $x = -1$ **c** $x = -3$

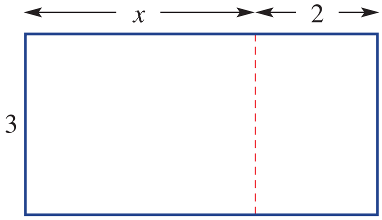
3 Consider the isosceles triangle shown.



a Write an expression for the perimeter of the triangle.
b Find the perimeter when $x = 3$ and $y = 2$.

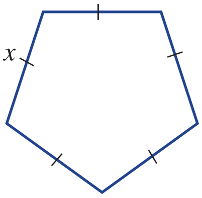
Example 16a **4** Pens cost $3 each.
a Write an expression for the total cost of n pens.
b If $n = 12$, find the total cost.

Example 16b **5** **a** Write an expression for the total area of the shape shown.
b If $x = 9$, what is the area?



Example 16c **6** An electrician charges a call-out fee of $30 and $90 per hour.
Which of the following represents the total cost for x hours?
A $x(30 + 90)$ **B** $30x + 90$ **C** $30 + 90x$ **D** $120x$

7 **a** Give an expression for the perimeter of this regular pentagon.
b If each side length is doubled, what would be the perimeter?
c If each side length is increased by 3, write a new expression for the perimeter.



PROBLEM-SOLVING AND REASONING	8, 9, 12	8–10, 12, 13	9–11, 13, 14

8 An indoor soccer pitch costs $40 per hour to hire plus a $30 booking fee.
a Write an expression for the cost of hiring the pitch for x hours.
b Hence, find the cost of hiring the pitch for a round-robin tournament that lasts 8 hours.

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9 A plumber says that the cost, in dollars, to hire her for x hours is $50 + 60x$.

- **a** What is her call-out fee?
- **b** How much does she charge per hour?
- **c** If you had \$200, what is the longest period you could hire the plumber?

10 A repairman says the cost, in dollars, to hire his services for x hours is $20(3 + 4x)$.

- **a** How much would it cost to hire him for 1 hour?
- **b** Expand the expression he has given you.
- **c** Hence, state:
  - **i** his call-out fee
  - **ii** the amount he charges per hour.

11 Three deals are available at a fair.

Deal 1: Pay \$10, rides cost \$4 each.
Deal 2: Pay \$20, rides cost \$1 each.
Deal 3: Pay \$30, all rides are free.

- **a** Write an expression for the total cost of n rides using deal 1. (The total cost includes the entry fee of \$10.)
- **b** Write an expression for the total cost of n rides using deal 2.
- **c** Write an expression for the total cost of n rides using deal 3.
- **d** Which of the three deals is best for someone going on just two rides?
- **e** Which of the three deals is best for someone going on 20 rides?
- **f** Fill in the gaps:
  - **i** Deal 1 is best for people wanting up to _____ rides.
  - **ii** Deal 2 is best for people wanting between _____ and _____ rides.
  - **iii** Deal 3 is best for people wanting more than _____ rides.

12 In a particular city, taxis charge \$4 to pick up a passenger (flag fall) and then \$2 per minute of travel. Three drivers have different ways of calculating the total fare.

Russell adds 2 to the number of minutes travelled and doubles the result.
Jessie doubles the number of minutes travelled and then adds 4.

Arash halves the number of minutes travelled, adds 1 and then quadruples the result.

- **a** Write an expression for the total cost of travelling x minutes in:
  - **i** Russell's taxi
  - **ii** Jessie's taxi
  - **iii** Arash's taxi
- **b** Prove that all three expressions are equivalent by expanding them.
- **c** A fourth driver starts by multiplying the number of minutes travelled by 4 and then adding 8. What should she do to this result to calculate the correct fare?

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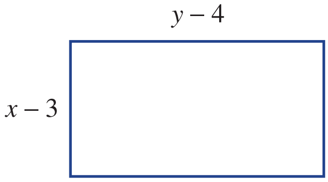
13 Roberto draws a rectangle with unknown dimensions. He notes that the area is $(x - 3)(y - 4)$.



- **a** If $x = 5$ and $y = 7$, what is the area?
- **b** What is the value of $(x - 3)(y - 4)$ if $x = 1$ and $y = 1$?
- **c** Roberto claims that this proves that if $x = 1$ and $y = 1$ then his rectangle has an area of 6. What is wrong with his claim?
  (*Hint*: try to work out the rectangle's perimeter.)

14 Tamir notes that whenever he hires an electrician, they charge a call-out fee $\$F$ and an hourly rate of $\$H$ per hour.

- **a** Write an expression for the cost of hiring an electrician for 1 hour.
- **b** Write an expression for the cost of hiring an electrician for 2 hours.
- **c** Write an expression for the cost of hiring an electrician for 30 minutes.
- **d** How much does it cost to hire an electrician for t hours?

ENRICHMENT	–	–	15

Ticket sales

15 At a carnival there are six different deals available to reward loyal customers.

Deal	Entry cost ($)	Cost per ride ($)
A	79	0
B	50	2
C	31	4
D	18	6
E	7	8
F	0	10

The queue consists of 100 customers. The first customer knows they will go on one ride, the second will go on two rides, and the pattern continues, with the 100th customer wanting to go on 100 rides. Assuming that each customer can work out their best deal, how many of each deal will be sold?

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1J Index laws for multiplication and division

Interactive
Widgets
HOTsheets
Walkthrough

Recall that x^2 means $x \times x$ and x^3 means $x \times x \times x$. **Index notation** provides a convenient way to describe repeated multiplication.

index or exponent or power

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3$$

base

Notice that $3^5 \times 3^2 = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{3^5} \times \underbrace{3 \times 3}_{3^2}$

which means that $3^5 \times 3^2 = 3^7$.

Similarly it can be shown that $2^6 \times 2^5 = 2^{11}$.

When dividing, note that:

$$\frac{5^{10}}{5^7} = \frac{5 \times 5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}$$
$$= 5 \times 5 \times 5$$

So $5^{10} \div 5^7 = 5^3$.

Let's start: Comparing powers

- Arrange these numbers from smallest to largest.
  $2^3, 3^2, 2^5, 4^3, 3^4, 2^4, 4^2, 5^2, 1^{20}$
- Did you notice any patterns?
- If all the bases were negative, how would that change your arrangement from smallest to largest? For example, 2^3 becomes $(-2)^3$.

Key ideas

- **index** or **exponent** or **power**

  $$5^n = \underbrace{5 \times 5 \times \ \dots \times 5}$$

  base The number 5 appears n times.

  For example, $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$.
- An expression such as 4×5^3 can be written in **expanded form** as $4 \times 5 \times 5 \times 5$.
- The **index law for multiplying terms** with the same base: $3^m \times 3^n = 3^{m+n}$
  For example, $3^4 \times 3^2 = 3^6$.
- The **index law for dividing terms** with the same base: $3^m \div 3^n = \dfrac{3^m}{3^n} = 3^{m-n}$
  For example, $3^8 \div 3^5 = 3^3$.

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Example 17 Using the index law for multiplication

Simplify the following using the index law for multiplication.

a $5^3 \times 5^7 \times 5^2$ **b** $5^3 \times 5^4$ **c** $6^5 \times 6 \times 6^3$

SOLUTION	EXPLANATION
a $5^3 \times 5^7 \times 5^2 = 5^{12}$	$3 + 7 + 2 = 12$, so the base 5 appears 12 times, giving 5^{12}.
b $5^3 \times 5^4 = 5^7$	$3 + 4 = 7$, so $5^3 \times 5^4 = 5^7$.
c $6^5 \times 6 \times 6^3 = 6^5 \times 6^1 \times 6^3 = 6^9$	Write 6 as 6^1, then add the powers. $5 + 1 + 3 = 9$, so the final result is 6^9.

Example 18 Using the index law for division

Simplify the following using the index law for division.

a $\dfrac{5^7}{5^3}$ **b** $\dfrac{6^{20}}{6^5}$ **c** $\dfrac{10 \times 5^6}{4 \times 5^2}$

SOLUTION

a
$$\frac{5^7}{5^3} = 5^4$$

EXPLANATION

Considering
$$\frac{5^7}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5}}$$
$$= 5^4$$
or just $7 - 3 = 4$.

b
$$\frac{6^{20}}{6^5} = 6^{15}$$

Use the second index law, so $20 - 5 = 15$.

c
$$\frac{10 \times 5^6}{4 \times 5^2} = \frac{10}{4} \times \frac{5^6}{5^2}$$

First separate the numbers into a separate fraction.

$$= \frac{5}{2} \times \frac{5^4}{1}$$

Cancel the common factor of 2 and use the second index law.

$$= \frac{5^5}{2}$$

Combine the result as a single fraction.

Exercise 1J

UNDERSTANDING AND FLUENCY	1–6, 7–8(½)	4–7, 8–9(½)	7, 8–9(½)

1 Fill in the gaps: In the expression 5^7 the base is ____ and the exponent is ____.

2 Which of the following expressions is the same as 3^5?

A 3×5 **B** $3 \times 3 \times 3 \times 3 \times 3$

C $5 \times 5 \times 5$ **D** $5 \times 5 \times 5 \times 5 \times 5$

3 Which of the following is the same as $4 \times 4 \times 4$?

A 4^4 **B** 4^3 **C** 3^4 **D** 4×3

4 **a** Calculate the value of:

i 2^2 **ii** 2^3 **iii** 2^5 **iv** 2^6

b Is $2^2 \times 2^3$ equal to 2^5 or 2^6?

5 **a** Calculate the value of:

i 5^6 **ii** 5^2 **iii** 5^3 **iv** 5^4

b Is $5^6 \div 5^2$ equal to 5^3 or 5^4?

6 **a** Write 8^3 in expanded form.

b Write 8^4 in expanded form.

c Write the result of multiplying $8^3 \times 8^4$ in expanded form.

d Which of the following is the same as $8^3 \times 8^4$?

A 8^{12} **B** 8^5 **C** 8^7 **D** 8^1

Example 17a

7 Simplify the following, giving your answers in index form.

a $4^3 \times 4^5$

b $3^{10} \times 3^2$

c $2^{10} \times 2^5 \times 2^3$

d $7^2 \times 7 \times 7^3$

Example 17b,c

8 Simplify the following using the index law for multiplication.

a $2^3 \times 2^4$ **b** $5^2 \times 5^4$

c $7^{10} \times 7^3$ **d** $3^7 \times 3^2$

e $3^2 \times 3^4 \times 3^3$ **f** $7^2 \times 7^4 \times 7^3$

g $2^2 \times 2^3 \times 2^4$ **h** $2^{10} \times 2^{12} \times 2^{14}$

i 10×10^3 **j** $6^2 \times 6$

k $5^4 \times 5^3 \times 5$ **l** $2^2 \times 2 \times 2$

Example 18

9 Simplify the following using the index law for division.

a $\frac{3^5}{3^2}$ **b** $\frac{12^{10}}{12^4}$ **c** $\frac{5^{10}}{5^3}$ **d** $\frac{2^{10}}{2}$

e $\frac{7^5}{7^3}$ **f** $\frac{3^{10}}{3^5}$ **g** $\frac{2^3 \times 5^{10}}{2^2 \times 5^4}$ **h** $\frac{2^4 \times 5^3}{2^2}$

PROBLEM-SOLVING AND REASONING	10–12	10, 12, 13	11–14

10 Tamir enters $2^{10000} \div 2^{9997}$ into his calculator and gets the error message ‘Number Overflow’ because 2^{10000} is too large.

a According to the second index law, what does $2^{10000} \div 2^{9997}$ equal? Give your final answer as a number.

b Find the value of $(5^{2000} \times 5^{2004}) \div 5^{4000}$.

c What is the value of $\frac{3^{700} \times 3^{300}}{3^{1000}}$?

11 A student tries to simplify $3^2 \times 3^4$ and gets the result 9^6.

a Use a calculator to verify this is incorrect.

b Write out $3^2 \times 3^4$ in expanded form, and explain why it is not the same as 9^6.

c Explain the mistake the student has made in attempting to apply the first index law.

12 Recall that $(-3)^2$ means $-3 \times (-3)$, so $(-3)^2 = 9$.

a Evaluate:

i $(-2)^2$

ii $(-2)^3$

iii $(-2)^4$

iv $(-2)^5$

b Complete the following generalisations.

i A negative number to an even power is _____________.

ii A negative number to an odd power is _____________.

c Given that $2^{10} = 1024$, find the value of $(-2)^{10}$.

13 **a** Use the index law for division to write $\frac{5^3}{5^3}$ in index form.

b Given that $5^3 = 125$, what is the numerical value of $\frac{5^3}{5^3}$?

c According to this, what is the value of 5^0? Check whether this is also the result your calculator gives.

d What is the value of 12^0? Check with a calculator.

14 **a** If $\frac{3^a}{3^b} = 9$, what does this tell you about the value of a and b?

b Given that $\frac{2^a}{2^b} = 8$, find the value of $\frac{5^a}{5^b}$.

ENRICHMENT – – 15–17

Simplifying expressions with non-numerical bases

15 Index laws can be applied to algebraic bases; i.e. $x^7 \times x^4 = x^{11}$ and $x^9 \div x^3 = x^6$.
Apply the index laws to each of the following to simplify each one.

a $m^3 \times m^2$

b $x^8 \times x^2$

c $x^{12} \div x^2$

d $x^{12} \div x^6$

e $a^{12} \times a^7$

f $m^3 \times m$

g $m \times m^4 \times m^2$

h $a^{10} \div a^7$

i $w^7 \times w^4 \times w$

j $x^{15} \div x^9$

16 Simplify the following.

a $\dfrac{5^3x^4y^7 \times 5^{10}xy^3}{5^2xy \times 5x^3y^7 \times 5^7}$

b $\dfrac{a^2a^4a^6a^8a^{10}}{a^1a^3a^5a^7a^9}$

c $\dfrac{ab^2c^3d^4e^5}{bc^2d^3e^4}$

d $1q \times 2q^2 \times 3q^3 \times 4q^4 \times 5q^5$

17 Simplify the following expressions involving algebraic fractions.

a $\dfrac{5^3}{2^2x^5} \times \dfrac{2^4}{5^2x}$

b $\dfrac{7a^3b^2}{2c^5} \div \dfrac{49a^2b}{2^3c^{20}}$

c $10x^3y^5 \div \dfrac{1}{-10x^4y^3}$

d $\dfrac{12x^2y^5}{21x^3} \times \dfrac{7x^4y}{3y^6}$

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1K The zero index and power of a power

Interactive
Widgets
HOTsheets
Walkthrough

Consider what the expanded form of $(5^3)^4$ would be:

$$\begin{aligned}(5^3)^4 &= 5^3 \times 5^3 \times 5^3 \times 5^3\\ &= 5 \times 5 \times 5 \quad \times \quad 5 \times 5 \times 5 \quad \times \quad 5 \times 5 \times 5 \quad \times \quad 5 \times 5 \times 5\\ &= 5^{12}\end{aligned}$$

Similarly:

$$\begin{aligned}(6^4)^2 &= 6^4 \times 6^4\\ &= 6 \times 6 \times 6 \times 6 \quad \times \quad 6 \times 6 \times 6 \times 6\\ &= 6^8\end{aligned}$$

This leads us to an index law: $(a^m)^n = a^{mn}$.

When a number (other than zero) is raised to the power of 0 the result is 1.

This can be seen by the pattern in this table.

4^4	4^3	4^2	4^1	4^0
256	64	16	4	1

÷4 ÷4 ÷4 ÷4

Let's start: How many factors?

The number 7 has two factors (1 and 7) and the number 7^2 has three factors (1, 7 and 49).

- Which of these has the most factors?

  7^5

  $7^2 \times 7^3$

  $(7^2)^3$

  $\frac{7^{10}}{7^6}$

- Which has more factors: 7^{10} or 10^7? Compare your answers with others in your class.

Key ideas

- When a number (other than zero) is raised to the power of 0 the result is 1. For example, $5^0 = 1$ and $8^0 = 1$.
- A power of a power can be simplified by multiplying indices. For example, $(6^2)^5 = 6^{10}$.
- Expressions involving powers can be expanded, so $(3 \times 5)^4 = 3^4 \times 5^4$ and $(2 \times 5)^{10} = 2^{10} \times 5^{10}$.

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Example 19 Using the index law for power of a power

Simplify the following expressions using the index laws.

a $(2^3)^5$

b $(5^2)^4 \times (5^3)^2$

SOLUTION	EXPLANATION
a $(2^3)^5 = 2^{15}$	$3 \times 5 = 15$, so we can apply the index law easily.
b $(5^2)^4 \times (5^3)^2 = 5^8 \times 5^6$	Apply the index law with $2 \times 4 = 8$ and $3 \times 2 = 6$.
$= 5^{14}$	Apply the first index law: $8 + 6 = 14$.

Example 20 Working with the zero index

Simplify the following expressions using the index laws.

a $4^0 \times 8$

b $4^0 \times 8^0$

SOLUTION	EXPLANATION
a $4^0 \times 8 = 1 \times 8$	Any number to the power of 0 equals 1, so $4^0 = 1$.
$= 8$	
b $4^0 \times 8^0 = 1 \times 1$	$4^0 = 1$ and $8^0 = 1$.
$= 1$	

Exercise 1K

UNDERSTANDING AND FLUENCY	1–6, 7–8(½)	4, 5–7(½)	5–7(½)

1 Which one of the following is equivalent to $(5^3)^2$?

A $5 \times 5 \times 5$ **B** $5^3 \times 5^3$ **C** $5^2 \times 5^2$ **D** $5 \times 3 \times 2$

2 Which of the following is equivalent to $(3x)^2$?

A $3 \times x$ **B** $3 \times x \times x$ **C** $3 \times x \times 3 \times x$ **D** $3 \times 3 \times x$

3 **a** Copy and complete the table below.

3^5	3^4	3^3	3^2	3^1	3^0
243	81				

b What is the pattern in the bottom row from one number to the next?

4 **a** Calculate the value of:

i $(4^2)^3$ **ii** 4^5 **iii** 4^6 **iv** 4^{23}

b Is $(4^2)^3$ equal to 4^5, 4^6 or 4^{23}?

Example 20a

5 Simplify the following.

a $(2^3)^4$ **b** $(3^2)^8$ **c** $(7^4)^9$

d $(6^3)^3$ **e** $(7^8)^3$ **f** $(2^5)^{10}$

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Example 20b

6 Simplify the following using the index laws.

a $(2^3)^2 \times (2^5)^3$ **b** $(5^2)^6 \times (5^3)^2$ **c** $(7^4)^2 \times (7^5)^3$

d $(3^3)^6 \times (3^2)^2$ **e** $(2^3)^2 \times (2^4)^3$ **f** $(11^2)^6 \times (11^2)^3$

g $\dfrac{(5^3)^4}{5^2}$ **h** $\dfrac{(11^7)^2}{(11^3)^2}$ **i** $\dfrac{(11^5)^3}{11^2}$

j $\dfrac{(2^2)^{10}}{(2^3)^2}$ **k** $\dfrac{3^{20}}{(3^3)^5}$ **l** $\dfrac{(11^2)^{10}}{(11^3)^6}$

Example 19

7 Simplify the following.

a 3^0 **b** 5^0 **c** $3^0 + 5^0$

d $3 + 5^0$ **e** $(3 + 5)^0$ **f** $3^0 \times 5^0$

g $(3 \times 5)^0$ **h** 3×5^0 **i** $(35 \times 53)^0$

PROBLEM-SOLVING AND REASONING	8, 12	8–10, 12, 13	9–11, 13–15

8 Find the missing value that would make the following simplifications correct.

a $(5^3)^{\square} = 5^{15}$ **b** $(2^{\square})^4 = 2^{12}$

c $(3^2)^3 \times 3^{\square} = 3^{11}$ **d** $(7^4)^{\square} \times (7^3)^2 = 7^{14}$

9 **a** Use the fact that $(5^2)^3 = 5^6$ to simplify $\left((5^2)^3\right)^4$.

b Simplify $\left((5^3)^4\right)^5$.

c Put the following numbers into ascending order. (You do not need to calculate the actual values.)

2^{100}, $(2^7)^{10}$, $\left((2^5)^6\right)^7$, $\left((2^3)^4\right)^5$

10 **a** How many zeros does each of the following numbers have?

i 10^2 **ii** 10^5 **iii** 10^6

b How many zeros does the number $(10^5 \times 10^6 \times 10^7)^3$ have?

11 **a** Simplify $x^3 \times x^4$.

b Simplify $(x^3)^4$.

c Find the two values of x that make $x^3 \times x^4$ and $(x^3)^4$ equal.

12 For this question you will be demonstrating why a^0 should equal 1 for any value of a other than zero.

a State the value of $\dfrac{5^2}{5^2}$.

b Use the index law for division to write $\dfrac{5^2}{5^2}$ as a power of 5.

c Use this method to demonstrate that 3^0 should equal 1.

d Use this method to demonstrate that 100^0 should equal 1.

e Explain why you cannot use this method to show that 0^0 should equal 1.

13 Rose is using her calculator and notices that $(2^3)^4 = (2^6)^2$.

a Explain why this is the case.

b Which of the following are also equal to $(2^3)^4$?

A $(2^4)^3$ **B** $(2^2)^6$ **C** $(4^2)^3$ **D** $(4^3)^2 \times (6^2)^2$

c Freddy claims that $(2^5)^6$ can be written in the form $(4^{\square})^{\square}$. Find one way to fill in the two missing values.

14 **a** According to the index laws, what number is equal to $(9^{0.5})^2$?
  b What positive number makes the equation $x^2 = 9$ true?
  c What should $9^{0.5}$ equal according to this? Check using a calculator.
  d Use this observation to predict the value of $36^{0.5}$.

15 Alexis notices that $\dfrac{(a^3)^2 \times a^4}{(a^2)^5}$ is always equal to 1, regardless of the value of a.
  a Simplify the expression above.
  b Give an example of two other expressions that will always equal 1 because of the index laws.

ENRICHMENT	–	–	16–18

Simplifying expressions with non-numerical bases

16 Use your knowledge of index laws to simplify the following.
  a x^0
  b $5 + 5^0$
  c $(a^7)^2$
  d $(2w^6)^2$
  e m^0
  f $7a^0$
  g $(a^7)^2$
  h $(3a^7)^2$
  i $(3a)^0$
  j $3a^0$

17 Simplify the following expressions, which have non-numerical bases.
  a $(x^7)^2$
  b $(m^8)^3$
  c $(x^2)^5$
  d $(a^7)^2$
  e $(m^8)^2$
  f $(m^6n^2)^2$
  g $(2a^7)^3$
  h $(5m^4)^2$
  i $(6x^4)^2$
  j $(6a^2b^5)^3$

18 Simplify the following using the index laws.
  a $\dfrac{(5x^2)^3 \times (5x^3)^4}{(5x^6)^3}$
  b $\dfrac{(x^2)^4}{x^3} \times \dfrac{x^7}{(x^2)^2}$
  c $\dfrac{(x^2y^3)^4 \times (x^3y^2)^5}{(xy)^7 \times (x^2y)^6}$
  d $\dfrac{(a^2b^3c^4)^{10}}{a^{10}b^{20}c^{30}} \div \dfrac{a^3}{b^2}$
  e $\dfrac{(x^{20}y^{10})^5}{(x^{10}y^{20})^2}$

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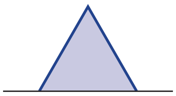
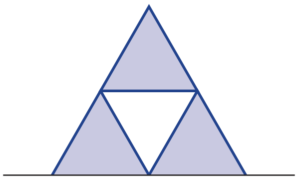
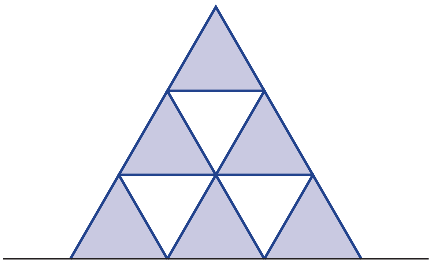
Card pyramids

Using a deck of playing cards, build some pyramids on your desk like the ones illustrated below.

		
Pyramid 1 (One-triangle pyramid) (Two cards)	Pyramid 2 (Three-triangle pyramids) (Seven cards)	Pyramid 3 (Six-triangle pyramids) (Fifteen cards)

1 Copy and complete this table.

Number of triangle pyramids on base	1	2	3	5			
Total number of triangle pyramids	1	3				45	55
Total number of cards required	2		15		100		

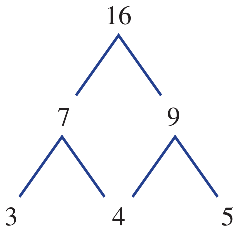
2 Describe the number of pyramids in, and the number of cards required for, pyramid 20 (20 pyramids on the base). How did you get your answer?

3 If you had 10 decks of playing cards, what is the largest tower you could make? Describe how you obtained your answer.

Number pyramids

Number pyramids with a base of three consecutive numbers

1 Can you explain how this number pyramid is constructed?



2 Draw a similar number pyramid starting with the number 4 on the left of the base.

3 Draw a similar number pyramid that has 44 as its top number. Remember the base of the pyramid must be consecutive numbers.

4 Can you draw a similar number pyramid that has 48 as its top number? Explain your answer.

5 Draw several of these pyramids to investigate how the top number is related to the starting value.
- Set up a table showing starting values and top numbers.
- Can you work out an algebraic rule that calculates the top number when given the starting number?

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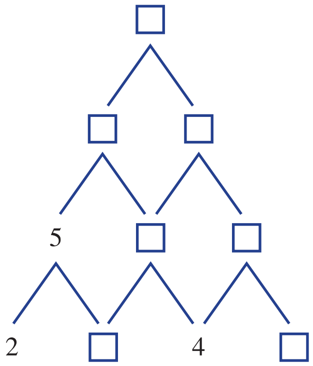
6 Draw a number pyramid that has a base row of n, $n + 1$ and $n + 2$. What is the algebraic expression for the top number? Check this formula using some other number pyramids.

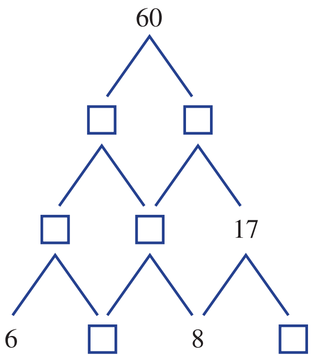
7 What is the sum of all the numbers in a pyramid with base row -10, -9, -8?

8 Determine an expression for the sum of all the numbers in a pyramid starting with n on the base.

Number pyramids with four consecutive numbers on the base

1 Copy and complete the following number pyramids.





2 Investigate how the top number is related to the starting number. Can you show this relationship using algebra?

3 Write the sequence of all the possible top numbers less than 100.

4 What patterns can you see in this sequence of top numbers? Can you find some way of showing these patterns using algebraic expressions? Let the bottom row start with n. (In the examples above, $n = 6$ and $n = 2$.)

Number pyramids with many consecutive numbers on the base

1 Determine the algebraic rule for the value of the top number for a pyramid with a base of six consecutive numbers starting with n.

2 List the algebraic expressions for the first number of each row for several different-sized pyramids all starting with n. What patterns can you see occurring in these expressions for:
- the coefficients of n?
- the constants?

3 What is the top number in a pyramid with a base of 40 consecutive numbers, starting with 5?

4 Write an expression for the top number if the base has 1001 consecutive numbers, starting with n.

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Puzzles and challenges

1 Five consecutive even integers have $2m + 2$ as the middle integer. Find two simplified equivalent expressions for the sum of these five integers.

2 Rearrange the order of the five expressions $4(a + 1)$, $6a - 5$, $2 - a$, $a - 7$, $6 - 2a$ so that the sum of the first three expressions and the sum of the last three expressions are both equal to $3(a + 1)$.

3 Write this list 16^{1000}, 8^{1334}, 4^{1999}, 2^{4001} in ascending order.

4 Find the largest value.

- **a** If m can be any number, what is the largest value that $10 - m(m + 5)$ could have?
- **b** If $x + y$ evaluates to 15, what is the largest value that $x \times y$ could have?
- **c** If a and b are chosen so that $a^2 + b^2$ is equal to $(a + b)^2$, what is the largest value of $a \times b$?

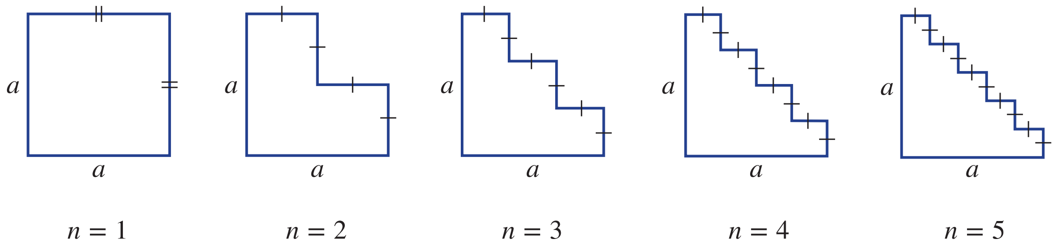
5 Simplify these algebraic expressions.

- **a** $\frac{a}{5} + \frac{a + 1}{6} - \frac{a}{2}$
- **b** $\frac{x - 1}{3} - \frac{2x - 3}{7} + \frac{x}{6}$

6 The following three expressions all evaluate to numbers between 1 and 100, but most calculators cannot evaluate them. Find their values using the index laws.

- **a** $\frac{2^{1001} \times 2^{2002}}{(2^{150})^{20}}$
- **b** $\frac{5^{1000} \times 3^{1001}}{15^{999}}$
- **c** $\frac{8^{50} \times 4^{100} \times 2^{200}}{(2^{250})^2 \times 2^{48}}$

7 Consider the following pattern.



The perimeter for the shape when $n = 1$ is given by the expression $4a$ and the area is a^2.

- **a** Give expressions for the perimeter and area of the other shapes shown above and try to find a pattern.
- **b** If $a = 6$ and $n = 1000$, state the perimeter and give the approximate area.

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Algebraic techniques 2 and indices

Pronumeral: a letter that stands for one or more numbers

Language

+	−	×	÷
sum more than added increased	difference less than minus decreased	product times double (2×) twice (2×) triple (3×)	quotient divide one-third one-half quarter

× ÷ not used in algebraic expressions

an expression

terms

$5x$ $-7xy$ -8

5 is the coefficient of x

constant term is −8

−7 is the coefficient of xy

Algebraic terms

Concise form		Expanded form
$5x^2y$	→	$5xxy$
$-2ab^2$	→	$-2abb$
$7p^2q^3$	→	$7ppqqq$
$-\frac{4ab^2}{5c^3}$	→	$-\frac{4abb}{5ccc}$

Like terms

Variables have identical expanded form.

$6a^2m \longrightarrow 6aam$

$-2a^2m \longrightarrow -2aam$

$5ma^2 \longrightarrow 5maa = 5aam$

$6a^2m, -2a^2m$ and $5ma^2$ are **like terms**

$ab = ba$

Adding and subtracting like terms

- Count 'how many'.
- Don't change variables.

$\boxed{-6a}\ \boxed{+4a^2}\ \boxed{-3}\ \boxed{+9a^2}\ \boxed{-12a}$

sign in front belongs to term

$= -6a - 12a + 4a^2 + 9a^2 - 3$

$= -18a + 13a^2 - 3$

a stays the same

a^2 stays the same

Expanding brackets

$3(x + y) = x + y + x + y + x + y$

$= 3x + 3y$

Distributive law

$a(b + c) = ab + ac$

$5(2a + m) = 5(2a) + 5(m)$

$= 10a + 5m$

$7(k - 3a) = 7(k) - 7(3a)$

$= 7k - 21a$

$a(b - c) = ab - ac$

$5(2a - 3) - 7(4 + a)$

$= 5(2a) - 5(3) - 7(4) + -7(a)$

$= 10a - 15 - 28 - 7a$

$= 3a - 43$

Factorising

$12x + 6a$ (HCF = 6)

$= 6 \times 2x + 6 \times a$

$= 6(2x + a)$

$12a^2m + 8am^2$

$= 12aam + 8amm$ HCF $= 4am$

$= 4am \times 3a + 4am \times 2m$

$= 4am(3a + 2m)$

Substitution

'evaluate' 'substitute'

replace variables with numbers and calculate answer

$\frac{7a}{4} + 3(a + b) + 4b^2$

$a = 8$ $b = 2$

$= \frac{7 \times 8}{4} + 3 \times (8 + 2) + 4 \times 2 \times 2$

$= 14 + 30 + 16$

$= 60$

Equivalent expressions

	$7 - 3x$	$2 - 3x + 5$
$x = 2$	$7 - 6 = 1$	$2 - 6 + 5 = 1$
$x = 10$	$7 - 30 = -23$	$2 - 30 + 5 = -23$

$7 - 3x = 2 - 3x + 5$

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Algebraic techniques 2 and indices

Algebraic fractions

Adding and subtracting

$$\frac{3a}{2}+\frac{7m}{5}=\frac{3a\times 5}{2\times 5}+\frac{7m\times 2}{5\times 2}$$
$$=\frac{15a}{10}+\frac{14m}{10}$$
$$=\frac{15a+14m}{10}$$

'unit' is tenths

Multiplying

$$\frac{7\overset{1}{\cancel{a}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{4}{\cancel{12}}m}{\cancel{a}}$$
$$=28m$$
$$\frac{a}{a}=1$$

Dividing

$$\frac{3a}{4}\div\frac{2}{3}$$
$$=\frac{3a}{4}\times\frac{3}{2}$$
$$=\frac{9a}{8}$$

$$\frac{15x}{y}\div 20x$$
$$=\frac{15x}{y}\times\frac{1}{20x}$$
$$=\frac{\overset{3}{\cancel{15}}\overset{1}{\cancel{x}}}{y}\times\frac{1}{\underset{4}{\cancel{20}}\underset{1}{\cancel{x}}}$$
$$=\frac{3}{4y}$$

$$a\div a=a\times\frac{1}{a}=\frac{a}{a}=1$$

Index notation

exponent or power or index

$$\underset{\text{base}}{5}^{3}=\underbrace{5\times 5\times 5}_{\text{5 appears 3 times}}$$

Index laws

1. $5^m\times 5^n=5^{m+n}$
2. $5^m\div 5^n=\dfrac{5^m}{5^n}=5^{m-n}$
3. $(5^m)^n=5^{mn}$

Examples

1. $3^2\times 3^5=3^7$
2. $\dfrac{12^5}{12^2}=12^3$
3. $(2^3)^4=2^{12}$

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Multiple-choice questions

1 Consider the expression $5a^2 - 3b + 8$. Which one of the following statements is true?

A The coefficient of a is 5.
B It has five terms.
C The constant term is 8.
D The coefficient of b is 5.
E The coefficient of a^2 is 10.

2 Half the sum of double x and 3 can be written as:

A $\frac{1}{2} \times 2x + 3$ **B** $\frac{2x + 6}{2}$ **C** $x + 6$ **D** $\frac{2x + 3}{2}$ **E** $\frac{2(x - 3)}{2}$

3 If $n + 2$ is an odd integer, the next odd integer can be written as:

A $n + 3$ **B** $n + 4$ **C** $n + 5$ **D** $n + 1$ **E** n

4 Find the value of $5 - 4a^2$ given $a = 2$.

A 3 **B** -3 **C** 21 **D** -11 **E** 13

5 $3 \times x \times y$ is equivalent to:

A $3x + y$ **B** xy **C** $3 + x + y$ **D** $3x + 3y$ **E** $xy + 2xy$

6 $\frac{12ab}{24a^2}$ can be simplified to:

A $2ab$ **B** $\frac{2a}{b}$ **C** $\frac{b}{2a}$ **D** $\frac{ab}{2}$ **E** $\frac{b}{2}$

7 The expanded form of $2x(3 + 5y)$ is:

A $6x + 5y$ **B** $3x + 5y$ **C** $6x + 5xy$ **D** $6 + 10y$ **E** $6x + 10xy$

8 Simplifying $3a \div 6b$ gives:

A 2 **B** $\frac{a}{b}$ **C** $\frac{2a}{b}$ **D** $\frac{ab}{2}$ **E** $\frac{a}{2b}$

9 $5^7 \times 5^4$ is equal to:

A 25^{11} **B** 5^{28} **C** 25^3 **D** 5^3 **E** 5^{11}

10 The factorised form of $3a^2 - 6ab$ is:

A $3a^2(1 - 2b)$ **B** $3a(a - 2b)$ **C** $3a(a - b)$ **D** $6a(a - b)$ **E** $3(a^2 - 2ab)$

Short-answer questions

1 State whether each of the following is true or false.

a The constant term in the expression $5x + 7$ is 5.
b $16xy$ and $5yx$ are like terms.
c The coefficient of d in the expression $6d^2 + 7d + 8abd + 3$ is 7.
d The highest common factor of $12abc$ and $16c$ is $2c$.
e The coefficient of xy in the expression $3x + 2y$ is 0.

2 For the expression $6xy + 2x - 4y^2 + 3$, state:

a the coefficient of x
b the constant term
c the number of terms
d the coefficient of xy

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3 Substitute the following values of a to evaluate the expression $8 - a + 2a^2$.

a -1 **b** 2 **c** -3 **d** 0

4 Substitute $x = 2$ and $y = -3$ into each of the following.

a $2y + 3$ **b** $3x + y$ **c** $xy + y$

d $4x - 2y$ **e** $\frac{5xy}{6} + 2$ **f** $\frac{-3x + 2y}{x + y}$

5 For what value of x is $3 - x$ equal to $x - 3$?

6 Simplify each of these expressions by collecting like terms.

a $7m + 9m$ **b** $3a + 5b - a$ **c** $x^2 - x + x^2 + 1$

d $5x + 3y + 2x + 4y$ **e** $7x - 4x^2 + 5x^2 + 2x$ **f** $-8m + 7m + 6n - 18n$

7 Simplify:

a $9a \times 4b$

b $30 \times x \times y \div 2$

c $-8x \times 4y \div (-2)$

8 Copy and complete the following equivalences.

a $3x + 4y = 8x - \square + 4y$ **b** $3ab \times \square = -12abc$

c $\frac{3x}{4} = \frac{\square}{20}$ **d** $9a^2b \div \square = 3a$

9 Express each of the following in their simplest form.

a $\frac{5x}{12} - \frac{x}{6}$ **b** $\frac{2a}{5} + \frac{b}{15}$ **c** $\frac{6x}{5} \times \frac{15}{2x^2}$ **d** $\frac{4a}{7} \div \frac{8a}{21b}$

10 Expand and simplify when necessary.

a $3(x - 4)$ **b** $-2(5 + x)$ **c** $k(3l - 4m)$ **d** $2(x - 3y) + 5x$

e $7 - 3(x - 2)$ **f** $10(1 - 2x)$ **g** $4(3x - 2) + 2(3x + 5)$

11 Factorise fully.

a $2x + 6$ **b** $24 - 16g$ **c** $12x + 3xy$ **d** $7a^2 + 14ab$

12 By factorising first, simplify the following fractions.

a $\frac{5a + 10}{5}$ **b** $\frac{12x - 24}{x - 2}$ **c** $\frac{16p}{64p + 48pq}$

13 Find the missing values.

a $7^5 \times 7^2 = 7^{\square}$ **b** $5^4 \div 5 = 5^{\square}$ **c** $(3^2)^3 = 3^{\square}$ **d** $3^0 = \square$

14 Use the index laws to simplify each of the following expressions.

a $5^2 \times 5^5$ **b** $6^7 \times 6$ **c** $\frac{7^5}{7^3}$

d $\frac{5^6}{5^5}$ **e** $(2^3)^4$ **f** $(6^2)^3$

15 Simplify:

a 6×5^0 **b** $(6 \times 5)^0$ **c** $6^0 \times 5$

Extended-response questions

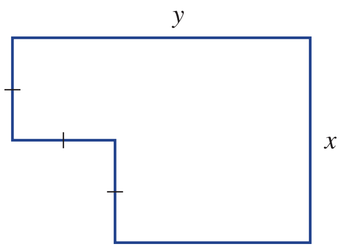
1 Two bus companies have different pricing structures.

Company A: \$120 call-out fee, plus \$80 per hour

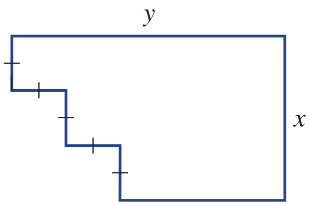
Company B: \$80 call-out fee, plus \$100 per hour

- **a** Write an expression for the total cost \$$A$ of travelling n hours with company A.
- **b** Write an expression for the total cost \$$B$ of travelling for n hours with company B.
- **c** Hence, state the cost of travelling for three hours with each company.
- **d** For how long would you need to hire a bus to make company A the cheaper option?
- **e** A school principal cannot decide which bus company to choose and hires three buses from company A and two buses from company B. Give an expanded expression to find the total cost for the school to hire the five buses for n hours.
- **f** If the trip lasts for five hours, how much does it cost to hire the five buses for this period of time?

2 Consider the floor plan shown.



- **a** Write an expanded expression for the floor's area in terms of x and y.
- **b** Hence, find the floor's area if $x = 6$ metres and $y = 7$ metres.
- **c** Write an expression for the floor's perimeter in terms of x and y.
- **d** Hence, find the floor's perimeter if $x = 6$ metres and $y = 7$ metres.
- **e** Another floor plan is shown below. Write an expression for the floor's area and an expression for its perimeter.



- **f** **i** By how much does the area differ in the two floor plans?
  - **ii** By how much does the perimeter differ in the two floor plans?

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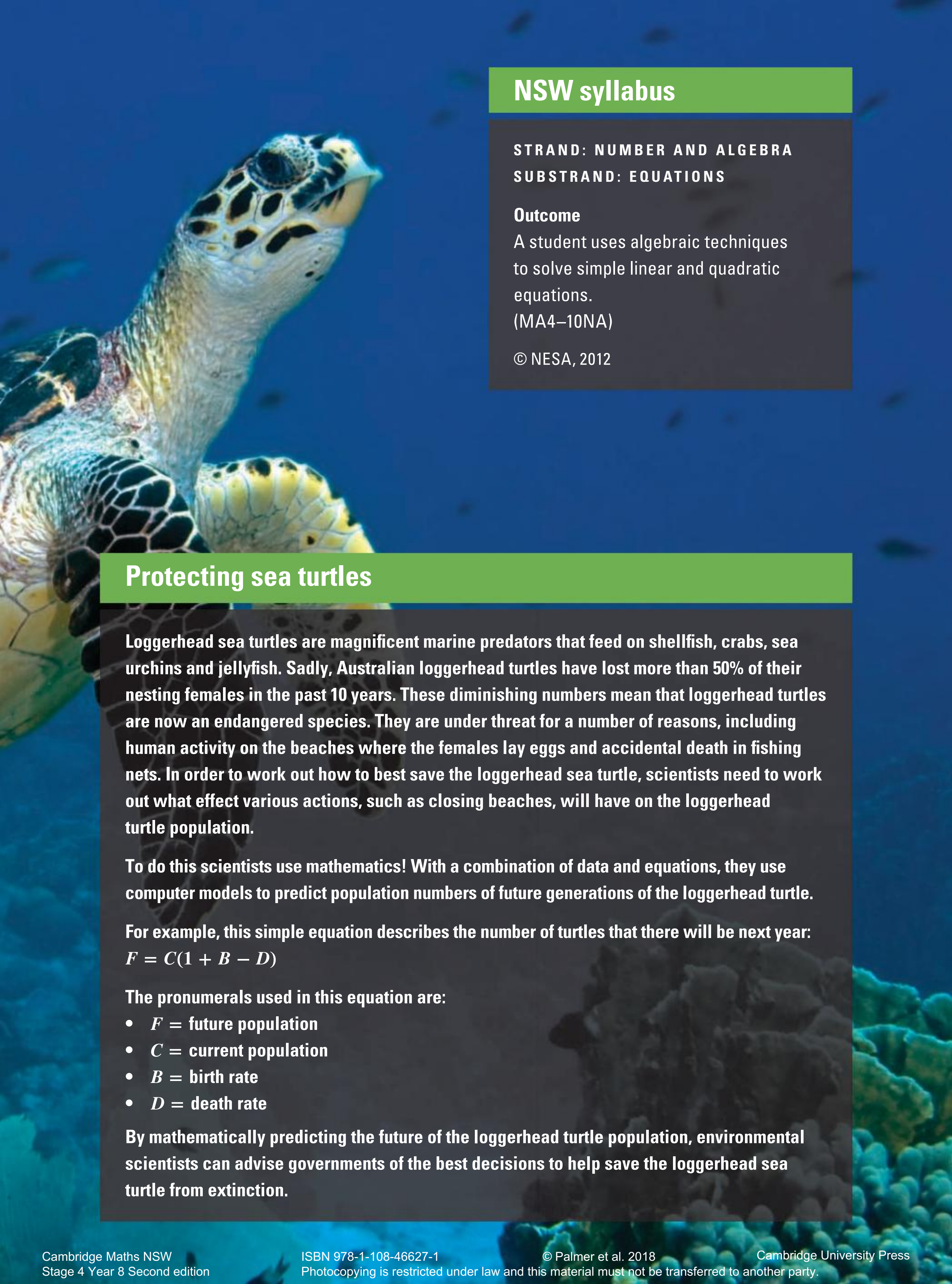
Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

2

Equations 2

What you will learn



NSW syllabus

STRAND: NUMBER AND ALGEBRA

SUBSTRAND: EQUATIONS

Outcome

A student uses algebraic techniques to solve simple linear and quadratic equations.
(MA4–10NA)

Protecting sea turtles

Loggerhead sea turtles are magnificent marine predators that feed on shellfish, crabs, sea urchins and jellyfish. Sadly, Australian loggerhead turtles have lost more than 50% of their nesting females in the past 10 years. These diminishing numbers mean that loggerhead turtles are now an endangered species. They are under threat for a number of reasons, including human activity on the beaches where the females lay eggs and accidental death in fishing nets. In order to work out how to best save the loggerhead sea turtle, scientists need to work out what effect various actions, such as closing beaches, will have on the loggerhead turtle population.

To do this scientists use mathematics! With a combination of data and equations, they use computer models to predict population numbers of future generations of the loggerhead turtle.

For example, this simple equation describes the number of turtles that there will be next year:
$F = C(1 + B - D)$

The pronumerals used in this equation are:
- $F =$ future population
- $C =$ current population
- $B =$ birth rate
- $D =$ death rate

By mathematically predicting the future of the loggerhead turtle population, environmental scientists can advise governments of the best decisions to help save the loggerhead sea turtle from extinction.

Pre-test

1 Simplify these algebraic expressions.

a $9m + 2m$
b $4a - 3a$
c $7n + 3n - n$
d $8a + 2a - 10$
e $4x - 7x$
f $4p \times 3q$

2 Expand these algebraic expressions using the distributive law.

a $3(m + 4)$
b $2(a + b)$
c $3(x + 7)$
d $-2(x - 3)$

3 Expand and simplify.

a $3(m + 5) + 2m$
b $7(2x + 3) + 3x$

4 Simplify:

a $m + 3 - m$
b $a + 6 - 6$
c $3x \div 3$
d $-m \times (-1)$

5 I think of a number, double it, and then add 3 to get 27. What is the number?

6 Find the missing number to make the following equations true.

a $4 + \square = 12$
b $6 \times \square = 24$
c $6 - \square = 8$

7 Complete each statement.

a $6 + \square = 0$
b $8 \times \square = 1$
c $4a \div \square = a$
d $\frac{x}{3} \times \square = x$

8 Solve each of the following equations by inspection or using guess and check.

a $x + 8 = 12$
b $4x = 32$
c $m - 6 = -2$
d $3m = 18$

9 Copy and complete this working to find the solution.

a

$$\begin{array}{ccc} & 2x - 1 = 15 & \\ +1 \downarrow & & \downarrow +1 \\ & 2x = \underline{\qquad} & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & x = \underline{\qquad} & \end{array}$$

b

$$\begin{array}{ccc} & \dfrac{x + 1}{3} = 5 & \\ \times 3 \downarrow & & \downarrow \times 3 \\ & x + 1 = \underline{\qquad} & \\ \downarrow & & \downarrow \\ & x = \underline{\qquad} & \end{array}$$

c

$$\begin{array}{ccc} & \dfrac{5x}{2} - 3 = 7 & \\ +3 \downarrow & & \downarrow +3 \\ & \dfrac{5x}{2} = \underline{\qquad} & \\ \downarrow & & \downarrow \\ & 5x = \underline{\qquad} & \\ \downarrow & & \downarrow \\ & x = \underline{\qquad} & \end{array}$$

d

$$\begin{array}{ccc} & 2(x + 2) = 8 & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & x + 2 = \underline{\qquad} & \\ \downarrow & & \downarrow \\ & x = \underline{\qquad} & \end{array}$$

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2A Reviewing equations REVISION

Interactive
Widgets
HOTsheets
Walkthrough

An equation is a statement that two things are equal, such as:

$2 + 2 = 4$
$7 \times 5 = 30 + 5$
$4 + x = 10 + y$

It consists of two expressions separated by the equals sign (=), and it is considered true if the left-hand side and right-hand side are equal. True equations include $7 + 10 = 20 - 3$ and $8 = 4 + 4$; examples of false equations are $2 + 2 = 7$ and $10 \times 5 = 13$.

If an equation has a pronumeral in it, such as $3 + x = 7$, then a **solution** to the equation is a value to substitute for the pronumeral to form a true equation. In this case, a solution is $x = 4$ because $3 + 4 = 7$ is a true equation.

Let's start: Solving the equations

- Find a number that would make the equation $25 = b \times (10 - b)$ true.
- How can you prove that this value is a solution?
- Try to find a solution to the equation $11 \times b = 11 + b$.

Key ideas

- An **equation** is a mathematical statement that two expressions are equal, such as $4 + x = 32$. It could be true (e.g. $4 + 28 = 32$) or false (e.g. $4 + 29 = 32$).
- A pronumeral in an equation is sometimes called an **unknown**.
- An equation has a left-hand side (LHS) and a right-hand side (RHS).
- A **solution** to an equation is a value that makes an equation true. The process of finding a solution is called **solving**. In an equation with a pronumeral, the pronumeral is sometimes called an **unknown**.
- An equation could have no solutions or it could have one or more solutions.

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Example 1 Classifying equations as true or false

For each of the following equations, state whether they are true or false.

a $3 + 8 = 15 - 4$
b $7 \times 3 = 20 + 5$
c $x + 20 = 3 \times x$, if $x = 10$

SOLUTION	EXPLANATION
a True	Left-hand side (LHS) is $3 + 8$, which is 11. Right-hand side (RHS) is $15 - 4$, which is also 11. Since LHS equals RHS, the equation is true.
b False	$\text{LHS} = 7 \times 3 = 21$ $\text{RHS} = 20 + 5 = 25$ Since LHS and RHS are different, the equation is false.
c True	If $x = 10$ then $\text{LHS} = 10 + 20 = 30$. If $x = 10$ then $\text{RHS} = 3 \times 10 = 30$. LHS equals RHS, so the equation is true.

Example 2 Stating a solution to an equation

State a solution to each of the following equations.

a $4 + x = 25$
b $5y = 45$
c $26 = 3z + 5$

SOLUTION	EXPLANATION
a $x = 21$	We need to find a value of x that makes the equation true. Since $4 + 21 = 25$ is a true equation, $x = 21$ is a solution.
b $y = 9$	If $y = 9$ then $5y = 5 \times 9 = 45$, so the equation is true.
c $z = 7$	If $z = 7$ then $3z + 5 = 3 \times 7 + 5$ $= 21 + 5$ $= 26$ Note: The fact that z is on the right-hand side of the equation does not change the procedure.

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Example 3 Writing equations from a description

Write equations for the following scenarios.

a The number k is doubled, then 3 is added and the result is 52.

b Akira works n hours, earning \$12 per hour. The total she earned was \$156.

SOLUTION	EXPLANATION
a $2k + 3 = 52$	The number k is doubled, giving $k \times 2$. This is the same as $2k$. Since 3 is added, the left-hand side is $2k + 3$, which must be equal to 52 according to the description.
b $12n = 156$	If Akira works n hours at \$12 per hour, the total amount earned is $12 \times n$, or $12n$.

Exercise 2A REVISION

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 8(½)	3, 4, 5–6(½), 7, 8(½)	6–8(½)

Example 1a,b

1 Classify these equations as true or false.

a $5 \times 3 = 15$
b $7 + 2 = 12 + 3$
c $5 + 3 = 16 \div 2$
d $8 - 6 = 6$
e $4 \times 3 = 12 \times 1$
f $2 = 8 - 3 - 3$

2 If the value of x is 3, what is the value of the following?

a $10 + x$
b $3x$
c $5 - x$
d $6 \div x$

3 State the value of the missing number to make the following equations true.

a $5 + \square = 12$
b $10 \times \square = 90$
c $\square - 3 = 12$
d $3 + 5 = \square$

4 Consider the equation $15 + 2x = x \times x$.

a If $x = 5$, find the value of $15 + 2x$.
b If $x = 5$, find the value of $x \times x$.
c Is $x = 5$ a solution to the equation $15 + 2x = x \times x$?
d Give an example of another equation that has $x = 5$ as a solution.

Example 1c

5 If $x = 2$, state whether the following equations are true or false.

a $7x = 8 + 3x$
b $10 - x = 4x$
c $3x = 5 - x$
d $x + 4 = 5x$
e $10x = 40 \div x$
f $12x + 2 = 15x$

6 If $a = 3$, state whether the following equations are true or false.

a $7 + a = 10$
b $2a + 4 = 12$
c $8 - a = 5$
d $4a - 3 = 9$
e $7a + 2 = 8a$
f $a = 6 - a$

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7 Someone has attempted to solve the following equations. State whether the solution is correct (C) or incorrect (I).

a $5 + 2x = 4x - 1$, proposed solution: $x = 3$
b $4 + q = 3 + 2q$, proposed solution: $q = 10$
c $13 - 2a = a + 1$, proposed solution: $a = 4$
d $b \times (b + 3) = 4$, proposed solution, $b = -4$

Example 2

8 State a solution to each of the following equations.

a $5 + x = 12$
b $3 = x - 10$
c $4v + 2 = 14$
d $17 = p - 2$
e $10x = 20$
f $16 - x = x$
g $4u + 1 = 29$
h $7k = 77$
i $3 + a = 2a$

PROBLEM-SOLVING AND REASONING	9, 10, 14	9–12, 14, 15	11–14, 16

Example 3

9 Write equations for each of the following problems. You do not need to solve the equations.

a A number x is doubled and then 7 is added. The result is 10.
b The sum of x and half of x is 12.
c Aston's age is a. His father, who is 25 years older, is twice as old as Aston.
d Fel's height is h cm and her brother Pat is 30 cm taller. Pat's height is 147 cm.
e Coffee costs \$$c$ per cup and tea costs \$$t$. Four cups of coffee and three cups of tea cost a total of \$21.
f Chairs cost \$$c$ each. To purchase eight chairs and a \$2000 table costs a total of \$3600.

10 Find the value of the number in the following statements.

a A number is tripled to obtain the result 21.
b Half of a number is 21.
c Six less than a number is 7.
d A number is doubled and the result is −16.
e Three-quarters of a number is 30.
f Six more than a number is −7.

11 Berkeley buys x kg of oranges at \$3.20 per kg. He spends a total of \$9.60.

a Write an equation involving x to describe this situation.
b State a solution to this equation.

12 Emily's age in 10 years' time will be triple her current age. She is currently E years old.

a Write an equation involving E to describe this situation.
b Find a solution to this equation.
c How old is Emily now?
d How many years will she have to wait until she is four times her current age?

13 Find two possible values of t that make the equation $t(10 - t) = 21$ true.

14 **a** Why is $x = 3$ a solution to $x^2 = 9$?
b Why is $x = -3$ a solution to $x^2 = 9$?
c Find the two solutions to $x^2 = 64$. (*Hint*: one is negative.)
d Explain why $x^2 = 0$ has only one solution but $x^2 = 1$ has two.
e Explain why $x^2 = -9$ has no solutions. (*Hint*: consider positive and negative multiplication.)

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15 **a** Explain why the equation $x + 3 = x$ has no solutions.

b Explain why the equation $x + 2 = 2 + x$ is true, regardless of the value of x.

c Show that the equation $x + 3 = 10$ is sometimes true and sometimes false.

d Classify the following equations as always true (A), sometimes true (S) or never true (N).

- **i** $x + 2 = 10$
- **ii** $5 - q = q$
- **iii** $5 + y = y$
- **iv** $10 + b = 10$
- **v** $2 \times b = b + b$
- **vi** $3 - c = 10$
- **vii** $3 + 2z = 2z + 1$
- **viii** $10p = p$
- **ix** $2 + b + b = (b + 1) \times 2$

e Give a new example of another equation that is always true.

16 **a** The equation $p \times (p + 2) = 3$ has two solutions. State the two solutions. (*Hint*: one of them is negative.)

b How many solutions are there for the equation $p + (p + 2) = 3$?

c Try to find an equation that has three solutions.

ENRICHMENT	–	–	17, 18

More than one unknown

17 **a** There are six equations in the square below. Find the values of a, b, c, d and e to make all six equations true.

a	$+$	12	$=$	22
$\times$		$\div$		$-$
2	$\times$	b	$=$	c
$=$		$=$		$=$
d	$\div$	e	$=$	10

b Find the value of f that makes the equation $a \times b \times e = c \times d \times f$ true.

18 For each of the following pairs of equations, find values of c and d that make both equations true. More than one answer may be possible.

- **a** $c + d = 10$ and $cd = 24$
- **b** $c - d = 8$ and $c + d = 14$
- **c** $c \div d = 4$ and $c + d = 30$
- **d** $cd = 0$ and $c - d = 7$

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2B Equivalent equations REVISION

Interactive
Widgets
HOTsheets
Walkthrough

If we have an equation, we can obtain an **equivalent** equation by performing the same operation to both sides. For example, if we have $2x + 4 = 20$, we can add 3 to both sides to obtain $2x + 7 = 23$.

The new equation will be true for exactly the same values of x as the old equation. This observation helps us to solve equations systematically. For example, $2x + 4 = 20$ is equivalent to $2x = 16$ (subtract 4 from both sides), and this is equivalent to $x = 8$ (divide both sides by 2). The bottom equation is true only if x has the value 8, so this means the solution to the equation $2x + 4 = 20$ is $x = 8$. We write this as:

$$\begin{tikzcd}
2x + 4 = 20 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
2x = 16 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x = 8
\end{tikzcd}$$

Let's start: Attempted solutions

Below are three attempts at solving the equation $4x - 8 = 40$. Each has a problem.

Attempt 1

$$\begin{tikzcd}
4x - 8 = 40 \arrow[d, bend right, "+8"'] \arrow[d, bend left, "+8"] \\
4x = 48 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
x = 44
\end{tikzcd}$$

Attempt 2

$$\begin{tikzcd}
4x - 8 = 40 \arrow[d, bend right, "+8"'] \arrow[d, bend left, "-8"] \\
4x = 32 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
x = 8
\end{tikzcd}$$

Attempt 3

$$\begin{tikzcd}
4x - 8 = 40 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
x - 8 = 10 \arrow[d, bend right, "+8"'] \arrow[d, bend left, "+8"] \\
x = 18
\end{tikzcd}$$

- Can you prove that these results are not the correct solutions to the equation above?
- For each one, find the mistake that was made.
- Can you solve $4x - 8 = 40$ systematically?

Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side and right-hand side of the equation.
- To solve an equation **systematically**, repeatedly find an equivalent equation that is simpler.
  For example:

$$\begin{tikzcd}
5x + 2 = 32 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\
5x = 30 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
x = 6
\end{tikzcd}$$

- To check a solution, substitute the unknown's value to see if the equation is true.
  For example: LHS $= 5(6) + 2$ and RHS $= 32$.

Example 4 Finding equivalent equations

Show the result of applying the given operation to both sides of these equations.

a $8y = 40\ [\div 8]$ **b** $10 + 2x = 36\ [-10]$ **c** $5a - 3 = 12\ [+3]$

SOLUTION

a

$$\begin{tikzcd} 8y = 40 \arrow[d, bend right, "\div 8"'] \arrow[d, bend left, "\div 8"] \\ y = 5 \end{tikzcd}$$

b

$$\begin{tikzcd} 10 + 2x = 36 \arrow[d, bend right, "-10"'] \arrow[d, bend left, "-10"] \\ 2x = 26 \end{tikzcd}$$

c

$$\begin{tikzcd} 5a - 3 = 12 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\ 5a = 15 \end{tikzcd}$$

EXPLANATION

Write out the equation and then divide both sides by 8.

$40 \div 8$ is 5 and $8y \div 8$ is y.

Write out the equation and then subtract 10 from both sides.

$36 - 10$ is 26.

$10 + 2x - 10$ is $2x$.

Write out the equation and then add 3 to both sides.

$12 + 3$ is 15.

$5a - 3 + 3$ is $5a$.

Example 5 Solving equations systematically

Solve the following equations and check the solution by substituting.

a $x - 4 = 16$ **b** $2u + 7 = 17$ **c** $40 - 3x = 22$

SOLUTION

a

$$\begin{tikzcd} x - 4 = 16 \arrow[d, bend right, "+4"'] \arrow[d, bend left, "+4"] \\ x = 20 \end{tikzcd}$$

So the solution is $x = 20$.

b

$$\begin{tikzcd} 2u + 7 = 17 \arrow[d, bend right, "-7"'] \arrow[d, bend left, "-7"] \\ 2u = 10 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ u = 5 \end{tikzcd}$$

So the solution is $u = 5$.

c

$$\begin{tikzcd} 40 - 3x = 22 \arrow[d, bend right, "-40"'] \arrow[d, bend left, "-40"] \\ -3x = -18 \arrow[d, bend right, "\div -3"'] \arrow[d, bend left, "\div -3"] \\ x = 6 \end{tikzcd}$$

So the solution is $x = 6$.

EXPLANATION

By adding 4 to both sides of the equation, we get an equivalent equation.

Check: $20 - 4 = 16$ ✓

To remove the $+7$, we subtract 7 from both sides. Finally, we divide by 2 to reverse the $2u$. Remember that $2u$ means $2 \times u$.

Check: $2(5) + 7 = 10 + 7 = 17$ ✓

We subtract 40 from both sides to remove the 40 at the start of the LHS.

Since $-3 \times x = -18$, we divide by -3 to get the final solution.

Check: $40 - 3(6) = 40 - 18 = 22$ ✓

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Exercise 2B **REVISION**

UNDERSTANDING AND FLUENCY	1–6, 7–9(½)	2–4, 5–10(½)	6–10(½)

1 For each of the following equations add 4 to both sides to obtain an equivalent equation.

a $3x = 10$ **b** $7 + k = 14$ **c** $5 = 2x$

2 For each equation fill in the blank to get an equivalent equation.

a
$$\begin{tikzcd} 5x = 10 \arrow[d, bend right, "+2"'] \arrow[d, bend left, "+2"] \\ 5x + 2 = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} 10 - 2x = 20 \arrow[d, bend right, "+5"'] \arrow[d, bend left, "+5"] \\ 15 - 2x = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} 3q + 4 = \_\_ \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 3q = 12 \end{tikzcd}$$

d
$$\begin{tikzcd} 7z + 12 = 4z + 10 \arrow[d, bend right, "-10"'] \arrow[d, bend left, "-10"] \\ 7z + 2 = \_\_ \end{tikzcd}$$

3 Consider the equation $4x = 32$.

a Copy and complete the following working.

$$\begin{tikzcd} 4x = 32 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x = \_\_ \end{tikzcd}$$

b What is the solution to the equation $4x = 32$?

4 To solve the equation $10x + 5 = 45$, which of the following operations would you first apply to both sides?

A divide by 5 **B** subtract 5 **C** divide by 10 **D** subtract 45

Example 4

5 For each equation, show the result of applying the given operation to both sides.

a $10 + 2x = 30$ [−10] **b** $4 + q = 12$ [−2] **c** $13 = 12 - q$ [+5]

d $4x = 8$ [×3] **e** $7p = 2p + 4$ [+6] **f** $3q + 1 = 2q + 1$ [−1]

6 Copy and complete the following to solve the given equation systematically.

a
$$\begin{tikzcd} 10x = 30 \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ x = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} q + 5 = 2 \arrow[d, bend right, "-5"'] \arrow[d, bend left, "-5"] \\ \_\_ = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} 4x + 2 = 22 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 4x = \_\_ \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ \_\_ = \_\_ \end{tikzcd}$$

d
$$\begin{tikzcd} 30 = 4p + 2 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ \_\_ = \_\_ \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ \_\_ = \_\_ \end{tikzcd}$$

e
$$\begin{tikzcd} 20 - 4x = 8 \arrow[d, bend right, "-20"'] \arrow[d, bend left, "-20"] \\ -4x = -12 \arrow[d, bend right, "\div -4"'] \arrow[d, bend left, "\div -4"] \\ x = \_\_ \end{tikzcd}$$

f
$$\begin{tikzcd} p \div 3 + 6 = 8 \arrow[d, bend right, "-6"'] \arrow[d, bend left, "-6"] \\ p \div 3 = 2 \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ \_\_ = \_\_ \end{tikzcd}$$

Example 5a

7 Solve the following equations systematically.

a $a + 5 = 8$ **b** $t \times 2 = 14$

c $7 = q - 2$ **d** $11 = k + 2$

e $19 = x + 9$ **f** $-30 = 3h$

g $-36 = 9l$ **h** $g \div 3 = -3$

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Example 5b

8 Solve the following equations systematically. Check your solutions using substitution.

a $5 + 9h = 32$
b $9u - 6 = 30$
c $13 = 5s - 2$
d $-18 = 6 - 3w$
e $-12 = 5x + 8$
f $-44 = 10w + 6$
g $8 = -8 + 8a$
h $4y - 8 = -40$

Example 5c

9 Solve the following equations systematically and check your solutions.

a $20 - 4d = 8$
b $34 = 4 - 5j$
c $21 - 7a = 7$
d $6 = 12 - 3y$
e $13 - 8k = 45$
f $44 = 23 - 3n$
g $13 = -3b + 4$
h $-22 = 14 - 9b$

10 The following equations do not have whole number solutions. Solve the following equations systematically, giving each solution as a fraction.

a $2x + 3 = 10$
b $5 + 3q = 6$
c $12 = 10b + 7$
d $15 = 10 + 2x$
e $15 = 10 - 2x$
f $13 + 2p = -10$
g $22 = 9 + 5y$
h $12 - 2y = 15$

PROBLEM-SOLVING AND REASONING	11, 15	11–13, 15, 16	12–14, 16, 17

11 For each of the following, write an equation and solve it systematically.

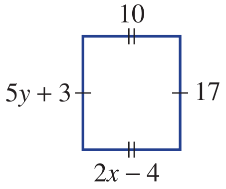
a The sum of p and 8 is 15.
b The product of q and -3 is 12.
c Four is subtracted from double the value of k and the result is 18.
d When r is tripled and 4 is added, the result is 34.
e When x is subtracted from 10, the result is 6.
f When triple y is subtracted from 10, the result is 16.

12 Solve the following equations systematically. More than two steps are involved.

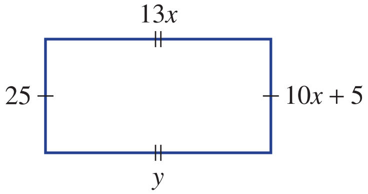
a $14 \times (4x + 2) = 140$
b $8 = (10x - 4) \div 2$
c $-12 = (3 - x) \times 4$

13 The following shapes are rectangles. By solving equations systematically, find the value of the variables. Some of the answers will be fractions.

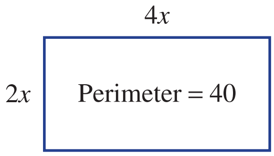
a



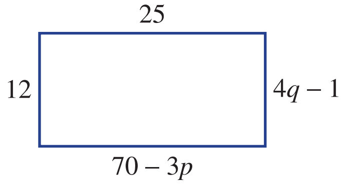
b



c



d



14 Sidney works for 10 hours at the normal rate of pay (\$$x$ per hour), and then the next three hours at double that rate. If he earns a total of \$194.88, write an equation and solve it to find his normal hourly rate.

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15 **a** Prove that $7x + 4 = 39$ and $-2x + 13 = 3$ are equivalent by filling in the missing steps.

$$\begin{tikzcd}
7x + 4 = 39 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
7x = 35 \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\
\_\_\_ = \_\_\_ \arrow[d, bend right, "\times -2"'] \arrow[d, bend left, "\times -2"] \\
\_\_\_ = \_\_\_ \arrow[d, bend right, "+13"'] \arrow[d, bend left, "+13"] \\
-2x + 13 = 3
\end{tikzcd}$$

b Prove that $10k + 4 = 24$ and $3k - 1 = 5$ are equivalent.

16 **a** Prove that $4x + 3 = 11$ and $2x = 4$ are equivalent. Try to use just two steps to get from one equation to the other.

b Are the equations $5x + 2 = 17$ and $x = 5$ equivalent?

c Prove that $10 - 2x = 13$ and $14x + 7 = 20$ are not equivalent, no matter how many steps are used.

17 A student has taken the equation $x = 5$ and performed some operations to both sides:

$$\begin{tikzcd}
x = 5 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\
4x = 20 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\
4x + 3 = 23 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
(4x + 3) \times 2 = 46
\end{tikzcd}$$

a Solve $(4x + 3) \times 2 = 46$ systematically.

b Describe how the steps you used in your solution compare with the steps used by the student.

c Give an example of another equation that has $x = 5$ as its solution.

d Explain why there are infinitely many different equations with the solution $x = 5$.

ENRICHMENT	–	–	18

Dividing whole expressions

18 It is possible to solve $2x + 4 = 20$ by first dividing both sides by 2, as long as every term is divided by 2. So you could solve it in either of these fashions.

$$\begin{tikzcd}
2x + 4 = 20 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
2x = 16 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x = 8
\end{tikzcd}$$

$$\begin{tikzcd}
2x + 4 = 20 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x + 2 = 10 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\
x = 8
\end{tikzcd}$$

Note that $2x + 4$ divided by 2 is $x + 2$, not $x + 4$. Use this method of dividing first to solve the following equations, and then check that you get the same answer as if you subtracted first.

a $2x + 6 = 12$

b $4x + 12 = 16$

c $10x + 30 = 50$

d $2x + 5 = 13$

e $5x + 4 = 19$

f $3 + 2x = 5$

g $7 = 2x + 4$

h $10 = 4x + 10$

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2C Equations with fractions

Interactive

Widgets

HOTsheets

Walkthrough

Recall from algebra that a fraction such as $\frac{x}{3}$ represents $x \div 3$. This means that to solve an equation with $\frac{x}{3}$ on one side, we should first multiply both sides by 3. For example:

$$\begin{tikzcd} \frac{x}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x = 30 \end{tikzcd} \qquad \begin{tikzcd} 20 = \frac{x}{5} \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ 100 = x \end{tikzcd}$$

$$\therefore x = 100$$

Let's start: Practising with fractions

- If x is a number greater than 1, evaluate these expressions and put them into ascending order (smallest to largest):

$$\frac{2x+1}{2} \qquad 2\left(\frac{x}{2}+1\right) \qquad \frac{2}{x+1} \qquad \frac{2+2x}{2} \qquad 2\left(x+\frac{1}{2}\right)$$

- Investigate how the order might change if x could be a number less than 1.

Key ideas

- $\frac{a}{b}$ means $a \div b$.
- To solve an equation with a fraction on one side, multiply both sides by the denominator.
  For example:

$$\begin{tikzcd} \frac{q}{4} = 12 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ q = 48 \end{tikzcd}$$

Example 6 Solving equations with fractions

Solve the following equations systematically.

a $\frac{4x}{3} = 8$ **b** $\frac{4y+15}{9} = 3$ **c** $4 + \frac{5x}{2} = 29$ **d** $7 - \frac{2x}{3} = 5$

SOLUTION	EXPLANATION
a $\begin{tikzcd} \frac{4x}{3} = 8 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 4x = 24 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x = 6 \end{tikzcd}$	Multiplying both sides by 3 removes the denominator of 3. Both sides are divided by 4 to solve the equation.

Example continues on next page

Continued from previous page

b

$$\begin{tikzcd}
\frac{4y + 15}{9} = 3 \arrow[d, bend right=60, "\times 9"'] \arrow[d, bend left=60, "\times 9"] \\
4y + 15 = 27 \arrow[d, bend right=60, "-15"'] \arrow[d, bend left=60, "-15"] \\
4y = 12 \arrow[d, bend right=60, "\div 4"'] \arrow[d, bend left=60, "\div 4"] \\
y = 3
\end{tikzcd}$$

Multiplying both sides by 9 removes the denominator of 9.

The equation $4y + 15 = 27$ is solved in the usual fashion (subtract 15, divide by 4).

c

$$\begin{tikzcd}
4 + \frac{5x}{2} = 29 \arrow[d, bend right=60, "-4"'] \arrow[d, bend left=60, "-4"] \\
\frac{5x}{2} = 25 \arrow[d, bend right=60, "\times 2"'] \arrow[d, bend left=60, "\times 2"] \\
5x = 50 \arrow[d, bend right=60, "\div 5"'] \arrow[d, bend left=60, "\div 5"] \\
x = 10
\end{tikzcd}$$

We must subtract 4 first because we do not have a fraction by itself on the left-hand side. Once there is a fraction by itself, multiply by the denominator (2).

d

$$\begin{tikzcd}
7 - \frac{2x}{2} = 5 \arrow[d, bend right=60, "-7"'] \arrow[d, bend left=60, "-7"] \\
-\frac{2x}{3} = -2 \arrow[d, bend right=60, "\times -1"'] \arrow[d, bend left=60, "\times -1"] \\
\frac{2x}{3} = 2 \arrow[d, bend right=60, "\times 3"'] \arrow[d, bend left=60, "\times 3"] \\
2x = 6 \arrow[d, bend right=60, "\div 2"'] \arrow[d, bend left=60, "\div 2"] \\
x = 3
\end{tikzcd}$$

Subtract 7 first to get a fraction. When both sides are negative, multiplying (or dividing) by −1 makes them both positive.

Exercise 2C

UNDERSTANDING AND FLUENCY	1–3, 4–5(½)	2, 3, 4–5(½)	4–5(½)

1 **a** If $x = 4$, find the value of $\frac{x}{2} + 6$.

b If $x = 4$, find the value of $\frac{x + 6}{2}$.

c Are $\frac{x}{2} + 6$ and $\frac{x + 6}{2}$ equivalent expressions?

2 Fill in the missing steps to solve these equations.

a

$$\begin{tikzcd}
\frac{x}{3} = 10 \arrow[d, bend right=60, "\times 3"'] \arrow[d, bend left=60, "\times 3"] \\
x = \_\_
\end{tikzcd}$$

b

$$\begin{tikzcd}
\frac{m}{5} = 2 \arrow[d, bend right=60, "\times 5"'] \arrow[d, bend left=60, "\times 5"] \\
m = \_\_
\end{tikzcd}$$

c

$$\begin{tikzcd}
11 = \frac{q}{2} \arrow[d, bend right=60, "\square"'] \arrow[d, bend left=60, "\square"] \\
\_\_ = q
\end{tikzcd}$$

d

$$\begin{tikzcd}
\frac{p}{10} = 7 \arrow[d, bend right=60, "\square"'] \arrow[d, bend left=60, "\square"] \\
p = \_\_
\end{tikzcd}$$

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3 Match each of these equations with the correct first step to solve it.

a $\frac{x}{4} = 7$

b $\frac{x - 4}{2} = 5$

c $\frac{x}{2} - 4 = 7$

d $\frac{x}{4} + 4 = 3$

A Multiply both sides by 2.

B Add 4 to both sides.

C Multiply both sides by 4.

D Subtract 4 from both sides.

Example 6a

4 Solve the following equations systematically.

a $\frac{b}{5} = 4$

b $\frac{g}{10} = 2$

c $\frac{a}{5} = 3$

d $\frac{k}{6} = 3$

e $\frac{2l}{5} = 8$

f $\frac{7w}{10} = -7$

g $\frac{3s}{2} = -9$

h $\frac{5v}{4} = 15$

i $\frac{3m}{7} = 6$

j $\frac{2n}{7} = 4$

k $\frac{-7j}{5} = 7$

l $\frac{-6f}{5} = -24$

Example 6b,c,d

5 Solve the following equations systematically. Check your solutions by substituting.

a $\frac{t - 8}{2} = -10$

b $\frac{h + 10}{3} = 4$

c $\frac{a + 12}{5} = 2$

d $\frac{c - 7}{2} = -5$

e $-1 = \frac{s - 2}{8}$

f $\frac{5j + 6}{8} = 2$

g $3 = \frac{7v}{12} + 10$

h $\frac{4n}{9} - 6 = -2$

i $\frac{7q + 12}{5} = -6$

j $-4 = \frac{f - 15}{3}$

k $15 = \frac{3 - 12l}{5}$

l $9 - \frac{4r}{7} = 5$

m $-6 = \frac{5x - 8}{-7}$

n $\frac{5u - 7}{-4} = -2$

o $\frac{5k + 4}{-8} = -3$

p $20 = \frac{3 + 13b}{-7}$

q $\frac{7m}{12} - 12 = -5$

r $4 + \frac{-7y}{8} = -3$

s $4 = \frac{p - 15}{-3}$

t $\frac{g - 3}{5} = -1$

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PROBLEM-SOLVING AND REASONING	6, 9	6, 7, 9, 10	6(½), 7–9, 10(½)

6 For the following puzzles, write an equation and solve it to find the unknown number.

- **a** A number x is divided by 5 and the result is 7.
- **b** Half of y is -12.
- **c** A number p is doubled and then divided by 7. The result is 4.
- **d** Four is added to x. This is halved to get a result of 10.
- **e** x is halved and then 4 is added to get a result of 10.
- **f** A number k is doubled and then 6 is added. This result is halved to obtain -10.

7 The average of two numbers can be found by adding them and then dividing the result by 2.

- **a** If the average of x and 5 is 12, what is x? Solve the equation $\dfrac{x + 5}{2} = 12$ to find out.
- **b** The average of 7 and p is -3. Find p by writing and solving an equation.
- **c** The average of a number and double that number is 18. What is that number?
- **d** The average of $4x$ and 6 is 19. What is the average of $6x$ and 4? (*Hint*: find x first.)

8 A restaurant bill of \$100 is to be paid. Blake puts in one-third of the amount in his wallet, leaving \$60 to be paid by the other people at the table.

- **a** Write an equation to describe this situation, if b represents the amount in Blake's wallet before he pays.
- **b** Solve the equation systematically and, hence, state how much money Blake has in his wallet.

9 When solving $\dfrac{2x}{3} = 10$ we first multiply by the denominator, but we could have written $2\left(\dfrac{x}{3}\right) = 10$ and divided both sides by 2.

- **a** Solve $2\left(\dfrac{x}{3}\right) = 10$.
- **b** Is the solution the same as the solution for $\dfrac{2x}{3} = 10$ if both sides are first multiplied by 3?
- **c** Solve $\dfrac{147q}{13} = 1470$ by first:
  - **i** multiplying both sides by 13
  - **ii** dividing both sides by 147
- **d** What is one advantage in dividing first rather than multiplying?
- **e** Solve the following equations.
  - **i** $\dfrac{20p}{14} = 40$
  - **ii** $\dfrac{13q}{27} = -39$
  - **iii** $\dfrac{-4p}{77} = 4$
  - **iv** $\dfrac{123r}{17} = 246$

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10 To solve an equation with a variable as the denominator we can first multiply both sides by that variable.

$$\begin{tikzcd} \frac{30}{x} = 10 \arrow[d, bend right, "\times x"'] \arrow[d, bend left, "\times x"] \\ 30 = 10x \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ 3 = x \end{tikzcd}$$

Use this method to solve the equations.

a $\frac{12}{x} = 2$

b $\frac{-15}{x} = -5$

c $\frac{1}{x} + 3 = 4$

d $4 + \frac{20}{x} = 14$

e $\frac{16}{x} + 1 = 3$

f $5 = \frac{-10}{x} + 3$

ENRICHMENT	–	–	11, 12

Fractional solutions

11 Solve the following equations. Note that the solutions should be given as fractions.

a $\frac{4x + 3}{5} = 12$

b $\frac{8 + 3x}{5} = 6$

c $7 = \frac{x}{4} + \frac{1}{3}$

d $2 = \frac{10 - 3x}{4}$

12 Recall from Section 1E (Adding and subtracting algebraic fractions) that algebraic fractions can be combined by finding a common denominator. For example:

$$\begin{aligned} \frac{2x}{3} + \frac{5x}{4} &= \frac{8x}{12} + \frac{15x}{12} \\ &= \frac{23x}{12} \end{aligned}$$

Use this simplification to solve the following equations.

a $\frac{2x}{3} + \frac{5x}{4} = 46$

b $\frac{x}{5} + \frac{x}{6} = 22$

c $10 = \frac{x}{2} + \frac{x}{3}$

d $4 = \frac{x}{2} - \frac{x}{3}$

e $\frac{6x}{5} + \frac{2x}{3} = 28$

f $4 = \frac{3x}{7} - \frac{x}{3}$

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2D Equations with pronumerals on both sides

Interactive

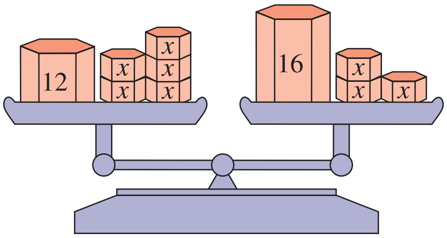
Widgets

HOTsheets

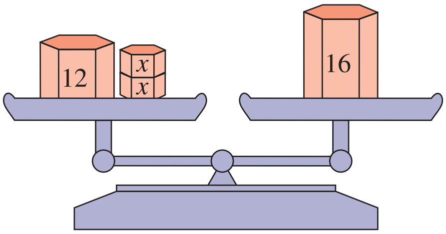
Walkthrough

So far all the equations we have considered involved a pronumeral either on the left-hand side (e.g. $2x + 3 = 11$) or on the right side (e.g. $15 = 10 - 2x$). But how can you solve an equation with pronumerals on both sides (e.g. $12 + 5x = 16 + 3x$)? The idea is to look for an equivalent equation with pronumerals on just one side.

The equation $12 + 5x = 16 + 3x$ can be thought of as balancing scales, as shown on the right.



Then $3x$ can be removed from both sides of this equation to get:



The resulting equation $12 + 2x = 16$ is straightforward to solve.

Let's start: Moving pronumerals

You are given the equation $11 + 5x = 7 + 3x$.

- Can you find an equivalent equation with x just on the left-hand side?
- Can you find an equivalent equation with x just on the right-hand side?
- Try to find an equivalent equation with $9x$ on the left-hand side.
- Do all of these equations have the same solution? Try to find it.

Key ideas

- If both sides of an equation have a pronumeral added or subtracted, the new equation will be equivalent to the original equation.
- If pronumerals are on both sides of an equation, add or subtract it so that the pronumeral appears on only one side. For example:

$$\begin{tikzcd} 10 + 5a = 13 + 2a \arrow[d, bend right, "-2a"'] \arrow[d, bend left, "-2a"] \\ 10 + 3a = 13 \end{tikzcd} \qquad \begin{tikzcd} 4b + 12 = 89 - 3b \arrow[d, bend right, "+3b"'] \arrow[d, bend left, "+3b"] \\ 7b + 12 = 89 \end{tikzcd}$$

- Sometimes it is wise to swap the left-hand side and right-hand side.

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Example 7 Solving equations with pronumerals on both sides

Solve the following equations.

a $7t + 4 = 5t + 10$ **b** $6x + 4 = 22 - 3x$ **c** $2u = 7u - 20$

SOLUTION

a

$$\begin{array}{ccc} & 7t + 4 = 5t + 10 & \\ -5t & & -5t \\ & 2t + 4 = 10 & \\ -4 & & -4 \\ & 2t = 6 & \\ \div 2 & & \div 2 \\ & t = 3 & \end{array}$$

b

$$\begin{array}{ccc} & 6x + 4 = 22 - 3x & \\ +3x & & +3x \\ & 9x + 4 = 22 & \\ -4 & & -4 \\ & 9x = 18 & \\ \div 9 & & \div 9 \\ & x = 2 & \end{array}$$

c

$$\begin{array}{ccc} & 2u = 7u - 20 & \\ -2u & & -2u \\ & 0 = 5u - 20 & \\ +20 & & +20 \\ & 20 = 5u & \\ \div 5 & & \div 5 \\ & 4 = u & \\ & \therefore u = 4 & \end{array}$$

EXPLANATION

Pronumerals are on both sides of the equation, so subtract $5t$ from both sides.

Once $5t$ is subtracted, the usual procedure is applied for solving equations.

$$\begin{aligned} \text{LHS} &= 7(3) + 4 & \text{RHS} &= 5(3) + 10 \\ &= 25 & &= 25 \ \checkmark \end{aligned}$$

Pronumerals are on both sides. To get rid of $3x$, we add $3x$ to both sides of the equation.

Alternatively, $6x$ could have been subtracted from both sides of the equation to get $4 = 22 - 9x$.

$$\begin{aligned} \text{LHS} &= 6(2) + 4 & \text{RHS} &= 22 - 3(2) \\ &= 16 & &= 16 \ \checkmark \end{aligned}$$

Choose to remove $2u$ by subtracting it.

Note that $2u - 2u$ is equal to 0, so the LHS of the new equation is 0.

$$\begin{aligned} \text{LHS} &= 2(4) & \text{RHS} &= 7(4) - 20 \\ &= 8 & &= 8 \ \checkmark \end{aligned}$$

Exercise 2D

UNDERSTANDING AND FLUENCY	1–3, 4–7(½)	2, 3, 4–7(½)	5–7

1 If $x = 3$, are the following equations true or false?

a $5 + 2x = 4x - 1$ **b** $7x = 6x + 5$

c $2 + 8x = 12x$ **d** $9x - 7 = 3x + 11$

2 Fill in the blanks for these equivalent equations.

a

$$\begin{array}{ccc} & 5x + 3 = 2x + 8 & \\ -2x & & -2x \\ & \underline{\qquad} = 8 & \end{array}$$

b

$$\begin{array}{ccc} & 9q + 5 = 12q + 21 & \\ -9q & & -9q \\ & \underline{\qquad} = 3q + 12 & \end{array}$$

c

$$\begin{array}{ccc} & 3p + 9 = 5 - 2p & \\ +2p & & +2p \\ & \underline{\qquad} = \underline{\qquad} & \end{array}$$

d

$$\begin{array}{ccc} & 15k + 12 = 13 - 7k & \\ +7k & & +7k \\ & \underline{\qquad} = \underline{\qquad} & \end{array}$$

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3 To solve the equation $12x + 2 = 8x + 16$, which one of the following first steps will ensure that x is only on one side of the equation?

A subtract 2
B subtract $8x$
C add $12x$
D subtract 16
E add $20x$

Example 7a

4 Solve the following equations systematically and check your solutions.

a $10f + 3 = 23 + 6f$
b $10y + 5 = 26 + 3y$
c $7s + 7 = 19 + 3s$
d $9j + 4 = 4j + 14$
e $2t + 8 = 8t + 20$
f $4 + 3n = 10n + 39$
g $4 + 8y = 10y + 14$
h $5 + 3t = 6t + 17$
i $7 + 5q = 19 + 9q$

Example 7b

5 Solve the following equations systematically, checking your solutions using substitution.

a $9 + 4t = 7t + 15$
b $2c - 2 = 4c - 6$
c $6t - 3 = 7t - 8$
d $7z - 1 = 8z - 4$
e $8t - 24 = 2t - 6$
f $2q - 5 = 3q - 3$
g $5x + 8 = 6x - 1$
h $8w - 15 = 6w + 3$
i $6j + 4 = 5j - 1$

Example 7c

6 Solve the following equations systematically. Your solutions should be checked using substitution.

a $1 - 4a = 7 - 6a$
b $6 - 7g = 2 - 5g$
c $12 - 8n = 8 - 10n$
d $2 + 8u = 37 + 3u$
e $21 - 3h = 6 - 6h$
f $37 - 4j = 7 - 10j$
g $13 - 7c = 8c - 2$
h $10 + 4n = 4 - 2n$
i $10a + 32 = 2a$
j $10v + 14 = 8v$
k $18 + 8c = 2c$
l $2t + 7 = 22 - 3t$
m $6n - 47 = 9 - 8n$
n $3n = 15 + 8n$
o $38 - 10l = 10 + 4l$

7 Solve the following equations systematically. Your answers should be given as fractions.

a $3x + 5 = x + 6$
b $5k - 2 = 2k$
c $3 + m = 6 + 3m$
d $9j + 4 = 5j + 14$
e $3 - j = 4 + j$
f $2z + 3 = 4z - 8$

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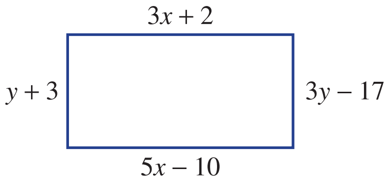
PROBLEM-SOLVING AND REASONING	8, 12	8–10, 12, 13	9–11, 13, 14

8 Write an equation and solve it systematically to find the unknown number in these problems.

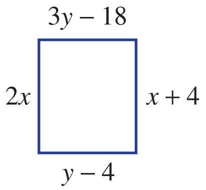
- **a** Doubling x and adding 3 is the same as tripling x and adding 1.
- **b** If z is increased by 9, this is the same as doubling the value of z.
- **c** The product of 7 and y is the same as the sum of y and 12.
- **d** When a number is increased by 10, this has the same effect as tripling the number and subtracting 6.

9 Find the value of x and y in the following rectangles.

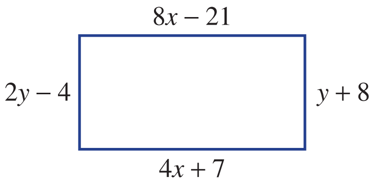
a



b



10 Find the area and the perimeter of this rectangle.



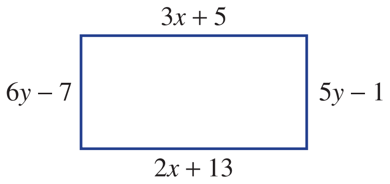
11 At a newsagency, Preeta bought four pens and a \$1.50 newspaper, while her husband Levy bought two pens and a \$4.90 magazine. To their surprise the cost was the same.

- **a** Write an equation to describe this, using p for the cost of a single pen.
- **b** Solve the equation to find the cost of a pen.
- **c** If Fred has a \$20 note, what is the maximum number of pens that he can purchase?

12 To solve the equation $12 + 3x = 5x + 2$ you can first subtract $3x$ or subtract $5x$.

- **a** Solve the equation above by first subtracting $3x$.
- **b** Solve the equation above by first subtracting $5x$.
- **c** What is the difference between the two methods?

13 Prove that the rectangular shape shown must be a square. (*Hint*: first find the values of x and y.)



14 **a** Try to solve the equation $4x + 3 = 10 + 4x$.

b This tells you that the equation you are trying to solve has no solutions (because $10 = 3$ is never true). Prove that $2x + 3 = 7 + 2x$ has no solutions.

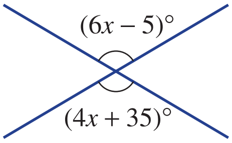
c Give an example of another equation that has no solutions.

ENRICHMENT	–	–	15

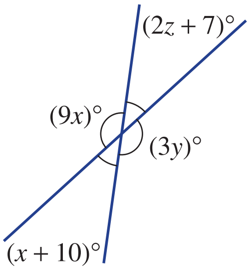
Geometric equations

15 Find the values of the pronumerals in the following geometric diagrams.

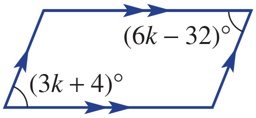
a



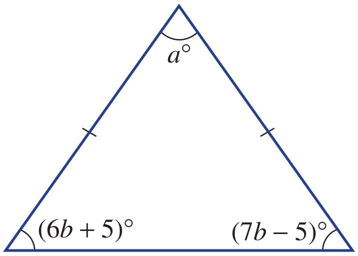
b



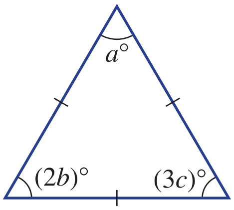
c



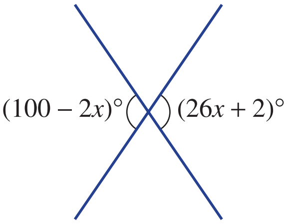
d



e



f



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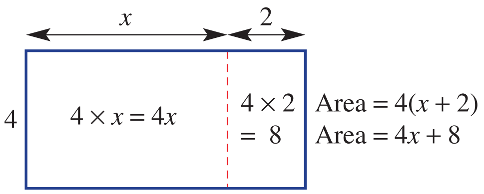
2E Equations with brackets

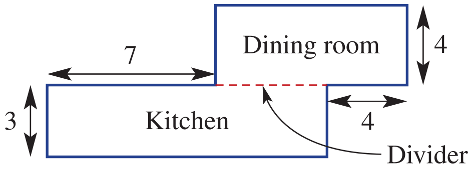
In Chapter 1 it was noted that expressions with brackets could be expanded by considering rectangle areas.

The diagram shows that $4(x + 2)$ and $4x + 8$ are equivalent. This becomes quite helpful when solving an equation like $4(x + 2) = 5x + 1$. We just solve $4x + 8 = 5x + 1$ using the techniques from the previous section.



Let's start: Architect's dilemma

In the house plans shown, the kitchen and dining room are separated by a dividing door.



- If the breadth of the divider is x, what is the area of the kitchen? What is the area of the dining room?
- Try to find the breadth of the divider if the areas of the two rooms are equal.
- Is it easier to solve $3(7 + x) = 4(x + 4)$ or $21 + 3x = 4x + 16$? Which of these did you solve when trying to find the breadth of the divider?

Key ideas

- To expand brackets, use the **distributive law**, which states that:
  - $a(b + c) = ab + ac$
    For example, $3(x + 4) = 3x + 12$.
  - $a(b - c) = ab - ac$
    For example, $4(b - 2) = 4b - 8$.
- **Like terms** are terms that contain exactly the same pronumerals and can be collected to simplify expressions. For example, $5x + 10 + 7x$ can be simplified to $12x + 10$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

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Example 8 Solving equations with brackets

Solve the following equations by first expanding any brackets.

a $3(p + 4) = 18$ **b** $-12(3q + 5) = -132$

c $4(2x - 5) + 3x = 57$ **d** $2(3k + 1) = 5(2k - 6)$

SOLUTION	EXPLANATION
a $\begin{array}{rcl} & 3(p+4) = 18 & \\ & 3p + 12 = 18 & \\ -12 & & -12 \\ & 3p = 6 & \\ \div 3 & & \div 3 \\ & p = 2 & \end{array}$	Use the distributive law to expand the brackets. Solve the equation by performing the same operations to both sides.
b $\begin{array}{rcl} & -12(3q+5) = -132 & \\ & -36q + (-60) = -132 & \\ & -36q - 60 = -132 & \\ +60 & & +60 \\ & -36q = -72 & \\ \div -36 & & \div -36 \\ & q = 2 & \end{array}$	Use the distributive law to expand the brackets. Simplify $-36q + (-60)$ to $-36q - 60$. Solve the equation by performing the same operations to both sides.
c $\begin{array}{rcl} & 4(2x-5) + 3x = 57 & \\ & 8x - 20 + 3x = 57 & \\ & 11x - 20 = 57 & \\ +20 & & +20 \\ & 11x = 77 & \\ \div 11 & & \div 11 \\ & x = 7 & \end{array}$	Use the distributive law to expand the brackets. Combine the like terms: $8x + 3x = 11x$. Solve the equation by performing the same operations to both sides.
d $\begin{array}{rcl} & 2(3k+1) = 5(2k-6) & \\ & 6k + 2 = 10k - 30 & \\ -6k & & -6k \\ & 2 = 4k - 30 & \\ +30 & & +30 \\ & 32 = 4k & \\ \div 4 & & \div 4 \\ & 8 = k & \\ & \therefore k = 8 & \end{array}$	Use the distributive law on both sides to expand the brackets. Solve the equation by performing the same operations to both sides.

Exercise 2E

UNDERSTANDING AND FLUENCY	1–5, 6–7(½)	1, 4, 5, 6–8(½)	5–9(½)

1 Fill in the missing numbers.

a $4(y + 3) = 4y + \square$ **b** $7(2p - 5) = \square p - 35$

c $2(4x + 5) = \square x + \square$ **d** $10(5 + 3q) = \square + \square q$

2 Match each expression **a**–**d** with its expanded form **A**–**D**.

a	$2(x + 4)$	**A**	$4x + 8$
b	$4(x + 2)$	**B**	$2x + 4$
c	$2(2x + 1)$	**C**	$2x + 8$
d	$2(x + 2)$	**D**	$4x + 2$

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3 Rolf is unsure whether $4(x + 3)$ is equivalent to $4x + 12$ or $4x + 3$.

a Fill out the table below.

x	0	1	2
$4(x + 3)$			
$4x + 12$			
$4x + 3$			

b What is the correct expansion of $4(x + 3)$?

4 Simplify the following expressions by collecting like terms.

a $4x + 3x$ **b** $7p + 2p + 3$ **c** $8x - 2x + 4$
d $2k + 4 + 5k$ **e** $3x + 6 - 2x$ **f** $7k + 21 - 2k$

Example 8a

5 Solve the following equations by first expanding the brackets.

a $2(4u + 2) = 52$ **b** $3(3j - 4) = 15$ **c** $5(2p - 4) = 40$
d $15 = 5(2m - 5)$ **e** $2(5n + 5) = 60$ **f** $26 = 2(3a + 4)$

Example 8b

6 Solve the following equations involving negative numbers.

a $-6(4p + 4) = 24$ **b** $-2(4u - 5) = 34$ **c** $-2(3v - 4) = 38$
d $28 = -4(3r + 5)$ **e** $-3(2b - 2) = 48$ **f** $-6 = -3(2d - 4)$

Example 8c

7 Solve the following equations by expanding and combining like terms.

a $4(3y + 2) + 2y = 50$ **b** $5(2l - 5) + 3l = 1$ **c** $4(5 + 3w) + 5 = 49$
d $49 = 5(3c + 5) - 3c$ **e** $28 = 4(3d + 3) - 4d$ **f** $58 = 4(2w - 5) + 5w$
g $23 = 4(2p - 3) + 3$ **h** $44 = 5(3k + 2) + 2k$ **i** $49 = 3(2c - 5) + 4$

Example 8d

8 Solve the following equations by expanding brackets on both sides.

a $5(4x - 4) = 5(3x + 3)$ **b** $6(4 + 2r) = 3(5r + 3)$ **c** $5(5f - 2) = 5(3f + 4)$
d $4(4p - 3) = 2(4 + 3p)$ **e** $2(5h + 4) = 3(4 + 3h)$ **f** $4(4r - 5) = 2(5 + 5r)$
g $4(3r - 2) = 4(2r + 3)$ **h** $2(2p + 4) = 2(3p - 2)$ **i** $3(2a + 1) = 11(a - 2)$

9 Solve the following equations systematically.

a $2(3 + 5r) + 6 = 4(2r + 5) + 6$ **b** $3(2l + 2) + 18 = 4(4l + 3) - 8$
c $2(3x - 5) + 16 = 3 + 5(2x - 5)$ **d** $3(4s + 3) - 3 = 3(3s + 5) + 15$
e $4(4y + 5) - 4 = 6(3y - 3) + 20$ **f** $3(4h + 5) + 2 = 14 + 3(5h - 2)$

PROBLEM-SOLVING AND REASONING	10, 11, 14	11, 12, 14, 15	11–13, 15, 16

10 Desmond notes that in 4 years' time his age when doubled will give the number 50. Desmond's current age is d.

a Write an expression for Desmond's age in 4 years' time.
b Write an expression for double his age in 4 years' time.
c Write an equation to describe the situation described above.
d Solve the equation to find his current age.

11 Rahda's usual hourly wage is $\$w$. She works for 5 hours at this wage and then three more hours at an increased wage of $\$(w + 4)$.

a Write an expression for the total amount Rahda earns for the 8 hours.
b Rahda earns \$104 for the 8 hours. Write and solve an equation to find her usual hourly wage.

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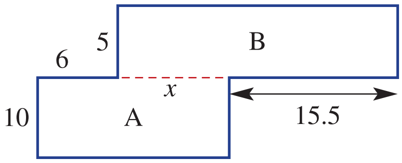
12 Kate’s age 5 years ago, when doubled, is equal to triple her age 10 years ago.

a Write an equation to describe this, using k for Kate’s current age.

b Solve the equation to find Kate’s current age.

13 Rectangles A and B have the same area.



a What is the value of x?

b State the perimeter of the shape shown above.

14 Abraham is asked how many people are in the room next door. He answers that if three more people walk in and then the room’s population is doubled, this will have the same effect as quadrupling the population and then 11 people leaving. Prove that what Abraham said cannot be true.

15 Ajith claims that three times his age 5 years ago is the same as nine times how old he will be next year. Prove that what Ajith is saying cannot be true.

16 A common mistake when expanding is to write $2(n + 3)$ as $2n + 3$. These are not equivalent, since, for example, $2(5 + 3) = 16$ and $2 \times 5 + 3 = 13$.

a Prove that they are never equal by trying to solve $2(n + 3) = 2n + 3$.

b Prove that $4(2x + 3)$ is never equal to $8x + 3$ but it is sometimes equal to $4x + 12$.

ENRICHMENT	–	–	17

Challenging expansions

17 Solve the following equations. Note that, in general, your answers will not be integers.

a $2(3x + 4) + 5(6x + 7) = 64x + 1$

b $-5(3p + 2) + 5(2p + 3) = -31$

c $-10(n + 1) + 20(2n + 13) = 7$

d $4(2q + 1) - 5(3q + 1) = 11q - 1$

e $x + 2(x + 1) + 3(x + 2) = 11x$

f $m - 2(m + 1) - 3(m - 1) = 2(1 - 4m)$

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2F Solving simple quadratic equations

Most of the equations you have worked with so far are called linear equations like $2x - 3 = 7$ and $5(a + 2) = 7(a - 1)$, where the power of the pronumeral is 1 and there is usually a single solution. Another type of equation is of the form $x^2 = c$ and this is an example of a simple quadratic equation. Note that the power of the pronumeral x is 2. Depending on the value of c, x can have zero, one or two solutions. These types of equations appear frequently in mathematics and in problems involving distance, area, graphs and motion.

Let's start: How many solutions?

Consider the equation $x^2 = c$. How many values of x can you think of that satisfy the equation when:

- $c = 0$?
- $c = 9$?
- $c = -4$?

What conclusions can you come to regarding the number of solutions for x depending on the value of c?

Key ideas

■ Simple quadratic equations of the form $x^2 = c$:

- $x^2 = 9$ has two solutions because 9 is a positive number.

$$x^2 = 9$$
$$x = \sqrt{9}, \qquad x = -\sqrt{9} \qquad \text{Note: } 3^2 = 9 \text{ and } (-3)^2 = 9.$$
$$x = 3, \qquad x = -3$$
$$x = \pm 3, \text{ where } \pm 3 \text{ represent both solutions.}$$

- $x^2 = 0$ has one solution $(x = 0)$ because $0^2 = 0$.
- $x^2 = -9$ has no solution because the square of any number is 0 or positive.

Example 9 Solving $x^2 = c$ when $c > 0$

Solve the following equations. Round to two decimal places in part **b** by using a calculator to assist.

a $x^2 = 81$ **b** $x^2 = 23$

SOLUTION	EXPLANATION
a $x = 9$ or $x = -9$	The equation has two solutions because 81 is a positive number. $9^2 = 81$ and $(-9)^2 = 81$.
b $x = \sqrt{23} = 4.80$ (to 2 decimal places) or $x = -\sqrt{23} = -4.80$ (to 2 decimal places)	The number 23 is not a perfect square so $\sqrt{23}$ can be rounded if required.

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Example 10 Stating the number of solutions

State the number of solutions for x in these equations.

a $x^2 = -3$ **b** $x^2 = 0$ **c** $x^2 = 7$

SOLUTION	EXPLANATION
a zero solutions	In $x^2 = c$, if $c < 0$ there are no solutions because any number squared is positive or zero.
b one solution	$x = 0$ is the only solution to $x^2 = 0$.
c two solutions	The equation has two solutions because $\sqrt{7}$ is a positive number.

Excersice 2F

UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	3, 4–6(½)	4–6(½)

1 **a** Calculate the following.
  - **i** 3^2 and $(-3)^2$
  - **ii** 6^2 and $(-6)^2$
  - **iii** 1^2 and $(-1)^2$
  - **iv** 10^2 and $(-10)^2$

b What do you notice about the answers to each pair?

2 **a** Use a calculator to multiply these numbers by themselves. Recall that a neg × neg = pos.
  - **i** -3
  - **ii** 7
  - **iii** 13
  - **iv** -8

b Did you obtain any negative numbers in part **a**?

3 Write in the missing numbers.

a $(-3)^2 =$ _____ and $3^2 = 9$ so if $x^2 = 9$ then $x =$ _____ or $x =$ _____.

b $(5)^2 =$ _____ and $(-5)^2 = 25$ so if $x^2 = 25$ then $x =$ _____ or $x =$ _____.

c $(11)^2 = 121$ and $(-11)^2 =$ ____ so if $x^2 = 121$ then $x =$ _____ or $x =$ _____.

Example 9a **4** Solve the following equations.
- **a** $x^2 = 4$
- **b** $x^2 = 49$
- **c** $x^2 = 100$
- **d** $x^2 = 64$
- **e** $x^2 = 1$
- **f** $x^2 = 144$
- **g** $x^2 = 36$
- **h** $x^2 = 121$
- **i** $x^2 = 169$
- **j** $x^2 = 256$
- **k** $x^2 = 900$
- **l** $x^2 = 10000$

Example 9b **5** Solve the following and round to two decimal places.
- **a** $x^2 = 6$
- **b** $x^2 = 12$
- **c** $x^2 = 37$
- **d** $x^2 = 41$
- **e** $x^2 = 104$
- **f** $x^2 = 317$
- **g** $x^2 = 390$
- **h** $x^2 = 694$

Example 10 **6** State the number of solutions for these equations.
- **a** $x^2 = 10$
- **b** $x^2 = 4$
- **c** $x^2 = 3917$
- **d** $x^2 = -4$
- **e** $x^2 = -94$
- **f** $x^2 = 0$
- **g** $a^2 = 0$
- **h** $y^2 = 1$

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PROBLEM-SOLVING AND REASONING	7, 8, 10	7, 9(½), 11, 12	9(½), 10, 12, 13

7 The area of a square is 25 m^2. Find its perimeter.

8 A square mirror has an area of 1 m^2. Find its perimeter.

9 By first dividing both sides by the coefficient of x^2, solve these simple quadratic equations.

a $2x^2 = 8$
b $3x^2 = 3$
c $5x^2 = 45$
d $-3x^2 = -12$
e $-2x^2 = -50$
f $7x^2 = 0$
g $-6x^2 = -216$
h $-10x^2 = -1000$

10 Explain why:

a $x^2 = 0$ has only one solution
b $x^2 = c$ has no solutions when $c < 0$.

11 Solve the equations with the given conditions.

a $x^2 = 16$ when $x > 0$
b $x^2 = 25$ when $x < 0$
c $x^2 = 49$ when $x < 0$
d $x^2 = 196$ when $x > 0$

12 The exact value solutions to $x^2 = 5$, for example, are written as $x = \sqrt{5}$ or $-\sqrt{5}$. Alternatively, we can write $x = \pm\sqrt{5}$.

Write the exact value solutions to these equations.

a $x^2 = 11$
b $x^2 = 17$
c $x^2 = 33$
d $x^2 = 156$

13 The triad $(a, b, c) = (3, 4, 5)$ satisfies the equation $a^2 + b^2 = c^2$ because $3^2 + 4^2 = 5^2$.

a Decide if the following triads also satisfy the equation $a^2 + b^2 = c^2$.
  i $(6, 8, 10)$
  ii $(5, 12, 13)$
  iii $(1, 2, 3)$
  iv $(-3, -4, -5)$
  v $(-2, -3, -6)$
  vi $(-8, 15, 17)$

b Can you find any triads, for which a, b, c are positive integers, that satisfy $a^3 + b^3 = c^3$?

ENRICHMENT	–	–	14

Solving more complex linear equations

14 Compare this linear and quadratic equation solution.

$$\begin{array}{ccc} & 3x - 1 = 11 & \\ +1 & & +1 \\ & 3x = 12 & \\ \div 3 & & \div 3 \\ & x = 4 & \end{array} \qquad \begin{array}{ccc} & 3x^2 - 1 = 11 & \\ +1 & & +1 \\ & 3x^2 = 12 & \\ \div 3 & & \div 3 \\ & x^2 = 4 & \\ & x = \pm 2 & \end{array}$$

Now solve these quadratic equations.

a $2x^2 + 1 = 9$
b $5x^2 - 2 = 3$
c $3x^2 - 4 = 23$
d $-x^2 + 1 = 0$
e $-2x^2 + 8 = 0$
f $7x^2 - 6 = 169$
g $4 - x^2 = 0$
h $27 - 3x^2 = 0$
i $38 - 2x^2 = -34$

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2G Formulas and relationships EXTENSION

Interactive
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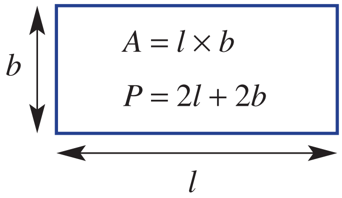
Walkthrough

Some equations are called formulas. A formula shows the relationship between two or more variables. For example, the area of a rectangle is related to its length and breadth, given by the formula $A = l \times b$, and its perimeter is given by $P = 2l + 2b$.



Although these are often used as a definition for the area and a definition for the perimeter, they are also just equations—two expressions written on either side of an equals sign.

Let’s start: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = l \times b$ and $P = 2l + 2b$.

- If $l = 10$ and $b = 7$, find the perimeter and the area.
- If $l = 2$ and $b = 8$, find the perimeter and the area.
- Notice that sometimes the area value is bigger than the perimeter value, and sometimes the area value is less than the perimeter value. If $l = 10$, is it possible to make the area and the perimeter values equal?
- If $l = 2$, can you make the area and the perimeter equal? Discuss.

Key ideas

- A **formula** or **rule** is an equation containing two or more pronumerals, one of which is the subject of the equation.
- The **subject** of a formula is a pronumeral that occurs by itself on the left-hand side. For example: V is the subject of $V = 3x + 2y$.
- To use a formula, substitute all the known values and then solve the equation to find the unknown value.

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Example 11 Applying a formula

Apply the formula for a rectangle's perimeter $P = 2l + 2b$ to find:

a P when $l = 4$ and $b = 7$

b l when $P = 40$ and $b = 3$

SOLUTION	EXPLANATION
a $P = 2l + 2b$	Write the formula.
$P = 2 \times 4 + 2 \times 7$	Substitute in the values for l and b.
$P = 22$	Simplify the result.
b $P = 2l + 2b$	Write the formula.
$40 = 2l + 2 \times 3$	Substitute in the values for P and b to obtain an equation.
$$\begin{tikzcd} 40 = 2l + 6 \arrow[d, "-6"', "-6"] \\ 34 = 2l \arrow[d, "\div 2"', "\div 2"] \\ 17 = l \end{tikzcd}$$	Solve the equation to obtain the value of l.

Exercise 2G EXTENSION

UNDERSTANDING AND FLUENCY	1–7	1, 3–9	5–9

1 **a** Substitute $x = 4$ into the expression $x + 7$.

b Substitute $a = 2$ into the expression $3a$.

c Substitute $p = 5$ into the expression $2p - 3$.

d Substitute $r = -4$ into the expression $7r$.

2 If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?

A $10 = 6 + x$ **B** $10 = 3m + 2$ **C** $2 = 3m + 10$ **D** $P = 30 + 2$

3 If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?

A $12 = 40 + Q$ **B** $L = 40 + 12$ **C** $12 = 410 + Q$ **D** $10 = 48 + Q$

Example 11a

4 Consider the rule $A = 4p + 7$.

a Find A if $p = 3$. **b** Find A if $p = 11$.

c Find A if $p = -2$. **d** Find A if $p = \frac{13}{2}$.

Example 11b

5 Consider the rule $U = 8a + 4$.

a Find a if $U = 44$. Set up and solve an equation.

b Find a if $U = 92$. Set up and solve an equation.

c If $U = -12$, find the value of a.

6 Consider the relationship $y = 2x + 4$.

a Find y if $x = 3$.

b By solving an appropriate equation, find the value of x that makes $y = 16$.

c Find the value of x if $y = 0$.

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7 Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

8 Assume that x and y are related by the equation $4x + 3y = 24$.

- **a** If $x = 3$, find y by solving an equation.
- **b** If $x = 0$, find the value of y.
- **c** If $y = 2$, find x by solving an equation.
- **d** If $y = 0$, find the value of x.

9 Consider the formula $G = k(2a + p) + a$.

- **a** If $k = 3$, $a = 7$ and $p = -2$, find the value of G.
- **b** If $G = 78$, $k = 3$ and $p = 5$, find the value of a.

PROBLEM-SOLVING AND REASONING	10, 13	10, 11, 13, 14	11, 12, 15, 16

10 The cost $\$C$ to hire a taxi for a trip of length d km is $C = 3 + 2d$.

- **a** Find the cost of a 10 km trip (i.e. for $d = 10$).
- **b** A trip has a total cost of \$161.
  - **i** Set up an equation by substituting $C = 161$.
  - **ii** Solve the equation systematically.
  - **iii** How far did the taxi travel? (Give your answer in km.)

11 In Rugby Union, 5 points are awarded for a try, 2 points for a conversion and 3 points for a penalty. A team's score is therefore $S = 5t + 2c + 3p$, where t is the number of tries, c is the number of conversions and p is the number of penalties.

- **a** Find the score if $t = 2$, $c = 1$ and $p = 2$.
- **b** If a team's score is 20 and they scored three tries and one penalty, how many conversions did they score?
- **c** In Rugby League, 4 points are awarded per try (t), 2 points for a conversion (c) and 1 point for a drop goal (d). Write a formula for the total score (S) of a Rugby League team.

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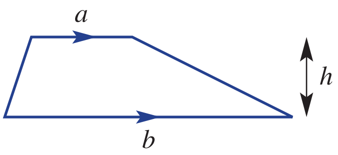
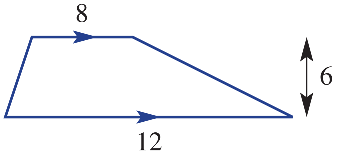
12 The formula for the area of a trapezium is $A = \frac{1}{2}h(a + b)$.

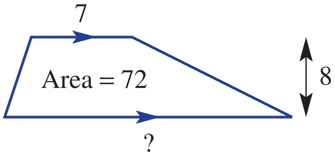
a Find the area of the trapezium shown at right.



b Find the value of h if $A = 20$, $a = 3$ and $b = 7$.



c Find the missing value in the trapezium shown at right.



13 Katy is a scientist who tries to work out the relationship between the volume of a gas, V mL, and its temperature, T°C. She takes a few measurements.

V	10	20
T	10	15

a What is a possible rule between V and T?
b Use your rule to find the volume at a temperature of 27°C.
c Prove that the rule $T = \frac{(V - 10)^2}{20} + 10$ would also work for Katy's results.

14 Consider the rule $G = 120 - 4p$.
a If p is between 7 and 11, what is the largest value of G?
b If p and G are equal, what value do they have?

15 Marie is a scientist who is trying to discover the relationship between the volume of a gas, V, its temperature, T, and its transparency, A. She takes a few measurements.

	Test 1	Test 2
V	10	20
A	2	5
T	15	12

Which one or more of the following rules are consistent with the experiment's results?

A $T = \frac{3V}{A}$ **B** $T = V + 2A$ **C** $T = 17 - A$

16 Temperatures in degrees Fahrenheit and degrees Celsius are related by the rule $F = 1.8C + 32$.
a By substituting $F = x$ and $C = x$, find a value such that the temperature in Fahrenheit and the temperature in Celsius are equal.
b By substituting $F = 2x$ and $C = x$, find a temperature in Celsius that doubles to give the temperature in Fahrenheit.
c Prove that there are no Celsius temperatures that can be multiplied by 1.8 to give the temperature in Fahrenheit.

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ENRICHMENT	–	–	17

Mobile phone plans

17 Two companies have mobile phone plans that factor in the number of minutes spent talking each month (t) and the total number of calls made (c).

Company A's cost, in cents: $A = 20t + 15c + 300$
Company B's cost, in cents: $B = 30t + 10c$

- **a** In one month 12 calls were made, totalling 50 minutes on the phone. Find the cost, in dollars, that company A and company B would have charged.
- **b** In another month, a company A user is charged \$15 (1500 cents) for making 20 calls. How long are these calls in total?
- **c** In another month, a company B user talks for 60 minutes in total and is charged \$21. What is the average length of these calls?
- **d** Briony notices one month that for her values of t and c, the two companies cost the same. Find a possible value of t and c that would make this happen.
- **e** Briony reveals that she made exactly 20 calls for the month in which the two companies' charges would be the same. How much time did she spend talking?

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2H Applications EXTENSION

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Although knowing how to solve equations is useful, it is important to be able to recognise when real-world situations can be thought of as equations. This is the case whenever it is known that two values are equal.

In this case, an equation can be constructed and solved. It is important to translate this solution into a meaningful answer within the real-world context.

Let's start: Sibling sum

Jerome and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 26, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if x is used for Jerome's age?
- How would the equation change if x is used for Jerome's sister's age instead?

Key ideas

- An equation can be used to describe any situation in which two values are equal.
- To solve a problem follow these steps.

1 Define pronumerals to stand for unknown numbers.

1 Let $x =$ Jerome's age.
$\therefore x + 4 =$ his sister's age

2 Write an equation to describe the problem.

2 $x + x + 4 = 32$

3 Solve the equation by inspection or systematically.

3
$$\begin{tikzcd} 2x + 4 = 32 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2x = 28 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x = 14 \end{tikzcd}$$

4 Make sure you answer the original question, including the correct units (e.g. dollars, years, cm).

4 Jerome is 14 years old and his sister is 18.

5 Check that your answer is reasonable.

Example 12 Solving a problem using equations

The weight of six identical books is 1.2 kg. What is the weight of one book?

SOLUTION

Let $b =$ weight of one book (in kg).

$6b = 1.2$

$$\begin{tikzcd} 6b = 1.2 \arrow[d, bend right, "\div 6"'] \arrow[d, bend left, "\div 6"] \\ b = 0.2 \end{tikzcd}$$

The books weigh 0.2 kg each, or 200 g each.

EXPLANATION

Define a pronumeral to stand for the unknown number.

Write an equation to describe the situation.

Solve the equation.

Answer the original question. It is not enough to give a final answer as 0.2; this is not the weight of a book, it is just a number.

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Example 13 Solving a harder problem using equations

Purchasing five apples and a $2.40 mango costs the same as purchasing seven apples and a mandarin that costs 60 cents. What is the cost of each apple?

SOLUTION

Let c = cost of one apple, in dollars.

$5c + 2.4 = 7c + 0.6$

$$\begin{array}{rcl} & 5c + 2.4 = 7c + 0.6 & \\ -5c \downarrow & & \downarrow -5c \\ & 2.4 = 2c + 0.6 & \\ -0.6 \downarrow & & \downarrow -0.6 \\ & 1.8 = 2c & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & 0.9 = c & \end{array}$$

Apples cost 90 cents each.

EXPLANATION

Define a pronumeral to stand for the unknown number.

Write an equation to describe the situation. Note that 60 cents must be converted to $0.6 to keep the units the same throughout the equation.

Solve the equation.

Answer the original question. It is not enough to give a final answer as 0.9; this is not the cost of an apple, it is just a number.

Example 14 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

SOLUTION

Let l = Luke's age.

$l + 2l = 60$

$$\begin{array}{rcl} & 3l = 60 & \\ \div 3 \downarrow & & \downarrow \div 3 \\ & l = 20 & \end{array}$$

Luke is 20 years old and Jane is 40 years old.

EXPLANATION

Define a pronumeral for the unknown value. Once Luke's age is found, we can double it to find Jane's age.

Write an equation to describe the situation. Note that Jane's age is $2l$ because she is twice as old as Luke.

Solve the equation by first combining like terms.

Answer the original question.

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Exercise 2H EXTENSION

UNDERSTANDING AND FLUENCY	1–6	2, 4–7	5–7

1 Match each of these worded descriptions (**a**–**e**) with an appropriate expression (**A**–**E**).

a	the sum of x and 3	**A**	$2x$
b	the cost of two apples if they cost \$$x$ each	**B**	$x + 1$
c	the cost of x oranges if they cost \$1.50 each	**C**	$3x$
d	triple the value of x	**D**	$x + 3$
e	one more than x	**E**	$1.5x$

2 For the following problems choose the equation to describe them.

a The sum of x and 5 is 11.

A $5x = 11$ **B** $x + 5 = 11$ **C** $x - 5 = 11$ **D** $11 - 5$

b The cost of four pens is \$12. Each pen costs \$$p$.

A $4 = p$ **B** $12p$ **C** $4p = 12$ **D** $12p = 4$

c Josh's age next year is 10. His current age is j.

A $j + 1 = 10$ **B** $j = 10$ **C** 9 **D** $j - 1 = 10$

d The cost of n pencils is \$10. Each pencil costs \$2.

A $n \div 10 = 2$ **B** 5 **C** $10n = 2$ **D** $2n = 10$

3 Solve the following equations.

a $5p = 30$ **b** $5 + 2x = 23$

c $12k - 7 = 41$ **d** $10 = 3a + 1$

Example 12

4 Jerry buys four cups of coffee for \$13.20.

a Choose a pronumeral to stand for the cost of one cup of coffee.

b Write an equation to describe the problem.

c Solve the equation systematically.

d Hence, state the cost of one cup of coffee.

5 A combination of six chairs and a table costs \$3000. The table alone costs \$1740.

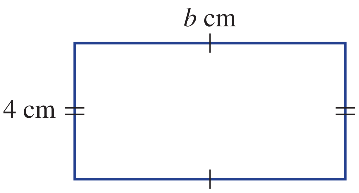
a Define a pronumeral for the cost of one chair.

b Write an equation to describe the problem.

c Solve the equation systematically.

d Hence, state the cost of one chair.

6 The perimeter of this rectangle is 72 cm.



a Write an equation to describe the problem, using b for the breadth.

b Solve the equation systematically.

c Hence, state the breadth of the rectangle.

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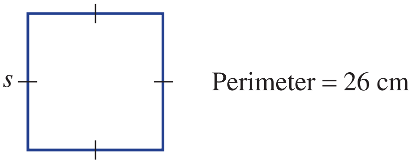
7 A plumber charges a \$70 call-out fee and \$52 per hour. The total cost of a particular visit is \$252.

- **a** Define a pronumeral to stand for the length of the visit, in hours.
- **b** Write an equation to describe the problem.
- **c** Solve the equation systematically.
- **d** State the length of the plumber's visit, giving your answer in minutes.

PROBLEM-SOLVING AND REASONING	9, 14	8–11, 14	11–15

Example 13 **8** A number is tripled, then 2 is added. This gives the same result as if the number were quadrupled. Set up and solve an equation to find the original number.

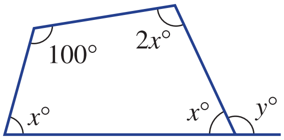
9 A square has a perimeter of 26 cm.



- **a** Solve an equation to find its side length.
- **b** Hence, state the area of the square.

Example 14 **10** Alison and Flynn's combined age is 40. Given that Flynn is 4 years older than Alison, write an equation and use it to find Alison's age.

11 Recall that in a quadrilateral the sum of all angles is 360°. Find the values of x and y in the diagram below.



12 The sum of three consecutive numbers is 357.

- **a** Use an equation to find the smallest of the three numbers.
- **b** What is the average of these three numbers?
- **c** If the sum of three consecutive numbers is 38 064, what is their average?

13 The breadth of a rectangular pool is 5 metres longer than the length. The perimeter of the pool is 58 metres.

- **a** Draw a diagram of this situation.
- **b** Use an equation to find the pool's length.
- **c** Hence, state the area of the pool.

14 The average of two numbers can be found by adding them and dividing by 2.

- **a** If the average of x and 10 is 30, what is the value of x?
- **b** If the average of x and 10 is 2, what is the value of x?
- **c** If the average of x and 10 is some number R, create a formula for the value of x.

15 Sometimes you are given an equation to solve a puzzle, but the solution of the equation is not actually possible for the situation. Consider these five equations.

- **A** $10x = 50$
- **B** $8 + x = 10$
- **C** $10 + x = 8$
- **D** $10x = 8$
- **E** $3x + 5 = x + 5$

- **a** You are told that the number of people in a room can be determined by solving an equation. Which of these equations could be used to give a reasonable answer?
- **b** If the length of an insect is given by the variable x cm, which of the equations could be solved to give a reasonable value of x?
- **c** Explain why equation **D** could not be used to find the number of people in a room but could be used to find the length of an insect.
- **d** Give an example of a puzzle that would make equation **C** reasonable.

ENRICHMENT	–	–	16

Unknown numbers

16 Find the unknown number using equations. The answers might not be whole numbers.

- **a** The average of a number and double the number is 25.5.
- **b** Adding 3 to twice a number is the same as subtracting 9 from half the number.
- **c** The average of a number and double the number gives the same result as adding 1 to the original number and then multiplying by one-third.
- **d** The product of 5 and a number is the same as the sum of 4 and twice the original number.
- **e** The average of five numbers is 7. When one more number is added, the average becomes 10. What number was added?

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2I Inequalities EXTENSION

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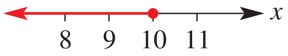
An **inequality** is like an equation but, instead of indicating that two expressions are equal, it indicates which of the two has the greater value.

For example, $2 + 4 < 7$, $3 \times 5 \geq 15$ and $x \leq 10$ are all inequalities. The first two are true, and the last one could be true or false depending on the value of x.

For instance, the numbers 9.8, 8.45, 7 and -120 all make this inequality true.

We could use a number line to represent all the values of x that make $x \leq 10$ a true statement.



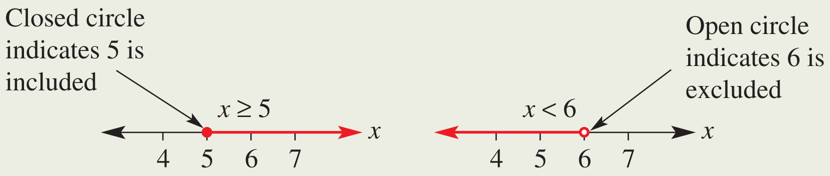
Let's start: Small sums

Two positive whole numbers are chosen: x and y. You are told that $x + y \leq 5$.

- How many possible pairs of numbers make this true? For example, $x = 2$ and $y = 1$ is one pair and it is different from $x = 1$ and $y = 2$.
- If $x + y \leq 10$, how many pairs are possible? Try to find a pattern rather than listing them all.
- If all you know about x and y is that $x + y > 10$, how many pairs of numbers could there be?

Key ideas

- An **inequality** is a statement of the form:
  - LHS > RHS (greater than). For example: $5 > 2$.
  - LHS ≥ RHS (greater than or equal to). For example: $7 \geq 7$ or $10 \geq 7$.
  - LHS < RHS (less than). For example: $2 < 10$.
  - LHS ≤ RHS (less than or equal to). For example: $5 \leq 5$ or $2 \leq 5$.
- Inequalities can be reversed: $3 < x$ and $x > 3$ are equivalent.
- Inequalities can be represented on a number line, using closed circles at the end points if the value is included, or open circles if it is excluded.



- A range can be represented as a segment on the number line, using appropriate closed and open end points.

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Example 15 Representing inequalities on a number line

Represent the following inequalities on a number line.

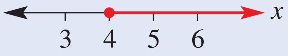
a $x \geq 4$

b $x < 6$

c $1 < x \leq 5$

SOLUTION

a $x \geq 4$

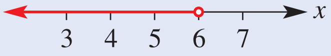


EXPLANATION

A circle is placed at 4 and then the arrow points to the right, towards all numbers greater than 4.

The circle is filled (closed) because 4 is included in the set.

b $x < 6$



A circle is placed at 6 and then the arrow points to the left, towards all numbers less than 6.

The circle is hollow (open) because 6 is not included in the set.

c $1 < x \leq 5$

Circles are placed at 1 and 5, and a line goes between them to indicate that all numbers in between are included.

The circle at 1 is open because the inequality is $<$ not $\leq$.

Example 16 Using inequalities to describe real-life situations

Describe the following situations as an inequality, using x to stand for the unknown quantity.

a Fatima is shorter than 160 cm.

b Craig is at least as old as Maria, who is 10.

c Roslyn's test score is between 40 and 50 inclusive.

SOLUTION	EXPLANATION
a $x < 160$	Using x to stand for Fatima's height, x must be less than 160.
b $x \geq 10$	Craig is at least 10, so his age is greater than or equal to 10.
c $40 \leq x \leq 50$	x is between 40 and 50. The word 'inclusive' tells us that 40 and 50 are both included, so $\leq$ is used (rather than $<$ if the word 'exclusive' is used).

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Exercise 2I EXTENSION

UNDERSTANDING AND FLUENCY	1–4, 5(½), 6	4, 5(½), 6, 7(½)	5(½), 6, 7(½)

1 Classify the following statements as true or false.

a $5 > 3$ **b** $7 < 5$ **c** $2 < 12$
d $13 < 13$ **e** $13 \leq 13$ **f** $4 \geq 2$

2 Match each of these inequalities (**a**–**d**) with the appropriate description (**A**–**D**).

a $x > 5$ **b** $x < 5$ **c** $x \geq 3$ **d** $x \leq 3$

A The number x is less than 5.
B The number x is greater than or equal to 3.
C The number x is less than or equal to 3.
D The number x is greater than 5.

3 For each of the following, state whether they make the inequality $x > 4$ true or false.

a $x = 5$ **b** $x = -2$ **c** $x = 4$ **d** $x = 27$

4 If $x = 12$, classify the following inequalities as true or false.

a $x > 2$ **b** $x < 11$ **c** $x \geq 13$ **d** $x \leq 12$

Example 15a,b

5 Represent the following inequalities on separate number lines.

a $x > 3$ **b** $x < 10$ **c** $x \geq 2$ **d** $x < 1$
e $x \geq 1$ **f** $x \geq 4$ **g** $x < -5$ **h** $x < -9$
i $x \leq 2$ **j** $x < -6$ **k** $x \geq -3$ **l** $x \leq 5$
m $10 > x$ **n** $2 < x$ **o** $5 \geq x$ **p** $-3 \leq x$

Example 15c

6 **a** List which of the following numbers make the inequality $2 \leq x < 7$ true.
8, 1, 3, 4, 6, 4.5, 5, 2.1, 7, 6.8, 2

b Represent the inequality $2 \leq x < 7$ on a number line.

7 Represent the following inequalities on separate number lines.

a $1 \leq x \leq 6$ **b** $4 \leq x < 11$ **c** $-2 < x \leq 6$ **d** $-8 \leq x \leq 3$
e $2 < x \leq 5$ **f** $-8 < x < -1$ **g** $7 < x \leq 8$ **h** $0 < x < 1$

PROBLEM-SOLVING AND REASONING	8, 11a,b	8, 10, 11	9–11, 12(½)

Example 16

8 For each of the following descriptions, choose an appropriate inequality from **A**–**H** below.

a Pranav is more than 12 years old.
b Marika is shorter than 150 cm.
c Matthew is at least 5 years old but he is younger than 10.
d The temperature outside is between −12°C and 10°C inclusive.

A $x < 150$ **B** $x < 12$ **C** $x > 12$ **D** $x \leq 150$
E $10 \leq x \leq -12$ **F** $-12 \leq x \leq 10$ **G** $5 \leq x < 10$ **H** $5 < x \leq 10$

9 It is known that Tim's age is between 20 and 25 inclusive, and Nick's age is between 23 and 27 inclusive.

a If t = Tim's age and n = Nick's age, write two inequalities to represent these facts.
b Represent both inequalities on the same number line.
c Nick and Tim are twins. What is the possible range of their ages? Represent this on a number line.

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10 At a certain school the following grades are awarded for different scores.

Score	$x \geq 80$	$60 \leq x < 80$	$40 \leq x < 60$	$20 \leq x < 40$	$x < 20$
Grade	A	B	C	D	E

a Convert the following scores into grades.

i 15 **ii** 79 **iii** 80 **iv** 60 **v** 30

b Emma got a B on one test, but her sister Rebecca got an A with just 7 more marks. What is the possible range for Emma's score?

c Hugh's mark earned him a C. If he had scored half this mark, what grade would he have earned?

d Alfred and Reuben earned a D and a C, respectively. If their scores were added together, what grade or grades could they earn?

e Michael earned a D and was told that if he doubled his mark he would have a B. What grade or grades could he earn if he got an extra 10 marks?

11 Sometimes multiple inequalities can be combined to a simpler inequality.

a Explain why the combination $x \geq 5, x \geq 7$ is equivalent to the inequality $x \geq 7$.

b Simplify the following pairs of inequalities to a single inequality.

i $x > 5, x \geq 2$ **ii** $x < 7, x < 3$ **iii** $x \geq 1, x > 1$

iv $x \leq 10, x < 10$ **v** $x > 3, x < 10$ **vi** $x > 7, x \leq 10$

c Simplify the following pairs of inequalities to a single inequality.

i $3 < x < 5, 2 < x < 7$ **ii** $-2 \leq x < 4, -2 < x \leq 4$

iii $7 < x \leq 10, 2 \leq x < 8$ **iv** $5 \leq x < 10, 9 \leq x \leq 11$

12 Some inequalities, when combined, have no solutions; some have one solution and some have infinitely many solutions. Label each of the following pairs using 0, 1 or ∞ (infinity) to state how many solutions they have.

a $x \geq 5$ and $x \leq 5$ **b** $x > 3$ and $x < 10$

c $x \geq 3$ and $x < 4$ **d** $x > 3$ and $x < 2$

e $-2 < x < 10$ and $10 < x < 12$ **f** $-3 \leq x \leq 10$ and $10 \leq x \leq 12$

g $x > 2.5$ and $x \leq 3$ **h** $x \geq -5$ and $x \leq -7$

ENRICHMENT	–	–	13, 14

Working within boundaries

13 If it is known that $0 \leq x \leq 10$ and $0 \leq y \leq 10$, which of the following inequalities must be true? Justify your answers.

a $x + y \leq 30$ **b** $2x \leq 20$

c $10 \leq 2y \leq 20$ **d** $x \times y \leq 100$

e $0 \leq x - y \leq 10$ **f** $x + 5y \leq 100$

14 If it is known that $0 \leq a \leq 10$, $0 \leq b \leq 10$ and $0 \leq c \leq 10$, what is the largest value that the following expressions could have?

a $a + b + c$ **b** $ab + c$

c $a(b + c)$ **d** $a \times b \times c$

e $a - b - c$ **f** $a - (b - c)$

g $3a + 4$ **h** $a - bc$

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2J Solving inequalities EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

Sometimes a problem arises in which an inequality is more complicated than something such as $x > 5$ or $y \le 40$. For instance, you could have the inequality $2x + 4 > 100$. To **solve** an inequality means to find all the values that make it true. For the inequality above, $x = 50$, $x = 90$ and $x = 10000$ are all part of the solution, but the solution is best described as $x > 48$, because any number greater than 48 will make the inequality true and any other number makes it false.

The rules for solving inequalities are very similar to those for equations: perform the same operation to both sides. The one exception occurs when multiplying or dividing by a negative number. We can do this, but we must flip the sign because of the following observation.

$$\begin{tikzcd} 5 > 2 \arrow[d, bend right, "\times -1"'] \arrow[d, bend left, "\times -1"] \\ -5 > -2 \end{tikzcd}$$

Incorrect method

$$\begin{tikzcd} 5 > 2 \arrow[d, bend right, "\times -1"'] \arrow[d, bend left, "\times -1"] \\ -5 < -2 \end{tikzcd}$$

Correct method

Let's start: Limousine costing

A limousine is hired for a wedding. The charge is a \$50 hiring fee plus \$200 per hour.

- If the total hire time is more than 3 hours, what can you say about the total cost?
- If the total cost is less than \$850 but more than \$450, what can you say about the total time the limousine was hired?

Key ideas

■ Given an inequality, an **equivalent** inequality can be obtained by:
- adding or subtracting an expression from both sides
- multiplying or dividing both sides by any positive number
- multiplying or dividing both sides by a negative number and reversing the inequality symbol
- swapping left-hand side and right-hand side and reversing the inequality symbol.

Example 1

$$\begin{tikzcd} 2x + 4 < 10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2x < 6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x < 3 \end{tikzcd}$$

No need to reverse.

Example 2

$$\begin{tikzcd} -4x + 2 < 6 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ -4x \boxed{<} 4 \arrow[d, bend right, "\div -4"'] \arrow[d, bend left, "\div -4"] \arrow[d] \\ x \boxed{>} -1 \end{tikzcd}$$

Sign is reversed because we are dividing by a negative number.

Example 3

$$\begin{tikzcd} 2 > x + 1 \arrow[d, bend right, "-1"'] \arrow[d, bend left, "-1"] \\ 1 \boxed{>} x \arrow[d] \\ x \boxed{<} 1 \end{tikzcd}$$

Sign is reversed because we have swapped the LHS & RHS.

Example 17 Solving inequalities

Solve the following inequalities.

a $5x + 2 < 47$ **b** $\dfrac{3 + 4x}{9} \geq 3$ **c** $15 - 2x > 1$

SOLUTION

a

$$\begin{tikzcd} 5x + 2 < 47 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 5x < 45 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x < 9 \end{tikzcd}$$

EXPLANATION

The inequality is solved in the same way as an equation is solved: 2 is subtracted from each side and then both sides are divided by 5. The sign does not change throughout.

b

$$\begin{tikzcd} \dfrac{3 + 4x}{9} \geq 3 \arrow[d, bend right, "\times 9"'] \arrow[d, bend left, "\times 9"] \\ 3 + 4x \geq 27 \arrow[d, bend right, "-3"'] \arrow[d, bend left, "-3"] \\ 4x \geq 24 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x \geq 6 \end{tikzcd}$$

The inequality is solved in the same way as an equation is solved. Both sides are first multiplied by 9 to eliminate 9 from the denominator.

c

$$\begin{tikzcd} 15 - 2x > 1 \arrow[d, bend right, "-15"'] \arrow[d, bend left, "-15"] \\ -2x > -14 \arrow[d, bend right, "\div -2"'] \arrow[d, bend left, "\div -2"] \\ x < 7 \end{tikzcd}$$

15 is subtracted from each side.

Both sides are divided by −2. Because this is a negative number, the inequality is reversed from > to <.

Exercise 2J EXTENSION

UNDERSTANDING AND FLUENCY	1–4, 5–7(½)	4, 5–7(½), 8	5–7(½), 8

1 If $x = 3$, classify the following inequalities as true or false.

a $x + 4 > 2$ **b** $5x \geq 10$

c $10 - x < 5$ **d** $5x + 1 < 16$

2 State whether the following choices of x make the inequality $2x + 4 \geq 10$ true or false.

a $x = 5$ **b** $x = 1$

c $x = -5$ **d** $x = 3$

3 **a** Copy and complete the following.

$$\begin{tikzcd} 2x < 8 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x < \_\_ \end{tikzcd}$$

b What is the solution to the inequality $2x < 8$?

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4 **a** Copy and complete the following.

$$\begin{tikzcd}
2x + 4 \geq 10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
2x \geq \underline{\qquad} \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x \geq \underline{\qquad}
\end{tikzcd}$$

b What is the solution to the inequality $2x + 4 \geq 10$?

c If $x = 7.1328$, is $2x + 4 \geq 10$ true or false?

Example 17a

5 Solve the following inequalities.

a $x + 9 > 12$

b $4l + 9 \geq 21$

c $8g - 3 > 37$

d $2r - 8 \leq 6$

e $9k + 3 > 21$

f $8s - 8 < 32$

g $8a - 9 > 23$

h $2 + n \geq 7$

i $9 + 2d \geq 23$

j $8 + 6h < 38$

k $10 + 7r \leq 24$

l $6 + 5y < 26$

Example 17b

6 Solve the following inequalities involving fractions.

a $\dfrac{d - 9}{2} > 10$

b $\dfrac{y + 4}{2} \leq 7$

c $\dfrac{x - 3}{4} > 2$

d $\dfrac{q + 4}{2} \leq 11$

e $\dfrac{2x + 4}{3} > 6$

f $\dfrac{7 + 3h}{2} < 5$

g $\dfrac{4 + 6p}{4} \geq 4$

h $\dfrac{8j + 2}{7} < 6$

Example 17c

7 Solve the following inequalities involving negative numbers. Remember to reverse the inequality when multiplying or dividing by a negative number.

a $6 - 2x < 4$

b $24 - 6s \geq 12$

c $43 - 4n > 23$

d $34 - 2j < 14$

e $2 - 9v \leq 20$

f $2 - 7j \leq 37$

g $48 - 8c \geq 32$

h $42 - 8h \leq 42$

i $7 - 8s > 31$

j $6 - 8v > 22$

k $10 - 4v \geq 18$

l $4 - 5v < 29$

8 Match the following inequalities (**a**–**d**) with their solutions depicted on a number line (**A**–**D**).

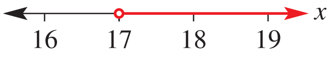
a $5x + 2 \geq 17$

b $\dfrac{x + 1}{6} > 3$

c $9(x + 4) < 45$

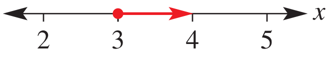
d $5 - 2x < 3$

A



B

C



D

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PROBLEM-SOLVING AND REASONING	9, 10, 12	9, 10, 12–14	10, 11, 13–15

9 Kartik buys four cartons of milk and a \$20 phone card. The total cost of his shopping was greater than \$25.

a If c is the cost of a carton of milk, write an inequality to describe the situation above.

b Solve the inequality to find the possible values of c.

c If the milk's cost is a multiple of 5 cents, what is the minimum price it could be?

10 In AFL football the score is given by $6g + b$, where g is the number of goals and b is the number of behinds. A team scored 4 behinds and their score was less than or equal to 36.

a Write an inequality to describe this situation.

b Solve the inequality.

c Given that the number of goals must be a whole number, what is the maximum number of goals that the team could have scored?

11 Recall that to convert degrees Celsius to degrees Fahrenheit the rule is $F = 1.8C + 32$. Pippa informs you that the temperature is between 59° and 68° Fahrenheit, inclusive.

a Solve $1.8C + 32 \geq 59$.

b Solve $1.8C + 32 \leq 68$.

c Hence, state the solution to $59 \leq 1.8C + 32 \leq 68$, giving your answer as a single inequality.

d Pippa later realises that the temperatures she gave you should have been doubled—the range is actually 118° to 136° Fahrenheit. State the range of temperatures in degrees Celsius, giving your answer as an inequality.

12 To say that a number x is positive is to say that $x > 0$.

a If $10x - 40$ is positive, find all the possible values of x. That is, solve $10x - 40 > 0$.

b Find all k values that make $2k - 6$ positive.

c If $3a + 6$ is negative and $10 - 2a$ is positive, what are the possible values of a?

d If $5a + 10$ is negative and $10a + 30$ is positive, what are the possible values of a?

13 **a** Prove that if $5x - 2$ is positive, then x is positive.

b Prove that if $2x + 6$ is positive, then $x + 5$ is positive.

c Is it possible that $10 - x$ is positive and $10 - 2x$ is positive but $10 - 3x$ is negative? Explain.

d Is it possible that $10 - x$ is positive and $10 - 3x$ is positive but $10 - 2x$ is negative? Explain.

14 A puzzle is given below with four clues.

Clue A: $3x > 12$
Clue B: $5 - x \leq 4$
Clue C: $4x + 2 \leq 42$
Clue D: $3x + 5 < 36$

a Two of the clues are unnecessary. State which two clues are not needed.

b Given that x is a whole number divisible by 4, what is the solution to the puzzle?

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15 Multiplying or dividing by a negative number can be avoided by adding the pronumeral to the other side of the equation. For example:

$$\begin{tikzcd}
-4x + 2 < 6 \arrow[d, bend right, "+4x"'] \arrow[d, bend left, "+4x"] \\
2 < 6 + 4x \arrow[d, bend right, "-6"'] \arrow[d, bend left, "-6"] \\
-4 < 4x \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
-1 < x
\end{tikzcd}$$

This can be rearranged to $x > -1$, which is the same as the answer obtained using the method shown in the Key ideas. Use this method to solve the following inequalities.

a $-5x + 20 < 10$
b $12 - 2a \geq 16$
c $10 - 5b > 25$
d $12 < -3c$

ENRICHMENT	–	–	16

Pronumerals on both sides

16 This method for solving inequalities allows both sides to have any expression subtracted from them. This allows us to solve inequalities with pronumerals on both sides. For example:

$$\begin{tikzcd}
12x + 5 \leq 10x + 11 \arrow[d, bend right, "-10x"'] \arrow[d, bend left, "-10x"] \\
2x + 5 \leq 11
\end{tikzcd}$$

This can then be solved by the usual method. If we end up with a pronumeral on the right-hand side, such as $5 < x$, the solution is rewritten as $x > 5$.

Solve the following inequalities.

a $12x + 5 \leq 10x + 11$
b $7a + 3 > 6a$
c $5 - 2b \geq 3b - 35$
d $7c - 5 < 10c - 11$
e $14k > 200 + 4k$
f $9g + 40 < g$
g $4(2a + 1) > 7a + 12$
h $2(3k - 5) \leq 5k - 1$
i $2(3p + 1) > 4(p + 2) + 3$

Tiling a pool edge

The Sunny Swimming Pool Company constructs rectangular pools, each 4 m in breadth with various lengths. There are non-slip square tiles, 50 cm by 50 cm, that can be used for the external edging around the pool perimeter where swimmers walk.

1 Draw a neat diagram illustrating the pool edge with one row of flat tiles bordering the perimeter of a rectangular pool 4 m in breadth and 5 m long.

2 Develop a table showing the dimensions of rectangular pools, each with breadth 4 m and ranging in length from 5 m to 10 m. Add a column for the total number of tiles required for each pool when one row of flat tiles borders the outside edge of the pool.

3 Develop an algebraic rule for the total number of tiles, T, required for bordering the perimeter of rectangular pools that are 4 m in breadth and x m long.

4 **a** Use your algebraic rule to form equations for each of the following total number of tiles when a single row of flat tiles is used for pool edging.
- **i** 64 tiles
- **ii** 72 tiles
- **iii** 80 tiles
- **iv** 200 tiles

b By manually solving each equation, determine the lengths of the various pools that use each of the above numbers of tiles.

5 Develop an algebraic rule for the total number of tiles, T, required for two rows of flat tiles bordering rectangular pools that are 4 m in breadth and x m long.

6 **a** Use your algebraic rule to form equations for each of the following total number of tiles when two rows of flat tiles are used for pool edging.
- **i** 96 tiles
- **ii** 120 tiles
- **iii** 136 tiles
- **iv** 248 tiles

b By manually solving each equation, determine the lengths of the pools that use these numbers of tiles.

7 Determine an algebraic rule for the total number of tiles, T, required for n rows of flat tiles bordering rectangular pools that are 4 m in breadth and x m in length.

8 Use this algebraic rule to form equations for each of the following pools, and then manually solve each equation to determine the length of each pool.

Pool	Breadth of pool 4 m	Length of pool, x m	Number of layers, n	Total number of tiles, T
A	4		3	228
B	4		4	288
C	4		5	500

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Hiring a tuk-tuk taxi

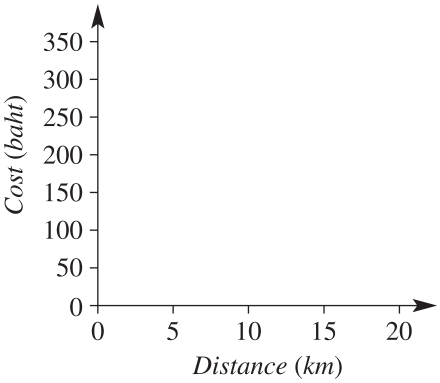
The cost of hiring a tuk-tuk taxi in Thailand is 50 baht hiring cost (flag fall) plus 15 baht every 2 km.

1 Using a table, show the costs of hiring a tuk-tuk taxi for 2, 4, 6, 8, 10 and 20 km.

2 On graph paper, draw a graph of the hiring cost (vertical axis) versus the distance (horizontal axis) travelled. Use the grid at right as a guide.

3 On your graph, rule lines to show the distance travelled for hiring costs of 125 baht and 170 baht.

4 Write an algebraic formula for the hiring cost, C, of an x km ride in a tuk-tuk taxi.



5 **a** Use your algebraic formula to form equations for each of the following trip costs.
  i 320 baht
  ii 800 baht
  iii 1070 baht
  b Use your equation-solving procedures to calculate the distance travelled for each trip.

6 A tourist is charged A\$100 for the 116 km trip from Pattaya Beach to the Bangkok International airport. If 1 baht can be bought for A\$0.03318, do you think that this is a fair price? Justify your answer with calculations.

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1 Find the unknown value in the following puzzles.

a A number is increased by 2, then doubled, then increased by 3 and then tripled. The result is 99.

b A number is doubled and then one-third of the number is subtracted. The result is 5 larger than the original number.

c In 5 years' time Alf will be twice as old as he was 2 years ago. How old is Alf now?

d The price of a shirt is increased by 10% for GST and then decreased by 10% during a sale. The new price is $44. What was the original price?

e One-third of a number is subtracted from 10 and then the result is tripled, giving the original number back again.

2 Consider the following 'proof' that $0 = 1$.

$$\begin{tikzcd}
2x + 5 = 3x + 5 \arrow[d, bend right, "-5"'] \arrow[d, bend left, "-5"] \\
2x = 3x \arrow[d, bend right, "\div x"'] \arrow[d, bend left, "\div x"] \\
2 = 3 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\
0 = 1
\end{tikzcd}$$

a Which step caused the problem in this proof? (*Hint*: consider the actual solution to the equation.)

b Prove that $0 = 1$ is equivalent to the equation $22 = 50$ by adding, subtracting, multiplying and dividing both sides.

3 Find all the values of x that would make both these inequalities false:

$19 - 2x < 5$ and $20 + x > 4x + 2$

4 The following six expressions could appear on either side of an equation. Using just two of the expressions, create an equation that has no solution.

$2x$ $3x + 1$ $7x + 4$ $4(x + 7)$ $2 + 3(x + 1)$ $2(3 + x) - 1$

5 A pair of scales only registers weights between 100 kg and 150 kg, but it allows more than one person to get on at a time.

a If three people weigh themselves in pairs and the first pair weighs 117 kg, the second pair weighs 120 kg and the third pair weighs 127 kg, what are their individual weights?

b If another three people weigh themselves in pairs and get weights of 108 kg, 118 kg and 130 kg, what are their individual weights?

c A group of four children, who all weigh less than 50 kg, weigh themselves in groups of three, getting the weights 122 kg, 128 kg, 125 kg and 135 kg. How much does each child weigh?

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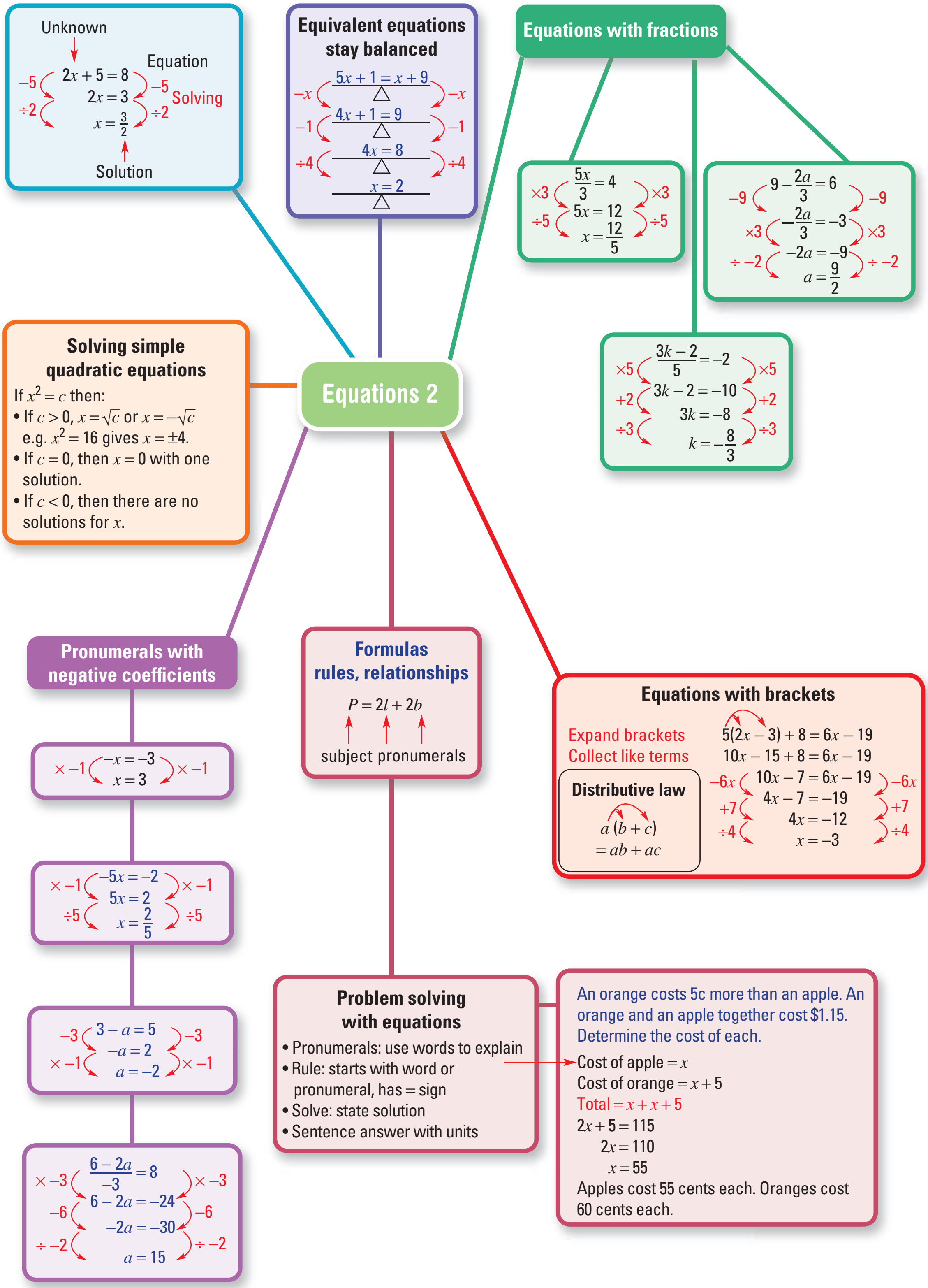
Chapter summary



Equations 2

Unknown

Equation

$2x + 5 = 8$

-5 $\quad$ -5

$2x = 3$

$\div 2$ $\quad$ $\div 2$

$x = \frac{3}{2}$

Solving

Solution

Equivalent equations stay balanced

$5x + 1 = x + 9$

$-x$ $\quad$ $-x$

$4x + 1 = 9$

-1 $\quad$ -1

$4x = 8$

$\div 4$ $\quad$ $\div 4$

$x = 2$

Equations with fractions

$\frac{5x}{3} = 4$

$\times 3$ $\quad$ $\times 3$

$5x = 12$

$\div 5$ $\quad$ $\div 5$

$x = \frac{12}{5}$

$9 - \frac{2a}{3} = 6$

-9 $\quad$ -9

$-\frac{2a}{3} = -3$

$\times 3$ $\quad$ $\times 3$

$-2a = -9$

$\div -2$ $\quad$ $\div -2$

$a = \frac{9}{2}$

$\frac{3k - 2}{5} = -2$

$\times 5$ $\quad$ $\times 5$

$3k - 2 = -10$

$+2$ $\quad$ $+2$

$3k = -8$

$\div 3$ $\quad$ $\div 3$

$k = -\frac{8}{3}$

Solving simple quadratic equations

If $x^2 = c$ then:

- If $c > 0$, $x = \sqrt{c}$ or $x = -\sqrt{c}$ e.g. $x^2 = 16$ gives $x = \pm 4$.
- If $c = 0$, then $x = 0$ with one solution.
- If $c < 0$, then there are no solutions for x.

Pronumerals with negative coefficients

$-x = -3$

$\times -1$ $\quad$ $\times -1$

$x = 3$

$-5x = -2$

$\times -1$ $\quad$ $\times -1$

$5x = 2$

$\div 5$ $\quad$ $\div 5$

$x = \frac{2}{5}$

$3 - a = 5$

-3 $\quad$ -3

$-a = 2$

$\times -1$ $\quad$ $\times -1$

$a = -2$

$\frac{6 - 2a}{-3} = 8$

$\times -3$ $\quad$ $\times -3$

$6 - 2a = -24$

-6 $\quad$ -6

$-2a = -30$

$\div -2$ $\quad$ $\div -2$

$a = 15$

Formulas rules, relationships

$P = 2l + 2b$

subject pronumerals

Equations with brackets

Expand brackets $\quad 5(2x - 3) + 8 = 6x - 19$

Collect like terms $\quad 10x - 15 + 8 = 6x - 19$

$10x - 7 = 6x - 19$

$-6x$ $\quad$ $-6x$

$4x - 7 = -19$

$+7$ $\quad$ $+7$

$4x = -12$

$\div 4$ $\quad$ $\div 4$

$x = -3$

Distributive law

$a(b + c)$

$= ab + ac$

Problem solving with equations

- Pronumerals: use words to explain
- Rule: starts with word or pronumeral, has = sign
- Solve: state solution
- Sentence answer with units

An orange costs 5c more than an apple. An orange and an apple together cost $1.15. Determine the cost of each.

Cost of apple $= x$

Cost of orange $= x + 5$

Total $= x + x + 5$

$2x + 5 = 115$

$2x = 110$

$x = 55$

Apples cost 55 cents each. Oranges cost 60 cents each.

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Multiple-choice questions

1 Which one of the following equations is true?

A $20 \div (2 \times 5) = 20 \div 10$
B $12 - 8 = 2$
C $5 \div 5 \times 5 = \frac{1}{5}$
D $15 + 5 \times 4 = 20 + 4$
E $10 - 2 \times 4 = 4 \times 2 - 5$

2 Which one of the following equations does not have the solution $x = 9$?

A $4x = 36$ **B** $x + 7 = 16$ **C** $\frac{x}{3} = 3$
D $x + 9 = 0$ **E** $14 - x = 5$

3 The solution to the equation $3a + 8 = 29$ is:

A $a = 21$ **B** $a = 12\frac{1}{3}$ **C** $a = 7$ **D** $a = 18$ **E** $a = 3$

4 'Three less than half a number is the same as four more than the number' can be expressed as an equation by:

A $\frac{x}{2} - 3 = 4x$ **B** $\frac{(x - 3)}{2} = x + 4$ **C** $\frac{x}{2} - 3 = x + 4$
D $\frac{x}{2} + 3 = x + 4$ **E** $\frac{x}{2} - 3 + 4$

5 The solution to the equation $-3(m + 4) = 9$ is:

A $m = 7$ **B** $m = -7$ **C** $m = -1$ **D** $m = 1$ **E** $m = -3$

6 If $12 + 2x = 4x - 6$, then x equals:

A 8 **B** 9 **C** 12 **D** 15 **E** 23

7 Which equation does NOT have the same solution as the others?

A $2x + 4 = 0$ **B** $2x = -4$ **C** $0 = 4 + 2x$ **D** $2x = 4$ **E** $2(x + 2) = 0$

8 Which one of the following equations has the solution $n = 10$?

A $4 - n = 6$ **B** $2n + 4 = 3n + 5$ **C** $50 - 4n = 90$
D $2(n + 5) = 3(n + 1)$ **E** $70 - 6n = n$

9 Malcolm solves an equation as follows.

$5 - 2x + 4 = 11$ line 1
$1 - 2x = 11$ line 2
$-2x = 10$ line 3
$x = -5$ line 4

Choose the correct statement.

A The only mistake was made in moving from line 1 to line 2.
B The only mistake was made in moving from line 2 to line 3.
C The only mistake was made in moving from line 3 to line 4.
D A mistake was made in moving from line 1 to line 2 and from line 3 to line 4.
E No mistakes were made.

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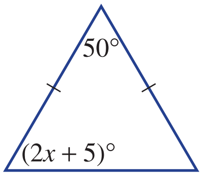
10 The value of x in this isosceles triangle is:



- **A** 30
- **B** 45
- **C** 62.5
- **D** 65
- **E** 130

Short-answer questions

1 Classify each of the following as true or false.

- **a** If $3x = 6$ then $x = 3$.
- **b** If $a = 21$ then $a + a - a = a$.
- **c** $5 \times 4 = 10 + 10$

2 Find the solutions to these equations.

- **a** $4m = 16$
- **b** $\frac{m}{3} = -4$
- **c** $9 - a = 10$
- **d** $10m = 2$
- **e** $2m + 6 = 36$
- **f** $a + a = 12$

3 Write an equation to represent each of the following statements. You do not need to solve the equations.

- **a** Double m plus 3 equals three times m.
- **b** The sum of n and four is multiplied by five; the answer is 20.
- **c** The sum of two consecutive even numbers, the first being x, is 74.

4 For each equation below, state the first operation you would apply to both sides.

- **a** $15 + 2x = 45$
- **b** $\frac{x}{2} - 5 = 6$
- **c** $3a + 3 = 2a + 10$

5 Solve:

- **a** $a + 8 = 12$
- **b** $6 - y = 15$
- **c** $2x - 1 = -9$
- **d** $5 + 3x = 17$
- **e** $20 - 4x = 12$
- **f** $8a - 8 = 0$

6 Solve the following equations systematically.

- **a** $\frac{m}{3} = -2$
- **b** $\frac{5x}{2} = 20$
- **c** $\frac{-2y}{3} = 12$
- **d** $-5 = \frac{k + 3}{11}$
- **e** $\frac{8 - 2w}{3} = 4$
- **f** $13 = \frac{2a - 8}{6}$

7 Solve these equations by first finding a common denominator.

- **a** $2 = \frac{x}{8} + \frac{1}{4}$
- **b** $\frac{5x}{3} - \frac{3x}{2} = 1$
- **c** $\frac{x}{2} - \frac{x}{4} = 10$

8 Solve these equations.

- **a** $8a - 3 = 7a + 5$
- **b** $2 - 3m = m$
- **c** $3x + 5 = 7x - 11$
- **d** $10 - 4a = a$
- **e** $2x + 8 = x$
- **f** $4x + 9 = 5x$

9 Solve the following equations by first expanding the brackets.

- **a** $2(x + 5) = 16$
- **b** $3(x + 1) = -9$
- **c** $18 = -2(2x - 1)$
- **d** $3(2a + 1) = 27$
- **e** $5(a + 4) = 3(a + 2)$
- **f** $2(3m + 5) = 16 + 3(m + 2)$
- **g** $8(3 - a) + 16 = 64$
- **h** $2x + 10 = 4(x - 6)$
- **i** $4(2x - 1) - 3(x - 2) = 10x - 3$

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10 **a** The sum of three consecutive numbers is 39. First write an equation and then find the value of the smallest number.

b Four times a number less 5 is the same as double the number plus 3. Write an equation to find the number.

c The difference between a number and three times that number is 17. What is the number?

11 Solve the following simple quadratic equations. Round to two decimal places where necessary.

a $x^2 = 4$ **b** $x^2 = 100$ **c** $a^2 = 49$

d $x^2 = 8$ **e** $y^2 = 39$ **f** $b^2 = 914$

12 State the number of solutions for x in these equations.

a $2x = 6$ **b** $x^2 = 16$ **c** $x^2 = 0$

d $5x^2 = 0$ **e** $4x - 1 = 3$ **f** $x^2 = -3$

Extended-response questions

1 To upload an advertisement to the www.searches.com.au website it costs \$20 and then 12 cents whenever someone clicks on it.

a Write a formula relating the total cost (\$$S$) and the number of clicks (n) on the advertisement.

b If the total cost is \$23.60, write and solve an equation to find out how many times the advertisement has been clicked on.

c To upload to the www.yousearch.com.au website it costs \$15 initially and then 20 cents for every click. Write a formula for the total cost, \$$Y$, when the advertisement has been clicked n times.

d If a person has at most \$20 to spend, what is the maximum number of clicks they can afford for their advertisement at yousearch.com.au?

e Set up and solve an equation to find the minimum number of clicks for which the total cost of posting an advertisement to searches.com.au is less than the cost of posting to yousearch.com.au.

2 Mahni plans to spend the next 12 weeks saving some of her income. She will save \$$x$ a week for the first 6 weeks and then \$$(2x - 30)$ a week for the following 6 weeks.

a Write an expression for the total amount Mahni saves over the 12 weeks.

b If Mahni managed to save \$213 in the first 6 weeks, how much did she save:

i in the first week? **ii** in the 7th week?

iii in total over the 12 weeks?

c If Mahni wishes to save a total of \$270, write and solve an equation to find out how much she would have to save in the first week.

d If Mahni wishes to save the same amount in the first 6 weeks as in the last 6 weeks, how much should she save each week?

e In the end Mahni decides that she does not mind exactly how much she saves but wants it to be between \$360 and \$450. State the range of x values that would achieve this goal, giving your answer as an inequality.

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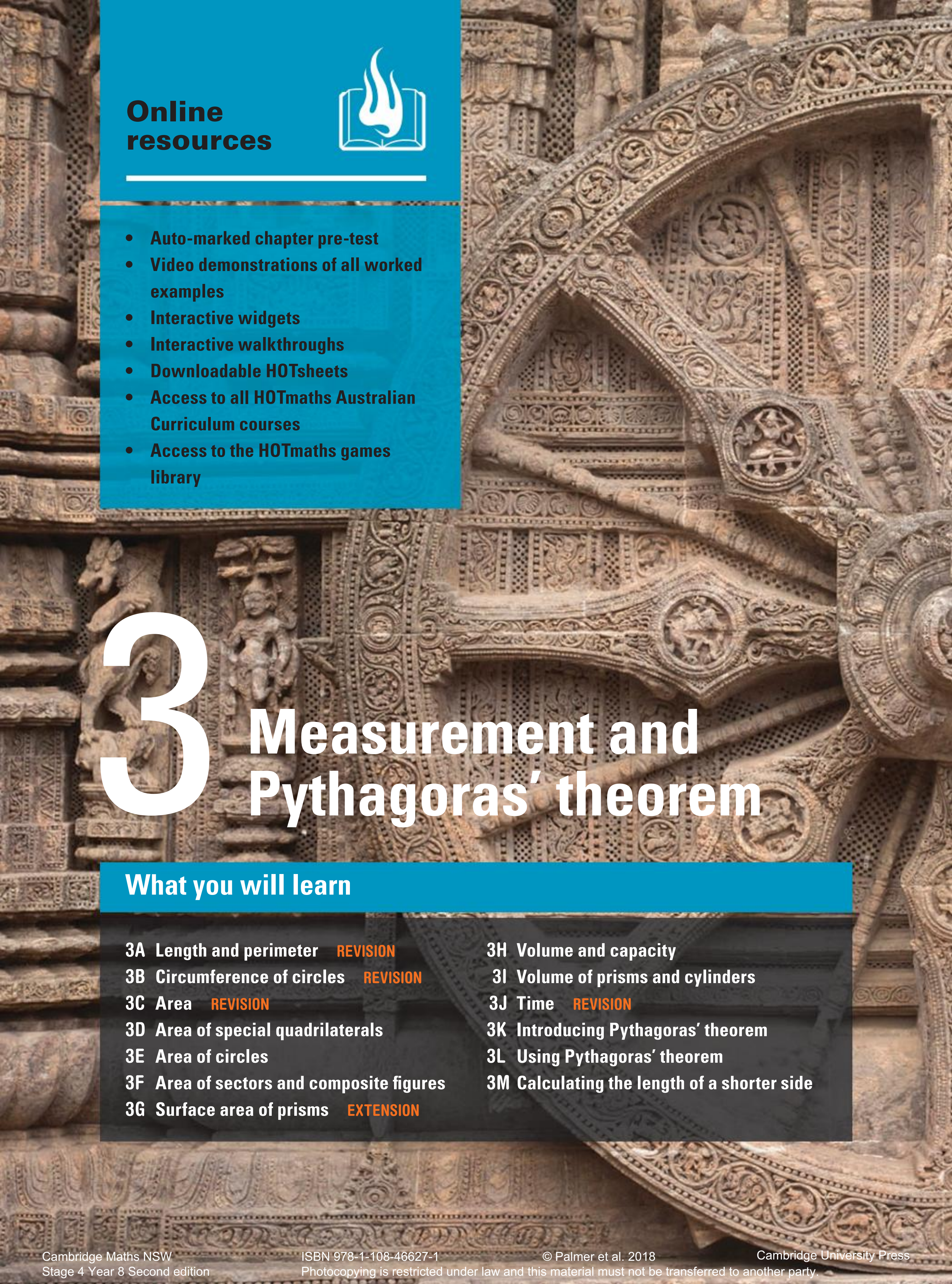


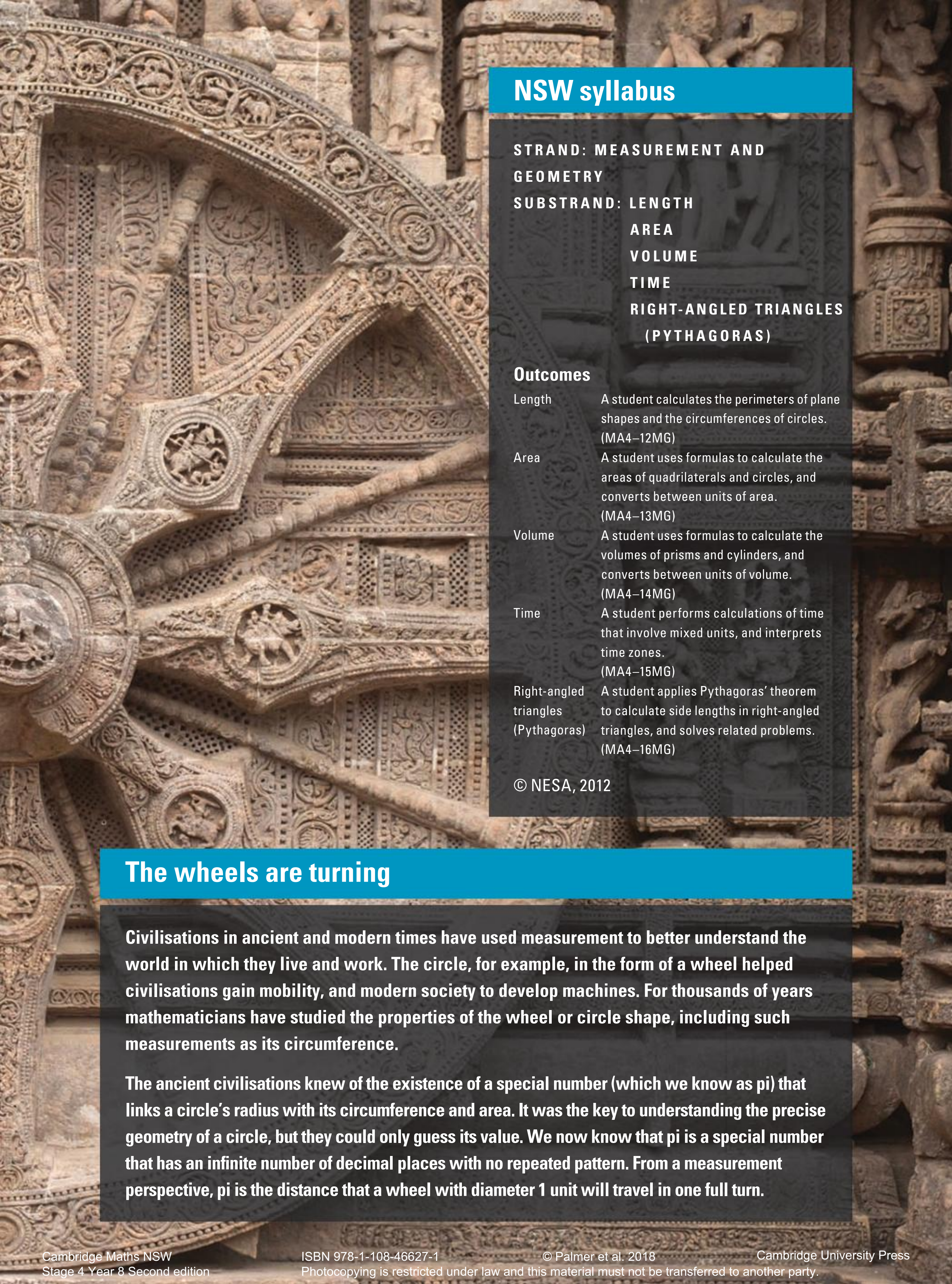
Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

3

Measurement and Pythagoras' theorem

What you will learn



NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY
SUBSTRAND: LENGTH
AREA
VOLUME
TIME
RIGHT-ANGLED TRIANGLES (PYTHAGORAS)

Outcomes

Length	A student calculates the perimeters of plane shapes and the circumferences of circles. (MA4–12MG)
Area	A student uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area. (MA4–13MG)
Volume	A student uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume. (MA4–14MG)
Time	A student performs calculations of time that involve mixed units, and interprets time zones. (MA4–15MG)
Right-angled triangles (Pythagoras)	A student applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems. (MA4–16MG)

The wheels are turning

Civilisations in ancient and modern times have used measurement to better understand the world in which they live and work. The circle, for example, in the form of a wheel helped civilisations gain mobility, and modern society to develop machines. For thousands of years mathematicians have studied the properties of the wheel or circle shape, including such measurements as its circumference.

The ancient civilisations knew of the existence of a special number (which we know as pi) that links a circle's radius with its circumference and area. It was the key to understanding the precise geometry of a circle, but they could only guess its value. We now know that pi is a special number that has an infinite number of decimal places with no repeated pattern. From a measurement perspective, pi is the distance that a wheel with diameter 1 unit will travel in one full turn.

1 Convert these measurements to the units shown in the brackets.

a 3 m (cm) **b** 20 cm (mm) **c** 1.8 km (m)

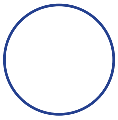
d 0.25 m (cm) **e** 35 mm (cm) **f** 4200 m (km)

g 500 cm (m) **h** 100 mm (m) **i** 2 minutes (seconds)

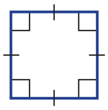
j 3 L (mL) **k** 4000 mL (L) **l** 3000 g (kg)

2 Name these shapes.

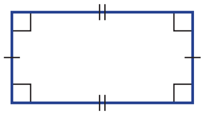
a



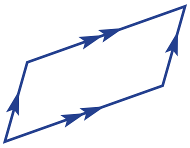
b



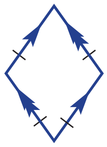
c



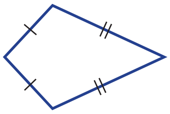
d



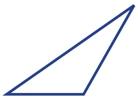
e



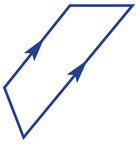
f



g



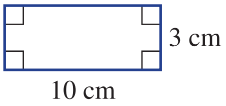
h



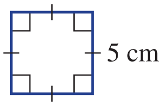
3 Find the area of these rectangles and triangles.

Remember: Area (rectangle) = $l \times b$ and Area (triangle) = $\frac{1}{2}bh$.

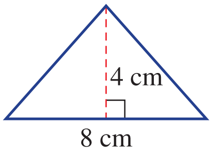
a



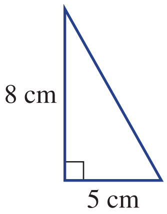
b



c

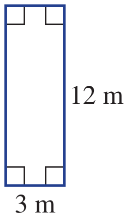


d

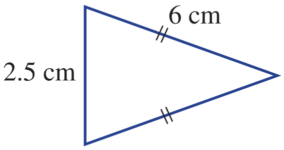


4 Find the perimeter of these shapes.

a



b



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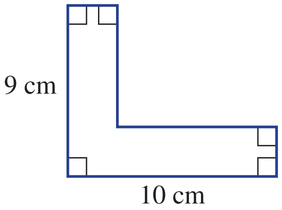


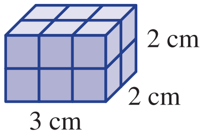
5 Evaluate the following.

a $\frac{1}{2} \times 5 \times 4$ **b** $\frac{1}{2}(2 + 7) \times 6$ **c** 5^2 **d** 11^2

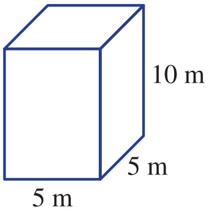
e $\frac{1}{2}(22 + 17) \times 3$ **f** $\sqrt{36}$ **g** $\sqrt{81}$ **h** $\sqrt{144}$

6 Using $V = l \times b \times h$, find the volume of these rectangular prisms.

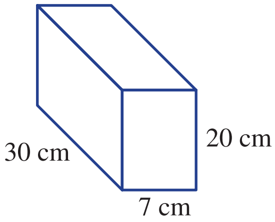
a



b



c



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3A Length and perimeter REVISION

Interactive
Widgets
HOTsheets
Walkthrough

For thousands of years, civilisations have found ways to measure length. The Egyptians, for example, used the cubit (length of an arm from the elbow to the tip of the middle finger); the Romans used the pace (5 feet); and the English developed their imperial system using inches, feet, yards and miles. The modern-day system used in Australia (and most other countries) is the metric system, which was developed in France in the 1790s and is based on the unit called the metre. We use units of length to describe the distance between two points, or the distance around the outside of a shape, called the perimeter.

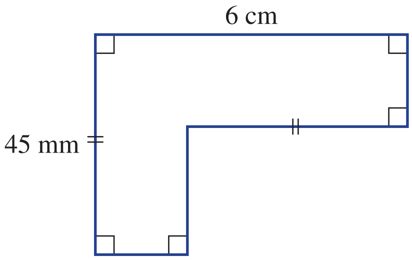
Let's start: Provide the perimeter

In this diagram some of the lengths are given. Three students are asked to find the perimeter.

- Will says that you cannot work out some lengths and so the perimeter cannot be found.
- Sally says that there is enough information and the answer is 9 + 12 = 21 cm.
- Greta says that there is enough information but the answer is 90 + 12 = 102 cm.

Who is correct?

Discuss how each person arrived at their answer.

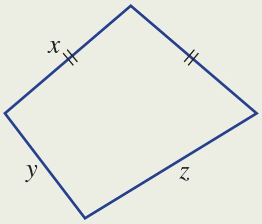


Key ideas

- The common metric units of length include the **kilometre** (km), the **metre** (m), the **centimetre** (cm) and the **millimetre** (mm).

$$\begin{tikzcd}
\text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"]
\end{tikzcd}$$

- **Perimeter** is the distance around a closed shape.
  - All units must be of the same type when calculating the perimeter.
  - Sides with the same type of markings (dashes) are of equal length.



$$P = 2x + y + z$$

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Example 1 Converting length measurements

Convert these lengths to the units shown in the brackets.

a 5.2 cm (mm) **b** 85 000 cm (km)

SOLUTION

a $5.2\text{ cm} = 5.2 \times 10$
$= 52\text{ mm}$

b $85\,000\text{ cm} = 85\,000 \div 100 \div 1000$
$= 0.85\text{ km}$

EXPLANATION

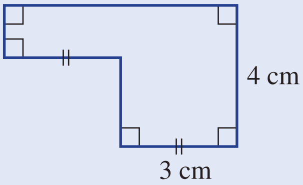
1 cm = 10 mm so multiply by 10.

$\text{cm} \xrightarrow{\times 10} \text{mm}$

1 m = 100 cm and 1 km = 1000 m, so divide by 100 and 1000.

Example 2 Finding perimeters

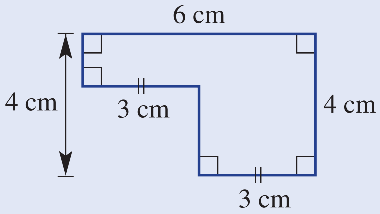
Find the perimeter of this shape.



SOLUTION

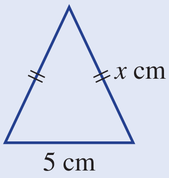
$P = 2 \times (3 + 3) + 2 \times 4$
$= 12 + 8$
$= 20\text{ cm}$

EXPLANATION



Example 3 Finding an unknown length

Find the unknown value x in this triangle if the perimeter is 19 cm.



$P = 19\text{ cm}$

SOLUTION

$2x + 5 = 19$
$2x = 14$
$x = 7$

EXPLANATION

$2x + 5$ makes up the perimeter.
$2x$ is the difference between 19 and 5.
If $2x = 14$ then $x = 7$ since $2 \times 7 = 14$.

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Exercise 3A REVISION

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	4, 5–7(½)	5–7(½)

1 Write the missing number in these sentences.

a There are ___ mm in 1 cm. **b** There are ___ cm in 1 m.

c There are ___ m in 1 km. **d** There are ___ cm in 1 km.

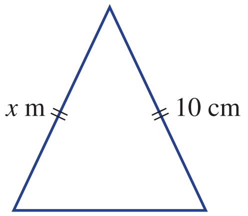
e There are ___ mm in 1 m. **f** There are ___ mm in 1 km.

2 Evaluate the following.

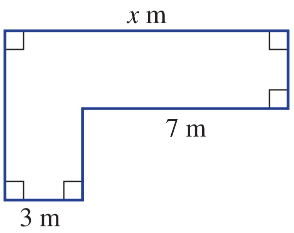
a 10×100 **b** 100×1000 **c** $10 \times 100 \times 1000$

3 Find the value of x in these diagrams.

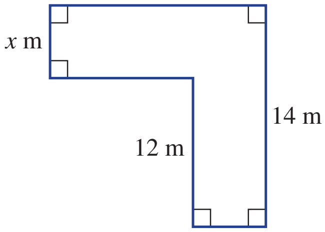
a



b



c



4 Find the perimeter of these quadrilaterals.

a square with side length 3 m

b rectangle with side lengths 4 cm and 7 cm

c rhombus with side length 2.5 mm

d parallelogram with side lengths 10 km and 12 km

e kite with side lengths 0.4 cm and 0.3 cm

f trapezium with side lengths 1.5 m, 1.1 m, 0.4 m and 0.6 m

Example 1

5 Convert these measurements to the units shown in the brackets.

a 3 cm (mm) **b** 6.1 m (cm) **c** 8.93 km (m) **d** 3 m (mm)

e 0.0021 km (m) **f** 320 mm (cm) **g** 9620 m (km) **h** 38 000 cm (km)

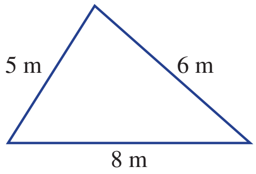
i 0.0043 m (mm) **j** 0.0204 km (cm) **k** 23 098 mm (m) **l** 342 000 cm (km)

m 194 300 mm (m) **n** 10 000 mm (km) **o** 0.02403 m (mm) **p** 994 000 mm (km)

Example 2

6 Find the perimeter of these shapes.

a



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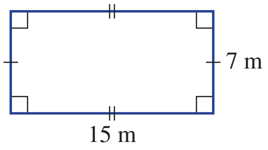

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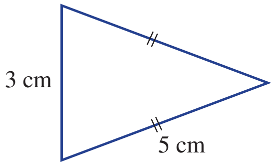

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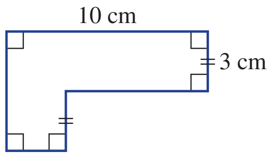

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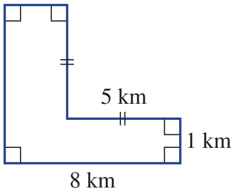

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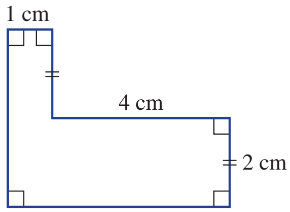

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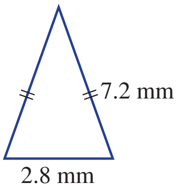

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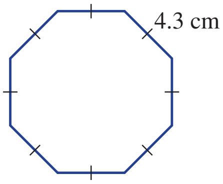
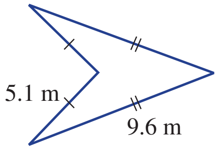


i



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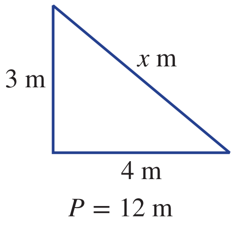
Example 3

7 Find the unknown value x in these shapes with the given perimeter (P).

a



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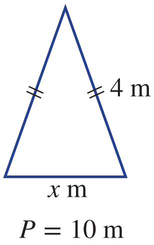

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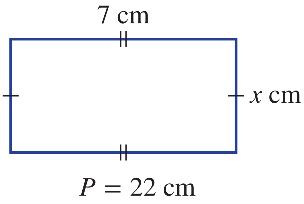

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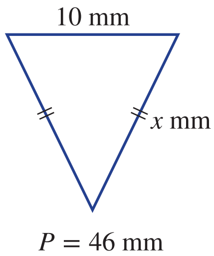
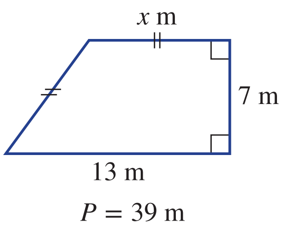


e



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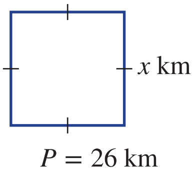
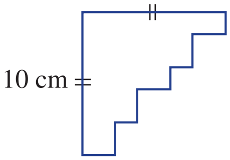


PROBLEM-SOLVING AND REASONING	8, 9, 13	8–11, 13	9–14

8 Find the perimeter of these shapes. Give your answers in cm. Assume that all angles that look right angled are 90°.

a



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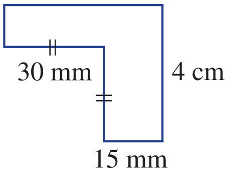

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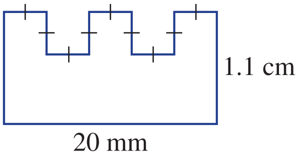

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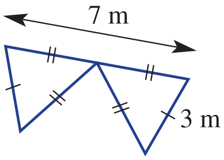

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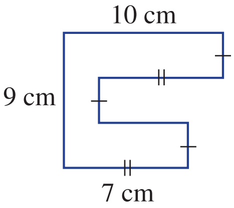

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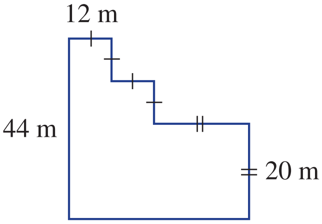
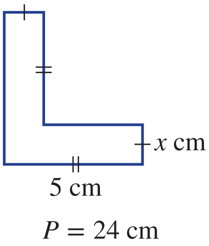


9 Find the unknown value x in these diagrams. All angles are 90°.

a



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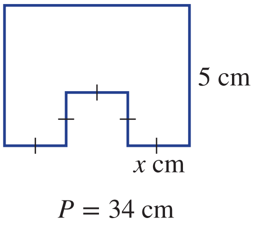

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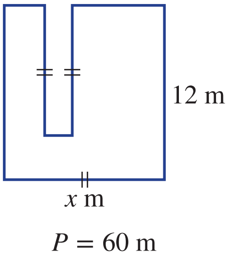

10 Jennifer needs to fence her country house block to keep her dog in. The block is a rectangle with length 50 m and breadth 42 m. Fencing costs $13 per metre. What will be the total cost of fencing?

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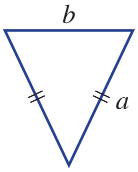
11 Gillian can jog 100 metres in 24 seconds. How long will it take her to jog 2 km? Assume Gillian can keep jogging at the same rate and give your answer in minutes.

12 A rectangular picture of length 65 cm and breadth 35 cm is surrounded by a frame of breadth 5 cm. What is the perimeter of the framed picture?

13 Write down rules using the given letters for the perimeter of these shapes (e.g. $P = a + 2b$). Assume that all angles that look right angled are 90°.

a



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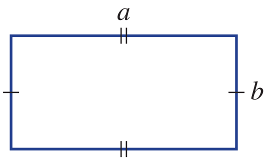

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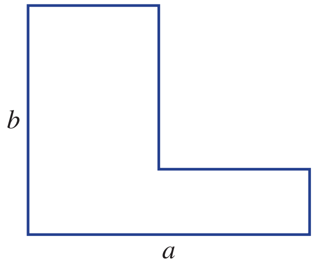
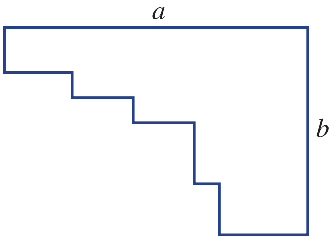


d



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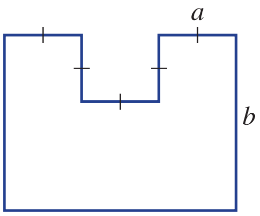

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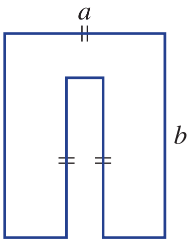
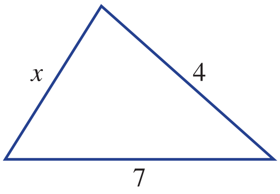


14 These shapes have perimeter P. Write a rule for x in terms of P (e.g. $x = P - 10$).

a



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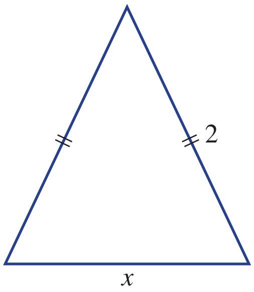

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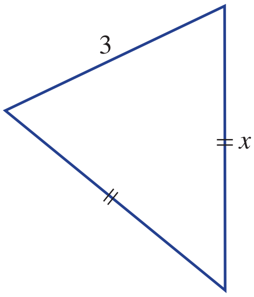

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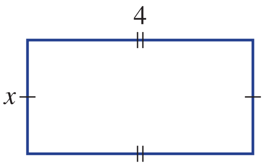

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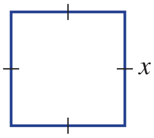

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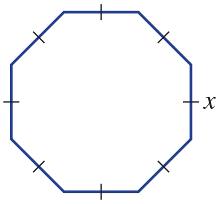

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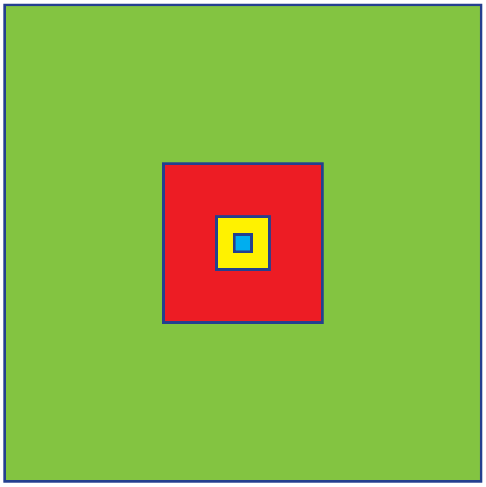
ENRICHMENT	–	–	15

Disappearing squares

15 A square is drawn with a particular side length. A second square is drawn inside the square so that its side length is one-third that of the original square. Then a third square is drawn, with side length of one-third that of the second square and so on.

