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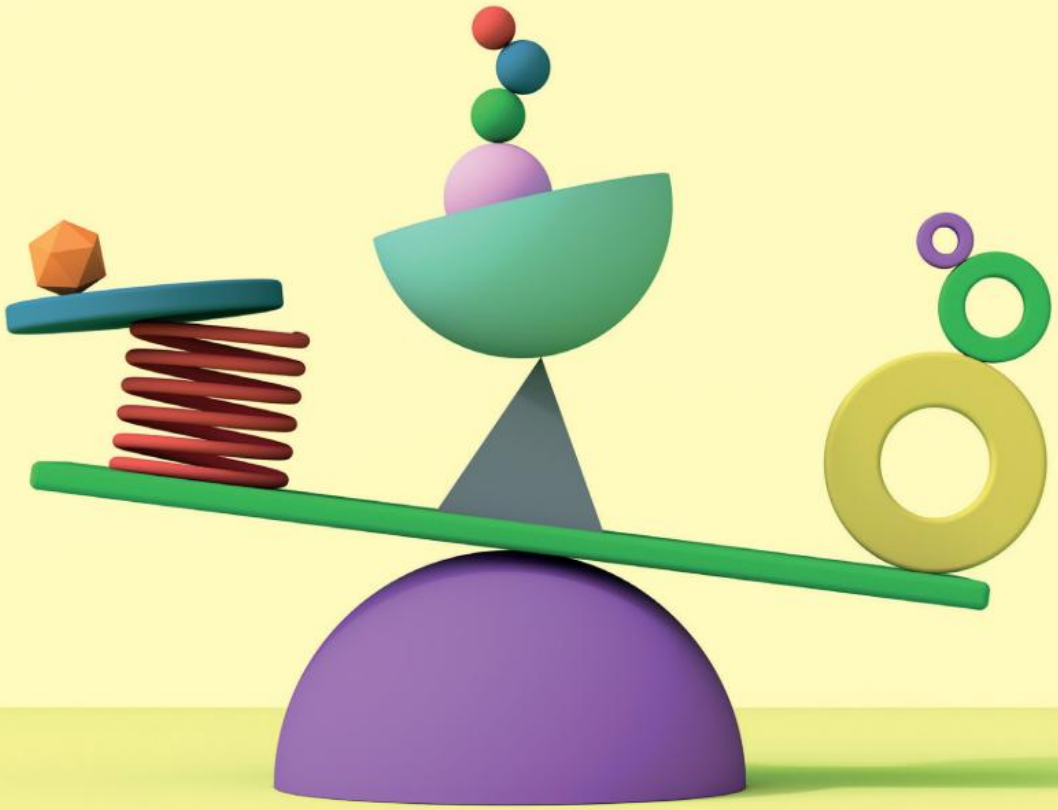
SECOND EDITION

OXFORD MATHS

8



Series Consultant: Thomas Christiansen
Helen Sorenson Alexander Blanksby
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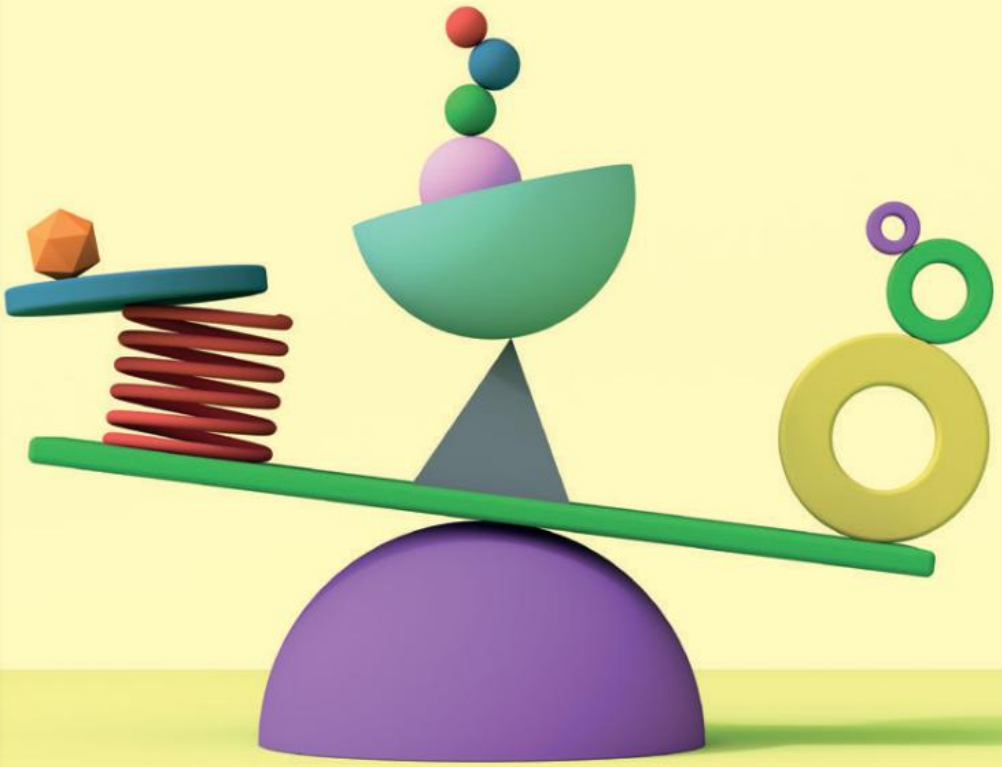

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Warning to First Nations Australians
Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

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CONTENTS

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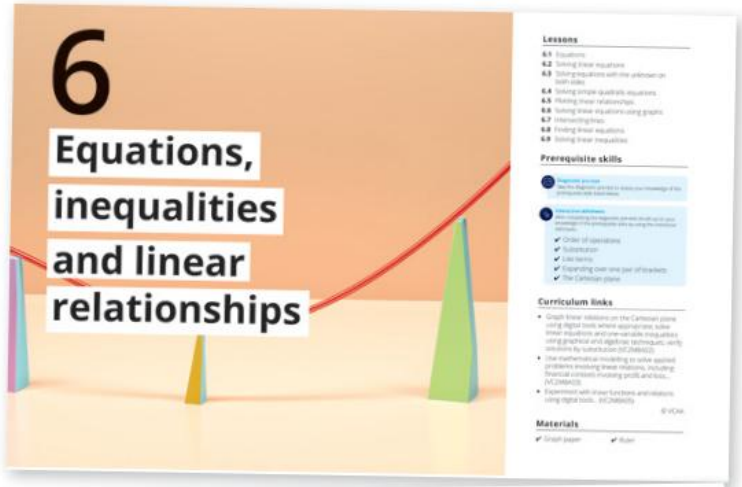
Introducing Oxford Maths 7–10 (Second edition)

Oxford Maths 7–10 Victorian Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

Key features of Student Books

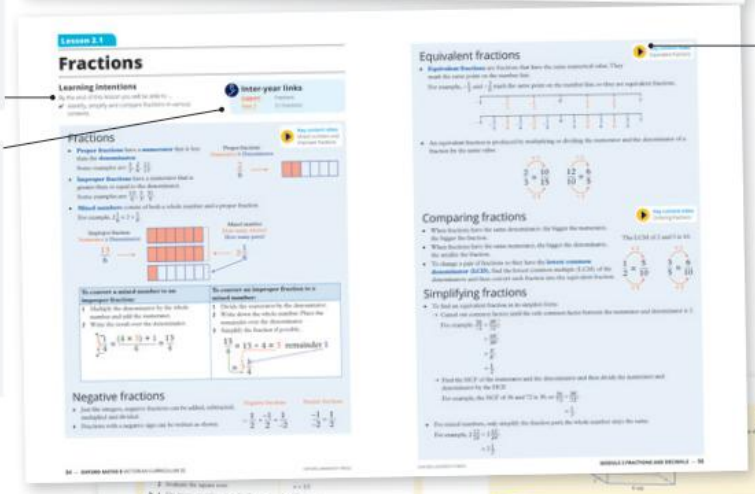
- > Complete access to all digital resources.
- > Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- > STEAM projects encourage inter-disciplinary thinking.
- > Semester reviews provide an opportunity to revise key concepts from each semester.
- > An illustrated glossary of newly introduced mathematics terms.



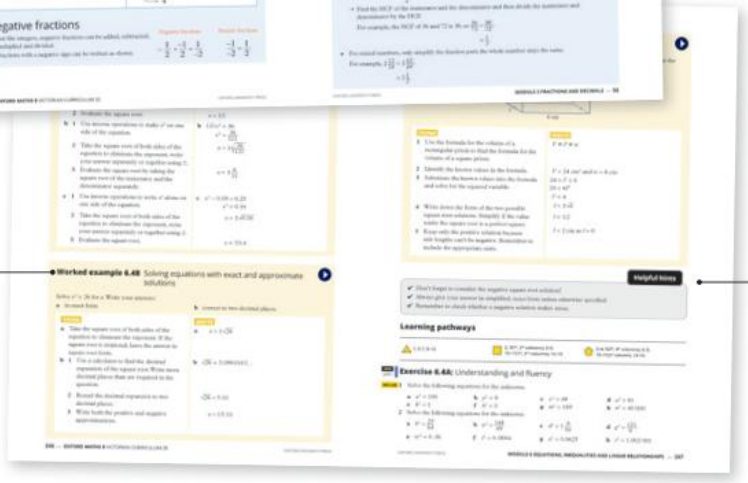
Each module opens with:
- **Prerequiste skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the Victorian Curriculum.
- **Materials** used to complete the exercises.

Learning intentions
- Signpost the foundational skills being developed in each lesson.

Inter-year links
- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.



New theory
- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life and aid understanding.



Worked examples
- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

Helpful hints
- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

Differentiated learning pathways

- Separate each exercise into three pathways, tailored for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every module.

Understanding and fluency

- Basic exercises dedicated to practicing key concepts.

Checkpoint

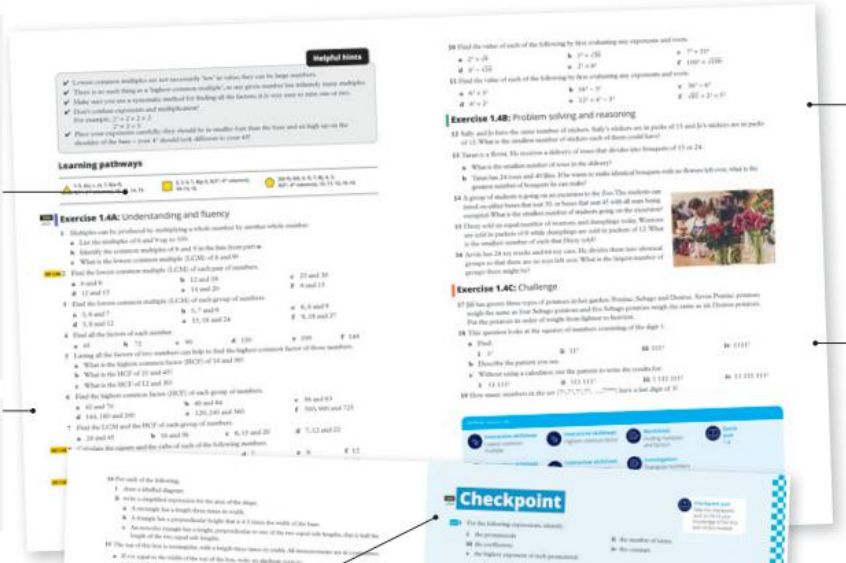
- A section at the middle of each module dedicated to surmising key skills and encourage memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

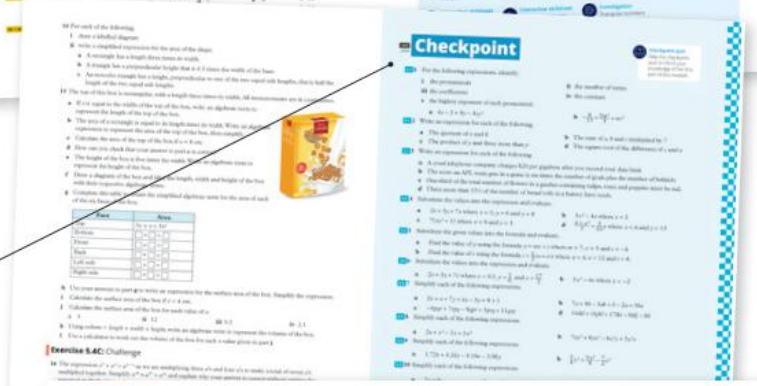
Module summary

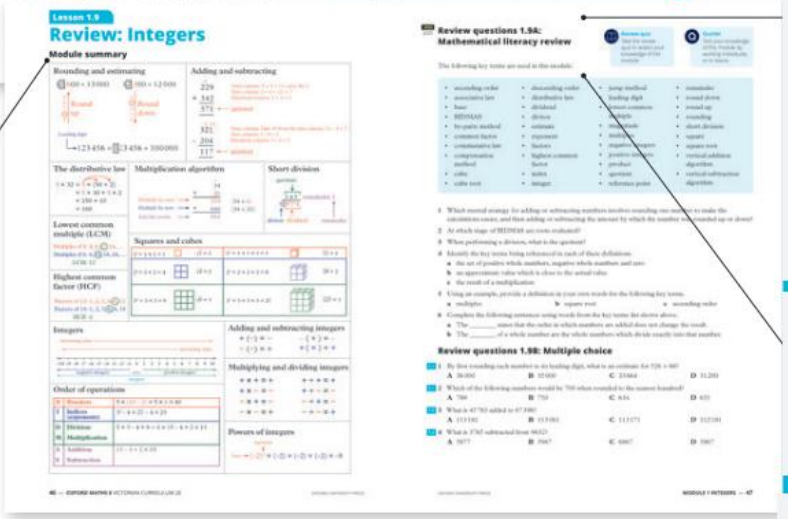
- Condense all the theory from each lesson into one accessible revision page.

Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.







Review questions

- Additional practice questions to further consolidate understanding at the end of each module.
- Reference to an online review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Mathematical literacy reviews

- A comprehensive literacy review assesses understanding of key terms.

Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem solve using design thinking.

Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.

Introducing Oxford Maths 7–10 (Second edition)

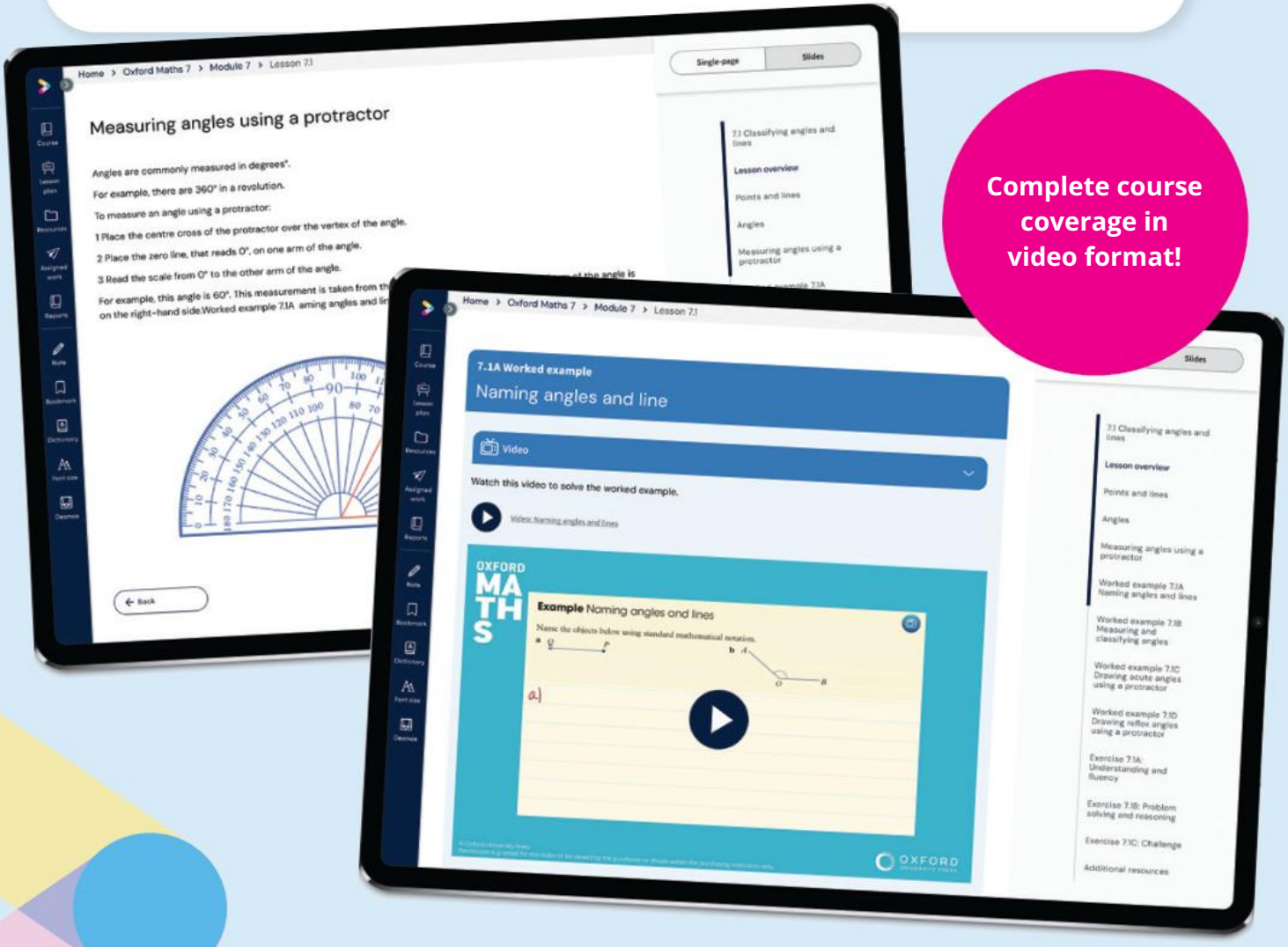
oxforddigital

This Student Book is supported by a range of digital resources and features for students and teachers on Oxford Digital.

Key features for students

Student digital access allows you to:

- **view** all the Student Book content in a fully accessible, reflowable format that's delivered in bite-sized chunks so you can work at you own pace
- **highlight**, **take notes**, **bookmark** pages, or define words with the built-in **Australian Oxford Dictionary**
- **watch** hundreds of short **content videos** and **worked example videos** to help you revise anything you don't understand, catch up on things you've missed or complete your homework
- **complete** thousands of interactive questions and quizzes as you work through the content and get the answers and results sent to you instantly
- **use** a suite of **Desmos calculators and graphing tools** to help you complete your work.

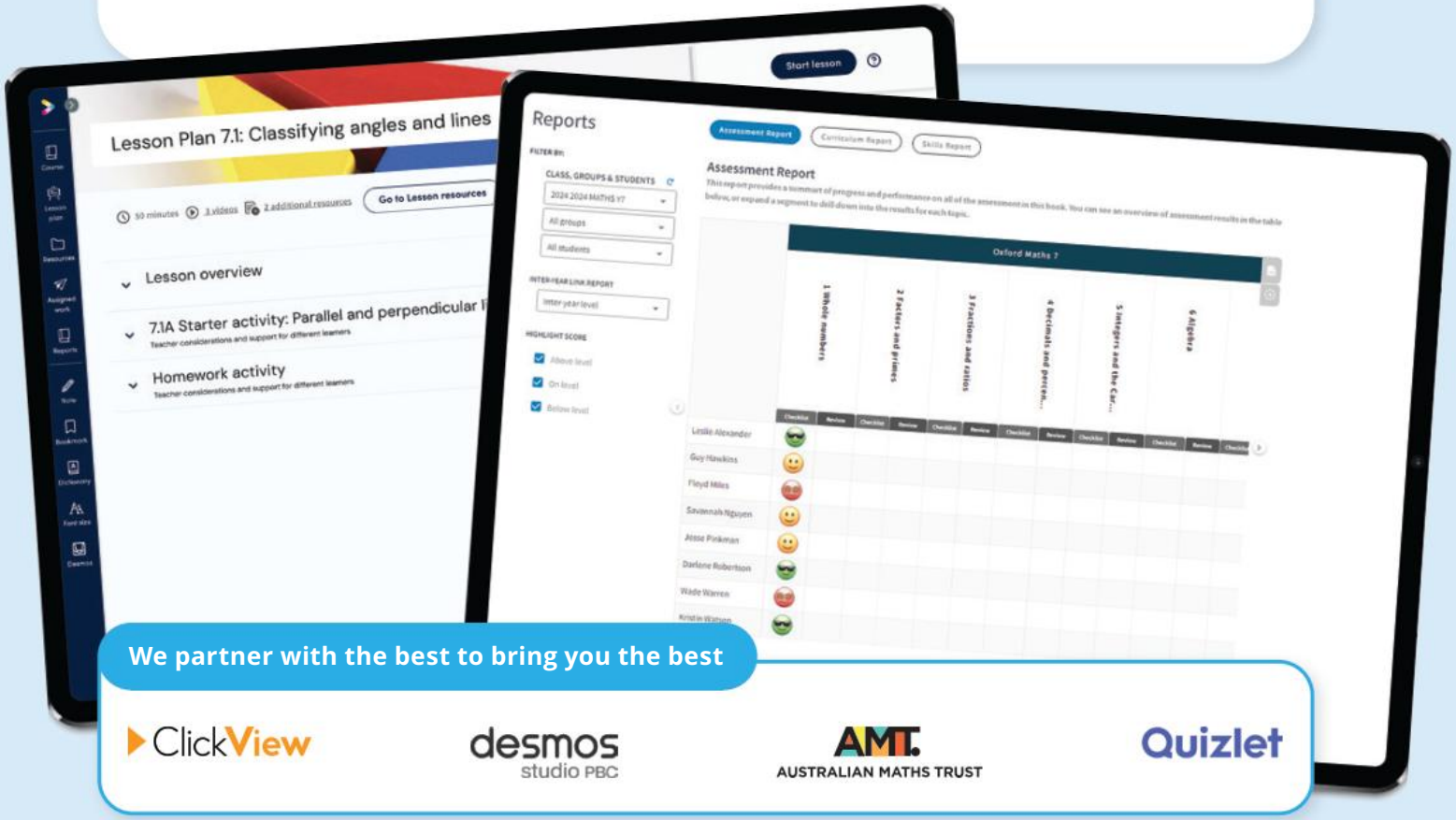


Complete course coverage in video format!

Key features for teachers

Teacher digital access allows you to:

- **elevate** your teaching and reduce planning and preparation time with **Live Lesson mode**. This Premium feature is an Australian first that lets you upgrade from traditional print-based lesson plans to **fully integrated, perfectly sequenced and timed interactive lessons complete with classroom activities** that drive engagement and participation
- **personalise** learning for every student and **differentiate** content based on student strengths and weaknesses. Assign support or extension resources to any student in any year level using **inter-year links**
- **identify gaps in understanding** with diagnostic quizzes that generate personalised learning pathway reports for every student based on ability. Learning pathway reports provide clear learning goals and exercises for every lesson including interactive resources designed to target misconceptions and give each student what they need to address them!
- **begin every lesson** with ready-made learning intentions and success criteria
- **revolutionise** your planning, marking and reporting with powerful analytics on student performance and progress:
  - **Assessment report** shows how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding
  - **Curriculum report** summarises student performance against specific curriculum content descriptors and curriculum codes
  - **Skills report** displays students' understanding of prerequisite skills in mathematics.



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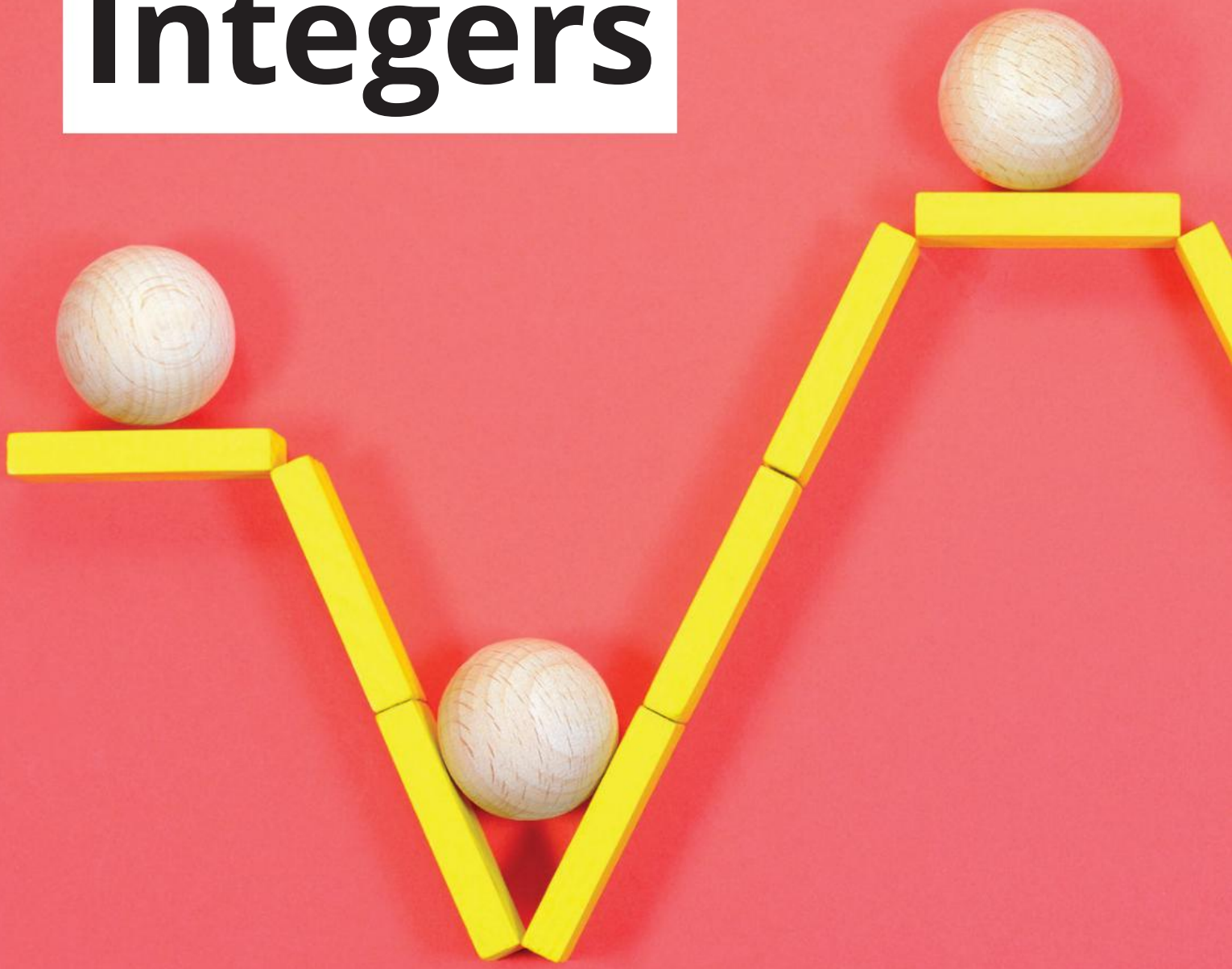
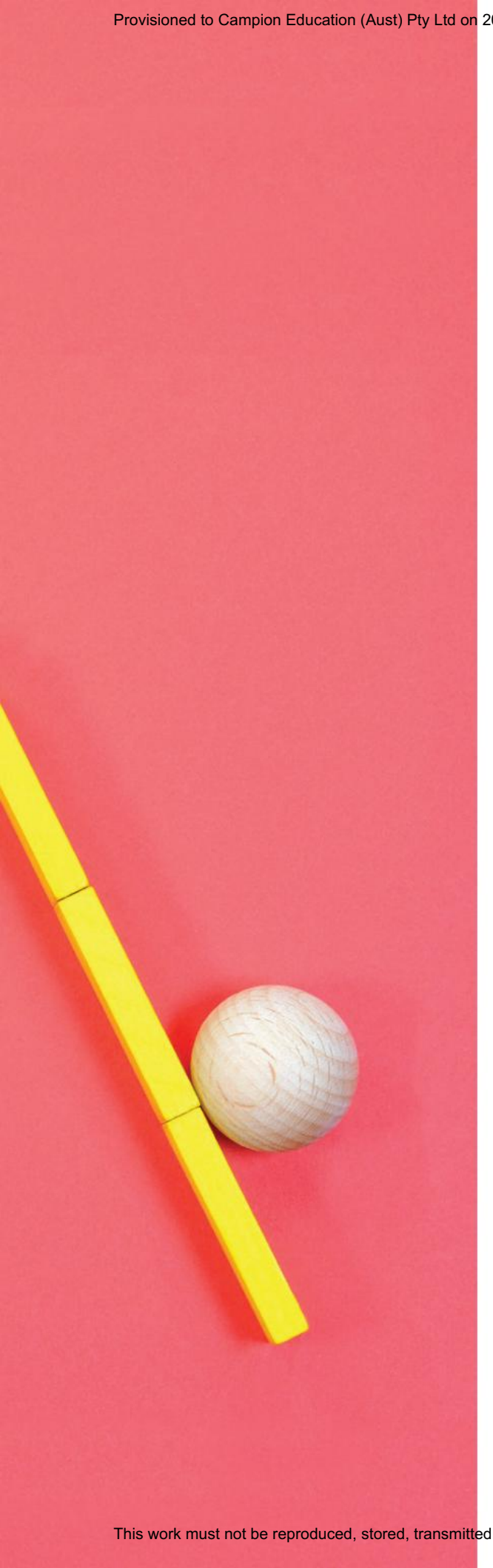
1

Integers





Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Adding and subtracting two-digit numbers
- ✔ Times tables
- ✔ Counting in multiples
- ✔ Number lines

Curriculum links

- Use the 4 operations with integers and with rational numbers, choosing and using efficient mental and written strategies, and digital tools where appropriate, and making estimates for these computations (VC2M8N04)

Materials

- ✔ Calculator

Lesson 1.1

Rounding and estimating

Learning intentions

By the end of this lesson you will be able to ...

✔ round whole numbers and estimate the results of simple calculations.

Inter-year links

Support	Place value
Year 7	1.1 Place value
Year 9	1.5 Scientific notation

Place value

Key content video
Place value

- Place value charts can be used to determine the value of each digit in a number.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	8	7	2	6	3

The place value of 3 is 3 000 000
The place value of 2 is 200 000
The place value of 8 is 80 000
The place value of 7 is 7000
The place value of 2 is 200
The place value of 6 is 60
The place value of 3 is 3

→ 3 2 8 7 2 6 3

Rounding

Key content video
Rounding and estimating

- **Rounding** a number means to replace it with an approximation that is simpler and easier to use. Using rounded numbers makes calculations less accurate.
  → 'Approximately equal to' is shown using the symbol '≈'.
- To round to the *nearest thousand*, place a box around the digit in the *thousands place*.
  → If the digit to the right of the box is equal to or greater than 5, **round up**.
  → If the digit to the right of the box is less than 5, **round down**.

$12\,600 \approx 13\,000$ (9, 8, 7, (6), 5 — Round up)

$17\,300 \approx 17\,000$ (4, (3), 2, 1, 0 — Round down)

- To round to a number's **leading digit**, place a box around the first digit and check if the second digit is less than, equal to or greater than 5.

Leading digit
$123\,456 = 123\,456 \approx 100\,000$

Estimations

- An **estimate** is an approximate value which is close to the actual value. We can estimate the answers of calculations by rounding.
  For example, a crowd of 10 213 on Day 1 and a crowd of 6156 on Day 2 at Bells Beach could be described as $10\,213 + 6156 \approx 10\,000 + 6000$, which is approximately 16 000 people. The actual number of people is 16 369.

Worked example 1.1A Rounding a number

Write an approximation for each number by rounding to the nearest hundred.

a 17 845 **b** 4565 **c** 992

THINK

a 1 Draw a box around the digit in the hundreds place.
2 Consider the digit to the right of the boxed digit. This digit is less than 5, so do not change the boxed digit.
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.

b 1 Draw a box around the digit in the hundreds place.
2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box.
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.

c 1 Draw a box around the digit in the hundreds place.
2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box. The boxed digit changes from 9 to 10, so write 0 in the boxed digit's place and add one to the place to the left.
3 Replace all digits to the right of the boxed digit with a zero.

WRITE

a 17 $\boxed{8}$45
17 $\boxed{8}$45

17 $\boxed{8}$00
17 845 ≈ 17 800

b 4$\boxed{5}$65
4$\boxed{5}$65

4$\boxed{6}$00
4565 ≈ 4600

c $\boxed{9}$92
$\boxed{9}$92

1$\boxed{0}$00
992 ≈ 1000

Worked example 1.1B Estimation by rounding

Estimate the result of each calculation by first rounding each number to its leading digit.

a 1307 + 4875 **b** 576 × 42 **c** 45 229 ÷ 5

THINK

1 Round each number to its leading digit.
2 Perform the calculation.
3 Write the answer using the '≈' symbol.

WRITE

a $\boxed{1}307 + \boxed{4}875 \approx 1000 + 5000$
$= 6000$
$1307 + 4875 \approx 6000$

b $\boxed{5}76 \times \boxed{4}2 \approx 600 \times 40$
$= 24\,000$
$576 \times 42 \approx 24\,000$

c $\boxed{4}5\,229 \div \boxed{5} \approx 50\,000 \div 5$
$= 10\,000$
$45\,229 \div 5 \approx 10\,000$

Helpful hints

- ✔ Remember that the place of a digit in a number indicates its value.
  For example, the 4 in 3452 represents 400, not 4.
- ✔ The digit 0 (zero) is important. It shows that there is nothing in that place value position and it keeps all other digits in the correct places.
- ✔ Be careful when rounding to a specific place value. If you place a box around the wrong digit your answer will not be correct. Use the place value chart to double-check if you are unsure.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1 000 000	100 000	10 000	1000	100	10	1

Learning pathways

 1–6, 7(a, c, e), 8(1st, 2nd columns), 9–11, 13–15, 18

 3–6, 7(b, d, f), 8(2nd, 3rd columns), 9–10(e–h), 12, 14, 16, 17, 19, 20

 3, 4, 6, 8(i–l), 10, 12, 16, 17, 20–22

ANS p520

Exercise 1.1A: Understanding and fluency

1 Decide whether each number is closer to 400 or 500.

a 438 **b** 477 **c** 462 **d** 455 **e** 433 **f** 449

2 Decide whether each number is closer to 6000 or 7000.

a 6789 **b** 6306 **c** 6010 **d** 6505 **e** 6880 **f** 6448

WE 1.1A **3** Consider the following numbers.

i 482 **ii** 6377 **iii** 56 026 **iv** 738 494 **v** 8075 **vi** 904 507

a Round each number to the nearest ten.

b Round each number to the nearest hundred.

4 Consider the following numbers.

i 36 428 **ii** 7510 **iii** 183 915 **iv** 50 703 **v** 6052 **vi** 825

a Round each number to the nearest thousand.

b Round each number to the nearest hundred.

5 Round each number to its leading digit.

a 77 **b** 42 **c** 81 **d** 347 **e** 160

f 555 **g** 909 **h** 2489 **i** 6902 **j** 22 117

6 Round each number to its leading digit.

a 94 **b** 99 **c** 952 **d** 92 949 **e** 9008 **f** 960

7 Give three examples of numbers that round to each of these approximations if you round to the leading digit.

a 40 **b** 700 **c** 3000 **d** 50 000 **e** 800 000 **f** 2 000 000

8 Calculate:

a 900 + 700 **b** 300 − 10 **c** 30 × 20

d 80 ÷ 4 **e** 500 × 40 **f** 800 + 3000

g 200 ÷ 5 **h** 6000 − 500 **i** 40 000 ÷ 20

j 7000 × 300 **k** 20 000 − 9000 **l** 5000 + 10 000

WE 1.1B **9** Estimate the result of each calculation by first rounding each number to its leading digit.

a 468 + 731 **b** 92 – 38 **c** 27 × 49 **d** 83 ÷ 2

e 582 × 17 **f** 245 + 6379 **g** 4512 ÷ 43 **h** 137 – 51

10 Estimate the result of each calculation by first rounding each number to its leading digit.

a 3694 ÷ 442 **b** 187 × 9364 **c** 7085 – 750 **d** 964 + 5803

e 8277 × 65 234 **f** 13 761 + 8036 **g** 94 113 ÷ 587 **h** 24 905 – 780

11 An office building has 35 782 panes of glass. Write an approximation for this number if you round to:

a the leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten

e the nearest ten thousand.

12 Russia is the largest country in the world, with an area of 17 098 242 km^2. Round this value to:

a its leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten thousand

e the nearest hundred thousand

f the nearest million.

Exercise 1.1B: Problem solving and reasoning

13 Ida is making a budget for her family's groceries. She decides to base her budget on the previous week's grocery bill, in which she spent the following:

- Supermarket $184
- Fish market $37
- Butcher $48
- Fruit and vegetable market $29

a Estimate to the nearest $10 the cost of the groceries for Ida's family last week.

b If Ida uses this figure for her budget, will it be an overestimation or underestimation?

14 Tina thinks 9648 rounded to its leading digit is 10 000. Ruby isn't sure, as she thinks the rounded number should have the same number of digits as the original number. Who is correct? Explain.

15 Daniel is saving for a quad bike. He has saved $60 per month for the past 28 months.

a Write an approximation for the number of months by rounding the value to its leading digit.

b Using your answer to part **a**, estimate the amount of money Daniel has saved.

c Does he have enough money to buy the quad bike?

d If he doesn't have enough, estimate for how many more months he needs to save.

16 Paving tiles cost $7 each. Antonia needs 385 tiles for her back patio.

a Write an approximation of the number of tiles Antonia needs by rounding to the leading digit.

b Estimate the cost of the tiles, using your answer to part **a**.

c Compare your estimate to the exact cost of the tiles.

17 The school fundraising committee wants to raise money by buying boxes of sunscreen from a wholesaler to sell for a profit. Each box has a wholesale price of $47.

a Estimate the number of boxes the committee could buy with $2000.

b If the committee used this estimate, without performing the exact calculation decide whether there would be money left over or money owed to the wholesaler. Explain your answer.

18 Australia has an area of 7 692 024 km^2 and Indonesia has an area of 1 904 560 km^2.

a Round each value to its leading digit.

b Use your approximations from part **a** to estimate:

i the difference in area between the two countries

ii the area of Australia compared to the area of Indonesia.

c Check how close your estimations from part **b** are to the exact answers.

19 People attending a State of Origin rugby league match entered the stadium through one of five gates. The number of people passing through each gate is shown.

Gate	A	B	C	D	E
Number of people	9361	10 758	12 196	8844	11 037

a Estimate the number of people attending the match by rounding the numbers at each gate to the leading digit.

b Calculate the exact number of people attending the match.

20 Australia is the sixth largest country in the world. The areas of the states and territories (including islands) of Australia are shown on the map.

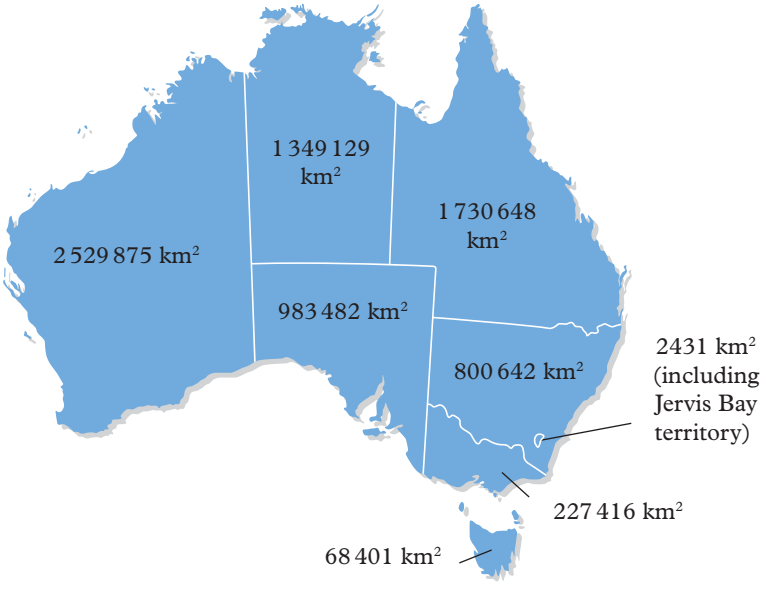
a Round each area to its leading digit.

b Estimate the area of Australia by adding up the approximate values from part **a**.

c The area of NSW, rounded to the nearest thousand, is 801 000 km^2. Round each of the other areas to the nearest thousand.

d Estimate the area of Australia by adding up the approximate values from part **c**.

e Compare the estimates you obtained in parts **b** and **d** with the actual area of Australia of 7 692 024 km^2.



Exercise 1.1C: Challenge

21 The area of Australia is 7 692 024 km^2 and the population at the end of 2020 was 25 694 393.

a Estimate the area per person for the entire population by rounding both numbers to the leading digit.

b New Zealand has an area of 268 021 km^2 and the population at the end of 2020 was 5 106 400. Estimate the area per person for New Zealand by rounding both numbers to the leading digit and compare it with that for Australia.

22 Estimate the number of words you say in a day.

Online resources:

- Interactive skillsheet: Place value
- Interactive skillsheet: Rounding whole numbers
- Quick quiz: 1.1
- Interactive skillsheet: Estimation by rounding
- Worksheet: Rounding whole numbers
- Investigation: An Australian road trip

Lesson 1.2

Adding and subtracting whole numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ add and subtract whole numbers.

Inter-year links

Support	Adding whole numbers
Year 7	1.2 Adding whole numbers
Year 7	1.3 Subtracting whole numbers

Key content video
Adding whole numbers

Addition properties

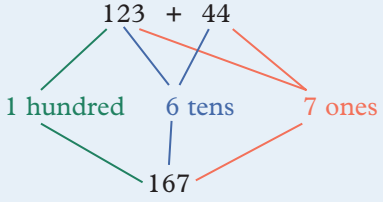
- The **associative law** of addition states that, regardless of how the numbers are grouped in an addition, the result does not change.
  For example, $(3 + 5) + 2 = 3 + (5 + 2)$.
- The **commutative law** of addition states that the order in which numbers are added does not change the result.
  For example, $2 + 8 = 8 + 2$.

Addition (+)	Subtraction (−)
• Sum	• Difference
• Plus	• Take away
• More than	• Less than
• Increase	• Fewer
• Total	• Reduce
• Together	• Minus

Mental strategies

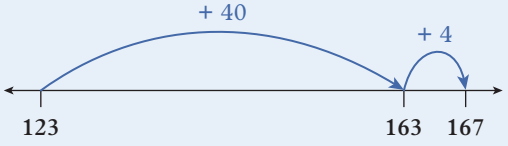
- The **by-parts method** involves adding the digits in each place value separately.

For example, $123 + 44 = 100 + (20 + 40) + (3 + 4)$
$= 100 + 60 + 7$
$= 167$



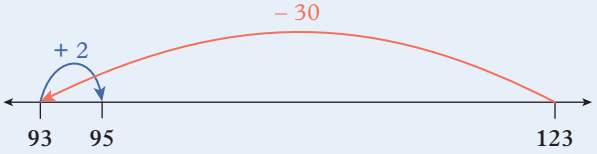
- The **jump method** involves breaking down one of the given numbers, then adding or subtracting each part of the number in stages.

For example, $123 + 44 = 123 + 40 + 4$
$= 163 + 4$
$= 167$



- The **compensation method** involves rounding one number to make the calculation easier. The amount by which the number was rounded up or down is then added or subtracted.

For example, $123 - 28 = 123 - 30 + 2$
$= 93 + 2$
$= 95$



Vertical addition algorithm

- Start by adding the digits in the ones column, then add the digits in the tens column, then the hundreds column and so on.

$$\begin{array}{r} {\scriptstyle 1} \\ 472 \\ +\ 846 \\ \hline 1318 \end{array}$$

Ones column: 2 + 6 = 8
Tens column: 7 + 4 = 11, carry the 10
Hundreds column: 4 + 8 + (1) = 13
← Answer

Vertical subtraction algorithm

Key content video
Subtracting whole numbers

- Start by subtracting the digits in the ones column, then subtract the digits in the tens column, then the hundreds column, and so on.
  - → If the subtraction in a particular column cannot be done, take a value from the column to the left.

$$\begin{array}{r} {\scriptstyle 3\ 7\ 11} \\ 3\not{8}\not{1} \\ -\ 145 \\ \hline 236 \end{array}$$

Ones column: Take 10 from the tens column
11 − 5 = 6
Tens column: 7 − 4 = 3
Hundreds column: 3 − 1 = 2
← Answer

Worked example 1.2A Using mental strategies to add two numbers

Work out each sum using the mental strategy given in brackets.

a 282 + 317 (by-parts method)
b 51 + 39 (jump method)
c 48 + 68 (compensation method)

THINK

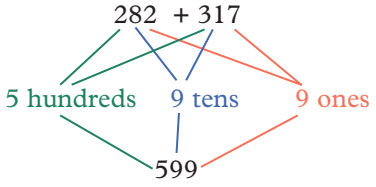
a **1** Add the hundreds together.
2 Add the tens together.
3 Add the ones together.
4 Combine these sums for the final answer.

b **1** Choose a starting number, then break down the second number into tens and ones.
2 Add the tens to the starting number, followed by the ones.

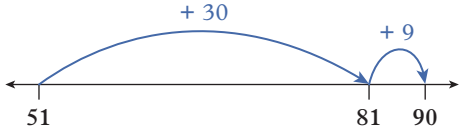
c **1** Choose a starting number, then round the second number to the nearest 10.
2 Add the rounded number to the starting number, then adjust for rounding.

WRITE

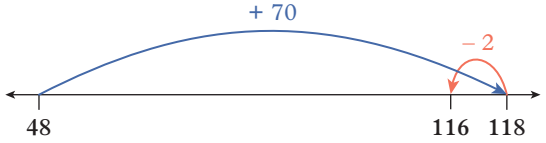
a 282 + 317 = 599



b 51 + 39 = 90



c 48 + 68 = 116



Worked example 1.2B Using the vertical addition algorithm

Calculate 781 + 965 using the vertical addition algorithm:
$$\begin{array}{r} 781 \\ +\ 965 \\ \hline \end{array}$$

THINK

1. Add the digits in the ones column: 1 + 5 = 6.
2. Add the digits in the tens column: 8 + 6 = 14.
   Write 4 in the tens column of the answer and write 1 above the hundreds column.
3. Add the digits in the hundreds column: 1 + 7 + 9 = 17.

WRITE

$$\begin{array}{r} {\scriptstyle 1} \\ 781 \\ +\ 965 \\ \hline 1746 \end{array}$$

Worked example 1.2C Using mental strategies to subtract two numbers

Work out each difference using the mental strategy given in brackets.

a 112 – 67 (jump method)

b 200 – 89 (compensation method)

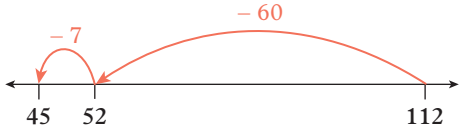
THINK

a
1. Start with the first number, then break down the second number into tens and ones.
2. Subtract the tens from that number, followed by the ones.

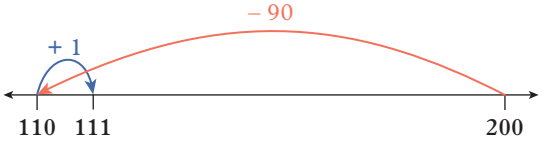
b
1. Start with the first number, then round the second number to the nearest 10.
2. Subtract the rounded number from the starting number, then adjust for rounding.

WRITE

a 112 – 67 is 45.



b 200 – 89 is 111.



Worked example 1.2D Using the vertical subtraction algorithm

Calculate 678 – 93 using the vertical subtraction algorithm:
$$\begin{array}{r} 678 \\ -\ 93 \\ \hline \end{array}$$

THINK

1. Subtract the digits in the ones column: 8 – 3 = 5.
2. Subtract the digits in the tens column: 7 – 9.
   Take 1 from the hundreds column by reducing the hundreds to 5 and increasing the tens to 17.
   Subtract the digits in the tens column: 17 – 9 = 8.
3. Subtract the digits in the hundreds column: 5 – 0 = 5. Recall that if there is no digit in a place then that place has a value of zero.

WRITE

$$\begin{array}{r} {\scriptstyle 5\ 17} \\ 6\not{7}8 \\ -\ 93 \\ \hline 585 \end{array}$$

Helpful hints

- ✔ Don't forget that addition is commutative – reordering sums can make them a lot easier!
- ✔ Subtraction is not commutative. Unlike addition problems, you cannot change the order of the numbers in subtraction problems. You must work through subtractions from left to right.
- ✔ Make sure that your place value columns are well aligned so that you add or subtract the correct digits.
- ✔ Read each question carefully. You may find some questions easy to calculate in your head.

Learning pathways

1–3, 4(a–c), 5(a, c), 6, 7, 8–10(b, c), 11, 13, 15	1a–d(ii, iii), 3(c, d), 4(d–f), 5(b, d), 6a–d(ii, iii), 8–10, 12, 14, 15, 17, 18(a)	1a–d(iv), 4(e, f), 5(c, d), 6a–d(iv), 9(c, d), 10(c, d), 12, 14–19

ANS p520

Exercise 1.2A: Understanding and fluency

WE 1.2A **1** There are many different ways to add two whole numbers.

a Use the by-parts method to work out each sum.

i 34 + 63 **ii** 89 + 72 **iii** 654 + 325 **iv** 1243 + 9654

b Use the jump method to work out each sum.

i 647 + 72 **ii** 768 + 51 **iii** 456 + 128 **iv** 3452 + 139

c Use the compensation method to work out each sum.

i 24 + 99 **ii** 472 + 56 **iii** 658 + 465 **iv** 7629 + 475

d Use any mental strategy to work out each sum.

i 87 + 90 **ii** 489 + 650 **iii** 8765 + 59 **iv** 5983 + 587

WE 1.2B **2** Calculate each sum using the vertical addition algorithm.

a $\begin{array}{r} 243 \\ +\ 715 \\ \hline \end{array}$ **b** $\begin{array}{r} 538 \\ +\ 154 \\ \hline \end{array}$ **c** $\begin{array}{r} 758 \\ +\ 461 \\ \hline \end{array}$ **d** $\begin{array}{r} 697 \\ +\ 805 \\ \hline \end{array}$

3 Use the vertical addition algorithm to calculate each of the following.

a 4765 + 476 **b** 7410 + 3065 **c** 16 239 + 2633 **d** 98 543 + 23 987

4 Use the vertical addition algorithm to calculate each of the following.

a 456 + 280 + 487 **b** 876 + 387 + 594 **c** 986 + 386 + 2769

d 56 + 875 + 3987 **e** 6954 + 389 + 23 **f** 7632 + 9520 + 3198

5 Evaluate:

a 56 871 + 34 981 **b** 54 870 + 29 528 + 43 679

c 5498 + 2873 + 8921 + 7629 **d** 397 + 287 + 888 + 396 + 492

WE 1.2C **6** There are many different ways to subtract one whole number from another whole number.

a Use the jump method to work out each difference.

i 93 – 30 **ii** 126 – 39 **iii** 145 – 51 **iv** 764 – 71

b Use the compensation method to work out each difference.

i 87 – 54 **ii** 763 – 68 **iii** 874 – 618 **iv** 433 – 67

c Use any mental strategy to work out each difference.

i 130 – 70 **ii** 5600 – 410 **iii** 390 – 180 **iv** 568 – 456

d Use any mental strategy to work out each problem.

i 79 + 39 – 56 **ii** 750 + 830 – 690 **iii** 78 + 565 – 120 **iv** 350 – 90 + 876

WE 1.2D **7** Calculate each difference using the vertical subtraction algorithm.

a $\begin{array}{r} 438 \\ -\ 399 \\ \hline \end{array}$ **b** $\begin{array}{r} 852 \\ -\ 576 \\ \hline \end{array}$ **c** $\begin{array}{r} 2356 \\ -\ 976 \\ \hline \end{array}$ **d** $\begin{array}{r} 3765 \\ -\ 387 \\ \hline \end{array}$

8 Use the vertical subtraction algorithm to calculate each of the following.

a 487 – 458 **b** 3865 – 376 **c** 4652 – 234 **d** 6798 – 478

9 Use the vertical subtraction algorithm to calculate each of the following.

a 7654 – 488 **b** 7421 – 6423 **c** 75 638 – 689 **d** 86 408 – 42 766

10 Calculate:

a 566 + 238 – 328 **b** 2776 + 5409 – 3111
c 76 409 + 45 629 – 83 888 **d** 6543 – 2389 + 4319

Exercise 1.2B: Problem solving and reasoning

11 Yiyan is the manager of five city car parks. The capacities of the car parks are 356, 821, 489, 398 and 450 cars. What is the total number of cars that can use Yiyan's car parks?

12 The population of Victoria is 6 503 941 and the population of Melbourne is 4 917 750.

a How many Victorians live outside Melbourne?
b How many people will have to move to Victoria for the Victorian population to be exactly 7 million?
c Within a particular year, 32 000 people move out of Victoria and 59 000 people move in. What will be the population of Victoria at the end of the year?

13 Two local libraries decide to merge their book collections to form a mega-library. Spring Hill Library has 27 356 books and Dove St Library has 54 237 books.

a How many books are there in total?
b The mega-library finds that there are two copies of 28 345 books so it is decided one copy of each will be given away. After the giveaway, how many books will there be in the new library?

14 Raf and Mac go on a road trip around Victoria. They combine expenses for the trip and Raf keeps a tally of the distances they travel and the money they spend.

Stage	Distance travelled (km)	Accommodation (2 nights)	Expenses (petrol, food, etc.)
Melbourne to Warrnambool	257	$292	$128
Warrnambool to Horsham	222	$320	$175
Horsham to Mildura	304	$228	$184
Mildura to Echuca	385	$248	$162
Echuca to Melbourne	222	$0	$85

a What was the total distance Raf and Mac travelled on their road trip?
b How much in total did they spend on accommodation?
c What were their total expenses, not including accommodation?
d Raf and Mac had each saved up $1000 for the trip. How much was left over when they returned to Melbourne?

15 Aya has 2437 subscribers to her YouTube channel and Akuac has 3762 subscribers.

a What is the total number of subscribers between the two girls?

b Aya and Akuac decide to combine their channels into one new channel, thus combining their subscribers.

If there are 1562 people who initially subscribed to both channels, how many subscribers will the new combined channel have? (Assuming channels can be merged and subscribers are not counted twice.)

16 The Australian Alps Walking Track goes from Mt Hotham to Mt Bogong in Victoria's high country. Jai and Rudra walk the track over 4 days in the summer. They walk the following distances per day: 13 km, 17 km, 14 km and 16 km.

a How long is the Australian Alps Walking Track?

b Mt Bogong is 1986 m above sea level and Mt Hotham is 1861 m above sea level.

What is the difference in height between the two mountains?

17 Shannon's shoe store has enough space for 568 pairs of shoes. If Shannon has 483 pairs of shoes in her store, how many does she need to order to be at full capacity?

Exercise 1.2C: Challenge

18 Look for a pattern to help you answer the following questions.

a What is the difference between the sum of the first five odd numbers (starting from 1) and the sum of the first five even numbers (starting from 2)?

b What is the difference between the sum of the first 2000 odd numbers (starting from 1) and the first 2000 even numbers (starting from 2)?

19 A cryptarithm is a type of mathematical puzzle in which numbers have been replaced by letters. There are six solutions to the cryptarithm below. How many of them can you find?

$$\begin{array}{r} \text{SEED} \\ +\text{WATER} \\ \hline \text{FRUIT} \end{array}$$

Online resources:

Interactive skillsheet
Adding and subtracting whole numbers

Investigation
Palindromic numbers

Quick quiz
1.2

Lesson 1.3

Multiplying and dividing whole numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ multiply and divide whole numbers.

Inter-year links

Support	Multiplying whole numbers
Year 7	1.4 Multiplying whole numbers
Year 7	1.5 Long multiplication
Year 7	1.6 Dividing whole numbers

Multiplication

Key content video
Multiplying whole numbers

- Repeated addition can be reduced to multiplication of **factors** to give a **product**.

$$\underbrace{3 + 3}_{\text{repeated addition}} = \overbrace{2 \times 3}^{\text{factors}} = \underbrace{6}_{\text{product}}$$

Multiplication strategies

- The associative law of multiplication states that, regardless of how the numbers are grouped, the result of a multiplication does not change.

  For example, $(2 \times 4) \times 6 = 2 \times (4 \times 6)$.
- The commutative law of multiplication states that the order in which numbers are multiplied does not change the results.

  For example, $3 \times 7 = 7 \times 3$.
- The **distributive law** states that multiplication can be 'distributed' across a bracket. This means that large numbers can be broken down into a sum of smaller numbers, each of which can then be multiplied separately.

  For example,

$$\begin{aligned} 5 \times 32 &= 5 \times (30 + 2) \\ &= 5 \times 30 + 5 \times 2 \\ &= 150 + 10 \\ &= 160 \end{aligned}$$

Multiplication algorithm

Key content video
Long multiplication

- When two numbers are multiplied, the first number is multiplied by the individual digits (from right to left) of the second number, considering the place value of the digits (ones, then tens, etc.). Then, the products are added together.

$$\begin{array}{rrl} & {\scriptstyle 2}\,34 & \\ \times & 26 & \\ \hline & 204 & (34 \times 6) \\ + & 680 & (34 \times 20) \text{ Don't forget the zero(s)} \\ \hline & 884 & \end{array}$$

Multiply by ones → 204 (34 × 6)
Multiply by tens → + 680 (34 × 20) Don't forget the zero(s)
Add the results → 884

Division

Key content video
Dividing whole numbers

- When performing a division, the **dividend** is the number that is being divided by another number.
- The **divisor** is the number by which the dividend is being divided.
- The **quotient** is the whole-number result of the division.
- The **remainder** is the part of the dividend that is left over when the number is divided by the divisor. The remainder can be zero.

$$13 \div 6 = 2 \text{ remainder } 1$$

dividend, divisor, quotient, remainder

Short division

- **Short division** is an algorithm used to simplify a division problem by breaking it down into easy steps.
- Start by dividing the first digit of the dividend by the divisor.

quotient: 1 4 5 remainder 1

$6\overline{)8^{2}7^{3}1}$

divisor, dividend, remainder

Multiplication (×)	Division (÷)
• Product	• Quotient
• Times	• Divide
• Multiply	• Shared
• Groups of	• Distributed

Worked example 1.3A Using strategies to multiply numbers

Work out each product using the multiplication strategy given in brackets.

a 9×34 (distributive law)

b $12 \times 4 \times 2$ (associative law)

THINK

a 1 Break down the second number into tens and ones.

2 Multiply each part of the second number by the first number.

3 Add the results together.

b 1 Use brackets to group pairs of numbers that are easy to multiply together and/or produce round numbers.

2 Multiply the numbers grouped in the brackets.

3 Multiply the result by the third number.

WRITE

a $9 \times 34 = 9 \times (30 + 4)$

$= 9 \times 30 + 9 \times 4$

$= 270 + 36$

$= 306$

b $12 \times 4 \times 2 = 12 \times (4 \times 2)$

$= 12 \times 8$

$= 96$

Worked example 1.3B Using the multiplication algorithm

Calculate the product using the multiplication algorithm: $\begin{array}{r} 359 \\ \times\ 72 \\ \hline \end{array}$

THINK

1. Set out the multiplication problem by lining up the digits according to their place value.
2. Multiply the first number by the ones of the second number: 359 by 2 ($359 \times 2 = 718$).
3. Multiply the first number by the tens of the second number: 359 by 70 ($359 \times 70 = 25\,130$).
4. Add the products and write the final answer at the bottom of the calculation.

WRITE

$$\begin{array}{r} {\scriptstyle 4\ 6} \\ {\scriptstyle 1\ 1} \\ 359 \\ \times\ \ 72 \\ \hline 718 \quad (359 \times 2) \\ +\ 25130 \quad (359 \times 70) \\ \hline 25848 \end{array}$$

$359 \times 72 = 25\,848$

Worked example 1.3C Using short division

Use short division to calculate the quotient and remainder of $1257 \div 8$.

THINK

1. Set up the problem using the division algorithm. The divisor is 8 and the dividend is 1257.
2. How many 8s go into 1? Write a zero above the 1.
3. How many 8s go into 12? Write 1 above the 2 on the quotient line. Put the remainder 4 in front of the next digit, 5.
4. How many 8s go into 45? Write a 5 above the 5. Put the remainder 5 in front of the next digit, 7.
5. How many 8s go into 57? Write 7 above the 7 on the quotient line. Work out the remainder. $7 \times 8 = 56$, so the remainder is $57 - 56 = 1$.

WRITE

$$\begin{array}{r} 0\ 1\ 5\ 7 \\ 8\overline{)1\ 2^{4}5^{5}7} \end{array}$$ remainder 1

$1257 \div 8 = 157$ remainder 1

Helpful hints

- ✔ When using the distributive law, don't forget to maintain place values using zeros.
  For example, $32 \times 3 = (30 \times 3) + (2 \times 3)$
  $32 \times 3 \neq (3 \times 3) + (2 \times 3)$
- ✔ Carefully line up digits by place value when setting up long multiplication problems to avoid making errors when adding the products together.
- ✔ Remember that division goes from left to right.
- ✔ BIDMAS is a helpful acronym when considering the order of operations for an equation.

Learning pathways

1–3(1st, 2nd columns), 4(a, c), 5–7, 8–9(a–c), 10-13, 17	1–3(2nd, 3rd columns), 4(b, d), 6(c, f), 7, 8–9(d–f), 10(c, d), 11, 14, 15, 17, 19	1(j–l), 2(g, i), 4(d), 6(d–f), 8–9(e, f), 10(c, d), 11, 14, 16, 18–21

ANS p521

Exercise 1.3A: Understanding and fluency

WE 1.3A **1** Use the distributive law to calculate each of the following.

a 3×42	**b** 6×102	**c** 203×7	**d** 305×9
e 17×98	**f** 83×99	**g** 24×95	**h** 41×19
i 62×101	**j** 29×1001	**k** 11×97	**l** 15×999

2 Use mental strategies to calculate each of the following.

a $17 \times 8 \times 3$	**b** $21 \times 5 \times 6$	**c** $3 \times 74 \times 2$	**d** $66 \times 8 \times 10$
e $4 \times 6 \times 25$	**f** $921 \times 5 \times 20$	**g** $35 \times 4 \times 5$	**h** $29 \times 250 \times 4$
i $200 \times 186 \times 5$			

3 Evaluate each product.

a 9×70	**b** 12×800	**c** 11×700	**d** 126×50
e 356×900	**f** 2765×3000	**g** 2763×9000	**h** $1674 \times 50\,000$

4 Evaluate each product.

a $60 \times 40 \times 300$	**b** $80 \times 700 \times 8000$	**c** $2400 \times 20 \times 200$	**d** $15\,000 \times 500 \times 50$

WE 1.3B **5** Multiply the following pairs of numbers using the multiplication algorithm.

a 589×76	**b** 498×49	**c** 4598×23
d 4623×32	**e** 9245×72	**f** 9430×520

6 Multiply the following pairs of numbers using the multiplication algorithm.

a 387×860	**b** 378×598	**c** 638×496
d 3769×458	**e** 4287×352	**f** 5923×5030

7 Consider the following products.

i 8462×95 **ii** 3299×87 **iii** $95\,687 \times 625$ **iv** $78\,438 \times 8345$

a Estimate these products by first rounding each number to its leading digit, then multiplying.

b Check how close your estimations are to the exact results by performing the calculations.

WE 1.3C **8** Use short division to calculate each quotient and remainder.

a $8\overline{)463}$	**b** $6\overline{)348}$	**c** $7\overline{)160}$
d $5\overline{)375}$	**e** $9\overline{)689}$	**f** $4\overline{)5682}$

9 Use short division to calculate each quotient and remainder.

a $5824 \div 4$	**b** $8399 \div 6$	**c** $4210 \div 3$
d $84\,276 \div 5$	**e** $56\,289 \div 9$	**f** $1489 \div 7$

10 Perform each division using short division.

a $\frac{143}{6}$ **b** $\frac{653}{8}$ **c** $\frac{246}{7}$ **d** $\frac{489}{3}$

Exercise 1.3B: Problem solving and reasoning

11 The average of a set of scores is the total sum of the scores divided by the number of scores. For example, the average of 25, 30 and 23 is $(25 + 30 + 23) \div 3$ or $\frac{25 + 30 + 23}{3}$.

a Find the sum of 19, 22, 24 and 27.

b Divide your answer from part **a** by 4 to find the average of 19, 22, 24 and 27.

12 The commutative law relates to the order of operations.

a Calculate:

i $3 + 6$ **ii** $6 + 3$

b Does the order matter when you add two numbers? Try two more examples.

c Does subtraction obey the commutative law? Try these calculations.

i $6 - 3$ **ii** $3 - 6$

d Does multiplication obey the commutative law? Try these.

i 6×3 **ii** 3×6

e Does division obey the commutative law? Try these.

i $6 \div 3$ **ii** $3 \div 6$

13 The associative law also relates to the order of operations.

a Calculate:

i $(5 + 6) + 7$ **ii** $5 + (6 + 7)$ **iii** $(5 + 7) + 6$

b Does it matter in which order you add three numbers? Try two more examples.

c Do subtraction, multiplication and division obey the associative law? Try some examples to help you decide.

14 The commutative and associative laws can make mental calculations easier.

a To calculate $54 + 118 + 16$, you can add any two of the numbers together first.

i Which two numbers are easiest to add together first? Explain.

ii Perform the calculation.

b To calculate $87 \times 25 \times 4$, you can multiply any two of the numbers together first.

i Which two numbers are easiest to multiply together first? Explain.

ii Perform the calculation.

c Explain how these laws can make some calculations easier to perform.

15 This question looks at the distributive law.

a Calculate these, then compare your answers.

i $4 \times (5 + 6)$

ii $4 \times 5 + 4 \times 6$

b Calculate these, then compare your answers.

i $6 \times (10 - 2)$

ii $6 \times 10 - 6 \times 2$

c Parts **a** and **b** are examples of the application of the distributive law. Describe this law.

d Show how the distributive law works by providing two more examples.

16 The distributive law can be used to make some multiplication calculations easier to perform.

a The calculation 16×24 can be written as $16 \times (20 + 4)$ or $16 \times 20 + 16 \times 4$.

i Calculate 16×20.

ii Calculate 16×4.

iii Add your results and write the answer to 16×24.

b The calculation 45×998 can be written as $45 \times (1000 - 2)$ or $45 \times 1000 - 45 \times 2$.

i Calculate 45×1000.

ii Calculate 45×2.

iii Subtract your results and write the answer to 45×998.

c Explain how the distributive law can be used to make some multiplication calculations easier to perform.

17 Ms Lee spends $480 per school year on coffee from The Blue Cup Café.

a If the school year is 40 weeks, how much does Ms Lee spend per week?

b If The Blue Cup Café has a special teacher price of $4 per cup, how many times per week does Ms Lee buy coffee?

c Mr Lin prefers the cheaper coffee at the service station over the road. He pays $2 per cup and buys coffee five days per week.

i How much does Mr Lin pay for coffee in one week?

ii How much does Mr Lin pay for coffee in a school year?

18 The Year 8 class captains are tasked with providing icy poles for the whole school at the swimming sports carnival. There are 1289 students at the school.

a How many packs of icy poles do the class captains have to buy if there are 20 in each pack, so that no student misses out?

b How many icy poles will be left over if every student has one?

c If the packs of icy poles cost $3 each, how much money do the class captains need?

19 Astrid is planning to hire a car for a road trip from Melbourne to Cairns. The distance she plans to travel is 2814 km. The car hire company informs Astrid that the car can travel an average of 7 kilometres per litre of fuel.

a Use this information to calculate how many litres of fuel Astrid will need to complete her journey.

b If Astrid's fuel tank is 50 litres, how many full tanks of fuel will she need for her journey?

c Astrid wants to make sure she overestimates the amount of money she will need to spend on fuel, so she rounds the price of fuel up to $3 per litre. Using this estimate, how much money should Astrid budget for her fuel expenses?

Exercise 1.3C: Challenge

20 Darcy makes a number of mini spring rolls. After eating one, she gives half the remainder to her sister. After eating another spring roll, she gives half of what is left to her brother. Darcy now has only five spring rolls left. How many did she start with?

21 A recipe for eight biscuits requires 20 g butter, 30 g sugar and 40 g rolled oats.

a How many grams of sugar do I need if I want to make 16 biscuits?

b How many biscuits can I make if I have 140 g butter, 150 g sugar and 160 g rolled oats?

Online resources:

Interactive skillsheet
Multiplying and dividing whole numbers

Investigation
Factorials

Quick quiz
1.3

Lesson 1.4

Multiples, factors, exponents and roots

Learning intentions

By the end of this lesson you will be able to ...

✔ find lowest common multiples and highest common factors, and evaluate squares, cubes and roots

Inter-year links

Support	Multiples and factors
Year 7	2.3 Multiples and the lowest common multiple
Year 9	1.1 Exponents

Multiples

Key content video
Multiples and the lowest common multiple

- The **multiples** of a whole number are the numbers produced by multiplying that number by another whole number.
  For example, some multiples of 3 are:
  $3 \times 1 = 3$
  $3 \times 2 = 6$
  $3 \times 3 = 9$
- The **lowest common multiple (LCM)** of two or more numbers is the common multiple of the numbers with the smallest value.

Multiples of 4: 4, 8, 12, 16, ...
Multiples of 6: 6, 12, 18, 24, ...
LCM: 12

Factors

Key content video
Factors and the highest common factor

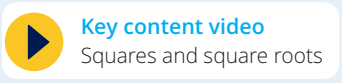
- The **factors** of a whole number are the whole numbers which divide exactly into that number.
  For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- The divisibility rules are helpful when looking for factors of large numbers.

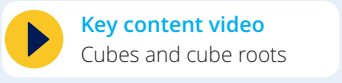
÷ 2 The last digit of the number is even.	÷ 3 The sum of all the digits in the number is divisible by 3.	÷ 4 The number made by the last two digits is divisible by 4.
÷ 5 The last digit is 5 or 0.	÷ 6 The number is divisible by 2 and 3.	÷ 7 There is no easy trick for 7.
÷ 8 The number made by the last three digits is divisible by 8.	÷ 9 The sum of all the digits in the number is divisible by 9.	÷ 10 The last digit is 0.

- The **highest common factor (HCF)** of two or more whole numbers is the **common factor** of the numbers with the greatest value.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18
HCF: 6

Exponents





- Repeated multiplications can be represented using exponents.

exponent
base → $2^3 = 2 \times 2 \times 2$

- The **base** is the number that is repeatedly multiplied.
- The **exponent** (or **index**) represents the number of times the base is used in the multiplication.
- The **square** of a number is the result of multiplying that number by itself. Squaring a number is the same as raising it to a power (or exponent) of 2 and is indicated by a superscript 2.

  For example, $5^2 = 5 \times 5 = 25$.
- The **cube** of a number is the result of multiplying that number by itself twice. Cubing a number is the same as raising it to a power (or exponent) of 3 and is indicated by a superscript 3.

  For example, $4^3 = 4 \times 4 \times 4 = 64$.

Roots

- To find the **square root** of a number, find the number that when squared, or has 'an exponent of 2', results in the original number.
- For positive numbers, there is an inverse relationship between squares and square roots.

  For example, $\sqrt{9} = 3$ because $3^2 = 9$.
- To find the **cube root** of a number, find the number that when cubed, or has 'an exponent of 3', results in the original number.
- There is an inverse relationship between cubes and cube roots.

  For example, $\sqrt[3]{8} = 2$ because $2^3 = 8$.
- When multiplying a number by a square root or cube root, we can write the number in front of the root without using a multiplication sign.

  For example, $5\sqrt{9}$ means the same as $5 \times \sqrt{9}$

Squares	Square roots	Cubes	Cube roots
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^2 = 49$	$\sqrt{49} = 7$	$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^2 = 64$	$\sqrt{64} = 8$	$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^2 = 81$	$\sqrt{81} = 9$	$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^2 = 100$	$\sqrt{100} = 10$	$10^3 = 1000$	$\sqrt[3]{1000} = 10$

Worked example 1.4A Finding the LCM and HCF

Determine:

a the LCM of 14 and 35

b the HCF of 24 and 38.

THINK

a **1** List the multiples of 14 and the multiples of 35.

2 Draw a box around the multiple with the lowest value that appears in both lists (the LCM).

b **1** List all the factors of 24 and all the factors of 38.

2 Draw a box around the factor with the highest value that appears in both lists (the HCF).

WRITE

a Multiples of 14: 14, 28, 42, 56, 70, …
Multiples of 35: 35, 70, 105, 140, …
The LCM is 70.

b Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 38: 1, 2, 19, 38
The HCF is 2.

Worked example 1.4B Calculating the square and cube

Calculate:

a the square of 5

b the cube of 5.

THINK

1 To 'square' 5, raise 5 to the power of 2.
To 'cube' 5, raise 5 to the power of 3.

2 Identify the base and the exponent and multiply the base by itself as many times as specified by the exponent.

3 Perform the multiplication.

WRITE

a $5^2 = 5 \times 5$
$= 25$

b $5^3 = 5 \times 5 \times 5$
$= (5 \times 5) \times 5$
$= 25 \times 5$
$= 125$

Worked example 1.4C Calculating the square root and cube root

Calculate the value of each of the following roots.

a $\sqrt{36}$

b $\sqrt[3]{64}$

THINK

a Use times tables to determine which number, multiplied by itself, is equal to the number under the square root symbol.

b Use times tables and estimation to determine which number, multiplied by itself three times, is equal to the number under the cube root symbol.

WRITE

a $6 \times 6 = 36$
So, $\sqrt{36} = 6$

b $4^3 = 4 \times 4 \times 4$
$= (4 \times 4) \times 4$
$= 16 \times 4$
$= 64$
So, $\sqrt[3]{64} = 4$

Helpful hints

- ✔ Lowest common multiples are not necessarily 'low' in value; they can be large numbers.
- ✔ There is no such thing as a 'highest common multiple', as any given number has infinitely many multiples.
- ✔ Make sure you use a systematic method for finding all the factors; it is very easy to miss one or two.
- ✔ Don't confuse exponents and multiplication!
  For example, $2^3 = 2 \times 2 \times 2$
  $2^3 \neq 2 \times 3$
- ✔ Place your exponents carefully; they should be in smaller font than the base and sit high up on the shoulder of the base – your 4^3 should look different to your 43!

Learning pathways

1–5, 6(a, c, e), 7, 8(a–f), 9(1st, 2nd columns), 10, 11, 14, 15	2, 3, 6, 7, 8(g–l), 9(3rd, 4th columns), 10–13, 16	3(d–f), 6(b, d, f), 7, 8(j, k, l), 9(3rd, 4th columns), 10, 11, 13, 15–19

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Exercise 1.4A: Understanding and fluency

1 Multiples can be produced by multiplying a whole number by another whole number.

a List the multiples of 6 and 9 up to 100.

b Identify the common multiples of 6 and 9 in the lists from part **a**.

c What is the lowest common multiple (LCM) of 6 and 9?

WE 1.4A **2** Find the lowest common multiple (LCM) of each pair of numbers.

a 6 and 8 **b** 12 and 18 **c** 25 and 30

d 12 and 15 **e** 14 and 20 **f** 9 and 15

3 Find the lowest common multiple (LCM) of each group of numbers.

a 5, 6 and 7 **b** 5, 7 and 9 **c** 6, 8 and 9

d 5, 8 and 12 **e** 15, 18 and 24 **f** 9, 18 and 27

4 Find all the factors of each number.

a 45 **b** 72 **c** 90 **d** 120 **e** 100 **f** 144

5 Listing all the factors of two numbers can help to find the highest common factor of those numbers.

a What is the highest common factor (HCF) of 18 and 36?

b What is the HCF of 21 and 45?

c What is the HCF of 12 and 30?

6 Find the highest common factor (HCF) of each group of numbers.

a 42 and 70 **b** 48 and 84 **c** 36 and 63

d 144, 180 and 200 **e** 120, 240 and 360 **f** 500, 900 and 725

7 Find the LCM and the HCF of each group of numbers.

a 24 and 45 **b** 16 and 36 **c** 6, 15 and 20 **d** 7, 12 and 22

WE 1.4B **8** Calculate the square and the cube of each of the following numbers.

a 3 **b** 2 **c** 10 **d** 7 **e** 6 **f** 12

g 8 **h** 4 **i** 1 **j** 9 **k** 11 **l** 14

WE 1.4C **9** Calculate the value of each of the following roots.

a $\sqrt{9}$ **b** $\sqrt[3]{8}$ **c** $\sqrt[3]{125}$ **d** $\sqrt{64}$

e $\sqrt{81}$ **f** $\sqrt[3]{216}$ **g** $\sqrt{100}$ **h** $\sqrt[3]{343}$

i $\sqrt[3]{512}$ **j** $\sqrt[3]{2197}$ **k** $\sqrt{196}$ **l** $\sqrt[3]{3375}$

10 Find the value of each of the following by first evaluating any exponents and roots.

a $2^3 \times \sqrt{9}$ **b** $3^3 + \sqrt{36}$ **c** $7^2 \times 10^3$

d $4^3 \div \sqrt{16}$ **e** $2^3 \times 8^2$ **f** $100^2 \times \sqrt{100}$

11 Find the value of each of the following by first evaluating any exponents and roots.

a $6^2 \times 3^3$ **b** $18^2 \div 3^2$ **c** $36^2 \div 6^3$

d $4^2 \times 2^1$ **e** $12^2 \times 4^2 \div 3^2$ **f** $\sqrt{81} \times 2^2 \times 5^2$

Exercise 1.4B: Problem solving and reasoning

12 Sally and Jo have the same number of stickers. Sally's stickers are in packs of 15 and Jo's stickers are in packs of 12. What is the smallest number of stickers each of them could have?

13 Taran is a florist. He receives a delivery of roses that divides into bouquets of 15 or 24.

a What is the smallest number of roses in the delivery?

b Taran has 24 roses and 40 lilies. If he wants to make identical bouquets with no flowers left over, what is the greatest number of bouquets he can make?

14 A group of students is going on an excursion to the Zoo. The students can travel on either buses that seat 30. or buses that seat 45 with all seats being occupied. What is the smallest number of students going on the excursion?

15 Dizzy sold an equal number of wontons and dumplings today. Wontons are sold in packets of 8 while dumplings are sold in packets of 12. What is the smallest number of each that Dizzy sold?

16 Arvin has 24 toy trucks and 64 toy cars. He divides them into identical groups so that there are no toys left over. What is the largest number of groups there might be?

Exercise 1.4C: Challenge

17 Jill has grown three types of potatoes in her garden: Pontiac, Sebago and Desiree. Seven Pontiac potatoes weigh the same as four Sebago potatoes and five Sebago potatoes weigh the same as six Desiree potatoes. Put the potatoes in order of weight from lightest to heaviest.

18 This question looks at the squares of numbers consisting of the digit 1.

a Find:

i 1^2 **ii** 11^2 **iii** 111^2 **iv** 1111^2

b Describe the pattern you see.

c Without using a calculator, use the pattern to write the results for:

i $11\,111^2$ **ii** $111\,111^2$ **iii** $1\,111\,111^2$ **iv** $11\,111\,111^2$

19 How many numbers in the set $\{7^1, 7^2, 7^3, 7^4, \ldots, 7^{2000}\}$ have a last digit of 3?

Online resources:

- **Interactive skillsheet** Lowest common multiple
- **Interactive skillsheet** Highest common factor
- **Worksheet** Finding multiples and factors
- **Quick quiz** 1.4
- **Interactive skillsheet** Exponents
- **Interactive skillsheet** Squares and square roots
- **Investigation** Triangular numbers

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

1.1 **1** Round each number to the nearest hundred.

a 576 **b** 934
c 7871 **d** 2839
e 45 621 **f** 76 618

1.1 **2** Estimate the result of each calculation by first rounding each number to its leading digit.

a 849 + 381 **b** 56 – 18
c 65 × 43 **d** 582 ÷ 16
e 493 × 28 **f** 825 + 9114
g 8402 ÷ 79 **h** 658 – 180

1.2 **3** Calculate each of the following.

a 64 + 98 **b** 745 + 832
c 6982 + 5176 **d** 28 + 743 + 6287
e 6287 + 341 + 89 **f** 3916 + 5328 + 6124

1.2 **4** Calculate each of the following.

a 63 – 45 **b** 582 – 96
c 846 – 757 **d** 5723 – 68
e 8743 – 374 **f** 7398 – 3572

1.2 **5** Calculate each of the following.

a 68 – 34 + 287 **b** 738 + 296 – 457
c 5921 + 4827 – 4277 **d** 83 422 + 82 113 – 57 329

1.3 **6** Calculate each of the following.

a 768 × 43 **b** 846 × 76
c 6214 × 57 **d** 9262 × 83
e 7248 × 82 **f** 6524 × 645

1.3 **7** Use short division to calculate each quotient and remainder.

a 395 ÷ 4 **b** 646 ÷ 7
c 8328 ÷ 5 **d** 76 398 ÷ 9
e 68 341 ÷ 11 **f** 653 789 ÷ 6

1.4 **8** Find the lowest common multiple (LCM) of each pair of numbers.

a 7 and 6 **b** 4 and 9
c 5 and 12 **d** 30 and 45
e 18 and 24 **f** 36 and 54

1.4 **9** Find the highest common factor (HCF) of each pair of numbers.

a 24 and 30 **b** 27 and 72
c 45 and 75 **d** 120 and 185
e 450 and 600 **f** 288 and 432

1.4 **10** Find the value of each of the following by first evaluating any exponents and roots.

a $6^2 + 4^3$ **b** $9^2 \div 3^2$
c $\sqrt{81} \times 4^2$ **d** $2^3 \div 8$
e $12^2 + 6^2 \div 3^2$ **f** $\sqrt{49} \times 3^3 + 8^2$

Lesson 1.5

Integers

Learning intentions

By the end of this lesson you will be able to ...

✔ understand and use negative numbers.

Inter-year links

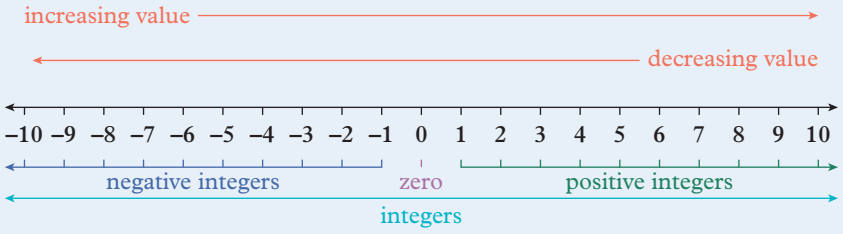
Support Negative numbers

Year 7 5.1 Negative numbers

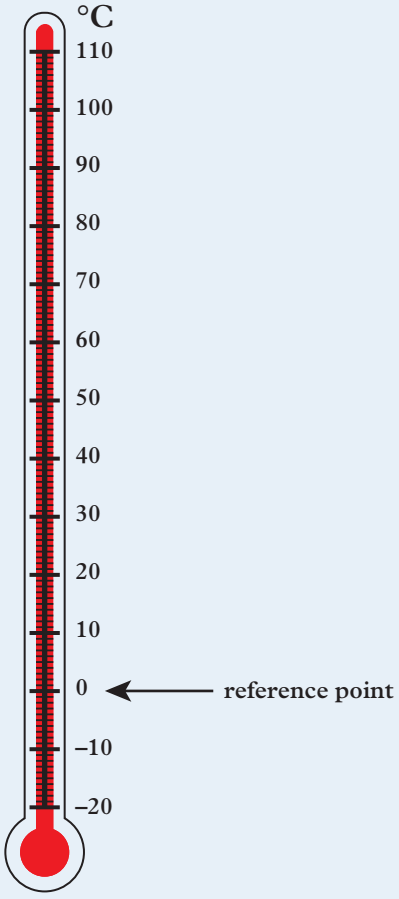
Integers

Key content video
Negative numbers

- The set of **integers** includes all positive whole numbers, negative whole numbers and zero.
- **Negative integers** are less than zero in value and are preceded by a negative sign, −, placed directly before the first digit.
- **Positive integers** are greater than zero in value and can be represented by using a positive sign, +. The positive sign can also be left out.

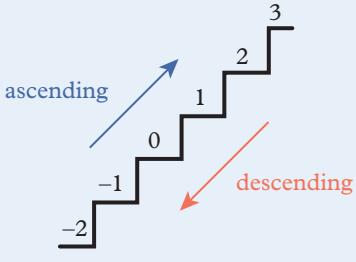


- In the real world, negative numbers are used to represent quantities that lie on a scale that goes below zero. A sign is used to indicate the direction relative to the **reference point** of zero. For example, a negative temperature indicates that the temperature has dropped below the reference point of 0°.
- The **magnitude** of a number is the value of the number ignoring the sign. For example, the magnitude of −3 is 3.



Ordering numbers

- Numbers are in **ascending order** when they are arranged in order of increasing value.
- Numbers are in **descending order** when they are arranged in order of decreasing value.

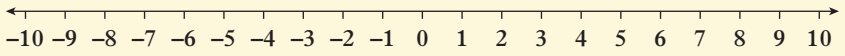


- 'Less than' is shown using the symbol '<'.
- 'Greater than' is shown using the symbol '>'.
- 'Equal to' is shown using the symbol '='.

Worked example 1.5A Identifying the greater of two integers

Identify which integer is greater in each pair of numbers. Use the following number line to help you.

a −5 and 2 **b** −4 and −7



THINK

a **1** Locate the two numbers on the number line and compare their positions. 2 is to the right of −5, so 2 is greater than −5.

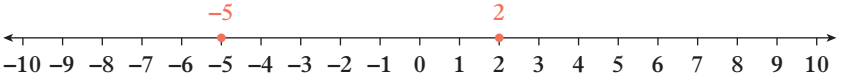
2 Use the 'greater than' symbol, >, in your answer.

b **1** Locate the two numbers on the number line and compare their positions. −4 is to the right of −7, so −4 is greater than −7.

2 Use the 'greater than' symbol, >, in your answer.

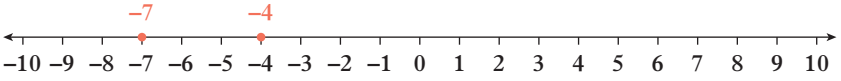
WRITE

a



$2 > -5$

b



$-4 > -7$

Worked example 1.5B Representing a situation with an integer

Provide an integer to represent each situation.

a The temperature is 15 degrees above zero.

b A fish swims 4 m below the surface of the water.

c Your bank account is overdrawn by $30.

THINK

a **1** The reference point is 0°.

2 Determine the sign of the integer based on the number's position relative to the reference point. '15 degrees above zero' can be indicated using 15.

b **1** The reference point is the water surface.

2 Determine the sign of the integer based on the number's position relative to the reference point. '4 m below' can be indicated using −4.

c **1** The reference point is $0.

2 Determine the sign of the integer based on the number's position relative to the reference point. 'Overdrawn by $30' can be indicated using −30.

WRITE

a 15

b −4

c −30

Helpful hints

- ✔ The negative equivalent of a positive number is called the additive inverse.
  For example, −4 is the additive inverse of 4.
- ✔ Don't get your 'greater than' and 'less than' symbols mixed up! The 'mouth' of the symbol is always trying to 'eat' the greater number.
  For example, $4 > 3$ and $3 < 4$.

Learning pathways

1–5, 6(a–f), 7(a, c), 8(a, c), 9, 10(a–d), 11, 12	4, 5, 6(g–l), 7(b, d), 8(b, d), 9, 10(e–h), 12–15	6(g–l), 7(c, d), 8(c, d), 9, 10(e–h), 13–16

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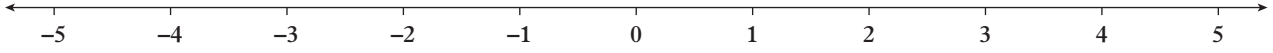
Exercise 1.5A: Understanding and fluency

1 As shown in the following number line, write:

a all the negative integers

b all the positive integers

c the integer that is neither positive nor negative.



WE 1.5A **2** Which integer is greater in each pair of numbers?

a 3 and 9 **b** 0 and 5 **c** −7 and 1

d 5 and −8 **e** 0 and −2 **f** −6 and −4

3 Which integer is smaller in each pair of numbers?

a 5 and 6 **b** 0 and 3 **c** −2 and 7

d 4 and −5 **e** 0 and −1 **f** −8 and −9

4 Which number is greater in each pair of numbers?

a 0 and −38 **b** −42 and 24 **c** 14 and −16

d 125 and −50 **e** −25 and −35 **f** −14 and 0

5 Which number is smaller in each pair of numbers?

a 7 and −70 **b** −21 and −12 **c** 55 and −68

d −40 and −50 **e** −354 and −345 **f** 450 and −450

6 Complete each number statement by writing < (is less than) or > (is greater than) in the space provided. You can draw a number line to help you think.

a −3 ____ 2 **b** −8 ____ −4 **c** 0 ____ −1

d 9 ____ −9 **e** −3 ____ −5 **f** −7 ____ −2

g 4 ____ −6 **h** 5 ____ −12 **i** 6 ____ −4

j −18 ____ −8 **k** −6 ____ −7 **l** −8 ____ 0

7 Write each list of integers in ascending order.

a 14, −20, 10, −7, 8, −11, −12 **b** −8, −88, 68, 8, −28, −18, −48

c −33, 42, −19, −41, 0, 6, 29 **d** 73, −82, 3, −140, 104, 145, −126

8 Write each list of numbers in descending order.

a −15, 71, −27, −10, 4, −9

b −1, −2, 5, 10, −11, −5

c 0, 5, −9, −14, 21, −19

d −11, 51, −71, −62, 71, 12, −72

9 List all the integers between, but not including, −7 and +4.

WE 1.5B **10** Provide an integer to represent each situation.

a The lift stops at the third floor below the ground floor.

b You have $2174 in the bank.

c The temperature inside a freezer is 18 degrees below zero.

d The shoreline of the Dead Sea, Jordan, is 408 m below sea level.

e The top of Mt Kilimanjaro, Tanzania, is 5895 m above sea level.

f A skydiver is 500 m above the ground.

g Your bank account is overdrawn by $46.

h A submarine is 40 m below sea level.

Exercise 1.5B: Problem solving and reasoning

11 Polar bears live close to the edge of sea ice in the Arctic, where in winter the temperatures range from around −45°C to 0°C. In summer, the temperatures range from −10°C to 10°C. The ocean temperature is about −2°C.

a Show −45°C, 0°C, −10°C, 10°C and −2°C on a number line.

b Is −10°C higher or lower than −45°C?

c Write the five temperatures in order from lowest to highest.

d A polar bear dips her paw into the water to catch a fish. On a day with an air temperature of −18°C, does the water feel warmer or colder than the air?

12 Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

a Ashwan checks his bank balance and sees he has $86 in his account. Write this as a positive integer.

b The bank allows Ashwan's account to be overdrawn. This means that he can withdraw more money than he actually has in his account. If he withdraws $100 at an ATM, how much does he owe the bank?

c Write his new bank balance as a negative integer.

13 Jasmine needs glasses for reading as she is long sighted (hypermetropic). The prescription for the lenses in her glasses is +2.50. Alex wears contact lenses because he is short sighted (myopic). His lenses have prescriptions of −3.50 for his left eye and −4.25 for his weaker right eye.

a If Justin has a prescription of +1.75, is he long or short sighted?

b If Elle has a prescription of −4.75, is she long or short sighted?

c How does Elle's prescription compare to Alex's? Who has weaker vision?

d What do you think the reference of zero means in this situation?

14 Bank statements usually show a transaction as a positive number if an amount is added or credited to the account, or as a negative number for withdrawals or debits.

Use the following bank statement to answer these questions.

Transaction	Balance
	+$52
+$23	+$75
−$35	+$40
−$51	−$11
+$34	+$23
+$13	+$36
−$49	−$13

a What is the largest amount:

i credited (added) to the account **ii** debited (subtracted) from the account?

b What is the smallest amount:

i credited to the account **ii** debited from the account?

c What does the final balance indicate about the account?

Exercise 1.5C: Challenge

15 One way to represent integers is with coloured dots. Use one blue dot to represent +1 and one red dot to represent −1.

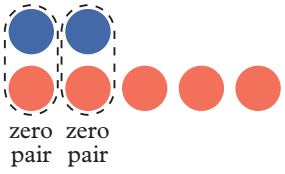
a How many red dots would you use to represent −3?

b Use the coloured dots to show each integer listed below. Draw each arrangement.

i 2 **ii** −4 **iii** 6 **iv** −5 **v** −1 **vi** 8

c One blue dot and one red dot together are called a zero pair. Explain how a zero pair represents 0.

d Explain how this arrangement of dots represents −3.

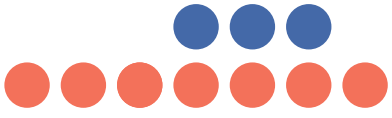


e What integer does each arrangement represent?

i

ii

iii



16 Find five consecutive numbers that add to −35.

Online resources:

Interactive skillsheet
Ordering positive and negative numbers

Worksheet
Comparing positive and negative numbers

Investigation
Comparing world temperatures

Quick quiz
1.5

Lesson 1.6

Adding and subtracting integers

Learning intentions

By the end of this lesson you will be able to ...

✔ add and subtract positive and negative integers.

Inter-year links

Year 7 — 5.2 Adding and subtracting positive numbers

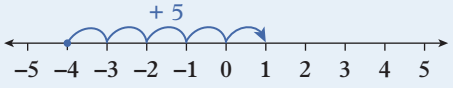
Adding and subtracting positive integers

Key content video: Adding and subtracting positive numbers

- When adding a positive integer, move to the right on the number line.

  For example, to calculate $-4 + 5$, start at -4 and move 5 steps to the right.

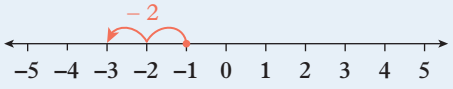
  $-4 + 5 = 1$



- When subtracting a positive integer, move to the left on the number line.

  For example, to calculate $-1 - 2$, start at -1 and move 2 steps to the left.

  $-1 - 2 = -3$



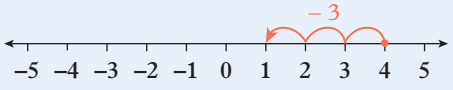
Adding and subtracting negative integers

Key content video: Adding and subtracting negative numbers

- When adding a negative integer, move to the left on the number line.

  For example, to calculate $4 + (-3)$, start at 4 and move 3 steps to the left.

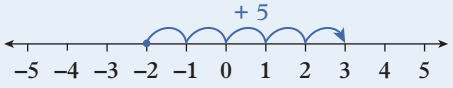
  $4 + (-3) = 4 - 3$
  $= 1$



- When subtracting a negative integer, move to the right on the number line.

  For example, to calculate $-2 - (-5)$, start at -2 and move 5 steps to the right.