- **a** What is the minimum number of squares that would need to be drawn in this pattern (including the starting square) if the innermost square has a perimeter of less than one-hundredth the perimeter of the outermost square?
- **b** Now imagine the situation is reversed and each square's perimeter is three times larger than the next smallest square. What is the minimum number of squares that would be drawn in total if the perimeter of the outermost square is to be at least 1000 times the perimeter of the innermost square?



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3B Circumference of circles REVISION

Since ancient times people have known about a special number that links a circle's diameter to its circumference. We know this number as pi (π). π is a mathematical constant that appears in formulas relating to circles, but it is also important in many other areas of mathematics. The actual value of π has been studied and approximated by ancient and more modern civilisations over thousands of years.

The Egyptians knew π was slightly more than 3 and approximated it to be $\frac{256}{81} \approx 3.16$. The Babylonians used $\frac{25}{8} \approx 3.125$ and the ancient Indians used $\frac{339}{108} \approx 3.139$.

It is believed that Archimedes of Syracus (287–212 BCE) was the first person to use a mathematical technique to evaluate π. He was able to prove that π was greater than $\frac{223}{71}$ and less than $\frac{22}{7}$. In 480 AD, the Chinese mathematician Zu Chongzhi showed that π was close to $\frac{335}{113} \approx 3.1415929$, which is accurate to seven decimal places.

Before the use of calculators, the fraction $\frac{22}{7}$ was commonly used as a good and simple approximation to π. Interestingly, mathematicians have been able to prove that π is an irrational number, which means that there is no fraction that can be found that is exactly equal to π. If the exact value of π was written down as a decimal, the decimal places would continue forever with no repeated pattern.

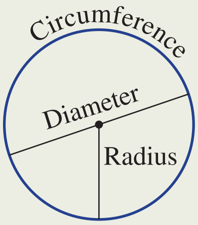
Let's start: Discovering pi

Steps:

1. Find a circular object like a dinner plate or a wheel.
2. Use a tape measure or string to measure the circumference (in mm).
3. Measure the diameter (in mm).
4. Use a calculator to divide the circumference by the diameter. (It should be about 3.14.)
5. Repeat steps 1 to 4 with a larger or smaller object.

Key ideas

- Features of a circle
  - **Diameter** (d) is the distance across the centre of a circle.
  - **Radius** (r) is the distance from the centre to the circle. Note: $d = 2r$.



- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi r$ or $C = \pi d$.
- **Pi** (π) ≈ 3.14159 (correct to five decimal places)
  - Common approximations include 3.14 and $\frac{22}{7}$.
  - A more precise estimate for pi can be found on most calculators or on the internet.

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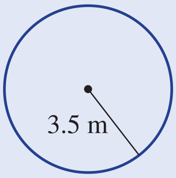
Example 4 Finding the circumference with a calculator

Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of π.

a



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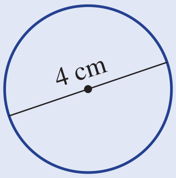
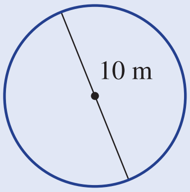


SOLUTION	EXPLANATION
a $C = 2\pi r$	Since r is given, you can use $C = 2\pi r$.
$= 2 \times \pi \times 3.5$	Alternatively, use $C = \pi d$ with $d = 7$.
$= 7\pi$	
$= 21.99$ m (to 2 d.p.)	Round off, as instructed.
b $C = \pi d$	Substitute $d = 4$ into the rule $C = \pi d$ or use
$= \pi \times 4$	$C = 2\pi r$ with $r = 2$.
$= 4\pi$	
$= 12.57$ cm (to 2 d.p.)	Round off, as instructed.

Example 5 Finding the circumference without a calculator

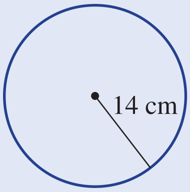
Calculate the circumference of these circles, using the given approximation of π.

a



$\pi = 3.14$

b



$\pi = \frac{22}{7}$

SOLUTION	EXPLANATION
a $C = \pi d$	Use $\pi = 3.14$ and multiply mentally. Move the
$= 3.14 \times 10$	decimal point one place to the right.
$= 31.4$ m	Alternatively, use $C = 2\pi r$ with $r = 5$.
b $C = 2\pi r$	Use $\pi = \frac{22}{7}$ and cancel the 14 with the 7 before calculating the final answer.
$= 2 \times \frac{22}{7} \times 14$	$2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2$
$= 88$ cm	

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Exercise 3B REVISION

UNDERSTANDING AND FLUENCY	1–7	4, 5(½), 6, 7	5(½), 6, 7

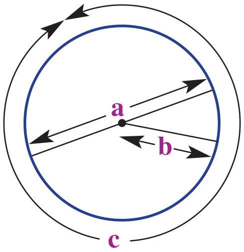
1 Evaluate the following using a calculator and round to two decimal places.

a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

2 Write down the value of π, correct to:

a one decimal place **b** two decimal places **c** three decimal places

3 Name the features of the circle shown at right.

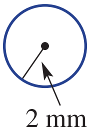


4 A circle has circumference (C) 81.7 m and diameter (d) 26.0 m, correct to one decimal place. Calculate $C \div d$. What do you notice?

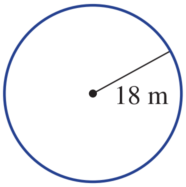
Example 4

5 Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of pi.

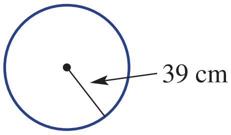
a



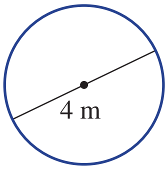
b



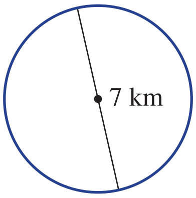
c



d



e



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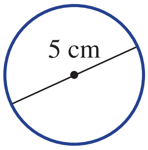
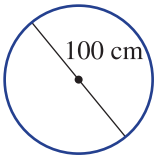


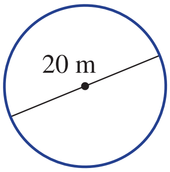
Example 5a

6 Calculate the circumference of these circles using $\pi = 3.14$.

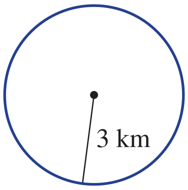
a



b



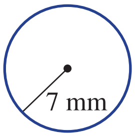
c



Example 5b

7 Calculate the circumference of these circles using $\pi = \frac{22}{7}$.

a



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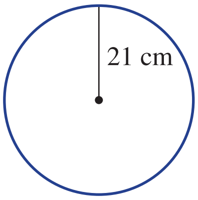

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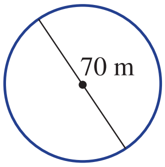


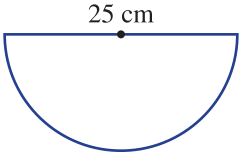
PROBLEM-SOLVING AND REASONING	8–10, 14	9–12, 14, 15	11–13, 15–17

8 A water tank has a diameter of 3.5 m. Find its circumference, correct to one decimal place.

9 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round your answer to the nearest whole number of metres.

10 These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to two decimal places.

a



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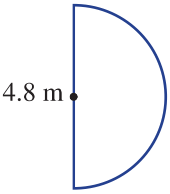

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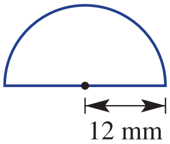
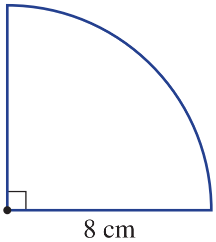


11 Calculate the perimeter of these diagrams, correct to two decimal places.

a



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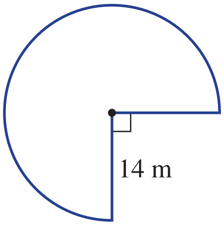

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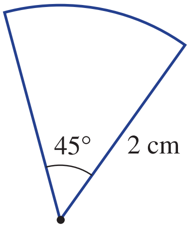
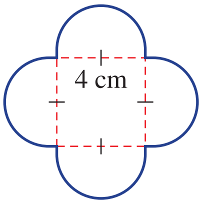
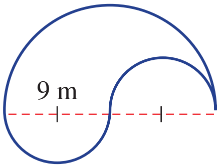


12 Calculate the perimeter of these shapes, correct to two decimal places.

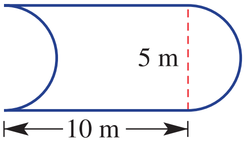
a



b



c



13 Here are some students' approximate circle measurements. Which students have incorrect measurements?

	r	*C*
Mick	4 cm	25.1 cm
Svenya	3.5 m	44 m
Andre	1.1 m	13.8 m

14 Explain why the rule $C = 2\pi r$ is equivalent to (i.e. the same as) $C = \pi d$.

15 It is more precise in mathematics to give 'exact' values for circle calculations in terms of π. For example, $C = 2 \times \pi \times 3$ gives $C = 6\pi$. This gives the final exact answer and is not written as a rounded decimal. Find the exact answers for Question **5** in terms of π.

16 Find the exact answers for Question **12** in terms of π.

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17 We know that $C = 2\pi r$ or $C = \pi d$.

a Rearrange these rules to write a rule for:

- **i** r in terms of C
- **ii** d in terms of C

b Use the rules you found in part **a** to find the following, correct to two decimal places.

- **i** the radius of a circle with circumference 14 m
- **ii** the diameter of a circle with circumference 20 cm

ENRICHMENT	–	–	18

Memorising pi

18 The box shows π correct to 100 decimal places. The Guinness World record for the most number of digits of π recited from memory is held by Rajveer Meena, an Indian student. He recited 70 000 digits, taking nearly 10 hours.

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679

Challenge your friends to see who can remember the most number of digits in the decimal representation of π.

Number of digits memorised	Report
10+	A good show
20+	Great effort
35+	Superb
50+	Amazing memory

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3C Area REVISION

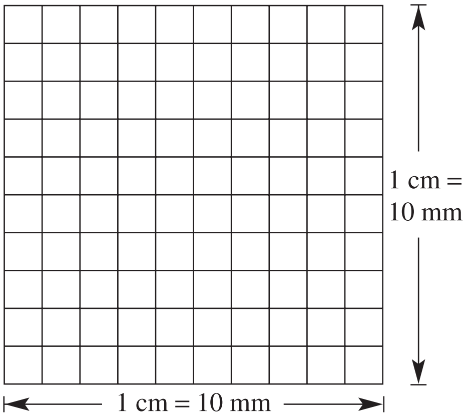
Interactive
Widgets
HOTsheets
Walkthrough

Area is a measure of surface and is often referred to as the amount of space contained inside a two-dimensional space. Area is measured in square units and the common metric units are square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), square kilometres (km^2) and hectares (ha). The hectare is often used to describe area of land, since the square kilometre for such areas is considered to be too large a unit and the square metre too small. A school football oval might be about 1 hectare, for example, and a small forest might be about 100 hectares.

Let's start: Squares of squares

Consider this enlarged drawing of one square centimetre divided into square millimetres.

- How many square millimetres are there on one edge of the square centimetre?
- How many square millimetres are there in total in 1 square centimetre?
- What would you do to convert between mm^2 and cm^2 or cm^2 and mm^2. Why?
- Can you describe how you could calculate the number of square centimetres in 1 square metre and how many square metres in 1 square kilometre? What diagrams would you use to explain your answer?



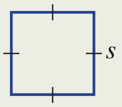
Key ideas

- The common metric units for area include:
  - square millimetres (mm^2)
  - square centimetres (cm^2)
  - square metres (m^2)
  - square kilometres (km^2)
  - hectares (ha)

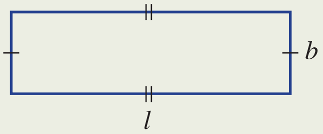
$$\begin{tikzcd} ha \arrow[r, bend left, "\times 10\,000"] & m^2 \arrow[l, bend left, "\div 10\,000"] \end{tikzcd}$$

$$\begin{tikzcd} km^2 \arrow[r, bend left, "\times 1000^2 = 1\,000\,000"] & m^2 \arrow[l, bend left, "\div 1000^2 = 1\,000\,000"] \arrow[r, bend left, "\times 100^2 = 10\,000"] & cm^2 \arrow[l, bend left, "\div 100^2 = 10\,000"] \arrow[r, bend left, "\times 10^2 = 100"] & mm^2 \arrow[l, bend left, "\div 10^2 = 100"] \end{tikzcd}$$

- Area of squares, rectangles and triangles
  - Square $A = S \times S = S^2$



  - Rectangle $A = l \times b = lb$



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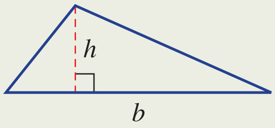
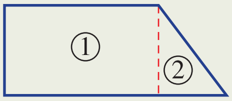
  - Triangle $A = \frac{1}{2} \times b \times h = \frac{1}{2}bh$

    The dashed line which gives the height is **perpendicular** (at right angles) to the base.



- Areas of **composite shapes** can be found by adding or subtracting the area of more basic shapes.



Example 6 Converting units of area

Convert these area measurements to the units shown in the brackets.

a $0.248\ \text{m}^2\ (\text{cm}^2)$ **b** $3100\ \text{mm}^2\ (\text{cm}^2)$

SOLUTION

a $0.248\ \text{m}^2 = 0.248 \times 10\,000$

$= 2480\ \text{cm}^2$

b $3100\ \text{mm}^2 = 3100 \div 100$

$= 31\ \text{cm}^2$

EXPLANATION

$1\ \text{m}^2 = 100^2\ \text{cm}^2 = 10\,000\ \text{cm}^2$

Multiply since you are changing to a smaller unit.

$$\begin{tikzcd} \text{m}^2 \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \end{tikzcd}$$

$1\ \text{cm}^2 = 10^2\ \text{mm}^2 = 100\ \text{mm}^2$

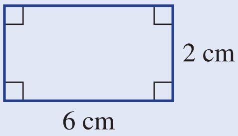
Divide since you are changing to a larger unit.

$$\begin{tikzcd} \text{cm}^2 & \text{mm}^2 \arrow[l, bend left, "\div 10^2"] \end{tikzcd}$$

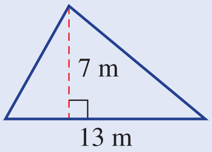
Example 7 Finding areas of rectangles and triangles

Find the area of these shapes.

a



b



SOLUTION

a $A = lb$

$= 6 \times 2$

$= 12\ \text{cm}^2$

b $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 7$

$= 45.5\ \text{m}^2$

EXPLANATION

Write the formula for the area of a rectangle and substitute $l = 6$ and $b = 2$.

Remember that the height is measured using a line that is perpendicular to the base.

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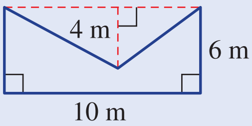
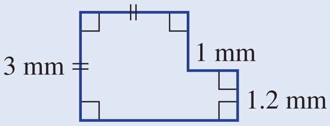
Example 8 Finding areas of composite shapes

Find the area of these composite shapes using addition or subtraction.

a



b



SOLUTION

a $A = lb - \frac{1}{2}bh$

$= 10 \times 6 - \frac{1}{2} \times 10 \times 4$

$= 60 - 20$

$= 40\text{ m}^2$

b $A = S^2 + lb$

$= 3^2 + 1.2 \times 1$

$= 9 + 1.2$

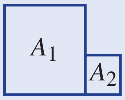
$= 10.2\text{ mm}^2$

EXPLANATION

The calculation is done by subtracting the area of a triangle from the area of a rectangle.

Rectangle − triangle

The calculation is done by adding the area of a rectangle to the area of a square.



Area $= A_1 + A_2$

Exercise 3C REVISION

UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	2, 3, 4–6(½)	5–6(½)

1 By considering the given diagrams, answer the questions.

a **i** How many mm^2 in 1 cm^2?

ii How many mm^2 in 4 cm^2?

iii How many cm^2 in 300 mm^2?

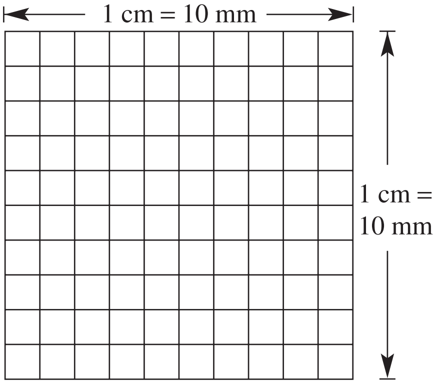
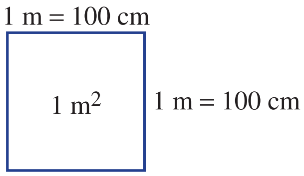


Diagram not to scale

b **i** How many cm^2 in 1 m^2?

ii How many cm^2 in 7 m^2?

iii How many m^2 in 40 000 cm^2?



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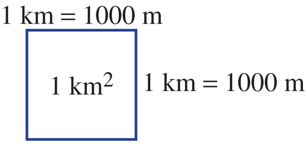
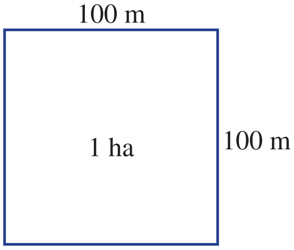
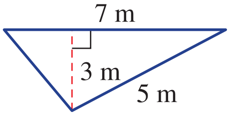
c **i** How many m^2 in 1 km^2?
ii How many m^2 in 5 km^2?
iii How many km^2 in 2 500 000 m^2?

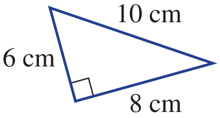


d **i** How many m^2 in 1 ha?
ii How many m^2 in 3 ha?
iii How many ha in 75 000 m^2?



2 Which length measurements would be used for the *base* and the *height* (in that order) to find the area of these triangles?

a


b


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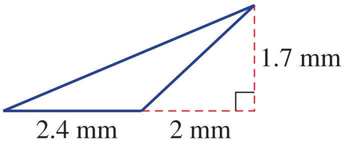


3 How many square metres are in 1 hectare?

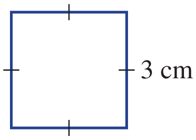
Example 6

4 Convert these area measurements to the units shown in the brackets.

a 2 cm^2 (mm^2)
b 7 m^2 (cm^2)
c 0.5 km^2 (m^2)
d 3 ha (m^2)
e 0.34 cm^2 (mm^2)
f 700 cm^2 (m^2)
g 3090 mm^2 (cm^2)
h 0.004 km^2 (m^2)
i 2000 cm^2 (m^2)
j 450 000 m^2 (km^2)
k 4000 m^2 (ha)
l 3210 mm^2 (cm^2)
m 320 000 m^2 (ha)
n 0.0051 m^2 (cm^2)
o 0.043 cm^2 (mm^2)
p 4802 cm^2 (m^2)
q 19 040 m^2 (ha)
r 2933 m^2 (ha)
s 0.0049 ha (m^2)
t 0.77 ha (m^2)
u 2.4 ha (m^2)

Example 7

5 Find the areas of these squares, rectangles and triangles.

a


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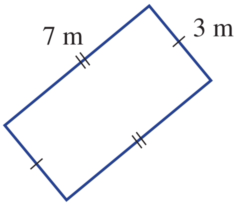
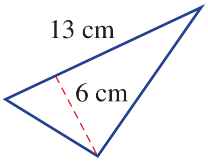
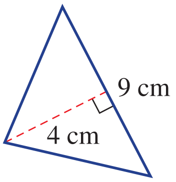
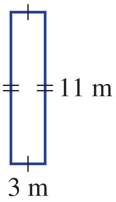
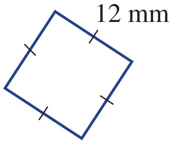
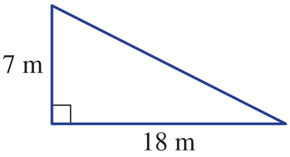


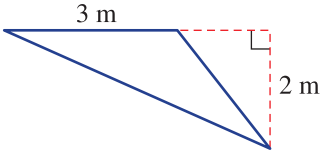
c


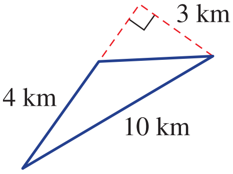
d


e


f


g


h


i


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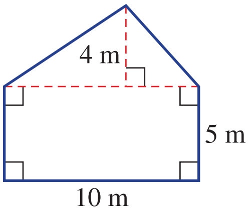
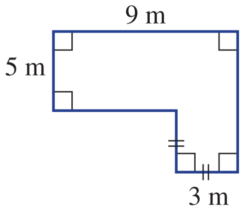
Example 8

6 Find the area of these composite shapes using addition or subtraction.

a



b



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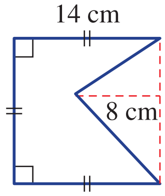
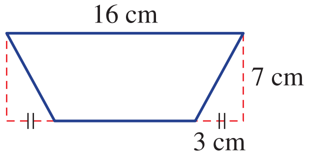
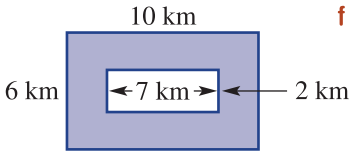


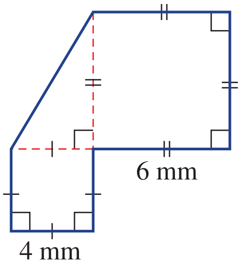
d



e



f



PROBLEM-SOLVING AND REASONING	7, 8, 13	7–10, 13, 14	7–15

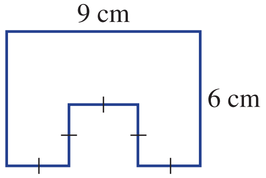
7 Use your knowledge of area units to convert these measurements to the units shown in the brackets.

a 0.2 m^2 (mm^2) **b** 0.000043 km^2 (cm^2) **c** 374000 cm^2 (km^2)

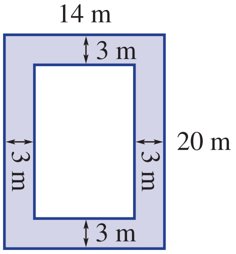
d 10920 mm^2 (m^2) **e** 0.0000002 ha (cm^2) **f** 6 km^2 (ha)

8 Find the area of these composite shapes. You may need to determine some side lengths first. Assume that all angles that look right angled are 90°.

a



b



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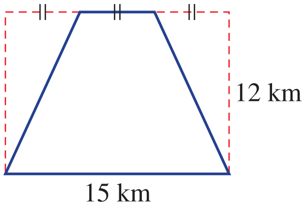

9 Find the side length of a square if its area is:

a 36 m^2 **b** 2.25 cm^2

10 **a** Find the area of a square if its perimeter is 20 m.

b Find the area of a square if its perimeter is 18 cm.

c Find the perimeter of a square if its area is 49 cm^2.

d Find the perimeter of a square if its area is 169 m^2.

11 A triangle has area 20 cm^2 and base 4 cm. Find its height.

12 Paint costs $12 per litre and can only be purchased in a full number of litres. One litre of paint covers an area of 10 m^2. A rectangular wall is 6.5 m long and 3 m high and needs two coats of paint. What will be the cost of paint for the wall?

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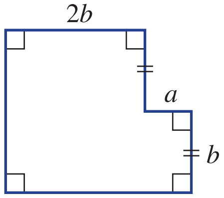
13 Write down expressions for the area of these shapes in simplest form using the letters a and b (e.g. $A = 2ab + a^2$).

a



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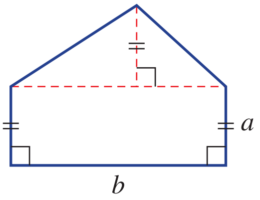
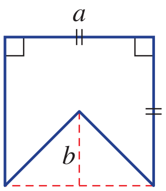


c



14 Using only whole numbers for length and breadth, answer the following questions.

- **a** How many distinct (different) rectangles have an area of 24 square units?
- **b** How many distinct squares have an area of 16 square units?

15 Write down rules for:

- **a** the breadth of a rectangle (b) with area A and length l
- **b** the side length of a square (l) with area A
- **c** the height of a triangle (h) with area A and base b

ENRICHMENT	–	–	16

The acre

16 Two of the more important imperial units of length and area that are still used today are the mile and the acre. Many of our country and city roads, farms and house blocks were divided up using these units.

Here are some conversions:

1 square mile = 640 acres
1 mile ≈ 1.609 344 km
1 hectare = 10 000 m^2

- **a** Use the given conversions to find:
  - **i** the number of square kilometres in 1 square mile (round to two decimal places)
  - **ii** the number of square metres in 1 square mile (round to the nearest whole number)
  - **iii** the number of hectares in 1 square mile (round to the nearest whole number)
  - **iv** the number of square metres in 1 acre (round to the nearest whole number)
  - **v** the number of hectares in 1 acre (round to one decimal place)
  - **vi** the number of acres in 1 hectare (round to one decimal place)
- **b** A dairy farmer has 200 acres of land. How many hectares is this? (Round your answer to the nearest whole number.)
- **c** A house block is 2500 m^2. What fraction of an acre is this? (Give your answer as a percentage rounded to the nearest whole number.)

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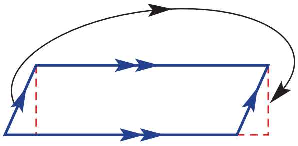
3D Area of special quadrilaterals

The formulas for the area of a rectangle and a triangle can be used to develop the area of other special quadrilaterals. These quadrilaterals include the parallelogram, the rhombus, the kite and the trapezium. Knowing the formulas for the area of these shapes can save a lot of time by dividing shapes into rectangles and triangles.

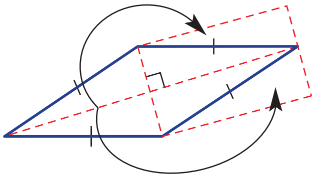
Let's start: Developing formulas

These diagrams contain clues as to how you might find the area of the shape using only what you know about rectangles and triangles. Can you explain what each diagram is trying to tell you?

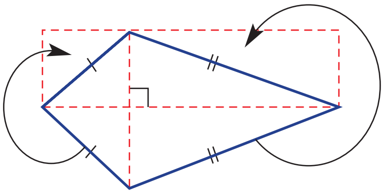
- Parallelogram



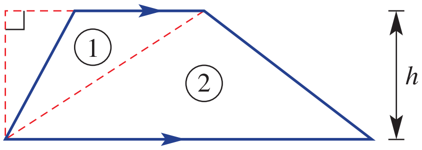
- Rhombus



- Kite



- Trapezium

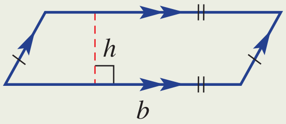


Key ideas

- Area of a **parallelogram**

  Area = base × perpendicular height

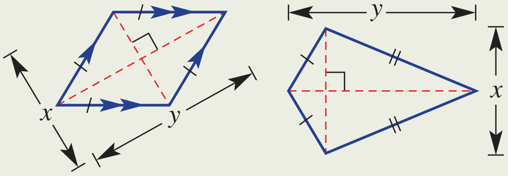
  or $A = bh$



- Area of a **rhombus** and the area of a **kite**

  Area $= \frac{1}{2} \times$ diagonal $x \times$ diagonal y

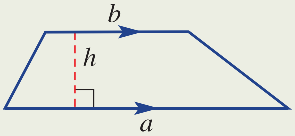
  or $A = \frac{1}{2}xy$



- Area of a **trapezium**

  Area $= \frac{1}{2} \times$ sum of parallel sides × perpendicular height

  or $A = \frac{1}{2}h(a + b)$



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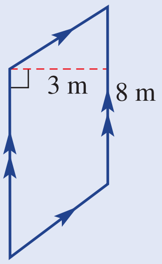
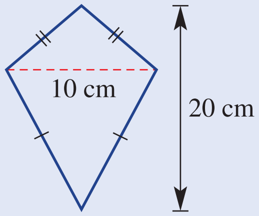
Example 9 Finding areas of special quadrilaterals

Find the area of these shapes.

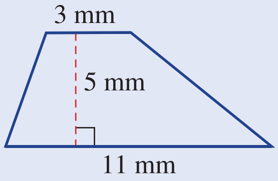
a



b



c



SOLUTION	EXPLANATION
a $A = bh$ $= 8 \times 3$ $= 24\text{ m}^2$	The height is measured at right angles to the base.
b $A = \frac{1}{2}xy$ $= \frac{1}{2} \times 10 \times 20$ $= 100\text{ cm}^2$	Use the formula $A = \frac{1}{2}xy$ since both diagonals are given. This formula can also be used for a rhombus.
c $A = \frac{1}{2}h(a + b)$ $= \frac{1}{2} \times 5 \times (11 + 3)$ $= \frac{1}{2} \times 5 \times 14$ $= 35\text{ mm}^2$	The two parallel sides are 11 mm and 3 mm in length. The perpendicular height is 5 mm.

Exercise 3D

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4	1, 3(½), 4	3(½), 4

1 Find the value of A using these formulas and given values.

a $A = bh\ (b = 2, h = 3)$

b $A = \frac{1}{2}xy\ (x = 5, y = 12)$

c $A = \frac{1}{2}h(a + b)\ (a = 2, b = 7, h = 3)$

d $A = \frac{1}{2}h(a + b)\ (a = 7, b = 4, h = 6)$

2 Complete these sentences.

a A perpendicular angle is __________ degrees.

b In a parallelogram, you find the area using a base and the ________________.

c The two diagonals in a kite or a rhombus are ________________.

d To find the area of a trapezium you multiply $\frac{1}{2}$ by the sum of the two ________________ sides and then by the ________________ height.

e The two special quadrilaterals that have the same area formula using diagonal lengths x and y are the ________________ and the ________________.

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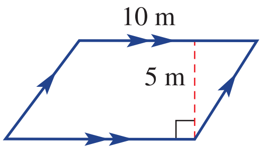
Example 9

3 Find the area of these special quadrilaterals. First give the name of the shape.

a



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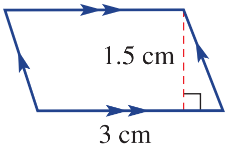

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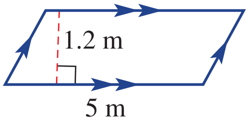

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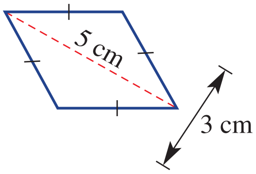

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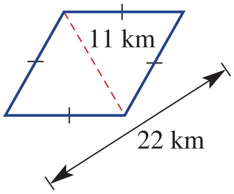
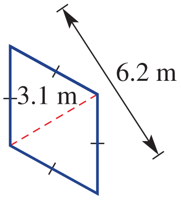


f



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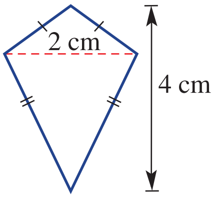
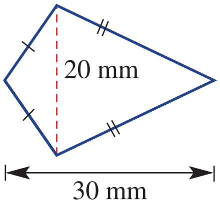
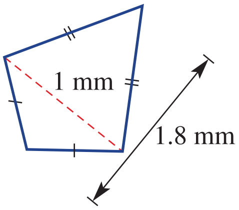


h



i



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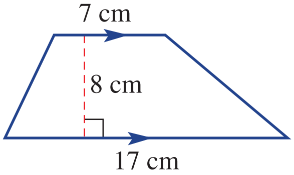

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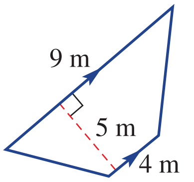
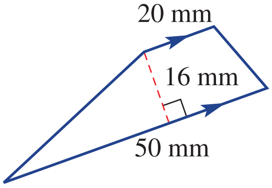
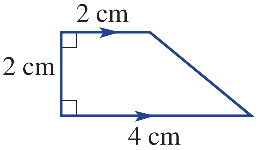


l



4 These trapeziums have one side at right angles to the two parallel sides. Find the area of each.

a



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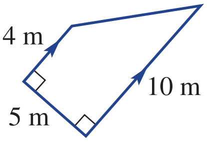

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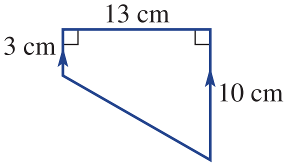
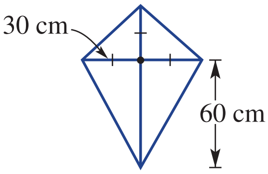


PROBLEM-SOLVING AND REASONING	5, 6, 9	5–7, 9, 10	6–8, 10, 11

5 A flying kite is made from four centre rods all connected near the middle of the kite, as shown. What area of plastic, in square metres, is needed to cover the kite?



6 A parallelogram has an area of 26 m^2 and its base length is 13 m. What is its perpendicular height?

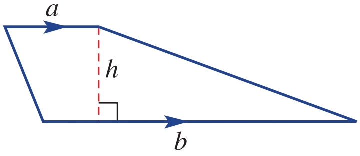
7 A landscape gardener charges \$20 per square metre of lawn. A lawn area is in the shape of a rhombus and its diagonals are 8 m and 14.5 m. What would be the cost of laying this lawn?

8 The parallel sides of a trapezium are 2 cm apart and one of the sides is three times the length of the other. If the area of the trapezium is 12 cm^2, what are the lengths of the parallel sides?

9 Consider this shape.

a What type of shape is it?

b Find its area if $a = 5$, $b = 8$ and $h = 3$.
All measurements are in cm.



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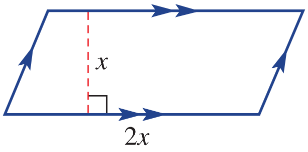
10 Write an expression for the area of these shapes in simplest form (e.g. $A = 2a + 3ab$).

a



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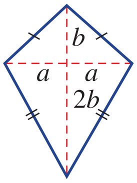

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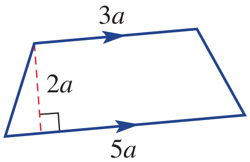
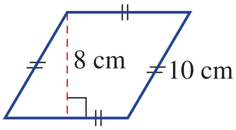


11 Would you use the formula $A = \frac{1}{2}xy$ to find the area of this rhombus? Explain.



ENRICHMENT	–	–	12, 13

Proof

12 Complete these proofs to give the formula for the area of a rhombus and a trapezium.

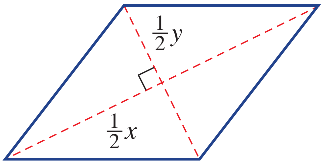
a Rhombus

$A = 4$ triangle areas

$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 4 \times \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_\_$



b Trapezium 1

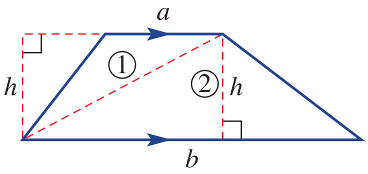
$A =$ Area triangle 1 + Area triangle 2

$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$

$= \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_\_ + \frac{1}{2} \times \_\_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_$



c Trapezium 2

$A =$ Area rectangle + Area triangle

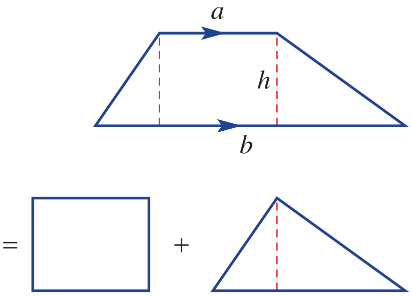
$= \text{length} \times \text{breadth} + \frac{1}{2} \times \text{base} \times \text{height}$

$= \_\_\_\_\_ \times \_\_\_\_\_ + \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_ - \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$



13 Design an A4 poster for one of the proofs in Question **12** to be displayed in your class.

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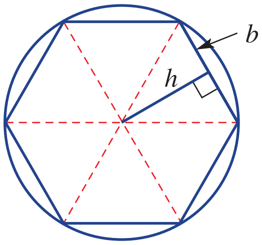
3E Area of circles

Interactive
Widgets
HOTsheets
Walkthrough

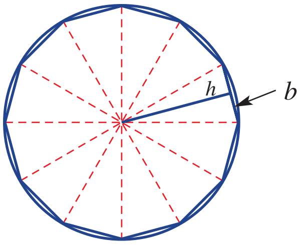
We know that the link between the perimeter of a circle and its radius has challenged civilisations for thousands of years. Similarly, people have studied the link between a circle's radius and its area.



Hexagon ($n = 6$)
$A = 6 \times \frac{1}{2}bh$

Archimedes (287–212 BCE) attempted to calculate the exact area of a circle using a particular technique involving limits. If a circle is approximated by a regular hexagon, then the approximate area would be the sum of the areas of six triangles with base b and height h.

So $A \approx 6 \times \dfrac{1}{2}bh$.



Dodecagon ($n = 12$)
$A = 12 \times \frac{1}{2}bh$

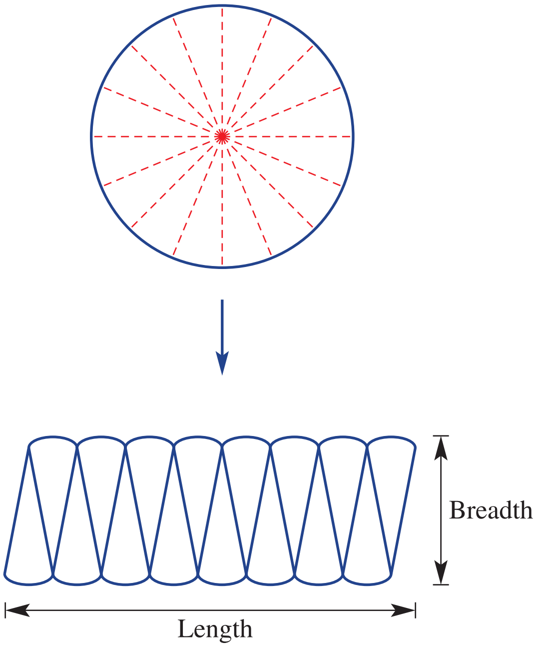
If the number of sides (n) on the polygon increases, the approximation would improve. If n approaches infinity, the error in estimating the area of the circle would diminish to zero.

Proof

$$
\begin{aligned}
A &= n \times \frac{1}{2}bh \\
&= \frac{1}{2} \times nb \times h \\
&= \frac{1}{2} \times 2\pi r \times r \\
&= \pi r^2
\end{aligned}
$$

(As n approaches ∞, nb limits to $2\pi r$ as nb is the perimeter of the polygon, and h limits to r.)

Let's start: Area as a rectangle



Imagine a circle cut into small sectors and arranged as shown.

Now try to imagine how the arrangement on the right would change if the number of sector divisions was not 16 (as shown) but a much higher number.

- What would the shape on the right look like if the number of sector divisions was a very high number? What would the length and breadth relate to in the original circle?
- Try to complete this proof.

  A = length × breadth

  $= \frac{1}{2} \times$ _____ $\times r$

  $=$ _____

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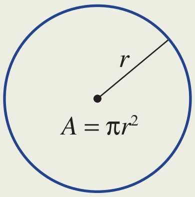
Key ideas

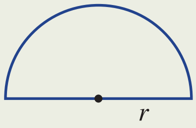
- The ratio of the area of a circle to the square of its radius is equal to π.

  i.e. $\frac{A}{r^2} = \pi$ so $A = \pi r^2$



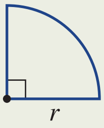
- A half circle is called a **semicircle**.

  $A = \frac{1}{2}\pi r^2$



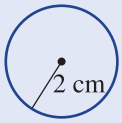
- A quarter circle is called a **quadrant**.

  $A = \frac{1}{4}\pi r^2$



Example 10 Finding circle areas using a calculator

Use a calculator to find the area of this circle, correct to two decimal places.

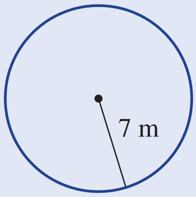


SOLUTION	EXPLANATION
$A = \pi r^2$ $= \pi \times 2^2$ $= 12.57\text{ cm}^2$ (to 2 d.p.)	Use the π button on the calculator and enter $\pi \times 2^2$ or $\pi \times 4$.

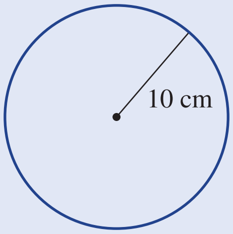
Example 11 Finding circle areas without technology

Find the area of these circles, using the given approximate value of π.

a $\pi = \frac{22}{7}$



b $\pi = 3.14$



SOLUTION	EXPLANATION
a $A = \pi r^2$	Always write the rule.
$= \frac{22}{7} \times 7^2$	Use $\pi = \frac{22}{7}$ and $r = 7$.
$= 154\text{ m}^2$	$\frac{22}{7} \times 7 \times 7 = 22 \times 7$
b $A = \pi r^2$	Use $\pi = 3.14$ and substitute $r = 10$.
$= 3.14 \times 10^2$	3.14×10^2 is the same as 3.14×100.
$= 314\text{ cm}^2$	

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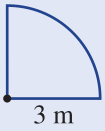
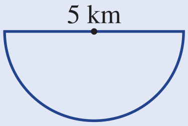
Example 12 Finding areas of semicircles and quadrants

Find the area of this quadrant and semicircle, correct to two decimal places.

a



b



SOLUTION

a

$$A = \frac{1}{4} \times \pi r^2$$
$$= \frac{1}{4} \times \pi \times 3^2$$
$$= 7.07\ \text{m}^2 \quad \text{(to 2 d.p.)}$$

b

$$r = \frac{5}{2} = 2.5$$
$$A = \frac{1}{2} \times \pi r^2$$
$$= \frac{1}{2} \times \pi \times 2.5^2$$
$$= 9.82\ \text{km}^2 \quad \text{(to 2 d.p.)}$$

EXPLANATION

The area of a quadrant is $\frac{1}{4}$ the area of a circle with the same radius.

The radius is half the diameter.

The area of a semicircle is $\frac{1}{2}$ the area of a circle with the same radius.

Exercise 3E

UNDERSTANDING AND FLUENCY	1–6	3, 4–6(½)	4–6(½)

1 Evaluate without the use of a calculator.

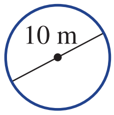
a 3.14×10 **b** 3.14×4 **c** $\frac{22}{7} \times 7$ **d** $\frac{22}{7} \times 7^2$

2 Use a calculator to evaluate these to two decimal places.

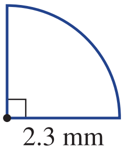
a $\pi \times 5^2$ **b** $\pi \times 13^2$ **c** $\pi \times 3.1^2$ **d** $\pi \times 9.8^2$

3 What is the length of the radius in these shapes? Assume that all angles that look right angled are 90°.

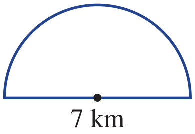
a



b



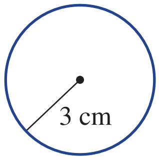
c



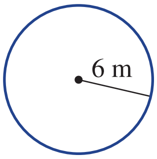
Example 10

4 Use a calculator to find the area of these circles, correct to two decimal places.

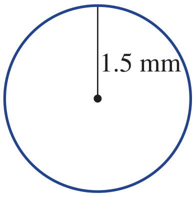
a



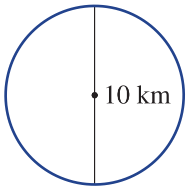
b



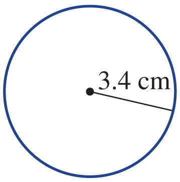
c



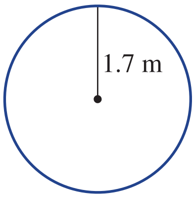
d



e



f



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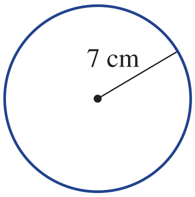
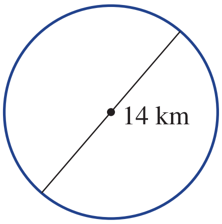
Example 11 **5** Find the area of these circles, using the given approximate value of π.

a



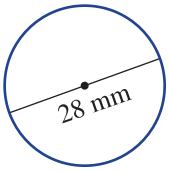
$\pi = \frac{22}{7}$

b



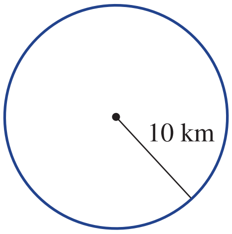
$\pi = \frac{22}{7}$

c



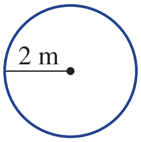
$\pi = \frac{22}{7}$

d



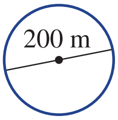
$\pi = 3.14$

e



$\pi = 3.14$

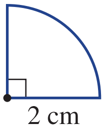
f



$\pi = 3.14$

Example 12 **6** Find the area of these quadrants and semicircles, correct to two decimal places.

a



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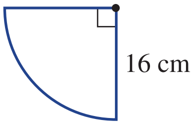
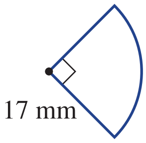
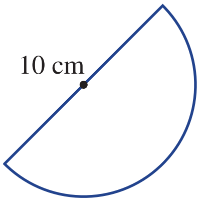


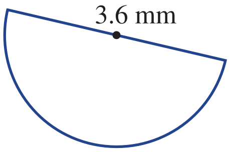
c



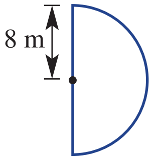
d



e



f



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PROBLEM-SOLVING AND REASONING	7, 8, 13	9–11, 13, 14	10, 12, 14, 15

7 A pizza tray has a diameter of 30 cm. Calculate its area to the nearest whole number of cm^2.

8 A tree trunk is cut to reveal a circular cross-section of radius 60 cm. Is the area of the cross-section more than 1 m^2 and, if so, by how much? Round your answer to the nearest whole number of cm^2.

9 A circular oil slick has a diameter of 1 km. The newspaper reported an area of more than 1 km^2. Is the newspaper correct?

10 Two circular plates have radii 12 cm and 13 cm. Find the difference in their area, correct to two decimal places.

11 Which has the largest area: a circle of radius 5 m, a semicircle of radius 7 m or a quadrant of radius 9 m?

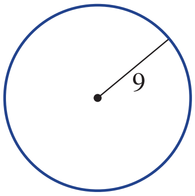
12 A square of side length 10 cm has a hole in the middle. The diameter of the hole is 5 cm. What is the area remaining? Round the answer to the nearest whole number.

13 A circle has radius 2 cm.

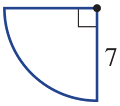
- **a** Find the area of the circle using $\pi = 3.14$.
- **b** Find the area if the radius is doubled to 4 cm.
- **c** What is the effect on the area if the radius is doubled?
- **d** What is the effect on the area if the radius is tripled?
- **e** What is the effect on the area if the radius is quadrupled?
- **f** What is the effect on the area if the radius is multiplied by n?

14 The area of a circle with radius 2 can be written exactly as $A = \pi \times 2^2 = 4\pi$. Write the exact area of these shapes.

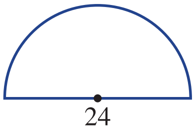
a



b



c



15 We know that the diameter d of a circle is twice the radius r; that is, $d = 2r$ or $r = \frac{1}{2}d$.

- **a** Substitute $r = \frac{1}{2}d$ into the rule $A = \pi r^2$ to find a rule for the area of a circle in terms of d.
- **b** Use your rule from part **a** to check that the area of a circle with diameter 10 m is 25π m^2.

ENRICHMENT	–	–	16

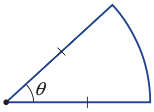
Reverse problems

16 Reverse the rule $A = \pi r^2$ to find the radius in these problems.

- **a** If $A = 10$, use your calculator to show that $r \approx 1.78$.
- **b** Find the radius of circles with these areas. Round the answer to two decimal places.
  - **i** 17 m^2
  - **ii** 4.5 km^2
  - **iii** 320 mm^2
- **c** Can you write a rule for r in terms of A? Check that it works for the circles defined in part **b**.

3F Area of sectors and composite figures

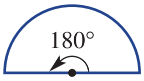
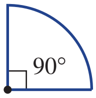
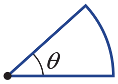
A slice of pizza or a portion of a round cake cut from the centre forms a shape called a sector.



The area cleaned by a windscreen wiper could also be thought of as a difference of two sectors with the same angle but different radii. Clearly the area of a sector depends on its radius, but it also depends on the angle between the two straight edges.

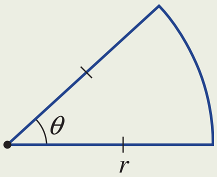
Let's start: The sector area formula

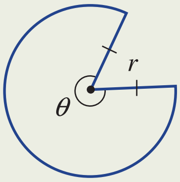
Complete this table to develop the rule for finding the area of a sector.

Angle	Fraction of area	Area rule	Diagram
180°	$\frac{180°}{360°} = \frac{1}{2}$	$A = \frac{1}{2} \times \pi r^2$	
90°	$\frac{90°}{360°} = \_\_\_$	$A = \_\_\_ \times \pi r^2$	
45°			
30°			
θ		$A = \_\_\_ \times \pi r^2$	

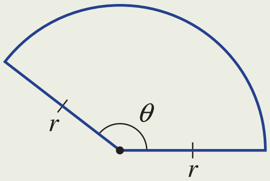
Key ideas

- A **sector** is formed by dividing a circle with two radii.

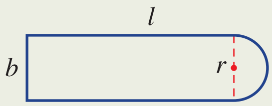




- A sector's area is determined by calculating a fraction of the area of a circle with the same radius.
  - Fraction is $\frac{\theta}{360}$.
  - Sector area $= \frac{\theta}{360} \times \pi r^2$



- The area of a **composite shape** can be found by adding or subtracting the areas of more basic shapes.



$A = lb + \frac{1}{2}\pi r^2$

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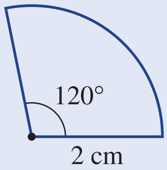
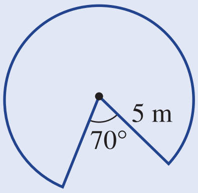
Example 13 Finding areas of sectors

Find the area of these sectors, correct to two decimal places.

a



b



SOLUTION

a $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{120}{360} \times \pi \times 2^2$

$= \frac{1}{3} \times \pi \times 4$

$= 4.19\text{ cm}^2$ (to 2 d.p.)

b $\theta = 360 - 70 = 290$

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{290}{360} \times \pi \times 5^2$

$= 63.27\text{ m}^2$ (to 2 d.p.)

EXPLANATION

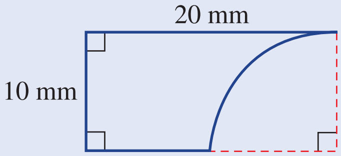
First write the rule for the area of a sector.

Substitute $\theta = 120$ and $r = 2$. Note that $\frac{120}{360}$ simplifies to $\frac{1}{3}$.

First calculate the angle inside the sector and remember that a revolution is 360°. Then substitute $\theta = 290$ and $r = 5$.

Example 14 Finding areas of composite shapes

Find the area of this composite shape, correct to the nearest square millimetre.



SOLUTION

$A = lb - \frac{1}{4}\pi r^2$

$= 20 \times 10 - \frac{1}{4} \times \pi \times 10^2$

$= 200 - 25\pi$

$= 121\text{ mm}^2$ (to nearest whole number)

EXPLANATION

The area can be found by subtracting the area of a quadrant from the area of a rectangle.

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Exercise 3F

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6(½)	3, 4(½), 5, 6(½)	4(½), 5, 6(½)

1 Simplify these fractions.

a $\frac{180}{360}$ **b** $\frac{90}{360}$ **c** $\frac{60}{360}$ **d** $\frac{45}{360}$

2 Evaluate the following using a calculator. Give your answer correct to two decimal places.

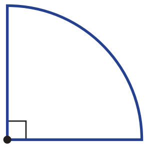
a $\frac{180}{360} \times \pi \times 2^2$

b $\frac{20}{360} \times \pi \times 7^2$

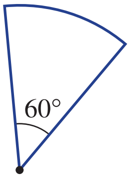
c $\frac{210}{360} \times \pi \times 2.3^2$

3 What fraction of a circle in simplest form is shown by these sectors?

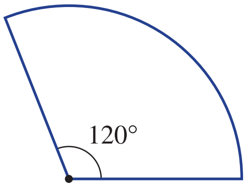
a



b



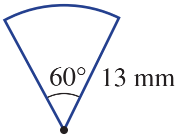
c



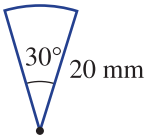
Example 13a

4 Find the area of these sectors, correct to two decimal places.

a



b



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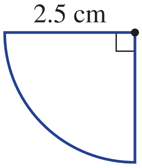
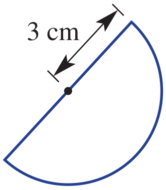


d



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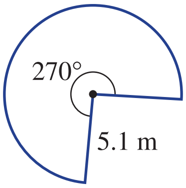

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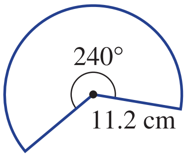

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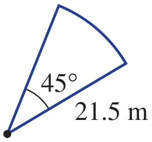

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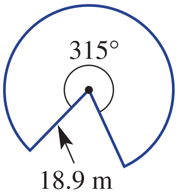
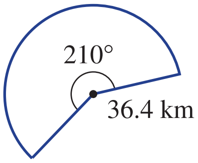


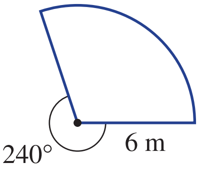
i



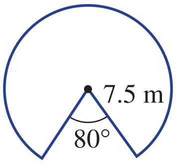
Example 13b

5 Find the area of these sectors, correct to two decimal places.

a



b



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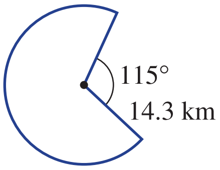

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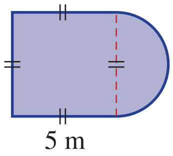
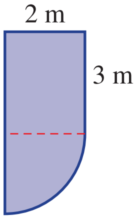
Example 14

6 Find the areas of these composite shapes using addition or subtraction. Round the answer to two decimal places. Assume that all angles that look right angled are 90°.

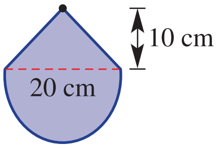
a



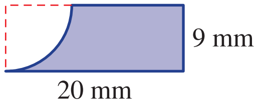
b



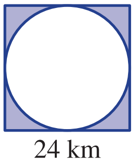
c



d



e



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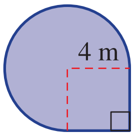

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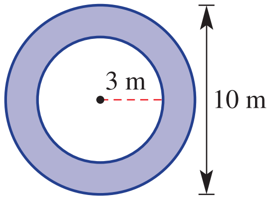

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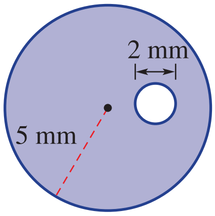

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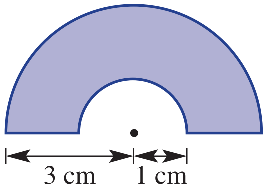
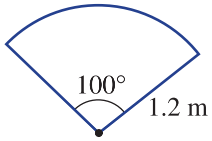


PROBLEM-SOLVING AND REASONING

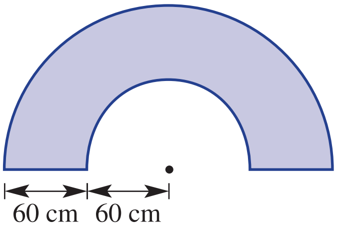
7, 8, 11(½)	8, 9, 11(½)	8–10, 11(½), 12

7 A simple bus wiper blade wipes an area over 100°, as shown. Find the area wiped by the blade, correct to two decimal places.



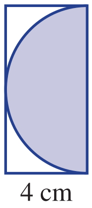
8 At Buy-by-the-sector Pizza they offer a sector of a 15 cm radius pizza with an angle of 45° or a sector of a 13 cm radius pizza with an angle of 60°. Which piece gives the bigger area and by how much? Round the answer to two decimal places.

9 An archway is made up of an inside and outside semicircle, as shown. Find the area of the arch, correct to the nearest whole cm^2.



10 What percentage of the total area is occupied by the shaded region in these diagrams? Round the answer to one decimal place. Assume that all angles that look right angled are 90°.

a



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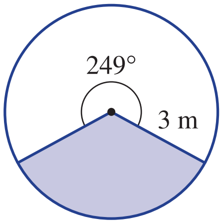
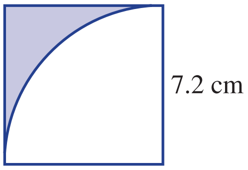
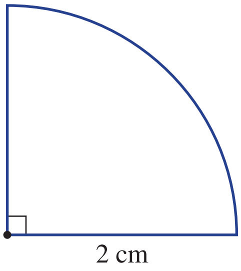


c



11 An exact area measure in terms of π might look like $\pi \times 2^2 = 4\pi$. Find the exact area of these shapes in terms of π. Simplify your answer.

a



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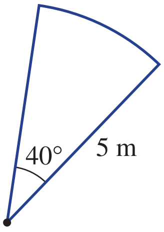

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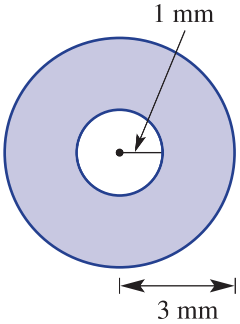

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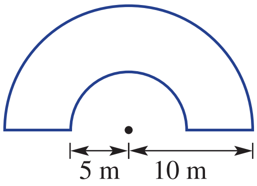

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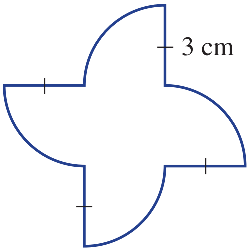

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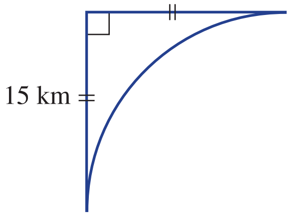
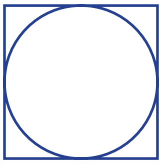


12 Consider the percentage of the area occupied by a circle inside a square and touching all sides, as shown.



a If the radius of the circle is 4 cm, find the percentage of area occupied by the circle. Round the answer to one decimal place.

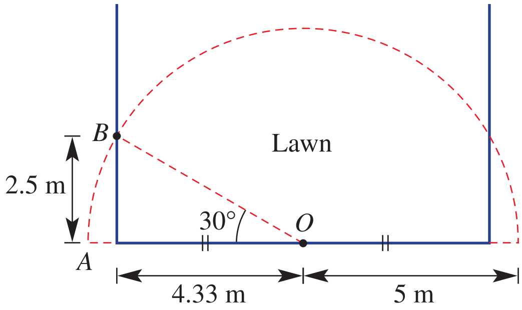
b Repeat part **a** for a radius of 10 cm. What do you notice?

c Can you prove that the percentage area is always the same for any radius *r*? (*Hint*: find the percentage area using the letter *r* for the radius.)

ENRICHMENT	–	–	13

Sprinkler waste

13 A rectangular lawn area has a 180° sprinkler positioned in the middle of one side, as shown.



a Find the area of the sector *OAB*, correct to two decimal places.

b Find the area watered by the sprinkler outside the lawn area, correct to two decimal places.

c Find the percentage of water wasted, giving the answer correct to one decimal place.

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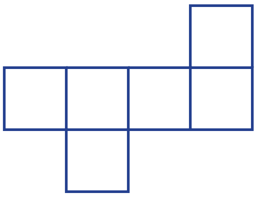
3G Surface area of prisms EXTENSION

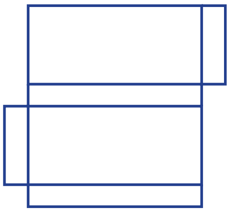
Many problems in three dimensions can be solved by looking at the problem or parts of the problem in two dimensions. Finding the surface area of a solid is a good example of this, as each face can usually be looked at in two-dimensional space. The approximate surface area of the walls of an unpainted house, for example, could be calculated by looking at each wall separately and adding to get a total surface area.

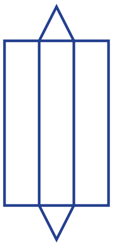
Let's start: Possible prisms

Here are three nets that fold to form three different prisms.

- Can you draw and name the prisms?
- Try drawing other nets of these prisms that are a different shape to the nets given here.

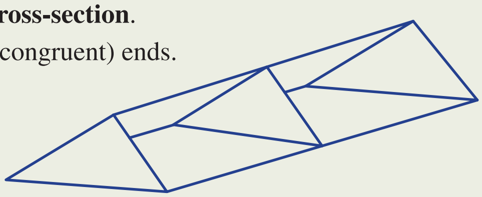




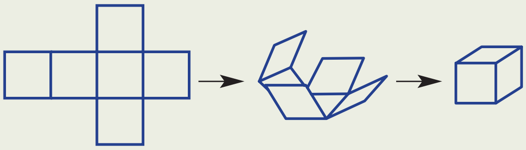


Key ideas

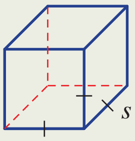
- A **prism** is a polyhedron with a constant (uniform) **cross-section**.
  - The cross-section is parallel to the two identical (congruent) ends.
  - The other sides are parallelograms (or rectangles for right prisms).



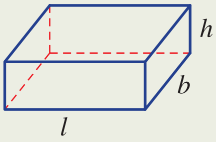
- A **net** is a two-dimensional representation of all the surfaces of a solid. It can be folded to form the solid.



- The **surface area** (A) of a prism is the sum of the areas of all its faces.



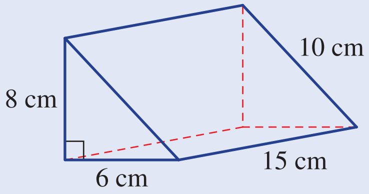
$A = 6s^2$



$A = 2lb + 2lh + 2bh$

Example 15 Calculating surface areas

Find the total surface area of this prism.



SOLUTION

Area of two triangular ends

$A = 2 \times \frac{1}{2} \times bh$

$= 2 \times \frac{1}{2} \times 6 \times 8$

$= 48\ \text{cm}^2$

Area of three rectangles

$A = (6 \times 15) + (8 \times 15) + (10 \times 15)$

$= 360\ \text{cm}^2$

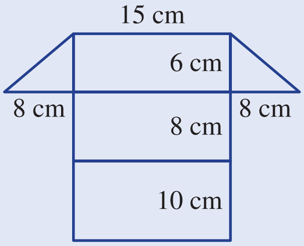
Surface area

$A = 48 + 360$

$= 408\ \text{cm}^2$

EXPLANATION

One possible net is:



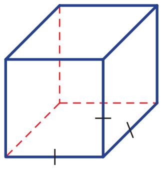
Work out the area of each shape or group of shapes and find the sum of their areas to obtain the surface area.

Exercise 3G EXTENSION

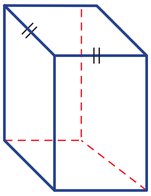
UNDERSTANDING AND FLUENCY	1–3, 4(½)	3, 4(½), 5	4(½), 5

1 How many faces are there on these prisms? Also name the types of shapes that make the different faces.

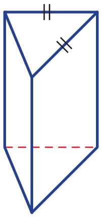
a



b

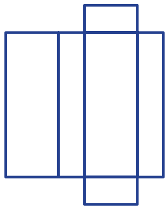


c



2 Match the net (**a–c**) to its solid (**A–C**).

a



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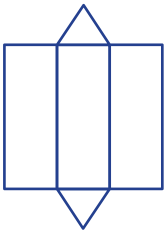

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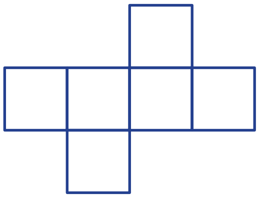
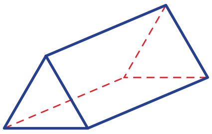
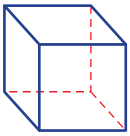


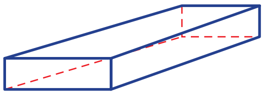
A



B



C



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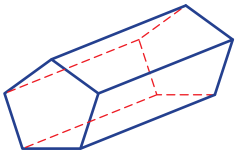
3 How many rectangular faces are on these solids?

a triangular prism **b** rectangular prism
c hexagonal prism **d** pentagonal prism

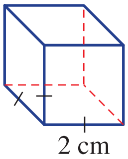
Pentagonal prism



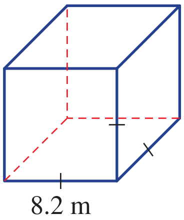
Example 15

4 Find the surface area of these prisms.

a



b



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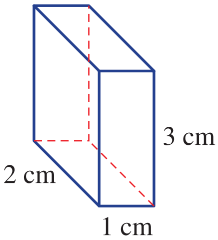

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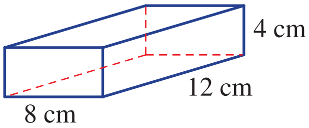

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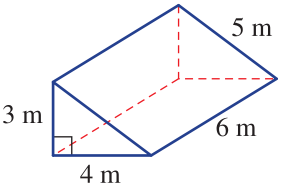

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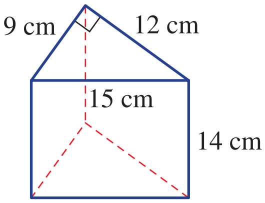
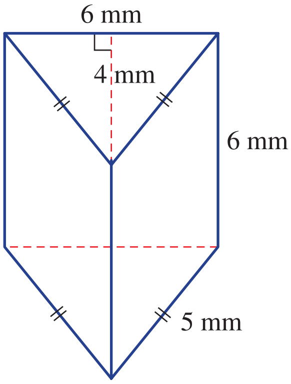


g



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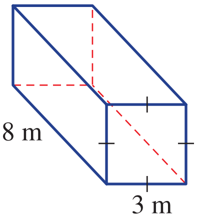

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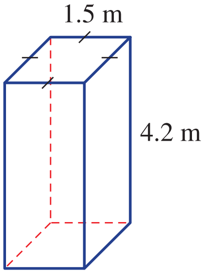
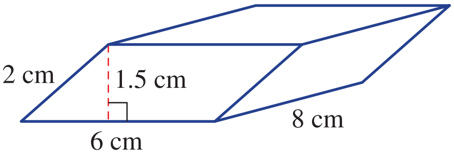


5 This prism has two end faces that are parallelograms.

a Use $A = bh$ to find the combined area of the two ends.

b Find the surface area of the prism.



PROBLEM-SOLVING AND REASONING	6, 7, 10	6–8, 10, 11	7–11

6 An open box (with no lid) is in the shape of a cube and is painted on the outside including the base. What surface area is painted if the side length of the box is 20 cm?

7 A book measuring 20 cm long, 15 cm wide and 3 cm thick is covered in plastic. What area of plastic is needed to cover 1000 books? Convert your answer to m^2.

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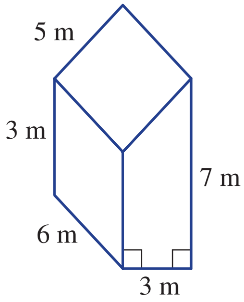
8 Find the surface area of these solids.

a



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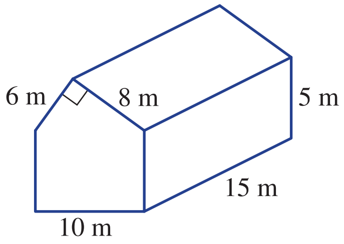
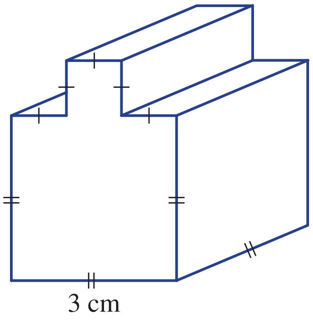
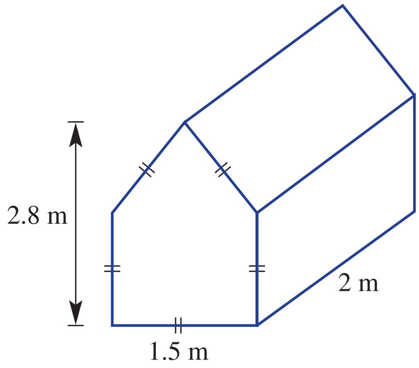


c

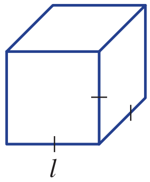


9 The floor, sides and roof of this tent are made from canvas at a cost of $5 per square metre. The tent's dimensions are shown in the diagram. What is the cost of the canvas for the tent?

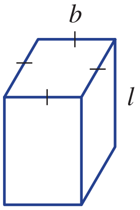


10 Write down the rule for the surface area for these prisms, in simplest form.

a



b



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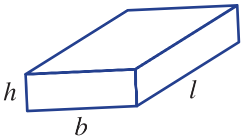


11 A cube of side length 1 cm has a surface area of 6 cm^2.

a What is the effect on the surface area of the cube if:

- **i** its side length is doubled?
- **ii** its side length is tripled?
- **iii** its side length is quadrupled?

b Do you notice a pattern from your answers to part **a**. What effect would multiplying the side length by a factor of n have on the surface area?

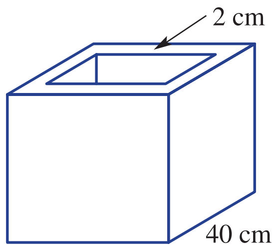
ENRICHMENT	–	–	12

The thick wooden box

12 An open box (with no lid) in the shape of a cube is made of wood that is 2 cm thick. Its outside side length is 40 cm.

a Find its inner surface area and its outer suface area.

b If the box was made with wood that is 1 cm thick, what would be the increase in surface area?



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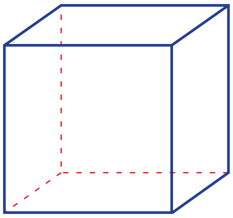
3H Volume and capacity

Volume is a measure of the space occupied by a three-dimensional object. It is measured in cubic units. Common metric units for volume given in abbreviated form include mm^3, cm^3, m^3 and km^3. We also use mL, L, kL and ML to describe volumes of fluids or gas. The volume of space occupied by a room in a house, for example, might be calculated in cubic metres (m^3) or the capacity of a fuel tanker might be measured in litres (L) or kilolitres (kL).

Let's start: Volume

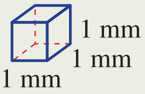


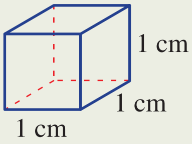
We all know that there are 100 cm in 1 m, but do you know how many cubic centimetres are in 1 cubic metre?

- Try to visualise 1 cubic metre—1 metre long, 1 metre wide and 1 metre high. Guess how many cubic centimetres would fit into this space.
- Describe a method for working out the exact answer. Explain how your method works.

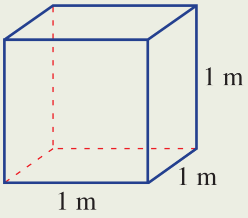
Key ideas

- **Volume** is measured in cubic units.
- The common metric units for volume include:
  - cubic millimetre (mm^3)
  - cubic centimetre (cm^3)
  - cubic metre (m^3)





(Not drawn to scale.)



- Conversions for volume

$$\begin{tikzcd}
km^3 \arrow[r, bend left, "\times 1000^3"] & m^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & cm^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & mm^3 \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

- **Capacity** is the volume of fluid or gas that a container can hold. Common metric units are:
  - millilitre (mL)
  - litre (L)
  - kilolitre (kL)
  - megalitre (ML)

$$\begin{tikzcd}
ML \arrow[r, bend left, "\times 1000"] & kL \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & L \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & mL \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

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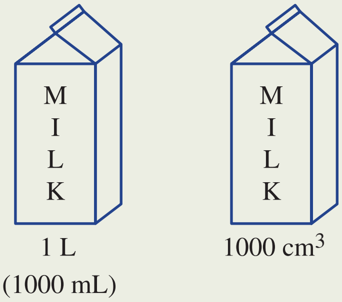
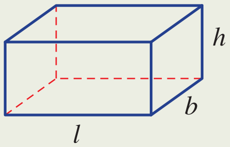
Key ideas

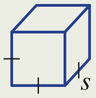
- Some common conversions are:
  - 1 mL = 1 cm^3
  - 1 L = 1000 mL
  - 1 kL = 1000 L = 1 m^3



- Volume of a rectangular prism
  - Volume = length × breadth × height
    $V = lbh$

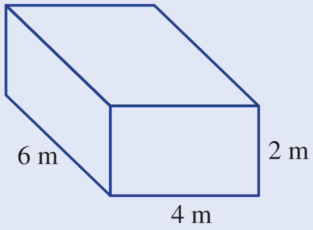


- Volume of a cube
  $V = s^3$



Example 16 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



SOLUTION	EXPLANATION
V = lbh $= 6 \times 4 \times 2$ $= 48$ m^3	First write the rule and then substitute for the length, breadth and height. Any order will do since $6 \times 4 \times 2 = 4 \times 6 \times 2 = 2 \times 4 \times 6$ etc.

Example 17 Calculating capacity

Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.

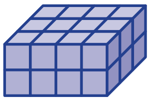
SOLUTION	EXPLANATION
$V = lbh$ $= 20 \times 10 \times 15$ $= 3000$ cm^3 $= 3000 \div 1000$ $= 3$ L	First calculate the volume of the container in cm^3. Then convert to litres using 1 L = 1000 cm^3.

Exercise 3H

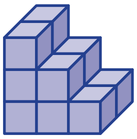
UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	3, 4–6(½)	4–6(½)

1 Count how many cubic units are shown in these solids.

a



b



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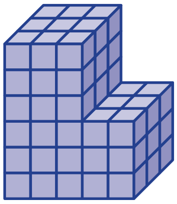


2 Find the missing number.

a $3 \times 4 \times 8 = \_\_\_$

b $\_\_\_ \times 5 \times 20 = 600$

c $8 \times \_\_\_ \times 12 = 192$

d $20 \times 2 \times \_\_\_ = 200$

3 Write the missing number in the following unit conversions.

a 1 L = ___ mL

b ___ kL = 1000 L

c 1000 kL = ___ ML

d 1 mL = ___ cm^3

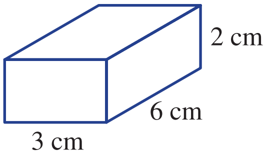
e 1000 cm^3 = ___ L

f 1 m^3 = ___ L

Example 16

4 Find the volume of these rectangular prisms.

a



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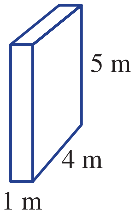

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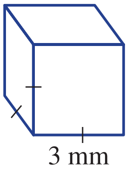

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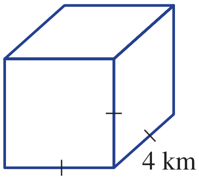

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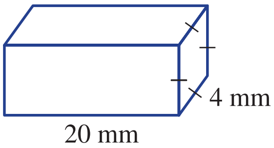

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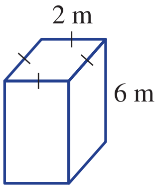


5 Convert the measurements to the units shown in the brackets.

a 2 L (mL)

b 5 kL (L)

c 0.5 ML (kL)

d 3000 mL (L)

e 4 mL (cm^3)

f 50 cm^3 (mL)

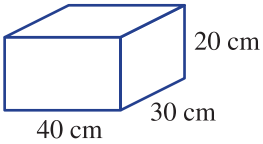
g 2500 cm^3 (L)

h 5.1 L (cm^3)

Example 17

6 Find the capacity of these containers, converting your answer to litres.

a



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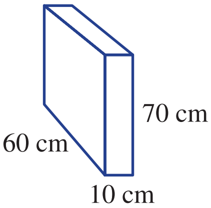

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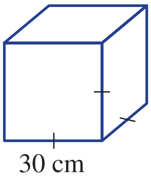

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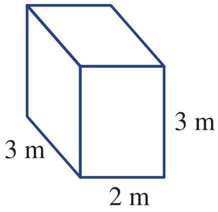

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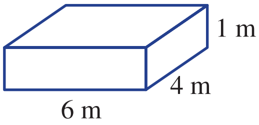

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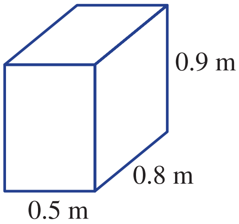

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PROBLEM-SOLVING AND REASONING	7–9, 12	8–10, 12, 13	9–11, 13, 14

7 An oil tanker has a capacity of 60 000 m^3.

a What is the ship's capacity in:

  i litres?

  ii kilolitres?

  iii megalitres?

b If the ship leaks oil at a rate of 300 000 litres per day, how long will it take for all the oil to leak out? Assume the ship started with full capacity.

8 Water is being poured into a fish tank at a rate of 2 L every 10 seconds. The tank is 1.2 m long by 1 m wide by 80 cm high. How long will it take to fill the tank? Give the answer in minutes.

9 A city skyscraper is a rectangular prism measuring 50 m long, 40 m wide and 250 m high.

a What is the total volume in m^3?

b What is the total volume in ML?

10 If 1 kg is the mass of 1 L of water, what is the mass of water in a full container that is a cube with side length 2 m?

11 Using whole numbers only, give all the possible dimensions of rectangular prisms with the following volume. Assume the units are all the same.

a 12 cubic units

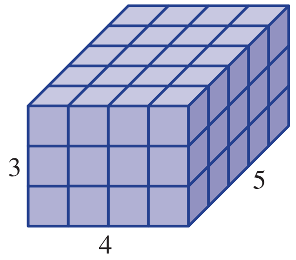
b 30 cubic units

c 47 cubic units

12 Explain why a rectangular prism of volume 46 cm^3 cannot have all its side lengths (length, breadth and height) as whole numbers greater than 1. Assume all lengths are in centimetres.

13 How many cubes with side lengths that are a whole number of centimetres have a capacity of less than 1 litre?

14 Consider this rectangular prism.



a How many cubes are in the base layer?

b What is the area of the base?

c What do you notice about the two answers from above? How can this be explained?

d If A represents the area of the base, explain why the rule $V = Ah$ can be used to find the volume of a rectangular prism.

e Could any side of a rectangular prism be considered to be the base when using the rule $V = Ah$? Explain.

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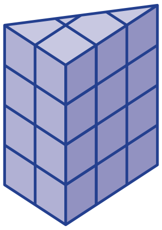
ENRICHMENT – – 15

Having rectangular prisms

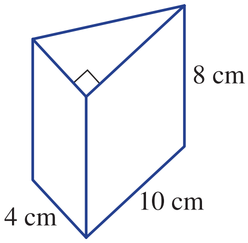
15 This question looks at using half of a rectangular prism to find the volume of a triangular prism.



a Consider this triangular prism.
  i Explain why this solid could be thought of as half a rectangular prism.
  ii Find its volume.

b Using a similar idea, find the volume of these prisms.

i



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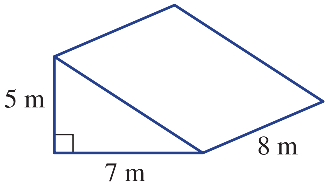

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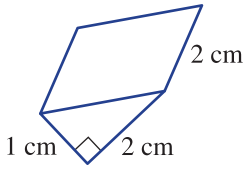

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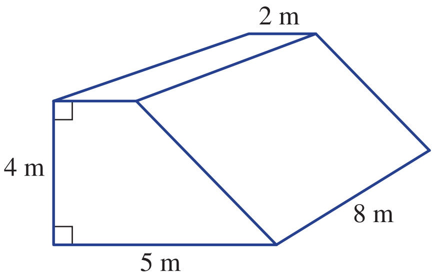

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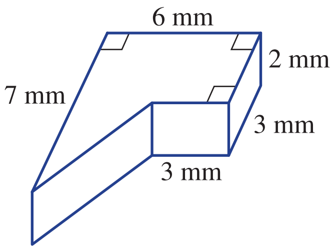

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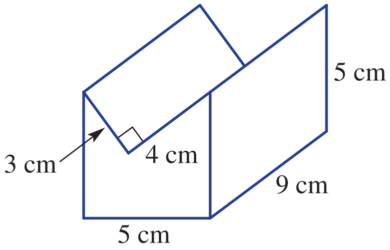

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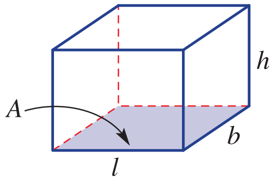
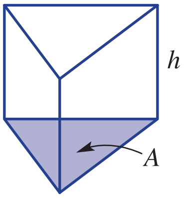
3I Volume of prisms and cylinders



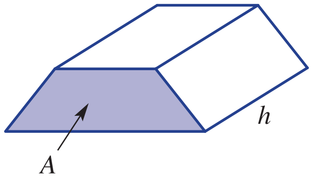
We know that for a rectangular prism its volume V is given by the rule $V = lbh$. Length × breadth (lb) gives the number of cubes on the base, but it also tells us the area of the base, A. So $V = lbh$ could also be written as $V = Ah$.

The rule $V = Ah$ can also be applied to prisms that have different shapes as their bases. One condition, however, is that the area of the base must represent the area of the cross-section of the solid. The height h is measured perpendicular to the cross-section. Note that a cylinder is *not* a prism as it does not have sides that are parallelograms; however, it can be treated like a prism when finding its volume because it has a constant cross-section, a circle.

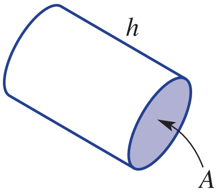
Here are some examples of two prisms and a cylinder with A and h marked.



Cross-section is a triangle



Cross-section is a trapezium



Cross-section is a circle

Let's start: Drawing prisms

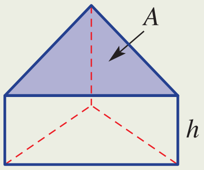
Try to draw prisms (or cylinders) that have the following shapes as their cross-sections.

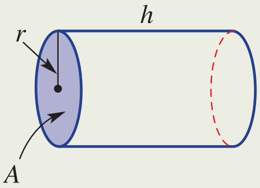
- circle
- trapezium
- parallelogram
- triangle
- pentagon

The cross-section of a prism should be the same size and shape along the entire length of the prism. Check this property on your drawings.

Key ideas

- A **prism** is a polyhedron with a constant (uniform) cross-section, with area A.
  - Its sides between the two congruent ends are parallelograms.
  - A right prism has rectangular sides between the congruent ends.
- Volume of a prism = Area of cross-section × perpendicular height or $V = Ah$.
- Volume of a **cylinder** $= Ah = \pi r^2 \times h = \pi r^2 h$
  So $V = \pi r^2 h$.





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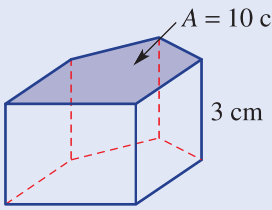
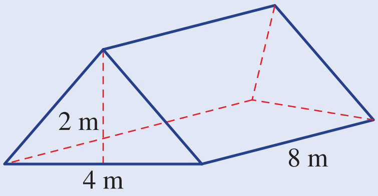
Example 18 Finding the volumes of prisms

Find the volumes of these prisms.

a



b



SOLUTION

a $V = Ah$

$= 10 \times 3$

$= 30\text{ cm}^3$

b $V = Ah$

$= \left(\frac{1}{2} \times 4 \times 2\right) \times 8$

$= 32\text{ m}^3$

EXPLANATION

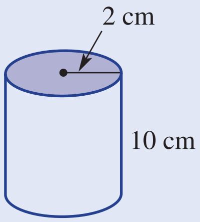
Write the rule and substitute the given values of A and h, where A is the area of the cross-section.

The cross-section is a triangle, so use $A = \frac{1}{2}bh$ with base 4 m and height 2 m.

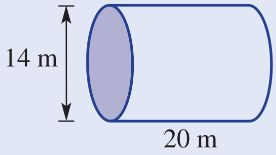
Example 19 Finding the volume of a cylinder

Find the volumes of these cylinders, rounding the answer to two decimal places.

a



b



SOLUTION

a $V = \pi r^2 h$

$= \pi \times 2^2 \times 10$

$= 125.66\text{ cm}^3$ (to 2 d.p.)

b $V = \pi r^2 h$

$= \pi \times 7^2 \times 20$

$= 3078.76\text{ m}^3$ (to 2 d.p.)

EXPLANATION

Write the rule and then substitute the given values for π, r and h.

Round as required.

The diameter is 14 m so the radius is 7 m.

Round as required.

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Exercise 3I

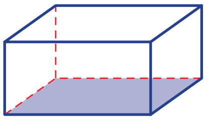
UNDERSTANDING AND FLUENCY	1–3, 4, 5–6(½)	2, 4, 5–6(½)	4, 5–6(½)

1 For these solids below:

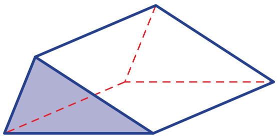
i State whether or not it is a prism.

ii If it is a prism, state the shape of its cross-section.

a



b



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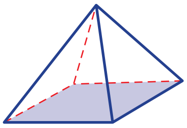
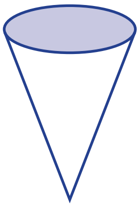
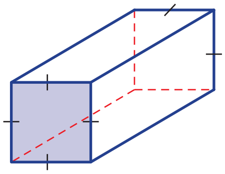


d



e



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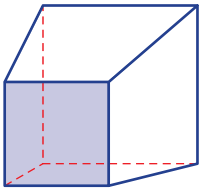
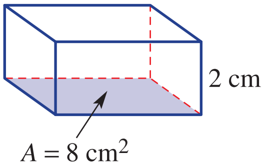
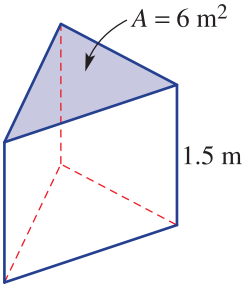


2 For these prisms and cylinder state the value of A and the value of h that could be used in the rule $V = Ah$ to find the volume of the solid.

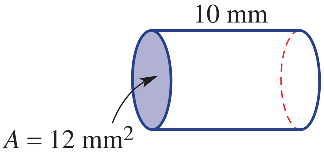
a



b



c



3 Evaluate the following.

a $2 \times 5 \times 4$

b $\frac{1}{2} \times 3 \times 4 \times 6$

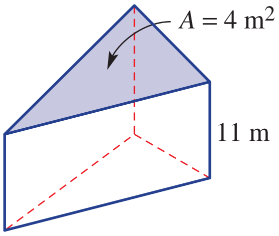
c $\frac{1}{2}(2 + 3) \times 4$

d $3.14 \times 10^2 \times 20$

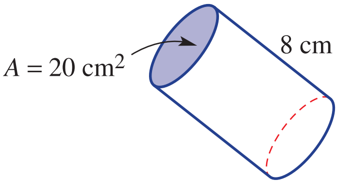
Example 18a

4 Find the volume of these solids using $V = Ah$.

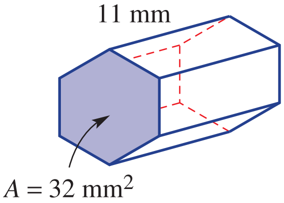
a



b



c



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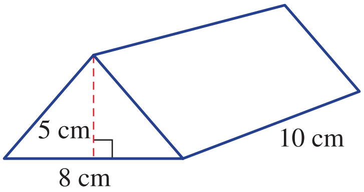
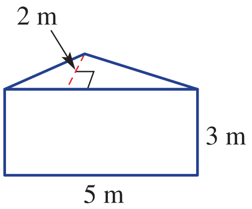
Example 18b

5 Find the volume of these prisms.

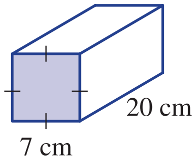
a



b



c



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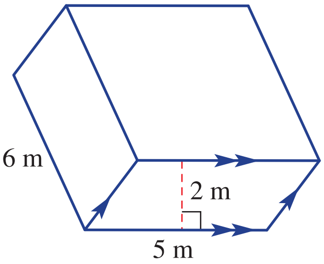

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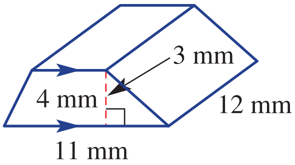

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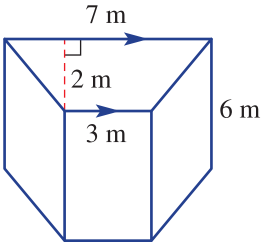
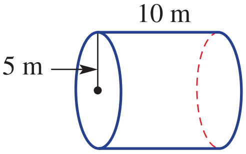


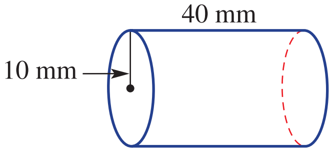
Example 19

6 Find the volume of these cylinders. Round the answer to two decimal places.

a



b



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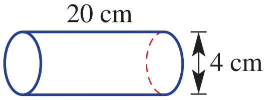

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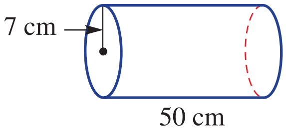

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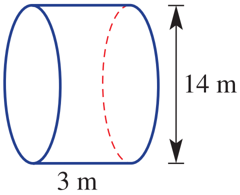

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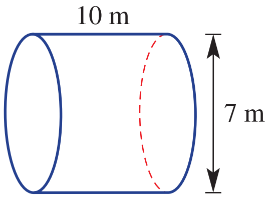


PROBLEM-SOLVING AND REASONING	7, 8, 12	8–10, 12, 13	9–11, 13, 14

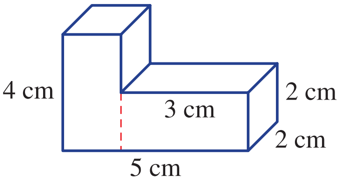
7 A cylindrical tank has a diameter of 3 m and height 2 m.

a Find its volume in m^3, correct to three decimal places.

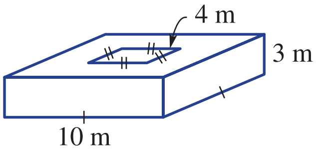
b What is the capacity of the tank in litres?

8 These solids are made up of more than one rectangular prism. Use addition or subtraction to find the volume of the composite solid.

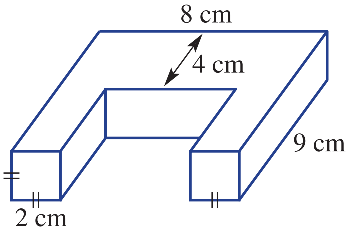
a



b



c



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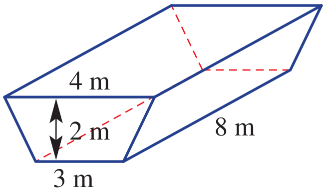
9 Susan pours water from a full 4 L container into a number of water bottles for a camp hike. Each water bottle is a cylinder with radius 4 cm and height 20 cm. How many bottles can be filled completely?

10 There are 80 liquorice cubes stacked in a cylindrical glass jar. The liquorice cubes have a side length of 2 cm and the glass jar has a radius of 5 cm and a height of 12 cm. How much air space remains in the jar of liquorice cubes? Give the answer correct to two decimal places.

11 A swimming pool is a prism with a cross-section that is a trapezium. The pool is being filled at a rate of 1000 litres per hour.

a Find the capacity of the pool in litres.

b How long will it take to fill the pool?



12 Using exact values (e.g. 10π cm^3) calculate the volume of cylinders with these dimensions.

a radius 2 m and height 5 m

b radius 10 cm and height 3 cm

c diameter 8 mm and height 9 mm

d diameter 7 m and height 20 m

13 A cylinder has a volume of 100 cm^3. Give three different combinations of radius and height measurements that give this volume. Give these lengths correct to two decimal places.

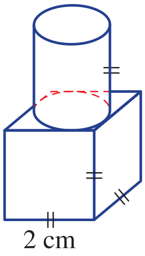
14 A cube has side length x metres and a cylinder has a radius also of x metres and height h. What is the rule linking x and h if the cube and the cylinder have the same volume?

ENRICHMENT – – 15

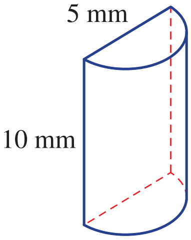
Complex composites

15 Use your knowledge of volumes of prisms and cylinders to find the volume of these composite solids. Round the answer to two decimal places where necessary.

a



b



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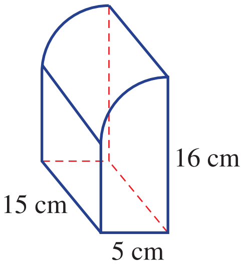

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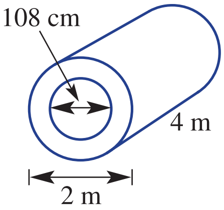

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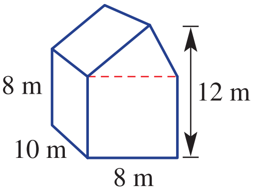

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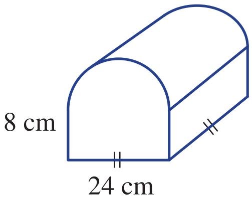

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3J Time REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Time in minutes and seconds is based on the number 60. Other units of time, including the day and year, are defined by the rate at which the Earth spins on its axis and the time that the Earth takes to orbit the Sun.

The origin of the units seconds and minutes dates back to the ancient Babylonians, who used a base 60 number system. The 24-hour day dates back to the ancient Egyptians, who described the day as 12 hours of day and 12 hours of night. Today, we use a.m. (*ante meridiem*, which is Latin for 'before noon') and p.m. (*post meridiem*, which is Latin for 'after noon') to represent the hours before and after noon (midday). During the rule of Julius Caesar, the ancient Romans introduced the solar calendar, which recognised that the Earth takes about $365\frac{1}{4}$ days to orbit the Sun. This gave rise to the leap year, which includes one extra day (in February) every 4 years.

Let's start: Knowledge of time

Do you know the answers to these questions about time and the calendar?

- When is the next leap year?
- Why do we have a leap year?
- Which months have 31 days?
- Why are there different times in different countries or parts of a country?
- What do BCE (or BC) and CE (or AD) mean on time scales?

Key ideas

- The standard unit of time is the **second** (s).
- Units of time include:
  - 1 **minute** (min) = 60 seconds (s)
  - 1 **hour** (h) = 60 minutes (min)
  - 1 **day** = 24 hours (h)
  - 1 **week** = 7 days
  - 1 **year** = 12 months

$$\begin{tikzcd}
\text{day} \arrow[r, bend left, "\times 24"] & \text{hour} \arrow[l, bend left, "\div 24"] \arrow[r, bend left, "\times 60"] & \text{minute} \arrow[l, bend left, "\div 60"] \arrow[r, bend left, "\times 60"] & \text{second} \arrow[l, bend left, "\div 60"]
\end{tikzcd}$$

- a.m. or p.m. is used to describe the 12 hours before and after noon (midday).
- **24-hour time** shows the number of hours and minutes after midnight.
  - 0330 is 3:30 a.m.
  - 1530 is 3:30 p.m.
- The 'degrees, minutes and seconds' button on a calculator can be used to convert a particular time into hours, minutes and seconds.
  For example: 4.42 hours = 4°25′12″ meaning 4 hours, 25 minutes and 12 seconds.

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Key ideas

- The Earth is divided into 24 time zones (one for each hour).
  - Twenty-four 15° lines of longitude divide the Earth into its time zones. Time zones also depend on a country's borders and its proximity to other countries. (See map on pages 166–7 for details.)
  - Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
  - Places east of Greenwich are ahead in time.
  - Places west of Greenwich are behind in time.
- Australia has three time zones:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.

Example 20 Converting units of time

Convert these times to the units shown in the brackets.

a 3 days (minutes)

b 30 months (years)

SOLUTION	EXPLANATION
a $3 \text{ days} = 3 \times 24 \text{ h}$ $= 3 \times 24 \times 60 \text{ min}$ $= 4320 \text{ min}$	1 day = 24 hours 1 hour = 60 minutes
b $30 \text{ months} = 30 \div 12 \text{ years}$ $= 2\frac{1}{2} \text{ years}$	There are 12 months in 1 year.

Example 21 Using 24-hour time

Write these times using the system given in the brackets.

a 4:30 p.m. (24-hour time)

b 1945 hours (a.m./p.m.)

SOLUTION	EXPLANATION
a 4:30 p.m. = 1200 + 0430 = 1630 hours	Since the time is p.m., add 12 hours to 0430 hours.
b 1945 hours = 7:45 p.m.	Since the time is after 1200 hours, subtract 12 hours.

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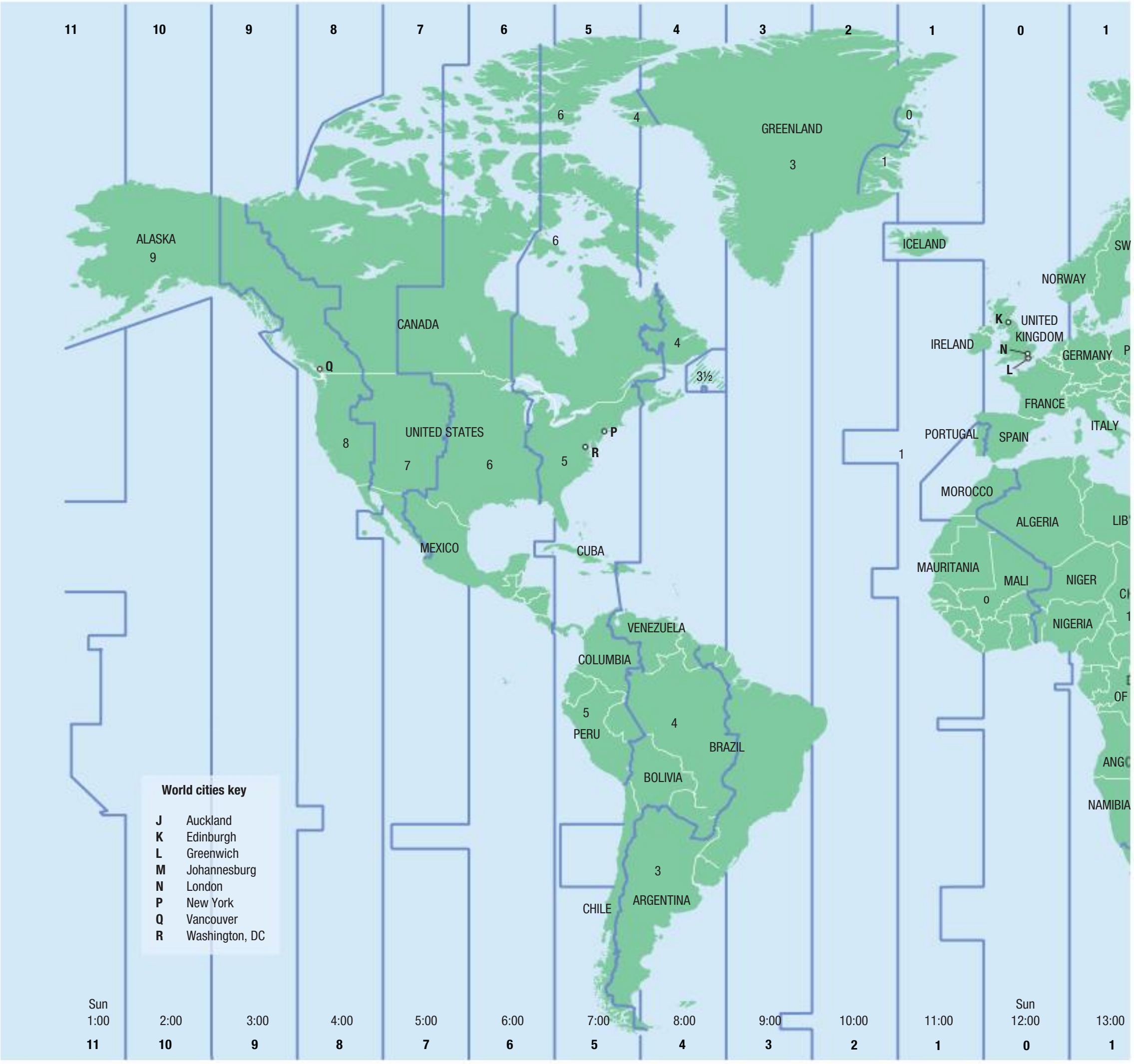

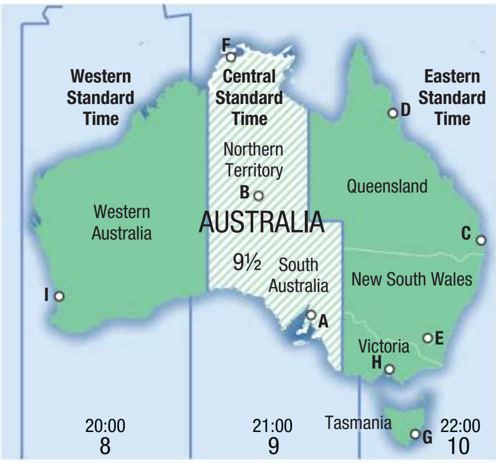

Australian cities key

A Adelaide
B Alice Springs
C Brisbane
D Cairns
E Canberra, ACT
F Darwin
G Hobart
H Melbourne
I Perth

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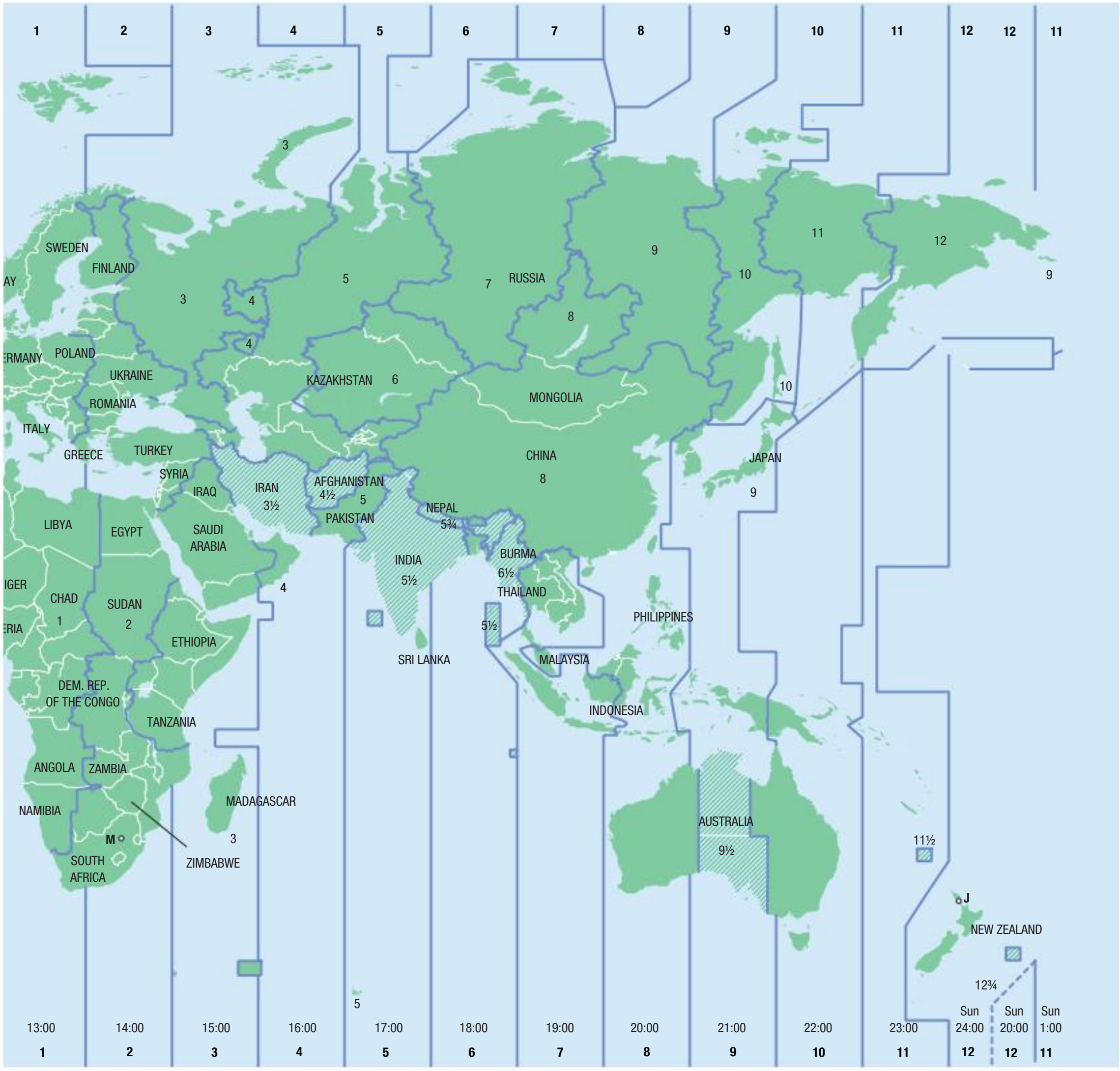
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SWEDEN
FINLAND
AY
ERMANY
POLAND
UKRAINE
ROMANIA
ITALY
GREECE
TURKEY
SYRIA
IRAQ
IRAN
3½
AFGHANISTAN
4½
5
PAKISTAN
NEPAL
5¾
INDIA
5½
SRI LANKA
BURMA
6½
THAILAND
5½
MALAYSIA
INDONESIA
PHILIPPINES
KAZAKHSTAN
6
RUSSIA
MONGOLIA
CHINA
8
JAPAN
9
10
11
12
9
LIBYA
EGYPT
SAUDI
ARABIA
IGER
CHAD
1
SUDAN
2
ERIA
ETHIOPIA
4
DEM. REP.
OF THE CONGO
TANZANIA
ANGOLA
ZAMBIA
NAMIBIA
MADAGASCAR
3
M
SOUTH
AFRICA
ZIMBABWE
AUSTRALIA
9½
11½
J
NEW ZEALAND
12¾
5
13:00
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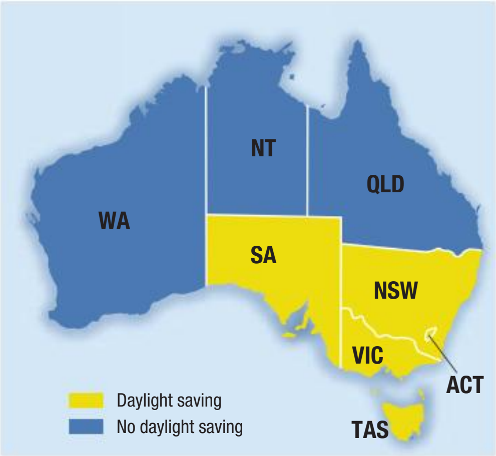
NT
QLD
WA
SA
NSW
VIC
ACT
TAS
🟨 Daylight saving
🟦 No daylight saving

Example 22 Using time zones

Coordinated Universal Time (UTC) and is based on the time in Greenwich, United Kingdom. Use the world time zone map (on pages 166–7) to answer the following.

a When it is 2 p.m. UTC, find the time in these places.
  i France
  ii China
  iii Queensland
  iv Alaska

b When it is 9:35 a.m. in New South Wales, Australia, find the time in these places.
  i Alice Springs
  ii Perth
  iii London
  iv central Greenland

SOLUTION	EXPLANATION
a i 2 p.m. + 1 hour = 3 p.m.	Use the time zone map to see that France is to the east of Greenwich and is in a zone that is 1 hour ahead.
ii 2 p.m. + 8 hours = 10 p.m.	From the time zone map, China is 8 hours ahead of Greenwich.
iii 2 p.m. + 10 hours = 12 a.m.	Queensland uses Eastern Standard Time, which is 10 hours ahead of Greenwich.
iv 2 p.m. – 9 hours = 5 a.m.	Alaska is to the west of Greenwich, in a time zone that is 9 hours behind.
b i 9:35 a.m. – $\frac{1}{2}$ hour = 9:05 a.m.	Alice Springs uses Central Standard Time, which is $\frac{1}{2}$ hour behind Eastern Standard Time.
ii 9:35 a.m. – 2 hours = 7:35 a.m.	Perth uses Western Standard Time, which is 2 hours behind Eastern Standard Time.
iii 9:35 a.m. – 10 hours = 11:35 p.m. (the day before)	UTC (time in Greenwich, United Kingdom) is 10 hours behind EST.
iv 9:35 a.m. – 13 hours = 8:35 p.m. (the day before)	Central Greenland is 3 hours behind UTC in Greenwich, so is 13 hours behind EST.

Exercise 3J REVISION

UNDERSTANDING AND FLUENCY	1–3, 4–6(½), 7, 8–9(½)	3, 4–6(½), 7, 8–10(½)	4–6(½), 7, 8–10(½), 11

1 From options **A** to **F**, match up the time units with the most appropriate description.

a single heartbeat
b 40 hours of work
c duration of a university lecture
d bank term deposit
e 200 m run
f flight from Australia to the UK

A 1 hour
B 1 minute
C 1 day
D 1 week
E 1 year
F 1 second

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2 Find the number of:

a seconds in 2 minutes
b minutes in 180 seconds
c hours in 120 minutes
d minutes in 4 hours
e hours in 3 days
f days in 48 hours
g weeks in 35 days
h days in 40 weeks

3 What is the time difference between these times?

a 12 p.m. and 6:30 p.m.
b 12 a.m. and 10:45 a.m.
c 12 a.m. and 4:20 p.m.
d 11 a.m. and 3:30 p.m.

Example 20

4 Convert these times to the units shown in the brackets.

a 3 h (min)
b 10.5 min (s)
c 240 s (min)
d 90 min (h)
e 6 days (h)
f 72 h (days)
g 1 week (h)
h 1 day (min)
i 14 400 s (h)
j 20 160 min (weeks)
k 2 weeks (min)
l 24 h (s)

5 Convert the following to hours and minutes. For parts **f** to **i**, use the 'degrees, minutes and seconds' button on a calculator to convert to hours, minutes and seconds.

a 2.5 hours
b 3.25 hours
c 1.75 hours
d 4.2 hours
e 2.6 hours
f 3.21 hours
g 2.38 hours
h 7.74 hours
i 6.03 hours

Example 21

6 Write these times using the system shown in the brackets.

a 1:30 p.m. (24-hour)
b 8:15 p.m. (24-hour)
c 10:23 a.m. (24-hour)
d 11:59 p.m. (24-hour)
e 0630 hours (a.m./p.m.)
f 1300 hours (a.m./p.m.)
g 1429 hours (a.m./p.m.)
h 1938 hours (a.m./p.m.)
i 2351 hours (a.m./p.m.)

7 Round these times to the nearest hour.

a 1:32 p.m.
b 5:28 a.m.
c 1219 hours
d 1749 hours

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8 What is the time difference between these time periods?

a 10:30 a.m. and 1:20 p.m.
b 9:10 a.m. and 3:30 p.m.
c 2:37 p.m. and 5:21 p.m.
d 10:42 p.m. and 7:32 a.m.
e 1451 and 2310 hours
f 1940 and 0629 hours

Example 22a

9 Use the time zone map on pages 166–7 to find the time in the following places, when it is 10 a.m. UTC.

a Spain
b Turkey
c Tasmania
d Darwin
e Argentina
f Peru
g Alaska
h Portugal

Example 22b

10 Use the time zone map on pages 166–7 to find the time in these places, when it is 3:30 p.m. in Victoria.

a United Kingdom
b Libya
c Sweden
d Perth
e Japan
f central Greenland
g Alice Springs
h New Zealand

11 What is the time difference between these pairs of places?

a United Kingdom and Kazakhstan
b South Australia and New Zealand
c Queensland and Egypt
d Peru and Angola (in Africa)
e Mexico and Germany

PROBLEM-SOLVING AND REASONING	12–14, 22	14–17, 22–23	17–21, 23–25

12 A scientist argues that dinosaurs died out 52 million years ago, whereas another says they died out 108 million years ago. What is the difference in their time estimates?

13 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

14 Adrian arrives at school at 8:09 a.m. and leaves at 3:37 p.m. How many hours and minutes is Adrian at school?

15 On a flight to Europe, Janelle spends 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, Malaysia, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

16 A phone plan charges 11 cents per 30 seconds. The 11 cents are added to the bill at the beginning of every 30-second block of time.

a What is the cost of a 70-second call?
b What is the cost of a call that lasts 6 minutes and 20 seconds?

17 A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

a per day
b per hour
c per minute
d per second (in cents)

18 A 2-hour football match starts at 2:30 p.m. Eastern Standard Time (EST) in Newcastle, NSW. What time will it be in the United Kingdom when the match finishes?

19 If the date is 29 March and it is 3 p.m. in Perth, what is the time and date in these places?

a Italy
b Alaska
c Chile

20 Monty departs on a 20-hour flight from Brisbane to London, United Kingdom, at 5 p.m. on 20 April. Give the time and date of his arrival in London.

21 Elsa departs on an 11-hour flight from Johannesburg, South Africa, to Perth at 6:30 a.m. on 25 October. Give the time and date of her arrival in Perth.

22 When there are 365 days in a year, how many weeks are there in a year? Round your answer to two decimal places.

23 **a** To convert from hours to seconds, what single number do you multiply by?
b To convert from days to minutes, what single number do you multiply by?
c To convert from seconds to hours, what single number do you divide by?
d To convert from minutes to days, what single number do you divide by?

24 Assuming there are 365 days in a year and my birthday falls on a Wednesday this year, on what day will my birthday fall in 2 years' time?

25 **a** Explain why you gain time when you travel from Australia to Europe.
b Explain why you lose time when you travel from Germany to Australia.
c Explain what happens to the date when you fly from Australia to Canada across the International Date Line.

ENRICHMENT	–	–	26

Daylight saving

26 Use the internet to investigate how daylight saving affects the time in some places. Write a brief report discussing the following points.

a **i** Name the States in Australia that use daylight saving.
  ii Name five other countries that use daylight saving.
b Describe how daylight saving works, why it is used and what changes have to be made to our clocks.
c Describe how daylight saving in Australia affects the time difference between time zones. Use New South Wales and Greece as an example.

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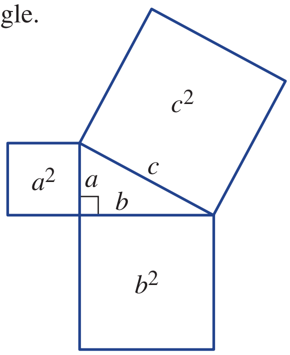
3K Introducing Pythagoras' theorem

Interactive
Widgets
HOTsheets
Walkthrough

Pythagoras was a philosopher in ancient Greece who lived in the sixth century BCE. He studied astronomy, mathematics, music and religion, but is most well known for the famous Pythagoras' theorem. Pythagoras was known to provide a proof for the theorem that bears his name, and methods to find Pythagorean triads, which are sets of three whole numbers that make up the sides of right-angled triangles.

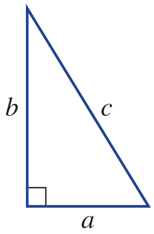
The ancient Babylonians, 1000 years before Pythagoras' time, and the ancient Egyptians also knew that there was a relationship between the sides of a right-angled triangle. Pythagoras, however, was able to clearly explain and prove the theorem using mathematical symbols. The ancient theorem is still one of the most commonly used theorems today.

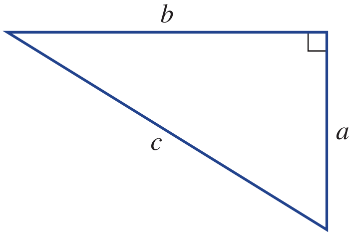
Pythagoras' theorem states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. An illustration of the theorem includes squares drawn on the sides of the right-angled triangle. The area of the larger square (c^2) is equal to the sum of the two smaller squares ($a^2 + b^2$).

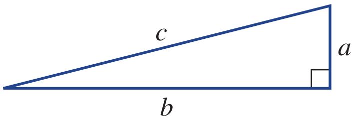


Let's start: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles, to the nearest mm. Then complete the table.





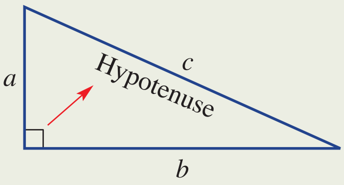


a	b	c	a^2	b^2	c^2

- Can you see any relationship between the numbers in the columns for a^2 and b^2 and the number in the column for c^2?
- Can you write this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it is unknown.
- Research how you can cut the two smaller squares (a^2 and b^2) to fit the pieces into the largest square (c^2).

Key ideas

- **The hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.



- **Pythagoras' theorem**
  - The square of the hypotenuse is the sum of the squares of the lengths of the other two shorter sides.
  - $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$
- A **Pythagorean triad** (or triple) is a set of three integers that satisfy Pythagoras' theorem.

Example 23 Checking Pythagorean triads

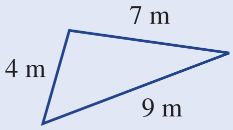
Decide if the following are Pythagorean triads.

a 6, 8, 10 **b** 4, 5, 9

SOLUTION	EXPLANATION
a $a^2 + b^2 = 62 + 82$ $= 36 + 64$ $= 100\,(=10^2)$ $\therefore$ 6, 8, 10 is a Pythagorean triad.	Let $a = 6$, $b = 8$ and $c = 10$ and check that $a^2 + b^2 = c^2$.
b $a^2 + b^2 = 4^2 + 5^2$ $= 16 + 25$ $= 41$ $\neq 9^2$ $\therefore$ 4, 5, 9 is not a Pythagorean triad.	$a^2 + b^2 = 41$ and $9^2 = 81$ so $a^2 + b^2 \neq c^2$.

Example 24 Deciding if a triangle has a right angle

Decide if this triangle has a right angle.



SOLUTION	EXPLANATION
$a^2 + b^2 = 42 + 72$ $= 16 + 49$ $= 65\ (\neq 9^2 = 81)$ So the triangle does not have a right angle.	Check to see if $a^2 + b^2 = c^2$. In this case $a^2 + b^2 = 65$ and $c^2 = 81$, so the triangle is not right angled.

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Exercise 3K

UNDERSTANDING AND FLUENCY	1–4, 5(½), 7(½)	5(½), 6, 7(½)	5(½), 6, 7(½)

1 Calculate these squares and sums of squares.

- **a** 3^2
- **b** 5^2
- **c** 12^2
- **d** 1.5^2
- **e** $2^2 + 4^2$
- **f** $3^2 + 7^2$
- **g** $6^2 + 11^2$
- **h** $12^2 + 15^2$

2 Decide if these equations are true or false.

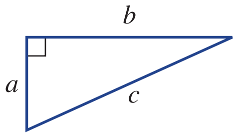
- **a** $2^2 + 3^2 = 4^2$
- **b** $6^2 + 8^2 = 10^2$
- **c** $7^2 + 24^2 = 25^2$
- **d** $5^2 - 3^2 = 4^2$
- **e** $6^2 - 3^2 = 2^2$
- **f** $10^2 - 5^2 = 5^2$

3 Write the missing words in this sentence.

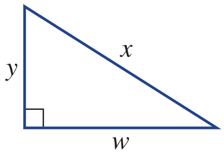
The ____________ is the longest side of a right-angled ____________.

4 Which letter marks the length of the hypotenuse in these triangles?

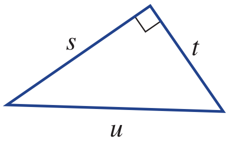
a



b



c



Example 23

5 Decide if the following are Pythagorean triads.

- **a** 3, 4, 6
- **b** 4, 2, 5
- **c** 3, 4, 5
- **d** 9, 12, 15
- **e** 5, 12, 13
- **f** 2, 5, 6
- **g** 9, 40, 41
- **h** 10, 12, 20
- **i** 4, 9, 12

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6 Complete this table and answer the questions.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5				
6	8	10				
8	15	17				

a Which two columns give equal results?

b What would be the value of c^2 if:

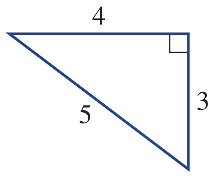
i $a^2 = 4$ and $b^2 = 9$? **ii** $a^2 = 7$ and $b^2 = 13$?

c What would be the value of $a^2 + b^2$ if:

i $c^2 = 25$? **ii** $c^2 = 110$?

7 Check that $a^2 + b^2 = c^2$ for all these right-angled triangles.

a



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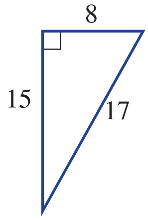

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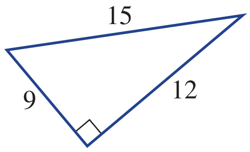

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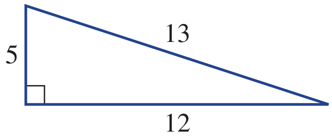
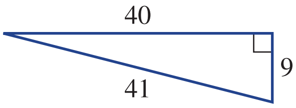
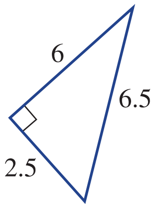


e



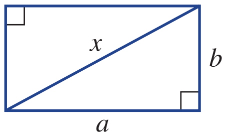
f



PROBLEM-SOLVING AND REASONING	8, 9, 11	8, 9, 11, 12	9–13

8 Write down Pythagoras' theorem using the letters given these diagrams.

a



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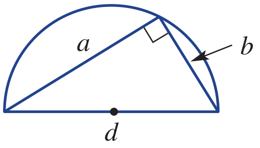

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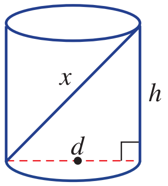


9 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

a Using $c = 40$, decide if $a^2 + b^2 = c^2$.

b Do you think the triangle formed by the mast and the cable is right angled? Give a reason.

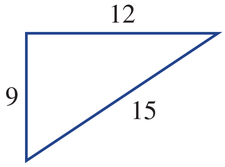
10 (3, 4, 5) and (5, 12, 13) are Pythagorean triads since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. Find 10 more Pythagorean triads, using whole numbers less than 100.

Extension: Find the total number of Pythagorean triads with whole numbers of less than 100.

Example 24

11 If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

a



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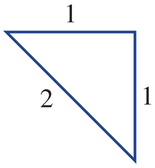
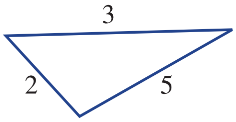
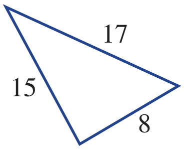


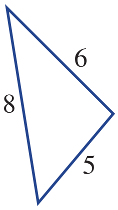
c



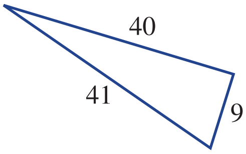
d



e



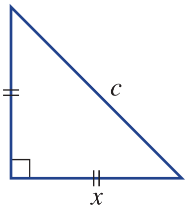
f



12 If $a^2 + b^2 = c^2$ is true, complete these statements.

a $c^2 - b^2 =$ ____ **b** $c^2 - a^2 =$ ____ **c** $c =$ ________

13 This triangle is isosceles. Write Pythagoras' theorem using the given letters. Simplify if possible.



ENRICHMENT	–	–	14

Pythagoras' theorem proof

14 There are many ways to prove Pythagoras' theorem, both algebraically and geometrically.

a Here is an incomplete proof of the theorem that uses this illustrated geometric construction.

Area of inside square $= c^2$

Area of 4 outside triangles $= 4 \times \frac{1}{2} \times$ base × height

$=$ ________

Total area of outside square $=$ (______ + ______)^2

$= a^2 + 2ab + b^2$

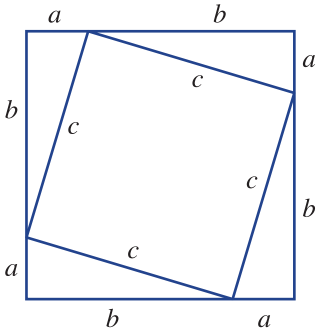
Area of inside square $=$ Area outside square $-$ Area of 4 triangles

$=$ ________ $-$ ________

$=$ ________

Comparing results from the first and last steps gives:

$c^2 =$ ________



b Use the internet to search for other proofs of Pythagoras' theorem. See if you can explain and illustrate them.

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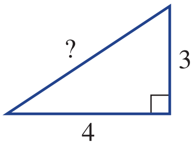
3L Using Pythagoras' theorem

From our understanding of algebra, we know that equations can be solved to find the value of an unknown. This is also the case for equations derived from Pythagoras' theorem where if two of the side lengths of a right-angled triangle are known, then the third can be found.

So if $c^2 = 3^2 + 4^2$, then $c^2 = 25$ and $c = 5$.

We also notice that if $c^2 = 25$, then $c = \sqrt{25} = 5$ (if $c > 0$).

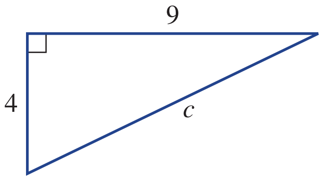
Pythagoras' theorem has a wide range of applications wherever right-angled triangles can be drawn.



Note that a number using a $\sqrt{\ }$ sign may not always result in a whole number. For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers and neither can be written as a fraction. These types of numbers are called surds and are a special group of numbers (irrational numbers) that are often approximated using rounded decimals.

Let's start: Correct layout

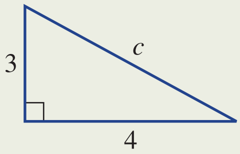
Three students who are trying to find the value of c in this triangle using Pythagoras' theorem write their solutions on a board. There are only very minor differences between each solution and the answer is written rounded to two decimal places. Which student has all the steps correct? Give reasons why the other two solutions are not laid out correctly.



Student 1	Student 2	Student 3
$c^2 = a^2 + b^2$ $= 4^2 + 9^2$ $= 97$ $= \sqrt{97}$ $= 9.85$	$c^2 = a^2 + b^2$ $= 4^2 + 9^2$ $= 97$ $\therefore\ c = \sqrt{97}$ $= 9.85$	$c^2 = a^2 + b^2$ $= 4^2 + 9^2$ $= 97$ $= \sqrt{97}$ $= 9.85$

Key ideas

- To find the length of the hypotenuse:



$$\begin{aligned} c^2 &= 3^2 + 4^2 \\ c^2 &= 9 + 16 \\ c^2 &= 25 \text{ (where } c \text{ is positive)} \\ \therefore c &= \sqrt{25} \\ &= 5 \end{aligned}$$

  - In this example, the equation $c^2 = 25$ has only one solution, not two, because c is a length.
- Note that the final step may not always result in a whole number.
  For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers.
- **Surds** are numbers that have a $\sqrt{\ }$ sign when written in simplest form.
  - They are not a whole number and cannot be written as a fraction.
  - Written as a decimal, the decimal places would continue forever with no repeated pattern (just like the number pi), so surds are irrational numbers.
  - Examples of surds are $\sqrt{2}$ and $\sqrt{5}$.

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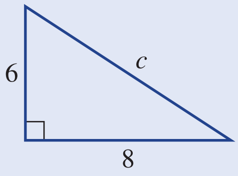
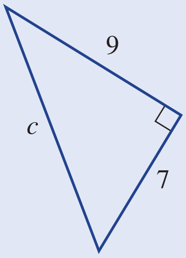
Example 25 Finding the length of the hypotenuse

Find the length of the hypotenuse for these right-angled triangles. Round the answer for part **b** to two decimal places.

a



b



SOLUTION

a $c^2 = a^2 + b^2$

$= 6^2 + 8^2$

$= 100$

$\therefore c = \sqrt{100}$

$= 10$

b $c^2 = a^2 + b^2$

$= 7^2 + 9^2$

$= 130$

$\therefore c = \sqrt{130}$

$= 11.40$ (to 2 d.p.)

EXPLANATION

Write the equation for Pythagoras' theorem and substitute the values for the shorter sides.

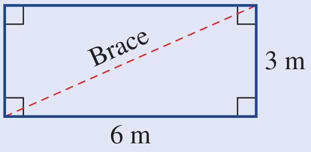
Find c by taking the square root.

First calculate the value of $7^2 + 9^2$.

$\sqrt{130}$ is a surd, so round the answer as required.

Example 26 Applying Pythagoras' theorem

A rectangular wall is to be strengthened by a diagonal brace. The wall is 6 m wide and 3 m high. Find the length of brace required, correct to the nearest cm.



SOLUTION

$c^2 = a^2 + b^2$

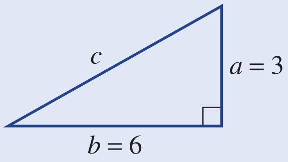
$= 3^2 + 6^2$

$= 45$

$c = \sqrt{45}$

$= 6.71$ m or 671 cm (nearest cm)

EXPLANATION



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Exercise 3L

UNDERSTANDING AND FLUENCY	1–5	3, 4–5(½)	4–5(½)

1 Decide if these numbers written with a $\sqrt{\ }$ sign simplify to a whole number. Answer yes or no.

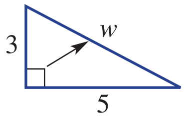
a $\sqrt{9}$ **b** $\sqrt{11}$ **c** $\sqrt{20}$ **d** $\sqrt{121}$

2 Round these surds correct to two decimal places, using a calculator.

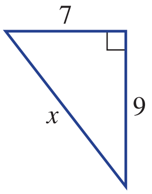
a $\sqrt{10}$ **b** $\sqrt{26}$ **c** $\sqrt{65}$ **d** $\sqrt{230}$

3 Write Pythagoras' theorem for each of the following triangles.

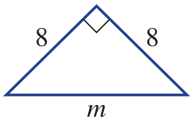
a



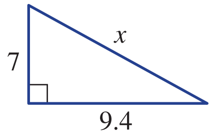
b



c



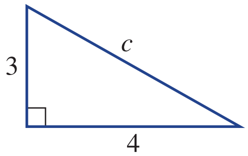
d



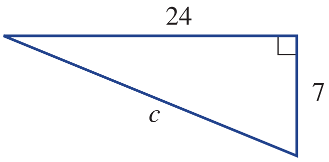
Example 25a

4 Find the length of the hypotenuse (c) of these right-angled triangles.

a



b



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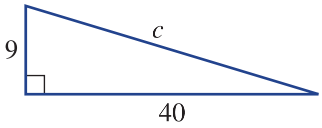

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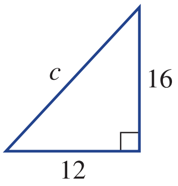

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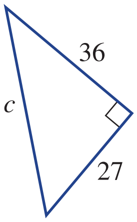

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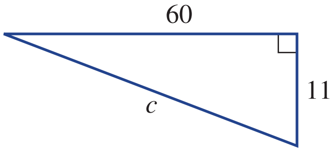
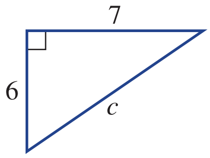


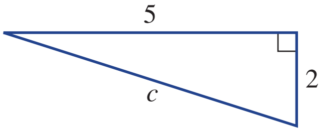
Example 25b

5 Find the length of the hypotenuse (c) of these right-angled triangles, correct to two decimal places.

a



b



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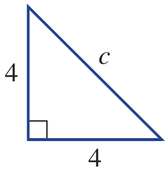

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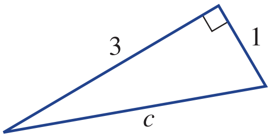

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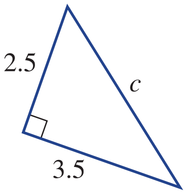

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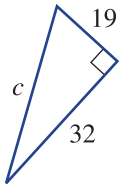
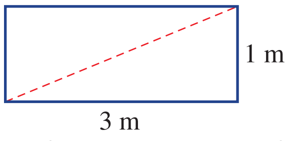


PROBLEM-SOLVING AND REASONING	6, 7, 10	7–10	7–11

Example 26

6 A rectangular board is to be cut along one of its diagonals. The board is 3 m wide and 1 m high. What will be the length of the cut, correct to the nearest cm?



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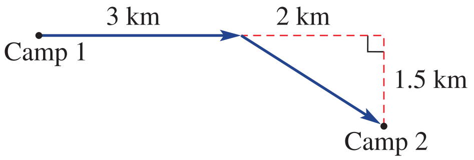
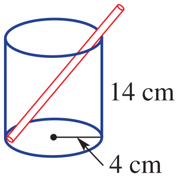
7 The size of a television screen is determined by its diagonal length. Find the size of a television screen that is 1.2 m wide and 70 cm high. Round the answer to the nearest cm.

8 Here is a diagram showing the path of a bushwalker from camp 1 to camp 2. Find the total distance, calculated to one decimal place.



9 A 20 cm straw sits in a cylindrical glass, as shown. What length of straw sticks above the top of the glass? Round the answer to two decimal places.



10 Explain the error in each set of working.

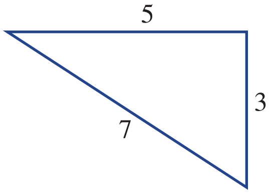
a $c^2 = 2^2 + 3^2$
$\therefore c = 2 + 3$
$= 5$

b $c^2 = 3^2 + 4^2$
$= 7^2$
$= 49$
$\therefore c = 7$

c $c^2 = 2^2 + 5^2$
$= 4 + 25$
$= 29$
$= \sqrt{29}$

11 Prove that these are not right-angled triangles.

a



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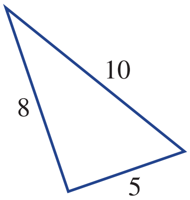

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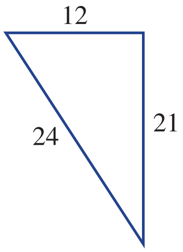
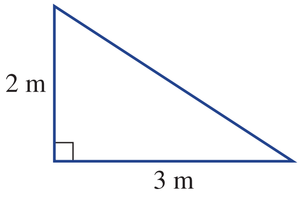


ENRICHMENT – – 12

Perimeter and Pythagoras

12 Find the perimeter of these shapes, correct to two decimal places.

a



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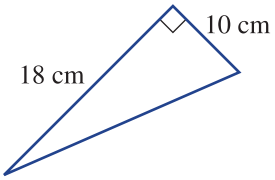

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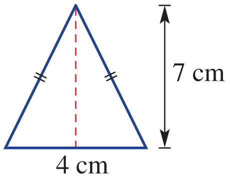

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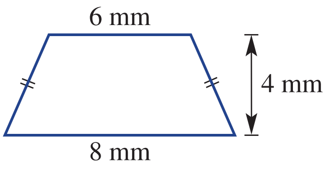

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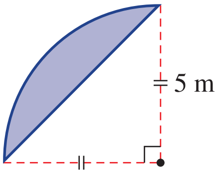

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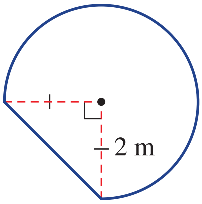

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3M Calculating the length of a shorter side

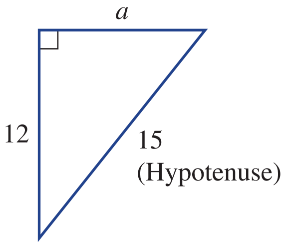
We know that if we are given the two shorter sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the hypotenuse. Generalising further, we can say that when given *any* two sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the third side.

Let's start: What's the setting out?

The triangle shown has a hypotenuse length of 15 and one of the shorter sides is of length 12. Here is the setting out to find the length of the unknown side a.

Can you fill in the missing gaps and explain what is happening at each step.

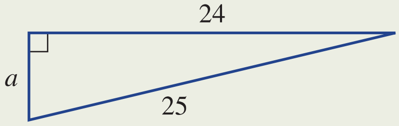
$$a^2 + b^2 = c^2$$
$$a^2 + \_\_\_^2 = \_\_\_^2$$
$$a^2 + \_\_\_ = \_\_\_$$
$$a^2 = \_\_\_ \text{ (Subtract \_\_\_ from both sides.)}$$
$$\therefore a = \sqrt{\_\_\_\_\_\_}$$
$$= \_\_\_$$



Key ideas

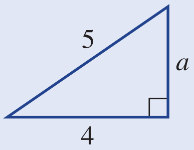
- Pythagoras' theorem can be used to find the length of the shorter sides of a right-angled triangle if the length of the hypotenuse and another side are known.
- Use subtraction to make the unknown the subject of the equation.
  For example:

$$a^2 + b^2 = c^2$$
$$a^2 + 24^2 = 25^2$$
$$a^2 + 576 = 625$$
$$a^2 = 49 \text{ (Subtract 576 from both sides.)}$$
$$\therefore a = \sqrt{49}$$
$$= 7$$



Example 27 Finding the length of a shorter side

Find the value of a in this right-angled triangle.



SOLUTION

$$a^2 + b^2 = c^2$$
$$a^2 + 4^2 = 5^2$$
$$a^2 + 16 = 25$$
$$a^2 = 9$$
$$\therefore a = \sqrt{9}$$
$$= 3$$

EXPLANATION

Write the equation for Pythagoras' theorem and substitute the known values.

Subtract 16 from both sides.

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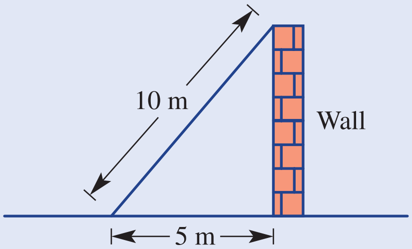
Example 28 Applying Pythagoras to find a shorter side

A 10 m steel brace holds up a concrete wall. The bottom of the brace is 5 m from the base of the wall. Find the height of the concrete wall, correct to two decimal places.



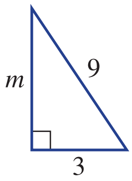
SOLUTION	EXPLANATION
Let a metres be the height of the wall.	Choose a letter (pronumeral) for the unknown height.
$a^2 + b^2 = c^2$	Substitute into Pythagoras' theorem.
$a^2 + 5^2 = 10^2$	
$a^2 + 25 = 100$	
$a^2 = 75$	Subtract 25 from both sides.
$a = \sqrt{75}$	$\sqrt{75}$ is the exact answer.
$= 8.66$ (to 2 d.p.)	Round as required.
The height of the wall is 8.66 metres.	Answer a worded problem using a full sentence.

Exercise 3M

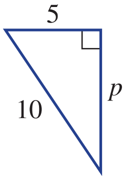
UNDERSTANDING AND FLUENCY	1–4	2, 3–4(½)	3–4(½)

1 Write Pythagoras' theorem for each given triangle.

a



b



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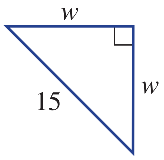

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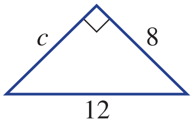
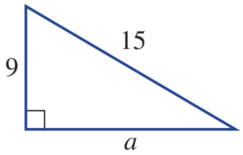


2 Copy and complete the missing steps.

a



$a^2 + b^2 = c^2$

$a^2 + 9^2 = \underline{\qquad}$

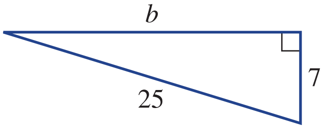
$a^2 + \underline{\qquad} = 225$

$a^2 = \underline{\qquad}$

$\therefore a = \sqrt{\underline{\qquad\quad}}$

$= \underline{\qquad}$

b



$a^2 + b^2 = c^2$

$7^2 + b^2 = \underline{\qquad}$

$\underline{\qquad} + b^2 = \underline{\qquad}$

$b^2 = 576$

$\therefore b = \sqrt{\underline{\qquad\quad}}$

$= \underline{\qquad}$

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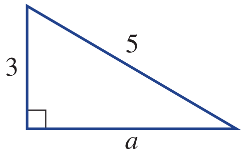
Example 27

3 Find the length of the unknown side in these right-angled triangles.

a



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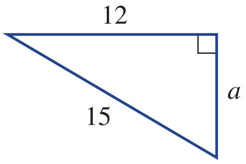

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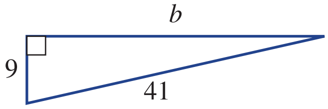

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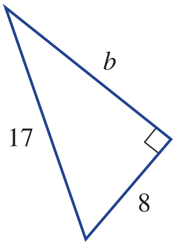

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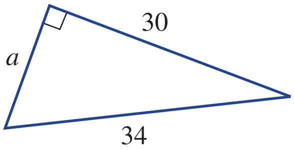

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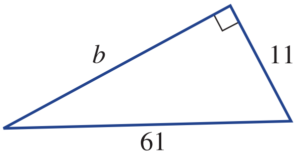
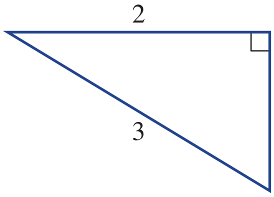


4 Find the length of the unknown side in these right-angled triangles, giving the answer correct to two decimal places.

a



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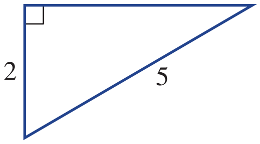

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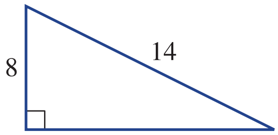

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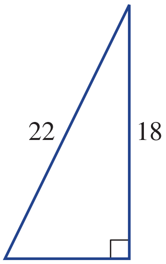

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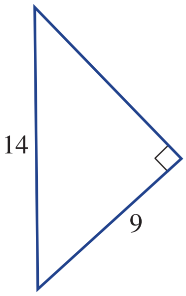

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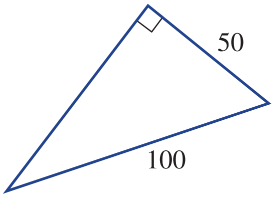
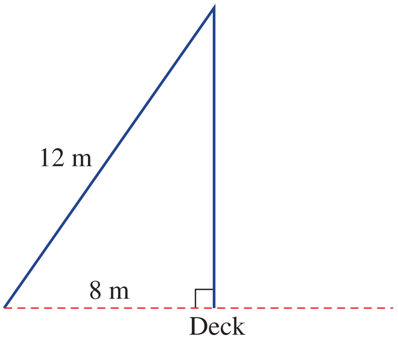


PROBLEM-SOLVING AND REASONING 5, 6, 9 5–7, 9, 10 6–8, 10, 11

Example 28

5 A yacht's mast is supported by a 12 m cable attached to its top. On the deck of the yacht, the cable is 8 m from the base of the mast. How tall is the mast? Round the answer to two decimal places.



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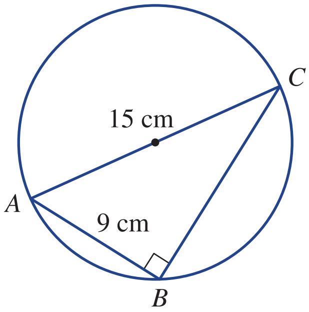
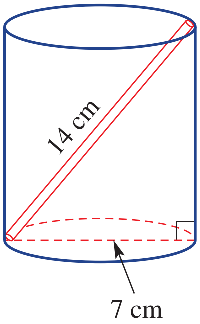
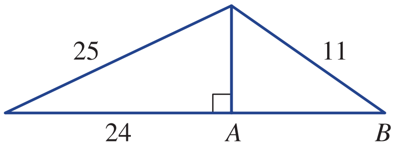
6 A circle's diameter AC is 15 cm and the chord AB is 9 cm. Angle ABC is 90°. Find the length of the chord BC.



7 A 14 cm drinking straw just fits into a can, as shown. The diameter of the can is 7 cm. Find the height of the can, correct to two decimal places.



8 Find the length AB in this diagram. Round to two decimal places.



9 Describe what is wrong with the second line of working in each step.

a $a^2 + 10 = 24$

$a^2 = 34$

b $a^2 = 25$

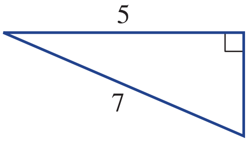
$= 5$

c $a^2 + 25 = 36$

$a^2 + 5 = 6$

10 The number $\sqrt{11}$ is an example of a surd that is written as an exact value. Find the surd that describes the exact lengths of the unknown sides of these triangles.

a



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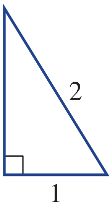

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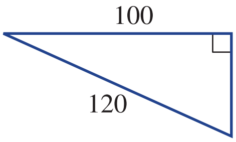

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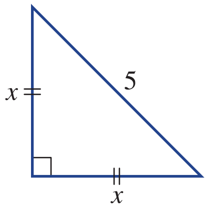
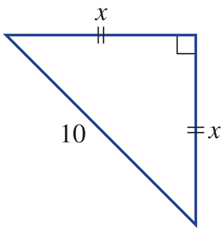
11 Show how Pythagoras' theorem can be used to find the unknown length in these isosceles triangles. Complete the solution for part **a** and then try the others. Round to two decimal places.

a



$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + x^2 &= 5^2 \\ 2x^2 &= 25 \\ x^2 &= \underline{\hspace{1cm}} \\ \therefore x &= \sqrt{\underline{\hspace{2cm}}} \end{aligned}$$

b



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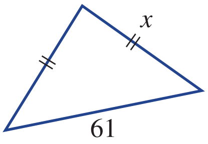

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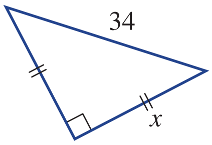


ENRICHMENT	–	–	12

Pythagorean families

12 (3, 4, 5) is called a Pythagorean triad because the numbers 3, 4 and 5 satisfy Pythagoras' theorem ($3^2 + 4^2 = 5^2$).

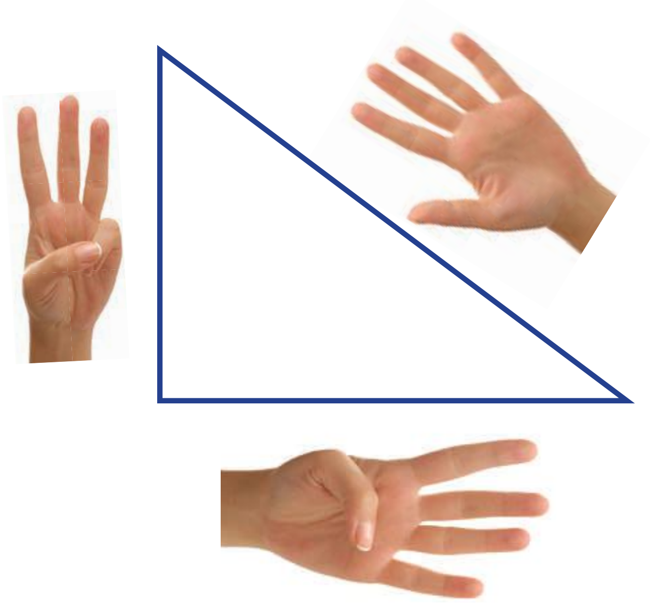
a Explain why (6, 8, 10) is also a Pythagorean triad.

b Explain why (6, 8, 10) is considered to be in the same family as (3, 4, 5).

c List three other Pythagorean triads in the same family as (3, 4, 5) and (6, 8, 10).

d Find another triad not in the same family as (3, 4, 5), but which has all three numbers less than 20.

e List five triads that are each the smallest triad of five different families.



3, 4, 5 is the best known of an infinite number of Pythagorean triads.

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Investigation

GMT and travel

As discussed in Section 3J, the world is divided into 24 time zones, which are determined loosely by each 15° meridian of longitude. World time is based on the time at a place called Greenwich near London, United Kingdom. This time is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT). Places east of Greenwich are ahead in time, and places west of Greenwich are behind.

In Australia, Western Standard Time is 2 hours behind Eastern Standard Time, and Central Standard Time is $\frac{1}{2}$ hour behind Eastern Standard Time. Use the world time zone map on pages 166–7 to answer these questions and to investigate how the time zones affect the time when we travel.

1 *East and west*

Name five countries that are:

a ahead of GMT
b behind GMT

2 *Noon in Greenwich*

When it is noon in Greenwich, what is the time in these places?

a Sydney
b Perth
c Darwin
d Washington, DC
e Auckland
f France
g Johannesburg
h Japan

3 *2 p.m. EST*

When it is 2 p.m. Eastern Standard Time (EST) on Wednesday, find the time and day in these places.

a Perth
b Adelaide
c London
d western Canada
e China
f United Kingdom
g Alaska
h South America

4 *Adjusting your watch*

a Do you adjust your watch forwards or backwards when you are travelling to these places?
  i India
  ii New Zealand

b In what direction should you adjust your watch if you are flying over the Pacific Ocean?

5 *Flight travel*

a You fly from Perth to Brisbane on a 4-hour flight that departed at noon. What is the time in Brisbane when you arrive?

b You fly from Melbourne to Edinburgh on a 22-hour flight that departed at 6 a.m. What is the time in Edinburgh when you arrive?

c You fly from Sydney to Los Angeles on a 13-hour flight that departed at 7:30 p.m. What is the time in Los Angeles when you arrive?

d Copy and complete the following table.

Departing	Arriving	Departure time	Flight time (hours)	Arrival time
Brisbane	Broome	7 a.m.	3.5	
Melbourne	London	1 p.m.	23	
Hobart	Adelaide		1.5	4 p.m.
London	Tokyo		12	11 p.m.
New York	Sydney		15	3 a.m.
Beijing	Vancouver	3:45 p.m.		7:15 p.m.

e Investigate how daylight saving alters the time in some time zones and why. How does this affect flight travel? Give examples.

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Pythagorean triads and spreadsheets

Pythagorean triads (or triples) can be grouped into families. The triad (3, 4, 5) is the base triad for the family of triads ($3k$, $4k$, $5k$). Here are some triads in this same family.

k	1	2	3
Triad	(3, 4, 5) (base triad)	(6, 8, 10)	(9, 12, 15)

1 Write down three more triads in the family ($3k$, $4k$, $5k$).

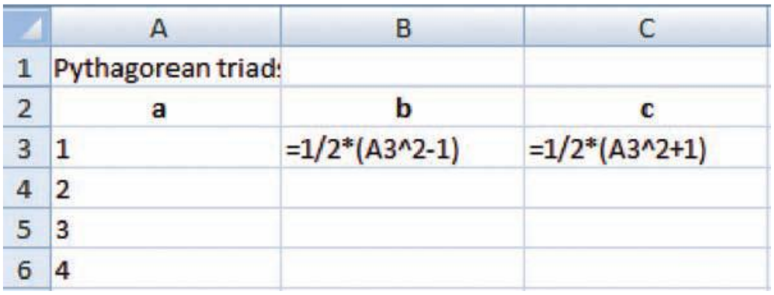
2 Write down three triads in the family ($7k$, $24k$, $25k$).

3 If ($3k$, $4k$, $5k$) and ($7k$, $24k$, $25k$) are two triad families, can you find three more families that have whole numbers less than 100?

4 Pythagoras discovered that if the smaller number in a base triad is a, then the other two numbers in the triad are given by the rules:

$$\frac{1}{2}(a^2 + 1) \text{ and } \frac{1}{2}(a^2 - 1)$$

Set up a spreadsheet to search for all the families of triads of whole numbers less than 200. Here is how a spreadsheet might be set up.



	A	B	C
1	Pythagorean triads		
2	a	b	c
3	1	=1/2*(A3^2-1)	=1/2*(A3^2+1)
4	2		
5	3		
6	4		

Fill down far enough so that c is a maximum of 200.

5 List all the base triads that have whole numbers (less than 200). How many are there?

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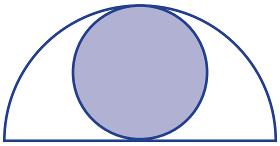
Puzzles and challenges

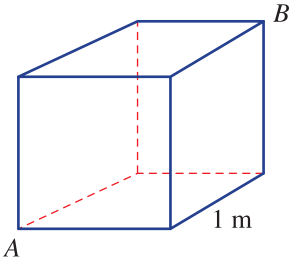
1 A cube has capacity 1 L. What are its dimensions in cm, correct to one decimal place?

2 A fish tank is 60 cm long, 30 cm wide, 40 cm high and contains 70 L of water. Rocks with a volume of 3000 cm^3 are placed into the tank. Will the tank overflow?

3 What proportion (fraction or percentage) of the semicircle does the full circle occupy?

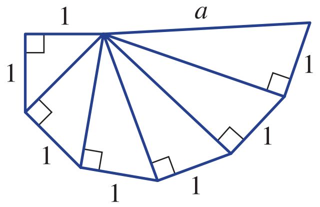


4 What is the distance *AB* in this cube? (Pythagoras' theorem is required.)



5 By what number do you multiply the radius of a circle to double its area?

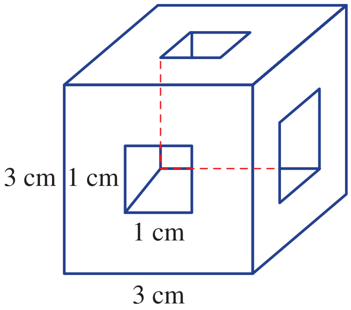
6 Find the exact value (as a surd) of *a* in this diagram. (Pythagoras' theorem is required.)



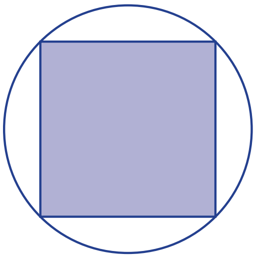
7 A cube of side length 3 cm has its core removed in all directions, as shown. Find:

a its inner surface area

b its outer surface area



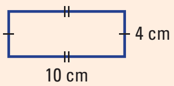
8 A square just fits inside a circle. What percentage of the circle is occupied by the square?



Measurement and Pythagoras' theorem

Length

Perimeter



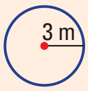
$P = 2 \times 10 + 2 \times 4$
$= 28$ cm

Units

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

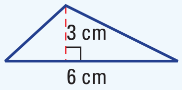
Circumference

$C = 2\pi r$ or πd
$= 2 \times \pi \times 3$
$= 6\pi$ m^2



Area

Triangle



$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 6 \times 3$
$= 9$ cm^2

Units

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "\times 1000^2"] & \text{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \text{mm}^2 \arrow[l, bend left, "\div 10^2"]
\end{tikzcd}$$

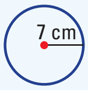
1 ha = 10 000 m^2

Quadrilaterals

- Square $A = s^2$
- Rectangle $A = lb$
- Parallelogram $A = bh$
- Rhombus $A = \frac{1}{2}xy$
- Kite $A = \frac{1}{2}xy$
- Trapezium $A = \frac{1}{2}h(a + b)$

Circle

$A = \pi r^2$
$= \pi \times 7^2$
$= 49\pi$ cm^2



Sector

$A = \frac{\theta}{360} \times \pi r^2$
$= \frac{280}{360} \times \pi \times 2^2$
$= 9.77$ m^2

Volume

Units

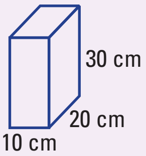
km^3, m^3, cm^3, mm^3

$$\begin{tikzcd}
\text{ML} \arrow[r, bend left, "\times 1000"] & \text{kL} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{mL} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

1 mL = 1 cm^3
1 m^3 = 1000 L

Rectangular prism

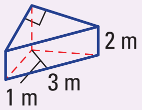
$V = lbh$
$= 10 \times 20 \times 30$
$= 6000$ cm^3
$= 6$ L

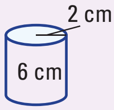


Prism and cylinders

$V = Ah$
$= \frac{1}{2} \times 3 \times 1 \times 2$
$= 3$ m^3

$V = \pi r^2 h$
$= \pi \times 2^2 \times 6$
$= 75.40$ cm^3



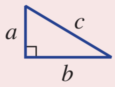


Time

1 min = 60 s
1 h = 60 min
0311 is 03:11 a.m.
2049 is 08:49 p.m.

Pythagoras' theorem

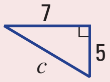
Theorem



$a^2 + b^2 = c^2$

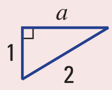
Finding *c*

$c^2 = 5^2 + 7^2$
$= 74$
$\therefore c = \sqrt{74}$



A shorter side

$a^2 + 1^2 = 2^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \sqrt{3}$



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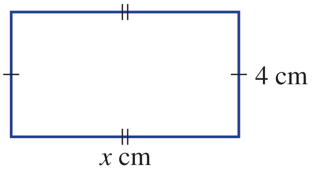
Multiple-choice questions

1 The perimeter of this rectangle is 20 cm. The unknown value x is:

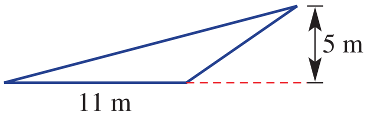


A 4 **B** 16 **C** 5 **D** 10 **E** 6

2 A wheel has a diameter of 2 m. Its circumference and area (in that order) are given by:

A π, π^2 **B** $2\pi, \pi$ **C** $4\pi, 4\pi$ **D** 2, 1 **E** 4, 4

3 The area of this triangle is:

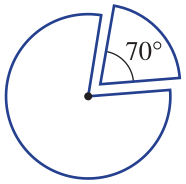


A 27.5 m^2 **B** 55 m **C** 55 m^2 **D** 110 m **E** 16 m^2

4 Using $\pi = 3.14$, the area of a circular oil slick with radius 100 m is:

A 7850 m^2 **B** 314 m^2 **C** 31 400 m^2 **D** 78.5 m^2 **E** 628 m^2

5 A sector of a circle is removed, as shown. The fraction of the circle remaining is:

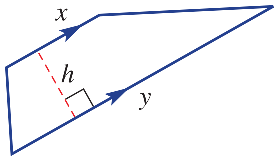


A 290 **B** $\frac{29}{36}$ **C** $\frac{7}{36}$ **D** $\frac{7}{180}$ **E** $\frac{3}{4}$

6 A cube has a volume of 64 cm^3. The length of each side is:

A 32 cm **B** $\frac{64}{3}$ cm **C** 16 cm **D** 8 cm **E** 4 cm

7 The rule for the area of the trapezium shown is:



A $\frac{1}{2}xh$ **B** $\frac{1}{2}(x + y)$ **C** $\frac{1}{2}xy$ **D** πxy^2 **E** $\frac{1}{2}h(x + y)$

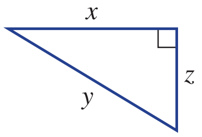
8 The volume of a rectangular prism is 48 cm^3. If its breadth is 4 cm and height 3 cm, its length would be:

A 3 cm **B** 4 cm **C** 2 cm **D** 12 cm **E** 96 cm

9 A cylinder has radius 7 cm and height 10 cm. Using $\pi = \frac{22}{7}$, its volume would be:

A 1540 cm^2 **B** 440 cm^3 **C** 440 L **D** 1540 cm^3 **E** 220 cm^3

10 The rule for Pythagoras' theorem for this triangle would be:



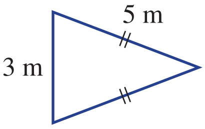
A $a^2 - b^2 = c^2$ **B** $x^2 + y^2 = z^2$ **C** $z^2 + y^2 = x^2$ **D** $x^2 + z^2 = y^2$ **E** $y = \sqrt{x^2 - z^2}$

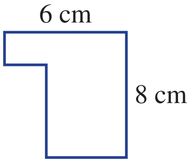
Short-answer questions

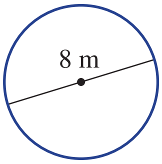
1 Convert these measurements to the units given in the brackets.

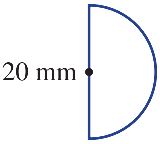
a 2 m (mm)
b 50 000 cm (km)
c 3 cm^2 (mm^2)
d 4000 cm^2 (m^2)
e 0.01 km^2 (m^2)
f 350 mm^2 (cm^2)
g 400 cm^3 (L)
h 0.2 m^3 (L)

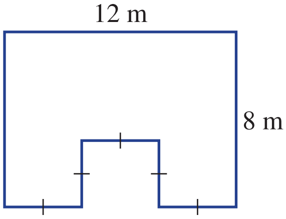
2 Find the perimeter/circumference of these shapes. Round the answer to two decimal places where necessary. Assume that all angles that look right angled are 90°.

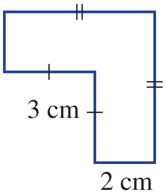
a


b


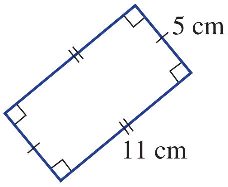
c


d


e


f


3 Find the area of these shapes. Round the answer to two decimal places where necessary.

a


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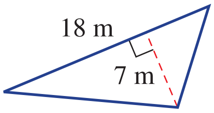

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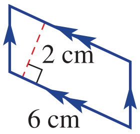

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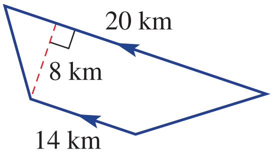
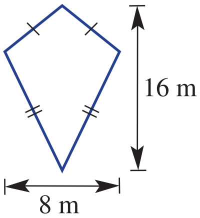
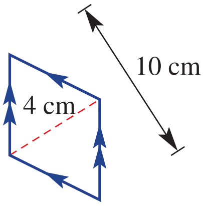
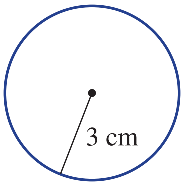
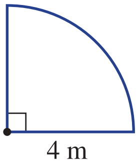
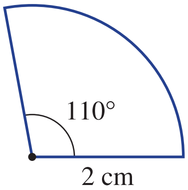


e


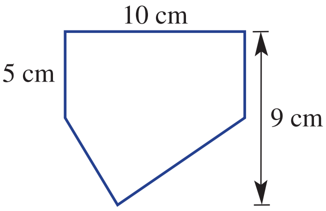
f


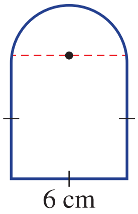
g


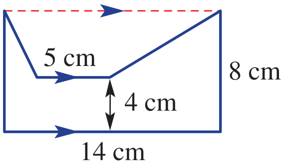
h


i


4 Find the area of these composite shapes. Assume that all angles that look right angled are 90°.

a


b


c


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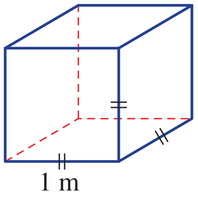
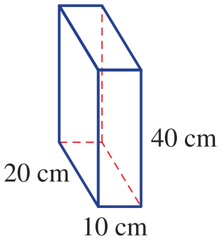
5 Find the volume of each prism, giving your answer in litres. Remember 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

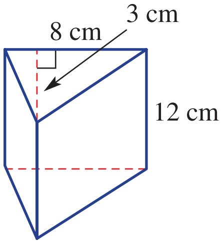
a



b

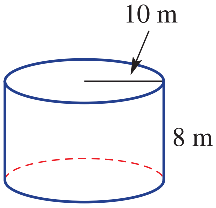


c

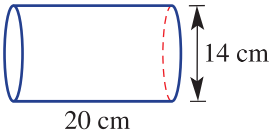


6 Find the volume of these cylinders, rounding the answer to two decimal places.

a



b



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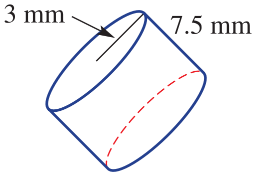


7 An oven is heated from 23°C to 310°C in 18 minutes and 37 seconds. It then cools by 239°C in 1 hour, 20 minutes and 41 seconds.

a Give the temperature:

i increase **ii** decrease

b What is the total time taken to heat and cool the oven?

c How much longer does it take for the oven to cool down than heat up?

8 **a** What is the time difference between 4:20 a.m. and 2:37 p.m. of the same day?

b Write 2145 hours in a.m./p.m. time.

c Write 11:31 p.m. in 24-hour time.

9 When it is 4:30 p.m. in Western Australia, state the time in each of these places.

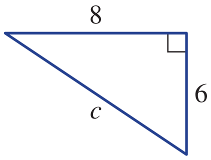
a New South Wales **b** Adelaide **c** United Kingdom

d China **e** Finland **f** South Korea

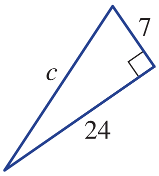
g Russia (eastern tip) **h** New Zealand

10 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round the answer to two decimal places in part **c**.

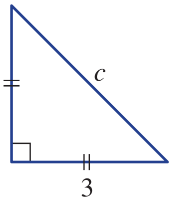
a



b

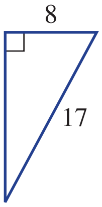


c

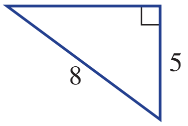


11 Use Pythagoras' theorem to find the unknown length in these right-angled triangles. Round the answer to two decimal places in parts **b** and **c**.

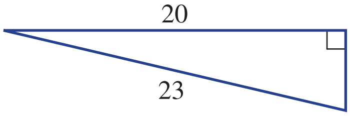
a



b



c



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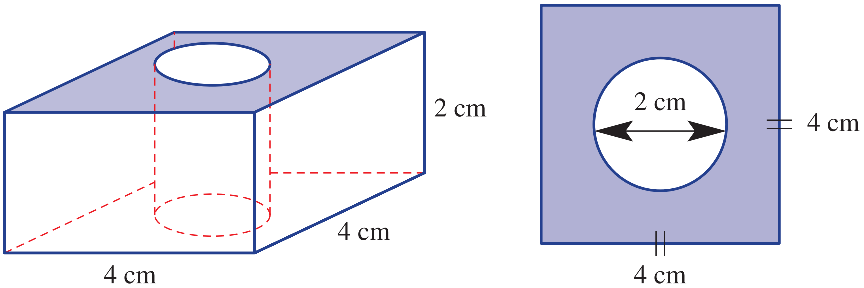
Extended-response questions

1 A company makes square nuts for bolts to use in building construction and steel structures. Each nut starts out as a solid steel square prism. A cylinder of diameter 2 cm is bored through its centre to make a hole. The nut and its top view are shown here.



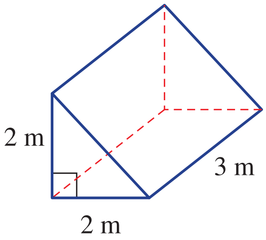
The company is interested in how much paint is required to paint the nuts. The inside surface of the hole is not to be painted. Round all answers to two decimal places where necessary.

a Find the area of the top face of the nut.
b Find the total outside surface area of the nut.
c If the company makes 10000 nuts, how many square metres of surface needs to be painted?

The company is also interested in the volume of steel used to make the nuts.

d Find the volume of steel removed from each nut to make the hole.
e Find the volume of steel in each nut.
f Assuming that the steel removed to make the hole can be melted and reused, how many nuts can be made from 1 L of steel?

2 A simple triangular shelter has a base width of 2 cm, a height of 2 m and a length of 3 m.



a Use Pythagoras's theorem to find the hypotenuse length of one of the ends of the tent. Round the answer to one decimal place.
b Assuming that all the faces of the shelter including the floor are covered with canvas material, what area of canvas is needed to make the shelter? Round the answer to the nearest whole number of square metres.
c Every edge of the shelter is to be sealed with a special tape. What length of tape is required? Round to the nearest whole number of metres.
d The shelter tag says that it occupies 10000 L of space. Show working to decide if this is true or false. What is the difference?

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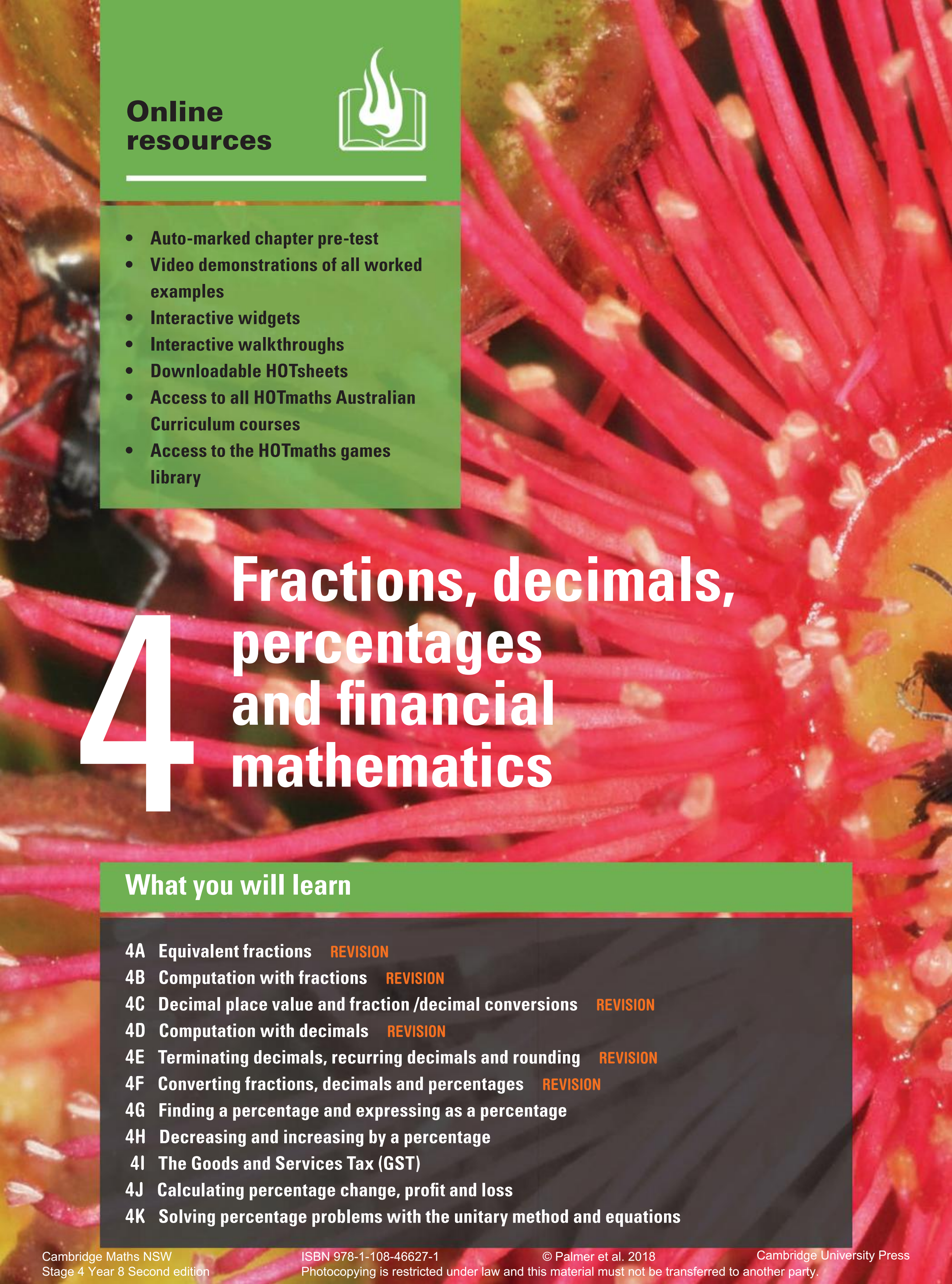

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

4

Fractions, decimals, percentages and financial mathematics

What you will learn

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NSW syllabus

STRAND: NUMBER AND ALGEBRA

SUBSTRANDS: FRACTIONS, DECIMALS AND PERCENTAGES

FINANCIAL MATHEMATICS

Outcomes

A student operates with fractions, decimals and percentages.
(MA4–5NA)

A student solves financial problems involving purchasing goods.
(MA4–6NA)

Phi and golden rectangles

Phi is a unique and mysterious decimal number approximately equal to 1.618. As of 2015, the value of phi has now been calculated to two trillion (2 000 000 000 000) decimal places. Phi's decimal places continue forever; no pattern has been found and it cannot be written as a fraction.

Golden rectangles are rectangles that have the proportion of length to width equal to phi : 1. This ratio is thought to be the most visually appealing proportion to the human eye. Proportions using the decimal phi are found in an astounding variety of places including ancient Egyptian pyramids, Greek architecture and sculptures, art, the nautilus shell, an ant, a dolphin, a beautiful human face, an ear, a tooth, the body, the graphic design of credit cards, websites and company logos, DNA and even the shape of the Milky Way galaxy.

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Pre-test

1 Match these words to the types of fractions given:

whole number, improper fraction, proper fraction, mixed numeral

a $1\frac{2}{5}$ **b** $\frac{3}{7}$ **c** $\frac{10}{2}$ **d** $\frac{7}{4}$

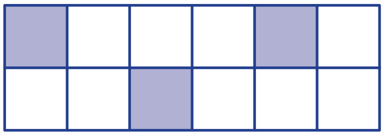
2 How many quarters are in 5 wholes?

3 Fill in the blanks.

a $\frac{3}{4} = \frac{75}{\square}$ **b** $\frac{3}{6} = \frac{\square}{2}$ **c** $1\frac{3}{5} = \frac{\square}{5}$ **d** $\frac{\square}{3} = \frac{16}{12}$

4 What fraction is shaded?

a



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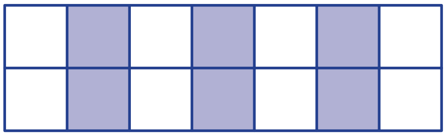


5 Match the fractions on the left-hand side to their decimal form on the right.

a $\frac{1}{2}$ **A** 3.75

b $\frac{1}{100}$ **B** 0.625

c $\frac{3}{20}$ **C** 0.01

d $3\frac{3}{4}$ **D** 0.5

e $\frac{5}{8}$ **E** 0.15

6 Find:

a $\frac{1}{2} + \frac{1}{4}$ **b** $0.5 + \frac{1}{2}$ **c** $3 - 1\frac{1}{3}$

d $0.3 + 0.2 + 0.1$ **e** $\frac{2.4}{2}$ **f** 0.5×6

7 For each of the following, write as:

i simple fractions

ii decimals

a 10% **b** 25% **c** 50% **d** 75%

8 Find 10% of:

a \$50 **b** \$66 **c** 8 km **d** 6900 m

9 Find:

a 25% of 40 **b** 75% of 24 **c** 90% of \$1

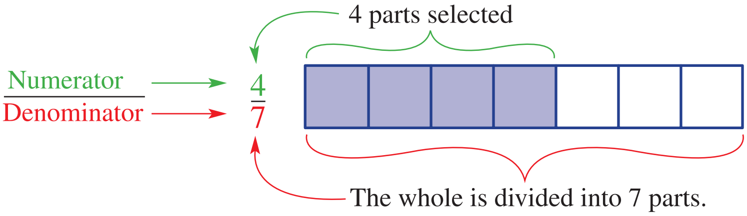
10 Copy and complete the following table.

Fraction	$\frac{3}{4}$			$\frac{2}{5}$				2
Decimal		0.2			0.99		1.6	
Percentage			15%			100%		

4A Equivalent fractions REVISION

Fractions are extremely useful in all practical situations whenever a proportion is required. Fractions are used by a chef measuring the ingredients for a cake, a builder measuring the ingredients for concrete, and a musician using computer software to create music.

A fraction is formed when a whole number or amount is divided into equal parts. The bottom number is referred to as the **d**enominator (**d**own) and tells you how many parts the whole is divided up into. The top number is referred to as the n**u**merator (**u**p) and tells you how many of the parts you have selected.



Equivalent fractions are fractions that represent equal portions of a whole amount and so are equal in value. The skill of generating equivalent fractions is needed whenever you add or subtract fractions with different denominators.

Let's start: Know your terminology

It is important to know and understand key terms associated with the study of fractions.

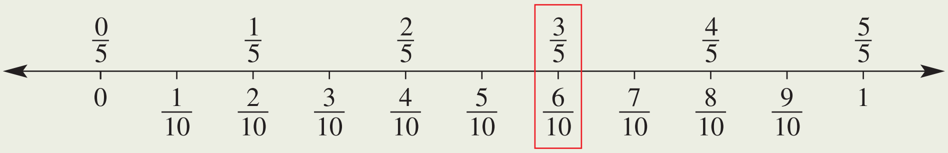
Working with a partner and using your previous knowledge of fractions, write a one-sentence definition or explanation for each of the following key terms. Also give an example of each.

- numerator
- equivalent fraction
- improper fraction
- multiples
- reciprocal
- lowest common multiple
- ascending
- composite number
- denominator
- proper fraction
- mixed numeral
- factors
- highest common factor
- descending
- lowest common denominator
- rational numbers
- irrational numbers

Key ideas

■ **Equivalent fractions** are equal in value. They mark the same place on a number line.

For example, $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent fractions.



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Key ideas

- Equivalent fractions are formed by multiplying or dividing a fraction by a number equal to 1, which can be written in the form $\frac{a}{a}$.

  For example:

  $$\frac{3}{4} \times 1 = \frac{3}{4} \times \frac{2}{2} = \frac{6}{8} \qquad \therefore \frac{3}{4} \text{ and } \frac{6}{8} \text{ are equivalent fractions.}$$

- Equivalent fractions are therefore produced by multiplying the numerator and the denominator by the same whole number.

  For example:

  $$\frac{2}{7} \underset{\times 5}{\overset{\times 5}{=}} \frac{10}{35} \qquad \therefore \frac{2}{7} \text{ and } \frac{10}{35} \text{ are equivalent fractions.}$$

- Equivalent fractions are also produced by dividing the numerator and the denominator by the same common factor.

  For example:

  $$\frac{6}{21} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{7} \qquad \therefore \frac{6}{21} \text{ and } \frac{2}{7} \text{ are equivalent fractions.}$$

- The **simplest form** of a fraction is an equivalent fraction with the lowest possible whole numbers in the numerator and denominator. This is achieved by dividing the numerator and the denominator by their highest common factor (HCF). In the simplest form of a fraction, the HCF of the numerator and the denominator is 1.

  For example:

  $$\frac{12}{18} \qquad \text{HCF of 12 and 18 is 6.}$$

  $$\frac{12}{18} \underset{\div 6}{\overset{\div 6}{=}} \frac{2}{3} \qquad \therefore \frac{12}{18} \text{ written in simplest form is } \frac{2}{3}.$$

  This technique is also known as 'cancelling'. It is important to understand that cancelling is dividing the numerator and denominator by the same number.

  $$\frac{12}{18} = \frac{2 \times \cancel{6}^{1}}{3 \times \cancel{6}^{1}} = \frac{2}{3}$$

  The HCF is cancelled (divided) from the numerator and the denominator.

  $\frac{6}{6}$ 'cancels' to 1 because 6 ÷ 6 = 1

- Two fractions are equivalent if they have the same numerator and denominator in simplest form.

Example 1 Generating equivalent fractions

Rewrite the following fractions with a denominator of 40.

a $\frac{3}{5}$ **b** $\frac{1}{2}$ **c** $\frac{7}{4}$ **d** $\frac{36}{120}$

SOLUTION	EXPLANATION
a $\frac{3}{5} = \frac{24}{40}$	Denominator has been multiplied by 8. Numerator must be multiplied by 8.
b $\frac{1}{2} = \frac{20}{40}$	Multiply numerator and denominator by 20.
c $\frac{7}{4} = \frac{70}{40}$	Multiply numerator and denominator by 10.
d $\frac{36}{120} = \frac{12}{40}$	Divide numerator and denominator by 3.

Example 2 Converting to simplest form

Write the following fractions in simplest form.

a $\frac{8}{20}$ **b** $\frac{25}{15}$

SOLUTION	EXPLANATION
a $\frac{8}{20} = \frac{2 \times \cancel{4}^{1}}{5 \times \cancel{4}_{1}} = \frac{2}{5}$	The HCF of 8 and 20 is 4. Both the numerator and the denominator are divided by the HCF of 4.
b $\frac{25}{15} = \frac{5 \times \cancel{5}^{1}}{3 \times \cancel{5}_{1}} = \frac{5}{3}$	The HCF of 25 and 15 is 5. The 5 is 'cancelled' from the numerator and the denominator.

Exercise 4A REVISION

UNDERSTANDING AND FLUENCY	1–3	3, 4–6(½), 7, 8, 9–10(½)	6(½), 7, 8, 9–10(½)

1 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $\frac{3}{5} = \frac{\square}{10} = \frac{12}{\square} = \frac{120}{\square}$

b $\frac{4}{7} = \frac{8}{\square} = \frac{\square}{70} = \frac{80}{\square}$

c $\frac{3}{9} = \frac{\square}{3} = \frac{2}{\square} = \frac{14}{\square}$

d $\frac{18}{24} = \frac{6}{\square} = \frac{\square}{4} = \frac{15}{\square}$

2 Which of the following fractions are equivalent to $\frac{2}{3}$?

$\frac{2}{30}$ $\frac{4}{9}$ $\frac{4}{6}$ $\frac{20}{30}$ $\frac{1}{2}$ $\frac{4}{3}$ $\frac{10}{15}$ $\frac{20}{36}$

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3 Are the following statements true or false?

a $\frac{1}{2}$ and $\frac{1}{4}$ are equivalent fractions.

b $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions.

c The fraction $\frac{8}{9}$ is written in its simplest form.

d $\frac{14}{21}$ can be simplified to $\frac{2}{3}$.

e $\frac{11}{99}$ and $\frac{1}{9}$ and $\frac{2}{18}$ are all equivalent fractions.

f $\frac{4}{5}$ can be simplified to $\frac{2}{5}$.

Example 1

4 Rewrite the following fractions with a denominator of 24.

a $\frac{1}{3}$ **b** $\frac{2}{8}$ **c** $\frac{1}{2}$ **d** $\frac{5}{12}$

e $\frac{5}{6}$ **f** $\frac{5}{1}$ **g** $\frac{3}{4}$ **h** $\frac{7}{8}$

5 Rewrite the following fractions with a denominator of 30.

a $\frac{1}{5}$ **b** $\frac{2}{6}$ **c** $\frac{5}{10}$ **d** $\frac{3}{1}$

e $\frac{2}{3}$ **f** $\frac{22}{60}$ **g** $\frac{5}{2}$ **h** $\frac{150}{300}$

6 Find the missing value to make the equation true.

a $\frac{2}{5}=\frac{\square}{15}$ **b** $\frac{7}{9}=\frac{14}{\square}$ **c** $\frac{7}{14}=\frac{1}{\square}$ **d** $\frac{21}{30}=\frac{\square}{10}$

e $\frac{4}{3}=\frac{\square}{21}$ **f** $\frac{8}{5}=\frac{80}{\square}$ **g** $\frac{3}{12}=\frac{\square}{60}$ **h** $\frac{7}{11}=\frac{28}{\square}$

7 State the missing numerators for the following sets of equivalent fractions.

a $\frac{1}{2}=\frac{\square}{4}=\frac{\square}{6}=\frac{\square}{10}=\frac{\square}{20}=\frac{\square}{32}=\frac{\square}{50}$

b $\frac{3}{2}=\frac{\square}{4}=\frac{\square}{8}=\frac{\square}{20}=\frac{\square}{30}=\frac{\square}{100}=\frac{\square}{200}$

c $\frac{2}{5}=\frac{\square}{10}=\frac{\square}{15}=\frac{\square}{20}=\frac{\square}{35}=\frac{\square}{50}=\frac{\square}{75}$

8 State the missing denominators for the following sets of equivalent fractions.

a $\frac{1}{3}=\frac{2}{\square}=\frac{4}{\square}=\frac{8}{\square}=\frac{10}{\square}=\frac{25}{\square}=\frac{100}{\square}$

b $\frac{1}{5}=\frac{2}{\square}=\frac{3}{\square}=\frac{4}{\square}=\frac{5}{\square}=\frac{10}{\square}=\frac{12}{\square}$

c $\frac{5}{4}=\frac{10}{\square}=\frac{15}{\square}=\frac{35}{\square}=\frac{55}{\square}=\frac{100}{\square}=\frac{500}{\square}$

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Example 2

9 Write the following fractions in simplest form.

a $\frac{3}{9}$ **b** $\frac{4}{8}$ **c** $\frac{10}{12}$ **d** $\frac{15}{18}$

e $\frac{11}{44}$ **f** $\frac{12}{20}$ **g** $\frac{16}{18}$ **h** $\frac{25}{35}$

i $\frac{15}{9}$ **j** $\frac{22}{20}$ **k** $\frac{120}{100}$ **l** $\frac{64}{48}$

10 Using your calculator, express the following fractions in simplest form.

a $\frac{24}{96}$ **b** $\frac{36}{108}$ **c** $\frac{165}{195}$ **d** $\frac{196}{476}$

e $\frac{565}{452}$ **f** $\frac{637}{273}$ **g** $\frac{225}{165}$ **h** $\frac{742}{224}$

PROBLEM-SOLVING AND REASONING	11, 14	11, 12, 14, 15	11–16

11 Three of the following eight fractions are not written in simplest form. Write down these three fractions and simplify them.

$\frac{17}{31}$ $\frac{14}{42}$ $\frac{5}{11}$ $\frac{51}{68}$ $\frac{23}{93}$ $\frac{15}{95}$ $\frac{1}{15}$ $\frac{13}{31}$

12 Group the following 12 fractions into six pairs of equivalent fractions.

$\frac{5}{11}$ $\frac{3}{5}$ $\frac{7}{21}$ $\frac{8}{22}$ $\frac{2}{7}$ $\frac{20}{50}$ $\frac{6}{21}$ $\frac{9}{15}$ $\frac{15}{33}$ $\frac{1}{3}$ $\frac{16}{44}$ $\frac{6}{15}$

13 A 24-hour swim-a-thon is organised to raise funds for a local charity. The goal is to swim 1500 laps during the 24-hour event.

After 18 hours a total of 1000 laps have been swum.

a On the basis of time, what fraction, in simplest form, of the swim-a-thon has been completed?

b On the basis of laps, what fraction, in simplest form, of the swim-a-thon has been completed?

c Are the swimmers on target to achieve their goal? Explain your answer using equivalent fractions.

14 **a** A particular fraction has a prime number in the numerator and a different prime number in the denominator. Using some examples, justify whether or not this fraction can be simplified.

b A fraction has a composite number in the numerator and a different composite number in the denominator. Using some examples, justify whether or not this fraction can be simplified.

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15 **a** A pizza is cut into eight equal pieces. Can it be shared by four people in such a way that no-one receives an equivalent fraction of the pizza?

b A pizza is cut into 12 equal pieces. Can it be shared by four people in such a way that no-one receives the same amount of the pizza?

c What is the least number of pieces into which a pizza can be cut so that four people can share it and each receive a different amount?

16 Are there a finite or infinite number of equivalent fractions of $\frac{1}{2}$? Write a one-paragraph response to justify your answer.

ENRICHMENT – – 17

Equivalent algebraic fractions

17 Algebraic fractions contain pronumerals (letters). $\frac{3a}{5}$ and $\frac{x}{y}$ are both examples of algebraic fractions.

a Find the value of the $\square$ to produce equivalent algebraic fractions.

i $\frac{2a}{3b} = \frac{4a}{\square}$

ii $\frac{x}{y} = \frac{\square}{5y}$

iii $\frac{3b}{20} = \frac{12b}{\square}$

iv $\frac{4de}{p} = \frac{\square}{3p}$

v $\frac{a}{b} = \frac{ac}{\square}$

vi $\frac{3k}{2t} = \frac{\square}{2tm}$

vii $\frac{4a}{5b} = \frac{\square}{20bc}$

viii $\frac{x}{y} = \frac{\square}{y^2}$

b Simplify the following algebraic fractions.

i $\frac{15b}{20}$

ii $\frac{4}{8y}$

iii $\frac{3x}{5x}$

iv $\frac{5y}{8xy}$

v $\frac{10p}{15qp}$

vi $\frac{120y}{12xy}$

vii $\frac{mnop}{mnpq}$

viii $\frac{15x}{5x^2}$

c Can an algebraic fraction be simplified to a non-algebraic fraction? Justify your answer by using examples.

d Can a non-algebraic fraction have an equivalent algebraic fraction? Justify your answer by using examples.

4B Computation with fractions REVISION

Interactive
Widgets
HOTsheets
Walkthrough

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing fractions. Proper fractions, improper fractions and mixed numerals will be considered for each of the four mathematical operations.

Let's start: You write the question

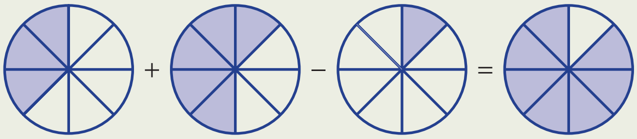
Here are six different answers: $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 2, -\frac{1}{4}$.

Your challenge is to write six different questions that will produce each of the six answers above.

Each question must use only the two fractions $\frac{1}{2}$ and $\frac{1}{4}$ and one operation (i.e. +, −, ×, ÷).

Key ideas

- **Adding and subtracting fractions**
  - When we add or subtract fractions, we count how many we have of a certain 'type' of fraction. For example, we could count eighths: 3 *eighths* plus 5 *eighths* minus 1 *eighth* equals 7 *eighths*. When we count 'how many' *eighths,* the answer must be in *eighths*.



$$\frac{3}{8} + \frac{5}{8} - \frac{1}{8} = \frac{7}{8}$$

  - Denominators *must* be the same before you can proceed with adding or subtracting fractions.
  - If the denominators are different, use the lowest common multiple (LCM) of the denominators to find equivalent fractions.
  - When the denominators are the same, simply add or subtract the numerators as required. The denominator remains the same.

1 Equivalent fractions

2 Add numerators

$$\frac{1}{2} + \frac{2}{3} \longleftarrow \text{Different denominators}$$

$$= \frac{3}{6} + \frac{4}{6} \longleftarrow \text{Same denominators}$$

$$= \frac{7}{6} \longleftarrow \text{Denominator stays the same}$$

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Key ideas

Common multiple (CM) and lowest common multiple (LCM)

- A common multiple is found by multiplying whole numbers together.
- The LCM is found by multiplying the whole numbers together and then dividing by the HCF of the whole numbers.

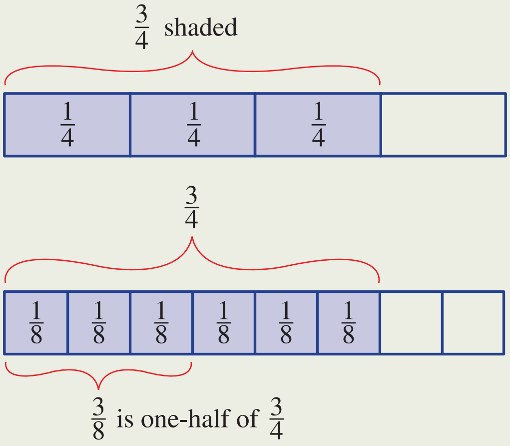
  For example, consider the whole numbers 4 and 6.

  A common multiple is 24 (i.e. 4×6). The LCM is 12 $\left(\text{i.e. } \dfrac{4 \times 6}{2}\right)$.
- The lowest common denominator (LCD) is the lowest common multiple (LCM) of the denominators.

Multiplying fractions

Example:

$$\begin{aligned} \frac{1}{2} \text{ of } \frac{3}{4} &= \frac{1}{2} \times \frac{3}{4} \\ &= \frac{3}{8} \end{aligned}$$



- Denominators *do not* need to be the same before you can proceed with fraction multiplication.
- Simply multiply the numerators together and multiply the denominators together.
- Mixed numerals must be converted to improper fractions before you can proceed.
- If possible, simplify or ‘cancel’ fractions before multiplying.

Example:

Multiply numerators together.

$$\frac{2}{3} \times \frac{5}{9} = \frac{2 \times 5}{3 \times 9} = \frac{10}{27}$$

Multiply denominators together.

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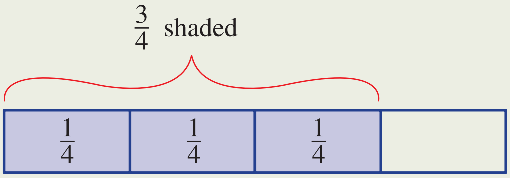
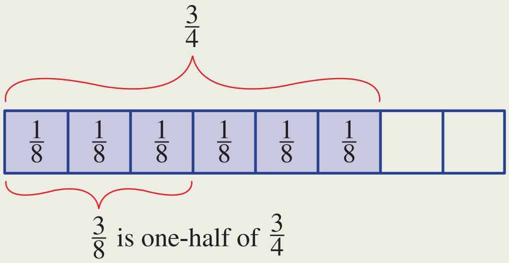
- **Dividing fractions**

  Example:





$\frac{3}{4} \div \frac{1}{4}$ is the same as ‘how many quarters $\left(\frac{1}{4}\right)$ are in $\frac{3}{4}$?’

$$\begin{aligned}\frac{3}{4} \div \frac{1}{4} &= \frac{3}{4} \times \frac{4}{1}\\ &= 3\end{aligned}$$

$\frac{3}{4} \div 2$ is the same as ‘one-half of $\frac{3}{4}$.’

$$\begin{aligned}\frac{3}{4} \div 2 &= \frac{3}{4} \times \frac{1}{2}\\ &= \frac{3}{8}\end{aligned}$$

  - Denominators *do not* need to be the same before you can proceed.
  - Mixed numerals must be converted to improper fractions before you can proceed.
  - To divide by a fraction multiply by its reciprocal.
  - The **reciprocal** of a fraction is found by swapping the numerator and the denominator. This is known as inverting the fraction.

    Proceed as for multiplying fractions.

    Example:

    $$\frac{3}{8} \div \frac{5}{4} = \frac{3}{8} \times \frac{4}{5} = \frac{12}{40} = \frac{3}{10}$$

    Instead of ÷, × by the reciprocal fraction.

- **Checking your answer**
  - Final answers should be written in simplest form.
  - It is common to write answers involving improper fractions as mixed numerals.

Summary

- **Adding and subtracting fractions**

  When the denominators are the same, simply add or subtract the numerators. The denominator remains the same.
- **Multiplying fractions**

  Cancel where possible, then multiply the numerators together and multiply the denominators together.
- **Dividing fractions**

  To divide by a fraction multiply by its reciprocal.

Example 3 Adding and subtracting fractions

Evaluate:

a $\frac{3}{5} + \frac{4}{5}$

b $\frac{5}{3} - \frac{3}{4}$

SOLUTION	EXPLANATION
a $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$ $= 1\frac{2}{5}$	The denominators are the same, therefore count the number of fifths by simply add the numerators. The final answer is written as a mixed numeral.
b $\frac{5}{3} - \frac{3}{4} = \frac{20}{12} - \frac{9}{12}$ $= \frac{11}{12}$	LCM of 3 and 4 is 12. Write equivalent fractions with a denominator of 12. The denominators are the same, so subtract the numerators.

Example 4 Adding and subtracting mixed numerals

Evaluate:

a $3\frac{5}{8} + 2\frac{3}{4}$

b $2\frac{1}{2} - 1\frac{5}{6}$

SOLUTION	EXPLANATION
a $3\frac{5}{8} + 2\frac{3}{4} = \frac{29}{8} + \frac{11}{4}$ $= \frac{29}{8} + \frac{22}{8}$ $= \frac{51}{8} = 6\frac{3}{8}$	Convert mixed numerals to improper fractions. The LCM of 8 and 4 is 8. Write equivalent fractions with LCD. Add numerators together, denominator remains the same. Convert the answer back to a mixed numeral.
Alternative method: $3\frac{5}{8} + 2\frac{3}{4} = 3 + 2 + \frac{5}{8} + \frac{3}{4}$ $= 5 + \frac{5}{8} + \frac{6}{8}$ $= 5 + \frac{11}{8} = 6\frac{3}{8}$	Add the whole number parts together. The LCM of 8 and 4 is 8. Write equivalent fractions with LCD. Add fractions parts together and simplify the answer.
b $2\frac{1}{2} - 1\frac{5}{6} = \frac{5}{2} - \frac{11}{6}$ $= \frac{15}{6} - \frac{11}{6}$ $= \frac{4}{6} = \frac{2}{3}$	Convert mixed numerals to improper fractions. The LCD of 2 and 6 is 6. Write equivalent fractions with LCD. Subtract numerators and simplify the answer.
Alternative method: $2\frac{1}{2} - 1\frac{5}{6} = 1\frac{1}{2} - \frac{5}{6}$ $= 1\frac{3}{6} - \frac{5}{6}$ $= 1 - \frac{2}{6}$ $= \frac{4}{6} = \frac{2}{3}$	Subtract the whole numbers. The LCM of 2 and 6 is 6. Subtract numerators. Subtract fraction from whole number. Simplify the answer.

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Example 5 Multiplying fractions

Evaluate:

a $\frac{2}{5} \times \frac{3}{7}$ **b** $\frac{8}{5} \times \frac{7}{4}$ **c** $3\frac{1}{3} \times 2\frac{2}{5}$

SOLUTION	EXPLANATION
a $\begin{aligned}\frac{2}{5} \times \frac{3}{7} &= \frac{2 \times 3}{5 \times 7}\\ &= \frac{6}{35}\end{aligned}$	Multiply the numerators together. Multiply the denominators together. The answer is in simplest form.
b $\begin{aligned}\frac{\overset{2}{\cancel{8}}}{5} \times \frac{7}{\underset{1}{\cancel{4}}} &= \frac{2 \times 7}{5 \times 1}\\ &= \frac{14}{5} = 2\frac{4}{5}\end{aligned}$	Cancel first. Then multiply numerators together and denominators together. Write the answer as an improper fraction.
c $\begin{aligned}3\frac{1}{3} \times 2\frac{2}{5} &= \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{5}}}\\ &= \frac{2 \times 4}{1 \times 1}\\ &= \frac{8}{1} = 8\end{aligned}$	Convert to improper fractions first. Simplify fractions by cancelling. Multiply ‘cancelled’ numerators and ‘cancelled’ denominators together. Write the answer in simplest form.

Example 6 Dividing fractions

Evaluate:

a $\frac{2}{5} \div \frac{3}{7}$ **b** $\frac{5}{8} \div \frac{15}{16}$ **c** $2\frac{1}{4} \div 1\frac{1}{3}$

SOLUTION	EXPLANATION
a $\begin{aligned}\frac{2}{5} \div \frac{3}{7} &= \frac{2}{5} \times \frac{7}{3}\\ &= \frac{14}{15}\end{aligned}$	Change ÷ sign to a × sign and invert the divisor (the second fraction). Proceed as for multiplication. Multiply numerators together and multiply denominators together.
b $\begin{aligned}\frac{5}{8} \div \frac{15}{16} &= \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} \times \frac{\cancel{16}^{2}}{\cancel{15}_{3}}\\ &= \frac{1}{1} \times \frac{2}{3}\\ &= \frac{2}{3}\end{aligned}$	Change ÷ sign to a × sign and invert the divisor (the second fraction). Proceed as for multiplication.
c $\begin{aligned}2\frac{1}{4} \div 1\frac{1}{3} &= \frac{9}{4} \div \frac{4}{3}\\ &= \frac{9}{4} \times \frac{3}{4}\\ &= \frac{27}{16} = 1\frac{11}{16}\end{aligned}$	Convert mixed numerals to improper fractions. Change ÷ sign to × sign and invert the divisor (the second fraction). Proceed as for multiplication.

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Exercise 4B REVISION

UNDERSTANDING AND FLUENCY	1–4, 5–11(½), 12–14	4, 5–11(½), 12–14	5–14(½)

The answers to questions 1 and 2 may be found in Key ideas.

1 **a** Which two operations require the denominators to be the same before proceeding?
b Which two operations do not require the denominators to be the same before proceeding?

2 **a** For which two operations must you first convert mixed numerals to improper fractions?
b For which two operations can you choose whether or not to convert mixed numerals to improper fractions before proceeding?

3 State the LCD for the following pairs of fractions.

a $\frac{1}{5}+\frac{3}{4}$ **b** $\frac{2}{9}+\frac{5}{3}$ **c** $\frac{11}{25}+\frac{7}{10}$ **d** $\frac{5}{12}+\frac{13}{8}$

4 Rewrite the following examples and fill in the empty boxes.

a $\frac{2}{3}+\frac{1}{4}$
$=\frac{8}{12}+\frac{\square}{12}$
$=\frac{11}{\square}$

b $\frac{7}{8}-\frac{9}{16}$
$=\frac{\square}{16}-\frac{9}{16}$
$=\frac{\square}{16}$

c $1\frac{4}{7}\times\frac{3}{5}$
$=\frac{\square}{7}\times\frac{3}{5}$
$=\frac{\square}{35}$

d $\frac{5}{7}\div\frac{2}{3}$
$=\frac{5}{7}\,\square\,\frac{3}{2}$
$=\frac{15}{\square}=\square\frac{\square}{14}$

Example 3

5 Evaluate:

a $\frac{1}{5}+\frac{2}{5}$ **b** $\frac{7}{9}-\frac{2}{9}$ **c** $\frac{5}{8}+\frac{7}{8}$ **d** $\frac{24}{7}-\frac{11}{7}$

e $\frac{3}{4}+\frac{2}{5}$ **f** $\frac{3}{10}+\frac{4}{5}$ **g** $\frac{5}{7}-\frac{2}{3}$ **h** $\frac{11}{18}-\frac{1}{6}$

Example 4

6 Evaluate:

a $3\frac{1}{7}+1\frac{3}{7}$ **b** $7\frac{2}{5}+2\frac{1}{5}$ **c** $3\frac{5}{8}-1\frac{2}{8}$ **d** $8\frac{5}{11}-7\frac{3}{11}$

e $5\frac{1}{3}+4\frac{1}{6}$ **f** $17\frac{5}{7}+4\frac{1}{2}$ **g** $6\frac{1}{2}-2\frac{3}{4}$ **h** $4\frac{2}{5}-2\frac{5}{6}$

Example 5a,b

7 Evaluate:

a $\frac{3}{5}\times\frac{1}{4}$ **b** $\frac{2}{9}\times\frac{5}{7}$ **c** $\frac{7}{5}\times\frac{6}{5}$ **d** $\frac{5}{3}\times\frac{8}{9}$

e $\frac{4}{9}\times\frac{3}{8}$ **f** $\frac{12}{10}\times\frac{5}{16}$ **g** $\frac{12}{9}\times\frac{2}{5}$ **h** $\frac{24}{8}\times\frac{5}{3}$

Example 5c

8 Evaluate:

a $2\frac{3}{4}\times1\frac{1}{3}$ **b** $3\frac{2}{7}\times\frac{1}{3}$ **c** $4\frac{1}{6}\times3\frac{3}{5}$ **d** $10\frac{1}{2}\times3\frac{1}{3}$

9 State the reciprocal of the following fractions.

a $\frac{5}{8}$ **b** $\frac{3}{2}$ **c** $3\frac{1}{4}$ **d** $1\frac{1}{11}$

10 Write the first step for the following division questions. (There is no need to find the answer.)

a $\frac{3}{13}\div\frac{9}{7}$ **b** $\frac{1}{2}\div\frac{1}{4}$ **c** $\frac{17}{10}\div\frac{3}{1}$ **d** $2\frac{1}{3}\div1\frac{3}{4}$

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Example 6a,b

11 Evaluate:

a $\frac{2}{9} \div \frac{3}{5}$ **b** $\frac{1}{3} \div \frac{2}{5}$ **c** $\frac{8}{7} \div \frac{11}{2}$ **d** $\frac{11}{3} \div \frac{5}{2}$

e $\frac{3}{4} \div \frac{6}{7}$ **f** $\frac{10}{15} \div \frac{1}{3}$ **g** $\frac{6}{5} \div \frac{9}{10}$ **h** $\frac{22}{35} \div \frac{11}{63}$

Example 6c

12 Evaluate:

a $1\frac{4}{7} \div 1\frac{2}{3}$ **b** $3\frac{1}{5} \div 8\frac{1}{3}$ **c** $3\frac{1}{5} \div 2\frac{2}{7}$ **d** $6\frac{2}{4} \div 2\frac{1}{6}$

13 Evaluate:

a $\frac{1}{2} \times \frac{3}{4} \div \frac{2}{5}$ **b** $\frac{3}{7} \div \frac{9}{2} \times \frac{14}{16}$ **c** $2\frac{1}{3} \div 1\frac{1}{4} \div 1\frac{3}{5}$ **d** $4\frac{1}{2} \times 3\frac{1}{3} \div 10$

14 For questions **7–13**, redo the first part of the question and the last part of the question on your calculator to check your answer is correct.

PROBLEM-SOLVING AND REASONING	15, 16, 19	15–17, 19, 20	17–21

15 Max and Tanya are painting two adjacent walls of equal area. Max has painted $\frac{3}{7}$ of his wall and Tanya has painted $\frac{2}{5}$ of her wall.

a What fraction of the two walls have Max and Tanya painted in total?

b What fraction of the two walls remains to be painted?

16 Eilish is required to write a 600-word literature essay. After working away for one hour, the word count on her computer shows that she has typed 240 words. What fraction of the essay does Eilish still need to complete? Write your answer in simplest form.

17 Vernald orders $12\frac{1}{4}$ kilograms of Granny Smith apples. Unfortunately $\frac{3}{7}$ of the apples are bruised and unusable. How many kilograms of good apples does Vernald have to make his apple pies?

18 Mary asks her dad to buy 18 bottles of soft drink for her party. Each bottle contains $1\frac{1}{4}$ litres. The glasses they have for the party can hold $\frac{1}{5}$ of a litre. How many glasses could be filled from the 18 bottles of soft drink?

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19 Fill in the empty boxes to make the following fraction equations true. More than one answer may be possible.

a $\frac{3}{\square} + \frac{2}{\square} = 1$

b $\frac{11}{7} - \frac{\square}{3} = \frac{5}{21}$

c $\frac{1}{\square} + \frac{\square}{3} = \frac{13}{15}$

d $5\frac{\square}{4} - 3\frac{\square}{2} = 1\frac{3}{4}$

20 Fill in the empty boxes to make the following fraction equations true. More than one answer may be possible.

a $\frac{\square}{4} \times \frac{5}{6} = \frac{\square}{12}$

b $\frac{\square}{21} \div \frac{3}{7} = 2$

c $\frac{2}{\square} + \frac{\square}{5} = \frac{2}{5}$

d $1\frac{1}{4} \times \square\frac{\square}{\square} = 4\frac{1}{12}$

21 Write what you think are the four key bullet points when computing with fractions. Commit these to memory and then prepare a one minute oral presentation to give to your classmates explaining your four key bullet points.

ENRICHMENT	–	–	22

How small, how close, how large?

22 You have five different fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ and four different operations $+, -, \times, \div$ at your disposal. You must use each fraction and each operation once and only once. You may use as many brackets as you need.

Here is your challenge:

a Produce an expression with the smallest possible positive answer.

b Produce an expression with an answer of 1, or as close to 1 as possible.

c Produce an expression with the largest possible answer.

An example of an expression using each of the five fractions and four operations is $\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right) \times \frac{1}{5} \div \frac{1}{6}$. This has an answer of $\frac{7}{10}$.

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4C Decimal place value and fraction/decimal conversions

REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Decimals are another way of representing 'parts of a whole'. They are an extension of our base 10 number system. The term *decimal* is derived from the Latin word *decem* meaning 'ten'.

A decimal point is used to separate the whole number and the fraction part. Decimals have been studied extensively in Year 7. In this section we revisit the concepts of comparing decimals and converting between decimals and fractions.

Let's start: Order 10

The following 10 numbers all contain a whole number part and a fraction part. Some are decimals and some are mixed numerals.

Work with a partner. Your challenge is to place the 10 numbers in ascending order.

$$4\frac{1}{11} \quad 3\frac{5}{6} \quad 3.3 \quad 3\frac{72}{100} \quad 3\frac{1}{3} \quad 2.85 \quad 3.09 \quad 2\frac{3}{4} \quad 3\frac{2}{5} \quad 3.9$$

Key ideas

- When dealing with decimal numbers, the place value table is extended to involve tenths, hundredths, thousandths etc.
- The number 517.364 means:

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
5	1	7	.	3	6	4
5×100	1×10	7×1	.	$3 \times \frac{1}{10}$	$6 \times \frac{1}{100}$	$4 \times \frac{1}{1000}$
500	10	7	.	$\frac{3}{10}$	$\frac{6}{100}$	$\frac{4}{1000}$

- **Comparing and ordering decimals**
  To compare two decimal numbers with digits in the same place value columns, you must compare the left-most digits first. Continue comparing digits as you move from left to right until you find two digits that are different.
  For example, compare 362.581 and 362.549.

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
3	6	2	.	5	8	1
3	6	2	.	5	4	9

Both numbers have identical digits in the hundreds, tens, units and tenths columns. Only when we get to the hundredths column is there a difference. The 8 is bigger than the 4 and therefore $362.581 > 362.549$.

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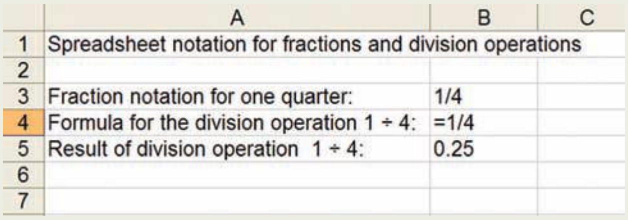
Key ideas

- **Converting decimals to fractions (non-calculator)**
  - Count the number of decimal places used.
  - This is the number of zeros that you must place in the denominator.
  - Simplify the fraction if required.

    For example: $0.64 = \frac{64}{100} = \frac{16}{25}$
- **Converting fractions to decimals (non-calculator)**
  - If the denominator is a power of 10, simply change the fraction directly to a decimal from your knowledge of its place value.

    For example: $\frac{239}{1000} = 0.239$
  - If the denominator is not a power of 10, try to find an equivalent fraction for which the denominator is a power of 10 and then convert to a decimal.

    For example: $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15$
  - A method that will always work for converting fractions to decimals is to divide the numerator by the denominator. This can result in terminating and recurring decimals and is covered in Section 4E.



	A	B	C
1	Spreadsheet notation for fractions and division operations		
2			
3	Fraction notation for one quarter:	1/4	
4	Formula for the division operation 1 ÷ 4:	=1/4	
5	Result of division operation 1 ÷ 4:	0.25	
6			
7			

A fraction is equivalent to a division operation. In spreadsheets and some calculators fractions and division operations are typed the same way, using a solidus or slash (/) between the numbers.

Example 7 Comparing decimals

Compare the following decimals and place the correct inequality sign between them.

57.89342 and 57.89631

SOLUTION

$57.89342 < 57.89631$

EXPLANATION

Digits are the same in the tens, units, tenths and hundredths columns.

Digits are first different in the thousandths column.

$\frac{3}{1000} < \frac{6}{1000}$

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Example 8 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.725 **b** 5.12

SOLUTION

a $\frac{725}{1000} = \frac{29}{40}$

b $5\frac{12}{100} = 5\frac{3}{25}$

EXPLANATION

Three decimal places, therefore three zeros in denominator.

$0.725 = 725$ thousandths

Two decimal places, therefore two zeros in denominator.

$0.12 = 12$ hundredths

Example 9 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{239}{100}$ **b** $\frac{9}{25}$

SOLUTION

a $\frac{239}{100} = 2\frac{39}{100} = 2.39$

b $\frac{9}{25} = \frac{36}{100} = 0.36$

EXPLANATION

Convert improper fraction to a mixed numeral.

Denominator is a power of 10.

$\frac{9}{25} = \frac{9 \times 4}{25 \times 4} = \frac{36}{100}$

Exercise 4C REVISION

UNDERSTANDING AND FLUENCY	1–6, 7(½), 8	4, 5(½), 6, 7–8(½), 9	5(½), 7–8(½), 9

Example 7

1 Compare the following decimals and place the correct inequality sign (< or >) between them.

a 36.485 37.123
b 21.953 21.864
c 0.0372 0.0375
d 4.21753 4.21809
e 65.4112 64.8774
f 9.5281352 9.5281347

2 Arrange the following sets of decimals in descending order.

a 3.625, 3.256, 2.653, 3.229, 2.814, 3.6521
b 11.907, 11.891, 11.875, 11.908, 11.898, 11.799
c 0.043, 1.305, 0.802, 0.765, 0.039, 1.326

3 Which of the following is the mixed numeral equivalent of 8.17?

A $8\frac{1}{7}$ **B** $8\frac{17}{10}$ **C** $8\frac{1}{17}$ **D** $8\frac{17}{1000}$ **E** $8\frac{17}{100}$

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4 Which of the following is the mixed numeral equivalent of 5.75?

A $5\frac{75}{10}$ **B** $5\frac{25}{50}$ **C** $5\frac{3}{4}$ **D** $5\frac{15}{20}$ **E** $5\frac{75}{1000}$

Example 8

5 Convert the following decimals to fractions in their simplest form.

a 0.31 **b** 0.537 **c** 0.815 **d** 0.96
e 5.35 **f** 8.22 **g** 26.8 **h** 8.512

6 Complete the following statements to convert the fractions to decimals.

a $\frac{3}{5} = \frac{6}{\square} = 0.\square$

b $\frac{11}{20} = \frac{\square}{100} = 0.5\square$

Example 9

7 Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $\frac{301}{1000}$ **c** $\frac{405}{100}$ **d** $\frac{76}{10}$
e $\frac{3}{25}$ **f** $\frac{7}{20}$ **g** $\frac{5}{2}$ **h** $\frac{7}{4}$

8 Using a calculator, convert the following decimals to fractions and fractions to decimals.

a 0.052 **b** 6.125 **c** 317.06 **d** 0.424
e $\frac{11}{40}$ **f** $\frac{3}{8}$ **g** $\frac{17}{25}$ **h** $\frac{29}{125}$

9 Convert the following mixed numerals to decimals and then place them in ascending order. Consider whether to use a calculator or not. Possibly a combination of mental arithmetic and some simple calculator keystrokes is the most efficient strategy.

$2\frac{2}{5}, \ 2\frac{1}{4}, \ 2\frac{3}{8}, \ 2\frac{7}{40}, \ 2\frac{9}{50}, \ 2\frac{3}{10}$

PROBLEM-SOLVING AND REASONING	10, 11, 13	11–14	12–14

10 The distances from Nam's locker to his six different classrooms are listed below.

• locker to room B5	(0.186 km)
• locker to gym	(0.316 km)
• locker to room A1	(0.119 km)
• locker to room C07	(0.198 km)
• locker to room P9	(0.254 km)
• locker to BW Theatre	(0.257 km)

List Nam's six classrooms in order of distance of his locker from the closest classroom to the one furthest away.

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11 The prime minister's approval rating is 0.35, and the opposition leader's approval rating is $\frac{3}{8}$. Which leader is ahead in the popularity polls and by how much?

12 Justin dug six different holes for planting six different types of fruit bushes and trees. He measured the dimensions of the holes and found them to be:

- **A** depth 1.31 m, width 0.47 m
- **B** depth 1.15 m, width 0.39 m
- **C** depth 0.85 m, width 0.51 m
- **D** depth 0.79 m, width 0.48 m
- **E** depth 1.08 m, width 0.405 m
- **F** depth 1.13 m, width 0.4 m

a List the holes in increasing order of depth.

b List the holes in decreasing order of width.

13 **a** Write a decimal that lies midway between 2.65 and 2.66.

b Write a fraction that lies midway between 0.89 and 0.90.

c Write a decimal that lies midway between 4.6153 and 4.6152.

d Write a fraction that lies midway between 2.555 and 2.554.

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14 Complete the following magic square using a mixture of fractions and decimals.

2.6		$1\frac{4}{5}$
	$\frac{6}{2}$	
4.2		

ENRICHMENT	–	–	15

Exchange rates

15 The table below shows a set of exchange rates between the US dollar (US$), the Great Britain pound (£), the Canadian dollar (C$), the euro (€) and the Australian dollar (A$).

	United States **US$**	Great Britain **£**	Canada **C$**	European Union **€**	Australia **A$**
US$	1	1.58781	0.914085	1.46499	0.866558
£	0.629795	1	0.575686	0.922649	0.545754
C$	1.09399	1.73705	1	1.60269	0.948006
€	0.682594	1.08383	0.623949	1	0.591507
A$	1.15399	1.83232	1.05484	1.69059	1

The following two examples are provided to help you to interpret the table.

- A$1 will buy US$0.866558.
- You will need A$1.15399 to buy US$1.

Study the table and answer the following questions.

a How many euros will A$100 buy?

b How many A$ would buy £100?

c Which country has the most similar currency rate to Australia?

d Would you prefer to have £35 or €35?

e C$1 has the same value as how many US cents?

f If the cost of living was the same in each country in terms of each country's own currency, list the five money denominations in descending order of value for money.

g A particular new car costs £30000 in Great Britain and $70000 in Australia. If it costs A$4500 to freight a car from Great Britain to Australia, which car is cheaper to buy? Justify your answer by using the exchange rates in the table.

h Research the current exchange rates and see how they compare to those listed in the table.

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4D Computation with decimals REVISION

Interactive
Widgets
HOTsheets
Walkthrough

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing decimals.

Reminder:

$$\underset{\text{dividend}}{\underset{\uparrow}{3.6}} \div \underset{\text{divisor}}{\underset{\uparrow}{2}} = \underset{\text{quotient}}{\underset{\uparrow}{1.8}}$$

Let's start: Match the phrases

There are eight different sentence beginnings and eight different sentence endings below. Your task is to match each sentence beginning with its correct ending. When you have done this, write the eight correct sentences in your workbook.

Sentence beginnings	**Sentence endings**
When adding or subtracting decimals	the decimal point appears to move two places to the right.
When multiplying decimals	the decimal point in the quotient goes directly above the decimal point in the dividend.
When multiplying decimals by 100	make sure you line up the decimal points.
When dividing decimals by decimals	the number of decimal places in the question must equal the number of decimal places in the answer.
When multiplying decimals	the decimal point appears to move two places to the left.
When dividing by 100	start by ignoring the decimal points.
When dividing decimals by a whole number	we start by changing the question so that the divisor is a whole number.

Key ideas

- **Adding and subtracting decimals**
  - Ensure digits are correctly aligned in similar place value columns.
  - Ensure the decimal points are lined up directly under one another.

$$37.56 + 5.231 \qquad \begin{array}{r} 37.560 \\ +\ \ 5.231 \\ \hline \end{array} \ \checkmark \qquad \begin{array}{l} 37.56 \\ 5.231 \\ \hline \end{array} \ \times$$

- **Multiplying and dividing decimals by powers of 10**
  - When multiplying, the decimal point appears to move to the *right* the same number of places as there are zeros in the multiplier.

$13.753 \times 100 = 1375.3$ $\qquad 13.\overset{\curvearrowright\curvearrowright}{75}3$

Multiply by 10 twice.

  - When dividing, the decimal point appears to move to the *left* the same number of places as there are zeros in the divisor.

$586.92 \div 10 = 58.692$ $\qquad 58\overset{\curvearrowleft}{6.}92$

Divide by 10 once.

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Key ideas

Multiplying decimals

- Initially ignore the decimal points and carry out routine multiplication.
- The decimal place is correctly positioned in the answer according to the following rule: ‘The number of decimal places in the answer must equal the total number of decimal places in the question.’

$$5.73 \times 8.6 \longrightarrow \begin{array}{r} 573 \\ \times\ \ 86 \\ \hline 49278 \end{array} \longrightarrow 5.\underline{7}\underline{3} \times 8.\underline{6} = 49.\underline{2}\underline{7}\underline{8}$$

(3 decimal places in question, 3 decimal places in answer)

Dividing decimals

The decimal point in the quotient goes directly above the decimal point in the **dividend**.

$$56.34 \div 3 \qquad \text{Divisor} \longrightarrow \begin{array}{r} 18.78 \\ 3\overline{)56.34} \end{array} \begin{array}{l} \longleftarrow \text{Quotient (answer)} \\ \longleftarrow \text{Dividend} \end{array}$$

We avoid dividing decimals by other decimals. Instead we change the **divisor** into a whole number. Of course, whatever change we make to the divisor we must also make to the dividend, so it is equivalent to multiplying by 1 and the value of the question is not changed.

We avoid $\longrightarrow$ 27.354 ÷ 0.02

Preferring to do $\longrightarrow$ 2735.4 ÷ 2

$$27.354 \div 0.02$$

Example 10 Adding and subtracting decimals

Calculate:

a 23.07 + 103.659 + 9.9

b 9.7 − 2.86

SOLUTION

a

$$\begin{array}{r} 23.070 \\ 103.659 \\ +\ \ \ 9.900 \\ \hline 136.629 \end{array}$$

b

$$\begin{array}{rrr} {}^{8}\cancel{9}. & {}^{16}\cancel{7} & {}^{1}\cancel{0} \\ -\ 2. & 8 & 6 \\ \hline 6. & 8 & 4 \end{array}$$

EXPLANATION

a Make sure all decimal points and places are correctly aligned directly under one another. Fill in missing decimal places with zeros.

Carry out the addition of each column, working from right to left.

b Align decimal points directly under one another and fill in missing decimal places with zeros.

Carry out subtraction following the same procedure as for subtraction of whole numbers.

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Example 11 Multiplying and dividing by powers of 10

Calculate:

a $9.753 \div 100$

b 27.58×10000

SOLUTION

a $9.753 \div 100 = 0.09753$

Dividing by 100, therefore decimal point appears to move two places to the left. Additional zeros are inserted as necessary.

.09.753

b $27.58 \times 10000 = 275800$

Multiplying by 10 000, therefore decimal point appears to move four places to the right. Additional zeros are inserted as necessary.

27.5800.

EXPLANATION

Example 12 Multiplying decimals

Calculate:

a 2.57×3

b 4.13×9.6

SOLUTION

a

$$\begin{array}{r} {}^{1}2\,{}^{2}5\;7 \\ \times \quad\;\; 3 \\ \hline 7\;\;7\;1 \end{array}$$

Perform multiplication ignoring decimal point. There are two decimal places in the question, so two decimal places in the answer.

$2.57 \times 3 = 7.71$

Estimation is less than 10 ($\approx 3 \times 3 = 9$).

b

$$\begin{array}{r} 413 \\ \times \;\; 96 \\ \hline 2478 \\ 37170 \\ \hline 39648 \end{array}$$

Ignore both decimal points.
Perform routine multiplication.
There is a total of three decimal places in the question, so there must be three decimal places in the answer.

$4.13 \times 9.6 = 39.648$

Estimation is about 40 ($\approx 4 \times 10 = 40$).

EXPLANATION

Example 13 Dividing decimals

Calculate:

a 35.756 ÷ 4

b 64.137 ÷ 0.03

SOLUTION

a 8.939

$$\begin{array}{r} 8.\,9\,3\,9 \\ 4\overline{)35.{}^{\color{red}3}7{}^{\color{red}1}5{}^{\color{red}3}6} \end{array}$$

b 64.137 ÷ 0.03

= 6413.7 ÷ 3 = 2137.9

$$\begin{array}{r} 2\,1\,3\,7.\,9 \\ 3\overline{)64{}^{\color{red}1}1{}^{\color{red}2}3.{}^{\color{red}2}7} \end{array}$$

EXPLANATION

Carry out division, remembering that the decimal point in the answer is placed directly above the decimal point in the dividend.

Instead of dividing by 0.03, multiply both the divisor and the dividend by 100.

Move each decimal point two places to the right.

Carry out the division question 6413.7 ÷ 3.

Exercise 4D REVISION

UNDERSTANDING AND FLUENCY	1, 2–4(½), 5, 6(½), 7, 8(½), 9, 10–11(½)	2–4(½), 5, 6(½), 7, 8(½), 9, 10–11(½)	5, 6(½), 7, 8(½), 9, 10–12(½)

1 Which of the following is correctly set up for the following addition?

5.386 + 53.86 + 538.6

A $$\begin{array}{r} 5.386 \\ 53.86 \\ \underline{+\ 538.6} \end{array}$$

B $$\begin{array}{r} 5.386 \\ 53.860 \\ \underline{+\ 538.600} \end{array}$$

C $$\begin{array}{rr} & 5.386 \\ & 53.86 \\ \underline{+} & \underline{538.6} \end{array}$$

D $$\begin{array}{r} 538 + 53 + 5 \\ \underline{+\ 0.386 + 0.86 + 0.6} \end{array}$$

Example 10

2 Calculate:

a 23.57 + 39.14 **b** 64.28 + 213.71 **c** 5.623 + 18.34 **d** 92.3 + 1.872

e 38.52 − 24.11 **f** 76.74 − 53.62 **g** 123.8 − 39.21 **h** 14.57 − 9.8

3 Calculate:

a 13.546 + 35.2 + 9.27 + 121.7 **b** 45.983 + 3.41 + 0.032 + 0.8942

c 923.8 + 92.38 + 9.238 + 0.238 **d** 4.572 + 0.0329 + 2.0035 + 11.7003

4 Calculate:

a 3.456 + 12.723 − 4.59 **b** 7.213 − 5.46 + 8.031

c 26.451 + 8.364 − 14.987 **d** 12.7 − 3.45 − 4.67

5 The correct answer to the problem 2.731 ÷ 1000 is:

A 2731 **B** 27.31 **C** 2.731 **D** 0.02731 **E** 0.002731

Example 11

6 Calculate:

a 36.5173 × 100 **b** 0.08155 × 1000 **c** 7.5 ÷ 10 **d** 3.812 ÷ 100

e 634.8 × 10 000 **f** 1.0615 ÷ 1000 **g** 0.003 × 10 000 **h** 0.452 ÷ 1000

7 If 56 × 37 = 2072, the correct answer to the problem 5.6 × 3.7 is:

A 207.2 **B** 2072 **C** 20.72 **D** 2.072 **E** 0.2072

Example 12

8 Calculate:

a 12.45 × 8 **b** 4.135 × 3 **c** 26.2 × 4.1 **d** 5.71 × 0.32

e 0.0023 × 8.1 **f** 300.4 × 2.2 **g** 7.123 × 12.5 **h** 81.4 × 3.59

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9 Which of the following divisions would provide the same answer as the division question 62.5314 ÷ 0.03?

A 625.314 ÷ 3 **B** 6253.14 ÷ 3 **C** 0.625314 ÷ 3 **D** 625 314 ÷ 3

Example 13a

10 Calculate:

a 24.54 ÷ 2 **b** 17.64 ÷ 3 **c** 0.0485 ÷ 5 **d** 347.55 ÷ 7
e 133.44 ÷ 12 **f** 4912.6 ÷ 11 **g** 2.58124 ÷ 8 **h** 17.31 ÷ 5

Example 13b

11 Calculate:

a 6.114 ÷ 0.03 **b** 0.152 ÷ 0.4 **c** 4023 ÷ 0.002 **d** 5.815 ÷ 0.5
e 0.02345 ÷ 0.07 **f** 16.428 ÷ 1.2 **g** 0.5045 ÷ 0.8 **h** 541.31 ÷ 0.4

12 Calculate:

a 13.7 + 2.59 **b** 35.23 − 19.71 **c** 15.4 × 4.3 **d** 9.815 ÷ 5
e 13.72 × 0.97 **f** 6.7 − 3.083 **g** 0.582 ÷ 0.006 **h** 7.9023 + 34.81

PROBLEM-SOLVING AND REASONING	13, 14, 17	14, 15, 17, 18	14–19

13 The heights of Mrs Buchanan's five grandchildren are 1.34 m, 1.92 m, 0.7 m, 1.5 m, and 1.66 m. What is the combined height of Mrs Buchanan's grandchildren?

14 If the rental skis at Mt Buller were lined up end to end, they would reach from the summit of Mt Buller all the way down to the entry gate at Mirimbah. The average length of a downhill ski is 1.5 m and the distance from the Mt Buller summit to Mirimbah is 18.3 km. How many rental skis are there?

15 Design your own question similar to question **14** and ask a classmate to solve it.

For example, you might use the number of cars placed bumper to bumper from Sydney to Brisbane.

Carry out some research so that your decimal lengths are approximately accurate.

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16 Joliet is a keen walker. She has a pedometer that shows she has walked 1 428 350 paces so far this year. Her average pace length is 0.84 metres. How many kilometres has Joliet walked so far this year? (Give your answer correct to the nearest kilometre.)

17 A steel pipe of length 7.234 m must be divided into four equal lengths. The saw blade is 2 mm thick. How long will each of the four lengths be?

18 If $a = 0.12$, $b = 2.3$ and $c = 3.42$, find:

a $a + b + c$ **b** $c - (a + b)$ **c** $a \times b \times c$ **d** $c \div a - b$

19 If $a = 0.1$, $b = 2.1$ and $c = 3.1$, without evaluating, which of the following alternatives would provide the biggest answer?

A $a + b + c$ **B** $a \times b \times c$ **C** $b \div a + c$ **D** $c \div a \times b$

ENRICHMENT – – 20

Target practice

20 In each of the following problems, you must come up with a starting decimal number(s) that will provide an answer within the target range provided when the nominated operation is performed.

For example: Find a decimal number that when multiplied by 53.24 will give an answer between 2.05 and 2.1.

You might like to use trial and error, or you might like to work out the question backwards.

Confirm these results on your calculator:

$0.03 \times 53.24 = 1.5972$ (answer outside target range—too small)

$0.04 \times 53.24 = 2.1296$ (answer outside target range—too large)

$0.039 \times 53.24 = 2.07636$ (answer within target range of 2.05 to 2.1)

Therefore, a possible answer is 0.039.

Try the following target problems. (Aim to use as few decimal places as possible.)

Question	Starting number	Operation (instruction)	Target range
1	0.039	× 53.24	2.05–2.1
2		× 0.034	100–101
3		÷ 1.2374	75.7–75.8
4		× by itself (square)	0.32–0.33
5		÷ (−5.004)	9.9–9.99

Try the following target problems. (Each starting number must have at least two decimal places.)

Question	Two starting numbers	Operation (instruction)	Target range
6	0.05, 3.12	×	0.1–0.2
7		÷	4.1–4.2
8		×	99.95–100.05
9		+	0.001–0.002
10		−	45.27

You might like to make up some of your own target problems.

4E Terminating decimals, recurring decimals and rounding

REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Not all fractions convert to the same type of decimal. For example:

$$\frac{1}{2} = 1 \div 2 = 0.5$$

$$\frac{1}{3} = 1 \div 3 = 0.33333\ldots$$

$$\frac{1}{7} = 1 \div 7 = 0.142857\ldots$$

Decimals that stop (or terminate) are known as terminating decimals, whereas decimals that continue on forever with some form of pattern are known as repeating or recurring decimals.

Let's start: Decimal patterns

Carry out the following short divisions without using a calculator and observe the pattern of digits that occur.

For example: $\frac{1}{11} = 1 \div 11 = \begin{array}{r} 0.\;0\;9\;0\;9\ldots \\ 11\overline{)1.\;{}^{1}0\,{}^{10}0\,{}^{1}0\,{}^{10}0\,{}^{1}0} \end{array} = 0.0909090909\ldots$

Try the following: $\frac{1}{3}, \frac{2}{7}, \frac{4}{9}, \frac{5}{11}, \frac{8}{13}, \frac{25}{99}$

Remember to keep adding zeros to the dividend until you have a repetitive pattern.

Which fraction has the longest repetitive pattern?

Key ideas

- A **terminating decimal** has a finite number of decimal places (i.e. it terminates).

  For example: $\frac{5}{8} = 5 \div 8 = 0.625 \quad \begin{array}{r} 0.\;6\;2\;5 \\ 8\overline{)5.\,{}^{5}0\,{}^{2}0\,{}^{4}0} \end{array} \longleftarrow$ Terminating decimal

- A **recurring decimal** (or **repeating decimal)** has an infinite number of decimal places with a finite sequence of digits that are repeated indefinitely.

  For example: $\frac{1}{3} = 1 \div 3 = 0.333\ldots \quad \begin{array}{r} 0.\;3\;3\;3\ldots \\ 3\overline{)1.\,{}^{1}0\,{}^{1}0\,{}^{1}0\,{}^{1}0} \end{array} \longleftarrow$ Recurring decimal

- All terminating and recurring decimals can be expressed as fractions. All numbers that can be expressed as a fraction (i.e. written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$) are known as 'rational' numbers.
- A convention is to use dots placed above the digits to show the start and finish of a repeating cycle of digits.

  For example: $0.55555\ldots = 0.\dot{5}$ and $0.3412412412\ldots = 0.\dot{3}41\dot{2}$

  Another convention is to use a horizontal line placed above the digits to show the repeating cycle of digits.

  For example: $0.55555\ldots = 0.\overline{5}$ and $0.3412412412\ldots = 0.\overline{3412}$

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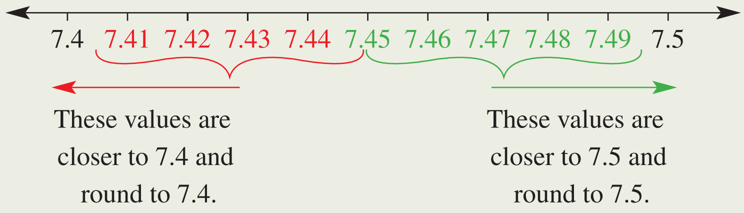
Key ideas

- **Rounding decimals** involves approximating a decimal number to fewer decimal places. When rounding, the **critical digit** is the digit immediately after the rounding digit.
  - If the critical digit is less than 5, the rounding digit is not changed.
  - If the critical digit is 5 or more, the rounding digit is increased by 1.
    For example: 51.34721 rounded to two decimal places is 51.35.
    The critical digit is 7, which is greater than 5, hence the rounding digit is increased by 1.

An illustration of rounding to one decimal place:
The decimals 7.41 to 7.44 are closer in value to 7.4 and will all round to 7.4.
The values 7.46 to 7.5 are closer in value to 7.5 and will all round to 7.5.
7.45 also rounds to 7.5.



Example 14 Writing terminating decimals

Convert the following fractions to decimals.

a $\frac{1}{4}$

b $\frac{7}{8}$

SOLUTION

a $\frac{1}{4} = 0.25$

b $\frac{7}{8} = 0.875$

EXPLANATION

$$\begin{array}{r} 0.\ 2\ \ 5 \\ 4\overline{)1.\ {}^{1}0\ {}^{2}0} \end{array}$$

$$\begin{array}{r} 0.\ 8\ \ 7\ \ 5 \\ 8\overline{)7.\ {}^{7}0\ {}^{6}0\ {}^{4}0} \end{array}$$

Example 15 Writing recurring decimals

Express the following fractions as recurring decimals.

a $\frac{2}{3}$

b $3\frac{5}{7}$

SOLUTION

a $\frac{2}{3} = 0.\dot{6}$

b $3\frac{5}{7} = 3.\dot{7}1428\dot{5}$ or $3.\overline{714285}$

EXPLANATION

$$\begin{array}{r} 0.\ 6\ \ 6\ldots \\ 3\overline{)2.\ {}^{2}0\ {}^{2}0\ {}^{2}0} \end{array}$$

$$\begin{array}{r} 0.\ 7\ \ 1\ \ 4\ \ 2\ \ 8\ \ 5\ \ 7\ldots \\ 7\overline{)5.\ {}^{5}0\ {}^{1}0\ {}^{3}0\ {}^{2}0\ {}^{6}0\ {}^{4}0\ {}^{5}0\ {}^{1}0} \end{array}$$

Example 16 Rounding decimals

Round each of the following to the specified number of decimal places.

a 15.35729 (three decimal places)
b 4.86195082 (four decimal places)

SOLUTION	EXPLANATION
a 15.35729 ≈ 15.357 (to 3 d.p.)	Critical digit is 2, which is less than 5. Rounding digit remains the same.
b 4.86195082 ≈ 4.8620 (to 4 d.p.)	Critical digit is 5, therefore increase rounding digit by 1. There is a carry-on effect, as the rounding digit was a 9.

Example 17 Rounding recurring decimals

Write $\frac{3}{7}$ as a decimal, correct to two decimal places.

SOLUTION	EXPLANATION
$\frac{3}{7} = 0.43$ (to 2 d.p.)	$\begin{array}{r} 0.\;4\;\;2\;\;8\;\;\; \\ 7\overline{)3.{}^{3}0\;{}^{2}0\;{}^{6}0\;{}^{4}0} \end{array}$

Exercise 4E REVISION

UNDERSTANDING AND FLUENCY	1–6, 7–8(½), 9	3–6, 7–8(½), 9, 10	5, 6, 7–8(½), 9, 10

1 State whether the following are terminating decimals (T) or recurring decimals (R).

a 5.47
b $3.1541\dot{5}\ldots$
c $8.\dot{6}$
d 7.1834
e 0.333
f $0.\dot{5}3\dot{4}$
g 0.5615
h 0.32727…

2 Express the following recurring decimals using the convention of dots or a bar to indicate the start and finish of the repeating cycle.

a 0.33333…
b 6.21212121…
c 8.5764444…
d 2.135635635…
e 11.2857328573…
f 0.003523523…

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Example 14

3 Convert the following fractions to decimals.

a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{1}{8}$ **d** $\frac{11}{20}$

Example 15

4 Express the following fractions as recurring decimals.

a $\frac{1}{3}$ **b** $\frac{5}{9}$ **c** $\frac{5}{6}$ **d** $\frac{8}{11}$

e $\frac{3}{7}$ **f** $\frac{5}{13}$ **g** $3\frac{2}{15}$ **h** $4\frac{6}{7}$

5 Write down the 'critical' digit (i.e. the digit immediately after the rounding digit) for each of the following.

a 3.5724 (rounding to three decimal places)
b 15.89154 (rounding to one decimal place)
c 0.004571 (rounding to four decimal places)
d 5432.726 (rounding to two decimal places)

6 State how you round decimal numbers on your calculator.

Example 16

7 Round each of the following to the specified number of decimal places, which is the number in the bracket.

a 0.76581 (3)
b 9.4582 (1)
c 6.9701 (1)
d 21.513426 (4)
e 0.9457 (2)
f 17.26 (0)
g 8.5974 (2)
h 8.10552 (3)

8 Write each of the following decimals, correct to two decimal places.

a 17.0071
b 0.9192
c 4.4444
d 14.5172
e 5.1952
f 78.9963
g 0.0015
h 2.6808

9 Round each of the following to the nearest whole number.

a 65.3197
b 8.581
c 29.631
d 4563.18

Example 17

10 Write each of the following fractions as decimals, correct to two decimal places.

a $\frac{6}{7}$ **b** $\frac{2}{9}$ **c** $\frac{4}{11}$ **d** $\frac{5}{12}$

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PROBLEM-SOLVING AND REASONING	11, 12, 15	12–15	12–16

11 Round each of the following to the number of decimal places specified in the brackets.

a 7.699951 (4) **b** 4.95953 (1) **c** 0.0069996 (5)

12 Simone and Greer are two elite junior sprinters. At the Queensland State championships, Simone recorded her personal best 100 m time of 12.77 seconds, while Greer came a close second with a time of 12.83 seconds.

a If the electronic timing equipment could display times only to the nearest second, what would be the time difference between the two sprinters?

b If the electronic timing equipment could record times only to the nearest tenth of a second, what would be the time difference between the two sprinters?

c What is the time difference between the two girls, correct to two decimal places?

d If the electronic timing system could measure accurately to three decimal places, what would be the quickest time that Simone could have recorded?

e Assume that Simone and Greer ran at a consistent speed throughout the 100 m race. Predict the winning margin (correct to the nearest centimetre).

13 When $\frac{3}{7}$ is expressed in decimal form, what is the digit in the 19th decimal place?

14 Express $\frac{1}{17}$ as a recurring decimal.

15 Frieda stated that she knew an infinite non-recurring decimal. Andrew said that was impossible. He was confident that all decimals either terminated or repeated and that there was no such thing as an infinite non-recurring decimal. Who is correct?

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16 Two students give the following answers in a short test on rounding. Both students have one particular misunderstanding. Study their answers carefully and write a comment to help correct each student's misunderstanding.

Rounding question		Student A	Student B
0.543	(2)	0.54 ✓	0.50 ✗
6.7215	(3)	6.721 ✗	6.722 ✓
5.493	(1)	5.5 ✓	5.5 ✓
8.2143	(3)	8.214 ✓	8.210 ✗
11.54582	(2)	11.54 ✗	11.55 ✓

ENRICHMENT	–	–	17

Will it terminate or recur?

17 Can you find a way of determining if a fraction will result in a terminating (T) decimal or a recurring (R) decimal?

a Predict the type of decimal answer for the following fractions, and then convert them to see if you are correct.

i $\frac{1}{8}$ **ii** $\frac{1}{12}$

iii $\frac{1}{14}$ **iv** $\frac{1}{15}$

v $\frac{1}{20}$ **vi** $\frac{1}{60}$

A key to recognising whether a fraction will result in a terminating or recurring decimal lies in the factors of the denominator.

b Write down the factors of the denominators listed above.

c From your observations, write down a rule that assists the recognition of when a particular fraction will result in a terminating or recurring decimal.

d Without evaluating, state whether the following fractions will result in terminating or recurring decimals.

i $\frac{1}{16}$ **ii** $\frac{1}{9}$

iii $\frac{1}{42}$ **iv** $\frac{1}{50}$

v $\frac{1}{75}$ **vi** $\frac{1}{99}$

vii $\frac{1}{200}$ **viii** $\frac{1}{625}$

4F Converting fractions, decimals and percentages REVISION

Per cent is Latin for 'out of 100'. One dollar equals 100 **cents** and one **century** equals 100 years. We come across percentages in many everyday situations. Interest rates, discounts, test results and statistics are just some of the common ways we deal with percentages.

Percentages are closely related to fractions. A percentage is another way of writing a fraction with a denominator of 100. Therefore, 63% means that if something is broken into 100 parts you would have 63 of them (i.e. $63\% = \frac{63}{100}$).

Let's start: Estimating percentages


Creamy soda


Lime


Orange


Cola


Lemon


Milkshake


Raspberry


Chocolate

- Estimate the percentage of drink remaining in each of the glasses shown above.
- Discuss your estimations with a partner.
- Estimate what percentage of the rectangle below has been shaded in.

- Use a ruler to draw several 10 cm × 2 cm rectangles. Work out an amount you would like to shade in, then use your ruler to measure precisely the amount to shade. Shade in this amount.
- Ask your partner to guess the percentage you shaded.
- Have several attempts with your partner and see if your estimation skills improve.

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Key ideas

- The symbol % means ‘per cent’. It comes from the Latin words *per centum*, which translates to ‘out of 100’.

  For example: 23% means 23 out of 100 or $\frac{23}{100} = 0.23$.

- **To convert a percentage to a fraction**
  - Change the % sign to a denominator of 100.
  - Simplify the fraction if required.

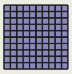
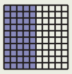
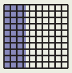
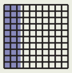
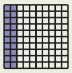
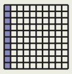
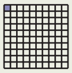
  For example: $35\% = \frac{35}{100} = \frac{7}{20}$

- **To convert a percentage to a decimal**
  - Divide by 100%. Therefore move the decimal point two places to the left.
    For example: $46\% = 46\% \div 100\% = 46 \div 100 = 0.46$

- **To convert a fraction to a percentage**
  - Multiply by 100%.

    For example: $\frac{1}{8} = \frac{1}{8} \times 100\% = \frac{1}{8} \times \frac{100}{1}\% = \frac{25}{2}\% = 12\frac{1}{2}\%$

- **To convert a decimal to a percentage**
  - Multiply by 100%. Therefore move the decimal point two places to the right.
    For example: $0.812 = 0.812 \times 100\% = 81.2\%$
  - Common percentages and their equivalent fractions and decimals are shown in the table below. It is helpful to know these.

Words	Diagram	Fraction	Decimal	Percentage
one whole		1	1	100%
one-half		$\frac{1}{2}$	0.5	50%
one-third		$\frac{1}{3}$	0.333... or $0.\dot{3}$	$33\frac{1}{3}\%$
one-quarter		$\frac{1}{4}$	0.25	25%
one-fifth		$\frac{1}{5}$	0.2	20%
one-tenth		$\frac{1}{10}$	0.1	10%
one-hundredth		$\frac{1}{100}$	0.01	1%

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Example 18 Converting percentages to fractions

Convert the following percentages to fractions or mixed numerals in their simplest form.

a 160%

b 12.5%

SOLUTION	EXPLANATION
a $160\% = \frac{160}{100}$	Change % sign to a denominator of 100.
$= \frac{8 \times 20}{5 \times 20}$	Cancel the HCF.
$= \frac{8}{5} = 1\frac{3}{5}$	Convert answer to a mixed numeral.
b $12.5\% = \frac{12.5}{100}$ or $= \frac{12.5}{100}$	Change % sign to a denominator of 100.
$= \frac{25}{200}$ $= \frac{125}{1000}$	Multiply numerator and denominator by 2 or by 10 to make whole numbers.
$= \frac{1}{8}$ $= \frac{1}{8}$	Simplify fraction by cancelling the HCF.

Example 19 Converting percentages to decimals

Convert the following percentages to decimals.

a 723%

b 13.45%

SOLUTION	EXPLANATION
a 723% = 7.23	723 ÷ 100 723. Decimal point appears to move two places to the left.
b 13.45% = 0.1345	13.45 ÷ 100 13.45

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Example 20 Converting fractions to percentages

Convert the following fractions and mixed numerals to percentages.

a $\frac{3}{5}$ **b** $\frac{7}{40}$ **c** $2\frac{1}{4}$ **d** $\frac{2}{3}$

SOLUTION	EXPLANATION
a $\frac{3}{5} \times 100\% = \frac{3}{{}_{1}\cancel{5}} \times \frac{{}^{20}\cancel{100}}{1}\%$ $= 60\%$ Alternatively: $\frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 60\%$	Multiply by 100%. Simplify by cancelling the HCF.
b $\frac{7}{40} \times 100\% = \frac{7}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$ $= \frac{35}{2}\% = 17\frac{1}{2}\%$	Multiply by 100%. Simplify by cancelling the HCF. Write the answer as a mixed numeral.
c $2\frac{1}{4} \times 100\% = \frac{9}{{}_{1}\cancel{4}} \times \frac{{}^{25}\cancel{100}}{1}\%$ $= 225\%$ Alternatively: $2 = 200\%$ and $\frac{1}{4} = 25\%$. So $2\frac{1}{4} = 225\%$.	Convert mixed numeral to improper fraction. Cancel and simplify.
d $\frac{2}{3} \times 100\% = \frac{2}{3} \times \frac{100}{1}\%$ $= \frac{200}{3}\% = 66\frac{2}{3}\%$ Alternatively: $\frac{1}{3} = 33\frac{1}{3}\%$ So $\frac{2}{3} = 66\frac{2}{3}\%$.	Multiply by 100%. Multiply numerators and denominators. Write answer as a mixed numeral.

Example 21 Converting decimals to percentages

Convert the following decimals to percentages.

a 0.458 **b** 17.5

SOLUTION	EXPLANATION
a $0.458 = 45.8\%$	$0.458 \times 100 \qquad 0.458$ The decimal point appears to move two places to the right.
b $17.5 = 1750\%$	$17.5 \times 100 \qquad 17.50$

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Exercise 4F REVISION

UNDERSTANDING AND FLUENCY	1, 2–3(½), 4, 5(½), 6, 7–9(½)	3(½), 4, 5(½), 6, 7–9(½), 10, 11(½)	5(½), 6, 7–9(½), 10, 11(½)

1 The fraction equivalent of 27% is:

A $\frac{2}{7}$ **B** $\frac{27}{100}$ **C** 2700 **D** $\frac{13.5}{10}$

Example 18a

2 Convert the following percentages to fractions or mixed numerals in their simplest form.

a 39% **b** 11% **c** 20% **d** 75%
e 125% **f** 70% **g** 205% **h** 620%

Example 18b

3 Convert the following percentages to fractions in their simplest form.

a $37\frac{1}{2}\%$ **b** 15.5% **c** $33\frac{1}{3}\%$ **d** $66\frac{2}{3}\%$
e 2.25% **f** 4.5% **g** $10\frac{1}{5}\%$ **h** 87.5%

4 The decimal equivalent of 37% is:

A 0.037 **B** 0.37 **C** 3.7 **D** 37.00

Example 19

5 Convert the following percentages to decimals.

a 65% **b** 37% **c** 158% **d** 319%
e 6.35% **f** 0.12% **g** 4051% **h** 100.05%

6 The percentage equivalent of $\frac{47}{100}$ is:

A 0.47% **B** 4.7% **C** 47% **D** 470%

Example 20 a, b

7 Convert the following fractions to percentages.

a $\frac{2}{5}$ **b** $\frac{1}{4}$ **c** $\frac{11}{20}$ **d** $\frac{13}{50}$
e $\frac{9}{40}$ **f** $\frac{17}{25}$ **g** $\frac{150}{200}$ **h** $\frac{83}{200}$

Example 20c

8 Convert the following mixed numerals and improper fractions to percentages.

a $2\frac{3}{4}$ **b** $5\frac{1}{5}$ **c** $\frac{7}{4}$ **d** $\frac{9}{2}$
e $3\frac{12}{25}$ **f** $1\frac{47}{50}$ **g** $\frac{77}{10}$ **h** $\frac{183}{20}$

Example 20d

9 Without using a calculator, convert the following fractions to percentages. Check your answers using a calculator.

a $\frac{1}{3}$ **b** $\frac{1}{8}$ **c** $\frac{1}{12}$ **d** $\frac{1}{15}$
e $\frac{3}{8}$ **f** $\frac{2}{7}$ **g** $\frac{3}{16}$ **h** $\frac{27}{36}$

10 The percentage equivalent of 0.57 is:

A 57% **B** 5.7% **C** 570% **D** 0.57%

Example 21

11 Without using a calculator, convert the following decimals to percentages. Check your answers using a calculator.

a 0.42 **b** 0.17 **c** 3.541 **d** 11.22
e 0.0035 **f** 0.0417 **g** 0.01 **h** 1.01

PROBLEM-SOLVING AND REASONING	12, 13, 17	13–15, 17, 18	14–18

12 Which value is larger?

A 0.8 of a large pizza **B** 70% of a large pizza **C** $\frac{3}{4}$ of a large pizza

13 Complete the following conversion tables involving common fractions, decimals and percentages.

a

Fraction	Decimal	%
$\frac{1}{4}$		
$\frac{2}{4}$		
$\frac{3}{4}$		
$\frac{4}{4}$		

b

Fraction	Decimal	%
	$0.\dot{3}$	
	$0.\dot{6}$	
	$0.\dot{9}$	

c

Fraction	Decimal	%
		20%
		40%
		60%
		80%
		100%

14 Complete the following conversion tables.

a

Fraction	Decimal	%
		15%
	0.24	
$\frac{3}{8}$		
$\frac{5}{40}$		
	0.7	
		62%

b

Fraction	Decimal	%
$\frac{11}{5}$		
	0.003	
		6.5%
		119%
	4.2	
$\frac{5}{6}$		

15 A cake is cut into eight equal pieces. What fraction, what percentage and what decimal does one slice of the cake represent?

16 The Sharks team has won 13 out of 17 games for the season to date. The team still has three games to play. What is the smallest and the largest percentage of games the Sharks could win for the season?

17 **a** Explain why multiplying by 100% is the same as multiplying by 1.

b Explain why dividing by 100% is the same as dividing by 1.

18 Let A, B, C and D represent different digits.

a Convert BC% to a fraction.

b Convert CD.B% to a decimal.

c Convert A.BC to a percentage.

d Convert D.DBCC to a percentage.

e Convert $\frac{A}{D}$ to a percentage.

f Convert B$\frac{C}{A}$ to a percentage.

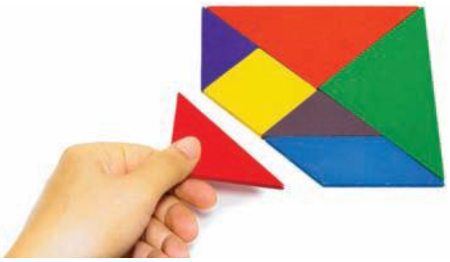
ENRICHMENT	–	–	19

Tangram percentages

19 You are most likely familiar with the ancient Chinese puzzle known as a tangram.

A tangram consists of seven geometric shapes (tans), as shown.

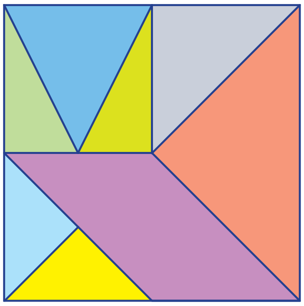
The tangram puzzle is precisely constructed using vertices, midpoints and straight edges.



a Express each of the separate tan pieces as a percentage, a fraction and a decimal amount of the entire puzzle.

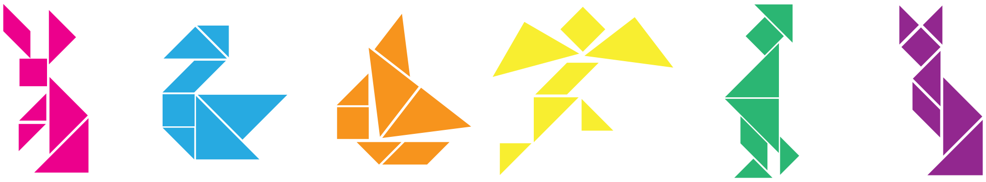
b Check your seven tans add up to a total of 100%.

c Starting with a square, design a new version of a ‘modern’ tangram puzzle. You must have at least six pieces in your puzzle. An example of a modern puzzle is shown.



d Express each of the separate pieces of your new puzzle as a percentage, a fraction and a decimal amount of the entire puzzle.

e Separate pieces of tangrams can be arranged to make more than 300 creative shapes and designs, some of which are shown below. You may like to research tangrams and attempt to make some of the images.



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4G Finding a percentage and expressing as a percentage

Showing values in percentages ‘standardises’ the value and makes it easier for comparisons to be made. For example, Huen’s report card could be written as marks out of each total or in percentages:

French test $\frac{14}{20}$
German test $\frac{54}{75}$

French test 70%
German test 72%

It is clear that Huen’s German score was the stronger result. Comparison is easier when proportions or fractions are written as percentages (equivalent fractions with denominator of 100). Expressing one number as a percentage of another number is the technique covered in this section.

Another common application of percentages is to find a certain percentage of a given quantity. Throughout your life you will come across many examples in which you need to calculate percentages of a quantity. Examples include retail discounts, interest rates, personal improvements, salary increases, commission rates and more.

Let’s start: What percentage has passed?

Answer the following questions.

- What percentage of your day has passed?
- What percentage of the current month has passed?
- What percentage of the current season has passed?
- What percentage of your school year has passed?
- What percentage of your school education has passed?
- If you live to an average age, what percentage of your life has passed?
- When you turned 5, what percentage of your life was 1 year?
- When you are 40, what percentage of your life will 1 year be?

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Key ideas

- **To express one quantity as a percentage of another**
  1. Write a fraction with the ‘part amount’ as the numerator and the ‘whole amount’ as the denominator.
  2. Convert the fraction to a percentage by multiplying by 100%.
     For example: Express a test score of 14 out of 20 as a percentage.
     14 is the ‘part amount’ that you want to express as a percentage out of 20, which is the ‘whole amount’.

$$\frac{14}{20} \times 100\% = \frac{14}{{}_{1}\cancel{20}} \times \frac{{}^{5}\cancel{100}}{1}\% = 70\% \qquad \text{Alternatively, } \frac{14}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{70}{100} = 70\%.$$

$$\therefore \frac{14}{20} = 70\%$$

- **To find a certain percentage of a quantity**
  1. Express the required percentage as a fraction.
  2. Change the ‘of’ to a multiplication sign.
  3. Express the number as a fraction.
  4. Follow the rules for multiplication of fractions.
     For example: Find 20% of 80.

$$20\% \text{ of } 80 = \frac{20}{100} \times \frac{80}{1} = \frac{{}^{4}\cancel{20}}{{}_{1}\cancel{5}\,\cancel{100}} \times \frac{{}^{4}\cancel{80}}{1} = 16 \qquad \text{Alternatively, } 20\% \text{ of } 80 = \frac{1}{5} \times 80$$

$$= 80 \div 5$$

$$= 16$$

Example 22 Expressing one quantity as a percentage of another

Express each of the following as a percentage.

a 34 out of 40 **b** 13 out of 30

SOLUTION	EXPLANATION
a $\frac{34}{40} \times \frac{100}{1}\% = \frac{17}{20} \times \frac{100}{1}\%$	Write as a fraction, with the first quantity as the numerator and second quantity as the denominator.
$= \frac{17}{1} \times \frac{5}{1}\%$	Multiply by 100%.
$= 85\%$	Cancel and simplify.
b $\frac{13}{30} \times \frac{100}{1}\% = \frac{13}{3} \times \frac{10}{1}\%$	Write quantities as a fraction and multiply by 100%.
$= \frac{130}{3}\%$	Cancel and simplify.
$= 43\frac{1}{3}\%$ or $43.\dot{3}\%$	Express the percentage as a mixed numeral or a recurring decimal.

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Example 23 Converting units before expressing as a percentage

Express:

a 60 cents as a percentage of \$5

b 2 km as a percentage of 800 m

SOLUTION

a $$\frac{60}{500} \times \frac{100}{1}\% = \frac{60}{5}\%$$
$$= 12\%$$

b $$\frac{2000}{800} \times \frac{100}{1}\% = \frac{2000}{8}\%$$
$$= 250\%$$

EXPLANATION

a Convert \$5 to 500 cents.

Write quantities as a fraction and multiply by 100%.

Cancel and simplify.

Alternatively, $\frac{60}{500} = \frac{6}{50} = \frac{12}{100} = 12\%$.

b Convert 2 km to 2000 m.

Write quantities as a fraction and multiply by 100%.

Cancel and simplify.

Alternatively, $\frac{2000}{800} = \frac{200}{8} = \frac{50}{2} = \frac{25}{10} = \frac{250}{100} = 250\%$.

Example 24 Finding a certain percentage of a quantity

Find:

a 25% of 128

b 155% of 60

SOLUTION

a $$25\% \text{ of } 128 = \frac{25}{100} \times \frac{128}{1}$$
$$= \frac{1}{4} \times \frac{128}{1} = 32$$

b $$155\% \text{ of } 60 = \frac{155}{100} \times \frac{60}{1}$$
$$= \frac{155}{5} \times \frac{3}{1}$$
$$= \frac{31}{1} \times \frac{3}{1} = 93$$

EXPLANATION

a Write the percentage as a fraction over 100.

Cancel and simplify.

Alternatively, 25% of 128 $= \frac{1}{4}$ of $128 = 128 \div 4 = 32$.

b Write the percentage as a fraction over 100.

Cancel and simplify.

Exercise 4G

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 6, 9–10(½)	3–10(½)	6–11(½)

1 The correct working line to express 42 as a percentage of 65 is:

A $\frac{42}{100} \times 65\%$ **B** $\frac{65}{42} \times 100\%$ **C** $\frac{100}{42} \times 65\%$ **D** $\frac{42}{65} \times 100\%$

2 The correct working line to find 42% of 65 is:

A $\frac{42}{100} \times 65$ **B** $\frac{65}{42} \times 100$ **C** $\frac{100}{42} \times 65$ **D** $\frac{42}{65} \times 100$

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Example 22

3 Express each of the following as a percentage.

a 20 out of 25 **b** 13 out of 20 **c** 39 out of 50
d 12 out of 40 **e** 26 out of 130 **f** 44 out of 55
g 24 out of 60 **h** 6 out of 16 **i** 11 out of 30
j 85 out of 120 **k** 17 out of 24 **l** 34 out of 36

4 Express the first quantity as a percentage of the second quantity, giving answers in fractional form where appropriate.

a 3, 10 **b** 9, 20 **c** 25, 80 **d** 15, 18
e 64, 40 **f** 82, 12 **g** 72, 54 **h** 200, 75

5 Express the first quantity as a percentage of the second quantity, giving answers in decimal form, correct to two decimal places where appropriate.

a 2, 24 **b** 10, 15 **c** 3, 7 **d** 18, 48
e 56, 35 **f** 15, 12 **g** 9, 8 **h** 70, 30

Example 23

6 Express:

a 40 cents as a percentage of \$8
b 50 cents as a percentage of \$2
c 3 mm as a percentage of 6 cm
d 400 m as a percentage of 1.6 km
e 200 g as a percentage of 5 kg
f 8 km as a percentage of 200 m
g 1.44 m as a percentage of 48 cm
h \$5.10 as a percentage of 85 cents

7 Express each quantity as a percentage of the total.

a 28 laps of a 50-lap race completed
b saved \$450 towards a \$600 guitar
c 172 fans in a train carriage of 200 people
d level 7 completed of a 28-level video game
e 36 students absent out of 90 total
f 14 km mark of a 42 km marathon

8 Copy and complete the following sentences.

a Finding 1% of a quantity is the same as dividing the quantity by ______.
b Finding 10% of a quantity is the same as dividing the quantity by ______.
c Finding 20% of a quantity is the same as dividing the quantity by ______.
d Finding 50% of a quantity is the same as dividing the quantity by ______.

Example 24a

9 Find:

a 50% of 36 **b** 20% of 45 **c** 25% of 68 **d** 32% of 50
e 5% of 60 **f** 2% of 150 **g** 14% of 40 **h** 70% of 250
i 15% of 880 **j** 45% of 88 **k** 80% of 56 **l** 92% of 40

Example 24b

10 Find:

a 130% of 10 **b** 200% of 40 **c** 400% of 25 **d** 155% of 140
e 125% of 54 **f** 320% of 16 **g** 105% of 35 **h** 118% of 60

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11 Find:

a 20% of 90 minutes
b 15% of $5
c 30% of 150 kg
d 5% of 1.25 litres
e 40% of 2 weeks
f 75% of 4.4 km

PROBLEM-SOLVING AND REASONING	12, 13, 17	13–15, 17, 18	14–16, 18–20

12 Find:

a $33\frac{1}{3}$% of 16 litres of orange juice
b $66\frac{2}{3}$% of 3000 marbles
c $12\frac{1}{2}$% of a $64 pair of jeans
d 37.5% of 120 donuts

13 In a survey, 35% of respondents said they felt a penalty was too lenient and 20% felt it was too harsh. If there were 1200 respondents, how many felt the penalty was appropriate?

14 Four students completed four different tests. Their results were:
Maeheala: 33 out of 38 marks
Wasim: 16 out of 21 marks
Francesca: 70 out of 85 marks
Murray: 92 out of 100 marks
Rank the students in decreasing order of test percentage.

15 Jasper scored 22 of his team's 36 points in the basketball final. What percentage of the team's score did Jasper shoot? Express your answer as a fraction and as a recurring decimal.

16 Due to illness, Vanessa missed 15 days of the 48 school days in Term 1. What percentage of classes did Vanessa attend in Term 1?

17 Eric scored 66% on his most recent Mathematics test. He has studied hard and is determined to improve his score on the next topic test, which will be out of 32 marks. What is the least number of marks Eric can score to improve on his previous test score?

18 Calculate 40% of $60 and 60% of $40. What do you notice? Can you explain your findings?

19 Which of the following alternatives would calculate a% of b?

A $\frac{100}{ab}$
B $\frac{ab}{100}$
C $\frac{a}{100b}$
D $\frac{100a}{b}$

20 Which of the following alternatives would express x as a percentage of y?

A $\frac{100}{xy}$
B $\frac{xy}{100}$
C $\frac{x}{100y}$
D $\frac{100x}{y}$

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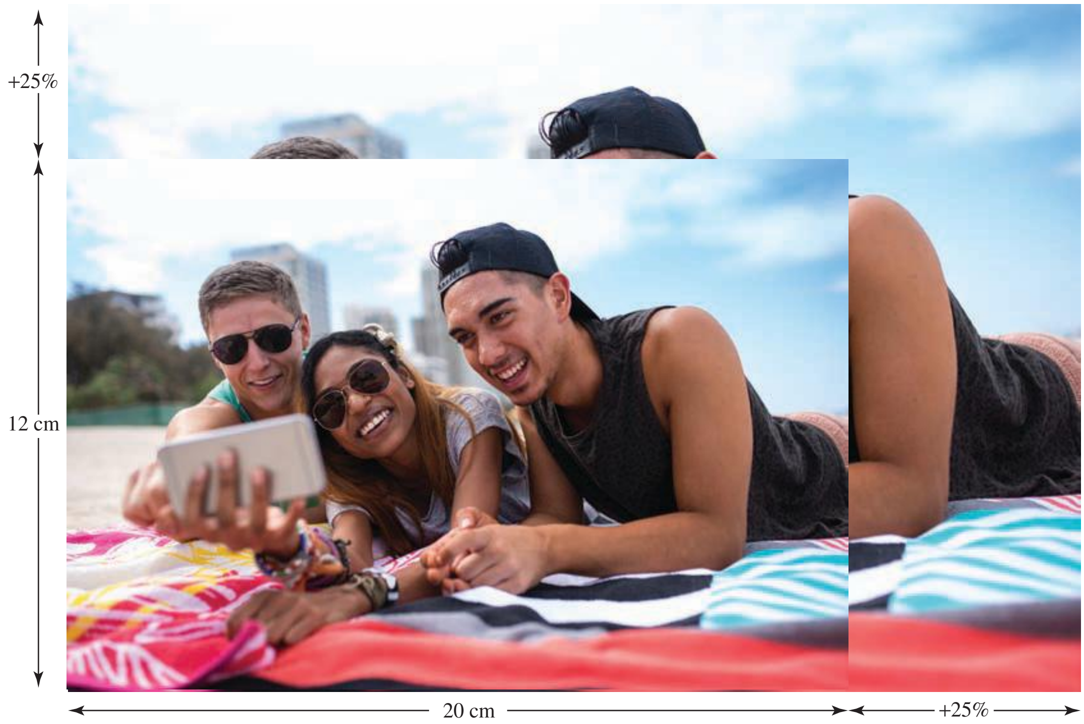
ENRICHMENT	–	–	21

Two-dimensional percentage increases



21 A rectangular photo has dimensions 12 cm by 20 cm.

a What is the area of the photo in cm^2?

The dimensions of the photo are increased by 25%.

b What is 25% of 20 cm?

c What are the new dimensions of the photo?

d What is the new area of the photo in cm^2?

e What is the increase in the area of the photo?

f What is the percentage increase in the area of the photo?

g What effect did a 25% increase in dimensions have on the area of the photo?

h Can you think of a quick way of calculating the percentage increase in the area of a rectangle for a given increase in each dimension?

i What would be the percentage increase in the area of a rectangle if the dimensions were:

- **i** increased by 10%?
- **ii** increased by 20%?
- **iii** increased by $33\frac{1}{3}\%$?
- **iv** increased by 50%?

You might like to draw some rectangles of particular dimensions to assist your understanding of the increase in percentage area.

j What percentage increase in each dimension would you need to exactly double the area of the rectangle?

k You might like to explore the percentage increase in the volume of a three-dimensional shape when each of the dimensions is increased by a certain percentage.

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4H Decreasing and increasing by a percentage

Percentages are regularly used when dealing with money. Here are some examples.

- Decreasing an amount by a percentage (Discount)
  All items in the store are reduced by 30% for the three-day sale.
  The value of the car is depreciating at a rate of 18% per annum.
- Increasing an amount by a percentage (Mark-up)
  A retail shop marks up items by 25% of the wholesale price.
  The professional footballer's new contract was increased by 40%.

When dealing with questions involving money, you generally round your answers to the nearest cent. Do this by rounding, correct to two decimal places. For example: \$356.4781 rounds to \$356.48 (suitable if paying by credit card).

As our smallest coin is the 5 cent coin, on many occasions it will be appropriate to round your answer to the nearest 5 cents. For example: \$71.12 rounds to \$71.10 (suitable if paying by cash).

Let's start: Original value $\pm$ % change $=$ new value

The table below consists of three columns: the original value of an item, the percentage change that occurs and the new value of the item. However, the data in each of the columns have been mixed up. Your challenge is to rearrange the data in the three columns so that each row is correct.

This is an example of a correct row.

Original value	Percentage change	New value
\$65.00	Increase by 10%	\$71.50

Rearrange the values in each column in this table so that each row is correct.

Original value	Percentage change	New value
\$102.00	Increase by 10%	\$73.50
\$80.00	Increase by 5%	\$70.40
\$58.00	Decreased by 2%	\$76.50
\$64.00	Decrease by 25%	\$78.40
\$70.00	Increase by 30%	\$73.80
\$82.00	Decrease by 10%	\$75.40

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- **Decreasing by a percentage**

  The original price is 100%.

  If the decrease is 5%, the new value is 95% of the original value.

  Words used to describe this are: *discount, sale, mark-down, decrease, loss, deflation* and *depreciation.*

  There are two ways to decrease by 5%:

Method 1:	Method 2:
Calculate 5%.	Subtract 5% from 100% to give 95%.
Subtract that amount from the value.	Calculate 95% of the value.

- **Increasing by a percentage**

  The original price is 100%.

  If the increase is 5%, the new value is 105% of the original value.

  Words used to describe this are: **mark-up, increase, profit, inflation, surcharge** and **appreciation**.

Method 1:	Method 2:
Calculate 5%.	Add 5% to 100% to give 105%.
Add that amount to the value.	Calculate 105% of the value.

- **In the retail industry:**
  - **Cost price** is the amount the retailer or shopkeeper pays for the goods.
  - **Selling price, marked-up price** or **retail price** is the advertised amount for which the customer can purchase the goods.
  - **Discounted price** or **marked-down price** is an amount that the retailer is willing to accept.
  - **Profit (or loss)** is the difference between the cost price and the price for which the item is sold.

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Example 25 Finding new values

Find the new value when:

a \$160 is increased by 40% **b** \$63 is decreased by 20%

SOLUTION	EXPLANATION
a $40\%\text{ of }\$160 = \frac{40}{100} \times \frac{160}{1} = \$64$	Calculate 40% of \$160. Cancel and simplify.
New price $= \$160 + \$64$ $= \$224$	New price = original price + increase
Alternative method:	
$100\% + 40\% = 140\% = 1.4$ $\$160 \times 1.4 = \$224$	The new value is 140% of the old value.
b $20\%\text{ of }\$63 = \frac{20}{100} \times \frac{63}{1} = \$12.60$	Calculate 20% of \$63. Cancel and simplify.
New price $= \$63 - \$12.60$ $= \$50.40$	New price = original price − decrease
Alternative method: $100\% - 20\% = 80\% = 0.8$ $\$63 \times 0.8 = \$50.40$	

Example 26 Calculating discounts or mark-ups

a Find the cost of a \$860 television that has been discounted by 25%.

b Find the cost of a \$250 microwave oven that has been marked up by 12%.

SOLUTION	EXPLANATION
a Discount $= 25\%\text{ of }\$860$ $= \frac{25}{100} \times \frac{860}{1} = \$215$	Calculate 25% discount. Cancel and simplify.
Selling price $= \$860 - \$215$ $= \$645$	Selling price = cost price − discount
Alternative method: $100\% - 25\% = 75\% = 0.75$ $\$860 \times 0.75 = \$645$	
b Mark-up $= 12\%\text{ of }\$250$ $= \frac{12}{100} \times \frac{250}{1} = \$30$	Calculate 12% of \$250. Cancel and simplify.
Selling price $= \$250 + \$30$ $= \$280$	Selling price = cost price + mark-up
Alternative method: $100\% + 12\% = 112\% = 1.12$ $\$250 \times 1.12 = \$280$	

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Exercise 4H

UNDERSTANDING AND FLUENCY	1–4, 5–7(½)	4, 5–7(½), 8	6–7(½), 8

1 Calculate the new price when:
- **a** an item marked at \$15 is discounted by \$3
- **b** an item marked at \$25.99 is marked up by \$8
- **c** an item marked at \$17 is reduced by \$2.50
- **d** an item marked at \$180 is increased by \$45

2 A toy store is having a sale in which everything is discounted by 10% of the recommended retail price (RRP). A remote-control car is on sale and has a RRP of \$120.
- **a** Calculate the discount on the car (i.e. 10% of \$120).
- **b** Calculate the selling price of the car (i.e. RRP – discount).

3 Shop A is advertising a watch for \$350 with a discount of \$80. Shop B is advertising the same watch for \$400 with a 30% discount. Calculate the selling price of the watch in each shop.

4 Calculate the new price when:
- **a** an item marked at \$80 is discounted by 50%
- **b** an item marked at \$30 is marked up by 20%
- **c** an item marked at \$45 is reduced by 10%
- **d** an item marked at \$5 is increased by 200%

Example 25

5 Find the new value when:
- **a** \$400 is increased by 10%
- **b** \$240 is increased by 15%
- **c** \$80 is decreased by 20%
- **d** \$42 000 is decreased by 2%
- **e** \$5000 is increased by 8%
- **f** \$60.60 is increased by 60%
- **g** \$15 is decreased by 10%
- **h** \$84 is decreased by 40%

Example 26

6 Find the cost of the following.
- **a** a \$600 television that has been discounted by 20%
- **b** a \$150 ripstik that has been reduced by 15%
- **c** a \$52 jumper that has depreciated by 25%
- **d** an \$80 framed Pink poster that has been marked up by 30%
- **e** a \$14 meal that has been increased by 10%
- **f** a \$420 stereo that has been marked up by 50%

7 Calculate the selling prices of the following items if they are to be reduced by 25%.
- **a** \$16 thongs
- **b** \$32 sunhat
- **c** \$50 sunglasses
- **d** \$85 swimsuit
- **e** \$130 boogie board
- **f** \$6.60 surfboard wax

8 Using a calculator, calculate the selling prices of the following items.
- **a** a \$450 Blu-ray player reduced by 12%
- **b** a \$12 990 second-hand car reduced by 15%
- **c** a \$675 surfboard marked up by 24%
- **d** a \$725 000 home with an additional 3% selling fee

PROBLEM-SOLVING AND REASONING	9, 10, 14	10–12, 14, 15	11–15

9 Shop C and shop D purchase Extreme Game packages at a cost price of $60.
Shop C has a mark-up of $20 for retailers and shop D has a mark-up of 25%.
Calculate the selling price for the Extreme Game package at each shop.

10 A retail rug store marks up all items by 25% of the wholesale price. The wholesale price of a premier rug is $200 and for a luxury rug is $300. What is the customer price for the two different types of rugs?

11 A bookstore is offering a discount of 10%. Jim wants to buy a book with a RRP of $49.90. How much will it cost him if he pays by cash? How much will it cost him if he pays by credit card?

12 Shipshape stores are having a sale and reducing all items by 20%. Gerry purchases the following items, which still have their original price tags: jeans $75, long-sleeved shirt $94, T-shirt $38 and shoes $125. What is Gerry's total bill at the sale?

13 At the end of the winter season an outdoors store has a 20% discount on all items in the store. Two weeks later they are still heavily overstocked with ski gear and so they advertise a further 40% off already discounted items. Calculate the new selling price of a pair of ski goggles with a RRP of $175.00.

14 Georgie desperately wants to buy a new mountain bike that has a RRP of $350. She has only $220. Fortunately, her local bike store regularly has sales during which the discounts are multiples of 5%. What is the smallest discount that will enable Georgie to purchase the bike?

15 Patrick wishes to purchase a trail bike and has visited three different stores. For the same model he has received a different deal at each store.
Pete's Trail Bikes has the bike priced at $2400 and will not offer any discount.
Eastern Bikers has the bike priced at $2900 but will offer a 20% discount.
City Trail Bikes has the bike priced at $2750 and will offer a 15% discount.
What is the cheapest price for which Patrick can purchase his trail bike and which store is offering the best deal?

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ENRICHMENT	–	–	16

Commission

16 Many sales representatives are paid by commission. This means their wage is calculated as a certain percentage of their sales. For example, Ben, a car salesman is paid 5% commission. In one week Ben sold three cars for a total value of \$42 000. His wage for that week was 5% of \$42 000 = \$2100. If in the next week Ben sells no cars, he would receive no wage.

a Calculate the following amounts.
  - **i** 10% commission on sales of \$850
  - **ii** 3% commission on sales of \$21 000
  - **iii** 2.5% commission on sales of \$11 000
  - **iv** 0.05% commission on sales of \$700 000

Generally sales representatives can be paid by commission only, by an hourly rate only, or by a combination of an hourly rate and a percentage commission. The combination is common, as it provides workers with the security of a wage regardless of sales, but also the added incentive to boost wages through increased sales.

Solve the following three problems involving commission.

b Stuart sells NRL records. He earns \$8.50 per hour and receives 5% commission on his sales. Stuart worked for 5 hours at the Brisbane Broncos vs Canberra Raiders match and sold 320 records at \$4 each. How much did Stuart earn?

c Sam, Jack and Justine are all on different pay structures at work. Sam is paid an hourly rate of \$18 per hour. Jack is paid an hourly rate of \$15 per hour and 4% commission. Justine is paid by commission only at a rate of 35%.
Calculate their weekly wages if they each worked for 40 hours and each sold \$2000 worth of goods.

d Clara earns an hourly rate of \$20 per hour and 5% commission. Demi earns an hourly rate of \$16 per hour and 10% commission. They each work a 40-hour week. In one particular week, Clara and Demi both sell the same value of goods and both receive the same wage. How much did they sell and what was their wage?

e Do you know anyone who receives a commission as part of their salary? Interview someone and understand the structure of their salary. Would you like to be paid a set salary or like the added incentive of commission? Summarise your findings and write a brief report.

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4I The Goods and Services Tax (GST)

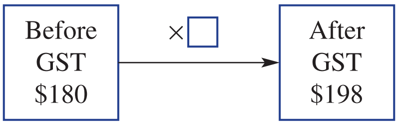
The Goods and Services Tax or GST is a broad-based tax on most goods and services sold or consumed in Australia.

The advertised price of the goods in shops, restaurants and other businesses must include the GST. At present in Australia the GST is set at 10%.

Not all goods and services are taxed under the GST. Items that are exempt from the goods and services tax include: most basic foods, some education courses and some medical and healthcare products and services.

Let's start: GST

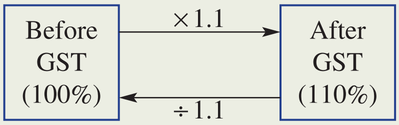
Look at the prices before and after GST is included.



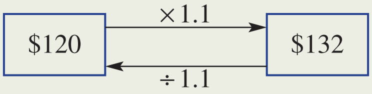
- How much GST was paid?
- What percentage is the GST?
- What number could be placed in the box?
- If the GST was 12%, what number would go in the box?

Key ideas

- The GST is 10% of the usual sale price. It is paid by the consumer and passed onto the government by the supplier.
- The final advertised price, inclusive of the GST, represents 110% of the value of the product: the cost (100%) plus the GST of 10% gives 110%.
- The **unitary method** can be used to find the GST included in the price of an item or the pre-tax price.
  It involves finding the value of 'one unit', usually 1%, then using this information to answer the question.
- Alternatively, this chart could be used:



For example:



Example 27 Calculating the GST

Calculate the GST payable on:

a a table that a manufacturer values at $289

b a bill from a landscape gardener of $2190

SOLUTION	EXPLANATION
a $\frac{10}{100} \times 289 = 28.9$	GST is 10% of the value.
The GST is $28.90.	Find 10% of $289.
b $\frac{10}{100} \times 2190 = 219$ The GST is $219.	Find 10% of $2190.

Example 28 Using the unitary method to find the full amount

A bike has GST of $38 added to its price. What is the price of the bike:

a before the GST? **b** after the GST?

SOLUTION	EXPLANATION
a 10% = 38 1% = 3.8	Divide by 10 to find the value of 1%.
100% = 380 Before the GST was added, the cost of the bike was $380.	Multiply by 100 to find 100%.
b After the GST, the price is $418.	Add the GST to the pre-GST price. $380 + $38 = $418

Example 29 Using the unitary method to find the pre-GST price

The final price at a café, including the 10% GST, is $137.50. What is the pre-GST price of the meal?

SOLUTION	EXPLANATION
110% = 137.50 1% = 1.25 100% = 125	The GST adds 10% to the cost of the meal. 100% + 10% = 110% ∴ Final price is 110%.
The pre-GST cost is $125.	Divide by 110 to find the value of 1%.
Alternative method:	Multiply by 100 to find 100%.
$137.50 ÷ 1.1 = $125	Dividing by 1.1 gives the original price, excluding GST.

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Exercise 4I

UNDERSTANDING AND FLUENCY	1–7	3–4(½), 5, 7–8(½)	5, 7–8(½), 9

1 Without using a calculator, evaluate the following.

a 10% of \$50 **b** 10% of \$160
c 10% of \$250 **d** 10% of \$700
e 10% of \$15 **f** 10% of \$88
g 10% of \$5 **h** 10% of \$2.50

2 Complete this table, without using a calculator.

	Price (no GST)	10% GST	Price (inc. GST)
a	\$100		
b	\$50		
c	\$150		
d	\$5		
e	\$1		
f	\$120		

Example 27

3 Calculate the GST payable on goods priced at:

a \$680 **b** \$4000 **c** \$550
d \$28 **e** \$357 **f** \$5.67

Example 28

4 Calculate the final price, including the GST, on items priced at:

a \$700 **b** \$3000 **c** \$450
d \$34 **e** \$56 700 **f** \$4.90

5 If the GST is \$8, what was the original price?

6 The final advertised price of a pair of shoes is \$99. The GST included in this price is:

A \$9.90 **B** \$90 **C** \$9 **D** \$89.10

Example 29

7 The final price to the consumer includes the 10% GST. Calculate the pre-GST price if the final price is:

a \$220 **b** \$66 **c** \$8800
d \$121 **e** \$110 **f** \$0.99

8 Calculate the pre-GST price if the final cost to the consumer is:

a \$352 **b** \$1064.80 **c** \$506
d \$52.25 **e** \$10 791 **f** \$6.16

9 Copy and complete the table.

Pre-GST price	10% GST	Final cost including the 10% GST
\$599		
	\$68	
	\$70	
		\$660
\$789		
	\$89.20	
		\$709.50
		\$95.98

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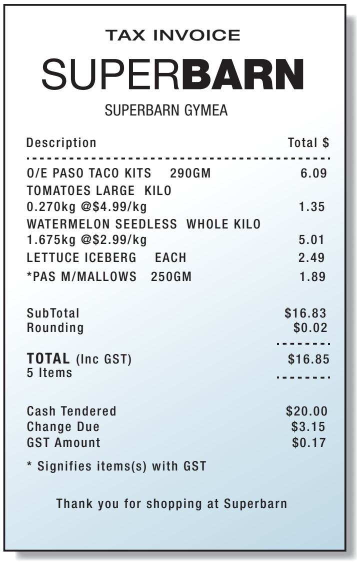
PROBLEM-SOLVING AND REASONING	10, 13	10, 11, 13, 14	12–15

10 Here are three real-life receipts (with the names of shops changed). GST rate is 10%. Answer the following based on each one.

Superbarn

a How much was spent at Superbarn?
b How many kilograms of tomatoes were bought?
c Which item included the GST, and how do you tell by looking at the receipt?
d What was the cost of the item if the GST is not included?



TAX INVOICE

SUPERBARN

SUPERBARN GYMEA

Description	Total $
O/E PASO TACO KITS 290GM	6.09
TOMATOES LARGE KILO 0.270kg @$4.99/kg	1.35
WATERMELON SEEDLESS WHOLE KILO 1.675kg @$2.99/kg	5.01
LETTUCE ICEBERG EACH	2.49
*PAS M/MALLOWS 250GM	1.89
SubTotal	$16.83
Rounding	$0.02
TOTAL (Inc GST) 5 Items	$16.85
Cash Tendered	$20.00
Change Due	$3.15
GST Amount	$0.17

* Signifies items(s) with GST

Thank you for shopping at Superbarn

Gymea Fruit Market

a What was the cost of bananas per kilogram?
b On what date was the purchase made?
c What does ROUND mean?
d What was the total paid for the items?
e How much tax was included in the bill?
f What percentage of the bill was the tax?

GYMEA FRUIT MARKET

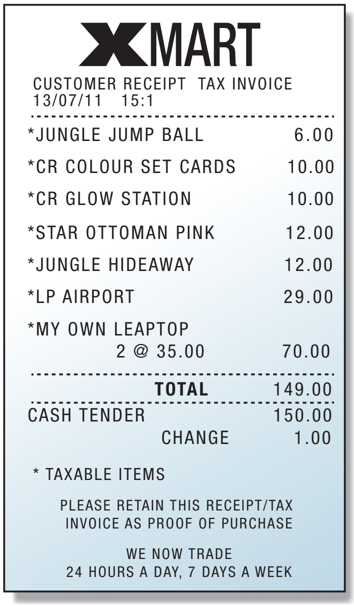
HAVE A NICE DAY

DATE 05/07/2011 TUES TIME 11:21

0.090 KG @ $14.99/kg BANANA SUGAR	$1.35
SL MUSHROOM	$2.49
PISTACCHIO 11	$6.00
ROUND	$0.01
TOTAL	$9.85
CASH	$9.85
TAX 1	$0.55

Xmart

a How many toys were purchased?
b What was the cost of the most expensive item?
c Which of the toys attracted GST?
d How much GST was paid in total?
e What percentage of the total bill was the GST?



XMART

CUSTOMER RECEIPT TAX INVOICE
13/07/11 15:1

*JUNGLE JUMP BALL	6.00
*CR COLOUR SET CARDS	10.00
*CR GLOW STATION	10.00
*STAR OTTOMAN PINK	12.00
*JUNGLE HIDEAWAY	12.00
*LP AIRPORT	29.00
*MY OWN LEAPTOP 2 @ 35.00	70.00
TOTAL	149.00
CASH TENDER	150.00
CHANGE	1.00

* TAXABLE ITEMS

PLEASE RETAIN THIS RECEIPT/TAX
INVOICE AS PROOF OF PURCHASE

WE NOW TRADE
24 HOURS A DAY, 7 DAYS A WEEK

11 A plumber's quote for installing a dishwasher is $264, including the GST.

a Use the unitary method to calculate the GST included in this price.
b Calculate the pre-GST price for installing the dishwasher.
c Divide $264 by 11. What do you notice? Explain why this works.
d Divide $264 by 1.1. Why does this give the same answer as part **b**?

12 Use the technique outlined in question **11** to find the GST already paid on goods and services costing:

a $616 **b** $1067 **c** $8679 **d** $108.57

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13 The cost of a lounge suite is $990 and includes $90 in GST. Find 10% of $990 and explain why it is more than the GST included in the price.

14 Explain why adding 10% GST and then applying a 10% discount does not give you the pre-GST price.

15 Other countries around the world also have a GST. In Canada the GST is 5%, in New Zealand it is 15% and in Singapore it is 7%.
Explain how you could find the pre-GST price on items sold in each of these countries.

ENRICHMENT	–	–	16

GST and the manufacturing process

The final consumer of the product pays GST. Consider the following.

A fabric merchant sells fabric at $66 (including the 10% GST) to a dressmaker. The merchant makes $60 on the sale and passes the $6 GST onto the Australian Tax Office (ATO).

A dressmaker uses the fabric and sells the product onto a retail store for $143 (this includes the $13 GST). As the dressmaker has already paid $6 in GST when they bought the fabric, they have a GST tax credit of $6 and they pass on the $7 to the ATO.

The retailer sells the dress for $220, including $20 GST. As the retailer has already paid $13 in GST to the dressmaker, they are required to forward the extra $7 to the ATO.

The consumer who buys the dress bears the $20 GST included in the price.

16 Copy and complete the following.

Raw material $110 (includes the 10% GST)

GST on sale = ________
GST credit = $0
Net GST to pay = ________

Production stage $440 (includes the 10% GST)

GST on sale = ________
GST credit = ________
Net GST to pay = ________

Distribution stage $572 (includes the 10% GST)

GST on sale = ________
GST credit = ________
Net GST to pay = ________

Retail stage $943.80 (includes the 10% GST)

GST on sale = ________
GST credit = ________
Net GST to pay = ________

GST paid by the final consumer = ______________

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4J Calculating percentage change, profit and loss

If you sell something for more than you paid for it, you have made a profit. On the other hand, if you sell something for less than you paid for it, you have made a loss.

Retailers, business people and customers are all interested to know the percentage profit or the percentage loss that has occurred. In other words, they are interested to know about the percentage change. The percentage change provides valuable information over and above the simple value of the change.

For example:

Hat was $40
Discount $5
Now $35

Cap was $8
Discount $5
Now $3

The change for each situation is exactly the same, a $5 discount. However, the percentage change is very different.

Hat was $40. Discount 12.5%. Now $35.

Cap was $8. Discount 62.5%. Now $3.

For the sunhat, there is a 12.5% change, and for the cap there is a 62.5% change. In this case, the percentage change would be known as a **percentage discount**.

Let's start: Name the acronym

What is an acronym?

How many of the following nine business and finance-related acronyms can you name?

- RRP
- CBD
- GDP
- GST
- COD
- IOU
- ATM
- EFTPOS
- ASX

Can you think of any others?

How about the names of two of the big four Australian banks: NAB and ANZ?

How do these acronyms relate to percentages?

The following three acronyms are provided for fun and interest only. Do you know what they stand for?

- SCUBA diving
- LASER gun
- BASE jumping

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Key ideas

- **Profit** = selling price − cost price
- **Loss** = cost price − selling price
- Calculating a percentage change involves the technique of expressing one quantity as a percentage of another (see Section 4G).

$$\textbf{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100\%$$

$$\textbf{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$$

$$\textbf{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$$

Example 30 Calculating percentage change

Calculate the percentage change (profit/loss) when:

a \$25 becomes \$32

b \$60 becomes \$48

SOLUTION	EXPLANATION
a Profit = \$7	Profit = \$32 − \$25
$\% \text{ Profit} = \frac{7}{25} \times \frac{100}{1}\%$ $= 28\%$	$\text{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$
Alternatively, $\frac{7}{25} = \frac{28}{100} = 28\%$.	
b Loss = \$12	Loss = \$60 − \$48
$\% \text{ Loss} = \frac{12}{60} \times \frac{100}{1}\%$ $= 20\%$	$\text{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$
Alternatively, $\frac{12}{60} = \frac{1}{5} = \frac{20}{100} = 20\%$.	

Example 31 Solving worded problems

Ross buys a ticket to a concert for \$125, but later is unable to go. He sells it to his friend for \$75. Calculate the percentage loss Ross made.

SOLUTION	EXPLANATION
Loss = \$125 − \$75 = \$50	Loss = cost price − selling price
$\% \text{ Loss} = \frac{50}{125} \times \frac{100}{1}\%$ $= 40\%$	$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$
Ross made a 40% loss on the concert ticket.	

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Exercise 4J

UNDERSTANDING AND FLUENCY	1–4, 5(½), 6, 7	3, 4, 5(½), 6, 7	6, 7

1 Calculate the profit made in each of the following situations.

- **a** Cost price = \$14, Sale price = \$21
- **b** Cost price = \$75, Sale price = \$103
- **c** Cost price = \$25.50, Sale price = \$28.95
- **d** Cost price = \$499, Sale price = \$935

2 Calculate the loss made in each of the following situations.

- **a** Cost price = \$22, Sale price = \$9
- **b** Cost price = \$92, Sale price = \$47
- **c** Cost price = \$71.10, Sale price = \$45.20
- **d** Cost price = \$1121, Sale price = \$874

3 Which of the following is the correct formula for working out percentage change?

- **A** $\% \text{ Change} = \dfrac{\text{change}}{\text{original value}}$
- **B** $\% \text{ Change} = \dfrac{\text{original value}}{\text{change}} \times 100\%$
- **C** $\% \text{ Change} = \text{change} \times 100\%$
- **D** $\% \text{ Change} = \dfrac{\text{change}}{\text{original value}} \times 100\%$

4 The formula used for calculating percentage loss is: $\% \text{ Loss} = \dfrac{\text{loss}}{\text{cost price}} \times 100\%$.

Which of the following would therefore be the formula for calculating the percentage discount?

- **A** $\% \text{ Discount} = \dfrac{\text{discount}}{\text{cost price}} \times 100\%$
- **B** $\% \text{ Discount} = \dfrac{\text{cost price}}{\text{discount}} \times 100\%$
- **C** $\% \text{ Discount} = \text{discount} \times 100\%$
- **D** $\% \text{ Discount} = \dfrac{\text{discount}}{100\%} \times \text{cost price}$

Example 30

5 Find the percentage change (profit or loss) when:

- **a** \$20 becomes \$36
- **b** \$10 becomes \$13
- **c** \$40 becomes \$30
- **d** \$25 becomes \$21
- **e** \$12 becomes \$20
- **f** \$8 becomes \$11
- **g** \$6 becomes \$4
- **h** \$150 becomes \$117

6 Find the percentage change (increase or decrease) when:

- **a** 15°C becomes 18°C
- **b** 18°C becomes 15°C
- **c** 4°C becomes 24°C
- **d** 12°C becomes 30°C

7 Find the percentage change in population when:

- **a** a town of 4000 becomes a town of 5000
- **b** a city of 750 000 becomes a city of 900 000
- **c** a country of 5 000 000 becomes a country of 12 000 000

PROBLEM-SOLVING AND REASONING	8, 9, 13	8–11, 13, 14	10–12, 14, 15

Example 31

8 Gari buys a ticket to a concert for $90, but is unable to go. She sells it to her friend for $72. Calculate the percentage loss Gari made.

9 Estelle purchased a piece of sporting memorabilia for $120. Twenty years later she sells it for $900. Calculate the percentage profit Estelle has made.

10 Xavier purchased materials for $48 and made a dog kennel. He later sold the dog kennel for $84.

a Calculate the profit Xavier made.

b Calculate the percentage profit Xavier made.

11 Gemma purchased a $400 foal, which she later sold for $750.

a Calculate the profit Gemma made.

b Calculate the percentage profit Gemma made.

12 Lee-Sen purchased a $5000 car, which she later sold for $2800.

a Calculate Lee-Sen's loss.

b Calculate Lee-Sen's percentage loss.

13 Explain why an increase of 10% followed by a decrease of 10% on the new price gives an answer that is less than the original amount.

14 Which of the following stores is offering a larger percentage discount?
Store A: Jeans originally selling for $60, now on sale for $51
Store B: Jeans originally selling for $84, now on sale for $73.50

15 The circulation of a student newspaper at Burrough High School was 60 copies in Term 1, 120 copies in Term 2, 200 copies in Term 3 and 360 copies in Term 4.

a Calculate the percentage growth in circulation between:

- **i** Term 1 and Term 2
- **ii** Term 2 and Term 3
- **iii** Term 3 and Term 4

b In which term was there the greatest percentage growth in circulation?

c What was the overall percentage growth in circulation over the course of the year?

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ENRICHMENT	–	–	16

Australia's population

16 Australia's population in early 2010 was 22 003 926 and the percentage population growth was 1.8% per year.

a Given this population size and percentage growth, how many more Australians would there be after 1 year?

b How many would there be after:

i 2 years? **ii** 5 years? **iii** 10 years?

The Australian Bureau of Statistics carries out comprehensive population projections. One such projection assumes the following growth rates.

- One birth every 1 minute and 44 seconds.
- One death every 3 minutes and 39 seconds.
- A net gain of one international migrant every 1 minute and 53 seconds.
- An overall total population increase of one person every 1 minute and 12 seconds.

c Calculate the percentage growth per annum, using the 2010 population of 22 003 926 and the projected total population increase of one person every 1 minute and 12 seconds. Give your answer correct to one decimal place.

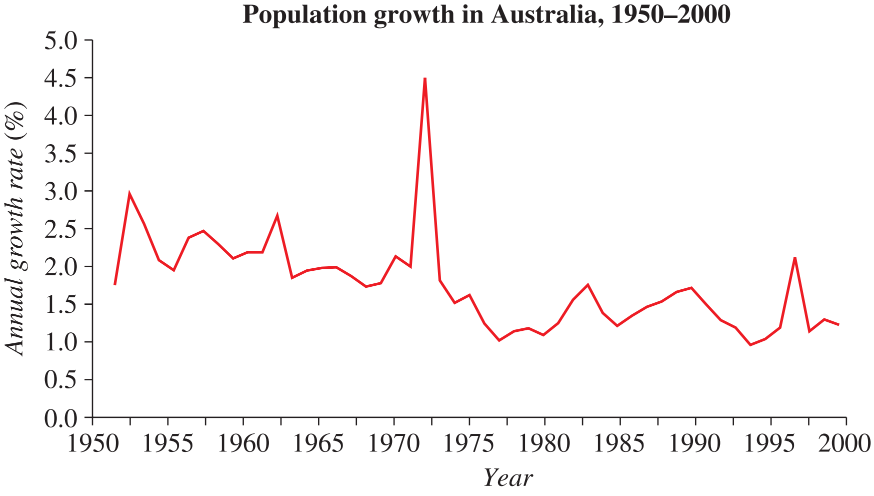
d Find out the current population of Australia and the population 10 years ago, and work out the percentage growth over the past decade.

e How does Australia's percentage growth compare with that of other countries?

f Find out the current life expectancy of an Australian male and female. Find out their life expectancy 50 years ago. Calculate the percentage increase in life expectancy over the past 50 years.

g A key factor in population growth is the total fertility rate (TFR) (i.e. the number of babies per woman). For Australia the TFR in 2010 was at 1.74 expected births per woman. The peak of Australia's TFR over the past 100 years was 3.6 children per woman in 1961. What is the percentage decrease in TFR from 1961 to 2010?

h Carry out some of your own research on Australia's population.



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4K Solving percentage problems with the unitary method and equations

In the GST section, we did calculations like this:

$$\begin{aligned} 10\% &= 38 \\ 1\% &= 3.8 \\ 100\% &= 380 \end{aligned}$$

Once we know what 1% is worth, we can find any percentage amount. This is called the *unitary method*.

Let's start: Using the unitary method

By first finding the value of '1 unit', answer the following questions using mental arithmetic only.

1 Four tickets to a concert cost $100. How much will three tickets cost?
2 Ten workers can dig 40 holes in an hour. How many holes can seven workers dig in an hour?
3 Six small pizzas cost $54. How much would 10 small pizzas cost?
4 If eight pairs of socks cost $64, how much would 11 pairs of socks cost?
5 Five passionfruit cost $2.00. How much will nine passionfruit cost?
6 If a worker travels 55 km in five trips from home to the worksite, how far will they travel in seven trips?

Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to answer the question.
- When dealing with percentages, finding 'one unit' corresponds to finding one per cent (1%).
- Once the value of 1% of an amount is known, it can be multiplied to find the value of any desired percentage.

Example 32 Using the unitary method to find the full amount

If 8% of an amount of money is $48, what is the full amount of money?

SOLUTION	EXPLANATION
$\div 8 \; 8\%$ of amount is \$48. $\div 8$	
1% of amount is \$6.	Divide by 8 to find the value of 1%.
$\times 100 \; 100\%$ of amount is \$600. $\times 100$	Multiply by 100 to find the value of 100%.
Full amount of money is \$600.	
Alternative solution:	
$8\% \text{ of } A = \$48$	Form an equation.
$\div 0.08 \; 0.08 \times A = 48 \; \div 0.08$	
$A = 48 \div 0.08$	Divide both sides by 0.08.
$A = 600$	

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Example 33 Using the unitary method to find a new percentage

If 11% of the food bill was $77, how much is 25% of the food bill?

SOLUTION	EXPLANATION
11% of food bill is $77.	
÷11 ÷11	
∴ 1% of food bill is $7.	Divide by 11 to find the value of 1%.
×25 ×25	
∴ 25% of food bill is $175.	Multiply by 25 to find the value of 25%.
Alternative solution:	
$11\% \text{ of B} = \$77$	
$0.11 \times \text{B} = 77$	Form an equation.
$\text{B} = 77 \div 0.11$	Solve the equation.
$\text{B} = 700$	
$0.25 \times \text{B} = 175$	We need 25% of B.

Example 34 Using the unitary method to find the original price

A pair of shoes has been discounted by 20%. If the sale price is $120, what was the original price of the shoes?

SOLUTION	EXPLANATION
Only paying 80% of original price:	20% discount, so paying (100 – 20)%.
∴ 80% of original price is $120.	Divide by 80 to find the value of 1%.
÷80 ÷80	
∴ 1% of original price is $1.50.	
×100 ×100	
∴ 100% of original price is $150.	Multiply by 100 to find the value of 100%.
The original price of the shoes was $150.	
Alternative solution:	
$80\% \text{ of P} = \$120$	
$0.8 \times \text{P} = 120$	Form an equation.
$\text{P} = 120 \div 0.8$	Solve the equation.
$\text{P} = 150$	

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Exercise 4K

UNDERSTANDING AND FLUENCY	1–8	2–4, 5(½), 6–9	4, 5(½), 6, 7(½), 8, 9

1 If 10% of an amount of money is $75. How much is 1% of that amount?

A $1 **B** $7.50 **C** $75 **D** $750

2 If 10% of an amount of money is $22. How much is 100% of that amount?

A $100 **B** $2.20 **C** $22 **D** $220

3 Which alternative best describes the unitary method when dealing with percentages?

A Work out the factor required to multiply the given percentage to make 100%.
B Find the value of 1% and then multiply to find the value of percentage required.
C Once you find the full amount, or 100% of the amount, this is known as 'the unit'.
D Find what percentage equals $1 and then find the given percentage.

Example 32

4 If 6% of an amount of money is $30, how much is the full amount of money?

5 Calculate the full amount of money for each of the following.

a 3% of an amount of money is $27
b 5% of an amount of money is $40
c 12% of an amount of money is $132
d 60% of an amount of money is $300
e 8% of an amount of money is $44
f 6% of an amount of money is $15

Example 33

6 If 4% of the total bill is $12, how much is 30% of the bill?

7 Calculate:

a 20% of the bill, if 6% of the total bill is $36
b 80% of the bill, if 15% of the total bill is $45
c 3% of the bill, if 40% of the total bill is $200
d 7% of the bill, if 25% of the total bill is $75

8 What is the total volume if 13% of the volume is 143 litres?

9 What is the total mass if 120% of the mass is 720 kg?

PROBLEM-SOLVING AND REASONING	10, 11(½), 14	11(½), 12, 14, 15	11(½), 12, 13, 15, 16

Example 34

10 A necklace in a jewellery store has been discounted by 20%. If the sale price is $240, what was the original price of the necklace?

11 Find the original price of the following items.

a A pair of jeans discounted by 40% has a sale price of $30.
b A hockey stick discounted by 30% has a sale price of $105.
c A second-hand computer discounted by 85% has a sale price of $90.
d A second-hand textbook discounted by 80% has a sale price of $6.
e A standard rose bush discounted by 15% has a sale price of $8.50.
f A motorbike discounted by 25% has a sale price of $1500.

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12 Forty per cent of workers for a large construction company prefer to have an early lunch and 25% of workers prefer to work through lunch and leave an hour earlier at the end of the day. All other workers prefer a late lunch. If 70 workers prefer a late lunch, how many workers are employed by the construction company?

13 Daryl receives an amount of money from his grandparents for his birthday. He spends 70% of the money buying a new music CD. He then spends 50% of the remaining money buying more credit for his mobile phone. After these two purchases, Daryl has $6 remaining. How much money did Daryl's grandparents give him for his birthday?

14 If 22% of an amount is $8540, which of the following would give the value of 1% of the amount?

A $8540 × 100 **B** $8540 ÷ 100 **C** $8540 × 22 **D** $8540 ÷ 22

15 If y% of an amount of money is $8, how much is the full amount of money?

16 If C% of an amount of money is $$D$, how much is F% of the amount of money?

ENRICHMENT	–	–	17

Lots of ups and downs

17 A shirt is currently selling for a price of $120.

- **a** Increase the price by 10%, then increase that price by 10%. Is this the same as increasing the price by 20%?
- **b** Increase the price of $120 by 10%, then decrease that price by 10%. Did the price go back to $120?
- **c** The price of $120 is increased by 10%. By what percentage must you decrease the price for it to go back to $120?
- **d** If the price of $120 goes up by 10% at the end of the year, how many years does it take for the price to double?

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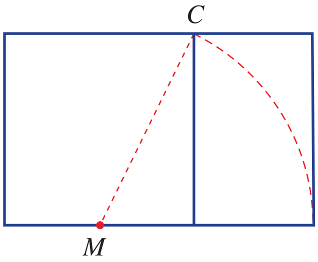
Investigation

Phi, the golden number

Constructing a golden rectangle

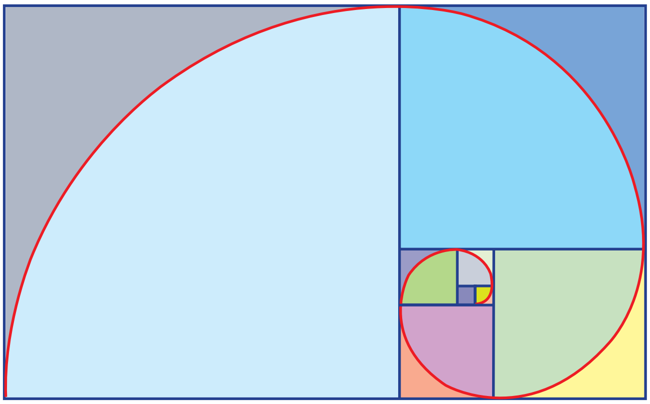
You will need a ruler, a pencil, a pair of compasses and a sheet of A4 paper.

1. Rule a square of side length about 15 cm on the left side of your paper.
2. Rule a line from the midpoint (*M*) of one side of the square to the opposite corner (*C*), as shown.
3. With your compass point on *M* and the compass pencil on *C* (radius *MC*), draw an arc from *C*, as shown.
4. Extend the base of the square to meet this arc. This new length is the base length of a golden rectangle.
5. Complete the golden rectangle and erase the vertical line from *C* and the arc.
6. You now have two golden rectangles, one inside the other.



Constructing a golden spiral

1. In your smaller golden rectangle, rule the largest square that can be drawn next to the first square.
2. Continue adding squares in each smaller golden rectangle until there are at least seven adjacent squares arranged as shown below.
3. In each square, mark the corners that are closest to the smallest square.
4. Using a compass with the point on these corners, draw arcs equal to the side of each square.
5. Colour your golden spiral pattern.



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Calculating phi

Phi is the ratio of the length to the breadth (height) of a golden spiral or a golden rectangle.

In the design you have drawn there are several golden rectangles.

Measure the length and breadth of each golden rectangle and calculate the value of phi (length divided by breadth) for each.

Work out the average (mean) of all your calculations of phi. Compare your average with the actual value of $\frac{1 + \sqrt{5}}{2} = 1.61\,803\ldots$

Golden rectangles in the human body

Investigate some of your own measurements and see how close they are to the golden ratio of phi $: 1 \approx 1.6 : 1$.
Golden ratios in the human body include:
- total height : head to fingertips
- total height : navel (belly button) to floor
- height of head : width of head
- shoulder to fingertips : elbow to fingertips
- length of forearm : length of hand
- hip to floor : knee to floor
- length of longest bone of finger : length of middle bone of same finger
- eyes to the bottom of the chin : width of ‘relaxed’ smile
- width of two centre teeth : height of centre tooth

Research and class presentation

Research the internet to discover examples of golden rectangles in nature, human anatomy, architecture, art or graphic design. Present your findings to your class using a PowerPoint presentation or a poster.

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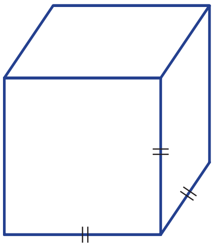
Puzzles and challenges

1 The side length of a cube is increased by 20%. As a percentage, by how much does its volume change?



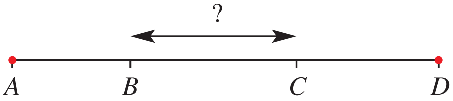
2 The cost of an item in a shop increased by 20% and was later decreased by 20% at a sale. Express the final price as a percentage of the original price.

3 Evaluate $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{999}{1000}$ without using a calculator.

4 Some prize money is shared between three students. Abby receives $\frac{1}{4}$ of the prize, Evie $\frac{1}{3}$ and Molly the remainder. If Molly is given \$20, what is the total value of the prize money?

5 Freshly squeezed orange juice contains 85% water. A food factory has 100 litres of fresh orange juice which is then concentrated by removing 80% of the water. What percentage of water is in the concentrated orange juice?

6 On line segment AD, AC is $\frac{2}{3}$ of AD and BD is $\frac{3}{4}$ of AD. What fraction of AD is BC?



7 Fifty people surveyed said they like apples or bananas or both. Forty per cent of the people surveyed like apples and 70% of the people like bananas. What percentage of people like only apples?

8 A bank balance of \$100 has 10% of its value added to it at the end of every year, so that at the end of the second year the new balance is \$110 + \$11 = \$121.
How many full years will it take for the balance to be more than \$10 000?

9 From the time Lilli gets up until her bus leaves at 8:03 a.m., she uses one-fifth of the time having a shower and getting dressed, then one-quarter of the remaining time packing her lunch and school bag, one-third of the remaining time having breakfast and then one-half of the remaining time practising the piano, which she finishes at exactly 7:45 a.m. At what time does Lilli get up?

10 Four numbers, $a, b, c, d,$ are evenly spaced along a number line. Without using a calculator, find the values of b and c as fractions, given that:

a $a = -1\frac{1}{3}, d = \frac{1}{6}$

b $a = 1\frac{7}{20}, d = 2\frac{1}{5}$

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Numerator

2 parts selected

$\frac{2}{3} =$ 

3 parts in one whole

Denominator

$$-4 + 3 - \frac{1}{8} + \frac{5}{12}$$
$$= -1 - \frac{1}{8} \times \frac{3}{3} + \frac{5}{12} \times \frac{2}{2}$$
$$= -1 - \frac{3}{24} + \frac{10}{24}$$
$$= -1 + \frac{7}{24}$$
$$= -\frac{24}{24} + \frac{7}{24}$$
$$= -\frac{17}{24}$$

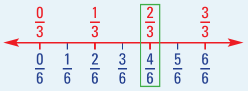
OR

$$-4\frac{1}{8} - \left(-3\frac{5}{12}\right)$$
$$= -4\frac{1}{8} + 3\frac{5}{12}$$
$$= -\frac{33}{8} \times \frac{3}{3} + \frac{41}{12} \times \frac{2}{2}$$
$$= -\frac{99}{24} + \frac{82}{24}$$
$$= -\frac{17}{24}$$

Adding and subtracting fractions

- Equivalent fractions
- 'Unit' is the denominator.
- Add/subtract numerators.
- Keep denominator the same.

Equivalent fractions represent equal proportions of one whole.



Simplest form $\boxed{\frac{2}{3}} \overset{\times 2}{\underset{\times 2}{=}} \frac{4}{6} \overset{\times 5}{\underset{\times 5}{=}} \frac{20}{30} \overset{\div 2}{\underset{\div 2}{=}} \frac{10}{15} \overset{\div 5}{\underset{\div 5}{=}} \frac{2}{3}$

Equivalent fractions have the same simplest form and are equal in value.

Lowest common denominator (LCD) is lowest common multiple (LCM).

$$\frac{3}{35} + \frac{1}{40}$$
$$= \frac{8}{8} \times \frac{3}{5 \times 7} + \frac{1}{5 \times 8} \times \frac{7}{7}$$
$$= \frac{24}{280} + \frac{7}{280}$$
$$= \frac{31}{280}$$

HCF of 35 and 40 is 5.

Negative fractions

$\frac{5}{6} + \left(-\frac{1}{6}\right)$

↓

Subtract opposite

↓

$= \frac{5}{6} - \frac{1}{6}$

$\frac{5}{6} - \left(-\frac{1}{6}\right)$

↓

Add opposite

↓

$= \frac{5}{6} + \frac{1}{6}$

Fractions

Mixed ⟶ Improper

$$2\frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$$

Dividing fractions

- Mixed numerals ⟶ Improper fractions
- Denominators do not need to be the same.
- Multiply by reciprocal of divisor.
- Cancel: Divide common factors in numerator and denominator.

Multiplying fractions

- Mixed numerals ⟶ Improper fractions
- Denominators do not need to be the same.
- Cancel: Divide common factors in numerator and denominator.

Improper ⟶ Mixed

$$\frac{40}{15} = 2\frac{10}{15} = 2\frac{\cancel{5} \times 2}{\cancel{5} \times 3} = 2\frac{2}{3}$$

OR

$$\frac{40}{15} = \frac{\cancel{5} \times 8}{\cancel{5} \times 3} = \frac{8}{3} = 2\frac{2}{3}$$

Negative fractions

$$\frac{2}{3} \div \left(-\frac{1}{6}\right)$$
$$= \frac{2}{3} \times \left(-\frac{1}{6}\right)$$

Different signs

↓

Negative answer

$$= -\frac{12}{3}$$
$$= -4$$

$$-\frac{5}{8} \div \left(-\frac{3}{8}\right)$$
$$= -\frac{5}{8} \times \left(-\frac{8}{3}\right)$$

Same signs

↓

Positive answer

$$= \frac{5}{3}$$
$$= 1\frac{2}{3}$$

$$-\frac{7}{12} \div \left(-4\frac{2}{3}\right)$$
$$= -\frac{7}{12} \div \left(-\frac{14}{3}\right)$$
$$= -\frac{\cancel{7}^{1}}{\cancel{12}_{4}} \times -\frac{\cancel{3}^{1}}{\cancel{14}_{2}}$$
$$= \frac{1}{8}$$

Reciprocal of $-\frac{14}{3}$ is $-\frac{3}{14}$

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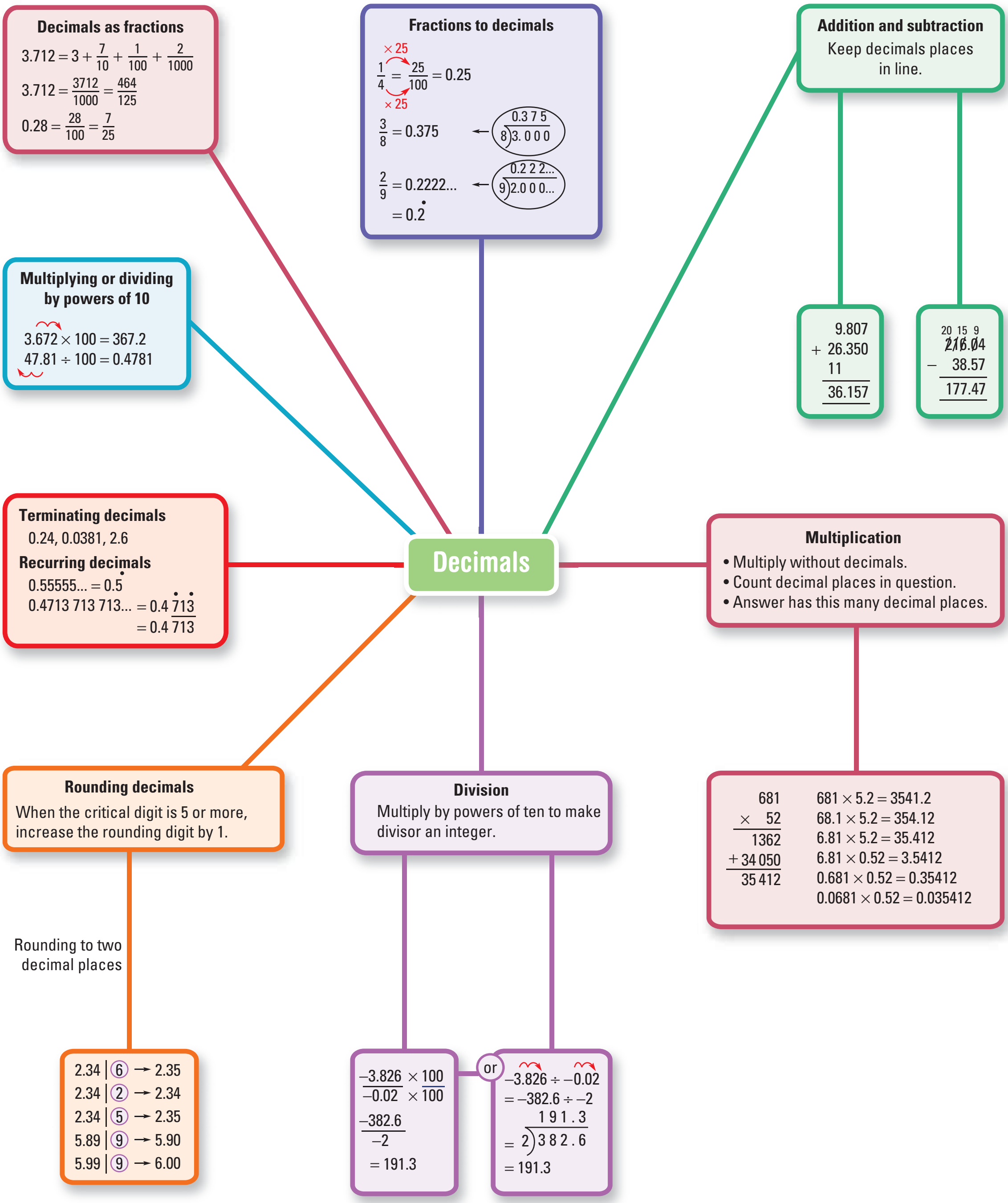

Decimals

Decimals as fractions

$3.712 = 3 + \frac{7}{10} + \frac{1}{100} + \frac{2}{1000}$

$3.712 = \frac{3712}{1000} = \frac{464}{125}$

$0.28 = \frac{28}{100} = \frac{7}{25}$

Fractions to decimals

$\frac{1}{4} \overset{\times 25}{\underset{\times 25}{=}} \frac{25}{100} = 0.25$

$\frac{3}{8} = 0.375 \leftarrow 8\overline{)3.000}$ with quotient 0.375

$\frac{2}{9} = 0.2222\ldots \leftarrow 9\overline{)2.000\ldots}$ with quotient $0.222\ldots$

$= 0.\dot{2}$

Addition and subtraction

Keep decimals places in line.

$$\begin{array}{r} 9.807 \\ +\ 26.350 \\ 11 \\ \hline 36.157 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 20\ 15\ \ 9} \\ \cancel{2}\cancel{1}\cancel{6}.\cancel{0}4 \\ -\ 38.57 \\ \hline 177.47 \\ \hline \end{array}$$

Multiplying or dividing by powers of 10

$3.672 \times 100 = 367.2$

$47.81 \div 100 = 0.4781$

Terminating decimals

0.24, 0.0381, 2.6

Recurring decimals

$0.55555\ldots = 0.\dot{5}$

$0.4713\,713\,713\ldots = 0.4\dot{7}1\dot{3}$

$= 0.4\overline{713}$

Multiplication

- Multiply without decimals.
- Count decimal places in question.
- Answer has this many decimal places.

$$\begin{array}{r} 681 \\ \times\ \ 52 \\ \hline 1362 \\ +\,34\,050 \\ \hline 35\,412 \end{array}$$

$681 \times 5.2 = 3541.2$

$68.1 \times 5.2 = 354.12$

$6.81 \times 5.2 = 35.412$

$6.81 \times 0.52 = 3.5412$

$0.681 \times 0.52 = 0.35412$

$0.0681 \times 0.52 = 0.035412$

Rounding decimals

When the critical digit is 5 or more, increase the rounding digit by 1.

Rounding to two decimal places

$2.34 \mid ⑥ \rightarrow 2.35$

$2.34 \mid ② \rightarrow 2.34$

$2.34 \mid ⑤ \rightarrow 2.35$

$5.89 \mid ⑨ \rightarrow 5.90$

$5.99 \mid ⑨ \rightarrow 6.00$

Division

Multiply by powers of ten to make divisor an integer.

$\frac{-3.826}{-0.02} \times \frac{100}{100}$

$\frac{-382.6}{-2}$

$= 191.3$

or

$-3.826 \div -0.02$

$= -382.6 \div -2$

$= 2\overline{)382.6}$ with quotient 191.3

$= 191.3$

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Percentages

means 'out of 100'

Fraction to percentage

$\frac{5}{8} \times 100\% = 62.5\%$

Decimal to percentage

$0.362 \times 100\% = 36.2\%$

Goods and Services Tax (GST)

Current GST rate in Australia is 10%.

$$\begin{tikzcd} \text{Ⓐ Price (excluding GST)} \arrow[r, bend left, "\times 1.1"] & \text{Ⓑ Price (including GST)} \arrow[l, bend left, "\div 1.1"] \end{tikzcd}$$

GST = 10% of A or B – A or B ÷ 11

Percentage of a quantity

$7\% \text{ of } \$40 = \frac{7}{10\cancel{0}} \times 4\cancel{0} = \$2.80$

Percentage to fraction

$56\% = \frac{56}{100} = \frac{14 \times \overset{1}{\cancel{4}}}{25 \times \cancel{4}} = \frac{14}{25}$

'A' as a percentage of 'B'

$\frac{A}{B} \times 100\%$

25 g as a percentage of 1 kg

$\frac{25}{1000} \times 100\% = 2.5\%$

Same units

Memorise

$50\% = \frac{1}{2} = 0.5$

$33\frac{1}{3}\% = \frac{1}{2} = 0.\dot{3}$

$25\% = \frac{1}{4} = 0.25$

$20\% = \frac{1}{5} = 0.2$

$10\% = \frac{1}{10} = 0.1$

$12\frac{1}{2}\% = \frac{1}{8} = 0.125$

$66\frac{2}{3}\% = \frac{2}{3} = 0.\dot{6}$

$75\% = \frac{3}{4} = 0.75$

Percentage profit

$= \frac{\text{profit}}{\text{cost price}} \times 100\%$

Percentage loss

$= \frac{\text{loss}}{\text{cost price}} \times 100\%$

Percentage increase

- mark-up
- profit

Selling price = cost price + mark-up (profit)

$12\frac{1}{2}\%$ mark-up on \$300

- Increase $= \frac{12\frac{1}{2}}{100} \times 300 = \$37.50$
- New price $= 300 + 37.50$
  $= \$337.50$

Alternatively:
$100\% + 12.5\% = 112.5\%$
$= 1.125$
$\$300 \times 1.125 = \$337.50$

Percentage decrease

- Discount
- Reduce
- Depreciate

New price = original price – decrease

\$1700 TV discounted by 10%

Discount $= 1700 \times \frac{10}{100} = \$170$

Sale price $= 1700 - 170$
$= \$1530$

Unitary method

1 unit is 1%

If … find 80%

$$\begin{tikzcd} 6\% \text{ is } \$420 \arrow[d, "\div 6"'] \arrow[d, "\div 6"] \\ 1\% \text{ is } \$70 \arrow[d, "\times 80"'] \arrow[d, "\times 80"] \\ 80\% \text{ is } \$5600 \end{tikzcd}$$

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Multiple-choice questions

1 0.36 expressed as a fraction is:

A $\frac{36}{10}$ **B** $\frac{36}{100}$ **C** $\frac{3}{6}$ **D** $\frac{9}{20}$ **E** $\frac{36}{1000}$

2 $\frac{2}{11} + \frac{5}{8}$ is equal to:

A $\frac{7}{19}$ **B** $\frac{10}{19}$ **C** $\frac{10}{88}$ **D** $\frac{31}{44}$ **E** $\frac{71}{88}$

3 When 21.63 is multiplied by 13.006, the number of decimal places in the answer is:

A 2 **B** 3 **C** 4 **D** 5 **E** 6

4 $\frac{124}{36}$ is the same as:

A 88 **B** $\frac{34}{9}$ **C** $3\frac{4}{9}$ **D** 3.49 **E** $\frac{69}{20}$

5 When $5\frac{1}{3}$ is written as an improper fraction, its reciprocal is:

A $\frac{1}{53}$ **B** 53 **C** $\frac{16}{3}$ **D** $\frac{3}{16}$ **E** $\frac{3}{15}$

6 Which decimal has the largest value?

A 6.0061 **B** 6.06 **C** 6.016 **D** 6.0006 **E** 6.006

7 9.46×10^5 is the same as:

A 94 600 000 **B** 946 000 **C** 94 605 **D** 0.0000946 **E** 9460

8 75% of 84 is the same as:

A $\frac{84}{75} \times 3$ **B** $\frac{84}{3} \times 4$ **C** $84 \times 100 \div 75$ **D** $\frac{(0.75 \times 84)}{100}$ **E** $\frac{84}{4} \times 3$

9 590% is the same as:

A 59.0 **B** 0.59 **C** 5.9 **D** 0.059 **E** 5900

10 $790 increased by 15% gives:

A $118.50 **B** $908.50 **C** $671.50 **D** $805 **E** $113.50

11 A camera has $87 of GST included in its price. What is the price of the camera including the GST?

A $870 **B** $95.70 **C** $790 **D** $1052.70 **E** $957

Short-answer questions

1 Copy and complete:

a $\frac{7}{20} = \frac{\square}{60}$ **b** $\frac{25}{40} = \frac{5}{\square}$ **c** $\frac{350}{210} = \frac{\square}{6}$

2 Simplify:

a $\frac{25}{45}$ **b** $\frac{36}{12}$ **c** $\frac{102}{12}$

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3 Evaluate each of the following.

a $\frac{5}{11}+\frac{2}{11}$
b $\frac{7}{8}-\frac{3}{4}$
c $3-1\frac{1}{4}$
d $3\frac{1}{4}-1\frac{2}{3}$
e $2\frac{2}{5}+3\frac{1}{4}$
f $1\frac{1}{2}+2\frac{2}{3}-\frac{3}{5}$

4 Evaluate:

a $\frac{2}{3}\times 12$
b $\frac{3}{7}\times 1\frac{1}{12}$
c $2\frac{1}{3}\times 6$
d $3\div\frac{1}{2}$
e $\frac{2}{3}\div 12$
f $1\frac{1}{2}\div\frac{3}{4}$

5 Evaluate each of the following.

a $\frac{2}{3}-\frac{1}{5}$
b $\frac{3}{4}\times\frac{1}{5}$
c $\left(\frac{3}{5}\right)^2$
d $\frac{3}{4}-\left(-\frac{1}{5}\right)$
e $\frac{5}{3}\div\left(-\frac{1}{3}\right)$
f $6\frac{1}{4}+1\frac{1}{3}$

6 Insert >, < or = to make each of the statements true.

a $\frac{11}{20}$ ☐ 0.55
b $\frac{2}{3}$ ☐ 0.7
c 0.763 ☐ 0.7603

7 Evaluate:

a $12.31 + 2.34 + 15.73$
b $14.203 - 1.4$
c 569.74×100
d 25.14×2000
e 7.4×10^4
f $5 - 2.0963$

8 Calculate:

a 2.67×4
b 2.67×0.04
c 1.2×12
d $1.02 \div 4$
e $1.8 \div 0.5$
f $9.856 \div 0.05$

9 Round these decimals to three decimal places.

a 0.666831
b 3.57964
c 0.00549631

10 Copy and complete this table of conversions.

0.1					0.75		
	$\frac{1}{100}$			$\frac{1}{4}$		$\frac{1}{3}$	$\frac{1}{8}$
		5%	50%				

11 Express each of the following as a percentage.

a \$35 out of \$40
b 6 out of 24
c \$1.50 out of \$1
d 16 cm out of 4 m
e 15 g out of $\frac{1}{4}$ kg

12 Find:

a 30% of 80
b 15% of \$70
c $12\frac{1}{2}$% of 84

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13 **a** Increase \$560 by 10%.

b Decrease \$4000 by 18%.

c Increase \$980 by 5% and then decrease the result by 5%. Calculate the overall percentage loss.

14 A new plasma television is valued at \$3999. During the end-of-year sale, it is discounted by 9%. What is the discount and the sale price?

15 Jenni works at her local pizza takeaway restaurant and is paid \$7.76 per hour. When she turns 16, her pay will increase to \$10 an hour. What will be the percentage increase in her pay (to the nearest per cent)?

16 An investment of \$22 000 earns \$2640 in interest over 2 years. What is the flat interest rate per year?

17 Johan saves 15% of his weekly wage. He saved \$5304 during the year. Calculate Johan's weekly wage.

18 A dress cost \$89.10 after a 10% discount. How much was saved on the original price of the dress?

19 If 5% of an amount equals 56, what is 100% of the amount?

20 A shopping receipt quotes an A4 folder as costing \$3.64 including 10% GST. What is the cost of the folder pre-GST, correct to the nearest cent?

21 Copy and complete the table.

Pre-GST price	10% GST	Final cost including the 10% GST
\$250		
	\$45	
	\$8	
		\$737

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Extended-response question

The following table shows the value of A$1 (one Australian dollar) in foreign currency.

Indian rupee (INR)	42
Singapore dollar (SGD)	1.25
Thai baht (THB)	30
Hong Kong dollar (HKD)	7

Genevieve is planning an extended holiday to Asia, where she plans to visit India, Singapore, Phuket and Hong Kong.

a She has decided to convert some Australian dollars to each of the currencies above before she flies out. How much of each currency will Genevieve receive if she converts A$500 to each currency?

b During the exchange Genevieve must pay a fee of 1.5% for each transaction. How much does she pay in fees (in A$)?

c **i** The day after she leaves, the exchange rate for Indian rupees is A$1 = 43.6 INR. How much more would Genevieve have received if she had waited until arriving in India to convert her Australian dollars? (Ignore transaction fees.)

  ii Express this as a percentage (to one decimal place).

d **i** On her return to Australia, Genevieve has in her wallet 1000 INR, 70 SGD and 500 THB. Using the same exchange rate, how much is this in A$?

  ii Genevieve must also pay the 1.5% transaction fee. How much does she receive in A$, and is it enough to buy a new perfume at the airport for $96?

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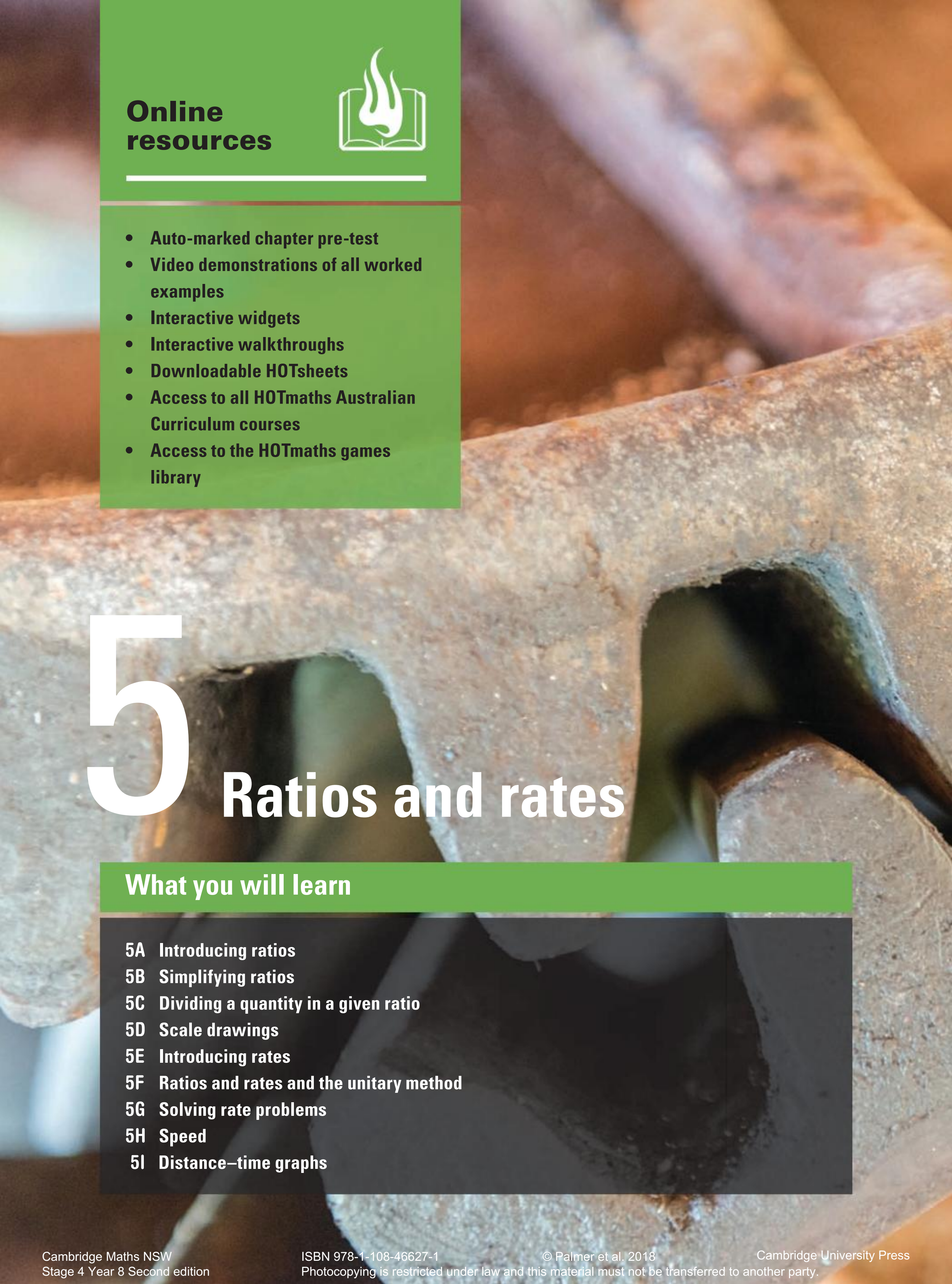

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

5

Ratios and rates

What you will learn

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: RATIOS AND RATES
FINANCIAL MATHEMATICS

Outcomes

A student operates with ratios and rates, and explores their graphical representation.
(MA4–7NA)
A student solves financial problems involving purchasing goods.
(MA4–6NA)

Bicycle gear ratios

The bicycle is renowned as the most energy-efficient means of human transport. A bicycle with a range of gears is an extremely versatile and enjoyable machine. The 'gear ratio' of a bicycle is the ratio of pedal rotations compared to the rate at which the wheels rotate. Manufacturers apply the mathematics of gear ratios to provide cyclists with the range of gears that is most useful for the style of bike.

Have you ever tried to ride up a steep hill in too high a gear or reached the bottom of a hill while still in a very low gear? A bicycle rider has a comfortable range of pressures and rotation rates that they can apply to the pedals. Gears make it easier to stay in this comfortable range over a wide variety of riding conditions, such as BMX, downhill racing, a fast time trial for racing bikes on flat ground, or climbing steep, rough trails on a mountain bike.

Unicycles have only one wheel and so have a ratio of 1 : 1 for pedal turns to wheel turns. Unicycles can't reach high speeds because it is very difficult to rotate pedals more than about 100 rpm; however, riding a unicycle is fun and it requires strength and agility to remain stable, especially when playing unicycle hockey!

1 Express each fraction in its simplest form.

a $\frac{2}{4}$ **b** $\frac{6}{12}$

c $\frac{8}{24}$ **d** $\frac{15}{20}$

e $\frac{8}{12}$ **f** $\frac{16}{32}$

g $\frac{40}{60}$ **h** $\frac{200}{350}$

2 Copy and complete:

a $\frac{2}{5} = \frac{4}{\square}$ **b** $\frac{4}{7} = \frac{\square}{28}$ **c** $\frac{3}{2} = \frac{12}{\square}$

3 Convert:

a 5 m = ____ cm

b 6 km = ____ m

c 500 cm = _____ m

d 80 mm = ____ cm

e 120 cm = ____ m

f 15 000 m = ____ km

4 Express the first quantity as a fraction of the second.

a 50 cm, 1 m **b** 15 minutes, 1 hour

c 200 g, 1 kg **d** 40 minutes, 2 hours

e 40 m, $\frac{1}{2}$ km **f** 50 cm, $\frac{1}{2}$ m

5 Find:

a $\frac{2}{5}$ of $40 **b** $\frac{1}{3}$ of 60 m **c** $\frac{2}{3}$ of 24 cm

d $\frac{3}{4}$ of 800 m **e** $1\frac{1}{2}$ lots of $5 **f** $\frac{1}{8}$ of $100

6 The cost of 1 kg of bananas is $4.99. Find the cost of:

a 2 kg **b** 5 kg **c** 10 kg **d** $\frac{1}{2}$ kg

7 Kevin walks 3 km in 1 hour. How far did he walk in 30 minutes?

8 Tao earns $240 for working 8 hours. How much did Tao earn each hour?

9 A car travels at an average speed of 60 km per hour.

a How far does it travel in the following times?

  i 2 hours

  ii 5 hours

  iii $\frac{1}{2}$ hour

b How long would it take to travel the following distances?

  i 180 km

  ii 90 km

  iii 20 km

c If the car's speed was 70 km per hour, how many minutes would it take to travel 7 km?

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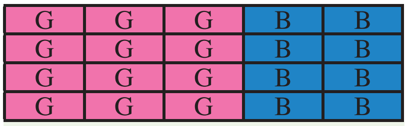
5A Introducing ratios

So far you have used fractions, decimals and percentages to describe situations like the seating plan in this diagram.



G	G	G	B	B
G	G	G	B	B
G	G	G	B	B
G	G	G	B	B

The seating plan for my class
(G is 1 girl and B is 1 boy.)

In this chapter you will learn how to use ratio to compare two or more quantities.

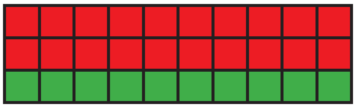
The simplest ratio is 1 is to 1.

In this diagram the ratio of **red squares** to **green squares** is **1** : **1**.

This is pronounced ‘1 is to 1’.

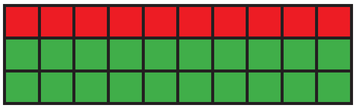
Half of the squares are red and **half are green**.

In this diagram, the ratio of **red squares** to **green squares** is **2** : **1**.



Two-thirds are red and **one-third is green**.

In this diagram, the ratio of **red squares** to **green squares** is **1** : **2**.



One-third is red and **two-thirds are green**.

Let’s start: Seating plan

Consider the seating plan at the top of the page.

- What fraction of the class are girls?
- What fraction of the class are boys?
- What is the ratio of girls to boys?
- What is the ratio of boys to girls?

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Key ideas

- A **ratio** shows the relationship between two or more amounts.
  - The amounts are separated by a colon (:).
  - The amounts are measured using the same units.
    For example, 1 mm : 100 mm is written as 1 : 100.
  - The ratio 1 : 100 is read as 'the ratio of 1 is to 100'.
  - The order in which the numbers are written is important.
    For example, teachers : students = 1 : 20 means '1 teacher for every 20 students'.
- It is possible to have three or more numbers in a ratio.
  For example, flour : water : milk = 2 : 3 : 1.
- All the amounts in a ratio can be multiplied (or divided) by the same number to give an **equivalent ratio**. For example:

$$\begin{tikzcd} 1 \arrow[d, bend right, "\times 2"'] & : & 3 \arrow[d, bend left, "\times 2"] \\ 2 & : & 6 \end{tikzcd} \quad \begin{tikzcd} 1 \arrow[d, bend right, "\times 10"'] & : & 3 \arrow[d, bend left, "\times 10"] \\ 10 & : & 30 \end{tikzcd} \quad \begin{tikzcd} 8 \arrow[d, bend right, "\div 2"'] & : & 12 \arrow[d, bend left, "\div 2"] \\ 4 & : & 6 \end{tikzcd} \quad \begin{tikzcd} 8 \arrow[d, bend right, "\div 4"'] & : & 12 \arrow[d, bend left, "\div 4"] \\ 2 & : & 3 \end{tikzcd}$$

Example 1 Writing ratios

A sample of mixed nuts contains 5 cashews, 12 peanuts and 2 macadamia nuts.
Write down:

a the ratio of cashews to peanuts to macadamias
b the ratio of cashews to the total number of nuts
c the ratio of peanuts to other nuts

SOLUTION	EXPLANATION
a 5 : 12 : 2	cashews : peanuts : macadamias
b 5 : 19	5 cashews, total nuts 19
c 12 : 7	12 peanuts, other nuts 7

Example 2 Producing equivalent ratios

Complete each pair of equivalent ratios.

a $4:9 = 16:\square$ **b** $30:15 = \square:5$

SOLUTION **EXPLANATION**

a

$$\begin{tikzcd} 4 \arrow[d, bend right, "\times 4"'] & : & 9 \arrow[d, bend left, "\times 4"] \\ 16 & : & \boxed{36} \end{tikzcd}$$

Both numbers are multiplied by 4.

b

$$\begin{tikzcd} 30 \arrow[d, bend right, "\div 3"'] & : & 15 \arrow[d, bend left, "\div 3"] \\ \boxed{10} & : & 5 \end{tikzcd}$$

Both numbers are divided by 3.

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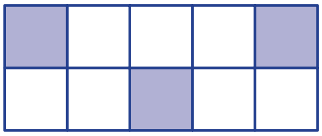
Exercise 5A

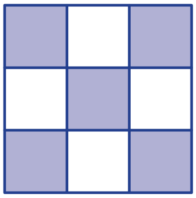
UNDERSTANDING AND FLUENCY	1–6, 7(½), 8	3–6, 7(½), 8, 9	4–9(½)

1 Write down the ratio of shaded parts to unshaded parts for each grid.

a

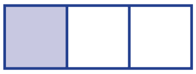


b

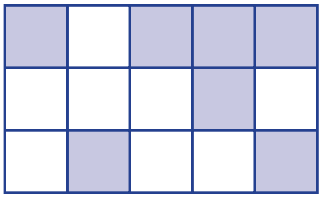


2 Write down the ratio of shaded parts to total parts for each grid.

a



b



3 Express each of the following pairs of quantities as a ratio.

a 9 goals to 4 goals
b 52 litres to 17 litres
c 7 potatoes to 12 carrots
d 3 blue marbles to 5 green marbles

Example 1

4 A bag contains 5 green marbles, 7 blue marbles and 3 yellow marbles.
Write down the ratio of:

a green marbles to yellow marbles
b blue marbles to total marbles
c yellow to blue to green marbles
d green to yellow to blue marbles

5 Over the past fortnight, it has rained on eight days and it has snowed on three days.
Write down the ratio of:

a rainy days to snowy days
b snowy days to total days
c fine days to rainy and snowy days
d rainy days to non-rainy days

6 In a box of 40 flavoured icy poles there are 13 green, 9 lemonade, 11 raspberry and 7 orange icy poles. Write down the ratio of:

a green icy poles to orange icy poles
b raspberry icy poles to lemonade icy poles
c the four different flavours of icy poles
d green and orange icy poles to raspberry and lemonade icy poles

Example 2

7 Complete each pair of equivalent ratios.

a $1:3 = 4:\square$ **b** $1:7 = 2:\square$ **c** $2:5 = \square:10$ **d** $3:7 = \square:21$
e $5:10 = 1:\square$ **f** $12:16 = 3:\square$ **g** $12:18 = \square:3$ **h** $20:50 = \square:25$
i $4:7 = 44:\square$ **j** $14:17 = 42:\square$ **k** $27:6 = \square:2$ **l** $121:66 = \square:6$

8 Complete each pair of equivalent ratios.

a $2:3:5 = 4:\square:\square$ **b** $4:12:16 = \square:6:\square$
c $1:7:9 = \square:\square:63$ **d** $22:110:66 = 11:\square:\square$

9 Write three equivalent ratios for each of the following ratios.

a $1:2$ **b** $2:5$ **c** $8:6$ **d** $9:3$

PROBLEM-SOLVING AND REASONING	10, 11, 15	11–13, 15, 16	12–14, 16–18

10 Sort the following ratios into three pairs of equivalent ratios.
$2:5$, $6:12$, $7:4$, $1:2$, $4:10$, $70:40$

11 During a recent dry spell, it rained on only three days during the month of September.

a What is the ratio of wet days to total days for the month of September?
b Write equivalent ratios for a total of 10 days and 100 days.

12 On their way to work, Andrew passes 15 sets of traffic lights and Pauline passes 10 sets of traffic lights. One morning Andrew was stopped by 12 red traffic lights. How many green traffic lights would Pauline need to pass through to have the equivalent ratio of red to green traffic lights as Andrew?

13 Write the ratio of vowels to consonants for each of the following words.

a Queensland **b** Canberra
c Wagga Wagga **d** Australia

14 Name any Australian states that have a vowel to consonant ratio of $1:1$.

15 Can 10 people be divided into two groups with a ratio of $1:2$? Explain.

16 Bertrand wins one-third of his games of tennis this season. Write down his win to loss ratio.

17 a What is the ratio of black keys to white keys in one octave of the piano?
b What is the ratio of black keys to white keys for the entire 88 keys of the piano?
c Are these equivalent ratios?

18 Write the missing expression.

a $2:x = 6:\_\_\_$ **b** $2:x = 5:\_\_\_$
c $y:2x = \_\_\_:8x$ **d** $12xy:6y = \_\_\_:1$

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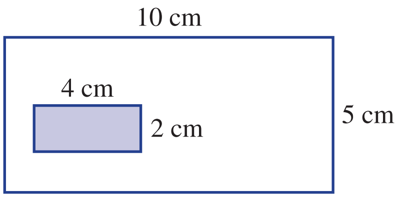
ENRICHMENT	–	–	19–21

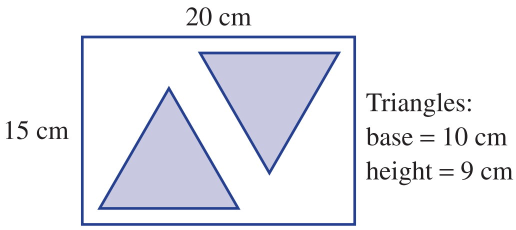
Area ratios

19 Using the dimensions provided, find the ratio of the shaded area to the unshaded area for each of the following diagrams.

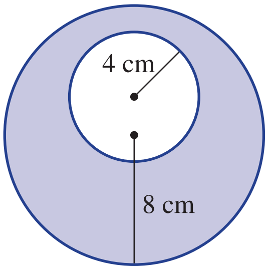
a



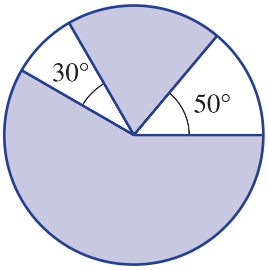
b



c



d



20 Estimate the ratio of the shaded floor to the unshaded floor in this photo.

21 Design your own diagram for which the ratio of shaded area to unshaded area is:

a 1 : 3

b 1 : 7

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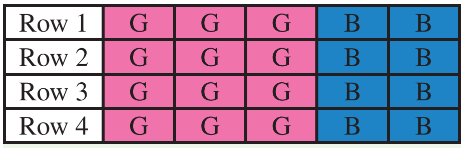
5B Simplifying ratios

Using the diagram below, there are several equivalent ways to write the ratio of girls to boys.



Row 1	G	G	G	B	B
Row 2	G	G	G	B	B
Row 3	G	G	G	B	B
Row 4	G	G	G	B	B

The seating plan for my class
(G is 1 girl and B is 1 boy.)

	Ratio of girls to boys
Using the whole class	$12:8$
Using three rows	$9:6$
Using two rows	$6:4$
Using one row	$3:2$
Using one boy	$1.5:1$
Using one girl	$1:\frac{2}{3}$

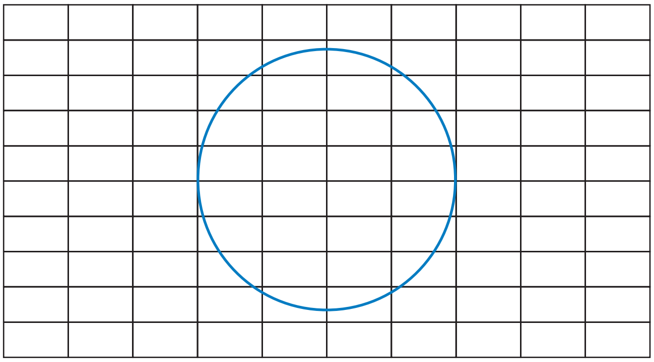
In these equivalent ratios, the *simplest form* is $3:2$ because:

- both numbers are whole numbers, and
- the highest common factor (HCF) is 1.

In this class, there are 3 girls for every 2 boys.

Let's start: The national flag of Rationia guessing competition

The mathemagical nation of Rationia has a new national flag.



Without doing any calculations, use a simplified ratio such as $2:3$ to *estimate* the following ratios.

- area inside blue circle : area outside blue circle
- area inside blue circle : total area of flag

Compare your estimates with those of others in your class.

Maybe there will be a prize for the best estimator!

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Key ideas

- The quantities in ratios must be expressed using the same unit.
  For example, 20 minutes : 1 hour = 20 minutes : 60 minutes
  = 20 : 60
- **Simplifying ratios**
  A ratio is simplified by dividing all numbers in the ratio by their highest common factor (HCF).
  For example, the ratio 15 : 25 can be simplified to 3 : 5.

$$\begin{tikzcd} 15:25 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 3:5 \end{tikzcd}$$

- Ratios in simplest form use whole numbers only.
- In the following list of equivalent ratios, 2 : 3 is expressed in simplest form.

$$2:3 = 1:1\frac{1}{2} = 5:7.5 = 4:6 = 20:30$$

Example 3 Simplifying ratios

Simplify the following ratios.

a 7 : 21

b 450 : 200

SOLUTION	EXPLANATION
a $\begin{tikzcd} 7:21 \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\ 1:3 \end{tikzcd}$	HCF of 7 and 21 is 7. Divide both quantities by 7.
b $\begin{tikzcd} 450:200 \arrow[d, bend right, "\div 50"'] \arrow[d, bend left, "\div 50"] \\ 9:4 \end{tikzcd}$	HCF of 450 and 200 is 50. Divide both quantities by 50.

Example 4 Simplifying ratios involving fractions

Simplify the following ratios.

a $\frac{3}{5}:\frac{1}{2}$

b $2\frac{1}{3}:1\frac{1}{4}$

SOLUTION	EXPLANATION
a $\begin{tikzcd} \frac{3}{5}:\frac{1}{2} \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ 6:5 \end{tikzcd}$ Alternatively, $\frac{3}{5}:\frac{1}{2} = \frac{6}{10}:\frac{5}{10} = 6:5$.	LCD of 5 and 2 is 10. Multiply both quantities by 10.
b $\begin{tikzcd} \frac{7}{3}:\frac{5}{4} \arrow[d, bend right, "\times 12"'] \arrow[d, bend left, "\times 12"] \\ 28:15 \end{tikzcd}$	Convert mixed numerals to improper fractions. LCD of 3 and 4 is 12. Multiply both quantities by 12.

Example 5 Changing quantities to the same units

First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 4 mm to 2 cm **b** 25 minutes to 2 hours

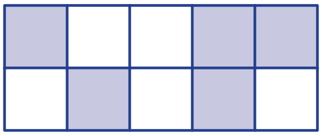
SOLUTION	EXPLANATION
a 4 mm to 2 cm = 4 mm to 20 mm	2 cm = 20 mm
= 4 : 20	Once in same unit, write as a ratio.
= 1 : 5	Simplify ratio by dividing by HCF of 4.
b 25 minutes to 2 hours	2 hours = 120 minutes
= 25 minutes to 120 minutes	Once in same unit, write as a ratio.
= 25 : 120	Simplify ratio by dividing by HCF of 5.
= 5 : 24	

Exercise 5B

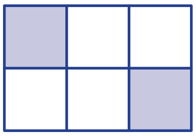
UNDERSTANDING AND FLUENCY	1–4, 5–7(½), 9(½)	3, 4, 5–9(½)	5–9(½)

1 Write down the ratio of shaded parts to unshaded parts for each of the following, in simplest form.

a



b



2 To express the ratio 4 : 16 in simplest form, you would:

A multiply both quantities by 2
B subtract 4 from both quantities
C divide both quantities by 2
D divide both quantities by 4

3 Decide which of the following ratios is not written in simplest form.

A 1 : 5 **B** 3 : 9 **C** 2 : 5 **D** 11 : 17

4 Decide which of the following ratios is written in simplest form.

A 2 : 28 **B** 15 : 75 **C** 14 : 45 **D** 13 : 39

Example 3

5 Simplify the following ratios.

a 2 : 8 **b** 10 : 50 **c** 4 : 24 **d** 6 : 18
e 8 : 10 **f** 25 : 40 **g** 21 : 28 **h** 24 : 80
i 18 : 14 **j** 26 : 13 **k** 45 : 35 **l** 81 : 27
m 51 : 17 **n** 20 : 180 **o** 300 : 550 **p** 150 : 75
q 1200 : 100 **r** 70 : 420 **s** 200 : 125 **t** 90 : 75

6 Simplify the following ratios.

a 2 : 4 : 6 **b** 12 : 21 : 33 **c** 42 : 60 : 12 **d** 85 : 35 : 15
e 12 : 24 : 36 **f** 100 : 300 : 250 **g** 270 : 420 : 60 **h** 24 : 48 : 84

Example 4a

7 Simplify the following ratios.

a $\frac{1}{3}:\frac{1}{2}$ **b** $\frac{1}{4}:\frac{1}{5}$ **c** $\frac{2}{5}:\frac{3}{4}$ **d** $\frac{2}{7}:\frac{1}{5}$
e $\frac{3}{8}:\frac{1}{4}$ **f** $\frac{7}{10}:\frac{4}{5}$ **g** $\frac{11}{10}:\frac{2}{15}$ **h** $\frac{9}{8}:\frac{7}{12}$

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Example 4b

8 Simplify the following ratios. Use a calculator to check your answers.

a $1\frac{1}{2}:\frac{3}{4}$ **b** $2\frac{1}{5}:\frac{3}{5}$ **c** $3\frac{1}{3}:1\frac{2}{5}$ **d** $4\frac{1}{3}:3\frac{3}{4}$

Example 5

9 First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 12 mm to 3 cm
b 7 cm to 5 mm
c 120 m to 1 km
d 60 mm to 2.1 m
e 3 kg to 450 g
f 200 g to 2.5 kg
g 2 tonnes to 440 kg
h 1.25 L to 250 mL
i 400 mL to 1 L
j 20 minutes to 2 hours
k 3 hours to 15 minutes
l 3 days to 8 hours
m 180 minutes to 2 days
n 8 months to 3 years
o 4 days to 4 weeks
p 8 weeks to 12 days
q 50 cents to $4
r $7.50 to 25 cents

PROBLEM-SOLVING AND REASONING 10, 11, 14 | 11, 12, 14, 15 | 11–13, 15–17

10 Which of these ratios is the simplest form of the ratio $\frac{1}{2}:2$?

A $2:\frac{1}{2}$ **B** $1:4$ **C** $\frac{1}{4}:1$ **D** $1:1$

11 Kwok was absent from school on eight days in Term 1. There were 44 school days in Term 1. Write the following ratios in simplest form.

a ratio of days absent to total days
b ratio of days present to total days
c ratio of days absent to days present

12 Over the past four weeks, the Paske family has eaten takeaway food for dinner on eight occasions. They had hamburgers twice, fish and chips three times, and pizza three times. Every other night they had home-cooked dinners. Write the following ratios in simplest form.

a ratio of hamburgers to fish and chips to pizza
b ratio of fish and chips to pizza
c ratio of takeaway dinners to home-cooked dinners
d ratio of home-cooked dinners to total dinners

13 When Lisa makes fruit salad for her family, she uses 5 bananas, 5 apples, 2 passionfruit, 4 oranges, 3 pears, 1 lemon (for juice) and 20 strawberries.

a Write the ratio of the fruits in Lisa's fruit salad.
b Lisa wishes to make four times the amount of fruit salad to take to a party. Write an equivalent ratio that shows how many of each fruit Lisa would need.
c Write these ratios in simplest form:
  i bananas to strawberries **ii** strawberries to other fruits

14 Feng incorrectly simplified 12 cm to 3 mm as a ratio of 4 : 1. What was Feng's mistake and what is the correct simplified ratio?

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15 Mariah has \$4 and Rogan has 50 cents. To write a ratio in simplest form, values must be written in the same unit.

- **a** First convert both units to dollars, and then express the ratio of Mariah's money to Rogan's money in simplest form.
- **b** As an alternative, convert both units to cents and express the ratio in simplest form. Do you arrive at the same simplified ratio?

16 **a** Write two quantities of time, in different units, which have a ratio of $2:5$.

- **b** Write two quantities of distance, in different units, which have a ratio of $4:3$.

17 Simplify the following ratios.

a $2a:4b$	**b** $15x:3y$	**c** $a:a^2$
d $5f:24f$	**e** $hk:3k$	**f** $24xyz:60yz$

ENRICHMENT	–	–	18

Aspect ratios

18 Aspect ratio is the relationship between the width and height of the image as displayed on a screen.

- **a** A standard analogue television has an aspect ratio of $1.\dot{3}:1$. Write this ratio in simplest form.
- **b** A high definition digital television has an aspect ratio of $1.\dot{7}:1$. Write this ratio in simplest form.
- **c** Although these two ratios are clearly not equivalent, there is a connection between the two. What is it?
- **d** The digital television aspect ratio of $1.\dot{7}:1$ was chosen as the best compromise to show widescreen movies on television. Many major films are presented in Panavision, which has an aspect ratio of $2.35:1$. Write this ratio in simplest form.
- **e** Investigate the history of aspect ratio in films and television.
- **f** Research how formats of unequal ratios are converted through the techniques of cropping, zooming, letterboxing, pillarboxing or distorting.
- **g** Investigate aspect ratios of other common media.
  - **i** Find the aspect ratio for several different newspapers.
  - **ii** Find the aspect ratio for several common sizes of photographs.
  - **iii** Find the aspect ratio for a piece of A4 paper. (A surd is involved!)

5C Dividing a quantity in a given ratio

Interactive
Widgets
HOTsheets
Walkthrough

When two people share an amount of money they usually split it evenly. This is a 1 : 1 ratio or a 50–50 split. Each person gets half the money.

Sometimes one person deserves a larger share than the other. The diagram below shows $20 being divided in the ratio 3 : 2. Gia's share is $12 and Ben's is $8.

Gia's share			Ben's share	
$1	$1	$1	$1	$1
$1	$1	$1	$1	$1
$1	$1	$1	$1	$1
$1	$1	$1	$1	$1

Gia and Ben divided $20 in the ratio 3 : 2.

Let's start: Is there a shortcut?

Take 15 counters or blocks and use them to represent $1 each.

Working with a partner:

- Share the $15 evenly. How much did each person get?
- Start again. This time share $15 in the ratio 2 : 1, like this: '2 for me, 1 for you; 2 for me, 1 for you…'. How much did each person get?
- Start again. This time share $15 in the ratio 3 : 2, like this: '3 for me, 2 for you; 3 for me, 2 for you…'. How much did each person get?
- Is there a shortcut? Without using the counters, can you work out how to share $15 in the ratio 4 : 1?
- Does your shortcut work correctly for the example at the top of this page?
- Have a class discussion about different shortcuts.
- Use your favourite shortcut (and a calculator) to divide $80 in the ratio 3 : 2. Write down the steps you used.

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Key ideas

- When Gia and Ben divide \$20 in the ratio 3 : 2, for every \$3 given to Gia, \$2 is given to Ben.
- There are various ways to do the calculation, such as:
  - Using the unitary method to find each part
    3 : 2 implies 3 parts and 2 parts, which makes 5 parts.
    \$20 divided by 5 gives \$4 for each part.
    Gia's share is 3 × \$4 = \$12.
    Ben's share is 2 × \$4 = \$8.

$$\begin{tikzcd}[row sep=small]
5 \text{ parts} = \$20 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
1 \text{ part} = \$4 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\
3 \text{ parts} = \$12
\end{tikzcd}$$

  - Using fractions of the amount
    3 : 2 implies 3 parts and 2 parts, which makes 5 parts.
    Gia gets 3 of the 5 parts (i.e. three-fifths of \$20).
    Ben gets 2 of the 5 parts (i.e. two-fifths of \$20).

    Gia's share is: $\frac{3}{5} \times \$20 = \$20 \div 5 \times 3 = \$12$

    Ben's share is: $\frac{2}{5} \times \$20 = \$20 \div 5 \times 2 = \$8$

Example 6 Dividing a quantity in a particular ratio

Divide 54 m in the ratio 4 : 5.

SOLUTION

Unitary method

Total number of parts = 9

$$\begin{tikzcd}[row sep=small]
9 \text{ parts} = 54 \text{ m} \arrow[d, bend right, "\div 9"'] \arrow[d, bend left, "\div 9"] \\
1 \text{ part} = 6 \text{ m} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\
4 \text{ parts} = 24 \text{ m}
\end{tikzcd}
\qquad
\begin{tikzcd}
1 \text{ part} = 6 \text{ m} \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\
5 \text{ parts} = 30 \text{ m}
\end{tikzcd}$$

54 m divided in the ratio 4 : 5 is 24 m and 30 m.

Fractions method

$$\frac{4}{9} \text{ of } 54 = \frac{4}{9} \times \frac{54}{1} = 24$$

$$\frac{5}{9} \text{ of } 54 = \frac{5}{9} \times \frac{54}{1} = 30$$

54 m divided in the ratio 4 : 5 is 24 m and 30 m.

EXPLANATION

Total number of parts = 4 + 5 = 9
Value of 1 part = 54 m ÷ 9 = 6 m

Check numbers add to total:
24 + 30 = 54

$$\text{Fraction} = \frac{\text{number in ratio}}{\text{total number of parts}}$$

Check numbers add to total:
24 + 30 = 54

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Example 7 Dividing a quantity in a ratio with three terms

Divide $300 in the ratio 2 : 1 : 3.

SOLUTION	EXPLANATION
Unitary method	
Total number of parts = 6	Total number of parts = 2 + 1 + 3 = 6
6 parts = $300 ÷6 ÷6 1 part = $50 ×2 ×2 2 parts = $100 1 part = $50 ×3 ×3 3 parts = $150	Value of 1 part = $300 ÷ 6 = $50
$300 divided in the ratio 2 : 1 : 3 is $100, $50 and $150.	Check numbers add to total: $100 + $50 + $150 = $300
Fractions method	
$\frac{2}{6}$ of 300 = $\frac{2}{6} \times \frac{300}{1} = 100$	Fraction = $\frac{\text{number in ratio}}{\text{total number of parts}}$
$\frac{1}{6}$ of 300 = $\frac{1}{6} \times \frac{300}{1} = 50$	
$\frac{3}{6}$ of 300 = $\frac{3}{6} \times \frac{300}{1} = 150$	
$300 divided in the ratio 2 : 1 : 3 is $100, $50 and $150.	Check numbers add to total: $100 + $50 + $150 = $300

Example 8 Finding a total quantity from a given ratio

The ratio of boys to girls at Birdsville College is 2 : 3. If there are 246 boys at the school, how many students attend Birdsville College?

SOLUTION	EXPLANATION
Unitary method	
2 parts = 246 ÷2 ÷2 1 part = 123 ×5 ×5 5 parts = 615	Ratio of boys : girls is 2 : 3. Boys have ‘2 parts’ = 246 Value of 1 part = 246 ÷ 2 = 123 Total parts = 2 + 3 = 5 parts
615 students attend Birdsville College.	Check: 5 parts = 5 × 123 = 615
Equivalent ratios method	
boys : girls = 2 : 3 ×123 ×123 = 246 : 369	Use equivalent ratios. Multiply each quantity by 123.
615 students attend Birdsville College.	Total number of students = 246 boys + 369 girls = 615

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Exercise 5C

UNDERSTANDING AND FLUENCY	1–4, 5(½), 6, 8	3, 4, 5(½), 7, 8(½)	5(½), 7, 8(½)

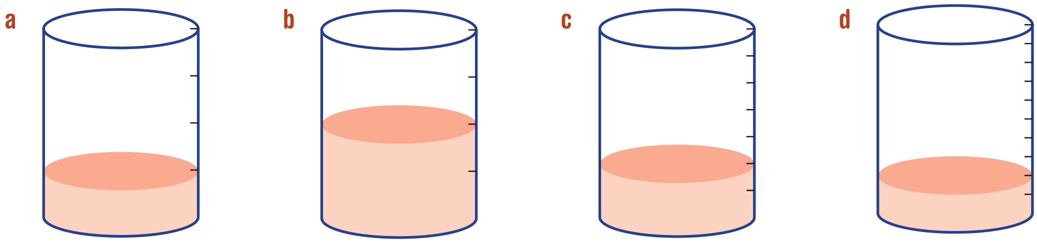
1 Find the total number of parts in the following ratios.

a 3 : 7 **b** 1 : 5 **c** 11 : 3 **d** 2 : 3 : 4

2 The ratio of girls to boys in a class is 3 : 5.

a What fraction of the class is girls?
b What fraction of the class is boys?

3 The diagram shows four glasses that contain different amounts of cordial. Water is then added to fill each glass. For each drink shown, what is the ratio of cordial to water?



4 What fraction of each of the drinks above is cordial?

Example 6

5 Divide:

a \$60 in the ratio 2 : 3
b \$110 in the ratio 7 : 4
c \$1000 in the ratio 3 : 17
d 48 kg in the ratio 1 : 5
e 14 kg in the ratio 4 : 3
f 360 kg in the ratio 5 : 7
g 72 m in the ratio 1 : 2
h 40 m in the ratio 3 : 5
i 155 m in the ratio 4 : 1

6 Share \$400 in the ratio:

a 1 : 3 **b** 2 : 3 **c** 3 : 5 **d** 9 : 11

7 Share \$6600 in the ratio:

a 4 : 7 **b** 2 : 3 **c** 24 : 76 **d** 17 : 13

Example 7

8 Divide:

a \$200 in the ratio 1 : 2 : 2
b \$400 in the ratio 1 : 3 : 4
c 12 kg in the ratio 1 : 2 : 3
d 88 kg in the ratio 2 : 1 : 5
e 320 kg in the ratio 12 : 13 : 15
f \$50 000 in the ratio 1 : 2 : 3 : 4

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PROBLEM-SOLVING AND REASONING	9, 10, 14, 17	9–17	12–15, 18–20

9 Solve the following ratio problems.
 a The required staff to student ratio for an excursion is 2:15. If 340 people attended the excursion, how many teachers were there?
 b The ratio of commercials to actual show time for a particular TV channel is 2:3. How many minutes of actual show were there in 1 hour?
 c A rectangle has length and breadth dimensions in the ratio 3 : 1. If a particular rectangle has a perimeter of 21 m, what is its breadth?
 d Walter and William have a height ratio of 7:8. If their combined height is 285 cm, how tall is Walter?

10 Evergreen Fertiliser is made up of the three vital nutrients nitrogen, potassium and phosphorus in the ratio 4:5:3. How much of each nutrient is present in a 1.5 kg bag?

11 The angles of a triangle are in the ratio 2:3:4. Find the size of each angle.

12 Three friends, Cam, Holly and Seb, share a prize of $750 in the ratio 3:4:8. How much more money does Seb receive than Cam?

13 Trudy and Bella make 80 biscuits. If Trudy makes 3 biscuits in the time that Bella makes 2 biscuits, how many biscuits does Trudy make?

Example 8

14 In Year 8, the ratio of boys to girls is 5:7. If there are 140 girls in Year 8, what is the total number of students in Year 8?

15 A light rye bread requires a ratio of wholemeal flour to plain flour of 4:3. A baker making a large quantity of loaves uses 126 cups of plain flour. What is the total amount of flour used by the baker?

16 A textbook contains three chapters and the ratio of pages in these chapters is 3:2:5. If there are 24 pages in the smallest chapter, how many pages are in the textbook?

17 The ratio of the cost of a shirt to the cost of a jacket is 2:5. If the jacket costs $240 more than the shirt, find the cost of the shirt and the cost of the jacket.

18 In a class of 24 students the ratio of girls to boys is 1:2.
 a How many more girls are needed to make the ratio 1:1?
 b If 4 more girls and 4 more boys join the class, what will be the new ratio?
 c On one day, the ratio is 3:7. How many boys and how many girls are absent?

19 A right-angled triangle has its two acute angles in the ratio 1:2. What is the size of the smallest angle?

20 A triangle has angles in the ratio 1 : 1 : 4. What type of triangle is it and what are the sizes of its three angles?

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ENRICHMENT	–	–	21

A fair share

21 Three students, Ramshid, Tony and Manisha, enter a group Geography competition.

- Ramshid spends 5 hours preparing the PowerPoint presentation.
- Tony spends 3 hours researching the topic.
- Manisha spends $2\frac{1}{2}$ hours designing the poster.

Their group wins second place in the competition and receive a prize of $250.
Ramshid, Tony and Manisha decide to share the prize money in the ratio 3 : 2 : 1.

a How much money does each student receive? Round the answer to the nearest cent.

b If the prize money is divided up according to the time spent on the project, what would be the new ratio? Write the ratio in whole numbers.

c How much money does each student receive with the new ratio? Round the answer to the nearest cent.

d Although she spent the least time on the project, Manisha would prefer to divide the prize according to time spent. How much more money does Manisha receive with the new ratio?

e Tony prefers that the original ratio remains. How much is Tony better off using the original ratio?

f Which ratio would Ramshid prefer and why?

g The group ends up going with a ratio based on time spent but then round the amounts to the nearest $10. How much prize money does each student receive?

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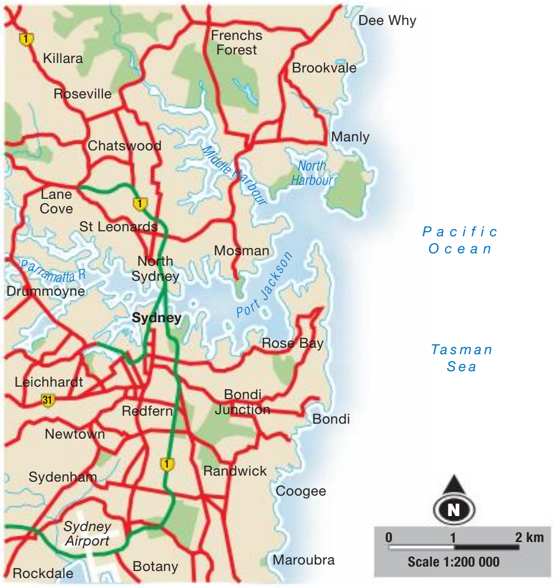
5D Scale drawings

Scale drawings are a special application of ratios. The purpose of a scale drawing is to provide accurate information about an object which has dimensions that are either too large or too small to be shown on a page.

When a scale drawing is exactly the same size as its real-life object, it has a scale of 1 : 1 and is congruent (i.e. identical) to the object it represents. However, in most cases this is impractical and so a smaller scale drawing or model is used. The map shows a scale to indicate the relationship of the map distance to the actual distance. The scale is displayed as a ratio.

Three common travel maps with their scales are shown below.



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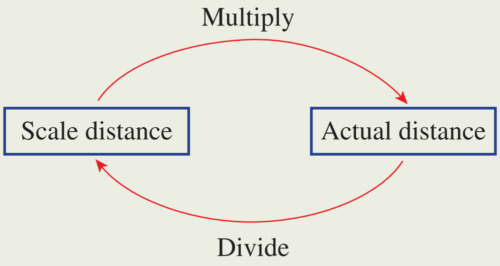
Let's start: Scaling down and scaling up

- Brainstorm specific examples where you have come across scale drawings.
  Think of examples where a scale drawing has been used to show very large objects or distances, and also think of examples where a scale drawing has been used to show very small objects.
- Share your list of examples with a classmate.
- As a class, make a list on the board of specific examples of scale drawings.
- What are some examples of common scales that are used?

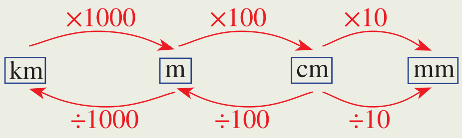
Key ideas

- A **scale drawing** has exactly the same shape as the original object, but a different size.
  All scale drawings should indicate the scale ratio.
- The scale on a drawing is written as a scale ratio:
  drawing length: actual length
  Drawing length represents the length on the diagram, map or model.
  Actual length represents the real length of the object.
- A **scale ratio** of 1 : 100 means the actual or real lengths are 100 times greater than the lengths measured on the diagram.
- A scale ratio of 20 : 1 means the scaled lengths on the model are 20 times greater than the actual or real lengths.
- It is helpful if scales begin with a 1. Then the second number in the ratio can be referred to as the **scale factor**. Scale ratios that do not start with a 1 can be converted using equivalent ratios.
- To convert a scaled distance to an actual distance, you multiply by the scale factor.
- To convert an actual (real) distance to a scaled distance, you divide by the scale factor.



- **Conversion of units**. It is important to remember how to correctly convert measurements of length when dealing with scales.

$$1 \text{ km} = 1000 \text{ m}$$
$$1 \text{ m} = 100 \text{ cm}$$
$$1 \text{ cm} = 10 \text{ mm}$$



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Example 9 Converting scale distance to actual distance

A map has a scale of 1 : 20 000.
Find the actual distance for each scaled distance.

a 2 cm **b** 5 mm **c** 14.3 cm

SOLUTION	EXPLANATION
a Actual distance = 2 cm × 20 000	Scale factor = 20 000
= 40 000 cm	Multiply scaled distance by scale factor.
= 400 m (or 0.4 km)	Convert answer into sensible units.
b Actual distance = 5 mm × 20 000	Multiply scaled distance by scale factor.
= 100 000 mm	Convert answer into sensible units.
= 100 m	
c Actual distance = 14.3 cm × 20 000	Multiply scaled distance by scale factor.
= 286 000 cm	Shortcut: × 2, then × 10 000.
= 2.86 km	Convert answer into sensible units.

Example 10 Converting actual distance to scaled distance

A model boat has a scale of 1 : 500.
Find the scaled length for each of these actual lengths.

a 50 m **b** 75 cm **c** 4550 mm

SOLUTION	EXPLANATION
a Scaled distance = 50 m ÷ 500	Scale factor = 500
= 5000 cm ÷ 500	Convert metres to centimetres.
= 10 cm (0.1 m)	Divide actual distance by scale factor.
	Convert answer into sensible units.
b Scaled distance = 75 cm ÷ 500	Convert centimetres to millimetres.
= 750 mm ÷ 500	Divide actual distance by scale factor.
= 1.5 mm	
c Scaled distance = 4550 mm ÷ 500	Divide actual distance by scale factor.
= 45.5 mm ÷ 5	Shortcut: ÷ 100, then ÷ 5 (or vice versa).
= 9.1 mm	

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Example 11 Determining the scale factor

State the scale factor in the following situations.

a A length of 4 mm on a scale drawing represents an actual distance of 50 cm.

b An actual length of 0.1 mm is represented by 3 cm on a scaled drawing.

SOLUTION	EXPLANATION
a Scale ratio $= 4\text{ mm} : 50\text{ cm}$	Write the ratio drawing length : actual length.
$= 4\text{ mm} : 500\text{ mm}$	Convert to ‘like’ units.
Scale ratio $= 4 : 500$	Write the scale ratio without units.
$= 1 : 125$	Divide both numbers by 4.
Scale factor $= 125$	Ratio is now in the form 1 : scale factor. The actual size is 125 times larger than the scaled drawing.
b Scale ratio $= 3\text{ cm} : 0.1\text{ mm}$	Write the ratio drawing length : actual length.
$= 30\text{ mm} : 0.1\text{ mm}$	Convert to ‘like’ units.
Scale ratio $= 30 : 0.1$	Write the scale ratio without units.
$= 300 : 1$	Multiply both numbers by 10.
$= 1 : \frac{1}{300}$	Divide both numbers by 300.
Scale factor $= \frac{1}{300}$	Ratio is now in the form 1 : scale factor. The actual size is 300 times smaller than the scaled drawing.

Exercise 5D

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 7	3, 4, 5–6(½), 7, 8(½)	5–8(½)

1 A map has a scale of 1 : 50 000, therefore how many centimetres in real life does 1 cm on the map equal?

2 A map has a scale of 1 : 100. How many metres on the map does 300 m in real life equal?

3 Convert 10 000 cm to:

a mm **b** m **c** km

4 Convert 560 m to:

a km **b** cm **c** mm

Example 9

5 Find the actual distance for each of the following scaled distances. Give your answer in an appropriate unit.

a	scale 1 : 10 000	**i** 2 cm	**ii** 4 mm	**iii** 7.3 cm
b	scale 1 : 20 000	**i** 80 cm	**ii** 18.5 mm	**iii** 1.25 m
c	scale 1 : 400	**i** 16 mm	**ii** 72 cm	**iii** 0.03 m
d	scale 1 : 300	**i** 5 mm	**ii** 8.2 cm	**iii** 7.1 m
e	scale 1 : 2	**i** 44 m	**ii** 310 cm	**iii** 2.5 mm
f	scale 1 : 5	**i** 4 m	**ii** 24 cm	**iii** 155 mm
g	scale 1 : 0.5	**i** 12 cm	**ii** 3.2 mm	**iii** 400 m
h	scale 100 : 1	**i** 3 cm	**ii** 11.5 km	**iii** 81.5 cm

Example 10

6 Find the scaled length for each of these actual lengths.

a scale 1 : 200 **i** 200 m **ii** 4 km **iii** 60 cm
b scale 1 : 500 **i** 10 000 m **ii** 1 km **iii** 75 cm
c scale 1 : 10 000 **i** 1350 m **ii** 45 km **iii** 736.5 m
d scale 1 : 20 000 **i** 12 km **ii** 1800 m **iii** 400 mm
e scale 1 : 250 000 **i** 5000 km **ii** 750 000 m **iii** 1250 m
f scale 1 : 10 **i** 23 m **ii** 165 cm **iii** 6.15 m
g scale 1 : 0.1 **i** 30 cm **ii** 5 mm **iii** 0.2 mm
h scale 200 : 1 **i** 1 mm **ii** 7.5 cm **iii** 8.2 mm

Example 11

7 Determine the scale ratio for each of the following.

a 2 mm on a scale drawing represents an actual distance of 50 cm.
b 4 cm on a scale drawing represents an actual distance of 2 km.
c 1.2 cm on a scale drawing represents an actual distance of 0.6 km.
d 5 cm on a scale drawing represents an actual distance of 900 m.
e An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.
f An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

8 Convert the two measurements provided in each scale into the same unit, and then write the scale as a ratio of two numbers in simplest form.

a 2 cm : 200 m **b** 5 mm : 500 cm **c** 12 mm : 360 cm
d 4 mm : 600 m **e** 4 cm : 5 m **f** 1 cm : 2 km
g 28 mm : 2800 m **h** 3 cm : 0.6 mm **i** 1.1 m : 0.11 mm

PROBLEM-SOLVING AND REASONING

9–11, 15	10–13, 15–17	11–14, 17–19

9 A model city has a scale ratio of 1 : 1000.

a Find the actual height of a skyscraper that has a scaled height of 8 cm.
b Find the scaled length of a train platform that is 45 m long in real life.

10 Blackbottle and Toowoola are 17 cm apart on a map with a scale of 1 : 50 000. How far apart are the towns in real life?

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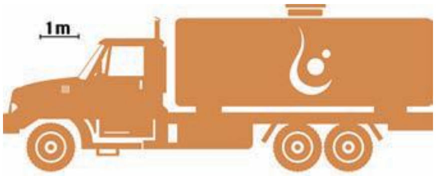
11 Using the house plans shown, state the actual dimensions of the following rooms.

a bedroom 1

b family room

c patio

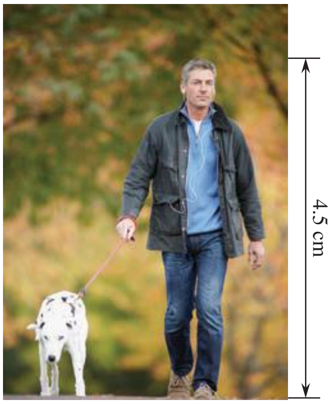
12 From the scaled drawing below, calculate the actual length and height of the truck, giving your answer to the nearest metre.



13 Jackson enjoying a walk with his dog. Jackson is 1.8 m tall in real life.

a What is the scale of the photograph?

b How tall is Jackson's dog in real life?