  $-2 - (-5) = -2 + 5$
  $= 3$



- Adding a negative number can be simplified to subtracting its value. $+(-) = -$
- Subtracting a negative number can be simplified to adding its value. $-(-) = +$

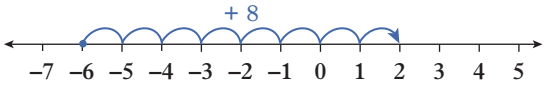
Worked example 1.6A Adding and subtracting positive numbers

Evaluate:

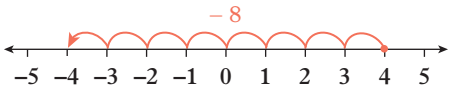
a $-6+8$ **b** $4-8$

THINK

a Start at the first number, then add the second number by moving right.



b Start at the first number, then subtract the second number by moving left.



WRITE

a $-6+8=2$

b $4-8=-4$

Worked example 1.6B Adding and subtracting negative numbers

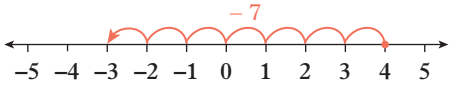
Evaluate:

a $4+(-7)$ **b** $-2-(-3)$

THINK

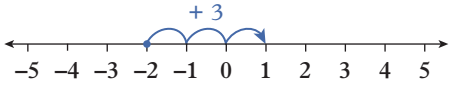
a 1 Simplify the calculation. Remember that $+(-)=-$.

2 Start at the first number, then subtract the value by moving left.



b 1 Simplify the calculation. Write the problem in a simpler equivalent form by replacing $-(-)$ with $+$.

2 Start at the first number, then add the value by moving right.



WRITE

a $4+(-7)=4-7$
$=-3$

b $-2-(-3)=-2+3$
$=1$

Helpful hints

- ✔ The negative sign is identical to the minus sign. The function of the symbol depends on its context.

Example	Function of '−'	Read as
-8	Indicates the number is less than zero in value	'negative eight'
$2-8$	Indicates the operation of subtraction	'two minus eight'
$-8+2$	Indicates the number is less than zero in value	'negative eight plus two'

- ✔ Don't make assumptions about the sign of the answer – subtraction can produce positive numbers and addition can produce negative numbers!
- ✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$+(-)=-$ $-(-)=+$ $+(+)=+$ $-(+)=-$

Learning pathways

 1–2, 3–4(a–e), 5(a–c), 6(a–e), 7(a), 8(a,c,e), 10, 12, 13, 16, 17

 3–4(e–i), 5(d–f), 7(b), 8(b,d,f), 9(b,d), 11, 12, 14, 16, 18

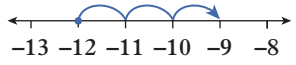
 7–9, 12–18

ANS p523

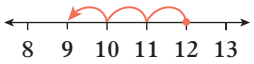
Exercise 1.6A: Understanding and fluency

1 Match each diagram (**A**, **B**, **C**) to the appropriate problem (**a**, **b**, **c**).

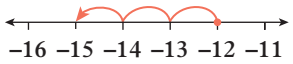
A



B



C



a $-12 - 3$ **b** $-12 + 3$ **c** $12 - 3$

2 Calculate each result for question **2**.

3 Decide whether each result is positive, negative or zero.

a $-3 - 7$ **b** $16 + 14$ **c** $8 - 15$ **d** $-13 + 13$ **e** $-11 + 18$

f $19 - 12$ **g** $-17 + 4$ **h** $-14 - 10$ **i** $17 - 20$ **j** $-15 + 16$

WE 1.6A **4** Evaluate.

a $-4 + 7$ **b** $1 + 5$ **c** $8 - 3$ **d** $-2 - 6$ **e** $-9 + 5$

f $3 - 6$ **g** $-5 - 1$ **h** $7 - 2$ **i** $2 - 6$ **j** $-8 - 3$

5 Simplify the following calculations by using + or − to fill in the boxes.

a $-3 - (+7) = -3 \square 7$ **b** $1 + (+6) = 1 \square 6$ **c** $-4 + (-5) = -4 \square 5$

d $2 - (-4) = 2 \square 4$ **e** $5 - (+9) = 5 \square 9$ **f** $-6 - (-8) = -6 \square 8$

WE 1.6B **6** Evaluate.

a $-4 - (-7)$ **b** $9 + (-3)$ **c** $2 - (+6)$ **d** $-5 - (-5)$ **e** $-8 + (+7)$

f $6 + (-1)$ **g** $-3 - (-4)$ **h** $4 - (+9)$ **i** $2 - (-7)$ **j** $-1 + (+1)$

7 Complete each addition table.

a

+	−5	−3	0	1	4
−3				−2	
−2					
−1			−1		
3					
6	1				

b

+	−10	−14	−7	13	19
11					
14					
−10					
−22					
−18					

8 Calculate:

a $-2 + 5 + 7$ **b** $3 - 9 + 2$ **c** $6 - 1 - 8$

d $8 + (-6) - (-10)$ **e** $-22 - (+5) + (+13)$ **f** $-14 - (-8) + (-25)$

9 Evaluate each of the following using the commutative and associative laws.

a $14 - 5 - (-6)$ **b** $10 - 13 + (-7)$

c $8 - 497 + (-128 - 3)$ **d** $-15 + 528 + 365 - 28$

Exercise 1.6B: Problem solving and reasoning

10 You enter a lift at the 15th floor and travel down 19 floors. Write down and evaluate a calculation that can be used to determine which level you finish at.

11 Sarah enters a lift at the third floor and travels down nine floors and then up five floors. Write down and evaluate a calculation that can be used to determine what floor she finishes at.

12 What is the difference between a temperature of −18°C and a temperature of 33°C?

13 Jess owes her brother $60.

a Write this amount as an integer.

b She pays him $35. Write an addition problem to work out the amount she still owes him.

c How much does Jess still owe her brother?

14 A bungee jumper dives from a platform (position A) 50 m above the water. He dips 2 m under the water (position B) before rebounding to a height 18 m below the platform (position C).

a If the reference is the surface of the water, write positions A, B and C as integers.

b Write an expression to find the distance between:

i A and B **ii** B and C **iii** A and C.

c Calculate each distance.

d What is the largest distance for part **c**? Explain why that is the largest distance.

15 A deep sea submersible is lowered into the ocean from a ship's deck 15 m above sea level. The driverless vessel is on a mission to take photos of deep sea animals.

For each part, write an addition or subtraction problem and then calculate the answer.

a A deep sea anglerfish is 900 m below sea level. What vertical distance has the submersible travelled from the deck of the ship to reach this fish?

b A deep sea amphipod is 2500 m below sea level. How much further has the submersible travelled vertically to reach it?

c What vertical distance does the submersible now need to travel to return to the deck of the ship?

d When the ship gets back to the dock, the crane on board the ship lowers the submersible onto the dock. The crane lifts the submersible 2 m above the ship's deck, moves it horizontally at that height and then lowers it 8 m onto the dock. What is the height of the dock above sea level?

Exercise 1.6C: Challenge

16 What number should be subtracted from each of these to give the result of 20?

a 65 **b** −10 **c** 1281 **d** −347 **e** −1000 **f** −3240

17 Find the missing integer to make each number sentence true.

a $-54 + 33 + \square = 12$ **b** $121 - \square - 50 = -6$

c $-300 + 225 - \square = 50$ **d** $\square - 58 + 73 = -25$

18 At the start of March, Tyra's bank account shows a balance of −$310. At the end of the month, it shows a balance of −$247.

a Calculate the difference between the two account balances. Show the subtraction problem you used to calculate the result.

b Interpret the result to part **a**.

Hint: What does this result mean?

Online resources:

- **Interactive skillsheet** Adding and subtracting with negative numbers
- **Worksheet** Simplifying addition and subtraction calculations
- **Investigation** Create a subtraction
- **Quick quiz** 1.6

Lesson 1.7

Multiplying and dividing integers

Learning intentions

By the end of this lesson you will be able to ...

✔ multiply and divide positive and negative integers.

Inter-year links

Support Multiplying whole numbers

Year 7 1.4 Multiplying whole numbers

Key content video
Multiplying and dividing integers

Multiplying integers

- Multiplying two integers can give a result that is positive or negative (or zero), according to these rules.
  - → The product of two positive integers is a positive integer.
  - → The product of a positive integer and a negative integer is a negative integer.
  - → The product of two negative integers is a positive integer.

$$+ \times + = +$$
$$+ \times - = -$$
$$- \times + = -$$
$$- \times - = +$$

Dividing integers

- Division of integers follows the same rules as multiplication of integers.
  - → The quotient of two positive integers is a positive integer.
  - → The quotient of a positive integer and a negative integer is a negative integer.
  - → The quotient of two negative integers is a positive integer.

$$+ \div + = +$$
$$+ \div - = -$$
$$- \div + = -$$
$$- \div - = +$$

Exponents of negative numbers

- When applying exponents to negative numbers, the sign needs to be taken into account.

exponent

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

  - → The square of a negative number is positive.
    For example, $(-4)^2 = 16$.
  - → The cube of a negative number is negative.
    For example, $(-4)^3 = -64$.

Worked example 1.7A Multiplying integers

Evaluate each product.

a -4×7 **b** $-9 \times (-5)$

THINK	WRITE
a 1 Determine whether the result is positive or negative. The signs are different, so the result is negative.	**a** $4 \times 7 = 28$
2 Calculate the product.	So, $-4 \times 7 = -28$
b 1 Determine whether the result is positive or negative. The signs are the same, so the result is positive.	**b** $9 \times 5 = 45$
2 Calculate the product.	So, $-9 \times (-5) = 45$

Worked example 1.7B Dividing integers

Evaluate each quotient.

a $-30 \div (-5)$ **b** $28 \div (-7)$

THINK	WRITE
a 1 Decide whether the result is positive or negative. The signs are the same, so the result is positive.	**a** $30 \div 5 = 6$
2 Calculate the quotient.	So, $-30 \div (-5) = 6$
b 1 Decide whether the result is positive or negative. The signs are different, so the result is negative.	**b** $28 \div 7 = 4$
2 Calculate the quotient.	So, $28 \div (-7) = -4$

Worked example 1.7C Multiplying three integers

Evaluate the product $-3 \times (-5) \times (-2)$.

THINK	WRITE
1 Work from left to right. Multiply the first two numbers and determine the sign. As the signs are the same, the result is positive.	$-3 \times (-5) = 15$
2 Multiply the result by the third number and determine the sign. As the signs are different, the result is negative.	$15 \times (-2) = -30$ $-3 \times (-5) \times (-2) = -30$

Helpful hints

- ✔ Be careful when determining the sign of a result when multiplying or dividing numbers.
- ✔ If the signs are the same, then the result is positive.
- ✔ If the signs are different, then the result is negative.

Learning pathways

1(a,c), 2–3(1st, 2nd columns), 4(a,c,e), 5–6(1st, 2nd columns), 7(a), 9–10(a,d), 11–12(a,c,e), 13–15, 18–19	5–6, 7(b), 8, 9–10(d–f), 11–12(a,c,e), 14, 15, 17–19, 20(a, b)	5(i–l), 7(b), 8, 9(d–f), 10(b, d, e), 11(c, d), 12(d–f), 16-21

ANS p524

Exercise 1.7A: Understanding and fluency

1 Express these repeated additions as multiplications.

a $9 + 9 + 9 + 9 + 9$ **b** $(-7) + (-7) + (-7) + (-7) + (-7) + (-7)$

c $-(6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6)$ **d** $-[(-3) + (-3) + (-3) + (-3)]$

WE 1.7A **2** Evaluate each product.

a $3 \times (-4)$ **b** 7×5 **c** -6×2 **d** $-3 \times (-9)$

e -1×8 **f** $4 \times (-6)$ **g** $-8 \times (-9)$ **h** 2×10

i -4×11 **j** $7 \times (-1)$ **k** $-2 \times (-4)$ **l** 9×5

WE 1.7B **3** Evaluate each quotient.

a $15 \div (-3)$ **b** $-32 \div (-8)$ **c** $24 \div 4$ **d** $-63 \div 9$

e $-17 \div (-1)$ **f** $48 \div (-6)$ **g** $-56 \div 8$ **h** $-81 \div (-9)$

i $42 \div (-7)$ **j** $-12 \div 1$ **k** $-20 \div (-2)$ **l** $72 \div (-8)$

4 Evaluate each quotient.

a $\frac{-48}{6}$ **b** $\frac{60}{-5}$ **c** $\frac{-51}{-3}$ **d** $\frac{150}{10}$ **e** $\frac{-200}{-25}$ **f** $\frac{57}{-19}$

5 Calculate each of the following.

a $5 \times (-7)$ **b** $-9 \div 3$ **c** $-28 \div (-4)$ **d** -10×6

e $36 \div (-9)$ **f** $-15 \times (-4)$ **g** $-100 \div 20$ **h** $18 \times (-2)$

i $-3 \times (-30)$ **j** $25 \div (-25)$ **k** -1×38 **l** $-45 \div (-1)$

6 Complete each statement.

a $-15 \times \square = 45$ **b** $56 \div \square = -8$ **c** $\square \times (-4) = 36$

d $\square \div (-25) = -4$ **e** $-140 \div \square = 2$ **f** $\square \times 16 = -48$

7 Complete each multiplication table.

a

×	−2	−1	0	1	2
−4	8			−4	
−2					
0			0		
2					
4		−4			
6					

b

×		−20	−10		20
7				70	
			−50		
0					
−2	50				
		80			
					−120

8 A short way of writing $-1 \times (-2)$ is $-(-2)$. Write each problem as the product of two numbers and then calculate the result.

a $-(-7)$ **b** $-(+12)$ **c** $-(-88)$ **d** $-(+25)$ **e** $-(8 - 5)$ **f** $-(-3 + 2)$

WE 1.7C **9** Evaluate each product.

a $-4 \times 3 \times (-5)$ **b** $3 \times (-2) \times 7$ **c** $6 \times (-1) \times 9$

d $8 \times 2 \times (-3)$ **e** $-5 \times (-2) \times (-13)$ **f** $-7 \times 3 \times (-4)$

10 Evaluate each statement.

a $-2 \times 2 \times (-3) \times 3$ **b** $20 \div (-2) \div (-2) \div (-1)$ **c** $-4 \times (-1) \times 3 \times (-2)$

d $-18 \div (-2) \times 4 \div (-6)$ **e** $9 \times (-2) \div 3 \times (-5)$ **f** $-7 \times (-4) \div 2 \times (-3)$

11 Calculate the value of the following squares and cubes.

a $(-10)^2$ **b** $(-2)^2$ **c** $(-4)^2$ **d** $(-3)^3$ **e** $(-5)^3$ **f** $(-10)^3$

12 Determine the sign that each of the following expressions will have when evaluated.

a $-2 \times 4 \times (-5)$ **b** $-3 \times (-1)^2 \times 7$ **c** $-9 \div (-1)^3 \times 3$

d $6 \times 2 \div (-3)$ **e** $-10 \div (-100) \times (-1000)$ **f** $-(-1) \times 2 \div (-3)$

Exercise 1.7B: Problem solving and reasoning

13 Complete each set of problems. Describe the pattern you see.

a

$2 \times 5 =$ _____
$2 \times 4 =$ _____
$2 \times 3 =$ _____
$2 \times 2 =$ _____
$2 \times 1 =$ _____
$2 \times 0 =$ _____
$2 \times (-1) =$ _____
$2 \times (-2) =$ _____
$2 \times (-3) =$ _____
$2 \times (-4) =$ _____
$2 \times (-5) =$ _____

b

$-3 \times 5 =$ _____
$-3 \times 4 =$ _____
$-3 \times 3 =$ _____
$-3 \times 2 =$ _____
$-3 \times 1 =$ _____
$-3 \times 0 =$ _____
$-3 \times (-1) =$ _____
$-3 \times (-2) =$ _____
$-3 \times (-3) =$ _____
$-3 \times (-4) =$ _____
$-3 \times (-5) =$ _____

c

$-4 \times (-5) =$ _____
$-4 \times (-4) =$ _____
$-4 \times (-3) =$ _____
$-4 \times (-2) =$ _____
$-4 \times (-1) =$ _____
$-4 \times 0 =$ _____
$-4 \times 1 =$ _____
$-4 \times 2 =$ _____
$-4 \times 3 =$ _____
$-4 \times 4 =$ _____
$-4 \times 5 =$ _____

d

$5 \times (-5) =$ _____
$5 \times (-4) =$ _____
$5 \times (-3) =$ _____
$5 \times (-2) =$ _____
$5 \times (-1) =$ _____
$5 \times 0 =$ _____
$5 \times 1 =$ _____
$5 \times 2 =$ _____
$5 \times 3 =$ _____
$5 \times 4 =$ _____
$5 \times 5 =$ _____

14 Dividing is the inverse operation to multiplying.

a Complete each statement.

i $2 \times 3 =$ ☐ so ☐ $\div 2 = 3$ or ☐ $\div 3 = 2$

ii $2 \times (-3) =$ ☐ so ☐ $\div 2 = -3$ or ☐ $\div (-3) = 2$

iii $-2 \times 3 =$ ☐ so ☐ $\div (-2) = 3$ or ☐ $\div 3 = -2$

iv $-2 \times (-3) =$ ☐ so ☐ $\div (-2) = -3$ or ☐ $\div (-3) = -2$

b Does division follow the same pattern as multiplication to give positive or negative results?

c Is the result positive or negative for each of these division types?

i positive ÷ positive **ii** negative ÷ negative

iii positive ÷ negative **iv** negative ÷ positive

15 Zero is the only integer that is neither positive nor negative.

a Using a calculator, determine:

i 8×0 **ii** -15×0 **iii** 0×65 **iv** $0 \times (-72)$

b What do you notice when you multiply any integer by zero?

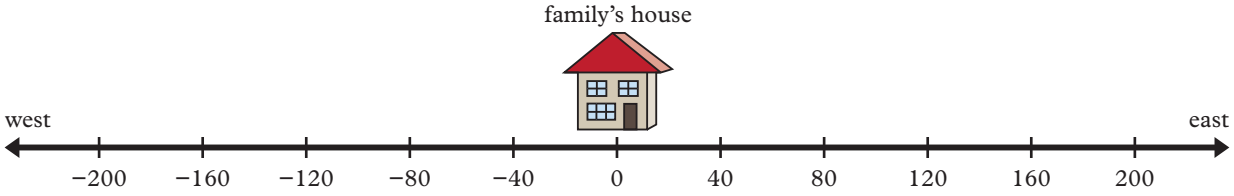
c Using a calculator, determine:

i $0 \div (-9)$ **ii** $0 \div 26$ **iii** $0 \div (-11)$ **iv** $0 \div 39$

d What do you notice when you divide zero by any non-zero integer?

16 Astro the dog has escaped from his yard. His owners decide to search along the road which runs east–west outside the house. This number line can be used to represent the road outside the family's house, with the house at zero.

a Tyler and Rhys walk 120 m west from the house and call Astro's name. What is their position on the number line?



b Sophie and Imogen walk 80 m east from the house and call Astro's name. What is their position on the number line?

c What is the distance between the pairs of searchers?

d Imogen now walks a further 30 m east. What is her new position on the number line?

e Rhys now walks 160 m east. What is his new position on the number line?

f What is the distance between:

i Sophie and Rhys **ii** Tyler and Imogen?

g Natalie starts at the house, then walks twice as far as Sophie but in the same direction as Tyler. What is her position on the number line?

h Hayden starts at the house, then walks one-third of the distance walked by Tyler and in the same direction. What is his position on the number line?

i A neighbour has found Astro in her yard, 60 m west of the family's house. Who is closest to Astro? How far and in which direction does he or she need to walk to collect the dog?

j If each member of the family now walks home, calculate the distance each person walked from the start of the search.

17 Consider $(+3)^2$, $(-3)^2$, 3^2 and -3^2.

a Which of these produce the same result? **b** Explain how $(-3)^2$ is different from -3^2.

c Decide whether each pair produces the same result.

i $(-4)^3$ and -4^3 **ii** $(-2)^4$ and -2^4 **iii** $(-1)^6$ and -1^6 **iv** $(-10)^5$ and -10^5

d Describe the general pattern you have seen.

18 What integer(s), when squared, give(s) each result?

a 49 **b** 81 **c** 1 **d** 4 **e** 64 **f** 100

19 What integer(s), when cubed, give(s) each result?

a 27 **b** −125 **c** 64 **d** −1 **e** −64 **f** −1000

Exercise 1.7C: Challenge

20 Find two integers that have:

a a sum of −9 and a product of 18 **b** a sum of 4 and a product of −45

c a sum of −5 and a product of −66 **d** a sum of 10 and a product of −56

e a difference of 2 and a product of 24 **f** a difference of 8 and a product of 9.

21 The product of 4 integers is 50. What are the possible sums of these 4 integers if they are:

a all positive **b** all negative **c** a mix of positive and negative?

Online resources:

- **Interactive skillsheet** Multiplying and dividing negative numbers
- **Interactive skillsheet** Exponents of negative numbers
- **Worksheet** Multiplying and dividing integers
- **Worksheet** Powers of positive and negative numbers
- **Investigation** Multiplication triangulation
- **Quick quiz** 1.7

Lesson 1.8

Order of operations

Learning intentions

By the end of this lesson you will be able to ...

✔ use the order of operations to correctly evaluate expressions.

Inter-year links

Support Year 7 — Order of operations — 1.8 Order of operations

Order of operations

Key content video: Order of operations

- **'BIDMAS'** can be used to help remember the order of operations.

B	**Brackets**	Operations inside brackets are always performed first. For example, $2^2 \times (4 - 6) - 8 \div (-2) = 2^2 \times (-2) - 8 \div (-2)$.
I	**Indices (exponents)**	Next, evaluate exponents and roots. For example, $2^2 \times (-2) - 8 \div (-2) = 4 \times (-2) - 8 \div (-2)$.
D **M**	**Division** **Multiplication**	Then, working from **left to right**, perform any multiplication or division before adding or subtracting. For example, $4 \times (-2) - 8 \div (-2) = -8 - (-4)$.
A **S**	**Addition** **Subtraction**	Finally, working from **left to right**, perform any addition and subtraction. For example, $-8 - (-4) = -8 + 4 = -4$.

- Operations at the same ranking are performed in order from left to right.

  For example,

$$5 + 8 - 2 = 13 - 2 \quad \text{and} \quad -20 \times \frac{3}{10} = -\frac{60}{10}$$
$$= 11 \qquad\qquad = -6.$$

- Where there is more than one set of brackets in the problem, perform the operations inside the innermost brackets first.

  For example,

$$\begin{aligned}[2 \times (5 - 3)] - 7 &= [2 \times (5 - 3)] - 7 \\ &= (2 \times 2) - 7 \\ &= 4 - 7 \\ &= -3.\end{aligned}$$

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)
• Sum	• Difference	• Product	• Quotient
• Plus	• Take away	• Times	• Divide
• More than	• Less than	• Multiply	• Shared
• Increase	• Fewer	• Groups of	• Distributed
• Total	• Reduce		
• Together	• Minus		

Worked example 1.8A Order of operations

Evaluate each of the following.

a $-3 + 4 \times (-2)$ **b** $7 - (-15) \div 5$ **c** $(-2)^3 \times 3 + 5$

THINK

a 1 Perform the multiplication and determine the sign of the result.

2 Simplify the signs and perform the subtraction.

b 1 Perform the division and determine the sign of the result.

2 Simplify the signs and perform the addition.

c 1 Apply the exponent and determine the sign of the result.

2 Perform the multiplication and determine the sign of the result.

3 Perform the addition.

WRITE

a $-3 + 4 \times (-2) = -3 + (-8)$

$= -3 - 8$

$= -11$

b $7 - (-15) \div 5 = 7 - (-3)$

$= 7 + 3$

$= 10$

c $(-2)^3 \times 3 + 5 = -8 \times 3 + 5$

$= -24 + 5$

$= -19$

Worked example 1.8B Order of operations with brackets

Evaluate each of the following.

a $-18 \div (4 - 10) + 2 \times 7$ **b** $6 - [-12 \div (9 - 5)]^2 - 2$

THINK

a 1 Perform any operations inside brackets first.

2 Working from left to right, perform division and multiplication before addition.

3 Perform the addition.

b 1 There are two sets of brackets. Perform the operation in the innermost set of brackets first.

2 Perform the operation in the remaining set of brackets.

3 Apply the exponent before addition and subtraction.

4 Working from left to right, perform the subtraction.

WRITE

a $-18 \div (4 - 10) + 2 \times 7 = -18 \div -6 + 2 \times 7$

$= 3 + 2 \times 7$

$= 3 + 14$

$= 17$

b $6 - [-12 \div (9 - 5)]^2 - 2 = 6 - [-12 \div 4]^2 - 2$

$= 6 - [-3]^2 - 2$

$= 6 - 9 - 2$

$= -3 - 2$

$= -5$

Helpful hints

- ✔ Remember to use BIDMAS rather than working from left to right.
- ✔ It is helpful to determine the sign at each step of a calculation before progressing.
- ✔ For multiplying and dividing integers, recall the simplification rules for calculations involving two numbers.
  - If the signs are the same, then the result is positive.
  - If the signs are different, then the result is negative.

  $+(-) = -$ $\quad -(-) = +$ $\quad +(+) = +$ $\quad -(+) = -$
- ✔ For adding and subtracting integers, recall the simplification rules. You can also use the number line to help you.

Learning pathways

1–3, 4(1st column), 5–10, 12, 13(a, b) | 3, 4(2nd column), 5–9, 11, 13–16 | 4(2nd column), 5–8, 13, 14, 16–19

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Exercise 1.8A: Understanding and fluency

1 For each calculation, list the operations in the correct order in which they should be completed.

a $-28 - 3 \times 7 + 11$ **b** $50 + (34 - 19) \div (-5)$ **c** $12 + 3 \times \sqrt{4} - (-41)$

d $-20 - 2 \times 6 + 3^2$ **e** $8 + [9 - (-3 + 2)]$ **f** $24 \div 6 + [5 + (-1)]^2$

WE 1.8A **2** Evaluate each of the following.

a $16 + 28 \div 4$ **b** $60 - 2 \times 10$ **c** $27 \div 3 + 6 \times 11$

d $22 + 30 \times 2^3 \div 15$ **e** $10 \times 8 - 7 \times \sqrt{9}$ **f** $6^2 - 8 \times 3 + 35$

3 Evaluate each of the following.

a $7 - 2 \times 8$ **b** $-1 + 16 \div 16$ **c** $28 - 3 \times 11$

d $-6 + 5 \times \sqrt[3]{27} - 2$ **e** $-5 + [-9 + (-6 + 4)]$ **f** $42 \div (-7) + [5 \times (-2)^2]$

WE 1.8B **4** Evaluate each of the following.

a $72 \div (7 + 2) - 2 \times 3$ **b** $(27 - 14) \times (19 + 11)$

c $11 - (4 \times 2 - 19)$ **d** $(-3 \times 7 + 1) \div (8 - 12)$

e $9^2 + 7 \times (34 - 28)$ **f** $10 \times 8 - 4 \times (3^2 + \sqrt[3]{8})$

g $60 \div [6 + (3 \times 5 - 1)] + 48$ **h** $-12 + [-48 \div (28 - 26)]^2 - 9$

5 Evaluate each of the following.

a $(-3)^2 \times (-14 + 10) \div (-6)$ **b** $(-19 + 16)^2 + (-23 + 17)^3$ **c** $(-2)^3 \times (-4) \div (-8)$

d $(-2)^2 \times (-2^2)$ **e** $(120 \div 6) - (-10) \div \sqrt{4}$ **f** $[-3 + (150 \div 50)] \times (-7)^2 \div 5^2$

6 Evaluate each of the following using the commutative and associative laws.

a $-10 \div 8 \times 4$ **b** $17 - (-13 + 7)$

c $(-9) \times 5 + 11 \times (-9)$ **d** $4 \times 8 \times (-125) \times 25$

e $48 \times 75 \times 81 \div (-25) \div 24 \div (-27)$ **f** $365 \times (-23) + (-365) \times 77$

g $-64 + 64 \times 101$ **h** $-63 \times (-257) + 630 \times (-25)$

7 Use the correct order of operations to determine the value of the following expressions. Hint: The root sign is a grouping symbol, so operations contained within the root sign should be completed before applying the root.

a $4(\sqrt{25} + 15)$ **b** $(\sqrt{36} - \sqrt{25})^3$ **c** $2\sqrt{5^2 - 4^2}$ **d** $3(\sqrt{13^2 - 12^2})^2$

8 Identify the mistakes that have been made in the following calculations.

a $(-10 + 18)^2 = -64$ **b** $-15 + 5 \times 5 = -50$ **c** $-18 \div 3 + \sqrt{36} = -2$

d $(4 - 20) \div (-4) = -4$ **e** $(-4)^3 \div 8 = 8$ **f** $-5 \times 6 + 90 \div 10 = 39$

9 Find the correct answers to the problems in question **8**.

Exercise 1.8B: Problem solving and reasoning

10 In which of the following do the brackets not change the value of $-2^2 + 5 \times 3^2 - 2 \times \sqrt[3]{8} - 7$?

A $(-2)^2 + 5 \times 3^2 - 2 \times \sqrt[3]{8} - 7$ **B** $(-2^2 + 5) \times 3^2 - 2 \times \sqrt[3]{8} - 7$

C $-2^2 + (5 \times 3^2) - 2 \times \sqrt[3]{8} - 7$ **D** $-2^2 + 5 \times (3^2 - 2) \times \sqrt[3]{8} - 7$

11 You have $195 to buy three shirts priced at $58 each.

a Write a calculation to show how to work out how much money you will have left over.

b Estimate how much money is left over by rounding each number to its leading digit then performing the calculation.

c Perform the calculation in part **a** and compare your estimate to the exact amount of money you have left over.

12 Using the images shown, calculate the cost of buying lunch for the class: 14 salad rolls, 11 samosa packs, 9 flavoured milks, 16 orange juices and 25 apples. Show the numbers and operations you used in the calculation.

$7 $6 for three $4 $5 $1





13 When standing with outstretched arms above her head, the height of an Olympic diver is 3 m. She dives from the 10 m platform and touches the bottom of the pool with her outstretched fingers. The depth of the pool is 6 m.

a If the pool surface is the reference point for zero, write the maximum height of the diver above the pool surface as a directed number.

b Write the depth of the pool as a directed number.

c Write a subtraction problem to calculate the vertical distance covered by the diver. Hint: Find the difference between the two positions.

d What vertical distance has the diver covered?

14 The daily maximum and minimum temperatures at Thredbo were recorded over a week.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Maximum temperature (°C)	8	4	−3	−2	−1	0	1
Minimum temperature (°C)	2	−3	−5	−6	−6	−3	0

a Calculate the difference between the minimum and maximum temperatures for each day.

b Which day had the biggest range of temperatures?

c Calculate the average (mean) of:

i the maximum temperatures **ii** the minimum temperatures.

d What is the difference between the average minimum and average maximum temperatures?

15 This incomplete statement shows some deposits and withdrawals made to Martin's bank account.

Date	Reference	Transaction	Balance
30 April	–	–	+$289
2 May	Tony's Newsagent	+$132	
15 May	Movie Place	−$17	
21 May	Bicycle City	−$500	
24 May	Tony's Newsagent	+$230	
26 May	DD Online Music Store	−$38	
27 May	Sports and Stuff Pty Ltd		+$69

a Is a transaction of +$132 a deposit or a withdrawal?

b Is a transaction of −$17 a deposit or a withdrawal?

c What does it mean if the balance in Martin's account is:

i positive **ii** negative?

d Find the account balance after the transaction made on 21 May.

e Find the balance after the transaction made on 24 May.

f What transaction is made on 27 May so that the account balance is +$69?

g Was Martin's account overdrawn at any stage? What penalty do banks have for an account that is overdrawn?

Exercise 1.8C: Challenge

16 If the sum of two numbers is zero, is the product of the numbers positive or negative? Explain.

17 Find three integers whose sum is −5 and product is 120.

18 Evaluate the following.

a $6 + \cfrac{52}{6 + \cfrac{63}{3 + \cfrac{48}{3 + \cfrac{55}{3 + \cfrac{96}{8 + \cfrac{40}{8 + \cfrac{4}{1 + \cfrac{5}{1 + \cfrac{8}{2}}}}}}}}}$

b $4 - \cfrac{-64}{10 - \cfrac{52}{15 - \cfrac{132}{-3 - \cfrac{-54}{-1 - \cfrac{50}{3 - \cfrac{7}{3 - \cfrac{36}{4 - \cfrac{10}{1 - \cfrac{24}{8}}}}}}}}}$

19 The 'Four Fours' problem uses four of the digit '4' and a combination of operations and brackets to write calculations that are equal to each integer.

For example, 4 + 4 + 4 + 4 = 16.

a Write a series of calculations, using four '4's and any combination of the operations +, −, ×, ÷ and brackets, that are equal to each of the integers 0 to 20. Are all of these integers possible using these operations?

b If we include the square root operation, $\sqrt{4}$, can you write a calculation for each of the integers 1 to 20? If you couldn't find all the numbers in part **a**, see if you can find them using the square root operation.

c The factorial, $n!$, is an operation that multiplies each of the integers from a number, n, down to 1.

For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

Using the factorial with the above operations, which numbers are now possible?

d Explain how you could use your results above to write calculations that are equal to each of the integers from −20 to −1.

Online resources:

- Interactive skillsheet: Order of operations
- Worksheet: Working with positive and negative numbers
- Investigation: Converting temperatures
- Quick quiz: 1.8

Lesson 1.9

Review: Integers

Module summary

Rounding and estimating

$12\,600 \approx 13\,000$ (9, 8, 7, (6), 5 → Round up)

$12\,300 \approx 12\,000$ (4, (3), 2, 1, 0 → Round down)

Leading digit

$123\,456 = [1]23\,456 \approx 100\,000$

Adding and subtracting

$$\begin{array}{r} {\scriptstyle 1} \\ 229 \\ +\ 342 \\ \hline 571 \end{array}$$ ← answer

Ones column: 9 + 2 = 11, carry the 1
Tens column: 2 + 4 + (1) = 7
Hundreds column: 2 + 3 = 5

$$\begin{array}{r} {\scriptstyle 1\ 11} \\ 321 \\ -\ 204 \\ \hline 117 \end{array}$$ ← answer

Ones column: Take 10 from the tens column, 11 − 4 = 7
Tens column: 1 − 0 = 1
Hundreds column: 3 − 2 = 1

The distributive law

$5 \times 32 = 5 \times (30 + 2)$
$= 5 \times 30 + 5 \times 2$
$= 150 + 10$
$= 160$

Multiplication algorithm

$$\begin{array}{r} {\scriptstyle 2} \\ 34 \\ \times\ 26 \\ \hline 204 \\ +\ 680 \\ \hline 884 \end{array}$$

Multiply by ones → 204 (34 × 6)
Multiply by tens → 680 (34 × 20)
Add the results → 884

Short division

quotient: 145 remainder 1

$6\overline{)8^{2}7^{3}1}$

divisor: 6; dividend: 871; remainder: 1

Lowest common multiple (LCM)

Multiples of 4: 4, 8, (12), 16, …
Multiples of 6: 6, (12), 18, 24, …
LCM: 12

Highest common factor (HCF)

Factors of 12: 1, 2, 3, 4, (6), 12
Factors of 18: 1, 2, 3, (6), 9, 18
HCF: 6

Squares and cubes

$1^2 = 1 \times 1 = 1$	□	$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$		$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$		$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$		$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$		$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$		$\sqrt[3]{27} = 3$

Integers

increasing value →
← decreasing value

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

negative integers | zero | positive integers

integers

Adding and subtracting integers

$+\,(-) = -$ $\quad -\,(+) = -$
$-\,(-) = +$ $\quad +\,(+) = +$

Multiplying and dividing integers

$+ \times + = +$ $\quad + \div + = +$
$+ \times - = -$ $\quad + \div - = -$
$- \times + = -$ $\quad - \div + = -$
$- \times - = +$ $\quad - \div - = +$

Order of operations

B	Brackets	$5 \times (10 - 2) = 5 \times 8 = 40$
I	Indices (exponents)	$3^3 - 4 = 27 - 4 = 23$
D M	Division Multiplication	$5 \times 3 - 4 + 8 \div 4 = 15 - 4 + 2 = 13$
A S	Addition Subtraction	$15 - 4 + 2 = 13$

Powers of integers

exponent

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

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Review questions 1.9A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- ascending order
- associative law
- base
- BIDMAS
- by-parts method
- common factor
- commutative law
- compensation method
- cube
- cube root
- descending order
- distributive law
- dividend
- divisor
- estimate
- exponent
- factors
- highest common factor
- index
- integer
- jump method
- leading digit
- lowest common multiple
- magnitude
- multiples
- negative integers
- positive integers
- product
- quotient
- reference point
- remainder
- round down
- round up
- rounding
- short division
- square
- square root
- vertical addition algorithm
- vertical subtraction algorithm

1 Which mental strategy for adding or subtracting numbers involves rounding one number to make the calculations easier, and then adding or subtracting the amount by which the number was rounded up or down?

2 At which stage of BIDMAS are roots evaluated?

3 When performing a division, what is the quotient?

4 Identify the key terms being referenced in each of these definitions.

a the set of positive whole numbers, negative whole numbers and zero

b an approximate value which is close to the actual value

c the result of a multiplication

5 Using an example, provide a definition in your own words for the following key terms.

a multiples **b** square root **c** ascending order

6 Complete the following sentences using words from the key terms list shown above.

a The _____ states that the order in which numbers are added does not change the result.

b The _____ of a whole number are the whole numbers which divide exactly into that number.

Review questions 1.9B: Multiple choice

1.1 **1** By first rounding each number to its leading digit, what is an estimate for 526×68?

A 36 000 **B** 35 000 **C** 33 664 **D** 31 200

1.1 **2** Which of the following numbers would be 700 when rounded to the nearest hundred?

A 789 **B** 750 **C** 634 **D** 651

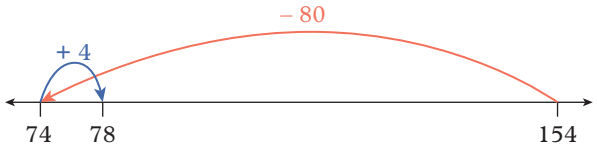
1.2 **3** What is 45 783 added to 67 398?

A 113 181 **B** 113 081 **C** 113 171 **D** 112 181

1.2 **4** What is 3765 subtracted from 9632?

A 5877 **B** 5967 **C** 6867 **D** 5867

1.2 **5** Which calculation is being displayed in this diagram of the compensation method?



A 154 – 76 = 78
B 154 – 84 = 78
C 154 – 84 = 74
D 74 + 78 = 154

1.3 **6** What is 144 multiplied by 212?
A 356 **B** 30 536 **C** 30 000 **D** 30 528

1.3 **7** What is 672 divided by 12?
A 58 **B** 57 **C** 56 **D** 55

1.3 **8** What are the quotient and remainder when 1175 is divided by 7?
A quotient = 166, remainder = 1
B quotient = 167, remainder = 1
C quotient = 168, remainder = 1
D quotient = 167, remainder = 6

1.4 **9** Which of the following is the HCF of 36 and 90?
A 3 **B** 9 **C** 36 **D** 18

1.4 **10** Which of the following represents the square root of 36?
A $\sqrt{36}$ **B** $\sqrt{6}$ **C** 36^2 **D** 6^2

1.5 **11** Which of the following numbers is smaller than −25?
A −18 **B** 2 **C** −30 **D** 0

1.5 **12** Which list of integers is written in descending order?
A 14, 11, 6, 0, −3, −5, −22
B 14, 11, 6, 0, −22, −5, −3
C −22, 14, 11, 6, −5, −3, 0
D −22, −5, −3, 0, 6, 11, 14

1.6 **13** What integer when added to 12 gives −7?
A −19 **B** −5 **C** 5 **D** 19

1.6 **14** What is −24 − 20?
A −4 **B** 4 **C** −44 **D** 44

1.6 **15** What does −35 − (−47) simplify to?
A −35 − 47 **B** 35 − 47 **C** 35 + 47 **D** −35 + 47

1.7 **16** What is 8 ÷ (−2)?
A −16 **B** −4 **C** 4 **D** 16

1.7 **17** What pair of integers gives a sum of −5 and a product of −36?
A −4 and 9 **B** −9 and 4 **C** 4 and 9 **D** −9 and −4

1.7 **18** Which gives the largest result?
A $(-2)^6$ **B** 6^2 **C** $(-1)^{10}$ **D** $(-4)^3$

1.8 **19** Which of these problems does *not* equal 5?
A −4 − 11 + 16 − 2 + 6
B 8 − 13 − 1 + 17 − 6
C −7 + 5 − 9 + 22 − 6
D 5 − 12 + 3 − 7 + 6

1.8 **20** What operation is performed first in $6 \times (12 - 5) + 8^2 \div 4$?
A 12 − 5 **B** 8^2 **C** 6 × 12 **D** 64 ÷ 4

Review questions 1.9C: Short answer

1.1 **1** Round each number to its leading digit.

a 236 **b** 67 145 **c** 3890 **d** 149 046

1.1 **2** Estimate the result by first rounding each number to its leading digit.

a 12 345 + 3648 **b** 94 501 − 32 566 **c** 394 × 338 **d** 18 654 ÷ 425

1.2 **3** Three houses in one street were all sold on the same day. The selling prices were $765 340, $875 900 and $655 000. What was the total price of all three houses?

1.2 **4** The City of Newcastle has 390 519 residents, while the Blue Mountains region has 78 121 residents. What is the difference in the number of residents between the two local government areas?

1.3 **5** Quinn sold 14 bouquets of roses today. If each bouquet has 12 roses, how many roses in total did Quinn sell?

1.3 **6** Tyler has saved up $165. He intends to spend the money on $15 t-shirts. How many will he be able to buy?

1.4 **7** Aida sold equal numbers of jam donuts and cinnamon donuts today. Jam donuts are sold in boxes of six, while cinnamon donuts are sold in packets of nine. What is the smallest number of each that Aida sold?

1.4 **8** Find the value of the following.

a 6^3 **b** 2^7

c 8 squared **d** 11^2

1.4 **9** Find the value of the following.

a square root of 4 **b** $\sqrt{81}$ **c** cube root of 27 **d** $\sqrt[3]{125}$

1.5 **10** Use the correct inequality sign to make each statement true.

a $-5 \square 2$ **b** $-5 \square -2$ **c** $7 \square -4$ **d** $4 \square -7$

1.5 **11** Write each list of integers in ascending order.

a 24, −40, 20, −14, 16, −22, −24

b −17, 21, −27, −35, 0, 9, 63

c −6, −66, 86, 6, −26, −16, −46

d 65, −87, 3, −300, 190, 45, −409

1.6 **12** Calculate:

a −5 + (−4) **b** 2 + (7) **c** 22 + (−34) **d** −50 + (69)

1.6 **13** Tamara's bank account shows a balance of −$28. If she deposits $150, what is her new account balance?

1.6 **14** Overnight, the minimum temperature in Alice Springs was −3°C. By 2 pm, the temperature had climbed to a maximum of 18°C. Calculate the difference between the minimum and maximum temperatures.

1.6 **15** Calculate:

a −8 + 7 **b** −4 − 6 **c** 17 − 25

d −44 + 34 **e** −66 + 66 **f** −50 − 50

1.7 **16** Calculate:

a 7 × (−8) **b** −5 × (−9) **c** −36 ÷ 4 **d** −100 ÷ (−20)

e −12 × 0 **f** 4 × (−15) **g** $\frac{18}{-6}$ **h** $\frac{-42}{-3}$

1.7 **17** Calculate:

a −6 × 3 × (−10) **b** [20 ÷ (−4)] × 7 **c** [−18 ÷ (−3)] × 2 **d** −5 × (−2) × (−4)

1.7 **18** Calculate:

a $(-2)^3$ **b** $(-3)^2$ **c** $(-1)^3$ **d** $(-10)^2$

1.8 **19** Calculate:

a $33 + (-40) \times 3 \div 10$

b $75 - 4 \times 11 + 6^2 \div (-4)$

c $-10 \times 7 - 5 \times [(-4)^2 - 4]$

d $-120 \div [-9 + (4 \times 8 - 11)]$

1.8 **20** Calculate:

a $-15 \times (-4 + 16)$

b $(-4)^2 - (-3)^3$

c $[-6 \times 7 + (-10 + 7)^2] \div 3$

d $[-1 + (-2)^2 + (-3)^3] \div 3$

Review questions 1.9D: Mathematical modelling

1 Jez is the elephant keeper at the zoo. He has two elephants to take care of: Mali and Man Jai. Mali weighs $3^3 \times 10^2$ kg while Man Jai weighs $2 \times 3^2 \times 10^2$ kg.

a What is the weight of each elephant as a basic numeral?

b What is the difference in weight between the two elephants?

c What is the sum of the weights of the two elephants?

d What is the HCF between the weights of the elephants?

e Jez has to divide the food for the elephants according to their weight.

i The elephants need about one-tenth of their weight in food. How much food does Jez have to provide?

ii How much food does each elephant need?

f Man Jai gets sick and stops eating for a few days. As a result, he loses 5 kg each day. Jez writes the following expression to show Man Jai's weight after his illness: $2 \times 3^2 \times 10^2 - 5^n$ where n is the number of days he is sick. For example, if Man Jai is sick for 2 days, then Jez calculates his weight as $2 \times 3^2 \times 10^2 - 5^2$.

i Calculate Man Jai's weight after his illness using Jez's expression for 1, 2 and 3 days.

ii Explain the problem with Jez's expression.

iii Rewrite the expression so that it gives the correct weight of Man Jai after his illness.

2 The population figures for each state and territory of Australia from the 2021 Census are displayed in this table.

State or territory	Population
New South Wales	8 072 163
Victoria	6 503 491
Queensland	5 156 138
Western Australia	2 660 026
South Australia	1 781 516
Tasmania	557 571
Australian Capital Territory	454 499
Northern Territory	232 605

a Determine the actual difference in population between NSW and Victoria.

b Round each population figure to the leading digit.

c Use your answers from part **b** to answer the following questions.

i Estimate the difference in population between NSW and Victoria. Compare this to part **a**.

ii Estimate the population of Australia.

3 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minimum temperature (°C)	1	−2	−2	−3	−5	−2	−1
Maximum temperature (°C)	8	5	9	4	−1	5	5

a Which day had:

i the highest temperature

ii the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for each day.

c Which day had the biggest range of temperatures?

d Calculate the average of the minimum and maximum temperatures.

e What is the difference between the average minimum and average maximum temperatures?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Round whole numbers ☐ Estimate the results of simple calculations ☐ Assess the accuracy of estimations	☐ Go back to Lesson 1.1 Rounding and estimating
☐ Use mental strategies for addition and subtraction ☐ Use the vertical addition algorithm ☐ Use the vertical subtraction algorithm	☐ Go back to Lesson 1.2 Adding and subtracting whole numbers
☐ Use mental strategies for multiplication ☐ Calculate products using the multiplication algorithm ☐ Calculate quotients using short division	☐ Go back to Lesson 1.3 Multiplying and dividing whole numbers
☐ Find the lowest common multiple of two or more numbers ☐ Find the highest common multiple of two or more numbers ☐ Evaluate squares and cubes of numbers ☐ Evaluate square roots and cube roots	☐ Go back to Lesson 1.4 Multiples, factors, exponents and roots
☐ Compare the value of positive and negative numbers ☐ Use negative numbers to represent real-life situations	☐ Go back to Lesson 1.5 Integers
☐ Solve problems involving addition and subtraction of positive and negative numbers	☐ Go back to Lesson 1.6 Adding and subtracting integers
☐ Solve problems involving multiplication and division of negative numbers ☐ Solve problems involving exponents of negative numbers	☐ Go back to Lesson 1.7 Multiplying and dividing integers
☐ Order operations in calculations involving two or more operations ☐ Evaluate expressions including integers using the order of operations	☐ Go back to Lesson 1.8 Order of operations

2 Fractions and decimals

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Place value
- ✔ Number lines
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying and dividing whole numbers
- ✔ Order of operations

Curriculum links

- Recognise irrational numbers in applied contexts, including π and numbers that develop from the square root of positive real numbers that are not perfect squares, and recognise that irrational numbers cannot develop from the division of integer values by natural numbers (VC2M8N01)
- Convert between fractions and terminating or recurring decimals, using digital tools as appropriate (VC2M8N03)
- Use the 4 operations with integers and with rational numbers, choosing and using efficient mental and written strategies, and digital tools where appropriate, and making estimates for these computations (VC2M8N04)

Materials

- ✔ Calculator

Lesson 2.1

Fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ identify, simplify and compare fractions in various contexts.

Inter-year links

Support Year 7 — Fractions: 3.1 Fractions

Fractions

Key content video: Mixed numbers and improper fractions

- **Proper fractions** have a **numerator** that is less than the **denominator**.
  Some examples are: $\frac{3}{5}$, $\frac{2}{6}$, $\frac{11}{15}$.
- **Improper fractions** have a numerator that is greater than or equal to the denominator.
  Some examples are: $\frac{13}{6}$, $\frac{5}{5}$, $\frac{31}{8}$.
- **Mixed numbers** consist of both a whole number and a proper fraction.
  For example, $2\frac{1}{6} = 2 + \frac{1}{6}$.

Proper fractions
Numerator < Denominator

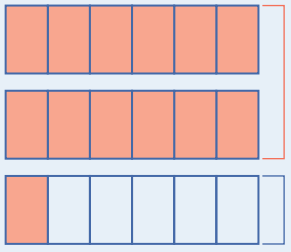
$\frac{2}{6}$ →

Improper fraction
Numerator ≥ Denominator

$\frac{13}{6}$ →

Mixed number
How many wholes?
How many parts?

← $2\frac{1}{6}$



To convert a mixed number to an improper fraction:	To convert an improper fraction to a mixed number:
1 Multiply the denominator by the whole number and add the numerator. 2 Write the result over the denominator. $3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$	1 Divide the numerator by the denominator. 2 Write down the whole number. Place the remainder over the denominator. 3 Simplify the fraction if possible. $\frac{13}{4} = 13 \div 4 = 3$ remainder 1 $= 3\frac{1}{4}$

Negative fractions

- Just like integers, negative fractions can be added, subtracted, multiplied and divided.
- Fractions with a negative sign can be written as shown.

Negative fractions

$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

Positive fractions

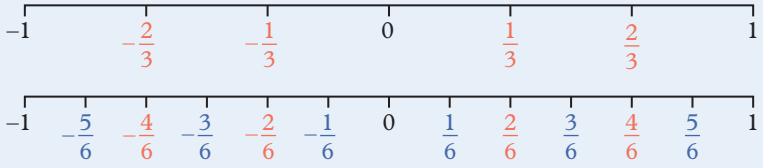
$\frac{-1}{-2} = \frac{1}{2}$

Equivalent fractions

Key content video
Equivalent fractions

- **Equivalent fractions** are fractions that have the same numerical value. They mark the same point on the number line.

  For example, $-\frac{1}{3}$ and $-\frac{2}{6}$ mark the same point on the number line, so they are equivalent fractions.



- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.

$$\frac{2}{3} \overset{\times 5}{=} \frac{10}{15} \qquad \frac{12}{10} \overset{\div 2}{=} \frac{6}{5}$$

Comparing fractions

Key content video
Ordering fractions

- When fractions have the same denominator, the bigger the numerator, the bigger the fraction.
- When fractions have the same numerator, the bigger the denominator, the smaller the fraction.
- To change a pair of fractions so they have the **lowest common denominator (LCD)**, find the lowest common multiple (LCM) of the denominators and then convert each fraction into the equivalent fraction.

The LCM of 2 and 5 is 10.

$$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10} \qquad \frac{3}{5} \overset{\times 2}{=} \frac{6}{10}$$

Simplifying fractions

- To find an equivalent fraction in its simplest form:
  - → Cancel out common factors until the only common factor between the numerator and denominator is 1.

    For example, $\frac{36}{72} = \frac{\cancel{36}^{18}}{\cancel{72}^{36}}$

    $= \frac{\cancel{18}^{3}}{\cancel{36}^{6}}$

    $= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$

    $= \frac{1}{2}$.
  - → Find the HCF of the numerator and the denominator and then divide the numerator and denominator by the HCF.

    For example, the HCF of 36 and 72 is 36, so $\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}}$

    $= \frac{1}{2}$.
- For mixed numbers, only simplify the fraction part; the whole number stays the same.

  For example, $2\frac{12}{24} = 2\frac{\cancel{12}^{1}}{\cancel{24}^{2}}$

  $= 2\frac{1}{2}$.

Worked example 2.1A Writing equivalent fractions

Fill the gaps to produce equivalent fractions.

a $\frac{3}{7} = \frac{18}{\square}$ **b** $\frac{91}{35} = \frac{\square}{5}$ **c** $\frac{-2}{11} = \frac{\square}{33}$

THINK

1. Look for a relationship between the two numerators or the two denominators. Consider if the equivalent fraction has been found by using multiplication or division.
2. Use the same relationship to obtain the denominator or numerator of the second fraction.
3. Ensure that the sign of the equivalent fraction matches the first fraction.

WRITE

a $\frac{3}{7} \overset{\times 6}{=} \frac{18}{\square}$

$\frac{3}{7} \underset{\times 6}{\overset{\times 6}{=}} \frac{18}{42}$

b $\frac{91}{35} = \frac{\square}{5}$ ($\div 7$)

$\frac{91}{35} \underset{\div 7}{\overset{\div 7}{=}} \frac{13}{5}$

c $\frac{-2}{11} = \frac{\square}{33}$ ($\times 3$)

$\frac{-2}{11} \underset{\times 3}{\overset{\times 3}{=}} \frac{-6}{33}$

Worked example 2.1B Ordering fractions by finding the lowest common denominator

Determine which fraction is greater. Write your answer as an inequality.

a $\frac{5}{6} \square \frac{7}{9}$ **b** $-\frac{7}{2} \square -\frac{13}{4}$

THINK

1. Find the LCD of the denominators.
2. For each fraction, find the equivalent fraction with a denominator equal to the LCD.
3. Compare the fractions with the same denominator and write the answer using the original fractions.

WRITE

a 18 is the LCD.

$\frac{5}{6} \overset{\times 3}{=} \frac{15}{18}$ $\quad \frac{7}{9} \overset{\times 2}{=} \frac{14}{18}$

$\frac{15}{18} > \frac{14}{18}$; therefore, $\frac{5}{6} > \frac{7}{9}$.

b 4 is the LCD.

$-\frac{7}{2} \overset{\times 2}{=} -\frac{14}{4}$

$-\frac{14}{4} < -\frac{13}{4}$; therefore, $-\frac{7}{2} < -\frac{13}{4}$.

Worked example 2.1C Simplifying fractions

Write each fraction in its simplest form.

a $\frac{24}{40}$ **b** $2\frac{35}{105}$

THINK

a 1 Find a common factor of 24 and 40. 2 is a common factor.
2 Divide both the numerator and the denominator by 2.
3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

b 1 Find the HCF of the numerator and the denominator of the fraction component.
2 Leave the whole number as it is and divide both the numerator and the denominator by the HCF 35.

WRITE

a $\frac{24}{40} = \frac{\cancel{24}^{12}}{\cancel{40}^{20}}$
$= \frac{\cancel{12}^{6}}{\cancel{20}^{10}}$
$= \frac{\cancel{6}^{3}}{\cancel{10}^{5}}$
$= \frac{3}{5}$

b HCF = 35
$2\frac{35}{105} = 2\frac{\cancel{35}^{1}}{\cancel{105}^{3}}$
$= 2\frac{1}{3}$

Helpful hints

- ✔ Remember that 'equivalent' is another way of saying 'equal to'.
- ✔ Each fraction has infinitely many equivalent fractions. If the only common factor between the numerator and the denominator is 1, the fraction is in its simplest form.
  For example, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ and so on.
- ✔ Use the divisibility rules from Module 1. The larger the common factor, the faster you'll find the equivalent fraction in its simplest form.

Learning pathways

1–3, 4(a, c, e, f, k), 5, 6–9(1st, 2nd columns), 10, 11, 12(1st column), 13, 14, 15(a, b), 16(a, b, f, h), 18	1(b, d, e), 4(b, d, g, h, l), 6–9(2nd, 3rd columns), 10–11(b, c), 12(2nd column), 15(c, d), 16(c, d, e, g), 17, 19, 20(a)	4(i–l), 6(3rd column), 7(g, h), 8–9(3rd column), 10–11(b, c), 12(g–l), 16(e, g, h), 19–21

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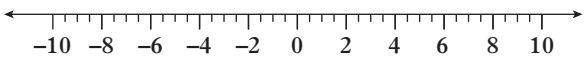
Exercise 2.1A: Understanding and fluency

1 Write all equivalent positive or negative fractions, showing the possible positions of the negative sign.

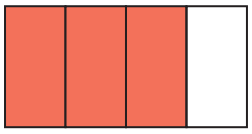
a $\frac{1}{-2}$ **b** $\frac{-10}{3}$ **c** $\frac{-5}{-6}$ **d** $\frac{-2}{-3}$ **e** $-\frac{2}{5}$ **f** $-\frac{-3}{-7}$

2 Show the position of each number on the following number line.



a −3 **b** $\frac{5}{2}$ **c** $-\frac{1}{2}$ **d** 5 **e** $7\frac{3}{4}$ **f** $-4\frac{3}{4}$

3 Consider the rectangle below.



a What fraction of the rectangle has been shaded?

b Draw a rectangle that is divided into 12 equal sections. Shade the rectangle so that it represents an equivalent fraction to that in part **a**.

c Write the equivalent fraction for part **b**.

WE 2.1A **4** Fill in the boxes below to produce equivalent fractions.

a $\frac{3}{10} = \frac{6}{\square}$ **b** $\frac{2}{7} = \frac{\square}{35}$ **c** $\frac{18}{33} = \frac{\square}{11}$ **d** $\frac{16}{48} = \frac{2}{\square}$

e $-\frac{4}{5} = -\frac{\square}{30}$ **f** $\frac{-8}{36} = \frac{-2}{\square}$ **g** $-1\frac{3}{8} = \frac{\square}{40}$ **h** $3\frac{3}{4} = \frac{75}{\square}$

i $\frac{1}{4} = \frac{2}{\square} = \frac{\square}{20} = \frac{8}{\square} = \frac{20}{\square} = \frac{\square}{100}$

j $-\frac{3}{7} = \frac{-15}{\square} = \frac{\square}{49} = \frac{\square}{70} = \frac{-300}{\square} = -\frac{\square}{210}$

k $1\frac{1}{5} = \frac{\square}{10} = \frac{60}{\square} = \frac{\square}{250} = \frac{\square}{45} = \frac{72}{\square}$

l $-2\frac{1}{3} = -\frac{14}{\square} = \frac{\square}{15} = \frac{49}{\square} = -\frac{\square}{9} = -\frac{\square}{300}$

5 Consider the list of fractions below.

$\frac{6}{10}, \frac{20}{15}, \frac{8}{20}, \frac{7}{10}, \frac{28}{21}, \frac{22}{55}, \frac{4}{10}, \frac{16}{9}, 1\frac{4}{12}, \frac{10}{25}, \frac{18}{45}, 2\frac{6}{15}$

Which of the fractions are equivalent to:

a $\frac{2}{5}$ **b** $\frac{4}{3}$

WE 2.1B **6** Fill in the boxes below using inequality symbols.

Hint: Write equivalent fractions with the same denominator.

a $\frac{3}{8} \square \frac{1}{2}$ **b** $\frac{4}{7} \square \frac{5}{14}$ **c** $\frac{-9}{10} \square \frac{-4}{5}$

d $-\frac{5}{12} \square -\frac{2}{3}$ **e** $1\frac{5}{6} \square 1\frac{19}{24}$ **f** $\frac{17}{15} \square \frac{7}{3}$

g $-\frac{2}{3} \square -\frac{3}{4}$ **h** $\frac{5}{6} \square \frac{4}{5}$ **i** $-\frac{11}{8} \square -\frac{5}{3}$

j $2\frac{3}{4} \square \frac{17}{6}$ **k** $-\frac{29}{3} \square -9\frac{1}{2}$ **l** $3\frac{5}{12} \square \frac{25}{8}$

7 Decide which fraction is greater in each pair.

a $\frac{5}{9}, \frac{7}{9}$ **b** $\frac{11}{4}, \frac{9}{4}$ **c** $-2\frac{3}{5}, -\frac{12}{5}$

d $-\frac{43}{6}, -7\frac{5}{6}$ **e** $3\frac{7}{11}, \frac{37}{11}$ **f** $-\frac{29}{2}, -8\frac{1}{2}$

g $4\frac{2}{3}, \frac{16}{3}$ **h** $-\frac{55}{9}, -6\frac{2}{9}$ **i** $3\frac{5}{7}, \frac{25}{7}$

WE 2.1C **8** Write each fraction in its simplest form.

a $\frac{8}{10}$ **b** $\frac{14}{21}$ **c** $\frac{42}{54}$

d $\frac{45}{72}$ **e** $\frac{40}{12}$ **f** $\frac{55}{30}$

9 Write each fraction in its simplest form.

a $\frac{6}{15}$ **b** $-\frac{33}{77}$ **c** $-\frac{18}{45}$

d $-\frac{30}{18}$ **e** $\frac{32}{36}$ **f** $\frac{18}{42}$

g $3\frac{24}{32}$ **h** $-5\frac{35}{63}$ **i** $4\frac{9}{33}$

j $-1\frac{24}{42}$ **k** $2\frac{15}{40}$ **l** $-7\frac{45}{54}$

10 Write each list of fractions in ascending order.

a $\frac{2}{5}, \frac{7}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$ **b** $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$ **c** $-1\frac{1}{2}, -\frac{2}{5}, -1\frac{9}{10}, -\frac{9}{2}, -\frac{3}{10}, -\frac{9}{5}$

11 Write each list of fractions in descending order.

a $\frac{13}{11}, \frac{5}{11}, \frac{19}{11}, \frac{9}{11}, \frac{3}{11}, 1\frac{5}{11}$ **b** $-\frac{2}{3}, -1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, -\frac{4}{3}, -1\frac{1}{4}$ **c** $\frac{5}{6}, -2\frac{1}{6}, -\frac{11}{6}, \frac{1}{6}, -\frac{5}{12}, 1\frac{1}{3}$

12 Write each amount as a fraction of the total amount in simplest form. Hint: Make sure the units are the same before you simplify your fraction.

a 15 kg out of 25 kg **b** 80 m out of 360 m **c** \$35 out of \$120

d 18 seconds out of 60 seconds **e** 50c out of \$3 **f** 14 days out of 5 weeks

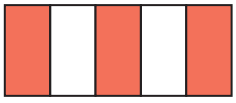
g 20 mm out of 7 cm **h** 300 g out of 2 kg **i** 28 seconds out of 1 minute

j 75c out of \$4.25 **k** 20 cm out of 6 m **l** 16 minutes out of 2 hours

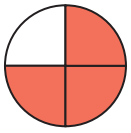
Exercise 2.1B: Problem solving and reasoning

13 Match one of the fractions $\frac{3}{4}, \frac{2}{3}, \frac{3}{5}$ or $\frac{5}{8}$ to each diagram.

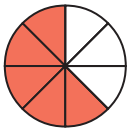
a



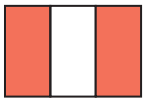
b



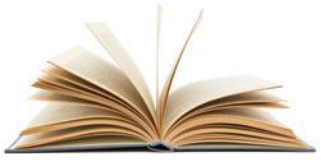
c



d



14 If you have read 56 out of 224 pages of a novel, what fraction have you read?



15 Consider these two rows of a multiplication grid.

	1	2	3	4	5	6	7	8	9
× 3	3	6	9	12	15	18	21	24	27
× 4	4	8	12	16	20	24	28	32	36

a Write numbers in each column of the table as a fraction by making the first number the numerator and the second the denominator. For example, the first column will be $\frac{3}{4}$.

b What do you notice in your answer to part **a**?

c Consider these two rows.

	1	2	3	4	5	6	7	8	9
× 7	7	14	21	28	35	42	49	56	63
× 5	5	10	15	20	25	30	35	40	45

Write numbers in each column of the table as a fraction by making the first number the numerator and the second the denominator. For example, the first column will be $\frac{7}{5}$.

d What do you notice in your answer to part **c**?

16 Find the fraction that is halfway between each pair of fractions.

a $\frac{3}{7}$ and $\frac{5}{7}$ **b** $-\frac{5}{11}$ and $-\frac{9}{11}$ **c** $\frac{3}{8}$ and $\frac{5}{8}$ **d** $-\frac{1}{3}$ and $-\frac{5}{9}$

e $\frac{4}{15}$ and $\frac{2}{5}$ **f** $\frac{1}{2}$ and $-\frac{3}{4}$ **g** $-1\frac{1}{4}$ and $1\frac{1}{3}$ **h** $2\frac{2}{3}$ and $3\frac{1}{5}$

17 Brodie scored 25 out of 30 on the first maths test, 21 out of 28 on the second, and 24 out of 36 on the third. Order Brodie's tests from best performance to worst.

18 Tony and Lisbeth hit a target with a ball with the same level of accuracy. Tony hit the target four times in five shots. Lisbeth hit the target 12 times. Use equivalent fractions to work out how many shots Lisbeth took.

19 Fractions can be compared in several ways:

1 comparing with a common denominator or the lowest common denominator

2 comparing with a common numerator or the lowest common numerator

3 comparing with a benchmark such as 1 or $\frac{1}{2}$.

For each of the following pairs of fractions:

i decide which strategy would be the most efficient to compare the pair and explain why you chose that strategy

ii state which fraction is greater by writing an appropriate inequality.

a $\frac{4}{7}, \frac{7}{15}$ **b** $\frac{15}{17}, \frac{15}{4}$ **c** $\frac{99}{100}, \frac{100}{101}$

d $1\frac{1}{8}, 1\frac{1}{11}$ **e** $1\frac{3}{5}, 1\frac{2}{3}$ **f** $\frac{77}{10}, \frac{39}{5}$

Exercise 2.1C: Challenge

20 The number of faults out of the total number of serves was recorded for each player in a tennis match.

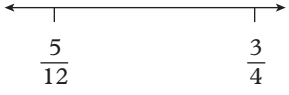
	Naomi	Ash
Total number of serves	119	102
Number of serving faults	14	10

a Who had the better success rate in serving?

b If Naomi served at the same fault rate in her next match, how many faults did she make out of 85 serves?

c If Ash served at the same fault rate in her next match and made 15 faults, what was her total number of serves?

21 a Which one of the following fractions could not be placed between the fractions marked on the number line?



A $\frac{3}{7}$ **D** $\frac{31}{41}$ **B** $\frac{5}{11}$ **C** $\frac{48}{96}$

b Explain how you checked each fraction.

Online resources:

- **Interactive skillsheet** Visual representations of fractions
- **Investigation** Cross country fractions
- **Worksheet** Equivalent fractions
- **Quick quiz** 2.1
- **Interactive skillsheet** Equivalent fractions

Lesson 2.2

Adding and subtracting fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ add and subtract fractions using efficient mental and written strategies.

Inter-year links

Support Year 7 — Adding and subtracting fractions: 3.5 Adding and subtracting fractions

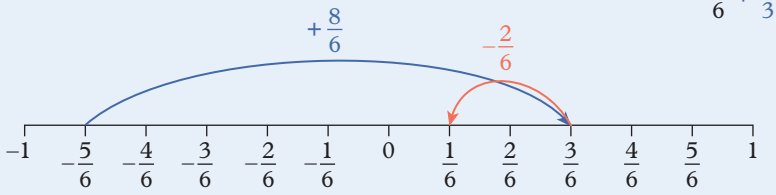
Adding and subtracting fractions

Key content video: Adding and subtracting fractions

Key content video: Working with negative fractions

- To add and subtract fractions:
  1. Convert all fractions into equivalent fractions with a common denominator.
  2. Add or subtract the numerators and keep the same denominator.
  3. Simplify the fraction.

$$-\frac{5}{6}+\frac{4}{3}-\frac{1}{3}=\frac{-5+8-2}{6}$$
$$=\frac{1}{6}$$



- To add or subtract mixed numbers, first convert the mixed numbers to improper fractions. Convert the sum or difference back to a mixed number and simplify your answer.
- Simplify the calculation by considering the operation and the sign applied to each fraction.

$$+\,(-)=-$$
$$-\,(+)=-$$
$$-\,(-)=+$$
$$+\,(+)=+$$

Worked example 2.2A Adding and subtracting fractions with common denominators

Determine the result of each sum or difference.

a $\frac{2}{7}+\frac{8}{7}$ **b** $\frac{13}{6}-\frac{4}{6}$

THINK

a Add the numerators and keep the same denominator.

b 1 Subtract the numerators and keep the same denominator.

2 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{2}{7}+\frac{8}{7}=\frac{10}{7}$

b $\frac{13}{6}-\frac{4}{6}=\frac{9^{3}}{6_{2}}$

$$=\frac{3}{2}$$

Worked example 2.2B Adding and subtracting fractions with different denominators

Determine the result of each sum or difference.

a $\frac{5}{6}+\frac{1}{2}$ **b** $\frac{11}{5}-\frac{6}{4}$

THINK

a 1 Convert into equivalent fractions with a common denominator. A common multiple of 2 and 6 is 6.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b 1 Convert into equivalent fractions with a common denominator. A common multiple of 5 and 4 is 20.

2 Subtract the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{5}{6}+\frac{1}{2}=\frac{5}{6}+\frac{1\times 3}{2\times 3}$

$=\frac{5}{6}+\frac{3}{6}$

$=\frac{8^{4}}{6^{3}}$

$=\frac{4}{3}$

b $\frac{11}{5}-\frac{6}{4}=\frac{11\times 4}{5\times 4}-\frac{6\times 5}{4\times 5}$

$=\frac{44}{20}-\frac{30}{20}$

$=\frac{14^{7}}{20^{10}}$

$=\frac{7}{10}$

Worked example 2.2C Adding and subtracting negative fractions

Determine the result of each sum or difference.

a $-\frac{12}{5}-\left(-\frac{7}{5}\right)$ **b** $-\frac{2}{3}+\left(-\frac{1}{2}\right)$

THINK

a 1 Simplify the calculation. Remember that subtracting a negative number is the same as adding a positive number.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b 1 Simplify the calculation. Remember that adding a negative number is the same as subtracting a positive number.

2 Convert into equivalent fractions with a common denominator. A common multiple of 3 and 2 is 6.

3 Subtract the numerators and keep the same denominator.

WRITE

a $-\frac{12}{5}-\left(-\frac{7}{5}\right)=-\frac{12}{5}+\frac{7}{5}$

$=-\frac{5^{1}}{5^{1}}$

$=-1$

b $-\frac{2}{3}+\left(-\frac{1}{2}\right)=-\frac{2}{3}-\frac{1}{2}$

$=-\frac{2\times 2}{3\times 2}-\frac{1\times 3}{2\times 3}$

$=-\frac{4}{6}-\frac{3}{6}$

$=-\frac{7}{6}$

Worked example 2.2D Adding and subtracting mixed numbers

Determine the result of each sum or difference.

a $2\frac{1}{3}+\frac{5}{6}$ **b** $1\frac{3}{4}-2\frac{5}{12}$

THINK

a **1** Convert the mixed numbers into improper fractions.

2 Convert into equivalent fractions with a common denominator. A common multiple of 3 and 6 is 6.

3 Add the numerators and keep the same denominator.

4 Convert the answer into a mixed number and simplify the fraction.

b **1** Convert the mixed numbers into improper fractions.

2 Convert into equivalent fractions with a common denominator. A common multiple of 4 and 12 is 12.

3 Subtract the numerators and keep the same denominator.

4 Simplify the fraction.

WRITE

a $2\frac{1}{3}+\frac{5}{6}=\frac{7}{3}+\frac{5}{6}$

$=\frac{7\times 2}{3\times 2}+\frac{5}{6}$

$=\frac{14}{6}+\frac{5}{6}$

$=\frac{19}{6}$

$=3\frac{1}{6}$

b $1\frac{3}{4}-2\frac{5}{12}=\frac{7}{4}-\frac{29}{12}$

$=\frac{7\times 3}{4\times 3}-\frac{29}{12}$

$=\frac{21}{12}-\frac{29}{12}$

$=-\frac{\cancel{8}^{2}}{\cancel{12}^{3}}$

$=-\frac{2}{3}$

Helpful hints

- ✔ Only add or subtract the numerators of fractions; the denominator stays the same.

$$\frac{5}{7}+\frac{1}{7}=\frac{5+1}{7}=\frac{6}{7}$$

$$\frac{5}{7}-\frac{1}{7}=\frac{5-1}{7}=\frac{4}{7}$$

- ✔ You can reduce the complexity of an addition or subtraction by finding the lowest common denominator. Always try to simplify first so that your lowest common multiple will be as small as possible.
- ✔ Adding does not always make a number larger. For example, adding a negative number will give a smaller result.
- ✔ Subtracting does not always make a number smaller. For example, subtracting a negative number will give a larger result.
- ✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$$+\,(-)=-$$
$$-\,(+)=-$$
$$-\,(-)=+$$
$$+\,(+)=+$$

Learning pathways

1–7, 8(a, b, c, f), 9, 10(a), 11(a–d), 14, 15	4, 5, 7, 8, 9, 12(b), 13(e–h), 14(a, b, c), 15, 16, 18, 19	4–5(e–h), 7–10(e–h), 11, 12(b, c), 13(g, h), 14(d–f), 16–20

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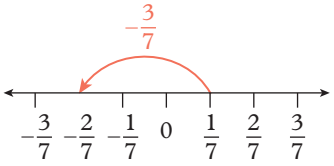
Exercise 2.2A: Understanding and fluency

1 Consider $\left(+\frac{1}{7}\right) + \left(-\frac{3}{7}\right)$.



a Explain why this can be simplified to $\frac{1}{7} - \frac{3}{7}$.

b Use this number line to calculate the result.

WE 2.2A **2** Evaluate the result of each sum or difference.

a $\frac{5}{17} + \frac{8}{17}$ **b** $\frac{7}{10} - \frac{3}{10}$ **c** $\frac{3}{8} + \frac{5}{8}$ **d** $\frac{6}{3} - \frac{5}{3}$

3 Find the lowest common denominator for each pair of fractions.

a $\frac{4}{5}$ and $\frac{2}{3}$ **b** $\frac{3}{7}$ and $\frac{1}{14}$ **c** $\frac{6}{10}$ and $\frac{4}{15}$ **d** $\frac{3}{8}$ and $\frac{5}{12}$

e $\frac{2}{15}$ and $\frac{2}{3}$ **f** $\frac{3}{4}$ and $\frac{2}{5}$ **g** $\frac{1}{6}$ and $\frac{3}{4}$ **h** $\frac{4}{9}$ and $\frac{1}{6}$

WE 2.2B **4** Evaluate the result of each sum or difference.

a $\frac{5}{22} + \frac{8}{11}$ **b** $\frac{3}{4} - \frac{2}{5}$ **c** $\frac{9}{10} + \frac{2}{3}$ **d** $\frac{3}{8} - \frac{1}{12}$

e $\frac{11}{15} - \frac{7}{10}$ **f** $\frac{5}{6} + \frac{3}{4}$ **g** $\frac{5}{6} - \frac{4}{9}$ **h** $\frac{3}{8} + \frac{2}{5}$

WE 2.2C **5** Evaluate the result of each sum or difference.

a $\frac{1}{7} - \left(-\frac{3}{7}\right)$ **b** $-\frac{2}{3} + \left(-\frac{5}{3}\right)$ **c** $\frac{5}{8} - \left(+\frac{7}{8}\right)$ **d** $-\frac{4}{5} + \left(+\frac{2}{5}\right)$

e $-\frac{1}{3} + \left(-\frac{4}{3}\right)$ **f** $-\frac{3}{4} - \left(+\frac{1}{2}\right)$ **g** $-\frac{1}{3} - \left(-\frac{5}{9}\right)$ **h** $\frac{2}{5} + \left(-\frac{3}{2}\right)$

6 Evaluate the result of each sum or difference.

a $-\frac{4}{5} + \frac{3}{5}$ **b** $\frac{2}{7} - \frac{5}{7}$ **c** $-\frac{3}{11} - \frac{6}{11}$ **d** $-\frac{4}{3} + \frac{1}{6}$ **e** $\frac{2}{7} - \frac{1}{2}$ **f** $-\frac{3}{4} + \frac{7}{8}$

WE 2.2D **7** Evaluate each problem and simplify where possible.

a $1\frac{1}{4} - \frac{7}{8}$ **b** $2\frac{3}{5} + \frac{13}{20}$ **c** $2\frac{2}{3} + 1\frac{4}{5}$ **d** $3\frac{7}{11} - 1\frac{1}{2}$

e $4\frac{3}{4} + 2\frac{2}{3}$ **f** $5\frac{5}{6} - 3\frac{2}{15}$ **g** $1\frac{2}{3} - 2\frac{1}{3}$ **h** $-4\frac{3}{5} - 1\frac{1}{10}$

8 Evaluate each problem and simplify where possible.

a $\frac{2}{3} - \frac{4}{3} + \frac{1}{3}$ **b** $-\frac{3}{4} + \frac{1}{4} + \frac{1}{2}$ **c** $\frac{2}{3} - \frac{5}{6} - \frac{1}{2}$ **d** $-\frac{3}{5} - \frac{1}{4} + \frac{7}{10}$

e $\frac{5}{9} - \frac{2}{3} + \frac{1}{2}$ **f** $5\frac{2}{7} - 9\frac{6}{7} + 1\frac{5}{7}$ **g** $-4\frac{3}{4} + 3\frac{5}{8} + 2\frac{7}{12}$ **h** $10\frac{3}{10} - 4\frac{3}{4} - 5\frac{3}{5}$

9 Evaluate each problem and simplify where possible.

a $1 - \frac{2}{3}$ **b** $3 - \frac{4}{3}$ **c** $1 + \frac{4}{3}$ **d** $-2 - \frac{5}{2}$

e $-\frac{4}{3} - 2$ **f** $\frac{2}{3} - 3$ **g** $2\frac{2}{3} - 3$ **h** $1\frac{5}{6} - 2$

10 Evaluate each problem and simplify where possible.

a $\frac{2}{3} - \left(\frac{4}{3} + \frac{5}{3}\right)$ **b** $\frac{2}{5} - \left(\frac{4}{5} - \frac{3}{5}\right)$ **c** $-\frac{1}{5} + \left(\frac{4}{3} - \frac{1}{2}\right)$ **d** $\frac{2}{3} - \frac{4}{3} - \left(\frac{1}{2} + \frac{2}{3}\right)$

e $\frac{1}{3} - \left(\frac{4}{3} + 1\right)$ **f** $\frac{5}{3} - \left(\frac{4}{3} - 3\right)$ **g** $2 - \left(-\frac{4}{5} - \frac{1}{5}\right)$ **h** $-\frac{2}{7} + \frac{4}{7} - \left(-\frac{1}{6} - \frac{2}{7}\right)$

Exercise 2.2B: Problem solving and reasoning

11 Nine numbers are arranged in a square.

$\frac{4}{5}$	$\frac{1}{10}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{1}{2}$	$\frac{7}{10}$
$\frac{2}{5}$	$\frac{9}{10}$	$\frac{1}{5}$

a Add the three numbers in:

i the first row **ii** the second row **iii** the third row.

b Add the three numbers in:

i the first column **ii** the second column **iii** the third column.

c Add the three numbers along each diagonal.

d What do you notice about your answers to parts **a**, **b** and **c**?

e Your answer to part **d** is the reason why the nine numbers form a magic square. The sum in each row, column and diagonal is called the magic sum. What is the magic sum for this set of numbers?

12 Copy and complete each magic square. Hint: First work out the magic sum.

a

		$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$

b

		$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	
$\frac{1}{3}$		

c

		$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$

13 Consider the following sums and differences. Without performing the calculation, state whether the sum or difference is positive, negative or zero.

a $\left(-\frac{2}{5}\right)+\frac{3}{4}$ **b** $\left(-\frac{7}{10}\right)-\left(-\frac{3}{10}\right)$ **c** $\frac{9}{11}-\left(-\frac{51}{17}\right)$ **d** $\frac{2}{3}+\left(-\frac{4}{6}\right)$

e $\left(-\frac{13}{25}\right)+\left(-\frac{81}{15}\right)$ **f** $\left(-\frac{5}{4}\right)-\left(+\frac{7}{3}\right)$ **g** $\left(-\frac{12}{5}\right)-\left(-\frac{36}{15}\right)$ **h** $-\frac{1}{77}-\frac{9}{15}$

14 We can take advantage of negative fractions to add and subtract mixed numbers without first converting to improper fractions by using the associative, commutative and distributive laws.

For example: $2\frac{1}{3}-1\frac{2}{3}=\left(2+\frac{1}{3}\right)-\left(1+\frac{2}{3}\right)$

$=2+\frac{1}{3}-1-\frac{2}{3}$

$=(2-1)+\left(\frac{1}{3}-\frac{2}{3}\right)$

$=1+\left(-\frac{1}{3}\right)$

$=\frac{2}{3}$

Calculate the following sums and differences without converting to improper fractions. Give your answer as a mixed number.

a $5\frac{7}{8}-4\frac{3}{8}$ **b** $5\frac{3}{8}-4\frac{7}{8}$ **c** $4\frac{7}{8}-5\frac{3}{8}$

d $4\frac{3}{8}-5\frac{7}{8}$ **e** $5\frac{7}{8}+4\frac{3}{8}$ **f** $-5\frac{7}{8}+\left(-4\frac{3}{8}\right)$

15 In subtraction of fractions problems, when is it better to convert to improper fractions first?

16 Craig wants to combine two open cartons of milk into one 2 L container. If one carton has $\frac{1}{3}$ L and the other has $1\frac{3}{4}$ L, will the 2 L container be large enough? Explain.

17 On Monday evening, Eleanor spent $1\frac{1}{4}$ hours on the computer researching an assignment for school, Adam spent $2\frac{1}{2}$ hours on the computer catching up with friends on Facebook, and their mother spent $\frac{5}{6}$ of an hour typing up a report for work. How much time was spent on the computer on Monday evening?

18 One-fifth of the student population at a local high school ride to school, one-tenth walk to school and two-thirds catch public transport. The remainder are driven to school.

What fraction of students:

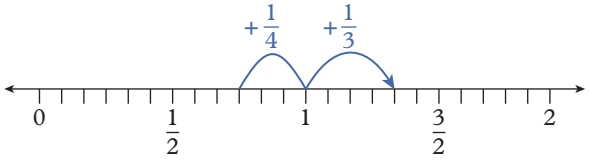
a ride or walk to school

b catch public transport, ride or walk to school

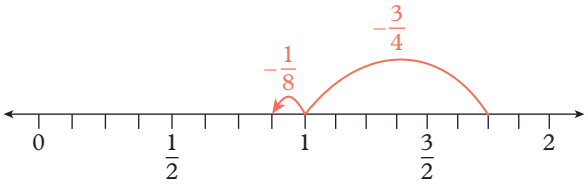
c are driven to school?

19 Match each of the number lines below (**a** to **e**) with the corresponding calculation (**A** to **E**). There is one set of calculations and one number line that do not have a match. For these, write a corresponding calculation and draw a corresponding number line diagram.

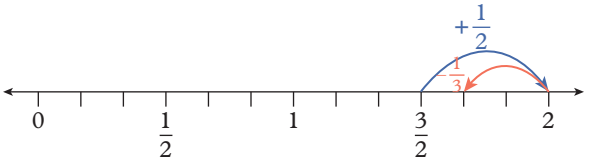
a



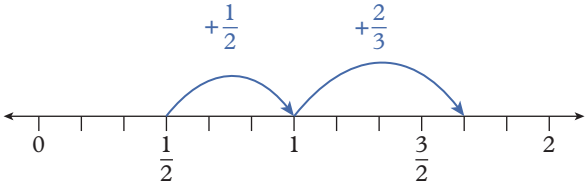
b



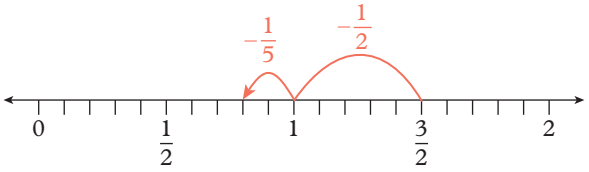
c



d



e



A $\frac{1}{2}+\frac{7}{6}=\left(\frac{1}{2}+\frac{3}{6}\right)+\frac{4}{6}$
$=1+\frac{2}{3}$
$=1\frac{2}{3}$

B $\frac{3}{4}+\frac{7}{12}=\left(\frac{3}{4}+\frac{3}{12}\right)+\frac{4}{12}$
$=1+\frac{1}{3}$
$=1\frac{1}{3}$

C $\frac{3}{2}-\frac{7}{10}=\left(\frac{3}{2}-\frac{5}{10}\right)-\frac{2}{10}$
$=1-\frac{1}{5}$
$=\frac{4}{5}$

D $\frac{7}{4}-\frac{7}{8}=\left(\frac{7}{4}-\frac{6}{8}\right)-\frac{1}{8}$
$=1-\frac{1}{8}$
$=\frac{7}{8}$

E $\frac{3}{4}-\frac{1}{12}=\left(\frac{3}{4}+\frac{3}{12}\right)-\frac{4}{12}$
$=1-\frac{1}{3}$
$=\frac{2}{3}$

Exercise 2.2C: Challenge

20 Consider the following calculations. Determine which one is not equivalent to the others.

A $\left(-\frac{5}{3}\right)+\left(+\frac{4}{7}\right)-\left(-\frac{7}{11}\right)$

B $\left(-\frac{5}{3}\right)-\left(-\frac{4}{7}\right)+\left(+\frac{7}{11}\right)$

C $\left(+\frac{4}{8}\right)-\left(+\frac{5}{3}\right)+\left(+\frac{4}{7}\right)-\left(-\frac{7}{11}\right)+\left(-\frac{33}{66}\right)$

D $\left(-\frac{7}{11}\right)-\left(-\frac{4}{7}\right)+\left(-\frac{5}{3}\right)$

E $\left(\frac{-5}{3}\right)+\left(\frac{-4}{-7}\right)-\left(\frac{7}{-11}\right)$

Online resources:

- **Interactive skillsheet** Adding and subtracting fractions
- **Investigation** Puzzling pumpkins
- **Quick quiz** 2.2

Lesson 2.3

Multiplying and dividing fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ multiply and divide fractions using efficient mental and written strategies.

Inter-year links

Year 7	3.6 Multiplying fractions
Year 7	3.7 Dividing fractions

Multiplying fractions

Key content video
Multiplying fractions

- To multiply two fractions together:
  1. Look diagonally and vertically for any common factors between the numerators and denominators. Cancel any common factors in the numerators and denominators by dividing both numbers by the highest common factor (HCF).
  2. Multiply the numerators together and the denominators together.

$$\frac{2}{3}\times\frac{6}{8}=\frac{2\times 6^{2}}{3^{1}\times 8}$$ Diagonally

$$=\frac{2^{1}\times 2}{1\times 8^{4}}$$ Diagonally

$$=\frac{1\times 2}{1\times 4}$$

$$=\frac{2^{1}}{4^{2}}$$ Vertically

$$=\frac{1}{2}$$

- When multiplying mixed numbers, convert them into improper fractions, multiply across and give the result as a mixed number.
- Whole numbers can be written as fractions with a denominator of 1. For example, $8=\frac{8}{1}$.
- When asked to find a fraction 'of' a whole number or fraction, replace the 'of' with '×'. For example, $\frac{2}{3}$ of 20 is the same as $\frac{2}{3}\times 20$.

Dividing fractions by fractions

Key content video
Dividing fractions

- When a fraction is multiplied by its **reciprocal**, or multiplicative inverse, the result is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ as $\frac{2}{3}\times\frac{3}{2}=\frac{6}{6}=1$.
- To find the reciprocal of a proper or improper fraction, invert the fraction by swapping the numerator and the denominator.
- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction. Keep the first fraction the same, and multiply by the reciprocal of the second fraction.

$$\frac{3}{4}\div\frac{3}{8}=\frac{3}{4}\times\frac{8}{3}$$

- For both multiplying and dividing fractions:
  - → If the signs are the same, then the result is positive.
  - → If the signs are different, then the result is negative.

Reciprocal 5 $\frac{1}{5}$ Reciprocal

Reciprocal $\frac{3}{4}$ $\frac{4}{3}$ Reciprocal

$+\times+=+$	$+\div+=+$
$+\times-=-$	$+\div-=-$
$-\times+=-$	$-\div+=-$
$-\times-=+$	$-\div-=+$

Worked example 2.3A Multiplying fractions

Determine the result of each product.

a $\frac{8}{9} \times \frac{5}{6}$ **b** $\frac{23}{9} \times 3$ **c** $1\frac{2}{3} \times 2\frac{7}{10}$

THINK

a 1 Simplify the 8 and 6 by dividing both numbers by 2.

2 Multiply the numerators together, then multiply the denominators together.

b 1 Write 3 as a fraction with a denominator of 1.

2 Simplify the 9 and 3 by dividing both numbers by 3.

3 Multiply the numerators together, then multiply the denominators together.

c 1 Convert each mixed number into an improper fraction.

2 Simplify the 5 and 10 by dividing both numbers by 5 and then cancel the 3 and 27 by dividing both numbers by 3.

3 Multiply the numerators together, then multiply the denominators together.

4 Convert the improper fraction to a mixed number.

WRITE

a $\frac{8}{9} \times \frac{5}{6} = \frac{\cancel{8}^{4}}{9} \times \frac{5}{\cancel{6}_{3}}$

$= \frac{4 \times 5}{9 \times 3}$

$= \frac{20}{27}$

b $\frac{23}{9} \times 3 = \frac{23}{9} \times \frac{3}{1}$

$= \frac{23}{\cancel{9}^{3}} \times \frac{\cancel{3}^{1}}{1}$

$= \frac{23 \times 1}{3 \times 1}$

$= \frac{23}{3}$

c $1\frac{2}{3} \times 2\frac{7}{10} = \frac{5}{3} \times \frac{27}{10}$

$= \frac{\cancel{5}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}_{2}}$

$= \frac{1 \times 9}{1 \times 2}$

$= \frac{9}{2}$

$= 4\frac{1}{2}$

Worked example 2.3B Dividing fractions

Determine the result of each quotient.

a $\frac{1}{7} \div \frac{3}{5}$ **b** $\frac{4}{5} \div 6$ **c** $3\frac{3}{8} \div 2\frac{1}{4}$

THINK

a 1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Multiply the numerators together and then multiply the denominators together.

WRITE

a $\frac{1}{7} \div \frac{3}{5} = \frac{1}{7} \times \frac{5}{3}$

$= \frac{1 \times 5}{7 \times 3}$

$= \frac{5}{21}$

b 1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Cancel out any common factors.

3 Multiply the numerators together, then multiply the denominators together.

c 1 Convert each mixed number to an improper fraction.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel out any common factors.

4 Multiply the numerators together, then multiply the denominators together.

5 Change the improper fraction to a mixed number.

b $\frac{4}{5} \div 6 = \frac{4}{5} \times \frac{1}{6}$

$= \frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{6}^{3}}$

$= \frac{2 \times 1}{5 \times 3}$

$= \frac{2}{15}$

c $3\frac{3}{8} \div 2\frac{1}{4} = \frac{27}{8} \div \frac{9}{4}$

$= \frac{27}{8} \times \frac{4}{9}$

$= \frac{\cancel{27}^{3}}{8^{2}} \times \frac{\cancel{4}^{1}}{\cancel{9}^{1}}$

$= \frac{3 \times 1}{2 \times 1}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

Worked example 2.3C Multiplying and dividing negative fractions

Determine the result of each product or quotient.

a $-\frac{2}{9} \times \left(-\frac{13}{4}\right)$ **b** $\frac{12}{7} \div \left(-\frac{3}{14}\right)$ **c** $-2\frac{3}{4} \div \left(-4\frac{1}{2}\right)$

THINK

a 1 Determine whether the result is positive or negative. The signs are the same, so the result is positive.

2 Simplify the 2 and 4 by dividing both numbers by 2.

3 Multiply the numerators together, then multiply the denominators together.

b 1 Determine whether the result is positive or negative. The signs are different, so the result is negative.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel out any common factors.

4 Multiply the numerators together, then multiply the denominators together.

WRITE

a $\frac{\cancel{2}^{1}}{9} \times \frac{13}{\cancel{4}^{2}} = \frac{13}{18}$

So, $-\frac{2}{9} \times \left(-\frac{13}{4}\right) = \frac{13}{18}$

b $\frac{12}{7} \div \frac{3}{14} = \frac{\cancel{12}^{4}}{\cancel{7}^{1}} \times \frac{\cancel{14}^{2}}{\cancel{3}^{1}}$

$= \frac{8}{1}$

$= 8$

So, $\frac{12}{7} \div \left(-\frac{3}{14}\right) = -8$

c **1** Determine whether the result is positive or negative. The signs are the same, so the result is positive.

2 Convert each mixed number to an improper fraction.

3 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

4 Cancel out any common factors.

5 Multiply the numerators together, then multiply the denominators together.

c $2\frac{3}{4} \div 4\frac{1}{2} = \frac{11}{4} \div \frac{9}{2}$

$= \frac{11}{\cancel{4}^{2}} \times \frac{\cancel{2}^{1}}{9}$

$= \frac{11}{18}$

So, $-2\frac{3}{4} \div \left(-4\frac{1}{2}\right) = \frac{11}{18}$

Helpful hints

- ✔ You don't need a common denominator when multiplying and dividing fractions.
- ✔ Don't forget to change any whole number into a fraction with a denominator of 1.
- ✔ Make sure to simplify your fractions! You can cancel common factors vertically or diagonally.
- ✔ The phrase 'Keep, Change, Flip' can help you remember how to turn a 'dividing by a fraction' problem into a multiplication problem.
- ✔ When working with fractions, multiplying does not always make a number larger and dividing does not always make a number smaller.