14 A scale length of $5\frac{1}{2}$ cm is equal to an actual distance of 44 km.

a How many kilometres does a scale length of 3 cm equal?

b How many kilometres does a scale length of 20 cm equal?

c How many centimetres does an actual distance of 100 km equal?

15 Group the six ratios below into pairs of equivalent ratios.

1 : 0.01 25 : 1 20 : 1 1 : 0.05 100 : 1 50 : 2

16 Which of the following scales would be most appropriate if you wanted to make a scale model of:

a a car?

b your school grounds?

c Mt Kosciuszko?

A 1 : 10 000 **B** 1 : 1000 **C** 1 : 100 **D** 1 : 10

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17 A cartographer (map maker) wants to make a map of a mountanious area that is 64 km^2 (side length of 8 km). She decides to use a scale of 1 : 1000. Describe the problem she is going to have.

18 A scientist says that the image of a 1 mm long insect is magnified by a scale of 1 : 1000 through his magnifying glass. Explain what might be wrong with the scientist's ratio.

19 Obtain a map of Australia.

- **a** What is the scale of your map?
- **b** Which two capital cities in Australia are the furthest apart? State the distance.
- **c** Which two capital cities in Australia are closest together? State the distance.
- **d** Which capital city is furthest away from Sydney? State the distance.
- **e** Which capital city is closest to Adelaide? State the distance.
- **f** Check the accuracy of your answers on the internet.

ENRICHMENT	–	–	20

Scaled drawing

20 Design a scaled drawing of one of the following.

- your classroom and the associated furniture
- your bedroom and the associated furniture
- a room in your house (bathroom, family room, garage …)

Your scaled drawing should fit comfortably onto an A4 page.

- **a** Measure the dimensions of your chosen room and the dimensions of an A4 page, and determine the most appropriate scale for your diagram.
- **b** Show the size and location of doors and windows, where appropriate.
- **c** Produce a second scale diagram of your room, but show a different arrangement of the furniture.

Can you produce the scale diagram on a computer software package?

5E Introducing rates

Interactive
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Walkthrough

If you were to monitor what you said each day, you might well find that you speak about rates many times per day!

A **ratio** shows the relationship between the same type of quantities with the same units, but a **rate** shows the relationship between two different types of quantities with different units.

The following are all examples of rates.

- Cost of petrol is $1.45 per litre.
- Rump steak is on special for $18/kg.
- Dad drove to school at an average speed of 52 km/h.
- After the match, your heart rate was 140 beats/minute.

Unlike ratios, a **rate** compares different types of quantities, and so units must be shown.
For example:

- The ratio of boys to girls in a school is 4 : 5.
- The average rate of growth of an adolescent boy is 12 cm/year.

Let's start: State the rate

For each of the following statements, write down a corresponding rate.

- The Lodges travelled 400 km in 5 hours.
- Gary is paid $98 for a 4-hour shift at work.
- Felicity spent $600 on a two-day shopping spree.
- Max has grown 9 cm in the past three months.
- Vuong paid $37 for half a cubic metre of crushed rock.
- Paul cycled a total distance of 350 km for the week.

What was the rate (in question/minute) at which you answered these questions?

Key ideas

- Ratios compare quantities measured in the same units, whereas rates compare quantities measured in different units.
- All rates must include units for each quantity.
- The two different units are separated by a solidus or slash '/', which is the mathematical symbol for 'per'. For example: 20 km/h = 20 km per hour = 20 km for each hour
- It is convention to write rates in their simplest form. This involves writing the rate for only one unit of the second quantity.

  For example: spent $45 in 3 hours

  $$\begin{array}{rcll} & = & \text{\$45 in 3 hours} & \leftarrow \text{Non-simplified rate} \\ \div 3 & & & \div 3 \\ & = & \text{\$15 in 1 hour} & \leftarrow \text{Simplified rate (\$15/h)} \end{array}$$

- The **average rate** is calculated by dividing the total change in one quantity by the total change in the second quantity.

  For example: reading a 400-page book in 4 days

  $$\begin{array}{rcll} \text{Average reading rate} & = & \text{400 pages in 4 days} & \\ \div 4 & & & \div 4 \\ & = & \text{100 pages in 1 day} & \end{array}$$

  Therefore, average reading rate = 100 pages/day.

Example 12 Writing simplified rates

Express each of the following as a simplified rate.

a 12 students for two teachers

b $28 for 4 kilograms

SOLUTION

a 6 students/teacher

b $7/kg

EXPLANATION

$$\begin{tikzcd} \text{12 students for 2 teachers} \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ \text{6 students for 1 teacher} \end{tikzcd}$$

$$\begin{tikzcd} \text{\$28 for 4 kg} \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ \text{\$7 for 1 kg} \end{tikzcd}$$

Example 13 Finding average rates

Tom was 120 cm tall when he turned 10 years old. He was 185 cm tall when he turned 20 years old. Find Tom's average rate of growth per year between 10 and 20 years of age.

SOLUTION

$$\begin{tikzcd} \text{65 cm in 10 years} \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ \text{6.5 cm in 1 year} \end{tikzcd}$$

Average rate of growth = 6.5 cm/year

EXPLANATION

Growth = 185 − 120 = 65 cm

Divide both numbers by 10.

Exercise 5E

UNDERSTANDING AND FLUENCY	1–3, 4	3, 4(½), 5	4–5(½)

1 Which of the following are examples of rates?

A $5.50
B 180 mL/min
C $60/h
D $\frac{5}{23}$
E 4.2 runs/over
F 0.6 g/L
G 200 cm^2
H 84c/L

2 Match each rate in the first column with its most likely rate in the second column.

employee's wage	90 people/day
speed of a car	$2100/m^2
cost of building a new home	68 km/h
population growth	64 beats/min
resting heart rate	$15/h

3 Write typical units for each of the following rates.

a price of sausages
b petrol costs
c typing speed
d goal conversion rate
e energy nutrition information
f water usage in the shower
g pain relief medication
h cricket team's run rate

Example 12

4 Write each of the following as a simplified rate.

- **a** 12 days in 4 years
- **b** 15 goals in 3 games
- **c** \$180 in 6 hours
- **d** \$17.50 for 5 kilograms
- **e** \$126 000 to purchase 9 acres
- **f** 36 000 cans in 8 hours
- **g** 12 000 revolutions in 10 minutes
- **h** 80 mm rainfall in 5 days
- **i** 60 minutes to run 15 kilometres
- **j** 15 kilometres run in 60 minutes

Example 13

5 Find the average rate of change for each situation.

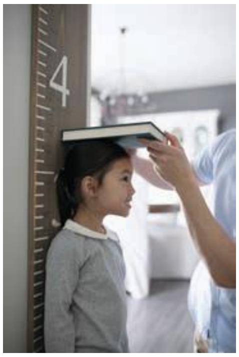
- **a** Relma drove 6000 kilometres in 20 days.
- **b** Fiora saved \$420 over three years.
- **c** A cricket team scored 78 runs in 12 overs.
- **d** Saskia grew 120 centimetres in 16 years.
- **e** Russell gained 6 kilograms in 4 years.
- **f** The temperature dropped 5°C in 2 hours.

PROBLEM-SOLVING AND REASONING	6–9, 12	8–10, 12, 13	9–13

6 A dripping tap fills a 9-litre bucket in 3 hours.

- **a** What is the dripping rate of the tap in litres/hour?
- **b** How long will it take the tap to fill a 21-litre bucket?

7 Martine grew an average rate of 6 cm/year for the first 18 years of her life. If Martine was 50 cm long when she was born, how tall was Martine when she turned 18?



8 If 30 salad rolls are bought to feed 20 people at a picnic, and the total cost is \$120, find the following rates.

- **a** salad rolls/person
- **b** cost/person
- **c** cost/roll

9 The number of hours of sunshine was recorded each day for one week in April. The results are listed.

Monday 6 hours, Tuesday 8 hours, Wednesday 3 hours, Thursday 5 hours, Friday 7 hours, Saturday 6 hours, Sunday 7 hours

- **a** Find the average number of hours of sunshine:
  - **i** per weekday
  - **ii** per weekend day
  - **iii** per week
  - **iv** per day
- **b** Given the rates above, how many hours of sunshine would you expect in April?

10 Harvey finished a 10 kilometre race in 37 minutes and 30 seconds. Jacques finished a 16 kilometre race in 53 minutes and 20 seconds. Calculate the running rate of each runner. Which runner has a faster running pace?

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11 The Tungamah Football Club had 12 000 members. After five successful years and two premierships, they now have 18 000 members.

- **a** What has been the average rate of membership growth per year for the past 5 years?
- **b** If this membership growth rate continues, how many more years will it take for the club to have 32 000 members?

12 **a** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:

  - **i** km/L
  - **ii** L/km

- **b** How can you convert km/L to L/km?

13 The Teleconnect telecommunications company has a variable call charge rate for phone calls of up to 30 minutes. The charges are 50c/min for the first 10 minutes, 75c/min for the second 10 minutes and $1/min for the third 10 minutes.

- **a** Find the cost of phone calls of these given lengths.
  - **i** 8 minutes
  - **ii** 13 minutes
  - **iii** 24 minutes
  - **iv** 30 minutes
- **b** What is the average charge rate per minute for a 30-minute call?

Connectplus, a rival telecommunications company, charges a constant call rate of 60c/minute.

- **c** If you normally make calls that are 15 minutes long, which company has the better deal for you?
- **d** If you normally make calls that are 25 minutes long, which company has the better deal for you?
- **e** What is the length of phone call for which both companies would charge the same amount?

ENRICHMENT	–	–	14

Target 155

14 In Victoria, due to drought conditions, the State government urged all residents to save water. The goal was set for each Victorian to use no more than 155 litres of water per day.

- **a** How many people live in your household?
- **b** According to the Victorian government, how many litres of water can your household use per day?
- **c** Perform some experiments and calculate the following rates of water flow.
  - **i** shower rate (L/min)
  - **ii** washing machine (L/load)
  - **iii** hose (L/min)
  - **iv** toilet (L/flush, L/half flush)
  - **v** running tap (L/min)
  - **vi** drinking water (L/day)
  - **vii** dishwasher (L/wash)
  - **viii** water for food preparation (L/day)
- **d** Estimate the average daily rate of water usage for your household.
- **e** Ask your parents for a recent water bill and find out what your family household water usage rate was for the past three months.

Before the initiative, Victorians were using an average of 164 litres/day/person. Twelve months after the initiative, Victorians were using 151 litres/day/person.

- **f** How much water per year for the state of Victoria does this saving of 13 litres/day/person represent?
- **g** What is the rate at which your family is charged for its water?

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5F Ratios and rates and the unitary method

Interactive
Widgets
HOTsheets
Walkthrough

The concept of solving problems using the unitary method was introduced in Chapter 4. The unitary method involves finding the value of 'one unit'. This is done by dividing the amount by the given quantity. Once the value of 'one unit' is known, multiplying will find the value of the number of units required.

Let's start: Finding the value of '1 unit'

For each of the following, find the value of one unit or one item.

- eight basketballs cost \$200.
- four cricket bats cost \$316.
- 5 kg of watermelon cost \$7.50.

For each of the following, find the rate per one unit.

- Car travelled 140 km in 2 hours.
- 1000 L of water leaked out of the tank in 8 hours.
- \$51 was the price paid for 3 kg of John Dory fish.

For each of the following, find the value of one 'part'.

- Ratio of books to magazines read was 2 : 5. Milli has read 14 books.
- Ratio of pink to red flowers is 7 : 11. A total of 330 red flowers are in bloom.
- Ratio of girls to boys is 8 : 5. There are 40 girls in a group.

Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to solve the problem.
- When dealing with **ratios**, find the value of one 'part' of the ratio.
  For example: Ratio of cars to motorbikes is 35 : 2 and there are six motorbikes.

$$\begin{tikzcd} 2\text{ parts} = 6\text{ motorbikes} \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ 1\text{ part} = 3\text{ motorbikes} \end{tikzcd}$$

- When dealing with **rates**, find the value of the rate per one 'unit'.
  For example: Pedro earned \$64 for a 4-hour shift at work.
  Therefore, wage rate = \$64 per 4 hours = \$16 per hour = \$16/h.
- Once the value of one 'part' or the rate per one 'unit' is known, the value of any number of parts or units can be found by multiplying.
- The technique of dividing and/or multiplying values in successive one-step calculations can be applied to the concept of converting rates from a set of given units to a different set of units.

Example 14 Reviewing the unitary method

Andrea travels 105 km in seven identical car trips from home to school. How far would she travel in 11 such car trips?

SOLUTION

$$\begin{tikzcd} 7 \text{ car trips} = 105 \text{ km} \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\ 1 \text{ car trip} = 15 \text{ km} \arrow[d, bend right, "\times 11"'] \arrow[d, bend left, "\times 11"] \\ 11 \text{ car trips} = 165 \text{ km} \end{tikzcd}$$

Andrea travels 165 km.

EXPLANATION

Find the value of 1 unit by dividing both quantities by 7.

Solve the problem by multiplying both quantities by 11.

Example 15 Best buys

The same water is sold in two different-sized bottles:

750 mL for \$5 or 1.25 L for \$6.20

What is the difference between their ‘per litre’ price?

SOLUTION

$$\begin{tikzcd} 750 \text{ mL for } \$5 \arrow[d, bend right, "\div 7.5"'] \arrow[d, bend left, "\div 7.5"] \\ 100 \text{ mL for } \$0.667 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ 1000 \text{ mL } (1\text{ L}) \text{ for } \$6.67 \end{tikzcd}$$

\$6.67 per L (to nearest cent)

1.25 L for \$6.20

$$\begin{tikzcd} 1250 \text{ mL for } \$6.20 \arrow[d, bend right, "\div 12.5"'] \arrow[d, bend left, "\div 12.5"] \\ 100 \text{ mL for } \$0.496 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ 1000 \text{ mL } (1 \text{ L}) \text{ for } \$4.96 \end{tikzcd}$$

$\$6.67 - \$4.96 = \$1.71$

The difference is \$1.71 per litre.

EXPLANATION

$750 \div 100 = 7.5$

Calculate the price for 100 mL.

Multiply by 10.

$1250 \div 100 = 12.5$

Calculate the price for 100 mL.

Multiply by 10.

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Example 16 Solving rate problems using the unitary method

A truck uses 4 L of petrol to travel 36 km. How far will it travel if it uses 70 L of petrol?

SOLUTION

Rate of petrol consumption

36 km for 4 L
÷4 ÷4
9 km for 1 L
×70 ×70
630 km for 70 L

Truck will travel 630 km on 70 L.

EXPLANATION

Find the petrol consumption rate of 1 unit by dividing both quantities by 4.
Solve the problem by multiplying both quantities by 70.

Example 17 Converting units using the unitary method

Melissa works at the local supermarket and earns $57.60 for a 4-hour shift.
How much does she earn in c/min?

SOLUTION

Wage = $57.60 for 4 hours
÷4 ÷4
$14.40 for 1 hour
1440c mL for 60 minutes
÷60 ÷60
24c for 1 minute

Melissa earns 24c/min.

EXPLANATION

Write down Melissa's wage rate.
Find the rate of $ per 1 hour.

Convert $ to cents, and hours to minutes.
Divide rate by 60 to find rate of cents per minute.

Exercise 5F

UNDERSTANDING AND FLUENCY	1–6, 8	4–7, 8(½)	5–6(½), 7, 8(½)

Example 14

1 If 8 kg of chicken fillets cost $72, how much would 3 kg of chicken fillets cost?

2 If one dozen tennis balls cost $9.60, how much would 22 tennis balls cost?

3 If three pairs of socks cost $12.99, how much would 10 pairs of socks cost?

4 If 500 g of mince meat costs $5.50, how much would 4 kg of mince meat cost?

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Example 15

5 **a** A bag of ice costs \$3.99. Three bags cost \$10. How much extra does it cost to buy three bags rather than two?

b One shop sells 2 litres of soft drink for \$4.00. It is not cold. Another shop sells it cold for \$6.60. How much extra, per litre, is the second shop charging?

c The normal price for a 700 gram can of dog food is \$2.19. When it is on special you can buy five cans for \$9. Is this cheaper 'per kilogram' than buying the 1.2 kg can for \$3.69?

d A convenience store sells a 255 gram box of Dodo Pops for \$4.79. In the supermarket you can buy 650 grams for \$7. How much is this, in dollars per kilogram, in each shop?

e Fifty grams of Monaco Coffee costs \$5.59. How much (per kilogram) is saved by buying 400 grams for \$18?

f Bottles of juice sell for \$3.49 each for 345 mL. A box that holds 24 bottles costs \$37.95. How much cheaper is the 'per litre' price of the box?

Example 16

6 Solve the following rate problems.

a A tap is dripping at a rate of 200 mL every 5 minutes. How much water drips in 13 minutes?

b A professional footballer scores an average of three goals every six games. How many goals is he likely to score in a full season of 22 games?

c A snail travelling at a constant speed travels 400 mm in 8 minutes. How far does it travel in 7 minutes?

d A computer processor can process 500 000 kilobytes of information in 4 seconds. How much information can it process in 15 seconds?

7 Leonie, Spencer and Mackenzie have just won a prize. They decide to share it in the ratio 4 : 3 : 2. If Spencer receives \$450, how much do Leonie and Mackenzie receive, and what was the total value of the prize?

Example 17

8 Convert the following rates into units given in the brackets.

a \$15/h (c/min)
b \$144/h (c/s)
c 3.5 L/min (L/h)
d 20 mL/min (L/h)
e 0.5 kg/month (kg/year)
f 120 g/day (kg/week)
g 60 g/c (kg/\$)
h \$38/m (c/mm)
i 108 km/h (m/s)
j 14 m/s (km/h)

PROBLEM-SOLVING AND REASONING	9, 10, 14a	10–14	10–15

9 The Mighty Oats breakfast cereal is sold in boxes of three different sizes: small (400 g) for \$5.00, medium (600 g) for \$7.20, large (750 g) for \$8.25

a Find the value of each box in \$/100 g.

b What is the cheapest way to buy a minimum of 4 kg of the cereal?

10 Gemma runs 70 metres in 8.4 seconds. If she maintains the same average speed, in what time will she run 100 metres?

11 Zana's hair grew 6 cm in 5 months.

a Find Zana's average rate of hair growth in cm/month and in m/year.

b How long would it take for Zana's hair to grow 30 cm?

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12 Brontë can paint 15 m^2 in 20 minutes.

- **a** What is the rate at which Brontë paints in m^2/h?
- **b** What area can Brontë paint in 20 hours?
- **c** Brontë must paint 1000 m^2 in 20 hours. Find the rate at which she will need to paint in m^2/min.

13 If x donuts cost $\$y$:

- **a** How much would one donut cost?
- **b** How much would one dozen donuts cost?
- **c** How much would z donuts cost?

14 **a** A triangle has side lengths in the ratio $3:5:4$. If the longest side is 35 cm, find the lengths of the other two sides and the perimeter of the triangle.

- **b** A triangle has side lengths in the ratio $x:(x+3):(x-2)$. If the shortest side is 17 cm, find the lengths of the other two sides and the perimeter of the triangle.

15 In a faraway galaxy, a thriving alien colony uses the following units.

For money they have puks and paks: 1 puk (pu) = 1000 pak (pa)
For length they have doits and minidoits: 1 doit (D) = 80 minidoits (mD)

Polynaute rope is priced at 4 pu/D. Find the cost of the rope in terms of pa/mD.

ENRICHMENT	–	–	16

Where will we meet?

16 Phil lives in Perth and his friend Werner lives in Sydney. The distance, by road, between their two houses is 4200 km (rounded to the nearest 100 km).

Phil decides to drive to Sydney and Werner decides to drive to Perth. They leave home at the same time and travel the same route, but in opposite directions.

Phil drives at a constant speed of 75 km/h and Werner drives at a constant speed of 105 km/h.

- **a** Will they meet on the road at a spot closer to Sydney or closer to Perth?
- **b** How long will it take Phil to travel to Sydney?
- **c** How long will it take Werner to travel to Perth?
- **d** State the location of each friend after they have been driving for 15 hours.
- **e** At what location (distance from Sydney and/or Perth) will they meet?

When they meet, Phil decides to travel in Werner's car and they drive back to Werner's home at an average speed of 105 km/h.

- **f** How long does it take Phil to travel to Sydney?
- **g** Design a similar problem for two friends travelling at different constant speeds between two different capital cities in Australia.

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5G Solving rate problems

Interactive
Widgets
HOTsheets
Walkthrough

'One thing that is certain in life is that change is inevitable.'

We are constantly interested in rates of change and how things change over a period of time. We are also regularly faced with problems involving specific rates. Strong arithmetic skills and knowing whether to multiply or divide by the given rate allows many rate problems to be solved quickly and accurately.

Over the next few days, keep a record of any rates you observe or experience. Look out for the solidus or slash '/' sign and listen for the 'per' word.

Let's start: Estimate the rate

For each of the following statements, estimate a corresponding rate.

- **Commercial rate:** The number of commercials on TV per hour
- **Typing rate:** Your typing speed in words per minute
- **Laughing rate:** The number of times a teenager laughs per hour
- **Growth rate:** The average growth rate of a child from 0 to15 years of age
- **Running rate:** Your running pace in metres per second
- **Homework rate:** The number of subjects with homework per night
- **Clapping rate:** The standard rate of audience clapping in claps per minute
- **Thank you rate:** The number of opportunities to say thank you per day

Compare your rates. Which rate is the 'highest'? Which rate is the 'lowest'? Discuss your answers.

Key ideas

- When a rate is provided, a change in one quantity implies that an equivalent change must occur in the other quantity.
  For example: Patrick earns \$20/hour. How much will he earn for working 6 hours?

$$\begin{tikzcd}
\$20 \text{ for } 1 \text{ hour} \arrow[d, bend right, "\times 6"'] \arrow[d, bend left, "\times 6"] \\
\$120 \text{ for } 6 \text{ hours}
\end{tikzcd}$$

  For example: Patrick earns \$20/hour. How long will it take him to earn \$70?

$$\begin{tikzcd}
\$20 \text{ for } 1 \text{ hour} \arrow[d, bend right, "\times 3\frac{1}{2}"'] \arrow[d, bend left, "\times 3\frac{1}{2}"] \\
\$70 \text{ for } 3\frac{1}{2} \text{ hours}
\end{tikzcd}$$

- Carefully consider the units involved in each question and answer.

Example 18 Solving rate problems

a Rachael can touch type at 74 words/minute. How many words can she type in 15 minutes?

b Leanne works in a donut van and sells, on average, 60 donuts every 15 minutes. How long is it likely to take her to sell 800 donuts?

SOLUTION

a

$$\begin{tikzcd} \text{74 words in 1 minute} \arrow[d, bend right, "\times 15"'] \arrow[d, bend left, "\times 15"] \\ \text{1110 words in 15 minutes} \end{tikzcd}$$

Rachael can type 1110 words in 15 minutes.

b

$$\begin{tikzcd} \text{60 donuts in 15 minutes} \arrow[d, bend right, "\div 15"'] \arrow[d, bend left, "\div 15"] \\ \text{4 donuts in 1 minute} \arrow[d, bend right, "\times 200"'] \arrow[d, bend left, "\times 200"] \\ \text{800 donuts in 200 minutes} \end{tikzcd}$$

Leanne is likely to take 3 hours and 20 minutes to sell 800 donuts.

EXPLANATION

$$\begin{array}{r} 74 \\ \times 15 \\ \hline 370 \\ +740 \\ \hline 1110 \end{array}$$

Selling rate = 60 donuts/15 minutes

Divide both quantities by HCF of 15.

Multiply both quantities by 200.

Convert answer to hours and minutes.

Example 19 Solving combination rate problems

Three water hoses from three different taps are used to fill a large swimming pool. The first hose alone takes 200 hours to fill the pool. The second hose alone takes 120 hours to fill the pool and the third hose alone takes only 50 hours to fill the pool. How long would it take to fill the pool if all three hoses are used?

SOLUTION

In 600 hours:

Hose 1 would fill 3 pools.

Hose 2 would fill 5 pools.

Hose 3 would fill 12 pools.

Therefore, in 600 hours the three hoses together would fill 20 pools.

$$\text{Filling rate} = \begin{tikzcd} \text{600 hours for 20 pools} \arrow[d, bend right, "\div 20"'] \arrow[d, bend left, "\div 20"] \\ \text{30 hours for 1 pool} \end{tikzcd}$$

It would take 30 hours to fill the pool if all three hoses are used.

EXPLANATION

LCM of 200, 120 and 50 is 600.

Hose 1 = 600 h ÷ 200 h/pool = 3 pools

Hose 2 = 600 h ÷ 120 h/pool = 5 pools

Hose 3 = 600 h ÷ 50 h/pool = 12 pools

Together = 3 + 5 + 12 = 20 pools filled.

Simplify rate by dividing by HCF.

Exercise 5G

UNDERSTANDING AND FLUENCY	1–7	2–8	4–8

1 Fill in the gaps.

a
$$\begin{tikzcd} \text{60 km in 1 hour} \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ \text{180 km in \_\_\_\_} \end{tikzcd}$$

b
$$\begin{tikzcd} \text{\$25 in 1 hour} \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\_\_\_\_"] \\ \text{\$125 in \_\_\_\_} \end{tikzcd}$$

c
$$\begin{tikzcd} \text{7 questions in 3 minutes} \arrow[d, bend right, "\_\_\_\_"'] \arrow[d, bend left, "\_\_\_\_"] \\ \text{70 questions in \_\_\_\_} \end{tikzcd}$$

d
$$\begin{tikzcd} \text{120 litres in 1 minute} \arrow[d, bend right, "\_\_\_\_"'] \arrow[d, bend left, "\_\_\_\_"] \\ \text{\_\_\_\_ in 6 minutes} \end{tikzcd}$$

2 Fill in the gaps.

a
$$\begin{tikzcd} \text{\$36 for 3 hours} \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ \text{\_\_\_\_ for 1 hour} \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\_\_\_\_"] \\ \text{\_\_\_\_ for 5 hours} \end{tikzcd}$$

b
$$\begin{tikzcd} \text{150 rotations in 5 minutes} \arrow[d, bend right, "\_\_\_\_"'] \arrow[d, bend left, "\_\_\_\_"] \\ \text{\_\_\_\_\_\_\_\_ in 1 minute} \arrow[d, bend right, "\_\_\_\_"'] \arrow[d, bend left, "\_\_\_\_"] \\ \text{\_\_\_\_\_\_\_\_ in 7 minutes} \end{tikzcd}$$

Example 18a

3 A factory produces 40 plastic bottles/minute.

a How many bottles can the factory produce in 60 minutes?

b How many bottles can the factory produce in an 8 hour day of operation?

4 Mario is a professional home painter. When painting a new home, he uses an average of 2.5 litres of paint per hour. How many litres of paint would Mario use in a week if he paints for 40 hours?

Example 18b

5 A truck travels 7 km per litre of fuel. How many litres are needed for the truck to travel 280 km?

6 Daniel practises his guitar for 40 minutes every day.

a How many days will it take him to log up 100 hours of practice?

b How many days will it take him to log up 10 000 hours of practice?

c If Daniel practises six days per week for 50 weeks per year, how many years will it take him to log up 10 000 hours of practice?

7 A flywheel rotates at a rate of 1500 revolutions per minute.

a How many revolutions does the flywheel make in 15 minutes?

b How many revolutions does the flywheel make in 15 seconds?

c How long does it take for the flywheel to complete 15 000 revolutions?

d How long does it take for the flywheel to complete 150 revolutions?

8 Putra is an elite rower. When training, he has a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.

a How many times does Putra's heart beat during a 30 minute workout?

b How many times does Putra's heart beat during 30 minutes of 'rest'?

c If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

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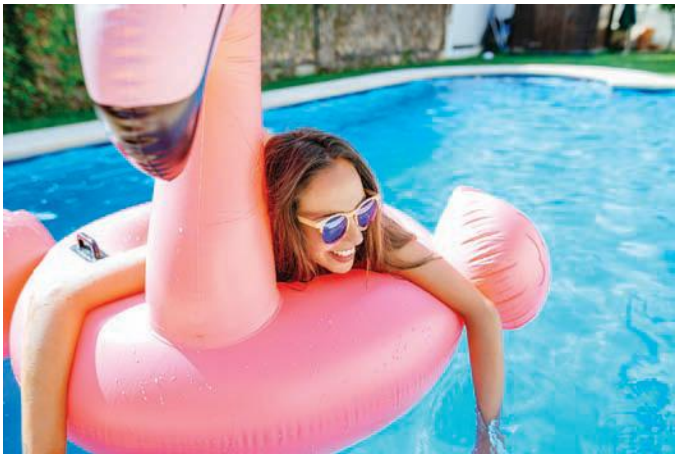
PROBLEM-SOLVING AND REASONING	9, 10, 13	10–15	10–12, 14–17

9 What is the cost of paving a driveway that is 18 m long and 4 m wide, if the paving costs \$35 per square metre?

10 A saltwater swimming pool requires 2 kg of salt to be added for every 10 000 litres of water. Joan's swimming pool is 1.5 metres deep, 5 metres wide and 15 metres long. How much salt will she need to add to her pool?



11 The Bionic Woman gives Batman a 12 second start in a 2 kilometre race. If the Bionic Woman runs at 5 km/min, and Batman runs at 3 km/min, who will win the race and by how many seconds?

12 At a school camp there is enough food for 150 students for five days.

a How long would the food last if there were only 100 students?

b If the food ran out after only four days, how many students attended the camp?

Example 19

13 Michelle can complete a landscaping job in six days and Danielle can complete the same job in four days. Working together, in how many days could they complete the job?

14 Three bricklayers, Maric, Hugh and Ethan, are cladding a new home. If Maric works alone, the job would take him eight days to complete. If Hugh works alone, the job would take him six days to complete, and if Ethan works by himself, the job would take him 12 days to complete.

a If the three men work together, how long will it take them to complete the job?

b What fraction of the house will each bricklayer complete?

15 Four cans of dog food will feed three dogs for one day.

a How many cans are needed to feed 10 dogs for six days?

b How many dogs can be fed for nine days from 60 cans?

c For how many days will 40 cans feed two dogs?

16 If it takes four workers, 4 hours to dig four holes, how long would it take two workers to dig six holes?

17 State the units required for the answer to each of the following rate calculations.

a \$205/kg × 48 kg

b 62 s × 12 m/s

c 500 beats ÷ 65 beats/min (bpm)

d 4000 revolutions ÷ 120 rev/min

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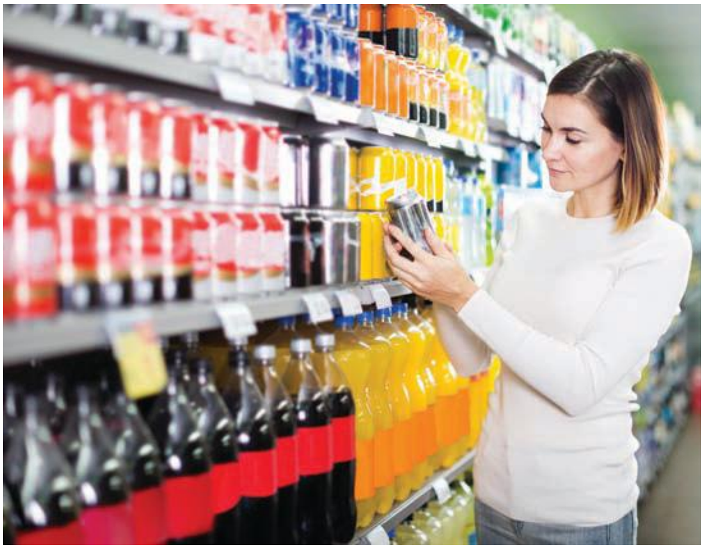
ENRICHMENT	–	–	18

Value for money

18 Soft drink can be purchased from supermarkets in a variety of sizes.
Below are the costs for four different sizes of a certain brand of soft drink.



600 mL 'buddies'	1.25 L bottles	2 L bottles	10 × 375 mL cans
\$2.70 each	\$1.60 each	\$2.20 each	\$6.00 per pack

a Find the economy rate (in \$/L) for each size of soft drink.

b Find and compare the cost of 30 litres of soft drink purchased entirely in each of the four different sizes.

c If you have only \$60 to spend on soft drink for a party, what is the difference between the greatest amount and the least amount of soft drink you could buy? Assume you have less than \$1.60 left.

Most supermarkets now include on the price tag the economy rate of each item to allow customers to compare value for money.

d Carry out some research at your local supermarket on the economy rates of a particular food item with a range of available sizes (such as drinks, breakfast cereals, sugar, flour). Write a report on your findings.

5H Speed

Interactive
Widgets
HOTsheets
Walkthrough

A rate that we come across almost every day is speed. **Speed** is the rate of distance travelled per unit of time.

On most occasions, speed is not constant and therefore we are interested in the average speed of an object. Average speed is calculated by dividing the distance travelled by the time taken.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Given the average speed formula, we can tell that all units of speed must have a unit of length in the numerator, followed by a unit of time in the denominator. Therefore, 'mm/h' is a unit of speed and could be an appropriate unit for the speed of a snail!

The most common units of speed are m/s and km/h.

Let's start: Which is faster?

With a partner, determine the faster of the two listed alternatives.

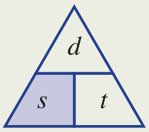
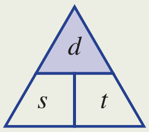
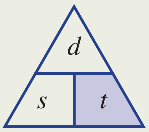
a	Car A travelling at 10 m/s	Car B travelling at 40 km/h
b	Walker C travelling at 4 km/h	Walker D travelling at 100 m/min
c	Jogger E running at 1450 m/h	Jogger F running at 3 m/s
d	Plane G flying at 700 km/h	Plane H flying at 11 km/min

Key ideas

- **Speed** is a measure of how fast an object is travelling.
- If the speed of an object does not change over time, the object is travelling at a **constant speed**. 'Cruise control' helps a car travel at a constant speed.
- When speed is not constant, due to acceleration or deceleration, we are often interested to know the **average speed** of the object.
- Average speed is calculated by the formula:

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}} \quad \text{or} \quad s = \frac{d}{t}$$

- Depending on the unknown value, the formula above can be rearranged to make d or t the subject. The three formulas involving s, d, and t are:

$s = \frac{d}{t}$	$d = s \times t$	$t = \frac{d}{s}$
		

- Care must be taken with units for speed, and sometimes units will need to be converted. The most common units of speed are m/s and km/h.

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Example 20 Finding average speed

Find the average speed in km/h of:

a a cyclist who travels 140 km in 5 hours

b a runner who travels 3 km in 15 minutes

SOLUTION

a $s = \frac{d}{t}$

$= \frac{140\text{ km}}{5\text{ h}}$

$= 28\text{ km/h}$

Alternative unitary method:

$$\begin{tikzcd} \text{140 km in 5 hours} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ \text{28 km in 1 hour} \end{tikzcd}$$

Average speed = 28 km/h

b $s = \frac{d}{t}$

$= \frac{3\text{ km}}{15\text{ min}}$

$= 3 \div \frac{1}{4}$

$= 3 \times 4 = 12\text{ km/h}$

Alternative unitary method:

$$\begin{tikzcd} \text{3 km in 15 minutes} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ \text{12 km in 60 minutes} \end{tikzcd}$$

Average speed = 12 km/h

EXPLANATION

The unknown value is speed.

Write the formula with s as the subject.

Distance travelled = 140 km

Time taken = 5 h

Speed unit is km/h.

Write down the rate provided in the question.

Divide both quantities by 5.

Distance travelled = 3 km

Time taken = 15 min or $\frac{1}{4}$ h

Dividing by $\frac{1}{4}$ is the same as multiplying by $\frac{4}{1}$.

Write down the rate provided in the question.

Multiply both quantities by 4.

Example 21 Finding the distance travelled

Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.

SOLUTION

$d = s \times t$

$= 95\text{ km/h} \times 15\text{ h}$

$= 1425\text{ km}$

Alternative unitary method:

$$\begin{tikzcd} \text{95 km in 1 hour} \arrow[d, bend right, "\times 15"'] \arrow[d, bend left, "\times 15"] \\ \text{1425 km in 15 hours} \end{tikzcd}$$

Truck travels 1425 km in 15 hours.

EXPLANATION

The unknown value is distance.

Write the formula with d as the subject.

Distance unit is km.

Write the rate provided in the question.

Multiply both quantities by 15.

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Example 22 Finding the time taken

Find the time taken by a hiker walking at 4 km/h to travel 15 km.

SOLUTION

$$\begin{aligned} t &= \frac{d}{s} \\ &= \frac{15 \text{ km}}{4 \text{ km/h}} \\ &= 3.75 \text{ h} \\ &= 3 \text{ h } 45 \text{ min} \end{aligned}$$

Alternative unitary method:

$$\begin{tikzcd} 4 \text{ km} \arrow[d, bend right, "\div 4"'] & \text{in } 1 \text{ hour} \arrow[d, bend left, "\div 4"] \\ 1 \text{ km} \arrow[d, bend right, "\times 15"'] & \text{in } \frac{1}{4} \text{ hour} \arrow[d, bend left, "\times 15"] \\ 15 \text{ km} & \text{in } \frac{15}{4} \text{ hour} \end{tikzcd}$$

It takes 3 h 45 min to travel 15 km.

EXPLANATION

The unknown value is time.

Write the formula with t as the subject.

The time unit is h.

Leave answer as a decimal or convert to hours and minutes.

$0.75 \text{ h} = 0.75 \times 60 = 45 \text{ min}$

Express the rate as provided in the question.

Divide both quantities by 4.

Multiply both quantities by 15.

Example 23 Converting units of speed

a Convert 72 km/h to m/s.

b Convert 8 m/s to km/h.

SOLUTION

a 72 km in 1 hour

$$\begin{tikzcd} 72\,000 \text{ m} \arrow[d, bend right, "\div 60"'] & \text{in } 60 \text{ minutes} \arrow[d, bend left, "\div 60"] \\ 1200 \text{ m} & \text{in } 1 \text{ minute} \end{tikzcd}$$

$$\begin{tikzcd} 1200 \text{ m} \arrow[d, bend right, "\div 60"'] & \text{in } 60 \text{ seconds} \arrow[d, bend left, "\div 60"] \\ 20 \text{ m} & \text{in } 1 \text{ second} \end{tikzcd}$$

$\therefore$ 72 km/h = 20 m/s

b 8 m in 1 second

$$\begin{tikzcd} 8 \text{ m} \arrow[d, bend right, "\times 60"'] & \text{in } 1 \text{ second} \arrow[d, bend left, "\times 60"] \\ 480 \text{ m} \arrow[d, bend right, "\times 60"'] & \text{in } 1 \text{ minute} \arrow[d, bend left, "\times 60"] \\ 28\,800 \text{ m} & \text{in } 1 \text{ hour} \end{tikzcd}$$

$\therefore$ 28.8 km in 1 hour

EXPLANATION

Express rate in kilometres per hour.

Convert km to m and hour to minutes.

Divide both quantities by 60.

Convert 1 minute to 60 seconds.

Divide both quantities by 60.

Shortcut for converting km/h → m/s ÷ 3.6.

Express rate in metres per second.

Multiply by 60 to find distance in 1 minute.

Multiply by 60 to find distance in 1 hour.

Convert metres to kilometres.

Shortcut: m/s × 3.6 → km/h

$8 \text{ m/s} \times 3.6 = 28.8 \text{ km/h}$

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Exercise 5H

UNDERSTANDING AND FLUENCY	1–9	3, 4, 5(½), 7, 8, 10	5–9(½), 10

1 Which of the following is not a unit of speed?

A m/s **B** km/h **C** cm/h **D** L/kg **E** m/min

2 If average speed $= \dfrac{\text{distance travelled}}{\text{time taken}}$ then the distance travelled must equal:

A average speed × time taken **B** $\dfrac{\text{average speed}}{\text{time taken}}$ **C** $\dfrac{\text{time taken}}{\text{average speed}}$

3 If average speed $= \dfrac{\text{distance travelled}}{\text{time taken}}$ then time taken must equal:

A distance travelled × average speed **B** $\dfrac{\text{distance travelled}}{\text{average speed}}$ **C** $\dfrac{\text{average speed}}{\text{distance travelled}}$

4 If an object travels 800 metres in 10 seconds, the average speed of the object is:

A 8000 m/s **B** 800 km/h **C** 80 km/h **D** 80 m/s

Example 20

5 Calculate the average speed of:

a a sprinter running 200 m in 20 seconds
b a skateboarder travelling 840 m in 120 seconds
c a car travelling 180 km in 3 hours
d a truck travelling 400 km in 8 hours
e a train travelling 60 km in 30 minutes
f a tram travelling 15 km in 20 minutes

Example 21

6 Calculate the distance travelled by:

a a cyclist travelling at 12 m/s for 90 seconds
b an ant travelling at 2.5 m/s for 3 minutes
c a bushwalker who has walked for 8 hours at an average speed of 4.5 km/h
d a tractor ploughing fields for 2.5 hours at an average speed of 20 km/h

Example 22

7 Calculate the time taken by:

a a sports car to travel 1200 km at an average speed of 150 km/h
b a bus to travel 14 km at an average speed of 28 km/h
c a plane to fly 6900 km at a constant speed of 600 km/h
d a ball moving through the air at a speed of 12 m/s to travel 84 m

Example 23a

8 Convert the following speeds to m/s.

a 36 km/h
b 180 km/h
c 660 m/min
d 4 km/s

Example 23b

9 Convert the following speeds to km/h.

a 15 m/s
b 2 m/s
c 12 m/min
d 1 km/s

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10 Complete the following table.

	Speed (s)	Distance (d)	Time (t)
a		50 km	2 h
b	30 m/s	1200 m	
c	5 km/h		12 h
d		600 m	5 min
e	210 km/h		20 min
f	100 m/s	10 km	

PROBLEM-SOLVING AND REASONING	11, 12, 17	12–15, 17, 18	13–16, 18, 19

11 A plane is flying at a cruising speed of 900 km/h. How far will the plane travel from 11:15 a.m. to 1:30 p.m. on the same day?

12 The wheels on Charlie's bike have a circumference of 1.5 m. When Charlie is riding fastest, the wheels rotate at a speed of five turns per second.

a What is the fastest speed Charlie can ride his bike, in km/h?

b How far would Charlie travel in 5 minutes at his fastest speed?

13 The back end of a 160 metre-long train disappears into a 700 metre-long tunnel. Twenty seconds later the front of the train emerges from the tunnel. Determine the speed of the train in m/s.

14 Anna rode her bike to school one morning, a distance of 15 km, at an average speed of 20 km/h. It was raining in the afternoon, so Anna decided to take the bus home. The bus trip home took 30 minutes. What was Anna's average speed for the return journey to and from school?

15 In Berlin 2009, Jamaican sprinter Usain Bolt set a new 100 m world record time of 9.58 seconds. Calculate Usain Bolt's average speed in m/s and km/h for this world record. (Round the answers correct to one decimal place.)

16 The Ghan train is an Australian icon. You can board The Ghan in Adelaide and 2979 km later, after travelling via Alice Springs, you arrive in Darwin. (Round the answers correct to one decimal place.)

a If you board The Ghan in Adelaide on Sunday at 2:20 p.m. and arrive in Darwin on Tuesday at 5:30 p.m., what is the average speed of the train journey?

b There are two major rest breaks. The train stops for $4\frac{1}{4}$ hours at Alice Springs and 4 hours at Katherine. Taking these breaks into account, what is the average speed of the train when it is moving?

17 Write two speeds in km/h that are between 40 m/s and 45 m/s.

18 Nina, Shanti and Belle run a 1000 m race at a constant speed. When Nina crosses the finish line first, she is 200 m ahead of Shanti and 400 m ahead of Belle. When Shanti crosses the finish line, how far ahead of Belle is she?

19 Julie and Jeanette enjoy finishing their 6 km morning run together. Julie runs at an average speed of 10 km/h and Jeanette runs at an average speed of 3 m/s. If Julie leaves at 8 a.m., at what time should Jeanette leave if they are to finish their run at the same time?

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ENRICHMENT	–	–	20

Speed research

20 Carry out research to find answers to the following questions.

Light and sound

a What is the speed of sound in m/s?
b What is the speed of light in m/s?
c How long would it take sound to travel 100 m?
d How long would it take light to travel 100 km?
e How many times quicker is the speed of light than the speed of sound?
f What is a Mach number?

Spacecraft

g What is the escape velocity needed by a spacecraft to 'break free' of Earth's gravitational pull? Give this answer in km/h and also km/s.
h What is the orbital speed of planet Earth? Give your answer in km/h and km/s.
i What is the average speed of a space shuttle on a journey from Earth to the International Space Station?

Knots

Wind speed and boat speed are often given in terms of knots (kt).

j What does a knot stand for?
k What is the link between nautical miles and a system of locating positions on Earth?
l How do you convert a speed in knots to a speed in km/h?

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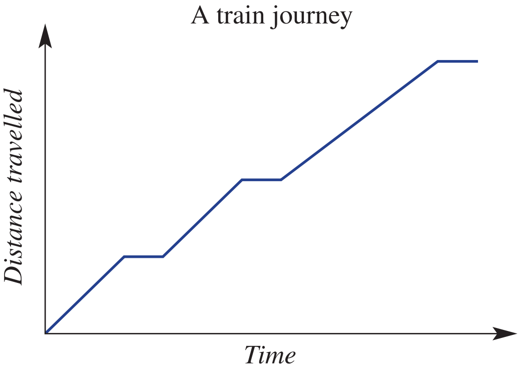
5I Distance–time graphs

Distance–time graphs show the distance on the vertical axis and the time on the horizontal axis. When an object moves at a constant speed, then the graph of distance vs time will be a straight line segment.
The steepness of a line segment shows the speed of that part of the journey. A flat line segment shows that there is no movement for a given time. Several different line segments joined together can make up a total journey.

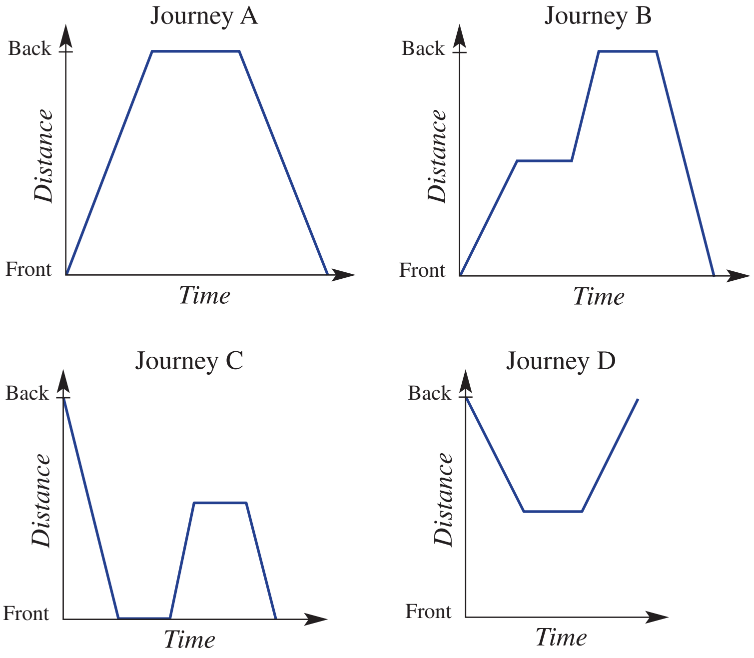
For example, a train journey could be graphed with a series of sloping line segments showing travel between stations and flat line segments showing when the train has stopped at stations.



Note: the acceleration and deceleration phases are not shown on this simplified distance–time graph.

Let's start: Matching graphs and journeys

A teacher draws four distance–time graphs on the whiteboard and some students volunteer to follow a graph for a walking journey between the front and back of the classroom. Work in pairs and match each graph with the student who correctly walks that journey.



- Ella walks from the back of the room, stops for a short time and then turns around and walks to the back of the room again.
- Jasmine walks from the front of the room to the back of the room, where she stops for a short time. She then turns and walks to the front of the room.
- Lucas walks from the back to the front of the room, where he stops for a short time. He then turns and walks halfway to the back, briefly stops and then turns again and walks to the front.
- Riley walks from the front of the room, stops briefly partway and then completes his walk to the back of the room, where he stops for a short time. He then turns and walks to the front.

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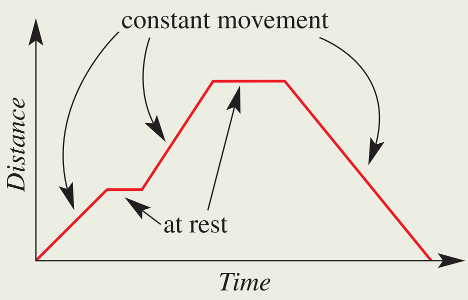
Key ideas

- To draw a graph of a journey, use time on the horizontal axis and distance on the vertical axis.
- Graphs of distance versus time usually consist of **line segments**.

A distance–time graph



- Each segment shows whether the object is moving or at rest.
- The steepness of a line segment shows the speed.
- Steeper lines show greater speed than less steep lines. Horizontal lines show that the person or vehicle is stationary.
- $\text{Speed} = \frac{\text{distance}}{\text{time}}$

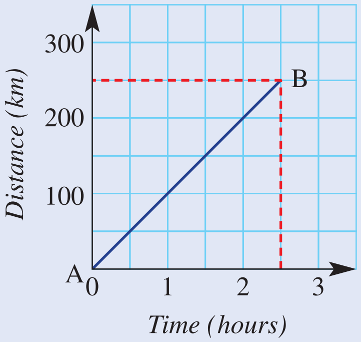
Example 24 Reading information from a graph

This distance–time graph shows the journey of a car from one town (A) to another (B).

a How far did the car travel?

b How long did it take the car to complete the journey?

c What was the average speed of the car?



SOLUTION	EXPLANATION
a 250 km	Draw an imaginary line from point B to the vertical axis; i.e. 250 km.
b 2.5 hours	Draw an imaginary line from point B to the horizontal axis; i.e. 2.5 hours.
c $s = \frac{d}{t}$ $= \frac{250\text{ km}}{2.5\text{ h}}$ $= 100\text{ km/h}$	$\text{Speed} = \frac{\text{distance}}{\text{time}}$ Distance = 250 km Time = 2.5 hours

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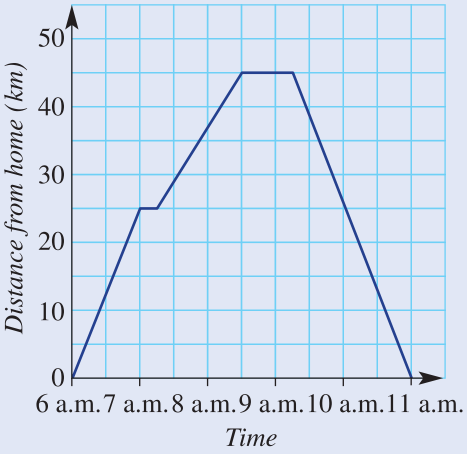
Example 25 Calculations from a distance–time graph

This distance–time graph shows Noah's cycle journey from home one morning.

a Find Noah's speed in the first hour of the journey.
b How many minutes was Noah's first rest break?
c Find Noah's speed between 7:15 a.m. and 8:30 a.m.
d At what time did Noah start and finish his second rest break?
e At approximately what time had Noah ridden a total of 65 km?
f Find Noah's speed on the return journey, to one decimal place.
g What was Noah's average speed for the whole journey?



SOLUTION	EXPLANATION
a 25 km/h	The first hour is 6 a.m. to 7 a.m. Noah cycles 25 km in one hour.
b 15 minutes	The first horizontal line segment is at 7 a.m. to 7:15 a.m.
c $s = \frac{d}{t}$ $= \frac{20\text{ km}}{1.25\text{ h}}$ $= 16$ km/h	Distance travelled = 45 km − 25 km Time taken = 7:15 a.m. to 8:30 a.m. = 1.25 hours
d 8:30 a.m. to 9:15 a.m.	A horizontal line segment shows that Noah is stopped.
e 10 a.m.	45 km plus 20 km of the return trip or 25 km from home.
f $s = \frac{d}{t}$ $= \frac{45\text{ km}}{1.75\text{ h}}$ $= 25.7$ km/h	Distance travelled = 45 km Time taken = 9:15 a.m. to 11 a.m. = 1.75 hours
g $s = \frac{d}{t}$ $= \frac{90\text{ km}}{5\text{ h}}$ $= 18$ km/h	Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$ Distance travelled $= 2 \times 45 = 90$ km Time taken = 6 a.m. to 11 a.m. = 5 hours

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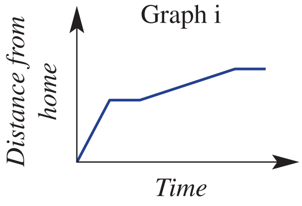
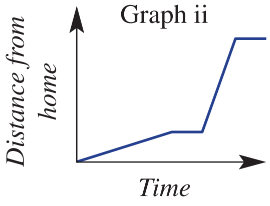
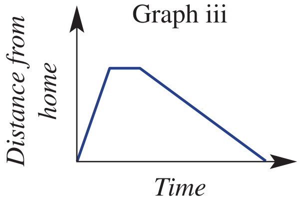
Exercise 5I

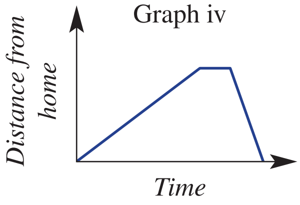
UNDERSTANDING AND FLUENCY	1–5	3–6	4–6

1 Each of these distance–time graphs show a person’s journey from home. Match each graph with the correct description of that journey.









Journey A: A man walks slowly away from home, stops for a short time, then continues walking away from home at a fast pace and finally stops again.

Journey B: A girl walks slowly away from home and then stops for a short time. She then turns around and walks at a fast pace back to her home.

Journey C: A boy walks quickly away from home, briefly stops, then continues slowly walking away from home and finally stops again.

Journey D: A woman walks quickly away from home and then stops briefly. She then turns around and slowly walks back home.

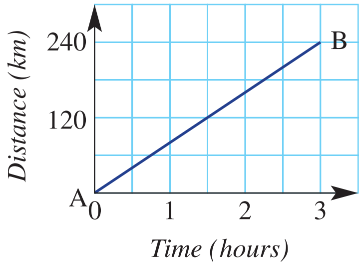
Example 24

2 This graph shows a train journey from one town (A) to another town (B).

a How far did the train travel?

b How long did it take to complete this journey?

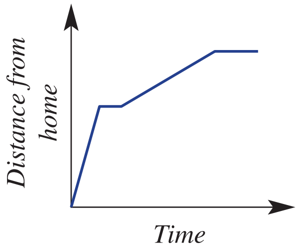
c What was the average speed of the train?



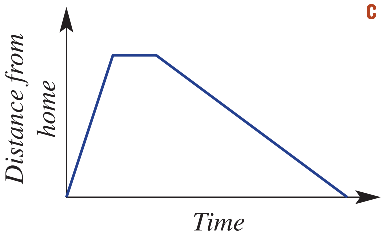
3 Each of these distance–time graphs shows a person’s journey. Using sentences, explain the meaning of each straight line segment, describing whether the person:

- travelled slowly or quickly or was stopped
- travelled away from home or towards home

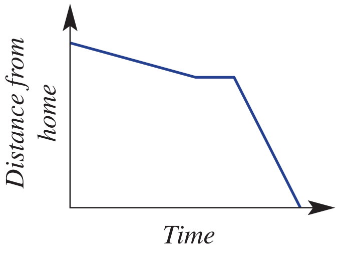
a



b



c



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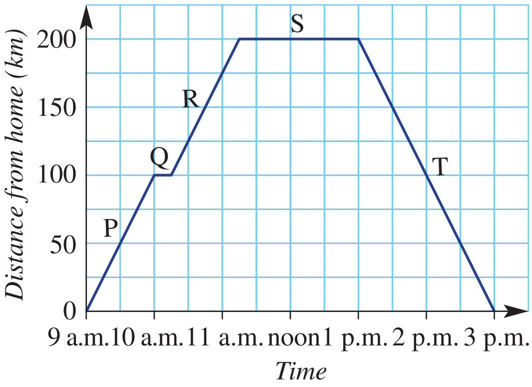
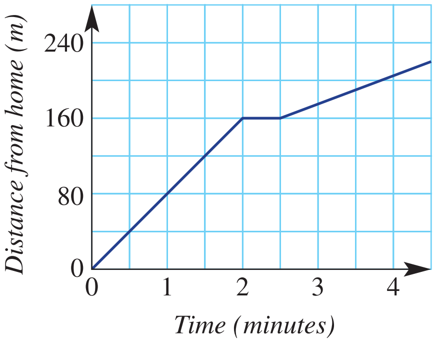
4 The Wilson family drove from their home to a relative's place for lunch and then back home again. This distance–time graph shows their journey.



For each description below, choose the line segment of the graph that matches it.

- **a** A $\frac{1}{4}$ hour rest break is taken from 10 a.m. to 10:15 a.m.
- **b** The Wilson's drove 100 km in the first hour.
- **c** The car is stopped for $1\frac{3}{4}$ hours.
- **d** The car travels from 100 km to 200 km away from home in this section of the journey.
- **e** At the end of this segment, the Wilsons start driving back to their home.
- **f** The Wilsons travel from 1 p.m. to 3 p.m. without stopping.
- **g** At the end of this segment, the Wilsons have arrived back home again.

5 This distance–time graph shows Levi's walk from his home to school one morning.



- **a** How far did Levi walk in the first minute?
- **b** How long did it take Levi to walk 120 m from home?
- **c** How far had Levi walked when he stopped to talk to a friend?
- **d** How long did Levi stop for?
- **e** How far had Levi walked after 3 minutes?
- **f** How far was the school from Levi's home?

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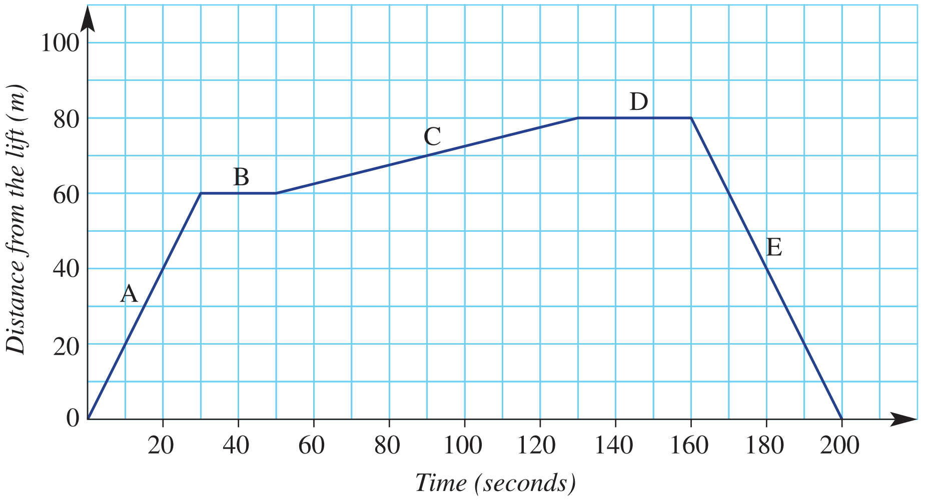
Example 25

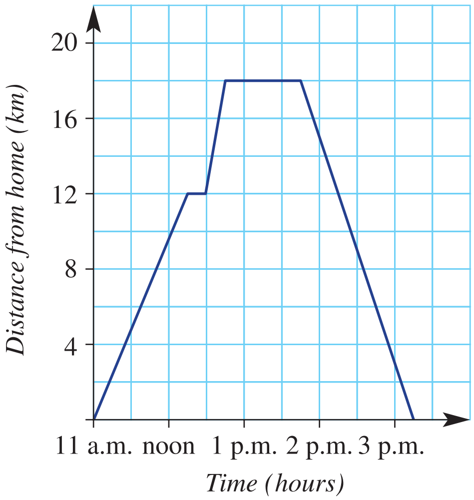
6 This distance–time graph shows Ava's short walk in a shopping centre from the lift and back to the lift.



- **a** Which line segments show that Ava was stopped?
- **b** For how long was Ava not walking?
- **c** What distance had Ava travelled by 20 seconds?
- **d** How long did Ava take to walk 70 m?
- **e** When did Ava turn around to start to walk back towards the lift?
- **f** At what times was Ava 60 m from the lift?
- **g** What was the total distance that Ava walked from the lift and back again?
- **h** For which section of Ava's walk does the line segment have the flattest slope? What does this tell you about Ava's speed for this section?
- **i** For which section of Ava's walk is the line segment steepest? What does this tell you about Ava's speed for this section?

PROBLEM-SOLVING AND REASONING	7, 10, 11	7–12	10–14

7 Enzo cycles from home to a friend's place. After catching up with his friend he then cycles back home. On the way to his friend's place, Enzo stopped briefly at a shop. His journey is shown on this distance–time graph.



- **a** At what time did Enzo arrive at the shop?
- **b** Find Enzo's speed (to one decimal place) between his home and the shop.
- **c** Find Enzo's speed when cycling between the shop and his friend's place.
- **d** What time did Enzo arrive at and leave his friend's place?
- **e** What was the total time that Enzo was stopped on this journey?
- **f** At what times was Enzo 12 km from home?
- **g** At what time had Enzo ridden a total of 30 km?
- **h** At what speed did Enzo travel when returning home from his friend's place?
- **i** Calculate Enzo's average speed (to one decimal place) over the whole journey.

8 Draw a separate distance–time travel graph for each of these journeys. Assume each section of a journey was at a constant speed.

a A motor bike travelled 200 km in 2 hours.

b A car travelled 100 km in 1 hour, stopped for $\frac{1}{2}$ hour and then travelled a further 50 km in $\frac{1}{2}$ hour.

c In a 90 minute journey, a cyclist travelled 18 km in 45 minutes, stopped for 15 minutes and then returned home.

9 A train travels 5 km in 8 minutes, stops at a station for 2 minutes, travels 12 km in 10 minutes, stops at another station for 2 minutes and then completes the journey by travelling 10 km in 15 minutes.

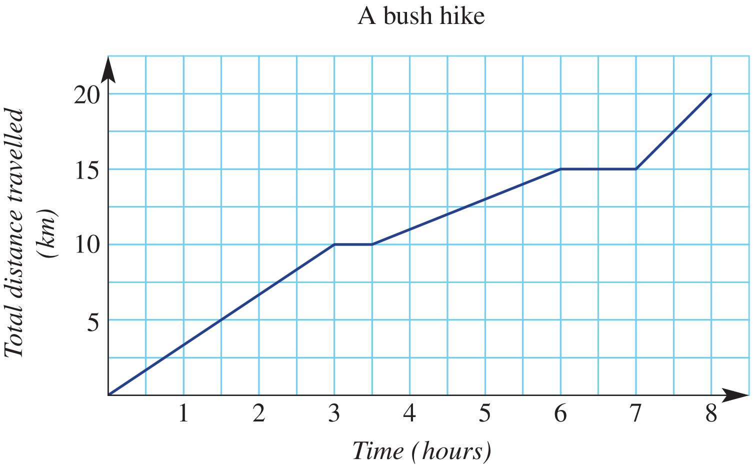
a Calculate the speed of the train in km/h (to one decimal place) for each section of the journey.

b Explain why the average speed over the whole journey is not the average of these three speeds.

c Calculate the average speed of the train in km/h (to one decimal place) over the whole journey.

d On graph paper, draw an accurate distance–time graph for the journey.

10 A one day, 20 km bush hike is shown by this graph of 'total distance travelled' versus time.



a What were the fastest and slowest speeds (in km/h) of the hikers? Suggest a feature of the hiking route that could have made these speeds so different.

b Write a story of the journey shown by this graph. In your story, use sentences to describe the features of the hike shown by each straight line segment of the graph, including the time taken and distance travelled. Also include a sentence comparing the average speeds for the different sections of the hike.

c Suppose the hikers turn back after their $\frac{1}{2}$ hour rest at 10 km and return to where they started from. Redraw the graph above, showing the same sections with the same average speeds but with 'distance from start' vs time.

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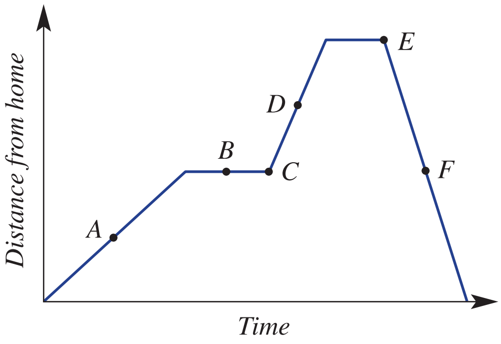
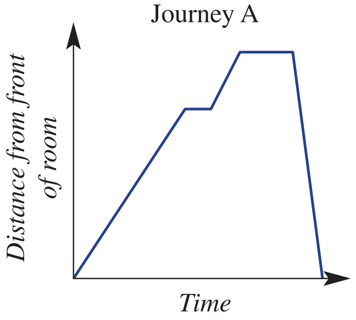
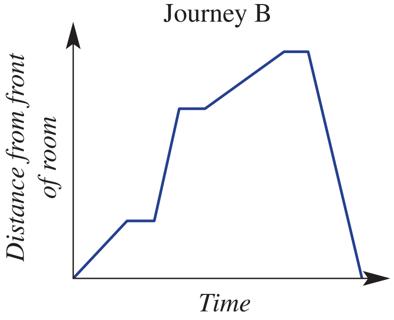
11 This distance–time graph shows Deanna's bike journey from home one morning. Choose true or false for each of these statements and give a reason for your answer.

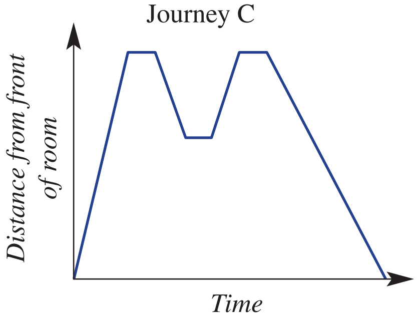


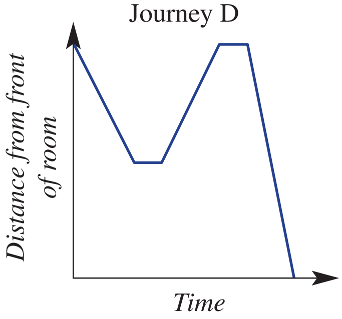
- **a** Deanna was the same distance from home at B and F.
- **b** Deanna was not cycling at B.
- **c** Deanna was cycling faster at A than she was at D.
- **d** Deanna was facing the same direction at C and F.
- **e** Deanna was cycling faster at F than she was at A.
- **f** Deanna was further from home at F than at D.
- **g** Deanna had ridden further at F than at D.

12 Match each distance–time graph with the student below who correctly walked the journey shown by that graph. Explain the reasons for your choice.


Journey A


Journey B


Journey C


Journey D

Adam: Walks from the front, stops, a fast walk, stops, slow walk to the back, stops, and walks to the front.

Max: Walks from the back, stops, walks to the front, stops, walks to the back.

Ruby: Walks from the back, stops halfway, turns, walks to the back, stops, walks to the front.

Conner: Fast walk from front to back, stops, walks towards the front, stops, walks to the back, stops, slower walk to the front.

Isla: Walks from the front, stops, walks to the back, stops, walks at a fast pace to the front.

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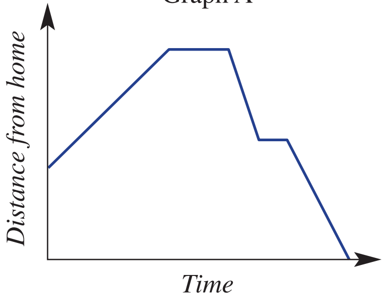
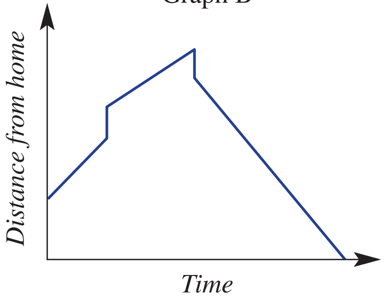
13 One afternoon, Sienna cycled from school to a friend's place, then to the library and finally she cycled home. Which of these distance–time graphs are appropriate to show Sienna's journey? Explain why.

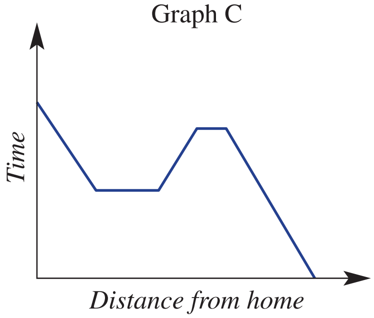
Graph A



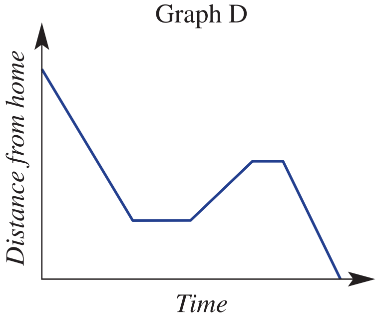
Graph B



Graph C

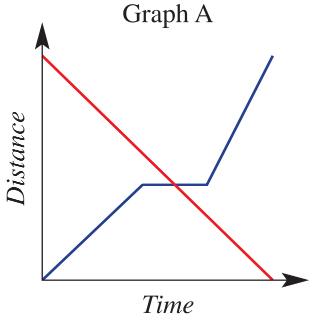


Graph D

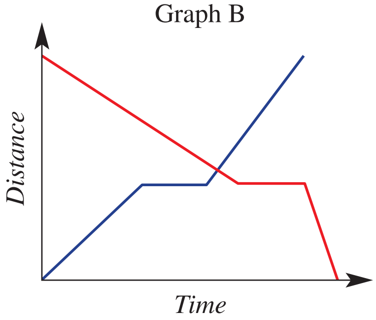


14 Jayden and Cooper cycled towards each other along the same track. Jayden was resting when Cooper caught up to him and stopped for a chat, and then they both continued their ride. Which of these distance–time graphs show their journeys? Explain why.

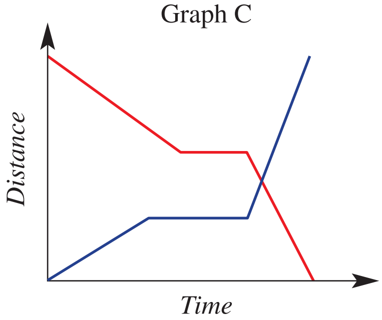
Graph A



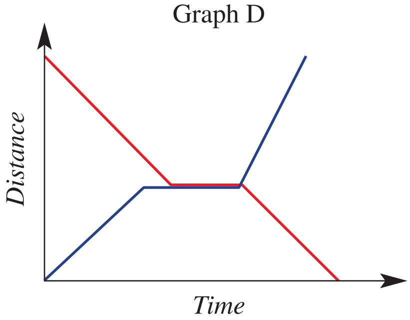
Graph B



Graph C



Graph D



ENRICHMENT	–	–	15

More than one journey

15 Northbrook is 160 km north of Gurang. Archie leaves Gurang to drive north and at the same time Heidi leaves Northbrook to drive south. After 40 minutes, Archie is halfway to Northbrook when his car collides with a tree. Five minutes later Heidi sees Archie's car and stops. Heidi immediately calls an ambulance, which comes from 15 km away and arrives in 10 minutes.

- **a** Using graph paper, draw a distance–time graph to show the journey of the two cars and the ambulance.
- **b** A tow-truck and the police also arrive at the accident. Write the rest of the story and graph the completed journey for the all vehicles involved that day. Use colours and include a key.
- **c** Explain the journeys shown in your graph to a classmate.
- **d** Would any of the drivers have been charged for speeding if it was a 110 km/h speed limit?

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Fun run investigation

Three maths teachers, Mrs M, Mr P and Mr A, trained very hard to compete in a Brisbane Fun Run. The 10.44 km route passed through the botanical gardens, along the Brisbane River to New Farm Park, and back again.

Their times and average stride lengths were:

- Mrs M: 41 minutes, 50 seconds: 8 strides per 9 m
- Mr P: 45 minutes, 21 seconds: 7 strides per 9 m
- Mr A: 47 minutes, 6 seconds: 7.5 strides per 9 m

Copy and complete the following table and determine which rates are the most useful representations of fitness. Give the answers rounded to one decimal place. Justify your conclusions.

Running rates	Mrs M	Mr P	Mr A	World record 10 km runners	Your family member or friend
Seconds per 100 m					
Seconds per km					
Metres per minute					
Km per hour					
Strides per 100 m					
Strides per minute					
Strides per hour					

Fitness investigation

1 Using a stopwatch, measure your resting heart rate in beats per minute.

2 Run on the spot for one minute and then measure your working heart rate in beats per minute.

3 Using a stopwatch, time yourself for a 100 m run and also count your strides. At the end, measure your heart rate in beats per minute. Also calculate the following rates.
  - your running rate in m/s, m/min and km/h
  - your running rate in time per 100 m and time per km
  - your rate of strides per minute, strides per km and seconds per stride

4 Run 100 m four times without stopping and, using a stopwatch, record your cumulative time after each 100 m.
  - Organise these results into a table.
  - Draw a graph of the distance you ran (vertical) vs time taken (horizontal).
  - Calculate your running rate in m/min for each 100 m section.
  - Calculate your overall running rate in m/min for the 400 m.
  - Explain how and why your running rates changed over the 400 m.

5 Try sprinting really fast over a measured distance and record the time. Calculate your sprinting rate in each of the following units.
  - minutes per 100 m
  - metres per minute
  - time per km
  - km per hour

6 Research the running rate of the fastest schoolboy and schoolgirl in Australia. How do their sprinting rates compare with the running rates of Australian Olympian athletes?

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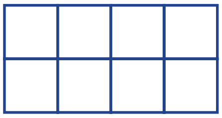
1 This diagram is made up of eight equal-sized squares.



How many squares need to be shaded if the ratio of shaded squares to unshaded squares is:

a 1 : 3? **b** 2 : 3? **c** 1 : 2?

2 Bottle A has 1 L of cordial drink with a cordial to water ratio of 3 : 7. Bottle B has 1 L of cordial drink with a cordial to water ratio of 1 : 4. The drink from both bottles is combined to form a 2 L drink. What is the new cordial to water ratio?

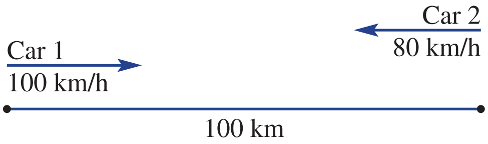
3 Brothers Marco and Matthew start riding from home into town, which is 30 km away. Marco rode at 10 km/h and Matthew took 20 minutes longer to complete the trip. Assuming that they both rode at a constant speed, how fast was Matthew riding?

4 **a** If one person takes 1 hour to dig one post hole, how long will it take two people to dig two post holes?

b If three people take 3 hours to make three wooden train sets, how many train sets can six people make in 6 hours?

5 At a market you can trade two cows for three goats or three goats for eight sheep. How many sheep are three cows worth?

6 Two cars travel toward each other on a 100 km straight stretch of road. They leave at opposite ends of the road at the same time. The cars' speeds are 100 km/h and 80 km/h. How long does it take for the cars to pass each other?



7 A river is flowing downstream at a rate of 1 km/h. In still water Portia can swim at a rate of 2 km/h. Portia dives into the river and swims downstream, then turns around and swims back to the starting point, taking 0.5 hours in total. How far did she swim?

8 Henrik walks at 4 km/h for time t_1, and then runs at 7 km/h for time t_2. He travels a total of 26 km. If he had run for the same time that he had walked (t_1) and walked for the same time that he had run (t_2), then he would have travelled 29 km. What was the total time Henrik spent walking and running?

9 A top secret goo has been developed so that it doubles in volume every 60 seconds. At midnight a small amount is placed in a beaker and its growth is observed. At 1 a.m. the beaker is full. At exactly what time did the goo fill only $\frac{1}{8}$ of the beaker?

10 A car averages 60 km/h for 20 km and then increases its speed, averaging 80 km/h for the next 20 km. Calculate the car's average speed (to two decimal places) for the full 40 km trip.

11 Shari travels at 50 km/h for 10 km. For the next 10 km she wants to drive fast enough to make her average speed 100 km/h over the 20 km trip. Advise Shari about reaching this average speed.

Puzzles and challenges

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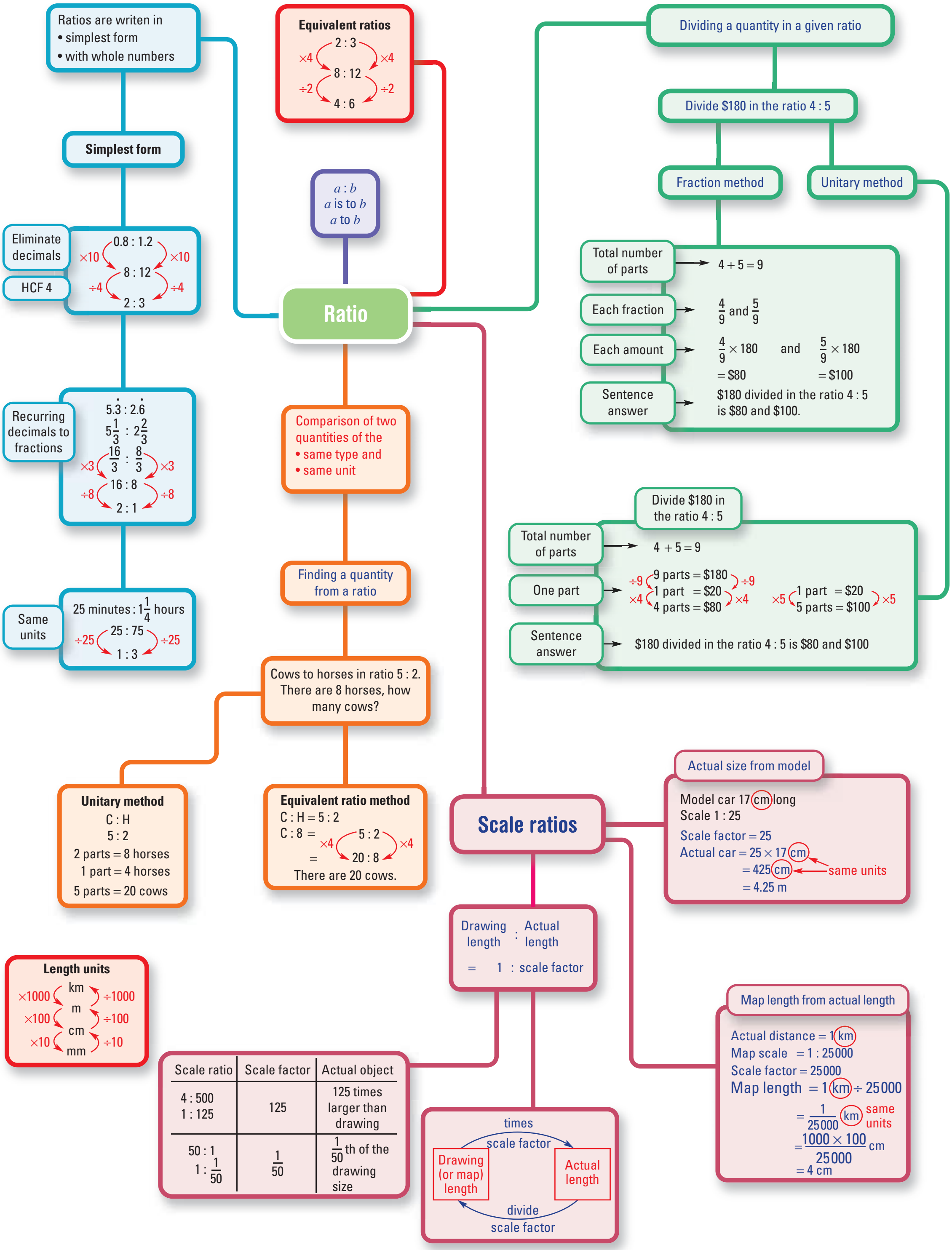
Chapter summary



Ratios are writen in
- simplest form
- with whole numbers

Simplest form

Eliminate decimals

HCF 4

0.8 : 1.2 ×10 ×10
8 : 12 ÷4 ÷4
2 : 3

Recurring decimals to fractions

$5.\dot{3} : 2.\dot{6}$
$5\frac{1}{3} : 2\frac{2}{3}$
$\frac{16}{3} : \frac{8}{3}$ ×3 ×3
16 : 8 ÷8 ÷8
2 : 1

Same units

25 minutes : $1\frac{1}{4}$ hours
25 : 75 ÷25 ÷25
1 : 3

Equivalent ratios

2 : 3 ×4 ×4
8 : 12 ÷2 ÷2
4 : 6

a : *b*
a is to *b*
a to *b*

Ratio

Comparison of two quantities of the
- same type and
- same unit

Finding a quantity from a ratio

Cows to horses in ratio 5 : 2. There are 8 horses, how many cows?

Unitary method
C : H
5 : 2
2 parts = 8 horses
1 part = 4 horses
5 parts = 20 cows

Equivalent ratio method
C : H = 5 : 2
C : 8 = 5 : 2 ×4 ×4
= 20 : 8
There are 20 cows.

Length units
km ×1000 ÷1000
m ×100 ÷100
cm ×10 ÷10
mm

Dividing a quantity in a given ratio

Divide $180 in the ratio 4 : 5

Fraction method

Unitary method

Total number of parts → 4 + 5 = 9
Each fraction → $\frac{4}{9}$ and $\frac{5}{9}$
Each amount → $\frac{4}{9} \times 180$ and $\frac{5}{9} \times 180$
= $80 = $100
Sentence answer → $180 divided in the ratio 4 : 5 is $80 and $100.

Divide $180 in the ratio 4 : 5
Total number of parts → 4 + 5 = 9
One part → ÷9 9 parts = $180 ÷9
×4 1 part = $20 ×4
4 parts = $80
×5 1 part = $20 ×5
5 parts = $100
Sentence answer → $180 divided in the ratio 4 : 5 is $80 and $100

Scale ratios

Actual size from model
Model car 17 cm long
Scale 1 : 25
Scale factor = 25
Actual car = 25 × 17 cm
= 425 cm ← same units
= 4.25 m

Drawing length : Actual length
= 1 : scale factor

Map length from actual length
Actual distance = 1 km
Map scale = 1 : 25000
Scale factor = 25000
Map length = 1 km ÷ 25000
= $\frac{1}{25000}$ km same units
= $\frac{1000 \times 100}{25000}$ cm
= 4 cm

Scale ratio	Scale factor	Actual object
4 : 500 1 : 125	125	125 times larger than drawing
50 : 1 1 : $\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{50}$ th of the drawing size

times scale factor
Drawing (or map) length
Actual length
divide scale factor

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Rates

Comparing two quantities with different units

Average rates

720 km driven in 10 hours

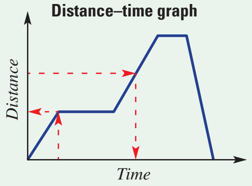
Average speed $= \frac{720}{10}$

$= 72$ km/h

Simplified rates

- 25 km/h many km per one hour?
- \$12/kg many \$ per one kg?

Distance–time graph

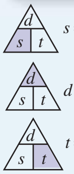


- Steeper lines show greater speed than less steep lines.
- Flat segment means the object is at rest.

Reading the graph:

- Start on given distance. Move across to line, then down to time scale (or in reverse).

s : speed
d : distance
t : time



$s = \frac{d}{t}$

$d = s \times t$

$t = \frac{d}{s}$

Unitary method to change units

For an 18 km jog in $1\frac{1}{2}$ hours determine average rate in m/min.

$\div 1\frac{1}{2}$ 18 km in $1\frac{1}{2}$ hours $\div 1\frac{1}{2}$ ← 1 part

$\frac{18}{1\frac{1}{2}}$ km in 1 hour

$\div 60$ 12 000 m in 60 min $\div 60$ ← 1 part

200 m in 1 min

Average speed = 200 m/min

Unitary method with rates

4 L paint to cover 80 m^2

? L to cover 165 m^2

$\div 80$ 4 L for 80 m^2 $\div 80$ ← 1 part

$\frac{4}{80}$ L for 1 m^2

$\times 165$ $\frac{4}{80} \times$ 165 for 165 m^2 $\times 165$

8.25 L for 165 m^2

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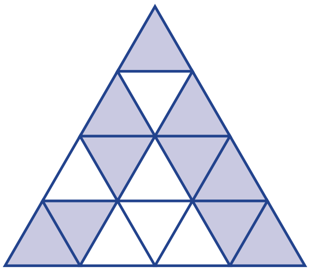
Multiple-choice questions

1 A school has 315 boys, 378 girls and 63 teachers. The ratio of students to teachers is:

A 11 : 1 **B** 1 : 11 **C** 5 : 6 **D** 6 : 5 **E** 7 : 5

2 Find the ratio of the shaded area to the unshaded area in this triangle.



A 3 : 5 **B** 8 : 5 **C** 5 : 3 **D** 5 : 8 **E** 4 : 3

3 The ratio 500 mm to $\frac{1}{5}$ m is the same as:

A 50 : 2 **B** 2500 : 1 **C** 2 : 5 **D** 2 : 50 **E** 5 : 2

4 The ratio $1\frac{1}{2} : \frac{3}{4}$ simplifies to:

A 2 : 1 **B** 1 : 2 **C** 3 : 4 **D** 4 : 3 **E** 3 : 2

5 \$750 is divided in the ratio 1 : 3 : 2. The smallest share is:

A \$250 **B** \$125 **C** \$375 **D** \$750 **E** \$75

6 The ratio of the areas of two triangles is 5 : 2. The area of the larger triangle is 60 cm^2. What is the area of the smaller triangle?

A 12 cm^2 **B** 24 cm^2 **C** 30 cm^2 **D** 17 cm^2 **E** 34 cm^2

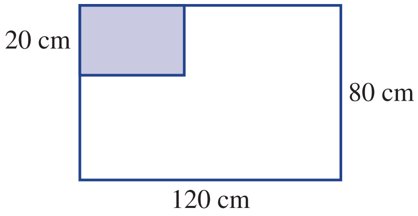
7 Callum fills his car with 28 litres of petrol at 142.7 cents per litre. His change from \$50 cash is:

A \$10 **B** \$39.95 **C** \$71.35 **D** \$40 **E** \$10.05

8 45 km/h is the same as:

A 0.25 m/s **B** 25 m/s **C** 12.5 m/s **D** 75 m/s **E** 125 m/s

9 A flag is created by enlarging the shaded rectangle, as shown. What is the length of the original rectangle?



A 20 cm **B** 30 cm **C** 40 cm **D** 13 cm **E** 10 cm

10 On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?

A 1 : 5 **B** 1 : 5000 **C** 1 : 50 000 **D** 1 : 5 000 000 **E** 1 : 50

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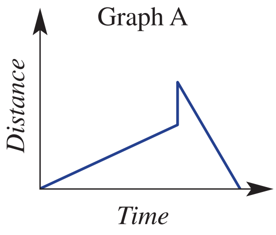
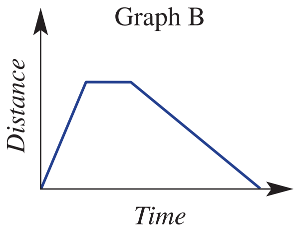
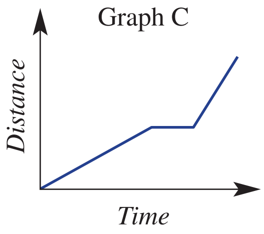
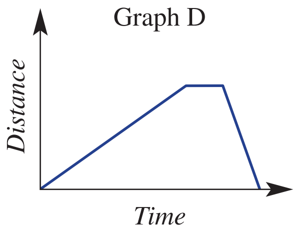
11 A toddler walked slowly away from his mum, had a short stop, then turned and walked quickly back to his mum again. In each graph below, the distance is measured from the toddler's mum. Choose the correct distance–time graph for this journey.









Short-answer questions

1 In Lao's pencil case there are six coloured pencils, two black pens, one red pen, five textas, three lead pencils and a ruler. Find the ratio of:

- **a** lead pencils to coloured pencils
- **b** black pens to red pens
- **c** textas to all pencils

2 True or false?

- **a** $1:4 = 3:6$
- **b** The ratio $2:3$ is the same as $3:2$.
- **c** The ratio $3:5$ is written in simplest form.
- **d** 40 cm : 1 m is written as $40:1$ in simplest form.
- **e** $1\frac{1}{4}:2 = 5:8$

3 Copy and complete.

- **a** $4:50 = 2:\square$
- **b** $1.2:2 = \square:20$
- **c** $\frac{2}{3}:4 = 1:\square$
- **d** $1:\square:5 = 5:15:25$

4 Simplify the following ratios.

- **a** $10:40$
- **b** $36:24$
- **c** $75:100$
- **d** $8:64$
- **e** $27:9$
- **f** $5:25$
- **g** $6:4$
- **h** $52:26$
- **i** $6b:9b$
- **j** $8a:4$
- **k** $\frac{2}{7}:\frac{5}{7}$
- **l** $1\frac{1}{10}:\frac{2}{10}$
- **m** $2\frac{1}{2}:\frac{3}{4}$
- **n** $12:36:72$

5 Simplify the following ratios by first changing to the same units.

- **a** 2 cm : 8 mm
- **b** 5 mm : 1.5 cm
- **c** 3 L : 7500 mL
- **d** 30 min : 1 h
- **e** 400 kg : 2 t
- **f** 6 h : 1 day
- **g** 120 m : 1 km
- **h** 45 min : $2\frac{1}{2}$ h

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6 Divide:

a \$80 in the ratio 7 : 9
b 200 kg in the ratio 4 : 1
c 40 m in the ratio 6 : 2
d \$1445 in the ratio 4 : 7 : 6
e \$1 in the ratio 3 : 1 : 1

7 **a** The ratio of the cost price of a TV to its retail price is 5 : 12. If its cost price is \$480, calculate its retail price.

b The ratio of Taylah's height to Ben's height is 12 : 17. If the difference in their heights is 60 cm, how tall is Taylah?

c Orange juice, pineapple juice and guava juice are mixed in the ratio 4 : 3 : 2. If 250 mL of guava juice is used, how many litres of drink does this make?

8 Express each rate in simplest form.

a 10 km in 2 hours
b \$650 for 13 hours
c 2800 km in 20 days

9 Copy and complete.

a 400 km on 32 litres = ______ km/L
= ______ L/100 Km

b 5 grams in 2 min = ______ g/min
= ______ g/h

c \$1200 in $\frac{1}{2}$ day = ______ \$/day
= ______ \$/h

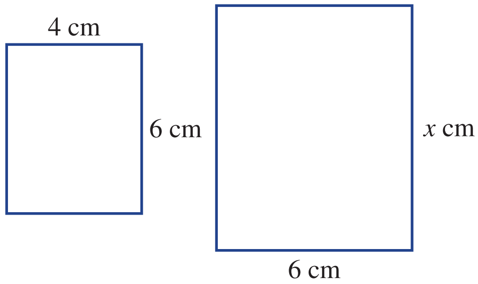
10 For a scale of 1 : 1000, find the real length (in metres) if the scale length is given as:

a 0.0002 m
b 2.7 cm
c 140 mm

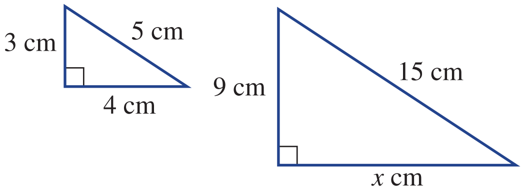
11 Two cities are 50 km apart. How many millimetres apart are they on a map that has a scale of 1 : 100 000?

12 Find the scale factor for the following pairs of similar figures, and find the value of x.

a



b



13 Copy and complete.

a \$120/h = _____ c/min
b 6 m/s = _____ km/h
c 720 km/h = _____ m/s

14 **a** A truck uses 12 litres of petrol to travel 86 km. How far will it travel on 42 litres of petrol?

b Samira earns \$267.20 for a 12-hour shift. How much will she earn for a 7-hour shift?

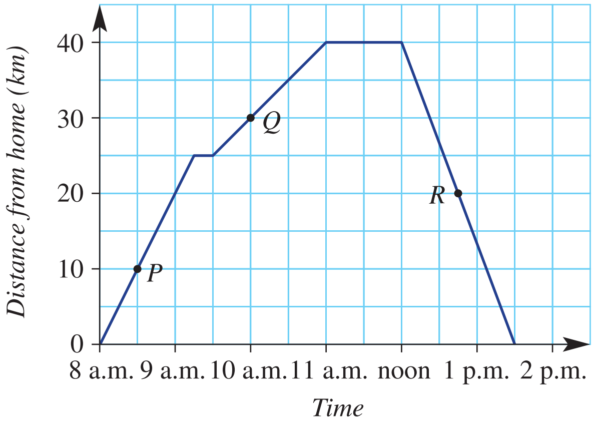
c Tap 1 fills the pool in 12 hours, while tap 2 fills the same pool in 15 hours. How long does it take to fill this pool if both taps are used?

15 **a** Sandra drives to her mother's house. It takes 45 minutes. Calculate Sandra's average speed if her mother lives 48 km away.

b How long does it take Ari to drive 180 km along the freeway to work if he manages to average 100 km/h for the trip?

c How far does Tarik ride his bike if he rides at 4.5 km/h for 90 minutes?

16 This distance–time graph shows Mason's cycle journey from home one morning.



- **a** How far was Mason from home at 8:45 a.m.?
- **b** What was Mason's speed in the first hour of the journey?
- **c** How many minutes was Mason's first rest break?
- **d** What was the furthest distance that Mason was from his home?
- **e** Was Mason further away from home at point Q or point R?
- **f** When had Mason ridden 60 km?
- **g** How far did Mason ride between points P and R?
- **h** Describe each section of Mason's journey from 9:30 a.m. to the finish. Include the distances, times, speeds and any features.

Extended-response question

The Harrison family and the Nguyen family leave Wahroonga, Sydney, at 8 a.m. on Saturday morning for a holiday in Coffs Harbour, which is 500 km north of Sydney.

The Harrisons' 17-year-old son drives for the first 2 hours at 90 km/h. They then stop for a rest of $1\frac{1}{2}$ hours. Mr Harrison drives the rest of the way, arriving at 3 p.m.

The Nguyen family drives the first 300 km in 3 hours and then have a break for 1 hour. Then the Nguyens continue with no more stops and arrive in Coffs Harbour at 2:15 p.m.

- **a** Using graph paper, draw a distance–time graph showing both journeys.
- **b** How far apart were the two families at 10 a.m.?
- **c** At what times did the Nguyens and Harrisons arrive at the halfway point?
- **d** What was the Nguyen's speed for the last section of their trip (correct to one decimal place)?
- **e** Calculate the average speed of the Nguyen's journey.
- **f** At what time did the Harrisons resume their journey after their rest stop?
- **g** How many kilometres did the Harrisons still have to cover after their rest break before arriving in Coffs Harbour?
- **h** For how long did Mr Harrison drive and at what speed?
- **i** How far south of Coffs Harbour were the Harrisons when the Nguyen family arrived there?
- **j** Calculate the cost of the petrol for each family's trip if petrol cost is 139 c/L and the Harrison car's consumption is 9 L/100 km, and the Nguyen's car uses 14 L/100 km.

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Chapter 1: Algebraic techniques 2 and indices

Multiple-choice questions

1 $8^3 \times 8^4$ is equal to:

A 8^{-1} **B** 64^7 **C** 8^7 **D** 8^{12} **E** 64^{12}

2 $4x + 5 + 3x$ is equal to:

A $8x + 4$ **B** $12x$ **C** $12 + x^2$ **D** $2x + 12$ **E** $7x + 5$

3 $12m + 18$ factorises to:

A $2(6m - 9)$ **B** $-6(2m - 3)$ **C** $6(3 - 2m)$ **D** $6(2m + 3)$ **E** $12(m + 6)$

4 $5a + 5 - 4a - 4 - a - 1$ equals:

A $a - 1$ **B** 0 **C** $2 - a$ **D** $a + 1$ **E** a

5 Which answer is NOT equivalent to $(m \times n) \div (p \times q)$?

A $\frac{mn}{pq}$ **B** $m \times \frac{n}{pq}$ **C** $\frac{m}{p} \times \frac{n}{q}$ **D** $\frac{mnq}{p}$ **E** $mn \times \frac{1}{pq}$

Short-answer questions

1 Write an expression for:

a the sum of p and q **b** the product of p and 3

c half the square of m **d** the sum of x and y, divided by 2

2 If $a = 6$, $b = 4$ and $c = -1$, evaluate:

a $a + b + c$ **b** $ab - c$ **c** $a(b^2 - c)$

d $3a^2 + 2b$ **e** abc **f** $\frac{ab}{c}$

3 Simplify each algebraic expression.

a $4 \times 6k$ **b** $a + a + a$ **c** $a \times a \times a$ **d** $7p \div 14$

e $3ab + 2 + 4ab$ **f** $7x + 9 - 6x - 10$ **g** $18xy \div 9x$ **h** $m + n - 3m + n$

4 Simplify:

a $\frac{5xy}{5}$ **b** $\frac{3x}{7} - \frac{2x}{7}$ **c** $\frac{w}{5} + \frac{w}{2}$ **d** $3a + \frac{a}{2}$

5 Simplify:

a $\frac{m}{5} \times \frac{5}{6}$ **b** $\frac{ab}{7} \div \frac{1}{7}$ **c** $\frac{m}{3} \times \frac{n}{2} \div \frac{mn}{4}$

6 Expand, and simplify where necessary.

a $6(2m - 3)$ **b** $10 + 2(m - 3)$ **c** $5(A + 2) + 4(A - 1)$

7 Factorise:

a $18a - 12$ **b** $6m^2 + 6m$ **c** $-8m^2 - 16mn$

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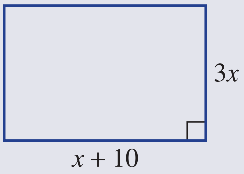
8 For the rectangle shown, write an expression for its:

a perimeter
b area

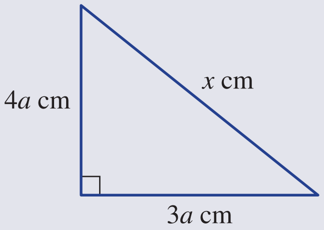


9 Simplify the following, leaving your answer in index form.

a $5^7 \times 5^2$ **b** $3^8 \times 3^2$ **c** $6^7 \times 6$
d $4^{10} \div 4^3$ **e** $7^{12} \div 7^6$ **f** $6^7 \div 6$

10 **a** $(5^2)^3$ **b** $(2^3)^4$ **c** 12^0 **d** 5×4^0

Extended-response question



a Write an expression for the perimeter of this triangle.
b Write an expression for the area of this triangle.
c Use Pythagoras' theorem to find a relationship between x and a.
d Use your relationship to write an expression for the perimeter in terms of only a.
e If the perimeter equals 72 cm, what is the area of this triangle?

Chapter 2: Equations 2

Multiple-choice questions

1 The sum of a number and three is doubled. The result is 12. This can be written as:

A $x + 3 \times 2 = 12$ **B** $2x + 6 = 12$ **C** $2x + 3 = 12$
D $x + 3 = 24$ **E** $x + 3 = 12$

2 The solution to the equation $2m - 4 = 48$ is:

A $m = 8$ **B** $m = 22$ **C** $m = 20$ **D** $m = 26$ **E** $m = 24$

3 The solution to the equation $-5(m - 4) = 30$ is:

A $m = -10$ **B** $m = 10$ **C** $m = 2$ **D** $m = \frac{-34}{5}$ **E** $m = -2$

4 Which equation has NO solution?

A $2x = 9$ **B** $x^2 = 4$ **C** $x^2 = -9$ **D** $2x = x$ **E** $x + 1 = 0$

5 Which equation has two solutions?

A $2x = 9$ **B** $x^2 = 4$ **C** $x^2 = -9$ **D** $2x = x$ **E** $x + 1 = 0$

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Short-answer questions

1 Solve each of these equations.

- **a** $3w = 27$
- **b** $12 = \frac{m}{6}$
- **c** $4 - x = 3$
- **d** $4a + 2 = 10$
- **e** $2w + 6 = 0$
- **f** $\frac{x}{5} - 1 = 6$

2 Solve:

- **a** $6 = 4 - 4m$
- **b** $3a + 4 = 7a + 8$
- **c** $3(x + 5) = 15$
- **d** $\frac{x}{5} + \frac{x}{3} = 1$
- **e** $2(5 - a) = 3(2a - 1)$
- **f** $\frac{a + 7}{2a} = 3$

3 Double a number less 3 is the same as 9. What is the number?

4 A father is currently six times as old as his son. In 10 years' time his son will be 20 years younger than his dad. How old is the son now?

5 Solve the following equations where possible.

- **a** $x^2 = 9$
- **b** $x^2 = 36$
- **c** $a^2 = 144$
- **d** $y^2 = 0$
- **e** $b^2 = -4$
- **f** $c^2 = -100$

6 The formula $F = ma$ relates force, mass and acceleration.

- **a** Find F if $m = 10$ and $a = 3$.
- **b** Find m if $a = 5$ and $F = 20$.

Extended-response question

EM Publishing has fixed costs of \$1500 and production costs of \$5 per book. Each book has a retail price of \$17.

- **a** Write an equation for the cost (C) of producing n books.
- **b** Write an equation for the revenue (R) for selling n books.
- **c** A company 'breaks even' when the cost of production equals the revenue from sales. How many books must the company sell to break even?
- **d** Write an equation for the profit (P) of selling n books.
- **e** Calculate the profit if 200 books are sold.
- **f** What is the profit if 100 books are sold? Explain the significance of this answer.

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Chapter 3: Measurement and Pythagoras' theorem

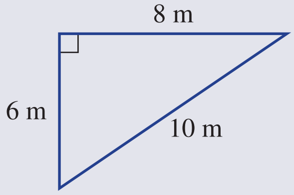
Multiple-choice questions

1 A cube has a volume of 8 cubic metres. The area of each face of the cube is:

A 2 m^2
B 4 m^2
C 24 m^2
D 384 m^2
E 512 m^2

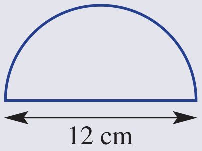
2 The area of this triangle is:

A 48 m^2
B 24 m^2
C 30 m^2
D 40 m^2
E 480 m^2



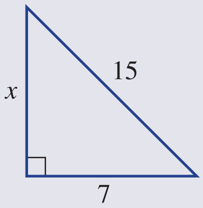
3 The perimeter of this semicircle is closest to:

A 38 cm
B 30 cm
C 19 cm
D 31 cm
E 37 cm



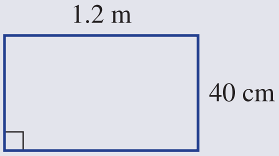
4 The value of x in this triangle is closest to:

A 176
B 14
C 274
D 17
E 13



5 The area of this rectangle is:

A 48 m^2
B 48 000 cm^2
C 480 cm^2
D 0.48 m^2
E 4.8 m^2



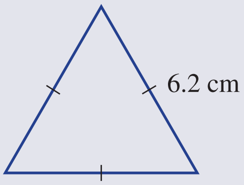
Short-answer questions

1 Complete these conversions.

a 5 m = ______ cm
b 1.8 m = ______ cm
c 9 m^2 = ______ cm^2
d 1800 mm = ______ m
e 4 L = ______ cm^3
f $\frac{1}{100}$ km^2 = ______ m^2

2 Find the perimeter of these shapes.

a



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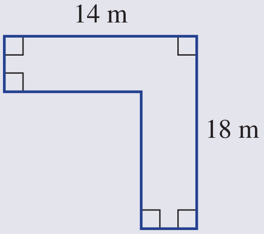
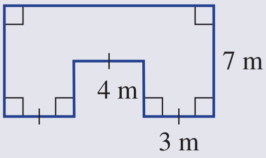


c



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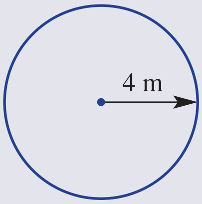
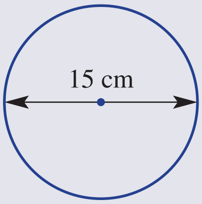
3 Find, correct to two decimal places:

i the circumference **ii** the area

a



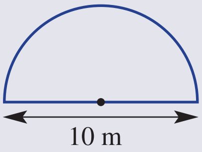
b



4 Find, correct to two decimal places:

i the perimeter **ii** the area

a



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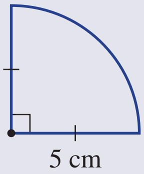
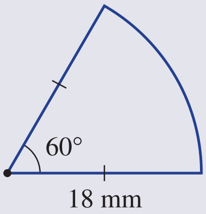
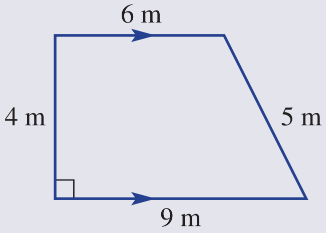


c



5 Find the area of these shapes.

a



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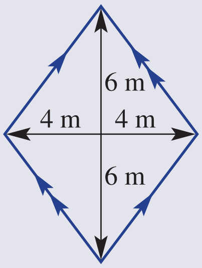
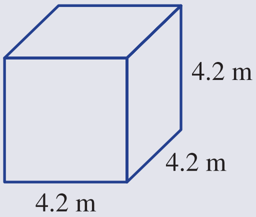
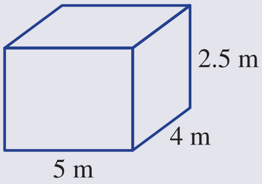


6 Find the volume of these solids.

a



b



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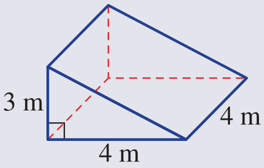
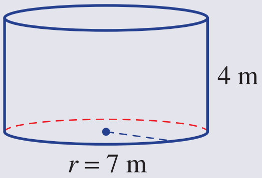


7 Find the volume of each cylinder, correct to two decimal places.

a



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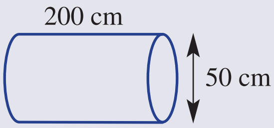
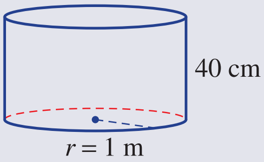
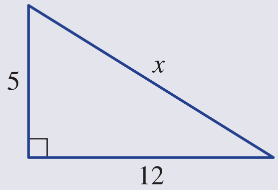


c

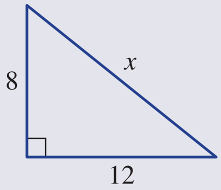


8 Find the value of x in these triangles. Round to two decimal places for part **b**.

a



b



9 Write these times using 24-hour time.

a 3:30 p.m.

b 7:35 a.m.

Extended-response question

A square sheet of metal measuring 15 m by 15 m has equal squares of sides x m cut from each corner, as shown. It is then folded to form an open tray.

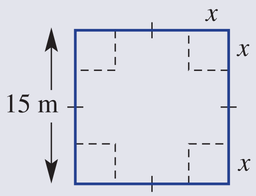
a What is the length of the base of the tray? Write an expression.

b What is the height of the tray?

c Write an expression for the volume of the tray.

d If $x = 1$, find the volume of the tray.

e What value of x do you think produces the maximum volume?



Chapter 4: Fractions, decimals, percentages and financial mathematics

Multiple-choice questions

1 $\frac{150}{350}$ simplifies to:

A $\frac{6}{14}$ **B** $\frac{3}{70}$ **C** $\frac{15}{35}$ **D** $\frac{3}{7}$ **E** $\frac{75}{175}$

2 Sienna spends $\frac{3}{7}$ of her \$280 income on clothes and saves the rest. She saves:

A \$470 **B** \$120 **C** \$160 **D** \$2613 **E** \$400

3 0.008×0.07 is equal to:

A 0.056 **B** 0.0056 **C** 0.00056 **D** 56 **E** 0.56

4 0.24 expressed as a fraction is:

A $\frac{1}{24}$ **B** $\frac{25}{6}$ **C** $\frac{12}{5}$ **D** $\frac{24}{10}$ **E** $\frac{6}{25}$

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5 If 5% of $x = y$, then 10% of $2x$ equals:

A $4y$ **B** $2y$ **C** $\frac{1}{2}y$ **D** $10y$ **E** $5y$

6 When paying for a meal at a local café the bill is $97.90. The GST included in this cost is:

A $8.90 **B** $9.79 **C** $89 **D** $88.11 **E** $8.11

Short-answer questions

1 Copy and complete these equivalent fractions.

a $\frac{3}{5} = \frac{\square}{30}$ **b** $\frac{\square}{11} = \frac{5}{55}$ **c** $1\frac{4}{6} = \frac{\square}{3}$

2 Evaluate each of the following.

a $\frac{3}{4} - \frac{1}{2}$ **b** $\frac{4}{5} + \frac{3}{5}$ **c** $1\frac{1}{2} + 1\frac{3}{4}$

d $\frac{4}{7} - \frac{2}{3}$ **e** $\frac{4}{9} \times \frac{3}{4}$ **f** $1\frac{1}{2} \times \frac{3}{5}$

3 Write the reciprocal of:

a $\frac{2}{5}$ **b** 8 **c** $4\frac{1}{5}$

4 Evaluate:

a $2\frac{1}{2} \times 1\frac{4}{5}$ **b** $1\frac{1}{2} \div 2$ **c** $1\frac{1}{2} \times \frac{1}{4} \div \frac{3}{5}$

5 Calculate each of the following:

a $3.84 + 3.09$ **b** $10.85 - 3.27$ **c** $12.09 \div 3$

d $6.59 - 0.2 \times 0.4$ **e** 96.37×40 **f** $15.84 \div 0.02$

6 Evaluate:

a 5.3×103 **b** 9.6×105 **c** $61.4 \div 100$

7 Copy and complete this table of decimals, fractions and percentages.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$				
Decimal								0.99	0.005
Percentage						80%	95%		

8 Find:

a 10% of 56 **b** 12% of 98 **c** 15% of 570 m

d 99% of $2 **e** $12\frac{1}{2}$% of $840 **f** 58% of 8500 g

9 **a** Increase \$560 by 25%.
  b Decrease \$980 by 12%.
  c Increase \$1 by 8% and then decrease the result by 8%.

10 A \$348 dress sells for \$261. This represents a saving of x%. What is the value of x?

11 Complete the table of pre- and post-GST prices; remember in Australia the GST is 10%.

Pre-GST	GST	Post-GST
\$550.00		
	\$7.00	
	\$47.50	
		\$962.50

Extended-response question

A laptop decreases in value by 15% a year.

a Find the value of a \$2099 laptop at the end of:
  i 1 year **ii** 2 years **iii** 3 years

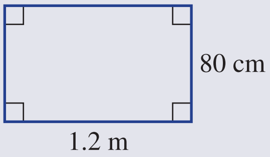
b After how many years is the laptop worth less than \$800?

c Is the laptop ever going to have a value of zero dollars? Explain.

Chapter 5: Ratios and rates

Multiple-choice questions

1 The ratio of the length to the breadth of this rectangle is:
  A 12 : 80 **B** 3 : 20 **C** 3 : 2
  D 20 : 3 **E** 2 : 3



2 Simplify the ratio 500 cm to $\frac{3}{4}$ km.
  A 2 : 3 **B** 1 : 150 **C** 3 : 2 **D** 150 : 1 **E** 3 : 4

3 \$18 is divided in the ratio 2 : 3. The larger part is:
  A \$10.80 **B** \$7.20 **C** \$3.60 **D** \$12 **E** \$14.40

4 Calvin spent \$3 on his mobile phone for every \$4 he spent on music. Calvin spent \$420 on his phone last year. How much did he spend on music for the same year?
  A \$560 **B** \$315 **C** \$140 **D** \$240 **E** \$525

5 A boat sailed 30 kilometres in 90 minutes. What was the average speed of the boat?
  A 15 km/h **B** 45 km/h **C** 3 km/h **D** 20 km/h **E** 35 km/h

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Short-answer questions

1 Simplify these ratios.

a 24 to 36
b 15 : 30 : 45
c 0.6 m to 70 cm
d 15 cents to $2
e $\frac{3}{4}$ to 2
f 60 cm to 2 m

2 **a** Divide 960 cm in the ratio 3 : 2.
b Divide $4000 in the ratio 3 : 5.
c Divide $8 in the ratio 2 : 5 : 3.

3 A 20 metre length of wire is used to fence a rectangular field with dimensions in the ratio 3 : 1. Calculate the area of the field.

4 A business has a ratio of profit to costs of 5 : 8. If the costs were $12 400, how much profit was made?

5 Complete these rates.

a 5 g/min = ___ g/h
b $240 in 8 hours = $___ /h
c 450 km in $4\frac{1}{2}$ h = ___ km/h

6 A shop sells $1\frac{1}{2}$ kg bags of apples for $3.40. Find the cost of a 1 kilogram at this rate.

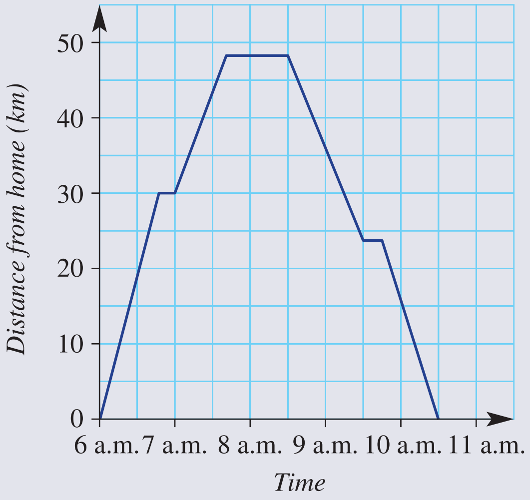
7 A car travels the 1035 km from southern Sydney to Melbourne in 11.5 hours. Calculate its average speed.

Extended-response questions

1 A small car uses 30 litres of petrol to travel 495 km.

a At this rate, what is the maximum distance the small car can travel on 45 litres of petrol?
b What is the average distance travelled per litre?
c Find the number of litres used to travel 100 km, correct to one decimal place.
d Petrol costs 134.9 cents/litre. Find the cost of petrol for the 495 km trip.
e A larger car uses 42 litres of petrol to travel 378 km. The smaller car holds 36 litres of petrol, whereas the larger car holds 68 litres. How much further can the larger car travel on a full tank of petrol?

2 The training flight of a racing pigeon is shown in this distance–time graph.



a How long does the pigeon take to fly the first 20 km?

b How long is each of the rest stops?

c How far has the pigeon flown by 7:15 a.m.?

d At what times is the pigeon 30 km from home?

e Calculate the pigeon's speed for each flight section of the journey, listing them in order.

f What is the furthest distance the pigeon was from home?

g How far does the pigeon fly between 7:30 a.m. and 9:15 a.m?

h At what time has the pigeon flown a total of 80 km?

i Calculate the pigeon's average speed for the flight (correct to one decimal place).

j Describe the pigeon's flight from 6 a.m to 7:45 a.m., including the starting and finishing times, the distances flown and the speed of each section.

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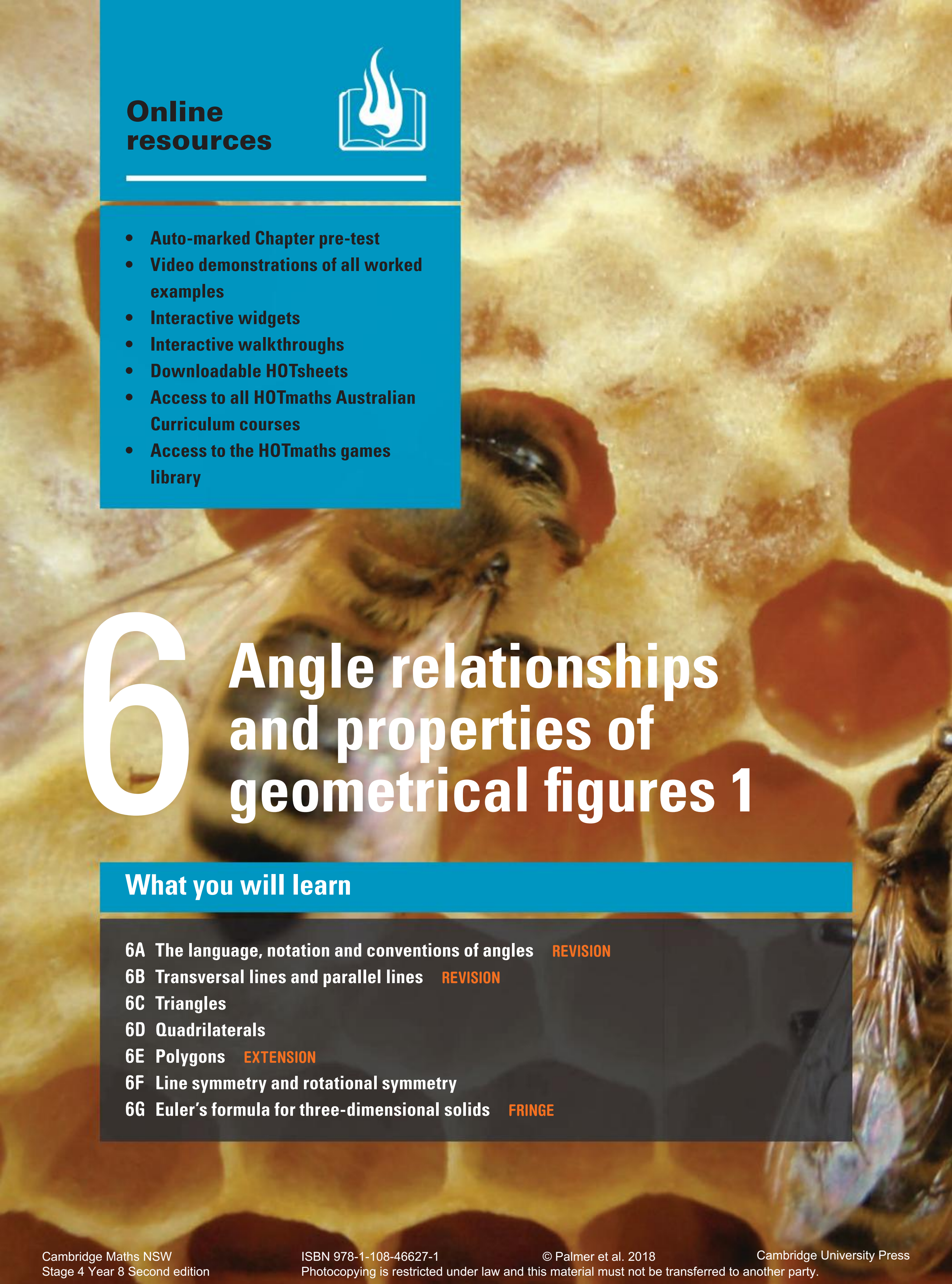


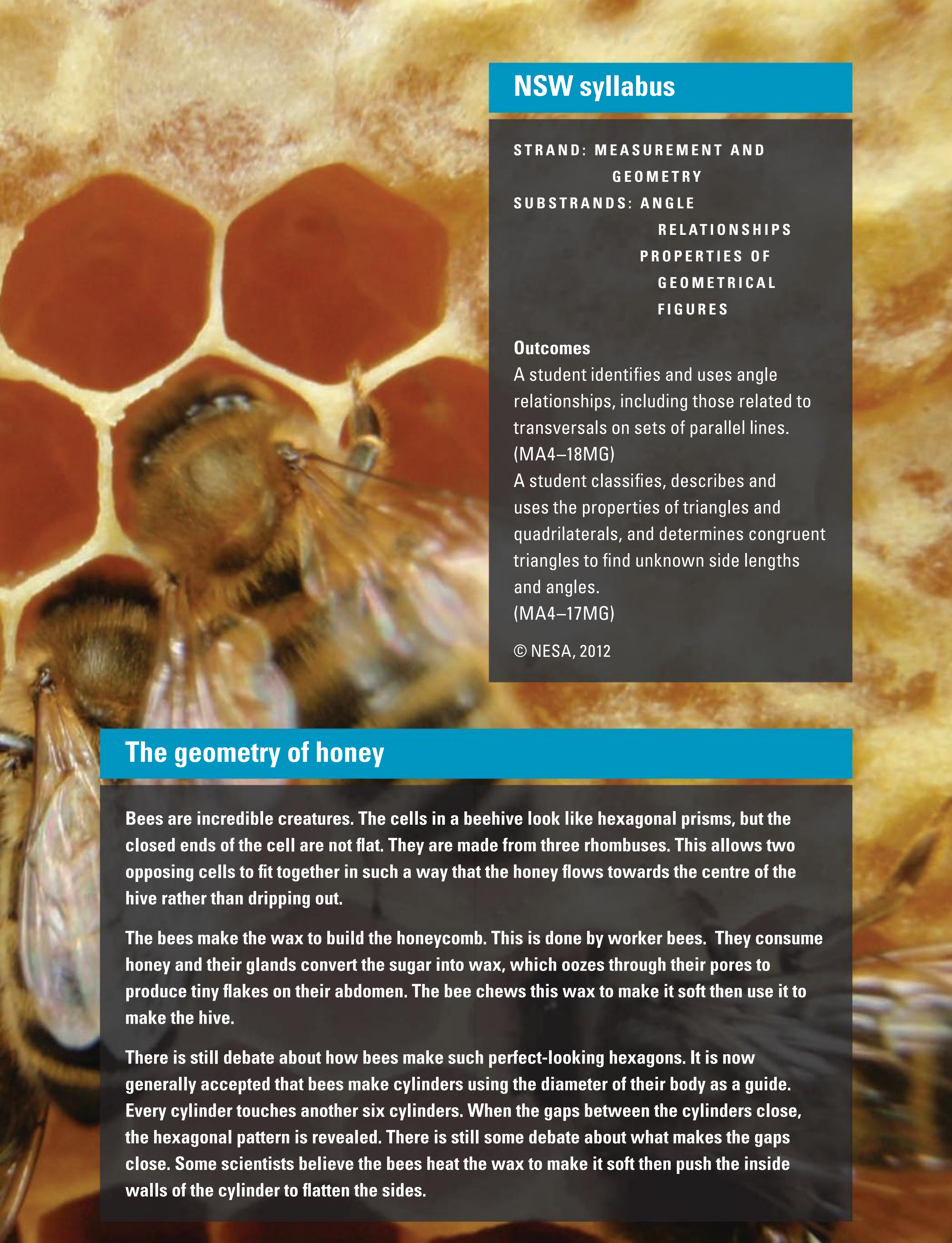
Online resources

- Auto-marked Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

6

Angle relationships and properties of geometrical figures 1

What you will learn



NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY

SUBSTRANDS: ANGLE RELATIONSHIPS

PROPERTIES OF GEOMETRICAL FIGURES

Outcomes

A student identifies and uses angle relationships, including those related to transversals on sets of parallel lines.
(MA4–18MG)

A student classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles.
(MA4–17MG)

The geometry of honey

Bees are incredible creatures. The cells in a beehive look like hexagonal prisms, but the closed ends of the cell are not flat. They are made from three rhombuses. This allows two opposing cells to fit together in such a way that the honey flows towards the centre of the hive rather than dripping out.

The bees make the wax to build the honeycomb. This is done by worker bees. They consume honey and their glands convert the sugar into wax, which oozes through their pores to produce tiny flakes on their abdomen. The bee chews this wax to make it soft then use it to make the hive.

There is still debate about how bees make such perfect-looking hexagons. It is now generally accepted that bees make cylinders using the diameter of their body as a guide. Every cylinder touches another six cylinders. When the gaps between the cylinders close, the hexagonal pattern is revealed. There is still some debate about what makes the gaps close. Some scientists believe the bees heat the wax to make it soft then push the inside walls of the cylinder to flatten the sides.

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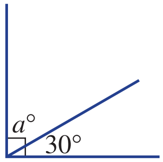
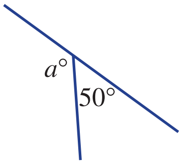
Pre-test

1 Find the value of *a* in these diagrams.

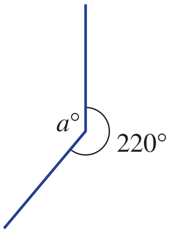
a



b



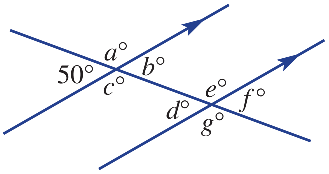
c



2 Name these angles as acute, right, obtuse, straight, reflex or revolution.

a 360°
b 90°
c 37°
d 149°
e 180°
f 301°

3 This diagram includes a pair of parallel lines and a third line called a transversal.



a What is the value of *a*?
b Which pronumerals are equal in value to *a*?
c Which pronumerals are equal in value to 50?

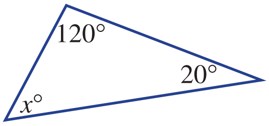
4 Name the triangle that best fits the description. Choose from *scalene*, *isosceles*, *equilateral*, *acute*, *right* or *obtuse*. Draw an example of each triangle to help.

a one obtuse angle
b two equal-length sides
c all angles acute
d three different side lengths
e three equal 60° angles
f one right angle

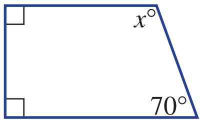
5 Name the six special quadrilaterals. Draw an example of each.

6 Find the value of *x* in these shapes, using the given angle sum.

a Angle sum = 180°

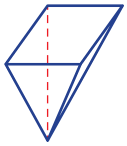


b Angle sum = 360°

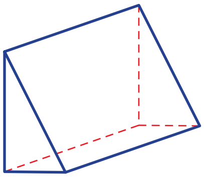


7 Name each solid as a cube, a triangular prism or a pyramid.

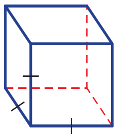
a



b



c



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6A The language, notation and conventions of angles REVISION

Interactive
Widgets
HOTsheets
Walkthrough

For more than 2000 years school geometry has been based on the work of Euclid, the Greek mathematician who lived in Egypt in about 300 BCE. Before this time, the ancient civilisations had demonstrated and documented an understanding of many aspects of geometry, but Euclid was able to produce a series of 13 books called the *Elements*, which contained a staggering 465 propositions. This great work is written in a well-organised and structured form, carefully building on solid mathematical foundations. The most basic of these foundations, called axioms, are fundamental geometric principles from which all other geometry can be built. There are five axioms described by Euclid:

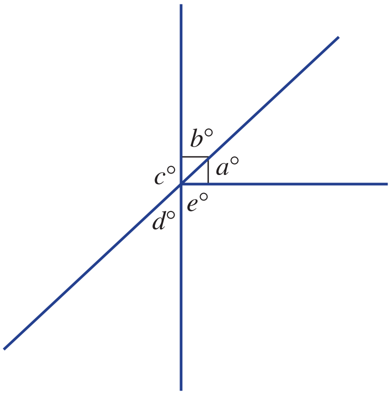
- Any two points can be joined by a straight line.
- Any finite straight line (segment) can be extended in a straight line.
- A circle can be drawn with any centre and any radius.
- All right angles are equal to each other.
- Given a line and a point not on the line, there is only one line through the given point and in the same plane that does not intersect the given line.

These basic axioms are considered to be true without question and do not need to be proven. All other geometrical results can be derived from these axioms.

Let's start: Create a sentence or definition

The five pronumerals a, b, c, d and e represent the number of degrees in the five angles in this diagram. Can you form a sentence using two or more of these pronumerals and one of the following words? Using simple language, what is the meaning of each of your sentences?

- supplementary
- revolution
- adjacent
- complementary
- vertically opposite
- right



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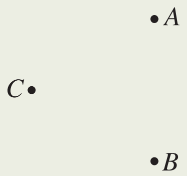
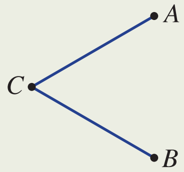
Key ideas

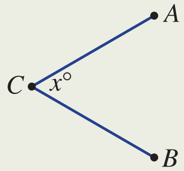
- A **point** represents a position.
  - It is shown using a dot and generally labelled with an upper case letter.
  - The diagram shows points *A*, *B* and *C*.



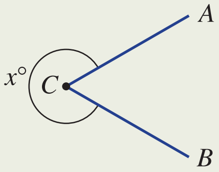
- This diagram shows **intervals** *AC* and *CB*. These are sometimes called **line segments**.
  - *AC* and *CB* form two **angles**. One is **acute** and one is **reflex**.
  - *C* is called the **vertex**. The plural is **vertices**.

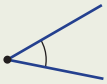
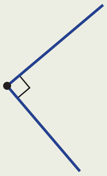
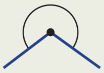
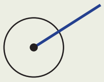


- This diagram shows acute angle *ACB*. It can be written as: $\angle C$ or $\angle ACB$ or $\angle BCA$ or $A\hat{C}B$ or $B\hat{C}A$
  - *CA* and *CB* are sometimes called **arms**.
  - The pronumeral *x* represents the number of degrees in the angle.



- This diagram shows reflex $\angle ACB$.



Type of angle	Size of angle	Diagram
acute	greater than 0° but less than 90°	
right	exactly 90°	
obtuse	greater than 90° but less than 180°	
straight	exactly 180°	
reflex	greater than 180° but less than 360°	
revolution	exactly 360°	

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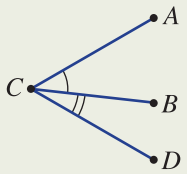
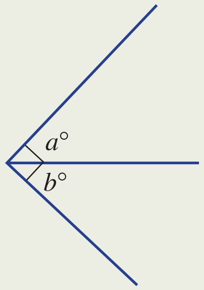
- This diagram shows two angles sharing a vertex and an arm. They are called **adjacent** angles.



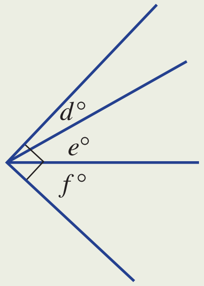
- This diagram shows two angles in a right angle. They are adjacent complementary angles. $a°$ is the complement of $b°$.

$$a + b = 90$$



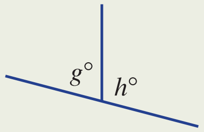
- It is possible to have three or more angles in a right angle. They are not complementary.

$$d + e + f = 90$$



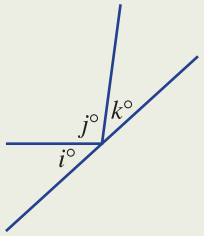
- This diagram shows two angles on a straight line. They are **adjacent supplementary** angles. $g°$ is the **supplement** of $h°$.

$$g + h = 180$$



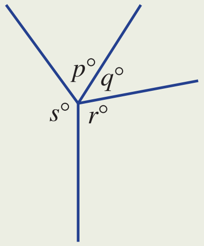
- It is possible to have three or more angles on a straight line. They are not supplementary.

$$i + j + k = 180$$



- This diagram shows **angles at a point** and **angles in a revolution**.

$$p + q + r + s = 360$$



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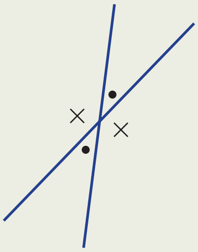
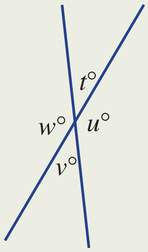
Key ideas

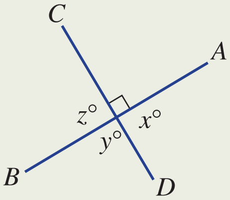
- When two straight lines meet they form two pairs of **vertically opposite angles**. Vertically opposite angles are equal.





$t = v$
$u = w$

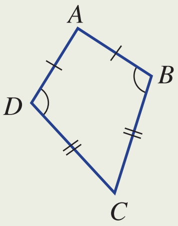
- If one of the four angles in vertically opposite angles is a right angle, then all four angles are right angles.



$x = 90$
$y = 90$
$z = 90$

  - AB and CD are **perpendicular lines**. This is written as $AB \perp CD$.

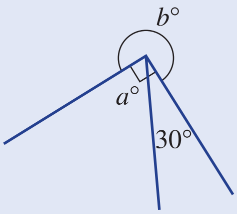
- The markings in these diagrams indicate that:
  - $AB = AD$
  - $BC = CD$
  - $\angle ABC = \angle ADC$



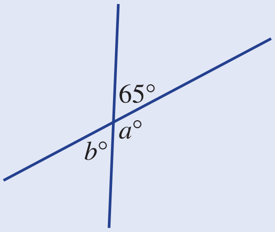
Example 1 Finding the value of pronumerals in geometrical figures

Determine the value of the pronumeral in these diagrams, giving reasons.

a



b

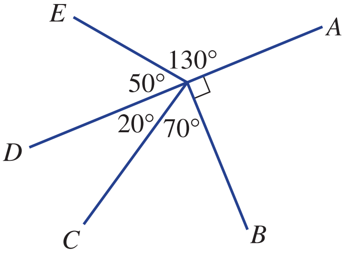


SOLUTION	EXPLANATION
a $a + 30 = 90$ (angles in a right angle) $a = 60$	$a°$ and $30°$ are adjacent complementary angles.
$b + 90 = 360$ (angles in a revolution) $b = 270$	$b°$ and $90°$ make a revolution.
b $a + 65 = 180$ (angles on a straight line) $a = 115$	$a°$ and $65°$ are adjacent supplementary angles.
$b = 65$ (vertically opposite angles)	$b°$ and $65°$ are vertically opposite angles.

Exercise 6A REVISION

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6	3, 4(½), 5, 6, 7(½)	4–5(½), 6, 7(½)

1 Complete these sentences for this diagram.



a 20° and 70° are ________________ angles.

b 20° is the ____________________ of 70°.

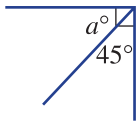
c 130° and 50° are ________________ angles.

d 130° is the ____________________ of 50°.

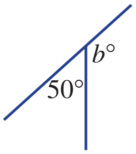
e The five angles in the diagram form a complete ________________.

2 State the value of the pronumeral in these diagrams.

a



b



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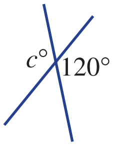

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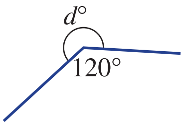
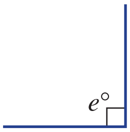
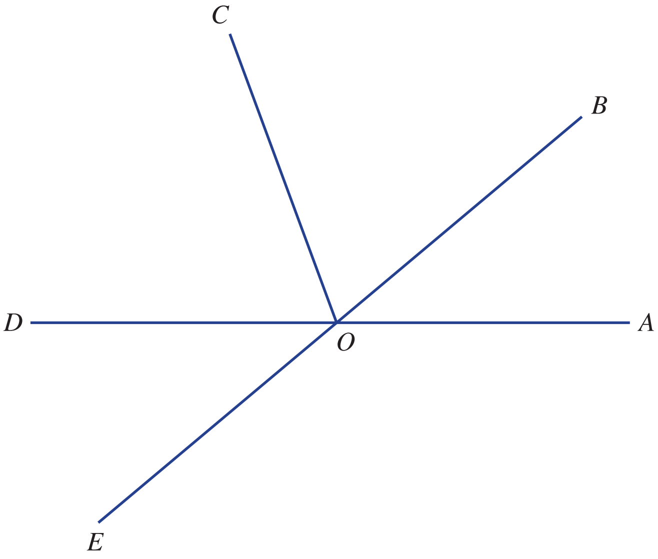


e



3 Estimate the size of these angles, then measure with a protractor.



a $\angle AOB$ **b** $\angle AOC$ **c** reflex $\angle AOE$

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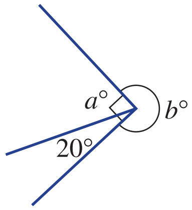
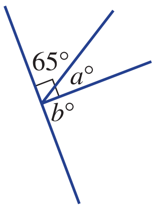
Example 1

4 Determine the value of the pronumerals, giving reasons.

a



b



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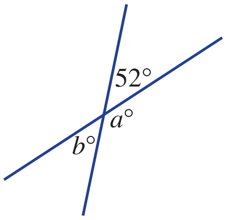

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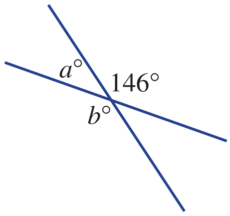
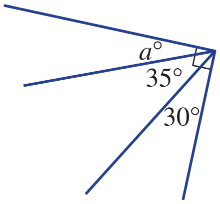
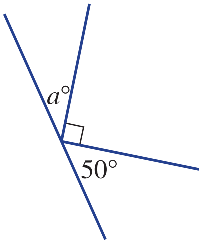


e



f



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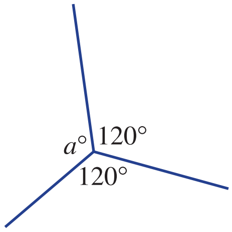
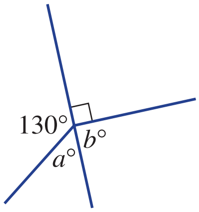
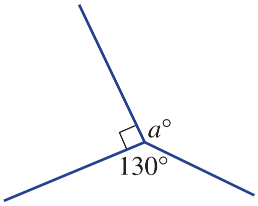


h



i



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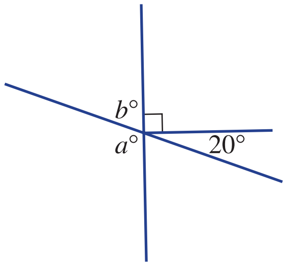
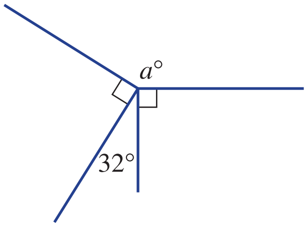
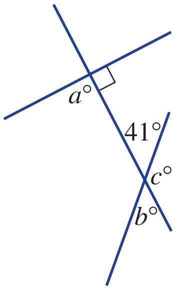


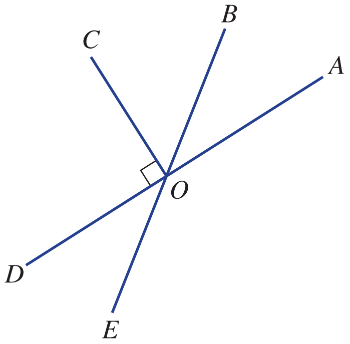
k



l



5 Consider the given diagram.



a Name an angle that is:

- **i** vertically opposite to $\angle AOB$
- **ii** complementary to $\angle BOC$
- **iii** supplementary to $\angle AOE$
- **iv** supplementary to $\angle AOC$

b Copy and complete:

- **i** $AD \perp$ _____
- **ii** _____ $\perp OD$

6 What is the complement of the following?

a 30° **b** 80° **c** 45° **d** 17°

7 What is the supplement of the following?

a 30° **b** 80° **c** 45° **d** 17°
e 120° **f** 95° **g** 135° **h** 167°

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PROBLEM-SOLVING AND REASONING	8, 9(½), 11	8–12	9(½), 10, 12, 13

8 A round birthday cake is cut into sectors for nine friends (including Jack) at Jack's birthday party. After the cake is cut, there is no cake remaining. What will be the angle at the centre of the cake for Jack's piece if:

a everyone receives an equal share?

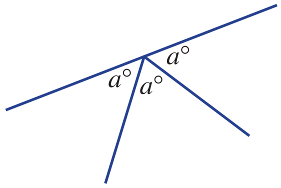
b Jack receives twice as much as everyone else? (In parts **b**, **c** and **d** assume his friends have an equal share of the rest.)

c Jack receives four times as much as everyone else?

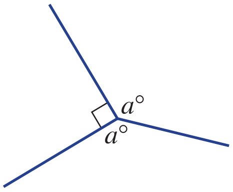
d Jack receives ten times as much as everyone else?

9 Find the value of a in these diagrams.

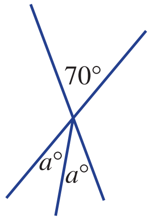
a



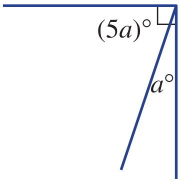
b



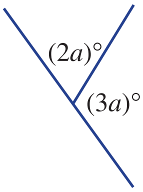
c



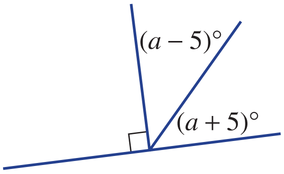
d



e

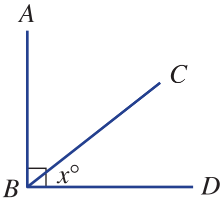


f



10 Write down an algebraic expression for the size of the $\angle ABC$.

a



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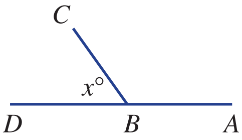
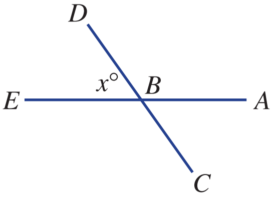


c



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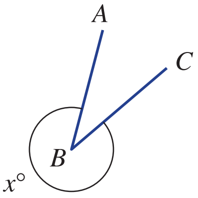

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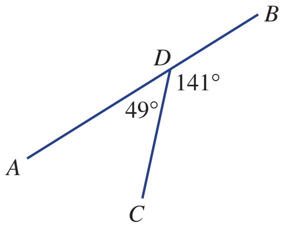
11 Explain what is wrong with these diagrams.

a



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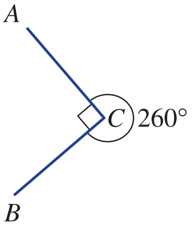
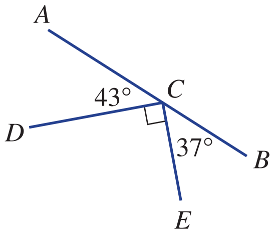
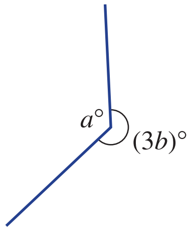


c

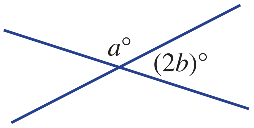


12 Write down an equation (e.g. $2a + b = 90$) for these diagrams.

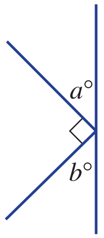
a



b



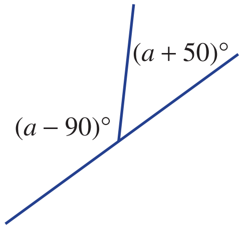
c



13 Consider this diagram (not drawn to scale).

a Calculate the value of a.

b Explain what is wrong with the way the diagram is drawn.

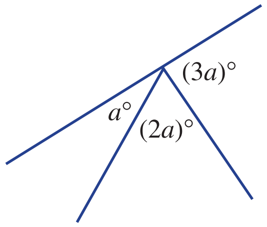


ENRICHMENT – – 14

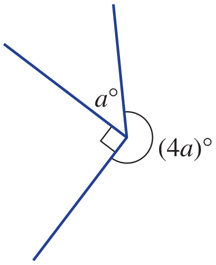
Geometry with equations

14 Equations can be helpful in solving geometric problems in which more complex expressions are involved. Find the value of a in these diagrams.

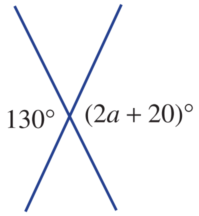
a



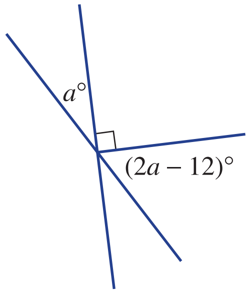
b



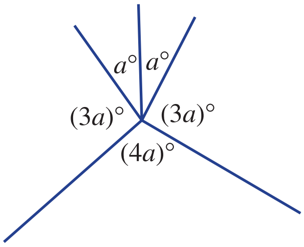
c



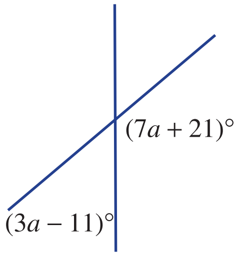
d



e



f



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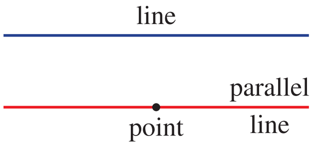
6B Transversal lines and parallel lines REVISION

Euclid's fifth axiom is: Given a line (shown in blue) and a point not on the line, there is only one line (shown in red) through the given point and in the same plane that does not intersect the given line.

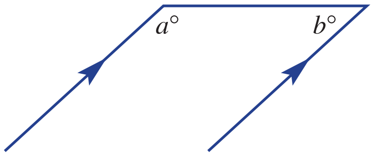


In simple language, Euclid's fifth axiom says that parallel lines do not intersect.

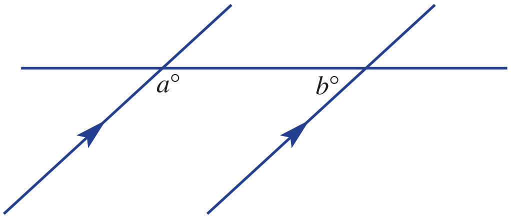
All sorts of shapes and solids both in the theoretical and practical worlds can be constructed using parallel lines. If two lines are parallel and are cut by a third line, called a transversal, special pairs of angles are created.

Let's start: Hidden transversals

This diagram can often be found as part of a shape such as a parallelogram or other more complex diagram.



To see the relationship between a and b more easily, you can extend the lines to form this second diagram. In this new diagram, you can now see the pair of parallel lines and the relationships between all the angles.



- Copy the new diagram.
- Label each of the eight angles formed with the pronumeral a or b, whichever is appropriate.
- What is the relationship between a and b? Can you explain why?

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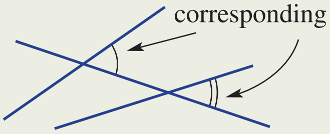
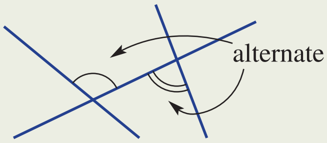
Key ideas

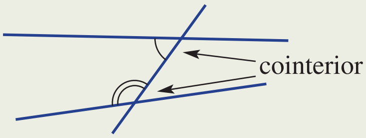
- A **transversal** is a line cutting two or more other lines.
- When a transversal crosses two or more lines, pairs of angles can be:
  - **corresponding** (in matching positions)



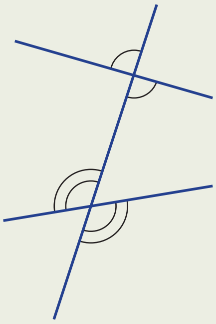
  - **alternate** (on opposite sides of the transversal and inside the other two lines)



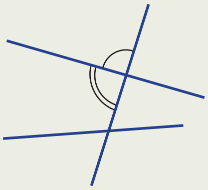
  - **cointerior** (on the same side of the transversal and inside the other two lines)



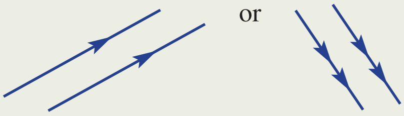
  - vertically opposite



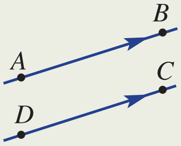
  - angles on a straight line



- Lines are parallel if they do not intersect.
  - Parallel lines are marked with the same number of arrows.

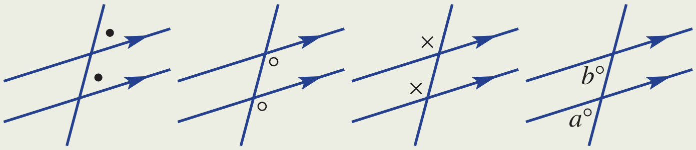


  - In the diagram at right, it is acceptable to write $AB \parallel DC$ or $BA \parallel CD$ but *not* $AB \parallel CD$.



- When two parallel lines are cut by a transversal:
  - the corresponding angles are equal

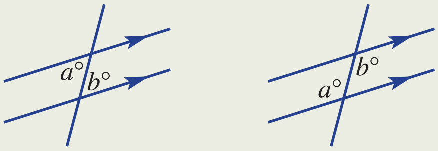
    Note: There are four pairs of corresponding angles.



$a = b$

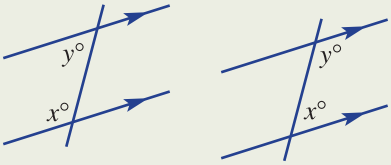
Key ideas

- the alternate angles are equal
  Note: There are two pairs of alternate angles.



$a = b$

- the cointerior angles are supplementary (sum to 180°).
  Note: There are two pairs of cointerior angles.

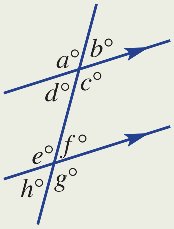


$x + y = 180$

■ The eight angles can be grouped in the following way.
In this diagram:

$a = c = e = g$

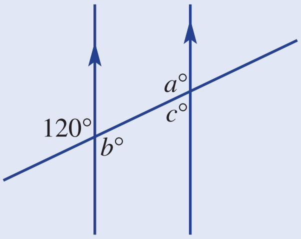
$b = d = f = h$



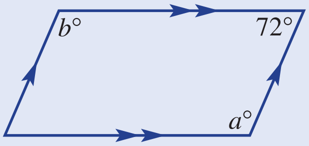
Example 2 Finding angles on parallel lines

Find the value of the pronumerals in these diagrams, giving reasons.

a



b



SOLUTION	EXPLANATION
a $a = 120$ (corresponding angles on parallel lines)	Corresponding angles on parallel lines are equal.
$b = 120$ (vertically opposite angles)	Vertically opposite angles are equal.
$c + 120 = 180$ (cointerior angles on parallel lines) $c = 60$	$b°$ and $c°$ are cointerior angles and sum to 180°. Alternatively, look at $a°$ and $c°$, which are adjacent supplementary angles.
b $a + 72 = 180$ (cointerior angles on parallel lines) $a = 108$ $b + 72 = 180$ (cointerior angles on parallel lines) $b = 108$	The pairs of angles are cointerior, which are supplementary because the lines are parallel.

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Exercise 6B REVISION

UNDERSTANDING AND FLUENCY	1–5	2–6(½)	3–6(½)

1 Two parallel lines are cut by a transversal. Write the missing word (equal or supplementary).

a Corresponding angles are _______________.

b Cointerior angles are _______________.

c Alternate angles are _______________.

2 Name the angle that is:

a corresponding to ∠*ABF*

b corresponding to ∠*BCG*

c alternate to ∠*FBC*

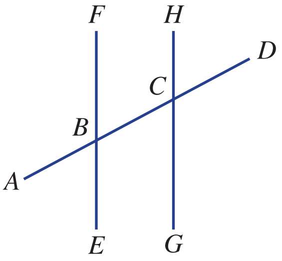
d alternate to ∠*CBE*

e cointerior to ∠*HCB*

f cointerior to ∠*EBC*

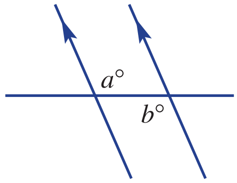
g vertically opposite to ∠*ABE*

h vertically opposite to ∠*HCB*

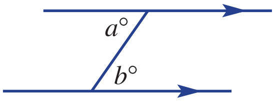


3 State whether the following pairs of marked angles are corresponding, alternate or cointerior. For each diagram write down the relationship between $a°$ and $b°$.

a



b



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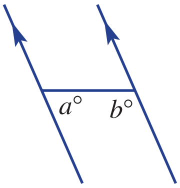

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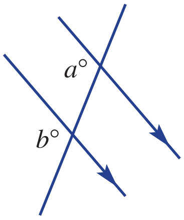

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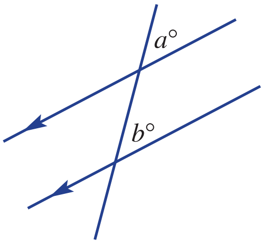

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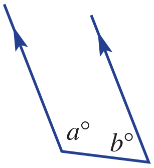
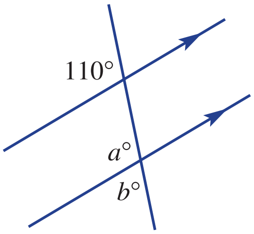


Example 2a

4 Find the value of the pronumerals in these diagrams, giving reasons.

a



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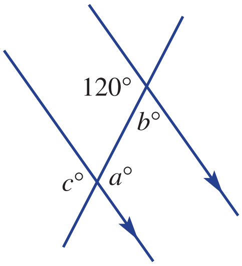

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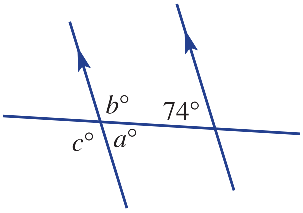
c



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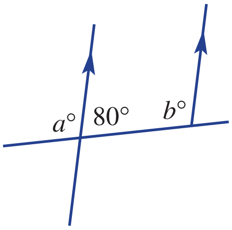

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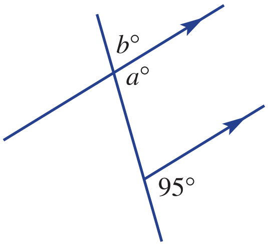

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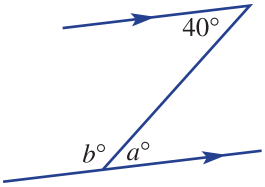
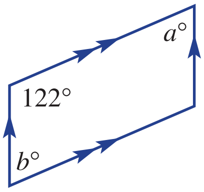


Example 2b

5 Find the value of the pronumerals in these diagrams, giving reasons.

a



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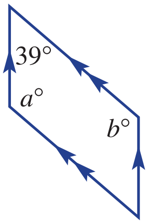

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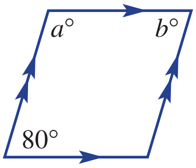

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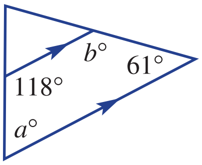

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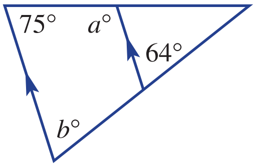

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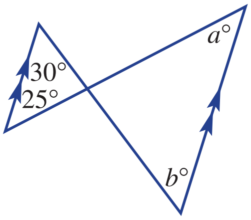
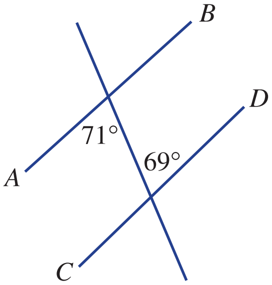


6 For each of the following diagrams decide if *AB* and *CD* are parallel. Explain each answer.

a



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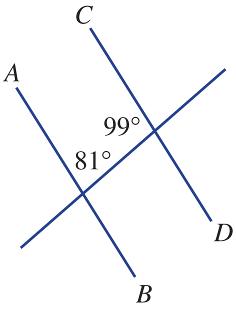

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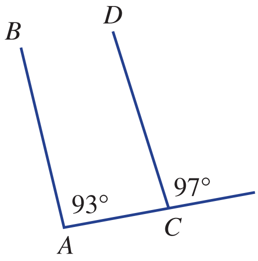

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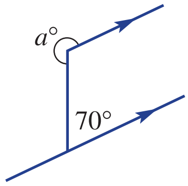
PROBLEM-SOLVING AND REASONING	7–8(½)	7–8(½), 9	7–8(½), 9

7 Find the value of a in these diagrams.

a



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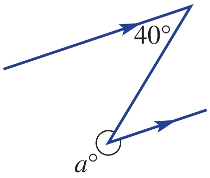

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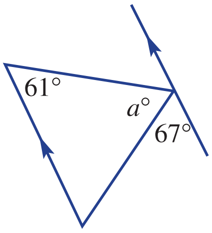

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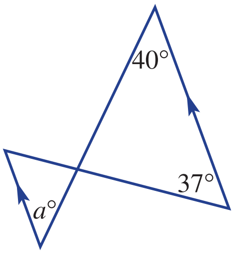

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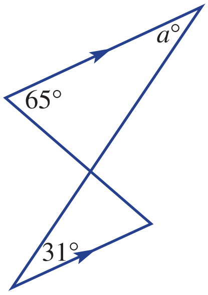

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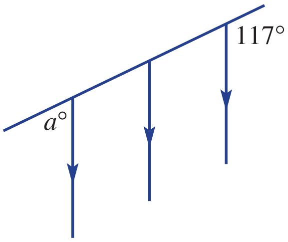
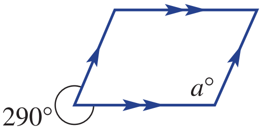


g



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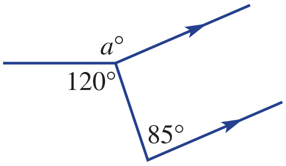
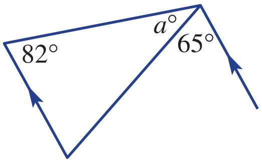
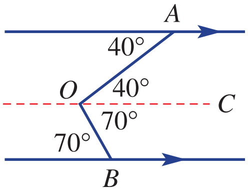


i

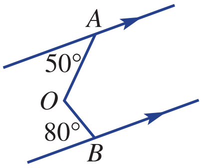


8 Sometimes parallel lines can be added to a diagram to help find an unknown angle. For example, $\angle AOB$ can be found in the diagram at right by first drawing the dashed line and finding $\angle AOC$ $(40°)$ and $\angle COB$ $(70°)$. So $\angle AOB = 40° + 70° = 110°$.

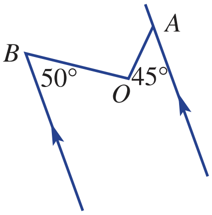


Apply a similar technique to find $\angle AOB$ in these diagrams.

a



b



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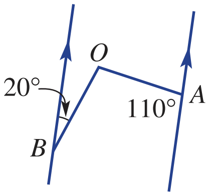

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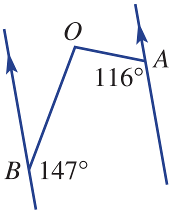

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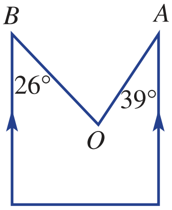

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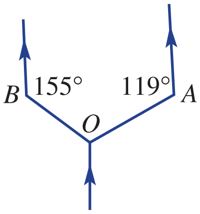

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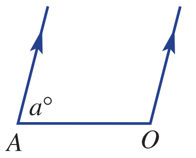
9 Write an algebraic expression for the size of $\angle AOB$.

a



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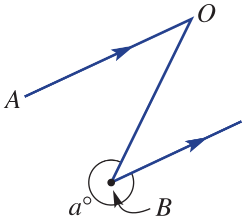

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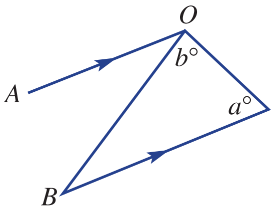
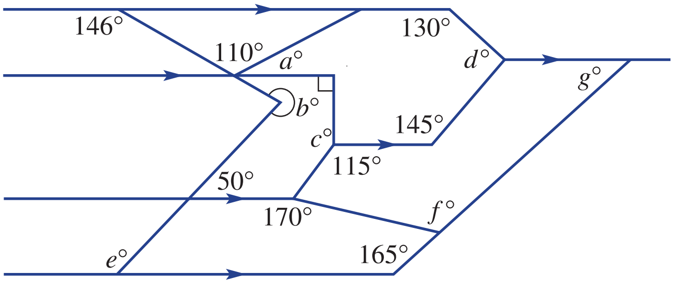


ENRICHMENT	–	–	10

Pipe networks

10 A plan for a natural gas plant includes many intersecting pipe lines, some of which are parallel. Help the designers finish the plans by calculating the values of a, b etc.



6C Triangles

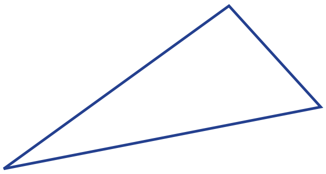
Interactive
Widgets
HOTsheets
Walkthrough

A triangle is a polygon with three sides. As a real-life object, the triangle is a very rigid shape and this leads to its use in the construction of houses and bridges. It is one of the most commonly used shapes in design and construction.

Knowing the properties of triangles can help to solve many geometrical problems, and this knowledge can be extended to explore other more complex shapes.

Let's start: Illustrating the angle sum and the triangle inequality

You can complete this task using a pencil and ruler or using dynamic geometry software.

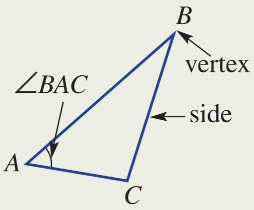


- Draw any triangle and measure each interior angle and each side.
- Add all three angles to find the angle sum of your triangle.
- Compare your angle sum with the results of others. What do you notice?
- Add the two shorter sides together and compare that sum to the longest side.
- Is the longest side opposite the largest angle?

If dynamic geometry is used, drag one of the vertices to alter the sides and the interior angles. Now check to see if your conclusions remain the same.

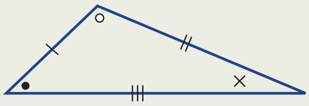
Key ideas

- A **triangle** has:
  - 3 sides: AB, AC, BC
  - 3 vertices (the plural of 'vertex'): A, B, C
  - 3 interior angles: $\angle ABC$, $\angle BAC$, $\angle ACB$

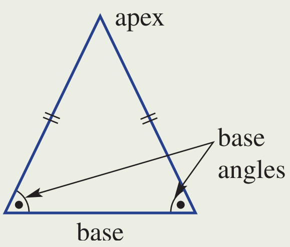


- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

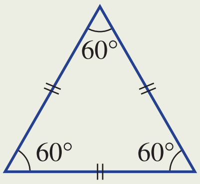
Scalene



Isosceles



Equilateral



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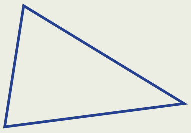
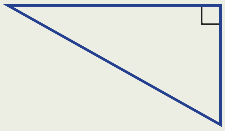
Key ideas

- Triangles classified by interior angles

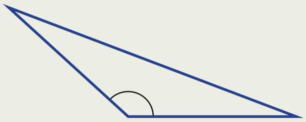
Acute-angled
(all angles acute)



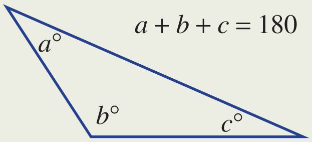
Right-angled
(one right angle)



Obtuse-angled
(one obtuse angle)

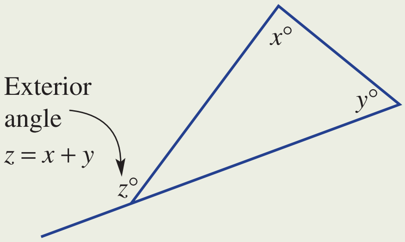


- The angle sum of a triangle is 180°.



$a + b + c = 180$

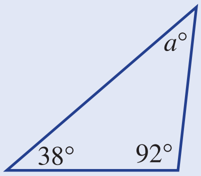
- The **exterior angle theorem:**
  The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



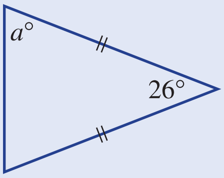
Example 3 Using the angle sum of a triangle

Find the value of a in these triangles.

a



b



SOLUTION

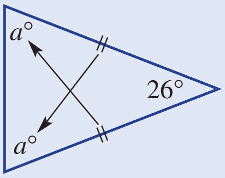
a $a + 38 + 92 = 180$ (angle sum of triangle)
$a + 130 = 180$
$\therefore a = 50$

b $a + a + 26 = 180$
$2a + 26 = 180$ (angle sum of a triangle)
$2a = 154$
$\therefore a = 77$

EXPLANATION

The angle sum of the three interior angles of a triangle is 180°. Also 38 + 92 = 130 and 180 − 130 = 50.

The angles opposite the equal sides are equal.



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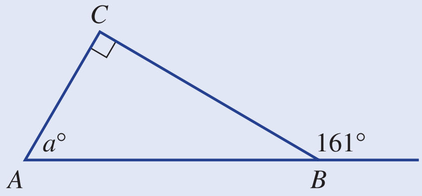
Example 4 Using the exterior angle theorem

Find the value of a.



SOLUTION

$a + 90 = 161$ (exterior angle of $\triangle ABC$)

$a = 71$

Alternatively:

$\angle ABC + 161 = 180$ (angles on a straight line)

$\angle ABC = 19$

$a + 19 + 90 = 180$ (angle sum of a triangle)

$a = 71$

EXPLANATION

Use the exterior angle theorem for a triangle. The exterior angle (161°) is equal to the sum of the two opposite interior angles.

Alternatively, find $\angle ABC$ (19°), then use the triangle angle sum to find the value of a.

Exercise 6C

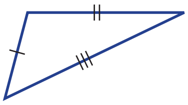
UNDERSTANDING AND FLUENCY	1–6, 7(½)	2, 3, 4(½), 5–6, 7(½)	4(½), 5–6, 7(½)

1 Give the common name of a triangle with these properties.

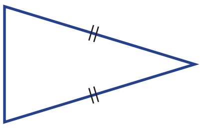
- **a** one right angle
- **b** two equal side lengths
- **c** all angles acute
- **d** all angles 60°
- **e** one obtuse angle
- **f** three equal side lengths
- **g** two equal angles
- **h** three different side lengths

2 Classify these triangles as scalene, isosceles or equilateral.

a



b



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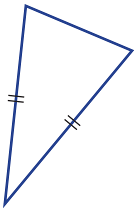

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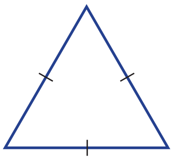
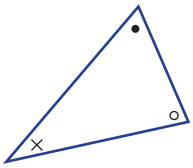
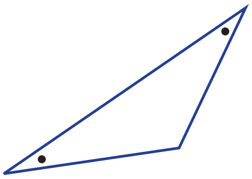


e

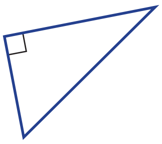


f

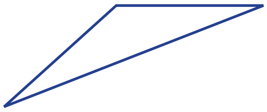


3 Classify these triangles as acute, right or obtuse.

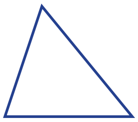
a



b



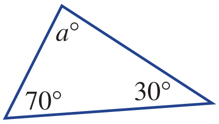
c



Example 3a

4 Use the angle sum of a triangle to help find the value of the pronumeral in these triangles.

a



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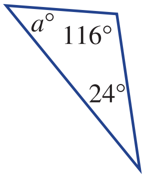

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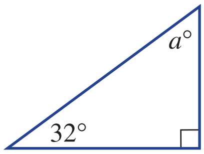

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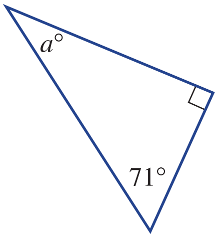

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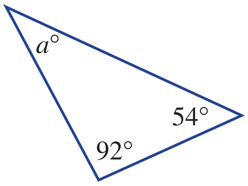

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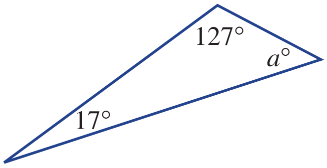
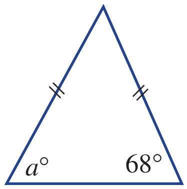


5 These triangles are isosceles. Find the value of a.

a



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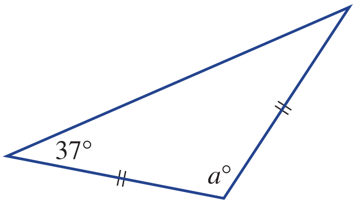

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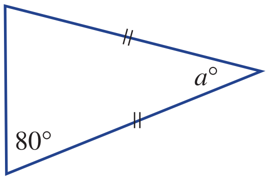
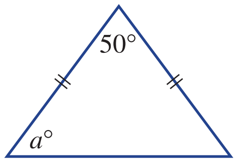


Example 3b

6 Find the value of a in these isosceles triangles.

a



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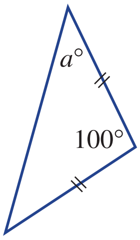

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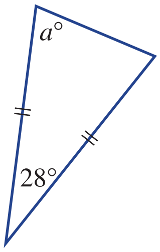
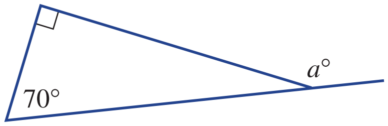


Example 4

7 Find the value of a.

a



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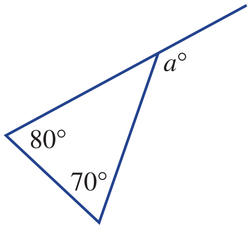

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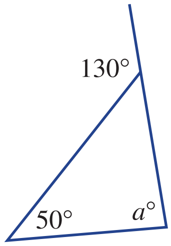

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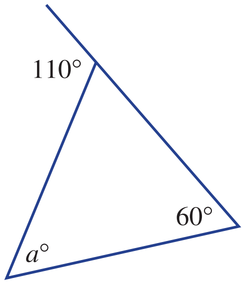

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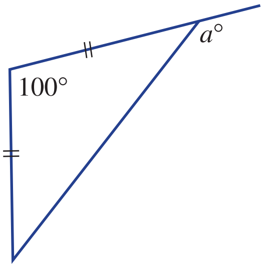
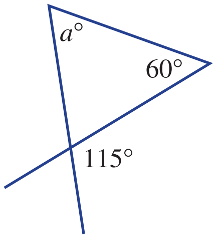


f



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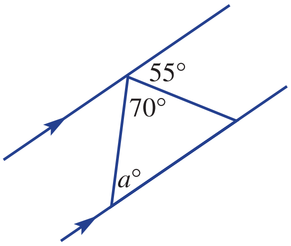
PROBLEM-SOLVING AND REASONING	8, 9, 11	8, 9, 11, 12	9, 10, 12–15

8 Decide if it is possible to draw a triangle with the given description. Draw a diagram to support your answer.

a right and scalene
b obtuse and equilateral
c right and isosceles
d acute and isosceles
e acute and equilateral
f obtuse and isosceles

9 Use your knowledge of parallel lines and triangles to find the unknown angle *a*.

a



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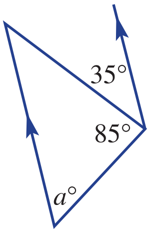

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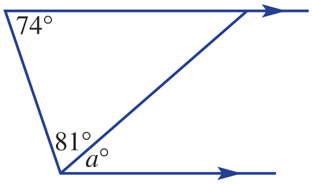
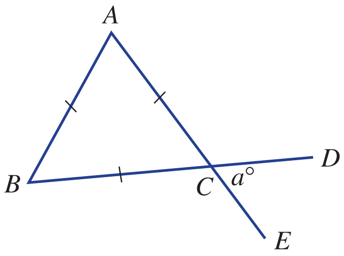


10 Find the value of *a* in these diagrams, giving reasons.

a



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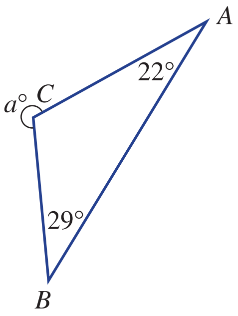

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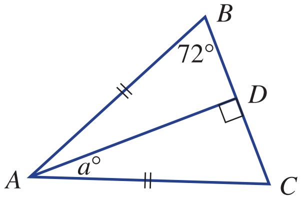

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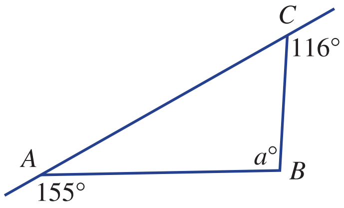

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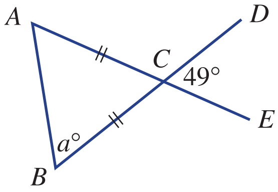

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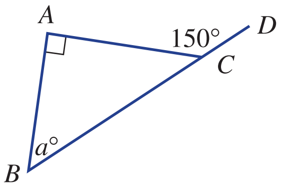
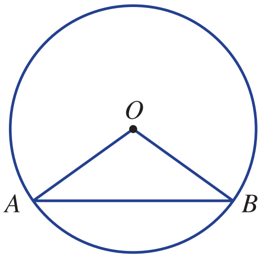
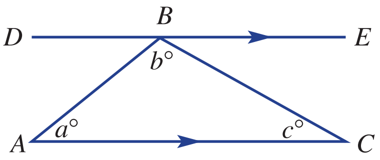


11 A triangle is constructed using two radii and a chord.



a What type of triangle is $\triangle AOB$? Explain.
b Name two angles that are equal.
c Find $\angle ABO$ if $\angle BAO$ is 30°.
d Find $\angle AOB$ if $\angle OAB$ is 36°.
e Find $\angle ABO$ if $\angle AOB$ is 100°.

12 To prove that the angle sum of a triangle is 180°, work through these steps with the given diagram.

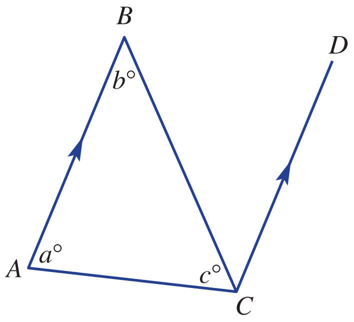


a Using the pronumerals a, b or c, give the value of these angles and state a reason.
  i $\angle ABD$ **ii** $\angle CBE$
b What is true about the three angles $\angle ABD$, $\angle ABC$ and $\angle CBE$ and why?
c What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the angle sum of the triangle ABC?

13 Prove that a triangle cannot have two right angles.

14 Prove that an equilateral triangle must have 60° angles.

15 A different way of proving the angle sum of a triangle is to use this diagram.



a Give a reason why $\angle BCD = b$.
b What do you know about the two angles $\angle BAC$ and $\angle ACD$ and why?
c What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the triangle ABC?

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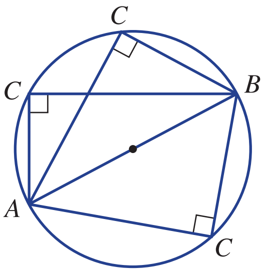
ENRICHMENT	–	–	16

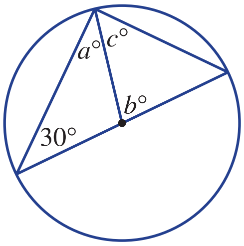
Angle in a semicircle

16 The angle sum of a triangle can be used to prove other theorems, one of which relates to the angle in a semicircle. This theorem says that $\angle ACB$ in a semicircle is always 90°, where AB is a diameter.

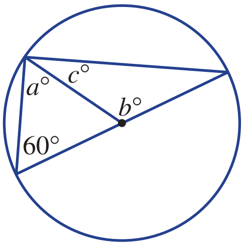


a Use your knowledge of isosceles triangles to find the value of a, b and c in this circle.

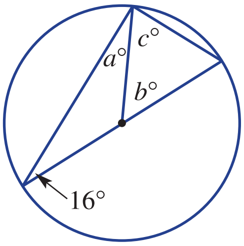
b What do you notice about the sum of a and c?



c Repeat parts **a** and **b** above for this circle.



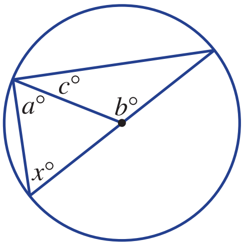
d Repeat parts **a** and **b** above for this circle.



e What do you notice about the sum of a and c for all the circles above?

f Prove this result generally by finding:

- **i** a, b and c in terms of x
- **ii** the value of $a + c$



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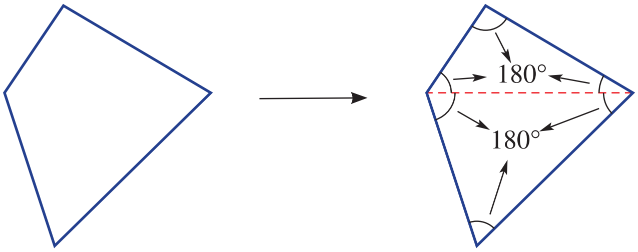
6D Quadrilaterals

Quadrilaterals are four-sided polygons. All quadrilaterals have the same angle sum, but other properties depend on such things as pairs of sides of equal length, parallel sides and lengths of diagonals. All quadrilaterals can be drawn as two triangles and, since the six angles inside the two triangles make up the four angles of the quadrilateral, the angle sum is 2 × 180° = 360°.



Some quadrilaterals possess special properties and can therefore be classified as one or more of the following:

- kite
- trapezium
- parallelogram
- rhombus
- rectangle
- square

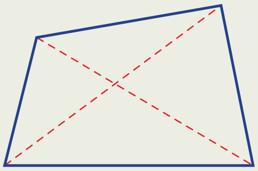
Let's start: Which quadrilateral?

Name all the different quadrilaterals you can think of that have the properties listed below. There may be more than one quadrilateral for each property listed. Draw each quadrilateral to illustrate the shape and its features.

- the opposite sides are parallel
- the opposite sides are equal
- the adjacent sides are perpendicular
- the opposite angles are equal

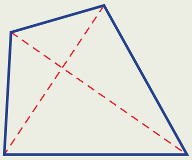
Key ideas

- Every quadrilateral has two diagonals.
  - In some quadrilaterals the diagonals bisect each other (i.e. cut each other in half).



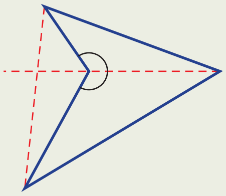
- **Quadrilaterals** can be convex or non-convex.
  - Convex quadrilaterals have all vertices pointing outwards.
  - Non-convex (or concave) quadrilaterals have one vertex pointing inwards.
  - Both diagonals of convex quadrilaterals lie inside the figure.

Convex



All interior angles less than 180°

Non-convex



One reflex interior angle

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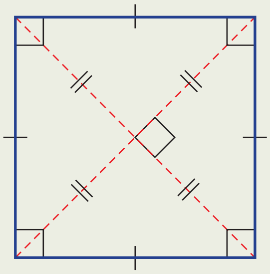
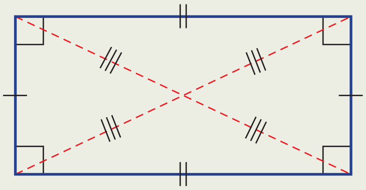
Key ideas

- Special quadrilaterals

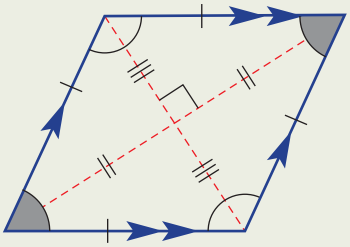
Square



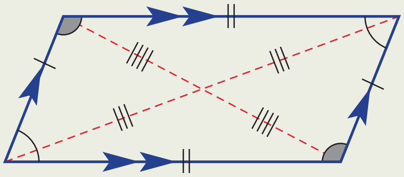
Rectangle



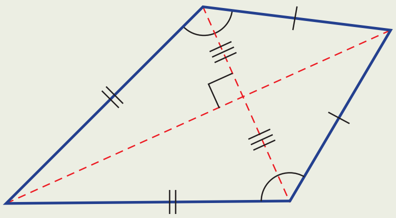
Rhombus



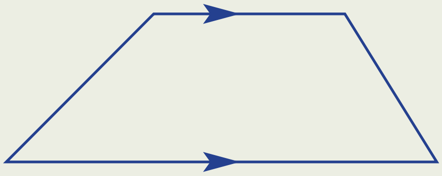
Parallelogram



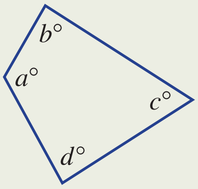
Kite

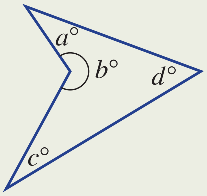


Trapezium



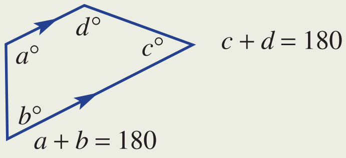
- The angle sum of any quadrilateral is 360°.





$a + b + c + d = 360$

- Quadrilaterals with parallel sides contain two pairs of cointerior angles.



$c + d = 180$

$a + b = 180$

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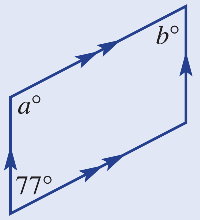
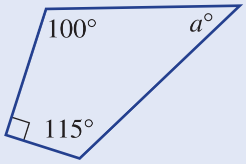
Example 5 Using the angle sum of a quadrilateral

Find the value of the pronumerals in these quadrilaterals.

a



b



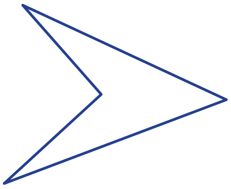
SOLUTION	EXPLANATION
a $a + 77 = 180$ (cointerior angles in parallel lines)	Two angles inside parallel lines are cointerior and therefore add to 180°.
$a = 103$	Opposite angles in a parallelogram are equal.
$b = 77$ (opposite angles in parallelogram)	
b $a + 100 + 90 + 115 = 360$ (angle sum of a quadrilateral)	The sum of angles in a quadrilateral is 360°.
$a + 305 = 360$	
$a = 55$	

Exercise 6D

UNDERSTANDING AND FLUENCY	1–4	2, 3, 4(½), 5	3, 4(½), 5

1 Decide if these quadrilaterals are convex or non-convex.

a



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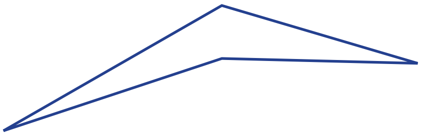
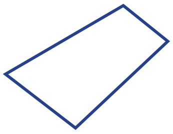


c



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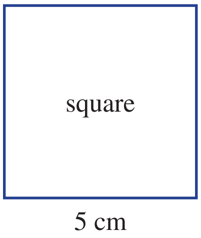
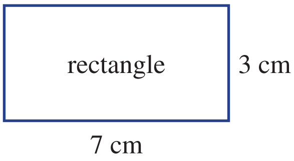
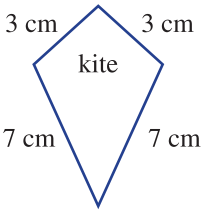
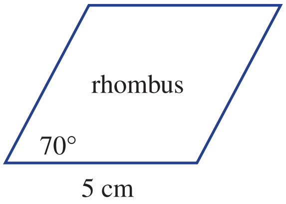
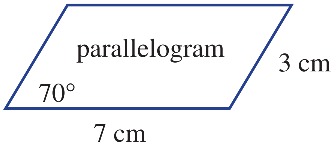
2 Use a ruler and protractor to make a neat and accurate drawing of these special quadrilaterals.

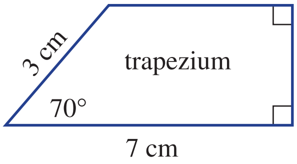












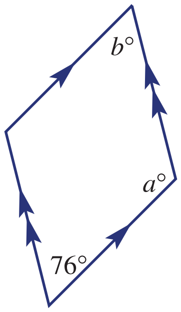
Use your shapes to write YES in the cells in the table, for the statements that are definitely true.

Property	Trapezium	Kite	Parallelogram	Rectangle	Rhombus	Square
The opposite sides are parallel						
All sides are equal						
The adjacent sides are perpendicular						
The opposite sides are equal						
The diagonals are equal						
The diagonals bisect each other						
The diagonals bisect each other at right angles						
The diagonals bisect the angles of the quadrilateral						

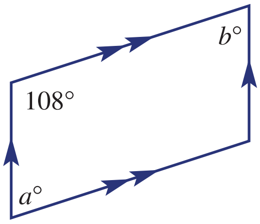
Example 5a

3 Find the value of the pronumerals in these quadrilaterals.

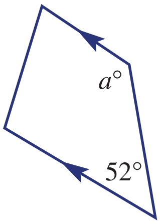
a



b



c



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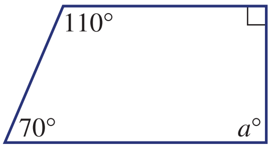
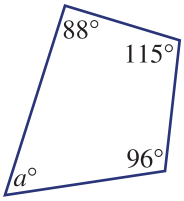
Example 5b

4 Use the quadrilateral angle sum to find the value of *a* in these quadrilaterals.

a



b



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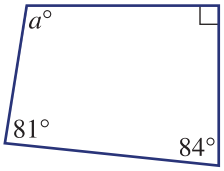

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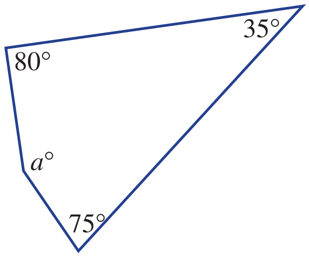
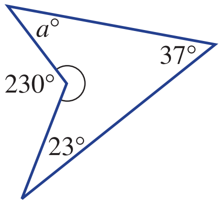


e



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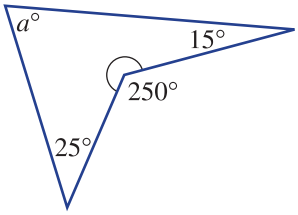
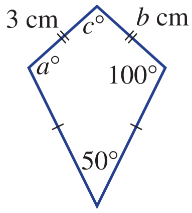


5 By considering the properties of the given quadrilaterals, give the values of the pronumerals.

a



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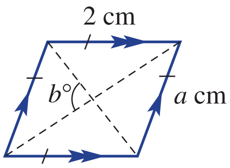

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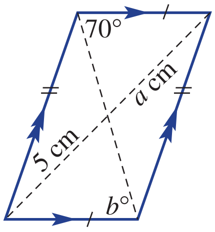
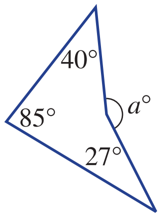


PROBLEM-SOLVING AND REASONING	6, 8	6, 8, 9	6(½), 7, 9, 10

6 Use your knowledge of geometry from the previous sections to find the values of *a*.

a



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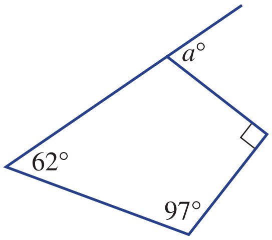

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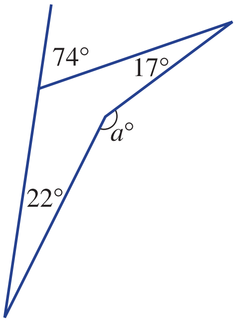
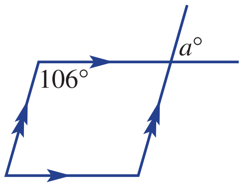


d



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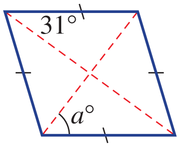

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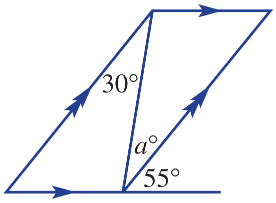

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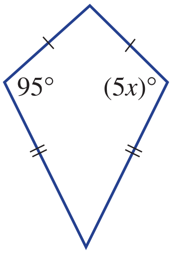
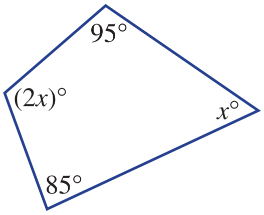
7 Use your algebra skills to find the value of x in these diagrams.

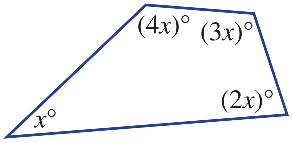
a



b



c



8 The word ‘bisect’ means to cut in half.

a Which quadrilaterals have diagonals that bisect each other?

b Which quadrilaterals have diagonals that bisect all their interior angles?

9 By considering the properties of special quadrilaterals, decide if the following are always true.

a A square is a type of rectangle.

b A rectangle is a type of square.

c A square is a type of rhombus.

d A rectangle is a type of parallelogram.

e A parallelogram is a type of square.

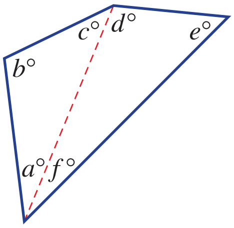
f A rhombus is a type of parallelogram.

10 Is it possible to draw a non-convex quadrilateral with two or more interior reflex angles? Explain and illustrate.

ENRICHMENT	–	–	11

Proof

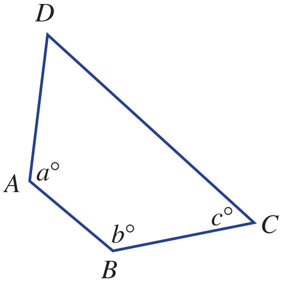
11 Complete these proofs of two different angle properties of quadrilaterals.



a Angle sum $= a + b + c +$ _____ $+$ _____ $+$ _____

$=$ _____ $+$ _____ (angle sum of a triangle)

$=$ _____



b $\angle ADC +$ _____ $+$ _____ $+$ _____ $= 360°$ (angle sum of a quadrilateral)

Reflex $\angle ADC = 360 - \angle ADC$

$= 360 - (360 - ($_____ $+$ _____ $+$ _____$))$

$=$ _____ $+$ _____ $+$ _____

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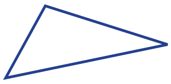
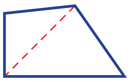
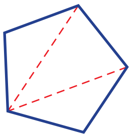
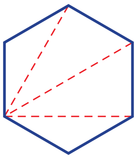
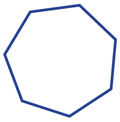
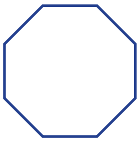
6E Polygons EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

The word 'polygon' comes from the Greek words *poly*, meaning 'many', and *gonia*, meaning 'angles'. The number of interior angles equals the number of sides, and the angle sum of each type of polygon depends on this number. Also, there exists a general rule for the angle sum of a polygon with n sides, which we will explore in this section.

Let's start: Developing the rule

The following procedure uses the fact that the angle sum of a triangle is 180°, which was proved in an earlier section. Complete the table and try to write the general rule in the final row.

Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180^\circ = 180^\circ$
Quadrilateral 	4	2	$\_\_\_\_ \times 180^\circ = \_\_\_\_$
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		$(\_\_\_) \times 180^\circ$

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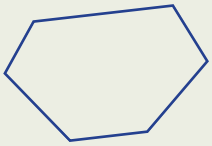
Key ideas

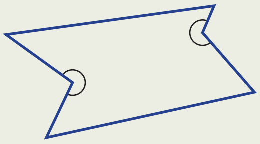
- **Polygons** are shapes with straight sides. They can be convex or non-convex.
  - Convex polygons have all vertices pointing outwards.
  - Non-convex (or concave) polygons have at least one vertex pointing inwards and at least one reflex interior angle.

Convex



This means all interior angles are less than 180°

Non-convex

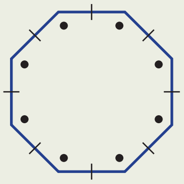


This means there is at least one reflex interior angle

- Polygons are named according to their number of sides.

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- The angle sum S of a polygon with n sides is given by the rule: $S = (n - 2) \times 180°$
- A **regular polygon** has sides of equal length and equal interior angles.



A regular octagon

Example 6 Finding the angle sum

Find the angle sum of a heptagon.

SOLUTION	EXPLANATION
$S = (n - 2) \times 180°$	A heptagon has 7 sides so $n = 7$.
$= (7 - 2) \times 180°$	
$= 5 \times 180°$	Simplify $(7 - 2)$ before multiplying by 180°.
$= 900°$	

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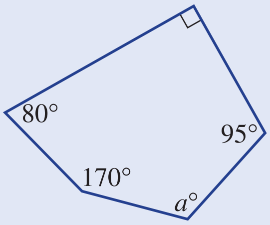
Example 7 Finding angles in polygons

Find the value of a in this pentagon.



SOLUTION

$$\begin{aligned} S &= (n - 2) \times 180^\circ \\ &= (5 - 2) \times 180^\circ \\ &= 540^\circ \end{aligned}$$

$$\begin{aligned} a + 170 + 80 + 90 + 95 &= 540 \\ a + 435 &= 540 \\ a &= 105 \end{aligned}$$

EXPLANATION

First calculate the angle sum of a pentagon using $n = 5$.

Sum all the angles and set this equal to the angle sum of 540°. Subtract 435 from both sides.

Example 8 Finding interior angles of regular polygons

Find the size of an interior angle in a regular octagon.

SOLUTION

$$\begin{aligned} S &= (n - 2) \times 180^\circ \\ &= (8 - 2) \times 180^\circ \\ &= 1080^\circ \end{aligned}$$

$$\begin{aligned} \text{Angle size} &= 1080^\circ \div 8 \\ &= 135^\circ \end{aligned}$$

EXPLANATION

First calculate the angle sum of a octagon using $n = 8$.

All eight angles are equal in size so divide the angle sum by 8.

Exercise 6E EXTENSION

UNDERSTANDING AND FLUENCY	1–7	4, 5–6(½), 7	5–7(½)

1 State the number of sides of these polygons.

a hexagon **b** quadrilateral **c** decagon
d heptagon **e** pentagon **f** dodecagon

2 Evaluate $(n - 2) \times 180^\circ$ if:

a $n = 6$ **b** $n = 10$ **c** $n = 22$

3 What is the common name given to these polygons?

a regular quadrilateral **b** regular triangle

4 Regular polygons have equal interior angles. Find the size of an interior angle for these regular polygons with the given angle sum.

a pentagon (540°) **b** decagon (1440°) **c** octagon (1080°)

Example 6

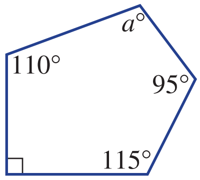
5 Find the angle sum of these polygons.

- **a** hexagon
- **b** nonagon
- **c** heptagon
- **d** 15-sided polygon
- **e** 45-sided polygon
- **f** 102-sided polygon

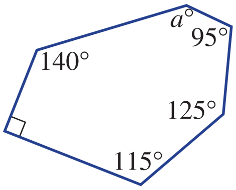
Example 7

6 Find the value of *a* in these polygons.

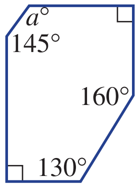
a



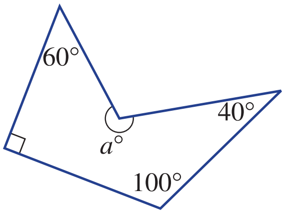
b



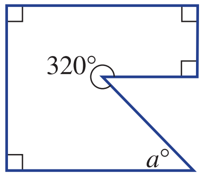
c



d



e



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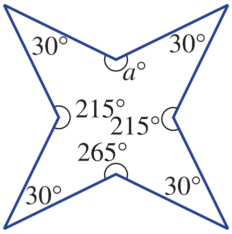


Example 8

7 Find the size of an interior angle of these regular polygons. Round the answer to one decimal place where necessary.

- **a** regular pentagon
- **b** regular heptagon
- **c** regular undecagon
- **d** regular 32-sided polygon

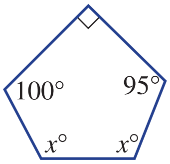
PROBLEM-SOLVING AND REASONING	8, 10	8, 9(½), 10	8, 9(½), 11

8 Find the number of sides of a polygon with the given angle sums.

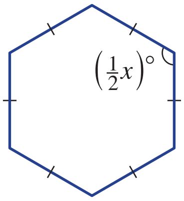
- **a** 1260°
- **b** 2340°
- **c** 3420°
- **d** 29 700°

9 Find the value of *x* in these diagrams.

a



b



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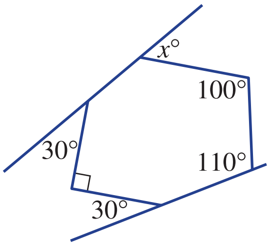

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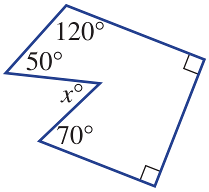

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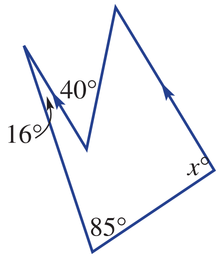
e



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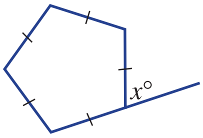


10 Consider a regular polygon with a very large number of sides (n).

- **a** What shape does this polygon look like?
- **b** Is there a limit to the size of a polygon angle sum or does it increase to infinity as n increases?
- **c** What size does each interior angle approach as n increases?

11 Let S be the angle sum of a regular polygon with n sides.

- **a** Write a rule for the size of an interior angle in terms of S and n.
- **b** Write a rule for the size of an interior angle in terms of n only.
- **c** Use your rule to find the size of an interior angle of these polygons. Round to two decimal places where appropriate.
  - **i** regular dodecagon
  - **ii** 82-sided regular polygon

ENRICHMENT	–	–	12, 13

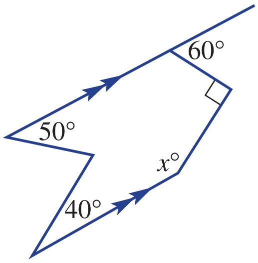
Unknown challenges

12 Find the number of sides of a regular polygon if each interior angle is

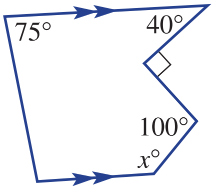
a 120° **b** 162° **c** 147.272727…°

13 With the limited information provided, find the value of x in these diagrams.

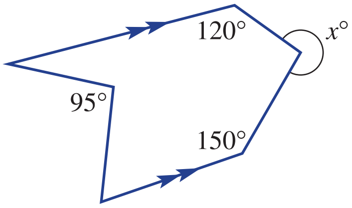
a



b



c



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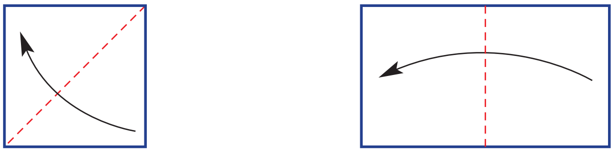
6F Line symmetry and rotational symmetry

You see many symmetrical geometrical shapes in nature. The starfish and sunflower are two examples. Shapes such as these may have two types of symmetry: line and rotational.

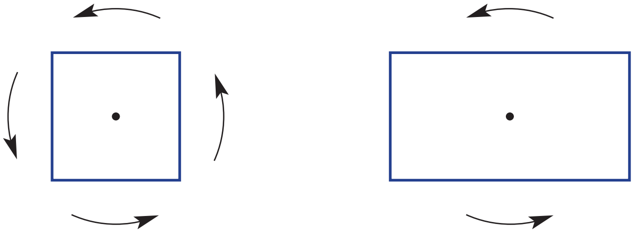
Let's start: Working with symmetry

On a piece of paper draw a square (with side length of about 10 cm) and a rectangle (with length of about 15 cm and breadth of about 10 cm), then cut them out.

- How many ways can you fold each shape in half so that the two halves match exactly? The number of creases formed will be the number of lines of symmetry.



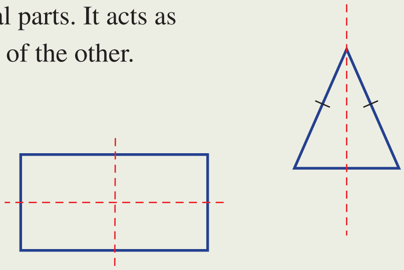
- Now locate the centre of each shape and place a sharp pencil on this point. Rotate the shape 360°. How many times does the shape make an exact copy of itself in its original position? This number describes the rotational symmetry.



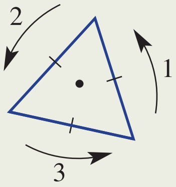
Key ideas

- An axis or **line of symmetry** divides a shape into two equal parts. It acts as a mirror line, with each half of the shape being a reflection of the other.

  This isosceles triangle has one line (axis) of symmetry.
  The plural of axis is axes.
  A rectangle has two lines (axes) of symmetry.



- The order of rotational symmetry refers to the number of times a figure coincides with its original position in turning through one full rotation. For example, the order of rotational symmetry for an equilateral triangle is 3, whereas for rectangle it is 2.



- We say that there is no rotational symmetry if the order of **rotational symmetry** is 1. Right-angle triangles do not have rotational symmetry.

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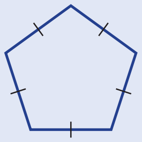
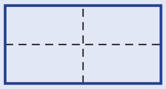
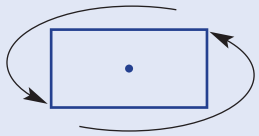
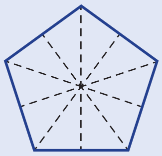
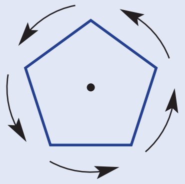
Example 9 Finding the symmetry of shapes

Give the number of lines of symmetry and the order of rotational symmetry for each of these shapes.

a rectangle

b regular pentagon



SOLUTION	**EXPLANATION**
a 2 lines of symmetry	
rotational symmetry: order 2	
b 5 lines of symmetry	
rotational symmetry: order 5	

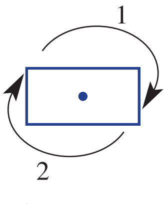
Exercise 6F

UNDERSTANDING AND FLUENCY	1–8	2–8	4–8

1 How many ways could you fold each of these shapes in half so that the two halves match exactly? (To help you solve this problem, try cutting out the shapes and folding them.)

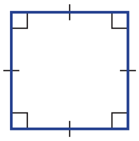
- **a** square
- **b** rectangle
- **c** equilateral triangle
- **d** isosceles triangle
- **e** rhombus
- **f** parallelogram

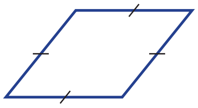
2 For the shapes listed in Question **1**, imagine rotating them 360° about their centre. How many times do you make an exact copy of the shape in its original position?



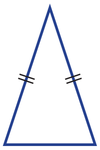
Example 9

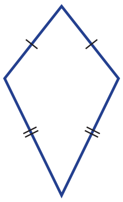
3 Give the number of lines of symmetry and the order of rotational symmetry for each shape.

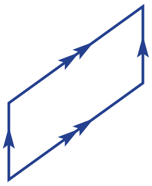
a


b


c


d


e


f


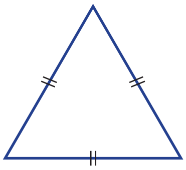
4 Name a type of triangle that has the following properties.

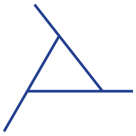
- **a** three lines of symmetry and order of rotational symmetry 3
- **b** one line of symmetry and no rotational symmetry
- **c** no line or rotational symmetry

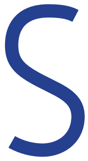
5 List the quadrilaterals that have these properties.

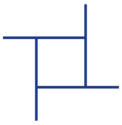
- **a** number of lines of symmetry
  - **i** 1 **ii** 2 **iii** 3 **iv** 4
- **b** order of rotational symmetry
  - **i** 1 **ii** 2 **iii** 3 **iv** 4

6 State the number of lines of symmetry and order of rotational symmetry for each of the following.

a


b


c


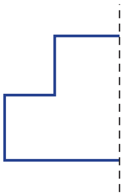
d


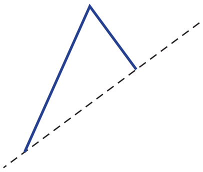
7 Of the capital letters of the alphabet shown here, state which have:

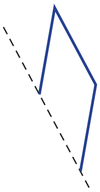
- **a** 1 line of symmetry
- **b** 2 lines of symmetry
- **c** rotational symmetry of order 2

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

8 Complete the other half of these shapes for the given axis of symmetry.

a


b


c


PROBLEM-SOLVING AND REASONING	9, 11	9–11	10–12

9 Draw the following shapes, if possible.

- **a** a quadrilateral with no lines of symmetry
- **b** a hexagon with one line of symmetry
- **c** a shape with seven lines of symmetry and rotational symmetry of order 7
- **d** a diagram with no line of symmetry but rotational symmetry of order 3
- **e** a diagram with one line of symmetry and no rotational symmetry

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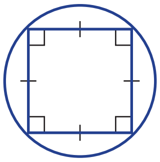
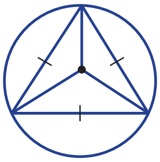
10 These diagrams are made up of more than one shape. State the number of lines of symmetry and the order of rotational symmetry.

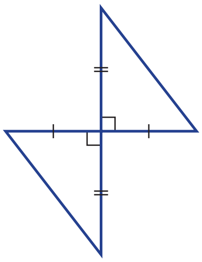
a



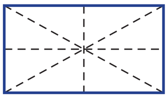
b



c

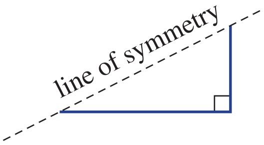


11 Many people think a rectangle has four lines of symmetry, including the diagonals.



a Complete the other half of the diagram on the right to show that this is not true.

b Using the same method as that used in part **a**, show that the diagonals of a parallelogram are not lines of symmetry.



12 A trapezium has one pair of parallel sides.

a State whether trapeziums always have:

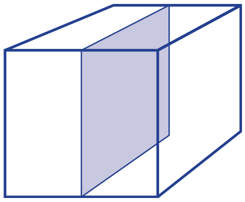
- **i** line symmetry
- **ii** rotational symmetry

b What type of trapezium will have one line of symmetry? Draw one.

ENRICHMENT	–	–	13

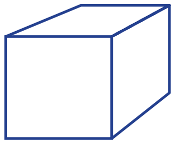
Symmetry in 3D

13 Some solid objects also have symmetry. Rather than line symmetry, they have **plane symmetry**. This cube shows one plane of symmetry, but there are more that could be drawn.

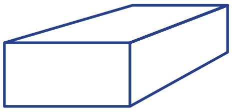


State the number of planes of symmetry for each of these solids.

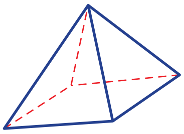
a cube



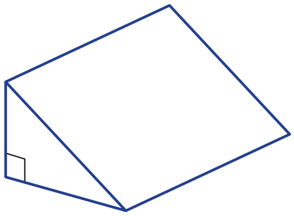
b rectangular prism



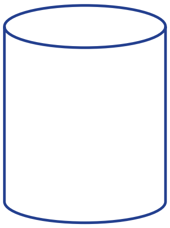
c right square pyramid



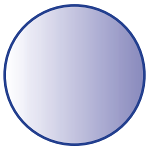
d right triangular prism



e cylinder



f sphere



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6G Euler's formula for three-dimensional solids FRINGE

A solid occupies three-dimensional space and can take on all sorts of shapes. The outside surfaces could be flat or curved and the number of surfaces will vary depending on the properties of the solid. A solid with all flat surfaces is called a polyhedron, plural *polyhedra* or *polyhedrons*. The word 'polyhedron' comes from the Greek words *poly*, meaning 'many', and *hedron*, meaning 'faces'.

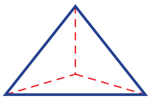
Let's start: Developing Euler's formula

Create a table with each polyhedron listed below in its own row in column 1 (see below). Include the name and a drawing of each polyhedron. Add columns to the table for faces (F), vertices (V), edges (E) and faces plus vertices added together ($F + V$).

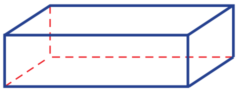
Polyhedron	Drawing	Faces (F)	Vertices (V)	Edges (E)	$F + V$

Count the faces, vertices and edges for each polyhedron and record the numbers in the table.

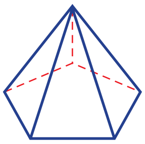
Tetrahedron



Hexahedron



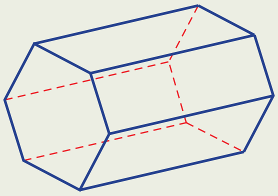
Pentagonal pyramid



- What do you notice about the numbers in the columns for E and $F + V$?
- What does this suggest about the variables F, V and E? Can you write a formula?
- Add rows to the table, draw your own polyhedra and test your formula by finding the values for F, V and E.

Key ideas

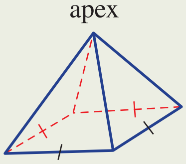
- A **polyhedron** (plural: polyhedra) is a closed solid with flat surfaces (faces), vertices and edges.
  - Polyhedra can be named by their number of faces. For example, tetrahedron (4 faces), pentahedron (5 faces) and hexahedron (6 faces).
- **Euler's formula** for polyhedra with F faces, V vertices and E edges is given by: $E = F + V - 2$
- **Prisms** are polyhedra with two identical (congruent) ends. The congruent ends define the **cross-section** of the prism and also its name. The other faces are parallelograms. If these faces are rectangles, as shown, then the solid is a **right prism**.



Hexagonal prism

Key ideas

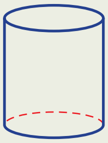
- **Pyramids** are polyhedra with a base face and all other faces meeting at the same vertex point called the apex. They are named by the shape of the base.



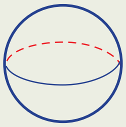
Square pyramid

- Some solids have **curved surfaces**. Common examples include:

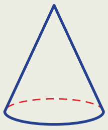
Cylinder



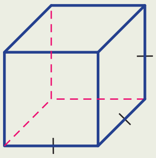
Sphere



Cone



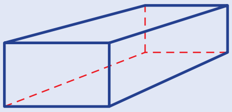
- A **cube** is a hexahedron with six square faces.



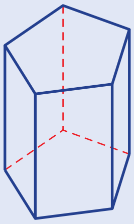
Example 10 Classifying solids

a Classify these solids by considering the number of faces.

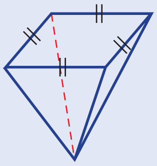
i



ii



iii



b Name these solids as a type of prism or pyramid (e.g. hexagonal prism or hexagonal pyramid).

SOLUTION	EXPLANATION
a **i** hexahedron	The solid has six faces.
ii heptahedron	The solid has seven faces.
iii pentahedron	The solid has five faces.
b **i** rectangular prism	It has two rectangular ends with rectangular sides.
ii pentagonal prism	It has two pentagonal ends with rectangular sides.
iii square pyramid	It has a square base and four triangular faces meeting at an apex.

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Example 11 Using Euler's formula

Use Euler's formula to find the number of faces on a polyhedron that has 10 edges and six vertices.

SOLUTION	EXPLANATION
$E = F + V - 2$ $10 = F + 6 - 2$ $10 = F + 4$ $F = 6$	Write down Euler's formula and make the appropriate substitutions. Solve for F, which represents the number of faces.

Exercise 6G FRINGE

UNDERSTANDING AND FLUENCY	1–10	4, 5–6(½), 7–11	5–6(½), 7–11

1 Write the missing word in these sentences.

a A polyhedron has faces, ____________ and edges.

b A heptahedron has ______ faces.

c A prism has two ____________ ends.

d A pentagonal prism has ____________ faces.

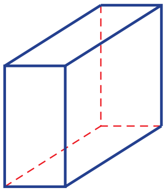
e The base of a pyramid has eight sides. The pyramid is called a ____________ pyramid.

2 Find the value of the pronumeral in these equations.

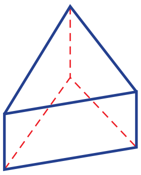
a $E = 10 + 16 - 2$ **b** $12 = F + 7 - 2$ **c** $12 = 6 + V - 2$

3 Count the number of faces, vertices and edges (in that order) on these polyhedra.

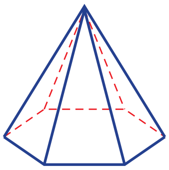
a



b



c



4 Which of these solids are polyhedra (i.e. have only flat surfaces)?

A cube **B** pyramid **C** cone **D** sphere

E cylinder **F** rectangular prism **G** tetrahedron **H** hexahedron

Example 10a

5 Name a polyhedron that has the given number of faces.

a 6 **b** 4 **c** 5 **d** 7

e 9 **f** 10 **g** 11 **h** 12

6 How many faces do you think these polyhedra have?

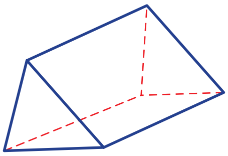
a octahedron **b** hexahedron **c** tetrahedron **d** pentahedron

e heptahedron **f** nonahedron **g** decahedron **h** undecahedron

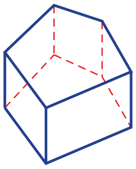
Example 10b

7 Name these prisms.

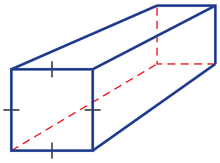
a



b



c



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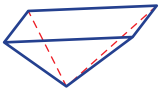
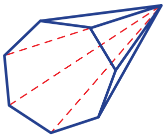
Example 10b

8 Name these pyramids.

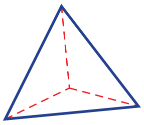
a



b



c



Example 11

9 **a** Copy and complete this table.

Solid	**Number of faces (F)**	**Number of vertices (V)**	**Number of edges (E)**	$F + V$
cube				
square pyramid				
tetrahedron				
octahedron				

b Compare the number of edges (E) with the value $F + V$ for each polyhedron. What do you notice?

10 Use Euler’s formula to calculate the missing numbers in this table.

Faces (F)	**Vertices (V)**	**Edges (E)**
6	8	___
___	5	8
5	___	9
7	___	12
___	4	6
11	11	___

11 **a** A polyhedron has 16 faces and 12 vertices. How many edges does it have?

b A polyhedron has 18 edges and 9 vertices. How many faces does it have?

c A polyhedron has 34 faces and 60 edges. How many vertices does it have?

PROBLEM-SOLVING AND REASONING	12, 15	12, 13, 15, 16	13, 14, 16–18

12 Decide if the following statements are true or false. Make drawings to help.

a A tetrahedron is a pyramid.

b All solids with curved surfaces are cylinders.

c A cube and a rectangular prism are both hexahedrons.

d A hexahedron can be a pyramid.

e There are no solids with zero vertices.

f There are no polyhedra with three surfaces.

g All pyramids will have an odd number of faces.

13 Decide if it is possible to cut the solid using a single straight cut, to form the new solid given in the brackets.

a cube (rectangular prism)

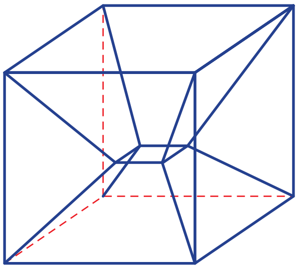
b square-based pyramid (tetrahedron)

c cylinder (cone)

d octahedron (pentahedron)

e cube (heptahedron)

14 This solid is like a cube but is open at the top and bottom and there is a square hole in the middle forming a tunnel. Count the number of faces (F), vertices (V) and edges (E) then decide if Euler's formula is true for such solids.



15 **a** A cuboid is a common name for a solid with six rectangular faces. Name the solid in two other different ways.

b A pyramid has base with 10 sides. Name the solid in two ways.

16 Rearrange Euler's formula.

a Write V in terms of F and E.

b Write F in terms of V and E.

17 Show that Euler's formula applies for these solids.

a heptagonal pyramid **b** octagonal prism **c** octahedron

18 Are the following statements true or false?

a For all pyramids, the number of faces is equal to the number of vertices.

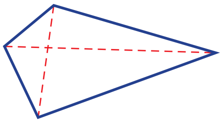
b For all convex polyhedra, the sum $E + V + F$ is even.

ENRICHMENT – – 19

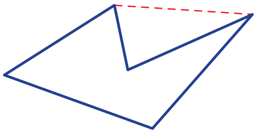
Convex solids

19 Earlier you learnt that a convex polygon will have all interior angles less than 180°. Notice also that all diagonals in a convex polygon are drawn *inside* the shape.

Convex polygon

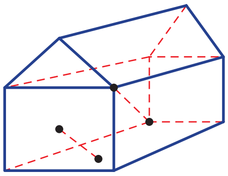


Non-convex polygon

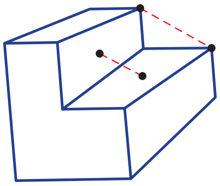


Solids can also be classified as convex or non-convex.

Convex solid



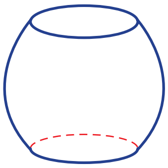
Non-convex solid



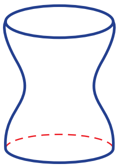
To test for a non-convex solid, join two vertices or two faces with a line segment that passes outside the solid.

a Decide if these solids are convex or non-convex.

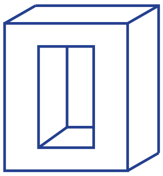
i



ii



iii



b Draw your own non-convex solids and check by connecting any two vertices or faces with a line segment outside the solid.

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Constructions

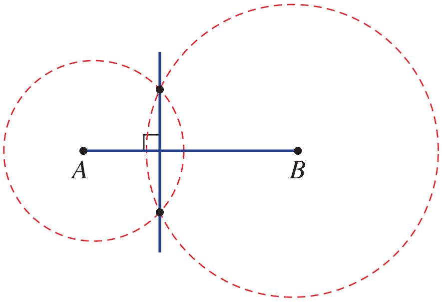
Geometric construction involves a precise set of mathematical and geometric operations that do not involve any approximate measurements or other guess work. The basic tools for geometric construction are a pair of compasses, a straight edge and a pencil or drawing pen. Computer geometry or drawing packages can also be used, and include digital equivalents of these tools.

For the following constructions use only a pair of compasses, a straight edge and a pencil. Alternatively, use computer geometry software and adapt the activities where required.

Perpendicular line

Construct:

a a segment *AB*
b a circle with centre *A*
c a circle with centre *B*
d a line joining the intersection points of the two circles



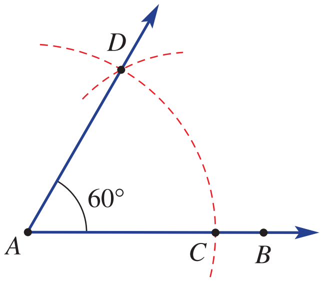
Perpendicular bisector

Repeat the construction for a perpendicular line, but ensure that the two circles have the same radius. If computer geometry is used, use the length of the segment *AB* for the radius of both circles.

A 60º angle

Construct:

a a ray *AB*
b an arc with centre *A*
c the intersection point *C*
d an arc with centre *C* and radius *AC*
e a point *D* at the intersection of the two arcs
f a ray *AD*



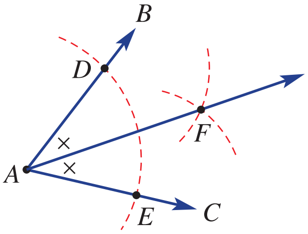
Equilateral triangle

Repeat the construction for a 60° angle, then construct the segment *CD*.

Angle bisector

Construct:

a any angle ∠*BAC*
b an arc with centre *A*
c the two intersection points *D* and *E*
d two arcs of equal radius with centres at *D* and *E*
e the intersection point *F*
f the ray *AF*



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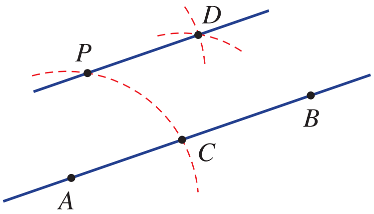
Parallel line through a point

Construct:

a a line AB and point P
b an arc with centre A and radius AP
c the intersection point C
d an arc with centre C and radius AP
e an arc with centre P and radius AP
f the intersection point D
g the line PD



Rhombus

Repeat the construction for a parallel line through a point and construct the segments AP and CD.

Construction challenges

For a further challenge try to construct these objects. No measurement is allowed.

a 45° angle
b square
c perpendicular line at the end of a segment
d parallelogram
e regular hexagon

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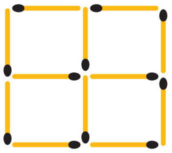
1 This shape includes 12 matchsticks. (To solve these puzzles all matches remaining must connect to other matches at both ends.)

a Remove two matchsticks to form two squares.

b Move three matchsticks to form three squares.

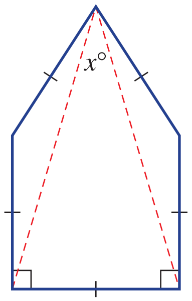


2 **a** Use nine matchsticks to form five equilateral triangles.

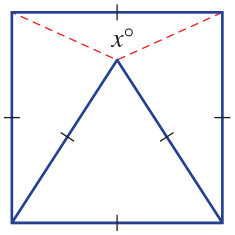
b Use six matchsticks to form four equilateral triangles.

3 Find the value of *x* in these diagrams.

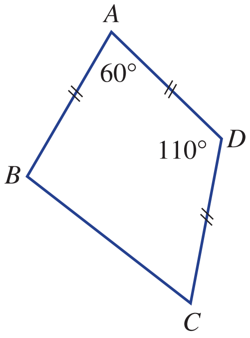
a



b

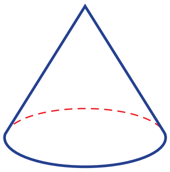


4 Find the size of $\angle ABC$ in this quadrilateral.



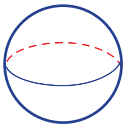
5 Is it possible to draw a net for these solids? If so, draw the net.

a



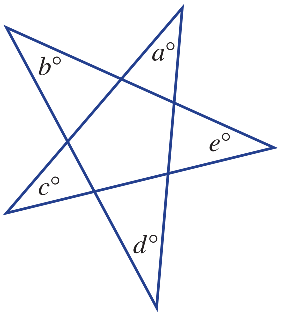
Cone

b



Sphere

6 Find the value of $a + b + c + d + e$ in this star. Give reasons for your answer.



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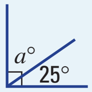
Chapter summary

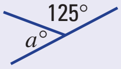
Angle relationships and properties of geometrical figures 1

Angles in a right angle



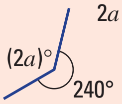
$a + 25 = 90$
$a = 65$

Angles on a straight line



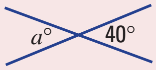
$a + 125 = 180$
$a = 55$

Angles in a revolution



$2a + 240 = 360$
$2a = 120$
$a = 60$

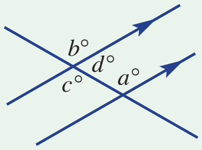
Vertically opposite angles



$a = 40$

Parallel lines

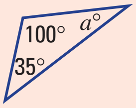
- Corresponding angles are equal ($a = b$)
- Alternate angles are equal ($a = c$)
- Cointerior angles are supplementary ($a + d = 180$)



Triangles

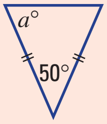
Angle sum = 180°

Scalene



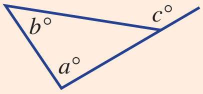
$a + 135 = 180$
$a = 45$

Isosceles

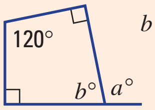


$2a + 50 = 180$
$2a = 130$
$a = 65$

Exterior angles



$c = a + b$



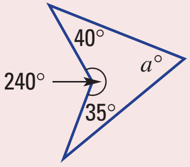
$b + 300 = 360$
$b = 60$
$a = 120$

Quadrilaterals

(square, rectangle, rhombus, parallelogram, kite, trapezium)

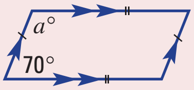
Angle sum = 360°

Non-convex



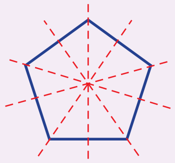
$a + 315 = 360$
$a = 45$

Parallelogram



$a + 70 = 180$
$a = 110$

Symmetry



regular pentagon

- 5 lines of symmetry
- Rotational symmetry of order 5

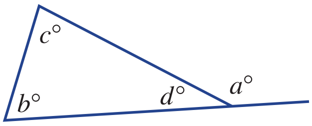
Multiple-choice questions

1 What is the name given to two angles that add to 90°?

A right **B** supplementary **C** revolutionary
D complementary **E** vertically opposite

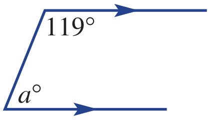
2 The value of a in this diagram is:

A $b + c$
B $c + d$
C $b + d$
D $180 - a$
E $d + 180$



3 The value of a in this diagram is:

A 45
B 122
C 241
D 119
E 61



4 Which quadrilateral has four equal sides and diagonals equal in length?

A kite **B** rhombus **C** square
D parallelogram **E** rectangle

5 The size of an interior angle of an equilateral triangle is:

A 30° **B** 45° **C** 60° **D** 120° **E** 180°

6 The size of an exterior angle on an equilateral triangle is:

A 60° **B** 120° **C** 180° **D** 100° **E** 360°

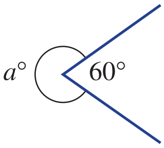
7 What is the complement of 60°?

A 15° **B** 30° **C** 120° **D** 300° **E** 360°

8 What is the supplement of 60° ?

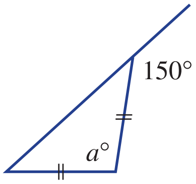
A 15° **B** 30° **C** 120° **D** 300° **E** 360°

9 What is the value of a?



A 15 **B** 30 **C** 120 **D** 300 **E** 360

10 What is the value of a ?



A 15 **B** 30 **C** 120 **D** 300 **E** 360

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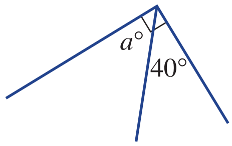
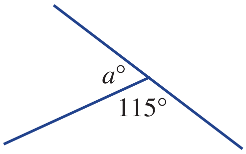
Short-answer questions

1 Find the value of a in these diagrams.

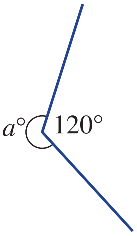
a



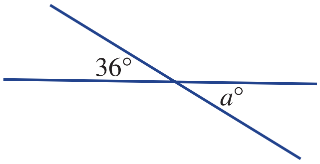
b



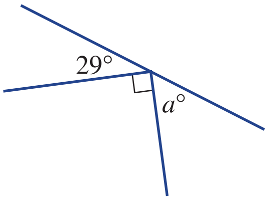
c



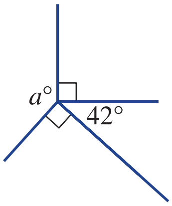
d



e

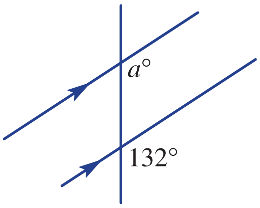


f

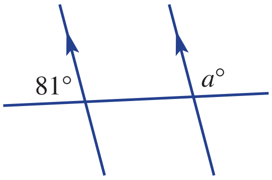


2 These diagrams include parallel lines. Find the value of a.

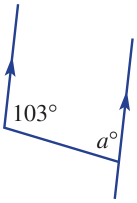
a



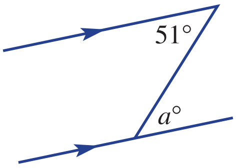
b



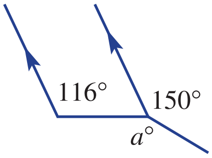
c



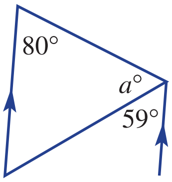
d



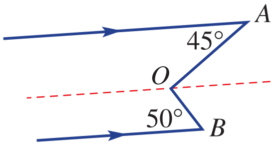
e



f

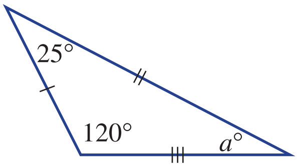


3 Use the dashed construction line to help find the size of $\angle AOB$ in this diagram.

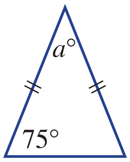


4 Use the side lengths and angle sizes to give a name for each triangle (e.g. right-angled isosceles) and find the value of a.

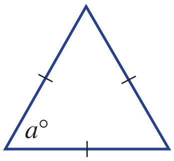
a



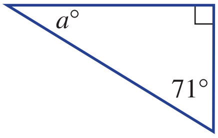
b



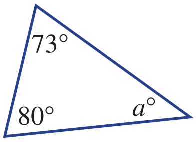
c



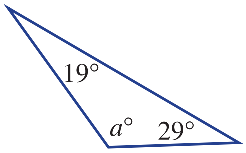
d



e



f



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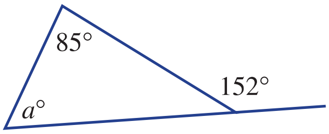
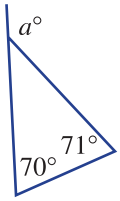
5 These triangles include exterior angles. Find the value of a.

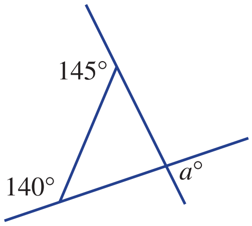
a



b

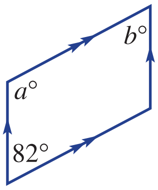


c

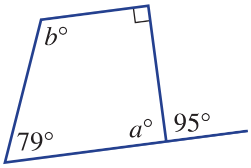


6 Find the value of a and b in these quadrilaterals.

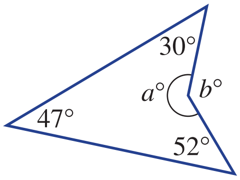
a



b



c



7 How many axes of symmetry do the following shapes possess?

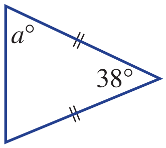
a rectangle **b** parallelogram **c** square **d** kite

8 What is the order of rotational symmetry for these shapes?

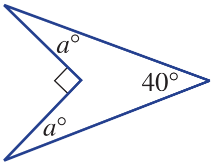
a rectangle **b** parallelogram **c** square **d** equilateral triangle

9 Find the value of a.

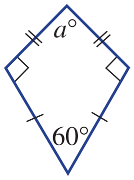
a



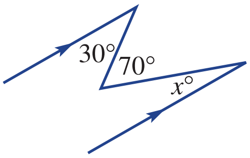
b



c

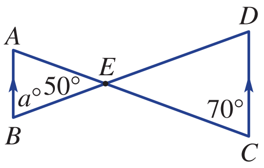


10 Find the value of x in this diagram.

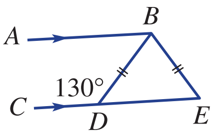


Extended-response questions

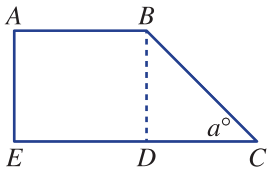
1 Find the value of a, giving reasons.



2 Find the size of $\angle ABE$, giving reasons.



3 In the diagram, $ABDE$ is a rectangle.
$AB = 5$ cm, $AE = 8$ cm, $EC = 13$ cm.
Find the value of a, giving reasons.



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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

7

Linear relationships 1

What you will learn

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: LINEAR RELATIONSHIPS

Outcome

A student creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane. (MA4–11NA)

Mining optimisation

The Australian mining companies of today spend millions of dollars planning and managing their mining operations. The viability of a mine depends on many factors, including availability, product price and quality, as well as environmental considerations.

Through a process called linear programming, many of these factors are represented using linear equations and straight-line graphs. From these equations and graphs an optimal solution can be found, which tells the company the most efficient and cost-effective way to manage all of the given factors. Such use of simple linear graphs can save companies many millions of dollars.

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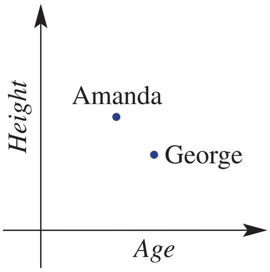
1 This graph shows the relationship between age and height of two people.



a Who is older? **b** Who is taller?

2 Write the x-coordinate for these points.

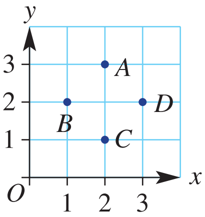
a (1, 2) **b** (1, 5)

c (0, −1) **d** (−3, 0)

3 Write the y-coordinate for these points.

a (1, 6) **b** (−4, −1) **c** (−3, 0) **d** (−4, 2)

4 The coordinates of the point A on this graph are (2, 3). What are the coordinates of these points?



a B **b** C **c** D

5 If $y = x + 4$, find the value of y when:

a $x = 3$ **b** $x = 0$ **c** $x = -2$

6 If $y = -2x - 3$, find the value of y when:

a $x = 2$ **b** $x = 0$ **c** $x = -3$

7 Complete the tables for the given rules.

a $y = 3x - 2$

x	−2	−1	0	1	2
y					

b $y = -x + 4$

x	−2	−1	0	1	2
y					

8 Find the rule for these tables of values.

a

x	0	1	2	3	4
y	0	3	6	9	12

b

x	−1	0	1	2	3
y	−1	1	3	5	7

c

x	−3	−2	−1	0	1
y	−11	−8	−5	−2	1

d

x	0	1	2	3	4	5
y	5	4	3	2	1	0

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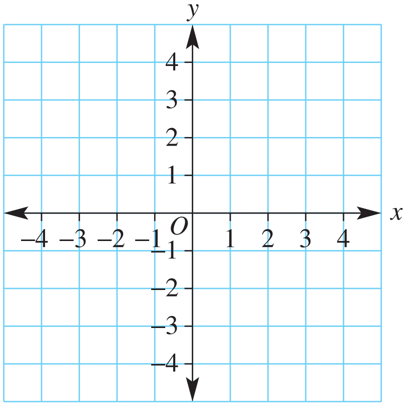
7A The Cartesian plane

On a number plane, a pair of coordinates gives the exact position of a point. The number plane is also called the Cartesian plane after its inventor, René Descartes, who lived in France in the 17th century. The number plane extends both a horizontal axis (x) and vertical axis (y) to include negative numbers. The point where these axes cross over is called the origin and it provides a reference point for all other points on the plane.

Let's start: Make the shape

In groups or as a class, see if you can remember how to plot points on a number plane. Then decide what type of shape is formed by each set.

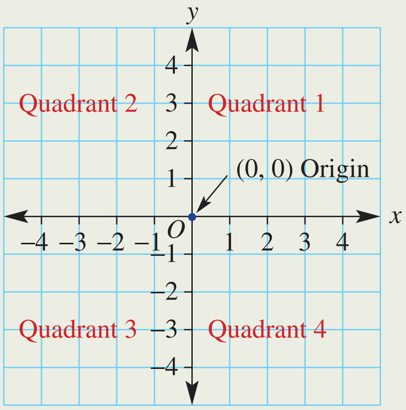


- $A(0, 0)$, $B(3, 1)$, $C(0, 4)$
- $A(-2, 3)$, $B(-2, -1)$, $C(-1, -1)$, $D(-1, 3)$
- $A(-3, -4)$, $B(2, -4)$, $C(0, -1)$, $D(-1, -1)$

Discuss the basic rules for plotting points on a number plane.

Key ideas

- The **Cartesian plane** includes a vertical y-axis and a horizontal x-axis intersecting at right angles. It is also called a number plane.
  - There are four **quadrants**, labelled as shown.
- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first, followed by the y-coordinate.
- The point $(0, 0)$ is called the **origin**.
- $(x, y) = \left(\begin{array}{c}\text{horizontal}\\ \text{units from}\\ \text{origin}\end{array}, \begin{array}{c}\text{vertical}\\ \text{units from}\\ \text{origin}\end{array}\right)$



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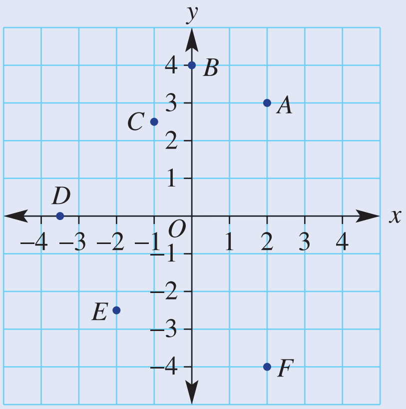
Example 1 Plotting points

Draw a number plane extending from −4 to 4 on both axes, then plot and label these points.

a $A(2, 3)$	**b** $B(0, 4)$	**c** $C(-1, 2.5)$
d $D(-3.5, 0)$	**e** $E(-2, -2.5)$	**f** $F(2, -4)$

SOLUTION



EXPLANATION

The x-coordinate is listed first followed by the y-coordinate.

For each point start at the origin (0, 0) and move left or right or up and down to suit both x- and y-coordinates.

For point $C(-1, 2.5)$, for example, move 1 to the left and 2.5 up.

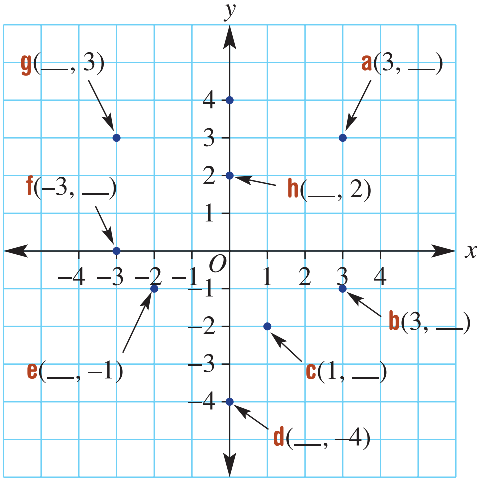
Exercise 7A

UNDERSTANDING AND FLUENCY	1–3, 4(½)	3, 4(½), 5a	4(½), 5b

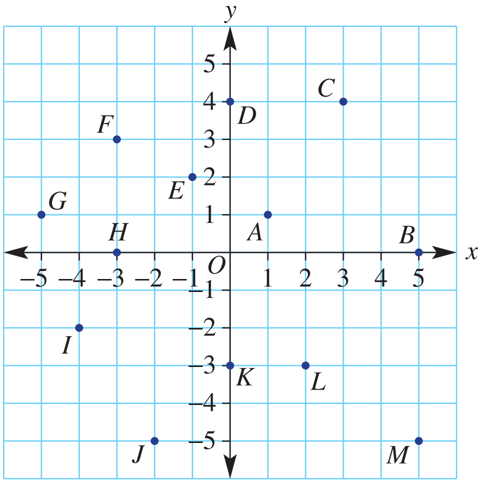
1 Refer to the Key ideas and complete each of the following sentences.

- **a** The coordinates of the origin are ________.
- **b** The vertical axis is called the ________-axis.
- **c** The quadrant that has positive coordinates for both x and y is the ________ quadrant.
- **d** The quadrant that has negative coordinates for both x and y is the ________ quadrant.
- **e** The point $(-2, 3)$ has x-coordinate ________.
- **f** The point $(1, -5)$ has y-coordinate ________.

2 Write the missing number for the coordinates of the points **a**–**h**.



3 Write the coordinates of the points labelled A to M.



Example 1 **4** Draw a number plane extending from −4 to 4 on both axes and then plot and label these points.

a $A(4, 1)$	**b** $B(2, 3)$
c $C(0, 1)$	**d** $D(-1, 3)$
e $E(-3, 3)$	**f** $F(-2, 0)$
g $G(-3, -1)$	**h** $H(-1, -4)$
i $I(0, -2.5)$	**j** $J(0, 0)$
k $K(3.5, -1)$	**l** $L(1.5, -4)$
m $M(-3.5, -3.5)$	**n** $N(-3.5, 0.5)$
o $O(3.5, 3.5)$	**p** $P(2.5, -3.5)$

5 Using a scale extending from −5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

a $(-2, -2), (2, -2), (2, 2), (1, 3), (1, 4), \left(\frac{1}{2}, 4\right), \left(\frac{1}{2}, 3\frac{1}{2}\right), (0, 4), (-2, 2), (-2, -2)$

b $(2, 1), (0, 3), (-1, 3), (-3, 1), (-4, 1), (-5, 2), (-5, -2), (-4, -1), (-3, -1), (-1, -3),$ $(0, -3), (2, -1), (1, 0), (2, 1)$

PROBLEM-SOLVING AND REASONING	6–7(½), 9	6–7(½), 9, 10	7–8(½), 10, 11

6 One point in each set is not ‘in line’ with the other points. Name the point in each case.

a $A(1, 2), B(2, 4), C(3, 4), D(4, 5), E(5, 6)$

b $A(-5, 3), B(-4, 1), C(-3, 0), D(-2, -3), E(-1, -5)$

c $A(-4, -3), B(-2, -2), C(0, -1), D(2, 0), E(3, 1)$

d $A(6, -4), B(0, -1), C(4, -3), D(3, -2), E(-2, 0)$

7 Each set of points forms a basic shape. Describe the shape without drawing a graph, if you can.

a $A(-2, 4), B(-1, -1), C(3, 0)$

b $A(-3, 1), B(2, 1), C(2, -6), D(-3, -6)$

c $A(-4, 2), B(3, 2), C(4, 0), D(-3, 0)$

d $A(-1, 0), B(1, 3), C(3, 0), D(1, -9)$

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8 The midpoint of a line segment (or interval) is the point that cuts the segment in half. Find the midpoint of the line segment joining these pairs of points.

- **a** (1, 3) and (3, 5)
- **b** (−4, 1)and (−6, 3)
- **c** (−2, −3) and (0, −2)
- **d** (3, −5) and (6, −4)
- **e** (−2.5, 1) and (2.5, 1)
- **f** (−4, 1.5) and (0, 0.5)

9 List all the points, using only integer values of x and y that lie on the line segment joining these pairs of points.

- **a** (1, −3) and (1, 2)
- **b** (−2, 0) and (3, 0)
- **c** (−3, 4) and (2, −1)
- **d** (−3, −6) and (3, 12)

10 If (a, b) is a point on a number plane, name the quadrant or quadrants that matches the given description.

- **a** $a > 0$ and $b < 0$
- **b** $a < 0$ and $b > 0$
- **c** $a < 0$
- **d** $b < 0$

11 A set of points has coordinates $(0, y)$, where y is any number. What does this set of points represent?

ENRICHMENT	–	–	12, 13

Distances between points

12 Find the distance between these pairs of points.

- **a** (0, 0) and (0, 10)
- **b** (0, 0) and (−4, 0)
- **c** (−2, 0) and (5, 0)
- **d** (0, −4) and (0, 7)
- **e** (−1, 2) and (5, 2)
- **f** (4, −3) and (4, 1)

13 When two points are not aligned vertically or horizontally, Pythagoras' theorem can be used to find the distance between them. Find the distance between these pairs of points.

- **a** (0, 0) and (3, 4)
- **b** (0, 0) and (5, 12)
- **c** (−3, −4) and (4, 20)
- **d** (1, 1) and (4, −1)
- **e** (−1, −2) and (2, 7)
- **f** (−3, 4) and (3, −1)

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7B Using rules, tables and graphs to explore linear relationships

From our earlier study of formulas we know that two (or more) variables that have a relationship can be linked by a rule. A rule with two variables can be represented on a graph to illustrate this relationship. The rule can be used to generate a table that shows coordinate pairs (x, y). The coordinates can be plotted to form the graph. Rules that give straight-line graphs are described as being **linear**.

For example, the rule linking degrees Celsius (°C) with degrees Fahrenheit (°F) is given by $C = \frac{5}{9}(F - 32)$ and gives a straight-line graph.

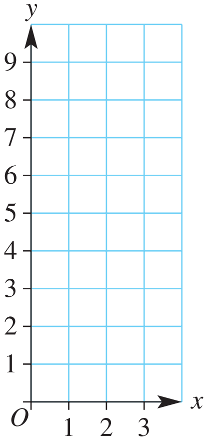
Let's start: They're not all straight

Not all rules give a straight-line graph. Here are three rules that can be graphed to give lines or curves.

1 $y = \frac{6}{x}$ **2** $y = x^2$ **3** $y = 2x + 1$

- In groups, discuss which rule(s) might give a straight-line graph and which might give curves.
- Use the rules to complete the given table of values.

x	1	2	3
$y = \frac{6}{x}$			
$y = x^2$			
$y = 2x + 1$			



- Discuss how the table of values can help you decide which rule(s) give a straight line.
- Plot the points to see if you are correct.

Key ideas

- A **rule** is an equation that describes the relationship between two or more variables.
- A **linear** relationship will result in a straight-line graph.
- For two variables, a linear rule is often written with y as the subject. For example: $y = 2x - 3$ or $y = -x + 7$.
- One way to graph a linear relationship using a formula or rule is to follow these steps.
  1. Construct a table of values finding a y-coordinate for each given x-coordinate by substituting each x-coordinate into the rule.
  2. Plot the points given in the table on a set of axes.
  3. Draw a line through the points to complete the graph.

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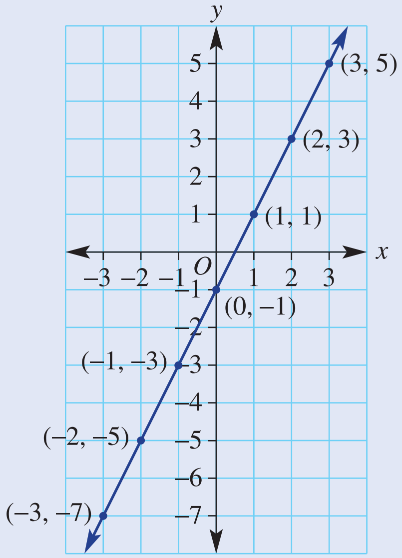
Example 2 Plotting a graph from a rule

For the rule $y = 2x - 1$ construct a table and draw a graph.

SOLUTION

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5



EXPLANATION

Substitute each x-coordinate in the table into the rule to find the y-coordinate.

Plot each point (−3, −7), (−2, −5) … and join them to form the straight-line graph.

Example 3 Checking if a point lies on a line

Decide if the points (1, 3) and (−2, −4) lie on the graph of $y = 3x$.

SOLUTION

Substitute (1, 3).

$y = 3x$

$\text{LHS} = y \qquad \text{RHS} = 3x$

$\quad = 3 \qquad\qquad = 3 \times 1$

$\qquad\qquad\qquad = 3$

LHS = RHS

So (1, 3) is on the line.

Substitute (−2, −4).

$y = 3x$

$\text{LHS} = y \qquad \text{RHS} = 3x$

$\quad = -4 \qquad\qquad = 3 \times -2$

$\qquad\qquad\qquad = -6$

$\text{LHS} \neq \text{RHS}$

So (−2, −4) is not on the line.

EXPLANATION

Substitute (1, 3) into the rule for the line.
The point satisfies the equation, so the point is on the line.

Substitute (−2, −4) into the rule for the line.
The point does not satisfy the equation, so the point is not on the line.

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Exercise 7B

UNDERSTANDING AND FLUENCY	1–5	3, 4–5(½)	4–5 (½)

1 For the rule $y = 2x + 3$ find the y-coordinate for these x-coordinates.

a 1 **b** 2 **c** 0 **d** -1
e -5 **f** -7 **g** 11 **h** -12

2 Write the missing number in these tables for the given rules.

a $y = 2x$

x	0	1	2	3
y	0		4	6

b $y = x - 3$

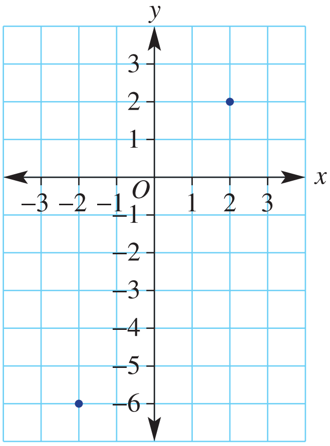
x	-1	0	1	2
y	-4	-3		-1

c $y = 5x + 2$

x	-3	-2	-1	0
y		-8	-3	2

3 Complete the graph to form a straight line from the given rule and table. Two points have been plotted for you.
$y = 2x - 2$

x	-2	-1	0	1	2
y	-6	-4	-2	0	2



Example 2

4 For each rule construct a table like the one shown here.

x	-3	-2	-1	0	1	2	3
y							

a $y = x + 1$ **b** $y = x - 2$ **c** $y = 2x - 3$ **d** $y = 2x + 1$
e $y = -2x + 3$ **f** $y = -3x - 1$ **g** $y = -x$ **h** $y = -x + 4$

Example 3

5 Decide if the given points lie on the graph with the given rule.

a Rule: $y = 2x$ Points: **i** (2, 4) and **ii** (3, 5)
b Rule: $y = 3x - 1$ Points: **i** (1, 1) and **ii** (2, 5)
c Rule: $y = 5x - 3$ Points: **i** $(-1, 0)$ and **ii** (2, 12)
d Rule: $y = -2x + 4$ Points: **i** (1, 2) and **ii** (2, 0)
e Rule: $y = 3 - x$ Points: **i** (1, 2) and **ii** (4, 0)
f Rule: $y = 10 - 2x$ Points: **i** (3, 4) and **ii** (0, 10)
g Rule: $y = -1 - 2x$ Points: **i** $(2, -3)$ and **ii** $(-1, 1)$

PROBLEM-SOLVING AND REASONING	6, 9	6, 7(½), 9, 10	7, 8, 10, 11

6 For x-coordinates from -3 to 3, construct a table and draw a graph for these rules. For parts **c** and **d** remember that subtracting a negative number is the same as adding its opposite. For example, $3-(-2)=3+2$.

a $y=\frac{1}{2}x+1$ **b** $y=-\frac{1}{2}x-2$ **c** $y=3-x$ **d** $y=1-3x$

7 For the graphs of these rules, state the coordinates of the two points at which the line cuts the x- and y-axes.

a $y=x+1$ **b** $y=2-x$ **c** $y=2x+4$

d $y=10-5x$ **e** $y=2x-3$ **f** $y=7-3x$

8 The rules for two lines are $y=x+2$ and $y=5-2x$. At what point do they intersect?

9 **a** What is the minimum number of points needed to draw a graph of a straight line?

b Draw the graph of these rules by plotting only two points. Use $x=0$ and $x=1$.

i $y=\frac{1}{2}x$ **ii** $y=2x-1$

10 **a** The graphs of $y=x$, $y=3x$ and $y=-2x$ all pass through the origin (0, 0). Explain why.

b The graphs of $y=x-1$, $y=3x-2$ and $y=5-2x$ do not pass through the origin (0, 0). Explain why.

11 The y-coordinates of points on the graphs of the rules in Question **4** parts **a** to **d** increase as the x-coordinates increase. Also the y-coordinates of points on the graphs of the rules in Question **4** parts **e** to **h** decrease as the x-coordinates increase.

a What do the rules in Question **4** parts **a** to **d** have in common?

b What do the rules in Question **4** parts **e** to **h** have in common?

c What feature of a rule tells you that a graph increases or decreases as the x-coordinate increases?

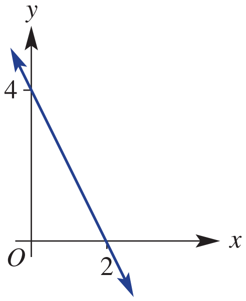
ENRICHMENT	–	–	12

Graphing lines using the x- and y-intercepts

12 A sketch of a straight-line graph can be achieved by finding only the x- and y-intercepts and labelling these on the graph. The y-intercept is the y value of the point where $x=0$. The x-intercept is the x value of the point where $y=0$.

For example: $y=4-2x$

y-intercept ($x=0$)	x-intercept ($y=0$)
$y=4-2\times 0$	$0=4-2x$
$y=4$	$-4=-2x$
	$2=x$



Sketch graphs of these rules, using the method outlined above.

a $y=x+4$ **b** $y=2x-4$ **c** $y=5-x$ **d** $y=-1+2x$

e $y=7x-14$ **f** $y=5-3x$ **g** $y=3-2x$ **h** $3y-2x=6$

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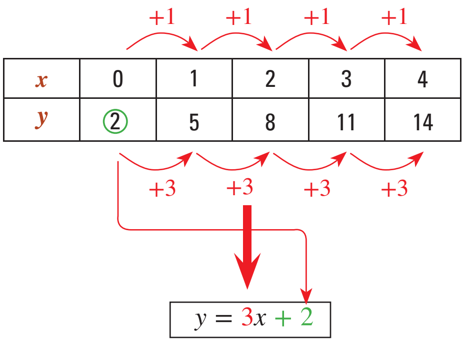
7C Finding the rule using a table of values

An equation is the best way to represent a linear relationship. In this section you will learn how to use the points in the table of values to write the rule in words and as an equation.

In the table below, note that:

- The numbers in the top row are increasing by 1.
- The numbers in the bottom row are increasing by 3.
- 3 divided by 1 gives 3.
- The number in the bottom row under the 0 is +2.



x	0	1	2	3	4
y	②	5	8	11	14

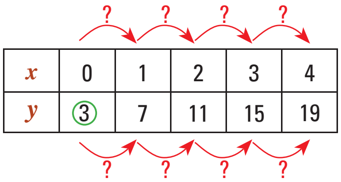
$y = 3x + 2$

From table of values to equation

These observations make it easy to 'find the rule'.
Look at the red and green numbers!

Let's start: What's my rule?

Consider the table below.



x	0	1	2	3	4
y	③	7	11	15	19

- Are the numbers in the top row increasing by 1? If yes, that is great!
- What is the pattern in the bottom row?
- What number is under the 0?
- The rule is one of the following equations. Which one is correct?

  A $y = 3x + 3$ **B** $y = 3x + 4$ **C** $y = 4x + 3$ **D** $y = 4x + 4$
- Copy and complete the following sentence.

  To find the value of y, choose a value for x and multiply by ________.

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Key ideas

- It is possible to use the table of values to express 'the rule' as an equation.
  - The equations usually look like these:

    $y = 3x + 2 \qquad y = 3x - 1 \qquad y = 3x \qquad y = x + 3$

- If the numbers in the top row of the table are in sequence, the following hints make it simple to find the equation.

+1 +1 +1 +1

x	0	1	2	3	4
y	−1	3	7	11	15

+4 +4 +4 +4

  - The numbers in the top row are increasing by 1.
  - The numbers in the bottom row are increasing by 4.
  - 4 divided by 1 gives 4.
  - The number in the bottom row under the 0 is −1.
  - The equation is $y = 4x - 1$.
  - In words, the rule is: To find the value of y, multiply x by 4 and then subtract 1.

- A rule must be true for every pair of numbers in the table of values. In the table above, if any number in the top row is multiplied by 4 and then reduced by 1, the result is the number below it, in the bottom row.
  - If the value of y when $x = 0$ is not given in the table, substitute another pair of coordinates to find the value of the constant.

x	2	3	4	5
y	5	7	9	11

+2 +2 +2

$y = 2x + \square$

$5 = 2 \times 2 + \square$ substituting (2, 5)

So $\square = 1$.

Alternatively, extend the table to the left so that 0 appears in the top row.

x	0	1	2	3	4	5
y	1	3	5	7	9	11

−2 −2

- The following names may be applied to the numbers in the equation.

$$y = \boxed{3}x \boxed{+ 2}$$

coefficient of x is 3

constant is 2

- In later sections, the coefficient of x is called the gradient.

Example 4 Finding rules from tables

Find the rule for these tables of values.

a

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

b

x	3	4	5	6	7
y	−5	−7	−9	−11	−13

SOLUTION

a Coefficient of x is 3.

Constant is −2.

$y = 3x - 2$

b Coefficient of x is −2.

$y = -2x + \square$

Substitute (3, −5).

$-5 = -2 \times 3 + \square$

$-5 = -6 + \square$

So $\square = 1$.

$y = -2x + 1$

EXPLANATION

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

+3 +3 +3 +3

$y = 3x - 2$

x	3	4	5	6	7
y	−5	−7	−9	−11	−13

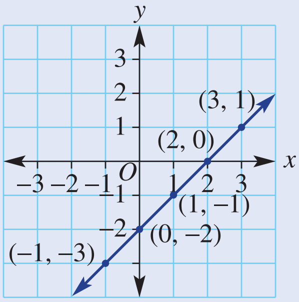
−2 −2 −2 −2

$y = -2x + \square$

To find the constant, substitute a point and choose the constant so that the equation is true. This can be done mentally.

Example 5 Finding a rule from a graph and write the rule in words

Find the rule for this graph by first constructing a table of (x, y) values. Write the rule in words.



SOLUTION

x	−1	0	1	2	3
y	−3	−2	−1	0	1

Coefficient of x is 1.

When $x = 0$, $y = -2$

$y = x - 2$

Rule in words: To find a y value, start with x and then subtract 2.

EXPLANATION

Construct a table using the points given on the graph.

Change in y is 1 for each increase by 1 in x.

x	−1	0	1	2	3
y	−3	−2	−1	0	1

+1 +1 +1 +1

$y = 1x - 2$ or $y = x - 2$

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Exercise 7C

UNDERSTANDING AND FLUENCY	1–6	3, 4–6(½)	5–6(½)

1 By how much does y increase for each increase by 1 in x? If y is decreasing, give a negative answer.

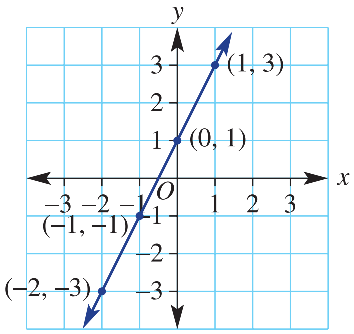
a

x	−2	−1	0	1	2
y	−1	1	3	5	7

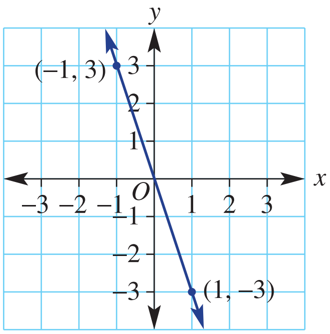
b

x	−3	−2	−1	0	1
y	4	3	2	1	0

c



d



2 For each of the tables and graphs in Question **1**, state the value of y when $x = 0$.

3 Find the missing number in these equations.

a $4 = 2 + \square$

b $3 = 5 + \square$

c $-1 = 2 + \square$

d $5 = 3 \times 2 + \square$

e $-2 = 2 \times 4 + \square$

f $-10 = 2 \times (-4) + \square$

g $-8 = -4 \times 2 + \square$

h $-12 = -5 \times 3 + \square$

i $16 = -4 \times (-3) + \square$

Example 4a

4 Find the rule for these tables of values.

a

x	−2	−1	0	1	2
y	0	2	4	6	8

b

x	−2	−1	0	1	2
y	−7	−4	−1	2	5

c

x	−3	−2	−1	0	1
y	4	3	2	1	0

d

x	−1	0	1	2	3
y	8	6	4	2	0

Example 4b

5 Find the rule for these tables of values.

a

x	1	2	3	4	5
y	5	9	13	17	21

b

x	−5	−4	−3	−2	−9
y	−13	−11	−9	−7	−5

c

x	5	6	7	8	9
y	−12	−14	−16	−18	−20

d

x	−6	−5	−4	−3	−2
y	10	9	8	7	6

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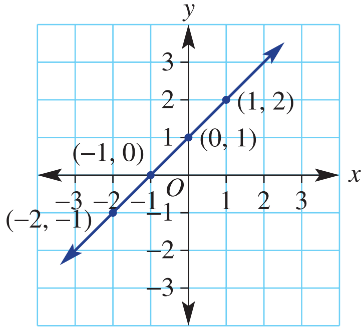
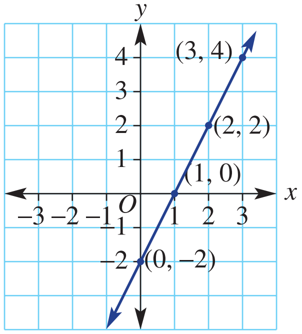
Example 5

6 Find the rule for these graphs by first constructing a table of (x, y) values. Write the rule in words.

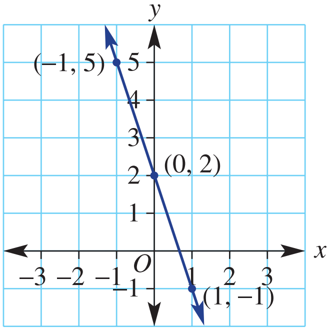
a



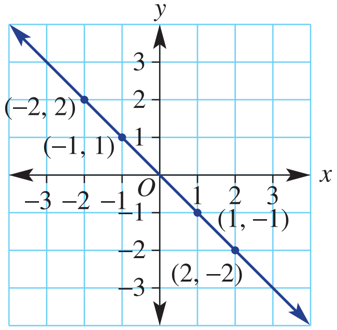
b



c



d



PROBLEM-SOLVING AND REASONING	7, 8, 11	7–9, 11, 12	8–10, 12–14

7 Find the rule for these sets of points. Try to do it without drawing a graph or table.

a (1, 3), (2, 4), (3, 5), (4, 6)

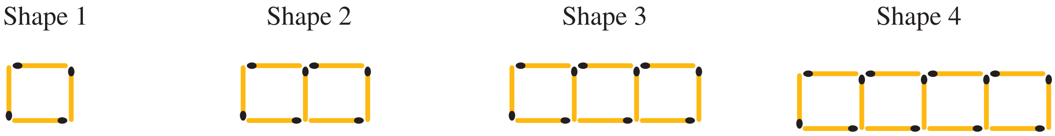
b (−3, −7), (−2, −6), (−1, −5), (0, −4)

c (−1, −3), (0, −1), (1, 1), (2, 3)

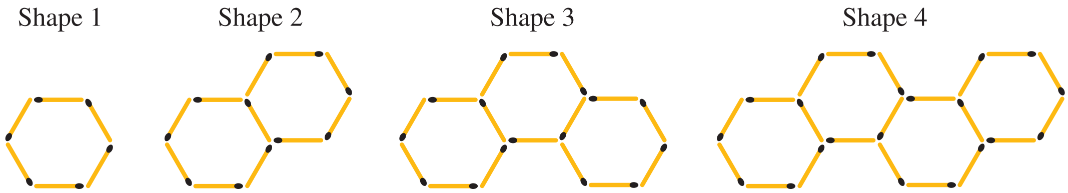
d (−2, 3), (−1, 2), (0, 1), (1, 0)

8 Write a rule for these matchstick patterns. Write the rule in words.

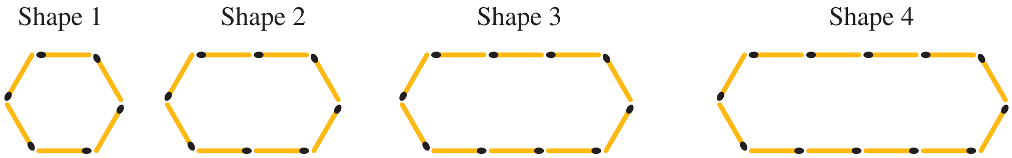
a x = number of squares y = number of matchsticks



b x = number of hexagons y = number of matchsticks



c x = number of matchsticks on top row y = number of matchsticks



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9 A graph passes through the two points $(0, -2)$ and $(1, 6)$. What is the rule of the graph?

10 A graph passes through the two points $(-2, 3)$ and $(4, -3)$. What is the rule of the graph?

11 The rule $y = -2x + 3$ can be written as $y = 3 - 2x$. Write these rules in a similar form.

a $y = -2x + 5$ **b** $y = -3x + 7$ **c** $y = -x + 4$ **d** $y = -4x + 10$

12 A straight line has two points: $(0, 2)$ and $(1, b)$.

a Write an expression for the coefficient of x in the rule linking y and x.

b Write the rule for the graph in terms of b.

13 A straight line has two points: $(0, a)$ and $(1, b)$.

a Write an expression for the coefficient of x in the rule linking y and x.

b Write the rule for the graph in terms of a and b.

14 In Question **8a** you can observe that three extra matchsticks are needed for each new shape and one matchstick is needed to complete the first square (so the rule is $y = 3x + 1$). In a similar way, describe how many matchsticks are needed for the shapes in:

a Question **8b** **b** Question **8c**

ENRICHMENT	–	–	15

Skipping x values

15 Consider this table of values.

x	−2	0	2	4
y	−4	−2	0	2

a The increase in y for each unit increase in x is not 2. Explain why.

b If the pattern is linear, state the increase in y for each increase by 1 in x.

c Write the rule for the relationship.

d Find each rule for these tables.

i

x	−4	−2	0	2	4
y	−5	−1	3	7	11

ii

x	−3	−1	1	3	5
y	−10	−4	2	8	14

iii

x	−6	−3	0	3	6
y	15	9	3	−3	−9

iv

x	−10	−8	−6	−4	−2
y	20	12	4	−4	−12

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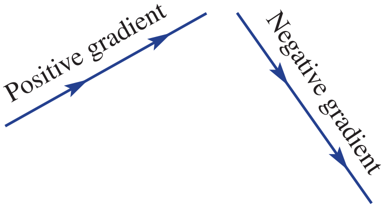
7D Gradient EXTENSION

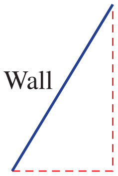
The gradient of a line is a measure of how steep the line is. The steepness or slope of a line depends on how far it rises or falls over a given horizontal distance. This is why the gradient is calculated by dividing the vertical rise by the horizontal run between two points. Lines that rise (from left to right) have a positive gradient and lines that fall (from left to right) have a negative gradient.



Let's start: Which is the steepest?

At a children's indoor climbing centre there are three types of sloping walls to climb. The blue wall rises 2 metres for each metre across. The red wall rises 3 metres for every 2 metres across, and the yellow wall rises 7 metres for every 3 metres across.

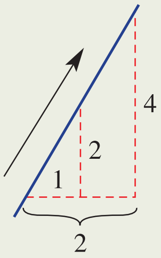
- Draw a diagram showing the slope of each wall.
- Label your diagrams with the information given above.
- Discuss which wall might be the steepest, giving reasons.
- Discuss how it might be possible to accurately compare the slope of each wall.



Key ideas

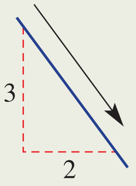
- The **gradient** is a measure of **slope**.
  - It is the increase in y as x increases by 1.
  - It is the ratio of the change in y over the change in x.
  - **Gradient** $= \dfrac{\textbf{rise}}{\textbf{run}}$
  - Rise = change in y
  - Run = change in x
  - The run is always considered to be positive when moving from left to right on the Cartesian plane.
  - The two diagrams show a positive rise and a negative rise.

Positive gradient



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = \frac{2}{1} = 2$$

Negative gradient



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2} = -\frac{3}{2}$$

- A gradient is negative if y decreases as x increases. The rise is considered to be negative.
- The gradient of a horizontal line is zero.
- The gradient of a vertical line is undefined, which means there is no numerical value assigned to this gradient.

$$\text{Gradient} = \frac{0}{2} = 0$$

Gradient $= \frac{2}{0}$, which is undefined.

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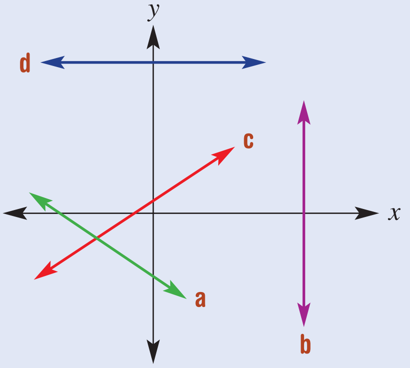
Example 6 Defining a type of gradient

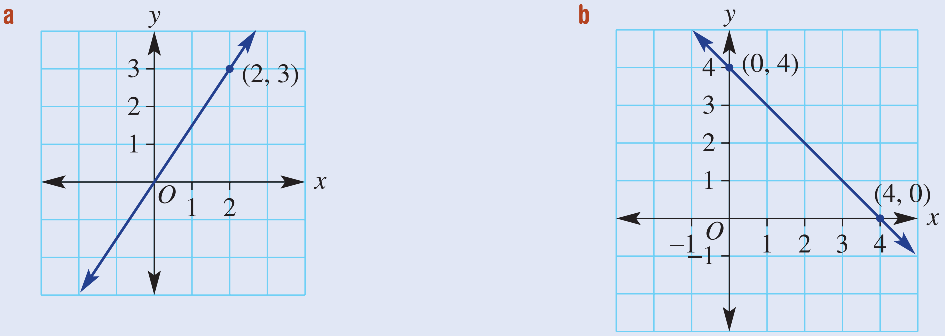
Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



SOLUTION		**EXPLANATION**
a	negative gradient	As x increases, y decreases.
b	undefined gradient	The line is vertical.
c	positive gradient	y increases as x increases.
d	zero gradient	There is no increase or decrease in y as x increases.

Example 7 Finding the gradient from a graph

Find the gradient of these lines.

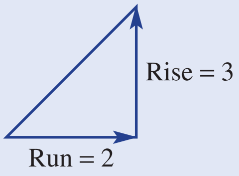


SOLUTION

EXPLANATION

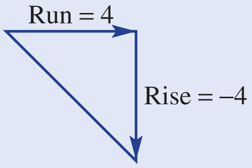
a $$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{3}{2} \text{ or } 1.5$$

The rise is 3 for every 2 across to the right.



b $$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-4}{4}$$
$$= -1$$

The y value falls 4 units while the x value increases by 4.



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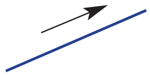
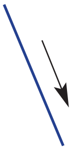
Exercise 7D **EXTENSION**

UNDERSTANDING AND FLUENCY	1, 2, 3–4(½), 5–6(½)	3–6(½)	5–6(½)

1 Decide if the gradients of these lines are positive or negative.

a


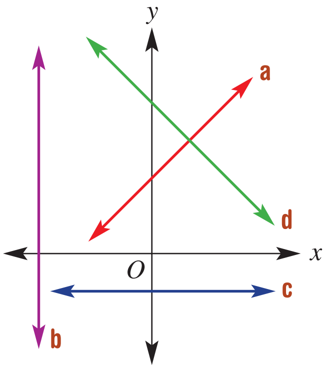
b


c


d


Example 6

2 Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



3 Simplify these fractions.

a $\frac{4}{2}$

b $\frac{12}{4}$

c $\frac{6}{4}$

d $\frac{14}{7}$

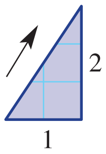
e $-\frac{6}{3}$

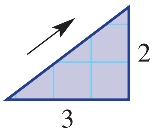
f $-\frac{20}{4}$

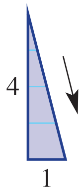
g $-\frac{16}{6}$

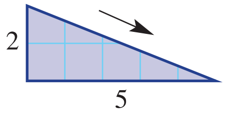
h $-\frac{15}{9}$

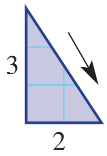
4 Write down the gradient $\frac{\text{rise}}{\text{run}}$ for each of these slopes.

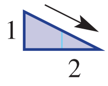
a


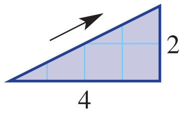
b


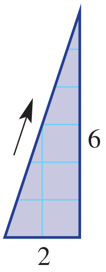
c


d


e


f


g


h


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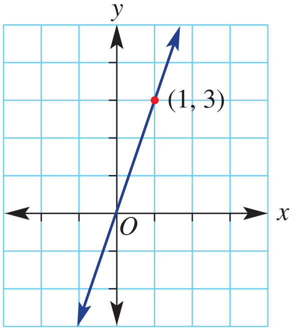
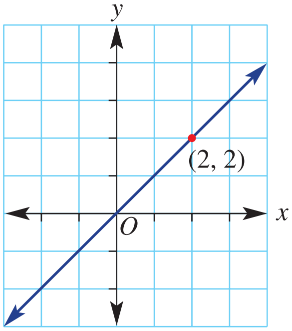
Example 7a **5** Find the gradient of these lines. Use Gradient $= \frac{\text{rise}}{\text{run}}$.

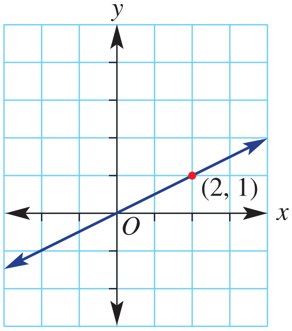
a



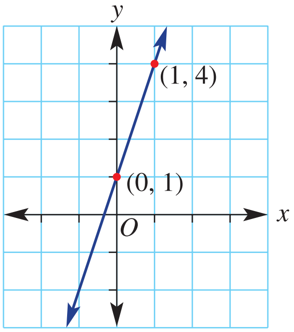
b



c



d



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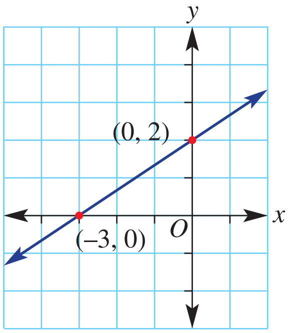

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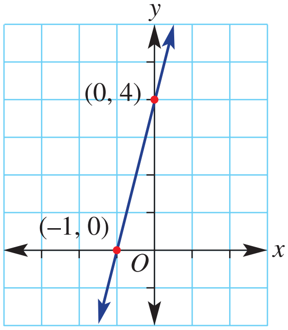
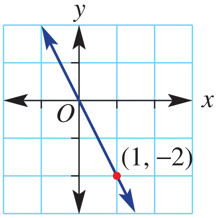


Example 7b **6** Find the gradient of these lines.

a



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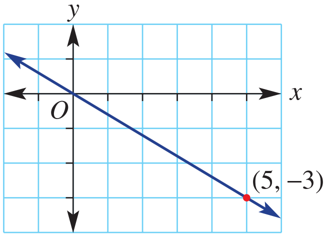

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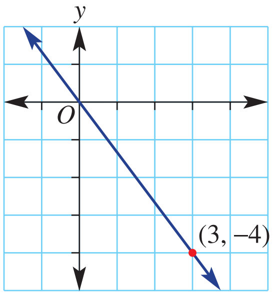

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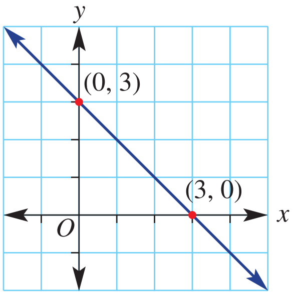

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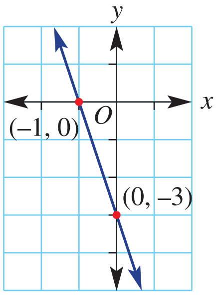

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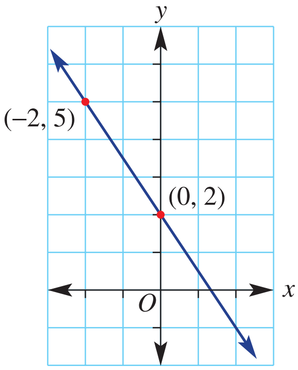

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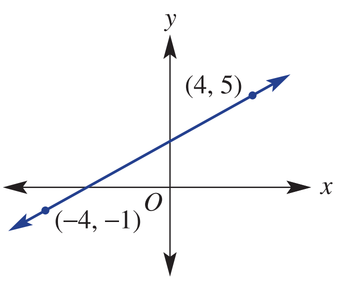
PROBLEM-SOLVING AND REASONING	7, 8, 11	8, 9, 11, 12	8–10, 12, 13

7 Abdullah climbs a rocky slope that rises 12 m for each 6 metres across. His friend Jonathan climbs a nearby grassy slope that rises 25 m for each 12 m across. Which slope is steeper?

8 A submarine falls 200 m for each 40 m across and a torpedo falls 420 m for each 80 m across in pursuit of the submarine. Which has the steeper gradient, the submarine or torpedo?

9 Find the gradient of these lines. You will need to first calculate the rise and run.

a



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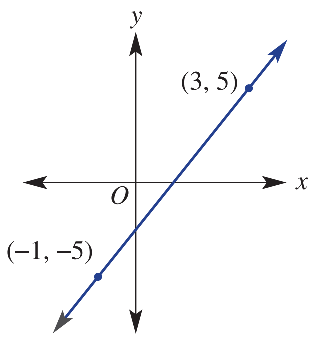

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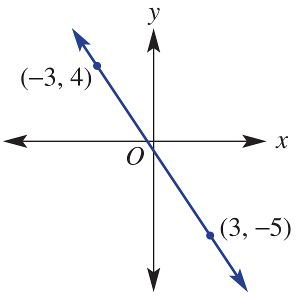

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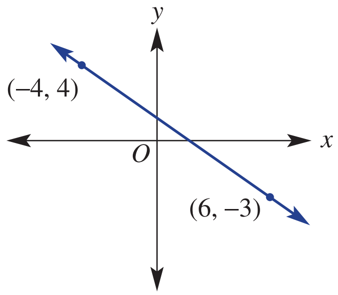

10 Find the gradient of the line joining these pairs of points.

a (0, 2) and (2, 7)

b (0, −1) and (3, 4)

c (−3, 7) and (0, −1)

d (−5, 6) and (1, 2)

e (−2, −5) and (1, 3)

f (−5, 2) and (5, −1)

11 A line with gradient 3 joins (0, 0) with another point, A.

a Write the coordinates of three different positions for A, using positive integers for both the x- and y-coordinates.

b Write the coordinates of three different positions for A using negative integers for both the x- and y-coordinates.

12 A line joins the point (0, 0) with the point (a, b) with a gradient of 2.

a If $a = 1$, find b.

b If $a = 5$, find b.

c Write an expression for b in terms of a.

d Write an expression for a in terms of b.

13 A line joins the point (0, 0) with the point (a, b) with a gradient of $-\frac{1}{2}$.

a If $a = 1$, find b.

b If $a = 3$, find b.

c Write an expression for b in terms of a.

d Write an expression for a in terms of b.

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ENRICHMENT – – 14

Rise and run as differences

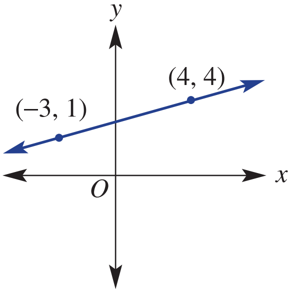
14 The run and rise between two points can be calculated by finding the difference between the pairs of x- and pairs of y-coordinates.

For example:

Rise $= 4 - 1 = 3$

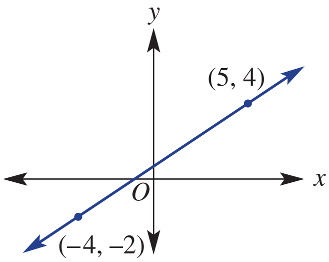
Run $= 4 - (-3) = 7$

Gradient $= \frac{3}{7}$



Use this method to find the gradient of the line joining the given points.

a



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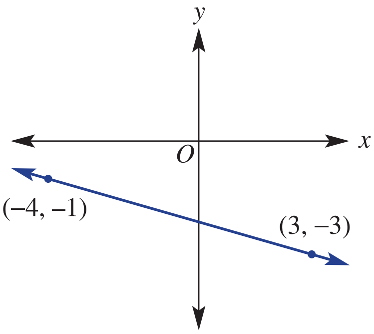

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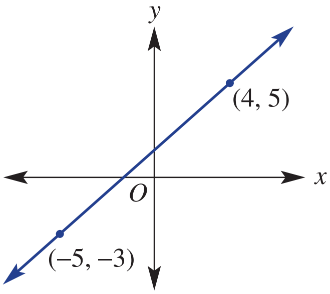

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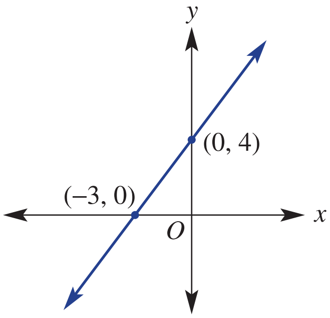

e $(-3, 1)$ and $(4, -2)$

f $(-2, -5)$ and $(1, 7)$

g $(5, -2)$ and $(-4, 6)$

h $\left(-\frac{1}{2}, 2\right)$ and $\left(\frac{9}{2}, -3\right)$

i $\left(-4, \frac{2}{3}\right)$ and $\left(3, -\frac{4}{3}\right)$

j $\left(-\frac{3}{2}, \frac{2}{3}\right)$ and $\left(4, -\frac{2}{3}\right)$

k $\left(-\frac{7}{4}, 2\right)$ and $\left(\frac{1}{3}, \frac{1}{2}\right)$

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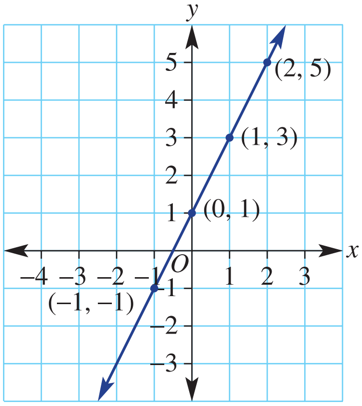
7E Gradient–intercept form EXTENSION

From previous sections in this chapter you may have noticed some connections between the numbers that make up a rule for a linear relationship and the numbers that are the gradient and y-coordinate with $x = 0$ (the y-intercept). This is no coincidence. Once the gradient and y-intercept of a graph are known, the rule can be written down without further analysis.

Let's start: What's the connection?

To explore the connection between the rule for a linear relationship and the numbers that are the gradient and the y-intercept, complete the missing details for the graph and table below.



x	−1	0	1	2
y	−1	1		

+2

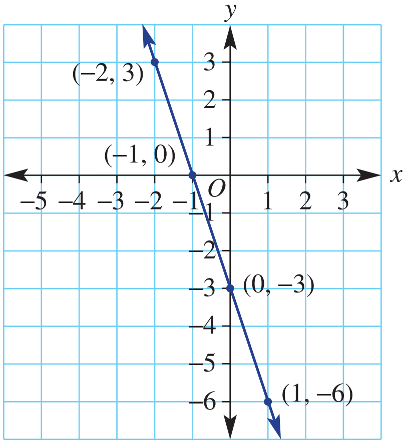
$$y = \square \times x + \square$$

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \_\_\_$$

y-intercept = ___

- What do you notice about the numbers in the rule, including the coefficient of x and the constant (below the table), and the numbers for the gradient and y-intercept?
- Complete the details for this new example below to see if your observations are the same.



x	−2	−1	0	1
y				

$$y = \square \times x + \square$$

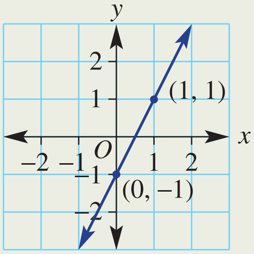
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \_\_\_$$

y-intercept = ___

Key ideas

- The rule for a straight-line graph is given by $y = mx + b$ or $y = mx + c$, where:
  - m is the gradient
  - b or c is the **y-intercept**



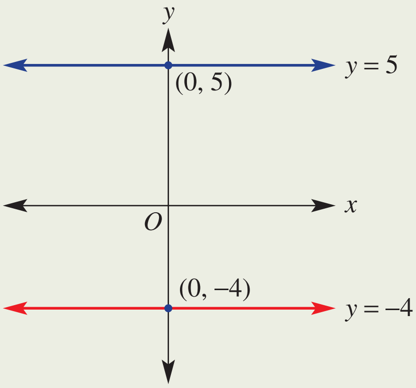
$m = \frac{2}{1} = 2$

$b = -1$

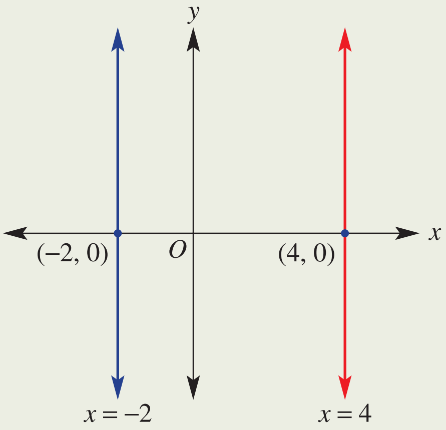
So $y = mx + b$ becomes:

$y = 2x - 1$

- A horizontal line (parallel to the x-axis) has the rule $y = b$ or $y = c$.
  Examples of horizontal lines include $y = 5$, $y = -4$ and $y = 0$ (which is the x-axis). They all have zero gradient.



- A vertical line (parallel to the y-axis) has the rule $x = k$.
  Examples of vertical lines include $x = 4$, $x = -2$ and $x = 0$ (which is the y-axis). Their gradient is undefined.



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Example 8 Stating the gradient and y-intercept from the rule

Write down the gradient and y-intercept for the graphs of these rules.

a $y = 2x + 3$

b $y = \frac{1}{3}x - 4$

SOLUTION

a $y = 2x + 3$

gradient: $m = 2$

y-intercept: $b = 3$

b $y = \frac{1}{3}x - 4$

gradient: $m = \frac{1}{3}$

y-intercept: $b = -4$

EXPLANATION

The coefficient of x is 2 and this number is the gradient.

The y-intercept is the constant.

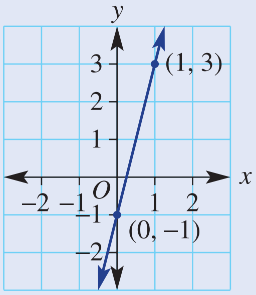
The gradient (m) is the coefficient of x.

Remember that $y = \frac{1}{3}x - 4$ is the same as $y = \frac{1}{3}x + (-4)$ so the constant is -4.

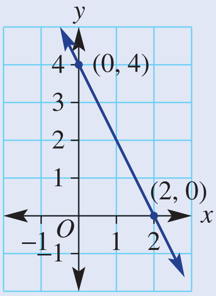
Example 9 Finding a rule from a graph

Find the rule for these graphs by first finding the values of m and the y-intercept.

a



b



SOLUTION

a $m = \frac{\text{rise}}{\text{run}}$

$= \frac{4}{1} = 4$

$b = -1$

$y = 4x - 1$

b $m = \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{2}$

$= -2$

$b = 4$

$y = -2x + 4$

EXPLANATION

Between $(0, -1)$ and $(1, 3)$ the rise is 4 and the run is 1.

The line cuts the y-axis at -1.

Substitute the value of m and b into $y = mx + b$.

Between $(0, 4)$ and $(2, 0)$, y falls by 4 units as x increases by 2.

The line cuts the y-axis at 4.

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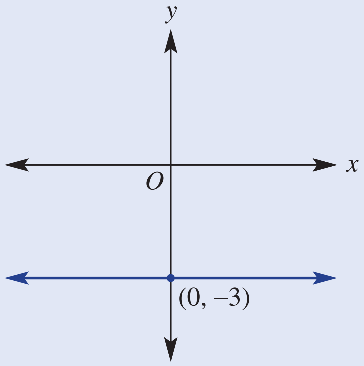
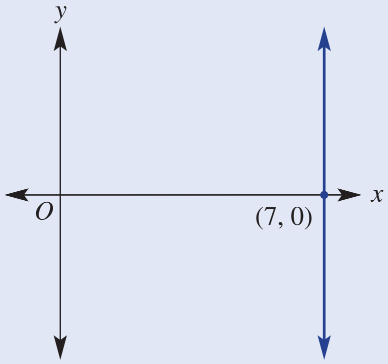
Example 10 Finding rules for horizontal and vertical lines

Write the rule for these horizontal and vertical lines.

a



b



SOLUTION	EXPLANATION
a $y = -3$	All points on the line have a y value of -3 and the gradient is 0.
b $x = 7$	Vertical lines take the form $x = k$. Every point on the line has an x value of 7.

Exercise 7E EXTENSION

UNDERSTANDING AND FLUENCY	1–6(½)	2–7(½)	3–7(½)

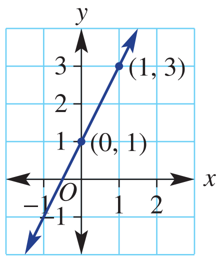
1 Substitute the given value of m and b into $y = mx + b$ to write a rule.

a $m = 2$ and $b = 3$ **b** $m = 4$ and $b = 2$ **c** $m = -3$ and $b = 1$

d $m = -5$ and $b = -3$ **e** $m = 2$ and $b = -5$ **f** $m = -10$ and $b = -11$

2 For these graphs write down the y-intercept and find the gradient using $\frac{\text{rise}}{\text{run}}$.

a



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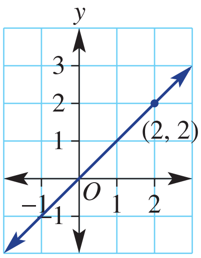
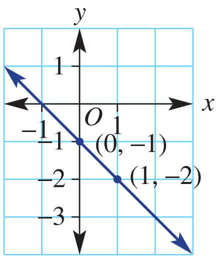
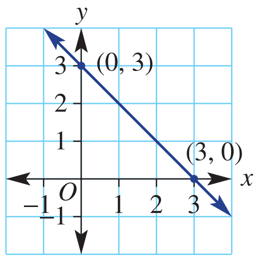


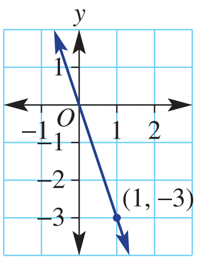
c



d



e



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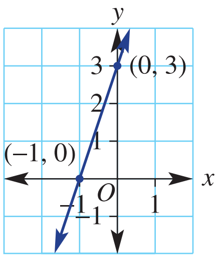

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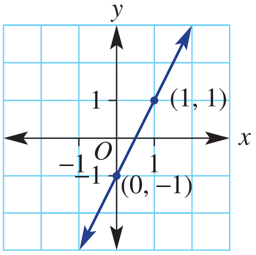
Example 8 **3** Write down the gradient and y-intercept for the graphs of these rules.

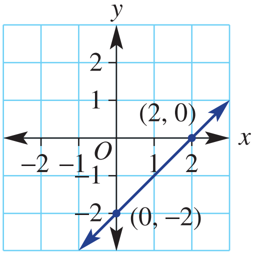
a $y = 4x + 2$
b $y = 3x + 7$
c $y = \frac{1}{2}x + 1$
d $y = \frac{2}{3}x + \frac{1}{2}$
e $y = -2x + 3$
f $y = -4x + 4$
g $y = -7x - 1$
h $y = -3x - 7$
i $y = -x - 6$
j $y = \frac{-1}{2}x + 5$
k $y = \frac{-2}{3}x - \frac{1}{2}$
l $y = \frac{3}{7}x - \frac{4}{5}$

Example 9a **4** Find the rule for these graphs by first finding the values of m and b.

a



b



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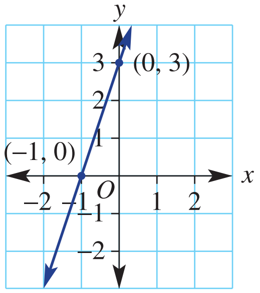

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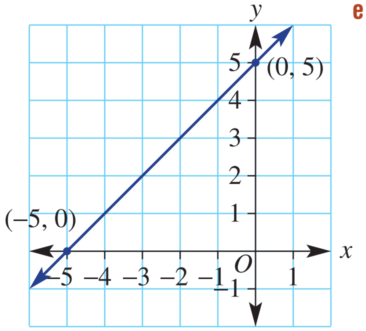

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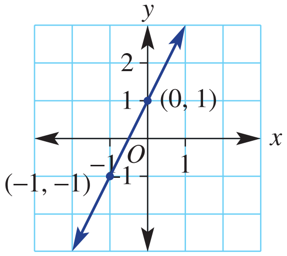

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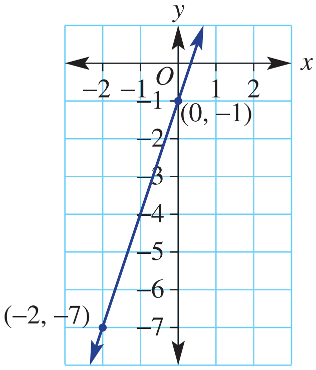
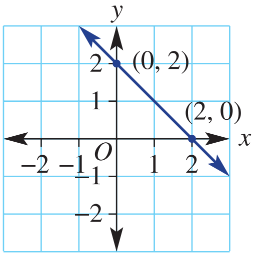


Example 9b **5** Find the rule for these graphs by first finding the values of m and b.

a



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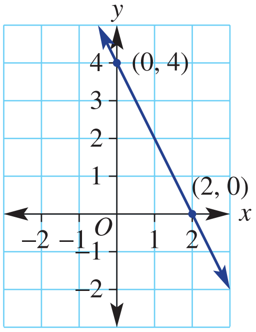

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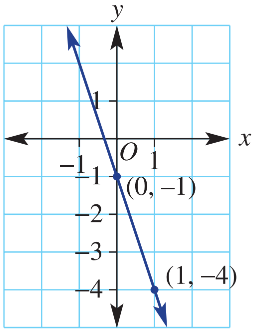

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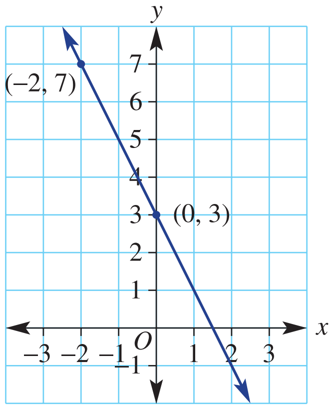

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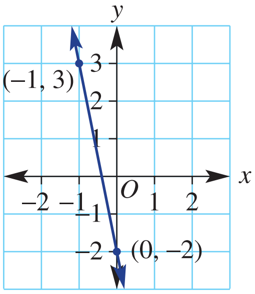

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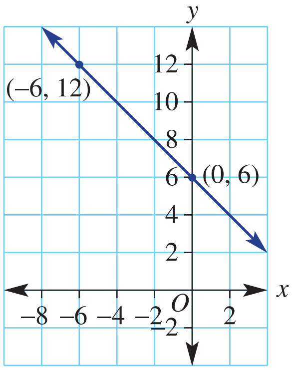

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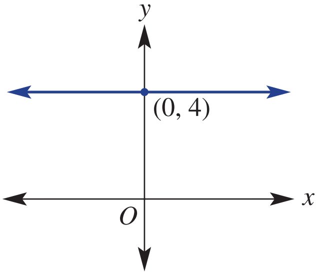
Example 10

6 Write the rule for these horizontal and vertical lines.

a



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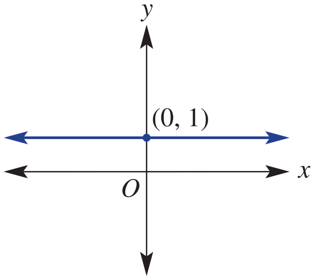

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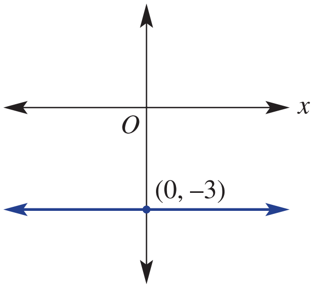

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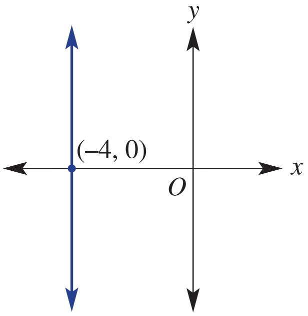

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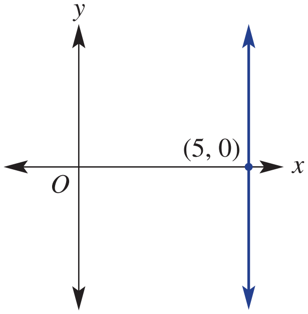

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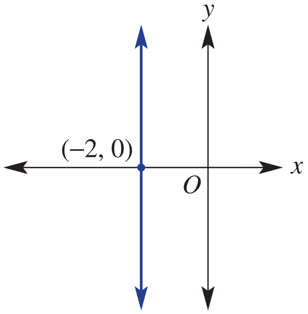


7 Sketch horizontal or vertical graphs for these rules.

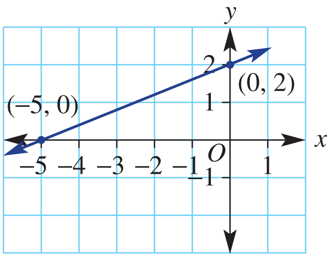
a $y = 2$ **b** $y = -1$ **c** $y = -4$ **d** $y = 5$

e $x = -3$ **f** $x = 4$ **g** $x = 1$ **h** $x = -1$

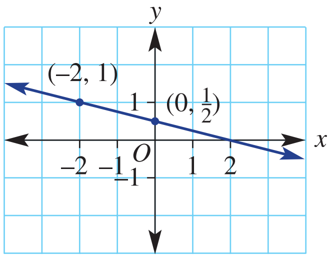
PROBLEM-SOLVING AND REASONING 8, 9, 12 | 8–10, 12, 13 | 9–11, 13, 14

8 These graphs have rules that involve fractions. Find m and b and write the rule.

a



b



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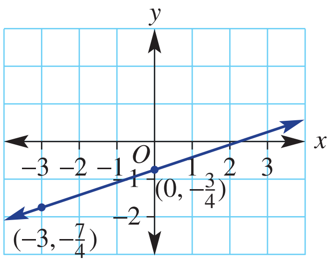

9 Find the rule for the graph of the lines connecting these pairs of points.

a (0, 0) and (2, 6) **b** (−1, 5) and (0, 0)

c (−2, 5) and (0, 3) **d** (0, −4) and (3, 1)

10 A line passes through the given points. Note that the y-intercept is not given. Find m and b and write the linear rule. A graph may be helpful.

a (−1, 1) and (1, 5) **b** (−2, 6) and (2, 4)

c (−2, 4) and (3, −1) **d** (−5, 0) and (2, 14)

11 Find the rectangular area enclosed by these sets of lines.

a $x = 4, x = 1, y = 2, y = 7$ **b** $x = 5, x = -3, y = 0, y = 5$

12 **a** Explain why the rule for the x-axis is given by $y = 0$.

b Explain why the rule for the y-axis is given by $x = 0$.

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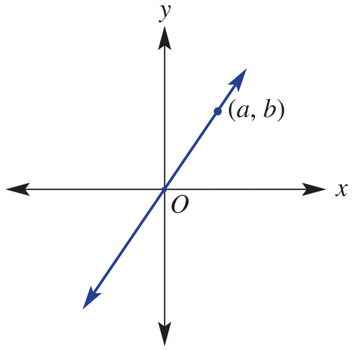
13 Write the rule for these graphs. Your rule could include the pronumerals *a* and/or *b*.

a



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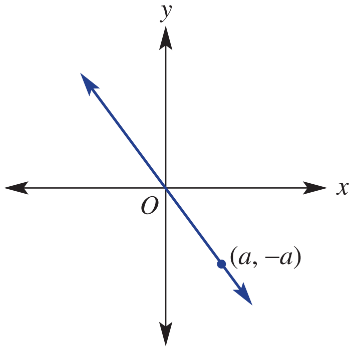
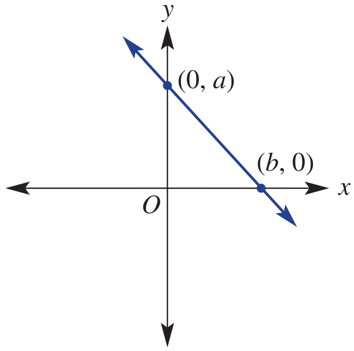


c



14 Some rules for straight lines may not be written in the form $y = mx + b$. The rule $2y + 4x = 6$, for example, can be rearranged to $2y = -4x + 6$ and then to $y = -2x + 3$. So $m = -2$ and $b = 3$. Use this idea to find m and b for these rules.

a $2y + 6x = 10$ **b** $3y - 6x = 9$ **c** $2y - 3x = 8$ **d** $x - 2y = -6$

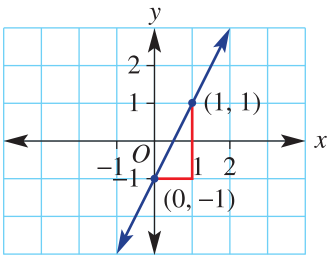
ENRICHMENT – – 15

Sketching with m and b

15 The gradient and y-intercept can be used to sketch a graph without the need to plot more than two points.

For example, the graph of the rule $y = 2x - 1$ has $m = 2\left(= \frac{2}{1}\right)$ and $b = -1$. By plotting the point

$(0, -1)$ for the y-intercept and moving 1 to the right and 2 up for the gradient, a second point $(1, 1)$ can be found.



Use this idea to sketch the graphs of these rules.

a $y = 3x - 1$ **b** $y = 2x - 3$ **c** $y = -x + 2$ **d** $y = -3x - 1$

e $y = 4x$ **f** $y = -5x$ **g** $y = \frac{1}{2}x - 2$ **h** $y = -\frac{3}{2}x + 1$

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7F The *x*-intercept EXTENSION

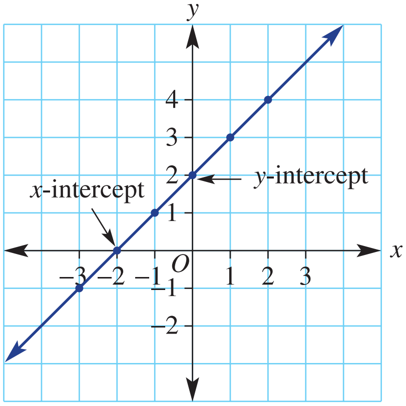
Interactive
Widgets
HOTsheets
Walkthrough

We know that the *y*-intercept is determined by the point where a line cuts the *y*-axis. This is also the value of *y* for the rule where $x = 0$. Similarly, the *x*-intercept can be found by letting $y = 0$. This can be viewed in a table of values or found using the algebraic method.

x-intercept where $y = 0$

x	−3	−2	−1	0	1	2
y	−1	0	1	2	3	4

y-intercept where $x = 0$



Let's start: Discover the method

These rules all give graphs that have *x*-intercepts at which $y = 0$.

A $y = 2x - 2$ **B** $y = x - 5$ **C** $y = 3x - 9$ **D** $y = 4x + 3$

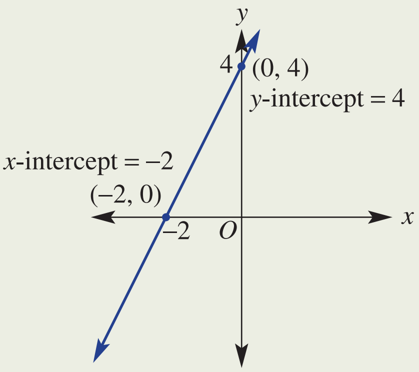
- First try to guess the *x*-intercept using a trial and error (guess and check) method. Start by asking what value of *x* makes $y = 0$.
- Discuss why the rule for **D** is more difficult to work with than the others.
- Can you describe an algebraic method that will give the *x*-intercept for any rule? How would you show your working for such a method?

Key ideas

- The ***x*-intercept** is the *x* value of the point on a graph where $y = 0$.
- Find the *x*-intercept by substituting $y = 0$ into the rule. Solve the equation by inspection or systematically. For example:

$$\begin{array}{rrcll} & y &=& 2x + 4 & \\ -4 & 0 &=& 2x + 4 & -4 \\ \div 2 & -4 &=& 2x & \div 2 \\ & -2 &=& x & \end{array}$$

$\therefore$ *x*-intercept is −2.



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Example 11 Finding the x-intercept

For the graphs of these rules, find the x-intercept.

a $y = 3x - 6$

b $y = -2x + 1$

SOLUTION

a

$$\begin{array}{rrcll} & y &=& 3x - 6 & \\ & 0 &=& 3x - 6 & \\ +6 & 6 &=& 3x & +6 \\ \div 3 & 2 &=& x & \div 3 \end{array}$$

$\therefore$ x-intercept is 2.

b

$$\begin{array}{rrcll} & y &=& -2x + 1 & \\ & 0 &=& -2x + 1 & \\ -1 & -1 &=& -2x & -1 \\ \div 2 & \frac{1}{2} &=& x & \div 2 \end{array}$$

$\therefore$ x-intercept is $\frac{1}{2}$.

EXPLANATION

a

Substitute $y = 0$ into the rule.

Add 6 to both sides.

Divide both sides by 3.

b

Substitute $y = 0$ into the rule.

Subtract 1 from both sides.

Divide both sides by -2.

Example 12 Sketching with intercepts

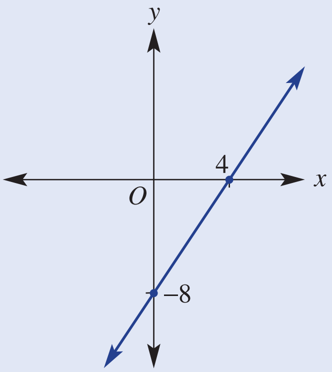
Find the x- and y-intercepts and then sketch the graph of the rule $y = 2x - 8$.

SOLUTION

$$\begin{array}{rrcll} & y &=& 2x - 8 & \\ & 0 &=& 2x - 8 & \\ +8 & 8 &=& 2x & +8 \\ \div 2 & 4 &=& x & \div 2 \end{array}$$

x-intercept is 4.

y-intercept is –8.



EXPLANATION

Substitute $y = 0$ into the rule.

Add 8 to both sides.

Divide both sides by 2.

The y-intercept is the value of b in $y = mx + b$.

Alternatively, substitute $x = 0$ to get $y = -8$.

Sketch by showing the x- and y-intercepts.

There is no need to show a grid.

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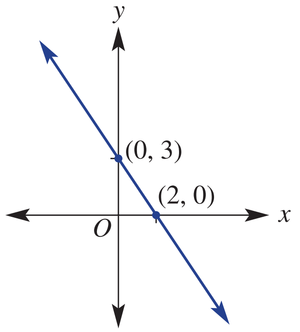
Exercise 7F EXTENSION

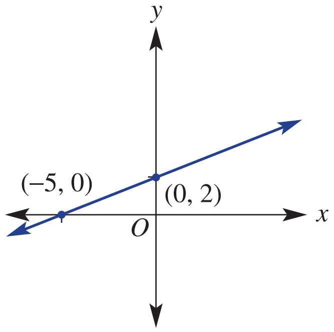
UNDERSTANDING AND FLUENCY 1, 2, 3–5(½) 3–5(½) 4–5(½)

1 Look at these graphs and write down the x-intercept (i.e. the value of x where $y = 0$).

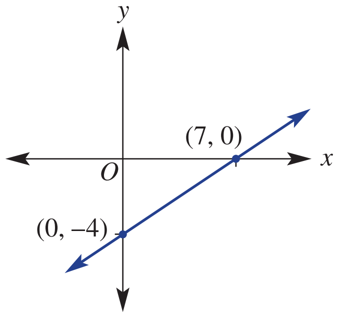
a



b



c



2 For each of these tables state the value of x that gives a y value of zero.

a

x	−2	−1	0	1	2
y	2	0	−2	−4	−6

b

x	1	2	3	4	5
y	3	2	1	0	−1

c

x	−4	−3	−2	−1	0
y	8	4	0	−4	−8

d

x	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
y	−2	0	2	4	6

3 Solve each of these equations for x.

a $0 = x - 4$ **b** $0 = x + 2$ **c** $0 = x + 5$

d $0 = 2x + 10$ **e** $0 = 3x - 12$ **f** $0 = 4x + 28$

g $0 = 3x + 1$ **h** $0 = 5x + 2$ **i** $0 = -2x - 4$

j $0 = -5x - 20$ **k** $0 = -x + 2$ **l** $0 = -x + 45$

Example 11

4 For the graphs of these rules, find the x-intercept.

a $y = x - 1$ **b** $y = x - 6$ **c** $y = x + 2$

d $y = 2x - 8$ **e** $y = 4x - 12$ **f** $y = 3x + 6$

g $y = 2x + 20$ **h** $y = -2x + 4$ **i** $y = -4x + 8$

j $y = -2x - 4$ **k** $y = -x - 7$ **l** $y = -x + 11$

Example 12

5 Find the x- and y-intercepts and then sketch the graphs of these rules.

a $y = x + 1$ **b** $y = x - 4$ **c** $y = 2x - 10$

d $y = 3x + 9$ **e** $y = -2x - 4$ **f** $y = -4x + 8$

g $y = -x + 3$ **h** $y = -x - 5$ **i** $y = -3x - 15$

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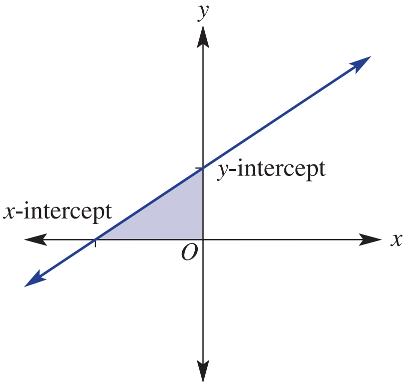
PROBLEM-SOLVING AND REASONING	6, 7, 10	6–8, 10, 11	7–9, 11, 12

6 Find the area of the triangle enclosed by the x-axis, the y-axis and the given line. You will first need to find the x- and y-intercepts.



a $y = 2x + 4$ **b** $y = 3x - 3$ **c** $y = -x + 5$ **d** $y = -4x - 8$

7 Write the rule for a graph that matches the given information.

a y-intercept 4, x-intercept -4

b y-intercept -1, x-intercept 2

c x-intercept -2, gradient 3

d x-intercept 5, gradient -5

8 The height of water (H cm) in a tub is given by $H = -2t + 20$, where t is the time in seconds.

a Find the height of water initially (i.e. at $t = 0$).

b How long will it take for the tub to empty?

9 The amount of credit (C cents) on a phone card is given by the rule $C = -t + 200$, where t is time in seconds.

a How much credit is on the card initially (i.e. at $t = 0$)?

b For how long can you use the phone card before the money runs out?

10 Some lines have no x-intercept. What type of lines are they? Give two examples.

11 Decide if the x-intercept will be positive or negative for $y = mx + b$ under these conditions.

a m is positive and b is positive (e.g. $y = 2x + 4$)

b m is positive and b is negative (e.g. $y = 2x - 4$)

c m is negative and b is negative (e.g. $y = -2x - 4$)

d m is negative and b is positive (e.g. $y = -2x + 4$)

12 Write the rule for the x-intercept if $y = mx + b$. Your answer will include the pronumerals m and b.

ENRICHMENT	–	–	13

Using $ax + by = d$

13 The x-intercept can be found even when the rule for the graph is given in any form. Substituting $y = 0$ starts the process, whatever the form of the rule. Find the x-intercept for the graphs of these rules.

a $x + y = 6$ **b** $3x - 2y = 12$ **c** $y - 2x = 4$

d $2y - 3x = -9$ **e** $y - 3x = 2$ **f** $3y + 4x = 6$

g $5x - 4y = -10$ **h** $2x + 3y = 3$ **i** $y - 3x = -1$

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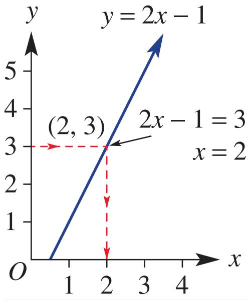
7G Solving linear equations using graphical techniques

The rule for a straight line shows the connection between the *x*- and *y*-coordinate of each point on the line. We can substitute a given *x*-coordinate into the rule to calculate the *y*-coordinate. When we substitute a *y*-coordinate into the rule, it makes an equation that can be solved to give the *x*-coordinate. So, for every point on a straight line, the value of the *x*-coordinate is the solution to a particular equation.

The point of intersection of two straight lines is the shared point where the lines cross over each other. This is the only point with coordinates that satisfy both equations; that is, makes both equations true (LHS = RHS).



For example, the point $(2, 3)$ on the line $y = 2x - 1$ shows us that when $2x - 1 = 3$, the solution is $x = 2$.

Let's start: Matching equations and solutions

When a value is substituted into an equation and it makes the equation true (LHS = RHS), then that value is a solution to that equation.

a From the lists below, match each equation with a solution. Some equations have more than one solution.

Equations	
$2x - 4 = 8$	$y = x + 4$
$3x + 2 = 11$	$y = 2x - 5$
$y = 10 - 3x$	$5x - 3 = 2$

Possible solutions			
$x = 1$	$(1, 5)$	$x = 2$	$(3, 1)$
$x = -1$	$x = 6$	$(2, -1)$	$(2, 6)$
$(-2, -9)$	$(-2, 16)$	$x = 3$	$(2, 4)$

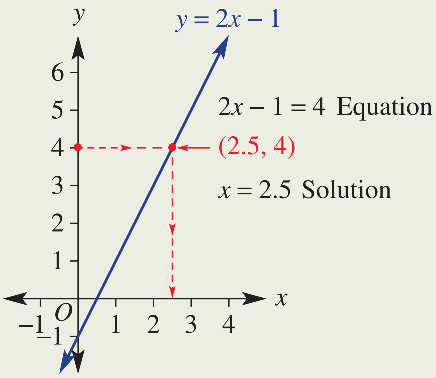
b Which two equations share the same solution and what is this solution?

c List the equations that have only one solution. What is a common feature of these equations?

d List the equations that have more than one solution. What is a common feature of these equations?

Key ideas

- The *x*-coordinate of each point on the graph of a straight line is a solution to a particular linear equation.
  - A particular linear equation is formed by substituting a chosen *y*-coordinate into a linear relationship. For example, if $y = 2x - 1$ and $y = 4$, then the linear equation is $2x - 1 = 4$.
  - The solution to this equation is the *x*-coordinate of the point with the chosen *y*-coordinate. For example, the point $(2.5, 4)$ shows that $x = 2.5$ is the solution to $2x - 1 = 4$.



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- A point (x, y) is a solution to the equation for a line if its coordinates make the equation true.
  - An equation is true when LHS = RHS after the coordinates are substituted.
  - Every point on a straight line is a solution to the equation for that line.
  - Every point that is not on a straight line is not a solution to the equation for that line.
- The point of intersection of two straight lines is the only solution that satisfies both equations.
  - The point of intersection is the shared point where two straight lines cross each other.
  - This is the only point with coordinates that makes both equations true.

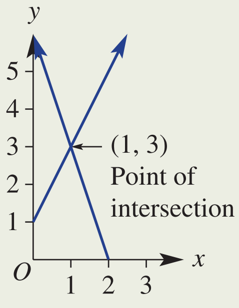
    For example, (1, 3) is the only point that makes both $y = 6 - 3x$ and $y = 2x + 1$ true.

    Substituting (1, 3):

    $y = 6 - 3x$ $y = 2x + 1$

    $3 = 6 - 3 \times 1$ $3 = 2 \times 1 + 1$

    $3 = 3$ (True) $3 = 3$ (True)



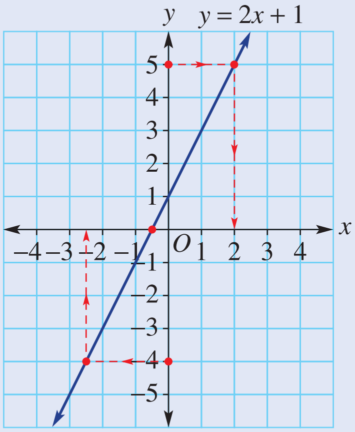
Example 13 Using a linear graph to solve an equation

Use the graph of $y = 2x + 1$, shown here, to solve each of the following equations.

a $2x + 1 = 5$

b $2x + 1 = 0$

c $2x + 1 = -4$



SOLUTION	EXPLANATION
a $x = 2$	Locate the point on the line with y-coordinate 5. The x-coordinate of this point is 2, so $x = 2$ is the solution to $2x + 1 = 5$.
b $x = -0.5$	Locate the point on the line with y-coordinate 0. The x-coordinate of this point is −0.5, so $x = -0.5$ is the solution to $2x + 1 = 0$.
c $x = -2.5$	Locate the point on the line with y-coordinate −4. The x-coordinate of this point is −2.5, so $x = -2.5$ is the solution to $2x + 1 = -4$.

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Example 14 Using the point of intersection of two lines to solve an equation

Use the graph of $y = 4 - x$ and $y = 2x + 1$, shown here, to answer these questions.

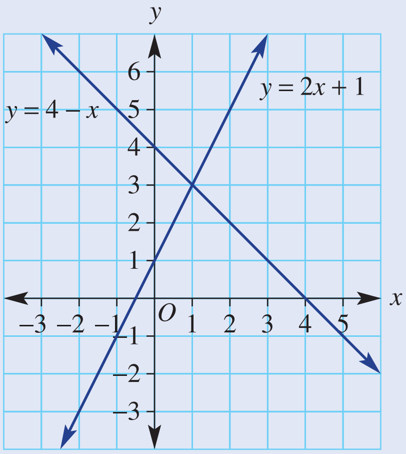
a Write two equations that each have $x = -2$ as a solution.

b Write four solutions (x, y) for the line with equation $y = 4 - x$.

c Write four solutions (x, y) for the line with equation $y = 2x + 1$.

d Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.

e Solve the equation $4 - x = 2x + 1$.



SOLUTION	EXPLANATION
a $4 - x = 6$ $2x + 1 = -3$	$(-2, 6)$ is on the line $y = 4 - x$ so $4 - x = 6$ has solution $x = -2$. $(-2, -3)$ is on the line $y = 2x + 1$, so $2x + 1 = -3$ has solution $x = -2$.
b $(-2, 6)(-1, 5)(1, 3)(4, 0)$	Many correct answers. Each point on the line $y = 4 - x$ is a solution to the equation for that line.
c $(-2, -3)(0, 1)(1, 3)(2, 5)$	Many correct answers. Each point on the line $y = 2x + 1$ is a solution to the equation for that line.
d $(1, 3)$ $\quad$ $(1, 3)$ $y = 4 - x$ $\quad$ $y = 2x + 1$ $3 = 4 - 1$ $\quad$ $3 = 2 \times 1 + 1$ $3 = 3$ True $\quad$ $3 = 3$ True	The point of intersection $(1, 3)$ is the solution that satisfies both equations. Substitute $(1, 3)$ into each equation and show that it makes a true equation (LHS = RHS).
e $x = 1$	The solution to $4 - x = 2x + 1$ is the x-coordinate at the point of intersection. The value of both rules is equal for this x-coordinate.

Exercise 7G

UNDERSTANDING AND FLUENCY	1–8	3–7(½), 8	5–9

1 Use the given rule to complete this table, and then plot and join the points to form a straight line $y = 2x - 1$.

x	−2	−1	0	1	2
y					

2 Substitute each given y-coordinate into the rule $y = 2x - 3$, and then solve the equation algebraically to find the x-coordinate.

i $y = 7$ $\quad$ **ii** $y = -5$

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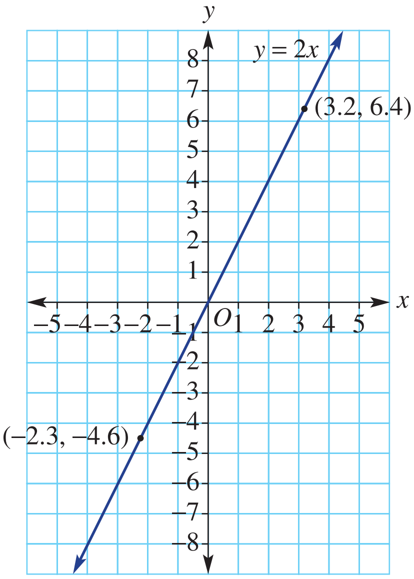
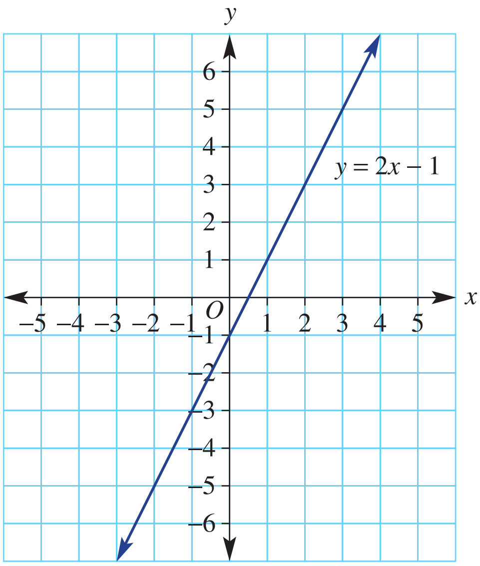
3 State the coordinates (x, y) of the point on this graph of $y = 2x$, where:

a $2x = 4$ (i.e. $y = 4$)
b $2x = 6.4$
c $2x = -4.6$
d $2x = 7$
e $2x = -14$
f $2x = 2000$
g $2x = 62.84$
h $2x = -48.602$
i $2x =$ *any number* (worded answer)



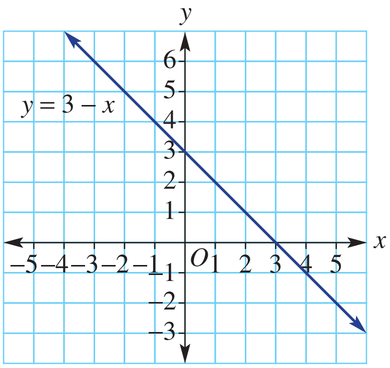
Example 13 **4** Use the graph of $y = 2x - 1$, shown here, to find the solution to each of these equations.

a $2x - 1 = 3$
b $2x - 1 = 0$
c $2x - 1 = 5$
d $2x - 1 = -6$
e $2x - 1 = -4$



5 Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.

a $3 - x = 5.5$
b $3 - x = 0$
c $3 - x = 3.5$
d $3 - x = -1$
e $3 - x = -2$



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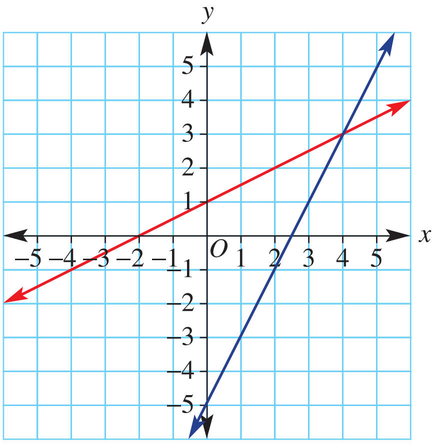
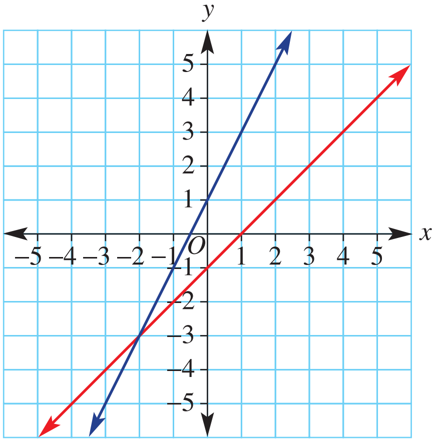
6 For each of these graphs write down the coordinates of the point of intersection (i.e. the point where the lines cross over each other).

a



b



7 Graph each pair of lines on the same set of axes and read off the point of intersection.

a $y = 2x - 1$

x	−2	−1	0	1	2	3
y						

$y = x + 1$

x	−2	−1	0	1	2	3
y						

b $y = -x$

x	−2	−1	0	1	2	3
y						

$y = x + 2$

x	−2	−1	0	1	2	3
y						

8 Use digital technology to sketch a graph of $y = 1.5x - 2.5$ for x and y values between −7 and 7. Use the graph to solve each of the following equations. Round answers to two decimal places.

a $1.5x - 2.5 = 3$

b $1.5x - 2.5 = -4.8$

c $1.5x - 2.5 = 5.446$

9 Use digital technology to sketch a graph of each pair of lines and find the coordinates of the points of intersection. Round answers to two decimal places.

a $y = 0.25x + 0.58$ and $y = 1.5x - 5.4$

b $y = 2 - 1.06x$ and $y = 1.2x + 5$

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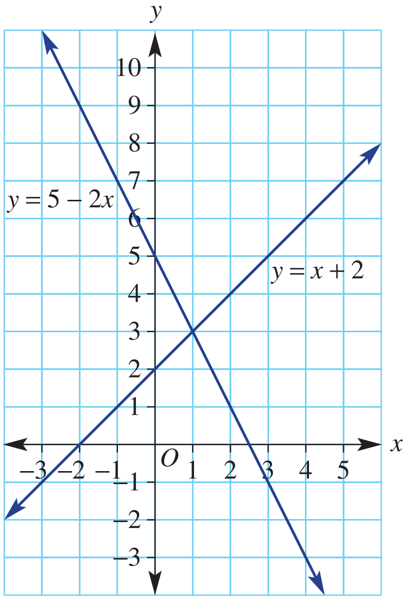
PROBLEM-SOLVING AND REASONING	10, 11, 14	10–12, 14	11–13, 15

Example 14

10 Use the graph of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.

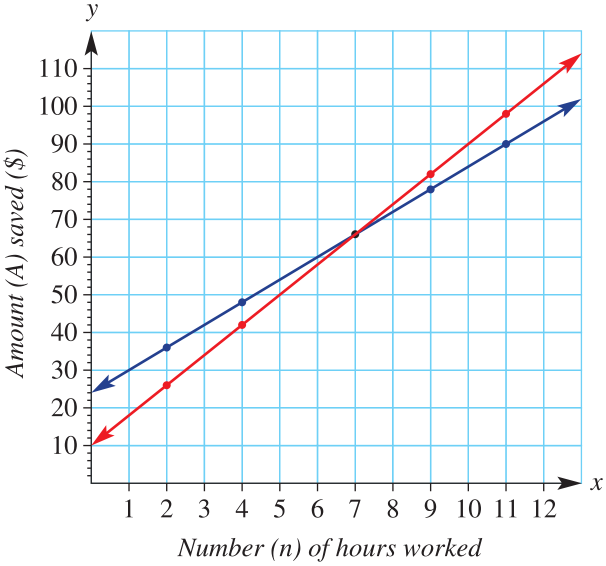


- **a** Write two equations that each have $x = -1$ as a solution.
- **b** Write four solutions (x, y) for the equation $y = 5 - 2x$.
- **c** Write four solutions (x, y) for the equation $y = x + 2$.
- **d** Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.
- **e** Solve the equation $5 - 2x = x + 2$ from the graph.

11 Jayden and Ruby are saving all their money for the school ski trip.

- Jayden has saved \$24 and earns \$6 per hour mowing lawns.
- Ruby has saved \$10 and earns \$8 per hour babysitting.

This graph shows the total amount (A), in dollars, of their savings for the number (n) of hours worked.



- **a** Here are two rules for calculating the amount (A) saved for working for n hours:
  $A = 10 + 8n$ and $A = 24 + 6n$
  Which rule applies to Ruby and which to Jayden? Explain why.
- **b** Use the appropriate line on the graph above to find the solution to the following equations.
  - **i** $10 + 8n = 42$
  - **ii** $24 + 6n = 48$
  - **iii** $10 + 8n = 66$
  - **iv** $24 + 6n = 66$
  - **v** $10 + 8n = 98$
  - **vi** $24 + 6n = 90$
- **c** From the graph write three solutions (n, A) that satisfy $A = 10 + 8n$.
- **d** From the graph write three solutions (n, A) that satisfy $A = 24 + 6n$.
- **e** Write the solution (n, A) that is true for both Ruby's and Jayden's equations, and show that it satisfies both equations.
- **f** From the graph find the solution to the equation $10 + 8n = 24 + 6n$ (i.e. find the value of n that makes Ruby's and Jayden's savings equal to each other).
- **g** Explain how many hours have been worked and what their savings are at the point of intersection of the two lines.

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12 Jessica and Max have a 10 second running race.

- Max runs at 6 m/s.
- Jessica is given a 10 m head-start and runs at 4 m/s.

a Copy and complete this table showing the distance run by each athlete.

Time (t), in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance (d), in metres	0										
Jessica's distance (d), in metres	10										

b Plot these points on a distance–time graph and join them to form two straight lines, labelling them 'Jessica' and 'Max'.

c Find the rule linking distance d and time t for Max.

d Using the rule for Max's race, write an equation that has the solution:

i $t = 3$ **ii** $t = 5$ **iii** $t = 8$

e Find the rule linking distance d and time t for Jessica.

f Using the rule for Jessica's race, write an equation that has the solution:

i $t = 3$ **ii** $t = 5$ **iii** $t = 8$

g Write the solution (t, d) that is true for both distance equations and show that it satisfies both equations.

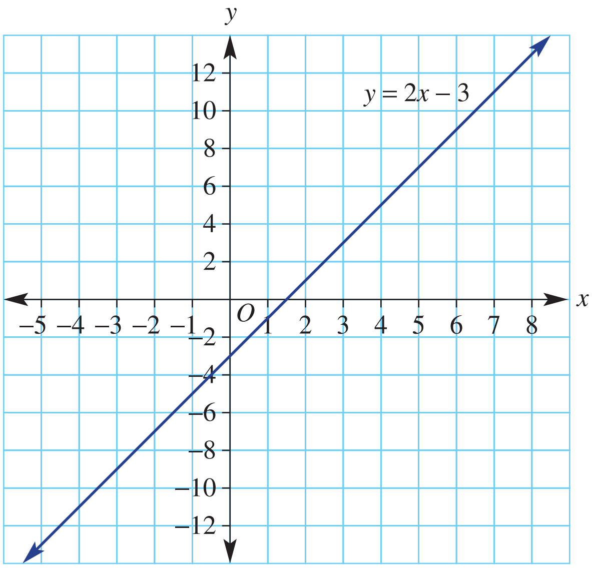
h Explain what is happening in the race at the point of intersection, and for each athlete state the distance from the starting line and time taken.

13 Some equations can be rearranged so that a given graph can be used to find a solution. For example, $4x - 1 = 2(x - 3)$ can be solved using the graph of $y = 2x - 3$ with this rearrangement:

$$\begin{array}{rcl} & 4x - 1 = 2(x - 3) & \\ & 4x - 1 = 2x - 6 & \\ -2x \downarrow & & \downarrow -2x \\ & 2x - 1 = \quad -6 & \\ -2 \downarrow & & \downarrow -2 \\ & 2x - 3 = -8 & \end{array}$$

$y = -8$ on the graph gives the solution $x = -2.5$.

Rearrange the following equations so the left side of each is $2x - 3$, and then use this graph of $y = 2x - 3$ to find each solution.



a $4x - 1 = 2(x - 5)$

b $5x + 7 = 3(x + 4)$

c $3 = 6 - 2(x - 3)$

d $3 + 4x - 1 = x + 14 + x - 4$

e $3(x - 3) - 2x = 4x + 3 - 5x$

f $4(x - 1) + x = 5x - 7 - 2x$

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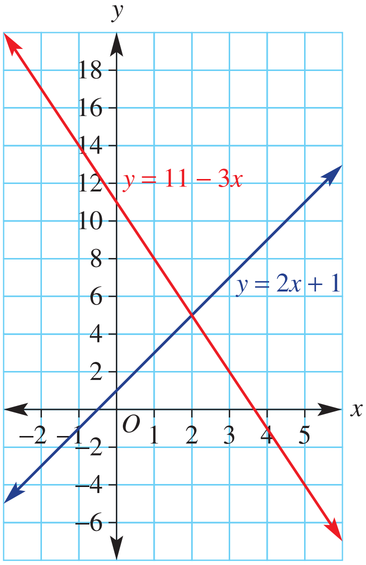
14 This graph shows two lines with equations $y = 11 - 3x$ and $y = 2x + 1$.



a Copy and complete the coordinates of each point that is a solution for the given linear equation.

i $y = 11 - 3x$

$(-2, ?), (-1, ?)(0, ?)(1, ?)(2, ?)(3, ?)(4, ?)(5, ?)$

ii $y = 2x + 1$

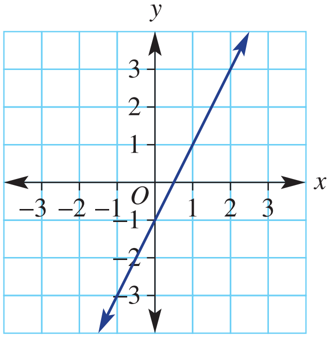
$(-2, ?), (-1, ?)(0, ?)(1, ?)(2, ?)(3, ?)(4, ?)(5, ?)$

b State the coordinates of the point of intersection and show it is a solution to both equations.

c Explain why the point of intersection is the only solution that satisfies both equations.

15 Here is a table and graph for the line $y = 2x - 1$.

x	−1	−0.5	0	0.5	1	1.5	2
y	−3	−2	−1	0	1	2	3



a Luke says: "Only seven equations can be solved from points on this line because the y values must be whole numbers."

i What are three of the equations and their solutions that Luke could be thinking of?

ii Is Luke's statement correct? Explain your conclusion with some examples.

b Chloe says: "There are sixty equations that can be solved from points on this line because y values can go to one decimal place."

i What are three extra equations and their solutions that Chloe might have thought of?

ii Is Chloe's statement correct? Explain your conclusion with some examples.

c Jamie says: "There are an infinite number of equations that can be solved from points on a straight line."

i Write the equations that can be solved from these two points: (1.52, 2.04) and (1.53, 2.06).

ii Write the coordinates of two more points with x-coordinates between the first two points and write the equations that can be solved from these two points.

iii Is Jamie's statement correct? Explain your reasons.

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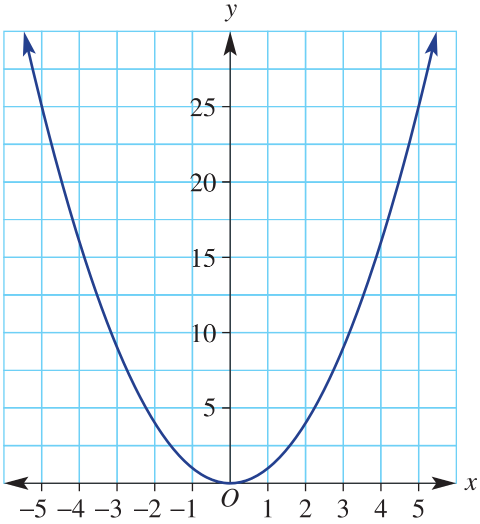
ENRICHMENT	–	–	16

More than one solution

16 **a** Use this graph of $y = x^2$ to solve the following equations.

- **i** $x^2 = 4$
- **ii** $x^2 = 9$
- **iii** $x^2 = 16$
- **iv** $x^2 = 25$



b Explain why there are two solutions to each of the equations in part **a** above.

c Use digital technology to graph $y = x^2$ and graphically solve the following equations, rounding answers to two decimal places.

- **i** $x^2 = 5$
- **ii** $x^2 = 6.8$
- **iii** $x^2 = 0.49$
- **iv** $x^2 = 12.75$
- **v** $x^2 = 18.795$

d Give one reason why the graph of $y = x^2$ does *not* give a solution to the equation $x^2 = -9$.

e List three more equations of the form $x^2 =$ 'a number' that *can't* be solved from the graph of $y = x^2$.

f List the categories of numbers that *will* give a solution to the equation: $x^2 =$ 'a number'.

g Graph $y = x + 2$ and $y = x^2$ on the same screen and graphically solve $x^2 = x + 2$ by finding the x values of the points of intersection.

h Use digital technology to solve the following equations using graphical techniques. Round answers to two decimal places.

- **i** $x^2 = 3x + 16$
- **ii** $x^2 = 27 - 5x$
- **iii** $x^2 = 2x - 10$
- **iv** $x^2 = 6x - 9$

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7H Applying linear graphs EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

Rules and graphs can be used to help analyse many situations in which there is a relationship between two variables. If the rate of change of one variable with respect to another is constant, then the relationship will be linear and a graph will give a straight line. For example, if a pile of dirt being emptied out of a pit increases at a rate of 12 tonnes per hour, then the graph of the mass of dirt over time would be a straight line. For every hour, the mass of dirt increases by 12 tonnes.

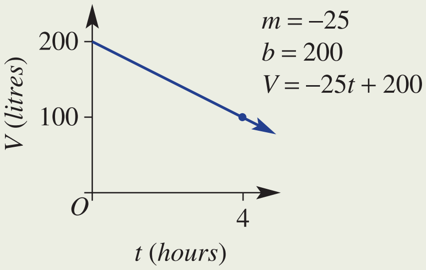
Let's start: Water storage

The volume of water in a tank starts at 1000 litres and with constant rainfall the volume of water increases by 2000 litres per hour for 5 hours.

- Describe the two related variables in this situation.
- Discuss whether or not the relationship between the two variables is **linear**.
- Use a table and a graph to illustrate the relationship.
- Find a rule that links the two variables and discuss how your rule might be used to find the volume of water in the tank at a given time.

Key ideas

- If the rate of change of one variable with respect to another is constant, then the relationship between the two variables is **linear**.
- When applying straight-line graphs, choose letters to replace x and y to suit the variables. For example, V for volume and t for time.
- $y = mx + b$ can be used to help find the linear rule linking two variables.



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Example 15 Linking distance with time

A hiker walks at a constant rate of 4 kilometres per hour for 4 hours.

a Draw a table of values, using t for time in hours and d for distance in kilometres. Use t between 0 and 4.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking d with t.

d Use your rule to find the distance travelled for 2.5 hours of walking.

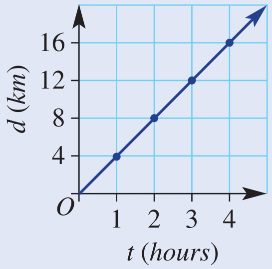
e Use your rule to find the time taken to travel 8 km.

SOLUTION

a

t	0	1	2	3	4
d	0	4	8	12	16

b



c $m = \frac{4}{1} = 4$

$b = 0$

$y = mx + b$

$d = 4t + 0$

So $d = 4t$.

d $d = 4t$

$= 4 \times 2.5$

$= 10$

The distance is 10 km after 2.5 hours of walking.

e $d = 4t$

$8 = 4t$

$2 = t$

It takes 2 hours to travel 8 km.

EXPLANATION

a d increases by 4 for every increase in t by 1.

b Plot the points on a graph, using a scale that matches the numbers in the table.

c Using $y = mx + b$, find the gradient (m) and the y-intercept (b). Replace y with d and x with t.

d Substitute $t = 2.5$ into your rule and find the value for d.

e Substitute $d = 8$ into your rule, then divide both sides by 4.

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Example 16 Applying graphs when the rate is negative

The initial volume of water in a dish in the Sun is 300 mL. The water evaporates and the volume decreases by 50 mL per hour for 6 hours.

a Draw a table of values, using t for time in hours and V for volume in millilitres.
b Draw a graph by plotting the points given in the table in part **a**.
c Write a rule linking V with t.
d Use your rule to find the volume of water in the dish after 4.2 hours in the Sun.
e Use your rule to find the time taken for the volume to reach 75 mL.

SOLUTION

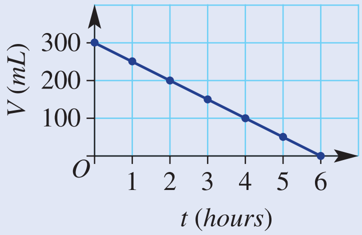
EXPLANATION

a

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

The volume starts at 300 millilitres and decreases by 50 millilitres every hour.

b



Use numbers from 0 to 300 on the V-axis and from 0 to 6 on the t-axis to accommodate all the numbers in the table.

c $m = \frac{-50}{1} = -50$

The gradient m in $y = mx + b$ is given by $\frac{\text{rise}}{\text{run}} = \frac{-50}{1}$.

$b = 300$
$y = mx + b$
$V = -50t + 300$

b is the y-intercept $= 300$

d $V = -50t + 300$
$= -50 \times 4.2 + 300$
$= 90$

The volume of water in the dish is 90 millilitres after 4.2 hours.

Substitute $t = 4.2$ into your rule to find V.

e $V = -50t + 300$
$75 = -50t + 300$
$-225 = -50t$
$4.5 = t$

It takes 4.5 hours for the volume to reach 75 mL.

Substitute $V = 75$ into your rule.
Subtract 300 from both sides.

Divide both sides by -50.

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Exercise 7H EXTENSION

UNDERSTANDING AND FLUENCY	1–7	4–8	6–8

1 A rule linking distance d and time t is given by $d = 10t + 5$. Use this rule to find the value of d for the given values of t.

a $t = 1$ **b** $t = 4$ **c** $t = 0$ **d** $t = 12$

2 Write down the gradient (m) and the y-intercept (b) for these rules.

a $y = 3x + 2$ **b** $y = -2x - 1$ **c** $y = 4x + 2$

d $y = -10x - 3$ **e** $y = -20x + 100$ **f** $y = 5x - 2$

3 The height (in cm) of fluid in a flask increases at a rate of 30 cm every minute starting at 0 cm. Find the height of fluid in the flask at these times.

a 2 minutes **b** 5 minutes **c** 11 minutes

4 The volume of gas in a tank decreases from 30 L by 2 L every second. Find the volume of gas in the tank at these times.

a 1 second **b** 3 seconds **c** 10 seconds

Example 15

5 A jogger runs at a constant rate of 6 kilometres per hour for 3 hours.

a Draw a table of values, using t for time in hours and d for distance in kilometres. Use t between 0 and 3.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking d with t.

d Use your rule to find the distance travelled for 1.5 hours of jogging.

e Use your rule to find how long it takes to travel 12 km.

6 A paddle steamer moves up the Murray River at a constant rate of 5 kilometres per hour for 8 hours.

a Draw a table of values, using t for time in hours and d for distance in kilometres. Use t between 0 and 8.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking d with t.

d Use your rule to find the distance travelled after 4.5 hours.

e Use your rule to find how long it takes to travel 20 km.

Example 16

7 The volume of water in a sink is 20 litres. The plug is pulled out and the volume decreases by 4 litres per second for 5 seconds.

a Draw a table of values, using t for time in seconds and V for volume in litres.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking V with t.

d Use your rule to find the volume of water in the sink 2.2 seconds after the plug is pulled.

e Use your rule to find how long it takes for the volume to fall to 8 litres.

8 A weather balloon at a height of 500 metres starts to descend at a rate of 125 metres per minute for 4 minutes.

a Draw a table of values, using t for time in minutes and h for height in metres.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking h with t.

d Use your rule to find the height of the balloon after 1.8 minutes.

e Use your rule to find how long it takes for the balloon to fall to a height of 125 metres.

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PROBLEM-SOLVING AND REASONING	9, 12	9, 10, 12	10–13

9 A BBQ gas bottle starts with 3.5 kg of gas. Gas is used at a rate of 0.5 kg per hour for a long lunch.

a Write a rule for the mass of gas, M, in terms of time, t.

b How long will it take for the gas bottle to empty?

c How long will it take for the mass of the gas in the bottle to reduce to 1.25 kg?

10 A cyclist races 50 km at an average speed of 15 km per hour.

a Write a rule for the distance travelled d in terms of time t.

b How long will it take the cyclist to travel 45 km?

c How long will the cyclist take to complete the 50 km race? Give your answer in hours and minutes.

11 An oil well starts to leak and the area of an oil slick increases by 8 km^2 per day. How long will it take the slick to increase to 21 km^2? Give your answer in days and hours.

12 The volume of water in a tank (in litres) is given by $V = 2000 - 300t$, where t is in hours.

a What is the initial volume?

b Is the volume of water in the tank increasing or decreasing? Explain your answer.

c At what rate is the volume of water changing?

13 The cost of a phone call is 10 cents plus 0.5 cents per second.

a Explain why the cost, c cents, of the phone call for t seconds is given by $c = 0.5t + 10$.

b Explain why the cost, C dollars, of the phone call for t seconds is given by $C = 0.005t + 0.1$.

ENRICHMENT	–	–	14

Danger zone

14 Two small planes take off and land at the same airfield. One plane takes off from the runway and gains altitude at a rate of 15 metres per second. At the same time, the second plane flies near the runway and reduces its altitude from 100 metres at rate of 10 metres per second.

a Draw a table of values, using t between 0 and 10 seconds and h for height in metres, of both planes.

t (s)	0	1	2	3	4	5	6	7	8	9	10
h_1 (m)											
h_2 (m)											

b On the one set of axes draw a graph of the height of each plane during the 10-second period.

c How long does it take for the second plane to touch the ground?

d Write a rule for the height of each plane.

e At what time are the planes at the same height?

f At what time is the first plane at a height of 37.5 metres?

g At what time is the second plane at a height of 65 metres?

h At the same time, a third plane at an altitude of 150 metres descends at a rate of 25 metres per second. Will all three planes ever be at the same height at the same time? What are the heights of the three planes at the 4-second mark?

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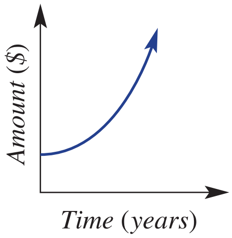
7I Non-linear graphs

Not all relationships between two variables are linear. The amount of money invested in a compound interest account, for example, will not increase at a constant rate. Over time, the account balance will increase more rapidly, meaning that the graph of the relationship between *Amount* and *Time* will be a curve and not a straight line.



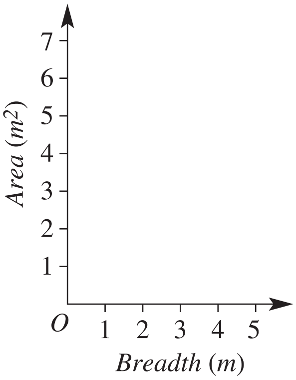
Let's start: The fixed perimeter playpen

Imagine you have 10 metres of fencing material to make a rectangular playpen.

- List some possible dimensions of your rectangle.
- What is the area of the playpen for some of your listed dimensions?
- Complete this table, showing all the positive integer dimensions of the playpen.

Breadth (m)	1	2	3	4
Length (m)		3		
Area (m^2)		6		

- Plot the *Area* against *Breadth* to form a graph.
- Discuss the shape of your graph.
- Discuss the situation and graphical points when the breadth is 1 m or 4 m.
- What dimensions would deliver a maximum area and explain how your graph helps determine this?

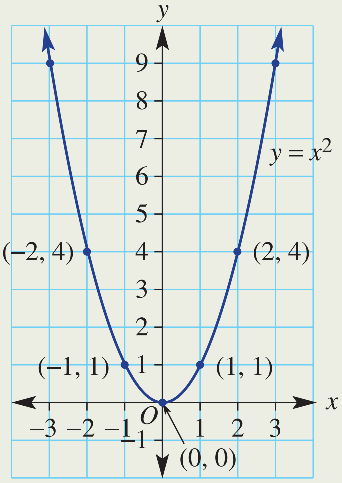


Key ideas

- To plot **non-linear** curves when given their rule, follow these steps.
  - Construct a table of values using the rule.
  - Plot the points on a set of axes.
  - Join the plotted points to form a smooth curve.
- The graph of $y = x^2$ is an example of a non-linear graph called a **parabola**.

  The table shows five points that lie on the parabola. In each point, the y value is the square of the x value.

x	−2	−1	0	1	2
y	4	1	0	1	4



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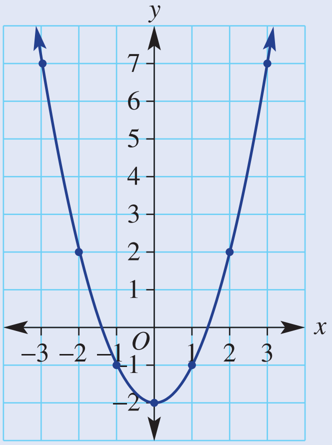
Example 17 Plotting a non-linear relationship

Plot points to draw the graph of $y = x^2 - 2$, using a table.

SOLUTION

x	−3	−2	−1	0	1	2	3
y	7	2	−1	−2	−1	2	7



EXPLANATION

Find the value of y by substituting each value of x into the rule.

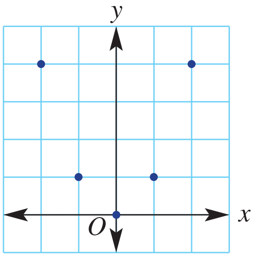
Plot the points and join with a smooth curve. The curve is called a parabola.

Exercise 7I

UNDERSTANDING AND FLUENCY	1–4	3, 4	4(½)

1 Copy these graphs and then join the points to form smooth curves.

a



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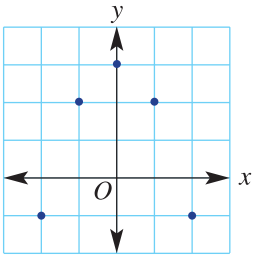
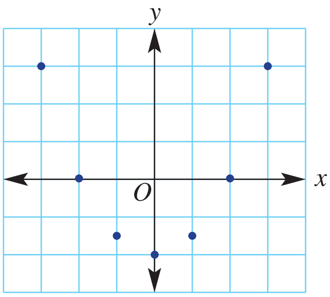
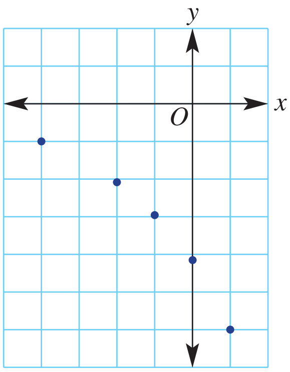


c



d



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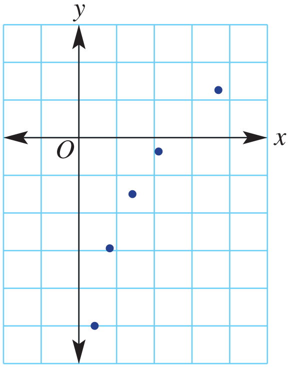
e



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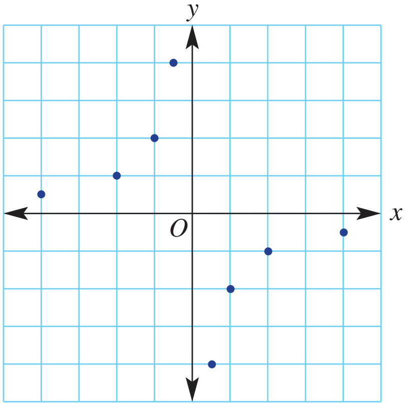


2 If $y = x^2 - 1$, find the value of y for these x values.

a $x = 0$

b $x = 3$

c $x = 2$

d $x = -4$

3 If $y = 2^x$, find the value of y for these x values.

a $x = 1$

b $x = 2$

c $x = 3$

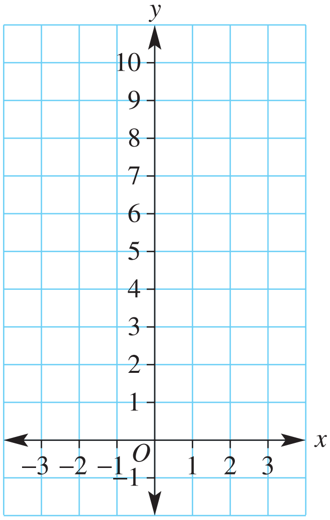
d $x = 4$

Example 17

4 Plot points to draw the graph of each of the given rules. Use the table and set of axes as a guide.

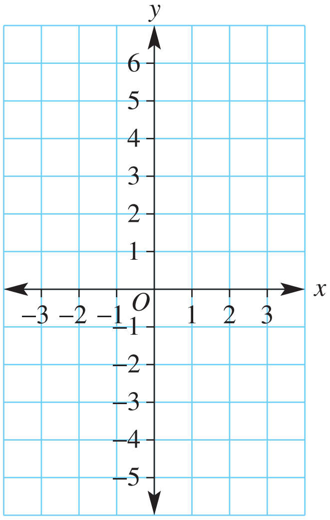
a $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9						



b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5						



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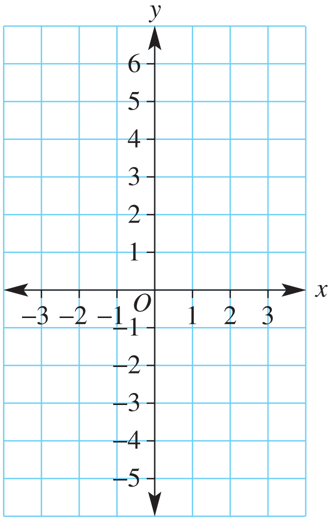
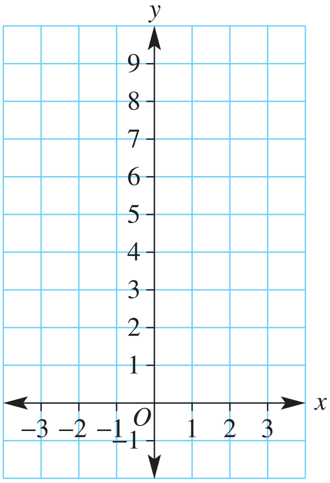
c $y = 5 - x^2$

x	-3	-2	-1	0	1	2	3
y		1					



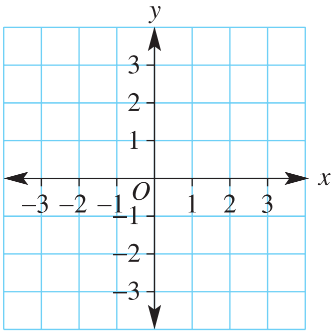
d $y = 2^x$

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1			



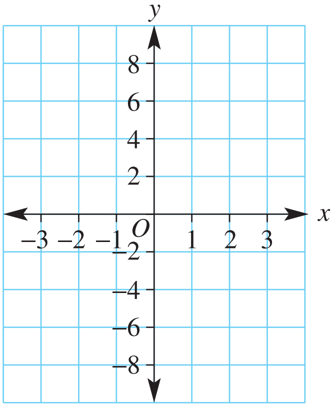
e $y = \frac{1}{x}$

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y			-2	2		



f $y = x^3$

x	-2	-1	0	1	2
y					



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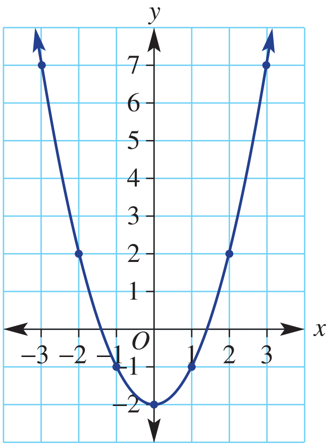
PROBLEM-SOLVING AND REASONING	5, 7	5–7	5–7

5 Find a rule for these non-linear graphs.

a



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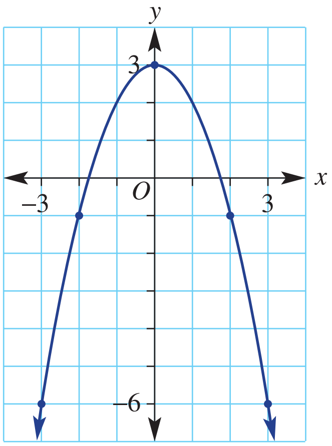

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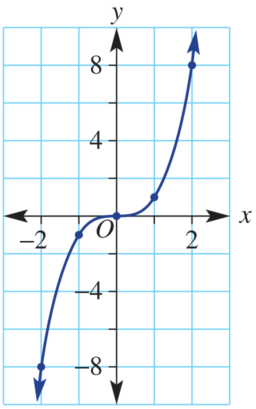

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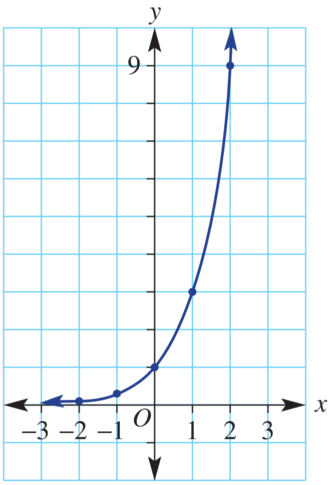
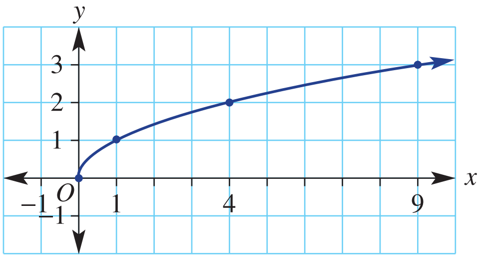
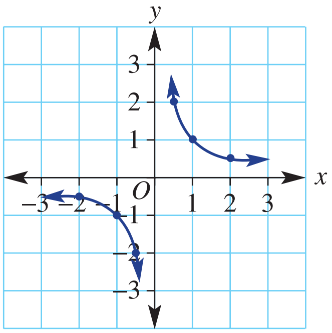


e



f



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6 What are the lowest and highest values of y on a graph of these rules if x ranges from -3 to 3?

a $y = x^2$
b $y = x^2 - 10$
c $y = 6 - x^2$
d $y = x^3$
e $y = -x^3 - 1$
f $y = (x - 1)^3$

7 Here are some rules classified by the name of their graph.

Line	**Parabola**	**Hyperbola**	**Exponential**
$y = 2x - 3$	$y = x^2$	$y = \frac{1}{x}$	$y = 3^x$
$y = -5x + 1$	$y = 3 - x^2$	$y = -\frac{5}{x}$	$y = 4^x - 3$

Name the type of graph that is produced by each of these rules.

a $y = x^2 + 7$
b $y = -2x + 4$
c $y = \frac{2}{x}$
d $y = 7^x$
e $y = -\frac{3}{x} + 1$
f $y = 4 - x^2$
g $y = 1 - x$
h $y = (0.5)^x$

ENRICHMENT	–	–	8

Families of parabolas

8 For each family of parabolas, plot graphs by hand or use technology to draw each set on the same set of axes. Then describe the features of each family. Describe the effect on the graph when the number a changes.

a Family 1: $y = ax^2$ Use $a = \frac{1}{2}$, $a = 1$, $a = 2$ and $a = 3$.
b Family 2: $y = -ax^2$ Use $a = \frac{1}{2}$, $a = 1$, $a = 2$ and $a = 3$.
c Family 3: $y = x^2 + a$ Use $a = -3$, $a = -1$, $a = 0$, $a = 2$ and $a = 5$.
d Family 4: $y = (x - a)^2$ Use $a = -2$, $a = 0$, $a = 1$ and $a = 4$.

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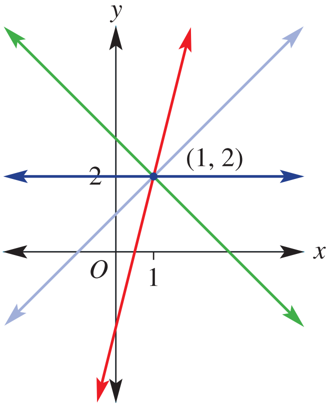
Families of straight lines

A set of lines are said to be in the same family if the lines have something in common. For example, four lines that pass through the point (1, 2) could be called a family.



The parallel family

1 Complete the table for these rules.

- **a** $y_1 = 2x - 5$
- **b** $y_2 = 2x - 2$
- **c** $y_3 = 2x$
- **d** $y_4 = 2x + 3$

x	−3	−2	−1	0	1	2	3
y_1							
y_2							
y_3							
y_4							

2 Plot the points given in your table to draw graphs of the four rules in part **a** on the one set of axes. Label each line with its rule.

3 What do you notice about the graphs of the four rules? Describe how the numbers in the rule relate to its graph.

4 How would the graphs for the rules $y = 2x + 10$ and $y = 2x - 7$ compare with the graphs you have drawn above? Explain.

The point family

1 Complete the table for these rules.

- **a** $y_1 = x + 1$
- **b** $y_2 = 2x + 1$
- **c** $y_3 = 1$
- **d** $y_4 = -x + 1$
- **e** $y = -\frac{1}{2}x + 1$

x	−3	−2	−1	0	1	2	3
y_1							
y_2							
y_3							
y_4							
y_5							

2 Plot the points given in your table to draw graphs of the five rules in part **a** on one set of axes. Label each line with its rule.

3 What do you notice about the graphs of the five rules? Describe how the numbers in the rule relate to its graph.

4 How would the graphs for the rules $y = 3x + 1$ and $y = -\frac{1}{3}x + 1$ compare with the graphs you have drawn above? Explain.

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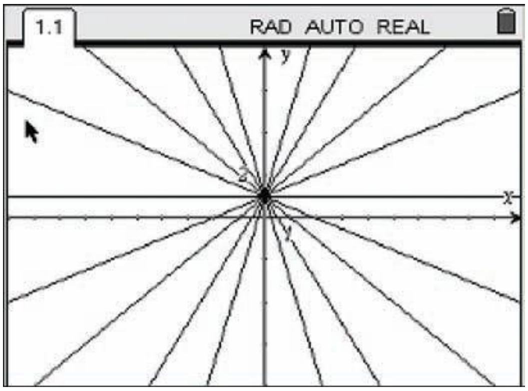
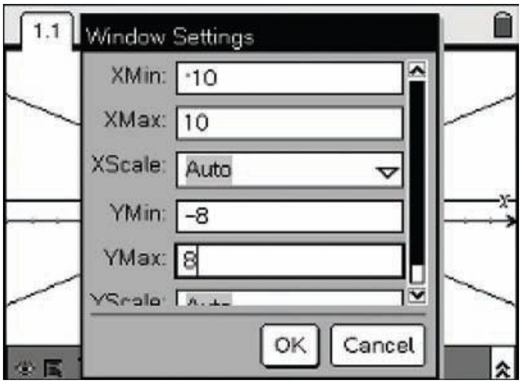
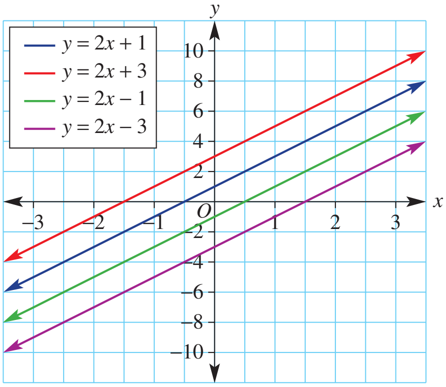
Exploring families with technology

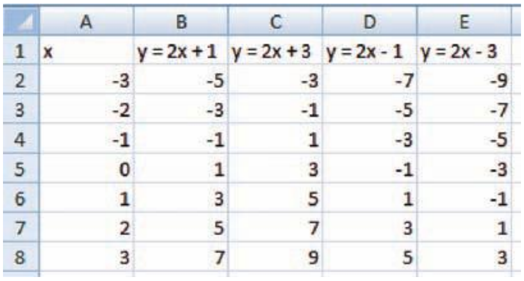
Graphics or CAS calculators, graphing software and spreadsheets are useful tools to explore families of straight lines. Here are some screenshots showing the use of technology.




1.1 Window Settings

XMin: -10
XMax: 10
XScale: Auto
YMin: -8
YMax: 8



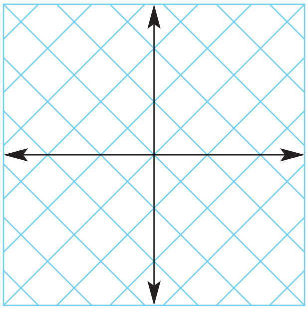


	A	B	C	D	E
1	x	y = 2x + 1	y = 2x + 3	y = 2x - 1	y = 2x - 3
2	-3	-5	-3	-7	-9
3	-2	-3	-1	-5	-7
4	-1	-1	1	-3	-5
5	0	1	3	-1	-3
6	1	3	5	1	-1
7	2	5	7	3	1
8	3	7	9	5	3

1 Choose one type of technology and sketch the graphs for the two families of straight lines shown in the previous two sections: the ‘parallel family’ and the ‘point family’.

2 Use your chosen technology to help design a family of graphs that produces the patterns shown. Write down the rules used and explain your choices.

a



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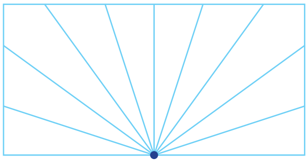

3 Make up your own design, then use technology to produce it. Explain how your design is built and give the rules that make up the design.

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Puzzles and challenges

1 A trekker hikes down a track at 3 km per hour. Two hours later, a second trekker sets off on the same track at 5 km per hour. How long is it before the second trekker catches up with the first?

2 Find the rules for the non-linear relationships with these tables.

a

x	−2	−1	0	1	2
y	1	−2	−3	−2	1

b

x	−2	−1	0	1	2
y	6	9	10	9	6

c

x	0	1	4	9	16
y	1	2	3	4	5

d

x	−3	−2	−1	0	1
y	−30	−11	−4	−3	−2

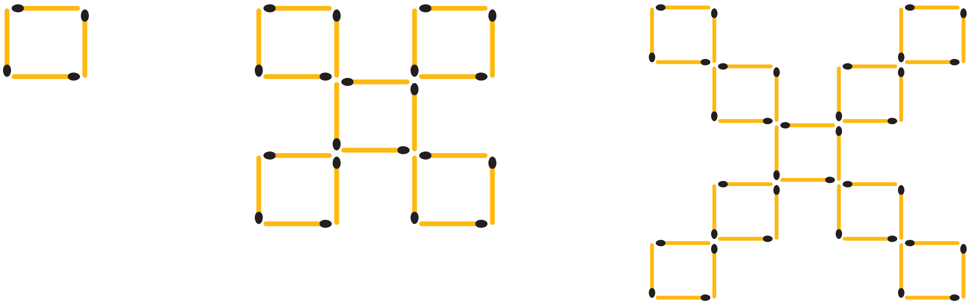
3 A line with a gradient of 3 intersects another line at right angles. Find the gradient of the other line.

4 Two cars travel toward each other on a 100 km stretch of road. One car travels at 80 km per hour and the other at 70 km per hour. If they set off at the same time, how long will it be before the cars meet?

5 Find the y-intercept of a line joining the two points $(-1, 5)$ and $(2, 4)$.

6 Find the rule of a line that passes through the two points $(-3, -1)$ and $(5, 3)$.

7 Find the number of matchsticks needed in the 100th diagram in the pattern given below. The first three diagrams in the pattern are given.



8 At a luxury car hire shop, a Ferrari costs \$300 plus \$40 per hour. A Porsche costs \$205 plus \$60 per hour. What hire time makes both cars the same cost? Give the answer in hours and minutes.

9 Find the area of the figure enclosed by the four lines:
$x = 6, y = 4, y = -2$ and $y = x + 5$.

10 A rectangle $ABCD$ is rotated about A 90° clockwise.

a In two different ways, explain why the diagonal AC is perpendicular to its image $A'C'$.

b If $AB = p$ and $BC = q$, find the simplified product of the gradients of AC and $A'C'$.

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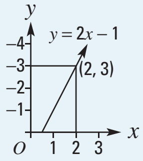
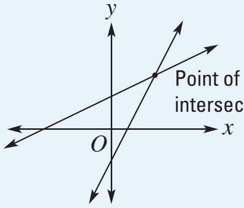
Linear relationships 1



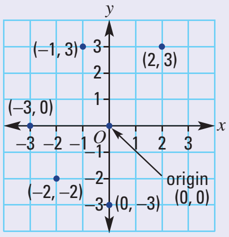
To graphically solve $2x - 1 = 3$:

- Locate point with $y = 3$.
- x-coordinate is solution, i.e. $x = 2$.



The point of intersection is the only solution that satisfies both linear equations.

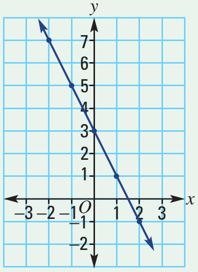
Cartesian plane (number plane)



Rules, tables and graphs

$y = -2x + 3$

x	–2	–1	0	1	2
y	7	5	3	1	–1



Finding the rule from tables

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

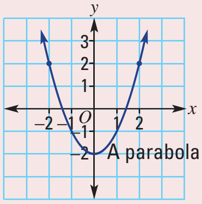
+3 +3 +3 +3

$y = \square \times x + \square$
$= 3x + (-2)$
$= 3x - 2$

Non-linear graphs

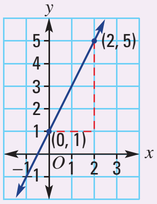
$y = x^2 - 2$

x	–2	–1	0	1	2
y	2	–1	–2	–1	2



$y = mx + b$

Gradient y-intercept



$m = \frac{4}{2} = 2$
$b = 1$
$y = 2x + 1$

Applications

- Distance increases by 20 km per hour.

t	0	1	2	3
d	0	20	40	60

$d = 20t$

- Volume decreases from 1000 L by 200 L per minute.

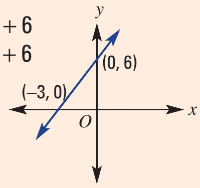
t	0	1	2	3	4	5
V	1000	800	600	400	200	0

$V = -200t + 1000$

x-intercept

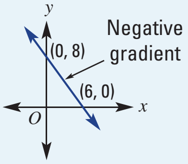
$y = 0$

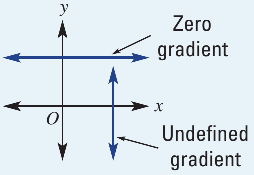
$y = 2x + 6$
$0 = 2x + 6$
$-6 = 2x$
$-3 = x$



Gradient

$\frac{\text{rise}}{\text{run}} = -\frac{8}{6}$
$= -\frac{4}{3}$





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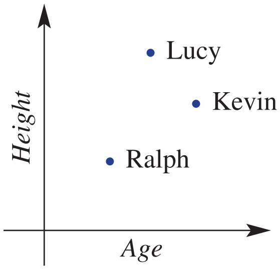
Multiple-choice questions

1 This graph shows the relationship between the height and age of three people. Who is the tallest person?



A Ralph
B Lucy
C Kevin
D Lucy and Ralph together
E Kevin and Lucy together

2 The name of the point (0, 0) on a number (Cartesian) plane is:

A y-intercept
B gradient
C origin
D axis
E x-intercept

3 Which point is not in line with the other points: $A(-2, 3)$, $B(-1, 2)$, $C(0, 0)$, $D(1, 0)$, $E(2, -1)$?

A A
B B
C C
D D
E E

4 Which of the points $A(1, 2)$, $B(2, -1)$ or $C(3, -4)$ lie on the line $y = -x + 1$?

A C
B A and C
C A
D B
E none

5 A rule gives this table of values.

x	−2	−1	0	1	2
y	3	2	1	0	−1

Which point lies on the y-axis?

A $(-2, 3)$
B $(-1, 2)$
C $(0, 1)$
D $(1, 0)$
E $(2, -1)$

6 Which line passes through the point $(3, 2)$?

A $y = 2x + 1$
B $y = 2x - 1$
C $y = 2x$
D $y = 2x - 4$
E $y = 2x + 4$

7 Which line does not pass through $(3, 2)$?

A $y = x + 1$
B $y = x - 1$
C $y = 5 - x$
D $y = 3x - 7$
E $y = 8 - 2x$

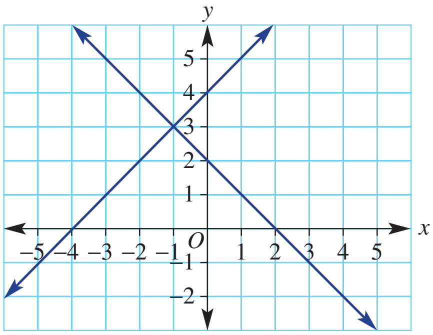
8 Which line passes through the y-axis at $(0, 3)$?

A $y = 3x$
B $y = x - 3$
C $y = 3x + 1$
D $y = -3x$
E $y = x + 3$

9 Consider the line $y = 3x + 2$. There is a point on the curve $(a, 0)$. The value of a is:

A 0
B 2
C 3
D $-\frac{2}{3}$
E $\frac{2}{3}$

10 Which point is the point of intersection of the two lines graphed here?



A $(-4, 0)$
B $(2, 0)$
C $(0, 4)$
D $(-1, 3)$
E $(0, 2)$

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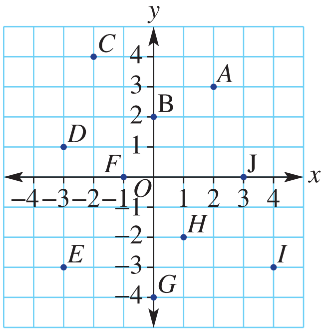
Short-answer questions

1 Write the coordinates of all the points *A*–*J* in the graph below.



2 For each rule create a table using x values from −3 to 3 and plot to draw a straight-line graph.

a $y = 2x$ **b** $y = 3x - 1$ **c** $y = 2x + 2$

d $y = -x + 1$ **e** $y = -2x + 3$ **f** $y = 3 - x$

3 Write the rule for these tables of values.

a

x	−2	−1	0	1	2
y	−3	−1	1	3	5

b

x	−2	−1	0	1	2
y	−4	−1	2	5	8

c

x	3	4	5	6	7
y	6	7	8	9	10

d

x	−3	−2	−1	0	1
y	4	3	2	1	0

e

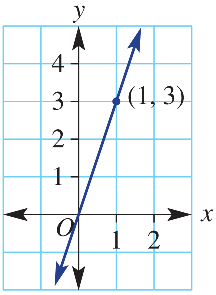
x	−1	0	1	2	3
y	3	−1	−5	−9	−13

f

x	−6	−5	−4	−3	−2
y	8	7	6	5	4

4 Find the rule from these graphs.

a



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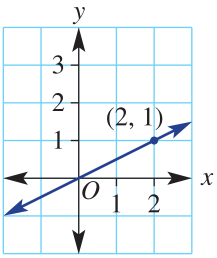

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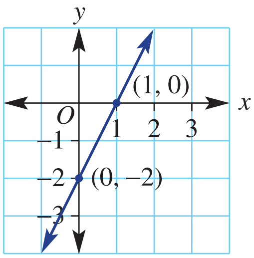

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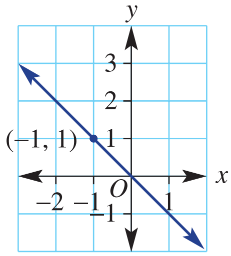

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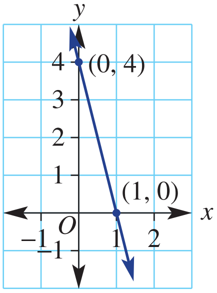
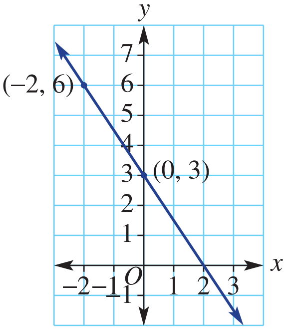


f



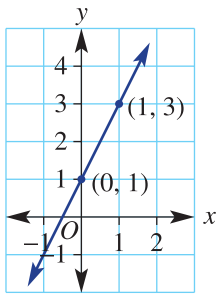
5 Find the equation of the line joining these pairs of points.

a (0, 0) and (3, −12) **b** (−4, 2) and (0, 0)

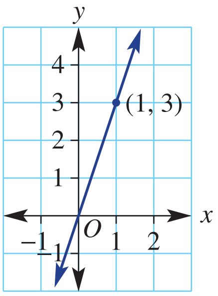
c (1, 1) and (4, 4) **d** (−5, 3) and (1, −9)

6 Write the rule for these graphs.

a



b



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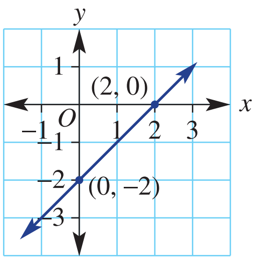

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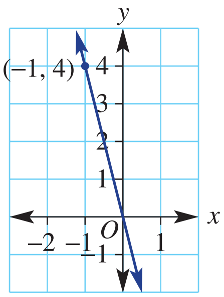

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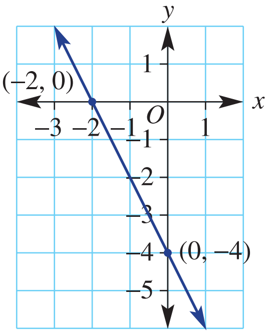

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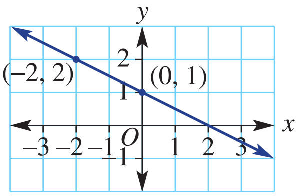


7 **a** Consider this table:

x	0	1	2	3	4
y	5	8	11	14	17

Complete this sentence:

To find a value of *y*, ____________________ *x* by _____________ and then _______________.

b Repeat part **a** for these tables.

i

x	0	1	2	3
y	2	8	14	20

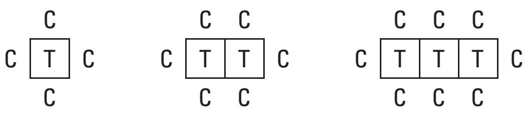
ii

x	0	1	2	3
y	6	8	10	12

8 A café arranges its tables like this:

T = table

C = chairs



a Complete the table.

T	1	2	3	4	5
C					

b Complete the sentence:

To find the number of chairs, multiply ____.

c Complete the rule: $C =$

d How many chairs would be needed for ten tables?

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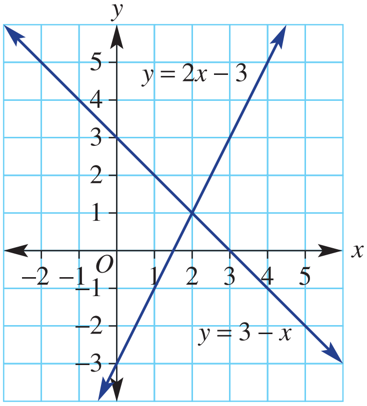
9 **a** Use the appropriate line on this graph to find the solution to the following linear equations.

i $3 - x = 4$
ii $2x - 3 = -1$
iii $3 - x = 2$
iv $2x - 3 = 4$
v $3 - x = -1.5$

b Write the coordinates of the point of intersection of these two lines.

c Show whether $(-1, 4)$ is a solution or not a solution to $y = 3 - x$ and $y = 2x - 3$.

d Show that the point of intersection is a solution to both equations $y = 3 - x$ and $y = 2x - 3$.



10 Using a table with x values between -2 and 2, draw a smooth curve for the non-linear graphs of these rules.

a $y = x^2 - 2$
b $y = x^3$
c $y = \frac{2}{x}$

Extended-response questions

1 A seed sprouts and the plant grows 3 millimetres per day in height for 6 days.

a Construct a table of values, using t for time in days and h for height in millimetres.

b Draw a graph using the points from your table. Use t on the horizontal axis.

c Find a rule linking h with t.

d Use your rule to find the height of the plant after 3.5 days.

e If the linear pattern continued, what would be the height of the plant after 10 days?

f How long will it be before the plant grows to 15 millimetres in height?

2 A speed boat at sea is initially 12 km from a distant buoy. The boat travels towards the buoy at a rate of 2 km per minute. The distance between the boat and the buoy will therefore decrease over time.

a Construct a table showing t for time, in minutes, and d for distance to the buoy, in kilometres.

b Draw a graph using the points from your table. Use t on the horizontal axis.

c How long does it take the speed boat to reach the buoy?

d What is the gradient of the line drawn in part **b**?

e Find a rule linking d with t.

f Use your rule to find the distance from the buoy at the 2.5 minute mark.

g How long does it take for the distance to reduce to 3.5 km?

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Online resources

- Auto-marked chapter
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

8

Transformations and congruence

What you will learn

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NSW syllabus

STRANDS: NUMBER AND ALGEBRA
MEASUREMENT AND GEOMETRY
SUBSTRANDS: LINEAR RELATIONSHIPS
PROPERTIES OF GEOMETRICAL FIGURES

Outcomes

A student creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane.
(MA4–11NA)

A student classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles.
(MA4–17MG)

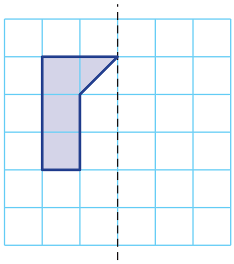
Symmetrical architecture

Geometry is at the foundation of design and architecture. Many public and private residences have been designed and built with a strong sense of geometry.

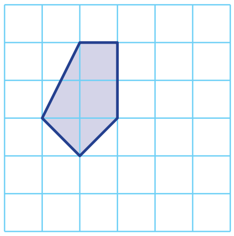
The Pantheon is about 2000 years old and is one of the oldest buildings in Rome. Its rectangular portico is supported by eight cylindrical columns. Along with the triangular roof, these provide perfect line symmetry at the entrance of the building. Inside the main dome the symmetry changes. At the centre of the dome roof is a large hole that lets in the sunlight. The height of the dome is the same as its width, so a sphere of the same diameter would fit perfectly under the dome.

1 Complete the simple transformations of the given shapes as instructed.

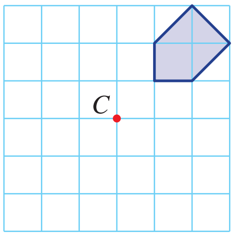
a Reflect this shape over the mirror line.



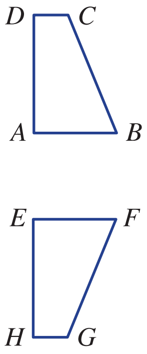
b Shift this shape 2 units to the right and 1 unit down.



c Rotate this shape 180° about point *C*.



2 Consider this pair of quadrilaterals.



a Name the vertex, *E*, *F*, *G* or *H*, that matches (corresponds to) vertex:

i *B* **ii** *D*

b Name the angle, ∠*A*, ∠*B*, ∠*C* or ∠*D*, that matches (corresponds to) angle:

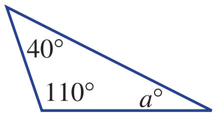
i ∠*E* **ii** ∠*G*

c Name the side, *EF*, *FG*, *GH* or *HE*, that matches (corresponds to) side:

i *CD* **ii** *BC*

3 Find the unknown angle in these triangles.

a



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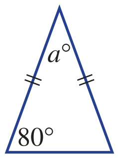

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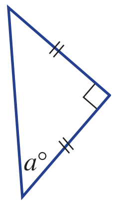

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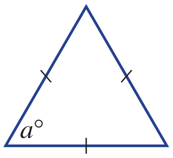

4 Which of the special quadrilaterals definitely fit the description?

A square
B rectangle
C rhombus
D parallelogram
E kite
F trapezium

a Opposite sides are of equal length.

b It has at least one pair of equal opposite angles.

c Diagonals are of equal length.

d Diagonals intersect at right angles.