Keep Change Flip

$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$

Learning pathways

1–5(1st, 2nd columns), 6–9, 13–15, 16(a), 18(a, b)

1–5(3rd, 4th columns), 6–9, 13, 15, 16(a, b, c), 17(a, c), 18, 19(a, b), 20

2(e–h), 5, 9, 15–17, 18(c, d), 19, 20, 21

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Exercise 2.3A: Understanding and fluency

1 Write the equivalent multiplication equation for each division equation by using the reciprocal.

a $\frac{9}{20} \div \frac{3}{5}$ **b** $\frac{9}{20} \div \frac{7}{15}$ **c** $\frac{20}{9} \div \frac{15}{7}$ **d** $\frac{20}{9} \div \frac{7}{15}$

e $\frac{2}{9} \div \frac{7}{4}$ **f** $\frac{2}{9} \div \frac{5}{21}$ **g** $\frac{9}{2} \div \frac{21}{5}$ **h** $\frac{9}{2} \div \frac{5}{21}$

WE 2.3A **2** Evaluate each product. Hint: Simplify before multiplying.

a $\frac{3}{8} \times \frac{2}{5}$ **b** $\frac{9}{10} \times \frac{4}{3}$ **c** $\frac{18}{25} \times \frac{35}{36}$ **d** $\frac{32}{49} \times \frac{21}{40}$

e $\frac{6}{375} \times \frac{25}{20}$ **f** $\frac{64}{12} \times \frac{27}{16}$ **g** $\frac{10}{77} \times \frac{242}{33}$ **h** $\frac{75}{70} \times \frac{42}{45}$

3 Evaluate each product.

a $1\frac{7}{8} \times \frac{3}{5}$ **b** $3\frac{1}{5} \times 1\frac{1}{4}$ **c** $9\frac{1}{6} \times 1\frac{1}{11}$ **d** $\frac{5}{18} \times 2\frac{7}{10}$

e $5\frac{1}{3} \times 2\frac{2}{5}$ **f** $\frac{3}{4} \times 8$ **g** $7 \times \frac{3}{8}$ **h** $2\frac{1}{6} \times 15$

i $42 \times 3\frac{9}{21}$ **j** $12 \times 5\frac{1}{60}$ **k** $\frac{12}{23} \times 3\frac{1}{3}$ **l** $1\frac{12}{13} \times \frac{5}{3}$

WE 2.3B **4** Evaluate each quotient.

a $\frac{5}{7} \div \frac{3}{2}$ **b** $\frac{3}{14} \div \frac{12}{35}$ **c** $\frac{9}{28} \div \frac{6}{7}$ **d** $\frac{15}{22} \div \frac{5}{33}$

e $\frac{21}{20} \div \frac{3}{20}$ **f** $\frac{17}{42} \div \frac{17}{6}$ **g** $\frac{27}{40} \div \frac{15}{40}$ **h** $\frac{12}{5} \div \frac{3}{50}$

5 Evaluate each quotient.

a $2\frac{3}{4} \div \frac{5}{12}$ **b** $1\frac{5}{8} \div 2\frac{8}{9}$ **c** $2\frac{1}{3} \div 4\frac{2}{3}$ **d** $\frac{50}{63} \div 3\frac{4}{7}$

e $2\frac{1}{3} \div 1\frac{5}{9}$ **f** $\frac{12}{7} \div 6$ **g** $\frac{17}{4} \div 7$ **h** $12 \div \frac{6}{7}$

i $7 \div \frac{17}{4}$ **j** $20\frac{25}{27} \div 5$ **k** $\frac{2}{13} \div 1\frac{1}{3}$ **l** $2\frac{3}{14} \div \frac{5}{7}$

WE 2.3C **6** Evaluate each of the following.

a $-\frac{6}{11} \times \left(-\frac{2}{3}\right)$ **b** $\frac{4}{5} \times \left(-\frac{5}{12}\right)$ **c** $-\frac{15}{8} \times \frac{16}{25}$

d $-\frac{12}{35} \div \left(-\frac{7}{9}\right)$ **e** $-\frac{5}{6} \div \frac{10}{7}$ **f** $\frac{4}{9} \times \left(-\frac{5}{12}\right)$

g $-4\frac{1}{2} \times 3\frac{1}{3}$ **h** $-2\frac{1}{6} \div \left(-3\frac{1}{4}\right)$ **i** $7\frac{2}{3} \div \left(-5\frac{1}{9}\right)$

7 When dividing two fractions with the same denominator, the question can be simplified as finding the quotient of the two numerators. For example, $\frac{7}{9} \div \frac{2}{9} = 7 \div 2 = \frac{7}{2}$. Evaluate each quotient below using this method.

a $\frac{1}{2} \div \frac{3}{2}$ **b** $\frac{7}{12} \div \frac{5}{12}$ **c** $\frac{1}{10} \div \frac{3}{5}$ **d** $\frac{3}{4} \div \frac{3}{8}$

e $\frac{5}{8} \div \left(-\frac{3}{2}\right)$ **f** $\left(-\frac{3}{2}\right) \div \left(-\frac{1}{4}\right)$ **g** $\frac{-39}{100} \div \frac{39}{-20}$ **h** $\frac{17}{-20} \div \frac{-34}{100}$

8 Evaluate each problem and simplify where possible.

a $\frac{2}{3} \times \left(\frac{4}{3} \div \frac{5}{3}\right)$ **b** $\frac{2}{5} \div \left(\frac{4}{5} \times \frac{3}{5}\right)$ **c** $-\frac{1}{5} \times \left(-\frac{4}{3} \div \frac{1}{2}\right)$ **d** $\frac{2}{3} \times \frac{4}{3} \times \left(-\frac{1}{2}\right) \div \frac{2}{3}$

e $-\frac{1}{3} \times \left(\frac{4}{-3}\right) \div \frac{7}{2}$ **f** $\frac{5}{3} \div \left(\frac{4}{3} \div 3\right)$ **g** $2\frac{1}{2} \div \left(-\frac{4}{3} \div \frac{1}{3}\right)$ **h** $-\frac{2}{3} \div \left(\frac{4}{3} \times \left(\frac{1}{2} \div \frac{2}{3}\right)\right)$

9 Evaluate the result of each of the following. Hint: Recall your knowledge of BIDMAS.

a $\left(\frac{6}{11} + \frac{2}{3}\right) \div \frac{5}{6}$ **b** $3\frac{1}{4} - 2\frac{1}{2} \times \frac{7}{10}$ **c** $\frac{8}{9} \times \left(\frac{4}{8} - \frac{3}{4}\right) \div 2\frac{2}{3}$ **d** $2\frac{3}{5} \times 3\frac{1}{3} + \frac{2}{11} \times 1\frac{3}{8}$

e $-\frac{5}{9} \times \frac{3}{5} - \frac{2}{3}$ **f** $\frac{1}{2} + \frac{4}{7} \div -\frac{2}{5}$ **g** $\frac{4}{9} \times \frac{3}{8} - \frac{5}{3} \times \frac{7}{10}$ **h** $-\frac{5}{2} \times \left(\frac{7}{15} - \frac{3}{5}\right)$

Exercise 2.3B: Problem solving and reasoning

10 The quotient between a fraction and an integer can be evaluated in two ways, for each of the following:

i $\frac{6}{7} \div 2$ **ii** $\frac{5}{2} \div 4$ **iii** $\frac{25}{16} \div -5$ **iv** $-\frac{5}{8} \div 3$

a Evaluate the quotients by dividing the numerator by the integer. For example, $\frac{10}{13} \div 5 = \frac{10 \div 5}{13} = \frac{2}{13}$.

b Evaluate the quotients by multiplying the denominator of the fraction by the integer.

For example, $\frac{10}{13} \div 3 = \frac{10}{13} \times \frac{1}{3} = \frac{10}{13 \times 3} = \frac{10}{39}$.

c Do your answers in **a** match with your answers in **b**? When is it easier to use method **a** rather than method **b**?

11 Leanne has 20 red buttons amongst other coloured buttons to make an art piece. Leanne wants the red buttons to be $\frac{5}{6}$ of the entire design.

a How many buttons in total are needed for Leanne's art piece?

b Leanne stumbles across a box containing more red buttons and decides to make the art piece $\frac{1}{4}$ larger than its original size. How many red buttons are needed for the increased art piece?

c What fraction is the number of buttons in the increased art piece out of the number of buttons in the original-size art piece?

12 Grapes are to be sold in $2\frac{1}{2}$ kg bags.

a How many bags must Tricia pack if she has a 20 kg crate of grapes?

b From another crate, Tricia packs 11 bags and has $1\frac{1}{2}$ kg of grapes left over. How many kilograms of grapes were in the second crate?

13 A quantity can increase or decrease by a fraction.

a Calculate each amount.

i $\frac{1}{2}$ of 14 hours **ii** $\frac{3}{4}$ of \$48 **iii** $\frac{2}{5}$ of 35 kg **iv** $\frac{4}{9}$ of 72 cm

b Use your answers in part **a** to find the amount after the following change.

i increase 14 hours by $\frac{1}{2}$ of the original amount.

ii increase \$48 by $\frac{3}{4}$ of the original amount.

iii decrease 35 kg by $\frac{2}{5}$ of the original amount.

vi decrease 72 cm by $\frac{4}{9}$ of the original amount.

c To increase a quantity by a fraction, add the fraction to 1 and multiply the sum with the original amount. For example, to increase 14 hours by $\frac{1}{2}$, multiply 14 by $\left(1 + \frac{1}{2}\right)$. To decrease a quantity by a fraction, subtract the fraction from 1 and multiply the difference with the original amount. For example, to decrease 14 hours by $\frac{1}{2}$, multiply 14 by $\left(1 - \frac{1}{2}\right)$. Use this method to verify your answers in part **b**.

d Calculate the following amount in the units shown in brackets.

i increase 3 weeks by $\frac{2}{7}$ of the original amount (days)

ii decrease 2 cm by $\frac{3}{5}$ of the original amount (mm)

14 The seating capacity of an interstate train carriage is 88 people. If the carriage is three-quarters full, how many people are seated in the train carriage?

15 To bake 12 chocolate cupcakes, you need these ingredients.

Chocolate cupcakes

(makes 12)

100 g butter

$\frac{1}{2}$ cup sugar

2 large eggs

$1\frac{1}{2}$ cups flour

1 teaspoon baking powder

$\frac{1}{2}$ cup milk

$\frac{3}{4}$ cup cocoa powder

1 cup boiling water

Chocolate icing

$1\frac{1}{2}$ cups icing sugar

150 g butter

120 g chocolate

$\frac{1}{2}$ cup thickened cream

a Work out the quantities of each ingredient (excluding the icing) if you want to bake:

i 24 cupcakes **ii** 6 cupcakes **iii** 8 cupcakes.

b Finn bakes 12 cupcakes but before he can ice them, one-quarter are eaten. Adjust the ingredient list for the chocolate icing so it makes only enough for the remaining cupcakes.

16 We can evaluate powers of fractions by using repeated multiplication of fractions.

For example, $\left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1^2}{3^2} = \frac{1}{9}$.

Evaluate the following. Hint: Be careful with powers of negative fractions and write mixed numbers as improper fractions.

a $\left(\frac{2}{5}\right)^3$ **b** $\left(-\frac{2}{3}\right)^2$ **c** $\left(-\frac{2}{3}\right)^3$ **d** $\left(\frac{9}{7}\right)^2$ **e** $\left(3\frac{1}{3}\right)^3$

17 Since repeated multiplication of fractions repeatedly multiplies the numerator and repeatedly multiplies the denominator, we can reverse the process to find the square root of a fraction by finding the square root of the numerator and the square root of the denominator.

For example, $\sqrt{\frac{4}{9}} = \sqrt{\frac{2^2}{3^2}} = \sqrt{\frac{2}{3} \times \frac{2}{3}} = \frac{2}{3}$.

Evaluate the following. Hint: Write mixed numbers as improper fractions and simplify first.

a $\sqrt{\frac{16}{25}}$ **b** $\sqrt{\frac{49}{9}}$ **c** $\sqrt{\frac{81}{100}}$ **d** $\sqrt{\frac{50}{18}}$ **e** $\sqrt{3\frac{13}{36}}$

18 Mixed numbers can be multiplied without first writing them as improper fractions by using the distributive property.

For example,

$$\begin{aligned} 8 \times 2\frac{1}{2} &= 8 \times \left(2 + \frac{1}{2}\right) \\ &= 8 \times 2 + 8 \times \frac{1}{2} \\ &= 16 + 4 \\ &= 20. \end{aligned}$$

Multiply the following by using the distributive property. Write your answers using mixed numbers where necessary.

a $14 \times 2\frac{4}{7}$ **b** $30 \times 4\frac{9}{10}$ **c** $\frac{20}{3} \times 6\frac{2}{5}$ **d** $8 \times 4\frac{3}{5}$

Exercise 2.3C: Challenge

19 Multiplying two mixed numbers can be performed by using the distributive law three times.

For example,

$$\begin{aligned} 8\frac{1}{2} \times 2\frac{1}{2} &= 8\frac{1}{2} \times \left(2 + \frac{1}{2}\right) \\ &= \left(8\frac{1}{2} \times 2\right) + \left(8\frac{1}{2} \times \frac{1}{2}\right) \\ &= \left(8 \times 2 + \frac{1}{2} \times 2\right) + \left(8 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}\right) \\ &= (16 + 1) + \left(4 + \frac{1}{4}\right) \\ &= 17 + 4\frac{1}{4} \\ &= 21\frac{1}{4}. \end{aligned}$$

Multiply the following by using the distributive property. Write your answers using mixed numbers where necessary.

a $2\frac{3}{4} \times 6\frac{8}{9}$ **b** $14\frac{7}{15} \times 5\frac{3}{7}$ **c** $4\frac{1}{5} \times 9\frac{1}{3}$ **d** $3\frac{7}{12} \times 5\frac{3}{14}$

20 Explain why $3\frac{9}{10} \div 5\frac{2}{3} = \left(3 \div 5\frac{2}{3}\right) + \left(\frac{9}{10} \div 5\frac{2}{3}\right)$ but $3\frac{9}{10} \div 5\frac{2}{3} \neq \left(3\frac{9}{10} \div 5\right) + \left(3\frac{9}{10} \div \frac{2}{3}\right)$ by referring to the distributive property and the reciprocal.

21 Evaluate the result of each of the following.

a $\dfrac{-\frac{3}{4} \div \sqrt{\frac{9}{16}}}{2 - \frac{3}{5}}$ **b** $\dfrac{35}{4} \div \dfrac{2\frac{9}{13} \div 3}{1 - \frac{3}{4}}$

Online resources:

- Interactive skillsheet: Multiplying and dividing fractions
- Worksheet: Performing the four operations on fractions
- Investigation: Fractions on the farm
- Quick quiz: 2.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

2.1 **1** Fill the gaps to produce equivalent fractions.

a $\frac{6}{} = \frac{48}{80}$ **b** $\frac{}{20} = 2\frac{3}{4}$

c $\frac{63}{18} = \frac{}{8}$ **d** $\frac{6}{5} = \frac{-18}{}$

2.1 **2** Simplify the following fractions.

a $\frac{12}{15}$ **b** $\frac{60}{45}$

c $8\frac{16}{64}$ **d** $\frac{35}{-63}$

2.1 **3** Which fraction in each given pair is greater? Write your answer as an inequality.

a $\frac{2}{3} \square \frac{3}{4}$ **b** $\frac{7}{9} \square -\frac{8}{9}$

c $-\frac{17}{10} \square -\frac{9}{5}$ **d** $\frac{30}{36} \square \frac{36}{54}$

2.1 **4** Determine the LCD for each fraction pair.

a $\frac{3}{4}$ and $\frac{2}{3}$ **b** $\frac{5}{9}$ and $\frac{1}{6}$

c $\frac{3}{8}$ and $\frac{5}{12}$ **d** $\frac{7}{11}$ and $\frac{31}{33}$

2.2 **5** Evaluate the following sums.

a $\frac{2}{3} + \frac{2}{5}$ **b** $-\frac{7}{9} + 4\frac{2}{3}$

c $\frac{43}{10} + \left(-2\frac{3}{50}\right)$ **d** $-\frac{9}{7} + \left(-\frac{9}{4}\right)$

2.2 **6** Evaluate the following differences.

a $\frac{2}{3} - \frac{3}{4}$ **b** $-4\frac{1}{5} - 2\frac{8}{15}$

c $\frac{3}{16} - \left(-\frac{17}{24}\right)$ **d** $-3\frac{1}{3} - \left(-2\frac{1}{7}\right)$

2.3 **7** Evaluate the following products.

a $\frac{12}{5} \times \frac{25}{6}$ **b** $-\frac{4}{3} \times \frac{8}{9}$

c $4\frac{2}{5} \times \left(-\frac{1}{2}\right)$ **d** $-\frac{8}{6} \times \left(-3\frac{1}{3}\right)$

2.3 **8** Determine the reciprocal of the following.

a $\frac{3}{5}$ **b** $\frac{1}{7}$

c 9 **d** $5\frac{2}{3}$

2.3 **9** Evaluate the following quotients.

a $\frac{12}{35} \div \frac{4}{7}$ **b** $-2\frac{4}{5} \div \frac{2}{5}$

c $\frac{6}{7} \div \left(-1\frac{7}{8}\right)$ **d** $-\frac{3}{2} \div \left(-\frac{27}{16}\right)$

2.3 **10** Evaluate the following using the order of operations.

a $\frac{3}{5} + \frac{6}{5} \times \frac{2}{5}$ **b** $-\frac{2}{3} - \frac{11}{15} + \frac{7}{10}$

c $\frac{4}{9} + \left(-\frac{6}{5}\right) \div \frac{8}{15}$ **d** $\frac{9}{10} \div \left(-\frac{5}{3}\right) \times \left(-\frac{2}{7}\right)$

2.3 **11** Find:

a $\frac{5}{8}$ of 104 GB **b** $\frac{3}{4}$ of 10 L

c $1\frac{1}{2}$ of 500 g **d** $\frac{17}{20}$ of \$5.00

Lesson 2.4

Decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ order decimals, round decimals, and convert between fractions and decimals.

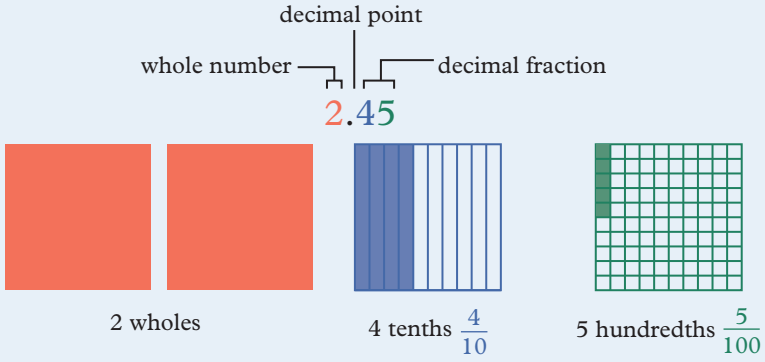
Inter-year links

Support	Fractions and decimals
Year 7	4.1 Decimals
Year 7	4.2 Ordering and rounding decimals

Decimals

Key content video
Understanding decimals

- A **decimal** is a number whose whole number part and fraction part are separated by a decimal point.



- The value of each digit depends on the place or position of the digit in the number.
  For example, the decimal 1234.5678 is shown in the place value table.

Thousands 1000	Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
1	2	3	4	.	5	6	7	8

→ Each place value to the right is 10 times smaller than the place before.
→ Each place value to the left is 10 times larger than the place before.

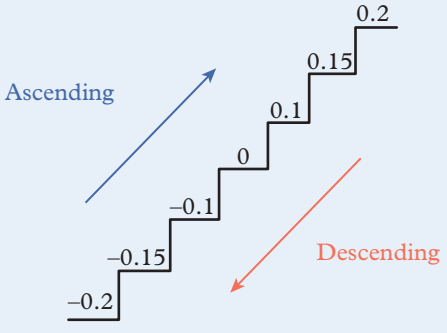
Key content video
Ordering and rounding decimals

Ordering numbers

- To order decimals consider the digits from left to right.

  For example, 5.673 and 5.689 have the same whole number and tenth values. 8 hundredths is greater than 7 hundredths, so 5.673 < 5.689.

Ones	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
5	.	6	7	3
5	.	6	8	9



Rounding

- A decimal can be rounded to a given number of decimal places by considering the next digit to the right.
  For example, both numbers shown have been rounded to two decimal places.

$1.2395 \approx 1.24$ (9, 8, 7, 6, 5 — Round up)

$1.2325 \approx 1.23$ (4, 3, 2, 1, 0 — Round down)

Converting between fractions and decimals

- To convert from a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Write 10, 100, 1000 … with as many zeros as there are digits after the decimal point in the denominator.

0.5	5 tenths	$\frac{5}{10}$
0.55	55 hundredths	$\frac{55}{100}$
0.555	555 thousandths	$\frac{555}{1000}$

- To convert from a fraction to a decimal, **trailing zeros** are added following digits after the decimal point in the dividend until there is no remainder. Trailing zeros do not change the value of the number.

Align decimal points

Dividend → 1, Divisor → 8 = 8)1.0²0⁴0 = 0.125

Trailing zeros

Worked example 2.4A Ordering decimals

Arrange the following in ascending order.
5.74, 5.7, 5.47, 5.04, 5.407, 4.57

THINK

1 Compare the whole number parts. There are five numbers with a whole number of 5 and one with a whole number of 4.

2 Compare the tenths values of the numbers with the same whole number part. 7 tenths is greater than 4 tenths and 4 tenths is greater than 0 tenths.

3 Compare the hundredths values of the numbers with the same whole number and tenths parts. 7 hundredths is greater than 0 hundredths and 4 hundredths is greater than 0 hundredths. When a place value does not contain a digit, its value is zero.

4 Write the decimals in ascending order, from smallest to largest.

WRITE

4.57, 5.74, 5.7, 5.47, 5.04, 5.407

4.57, 5.04, 5.47, 5.74, 5.407, 5.7

4.57, 5.04, 5.47, 5.74, 5.407, 5.7

4.57, 5.04, 5.407, 5.47, 5.7, 5.74

Worked example 2.4B Rounding decimals

Round each number to two decimal places.

a 5.7323 **b** −12.099 76

THINK	WRITE
a **1** Draw a box around the second decimal place.	**a** $5.7\boxed{3}23$
2 Look at the digit to the right of the box. The digit to the right is a 2. Do not change the digit in the box.	$5.7\boxed{3}2\,3$
3 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.	$5.7323 \approx 5.73$
b **1** Draw a box around the second decimal place.	**b** $-12.0\boxed{9}9\,76$
2 Look at the digit to the right of the box. The digit to the right is a 9. Add 1 to the digit in the box.	$-12.0\boxed{9}9\,76$
3 As the boxed digit changes from 9 to 10, write zero in the boxed digit's place and add one to the place to the left.	$-12.1\boxed{0}$
4 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same. Keep the zero in the second decimal place as the question is asking for two decimal places.	$-12.099\,76 \approx -12.10$

Worked example 2.4C Converting a decimal to a fraction

Write the following decimals as fractions.

a 1.4 **b** −0.52 **c** 10.068

THINK	WRITE
1 Identify any whole numbers to the left of the decimal point.	**a** $1.4 = 1\frac{4}{10}$
2 Place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point in the decimal fraction part of the number.	$= 1\frac{\cancel{4}^{2}}{\cancel{10}^{5}}$
3 Simplify the fraction by finding common factors or the HCF of the numerator and denominator.	$= 1\frac{2}{5}$
	b $-0.52 = -\frac{52}{100}$
	$= -\frac{\cancel{52}^{13}}{\cancel{100}^{25}}$
	$= -\frac{13}{25}$
	c $10.068 = 10\frac{68}{1000}$
	$= 10\frac{\cancel{68}^{17}}{\cancel{1000}^{250}}$
	$= 10\frac{17}{250}$

Worked example 2.4D Converting fractions to decimals

Write $\frac{3}{8}$ as a decimal.

THINK

1 As the denominator of the fraction is not a power of 10, write the fraction as a division problem with a decimal point after the numerator.

2 Work out the division using short division. Include trailing zeros after the decimal point until you can divide exactly without a remainder. Align the decimal points.

WRITE

$\frac{3}{8} = 3 \div 8$

$$8\overline{)3.^{3}0^{6}0^{4}0} \quad 0.375$$

$\frac{3}{8} = 0.375$

Helpful hints

- ✔ Decimals and fractions are actually two different ways of writing the same thing!
- ✔ Zeros between non-zero digits in a decimal (called placeholder zeros) must never be left out, or the value of the number will be changed.
- ✔ Zeros at the end of a decimal (called trailing zeros) do not change the value of the number.
- ✔ If you have trouble determining the place value of a digit, you can draw the place value table to help you.

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
1	2	.	3	4	5	6

- ✔ You may think that 0.000 19 is greater than 0.2 because there are more digits shown. Remember to compare from left to right. As 0 tenths is smaller than 2 tenths, 0.2 is greater than 0.000 19.
- ✔ Make sure to consider the real life context before rounding decimals.
  For example, if a chip packet costs $1 and I have $1.50, don't tell me that I can buy 1.5 chip packets – I can only buy 1!

Learning pathways

1–6, 7(a–c, g, i), 8(a–f), 9(a-d, g), 10(a–f, h), 11(a–e), 12(a–d, g, j), 13, 14, 17, 19	2(d–f), 4–5(b, d), 7(d–i), 8(g–l), 9(e–h), 10(g–l), 11(f–h), 12(e–h, k, l), 15, 16, 18, 20, 21	2(e, f), 4(d), 5(c, d), 7–8(3rd column), 9(g, h), 10(j–l), 11(g, h), 12(i–l), 15, 18, 20–23

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Exercise 2.4A: Understanding and fluency

1 Write the following numbers into the place value chart below.

a 0.0065 **b** 0.6500 **c** 6.005 **d** 60.5

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
		.				

2 Determine the place value of 5 in each number. Refer to the place value chart to help you.

a 4.705 **b** 59.300 **c** 605.9

d 1740.50 **e** 1.3745 **f** 88.352 00

3 How many decimal places does each number in question **2** have?

WE 2.4A **4** Write each list of numbers in ascending order.

a 0.214, 0.142, 0.2, 0.41, 0.104, 0.14

b 8.702, 9.87, 8.072, 9.782, 8.2, 8.97

c 150.4, –10.45, 15.04, 10.54, 105.4, –15.40

d –799.8, –800.07, 800.7, –800.8, 799.08, –800.08

5 Write each list of numbers in descending order.

a 13.7, 13.07, 13.173, 13.31, 13.137, 13.713

b 0.658, 0.6058, 0.6508, 0.685, 0.6085, 0.65

c –50.5050, 5.5555, 5.550 00, 555.55, 5.005 00, –50.0505

d 100, –99.90, –100, 999.099, –99.009, 999.90

6 Explain why 9.46 is larger than 9.452 even though 9.452 has more digits.

WE 2.4B **7** Round each number to two decimal places.

a 25.679 **b** −673.8234 **c** 0.784 231

d −101.0157 **e** 8148.255 **f** −54.2618

g 330.406 **h** −4.854 793 **i** 15.9999

8 Round each decimal to the number of decimal places shown in the brackets.

a 57.234 (2) **b** 8.9137 (3) **c** 92.087 12 (4)

d −0.756 825 (3) **e** −135.152 79 (1) **f** 65.071 226 33 (5)

g 7.510 429 (3) **h** −208.009 862 (4) **i** 1.727 727 727 (5)

j 845.845 845 (1) **k** −0.123 456 78 (4) **l** −0.123 456 78 (7)

WE 2.4C **9** Write the following decimals as fractions.

a 1.03 **b** 2.9 **c** 3.007 **d** −0.0001

e 10.7 **f** −0.000 03 **g** 12.000 009 **h** −0.000 000 7

10 Write each number as a fraction in simplest form.

a 0.12 **b** −0.54 **c** 0.803

d −0.04 **e** 0.025 **f** −0.0006

g −0.74 **h** 0.252 **i** −0.0015

j −0.048 **k** 0.000 17 **l** −0.85

11 Write each fraction as a decimal.

a $\frac{7}{10}$ **b** $-\frac{29}{100}$ **c** $\frac{451}{1000}$ **d** $-\frac{2047}{10\,000}$

e $\frac{4}{100}$ **f** $-\frac{9}{1000}$ **g** $\frac{5}{10\,000}$ **h** $-\frac{63}{1000}$

WE 2.4D **12** Write each fraction as a decimal.

a $\frac{4}{5}$ **b** $\frac{5}{8}$ **c** $-\frac{1}{2}$ **d** $\frac{5}{4}$

e $\frac{1}{8}$ **f** $-\frac{2}{5}$ **g** $\frac{7}{20}$ **h** $-\frac{9}{25}$

i $\frac{1}{40}$ **j** $-\frac{3}{5}$ **k** $\frac{11}{50}$ **l** $-\frac{5}{16}$

Exercise 2.4B: Problem solving and reasoning

13 Wendy thinks −8.2735 rounded to three decimal places is −8.273. Rosa thinks it is −8.274. Who is correct? Explain.

14 Jayden thinks that 24.7963 rounded to two decimal places is 24.80. Oscar thinks it is 24.71. Who is correct? Explain.

15 Peter thinks that 15.998 33 rounded to two decimal places is 16. Kota thinks it is 16.00. Who is correct? Explain.

16 Sabine needs to round 2.448 to one decimal place. First, Sabine rounds 2.448 to two decimal places to get 2.45. Sabine then rounds to one decimal place to get 2.5. Explain what Sabine has done wrong.

17 The length of a mosquito is measured to be 0.68 cm. Write this value correct to one decimal place.

18 Six students' heights were measured as 1.651 m, 1.63 m, 1.568 m, 1.615 m, 1.58 m and 1.657 m.

a Which is the smallest measurement?
b Which is the largest measurement?
c Write the list of measurements in ascending order.
d Round each measurement to two decimal places.
e Write your list from part **d** in ascending order. What do you notice?
f Round each measurement to one decimal place.
g Write your list from part **f** in ascending order. What do you notice?

19 The width of a human hair is measured to be 0.018 35 mm. Write this value as an approximation to:

a two decimal places **b** three decimal places **c** four decimal places.

20 The diameter of an optical fibre is 0.2845 mm.

a Round this value to:
i one decimal place **ii** two decimal places **iii** three decimal places.
b Write the exact diameter as a fraction in simplest form.

21 Round each amount to the nearest five cents.

a $42.37 **b** $207.62 **c** $7.24
d $15.03 **e** $60.71 **f** $99.99

Exercise 2.4C: Challenge

22 Use each digit from 0 to 9 only once to write a number with nine decimal places such that when rounded to an even number of decimal places, the number rounds up, and when rounded to an odd number of decimal places, the number rounds down.

□.□□□□□□□□□

23 Write the interval of values that could have been rounded to the given value in the middle in the following form. Hint: For part **a**, consider what is the smallest number with two decimal places that could round up to 4.7 and the biggest number that could round down to 4.7.

a □.□□ ≤ 4.7 < □.□□ **b** □□.□□□ ≤ 12.75 < □□.□□□

Online resources:

- Interactive skillsheet: Ordering decimals
- Interactive skillsheet: Rounding decimals
- Investigation: Decimal history
- Quick quiz: 2.4
- Interactive skillsheet: Fractions and decimals
- Worksheet: Decimal places and rounding

Lesson 2.5

Adding and subtracting decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ add and subtract decimals using efficient mental and written strategies.

Inter-year links

Support Year 7 — Adding and subtracting decimals; 4.3 Adding and subtracting decimals

Adding and subtracting decimals

Key content video: Adding and subtracting decimals

Key content video: Working with negative decimals

- Decimals can be added or subtracted in the same way as whole numbers using the vertical addition and subtraction algorithms. Add or subtract digits in each column from right to left.

$$\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \end{array}$$

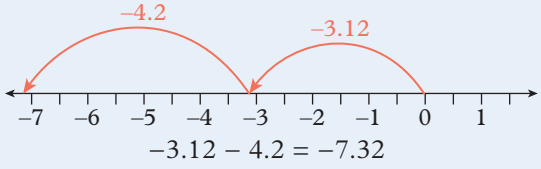
- Make sure that the decimal points are aligned and the digits are in the correct place value columns.
- When working with negative decimals, simplify the calculation by considering the operation and whether each decimal is positive or negative.

$$+\,(+) = +$$
$$-\,(+) = -$$
$$+\,(-) = -$$
$$-\,(-) = +$$

- Use the commutative, associative and distributive laws to rewrite calculations so they are easier to evaluate, including using a vertical algorithm where necessary.
  - → It may be helpful to use a number line to determine whether the result of an operation is positive or negative.

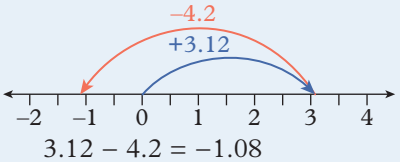
  For example:

$$\begin{aligned} -3.12 - 4.2 &= -1 \times (3.12 + 4.2) \\ &= -1 \times 7.32 \\ &= -7.32 \end{aligned}$$



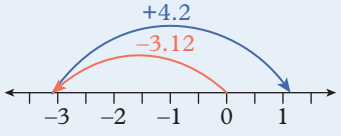
$-3.12 - 4.2 = -7.32$

$$\begin{aligned} 3.12 - 4.2 &= -(4.2 - 3.12) \\ &= -1.08 \end{aligned}$$



$3.12 - 4.2 = -1.08$

$$\begin{aligned} -3.12 + 4.2 &= +(4.2 - 3.12) \\ &= +1.08 \end{aligned}$$



$-3.12 + 4.2 = (+)1.08$

Worked example 2.5A Adding and subtracting decimals using the vertical algorithm

Use vertical addition and vertical subtraction to calculate each result.

a $14.507 + 8.63$ **b** $45.93 - 30.498$

THINK

1 Estimate the result by rounding to the nearest whole number.

2 Write the numbers underneath each other. Ensure the decimal points and digits are aligned.

3 Replace blank spaces with additional zeros so that each number contains the same number of decimal places.

4 Add or subtract the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.

5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.

WRITE

a $14.507 + 8.63 \approx 15 + 9$
$= 24$

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ \ \ } \\ 14.507 \\ +\ 08.630 \\ \hline 23.137 \end{array}$$

$14.507 + 8.63 = 23.137$
$24 \approx 23.137$

b $45.93 - 30.498 \approx 46 - 30$
$= 16$

$$\begin{array}{r} {\scriptstyle \ \ \ \ \ 8\ 2\ 10} \\ {\scriptstyle 12} \\ 45.9\not{3}\not{0} \\ -\ 30.498 \\ \hline 15.432 \end{array}$$

$45.93 - 30.498 = 15.432$
$16 \approx 15.432$

Worked example 2.5B Adding decimals with negative signs

Evaluate the following sums.

a $-3.256 + 5.89$ **b** $-1.68 + (-3.45)$

THINK

a 1 Use the commutative law of addition to rewrite the expression in a simpler equivalent form.

2 Use the vertical algorithm to evaluate the new expression.

3 Check your answer by rounding each number in the original expression to the nearest whole number.

WRITE

a $-3.256 + 5.89 = 5.89 - 3.256$

$$\begin{array}{r} {\scriptstyle \ \ \ \ \ 8\ 10} \\ 5.8\not{9}\not{0} \\ -3.256 \\ \hline 2.634 \end{array}$$

$-3.256 + 5.89 = 2.634$
$-3.256 + 5.89 \approx -3 + 6$
$= 3$
$3 \approx 2.634$

b 1 Use the distributive law of multiplication to rewrite the expression as a product of −1 and a sum of two positive decimals.

2 Calculate the sum of the positive decimals using the vertical addition algorithm.

3 Multiply the sum by −1 to find the result of the original expression.

4 Check your answer by rounding each number in the original expression to the nearest whole number.

b $-1.68 + (-3.45) = -1 \times (1.68 + 3.45)$

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 1.68 \\ +3.45 \\ \hline 5.13 \end{array}$$

$$\begin{aligned} -1.68 + (-3.45) &= -(1.68 + 3.45) \\ &= -5.13 \end{aligned}$$

$$\begin{aligned} -1.68 + (-3.45) &\approx -2 + (-3) \\ &= -5 \end{aligned}$$

$$-5 \approx -5.13$$

Worked example 2.5C Subtracting decimals with positive and negative signs

Evaluate the following differences.

a $7.58 - 8.6$ **b** $4.206 - (-13.797)$

THINK

a 1 Use the distributive law of multiplication to rewrite the expression as a product of −1 and a positive difference.

2 Subtract the smaller decimal from the larger decimal using the vertical subtraction algorithm.

3 Multiply the difference by −1 to find the result of the original expression.

4 Check your answer by rounding each number in the original expression to the nearest whole number.

b 1 Simplify the signs.

2 Add the two decimals together using the vertical addition algorithm.

3 Check your answer by rounding each number in the original expression to the nearest whole number.

WRITE

a $7.58 - 8.6 = -1 \times (8.6 - 7.58)$

$$\begin{array}{r} {\scriptstyle 5\ 10} \\ 8.\not{6}\not{0} \\ -\ 7.58 \\ \hline 1.02 \end{array}$$

$7.58 - 8.6 = -1.02$

$$\begin{aligned} 7.58 - 8.6 &\approx 8 - 9 \\ &= -1 \end{aligned}$$

$$-1 \approx -1.02$$

b $4.206 - (-13.797) = 4.206 + 13.797$

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ 1} \\ 13.797 \\ +\ 4.206 \\ \hline 18.003 \end{array}$$

$4.206 - (-13.797) = 18.003$

$$\begin{aligned} 4.206 - (-13.797) &\approx 4 - (-14) \\ &= 18 \end{aligned}$$

$$18 \approx 18.003$$

Helpful hints

- ✔ Rounding to the nearest whole number is a good way to check if your result makes sense.
- ✔ Make sure that the decimal points in each number are aligned when adding or subtracting decimals. For example,

$$\begin{array}{r} 100\,000.3 \\ -\quad 12.000\,003 \\ \hline \end{array}$$

- ✔ When adding trailing zeros after the decimal, you are not changing the number. You are just writing a zero that was already implicitly there.

Learning pathways

1, 2(a–c), 3, 4–7(1st column), 8–11, 12(a, b), 14, 15(a)	1, 2(d–f), 3, 4–8(2nd column), 11, 12(a, c), 13, 15, 16(a, b)	2(d–f), 4–7(2nd column), 8(d), 12(c, d), 13, 16(c, d), 17, 18

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Exercise 2.5A: Understanding and fluency

WE 2.5A **1** Use the vertical addition algorithm to calculate each result.

a $\begin{array}{r} 5.27 \\ +4.83 \\ \hline \end{array}$ **b** $\begin{array}{r} 18.9 \\ +\ 7.35 \\ \hline \end{array}$ **c** $\begin{array}{r} 0.085 \\ 1.32 \\ +0.647 \\ \hline \end{array}$ **d** $\begin{array}{r} 17.378 \\ 6.003 \\ +\ 5.999 \\ \hline \end{array}$

2 Use the vertical addition algorithm to calculate each result.

a 23.82 + 17.13 + 124.06 + 82.95
b 5.613 + 9.27 + 11.3856 + 7.5
c 324.7 + 76.29 + 4.93 + 285.064
d 74.0106 + 2.93 + 28.738 + 0.9167
e 0.0034 + 0.21 + 0.024 + 0.0402
f 99.999 + 14.24 + 4.748 + 0.0284

3 Use the vertical subtraction algorithm to calculate each result.

a $\begin{array}{r} 158.29 \\ -\ 75.46 \\ \hline \end{array}$ **b** $\begin{array}{r} 2051.384 \\ -\ 1265.73 \\ \hline \end{array}$ **c** $\begin{array}{r} 0.35 \\ -\ 0.2156 \\ \hline \end{array}$ **d** $\begin{array}{r} 70.308 \\ -\ 44.057 \\ \hline \end{array}$

4 Use the vertical subtraction algorithm to calculate each result.

a 9.104 − 6.3728
b 265.7 − 88.504
c 5061.03 −1266.735
d 73 412 − 8743.56
e 9000 − 0.555
f 6.5677 − 4.5678

WE 2.5B **5** Evaluate the following sums.

a 3.2 + (−5.8)
b −3.2 + (−5.8)
c −3.2 + 5.8
d 51.294 + (−3.028)
e −4.1928 + 0.285
f −0.048 + (−495.19)

WE 2.5C **6** Evaluate the following differences.

a 4.78 − 8.48
b −4.78 − 8.48
c −4.78 − (−8.48)
d −3.541 − 58.242
e −274.28 − (−0.4827)
f 9.402 − 10.611

7 Evaluate the following.

a 5.8 − 9.9 + 3.2
b 19.53 + 3.78 − 32.05
c 100 − 210.15 + 27.3
d 0.0032 + 0.1 − 0.352

8 Evaluate the following.

a $3.7 - (5.2 + 6.1)$

b $4.8 - (9.7 - 10.11)$

c $17.3 - (12.4 - 30.8)$

d $-85.23 - (0.0573 + 2.902\,04)$

Exercise 2.5B: Problem solving and reasoning

9 The circuit of a fun run has five different stages. The lengths of the stages are: 1.435 km, 2.790 km, 1.902 km, 2.285 km and 3.613 km. First estimate the distance around the full circuit by rounding each length to the nearest whole number and then calculate the exact distance.

10 Lee Redmond once held the record for the longest fingernails on both hands (see table), after not cutting them for 30 years.

	Fingernail lengths (cm)	
	Right hand	**Left hand**
Thumb	76.4	80.0
Index finger	72.3	76.4
Middle finger	74.1	76.7
Ring finger	73.6	76.2
Little finger	71.6	73.6

a What is the total length of the fingernails on:

i her right hand

ii her left hand?

b Which hand has the longer fingernails? Suggest a reason for this.

c What is the total length of the fingernails on both hands?

11 Consider the calculation $7.2 - 9.4 + 2.3$.

a Which of the following is not equivalent to the original calculation?

A $(7.2 - 9.4) + 2.3$

B $(7.2 + 2.3) - 9.4$

C $7.2 - (9.4 - 2.3)$

D $7.2 - (9.4 + 2.3)$

E $2.3 - (9.4 - 7.2)$

b Explain why the answer to part **a** is not equivalent to $7.2 - 9.4 + 2.3$ and write a similar expression that would be equivalent.

c For any calculation involving only addition and subtraction, if all subtractions are performed first, will the answer always be correct? Explain why or why not.

12 Calculate the following sums and differences by using addition and subtraction of the equivalent fractions.

For example, $12.3 + 4.21 = \frac{123}{10} + \frac{421}{100} = \frac{1230}{100} + \frac{421}{100} = \frac{1651}{100} = 16.51$.

a $-17.32 + 4.0032$

b $9.999 + (-50.91)$

c $-0.003 - (-0.0502)$

d $-90.06 - 73.0012$

13 Jimmy currently has $32.50 saved up. He wants to buy a new game that costs $69.99. Jimmy's parents agree to pay the difference if Jimmy pays them back. Jimmy earns $5.50 a week in pocket money but can increase it by completing various chores. Each chore has a limit of how often Jimmy can complete it for pocket money. The chores and how much Jimmy earns from them are listed below.

- unload and hang the washing up to dry, $1.10 twice a week
- fold the laundry and put it away, $0.90 twice a week
- take out the rubbish bin, $0.45 daily
- dust the shelves and bookcases, $1.70 weekly
- help prepare dinner, $2.10 daily

a Can Jimmy pay his parents back in two weeks? State how much Jimmy still owes or how much extra he earns.

b If Jimmy wants to save up for a $79.99 game after he has paid his parents back, what is the minimum number of additional weeks it will take?

14 Look back over your calculations for adding decimals together. What do you notice about the number of decimal places in the calculation and the number of decimal places in the result? Determine the number of decimal places in the result of a sum where a number with x decimal places is added to a number with y decimal places, where x is greater than or equal to y.

15 a Use the fact that $-(3.12 + 4.2) = -3.12 + (-4.2)$ by the distributive law to help explain why we can add 3.12 and 4.2 to calculate $-3.12 - 4.2$.

b Use the fact that $-(4.2 - 3.12) = -4.2 - (-3.12)$ by the distributive law to explain why we can subtract 3.12 from 4.2 to calculate $3.12 - 4.2$.

16 Subtracting a negative number can be rewritten as adding the additive inverse. The associative and commutative laws allow us to rearrange the order of calculations. We can then add all of the positive numbers together, then add all of the negative numbers together. The result can be determined by finding the difference between the two sums. For example,

$$\begin{aligned} & -3.1 + (-2.4) + 4.9 - (-9.3) - 6.2 \\ &= (-3.1) + (-2.4) + (+4.9) + (+9.3) + (-6.2) \\ &= [(+4.9) + (+9.3)] + [(-3.1) + (-2.4) + (-6.2)] \\ &= 14.2 + (-11.7) \\ &= 2.5 \end{aligned}$$

Write the following as the difference of the total positive and negative sums by making use of the associative and commutative laws, then evaluate.

a $9.2 - 7.2 + (-4.1) - (-4.8) + 7 - 2.6$

b $-28.19 - (-49.18) + 99.24 + (-72.09) - (-13.25)$

c $0.028 - 0.18 - (-0.291) + (-2.59) - 12.01 + 9$

d $-70.18 - 81.8 + 29.09 - (-45.29) - 29.90 + 17.7$

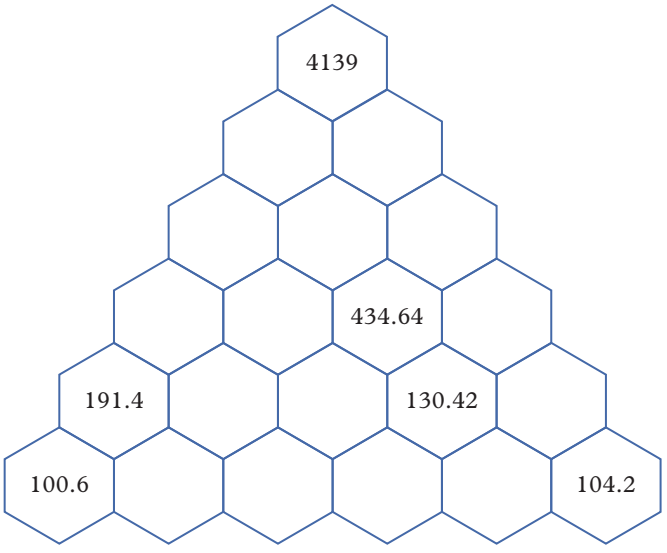
17 You begin by facing north. You turn 197.35° clockwise, then 87.194° anti-clockwise, then 3.813° clockwise, then 99.24° clockwise, then 100.01° anti-clockwise, and finally 32.021° anti-clockwise.

a In what direction (clockwise or anti-clockwise) and by how many degrees are you now facing relative to north?

b How many degrees did you turn in total, regardless of direction?

Exercise 2.5C: Challenge

18 Fill in the blank boxes with the correct number. Every number is formed by adding the number left below and right below in the box. Your algebra skills may be useful for solving this problem.



Online resources:

Interactive skillsheet
Adding and subtracting decimals

Investigation
How large is your hand?

Quick quiz
2.5

Lesson 2.6

Multiplying and dividing decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ multiply and divide decimals using efficient mental and written strategies.

Inter-year links

Support	Multiplying decimals
Year 7	4.4 Multiplying decimals
Year 7	4.5 Dividing decimals

Key content video
Multiplying decimals

Multiplying and dividing by powers of 10

- Multiplying a number by 10 increases each digit's place value by 1 column. Move the decimal point one place value space to the right and insert a zero where necessary.

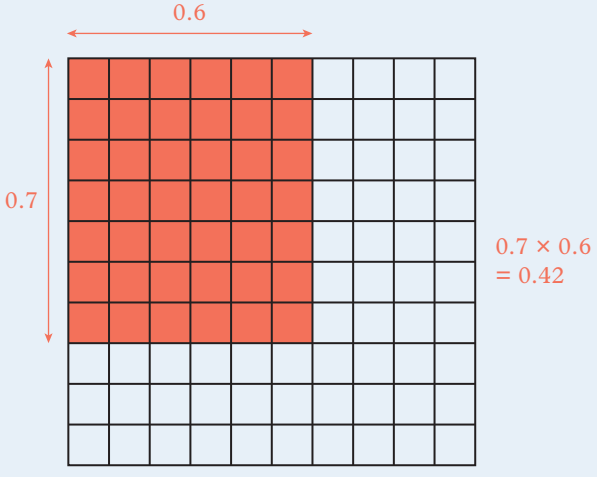
$$5.23 \times 10 = 52.3$$
$$5.23 \times 100 = 523.$$
$$5.23 \times 1000 = 5230.$$

- Dividing a number by 10 decreases each digit's place value by 1 column. Move the decimal point one place value space to the left and insert a zero where necessary.

$$5.23 \div 10 = 0.523$$
$$5.23 \div 100 = 0.0523$$
$$5.23 \div 1000 = 0.005\,23$$

Multiplying decimals

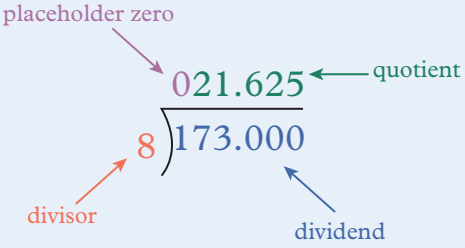
- Decimals can be multiplied in the same way as whole numbers.
- The result has the same number of decimal places as the total number of decimal places in the original problem.
  1 Count the decimal places in the problem.
  2 Multiply the decimals as whole numbers.
  3 Write the answer with this same number of decimal places. Add zeros where necessary.



Dividing a decimal by a whole number

Key content video
Dividing a decimal by a whole number

- Align the decimal point in the answer with the one in the dividend.
- Trailing zeros are added after the decimal point in the dividend until there is no remainder.

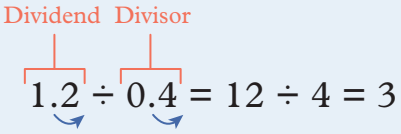


Dividing a decimal by a decimal

Key content video
Dividing a decimal by a decimal

- When you divide by a decimal, you can create an equivalent problem by moving the decimal point in both the dividend and divisor the same number of spaces to the right of the decimal point, or multiplying by $\frac{10}{10}, \frac{100}{100}, \frac{1000}{1000}, \ldots$ (You are creating equivalent decimal fractions!)

1 Convert the divisor to a whole number by moving the decimal point to the right.
2 Move the decimal point the same number of spaces to the right on the dividend.
3 Perform short division and add trailing zeros when necessary. Make sure the decimal points are aligned.



Worked example 2.6A Multiplying and dividing by powers of 10

Calculate:

a 2.27×100

b $3.14 \div 1000$

THINK

a When multiplying by 100, move the decimal point two place value spaces to the right. Don't include the decimal point if there are no digits other than zero after it.

b When dividing by 1000, move the decimal point three place value spaces to the left. Add zeros where necessary.

WRITE

a 2.27
$2.27 \times 100 = 227$

b 0003.14
$3.14 \div 1000 = 0.003\,14$

Worked example 2.6B Multiplying decimals

Use long multiplication to calculate 20.45×-3.9.

THINK

1. Count the decimal places in the problem. There are three decimal places in total.
2. Multiply the decimals as whole numbers using long multiplication.
3. Write the answer with this same number of decimal places. Add zeros where necessary. Remember that a positive number multiplied by a negative number is a negative number, so write the answer with a negative sign.

WRITE

20.45×-3.9

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ {\scriptstyle 4\ 4} \\ 2045 \\ \times\ \ 39 \\ \hline 18405 \\ +61350 \\ \hline 79755 \end{array}$$

$79\,755$

$20.45 \times -3.9 = -79.755$

Worked example 2.6C Dividing a decimal by a whole number

Use short division to calculate $-37.45 \div 4$.

THINK

1. Evaluate the division in the same way as for whole numbers, using short division.
2. Align the decimal point and the digits that have the same place value.
3. Add trailing zeros after the last digit in the dividend to obtain an exact decimal answer without a remainder.
4. Write the answer with the appropriate sign. Remember that a negative divided by a positive is a negative.

WRITE

$$\begin{array}{r} 9.3625 \\ \hline 4\overline{)37.^{1}4^{2}5^{1}0^{2}0} \end{array}$$

$-37.45 \div 4 = -9.3625$

Worked example 2.6D Dividing by decimals

Use short division to calculate $-9.46 \div -0.5$.

THINK

1. Convert the divisor to a whole number by moving the decimal point to the right.
2. Move the decimal point the same amount of spaces to the right on the dividend. Add a placeholding zero where necessary.
3. Perform short division and add trailing zeros where necessary. Make sure the decimal points are aligned.
4. Remember that a negative divided by a negative is a positive.

WRITE

$-9.46 \div -0.5 = -94.6 \div -5$

$$\begin{array}{r} 18.92 \\ \hline 5\overline{)9^{4}4.^{4}6^{1}0} \end{array}$$

$-9.46 \div -0.5 = 18.92$

Helpful hints

- ✔ Moving the decimal point of the dividend and the divisor one place to the right is the same as multiplying by $\frac{10}{10}$.
- ✔ Remember that trailing zeros should be added to the dividend until you can divide exactly, without a remainder.

Learning pathways

1–5, 6(a, b, d, e, g), 7, 9(a, c, d), 10(a–c), 12(a, b), 13, 14	1, 2, 3–4(e–h), 6(c, d, f, h, i), 7, 9(b, e, f), 10, 11, 12(c), 15–17(a, c)	3–4(c, f, h), 6(d, f, g, i), 7, 9(c, e, f), 10(e, f), 12(c), 15(b, d), 16–20

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Exercise 2.6A: Understanding and fluency

WE 2.6A **1** Evaluate the products and quotients.

a 0.0052×10 **b** 10.4×100 **c** $92.1 \div 10$

d $340 \div 10\,000$ **e** $0.057 \div 100$ **f** $45.78 \div 1000$

WE 2.6B **2** Use multiplication to calculate each result. Remember to estimate your result first, and to compare it with your answer.

a 24.35×8 **b** 305.6×4 **c** $17\,352.7 \times 3$ **d** $0.327\,51 \times 9$

3 Use long multiplication to calculate each result.

a 8.27×5.3 **b** -12.86×2.4 **c** $-235.2 \times (-9.7)$ **d** 62.513×1.8

e $9.2 \times (-0.46)$ **f** $3.8927 \times (-0.52)$ **g** $-56.361 \times (-0.14)$ **h** 157.93×7.12

WE 2.6C **4** Use short division to calculate each result.

a $72.596 \div 4$ **b** $6.724 \div (-5)$ **c** $-58.2743 \div 7$ **d** $65.0955 \div 3$

e $-4.73 \div (-8)$ **f** $0.627 \div (-4)$ **g** $0.1536 \div 6$ **h** $-138.0714 \div 5$

5 Dividing a decimal by another decimal is easier if you write an equivalent division problem where the divisor is a whole number.

a In $3.568 \div 0.4$, the divisor is 0.4. Should you multiply 0.4 by 10, 100 or 1000 to produce 4?

b Complete: $3.568 \div 0.4 =$ _____ $\div 4$

c Why is it important to multiply each decimal in the division problem by the same number?

d Use your answer to part **b** to work out the division.

6 Write each calculation as an equivalent division problem where the divisor is a whole number.

a $7.24 \div 0.2$ **b** $4.902 \div (-0.6)$ **c** $19.764 \div 0.03$

d $-1.728 \div (-0.08)$ **e** $1.4368 \div 0.004$ **f** $-7.491 \div 0.06$

g $4.872 \div (-1.5)$ **h** $2.808 \div 0.12$ **i** $-0.1638 \div (-0.009)$

WE 2.6D **7** Calculate the result of each division problem in question **6**.

8 Evaluate the following.

a $0.62 \times (1.25 \div 0.05)$ **b** $-0.063 \div (0.7 \div 35)$

c $-1.41 \times (-3.2 \div 0.02)$ **d** $2.13 \times 0.06 \times (-0.2) \div 0.05$

e $-33.3 \times (-0.2 \div 1.25)$ **f** $5.25 \div (0.1 \div 2)$

g $3.21 \div (-0.18 \div 0.6)$ **h** $-14.4 \div (0.03 \times (-10 \div 0.25))$

9 Evaluate the following.

a $(15.73 + 2.9) \times 1.5 - 26.07$

b $72.816 \div 8 + 4.9 \times 0.5$

c $6.9 + 0.528 \div 0.04 - 9.3$

d $12.08 - \dfrac{3.4 + 0.5}{4} + 8.217$

e $-3.2 \times (-9.51) + (-91.5) \div 1.5$

f $\dfrac{0.31 \times 0.031}{5} - \dfrac{5}{-0.25 \times (-0.4)}$

Exercise 2.6B: Problem solving and reasoning

10 Calculate the cost of each item. Write your answers correct to the nearest cent.

a 4 kg of frozen berries at $7.35 per kg

b 3 kg of watermelon at $1.95 per kg

c 0.6 kg of snow peas at $5.62 per kg

d 2.5 kg of potatoes at $3.95 per kg

e 0.55 kg of bok choy at $8.99 per kg

f 1.74 kg of green beans at $6.89 per kg

11 Estimate the costs related to the items listed in question **10**.

a Estimate the total cost of all items by first rounding the cost per kg of each item to the nearest dollar. Round your final answer to the nearest dollar.

b Find the total cost of these items. Round your answer to the nearest five cents.

c How much change would you receive if you paid with a $100 note?

12 Use division to answer the following questions.

a Calculate the cost of one item in each case.

i nine bread rolls cost $5.40

ii four cartons of milk cost $11.56

iii five DVDs cost $94.75

b Calculate the cost of 1 kg in each case.

i 3 kg of tomatoes cost $13.95

ii 7 kg of sausages cost $47.95

iii 6 kg of dog food cost $81.66

c Calculate the cost of 100 g in each case.

i 200 g of salami cost $25.90

ii 500 g of cheese cost $7.95

iii 300 g of cashews cost $5.85

13 A box of 10 coloured pencils costs $7.50. The pencils can also be bought individually for 72 cents each.

a Find the cost per pencil in each case.

b Which option provides better value (or is the 'best buy')? Explain.

14 William bought a 400 g pack of mushrooms for $3.69. Kate bought 300 g of loose mushrooms for $2.98. In each of these calculations, round your answer to the nearest cent.

a Calculate the amount paid by each of them for 100 g of mushrooms.

b Who made the best buy? Explain.

c Why does a store offer both options?

15 Calculate the following products and quotients by using multiplication and division of fractions.

For example, $12.3 \times 0.02 = \frac{123}{10} \times \frac{2}{100}$ and $0.02 \div 0.4 = \frac{2}{100} \div \frac{4}{10}$

$= \frac{246}{1000}$ $= \frac{2}{100} \times \frac{10}{4}$

$= 0.246$ $= \frac{1}{20}$

$= \frac{5}{100}$

$= 0.05$

a -0.17×0.0032 **b** $9.999 \times (-5.91)$ **c** $-0.003 \div (-7.5)$ **d** $-9.6 \div 0.0012$

16 We can evaluate powers of decimals by using repeated multiplication of decimals. However, evaluating the power of a decimal can be made easier by first writing the decimal as a fraction. For example,

$(-1.2)^2 = \left(-\frac{12}{10}\right)^2 = \left(-\frac{12}{10}\right) \times \left(-\frac{12}{10}\right) = \frac{144}{100} = 1.44$ and

$(-0.2)^3 = \left(-\frac{2}{10}\right)^3 = \left(-\frac{2}{10}\right) \times \left(-\frac{2}{10}\right) \times \left(-\frac{2}{10}\right) = -\frac{8}{1000} = -0.008$

Evaluate the following powers.

a $(-0.4)^3$ **b** $(-0.001)^3$ **c** $(-0.09)^2$ **d** $(2.5)^3$

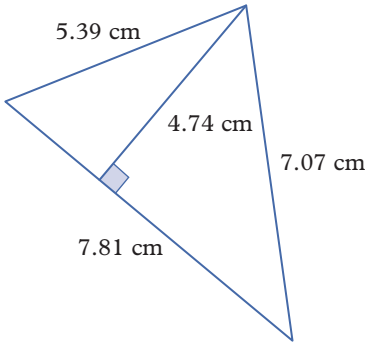
17 Rewrite the following division problems as multiplications by the reciprocal, then evaluate the quotient.

a $6.3 \div 0.5$ **b** $9.2 \div 0.2$ **c** $0.49 \div 0.25$

d $7.4 \div 1.25$ **e** $0.041 \div 2.5$ **f** $10.21 \div 6.25$

Exercise 2.6C: Challenge

18 Calculate the area and perimeter of the triangle shown.



19 Evaluate the following.

a $-2.25 \times \left(-\frac{12}{5} \div 0.05\right)$ **b** $-0.03 \div \left(-\frac{1}{3} \times 0.7\right)$ **c** $-1.41 \times \left(-3.2 \div \frac{1}{6}\right)$

d $2.2 \times -0.06 \times (-0.2)^3 \div \frac{2}{25}$ **e** $-20.5 - \left(-\frac{1}{5} \div 1.25\right)$ **f** $5\frac{1}{4} \div \left(12.5 \times \left(-\frac{2}{25}\right)\right)$

20 Use the digits 0 to 9 no more than once each to make the result of this calculation as close to 0 as possible.

□.□□□ − □.□□ × □.□□

Online resources:

- **Interactive skillsheet** Multiplying and dividing decimals
- **Worksheet** Performing the four operations on decimals
- **Investigation** Let's party!
- **Quick quiz** 2.6

Lesson 2.7

Terminating, non-terminating and recurring decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and represent rational and irrational numbers using appropriate notations.

Inter-year links

Support	Fractions, decimals and percentages
Year 7	4.7 Fractions, decimals and percentages

Key content video
Terminating, non-terminating and recurring decimals

Terminating decimals

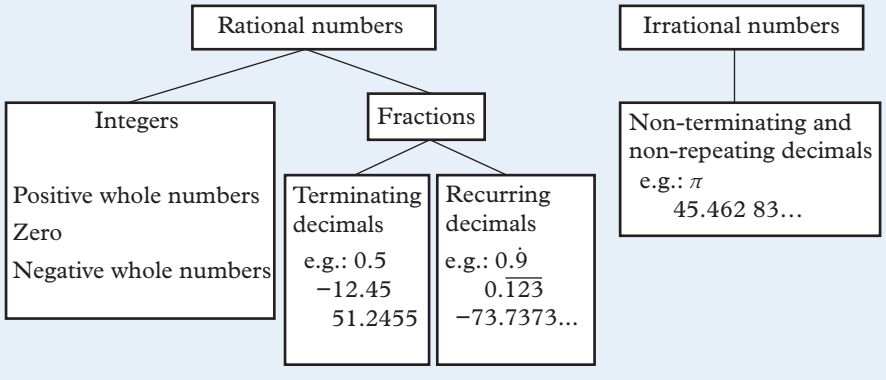
- A **terminating decimal** 'terminates' or stops after a number of decimal places.
  For example: 24.3, 0.49, 5.768 234

Non-terminating decimals

- A **non-terminating decimal** has an endless (infinite) number of decimal places.
  For example: 17.326 35…, 0.567 743…, 0.788 888…
  → The ellipses (three full stops) written after the last decimal place indicate that an endless number of digits follow. If there is a clear pattern in the preceding digits, then the pattern continues.

Recurring decimals

- A **recurring decimal** is a special type of non-terminating decimal. The digits after the decimal point form a recurring or repeating pattern.
  For example: 8.999 999…, 42.671 671 671…, 0.582 323 232…
- A recurring decimal can be written in an abbreviated form using dots or a vinculum (bar) over repeating digits.
  → If a single digit is repeated, place a bar over the digit that repeats forever.
  For example, $8.999\,999... = 8.\dot{9}$
  → If more than one digit is repeated, place a dot over the first and last digits in the repeating pattern, or place a bar over the digits in the repeating pattern.
  For example, $0.582\,323\,23... = 0.58\dot{2}\dot{3}$ and $42.671\,671... = 42.\overline{671}$
- **Rational numbers** are numbers that can be written exactly as fractions; both the numerator and denominator need to be integers and the denominator is non-zero. This includes whole numbers, fractions, terminating decimals and recurring decimals.
- **Irrational numbers** are numbers that cannot be written exactly as fractions. These are non-terminating decimals that are not recurring decimals; for example, π (or '**pi**').



Worked example 2.7A Writing recurring decimals in abbreviated form

Write each recurring decimal in abbreviated form using dots or a bar.

a $7.682\,222\,222\ldots$ **b** $-24.503\,150\,315\,031\ldots$

THINK

a Look for a repeating pattern of digits. Since there is only one digit that repeats, show a dot (or a bar) above the 2.

b Look for a repeating pattern of digits. Show a bar above 5031 (or show a dot above the first and last digit in the pattern).

WRITE

a $7.682\,222\,222\ldots$
$= 7.68\dot{2}$

b $-24.503\,150\,315\,031\ldots$
$= -24.\overline{5031}$ (or $-24.\dot{5}03\dot{1}$)

Worked example 2.7B Writing recurring decimals to a number of decimal places

Write each recurring decimal to show twelve decimal places.

a $0.\dot{4}$ **b** $-159.8\overline{372}$

THINK

a 1 Identify the repeating digit. The digit 4 has a dot above it so it is a repeating digit.

2 Write the decimal to twelve decimal places and indicate that more digits follow.

b 1 Identify the repeating digits. The digits 372 have a bar above them, so all three digits are repeating.

2 Write the decimal to twelve decimal places and indicate that more digits follow.

WRITE

a $0.\dot{4}$
$= 0.444\,444\,444\,444\ldots$

b $-159.8\overline{372}$
$= -159.837\,237\,237\,237\ldots$

Worked example 2.7C Expressing fractions as recurring decimals

Convert $\frac{5}{3}$ to a decimal by dividing the numerator by the denominator.

THINK

1 Write the fraction as a division problem with a decimal point after the numerator.

2 Perform short division and add trailing zeros until a pattern occurs. Make sure the decimal points are aligned.

3 Write the decimal using the abbreviated form with a dot (or a bar) above 6.

WRITE

$\frac{5}{3} = 5 \div 3$

$$3\overline{)5.^{2}0^{2}0^{2}0^{2}0} \quad 1.\ 6\ \ 6\ \ 6\ldots$$

$\frac{5}{3} = 1.\dot{6}$

Worked example 2.7D Expressing mixed numbers as recurring decimals

Convert $-1\frac{2}{7}$ to a decimal by first converting to an improper fraction and then dividing the numerator by the denominator.

THINK

1. Convert the mixed number to an improper fraction.
2. Write the fraction as a division problem with a decimal point after the numerator.
3. Perform short division and add trailing zeros until a pattern occurs. Make sure the decimal points are aligned.
4. Write the decimal using the abbreviated form with a bar above the six recurring digits.

WRITE

$-1\frac{2}{7} = -\frac{9}{7}$

$= -9 \div 7$

$$7\overline{)-9.^{2}0^{6}0^{4}0^{5}0^{1}0^{3}0^{2}0^{6}0^{4}0^{5}0} \quad -1.\,2\,8\,5\,7\,1\,4\,2\,8\,5\,7\ldots$$

$-1\frac{2}{7} = -1.\overline{285\,714}$

Helpful hints

- ✔ There are only two options for fractions when they are converted to decimals. Either they terminate or they are recurring. There are no fractions with an integer numerator and denominator that do not terminate and have no recurring pattern. Such decimals can never be written as fractions.
- ✔ Some people think that decimals are more 'exact' than fractions. However, mathematicians prefer fractions as you cannot tell if a decimal has been rounded, truncated or only measured to a number of decimal places.
- ✔ Zero is a rational number, as $0 = \frac{0}{1}$. In fact, all integers are rational numbers.
- ✔ There is no limit on the number of decimals that can make a repeating pattern in a recurring decimal. For example, $\frac{1}{7} = 0.\overline{142\,857}$ has a pattern of six repeating digits.

Learning pathways

1–3, 4–5(a–h), 6, 7(a, c, f), 8, 9, 10(a–f), 11, 14	3, 4–5(i–l), 6–8, 9, 10(g–k), 12–15	3, 4(j–l), 5(i–l), 6–7(f, g, h), 9, 10(g, h, k, l), 12–16

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Exercise 2.7A: Understanding and fluency

1 State whether each decimal is terminating (T) or non-terminating (N).

a 4.572 35…	**b** 56.803	**c** 21.577 77…	**d** −8.561 395 62
e −0.451 730 69…	**f** 17.565 656 5…	**g** −0.123 412 341 234…	**h** −0.787 878 78
i −30.082 174…	**j** 291.291 291 2	**k** 0.999…	**l** −5.043 127

2 Which of the numbers in question **1** are recurring decimals?

3 Explain the differences between terminating, non-terminating and recurring decimals.

WE 2.7A **4** Write each recurring decimal in abbreviated form using dots or a bar.

a 0.999 999… **b** −4.363 636 3… **c** 87.255 555… **d** 3.476 476 476…

e −0.738 222 222… **f** 16.196 666 66… **g** 483.888 88… **h** −6.729 191 919…

i −0.183 673 673 67… **j** 1.356 283 562 835… **k** 28.707 070 7… **l** −9.230 714 444 44…

WE 2.7B **5** Write each recurring decimal to show twelve decimal places.

a $0.\dot{1}$ **b** $5.3\dot{7}$ **c** $-14.\overline{82}$ **d** $-21.5\dot{6}\dot{9}$

e $8.\overline{104}$ **f** $0.\overline{6513}$ **g** $-232.68\dot{4}$ **h** $44.761\,\overline{359}$

i $-94.570\dot{2}$ **j** $-1.\overline{739\,24}$ **k** $11.812\dot{1}$ **l** $-6.9\overline{04}$

WE 2.7C **6** Convert each fraction to a decimal by dividing the numerator by the denominator.

a $\frac{7}{10}$ **b** $-\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{5}{4}$

e $\frac{2}{11}$ **f** $-\frac{5}{8}$ **g** $\frac{8}{3}$ **h** $-\frac{3}{4}$

WE 2.7D **7** Convert each fraction to a decimal.

a $-1\frac{1}{8}$ **b** $1\frac{2}{5}$ **c** $2\frac{5}{6}$ **d** $-1\frac{1}{7}$

e $3\frac{3}{5}$ **f** $-4\frac{1}{9}$ **g** $1\frac{8}{11}$ **h** $-2\frac{5}{12}$

8 Consider the fractions and your answers to question **6**.

a Which fractions produce:

i terminating decimals **ii** non-terminating decimals?

b Which fractions produce recurring decimals?

c Do any of the fractions produce a non-terminating decimal that isn't a recurring decimal?

9 Numbers that can be written exactly as fractions with integer numerator and denominator where the denominator is non-zero are rational numbers.

a Explain why whole numbers and terminating decimals are rational numbers.

b Are recurring decimals rational numbers? Explain your answer.

10 Classify each number as rational or irrational.

a 4.6 **b** 0.121 212 12… **c** 0.5871 **d** 2.357 896 21…

e 35 **f** 734.946 518 24… **g** $0.872\,\overline{503}$ **h** 96.734 25

i $\frac{3}{17}$ **j** 500 000.000… **k** 0.117 843 26… **l** $10\frac{3}{4}$

11 **a** Convert each fraction to a decimal.

i $\frac{1}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{3}{9}$ **iv** $\frac{4}{9}$

b What type of decimal do you get?

c Use the pattern from part **a** to write these as decimals.

i $\frac{5}{9}$ **ii** $\frac{6}{9}$ **iii** $\frac{7}{9}$ **iv** $\frac{8}{9}$

12 All fractions have patterns in their decimal expansion.

a Write down the first ten multiples of 9.

b Convert each fraction to a decimal.

i $\frac{1}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{3}{11}$ **iv** $\frac{4}{11}$

c Describe the pattern in the decimal expansion of the elevenths. Check your pattern holds by writing and then checking the decimal expansion of $\frac{9}{11}$.

Exercise 2.7B: Problem solving and reasoning

13 It is possible to determine if a fraction will have a recurring decimal or not without performing the division.

a Complete the table below.

Unit fraction	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{11}$	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{12}$
Decimal	$0.\dot{1}$	0.1	0.2								
Denominator's prime factorisation	3^2	2×5	5								
Is 2 and/or 5 a factor of the denominator?	No	Yes									
Any prime factors other than 2 and/or 5?	Yes	No									
Is the decimal terminating?	No										

b According to the table, when will a unit fraction have a terminating decimal?

c Use your rule from part **b** to decide if these unit fractions will have a terminating decimal or not. Convert each simplified fraction to a decimal to check your answers.

i $\frac{1}{25}$ **ii** $\frac{1}{24}$ **iii** $\frac{1}{18}$ **iv** $\frac{1}{40}$ **v** $\frac{1}{125}$

d Your rule from part **b** extends to all simplified fractions. Use the rule to decide if these fractions will have a terminating decimal or not. Remember to simplify your fractions first! Convert each simplified fraction to a decimal to check your answers.

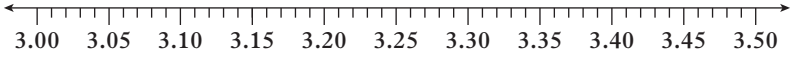
i $\frac{24}{7}$ **ii** $\frac{7}{32}$ **iii** $\frac{21}{30}$ **iv** $\frac{4}{26}$ **v** $\frac{42}{15}$

14 Before the use of calculators and computers, many mathematicians used a fraction as an estimate for the value of π. Some examples are shown in the table below.

Place, person and era of origin	**Fraction estimate**	**Decimal approximation (to six decimal places)**
Ancient Egypt (1700 BCE)	$\frac{256}{81}$	
Ancient Greece, Archimedes (287–212 BCE)	$\frac{223}{71}$	
	$\frac{22}{7}$	
China, Zu Chongzhi (429–500 CE)	$\frac{355}{113}$	
India, Bhāskara II (1114–1185 CE)	$\frac{3927}{1250}$	
Italy, Leonardo of Pisa (1170–1250 CE)	$\frac{12\,960}{4126}$	

a Use a calculator to complete the third column of the table.

b Locate π on the following number line.



c Find the symbol π on your calculator and obtain a decimal approximation for π. Which value in the third column is closest to the decimal from the calculator?

d Archimedes proved that π was between $\frac{223}{71}$ and $\frac{22}{7}$. Is it possible to use a computer to write the *exact* value of π as a fraction or a decimal? Explain.

Exercise 2.7C: Challenge

15 Recurring decimals can be written as fractions. For example, $0.\dot{6}$ is equivalent to $\frac{2}{3}$. You can verify this using short division or a calculator. To write a recurring decimal as a fraction, there is a series of steps you can follow.

a In the calculation below, match each step (**1–6**) with its corresponding line of working (**I–VI**) and then write the lines of working in the correct order.

Steps:	**Lines of working:**
1 Let x represent the recurring decimal.	**I** $0.\dot{6} = \frac{2}{3}$
2 Multiply the recurring decimal by 10 to find an equation for $10x$.	**II** $x = \frac{6}{9}$
3 Form a new subtraction equation from the two equations for x and $10x$.	**III** $10x = 6.666\,666\ldots$
4 Write the simplified equation that results from the subtraction.	**IV** $x = 0.666\,666\ldots$
5 Divide both sides of the equation by 9 to obtain x on the left-hand side of the equation and a fraction on the right-hand side.	**V** $10x - x$ $= 6.666\,666\ldots - 0.666\,666\ldots$
6 Write the recurring decimal as a fraction, in simplest form.	**VI** $9x = 6$

b Why did you multiply x by 10 in the calculation above? How did this help you obtain a whole number on the right-hand side of the equation in step **4**?

c Use the method in part **a** to write each of these recurring decimals as a faction.

i $0.\dot{2}$ **ii** $0.\dot{7}$ **iii** $2.\dot{3}$ **iv** $5.\dot{6}$

16 Recurring decimals can be written as fractions.

a Use a calculator to convert each fraction to a decimal. Show at least eight decimal places.

i $\frac{1}{9}$ **ii** $\frac{1}{99}$ **iii** $\frac{1}{999}$ **iv** $\frac{1}{9999}$

b Write each recurring decimal in abbreviated form with dots or a bar.

c We can write recurring decimals as fractions using denominators of 9, 99, 999, 9999, … where the number of nines in the denominator tells you how many digits recur.

For example, $\frac{34}{99} = 34 \times \frac{1}{99}$ and $\frac{1234}{9999} = 1234 \times \frac{1}{9999}$

$= 34 \times 0.\overline{01}$ $= 1234 \times 0.\overline{0001}$

$= 0.343434...$ $= 0.12341234...$

$= 0.\overline{34}$ $= 0.\overline{1234}$

Write the following fractions as recurring decimals without using division.

i $\frac{12}{99}$ **ii** $\frac{765}{999}$ **iii** $\frac{12}{9999}$ **iv** $\frac{6}{9}$

d Write each of the following recurring decimals as a fraction.

i $0.\overline{59}$ **ii** $0.\overline{401}$ **iii** $0.\dot{3}$ **iv** $6.\overline{23}$

If the recurrence does not begin at the tenths place, we can multiply the denominator of the fraction by powers of ten to start the recurrence in the hundredths, thousandths or ten thousandths place.

For example, $\frac{1}{9} = 0.\dot{1}$, $\frac{1}{90} = 0.0\dot{1}$, $\frac{1}{900} = 0.00\dot{1}$, $\frac{1}{99} = 0.\overline{01}$, $\frac{1}{999} = 0.0\overline{01}$

e Write each of the following recurring decimals as a fraction.

i $0.0\overline{59}$ **ii** $0.0\overline{401}$ **iii** $0.000\dot{3}$ **iv** $0.6\overline{23}$

Online resources:

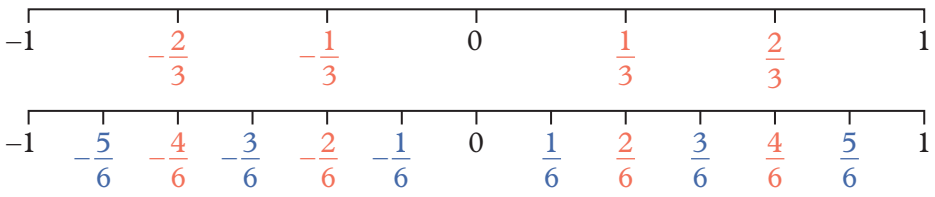
- **Interactive skillsheet** Terminating, non-terminating and recurring decimals
- **Worksheet** Writing recurring decimals
- **Investigation** Let the cycle begin!
- **Quick quiz** 2.7

Lesson 2.8

Review: Fractions and decimals

Module summary

Equivalent fractions



$\frac{2}{3} = \frac{10}{15}$ (×5) $\qquad \frac{12}{10} = \frac{6}{5}$ (÷2)

The HCF of 36 and 72 is 36.

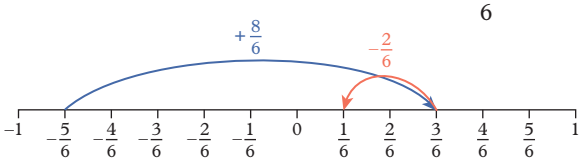
$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}} = \frac{1}{2}$

Adding and subtracting fractions

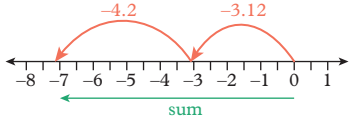
1 Convert into equivalent fractions with a common denominator.
2 Add or subtract the numerators and keep the same denominator.

$-\frac{5}{6} + \frac{4}{3} - \frac{1}{3} = -\frac{5}{6} + \frac{8}{6} - \frac{2}{6}$

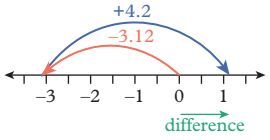
$= \frac{1}{6}$



Adding and subtracting decimals



$-\left(+\begin{array}{r} 3.12 \\ 4.20 \\ \hline 7.32 \end{array}\right)$



$+\left(\begin{array}{r} {}^{1}\cancel{4}.{}^{10}\cancel{2}\,0 \\ -\,3.1\,2 \\ \hline 1.0\,8 \end{array}\right)$

Fractions to decimals

Align decimal points

Dividend → $\frac{1}{8}$ = Divisor → $8\overline{)1.0^{2}0^{4}0}$ $\quad 0.1\,2\,5$

Trailing zeros

Multiplying fractions

$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}^{1}}{\cancel{3}^{1} \times 4}$

$= \frac{\cancel{2}^{1}}{\cancel{4}^{2}}$

$= \frac{1}{2}$

Diagonally

Vertically

Multiplying decimals

2.75 ← two decimal places
×0.03 ← two decimal places

$\begin{array}{r} {}^{2\,1}275 \\ \times \quad 3 \\ \hline 825 \end{array}$

2.75 × 0.03 = 0.0825 ← four decimal places

Dividing by a whole number

$9\overline{)172.44}$ $\quad 19.16$ ← quotient (the answer)

divisor (the number we are dividing by)

dividend (the number being divided)

Reciprocals

$\frac{3}{4} \quad \frac{4}{3}$

Dividing fractions

$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$

Multiplying and dividing by 10s

5.23 × 10 = 52.3
5.23 × 100 = 523.0
5.23 ÷ 10 = 0.523
5.23 ÷ 100 = 0.0523

Lowest common denominators

The LCM of 2 and 5 is 10.

$\frac{1}{2} = \frac{5}{10}$ (×5) $\qquad \frac{3}{5} = \frac{6}{10}$ (×2)

Dividing by a decimal

Dividend Divisor

$1.2 \div 0.4 = 12 \div 4 = 3$

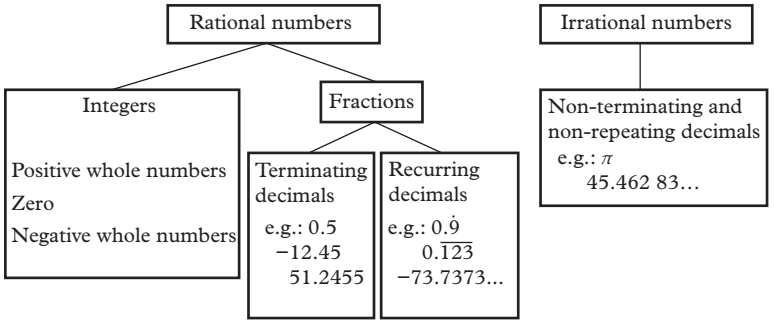
Improper fractions and mixed numbers

$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$

$\frac{13}{4} = 13 \div 4 = 3$ remainder 1

$= 3\frac{1}{4}$

Rational and irrational numbers



ANS p532

Review questions 2.8A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- decimal
- denominator
- equivalent fraction
- improper fraction
- irrational number
- lowest common denominator
- mixed number
- non-terminating decimal
- numerator
- pi
- proper fraction
- rational number
- reciprocal
- recurring decimal
- sign
- terminating decimal
- trailing zeros

1 Fill in 'lowest' or 'highest' in the following frequently used terms.

a _____ common multiple

b _____ common denominator

c _____ common factor

2 Briefly explain why when simplifying a fraction, the whole number part of a mixed number stays the same.

3 The phrase 'Keep, Change, Flip' can help you to turn a 'dividing by a fraction' problem into a multiplication problem. Explain what has been kept, changed and flipped.

4 Identify whether the following statements are true or false. If a statement is false, briefly explain why.

a The reciprocals of whole numbers are unit fractions.

b Multiplying by a fraction makes a number smaller.

c All decimals can be written as fractions.

d All fractions can be written as decimals.

5 Briefly explain the difference between the following terms using examples.

a digit, decimal place

b placeholder zero, trailing zero

c rational number, irrational number

Review questions 2.8B: Multiple choice

2.1 **1** What missing value in $\frac{7}{9} = \frac{49}{\square}$ will produce an equivalent fraction?

A 51 **B** 63 **C** 81 **D** 441

2.1 **2** Which of the following is incorrect?

A $\frac{5}{3} > \frac{4}{3}$ **B** $\frac{3}{5} < \frac{3}{4}$ **C** $\frac{2}{3} > \frac{32}{47}$ **D** $\frac{7}{9} < \frac{9}{11}$

2.2 **3** What is the lowest common denominator for $\frac{3}{8} - \frac{1}{6}$?

A 48 **B** 16 **C** 24 **D** 2

2.2 **4** What is $\frac{1}{5} - \frac{4}{5}$?

A -3 **B** $-\frac{3}{5}$ **C** $\frac{3}{5}$ **D** $\frac{5}{5}$

2.3 **5** Which problem has a positive answer?

A $-\frac{2}{3} + \left(-\frac{3}{4}\right)$ **B** $-\frac{2}{5} \times \frac{1}{2}$ **C** $-\frac{5}{2} - \left(-\frac{6}{7}\right)$ **D** $-\frac{1}{7} \div \left(-\frac{8}{9}\right)$

2.3 **6** What is $\frac{9}{10} \div 1\frac{1}{5}$?

A $\frac{9}{2}$ **B** $\frac{3}{4}$ **C** $\frac{27}{25}$ **D** $\frac{21}{10}$

2.4 **7** What is −3.125 84 rounded to three decimal places?

A −3.125 **B** −3.13 **C** −3.128 **D** −3.126

2.5 **8** What is −5 + 2.38?

A −7.38 **B** −7.62 **C** −2.62 **D** −2.38

2.6 **9** How many decimal places are there in the answer to 26.018 × 15.4?

A 4 **B** 3 **C** 2 **D** 8

2.7 **10** Which number is an example of a recurring decimal?

A 32.85 **B** −7.757 575 **C** −6.666... **D** 3.654 864 321...

2.7 **11** Which of the following is *not* equivalent to $\frac{5}{33}$?

A $0.\overline{15}$ **B** $0.\dot{1}\dot{5}$ **C** $0.1\dot{5}$ **D** $\frac{15}{99}$

Review questions 2.8C: Short answer

2.1 **1** Write each fraction in its simplest form.

a $\frac{35}{60}$ **b** $-\frac{30}{16}$ **c** $-3\frac{18}{32}$ **d** $5\frac{48}{72}$

2.2 **2** Evaluate:

a $\frac{3}{7}+\frac{2}{5}$ **b** $\frac{11}{15}-\frac{3}{10}$ **c** $5\frac{2}{3}+\frac{4}{11}$ **d** $6\frac{2}{5}-3\frac{4}{7}$

2.3 **3** Evaluate:

a $\frac{12}{25}\times\frac{5}{14}$ **b** $\frac{7}{18}\div\frac{7}{9}$ **c** $6\frac{1}{4}\times3\frac{2}{5}$ **d** $3\frac{1}{2}\div1\frac{5}{9}$

2.3 **4** Evaluate:

a $-\frac{10}{9}\times\left(-\frac{27}{35}\right)$ **b** $\frac{15}{11}\div\left(-\frac{90}{44}\right)$ **c** $-3\frac{5}{6}\times8\frac{2}{3}$ **d** $-2\frac{4}{5}\div\left(-5\frac{4}{9}\right)$

2.3 **5** Evaluate:

a $-\frac{3}{4}-\left(-\frac{1}{2}\right)$ **b** $\frac{2}{5}-\left(+\frac{1}{3}\right)$ **c** $\frac{3}{7}-1\frac{2}{7}$

d $-2\frac{1}{6}+6\frac{2}{3}$ **e** $\frac{1}{2}\div\frac{7}{4}-\frac{2}{5}$ **f** $-\frac{1}{8}+\frac{2}{3}\times\left(-\frac{9}{8}\right)$

2.3 **6** Find:

a $\frac{2}{5}$ of 60 km/h **b** $\frac{1}{2}$ of $\frac{2}{3}$ of a cup

c $3\frac{1}{3}$ of 21 metres **d** $\frac{18}{60}$ of 10 minutes

2.4 **7** Write in ascending order.

a 0.123, 0.120, 0.213, 0.0012, 0.231, 0.2 **b** 8.916, 9.6, 8.16, 8.9, 9.16, 9.916

c 0.05, 5, −0.5, −5, 0.50, −0.05 **d** −0.421, −2.41, −10.42, −14.2, −12.04, −2.14

2.4 **8** Write in descending order.

a 0.017, 0.7, 0.71, 0.0701, 0.717, 0.107 **b** 2.127, 2.1, 2.721, 2.27, 2.72, 2.7

c −0.89, 89, 90, −9.8, −0.899, 80.9 **d** −0.9, −1, −0.01, −0.99, −0.099, −0.1

2.4 **9** Round each decimal to the number of decimal places shown in brackets.

a 13.216 51 (2) **b** −0.001 215 (4)

c 54.545 545 (3) **d** −181.182 154 136 (1)

2.5 **10** Evaluate:

a 23.56 + 45.98 + 74.13 **b** 53.15 − 29.67

c 123.5 + 46.12 + 9.0142 + 87.15 **d** 1284.068 − 294.4687

2.5 **11** Evaluate:

a −5.42 + 1.59 **b** −8.30 − (−12.482)

c 49.207 + (−51.52) **d** 99.29 − (−15.490)

2.6 **12** Evaluate:

a 6.3128×4 **b** $65.589 \div (-6)$

c -3.46×5.7 **d** $-468.268 \times (-0.89)$

2.7 **13** For each of the following decimals:

a Decide whether the decimal is terminating or non-terminating.

i 0.123123123 **ii** 6.562562...

iii −5.5 **iv** 3.645841235...

b Decide if the decimal is recurring.

2.7 **14** Write each decimal in short form using dots or a horizontal bar.

a 0.333333... **b** 5.385385...

c −45.685757575... **d** −0.092451451...

2.7 **15** Write each number to show twelve decimal places.

a $-0.2\dot{3}$ **b** $3.\overline{27}$ **c** $46.35\dot{8}\dot{7}$ **d** $-17.36\overline{485}$

2.7 **16** Write each fraction as a recurring decimal using dots or a horizontal bar.

a $\frac{1}{30}$ **b** $-\frac{4}{7}$ **c** $-9\frac{5}{9}$ **d** $\frac{17}{11}$

Review questions 2.8D: Mathematical modelling

1 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Min. temp. (°C)	0.7	−2.2	−0.9	−3.4	−5.1	−1.8	0.2
Max. temp. (°C)	6.5	4.1	7.8	2.5	−0.6	3.9	5.3

a Which day had:

i the highest temperature

ii the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for Monday.

c Calculate the difference between the minimum and maximum temperatures for each other day.

d Which day had the biggest range of temperatures?

e Calculate the average of the following, correct to one decimal place.

i the minimum temperatures

ii the maximum temperatures

f What is the difference between the average minimum and average maximum temperatures?

2 Rob, Carina, Belinda and Mario want to buy Alicia a bunch of white roses for her birthday.

a Rob has $10.35, Carina has $8.25, Belinda has $9.90 and Mario has $10.80. How much money do they have altogether?

b Their local florist, Fantastic Flowers, has white roses for $2.99 each. How many white roses can they buy?

c How much change will they have left over?

d Fantastic Flowers has a special offer that if you buy a dozen white roses, you get $\frac{1}{4}$ of the total price off.

i How much would a dozen white roses cost without the discount?

ii How much would a dozen white roses cost with the discount?

iii Mario bargains with the storekeeper and manages to buy 18 roses for the price of 12 roses. Is this more discount compared with the special offer?

iv How much is a single rose with this offer?

3 A school has four house teams for sports events: Alpha, Beta, Gamma and Delta, and every student is assigned into one of the four houses. The school tries to have as close to one-quarter of students in each house every year. In 2022, the school had 1596 students.

a In 2022, Alpha house had exactly $\frac{1}{4}$ of the students in the school. How many students were in Alpha house?

b In 2022, Beta house had $\frac{9}{38}$ of the students in the school.

i How many students were in Beta house?

ii If $\frac{11}{21}$ of Beta house are girls, how many girls are in Beta house?

c In 2022, Gamma house had 402 students.

i What fraction of students were in Gamma house?

ii How many students did Gamma house have above or below one-quarter of the students at the school?

d In 2022:

i how many students were in Delta house

ii what fraction of the school were in Delta house?

e In 2023, Delta house had 416 students. If Delta house had $\frac{13}{50}$ of the students at the school that year, how many students went to the school in 2023?

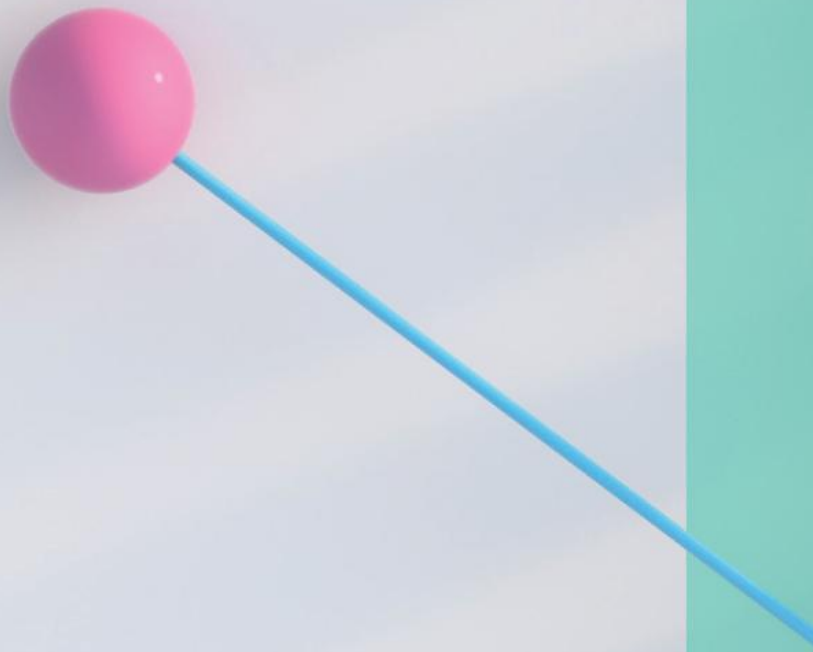
Checklist

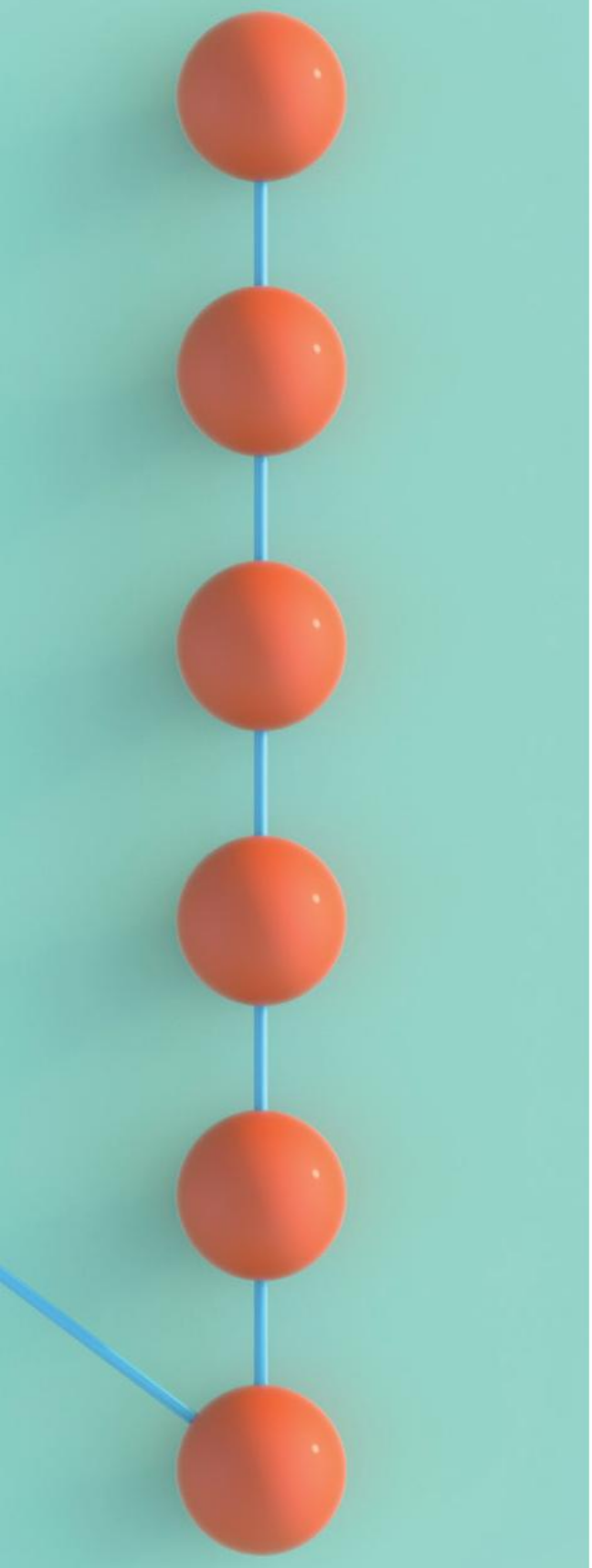
Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify and find equivalent fractions ☐ Compare fractions ☐ Simplify fractions	☐ Go back to Lesson 2.1 Fractions
☐ Add and subtract fractions with different denominators ☐ Add and subtract mixed numbers ☐ Add and subtract positive and negative fractions	☐ Go back to Lesson 2.2 Adding and subtracting fractions
☐ Multiply and divide mixed numbers ☐ Multiply and divide positive and negative fractions ☐ Determine fractions of an amount	☐ Go back to Lesson 2.3 Multiplying and dividing fractions
☐ Compare and order decimals ☐ Round decimals to a specified number of decimal places ☐ Convert between fractions and decimals	☐ Go back to Lesson 2.4 Decimals
☐ Add and subtract positive and negative decimals	☐ Go back to Lesson 2.5 Adding and subtracting decimals
☐ Multiply positive and negative decimals ☐ Divide positive and negative decimals	☐ Go back to Lesson 2.6 Multiplying and dividing decimals
☐ Identify decimals as terminating, non-terminating and recurring ☐ Convert fractions to recurring decimals ☐ Write non-terminating and recurring decimals using appropriate notation	☐ Go back to Lesson 2.7 Terminating, non-terminating and recurring decimals

3

Percentages, ratios and rates





Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Visual representation of fractions
- ✔ Equivalent fractions
- ✔ Multiplying and dividing fractions
- ✔ Multiplying and dividing decimals
- ✔ Rounding decimals

Curriculum links

- Solve problems involving the use of percentages, including percentage increases and decreases... (VC2M8N05)
- Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts involving profit and loss... (VC2M8N06)
- Recognise and use rates to solve problems involving the comparison of 2 related quantities of different units of measure (VC2M8M05)
- Use mathematical modelling to solve practical problems involving ratios and rates... (VC2M8M07)

Materials

- ✔ Calculator

Lesson 3.1

Percentages, fractions and decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ comparing and converting between percentages, fractions and decimals.