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8A Reflection

When an object is shifted from one position to another, rotated about a point, reflected over a line or enlarged by a scale factor, we say the object has been **transformed**. The names of these types of transformations are **reflection**, **translation**, **rotation** and **dilation**.

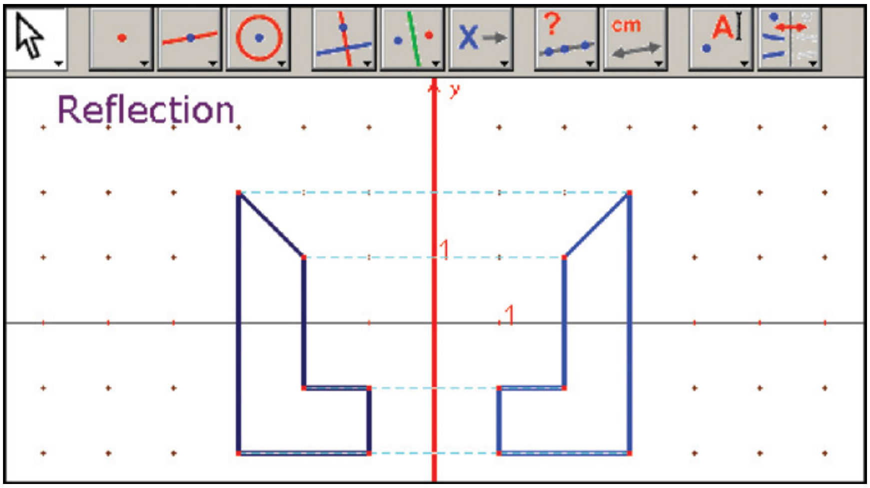
The first three of these transformations is called **isometric transformations** because the object's geometric properties are unchanged and the transformed object will be congruent to the original object. The word 'isometric' comes from the Greek words *isos* meaning 'equal' and *metron* meaning 'measure'. Dilation (or enlargement) results in a change in the size of an object to produce a 'similar' figure and this will be studied later in this chapter. The first listed transformation, reflection, can be thought of as the creation of an image over a mirror line.

Reflection creates an image reversed as in a mirror or in water.

Let's start: Visualising the image

This activity can be done individually by hand on a page, in a group using a whiteboard or using dynamic geometry projected onto a whiteboard.

- Draw any shape with straight sides.
- Draw a vertical or horizontal mirror line outside the shape.
- Try to draw the reflected image of the shape in the mirror line.
- If dynamic geometry is used, reveal the precise image (the answer) using the Reflection tool to check your result.
- For a further challenge, redraw or drag the mirror line so it is not horizontal or vertical. Then try to draw the image.



Dynamic geometry software provides a reflection tool.

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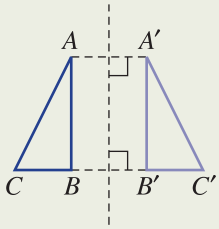
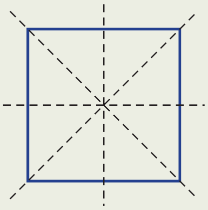
Key ideas

- **Reflection** is an **isometric transformation** in which the size of the object is unchanged.
- The **image** of a point A is denoted A'.
- Each point is reflected at right angles to the **mirror line**.
- The distance from a point A to the mirror line is equal to the distance from the image point A' to the mirror line.
- When a line of symmetry is used as a reflection line, the image looks the same as the original figure.
- When a point is reflected across the x-axis:
  - the x-coordinate remains unchanged
  - the y-coordinate undergoes a sign change.

  For example, (5, 3) becomes (5, −3); that is, the y values are equal in magnitude but opposite in sign.
- When the point is reflected across the y-axis:
  - the x-coordinate undergoes a sign change
  - the y-coordinate remains unchanged.

  For example, (5, 3) becomes (−5, 3).

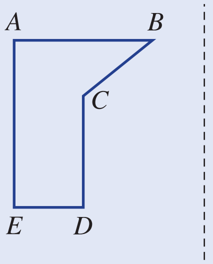




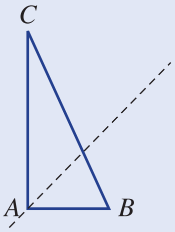
Example 1 Drawing reflected images

Copy the diagram and draw the reflected image over the given mirror line.

a

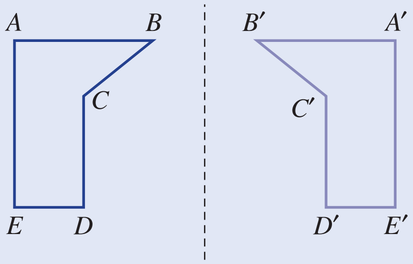


b



SOLUTION

a



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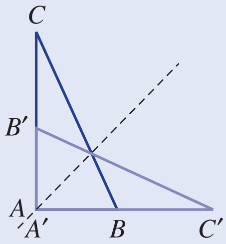

EXPLANATION

Reflect each vertex point at right angles to the mirror line. Join the image points to form the final image.

Reflect points A, B and C at right angles to the mirror line to form A', B' and C'. Note that A' is in the same position as A, as it is on the mirror line. Join the image points to form the image triangle.

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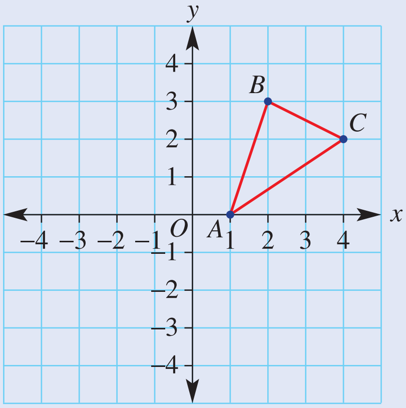
Example 2 Using coordinates

State the coordinates of the vertices A', B' and C' after this triangle is reflected in the given axes.

a x-axis **b** y-axis

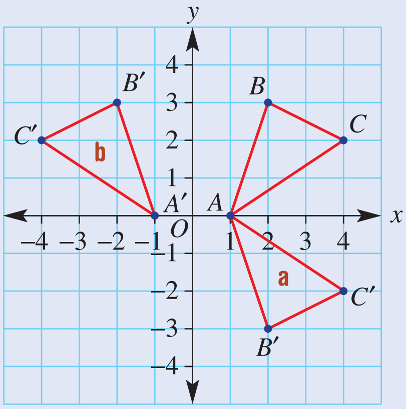


SOLUTION

a $A' = (1, 0)$
$B' = (2, -3)$
$C' = (4, -2)$

b $A' = (-1, 0)$
$B' = (-2, 3)$
$C' = (-4, 2)$

EXPLANATION

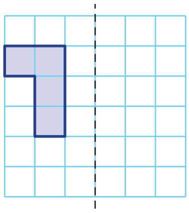


Exercise 8A

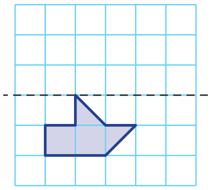
UNDERSTANDING AND FLUENCY	1–5	2, 3, 4(½), 5, 6	3–4(½), 5, 6

1 Use the grid to precisely reflect each shape in the given mirror line.

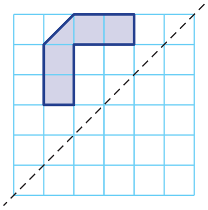
a



b



c



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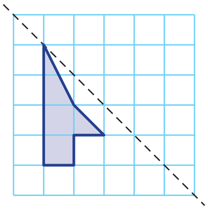

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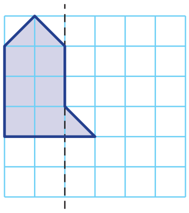

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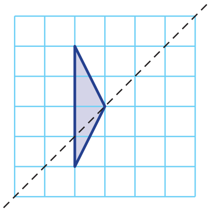

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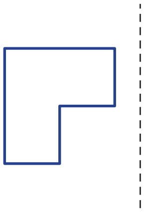
Example 1a

2 Copy the diagram and draw the reflected image over the given mirror line.

a



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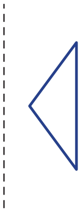

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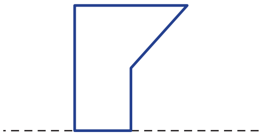

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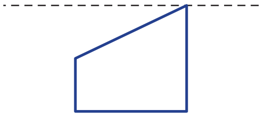

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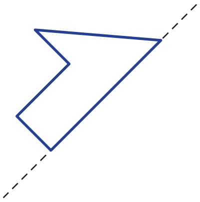

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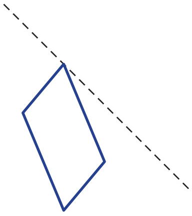
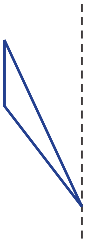


Example 1b

3 Copy the diagram and draw the reflected image over the given mirror line.

a



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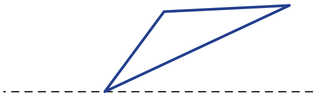

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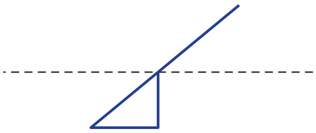

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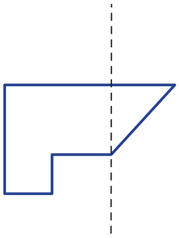

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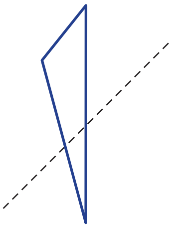

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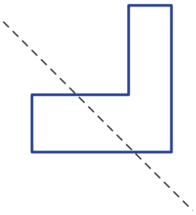
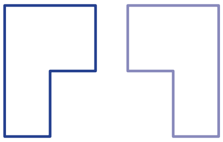


4 Copy the diagram and accurately locate and draw the mirror line. Alternatively, pencil in the line on this page.

a



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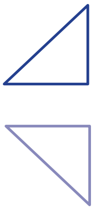

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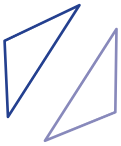

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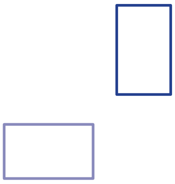

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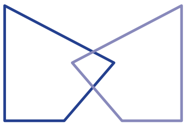

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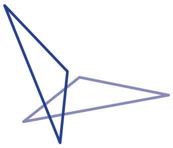

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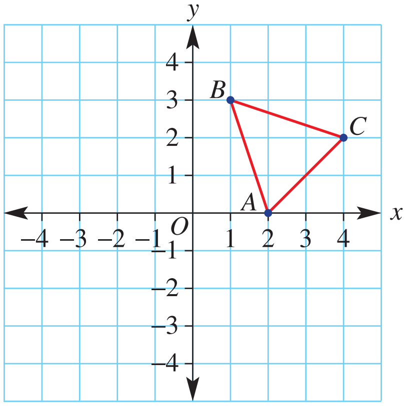
Example 2

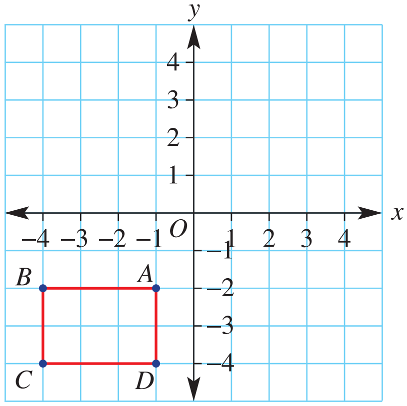
5 State the coordinates of the vertices A', B' and C' after the triangle is reflected in the given axes.

a x-axis **b** y-axis



6 State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

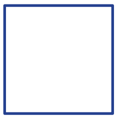
a x-axis **b** y-axis



PROBLEM-SOLVING AND REASONING	7, 9, 11	7, 8(½), 9–11	7, 8(½), 11–14

7 How many lines of symmetry do these shapes have?

a

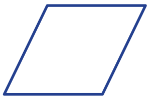


square

b

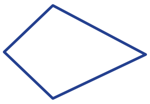
rectangle

c



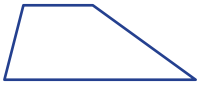
rhombus

d



kite

e

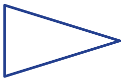


trapezium

f

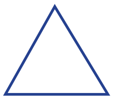
parallelogram

g



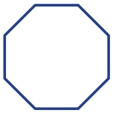
isosceles triangle

h



equilateral triangle

i



regular octagon

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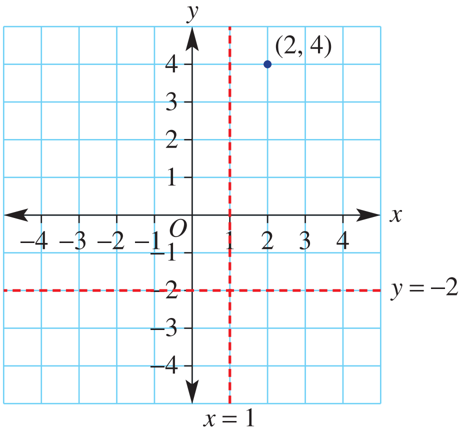
8 A point (2, 4) is reflected in the given horizontal or vertical line. State the coordinates of the image point. As an example, the graph of the mirror lines $x = 1$ and $y = -2$ are shown here.



The mirror lines are:

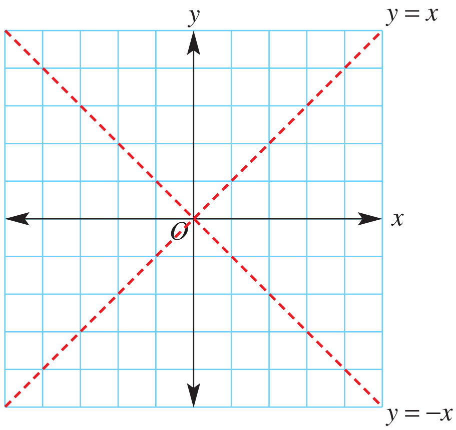
a $x = 1$
b $x = 3$
c $x = 0$
d $x = -1$
e $x = -4$
f $x = -20$
g $y = 3$
h $y = -2$
i $y = 0$
j $y = 1$
k $y = -5$
l $y = -37$

9 A shape with area 10 m^2 is reflected in a line. What is the area of the image shape? Give a reason for your answer.

10 How many lines of symmetry does a regular polygon with n sides have? Write an expression.

11 A point is reflected in the x-axis, then in the y-axis and finally in the x-axis again. What single reflection could replace all three reflections?

12 Two important lines of reflection on the coordinate plane are the line $y = x$ and the line $y = -x$, as shown.



a Draw the coordinate plane shown here. Draw a triangle with vertices $A(-1, 1)$, $B(-1, 3)$ and $C(0, 3)$. Then complete these reflections.
  i Reflect triangle ABC in the y-axis.
  ii Reflect triangle ABC in the x-axis.
  iii Reflect triangle ABC in the line $y = x$.
  iv Reflect triangle ABC in the line $y = -x$.

b Draw a coordinate plane and a rectangle with vertices $A(-2, 0)$, $B(-1, 0)$, $C(-1, -3)$ and $D(-2, -3)$. Then complete these reflections.
  i Reflect rectangle $ABCD$ in the y-axis.
  ii Reflect rectangle $ABCD$ in the x-axis.
  iii Reflect rectangle $ABCD$ in the line $y = x$.
  iv Reflect rectangle $ABCD$ in the line $y = -x$.

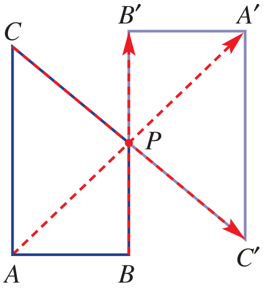
13 Points are reflected in a mirror line but do not change position. Describe the position of these points in relation to the mirror line.

14 Research to find pictures, diagrams, cultural designs and artwork that contain line symmetry. Make a poster or display them electronically.

ENRICHMENT – – 15

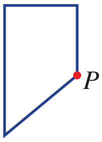
Reflection through a point

15 Rather than completing a reflection in a line, it is possible to reflect an object through a point. An example of a reflection through point P is shown here. A goes to A', B goes to B' and C goes to C' through P.

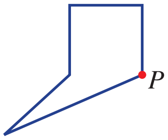


a Draw and reflect these shapes through the point P.

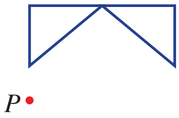
i



ii

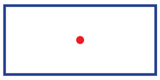


iii

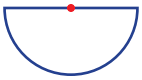


b Like line symmetry, shapes can have point symmetry if they can be reflected onto themselves through a point. Decide if these shapes have any point symmetry.

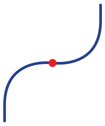
i



ii



iii



c How many special quadrilaterals can you name that have point symmetry?

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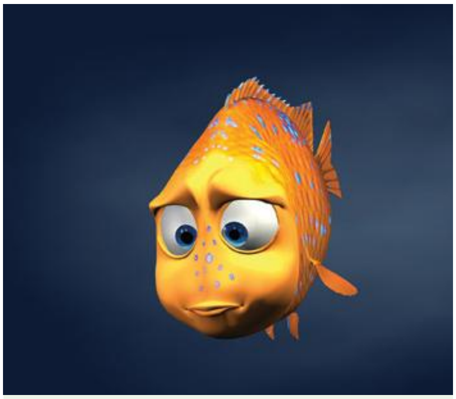
8B Translation

Interactive
Widgets
HOTsheets
Walkthrough

Translation is a shift of every point on an object in a given direction and by the same distance. The direction and distance is best described by the use of a translation vector. This vector describes the overall direction using a horizontal component (for shifts left and right) and a vertical component (for shifts up and down). Negative numbers are used for translations to the left and down.

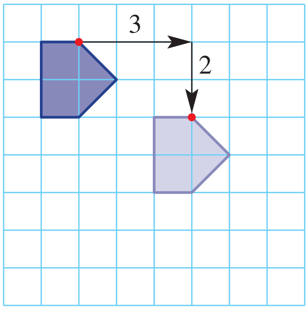
Designers of animated movies translate images in many of their scenes. Computer software is used and translation vectors help to define the specific movement of the objects and characters on the screen.



Animated characters move through a series of translations.

Let's start: Which is further?

Consider this shape on the grid. The shape is translated by the vector $\begin{bmatrix}3\\-2\end{bmatrix}$ (meaning 3 to the right and 2 down).



Now consider the shape being translated by these different vectors.

a $\begin{bmatrix}3\\-3\end{bmatrix}$ **b** $\begin{bmatrix}-1\\-4\end{bmatrix}$ **c** $\begin{bmatrix}5\\0\end{bmatrix}$ **d** $\begin{bmatrix}-2\\4\end{bmatrix}$

- By drawing and looking at the image from each translation, which vector do you think takes the shape furthest from its original position?
- Is there a way that you can calculate the exact distance? How?

Key ideas

- **Translation** is an isometric transformation that involves a shift by a given distance in a given direction.
- A **vector** $\begin{bmatrix}a\\b\end{bmatrix}$ can be used to describe the distance and direction of a translation.

right 2

vector $\begin{bmatrix}2\\-3\end{bmatrix}$

down 3

left 1

vector $\begin{bmatrix}-1\\4\end{bmatrix}$ up 4

  - If a is positive, you shift to the right.
  - If a is negative, you shift to the left.
  - If b is positive, you shift up.
  - If b is negative, you shift down.
- The image of a point A is denoted A'.

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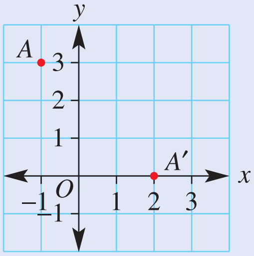
Example 3 Finding the translation vector

State the translation vector that moves the point $A(-1, 3)$ to $A'(2, 0)$.



SOLUTION

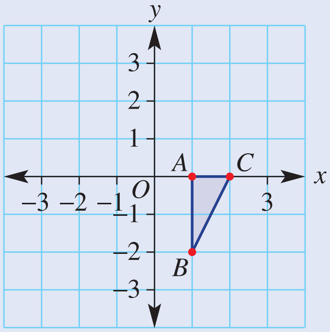
$$\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

EXPLANATION

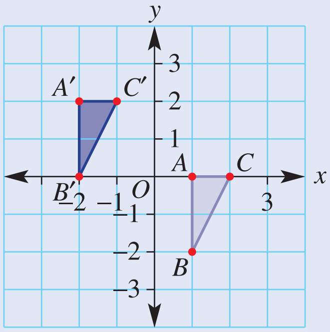
To shift A to A', move 3 units to the right and 3 units down.

Example 4 Drawing images using translation

Draw the image of the triangle ABC after a translation by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.



SOLUTION



EXPLANATION

First translate each vertex, A, B and C, 3 spaces to the left, and then 2 spaces up.

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Exercise 8B

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	3, 4, 5–7(½)	5–7(½)

1 Use the words left, right, up or down, to complete these sentences.

a The vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ means to move 2 units to the ______ and 4 units ______.

b The vector $\begin{bmatrix} -5 \\ 6 \end{bmatrix}$ means to move 5 units to the ______ and 6 units ______.

c The vector $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ means to move 3 units to the ______ and 1 unit ______.

d The vector $\begin{bmatrix} -10 \\ -12 \end{bmatrix}$ means to move 10 units to the ______ and 12 units ______.

2 Write the vector that describes these transformations.

a 5 units to the right and 2 units down

b 2 units to the left and 6 units down

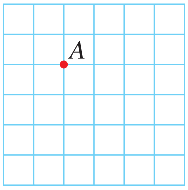
c 7 units to the left and 4 units up

d 9 units to the right and 17 units up

3 Decide if these vectors describe vertical or horizontal translation.

a $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$ **c** $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$ **d** $\begin{bmatrix} -6 \\ 0 \end{bmatrix}$

4 Copy and show on separate grids the position of A' after applying each of the following vector translations.



a $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ **b** $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ **c** $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ **d** $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$

Example 3

5 Write the vector that takes each point to its image. Use a grid to help you.

a $A(2, 3)$ to $A'(3, 2)$

b $B(1, 4)$ to $B'(4, 3)$

c $C(-2, 4)$ to $C'(0, 2)$

d $D(-3, 1)$ to $D'(-1, -3)$

e $E(-2, -4)$ to $E'(1, 3)$

f $F(1, -3)$ to $F'(-2, 2)$

g $G(0, 3)$ to $G'(2, 0)$

h $H(-3, 5)$ to $H'(0, 0)$

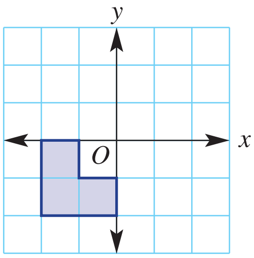
i $I(5, 2)$ to $I'(-15, 10)$

j $J(-3, -4)$ to $J'(-12, -29)$

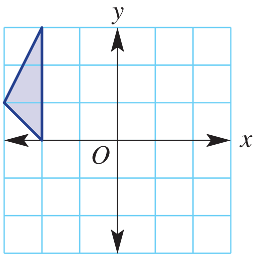
Example 4

6 Copy the diagrams and draw the image of the shapes translated by the given vectors.

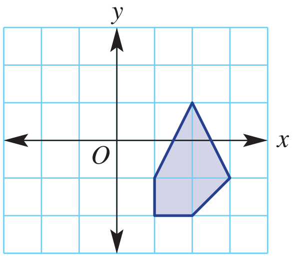
a $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$



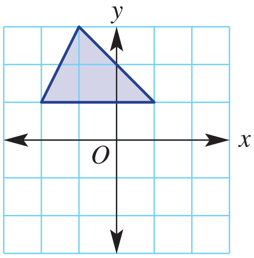
b $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$



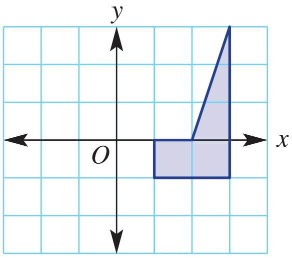
c $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$



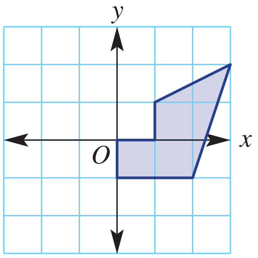
d $\begin{bmatrix} 0 \\ -3 \end{bmatrix}$



e $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$



f $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$



7 Write the coordinates of the image of the point $A(13, -1)$ after a translation by the given vectors.

a $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
b $\begin{bmatrix} 8 \\ 0 \end{bmatrix}$
c $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$
d $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$
e $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$
f $\begin{bmatrix} -10 \\ -5 \end{bmatrix}$
g $\begin{bmatrix} -2 \\ -8 \end{bmatrix}$
h $\begin{bmatrix} 6 \\ -9 \end{bmatrix}$
i $\begin{bmatrix} 12 \\ -3 \end{bmatrix}$
j $\begin{bmatrix} -26 \\ 14 \end{bmatrix}$
k $\begin{bmatrix} -4 \\ 18 \end{bmatrix}$
l $\begin{bmatrix} -21 \\ -38 \end{bmatrix}$

PROBLEM-SOLVING AND REASONING	8, 9, 11	8, 9, 11	9–12

8 Which vector from each set takes an object the greatest distance from its original position? You may need to draw diagrams to help, but you should not need to calculate distances.

a $\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \end{bmatrix}$

b $\begin{bmatrix} -1 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

9 A car makes its way around a city street grid. A vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ represents travelling 200 m east and 300 m north.

a Find how far the car travels if it follows these vectors:

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -4 \end{bmatrix}$

b What vector takes the car back to the origin (0, 0), assuming it started at the origin?

10 A point undergoes the following multiple translations with these given vectors. State the value of x and y of the vector that would take the image back to its original position.

a $\begin{bmatrix}3\\4\end{bmatrix}, \begin{bmatrix}-1\\-2\end{bmatrix}, \begin{bmatrix}x\\y\end{bmatrix}$

b $\begin{bmatrix}2\\5\end{bmatrix}, \begin{bmatrix}-7\\2\end{bmatrix}, \begin{bmatrix}-1\\-3\end{bmatrix}, \begin{bmatrix}x\\y\end{bmatrix}$

c $\begin{bmatrix}0\\4\end{bmatrix}, \begin{bmatrix}7\\0\end{bmatrix}, \begin{bmatrix}-4\\-6\end{bmatrix}, \begin{bmatrix}x\\y\end{bmatrix}$

d $\begin{bmatrix}-4\\20\end{bmatrix}, \begin{bmatrix}12\\0\end{bmatrix}, \begin{bmatrix}-36\\40\end{bmatrix}, \begin{bmatrix}x\\y\end{bmatrix}$

11 A reverse vector takes a point in the reverse direction by the same distance. Write the reverse vectors of these vectors.

a $\begin{bmatrix}3\\-2\end{bmatrix}$

b $\begin{bmatrix}-5\\0\end{bmatrix}$

c $\begin{bmatrix}x\\y\end{bmatrix}$

d $\begin{bmatrix}-x\\-y\end{bmatrix}$

12 These translation vectors are performed on a shape in succession (one after the other). What is a single vector that would complete all transformations for each part in one go?

a $\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}-3\\-4\end{bmatrix}, \begin{bmatrix}0\\3\end{bmatrix}$

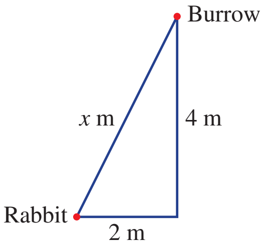
b $\begin{bmatrix}6\\4\end{bmatrix}, \begin{bmatrix}6\\-2\end{bmatrix}, \begin{bmatrix}-11\\0\end{bmatrix}$

c $\begin{bmatrix}a\\b\end{bmatrix}, \begin{bmatrix}c\\-a\end{bmatrix}, \begin{bmatrix}-a\\a-c\end{bmatrix}$

ENRICHMENT – – 13

How many options for the rabbit?

13 A fox spots a rabbit on open ground. The rabbit has 1 second to find a burrow before being caught by the fox. The rabbit can run a maximum of 5 metres in 1 second.



a Use Pythagoras' theorem to check that the distance x m in this diagram is less than 5 m.

b The rabbit runs a distance and direction described by the translation vector $\begin{bmatrix}-4\\3\end{bmatrix}$. Will the rabbit make it safely to the burrow?

c The rabbit's initial position is (0, 0) and there are burrows at every point that has integers as its coordinates; for example, (2, 3) and (−4, 1). How many burrows can it choose from to avoid the fox before its 1 second is up? Draw a diagram to help illustrate your working.

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8C Rotation

Interactive
Widgets
HOTsheets
Walkthrough

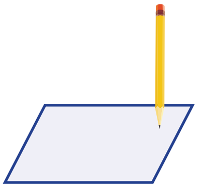
When the arm of a crane moves left, right, up or down, it undergoes a rotation about a fixed point. This type of movement is a transformation called a **rotation**. The pivot point on a crane would be called the **centre of rotation** and all other points on the crane's arm move around this point by the same angle in a circular arc.

Every point on this crane's arm rotates around the pivot point when it moves.

Let's start: Parallelogram centre of rotation

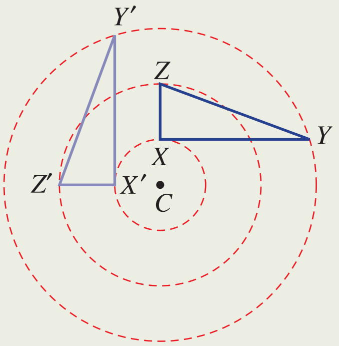
This activity will need a pencil, paper, ruler and scissors.

- Accurately draw a large parallelogram on a separate piece of paper and cut it out.
- Place the tip of a pencil at any point on the parallelogram and spin the shape around the pencil.
- At what position do you put the pencil tip to produce the largest circular arc?
- At what position do you put the pencil tip to produce the smallest circular arc?
- Can you rotate the shape by an angle of less than 360° so that it exactly covers the area of the shape in its original position? Where would you put the pencil to illustrate this?

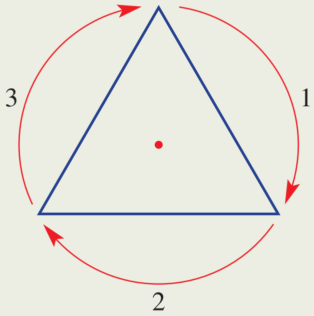


Key ideas

- **Rotation** is an isometric transformation about a centre point and by a given angle.
- An object can be rotated clockwise ↻ or anticlockwise ↺.
- Each point is rotated on a circular arc about the **centre of rotation**, *C*.
  - For example, this diagram shows a 90° anticlockwise rotation about the point *C*.



- A shape has **rotational symmetry** if it can be rotated about a centre point to produce an exact copy covering the entire area of the original shape.
  - The number of times the shape can make an exact copy in a 360° rotation is called the **order of rotational symmetry**. If the order of rotation is 1, then it is said that the shape has no rotational symmetry.
  - For example, this equilateral triangle has rotational symmetry of order 3.
  - All squares have order 4 but rectangles have order 2.



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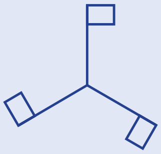
Example 5 Finding the order of rotational symmetry

Find the order of rotational symmetry for these shapes.

a

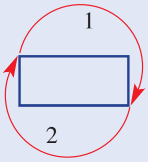
b



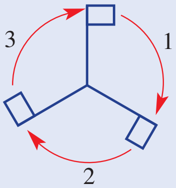
SOLUTION

a Order of rotational symmetry = 2

EXPLANATION



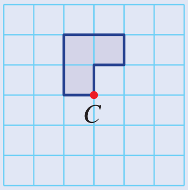
b Order of rotational symmetry = 3



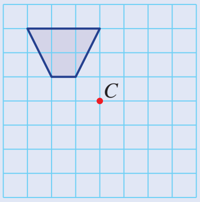
Example 6 Drawing a rotated image

Rotate these shapes about the point C by the given angle and direction.

a clockwise by 90°

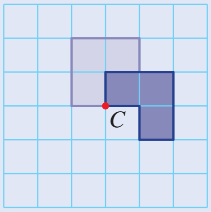


b 180°



SOLUTION

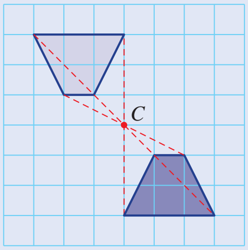
a



EXPLANATION

Take each vertex point and rotate about C by 90°, but it may be easier to visualise a rotation of some of the sides first. Horizontal sides will rotate to vertical sides in the image and vertical sides will rotate to horizontal sides in the image.

b



You can draw a dashed line from each vertex through C to a point at an equal distance on the opposite side.

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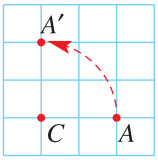
Exercise 8C

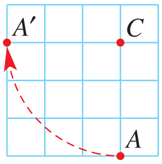
UNDERSTANDING AND FLUENCY	1–6	2, 3, 4(½), 5, 6(½), 7	3, 4(½), 5, 6(½), 7

1 Point A has been rotated to its image point A'. For each part state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

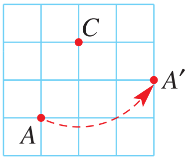
a



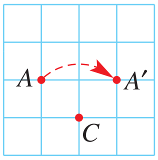
b



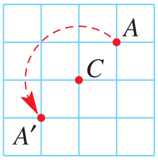
c



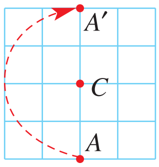
d



e



f



2 Complete these sentences.

a A rotation clockwise by 90° is the same as a rotation anticlockwise by ______.

b A rotation anticlockwise by 180° is the same as a rotation clockwise by ______.

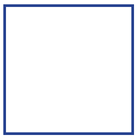
c A rotation anticlockwise by ______ is the same as a rotation clockwise by 58°.

d A rotation clockwise by ______ is the same as a rotation anticlockwise by 296°.

Example 5

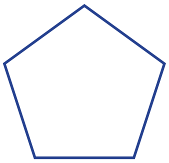
3 Find the order of rotational symmetry for these shapes.

a

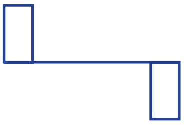


b

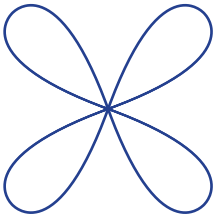
c



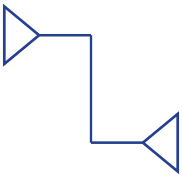
d



e



f



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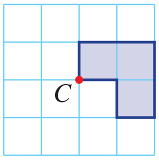
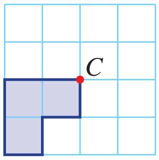
Example 6a

4 Rotate these shapes about the point C by the given angle and direction.

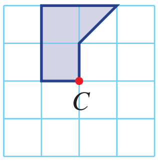
a clockwise by 90°



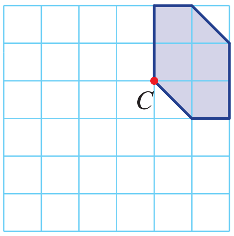
b anticlockwise by 90°



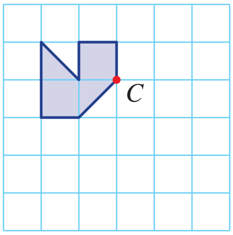
c anticlockwise by 180°



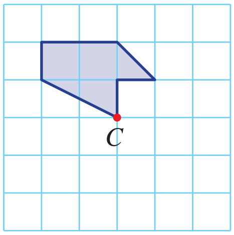
d clockwise by 90°



e anticlockwise by 180°



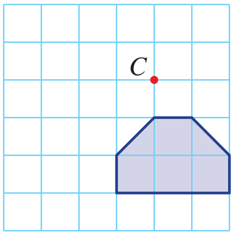
f clockwise by 180°



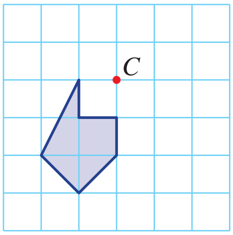
Example 6b

5 Rotate these shapes about the point C by the given angle and direction.

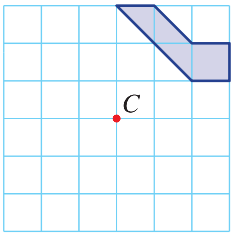
a clockwise by 90°



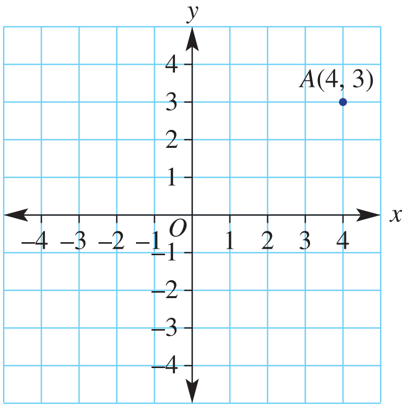
b anticlockwise by 90°



c anticlockwise by 180°



6 The point $A(4, 3)$ is rotated about the origin $C(0, 0)$ by the given angle and direction. Give the coordinates of A'.



a 180° clockwise

b 180° anticlockwise

c 90° clockwise

d 90° anticlockwise

e 270° clockwise

f 270° anticlockwise

g 360° clockwise

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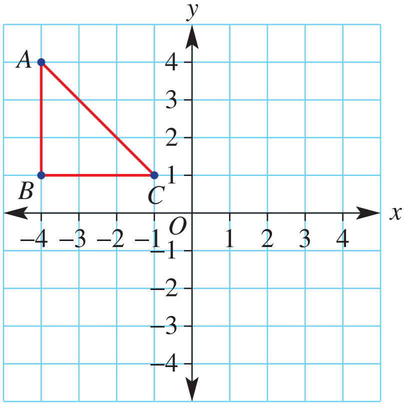
7 The triangle shown here is rotated about $(0, 0)$ by the given angle and direction. Give the coordinates of the image points A', B' and C'.

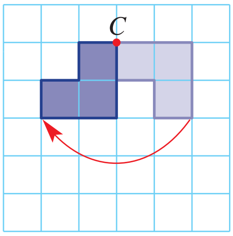


a 180° clockwise
b 90° clockwise
c 90° anticlockwise

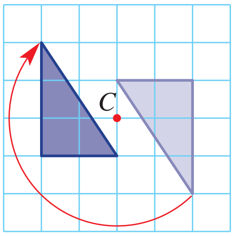
PROBLEM-SOLVING AND REASONING	8, 9, 11	8, 9, 11, 12	9, 10, 12, 13

8 By how many degrees have these shapes been rotated?

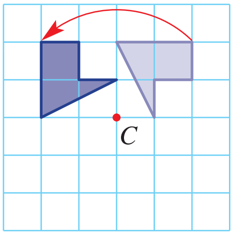
a



b



c



9 Which capital letters of the alphabet (A, B, C, …, Z) have rotational symmetry of order 2 or more?

10 Draw an example of a shape that has these properties.
a rotational symmetry of order 2 with no line symmetry
b rotational symmetry of order 6 with 6 lines of symmetry
c rotational symmetry of order 4 with no line symmetry

11 What value of x makes these sentences true?
a Rotating x degrees clockwise has the same effect as rotating x degrees anticlockwise.
b Rotating x degrees clockwise has the same effect as rotating $3x$ degrees anticlockwise.

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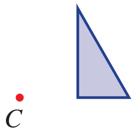
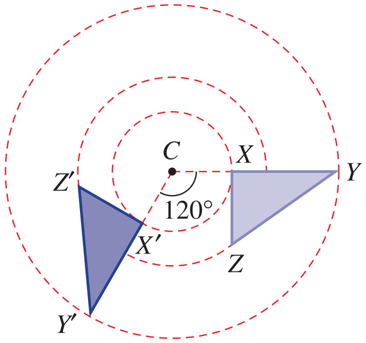
12 When working without a grid or without 90° angles, a protractor and a pair of compasses are needed to accurately draw images under rotation. This example shows a rotation of 120° about C.



a Copy this triangle with centre of rotation C onto a sheet of paper.

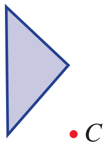


b Construct three circles with centre C and passing through the vertices of the triangle.

c Use a protractor to draw an image after these rotations.

- **i** 120° anticlockwise
- **ii** 100° clockwise

13 Make a copy of this diagram and rotate the shape anticlockwise by 135° about point C. You will need to use a pair of compasses and a protractor, as shown in Question **12**.

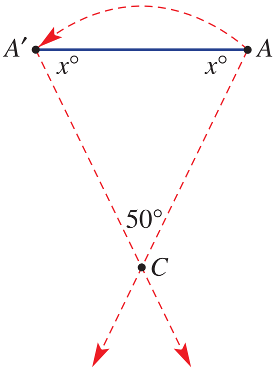


ENRICHMENT – – 14

Finding the centre of rotation

14 Finding the centre of rotation when the angle is known involves the calculation of an angle inside an isosceles triangle. For the rotation shown, the angle of rotation is 50°. The steps are given as:

- **i** Calculate $\angle CAA'$ and $\angle CA'A$.
  ($2x + 50 = 180$, so $x = 65$.)
- **ii** Draw the angles $\angle AA'C$ and $\angle A'AC$ at 65°, using a protractor.
- **iii** Locate the centre of rotation C at the point of intersection of AC and $A'C$.



a On a sheet of paper draw two points, A and A', about 4 cm apart. Follow the steps above to locate the centre of rotation if the angle of rotation is 40°.

b Repeat part **a** using an angle of rotation of 100°.

c When a shape is rotated and the angle is unknown, there is a special method for accurately pinpointing the centre of rotation. Research this method and illustrate the procedure using an example.

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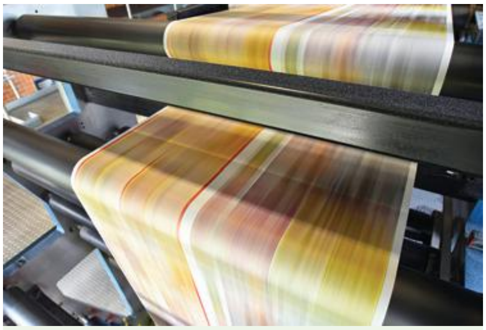
8D Congruent figures

Interactive
Widgets
HOTsheets
Walkthrough

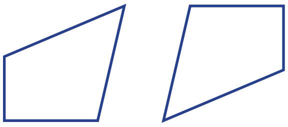
If two or more objects are identical in size and shape, we say they are congruent. In the diagram, the green shape is exactly the same shape and size as the other three pieces.



Each brochure being printed is identical, so the images on them are congruent.

Let's start: Are they congruent?

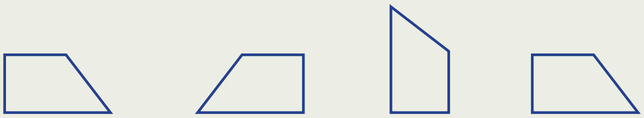
Here are two shapes. To be congruent they need to be exactly the same shape and size.



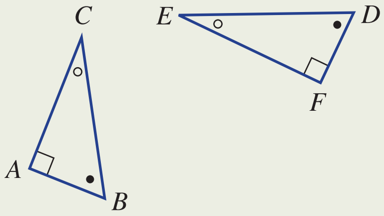
- Do you think they look congruent? Give reasons.
- What measurements could be taken to help establish whether or not they are congruent?
- Can you just measure lengths or do you need to measure angles as well? Discuss.

Key ideas

- A **figure** is a shape, diagram or illustration.
- **Congruent figures** have the same size and shape. They also have the same area. They are identical in every way.



- The image of a figure that is reflected, translated or rotated is congruent to the original figure.
- Corresponding (matching) parts of a figure have the same geometric properties.
  - Vertex C corresponds to vertex E.
  - Side AB corresponds to side FD.
  - $\angle B$ corresponds to $\angle D$.



- A congruence statement can be written using the symbol $\equiv$. For example, $\triangle ABC \equiv \triangle FDE$.
  - The symbol $\cong$ can also be used for congruence.
  - In a congruence statement vertices are named in matching order. For example, $\triangle ABC \equiv \triangle FDE$ *not* $\triangle ABC \equiv \triangle FDE$ because B matches D.
- Two circles are congruent if they have equal radii.

Example 7 Naming corresponding pairs

These two quadrilaterals are congruent. Name the objects in quadrilateral *EFGH* that correspond to these objects in quadrilateral *ABCD*.

a vertex *C* **b** side *AB* **c** $\angle C$

SOLUTION

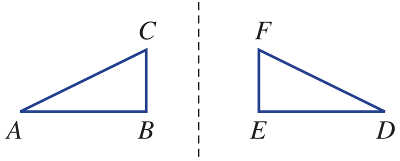
EXPLANATION

a vertex *G*	*C* sits opposite *A* and $\angle A$ is the smallest angle. *G* sits opposite *E* and $\angle E$ is also the smallest angle.
b side *EH*	Sides *AB* and *EH* are both the longest sides of their respective shapes. *A* corresponds to *E* and *B* corresponds to *H*.
c $\angle G$	$\angle C$ and $\angle G$ are both the largest angle in their corresponding quadrilateral.

Exercise 8D

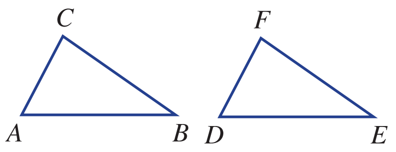
UNDERSTANDING AND FLUENCY	1–5	3, 4, 6	5, 6

1 In this diagram $\triangle ABC$ has been reflected to give the image triangle $\triangle DEF$.



- **a** Is $\triangle DEF$ congruent to $\triangle ABC$; i.e. is $\triangle DEF \equiv \triangle ABC$?
- **b** Name the vertex on $\triangle DEF$ that corresponds to:
  - **i** *A* **ii** *B* **iii** *C*
- **c** Name the side on $\triangle DEF$ that corresponds to:
  - **i** *AB* **ii** *BC* **iii** *AC*
- **d** Name the angle in $\triangle DEF$ that corresponds to:
  - **i** $\angle B$ **ii** $\angle C$ **iii** $\angle A$

2 In this diagram $\triangle ABC$ has been translated to give the image triangle $\triangle DEF$.



- **a** Is $\triangle DEF$ congruent to $\triangle ABC$; i.e. is $\triangle DEF \equiv \triangle ABC$?
- **b** Name the vertex on $\triangle DEF$ that corresponds to:
  - **i** *A* **ii** *B* **iii** *C*
- **c** Name the side on $\triangle DEF$ that corresponds to:
  - **i** *AB* **ii** *BC* **iii** *AC*
- **d** Name the angle in $\triangle DEF$ that corresponds to:
  - **i** $\angle B$ **ii** $\angle C$ **iii** $\angle A$

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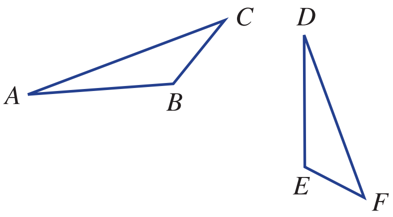
3 In this diagram $\triangle ABC$ has been rotated to give the image triangle $\triangle DEF$.



a Is $\triangle DEF$ congruent to $\triangle ABC$; i.e. is $\triangle DEF \equiv \triangle ABC$?

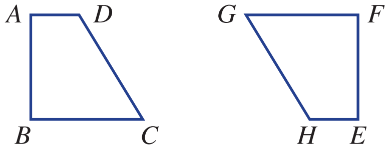
b Name the vertex on $\triangle DEF$ that corresponds to:
  i A **ii** B **iii** C

c Name the side on $\triangle DEF$ that corresponds to:
  i AB **ii** BC **iii** AC

d Name the angle in $\triangle DEF$ that corresponds to:
  i $\angle B$ **ii** $\angle C$ **iii** $\angle A$

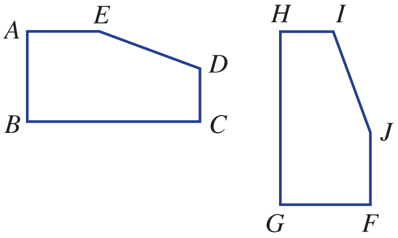
Example 7

4 These two quadrilaterals are congruent. Name the object in quadrilateral $EFGH$ that corresponds to these objects in quadrilateral $ABCD$.



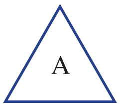
a **i** A **ii** D
b **i** AD **ii** CD
c **i** $\angle C$ **ii** $\angle A$

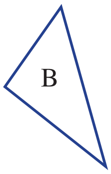
5 These two pentagons are congruent. Name the object in pentagon $FGHIJ$ that corresponds to these objects in pentagon $ABCDE$.

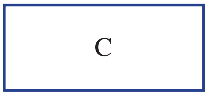


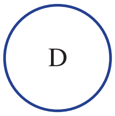
a **i** A **ii** D
b **i** AE **ii** CD
c **i** $\angle C$ **ii** $\angle E$

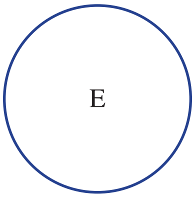
6 From all the shapes shown here, find three pairs that are congruent.

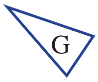


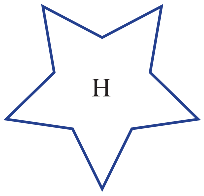


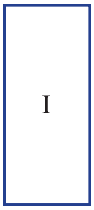


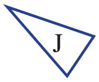


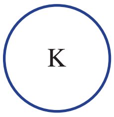


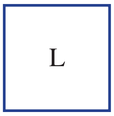


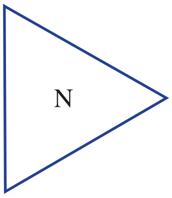












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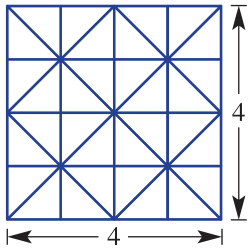
PROBLEM-SOLVING AND REASONING	7, 8, 10	7, 8, 10, 11	7, 9, 11–13

7 Using the lines in the diagram as the sides of triangles, how many triangles are there that have:



a area $\frac{1}{2}$?

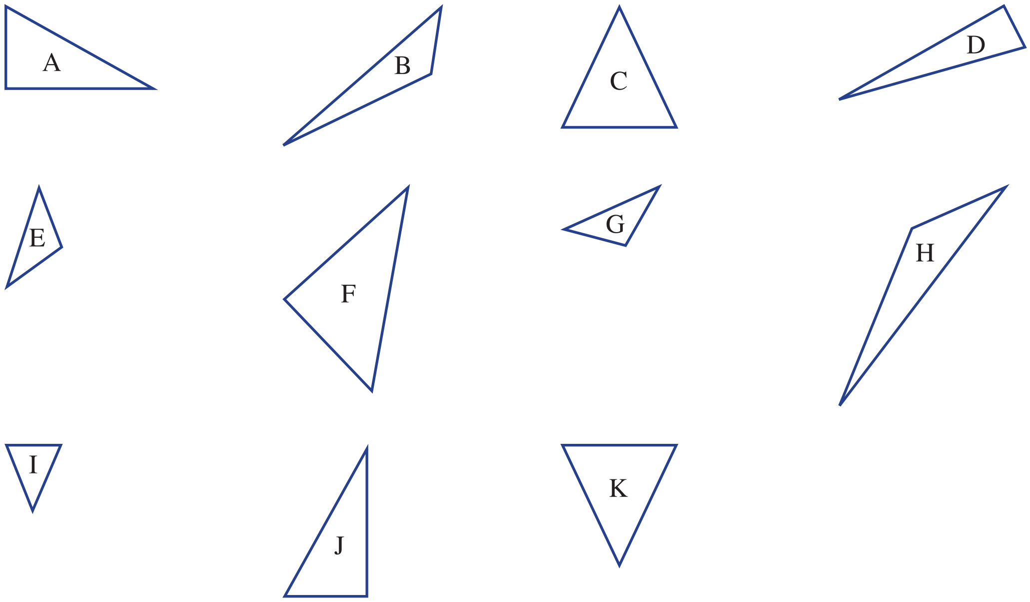
b area 1?

c area 2?

d area 4?

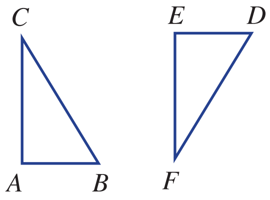
e area 8?

8 List the pairs of the triangles below that look congruent.

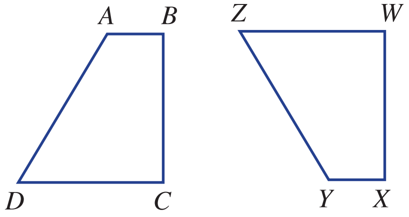


9 Write the pairs of corresponding vertices for these congruent shapes; for example, (A, E), (B, D).

a



b



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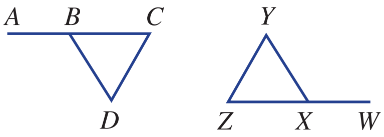

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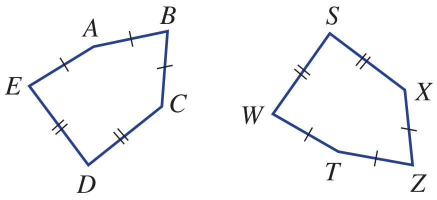
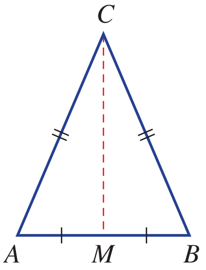


10 An isosceles triangle is cut as shown, using the midpoint of AB.

a Name the two triangles formed.

b Will the two triangles be congruent? Explain.



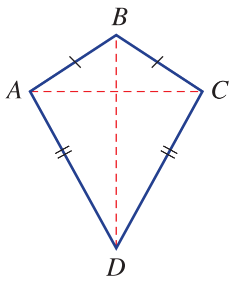
11 A kite $ABCD$ has diagonals AC and BD.

a The kite is cut using the diagonal BD.

- **i** Name the two triangles formed.
- **ii** Will the two triangles be congruent? Explain.

b The kite is cut using the diagonal AC.

- **i** Name the two triangles formed.
- **ii** Will the two triangles be congruent? Explain.



12 If a parallelogram is cut by either diagonal will the two triangles be congruent?

13 What is the condition for two circles to be congruent?

ENRICHMENT	–	–	14

Combined congruent transformations

14 Describe the combination of transformations (reflections, translations and/or rotations) that map each shape to its image under the given conditions. The reflections that are allowed include only those in the x- and y-axes and rotations will use $(0, 0)$ as its centre.

a A to A' with a reflection and then a translation

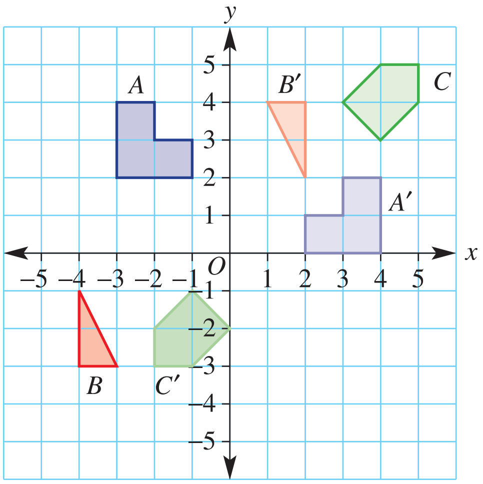
b A to A' with a rotation and then a translation

c B to B' with a rotation and then a translation

d B to B' with two reflections and then a translation

e C to C' with two reflections and then a translation

f C to C' with a rotation and then a translation



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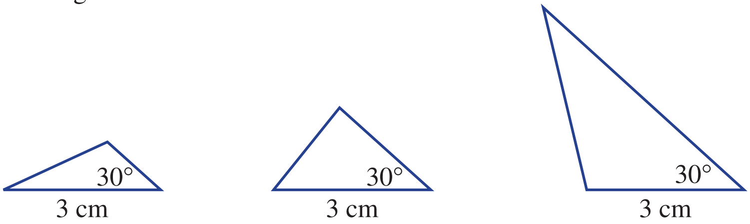
8E Congruent triangles

Interactive
Widgets
HOTsheets
Walkthrough

Imagine the sorts of design and engineering problems we would face if we could not guarantee that two objects, such as window panes or roof truss frames, were not the same size or shape. Furthermore, it might not be possible or practical to measure every length and angle to test for congruence. In the case of triangles, it is possible to consider only a number of pairs of sides or angles to test whether or not they are congruent.

Let's start: How much information is enough?

When given one corresponding angle (say 30°) and one corresponding equal side length (say 3 cm), it is clearly not enough information to say two triangles are congruent. This is because more than one triangle can be drawn with the given information.



Decide if the following information is enough to determine if two triangles are congruent. If you can draw two non-identical triangles, then there is not enough information. If you can draw only one unique triangle, then you have the conditions for congruence.

- $\triangle ABC$ with $AC = 4$ cm and $\angle C = 40°$
- $\triangle ABC$ with $AB = 5$ cm and $AC = 4$ cm
- $\triangle ABC$ with $AB = 5$ cm, $AC = 4$ cm and $\angle A = 45°$
- $\triangle ABC$ with $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm
- $\triangle ABC$ with $AB = 4$ cm, $\angle A = 40°$ and $\angle B = 60°$

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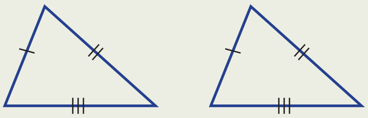
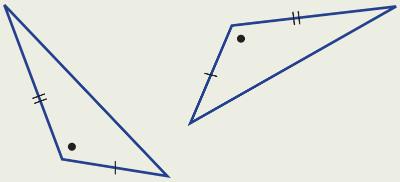
Key ideas

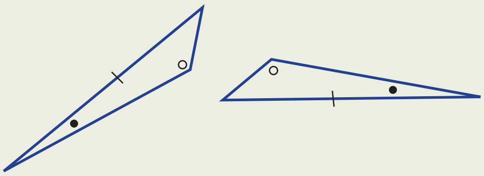
- There are four 'tests' that can be used to decide if two triangles are congruent.
- Two triangles are congruent when:
  - the **three sides of a triangle** are respectively equal to the **three sides of another triangle (SSS test)**



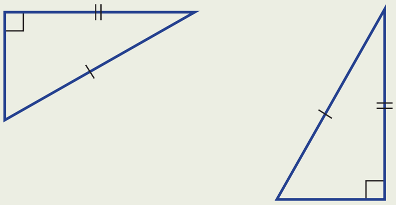
  - **two sides and the included angle** of a triangle are respectively equal to **two sides and the included angle of another triangle (SAS test)**



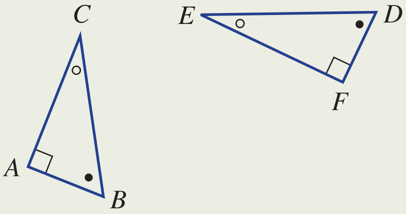
  - **two angles and one side** of a triangle are respectively equal to **two angles and the matching side of another triangle (AAS test)**



  - the **hypotenuse and a second side of a right-angled triangle** are respectively equal to the **hypotenuse and a second side of another right-angled triangle (RHS test)**



- Corresponding (matching) parts of two figures have the same geometric properties.
  - Vertex C corresponds to vertex E.
  - Side AB corresponds to side FD.
  - $\angle B$ corresponds to $\angle D$.



- A congruence statement can be written using the symbol $\equiv$. For example, $\triangle ABC \equiv \triangle FDE$.
  - This is read as '$\triangle ABC$ is congruent to $\triangle FDE$'.
  - In a congruence statement, vertices are named in matching order. For example, $\triangle ABC \equiv \triangle FDE$ *not* $\triangle ABC \equiv \triangle DEF$ because B matches D.

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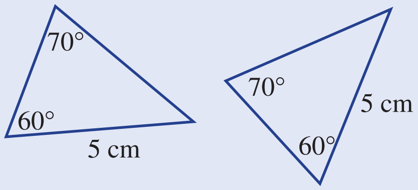
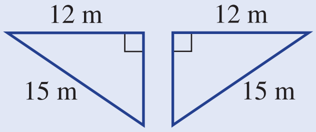
Example 8 Choosing a test for congruence

Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these pairs of triangles?

a



b



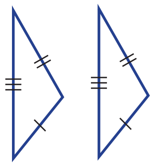
SOLUTION	EXPLANATION
a AAS	There are two equal angles and one pair of equal corresponding sides. The side that is 5 cm is adjacent to the 60° angle on both triangles.
b RHS	There is a pair of right angles with equal hypotenuse lengths. A second pair of corresponding sides are also of equal length.

Excercise 8E

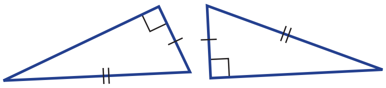
UNDERSTANDING AND FLUENCY	1–3, 4	2–4	3–4(½), 5

1 Pick the congruence test (SSS, SAS, AAS or RHS) that matches each pair of congruent triangles.

a



b



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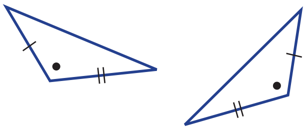

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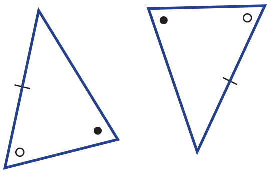
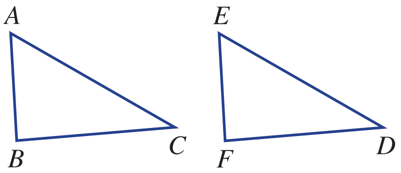


2 Write a congruence statement (e.g. $\triangle ABC \equiv \triangle DEF$) for these pairs of congruent triangles. Try to match vertices.

a



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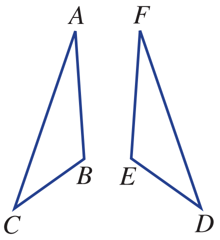

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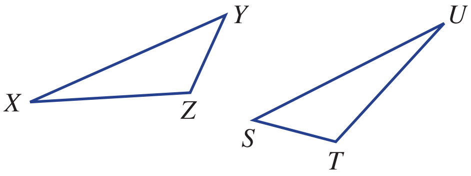

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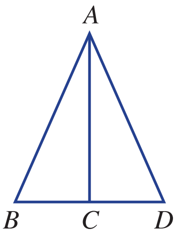

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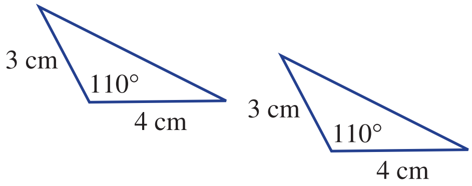
Example 8

3 Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these triangles?

a



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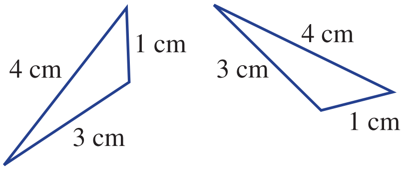

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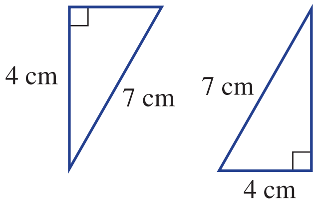

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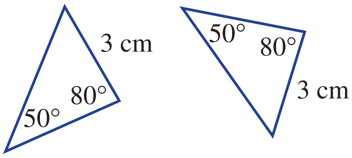
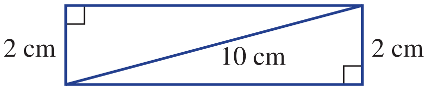
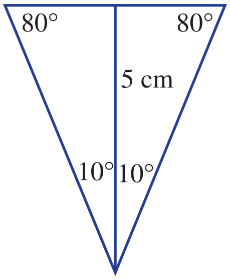


e

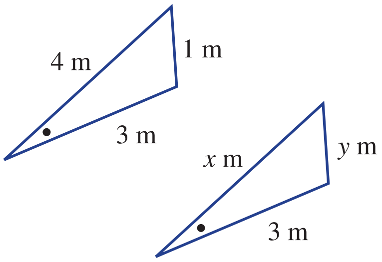


f



4 These pairs of triangles are congruent. Find the values of the pronumerals.

a



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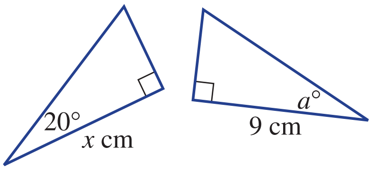

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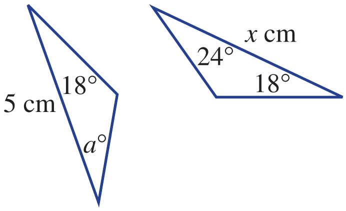

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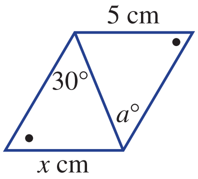

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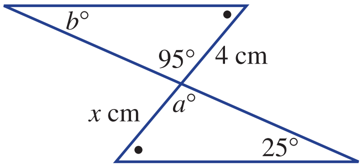

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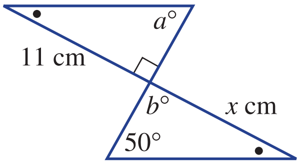

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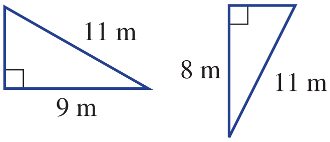
5 Decide if these pairs of triangles are congruent. If they are, write the test (SSS, SAS, AAS or RHS).

a


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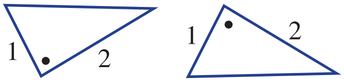

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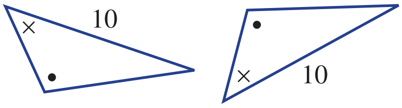

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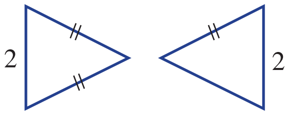

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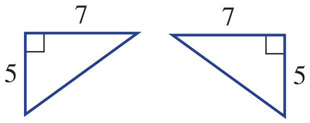

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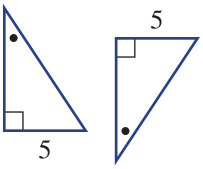
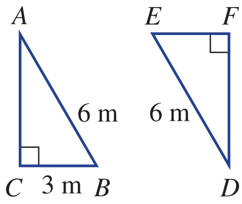


PROBLEM-SOLVING AND REASONING	6, 10a	6–8, 10a,b	7–10

6 Decide which piece of information from the given list needs to be added to the diagram if the two triangles are to be congruent.

$\angle B = 30^\circ$, $\angle C = 20^\circ$, $EF = 3\,\text{m}$, $FD = 3\,\text{m}$, $AB = 6\text{ cm}$, $AC = 6\text{ cm}$, $\angle C = 20^\circ$, $\angle A = 20^\circ$

a


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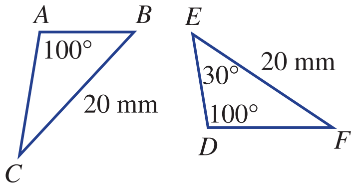

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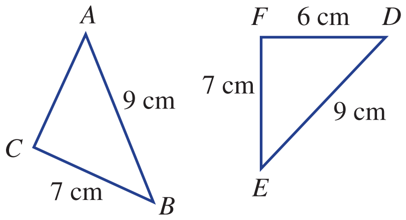

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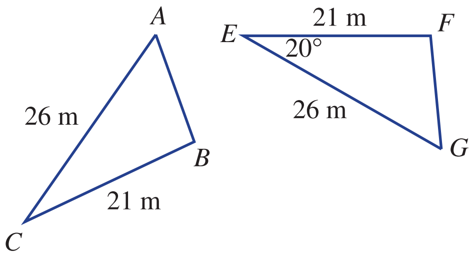
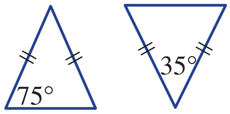


7 Decide if each pair of triangles is congruent. You may first need to use the angle sum of a triangle to help calculate some of the angles.

a


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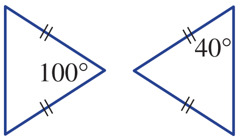

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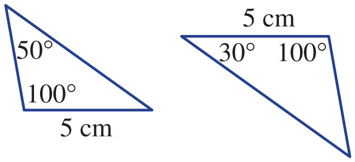

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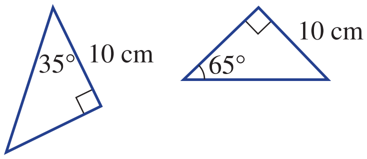

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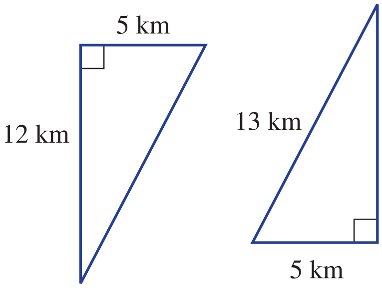
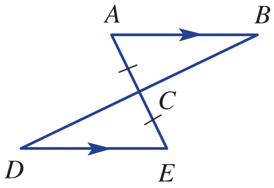
8 Two adjoining triangular areas of land have the dimensions shown. Use Pythagoras' theorem to help decide if the two areas of land are congruent.



9 **a** Explain why SSA is not sufficient to prove that two triangles are congruent. Draw diagrams to show your reasoning.

b Explain why AAA is not sufficient to prove that two triangles are congruent. Draw diagrams to show your reasoning.

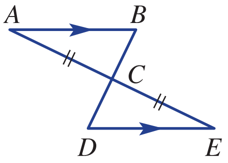
10 Here is a suggested proof showing that the two triangles in this diagram are congruent.



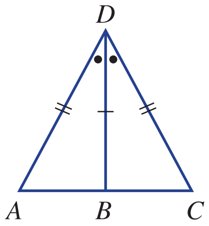
In $\triangle ABC$ and $\triangle EDC$:
$\angle CAB = \angle CED$ (alternate angles in parallel lines)
$\angle ACB = \angle ECD$ (vertically opposite angles)
$AC = EC$ (given equal and corresponding sides)
$\therefore \triangle ABC \equiv \triangle EDC$ (AAS)

Write a proof (similar to the above) for these pairs of congruent triangles. Any of the four tests (SSS, SAS, AAS or RHS) may be used.

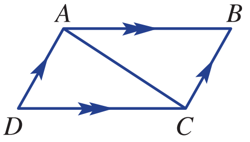
a



b



c



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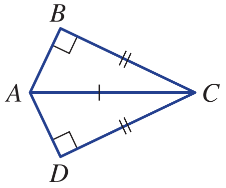

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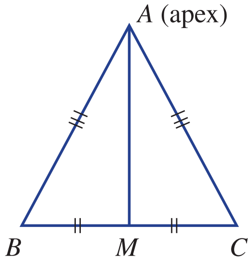
ENRICHMENT	–	–	11

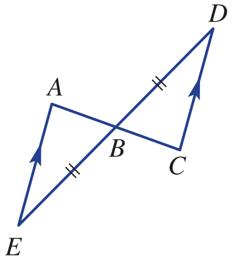
Proof challenge

11 Write a logical set of reasons (proof) as to why the following are true. Refer to Question **10** for an example of how to set out a simple proof.

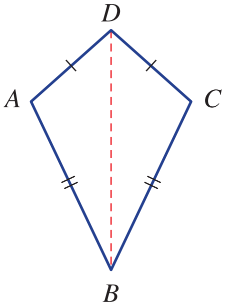
a The segment joining the apex of an isosceles triangle to the midpoint M of the base BC is at right angles to the base; that is, prove $\angle AMB = \angle AMC = 90°$.



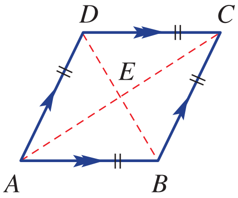
b The segment $AC = 2AB$ in this diagram.



c A kite has one pair of equal opposite angles.



d The diagonals of a rhombus intersect at right angles.



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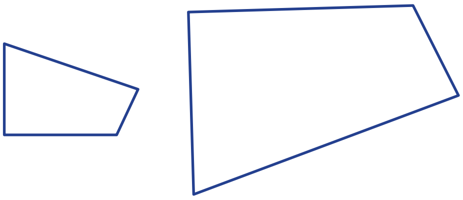
8F Similar figures EXTENSION

When you look at an object through a telescope or a pair of binoculars you expect the image to be much larger than the one you would see with the naked eye. Both images would be the same in every way except for their size. Such images in mathematics are called similar figures. Images that have resulted in a reduction in size (rather than an enlargement in size) are also considered to be similar figures.

An item through a telescope is 'similar' to how it appears to the naked eye.

Let's start: Are they similar?

Here are two quadrilaterals.

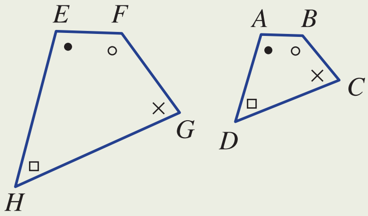


- Do they look similar in shape? Why?
- How could you use a protractor to help decide if they are similar in shape? What would you measure and check? Try it.
- How could you use a ruler to help decide if they are similar in shape? What would you measure and check? Try it.
- Describe the geometrical properties that similar shapes have. Are these types of properties present in all pairs of shapes that are 'similar'?

Key ideas

- **Similar figures** have the same shape but can be of different size.
- All corresponding angles are equal.
- All corresponding sides are in the same ratio.
- The ratio of sides is often written as a fraction, as shown below. It is often called the **scale factor**.

  Scale factor $= \frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD} = \frac{HE}{DA}$
- If the scale factor is 1, the figures are congruent.



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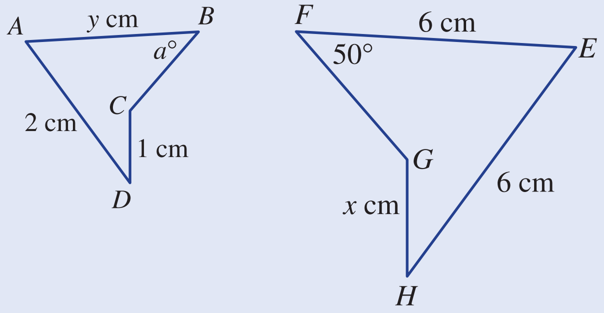
Example 9 Identifying corresponding features

For the following pairs of similar figures complete these tasks.



a List the pairs of corresponding sides.
b List the pairs of corresponding angles.
c Find the scale factor.
d Find the values of the pronumerals.

SOLUTION	EXPLANATION
a $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$	Pair up each vertex, noticing that G corresponds with C, H corresponds with D and so on.
b $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$	$\angle G$ and $\angle C$ are clearly the largest angles in their respective shapes. Match the other angles in reference to these angles.
c $\frac{HE}{DA} = \frac{6}{2} = 3$	HE and DA are corresponding sides both with given measurements. Divide the larger by the smaller.
d $a = 50$ $x = 3 \times 1 = 3$ cm $y = 6 \div 3 = 2$ cm	$\angle B$ and $\angle F$ are equal corresponding angles. The scale factor $\frac{HE}{DA} = 3$, so $\frac{GH}{CD}$ should also equal 3. Alternatively, say that GH is three times the length CD. Similarly, EF should be three times the length AB (y cm).

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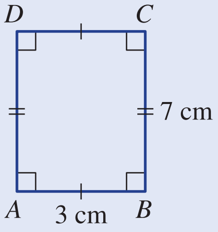
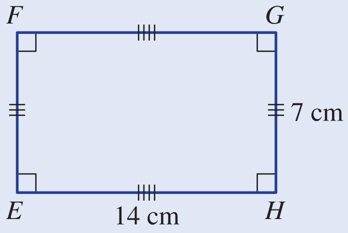
Example 10 Deciding if shapes are similar

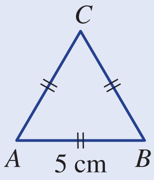
Decide if these shapes are similar by considering corresponding angles and the ratio of sides.

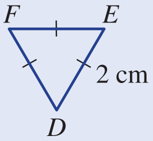
a





b





SOLUTION	EXPLANATION
a All corresponding angles are equal.	All angles are 90° and so corresponding angles are equal.
$\frac{EF}{BC} = \frac{14}{7} = 2$ $\frac{GH}{AB} = \frac{7}{3} = 2.\dot{3}$	Match pairs of sides to find scale factors.
Shapes are not similar because scale factors are not equal.	The scale factor needs to be equal if the shapes are to be similar.
b All angles are 60°.	All angles in an equilateral triangle are 60°.
$\frac{AB}{DE} = \frac{5}{2} = 2.5$ $\frac{BC}{EF} = \frac{5}{2} = 2.5$ $\frac{CA}{FD} = \frac{5}{2} = 2.5$	All scale factors are equal, so the triangles are similar.
Triangles are similar because corresponding sides are in the same ratio.	

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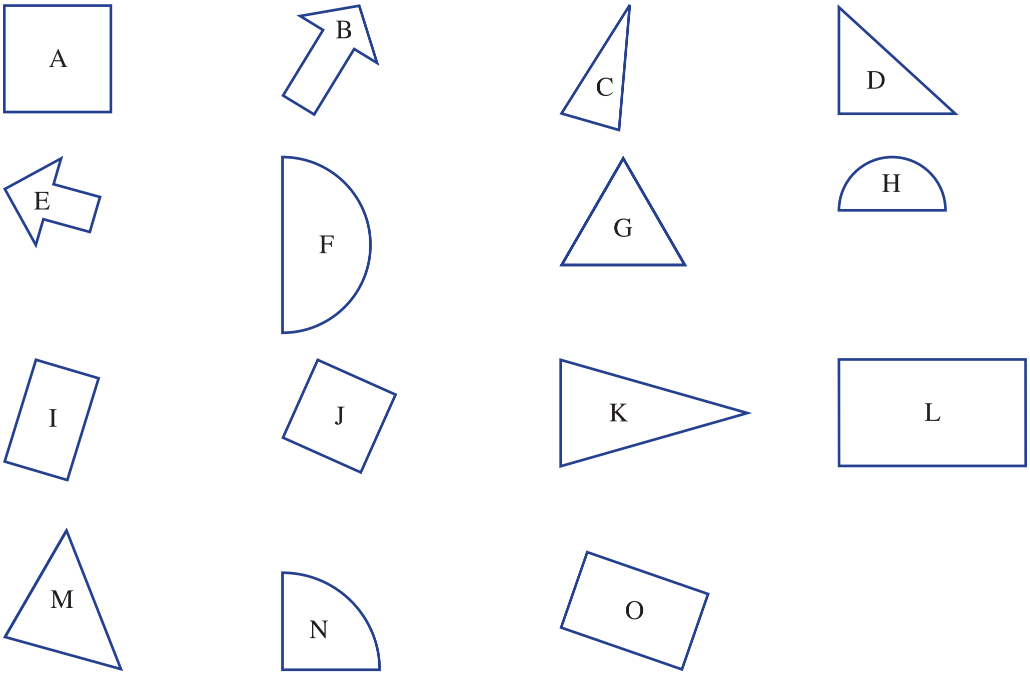
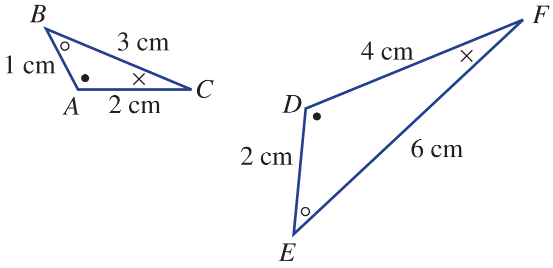
Exercise 8F EXTENSION

UNDERSTANDING AND FLUENCY	1–5	2–5	4, 5

1 List any pairs of shapes that look similar (i.e. same shape but different size).



2 Consider these two triangles.



a Name the angle on the larger triangle that corresponds to:

i $\angle A$ **ii** $\angle B$ **iii** $\angle C$

b Name the side on the smaller triangle that corresponds to:

i DE **ii** EF **iii** FD

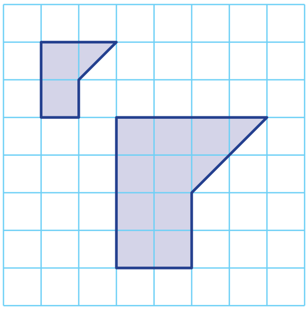
c Find these side ratios.

i $\frac{DE}{AB}$ **ii** $\frac{EF}{BC}$ **iii** $\frac{FD}{CA}$

d Would you say that the two triangles are similar? Why or why not?

3 Decide if the pairs of shapes on these grids are similar. If so state the scale factor.

a



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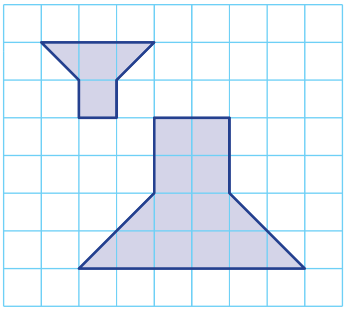
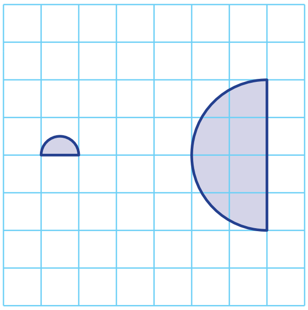


c



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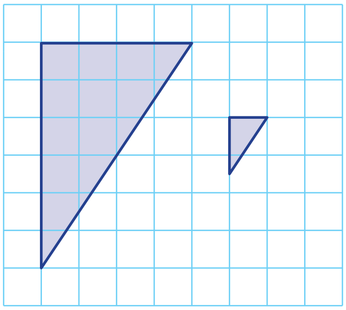


Example 9

4 For the following pairs of similar figures, complete these tasks.

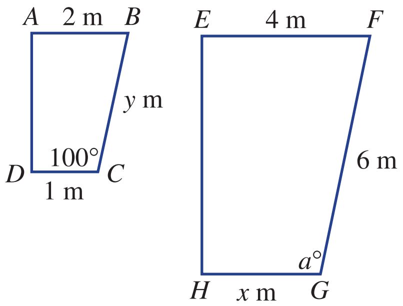
i List the pairs of corresponding sides.

ii List the pairs of corresponding angles.

iii Find the scale factor.

iv Find the values of the pronumerals.

a



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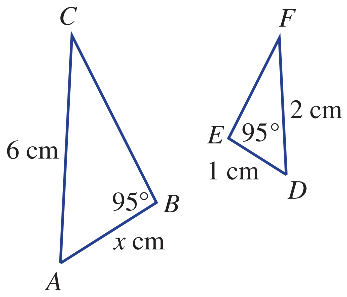

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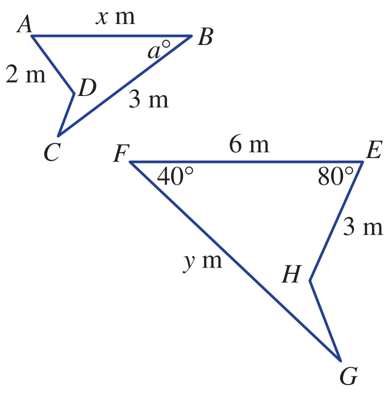

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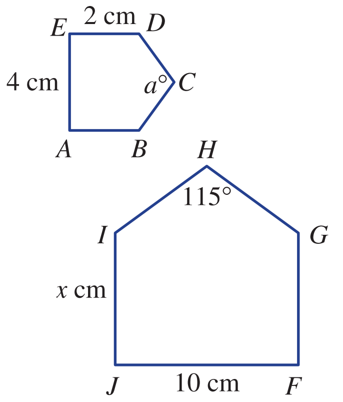

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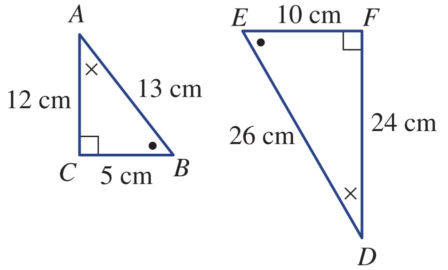
Example 10

5 Decide if these shapes are similar by considering corresponding angles and the ratios of sides.

a



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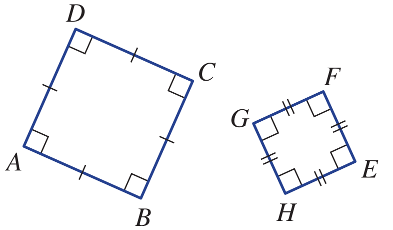

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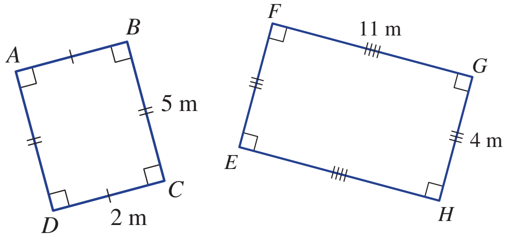

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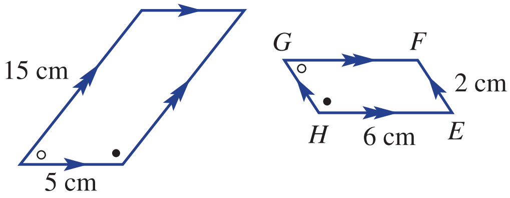

PROBLEM-SOLVING AND REASONING	6, 7, 10	7–11	7–10, 12

6 Two rectangular picture frames are similar in shape. A corresponding pair of sides are of length 50 cm and 75 cm. The other side length on the smaller frame is 70 cm. Find the perimeter of the larger frame.

7 Two similar triangles have a scale factor of $\frac{7}{3}$. If the larger triangle has a side length of 35 cm, find the length of the corresponding side on the smaller triangle.

8 One circle has a perimeter of 10π cm and another has an area of 16π cm^2. Find the scale factor of the diameters of the two circles.

9 A square photo of area 100 cm^2 is enlarged to an area of 900 cm^2. Find the scale factor of the side lengths of the two photos.

10 Are the following statements true or false? Give reasons for your answers.

- **a** All squares are similar.
- **b** All rectangles are similar.
- **c** All equilateral triangles are similar.
- **d** All isosceles triangles are similar.
- **e** All rhombuses are similar.
- **f** All parallelograms are similar.
- **g** All kites are similar.
- **h** All trapeziums are similar.
- **i** All circles are similar.

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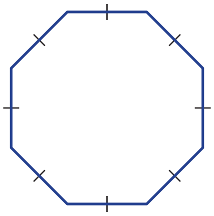
11 **a** If a regular polygon, such as this regular octagon, is enlarged, do the interior angles change?



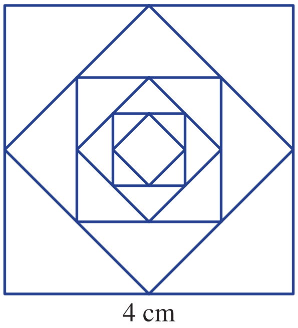
b Are all polygons with the same number of sides similar? Give reasons.

12 If a square is similar to another by a scale factor of 4, what is the ratio of the area of small square to the area of large square?

ENRICHMENT	–	–	13

Square designs

13 A square design consists of a series of squares, as shown. An inner square is formed by joining the midpoints of the sides of the next largest square.



a Use Pythagoras' theorem to find the side length of:

  i the second largest square

  ii the third largest square

b Find the scale factor of a pair of consecutive squares; for example, first and second or second and third.

c If the side length of the outside square is x cm, show that the scale factor of consecutive squares is equal to the result found in part **b**.

d What is the scale factor of a pair of alternate squares; for example, first and third or second and fourth?

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8G Similar triangles EXTENSION

Finding the approximate height of a tree or the width of a gorge using only simple equipment is possible without actually measuring the distance directly. Similar triangles can be used to calculate distances without the need to climb the tree or cross the gorge. It is important, however, to ensure that if the mathematics of similar triangles is going to be used, then the two triangles are in fact similar. We learnt earlier that there were four tests that help to determine if two triangles are congruent. Similarly there are four tests that help establish whether or not triangles are similar. Not all side lengths or angles are required to prove that two triangles are similar.

Let's start: How much information is enough?

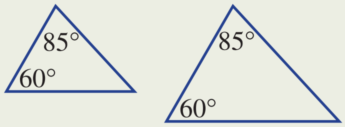
Given a certain amount of information, it may be possible to draw two triangles that are guaranteed to be similar in shape.

Decide if the information given is enough to guarantee that the two triangles will always be similar. If you can draw two triangles ($\triangle ABC$ and $\triangle DEF$) that are not similar, then there is not enough information provided.

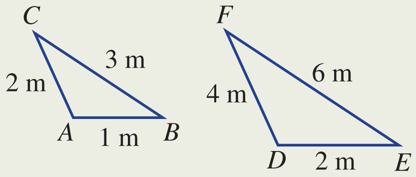
- $\angle A = 30°$ and $\angle D = 30°$
- $\angle A = 30°$, $\angle B = 80°$ and $\angle D = 30°$, $\angle E = 80°$
- $AB = 3$ cm, $BC = 4$ cm and $DE = 6$ cm, $EF = 8$ cm
- $AB = 3$ cm, $BC = 4$ cm, $AC = 5$ cm and $DE = 6$ cm, $EF = 8$ cm, $DF = 10$ cm
- $AB = 3$ cm, $\angle A = 30°$ and $DE = 3$ cm, $\angle D = 30°$
- $AB = 3$ cm, $AC = 5$ cm, $\angle A = 30°$ and $DE = 6$ cm, $DF = 10$ cm, $\angle D = 30°$

Key ideas

- Two triangles are similar when:
  - Two angles of a triangle are equal to two angles of another triangle.



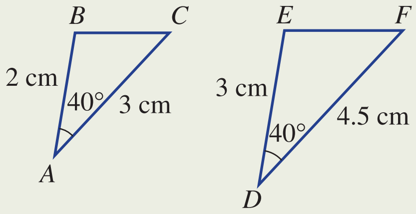
  - Three sides of a triangle are proportional to three sides of another triangle.



$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = 2$$

Scale factor

  - Two sides of a triangle are proportional to two sides of another triangle, and the included angles are equal.



$$\frac{DE}{AB} = \frac{DF}{AC} = 1.5$$

Scale factor

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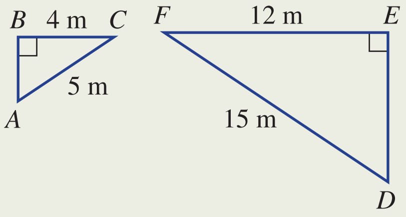
Key ideas

- The hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and second side of another right-angled triangle.



$$\frac{DF}{AC} = \frac{EF}{BC} = 3$$

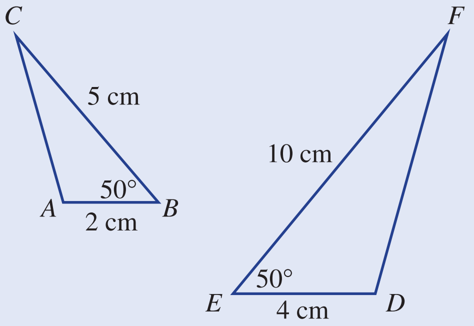
Scale factor

- If two triangles $\triangle ABC$ and $\triangle DEF$ are similar, we write $\triangle ABC \;|||\; \triangle DEF$.
- Abbreviations such as AAA are generally *not* used for similarity in NSW.

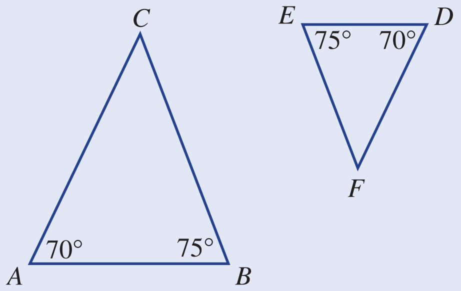
Example 11 Deciding if two triangles are similar

Decide, with reasons, why these pairs of triangles are similar.

a



b



SOLUTION	EXPLANATION
a $\frac{ED}{AB} = \frac{4}{2} = 2$ $\angle B = \angle E = 50^\circ$ $\frac{EF}{BC} = \frac{10}{5} = 2$ $\therefore \triangle ABC$ is similar to $\triangle DEF$, using SAS.	Work out the ratio of the two pairs of corresponding sides to see if they are equal. Note that the angle given is the included angle between the two given pairs of corresponding sides.
b $\angle A = \angle D$ $\angle B = \angle E$ $\therefore \triangle ABC$ is similar to $\triangle DEF$, using AAA.	Two pairs of equal angles are given. This implies that the third pair of angles are also equal and that the triangles are similar.

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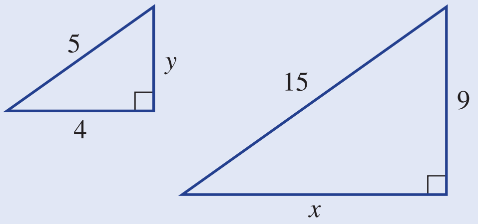
Example 12 Finding a missing length

Given that these pairs of triangles are similar, find the value of the pronumerals.



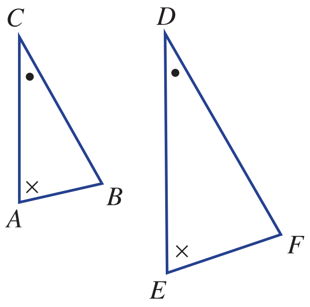
SOLUTION	EXPLANATION
Scale factor $= \frac{15}{5} = 3$	First find the scale factor.
$x = 4 \times 3 = 12$ $y = 9 \div 3 = 3$	Multiply or divide by the scale factor to find the values of the pronumerals.

Exercise 8G EXTENSION

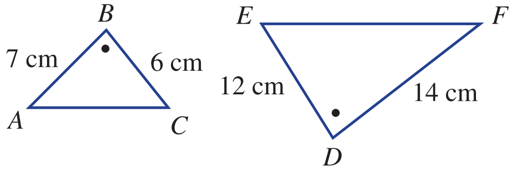
UNDERSTANDING AND FLUENCY	1–4	2–4	3, 4

1 Write a similarity statement for these pairs of similar triangles; for example, $\triangle ABC \,|||\, \triangle DEF$. Be careful with the order of letters and make sure each letter matches the corresponding vertex.

a



b



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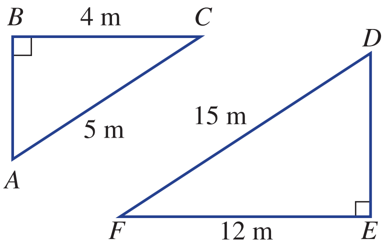

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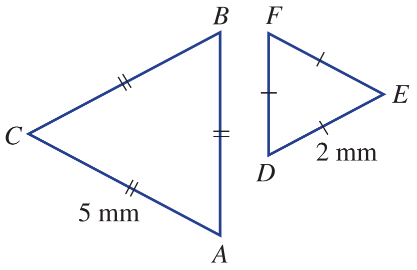

2 For the pairs of triangles in Question **1**, which of the four similar triangle tests would be used to show their similarity? See the Key ideas for a description of each test. There is one pair for each test.

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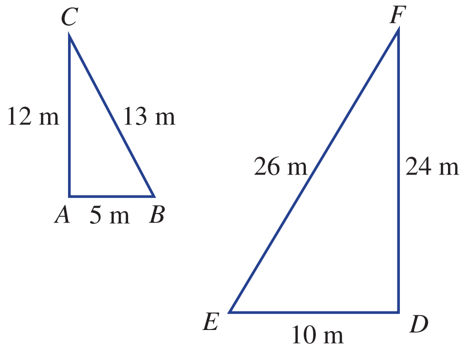
Example 11

3 Decide, with reasons, why these pairs of triangles are similar.

a



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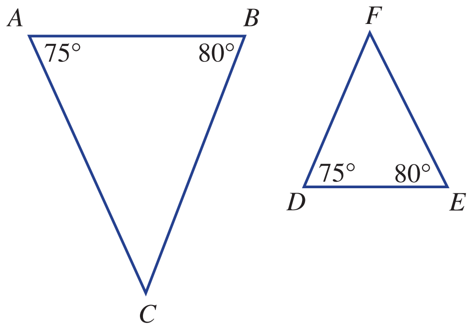

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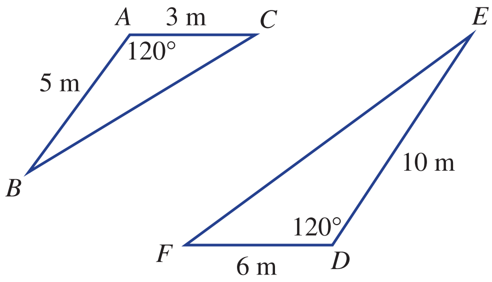

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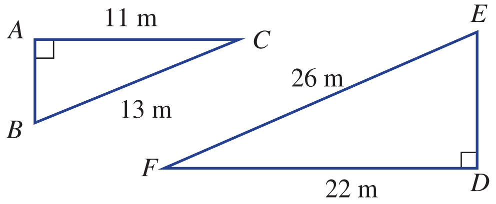
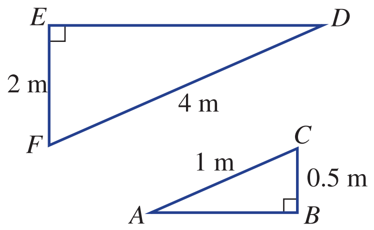


e



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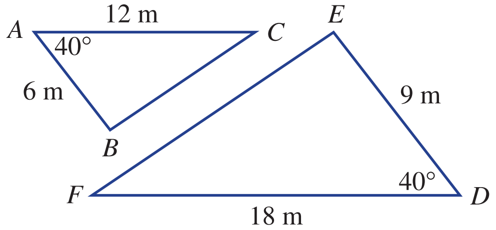

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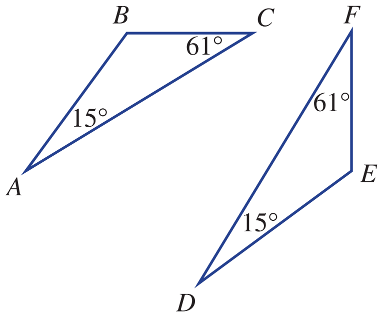

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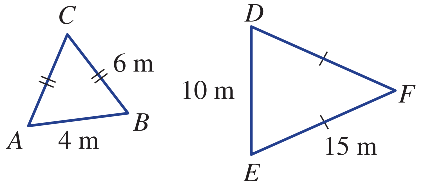

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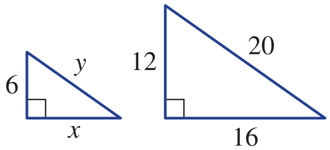
Example 12

4 Given that these pairs of triangles are similar, find the value of the pronumerals.

a



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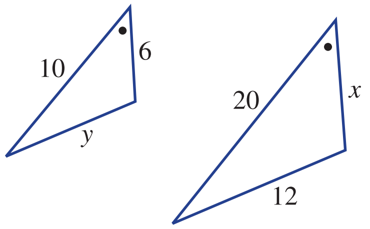
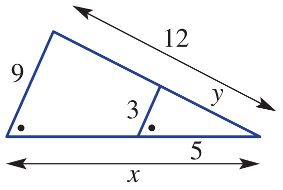


c



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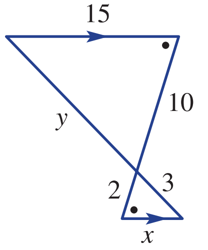
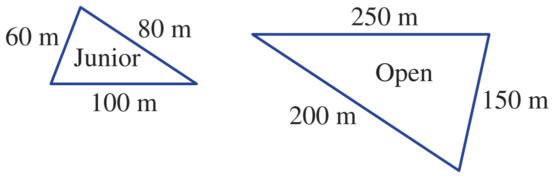


PROBLEM-SOLVING AND REASONING	5–7, 9	5–7, 9, 10	6–8, 10, 11

5 Two triangular tracks are to be used for the junior and open divisions for a school event.



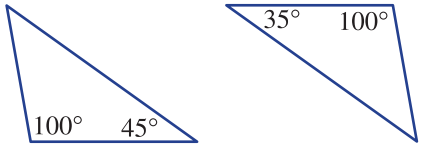
a Work out the scale factor for each of the three corresponding pairs of sides.

b Are the two tracks similar in shape? Give a reason.

c How many times would a student need to run around the junior track to cover the same distance as running one lap of the senior track?

6 By using the angle sum of a triangle decide if the given pairs of triangles are similar.

a



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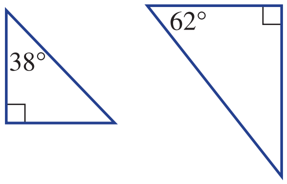

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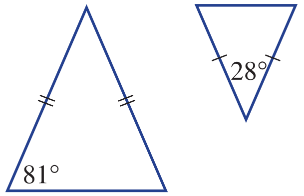

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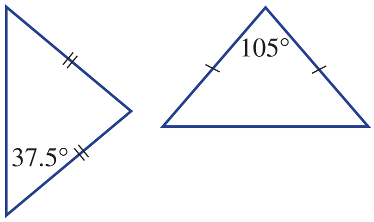
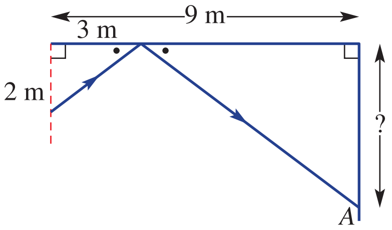


7 In a game of minigolf, a ball bounces off a wall, as shown. The ball proceeds and hits another wall at point A. How far down the right side wall does the ball hit?



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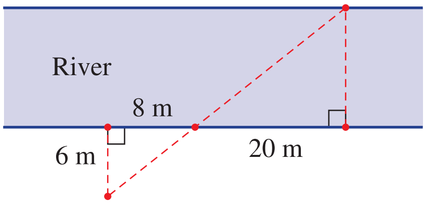
8 Trees on a river bank are to be used as markers to form two triangular shapes. Each tree is marked by a red dot, as shown in the diagram.

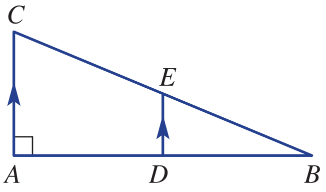


- **a** Are the two triangles similar?
- **b** Calculate the scale factor.
- **c** Calculate how far it is across the river.

9 Explain why only two pairs of angles are required to prove that two triangles are similar.

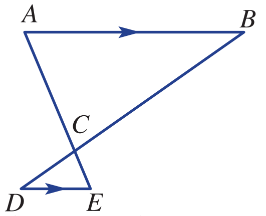
10 Give reasons why the pairs of triangles in these diagrams are similar. If an angle or side is common to both triangles, use the word 'common' in your reasoning.

a



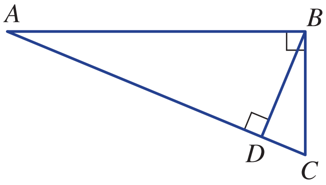
$\triangle ABC$ and $\triangle DBC$

b



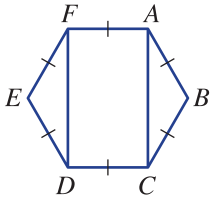
$\triangle ABC$ and $\triangle EDC$

c



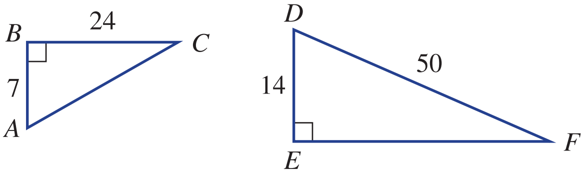
$\triangle ABC$ and $\triangle ADE$

d



$\triangle ABC$ and $\triangle FED$

11 Show how Pythagoras' theorem can be used to prove that these two triangles are similar.

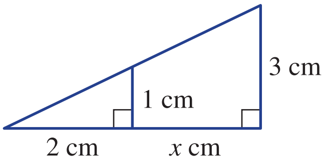


ENRICHMENT – – 12

Tricky unknowns

12 The pairs of triangles here show a length, x cm, which is not the length of a full side of a triangle. Find the value of x in each case.

a



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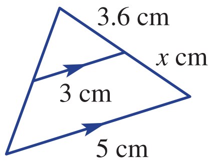

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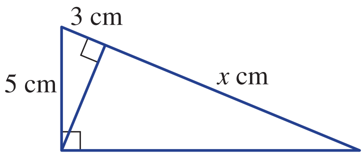

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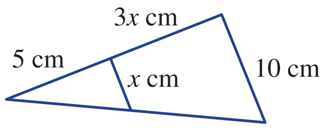

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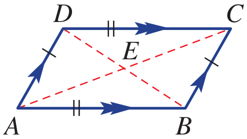
8H Using congruent triangles to establish properties of quadrilaterals

The properties of special quadrilaterals, including the parallelogram, rhombus, rectangle, square, trapezium and kite, can be examined more closely using congruence. By drawing the diagonals and using the tests for the congruence of triangles, we can prove many of the properties of these special quadrilaterals.

Let's start: Do the diagonals bisect each other?

Here is a parallelogram with two pairs of parallel sides. Let's assume also that opposite sides are equal (a basic property of a parallelogram, which we prove later in this exercise).

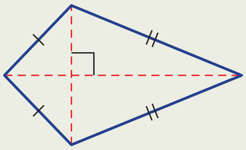


- First locate $\triangle ABE$ and $\triangle CDE$.
- What can be said about the angles $\angle BAE$ and $\angle DCE$?
- What can be said about the angles $\angle ABE$ and $\angle CDE$?
- What can be said about the sides AB and DC?
- Now what can be said about $\angle ABE$ and $\angle CDE$? Discuss the reasons.
- What does this tell us about where the diagonals intersect? Do they **bisect** each other and why? (To bisect means to cut in half.)

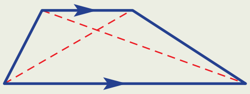
Key ideas

This is a summary of the properties of the special quadrilaterals.

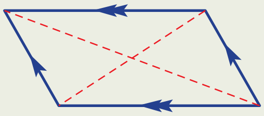
- Kite: A quadrilateral with two pairs of adjacent sides equal.
  - Two pairs of adjacent sides of a kite are equal.
  - One diagonal of a kite bisects the other diagonal.
  - One diagonal of a kite bisects the opposite angles.
  - The diagonals of a kite are perpendicular.



- Trapezium: A quadrilateral with at least one pair of parallel sides
  - At least one pair of sides of a trapezium are parallel.



- Parallelogram: A quadrilateral with both pairs of opposite sides parallel
  - The opposite sides of a parallelogram are parallel.
  - The opposite sides of a parallelogram are equal.
  - The opposite angles of a parallelogram are equal.
  - The diagonals of a parallelogram bisect each other.



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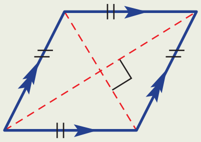
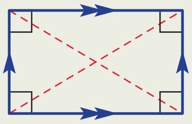
Key ideas

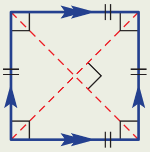
- Rhombus: A parallelogram with two adjacent sides equal in length
  - The opposite sides of a rhombus are parallel.
  - All sides of a rhombus are equal.
  - The opposite angles of a rhombus are equal.
  - The diagonals of a rhombus bisect the vertex angles.
  - The diagonals of a rhombus bisect each other.
  - The diagonals of a rhombus are perpendicular.



- Rectangle: A parallelogram with a right angle
  - The opposite sides of a rectangle are parallel.
  - The opposite sides of a rectangle are equal.
  - All angles at the vertices of a rectangle are 90°.
  - The diagonals of a rectangle are equal.
  - The diagonals of a rectangle bisect each other.

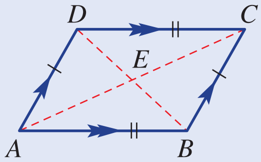


- Square: A rectangle with two adjacent sides equal
  - Opposite sides of a square are parallel.
  - All sides of a square are equal.
  - All angles at the vertices of a square are 90°.
  - The diagonals of a square bisect the vertex angles.
  - The diagonals of a square bisect each other.
  - The diagonals of a square are perpendicular.



Example 13 Proving that diagonals of a parallelogram bisect each other

Prove that the diagonals of a parallelogram bisect each other and give reasons.



SOLUTION

$\angle BAE = \angle DCE$ (alternate angles in parallel lines)

$\angle ABE = \angle CDE$ (alternate angles in parallel lines)

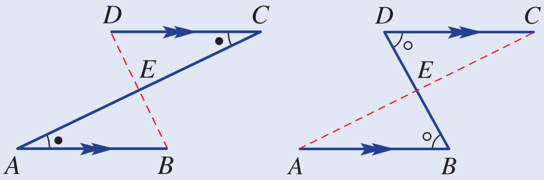
$AB = CD$ (given equal side lengths)

$\therefore \triangle ABE \equiv \triangle CDE$ (AAS)

$\therefore BE = DE$ and $AE = CE$

$\therefore$ Diagonals bisect each other.

EXPLANATION



The two triangles are congruent, using the AAS test.

Since $\triangle ABE$ and $\triangle CDE$ are congruent the corresponding sides are equal.

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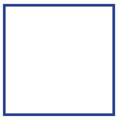
Exercise 8H

UNDERSTANDING AND FLUENCY	1–7	5–8	6–9

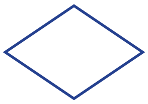
1 By drawing a single line, show how the following quadrilaterals can be made into two congruent triangles.

a

b



c



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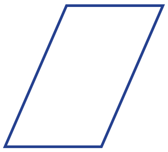
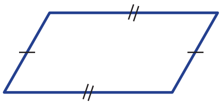


2 Which property of a parallelogram is illustrated in each diagram below?

a



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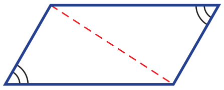

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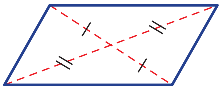

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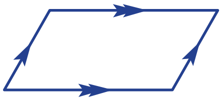


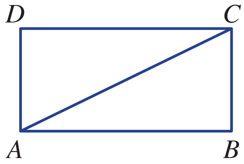
3 Write down four properties of:

a a rhombus

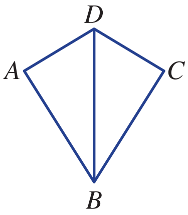
b a square

4 Name the side (e.g. AB) that is common to both triangles in each diagram.

a



b



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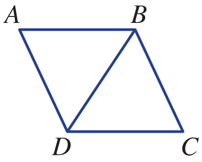
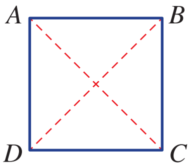


5 Given $ABCD$ is a square, complete the following proof to show that the diagonals of a square are equal.



In $\triangle ABC$ and $\triangle BCD$:

$\angle ABC =$ ________ (angles in a ______ are equal)

$AB = BC$ (____________________)

______ is common.

$\therefore \triangle ABC \equiv \triangle BCD$ (S ____ ____)

$\therefore AC =$ ________ (matching sides in ________ triangles are equal)

and the ____________ of a square are equal in length.

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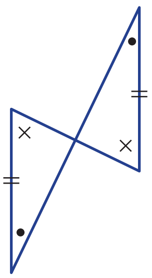
6 Which of the four tests for congruence of triangles would be used to prove that each pair of triangles is congruent?

a



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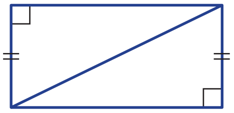

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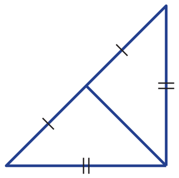
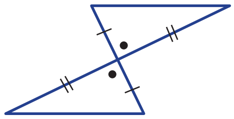
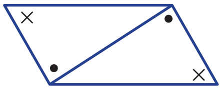


d



e



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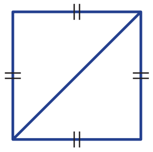
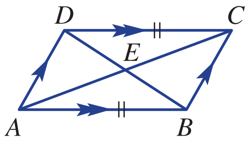


Example 13 **7** Prove, giving reasons, that the diagonals in a parallelogram bisect each other. You may assume here that opposite sides are equal, so use $AB = CD$. Complete the proof by following these steps.



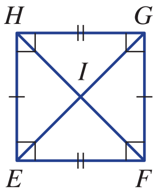
Step 1. List the pairs of equal angles in $\triangle ABE$ and $\triangle CDE$, giving reasons why they are equal.

Step 2. List the pairs of equal sides in $\triangle ABE$ and $\triangle CDE$, giving reasons why they are equal.

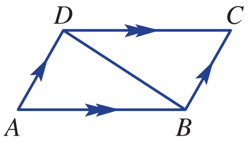
Step 3. Write $\triangle ABE \equiv \triangle CDE$ and give the reason SSS, SAS, AAS or RHS.

Step 4. State that $BE = DE$ and $AE = CE$ and give a reason.

8 Prove (as in Example 13) that the diagonals in a square, $EFGH$, bisect each other.



9 A parallelogram, $ABCD$, has two pairs of parallel sides.



a What can be said about $\angle ABD$ and $\angle CDB$ and give a reason?

b What can be said about $\angle BDA$ and $\angle DBC$ and give a reason?

c Which side is common to both $\triangle ABD$ and $\triangle CDB$?

d Which congruence test would be used to show that $\triangle ABD \equiv \triangle CDB$?

e If $\triangle ABD \equiv \triangle CDB$, what can be said about the opposite sides of a parallelogram?

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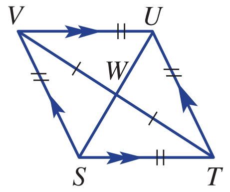
PROBLEM-SOLVING AND REASONING	10, 11, 13	10, 11, 13, 15	12–16

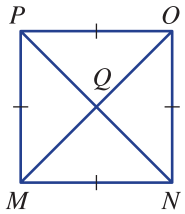
10 For this rhombus assume that $VW = WT$.

a Give reasons why $\triangle VWU \equiv \triangle TWU$.

b Give reasons why $\angle VWU = \angle TWU = 90°$.



11 For this square assume that $MQ = QO$ and $NQ = PQ$.



a Give reasons why $\triangle MNQ \equiv \triangle ONQ$.

b Give reasons why $\angle MQN = \angle OQN = 90°$.

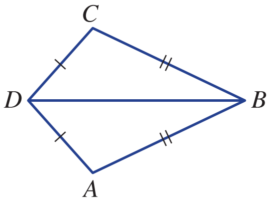
c Give reasons why $\angle QMN = 45°$.

12 Use the information in this kite to prove these results.

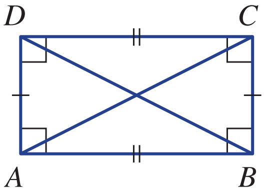
a $\triangle ABD \equiv \triangle CBD$

b $\angle DAB = \angle DCB$

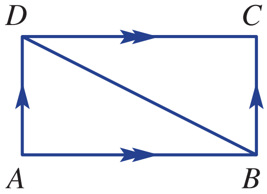
c $\angle ADB = \angle CDB$



13 Use Pythagoras' theorem to prove that the diagonals in a rectangle are equal in length. You may assume that opposite sides are equal.



14 Use the steps outlined in Question **6** to show that opposite sides of a rectangle, $ABCD$, are equal. Give all reasons.



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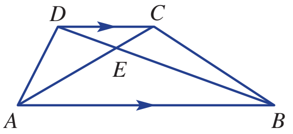
15 A trapezium, $ABCD$, has one pair of parallel sides.

a Which angle is equal to $\angle BAE$?

b Which angle is equal to $\angle ABE$?

c Explain why $\triangle ABE$ is not congruent to $\triangle CDE$.

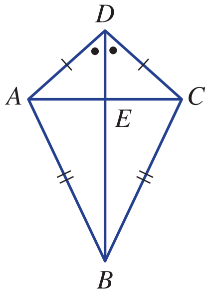


16 Use the information given for this kite to give reasons for the following properties. You may assume that $\angle ADE = \angle CDE$, as marked.

a $\angle DAE = \angle DCE$

b $\triangle AED \equiv \triangle CED$

c $\angle AED = \angle CED = 90°$

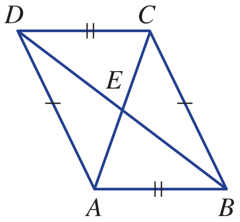


ENRICHMENT	–	–	17

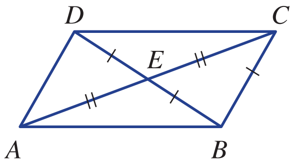
Prove the converse

17 Prove the following results, which are the converse (reverse) of some of the proofs completed earlier in this exercise.

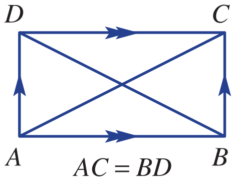
a If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram; that is, show that the quadrilateral has two pairs of parallel sides.



b If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



c If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.



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Investigation

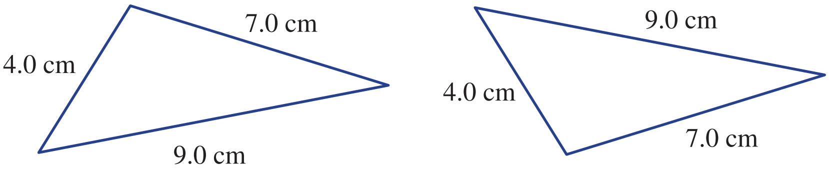
Investigating congruence with dynamic geometry software

Computer dynamic geometry is ideal for these activities but geometrical instruments could also be used.

Conditions for congruent triangles

1 SSS

- **a** Construct a triangle with side lengths 4 cm, 7 cm and 9 cm. Measure the given side lengths using one decimal place, and if using dynamic geometry adjust the triangle to suit. Use the measure tool for the side lengths.
- **b** Construct a second triangle with the same side lengths.
- **c** Do your two triangles look congruent? Measure the angles to help check.
- **d** Can you draw another triangle given the same information so that it does not look congruent to the others?



2 SAS

- **a** Construct a triangle with side lengths 4 cm and 7 cm and an included angle of 30°. Measure the given side lengths and the angle using one decimal place, and if using dynamic geometry adjust the triangle to suit.
- **b** Construct a second triangle with the same side lengths and included angle.
- **c** Do your two triangles look congruent? Measure the missing angle and side lengths to help check.
- **d** Can you draw another triangle given the same information so that it does not look congruent to the others?

3 AAS

- **a** Construct a triangle with two angles, 30° and 50°, and one side length of 6 cm. Measure the given side length and the angles using one decimal place, and if using dynamic geometry adjust the triangle to suit.
- **b** Construct a second triangle with the same side length and angles.
- **c** Do your two triangles look congruent? Measure the missing side length and angle to help check.
- **d** Can you draw another triangle given the same information so that it does not look congruent to the others?

4 RHS

- **a** Construct a triangle with angles of 90° and 50°, hypotenuse of length 6 cm and other side of length 4 cm. Measure the two given side lengths and the angle using one decimal place, and if using dynamic geometry adjust the triangle to suit.
- **b** Construct a second triangle with the same two side lengths and angles.
- **c** Do your two triangles look congruent? Measure the missing side length and angles to help check.
- **d** Can you draw another triangle given the same information so that it does not look congruent to the others?

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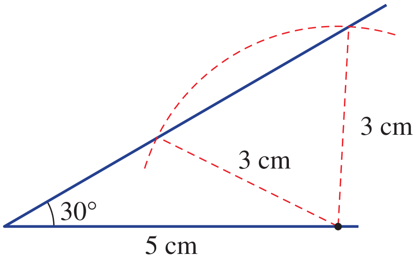
Exploring SSA

With the SAS test, the included angle must be between the two given sides. This investigation will explore a triangle using SSA.

1 Construct a triangle with sides 5 cm and 3 cm and one angle 30° (not an included angle).

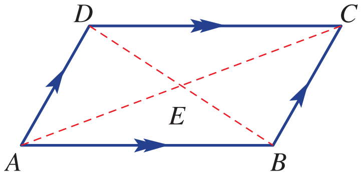
2 Can you construct a second triangle using the same information that is not congruent to the first?

3 Consider this diagram. Use it to help explain why SSA is not a test for congruence of two triangles.



Properties of quadrilaterals

1 Construct a parallelogram including its diagonals, as shown. Use the parallel line tool to create the parallel sides.



2 Measure the sides and angles of $\triangle ABE$ and $\triangle CDE$. What do you notice?

3 Now drag one of the vertices of the parallelogram. Does this change you conclusion from part b?

4 Explore other quadrilaterals by constructing their diagonals and looking for congruent triangles. Write up a brief summary of your findings.

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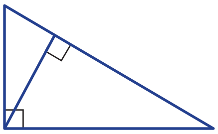
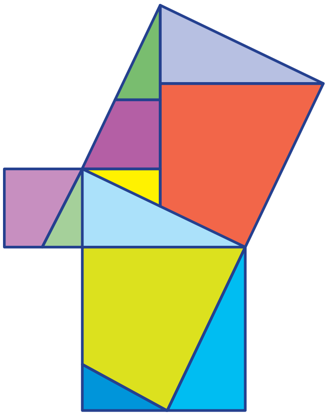
Puzzles and challenges

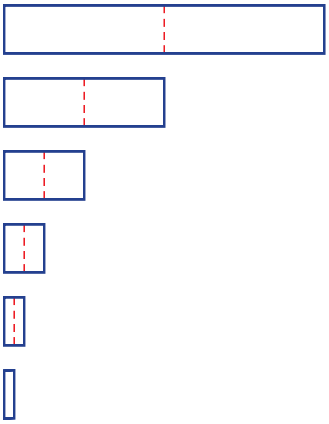
1 How many similar triangles are there in this figure? Give reasons.



2 Can you fit the shapes in the two smaller squares into the largest square? What does this diagram illustrate?
Try drawing or constructing the design and then use scissors to cut out each shape.

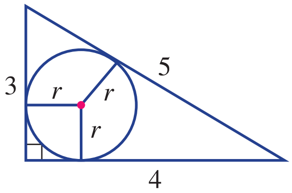


3 A strip of paper is folded five times in one direction only. How many creases will there be in the original strip when it is unfolded?

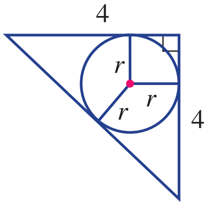


4 Use congruent triangles to find the radius, r, in these diagrams.

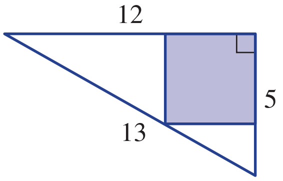
a



b



5 What is the side length of the largest square that can be cut out of this triangle?
Use similar triangles.



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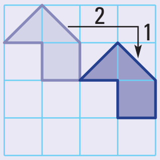
Transformations and congruence

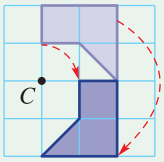
Translation

Vector $\begin{bmatrix}2\\-1\end{bmatrix}$

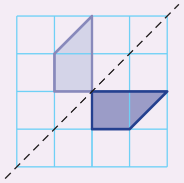


Rotation

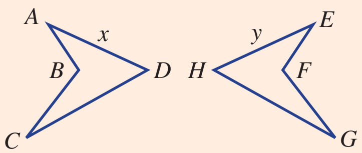
Clockwise by 90°



Reflection



Congruent figures

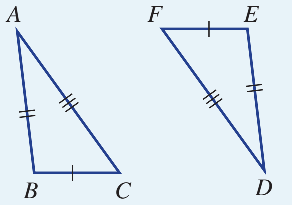


$ABCD \equiv EFGH \qquad x = y$

$AB = EF$

$\angle C = \angle G$

Congruent triangles



$\triangle ABC \equiv \triangle DEF$

$AB = DE,\ BC = EF,\ AC = DF$

$\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$

Tests: SSS, SAS, AAS, RHS

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Multiple-choice questions

1 The number of lines of symmetry in a regular pentagon is:

A 10 **B** 5 **C** 2 **D** 1 **E** 0

2 Which vector describes a translation of 5 units to the left and 3 units up?

A $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$ **B** $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ **C** $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ **D** $\begin{bmatrix} -5 \\ 3 \end{bmatrix}$ **E** $\begin{bmatrix} -5 \\ -3 \end{bmatrix}$

3 The point $A(-3, 4)$ is translated to the point A' by the vector $\begin{bmatrix} 6 \\ -4 \end{bmatrix}$. The coordinates of point A' are:

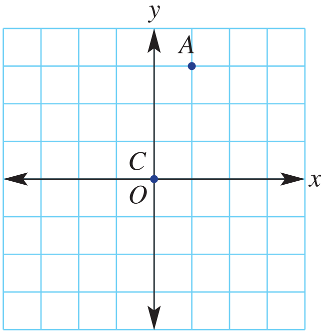
A (3, 8) **B** (−9, 8) **C** (3, 0) **D** (0, 3) **E** (−9, 0)

4 An anticlockwise rotation of 125° about a point is the same as a clockwise rotation about the same point of:

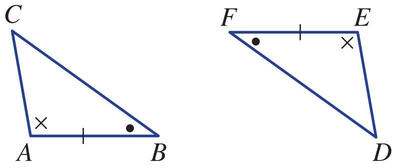
A 235° **B** 65° **C** 55° **D** 135° **E** 245°

5 Point $A(1, 3)$ is rotated clockwise about C by 90° to A'.
The coordinates of A' are:

A (3, 0) **B** (3, −1) **C** (−3, 1)
D (−1, 3) **E** (3, 1)



Questions **6**, **7**, **8** *and* **9** *relate to this pair of congruent triangles.*



6 The angle on $\triangle DEF$ that corresponds to $\angle A$ is:

A $\angle C$ **B** $\angle B$ **C** $\angle F$ **D** $\angle D$ **E** $\angle E$

7 If $AC = 5$ cm, then ED is equal to:

A 5 cm **B** 10 cm **C** 2.5 cm **D** 15 cm **E** 1 cm

8 A congruent statement with ordered vertices for the triangles is:

A $\triangle ABC \equiv \triangle FED$ **B** $\triangle ABC \equiv \triangle EDF$
C $\triangle ABC \equiv \triangle DFE$ **D** $\triangle ABC \equiv \triangle DEF$
E $\triangle ABC \equiv \triangle EFD$

9 Which of the four tests would be chosen to show that these two triangles are congruent?

A AAS **B** RHS **C** SAS **D** AAA **E** SSS

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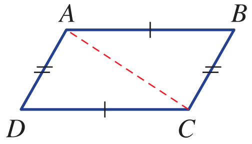
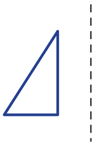
10 *ABCD* is a parallelogram. Based on the information supplied in the diagram, which test could be used most easily to prove that $\triangle ABC \equiv \triangle CDA$.

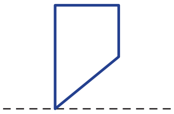


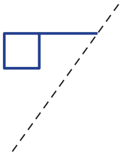
A AAS **B** RHS **C** SAS
D AAA **E** SSS

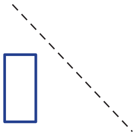
Short-answer questions

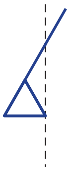
1 Copy these shapes and draw the reflected image over the mirror line.

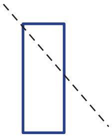
a


b


c


d


e


f


2 The triangle $A(1, 2)$, $B(3, 4)$, $C(0, 2)$ is reflected in the given axis. State the coordinates of the image points A', B' and C'.

a x-axis

b y-axis

3 How many lines of symmetry do these shapes have?

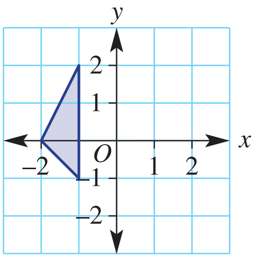
a square **b** isosceles triangle
c rectangle **d** kite
e regular hexagon **f** parallelogram

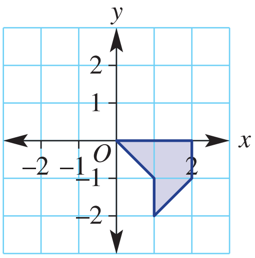
4 Write the vectors that translate each point A to its image A'.

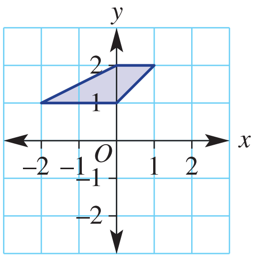
a $A(2, 5)$ to $A'(3, 9)$ **b** $A(-1, 4)$ to $A'(2, -2)$
c $A(0, 7)$ to $A'(-3, 0)$ **d** $A(-4, -6)$ to $A'(0, 0)$

5 Copy these shapes and draw the translated image using the given translation vector.

a $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ **b** $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$ **c** $\begin{bmatrix} 0 \\ -3 \end{bmatrix}$







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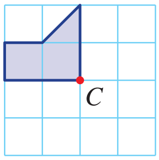
6 What is the order of rotational symmetry of these shapes?

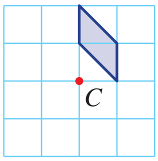
a equilateral triangle **b** parallelogram **c** isosceles triangle

7 Rotate these shapes about the point *C* by the given angle.

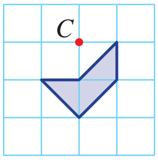
a clockwise 90°



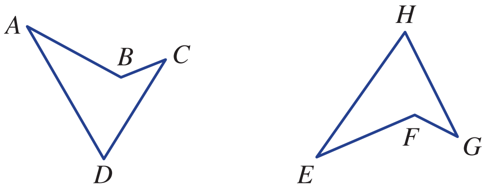
b clockwise 180°



c anticlockwise 270°



8 For these congruent quadrilaterals, name the object in quadrilateral *EFGH* that corresponds to the given object in quadrilateral *ABCD*.



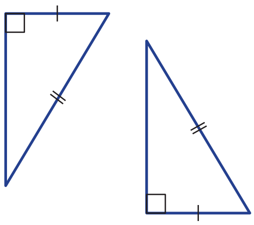
a **i** *B* **ii** *C*

b **i** *AD* **ii** *BC*

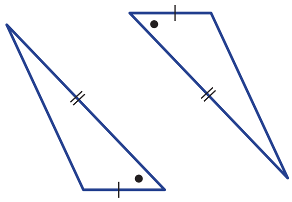
c **i** ∠*C* **ii** ∠*A*

9 Which of the tests SSS, SAS, AAS or RHS would you choose to explain the congruence of these pairs of triangles?

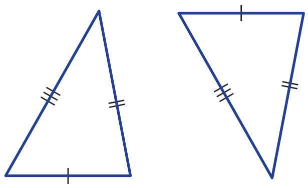
a



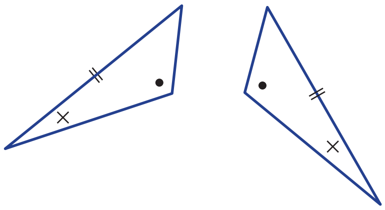
b



c

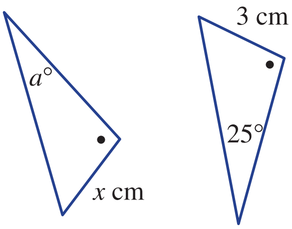


d

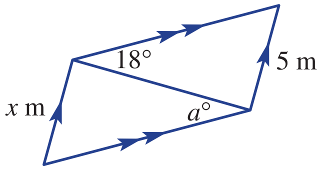


10 Find the values of the pronumerals for these congruent triangles.

a



b



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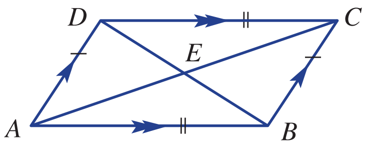
11 This quadrilateral is a parallelogram with two pairs of parallel sides. You can assume that $AB = DC$, as shown.

a Prove that $\triangle ABE \equiv \triangle CDE$.

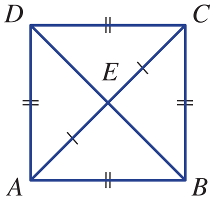
b Explain why BD and AC bisect each other.



12 This quadrilateral is a square with $AE = EC$.

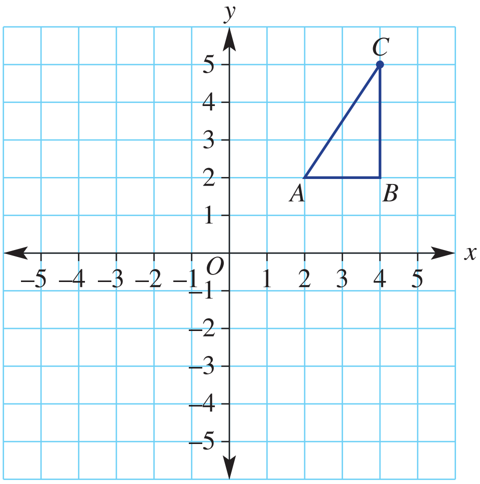
a Prove that $\triangle ABE \equiv \triangle CBE$.

b Explain why the diagonals intersect at right angles.



Extended-response question

The shape on this set of axes is to be transformed by a succession of transformations. The image of the first transformation is used to start the next transformation. For each set of transformations write down the coordinates of the vertices A', B' and C' of the final image.



Parts **a**, **b** and **c** are to be treated as separate questions.

a Set 1

- **i** reflection in the x-axis
- **ii** translation by the vector $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$
- **iii** rotation about (0, 0) by 180°

b Set 2

- **i** rotation about (0, 0) clockwise by 90°
- **ii** reflection in the y-axis
- **iii** translation by the vector $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

c Set 3

- **i** reflection in the line $y = -x$
- **ii** translation by the vector $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$
- **iii** rotation about (1, 0) by 90° clockwise

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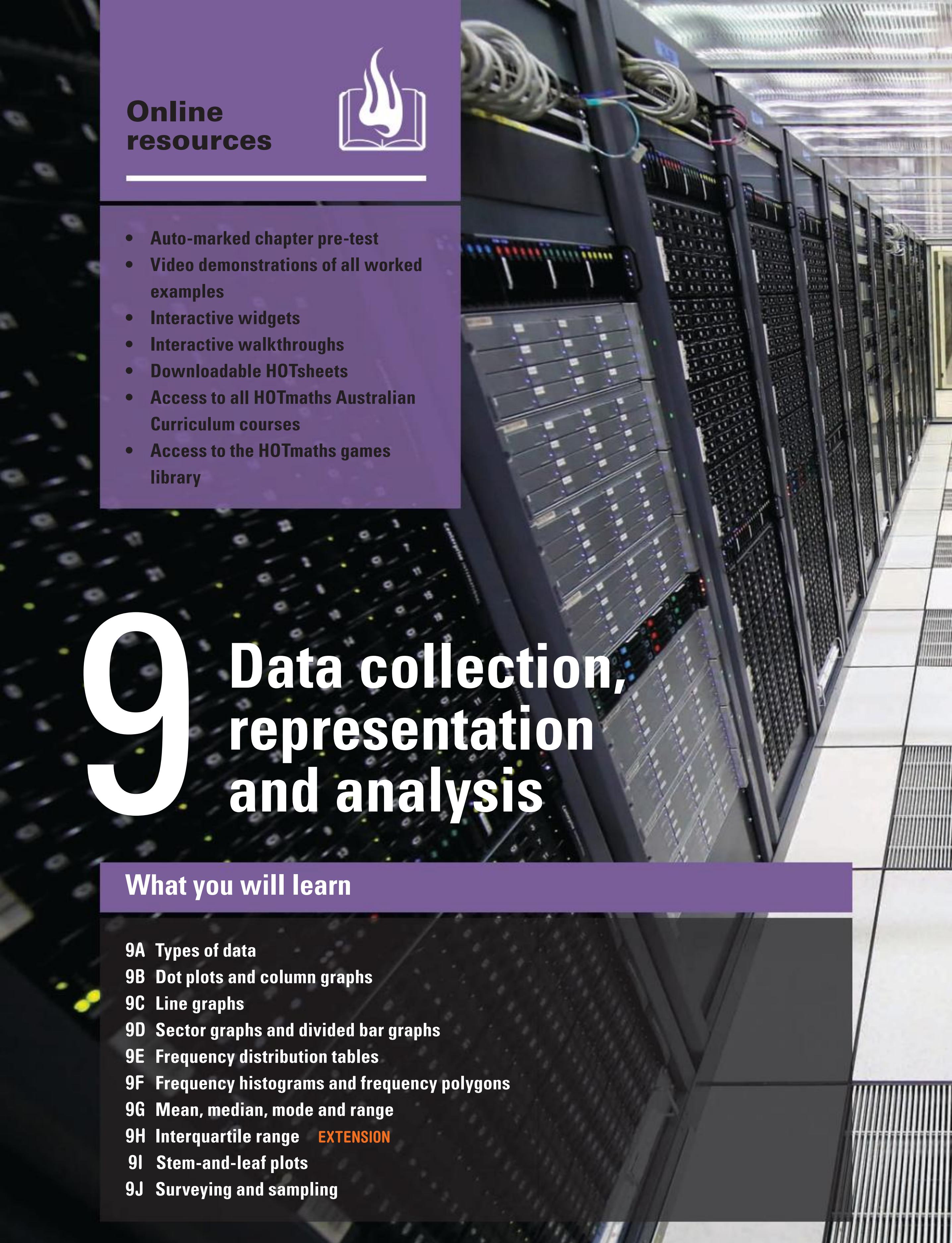


Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

9

Data collection, representation and analysis

What you will learn

NSW syllabus

STRAND: STATISTICS AND PROBABILITY

SUBSTRANDS: DATA COLLECTION AND REPRESENTATION
SINGLE VARIABLE DATA ANALYSIS

Outcomes

A student collects, represents and interprets single sets of data, using appropriate statistical displays. (MA4–19SP)

A student analyses single sets of data using measures of location and range. (MA4–20SP)

Search engine statistics

Search engines such as Google not only find web pages but also analyse and categorise searches. If the entire world's Google searches for one month (thousands of millions) were listed in a single document it would be an incredible list filling millions of pages, but it would be difficult to make conclusions about such a vast amount of data.

Search engines employ computer software engineers who are also highly skilled in mathematics, especially in statistics. In the case of Google, they organise worldwide searches into categories and present comparisons using graphs. This provides much more interesting and useful information for groups such as online shops, politicians, the entertainment industry, radio and TV stations, restaurants, airline companies and professional sports groups.

Using 'Google Insights for Search', graphical comparisons can be made between the frequencies of various types of searches within any country, state or city in the world. For example, which topics are of more interest (e.g. sport or news)? On which weekdays is shopping of more interest than sport? Which Australian state had the most searches about travel in the past year? Which Australian city has the highest percentage of people who buy books online? This information can be quite helpful for businesses wanting to know who their target audiences are.

In this chapter you will develop skills for organising data into categories, graphs and summaries so that useful comparisons can be made and future trends determined.

1 Arrange the following in ascending (increasing) order.

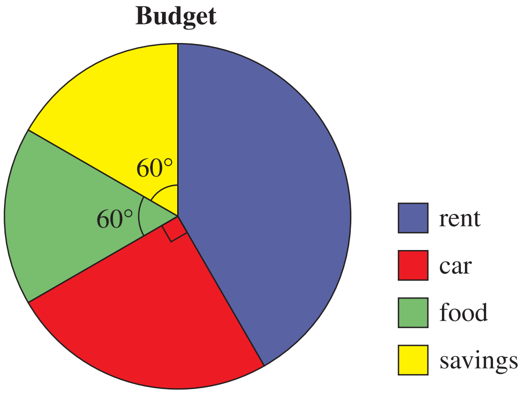
- **a** 2, 4, 10, 7, 1, 0, 6, 14, 9
- **b** 101, 20, 30.6, 204, 36, 100
- **c** 1.2, 1.9, 2.7, 1.7, 3.5, 3.2

2 Write down the total and the mean (average) for each of the sets below.

- **a** 4, 6, 8, 10 and 12
- **b** 15, 17, 19, 19 and 24
- **c** 0.6, 0.6, 0.6, 0.7 and 0.8

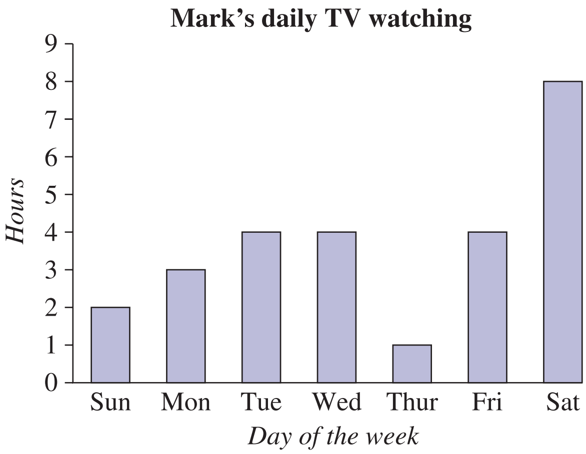
3 Use the information in the sector graph to answer the following questions.

- **a** What fraction of the income was spent on food?
- **b** What is the size of the angle for the rent sector?
- **c** If \$420 is saved each month, find how much is spent on:
  - **i** food
  - **ii** the car



4

- **a** How many hours of television were watched on Wednesday?
- **b** How many hours of television were watched on Monday?
- **c** On which day was the most television watched?
- **d** How many hours of television were watched over the week shown?
- **e** What fraction of Saturday was spent watching television?



5 Smarties can be purchased in small boxes. The small boxes are sold in a bag. The boxes in one bag contained the following smarties: 12, 11, 13, 12, 11, 12, 14, 10, 12, 13, 14.

- **a** How many boxes were in the bag?
- **b** Arrange the scores in ascending order.
- **c** Draw a dot plot to represent the data.
- **d** What is the most common number of smarties in a box?
- **e** What is the mean number of smarties in a box?

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9A Types of data

People collect or use data almost every day. Athletes and sports teams look at performance data, customers compare prices at different stores, investors look at daily interest rates, and students compare marks with other students in their class. Companies often collect and analyse data to help produce and promote their products to customers and to make predictions about the future.

Let's start: Collecting data – Class discussion

Consider, as a class, the following questions and discuss their implications.

- Have you or your family ever been surveyed by a telemarketer at home? What did they want? What time did they call?
- Do you think that telemarketers get accurate data? Why or why not?
- Why do you think companies collect data this way?
- If you wanted information about the most popular colour of car sold in NSW over the course of a year, how could you find out this information?

Key ideas

- In statistics, a **variable** is something measurable or observable that is expected to change over time or between individual observations. It can be numerical or categorical.
  - **Numerical** (**quantitative**), which can be discrete or continuous.
    - **Discrete numerical** – data that can only be particular numerical values. For example, the number of pets in a household (could be 0, 1, 2, 3 but not values in between like 1.3125).
    - **Continuous numerical** – data that can take any value in a range. Variables such as heights, weights and temperatures are all continuous. For instance, someone could have a height of 172 cm, 172.4 cm or 172.215 cm (if it can be measured accurately).
  - **Categorical** – data that is not numerical, such as colours, gender and brands of cars, are all examples of categorical data. In a survey, categorical data comes from answers that are given as words (e.g. 'yellow' or 'female') or ratings (e.g. 1 = dislike, 2 = neutral, 3 = like).

- Data can be collected from primary or secondary sources.
  - Data from a **primary source** is firsthand information collected from the original source by the person or organisation needing the data. For example, a survey an individual student conducts or census data collected by the Bureau of Statistics.
  - Data from a **secondary source** has been collected, published and possibly summarised by someone else before we use it. Data collected from newspaper articles, textbooks or internet blogs represents secondary source data.

- Samples and populations
  - When an entire population (e.g. a Maths class, a company, or a whole country) is surveyed, it is called a **census**.
  - When a subset of the population is surveyed, it is called a **sample**. Samples should be randomly selected and large enough to represent the views of the overall population.

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Example 1 Classifying variables

Classify the following variables as categorical, discrete numerical or continuous numerical.

a the gender of a newborn baby
b the length of a newborn baby

SOLUTION

a categorical

b continuous numerical

EXPLANATION

As the answer is ‘male’ or ‘female’ (a word, not a number) the data is categorical.

Length is a measurement, so all numbers are theoretically possible.

Example 2 Collecting data from primary and secondary sources

Decide whether a primary source or a secondary source is suitable for collection of data on each of the following and suggest a method for its collection.

a the average income of Australian households
b the favourite washing powder or liquid for households in Australia

SOLUTION

a primary source by looking at the census data

b secondary data source using the results from a market research agency

EXPLANATION

The population census held every 5 years in Australia is an example of a primary data source collection and will have this information.

A market research agency might collect these results using a random phone survey. Obtaining a primary source would involve conducting the survey yourself but it is unlikely that the sample will be large enough to be suitable.

Exercise 9A

UNDERSTANDING AND FLUENCY	1–5	3–6	4–6

1 Match each word on the left to its meaning on the right.

a Sample
b categorical
c discrete numerical
d primary source
e continuous numerical
f population

A only takes on particular numbers within a range
B a complete set of data
C a smaller group taken from the population
D data grouped in categories; words like ‘male’ and ‘female’
E data collected first hand
F can take on any number in a range

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Example 1

2 Classify the following as categorical or numerical.

- **a** The eye colour of each student in your class.
- **b** The date of the month each student was born (e.g. the 9th of a month).
- **c** The weight of each student when they were born.
- **d** The brands of airplanes landing at Sydney's international airport.
- **e** The temperature of each classroom.
- **f** The number of students in each classroom during period 1 on Tuesday.

What type of variable is the eye colour of the students in your class?

3 Give an example of:

- **a** discrete numerical data
- **b** continuous numerical data
- **c** categorical data

Example 1

4 Classify the following variables as categorical, continuous numerical or discrete numerical data.

- **a** the number of cars per household
- **b** the weights of packages sent by Australia Post on the 20th of December
- **c** the highest temperature of the ocean each day
- **d** the favourite brand of chocolate of the teachers at your school
- **e** the colours of the cars in the school car park
- **f** the brands of cars in the school car park
- **g** the number of letters in different words on a page
- **h** the number of advertisements in a time period over each of the free-to-air channels
- **i** the length of time spent doing this exercise
- **j** the arrival times of planes at JFK airport
- **k** the daily pollution levels on a particular highway
- **l** the number of SMS messages sent by an individual yesterday
- **m** the times for the 100 m freestyle event at the world championships over the past 10 years
- **n** the number of Blu-ray discs someone owns
- **o** the brands of cereals available at the supermarket
- **p** marks awarded on a maths test
- **q** the star rating on a hotel or motel
- **r** the censorship rating on a movie showing at the cinema

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5 Is observation or a sample or a census the most appropriate way to collect data on each of the following?

- **a** the arrival times of trains at Central Station during a day
- **b** the arrival times of trains at Central Station over the year
- **c** the heights of students in your class
- **d** the heights of all Year 8 students in the school
- **e** the heights of all Year 8 students in NSW
- **f** the number of plastic water bottles sold in a year
- **g** the religion of Australian families
- **h** the number of people living in each household in your class
- **i** the number of people living in each household in your school
- **j** the number of people living in each household in Australia
- **k** the number of native Australian birds found in a suburb
- **l** the number of cars travelling past a school between 8 a.m. and 9 a.m. on a school day
- **m** the money spent at the canteen by students during a week
- **n** the average income of Australians
- **o** the ratings of TV shows

Example 2

6 Identify whether a primary or secondary source is suitable for the collection of data on the following.

- **a** the number of soft drinks bought by the average Australian family in a week
- **b** the age of school leavers in Far North Queensland
- **c** the number of cigarettes consumed by school-age students in a day
- **d** the highest level of education obtained by the adults in a household
- **e** the reading level of students in Year 8 in Australia

PROBLEM-SOLVING AND REASONING	7, 8, 13, 14	8–10, 13, 14	10–16

7 Give a reason why someone might have trouble obtaining reliable and representative data using a primary source to find the following.

- **a** the temperature of the Indian Ocean over the course of a year
- **b** the religions of Australian families
- **c** the average income of someone in India
- **d** drug use by teenagers within a school
- **e** the level of education of different cultural communities within NSW

8 Secondary sources are already published data that is then used by another party in their own research. Why is the use of this type of data not always reliable?

9 When obtaining primary source data you can survey the population or a sample.

- **a** Explain the difference between a ‘population’ and a ‘sample’ when collecting data.
- **b** Give an example of a situation where you should survey a population rather than a sample.
- **c** Give an example of a situation where you should survey a sample rather than a population.

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10 A Likert-type scale is for categorical data where items are assigned a number. For example, the answer to a question could be 1 = dislike, 2 = neutral, 3 = like.

- **a** Explain why the data collected is categorical even though the answers are given as numbers.
- **b** Give examples of a Likert-type scale for the following categorical data. You might need to reorder some of the options.
  - **i** strongly disagree, somewhat disagree, somewhat agree, strongly agree
  - **ii** excellent, satisfactory, poor, strong
  - **iii** never, always, rarely, usually, sometimes
  - **iv** strongly disagree, neutral, strongly agree, disagree, agree

11 A sample should be representative of the population it reports on. For the following surveys, describe who might be left out and how this might introduce a bias.

- **a** a telephone poll with numbers selected from a phone book
- **b** a postal questionnaire
- **c** door-to-door interviews during the weekdays
- **d** a Girlfriend magazine poll
- **e** a Facebook survey

12 Another way to collect primary source data is by direct observation. For example, the colour of cars travelling through an intersection (categorical data) is best obtained in this way rather than through a questionnaire.

- **a** Give another example of a variable for which data could be collected by observation.
- **b** Explain how you could estimate the proportion of black cars parked at a large shopping centre car park without counting every single one.

13 When conducting research on Indigenous Australians, the elders of the community are often involved. Explain why the community is needed to be involved in the research process.

14 Television ratings are determined by surveying a sample of the population.

- **a** Explain why a sample is taken rather than conducting a census.
- **b** What would be a limitation of the survey results if the sample included 50 people nationwide?
- **c** If a class census is taken on which (if any) television program students watched from 7:30 p.m. to 8:30 p.m. last night, why might the results be different to the official ratings?
- **d** Research how many people are sampled by Nielsen Television Audience Measurement in order to get an accurate idea of viewing habits yet sticking within practical limitations.

15 Australia's census surveys the entire population every 5 years.

- **a** Why might Australia not conduct a census every year?
- **b** Over 40% of all Australians were born overseas or had at least one of their parents born overseas. How does this impact the need to be culturally sensitive when designing and undertaking a census?
- **c** The census can be filled out on a paper form or using the internet. Given that the data must be collated in a computer eventually, why does the government still allow paper forms to be used?
- **d** Why might a country like India or China conduct their national census every 10 years?

16 Write a sentence explaining why two different samples taken from the same population can produce different results. How can this problem be minimised?

ENRICHMENT	–	–	17

Collecting a sample

17 **a** Use a random generator on your calculator or computer to record the number of times the number 1 to 5 appears (you could even use a die by re-rolling whenever you get a 6) out of 50 trials. Record the data.

- **i** Tabulate your results.
- **ii** Compare the results with those of others in the class.
- **iii** Explain why any differences might occur.

b Choose a page at random from a novel or a webpage and count how many times each vowel (A, E, I, O, U) occurs. Assign each vowel the following value A = 1, E = 2, I = 3, O = 4, U = 5 and tabulate your results.

- **i** Why are the results different from those in part **a**?
- **ii** How might the results for the vowels vary depending on the webpage or novel chosen?

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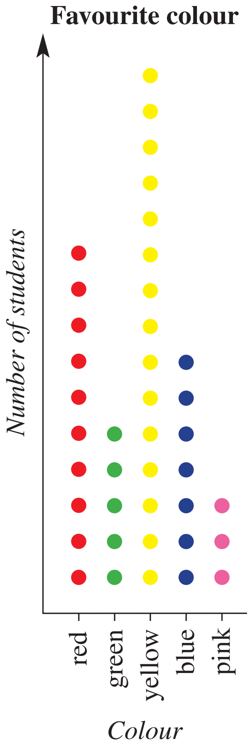
9B Dot plots and column graphs

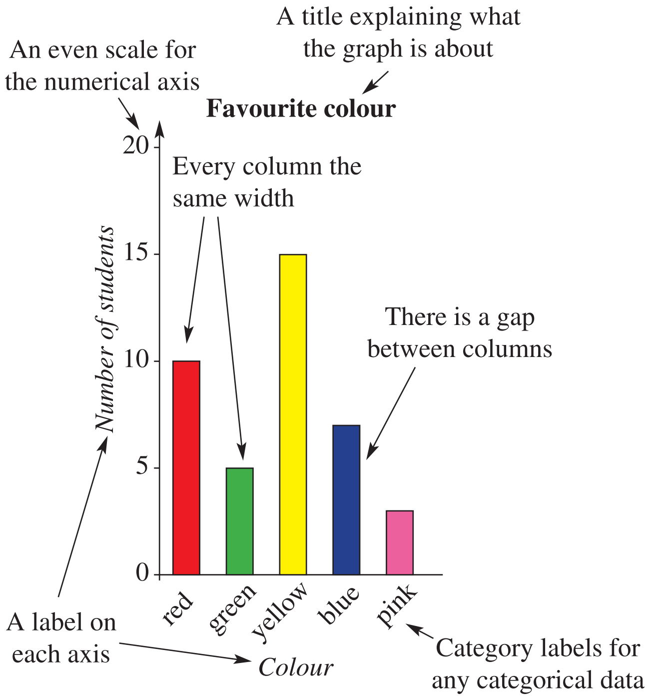
Interactive
Widgets
HOTsheets
Walkthrough

Numerical and categorical data can be shown graphically using **dot plots**, where each value is represented as a filled circle. More commonly, it is represented using **column graphs**, where the height of each column represents a number. Column graphs can be drawn vertically or horizontally.

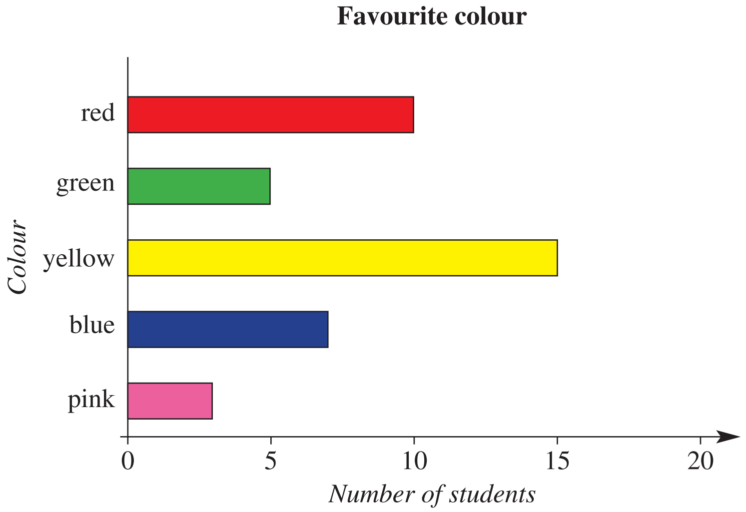
Consider a survey of students who are asked to choose their favourite colour from five possibilities. The results could be represented as a dot plot or as a column graph.



Represented as a dot plot



As a column graph (vertical)



As a column graph (horizontal)

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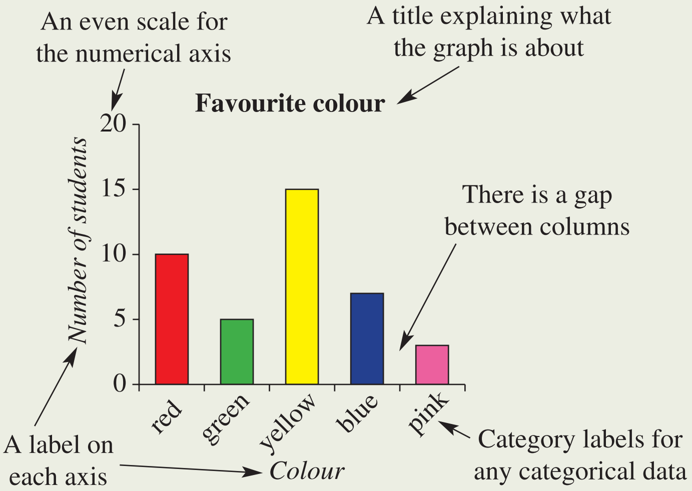
Let's start: Favourite colours

Survey the class to determine each student's favourite colour from the possibilities red, green, yellow, blue and pink.

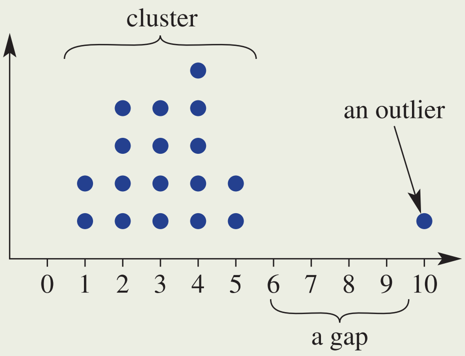
- Each student should draw a column graph or a dot plot to represent the results.
- What are some different ways that the results could be presented into a column graph? (There are more than 200 ways.)

Key ideas

- A **dot plot** can be used to display data, where each dot represents one **data value**. Dots must be aligned horizontally so the height of different columns can be seen easily.
- A **column graph** is an alternative way to show data in different categories, and is useful when more than a few items of data are present.
- Column graphs can be drawn vertically or horizontally. Horizontal column graphs can also be called a bar graph.
- Graphs should have the following features.



- Any numerical axis must be drawn to scale, usually starting from zero.
- An **outlier** is a value that is noticeably distinct from the main cluster of points.



- The mode is the most common response. It can be seen in the tallest column. In the graphs above, the modes are yellow and 4.
- The median is the middle value when the values are sorted from lowest to highest. If the values are 1, 3, 5, 9, 11, then the median is 5.

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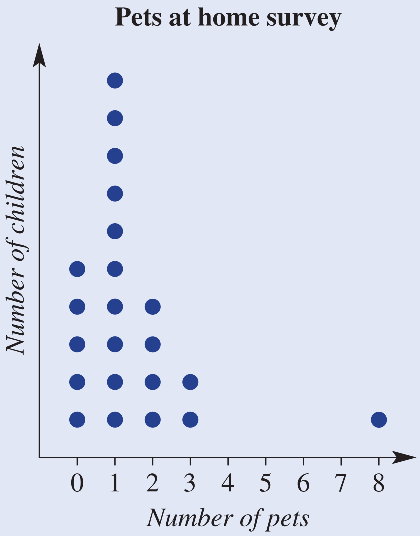
Example 3 Interpreting a dot plot

Some children were asked the following question in a survey: “How many pets do you have at home?” The responses are shown in the dot plot below.



a Use the graph to state how many children have 2 pets.
b How many children participated in the survey?
c What is the range of values?
d What is the median number of pets?
e What is the outlier?
f What is the mode?

SOLUTION	EXPLANATION
a 4 children	There are 4 dots in the ‘2 pets’ category, so 4 children have 2 pets.
b 22 children	The total number of dots is 22.
c $8 - 0 = 8$	Range = highest − lowest In this case, highest $= 8$, lowest $= 0$.
d 1 pet	As there are 22 children, the median is the average of the 11th and 12th value. In this case, the 11th and 12th values are both 1.
e the child with 8 pets	The main cluster of children has between 0 and 3 pets, but the person with 8 pets is significantly outside this cluster.
f 1 pet	The most common response was 1. It has the highest column of dots.

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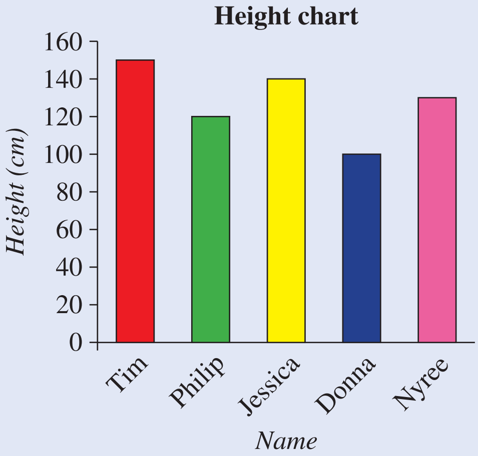
Example 4 Constructing a column graph

Draw a column graph to represent the following people's heights.

Name	Tim	Philip	Jessica	Donna	Nyree
Height (cm)	150	120	140	100	130

SOLUTION



EXPLANATION

First decide which scale goes on the vertical axis. Maximum height = 150 cm, so axis goes from 0 cm to 160 cm (to allow a bit above the highest value).

Remember to include all the features required, including axes labels and a graph title.

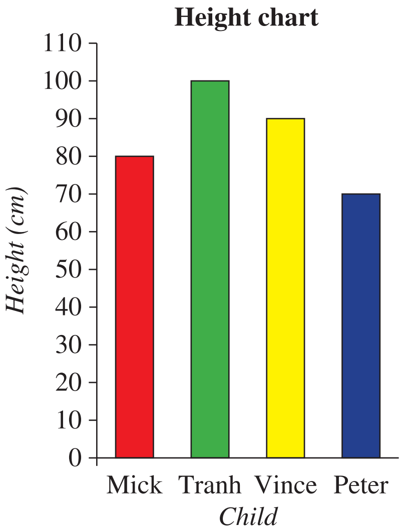
Columns should be of equal width so as not to show bias or be misleading.

Exercise 9B

UNDERSTANDING AND FLUENCY	1–7	2–8	3–8

1 The graph opposite shows the heights of four boys. Answer true or false to each of the following statements.

- **a** Mick is 80 cm tall.
- **b** Vince is taller than Tranh.
- **c** Peter is the shortest of the four boys.
- **d** Tranh is 100 cm tall.
- **e** Mick is the tallest of the four boys.



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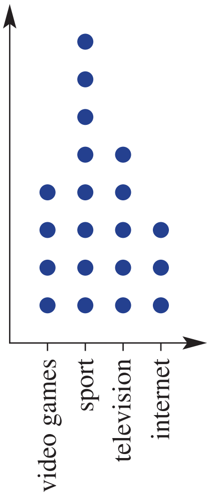
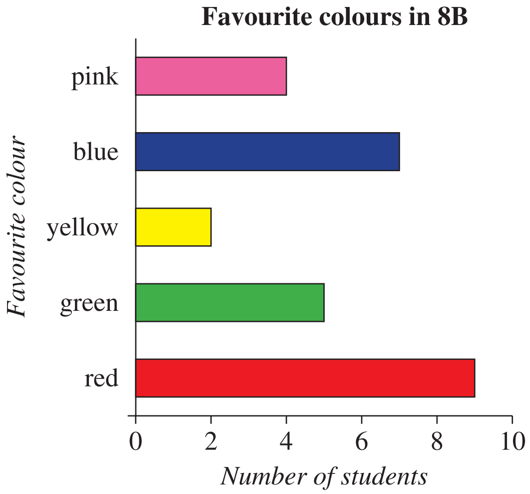
Example 3 **2** The favourite after-school activity of a number of Year 8 students is recorded in the dot plot below.



a How many students have chosen television as their favourite activity?
b How many students have chosen surfing the internet as their favourite activity?
c What is the most popular after-school activity for this group of students?
d How many students participated in the survey?

3 From a choice of pink, blue, yellow, green or red, each student of Year 8B chose their favourite colour. The results are graphed below.



a How many students chose yellow?
b How many students chose blue?
c What is the most popular colour?
d How many students participated in the class survey?
e Represent these results as a dot plot.

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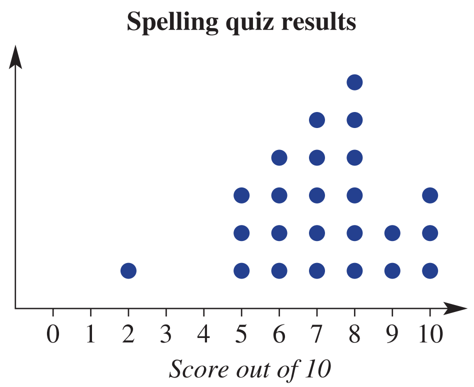
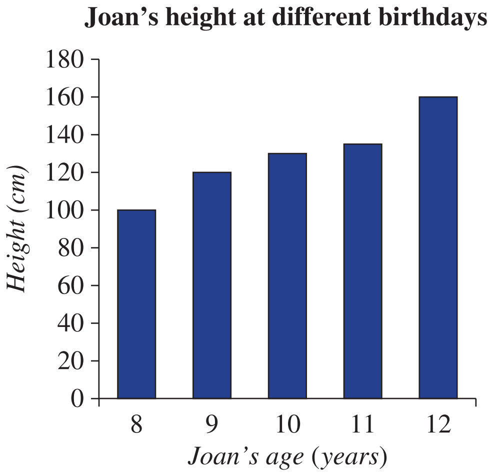
4 In a Year 4 class, the results of a spelling quiz are presented as a dot plot.

- **a** What is the most common score in the class?
- **b** How many students participated in the quiz?
- **c** What is the range of scores achieved?
- **d** What is the median score?
- **e** Identify the outlier.
- **f** What is the mode?
- **g** Are there any gaps in this set of data?



5 Joan has graphed her height at each of her past five birthdays.



- **a** How tall was Joan on her 9th birthday?
- **b** How much did Joan grow between her 8th birthday and 9th birthday?
- **c** How much did Joan grow between her 8th and 12th birthdays?
- **d** How old was Joan when she had her biggest growth spurt?

Example 4

6 Draw a column graph to represent each of these boys' heights at their birthdays.

a Mitchell

Age (years)	Height (cm)
8	120
9	125
10	135
11	140
12	145

b Fatu

Age (years)	Height (cm)
8	125
9	132
10	140
11	147
12	150

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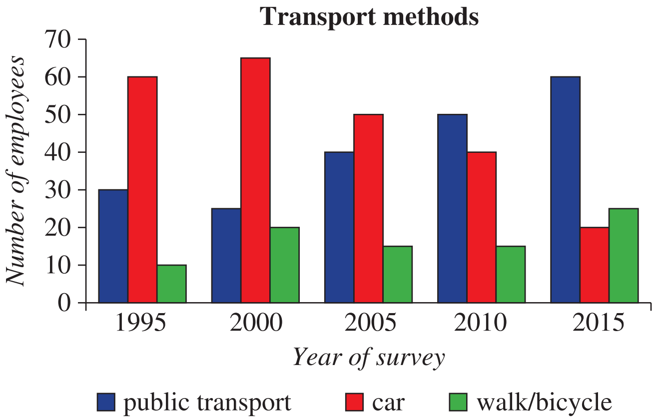
7 Every 5 years, a company in the city conducts a transport survey of people’s preferred method of getting to work in the mornings. The results are graphed below.



a Copy the following table into your workbook and complete it, using the graph.

	1995	2000	2005	2010	2015
Use public transport	30				
Drive a car	60				
Walk or cycle	10				

b In which year(s) is public transport the most popular option?

c In which year(s) are more people walking or cycling to work than driving?

d Give a reason why the number of people driving to work has decreased.

e What is one other trend that you can see from looking at this graph?

8 **a** Draw a column graph to show the results of the following survey of the number of boys and girls born at a certain hospital. Put time (years) on the horizontal axis.

	2000	2001	2002	2003	2004	2005
Number of boys born	40	42	58	45	30	42
Number of girls born	50	40	53	41	26	35

b During which year(s) were there more girls born than boys?

c Which year had the fewest number of births?

d Which year had the greatest number of births?

e During the entire period of the survey, were there more boys or girls born?

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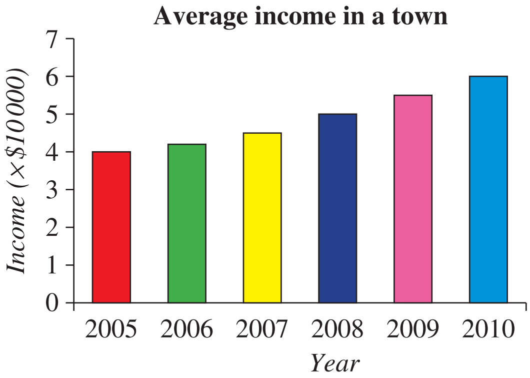
PROBLEM-SOLVING AND REASONING	9, 10, 12	10–12	10–13

9 The average income of adults in a particular town is graphed over a 6-year period.

a Describe in one sentence what has happened to the income over this period of time.
b Estimate what the income in this town might have been in 2004.
c Estimate what the average income might be in 2020 if this trend continues.

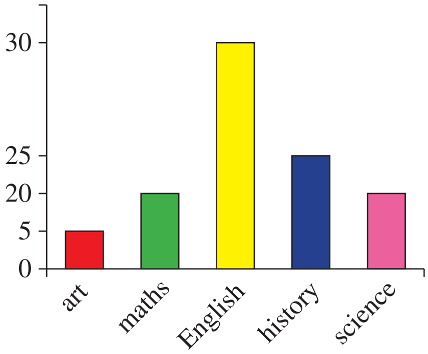


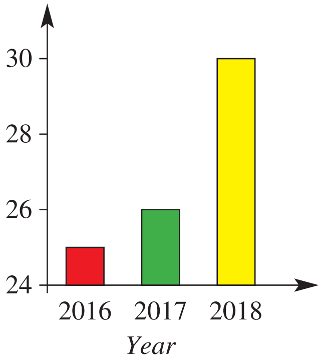
10 Explain why it is important to align dot points in a dot plot. Illustrate your explanation with two dot plots of the set of data below.

Activity	netball	dancing	tennis	chess
Number of students	5	3	2	4

11 A survey is conducted of students' favourite subjects from a choice of art, maths, English, history and science. Someone has attempted to depict the results in a column graph.

a What is wrong with the scale on the vertical axis?
b Give at least two other problems with this graph.
c Redraw the graph with an even scale and appropriate labels.
d The original graph makes maths look twice as popular as art, based on the column size. According to the survey, how many times more popular is maths?
e The original graph makes English look three times more popular than maths. According to the survey, how many times more popular is English?
f An English teacher wishes to show how much his subject has grown in popularity and draws the graph shown. Describe how starting the vertical axis from 24 makes the graph look different from starting the axis at 0.





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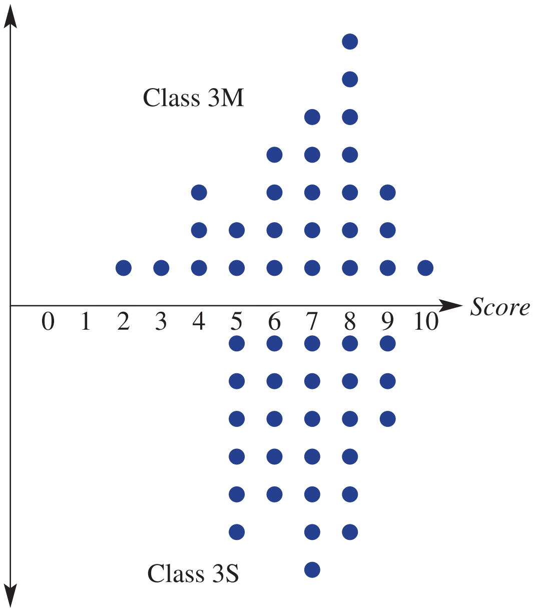
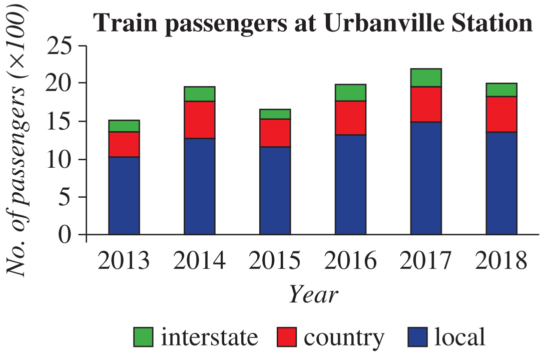
12 Mr Martin and Mrs Stevensson are the two Year 3 teachers at a school. For the latest arithmetic quiz, they have plotted their students' scores on a special dot plot called a parallel dot plot, shown below.

- **a** What is the median score for class 3M?
- **b** What is the median score for class 3S?
- **c** State the range of scores for each class.
- **d** Based on this test, which class has a greater spread of arithmetic abilities?
- **e** If the two classes competed in an arithmetic competition, where each class is allowed only one representative, which class is more likely to win? Justify your answer.



13 At a central city train station, three types of services run: local, country and interstate. The average number of passenger departures during each week is shown in the stacked column graph.



- **a** Approximately how many passenger departures per week were there in 2013?
- **b** Approximately how many passenger departures were there in total during 2018?
- **c** Does this graph suggest that the total number of passenger departures has increased or decreased during the period 2013–18?
- **d** Approximately how many passengers departed from this station in the period 2013–18? Explain your method clearly and try to get your answer within 10 000 of the actual number.

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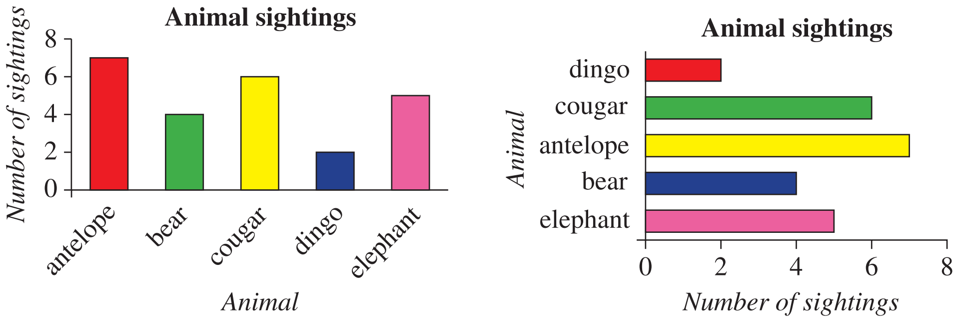
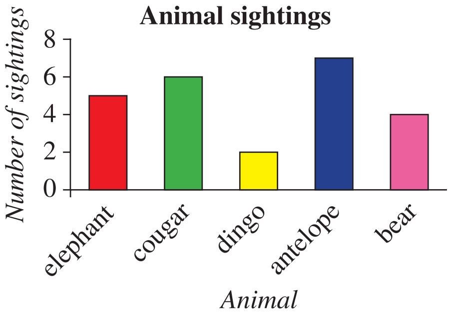
ENRICHMENT	–	–	14

How many ways?

14 As well as being able to draw a graph horizontally or vertically, the order of the categories can be changed. For instance, the following three graphs all represent the same data.





How many different column graphs could be used to represent the results of this survey? (Assume that you can change only the order of the columns, and the horizontal or vertical layout.) Try to list the options systematically to help with your count.

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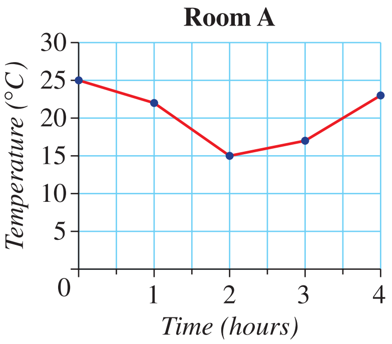
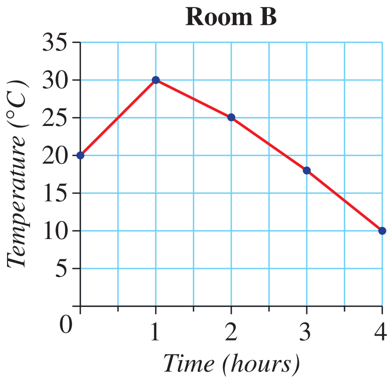
9C Line graphs

A **line graph** is a connected set of points joined with straight line segments. The variables on both axes should be continuous numerical data. It is often used when a measurement varies over time, in which case time is conventionally listed on the horizontal axis. One advantage of a line graph over a series of disconnected points is that it can often be used to estimate values that are unknown.

Let's start: Room temperature

As an experiment, the temperature in two rooms is measured hourly over a period of time. The results are graphed below.

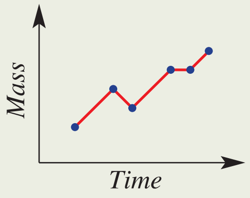




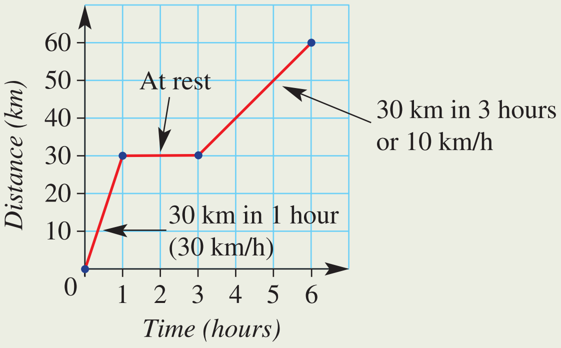
- Each room has a heater and an air conditioner to control the temperature. At what point do you think these were switched on and off in each room?
- For each room, what is the approximate temperature 90 minutes after the start of the experiment?
- What is the proportion of time that room A is hotter than room B?

Key ideas

- A **line graph** consists of a series of points joined by straight line segments. The variables on both axes should be **continuous** numerical data.
- Time is often shown on the horizontal axis:



- A common type of line graph is the distance–time graph studied in Chapter 5. This type of graph is commonly referred to as a travel graph.
  - Time is shown on the horizontal axis.
  - Distance is shown on the vertical axis.
  - The slope or gradient of the line indicates the rate at which the distance is changing over time. This is called **speed**.



Example 5 Drawing a line graph

The temperature in a room is noted at hourly intervals.

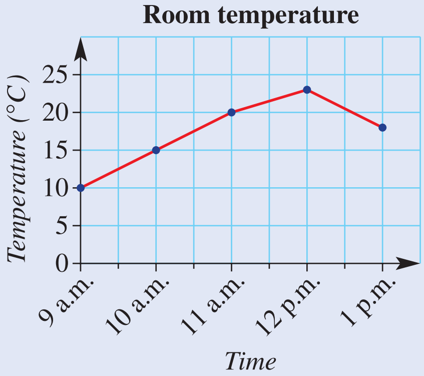
Time	9 a.m.	10 a.m.	11 a.m.	12 p.m.	1 p.m.
Temperature (°C)	10	15	20	23	18

a Present the results as a line graph.

b Use your graph to estimate the room temperature at 12:30 p.m.

SOLUTION

a



b about 20°C

EXPLANATION

- The vertical axis is from 0 to 25. The scale is even (i.e. increasing by 5 each time).
- Dots are placed for each measurement and joined with straight line segments.

By looking at the graph halfway between 12 p.m. and 1 p.m. an estimate is formed.

Example 6 Interpreting a travel graph

This travel graph shows the distance travelled by a cyclist over 5 hours.

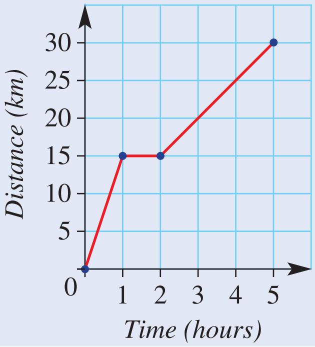
a How far did the cyclist travel in total?

b How far did the cyclist travel in the first hour?

c What is happening in the second hour?

d When is the cyclist travelling the fastest?

e In the fifth hour, how far does the cyclist travel?



SOLUTION	EXPLANATION
a 30 km	The right end point of the graph is at (5, 30).
b 15 km	At time equals 1 hour, the distance covered is 15 km.
c at rest	The distance travelled does not increase in the second hour.
d in the first hour	This is the steepest part of the graph.
e 5 km	In the last 3 hours, the distance travelled is 15 km, so in 1 hour, 5 km is travelled.

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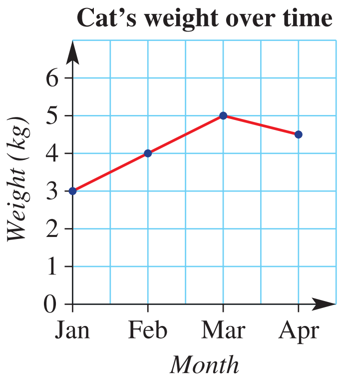
Exercise 9C

UNDERSTANDING AND FLUENCY 1–6 2–7 3–7

1 The line graph shows the weight of a cat over a 3-month period. It is weighed at the start of each month. What is the cat's weight at the start of:

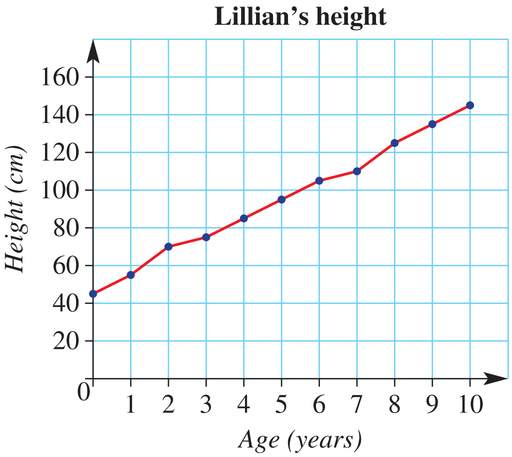
a January?
b February?
c March?
d April?



Example 5a **2** A dog is weighed over a period of 3 months. Draw a line graph of its weight.
January: 5 kg, February: 6 kg, March: 8 kg, April: 7 kg.

Example 5b **3** The graph shows Lillian's height over a 10-year period from when she was born.

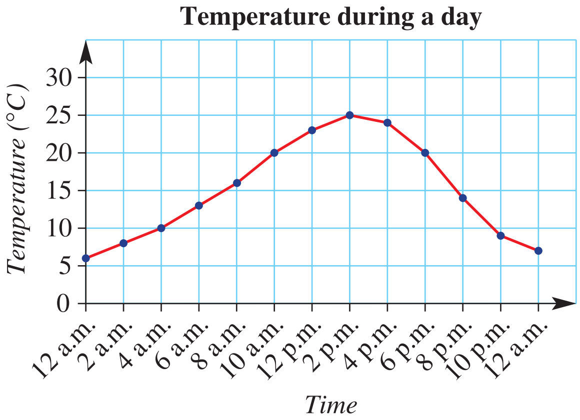
a What was Lillian's height when she was born?
b What was Lillian's height at the age of 7 years?
c At what age did she first reach 130 cm tall?
d How much did Lillian grow in the year when she was 7 years old?
e Use the graph to estimate her height at the age of $9\frac{1}{2}$ years.



4 Consider the following graph, which shows the outside temperature over a 24-hour period that starts at midnight.

a What was the temperature at midday?
b When was the hottest time of the day?
c When was the coolest time of the day?
d Use the graph to estimate the temperature at these times of the day.

i 4 a.m. **ii** 9 a.m.
iii 1 p.m. **iv** 3:15 p.m.



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5 Oliver weighed his pet dog on the first day of every month. He obtained the following results.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Weight (kg)	7	7.5	8.5	9	9.5	9	9.2	7.8	7.8	7.5	8.3	8.5

a Draw a line graph showing this information, make sure the vertical axis has an equal scale from 0 kg to 10 kg.

b Describe any trends or patterns that you see.

c Oliver put his dog on a weight loss diet for a period of 3 months. When do you think the dog started the diet? Justify your answer.

d Use a digital technology (e.g. a spreadsheet) to construct a line graph of the dog's weight.

Example 6

6 This travel graph shows the distance travelled by a van over 6 hours.

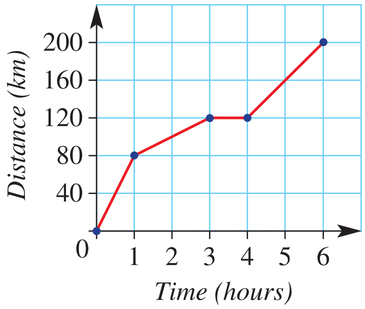
a How far did the van travel in total?

b How far did the van travel in the first hour?

c What is happening in the fourth hour?

d When is the van travelling the fastest?

e In the sixth hour, how far does the van travel?

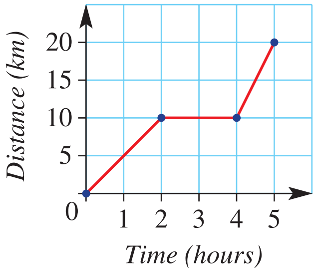


7 This travel graph shows the distance travelled by a bushwalker over 5 hours.

a For how long was the bushwalker at rest?

b How far did the bushwalker walk in the second hour?

c During which hour did the bushwalker walk the fastest?



PROBLEM-SOLVING AND REASONING	8, 10	8, 10	9, 10

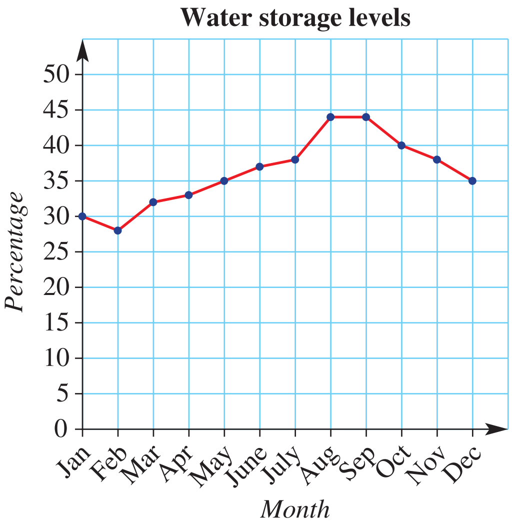
8 The water storage levels for a given city are graphed based on the percentage of water available. For this question, assume that the amount of water that people use in the city does not change from month to month.

a During which month did it rain the most in this city?

b At what time(s) in the year did the water storage fall below 40%?

c From August to September, it rained a total of 20 megalitres of water. How much water did the people in the city use during this period?

d Is it more likely that this city is located in the Northern Hemisphere or the Southern Hemisphere? Justify your answer.



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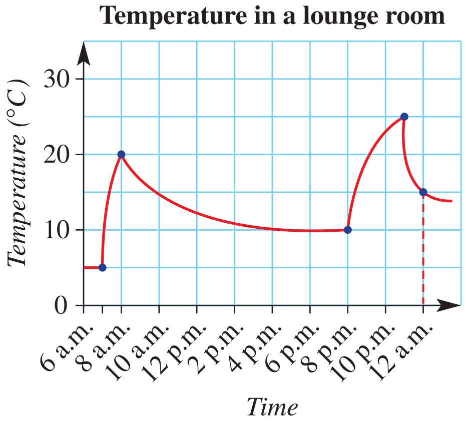
9 The temperature in a lounge room is measured frequently throughout a particular day. The results are presented in a line graph, as shown. The individual points are not indicated on this graph to reduce clutter.



a Twice during the day the heating was switched on. At what times do you think this happened? Explain your reasoning.

b When was the heating switched off? Explain your reasoning.

c The house has a single occupant, who works during the day. Describe when you think that person is:

- **i** waking up
- **ii** going to work
- **iii** coming home
- **iv** going to bed

d These temperatures were recorded during a cold winter month. Draw a graph that shows what the lounge room temperature might look like during a hot summer month. Assume that the room has an air conditioner, which the person is happy to use when at home.

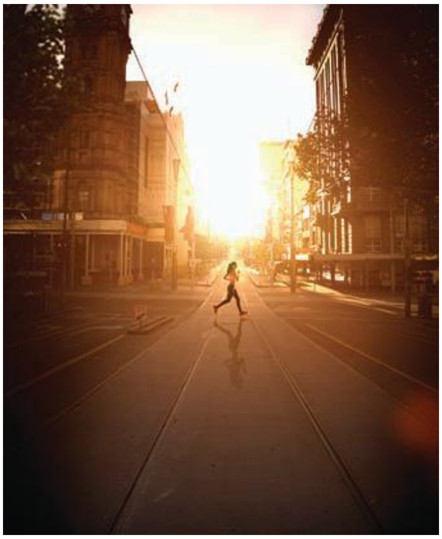
10 Draw travel graphs to illustrate the following journeys.

a A car travels:

- 120 km in the first two hours
- 0 km in the third hour
- 60 km in the fourth hour
- 120 km in the fifth hour

b A jogger runs:

- 12 km in the first hour
- 6 km in the second hour
- 0 km in the third hour
- at a rate of 6 km per hour for two hours



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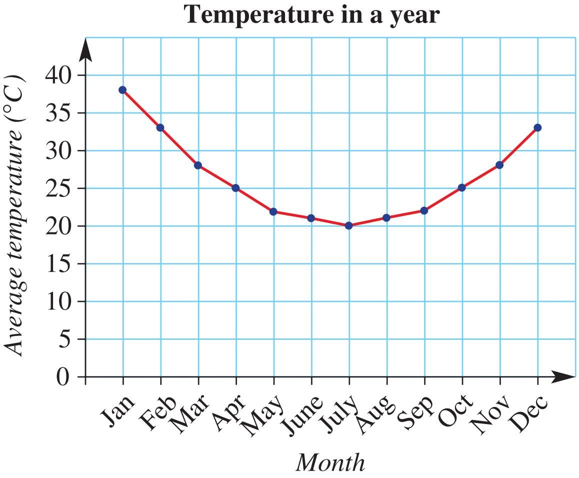
ENRICHMENT	–	–	11

Which hemisphere?

11 The following line graph shows the average monthly temperature in a city.



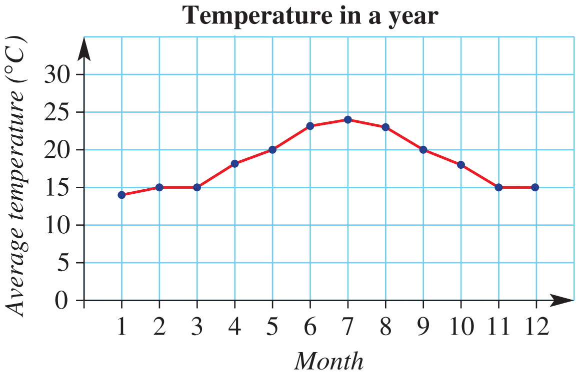
a Is this city in the Northern or Southern Hemisphere? Explain why.

b Is this city close to the equator or far from the equator? Explain why.

c Use a digital technology (e.g. a spreadsheet) to redraw the graph to start the 12-month period at July and finish in June.

d Describe how the new graph's appearance is different from the one shown above.

e In another city, somebody graphs the temperature over a 12-month period, as shown below. In which hemisphere is this city likely to be? Explain your answer.