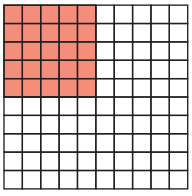
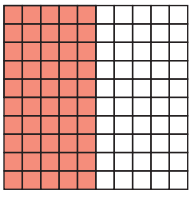
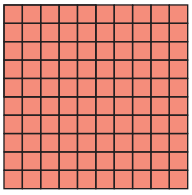
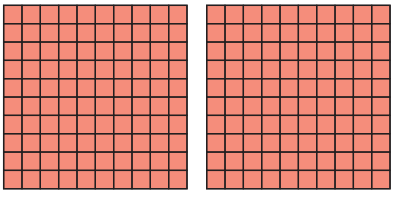
Inter-year links

Support Year 7 — Fractions, decimals and percentages; 4.7 Fractions, decimals and percentages

Percentages, fractions and decimals

Key content video: Fractions, decimals and percentages

- A **percentage** is a proportion of a whole that has been divided into one hundred parts.
- The term 'per cent' means 'per hundred' or 'out of 100'. The symbol for percentage is %.
- Percentages, fractions and decimals are different ways of representing the same value.

Percentage	Fraction	Decimal	
25%	$\frac{25}{100} = \frac{1}{4}$	0.25	
50%	$\frac{50}{100} = \frac{1}{2}$	0.5	
100%	$\frac{100}{100} = 1$	1	
200%	$\frac{200}{100} = 2$	2	

	Fraction	Decimal
Percentage to...	Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100}$ $= \frac{3}{20}$	Divide the percentage by 100. $23.4\% = 23.4 \div 100$ $= 0.234$

	Percentage	Decimal
Fraction to…	Multiply the fraction by 100% and simplify. $\frac{1}{3} \times 100\% = \frac{100}{3}\%$ $= 33\frac{1}{3}\%$	Divide the numerator by the denominator. Add trailing zeros as required. Align decimal points Dividend → $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ ← Divisor $0.1\ 2\ 5$ Trailing zeros

	Percentage	Fraction
Decimal to…	Multiply the decimal by 100 and include the percentage symbol %. $0.65 = 65\%$	Place the decimal as the numerator of the fraction with a denominator of 10, 100, 1000, … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100} = \frac{13}{20}$

Worked example 3.1A Converting from a percentage

Write each percentage as a fraction in its simplest form and as a decimal.

a 220% **b** 25.3%

THINK

1 To convert a percentage to a fraction, divide by 100 and simplify the fraction.

2 To write a percentage as a decimal, divide the percentage by 100.

WRITE

a Fraction: $220\% = \frac{\cancel{220}^{11}}{\cancel{100}^{5}}$

$= \frac{11}{5}$

Decimal: $220\% = 2.2$

b Fraction: $25.3\% = \frac{25.3}{100}$

$= \frac{253}{1000}$

Decimal: $25.3\% = 0.253$

Worked example 3.1B Converting from a decimal

Write each decimal as a percentage and as a fraction in its simplest form.

a 3.2 **b** 0.0125

THINK

1 To convert a decimal to a percentage, multiply by 100%. 'Move' the decimal point two places to the right and insert a place-holding zero in the empty space if necessary.

2 To convert a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000, … with as many zeros as there are digits after the decimal point and simplify.

WRITE

a Percentage: $3.2 = 320\%$

Fraction: $3.2 = 3\frac{\cancel{2}^{1}}{\cancel{10}^{5}}$

$= 3\frac{1}{5}$

b Percentage: $0.0125 = 1.25\%$

Fraction: $0.0125 = \frac{\cancel{125}^{1}}{\cancel{10\,000}^{80}}$

$= \frac{1}{80}$

Worked example 3.1C Converting from a fraction

Write each fraction as a percentage and as a decimal.

a $\frac{27}{300}$ **b** $4\frac{2}{3}$

THINK

a 1 Multiply the fraction by 100% and divide the numerator by the denominator to simplify.

2 Convert the fraction to an equivalent fraction with a denominator of 100 and then convert to a decimal by 'moving' the decimal point of the numerator two places to the left.

b 1 For mixed numbers, multiply the whole number and the fraction by 100%. Divide the numerator by the denominator to simplify, then add both values together.

2 Express the mixed number as a whole number plus a fraction. If you cannot easily determine an equivalent fraction with a denominator of 100, use short division to convert the fraction to a decimal. Add both values together again.

WRITE

a Percentage: $\frac{27}{300} = \frac{27}{300} \times 100\%$

$= \frac{2700}{300}\%$

$= 9\%$

Decimal: $\frac{27}{300} \overset{\div 3}{=} \frac{9}{100}$

$= 0.09$

b Percentage: $4\frac{2}{3} = 4\frac{2}{3} \times 100\%$

$= 400\% + \frac{200}{3}\%$

$= 400\% + 66\frac{2}{3}\%$

$= 466\frac{2}{3}\%$

Decimal: $4\frac{2}{3} = 4 + \frac{2}{3}$

$= 4 + 3\overline{)2.^{2}0^{2}0^{2}0}$ (0. 6 6 6...)

$= 4.\dot{6}$

Worked example 3.1D Comparing fractions, decimals and percentages

Complete each statement by writing < (is less than) or > (is greater than) in the space provided.

a $\frac{3}{4}$ _____ 0.7 **b** 35% _____ $\frac{2}{5}$

THINK

a 1 Convert one term into the same format as the other term. In this instance we will convert the fraction into a decimal.

2 Compare the two terms in the same format to determine which one is greater.

3 Complete the statement by using either > or <.

b 1 Convert one term into the same format as the other term. In this instance we will convert the fraction into a percentage.

2 Compare the two terms in the same format to determine which one is greater.

3 Complete the statement by using either > or <.

WRITE

a $\frac{3}{4} = \frac{75}{100}$

$= 0.75$

$0.75 > 0.7$

Therefore, $\frac{3}{4} > 0.7$

b $\frac{2}{5} = \frac{2}{5} \times 100\%$

$= \frac{200}{5}\%$

$= 40\%$

$35\% < 40\%$

Therefore, $35\% < \frac{2}{5}$

Helpful hints

- ✔ Remember that one whole equals 100%. If you have a percentage that is greater than 100% you have more than one whole.
- ✔ It's easier to convert from a fraction to a decimal when the fraction has a denominator of 100, before you simplify your answer.
- ✔ Don't forget to simplify your fractions.
- ✔ It's usually quicker to compare and order percentages or decimals than it is to compare and order fractions.
- ✔ As all percentages can be converted to fractions, percentages are also rational numbers.

Learning pathways

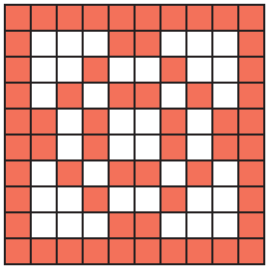
1–13, 14(a–d), 15–17, 20(a-d)	3(d–f), 4–7, 8(b, d), 9–14(e–h), 15, 18, 19(a, b), 20–22	5, 7, 9–14(e–h), 19–24

ANS p533

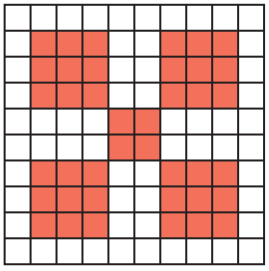
Exercise 3.1A: Understanding and fluency

1 What percentage of each large square has been shaded?

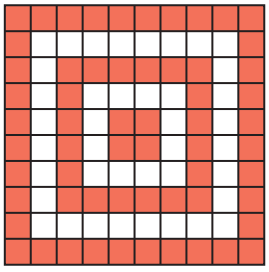
a



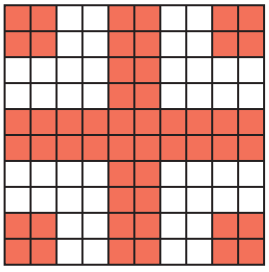
b



c



d



2 Write the percentage of each square in question **1** that is *not* shaded.

3 Calculate the percentage for each situation.

a Huynh has downloaded 76% of a song onto her iPhone. What percentage remains to be downloaded?

b If the selling price of an item is reduced by 23%, what percentage of the original price is to be paid?

c The GWS Giants lose 17% of their games. What percentage do they win (assuming no draws)?

d Forty-two per cent of homes were affected by floods. What percentage of homes were not affected?

e Morgan received 93% on her Japanese test. What percentage of marks did she lose?

f It is estimated that 54% of Australians do no exercise on a regular basis. What percentage of Australians exercise on a regular basis?

WE 3.1A **4** Write each percentage as a fraction in its simplest form and as a decimal.

a 17%	**b** 21%	**c** 35%	**d** 8%
e 215%	**f** 98%	**g** 174%	**h** 28%

5 Write each percentage as a fraction in its simplest form and as a decimal.

a 7% **b** 3.1% **c** 162% **d** 72.1%

e 347% **f** 81.09% **g** 10.57% **h** 6.78%

WE 3.1B **6** Write each decimal as a percentage and as a fraction in its simplest form.

a 0.71 **b** 0.28 **c** 0.05 **d** 0.08

e 0.9 **f** 0.31 **g** 0.54 **h** 0.95

7 Write each decimal as a percentage and as a fraction in its simplest form.

a 0.369 **b** 0.248 **c** 10.06 **d** 0.81

e 7.057 **f** 1.859 **g** 12.1 **h** 0.2446

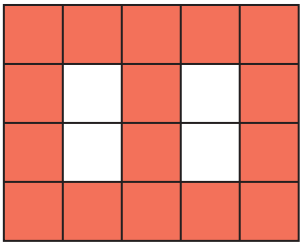
8 For each figure:

i write how many sections are shaded as a fraction of the total number of sections

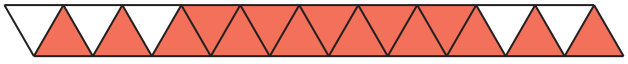
ii write this fraction as an equivalent fraction with a denominator of 100

iii write the percentage of sections that are shaded.

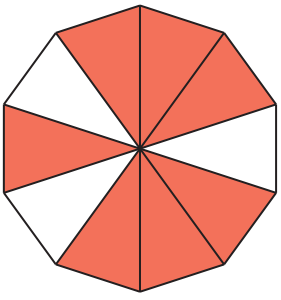
a



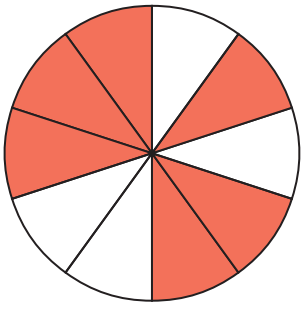
b



c



d



WE 3.1C **9** Write each fraction as a decimal and percentage by first finding an equivalent fraction that has a denominator of 100.

a $\frac{3}{5}$ **b** $\frac{7}{10}$ **c** $\frac{17}{200}$ **d** $\frac{23}{25}$

e $\frac{13}{10}$ **f** $\frac{3}{2}$ **g** $\frac{11}{400}$ **h** $\frac{7}{2}$

10 Write each mixed number as a percentage. Hint: Whole numbers are multiples of 100%.

a $2\frac{1}{4}$ **b** $6\frac{1}{200}$ **c** $7\frac{3}{10}$ **d** $3\frac{3}{4}$

e $4\frac{7}{25}$ **f** $5\frac{4}{5}$ **g** $1\frac{12}{500}$ **h** $13\frac{13}{50}$

11 Write each fraction as a percentage and a decimal.

a $\frac{5}{8}$ **b** $\frac{27}{40}$ **c** $5\frac{1}{3}$ **d** $\frac{3}{16}$

e $\frac{11}{125}$ **f** $3\frac{2}{9}$ **g** $\frac{11}{15}$ **h** $1\frac{3}{7}$

12 Write each percentage as a fraction in simplest form and as a decimal.

a $\frac{1}{4}\%$ **b** $\frac{11}{3}\%$ **c** $\frac{5}{8}\%$ **d** $2\frac{1}{2}\%$

e $8\frac{4}{5}\%$ **f** $15\frac{2}{3}\%$ **g** $3\frac{5}{9}\%$ **h** $13\frac{1}{6}\%$

WE 3.1D **13** Complete each statement by writing < (is less than) or > (is greater than) in the space provided.

a $0.46 \_\_\_\_ \frac{1}{2}$ **b** $\frac{3}{5} \_\_\_\_ 57\%$ **c** $0.95 \_\_\_\_ 71\%$ **d** $59\% \_\_\_\_ \frac{4}{7}$

e $\frac{5}{4} \_\_\_\_ 122\%$ **f** $252\% \_\_\_\_ 21.2$ **g** $13.4 \_\_\_\_ 1\frac{3}{4}$ **h** $2\frac{1}{6} \_\_\_\_ 206\%$

Exercise 3.1B: Problem solving and reasoning

14 Write a percentage to describe these situations by first finding an equivalent fraction that has a denominator of 100.

a Andrew ate three out of five cupcakes.

b Nine out of twenty people went home.

c Seventeen out of twenty-five people voted yes.

d Mark knew 46 people out of 200.

e Steffie obtained a score of 17 out of 20 for the quiz.

f The movie critic rated the movie one out of five stars.

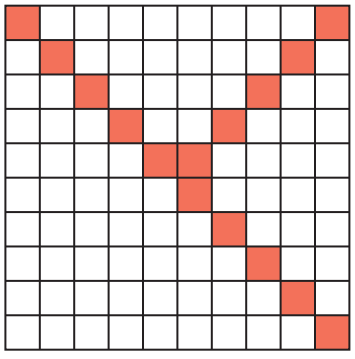
g A dam with a capacity of 600 megalitres currently holds 144 megalitres.

h Seventy-five of the 250 people surveyed were happy with their mobile phone provider.

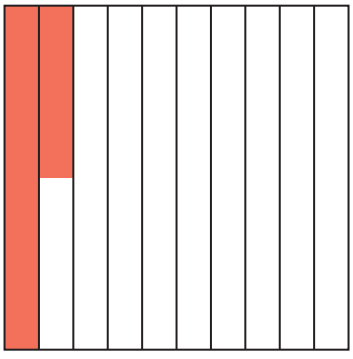
15 What percentage of its original value is a car worth if it depreciates (loses value) by these amounts?

a 20% **b** 19% **c** 8% **d** 12% **e** 28.5% **f** $33\frac{1}{3}\%$

16 Thomas and Matthew were asked to draw a diagram representing 15%. Their answers are shown below.



Matthew



Thomas

a Which diagram is correct? Explain your reasoning.

b Show another way of representing 15% in a diagram.

17 These containers each contain 50% of their total capacity.

Explain how each container holds a different volume of liquid, but they are the same percentage full.

18 A rational number is a number that can be written exactly as a fraction with the numerator and denominator both being integers. Explain why all percentages are rational numbers.

19 Arrange the following lists in ascending order by first converting all terms to either a fraction, decimal or percentage.

a $\frac{7}{5}$, 0.89, $\frac{3}{4}$, 200%, $\frac{4}{5}$, $1.\dot{3}$

b 4.9, $\frac{13}{2}$, 560%, 7.2, $4\frac{3}{10}$, $\frac{22}{4}$

c $\frac{16}{5}$, $\frac{3}{2}$, 340%, 0.28, $1\frac{4}{5}$, 65%

20 Percentages can be added and subtracted like other numbers. Calculate:

a 17% + 49%

b $23\% + 32\frac{1}{2}\%$

c $53\frac{1}{2}\% + 16\frac{1}{2}\% + 9\frac{2}{3}\%$

d 7.9% + 28.7% + 46.5%

e 92% − 68%

f $16\frac{4}{5}\% - 12\frac{2}{5}\%$

g 37.4% + 52.9% − 15.6%

h 75.95% + 5.28% − 32.13% − 28.66%

21 A packet of gumballs is emptied onto a table. See the gumballs on the right.

a Write the number of blue gumballs as a fraction of the total.

b Convert this to a decimal and round to four decimal places.

c Convert this decimal to a percentage. What percentage of the gumballs are blue?

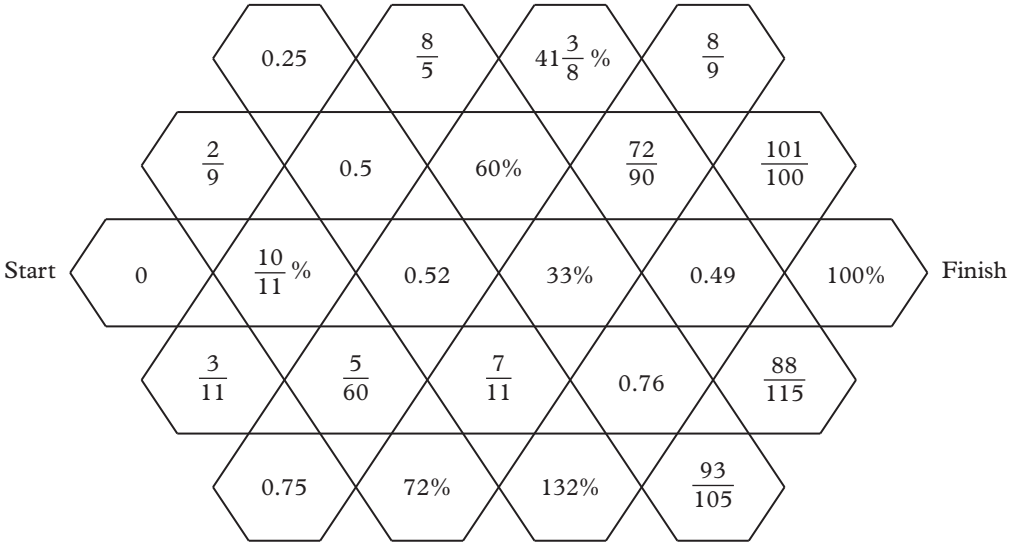
d What percentage of the gumballs, correct to two decimal places, is:

i orange **ii** green **iii** red or yellow **iv** not red?

22 Marigo received a $5\frac{1}{4}\%$ pay rise. What fraction of her salary does this represent?

Exercise 3.1C: Challenge

23 Starting at zero in this diagram, find the shortest path to 100%, moving in ascending order.



24 Showing examples, explain how you could represent the percentages below using:

a a rectangle of length 10 cm and width 2 cm **b** a grid of 200 squares.

80% 92% 25% 17% 43% 66%

Online resources:

Interactive skillsheet Fractions, decimals and percentages

Worksheet Converting percentages, fractions and decimals

Investigation Best one - percentages, decimals or fractions

Quick quiz 3.1

Lesson 3.2

Calculating percentages

Learning intentions

By the end of this lesson you will be able to ...

✔ perform calculations involving percentages.

Inter-year links

Support Calculating percentages

Year 7 4.8 Calculating percentages

Calculating percentages

Key content video
Calculating percentages

- To express a quantity as a percentage of another quantity:
  1. Make sure that both quantities are expressed in the same units.
  2. Form a fraction with the numerator as the quantity you want to express as a percentage. Simplify the fraction if possible.
  3. Write the fraction as a percentage.

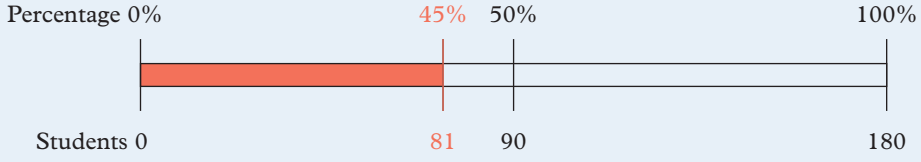
  For example, 81 students as a percentage of 180 students:

$$\frac{81}{180} = \frac{9}{20}$$
$$= \frac{45}{100}$$
$$= 45\%$$

- To calculate a percentage *of* a quantity, write the percentage as a decimal and multiply by the quantity.

  For example, 45% of 180 students like ice cream more than frozen yoghurt.

$$45\% \text{ of } 180 = 0.45 \times 180$$
$$= 81 \text{ students}$$



Percentage increase and decrease

Key content video
Percentage increases and decreases

- A **percentage increase** occurs when a quantity is increased by a percentage.
- A **percentage decrease** occurs when a quantity is decreased by a percentage.

- To increase a quantity by a percentage:
  1. Add the percentage to 100% and convert this new percentage to a decimal.
  2. Multiply the quantity to be increased by the decimal.

  For example, to increase a quantity by 50% multiply it by 1.5 (100% + 50% = 150%).
- To decrease a quantity by a percentage:
  1. Subtract the percentage from 100% and convert this new percentage to a decimal.
  2. Multiply the quantity to be decreased by the decimal.

  For example, to decrease a quantity by 40% multiply it by 0.6 (100% − 40% = 60%).

Percentage error

- The error of a measurement is the positive difference between the true value and the measured value.

  For example, if a person is 120 cm tall and their height is measured to be 123 cm, then the error in the measurement is 3 cm.
- The **percentage error** is the error of a measurement written as a percentage of the true value of the quantity.

  For example, the percentage error from the example above is:

$$\frac{3 \text{ cm}}{120 \text{ cm}} = \frac{1}{40}$$
$$= \frac{2.5}{100}$$
$$= 2.5\%$$

Worked example 3.2A Expressing a quantity as a percentage of another

a Write 60 as a percentage of 150.

b Write 90 cents as a percentage of $12.

THINK

a **1** Express the amount as a fraction of the total and simplify.

2 To convert a fraction to a decimal or percentage, write the fraction with a denominator of 100.

3 Write your answer.

b **1** Convert $12 to 1200 cents and write 90 cents as a fraction out of 1200.

2 To convert a fraction to a decimal or percentage, write the fraction with a denominator of 100.

3 Write your answer using the units from the question.

WRITE

a $\frac{60}{150} \overset{\div 30}{=} \frac{2}{5}$

$= \frac{40}{100}$

$= 40\%$

60 is 40% of 150.

b $\frac{90}{1200} \overset{\div 12}{=} \frac{7.5}{100}$

$= 7.5\%$

90 cents is 7.5% of $12.

Worked example 3.2B Calculating a percentage of a quantity

Calculate 7% of 220.

THINK

1 Convert the percentage to a decimal.

2 Multiply by the quantity.

3 Write your answer.

WRITE

7% of 220 = 220 × 0.07

= 15.4

7% of 220 is 15.4.

Worked example 3.2C Changing a quantity by a percentage

a Increase 4500 by 40%.

b Decrease 240 by 15%.

THINK

a 1 Add the percentage to 100% and convert the result to a decimal.

2 Multiply the quantity to be increased by the decimal.

b 1 Subtract the percentage from 100% and convert the result to a decimal.

2 Multiply the quantity to be decreased by the decimal.

WRITE

a Increase 4500 by 40%
$= 4500 \times (100\% + 40\%)$
$= 4500 \times 140\%$
$= 4500 \times 1.4$
$= 6300$

b Decrease 240 by 15%
$= 240 \times (100\% - 15\%)$
$= 240 \times 85\%$
$= 240 \times 0.85$
$= 204$

Worked example 3.2D Finding the original amount after a change

a After a 60% increase, a number is now 800. What was the original amount?

b After a 20% decrease, a number is now 160. What was the original amount?

THINK

a 1 To increase a number by 60%, multiply by 160%. To reverse the process, divide by 160%.

2 Convert this new percentage to a decimal.

3 Divide the new amount by the decimal to calculate the original amount.

b 1 To decrease a number by 20%, multiply by 80%. To reverse the process, divide by 80%.

2 Convert this new percentage to a decimal.

3 Divide the new amount by the decimal to calculate the original amount.

WRITE

a Original amount $= 800 \div (100\% + 60\%)$
$= 800 \div 160\%$
$= 800 \div 1.6$
$= 500$

b Original amount $= 160 \div (100\% - 20\%)$
$= 160 \div 80\%$
$= 160 \div 0.8$
$= 200$

Helpful hints

✔ You can use fractions or decimals to find the percentage of a quantity. Both methods give the same answer. For example, this is an alternative way to solve **Worked example 3.2A** part **a**.

$$\frac{60}{\cancel{150}^{3}} \times \frac{\cancel{100}^{2}\%}{1} = \frac{\cancel{60}^{20}}{\cancel{3}^{1}} \times \frac{2\%}{1}$$
$$= \frac{20}{1} \times \frac{2}{1}\%$$
$$= 40\%$$

✔ Writing a quantity as a percentage of a total and finding a percentage of a total are not the same, they are the opposite of each other.

20 as a percentage of 40	50% of 40
$\frac{20}{40} = 50\%$	$50\% \times 40 = 20$

Learning pathways

 1–7, 8–9(a–d), 10–12, 14, 15

 3–7, 8–9(e–h), 10, 11, 13, 15, 17-19

 5, 8(g-l), 11, 12, 16-21

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Exercise 3.2A: Understanding and fluency

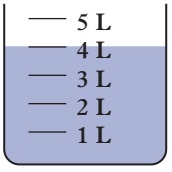
WE 3.2A **1** Write these amounts.

a 20 as a percentage of 80
b 45 as a percentage of 50
c 19 as a percentage of 25
d 90 as a percentage of 120
e 300 as a percentage of 250
f 135 as a percentage of 108

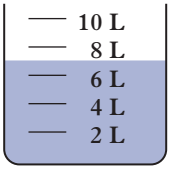
2 Calculate the percentage of the measuring jar:

i that has been filled
ii that is empty.

a



b



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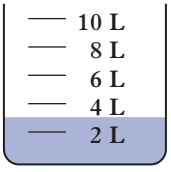

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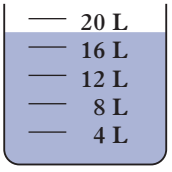


3 Write each proportion:

i as a fractional percentage
ii as a decimal percentage.

a 140 as a percentage of 800
b 72 as a percentage of 36 000
c 46 as a percentage of 125
d 57 as a percentage of 200
e 58 as a percentage of 80
f 282 as a percentage of 48

4 Write each amount as a percentage of the total.

a $1288 of a $5600 loan is repaid.
b 10.55 km of the 42.2 km marathon has been completed.
c 42 of the 120 lots of land have been sold.
d 63 likes have been received out of 90 views on YouTube.
e 3 of 40 training sessions have been missed.

5 Write each of these as a percentage.

a 50 g out of 4 kg
b 48 minutes out of 8 hours
c 9 months out of 3 years
d 18 hours out of one day
e 35 cents out of $3.50
f 90 minutes out of 24 hours
g 60 mL out of 2 L
h 800 mm out of 3.2 m

WE 3.2B **6** Calculate:

a 10% of 840
b 42% of 1800
c 6% of 150
d 18% of 6400
e 125% of 580
f 9% of 1250

7 Calculate:

a 63% of 720 m
b 150% of 90 seconds
c 35% of 660 cm
d 12% of 840 L
e 40% of 190 buttons
f 10.5% of 112 mL

8 Calculate:

a 30% of $1200
b 25% of $5000
c 40% of $250
d 15% of $10 500
e 8% of $200
f 21% of $2745
g 13% of $850
h 150% of $299
i $5\frac{1}{2}$% of $1260
j $10\frac{1}{4}$ % of $1260
k 20.75% of $199
l 450% of $820

WE 3.2C **9** Increase or decrease each amount by the given percentage. Where appropriate, round answers to two decimal places.

a Increase $3700 by 20%
b Decrease 120 kg by 5%
c Increase $980 by 5.2%
d Decrease $850 by 7.5%
e Increase $87.50 by 1.75%
f Decrease 75 minutes by 10%
g Increase 5.2 km by 12.4%
h Decrease 516 L by 26%

WE 3.2D **10** Calculate the original amount if:

a after a 25% increase a number is now 500
b after a 30% decrease a number is now 595
c after a 16% decrease a number is now 2688
d after a 50% increase a number is now 225
e after a 70% increase a number is now 459
f after a $33\frac{1}{3}$% increase a number is now 1200.

11 Calculate the new amount if:

a a score of 400 was increased by 15%
b 2500 L of water in a tank was decreased by 30%
c a playlist of 60 songs was increased by 75%
d the waiting time of 15 minutes was decreased by 40%
e the duration of a lecture 48 minutes long was increased by 37.5%.
f a marathon runner's personal running record of 4 hours 40 minutes was decreased by 5%.

Exercise 3.2B: Problem solving and reasoning

12 The tallest horse ever recorded was a Belgian draft horse named Big Jake, who measured 210 cm tall. The smallest horse ever recorded was a dwarf miniature horse named Thumbelina who measured 44.5 cm tall. As a comparison, an average horse is about 165 cm tall and an average Labrador dog is about 60 cm tall.

a Write Thumbelina's height as a fraction of Big Jake's in simplest form.
b Convert the answer from part **a** to a percentage rounded to two decimal places.
c Find the percentage, correct to two decimal places, of:
  i Thumbelina's height compared to an average horse's height
  ii Thumbelina's height compared to an average Labrador's height
  iii Big Jake's height compared to an average horse's height.
d The heights of horses and dogs are measured at their withers (shoulders) because this height does not change when the animal moves, like the height of the head can.
  i Estimate how tall these two horses would be measured to the top of their heads.
  ii How would this change the percentage you calculated in part **b**?

13 There are 50 cupcakes on a platter.

a By midday, 20% of the cupcakes have been eaten. How many cupcakes are left over?
b By the evening, a further 20% of the cupcakes have been eaten. How many cupcakes have been eaten since midday?
c Explain why the amounts obtained in parts **a** and **b** are different.
d Assuming no more cupcakes are eaten, how many remain at the end of the day?

14 For each situation:

i find the amount of change that has occurred

ii state whether it is an increase or decrease

iii calculate the increase or decrease as a percentage of the original.

a A room that began with 25 people now has 31 people.

b A shelf that had 40 chocolate bars now has only 5 chocolate bars.

c Stacey moves house and instead of walking 1.5 km to school, she walks 2 km.

d Ahmed was 150 cm tall in Year 7 and is now 165 cm tall.

e A jogger improves her lap time from 20 minutes to 18 minutes.

15 Tom measured the length of a table to be 43.6 cm when the actual length is 42.6 cm. Lisa measured the length of a bench to be 85.4 cm when the actual length is 86.4 cm.

a What is the size of the error made by each student?

b Calculate the percentage error for each student by finding the size of the error as a percentage of the actual length. Who has the greater percentage error and why?

16 Arthur is studying caterpillars for a science experiment. He measures one caterpillar as being 3.2 cm long. The actual length of the caterpillar is 3.15 cm. Calculate the error as a percentage of the actual length of the caterpillar.

17 In 2013 the wombat population in Epping Forest National Park was 196. The wombat population has been growing by 8% each year. How many wombats would be expected in the park in 2025 if this rate of growth continues?

18 The Australia Koala Foundation estimated that the koala population decreased 30% between 2018 and 2021. The minimum and maximum number of koalas in 2021 were 32 065 and 57 920, respectively.

a What is the maximum and minimum number of koalas you would expect for 2018?

b If this trend continues, what is the maximum number of koalas you would expect in 2024?

c If this trend continues, what is the minimum number of koalas you would expect in 2025?

Exercise 3.2C: Challenge

19 For each situation, find the original number if:

a 50% of the original number is 32

b 20% of the original number is 19.6

c 70% of the original number is 322.

20 Answer the following questions regarding percentage increase.

a The sides of a square increase by 20%. What is the percentage increase in area?

b The sides of a triangle increase by 20%. What is the percentage increase in area?

c Make a statement about the results you obtained in parts **a** and **b**.

d Investigate another shape and then make a general statement about the percentage increase in area when the dimensions of a shape are increased by 20%.

21 Jamie has 5 kg of flour. He uses 20% of the flour to bake some bread, and then he uses 5% of the remaining flour to bake some biscuits.

a How many kilograms of flour remained after he baked the bread?

b How much flour did he use to bake the biscuits?

c Calculate the total percentage of the flour that has been used after baking the bread and biscuits.

Online resources:

- **Interactive skillsheet** Calculating percentages
- **Worksheet** Calculating percentages
- **Investigation** Of and As – what's the diff?
- **Quick quiz** 3.2

Lesson 3.3

Mark-ups and discounts

Learning intentions

By the end of this lesson you will be able to ...

✔ perform financial calculations involving percentages.

Inter-year links

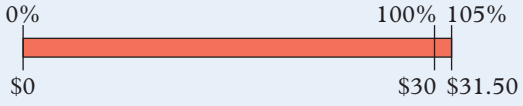
Support	Calculating percentages
Year 7	4.8 Calculating percentages
Year 9	9.2 Other forms of income
Year 10	12.1 Earning money

Mark-ups, discounts and GST

Key content video
Mark-ups and discounts

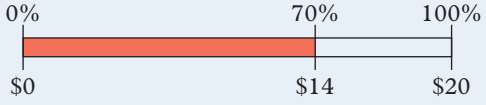
- The **cost price** (wholesale price) is the price paid to create or obtain a good or service before it is sold to a consumer.
- The **selling price** is the price that a product or service is sold at by the seller.
- A **mark-up** is the amount a cost price is increased by to give a profit and is usually expressed as a percentage.
- Finding the price after a mark-up is the same as increasing a quantity by a given percentage.
  1. Add the percentage to 100% and convert this new percentage to a decimal.
  2. Multiply the cost price by the decimal.

  Increase \$30 by 5% = (100% + 5%) × \$30
  = 105% × \$30
  = 1.05 × \$30
  = \$31.50

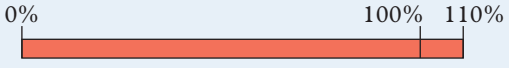


- A **discount** is the amount a selling price is decreased by to sell at a lower price and is usually expressed as a percentage.
- Finding the price after a discount is the same as decreasing a quantity by a given percentage.
  1. Subtract the percentage from 100% and convert this new percentage to a decimal.
  2. Multiply the selling price by the decimal.

  Decrease \$20 by 30% = (100% − 30%) × \$20
  = 70% × \$20
  = 0.7 × \$20
  = \$14



- **GST** (Goods and Services Tax) was introduced by the Australian Government in 1999. Currently, the rate is 10% of the selling price of a product or service.
- GST is added to the price of a good or service by performing a percentage increase of 10% to the selling price of a product or service.
  → The process of calculating the total amount paid by a customer is the same as calculating a 10% mark-up on the selling price.



The unitary method

Key content video
The unitary method with percentages

- The **unitary method** can be used to solve percentage increase or decrease questions when the original amount is not known. The unitary method works by first calculating the value of a single unit, in this case 1%, and then using this value to calculate an unknown amount, in this case 100%.

  For example, if after a discount of 15% a television has a retail price of \$765, you can divide the retail price by 85 (the remaining percentage) to calculate 1% of the original price. This can then be multiplied by 100 to give the original price.

$$1\% \text{ of original price} = \frac{\$765}{85}$$
$$= \$9$$
$$\text{Original price } (100\%) = \$9 \times 100$$
$$= \$900$$

Worked example 3.3A Calculating mark-ups and discounts

Calculate the final selling price of:

a a $350 camera with a 25% mark-up

b a $299 pair of shoes discounted by 30%

c a $418 monitor with GST.

THINK	WRITE
a 1 Mark-ups are the same as percentage increases. Add the percentage to 100% and convert this new percentage to a decimal.	**a** final price = (100% + 25%) × $350
2 Multiply the amount to be increased by the decimal.	= 125% × $350
3 Write your answer using the correct units.	= 1.25 × $350
	= $437.50
b 1 Discounts are the same as percentage decreases. Subtract the percentage from 100% and convert this new percentage to a decimal.	**b** final price = (100% − 30%) × $299
2 Multiply the amount to be decreased by the decimal.	= 70% × $299
3 Write your answer using the correct units.	= 0.7 × $299
	= $209.30
c 1 GST is a tax applied to goods, increasing the price of a good or service by 10%. Add the percentage to 100% and convert this new percentage to a decimal.	**c** final price = (100% + 10%) × $418
2 Multiply the amount to be increased by the decimal.	= 110% × $418
3 Write your answer using the correct units.	= 1.1 × $418
	= $459.80

Worked example 3.3B Calculating the original price

Calculate the original price of:

a a mountain bike, if the selling price is $904 and the mark-up is 60%

b a notebook, if the selling price is $1125 and the discount is 25%.

THINK	WRITE
a 1 To increase a number by 60%, multiply by 160%. To reverse the process, divide by 160%.	**a** original price = $904 ÷ (100% + 60%)
2 Convert this new percentage to a decimal.	= $904 ÷ 160%
3 Divide the selling price by the decimal to calculate the original price.	= $904 ÷ 1.6
	= $565
b 1 To decrease a number by 25%, multiply by 75%. To reverse the process, divide by 75%.	**b** original price = $1125 ÷ (100% − 25%)
2 Convert this new percentage to a decimal.	= $1125 ÷ 75%
3 Divide the selling price by the decimal to calculate the original price.	= $1125 ÷ 0.75
	= $1500

Worked example 3.3C Using the unitary method to calculate the original price

After a mark-up of 80%, a television retails for $1800. Use the unitary method to calculate the original price.

THINK

1 180% of the original price is $1800, so divide $1800 by 180 to calculate 1% of the original price.

2 Multiply the value for 1% of the original price by 100 to determine the original price.

WRITE

a 1% of original price $= \frac{\$1800}{180}$

$= \$10$

b original price $= \$10 \times 100$

$= \$1000$

Helpful hints

- ✔ A discount will result in a value less than the original selling price.
- ✔ A mark-up will result in a value greater than the cost price.

Learning pathways

1(a, c, e, g), 2(a-e), 3-4(1st column), 5, 6(a-d), 7(a, b), 8, 10-15

1(b, d, f, h), 2(e-i), 3-4(2nd column), 6(d-g), 7(c, d), 9, 10, 14-19

2(g-i), 4(2nd column), 6(d-g), 7(c, d), 10, 13, 16-20

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Exercise 3.3A: Understanding and fluency

1 Calculate each of these percentages.

a 10% of $360.50
b 25.25% of $4200
c 20% of $550.75
d 120% of $400
e 190% of $850
f 7.3% of $960
g 32% of $729
h 115.09% of $2900

WE 3.3A 2 Calculate the final selling prices.

a a 30% mark-up on $1200
b a 25% discount on $449
c a 40% discount on $2250
d a 75% mark-up on $149
e a 47% mark-up on $645
f a 38% discount on $750
g a 54% discount on $1200
h a 25% discount on $1050
i an 85% mark-up on $175

3 Lakmali works at a clothing store that applies a 45% mark-up on all items. What is the final selling price of:

a a pair of jeans with cost price $98
b a t-shirt with cost price $25
c a hoodie with cost price $65
d a shirt with cost price $102?

4 Calculate the prices paid for these items after GST is added, rounding to the nearest cent where appropriate.

a dining table and chairs: $1285
b services provided by a plumber: $240
c insurance purchased for a car: $601.45
d five 3 m lengths of timber at $6.50 per metre
e electricity service and supply charge: $314.65
f membership at a gymnasium at $72.95 per month

5 For each of these, determine:

i the selling price **ii** the mark-up or discount amount.

a A camera was purchased for $120 and sold later at a mark-up of 62%.

b A laptop originally marked at $1198 is offered for sale at a discount of 35%.

c Work tools each marked at $49.90 are offered for sale with a 15% discount.

WE 3.3B **6** Calculate the original price in each of these sales. Where necessary, round your answer to the nearest five cents.

a A mobile phone sells for $450 after a mark-up of 50%.

b A pair of sports shorts sells for $25 after a discount of 20%.

c Eyeliner sells for $11.85 following a 15% discount.

d A hardware store sells an electric chainsaw for $169 after it is marked up by 95%.

e A furniture store offers a leather lounge suite for sale for $9995 after a discount of 12.5%.

f Fitness equipment retails for $1499 following a 140% mark-up.

g The digital copy of a video game is on sale for $4.99 after a 90% discount.

7 The following items each include GST in the price. Calculate the pre-GST price, rounding to the nearest cent where appropriate.

a telephone and internet services $155.65 **b** computer accessories purchased for $235.95

c garden maintenance provided for $182 **d** a necklace bought at a jewellery store for $120.50

8 The selling price of an item is also known as the retail price. Michael plans to buy a new external hard drive to store his games. The hard drive has a retail price of $157.95, but he receives a 12.5% discount because he has a customer loyalty card.

a If the discount is 12.5%, what percentage of the retail price will Michael pay?

b Calculate the price Michael pays after the discount. Round your answer to the nearest five cents.

9 The following represent the original prices and the percentage discount amounts offered on some goods. In each case, calculate the following, rounding your answers to the nearest cent:

i the selling price after the discount **ii** the discount amount.

a $500; 12% discount **b** $179.50; 15% discount

c $249; 8% discount **d** $895.95; 4% discount

WE 3.3C **10** Use the unitary method to calculate the original prices. Round your answers to the nearest five cents where necessary.

a a bicycle retailing for $1189 after a mark-up of 45%

b a mobile phone retailing for $792 after a mark-up of 32%

c a box of chocolates retailing for $3.10 after a mark-up of 120%

d a sofa retailing for $1400 after a mark-up of 25%

Exercise 3.3B: Problem solving and reasoning

11 Melinda makes jewelled earrings and adds an 85% mark-up to her costs when determining the retail prices. Each individual earring contains a metal hook that costs $8.50 and three decorative stones that each cost $4.60.

a How much does it cost Melinda to make each pair of these earrings?

b What is the value of the mark-up?

c How much would Melinda advertise these pairs of jewelled earrings for?

12 A manufacturer advertises their football boots for a wholesale price of $89.90. A sports store plans to sell these boots to the public at a mark-up of 110%.

a If the mark-up is 110%, what percentage of the wholesale price will a member of the public pay for these boots?

b Calculate the retail price for these boots correct to the nearest five cents.

13 The following represent the wholesale prices and the percentage mark-up amounts offered on some goods. In each case, calculate the following, rounding your answers to the nearest cent:

i the retail price after the mark-up **ii** the mark-up amount.

a $620; 24% mark-up **b** $89.95; 45% mark-up

c $1269; 80% mark-up **d** $450.50; 85.5% mark-up

14 A girl's bicycle is reduced to $198 after a discount of 20%. To determine the original price, Jane thinks that she needs to calculate 20% of $198 and add the result to $198. Tim thinks that Jane has it wrong and that the calculation is more complex. Which person do you think is correct? Show working to support your answer.

15 Reconsider the scenario in question **14**. What was the price of the bike before the discount?

16 Julian accidentally discounted a $699 food mixer by 20% instead of the advertised 30%. Once he realised his mistake he took a further 10% off the discounted price. The customer claimed that this would not be the same as reducing the original price of the mixer by 30%. Who is correct? Show workings to support your answer.

17 Switched On Electrics is having a 'Scratch and Dent' sale. Any damaged stock will be discounted by 35%. A refrigerator is scratched and discounted to $1820.

a What was the original price of the refrigerator?

b How much money is saved because of the scratch?

18 A grocery store applies a 30% mark-up on all its goods. A GST of 10% is then added to this marked-up price.

a Calculate the selling price of each item once the mark-up and GST have been added.

i toothpaste, 160 g: cost price $3.88 **ii** tuna, 95 g: cost price $1.24

iii instant coffee: cost price $1.99 **iv** oil, 4 L: cost price $17.99

v rice crackers, 90 g: originally $2.49 **vi** raisin toast, 520 g: originally $4.59

b Compare the cost price with the selling price of each item and calculate the actual percentage increase correct to two decimal places.

c Explain why increasing a price by 30% then by another 10% is different from increasing it by 40%.

19 Consider the two statements: 'All stock discounted by 45%' and 'Stock discounted by up to 45%'. Explain whether these statements mean the same thing.

Exercise 3.3C: Challenge

20 David bought a new computer and paid $500 after several discounts. The computer was in a 45% off sale and his store membership rewarded him with an extra 10% off purchases more than $100. He also used a credit of $75 from the return of a faulty item.

a If the discount percentages were applied sequentially and then the store credit was applied, how much did the computer cost originally?

b If the store credit was applied first and then the discount percentages, how much did it originally cost?

c If the discounts were added and the total percentage was applied, and then the store credit was applied, how much did it cost originally?

d If the percentage discounts were applied sequentially, does it matter in which order they were applied? Explain why or why not.

Online resources:

Interactive skillsheet
Mark-ups, discounts and GST

Investigation
Inflating growth

Quick quiz
3.3

Lesson 3.4

Profit and loss

Learning intentions

By the end of this lesson you will be able to ...

✔ solve problems involving profit and loss.

Inter-year links

Support Calculating percentages

Year 7 4.8 Calculating percentages

Profit and loss

Key content video
Profit and loss

- A **profit** occurs when the selling price is greater than the cost price.

  Profit = selling price – cost price
- The **percentage profit** usually refers to the profit as a percentage of the cost price.

  $$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

  For example, Sam made a profit of $20 when he sold a bike he bought for $100.

  $$\begin{aligned}\text{Percentage profit} &= \frac{\text{profit}}{\text{cost price}} \\ &= \frac{20}{100} \\ &= 20\%\end{aligned}$$
- A mark-up and percentage profit are both calculated as the percentage amount that the cost price is increased by to give the selling price.
- A **loss** occurs when the selling price is less than the cost price.

  Loss = cost price – selling price
- The **percentage loss** usually refers to the loss as a percentage of the cost price.

  $$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

  For example, Greg made a loss of $120 when he sold the phone he bought for $499.

  $$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{cost price}} \\ &= \frac{120}{499} \\ &= 24.05\%\end{aligned}$$
- A discount is the amount a selling price is decreased by to sell at a lower price, whereas a percentage loss is calculated as the percentage amount that a cost price is decreased by to give the selling price.
- Percentage profit and loss calculations are usually in relation to the cost price, but can also be given in relation to the selling price.
- **Revenue** is the selling price multiplied by the number of items (or services) sold.

Worked example 3.4A Calculating profit and loss

A television initially bought for $800 is later sold for $950.

a State if a profit or loss has been made and determine the amount.

b Write the profit or loss amount as a percentage of the cost price.

THINK

a The selling price is more than the cost price, so a profit has been made. Find the difference.

b Write the profit amount as a percentage of the cost price. Write the comparison as a fraction and convert it to a percentage.

WRITE

a A profit has been made.

Profit = selling price − cost price
= \$950 − \$800
= \$150

b Percentage profit $= \frac{\cancel{150}^{3}}{\cancel{800}^{16}}$

$= \frac{3}{16} \times 100\%$

$= \frac{3}{\cancel{16}^{4}} \times \frac{\cancel{100}^{25}}{1}\%$

$= 18.75\%$

The television sold for an 18.75% profit.

Helpful hints

- ✔ Remember that you can only generate a profit when the selling price is greater than the cost price, and you can only generate a loss when the selling price is less than the cost price.
  - → When the result (selling price minus the cost price) is positive, there is a profit.
  - → When the result (selling price minus the cost price) is negative, there is a loss.
- ✔ When writing a percentage, make sure to carefully check what amount you are writing the percentage of.

Learning pathways

1–5, 7, 8, 10, 12, 14, 15, 20 | 2, 3, 6, 7, 9–11, 13, 16, 18, 19, 22 | 2, 3, 7, 10, 11, 16, 17, 18, 19, 21-24

ANS p535

Exercise 3.4A: Understanding and fluency

You can use your calculator for all questions in this section unless otherwise specified.

1 Determine the amount of profit or loss (in dollars) for each of the following.

a cost price: \$35, selling price: \$45
b cost price: \$82, selling price: \$68
c cost price: \$92.50, selling price: \$87.95
d cost price: \$299.98, selling price: \$145.50

WE 3.4A **2** For each of the following scenarios:

i state whether a profit or loss has been made and determine the amount
ii write the profit or loss amount as a percentage of the cost price, correct to two decimal places.

a Shoes are bought for \$240 and later sold for \$180.
b A greengrocer buys a box of cherries for \$2.50 and sells them for \$9.80.
c An investor buys shares for \$5.20 and sells them for \$4.80.
d A car is purchased brand new for \$24 640 and sold for \$19 250.
e Coins are purchased in a set for \$120 and sold for \$350.
f A novel is purchased for \$29.95 and sold for \$8.

3 Consider your answers from question **2** part **ii**.

a Re-calculate the percentages if they are now based on the selling price.
b How do the percentage amounts change if the percentages are based on the selling price instead of the cost price?

4 Calculate the percentage profit or loss on the cost price for each part in question **1**. Round your answers to two decimal places.

5 Xiao pays $198 for her wireless headphones and sells them to a friend for $150 when a new model comes out.

a Did Xiao make a profit or a loss? State the amount of profit or loss.

b Write the amount in part **a** as a percentage of the cost price.

6 As Benjamin became more successful at his BMX racing, he chose to sell his bike to buy a better model. The bike, which had cost him $240, was sold to a fellow competitor at a percentage profit of 5%.

a How much did Benjamin sell the bike for?

b The new bike Benjamin plans to buy will cost him $900. Write this as a percentage of the cost of his original bike.

7 For each of the situations below:

i state the value of the profit or loss

ii write the profit or loss as a percentage of the cost price (rounded to the nearest 1%).

a cost price: $48, selling price: $34

b cost price: $112.50, selling price: $240

c cost price: $35.90, selling price: $85.95

d cost price: $1649, selling price: $1238

e cost price: $29 895, selling price: $17 500

f cost price: $156 985, selling price: $425 850

8 John buys pears at the orchards for $2.95 per kilogram to sell at his market stall.

a How much does John mark up the cost of the pears per kilogram (see photo)?

b Write the mark-up amount as a percentage of the price John pays for the pears. State the mark-up as a percentage to the nearest 1%.

9 A wireless printer is initially priced at $249.95 and is offered for sale at a discounted price of $222.50.

a Calculate the loss on the sale of the printer.

b Write the loss as a percentage of the initial price to the nearest 1%.

10 The revenue a company earns is the total amount of money received from sales.

Revenue = selling price × sales

Profit per sale is the difference in the selling price and cost price, which can be multiplied by the number of sales to determine the total profit.

For example, if the original price is $5, the selling price is $8, and 20 are sold, then:

Revenue = $8 × 20 = $160

Profit = ($8 − $5) × 20 = $3 × 20 = $60

For each of the following, calculate the:

i revenue

ii total profit.

a cost price $2, selling price $5, 50 sold

b cost price $8, selling price $20, 100 sold

c cost price $10, selling price $40, 25 sold

d cost price $1.50, selling price $4.75, 45 sold

e cost price $7.99, selling price $13.99, 16 sold

f cost price 24c, selling price $1.99, 247 sold

Exercise 3.4B: Problem solving and reasoning

11 Joseph sells remote-controlled cars in his store. He knows that identical cars are being sold by a competitor for $65 and wants to beat their price. Joseph can purchase these cars from a wholesaler for $28 per car.

a Joseph aims to make a 150% profit on the sale of each car and must add 10% for GST. Do you think this profit margin is a suitable pricing strategy? Briefly explain.

b Using whole number values, what is the maximum percentage increase Joseph should apply to the wholesale price to beat the competitor's price? Remember to add the GST charge.

c Why is it necessary to consider a maximum percentage increase rather than any percentage increase Joseph wishes to apply?

12 A small share portfolio was purchased at a price of $1200 and sold 12 months later for $1500.

a Write the profit as a percentage of the original price.

b Write the final selling price as a percentage of the original purchase price.

c Compare the percentage increase in part **a** with the answer in part **b**. Briefly explain how they are related.

13 A car is bought for $20 000 and sold 6 months later for $16 000.

a Write the loss as a percentage of the original price paid for the car.

b Write the final selling price as a percentage of the original purchase price.

c Compare the percentage decrease in part **a** with the answer in part **b**. Briefly explain how they are related.

14 A sofa is bought for $600 and sold 6 months later for $450.

a Write the final selling price as a percentage of the original purchase.

b Use your answers from part **a** and question **13** to find the percentage loss.

15 What percentage increases or decreases match the following profits or losses?

a doubling your money **b** tripling your money **c** breaking even

d halving your money **e** quadrupling your money **f** losing all your money

16 The finance report on the nightly news displays the daily movement in the cost of common commodities. If the values given in this table represent the end-of-day trading figures, what were the values at the start of the day's trading?

Commodity	Final price $	Movement %
Gold	1732.95	↓ 0.5
Silver	33.56	↓ 1.4
Oil	102.46	↑ 0.3
Copper	3.71	↑ 1.2

↓ represents a decrease in price
↑ represents an increase in price

17 Olivia purchases a large block of land on which she plans to build four townhouses. The land costs $645 000 and the cost for each house is $230 000 (including plans, permits and other related charges). The project takes 2 years to complete and Olivia is charged $2300 in council rates each year. The real estate agency earns a commission of 1.75% for the sale of each townhouse. The four houses sell for $485 000, $490 000, $472 000 and $461 000, respectively.

a What were the total expenses accrued by Olivia before the sale of the townhouses?

b From the total sales, how much of the money:

i goes to Olivia **ii** is earned as commission by the real estate agency?

c Does Olivia make a profit or a loss?

d Write your answer from part **c** as a percentage of Olivia's total expenses. Do not include the real estate agency's commission.

18 Mario runs a hairdressing business from his home and sells shampoos, hair treatment and styling products to his customers. On all product sales, he plans to make a profit of 80% of the wholesale price he pays for the goods. On top of this, he knows he must add an additional 10% for GST. Mario believes he can determine the selling price by simply adding 90% to the wholesale price.

a A jar of styling gel has a wholesale price of $8.50. What is the price after Mario's profit mark-up?

b What is the selling price after GST is added?

c Increase $8.50 by 90% and compare your answer with the answer from part **b**. Is Mario's method of calculating the selling price correct? Why or why not?

19 For each of the following, write, correct to two decimal places where appropriate:

i the profit as a percentage of the selling price

ii the selling price as a percentage of the profit

iii the selling price as a percentage of the cost price

iv the percentage mark-up from the cost price to the selling price.

a cost price \$2, selling price \$5

b cost price \$8, selling price \$20

c cost price \$10, selling price \$40

d cost price \$1.50, selling price \$4.75

e cost price \$7.99, selling price \$13.99

f cost price 24c, selling price \$1.99

20 Determine the missing values in the table.

Cost price	Selling price	Cost price as a percentage of selling price	Number of sales	Revenue	Total profit	Profit as a percentage of the revenue
\$3.60	\$12		35			
	\$15		20		\$200	
	\$21			\$777	\$296	
			150	\$375	\$210	
			25	\$875		26%
			125		\$320	32%
	\$99	51%	8			
\$0.28		25%	271			
	\$32				\$145.92	24%
	\$6		15			40%

21 Percentage calculations can be described in different ways.

a Explain why finding 70% of a value is equal to decreasing the value by 30%.

b Explain why finding 130% of a value is equal to increasing the value by 30%.

22 A small business said they earned \$10 000 this week. They spent \$12 000 to make the sales.

a If the \$10 000 is the profit the business made, how much revenue did they earn?

b If the \$10 000 is the revenue the business earned, how much profit or loss did they make?

Exercise 3.4C: Challenge

23 Consider the following situations.

a A jar of styling gel has a wholesale price of \$8.50. Mario likes to make a profit of 80% on his wholesale prices and then adds 10% for GST. What single calculation can Mario perform to determine his selling price for a jar of styling gel?

b A motorbike sells for \$1200 after a mark-up of 60% followed by GST being added. What single calculation can be performed to determine the wholesale price of the motorbike?

c GST is added to a price and then the item is discounted by 25% to sell for \$400. What single calculation can be used to determine the original pre-GST price?

24 A company marks up their product's cost by 128% to its selling price. What percentage will the profit be of the revenue, correct to two decimal places?

Online resources:

Interactive skillsheet
Percentage profit and percentage loss

Worksheet
Financial calculations involving percentages

Quick quiz
3.4

ANS p536

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

3.1 **1** Write each percentage as a fraction in its simplest form and as a decimal.

a 25% **b** 76% **c** 4% **d** 0.5% **e** 120% **f** 2.56%

3.1 **2** Write each decimal as a percentage and as a fraction in its simplest form.

a 0.63 **b** 0.12 **c** 0.85 **d** 3.3 **e** 0.006 **f** 7.5

3.1 **3** Write each fraction as a decimal and a percentage.

a $\frac{3}{8}$ **b** $\frac{13}{40}$ **c** $5\frac{2}{3}$ **d** $\frac{5}{16}$ **e** $\frac{11}{25}$ **f** $8\frac{1}{6}$

3.2 **4** Write each of these as a percentage.

a 40 g out of 200 g **b** 45 minutes out of 180 minutes **c** 6 months out of 4 years
d 3 hours out of one day **e** 60 cents out of $3.00 **f** 960 mm out of 72 m

3.2 **5** Calculate:

a 23% of 580 L **b** 120% of $1500 **c** 2.8% of 700 cm
d 18% of 1700 m **e** 61% of 800 beans **f** $5\frac{1}{2}$% of $3500

3.2 **6** Increase or decrease each amount by the given percentage. Where appropriate, round answers to two decimal places.

a Increase $9600 by 20%. **b** Decrease 800 km by 12.5%. **c** Increase $4500 by 9.5%.
d Decrease $45.80 by 7.5%. **e** Increase 8.3 km by 1.75%. **f** Decrease 120 minutes by 10%.

3.3 **7** Calculate the selling price of these (rounded to the nearest 5 cents).

a a 60% mark-up on $1800 **b** a 15% discount on $950 **c** a 20% discount on $9870
d a 25% mark-up on $765 **e** a 21.5% mark-up on $954 **f** a 28% discount on $1250

3.3 **8** For each item, calculate:

i the GST to be added
ii the price it will be sold for (round to the nearest five cents).

a novel $32 **b** jeans $85 **c** laptop $1250
d pen $3.50 **e** purse $26 **f** chair $35

3.3 **9** Calculate the original price, if:

a 40% of an amount is $180 **b** 35% of an amount is $3430
c 16% of an amount is $1376 **d** 12% of an amount is $1014.60
e 85% of an amount is $595 **f** 24% of an amount is $6072.60.

3.4 **10** For each of the following scenarios:

i state whether a profit or loss has been made and determine the amount
ii write the profit or loss amount as a percentage of the cost price, correct to two decimal places where appropriate.

a A car is bought for $18 500 and sold a year later for $16 750.
b A vintage dress is bought for $15 and sold for $75.
c A trading card is purchased for $12 and sold for $100.
d A diamond ring is purchased for $3999 and is later sold for $2500.

3.4 **11** For each of the following, calculate the:

i revenue **ii** total profit each item makes.

a cost price $7, selling price $11, 70 sold **b** cost price $0.80, selling price $1.50, 330 sold
c cost price $5.50, selling price $12.50, 55 sold **d** cost price $18, selling price $27.50, 12 sold

Lesson 3.5

Ratios

Learning intentions

By the end of this lesson you will be able to ...

✔ write, use and simplify ratios.

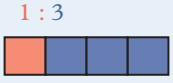
Inter-year links

Support	Equivalent fractions
Year 7	3.8 Ratios
Year 9	6.1 Ratios and rates

Ratios

Key content video
Understanding ratios

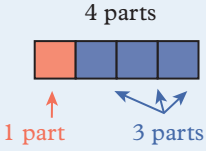
- A **ratio** is a comparison of two or more quantities with the same units.
  A ratio must be written in the order of the given worded descriptions.
  For example, the ratio 1 : 3 reads '1 part compared to 3 parts' or '1 to 3'.



The ratio of red blocks to blue blocks is 1 : 3.

- Ratios can be written as fractions where the numerator is the number in the ratio and the denominator is the total number of parts.

$\frac{1}{4}$ of the squares are red

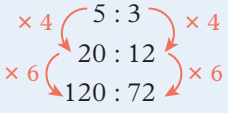


$\frac{3}{4}$ of the squares are blue

Equivalent ratios

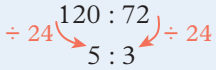
Key content video
Equivalent ratios

- Numbers in a ratio can be multiplied or divided by the same value to create an **equivalent ratio**.



Simplifying ratios

- A ratio is written in simplest form when all parts of the ratio are integers and the only common factor between all parts in the ratio is 1.



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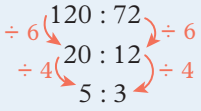
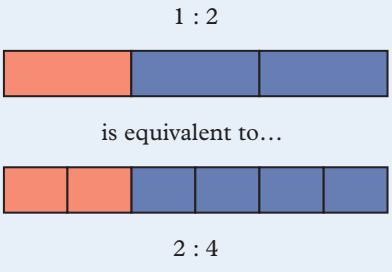
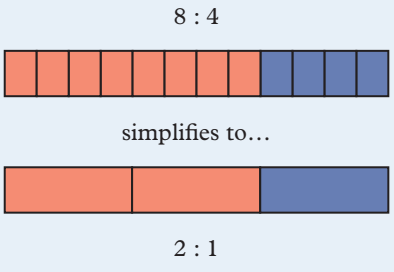






Comparisons with decimals and fractions

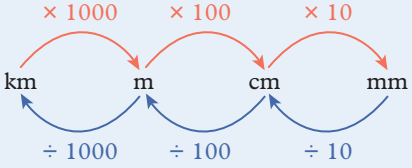
- Ratios containing decimals can be written as whole number ratios by multiplying each part of the ratio by a power of 10 (that is, 10, 100, 1000, etc.) to form whole numbers.

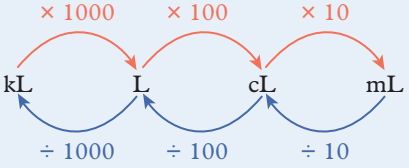
$$\begin{matrix} \times 10 \\ \\ = \end{matrix}\begin{matrix} 3.4 : 1.3 \\ \\ 34 : 13 \end{matrix}\begin{matrix} \times 10 \\ \\ \end{matrix}$$

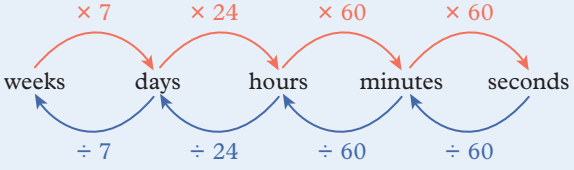
- Ratios containing fractions can be written as whole number ratios by first writing equivalent fractions with the lowest common denominator (LCD) and then multiplying each fraction by the value of the LCD to form whole numbers.
  - → Mixed numbers must first be converted to improper fractions. $2\frac{1}{4} : 1\frac{1}{3} = \frac{9}{4} : \frac{4}{3} = \frac{9}{12} : \frac{16}{12} = 9 : 16$

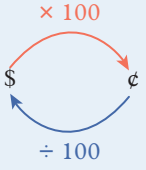
Converting units

- Before writing a ratio, the numbers must be in the same unit of measurement.
- Converting between units of measurement is important when calculating ratios.









Worked example 3.5A Part-to-part versus part-to-whole comparisons

Purple paint is made by mixing 4 parts of blue paint with 5 parts of red paint.

a Write a ratio comparing the blue to red paint.

b Write a ratio comparing the blue to purple paint.

c What fraction of purple paint is made up of blue paint?

d What fraction of purple paint is made up of red paint?

THINK

a Write the numbers as a ratio in the order they appear in the question.

b Add up the total number of parts in the purple paint. Write the numbers as a ratio in the order they appear in the question.

c Write a fraction showing the numerator as the blue paint and the denominator as the total number of parts.

d Write a fraction showing the numerator as the red paint and the denominator as the total number of parts.

WRITE

a blue : red
4 : 5

b 4 + 5 = 9
blue : purple
4 : 9

c blue paint = $\frac{4}{9}$

d red paint = $\frac{5}{9}$

Worked example 3.5B Working with equivalent ratios

Find the value of a if $3 : 5 = 21 : a$.

THINK

1. Compare the first number in each ratio, 3 and 21. What should 3 be multiplied by to give 21? 3 multiplied by 7 is 21.
2. As the ratios are equivalent, 5 multiplied by 7 is a.
3. Identify the value of a.

WRITE

$\times 7$ (3 : 5) $\times 7$
$= 21 : a$

$5 \times 7 = a$

$a = 35$

Worked example 3.5C Simplifying ratios involving decimals and fractions

Write each comparison as a ratio in simplest form.

a 4.5 to 2.75 **b** $2\frac{1}{6}$ to $\frac{5}{8}$

THINK

a
1. Write the comparison as a ratio.
2. Multiply each number by 100 to obtain whole numbers.
3. Simplify the ratio by dividing each number in the ratio by the HCF of 25.

b
1. Convert the mixed number to an improper fraction. Write the comparison as a ratio.
2. Identify the lowest common denominator (LCD) of the two fractions. The LCD is 24. Write each fraction as an equivalent fraction with a denominator of 24.
3. Multiply each fraction by 24 to obtain whole numbers.
4. Check if it can be simplified.

WRITE

a $\times 100$ (4.5 : 2.75) $\times 100$
$= 450 : 275$
$\div 25$ $= 18 : 11$ $\div 25$

b $2\frac{1}{6} = \frac{13}{6}$

$\frac{13}{6} : \frac{5}{8} = \frac{52}{24} : \frac{15}{24}$
$\times 24$ $= 52 : 15$ $\times 24$

Worked example 3.5D Writing ratios with different units

Write each comparison as a ratio.

a 7 seconds to 3 minutes **b** 0.05 km to 23 cm to 6.73 m

THINK

a
1. Convert the quantities to the same unit. Convert minutes into seconds by multiplying by the conversion factor, 60.
2. Write the numbers as a ratio in the order they appear in the question. Do not include units.

b
1. Convert the quantities to the same unit. Convert km to cm and m to cm by multiplying by the respective conversion factors, 100 000 and 100.
2. Write the numbers as a ratio in the order they appear in the question. Do not include units.

WRITE

a 3 mins = 180 seconds

7 : 180

b 0.05 km = 5000 cm
6.73 m = 673cm
5000 : 23 : 673

Helpful hints

- ✔ Don't mix up the order of the numbers in a ratio. It tells you the order of the quantities you are comparing.
- ✔ Ratios are written in comparison order, not in size order.
- ✔ When writing ratios, consider if you are comparing a part-to-part or a part-to-whole.
  For example, in the diagram shown, you can make both these comparisons:

  The ratio of red to blue tiles is 2 : 3.

  The ratio of blue tiles to the whole shape is 3 : 5.

- ✔ It is okay to compare quantities that contain fractions or decimals in a ratio; however, ratios in their simplest form never contain fractions or decimals.

Learning pathways

1-2, 3(a-h), 5(a-h), 6(1st, 2nd columns), 7(a-h), 8(1st column), 9, 11, 12, 16, 18	3(i-p), 5(e-l), 6(2nd, 3rd columns), 7(i-n), 8(2nd column), 10, 11, 13, 14, 17, 19	4(b, d, f, h), 5, 6(3rd, 4th columns), 7(i-n), 8(3rd column), 13, 15, 17, 19-21

ANS p537

Exercise 3.5A: Understanding and fluency

WE 3.5A **1** Green paint is made by mixing 3 parts blue paint with 4 parts yellow paint.

a Write a ratio comparing blue to yellow paint.
b Write a ratio comparing blue to green paint.
c What fraction of green paint is made up of blue paint?
d What fraction of green paint is made up of yellow paint?

2 A showbag contains 11 Cherry Ripes, 9 Flakes, 7 Picnics, 5 Time Outs and 8 Turkish Delights.

a Write the ratio comparing the number of:
i Cherry Ripes to Turkish Delights to Picnics
ii Flakes to the total number of chocolate bars.

b Write the number of:
i Time Outs and Turkish Delights as a fraction of the total number of chocolate bars
ii Flakes and Picnics as a percentage of the total number of chocolate bars.

WE 3.5B **3** Find the value of each pronumeral.

a $5 : 9 = 25 : a$	**b** $15 : 50 = 3 : b$	**c** $9 : 2 = c : 16$	**d** $8 : 36 = d : 9$
e $e : 55 = 19 : 11$	**f** $f : 4 = 42 : 24$	**g** $2 : g = 22 : 66$	**h** $20 : h = 5 : 2$
i $a : 24 = 20 : 30$	**j** $b : 52 = 20 : 13$	**k** $13 : c = 65 : 5$	**l** $25 : d = 10 : 14$
m $48 : 32 = 18 : e$	**n** $33 : 44 = f : 12$	**o** $33 : 54 = g : 72$	**p** $15 : 20 = 36 : h$

4 Fill in the boxes to complete the equivalent ratios.

a $2 : 5 = \square : 20 = 20 : \square$
b $6 : 7 = \square : 35 = 54 : \square$
c $\square : 360 = 70 : 60 = 7 : \square$
d $225 : \square = 9 : 12 = \square : 4$
e $\square : 104 = 68 : \square = 17 : 13$
f $\square : 48 = 75 : \square = 15 : 4$
g $20 : \square = 40 : 96 = \square : 240$
h $\square : 636 = 72 : 212 = 18 : \square$

5 Which pairs of ratios are equivalent? Explain how you know whether each pair is equivalent.

a 1 : 2 and 3 : 6	**b** 8 : 5 and 2 : 3	**c** 4 : 10 and 2 : 5	**d** 2 : 9 and 10 : 45
e 16 : 25 and 4 : 5	**f** 63 : 42 and 9 : 7	**g** 2 : 3 and 22 : 34	**h** 15 : 9 and 5 : 3
i 1 : 5 and 15 : 30	**j** 17 : 51 and 68 : 204	**k** 49 : 64 and 7 : 8	**l** 27 : 18 and 3 : 2

WE 3.5C **6** Write each ratio in simplest form.

a 7 : 84 **b** 9 : 12 **c** 106 : 36 **d** 22 : 55

e 2.3 to 5.9 **f** 7.2 to 8.4 **g** 1.05 to 3.15 **h** 8.1 to 1.75

i $\frac{1}{6}$ to $\frac{1}{4}$ **j** $\frac{2}{3}$ to $\frac{3}{4}$ **k** $\frac{1}{3}$ to $\frac{2}{5}$ **l** $\frac{3}{7}$ to $\frac{2}{11}$

m $2\frac{1}{8}$ to $\frac{2}{5}$ **n** $3\frac{1}{5}$ to 4 **o** 0.25 to $\frac{1}{8}$ **p** $\frac{3}{4}$ to 0.125

7 Write each comparison as a ratio in the given order.

a 20 seconds to 33 seconds **b** 95 kg to 57 kg

c 113 m to 167 m **d** 256 mL to 175 mL

e \$499 to \$575 **f** 23 days to 2 days

g 4 months to 13 months **h** 19 cans to 21 cans

i 42 km to 117 km **j** 16 L to 39 L

k 19 wins to 11 losses **l** 23 losses to 7 wins to 3 draws

m 15 minutes to 17 minutes to 19 minutes **n** 5 red to 7 blue to 11 yellow

WE 3.5D **8** Write each comparison as a ratio in simplest form.

a 13 days to 3 weeks **b** 2 hours to 17 minutes **c** 27 cents to \$5

d 4000 mL to 3 L **e** 1.7 km to 793 m **f** 7 months to 3 years

g 509 g to 1.3 kg **h** 65 mL to 3.2 L **i** 17 mm to 2.34 m

j $\frac{2}{3}$ hours to 0.8 hours **k** 0.9 km to 550 m **l** $1\frac{3}{4}$ minutes to 420 seconds

m 0.05 km to 23 cm to 6.73 m **n** 5631 g to 1.753 kg to 1 tonne

Exercise 3.5B: Problem solving and reasoning

9 Write each quantity as a ratio in simplest form.

a The number of students in a school is 975 while the number of teachers is 75.

b Mix 25 mL of cordial with 225 mL of water.

c 8500 Diamonds fans and 4500 Silver Ferns fans watched a netball match between the two teams.

d I have 8 gold bracelets and 14 silver bracelets.

10 Eric paid a deposit of \$704.70 for his overseas trip, which cost \$4698. Write the ratio, in simplest form, of the deposit paid to the amount owing.

11 Jasmine is helping her mother make pancakes for her class end-of-term breakfast. The ingredients are listed on the right and feed 10 people.

a Which ingredients can be written as a ratio? Clearly explain your choices.

b Write a ratio for the ingredients identified in part **a**.

c Repeat part **b** if 50 pancakes are required.

Pancakes
(serves 10)

500 mL full-cream milk
300 g self-raising flour
2 eggs
60 g butter
2 tablespoons sugar

12 The ratio of rainy days to sunny days in a month was 2 : 3. If there were 18 sunny days, how many rainy days were there?

13 A manager finds that in a box of 50 nectarines, 18% have blemishes. Write the:

a ratio of the numbers of unblemished nectarines to blemished nectarines

b fraction of the total number of nectarines that have blemishes.

14 The mark-up on a pair of jeans purchased for \$80 is 315%. What is the ratio of the cost price to the selling price?

15 Sofia's win–loss ratio against Bianca is 4 : 5. What does this mean?

16 Consider the height of Melbourne's Rialto Tower (270 m) and Taiwan's Taipei 101 (1670 feet).

a Explain why the ratio for the heights of Rialto Tower and Taipei 101 cannot be written as 270 : 1670.

b What needs to be done to the measurements before a comparison can be made?

c Write the ratio of the Rialto Tower's height to Taipei 101's height to the nearest metre. Hint: 1 foot = 0.3048 m

17 Whiter Than White bleach recommends three capfuls of bleach per $2\frac{1}{2}$ L of water.

a Can the two quantities be written as a ratio as they are?

b If a capful is 19 mL, what is the ratio of the quantities of bleach to water?

c Sharon thinks the solution of bleach and water is quite strong. Suggest an alternative ratio which would give a weaker solution. Explain your logic.

18 Gordon bought shares in a Telco company for $8.20 and sold them for $40.80. Find the ratio, in simplest form, of:

a the cost price to the selling price

b the cost price to the difference between the selling price and the cost price.

19 Imelda purchased a pair of shoes that had been discounted by 60%. The shoes were originally priced at $850. Find the ratio, in simplest form, of:

a the cost price to the sale price

b the discount compared to the cost price

c the discount compared to the sale price.

20 The Golden Ratio is an irrational number that is not quite as famous as π, but it also uses a Greek letter for its symbol – Φ(phi). Many artists and architects believe that rectangles whose length and width are in the Golden Ratio make the most pleasing shape. We can define the Golden Ratio as the ratio of a shorter length, y, and a longer length, x, such that x divided by y is also equal to the sum of the two lengths divided by the longer length.

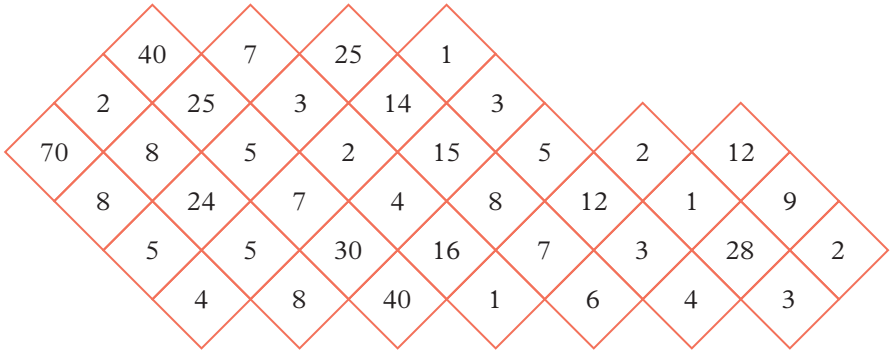
a Write out the two equations that define the Golden Ratio, Φ. When the shorter length, y, is equal to 1, what do you notice about the value of the Golden Ratio, Φ, and the longer length, x?

b Combine your equations from part **a** to create a new equation that does not contain x.

c The Golden Ratio, correct to 6 decimal places, is equal to 1.618 034. Substitute this value into your equation.

Exercise 3.5C: Challenge

21 Using only the numbers below, find 10 pairs of equivalent ratios. You must use all the numbers in the diagram and each number can only be used once. You may like to present your answers in a table like the one on the right.



1st ratio	2nd ratio

Online resources:

- Interactive skillsheet: Ratios in simplest form
- Worksheet: Writing ratios
- Investigation: Motorbike dilemmas
- Quick quiz: 3.5
- Interactive skillsheet: Converting between units of length
- Worksheet: Equivalent ratios
- Desmos activity: Paint

Lesson 3.6

Modelling with ratios

Learning intentions

By the end of this lesson you will be able to ...

✔ solve problems involving ratios.

Inter-year links

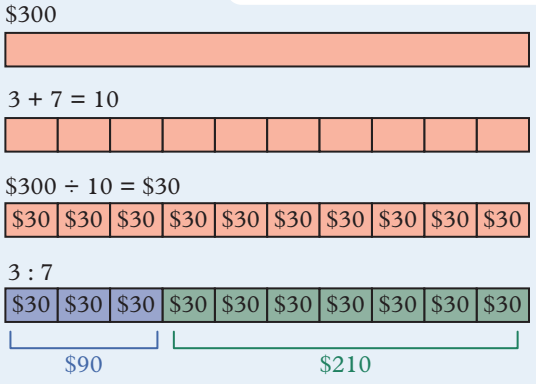
Support	Dividing a decimal by a whole number
Year 7	4.5 Dividing decimals
Year 9	6.1 Ratios and rates

Dividing a quantity into a given ratio

Key content video
Dividing a quantity into a given ratio

- When dividing a quantity in a given ratio, follow these steps:
  1. Find the total number of parts in the ratio.
  2. Divide the quantity by the total number of parts.
  3. Multiply the result of the previous step by each section of the ratio.

  For example, dividing \$300 into 3 : 7 gives \$90 and \$210.



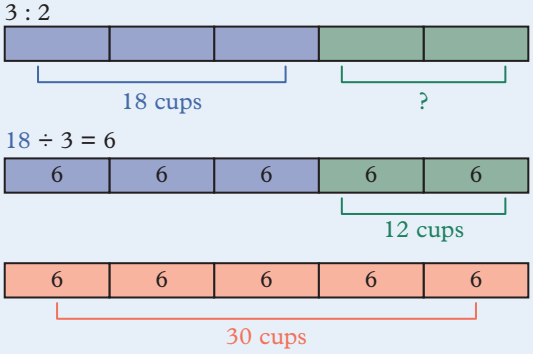
- Remember to include the same units with your answer.
- One way to check that you have divided the quantity correctly is to add the individual amounts to see whether the result is the same as the original quantity.

  For example, \$90 + \$210 = \$300.

Finding a quantity from a given ratio

- The unitary method can be used to find a quantity from a given ratio:
  1. Determine the value of 1 part of the ratio. Divide the given quantity by the number of parts in the corresponding side of the ratio.
  2. When the value of 1 part is found, any part of the ratio and the whole can be found.

  For example, if the ratio of rice to water in a recipe is 3 : 2 and a chef uses 18 cups of rice:
  - → 18 cups of rice is related to the 3 parts in the ratio.
  - → 18 divided by 3 gives 1 part equal to 6 cups.
  - → There are 2 parts water to 3 parts rice, so the water required to match the rice is 12 cups.
  - → 30 cups in total are required for this recipe.



Worked example 3.6A Dividing a quantity into a ratio

Divide 4800 in the ratio 3 : 1.

THINK

1. Find the total number of parts in the ratio.
2. Divide the quantity by the total number of parts.
3. Multiply 1200 by the number of parts in each section of the ratio.

WRITE

Number of parts = 3 + 1
= 4
4800 ÷ 4 = 1200

1200 × 3 = 3600
1200 × 1 = 1200

The ratio 3 : 1 divides 4800 into 3600 and 1200.

Worked example 3.6B Dividing a decimal quantity into a ratio

Divide $20.25 into the ratio 4 : 5.

THINK

1. Find the total number of parts in the ratio.
2. Divide the quantity by the total number of parts.
3. Multiply 2.25 by the number of parts in each section of the ratio. Remember to give your answer in dollars.

WRITE

Total number of parts = 4 + 5
= 9
20.25 ÷ 9 = 2.25
2.25 × 4 = 9
2.25 × 5 = 11.25
The ratio 4 : 5 divides $20.25 into $9 and $11.25.

Worked example 3.6C Dividing a fraction quantity into a ratio

Divide $\frac{11}{45}$ into the ratio 2 : 3.

THINK

1. Find the total number of parts in the ratio.
2. Divide the quantity by the total number of parts.
3. Multiply $\frac{11}{225}$ by the number of parts in each section of the ratio. Remember to simplify fractions where possible.

WRITE

Total number of parts = 2 + 3
= 5
$\frac{11}{45} \div 5 = \frac{11}{45} \times \frac{1}{5}$
$= \frac{11}{225}$
$\frac{11}{225} \times 2 = \frac{22}{225}$
$\frac{11}{225} \times 3 = \frac{33}{225}$
$= \frac{11}{75}$
The ratio 2 : 3 divides $\frac{11}{45}$ into $\frac{22}{225}$ and $\frac{11}{75}$.

Worked example 3.6D Finding a quantity from a given ratio

The ratio of cats to dogs in a neighbourhood is 11 : 6. If there are 96 dogs in the neighbourhood:

a how many cats are there

b how many cats and dogs are there in total?

THINK	WRITE
a **1** Determine the total number of parts in the ratio.	**a** 11 + 6 = 17
2 Determine the number of pets in one part of the ratio by dividing the given number of pets (102) by the number of parts in this section (6).	96 ÷ 6 = 16
3 Multiply the value from step **2** by the number of parts the cats represent.	16 × 11 = 176 There are 176 cats in the neighbourhood.
b Add the number of cats to the number of dogs.	**b** 176 + 96 = 272 There are 272 cats and dogs in total.

Helpful hints

- ✔ Always add the parts in a ratio together to find out how many parts there are in total.
- ✔ If you need to refresh your skills on multiplying and dividing decimals and fractions, refer to Module 2.
- ✔ You can consider the process of finding one part as simplifying. From **Worked example 3.6D**:

6 parts = 102 dogs
÷ 6 ↓ ÷ 6
1 part = 17 dogs

Learning pathways

1–10, 12–15, 17, 22	4–7, 10, 11, 13, 15, 16, 18, 19, 21, 23, 24(a-d)	5–7, 13, 15, 18, 20-25

ANS p538

Exercise 3.6A: Understanding and fluency

1 Divide 500 into the following ratios. Use the diagram to help you.

a 1 : 9 **b** 2 : 8 **c** 3 : 7 **d** 6 : 4 **e** 3 : 2 **f** 4 : 1

2 Use the diagram in question **1** to divide 35 cups of water into each ratio.

a 3 : 2 **b** 4 : 1 **c** 7 : 3 **d** 9 : 1 **e** 3 : 5 : 2 **f** 3 : 4 : 3

WE 3.6A **3** Divide $9200 into each ratio.

a 3 : 2 **b** 5 : 3 **c** 3 : 1 **d** 1 : 1 **e** 21 : 25 **f** 2 : 8

4 Divide $240 into each ratio.

a 1 : 2 **b** 9 : 1 **c** 2 : 10 **d** 10 : 6 **e** 3 : 1 : 4 **f** 7 : 2 : 1

WE 3.6B **5** Divide $3.60 into each ratio.

a 3 : 1 **b** 1 : 8 **c** 2 : 3 : 7

WE 3.6C **6** Divide $\frac{8}{15}$ into each ratio.

a 2 : 1 **b** 3 : 2 **c** 6 : 5 : 4

WE 3.6D **7** The ratio of fiction to non-fiction books in a library is 7 : 8. If there are 3500 fiction books in the library:

a how many non-fiction books are there **b** how many books are there in total?

8 The ratio of tomato seedlings to chilli seedlings in Paul's garden is 5 : 7. If there are 21 chilli seedlings:

a how many tomato seedlings are there **b** how many seedlings are there in total?

9 The ratio of blue pens to black pens to red pens in a classroom is 7 : 4 : 1. If there are 24 black pens in the classroom, how many pens are there altogether?

10 For best results, the instructions on a new cleaning product recommend the ratio of detergent to water be 2 : 7.

a How many litres of water are required if 300 mL of detergent are used?

b How many millilitres of detergent are required if 28 L of water are used?

11 A two-stroke fuel mixture is made by mixing oil and petrol in the ratio of 1 : 25.

a How many litres of petrol should be added to 850 mL of oil?

b How many millilitres of oil should be added to 15 L of petrol?

12 The ratio of junior to senior students at an interschool football competition is 5 : 2. If there are 224 students at the training session, how many are senior students?

Exercise 3.6B: Problem solving and reasoning

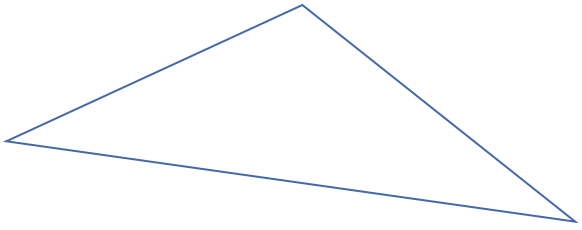
13 The ratio of Year 7s to Year 8s in a school orchestra is 5 : 4. If there are 15 Year 7s in the class, find:

a how many Year 8s are in the orchestra

b how many Year 7 and 8 students there are in total in the orchestra.

14 The size of the angles in a triangle are in the ratio 3 : 4 : 2. What is the size of each angle? Hint: The sum of angles in any triangle is 180°.

15 The side lengths of a scalene triangle are in the ratio 7 : 8 : 5.



a If the shortest side length is 17.4 cm, find the length of the longest side.

b Find the perimeter of the scalene triangle.

16 Nadia divides her pocket money for April in the ratio 3 : 2 : 1 to cover buying a gift, entertainment expenses and savings. Calculate the amount she has allocated for entertainment expenses if her total pocket money for April is $84.

17 Anzac biscuits date back at least 100 years. One of the earliest recipes is shown on the right. The 'dry ingredients' of oats, flour and sugar are mixed together first.

2 cups rolled oats
1 cup plain flour
½ cup sugar
125 g butter
1 tablespoon golden syrup
2 tablespoons boiling water

a What is the ratio of oats to flour to sugar?

b What fraction of the dry ingredients is flour?

c This recipe makes 12 biscuits. Sam decides to make Anzac biscuits for his year level. Sam combines the dry ingredients together and has 35 cups' worth. How many biscuits is he planning to make?

18 Lydia, Sylvia and Julia are three sisters, aged 16, 14 and 12, respectively. Their grandparents give them $210 and tell them to divide it amongst the three of them. Lydia suggests the girls divide the money between them according to the ratio of their ages. Julia objects to this and suggests the money should be divided in thirds. Sylvia says she is happy either way. Why do you think Sylvia feels this way? Use evidence to support your answer.

19 Jia is the treasurer of her netball club which has $200 in its budget. She has decided that $\frac{3}{20}$ of the budget will be spent on a present for the coach, $\frac{1}{4}$ of the budget will be spent on trophies for all the players and the rest of the budget will be spent on snacks at the games.

- **a** What fraction of the budget will be spent on snacks?
- **b** Write a ratio representing the money Jia will spend on the present, trophies and snacks.
- **c** Find the percentage that is allocated to each part of the budget.
- **d** Find how much money Jia will spend on each part of the budget.

20 Cathy has a collection of gems that consists of 14 sapphires, 4 diamonds and 10 emeralds.

- **a** What is the ratio of sapphires to diamonds to emeralds?
- **b** Cathy sends her collection to be authenticated and discovers two of the emeralds are actually sapphires. What is the new ratio of sapphires to diamonds to emeralds?
- **c** Cathy trades two emeralds with her friend in exchange for a diamond. What is the new ratio of sapphires to diamonds to emeralds?
- **d** Cathy decides to make a necklace with four of each gem from her collection. What is the ratio of sapphires to diamonds to emeralds in the resulting reduced collection?

21 The perimeter of a rectangular computer screen is 228 cm, with a width-to-height ratio of 12 : 7.

- **a** Calculate the width and height of the screen.
- **b** Calculate the area of the screen.

22 The ratio of blue and red paint in a mixture is 1 : 3. If there are 10 litres more red paint than blue paint, how much blue paint is used?

23 Markus is collecting marbles. If the ratio of red marbles to white marbles is 3 : 10, and the ratio of white marbles to blue marbles is 8 : 7, what is the ratio of red marbles : white marbles: blue marbles?

Exercise 3.6C: Challenge

24 To create two distinct shades of pink paint, the following ratios of white to red paint are required: 4 : 1 and 7 : 1. A 4 L can in each shade is purchased.

- **a** Calculate the amounts (in mL) of white and red paint in each can.
- **b** The two cans are accidentally mixed together. What is the new ratio of white to red paint?
- **c** Write the amount of red paint as a fraction, in simplest form, of the mixed paint.
- **d** Write the amount of white paint as a percentage of the mixed paint.
- **e** The mixed paint is divided evenly into two cans. How much red and white paint would have to go into each can to return the cans to the two original shades of paint?

25 An aquarium has fish and amphibians. The amphibians are frogs and newts. There are 40 more fish than amphibians. The ratio of frogs to newts to fish is 2 : 5 : 11. How many animals does the aquarium have?

Online resources:

- **Interactive skillsheet** Dividing a quantity into a ratio
- **Interactive skillsheet** Finding a quantity from a given ratio
- **Worksheet** Dividing a quantity into a given ratio
- **Investigation** On the ropes
- **Quick quiz** 3.6

Rates

Learning intentions

By the end of this lesson you will be able to ...

✔ solve problems involving rates.

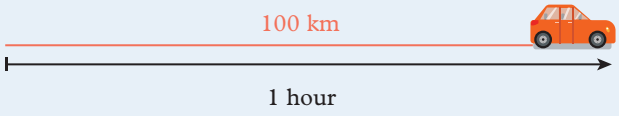
Inter-year links

Year 7	4.5 Dividing decimals
Year 9	6.1 Ratios and rates
Year 9	9.3 Taxation

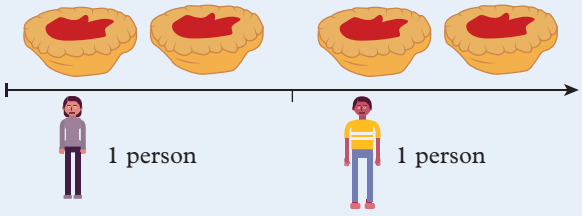
Rates

Key content video
Rates

- A **rate** is a comparison between two or more different quantities with different units. It is a measure of how a quantity changes for each unit of another quantity.
- An average rate can be calculated by dividing the total change in one quantity by the total change in the other quantity.
- The unit of a rate is two units separated by the word 'per' (meaning 'for each') or the symbol /. For example,
  - → 100 km/h is the speed at which 100 km is covered in one hour.

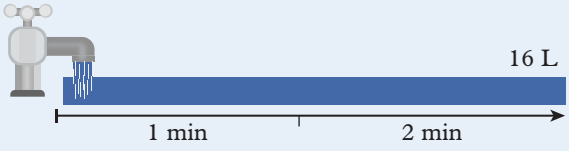


  - → 2 meat pies per person means each person gets 2 meat pies.



  - → $500 per week means someone receives $500 each week.

  - → 8 L/min means that for each minute, 8 litres flow.



- Order is important when writing a rate.
- Rates show the change of one quantity over another quantity. If the rate and the change in one of the quantities are known, we can calculate the change in the other quantity. For example, speed is a rate comparing distance and time.
  - → If the speed and distance travelled are known, we can calculate the duration of time.
  - → If the speed and duration of time are known, we can calculate the distance travelled.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{time} = \frac{\text{distance}}{\text{speed}} \qquad \text{distance} = \text{speed} \times \text{time}$$

Simplified rates

- Quantities in a rate can be multiplied or divided by the same value to find an **equivalent rate**.
- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.
  1. Write the rate as a fraction including the units.
  2. Simplify the fraction so that the denominator is 1.
  3. Write your answer as a rate using the '/' to indicate 'per'.

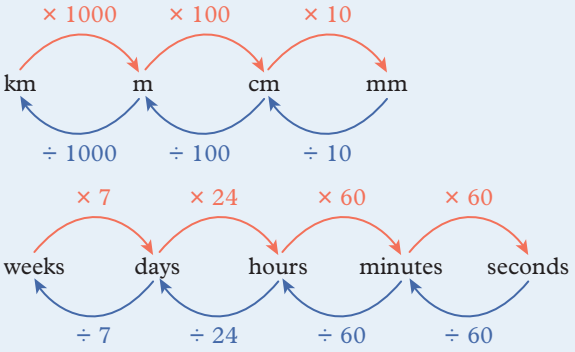
$$\frac{36\text{ km}}{12\text{ h}} \overset{\div 12}{=} \frac{3\text{ km}}{1\text{ h}} = 3\text{ km/h}$$

Converting units of rates

The same rate can be written in different units, remember to convert the units of measurements when calculating rates.

For example,

$$\begin{aligned}\frac{3\text{ km}}{\text{h}} &= \frac{3\text{ km}}{1\text{ h}}\\ &= \frac{3 \times 1000\text{ m}}{1 \times 60\text{ min}}\\ &= \frac{3 \times 1000\text{ m}}{1 \times 60 \times 60\text{ s}}\\ &= \frac{3 \times 1000}{1 \times 60 \times 60}\text{ m/s}\\ &= \frac{3000}{3600}\text{ m/s}\\ &= \frac{5}{6}\text{ m/s}\end{aligned}$$



Worked example 3.7A Writing statements as rates in simplest form

Write each statement as a rate in simplest form.

a 30 m in 5 seconds

b $54 for 40 L of petrol

THINK

a 1 Write the two quantities as a fraction including the units.

2 Simplify the fraction so that the denominator is 1. For the rate to be in simplest form, the second quantity needs be 1. Divide both quantities by 5.

3 Write your answer as a rate using the '/' to indicate 'per'.

b 1 Write the two quantities as a fraction including the units.

2 Simplify the fraction so that the denominator is 1. Divide both quantities by 40.

3 Write your answer as a rate using the '/' to indicate 'per'. The dollar sign '$' must be written before the number.

WRITE

a 30 m in 5 seconds $= \frac{30\text{ m}}{5\text{ s}}$

$$\frac{30\text{ m}}{5\text{ s}} \overset{\div 5}{=} \frac{6\text{ m}}{1\text{ s}}$$

$\frac{6\text{ m}}{1\text{ s}} = 6\text{ m/s}$

b $54 for 40 L of petrol $= \frac{\$54}{40\text{ L}}$

$$\frac{\$54}{40\text{ L}} \overset{\div 40}{=} \frac{\$1.35}{1\text{ L}}$$

$\frac{\$1.35}{1\text{ L}} = \$1.35\text{/L}$

Worked example 3.7B Solving problems using rates

Will loves to read. He can read 56 pages in half an hour.

a How many pages can he read in 2 hours?

b How many pages can he read in 3.5 hours?

c How long will it take him to read 1008 pages?

THINK

a 1 Write the two quantities as a fraction including the units. Simplify the fraction so that the denominator is 1.

2 Multiply the hourly rate by 2 to work out the new amount. The number of pages Will can read in 2 hours is found in the numerator of the equivalent rate.

b Multiply the hourly rate by 3.5 to work out the new amount. The number of pages Will can read in 3.5 hours is found in the numerator of the equivalent rate.

c Find an equivalent rate with 1008 pages as the numerator. Will can read 112 pages in 1 hour. Divide 1008 by 112 to find the value required to find the equivalent rate. Multiply the numerator and denominator of the original rate by this value to find the equivalent rate. The time taken is the denominator of the equivalent rate.

WRITE

a $56 \text{ pages per half hour} = \frac{56 \text{ pages}}{0.5 \text{ hour}}$

$= \frac{112 \text{ pages}}{1 \text{ hour}}$

$\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 2}{=} \frac{224 \text{ pages}}{2 \text{ hours}}$

Will can read 224 pages in 2 hours.

b $\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 3.5}{=} \frac{392 \text{ pages}}{3.5 \text{ hours}}$

Will can read 392 pages in 3.5 hours.

c $\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 9}{=} \frac{1008 \text{ pages}}{9 \text{ hours}}$

It takes Will 9 hours to read 1008 pages.

Worked example 3.7C Converting units of rates

Write 2.2 m/s in km/h.

THINK

1 Write the rate as a fraction where the denominator is 1. Include the units in both the denominator and the numerator to keep track of the conversion.

2 Write the quantity in the numerator in the desired unit using the conversion factor 1000 m =1 km.

3 Write the quantity in the denominator in the desired unit using the conversion factor 1 h = 60 mins = 3600 seconds.

4 Rewrite the fraction as a rate with the desired unit. Simplify the fraction.

WRITE

$2.2 \text{ m/s} = \frac{2.2 \text{ m}}{1 \text{ s}}$

$= \frac{2.2 \div 1000 \text{ km}}{1 \text{ s}}$

$= \frac{2.2 \div 1000 \text{ km}}{1 \div (60 \times 60) \text{ h}}$

$= \frac{2.2 \div 1000}{1 \div (60 \times 60)} \text{ km/h}$

$= \frac{2.2 \times 60 \times 60}{1 \times 1000} \text{ km/h}$

$= 7.92 \text{ km/h}$

Helpful hints

- ✔ Make sure that you identify the units of the rate. It's important to understand the units because they determine how you work with them.
- ✔ Choose multiplication or division to manipulate your rate to give your desired units.
- ✔ Don't forget to include units in your answer. Using units in your working helps to ensure they also appear in your answer.

Learning pathways

△ 1–3(1st column), 4–6, 9–11, 13–15, 17 □ 3(2nd column), 4, 6–8, 10–12, 16–18 ⬠ 4, 7, 8, 10, 15, 18–22

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Exercise 3.7A: Understanding and fluency

1 Write each statement as a rate with the appropriate unit.

a 60 km in each hour **b** 591 L in each minute **c** four quarters in each game
d $16 for each hour **e** 45 cm each year **f** five patients each hour
g $1.49 for each day **h** $28.99 for each month **i** $5.80 for each box
j 0.5 kg for each week **k** 290 mm each year **l** 14 km for each litre

2 What rate unit could be used in each situation?

a a cheetah chasing its prey **b** Usain Bolt sprinting 100 m **c** the cost of 10 apples
d the cost of tiling the bathroom **e** filling a bath with water **f** driving on a freeway

WE 3.7A **3** Write each statement as a rate in simplest form.

a 200 m in 4 minutes **b** 6 L in three bottles **c** 80 cents for 20 seconds
d 135 students with nine teachers **e** 468 seats in 18 rows **f** 100 newspapers in four bundles
g 68 runs in 10 overs **h** $20 for eight muffins **i** 102 goals in 15 games
j 39 996 tickets sold in 9 minutes **k** 77 goals over 22 games **l** area of 33.6 m^2 needs 6.4 L of paint

4 Write a rate in its simplest form to represent each situation. Give your answers to two decimal places where appropriate.

a A car uses 18 L of petrol to travel 252 km.
b The cost of 15 mangoes is $29.85.
c A bath fills up to 54 L in $4\frac{1}{2}$ minutes.
d A runner takes $2\frac{3}{4}$ hours to complete a 42.2 km marathon.
e Victoria's population is 6 865 358 and its area is 227 444 km^2.
f Light travels 1080 million km in 60 minutes.

WE 3.7B **5** Lina can type 1200 words in half an hour.

a How many words can she type in 1 hour?
b How many words can she type in 20 minutes?
c How long does it take Lina to type 1800 words?

6 Boris earns $182.40 for working 12 hours.

a How much does he earn for working 1 hour?
b How much does he earn for working 40 hours?
c How long does Boris have to work to earn $1459.20?

7 Sound travels 23 160 m in 60 seconds.

a How far does sound travel in 1 second?

b How far does sound travel in 1 hour?

c How long does it take for sound to travel 772 000 m?

8 Fabian worked 22 hours last weekend and received $396. At this rate, how much will he earn if he works $19\frac{3}{4}$ hours next weekend?

9 Calculate the average speed for each situation, rounding your answer to two decimal places where appropriate.

a a bus travels 876 km in 16 hours

b a jet travels 1965 km in 2.3 hours

c a cyclist rides 78 km in 2.4 hours

d an athlete runs 10 km in 26.44 minutes

e James Magnussen took 47.71 seconds to swim 100 m freestyle

f Libby Trickett took 52.88 seconds to swim 100 m freestyle

Exercise 3.7B: Problem solving and reasoning

WE 3.7C **10** Convert the following rates to the units in the brackets.

a $1.2/L (dollar/mL)

b $25/g (dollar/kg)

c 35 km/h (m/s)

d 5 cm/s (km/h)

e 200 g/mL (kg/L)

f 14.2 kg/L (g/mL)

11 Alexis and Kym arrange to meet at the running track to train for an athletics carnival. They cycle to the track, leaving their homes at the same time. It takes Alexis 8 minutes to travel 2800 m from her home to the track. Kym takes 18 minutes, after travelling a distance of 6.12 km.

Alexis thought she was faster, but Kym was not convinced.

a Calculate each girl's average speed.

b Who was fastest? How do you know?

c What unit did you use for speed? Explain.

12 Max's team scored 230 runs in 40 overs. William's team scored 280 runs in 50 overs.

a Calculate each team's run rate per over.

b Which team had a better run rate?

13 Leandro's resting heart rate is 14 beats in 15 seconds.

a Heart rate is often expressed in beats per minute (bpm). Write Leandro's heart rate in bpm.

b How many minutes does it take for Leandro's heart to beat 1512 times?

14 Victor takes 48 minutes to run 15 km.

a What distance does he cover per minute?

b If Victor is able to maintain this rate, what distance would he cover in these times?

i 55 min **ii** 60 min **iii** 1 hour 25 min **iv** 1.5 h

c How long would it take Victor to cover these distances?

i 625 m **ii** 31.25 km **iii** 25 km **iv** 2 km

15 Compare each pair of grocery items and work out which option is the better value for money.

a 5 kg of potatoes for $4.95 or 7.5 kg for $7.15

b a packet of four batteries for $3.99 or a packet of ten for $9.89

c a 500 g tin of coffee for $9 or a 150 g jar for $2.99

d a 440 g tin of peaches for $1.85 or a 650 g tin for $2.56

16 A phone company offers its customers two plans.

- Plan A: connection fee of 40 cents and then 40 cents per minute or part thereof
- Plan B: connection fee of 50 cents and then 38 cents per minute or part thereof

a Calculate the cost of a 1-minute call with each plan. Which offers the best deal?

b Calculate the cost of a 3-minute call with each plan. Which offers the best deal?

c Calculate the cost of a 5-minute call with each plan. Which offers the best deal?

d Calculate the cost of a 6-minute 23-second call with each plan. Which offers the best deal?

e Use your answers from parts **a–d** to explain when it would be best to use each plan.

17 Martha has been comparing the prices of goods in Italy with those in Australia. She looks up the exchange rate and finds that \$1 = €0.64 (Euro). A bag costs €80 online and Martha has a budget of \$120. Can she afford it?

18 Density is a substance's mass per unit of volume. 180 g of copper is mixed with 105 g of zinc to make an alloy. The density of copper is 9 g/cm^3 and the density of zinc is 7 g/cm^3.

a Determine the mass of the alloy.

b Determine the volume of the alloy.

c Determine the density of the alloy, correct to four decimal places.

19 When repaying a loan from a bank, you pay interest on top of the amount of the loan. The amount of interest, I, depends on the original amount you borrowed, P, the interest rate charged per year, r, and the time, T, it takes to repay the loan in years. The interest rate, r, is often written as a percentage. For example, $r = 5\%$ per annum means you need to pay 5% of the total amount you borrowed as interest per year.

a If you borrow 100 dollars from a bank with an interest rate of 5% per annum, how much interest do you need to pay?

b If you borrow 1000 dollars from a bank with an interest rate of 5% per annum for two years, how much interest do you need to pay in total?

c Write a formula to show the relationship between I, P, r and T.

Exercise 3.7C: Challenge

20 It takes 6 window washers 6 minutes to wash 6 windows. How long does it take 12 window washers to wash 12 windows?

21 Siv starts rowing across a 400 m wide river at a speed of $\frac{14}{9}$ m/s. At the same time Riley starts swimming from the other side of the river at a rate of 100 metres every 2 minutes. How far from Siv's starting point do they meet?

22 In Australia, workers pay income tax on their income to the government. The amount they pay depends on the amount they earn. The amount of income tax paid in the 2024–2025 financial year is outlined in the following table.

Taxable income	Tax on this income
0 – \$18 200	Nil (\$0)
\$18 201 – \$45 000	16 cents for each \$1 over \$18 200
\$45 001 – \$135 000	\$4288 plus 30 cents for each \$1 over \$45 000
\$135 001 – \$190 000	\$31 288 plus 37 cents for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45 cents for each \$1 over \$190 000

Source: https://www.ato.gov.au/rates/individual-income-tax-rates/

a For a worker earning \$46 000 a year, how much of their income will be taxed at the rate 30 cents per \$1?

b Calculate the tax the worker in part **b** needs to pay.

c Calculate the amount of tax paid by someone earning

i \$30 000 **ii** \$65 000 **iii** \$145 000 **iv** \$205 000.

d What do the amounts \$4288, \$31 288 and \$51 638 represent? Show how they are calculated.

Online resources:

Interactive skillsheet Rates | Worksheet Writing rates | Investigation Solution rates | Desmos activity Click battle | Quick quiz 3.7

Lesson 3.8

Review: Percentages, ratios and rates

Module summary

Percentages, fractions and decimals

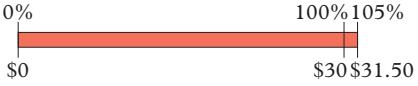
	Fraction	Decimal
Percentage to...	$15\% = \frac{15}{100} = \frac{3}{20}$	$23.4\% = \frac{23.4}{100} = 23.4 \div 100 = 0.234$

	Percentage	Decimal
Fraction to...	$\frac{1}{3} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$	Align decimal points. $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ (Dividend, Divisor, Trailing zeros) $= 0.1\,2\,5$

	Percentage	Fraction
Decimal to...	$0.65 = 65\%$	$0.65 = \frac{65}{100} = \frac{13}{20}$

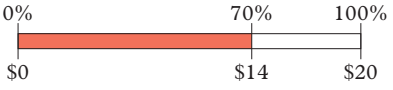
Mark-ups (percentage increase)

Increase \$30 by 5% = \$30 × (100% + 5%)
= \$30 × 105%
= \$30 × 1.05
= \$31.50



Discounts (percentage decrease)

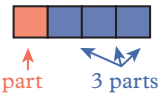
Decrease \$20 by 30% = \$20 × (100% – 30%)
= \$20 × 70%
= \$20 × 0.7
= \$14



Ratios

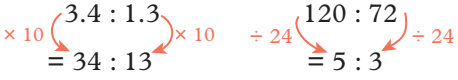
1 : 3

$\frac{1}{4}$ of the squares are red



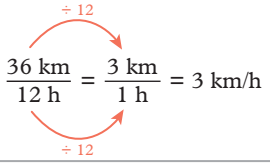
$\frac{3}{4}$ of the squares are blue

Simplifying ratios



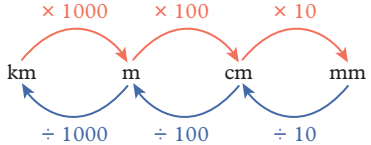
Equivalent ratios

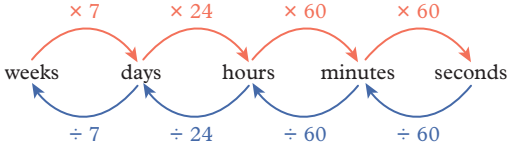
Rates

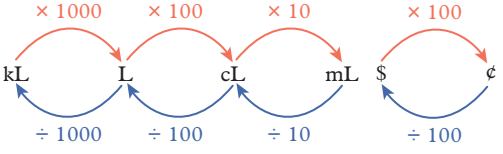


Units in ratios and rates

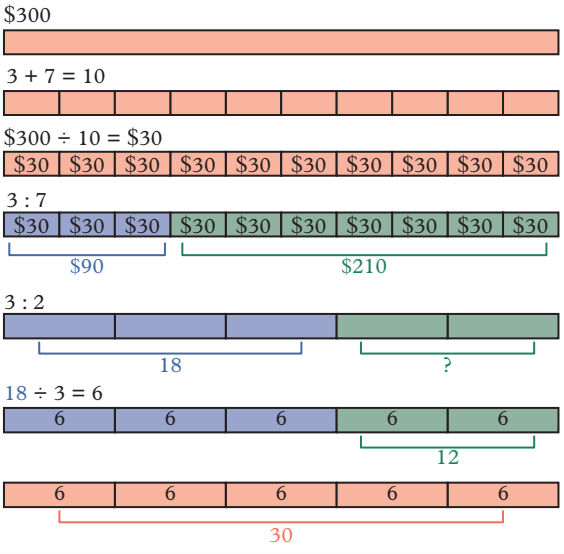
- In a ratio, all quantities must have the same units. In rates, two quantities with different units are compared.
- When converting the units that a rate is expressed in, write the rate as a fraction with the units included in the numerator and denominator, and then apply the appropriate conversion factors.







Modelling with ratios



Profit and loss

- Profit = selling price – cost price
- Percentage profit = $\frac{\text{profit}}{\text{cost/selling price}} \times 100\%$
- Loss = cost price – selling price
- Percentage loss = $\frac{\text{loss}}{\text{cost/selling price}} \times 100\%$
- Revenue = selling price × number of items sold

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Review questions 3.8A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- cost price
- discount
- equivalent ratio
- GST
- loss
- mark-up
- percentage
- percentage decrease
- percentage error
- percentage increase
- percentage loss
- percentage profit
- profit
- rate
- ratio
- revenue
- selling price
- unitary method

1 Which rate is calculated by dividing distance by time?

2 Explain the difference between a ratio and a rate.

3 Explain the difference between profit and loss.

4 Identify the key terms that are being referenced in each of these definitions.

a how a quantity changes for each unit of another quantity

b the price paid to create or obtain a good or service before it is sold to a consumer

5 Using an example, provide a definition in your own words for the following key terms.

a mark-up

b revenue

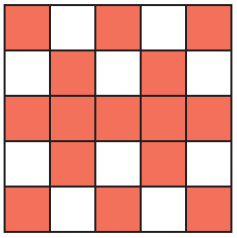
6 Complete the following sentences using words from the key terms list.

a A ratio is written in _____ when the only common factor between the terms in the ratio is 1.

b A _____ is a proportion of a whole that has been divided into one hundred parts.

Review questions 3.8B: Multiple choice

3.1 **1** What percentage of the figure on the right is shaded?



A 15%

B 40%

C 50%

D 60%

3.1 **2** What is $\frac{1}{5}$ written as a percentage?

A 10% **B** 15% **C** 20% **D** 25%

3.1 **3** Which value is *not* the same as 75%?

A 0.75 **B** 75 **C** $\frac{75}{100}$ **D** $\frac{3}{4}$

3.2 **4** What is 45% of 360?

A 162 **B** 198 **C** 162% **D** 315

3.3 **5** After a 32% discount, what is the selling price of an item originally marked $480?

A $153.60 **B** $326.40 **C** $448 **D** $633.60

3.4 **6** Which of the following statements is correct?

A A profit occurs when the cost price is greater than the selling price.

B A discount increases the selling price of an item or service.

C A loss occurs when the cost price is less than the selling price.

D Revenue is found by multiplying the selling price by the number of items sold.

3.5 **7** Written as a ratio, what is 3 hours to 37 minutes?

A 3 : 37 **B** 37 : 3 **C** 37 : 180 **D** 180 : 37

3.5 **8** Which ratio is 16 : 28 : 12 written in simplest form?

A 4 : 7 : 3 **B** 3 : 7 : 4 **C** 8 : 14 : 6 **D** 6 : 18 : 2

3.5 **9** What is the value of a in $3 : 7 = a : 35$?

A 7 **B** 14 **C** 15 **D** 21

3.6 **10** When $3600 is divided in the ratio 1 : 3, what are the two parts?

A $1200 and $2400 **B** $900 and $2700

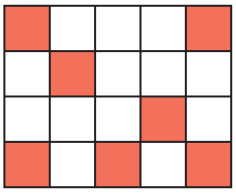
C $1500 and $2100 **D** $1200 and $2700

3.7 **11** What is 120 metres in 24 seconds written as a rate in its simplest form?

A $\frac{120}{24}$ m/s **B** $\frac{24}{120}$ m/s **C** $\frac{1}{5}$ m/s **D** 5 m/s

Review questions 3.8C: Short answer

3.1 **1** Consider this figure.



a How many shaded sections are there?

b Write the number of shaded sections as a fraction of the total number of sections.

c Write the fraction in part **b** as an equivalent fraction with a denominator of 100.

d Write the shaded amount as a percentage.

3.1 **2** Write each percentage as a decimal.

a 23% **b** 62% **c** 16.52% **d** 291.6%

3.1 **3** Write each decimal as a percentage.

a 0.32 **b** 7.62 **c** 31.625 **d** 0.0003

3.1 **4** Write each fraction as a percentage.

a $\frac{3}{4}$ **b** $\frac{7}{10}$ **c** $\frac{17}{20}$ **d** $\frac{15}{32}$

3.2 **5** Write:

a 20 as a percentage of 50 **b** 24 as a percentage of 150 **c** 58 as a percentage of 125.

3.2 **6** Calculate:

a 10% of 340 **b** 25% of 62

c 150% of 1500 **d** 37% of 2400.

3.3 **7** Calculate the selling price after each mark-up or discount.

a a 10% mark-up on $950

b a 44% mark-up on $628

c a 28% discount on $1255

3.3 **8** Calculate the percentage mark-up or discount.

a a radio with cost price $40 and selling price $55

b a book with cost price $35 and selling price $21

c shares with cost price $2.98 and selling price $4.52

d a TV with cost price $1400 and selling price $958

3.4 **9** Determine the amount of profit or loss in each of the following situations.

a cost price: $7.35, selling price: $12.99

b cost price: $24.25, selling price: $20.50

c cost price: $45, selling price: $78.20

3.4 **10** Write the profit or loss amount for each item in question **9** as a percentage of the cost price (rounded to the nearest %).

3.5 **11** Write each comparison as a ratio.

a 35 kg to 47 kg **b** 7 weeks to 31 weeks **c** 8 months to 3 years **d** 45 cents to $1.25

3.5 **12** Simplify each ratio.

a 12 : 44 **b** 64 : 108 **c** 16 : 32 : 96 **d** 15 : 75 : 105

3.5 **13** Calculate the value of each pronumeral.

a $6 : 7 = a : 49$ **b** $13 : 4 = 52 : b$ **c** $c : 48 = 3 : 12$ **d** $15 : d = 3 : 7$

3.6 **14** Calculate the total number of parts for each ratio.

a 3 : 5 **b** 12 : 5 **c** 4 : 5 : 8 **d** 6 : 10 : 15

3.6 **15** Divide $1500 in each ratio.

a 2 : 3 **b** 3 : 7 **c** 3 : 4 : 8 **d** 4 : 5 : 11

3.7 **16** Write each statement as a rate with an appropriate unit.

a 100 km per hour **b** 120 L in 40 seconds

c $140 for 8 hours of work **d** 26 books read in 6 months

3.7 **17** Write each statement as a rate in simplest form.

a a football team scored 180 points in 120 minutes of play

b the cost of 45 L of petrol was $62.55

c a 375 mL can of drink costs $1.80

d a worker earned $218.40 for 13 hours of work

Review questions 3.8D: Mathematical modelling

1 A market stall holder bought a variety of coloured 'hoodies' to sell. Each hoodie cost the stall holder $20 and she intends to sell them to the public at a mark-up of 65%.

a Write the percentage mark-up value as a:

i fraction in simplest form

ii decimal.

b Calculate the value of the mark-up on each hoodie.

c What price will the hoodies be?

d If the stall holder sells 35 hoodies on the first day, what is the difference between the total cost price of the hoodies and the total amount they sell for?

e To improve the sales of the hoodies for her second day at the market, the stall holder decides to offer a 10% discount on the selling price.

i What is the discount amount for each hoodie?

ii What is the new selling price?

iii What profit does the stall holder make on the sale of each hoodie?

iv The stall holder sells 55 hoodies at the discounted amount. Write the profit from the sale of these as a percentage of the cost she paid for them.

2 Kris is doing some landscaping in her backyard. She buys 3 cubic metres of Terracotta mulch at $75 per cubic metre and 5 cubic metres of Black mulch at $60 per cubic metre.

a What is the ratio of Terracotta mulch to Black mulch in simplest form?

b What price per cubic metre (to the nearest 5 cents) did Kris pay overall for mulch?

c For her rock garden, Kris decides she needs to buy River Rocks at $170 per cubic metre and Honeycomb Rocks at $100 per cubic metre. The ratio of River Rocks to Honeycomb Rocks is 2 : 3 and Kris buys 1.6 cubic metres of River Rocks.

i How many cubic metres of Honeycomb Rocks does she buy?

ii What is the total cost of the rocks?

d Kris buys a total of 36 plants to put in her garden. The three types of plants are Grevilleas, Callistemons and Banksias in the ratio 2 : 3 : 4.

i How many Callistemon plants does she buy?

ii How many Banksia plants does she buy?

iii Kris decides to increase the number of plants by 25%, while keeping the same ratio. How many extra of each plant does she need to buy?

3 Khadija's go-to trail mix recipe is shown on the right.

a Find the ratio of nuts to seeds in simplest form.

b Find the ratio of nuts to fruits in simplest form.

c How much chocolate would Khadija need for 12 cups of trail mix?

d Khadija decides to leave out the fruit and chocolate but wants to have the same amount of trail mix as the original recipe. How much of each of the nuts and seeds does she need if she wishes to keep the same ratio?

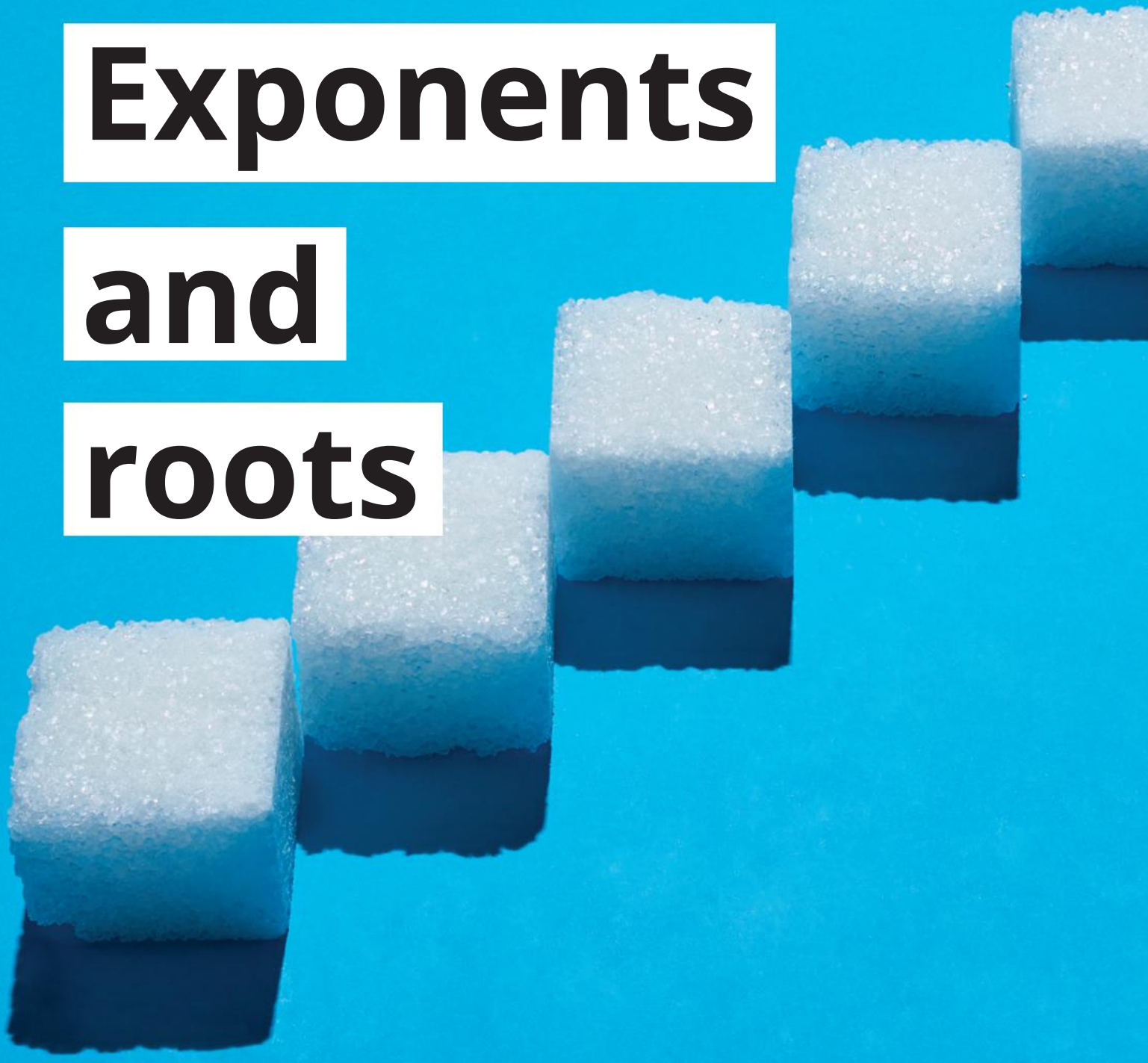
Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert between percentages, fractions and decimals ☐ Compare and order percentages, fractions and decimals	☐ Go back to Lesson 3.1 Percentages, fractions and decimals
☐ Express a quantity as a percentage of another quantity ☐ Calculate a percentage of a quantity ☐ Increase and decrease a quantity by a percentage	☐ Go back to Lesson 3.2 Calculating percentages
☐ Calculate mark-ups and discounts including GST ☐ Use the unitary method to solve financial problems	☐ Go back to Lesson 3.3 Mark-ups and discounts
☐ Calculate the profit or loss as an amount ☐ Calculate the profit or loss as a percentage	☐ Go back to Lesson 3.4 Profit and loss
☐ Write ratios in their simplest form ☐ Simplify ratios involving decimals and fractions ☐ Use equivalent ratios to find unknown quantities	☐ Go back to Lesson 3.5 Ratios
☐ Divide a quantity into a given ratio ☐ Use the unitary method to find a quantity from a given ratio	☐ Go back to Lesson 3.6 Modelling with ratios
☐ Write rates in their simplest form ☐ Use rates to solve problems ☐ Convert rates to different units	☐ Go back to Lesson 3.7 Rates

4

Exponents and roots



Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Divisibility rules
- ✔ Order of operations
- ✔ Prime and composite numbers
- ✔ Multiplying and dividing fractions
- ✔ Multiplying decimals

Curriculum links

- Recognise irrational numbers in applied contexts, including π and numbers that develop from the square root of positive real numbers that are not perfect squares, and recognise that irrational numbers cannot develop from the division of integer values by natural numbers (VC2M8N01)
- Establish and apply the exponent laws with positive integer exponents and the zero exponent, using exponent notation with numbers (VC2M8N02)

Materials

- ✔ Calculator

Lesson 4.1

Exponents

Learning intentions

By the end of this lesson you will be able to ...

✔ convert numbers between exponent form and expanded form, and use factor trees for prime factorisation.

Inter-year links

Year 7	1.7 Exponents and square roots
Year 9	1.1 Exponents
Year 10	1.1 Exponents

Exponent notation

Key content video
Exponents

- **Exponent form** is used to represent repeated multiplication.
  - → A number in exponent form is also known as a **power**.
  - → When an expression is written out in full it is said to be in **expanded form**.
  - → The actual value of the number is called the **basic numeral**.

base exponent

exponent form $3^4 = 3 \times 3 \times 3 \times 3$ expanded form

$= 81$ basic numeral

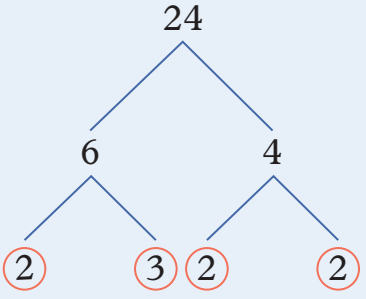
- The base is the number that is repeatedly multiplied.
- The exponent (or index) indicates how many times the base appears in expanded form.
  - → 3^4 is read as '3 to the power of 4'.
- If no exponent is shown, the implied exponent is 1.
  For example, $2 = 2^1$.
- An exponent only applies to the value directly preceding it. For example, in -2^4 the exponent only applies to the '2', not to '−2'. Therefore, $-2^4 = -2 \times 2 \times 2 \times 2 = -16$.
  - → For an exponent to be applied to a negative number, the negative number must be placed in brackets. For example, $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$.
  - → Placing brackets around a fraction indicates that the whole fraction is the base, not just the numerator.
  For example, $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$.
- When applying exponents to negative numbers, the sign of the basic numeral needs to be considered.
  - → If the base is negative and the exponent is an even number, the basic numeral is positive.
  - → If the base is negative and the exponent is an odd number, the basic numeral is negative.

  For example, the basic numeral for $(-3)^5$ is negative because the base is negative and the exponent is an odd number.
  $(-3) \times (-3) \times (-3) \times (-3) \times (-3) = -243$

Prime factorisation

Key content video
Prime factorisation

- The **prime factorisation** of an integer is the unique product of the **prime numbers** that make up that integer.
  - → Prime factorisation is often expressed in exponent form with the bases listed in ascending order.
  For example, the prime factorisation of $24 = 2^3 \times 3$.
- Prime factorisation can be performed using **factor trees**. In factor trees, **composite numbers** are broken down into pairs until only prime numbers remain.



Worked example 4.1A Calculating the value of a number in exponent form

Write 2^5 in expanded form and calculate its value.

THINK

1 Identify the base and the exponent. Write the base in expanded form by multiplying five twos together.

2 Perform the multiplication.

WRITE

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$= 32$

Worked example 4.1B Prime factorisation using factor trees

Use factor trees to express each number as the product of its prime factors. Write your answer in exponent form.

a 18 **b** 225

THINK

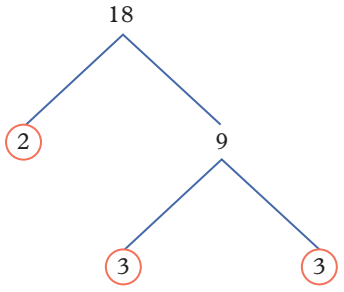
1 Divide the composite number by its lowest prime factor to identify a factor pair. The lowest prime factor of even numbers is always 2. Remember that if the sum of all the digits in a number is divisible by 3, that number is also divisible by 3.

2 Repeat step **1** until each branch ends in a prime number.

3 Write the initial composite number as a product of its prime factors. Write the answer in exponent form and list the bases in ascending order.

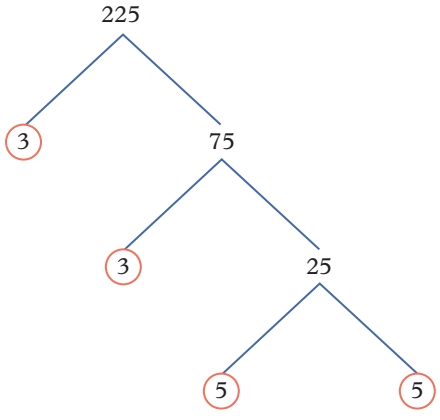
WRITE

a



$18 = 2 \times 3 \times 3$
$= 2 \times 3^2$

b



$225 = 3 \times 3 \times 5 \times 5$
$= 3^2 \times 5^2$

Helpful hints

- ✔ Take care when writing exponents – they should be written smaller than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34.
- ✔ Remember that calculating exponents and multiplication are different operations.
  For example, $3^2 \neq 3 \times 2$, $3^2 = 3 \times 3$.
- ✔ Read questions involving exponents carefully. You may be asked to:
  - → Write a number in expanded form; for example, $3^2 = 3 \times 3$
  - → Calculate or evaluate; for example, $3^2 = 9$

Learning pathways

1–10, 11(a, c), 12, 14, 19	3, 5–10, 11(b, c), 13, 15, 17, 18, 20, 21, 24	3, 5–7, 10, 11(c, d), 13, 16–18, 20–25

ANS p539

Exercise 4.1A: Understanding and fluency

1 The number 5^3 is written in exponent form.

- **a** State the value of the base.
- **b** State the value of the exponent.
- **c** Write the number in expanded form, as a repeated multiplication.
- **d** Perform the repeated multiplication to convert 5^3 into a basic numeral.

WE 4.1A **2** Write each number in expanded form and calculate its value.

a 2^5 **b** 4^6 **c** 1^9 **d** 3^5 **e** 10^7
f 2^8 **g** 5^4 **h** 7^5 **i** 8^3 **j** 6^4

3 Complete the table below.

	Exponent form	Base	Exponent	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	
b	2^5	2			32
c			3	$8 \times 8 \times 8$	
d				$6 \times 6 \times 6 \times 6 \times 6$	

4 Which of the following options is the expanded form of $(-4)^5$?

A $5 \times 5 \times 5 \times 5$
B $(-4) \times 4 \times 4 \times 4 \times 4$
C $(-4) \times (-4) \times (-4) \times (-4)$
D $(-4) \times (-4) \times (-4) \times (-4) \times (-4)$

5 Write each number in expanded form and calculate its value.

a $(-6)^3$ **b** $(-7)^4$ **c** $(-5)^4$ **d** $(-1)^6$ **e** $(-4)^5$ **f** $(-10)^5$

6 Write each number in expanded form and calculate its value.

a 0.8^2 **b** 3.4^3 **c** $(-0.5)^4$ **d** 0.3^4 **e** 0.1^5 **f** $(-0.2)^7$

7 Write each number in expanded form and calculate its value.

a $\left(\frac{1}{4}\right)^3$ **b** $\left(-\frac{7}{9}\right)^2$ **c** $\left(\frac{5}{6}\right)^4$ **d** $\left(\frac{3}{10}\right)^4$ **e** $\left(-\frac{1}{2}\right)^5$ **f** $\left(3\frac{1}{3}\right)^6$

8 Use a calculator to evaluate the following numbers.

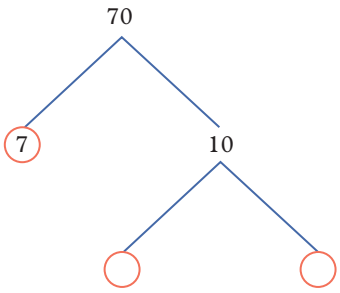
a 57^4 **b** 101^3 **c** $(-13)^7$

d 6.85^3 **e** $(-1.9)^4$ **f** 12.4^5

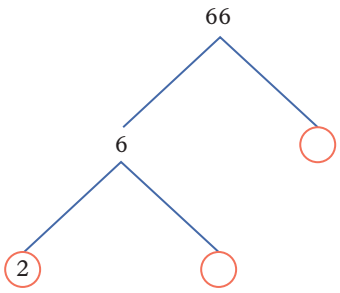
g $\left(\frac{18}{23}\right)^3$ **h** $\left(\frac{7}{11}\right)^6$ **i** $\left(-\frac{3}{5}\right)^9$

9 Complete each factor tree, then write the prime factorisation of each composite number in exponent form.

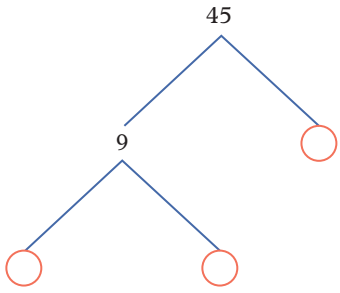
a



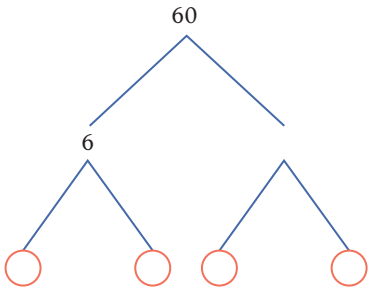
b



c



d



WE 4.1B **10** Use a factor tree to express each number as the product of its prime factors. Write your answer in exponent form.

a 75 **b** 48 **c** 90 **d** 126 **e** 288 **f** 1092

Exercise 4.1B: Problem solving and reasoning

11 Use the correct order of operations to complete the following calculations.

a $2^5 + 8^3 \div 4$ **b** $5^4 \times 0.2 + 3^4$ **c** $5.8 - (1.2)^2 + (0.1)^4$ **d** $\frac{7}{10} + \left(\frac{2}{5}\right)^3 \times 6\frac{1}{4}$

12 By collecting the like terms, write each of these multiplications in exponent form.

a $(-5) \times (-5) \times (-5) \times (-9) \times (-9) \times (-9) \times (-9)$

b $(-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times 3 \times 3 \times 3$

c $7 \times 7 \times (-6) \times (-6) \times (-6) \times (-6) \times (-6) \times (-6)$

d $(-8) \times (-8) \times (-8) \times (-8) \times (-10) \times (-10)$

13 Write each expression in expanded form and calculate its value.

a $(-2)^3 \times (-4)^2$ **b** $(-5)^2 \times 3^4$ **c** $(-10)^3 \times (-2)^5$

d $(-3)^5 \times (-1)^2$ **e** $(-6)^2 \times (-2)^3$ **f** $(-1)^4 \times 3^2$

14 This question explores powers of -1.

a Calculate each of these by first writing them in expanded form.

i $(-1)^1$ **ii** $(-1)^2$ **iii** $(-1)^3$ **iv** $(-1)^4$ **v** $(-1)^5$ **vi** $(-1)^6$

b Which of these expressions give a positive result?

c Which of these expressions give a negative result?

d Explain your observations.

e Predict the result for:

i $(-1)^{13}$ **ii** $(-1)^{32}$ **iii** $(-1)^{100}$ **iv** $(-1)^{203}$ **v** $(-1)^{188}$ **vi** $(-1)^{555}$

15 Without calculating each result, predict whether the basic numeral is positive or negative.

a $(-2)^9$ **b** $(-15)^6$ **c** $(-6)^7$ **d** $(-100)^4$

16 The following pattern details the prime factorisation of powers of 20.

$20 = 2^2 \times 5$

$20^2 = 2^4 \times 5^2$

$20^3 = 2^6 \times 5^3$

a Use the pattern to represent the prime factorisation of 20^4 and 20^5.

b Find a general rule for the prime factorisation of 20^n.

17 Use the fact that $84 = 2^2 \times 3 \times 7$ to find the prime factorisations of the following numbers.

a 168 **b** 42 **c** 28 **d** 420

18 Use the fact that $980 = 2^2 \times 5 \times 7^2$ to determine which numbers have the following prime factorisations.

a $2 \times 5 \times 7^2$ **b** $2^2 \times 5 \times 7$ **c** $2^3 \times 5 \times 7^2$ **d** $2^2 \times 5^2 \times 7^2$

19 For the six weeks of the school holidays, Sarah's parents offer to pay her $30 per week for doing some extra jobs around the house.

a How much would Sarah earn over the holidays if she agrees to this arrangement?

b Instead, Sarah suggests that her parents pay her $3 in the first week, then three times as much as the previous week in each of the following weeks.

i How much would Sarah earn in each week of the holidays under her suggested arrangement?

ii Compare the total amount that Sarah could earn over the school holidays from each arrangement. Which plan do you think Sarah's parents will agree to?

iii If Sarah's parents agree to her suggested arrangement over a longer period, express the amount that Sarah would earn in her eleventh week using exponent notation.

iv Calculate how much Sarah would earn in the eleventh week of her arrangement.

20 The size of a colony of bacteria containing 1000 cells doubles every hour.

a How large is the colony after:

i one hour **ii** two hours **iii** three hours?

b Use your answers to part **a** to complete the second row of the following table.

Number of hours	1	2	3	4	5
Size of colony					
Number of times larger than original amount		4		16	

c How many times larger than the original colony is the new colony after:

i one hour **ii** two hours **iii** three hours?

d Use your answers to part **c** to complete the third row of the table.

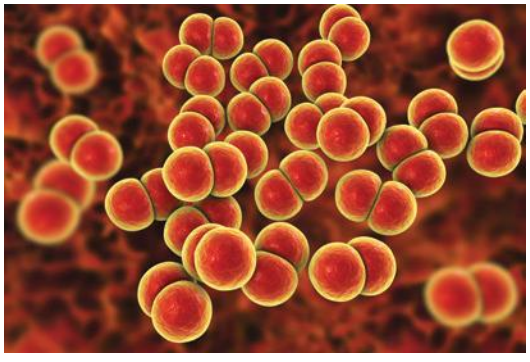
e Express each of the numbers in the third row of the table in exponent form with a base of 2.

f In exponent form, how many times larger than the original colony is the new colony after:

i 6 hours

ii 10 hours

iii 24 hours?



g Use a calculator to work out how many times larger the colony will be after 24 hours.

h Use your answer to part **g** to determine the number of cells in the colony after 24 hours.

21 Prime factorisation can be used to find the highest common factor (HCF) of two or more numbers.

- **a** Write 144 as the product of its prime factors.
- **b** Write 270 as the product of its prime factors.
- **c** Identify the prime factors that are shared by both numbers, then multiply those factors together for the HCF.
- **d** Use prime factorisation to find the HCF of the following pairs of numbers.
  - **i** 117 and 182
  - **ii** 420 and 132
  - **iii** 495 and 220
  - **iv** 432 and 405

22 Prime factorisation can also be used to find the lowest common multiple (LCM) of two or more numbers.

- **a** Write 30 as the product of its prime factors.
- **b** Write 63 as the product of its prime factors.
- **c** Multiply together the highest exponent of each prime factor from parts **a** and **b** to find the LCM of 30 and 63.
- **d** Use prime factorisation to find the LCM of each of the following pairs of numbers.
  - **i** 36 and 70
  - **ii** 18 and 96
  - **iii** 25 and 105
  - **iv** 32 and 60

23 We can also determine the number of factors a number has by using prime factorisation.

- **a** Write 150 as a product of its prime factors.
- **b** Add 1 to each of the exponents, and then multiply the exponents together. This will give the total number of factors of 150.
- **c** Write the factors of 150 to confirm your answer to part **b**.
- **d** Use prime factorisation to determine the number of factors each of the following numbers have.
  - **i** 120
  - **ii** 80
  - **iii** 44
  - **iv** 72

Exercise 4.1C: Challenge

24 A single parasite invades a host. The number of parasites doubles every 3 days. How many days until there are over 1 million parasites in the host?

25 The half-life of a chemical substance refers to the time it takes for the amount of that chemical to decrease by a half. The half-life of caffeine in your body is 6 hours. If a single shot of coffee contains c milligrams of caffeine and you drink a single-shot cup of coffee at 8 am, then a double-shot cup of coffee at 2 pm, how much caffeine is still in your body at 8 pm in terms of c?

Online resources:

- Interactive skillsheet: Exponents
- Interactive skillsheet: Prime factorisation
- Investigation: What are the odds?
- Desmos activity: Circles
- Quick quiz: 4.1

Lesson 4.2

Products and quotients of powers

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify products and quotients of numbers with the same base.

Inter-year links

Year 7 3.2 Equivalent fractions
Year 9 1.2 Products and quotients of powers
Year 10 1.1 Exponents

The exponent laws

Key content video
Products and quotients of powers

- Simplification involves reducing the complexity of an expression.
- The **exponent laws** can be used to simplify expressions involving numbers in exponent form.
- Exponent laws only apply to numbers with the *same base*.

Product of powers law

- When multiplying numbers in exponent form with the same base, add the exponents.
  → This can be generalised as $a^m \times a^n = a^{m+n}$.

$$\begin{aligned} 2^3 \times 2^5 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^{3+5} \\ &= 2^8 \end{aligned}$$

Quotient of powers law

- When dividing numbers in exponent form with the same base, subtract the second exponent from the first exponent.
  → This can be generalised as $a^m \div a^n = a^{m-n}$.
- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the exponent of the number in the denominator from the exponent of the number in the numerator.

$$\begin{aligned} 2^5 \div 2^3 &= (2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2) \\ &= 2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \div (\cancel{2} \times \cancel{2} \times \cancel{2}) \\ &= 2^{5-3} \\ &= 2^2 \end{aligned}$$

$$\begin{aligned} \frac{2^5}{2^3} &= \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\ &= \frac{2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} \\ &= 2^{5-3} \\ &= 2^2 \end{aligned}$$

Worked example 4.2A Simplifying products using an exponent law

Use an exponent law to write the following products as a single power with the given base.

a $4^6 \times 4^4$ **b** 9×9^7

THINK

1 Check that the bases are the same.

2 Write the base and add the exponents. Remember that $9 = 9^1$.

WRITE

a $4^6 \times 4^4 = 4^{(6+4)}$
$= 4^{10}$

b $9 \times 9^7 = 9^{(1+7)}$
$= 9^8$

Worked example 4.2B Simplifying quotients using an exponent law

Use an exponent law to write the following quotients as a single power with the given base.

a $3^{11} \div 3^{10}$ **b** $\frac{8^5}{8^2}$

THINK

a 1 Check that the bases are the same.
2 Write the base and subtract the second exponent from the first exponent. Simplify the answer by leaving out the exponent of 1.

b 1 Check that the bases are the same.
2 Write the base and subtract the exponent of the number in the denominator from the exponent of the number in the numerator.

WRITE

a $3^{11} \div 3^{10} = 3^{(11-10)}$
$= 3^1$
$= 3$

b $\frac{8^5}{8^2} = 8^{(5-2)}$
$= 8^3$

Helpful hints

✔ 'Simplify' and 'evaluate' are two different command terms:
→ To simplify a calculation means to reduce the complexity of the equation.
→ To evaluate a calculation means to convert it into a basic numeral.

Learning pathways

1–8, 9–11(a–c), 13	5–8, 9–11(d–f), 12, 14(a, b)	5–7, 9–11(d–f), 12, 14

ANS p541

Exercise 4.2A: Understanding and fluency

WE 4.2A **1** Use an exponent law to write the following products as a single power with the given base.

a $3^3 \times 3^2$ **b** $4^5 \times 4^3$ **c** $6^2 \times 6^4$
d $5^3 \times 5^6$ **e** $8^2 \times 8$ **f** 2×2^4

2 Evaluate each of your answers to question **1** by converting each power into a basic numeral.

WE 4.2B **3** Use an exponent law to write the following quotients as a single power with the given base.

a $9^{14} \div 9^{11}$ **b** $4^6 \div 4^5$ **c** $6^4 \div 6$
d $\frac{4^7}{4^2}$ **e** $\frac{10^8}{10^5}$ **f** $\frac{6^{11}}{6^7}$

4 Evaluate each of your answers to question **3**.

5 Use an exponent law to first simplify, then evaluate each of the following expressions.

a $2^5 \times 2^6$ **b** $5^2 \times 5^4$ **c** $(-3)^4 \times (-3)^3$ **d** $4^6 \times 4^2$
e $5^7 \div 5^4$ **f** $\frac{(-3)^9}{(-3)^3}$ **g** $9^6 \div 9^3$ **h** $2^{12} \div 2^5$
i $(-6)^3 \times (-6)^2$ **j** $\frac{8^5}{8^4}$ **k** $10^4 \times 10^5$ **l** $\frac{7^8}{7^6}$

6 The exponent laws also apply to non-integer bases. Use an exponent law to write each of the following expressions as a single power with the given base.

a $1.5^3 \times 1.5^6$ **b** $0.7^4 \times 0.7^2$ **c** $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4$ **d** $\left(\frac{7}{5}\right)^6 \times \left(\frac{7}{5}\right)^8$

e $(-9.2)^8 \div (-9.2)^5$ **f** $0.63^{11} \div 0.63^4$ **g** $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^2$ **h** $\left(-\frac{5}{2}\right)^9 \div \left(-\frac{5}{2}\right)^7$

Exercise 4.2B: Problem solving and reasoning

7 Determine the value of the missing exponent in the equations below.

a $7^5 \times 7^{\square} = 7^{15}$ **b** $6^{\square} \times 6 = 6^6$ **c** $2^{13} \div 2^{\square} = 2^5$

d $\frac{9^7}{9^{\square}} = 9^3$ **e** $\frac{9^{\square}}{9^7} = 9^{14}$ **f** $\frac{12^{\square}}{12^{15}} = 144$

8 Complete the following equations by filling in the missing exponents.

a $5^2 \times 5^5 \times 5^4 = 5^7 \times 5^4$
$= 5^{\square}$

b $20^8 \times 20 \times 20^{10} = 20^{\square} \times 20^{10}$
$= 20^{\square}$

c $8^{32} \div 8^7 \div 8^{10} = 8^{25} \div 8^{\square}$
$= 8^{\square}$

d $7^{24} \div 7^6 \div 7 = 7^{\square} \div 7^{\square}$
$= 7^{\square}$

9 Use an exponent law to write these expressions as a single power with the given base.

a $2^5 \times 2^4 \times 2^3$ **b** $3^3 \times 3 \times 3^2$ **c** $10^3 \times 10^2 \times 10^5$

d $4^2 \times 4^4 \times 4^3$ **e** $5^7 \times 5^3 \times 5^6$ **f** $9 \times 9^5 \times 9^8$

10 Use an exponent law to write these expressions as a single power with the given base.

a $(6^{10} \div 6^5) \div 6^3$ **b** $(7^{12} \div 7^2) \div 7^6$ **c** $(12^{16} \div 12^9) \div 12^6$

d $(8^{10} \div 8^3) \div 8^3$ **e** $(5^{15} \div 5^6) \div 5^8$ **f** $(11^{11} \div 11) \div 11^5$

11 Use exponent laws to write these calculations as single powers with the given base.

a $(3^3 \times 3^5) \div 3^6$ **b** $(5^2 \times 5^4) \div 5^3$ **c** $(3^9 \div 3^4) \times 3^2$

d $(7^7 \div 7^6) \times 7^4$ **e** $(11^2 \times 11^3 \times 11^4) \div 11^5$ **f** $(13^4 \times 13^4 \times 13^8) \div 13^{10}$

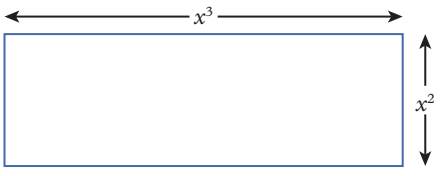
12 Light-years are a distance unit defined by the distance that light travels in one year. One light-year is approximately 10^{13} km. Icarus is one of the furthest stars from Earth to have been detected so far.
If Icarus is 10^{23} km from Earth, how many light-years is Icarus from Earth? Express your answer in exponent form.

13 Identify and explain the mistake made in each of the following equations, then give the correct value of the right-hand side in each instance.

a $5^8 \div 5^2 = 5^4$ **b** $2^3 \times 2^3 = 2^9$ **c** $\frac{4^7}{4^7} = 4^1$ **d** $9^8 \div 9^{10} = 9^2$

Exercise 4.2C: Challenge

14 The exponent laws can also be applied to terms containing pronumerals.



a Express the area of this rectangle as a product.

b Simplify your answer to part **a**.

c If the area of the rectangle is 7776 mm^2, what are the dimensions of the rectangle?

Online resources:

Interactive skillsheet Products and quotients of powers

Worksheet Multiplying and dividing using exponent form

Investigation Growing cubes

Quick quiz 4.2

Lesson 4.3

Raising exponents and the zero exponent

Learning intentions

By the end of this lesson you will be able to ...

✔ use the power of a power law, and use the zero exponent.

Inter-year links

Year 9 1.3 Raising exponents and the zero exponent

Year 10 1.1 Exponents

Power of a power law

Key content video: Raising exponents and the zero exponent

- A number in exponent form raised to another exponent can be simplified by multiplying the exponents.
  → This can be generalised as $(a^m)^n = a^{m \times n}$.

$$(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$$
$$= 2^{(3+3+3+3+3)}$$
$$= 2^{(3 \times 5)}$$
$$= 2^{15}$$

Expressions involving brackets

- When a product or quotient is in brackets, every number inside the brackets should have its exponent multiplied by the exponent outside the brackets.

$$(2 \times 3)^5 = 2^5 \times 3^5$$
$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$$

The zero exponent

- Excluding 0, any number raised to the power of 0 is equal to 1.
  → This can be generalised as $a^0 = 1$.

$$2^0 = 1$$

Worked example 4.3A Raising an exponent by another exponent

Use an exponent law to write $(3^4)^5$ as a single power with the given base.

THINK

Write the base and multiply the exponents.

WRITE

$$(3^4)^5 = 3^{(4 \times 5)}$$
$$= 3^{20}$$

Worked example 4.3B Distributing exponents across brackets

Simplify the following expressions by removing the brackets.

a $(4 \times 7)^2$ **b** $\left(\frac{5}{2}\right)^4$

THINK

Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that base numbers that do not have a stated exponent have an exponent of 1, so $4 = 4^1$.

WRITE

a $(4 \times 7)^2 = 4^{(1 \times 2)} \times 7^{(1 \times 2)}$
$= 4^2 \times 7^2$

b $\left(\frac{5}{2}\right)^4 = \frac{5^{(1 \times 4)}}{2^{(1 \times 4)}} = \frac{5^4}{2^4}$

Worked example 4.3C Evaluating calculations involving the zero exponent

Evaluate:

a 23^0 **b** $8^2 \div 8^2$

THINK

a Any number, excluding 0, with an exponent of 0 is equal to 1.

b When dividing numbers in exponent form with the same base, subtract the second exponent from the first exponent.

WRITE

a $23^0 = 1$

b $8^2 \div 8^2 = 8^{(2-2)}$
$= 8^0$
$= 1$

Helpful hints

- ✔ Take care not to mix up the exponent laws.
  - → across a multiplication sign, add exponents
  - → across a division sign, subtract exponents
  - → across brackets, multiply exponents
- ✔ Remember that $2^0 = 1$, *not* 0.

Exponent law	Example
Product of powers law	$2^5 \times 2^3 = 2^{5+3}$
Quotient of powers law	$2^5 \div 2^3 = 2^{5-3}$
Power of a power law	$(2^5)^3 = 2^{5\times 3}$

Learning pathways

1–7, 8(a–c), 12	1, 2, 4–6, 8, 10–13	1, 2, 5, 8, 9, 11–15

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Exercise 4.3A: Understanding and fluency

WE 4.3A **1** Use an exponent law to write the following expressions as a single power with the given base.

a $(6^4)^5$ **b** $(2^6)^2$ **c** $(7^4)^3$
d $(11^2)^4$ **e** $(8^5)^3$ **f** $(3^7)^2$

WE 4.3B **2** Simplify the following expressions by removing the brackets.

a $(5 \times 9)^3$ **b** $\left(\frac{3}{4}\right)^6$ **c** $\left(\frac{9}{5}\right)^4$
d $(11 \times 4)^7$ **e** $\left(\frac{7}{10}\right)^3$ **f** $(3 \times 8)^9$

WE 4.3C **3** Evaluate the following powers.

a 4^0 **b** $(-15)^0$ **c** 144^0

4 Use exponent laws to first simplify, then evaluate the following expressions.

a 3.2^0 **b** $(2 \times 3)^0$ **c** $\left(\frac{7}{8}\right)^0$
d $9^3 \div 9^3$ **e** $\frac{8^5}{8^5}$ **f** $\frac{4}{4}$

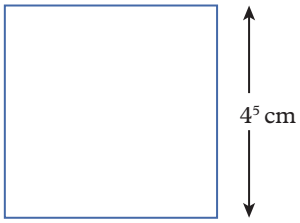
5 Use exponent laws to simplify each of the following expressions by removing the brackets.

a $(0.5^4)^3$ **b** $(1.2^8)^0$ **c** $(3.8^5)^5$
d $(2^5 \times 2.5^2)^4$ **e** $(0.7^7 \times 5.4^3)^0$ **f** $(4.8^0 \times 6.6^6)^3$

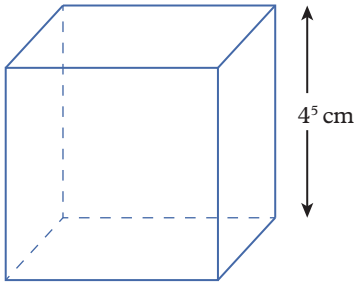
Exercise 4.3B: Problem solving and reasoning

6 Writing your answer in exponent form, determine the value of:

a the area of this square



b the volume of this cube.



7 Determine the value of the missing exponents in the following equations.

a $(7^3)^{\square} = 7^{18}$ **b** $(9^{\square})^5 = 9^{20}$ **c** $(8^4)^{\square} = 1$

d $(6 \times 8)^{\square} = 6^5 \times 8^5$ **e** $\left(\frac{4}{15}\right)^{\square} = \frac{4^{10}}{15^{10}}$ **f** $\left(\frac{2}{5}\right)^{\square} = 1$

8 Complete the following equations by filling in the missing exponent.

a $(2^2)^{\square} = 64$ **b** $(2 \times 3)^{\square} = 36$ **c** $\left(\frac{1}{5}\right)^{\square} = \frac{1}{125}$

d $(4^{\square})^3 = 64$ **e** $(3 \times 3)^{\square} = 81$ **f** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

9 Without evaluating, list the following numbers in ascending order.

3^{14}, $(3^2)^5$, $(3^4)^3$, $(3^{15})^0$, $((3^2)^3)^3$

10 Consider the expression $\frac{2^5}{2^5}$.

a Calculate the value of the expression.

b Use an exponent law to simplify the expression. Leave your answer in exponent form.

c Use this method to write a calculation showing that $15^0 = 1$.

11 Consider the sequence 3^5, 3^4, 3^3, 3^2, 3^1.

a Calculate the values of the numbers in this sequence.

b Use this sequence to explain why $3^0 = 1$.

12 Decide whether the following numerical statements are true or false.

a $(5^3)^4 = (5^2)^6$ **b** $(2^4)^6 = (2^8)^3$

c $(4 \times 5)^6 = (20^3)^3$ **d** $(2 \times 3)^8 = (6^2)^4$

Exercise 4.3C: Challenge

13 Find the value of x in the following equations.

a $(7^x)^5 = 7^{20}$

b $(7^x \times 7^x)^5 = 7^{20}$

14 Show that $(2^3)^6 = 4^9$.

15 If $x^y = y^x$ and $x \neq y$, what are the potential, positive integer values of x and y?

Online resources:

- **Interactive skillsheet** Raising exponents and the zero exponent
- **Investigation** Bert and Ernie's exponent laws
- **Quick quiz** 4.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

4.1 **1** Write each number in expanded form and calculate its value.

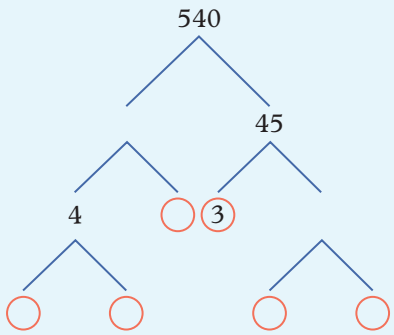
a 2^4 **b** 3^4 **c** $(-5)^3$ **d** $(-10)^6$

4.1 **2** Write each number in expanded form and calculate its value.

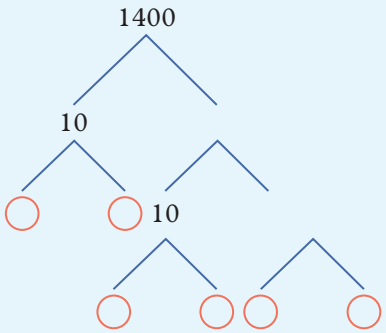
a 2.5^4 **b** $(-0.7)^5$ **c** $\left(\frac{2}{3}\right)^3$ **d** $\left(-\frac{1}{5}\right)^4$

4.1 **3** Complete each factor tree, then write the prime factorisation of each number in exponent form.

a



b



4.1 **4** Use a factor tree to express each number as the product of its prime factors. Write your answer in exponent form.

a 24 **b** 72 **c** 195 **d** 350

4.1 **5** A scientist studies a colony of bacteria in a petri dish and notes the following information.

Number of minutes	0	1	2	3	4	5
Size of colony	10	30	90			
Number of times as large as original amount	1	3	9			

a Complete the table.

b Express each of the numbers in the third row of the table in exponent form with a base of 3.

c In exponent form, how many times larger than the original colony will the new colony be after:

i 10 minutes **ii** 1 hour **iii** 24 hours?

d Use a calculator to determine how many bacteria will be in the colony after 10 minutes.

4.2 **6** Use an exponent law to write the following products as a single power with the given base.

a $2^5 \times 2^3$ **b** $7^{13} \times 7^7$ **c** $11^5 \times 11$ **d** $9^9 \times 9^9$

4.2 **7** Use an exponent law to write the following quotients as a single power with the given base.

a $3^{10} \div 3^6$ **b** $10^{11} \div 10$ **c** $\frac{14^6}{14^2}$ **d** $\frac{8^{17}}{8^9}$

4.2 **8** Simplify then evaluate each of the following expressions.

a $2^2 \times 2^4$ **b** $5^3 \times 5$ **c** $6^7 \div 6^5$ **d** $\frac{10^{13}}{10^6}$

4.3 **9** Use an exponent law to write the following expressions as a single power with the given base.

a $(4^3)^6$ **b** $(9^2)^3$ **c** $(13^{10})^4$ **d** $(10^{12})^8$

4.3 **10** Simplify the following expressions by removing the brackets.

a $(3 \times 5)^6$ **b** $\left(\frac{2}{7}\right)^4$ **c** $(9 \times 15)^5$ **d** $\left(\frac{9}{8}\right)^{10}$

4.3 **11** Evaluate the following expressions.

a 16^0 **b** $(4 \times 5)^0$ **c** $8^4 \div 8^4$ **d** $\left(\frac{10}{3}\right)^0$

Lesson 4.4

Combining the exponent laws

Learning intentions

By the end of this lesson you will be able to ...

✔ apply a combination of the exponent laws to simplify complex calculations.

Inter-year links

Support	Order of operations
Year 7	1.8 Order of operations
Year 10	1.1 Exponents

Using multiple exponent laws to simplify complex expressions

Key content video
Combining the exponent laws

- Multiple exponent laws can be used to simplify complex expressions.
- The order of operations also applies to the simplification of expressions containing exponents. Calculations contained in grouping symbols should be simplified first.

	Exponent law
Product of powers law	$a^m \times a^n = a^{m+n}$
Quotient of powers law	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$
Power of a power law	$(a^m)^n = a^{m \times n}$
Zero exponent	$a^0 = 1$
Distributing exponents	$(a \times b)^m = a^m \times b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Worked example 4.4A Using exponent laws to simplify complex expressions

Use exponent laws to write these expressions as a single power with the given base.

a $(2^3)^3 \times (2^2)^{11}$ **b** $\frac{3^7 \times 3^2}{3^5}$

THINK

a 1 To raise the exponent, write each base and multiply the exponents.

2 Check that the bases are the same. To simplify the product, write the base and add the exponents.

b 1 The fraction line groups the terms in the numerator and the denominator, so the expressions in the numerator and denominator should be simplified before dividing the numerator by the denominator. The bases in the numerator are the same, so to simplify the numerator, write the base and add the exponents.

2 The bases in the numerator and the denominator are the same, so to simplify the fraction, write the base and subtract the exponent of the number in the denominator from the exponent of the number in the numerator.

WRITE

a $(2^3)^3 \times (2^2)^{11} = 2^{(3 \times 3)} \times 2^{(2 \times 11)}$
$= 2^9 \times 2^{22}$
$= 2^{(9+22)}$
$= 2^{31}$

b $\frac{3^7 \times 3^2}{3^5} = \frac{3^{(7+2)}}{3^5}$
$= \frac{3^9}{3^5}$
$= 3^{(9-5)}$
$= 3^4$

Worked example 4.4B Simplifying calculations by writing numbers in exponent form

Consider the expression $\frac{1024}{32 \times 8}$.

a Write each of the numbers in the expression as a prime factor in exponent form.

b Simplify the expression using the exponent laws.

c Evaluate the expression.

THINK

a Rewrite each number in exponent form.

b Rewrite the fraction. All numbers are expressed in exponent form with a base of 2, so the expression can be simplified using the exponent laws. Simplify the denominator by adding the exponents, then simplify the fraction by subtracting the exponent of the number in the denominator from the exponent of the number in the numerator.

c Calculate the value of the expression by performing the multiplication.

WRITE

a $8 = 2^3, 32 = 2^5, 1024 = 2^{10}$

$$\frac{1024}{32 \times 8} = \frac{2^{10}}{2^5 \times 2^3}$$

b $$\frac{2^{10}}{2^5 \times 2^3} = \frac{2^{10}}{2^{(5+3)}}$$
$$= \frac{2^{10}}{2^8}$$
$$= 2^{(10-8)}$$
$$= 2^2$$

c $2^2 = 2 \times 2$
$= 4$

Helpful hints

Always check that the bases are the same before simplifying products and quotients of numbers in exponent form!

Learning pathways

1–5, 6(a, c), 7(a, b), 8, 11	1–6, 7(c, d, e), b, c 8(b, d, f, h), 9–11, 13	3–6, 7(c, d, e), 8(b, d, f, h), 9–14

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Exercise 4.4A: Understanding and fluency

WE 4.4A

1 Use exponent laws to write these expressions as a single power with the given base.

a $(3^2)^3 \times (3^4)^2$ **b** $(9^5)^4 \times (9^3)^7$ **c** $(12^3)^{12} \times (12^4)^0$ **d** $(8^0)^5 \times (8^0)^{21}$

2 Use exponent laws to write these expressions as a single power with the given base.

a $\frac{4^5 \times 4^3}{4^6}$ **b** $\frac{7^6 \times 7^6}{7^{10}}$ **c** $\frac{6^4 \times 6^7}{6^0}$ **d** $\frac{11^8 \times 11^3}{11^5}$

e $\frac{5^6 \times 5^{10}}{5^9 \times 5^3}$ **f** $\frac{8^7 \times 8^2}{8 \times 8^5}$ **g** $\frac{2^8 \times 2^6}{2^0 \times 2^4}$ **h** $\frac{4^{12} \times 4^6}{4^{12} \times 4^3}$

3 Write these expressions as a single power with the given base.

a $\frac{(3^2)^4 \times 3^5}{3^4 \times 3^3}$ **b** $\frac{3^5 \times 3^6 \times 3^7}{3^{11} \times 3^2}$ **c** $\frac{4^{14} \times 4^8}{4 \times (4^5)^0}$ **d** $\frac{2^{13} \times (2^4)^3 \times 2^5}{2^{12} \times 2^{10}}$

4 Complete this table.

Base 2	**Exponent form**	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	Basic numeral	2	4	8							
Base 3	**Exponent form**	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	Basic numeral	3	9	27							

WE 4.4B **5** Use your completed table from question **4** to help you answer this question.

a Write each of the numbers in the following expressions as a prime factor in exponent form.

i 32×16 **ii** $1024 \div 256$ **iii** $59\,049 \div 729$ **iv** $\dfrac{128 \times 32}{16}$

v $\dfrac{27 \times 243}{2187}$ **vi** $64 \times 256 \div 512$ **vii** $\dfrac{512 \times 64}{128 \times 16}$ **viii** $\dfrac{128 \times 1024}{256 \times 64}$

b Simplify the expressions using the exponent laws.

c Evaluate the expressions.

6 Evaluate each of the following expressions without a calculator.

a $512 \times 32 \div 64$ **b** $9 \times 243 \div 27$ **c** $\dfrac{32 \times 256}{8 \times 16}$ **d** $\dfrac{6561 \times 19\,683}{729 \times 81}$

Exercise 4.4B: Problem solving and reasoning

7 Use exponent laws to simplify, then evaluate the following expressions.

a $\dfrac{3^8 \times 2^6}{3^6 \times 2^3}$ **b** $\dfrac{6^4 \times 4^7}{6^3 \times 4^5}$ **c** $\dfrac{8^0 \times 9^8 \times 10^5}{9^7 \times 10^2}$ **d** $\dfrac{10^7 \times 8^4}{10 \times 8^2 \times 7^0}$

8 Fill in the missing exponents in the following equations.

a $(2^3)^4 \times (2^4)^{\square} = 2^{20}$ **b** $\dfrac{6^7 \times 6^7}{6^{\square}} = 6^2$ **c** $\dfrac{3^{\square} \times 3^2}{3^6} = 1$ **d** $(15^{\square})^8 \times (15^{10})^2 = 15^{20}$

e $\dfrac{(5^2)^4}{5^{\square}} = 5^3$ **f** $\dfrac{4 \times 4^{16}}{4^7 \times 4^{\square}} = 4$ **g** $(9^{\square} \times 9^4)^2 = 9^{14}$ **h** $\dfrac{(11^{\square})^6 \times 11^6}{11^8 \times 11^7} = 11^9$

9 Explain why $(-5)^2 \neq -(5)^2$, but $(-5)^3 = -(5)^3$.

10 Without evaluating, decide whether the following numerical statements are true or false.

a $(-9)^4 \neq -(9)^4$ **b** $(-8)^5 = -(2^3)^5$ **c** $(-2 \times 3)^9 \neq -(6^3)^3$ **d** $(-3 \times 5)^{16} = -(15^3)^8$

11 Use the order of operations to evaluate the following expressions.

a $(2^3 \times 2^2) + (4^3)^0$ **b** $(6^3 \div 6^2 \times 3^0)^2$ **c** $\dfrac{3^5 \div 3^3}{4^6}$

12 We can use the distributive law to expand expressions in exponent form involving brackets.
For example, $2^2(2^3 + 2^5) = 2^2 \times 2^3 + 2^2 \times 2^5$.

i Use the distributive law to expand, and then simplify the following expressions.

ii Evaluate the expressions.

a $3^3(3^2 + 3^0)$ **b** $5^2(5^4 + 5^2)$ **c** $7^3(7^3 + 7^0)$

Exercise 4.4C: Challenge

13 Use exponent laws to find the value of x in $\dfrac{(9^x)^3 \times (9^4)^x}{(9^x)^2} = 9^{45}$.

14 Evaluate $\dfrac{96^4 \times 1296^3}{864^5}$ by first writing each base as the product of its prime factors.

Online resources:


Interactive skillsheet
Exponent laws


Investigation
Bases of 4 and 5


Quick quiz
4.4

Lesson 4.5

Roots

Learning intentions

By the end of this lesson you will be able to ...

✔ evaluate roots of rational numbers.

Inter-year links

Year 7	1.7 Exponents and square roots
Year 9	1.6 Rational and irrational numbers
Year 10	2.1 Surds [10A]

Roots

Key content video
Roots and surds

- The **n^{th} root** of a number, x, is the number whose n^{th} power is x.
- Finding the n^{th} root of a number and raising a number to the n^{th} power are inverse operations.
- Roots are indicated by using the **radical sign**, $\sqrt{}$. If a number appears at the top left of the radical sign, this shows which root is being specified. For example, $\sqrt[4]{}$ is specifying the fourth root.
  → If a root has no stated number at the top left of the radical sign, $\sqrt{}$, then it is specifying the second root, which is the square root.
- The square root, $\sqrt{}$, of a number is the number that, when squared, results in the original number.
  For example, $\sqrt{16} = 4$ because $4^2 = 16$.
- The cube root, $\sqrt[3]{}$, of a number is the number that, when cubed, results in the original number.
  For example $\sqrt[3]{8} = 2$ because $2^3 = 8$.
- When a number appears directly before a root, then the root is being multiplied by that number.
  For example, $4\sqrt[3]{2} = 4 \times \sqrt[3]{2}$.

Surds

- A root of a number that produces an irrational number (a non-terminating, non-recurring decimal) is called a **surd**.
  For example, $\sqrt{2}$ is a surd because $\sqrt{2} = 1.414\,21...$
  → A calculator can be used to determine whether a root is a surd or not.

Exact and approximate solutions

- When solving problems involving square or cube roots, solutions that contain surds can be given as either an approximate value (by rounding) or as an exact value (by keeping the surd in the solution).

Worked example 4.5A Identifying surds

Use a calculator to determine whether the following roots are surds or not.

a $\sqrt{0.36}$ **b** $\sqrt{1.25}$

THINK

a $\sqrt{0.36} = 0.6$, which is a terminating decimal, so $\sqrt{0.36}$ is a rational number.

b $\sqrt{1.25} = 1.118\,03...$, which is a non-terminating, non-recurring decimal, so $\sqrt{1.25}$ is an irrational number.

WRITE

a $\sqrt{0.36}$ is not a surd.

b $\sqrt{1.25}$ is a surd.

Helpful hints

- ✔ As with indices, take care when writing cube roots. In a cube root, the '3' should be small and sit high up on the shoulder of the radical sign to avoid confusion between $\sqrt[3]{8}$ and $3\sqrt{8}$.
- ✔ Remember that if no number is specified to the top left of the root sign, it stands for 'square root'.

  For example, $\sqrt{4} = 2$.
- ✔ A radical sign is a grouping symbol, so operations contained within a radical sign should be completed before applying the root.

  For example: $\sqrt{16 + 9} = \sqrt{25}$
  $= 5$.

Learning pathways

1–5, 7, 8, 11, 12(a, b) | 2–5, 6–9, 11, 12, 15 | 2–6, 8, 10, 12–15

ANS p542

Exercise 4.5A: Understanding and fluency

1 Use a calculator to evaluate the following square roots.

a $\sqrt{196}$ **b** $\sqrt{256}$ **c** $\sqrt{1089}$
d $\sqrt{26\,244}$ **e** $\sqrt{2.25}$ **f** $\sqrt{53.29}$
g $\sqrt{67.24}$ **h** $\sqrt{475.24}$ **i** $\sqrt{25.3009}$

WE 4.5A **2** Use a calculator to determine whether the following roots are surds or not.

a $\sqrt{144}$ **b** $\sqrt{20}$ **c** $\sqrt{54.76}$
d $\sqrt{331\,776}$ **e** $\sqrt{8.3}$ **f** $\sqrt{0.49}$

3 Use the given facts to evaluate the following roots without a calculator.

a $\sqrt[4]{16}$, given that $2^4 = 16$ **b** $\sqrt[4]{625}$, given that $5^4 = 625$
c $\sqrt[4]{81}$, given that $3^4 = 81$ **d** $\sqrt[4]{10\,000}$, given that $10^4 = 10\,000$

4 Calculators can be used to determine whether a root is a surd or not.

a Use a calculator to determine whether the following roots are surds or not.
i $\sqrt[3]{3375}$ **ii** $\sqrt[3]{68.921}$ **iii** $\sqrt[3]{99}$
iv $\sqrt[4]{2401}$ **v** $\sqrt[4]{73}$ **vi** $\sqrt[4]{0.1}$

b Write the surds from part **a** as basic numerals, correct to four decimal places.

5 To evaluate the square root of a fraction, take the square root of the numerator and the denominator individually.

For example, $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$.

Evaluate the following fractional square roots.

a $\sqrt{\frac{1}{16}}$ **b** $\sqrt{\frac{1}{64}}$ **c** $\sqrt{\frac{4}{25}}$ **d** $\sqrt{\frac{9}{49}}$ **e** $\sqrt{\frac{81}{100}}$ **f** $\sqrt{\frac{36}{121}}$

6 Evaluate the following expressions, giving your answers as:

i exact values **ii** approximate values correct to two decimal places.

a $\sqrt{3^3 + 2^2}$ **b** $\sqrt{5 \times 4^3 - 2}$ **c** $\sqrt[3]{39 - 3^3}$ **d** $\sqrt[3]{(3^2)^2}$

Exercise 4.5B: Problem solving and reasoning

7 Taking the square root of a number is the inverse operation of squaring a number.

a Write $(2^3)^2$ as a single power with a base of 2.

b Use your answer to part **a** to write $\sqrt{2^6}$ as a single power with a base of 2.

c Evaluate $\sqrt{2^6}$.

8 Identify and explain the errors in the following calculations. Then change the right-hand side to make the equation a true statement.

a $\sqrt{16} = 8$ **b** $\sqrt[3]{64} = 8$

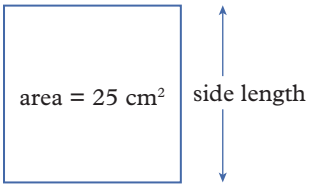
9 Elle uses a calculator to evaluate $\sqrt{50}$ as 7.071 067 811. She says that $\sqrt{50}$ is a rational number as it can be written as $\frac{7\,071\,067\,811}{1\,000\,000\,000}$. Explain why Elle is incorrect by referring to the number of digits a calculator displays.

10 Squaring a number and taking the square root of a number are inverse operations.

a What is the value of $(\sqrt{2})^2$?

b Use your answer to part **a** to write $(\sqrt{2})^3$ in the form $a\sqrt{b}$.

11 Consider the square on the right with an area of 25 cm^2.



a Explain how you can calculate the side length of the square.

b Calculate the side length of a square with area:

i 169 cm^2 **ii** 200 m^2 **iii** 268.96 km^2.

c Which of your answers is a surd?

12 Raising a negative number to an odd power results in a negative number.

a If $(-3)^3 = -27$, what is $\sqrt[3]{-27}$? **b** If $(-2)^5 = -32$, what is $\sqrt[5]{-32}$?

c Explain why $\sqrt{-4}$ is not a positive or negative number.

13 Consider the following pairs of expressions.

i $\sqrt{4} \times \sqrt{16}$ and $\sqrt{64}$ **ii** $\sqrt{9} \times \sqrt{36}$ and $\sqrt{324}$ **iii** $\sqrt{25} \times \sqrt{16}$ and $\sqrt{400}$

a Calculate the value of each expression.

b Use your answers to part **a** to write an equation linking $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{ab}$ where $a, b > 0$.

14 Use your answer from question **13** part **b** to simplify and evaluate the following square roots without using a calculator.

a $\sqrt{324}$ **b** $\sqrt{676}$ **c** $\sqrt{1936}$ **d** $\sqrt{5184}$

Exercise 4.5C: Challenge

15 Prime factorisation can be used to determine if a number is a perfect square.

For example, 196 is a perfect square because the prime factors of 196 can be divided into two identical groups.

$$\begin{aligned} 196 &= 2 \times 2 \times 7 \times 7 \\ &= 2 \times 7 \times 2 \times 7 \\ &= 14 \times 14 \\ &= 14^2 \end{aligned}$$

a Use prime factorisation to determine whether the following numbers are perfect squares, and find the square root of each perfect square.

i 168 **ii** 289 **iii** 576 **iv** 786

b Use prime factorisation to determine whether the following numbers are perfect cubes, and find the cube root of each perfect cube.

i 216 **ii** 512 **iii** 730 **iv** 1331

Online resources:

- **Interactive skillsheet** Roots
- **Worksheet** Squares and square roots of fractions
- **Investigation** Estimating square roots and cube roots
- **Quick quiz** 4.5

Lesson 4.6

Review: Exponents and roots

Module summary

Terminology

base exponent

expanded form

exponent form $3^4 = 3 \times 3 \times 3 \times 3$

$= 81$

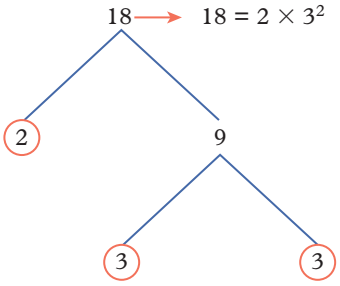
basic numeral

3^2 can be read as '3 to the power of 2', '3 squared' or 'the square of 3'.

4^3 can be read as '4 to the power of 3', '4 cubed' or 'the cube of 4'.

Prime factorisation

The prime factorisation of an integer is the product of the primes that make up that integer.



Exponent laws

	Exponent law	Example
Product of powers law	$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^{5+3} = 2^8$
Quotient of powers law	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$	$\frac{2^8}{2^5} = 2^8 \div 2^5 = 2^{8-5} = 2^3$
Power of a power law	$(a^m)^n = a^{m \times n}$	$(2^3)^5 = 2^{3 \times 5} = 2^{15}$
Zero exponent	$a^0 = 1$	$8^2 \div 8^2 = 8^{(2-2)} = 8^0 = 1$

Combining exponent laws

Multiple exponent laws can be used to simplify complex calculations.

$$\frac{3^7 \times 3^2}{3^5} = \frac{3^{(7+2)}}{3^5} = \frac{3^9}{3^5} = 3^{(9-5)} = 3^4$$

$$(2^3)^3 \times (2^2)^{11} = 2^{(3 \times 3)} \times 2^{(2 \times 11)} = 2^9 \times 2^{22} = 2^{(9+22)} = 2^{31}$$

Expressions involving brackets

Multiply every exponent inside the brackets by the exponent outside the brackets.

$$(a \times b)^m = a^m \times b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Roots

Finding the n^{th} root of a number and raising a number to the n^{th} power are inverse operations.

$\sqrt{\ }$ = square root $\sqrt[n]{\ }$ = n^{th} root

If $\sqrt[n]{x} = y$ then $y^n = x$

$4\sqrt[3]{2} = 4 \times \sqrt[3]{2}$

Surds

- Irrational numbers are non-recurring and non-terminating decimals.
- A root that produces an irrational number is a surd.

$\sqrt{2} = 1.41421...$

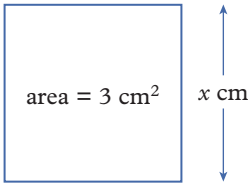
Exact and approximate solutions

Solutions that contain surds can be given as exact or approximate values.

For example, when finding the side length x cm of a square with an area of 3 cm^2:

$\rightarrow x = \sqrt{3}$ (exact value)

$\rightarrow x \approx 1.73$ (approximate value).



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Review questions 4.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- base
- basic numeral
- composite number
- exponent
- expanded form
- exponent form
- exponent laws
- exponent notation
- factor trees
- irrational number
- n^{th} root
- order of operations
- power
- power of a power law
- prime factorisation
- prime number
- product of powers law
- quotient of powers law
- radical sign
- rational number
- simplification
- surd

1 When an expression is written using exponent notation, in what form is it?

2 What can be used to help determine the prime factorisation of an integer?

3 If a root is an irrational number, what is it also known as?

4 Identify the key terms being referenced in each of these definitions.

a the correct sequence of conducting mathematics operations

b a non-terminating, non-repeating decimal

5 Using an example, provide a definition in your own words for the following key terms.

a quotient of powers law

b composite number

6 Complete the following sentences using words from the key terms list.

a The _____ is the number that is being repeatedly multiplied when a number is written in exponent form.

b _____ involves reducing the complexity of an expression.

Review questions 4.6B: Multiple choice

4.1 **1** 3^4 is equal to:

A 81 **B** 64 **C** 34 **D** 12

4.1 **2** $(-4)^3$ is equal to:

A 64 **B** -64 **C** 16 **D** -16

4.2 **3** Which of the following is equal to $3^5 \times 3^4$?

A 3^{20} **B** 3^0 **C** 3 **D** 3^9

4.2 **4** Which of the following is equal to $\frac{12^6}{12^2}$?

A 12 **B** 12^3 **C** 12^4 **D** 12^8

4.3 **5** Evaluate $\left(\frac{2}{5}\right)^3$.

A $\frac{8}{5}$ **B** $\frac{6}{5}$ **C** $\frac{6}{15}$ **D** $\frac{8}{125}$

4.3 **6** Which of the following expressions does not equal 1?

A 10^0 **B** $10^0 \times 10$ **C** $\frac{10^3}{10^3}$ **D** $(10^3)^0$

4.4 **7** Which of the following is equal to $(5^2)^7 \times (5^6)^4$?

A 54 **B** 5^7 **C** 5^{19} **D** 5^{38}

4.4 **8** The missing exponent in the equation $\frac{12^3 \times 12^{\square}}{12^4} = 12^5$ is:

A 3 **B** 4 **C** 5 **D** 6

4.5 **9** Which of the following is equal to $\sqrt{\frac{25}{64}}$?

A $\frac{5}{64}$ **B** $\frac{5}{8}$ **C** $\frac{25}{8}$ **D** $\frac{25}{64}$

4.5 **10** Which of the following roots is a surd?

A $\sqrt{8}$ **B** $\sqrt{16}$ **C** $\sqrt{64}$ **D** $\sqrt[3]{27}$

Review questions 4.6C: Short answer

4.1 **1** Write each number in expanded form, then calculate its value.

a 3^3 **b** 2^4 **c** 8^2 **d** 10^5

4.1 **2** Write each number in expanded form and calculate its value.

a 2.2^4 **b** $\left(\frac{3}{7}\right)^3$ **c** $(-0.9)^5$ **d** $\left(-\frac{4}{5}\right)^4$

4.1 **3** Use a factor tree to find the prime factorisation of the following numbers. Write your answer in exponent form.

a 45 **b** 124

4.2 **4** Use an exponent law to simplify:

a $3^5 \times 3^7$ **b** $2^7 \times 2^3$ **c** $12^3 \times 12$ **d** $4^6 \times 4^2 \times 4^3$

4.2 **5** Write the following expressions as a single power with the given base.

a $6^8 \div 6^2$ **b** $5^7 \div 5^7$ **c** $\frac{10^{15}}{10^8}$ **d** $\frac{7^2}{7}$

4.3 **6** Simplify the following expressions by removing the brackets.

a $(9^5)^3$ **b** $(6 \times 3)^{10}$ **c** $\left(\frac{3}{10}\right)^3$ **d** $\left(\frac{2 \times 7}{9}\right)^8$

4.3 **7** Evaluate the following expressions.

a 4×4^0 **b** $(7 \times 5)^0$ **c** $\frac{20^{10}}{20^{10}}$ **d** $\left(\frac{5}{5^0}\right)^2$

4.4 **8** Simplify, then evaluate each of the following expressions.

a $\frac{7^6 \times 7^5}{7^9}$ **b** $\frac{2^8 \times (2^4)^2 \times 2^3}{2^{14}}$

4.4 **9** For each of the following expressions:

i write each basic numeral in the expression in exponent form

ii simplify the expression using the exponent laws

iii evaluate the expression.

a $\frac{5^4}{25}$ **b** $\frac{3^2 \times 81}{27}$ **c** $\frac{64 \times 256}{4 \times 16}$ **d** $16 \times (4^2)^2$

4.5 **10** Calculators can be used to determine whether a root is a surd or not.

a Classify each of the following numbers as a surd or not a surd.

i $\sqrt{529}$ **ii** $\sqrt{700}$ **iii** $\sqrt{156.25}$ **iv** $\sqrt[3]{42}$

b Write the surds from part **a** as basic numerals correct to two decimal places.

4.5 **11** Evaluate the following expressions.

a $\sqrt{4} + 2 \times \sqrt[3]{8}$ **b** $3^3 - 5 \times \sqrt{16}$ **c** $(4^3 + 2) \times \sqrt{25}$ **d** $\frac{(4^2 \times 4^2 + 2 \times \sqrt{9})}{(3 + 12^0)}$

Review questions 4.6D: Mathematical modelling

1 A radioactive metal has a half-life of 1 day. This means that each day, half the mass of the metal disintegrates into the atmosphere as a result of radioactive decay.
A physicist observes a 240 gram piece of the metal over a number of days and records the following data.

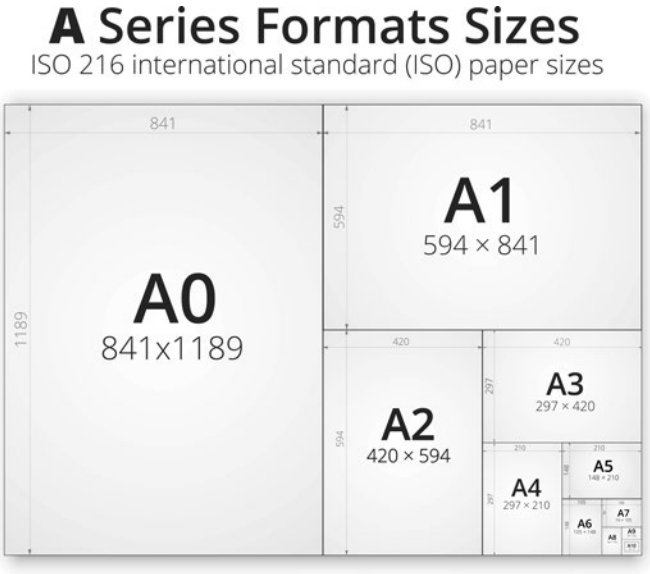
Number of days	Mass of metal (grams)	$\frac{\text{mass of metal}}{\text{original mass of metal}}$
0	240	
1		$\frac{1}{2}$
2		
3		

a Complete the table.
b Express each of the numbers in the third row in exponent form with a base of $\frac{1}{2}$.
c Write an expression in exponent form for the amount of metal remaining after 1 week.
d Use an exponent law to simplify, then evaluate your answer to part **c**. Express your answer as a fraction.
e How much metal will be left after 1 week? Express your answer as a decimal.
f How long until there is less than 1 gram of metal remaining?

2 Consider the fraction $\frac{490 \times 36^2}{756 \times 507^0}$.

a Use prime factorisation to rewrite the fraction, expressing each basic numeral as a product of prime factors.
b Simplify the fraction using exponent laws.
c Evaluate your answer to part **b**.

3 The thickness of a piece of A0 sized paper is 0.05 mm.
a When a piece of A0 paper is folded in half once, the size becomes A1. What is the thickness of the A1 paper?
b When a piece of A0 paper is folded in half twice, the size becomes A2. What is the thickness of the A2 paper?
c Determine the thickness of A6 paper if the A0 paper is folded in half six times to create the A6 size.
d If the thickness of a piece of paper is a, and the paper is folded in half n times, what is the thickness of the folded paper?