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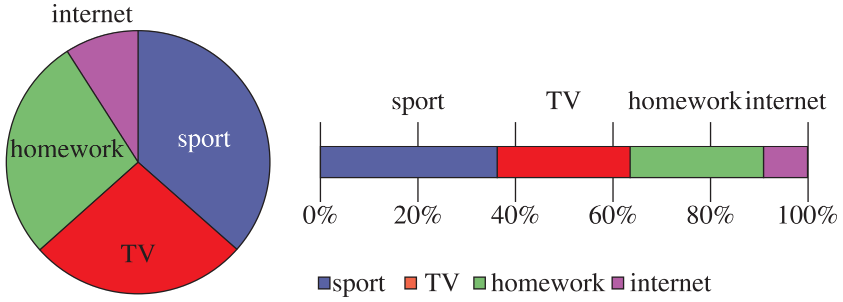
9D Sector graphs and divided bar graphs

Interactive
Widgets
HOTsheets
Walkthrough

A sector graph (also called a pie chart) consists of a circle divided into different sectors or ‘slices of pie’, where the size of each sector indicates the proportion occupied by any given item. A divided bar graph is a rectangle divided into different rectangles or ‘bars’, where the size of each rectangle indicates the proportion of each item. Both types of graphs are suitable for categorical but not numerical data.

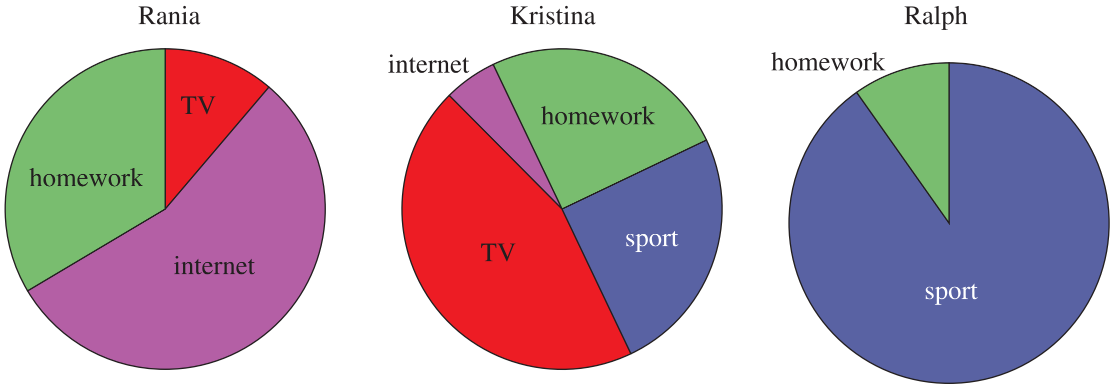
If a student is asked to describe how much time they spend each evening doing different activities, they could present their results as either type of graph:



From both graphs, it is easy to see that most of the student’s time is spent playing sport and the least amount of time is spent using the internet.

Let’s start: Student hobbies

Rania, Kristina and Ralph are asked to record how they spend their time after school. They draw the following graphs.



- Based on these graphs alone, describe each student in a few sentences.
- Justify your descriptions based on the graphs.

Key ideas

- To calculate the size of each section of the graph, divide the value in a given category by the sum of all category values. This gives the category’s proportion or fraction.
- To draw a **sector graph** (also called a **pie chart**), multiply each category’s proportion or fraction by 360° and draw a sector of that size.
- To draw a **divided bar graph**, multiply each category’s proportion or fraction by the total width of the rectangle and draw a rectangle of that size.

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Example 7 Drawing a sector graph and a divided bar graph

On a particular Saturday, Sanjay measured the number of hours he spent on different activities.

TV	internet	sport	homework
1 hour	2 hours	4 hours	3 hours

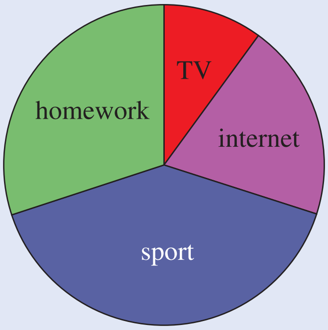
Represent the table as:

a a sector graph

b a divided bar graph

SOLUTION

a

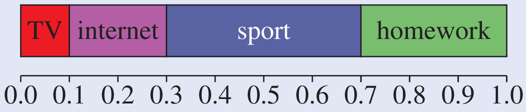


EXPLANATION

The total amount of time is $1 + 2 + 4 + 3 = 10$ hours. Then we can calculate the proportions and sector sizes:

Category	Proportion	Sector size (°)
TV	$\frac{1}{10} = 0.1 = 10\%$	$\frac{1}{10} \times 360 = 36$
internet	$\frac{2}{10} = 0.1 = 20\%$	$\frac{2}{10} \times 360 = 72$
sport	$\frac{4}{10} = 0.4 = 40\%$	$\frac{4}{10} \times 360 = 144$
homework	$\frac{3}{10} = 0.3 = 30\%$	$\frac{3}{10} \times 360 = 108$

b



Using the same proportions calculated above, make sure that each rectangle takes up the correct amount of space. For example, if the total width is 15 cm, then sport occupies $\frac{2}{5} \times 15 = 6$ cm.

Exercise 9D

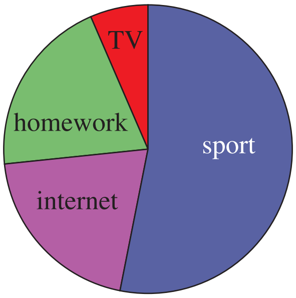
UNDERSTANDING AND FLUENCY	1–4	2–5	3–5

1 Jasna graphs a sector graph of how she spends her leisure time.

a What does Jasna spend the most time doing?

b What does Jasna spend the least time doing?

c Does she spend more or less than half of her time playing sport?



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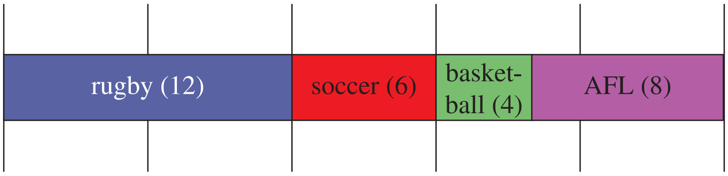
2 Thirty students are surveyed to find out their favourite sport and their results are graphed below.



a What is the most popular sport for this group of students?
b What is the least popular sport for this group of students?
c What fraction of the students has chosen soccer as their favourite sport?
d What fraction of the students has chosen either rugby or AFL?

Example 7a

3 A group of passengers arriving at an airport is surveyed to establish which countries they have come from. The results are presented below.

Country	China	United Kingdom	USA	France
No. of passengers	6	5	7	2

a What is the total number of passengers who participated in the survey?
b What proportion of the passengers surveyed have come from the following countries? Express your answer as a fraction.
 i China **ii** United Kingdom **iii** USA **iv** France
c On a sector graph, determine the angle size of the sector representing:
 i China **ii** United Kingdom **iii** USA **iv** France
d Draw a sector graph showing the information calculated in part **c**.

Example 7b

4 A group of students in Years 7 and 8 is polled on their favourite colour, and the results are shown at right.

Colour	**Year 7 votes**	**Year 8 votes**
red	20	10
green	10	4
yellow	5	12
blue	10	6
pink	15	8

a Draw a sector graph to represent the Year 7 colour preferences.
b Draw a different sector graph to represent the Year 8 colour preferences.
c Describe two differences between the charts.
d Construct a divided bar graph that shows the popularity of each colour across the total number of Years 7 and 8 students combined.

5 Consider the following results of a study on supermarket shopping habits.

Items	food	drinks	household items	other
Proportion of money spent	50%	25%	20%	5%

a Represent this information in a divided bar graph.
b Graph this information as a sector graph.
c Use a digital technology (e.g. a spreadsheet) to create each type of graph and describe how your answers to parts **a** and **b** differ from the technology's output.

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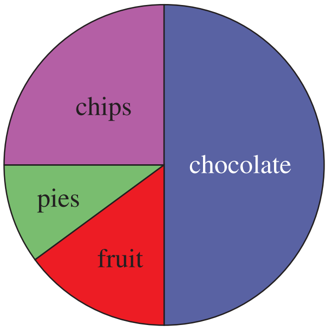
PROBLEM-SOLVING AND REASONING | 6, 9, 10 | 7, 8, 10 | 9–12

6 A group of Year 7 students was polled on their favourite foods, and the results are shown in this sector graph.



a If 40 students participated in the survey, find how many of them chose:

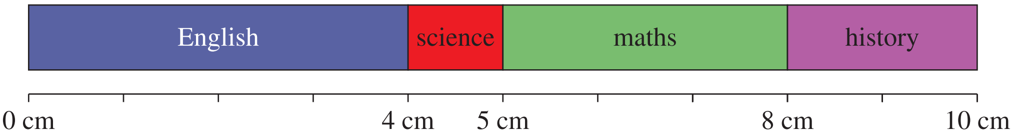
i chocolate **ii** chips **iii** fruit **iv** pies

b Health experts are worried about what these results mean. They would like fruit to appear more prominently in the sector graph, and to not have the chocolate sector next to the chips. Redraw the sector graph so this is the case.

c Another 20 students were surveyed. Ten of these students chose chocolate and the other 10 chose chips. Their results are to be included in the sector graph. Of the four sectors in the graph, state which sector will:

i increase in size **ii** decrease in size **iii** stay the same size

7 Yakob has asked his friends what is their favourite school subject, and he has created the following divided bar graph from the information.



a Calculate the percentage of the whole represented by:

i English **ii** maths **iii** history

b If Yakob surveyed 30 friends, state how many of them like:

i maths best **ii** history best **iii** either English or science best

c Redraw these results as a sector graph.

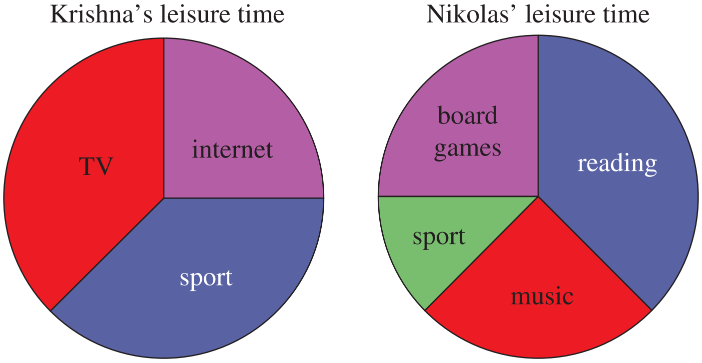
8 Friends Krishna and Nikolas have each graphed their leisure habits, as shown below.

a Which of the two friends spends more of their time playing sport?

b Which of the two friends does more intellectual activities in their leisure time?

c Krishna has only 2 hours of leisure time each day because he spends the rest of his time doing homework. Nikolas has 8 hours of leisure time each day. How does this affect your answers to parts **a** and **b** above?

d Given that Krishna's TV time and sport time are equal, what percentage of his leisure time does he spend watching TV?



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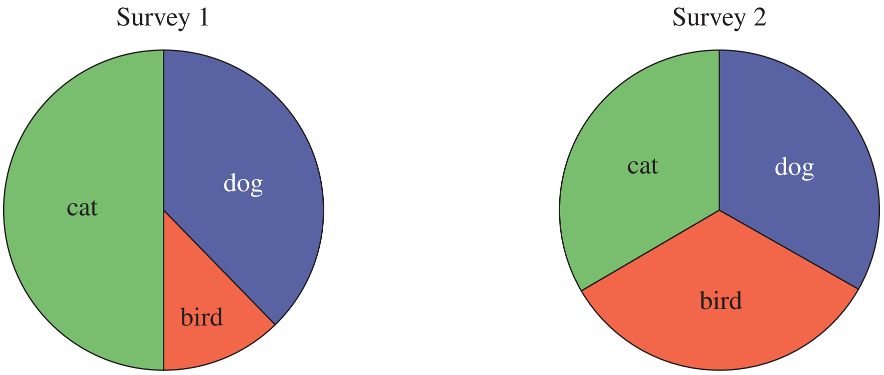
9 In two surveys, people were asked what is their favourite pet animal.



a If 16 people participated in survey 1, how many chose a dog?

b If 30 people participated in survey 2, how many chose a bird?

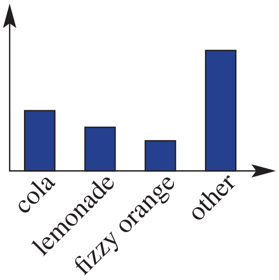
c Jason claims that 20 people participated in survey 1. Explain clearly why this cannot be true.

d Jaimee claims that 40 people participated in survey 2. Explain clearly why this cannot be true.

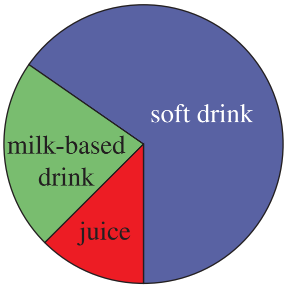
e In actual fact, the same number of people participated for each survey. Given that fewer than 100 people participated, how many participants were there? Give all the possible answers.

10 Explain why you can use a sector graph for categorical data but you cannot use a line graph for categorical data.

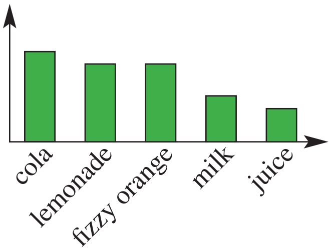
11 Three different surveys are conducted to establish whether soft drinks should be sold in the school canteen.



Survey 1: Favourite drink



Survey 2: Favourite type of drink



Survey 3: Sugar content per drink

a Which survey's graph would be the most likely to be used by someone who wished to show the financial benefit to the cafeteria of selling soft drinks?

b Which survey's graph would be the most likely to be used by someone who wanted to show there was not much desire for soft drink?

c Which survey's graph would be the most likely to be used by a person wanting to show how unhealthy soft drink is?

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12 The 'water footprint' of different foods refers to the volume of fresh water that is used to produce the food. The water footprint of some foods is shown in the table below.

Food	bread	cheese	chicken	cucumber	lettuce	milk	potato	rice
Footprint (L/kg)	1608	3178	4325	353	237	1800	287	2497

a What type(s) of graph could be used for the data above? Justify your choice(s).

b Choose a suitable type of graph and depict the above numbers graphically.

c How is a food's water footprint related to how sustainable it is to produce?

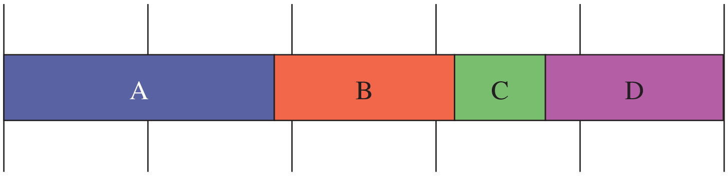
d Estimate how many litres of water would be used for a chicken burger. Include your estimates of each item's weight.

e Another way to present the data is to say how many grams of each food is made from 1 kilolitre of water. Redraw the table above with a row for 'water efficiency', in g/kL.

ENRICHMENT	–	–	13

Rearranging graphs

13 Consider the divided bar graph shown below.



a Show how this graph will look if the segments are placed in the order C, D, A, B (from left to right).

b In how many different ways could this divided bar graph be drawn (counting ABCD and CDAB as two different ways)?

c If this bar graph is redrawn as a sector graph, how many ways could the segments be arranged? Try to list them systematically. Do not consider two sector graphs to be different if one is just a rotation of another.

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9E Frequency distribution tables

Interactive
Widgets
HOTsheets
Walkthrough

Often the actual values in a set of data are not required—just knowing how many numbers fall into different ranges is often all the information that is needed. A **frequency distribution table** allows us to do this by listing how common the different values are.

Frequency distribution tables can be used for listing particular values or ranges of values.

Number of cars	Frequency
0	10
1	12
2	5
3	3

Age	Frequency
0–4	7
5–9	12
10–14	10
15–19	11

Let's start: Subject preferences

- Survey a group of peers to find their favourite school subject out of maths, English, science, music and sport.
- Represent your results in a table like the one below.

	Maths	English	Science	Music	Sport
Tally	𝍸	𝍸|	𝍸|||	||||	||
Frequency	5	6	8	4	2

- How would you expect the results to differ for different classes at your school, or for different schools?

Frequency tables can be used to show students' subject preferences.

Key ideas

- A **tally** is a tool used for counting as results are gathered. Numbers are written as vertical lines with every 5th number having a cross through a group of lines. For example, 4 is |||| and 7 is 𝍸||.
- A **frequency distribution table** shows how common a certain value is in a frequency column. A tallying column is also often used as data is gathered.
- The items can be individual values or intervals of values.

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Example 8 Interpreting tallies

a The different car colours along a quiet road are noted. Convert the following tally into a frequency distribution table.

White	Black	Blue	Red	Yellow
|||	𝍸𝍸|||	𝍸𝍸𝍸||	𝍸|	𝍸||||

b Hence, state how many red cars were spotted.

SOLUTION

a

Colour	white	black	blue	red	yellow
Frequency	3	13	17	6	9

b Six red cars were spotted.

EXPLANATION

Each tally is converted into a frequency.
For example, black is two groups of 5 plus 3, giving 10 + 3 = 13.

This can be read directly from the table.

Example 9 Constructing tables from data

Put the following data into a frequency distribution table: 1, 4, 1, 4, 1, 2, 3, 4, 6, 1, 5, 1, 2, 1.

SOLUTION

Number	1	2	3	4	5	6
Tally	𝍸|	||	|	|||	|	|
Frequency	6	2	1	3	1	1

EXPLANATION

Construct the tally as you read through the list. Then go back and convert the tally to frequencies.

Exercise 9E

UNDERSTANDING AND FLUENCY	1–5	2–6	3–6

1 The table below shows survey results for students' favourite colours.

Colour	Frequency
red	5
green	2
orange	7
blue	3

Classify the following as true or false.

a Five people chose red as their favourite colour.
b Nine people chose orange as their favourite colour.
c Blue is the favourite colour of three people.
d More people chose green than orange as their favourite colour.

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2 Fill in the blanks.

a The tally |||| represents the number ____.

b The tally 𝍸|| represents the number ____.

c The tally ____ represents the number 2.

d The tally ____ represents the number 11.

Example 8

3 A basketball player’s performance in one game is recorded in the following table.

	Passes	Shots at goal	Shots that go in	Steals
Tally	|||	𝍸𝍸|||	𝍸||	||
Frequency				

a Copy and complete the table, filling in the frequency row.

b How many shots did the player have at the goal?

c How many shots went in?

d How many shots did the player miss during the game?

Example 9

4 A student surveys her class to ask how many people are in their family.
The results are:

6, 3, 3, 2, 4, 5, 4, 5, 8, 5, 4, 8, 6, 7, 6, 5, 8, 4, 7, 6

a Construct a frequency distribution table. Include a tally row and a frequency row.

b How many students have exactly five people in their family?

c How many students have at least six people in their family?

5 Braxton surveyed a group of people to find out how much time they spent watching television last week. They gave their answers rounded to the nearest hour.

Number of hours	0–4	5–9	10–14	15–19	20–24	25 or more
Tally	𝍸|||	𝍸𝍸||	𝍸𝍸𝍸	𝍸||||	||||	||

a Draw a frequency distibution table of Braxton’s results, converting the tallies to numbers.

b How many people did Braxton survey?

c How many people spent 15–19 hours watching television last week?

d How many people watched television for less than 5 hours last week?

6 The heights of a group of 21 people are shown below, given to the nearest cm.

174	179	161	132	191	196	138	165	151	178	189
147	145	145	139	157	193	146	169	191	145	

a Copy and complete the frequency distribution table below.

Height (cm)	130–139	140–149	150–159	160–169	170–179	180–189	190+
Tally							
Frequency							

b How many people are in the range 150–159 cm?

c How many people are 180 cm or taller?

d How many people are between 140 cm and 169 cm tall?

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PROBLEM-SOLVING AND REASONING	7, 8, 10	7, 8, 10, 11	8, 9, 11, 12

7 A tennis player records the number of double faults they serve per match during one month.

Double faults	0	1	2	3	4	5
Frequency	4	2	1	0	2	1

a How many matches did they play in total during the month?

b How many times did they serve exactly one double fault?

c In how many matches did they serve no double faults?

d How many double faults did they serve in total during the month?

8 Match each of these datasets with the correct column (A, B, C or D) in the frequency distribution table shown below.

a 1, 1, 2, 3, 3

b 1, 2, 2, 2, 3

c 1, 1, 1, 2, 3

d 1, 2, 3, 3, 3

Number	**A**	**B**	**C**	**D**
1	3	2	1	1
2	1	1	1	3
3	1	2	3	1

9 Five different classes are in the same building in different rooms. The ages of students in each room are recorded in the frequency distribution table below.

Age	**Room A**	**Room B**	**Room C**	**Room D**	**Room E**
12	3	2	0	0	0
13	20	18	1	0	0
14	2	4	3	0	10
15	0	0	12	10	11
16	0	0	12	10	11
17	0	0	0	1	0

a How many students are in the building?

b How many 14-year-olds are in the building?

c How many students are in room B?

d What is the total age of all the people in room B?

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10 Some exam results are presented in the frequency distribution table below.

Score	**0–9**	**10–19**	**20–29**	**30–39**	**40–49**	**50–59**	**60–69**	**70–79**	**80–89**	**90–100**
Frequency	0	0	3	1	2	5	8	12	10	2

a Redraw the table so that the intervals are of width 20 rather than 10 (i.e. so the first column is 0–19, the second is 20–39, and so on).

b Show how the table would look if it were drawn with the intervals 0–29, 30–59, 60–89, 90–100.

c Explain why it is not possible to fill out the frequency distribution table below without additional information.

Score	0–24	25–49	50–74	75–100
Frequency				

d You are told that two students got less than 25, and 15 students got scores between 60 and 74. Use this information to fill out the frequency distribution table above.

e What does the frequency row add to in all of the tables you have drawn?

11 **a** How could the stem-and-leaf plot below be represented as a frequency distribution table using intervals 10–19, 20–29 and 30–39? (Remember that in a stem-and-leaf plot 1 | 2 represents the number 12, 3 | 5 represents 35, and so on.)

Stem	**Leaf**
1	258
2	1799
3	557789

b Give an example of a different stem-and-leaf plot that would give the same frequency distribution table.

c Which contains more information: a stem-and-leaf plot or a frequency distribution table?

d When would a frequency distribution table be more appropriate than a stem-and-leaf plot?

12 An AFL player notes the number of points he scores in his 22-week season.

Points	**Frequency**
0–4	3
5–9	11
10–14	6
15–19	1
20–24	1

a Write out one possible list, showing the scores he got for each of the 22 weeks.

b The list you wrote has a number from 10 to 14 written at least twice. Explain why this must be the case, regardless of what list you chose.

c Your list contains a number from 5 to 9 at least three times. Why must this be the case?

d Apart from the two numbers in parts **b** and **c**, is it possible to make a list that has no other repeated values?

e Is it possible to play 22 games and always score a different number of points?

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ENRICHMENT	–	–	13

Homework puzzle

13 Priscilla records the numbers of hours of homework she completes each evening from Monday to Thursday. Her results are shown in this frequency distribution table.

Number of hours	Frequency
1	1
2	1
3	2

a One possibility is that Priscilla worked 3 hours on Monday, 2 hours on Tuesday, 3 hours on Wednesday and 1 hour on Thursday. Give an example of another way her time could have been allocated for the four nights.

b If Priscilla spent at least as much time doing homework each night as on the previous night, how long did she spend on Tuesday night?

c How many ways are there of filling in the table below to match her description?

Monday	Tuesday	Wednesday	Thursday
☐ hours	☐ hours	☐ hours	☐ hours

d If you know Priscilla spent two hours on Monday night, how many ways are there to fill in the table?

e If you know Priscilla spent three hours on Monday night, how many ways are there to fill in the table?

f Priscilla's brother Joey did homework on all five nights. On two nights he worked for 1 hour, on two nights he worked for 2 hours and on one night he worked for 3 hours. In how many ways could the table below be filled in to match his description?

Monday	Tuesday	Wednesday	Thursday	Friday
☐ hours	☐ hours	☐ hours	☐ hours	☐ hours

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9F Frequency histograms and frequency polygons

Interactive
Widgets
HOTsheets
Walkthrough

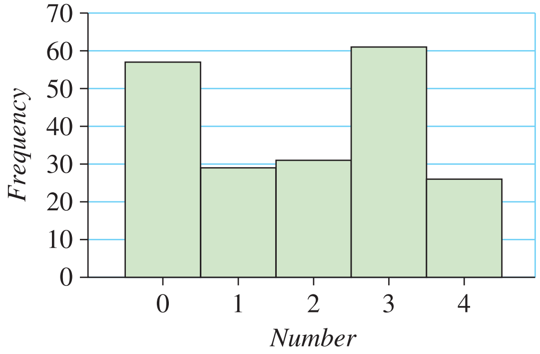
Frequency histograms and **frequency polygons** are graphical representations of a frequency distribution table so that patterns can be observed more easily.

For example, the data below is represented as a frequency distribution table, a histogram and a frequency polygon.

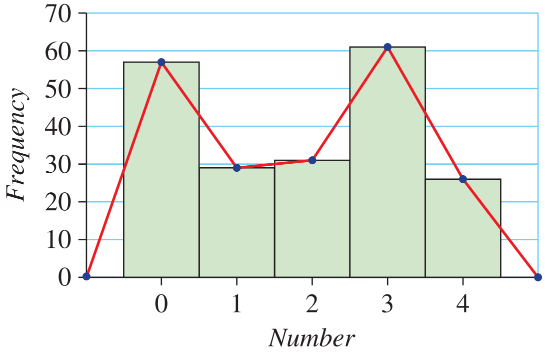
As a table

Number	Frequency
0	57
1	29
2	31
3	61
4	26

As a frequency histogram



As a frequency polygon

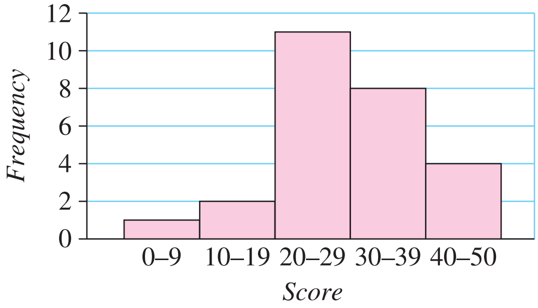


At a glance you can see from the histogram that 0 and 3 are about twice as common as the other values. This is harder to read straight from the table. Histograms are often used in digital cameras and photo editing software to show the brightness of a photo.

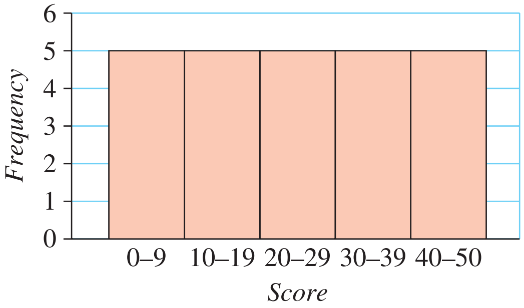
Let's start: Test analysis

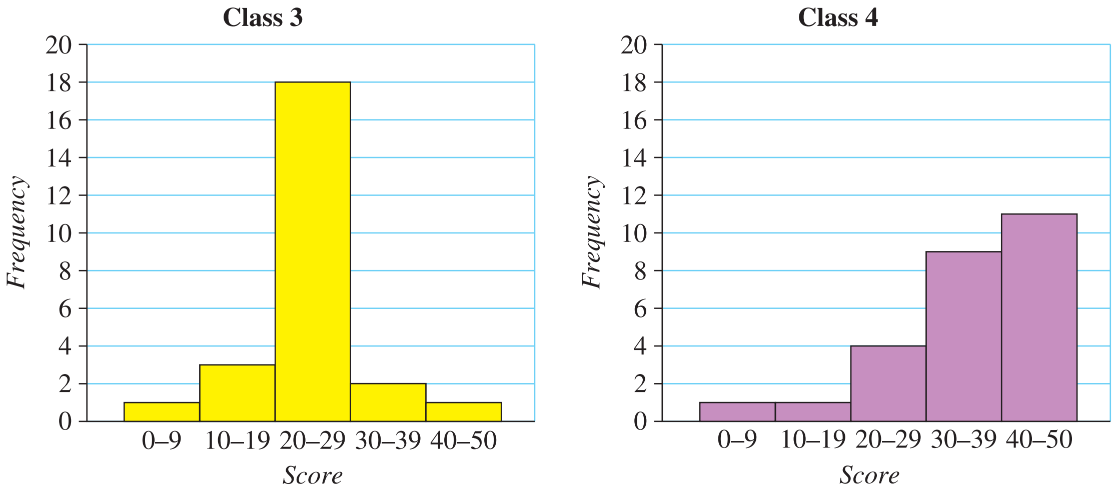
The results for some end-of-year tests are shown for four different classes, in four different histograms below.

Class 1



Class 2



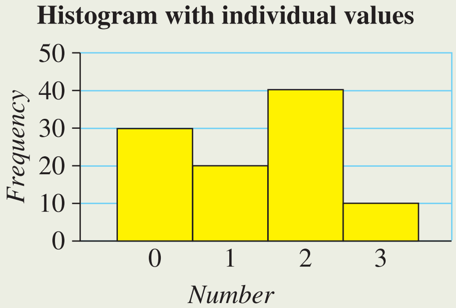


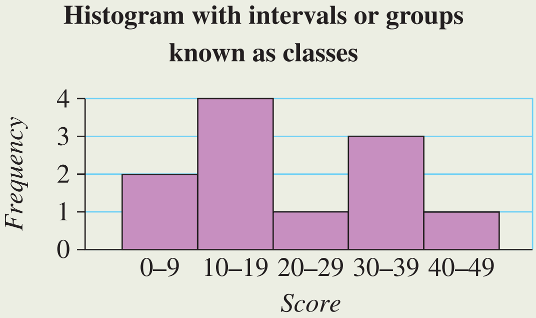
- Describe each class on the basis of these graphs.
- Which class has the highest average score?
- Which class has the highest overall score?
- Which class would be the easiest to teach and which would be the hardest, do you think?

Key ideas

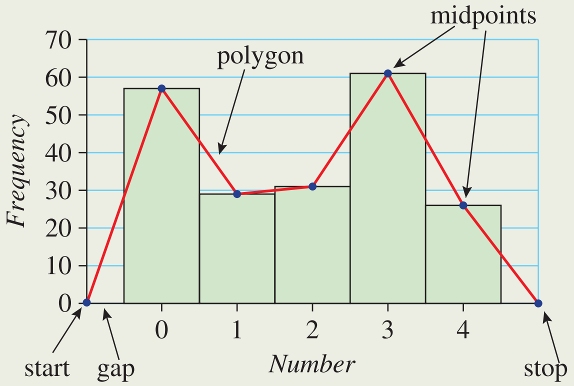
- A **histogram** is a graphical representation of a frequency distribution table. It can be used when the items are numerical.
- The vertical axis (y-axis) is used to represent the frequency of each item.
- Columns are placed next to one another with no gaps in between.
- Columns are of equal width with the score or class centre in the middle of each column.

For example:





- A half-column width space is placed between the vertical axis and the first column.
- A frequency polygon begins and ends on the horizontal axis and joins the midpoints of the tops of the columns.



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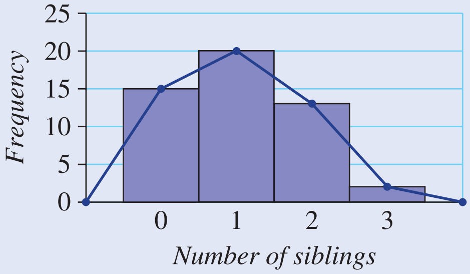
Example 10 Constructing histograms from tables

Represent the frequency distribution table below as a histogram and a polygon. Ensure an appropriate scale and label is chosen for each axis.

Number of siblings	Frequency
0	15
1	20
2	13
3	2

SOLUTION



EXPLANATION

The scale 0–25 is chosen to fit the highest frequency (20) on the vertical axis, and numbers 0–3 on the horizontal, to include all values.

Each different number of siblings in the frequency distribution table is given a column in the graph with a half-column width gap at the left.

Note the starting and ending point of the polygon.

Exercise 9F

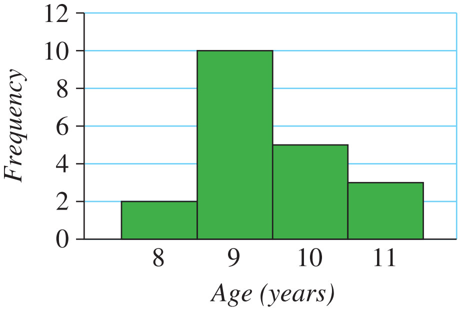
UNDERSTANDING AND FLUENCY	1–4	2–4	3, 4

1 The histogram shows the ages of people in an art class.

a How many 8-year-olds are in this class?

b What is the most common age for students in this class?

c What is the age of the oldest person in the class?

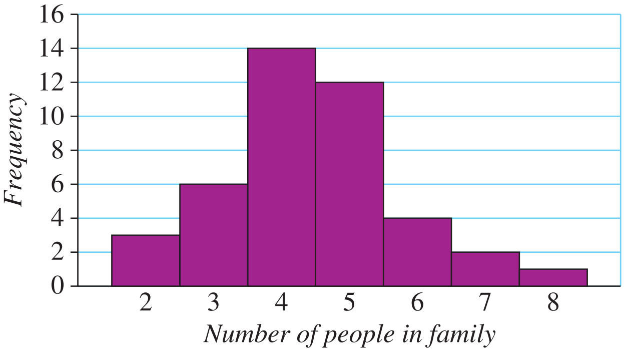


2 A survey is conducted of the number of people in different families. The results are shown.

a What is the most likely number of people in a family, on the basis of this survey?

b How many people responding to the survey said they had a family of 6?

c What is the least likely number (from 2 to 8) of people in a family, on the basis of this survey?



d Fill in the gaps:

By joining the centre of each column in the histogram we create a _____________ _____________.

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Example 10 **3** Represent the following frequency distribution tables as histograms. Ensure that appropriate scales and labels are chosen for the axes.

a

Number	Frequency
0	5
1	3
2	5
3	2
4	4

b

Number	Frequency
0	3
1	9
2	3
3	10
4	7

c

Age	Frequency
12	15
13	10
14	25
15	20
16	28

d

Number of cars	Frequency
0	4
1	5
2	4
3	2

4 For the following sets of data:

i Create a frequency distribution table. **ii** Hence, draw a histogram and a polygon.

a 1, 2, 5, 5, 3, 4, 4, 4, 5, 5, 5, 1, 3, 4, 1

b 5, 1, 1, 2, 3, 2, 2, 3, 3, 4, 3, 3, 1, 1, 3

c 4, 3, 8, 9, 7, 1, 6, 3, 1, 1, 4, 6, 2, 9, 7, 2, 10, 5, 5, 4

d 60, 52, 60, 59, 56, 57, 54, 53, 58, 56, 58, 60, 51, 52, 59, 59, 52, 60, 50, 52

PROBLEM-SOLVING AND REASONING

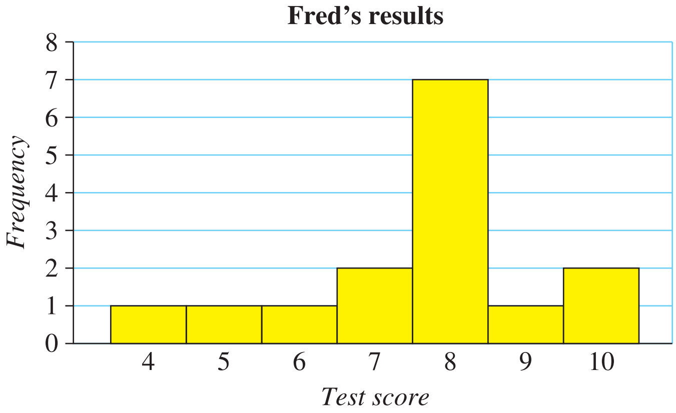
5, 6	6–8	7–10

5 Edwin records the results for his spelling tests out of 10. They are:

3, 9, 3, 2, 7, 2, 9, 1, 5, 7, 10, 6, 2, 6, 4.

a Draw a histogram for Edwin's results.

b Is Edwin a better or a worse speller generally than Fred, whose results are given by the histogram shown below?



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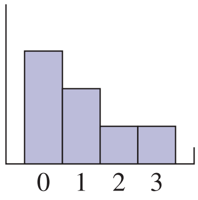
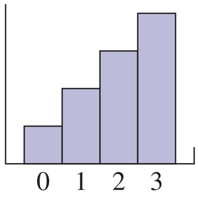
6 Some tennis players count the number of aces served in different matches. Match up the histograms (**a**–**d**) with the descriptions (**A**–**D**).

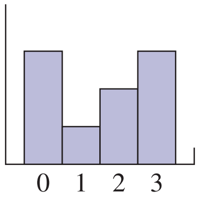
a



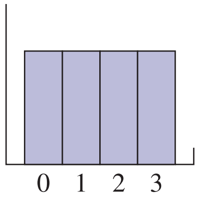
b



c

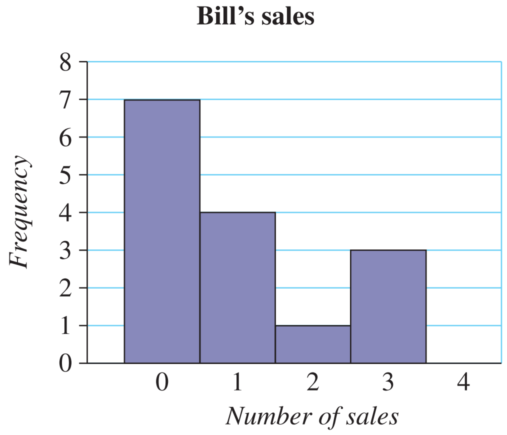


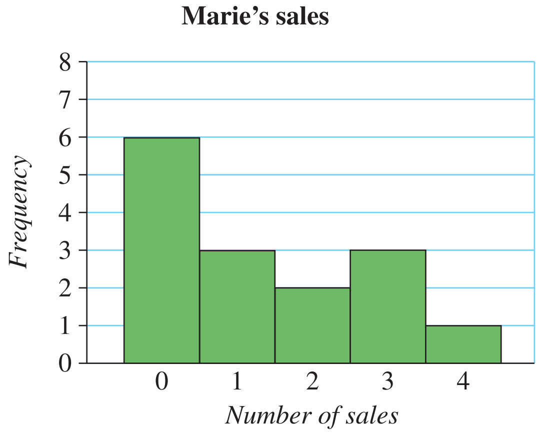
d

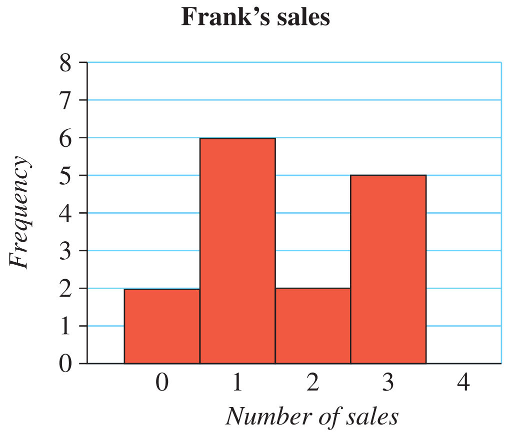


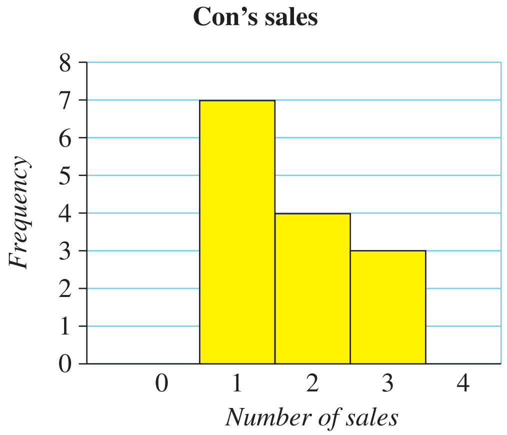
A Often serves aces.
B Generally serves 3 aces or 0 aces.
C Serves a different number of aces in each match.
D Rarely serves aces.

7 A car dealership records the number of sales each salesperson makes per day over three weeks.


Bill's sales


Marie's sales


Frank's sales


Con's sales

a Which salesperson holds the record for the greatest number of sales in one day?
b Which salesperson made a sale every day?
c Over the whole period, which salesperson made the most sales in total?
d Over the whole period, which salesperson made the fewest sales in total?
e On the same set of axes draw all four salesperson's frequency polygons (only draw the line joining the tops, not with columns).
f Explain why a frequency polygon can be more useful for comparing data than a histogram. (*Hint*: consider what multiple histograms would look like on the same axes.)

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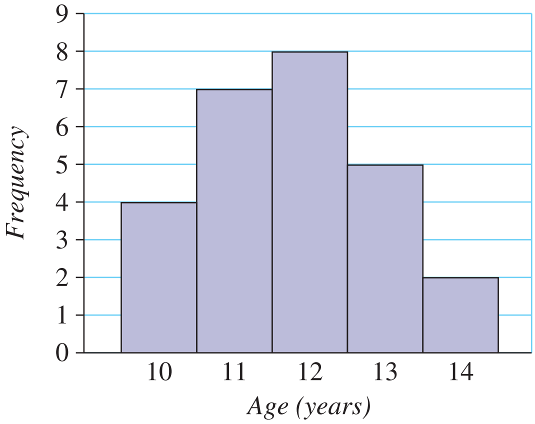
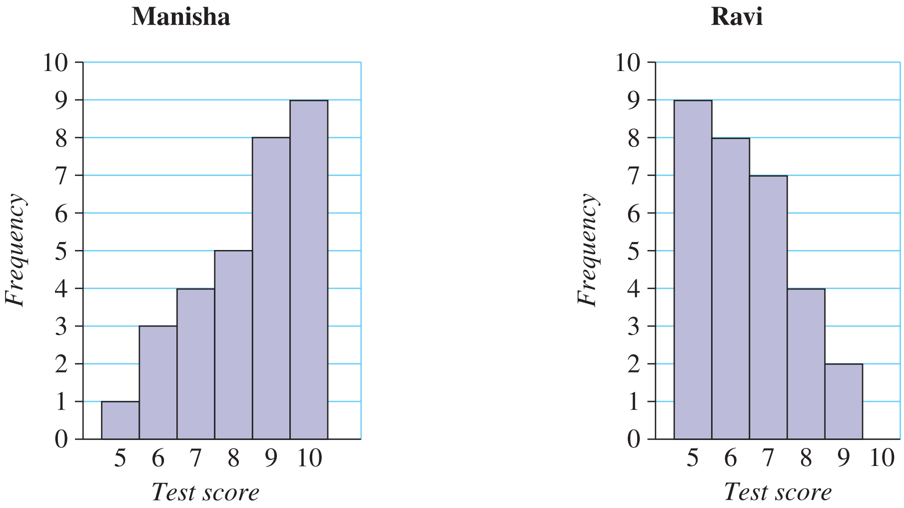
8 This histogram shows the ages of a group of people in a room.

- **a** Describe a histogram that shows the ages of this same group of people in 12 years' time.
- **b** Describe a histogram that shows the ages of this same group of people 12 years ago. Include a rough sketch.

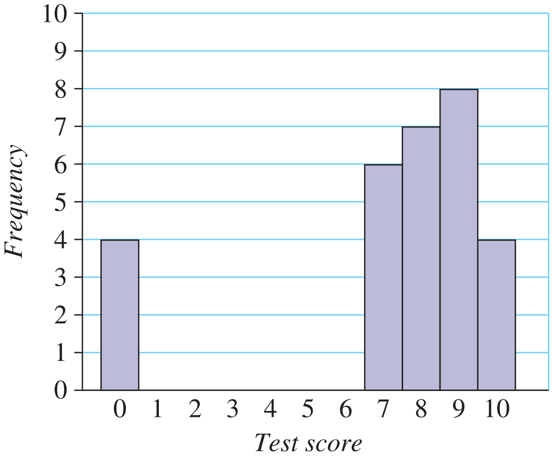


9 Explain why a half-column width space is necessary between the vertical axis and the first column of a histogram. You may search the internet for the answer.

10 Two students have each graphed a histogram that shows their results for a number of spelling tests. Each test is out of 10.



- **a** Does Manisha's graph show that her spelling is improving over the course of the year? Explain.
- **b** Ravi's spelling has actually improved consistently over the course of the year. Give an example of a list of the scores he might have received for the 30 weeks so far.
- **c** A third student has the results shown at right. What is a likely explanation for the '0' results?



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ENRICHMENT – – 11

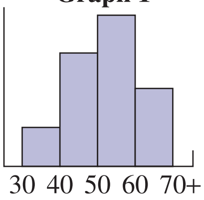
Heights, weights and ages mix-up

11 Three students survey different groups of people to find out their heights, weights and ages. Unfortunately they have mixed up all the graphs they obtained.

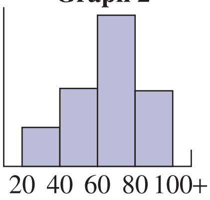
a Copy and complete the table below, stating which graph corresponds to which set of data.

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
primary school classroom	Graph 4		
shopping centre			
teachers common room			

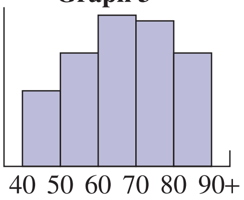
Graph 1



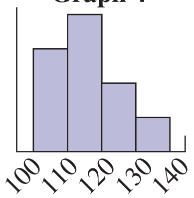
Graph 2



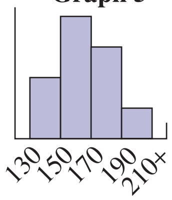
Graph 3



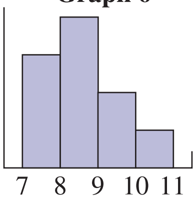
Graph 4



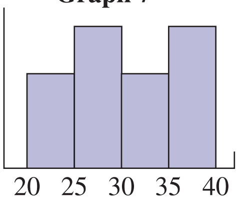
Graph 5



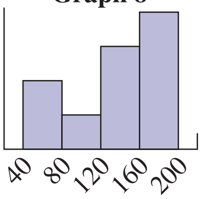
Graph 6



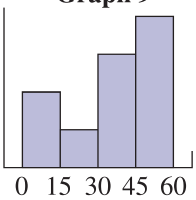
Graph 7



Graph 8



Graph 9



b Show with rough sketches how the height, weight and age graphs would look for:

- **i** people in a retirement village
- **ii** students at a secondary school
- **iii** guests at a 30-year high school reunion

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9G Mean, median, mode and range

Interactive
Widgets
HOTsheets
Walkthrough

Graphs and tables are very useful ways to represent datasets, but summary statistics are also useful for giving others an impression of where the data is centred and the amount of variety within the dataset. The mean, median, mode and range are useful tools for this purpose.

Let's start: Class summary

For each student in the class, find their height, their age, and how many siblings they have.

- Which of these three sets would you expect to have the largest range?
- Which of these three sets would you expect to have the smallest range?
- What do you think is the mean height of students in the class? Can you calculate it?

Key ideas

- Statisticians use **summary statistics** to highlight important aspects of a dataset. These are summarised below.
- Some summary statistics are called **measures of location**.
  - The two most commonly used measures of location are the **mean** and the **median**. These are also called '**measures of centre**' or '**measures of central tendency**'. Mean and median can only be applied to numerical data.
  - The **mean** is sometimes called the '**arithmetic mean**' or the '**average**'. The formula used for calculating the mean, $\bar{x}$, is:

$$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

For example, in the following dataset

5	7	2	5	1

the mean is $\dfrac{5 + 7 + 2 + 5 + 1}{5} = 20 \div 5 = 4$.

  - The **median** divides an ordered dataset into two sets, each of which contain the same number of data values. It is often called the 'middle value'. The median is found by firstly ensuring the data values are in ascending order, then selecting the 'middle' value.

If the number of values is odd, simply choose the one in the middle.

2 2 3 ④ 6 9 9

Median = 4

If the number of values is even find the average of the two in the middle.

2 2 3 ④ ⑦ 9 99

Median = (4 + 7) ÷ 2 = 5.5

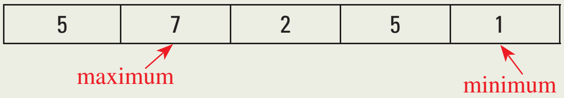
  - The **mode** of a dataset is the most frequently occurring data value. There can be more than one mode. When there are two modes, the data is said to be bimodal. The mode can be for categorical data. It can also be used for numerical data, but it may not be an accurate measure of centre. For example, in the dataset below the mode is 10 and it is also the largest data value.

1	2	3	10	10

- Some summary statistics are called **measures of spread**.
  - The **range** is a measure of spread.
  - The range of a dataset is given by the formula:

    Range = maximum data value − minimum data value

    For example, in the dataset below the range is 7 − 1 = 6



  - In the future you will encounter some other measures of spread, such as interquartile range and standard deviation.

Example 11 Finding the range, mean, median and mode

Consider the ages (in years) of seven people who are surveyed in a shop:

15, 31, 12, 47, 21, 65, 12

a Find the range of values.

b Find the mean of this set of data.

c Find the median of this set of data.

d Find the mode of this set of data.

e If another person is surveyed who is 29 years old, what will be the new median?

SOLUTION	EXPLANATION
a range = 65 − 12 = 53	Highest number = 65, lowest number = 12 The range is the difference.
b mean = 203 ÷ 7 = 29	Sum of values = 15 + 31 + 12 + 47 + 21 + 65 + 12 = 203 Number of values = 7
c values: 12, 12, 15, **21**, 31, 47, 65 median = 21	Place the numbers in ascending order and see that the middle value is 21.
d mode = 12	The most common value is 12.
e values: 12, 12, 15, **21**, **29**, 31, 47, 65 new median = (21 + 29) ÷ 2 = 25	Place the numbers in ascending order. Now there are two middle values (21 and 29), so the median is $\frac{21+29}{2} = \frac{50}{2} = 25$.

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Exercise 9G

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6	4(½), 5–7	4(½), 5–7

Example 11a

1 Consider the set of numbers 1, 5, 2, 10, 3.

- **a** Write the numbers from smallest to largest.
- **b** State the largest number.
- **c** State the smallest number.
- **d** What is the range?

2 State the range of the following sets of numbers.

- **a** 2, 10, 1, 3, 9
- **b** 6, 8, 13, 7, 1
- **c** 0, 6, 3, 9, 1
- **d** 3, 10, 7, 5, 10

Example 11b–d

3 For the set of numbers 1, 5, 7, 7, 10, find the:

- **a** total of the numbers when added
- **b** mean
- **c** median
- **d** mode

Example 11e

4 For each of the following sets of data, calculate the:

- **i** range
- **ii** mean
- **iii** median
- **iv** mode

- **a** 1, 7, 1, 2, 4
- **b** 2, 2, 10, 8, 13
- **c** 3, 11, 11, 14, 21
- **d** 25, 25, 20, 37, 25, 24
- **e** 1, 22, 10, 20, 33, 10
- **f** 55, 24, 55, 19, 15, 36
- **g** 114, 84, 83, 81, 39, 12, 84
- **h** 97, 31, 18, 54, 18, 63, 6

5 The number of aces that a tennis player serves per match is recorded over eight matches.

Match	1	2	3	4	5	6	7	8
Number of aces	11	18	11	17	19	22	23	12

- **a** What is the mean number of aces the player serves per match? Round your answer to one decimal place.
- **b** What is the median number of aces the player serves per match?
- **c** What is the range of this set of data?

6 Brent and Ali organise their test marks, for a number of topics in Maths, in a table.

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Test 10
Brent	58	91	91	75	96	60	94	100	96	89
Ali	90	84	82	50	76	67	68	71	85	57

- **a** Which boy has the higher mean?
- **b** Which boy has the higher median?
- **c** Which boy has the smaller range?
- **d** Which boy do you think is better at tests? Explain why.

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7 Bernadette writes down how many hours she works each day for one week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of hours	8	10	8	7	9

a What is the mean number of hours Bernadette worked each day?
b What is the median number of hours Bernadette worked each day?
c What is the mode number of hours Bernadette worked each day?
d If the number of hours worked on Tuesday changed to 20 (becoming an outlier), which measure of centre would be changed?

PROBLEM-SOLVING AND REASONING	8, 9, 13, 14	8–10, 13, 14	10–16

8 The set 3, 7, 9, 10 has one extra number added to it, and this causes the mean to be doubled. What is the number?

9 Alysha's tennis coach records how many double faults Alysha has served per match over a number of matches. Her coach presents the results in a table.

Number of double faults	0	1	2	3	4
Number of matches with this many double faults	2	3	1	4	2

a In how many matches does Alysha have no double faults?
b In how many matches does Alysha have three double faults?
c How many matches are included in the coach's study?
d What is the total number of double faults scored over the study period?
e Calculate the mean of this set of data, correct to one decimal place.
f What is the range of the data?

10 A soccer goalkeeper recorded the number of saves he makes per game during a season. He presents his records in a table.

Number of saves	0	1	2	3	4	5
Number of games	4	3	0	1	2	2

a How many games did he play that season?
b What is the mean number of saves this goalkeeper made per game? Use the $\bar{x}$ button on your calculator to check your answer.
c What is the most common number of saves that the keeper had to make during a game?

11 Give an example of a set of data with:
a a mean of 10 and a range of 2
b a median of 10 and a range of 5
c a range of 100 and a mean of 50
d a mean of 6, a median of 7 and a mode of 5

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12 The set 1, 2, 5, 5, 5, 8, 10, 12 has a mode of 5 and a mean of 6.

a If a set of data has a mode of 5 and a mean of 6, what is the smallest number of values the set could have? Give an example.

b Is it possible to make a dataset for which the mode is 5, the mean is 6 and the range is 20? Explain your answer.

13 A spreadsheet is a useful tool to calculate means, medians and ranges. For example, =AVERAGE(A1:A6) can be used to find the mean of the cells A1 to A6, and =MEDIAN(A1:A6) to find its median. To find the range use =MAX(A1:A6)−MIN(A1:A6) in a spreadsheet. Type the values 1, 3, 5, 10, 10, 13 into cells A1 to A6.

a Find the mean, median and range, using your spreadsheet.

b If each number is increased by 5, state the effect this has on the:

i mean **ii** median **iii** range

c If each of the original numbers is doubled, state the effect this has on the:

i mean **ii** median **iii** range

d Is it possible to include extra numbers and keep the same mean, median and range? Try to expand this set to at least 10 numbers, but keep the same values for the mean, median and range. Use your spreadsheet to help, but remember to change the cells from A1:A6.

14 **a** Two whole numbers are chosen with a mean of 10 and a range of 6. What are the numbers?

b Three whole numbers are chosen with a mean of 10 and a range of 2. What are the numbers?

c Three whole numbers are chosen with a mean of 10 and a range of 4. Can you determine the numbers? Try to find more than one possibility.

15 Explain why you can calculate the mode for numerical or categorical data but you can only calculate the mean, median and range from numerical data.

16 Magda has presented her weekly spelling test scores as a frequency distribution table.

Score	4	5	6	7	8	9	10
Frequency	1	5	5	11	12	5	1

This is easier than writing out the results as 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, …, 9, 10.

a State the range of scores and the mean score, correct to two decimal places.

b If the frequency of each score is reduced by one, describe the effect this will have on the range and the mean of the scores.

ENRICHMENT – – 17, 18

Mean challenges

17 For the set of numbers 1, 2, 3, …, 100, find the mean. (*Hint*: you can find this answer without a spreadsheet but you could check your answer using an appropriate formula.)

18 **a** Give an example of a set of numbers with the following properties.

i mean = median = mode

ii mean > median > mode

iii mode > median > mean

iv median < mode < mean

b If the range of a set of data is 1, is it still possible to find datasets for each of parts **i** to **iv** above?

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9H Interquartile range EXTENSION

Sometimes it is useful to know a number that describes the amount of spread in a dataset. Two commonly used measures of spread are range and interquartile range.

Interactive

Widgets

HOTsheets

Walkthrough

Let's start: Golfing

Three golfers record their scores over five games. In golf, lower scores are better.

Alfred: 82, 87, 80, 112, 79
Brenton: 90, 89, 91, 92, 89
Charlie: 72, 103, 94, 83, 80

- Which golfer do you think is best? Explain.
- Which golfer is most consistent? Explain.
- Alfred's range of values $(112 - 79 = 33)$ is larger than Charlie's $(103 - 72 = 31)$. Discuss whether this means that Charlie is a more consistent golfer than Alfred.

Key ideas

- The range and interquartile range are summary statistics known as 'measures of spread'.
- Around the world, there are several different methods used for finding the **quartiles** and the **interquartile range** (IQR). This is one of those methods:
  - Sort the data into ascending order.
  - **If the number of values is odd**, the median will be one of those values. The values below and above the median form two groups of equal size.
  - **If the number of values is even**, it is easier to form two groups of equal size.
  - Split the data into two equal-size groups.
  - The median of the lower half is called the **lower quartile**.
  - The median of the upper half is called the **upper quartile**.
  - IQR = upper quartile − lower quartile

For example: $\overbrace{1 \quad 5 \quad \underline{7 \quad 9} \quad 10 \quad 11}^{\text{lower half}} \;\Big|\; \overset{\text{upper half}}{13 \quad 17 \quad \underline{19 \quad 20} \quad 23 \quad 28}$

Lower quartile
$= (7 + 9) \div 2$
$= 8$

Upper quartile
$= (19 + 20) \div 2$
$= 19.5$

$$\begin{aligned} \text{IQR} &= 19.5 - 8 \\ &= 11.5 \end{aligned}$$

- Outliers affect the range but not the IQR.

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Example 12 Finding the IQR

Find the interquartile range of the following sets of data.

a 1, 15, 8, 2, 13, 10, 4, 14

b 2, 7, 11, 8, 3, 8, 10, 4, 9, 6, 8

SOLUTION	EXPLANATION
a Sorted: 1 2 4 8 | 10 13 14 15	Values must be sorted from lowest to highest.
1 ② ④ 8 10 ⑬ ⑭ 15 $\text{Median} = \frac{2 + 4}{2} = 3$ $\text{Median} = \frac{13 + 14}{2} = 13.5$	Lower quartile = median of the lower half. Upper quartile = median of the upper half.
$\text{IQR} = 13.5 - 3 = 10.5$	IQR = upper quartile − lower quartile
b Sorted: 2 3 4 6 7 ⑧ 8 8 9 10 11 2 3 ④ 6 7 8 8 ⑨ 10 11 Lower quartile Upper quartile $\text{IQR} = 9 - 4 = 5$	Sort the data and ignore the middle value (so there are two equal halves remaining). Lower quartile = median of the lower half. Upper quartile = median of the upper half. IQR = upper quartile − lower quartile

Exercise 9H EXTENSION

UNDERSTANDING AND FLUENCY	1–6, 7–9(½)	6, 7–9(½)	7–9(½)

1 Consider the set of numbers 1, 3, 2, 8, 5, 6.

- **a** State the largest number.
- **b** State the smallest number.
- **c** Hence, state the range by finding the difference of these two values.

2 Consider the set of numbers 1, 2, 4, 5, 5, 6, 7, 9, 9, 10.

- **a** State the median of 1, 2, 4, 5, 5.
- **b** State the median of 6, 7, 9, 9, 10.
- **c** Hence, state the interquartile range by finding the difference of these two values.

3 Consider the set of numbers 1, 5, 6, 8, 10, 12, 14.

- **a** State the median of the lower half (1, 5, 6).
- **b** State the median of the upper half (10, 12, 14).
- **c** Hence, state the interquartile range.

4 Complete these sentences.

- **a** The median of the lower half is called the _______ _______.
- **b** The difference between the highest and _______ values is the range.
- **c** In order to calculate the IQR, you should first _______ the numbers from lowest to highest.
- **d** The upper quartile is the _______ of the upper half of the sorted data.
- **e** When calculating the IQR, you remove the middle term if the number of values is _______.
- **f** The range and IQR are examples of a measure of _______.

5 Calculate the median of the following sets.

a 1, 3, 5, 6 **b** 2, 7, 8, 12, 14, 15 **c** 1, 4, 5, 6, 8, 10

6 Consider the set 8, 6, 10, 4, 3, 7, 5, 5.

a Write this list in ascending order.
b Calculate the lower quartile.
c Calculate the upper quartile.
d Calculate the IQR.

Example 12a

7 Find the IQR of the following sets.

a 1, 9, 11, 13, 28, 29 **b** 7, 9, 13, 16, 20, 28
c −17, −14, −12, −9, −8, 1, 9, 9, 12, 12 **d** −20, −20, −10, −3, −1, 0, 3, 5, 14, 18
e 0, 0, 4, 5, 7, 8, 8, 8, 11, 12, 17, 18 **f** 1, 9, 9, 9, 12, 18, 19, 20
g 0.28, 2.13, 3.64, 3.81, 5.29, 7.08 **h** 1.16, 2.97, 3.84, 3.94, 4.73, 6.14

Example 12b

8 Find the IQR of the following sets.

a 4, 4, 11, 16, 21, 27, 30 **b** 10, 11, 13, 22, 27, 30, 30
c −13, −13, −12, −3, 14, 15, 22, 23, 27 **d** −19, −17, −6, 0, 3, 3, 18, 23, 26
e 1, 1, 2, 9, 9, 12, 18, 18, 18 **f** 2, 2, 8, 9, 12, 14, 15, 16, 19
g 0.4, 0.9, 0.9, 1.9, 2.0, 3.9, 4.3, 4.4, 4.7 **h** 0.5, 1.1, 1.2, 1.4, 1.5, 2.1, 2.2, 2.4, 4.8

9 Find the IQR of the following sets. Remember to sort first.

a 0, 12, 14, 3, 4, 14 **b** 14, 0, 15, 18, 0, 3, 14, 7, 18, 12, 9, 5
c 6, 11, 3, 15, 18, 14, 13, 2, 16, 7, 7 **d** 6, 4, 6, 5, 14, 8, 10, 18, 16, 6
e 18, −15, 17, −15, −1, 2 **f** −12, −17, −12, 11, 15, −1
g −19, 8, 20, −10, 6, −16, 0, 14, 2, −2, 1 **h** −4, −9, 17, 7, −8, −4, −16, 4, 2, 5

PROBLEM-SOLVING AND REASONING	10, 11, 15	11–16	12–17

10 Gary and Nathan compare the number of runs they score in cricket over a number of weeks.

Gary: 17, 19, 17, 8, 11, 20, 5, 13, 15, 15
Nathan: 39, 4, 26, 28, 23, 18, 37, 18, 16, 20

a Calculate Gary's range.
b Calculate Nathan's range.
c Who has the greater range?
d Which cricketer is more consistent, on the basis of their ranges only?

11 Winnie and Max note the amount of water they drink (in litres) every day for a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Winnie	2.4	1.3	3	2.2	2	4	1.2
Max	1.5	2.3	0.8	1.2	3	2.5	4.1

a Calculate Winnie's IQR.
b Calculate Max's IQR.
c Who has the larger IQR?

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12 Over a 20-week period, Sara and Andy tally their spelling test results.

Score	0	1	2	3	4	5	6	7	8	9	10
Sara		𝍷𝍷		𝍷		𝍸	𝍷	𝍷𝍷	𝍷𝍷𝍷	𝍷	𝍸
Andy	𝍷				𝍷		𝍷𝍷𝍷𝍷		𝍸	𝍷𝍷	𝍸𝍷𝍷

- **a** Find the range for: **i** Sara and **ii** Andy.
- **b** Find the IQR for: **i** Sara and **ii** Andy.
- **c** On the basis of the range only, which student is more consistent?
- **d** On the basis of the IQR only, which student is more consistent?

13 Consider the set of numbers 2, 3, 3, 4, 5, 5, 5, 7, 9, 10.

- **a** Calculate the: **i** range and **ii** IQR.
- **b** If the number 10 is changed to 100, calculate the new: **i** range and **ii** IQR.
- **c** Explain why the IQR is a better measure of spread if there are outliers in a dataset.

14 Two unknown numbers are chosen with mean $= 10$ and range $= 4$. What is the product of the two unknown numbers?

15 Consider the set 1, 4, 5, 5, 6, 8, 11.

- **a** Find the range.
- **b** Find the IQR.
- **c** Is it ever possible for the IQR of a set to be larger than the range? Explain.
- **d** Is it possible for the IQR of a set to equal the range? Explain.

16 For a set of three numbers, what effect is there on the range if:

- **a** each number is increased by 10?
- **b** each number is doubled?

17 Why are the words ‘upper quartile’ and ‘lower quartile’ used? Think about what ‘quart’ might mean.

ENRICHMENT – – 18

Changing the frequency

18 Consider the data below, given as a frequency distribution table.

Number	1	2	3	4	5
Frequency	2	4	1	1	3

- **a** What is the range?
- **b** Calculate the IQR. (*Hint*: write the data as a list first.)
- **c** How would the range change if the frequency of each number were doubled?
- **d** How would the IQR change if the frequency of each number were doubled?
- **e** If the numbers themselves were doubled but the frequencies kept the same as in the table above, how would this change:
  - **i** the range? **ii** the IQR?

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9I Stem-and-leaf plots

A stem-and-leaf plot is a useful way of presenting numerical data in a way that allows trends to be spotted easily. Each number is split into a stem (the first digit or digits) and a leaf (the last digit).

	Stem	Leaf
53 is	5	3
78 is	7	8
125 is	12	5

By convention, leaves are shown in increasing order as you work away from the stem, and stems are shown in increasing order going down the page.

The advantage of presenting data like this comes when multiple numbers have the same stem. For example, the list 122, 123, 124, 124, 127, 129 can be represented as shown opposite.

Stem	Leaf
12	2 3 4 4 7 7 9

Let's start: Test score analysis

In a class, students' most recent test results out of 50 are recorded.

Test 1 results

43, 47, 50, 26, 38, 20, 25, 20, 50, 44,
33, 47, 47, 50, 37, 28, 28, 22, 21, 29

Test 2 results

Stem	Leaf
1	7
2	7 8
3	2 2 4 5 5 7 9
4	0 1 2 3 3 6 8 8
5	0 0

- For each test, try to find how many students:
  - achieved a perfect score (i.e. 50)
  - failed the test (i.e. less than 25)
  - achieved a mark in the 40s.
- If there are 100 test results that you wish to analyse, would you prefer a list or a stem-and-leaf plot?
- What is it that makes a stem-and-leaf plot easier to work with? Discuss.

Key ideas

- A stem-and-leaf plot is a way to display numerical data.
- Each number is usually split into a **stem** (the first digit or digits) and a **leaf** (the last digit). For example:

	Stem	Leaf
The number 7 is	0	7
The number 31 is	3	1
The number 152 is	15	2

3|1 means 31

- A key is usually connected to each stem and leaf plot to show the value of the stem and leaf. For example: 4|8 means 48 or 4|8 means 4.8.
- Leaves should be aligned vertically, listed in ascending order as you move away from the stem.
- Any outliers can be identified by looking at the lowest value or highest value to see if they are very different from all the other data values.

Example 13 Interpreting a stem-and-leaf plot

Average daily temperatures are shown for some different countries.

Stem	Leaf
1	3 6 6
2	0 0 1 2 5 5 6 8 9
3	0 2

2|5 means 25

a Write out the temperatures as a list.
b How many countries' temperatures are represented?
c What are the maximum and minimum temperatures?
d What is the range of temperatures recorded?
e What is the median temperature recorded?

SOLUTION	EXPLANATION
a 13, 16, 16, 20, 20, 21, 22, 25, 25, 26, 28, 29, 30, 32	Each number is converted from a stem and a leaf to a single number. For example, 1|3 is converted to 13.
b 14	The easiest way is to count the number of leaves – each leaf corresponds to one country.
c minimum = 13 maximum = 32	The first stem and leaf is 1|3 and the last stem and leaf is 3|2.
d range = 19	Range = maximum − minimum = 32 − 13 = 19
e median = 23.5	The middle value is halfway between the numbers 2|2 and 2|5, so median = $\frac{1}{2}(22 + 25) = 23.5$.

Example 14 Creating a stem-and-leaf plot

Represent this set of data as a stem-and-leaf plot:
23, 10, 36, 25, 31, 34, 34, 27, 36, 37, 16, 33

SOLUTION

Sorted: 10, 16, 23, 25, 27, 31, 33, 34, 34, 36, 36, 37

Stem	Leaf
1	0 6
2	3 5 7
3	1 3 4 4 6 6 7

2|5 means 25

EXPLANATION

Sort the list in increasing order so that it can be put directly into a stem-and-leaf plot.

Split each number into a stem and a leaf. Stems are listed in increasing order and leaves are aligned vertically, listed in increasing order down the page.

Add a key to show the value of the stem and leaf.

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Exercise 9I

UNDERSTANDING AND FLUENCY	1–6	3–7	4–7

1 The number 52 is entered into a stem-and-leaf plot.

a What digit is the stem?

b What digit is the leaf?

2 What number is represented by the following combinations?

a 3|9 **b** 2|7 **c** 13|4

3 In this stem-and-leaf plot, the smallest number is 35. What is the largest number?

Stem	Leaf
3	5 7 7 9
4	2 8
5	1 7

3|5 means 35

Example 13a-c

4 This stem-and-leaf plot shows the ages of people in a group.

Stem	Leaf
0	8 9
1	0 1 3 5 7 8
2	1 4

1|5 means 15 years old

a Write out the ages as a list.

b How many ages are shown?

c Answer true or false to each of the following.

i The youngest person is aged 10.

ii Someone in the group is 17 years old.

iii Nobody listed is aged 20.

iv The oldest person is aged 4.

Example 13d,a

5 For each of the stem-and-leaf plots below, state the range and the median.

a

Stem	Leaf
0	9
1	3 5 6 7 7 8 9
2	0 1 9

1|8 means 18

b

Stem	Leaf
1	1 4 8
2	1 2 4 4 6 8
3	0 3 4 7 9
4	2

3|7 means 37

c

Stem	Leaf
3	1 1 2 3 4 4 8 8 9
4	0 1 1 2 3 5 7 8
5	0 0 0

4|2 means 42

Example 14

6 Represent each of the following sets of data as a stem-and-leaf plot.

a 11, 12, 13, 14, 14, 15, 17, 20, 24, 28, 29, 31, 32, 33, 35

b 20, 22, 39, 45, 47, 49, 49, 51, 52, 52, 53, 55, 56, 58, 58

c 21, 35, 24, 31, 16, 28, 48, 18, 49, 41, 50, 33, 29, 16, 32

d 32, 27, 38, 60, 29, 78, 87, 60, 37, 81, 38, 11, 73, 12, 14

7 Represent each of the following datasets as a stem-and-leaf plot. (Remember: 101 is represented as 10|1.)

a 80, 84, 85, 86, 90, 96, 101, 104, 105, 110, 113, 114, 114, 115, 119

b 120, 81, 106, 115, 96, 98, 94, 115, 113, 86, 102, 117, 108, 91, 95

c 192, 174, 155, 196, 185, 178, 162, 157, 173, 181, 158, 193, 167, 192, 184, 187, 193, 165, 199, 184

d 401, 420, 406, 415, 416, 406, 412, 402, 409, 418, 404, 405, 391, 411, 413, 413, 408, 395, 396, 417

PROBLEM-SOLVING AND REASONING	8, 9, 13	8, 10, 12, 13	10–14

8 This back-to-back stem-and-leaf plot shows the ages of all the people in two shops. The youngest person in shop 1 is 15 (not 51).

For each statement below, state whether it is true in shop 1 only (1), shop 2 only (2), both shops (B) or neither shop (N).

Shop 1	Stem	Shop 2
5	1	6 7
7 7 5 3	2	4 5
	3	1
2	4	5

2 | 4 means 24 years old

a This shop has a 31-year-old person in it.
b This shop has six people in it.
c This shop has a 42-year-old person in it.
d This shop has a 25-year-old person in it.
e This shop has two people with the same age.
f This shop has a 52-year-old person in it.
g This shop has a 24-year-old person in it.
h This shop's oldest customer is an outlier.
i This shop's youngest customer is an outlier.

9 A company recorded the duration (in seconds) that visitors spent on its website's home page.

Stem	Leaf
0	2 4 6 8 9
1	0 0 1 2 8
2	2 7 9
3	
4	
5	8

2 | 7 means 27 years old

a How many visitors spent less than 20 seconds on the home page?
b How many visitors spent more than half a minute?
c How many visitors spent between 10 and 30 seconds?
d What is the outlier for this stem-and-leaf plot?
e The company wishes to summarise its results with a single number. 'Visitors spend approximately ____ on our home page.' What number could it use?

10 Two radio stations poll their audience to determine. their ages.

Station 1	Stem	Station 2
0	1	2 3 3 4 5 6 8 9
8 7	2	0 0 1 2 4 5 8 8
9 7 5 4 3 3	3	1 1 2
7 6 5 5 4 4 1	4	8
9 3 2 0	5	

3 | 1 means 31 years old

a Find the age difference between the oldest and youngest listener polled for:
  i station 1 **ii** station 2
b One of the radio stations plays contemporary music that is designed to appeal to teenagers, and the other plays classical music and broadcasts the news. Which radio station is most likely to be the one that plays classical music and news?
c Advertisers wish to know the age of the stations' audiences so that they can target their advertisements more effectively (e.g. to 38 to 58 year olds). Give a 20-year age range for the audience majority who listen to:
  i station 1 **ii** station 2

11 A group of boys and girls have their heights recorded in a back-to-back stem-and-leaf plot, shown here.

Girls	Stem	Boys
	10	
	11	
6 3 1	12	
8 4 3 2	13	8
7 6 5 4 0	14	3 4 7 9
6 4 1	15	0 1 2 4 6 8
2 0	16	2 3 6 8
3	17	

14 | 7 means 147 cm

a State the range of heights for:
  i boys **ii** girls

b Which gender has a bigger range?

c State the median height for:
  i boys **ii** girls

d Which gender has the larger median height?

e Which year level do you think these boys and girls are in? Justify your answer.

f Describe how you might expect this back-to-back stem-and-leaf plot to change if it recorded the heights of male and female Year 12 students.

12 A teacher has compiled her class' recent test scores out of 50 as a stem-and-leaf plot. However, some values are missing, as represented by the letters a, b, c and d.

Stem	Leaf
1	5
2	4 5 a 6 7 9
3	b 0 1 5
4	2 8 c
5	d

3 | 1 means 31

a How many students took the test?

b How many students passed the test (i.e. achieved a mark of 25 or higher)?

c State the possible values for each of the missing digits a to d.

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13 A stem-and-leaf plot is constructed showing the ages of all the people who attended a local farmer's market at a certain time of the day. However, the plot's leaves cannot be read.

Stem	Leaf
1	?
2	?
3	? ? ? ? ? ? ? ? ? ?
4	? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
5	? ? ?

- **a** For each of the following, either determine the exact answer or give a range of values the answer could take.
  - **i** How many people were at the market?
  - **ii** How many people aged in their 30s were at the market?
  - **iii** How old is the youngest person?
  - **iv** What is the age difference between the youngest and oldest person?
  - **v** How many people aged 40 or over were at the market?
  - **vi** How many people aged 35 or over were at the market?
- **b** Classify each of the following as true or false.
  - **i** The majority of people at the market were aged in their 30s or 40s.
  - **ii** There were five teenagers present.
  - **iii** Exactly two people were aged 29 years or under.
  - **iv** Two people in their 40s must have had the same age.
  - **v** Two people in their 30s must have had the same age.
  - **vi** Two people in their 20s could have had the same age.
- **c** Explain why it is possible to determine how many people were aged 40 or over, but not the number of people who were aged 40 or under.
- **d** It is discovered that the person under 20 years of age is an outlier for this market. What does that tell you about how old the next oldest person is?

14
- **a** Explain why it is important that leaves are aligned vertically. (*Hint*: consider how the overall appearance could be helpful with a large dataset.)
- **b** Why might it be important that data values are sorted in stem-and-leaf plots?

ENRICHMENT	–	–	15

Negative stem-and-leaf plots

15 Negative numbers can also be displayed in stem-and-leaf plots. This stem-and-leaf plot gives the average winter temperatures in 15 different cities.

Stem	Leaf
−2	4 4 9
−1	2 3 5 7
−0	5 8
0	3 4 6
1	5 8
2	3

−1|5 means −15°C

- **a** What are the minimum and maximum temperatures listed?
- **b** Find how many cities had average temperatures:
  - **i** between −10°C and 10°C
  - **ii** between −25°C and 5°C
  - **iii** below 5°C
- **c** Why is there a 0 row and a −0 row, even though 0 and −0 are the same number?
- **d** What is the average (or mean) of all the listed temperatures in the 15 cities? Give your answer correct to one decimal place.

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9J Surveying and sampling

To find information about a large number of people it is generally not possible to ask everybody to complete a survey, so instead a sample of the population is chosen and surveyed. It is hoped that the information given by this smaller group is representative of the larger group of people. Choosing the right sample size and obtaining a representative sample is harder than many people realise.

Let's start: Average word length

To decide how hard the language is in a book, you could try to calculate the average length of the words in it. Because books are generally too large, instead you can choose a smaller sample. For this exercise, you must decide or be assigned to the 'small sample', 'medium sample' or 'large sample' group. Then:

1 Pick a page from the book at random.

2 Find the average (mean) length of any English words on this page, choosing the first 10 words if you are in the 'small sample' group, the first 30 words for the 'medium sample' group and the first 50 words for the 'large sample' group.

Discuss as a class:

- Which group would have the best estimate for the average word length in the book?
- What are the advantages and disadvantages of choosing a large sample?
- Does this sample help to determine the average length of words in the English language?
- How could the results of a whole class be combined to get the best possible estimate for average word length in the book?
- If all students are allowed to choose the page on which to count words, rather than choosing one at random, how could this bias the results?

Key ideas

- A **population** is the set of all members of a group that is to be studied.
  For example: All the people in a town, looking at which local beach they prefer.
  All the kangaroos in a park, looking at the presence of any diseases.
- A **sample** is a subset (selected group) of a population.
  For example: 20 students selected from all Year 8 students in a school, looking at what their favourite football team is.
  One thousand people selected and called via telephone regarding their preferred political party.
- A **survey** can be conducted to obtain information about a large group by using a smaller sample. A survey conducted on an entire population is called a **census**.
- The accuracy of the survey's conclusion can be affected by:
  - the **sample size** (number of participants or items considered)
  - whether the sample is **representative** of the larger group, or **biased**, which can result in a sample mean significantly different from the population mean
  - whether there were any **measurement errors**, which could lead to **outliers**—values that are noticeably different from the other values.

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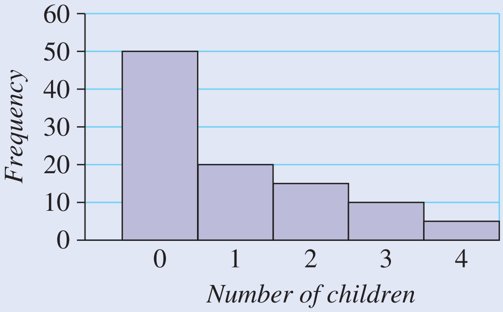
Example 15 Interpreting survey results

A survey is conducted asking 100 randomly selected adults how many children they have. The results are shown in this histogram.



a Assume that this sample is representative of the population.

- **i** What proportion of the adult population has two or more children?
- **ii** In a group of 9000 adults, how many would you expect to have four children?

b Which of the following methods of conducting the survey could lead to bias?

Method 1 Asking people waiting outside a childcare centre

Method 2 Randomly selecting people at a night club

Method 3 Choosing 100 adults at random from the national census and noting how many children they claimed to have.

SOLUTION	EXPLANATION
a **i** $\frac{3}{10}$	$15 + 10 + 5 = 30$ adults have two or more children. Proportion $= \frac{30}{100} = \frac{3}{10}$
ii 450	In the survey $\frac{5}{100} = \frac{1}{20}$ of the population have four children. $\frac{1}{20} \times 9000 = 450$
b Methods 1 and 2 could both lead to bias.	If someone is waiting outside a childcare centre they are more likely to have at least one child. If someone is at a night club they are likely to be a younger adult, and so less likely to have a child.

Exercise 9J

UNDERSTANDING AND FLUENCY 1–4 2–5 3–7

1 Marieko wishes to know the average age of drivers in her city. She could survey 10 of her friends, or survey 1000 randomly selected drivers.

- **a** Which of these options would give a more accurate result?
- **b** Which would be easier for Marieko to perform?

Example 15a

2 Ajith looks at a random sample of penguins and notes that of the 50 he sees, 20 of them have spots on their bodies.

- **a** What proportion of the population has spots?
- **b** If there are 5000 penguins in a region, on the basis of this sample how many would you expect to have spots on their bodies?
- **c** If there are 500 penguins in a region, how many would you expect to not have spots on their bodies?

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Example 15b

3 A survey is conducted asking people how many pets they own. You can assume it is a representative sample of the population.

Pets	0	1	2	3	4	Total
Responses	20	18	6	4	2	50

a Plot the results on a histogram.
b Of a group of 1000 people, how many of them would you expect to have no pets?
c Of a group of 5000 people, how many of them would you expect to have two or more pets?
d Why would conducting this survey outside a veterinary clinic cause a bias in the results?
e If someone replied to this survey that they had 100 pets, should their response be included? Describe how you could justify including or excluding this value.

4 Liwei attempts to find a relationship between people's ages and their incomes. He is considering some questions to put in his survey. For each question, decide whether it should be included in the survey, giving a brief explanation.
a What is your current age, in years?
b Are you rich?
c Are you old?
d How much money do you have?
e What is your name?
f How much money did you earn in the past year?
g How much money did you receive today?

5 For each of the following survey questions, give an example of an unsuitable location and time to conduct the survey if you wish to avoid a bias.
a A survey to find the average number of children in a car
b A survey to find how many people are happy with the current prime minister
c A survey to find the proportion of Australians who are vegetarians
d A survey to find the average amount spent on supermarket groceries

6 **a** Design a survey question to decide the average number of siblings of a Year 8 student in your school.
b Conduct the survey among a sample consisting of some students in your class, using a recording sheet to tally the results.
c Use your sample to estimate the average number of siblings.

7 **a** Explain the difference between a population and a sample when collecting data.
b Decide with justification whether a census or a randomly chosen sample is more appropriate to collect the following data.
  i The average age of all tennis players worldwide.
  ii The favourite movie within a circle of friends.
  iii The average amount of money spent by Year 8 students in the cafeteria at lunch today.
  iv The median house price of a small suburb.
  v The popularity of different television programs in NSW.

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PROBLEM-SOLVING AND REASONING	8, 11	8, 9, 11, 12	10–13

8 In a factory producing chocolate bars, a sample of bars is taken and automatically weighed to check whether they are between 50 and 55 grams. The results are shown in a frequency distribution table.

Weight (g)	49	50	51	52	53	54	55	108
Frequency	2	5	10	30	42	27	11	1

a Which weight value is an outlier?

b How could the automatic weighing mechanism have caused this measurement error?

c To find the spread of weights, the machine can calculate the range, or the IQR. Which would be a better value to use? Justify your answer.

d If measurement errors are not removed, would the mean or the median be a better guide to the ‘central weight’ of the bars?

9 A survey is being conducted to decide how many adults use mathematics later in life.

a If someone wanted to make it seem that most adults do not use mathematics, where and when could they conduct the survey?

b If someone wanted to make it seem that most adults use mathematics a lot, where and when could they conduct the survey?

c How could the survey be conducted to provide less biased results?

10 Describe an ethical issue that may arise from collecting and/or representing the following data.

a The presence of underlying medical conditions in job applications.

b The income of different families at a school.

c The average results of boys and girls in the final Year 12 maths exams.

d The safety features of a type of car as reported on a television program, where that car’s company is a station sponsor.

11 Evangeline intends to survey a group of students to investigate whether having older siblings affects how much they enjoy maths.

One possible survey form is below.

1) Do you have older siblings?________________________________

2) Do you enjoy maths? __________________________________

Some of the answers she receives are “I wish I didn’t”, “What’s a sibling?” and “Sometimes, if it’s a good topic.”

a Refine the questions in this survey to make them into clear Yes/No questions and show how your improved survey would look.

b When Evangeline collects the survey results, she uses a recording sheet with tallies like this:

Has older siblings	**Enjoys maths**
卌 ||||	卌 卌 |||

What is the problem with this recording sheet? (*Hint*: consider what she is trying to investigate.)

c Construct a new recording sheet that allows her to efficiently collect the data.

d After seeing a link, Evangeline wishes to know if people who have older siblings tend to like maths more, so she changes the second question to ‘How much do you like maths in general?”

Describe how she could make the possible answers on a scale from 1 to 5.

e Construct a recording sheet that would allow her to efficiently collect the results.

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12 Whenever a survey is conducted, even if the people being asked are randomly selected, bias can be introduced by the fact that some people will not answer the questions or return the surveys.

- **a** Assume 1000 surveys are mailed out to a random sample of people asking the question 'Do you think Australia should be a republic?' and 200 replies are received: 150 say 'yes' and 50 say 'no'.
    - **i** On the basis of the 200 people who returned the survey, what percentage are in favour of a republic?
    - **ii** If all 800 people who did not respond would have said 'yes', what percentage are in favour of a republic?
    - **iii** If all 800 people who did not respond would have said 'no', what percentage are in favour of a republic?
    - **iv** If the 1000 people receiving the survey are representative of the Australian population, what conclusion can be drawn about the popularity of a republic?
- **b** A survey is being conducted to decide how many people feel they are busy. Describe how bias is introduced by the people who agree to participate in the survey.
- **c** Give an example of another survey question that would cause a bias to be introduced simply on the basis of the people who participate.
- **d** Sometimes surveys ask the same question in different ways over a number of pages. Although this additional length makes it less likely that people will return the survey, why might the questioner wish to ask the same question in different ways?

13 A popular phrase is "Lies, damned lies, and statistics". Why is it important that someone making or conducting a survey behaves ethically?

ENRICHMENT	–	–	14

Media bias

14 Search a newspaper, magazine or website and find an example of a survey or poll that has been conducted.

- **a** Decide the following, with justifications.
    - **i** Can you tell if they chose a large enough sample for the survey?
    - **ii** Can you tell whether the sample chosen was representative or biased?
    - **iii** Would this newspaper, magazine or website have an incentive to choose or publish biased results?
    - **iv** How much are you relying on the publisher to behave ethically in choosing and displaying a representative sample?
- **b** Design a perfect survey to get the information that has been reported, and describe how you would choose the survey recipients.
- **c** Conduct the survey on a small random sample that is representative of the wider population, and compare your results with those reported.

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Investigation

School food survey

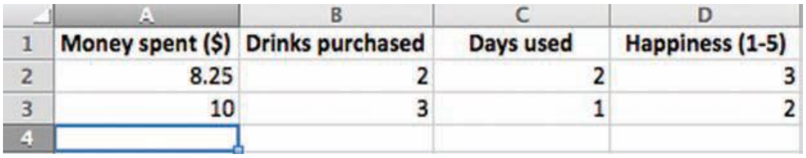
A new food company, GoodCheapFood, is thinking of taking over the running of your school's cafeteria. Before they invest in your school, they would like to know the eating and drinking habits of high school students, in particular:

- how much money students spend in the cafeteria (per week)
- how many drinks they purchase (per week)
- how many days of the week students use the cafeteria
- how happy students are with the current cafeteria, on a scale from 1 to 5.

1 Design a survey for GoodCheapFood that could be used to get the data they would like. Think about how the questions should be phrased to ensure that you get accurate results that are free of bias.

2 Explain why you cannot conduct a census of the school population and describe at least three different ways you could choose random samples of 20 students. For each sample, comment on whether a bias would be introduced and give an explanation for why selecting only Year 7 girls might result in significantly different results from selecting only Year 12 boys.

3 Conduct the survey on at least three different random samples of size 20 from within the school population and use a spreadsheet to tabulate the results. A separate sheet should be used for each of the samples and could be filled out as given below.



	A	B	C	D
1	Money spent ($)	Drinks purchased	Days used	Happiness (1-5)
2	8.25	2	2	3
3	10	3	1	2
4				

4 GoodCheapFood is interested in the range of the amount of money spent in the cafeteria in a week. Use a formula like = MAX(A2:A21)–MIN(A2:A21) to calculate the range for each of your samples, and give the results.

5 The company wishes to know the median number of drinks purchased. Explain why the median rather than the mean would be preferred by referring to any possible outliers, and then use a formula like = MEDIAN(B2:B21) to find the medians for each sample.

6 Find the mode number of days used in each of the samples and explain what this signifies, in plain English.

7 Find the mean happiness of students in each sample with a formula like = AVERAGE(D2:D21).

8 Explain why your three or more different random samples drawn have different summary statistics (mean, median, mode and range) even though they are all drawn from the school population.

9 GoodCheapFood is grateful that you have gone to this much effort. Present your findings as a brief summary, combining your samples to answer the four questions in which they were interested.

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1 Six numbers are listed in ascending order and some are removed. The mean and median are both 6, the mode is 2 and the range is 10. Fill in the missing numbers.

$$?, ?, 5, ?, ?, ?$$

2 A survey is conducted at a school and the results are presented as a sector graph. Find the minimum number of people who participated in the survey if the smallest sector has an angle of:

a 90° **b** 36° **c** 92° **d** 35°

3 In a class of 20 students, a poll was taken of the number of cars owned by each family. The median number of cars owned is 1.5 and the mean number is 1.4 cars. Complete the following table of the results.

Number of cars	0	1	2	3
Number of students	4			

4 Find a set of three numbers that have a range of 9, a mean of 10 and a median of 11.

5 The mean of a set of 20 numbers is 20. After the 21st number is added, the mean is now 21. What was the 21st number?

6 The median of a set of 10 numbers is 10. An even number is added, and the median is now 11 and the range is now 4. What number was added?

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Chapter summary

Data collection, representation and analysis

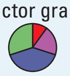
Graphs

Graphs

Column graph

Line graph

Sector graph



Divided bar graph

Stem-and-leaf plots

Stem	Leaf
0	1 1 3
1	2 5
2	0 3 8 7

Key
1|5 means 15 km

Frequency distribution tables

Grouped scores

Age range	Frequency
0–4	3
5–9	7
10–14	5
	$n = 15$

Single scores

Number of children Score	Number of families Tally	Frequency
0	|||	3
1	~~||||~~	5
2	~~||||~~ ~~||||~~ |	11
3	~~||||~~	5
4	~~||||~~ |	6
		$n = 30$

Measures of centre

numerical order →

0, 1, 4, 4, 5, 6, 9, 30

Median (middle) $= \frac{4+5}{2} = 4.5$

Mode (most frequent) = 4

Mean (average) $= \frac{59}{8} = 7.4$

Measures of spread

Range = 28 – 1 = 27

lower half | upper half

1 5 7 9 10 11 | 13 17 19 20 23 28

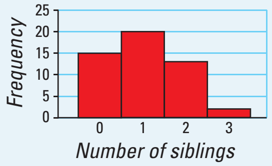
Lower quartile = Median = 8

Upper quartile = Median = 19.5

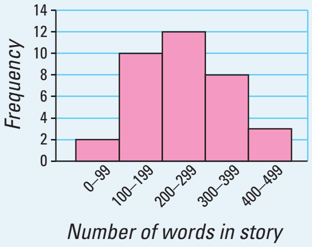
IQR = 19.5 – 8 = 11.5
Interquartile range

Histograms

Single scores

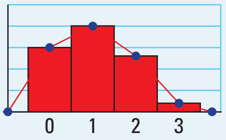


Grouped scores



Frequency polygons

Lines joining each column's centre



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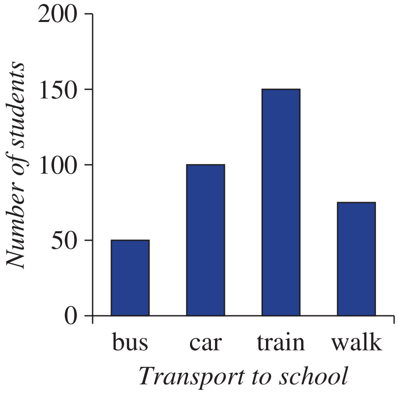
Multiple-choice questions

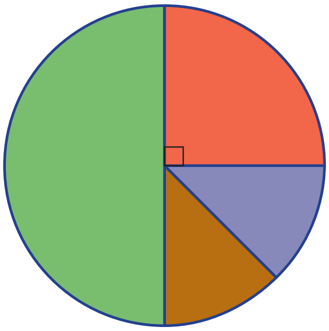
1 Using the information in the column graph, how many students don't walk to school?

A 75 **B** 150 **C** 300 **D** 375 **E** 100



2 The lollies in a bag are grouped by colour and the proportions shown in the sector graph. If there are equal numbers of blue and brown lollies, how many are blue, given that the bag contains 28 green ones?

A 112
B 7
C 56
D 14
E 28



3 The tally below shows the number of goals scored by a soccer team over a season.

Goals	0	1	2	3	4
Tally	卌||	卌	|||	|||	||

Which one of the following statements is true?

A In the first game the team scored five goals.
B In four of the games the team scored two goals.
C In most of their games they did not score any goals.
D The team had five games in which they scored one goal.
E The total number of goals scored for the season was 10.

4 The median number of goals scored by the soccer team above is:

A 0 **B** 1 **C** 2 **D** 3 **E** 4

5 Which is the best description of the mode in a set of test scores?

A the average of the scores
B the score in the middle
C the score with the highest frequency
D the difference between the highest and lowest score
E the lowest score

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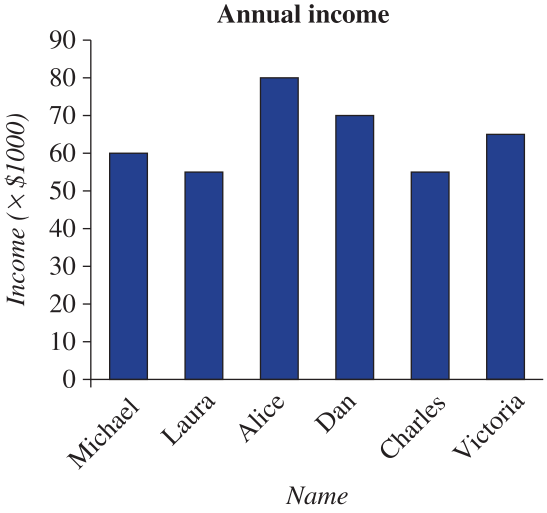
6 In the column graph shown, the highest income is earned by:

A Michael **B** Alice **C** Dan **D** Laura **E** Victoria



Question 7 relates to the following information.

The results of a survey are shown below.

Instrument learned	piano	violin	drums	guitar
Number of students	10	2	5	3

7 If the results above are presented as a sector graph, then the angle occupied by the drums sector is:

A 360° **B** 180° **C** 120° **D** 90° **E** 45°

8 The median of the numbers 2, 4, 7, 9, 11, 12 is:

A 7.5 **B** 7 **C** 9 **D** 8 **E** 11

Questions 9 and 10 relate to the following information.

In a class of 20 students, the number of days each student was absent over a 10-week period is recorded.

1, 0, 1, 2, 2, 3, 2, 4, 3, 0, 1, 1, 2, 3, 3, 3, 2, 2, 2, 2

9 The mode is:

A 0 **B** 1 **C** 2 **D** 3 **E** 4

10 The mean number of days a student was absent is:

A 1 **B** 2 **C** 1.95 **D** 3 **E** 39

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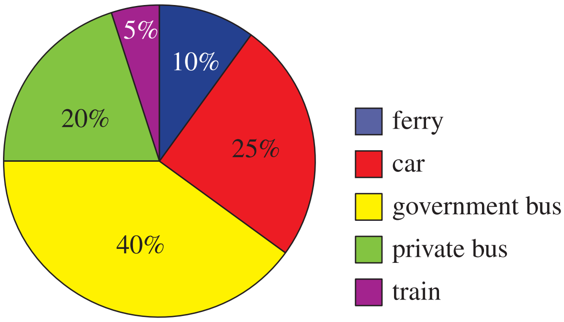
Short-answer questions



1 The sector graph shows the mode of transport office workers use to get to work every day.

- **a** Which mode of transport is the most popular?
- **b** Which mode of transport is the least popular?
- **c** What angle of the 360° sector graph is represented by ‘private buses’?
- **d** If 20 000 workers were surveyed, how many people travelled to work each day by train?
- **e** The year after this survey was taken, it was found that the number of people using government buses had decreased. Give a reason why this could have occurred.

2 Some students were asked how many hours of study they did before their half-yearly Maths exam. Their responses are represented in the table below.

0 hours	**1 hour**	**2 hours**	**3 hours**	**4 hours**
||	𝍸|	|||	|||	𝍸|||

- **a** How many students are in the class?
- **b** Convert the tally above into a frequency distribution table.
- **c** Draw a histogram to represent the results of the survey.
- **d** What proportion of the class did no study for the exam?
- **e** Calculate the mean number of hours the students in the class spent studying for the exam, giving the answer correct to one decimal place.

3
- **a** Rewrite the following data in ascending order.

  56 52 61 63 43 44 44 72 70 38 55
  60 62 59 68 69 74 84 66 53 71 64
- **b** What is the mode?
- **c** What is the median for these scores?
- **d** Calculate the range.

4 The ages of girls in an after-school athletics squad are shown in the table below.

- **a** State the total number of girls in the squad.
- **b** Display their ages in a histogram and draw a frequency polygon on top of it.
- **c** Calculate the mean age of the squad, correct to two decimal places.
- **d** What is the median age of the girls in the squad?

Age (years)	**Frequency**
10	3
11	8
12	12
13	4
14	3

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5 A group of teenagers were weighed and their weights recorded, to the nearest kilogram. The results are as follows.

56 64 72 81 84 51 69 69 63 57 59 68 72 73 72 80 78 61 61 70

57 53 54 65 61 80 73 52 64 66 66 56 50 64 60 51 59 69 70 85

a Find the highest and lowest weights.
b Create a grouped frequency distribution table using the groups 50–54, 55–59, 60–64 etc.
c Find:
  i the range
  ii the modal class
d Why is this sample not representative of the whole human population?

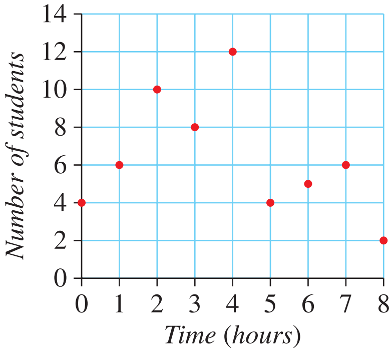
6 **a** Consider the data 5, 1, 7, 9, 1, 6, 4, 10, 12, 14, 6, 3. Find:
  i the mean
  ii the median
  iii the lower quartile
  iv the upper quartile
  v the interquartile range (IQR)
b Repeat for the data 6, 2, 8, 10, 2, 7, 5, 11, 13, 15, 7, 4. What do you notice?

7 In an attempt to find the average number of hours of homework that a Year 8 student completes, Samantha asks 10 of her friends in Year 8 how much homework they do.
a Explain two ways in which Samantha's sampling is inadequate to get the population average.
b If Samantha wished to convey to her parents that she did more than the average, how could she choose 10 people to bias the results in this way?

8 A Year 7 group was asked how many hours of television they watch in a week. The results are given in the table.
a How many students participated in the survey?
b What is the total number of hours of television watched?
c Find the mean number of hours of television watched.
d Show this information in a column graph.

TV watched (hours)	No. of students
8	5
9	8
10	14
11	8
12	5

9 The number of students in the library is recorded hourly, as displayed in the graph.
a How many students entered the library when it first opened?
b How many students were in the library at 8 hours after opening?
c If the library opens at 9 a.m., at what time are there the most number of students in the library?
d How many students were in the library at 4 p.m.?



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10 120 people were asked to nominate their favourite takeaway food from the list: chicken, pizza, hamburgers, Chinese. The results are given in the table.

Food	Frequency
chicken	15
pizza	40
hamburgers	30
Chinese	35

a If you want to show the data in a sector graph, state the angle needed to represent Chinese food.
b What percentage of people prefer hamburgers?
c Represent the results in a sector graph.

11 Consider the data 1, 4, 2, 7, 3, 2, 9, 12. State the:
a range **b** mean **c** median **d** mode

12 Consider the data 0, 4, 2, 9, 3, 7, 3, 12. State the:
a range **b** mean **c** median **d** mode

Extended-response questions

1 The number of rainy days experienced throughout a year in a certain town is displayed below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
No. of rainy days	10	11	3	7	2	0	1	5	6	9	7	5

a Show this information in a line graph.
b For how many days of the year did it rain in this town?

2 At a school camp, a survey was conducted to establish each student's favourite dessert.

Ice-cream	Yoghurt	Danish pastry	Jelly	Pudding	Cheesecake
10	5	2	7	4	12

a How many students participated in the survey?
b What is the most popular dessert selected?
c What is the probability that a randomly selected student chooses jelly as their favourite dessert?
d For each of the following methods listed below, state whether it would be a reasonable way of presenting the survey's results.
  i column graph **ii** line graph
  iii sector graph **iv** divided bar graph
e If the campers attend a school with 800 students, how many students from the entire school would you expect to choose pudding as their preferred dessert?

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Chapter 6: Angle relationships and properties of geometrical figures 1

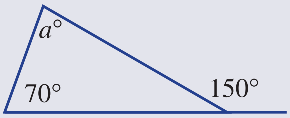
Multiple-choice questions

1 The supplement of 80° is:

A 10° **B** 100° **C** 280° **D** 20° **E** 80°

2 In this diagram a equals:

A 150 **B** 220 **C** 70 **D** 80 **E** 30

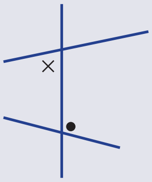


3 Each interior angle of an equilateral triangle is:

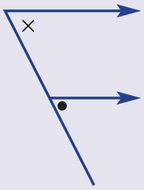
A 30° **B** 45° **C** 90° **D** 180° **E** 60°

4 Which diagram shows equal alternate angles?

A



B



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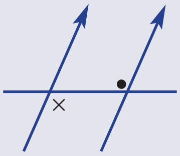

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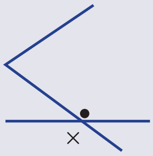
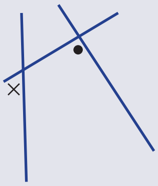


E



5 In which quadrilateral are the diagonals *definitely* equal in length?

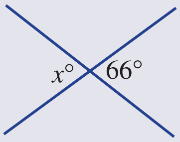
A rhombus **B** kite **C** rectangle

D trapezium **E** None of these options

Short-answer questions

1 Find the value of x.

a



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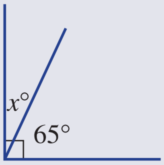

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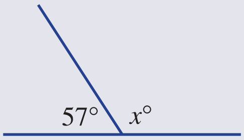

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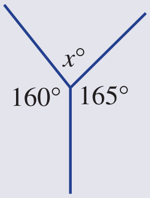

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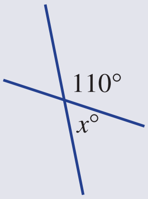

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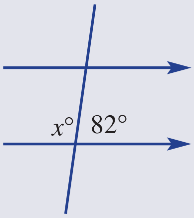

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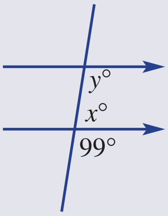
2 Find the value of each pronumeral.

a



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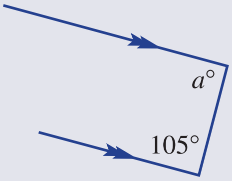

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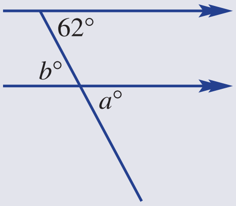

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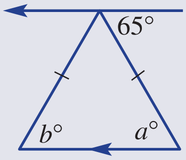

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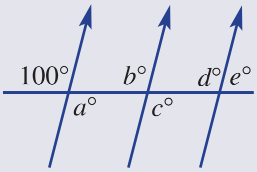

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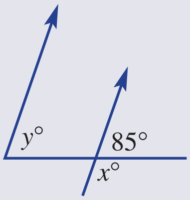
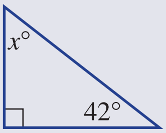


3 Find the value of the pronumeral in these triangles.

a



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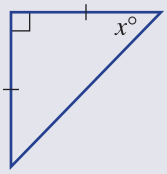

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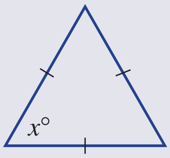

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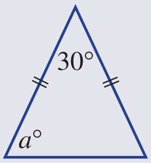

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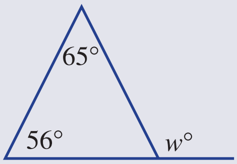

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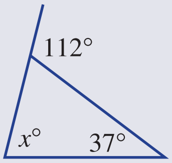
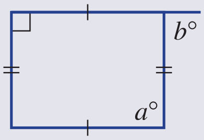
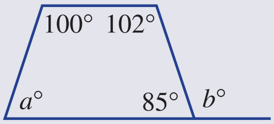


4 Find the value of *a* and *b* in these quadrilaterals.

a



b



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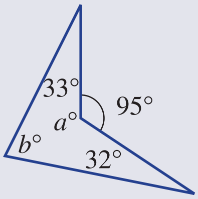
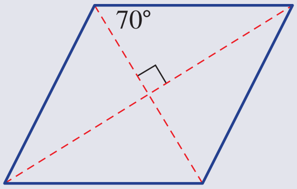


5 Find all the unknown angles in this rhombus.



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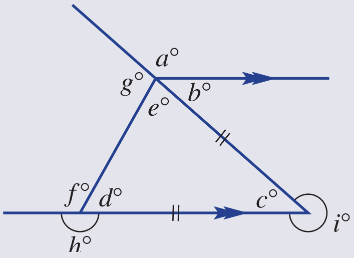
Extended-response question

If $a = 115$, find the size of each angle marked. Give a reason for each answer. Write your answers in the order you found them.



Chapter 7: Linear relationships 1

Multiple-choice questions

1 The point (2, −1) lies on the line with equation:

A $y = 3x - 1$ **B** $y = 3 - x$ **C** $x + y = 3$ **D** $y = 1 - x$ **E** $y = x + 1$

2 The coordinates of the point 3 units directly above the origin is:

A (0, 0) **B** (0, 3) **C** (0, −3) **D** (3, 0) **E** (−3, 0)

3 The rule for the table of values shown is:

A $y = 2x$ **B** $y = 2x + 2$ **C** $y = 2(x + 2)$
D $y = x + 4$ **E** $y = x + 2$

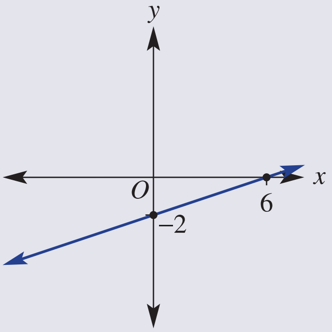
x	0	2	4
y	4	8	12

4 Which point lies on the line $y = 4x - 5$?

A (2, 0) **B** (2, 1) **C** (2, 2) **D** (3, 2) **E** (2, 3)

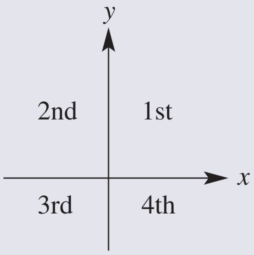
5 The equation of the line shown is:

A $y = 6x - 2$
B $y = 3x - 2$
C $y = x - 2$
D $3y = x - 6$
E $y = 3x - 6$



Short-answer questions

1 In which quadrant does each point lie?



a (5, 1) **b** (−3, 4) **c** (−5, −1) **d** (8, −3)

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2 **a** Complete these tables of values.

i $y = 2x + 1$

x	0	1	2	3
y				

ii $y = 4 - x$

x	0	1	2	3
y				

b Sketch each line on the same number plane and state the point of intersection of the two lines.

3 Consider the table of values:

x	0	1	2	3	4
y	−6	0	6	12	18

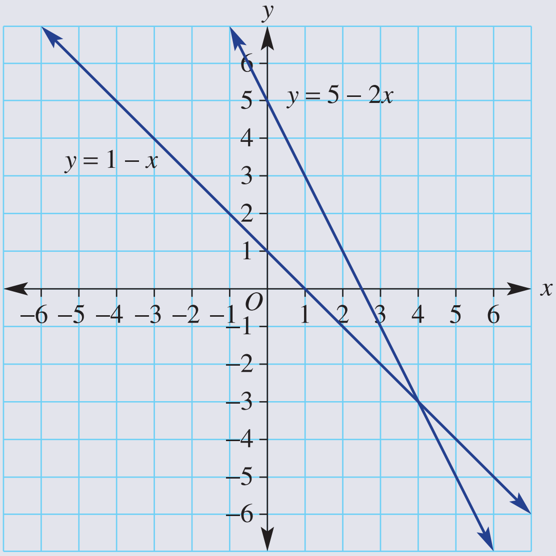
Complete this sentence: To find a y value, multiply x by ___________, then ___________ __________.

4 Find the rule for this table:

x	0	2	4	6
y	3	4	5	6

5 Sketch the curves $y = x^2$ and $y = 2x$ on the same number plane, by first completing a table of values with x from −2 to 3. Write down the point of intersection of the two graphs in the first quadrant.

Extended-response question



a Use the appropriate line on the graph above to find the solution to the following equations.

i $1 - x = 5$ **ii** $1 - x = 3.5$ **iii** $5 - 2x = 3$

iv $5 - 2x = -2$ **v** $1 - x = -5$

b Write the coordinates of the point of intersection of these two lines.

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c Show whether each point below is a solution or not a solution of the linear equation $y = 5 - 2x$.
  i (1, 3) **ii** (3, 1) **iii** (5, −5)

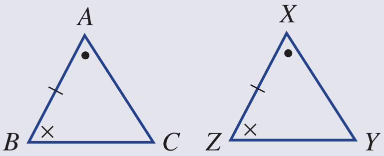
d State the point of intersection and show it is a solution to both linear equations $y = 5 - 2x$ and $y = 1 - x$.

Chapter 8: Transformations and congruence

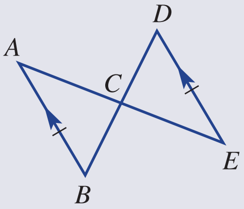
Multiple-choice questions

1 The number of lines of symmetry in a square is:
A 0 **B** 2 **C** 4 **D** 6 **E** 8

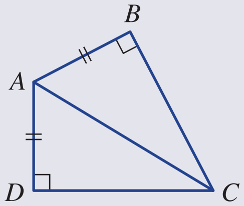
2 The side AC corresponds to:
A XZ **B** XY
C ZY **D** BC
E AX



3 A congruency statement for these triangles is:
A $\triangle ABC \equiv \triangle CED$ **B** $\triangle ABC \equiv \triangle CDE$
C $\triangle ABC \equiv \triangle ECD$ **D** $\triangle ABC \equiv \triangle DCA$
E $\triangle ABC \equiv \triangle EDC$



4 Which test is used to show triangle ABC is congruent to triangle ADC?
A SSS **B** SAS **C** AAS
D RHS **E** ASA



5 Which of the following codes is not enough to prove congruency for triangles?
A SSS **B** AAS **C** AAA **D** SAS **E** ASA

Short-answer questions

1 How many lines of symmetry does each of these shapes have?
a scalene triangle **b** rhombus **c** rectangle **d** semicircle

2 Write the vectors that translate each point P to its image P'.
a $P(1, 1)$ to $P'(3, 3)$ **b** $P(-1, 4)$ to $P'(-2, 2)$

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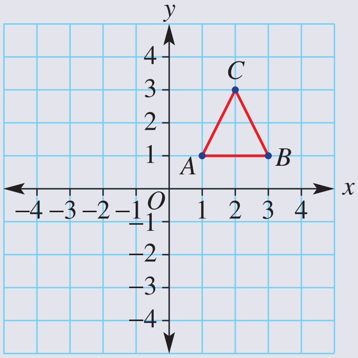
3 Triangle ABC is on a Cartesian plane, as shown. List the coordinates of the image points A', B' and C' after:

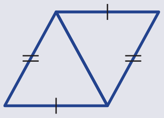
a a rotation 90° clockwise about (0, 0)

b a rotation 180° about (0, 0)

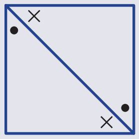


4 Write the congruency test that would be used to prove the following pairs of triangles are congruent.

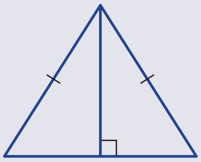
a



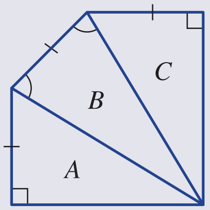
b



c



5 Which two triangles are congruent?



Extended-response question

A circle of radius 3 cm is enlarged so that the ratio of radii becomes 2 : 5.

a What is the diameter of the enlarged image?

b What is the ratio of the circumference of the original circle to its image?

c What is the ratio of their areas?

d If the ratio of a circle's area to its image becomes 9 : 100, what is the radius of the image?

Chapter 9: Data collection, representation and analysis

The following information is relevant for questions 1 and 2.

A survey asked 60 participants their favourite colour. The results are shown below.

blue	pink	green	purple	black
12	20	6	12	10

1 If a divided bar graph 10 cm long is constructed to display the data, the length required for purple would be:

A 12 cm **B** 2 cm **C** 1 cm **D** 5 cm **E** 3 cm

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2 If the results are shown as a sector graph, the size of the smallest sector would be:

A 36° **B** 6° **C** 10° **D** 60° **E** 30°

3 For the set of numbers 1, 5, 2, 10, 1, 6, the difference between the mode and median is:

A 5.5 **B** 2.5 **C** 3.5 **D** 3.2 **E** 2

4 For the set of numbers 3, 2, 1, 5, 7, 3, 1, 3, the mean is:

A 3 **B** 5 **C** 6 **D** 22.375 **E** 3.125

5 Which of the following sets of numbers has a mode of 3 and a mean of 2?

A 1, 1, 2, 2, 3 **B** 2, 2, 2, 4, 5 **C** 1, 2, 3, 4, 5 **D** 1, 3, 3, 3, 0 **E** 1, 2, 3, 3, 3

Short-answer questions

1 Find:

- **i** the mean
- **ii** the median
- **iii** the range of these datasets.

a 10, 15, 11, 14, 14, 16, 18, 12

b 1, 8, 7, 29, 36, 57

c 1.5, 6, 17.2, 16.4, 8.5, 10.4

2 For the dataset 1, 1, 3, 5, 8, 8, 9, 10, 12, 17, 24, 30, find:

a the median score

b the lower quartile

c the upper quartile

d the interquartile range

3 The mean mark for a Chemistry quiz for a class of 20 students is 16 out of 20. On the same quiz, another class of 15 students has a mean of 18 out of 20. What is the combined mean of all the students? Give your answer correct to one decimal place.

4 Calculate the mean and median for:

a 1, 2, 5, 10, 10, 13, 13, 18, 19, 24, 25, 28, 28, 30, 35

b

Score	Frequency
10	15
11	29
12	11
13	5

5 Draw a histogram for the frequency distribution table in Question **4b**.

6 This stem-and-leaf plot shows the ages of a group of people in a room.

a How many people are in the room?

b What are the ages of the youngest and oldest people?

c For the data presented in this plot, find the:

- **i** range
- **ii** median
- **iii** mode

Stem	Leaf
0	3 5
1	1 7 9
2	0 2 2 3
3	6 9
4	3 7

2|2 means 22

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7 A set of six scores is given as 17, 24, 19, 36, 22 and $\square$.

a Find the value of $\square$ if the mean of the scores is 23.

b Find the value of $\square$ if the median of the scores is 22.

c If the value of $\square$ is 14, and the score of 17 is removed from the set, calculate the new mean and median of the group.

8 The ages of 50 people at a party are shown in the table below.

Age (years)	0–9	10–19	20–29	30–39	40–49	50–59	Over 60
Frequency	3	7	1	28	6	2	3

How many people are aged:

a 0–9?

b 30 or older?

c in their twenties?

d not in their fifties?

Extended-response question

Two groups of students have their pulse rates recorded as beats per minute. The results are listed here:

Group A: 65, 70, 82, 81, 67, 74, $\square$, 81, 88, 84, 72, 65, 66, 81, 72, 68, 86, 86

Group B: 83, 88, 78, 60, 81, 89, 91, 76, 78, 72, 86, 80, 64, 77, 62, 74, 87, 78

a How many students are in group B?

b If the median pulse rate for group A is 76, what number belongs in the $\square$?

c What is the median pulse rate for group B?

d Which group has the largest range?

Chapter 1

Pre-test

1 **a** 32 **b** 14 **c** 12 **d** 30

2 **a** 4 **b** −3 **c** −11 **d** −3

3 **a** $x + 5$ **b** $m - 7$ **c** xy **d** $\frac{w}{2}$
e $2(p + q)$ **f** $\frac{1}{2}(p + q)$ or $\frac{p + q}{2}$

4 7.4

5 **a**

A	0	3	7	10
M	3	9	17	23

b

x	1	3	11	0
y	1	2	6	$\frac{1}{2}$

6 **a** 4 **b** −12 **c** 20 **d** 6

7 **a** 5×5 **b** $2 \times 2 \times 2 \times 2$
c $3 \times 3 \times 3$ **d** $-8 \times -8 \times -8$

8 **a** 125 **b** 41 **c** 16 **d** 23

9 **a** 5 **b** 6 **c** 10
d 3, 2 **e** 1

10 **a** 12 **b** 3 **c** 48

Exercise 1A

1 **a** $3a, 2b, 5c$ **b** **i** 3 **ii** 2 **iii** 5
c $2x + 5y + 8z$. Other answers are possible.

2 **a** 6 **b** **i** 5 **ii** 7 **iii** 1
c $x + 2y + 3z + 4w + 91k$. Other answers are possible.

3 **a** F **b** C **c** E **d** D
e A **f** B

4 **a** **i** 3 **ii** $7a, 2b, c$ **b** **i** 3 **ii** $19y, 52x, 32$
c **i** 2 **ii** $a, 2b$ **d** **i** 4 **ii** $7u, 3v, 2a, 123c$
e **i** 2 **ii** $10f, 2be$ **f** **i** 5 **ii** $9, 2b, 4c, d, e$
g **i** 4 **ii** $5, x^2y, 4abc, 2nk$
h **i** 4 **ii** $ab, 2bc, 3cd, 4de$

5 **a** 2 **b** 1 **c** 9 **d** −2
e 1 **f** 0 **g** 0 **h** −6
i −1 **j** −12 **k** −1 **l** −3

6 **a** $y + 7$ **b** $x - 3$ **c** $a + b$ **d** $4p$
e $4 - \frac{q}{2}$ **f** $10 + \frac{r}{3}$ **g** $2(b + c)$ **h** $b + 2c$
i $\frac{abc}{7}$ **j** $\frac{a}{4} + \frac{b}{2}$ **k** $\frac{x}{2y}$ **l** $a - \frac{b}{2}$
m k^2 **n** w^2 **o** $q - p$ **p** $y^2 - 3x$

7 **a** the sum of 3 and x **b** the sum of a and b
c the product of 4, b and c **d** double a is added to b
e b is subtracted from 4 and the result is doubled.
f b is doubled and the result is subtracted from 4.

8 **a** $7x$ **b** **i** $x - 3$ **ii** $7(x - 3)$

9 **a** $2p$ **b** $48p$ **c** $30p + 18(p + 20)$

10 **a** $4a$ **b** $7b$ **c** $5a + 5b$ **d** $\frac{7a + 7b}{2}$

11 $70 + 90x$

12 **a** $20 + 50t$ **b** $0.2 + \frac{t}{2}$ **c** $0.2 + 30t$

13 **a** true **b** true **c** false
d true **e** false **f** false

14 **a** true **b** true **c** false **d** true

15 Adding 5 and doubling are happening in an opposite order, so changing the result.

16 **a** 36 **b** 676 **c** t
d $121k$ **e** 67 108 863

Exercise 1B

1 15

2 8

3 30

4 **a** 14 **b** 30 **c** no

5 **a** 30 **b** 37 **c** 16 **d** 58
e 2 **f** −40 **g** −61 **h** −19

6 **a** 24 **b** −9 **c** 1 **d** −19
e 12 **f** 90 **g** 1 **h** −7
i 100 **j** 13 **k** 45 **l** 16

7 **a** 8 **b** 32 **c** 6 **d** −2
e 13 **f** −31

8 **a** 96 **b** 122 **c** 24 **d** −38
e 132 **f** 54

9 **a** E **b** E **c** N **d** E
e N **f** N **g** E **h** E

10 **a** $3 + x$ **b** $2 - a$ **c** $4t - 2t$ **d** $3u - 8$

11 7, 8, 9, 13, 103

12 $4y + 2x + 5$, $5 + 2x + 4y$, $2(x + 2y) + 5$. Other answers are possible.

13

x	3	1	0.25	6	−2	2
$4x + 2$	14	6	3	26	−6	10
$4 - 3x$	−5	1	3.25	−14	10	−2
$2x - 4$	2	−2	−3.5	8	−8	0

14 **a** **i** 9 **ii** 9 **iii** 49 **iv** 49
b 625
c The negative sign cancels out since neg × neg = pos.
d $(-3)^2$ means -3×-3 but -3^2 means $-(3 \times 3)$.

15 **a** Example: $24 \div (2 \times 3) = 4$ but $(24 \div 2) \times 3 = 36$
b No, this is only when the operation is the same throughout.
c No, e.g. $24 \div (6 \div 2) = 8$ but $(24 \div 6) \div 2 = 2$.

16 **a** yes **b** no **c** no
d yes **e** yes **f** no

17 **a** It is
b No, e.g. $(2 + 1)^2 = 9$ but $2^2 + 1^2 = 5$.
c Yes, substitute values
d No, e.g. $\sqrt{9} + \sqrt{16} = 7$ but $\sqrt{9 + 16} = 5$.
e Choosing $a = 0$ and $b = 0$

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18 a Multiplication is commutative (order unimportant).
b Adding a number to itself is double the number.
c A number subtracted from itself always results in zero.
d Dividing by 2 and multiplying by $\frac{1}{2}$ have the same effect.
e $a \times 3a$ means $a \times 3 \times a$, which is equivalent to $3 \times a \times a$ by commutative law and can be written as $3a^2$.
f Negative sign is lost when squaring because neg × neg = pos.

19

a	5	8	2	3	−20	10	−9	−6
b	2	2	1	7	10	−3	10	−13
$a + b$	7	10	3	10	−10	7	1	−19
$a + 2b$	9	12	4	17	0	4	11	−32
$a - b$	3	6	1	−4	−30	13	−19	7
$a - 2b$	1	4	0	−11	−40	16	−29	20

Exercise 1C

1 a L **b** L **c** L **d** N
e N **f** L **g** N **h** N
2 a L **b** L **c** N **d** L
e N **f** N
3 a 21 **b** 21 **c** true
4 a 23 **b** 84 **c** false
5 a 28
b i 12 **ii** 20 **iii** 28
c $7x$
6 a $5x$ **b** $19a$ **c** $9x$ **d** $7xy$ **e** $13uv$
f $14ab$ **g** $7ab$ **h** $16k$ **i** $10k$
7 a $9f + 12$ **b** $13x + 8y$ **c** $7a + 11b$
d $13a + 9b$ **e** $12 + 12x$ **f** $8a + 3b + 3$
g $14x + 30y$ **h** $21a + 4$ **i** $17x^2y + 5x$
j $13xy$ **k** $-x^2$ **l** $2a + 4b - 7ab$
m $10 + 9q - 4r$ **n** $9b + 2b^2$
8 a C **b** A **c** D **d** E **e** B
9 a $22x$ **b** $6y + 6 + 2x$ **c** $12a + 4b$
10 a \$13c **b** \$9nc
11 a 7, 2 **b** 6, 7 **c** 7, 9, 5 **d** 8, 6
12 From left to right, top to bottom: $2x$, $2y$, $3y$, $5x + y$, $2x + 2y$
13 9 ways
14 $-50a$
15 Both are equivalent to $17x + 7y$.
16 a If $a = 1$, $b = 2$: $4a + 3b = 10$, $7ab = 14$. Other answers are possible.
b Yes, for example if $a = 0$ and $b = 0$.
c No, they are equivalent.
17 a Yes, both are equivalent to $4x$.
b −13, −3, −2
18 a From top left to bottom right:
$a + 3$, $-a$, $2b + 4$, $2a$, $a + 1$, $7a + 2b$, 0
b Answers will vary.

Exercise 1D

1 B
2 a $\frac{3}{5}$ **b** $\frac{1}{3}$ **c** $\frac{3}{2}$ **d** $\frac{3}{5}$
3 B
4 a $3xy$ **b** $5abc$ **c** $12ab^2$ **d** $4ac^3$
5 a $63d$ **b** $10ab$ **c** $36x$ **d** $8abcd$
e $60abcd$ **f** $48abde$
6 a $24abc$ **b** a^2 **c** $3d^2$ **d** $10d^2e$
e $14x^2y$ **f** $10x^2y$ **g** $8x^2yz$ **h** $8a^2b^2cd$
i $18a^3b$ **j** $6xz^2$ **k** $-10xy^2z$ **l** $16xy^3$
7 a $2 \times 3 \times 3 \times x \times y$ **b** $2 \times 2 \times 3 \times a \times b \times c$
c $3 \times 5 \times x \times x$ **d** $3 \times 17 \times a \times b \times b$
8 a $\frac{1}{2}$ **b** $\frac{x}{2y}$ **c** $\frac{5x}{6}$ **d** $\frac{a}{4}$
e $\frac{x}{3}$ **f** $\frac{1}{6x}$ **g** $\frac{-x}{2yz^2}$ **h** $\frac{-2y}{3}$
i $\frac{-a}{2b}$ **j** $\frac{-7p}{q}$ **k** 7 **l** $\frac{3}{4z}$
9 a $\frac{x}{3}$ **b** $\frac{m}{n}$ **c** $\frac{10a}{12b} = \frac{5a}{6b}$ **d** $\frac{x^2}{9}$
e $\frac{a + b}{3}$ **f** $\frac{3}{4}$ **g** $4a$ **h** $\frac{a}{3b}$
10 a $8ab$ **b** $24x^2$ **c** $18xy$ **d** $17xy$
e $13a^2$ **f** $88xy$
11 a $2y$ **b** $3b^2$ **c** -2 **d** $28rs$
e $8ab^2$ **f** $-7x$
12 $18x^3$
13 a x^2 **b** $4x$
c $\frac{x^2}{4x} = \frac{x}{4} =$ one-quarter of side length
14 a no **b** $\frac{2a}{5}$ and $\frac{2}{5} \times a$ **c** $a = 1$, $a = -1$
15 a bc **b** $\frac{8a^2}{3}$ **c** $\frac{x}{6}$ **d** $16a$ **e** $\frac{1}{2}$ **f** $\frac{-2x}{y}$

Exercise 1E

1 a 7 **b** 4, 9 **c** algebraic **d** 7
2 a 15 **b** 20 **c** 42 **d** 6
3 a 4 **b** 12 **c** 4 **d** 30
4 a $\frac{7}{12}$ **b** $\frac{17}{35}$ **c** $\frac{3}{10}$ **d** $\frac{3}{20}$
5 a 15 **b** 20 **c** 20 **d** 12
6 a $2x$ **b** $6a$ **c** $16z$ **d** $15k$
7 a $\frac{3x}{4}$ **b** $\frac{7a}{3}$ **c** $\frac{3b}{5}$ **d** $\frac{5k}{3}$
e $\frac{5a}{6}$ **f** $\frac{9a}{20}$ **g** $\frac{7p}{10}$ **h** $\frac{3q}{4}$
i $\frac{29k}{35}$ **j** $\frac{16m}{15}$ **k** $\frac{47p}{30}$ **l** $\frac{5x}{8}$
8 a $\frac{2y}{5}$ **b** $\frac{5p}{13}$ **c** $\frac{8r}{7}$ **d** $\frac{6q}{5}$
e $\frac{p}{6}$ **f** $\frac{t}{15}$ **g** $\frac{7u}{22}$ **h** $\frac{11y}{6}$
i $\frac{-r}{6}$ **j** $\frac{-13u}{42}$ **k** $\frac{33u}{4}$ **l** $\frac{-29p}{132}$

9 **a** $\frac{13x}{3}$ **b** $\frac{7x}{2}$ **c** $\frac{11a}{5}$

d $\frac{2p}{3}$ **e** $\frac{100u + 9v}{30}$ **f** $\frac{7y - 4x}{10}$

g $\frac{4t + 7p}{2}$ **h** $\frac{x - 3y}{3}$ **i** $\frac{35 - 2x}{7}$

10 **a** $\frac{x}{3}$ **b** $\frac{x}{4}$ **c** $\frac{7x}{12}$

11 **a** $\frac{T}{4} + \frac{B}{2}$ **b** $\frac{T + 2B}{4}$ **c** 251 litres

12 $\frac{A - 40}{2}$

13 **a** For example, if $x = 12$, $\frac{x}{2} + \frac{x}{3} = 10$ and $\frac{5x}{6} = 10$.

b For example, if $x = 1$, $\frac{1}{4} + \frac{1}{5} \neq \frac{2}{9}$.

c No. If $x = 1$ they are different.

14 **a** **i** $\frac{x}{6}$ **ii** $\frac{x}{12}$ **iii** $\frac{x}{20}$ **iv** $\frac{x}{30}$

b Denominator is product of initial denominators, numerator is always x.

c $\frac{x}{10} - \frac{x}{11}$

15 **a** $\frac{2z}{3}$ **b** $\frac{7x}{10}$ **c** $\frac{29u}{8}$ **d** $\frac{29k}{12}$

e $\frac{3p - 12}{4}$ **f** $\frac{47u}{60}$ **g** $\frac{24 + j}{12}$ **h** $\frac{16t + 2r}{15}$

Exercise 1F

1 **a** 8 **b** 3 **c** 55 **d** 32

2 C

3 **a** $\frac{7}{15}$ **b** $\frac{2}{11}$ **c** $\frac{3}{22}$ **d** $\frac{1}{4}$

e $\frac{5}{6}$ **f** $\frac{3}{8}$ **g** $\frac{12}{17}$ **h** $\frac{4}{9}$

4 **a** $\frac{x}{3}$ **b** $\frac{5x}{2}$ **c** $\frac{a}{2}$ **d** $\frac{2}{5}$

5 **a** $\frac{2x}{15}$ **b** $\frac{a}{63}$ **c** $\frac{8a}{15}$ **d** $\frac{4c}{25}$

e $\frac{8ab}{15}$ **f** $\frac{21a^2}{10}$

6 **a** $\frac{7xy}{5}$ **b** $\frac{7bd}{15}$ **c** $\frac{6ab}{5c}$ **d** $\frac{18de}{7}$

e $\frac{1}{4}$ **f** $\frac{2}{3}$

7 **a** $\frac{15a}{4}$ **b** $\frac{14x}{15}$ **c** $\frac{18a}{5}$ **d** $\frac{7}{6x}$

e $\frac{6}{5y}$ **f** $\frac{x}{14}$ **g** $\frac{10a}{7}$ **h** $\frac{10b}{7c}$

i $\frac{3x}{10y}$ **j** $\frac{2y^2}{3x}$ **k** $\frac{5}{42x^2}$ **l** $\frac{14a^2}{5b}$

8 **a** $\frac{12x}{5}$ **b** $\frac{4x}{15}$ **c** $\frac{10}{x}$

d $\frac{4a}{3}$ **e** $\frac{7}{2x}$ **f** $\frac{2}{x}$

9 **a** $\$\frac{x}{2}$ **b** $\$\frac{x}{6}$

10 **a** xy m^2 **b** **i** $\frac{x}{2}$ m **ii** $\frac{3y}{4}$ m **iii** $\frac{3xy}{8}$ m^2 **c** $\frac{3}{8}$

11 **a** $\frac{3q}{2}$ **b** $\frac{x}{2}$ **c** 1 **d** $\frac{3x}{8}$

12 **a** 1 **b** x^2 **c** Both are $\frac{x}{3}$.

d $\frac{a}{bc}$ **e** $\frac{ac}{b}$

13 **a** **i** $\frac{11x}{30}$ **ii** $\frac{x}{30}$ **iii** $\frac{x^2}{30}$ **iv** $\frac{6}{5}$

b $\frac{x}{5} \div \frac{x}{6}$

14 **a** $1 \div \frac{a}{b} = \frac{1}{1} \times \frac{b}{a} =$ reciprocal of $\frac{a}{b}$ **b** $\frac{a}{b}$

15 **a** x^2

b $\frac{x^2}{4}$

c $\frac{9x^2}{25}$. Proportion is $\frac{9}{25} < \frac{1}{2}$.

d $\frac{49x^2}{100}$ or $0.49x^2$

e 0.707 (Exact answer is $\frac{1}{\sqrt{2}}$.)

Exercise 1G

1 **a** 10 **b** $5x$ **c** $10 + 5x$ **d** $10 + 5x$

2 $6 + 21x$

3 **a** $6a$ **b** 42 **c** $6a + 42$ **d** $6a + 42$

4 D

5 **a** 10 **b** 12 **c** 10 **d** 14

6 **a** $9a + 63$ **b** $4 + 2t$ **c** $8m - 80$ **d** $24 - 3v$

e $-45 - 5g$ **f** $-35b - 28$ **g** $-9u - 81$ **h** $-40 - 8h$

i $30 - 5j$ **j** $12 - 6m$ **k** $30 - 3b$ **l** $2c - 16$

7 **a** $8zk - 8zh$ **b** $6jk + 6ja$ **c** $4ur - 8uq$

d $6pc - 2pv$ **e** $10am + mv$ **f** $-2s^2 - 10sg$

g $-24gq - 3g^2$ **h** $-fn - 4f^2$ **i** $-8u^2 - 80ut$

j $-st - 5s^2$ **k** $2mh - 9m^2$ **l** $20aw - 40a^2$

8 **a** $65f + 70$ **b** $44x + 16$ **c** $15a + 32$ **d** $24v + 60$

e $76a + 70$ **f** $20q - 30$ **g** $32m - 30$ **h** $22m + 32$

9 **a** $6x + 4y + 8z$ **b** $14a - 21ab + 28ay$

c $8qz + 4aq + 10q$ **d** $-6 - 12k - 6p$

e $-5 - 25q + 10r$ **f** $-7kr - 7km - 7ks$

10 **a** $55d + 37$ **b** $115f + 54$ **c** $32j + 30$

d $75d + 32$ **e** $4j + 40$ **f** $68g + 52$

11 **a** $3(t + 4) = 3t + 12$ **b** $2(u - 3) = 2u - 6$

c $3(2v + 5) = 6v + 15$ **d** $2(3w - 2) = 6w - 4$

12 **a** D **b** A **c** B **d** E **e** C

13 **a** $5b + 3g$ **b** $2(5b + 3g) = 10b + 6g$

c $2(5b + 3g) + 4(b + g) = 14b + 10g$

d \$220

14 **a** $2(4a + 12b)$, $8(a + 3b)$. Other answers are possible.

b $2(2x + 4y)$. Other answers are possible.

c $4(3a - 2b)$. Other answers are possible.

d $3a(6b + 4c)$. Other answers are possible.

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15 a 1836 **b** 1836

c i 154 **ii** 352 **iii** 627 **iv** 869

d i 35 **ii** 399 **iii** 143 **iv** 39999

e $15^2 = 225$

$25^2 = 625$

$35^2 = 1225$

These numbers all end in 25.

In addition,

$15^2 = (10 + 5) \times (10 + 5) = 100 \times 2 + 25 = 225$

$25^2 = (20 + 5) \times (20 + 5) = 200 \times 3 + 25 = 625$

$35^2 = (30 + 5) \times (30 + 5) = 300 \times 4 + 25 = 1225$

If this pattern continues, it could be stated that

$(10n + 5)(10n + 5) = 100n(n + 1) + 25$

16 Both simplify to $8a + 6ab$.

17 $x^2 + 3xy + 2y^2$

18 $\frac{1}{3}(6(x + 2) - 6) - 2$ simplifies to $2x$.

19 a $\frac{5x + 2}{6}$ **b** $\frac{8x + 15}{15}$ **c** $\frac{5x - 2}{8}$

d $\frac{7x + 10}{12}$ **e** $\frac{7x + 3}{10}$ **f** $\frac{31x + 9}{35}$

Exercise 1H

1 a 1, 2, 4, 5, 10, 20 **b** 1, 2, 3, 4, 6, 12

c 1, 3, 5, 15 **d** 1, 3, 9, 27

2 2

3 a 6 **b** 5 **c** 20 **d** 2

4 a 12 **b** 35 **c** 12, 30 **d** $14a, 21b$

e 7 **f** 3 **g** $2, q$ **h** 4

5

$5x + 15$	25	50	35	15	0	−15
$5(x + 3)$	25	50	35	15	0	−15

6 a 5 **b** 4 **c** 9 **d** $7x$ **e** $2y$

f $11xy$ **g** $4r$ **h** $3a$ **i** p

7 a $3(x + 2)$ **b** $8(v + 5)$ **c** $5(3x + 7)$

d $5(2z + 5)$ **e** $4(10 + w)$ **f** $5(j - 4)$

g $3(3b - 5)$ **h** $4(3 - 4f)$ **i** $5(d - 6)$

8 a $2n(5c + 6)$ **b** $8y(3 + r)$ **c** $2n(7j + 5)$

d $4g(6 + 5j)$ **e** $2(5h + 2z)$ **f** $10(3u - 2n)$

g $8y(5 + 7a)$ **h** $3d(4 + 3z)$ **i** $3m(7h - 3x)$

j $7u(7 - 3b)$ **k** $14u(2 - 3b)$ **l** $3(7p - 2c)$

9 a $-3(x + 2)$ **b** $-5(x + 4)$ **c** $-5(2a + 3)$

d $-10(3p + 4)$ **e** $x(x + 6)$ **f** $k(k + 4)$

g $m(2m + 5)$ **h** $3p(2p + 1)$

10 a 5 **b** $4a + 12$

11 a $6x + 18$ **b** $6(x + 3)$ **c** $x + 3$

d $2x + 6$ **e** $3x + 9$

12 a $7x + 7$

b $7(x + 1)$

c Student prediction.

d $28(x + 1)$

13 $(x + 2)(y + 3)$

14 a i I eat fruit and I eat vegetables.

ii Rohan likes maths and Rohan likes English.

iii Petra has a computer and Petra has a television.

iv Hayden plays tennis and Hayden plays chess and Anthony plays tennis and Anthony plays chess.

b i I like sewing and cooking.

ii Olivia and Mary like ice-cream.

iii Brodrick eats chocolate and fruit.

iv Adrien and Ben like chocolate and soft drinks.

15 a $\frac{2}{5}$ **b** $\frac{7}{2}$ **c** $\frac{3c + 5}{1 + 2b}$

d $\frac{2a + b}{4c + 5d}$ **e** $\frac{5}{3}$ **f** $\frac{7}{9}$

g $\frac{7}{2}$ **h** $\frac{6}{4 + q}$ **i** 5

Exercise 1I

1 a 35 **b** 41 **c** 5

2 a 50 **b** 20 **c** 0

3 a $2x + y$ **b** 8

4 a $\$3n$ **b** \$36

5 a $3(x + 2)$ or $3x + 6$ **b** 33 units^2

6 C

7 a $5x$ **b** $10x$ **c** $5(x + 3)$ or $5x + 15$

8 a $30 + 40x$ **b** \$350

9 a \$50 **b** \$60 **c** 2 hours 30 minutes

10 a \$140 **b** $60 + 80x$ **c i** \$60 **ii** \$80

11 a $10 + 4n$ **b** $20 + n$ **c** 30 **d** deal 1

e deal 3 **f i** 3 **ii** 4, 10 **iii** 10

12 a i $2(x + 2)$ **ii** $2x + 4$ **iii** $4\left(\frac{x}{2} + 1\right)$

b All expand to $2x + 4$. **c** Divide by 2.

13 a 6 units^2 **b** 6 units^2

c Both dimensions are negative, so the shape cannot exist.

14 a $F + H$ **b** $F + 2H$ **c** $F + \frac{H}{2}$ **d** $F + tH$

15 3 buy deal F, 2 buy E, 1 buys D, 3 buy C, 5 buy B and 86 buy A.

Exercise 1J

1 5, 7 **2** B **3** B

4 a i 4 **ii** 8 **iii** 32 **iv** 64

b 2^5

5 a i 15625 **ii** 25 **iii** 125 **iv** 625

b 5^4

6 a $8 \times 8 \times 8$ **b** $8 \times 8 \times 8 \times 8$

c $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **d** C

7 a 4^8 **b** 3^{12} **c** 2^{18} **d** 7^6

8 a 2^7 **b** 5^6 **c** 7^{13} **d** 3^9

e 3^9 **f** 7^9 **g** 2^9 **h** 2^{36}

i 10^4 **j** 6^3 **k** 5^8 **l** 2^4

9 a 3^3 **b** 12^6 **c** 5^7 **d** 2^9

e 7^2 **f** 3^5 **g** 2×5^6 **h** $2^2 \times 5^3$

10 a $2^3 = 8$ **b** $5^4 = 625$ **c** 1

11 a Verify
b $3^2 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$, $9^6 = 9 \times 9 \times 9 \times 9 \times 9 \times 9$, which is obviously different
c The base should stay the same.
12 a i 4 **ii** -8 **iii** 16 **iv** -32
b i positive **ii** negative **c** 1024
13 a 5^0 **b** 1 **c** 1 **d** 1
14 a that $a - b = 2$ **b** 125
15 a m^5 **b** x^{10} **c** x^{10} **d** x^6
e a^{19} **f** m^4 **g** m^7 **h** a^3
i w^{12} **j** x^6
16 a $125xy^2$ **b** a^5 **c** $abcde$ **d** $120q^{15}$
17 a $\frac{20}{x^6}$ **b** $\frac{4abc^{15}}{7}$ **c** $-100x^7y^8$ **d** $\frac{4x^3}{3}$

Exercise 1K

1 B
2 C
3 a 27, 9, 3, 1 **b** divide by 3 each time
4 a i 4096 **ii** 1024 **iii** 4096
iv A very large number (7.04×10^{13})
b 4^6
5 a 2^{12} **b** 3^{16} **c** 7^{36} **d** 6^9
e 7^{24} **f** 2^{50}
6 a 2^{21} **b** 5^{18} **c** 7^{23} **d** 3^{22}
e 2^{18} **f** 11^{18} **g** 5^{10} **h** 11^8
i 11^{13} **j** 2^{14} **k** 3^5 **l** 11^2
7 a 1 **b** 1 **c** 2
d 4 **e** 1 **f** 1
g 1 **h** 3 **i** 1
8 a 5 **b** 3 **c** 5 **d** 2
9 a 5^{24} **b** 5^{60} **c** $((2^3)^4)^5, (2^7)^{10}, 2^{100}, ((2^5)^6)^7$
10 a i 2 **ii** 5 **iii** 6 **b** 54
11 a x^7 **b** x^{12} **c** $x = 0, x = 1$
12 a 1 **b** $5^{2-2} = 5^0$ **c** $3^0 - 3^{2-2} = \frac{3^2}{3^2} = \frac{9}{9} = 1$
d $100^0 = 100^{2-2} = \frac{100^2}{100^2} = \frac{10\,000}{10\,000} = 1$
e $\frac{0^2}{0^2}$ cannot be calculated (dividing by zero).
13 a Both equal 2^{12}. **b** B
c 3, 5. Other answers are possible.
14 a 9 **b** 3 **c** 3 **d** 6
15 a 1 **b** Many answers are possible.
16 a 1 **b** 6 **c** a^{14} **d** $4w^{12}$
e 1 **f** 7 **g** a^{14} **h** $9a^{14}$
i 1 **j** 3
17 a x^{14} **b** m^{24} **c** x^{10} **d** a^{14}
e m^{16} **f** $m^{12}n^4$ **g** $8a^{21}$ **h** $25m^8$
i $36x^8$ **j** $216a^6b^{15}$
18 a 5^4 **b** x^8 **c** x^4y^9 **d** $a^7b^{12}c^{10}$
e $x^{80}y^{10}$

Puzzles and challenges

1 $10m + 10 = 10(m + 1)$
2 Any list with $6 - 2a$ central; $2 - a$, $6a - 5$ together; $a - 7$, $4(a + 1)$ together. e.g. $2 - a$, $6a - 5$, $6 - 2a$, $a - 7$, $4(a + 1)$
3 $4^{1999}, 16^{1000}, 2^{4001}, 8^{1334}$
4 a $\frac{65}{4}$ **b** $\frac{225}{4}$ **c** 0
5 a $\frac{5 - 4a}{30}$ **b** $\frac{9x + 4}{42}$
6 a 8 **b** 45 **c** 4
7 a All perimeters $= 4a$
Areas: $a^2, \frac{3}{4}a^2, \frac{6}{9}a^2, \frac{10}{16}a^2, \frac{15}{25}a^2$
b $P = 24, A = \frac{9009}{500}$ or approximately 18.

Multiple-choice questions

1 C **2** D **3** B **4** D **5** E
6 C **7** E **8** E **9** E **10** B

Short-answer questions

1 a false **b** true **c** true
d false **e** true
2 a 2 **b** 3 **c** 4 **d** 6
3 a 11 **b** 14 **c** 29 **d** 8
4 a -3 **b** 3 **c** -9 **d** 14
e -3 **f** 12
5 3
6 a $16m$ **b** $2a + 5b$ **c** $2x^2 - x + 1$
d $7x + 7y$ **e** $9x + x^2$ **f** $-m - 12n$
7 a $36ab$ **b** $15xy$ **c** $16xy$
8 a $5x$ **b** $-4c$ **c** $15x$ **d** $3ab$
9 a $\frac{x}{4}$ **b** $\frac{6a + b}{15}$ **c** $\frac{9}{x}$ **d** $\frac{3b}{2}$
10 a $3x - 12$ **b** $-10 - 2x$ **c** $3kl - 4km$ **d** $7x - 6y$
e $13 - 3x$ **f** $10 - 20x$ **g** $18x + 2$
11 a $2(x + 3)$ **b** $8(3 - 2g)$ **c** $3x(4 + y)$ **d** $7a(a + 2b)$
12 a $a + 2$ **b** 12 **c** $\frac{1}{4 + 3q}$
13 a 7 **b** 3 **c** 6 **d** 1
14 a 5^7 **b** 6^8 **c** 7^2 **d** 5
e 2^{12} **f** 6^6
15 a 6 **b** 1 **c** 5

Extended-response questions

1 a $120 + 80n$ **b** $80 + 100n$
c A costs \$360, B costs \$380.
d any more than two hours
e $520 + 440n$ **f** \$2720

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2 **a** $xy - \frac{x^2}{4}$ **b** 33 m^2 **c** $2x + 2y$ **d** 26 m
e Area = $xy - \frac{x^2}{3}$, Perimeter = $2x + 2y$
f **i** Area is reduced by $\frac{x^2}{12}$
ii Perimeter remains the same

Chapter 2

Pre-test

1 **a** $11m$ **b** a **c** $9n$ **d** $10a - 10$
e $-3x$ **f** $12pq$
2 **a** $3m + 12$ **b** $2a + 2b$ **c** $3x + 21$ **d** $-2x + 6$
3 **a** $5m + 15$ **b** $17x + 21$
4 **a** 3 **b** a **c** x **d** m
5 12
6 **a** 8 **b** 4 **c** -2
7 **a** -6 **b** $\frac{1}{8}$ **c** 4 **d** 3
8 **a** 4 **b** 8 **c** 4 **d** 6
9 **a** 16, 8 **b** 15, 14 **c** 10, 20, 4 **d** 4, 2

Exercise 2A

1 **a** true **b** false **c** true **d** false
e true **f** true
2 **a** 13 **b** 9 **c** 2 **d** 2
3 **a** 7 **b** 9 **c** 15 **d** 8
4 **a** 25 **b** 25
c yes **d** e.g. $x + 2 = 7$ (other answers possible)
5 **a** true **b** true **c** false **d** false
e true **f** false
6 **a** true **b** false **c** true **d** true
e false **f** true
7 **a** C **b** I **c** C **d** C
8 **a** $x = 7$ **b** $x = 13$ **c** $v = 3$ **d** $p = 19$
e $x = 2$ **f** $x = 8$ **g** $u = 7$ **h** $k = 11$
i $a = 3$
9 **a** $2x + 7 = 10$ **b** $x + \frac{x}{2} = 12$
c $25 + a = 2a$ **d** $h + 30 = 147$
e $4c + 3t = 21$ **f** $8c + 2000 = 3600$
10 **a** 7 **b** 42 **c** 13 **d** -8 **e** 40 **f** -13
11 **a** $3.2x = 9.6$ **b** $x = 3$
12 **a** $E + 10 = 3E$ **b** $E = 5$
c 5 years old **d** 15
13 $t = 3, t = 7$
14 **a** $3^2 = 3 \times 3 = 9$ **b** $(-3)^2 = -3 \times -3 = 9$
c $x = 8, x = -8$
d 0 and -0 are the same number, but 1 and -1 are different numbers.
e A number multiplied by itself cannot be negative; e.g. neg. × neg = pos.
15 **a** No number can equal 3 more than itself.
b Addition is commutative.
c If $x = 7$ it is true, if $x = 6$ it is false.
d **i** S **ii** S **iii** N
iv S **v** A **vi** S
vii N **viii** S **ix** A
e $7 + x = x + 7$. Other answers are possible.
16 **a** $p = 1, p = -3$ **b** one
c Many answers possible, e.g. $(p - 2) \times (p - 3) \times (p - 5) = 0$ has 3 solutions.
17 **a** $a = 10, b = 6, c = 12, d = 20, e = 2$
b $f = \frac{1}{2}$
18 **a** $c = 4, d = 6$ (or $c = 6, d = 4$)
b $c = 11, d = 3$
c $c = 24, d = 6$
d $c = 7, d = 0$ (or $c = 0, d = -7$)

Exercise 2B

1 **a** $3x + 4 = 14$
b $11 + k = 18$
c $9 = 2x + 4$
2 **a** 12 **b** 25 **c** 16 **d** $4z$
3 **a** 8 **b** $x = 8$
4 B
5 **a** $2x = 20$ **b** $2 + q = 10$ **c** $18 = 17 - q$
d $12x = 24$ **e** $7p + 6 = 2p + 10$ **f** $3q = 2q$
6 **a** $x = 3$ **b** $q = -3$
c $x = 5$ **d** $p = 7$ (missing operation: ÷4)
e $x = 3$ **f** $p = 6$ (missing operation: ×3)
7 **a** $a = 3$ **b** $t = 7$ **c** $q = 9$ **d** $k = 9$
e $x = 10$ **f** $h = -10$ **g** $l = -4$ **h** $g = -9$
8 **a** $h = 3$ **b** $u = 4$ **c** $s = 3$ **d** $w = 8$
e $x = -4$ **f** $w = -5$ **g** $a = 2$ **h** $y = -8$
9 **a** $d = 3$ **b** $j = -6$ **c** $a = 2$ **d** $y = 2$
e $k = -4$ **f** $n = -7$ **g** $b = -3$ **h** $b = 4$
10 **a** $x = \frac{7}{2}$ **b** $q = \frac{1}{3}$ **c** $b = \frac{1}{2}$ **d** $x = \frac{5}{2}$
e $x = -\frac{5}{2}$ **f** $p = -\frac{23}{2}$ **g** $y = \frac{13}{5}$ **h** $y = -\frac{3}{2}$
11 **a** $p + 8 = 15, p = 7$ **b** $q \times -3 = 12, q = -4$
c $2k - 4 = 18, k = 11$ **d** $3r + 4 = 34, r = 10$
e $10 - x = 6, x = 4$ **f** $10 - 3y = 16, y = -2$
12 **a** $x = 2$ **b** $x = 2$ **c** $x = 6$
13 **a** $x = 7, y = \frac{14}{5}$
b $x = 2, y = 26$
c $x = \frac{10}{3}$
d $p = 15, q = \frac{13}{4}$
14 $10x + 6x = 194.88$, hourly rate = \$12.18

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15 a

$$\begin{tikzcd}
7x + 4 = 39 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
7x = 35 \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\
x = 5 \arrow[d, bend right, "\times -2"'] \arrow[d, bend left, "\times -2"] \\
-2x = -10 \arrow[d, bend right, "+13"'] \arrow[d, bend left, "+13"] \\
-2x + 13 = 3
\end{tikzcd}$$

b

$$\begin{tikzcd}
10k + 4 = 24 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
10k = 20 \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\
k = 2 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\
3k = 6 \arrow[d, bend right, "-1"'] \arrow[d, bend left, "-1"] \\
3k - 1 = 5
\end{tikzcd}$$

16 a

$$\begin{tikzcd}
4x + 3 = 11 \arrow[d, bend right, "-3"'] \arrow[d, bend left, "-3"] \\
4x = 8 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
2x = 4
\end{tikzcd}$$

b no

c The two equations have different solutions and so cannot be equivalent.

17 a $x = 5$

b Opposite operations from bottom to top.

c For example, $7 - 3x = -8$.

d There are infinitely many operations we could perform to both sides (e.g. add 1, and 2).

18 a $x = 3$ **b** $x = 1$ **c** $x = 2$ **d** $x = 4$

e $x = 3$ **f** $x = 1$ **g** $x = \frac{3}{2}$ **h** $x = 0$

Exercise 2C

1 a 8 **b** 5 **c** no

2 a 30 **b** 10 **c** ×2, 22 **d** ×10, 70

3 a C **b** A **c** B **d** D

4 a $b = 20$ **b** $g = 20$ **c** $a = 15$ **d** $k = 18$

e $l = 20$ **f** $w = -10$ **g** $s = -6$ **h** $v = 12$

i $m = 14$ **j** $n = 14$ **k** $j = -5$ **l** $f = 20$

5 a $t = -12$ **b** $h = 2$ **c** $a = -2$ **d** $c = -3$

e $s = -6$ **f** $j = 2$ **g** $v = -12$ **h** $n = 9$

i $q = -6$ **j** $f = 3$ **k** $l = -6$ **l** $r = 7$

m $x = 10$ **n** $u = 3$ **o** $k = 4$ **p** $b = -11$

q $m = 12$ **r** $y = 8$ **s** $p = 3$ **t** $g = -2$

6 a $\frac{x}{5} = 7, x = 35$ **b** $\frac{y}{2} = -12, y = -24$

c $\frac{2p}{7} = 4, p = 14$ **d** $\frac{x+4}{2} = 10, x = 16$

e $\frac{x}{2} + 4 = 10, x = 12$ **f** $\frac{2k+6}{2} = -10, k = -13$

7 a 19 **b** −13 **c** 12 **d** 26

8 a $100 - \frac{b}{3} = 60$ **b** \$120

9 a $x = 15$ **b** yes

c i $q = 130$ **ii** $q = 130$

d Keeps numbers smaller, so can be done without calculator.

e i $p = 28$ **ii** $q = -81$ **iii** $p = -77$ **iv** $r = 34$

10 a $x = 6$ **b** $x = 3$ **c** $x = 1$ **d** $x = 2$

e $x = 8$ **f** $x = -5$

11 a $x = \frac{57}{4}$ **b** $x = \frac{22}{3}$ **c** $x = \frac{80}{3}$ **d** $x = \frac{2}{3}$

12 a $x = 24$ **b** $x = 60$ **c** $x = 12$ **d** $x = 24$

e $x = 15$ **f** $x = 42$

Exercise 2D

1 a true **b** false **c** false **d** true

2 a $3x + 3$ **b** 5

c $5p + 9 = 5$ **d** $22k + 12 = 13$

3 B

4 a $f = 5$ **b** $y = 3$ **c** $s = 3$ **d** $j = 2$

e $t = -2$ **f** $n = -5$ **g** $y = -5$ **h** $t = -4$

i $q = -3$

5 a $t = -2$ **b** $c = 2$ **c** $t = 5$ **d** $z = 3$

e $t = 3$ **f** $q = -2$ **g** $x = 9$ **h** $w = 9$

i $j = -5$

6 a $a = 3$ **b** $g = 2$ **c** $n = -2$ **d** $u = 7$

e $h = -5$ **f** $j = -5$ **g** $c = 1$ **h** $n = -1$

i $a = -4$ **j** $v = -7$ **k** $c = -3$ **l** $t = 3$

m $n = 4$ **n** $n = -3$ **o** $l = 2$

7 a $x = \frac{1}{2}$ **b** $k = \frac{2}{3}$ **c** $m = -\frac{3}{2}$

d $j = \frac{5}{2}$ **e** $j = -\frac{1}{2}$ **f** $z = \frac{11}{2}$

8 a $2x + 3 = 3x + 1$ so $x = 2$ **b** $z + 9 = 2z$ so $z = 9$

c $7y = y + 12$ so $y = 2$ **d** $n + 10 = 3n - 6$ so $n = 8$

9 a $x = 6$ and $y = 10$ **b** $x = 4$ and $y = 7$

10 Area = 700 units^2, Perimeter = 110 units

11 a $4p + 1.5 = 2p + 4.9$ **b** \$1.70 **c** 11

12 a $x = 5$ **b** $x = 5$

c Variable appears on RHS if subtract $3x$.

13 $x = 8$, $y = 6$, so length = breadth = 29.

14 a No solutions.

b Subtract $2x$, then $3 = 7$ (impossible).

c $5x + 23 = 5x + 10$. Other answers are possible.

15 a $x = 20$ **b** $x = 17, y = 51, z = 10$

c $k = 12$ **d** $b = 10, a = 50$

e $a = 60, b = 30, c = 20$ **f** $x = 3.5$

Exercise 2E

1 a 12 **b** 14 **c** 8, 10 **d** 50, 30

2 a C **b** A **c** D **d** B

3 a

x	0	1	2
$4(x+3)$	12	16	20
$4x + 12$	12	16	20
$4x + 3$	3	7	11

b $4x + 12$

4 a $7x$ **b** $9p + 3$ **c** $6x + 4$ **d** $7k + 4$

e $x + 6$ **f** $5k + 21$

5 **a** $u = 6$ **b** $j = 3$ **c** $p = 6$ **d** $m = 4$
e $n = 5$ **f** $a = 3$

6 **a** $p = -2$ **b** $u = -3$ **c** $v = -5$ **d** $r = -4$
e $b = -7$ **f** $d = 3$

7 **a** $y = 3$ **b** $l = 2$ **c** $w = 2$ **d** $c = 2$
e $d = 2$ **f** $w = 6$ **g** $p = 4$ **h** $k = 2$
i $c = 10$

8 **a** $x = 7$ **b** $r = 5$ **c** $f = 3$ **d** $p = 2$
e $h = 4$ **f** $r = 5$ **g** $r = 5$ **h** $p = 6$
i $a = 5$

9 **a** $r = 7$ **b** $l = 2$ **c** $x = 7$ **d** $s = 8$
e $y = 7$ **f** $h = 3$

10 **a** $d + 4$ **b** $2(d + 4)$
c $2(d + 4) = 50$ **d** 21

11 **a** $5w + 3(w + 4)$ **b** $11.50

12 **a** $2(k - 5) = 3(k - 10)$ **b** 20

13 **a** 3.5 **b** 80

14 The equation is $2(x + 3) = 4x - 11$, which would mean 8.5 people next door.

15 Equation $3(x - 5) = 9(x + 1)$ has solution $x = -4$. He cannot be −4 years old.

16 **a** $2n + 6 = 2n + 3$ implies $6 = 3$.
b $8x + 12 = 8x + 3$ implies $12 = 3$, but if $x = 0$ then $4(2x + 3) = 4x + 12$ is true.

17 **a** $x = \frac{3}{2}$ **b** $p = \frac{36}{5}$ **c** $n = -\frac{81}{10}$ **d** $q = 0$
e $x = \frac{8}{5}$ **f** $m = \frac{1}{4}$

Exercise 2F

1 **a** **i** 9 and 9 **ii** 36 and 36
iii 1 and 1 **iv** 100 and 100
b They are equal.

2 **a** **i** 9 **ii** 49 **iii** 169 **iv** 64
b no

3 **a** 9, 3, −3 **b** 25, 5, −5 **c** 121, 11, −11

4 **a** ±2 **b** ±7 **c** ±10 **d** ±8
e ±1 **f** ±12 **g** ±6 **h** ±11
i ±13 **j** ±16 **k** ±30 **l** ±100

5 **a** ±2.45 **b** ±3.46 **c** ±6.08 **d** ±6.40
e ±10.20 **f** ±17.80 **g** ±19.75 **h** ±26.34

6 **a** 2 **b** 2 **c** 2 **d** 0
e 0 **f** 1 **g** 1 **h** 2

7 20 m **8** 4 m

9 **a** ±2 **b** ±1 **c** ±3 **d** ±2
e ±5 **f** 0 **g** ±6 **h** ±10

10 **a** $x = 0$ is the only number that squares to give 0.
b x^2 is positive for all values of x.

11 **a** 4 **b** −5 **c** −7 **d** 14

12 **a** $\pm\sqrt{11}$ **b** $\pm\sqrt{17}$ **c** $\pm\sqrt{33}$ **d** $\pm\sqrt{156}$

13 **a** **i** yes **ii** yes **iii** no
iv yes **v** no **vi** yes
b There are none.

14 **a** ±2 **b** ±1 **c** ±3
d ±1 **e** ±2 **f** ±5
g ±2 **h** ±3 **i** ±6

Exercise 2G

1 **a** 11 **b** 6 **c** 7 **d** −28

2 B **3** A

4 **a** $A = 19$ **b** $A = 51$ **c** $A = -1$ **d** $A = 33$

5 **a** $a = 5$ **b** $a = 11$ **c** $a = -2$

6 **a** $y = 10$ **b** $x = 6$ **c** $x = -2$

7 $m = 5.5$

8 **a** $y = 4$ **b** $y = 8$ **c** $x = 4.5$ **d** $x = 6$

9 **a** $G = 43$ **b** $a = 9$

10 **a** $23
b **i** $161 = 3 + 2d$ **ii** $d = 79$ **iii** 79 km

11 **a** 18 **b** 1 **c** $S = 4t + 2c + d$

12 **a** $A = 60$ **b** $h = 4$ **c** 11

13 **a** $T = \frac{V}{2} + 5$. Other answers are possible.
b 44 mL if using rule above. Other answers are possible.
c $\frac{(10 - 10)^2}{20} + 10 = 10$ and $\frac{(20 - 10)^2}{20} + 10 = 15$

14 **a** 92 **b** 24

15 A and C

16 **a** −40°C = −40°F
b 160°C = 320°F
c $1.8x = 1.8x + 32$ implies $0 = 32$.

17 **a** A charges $14.80, B charges $16.20.
b 45 minutes
c 2 minutes per call
d $t = 50$ and $c = 40$. Other answers are possible.
e 40 minutes

Exercise 2H

1 **a** D **b** A **c** E **d** C **e** B

2 **a** B **b** C **c** A **d** D

3 **a** $p = 6$ **b** $x = 9$ **c** $k = 4$ **d** $a = 3$

4 **a** Let c = cost of one cup. **b** $4c = 13.2$
c $c = 3.3$ **d** $3.30

5 **a** Let c = cost of one chair. **b** $6c + 1740 = 3000$
c $c = 210$ **d** $210

6 **a** $2(4 + b) = 72$ or $8 + 2b = 72$
b $b = 32$ **c** 32 cm

7 **a** Let t = time spent (hours). **b** $70 + 52t = 252$
c $t = 3.5$ **d** 210 minutes

8 2

9 **a** $4s = 26$, $s = 6.5$ **b** 42.25 cm^2

10 Alison is 18.

11 $x = 65$, $y = 115$

12 **a** 118
b 119
c 12 688

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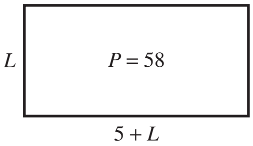
13 a



b 12 m **c** 204 m^2

14 a 50 **b** −6 **c** $x = 2R - 10$

15 a A, B and E **b** A, B and D

c Impossible to have 0.8 people in a room but can have 8 mm insect.

d The temperature this morning increased by 10°C and it is now 8°C. What was the temperature this morning? Other answers are possible.

16 a 17 **b** −8 **c** $\frac{2}{7}$

d $\frac{4}{3}$ **e** 25

Exercise 2I

1 a true **b** false **c** true

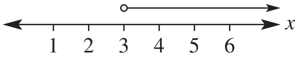
d false **e** true **f** true

2 a D **b** A **c** B **d** C

3 a true **b** false **c** false **d** true

4 a true **b** false **c** false **d** true

5 a



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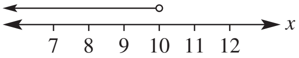

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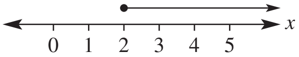

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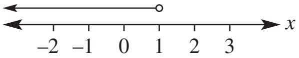

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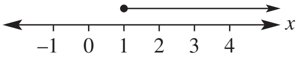

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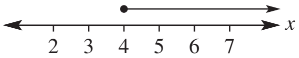

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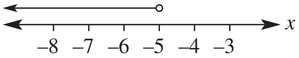

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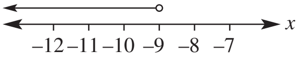

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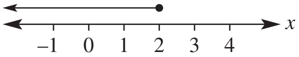

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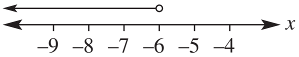

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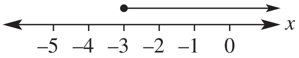

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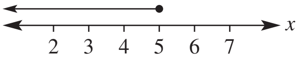

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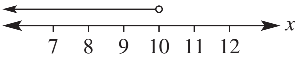

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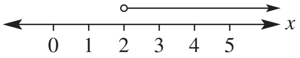

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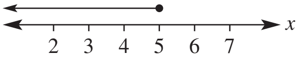

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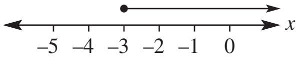
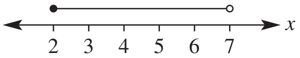
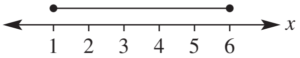


6 a 3, 4, 6, 4.5, 5, 2.1, 6.8, 2

b



7 a



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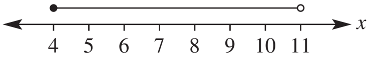

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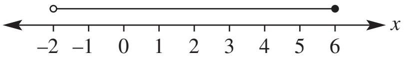

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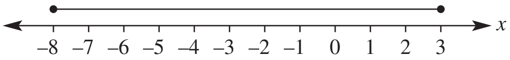

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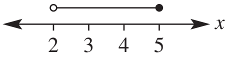

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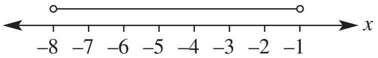

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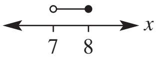

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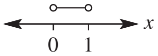
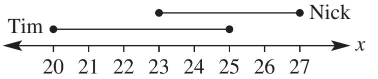


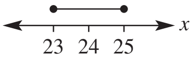
8 a C **b** A **c** G **d** F

9 a $20 \leqslant t \leqslant 25$ and $23 \leqslant n \leqslant 27$

b



c They are between 23 and 25 years old.



10 a **i** E **ii** B **iii** A **iv** B **v** D

b $73 \leqslant x < 80$ **c** D

d B or A **e** C only

11 a If x is larger than 7, it must automatically be larger than 5.

b **i** $x > 5$ **ii** $x < 3$ **iii** $x > 1$

iv $x < 10$ **v** $3 < x < 10$ **vi** $7 < x \leqslant 10$

c **i** $3 < x < 5$ **ii** $-2 < x < 4$

iii $7 < x < 8$ **iv** $9 \leqslant x < 10$

12 a 1 **b** ∞ **c** ∞ **d** 0

e 0 **f** 1 **g** ∞ **h** 0

13 a Must be true. **b** Must be true.

c Not necessarily (e.g. $y = 0$). **d** Must be true.

e Not necessarily (e.g. $x = 3$ and $y = 5$).

f Must be true.

14 a 30 **b** 110 **c** 200 **d** 1000

e 10 **f** 20 **g** 34 **h** 10

Exercise 2J

1 a true **b** true **c** false **d** false

2 a true **b** false **c** false **d** true

3 a 4 **b** $x < 4$

4 a 6, 3 **b** $x \geqslant 3$ **c** true

5 a $x > 3$ **b** $l \geqslant 3$ **c** $g > 5$ **d** $r \leqslant 7$
e $k > 2$ **f** $s < 5$ **g** $a > 4$ **h** $n \geqslant 5$
i $d \geqslant 7$ **j** $h < 5$ **k** $r \leqslant 2$ **l** $y < 4$

6 a $d > 29$ **b** $y \leqslant 10$ **c** $x > 11$ **d** $q \leqslant 18$
e $x > 7$ **f** $h < 1$ **g** $p \geqslant 2$ **h** $j < 5$

7 a $x > 1$ **b** $s \leqslant 2$ **c** $n < 5$ **d** $j > 10$
e $v \geqslant -2$ **f** $j \geqslant -5$ **g** $c \leqslant 2$ **h** $h \geqslant 0$
i $s < -3$ **j** $v < -2$ **k** $v \leqslant -2$ **l** $v > -5$

8 a C **b** A **c** B **d** D

9 a $4c + 20 > 25$ **b** $c > 1.25$ **c** \$1.30

10 a $6g + 4 \leqslant 36$ **b** $g \leqslant \frac{16}{3}$ **c** 5 goals

11 a $C \geqslant 15$ **b** $C \leqslant 20$
c $15 \leqslant C \leqslant 20$ **d** $47.\dot{7} \leqslant C \leqslant 57.\dot{7}$

12 a $x > 4$ **b** $k > 3$
c $a < -2$ **d** $-3 < a < -2$

13 a If $5x - 2 > 0$ then $x > \frac{2}{5}$, so x is positive.
b If $2x + 6 > 0$ then $x > -3$, so $x + 5 > 2$ is positive.
c Yes, for example if $x = 4$. **d** no

14 a clues B and D **b** $x = 8$

15 a $x > 2$ **b** $a \leqslant -2$ **c** $b < -3$ **d** $c < -4$

16 a $x \leqslant 3$ **b** $a > -3$ **c** $b \leqslant 8$ **d** $c > 2$
e $k > 20$ **f** $g < -5$ **g** $a > 8$ **h** $k \leqslant 9$
i $p > 4.5$

Puzzles and challenges

1 a 13 **b** 7.5 **c** 9 years
d \$44.44 **e** 15

2 a 2nd step or 3rd line (cannot divide by 0)
b

$$\begin{tikzcd}
0 = 1 \arrow[d, bend right, "\times 28"'] \arrow[d, bend left, "\times 28"] \\
0 = 28 \arrow[d, bend right, "+22"'] \arrow[d, bend left, "+22"] \\
22 = 50
\end{tikzcd}$$

3 $6 \leqslant x \leqslant 7$

4 $2x = 2(3 + x) - 1$ or $3x + 1 = 2 + 3(x + 1)$

5 a 65 kg, 62 kg, 55 kg **b** 70 kg, 60 kg, 48 kg
c 35 kg, 42 kg, 45 kg, 48 kg

Multiple-choice questions

1 A **2** D **3** C **4** C **5** B
6 B **7** D **8** E **9** A **10** A

Short-answer questions

1 a false **b** true **c** true

2 a $m = 4$ **b** $m = -12$ **c** $a = -1$ **d** $m = \frac{1}{5}$
e $m = 15$ **f** $a = 6$

3 a $2m + 3 = 3m$ **b** $5(n + 4) = 20$ **c** $x + x + 2 = 74$

4 a subtract 15 **b** add 5 **c** subtract $2a$

5 a $a = 4$ **b** $y = -9$ **c** $x = -4$ **d** $x = 4$
e $x = 2$ **f** $a = 1$

6 a $m = -6$ **b** $x = 8$ **c** $y = -18$ **d** $k = -58$
e $w = -2$ **f** $a = 43$

7 a $x = 14$ **b** $x = 6$ **c** $x = 40$

8 a $a = 8$ **b** $m = \frac{1}{2}$ **c** $x = 4$ **d** $a = 2$
e $x = -8$ **f** $x = 9$

9 a $x = 3$ **b** $x = -4$ **c** $x = -4$ **d** $a = 4$
e $a = -7$ **f** $m = 4$ **g** $a = -3$ **h** $x = 17$
i $x = 1$

10 a 12 **b** $4x - 5 = 2x + 3$ **c** 8.5

11 a ± 2 **b** ± 10 **c** ± 7
d ± 2.83 **e** ± 6.24 **f** ± 30.23

12 a 1 **b** 2 **c** 1 **d** 1 **e** 1 **f** 0

Extended-response questions

1 a $S = 20 + 0.12n$ **b** 30 times
c $Y = 15 + 0.2n$ **d** 25
e $20 + 0.12n = 15 + 0.2n$, $n = 62.5$, so 63 is the minimum number.

2 a $18x - 180$
b i \$35.50 **ii** \$41 **iii** \$459
c \$25 **d** \$30 **e** $30 \leqslant x \leqslant 35$

Chapter 3

Pre-test

1 a 300 cm **b** 200 mm **c** 1800 m **d** 25 cm
e 3.5 cm **f** 4.2 km **g** 5 m **h** 0.1 m
i 120 s **j** 3000 mL **k** 4 L **l** 3 kg

2 a circle **b** square **c** rectangle
d parallelogram **e** rhombus **f** kite
g triangle **h** trapezium

3 a 30 cm^2 **b** 25 m^2 **c** 16 cm^2 **d** 20 cm^2

4 a 30 m **b** 14.5 cm **c** 38 cm

5 a 10 **b** 27 **c** 25 **d** 121
e 58.5 **f** 6 **g** 9 **h** 12

6 a 12 cm^3 **b** 250 m^2 **c** 4200 cm^3

Exercise 3A

1 a 10 **b** 100 **c** 1000 **d** 100 000
e 1000 **f** 1 000 000

2 a 1000 **b** 100 000 **c** 1 000 000

3 a 10 **b** 10 **c** 2

4 a 12 m **b** 22 cm **c** 10 mm **d** 44 km
e 1.4 cm **f** 3.6 m

5 **a** 30 mm **b** 610 cm **c** 8930 m **d** 3000 mm

e 2.1 m **f** 32 cm **g** 9.62 km **h** 0.38 km

i 4.3 mm **j** 2040 cm **k** 23.098 m **l** 3.42 km

m 194.3 m **n** 0.01 km **o** 24.03 mm **p** 0.994 km

6 **a** 19 m **b** 44 m **c** 13 cm

d 32 cm **e** 28 km **f** 18 cm

g 17.2 mm **h** 34.4 cm **i** 29.4 m

7 **a** 5 **b** 2 **c** 4 **d** 18

e 9.5 **f** 6.5

8 **a** 40 cm **b** 17 cm **c** 7.8 cm **d** 2000 cm

e 46 cm **f** 17 600 cm

9 **a** 2 **b** 3 **c** 9

10 \$2392

11 8 min

12 240 cm

13 **a** $P = 2a + b$ **b** $P = 2a + 2b$

c $P = 2a + 2b$ **d** $P = 2a + 2b$

e $P = 8a + 2b$ **f** $P = 4a + 2b$

14 **a** $x = P - 11$ **b** $x = P - 4$ **c** $x = \dfrac{P-3}{2}$

d $x = \dfrac{P-8}{2}$ **e** $x = \dfrac{P}{4}$ **f** $x = \dfrac{P}{8}$

15 **a** 6 squares **b** 8 squares

Exercise 3B

1 **a** 15.71 **b** 40.84 **c** 18.85 **d** 232.48

2 **a** 3.1 **b** 3.14 **c** 3.142

3 **a** diameter **b** radius **c** circumference

4 Answer is close to pi.

5 **a** 12.57 mm **b** 113.10 m **c** 245.04 cm

d 12.57 m **e** 21.99 km **f** 15.71 cm

6 **a** 314 cm **b** 62.8 m **c** 18.84 km

7 **a** 44 mm **b** 132 cm **c** 220 m

8 11.0 m

9 12 566 m

10 **a** 64.27 cm **b** 12.34 m **c** 61.70 mm

11 **a** 28.57 cm **b** 93.97 m **c** 5.57 cm

12 **a** 25.13 cm **b** 56.55 m **c** 35.71 m

13 Svenya and Andre

14 $d = 2r$, so $2\pi r$ is the same as πd.

15 **a** 4π mm **b** 36π m **c** 78π cm

d 4π m **e** 7π km **f** 5π cm

16 **a** 8π cm **b** 18π m **c** $(5\pi + 20)$ m

17 **a** **i** $r = \dfrac{C}{2\pi}$ **ii** $d = \dfrac{C}{\pi}$

b **i** 2.23 m **ii** 6.37 cm

Exercise 3C

1 **a** **i** 100 **ii** 400 **iii** 3

b **i** 10 000 **ii** 70 000 **iii** 4

c **i** 1 000 000 **ii** 5 000 000 **iii** 2.5

d **i** 10 000 **ii** 30 000 **iii** 7.5

2 **a** 7 m, 3 m **b** 8 cm, 6 cm (or other way around)

c 2.4 mm, 1.7 mm

3 10 000

4 **a** 200 mm^2 **b** 70 000 cm^2 **c** 500 000 m^2

d 30 000 m^2 **e** 34 mm^2 **f** 0.07 m^2

g 30.9 cm^2 **h** 4000 m^2 **i** 0.2 m^2

j 0.45 km^2 **k** 0.4 ha **l** 32.1 cm^2

m 32 ha **n** 51 cm^2 **o** 4.3 mm^2

p 0.4802 m^2 **q** 1.904 ha **r** 0.2933 ha

s 49 m^2 **t** 7700 m^2 **u** 24 000 m^2

5 **a** 9 cm^2 **b** 21 m^2 **c** 39 cm^2

d 18 cm^2 **e** 33 m^2 **f** 144 mm^2

g 63 m^2 **h** 3 m^2 **i** 6 km^2

6 **a** 70 m^2 **b** 54 m^2 **c** 140 cm^2

d 91 cm^2 **e** 46 km^2 **f** 64 mm^2

7 **a** 200 000 mm^2 **b** 430 000 cm^2 **c** 0.0000374 km^2

d 0.01092 m^2 **e** 20 cm^2 **f** 600 ha

8 **a** 45 cm^2 **b** 168 m^2 **c** 120 km^2

9 **a** 6 m **b** 1.5 cm

10 **a** 25 m^2 **b** 20.25 cm^2 **c** 28 cm **d** 52 m

11 10 cm **12** \$48

13 **a** $A = 4b^2 + ab$ or $A = b(4b + a)$

b $A = 1.5ab$ or $A = \dfrac{3ab}{2}$

c $A = a^2 - \dfrac{ab}{2}$ or $A = a\left(a - \dfrac{b}{2}\right)$

14 **a** 4 **b** 1

15 **a** $b = \dfrac{A}{l}$ **b** $l = \sqrt{A}$ **c** $h = \dfrac{2A}{b}$

16 **a** **i** 2.59 km^2 **ii** 2 589 988 m^2 **iii** 259 ha

iv 4047 m^2 **v** 0.4 ha **vi** 2.5 acres

b 81 ha

c 62%

Exercise 3D

1 **a** 6 **b** 30 **c** 13.5 **d** 33

2 **a** 90° **b** height

c perpendicular **d** parallel, perpendicular

e rhombus, kite

3 **a** parallelogram, 50 m^2 **b** parallelogram, 4.5 cm^2

c parallelogram, 6 m^2 **d** rhombus, 7.5 cm^2

e rhombus, 121 km^2 **f** rhombus, 9.61 m^2

g kite, 4 cm^2 **h** kite, 300 mm^2

i kite, 0.9 mm^2 **j** trapezium, 96 cm^2

k trapezium, 32.5 m^2 **l** trapezium, 560 mm^2

4 **a** 6 cm^2 **b** 35 m^2 **c** 84.5 cm^2

5 0.27 m^2

6 2 m

7 \$1160

8 3 cm and 9 cm

9 **a** trapezium **b** 19.5 cm^2

10 **a** $A = 2x^2$ **b** $A = 3ab$ **c** $A = 8a^2$

11 No, use formula for parallelogram $A = bh$, as we already know these lengths.

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12 a

$$\begin{aligned} A &= 4 \text{ triangle areas} \\ &= 4 \times \frac{1}{2} \times \text{base} \times \text{height} \\ &= 4 \times \frac{1}{2} \times \frac{1}{2}x \times \frac{1}{2}y \\ &= \frac{1}{2}xy \end{aligned}$$

b

$$\begin{aligned} A &= \text{Area (triangle 1)} + \text{Area (triangle 2)} \\ &= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2 \\ &= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h \\ &= \frac{1}{2}ah + \frac{1}{2}bh \\ &= \frac{1}{2}h(a + b) \end{aligned}$$

c

$$\begin{aligned} A &= \text{Area(rectangle)} + \text{Area(triangle)} \\ &= \text{length} \times \text{breadth} + \frac{1}{2} \times \text{base} \times \text{height} \\ &= a \times h + \frac{1}{2} \times (b - a) \times h \\ &= ah + \frac{1}{2}bh - \frac{1}{2}ah \\ &= \frac{1}{2}ah + \frac{1}{2}bh \\ &= \frac{1}{2}h(a + b) \end{aligned}$$

Exercise 3E

1 a 31.4 **b** 12.56 **c** 22 **d** 154

2 a 78.54 **b** 530.93 **c** 30.19 **d** 301.72

3 a 5 m **b** 2.3 mm **c** 3.5 km

4 a 28.27 cm^2 **b** 113.10 m^2 **c** 7.07 mm^2
d 78.54 km^2 **e** 36.32 cm^2 **f** 9.08 m^2

5 a 154 cm^2 **b** 154 km^2 **c** 616 mm^2
d 314 km^2 **e** 12.56 m^2 **f** 31 400 m^2

6 a 3.14 cm^2 **b** 201.06 cm^2 **c** 226.98 mm^2
d 39.27 cm^2 **e** 5.09 mm^2 **f** 100.53 m^2

7 707 cm^2

8 yes, by 1310 cm^2

9 no ($A = 0.79$ km^2)

10 78.54 cm^2

11 circle of radius 5 m

12 80 cm^2

13 a 12.56 cm^2 **b** 50.24 cm^2
c quadrupled (×4) **d** multiplied by 9
e multiplied by 16 **f** multiplied by n^2

14 a 81π **b** $\frac{49\pi}{4}$ **c** 72π

15 a $A = \frac{\pi d^2}{4}$ **b** true

16 a true
b i 2.33 m **ii** 1.20 km **iii** 10.09 mm
c $r = \sqrt{\frac{A}{\pi}}$

Exercise 3F

1 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{6}$ **d** $\frac{1}{8}$

2 a 6.28 **b** 8.55 **c** 9.69

3 a $\frac{1}{4}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$

4 a 88.49 mm^2 **b** 104.72 mm^2 **c** 4.91 cm^2
d 14.14 cm^2 **e** 61.28 m^2 **f** 262.72 cm^2
g 181.53 m^2 **h** 981.93 m^2 **i** 2428.12 km^2

5 a 37.70 m^2 **b** 137.44 m^2 **c** 437.21 km^2

6 a 34.82 m^2 **b** 9.14 m^2 **c** 257.08 cm^2
d 116.38 mm^2 **e** 123.61 km^2 **f** 53.70 m^2
g 50.27 m^2 **h** 75.40 mm^2 **i** 12.57 cm^2

7 1.26 m^2

8 13 cm radius pizza by 0.13 cm^2

9 16 965 cm^2

10 a 78.5% **b** 30.8% **c** 21.5%

11 a π cm^2 **b** $\frac{25\pi}{9}$ m^2 **c** 8π mm^2
d $\frac{75\pi}{2}$ m^2 **e** $9\pi + 9$ cm^2 **f** $225 - \frac{225\pi}{4}$ km^2

12 a 78.5%
b 78.5%, same answers as for part **a**.
c Percentage area $= \frac{\pi r^2}{4} \div r^2 \times 100 = 25\pi \approx 78.5\%$

13 a 6.54 m^2 **b** 2.26 m^2 **c** 5.8%

Exercise 3G

1 a 6, squares
b 6 squares and rectangles
c 6, isosceles triangles and rectangles

2 a C **b** A **c** B

3 a 3 **b** 6 **c** 6 **d** 5

4 a 24 cm^2 **b** 403.44 m^2 **c** 22 cm^2
d 352 cm^2 **e** 84 m^2 **f** 612 cm^2
g 120 mm^2 **h** 114 m^2 **i** 29.7 m^2

5 a 18 cm^2 **b** 146 cm^2

6 2000 cm^2

7 81 m^2

8 a 138 m^2 **b** 658 m^2 **c** 62 cm^2

9 \$107.25

10 a $\text{SA} = 6l^2$ **b** $\text{SA} = 2b^2 + 4lb$
c $\text{SA} = 2bl + 2bh + 2lh$

11 a i quadrupled **ii** multiplied by 9 **iii** multiplied by 16
b multiplied by n^2

12 a 15 072 cm^2 **b** 456 cm^2

Exercise 3H

1 a 24 **b** 12 **c** 72

2 a 96 **b** 6 **c** 2 **d** 5

3 a 1000 **b** 1 **c** 1 **d** 1
e 1 **f** 1000

4 a 36 cm^3 b 20 m^3 c 27 mm^3 d 64 km^3
e 320 mm^3 f 24 m^3
5 a 2000 mL b 5000 L c 500 kL d 3 L
e 4 cm^3 f 50 mL g 2.5 L h 5100 cm^3
6 a 24 L b 42 L c 27 L d 18 000 L
e 24 000 L f 360 L
7 a i 60 000 000 L ii 60 000 kL iii 60 ML
b 200 days
8 80 minutes
9 a 500 000 m^3 b 500 ML
10 8000 kg
11 a (1, 1, 12) or (1, 2, 6) or (1, 3, 4) or (2, 2, 3)
b (1, 1, 30) or (1, 2, 15) or (1, 3, 10) or (1, 5, 6) or (2, 3, 5)
c (1, 1, 47)
12 Factors greater than 1 include only 2, 23 and 46, but 23 is prime so you cannot find 3 whole numbers that multiply to 46.
13 9
14 a 20 b 20
c Equal, the number of cubes on the base layer is the same as the number of squares on the base.
d If the number of cubes on the base layer is the same as the number of squares on the base, then Ah gives h layers of A cubes, giving the total.
e Yes, a rectangular prism could use 3 different bases.
15 a i By joining two of the same prisms together a rectangular prism could be formed.
ii 12 units^3
b i 160 cm^3 ii 140 m^3 iii 2 cm^3 iv 112 m^3
v 48 mm^3 vi 171 cm^3

Exercise 3I

1 a i prism ii rectangle
b i prism ii triangle
c i not a prism (pyramid)
d i not a prism (cone)
e i prism ii square
f i not a prism (truncated pyramid)
2 a 8, 2 b 6, 1.5 c 12, 10
3 a 40 b 36 c 10 d 6280
4 a 44 m^3 b 160 cm^3 c 352 mm^3
5 a 200 cm^3 b 15 m^3 c 980 cm^3 d 60 m^3
e 270 mm^3 f 60 m^3
6 a 785.40 m^3 b 12 566.37 mm^3 c 251.33 cm^3
d 7696.90 cm^3 e 461.81 m^3 f 384.85 m^3
7 a 14.137 m^3 b 14 137 L
8 a 28 cm^3 b 252 m^3 c 104 cm^3
9 3 (almost 4 but not quite)
10 302.48 cm^3
11 a 56 000 L b 56 hours
12 a 20π m^3 b 300π cm^3 c 144π mm^3 d 245π m^3
13 Answers may vary, an example is $r = 5$ cm and $h = 1.27$ cm.
14 $x = \pi h$

15 a 14.28 cm^3 b 98.17 mm^3 c 1119.52 cm^3
d 8.90 m^3 e 800 m^3 f 10 036.67 cm^3

Exercise 3J

1 a F b D c A
d E e B f C
2 a 120 s b 3 min c 2 h
d 240 min e 72 h f 2 days
g 5 weeks h 280 days
3 a 6 h 30 min b 10 h 45 min
c 16 h 20 min d 4 h 30 min
4 a 180 min b 630 s c 4 min
d 1.5 h e 144 h f 3 days
g 168 h h 1440 min i 4 h
j 2 weeks k 20 160 min l 86 400 s
5 a 2 h 30 min b 3 h 15 min
c 1 h 45 min d 4 h 12 min
e 2 h 36 min f 3 h 12 min 36 s
g 2 h 22 min 48 s h 7 h 44 min 24 s
i 6 h 1 min 48 s
6 a 1330 h b 2015 h c 1023 h
d 2359 h e 6:30 a.m. f 1 p.m.
g 2:29 p.m. h 7:38 p.m. i 11:51 p.m.
7 a 2 p.m. b 5 a.m.
c 1200 hours d 1800 hours
8 a 2 h 50 min b 6 h 20 min c 2 h 44 min
d 8 h 50 min e 8 h 19 min f 10 h 49 min
9 a 11 a.m. b 12 p.m. c 8 p.m. d 7:30 p.m.
e 7 a.m. f 5 a.m. g 1 a.m. h 10 a.m.
10 a 5:30 a.m. b 7:30 a.m. c 6:30 a.m. d 1:30 p.m.
e 2:30 p.m. f 2:30 a.m. g 3 p.m. h 5:30 p.m.
11 a 5 h b 2.5 h c 8 h d 6 h e 7 h
12 56 million years
13 17 min 28 s
14 7 h 28 min
15 23 h 15 min
16 a 33c b 143c or $1.43
17 a $900 b $90 c $1.50 d 2.5c
18 6:30 a.m.
19 a 8 a.m. 29 March
b 10 p.m. 28 March
c 3 a.m. 29 March
20 3 a.m. 21 April
21 11:30 p.m. 25 October
22 52.14 weeks
23 a 3600 b 1440 c 3600 d 1440
24 Friday
25 a You have to turn your clock back.
b You have to turn your clock forward.
c You adjust the date back one day.
26 Students' reports will vary.