4 Consider the following list of numbers.

1, 2, 4, 6, 9, 16, 20, 25

a Arrange the numbers in ascending order using the less than sign '<':

b Without using a calculator, determine which of the following roots are surds.

i $\sqrt{1}$ **ii** $\sqrt{2}$ **iii** $\sqrt{4}$ **iv** $\sqrt{6}$

v $\sqrt{9}$ **vi** $\sqrt{16}$ **vii** $\sqrt{20}$ **viii** $\sqrt{25}$

c Use a calculator to evaluate the roots in part **b**, correct to two decimal places where necessary.

d Arrange your answers from part **c** in ascending order using the less than sign '<'.

e Compare your answers to parts **a** and **d**. What do you notice?

f Use your answer from part **e** to complete the following number statements.

i $\sqrt{30}$ _____ 6 **ii** $\sqrt{18}$ _____ 4 **iii** $\sqrt{12}$ _____ 4 **iv** $\sqrt{8}$ _____ 3

v $\sqrt{130}$ _____ 10 **vi** $\sqrt{60}$ _____ 8 **vii** $\sqrt{155}$ _____ 12 **viii** $\sqrt{300}$ _____ 20

g Arrange the following numbers in ascending order using the less than sign '<':

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \frac{1}{16}, \frac{1}{20}, \frac{1}{25}$

h Without using a calculator, determine which of the following roots are surds.

i $\sqrt{1}$ **ii** $\sqrt{\left(\frac{1}{2}\right)}$ **iii** $\sqrt{\left(\frac{1}{4}\right)}$ **iv** $\sqrt{\left(\frac{1}{6}\right)}$

v $\sqrt{\left(\frac{1}{9}\right)}$ **vi** $\sqrt{\left(\frac{1}{16}\right)}$ **vii** $\sqrt{\left(\frac{1}{20}\right)}$ **viii** $\sqrt{\left(\frac{1}{25}\right)}$

i Use a calculator to evaluate the roots in part **h** correct to two decimal places where necessary.

j Arrange your answers from part **i** in ascending order using the less than sign '<'.

k Compare your answers to parts **g** and **j**. Does your answer from part **e** still hold?

l Use your answer from part **k** to complete the following number statements.

i $\sqrt{\left(\frac{1}{30}\right)}$ _____ $\frac{1}{5}$ **ii** $\sqrt{\left(\frac{1}{70}\right)}$ _____ $\frac{1}{8}$ **iii** $\sqrt{\left(\frac{1}{10}\right)}$ _____ $\frac{1}{3}$ **iv** $\sqrt{\left(\frac{1}{17}\right)}$ _____ $\frac{1}{5}$

v $\sqrt{\left(\frac{7}{30}\right)}$ _____ $\frac{1}{3}$ **vi** $\sqrt{\left(\frac{3}{10}\right)}$ _____ $\frac{2}{5}$ **vii** $\sqrt{\left(\frac{7}{20}\right)}$ _____ $\frac{2}{5}$ **viii** $\sqrt{\left(\frac{4}{13}\right)}$ _____ $\frac{1}{4}$

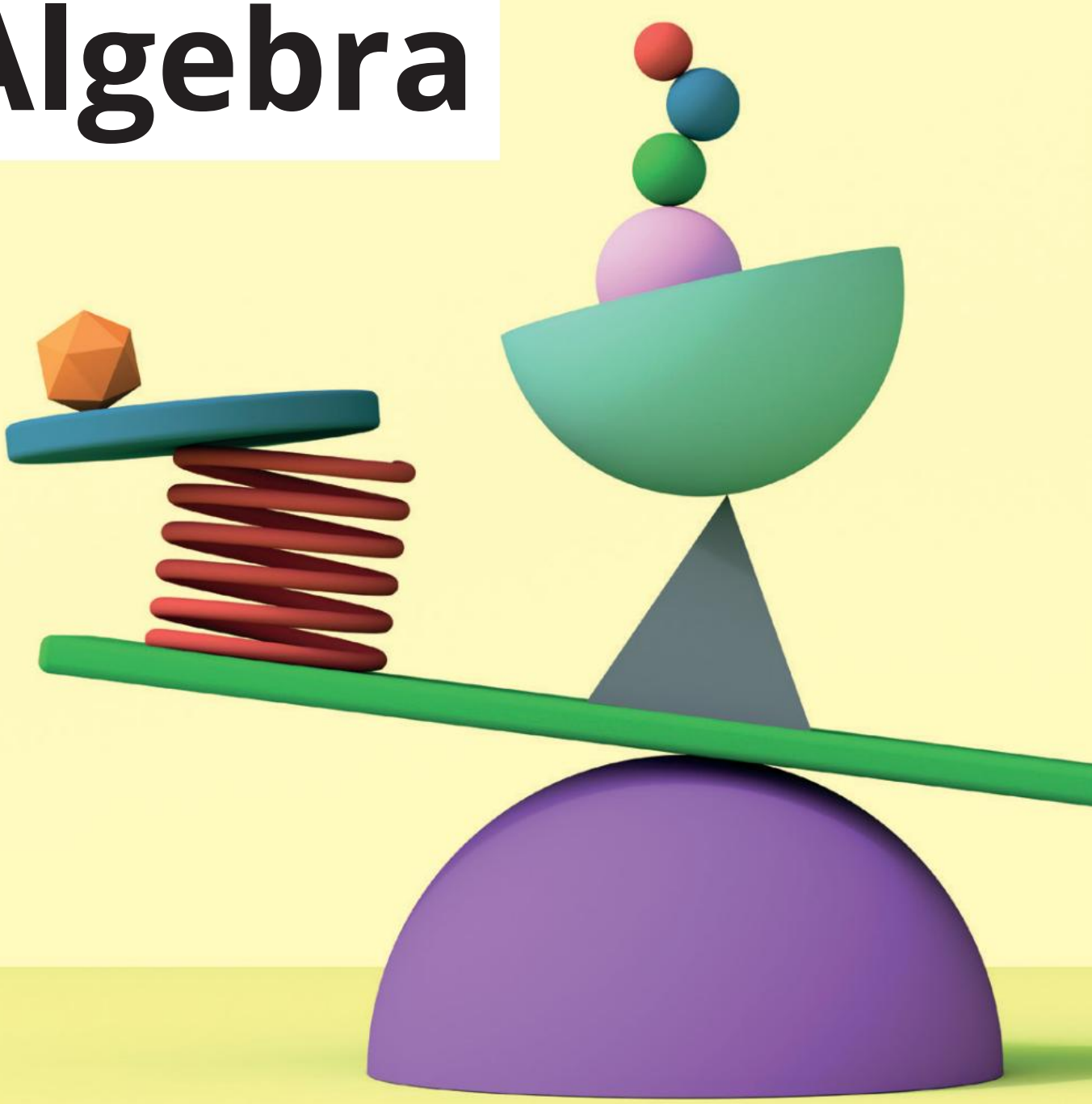
Checklist

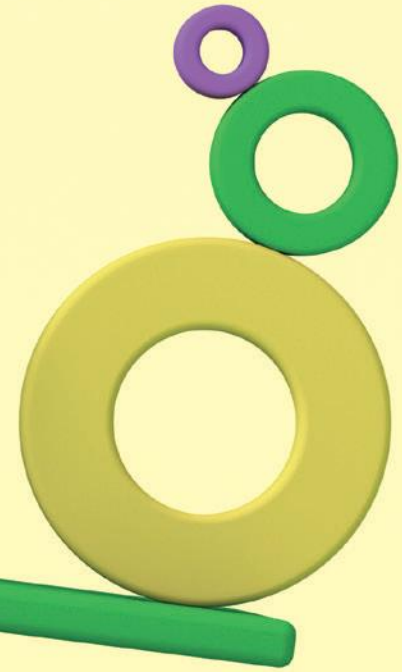
Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert numbers between exponent form and expanded form ☐ Calculate the value of numbers in exponent form ☐ Express integers as a product of their prime factors	☐ Go back to Lesson 4.1 Exponents
☐ Simplify products of numbers with the same base ☐ Simplify quotients of numbers with the same base	☐ Go back to Lesson 4.2 Products and quotients of powers
☐ Raise a number in exponent form by another exponent ☐ Evaluate expressions involving the zero exponent	☐ Go back to Lesson 4.3 Raising exponents and the zero exponent
☐ Use multiple exponent laws to simplify complex expressions	☐ Go back to Lesson 4.4 Combining the exponent laws
☐ Determine if a number is a surd or not ☐ Give solutions to problems involving surds as exact or approximate values	☐ Go back to Lesson 4.5 Roots

5

Algebra





Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Variables
- ✔ Grouping symbols
- ✔ Order of operations
- ✔ Highest common factor

Curriculum links

- Create, expand, factorise, rearrange and simplify linear expressions, applying the associative, commutative, identity, distributive and inverse properties (VC2M8A01)
- Use mathematical modelling to solve applied problems involving linear relations, including financial contexts involving profit and loss; formulate problems with linear functions, and choose a representation; interpret and communicate solutions in terms of the context, and review the appropriateness of the model (VC2M8A03)

Materials

- ✔ Calculator

Lesson 5.1

Pronumerals

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and write terms, expressions and equations.

Inter-year links

Support Writing formulas

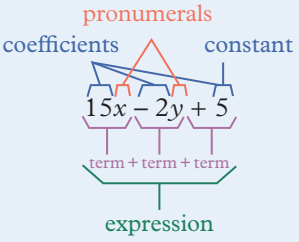
Year 7 6.2 Terms, expressions and equations

Pronumerals

- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an unknown number or a variable.
  - → An **unknown number** has one value.
  - → A **variable** is a quantity that can take on different values.

Terms and expressions

Key content video
Terms, expressions and equations



- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- A **term** is a single pronumeral, number or the product or quotient of numbers or pronumerals.
  - → In an expression without grouping symbols, terms are separated by plus and minus signs. These signs belong to the term on their right.
- A **coefficient** is the number multiplying any pronumerals in an algebraic term, usually written first.
  - → If the coefficient of a term is positive, then the term is said to be positive. Similarly, if the coefficient of a term is negative, then the term is said to be negative.
  - → A pronumeral without a number preceding it has a coefficient of 1 or −1, depending on whether the term is positive or negative.
- A **constant** is a term without any pronumerals. A constant term can be thought of as the coefficient of a variable with a zero exponent.

Algebraic notation

- Products are simplified by leaving out the multiplication sign and placing the number first.

  $7 \times x = 7x$ $\quad$ $7 \times (x + 2) = 7(x + 2)$
- When a pronumeral is multiplied by 1 or −1, the 1 is left out.

  $1 \times x = x$ $\quad$ $-1 \times x = -x$
- Quotients are represented using fractions.

  $x \div 7 = \frac{x}{7}$ $\quad$ $(x + 2) \div 7 = \frac{x + 2}{7}$
- Terms with fractional coefficients can be written in two ways.

  $\frac{x}{7} = \frac{1}{7}x$
- **Grouping symbols** include brackets, the fraction line in a quotient and the root sign. Grouping symbols are used to preserve order of operations in algebraic expressions.

  $7(x + 2)$

  $\frac{x + 2}{7}$

  $\sqrt{7(x + 2)}$

Worked example 5.1A Identifying terms, coefficients and constants

Consider the expression $4a + \frac{b}{2} - c - 11$.

a List the terms in the expression.

b List any variables in the expression.

c Identify the coefficient of each term that contains a variable.

d List any constant terms.

THINK	WRITE
a Identify the individual terms, which are individual numbers or pronumerals, and the products or quotients of numbers and pronumerals. Remember to include the plus and minus signs as they form part of the coefficients of each term.	**a** The terms are $4a$, $\frac{b}{2}$, $-c$ and -11.
b Variables are represented by pronumerals. List the pronumerals.	**b** The variables are a, b and c.
c Identify the number that comes before each pronumeral. Terms with fractional coefficients can be written in two ways. $\frac{b}{2} = \frac{1}{2}b$, so the coefficient of the b term is $\frac{1}{2}$. As the negative sign is part of the c term, the coefficient of c is -1.	**c** The coefficient of a is 4. The coefficient of b is $\frac{1}{2}$. The coefficient of c is -1.
d List the terms that do not contain pronumerals.	**d** The constant term is -11.

Worked example 5.1B Writing expressions

Write an expression for each statement.

a 5 less than a

b A quarter of b

THINK	WRITE
a Think about which operation is taking place. 'Less than' means subtract, so 5 less than a is a minus 5.	**a** $a - 5$
b 'A quarter' of b means b divided by 4. Simplify the quotient by rewriting it as a fraction. Terms with fractional coefficients can be written in two ways – either form is acceptable.	**b** $b \div 4$ $= \frac{b}{4}$ or $\frac{1}{4}b$

Worked example 5.1C Writing expressions using grouping symbols

Write an expression for each statement using grouping symbols where appropriate.

a Add 7 to b, then multiply the result by 4.

b The product of a and d, all divided by 2.

THINK	WRITE
a **1** Think about which operations are taking place. 'Add 7' means +7.	**a** $b + 7$
2 Look for words that refer to the order of operations. '*Then* multiply *the result* by 4' means the multiplication should be performed *after* the addition. Group $b + 7$ using brackets, then multiply the brackets by 4.	$(b + 7) \times 4$
3 Simplify the product by placing the number first and removing the multiplication sign.	$= 4(b + 7)$
b **1** Finding 'the product' means to multiply.	**b** $a \times d$
2 '*All* divided by 2' means the whole product should be divided by 2.	$(a \times d) \div 2$
3 Simplify the product and the quotient. The fraction line is a grouping symbol, so the brackets are no longer necessary.	$= \frac{ad}{2}$

Worked example 5.1D Writing an expression from a worded problem

Lee works casually as a nurse. He earns $35 an hour plus a bonus of $50 for each night shift.
Write an expression to represent the amount of money Lee earns for a night shift of t hours.

THINK	WRITE
1 Identify the variable in the problem.	t = number of hours in the shift
2 Think about which operations are taking place. If Lee earns $35 per hour, he will earn 35 times the number of hours he works. 'Plus' means add 50 to Lee's total earnings for a given night shift.	$35 \times t + 50$
3 Write an expression to represent the amount of money Lee will earn for a night shift of t hours. Simplify the product.	$= 35t + 50$

Helpful hints

- ✔ In the expression $5 + x$, the coefficient of x is 1, not 0.
- ✔ Watch out for negative coefficients in expressions containing two or more terms. In the expression $5 - 3x$, for example, the coefficient of x is -3.
- ✔ The order of operations changes with the placement of grouping symbols. Take care when placing brackets and writing fractions and roots! For example:
  $\frac{x+1}{2}$ is not the same as $\frac{x}{2} + 1$.

Learning pathways

 1–6, 7(b, c, f, h), 8–11, 13, 15

 1, 3, 5, 6, 8, 11, 12, 14, 16, 17, 19

 1, 3, 5, 8, 12, 16–20

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Exercise 5.1A: Understanding and fluency

WE 5.1A **1** For each of the following expressions:

i list the terms in the expression

ii list any variables in the expression

iii identify the coefficient of each term that contains a variable

iv list any constant terms.

a $6a + b - 1$ **b** $2xy - 3x$ **c** $11f - g + 15h - 10$ **d** $-\frac{9}{4}p$

e $\frac{3}{4}c + d + \frac{1}{3}$ **f** $\frac{x}{2} + \frac{y}{11} - \frac{7}{10} - \frac{z}{8}$ **g** $-\frac{m}{5} - \frac{n}{7} - 8p$ **h** $\frac{5x}{8} - 9y + \frac{z}{4} - \frac{2}{5}$

2 State whether each of these is an expression or an equation.

a $2a + 3$ **b** $x - 6 = 1$ **c** $7 = 5 + \frac{5}{6}h$

d $x + y + z = 9$ **e** $3a + b - 7$ **f** $k = \frac{4m + 1}{8}$

3 Answer true or false for each statement. For the statements that are false, provide a reason for your answer.

a $2x + \frac{1}{3} = 4$ is an expression.

b $5 - 12a + b$ has 3 terms.

c $7 - y$ is a term.

d $\frac{5}{7}$ is a term.

e The coefficient of $-d$ is 1.

f The coefficient of $\frac{b}{12}$ is 12.

g $-\frac{5f}{4}$ is an expression.

h The coefficient of $\frac{2x}{3}$ is $\frac{2}{3}$.

WE 5.1B **4** Write an expression for each statement.

a 5 more than a

b z divided by 3

c 23 less than y

d 50 times m

e The product of x and $\frac{2}{5}$

g Half of s

f Twice the value of p

h The sum of three-quarters of w and 5

5 Write an expression for each statement. Use x to represent the unknown number.

a A number is multiplied by 7 and then 2 is added.

b A number is divided by 3 and then 6 is subtracted.

c 10 less than three-quarters of a number

d The sum of 13 and a third of a number

e 28 less than the product of 15 and a number

f 5 less than half of a number

6 Write a statement to describe the order of operations in each of the following expressions.

a $c + 8$ **b** $11 - x$ **c** $\frac{x}{5}$

d $\frac{9}{10}q$ **e** $3x - 5$ **f** $\frac{10a}{3} + 16$

WE 5.1C **7** Write an expression for each statement using grouping symbols where appropriate.

a Add 6 to x, then multiply the result by 3.
b Subtract 10 from n, then divide the result by 7.
c The product of f and g, all multiplied by $\frac{3}{5}$
d 1 less than the square root of t
e The sum of b and 12, all divided by 15
f The square root of the sum of z and 9
g 7 less two-thirds of v
h Triple the sum of 33 and y

8 Write an expression to represent each of the following number sentences. Use x for the unknown number.

a Subtract 3 from a number, then multiply the result by 8
b The sum of a number and 20, all multiplied by 6
c 2 less than the product of $\frac{5}{6}$ and a number
d 12 more than four-fifths of a number
e Double the sum of a number and 10
f The sum of a number and 4 is divided by 12, then 16 is added to the result

Exercise 5.1B: Problem solving and reasoning

WE 5.1D **9** Amir has n songs on his phone. He downloads a further 18 songs. Write an expression to represent the number of songs he now has on his phone.

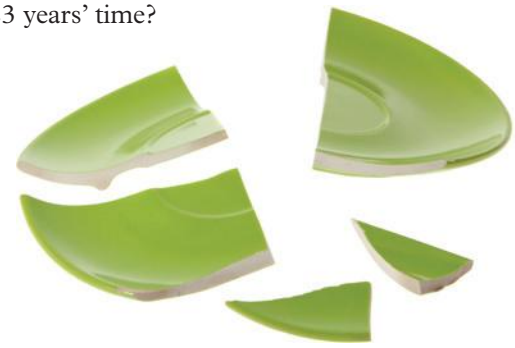
10 Four friends bought a lottery ticket together and agreed to share the prize evenly. If they win $\$c$ in total, how much does each person get?

11 Pritesh finds x old records in his shed and decides to sell them for \$8 each. Write an expression for the total price of the records.

12 Ash is running low on free space on his laptop. He deletes 10 GB (gigabytes) of data that he doesn't need anymore. Write an equation for the amount of data remaining on his laptop, y, compared to the original amount of data, x.

13 Hayden is y years old. Answer each of the following questions using an algebraic expression.

a How old was Hayden:
i 6 years ago
ii 10 years ago?
b How old will Hayden be in:
i 5 years' time
ii 23 years' time?
c How old is Hayden's sister if she is x years younger than him?
d How old is Hayden's uncle if he is twice as old as Hayden?



14 A kitchen hand is washing up c cups and p plates.

a How many items does he have to wash?
b If he breaks a plate, how many plates are left?
c After he breaks a plate, how many cups and plates does he now have to dry?

15 An art teacher orders n brushes from an online store that charges \$12 for delivery. The total cost of his order can be represented by the algebraic expression $8n + 12$.

a List the terms in the expression.
b What does the constant term represent?
c What does the coefficient of the term with n represent?
d What will the total cost of the order be if the art teacher orders 10 brushes?

16 Gillian hires a photobooth for a birthday party. The photobooth company charges $130 per hour and an extra flat fee of $25 for props and backdrops.

a Write an expression for the total cost of hiring the photobooth for t hours.

b How much will it cost Gillian to hire the photobooth, props and backdrops for 3 hours?

17 At the end of autumn, Ron has x chickens on his farm. 7 baby chickens hatch over winter. By the end of spring, more chickens have hatched. Ron now has double the number of chickens than he did at the end of winter.

a Write an expression for the total number of chickens on Ron's farm at the end of spring.

b If Ron had 13 chickens at the end of autumn, how many chickens did he have at the end of spring?

18 Naomi and Rupi are working on the following question:

'A container of x puzzle blocks is divided between three students. If each student has the same number of blocks and one block is left over, write an expression for the number of blocks each student has to work with.'

Naomi believes the answer is $\frac{x-1}{3}$ while Rupi thinks the answer should be $\frac{x+1}{3}$.

a Decide who is correct. Explain why their expression is correct.

b Rewrite the question to represent the other expression.

19 Consider the expression $\frac{4x-5}{9}$.

a Write a statement to represent the expression.

b Rewrite the expression as two separate fractions.

c What is the coefficient of the term with variable x?

d What is the value of the constant term?

Exercise 5.1C: Challenge

20 During a sale, a shop advertises 20% off the full price of t-shirts and 30% off the full price of pants. A customer buys x t-shirts that cost $\$y$ full price, and a pants that cost $\$b$ full price.

a Write a two-term expression for the total cost of the customer's purchase.

b If the customer bought twice as many t-shirts as pants, the t-shirts cost $20 each at full price and the pants cost $50 each at full price, rewrite the expression in terms of one pronumeral only.

c How many t-shirts and pairs of pants did the customer buy if they spent a total of $134?

Online resources:

- **Interactive skillsheet** Expressions and equations
- **Worksheet** Writing algebraic expressions
- **Desmos activity** Pool border problem
- **Quick quiz** 5.1
- **BLM** Algebra tiles
- **Investigation** Pronumerals in mark-ups and discounts

Lesson 5.2

Substitution

Learning intentions

By the end of this lesson you will be able to ...

✔ substitute values into and evaluate algebraic expressions.

Inter-year links

Year 7 6.4 Substitution

Substitution

Key content video
Substitution

- **Substitution** involves replacing each pronumeral in an algebraic expression with a number.
- For clarity, the number being substituted into an expression is enclosed in brackets when:
  - → the number is negative, or
  - → the expression involves multiplication or exponents.

Let $x = -2$

$5x^2 - x = 5(-2)^2 - (-2)$

- To **evaluate** an algebraic expression, substitute the known values of each pronumeral into the expression and complete the operations in the correct order.

Key content video
Order of operations in algebra

- A **formula** is an algebraic statement that describes the relationship between two or more variables. Usually, formulas are written with one variable on the left-hand side which does not appear on the right-hand side. This variable is called the **subject** of the formula. We say that the subject is written in terms of any variables on the right-hand side.

  For example, the formula for the area of a triangle is:

  $A = \frac{1}{2}bh$

  where A = area
  b = base length
  h = height.

  Here, A is the subject which is written in terms of b and h.

B	**Brackets**
I	**Indices or exponents**
D	**Division**
M	**Multiplication**
A	**Addition**
S	**Subtraction**

- Substitution can be used to find the value of an unknown variable when the values of all the other variables in a formula are known.

Worked example 5.2A Evaluating expressions

Evaluate each expression by substituting 3 for x.

a $\frac{x}{3} + 1$ **b** $4x^2 + 1$

THINK

a Replace x with 3 and complete the operations in the correct order. Division is performed before addition.

b Replace x with 3 and complete the operations. As the expression involves multiplication and exponents, enclose the 3 in brackets for clarity. Exponents are applied first, then multiplication, then addition.

WRITE

a $\frac{x}{3} + 1 = \frac{3}{3} + 1$
$= 1 + 1$
$= 2$

b $4x^2 + 1 = 4(3)^2 + 1$
$= 4(9) + 1$
$= 36 + 1$
$= 37$

Worked example 5.2B Substituting negative values into expressions

Evaluate each expression by letting $m = 5$ and $n = -2$.

a $3m - n$ **b** $2mn^2$

THINK

a Replace m with 5 and n with -2 and complete the operations in the correct order. Multiplication is performed before subtraction. Remember that subtracting a negative integer is equivalent to addition.

b Replace m with 5 and n with -2 and complete the operations. Exponents are applied before multiplication. Remember that the product of two negative numbers gives a positive number, so $(-2)^2 = (-2) \times (-2) = 4$.

WRITE

a $3m - n = 3(5) - (-2)$
$= 15 + 2$
$= 17$

b $2mn^2 = 2(5)(-2)^2$
$= 2(5)(4)$
$= 40$

Worked example 5.2C Evaluating expressions containing grouping symbols

Substitute $a = 3$ and $b = 4$ into each expression and evaluate.

a $7(a - b)$ **b** $\frac{2a + b}{5}$

THINK

a Replace a with 3 and b with 4 and complete the operations in the correct order. Bracketed operations are performed first, so complete the subtraction in the brackets before the multiplication. Remember that the product of a positive number and a negative number is a negative number.

b Replace a with 3 and b with 4 and complete the operations. The fraction line is a grouping symbol, so the expression in the numerator of the fraction should be evaluated before dividing through by the denominator.

WRITE

a $7(a - b) = 7(3 - 4)$
$= 7(-1)$
$= -7$

b $\frac{2a + b}{5} = \frac{2(3) + 4}{5}$
$= \frac{6 + 4}{5}$
$= \frac{10}{5}$
$= 2$

Worked example 5.2D Using formulas to find unknown values

Use the formula $y = 2\sqrt{x + 9}$ to find the value of y if $x = 7$.

THINK

To find the value of y, substitute $x = 7$ into the expression on the right-hand side of the equation and evaluate. Remember that the root sign is a grouping symbol, so the expression contained within the root sign should be evaluated before multiplying by 2.

WRITE

$y = 2\sqrt{x + 9}$
$= 2\sqrt{7 + 9}$
$= 2\sqrt{16}$
$= 2 \times 4$
$= 8$

Helpful hints

✔ Take care when substituting negative numbers into expressions. Writing calculations out in full will help you to simplify calculations involving negative numbers correctly.

$+ (+) = +$ $\quad + \times + = +$
$+ (-) = -$ $\quad + \times - = -$
$- (+) = -$ $\quad - \times + = -$
$- (-) = +$ $\quad - \times - = +$

Learning pathways

1–8, 9(a, b, e, f), 10, 11, 13 | 2, 3, 5, 6, 8, 9, 11, 12, 15 | 5, 6, 8, 9, 12, 14, 15

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Exercise 5.2A: Understanding and fluency

WE 5.2A **1** Evaluate each expression by substituting 4 for x.

a $x + 8$ **b** $\frac{x}{2}$ **c** $\sqrt{x}$ **d** $5x - 10$ **e** $\frac{2x}{5}$

f $5x^2$ **g** $4x^3 - 2$ **h** $2\sqrt{x} - 5$ **i** $5x^2 - x$ **j** $\frac{1}{2}x^2 - 8$

WE 5.2B **2** Evaluate each expression by letting $m = 4$ and $n = -3$.

a $m + n + 4$ **b** $2m + 5n$ **c** $7m - n$ **d** $\frac{1}{4}mn$

e $3m^2 + 6n^2$ **f** $\frac{1}{8}m^2 - 2n$ **g** $\frac{m^2n}{12}$ **h** $6mn + n^2$

3 Evaluate $3a + 5$ by substituting each value for a into the expression.

a $a = 2$ **b** $a = 7$ **c** $a = -1$ **d** $a = -3$

4 Evaluate $6y^2 - y$ by substituting each value for y.

a $y = -4$ **b** $y = 0$ **c** $y = \frac{1}{2}$ **d** $y = \frac{1}{3}$

WE 5.2C **5** Substitute $a = 5$ and $b = 2$ into each expression and evaluate.

a $10(a - b)$ **b** $\frac{2a + 2b}{7}$ **c** $2\sqrt{2a + 3b}$ **d** $\frac{3b + 1}{a}$

WE 5.2D **6** Use each of the following formulas to find the value of y if $x = 3$.

a $y = 5\sqrt{x + 6}$ **b** $y = \frac{4x - 15}{5}$ **c** $y = \frac{2}{9}(10 - x^2)$ **d** $y = \frac{1}{x} + \frac{5}{3}$

7 Find the value of y for each of these values of x using the formula $y = 2x + 7$.

a $x = 3$ **b** $x = -6$ **c** $x = 0$ **d** $x = \frac{1}{2}$

8 Find the value of k for each of these values of m using the formula $k = 4(m^2 - 3)$.

a $m = -2$ **b** $m = \frac{1}{4}$ **c** $m = 0$ **d** $m = -\frac{3}{4}$

9 Recall the rules for carrying out the four operations with negative numbers and evaluate each expression by letting $p = -6$, $q = 2$ and $r = -3$.

a $2(q - r)$ **b** $\sqrt{14q - 6p}$ **c** $q(p + r)$ **d** $\frac{r - p}{q}$

e $\frac{pq}{r}$ **f** $\frac{p^2 - q^2}{r}$ **g** $4(p^2 - q) - 5r$ **h** $-\frac{3p}{q - r}$

Exercise 5.2B: Problem solving and reasoning

10 The relationship between average speed, s, distance, d, and time, t, can be represented by the formula $s = \frac{d}{t}$.

a Find the average speed, in metres per second, of a bicycle that travels 120 m in 40 seconds.

b Find the average speed, in kilometres per hour, of a motorcyclist who travels 282 km in 3 hours.

11 The distance a skydiver falls through the air, d, in metres, after a given amount of time, t, in seconds, can be roughly calculated using the formula $d = 4.9t^2$.

a How far does the skydiver fall in the first second of his dive?

b How far does the skydiver fall:

i in the first 3 seconds of his dive **ii** in the first 5 seconds of his dive?

c Does the skydiver's speed increase or decrease over the first 5 seconds of his dive? Explain.

12 Use the formulas in the table to answer the questions below.

Shape	Area formula, A	Dimensions
Square	$A = l^2$	l = side length
Rectangle	$A = lw$	l = length w = width
Triangle	$A = \frac{1}{2}bh$	b = base length h = height

a What is the area of a square that has a side length of 2 cm?

b What is the area of a rectangle that has a length of 10 cm and a width of 7 cm?

c What is the area of a triangle that has a base length of 5 cm and a height of $\frac{5}{2}$ cm?

d For each of the following pairs of shapes, which has a greater area?

i a square with a side length of 13 cm, or a rectangle with a length of 5 cm and a width of 3 cm

ii a rectangle with a length of 10 cm and a width of $\frac{3}{5}$ cm, or a triangle with a base length of $\frac{5}{4}$ cm and a height of 15 cm

13 For each of the following:

i identify and explain the error made in each calculation **ii** determine the correct result.

a $2x + 6$

Let $x = 5$

$2x + 6 = 25 + 6$
$= 31$

b $3x^2 + 6$

Let $x = 1$

$3x^2 + 6 = 3(1)^2 + 6$
$= 9 + 6$
$= 15$

c $2^x + 4x$

Let $x = 0$

$2^x + 4x = 2^{(0)} + 2(0)$
$= 0 + 0$
$= 0$

d $x^2 - 4x$

Let $x = -3$

$x^2 - 4x = -(3)^2 - 4(-3)$
$= -9 + 12$
$= 3$

14 If \$$P$ is kept in a bank account that earns interest at a rate of r% per year, the amount of money in the bank account, \$$A$, after n years is given by: $A = P\left(1 + \frac{r}{100}\right)^n$.

a If \$1000 is kept in a bank account for 1 year at an interest rate of 5%, calculate how much money there will be in the account at the end of the year.

b \$2000 is invested in a bank account with an interest rate of 4% per year. Use a calculator to find the amount of money in the account after 5 years, correct to 2 decimal places.

Exercise 5.2C: Challenge

15 The salespeople at a certain car dealership are permitted to lower the selling price of different cars by different set amounts when bargaining with customers.

For a car marketed at \$$x$, the salespeople can lower the price by \$$y$. After this, they can take 5% off the reduced value. Finally, if the customer still insists on a lower price, they can take off a further \$$z$.

a Write a formula for the lowest possible selling price, \$$P$, of a car marketed at \$$x$.

b A customer haggles on the price of a hatchback marketed at \$19 990. The salesperson first reduces the price by \$1000, then reduces it by a further 5% and finally takes off \$500. What is the final price of the hatchback?

Online resources:

Interactive skillsheet Substitution

Worksheet Substitution

Investigation Arithmetic and geometric sequences

Quick quiz 5.2

Lesson 5.3

Adding and subtracting algebraic terms

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify algebraic expressions by collecting like terms.

Inter-year links

Year 9 2.1 Simplifying

Year 10 1.3 Simplifying

The laws of arithmetic

- All the laws of arithmetic apply to algebraic terms and expressions.

	Arithmetic	Algebra
The associative law	$(3 + 5) + 2 = 3 + (5 + 2)$ $(3 \times 5) \times 2 = 3 \times (5 \times 2)$	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
The commutative law	$3 + 2 = 2 + 3$ $3 \times 2 = 2 \times 3$	$a + b = b + a$ $a \times b = b \times a$

Like terms

- **Like terms** are terms that contain exactly the same pronumerals in expanded form.
  - → Following the commutative law, the order of pronumerals can be different in two like terms.
  - → It can be helpful to write terms containing powers out in expanded form to determine whether they are like terms.

    For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$ so a^2b and ab^2 are not like terms.

$3a^2b = 3 \times a \times a \times b$

exponent form — expanded form

Pronumeral(s)	Examples of like terms	Examples of unlike terms
x	$x, 3x, \frac{2}{3}x, 0.7x$ ...	$2, -3x^2, 4xy, 10y$
ab	$ba, -2ab, \frac{ab}{5}, 6.6ab$ ...	$2ab^2, -5a^2b, 25a^2b^2, \frac{7a}{2}, -9b$
m^2n	$-m^2n, 18m^2n, \frac{9m^2n}{4}, 0.05\,m^2n$ ...	$mn, -mn^2, 1.5m^2n^2, 4n, 5m^2$

Simplification by addition and subtraction of like terms

Key content video
Adding and subtracting algebraic terms

- Simplification involves reducing the complexity of an expression. This can be done by adding or subtracting like terms, multiplying terms or dividing terms.
- To simplify an algebraic expression by adding and subtracting like terms, or 'collecting like terms':
  1. Circle the like terms in the expression.
  2. Group the like terms together.
  3. Add and subtract the coefficients of each set of like terms.

$6ab + \frac{3}{5}a + ab - \frac{2}{5}a = 6ab + ab + \frac{3}{5}a - \frac{2}{5}a$

$= 7ab + \frac{1}{5}a$

Worked example 5.3A Identifying like terms

Identify and list the like terms in the following groups of terms.

a $36x, -x, 5y, \frac{3x}{4}, 0.5y, \frac{1}{2}y$

b $7xy, \frac{5}{2}yx, -2x, -4x^2, \frac{x^2}{6}, 10x$

THINK

a Circle the like terms, or terms that contain exactly the same pronumerals when written in expanded form. The values of the coefficients do not affect whether or not two or more terms are like terms.

b Circle the like terms. According to the commutative law, $xy = yx$, so $7xy$ and $\frac{5}{2}yx$ are like terms. Remember that $x^2 = x \times x$, so x and x^2 are *not* like terms.

WRITE

a $36x, -x, 5y, \frac{3x}{4}, 0.5y, \frac{1}{2}y$

The two sets of like terms in the list are:

$36x$, $-x$ and $\frac{3x}{4}$

$5y$, $0.5y$ and $\frac{1}{2}y$

b $7xy, \frac{5}{2}yx, -2x, -4x^2, \frac{x^2}{6}, 10x$

The three sets of like terms in the list are:

$7xy$ and $\frac{5}{2}yx$

$-2x$ and $10x$

$-4x^2$ and $\frac{x^2}{6}$

Worked example 5.3B Adding and subtracting like terms

Simplify each expression where possible.

a $10x - 3x + 3 + x$

b $-6xy + 2x - 3yx - \frac{x}{2}$

c $2 + xy^2 - 5x^2y - 12xy^2 - 7 - 2y^2x$

THINK

a **1** Circle the like terms in the expression.

2 Group the like terms together.

3 Add and subtract the coefficients of like terms. Note that '3' is a constant and there are no other constants in the expression, so 3 cannot be combined with any other terms.

b **1** Circle the like terms.

2 Group the like terms.

3 Add and subtract the coefficients of each set of like terms. Remember that fractional coefficients can be written in two ways, so $-\frac{x}{2} = -\frac{1}{2}x$.

c **1** Circle the like terms.

2 Group the like terms.

3 Add and subtract the coefficients of each set of like terms. Note that $xy^2 = x \times y \times y$, but $x^2y = x \times x \times y$, so xy^2 and x^2y are *not* like terms and $-5x^2y$ cannot be combined with any other terms.

WRITE

a $10x - 3x + 3 + x = 10x - 3x + x + 3$
$= 8x + 3$

b $-6xy + 2x - 3yx - \frac{x}{2} = -6xy - 3yx + 2x - \frac{x}{2}$
$= -9xy + \frac{3}{2}x$

c $2 + xy^2 - 5x^2y - 12xy^2 - 7 - 2y^2x = 2 - 7 + xy^2 - 12xy^2 - 2y^2x - 5x^2y$
$= -5 - 13xy^2 - 5x^2y$

Helpful hints

- ✔ For two terms to be like terms, the pronumeral components of the terms must be *exactly* the same, but following the commutative law, the order of the pronumerals can vary.
  For example, xy and yx are like terms.
- ✔ When adding or subtracting like terms, remember to only add or subtract the coefficients.
  For example, $3a^3 + 4a^3 = 7a^3$, not $7a^6$!

Learning pathways

1–4, 5(1st column), 6, 7(b, d, f, h), 8, 9(b, c, e), 10, 11(a–c)	1(b, d, f, h), 3, 5, 7, 9(d–f), 11–13, 14–16(a–c)	1(g, h), 3(g, h), 5, 7, 9, 14(b, d, f), 15–16(c, d), 17–19

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Exercise 5.3A: Understanding and fluency

WE 5.3A **1** Identify and list the like terms in the following groups of terms.

a $20x, 8y, -15y, y, -12x, 3x$

b $5x, \frac{3}{7}x, \frac{5}{7}y, -25x, 6y, \frac{x}{7}$

c $4xy, -yx, 0.1x, \frac{1}{8}xy, -\frac{yx}{10}, \frac{y}{15}$

d $-a, 5a^2, \frac{4}{9}a^2, 12.5a, 2.3a^2, -\frac{9a^2}{10}$

e $\frac{11}{18}p^3, \frac{p^2}{5}, 5.5p^3, p^3, -11p^2, -0.05p^2$

f $\frac{9ab}{10}, 7abc, -cab, \frac{5}{6}ab, 13cab, 7.2ba$

g $-9fg, 120f^2g, \frac{20fg^2}{3}, -\frac{14}{3}f^2g, 0.33fg^2, \frac{4fg}{5}$

h $\frac{2}{9}xy^2z, 0.095zyx, -12y^2xz, 18.6xyz, \frac{x^2y^2z}{24}, 99zy^2x$

2 Answer true or false to each statement. Explain why each false statement is incorrect.

a $2x$ and $-2x$ are like terms.

b $15a$ and $15a^2$ are like terms.

c $8p$ and $\frac{p}{5}$ are like terms.

d $\frac{5}{8}def$ and $5fed$ are like terms.

e $4ab$ and $6a^2b$ are like terms.

f $-3a^2b$ and $5ba^2$ are like terms.

3 Simplify each pair of like terms by adding or subtracting the coefficients of each term.

a $5x + 4x$

b $10kp - 2kp$

c $3a^2 - 7a^2$

d $5g - g$

e $yx - 3xy$

f $-5g^2h + 7g^2h$

g $\frac{3}{4}abc - \frac{1}{2}cab$

h $\frac{2xyz^2}{5} - \frac{2}{3}xz^2y$

4 Answer true or false to each statement. For the statements that are false, write the correct simplified expression.

a $8x + 6x$ simplifies to $14x$.

b $11k - k$ simplifies to 11.

c $3p + 6q$ simplifies to $9pq$.

d $5m^2 + 2n + m^2$ simplifies to $6m + 2n$.

e $2ab + 3a + 7ba$ simplifies to $12ab$.

f $\frac{1}{4}a + \frac{1}{3}c + \frac{1}{2}a + c$ simplifies to $\frac{3}{4}a + \frac{4}{3}c$.

g $x^2y + 2x + y$ cannot be simplified.

h $\frac{2}{9}p^2q - \frac{5}{6}p^2 + \frac{1}{6}qp^2$ cannot be simplified.

WE 5.3B **5** Simplify each expression where possible.

a $3b + 5 + 5b - 2b$

b $3n + 4mn - 9mn - 4n$

c $12 - 5c^2 - 8 - 3c^2$

d $4gh + 3hg - 3 - 6gh$

e $5pq - 6pq - 10q^2p + 8p^2q$

f $2x^2y + 10yx^2 - x^2y - 12xy^2$

g $\frac{5}{2}str + \frac{1}{3}st - \frac{1}{6}ts - \frac{2}{7}rst$

h $\frac{abc}{3} - \frac{2}{3}ab - \frac{5}{6}cab - 2 + \frac{ab}{6}$

i $\frac{g^3h}{5} - \frac{5}{3} + \frac{2}{10}g^3h + \frac{7}{2} + \frac{4g^2h}{5} - \frac{1}{15}gh$

j $\frac{5}{4}m^2n + \frac{1}{4}m^2n^4 + m^2n - \frac{3}{2} - m^2n^4 + \frac{5}{3}$

6 Consider the expression $7x^2 + 2xy + 9x^2 - yx$.

a Evaluate the expression by substituting $x = 3$ and $y = 7$ into the four terms.

b Simplify the expression by adding like terms.

c Check your answer to part **b** by substituting $x = 3$ and $y = 7$ into the simplified expression. Do you obtain the same answer as part **a**?

d Which expression was easier to evaluate? Explain.

7 Simplify then evaluate each expression by letting $a = 5$ and $b = 3$.

a $4a + 2a + 6b + 8b$

b $9a - 6a + 7b + 2b$

c $5ab + 4b - 2a - b$

d $6ab - ba - 7ba + a^2$

e $-4b^2 - b^2 + 8a^2 - a^2$

f $\frac{3}{4}a^2 - \frac{1}{2}a^2 + \frac{2}{3}b^2 - \frac{4}{3}b^2$

g $-ab + 3ab^2 - 10ba - 5b^2a$

h $7ab^2 + 2a^2b - 9ab^2 - 5ba^2$

i $\frac{3}{4}a^2b - 2ab^2 + \frac{a^2b}{5} - \frac{7}{6}ab^2$

Exercise 5.3B: Problem solving and reasoning

8 When grouping the like terms in an expression, Tania writes:

$$3x - 2y + 7x - 4y = 3x - 7x + 2y - 4y$$

a Identify and explain Tania's mistake.

b Show the correct grouping of like terms in the expression $3x - 2y + 7x - 4y$.

9 Simplify each expression by collecting like terms.

a $5x + 3y + 6y - 2x - 7y + 8x$

b $\frac{1}{4}y + \frac{1}{6} - \frac{1}{2}y - \frac{4}{3} + \frac{5}{4}y - y$

c $-2xy + 5yx + 4y - 8y + 2y - 3xy$

d $xy - x + 5yx - 3xy + 9x - 5yx$

e $8x^2y + 3yx^2 - 6xy^2 + 4 + 5yx^2 - 9$

f $\frac{3x^2y}{4} + 2xy^2 - 2x^2y - \frac{3}{5}yx^2 - \frac{xy^2}{2} + \frac{7}{4}y^2x - xy^2$

10 Paree is planning to build a rectangular garden bed.

a Draw a diagram of Paree's garden bed. If l = length and w = width, label each of the four sides of the diagram with the appropriate pronumeral.

b Write an expression for the perimeter of the garden bed by adding the lengths of each side.

c Simplify the expression by collecting like terms.

d If P = perimeter, write a formula for the perimeter of the garden bed.

e Use the formula to calculate the total perimeter of three different garden beds with the following dimensions.

i $l = 4$ m, $w = 3$ m

ii $l = 6.4$ m, $w = 2.8$ m

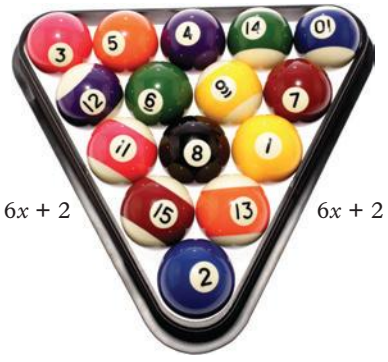
iii $l = 15\frac{1}{2}$ m, $w = 7\frac{1}{2}$ m

f Paree decides to plant a box hedge around the edge of her garden bed. She has enough plants to extend around a perimeter of 20 m. Suggest one possible set of length and width measurements for her garden bed.

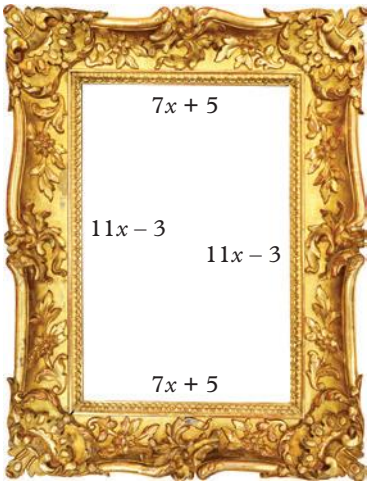
11 Write an expression for the perimeter of each of the objects shown below.

a length of outer edge of frame

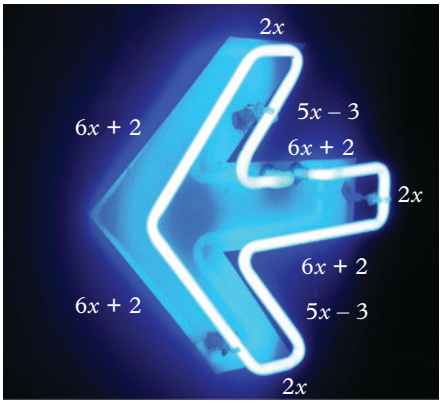
b perimeter of triangle



c perimeter of inner frame



d length of neon used



12 Calculate the perimeter for each object shown in question **11** if $x = 4$ cm.

13 Consider the following working out and explain the error made in each calculation. Give the correct simplified version of the left-hand side in each case.

a $a + a + a + a = a^4$

b $2x + 3y = 5xy$

c $9x + 4x = 13x^2$

d $8x - x = 8$

14 We can consider expressions as *like expressions* if they contain a factor of the exact same grouped expression.

For example, $2(x+3)$, $5(x+3)$, $\frac{x+3}{2}$ are like expressions because they all contain $(x+3)$ as a factor. However, $(x+3)$, $\frac{1}{x+3}$, $\sqrt{x+3}$ are not like expressions. Simplify the following by adding or subtracting like expressions.

a $3(2x+3)+9(2x+3)+4(2x+5)+7(2x+5)$

b $7(x-3y+4)-4(4-3y+x)+9(x+4-3y)-2(-3y+4+x)$

c $7(x+2)+2(x-2)-5(x+2)+6(x-2)$

d $\frac{6}{3x+1}+\frac{7}{3x+1}+\frac{8}{3x-1}+\frac{4}{3x-1}$

e $5\sqrt{7-2x}+4\sqrt{7+2x}-8\sqrt{7+2x}+2\sqrt{7-2x}$

f $-7\sqrt{a+b}+8(a+b)+\frac{2}{a+b}+6\sqrt{a+b}-\frac{5}{a+b}+(a+b)$

15 For each of the following statements, substitute the given values into each expression and evaluate them.

a $\frac{4}{3x}$ and $\frac{4}{3}x$

i $x=1$ **ii** $x=-1$ **iii** $x=2$

b $(-x)^2$ and $-x^2$

i $x=0$ **ii** $x=1$ **iii** $x=-1$

c $(-x)^3$ and $-x^3$

i $x=0$ **ii** $x=1$ **iii** $x=-1$

d $\frac{-3x}{5}$, $\frac{3x}{-5}$, $-\frac{3x}{5}$, and $\frac{-3x}{-5}$

i $x=1$ **ii** $x=-1$ **iii** $x=2$

Exercise 5.3C: Challenge

16 For each set of expressions, state whether the expressions are equivalent or not equivalent and provide reasons to support your claim.

a $\frac{4}{3x}$ and $\frac{4}{3}x$ **b** $(-x)^2$ and $-x^2$ **c** $(-x)^3$ and $-x^3$ **d** $\frac{-3x}{5}$, $\frac{3x}{-5}$, $-\frac{3x}{5}$ and $\frac{-3x}{-5}$

17 Simplify the following expression.

$2a^2b^7c^5d^5e^9f^{11}g^{14}+4f^{11}g^{14}b^7d^5c^5a^2e^9+6a^2e^9g^{14}b^5f^{11}c^5d^7+5b^7f^{11}a^2c^5e^9g^{14}d^5+3f^{11}g^{14}d^7c^5b^5a^2e^9$

18 Simplify the expression $1.\dot{6}x+0.1\dot{6}x+0.41\dot{6}x$.

19 The price of an adult ticket to a theme park is $\$x$. The price of a concession ticket is three-fifths of an adult ticket and the price of a child ticket is five-eighths of a concession ticket. Write an expression for the total cost for a group visiting the theme park with three adults, five concession card holders and two children.

Online resources:

- Interactive skillsheet: Like terms
- Worksheet: Adding like terms
- Worksheet: Like terms and algebra tiles
- Investigation: Help the animals
- Quick quiz: 5.3

Lesson 5.4

Multiplying algebraic terms

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify algebraic products by multiplying algebraic terms.

Inter-year links

Year 7 1.7 Exponents and square roots
Year 9 2.1 Simplifying
Year 10 1.3 Simplifying

Writing in exponent form

- Repeated multiplication of variables can be written in exponent form.

$$\underbrace{3 \times a \times a \times a \times b}_{\text{expanded form}} = \underbrace{3a^3b}_{\text{exponent form}}$$

Simplification by multiplication of algebraic terms

Key content video
Multiplying algebraic terms

- To multiply algebraic terms:
  1. Write each term in expanded form.
  2. Group the coefficients and like variables.
  3. Convert the term back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables, listed in alphabetical order.

$$\begin{aligned} 3a^3b \times (-2a^2b^2) &= 3 \times a \times a \times a \times b \times (-2) \times a \times a \times b \times b \\ &= 3 \times (-2) \times a \times a \times a \times a \times a \times b \times b \times b \\ &= -6a^5b^3 \end{aligned}$$

Worked example 5.4A Writing algebraic terms in exponent form

Simplify the following terms by expressing them in exponent form.

a $2 \times a \times a \times a$ **b** $-9 \times b \times a \times a$

THINK

a 1 Write any repeated multiplications in exponent form. Identify the exponent of each base by counting the number of times that base is repeated in expanded form. a is repeated 3 times in expanded form, so $a \times a \times a = a^3$.

2 Leave out the multiplication signs.

b 1 Write any repeated multiplications in exponent form.

2 Leave out the multiplication signs and list the variables in alphabetical order.

WRITE

a $2 \times a \times a \times a = 2 \times a^3$

$= 2a^3$

b $-9 \times b \times a \times a = -9 \times b \times a^2$

$= -9a^2b$

Worked example 5.4B Multiplying algebraic terms

Simplify the following products.

a $4ab \times 3c$ **b** $5a^2 \times (-a^4)$ **c** $-2p^2q^3 \times 7qp^5$

THINK

1 Write each term in expanded form. Remember the coefficient of $-a^4$ is -1.

2 Group the coefficients and variables.

3 Convert the term back into exponent form by counting the number of times each variable is repeated. Write the simplified coefficient first, followed by the variables, listed in alphabetical order.

WRITE

a $4ab \times 3c = 4 \times a \times b \times 3 \times c$
$= 4 \times 3 \times a \times b \times c$
$= 12abc$

b $5a^2 \times (-a^4) = 5 \times a \times a \times (-1) \times a \times a \times a \times a$
$= 5 \times (-1) \times a \times a \times a \times a \times a \times a$
$= -5a^6$

c $-2p^2q^3 \times 7qp^5 = -2 \times p \times p \times q \times q \times q \times 7 \times q \times p \times p \times p \times p \times p$
$= -2 \times 7 \times p \times p \times p \times p \times p \times p \times p \times q \times q \times q \times q$
$= -14p^7q^4$

Helpful hints

✔ Exponents only apply to the number or variable immediately to the left of the exponent.

For example, in the term $4gh^3$, the exponent of 3 only applies to the variable h, hence: $4gh^3 = 4 \times g \times h \times h \times h$

✔ Use brackets to indicate that the exponent applies to more than one number or variable.

For example, $(4gh)^3 = 4 \times g \times h \times 4 \times g \times h \times 4 \times g \times h$
$= 4 \times 4 \times 4 \times g \times g \times g \times h \times h \times h$
$= 64g^3h^3$

Learning pathways

1–6, 7(1st column), 8(b), 9–10(1st, 2nd columns), 11, 12(a), 13, 15(a–d)	2, 4(2nd column), 6, 7, 9–10(2nd, 3rd columns), 12, 14, 15	4(g–j), 6(e, f), 7(2nd column), 9–10(3rd column), 12, 14–17

ANS p546

Exercise 5.4A: Understanding and fluency

1 Simplify the following terms by determining the sign of the product, placing the coefficient first and leaving out the multiplication sign.

a $4 \times x$ **b** $6 \times k$ **c** $d \times 7$ **d** $m \times 5$ **e** $1 \times p$ **f** $-3 \times h$
g $a \times (-9)$ **h** $-y \times 10$ **i** $8 \times (-c)$ **j** $-1 \times e$ **k** $-2 \times (-f)$ **l** $-m \times (-3)$

2 Simplify the following terms.

a $3 \times a \times b$ **b** $4 \times m \times n$ **c** $p \times 2 \times t$ **d** $d \times (-8) \times c$ **e** $x \times y \times 9$
f $k \times f \times 5$ **g** $a \times b \times c$ **h** $h \times y \times e$ **i** $2 \times q \times 4$ **j** $m \times 3 \times (-6)$

3 Match each of the terms written in expanded form with its exponent form.

Expanded form	Exponent form
a $2 \times a \times a \times a \times b \times c$	**A** $2a^2bc$
b $2 \times a \times a \times b \times c$	**B** $2a^3bc$
c $2 \times a \times b \times b \times c \times c$	**C** $2ab^2c^2$

WE 5.4A **4** Simplify the following terms by expressing them in exponent form.

a $5 \times m \times m$
b $-2 \times a$
c $-1 \times x \times y$
d $n \times 2 \times n$
e $-6 \times d \times d \times c \times c$
f $7 \times a \times a \times c \times b$
g $s \times s \times t \times t \times t \times (-2)$
h $-10 \times g \times g \times g \times f \times f$
i $1 \times q \times p \times q \times r \times q \times r \times r$
j $z \times x \times y \times (-11) \times x \times x \times y \times z$

5 Complete each of the multiplication problems below.

a $2^3 \times 2^4$
$= 2 \times 2 \times 2 \times \square \times \square \times \square \times \square$
$= 2^7$

b $a^3 \times a^4$
$= a \times \square \times \square \times a \times \square \times \square \times \square$
$= \square$

c $m^2 \times m^6$
$= m \times \square \times \square \times \square \times \square \times \square \times \square \times \square$
$= \square$

d $x^5 \times x^2 \times x^3$
$= \square \times \square \times \square \times \square \times \square \times \square \times \square \times \square \times \square \times \square$
$= \square$

6 Simplify each of the expressions below.

a $a^2 \times a^3$
b $k^4 \times k$
c $c^4 \times c \times c^2$
d $e \times e^5 \times f^2 \times f^3$
e $a^3 \times d^5 \times a^2 \times d$
f $m \times n \times n^3 \times m^4 \times m^5 \times n$

WE 5.4B **7** Simplify the following products.

a $6a \times 2bc$
b $-a^2 \times h^3$
c $3wz \times 7xy$
d $6x^4 \times 2y^5$
e $7a^5 \times 3a^2$
f $-5p \times p^{11}$
g $8n \times 3n$
h $5x^6y^5 \times 3x^2y^5$
i $a^5b^4 \times (-a^3b^2)$
j $9xy \times (-2xz)$
k $\frac{1}{6}w^4x^3y^5 \times \frac{6}{5}x^2yz$
l $\frac{3}{7}a^3b^3c \times (-6ca^2b^4)$

8 Complete the multiplication tables below by multiplying each term listed in the left-hand column by each of the terms in the top row.

a

×	$4a$	3	$6m$	$2abc$
$3d$				
ab^2				
$5mn$				
$2ac$				

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$				
xy^2				
$6xy$				
$-7yz$				

9 Simplify each of the following products.

a $-2gh \times 5b \times 7k$ **b** $3g^4 \times 5h^3 \times 2g^6$ **c** $4b^3 \times 3b \times (-b^5)$

d $mn \times 6m \times 8q$ **e** $7gh \times (-3gh) \times fgh$ **f** $\frac{1}{2}m^7 \times (-\frac{3}{4}m^3n) \times \frac{3}{5}mn^2$

g $4a^2b^5 \times 3ac \times 2a^3$ **h** $-2y^5wx^2 \times 5x^5w^2y \times (-xy^2)$ **i** $\frac{5}{4}ab^9d \times (-3a^5b^4c) \times \frac{2}{3}d^3c^6a^2$

10 Complete the following products by finding the missing factor in each of the factor pairs.

a $2 \times \square = 6$ **b** $8 \times \square = -16$ **c** $\square \times k = 6k$ **d** $3 \times \square = 3x$

e $a \times \square = ab$ **f** $x \times \square = -7xy$ **g** $\square \times c = 5cd$ **h** $4a \times \square = 8ab^2$

i $\square \times 3mn = 15mn^4$ **j** $6e \times \square = -12e^3f$ **k** $\square \times 4l^2 = 20jk^2l^3$ **l** $5xy \times \square = -5x^2yz^5$

Exercise 5.4B: Problem solving and reasoning

11 There are a few conventions mathematicians follow when using fractional coefficients. Use your understanding of fractions and expanding expressions to answer the following.

a Which of the following is equal to $\frac{3}{2} \times 5$?

A $\frac{3 \times 5}{2 \times 5}$ **B** $\frac{3}{2 \times 5}$ **C** $\frac{3 \times 5}{2}$ **D** $\frac{3}{2} \times \frac{1}{5}$

b Which of the following is equal to $\frac{3}{2}x$?

A $\frac{3}{2} \times \frac{1}{x}$ **B** $\frac{3}{2x}$ **C** $\frac{3x}{2}$ **D** $\frac{3x}{2x}$

c State the coefficient of x and y for $\frac{2 + 7x - y}{5}$.

d Which of the following is equal to $3\frac{1}{2}$?

A $3 \times \frac{1}{2}$ **B** $3 \div \frac{1}{2}$ **C** $3 + \frac{1}{2}$ **D** $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

e Which of the following is equal to 32?

A $30 - 2$ **B** 3×2 **C** $3 \div 2$ **D** $30 + 2$

f Which of the following is equal to $3x$?

A $3 \times x$ **B** $3 \div x$ **C** $3 + x$ **D** $x \times x \times x$

g Using your answers from parts **c** and **d**, explain why mathematicians would choose not to use a mixed number, where possible, as a coefficient, such as $3\frac{1}{2}x$.

12 a Complete the following calculations.

i $(3x^2)^4 = (3x^2) \times (\square) \times (\square) \times (\square)$

$= 3 \times \square \times \square \times \square \times x^2 \times \square \times \square \times \square$

$= \square$

ii $\left(\frac{x^2}{3}\right)^4 = \left(\frac{x^2}{3}\right) \times \left(\frac{\square}{\square}\right) \times \left(\frac{\square}{\square}\right) \times \left(\frac{\square}{\square}\right)$

$= \frac{x^2 \times \square \times \square \times \square}{3 \times \square \times \square \times \square}$

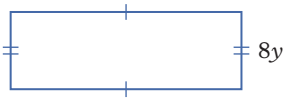
$= \frac{\square}{\square}$

b Use the method in part **a** to simplify the following expressions by writing them without brackets.

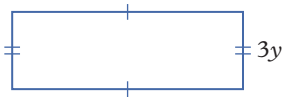
i $(2x^5)^6$ **ii** $\left(\frac{y^9}{2}\right)^5$ **iii** $(4a^2b^7)^3$ **iv** $\left(\frac{5u^6}{2v^5}\right)^4$

13 Write a simplified expression for the areas of the following shapes. Hint: See **Lesson 5.2** question **12** for formulas to find the area of each shape.

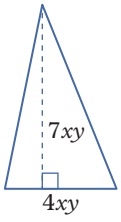
a



b



c



14 For each of the following,

i draw a labelled diagram

ii write a simplified expression for the area of the shape.

a A rectangle has a length three times its width.

b A triangle has a perpendicular height that is 4.3 times the width of the base.

c An isosceles triangle has a height, perpendicular to one of the two equal side lengths, that is half the length of the two equal side lengths.

15 The top of this box is rectangular, with a length three times its width. All measurements are in centimetres.

a If x is equal to the width of the top of the box, write an algebraic term to represent the length of the top of the box.

b The area of a rectangle is equal to its length times its width. Write an algebraic expression to represent the area of the top of the box, then simplify.

c Calculate the area of the top of the box if $x = 8$ cm.

d How can you check that your answer to part **c** is correct?

e The height of the box is five times the width. Write an algebraic term to represent the height of the box.

f Draw a diagram of the box and label the length, width and height of the box with their respective algebraic terms.

g Complete this table to obtain the simplified algebraic term for the area of each of the six faces of the box.

Face	Area
Top	$3x \times x = 3x^2$
Bottom	$\square \times \square = \square$
Front	$\square \times \square = \square$
Back	$\square \times \square = \square$
Left side	$\square \times \square = \square$
Right side	$\square \times \square = \square$

h Use your answers to part **g** to write an expression for the surface area of the box. Simplify the expression.

i Calculate the surface area of the box if $x = 4$ cm.

j Calculate the surface area of the box for each value of x.

i 5 **ii** 12 **iii** 0.5 **iv** 2.1

k Using *volume* = *length* × *width* × *height*, write an algebraic term to represent the volume of the box.

l Use a calculator to work out the volume of the box for each x value given in part **j**.

Exercise 5.4C: Challenge

16 The expression $a^3 \times a^4 = a^{3+4}$ as we are multiplying three a's and four a's to make a total of seven a's multiplied together. Simplify $a^{58} \times a^{97} \times a^{45}$ and explain why your answer is correct without writing the repeated multiplication out.

17 Simplify $2^{5a+1} \times 3^{8+10a} \times 2^{7a} \times 2^4 \times 3^{3a+8}$.

Online resources:

Interactive skillsheet
Multiplying terms

Worksheet
Writing terms in expanded form

Worksheet
Multiplying algebraic terms

Quick quiz
5.4

ANS p547

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

5.1 **1** For the following expressions, identify:

i the pronumerals
ii the number of terms
iii the coefficients
iv the constant
v the highest exponent of each pronumeral.

a $4x - 3 + 9y - 4xy^2$
b $-\frac{9}{13} + \frac{5m^2}{3} + nv^3$

5.1 **2** Write an expression for each of the following.

a The quotient of x and 8
b The sum of a, b and c multiplied by 7
c The product of y and three more than y
d The square root of the difference of c and a

5.1 **3** Write an expression for each of the following.

a A cruel telephone company charges $20 per gigabyte after you exceed your data limit.
b The score an AFL team gets in a game is six times the number of goals plus the number of behinds.
c One-third of the total number of flowers in a garden containing tulips, roses and poppies must be red.
d Three more than 15% of the number of bread rolls in a bakery have seeds.

5.2 **4** Substitute the values into the expression and evaluate.

a $2x + 3y + 7z$ where $x = 5$, $y = 6$ and $z = 8$
b $3x^2 - 4x$ where $x = 2$
c $7(xy^2 + 1)$ where $x = 9$ and $y = 3$
d $\frac{4 + x^2}{5} + \frac{3}{10}y$ where $x = 4$ and $y = 15$

5.2 **5** Substitute the given values into the formula and evaluate.

a Find the value of y using the formula $y = mx + c$ where $m = 7$, $x = 5$ and $c = -4$.
b Find the value of s using the formula $s = \frac{1}{2}(u + v)t$ where $u = 4$, $v = 12$ and $t = 8$.

5.2 **6** Substitute the values into the expression and evaluate.

a $2x + 3y + 7z$ where $x = 0.5$, $y = \frac{1}{6}$ and $z = \frac{17}{2}$
b $3x^2 - 4x$ where $x = -2$

5.3 **7** Simplify each of the following expressions.

a $2x + x + 7y + 4x - 3y + 9 + 1$
b $7a + 9b - 3ab + b - 2a + 5ba$
c $-6pqr + 7rpq - 9qpr + 5prq + 11qrp$
d $14ikl + 16jkl + 17lki - 8klj - kli$

5.3 **8** Simplify each of the following expressions.

a $2x + x^2 - 5x + 3x^2$
b $7xy^2 + 8yx^2 - 6x^2y + 5y^2x$

5.3 **9** Simplify each of the following expressions.

a $1.72x + 4.24y - 9.18x - 3.98y$
b $\frac{5}{4}x^2 + \frac{3x^2}{8} - \frac{7}{2}x^2$

5.4 **10** Simplify each of the following expressions.

a $7x \times 9y$
b $-3ac \times (-9bd)$
c $2(5gh)(-11pq)$
d $4xy \times abc \times 9uv$

5.4 **11** Simplify each of the following expressions.

a $12x^2 \times 3x^3$
b $7xy^2 \times 8x^3y^2$

5.4 **12** Simplify each of the following expressions.

a $0.12x \times 3.1x$
b $\frac{25x^8}{16} \times \frac{24}{15}x^4$

Lesson 5.5

Dividing algebraic terms

Learning intentions

By the end of this lesson you will be able to ...

✔ simplify expressions by dividing algebraic terms.

Inter-year links

Support	Equivalent fractions
Year 7	3.2 Equivalent fractions
Year 9	2.1 Simplifying
Year 10	1.3 Simplifying

Simplification by division of algebraic terms

Key content video
Dividing algebraic terms

- To divide algebraic terms:
  1. Write each term in expanded form.
  2. Divide the coefficients by the highest common factor (HCF).
  3. Cancel out repeated variables that appear in both the numerator and denominator.
  4. Convert the term back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

$$\frac{6a^5b^3}{3a^3b} = \frac{\cancel{6}^{2} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a \times \cancel{b} \times b \times b}{\cancel{3}^{1} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b}}$$
$$= \frac{2 \times a \times a \times b \times b}{1}$$
$$= 2a^2b^2$$

- Remember that quotients can be expressed as fractions.

$$x \div 7 = \frac{x}{7}$$

- Remember that a non-zero number divided by itself is equal to 1.

$$x \div x = x \times \frac{1}{x} = \frac{x}{x} = 1$$

Worked example 5.5A Dividing algebraic terms

Simplify the following quotients.

a $\frac{5a^2}{a}$ **b** $\frac{8a^3b}{2a^2b^2}$

THINK

1. Write each term in expanded form.
2. Divide the coefficients by the highest common factor (HCF).
3. Cancel out repeated variables that appear in both the numerator and denominator. Remember that any term divided by itself is equal to 1, so $\frac{a}{a} = 1$ and $\frac{b}{b} = 1$.
4. Convert the term(s) back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables, listed in alphabetical order.

WRITE

a $\frac{5a^2}{a} = \frac{5 \times a \times a}{a}$
$= \frac{5 \times a^{\cancel{1}} \times a}{a^{\cancel{1}}}$
$= \frac{5 \times a}{1}$
$= 5a$

b $\frac{8a^3b}{2a^2b^2} = \frac{8 \times a \times a \times a \times b}{2 \times a \times a \times b \times b}$
$= \frac{8^{\cancel{4}} \times a^{\cancel{1}} \times a^{\cancel{1}} \times a \times b^{\cancel{1}}}{2^{\cancel{1}} \times a^{\cancel{1}} \times a^{\cancel{1}} \times b^{\cancel{1}} \times b}$
$= \frac{4 \times a}{b}$
$= \frac{4a}{b}$

Helpful hints

- ✔ Take care when writing variables with a large exponent in expanded form. Make sure the number of times a variable is repeated in expanded form is equal to its exponent when written in exponent form.
- ✔ The divisibility rules can help you to find the HCF. Recall that a number is:
  - → divisible by 2 if the number is even
  - → divisible by 3 if the number's digits add to a multiple of three
  - → divisible by 5 if the number ends in zero or five.

Learning pathways

1, 2–3(a–j), 4–11 | 2–3(f–o), 5–13 | 3(f–o), 7–15

ANS p548

Exercise 5.5A: Understanding and fluency

1 Write the following fractions in their simplest form.

a $\frac{18}{9}$ **b** $\frac{27}{15}$ **c** $\frac{32}{12}$ **d** $\frac{125}{75}$ **e** $\frac{348}{66}$ **f** $\frac{729}{108}$

WE 5.5A

2 Simplify the following quotients.

a $\frac{5a}{5}$ **b** $\frac{15x}{3y}$ **c** $\frac{9d}{d}$ **d** $\frac{12w}{w}$ **e** $\frac{3m}{9m}$

f $\frac{4ab}{b}$ **g** $\frac{18c^3}{c^3}$ **h** $\frac{10b^5}{15b^5}$ **i** $\frac{4mnp}{24mn}$ **j** $\frac{35wx}{21wxy}$

k $\frac{6a^4b^2}{b^2}$ **l** $\frac{5m^3}{m^3n^4}$ **m** $\frac{6bcm^3}{9c}$ **n** $\frac{14s^3t^4}{7s^3}$ **o** $\frac{8h^5jk^2}{10h^5k^2}$

3 Simplify the following quotients.

a $\frac{7a^2}{7a}$ **b** $\frac{15x^3}{x^2y}$ **c** $\frac{10e}{e^2}$ **d** $\frac{14w^3}{w}$ **e** $\frac{12m^2n}{36m}$

f $\frac{9a^2b^2}{ab}$ **g** $\frac{2c^3}{c^2}$ **h** $\frac{8b^5}{12b^3}$ **i** $\frac{6m^3n^2p}{36mn}$ **j** $\frac{54w^5x^3}{36w^4x^2y}$

k $\frac{7a^4b^2}{a^2b}$ **l** $\frac{8mn^2}{m^3n^4}$ **m** $\frac{9bc^2m^3}{12cm}$ **n** $\frac{22st^4}{11s^3t}$ **o** $\frac{40h^5jk^2}{45h^3k}$

4 Complete the following division problems.

a $xy \div y$ **b** $8k \div 4$ **c** $18ab \div 2b$

d $9x^2y \div xy$ **e** $3mn^2 \div 12mn$ **f** $ab^2c^3d \div abc^2$

5 Simplify the following expressions.

a $\frac{-9xy}{3x}$ **b** $30a^3b^2 \div (-6ab^2)$ **c** $\frac{-12uv^3}{-8uv}$ **d** $-18rsv \div 21svp$

6 Simplify the following expressions.

a $\frac{x^4 \times x^3}{x^2}$ **b** $\frac{m^7 \times m^6}{m^9}$ **c** $\frac{6a^2 \times a^8}{a^4}$ **d** $\frac{n^5 \times n^7}{n^3 \times n^4}$ **e** $\frac{5d^6 \times d^3}{d^9}$

7 Fill in the blanks to complete the equation.

a $16m \div \square = 2m$ **b** $\frac{27jk}{\square} = -3k$ **c** $\square \div 2xy = 8x^2y$ **d** $\frac{\square}{10a^2b^3} = \frac{a}{5b}$

8 Simplify the following expressions.

a $\frac{8x^3y}{4x} + \frac{3x^2y^2}{y} - \frac{18x^5y^3}{3x^3y^2}$

b $\frac{16b^5a^3 - 3a^3b^5 + 7a^3b^5}{2a^2b}$

c $\frac{9x^2y}{3x^2} + \frac{16x^3y^5}{8x^2y^5} + \frac{36x^{11}y^9}{4x^{11}y^8} + \frac{15x^7y}{x^7} + \frac{33xy^{15}}{11y^{15}}$

d $\frac{4mn^2 + 15nm^2 - 6m^2n^2 + 17n^2m - 11m^2n}{mn}$

Exercise 5.5B: Problem solving and reasoning

9 Simplify the following quotients, then evaluate by substituting the given value(s).

a $\frac{42c^2d^6}{35c^5d^4}$ where $d = 5$ and $c = 6$

b $a^5z^3 \div az^2$ where $a = 2$ and $z = 4$

c $\frac{25pq}{5p}$ where $p = 3.5$ and $q = 8.1$

d $\frac{28x^4y^9}{4y^9x^4}$ where $x = 31$ and $y = 78$

10 Fabian and his father are laying cement pavers in the backyard to create a smooth surface for skateboarding. The pavers come in different sizes but are rectangular in shape and have a length twice their width.

a Let x represent the width of a paver in centimetres. Write the length in terms of x and hence write an algebraic term for the area of one paver.

b The ground to be paved is square with a length represented by $10x$ cm. Write an algebraic term for the area of the square to be paved.

c How many pavers are required to pave the ground?

d To check whether your answer to part **c** is correct, substitute a value for x. For example, let $x = 40$, and evaluate each of the following using this value of x.

i width of paver **ii** length of paver **iii** area of paver

iv length of the square to be paved **v** area of the square to be paved

e Use your answers to part **d iii** and **v** to calculate the number of pavers required. How does this compare to your answer to part **c**?

f Will the size of the pavers affect the number of pavers needed? Explain.

11 To divide two terms, we can multiply the first term by the reciprocal of the second term. Complete the following division problems.

a $\frac{3x}{8} \div \frac{5x}{4}$ **b** $\frac{9x^3}{4y} \div \frac{4x}{3y^2}$ **c** $\frac{x^2y^3}{z^5} \div \frac{1}{xyz}$ **d** $\frac{1}{ab^2c^3} \div a^2bc^3$

12 The lowest common multiple (LCM) of two numbers is the common multiple with the lowest value. When written as the product of prime factors, the LCM is the product of the highest power of each factor.

For example, the LCM of $2^3 \times 3^2 \times 5^6$ and $2^4 \times 3^5 \times 5$ is $2^4 \times 3^5 \times 5^6$. Similarly, the LCM of $x^3y^2z^6$ and x^4y^5z is $x^4y^5z^6$. Determine the LCM of the following:

a $2^9 \times 13^5 \times d^{12}$ and $2^{14} \times 13^2 \times d^6$

b $3^2 \times 7^4 \times a^9 \times b^2$ and $3^4 \times 7^2 \times a^5 \times b^6$

c $p^6q^7r^{11}s^6$ and $p^9q^2r^{15}s^2$

d $r^{42}s^{51}v^{99}$ and $r^{45}t^{78}u^{84}$

13 Simplify each expression.

a $5g \times 3h + \frac{8g^2h}{4g}$

b $\frac{3t \times 7u - 5 \times ut}{4t}$

c $8r^2 - 9r \times (2r - 7r)$

d $a^2 \times 2a - 10a^4 \div 2a^2 + \frac{6a^5}{3a^3} + 11a \times a$

Exercise 5.5C: Challenge

14 Simplify the following expression:

$$\frac{2x - 2x^2 \div 2x + 2x \times x - x^5 \div x^3}{3x + x^3 \div \left(5x \times \frac{x}{5}\right)}$$

15 Consider the following equation: $x^ay^b \div x^cy^d = x^ey^f$.

a Using six of the digits 1-9 only once, substitute values for a, b, c, d, e and f such that the following equation is correct and the value of $e + f$ is as large as possible. State the value of $e + f$.

b How do you know that this is the largest possible value of $e + f$?

Online resources:

Interactive skillsheet
Dividing terms

Worksheet
Dividing algebraic terms

Quick quiz
5.5

Lesson 5.6

Expanding

Learning intentions

By the end of this lesson you will be able to ...

✔ expand algebraic expressions.

Inter-year links

Year 7 6.3 Order of operations in algebra
Year 9 2.2 Expanding
Year 10 1.4 Expanding

The distributive law

- The distributive law applies to both numerical and algebraic expressions.

	Arithmetic	Algebra
The distributive law	$7(30 + 2) = 7 \times 30 + 7 \times 2$ $= 210 + 14$ $= 224$	$a(b + c) = ab + ac$

Expanding algebraic expressions

Key content video
Expanding algebraic expressions

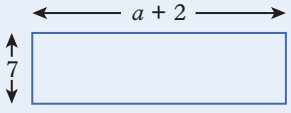
- To **expand** an algebraic product containing brackets using the distributive law:
  1 Multiply each term inside the brackets by the term outside the brackets.
  2 Simplify the results by performing any multiplication, addition and subtraction.

$$a(b + c) = a \times b + a \times c$$
$$= ab + ac$$

- If there are more than two terms in the expression inside the brackets, the distributive law continues to apply.
  For example, $a(b + c + d) = ab + ac + ad$.

The area model

- The use of the distributive law in algebra can be understood using calculations for the area of a rectangle. Recall that the formula for the area of a rectangle is Area = length × width.
  For example, consider a rectangle with length $a + 2$ and width 7.



  The area of the rectangle can be found by directly multiplying the length and width, or by splitting $a + 2$ into its two terms.
  → It may help to use a grid to complete both multiplications.

×	$a + 2$
7	$7(a + 2)$

×	a	2
7	$7a$	14

- The area of the rectangle is therefore equal to both $7(a + 2)$ and $7a + 14$, which implies that $7(a + 2) = 7a + 14$.

Worked example 5.6A Expanding by a whole number

Expand and simplify each expression using the distributive law.

a $6(a + 4)$ **b** $2(3x - y)$ **c** $3(2x + 4y + 5)$

THINK

a **1** Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications.

b **1** Multiply each term inside the brackets by the term outside the brackets. Note that the second term inside the brackets is negative.

2 Simplify the results by performing the multiplications.

c **1** Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications.

WRITE

a $6(a + 4) = 6 \times a + 6 \times 4$

$= 6a + 24$

b $2(3x - y) = 2 \times 3x + 2 \times (-y)$

$= 6x - 2y$

c $3(2x + 4y + 5) = 3 \times 2x + 3 \times 4y + 3 \times 5$

$= 6x + 12y + 15$

Worked example 5.6B Expanding by a variable

Expand and simplify each expression.

a $-7p(3q - 4)$ **b** $2m(5m - 4n)$ **c** $5a(a - 3b - 5)$

THINK

a **1** Multiply each term inside the brackets by the term outside the brackets. Enclose the second term in brackets to separate the plus sign and the negative coefficient of the p term.

2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.

b **1** Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications. Remember that $m \times m = m^2$.

c **1** Multiply each term inside the brackets by the term outside the brackets.

2 Simplify the results by performing the multiplications. Remember the product of a negative and positive number is negative.

WRITE

a $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$

$= -21pq + 28p$

b $2m(5m - 4n) = 2m \times 5m + 2m \times (-4n)$

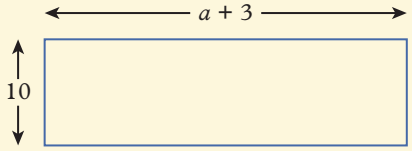
$= 10m^2 - 8mn$

c $5a(a - 3b - 5) = 5a \times a + 5a \times (-3b) + 5a \times (-5)$

$= 5a^2 - 15ab - 25a$

Worked example 5.6C Using the area model

Demonstrate that $10(a + 3) = 10a + 30$ by finding the area of the following rectangle in two different ways.



THINK

1 Write an expression for the area of the rectangle using the length and width as given in the question and using a grid. Remember to use brackets when multiplying the two expressions together.

2 Count the number of terms in the algebraic expression. There are two terms: a and 3. Draw a grid to represent splitting the rectangle into two smaller rectangles. Fill in the terms which need to be multiplied together in the first column and row in the grid.

3 Multiply the two pairs of terms and write the results in their simplest form.

4 Recognise that the area of the rectangle is equal to the sum of the two smaller rectangles.

WRITE

Area = length × width
$= 10 \times (a + 3)$
$= 10(a + 3)$

×	$a + 3$
10	$10(a + 3)$

×	a	3
10		

$10 \times a = 10a$
$10 \times 3 = 30$

×	a	3
10	$10a$	30

$10(a + 3) = 10a + 30$

Helpful hints

- ✔ When multiplying through by a negative term, use brackets for clarity.
  For example, $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
- ✔ Show all your working out to avoid making arithmetical errors and errors related to sign.
- ✔ After expanding expressions containing brackets, always simplify your results. Simplify, simplify, simplify!
- ✔ When expanding an expression containing brackets, make sure to multiply every term inside the brackets by the term outside the brackets.
  For example, $3(a + 4) = 3a + 12$, not $3a + 4$!

Learning pathways

1–8, 11, 12	2–4, 5(c, d), 6(b, d, f, h), 7, 8(c, f, g, h), 10, 14, 15, 17	2(2nd, 3rd columns), 3(e, f), 4, 6(b, d, f, h), 8(c, f, g, h), 10(c, d), 14–18

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Exercise 5.6A: Understanding and fluency

WE 5.6A **1** Expand and simplify each expression using the distributive law.

a $3(a + 2)$ **b** $5(k - 7)$ **c** $2(5 + a)$

d $8(a + b)$ **e** $4(2x + 6)$ **f** $2(-1 + 8m)$

g $7(5a + 3b)$ **h** $6(8c - 5d)$ **i** $3(-5m - 2n)$

WE 5.6B **2** Expand and simplify each expression.

a $a(b + 4)$ **b** $x(3 + w)$ **c** $d(c - e)$

d $p(2p + 3)$ **e** $k(1 - 4k)$ **f** $2x(4y + 3w)$

g $3x(1 - 4x)$ **h** $\frac{1}{3}m(2m + 7n)$ **i** $-3a(2a - 4b)$

j $-2e^2(-4e - 5)$ **k** $5a^2\left(\frac{1}{10}a^2 - 3ab\right)$ **l** $-11y(x^2 + y^2)$

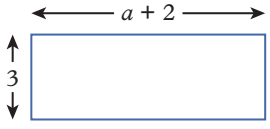
3 Use the distributive law to expand and simplify the following expressions.

a $2(3x + y + 7)$ **b** $5(2ab + 4d - 3)$ **c** $7(2c - d + 5ef)$

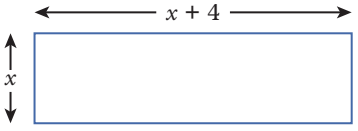
d $-6(3g - 2h + gh)$ **e** $-4(5k + 2km - 9)$ **f** $-3(2xy + 4 - 7w)$

WE 5.6C **4** Demonstrate that each statement is true by finding the area of each rectangle below in two different ways.

a $3(a + 2) = 3a + 6$



b $x(x + 4) = x^2 + 4x$



5 Apply the distributive law, then simplify by collecting any like terms to complete the following calculations.

a $5(a + 4) + 2(a + 7)$

$= 5 \times \square + 5 \times \square + 2 \times \square + 2 \times \square$

$= 5a + \square + 2a + \square$

$= \square a + \square$

b $3b(c + 2) + 2c(a + 5b)$

$= \square \times c + \square \times 2 + 2c \times \square + 2c \times \square$

$= 3bc + \square + \square + \square$

$= \square + \square + \square$

c $d(d + 8) + 6(d - 3)$

$= d \times \square + d \times \square + \square \times d + \square \times (-3)$

$= \square + 8d + \square - \square$

$= \square + \square - \square$

d $4k(2k + 3) - 2(k - 5)$

$= 4k \times \square + 4k \times \square + (-2) \times \square + (-2) \times (-5)$

$= 8k^2 + \square - \square + \square$

$= \square + \square + \square$

6 Expand and simplify each expression.

a $2(a + 6) + 5(a + 3)$ **b** $4(b - 3) + 6(b + 2)$

c $5m(n + 4) + 3n(k + 2m)$ **d** $7x(3 + y) + \frac{2}{3}y(8 - 3x)$

e $\frac{k}{4}(k - 1) + 5(k + 4)$ **f** $c(c + 4) - 7(c - 5)$

g $rst(2 + 5t) + 9r(4t + st)$ **h** $h(h^2 - 2) - 4h(h + 3)$

7 Complete the following expansion.

$-(a+2) = -1 \times (a+2)$

$= -1 \times \square - 1 \times \square$

$= -\square - \square$

8 Expand and simplify each expression.

a $6(p+3) + (p+2)$ **b** $3(d-2) - (d+8)$ **c** $x(x+2) + (x+5)$

d $k(k+4) - (3k+1)$ **e** $2e\left(e+\frac{1}{2}\right) - \left(e-\frac{3}{2}\right)$ **f** $4a(b+1) - (6+ab)$

g $\frac{1}{3}w(y-3) - (wy+2)$ **h** $mp(n-m) + (n-3m^2p)$

Exercise 5.6B: Problem solving and reasoning

9 Consider the expression $5a(3a+10)$.

a Calculate the value of the expression if $a = 4$ without expanding the expression.

b Expand the expression.

c Substitute $a = 4$ into your answer to part **b** and evaluate the resulting expression.

d Was it easier to evaluate the expression containing brackets or its expanded form? Explain.

10 Expand and then evaluate each expression by letting $a = 2$.

a $2a(a+5)$ **b** $3a(10-2a)$ **c** $5a(3a-13)$ **d** $\frac{a}{3}(6a-9)$

11 In three years, Emily will be twice as old as Chantelle.

a If Chantelle is currently c years old, write an expression for Chantelle's age in three years' time.

b Use your answer to part **a** to write an expression for Emily's age in three years' time.

c Use the distributive law to expand the expression for Emily's age in three years' time.

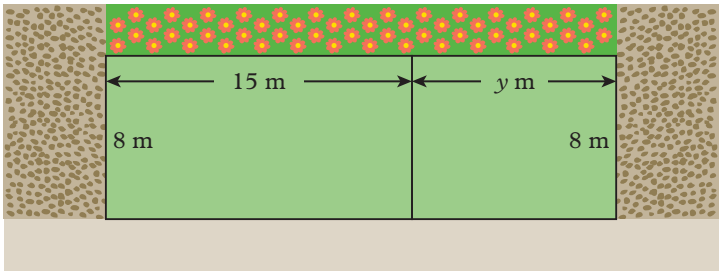
d Determine Emily's age in three years' time if Chantelle is currently 18 years old.

12 One of Andre's jobs is to mow the front lawn. The front lawn is rectangular and measures 15 m by 8 m.

a Calculate the area of lawn to be mown.

b A neighbour notices Andre is about to start mowing and asks Andre to mow his front lawn as well.

i Let y be equal to the width of the neighbour's front lawn. Write an expression for the area of lawn in the neighbour's front yard.



ii Add your answers to parts **a** and **b** to write an expression for the total area of lawn to be mown.

iii Now consider both lawns as one large rectangle. Draw a diagram of this rectangle and label the length and width measurements.

iv Use brackets to write an expression for the total area of the large rectangle.

v Expand the expression and compare the expanded expression to your answer to part **c**. Are the expressions equal? Why or why not?

13 Demonstrate that each of the following equations are correct by first drawing a suitable rectangle and then finding its area in two different ways.

a $6(k + 8) = 6k + 48$ **b** $4(2d + 9) = 8d + 36$ **c** $2p(p + 3) = 2p^2 + 6p$

14 The area model is a useful tool to understand expanding brackets but has some limitations.

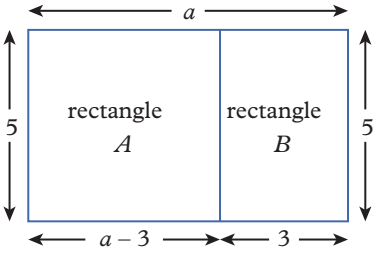
a Show that $5(a - 3) = 5a - 15$ using the same method as you did in question **4**.

b The grid multiplication you should have performed is correct, however, there is an issue with using area to show the two expressions are equal.

i What is the issue in your working in part **a**? Hint: Do both of the small rectangles exist?

ii Suggest one limitation of the area model.

c An alternative method is to use the following diagram.



i What are the length and width of rectangle A?

ii Use your answer from part **i** to write an expression for the area of rectangle A.

d Another way to calculate the area of rectangle A from part **c** is to subtract the area of rectangle B from the total area of the large rectangle.

i State the length and width of the large rectangle.

ii Use these length and width measurements to write an expression for the area of the large rectangle.

iii State the length and width of rectangle B.

iv Use these length and width measurements to write an expression for the area of rectangle B.

v Use your answers to parts **ii** and **iv** to write an expression for the area of rectangle A.

15 Draw a diagram to demonstrate $c(d - 5) = cd - 5c$.

16 If a and b are both positive integers, is $5a \times b + 3 < 5a(b + 3)$? Use expansion to justify your answer.

Exercise 5.6C: Challenge

17 Expand and simplify each expression.

a $3x^2(4x^3 - 7) - 5x^4(x + 2)$

b $a^3(ab + c - 1) + a^2(3ac - a^2b + a)$

c $2m(3m^2np^2 + 5mn) - 6n(m^2 + m^2p^2 - 4)$

d $5w(4w^2x - 3y) + 2y(9w - 6xy) - 4x(5w^3 - 3y^2)$

18 Evaluate each expression in question **17** using these values:

$a = -3, b = 7, c = 4, m = 2, n = -4, p = -1, w = -5, x = 2, y = -6$

Online resources:

- **Interactive skillsheet** Expanding over one pair of brackets
- **Worksheet** Using the distributive law to expand expressions
- **Worksheet** Expanding using algebra tiles
- **Investigation** Andrew's vegie patch
- **Desmos activity** Equivalent expressions
- **Quick quiz** 5.6

Lesson 5.7

Factorising

Learning intentions

By the end of this lesson you will be able to ...

✔ factorise algebraic expressions using the HCF.

Inter-year links

Support Multiples and factors

Year 7 2.4 Factors and the highest common factor

Year 9 2.3 Factorising using the HCF

Year 10 1.6 Factorising

The highest common factor (HCF) of algebraic terms

- A factor of an algebraic expression is an expression which divides exactly into the given expression. For example, 1, 2, $(x + 3)$ and $2(x + 3)$ are all factors of $2(x + 3)$.
- A common factor of two or more algebraic terms is a term that divides exactly into each of those terms.
- The highest common factor (HCF) of two or more algebraic terms is the 'highest' or 'largest' combination of coefficients, pronumerals and algebraic expressions that are factors of every term. The HCF of a set of algebraic terms can have both a numerical and a pronumeral component.

Set of algebraic terms	HCF
x, $7x$	x
$6m$, $12mn$, $8mp$	$2m$
$15a^3bc$, $9a^2c$	$3a^2c$

- To identify the HCF of a set of algebraic terms:
  1. Write each term as the product of its prime and algebraic factors.
  2. Circle the common factors.
  3. Multiply the common factors for the HCF.

$15a^3bc = 3 \times 5 \times a \times a \times a \times b \times c$

$9a^2c = 3 \times 3 \times a \times a \times c$

$\text{HCF} = 3 \times a \times a \times c$

$= 3a^2c$

Factorising algebraic expressions

Key content video
Factorising using the HCF

- **Factorising** an algebraic expression is the process of writing an algebraic expression as a product of its factors. Factorising is the opposite to expanding an algebraic expression.
- An algebraic expression is in simplest factorised form if the HCF of the factors is 1. For example, the simplest factorised form of $2x + 6$ is $2(x + 3)$, not $1(2x + 6)$.

expanding →

$7(a + 2) = 7 \times a + 7 \times 2 = 7a + 14$

← factorising

- The distributive law can be applied to factorise some algebraic expressions.
- To factorise an algebraic expression using the distributive law:
  1. Identify the HCF of all the terms in the expression.
  2. Write the HCF in front of the brackets.
  3. Inside the brackets, divide each term in the original expression by the HCF.
  4. Simplify the expression inside the brackets.

$7a + 14 = 7\left(\frac{7^1a}{7^1} + \frac{14^2}{7^1}\right)$

$= 7(a + 2)$

Original expression: $7a + 14$

HCF: 7

Factorised expression: $7(a + 2)$

Worked example 5.7A Identifying the highest common factor

Find the highest common factor (HCF) of each pair of terms.

a $2a$ and $5a$

b $12xy$ and $-18x^3$

THINK

a 1 Write each term as the product of its prime and algebraic factors.

2 Circle the common factors.

3 The only common factor is 'a', so the HCF is a.

b 1 Write each term as the product of its prime and algebraic factors. The negative sign can be represented using the factor '-1'.

2 Circle the common factors.

3 Multiply the common factors for the HCF. Simplify the product.

WRITE

a $2a = 2 \times (a)$

$5a = 5 \times (a)$

HCF $= a$

b $12xy = (2) \times 2 \times (3) \times (x) \times y$

$-18x^3 = -1 \times (2) \times (3) \times 3 \times (x) \times x \times x$

HCF $= 2 \times 3 \times x$

$= 6x$

Worked example 5.7B Factorising algebraic expressions

Factorise the following expressions.

a $8x + 28$

b $9xyz - 12wxy$

THINK

a 1 Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term in the original expression by the HCF.

4 Simplify the bracketed expression.

b 1 Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term in the original expression by the HCF. Note that the minus sign remains from the original expression.

4 Simplify the bracketed expression.

WRITE

a HCF $= 4$

$8x + 28 = 4\left(\frac{8^{2}x}{4^{1}} + \frac{28^{7}}{4^{1}}\right)$

$= 4(2x + 7)$

b HCF $= 3xy$

$9xyz - 12wxy = 3xy\left(\frac{9^{3}x^{1}y^{1}z}{3^{1}x^{1}y^{1}} - \frac{12^{4}wx^{1}y^{1}}{3^{1}x^{1}y^{1}}\right)$

$= 3xy(3z - 4w)$

Worked example 5.7C Factorising algebraic expressions containing exponents

Factorise each expression.

a $a^2 - 6a$ **b** $10m^3n + 15m^2n^2$

THINK

a **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term by the HCF. It is helpful to write the pronumerals with exponents in expanded form. Note that the minus sign remains from the original expression.

4 Simplify the bracketed expression.

b **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term by the HCF. It is sometimes helpful to write the pronumerals with exponents in expanded form. Alternatively, an exponent law can be used to simplify the exponents.

4 Simplify the bracketed expression.

WRITE

a HCF = a

$$a^2 - 6a = a\left(\frac{a^1 \times a}{a^1} - \frac{6 \times a^1}{a^1}\right)$$
$$= a(a - 6)$$

b HCF = $5m^2n$

$$10m^3n + 15m^2n^2 = 5m^2n\left(\frac{\cancel{10}^{2} \times m^1 \times m^1 \times m \times n^1}{\cancel{5}^{1} \times m^1 \times m^1 \times n^1} + \frac{\cancel{15}^{3} \times m^1 \times m^1 \times n^1 \times n}{\cancel{5}^{1} \times m^1 \times m^1 \times n^1}\right)$$
$$= 5m^2n(2m + 3n)$$

Helpful hints

- ✔ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.
- ✔ Expand your answer to check you have correctly factorised an expression.
- ✔ Make sure to divide all terms inside the brackets by the HCF.
  For example, $3x + 21 = 3(x + 7)$ not $3(x + 21)$!

Learning pathways

1, 2(1st column), 3, 4(1st, 2nd columns), 5, 6(1st, 2nd columns), 7, 8–9(1st column), 10–13	2(2nd column), 4(2nd, 3rd columns), 6(3rd, 4th columns), 7, 8–11(2nd column), 14–16	2(h, j, l, n), 4(3rd column), 6(4th column), 7, 8–11(2nd column), 14–18

ANS p549

Exercise 5.7A: Understanding and fluency

1 Match each pair of terms with their HCF.

Terms	**a** $15a$ and 10	**b** $5ab$ and $5a$	**c** $20a^2b$ and $10ab$	**d** $15a^5b^3$ and $30bc^2$
HCF	**A** $15b$	**B** $10ab$	**C** 5	**D** $5a$

WE 5.7A **2** Find the highest common factor (HCF) of each pair of terms.

a $4x$ and 12 **b** $10d$ and $-5d$ **c** $6j$ and $8jk$

d $3ab$ and $-3ab$ **e** $-9cd$ and $9ef$ **f** $7xy$ and $14yz$

g abc and bcd **h** $10abcd$ and $-14bcde$ **i** $6xy$ and $2y^2$

j $3c^2$ and $9b^2c$ **k** $30g^2h$ and $-24g^4$ **l** $4ab^3$ and $8bc^3$

m $2m^2n^2p^2$ and $3m^3p^2$ **n** $15r^2s^4t$ and $-20r^3s^4t^5$

3 Complete the following factorised expressions.

a $2x + 6 = 2\left(\frac{2x}{\square} + \frac{6}{\square}\right) = 2(\square + \square)$

b $8a - 36 = \square\left(\frac{8a}{4} - \frac{36}{4}\right) = \square(2a - 9)$

c $25mn + 30n = 5n\left(\frac{\square}{5n} + \frac{\square}{5n}\right) = 5n(\square + \square)$

d $kmn - mn = \square\left(\frac{kmn}{\square} - \frac{\square}{\square}\right) = \square(k - \square)$

e $12ef - 18df = 6f\left(\frac{\square}{\square} - \frac{\square}{\square}\right) = 6f(\square - \square)$

f $-26ab + 24cd = \square\left(\frac{-26ab}{\square} + \frac{\square}{\square}\right) = \square(13ab + \square)$

g $-abc + acd = -ac\left(\frac{\square}{\square} + \frac{\square}{\square}\right) = -ac(\square - \square)$

h $-21xyz - 28wxy = \square\left(\frac{\square}{\square} + \frac{\square}{\square}\right) = \square(\square + 4w)$

WE 5.7B **4** Factorise each expression.

a $4a + 8$ **b** $7e + 21$ **c** $45 + 5k$

d $3 + 12y$ **e** $27 - 9p$ **f** $24ab + 18b$

g $12st - 14t$ **h** $5mn + 10n$ **i** $3g - 9gh$

j $22xy - 4xz$ **k** $3ab + 3bc$ **l** $16mn - 8mp$

m $27xyz - 18wxy$ **n** $12kmn + 15mn$ **o** $-21efg + 24fgh$

5 Complete the following factorised expressions.

a $h^2 + 7h = h\left(\frac{h^2}{\square} + \frac{7h}{\square}\right) = h(\square + \square)$

b $10a + 5a^2 = \square\left(\frac{10a}{\square} + \frac{5a^2}{\square}\right) = \square(\square + a)$

c $6p^3 + 9p^5 = 3p^3\left(\frac{\square}{\square} + \frac{\square}{\square}\right) = 3p^3(\square + \square)$

d $6xy + 4x^2y = \square\left(\frac{6xy}{\square} + \frac{4x^2y}{\square}\right) = 2xy(\square + \square)$

e $mn^2 + 5mn = \square\left(\frac{\square}{\square} + \frac{5mn}{\square}\right) = \square(\square + 5)$

f $24efg^2 + 32fg^7h^3 = \square\left(\frac{24efg^2}{\square} + \frac{\square}{\square}\right) = \square(3e + \square)$

WE 5.7C **6** Factorise each expression.

a $c^2 + 4c$ **b** $6k - k^2$ **c** $a^2 - 2a$ **d** $3m^2 + 3m^4$

e $10p^3 + 11p^5$ **f** $4h - 7h^4$ **g** $6x^2y + 3x^3y^6$ **h** $5b^2c^5 - 10bc^2$

i $8mn^2 + 36m^2n^3$ **j** $27xyz^2 - 15x^2yz$ **k** $14a^3bc^{10} + 16a^2b^7c^4$ **l** $12e^2fg^2 + 18ef^5g^2$

7 Check your answers to question **6** by expanding each expression.

8 In some cases, it can be best to factorise using a HCF with a negative coefficient. Find the HCF that has a negative coefficient for each pair of terms.

a $-5a$ and -10
b $-2pq$ and $-6p$
c $-9hk$ and $-15jk$
d $-14rst$ and $-7st$
e $-4x^2$ and $4x$
f $-12m^3$ and $20m^2n^7p^5$
g $-x^2y$ and $-xy$
h $-8b^2c^2$ and $28abc^4$

9 Factorise each expression by finding the HCF that has a negative coefficient.

a $-4a - 12$
b $-6 - 8k$
c $-7m + 14mn$
d $-15b + 35ab$
e $-3st - 9tr$
f $-18abc - 24bcd$
g $-12x^{12} + 21x^{21}$
h $-h^2 - 2h$
i $-16x^2y^7 + 8x^5y^{10}$
j $-45ab^6 - 10a^{15}b^{10}c^8$
k $-42xyz^2 - 36x^4y^{13}z^2$
l $-11a^2bc^3 + 9ab^2c^9$

10 Find the HCF of the following sets of terms.

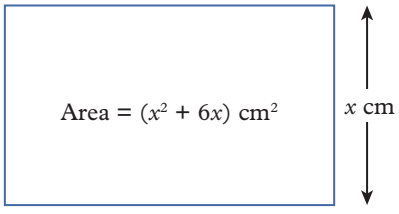
a $12a, 12b, 3a$
b $5abcd, 6bdfh, -7bdhp$
c $15rm, 30ms, 40nmq$
d $36ad, -42ab, -24ac, 18ae$
e $-8xyz, 4uxz, 16vzx, 32zwx$
f $21i^4j^2k, 49j^5i^2k^3, 28k^4i^3j^2$

11 Factorise the following expressions.

a $15p + 15q - 5$
b $6jk - 3ik - 10kl$
c $-8mnpq + 4jknp - 2pmkn$
d $27fg - 54gf + 45fh - 36hf$
e $17uvwx + 34xvu - 51vzxu - 68uv$
f $56a^8b^{12}c^4 + 32a^9b^7c^9 - 80a^7b^{11}c^7$

Exercise 5.7B: Problem solving and reasoning

12 Consider this rectangle, which has a width of x cm and an area of $(x^2 + 6x)$ cm^2.



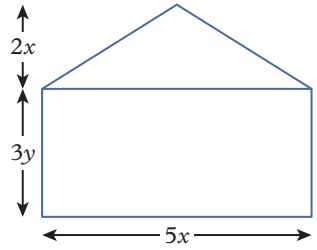
a Factorise the expression for the area of the rectangle.
b List the two factors that multiply to give the expression for the area.
c What is the expression for the length of this rectangle?
d Demonstrate you have the correct expression for the length of this rectangle by multiplying the length by the width to obtain the expression for its area.
e Another way of showing that you have the correct expression for the length of this rectangle is to use substitution. Try this method by letting $x = 3$.

13 Since factorising is the opposite of expanding, we can check our answers by doing the opposite manipulations to our answers. That is, expand after we factorise or factorise after we expand.

i Perform the following manipulations (expand or factorise).
ii Use the opposite manipulation (factorise or expand) to check if your answer is correct.

a Expand $8(4 - 3x)$.
b Factorise $7y + 49$.
c Expand $3b^3(2b^2 - 5b^5)$.
d Factorise $25a^2 - 125ab$.

14 Consider the composite shape shown below.



a Write an expression for the area of the composite shape.

b Factorise your answer to part **a**.

15 It is possible to take any term or expression out as a factor, even if it's not the highest common factor or even a common factor.

For example, to take 7 out as a factor of $x + 5y + 11z$, write 7 in front of $(x + 5y + 11z)$ and divide each term in the brackets by 7: $7\left(\frac{x}{7} + \frac{5y}{7} + \frac{11z}{7}\right)$. If possible, simplify the fractions.

For each of the expressions, take out the following terms as a factor.

i 6 **ii** -1 **iii** $\frac{1}{2}$ **iv** x **v** $-3x$

a $x + 5y + 11z$

b $2x - 3y + 6z$

c $18x^2 + 36x^3y - 54x^4z$

d $4xy - 6x - 10xz$

16 Each of the following problems appeared on a class test. Students were asked to write the expression in its simplest factorised form. Below are some calculations where a student has made a mistake.

i Describe the mistake in each calculation.

ii Factorise the expression correctly.

a $-18p + 12q = -6\left(\frac{-18p}{-6} + \frac{12q}{-6}\right) = -6(3p + 2q)$

b $8r - 16r^3 = 4r\left(\frac{8r}{4r} - \frac{16r^3}{4r}\right) = 4r(2 - 4r^2)$

c $20y^4 + 5y^3 = 5y^3\left(\frac{20y^4}{5y^3} + \frac{5y^3}{5y^3}\right) = 5y^3(4y)$

d $6x^3y + 7xy^3 = 6x^3y\left(\frac{6x^3y}{6x^3y} + \frac{7xy^3}{6x^3y}\right) = 6x^3y\left(1 + \frac{7y^2}{6x^2}\right)$

Exercise 5.7C: Challenge

17 Consider $y(3x + 2) + y(4x - 7)$ and $(x + 2)(3x + 2) + (x + 2)(4x - 7)$.

a Determine the HCF of each expression.

b Factorise and simplify each expression.

18 Factorise the expression $\frac{5}{8}x^{12}y^{19} - \frac{25}{8}y^{20}x^{21} + \frac{15}{8}x^{17}y^{17} - \frac{10}{8}y^{40}x^{32}$.

Online resources:

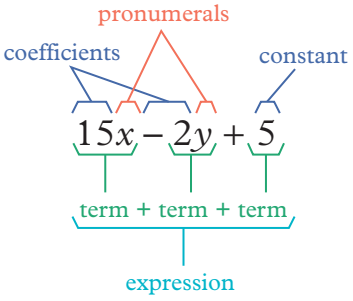
- Interactive skillsheet: Factorising using the HCF
- Worksheet: Factorising expressions
- Worksheet: Factorising using algebra tiles
- Quick quiz: 5.7

Lesson 5.8

Review: Algebra

Module summary

Terms and expressions



Algebraic notation

$7 \times x = 7x$

$7 \times (x + 2) = 7(x + 2)$

$1 \times x = x$

$x \div 7 = \frac{x}{7}$

$(x + 2) \div 7 = \frac{(x + 2)}{7}$

$\frac{x}{7} = \frac{1}{7}x$

Grouping symbols

$7(x + 2)$ $\quad \frac{x + 2}{7}$ $\quad 7\sqrt{x + 2}$

Substitution

Let $x = -2$

$5x^2 - x = 5(-2)^2 - (-2)$

Simplifying signs

$+(+) = +$ $\quad +\times + = +$

$+(-) = -$ $\quad +\times - = -$

$-(+) = -$ $\quad -\times + = -$

$-(-) = +$ $\quad -\times - = +$

Like terms

Pronumeral(s)	Examples of like terms
x	x, $3x$, $\frac{2}{3}x$, $0.7x$...
ab	ba, $-2ab$, $\frac{ab}{5}$, $6.6ab$...
m^2n	$-m^2n$, $18m^2n$, $\frac{9m^2n}{4}$, $0.05m^2n$...

Adding and subtracting like terms

$6ab + \frac{3}{5}a + ab - \frac{2}{5}a = 6ab + ab + \frac{3}{5}a - \frac{2}{5}a$

$= 7ab + \frac{1}{5}a$

Exponent and expanded form

$3a^2b = 3 \times a \times a \times b$

exponent form — expanded form

Expanding and factorising

expanding →

$7(a + 2) = 7a + 14$

← factorising

Expanding

$3(2a - 6) = 3 \times 2a + 3 \times (-6)$

$= 6a + (-18)$

$= 6a - 18$

Multiplying algebraic terms

$3a^3b \times (-2a^2b^2)$

$= 3 \times a \times a \times a \times b \times (-2) \times a \times a \times b \times b$

$= 3 \times (-2) \times a \times a \times a \times a \times a \times b \times b \times b$

$= -6a^5b^3$

Identifying the HCF

$15a^3bc = 3 \times 5 \times a \times a \times a \times b \times c$

$9a^2c = 3 \times 3 \times a \times a \times c$

HCF $= 3 \times a \times a \times c$

$= 3a^2c$

Dividing algebraic terms

$\frac{6a^5b^3}{3a^3b}$

$= \frac{6^2 \times a \times a \times a \times a \times a \times b \times b \times b}{3^1 \times a \times a \times a \times b}$

$= \frac{2 \times a \times a \times b \times b}{1}$

$= 2a^2b^2$

Factorising

$7a + 14 = 7\left(\frac{7^1a}{7^1} + \frac{14^2}{7^1}\right)$

$= 7(a + 2)$

Original expression: $7a + 14$

HCF: 7

Factorised expression: $7(a + 2)$

ANS p550

Review questions 5.8A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- associative law
- coefficient
- commutative law
- constant
- distributive law
- evaluate
- expand
- expanded form
- exponent form
- expression
- factorise
- formula
- grouping symbols
- like terms
- pronumeral
- subject
- substitution
- term
- unknown number
- variable

1 Write a statement to describe each of the following calculations. Pay attention to the order of operations.

a $2(a - b) - 4$ **b** $a - 2b - 4$ **c** $\frac{a - b}{2} - 4$

d $a - \left(\frac{b}{2} - 4\right)$ **e** $a - b^2$ **f** $(a - b)^2$

2 Write the associative and commutative laws for both addition and multiplication as well as the distributive law, using the pronumerals a, b, c.

3 What is the difference between a term and an expression?

4 Explain the relationship between factorising and expanding. Include examples in your answer.

Review questions 5.8B: Multiple choice

5.1 **1** What is the coefficient of the second term in the expression $10a^2 - 6a^2b + 13b - 15$?

A 6 **B** -6 **C** 13 **D** 15

5.1 **2** On a rainy day, the volume of water in a rainwater tank increases by 10 litres every hour. If there was originally 5 litres of water in the tank and it rains for t hours, this total volume of water in the tank is best represented by the expression:

A $15t$

B $10t$

C $10(t + 5)$

D $10t + 5$

5.2 **3** If $x = 4$ and $y = -3$, $2xy - 9y$ is equal to:

A 3 **B** 51 **C** -3 **D** -51

5.3 **4** Which list contains only like terms?

A $3a$, ab, $-5a$ **B** $4mn$, mn, $2nm$ **C** $-8p$, $6p^2$, $5p^3$ **D** $2cd$, de, $-7ce$

5.3 **5** Simplify $7m + 2n - 5m - 2n$.

A $2m$ **B** $12m + 4n$ **C** $2mn$ **D** $2m + 4n$

5.3 **6** What does $2x^2 - 3x^2$ simplify to?

A $5x^2$ **B** -1 **C** $-x^2$ **D** x^2

5.4 **7** What is the expanded form of $-5abc^2$?

A $-5 \times a \times a \times b \times b \times c \times c$

B $-5 \times a \times b \times c$

C $-5 \times a \times a \times b \times c$

D $-5 \times a \times b \times c \times c$

5.4 **8** Which of these is *not* equivalent to $24a^6b^3c^5$?

A $2a^2b^2 \times 3a^2c^3 \times 4a^2bc^2$ **B** $3a^4b \times 8b^2c \times a^2c^4$

C $3a^4c^5 \times 4a^2b \times 2ab^2$ **D** $bc^4 \times 6a^3 \times 4a^3b^2c$

5.5 **9** What does $\frac{4ab^2}{8ab}$ simplify to?

A $2b$ **B** $\frac{1}{2b}$ **C** $\frac{2}{b}$ **D** $\frac{b}{2}$

5.6 **10** When expanded, $5m(3m + 2n - 6)$ equals:

A $15m^2 + 2n - 6$ **B** $8m^2 + 7mn - 11m$

C $15m^2 + 10mn + 30m$ **D** $15m^2 + 10mn - 30m$

5.6 **11** Which expression simplifies to $12mn + 5m$?

A $m(3n - 4) + 2m(5n + 4)$ **B** $4n(2m - 3) + 2m(2n + 4)$

C $m(2n - 3) + 2m(5n + 4)$ **D** $3m(3n - 3) + m(3n + 4)$

5.7 **12** What is the HCF of $24ab^2c$ and $18bc$?

A $2bc$ **B** $3c$ **C** $6bc$ **D** $3ab$

Review questions 5.8C: Short answer

5.1 **1** Write an expression for each statement. Use m to represent the unknown number.

a A number is multiplied by 4 and then 3 is added.

b Half of the product of a number and 5.

c Add 3 to a number, then divide the result by 7.

d 6 less than a number all multiplied by $\frac{3}{5}$.

5.1 **2** Zoo entry is \$25 per adult, \$13 per child and \$20 per pensioner.
Write an expression for each of the following in dollars.

a the cost of entry for x adults

b the cost of entry for y children

c the cost of entry for p pensioners

d the total cost of entry for x adults, y children and p pensioners

5.2 **3** Evaluate each expression by substituting $a = 2$, $b = 5$ and $c = -3$.

a $7a + 2b$ **b** $5a^2 - 4bc$ **c** $-\frac{1}{3}(6ab - 12)$ **d** $\frac{b^2 - a^2}{c}$

5.2 **4** A cyclist is travelling with a constant acceleration, a m/s^2. Their final speed, v, is equal to the square root of the sum of the square of their initial speed, u, and twice the product of their acceleration and the distance they cover, s.

a Write a formula for the final speed of the cyclist.

b If the cyclist is travelling initially at 2 m/s and is accelerating at 3 m/s^2 for 42 m, what is the final speed?

5.3 **5** Identify and list the like terms in the following groups of terms.

a $6a, \frac{1}{7}a, b, -b, \frac{a}{10}, \frac{3}{8}b$ **b** $0.5x, 95y, \frac{1}{2}y, 1.7x, 10x, \frac{5}{6}x$

c $5de, -ed, \frac{2ef}{15}, 0.01de, 12fe, ef$ **d** $p^2, 2p^3, 5.1p^2, -15p^2, \frac{3p^3}{2}, \frac{3}{2}p^2$

e $\frac{9}{10}ab, 6.5abc, -6.5ba, 0.9cab, \frac{11}{20}bca, -ab$ **f** $\frac{4}{9}f^2g, \frac{gf^2}{3}, -23g^2f, -2.3g^2, \frac{6}{7}f^2, -3.14fg^2$

5.3 **6** Simplify the following expressions.

a $4m + 5n + 8m + 3n$ **b** $3cd + 2c + 5dc + 4$

c $2k^2 + 6 - 9k^2 - 1$ **d** $5ab^2 - 2a^2b + 4a^2b - 3ab^2$

e $2xyz + 5xy - 7zyx - yx + \frac{3}{2}xy - \frac{2}{5}y$ **f** $\frac{3}{4}pq^2 - 7q^2 - \frac{1}{2}q^2p - q^2 - pq^2$

5.4 **7** Simplify the following products.

a $x^6 \times x^8$ **b** $m \times m$

c $h^7 \times h \times h^3$ **d** $q^5 \times p^{10} \times q \times p^2$

e $x^5 \times y^{12} \times x \times x^{19} \times y \times y^2$ **f** $a \times c^9 \times b^{21} \times a^5 \times b^9 \times c^3$

5.4 **8** Simplify each product.

a $-5 \times f \times g$ **b** $p^2 \times 2s$ **c** $7g \times 8g^9$

d $3ab \times 8bc$ **e** $2m^5n^8 \times (-3m^7n^4)$ **f** $-2ef^2 \times \frac{2}{5}efg^2 \times (-3f^7g^{16})$

5.5 **9** Simplify the following quotients.

a $k^{11} \div k^7$ **b** $8c^9 \div c$ **c** $x^9y^2 \div x^6y$

d $m^6n^7 \div m^3n^7$ **e** $p^{20}q^8r^{20} \div p^9q^6r^2$ **f** $a^3b^{24}c^{11} \div a^2b^{12}c$

5.5 **10** Simplify the following quotients.

a $\frac{15a}{20}$ **b** $\frac{6cd}{2d}$ **c** $\frac{6abcd}{10bc}$

d $\frac{2e^2hk^2}{5ek}$ **e** $\frac{16x^{13}y^6}{28x^6y}$ **f** $\frac{2t^4 \times 4t^5}{8t^9}$

5.6 **11** Expand the following expressions.

a $5(a + 3)$ **b** $4(7b - c)$ **c** $\frac{c}{2}(6a + 10d)$ **d** $-6m(4m^2 + n)$

5.6 **12** Expand and simplify the following expressions.

a $3(k + 7) + -5(3k - 2)$ **b** $2y(y - 6) - (y + 4)$

5.7 **13** Find the highest common factor (HCF) of each pair of terms.

a 2 and $6a$ **b** $9p$ and $12pq$ **c** $10b^2$ and $-5bc$

d $16rst$ and $24st$ **e** $-a^4b^3c^{10}$ and $11a^2b$ **f** $-42mn^9p^8$ and $18m^6n^{12}p$

5.7 **14** Write the following expressions in their simplest factorised form.

a $14h - 18$ **b** $20ab + 10ac$ **c** $8c^2 + 12c$

d $-5efg - 5fgh$ **e** $15a^2b - 40ab$ **f** $18x^2y^5 - 24wx^{10}y^3$

Review questions 5.8D: Mathematical modelling

1 A roadside stall sells fresh free-range eggs daily. The price varies every week depending on the time of year and the number of eggs that were laid. Johnny shops at the stall daily to buy fresh eggs to always have a fresh supply. Johnny buys two more eggs than the day before each day for the entire week to have enough for the different meals he cooks on different days.

Let x be the number of eggs Johnny buys on Monday and let y be the price the stall sells eggs at for the week.

a Write an expression for the total cost for Johnny after a week as the sum of the costs of each day. Do not expand or simplify the expression.

b If the price of an egg is \$0.43 and Johnny buys 4 eggs on Monday, what was the total cost for the week?

c To make Johnny's life easier with budgeting, simplify the total cost expression from part **a** by expanding and collecting like terms.

d The next week, the price of an egg is \$0.48 and Johnny buys 3 eggs on Monday. Use the expression from part **c** to determine the total cost for the second week.

e Johnny thinks the expression will be even simpler if it is factorised. Factorise the expression from part **c**.

f For the third week, the price of an egg is \$0.39 and Johnny buys 5 eggs on Monday. Use the expression from part **e** to determine the total cost for the third week.

2 A gardening supplier is designing a rectangular prism–shaped planter box that has a length and width that are square numbers, but not necessarily the same square number, and a height that is a cube number.

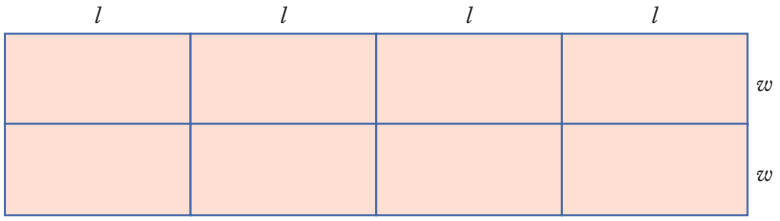
Let a be the number squared for the length, b be the number squared for the width, and c be the number cubed for the height.

a Write an expression for the perimeter of the base of one planter box.

b Write an expression for the area of the base of one planter box.

c Write an expression for the volume of one planter box.

d A gardener buys eight of these planter boxes and arranges them into two rows and four columns as shown.



i Write an expression for the perimeter of the base of the array of eight planter boxes.

ii Write an expression for the area of the base of the array of eight planter boxes.

iii Write an expression for the volume of the array of eight planter boxes.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify terms, variables, coefficients and constant terms ☐ Write algebraic expressions to represent worded problems	☐ Go back to Lesson 5.1 Pronumerals
☐ Substitute values into and evaluate expressions ☐ Substitute values into formulas to find unknown values	☐ Go back to Lesson 5.2 Substitution
☐ Identify like terms ☐ Simplify expressions by collecting like terms	☐ Go back to Lesson 5.3 Adding and subtracting algebraic terms
☐ Convert algebraic terms between expanded form and exponent form ☐ Simplify expressions by multiplying algebraic terms	☐ Go back to Lesson 5.4 Multiplying algebraic terms
☐ Simplify expressions by dividing algebraic terms	☐ Go back to Lesson 5.5 Dividing algebraic terms
☐ Expand algebraic expressions	☐ Go back to Lesson 5.6 Expanding
☐ Find the HCF of two or more algebraic terms ☐ Factorise algebraic expressions by identifying and dividing by the HCF	☐ Go back to Lesson 5.7 Factorising

Semester 1 review

Short answer

ANS p551

1 Round the following numbers according to the description in the brackets.

a 569 238.472 940 (correct to the nearest thousand)

b 569 238.472 940 (correct to the leading digit)

c 208 492.175 207 (correct to the nearest thousandth)

d 208 492.175 207 (correct to two decimal places)

e 0.000 047 19 (correct to the leading digit)

f 28 174.201 (correct to the nearest multiple of 20)

2 Fill in the gaps to produce equivalent fractions.

a $\frac{7}{8} = \frac{21}{\quad}$ **b** $\frac{-8}{10} = \frac{}{15}$ **c** $2\frac{4}{5} = \frac{70}{\quad}$ **d** $\frac{-1.2}{-0.07} = \frac{}{70}$

3 Simplify the following.

a $\frac{-90}{-120}$ **b** $\frac{300}{18}$ **c** $7\frac{45}{60}$ **d** 56 : 48

e 36 : 24 : 72 **f** 0.5 : 0.025 **g** $\frac{1}{9} : \frac{2}{3}$ **h** 150 km per 2.5 hours

4 Evaluate the following sums.

a $579 + 643 + 87$ **b** $14 + (-52)$ **c** $\frac{5}{9} + \frac{5}{6}$

d $12.41 + 9.015$ **e** $2^4 + 2^3$ **f** $3\frac{4}{5} + 6\frac{2}{3}$

5 Evaluate the following differences.

a $1041 - 828$ **b** $(-16) - (-23)$ **c** $\frac{1}{7} - \frac{5}{8}$

d $48.029 - 9.994$ **e** $3^4 - 3^2$ **f** $6\frac{3}{8} - 2\frac{7}{12}$

6 Evaluate the following products.

a 245×18 **b** -5×12 **c** $\frac{12}{10} \times \frac{25}{18}$

d $-4.2 \times (-31.8)$ **e** $2^2 \times 2^3$ **f** $3\frac{1}{5} \times 4\frac{1}{6}$

7 Evaluate the following quotients.

a $5144 \div 8$ **b** $-255 \div (-3)$ **c** $\frac{12}{25} \div \left(-\frac{9}{10}\right)$

d $0.042 \div 0.6$ **e** $3^8 \div 3^5$ **f** $4\frac{1}{12} \div 2\frac{5}{6}$

8 Evaluate the following exponents.

a 6^3 **b** 3^5 **c** $\left(\frac{5}{4}\right)^3$

d $(-0.2)^4$ **e** $\left(3\frac{1}{2}\right)^4$ **f** 7^0

9 Evaluate the following roots.

a $\sqrt{36}$ **b** $\sqrt[3]{8}$ **c** $\sqrt{\frac{81}{25}}$ **d** $\sqrt{0.49}$

10 Use the order of operations to evaluate the following expressions.

a $15 - 3^2 \times 8 \div 6 + 2$ **b** $25 \div (-5) - (-9) \times 7$

c $\frac{5}{6} - \frac{2}{3} \div \frac{4}{7} - \frac{3}{4}$ **d** $3.4 \times (2.5 + 3.7) - 8 \times 1.1^3$

11 Simplify the following.

a $5y + (-8xy) + 4x + 3yx + x + (-2y)$ **b** $12b - 8a - (-3b) - 7b$

c $2^4 \times 2^3$ **d** $7x^2y \times 5xy^5$

e $3^{12} \div 3^4$ **f** $\frac{24a^5b^6}{3b^2}$

g $(3^6)^5$ **h** $2^8 \times 3^{10} \div (2^2 \times 3^5)$

i $\frac{6x + 12}{3}$ **j** $\frac{12p^6q^3}{4p^2q^3} - \frac{18p^{12}q^8}{2p^8q^8}$

12 For the following expressions:

i round each value to the leading digit (first non-zero digit) and estimate the calculation

ii evaluate the actual result of the calculation

iii determine the percentage error between the estimation and the actual result, correct to two decimal places.

a $710\,284 \times 0.021\,83$ **b** $9107.4029 \div 2.145$

13 Write these lists of numbers in ascending order.

a 842.01, 8.4201, 824.01, 24.01, 24.1, 842.1 **b** $-5.31, 3.15, 5.13, -5.1, -5.031, 5.103$

c $\frac{7}{8}, \frac{3}{4}, \frac{4}{5}, \frac{11}{12}, \frac{9}{10}, \frac{2}{3}$ **d** $-\frac{3}{2}, \frac{6}{-2}, \frac{-3}{4}, \frac{-3}{-4}, \frac{4}{3}, -\frac{4}{3}$

14 Evaluate if the following are terminating or non-terminating. If they are non-terminating, determine if they are recurring or non-recurring.

a 2.33333… **b** 5.194 817 53…

c 2.849 **d** 0.001 212 121 2…

15 Complete the table below.

Fraction	Decimal/recurring decimal	Percentage
	1.45	
$\frac{7}{8}$		
	2.5	
$\frac{21}{5}$		
		$33.\dot{3}\% = 33\frac{1}{3}\%$
		108%
$\frac{4}{33}$		

16 Write the following.

a 12 games played as a fraction out of 15 games

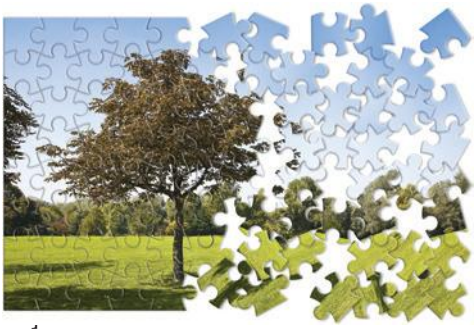
b 16 quests completed as a percentage out of 40 possible quests

c 8 episodes watched as a percentage out of 25 episodes

d the ratio of connected to unconnected puzzle pieces when 840 pieces are connected in a 1000-piece puzzle

e 57 eggs cooked in three weeks as a rate per week

f the average speed of a car that travels 276.25 km in 3.25 hours



17 Calculate:

a $\frac{5}{8}$ of 72

b 0.8 of 55

c 110% of 99

d 58 increased by 12%

e 120 decreased by 20%

f $2700 shared in the ratio 4 : 5

g the number of blue counters in a bag, if the bag has 35 red counters and the ratio of blue counters to red counters is 3 : 5

h the mass of a block if its density is 20 kg/m^3 and the volume of the block is 0.04 m^3

i the distance a bike will travel when ridden at 8 km/h for 30 minutes

j the time taken for someone walking at 2 m/s to walk 1 km.

18 Write an expression for the following:

a the difference of the squares of two numbers, p and q

b the sum of the product of 3 and a number, x, and the quotient of another number, y, and 12

c the quotient of 6 more than a number, x, and 6 less than the number x

d the square root of the number 7 times a number k

19 Expand and simplify:

a $8(4-3x)$ **b** $5-3(8x-11)+2x$ **c** $2b(6a+5b+3)$ **d** $5x^2y(6xy-9y^4)$

20 Factorise:

a $36x+27$ **b** $12a+36b-18c$ **c** $\frac{10}{3}p^3+\frac{25}{3}p^7$ **d** $-\frac{3}{8}u^2v+\frac{5}{4}uv^2$

21 Calculate the following:

a the cost price of a cupcake if it was marked up by 280% to be sold for \$2.10

b the cost price of a book if it was discounted by 40% to be sold for \$15.00

c the selling price of a bag of lollies that is bought at \$0.90 per bag and marked up by 120%

d the selling price of an indoor plant that was \$80 and is discounted by 20%

e the cost, including GST, of a meal that was \$45 not including GST

22 Calculate the LCM and HCF of the following.

a 18 and 30

b 210 and 700

c $2^4\times3^6\times5^4$ and $2^3\times3^9\times5^4$

d $2^7\times5^8\times7^4$ and $2^3\times3^8\times7\times11^2$

23 Substitute the value in brackets into each expression, then evaluate.

a $8d^2+5d+4$ $(d=10)$ **b** $\frac{3}{4}x+x^2$ $(x=-2)$

c $6y-\frac{2}{y}$ $\left(y=\frac{5}{3}\right)$ **d** $\frac{1.03-r^2}{r+0.7}$ $(r=0.5)$

24 State if the following are true or false.

a $(8-3)-4=8-(3-4)$ **b** $(120\div6)\div2=120\div(6\times2)$

c $(3^4)^5=3^{(4^5)}$ **d** $3\frac{1}{2}\times8=3\times8+\frac{1}{2}\times8$

e $4-(-3)=4+3$ **f** $3^8-3^4=3^2$

25 Express the following numbers as the product of their prime factors. Write your answer in exponent form.

a 120 **b** 875

Mathematical modelling

1 Barry owns a \$2 shop and uses a simple cash register where the prices of items must be entered manually. Part of the stock is fake flowers. He stocks three types of fake flowers.

- lower quality flowers that cost \$0.50 per flower
- medium quality flowers that cost \$1.25 per flower
- higher quality flowers that cost \$1.95 per flower

Barry marks up his stock 80% from the cost price to the selling price.

a Determine the selling price of the three types of flowers.

b A customer buys 5 lower quality flowers, 3 medium quality flowers, and 4 higher quality flowers. How much does the customer pay?

c Barry uses a different colour price tag each year to keep track of how long the stock has been in the store. Any item that has been in the store for two or more years is sold at a discount of 30%.

A customer buys 10 lower quality flowers, 8 medium quality flowers, and 5 higher quality flowers, all of which have been in the store for two or more years.

i How much does the customer pay?

ii How much more money would Barry have made if the flowers were sold at full price?

d Barry gets a delivery of 100 lower quality flowers, 75 medium quality flowers, and 50 higher quality flowers. The delivery cost was $20. Calculate:

i the total cost of purchasing the flowers including delivery

ii the difference between the total selling price and the total cost price if Barry sells all the flowers within two years.

e Another supplier sells their flowers for the following prices.

- lower quality flowers that cost $0.35 per flower
- medium quality flowers that cost $1.15 per flower
- higher quality flowers that cost $2.10 per flower
- delivery cost of $25 per shipment

How much cheaper or more expensive would Barry's purchase in part **d** be from this supplier?

2 A teacher notices a number of mistakes on a recent revision quiz.

i $372 + 654 = 926$ **ii** $1025 - 856 = 231$ **iii** $632 \times 72 = 5688$

iv $2760 \div 8 = 37$ **v** $4^4 = 16$ **vi** $\sqrt{16} = 8$

vii $\sqrt[3]{27} = 9$ **viii** $2^3 \times 2^2 = 2^6$ **ix** $3^3 \div 3^3 = 0$

a Determine the correct answer for each calculation.

b Calculate the percentage error between the correct and incorrect answers, correct to two decimal places.

c Modify the first number in each calculation so that the answers are correct.

3 Harrison was investigating the exponent laws and how they apply with algebra. He sets up the following pairs of products of prime factors:

i $2^5 \times 3^2 \times 5$ and $2^3 \times 3^5 \times 5^3$

ii $3^5 \times 2^2 \times 7$ and $3^3 \times 2^5 \times 7^3$

iii $3^5 \times 5^2 \times 7$ and $3^3 \times 5^5 \times 7^3$

iv a^5b^2c and $a^3b^5c^3$

a Determine the product of each pair. Leave your answer as the product of prime factors.

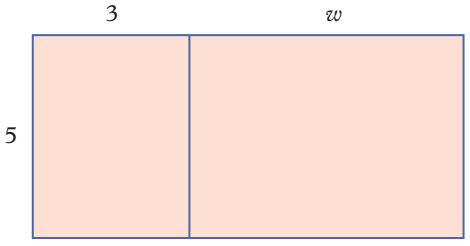
b Determine the quotient of each pair. Leave your answer as the product of prime factors.

c Determine the highest common factor of each pair. Leave your answer as the product of prime factors.

d Determine the lowest common multiple of each pair. Leave your answer as the product of prime factors.

4 Gavin is extending his patio. Currently, his patio is 5 metres by 3 metres.

a If Gavin wants to extend his patio by w metres, write an expression, using brackets, for the area of his new patio in m^2.



b Expand your answer to part **a**.

c If Gavin can only extend his patio to a total width of 8 m, what is the maximum area of his extension?

d Gavin wants to check the total area of the patio if he extends the patio by 1 m, 2 m, 3 m and 4 m. Calculate the area of the patio for each of these extension lengths.

e Gavin's partner wants to plant flowers along the two shorter sides and one of the longer sides of the patio.

i Write an expression for the total length of the flowerbed along these three sides of the patio.

ii Gavin digs a trench with a width x metres for the flowerbed. When Gavin's partner sees the flowerbed, he asks Gavin to double the width of the flowerbed. Use brackets to write an expression for the final area of the flowerbed in m^2.

iii Expand your answer to part **ii**.

iv If Gavin decides to extend the patio by 3 metres, and the flowerbed has a width of 0.5 metres, what will the total area of the flowerbed be?

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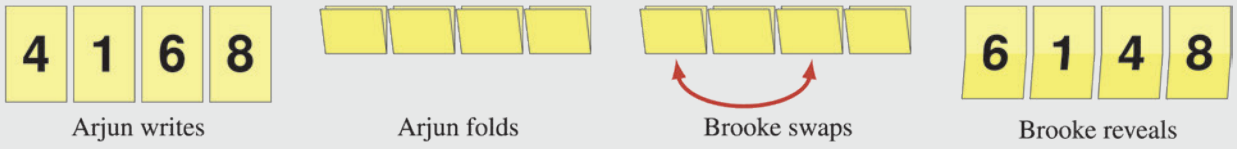
EXPLORATIONS 1

1 Note swap

Arjun and Brooke make up a game called 'note swap'. Arjun takes a number of sticky notes and places them in a line on the desk. While Brooke is not looking, he writes a single digit on each note to form a number, read left to right, that is divisible by the number of notes used. Arjun then folds each note in half so Brooke cannot see the digits. Brooke's job is to swap two sticky notes of her choosing and then reveal all the digits to form her own number, again reading left to right. If the new number is also divisible by the number of notes used, Brooke wins. If not, Arjun wins.



a Arjun uses 3 sticky notes. Explain why Brooke will always win, no matter how she swaps the notes.

b Arjun uses 4 sticky notes. What strategy should Brooke use to guarantee she wins?

c Arjun uses 7 sticky notes. Explain why swapping the first and last digit is a winning strategy for Brooke.

d From 2 to 12 inclusive, there is only one number of notes which allows Arjun to come up with a winning strategy. What is the number and what is his winning strategy?

e Show that Brooke has a winning strategy for all remaining cases from 2 to 12 inclusive.

f Do either Arjun or Brooke have a winning strategy for 14 notes?

2 Recurring themes

a By long or short division, show that $\frac{1}{7}$ as a decimal is $0.\overline{142857}$.

b Because every possible remainder from 1 to 6 occurs in the above calculation, it follows that the same pattern of recurring digits must also occur in the decimal forms of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$. Use this fact to write each of these fractions as a decimal, without doing any division.

c Convert $\frac{1}{14}$ to an equivalent (but unsimplified) fraction with a multiple of 10 in the denominator. Use this and part **b** to help you write $\frac{1}{14}$ as a decimal.

d What is $\frac{3}{14} - \frac{1}{5}$ as a simplified fraction? Use this to help you write 3 as a decimal.

e In part **d**, it made sense to subtract $\frac{1}{3}$ for two reasons: it is close to but slightly less than $\frac{3}{14}$, and its own decimal form terminates because 5 is a factor of 10.
Adapt the ideas above to write $\frac{5}{14}, \frac{9}{14}, \frac{11}{14}$ and $\frac{13}{14}$ as decimals.

f Find a clever way to work out $\frac{2}{63}$ as a decimal without doing the division.

3 Parcel packers

In the AMT warehouse, it takes 8 professional packers 15 hours to pack the parcels of competition papers to send to schools.

a Assuming that all professional packers work at the same constant rate and do not get in each other's way, how long would it take 12 professionals to pack all the parcels?

b The 12 professionals are joined by 8 trainee packers who can each work at only half the rate of a professional. How long would it take the professionals and trainees to pack all the parcels?

c Some professionals and 6 trainees started packing the parcels and would have taken 12 hours to complete the job. However, after $2\frac{1}{2}$ hours two professionals and all trainees had to leave. How many professionals were there? How much longer after the 8 packers left would the remaining packers take to finish the job?

d Suppose x professionals and y trainees require t hours to complete a job, but after T hours only X professionals and Y trainees continue. Find a formula for the number of hours needed by the remaining packers to finish the job.

4 Sharesies

Seven girls sit around a circular table. Each has a glass and some of the glasses contain water. Each glass can hold up to 720 mL. In turn, going clockwise around the table, each girl divides the water in her glass into six equal shares and adds these to the glasses of each of the other girls. No water is ever spilt.

a On one occasion, the first girl begins with a full glass. When all seven girls have completed their turns, the first girl ends up with a full glass again. How much water did the third girl have at the beginning?

b On another occasion, the total amount of water is 2.1 L. When all seven girls have completed their turns, each of them ends up with the same amount of water as they had originally. How much water did each girl have at the beginning?

c What is the answer to part **b** if each girl divides the water in her glass into seven equal parts and keeps one of the parts for herself?

5 Powerful stuff

a Without using a calculator, write the number 4096 as a:

i power of 2 **ii** power of 4 **iii** perfect square **iv** perfect cube.

b Explain why the number 1 has all of the properties in part **a**: it is a power of 2, a power of 4, a perfect square and a perfect cube. What is the next smallest number with all of these properties?

c Write the number $12^3 \times 18^5$ as the product of a perfect square and a perfect cube. Now find a second way to do this.

d Find all powers of 20 that *cannot* be written as the product of a perfect square and a perfect cube. What about powers of 24? Can you generalise your results?

e The sum $256^2 = 32^3 + 8^5$ shows a perfect square that can be written as the sum of a perfect cube and a fifth power. Another example is $3^2 = 2^3 + 1^5$, but we are not interested in the trivial case $0^2 = 0^3 + 0^5$.

i Find a perfect cube that can be written as the sum of a perfect square and a fifth power.

ii Find a fifth power that can be written as the sum of a perfect square and a perfect cube.

f Do some research to find out about *Pythagorean triples* and *Fermat's last theorem*.

i Can a fourth power be written as the sum of two perfect squares?

ii Can a sixth power be written as the sum of two perfect cubes?

6 House points

Maths teacher Mr Stan Deliver has his class practising mental arithmetic. Each student chooses a handful of consecutive numbers, none of which ends in zero. They add up the numbers and then divide the total by the last digit of each number. For each division which results in a whole number answer, the student earns a house point.

For example, Zeba chooses 23, 24 and 25 with total 72. She then calculates $72 \div 3$, $72 \div 4$ and $72 \div 5$. Two of these result in a whole number, so Zeba earns two house points.

a Andy chooses 15, 16 and 17. How many house points does he win? Is his score improved by adding one more number to his list?

b Basma chooses eight consecutive numbers. Explain why she always wins at least 2 house points. What is the smallest possible total of Basma's eight numbers if she only wins 2 house points?

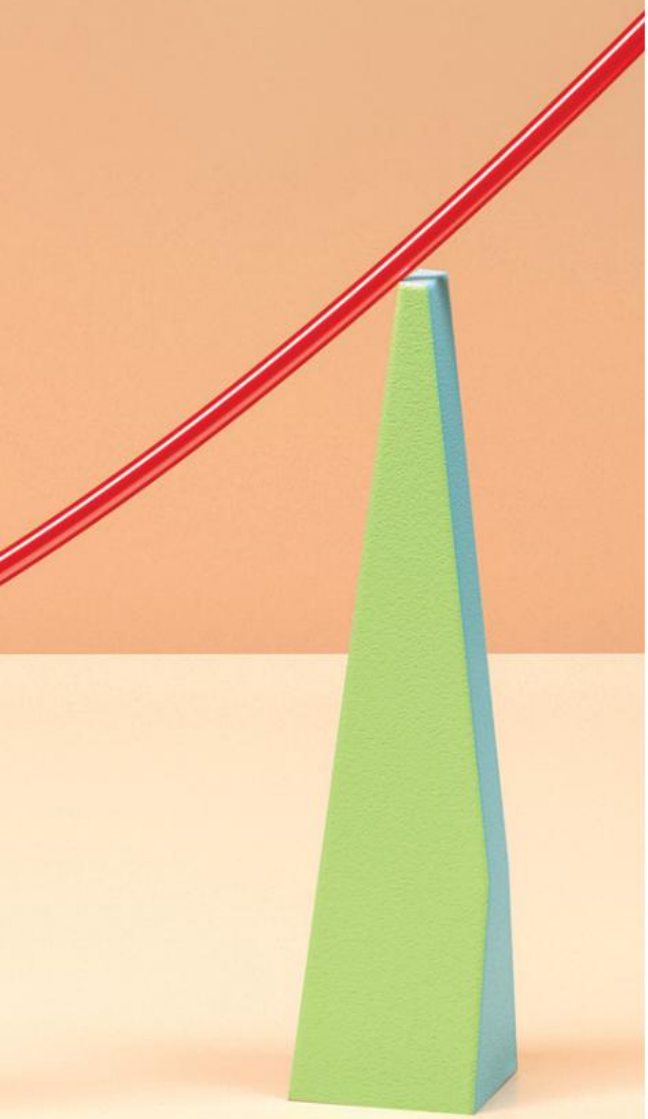
c Cheren chooses six consecutive numbers. Explain why she cannot win more than 2 house points. What is the smallest number of house points Cheren can win?

d Dmitri chooses nine consecutive numbers. Explain why he cannot win more than 5 house points. What is the smallest possible total of Dmitri's numbers if he does win the maximum 5 house points?

e Eli chooses well and wins a house point for every number in his list. What is the largest possible number of consecutive numbers he could have chosen? What is the smallest possible total of Eli's numbers?

6

Equations, inequalities and linear relationships



Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Order of operations
- ✔ Substitution
- ✔ Like terms
- ✔ Expanding over one pair of brackets
- ✔ The Cartesian plane

Curriculum links

- Graph linear relations on the Cartesian plane using digital tools where appropriate; solve linear equations and one-variable inequalities using graphical and algebraic techniques; verify solutions by substitution (VC2M8A02)
- Use mathematical modelling to solve applied problems involving linear relations, including financial contexts involving profit and loss... (VC2M8A03)
- Experiment with linear functions and relations using digital tools... (VC2M8A05)

Materials

- ✔ Graph paper
- ✔ Ruler

Lesson 6.1

Equations

Learning intentions

By the end of this lesson you will be able to...

✔ verify solutions and solve simple equations.

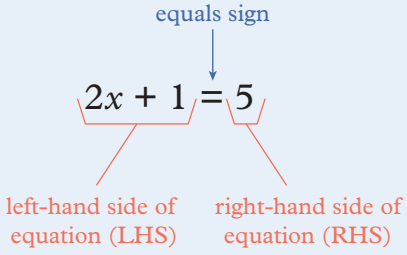
Inter-year links

Support Writing formulas

Year 7 6.5 Solving equations by inspection

Equations

- An **equation** is a mathematical statement that shows equivalence between the expression on the left- hand side (LHS) of the equation and the right-hand side (RHS) of the equation.
  → If two expressions are *not* equivalent, this is denoted using the 'not equal to' symbol, ≠.



Solving equations

Key content video
Solving equations by inspection

- To find a **solution** to an equation, find a value which, when substituted for the unknown number, makes the equation a true statement.
- Solutions are usually given with the pronumeral on the left-hand side and the value on the right-hand side of an equals sign, for example $x = 3$.
- To check if a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.
- To **solve by inspection**, examine the equation and mentally identify the number that when substituted for the unknown number makes the equation a true statement.

Worked example 6.1A Checking a solution

Determine whether $x = -3$ is the solution to the equation $2x + 5 = 1$.

THINK

1 Substitute $x = -3$ into the expression on the left-hand side of the equation and evaluate. Remember that the product of a positive and a negative number gives a negative number.
2 Determine whether the statement is true or false by comparing the value of the left- and right-hand sides of the equation.

WRITE

$2x + 5 = 1$
LHS $= 2(-3) + 5$
$= -6 + 5$
$= -1$
RHS $= 1$
LHS $\neq$ RHS

$x = -3$ is not the solution to $2x + 5 = 1$.

Worked example 6.1B Solving equations by inspection

Solve each equation by inspection.

a $a + 5 = 12$ **b** $7c = -35$

THINK	WRITE
a 1 Change the equation to a question. What number plus 5 is equal to 12?	**a** $a + 5 = 12$
2 Check the solution using substitution.	Let $a = 7$ LHS $= 7 + 5$ $= 12$ RHS $= 12$ LHS = RHS
3 State the solution to the equation.	$a = 7$
b 1 Change the equation to a question. What number, when multiplied by 7, gives –35? Since 7 is positive and –35 is negative, c must be a negative value so that the product, $7c$, is negative.	**b** $7c = -35$
2 Check the solution using substitution.	Let $c = -5$ LHS $= 7(-5)$ $= -35$ RHS $= -35$ LHS = RHS
3 State the solution to the equation.	$c = -5$

Worked example 6.1C Writing equations from number sentences

Write an equation to represent each of these number sentences. Use n for the unknown number in each instance.

a 3 more than a number is equal to 8.

b 5 less than a number is equal to −2.

c Twice a number is equal to 14.

THINK	WRITE
a Read through the number sentence from left to right. The left-hand side of the equation is '3 more than a number' or $n + 3$. The right-hand side of the equation is 8.	**a** $n + 3 = 8$
b Read through the number sentence from left to right. The left-hand side of the equation is '5 less than a number' or $n - 5$. The right-hand side of the equation is −2.	**b** $n - 5 = -2$
c Read through the number sentence from left to right. The left-hand side of the equation is 'twice a number' or $2n$. The right-hand side of the equation is 14.	**c** $2n = 14$

Worked example 6.1D Writing equations to represent worded problems

A rose bush grows three times as many roses as last year. Erin cuts 5 roses to fill a vase.

a If r roses grew on the bush last year, write an expression for the number of roses on the bush after Erin fills the vase.

b After Erin fills the vase, there are 13 roses left on the bush. Write an equation that can be used to determine how many roses grew on the bush last year.

c Solve the equation by inspection.

d How many roses grew on the bush last year?

THINK	WRITE
a Think about which operations are taking place. 'Three times' means multiply by 3. If Erin removes 5 roses to fill the vase, there will be 5 fewer roses on the bush.	**a** $3r - 5$
b If there are '13 roses left on the bush', the number of roses is equal to 13.	**b** $3r - 5 = 13$
c Consider what number makes the equation a true statement. Check the solution. If your initial guess is incorrect, try another value based on whether the result is greater or less than the right-hand side.	**c** Let $r = 6$ LHS $= 3r - 5$ $= 3(6) - 5$ $= 18 - 5$ $= 13$ RHS $= 13$ LHS = RHS $r = 6$
d Interpret the solution. As r = the number of roses that grew on the bush last year, 6 roses grew on the bush last year.	**d** 6 roses grew on the bush last year.

Helpful hints

- ✔ Remember to define your pronumerals before writing equations to represent a worded problem.
- ✔ If you have difficulty in solving equations by inspection, try using the 'guess, check and improve' method from Year 7. You can also try replacing pronumerals with a question mark or an empty box.
  For example, instead of $3 + x = 7$, you could write $3 + ? = 7$ or $3 + \square = 7$.
- ✔ When solving equations by inspection, you can often use addition, subtraction, multiplication or division facts to identify the solution.

Learning pathways

1–4, 5(1st, 2nd columns), 6, 8, 10–12,14,15,17–20	4, 5(2nd, 3rd columns), 7, 9–11, 13, 16–20, 22, 24	4, 5(3rd, 4th columns), 7, 10, 11, 15, 17–19, 21–25

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Exercise 6.1A: Understanding and fluency

1 Consider the difference between equations and expressions.

a State whether each of these is an equation or an expression.

i $5x + 4$ **ii** $8k = 24$ **iii** $14 - 3p = 5$

iv $\frac{3k - 1}{2}$ **v** $2 = 9 - x$ **vi** $4(7d + 3) = 8$

b Explain how you can tell the difference between an equation and an expression.

2 Substitute $x = 6$ into each equation. Does this value of x make the equation a true statement?

a $x + 3 = 9$ **b** $x - 5 = -1$ **c** $2x = 8$

d $\frac{x}{3} = 2$ **e** $4x + 5 = 19$ **f** $3x - 7 = 11$

3 Substitute $y = -9$ into each equation. Does this value of y make the equation a true statement?

a $y - 1 = -8$ **b** $3y = 27$ **c** $\frac{18}{y} = -2$

d $-5y = 45$ **e** $4y + 1 = -35$ **f** $2y + 6 = 24$

WE 6.1A **4** Determine if the value given in brackets for each pronumeral is the solution to the equation.

a $a - 5 = 6$ $(a = 11)$ **b** $4k = -16$ $(k = -4)$ **c** $m + 3 = 11$ $(m = 7)$ **d** $\frac{h}{5} = 2$ $(h = -10)$

e $6p - 4 = 2$ $(p = 1)$ **f** $3n + 7 = -1$ $(n = -2)$ **g** $\frac{b + 2}{3} = 2$ $(b = 7)$ **h** $15 - f = 16$ $(f = -1)$

WE 6.1B **5** Solve each equation by inspection.

a $a + 3 = 10$ **b** $b - 2 = 4$ **c** $3c = 24$ **d** $\frac{d}{4} = 3$

e $-4e = -8$ **f** $\frac{f}{3} = -1$ **g** $g - 5 = -4$ **h** $h + 6 = 6$

i $9 - i = 12$ **j** $8j = -40$ **k** $4 + k = 11$ **l** $l - 15 = 20$

m $\frac{24}{m} = 6$ **n** $-10n = 100$ **o** $\frac{x}{2} = -7$ **p** $p - 4 = -4$

q $8 = q + 2$ **r** $48 = -4r$ **s** $9 = 13 + s$ **t** $5 = \frac{t}{5}$

6 Which solution makes $2x + 5 = 13$ a true statement?

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

7 Which solution makes $3y - 1 = -7$ a true statement?

A $y = -7$ **B** $y = 4$ **C** $y = -2$ **D** $y = 2$

8 Which solution makes $\frac{m + 3}{8} = 1$ a true statement?

A $m = 5$ **B** $m = 6$ **C** $m = 7$ **D** $m = 8$

9 Which solution makes $\frac{k}{3} - 1 = 0$ a true statement?

A $k = 1$ **B** $k = 2$ **C** $k = 3$ **D** $k = 4$

WE 6.1C **10** For each of the number statements below:

i write an equation for the statement, using n as the unknown number

ii solve each equation by inspection.

a 4 added to a number equals 11.

b 3 subtracted from a number is the same as −2.

c 15 more than a number is equal to 21.

d 8 less than a number is equal to −4.

e Twice a number is equal to 14.

f One-quarter of a number is 9.

g The sum of a number and −7 is equal to −20.

h The product of a number and 11 is equal to 22.

i When a number is multiplied by 5, the result is 55.

j When a number is divided by 6, the result is −3.

11 Write an equation to represent each of these number sentences. Use x for the unknown number.

a A number is multiplied by 3, then 8 is added to give 11.

b A number is divided by 6, then 2 is subtracted to give 1.

c The sum of a number and 7 all divided by 2 is equal to 5.

d 9 less than a number all times 3 gives 3.

Exercise 6.1B: Problem solving and reasoning

WE 6.1D **12** Evie and her older brother Harry earn pocket money weekly. Harry receives $38 each week.

a Write an expression for the total amount of money they receive each week if Evie receives x dollars per week.

b Write an equation that can be used to find how much Evie receives each week if they receive a total of $62 per week.

c Solve the equation by inspection.

d How much pocket money does Evie receive each week?

13 Stella has 26 rolls of washi tape. A delivery containing more rolls of washi tape has just arrived.

a Write an expression for the total number of rolls of washi tape Stella will have if there are x rolls in the delivery.

b Write an equation that can be used to find how many rolls of washi tape are in the delivery if Stella has a total of 81 rolls of tape after the delivery.

c Solve the equation by inspection.

d How many rolls of washi tape were in the delivery?

14 Theo has invited some friends over for dinner. He has bought enough dumplings so that each person can eat 8.

a If x represents the number of people at Theo's dinner, write an equation that can be used to find the number of people if Theo bought a total of 96 dumplings.

b Solve the equation by inspection.

c How many people were at the dinner at Theo's house?

15 A bag of macadamia nuts is divided amongst four students.

a Define a pronumeral to represent the variable in this problem.

b Write an expression for the number of macadamia nuts each student receives.

c Write an equation for finding the number of macadamia nuts in the bag if each person receives nine macadamia nuts.

d Solve the equation by inspection to find the value of the pronumeral.

e How many macadamia nuts are in the bag?

16 The selection of truffles shown to the right costs a total of \$24.

a Assuming each truffle costs the same amount of money, write an equation that can be used to determine the cost of each truffle.

b Solve the equation by inspection to calculate the cost of one truffle.

17 We can solve two-step equations by inspection using more steps. For example, to solve $4x + 3 = 11$, follow these steps.

1 Identify the first operation applied to x using the order of operations. Here, this is multiplying x by 4. Substitute $y = 4x$ in the equation $4x + 3 = 11$.

2 Solve the equation $y + 3 = 11$ by inspection.

3 Substitute the result from step **2** into $y = 4x$ and solve the resulting equation by inspection.

$$\begin{aligned} y &= 4x \\ 4x + 3 &= 11 \\ y + 3 &= 11 \\ y &= 8 \\ 8 &= 4x \\ x &= 2 \end{aligned}$$

Solve the following equations using this method.

a $3a + 9 = 15$ **b** $5b - 3 = 2$ **c** $7c - 2 = 19$ **d** $2d + 10 = 10$

e $\frac{e+4}{5} = 2$ **f** $\frac{f-1}{3} = 3$ **g** $\frac{g-24}{2} = -10$ **h** $7 - \frac{h}{7} = 8$

18 Solve each equation by inspection using the method from question **17**.

a $4x + 1 = 21$ **b** $3x - 2 = 10$ **c** $7x + 1 = -6$ **d** $2x - 5 = 13$

e $\frac{x}{5} - 6 = -8$ **f** $\frac{x}{3} + 11 = 11$ **g** $\frac{x+9}{2} = 3$ **h** $\frac{x-4}{4} = -2$

19 Solve each of the equations from question **11** using the method from question **17**.

20 Nakkiah takes half of the fruit in a fruit bowl to basketball training, then her dad puts 3 new pieces of fruit into the bowl.

a If there are now 7 pieces of fruit in the bowl, write an equation to represent this problem.

b Solve the equation to calculate how many pieces of fruit were originally in the fruit bowl.

21 The number of cars in a car park changes during the day. At 11 am, there are twice as many cars as there were at 9 am. At 4 pm, there are 7 more cars in the car park than at 11 am.

a If there are 11 cars in the car park at 4 pm, write an equation that can be used to calculate how many cars were in the car park at 9 am.

b Use the equation to determine how many cars were in the car park at 9 am.

22 Tickets to Australia Zoo cost $25 per child and $40 per adult.

a Write an expression for the cost of c child tickets.

b Write an expression for the cost of a adult tickets.

c Write an equation for the number of child and adult tickets you could buy with $155.

d Poh's family is made up of 2 adults and 3 children. He thinks it will cost $155 to purchase tickets to Australia Zoo for his whole family. Use your equation to check if Poh's calculations are correct.

23 Sia starts a fitness routine and records the number of minutes she spends exercising each week. In the third week, Sia exercises 30 minutes more than she did in the first week. In the fifth week she exercises double the number of minutes she did in the third week.

a If Sia exercises for 270 minutes in the fifth week, write an equation that can be used to calculate how long she exercised in the first week.

b Use the equation to find how long Sia exercised in the first week.

24 Arvan records the amount of time he spends practising the guitar. Unfortunately, he spills some water on his record and some of the information is lost.

Day	Time spent practising	Comments
Monday		Good start!
Tuesday		Same as Monday
Wednesday		Double Monday!
Thursday		20 minutes less than yesterday
Friday		Same as Monday
Total	190 minutes	

Arvan thinks he can write an equation using the information in his table to find the missing amounts. Arvan's equation is $x + x + x + x + x = 190$.

a Explain why Arvan's equation is incorrect.

b Write a new equation that finds the number of minutes Arvan practises on Monday.

c Fill in the table for each day.

d Arvan's equation does calculate a certain value that could be useful for him. What is it?

Exercise 6.1C: Challenge

25 Lina visited the Milton Market one Sunday. At one stall she bought a handbag. At another stall she bought 3 t-shirts. At a food van she bought 3 donuts for $10. Lina went to the market with $100, came home with no money, and the handbag cost double the amount of all 3 t-shirts.

a Write an equation to solve the cost of the handbag.

b Solve the equation by inspection.

c Check your answer by substitution.

Online resources:

- **Interactive skillsheet** Writing formulas
- **Interactive skillsheet** Solving equations by inspection
- **Worksheet** Equations and algebra tiles
- **Worksheet** Solving equations by inspection
- **Investigation** Circus antics
- **Quick quiz** 6.1

Lesson 6.2

Solving linear equations

Learning intentions

By the end of this lesson you will be able to ...

✔ solve linear equations using inverse operations.

Inter-year links

Year 7 6.6 Solving equations using inverse operations

Year 9 3.1 Solving linear equations

Year 10 3.1 Solving linear equations

Equivalent equations

- **Equivalent equations** are equations that have the same solution (or set of solutions).
- When the same operation is performed on both sides of an equation, the equality between the two sides of the equation is preserved.

Inverse operations

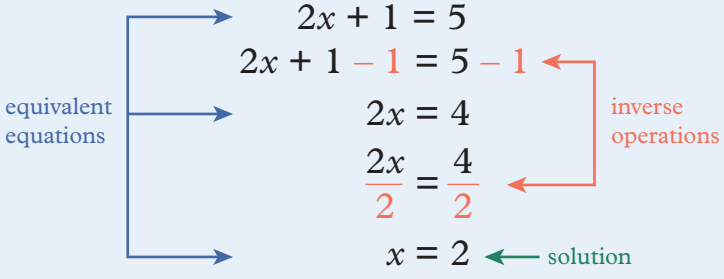
Key content video
Solving equations using inverse operations

- **Inverse operations**, or opposite operations, can reverse the effect of another operation.
  - → Adding and subtracting are inverse operations of each other.
  - → Multiplying and dividing are inverse operations of each another.

Operation	Inverse operation
+ 3	− 3
− 3	+ 3
× 3	÷ 3
÷ 3	× 3

- Inverse operations can be used to manipulate the expressions of an equation until we obtain an equation with the pronumeral by itself on one side of the equation.

  Each manipulation of the expressions produces an equivalent equation, with the simplest equivalent equation revealing the solution.
- To solve equations using inverse operations:
  1. Identify the operation(s) acting on the pronumeral.
  2. Identify the inverse operation(s) required to reverse the operation(s) and isolate x on one side of the equation.
  3. Apply the inverse operations to both sides of the equation or 'do the same to both sides' to maintain equivalence.



- For equations involving more than one operation, inverse operations must be performed in the *reverse order* to how they are applied to the pronumeral.
  - → For example, in the equation $2(x + 5) = 17$, the operations applied to x are addition (+ 5) followed by multiplication (× 2). This means that the first inverse operation to perform would be to reverse the multiplication (÷ 2), and the second would be to reverse the addition within the brackets (− 5).

Worked example 6.2A Solving one-step equations using inverse operations

Solve the following equations using inverse operations.

a $x + 3 = 11$

b $\frac{x}{5} = -8$

c $-4x = 16$

d $x - 12 = 7$

THINK	WRITE
a 1 Identify the operation acting on the pronumeral. In this case, 3 is being added to the x term.	**a** The operation acting on x is + 3.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, subtract 3 from both sides of the equation.	The inverse operation to + 3 is − 3. $x + 3 = 11$ $x + 3 - 3 = 11 - 3$
3 Simplify both sides of the equation by completing the inverse operation.	$x = 8$
4 Write the solution to the equation.	
b 1 Identify the operation acting on the pronumeral. In this case, the x term is being divided by 5.	**b** The operation acting on x is ÷ 5.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, multiply both sides of the equation by 5.	The inverse operation to ÷ 5 is × 5. $\frac{x}{5} = -8$ $\frac{x}{5} \times 5 = -8 \times 5$ $\frac{5^1 x}{5^1} = -40$
3 Simplify both sides of the equation by completing the inverse operation.	$x = -40$
4 Write the solution to the equation.	
c 1 Identify the operation acting on the pronumerals. In this case, the x term is being multiplied by −4.	**c** The operation acting on x is × (−4).
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, divide both sides of the equation by −4.	The inverse operation to × −4 is ÷ (−4). $-4x = 16$ $\frac{-4^1 x}{-4^1} = \frac{16^{-4}}{-4^1}$
3 Simplify both sides of the equation by completing the inverse operation.	$x = -4$
4 Write the solution to the equation.	
d 1 Identify the operation acting on the pronumerals. In this case, 12 is subtracted from the x term.	**d** The operation acting on x is −12.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, add 12 to both sides of the equation.	The inverse operation to −12 is +12. $x - 12 = 7$ $x - 12 + 12 = 7 + 12$
3 Simplify both sides of the operation by completing the inverse operation.	$x = 19$
4 Write the solution to the equation.	

Worked example 6.2B Solving two-step equations using inverse operations

Use inverse operations to solve the following equations.

a $10 - 4x = 26$ **b** $\frac{x+3}{7} = 6$

THINK

a 1 Apply the inverse operations to both sides of the equation in the reverse order to how they are applied to x. Reorder the terms so that the term containing the variable appears first. The operations applied to x are multiplication ($\times (-4)$) and then addition ($+ 10$). The first operation to perform is to reverse the addition ($- 10$). Apply the inverse operation to both sides of the equation.

2 The second operation to reverse is the multiplication, so divide both sides of the equation by -4 to obtain the solution. Remember that dividing a positive number by a negative number gives a negative number.

b 1 Apply the inverse operations to both sides of the equation in the reverse order to how they are applied to x. The operations applied to x are addition ($+ 3$) and then division ($\div 7$). The first operation to perform is to reverse the division ($\times 7$). Apply the inverse operation to both sides of the equation.

2 The second operation to reverse is the addition, so subtract 3 from both sides of the equation to obtain the solution.

WRITE

a
$$10 - 4x = 26$$
$$-4x + 10 = 26$$
$$-4x + 10 - 10 = 26 - 10$$
$$-4x = 16$$
$$\frac{-4^{1}x}{-4^{1}} = \frac{16^{-4}}{-4^{1}}$$
$$x = -4$$

b
$$\frac{x+3}{7} = 6$$
$$\frac{x+3}{7} \times 7 = 6 \times 7$$
$$\frac{7^{1}(x+3)}{7^{1}} = 42$$
$$x + 3 = 42$$
$$x + 3 - 3 = 42 - 3$$
$$x = 39$$

Helpful hints

✔ When solving linear equations, remember that any operation applied to one side of the equation must also be applied to the other side of the equation.

Learning pathways

1–11, 12(a–c), 13, 17(a–d)	4–8, 10, 11, 12(c, e, f), 13(c, d), 14, 15, 17(c, d, g, h),19	5–8, 10,12, 13(c, d), 16–20

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Exercise 6.2A: Understanding and fluency

1 Identify the inverse operation for each of the operations listed below.

a $+ 6$	**b** $\times 4$	**c** $- 7$	**d** $\div 3$
e $\times (-2)$	**f** $\div (-8)$	**g** $- 11$	**h** $\times (-20)$

2 Apply the inverse operation given in the brackets to both sides of the equation to solve each equation.

a $x + 6 = 19$ $(- 6)$	**b** $x - 13 = 29$ $(+ 13)$	**c** $11x = 132$ $(\div 11)$	**d** $\frac{x}{3} = 9$ $(\times 3)$
e $-5x = -5$ $(\div (-5))$	**f** $x - 1 = -2$ $(+ 1)$	**g** $\frac{x}{5} = -5$ $(\times 5)$	**h** $-12x = 36$ $(\div (-12))$

3 Identify the inverse operation required to solve each of these equations.

a $x - 6 = 22$ **b** $x + 9 = 11$ **c** $3x = 21$ **d** $\frac{x}{4} = 6$

e $x - 2 = 9$ **f** $\frac{x}{5} = -9$ **g** $7x = -49$ **h** $x + 1 = 12$

4 Identify the two inverse operations required (in order) to solve each of these equations.

a $3x + 3 = 24$ **b** $2x - 8 = 16$ **c** $\frac{x}{2} + 5 = 11$ **d** $5(x + 5) = 35$

e $5x - 11 = -36$ **f** $\frac{2x}{5} = 8$ **g** $9(x + 4) = -63$ **h** $\frac{x + 4}{3} = 10$

WE 6.2A **5** Solve the following equations using inverse operations.

a $x + 4 = 16$ **b** $x - 11 = 3$ **c** $6x = 42$ **d** $\frac{x}{2} = 5$

e $x + 5 = 5$ **f** $\frac{x}{11} = -2$ **g** $9x = -45$ **h** $x + 2 = -1$

i $-2x = 4$ **j** $-5x = 0$ **k** $x - 7 = -10$ **l** $-x = 17$

WE 6.2B **6** Use inverse operations to solve the following equations.

a $2x + 7 = 23$ **b** $9x - 4 = 5$ **c** $\frac{x}{6} + 3 = 7$ **d** $8(x - 2) = 32$

e $\frac{x + 2}{5} = 4$ **f** $\frac{x}{3} - 9 = 2$ **g** $\frac{x - 8}{2} = 6$ **h** $3(x + 7) = 27$

i $7x - 4 = 45$ **j** $\frac{x + 9}{5} = 3$ **k** $10x + 5 = 15$ **l** $2(x - 6) = 14$

7 Simplify the left-hand side of each equation and then solve the simplified equations using inverse operations.

a $-2x - 3x + 8 = 3$ **b** $x - 5x - 3 = 5$ **c** $-x + 11 + 3x = 19$ **d** $5x + 7 - 8x = 1$

e $-4(x - 3) + 2x = -18$ **f** $-8(x + 1) + 9 = -15$ **g** $-4(x - 1) + 3x = 13$ **h** $3(2 - x) + 2x = -4$

i $4x - 5(x + 1) = 17$ **j** $-5x + 2(x - 12) = 21$ **k** $-4x + 3(x - 1) = -2$ **l** $2(x - 1) - 3(x - 3) = 1$

8 The solution to a given equation is not always an integer. Solve the following equations and leave your answers as fractions.

a $5x = 1$ **b** $6x = 27$ **c** $3x - 5 = 12$

d $-5x + 7 = 3$ **e** $\frac{x - 2}{3} = \frac{4}{5}$ **f** $-\frac{2}{3}(x + 4) = 5$

9 Caleb decides that a quick way to find the solution to $x - 37 = 85$ is to subtract 37 from both sides of the equation. Is he correct? Explain.

Exercise 6.2B: Problem solving and reasoning

10 For each of the following:

i solve the equation by directly applying inverse operations

ii expand the brackets and then solve the equation using inverse operations

iii compare your solutions to parts **i** and **ii**.

a $3(x + 4) = 18$ **b** $-7(x - 2) = 14$ **c** $12(2 - x) = 36$ **d** $-9(5 - x) = 63$

11 Yuhan and Connor take different approaches to solving $\frac{2x}{3} = 4$ for x. Their approaches are shown on the right.

a Check each student's working. Has each student solved the equation correctly?

b Describe one advantage and one disadvantage of each approach.

Yuhan	Connor
$\frac{2x}{3} = 4$	$\frac{2x}{3} = 4$
$\frac{2x}{3} \times 3 = 4 \times 3$	$\frac{2}{3} \times x = 4$
$2x = 12$	$x = 4 \div \frac{2}{3}$
$\frac{2x}{2} = \frac{12}{2}$	$= 4 \times \frac{3}{2}$
$x = 6$	$= \frac{12}{2}$
	$= 6$

12 Solve the following questions:

a $\frac{4x}{7} = 8$ **b** $\frac{8x}{3} = -16$

c $-\frac{3}{5}x = 6$ **d** $\frac{2}{3}x = \frac{1}{2}$

e $\frac{2x}{9} = 0.4$ **f** $\frac{8x}{9} = \frac{5}{3}$

13 For each of the situations below:

i write an equation to represent each problem. **ii** solve the equation

a The temperature rises by 4°C in the first hour after sunrise and then doubles by midday. If it is 26°C at midday, what was the temperature at sunrise?

b Hiring a party venue costs $280. There is a $15 per person charge for finger food. If Marita has a budget of $1600, how many people can attend a party at this venue?

c Ryan is saving to buy a sound mixing board that costs $649. He saves $56 per month. If he currently has $257, in how many months will he be able to buy the mixing board?

d Rhys cooks some donuts. While they are still hot, his friends eat six of them. Half of the donuts that are left are iced and the other half are filled with jam. If there are 11 donuts filled with jam, how many donuts did Rhys make?

14 By the end of Tuesday, Jin is up to page 256 in her book. On Monday Jin read $\frac{1}{3}$ the number of pages she read on Tuesday. If Jin started Monday on page 216, how many pages did she read on Tuesday? Write an equation and solve it using inverse operations.

15 Henry plants x seedlings each week, but $\frac{1}{4}$ of the seedlings he plants die. At the end of 5 weeks Henry has 75 seedlings. Write an equation and solve it to find x.

16 Jenny's mum awards her 10 points for doing a job and deducts 5 points if she forgets to do a job. At the end of one week, Jenny has forgotten to do $1\frac{1}{2}$ times the number of jobs as she has remembered to do.

a Write an expression to represent the total number of points Jenny will have at the end of the week.

b Explain why you do not have enough information to determine the number of jobs that Jenny completed that week.

c Explain why Jenny will not end up with negative points.

d If Jenny ends up with 20 points, find how many jobs she was expected to do that week.

17 Solve each equation.

a $4(x + 2) - 5 = 15$ **b** $\frac{2x - 6}{5} = 4$ **c** $\frac{2x}{3} + 8 = 14$ **d** $\frac{5x + 1}{6} = -4$

e $\frac{6 - x}{5} = 2$ **f** $-\frac{x}{4} + 9 = 7$ **g** $9(x - 4) + 3 = 57$ **h** $\frac{4x}{7} - 3 = 5$

Exercise 6.2C: Challenge

18 For each of the equations below:

i identify the inverse operations (in order) required to solve the equation.

ii find the positive solutions to the equation.

a $3x^2 + 5 = 32$ **b** $4(x^2 - 2) = 56$ **c** $\sqrt{2x + 5} = 5$

19 Sophie is $\frac{1}{4}$ the age she will be in 36 years' time. How old is she now?

20 Giovanni solved the following equation and got $b = 12$ as his solution: $\frac{b + 12}{4} = 15$

a Prove by substitution that the solution is wrong.

b Identify the mistake.

c What is the correct solution?

Online resources:

- **Interactive skillsheet** Solving equations using inverse operations
- **Worksheet** Solving equations using algebra tiles
- **Worksheet** Solving equations using inverse operations
- **Investigation** Household bills
- **Quick quiz** 6.2

Lesson 6.3

Solving equations with the unknown on both sides

Learning intentions

By the end of this lesson you will be able to ...

✔ solve linear equations with the unknown on both sides.

Inter-year links

Year 7	6.6 Solving equations using inverse operations
Year 9	3.1 Solving linear equations
Year 10	3.1 Solving linear equations

Solving equations with the unknown on both sides

Key content video
Solving equations with the unknown on both sides

- As a pronumeral is just a letter standing in for a number, inverse operations can also be applied to terms containing a pronumeral.
- To solve an equation in which the unknown appears on both sides of the equation:
  1. Use inverse operations to remove the pronumeral term from one side of the equation.
  2. Solve the equation using inverse operations.
     For example, to remove $3x$ from the right-hand side of the equation to the right, subtract $3x$ from *both sides* of the equation.

$$\begin{aligned} 4x - 2 &= 3x + 1 \\ 4x - 3x - 2 &= 3x - 3x + 1 \\ x - 2 &= 1 \\ x - 2 + 2 &= 1 + 2 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 4x - 2 &= 3x + 1 && (-3x) \\ x - 2 &= 1 && (+2) \\ x &= 3 \end{aligned}$$

- When using inverse operations, it is important to show your working. This can be done by explicitly applying the inverse operations to both sides, or alternatively, by writing each inverse operation in brackets to the right of each equivalent equation.

Worked example 6.3A Solving equations with the unknown on both sides – subtracting the pronumeral term

Solve each equation using inverse operations.

a $6x - 3 = 5x + 7$ **b** $8x - 5 = 10x + 3$

THINK

1. Remove the pronumeral term from one side of the equation by subtracting the pronumeral term with the smaller coefficient from both sides of the equation.
2. Solve the equation using inverse operations.

WRITE

a
$$\begin{aligned} 6x - 3 &= 5x + 7 \\ 6x - 5x - 3 &= 5x - 5x + 7 \\ x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x &= 10 \end{aligned}$$

b
$$\begin{aligned} 8x - 5 &= 10x + 3 \\ 8x - 8x - 5 &= 10x - 8x + 3 \\ -5 &= 2x + 3 \\ -5 - 3 &= 2x + 3 - 3 \\ -8 &= 2x \\ \frac{-8^{-4}}{2^{1}} &= \frac{2^{1}x}{2^{1}} \\ -4 &= x \text{ or } x = -4 \end{aligned}$$

Worked example 6.3B Solving equations with the unknown on both sides – adding the pronumeral term

Find the value of x in the following equations.

a $4x + 7 = -2x - 23$ **b** $10 - 2x = 4x - 12$

THINK

1 Remove the negative pronumeral term from one side of the equation by adding the pronumeral term to both sides of the equation.

2 Solve the equation using inverse operations.

WRITE

a
$$4x + 7 = -2x - 23$$
$$4x + 2x + 7 = -2x + 2x - 23$$
$$6x + 7 = -23$$
$$6x + 7 - 7 = -23 - 7$$
$$6x = -30$$
$$\frac{\cancel{6}^{1}x}{\cancel{6}^{1}} = \frac{\cancel{-30}^{-5}}{\cancel{6}^{1}}$$
$$x = -5$$

b
$$10 - 2x = 4x - 12$$
$$10 - 2x + 2x = 4x + 2x - 12$$
$$10 = 6x - 12$$
$$10 + 12 = 6x - 12 + 12$$
$$22 = 6x$$
$$\frac{\cancel{22}^{11}}{\cancel{6}^{3}} = \frac{\cancel{6}^{1}x}{\cancel{6}^{1}}$$
$$\frac{11}{3} = x \text{ or } x = \frac{11}{3}$$

Helpful hints

- ✔ When solving an equation with the unknown on both sides, it doesn't matter which term you choose to add or subtract as long as the resulting equation has the unknown on only one side. If the resulting coefficient of the unknown is negative, remember to correctly apply the rules for multiplying or dividing by a negative number.
- ✔ To find the solution, the pronumeral does not have to appear on the left-hand side of the equation – as long as the pronumeral is by itself on one side of the equation, you have found the solution!

Learning pathways

1–3(1st, 2nd columns), 4–8, 10–12, 15	1–3(3rd, 4th columns), 4, 5, 7, 9, 12, 13, 15, 16(a, c, e)	2–3(3rd, 4th columns), 4, 7, 10, 12, 14–17

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Exercise 6.3A: Understanding and fluency

WE 6.3A **1** Solve each equation using the inverse operations.

a $9x - 2 = 8x + 11$	**b** $6x + 5 = 5x + 9$	**c** $2x + 7 = x + 12$	**d** $7x - 3 = 6x - 1$
e $4x - 9 = 5x - 3$	**f** $4x - 10 = 4 + 3x$	**g** $2x + 18 = 8 + 3x$	**h** $2 + 7x = 8x - 13$
i $3x + 9 = 5x - 7$	**j** $x - 12 = 1 + 2x$	**k** $11x - 5 = 10x + 3$	**l** $19x + 3 = 20x + 3$

2 Solve:

a $5a + 4 = 3a + 8$	**b** $4b - 1 = 2b + 7$	**c** $2c - 3 = 6c + 1$	**d** $3d + 10 = 7d + 2$
e $11e - 2 = 6 + 13e$	**f** $5f + 9 = 7f - 2$	**g** $2g + 6 = 5g - 4$	**h** $10 + 5h = h - 2$
i $7i + 2 = 3i + 12$	**j** $12 + j = 6j - 13$	**k** $8k + 3 = k - 3$	**l** $8l - 9 = 12l + 9$

WE 6.3B **3** Find the value of x in the following equations.

a $3x - 11 = -2x + 9$ **b** $2x + 1 = -5x - 13$ **c** $8x + 3 = -2x - 7$ **d** $3x - 8 = -6x + 19$

e $5x + 6 = -3x + 21$ **f** $3x - 5 = 7 - 9x$ **g** $-4x + 9 = -x - 9$ **h** $6 - x = x - 4$

i $29 - 2x = 5 - 6x$ **j** $2x + 2 = 30 - 4x$ **k** $43 - x = x + 1$ **l** $-12 - 5x = 24 - 13x$

4 For each of the equations below:

i solve the equation **ii** check your solution using substitution.

a $8x - 5 = 5x + 13$ **b** $2x - 5 = 15 - 3x$ **c** $4x - 15 = -4x + 17$ **d** $-2x + 12 = -x - 7$

e $4 + 7x = 5 + 12x$ **f** $-19 - 3x = 13 - x$ **g** $x - 6 = 9x + 10$ **h** $19 - 3x = 15 + 3x$

5 Consider the equation $2x + 3 + 4x + 1 = 3x + 7 + x - 5$.

a Simplify the equation by collecting like terms on each side.

b Solve the simplified equation.

6 Three students produce different steps of working to solve $6x + 11 = 2x - 13$.

Andy

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 8x + 11 &= -13 \\ 8x + 11 - 11 &= -13 - 11 \\ 8x &= -24 \\ \frac{8x}{8} &= \frac{-24}{8} \\ x &= -3 \end{aligned}$$

Kwame

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 &= 2x - 13 - 2x \\ 6x + 11 &= -13 \\ 6x + 11 - 11 &= -13 - 11 \\ 6x &= -24 \\ \frac{6x}{6} &= \frac{-24}{6} \\ x &= -4 \end{aligned}$$

Hayley

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= 13 \\ 4x + 11 - 11 &= 13 - 11 \\ 4x &= 2 \\ \frac{4x}{4} &= \frac{2}{4} \\ x &= \frac{1}{2} \end{aligned}$$

a Check each student's work. Identify and describe any mistakes.

b Solve the equation correctly. Show your working.

7 Solve each equation.

a $3p + 2p - 5 = p + 7$ **b** $5q + 2 + 3q + 4 = 13 + 2q + 5$

c $6r + 7 - 3r + 3 = r + 4$ **d** $4s + 5 - 2s + 1 = 26 - 3s$

e $6t + 2 + 4t + 9 = 2t + 3 + 3t - 2$ **f** $9 - 5u + 3u - 7 = 5 - 3u + 10$

Exercise 6.3B: Problem solving and reasoning

8 Selena and Hayden have the same amount of money. Selena buys 5 kg of cherries and has $2 left over. Hayden buys 2 kg of cherries and has $20 left over.

a If x represents the cost of 1 kg of cherries, which equation best suits this situation?

A $5x + 20 = 2x + 2$ **B** $5x + 2 = 2x + 20$

C $5x - 2 = 2x - 20$ **D** $5x - 20 = 2x - 2$

b Solve the equation to find the cost of 1 kg of cherries.

9 The number of biscuits in a box is unknown. However, Kayla can fill her biscuit tin with five more than the contents of three boxes or seven less than the contents of four boxes.

a If n represents the number of biscuits in a box, which equation best suits this situation?

A $3n - 5 = 4n - 7$ **B** $3n - 5 = 4n + 7$

C $3n + 5 = 4n + 7$ **D** $3n + 5 = 4n - 7$

b Solve the equation to find the number of biscuits in a box.

10 At the school canteen, the cost of three apples and five oranges is the same as the cost of seven apples and two oranges. Each orange costs 80 cents. Let a represent the cost of an apple in cents.

a Write an equation to represent this problem. **b** Solve the equation to find the cost of an apple.

11 Sean poses this problem to his neighbour: 'In four years, I will be three times as old as my cousin is now. Two years ago, I was twice my cousin's current age. Can you work out how old I am?' Let a represent the current age of Sean's cousin.

a One expression for Sean's current age is $3a - 4$. Write another expression for Sean's current age.

b Use the expressions in part **a** to write an equation.

c Solve the equation to determine the current age of Sean's cousin.

d How old is Sean?

12 Jada and Tom have an equal number of marbles. Jada has four small marbles and Tom has ten small marbles. Jada has three times as many large marbles as Tom. Let n represent the number of large marbles Tom has.

a Write an expression for how many marbles Tom has.

b Write an expression for how many marbles Jada has.

c Form an equation using these expressions.

d Solve the equation to find the value of n.

e How many large marbles does Tom have?

f How many large marbles does Jada have?

g If they agree to swap all their marbles, how many marbles will each person swap?

13 In an AFL match, each goal is worth 6 points and each behind is worth 1 point. An AFL match ends in a draw, with the home team scoring twice as many goals as the away team. The home team scored 3 behinds while the away team scored 27 behinds.

a Write an expression for the number of points scored by the away team using g to represent the number of goals scored by the away team.

b Write an expression for the number of points scored by the home team.

c Form an equation using these expressions.

d Solve the equation to find the value of g and hence list the final score of each team.

14 In rugby league, each try is worth 4 points and each penalty goal is worth 2 points. In a rugby league match, the away team scored twice as many points as the home team. The away team scored two penalty goals and three times as many tries as the home team. The home team scored four penalty goals.

a Let t represent the number of tries scored by the home team. Write an equation, in terms of t, to represent the situation.

b Solve the equation to find the value of t.

c List the number of tries scored by each team.

15 Consider the equation $5(x - 2) = 3(x + 8)$.

a Expand both sides of the equation.

b Solve the equation for x.

16 Solve each equation for x.

a $2(x - 5) = x + 19$

b $4(x - 1) = 2 - 2x$

c $3(x + 4) = 2(x + 9)$

d $6(x + 3) - x + 5 = 2(x + 1)$

e $4(2x - 3) = 3(7 - x)$

f $11(3x + 2) = 4(2x - 7)$

Exercise 6.3C: Challenge

17 For each of the equations below:

a explain how you would solve each equation

b find the positive solutions to the equation.

i $5(x^2 + 2) = 2(2x^2 + 7)$

ii $3(x^2 - 4) = 2(x^2 - 6)$

Online resources:

- **Interactive skillsheet** Solving equations with unknown on both sides
- **Worksheet** Solving more equations with algebra tiles
- **Investigation** Profit and loss
- **Quick quiz** 6.3

Lesson 6.4

Solving simple quadratic equations

Learning intentions

By the end of this lesson you will be able to ...

✔ solve simple quadratic equations.

Inter-year links

Year 7 1.7 Exponents and square roots
Year 9 4.1 Solving quadratic equations
Year 10 4.1 Solving quadratic equations

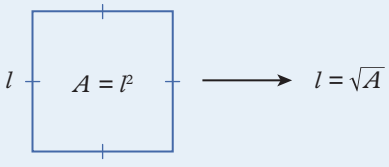
Solving simple quadratic equations

Key content video
Solving simple quadratic equations

- A single variable **quadratic equation** is an equation in which the highest exponent of the variable is 2.
- A simple quadratic equation is a single-variable equation in which the only exponent of the variable is 2.

Quadratic equations		Not quadratic equations
Simple	Not simple	
$x^2 = 9$	$x^2 + x = 9$	$x^3 = 9$
$5y^2 = 4$	$5y^2 = 4y$	$2y^5 = 4$
$25 - b^2 = 0$	$25b - b^2 = 0$	$4x^3 + 3x^2 + 2x + 1 = 8$
$9(z^2 - 4) = -1$	$9z(z - 4) = -1$	$2x - 1 = 5$

- The equation $x^2 = c$ has two solutions when c is positive: a positive solution, $x = \sqrt{c}$, and a negative solution, $x = -\sqrt{c}$.
- There are no real solutions to $x^2 = c$ if c is negative, because the square of both positive and negative numbers will always be positive.
- For $x^2 = 0$, $x = 0$ is the only solution.
- The solutions to a quadratic equation can be written as the square root of the number if it is an irrational number (this is called exact form), or it can be rounded to a given number of decimal places.
  - → This is because $(\sqrt{c})^2 = c$ and $(-\sqrt{c})^2 = c$.
  - → These solutions can be written as $x = \sqrt{c}$ or $x = -\sqrt{c}$, or together as $x = \pm\sqrt{c}$ (read 'x is equal to positive or negative $\sqrt{c}$').
    For example, if $x^2 = 11$, then $x = \pm\sqrt{11} \approx \pm 3.32$.
  - → Fractional square roots can be simplified using the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.
- Many formulas include squared variables. Given values for any other variables, use substitution to solve for possible values of the squared variable. For example, to determine the side length of a square given the area, substitute the area, A, into the formula $A = l^2$ and solve for l.
  - → It is important to consider whether a negative solution is possible in a particular context.
    For example, the solutions to $A = l^2$ are $\pm\sqrt{A}$, but length is positive so $l = \sqrt{A}$.



Worked example 6.4A Solving simple quadratic equations

Solve the following equations for x.

a $x^2 = 25$ **b** $121x^2 = 36$ **c** $x^2 - 0.09 = 0.25$

THINK	WRITE
a 1 Take the square root of both sides of the equation to eliminate the exponent, write your answer separately or together using $\pm$.	**a** $x^2 = 25$ $x = \pm\sqrt{25}$
2 Evaluate the square root.	$x = \pm 5$
b 1 Use inverse operations to make x^2 on one side of the equation.	**b** $121x^2 = 36$ $x^2 = \frac{36}{121}$
2 Take the square root of both sides of the equation to eliminate the exponent, write your answer separately or together using $\pm$.	$x = \pm\sqrt{\frac{36}{121}}$
3 Evaluate the square root by taking the square root of the numerator and the denominator separately.	$x = \pm\frac{6}{11}$
c 1 Use inverse operations to write x^2 alone on one side of the equation.	**c** $x^2 - 0.09 = 0.25$ $x^2 = 0.16$
2 Take the square root of both sides of the equation to eliminate the exponent, write your answer separately or together using $\pm$.	$x = \pm\sqrt{0.16}$
3 Evaluate the square root.	$x = \pm 0.4$

Worked example 6.4B Solving equations with exact and approximate solutions

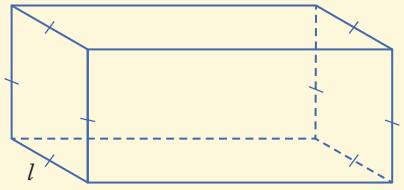
Solve $x^2 = 26$ for x. Write your answers:

a in exact form **b** correct to two decimal places.

THINK	WRITE
a Take the square root of both sides of the equation to eliminate the exponent. If the square root is irrational, leave the answer in square root form.	**a** $x = \pm\sqrt{26}$
b 1 Use a calculator to find the decimal expansion of the square root. Write more decimal places than are required in the question.	**b** $\sqrt{26} = 5.0990195\ldots$
2 Round the decimal expansion to two decimal places.	$\sqrt{26} \approx 5.10$
3 Write both the positive and negative approximations.	$x \approx \pm 5.10$

Worked example 6.4C Substituting into quadratic equations

Solve for the unknown side length of the following square prism with volume $V = 24$ cm^3. Recall that the volume of a rectangular prism is found using the formula $V = l \times w \times h$.



THINK	WRITE
1 Use the formula for the volume of a rectangular prism to find the formula for the volume of a square prism.	$V = l^2 \times w$
2 Identify the known values in the formula.	$V = 24$ cm^3 and $w = 6$ cm
3 Substitute the known values into the formula and solve for the squared variable.	$24 = l^2 \times 6$ $24 = 6l^2$ $l^2 = 4$
4 Write down the form of the two possible square root solutions. Simplify if the value under the square root is a perfect square.	$l = \pm\sqrt{4}$ $l = \pm 2$
5 Keep only the positive solution because side lengths can't be negative. Remember to include the appropriate units.	$l = 2$ cm as $l > 0$

Helpful hints

- ✔ Don't forget to consider the negative square root solution!
- ✔ Always give your answer in simplified, exact form unless otherwise specified.
- ✔ Remember to check whether a negative solution makes sense.

Learning pathways

1, 3–7, 9–13	2, 3(1st, 3rd columns), 6–9, 10–11(1st, 2nd columns), 12–15	2–4, 5(3rd, 4th columns), 6, 8, 10–11(3rd column), 13–16

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Exercise 6.4A: Understanding and fluency

WE 6.4A **1** Solve the following equations for the unknown.

a $x^2 = 100$ **b** $y^2 = 9$ **c** $z^2 = 49$ **d** $a^2 = 81$
e $k^2 = 1$ **f** $h^2 = 0$ **g** $m^2 = 189$ **h** $n^2 = 40\,000$

2 Solve the following equations for the unknown.

a $b^2 = \frac{25}{64}$ **b** $u^2 = \frac{144}{49}$ **c** $d^2 = 1\frac{9}{16}$ **d** $g^2 = \frac{121}{9}$
e $w^2 = 0.36$ **f** $i^2 = 0.0004$ **g** $x^2 = 0.0625$ **h** $r^2 = 1.002\,001$

3 Solve the following equations for the unknown.

a $x^2 - 5 = 116$ **b** $y^2 + 0.18 = 0.67$ **c** $z^2 - \frac{37}{20} = \frac{257}{80}$

d $5a^2 = 405$ **e** $\frac{b^2}{3} = 27$ **f** $\frac{4}{3}c^2 = \frac{3}{4}$

g $5m^2 + 13 = 333$ **h** $\frac{n^2 - 29}{5} = 23$ **i** $\frac{6}{p^2} - 5 = -4.76$

j $\frac{8}{3p^2} + 1 = \frac{35}{3}$ **k** $\frac{2}{q^2 - 5} = \frac{1}{2}$ **l** $\frac{39}{18r^2 + 5} - 4 = -1$

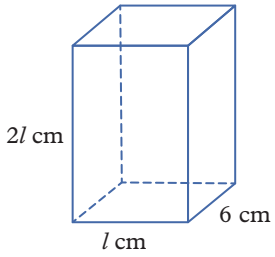
WE 6.4B **4** Solve the following equations for the unknown. Write your answers in exact form.

a $x^2 = 10$ **b** $y^2 = 91$ **c** $z^2 = 0.4$ **d** $i^2 = 1.6$

e $6j^2 = 13$ **f** $8k^2 - 7 = 49$ **g** $\frac{5}{3}m^2 - 1 = 0$ **h** $\frac{4n^2 - 2}{7} = 6$

5 Write your answers to question **4** correct to two decimal places.

WE 6.4C **6** Solve for the unknown side lengths of the following rectangular prism with volume $V = 300$ cm^3, length l cm, width w cm and height $2l$ cm. Recall that the volume of a rectangular prism is found using the formula $V = l \times w \times h$.



7 Recall that the formula for the area of a square with side length l is $A = l^2$.

a What is the formula for the total surface area of all six faces of a cube with side lengths l?

b Solve for the unknown side length l if the cube has a total surface area of 216 cm^2.

8 Substitute the given values into the formula and solve for the unknown. Write your answers correct to two decimal places.

a $v^2 = 2as + u^2$ for v where $a = 9$, $s = 80$ and $u = 5$ **b** $a^2 + b^2 = c^2$ for c where $a = 10$ and $b = 7$

c $v^2 = 2as + u^2$ for u where $v = 30$, $a = -5$ and $s = 80$ **d** $K = \frac{1}{2}mv^2$ for v where $K = 100$ and $m = 80$

e $\Delta = b^2 - 4ac$ for b where $\Delta = 25$, $a = 6$ and $c = 3$

Exercise 6.4B: Problem solving and reasoning

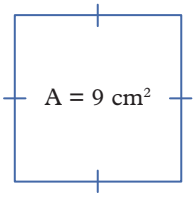
9 When working with measurements like length, area or volume, it is important to consider whether a negative solution to an equation makes sense.

a The formula $a^2 + b^2 = c^2$ gives the square of the longest side, c, given the lengths of the two perpendicular sides, a and b, in a right-angled triangle. How many solutions are there for the longest side, c? Explain.

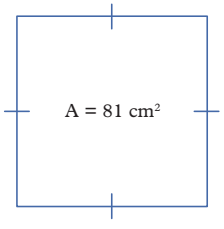
b The formula $A = \pi r^2$ gives the area, A, of a circle with a radius of length r. How many solutions are there for the radius, r? Explain.

10 Determine the side lengths of the following squares using the area formula $A = l^2$. Give exact answers, using the square root sign if required.

a



b



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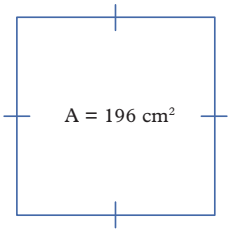

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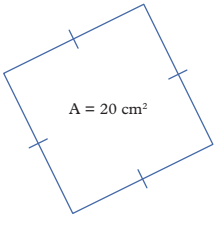

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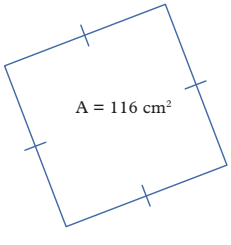

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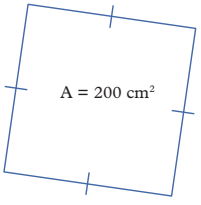
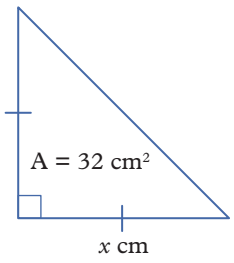


11 Determine the value of the unknown for the following triangles using the area formula $A = \frac{1}{2}bh$.

a



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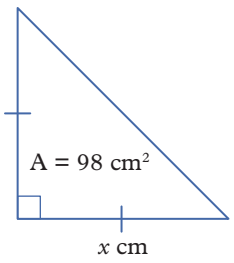

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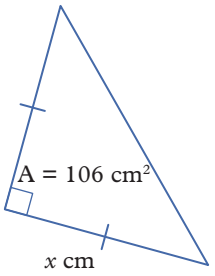

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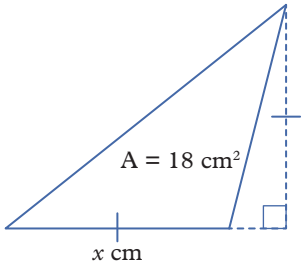

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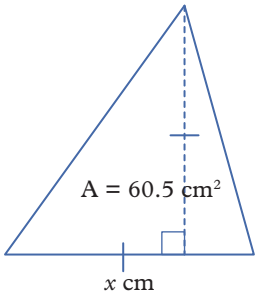

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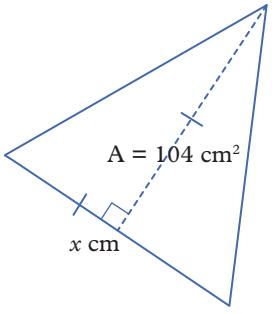

12 Antonina says that $x^2 + 25 = 0$ should have two solutions because 'the 25 is positive'.

a Explain why Antonina is incorrect.

b Explain why there are zero solutions to the equation $x^2 + 25 = 0$.