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Exercise 3K

1 **a** 9 **b** 25 **c** 144 **d** 2.25
e 20 **f** 58 **g** 157 **h** 369

2 **a** false **b** true **c** true **d** true
e false **f** false

3 hypotenuse, triangle

4 **a** c **b** x **c** u

5 **a** no **b** no **c** yes
d yes **e** yes **f** no
g yes **h** no **i** no

6

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5	9	16	25	25
6	8	10	36	64	100	100
8	15	17	64	225	289	289

$a^2 + b^2$ and c^2
b **i** 13 **ii** 20
c **i** 25 **ii** 110

7 **a** $3^2 + 4^2 = 5^2$ **b** $8^2 + 15^2 = 17^2$ **c** $9^2 + 12^2 = 15^2$
d $5^2 + 12^2 = 13^2$ **e** $9^2 + 40^2 = 41^2$ **f** $2.5^2 + 6^2 = 6.5^2$

8 **a** $a^2 + b^2 = x^2$ **b** $a^2 + b^2 = d^2$ **c** $d^2 + h^2 = x^2$

9 **a** no
b No, $a^2 + b^2 = c^2$ must be true for a right-angled triangle.

10 {(6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (21, 28, 35), (24, 32, 40), (27, 36, 45), (30, 40, 50), (33, 44, 55), (36, 48, 60), (39, 52, 65), (42, 56, 70), (45, 60, 75), (48, 64, 80), (51, 68, 85), (54, 72, 90), (57, 76, 95)}, {(5, 12, 13), (10, 24, 26), (15, 36, 39), (20, 48, 52), (25, 60, 65), (30, 72, 78), (35, 84, 91)}, {(7, 24, 25), (14, 48, 50), (21, 72, 75)}, {(8, 15, 17), (16, 30, 34), (24, 45, 51), (32, 60, 68), (40, 75, 85)}, {(9, 40, 41), (18, 80, 82)}, {(11, 60, 61)}, {(20, 21, 29), (40, 42, 58), (60, 63, 87)}. {(12, 35, 37), (24, 70, 74)}, {(28, 45, 53)}, {(33, 56, 65)}, {(16, 63, 65)}, {(48, 55, 73)}, {(13, 84, 85)}, {(36, 77, 85)}, {(39, 80, 89)}, {(65, 72, 97)}

11 **a** yes **b** no **c** no **d** yes
e no **f** yes

12 **a** a^2 **b** b^2 **c** $\sqrt{a^2 + b^2}$

13 $2x^2 = c^2$

14 **a** Area of triangle = c^2
Area of 4 outside triangles $= 4 \times \frac{1}{2} \times$ base $\times$ height $= 2ab$
Total area of outside square $= (a + b)^2 = a^2 + 2ab + b^2$
Area of inside square = Area (outside square) − 4 triangles
$= a^2 + 2ab + b^2 - 2ab$
$= a^2 + b^2$
Comparing results from the first and last steps gives
$c^2 = a^2 + b^2$.

Exercise 3L

1 **a** yes **b** no **c** no **d** yes

2 **a** 3.16 **b** 5.10 **c** 8.06 **d** 15.17

3 **a** $w^2 = 3^2 + 5^2$
b $x^2 = 9^2 + 7^2$
c $m^2 = 8^2 + 8^2$
d $x^2 = 7^2 + 9.4^2$

4 **a** 5 **b** 25 **c** 41
d 20 **e** 45 **f** 61

5 **a** 9.22 **b** 5.39 **c** 5.66
d 3.16 **e** 4.30 **f** 37.22

6 3.16 m or 316 cm

7 139 cm

8 5.5 km

9 3.88 cm

10 **a** 2nd line is incorrect, cannot take the square root of each term.
b 2nd line is incorrect, cannot add $3^2 + 4^2$ to get 7^2.
c Last line should say $\therefore c = \sqrt{29}$.

11 **a** $3^2 + 5^2 \neq 7^2$ **b** $5^2 + 8^2 \neq 10^2$ **c** $12^2 + 21^2 \neq 24^2$

12 **a** 8.61 m **b** 48.59 cm **c** 18.56 cm
d 22.25 mm **e** 14.93 m **f** 12.25 m

Exercise 3M

1 **a** $9^2 = m^2 + 3^2$ **b** $10^2 = 5^2 + p^2$
c $15^2 = w^2 + w^2$ **d** $12^2 = c^2 + 8^2$

2 **a** $a = 12$ **b** $b = 24$

3 **a** 4 **b** 9 **c** 40
d 15 **e** 16 **f** 60

4 **a** 2.24 **b** 4.58 **c** 11.49
d 12.65 **e** 10.72 **f** 86.60

5 8.94 m

6 12 cm

7 12.12 cm

8 8.49

9 **a** Should subtract not add 10.
b Should say $a = 5$.
c Cannot take the square root of each term.

10 **a** $\sqrt{24}$ **b** $\sqrt{3}$ **c** $\sqrt{4400}$

11 **a** 3.54 **b** 7.07 **c** 43.13 **d** 24.04

12 **a** $6^2 + 8^2 = 10^2$
b It is a multiple of (3, 4, 5).
c (9, 12, 15), (12, 16, 20), (15, 20, 25)
d (8, 15, 17)
e (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), (9, 40, 41), etc.

Puzzles and challenges

1 10 cm each side

2 Yes, 1 L will overflow.

3 $\frac{1}{2}$

4 $\sqrt{3} \approx 1.73$ m

5 $\sqrt{2}$
6 $\sqrt{7}$
7 **a** 24 cm^2 **b** 48 cm^2
8 63.66%

Multiple-choice questions

1 E **2** B **3** A **4** C **5** B
6 E **7** E **8** B **9** D **10** D

Short-answer questions

1 **a** 2000 mm **b** 0.5 km **c** 300 mm^2 **d** 0.4 m^2
e 10 000 m^2 **f** 3.5 cm^2 **g** 0.4 L **h** 200 L
2 **a** 13 m **b** 28 cm **c** 25.13 m
d 51.42 mm **e** 48 m **f** 20 cm
3 **a** 55 cm^2 **b** 63 m^2 **c** 12 cm^2
d 136 km^2 **e** 64 m^2 **f** 20 cm^2
g 28.27 cm^2 **h** 12.57 m^2 **i** 3.84 cm^2
4 **a** 70 cm^2 **b** 50.14 cm^2 **c** 74 cm^2
5 **a** 1000 L **b** 8 L **c** 0.144 L
6 **a** 2513.27 m^3 **b** 3078.76 cm^3 **c** 212.06 mm^3
7 **a** **i** 287°C **ii** 239°C
b 1 h 39 min 18 s
c 1 h 2 min 4 s
8 **a** 10 h 17 min
b 9:45 p.m.
c 2331 hours
9 **a** 6:30 p.m. **b** 6 p.m. **c** 8:30 a.m.
d 4:30 p.m. **e** 10:30 a.m. **f** 5:30 p.m.
g 8:30 p.m. **h** 8:30 p.m.
10 **a** 10
b 25
c 4.24
11 **a** 15
b 6.24
c 11.36

Extended-response questions

1 **a** 12.86 cm^2
b 57.72 cm^2
c 57.72 m^2
d 6.28 cm^3
e 25.72 cm^3
f 38 with some remainder
2 **a** 2.8 m
b 24 m^2
c 23 m
d No, 4000 L short.

Chapter 4

Pre-test

1 **a** mixed numeral **b** proper
c improper **d** improper
2 20
3 **a** 100 **b** 1 **c** 8 **d** 4
4 **a** $\frac{1}{4}$ **b** $\frac{3}{7}$
5 **a** D **b** C **c** E
d A **e** B
6 **a** $\frac{3}{4}$ **b** 1 **c** $1\frac{2}{3}$
d 0.6 **e** 1.2 **f** 3
7 **a** **i** $\frac{1}{10}$ **ii** 0.1 **b** **i** $\frac{1}{4}$ **ii** 0.25
c **i** $\frac{1}{2}$ **ii** 0.5 **d** **i** $\frac{3}{4}$ **ii** 0.75
8 **a** $5 **b** $6.60 **c** 0.8 km **d** 690 m
9 **a** 10 **b** 18 **c** 90 cents
10

Fraction	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{2}{5}$	$\frac{99}{100}$	1	$\frac{8}{5}$	2
Decimal	0.75	0.2	0.15	0.4	0.99	1.0	1.6	2.0
Percentage	75%	20%	15%	40%	99%	100%	160%	200%

Exercise 4A

1 **a** 6, 20, 200 **b** 14, 40, 140
c 1, 6, 42 **d** 8, 3, 20
2 $\frac{4}{6}, \frac{20}{30}, \frac{10}{15}$
3 **a** false **b** true **c** true
d true **e** true **f** false
4 **a** $\frac{8}{24}$ **b** $\frac{6}{24}$ **c** $\frac{12}{24}$ **d** $\frac{10}{24}$
e $\frac{20}{24}$ **f** $\frac{120}{24}$ **g** $\frac{18}{24}$ **h** $\frac{21}{24}$
5 **a** $\frac{6}{30}$ **b** $\frac{10}{30}$ **c** $\frac{15}{30}$ **d** $\frac{90}{30}$
e $\frac{20}{30}$ **f** $\frac{11}{30}$ **g** $\frac{75}{30}$ **h** $\frac{15}{30}$
6 **a** 6 **b** 18 **c** 2 **d** 7
e 28 **f** 50 **g** 15 **h** 44
7 **a** 2, 3, 5, 10, 16, 25
b 6, 12, 30, 45, 150, 300
c 4, 6, 8, 14, 20, 30
8 **a** 6, 12, 24, 30, 75, 300
b 10, 15, 20, 25, 50, 60
c 8, 12, 28, 44, 80, 400

9 **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{5}{6}$ **d** $\frac{5}{6}$

e $\frac{1}{4}$ **f** $\frac{3}{5}$ **g** $\frac{8}{9}$ **h** $\frac{5}{7}$

i $\frac{5}{3}$ **j** $\frac{11}{10}$ **k** $\frac{6}{5}$ **l** $\frac{4}{3}$

10 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{11}{13}$ **d** $\frac{7}{17}$

e $\frac{5}{4}$ **f** $\frac{7}{3}$ **g** $\frac{15}{11}$ **h** $\frac{53}{16}$

11 $\frac{14}{42} = \frac{1}{3}, \frac{51}{68} = \frac{3}{4}, \frac{15}{95} = \frac{3}{19}$

12 $\frac{5}{11} = \frac{15}{33}, \frac{3}{5} = \frac{9}{15}, \frac{7}{21} = \frac{1}{3}, \frac{8}{22} = \frac{16}{44}, \frac{2}{7} = \frac{6}{21}, \frac{20}{50} = \frac{6}{15}$

13 **a** $\frac{3}{4}$ **b** $\frac{2}{3}$

c No, $\frac{3}{4} = \frac{9}{12}$ of time complete. However, only $\frac{2}{3} = \frac{8}{12}$ of laps completed.

14 **a** Cannot be simplified, e.g. $\frac{3}{5}, \frac{7}{11}$. The HCF of the numerator and denominator is 1.

b Possibly; e.g. $\frac{15}{16}$, both are composite numbers, but HCF is 1 and therefore the fraction cannot be simplified. However, in $\frac{15}{18}$, both numbers are composite, HCF is 3 and therefore the fraction can be simplified to $\frac{5}{6}$.

15 **a** no **b** yes **c** 10

16 infinite

17 **a** **i** $6b$ **ii** $5x$ **iii** 80 **iv** $12de$

v bc **vi** $3km$ **vii** $16ac$ **viii** xy

b **i** $\frac{3b}{4}$ **ii** $\frac{1}{2y}$ **iii** $\frac{3}{5}$ **iv** $\frac{5}{8x}$

v $\frac{2}{3q}$ **vi** $\frac{10}{x}$ **vii** $\frac{o}{q}$ **viii** $\frac{3}{x}$

c Yes, $\frac{5x}{15x} = \frac{1}{3}$ **d** Yes, $\frac{1}{3} = \frac{a}{3a}$

Exercise 4B

1 **a** +, − **b** ×, ÷

2 **a** ×, ÷ **b** +, −

3 **a** 20 **b** 9 **c** 50 **d** 24

4 **a** 3, 12 **b** 14, 5 **c** 11, 33 **d** ×, 14, 1, 1

5 **a** $\frac{3}{5}$ **b** $\frac{5}{9}$ **c** $1\frac{1}{2}$ **d** $1\frac{6}{7}$

e $1\frac{3}{20}$ **f** $1\frac{1}{10}$ **g** $\frac{1}{21}$ **h** $\frac{4}{9}$

6 **a** $4\frac{4}{7}$ **b** $9\frac{3}{5}$ **c** $2\frac{3}{8}$ **d** $1\frac{2}{11}$

e $9\frac{1}{2}$ **f** $22\frac{3}{14}$ **g** $3\frac{3}{4}$ **h** $1\frac{17}{30}$

7 **a** $\frac{3}{20}$ **b** $\frac{10}{63}$ **c** $1\frac{17}{25}$ **d** $1\frac{13}{27}$

e $\frac{1}{6}$ **f** $\frac{3}{8}$ **g** $\frac{8}{15}$ **h** 5

8 **a** $3\frac{2}{3}$ **b** $1\frac{2}{21}$ **c** 15 **d** 35

9 **a** $\frac{8}{5}$ **b** $\frac{2}{3}$ **c** $\frac{4}{13}$ **d** $\frac{11}{12}$

10 **a** $\frac{3}{13} \times \frac{7}{9}$ **b** $\frac{1}{2} \times \frac{4}{1}$ **c** $\frac{17}{10} \times \frac{1}{3}$ **d** $\frac{7}{3} \times \frac{4}{7}$

11 **a** $\frac{10}{27}$ **b** $\frac{5}{6}$ **c** $\frac{16}{77}$ **d** $1\frac{7}{15}$

e $\frac{7}{8}$ **f** 2 **g** $1\frac{1}{3}$ **h** $3\frac{3}{5}$

12 **a** $\frac{33}{35}$ **b** $\frac{48}{125}$ **c** $1\frac{2}{5}$ **d** 3

13 **a** $\frac{15}{16}$ **b** $\frac{1}{12}$ **c** $1\frac{1}{6}$ **d** $1\frac{1}{2}$

14 Repeat answers

15 **a** $\frac{29}{70}$ **b** $\frac{41}{70}$

16 $\frac{3}{5}$

17 7 kg

18 112

19 Answers may vary.

a 5, 5 **b** 4 **c** 5, 2 **d** 1, 1

20 Answers may vary.

a 2, 5 **b** 18 **c** 10, 1 **d** $3\frac{4}{15}$

21 Answers may vary.

22 **a** $\left(\frac{1}{6} + \frac{1}{5} - \frac{1}{3}\right) \times \left(\frac{1}{4}\right) \div \left(\frac{1}{2}\right) = \frac{1}{60}$

b $\left(\frac{1}{2} \div \frac{1}{6} - \frac{1}{5} + \frac{1}{4}\right) \times \frac{1}{3} = 1\frac{1}{60}$

c $\frac{1}{2} \div \left(\frac{1}{5} - \frac{1}{6}\right) \times \left(\frac{1}{3} + \frac{1}{4}\right) = 8\frac{3}{4}$

Exercise 4C

1 **a** < **b** > **c** <

d < **e** > **f** >

2 **a** 3.6521, 3.625, 3.256, 3.229, 2.814, 2.653

b 11.908, 11.907, 11.898, 11.891, 11.875, 11.799

c 1.326, 1.305, 0.802, 0.765, 0.043, 0.039

3 E

4 C

5 **a** $\frac{31}{100}$ **b** $\frac{537}{1000}$ **c** $\frac{163}{200}$ **d** $\frac{24}{25}$

e $5\frac{7}{20}$ **f** $8\frac{11}{50}$ **g** $26\frac{4}{5}$ **h** $8\frac{64}{125}$

6 **a** 10, 6 **b** 55, 5

7 **a** 0.17 **b** 0.301 **c** 4.05 **d** 7.6

e 0.12 **f** 0.35 **g** 2.5 **h** 1.75

8 **a** $\frac{13}{250}$ **b** $6\frac{1}{8}$ **c** $317\frac{3}{50}$ **d** $\frac{53}{125}$

e 0.275 **f** 0.375 **g** 0.68 **h** 0.232

9 2.175, 2.18, 2.25, 2.3, 2.375, 2.4

10 A1, B5, C07, P9, BW Theatre, gym

11 Opposition leader is ahead by 0.025.

12 **a** D, C, E, F, B, A **b** C, D, A, E, F, B

13 **a** 2.655 **b** $\frac{179}{200}$ **c** 4.61525 **d** $2\frac{1109}{2000}$

ISBN 978-1-108-46627-1

14

2.6	4.6	$1\frac{4}{5}$
2.2	$\frac{6}{2}$	3.8
4.2	1.4	$3\frac{2}{5}$

15 a €59.15 b A\$183.23 c Canada
d £35 e 91.4c
f AUD, CAN, US, EURO, GB
g Cheaper to buy car in GB and freight to Australia.

Exercise 4D

1 B
2 a 62.71 b 277.99 c 23.963 d 94.172
e 14.41 f 23.12 g 84.59 h 4.77
3 a 179.716 b 50.3192 c 1025.656 d 18.3087
4 a 11.589 b 9.784 c 19.828 d 4.58
5 E
6 a 3651.73 b 81.55 c 0.75 d 0.03812
e 6 348 000 f 0.0010615 g 30 h 0.000452
7 C
8 a 99.6 b 12.405 c 107.42 d 1.8272
e 0.01863 f 660.88 g 89.0375 h 292.226
9 B
10 a 12.27 b 5.88 c 0.0097 d 49.65
e 11.12 f 446.6 g 0.322655 h 3.462
11 a 203.8 b 0.38 c 2 011 500 d 11.63
e 0.335 f 13.69 g 0.630625 h 1353.275
12 a 16.29 b 15.52 c 66.22 d 1.963
e 13.3084 f 3.617 g 97 h 42.7123
13 7.12 m
14 12 200 skis
15 Answers will vary.
16 1200 km
17 1807 mm, 1.807 m
18 a 5.84 b 1 c 0.94 392 d 26.2
19 D (65.1)
20 Answers may vary.

Exercise 4E

1 a T b R c R d T
e T f R g T h R
2 a $0.\dot{3}$ b $6.\dot{2}\dot{1}$ or $6.\overline{21}$
c $8.576\dot{4}$ d $2.1\dot{3}5\dot{6}$ or $2.1\overline{356}$
e $11.\dot{2}857\dot{3}$ or $11.\overline{28573}$ f $0.00\dot{3}5\dot{2}$ or $0.00\overline{352}$
3 a 0.6 b 0.75 c 0.125 d 0.55
4 a $0.\dot{3}$ b $0.\dot{5}$ c $0.8\dot{3}$ d $0.\overline{72}$
e $0.\overline{428571}$ f $0.\overline{384615}$ g $3.1\dot{3}$ h $4.\overline{857142}$
5 a 4 b 9 c 7 d 6
6 Answers will vary.
7 a 0.766 b 9.5 c 7.0 d 21.5134
e 0.95 f 17 g 8.60 h 8.106
8 a 17.01 b 0.92 c 4.44 d 14.52
e 5.20 f 79.00 g 0.00 h 2.68
9 a 65 b 9 c 30 d 4563
10 a 0.86 b 0.22 c 0.36 d 0.42
11 a 7.7000 b 5.0 c 0.00700
12 a 0 seconds b 0 seconds
c 0.06 seconds d 12.765 e 47 cm
13 4
14 $0.\overline{0588235294117647}$
15 Frieda is correct. Infinite, non-recurring decimals are known as surds. For example, $\sqrt{3} = 1.73205080\ldots$
16 Student A: When dealing with a critical digit of 5 they incorrectly round down rather than round up.
Student B: When rounding down, student B incorrectly replaces final digit with 0.
17 a i T ii R iii R
iv R v T vi R
b $8 = 2^3$, $12 = 2^2 \times 3$, $14 = 2 \times 7$, $15 = 3 \times 5$, $20 = 2^2 \times 5$, $60 = 2^2 \times 3 \times 5$
c Only denominators which have factors that are powers of 2 and/or 5 terminate.
d i T ii R iii R iv T
v R vi R vii T viii T

Exercise 4F

1 B
2 a $\frac{39}{100}$ b $\frac{11}{100}$ c $\frac{1}{5}$ d $\frac{3}{4}$
e $1\frac{1}{4}$ f $\frac{7}{10}$ g $2\frac{1}{20}$ h $6\frac{1}{5}$
3 a $\frac{3}{8}$ b $\frac{31}{200}$ c $\frac{1}{3}$ d $\frac{2}{3}$
e $\frac{9}{400}$ f $\frac{9}{200}$ g $\frac{51}{500}$ h $\frac{7}{8}$
4 B
5 a 0.65 b 0.37 c 1.58 d 3.19
e 0.0635 f 0.0012 g 40.51 h 1.0005
6 C
7 a 40% b 25% c 55% d 26%
e 22.5% f 68% g 75% h 41.5%
8 a 275% b 520% c 175% d 450%
e 348% f 194% g 770% h 915%
9 a $33\frac{1}{3}\%$ b $12\frac{1}{2}\%$ c $8\frac{1}{3}\%$ d $6\frac{2}{3}\%$
e $37\frac{1}{2}\%$ f $28\frac{4}{7}\%$ g $18\frac{3}{4}\%$ h 75%
10 A
11 a 42% b 17% c 354.1% d 1122%
e 0.35% f 4.17% g 1% h 101%

12 A

13 a

Fraction	Decimal	%
$\frac{1}{4}$	0.25	25%
$\frac{2}{4}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{4}{4}$	1	100%

b

Fraction	Decimal	%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}\%$
$\frac{3}{3}$	$0.\dot{9}$	100%

c

Fraction	Decimal	%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{5}{5}$	1	100%

14 a

Fraction	Decimal	%
$\frac{3}{20}$	0.15	15%
$\frac{6}{25}$	0.24	24%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{40}$	0.125	12.5%
$\frac{7}{10}$	0.7	70%
$\frac{31}{50}$	0.62	62%

b

Fraction	Decimal	%
$\frac{11}{5}$	2.2	220%
$\frac{3}{1000}$	0.003	0.3%
$\frac{13}{200}$	0.065	6.5%
$1\frac{19}{100}$	1.19	119%
$4\frac{1}{5}$	4.2	420%
$\frac{5}{6}$	$0.8\dot{3}$	$83\frac{1}{3}\%$

15 $\frac{1}{8}$, 12.5%, 0.125

16 65%, 80%

17 a $\times 100\% = \times\frac{100}{100} = \times 1$

b $\div 100\% = \div\frac{100}{100} = \div 1$

18 a $\frac{BC}{100}$ **b** 0. CDB **c** ABC%

d DDB.CC% **e** $\frac{100A}{D}\%$ **f** $\frac{100(BA + C)}{A}\%$

19 a 25%, 25%, $12\frac{1}{2}\%$, $12\frac{1}{2}\%$, $12\frac{1}{2}\%$, $6\frac{1}{4}\%$, $6\frac{1}{4}\%$

Exercise 4G

1 D

2 A

3 a 80% **b** 65% **c** 78% **d** 30%
e 20% **f** 80% **g** 40% **h** 37.5%
i $36\frac{2}{3}\%$ **j** $70\frac{5}{6}\%$ **k** $70\frac{5}{6}\%$ **l** $94\frac{4}{9}\%$

4 a 30% **b** 45% **c** $31\frac{1}{4}\%$ **d** $83\frac{1}{3}\%$
e 160% **f** $683\frac{1}{3}\%$ **g** $133\frac{1}{3}\%$ **h** $266\frac{2}{3}\%$

5 a 8.33% **b** 66.67% **c** 42.86% **d** 37.5%
e 160% **f** 125% **g** 112.5% **h** 233.33%

6 a 5% **b** 25% **c** 5% **d** 25%
e 4% **f** 4000% **g** 300% **h** 600%

7 a 56% **b** 75% **c** 86% **d** 25%
e 40% **f** $33\frac{1}{3}\%$

8 a 100 **b** 10 **c** 5 **d** 2

9 a 18 **b** 9 **c** 17 **d** 16
e 3 **f** 3 **g** 5.6 **h** 175
i 132 **j** 39.6 **k** 44.8 **l** 36.8

10 a 13 **b** 80 **c** 100 **d** 217
e 67.5 **f** 51.2 **g** 36.75 **h** 70.8

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11 **a** 18 minutes **b** $0.75 **c** 45 kg
d 62.5 mL **e** 5.6 days **f** 3.3 km

12 **a** $5\frac{1}{3}$ L **b** 2000 marbles
c $8 **d** 45 donuts

13 540

14 Murray, Maeheala, Francesca, Wasim

15 $61\frac{1}{9}\%$, $61.\dot{1}\%$

16 68.75%

17 22

18 $24, $24, They are the same. $\frac{40}{100} \times 60 = \frac{60}{100} \times 40$ as multiplication is commutative.

19 B

20 D

21 **a** 240 cm^2
b 5 cm
c 15 cm by 25 cm
d 375 cm^2
e 135 cm^2
f 56.25%
g 56.25% increase in area
h Multiply the percentage increase in each dimension.
$1.25 \times 1.25 = 1.5625 = 56.25\%$ increase
i **i** 21% **ii** 44% **iii** $\frac{7}{9}\%$ **iv** 125%
j 41.42% increase ($\sqrt{2}$)

Exercise 4H

1 **a** $12 **b** $33.99 **c** $14.50 **d** $225

2 **a** $12 **b** $108

3 shop A: $270, shop B: $280

4 **a** $40 **b** $36 **c** $40.50 **d** $15

5 **a** $440 **b** $276 **c** $64 **d** $41 160
e $5400 **f** $96.96 **g** $13.50 **h** $50.40

6 **a** $480 **b** $127.50 **c** $39 **d** $104
e $15.40 **f** $630

7 **a** $12 **b** $24 **c** $37.50 **d** $63.75
e $97.50 **f** $4.95

8 **a** $396 **b** $11 041.50
c $837 **d** $746 750

9 shop C: $80, shop D: $75

10 premier rug: $250, luxury rug: $375

11 cash $44.90: credit card: $44.91

12 $265.60

13 $84

14 40%

15 Eastern Bikers, $2320

16 **a** **i** $85 **ii** $630 **iii** $275 **iv** $350
b $106.50
c Sam: $720, Jack: $680, Justin: $700
d Sold: $3200, Wage: $960

Exercise 4I

1 **a** $5 **b** $16 **c** $25 **d** $70
e $1.50 **f** $8.80 **g** $0.50 **h** $0.25

2 **a** $10, $110 **b** $5, $55 **c** $15, $165
d $0.50, $5.50 **e** $0.10, $1.10 **f** $12, $132

3 **a** $68 **b** $400 **c** $55
d $2.80 **e** $35.70 **f** 57c

4 **a** $770 **b** $3300 **c** $495
d $37.40 **e** $62 370 **f** $5.39

5 $80

6 A

7 **a** $200 **b** $60 **c** $8000
d $110 **e** $100 **f** $0.90

8 **a** $320 **b** $968 **c** $460
d $47.50 **e** $9810 **f** $5.60

9

Pre-GST price	10% GST	Final cost, including the 10% GST
$599	$59.90	$658.90
$680	$68	$748
$700	$70	$770
$600	$60	$660
$789	$78.90	$867.90
$892	$89.20	$981.20
$645	$64.50	$709.50
$87.25	$8.73	$95.98

10

Superbarn	Gymea Fruit Market	Xmart
a 16.85 **b** 0.27 kg **c** marshmallows – marked with * **d** 1.89 – 0.17 = $1.72	**a** $14.99 per kg **b** 5th July 2011 **c** taking the cash amount to the nearest 5 cents **d** $9.85 **e** 55 cents **f** 5.6% (1 dec. pl.)	**a** 8 **b** $35 **c** all toys **d** $13.55 **e** 9.09% (2 dec. pl.)

11 **a** $24 **b** $240
c Dividing the post-GST price by 11 gives the GST included in the price, as 11 × 10% gives 110%.
d As 1.1 × 100 gives 110%, so dividing by 1.1 yields 100%.

12 **a** $56 **b** $97 **c** $789 **d** $9.87

13 Example, if the price is $100 it goes up to $110, but 10% of this is not $10, so it does not go back down to $100.

14 As the GST is found on the original 100% value of the item, 10% of the larger post-GST price will not give the same amount, as 10% of a larger amount gives a higher value each time. Remember the post-GST price is 110%.

15 Canada cost ÷ 105 × 100 or cost ÷ 1.05
New Zealand cost ÷ 115 × 100 or cost ÷ 1.15
Singapore cost ÷ 107 × 100 or cost ÷ 1.07

16

Raw material	$110	(includes the 10% GST) GST on sale = $10 GST credit = $0 Net GST to pay = $10
Production stage	$440	(includes the 10% GST) GST on sale = $40 GST credit = $10 Net GST to pay = $30
Distribution stage	$572	(includes the 10% GST) GST on sale = $52 GST credit = $40 Net GST to pay = $12
Retail stage	$943.80	(includes the 10% GST) GST on sale = $85.80 GST credit = $52 Net GST to pay = $33.80

GST paid by the final consumer = $85.80

Exercise 4J

1 **a** $7 **b** $28 **c** $3.45 **d** $436
2 **a** $13 **b** $45 **c** $25.90 **d** $247
3 D
4 A
5 **a** 80% profit **b** 30% profit
c 25% loss **d** 16% loss
e $66\frac{2}{3}\%$ profit **f** 37.5% profit
g $33\frac{1}{3}\%$ loss **h** 22% loss
6 **a** 20% increase **b** $16\frac{2}{3}\%$ decrease
c 500% increase **d** 150% increase
7 **a** 25% increase **b** 20% increase
c 140% increase
8 20% loss
9 650% profit
10 **a** $36 **b** 75% profit
11 **a** $350 **b** 87.5% profit
12 **a** $2200 **b** 44% loss
13 Example: Value = $200, because the 10% decrease is of a larger amount.
14 store A
15 **a** **i** 100% **ii** $66\frac{2}{3}\%$ **iii** 80%
b Term 2 **c** 500% growth
16 **a** 396 070
b **i** 22 803 196 **ii** 24 056 866 **iii** 26 301 344
c 2.0%

Exercise 4K

1 B **2** D **3** B **4** $500
5 **a** $900 **b** $800 **c** $1100 **d** $500
e $550 **f** $250
6 $90
7 **a** $120 **b** $240 **c** $15 **d** $21
8 1100 litres
9 600 kg
10 $300
11 **a** $50 **b** $150 **c** $600 **d** $30
e $10 **f** $2000
12 200
13 $40
14 D
15 $\frac{800}{y}$
16 $\frac{D}{C} \times F$ or $\frac{FD}{C}$
17 **a** no **b** No, it went to $118.80
c $9.\dot{0}\dot{9}\%$ **d** 8 years

Puzzles and challenges

1 72.8%
2 96%
3 $\frac{1}{1000}$
4 $48
5 53%
6 $\frac{5}{12}$
7 30%
8 49 years
9 6:33 a.m.
10 **a** $b = -\frac{5}{6}, c = -\frac{1}{3}$
b $b = 1\frac{19}{30}, c = 1\frac{11}{12}$

Multiple-choice questions

1 B **2** E **3** D **4** C
5 D **6** B **7** B **8** E
9 C **10** B **11** E

Short-answer questions

1 **a** 21 **b** 8 **c** 10
2 **a** $\frac{5}{9}$ **b** 3 **c** $\frac{17}{2}$
3 **a** $\frac{7}{11}$ **b** $\frac{1}{8}$ **c** $1\frac{3}{4}$
d $1\frac{7}{12}$ **e** $5\frac{13}{20}$ **f** $3\frac{17}{30}$
4 **a** 8 **b** $\frac{13}{28}$ **c** 14
d 6 **e** $\frac{1}{18}$ **f** 2
5 **a** $\frac{7}{15}$ **b** $\frac{3}{20}$ **c** $\frac{9}{25}$
d $\frac{19}{20}$ **e** −5 **f** $7\frac{7}{12}$

6 a = b < c >

7 a 30.38 b 12.803 c 56974 d 50280
e 74000 f 2.9037

8 a 10.68 b 0.1068 c 14.4 d 0.255
e 3.6 f 197.12

9 a 0.667 b 3.580 c 0.005

10

0.1	0.01	0.05	0.5	0.25	0.75	$0.\dot{3}$	0.125
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{8}$
10%	1%	5%	50%	25%	75%	$33\frac{1}{3}$%	12.5%

11 a 87.5% b 25% c 150%
d 4% e 6%

12 a 24 b $10.50 c 10.5

13 a $616 b $3280
c $977.55, overall percentage loss is 0.25%

14 $359.91, $3639.09

15 29% 16 6%

17 $680

18 $9.90

19 1120

20 $3.31

21

Pre-GST price	10% GST	Final cost, including the 10% GST
$250	$25	$275
$450	$45	$495
$80	$8	$88
$670	$67	$737

Extended-response question

a 21000 INR, 625 SGD, 15000 THB, 3500 HKD

b $30

c i 800 ii 3.8%

d i $96.48
ii $95.03, not enough to buy perfume.

Chapter 5

Pre-test

1 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{1}{3}$ d $\frac{3}{4}$
e $\frac{2}{3}$ f $\frac{1}{2}$ g $\frac{2}{3}$ h $\frac{4}{7}$

2 a 10 b 16 c 8

3 a 500 b 6000 c 5
d 8 e 1.2 f 15

4 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{1}{5}$
d $\frac{1}{3}$ e $\frac{2}{25}$ f 1

5 a $16 b 20 m c 16 cm
d 600 m e $7.50 f $12.50

6 a $9.98 b $24.95 c $49.90 d $2.50

7 1.5 km

8 $30/h

9 a i 120 km ii 300 km iii 30 km
b i 3 hours ii $1\frac{1}{2}$ hours iii $\frac{1}{3}$ hours
c 6 minutes

Exercise 5A

1 a 3 : 7 b 5 : 4

2 a 1 : 3 b 7 : 15

3 a 9 : 4 b 52 : 17 c 7 : 12 d 3 : 5

4 a 5 : 3 b 7 : 15 c 3 : 7 : 5 d 5 : 3 : 7

5 a 8 : 3 b 3 : 14 c 3 : 11 d 8 : 6

6 a 13 : 7 b 11 : 9
c 13 : 9 : 11 : 7 d 20 : 20 or 1 : 1

7 a 12 b 14 c 4 d 9
e 2 f 4 g 2 h 10
i 77 j 51 k 9 l 11

8 a 4 : 6 : 10 b 2 : 6 : 8
c 7 : 49 : 63 d 11 : 55 : 33

9 Answers may vary.
a 2 : 4, 3 : 6, 5 : 10 b 4 : 10, 20 : 50, 200 : 500
c 4 : 3, 16 : 12, 40 : 30 d 3 : 1, 6 : 2, 18 : 6

10 2 : 5 and 4 : 10, 6 : 12 and 1 : 2, 7 : 4 and 70 : 40

11 a 3 : 30 b 1 : 10, 10 : 100

12 2

13 a 4 : 6 b 3 : 5 c 4 : 6 d 5 : 4

14 Victoria, Tasmania, South Australia

15 No, 3 is not a factor of 10.

16 1 : 2

17 a 5 : 8 b 36 : 52 c No

18 a $3x$ b $2.5x$ c $4y$ d $2x$

19 a 4:21 b 3 : 7 c 3 : 1 d 7 : 2

20 2 : 3

Exercise 5B

1 a 1 : 1 b 1 : 2

2 D

3 B

4 C

5 a 1 : 4 b 1 : 5 c 1 : 6 d 1 : 3
e 4 : 5 f 5 : 8 g 3 : 4 h 3 : 10
i 9 : 7 j 2 : 1 k 9 : 7 l 3 : 1
m 3 : 1 n 1 : 9 o 6 : 11 p 2 : 1
q 12 : 1 r 1 : 6 s 8 : 5 t 6 : 5

6 a 1 : 2 : 3 b 4 : 7 : 11 c 7 : 10 : 2 d 17 : 7 : 3
e 1 : 2 : 3 f 2 : 6 : 5 g 9 : 14 : 2 h 2 : 4 : 7

7 a 2 : 3 b 5 : 4 c 8 : 15 d 10 : 7
e 3 : 2 f 7 : 8 g 33 : 4 h 27 : 14

8 a 2 : 1 b 11 : 3 c 50 : 21 d 52 : 45

9 **a** 2 : 5 **b** 14 : 1 **c** 3 : 25 **d** 1 : 35
e 20 : 3 **f** 2 : 25 **g** 50 : 11 **h** 5 : 1
i 2 : 5 **j** 1 : 6 **k** 12 : 1 **l** 9 : 1
m 1 : 16 **n** 2 : 9 **o** 1 : 7 **p** 14 : 3
q 1 : 8 **r** 30 : 1

10 B

11 **a** 2 : 11 **b** 9 : 11 **c** 2 : 9

12 **a** 2 : 3 : 3 **b** 1 : 1 **c** 2 : 5 **d** 5 : 7

13 **a** 5 : 5 : 2 : 4 : 3 : 1 : 20
b 20 : 20 : 8 : 16 : 12 : 4 : 80
c **i** 1 : 4 **ii** 1 : 1

14 Andrew did not convert amounts to the same units. Correct ratio is 40 : 1.

15 **a** 8 : 1 **b** 8 : 1, yes

16 Answers may vary.
a 24 minutes to 1 hour
b 2 kilometres to 1500 metres

17 **a** $a : 2b$ **b** $5x : y$ **c** $1 : a$
d 5 : 24 **e** $h : 3$ **f** $2x : 5$

18 **a** 4 : 3 **b** 16 : 9 **c** squared. $16 : 9 = 4^2 : 3^2$
d 47 : 20

Exercise 5C

1 **a** 10 **b** 6 **c** 14 **d** 9

2 **a** $\frac{3}{8}$ **b** $\frac{5}{8}$

3 **a** 1 : 3 **b** 1 : 1 **c** 2 : 5 **d** 1 : 4

4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{2}{7}$ **d** $\frac{1}{5}$

5 **a** \$24 and \$36 **b** \$70 and \$40
c \$150 and \$850 **d** 8 kg and 40 kg
e 8 kg and 6 kg **f** 150 kg and 210 kg
g 24 m and 48 m **h** 15 m and 25 m
i 124 m and 31 m

6 **a** \$100 and \$300 **b** \$160 and \$240
c \$150 and \$250 **d** \$180 and \$220

7 **a** \$2400 and \$4200 **b** \$2640 and \$3960
c \$1584 and \$5016 **d** \$3740 and \$2860

8 **a** \$40, \$80, \$80 **b** \$50, \$150, \$200
c 2 kg, 4 kg, 6 kg **d** 22 kg, 11 kg, 55 kg
e 96 kg, 104 kg, 120 kg
f \$5000, \$10 000, \$15 000, \$20 000

9 **a** 40 **b** 36 **c** 2.625 cm **d** 133 cm

10 nitrogen: 500 g, potassium: 625 g, phosphorus: 375 g

11 40°, 60°, 80°

12 \$250

13 48

14 240

15 294

16 120 pages

17 shirt \$160, jacket \$400

18 **a** 8 **b** 3 : 5
c 2 boys and 2 girls were absent or 5 boys and 9 girls.

19 30°

20 obtuse-angled isosceles triangle; 30, 30 and 120°

21 **a** Ramshid: \$125, Tony: \$83.33, Manisha: \$41.67
b 10 : 6 : 5
c Ramshid: \$119.05, Tony: \$71.43, Manisha: \$59.52
d \$17.85 **e** \$11.90
f Original ratio, as he receives \$5.95 more.
g Ramshid: \$120, Tony: \$70, Manisha: \$60

Exercise 5D

1 50 000 cm

2 3 m

3 **a** 100 000 mm **b** 100 m **c** 0.1 km

4 **a** 0.56 km **b** 56 000 cm **c** 560 000 mm

5 Numbers and units may vary.
a **i** 200 m **ii** 40 m **iii** 730 m
b **i** 16 km **ii** 370 m **iii** 25 km
c **i** 6.4 m **ii** 288 m **iii** 12 m
d **i** 150 cm **ii** 24.6 m **iii** 2.13 km
e **i** 88 m **ii** 620 cm **iii** 5 mm
f **i** 20 m **ii** 120 cm **iii** 77.5 cm
g **i** 6 cm **ii** 1.6 mm **iii** 200 m
h **i** 0.3 mm **ii** 115 m **iii** 8.15 mm

6 **a** **i** 1 m **ii** 20 m **iii** 3 mm
b **i** 20 m **ii** 2 m **iii** 1.5 mm
c **i** 13.5 cm **ii** 4.5 m **iii** 7.365 cm
d **i** 60 cm **ii** 9 cm **iii** 0.02 mm
e **i** 20 m **ii** 3 m **iii** 5 mm
f **i** 2.3 m **ii** 16.5 cm **iii** 61.5 cm
g **i** 3 m **ii** 5 cm **iii** 2 mm
h **i** 20 cm **ii** 15 m **iii** 1.64 m

7 **a** 1 : 250 **b** 1 : 50 000
c 1 : 50 000 **d** 1 : 18 000
e 7 : 1 or $1 : \frac{1}{7}$ **f** 600 : 1 or $1 : \frac{1}{600}$

8 **a** 1 : 10 000 **b** 1 : 1000 **c** 1 : 300
d 1 : 150 000 **e** 1 : 125 **f** 1 : 200 000
g 1 : 100 000 **h** 50 : 1 **i** 10 000 : 1

9 **a** 80 m **b** 4.5 cm

10 8.5 km

11 **a** 3.54 m by 2.46 m **b** 4.6 m by 4.6 m
c 7.69 m by 2 m

12 length 11 m, height 4 m

13 **a** 1 : 40 **b** 56 cm

14 **a** 24 km **b** 160 km **c** 12.5 cm

15 1 : 0.01 and 100 : 1, 25 : 1 and 50 : 2, 20 : 1 and 1 : 0.05

16 **a** car: D, 1 : 10
b school grounds: B, 1 : 1000
c Mt Kosciuszko: A, 1 : 10 000

17 With chosen scale, the map will be 8 m wide by 8 m high, which is too big to be practical.

18 The ratio provided is the wrong way around. It should be 1000 : 1.

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Exercise 5E

1 B, C, E, F, H
2 employee's wage: $15/h
speed of a car: 68 km/h
cost of building new home: $2100/m^2
population growth: 90 people/day
resting heart rate: 64 beats/min
3 **a** $/kg **b** $/L
c words per minute **d** goals/shots on goal
e kJ/serve or kJ/100 g **f** L/min
g mL/kg or mg/tablet **h** runs/over
4 **a** 3 days/year **b** 5 goals/game
c $30/h **d** $3.50/kg
e $14 000/acre **f** 4500 cans/hour
g 1200 revs/min **h** 16 mm rainfall/day
i 4 min/km **j** 0.25 km/min or 250 m/min
5 **a** 300 km/day **b** $140/year
c 6.5 runs/over **d** 7.5 cm/year
e 1.5 kg/year **f** dropped 2.5°C/hour
6 **a** 3 L/hour **b** 7 hours
7 158 cm
8 **a** 1.5 rolls/person **b** $6/person
c $4/roll
9 **a** **i** 5.8 hours/day **ii** 6.5 hours/day
iii 42 hours/week **iv** 6 hours/day
b 180 hours
10 Harvey: 3.75 min/km, Jacques: 3.33 min/km; Jacques
11 **a** 1200 members/year **b** 12 years
12 **a** **i** 9 km/L **ii** $\frac{1}{9}$ L/km
b Find the reciprocal.
13 **a** **i** $4 **ii** $7.25 **iii** $16.50 **iv** $22.50
b 75c/minute
c Teleconnect
d Connectplus
e $16\frac{2}{3}$ min or 16 min and 40 seconds

Exercise 5F

1 $27
2 $17.60
3 $43.30
4 $44
5 **a** $2.02
b $1.30/L
c yes ($2.57/kg vs $3.075/kg)
d $18.87/kg, $10.77/kg
e $66.80
f $5.53
6 **a** 520 mL **b** 11 goals
c 350 mm **d** 1 875 000 kilobytes
7 Leonie $600, Mackenzie $300, total $1350
8 **a** 25c/min **b** 4c/s **c** 210 L/h
d 1.2 L/h **e** 6 kg/year **f** 0.84 kg/week
g 6 kg/$ **h** 3.8c/mm **i** 30 m/s
j 50.4 km/h
9 **a** small: $1.25/100 g; medium: $1.20/100 g; large: $1.10/100 g
b 4 large, 1 medium, 1 small, $45.20
10 12 s
11 **a** 1.2 cm/month, 0.144 m/year
b 25 months
12 **a** 45 m^2/h **b** 900 m^2 **c** $\frac{5}{6}$ m^2/min
13 **a** $\frac{\$y}{x}$ **b** $\frac{\$12y}{x}$ **c** $\frac{\$zy}{x}$
14 **a** 21 cm, 28 cm, Perimeter = 84 cm
b 19 cm, 22 cm, Perimeter = 58 cm
15 50 pa/mD
16 **a** Perth **b** 56 hours **c** 40 hours
d Phil is 1125 km from Perth, Werner is 1575 km from Sydney.
e 2450 km from Sydney, 1750 km from Perth
f 46 hours and 40 minutes

Exercise 5G

1 **a** 3 hours **b** 5 hours, ×5
c ×10, 30 minutes, ×10 **d** ×6, 720 litres, ×6
2 **a** $12, $60, ×5
b ÷5, 30 rotations, ÷5, ×7, 210 rotations, ×7
3 **a** 2400 **b** 19 200
4 100 litres
5 40 litres
6 **a** 150 days **b** 15 000 days **c** 50 years
7 **a** 22 500 **b** 375 **c** 10 minutes **d** 6 seconds
8 **a** 3750 beats **b** 1380 beats **c** 80 minutes
9 $2520
10 22.5 kg
11 Bionic woman wins by 4 seconds.
12 **a** $7\frac{1}{2}$ days **b** 187 students
13 2.4 days
14 **a** $2\frac{2}{3}$ days
b Maric: $\frac{1}{3}$, Hugh: $\frac{4}{9}$, Ethan: $\frac{2}{9}$
15 **a** 80 cans **b** 5 dogs **c** 15 days
16 12 hours
17 **a** $ **b** m **c** min **d** min
18 **a** buddies: $4.50/L, 1.25 L bottles: $1.28/L, 2 L bottles: $1.10/L, cans: $1.60/L
b buddies: $135, 1.25 L bottles: $38.40, 2 L bottles: $33, cans: $48
c greatest amount = 54 L, least amount = 13.2 L

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Exercise 5H

1 D

2 A

3 B

4 D

5 **a** 10 m/s **b** 7 m/s **c** 60 km/h **d** 50 km/h
e 120 km/h or 2 km/min **f** 45 km/h or 0.75 km/min

6 **a** 1080 m **b** 450 m **c** 36 km **d** 50 km

7 **a** 8 hours **b** $\frac{1}{2}$ hour or 30 minutes
c 11.5 hours **d** 7 seconds

8 **a** 10 m/s **b** 50 m/s **c** 11 m/s **d** 4000 m/s

9 **a** 54 km/h **b** 7.2 km/h
c 0.72 km/h **d** 3600 km/h

10 **a** 25 km/h **b** 40 s **c** 60 km **d** 120 m/min
e 70 km **f** 100 s

11 2025 km

12 **a** 27 km/h **b** $2\frac{1}{4}$ km

13 27 m/s

14 24 km/h

15 10.4 m/s, 37.6 km/h

16 **a** 58.2 km/h **b** 69.4 km/h

17 Answers may vary; e.g. 150 km/h, 160 km/h

18 250 m

19 8:02:40; 2 minutes and 40 seconds after 8 a.m.

20 **a** 343 m/s **b** 299 792 458 m/s **c** 0.29 s
d 0.0003 s **e** 874 030
f how many times the speed of sound (mach 1 = speed of sound)
g 40 000 km/h, 11.11 km/s **h** 107 218 km/h, 29.78 km/s
i 69 km/h **j–l** Answers vary

Exercise 5I

1 Graph i = Journey C; Graph ii = Journey A;
Graph iii = Journey D; Graph iv = Journey B

2 **a** 240 km **b** 3 hours **c** 80 km/h

3 **a** A man travelled quickly away from home, then stopped for a short time. He then travelled slowly away from home and finally stopped again for a short time.
b A boy travelled quickly away from home, then stopped for a short time. He then turned around and travelled slowly back to his home.
c A girl travelled slowly towards home and part-way back stopped for a short time. She then travelled quickly back to her home.

4 **a** *Q* **b** *P* **c** *S* **d** *R*
e *S* **f** *T* **g** *T*

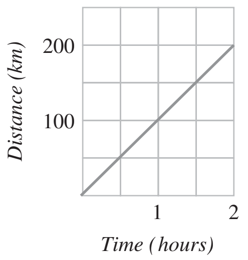
5 **a** 80 m **b** 1.5 minutes **c** 160 m
d $\frac{1}{2}$ minute **e** 180 m **f** 220 m

6 **a** segments B and D **b** 50 s
c 40 m **d** 90 s
e 160 s **f** 30–50 s and 170 s
g 160 m
h segment C, slowest walking speed
i segments A & E, fastest walking speed

7 **a** 12:15 p.m. **b** 9.6 km/h **c** 24 km/h
d 12:45 p.m., 1:45 p.m.
e 1 hour 15 minutes
f 12:15–12:30 p.m. and 2:15 p.m.
g 2:45 p.m. **h** 12 km/h **i** 8.5 km/h

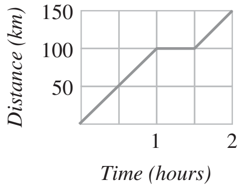
8 **a**

Motorbike journey



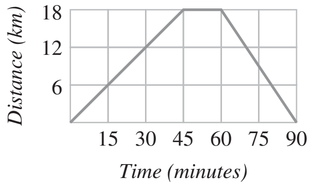
b

Car journey

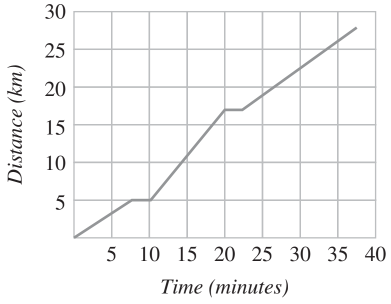


c

Cycle journey



9 **a** 37.5 km/h, 72 km/h, 40 km/h
b The three moving sections are over different time intervals and also the whole journey average must include the stops.
c 43.8 km/h
d



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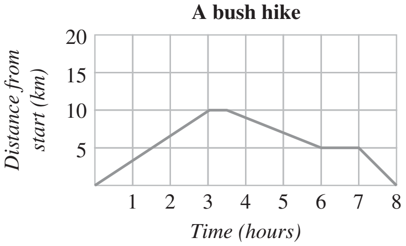
10 a 2 km/h (uphill) and 5 km/h (downhill).

b Many possible stories, for example:
A weekend bushwalk started with a 20-km hike on the first day. The first section of the hike was a 10 km, 3-hour hike to waterfalls where we had a half-hour rest. The hike up the mountain was steep and we progressed slowly, needing $2\frac{1}{2}$ hours to cover a distance of 5 km. On the summit of the mountain, we had a 1 hour rest. Finally we walked downhill for 5 km taking 1 hour to reach our campsite. On this last downhill section, we achieved our fastest average speed of 5 km/h compared to the initial average speed of 3.3 km/h and the slowest speed of 2 km/h when we were hiking up the mountain.

c



11 a True, as these points correspond to the same position on the distance scale.

b True, as this point is on a flat line segment.

c False, the line segment through A is flatter than the line segment through D.

d False, Deanna was riding away from home at C and turned towards home at E.

e True, as the line segment through F is steeper than the line segment through A.

f False, as point F corresponds to a smaller distance than point D from home.

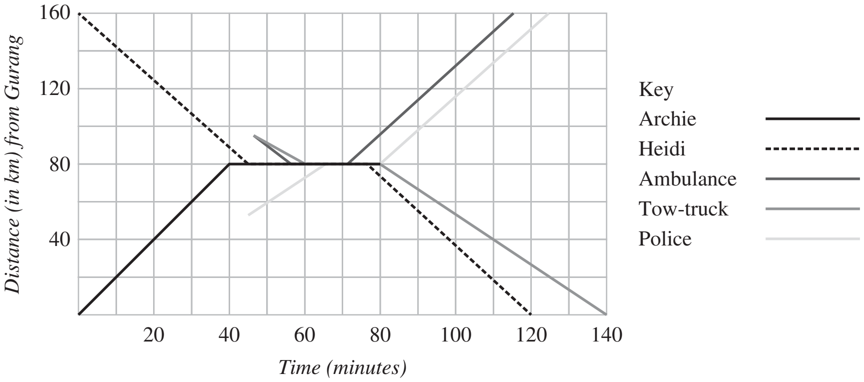
g True, point F is at a later time on Deanna's return trip home.

12 Journey A: Isla as she walks from the front and stops twice only.
Journey B: Adam as he has 3 stops and a very fast walk in one section.
Journey C: Conner as he walks to the back of the room twice.
Journey D: Ruby as her walk starts from the back.

13 Graph A: True, leaves from school (some distance from home), rides further away to her friend, then rides in the other direction and stops at library, which is on the way home.
Graph B: False, vertical lines are incorrect as it is impossible to change distance in zero time.
Graph C: False, distance must be on the vertical axis and time on the horizontal axis.
Graph D: True, the school is a long way from home, her friend is on the way home then Sienna must ride away from home to go to the library, then she changes direction and rides home.

14 Graph A: Incorrect as one rider didn't stop.
Graph B: Incorrect as riders not stopped at the same time.
Graph C: Incorrect as riders not stopped at the same place.
Graph D: Correct, Jayden stopped first, Cooper stopped same place same time, both left together.

15 a



b Many possible endings; for example:
The tow-truck also comes from 15 km away and arrives in 15 minutes. The police come from 25 km away and arrive in 20 minutes. The ambulance paramedics treat Archie for 17 minutes and then drive to Northbrook Hospital, arriving in 45 minutes. The tow-truck leaves the accident 10 minutes after the ambulance and arrives in Gurang after 1 hour. Heidi stays at the accident scene for 30 minutes and then drives to Northbrook in 45 minutes. The police leave at the same time as the tow-truck and drive to Northbrook in 45 minutes.

d Archie 120 km/h; Heidi 107 km/h; ambulance 120 km/h then 100 km/h; tow-truck 80 km/h; police 105 km/h, 107 km/h.
Yes, Archie could be charged for speeding; providing that the ambulance had flashing lights it would be legal for it to drive at 120 km/h.

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Puzzles and challenges

1 **a** 2 **b** $3\frac{1}{5}$ **c** $2\frac{2}{3}$

2 1 : 3

3 9 km/h

4 **a** 1 hour **b** 12 sets

5 12 sheep

6 $\frac{5}{9}$ hour

7 0.75 km

8 5 hours

9 12:57 a.m.

10 68.57 km/h

11 A 20 km trip at 100 km/h takes $\frac{1}{5}$ hour, time that Shari has already used up to drive the first 10 km. The second 10 km cannot be travelled in zero time; hence, it is impossible for Shari to achieve a 100 km/h average speed.

Multiple-choice questions

1 A **2** C **3** E **4** A
5 B **6** B **7** E **8** C
9 B **10** D **11** D

Short-answer questions

1 **a** 1 : 2 **b** 2 : 1 **c** 5 : 9

2 **a** false **b** false **c** true **d** false **e** true

3 **a** 25 **b** 12 **c** 6 **d** 3

4 **a** 1 : 4 **b** 3 : 2 **c** 3 : 4 **d** 1 : 8
e 3 : 1 **f** 1 : 5 **g** 3 : 2 **h** 2 : 1
i 2 : 3 **j** $2a$: 1 **k** 2 : 5 **l** 11 : 2
m 10 : 3 **n** 1 : 3 : 6

5 **a** 5 : 2 **b** 1 : 3 **c** 2 : 5 **d** 1 : 2
e 1 : 5 **f** 1 : 4 **g** 3 : 25 **h** 3 : 10

6 **a** \$35 : \$45 **b** 160 kg : 40 kg
c 30 m : 10 m **d** \$340 : \$595 : \$510
e 60c : 20c : 20c

7 **a** \$1152 **b** 144 cm **c** 1.125 L

8 **a** 5 km/h **b** \$50/h **c** 140 km/day

9 **a** 12.5 km/L, 8 L/100 km
b 2.5 g/min, 150 g/h
c \$2400/day, \$100/h

10 **a** 0.2 m **b** 27 m **c** 140 m

11 500 mm

12 **a** 1 : 1.5, $x = 9$ cm **b** 1 : 3, $x = 12$ cm

13 **a** 200c/min **b** 21.6 km/h
c 200 m/s

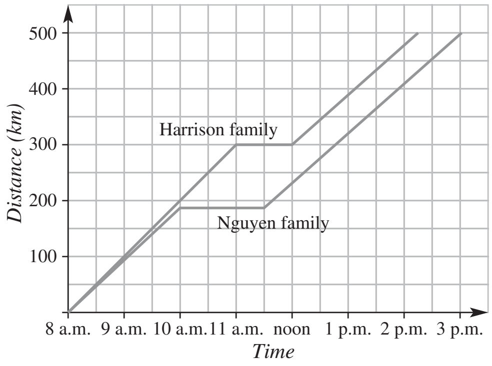
14 **a** 301 km **b** \$155.87 **c** $6\frac{2}{3}$ hours

15 **a** 64 km/h **b** 108 min **c** 6.75 km

16 **a** 15 km **b** 20 km/h **c** 15 minutes
d 40 km
e Q (30 km) further than R (20 km)
f 12:45 p.m.
g 50 km
h 9:30 a.m.–11 a.m.: 1.5 hours, 15 km, 10 km/h, riding away from home;
11 a.m.–12 p.m.: stopped for 1 hour, lunch break;
12 p.m.–1:30 p.m.: 1.5 hours, 40 km, 26.7 km/h, returning home.

Extended-response question

a



b 20 km

c Nguyens 10:30 a.m.; Harrisons 12:15 p.m.

d 88.9 km/h

e 80 km/h

f 11:30 a.m.

g 320 km

h $3\frac{1}{2}$ hours, 91.4 km/h

i 68.6 km

j Harrisons \$62.55; Nguyens \$97.30

Semester review 1

Algebraic techniques 2 and indices

Multiple-choice questions

1 C **2** E **3** D **4** B **5** D

Short-answer questions

1 **a** $p + q$ **b** $3p$ **c** $\frac{m^2}{2}$ **d** $\frac{x + y}{2}$

2 **a** 9 **b** 25 **c** 102 **d** 116
e -24 **f** -24

3 **a** $24k$ **b** $3a$ **c** a^3 **d** $\frac{p}{2}$
e $7ab + 2$ **f** $x - 1$ **g** $2y$ **h** $2n - 2m$

4 **a** xy **b** $\frac{x}{7}$ **c** $\frac{7w}{10}$ **d** $\frac{7a}{2}$

5 **a** $\frac{m}{6}$ **b** ab **c** $\frac{2}{3}$

6 **a** $12m - 18$ **b** $4 + 2m$ **c** $9A + 6$

7 **a** $6(3a - 2)$ **b** $6m(m + 1)$ **c** $-8m(m + 2n)$

8 **a** $8x + 20$ **b** $3x(x + 10)$

9 **a** 5^9 **b** 3^{10} **c** 6^8
d 4^7 **e** 7^6 **f** 6^6

10 **a** 5^6 **b** 2^{12} **c** 1 **d** 5

Extended-response question

a $(7a + x)$ cm
b $6a^2$ cm^2
c $x = 5a$
d $12a$ cm
e 216 cm^2

Equations 2

Multiple-choice questions

1 B **2** D **3** E **4** C **5** B

Short-answer questions

1 **a** $w = 9$ **b** $m = 72$ **c** $x = 1$ **d** $a = 2$
e $w = -3$ **f** $x = 35$
2 **a** $m = -\frac{1}{2}$ **b** $a = -1$ **c** $x = 0$ **d** $x = \frac{15}{8}$
e $a = \frac{13}{8}$ **f** $a = \frac{7}{5}$
3 6
4 4 years
5 **a** $x = \pm 3$ **b** $x = \pm 6$ **c** $a = \pm 12$
d $y = 0$ **e** no solution **f** no solution
6 **a** $F = 30$ **b** $m = 4$

Extended-response question

a $C = 5n + 1500$
b $R = 17n$
c 125
d $P = 12n - 1500$
e $900
f −$300 (a loss)

Measurement and Pythagoras' theorem

Multiple-choice questions

1 B **2** B **3** D **4** E **5** D

Short-answer questions

1 **a** 500 cm **b** 180 cm **c** 90 000 cm^2
d 1.8 m **e** 4000 cm^3 **f** 10 000 m^2
2 **a** 18.6 cm **b** 64 m **c** 40 m
3 **a** **i** 25.13 m **ii** 50.27 m^2
b **i** 47.12 cm **ii** 176.71 cm^2
4 **a** **i** 25.71 m **ii** 39.27 m^2
b **i** 17.85 cm **ii** 19.63 cm^2
c **i** 54.85 mm **ii** 169.65 mm^2
5 **a** 30 m^2 **b** 48 m^2
6 **a** $V = 74.088$ m^3 **b** $V = 50$ m^3 **c** $V = 24$ m^3
7 **a** 615.75 m^3 **b** 392 699.08 cm^3
c 1.26 m^3 or 1 256 637.06 cm^3
8 **a** 13 **b** 14.42
9 **a** 1530 **b** 0735

Extended-response question

a $15 - 2x$
b x
c $x(15 - 2x)^2$
d 169 m^3
e $x = 2.5$

Fractions, decimals, percentages and financial mathematics

Multiple-choice questions

1 D
2 C
3 C
4 E
5 A
6 A

Short-answer questions

1 **a** 18 **b** 1 **c** 5
2 **a** $\frac{1}{4}$ **b** $\frac{7}{5}$ **c** $3\frac{1}{4}$ **d** $-\frac{2}{21}$
e $\frac{1}{3}$ **f** $\frac{9}{10}$
3 **a** $\frac{5}{2}$ **b** $\frac{1}{8}$ **c** $\frac{5}{21}$
4 **a** $\frac{9}{2}$ **b** $\frac{3}{4}$ **c** $\frac{5}{8}$
5 **a** 6.93 **b** 7.58 **c** 4.03 **d** 6.51
e 3854.8 **f** 792
6 **a** 545.9 **b** 1008 **c** 0.614

7

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{19}{20}$	$\frac{99}{100}$	$\frac{1}{200}$
Decimal	0.25	0.5	0.2	$0.\dot{3}$	$0.\dot{6}$	0.8	0.95	0.99	0.005
Percentage	25%	50%	20%	$33.\dot{3}$%	$66.\dot{6}$%	80%	95%	99%	0.5%

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8 **a** 5.6 **b** 11.76 **c** 85.5 m **d** $1.98
e $105 **f** 4930 g
9 **a** $700 **b** $862.4 **c** $0.9936
10 25
11

Pre-GST	GST	Post-GST
$550	$55	$605
$70	$7	$77
$475	$47.50	$522.50
$875	$87.50	$962.50

Extended-response question

a **i** $1784.15 **ii** $1516.53 **iii** $1289.05
b 6 years
c No. There will always be 85% of the previous value.

Chapter 5: Ratios and rates

Multiple-choice questions

1 C **2** B **3** A **4** A **5** D

Short-answer questions

1 **a** 2 : 3 **b** 1 : 2 : 3 **c** 6 : 7
d 3 : 40 **e** 3 : 8 **f** 3 : 10
2 **a** 576 cm, 384 cm **b** $1500, $2500
c $1.60, $4, $2.40
3 18.75 m^2
4 $7750
5 **a** 300 g/h **b** $30/h **c** 100 km/h
6 $2.27
7 90 km/h

Extended-response questions

1 **a** 742.5 km **b** 16.5 km **c** 6.1 L
d $40.47 **e** 18 km
2 **a** 30 minutes
b 15 minutes, 45 minutes, 15 minutes
c 36 km
d 6:45 a.m.–7 a.m. and 9:15 a.m.
e 40 km/h, 24 km/h, 24 km/h, 32 km/h
f 48 km
g 24 km
h 10 a.m.
i 21.3 km/h
j First section: started at 6 a.m., finished at 6:45 a.m., flew 30 km at a speed of 40 km/h.
Second section: started at 6:45 a.m., finished at 7 a.m., flew 0 km, stopped/rested.
Third section: started at 7 a.m., finished at 7:45 a.m., flew 18 km at a speed of 24 km/h.

Chapter 6

Pre-test

1 **a** 60 **b** 130 **c** 140
2 **a** revolution **b** right **c** acute
d obtuse **e** straight **f** reflex
3 **a** 130 **b** (c, e, g) **c** (b, d, f)
4 **a** obtuse **b** isosceles **c** acute
d scalene **e** equilateral **f** right
5 square, rectangle, rhombus, parallelogram, kite, trapezium (examples of each shape will vary)
6 **a** 40 **b** 110
7 **a** pyramid **b** triangular prism **c** cube

Exercise 6A

1 **a** complementary **b** complement
c supplementary **d** supplement
e revolution
2 **a** 45 **b** 130 **c** 120
d 240 **e** 90
3 **a** 40° **b** 110° **c** 220°
4 **a** $a = 70$ (angles in a right angle)
$b = 270$ (angles in a revolution)
b $a = 25$ (angles in a right angle)
$b = 90$ (angles on a straight line)
c $a = 128$ (angles on a straight line)
$b = 52$ (vertically opposite angles)
d $a = 34$ (angles on a straight line)
$b = 146$ (vertically opposite angles)
e $a = 25$ (angles in a right angle)
f $a = 40$ (angles on a straight line)
g $a = 120$ (angles in a revolution)
h $a = 50$ (angles on a straight line)
$b = 90$ (angles on a straight line)
i $a = 140$ (angles in a revolution)
j $a = 110$ (vertically opposite angles)
$b = 70$ (angles on a straight line)
k $a = 148$ (angles in a revolution)
l $a = 90$ (angles on a straight line)
$b = 41$ (vertically opposite angles)
$c = 139$ (angles on a straight line)
5 **a** **i** $\angle DOE$ **ii** $\angle AOB$
iii $\angle DOE$ or $\angle AOB$ **iv** $\angle COD$
b **i** $AD \perp OC$ **ii** $OC \perp OD$
6 **a** 60° **b** 10° **c** 45° **d** 73°
7 **a** 150° **b** 100° **c** 135° **d** 163°
e 60° **f** 85° **g** 45° **h** 13°
8 **a** 40° **b** 72° **c** 120° **d** 200°
9 **a** 60 **b** 135 **c** 35
d 15 **e** 36 **f** 45
10 **a** $(90 - x)^\circ$ **b** $(180 - x)^\circ$
c x° **d** $(360 - x)^\circ$

11 a AB looks like a straight line but the angles do not add to 180°.

b The two angles make a revolution but they do not add to 360°.

c Same as part **a**.

12 a $a + 3b = 360$

b $a + 2b = 180$

c $a + b = 90$

13 a $a = 110$

b $(a + 50)°$ should be the larger looking angle.

14 a 30 **b** 54 **c** 55

d 34 **e** 30 **f** 17

Exercise 6B

1 a equal

b supplementary

c equal

2 a $\angle BCH$ **b** $\angle ABE$ **c** $\angle GCB$ **d** $\angle BCH$

e $\angle FBC$ **f** $\angle GCB$ **g** $\angle FBC$ **h** $\angle DCG$

3 a alternate $a = b$ **b** alternate $a = b$

c cointerior $a + b = 180°$ **d** corresponding $a = b$

e corresponding $a = b$ **f** cointerior $a + b = 180$

4 Note: there are other reasons that may be acceptable.

a $a = 110$ (corresponding angles on parallel lines)
$b = 70$ (angles on a straight line)

b $a = 120$ (alternate angles on parallel lines)
$b = 60$ (angles on a straight line)
$c = 120$ (corresponding angles on parallel lines)

c $a = 74$ (alternate angles on parallel lines)
$b = 106$ (cointerior angles on parallel lines)
$c = 106$ (vertically opposite angles)

d $a = 100$ (angles on a straight line)
$b = 100$ (corresponding angles on parallel lines)

e $a = 95$ (corresponding angles on parallel lines)
$b = 85$ (angles on a straight line)

f $a = 40$ (alternate angles on parallel lines)
$b = 140$ (angles on a straight line)

5 a $a = 58$ (cointerior angles on parallel lines)
$b = 58$ (cointerior angles on parallel lines)

b $a = 141$ (cointerior angles on parallel lines)
$b = 141$ (cointerior angles on parallel lines)

c $a = 100$ (cointerior angles on parallel lines)
$b = 80$ (cointerior angles on parallel lines)

d $a = 62$ (cointerior angles on parallel lines)
$b = 119$ (cointerior angles on parallel lines)

e $a = 105$ (cointerior angles on parallel lines)
$b = 64$ (corresponding angles on parallel lines)

f $a = 25$ (alternate angles on parallel lines)
$b = 30$ (alternate angles on parallel lines)

6 a No, the alternate angles are not equal.

b Yes, the cointerior angles are supplementary.

c No, the corresponding angles are not equal.

7 a 250 **b** 320 **c** 52

d 40 **e** 31 **f** 63

g 110 **h** 145 **i** 33

8 a 130° **b** 95° **c** 90°

d 97° **e** 65° **f** 86°

9 a $\angle AOB = (180 - a)°$

b $\angle AOB = (360 - a)°$

c $\angle AOB = (180 - a - b)°$

10 $a = 36, b = 276, c = 155, d = 85, e = 130, f = 155, g = 15$

Exercise 6C

1 a right-angled triangle **b** isosceles triangle

c acute-angled triangle **d** equilateral triangle

e obtuse-angled triangle **f** equilateral triangle

g isosceles triangle **h** scalene triangle

2 a scalene **b** isosceles **c** isosceles

d equilateral **e** scalene **f** isosceles

3 a right **b** obtuse **c** acute

4 a 80 **b** 40 **c** 58

d 19 **e** 34 **f** 36

5 a 68 **b** 106 **c** 20

6 a 65 **b** 40 **c** 76

7 a 160 **b** 150 **c** 80

d 50 **e** 140 **f** 55

8 a yes **b** no **c** yes

d yes **e** yes **f** yes

9 a 55 **b** 60 **c** 25

10 a $\angle ACB = 60°$ (angle in an equilateral triangle)
$\angle DCE = \angle ACB = 60°$ (vertically opposite angles)
$\therefore a = 60$

b $\angle ACB + 22° + 29° = 180°$ (angle sum of $\triangle ABC$)
$\therefore \angle ACB = 129°$
$a = 360 - 129 = 231$ (angles in a revolution)

c $\triangle ABC$ is isosceles ($AB = AC$)
$\angle BCA = \angle ABC = 72°$ (equal angles are opposite equal sides in $\triangle ABC$)
$a + 90 + 72 = 180$ (angle sum of $\triangle ABC$)
$a = 18$

d $\angle CAB = 25°$ (angles on a straight line)
$\angle ACB = 64°$ (angles on a straight line)
$a + 25 + 64 = 180$ (angle sum of $\triangle ABC$)
$a = 91$

e $\angle CAB = \angle CBA = a°$ (equal angles are opposite equal sides)
$\angle ACB = \angle DCE = 49°$ (vertically opposite angles)
$a + a + 49 = 180$ (angle sum of $\triangle ABC$)
$2a = 131$
$a = 65.5$

f $\angle ACB = 30°$ (angles on a straight line)
$a + 90 + 30 = 180$ (angle sum of $\triangle ABC$)
$a = 60$

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11 a Isosceles, the two radii are of equal length.
  b $\angle OAB$, $\angle OBA$
  c 30°
  d 108°
  e 40°

12 a i a, alternate angles in parallel lines
  ii c, alternate angles in parallel lines
  b They add to 180°, they are on a straight line.
  c $a + b + c = 180$, angles in a triangle add to 180°.

13 Hint: Let $a°$ be the third angle.

14 Hint: Let $a°$ be the size of each angle.

15 a alternate to $\angle ABC$ in parallel lines
  b supplementary, cointerior angles in parallel lines
  c $a + b + c = 180$, angles in a triangle add to 180°.

16 a $a = 30$, $b = 60$, $c = 60$
  b $a + c = 90$
  c $a = 60$, $b = 120$, $c = 30$, $a + c = 90$
  d $a = 16$, $b = 32$, $c = 74$, $a + c = 90$
  e $a + c = 90$
  f i $a = x$, $b = 2x$, $c = 90 - x$
  ii 90

Exercise 6D

1 a non-convex **b** non-convex **c** convex

2

Trapezium	Kite	Parallelogram	Rectangle	Rhombus	Square
		YES	YES	YES	YES
				YES	YES
			YES		YES
		YES	YES	YES	YES
			YES		YES
		YES	YES	YES	YES
				YES	YES
				YES	YES

3 a $a = 104$, $b = 76$
  b $a = 72$, $b = 72$
  c $a = 128$

4 a 90 **b** 61 **c** 105
  d 170 **e** 70 **f** 70

5 a $a = 100$, $b = 3$, $c = 110$
  b $a = 2$, $b = 90$
  c $a = 5$, $b = 70$

6 a 152 **b** 69 **c** 145
  d 74 **e** 59 **f** 30

7 a 19 **b** 60 **c** 36

8 a square, rectangle, rhombus and parallelogram
  b square, rhombus

9 a true **b** false **c** true
  d true **e** false **f** true

10 No, the two reflex angles would sum to more than 360° on their own.

11 a angle sum $= a + b + c + d + e + f = 180 + 180$ (angle sum of a triangle) $= 360°$
  b $\angle ADC = a + b + c = 360°$ (angle sum of a quadrilateral)

$$\begin{aligned}\text{Reflex } \angle ADC &= 360° - \angle ADC\\ &= 360° - (360° - (a + b + c))°\\ &= (a + b + c)°\end{aligned}$$

Exercise 6E

1 a 6 **b** 4 **c** 10
  d 7 **e** 5 **f** 12

2 a 720° **b** 1440° **c** 3600°

3 a square
  b equilateral triangle

4 a 108° **b** 144° **c** 135°

5 a 720° **b** 1260° **c** 900°
  d 2340° **e** 7740° **f** 18 000°

6 a 130 **b** 155 **c** 105
  d 250 **e** 40 **f** 265

7 a 108° **b** 128.6° **c** 147.3° **d** 168.75°

8 a 9 **b** 15 **c** 21 **d** 167

9 a 127.5 **b** 240 **c** 60
  d 60 **e** 79 **f** 72

10 a circle
  b Increases to infinity.
  c 180°

11 a $\dfrac{S}{n}$ **b** $\dfrac{180(n - 2)}{n}$
  c i 150° **ii** 175.61°

12 a 6 **b** 20 **c** 11

13 a 150 **b** 130 **c** 270

Exercise 6F

1 a 4 ways **b** 2 ways **c** 3 ways
  d 1 way **e** 2 ways **f** 0 ways

2 a 4 **b** 2 **c** 3
  d 1 **e** 2 **f** 2

3 a 4 and 4 **b** 2 and 2 **c** 2 and 2
  d 1 and 1 **e** 1 and 1 **f** 0 and 2

4 a equilateral **b** isosceles **c** scalene

5 a i kite **ii** rectangle, rhombus
  iii none **iv** square
  b i trapezium, kite
  ii rectangle, rhombus, parallelogram
  iii none
  iv square

6 a 3 and 3 **b** 0 and 3 **c** 0 and 2 **d** 0 and 4

7 a A, B, C, D, E, K, M, T, U, V, W, Y
  b H, I, O, X
  c H, I, O, S, X, Z

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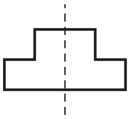
8 a



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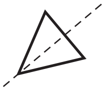

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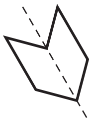
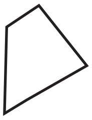


9 a



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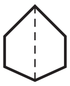

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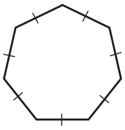

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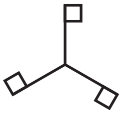

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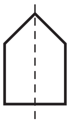
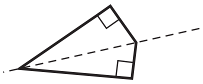


10 a 4 and 4 **b** 3 and 3 **c** 0 and 2

11 a



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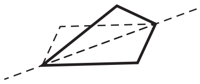
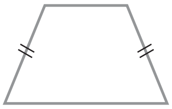


12 a **i** no **ii** no

b isosceles trapezium



13 a 9 **b** 3 **c** 4

d 1 **e** infinite **f** infinite

Exercise 6G

1 a vertices **b** seven **c** congruent

d seven **e** octagonal

2 a 24 **b** 7 **c** 8

3 a 6, 8, 12 **b** 5, 6, 9 **c** 7, 7, 12

4 A, cube; B, pyramid; F, rectangular prism; G, tetrahedron; H, hexahedron

5 a hexahedron **b** tetrahedron

c pentahedron **d** heptahedron

e nonahedron **f** decahedron

g undecahedron **h** dodecahedron

6 a 8 **b** 6 **c** 4 **d** 5

e 7 **f** 9 **g** 10 **h** 11

7 a triangular prism **b** pentagonal prism

c square prism

8 a rectangular pyramid **b** heptagonal pyramid

c triangular pyramid

9 a

Solid	Faces (F)	Vertices (V)	Edges (E)	$F + V$
cube	6	8	12	14
square pyramid	5	5	8	10
tetrahedron	4	4	6	8
octahedron	8	6	12	14

b $F + V$ is 2 more than E.

10

Faces (F)	Vertices (V)	Edges (E)
6	8	12
5	4	8
5	6	9
7	7	12
4	4	6
11	11	20

11 a 26 **b** 11 **c** 28

12 a true **b** false **c** true **d** true

e false (sphere) **f** true **g** false

13 a yes **b** yes **c** no **d** yes **e** yes

14 $F = 12$, $V = 12$ and $E = 24$ so Euler's rule does not apply.

15 a hexahedron, rectangular prism

b undecahedron, decagonal pyramid

16 a $V = E - F + 2$ **b** $F = E - V + 2$

17 a $V = 8$, $E = 14$, $F = 8$ and $V + F - 2 = E$

b $V = 16$, $E = 24$, $F = 10$ and $V + F - 2 = E$

c $V = 6$, $E = 12$, $F = 8$ and $V + F - 2 = E$

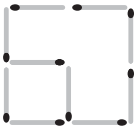
18 a true **b** true

19 a **i** convex **ii** non-convex **iii** non-convex

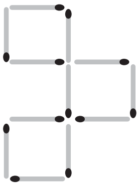
b Answers will vary.

Puzzles and challenges

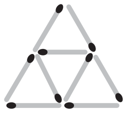
1 a



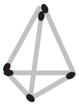
b



2 a



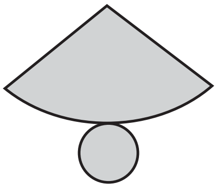
b



3 a 30 **b** 150

4 125°

5 a yes



Net

b no

6 180. Find each angle in the interior pentagon in terms of a, b, c, d and/or e. Then solve the sum equal to 540° for a pentagon.

Multiple-choice questions

1 D **2** A **3** E **4** C **5** C

6 B **7** B **8** C **9** D **10** C

Short-answer questions

1 **a** 50 **b** 65 **c** 240
d 36 **e** 61 **f** 138

2 **a** 132 **b** 99 **c** 77
d 51 **e** 146 **f** 41

3 95°

4 **a** obtuse-angled scalene, $a = 35$
b acute-angled, isosceles, $a = 30$
c acute-angled, equilateral, $a = 60$
d right-angled, scalene, $a = 19$
e acute-angled, scalene, $a = 27$
f obtuse-angled, scalene, $a = 132$

5 **a** 67 **b** 141 **c** 105

6 **a** $a = 98, b = 82$ **b** $a = 85, b = 106$
c $a = 231, b = 129$

7 **a** 2 **b** 0 **c** 4 **d** 1

8 **a** 2 **b** 2 **c** 4 **d** 3

9 **a** 71 **b** 25 **c** 120

10 40

Extended-response questions

1 $\angle BAE = \angle DCE = 70°$ (alternate angles in parallel lines)
$a + 70 + 50 = 180$ (angle sum of $\triangle ABE$)
$a = 60$

2 $\angle BDE = 180° - 30°$ (angles on a straight line)
$= 50°$
$\angle BED = \angle BDE = 50°$ (equal angles are opposite equal sides)
$\angle ABE + \angle BED = 180°$ (cointerior angles in parallel lines)
$\angle ABE + 50° = 180°$
$\angle ABE = 130°$

3 $ED = AB = 5$ (opposite sides of a rectangle)
$ED + DC = 13$
$5 + DC = 13$
$DC = 8$
$AE = BD = 8$ (opposite sides of a rectangle)
$\therefore \triangle BDC =$ isosceles
$\therefore \angle DBC = a°$ (equal angles are opposite equal sides)
Also $\angle BDE = 90°$ (angle in rectangle)
$\therefore \angle BDC = 90°$ (EC is a straight line)
$a + a + 90 = 180$ (angle sum of $\triangle BDC$)
$2a = 90$
$a = 45$

Chapter 7

Pre-test

1 **a** George **b** Amanda

2 **a** 1 **b** 1 **c** 0 **d** −3

3 **a** 6 **b** −1 **c** 0 **d** 2

4 **a** (1, 2) **b** (2, 1) **c** (3, 2)

5 **a** 7 **b** 4 **c** 2

6 **a** −7 **b** −3 **c** 3

7 **a**

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

b

x	−2	−1	0	1	2
y	6	5	4	3	2

8 **a** $y = 3x$ **b** $y = 2x + 1$
c $y = 3x - 2$ **d** $y = 5 - x$

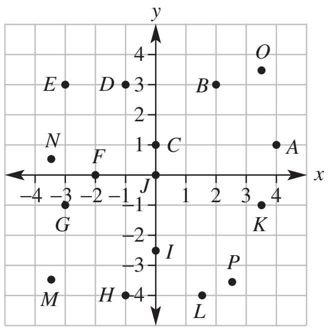
Exercise 7A

1 **a** (0, 0) **b** y
c 1st **d** 3rd
e −2 **f** −5

2 **a** 3 **b** −1 **c** −2 **d** 0
e −2 **f** 0 **g** −3 **h** 0

3 $A(1, 1), B(5, 0), C(3, 4), D(0, 4), E(-1, 2), F(-3, 3), G(-5, 1), H(-3, 0), I(-4, -2), J(-2, -5), K(0, -3), L(2, -3), M(5, -5)$

4



5 **a** house **b** fish

6 **a** B **b** C **c** E **d** D

7 **a** triangle **b** rectangle
c parallelogram **d** kite

8 **a** (2, 4) **b** (−5, 2) **c** (−1, −2.5)
d (4.5, −4.5) **e** (0, 1) **f** (−2, 1)

9 **a** (1, −2), (1, −1), (1, 0), (1, 1)
b (−1, 0), (0, 0), (1, 0), (2, 0)
c (−2, 3), (−1, 2), (0, 1), (1, 0)
d (−2, −3), (−1, 0), (0, 3), (1, 6), (2, 9)

10 **a** quadrant 4 **b** quadrant 2
c quadrants 2 and 3 **d** quadrants 3 and 4

11 a line on the y-axis

12 **a** 10 **b** 4 **c** 7 **d** 11
e 6 **f** 4

13 **a** 5 **b** 13 **c** 25 **d** $\sqrt{13}$
e $\sqrt{90}$ **f** $\sqrt{61}$

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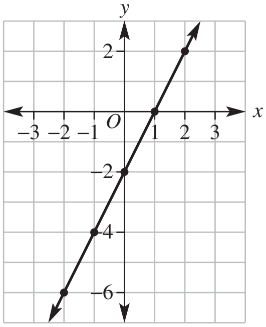
Exercise 7B

1 **a** 5 **b** 7 **c** 3 **d** 1
e −7 **f** −11 **g** 25 **h** −21

2 **a** 2 **b** −2 **c** −13

3



4 **a**

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

b

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1

c

x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3

d

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

e

x	−3	−2	−1	0	1	2	3
y	9	7	5	3	1	−1	−3

f

x	−3	−2	−1	0	1	2	3
y	8	5	2	−1	−4	−7	−10

g

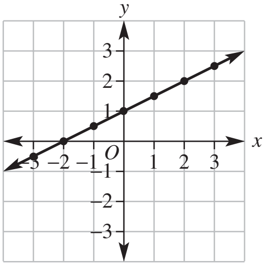
x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

h

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

5 **a** **i** yes **ii** no
b **i** no **ii** yes
c **i** no **ii** no
d **i** yes **ii** yes
e **i** yes **ii** no
f **i** yes **ii** yes
g **i** no **ii** yes

6 **a**



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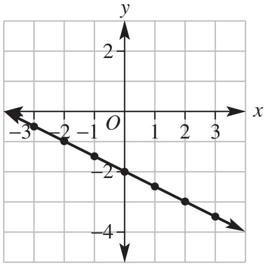

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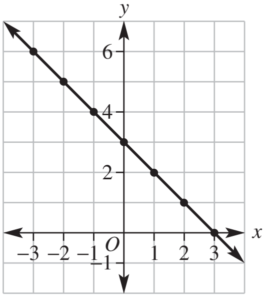

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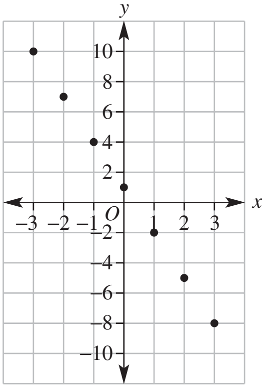
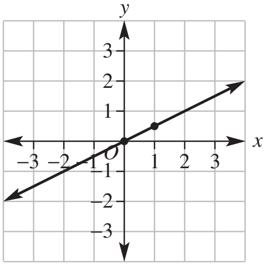


7 **a** (−1, 0), (0, 1) **b** (2, 0), (0, 2)
c (−2, 0), (0, 4) **d** (2, 0), (0, 10)
e (1.5, 0), (0, −3) **f** $\left(\frac{7}{3}, 0\right)$, (0, 7)

8 (1, 3)

9 **a** 2
b **i**



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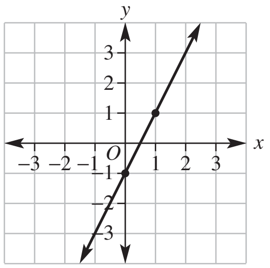
ii



10 a When $x = 0$, $y = 0$ for all rules.

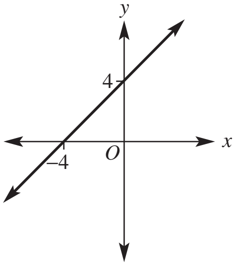
b All the rules have a constant so when $x = 0, y \neq 0$.

11 a The coefficient of x is positive.

b The coefficient of x is negative.

c the coefficient of x

12 a



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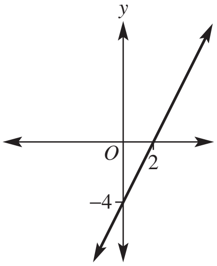

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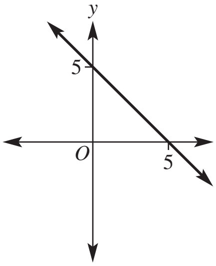

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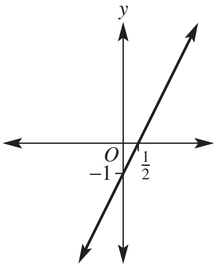

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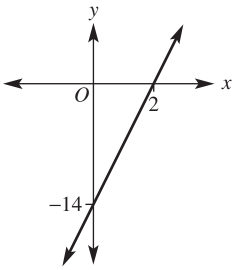

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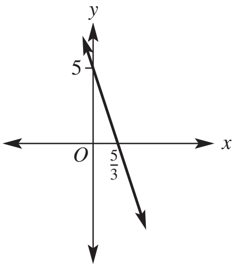
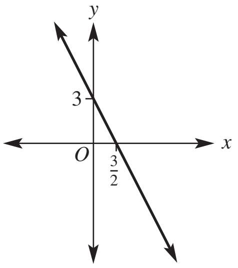
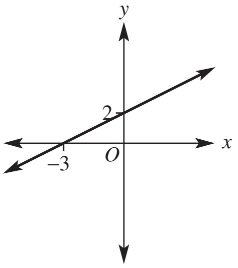


g



h



Exercise 7C

1 a 2 **b** −1 **c** 2 **d** −3

2 a 3 **b** 1 **c** 1 **d** 0

3 a 2 **b** −2 **c** −3

d −1 **e** −10 **f** −2

g 0 **h** 3 **i** 4

4 a $y = 2x + 4$ **b** $y = 3x - 1$

c $y = -x + 1$ **d** $y = -2x + 6$

5 a $y = 4x + 1$ **b** $y = 2x - 3$

c $y = -2x - 2$ **d** $y = -x + 4$

6 a $y = x + 1$. To find a y value start with x and then add 1.

b $y = 2x - 2$. To find a y value start with x, multiply by 2 and then subtract 2.

c $y = -3x + 2$. To find a y value start with x, multiply by −3 and then add 2.

d $y = -x$. To find a y value multiply x by −1.

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7 a $y = x + 2$ **b** $y = x - 4$
c $y = 2x - 1$ **d** $y = -x + 1$

8 a $y = 3x + 1$. To find the number of matchsticks, multiply the number of squares by 3 and then add 1.
b $y = 5x + 1$. To find the number of matchsticks, multiply the number of hexagons by 5 and then add 1.
c $y = 2x + 4$. To find the number of matchsticks, multiply the number in the top row by 2 and then add 4.

9 $y = 8x - 2$

10 $y = -x + 1$

11 a $y = 5 - 2x$ **b** $y = 7 - 3x$
c $y = 4 - x$ **d** $y = 10 - 4x$

12 a $b - 2$ **b** $y = (b - 2)x + 2$

13 a $b - a$ **b** $y = (b - a)x + a$

14 a 5 extra matchsticks are needed for each new shape and 1 matchstick is needed for the first hexagon, so the rule is $y = 5x + 1$.
b 2 extra matchsticks are needed for each new shape and 4 matchsticks are needed for the sides, so the rule is $y = 2x + 4$.

15 a x is not increasing by 1. **b** 1 **c** $y = x - 2$
d i $y = 2x + 3$ **ii** $y = 3x - 1$
iii $y = -2x + 3$ **iv** $y = -4x - 20$

Exercise 7D

1 a positive **b** negative **c** negative **d** positive

2 a positive **b** undefined **c** zero **d** negative

3 a 2 **b** 3 **c** $\frac{3}{2}$ **d** 2
e -2 **f** -5 **g** $-\frac{8}{3}$ **h** $-\frac{5}{3}$

4 a 2 **b** $\frac{2}{3}$ **c** -4 **d** $-\frac{2}{5}$
e $-\frac{3}{2}$ **f** $-\frac{1}{2}$ **g** $\frac{1}{2}$ **h** 3

5 a 3 **b** 1 **c** $\frac{1}{2}$
d 3 **e** $\frac{2}{3}$ **f** 4

6 a -2 **b** $-\frac{3}{5}$ **c** $-\frac{4}{3}$
d -1 **e** -3 **f** $-\frac{3}{2}$

7 grassy slope

8 torpedo

9 a $\frac{3}{4}$ **b** $\frac{5}{2}$ **c** $-\frac{3}{2}$ **d** $-\frac{7}{10}$

10 a $\frac{5}{2}$ **b** $\frac{5}{3}$ **c** $-\frac{8}{3}$
d $-\frac{2}{3}$ **e** $\frac{8}{3}$ **f** $-\frac{3}{10}$

11 Answers may vary. Examples:
a (1, 3), (2, 6), (3, 9) **b** (−1, −3), (−2, −6), (−3, −9)

12 a 2 **b** 10 **c** $b = 2a$ **d** $a = \frac{b}{2}$

13 a $-\frac{1}{2}$ **b** $-\frac{3}{2}$ **c** $b = -\frac{a}{2}$ **d** $a = -2b$

14 a $\frac{2}{3}$ **b** $-\frac{2}{7}$ **c** $\frac{8}{9}$ **d** $\frac{4}{3}$
e $-\frac{3}{7}$ **f** 4 **g** $-\frac{8}{9}$ **h** -1
i $-\frac{2}{7}$ **j** $-\frac{8}{33}$ **k** $-\frac{18}{25}$

Exercise 7E

1 a $y = 2x + 3$ **b** $y = 4x + 2$ **c** $y = -3x + 1$
d $y = -5x - 3$ **e** $y = 2x - 5$ **f** $y = -10x - 11$

2 a $b = 1, m = 2$ **b** $b = 0, m = 1$ **c** $b = -1, m = -1$
d $b = 3, m = -1$ **e** $b = 0, m = -3$ **f** $b = 3, m = 3$

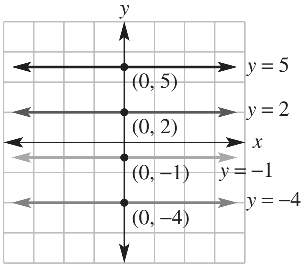
3 a $m = 4, b = 2$ **b** $m = 3, b = 7$
c $m = \frac{1}{2}, b = 1$ **d** $m = \frac{2}{3}, b = \frac{1}{2}$
e $m = -2, b = 3$ **f** $m = -4, b = 4$
g $m = -7, b = -1$ **h** $m = -3, b = -7$
i $m = -1, b = -6$ **j** $m = -\frac{1}{2}, b = 5$
k $m = -\frac{2}{3}, b = -\frac{1}{2}$ **l** $m = \frac{3}{7}, b = -\frac{4}{5}$

4 a $y = 2x - 1$ **b** $y = x - 2$ **c** $y = 3x + 3$
d $y = x + 5$ **e** $y = 2x + 1$ **f** $y = 3x - 1$

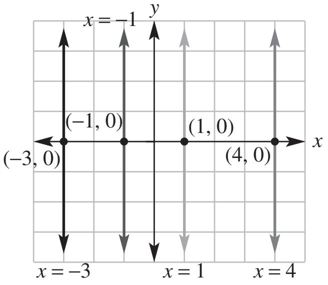
5 a $y = -x + 2$ **b** $y = -2x + 4$ **c** $y = -3x - 1$
d $y = -2x + 3$ **e** $y = -5x - 2$ **f** $y = -x + 6$

6 a $y = 4$ **b** $y = 1$ **c** $y = -3$
d $x = -4$ **e** $x = 5$ **f** $x = -2$

7 a, b, c, d



e, f, g, h



8 a $y = \frac{2}{5}x + 2$ **b** $y = -\frac{1}{4}x + \frac{1}{2}$ **c** $y = \frac{1}{3}x - \frac{3}{4}$

9 a $y = 3x$ **b** $y = -5x$
c $y = -x + 3$ **d** $y = \frac{5}{3}x - 4$

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10 a $y = 2x + 3$ **b** $y = -\frac{1}{2}x + 5$

c $y = -x + 2$ **d** $y = 2x + 10$

11 a 15 square units **b** 40 square units

12 a It is a horizontal line passing through $(0, 0)$.

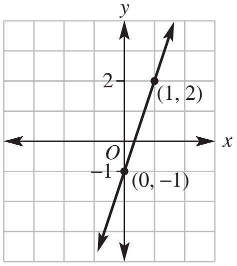
b It is a vertical line passing through $(0, 0)$.

13 a $y = \frac{b}{a}x$ **b** $y = -x$ **c** $y = -\frac{a}{b}x + a$

14 a $m = -3, b = 5$ **b** $m = 2, b = 3$

c $m = \frac{3}{2}, b = 4$ **d** $m = \frac{1}{2}, b = 3$

15 a



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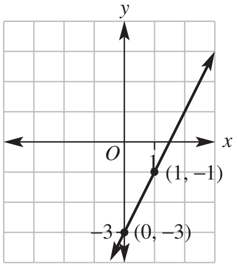

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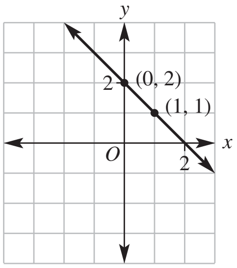

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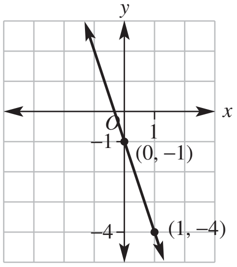

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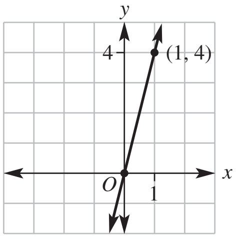

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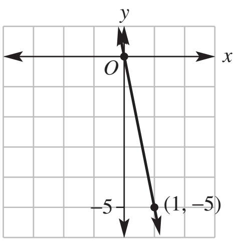
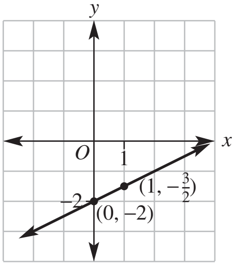
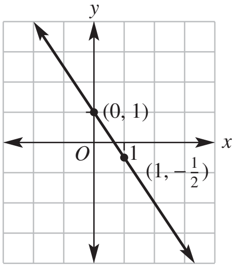


g



h



Exercise 7F

1 a 2 **b** −5 **c** 7

2 a −1 **b** 4 **c** −2 **d** 0

3 a 4 **b** −2 **c** −5 **d** −5

e 4 **f** −7 **g** $-\frac{1}{3}$ **h** $-\frac{2}{5}$

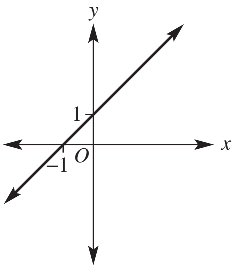
i −2 **j** −4 **k** 2 **l** 45

4 a 1 **b** 6 **c** −2 **d** 4

e 3 **f** −2 **g** −10 **h** 2

i 2 **j** −2 **k** −7 **l** 11

5 a



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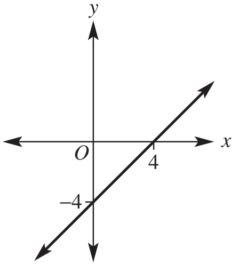

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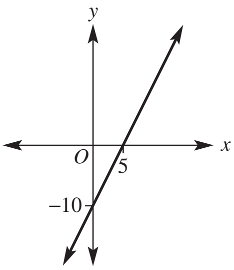
c



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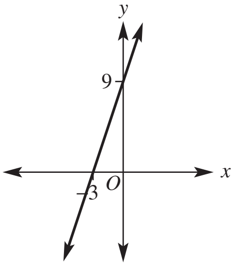

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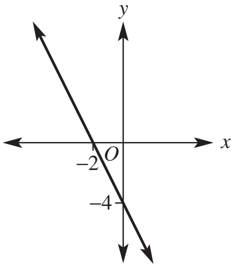

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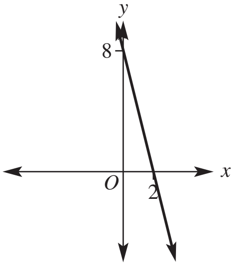

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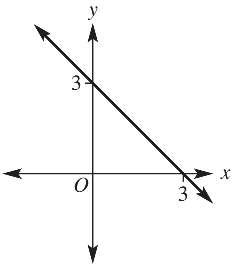

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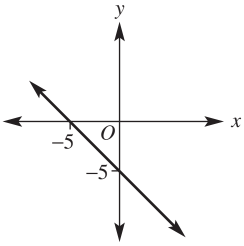
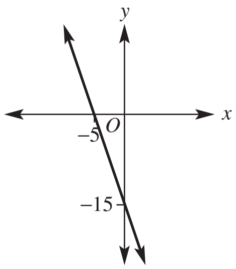


i



6 **a** 4 square units **b** 1.5 square units
c 12.5 square units **d** 8 square units

7 **a** $y = x + 4$ **b** $y = \frac{1}{2}x - 1$
c $y = 3x + 6$ **d** $y = -5x + 25$

8 **a** 20 cm **b** 10 seconds

9 **a** 200 cents **b** 200 seconds

10 horizontal lines; e.g. $y = 2$, $y = -5$

11 **a** negative **b** positive **c** negative **d** positive

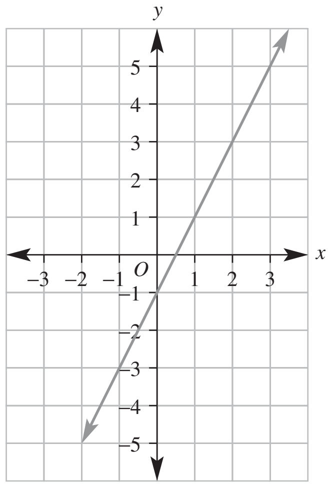
12 $x = -\frac{b}{m}$

13 **a** 6 **b** 4 **c** −2
d 3 **e** $-\frac{2}{3}$ **f** $\frac{3}{2}$
g −2 **h** $\frac{3}{2}$ **i** $\frac{1}{3}$

Exercise 7G

1

x	−2	−1	0	1	2
y	−5	−3	−1	1	3



2 **i** $x = 5$ **ii** $x = -1$

3 **a** (2, 4) **b** (3.2, 6, 4)
c (−2.3, −4, 6) **d** (3.5, 7)
e (−7, −14) **f** (1000, 2000)
g (31.42, 62.84) **h** (−24.301, −48.602)
i $\left(\frac{\textit{any number}}{2}, \textit{any number}\right)$

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4 a $x = 2$ **b** $x = 0.5$ **c** $x = 3$

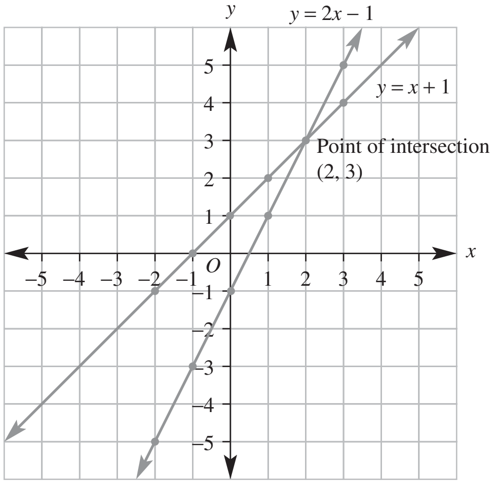
d $x = -2.5$ **e** $x = -1.5$

5 a $x = -2.5$ **b** $x = 3$ **c** $x = -0.5$

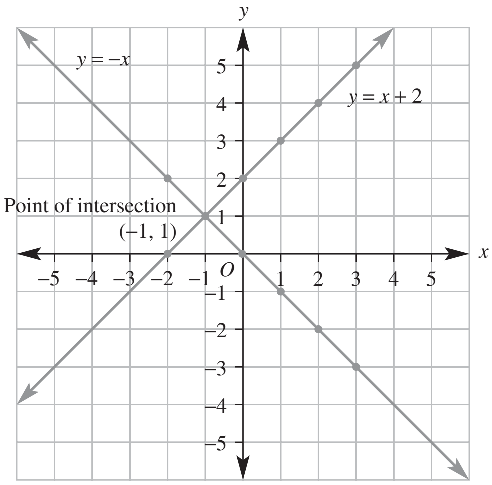
d $x = 4$ **e** $x = 5$

6 a (4, 3) **b** (−2, −3)

7 a



b



8 a $x = 3.67$ **b** $x = -1.53$ **c** $x = 5.30$

9 a (4.78, 1.78) **b** (−1.33, 3.41)

10 a $x + 2 = 1$ and $5 - 2x = 7$

b Any point that lies on the line is correct, e.g. (−2, 9)(0, 5)(1, 3)(2, 1)

c Any point that lies on the line is correct, e.g. (−2, 0)(0, 2)(1, 3)(3, 5)

d (1, 3)

$y = x + 2$ $y = 5 - 2x$
$3 = 1 + 2$ $3 = 5 - 2 \times 1$
$3 = 3$ True $3 = 3$ True

e $x = 1$

11 a $A = 10 + 8n$ applies to Ruby as she has \$10 to start with and adds to her savings by \$8 times the number (n) of hours worked. $A = 24 + 6n$ applies to Jayden as he has \$24 to start with and increases his savings by \$6 times the number (n) of hours worked.

b i $n = 4$ **ii** $n = 4$ **iii** $n = 7$

iv $n = 7$ **v** $n = 11$ **vi** $n = 11$

c Many solutions, e.g. (2, 26)(4, 42)(7, 66)(9, 82)(11, 98)

d Many solutions, e.g. (2, 36)(4, 48)(7, 66)(9, 78)(11, 90)

e (7, 66) (7, 66)

$A = 10 + 8n$ $A = 24 + 6n$
$66 = 10 + 8 \times 7$ $66 = 24 + 6 \times 7$
$66 = 10 + 56$ $66 = 24 + 42$
$66 = 66$ True $66 = 66$ True

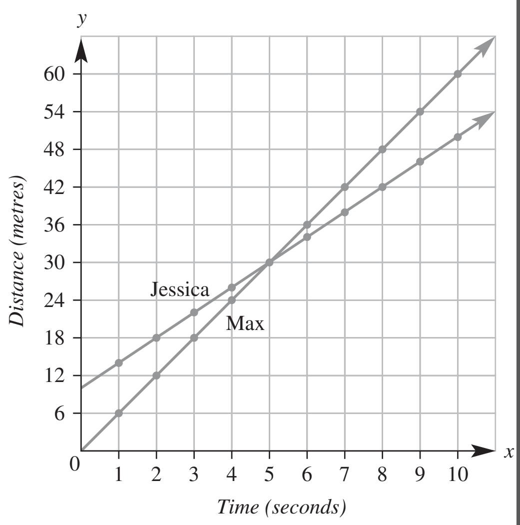
f $n = 7$

g Ruby and Jayden have both worked 7 hours and both have \$66 saved.

12 a

Time, in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance, in metres	0	6	12	18	24	30	36	42	48	54	60
Jessica's distance, in metres	10	14	18	22	26	30	34	38	42	46	50

b



c $d = 6t$

d i $6t = 18$ **ii** $6t = 30$ **iii** $6t = 48$

e $d = 10 + 4t$

f i $10 + 4t = 22$ **ii** $10 + 4t = 30$ **iii** $10 + 4t = 42$

g (5, 30) (5, 30)

$d = 6t$ $d = 10 + 4t$
$30 = 6 \times 5$ $30 = 10 + 4 \times 5$
$30 = 30$ True $30 = 30$ True

h Max catches up to Jessica. They are both 30 m from the starting line and have each run for 5 seconds.

13 a $2x - 3 = -12, x = -4.5$

b $2x - 3 = 2, x = 2.5$

c $2x - 3 = 6, x = 4.5$

d $2x - 3 = 5, x = 4$

e $2x - 3 = 9, x = 6$

f $2x - 3 = -6, x = -1.5$

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14 a i $(-2, 17)(-1, 14)(0, 11)(1, 8)(2, 5)(3, 2)(4, -1)(5, -4)$

ii $(-2, -3)(-1, -1)(0, 1)(1, 3)(2, 5)(3, 7)(4, 9)(5, 11)$

b $(2, 5)$

$y = 11 - 3x$	$y = 2x + 1$
$5 = 11 - 3 \times 2$	$5 = 2 \times 2 + 1$
$5 = 11 - 6$	$5 = 4 + 1$
$5 = 5$ True	$5 = 5$ True

c It is the only shared point.

15 a i Any answer with y as a whole number is correct, e.g. $2x - 1 = -3$, $2x - 1 = -1$, $2x - 1 = 3$

ii No. Many possible correct examples, e.g. $2x - 1 = 2.5$, $2x - 1 = -1.75$, $2x - 1 = 2.8$

b i Any answer with the y-value to one decimal place is correct, e.g. $2x - 1 = -2.6$, $2x - 1 = -1.8$, $2x - 1 = 0.7$

ii No. Many possible correct answers, e.g. $2x - 1 = 0.42$, $2x - 1 = -1.68$, $2x - 1 = 2.88$

c i $2x - 1 = 2.04$, $2x - 1 = 2.06$

ii Many possible correct answers, e.g. $(1.521, 2.042)$ $(1.529, 2.058)$; $2x - 1 = 2.042$, $2x - 1 = 2.058$

iii Yes, for every two points on a line another point can be found in between them, so there are an infinite number of points on a line. Also an infinite number of equations can be solved from the points on a straight line if the graph has a suitable scale (digitally possible).

16 a i $x = 2, x = -2$ **ii** $x = 3, x = -3$

iii $x = 4, x = -4$ **iv** $x = 5, x = -5$

b For each y-coordinate there are two different points, so two different solutions.

c i $x = 2.24, x = -2.24$ **ii** $x = 2.61, x = -2.61$

iii $x = 0.7, x = -0.7$ **iv** $x = 3.57, x = -3.57$

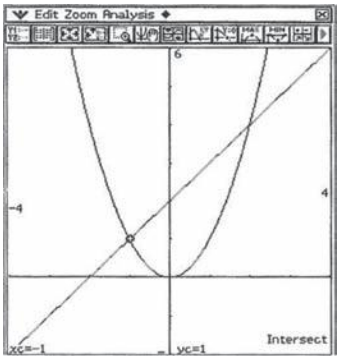
v $x = 4.34, x = -4.34$

d The graph of $y = x^2$ does not include a point where $y = -9$.

e Many correct answers all with x^2 equal to a negative number, e.g. $x^2 = -5, x^2 = -10, x^2 = -20$.

f Positive numbers or zero.

g $x = -1, x = 2$



h i $x = -2.77, x = 5.77$

ii $x = -8.27, x = 3.27$

iii no solution

iv $x = 3$

Exercise 7H

1 a 15 **b** 45 **c** 5 **d** 125

2 a $m = 3, b = 2$ **b** $m = -2, b = -1$

c $m = 4, b = 2$ **d** $m = -10, b = -3$

e $m = -20, b = 100$ **f** $m = 5, b = -2$

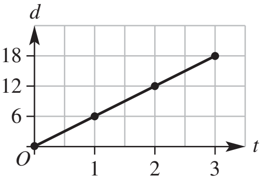
3 a 60 cm **b** 150 cm **c** 330 cm

4 a 28 L **b** 24 L **c** 10 L

5 a

t	0	1	2	3
d	0	6	12	18

b

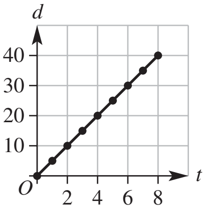


c $d = 6t$ **d** 9 km **e** 2 hours

6 a

t	0	1	2	3	4	5	6	7	8
d	0	5	10	15	20	25	30	35	40

b

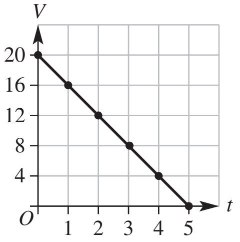


c $d = 5t$ **d** 22.5 km **e** 4 hours

7 a

t	0	1	2	3	4	5
v	20	16	12	8	4	0

b

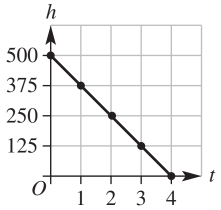


c $V = -4t + 20$ **d** 11.2 L **e** 3 seconds

8 a

t	0	1	2	3	4
h	500	375	250	125	0

b



c $h = -125t + 500$ **d** 275 m **e** 3 minutes

9 a $M = -0.5t + 3.5$ **b** 7 hours

c 4.5 hours

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10 a $d = 15t$ **b** 3 hours

c 3 hours 20 minutes

11 2 days 15 hours

12 a 2000 L

b Decreasing; it has a negative gradient.

c 300 L/h

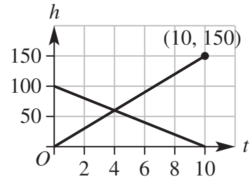
13 a Using cents, $m = \frac{1}{2}$ and $b = 10$.

b Using dollars, $m = 0.005$ and $b = 0.1$.

14 a

t	0	1	2	3	4	5	6	7	8	9	10
h_1	0	15	30	45	60	75	90	105	120	135	150
h_2	100	90	80	70	60	50	40	30	20	10	0

b



c 10 seconds

d $h = 15t, h = -10t + 100$

e at 4 seconds

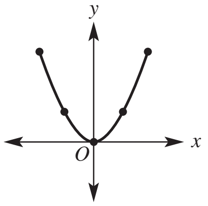
f at 2.5 seconds

g at 3.5 seconds

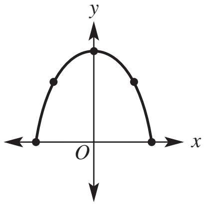
h no, 60 m, 60 m and 50 m

Exercise 7I

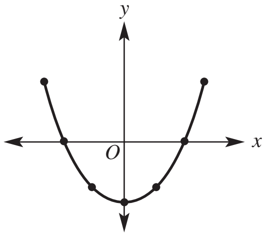
1 a



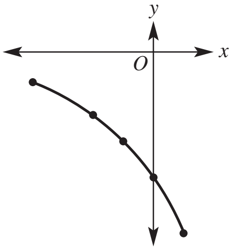
b



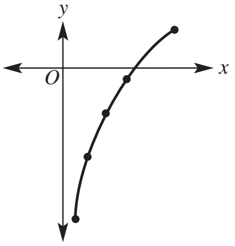
c



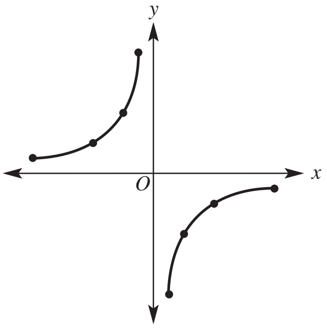
d



e



f

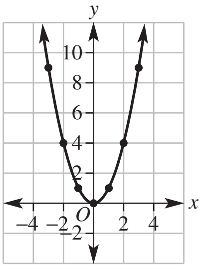


2 a −1 **b** 8 **c** 3 **d** 15

3 a 2 **b** 4 **c** 8 **d** 16

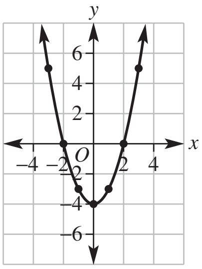
4 a $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



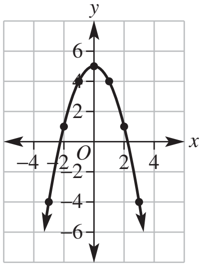
b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5



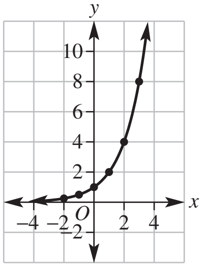
c $y = 5 - x^2$

x	−3	−2	−1	0	1	2	3
y	−4	1	4	5	4	1	−4



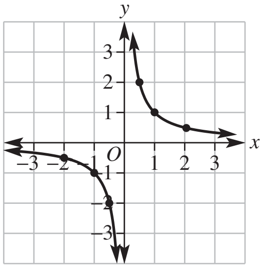
d $y = 2^x$

x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



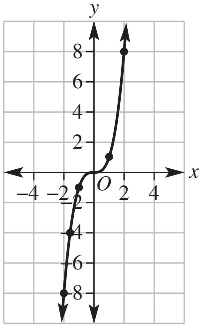
e $y = \frac{1}{x}$

x	−2	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	−1	−2	2	1	$\frac{1}{2}$



f $y = x^3$

x	−2	−1	0	1	2
y	−8	−1	0	1	8



5 **a** $y = x^2 - 2$ **b** $y = 3 - x^2$ **c** $y = x^3$

d $y = 3^x$ **e** $y = \sqrt{x}$ **f** $y = \frac{1}{x}$

6 **a** 0 and 9 **b** −10 and −1 **c** −3 and 6

d −27 and 27 **e** −28 and 26 **f** −64 and 8

7 **a** parabola **b** line **c** hyperbola

d exponential **e** hyperbola **f** parabola

g line **h** exponential

8 **a** Upright parabolas, as *a* increases the graphs become narrower.

b Inverted (upside down) parabolas, as *a* increases the graphs become narrower.

c Parabolas, as *a* increases the graphs shift up.

d Parabolas, as *a* increases the graphs shift right.

Puzzles and challenges

1 3 hours

2 **a** $y = x^2 - 3$ **b** $y = 10 - x^2$

c $y = \sqrt{x} + 1$ **d** $y = x^3 - 3$

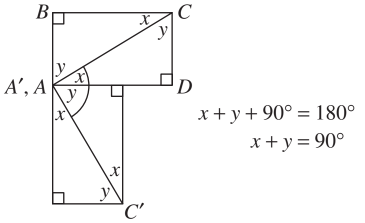
3 $-\frac{1}{3}$ **4** 40 min

5 $\frac{14}{3}$ **6** $y = \frac{1}{2}x + \frac{1}{2}$

7 1588 **8** 4 hours 45 minutes

9 60 units^2

10 **a** Diagonal *AC* has been rotated about *A* by 90° clockwise, so the angle between *AC* and *AC*′ is 90°.



b $\frac{p}{q} \times \frac{-q}{p} = -1$

Multiple-choice questions

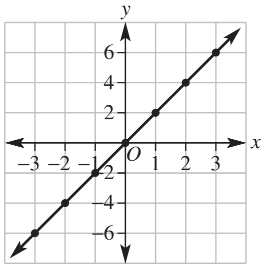
1 B **2** C **3** C **4** D **5** C

6 D **7** A **8** E **9** D **10** D

Short-answer questions

1 $A(2, 3), B(0, 2), C(-2, 4), D(-3, 1), E(-3, -3), F(-1, 0), G(0, -4), H(1, -2), I(4, -3), J(3, 0)$

2 **a**



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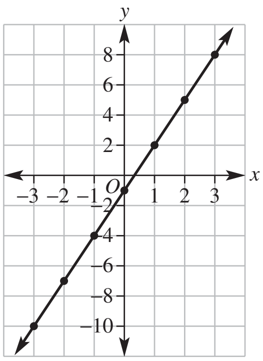
b



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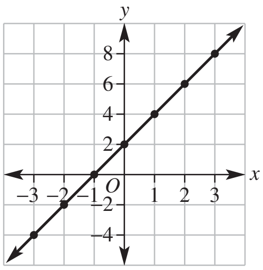
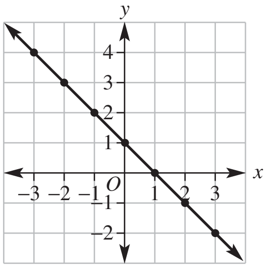
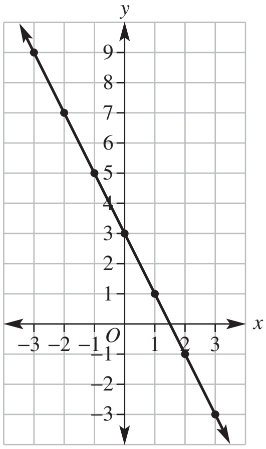


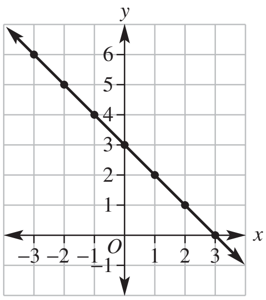
d



e



f



3 **a** $y = 2x + 1$ **b** $y = 3x + 2$ **c** $y = x + 3$
d $y = -x + 1$ **e** $y = -4x - 1$ **f** $y = -x + 2$

4 **a** $y = 3x$ **b** $y = \frac{x}{2}$ **c** $y = 2x - 2$
d $y = -x$ **e** $y = -4x + 4$ **f** $y = -\frac{3x}{2} + 3$

5 **a** $y = -4x$ **b** $-\frac{x}{2}$
c $y = x$ **d** $y = -2x - 7$

6 **a** $y = 2x + 1$ **b** $y = 3x$ **c** $y = x - 2$
d $y = -4x$ **e** $y = -2x - 4$ **f** $y = -\frac{1}{2}x + 1$

7 **a** multiply x by 3 and then add 5
b **i** multiply x by 6 and then add 2
ii multiply x by 2 and then add 6

8 **a**

T	1	2	3	4	5
C	4	6	8	10	12

b Multiply the number of tables by 2 and then add 2.
c $C = 2T + 2$
d 22

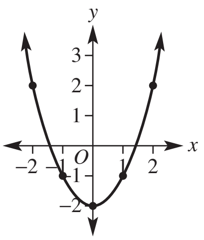
9 **a** **i** $x = -1$ **ii** $x = 1$ **iii** $x = 1$
iv $x = 3.5$ **v** $x = 4.5$
b (2, 1)
c

$y = 3 - x$	$y = 2x - 3$
$4 = 3 - -1$	$4 = 2 \times -1 - 3$
$4 = 3 + 1$	$4 = -2 - 3$
$4 = 4$ True	$4 = -5$ False
$(-1, 4)$ is a solution.	$(-1, 4)$ is not a solution..

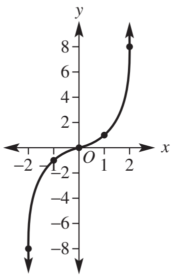
d

$y = 3 - x$	$y = 2x - 3$
$1 = 3 - 2$	$1 = 2 \times 2 - 3$
$1 = 1$ True	$1 = 1$ True

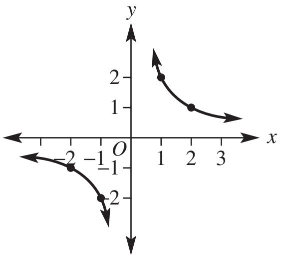
10 **a**



b



c



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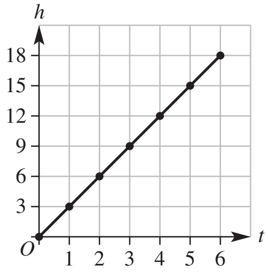
Extended-response questions

1 a

t	0	1	2	3	4	5	6
h	0	3	6	9	12	15	18

b

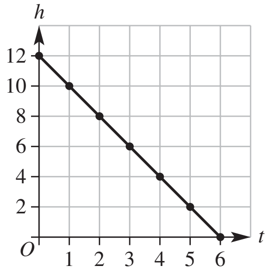


c $h = 3t$ **d** 10.5 mm **e** 30 mm **f** 5 days

2 a

t	0	1	2	3	4	5	6
h	12	10	8	6	4	2	0

b



c 6 minutes **d** −2

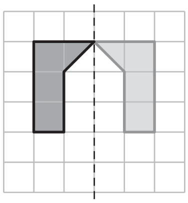
e $h = -2t + 12$ **f** 7 km

g 4 minutes 15 seconds

Chapter 8

Pre-test

1 a



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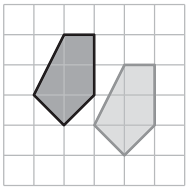

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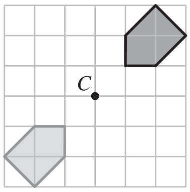


2 a i F **ii** H

b i $\angle A$ **ii** $\angle C$

c i GH **ii** FG

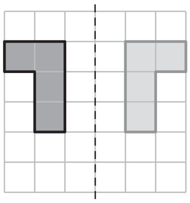
3 a 30° **b** 20° **c** 45° **d** 60°

4 a A, B, C, D **b** A, B, C, D, E

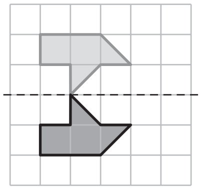
c A, B **d** A, C, E

Exercise 8A

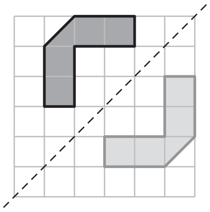
1 a



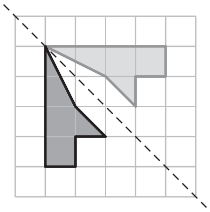
b



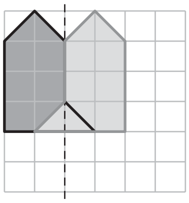
c



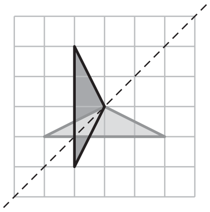
d



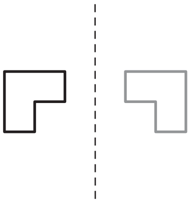
e



f



2 a



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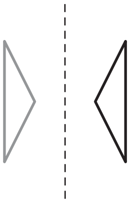

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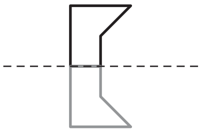

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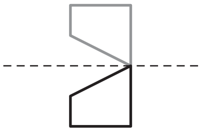
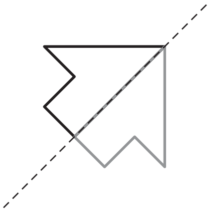
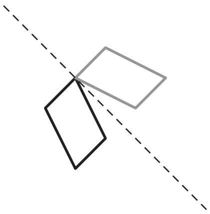


e



f



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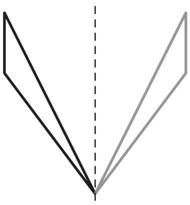

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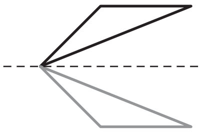

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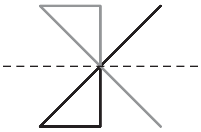

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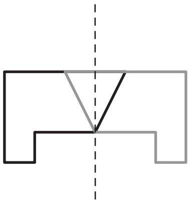

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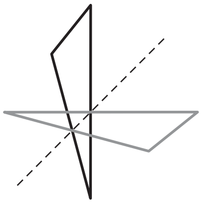
e



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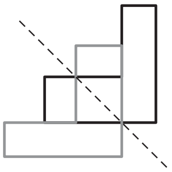
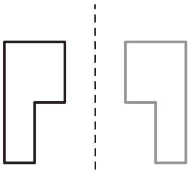


4 a



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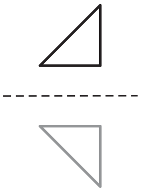

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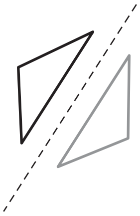

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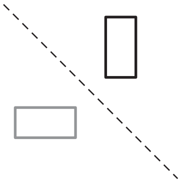

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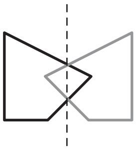

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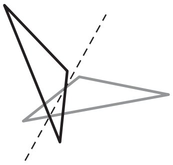


5 **a** $A'(2, 0)$, $B'(1, -3)$, $C'(4, -2)$

b $A'(-2, 0)$, $B'(-1, 3)$, $C'(-4, 2)$

6 **a** $A'(-1, 2)$, $B'(-4, 2)$, $C'(-4, 4)$, $D'(-1, 4)$

b $A'(1, -2)$, $B'(4, -2)$, $C'(4, -4)$, $D'(1, -4)$

7 **a** 4 **b** 2 **c** 2 **d** 1 **e** 0

f 0 **g** 1 **h** 3 **i** 8

8 **a** (0, 4) **b** (4, 4) **c** (−2, 4) **d** (−4, 4)

e (−10, 4) **f** (−42, 4) **g** (2, 2) **h** (2, −8)

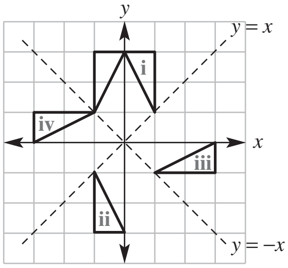
i (2, −4) **j** (2, −2) **k** (2, −14) **l** (2, −78)

9 10 m^2, the area is unchanged after reflection.

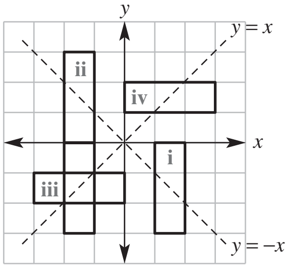
10 n

11 Reflection in the y-axis.

12 **a**



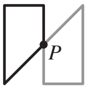
b



13 They are on the mirror line.

14 Answer may vary.

15 **a** **i**



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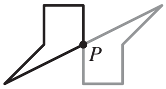
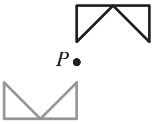


iii



b **i** yes **ii** no **iii** yes

c square, rectangle, rhombus, parallelogram

Exercise 8B

1 **a** right, up **b** left, up

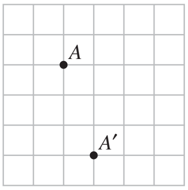
c right, down **d** left, down

2 **a** $\begin{bmatrix} 5 \\ -2 \end{bmatrix}$ **b** $\begin{bmatrix} -2 \\ -6 \end{bmatrix}$ **c** $\begin{bmatrix} -7 \\ 4 \end{bmatrix}$ **d** $\begin{bmatrix} 9 \\ 17 \end{bmatrix}$

3 **a** horizontal **b** vertical

c vertical **d** horizontal

4 **a**



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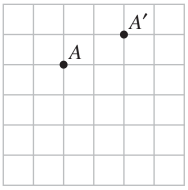

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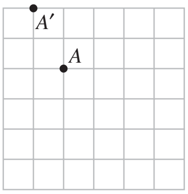

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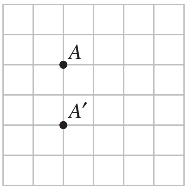
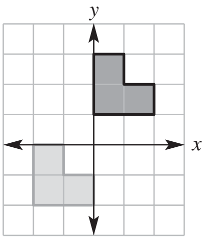


5 **a** $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ **b** $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ -2 \end{bmatrix}$ **d** $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$

e $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ **f** $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ **g** $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ **h** $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$

i $\begin{bmatrix} -20 \\ 8 \end{bmatrix}$ **j** $\begin{bmatrix} -9 \\ -25 \end{bmatrix}$

6 **a**



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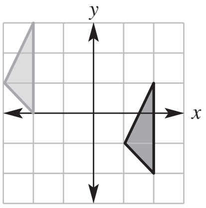

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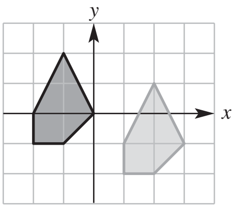

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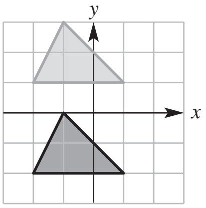

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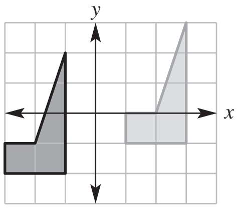
e



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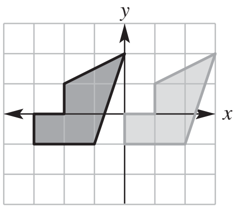


7 **a** (15, 2) **b** (21, −1) **c** (13, 6) **d** (9, 2)
e (11, −2) **f** (3, −6) **g** (11, −9) **h** (19, −10)
i (25, −4) **j** (−13, 13) **k** (9, 17) **l** (−8, −39)

8 **a** $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$ **b** $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

9 **a** 2.3 km (or 2300 m) **b** $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

10 **a** $x = -2, y = -2$ **b** $x = 6, y = -4$
c $x = -3, y = 2$ **d** $x = 28, y = -60$

11 **a** $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ **b** $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ **c** $\begin{bmatrix} -x \\ -y \end{bmatrix}$ **d** $\begin{bmatrix} x \\ y \end{bmatrix}$

12 **a** $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ **b** $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **c** $\begin{bmatrix} c \\ b - c \end{bmatrix}$

13 **a** $x \approx 4.47$ **b** just OK **c** 80 holes

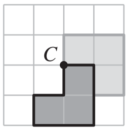
Exercise 8C

1 **a** anticlockwise, 90° **b** clockwise, 90°
c anticlockwise, 90° **d** clockwise, 90°
e anticlockwise, 180° **f** clockwise, 180°

2 **a** 270° **b** 180° **c** 302° **d** 64°

3 **a** 4 **b** 2 **c** 5 **d** 2
e 4 **f** 2

4 **a**



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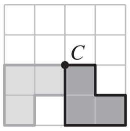

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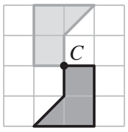

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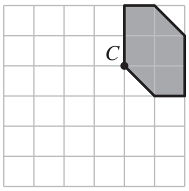

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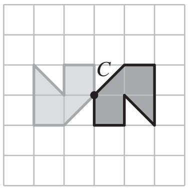

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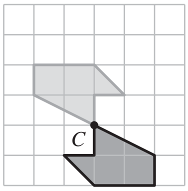
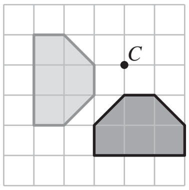
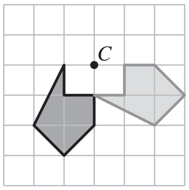


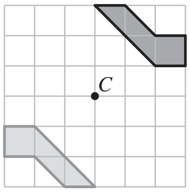
5 **a**



b



c



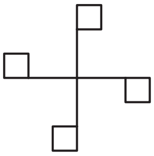
6 **a** (−4, −3) **b** (−4, −3) **c** (3, −4) **d** (−3, 4)
e (−3, 4) **f** (3, −4) **g** (4, 3)

7 **a** $A'(4, -4), B'(4, -1), C'(1, -1)$
b $A'(4, 4), B'(1, 4), C'(1, 1)$
c $A'(-4, -4), B'(-1, -4), C'(-1, -1)$

8 **a** 90° **b** 180° **c** 90°

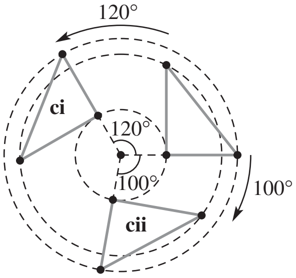
9 H, I, N, O, S, X and Z

10 Answers may vary. Examples are:
a parallelogram
b regular hexagon
c



11 **a** 180° **b** 90°

12



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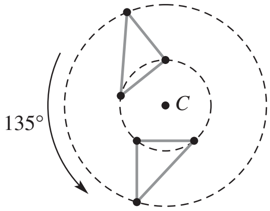

14 **a** use $x = 70°$ **b** use $x = 40°$

Exercise 8D

1 **a** yes
b **i** *D* **ii** *E* **iii** *F*
c **i** *DE* **ii** *EF* **iii** *DF*
d **i** ∠*E* **ii** ∠*F* **iii** ∠*D*

2 **a** yes
b **i** *D* **ii** *E* **iii** *F*
c **i** *DE* **ii** *EF* **iii** *DF*
d **i** ∠*E* **ii** ∠*F* **iii** ∠*D*

3 **a** yes
b **i** *D* **ii** *E* **iii** *F*
c **i** *DE* **ii** *EF* **iii** *DF*
d **i** ∠*E* **ii** ∠*F* **iii** ∠*D*

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4 a i E **ii** H
b i EH **ii** GH
c i $\angle G$ **ii** $\angle E$

5 a i F **ii** I
b i FJ **ii** HI
c i $\angle H$ **ii** $\angle J$

6 J and G; D and K; C and I

7 a 32 **b** 24 **c** 20 **d** 8 **e** 4

8 A and J; C and K; E and G

9 a $(A, E), (B, D), (C, F)$
b $(A, Y), (B, X), (C, W), (D, Z)$
c $(A, W), (B, X), (C, Z), (D, Y)$
d $(A, T), (B, Z), (C, X), (D, S), (E, W)$

10 a $\triangle AMC, \triangle BMC$
b Yes, all corresponding sides and angles will be equal.

11 a i $\triangle ABD, \triangle CBD$
ii Yes, all corresponding sides and angles will be equal.
b i $\triangle ABC, \triangle ACD$
ii No, sides and angles will not be equal.

12 yes

13 Equal radii.

14 a reflection in the y-axis then translation by the vector $\begin{bmatrix}1\\-2\end{bmatrix}$
b rotation anticlockwise about the origin $(0, 0)$ by 90° then translation by the vector $\begin{bmatrix}6\\3\end{bmatrix}$
c rotation about the origin $(0, 0)$ by 180° then translation by the vector $\begin{bmatrix}-2\\1\end{bmatrix}$
d reflection in the x-axis, reflection in the y-axis and translation by the vector $\begin{bmatrix}-2\\1\end{bmatrix}$
e reflection in the x-axis, reflection in the y-axis and translation by the vector $\begin{bmatrix}3\\2\end{bmatrix}$
f rotation about the origin $(0, 0)$ by 180° then translation by the vector $\begin{bmatrix}3\\2\end{bmatrix}$

Exercise 8E

1 a SSS **b** RHS **c** SAS **d** AAS

2 a $\triangle ABC \equiv \triangle EFD$ **b** $\triangle ABC \equiv \triangle FED$
c $\triangle XYZ \equiv \triangle UST$ **d** $\triangle ABC \equiv \triangle ADC$

3 a SAS **b** SSS **c** RHS **d** AAS
e RHS **f** AAS

4 a $x = 4, y = 1$ **b** $x = 9, a = 20$
c $x = 5, a = 24$ **d** $x = 5, a = 30$
e $x = 4, a = 95, b = 25$ **f** $x = 11, a = 50, b = 90$

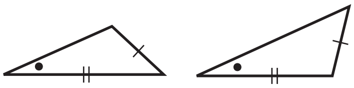
5 a no **b** yes, SAS **c** yes, AAS
d no **e** yes, SAS **f** yes, AAS

6 a $EF = 3\,\text{m}$ **b** $\angle B = 30°$
c $AC = 6\,\text{cm}$ **d** $\angle C = 20°$

7 a no **b** yes **c** yes **d** no

8 Yes, show SSS using Pythagoras' theorem.

9 a You can draw two different triangles with SSA.



b You can draw an infinite number of triangles with the same shape but of different size.

10 a $\angle CAB = \angle CED$ (alternate angles)
$\angle ACB = \angle ECD$ (vertically opposite angles)
$AC = EC$ (given equal and corresponding sides)
$\therefore \triangle ABC \equiv \triangle EDC$ (AAS)
b $BD = BD$ (given and common equal side)
$\angle ADB = \angle CDB$ (given and equal angles)
$AD = CD$ (given equal sides)
$\therefore \triangle ADB \equiv \triangle CDB$ (SAS)
c $\angle ACB = \angle CAD$ (alternate angles)
$\angle CAB = \angle ACD$ (alternate angles)
$AC = AC$ (given and common equal side)
$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)
d $\angle ABC = \angle ADC$ (given 90° angles)
$AC = AC$ (given and common equal side)
$BC = DC$ (given equal sides)
$\therefore \triangle ABC \equiv \triangle ADC$ (RHS)

11 a $AB = AC$ (given equal sides)
$BM = CM$ (given equal sides)
$AM = AM$ (given and common equal side)
$\therefore \triangle ABM \equiv \triangle ACM$ (SSS)
$\therefore \angle AMB = \angle AMC$
As $\angle AMB + \angle AMC = 180°$ then
$\angle AMB = \angle AMC = 90°$.
b $\angle AEB = \angle CDB$ (alternate angles)
$\angle EAB = \angle DCB$ (alternate angles)
$EB = BD$ (given equal sides)
$\therefore \triangle AEB \equiv \triangle CDB$ (AAS)
$\therefore AB = BC$ and $AC = 2AB$
c $AD = DC$ (given equal sides)
$AB = CB$ (given equal sides)
BD is a common side
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)
$\therefore \angle DAB = \angle DCB$
d $\triangle ACD \equiv \triangle ACB$ (SSS)
so $\angle DCA = \angle BCA$
Now $\triangle DCE \equiv \triangle BCE$ (AAS)
with $\angle CDE = \angle CBE$ (isosceles triangle)
So $\angle DEC = \angle BEC$
Since $\angle DEC$ and $\angle BEC$ are supplementary (sum to 180°)
So $\angle DEC = \angle BEC = 90°$
So diagonals intersect at right angles.

Exercise 8F

1 A and J; C and K; F and H; I and L

2 a i $\angle D$ **ii** $\angle E$ **iii** $\angle F$
b i AB **ii** BC **iii** CA
c i 2 **ii** 2 **iii** 2
d Yes, all side ratios are equal and all interior angles are equal.

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3 a yes, 2 **b** yes, 2 **c** yes, 4 **d** yes, 4

4 a i $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$
  ii $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$
  iii 2 **iv** $a = 100, x = 2, y = 3$
 b i $(AB, DE), (BC, EF), (CA, FD)$
  ii $(\angle A, \angle D), (\angle B, \angle E), (\angle C, \angle F)$
  iii $\frac{1}{3}$ **iv** $x = 3$
 c i $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$
  ii $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$
  iii 1.5 **iv** $a = 40, x = 4, y = 4.5$
 d i $(AB, FG), (BC, GH), (CD, HI), (DE, IJ), (EA, JF)$
  ii $(\angle A, \angle F), (\angle B, \angle G), (\angle C, \angle H), (\angle D, \angle I), (\angle E, \angle J)$
  iii 2.5 **iv** $a = 115, x = 5$

5 a Yes, all ratios are 2.
 b Yes, squares of different sizes.
 c No, ratios are not equal.
 d Yes, ratios are both 0.4 with equal angles.

6 360 cm

7 15 cm

8 1.25

9 3

10 a True, angles and side ratios will be equal.
 b False, side ratios may be different.
 c True, angles and side ratios will be equal.
 d False, angles and side ratios may be different.
 e False, angles may be different.
 f False, side ratios and angles may be different.
 g False, side ratios and angles may be different.
 h False, side ratios and angles may be different.
 i True, shape is always the same.

11 a no **b** No, they can have different shapes.

12 1 : 16

13 a i $\sqrt{8}$ **ii** 2
 b $\frac{2}{\sqrt{2}} = \sqrt{2}$ (small to big)
 c Using Pythagoras' theorem, the side length of the 2nd square is $\sqrt{\left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2} = \sqrt{\frac{x^2}{2}} = \frac{x}{\sqrt{2}}$.
 So the scale factor is $x \div \frac{x}{\sqrt{2}} = x \times \frac{\sqrt{2}}{x} = \sqrt{2}$.
 d 2 (small to big)

Exercise 8G

1 a $\triangle ABC \mathrel{|||} \triangle EFD$
 b $\triangle ABC \mathrel{|||} \triangle FDE$
 c $\triangle ABC \mathrel{|||} \triangle DEF$
 d $\triangle ABC \mathrel{|||} \triangle DEF$ (order does not matter)

2 a Two angles of $\triangle ABC$ are equal to two angle of $\triangle EFD$.
 b Two sides are proportional and the included angles are equal.
 c The hypotenuses are proportional and so is another pair of sides.
 d Three sides of $\triangle ABC$ are proportional to three sides of $\triangle DEF$.

3 a $\frac{DE}{AB} = \frac{10}{5} = 2$
 $\frac{DF}{AC} = \frac{24}{12} = 2$
 $\frac{EF}{BC} = \frac{26}{13} = 2$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 b $\angle A = \angle D$
 $\angle B = \angle E$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 c $\frac{DE}{AB} = \frac{10}{5} = 2$
 $\angle D = \angle A$
 $\frac{DF}{AC} = \frac{6}{3} = 2$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 d $\angle D = \angle A = 90^\circ$
 $\frac{EF}{BC} = \frac{26}{13} = 2$
 $\frac{DF}{AC} = \frac{22}{11} = 2$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 e $\angle E = \angle B = 90^\circ$
 $\frac{DF}{AC} = \frac{4}{1} = 4$
 $\frac{EF}{BC} = \frac{2}{0.5} = 4$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 f $\frac{DE}{AB} = \frac{9}{6} = 1.5$
 $\angle D = \angle A$
 $\frac{DF}{AC} = \frac{18}{12} = 1.5$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 g $\angle A = \angle D$
 $\angle C = \angle F$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

 h $\frac{DE}{AB} = \frac{10}{4} = 2.5$
 $\frac{DF}{AC} = \frac{15}{6} = 2.5$
 $\frac{EF}{BC} = \frac{15}{6} = 2.5$
 $\therefore \triangle ABC \mathrel{|||} \triangle DEF$

4 a $x = 8, y = 10$ **b** $x = 12, y = 6$
 c $x = 15, y = 4$ **d** $x = 3, y = 15$

5 a 2.5
 b yes (three pairs of sides are propotional)
 c 2.5

6 a yes **b** no **c** no **d** yes

7 4 m

8 a yes **b** 2.5 **c** 15 m

9 If two angles are known, then the third is automatically known using the angle sum of a triangle.

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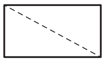
10 a $\angle A = \angle D$ (corresponding angles in parallel lines)
$\angle B = \angle B$ (common)
b $\angle ACB = \angle ECD$ (vertically opposite)
$\angle E = \angle A$ (alternate angles in parallel lines)
c $\angle A = \angle A$ (common)
$\angle ABC = \angle ADB$ (given)
d $EF = BA$ (equal sides)
$\angle E = \angle B$ (equal interior angles in regular polygons)
$DE = CB$ (equal sides)

11 Using Pythagoras' theorem $AC = 25$.
$\frac{DF}{AC} = \frac{50}{25} = 2$
$\frac{ED}{AB} = \frac{14}{7} = 2$
$\therefore \Delta ABC \;|||\; \Delta DEF$

12 a 4 **b** 2.4 **c** $\frac{16}{3}$ **d** $\frac{10}{3}$

Exercise 8H

1 (Other answers possible.)

a

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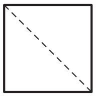
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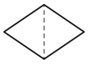
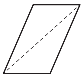

d


2 a Opposite sides of a parallelogram are equal.
b Opposite interior angles of a parallelogram are equal.
c Diagonals of a parallelogram bisect each other.
d Opposite sides of a parallelogram are parallel.

3 a All sides equal, opposite sides parallel, diagonals bisect at right angles, opposite interior angles are equal, diagonals bisect the interior angles through which they pass.
b All sides equal, opposite sides parallel, diagonals bisect at right angles, all angles equal, diagonals equal, diagonals bisect the interior angles through which they pass.

4 a AC **b** BD **c** DB

5 In $\triangle ABC$ and $\triangle BCD$
$\angle ABC = \angle BCD$ (angles in a square are equal)
$AB = BC$ (sides of a square are equal)
BC is common.
$\therefore \triangle ABC \equiv \triangle BCD$ (SAS)
$\therefore AC = BD$ (matching sides in congruent triangles are equal) and the diagonals of a square are equal in length.

6 a AAS **b** RHS **c** SSS
d SAS **e** AAS **f** SSS

7 $\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$\angle BAE = \angle DCE$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\triangle ABE \equiv \triangle CDE$ (AAS)
$BE = DE$ and $AE = CE$ because corresponding sides on congruent triangles are equal.

8 $\angle EFI = \angle GHI$ (alternate angles in parallel lines)
$\angle FEI = \angle HGI$ (alternate angles in parallel lines)
$EF = GH$ (given)
$\triangle EFI \equiv \triangle GHI$ (AAS)
$EI = GI$ and $FI = HI$ because corresponding sides on congruent triangles are equal.

9 a equal (alternate angles in parallel lines)
b equal (alternate angles in parallel lines)
c BD **d** AAS **e** They must be equal.

10 a $VU = TU$, $VW = TW$, UW is common.
So $\triangle VWU \equiv \triangle TWU$ by SSS.
b $\angle VWU = \angle TWU$ and since they add to 180° they must be equal and 90°.

11 a SSS (3 equal sides)
b They are equal and add to 180° so each must be 90°.
c Since $\triangle QMN$ is isosceles and $\angle MQN$ is 90° then $\angle QMN = 45°$.

12 a $AB = CB$, $AD = CD$ and BD is common.
So $\triangle ABD \equiv \triangle CBD$ by SSS.
b $\triangle ABD \equiv \triangle CBD$ so $\angle DAB = \angle DCB$
c $\triangle ABD \equiv \triangle CBD$ so $\angle ADB = \angle CDB$

13 Let $AD = BC = a$ and $AB = CD = b$.
Then show that both BD and AC are equal to $\sqrt{a^2 + b^2}$.

14 $\angle ABD = \angle CDB$ (alternate angles in parallel lines)
$\angle ADB = \angle CBD$ (alternate angles in parallel lines)
BD is common
So $\triangle ABD \equiv \triangle CDB$
So $AB = CD$ and $AD = BC$

15 a $\angle DCE$ **b** $\angle CDE$
c There are no pairs of equal sides.

16 a $\triangle ACD$ is isosceles.
b $AD = CD$, $\angle DAE = \angle DCE$ and $\angle ADE = \angle CDE$ (AAS)
c $\angle AED = \angle CED$ and sum to 180° so they are both 90°.

17 a First show that $\triangle ABD \equiv \triangle CDE$ by SSS.
So $\angle ABD = \angle CDE$ and $\angle ADB = \angle CBD$ and since these are alternate angles the opposite sides must be parallel.
b First show that $\triangle ABE \equiv \triangle CDE$ by SAS.
So $\angle ABE = \angle CDE$ and $\angle BAE = \angle DCE$ and since these are alternate angles the opposite sides must be parallel.
c First prove that $\triangle ABD \equiv \triangle BAC$ by SSS.
Now since $\angle DAB = \angle CBA$ and they are also cointerior angles in parallel lines then they must be 90°.

Puzzles and challenges

1 3, Reason is AAA for each pair with a right angle and a common angle.
2 Yes, illustrates Pythagoras' theorem using areas.
3 31
4 a $(3 - r) + (4 - r) = 5$, so $r = 1$
b $r = 4 - 2\sqrt{2}$
5 $\frac{60}{17}$

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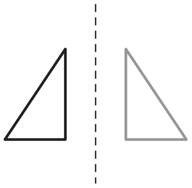
Multiple-choice questions

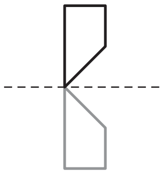
1 B **2** D **3** C **4** A **5** B
6 E **7** A **8** E **9** A **10** E

Short-answer questions

1 a



b



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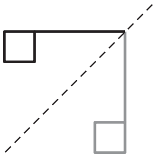
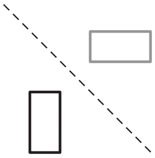
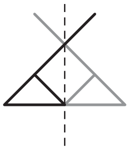


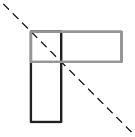
d



e



f

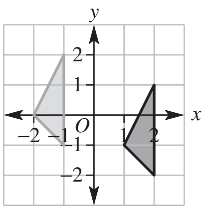


2 a $A'(1, -2), B'(3, -4), C'(0, -2)$
b $A'(-1, 2), B'(-3, 4), C'(0, 2)$

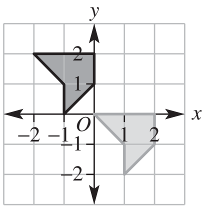
3 a 4 **b** 1 **c** 2 **d** 1
e 6 **f** 0

4 a $\begin{bmatrix}1\\4\end{bmatrix}$ **b** $\begin{bmatrix}3\\-6\end{bmatrix}$ **c** $\begin{bmatrix}-3\\-7\end{bmatrix}$ **d** $\begin{bmatrix}4\\6\end{bmatrix}$

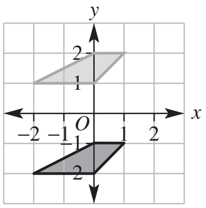
5 a



b

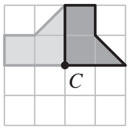


c

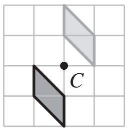


6 a 3 **b** 2
c no rotational symmetry

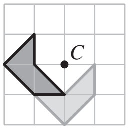
7 a



b



c



8 a i F **ii** G
b i EH **ii** FG
c i $\angle G$ **ii** $\angle E$

9 a RHS **b** SAS **c** SSS **d** AAS

10 a $x = 3, a = 25$ **b** $x = 5, a = 18$

11 a $\angle BAE = \angle DCE$ (alternate angles in parallel lines)
$\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\therefore \triangle ABE \equiv \triangle CDE$ (AAS)
b $AE = CE$ and $BE = ED$ so AC and BD bisect each other.

12 a $AE = CE$, $AB = CB$ and BE is common
$\therefore \triangle ABE \equiv \triangle CBE$ (SSS)
b $\angle AEB = \angle CEB$ because $\triangle ABE \equiv \triangle CBE$
and since $\angle AEB + \angle CEB = 180°$
$\angle AEB = \angle CEB = 90°$ so AC and BD bisect at right angles.

Extended-response question

1 a $A'(0, 1), B'(-2, 1), C'(-2, 4)$
b $A'(3, 1), B'(3, -1), C'(0, -1)$
c $A'(1, -1), B'(-1, -1), C'(-1, 2)$

Chapter 9

Pre-test

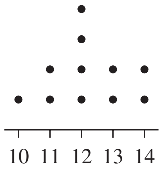
1 a 0, 1, 2, 4, 6, 7, 9, 10, 14 **b** 20, 30.6, 36, 100, 101, 204
c 1.2, 1.7, 1.9, 2.7, 3.2, 3.5

2 a Total = 40, average = 8
b Total = 94, average = 18.8
c Total = 3.3, average = 0.66

3 a $\frac{1}{6}$ **b** 150° **c i** \$420 **ii** \$630

4 a 4 **b** 3 **c** Saturday
d 26 **e** $\frac{1}{3}$

5 a 11
b 10, 11, 11, 12, 12, 12, 12, 13, 13, 14, 14
c



d 12 **e** $12.\dot{1}\dot{8}$

Exercise 9A

1 a C **b** D **c** A
d E **e** F **f** B

2 a categorical **b** numerical **c** numerical
d categorical **e** numerical **f** numerical

3 Answers will vary.

4 a discrete numerical
b continuous numerical
c continuous numerical
d categorical
e categorical
f categorical
g discrete numerical
h discrete numerical
i continuous numerical
j continuous numerical
k continuous numerical
l discrete numerical
m continuous numerical
n discrete numerical
o categorical
p discrete numerical
q categorical
r categorical

5 a observation
b sample of days using observation or secondary source records
c census of the class
d sample
e sample
f sample using secondary source data
g census (every 5 years this question appears on the population census)
h census of the class
i sample
j results from the population census
k observation
l observation
m sample
n census
o sample

6 a secondary – a market research company
b secondary – department of education data
c primary data collection via a sample
d secondary source using results from the census
e secondary source using NAPLAN results or similar

7 a Proximity to the Indian Ocean makes first hand collection of the data difficult.
b Too many people to ask and a sensitive topic means that using the census results as your source would be better.
c Extremely large population makes primary data difficult to collect.
d Sensitive topic might make student less keen to give honest and reliable answers.
e Cultural issues and the different cultural groups that exist in the community makes collection difficult.

8 The data is often collected by a market research company. It is not always possible to know how the data is collected, the areas it is collected from and whether there was a bias introduced in the surveys.

9 a A population is the entire group of people but a sample is a selection from within it.
b If the population is small enough (e.g. a class) or there is enough time/money to survey the entire population (e.g. national census).
c When it is too expensive or difficult to survey the whole population, e.g. television viewing habits of all of NSW.

10 a The answers stand for different categories and are not treated as numbers. They could have been A − E rather than 1 − 5.
b i 1 = strongly disagree, 2 = somewhat disagree, 3 = somewhat agree, 4 = strongly agree.
ii 1 = poor, 2 = satisfactory, 3 = strong, 4 = excellent.
iii 1 = never, 2 = rarely, 3 = sometimes, 4 = usually, 5 = always.
iv 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree.

11 a Excludes people who have only mobile numbers or who are out when phone is rung; could bias towards people who have more free time.
b Excludes people who do not respond to these types of mail outs; bias towards people who have more free time.
c Excludes working people; bias towards shift workers or unemployed.
d Excludes anyone who does not read this magazine; bias towards readers of Girlfriend.
e Excludes people who do not use Facebook; bias towards younger people or people with access to technology.

12 a For example, number of babies at a local playground. Other answers possible.
b Count a sample, e.g. just one floor of one car park.

13 It gives ownership and establishes trust where there may not have been any. It also ensures a deeper understanding of the process and need for honesty in the collection and use of any data.

14 a Too expensive and difficult to measure television viewing in millions of households.
b Not enough people – results can be misleading.
c Programs targeted at youth are more likely to be watched by the students.
d Research required.

15 a Too expensive and people might refuse to respond if it came too often.
b English as a second language can impact the collection of data (simple, unambiguous English is required). Some people from particular cultures may not be keen to share information about themselves.
c Some people cannot access digital technologies and they would be excluded from the results.
d Larger populations and a greater proportion of people in poverty can make census data harder to obtain.

16 Different people are chosen in the samples. Larger, randomly selected samples give more accurate guides.

17 a i Answers will vary.

ii Answers will vary.

iii Random processes give different results.

b i Different vowels have different frequencies of occurring.

ii If a high frequency word has an unusual range of vowels, e.g. a page or webpage on Mississippi.

Exercise 9B

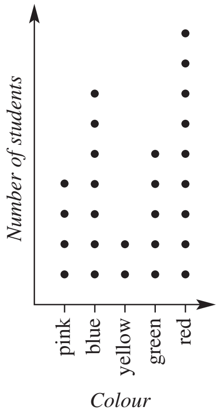
1 a true **b** false **c** true **d** true **e** false

2 a 5 **b** 3 **c** sport **d** 20

3 a 2 **b** 7 **c** red **d** 27

e

Favourite colour in 8B



4 a 8 **b** 24 **c** 8 **d** 7

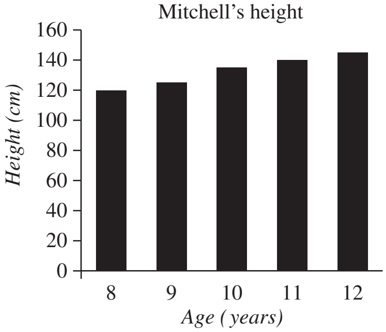
e 2 **f** 8 **g** yes (between 2 & 5)

5 a 120 cm **b** 20 cm

c 60 cm **d** 11 years old

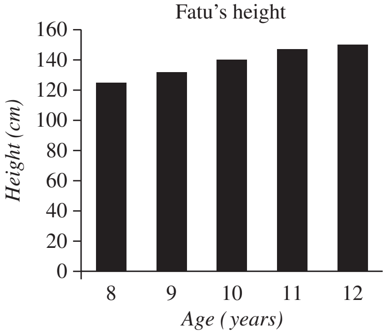
6 a

Mitchell's height



b

Fatu's height



7 a

	1995	2000	2005	2010	2015
Using public transport	30	25	40	50	60
Driving a car	60	65	50	40	20
Walking or cycling	10	20	15	15	25

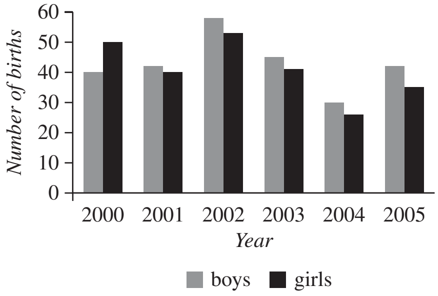
b 2010 and 2015

c 2015

d Environmental concerns; others answers possible.

e Public transport usage is increasing; other answers possible.

8 a



b 2000 **c** 2004 **d** 2002 **e** boys

9 a It has increased steadily. **b** approx. $38 000

c approx. $110 000–$130 000

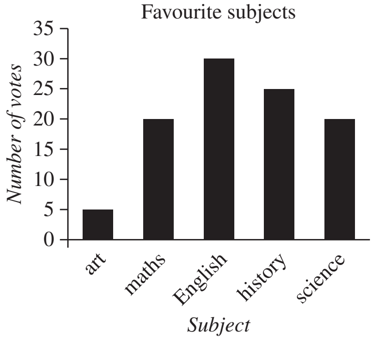
10 Helps to compare different categories quickly by comparing heights.

11 a It is unequal.

b The axes have no labels and it does not have a title.

c

Favourite subjects



d four times as popular

e one and a half times as popular

f Makes growth appear to have tripled when in fact it has only increased slightly.

12 a 7

b 7

c 3M : 8, 3S : 4

d 3M

e 3M because best student got 10.

13 a 1500 **b** 104 000 **c** increased

d approx. 590 000 passengers

14 240

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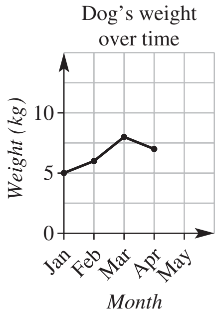
Exercise 9C

1 **a** 3 kg **b** 4 kg **c** 5 kg **d** 4.5 kg

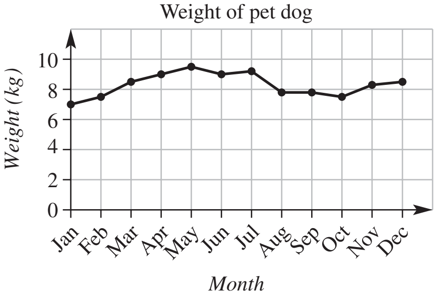
2



3 **a** 45 cm **b** 110 cm **c** 8 years
d 15 cm **e** 140 cm

4 **a** 23°C **b** 2 p.m. **c** 12 a.m.
d **i** 10°C **ii** 18°C **iii** 24°C **iv** 24.5°C

5 **a**



b Weight increases from January until July, then goes down suddenly.
c July 1
d Completed on computer

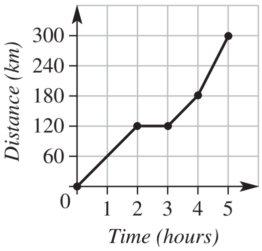
6 **a** 200 km **b** 80 km **c** At rest
d In the first hour **e** 40 km

7 **a** 2 hours **b** 5 km **c** Fifth hour

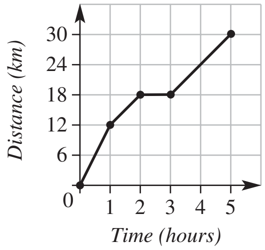
8 **a** July
b From October until the following July
c 20 megalitres because the level stayed the same
d Southern Hemisphere because winter occurs in middle of year.

9 **a** at 7 a.m. and 8 p.m. **b** at 8 a.m. and 11 p.m.
c **i** around 7 a.m. (heater goes on)
ii around 8 a.m. (turns heater off)
iii around 8 p.m. (heater put back on)
iv around 11 p.m. (heater turned off)
d Answers will vary.

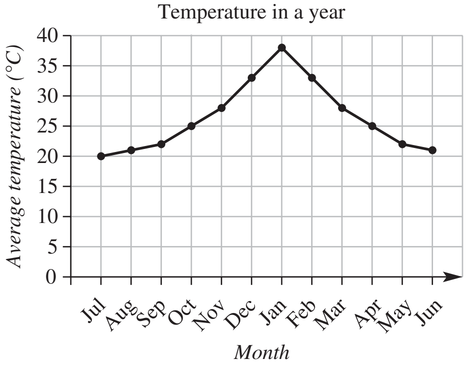
10 **a**



b



11 **a** The city is in Southern Hemisphere because it is hot in January/December.
b The city is quite close to the equator because the winter temperatures are reasonably high.
c



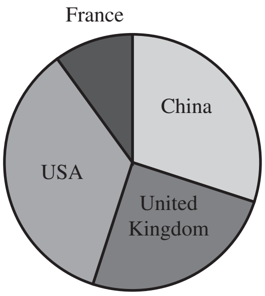
d Maximum occurs in the middle.
e It depends on what Month 1 means. If it means January, then this is the Northern Hemisphere. If it means June, then this is the Southern Hemisphere.

Exercise 9D

1 **a** playing sport
b watching TV
c more

2 **a** rugby **b** basketball **c** $\frac{1}{5}$ **d** $\frac{2}{3}$

3 **a** 20
b **i** $\frac{3}{10}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{20}$ **iv** $\frac{1}{10}$
c **i** 108° **ii** 90° **iii** 126° **iv** 36°
d



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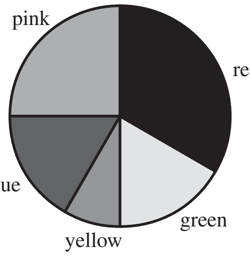
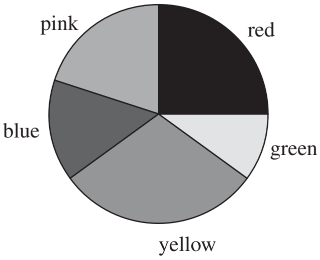
4 a

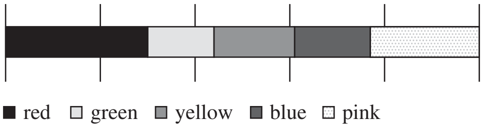


b

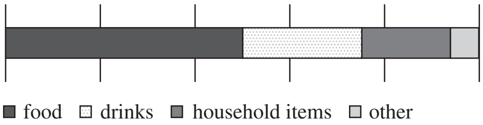


c Higher proportion of Year 7s like red; higher proportion of Year 8s likes yellow.

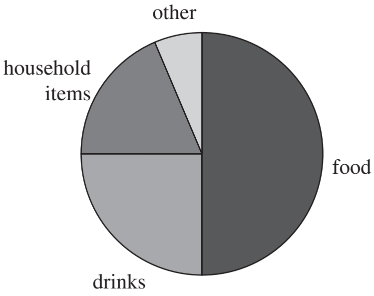
d



5 a



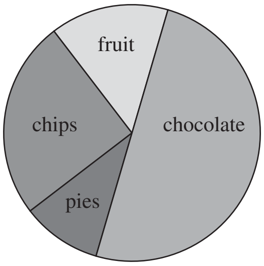
b



c Comparison required.

6 a i 20 **ii** 10 **iii** 6 **iv** 4

b

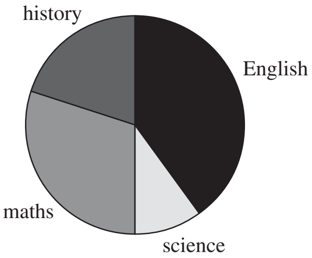


c i chips **ii** fruit and pies **iii** chocolate

7 a i 40% **ii** 30% **iii** 20%

b i 9 **ii** 6 **iii** 15

c



8 a Krishna

b Nikolas

c It means Nikolas also spends more time playing sport.

d 37.5°

9 a 6

b 10

c Bird was chosen by $\frac{1}{8}$, which would be 2.5 people.

d Each portion is $\frac{1}{3}$, but $\frac{1}{3}$ of 40 is not a whole number.

e 24, 48, 72 or 96 people participated in survey.

10 Need numbers for a meaningful axis but not for labels of each sector.

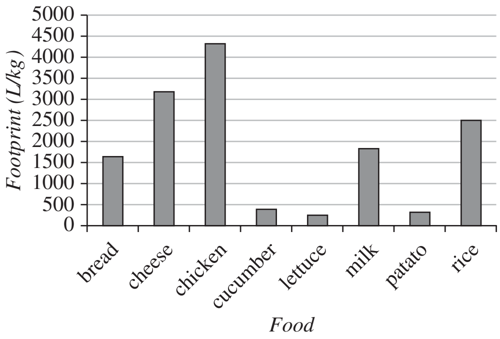
11 a Survey 2

b Survey 1

c Survey 3

12 a Column graph – categorical data. (Sector graph is inappropriate as not measuring proportions of a whole)

b Column graph



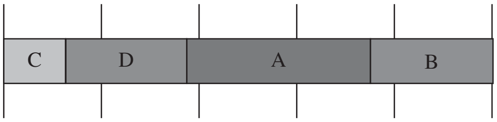
c As water becomes scarcer it is more difficult to produce these foods.

d Answers will vary.

e

	Bread	Cheese	Chicken	Cucumber	Lettuce	Milk	Potato	Rice
Efficiency (g/kL)	622	315	231	2833	4219	556	3484	400

13 a



b 24

c 6

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Exercise 9E

1 a true **b** false **c** true **d** false

2 a 4 **b** 7 **c** || **d** 𝍸𝍸|

3 a

	Passes	Shots at goal	Shots that go in	Steals
Frequency	3	13	7	2

b 13 **c** 7 **d** 6

4 a

People in family	2	3	4	5	6	7	8
Tally	|	||	||||	||||	||||	||	|||
Frequency	1	2	4	4	4	2	3

b 4 **c** 9

5 a

Number of hours	0–4	5–9	10–14	15–19	20–24	25 or more
Tally	8	12	15	9	4	2

b 50 **c** 9 **d** 8

6 a

Height (cm)	130–139	140–149	150–159	160–169	170–179	180–189	190+
Tally	|||	𝍸	||	|||	|||	|	||||
Frequency	3	5	2	3	3	1	4

b 2 **c** 5 **d** 10

7 a 10 **b** 2 **c** 4 **d** 17

8 a B **b** D **c** A **d** C

9 a 130 **b** 19 **c** 24 **d** 314

10 a

Score	0–19	20–39	40–59	60–79	80–100
Frequency	0	4	7	20	12

b

Score	0–29	30–59	60–89	90–100
Frequency	3	8	30	2

c It is unknown how many of the three people in the 20s got less than 25 and how many got more.

d

Score	0–24	25–49	50–74	75–100
Frequency	2	4	20	17

e 43

11 a

Range	10–19	20–29	30–39
Frequency	3	4	6

b Many possible answers. **c** a stem-and-leaf plot

d When individual numbers are not required but an overview is more important.

12 a Many possible answers.

b There are five possible values and it happened six times, so one value is repeated.

c Even if each score was achieved twice that would account for only 10 weeks (not 11).

d yes **e** yes

13 a Monday 3, Tuesday 2, Wednesday 1, Thursday 3

b 2 hours **c** 12 ways **d** 3 ways

e 6 ways **f** 30 ways

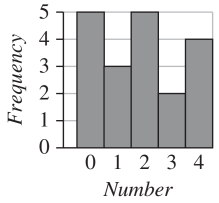
Exercise 9F

1 a 2 **b** 9 **c** 11 years old

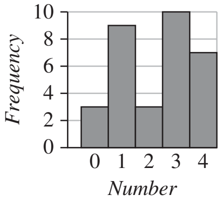
2 a 4 **b** 4

c 8 **d** frequency polygon

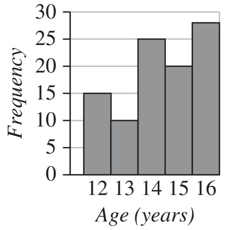
3 a



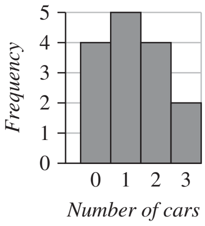
b



c



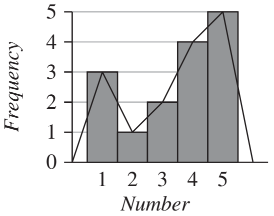
d



4 a i

Number	Frequency
1	3
2	1
3	2
4	4
5	5

ii



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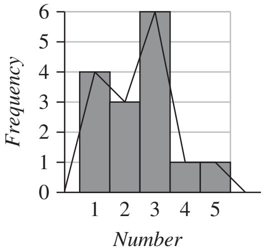
b **i**

Number	Frequency
1	4
2	3
3	6
4	1
5	1

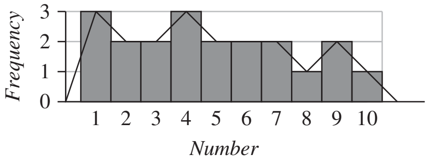
ii



c **i**

Number	Frequency
1	3
2	2
3	2
4	3
5	2
6	2
7	2
8	1
9	2
10	1

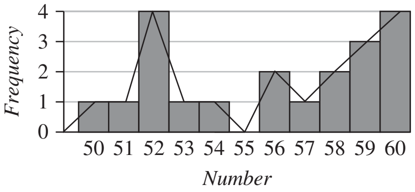
ii



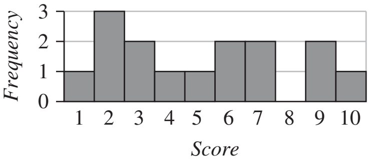
d **i**

Number	Frequency
50	1
51	1
52	4
53	1
54	1
55	0
56	2
57	1
58	2
59	3
60	4

ii



5 **a**

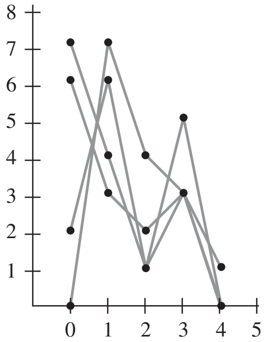


b Edwin is worse than Fred as most of Fred's scores are 8 or higher.

6 **a** D **b** A **c** B **d** C

7 **a** Marie **b** Con **c** Frank **d** Bill

e



Graph 1
(0, 7) – (1, 4) – (2, 1) – (3, 3) – (4, 0)
Graph 2
(0, 6) – (1, 3) – (2, 2) – (3, 3) – (4, 1)
Graph 3
(0, 2) – (1, 6) – (2, 2) – (3, 5) – (4, 0)
Graph 4
(0, 0) – (1, 7) – (2, 4) – (3, 3) – (4, 0)

f Allows you to see multiple frequency tables on same axes.

8 **a** It would look identical but the x-axes labels would start at 22 and go to 26.

b It would look just like the right half (12–14, but labelled 0–2).

9 So that there is a space for the frequency polygon to start.

10 **a** No, just that she is more likely to get higher marks than lower marks.

b 9 weeks of 5, then 8 weeks of 6, then 7 weeks of 7, then 4 weeks of 8, then 2 weeks of 9 out of 10

c They were absent from the test, or having a very bad day.

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11 a

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4	Graph 7	Graph 6
Shopping centre	Graph 8	Graph 2	Graph 9
Teachers, common room	Graph 5	Graph 3	Graph 1

b Answers will vary.

Exercise 9G

1 a 1, 2, 3, 5, 10 **b** 10 **c** 1 **d** 9
2 a 9 **b** 12 **c** 9 **d** 7
3 a 30 **b** 6 **c** 7 **d** 7
4 a i 6 **ii** 3 **iii** 2 **iv** 1
b i 11 **ii** 7 **iii** 8 **iv** 2
c i 18 **ii** 12 **iii** 11 **iv** 11
d i 17 **ii** 26 **iii** 25 **iv** 25
e i 32 **ii** 16 **iii** 15 **iv** 10
f i 40 **ii** 34 **iii** 30 **iv** 55
g i 102 **ii** 71 **iii** 83 **iv** 84
h i 91 **ii** 41 **iii** 31 **iv** 18
5 a 16.6 **b** 17.5 **c** 12
6 a Brent **b** Brent **c** Ali **d** Brent
7 a 8.4 **b** 8 **c** 8 **d** mean
8 43.5
9 a 2 **b** 4 **c** 12
d 25 **e** 2.1 **f** 4
10 a 12 **b** 2 **c** 0 saves
11 a $\{9, 11\}$; other solutions possible.
b $\{8, 10, 13\}$; other solutions possible.
c $\{0, 100\}$; other solutions possible.
d $\{0, 5, 5, 7, 8, 8.1, 8.9\}$; other solutions possible.
12 a three values; e.g. $\{5, 5, 8\}$
b yes; e.g. $\{0, 0, 5, 5, 5, 7, 20\}$
13 a mean: 7, median: 7.5, range: 12
b i increases by 5 **ii** increases by 5
iii no effect
c i doubles **ii** doubles **iii** doubles
d Yes; e.g. if the set were duplicated:
1, 1, 3, 3, 5, 5, 10, 10, 10, 10, 13, 13.
14 a 7 and 13 **b** 9, 10, 11
c They must be 8, 10 and 12.
15 Most frequent value makes sense even for categories but 'mean' requires adding (numbers) 'range' requires subtracting (numbers), and 'median' requires ordering (numbers).
16 a range = 6, mean = 7.18 **b** range = 4, mean = 7.21
17 50.5
18 a i 4, 4, 4; other solutions possible.
ii 1, 4, 3, 9, 1; other solutions possible.
iii 2, 3, 8, 10, 10; other solutions possible.
iv 1, 2, 3, 4, 4, 16; other solutions possible.
b It is possible. (Hint: Divide every number by the current range.)

Exercise 9H

1 a 8 **b** 1 **c** 7
2 a 4 **b** 9 **c** 5
3 a 5 **b** 12 **c** 7
4 a lower quartile **b** lowest
c sort (or order) **d** median
e odd **f** spread
5 a 4 **b** 10 **c** 5.5
6 a 3, 4, 5, 5, 6, 7, 8, 10 **b** 4.5
c 7.5 **d** 3
7 a 19 **b** 11 **c** 21 **d** 15
e 7 **f** 9.5 **g** 3.16 **h** 1.76
8 a 23 **b** 19 **c** 35 **d** 32
e 16.5 **f** 10.5 **g** 3.45 **h** 1.15
9 a 11 **b** 10.5 **c** 9 **d** 8
e 32 **f** 23 **g** 18 **h** 13
10 a 15 **b** 35 **c** Nathan **d** Gary
11 a 1.7 **b** 1.8 **c** Max
12 a i 9 **ii** 10
b i 4.5 **ii** 4.0
c Sara **d** Andy
13 a i 8 **ii** 4
b i 98 **ii** 4
c A single outlier greatly affects range whereas IQR is unaffected.
14 96
15 a 10 **b** 4
c No, the range is the largest difference between two numbers.
d Yes, for instance, for a set like 2, 2, 2, 6, 6, 6.
16 a no effect **b** range is doubled
17 The lower quartile is the number above the bottom quarter of values and the upper quartile is the number below the top quarter of values.
18 a 4 **b** 3
c It would stay the same.
d It would stay the same.
e i It would double.
ii It would double.

Exercise 9I

1 a 5 **b** 2
2 a 39 **b** 27 **c** 134
3 57
4 a 8, 9, 10, 11, 13, 15, 17, 18, 21, 24 **b** 10
c i false **ii** true **iii** true **iv** false
5 a range = 20, median = 17
b range = 31, median = 26
c range = 19, median = 40.5

6 a

Stem	Leaf
1	1 2 3 4 4 5 7
2	0 4 8 9
3	1 2 3 5

b

Stem	Leaf
2	0 2
3	9
4	5 7 9 9
5	1 2 2 3 5 6 8 8

c

Stem	Leaf
1	6 6 8
2	1 4 8 9
3	1 2 3 5
4	1 8 9
5	0

d

Stem	Leaf
1	1 2 4
2	7 9
3	2 7 8 8
6	0 0
7	3 8
8	1 7

7 a

Stem	Leaf
8	0 4 5 6
9	0 6
10	1 4 5
11	0 3 4 4 5 9

b

Stem	Leaf
8	1 6
9	1 4 5 6 8
10	268
11	3557
12	0

c

Stem	Leaf
15	5 7 8
16	2 5 7
17	3 4 8
18	1 4 4 5 7
19	2 2 3 3 6 9

d

Stem	Leaf
39	1 5 6
40	1 2 4 5 6 6 8 9
41	1 2 3 3 5 6 7 8
42	0

8 a 2 **b** B **c** 1 **d** B **e** 1
f N **g** 2 **h** 1 **i** N

9 a 10 **b** 1 **c** 8 **d** 58 **e** 10

10 a i 49 years **ii** 36 years
b radio station 1
c i 33 to 53 years **ii** 12 to 32 years

11 a i 30 cm **ii** 52 cm **b** girls
c i 152 cm **ii** 144.5 cm **d** boys
e perhaps Year 6 or 7
f Both would have a smaller range and a higher median.

12 a 15 **b** 13
c *a* is 5 or 6, *b* is 0, *c* is 8 or 9, *d* is 0.

13 a i 30 **ii** 10
iii between 10 and 19 years old
iv between 31 and 49 years
v 18 **vi** between 18 and 28
b i true **ii** false **iii** true **iv** true
v false **vi** false
c Cannot determine how many people are exactly aged 40 years.
d close to 30 years

14 a Easier to compare sizes of different stems visually.
b Helps to notice trends and to calculate median.

15 a minimum: –29°C, maximum: 23°C
b i 5 **ii** 10 **iii** 11
c Because –05 and 05 are different numbers.
d −5.2°C

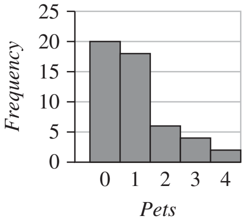
Exercise 9J

Some answers may vary.

1 a surveying 1000 randomly selected people
b surveying 10 friends

2 a $\frac{2}{5}$ **b** 2000 **c** 300

3 a



b 400 **c** 1200
d More likely that people will have pets if near a vet clinic.
e Discussion point.

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4 **a** Yes, it is required information.
b No, it is too vague or subjective.
c No, it is too vague or subjective.
d No, it addresses wealth but not income.
e No, it is irrelevant.
f Yes, it can be used to decide income.
g No, if it is not a pay day then results will be distorted.

5 **a** At midday on a Thursday on a major road.
b Outside a political party office.
c In a butcher's shop.
d At 11 p.m., when people will buy just a few items.

6 **a** 'How many siblings do you have?'
b–c Results will vary.

7 **a** Sample is a smaller group drawn from the population
b **i** sample (too big for a census)
ii census (small group)
iii census (possible to measure this for one lunch at most schools)
iv census (small group)
v sample (population too large)

8 **a** 108 g
b If two bars were on scales together.
c IQR as it is not affected by outlier
d median

9 **a** At a professional dance studio in the afternoon.
b At a university physics department.
c Choose a large random sample.

10 **a** Might discriminate against people with medical condition
b Might show favouritism to wealthier ones in hope of a donation
c Might use this to generalise about all boys versus all girls
d Temptation to present only favourable results or have a biased sample

11 **a** Answers vary.
b Does not allow the relationship to be seen.
c

		Enjoy maths?	
		Yes	No
Siblings?	Yes		
	No		

d 1 = strongly dislike
2 = dislike
3 = neutral
4 = like
5 = strongly like

e

		Rating				
		1	2	3	4	5
Siblings	Yes					
	No					

12 **a** **i** 75% **ii** 95% **iii** 15%
iv Between 15% and 95% of the population are in favour.
b Busy people are less likely to participate and will not be counted.
c 'Do you enjoy participating in surveys?'
d Helps correct for measurement error, caused by people answering either randomly or mistakenly.

13 Discussion point.

14 Answers vary.

Puzzles and challenges

1 2, 2, 7, 8, 12
2 **a** 4 **b** 10 **c** 90 **d** 72
3 6, 8, 2
4 5, 11, 14
5 41
6 12

Multiple-choice questions

1 C	**2** B	**3** D	**4** B	**5** C
6 B	**7** D	**8** D	**9** C	**10** C

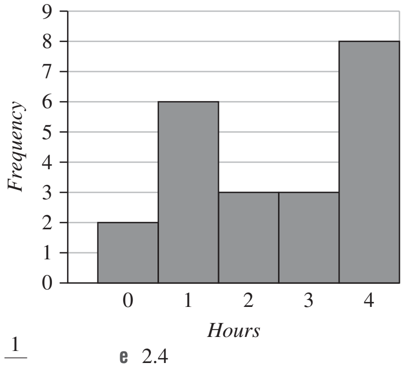
Short-answer questions

1 **a** government bus **b** train
c 72° **d** 1000
e Example: Prices went up for government buses.

2 **a** 22
b

Hours	0	1	2	3	4
Frequency	2	6	3	3	8

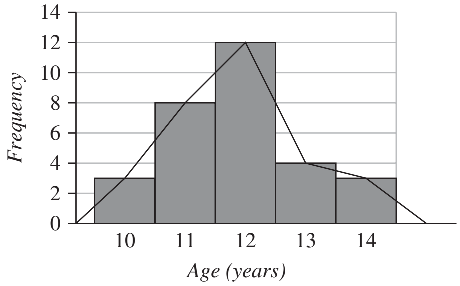
c



d $\frac{1}{11}$ **e** 2.4

3 **a** 38, 43, 44, 44, 52, 53, 55, 56, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 74, 84
b 44 **c** 61.5 **d** 46

4 **a** 30
b



c 11.87 years **d** 12

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5 **a** lowest: 50 kg, highest 85 kg

b

Weight	Frequency
50–54	6
55–59	6
60–64	8
65–69	7
70–74	7
75–79	1
80–85	5

c **i** 35 kg **ii** 60–64 kg

d Only teenagers were chosen, not including children or adults.

6 **a** **i** 6.5 **ii** 6 **iii** 3.5 **iv** 9.5 **v** 6

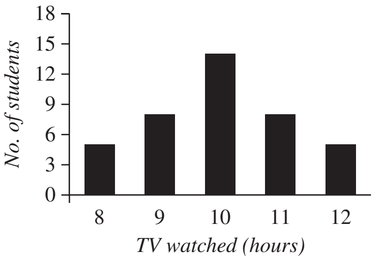
b Mean and median have increased by 1, but IQR is the same because all numbers have just increased by 1.

7 **a** Not enough people, and her friends might work harder (or less hard) than other students.

b She could choose 10 people who worked less hard than her.

8 **a** 40 **b** 400 **c** 10 hours

d



9 **a** four students

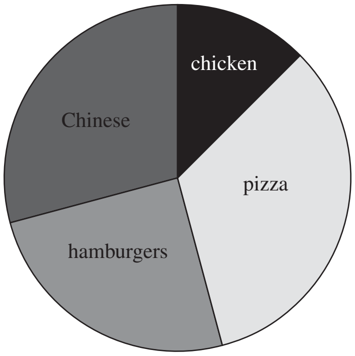
b two students

c 1 p.m.

d six students

10 **a** 105° **b** 25%

c



11 **a** 11 **b** 5 **c** 3.5 **d** 2

12 **a** 12

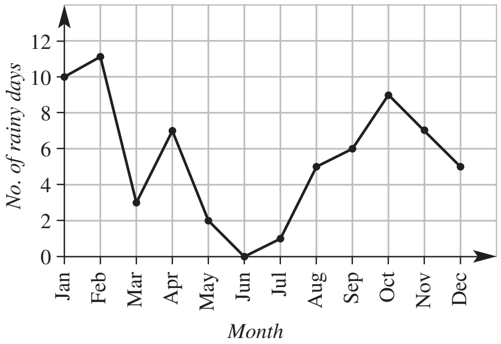
b 5

c 3.5

d 3

Extended-response questions

1 **a**



b 66

2 **a** 40 **b** cheesecake **c** $\frac{7}{40}$

d **i** yes **ii** no **iii** yes **iv** yes

e 80

Semester review 2

Angle relationships and properties of geometrical figures 1

Multiple-choice questions

1 B **2** D **3** E **4** C **5** C

Short-answer questions

1 **a** 66 **b** 25 **c** 123

d 35 **e** 70 **f** 98

2 **a** $x = 81, y = 99$ **b** $a = 75$

c $a = 62, b = 62$ **d** $a = 65, b = 65$

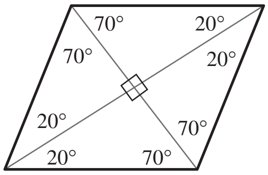
e $a = b = c = d = 100, e = 80$

f $x = 95, y = 85$

3 **a** 48 **b** 45 **c** 60 **d** 75 **e** 121 **f** 75

4 **a** $a = b = 90$ **b** $a = 73, b = 95$ **c** $a = 265, b = 30$

5



Extended-response question

$b = 65$ (supplementary to a)

$c = 65$ (alternate to b)

$d = e = 57.5$ (isosceles triangle)

$f = 122.5$ (supplementary to d)

$g = 122.5$ (revolution angle = 360)

$h = 180$ (straight angle)

$i = 295$ (revolution)

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Linear relationships 1

Multiple-choice questions

1 D
2 B
3 C
4 E
5 D

Short-answer questions

1 **a** 1st **b** 2nd **c** 3rd **d** 4th

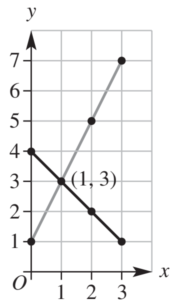
2 **a** **i**

x	0	1	2	3
y	1	3	5	7

ii

x	0	1	2	3
y	4	3	2	1

b

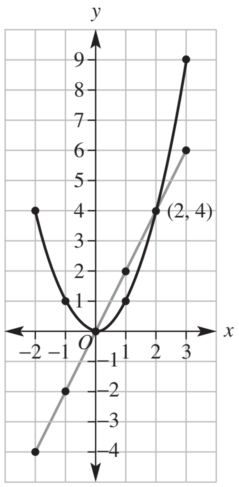


3 multiply x by 6, then subtract 6

4 $y = \frac{1}{2}x + 3$

5

x	−2	−1	0	1	2	3
$y = x^2$	4	1	0	1	4	9
$y = 2x$	−4	−2	0	2	4	6



Extended-response question

a **i** $x = -4$ **ii** $x = -2.5$ **iii** $x = 1$
iv $x = 3.5$ **v** $x = 6$

b $(4, -3)$

c **i** $y = 5 - 2x$
$3 = 5 - 2 \times 1$
$3 = 3$ True
$(1, 3)$ is a solution

ii $y = 5 - 2x$
$1 = 5 - 2 \times 3$
$1 = -1$ False
$(3, 1)$ is a not solution

iii $y = 5 - 2x$
$-5 = 5 - 2 \times 5$
$-5 = -5$ True
$(5, -5)$ is a solution

d The point is $(4, -3)$, so $x = 4$ and $y = -3$
$y = 5 - 2x$
LHS $= y$ RHS $= 5 - 2(4)$
$= -3$ $= -3$
$y = 1 - x$
LHS $= y$ RHS $= 1 - 4$
$= -3$ $= -3$
Therefore $(4, -3)$ satisfies both equations.

Transformations and congruence

Multiple-choice questions

1 C **2** B **3** E **4** D **5** C

Short-answer questions

1 **a** 0 **b** 2 **c** 2 **d** 1

2 **a** $\begin{bmatrix}2\\2\end{bmatrix}$
b $\begin{bmatrix}-1\\-2\end{bmatrix}$

3 **a** $A'(1, -1), B'(1, -3), C'(3, -2)$
b $A'(-1, -1), B'(-3, -1), C'(-2, -3)$

4 **a** SSS
b AAS
c RHS

5 A, C

Extended-response question

a 15 cm
b 2 : 5
c 4 : 25
d 10 cm

Data collection, representation and analysis

Multiple-choice questions

1 B **2** A **3** B **4** E **5** D

Short-answer questions

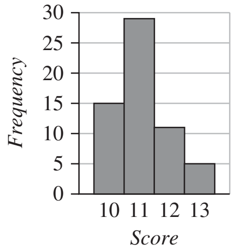
1 **a** **i** 13.75 **ii** 14 **iii** 8
b **i** 23 **ii** 18.5 **iii** 56
c **i** 10 **ii** 9.45 **iii** 15.7

2 **a** 8.5
b 4
c 14.5
d 10.5

3 16.9

4 **a** 17.4, 18
b 11.1, 11

5



6 **a** 13
b 3, 47
c **i** 44 **ii** 22 **iii** 22

7 **a** 20 **b** 22 **c** mean = 23, median = 22

8 **a** 3
b 39
c 1
d 48

Extended-response question

a 18
b 78
c 78
d group B

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