13 Solve the following equations for the unknown.

a $-8a^2 = -200$ **b** $-7b^2 = -305$ **c** $25 - c^2 = 9$

d $100 - d^2 = 25$ **e** $36 + 9e^2 = 81$ **f** $13 - 8f^2 = -1339$

14 The formula $L = \frac{gT^2}{4\pi^2}$ gives the length of a pendulum, L, in metres, given the time, T, it takes for the pendulum to swing, in seconds, and the force of gravity, $g = 9.8$ m/s^2 on Earth.

a Determine value(s) of T of a pendulum with a length of 25 m.

b Explain why you either accepted or rejected the negative solution.

Exercise 6.4C: Challenge

15 A quadratic equation in the form $(x + h)^2 = k$ can be solved for x by first finding the positive and negative roots of k, then subtracting h from both sides. For example:

$(x + 3)^2 = 25$

$x + 3 = \sqrt{25}$ or $x + 3 = -\sqrt{25}$

$x + 3 = 5$ or $x + 3 = -5$

$x = 2$ or $x = -8$.

Solve the following equations for x. Write your answers correct to two decimal places where required.

a $(x + 5)^2 = 36$ **b** $(x + 8)^2 = 64$ **c** $(x - 4)^2 = 100$

d $(x - 7)^2 = 49$ **e** $(x - 5)^2 = 20$ **f** $(x + 9)^2 = 90$

16 When solving equations like those in question **15**, we can write the exact form of both solutions as $x = -h \pm \sqrt{k}$. Solve the following equations for the unknown, leaving your answer in exact form.

a $(x + 5)^2 = 35$ **b** $(x + 8)^2 = 65$ **c** $(x - 4)^2 = 97$

d $(x - 7)^2 = 51$ **e** $(x - 5)^2 = 21$ **f** $(x + 9)^2 = 91$

Online resources:

Interactive skillsheet
Solving simple quadratic equations

Quick quiz
6.4

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

6.1 **1** Check if the given value of x is a solution to the following equations.

a $x + 2 = 4, x = 2$ **b** $6x = 30, x = 5$ **c** $\frac{1}{2}x = 4, x = 4$ **d** $\frac{1}{2}x + 2 = 4, x = 4$

6.1 **2** Solve the following equations by inspection.

a $3a = 12$ **b** $\frac{p}{3} = -7$ **c** $z + 8 = -9$

d $7a + 5 = 12$ **e** $\frac{2j}{5} = -6$ **f** $10y + 8 = 78$

6.1 **3** Write an equation to represent each of these number sentences. Use x for the unknown number.

a 6 added to a number gives 18.
b 10 less than a number is equal to -4.
c A number is multiplied by 4, then 9 is added to give 33.
d The sum of a number and 12 all divided by 3 is equal to 6.
e The sum of a number and 3 all multiplied by 8 is equal to 64.

6.2 **4** Solve the following equations using inverse operations.

a $x + 11 = 25$ **b** $x - 8 = 17$ **c** $12x = 96$

d $\frac{x}{7} = 5$ **e** $x + 23 = -18$ **f** $-6x = 54$

6.2 **5** Use inverse operations to solve the following equations.

a $3x + 8 = 20$ **b** $9x - 2 = 16$ **c** $\frac{x}{4} + 5 = 0$

d $6(x - 4) = 48$ **e** $\frac{x}{6} + 9 = 18$ **f** $\frac{x + 7}{2} = 10$

6.2 **6** Solve the following equations for x.

a $-6x + 4 = -20$ **b** $-8x - 3 = -51$ **c** $-3(x + 12) = 60$

d $\frac{x - 8}{6} = -4$ **e** $-9(x + 3) = 27$ **f** $\frac{x + 8}{3} = -11$

6.2 **7** Solve the following equations for x.

a $7x = 1$ **b** $-2x + 5 = 14$ **c** $\frac{5x}{6} = 7$

d $5(x + 3) = 21$ **e** $\frac{x - 4}{5} = \frac{2}{3}$ **f** $\frac{1}{3}(x + 6) = -5$

6.3 **8** Solve the following equations for x.

a $8x + 3 = 5x + 15$ **b** $9x + 4 = 6x + 13$ **c** $2x + 7 = x + 15$

d $10x - 8 = 18x + 14$ **e** $6x + 9 = 8x - 11$ **f** $3x - 6 = 12 + 9x$

6.3 **9** Solve the following equations for x.

a $9x - 8 = 13x + 13$ **b** $2x - 21 = 9x + 13$ **c** $-7x + 4 = 5x - 8$

d $4(x + 6) = 3x - 15$ **e** $7(x - 5) = 3x + 6$ **f** $5(x + 3) - 4x + 6 = 3(x + 7)$

6.4 **10** Solve the following equations for x.

a $x^2 = 4$ **b** $3x^2 = 48$ **c** $9x^2 = 16$

6.4 **11** Solve the following equations for x.

a $x^2 + 9 = 10$ **b** $x^2 + 7 = 7$ **c** $2x^2 + \frac{1}{9} = 1$

Lesson 6.5

Plotting linear relationships

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and plot linear relationships.

Inter-year links

Support	The Cartesian plane
Year 7	5.4 The Cartesian plane
Year 9	3.3 Plotting linear relationships
Year 10	3.3 Sketching linear graphs

The Cartesian plane and Cartesian coordinates

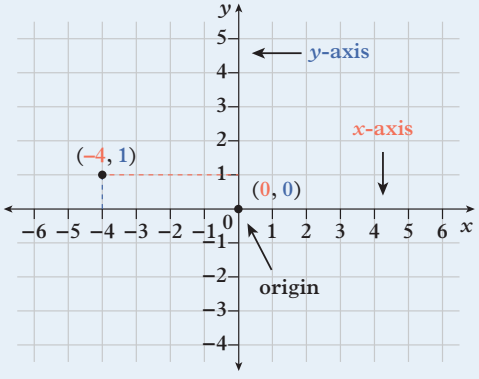
Key content video
Plotting linear relationships

- The **Cartesian plane** consists of a horizontal ***x*-axis** and a vertical ***y*-axis** that intersect at right angles at a point called the **origin**.
- **Coordinates** describe the position of a point on the Cartesian plane and consist of an ***x*-coordinate** and a ***y*-coordinate**.

$(-4, 1)$

x-coordinate: horizontal distance from the origin

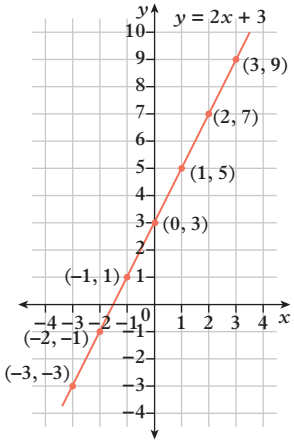
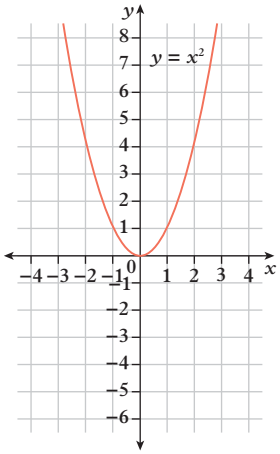
y-coordinate: vertical distance from the origin



Linear and non-linear relationships

- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of coordinates or a graph.
- **Linear relationships** produce straight line graphs called **linear graphs**.
- **Non-linear relationships** produce curved graphs called **non-linear graphs**.

For example, graphs of the linear relationship $y = 2x + 3$ and the non-linear relationship $y = x^2$ are shown below.

Linear relationship	Non-linear relationship
	

Drawing graphs

- Whereas a **plot** is composed of individual coordinate points, a **graph** is a continuous line made up of infinitely many coordinate points.
- Graphs can be drawn by plotting a series of coordinates. If there is a distinct pattern to the points, a straight line or smooth curve can be used to join the points together.

Worked example 6.5A Plotting points on the Cartesian plane

Plot the following points on the Cartesian plane and label each point with its coordinates.

a (2, 5) **b** (−5, −1)

THINK

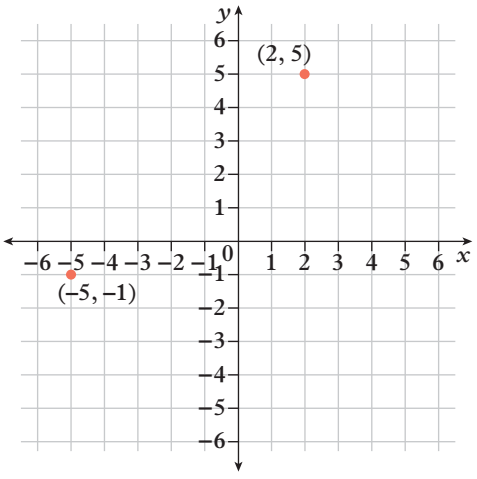
1 Draw a Cartesian plane. The x- and y-axes should extend far enough to accommodate the largest coordinates.

2 Mark each point with a dot and label the dot with its coordinates.

a The point (2, 5) has an x-coordinate of 2 and a y-coordinate of 5. Start at the origin, move 2 units to the right and 5 units up.

b The point (−5, −1) has an x-coordinate of −5 and a y-coordinate of −1. Start at the origin, move 5 units to the left and 1 unit down.

WRITE



Worked example 6.5B Using an equation to complete a table of values

Complete the table of values for $y = 2x - 1$.

x	−2	−1	0	1	2
y					

THINK

1 Substitute each value of x into the equation to find the corresponding value of y.

2 Complete the table of values.

WRITE

Let $x = -2$
$y = 2(-2) - 1$
$= -4 - 1$
$= -5$

Let $x = -1$
$y = 2(-1) - 1$
$= -2 - 1$
$= -3$

Let $x = 0$
$y = 2(0) - 1$
$= 0 - 1$
$= -1$

Let $x = 1$
$y = 2(1) - 1$
$= 2 - 1$
$= 1$

Let $x = 2$
$y = 2(2) - 1$
$= 4 - 1$
$= 3$

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

Worked example 6.5C Graphing linear relationships

Plot a graph of $y = 5 - x$ by first completing a table of values for integer values of x from -3 to 3.

THINK

1 Construct a table of values for x from -3 to 3. Substitute each value of x into the equation to find the corresponding values of y.

2 Write out the coordinates of each point listed in the table.

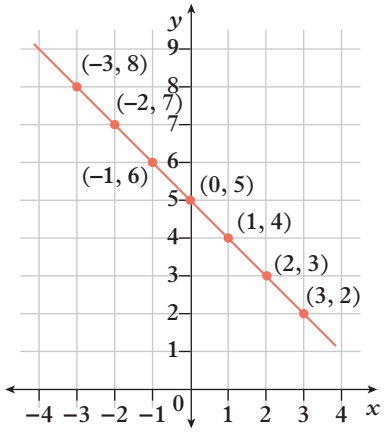
3 Plot the points on the Cartesian plane.

4 Join the points with a smooth line.

WRITE

x	-3	-2	-1	0	1	2	3
y	8	7	6	5	4	3	2

$(-3, 8), (-2, 7), (-1, 6), (0, 5), (1, 4), (2, 3), (3, 2)$



Worked example 6.5D Identifying linear and non-linear relationships

Use the tables of values to plot graphs and determine whether the relationships are linear or non-linear.

a

x	-2	-1	0	1	2
y	7	4	1	-2	-5

b

x	-2	-1	0	1	2
y	9	4	1	0	1

THINK

a 1 Write the coordinates of each point listed in the table.

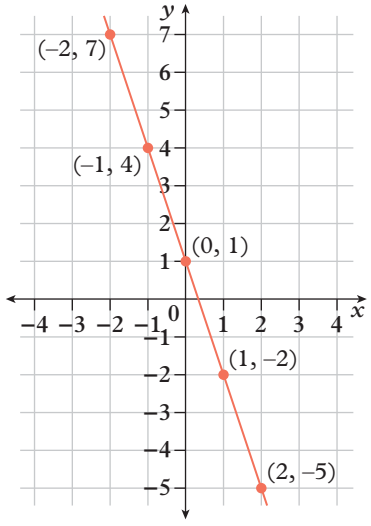
2 Plot the points on the Cartesian plane.

3 Join the coordinate points with a straight line or smooth curve.

4 The coordinates form a straight line, so the table of values represents a linear relationship.

WRITE

a $(-2, 7), (-1, 4), (0, 1), (1, -2), (2, -5)$



The table of values represents a linear relationship.

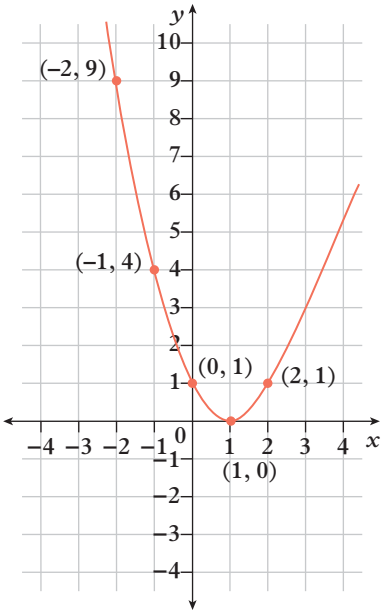
b **1** Write the coordinates of each point listed in the table.

2 Plot the points on the Cartesian plane.

3 Join the coordinate points with a straight line or smooth curve.

4 The coordinates do not form a straight line, so the table of values represents a non-linear relationship.

b (−2, 9), (−1, 4), (0, 1), (1, 0), (2, 1)



The table of values represents a non-linear relationship.

Helpful hints

- ✔ When plotting points, locate the value of the x-coordinate first, then move up or down to the value of the y-coordinate.
- ✔ When plotting a linear graph, use a ruler when joining the points with a straight line.
- ✔ Vertical and horizontal straight lines are also linear relationships.

Learning pathways

1–4, 5(1st, 2nd columns), 6–10, 12, 15	2–4, 5(2nd, 3rd columns), 6–12, 14–16	5(3rd, 4th columns), 6–11, 13, 15–17

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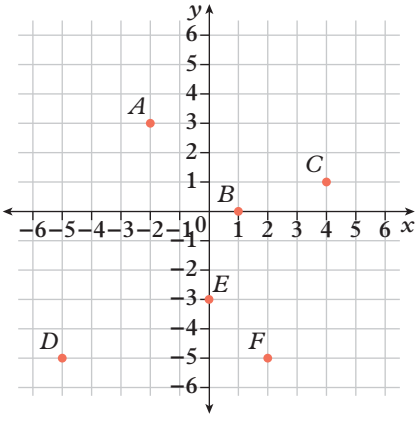
Exercise 6.5A: Understanding and fluency

1 Match the coordinates to the points shown on the Cartesian plane.

a (−2, 3) **b** (4, 1) **c** (2, −5)
d (1, 0) **e** (0, −3) **f** (−5, −5)



WE 6.5A **2** Plot the following points on the Cartesian plane and label each point with its coordinates.

a (3, 5) **b** (−4, −5) **c** (2, −2)
d (6, 0) **e** (−1, 4) **f** (0, 3)

WE 6.5B **3** Complete the table of values for each of the following linear equations.

a $y = x + 1$

x	−2	−1	0	1	2
y					

b $y = 2x + 2$

x	−3	−2	−1	0	1
y					

c $y = 3 - x$

x	−1	0	1	2	3
y					

d $y = 3x - 4$

x	−1	0	1	2	3
y					

4 Plot a graph of each of the linear relationships in question **3**.

WE 6.5C **5** Plot a graph of each linear relationship by first completing a table of values for integer values of x from −3 to 3.

a $y = x + 4$ **b** $y = x - 3$ **c** $y = -x$

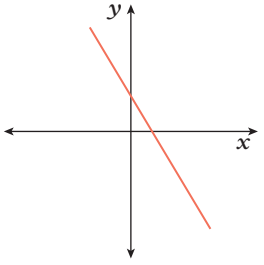
d $y = 2 - x$ **e** $y = 5 - x$ **f** $y = -x - 1$

g $y = 3x$ **h** $y = 2x + 3$ **i** $y = 4x - 2$

j $y = 3x + 1$ **k** $y = 6 - 3x$ **l** $y = 4 - 2x$

6 Determine whether the following relationships are linear or non-linear.

a



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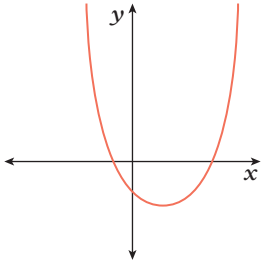
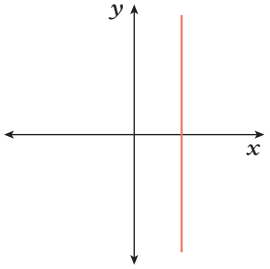


c



WE 6.5D **7** Use the tables of values to plot graphs and determine whether the following relationships are linear or non-linear.

a

x	−2	−1	0	1	2
y	0	1	2	3	4

b

x	−2	−1	0	1	2
y	8	5	2	−1	−4

c

x	0	1	2	3	4
y	0	2	5	10	17

8 Consider the relationships represented by the following tables of values.

a Plot the graph of each relationship using the table of values.

i $y = 2$

x	−2	−1	0	1	2
y	2	2	2	2	2

ii $y = -1$

x	−2	−1	0	1	2
y	−1	−1	−1	−1	−1

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

9 Consider the relationships represented by the following tables of values.

a Plot the graph of each relationship using the table of values.

i $x = 3$

x	3	3	3	3	3
y	−2	−1	0	1	2

ii $x = 0$

x	0	0	0	0	0
y	−2	−1	0	1	2

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

10 Without using a table of values, draw the graph of each relationship.

a $x = 2$ **b** $y = 4$ **c** $y = -3$ **d** $x = -1$ **e** $x = 5$ **f** $y = 0$

Exercise 6.5B: Problem solving and reasoning

11 The first difference of a table of values is the difference between values of y for consecutive values of x.

For example, in **Worked example 6.5D**, both tables of values show values of y for consecutive values of x from -2 to 2. In part **a**, the y-values always decrease by 3 as x increases by 1. As the first difference, -3, is constant, the table represents a linear relationship.

The table in part **b**, however, does not have a constant first difference. Instead, the difference between the y-values varies following the sequence 5,3,1,1. As the first difference is not constant, the table value of values represents a non-linear relationship.

Use this method to verify your solutions to question **7**.

12 When two variables are related, the variable that affects the other is called the *dependent variable* and the other variable is called the *independent variable*. Generally, the independent variable is shown in the first row in a table of values, and on the horizontal axis on the Cartesian plane.

Consider the following situation: Two hot air balloons lift off at the same time. Their heights over the first 10 seconds of flight are recorded in these tables.

Red balloon:

Time (s)	0	2	4	6	8	10
Height (m)	0	4	8	12	16	20

Blue balloon:

Time (s)	0	2	4	6	8	10
Height (m)	0	8	14	18	19	20

a What are the two variables shown in each table?

b In the hot air balloon scenario, which is the:

i independent variable **ii** dependent variable?

c Write the coordinates of the points listed in each table.

d Showing time on the horizontal axis and height on the vertical axis, plot the points on the Cartesian plane and join each set of points with a smooth line to produce two graphs. Label each graph.

e Use the graphs to estimate:

i the height of each balloon after 5 seconds **ii** the time it takes each balloon to reach a height of 7 metres.

f Which graph shows a linear relationship? Explain.

g Graphs can be used to make predictions. Which graph allows for more accurate predictions? Justify your choice.

13 This table shows the distance Jamie covers in each hour of a road trip in her car.

Time (h)	0	1	2	3	4	5
Distance (km)	0	80	160	240	320	400

a What is the independent variable in this relationship? What is the dependent variable?

b Plot a graph of this relationship and label the axes appropriately.

c Is the relationship linear? Explain.

d Use the graph to determine how far Jamie travelled in 2.5 hours.

e Use the graph to determine how long it took her to travel 260 km.

f If Jamie travelled for 7 hours in total, what total distance would she cover? What assumption do you need to make for this prediction?

g Write an equation to represent the relationship between the distance, d, Jamie travels and the time, t, it takes her to cover that distance. Explain your thinking.

h Use the equation to check your answer to part **f**.

14 The water consumption for a household can be described by the linear equation $w = 900d$, where w represents the amount of water (in L) used for the month and d represents the number of days from the start of the month.

a Identify the independent and dependent variables in this relationship.

b Plot a graph of this relationship for the first ten days of the month and label the axes appropriately.

c Is this relationship linear? Explain.

d Use the graph to determine the amount of water used in the first week of the month.

e A resident of the household wants to determine how many days it takes to consume 20 000 L of water.

i Should the resident use the equation or a graph to determine the time taken to consume 20 000 L of water? Justify your choice.

ii Determine how many days it takes to consume 20 000 L of water.

15 Gavin's water tank has sprung a leak and water is leaking out at a constant rate of 800 L/h. He used the following table to record how much water is left in the tank.

Time after the leakage started (h)	0	0.25	0.5	0.75	1
Water left in the tank (L)	2000	1800	1600	1400	1200

a How much water was in the tank before the leakage started?

b Write a formula to represent the relationship between the water left in the tank (w) and time (t).

c In this relationship, which is the:

i dependent variable **ii** independent variable?

d Plot a graph of this relationship and label the axes appropriately.

e Gavin called a plumber immediately after the leakage started. If it takes 3 hours for the plumber to travel to Gavin's place, how much water would be left in the tank by the time the plumber arrives?

16 The equation $y = kx$ gives a different linear relationship for each value of k.

a Plot the graphs of $y = kx$ for $k = 1$, $k = 2$ and $k = 3$ on the same set of axes.

b What do you notice about these lines? What can you say about their steepness? Where do they intercept the x-and y-axes?

c On the same set of axes used in part **b**, plot the graphs of $y = kx$ for $k = -1$, $k = -2$ and $k = -3$.

d What do you notice about the graphs of these lines? How do they compare to each other? How do they compare to the graphs drawn in part **a**?

e On a new set of axes, plot the graphs of $y = kx + 1$ for $k = 1$ and $k = 2$. What do you notice about how these graphs intercept the x- and y-axes? What do you notice about how they intersect each other?

f On a new set of axes, plot the graphs of $y = 2x$, $y = 2x + 1$ and $y = 2(x + 1)$. How do they compare to each other?

Exercise 6.5C: Challenge

17 Both of Lily's parents invested $1000 in a bank account when she was born. Lily's mum invested with 'compound interest' and her dad with 'simple interest'.

The following table shows the value of the investment over the years.

Lily's age, A (years)	**Mum's investment (M)**	**Dad's investment (D)**
0	$1000	$1000
1	$1100	$1120
2	$1210	$1240
3	$1331	$1360
4	$1464.10	$1480
5	$1610.51	$1600

a If you graphed M vs A and D vs A, which graph would be linear?

b Plot the graphs for both investments and use them to predict the value of the investments when Lily is 10.

c Based on this question, do you think compound interest or simple interest is a better choice? Are there other factors to consider?

Online resources:

- **Interactive skillsheet** Plotting graphs
- **Worksheet** Plotting linear relationships
- **Investigation** Graphing length and width of a rectangle of fixed area
- **Desmos activity** Mini golf marble slides
- **Quick quiz** 6.5

Lesson 6.6

Solving linear equations using graphs

Learning intentions

By the end of this lesson you will be able to ...

✔ solve linear equations using a graph.

Inter-year links

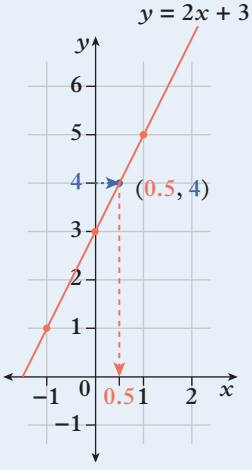
Support	Interpreting data
Year 7	5.4 The Cartesian plane
Year 9	3.5 Sketching linear graphs

Solving linear equations using graphs

Key content video
Solving linear equations using graphs

- Each point on a graph is made up of an x-coordinate and a corresponding y-coordinate. The equation of a graph describes the relationship between the x- and y-coordinates of all the points that make up the graph.
- The x-coordinate of each point on a graph is the solution to a specific equation related to the equation of the graph.

  For example, to solve the equation $2x + 3 = 4$ using the graph of $y = 2x + 3$, locate 4 on the y-axis, then trace across to the graph and down to the x-axis to find the corresponding value of x, which is 0.5.



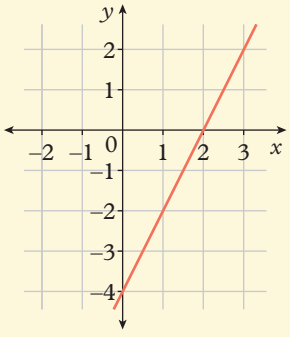
Worked example 6.6A Solving linear equations using a graph

Solve the following equations using the graph of $y = 2x - 4$.

a $2x - 4 = 2$

b $2x - 4 = 0$

c $2x - 4 = -1$



THINK

a Use the graph to find the value of x when $y = 2$. Locate 2 on the y-axis, trace across to the equation line, then down to the x-axis to find the corresponding value of x.

b Use the graph to find the value of x when $y = 0$. $y = 0$ is on the x-axis, so trace along the x-axis to the equation line to find the value of x when $y = 0$.

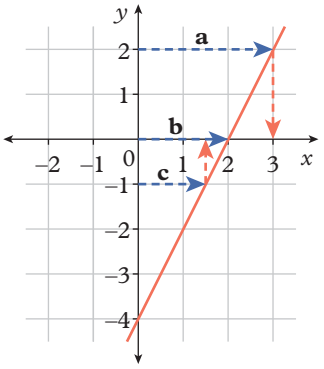
c Use the graph to find the value of x when $y = -1$. Locate -1 on the y-axis, trace across to the equation line, then down to the x-axis to find the corresponding value of x.

WRITE

a When $y = 2$, $x = 3$.
The solution to $2x - 4 = 2$ is $x = 3$.

b When $y = 0$, $x = 2$.
The solution to $2x - 4 = 0$ is $x = 2$.

c When $y = -1$, $x = 1.5$ or $1\frac{1}{2}$.
The solution to $2x - 4 = -1$ is $x = 1.5$ or $1\frac{1}{2}$.



Worked example 6.6B Solving linear equations by plotting graphs

Consider the following linear equations.

i $5 - x = 2$ **ii** $5 - x = -3$ **iii** $5 - x = 0$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y values that need to be included on the graph to solve each equation graphically.

c Use a table of values for x from -2 to 2 to plot the graph, and hence find the solution to each equation.

THINK

a Identify the required equation by considering the expression that is shared by all equations in the set.

b Each equation in the set is written in the form $5 - x = y$. List the value of y in each equation.

c **1** Construct a table of values for x is equal to $-2, -1, 0, 1, 2$.

2 Plot the coordinates on the Cartesian plane and join the points with a smooth line to draw the graph.

3 Use the graph to find the value of x for each y value listed in step **b**.

4 Write the solution to each equation.

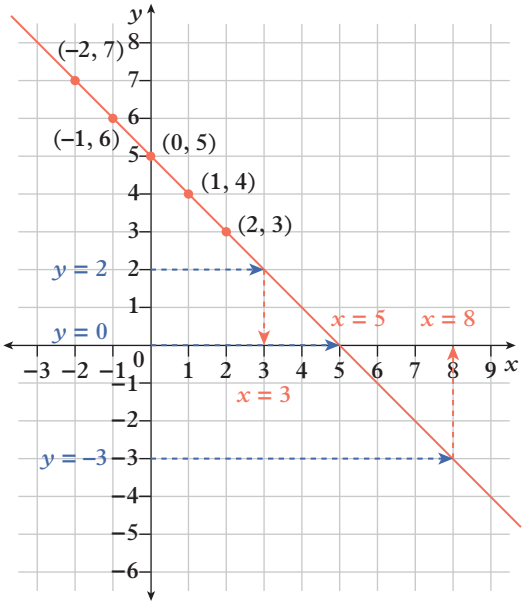
WRITE

a $y = 5 - x$

b $y = 2, -3$ and 0

c

x	-2	-1	0	1	2
y	7	6	5	4	3



i $x = 3$ **ii** $x = 8$ **iii** $x = 5$

Helpful hints

- ✔ The coordinates of graphs do not have to be integer values. For example, in the graph $y = 2x + 3$, $x = 1.5$, $x = 1.2$ and $x = -11.37$ all have a corresponding y-value.
- ✔ Although x and y are commonly used to represent the independent and dependent variables when working on the Cartesian plane, remember that variables can be represented using any pronumeral.
- ✔ When the value of a solution read from a graph is not an exact value, the graph can still be used to provide an estimate for a solution which can be calculated exactly by solving an equation.

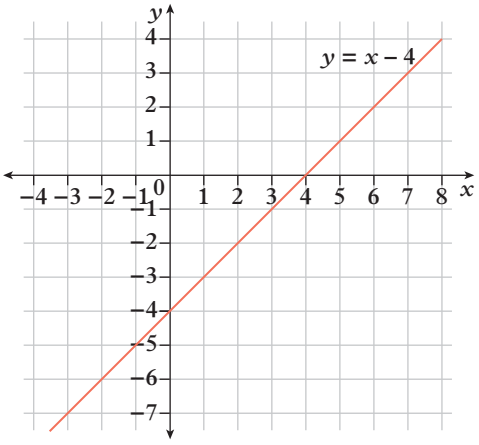
Learning pathways

1–5, 7, 9, 10	2–4, 5–8, 10–12	3, 4, 7, 8, 11–15

ANS p561

Exercise 6.6A: Understanding and fluency

1 Consider the graph of $y = x - 4$ shown to the right.



a Use the graph to find the value of x that corresponds to each of the following values of y.

i $y = 2$ **ii** $y = -3$ **iii** $y = -6$ **iv** $y = -2$

b Use your answers to part **a** to solve each equation.

i $x - 4 = 2$ **ii** $x - 4 = -3$ **iii** $x - 4 = -6$ **iv** $x - 4 = -2$

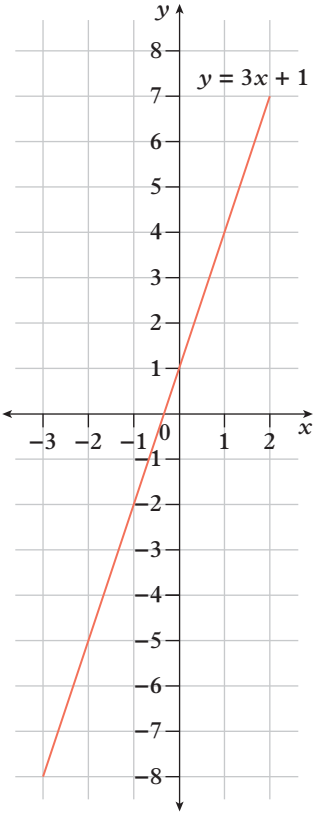
c Use the graph to solve the following equations.

i $x - 4 = 4$ **ii** $x - 4 = 0$ **iii** $x - 4 = -4$ **iv** $x - 4 = -6$

WE 6.6A **2** Solve the following equations using the graph of $y = 3x + 1$ shown to the right.

a $3x + 1 = 7$ **b** $3x + 1 = -2$ **c** $3x + 1 = 4$

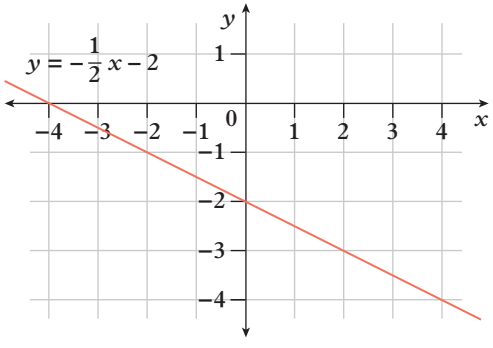
d $3x + 1 = 1$ **e** $3x + 1 = -5$ **f** $3x + 1 = -8$



3 Solve the following equations using the graph of $y = -\frac{1}{2}x - 2$ shown below.

a $-\frac{1}{2}x - 2 = -3$ **b** $-\frac{1}{2}x - 2 = -1$ **c** $-\frac{1}{2}x - 2 = -2$

d $-\frac{1}{2}x - 2 = 0$ **e** $-\frac{1}{2}x - 2 = -\frac{1}{2}$ **f** $-\frac{1}{2}x - 2 = -\frac{5}{2}$



4 Consider the linear relationship $y = 2 - x$.

a Plot the graph of $y = 2 -$ when x for x is equal to -3 to 3.

b Use the graph to solve the following set of equations.

i $2 - x = 1$ **ii** $2 - x = -1$ **iii** $2 - x = 0$

iv $2 - x = 2$ **v** $2 - x = 1.5$ **vi** $2 - x = 0.5$

WE 6.6B **5** Consider the following linear equations.

i $3x - 2 = 7$ **ii** $3x - 2 = -5$ **iii** $3x - 2 = -20$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y value that needs to be used to solve each equation.

c Plot the graph and hence find the solution to each equation.

6 Consider the following linear equations.

i $4x - 5 = 3$ **ii** $4x - 5 = -13$ **iii** $4x - 5 = -7$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y values that need to be included on the graph to solve each equation graphically.

c Plot the graph, and hence find the solution to each equation.

7 Solve each equation graphically.

a $1 - 2x = 3$ **b** $1 - 2x = -5$ **c** $1 - 2x = 7$ **d** $1 - 2x = -9$ **e** $1 - 2x = 9$ **f** $1 - 2x = -4$

8 Solve each equation graphically.

a $8x - 3 = 5$ **b** $8x - 3 = 13$ **c** $8x - 3 = 21$ **d** $8x - 3 = -3$ **e** $8x - 3 = -11$ **f** $8x - 3 = -7$

Exercise 6.6B: Problem solving and reasoning

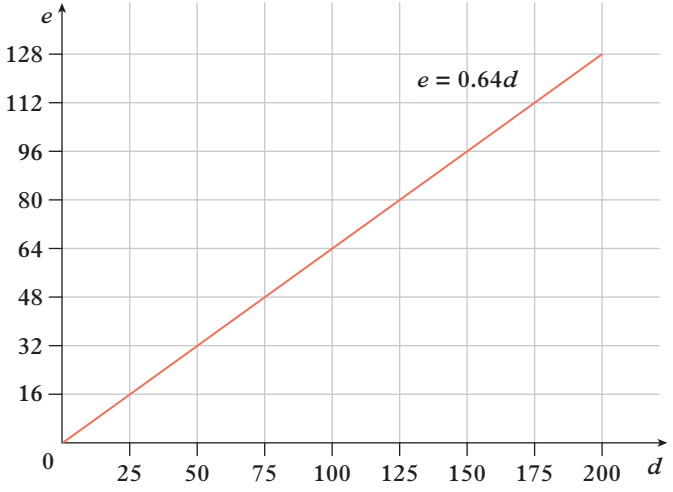
9 Consider the equation $-4x + 6 = -4$.

a Solve the equation using inverse operations.

b Solve the equation graphically.

c Describe one advantage and one disadvantage of the graphical solution method compared to the algebraic solution method.

10 Matteo has been comparing the prices of goods in Italy with those in Australia. To save time, he decides to plot a graph to convert Euros, e, to Australian dollars, d. He looks up the exchange rate and finds that \$1 = 0.64 Euros, or $e = 0.64d$.



a Use the graph to solve each of the following equations.

i $0.64d = 32$

ii $0.64d = 96$

iii $0.64d = 128$

b A designer belt costs 50 Euros.

i Write an equation that can be used to work out the cost of the belt in Australian dollars.

ii Use the graph to solve the equation. What is the equivalent cost of the belt in Australian dollars, to the nearest dollar?

c Matteo compiles a list of items for comparison.

shirt: 64 Euros
shoes: 100 Euros
jeans: 42 Euros
leather jacket: 140 Euros

i Write an equation that can be used to work out the equivalent cost of each item, in Australian dollars.

ii Use the graph to solve each equation, stating the value of d to the nearest whole number.

iii Write the cost of each item in Australian dollars.

11 In Australia, temperature is measured in degrees Celsius. Some countries, such as the USA, measure temperature in degrees Fahrenheit. The formula $F = \frac{9}{5}C + 32$ can be used to convert temperatures in degrees Celsius, C, into degrees Fahrenheit, F.

a Use the formula to complete the table of values below.

C	−25	0	25	50	75
F					

b Use the table of values to plot a graph of the relationship between degrees Celsius, C, and degrees Fahrenheit, F.

c Use the graph to solve each equation to find the value of C, rounded to the nearest integer.

i $\frac{9}{5}C + 32 = 40$ **ii** $\frac{9}{5}C + 32 = 104$ **iii** $\frac{9}{5}C + 32 = 14$

d Olivia is watching an American cooking show when the host gives the following advice on how to store food bought from the supermarket.

'Frozen meat should be stored in the freezer at −5°F, dairy products and vegetables should be stored in the fridge at about 37°F and tomatoes should be kept at room temperature, ideally around 68°F.'

i Write three equations that can be used to calculate the equivalent temperatures in degrees Celsius.

ii Use the graph to solve each equation, rounded to the nearest integer.

iii Rewrite the advice for an Australian audience.

e The host recommends keeping hot cooked food above 140°F. What temperature is this in degrees Celsius?

Exercise 6.6C: Challenge

12 Equations with the unknown on both sides can also be solved graphically. To solve $4x - 3 = 2x + 1$ graphically:

a Plot the graph of $y_1 = 4x - 3$ for x in the range −1 to 3.

b Plot the graph of $y_2 = 2x + 1$ on the same set of axes.

c Find the point where these lines intersect and write the coordinates of the point where $y_1 = y_2$.

d Write the solution to $4x - 3 = 2x + 1$.

e Check your solution by substituting the value for x into both sides of the equation.

13 To solve each of the linear equations below graphically:

i write the equations of the graphs to be plotted **ii** plot each pair of graphs on the same set of axes

iii read the solution to the equation off the graph **iv** use substitution to check your solution.

a $x - 2 = 7 - 2x$ **b** $2x + 4 = -3x - 6$

14 The number of lamingtons in a packet is unknown. However, Adam can fill a plastic container with two more than the contents of one packet or four less than the contents of two packets.

a Write an equation to represent this problem.

b Write the equations of the graphs that need to be plotted to solve the equation graphically.

c Solve the equation graphically.

d How many lamingtons are in a packet?

15 Solve each equation graphically.

a $3x - 4 = x - 1$

b $2x - 5 = -x - 5$

Online resources:

- **Interactive skillsheet** Solving linear equations graphically
- **Worksheet** Using graphs to solve linear equations
- **Investigation** Integer solutions to linear equations
- **Quick quiz** 6.6

Lesson 6.7

Intersecting lines

Learning intentions

By the end of this lesson you will be able to ...

✔ use points of intersection to solve linear equations.

Inter-year links

Year 7 5.4 The Cartesian plane

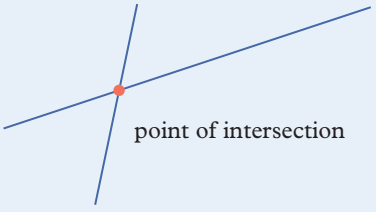
Year 9 3.5 Sketching linear graphs

Year 10 3.6 Simultaneous linear equations

Intersecting lines

Key content video
Intersecting lines

- When two or more graphs are drawn, they often intersect each other. The point where two or more lines intersect is called a **point of intersection**.
- A point of intersection can be identified visually on the Cartesian plane as the coordinates of the point where the lines intersect.
- A point is said to satisfy the equation of a line if the point lies on the line. To check if a point lies on the line, substitute the x- and y-coordinates into the equation of the line and verify that the left-hand side of the equation is equal to the right-hand side.
- A point of intersection satisfies the equations of all the intersecting lines.



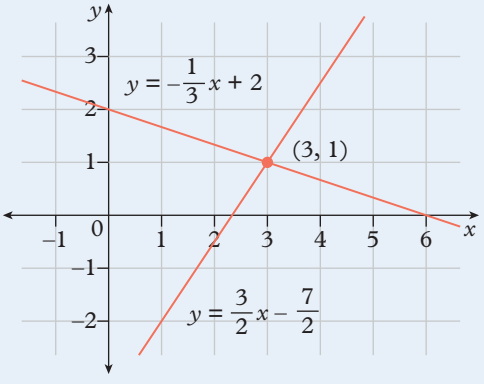
Solving linear equations

- A linear equation with the unknown on both sides can be solved by identifying the point of intersection of two linear relationships related to the original equation.
- To solve an equation with the unknown on both sides graphically:
  1. Identify the related linear relationship for each side of the equation.
  2. Plot both linear relationships on the same set of axes.
  3. Identify the point of intersection of the two lines. The solution to the equation is the x-coordinate of the point of intersection.
  4. Substitute the identified solution into the original equation and verify that the resulting equation is true.

For example, to solve the equation $\frac{3}{2}x - \frac{7}{2} = -\frac{1}{3}x + 2$, identify the two required linear relationships as $y = \frac{3}{2}x - \frac{7}{2}$ and $y = -\frac{1}{3}x + 2$. Plot both relationships on the same set of axes and identify the point of intersection as $(3, 1)$. Substitute the x-coordinate of the point of intersection into the original equation to verify the solution.

$$\text{LHS} = \frac{3}{2} \times 3 - \frac{7}{2} \qquad \text{RHS} = -\frac{1}{3} \times 3 + 2$$
$$= \frac{9}{2} - \frac{7}{2} \qquad = -1 + 2$$
$$= \frac{2}{2} \qquad = 1$$
$$= 1$$

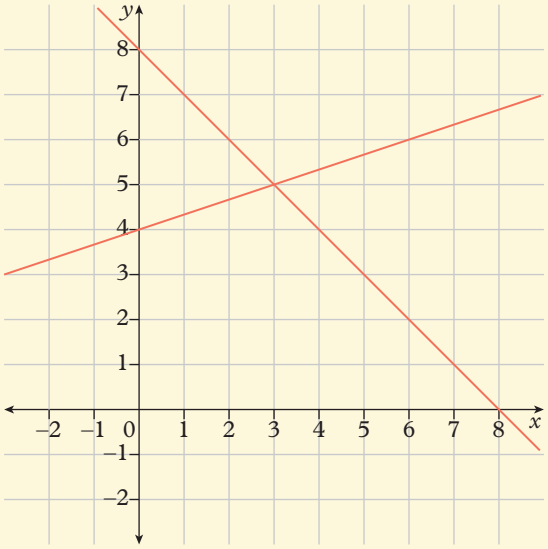
The solution to $\frac{3}{2}x - \frac{7}{2} = -\frac{1}{3}x + 2$ is $x = 3$.



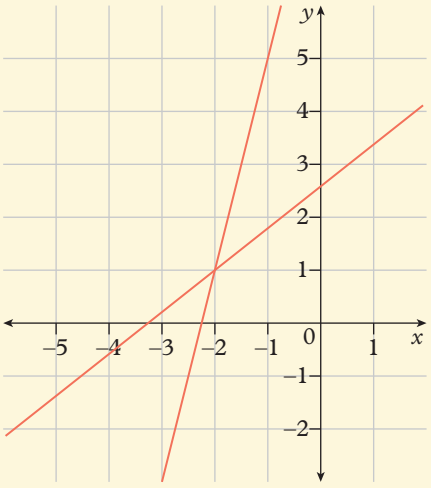
Worked example 6.7A Identifying a point of intersection graphically

For each pair of lines given, identify the point of intersection by writing its coordinates.

a



b



THINK

1 Identify the point where the two graphs intersect.

2 Write down the coordinates of that point.

WRITE

a (3, 5)

b (−2, 1)

Worked example 6.7B Identifying a point of intersection using a table of values

For each pair of equations, identify the point of intersection using the table of values given.

a $y = -x + 4$ and $y = 2x - 5$

x	0	1	2	3	4
$y = -x + 4$	4	3	2	1	0
$y = 2x - 5$	−5	−3	−1	1	3

b $y = -3x - 7$ and $y = -2x - 5$

x	−4	−3	−2	−1	0
$y = -3x - 7$					
$y = -2x - 5$					

THINK

a 1 Look for the column in the table where the two expressions have the same value.

2 State the point of intersection using the x-value and matching y-values.

b 1 Complete the table of values by substituting the x-values into each expression.

2 State the point of intersection using the x-value and matching y-values.

WRITE

a The column $x = 3$ contains the same value of y for each equation.

The point of intersection is (3, 1).

b

x	−4	−3	−2	−1	0
$y = -3x - 7$	5	2	−1	−4	−7
$y = -2x - 5$	3	1	−1	−3	−5

The column $x = -2$ contains the same value of y for each equation. The point of intersection is (−2, −1).

Worked example 6.7C Solving equations using points of intersection

For the equation $-2x + 1 = x - 2$:

a identify the two linear relationships required to solve the equation graphically

b solve the equation graphically

c verify the solution using substitution

d verify the solution by solving the equation algebraically.

THINK

a Equate each side of the equation to y to find the two linear relationships.

b 1 For both linear relationships, construct a table of values for values of x from -2 to 2.

2 Plot both sets of points on the same Cartesian plane and join each set of points with a smooth line to draw the graph.

3 Identify the point of intersection. The two lines intersect at $(1, -1)$.

4 The solution to the original equation is the x-coordinate of the point of intersection.

c Substitute the x- and y-coordinates into both sides of both equations.

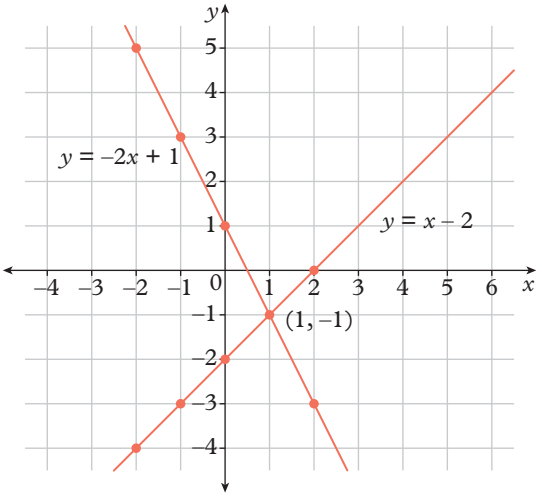
d Solve the equation using inverse operations.

WRITE

a $y = -2x + 1$ and $y = x - 2$

b

x	-2	-1	0	1	2
$y = -2x + 1$	5	3	1	-1	-3
$y = x - 2$	-4	-3	-2	-1	0



The solution is $x = 1$.

c
$$\begin{aligned} -2x + 1 &= x - 2 \\ \text{LHS} &= -2 \times 1 + 1 \\ &= -2 + 1 \\ &= -1 \\ \text{RHS} &= 1 - 2 \\ &= -1 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

d
$$\begin{aligned} -2x + 1 &= x - 2 && (+ 2x) \\ 1 &= 3x - 2 && (+ 2) \\ 3 &= 3x && (\div 3) \\ x &= 1 \end{aligned}$$

Helpful hints

- ✔ Intercepts and intersections sound similar. Make sure not to confuse the two! An intercept is an intersection with an axis rather than another graph.
- ✔ Use grid paper to ensure your graphs are drawn accurately so that points of intersection are easier to determine graphically.

Learning pathways

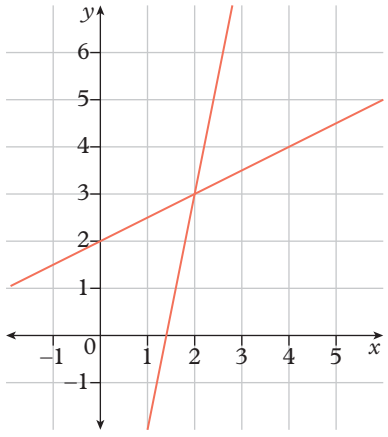
1–7, 9, 10	1–3(2nd column), 4–12	1–3(e, f), 4, 5(b, d, f), 6–8,11–13

ANS p563

Exercise 6.7A: Understanding and fluency

WE 6.7A **1** For each pair of lines given, identify the point of intersection by writing its coordinates.

a



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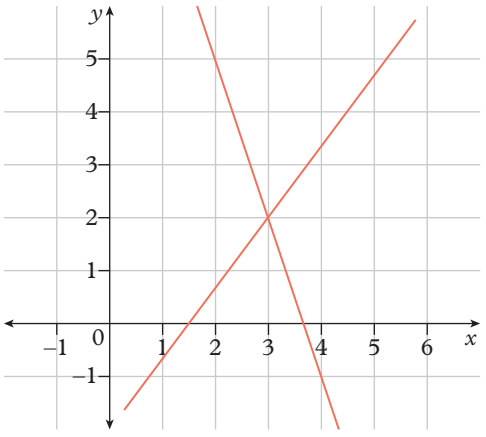
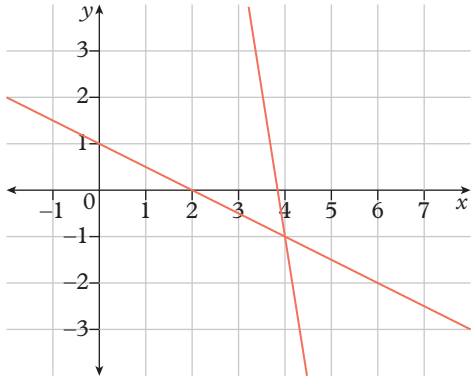


c



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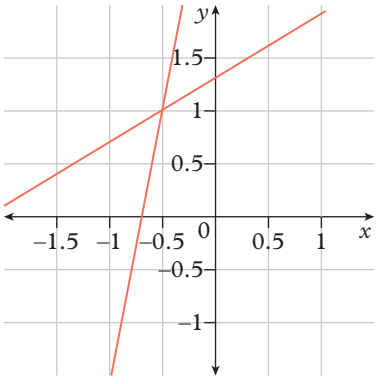
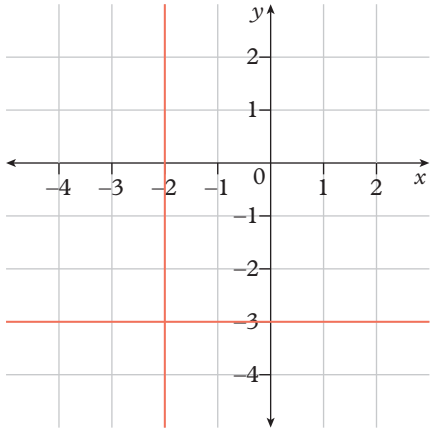
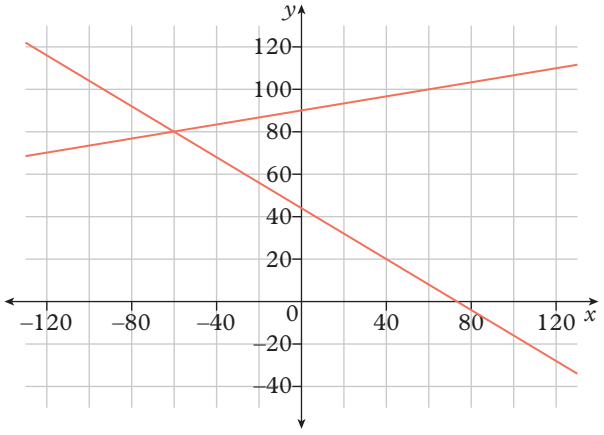


e



f



WE 6.7B **2** For each pair of equations, identify the point of intersection using the table of values given.

a $y = -2x + 11$ and $y = x - 1$

x	0	1	2	3	4
$y = -2x + 11$	11	9	7	5	3
$y = x - 1$	−1	0	1	2	3

b $y = 2x + 3$ and $y = -3x + 3$

x	−2	−1	0	1	2
$y = 2x + 3$	−1	1	3	5	7
$y = -3x + 3$	9	6	3	0	−3

c $y = 4x + 9$ and $y = 2x + 5$

x	−3	−2	−1	0	1
$y = 4x + 9$	−3	1	5	9	13
$y = 2x + 5$	−1	1	3	5	7

d $y = -7x - 9$ and $y = -2x - 4$

x	−3	−2	−1	0	1
$y = -7x - 9$	12	5	−2	−9	−16
$y = -2x - 4$	2	0	−2	−4	−6

e $y = 3x + 73$ and $y = -2x - 52$

x	−26	−25	−24	−23	−22
$y = 3x + 73$	−5	−2	1	4	7
$y = -2x - 52$	0	−2	−4	−6	−8

f $y = 3x + 16$ and $y = 2x + 21$

x	2	3	4	5	6
$y = 3x + 16$	22	25	28	31	34
$y = 2x + 21$	25	27	29	31	33

3 For each pair of equations, identify the point of intersection by first completing the table of values provided.

a $y = 3x - 2$ and $y = 2x - 1$

x	−1	0	1	2	3
$y = 3x - 2$					
$y = 2x - 1$					

b $y = 2x + 5$ and $y = -2x - 3$

x	−3	−2	−1	0	1
$y = 2x + 5$					
$y = -2x - 3$					

c $y = 7x - 16$ and $y = 4x - 10$

x	−2	−1	0	1	2
$y = 7x - 16$					
$y = 4x - 10$					

d $y = -2x - 6$ and $y = 6x + 2$

x	−3	−2	−1	0	1
$y = -2x - 6$					
$y = 6x + 2$					

e $y = -x - 23$ and $y = 2x + 55$

x	−27	−26	−25	−24	−23
$y = -x - 23$					
$y = 2x + 55$					

f $y = -3x - 43$ and $y = 4x - 78$

x	2	3	4	5	6
$y = -3x - 43$					
$y = 4x - 78$					

WE 6.7C **4** Consider the equation $4x - 8 = 8x - 20$.

a Plot the graphs of $y = 4x - 8$ and $y = 8x - 20$ on the same pair of axes.

b Identify the point of intersection of the two graphs.

c State the solution to the equation $4x - 8 = 8x - 20$.

d Verify your solution in part **c**:

i by substitution

ii by solving the equation $4x - 8 = 8x - 20$ algebraically.

5 Solve the following equations graphically.

a $-5x + 8 = 7x - 16$

b $-5x - 8 = -x + 4$

c $x + 3 = 5x + 11$

d $-5x - 24 = 2x + 4$

e $-x + 7 = 4x + 2$

f $-x - 4 = -3x - 16$

Exercise 6.7B: Problem solving and reasoning

6 Solve each of the following equations using a graph.

a $-10x - 4 = 5x + 11$ **b** $-3x - 18 = x + 2$ **c** $5x - 27 = 7x - 37$

d $x - 14 = -\frac{1}{2}x - 5$ **e** $-5x - 2 = x + 4$ **f** $x + 7 = \frac{3}{4}x + 6$

7 For each of the following pairs of equations:

i sketch the graph of both equations on the same pair of axes

ii identify the point of intersection.

iii identify an equation you could solve using parts **i** and **ii.**

a $y = 4x + 5$ and $y = -3x - 2$ **b** $y = 8x - 22$ and $y = -2x + 8$

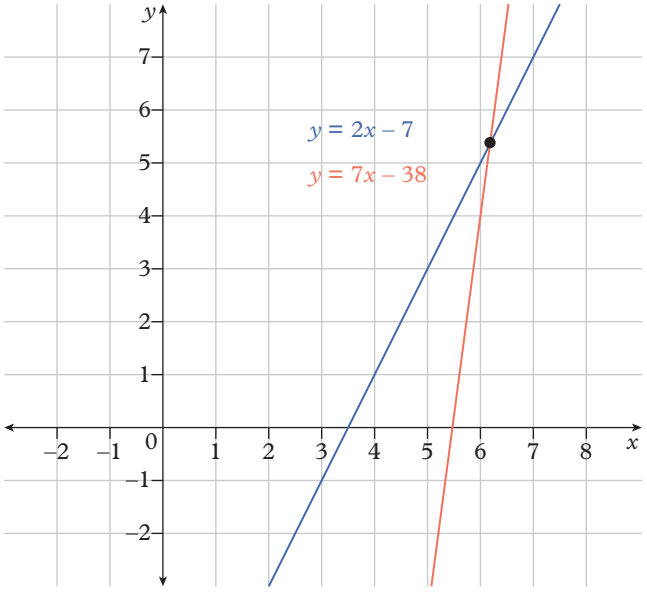
c $y = 5x + 26$ and $y = -3x - 22$ **d** $y = 3x + 3$ and $y = -x + 3$

8 Graph each of the equations below on the same set of axes. Use the graph to match two equations that intersect at each point of intersection A to F.

$A(-3, 3), B(0, 2), C(1, 1), D(-3, -1), E(-2, 1), F(0, 0)$

$y = -2x - 3, y = 2x + 5, y = -x, y = x, y = x + 2, y = -x + 2$

9 The coordinates of the intersection point of the graphs for $y = 2x - 7$ and $y = 7x - 38$ are not integers. Below is the graph of both equations.



a Complete the following sentences using the closest integer values.

i The x-coordinate of the intersection point lies between _____ and _____.

ii The y-coordinate of the intersection point lies between _____ and _____.

b Use a table of values, incrementing the x-coordinate by 0.1 each time, to determine the point of intersection of the two lines.

10 Use the steps from question **9** to find the intersection point of the following pairs of equations.

a $y = -4x + 29$ and $y = 6x - 44$ **b** $y = -5x + 29$ and $y = 5x - 22$

c $y = -3x + 22$ and $y = 7x - 69$ **d** $y = -5x + 45$ and $y = -10x + 84$

e $y = -2x - 8$ and $y = 8x + 47$ **f** $y = -x - 3$ and $y = 4x - 31$

11 Most of the pairs of lines you have considered up until now have had exactly one point of intersection.

a Draw an example of a pair of straight lines that do not intersect.

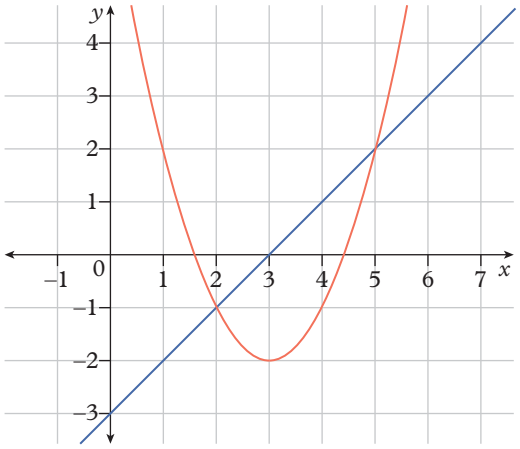
b What feature about these lines explains why they do not intersect?

12 Draw an example of a pair of straight lines that intersect at more than one point. What do you notice about this pair of lines?

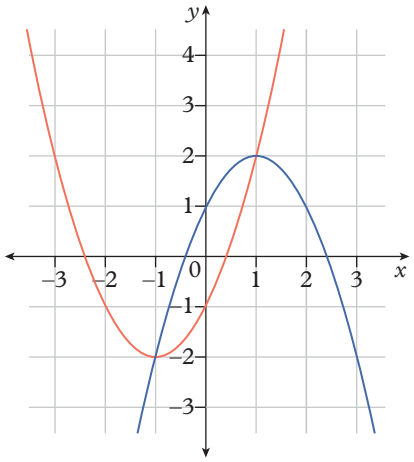
Exercise 6.7C: Challenge

13 Graphs of non-linear relations can also intersect each other. Along with intersecting once or not at all, they can have more than one intersection with each other. Identify the intersection points of the following pairs of graphs.

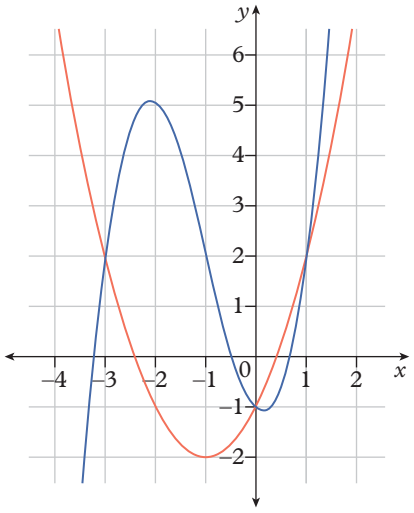
a



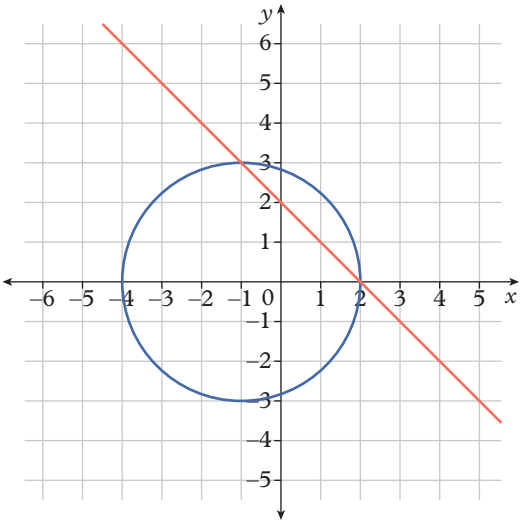
b



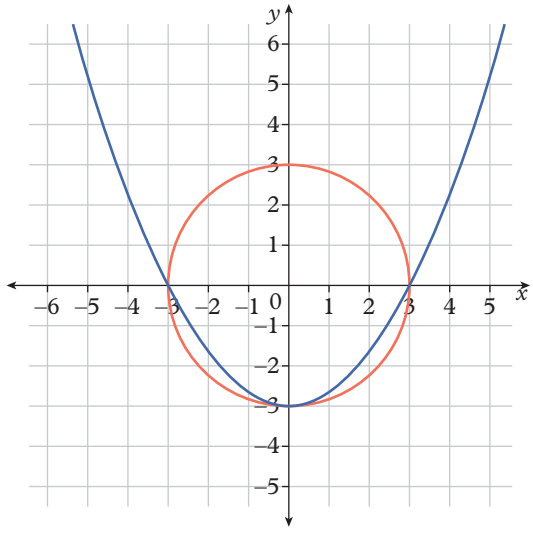
c



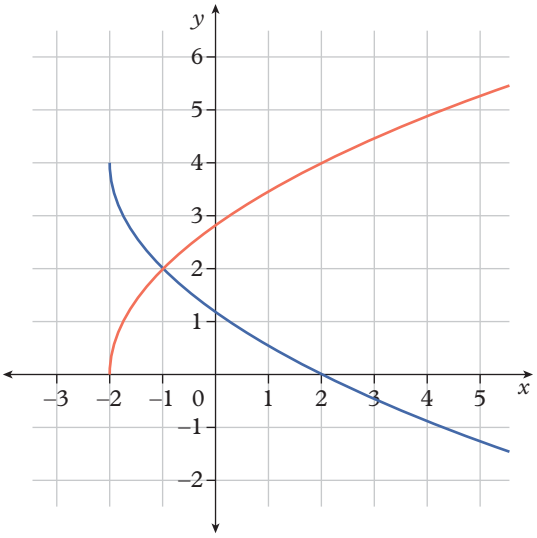
d



e



f



Online resources:

Interactive skillsheet
Intersecting lines

Desmos activity
Solutions to systems of linear equations

Quick quiz
6.7

Lesson 6.8

Finding linear equations

Learning intentions

By the end of this lesson you will be able to ...

✔ find the equation for a linear relationship.

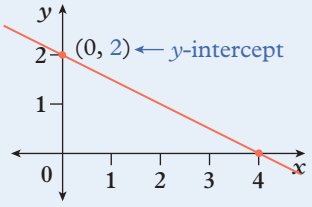
Inter-year links

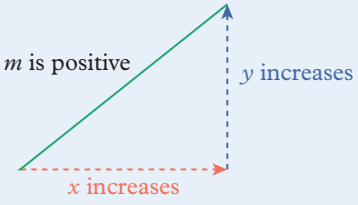
Year 7 5.4 The Cartesian plane
Year 9 3.6 Determining linear equations
Year 10 3.4 Determining linear equations

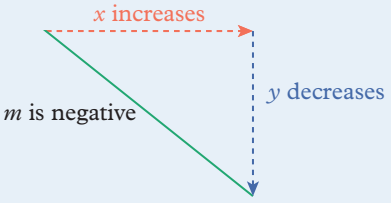
Finding the equation for a linear relationship

Key content video
Finding linear equations

- Linear equations can be written in the form: $y = mx + c$
- The constant, c, in $y = mx + c$ is equal to the value of y when $x = 0$. This is also known as the ***y*-intercept**.
- The coefficient, m, in $y = mx + c$ is the slope of the relationship, which is equal to the change in the value of y as x increases by 1.
  - → When y increases as x increases, m is positive.
  - → When y decreases as x increases, m is negative.
  - → The magnitude of m (the value ignoring the sign) indicates the steepness of the slope. A larger magnitude means a steeper slope.



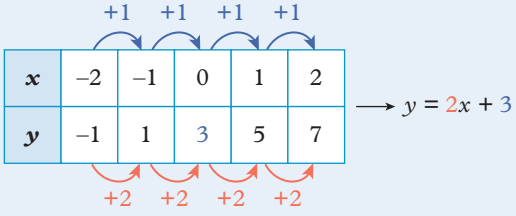




For example, the equation for the table of values below is $y = 2x + 3$ because:

→ each time the value of x increases by 1, the value of y increases by 2

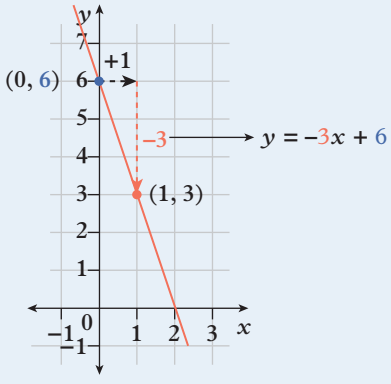
→ when $x = 0$, $y = 3$.



The equation for the graph below is $y = -3x + 6$ because:

→ each time the value of x increases by 1, the value of y decreases by 3

→ when $x = 0$, $y = 6$.



Worked example 6.8A Finding the linear equation for a table of values

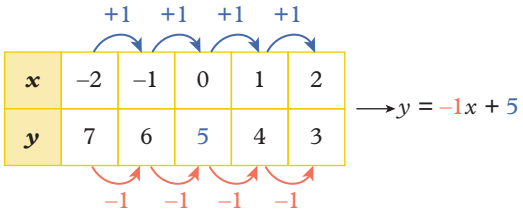
Use the table below to find the linear equation which represents the relationship between x and y.

x	−2	−1	0	1	2
y	7	6	5	4	3

THINK

1. The coefficient of x is equal to the change in y when x increases by 1. Each time the value of x increases by 1, the value of y decreases by 1, so the coefficient of x is $m = -1$.
2. The constant is equal to the value of y when $x = 0$. When $x = 0$, $y = 5$ so the constant $c = 5$.
3. Write the equation for the table.

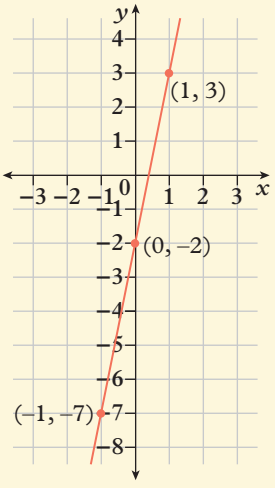
WRITE



The linear equation for the table is $y = -x + 5$.

Worked example 6.8B Finding the linear equation of a graph

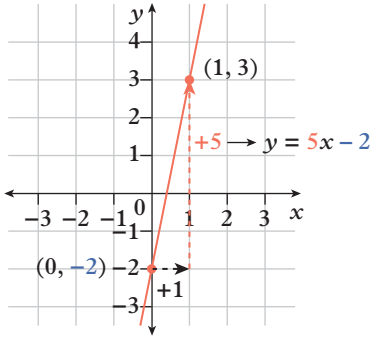
Find the equation of the graph below.



THINK

1. The coefficient of x is equal to the change in y when x increases by 1. Select two consecutive coordinate points and note the change in y when x increases by 1. Each time the value of x increases by 1, the value of y increases by 5, so the coefficient of x is $m = 5$.
2. The constant is equal to the value of y when $x = 0$. When $x = 0$, $y = -2$ so the constant $c = -2$.
3. Write the equation for the graph.

WRITE



The linear equation for the graph is $y = 5x - 2$.

Helpful hints

✔ If you're unsure about a question involving a table, plot the points and sketch the graph; this may help you find the equation!

Learning pathways

1–9, 10(a, c)	1–6, 8–12	1(d–f), 2–4, 8–13

ANS p564

Exercise 6.8A: Understanding and fluency

WE 6.8A **1** Use the following tables to find the linear equations which represent the relationships between x and y.

a

x	0	1	2	3	4
y	2	4	6	8	10

b

x	0	1	2	3	4
y	4	1	−2	−5	−8

c

x	−2	−1	0	1	2
y	−3	−1	1	3	5

d

x	−2	−1	0	1	2
y	1	−1	−3	−5	−7

e

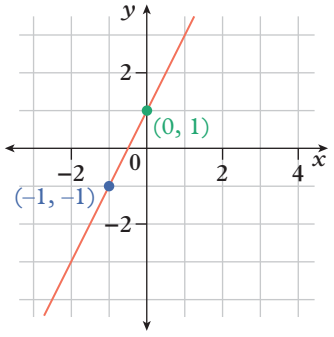
x	−4	−3	−2	−1	0
y	−4	−2	0	2	4

f

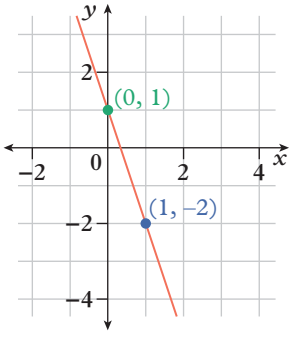
x	−4	−3	−2	−1	0
y	7	4	1	−2	−5

WE 6.8B **2** Find the equations for each of the graphs below.

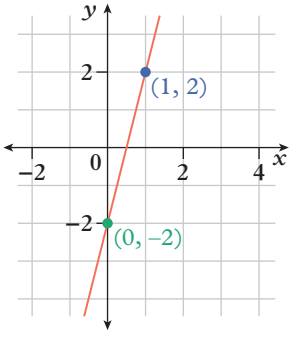
a



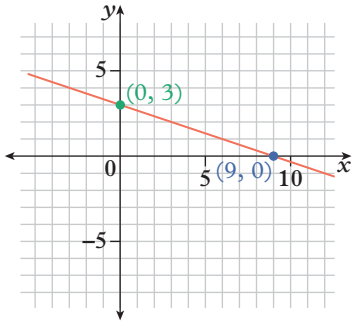
b



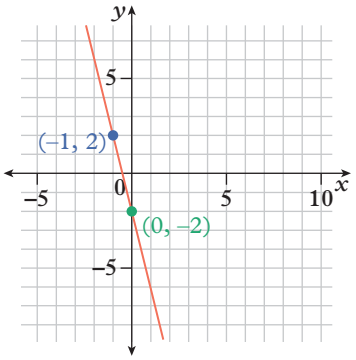
c



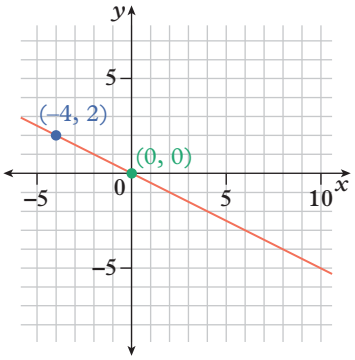
d



e



f



3 Sketch the graph of $y = 2x - 3$ by plotting the y-intercept and one other point.

4 Sketch the graph of $y = -3x + 2$ by plotting the y-intercept and one other point.

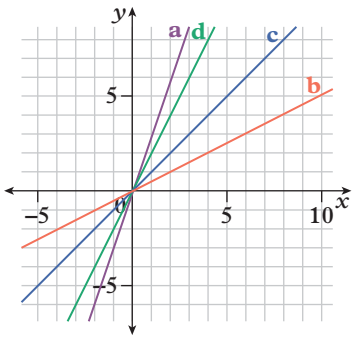
5 Match each graph with its equation.

I $y = 0.5x$

II $y = x$

III $y = 2x$

IV $y = 3x$



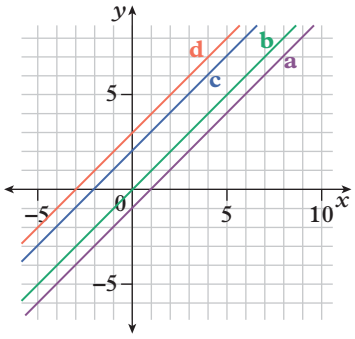
6 Match each graph with its equation.

I $y = x + 2$

II $y = x$

III $y = x - 1$

IV $y = x + 3$



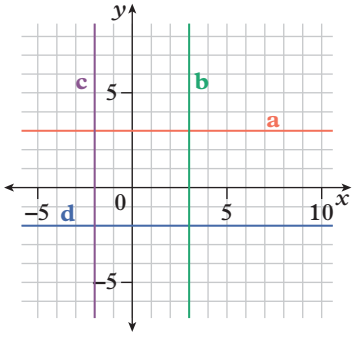
7 Match each graph with its equation.

I $y = 3$

II $x = 3$

III $y = -2$

IV $x = -2$



Exercise 6.8B: Problem solving and reasoning

8 Find the linear equations which are represented in the following tables.

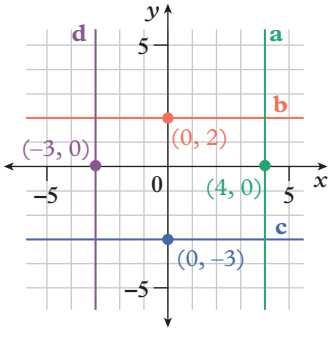
a

x	−4	−2	0	2	4
y	21	13	5	−3	−11

b

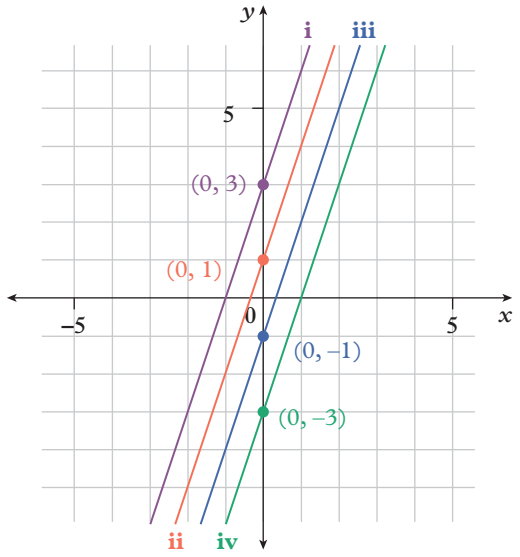
x	0	3	6	9	12
y	−5	−14	−23	−32	−41

9 Write the equations of these horizontal and vertical lines.

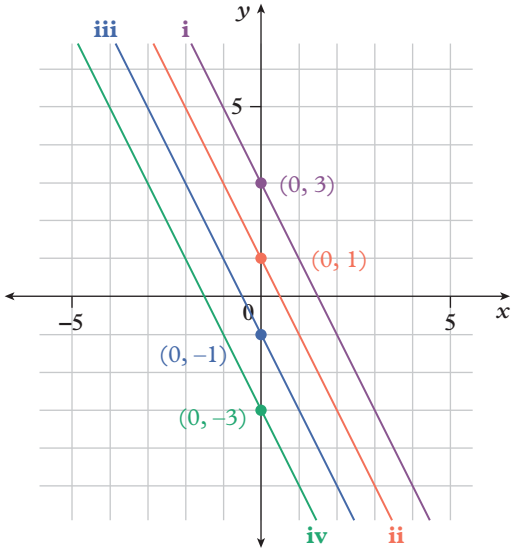


10 Write the equations of each of the linear graphs shown.

a



b



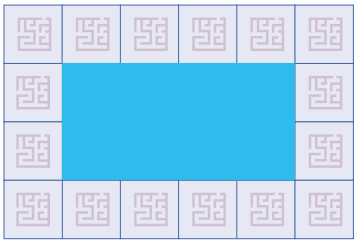
c What do the relationships in part **a** have in common?

d What do the relationships in part **b** have in common?

11 Consider your answers to question **10**. What can you say about linear graphs that have the same value of m in the equation $y = mx + c$?

Exercise 6.8C: Challenge

12 Tori used 16 tiles to make one row of tiles around her pool as shown. Tori wants to add more rows to the path and needs to know how many tiles she needs for each additional row.

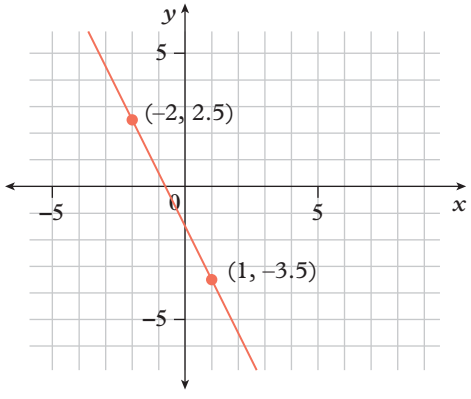


a Complete the following table.

Row	1	2	3	4
Tiles in the row	16			

b Use the table to write an equation that gives Tori the number of tiles she needs for each row.

13 Find the equation of this straight line.



Online resources:

- **Interactive skillsheet** Determining linear equations by inspection
- **Investigation** Up, up and away!
- **Desmos activity** Coin Capture: Lines
- **Quick quiz** 6.8

Lesson 6.9

Solving linear inequalities

Learning intentions

By the end of this lesson you will be able to ...

✔ solve and graph one-variable linear inequalities.

Inter-year links

Year 7 6.6 Solving equations using inverse operations

Year 9 3.2 Solving linear inequalities

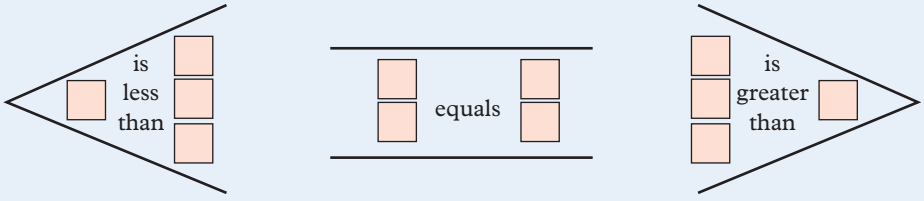
Year 10 3.7 Solving linear inequalities

Key content video
Solving and graphing simple linear inequalities

Linear inequalities

- An **inequality** is a mathematical statement that compares the values of two expressions. Inequalities are written using inequality signs, like the ones shown in the table below.

Inequality sign	Meaning	Example	Meaning of example
$<$	'is less than'	$x < 2$	x is less than 2
$>$	'is greater than'	$x > 2$	x is greater than 2
$\leq$	'is less than or equal to'	$x \leq 2$	x is less than or equal to 2
$\geq$	'is greater than or equal to'	$x \geq 2$	x is greater than or equal to 2



- A linear inequality is an inequality containing pronumerals only, raised to the power of 1, and where no two pronumerals are multiplied together.

Solving linear inequalities

- If the same operation is performed on both sides of an inequality sign, this results in an equivalent inequality of a different form. The possible values of the unknown remain the same.
- Inverse operations can be used to solve linear inequalities.

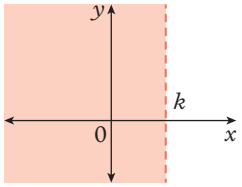
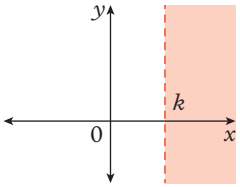
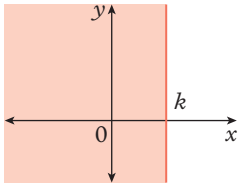
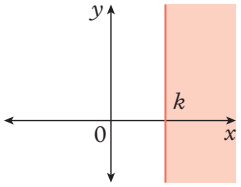
For example, $5 + 2x \leq 1 \quad (-5)$

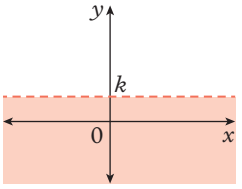
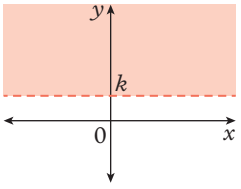
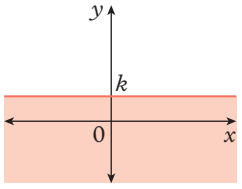
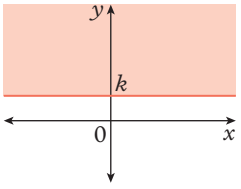
$2x \leq -4 \quad (\div 2)$

$x \leq -2$

Representing linear inequalities on the Cartesian plane

- Linear inequalities can be represented on the Cartesian plane using a boundary line and shading. On the Cartesian plane:
  - → a solid boundary line indicates that the points on the line are included, meaning the value is either 'less than or equal to' ($\leq$) or 'greater than or equal to' ($\geq$) the value represented by the boundary line on the Cartesian plane.
  - → a dotted boundary line indicates that the values on the line are not included, meaning the value is either 'less than' ($<$) or 'greater than' ($>$) the value represented by the boundary line on the Cartesian plane.

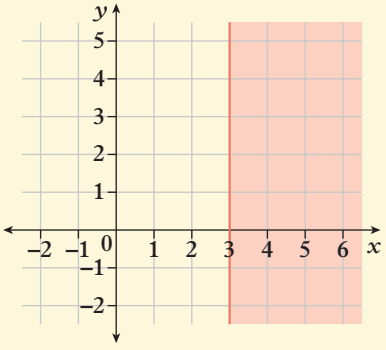
Inequality	$x < k$	$x > k$	$x \leq k$	$x \geq k$
Cartesian plane representation				
Meaning	x is less than k	x is greater than k	x is less than or equal to k	x is greater than or equal to k

Inequality	$y < k$	$y > k$	$y \leq k$	$y \geq k$
Cartesian plane representation				
Meaning	y is less than k	y is greater than k	y is less than or equal to k	y is greater than or equal to k

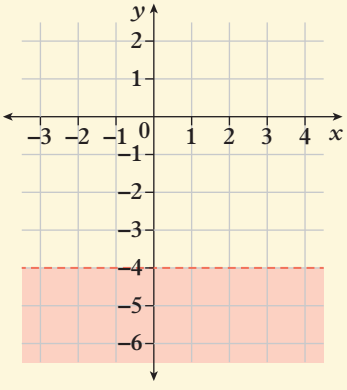
Worked example 6.9A Finding linear inequalities from a graph

Write the inequality represented by each of these graphs.

a



b



THINK

a The solid boundary line passes through $x = 3$, indicating that x can be equal to 3. The shaded area is on the right of the line, indicating that x can be any value greater than 3. So x is greater than or equal to 3.

b The dotted boundary line passes through $y = -4$, indicating that y cannot be equal to -4. The shaded area is below the line, indicating that y can be any value less than -4. So y is less than -4.

WRITE

a $x \geq 3$

b $y < -4$

Worked example 6.9B Solving one-variable linear inequalities

Solve each of these inequalities.

a $\frac{x}{4} - 3 \leq 0$ **b** $-2 > 5x + 3$ **c** $3(x + 6) \geq 12$

THINK

a 1 Apply the inverse operations to both sides in the correct order.

2 Check your solution. Substitute $x = 12$ into the LHS of the inequality. The result should equal the RHS. Then substitute any value for x that is less than 12 into the original inequality to check the inequality sign. The result should be less than or equal to the RHS.

b 1 Apply the inverse operations to both sides in the correct order.

2 Write the solution with x on the LHS. Reverse the inequality sign if needed.

3 Check your solution. Substitute $x = -1$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is less than -1 into the original inequality to check the inequality sign. The result should be greater than the LHS.

c 1 Apply the inverse operations to both sides in the correct order.

2 Write the solution with x on the LHS.

3 Check your solution. Substitute into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is greater than -2 into the original inequality to check the inequality sign. The result should be greater than the LHS.

WRITE

a $\frac{x}{4} - 3 \leq 0 \quad (+ 3)$

$\frac{x}{4} \leq 3 \quad (\times 4)$

$x \leq 12$

Check by substituting $x = 12$ into $\frac{x}{4} - 3 \leq 0$.

LHS $= \frac{12}{4} - 3$

$= 0$

$=$ RHS (For $x = 12$, LHS = RHS as required.)

Check by substituting a value less than 12 (for example, $x = 8$).

LHS $= \frac{8}{4} - 3$

$= -1$

$-1 < 0$ (For $x < 12$, LHS < RHS as required.)

b $-2 > 5x + 3 \quad (- 3)$

$-5 > 5x \quad (\div 5)$

$-1 > x$

$x < -1$

Check by substituting $x = -1$ into $-2 > 5x + 3$.

RHS $= 5(-1) + 3$

$= -2$

$=$ LHS (For $x = -1$, LHS = RHS as required.)

Check by substituting a value less than -1 (for example, $x = -2$).

RHS $= 5(-2) + 3$

$= -7$

$-7 < -2$ (For $x < -1$, LHS > RHS as required.)

c $3(x + 6) \geq 12 \quad (\div 3)$

$x + 6 \geq 4 \quad (- 6)$

$x \geq -2$

Check by substituting $x = -2$ into $3(x + 6) \geq 12$.

LHS $= 3(-2 + 6)$

$= 12$

$=$ RHS (For $x = -2$, LHS = RHS as required.)

Check by substituting a value greater than -2 (for example, $x = 0$).

LHS $= 3(0 + 6)$

$= 18$

$18 \geq 12$ (For $x = 0$, LHS ≥ RHS as required.)

Worked example 6.9C Solving and graphing one-variable linear inequalities

Solve and graph the following linear inequalities.

a $\frac{x}{3} + 4 \leq -2$ **b** $1 < 2x - 4$

THINK

a **1** Apply inverse operations to both sides in the correct order.

2 Draw a Cartesian plane, with the x-axis long enough to include $x = -18$. As x can be equal to -18, draw a solid vertical line at $x = -18$.

3 Shade the area to the left of $x = -18$ to represent $x \leq -18$.

b **1** Apply inverse operations to both sides in the correct order.

2 Write the solution with x on the LHS. Reverse the inequality sign if needed.

3 Draw a Cartesian plane, with the x-axis long enough to include $x = \frac{5}{2}$. As x cannot be equal to $\frac{5}{2}$, draw a dotted vertical line at $x = \frac{5}{2}$.

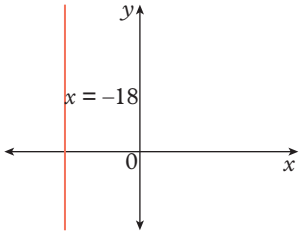
4 Shade the area to the right of $x = \frac{5}{2}$ to represent $x > \frac{5}{2}$.

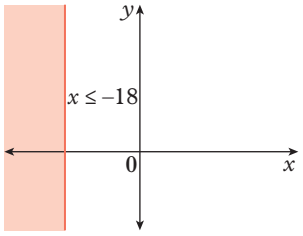
WRITE

a $\frac{x}{3} + 4 \leq -2$

$\frac{x}{3} \leq -6$ (-4)

$x \leq -18$ $(\times 3)$



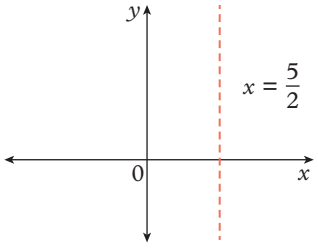


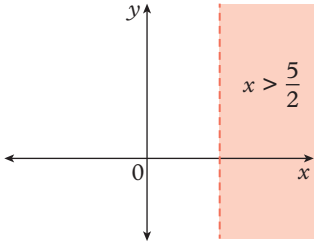
b $1 < 2x - 4$

$5 < 2x$ $(+4)$

$\frac{5}{2} < x$ $(\div 2)$

$x > \frac{5}{2}$





Helpful hints

✔ If you need to swap the LHS and RHS of an inequality, remember to reverse the inequality sign so that it is still pointing towards the lesser expression. For example, if $5 \leq x$, then $x \geq 5$.

Learning pathways

1, 2, 3(a, b, e, f), 4, 5(a–d), 6, 8, 9	2–3(b, d, f, h), 5–8, 10–12	2(b, d, f, h), 3(c, d, g, h), 6, 10–14

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Exercise 6.9A: Understanding and fluency

1 Write each of the following statements as an inequality.

a Two is less than three.

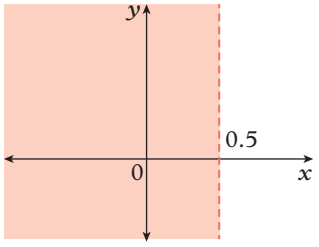
b Three is greater than two.

c Three is greater than or equal to two.

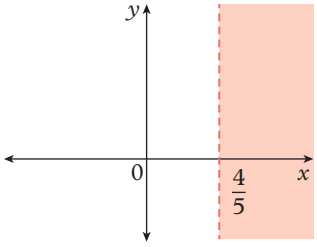
d Negative three is less than or equal to negative two.

WE 6.9A **2** Write the inequality represented on each of these graphs.

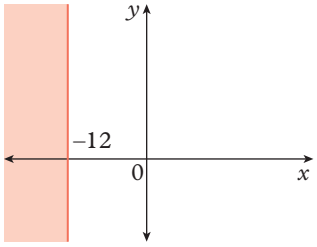
a



b



c



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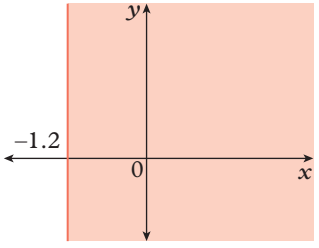

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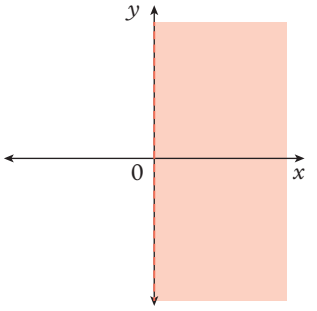
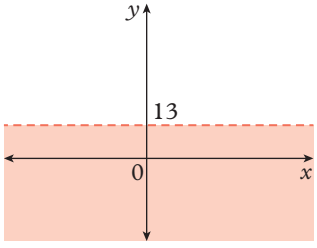


f



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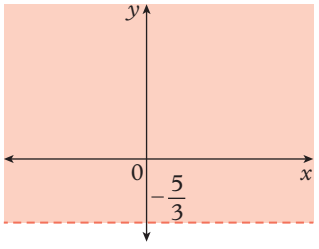
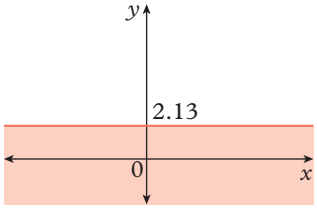
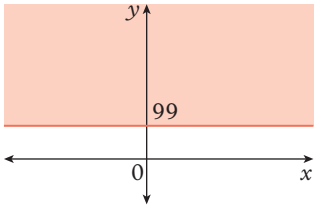


h



i



3 For each of these inequalities, show the values of x on the Cartesian plane.

a $x > 4$ **b** $x \leq 3$ **c** $x \geq -2$ **d** $x < 0$

e $y > 3$ **f** $y \leq -1$ **g** $y \geq -2$ **h** $y < 2$

4 Apply the operations below to each side of the following inequalities.

a $-4 < 9$

i add five **ii** subtract nine **iii** multiply by three **iv** divide by three

b $x \geq 6$

WE 6.9B **5** Solve each of these inequalities.

a $x + 5 > 9$ **b** $\frac{x}{10} \leq 3$ **c** $-20 > 4x$ **d** $\frac{x}{5} + 8 \leq 11$

e $2x - 9 \geq -5$ **f** $5(x + 3) \leq 25$ **g** $\frac{x - 8}{12} > -3$ **h** $5 < 10x + 14$

WE 6.9C **6** Solve each of these inequalities and show the solution on the Cartesian plane.

a $3x + 2 \geq -13$ **b** $\frac{2 + x}{5} > 3$ **c** $\frac{3x}{2} - 5 \leq 4$ **d** $\frac{x - 4}{2} < 5$

7 Explain how the number of solutions for x in linear equations (such as $ax + b = cx + d$) may or may not be different to the number of solutions for linear inequalities.

Exercise 6.9B: Problem solving and reasoning

8 Write an inequality statement to represent each of the following situations. Use a pronumeral for the unknown quantity each time.

a A house with a reserve price of $650 000 is sold at auction. What price could the house have sold for?

b To go on a theme park ride, a person must be over 97 cm tall. What could be the height of a person on this ride?

c The speed limit along a road is 60 km/h. At what speeds could a person legally travel along this road?

9 Todd is deciding how many packs of chewing gum he should buy. Each pack costs $3 and he has $20 in his wallet.

a Write an inequality to represent this situation and then solve it.

b List the number of packs of gum Todd could possibly buy.

10 Emily and Ethan are selling watermelons at a market. It costs $60 to set up the stall, and they agree to share the money they make evenly. Each watermelon is priced at $23.

a Write an expression for the amount of money they each will take home if they sell x watermelons.

b Emily and Ethan aim to sell enough watermelons to take home at least $100 each. Use your expression from part **a** to write an inequality for this situation.

c How many watermelons do they need to sell to meet their goal? Note that they only sell whole watermelons.

11 To make calls from Australia to friends and family travelling overseas, Jada has a pre-paid phone card with a credit of $50. For each call, she is charged a connection fee of 49c and a per minute rate depending on the country she is calling. The per minute charges are shown in the table on the right.

Country	Per minute rate
Egypt	$1.47
India	$1.35
Japan	$0.80
Morocco	$1.95

a For how many minutes could Jada talk to her friend travelling in Morocco and not go over her $50 limit?

b Another of Jada's friends is in Egypt. For how many minutes could Jada talk to her if she wants to save at least $25 of her $50 credit for another call?

Exercise 6.9C: Challenge

12 Solve each of these inequalities. Use substitution to check your solutions.

a $4x - 3 > 2x + 5$ **b** $7x + 1 \leq 3x - 7$ **c** $5x - 9 \geq 9 - x$ **d** $2x + 4 < 5x - 8$

e $3 - x > 4x - 2$ **f** $x + 11 \geq 7 - 3x$ **g** $8 - 3x \leq 18 - 5x$ **h** $1 - 6x > -3x - 5$

13 Solve each of these inequalities by first expanding to remove brackets.

a $2(x - 3) > x + 5$ **b** $4x - 7 \leq 3(x + 2)$ **c** $5(x + 1) \geq -3x - 11$

d $3(2x - 5) < 7x + 4$ **e** $9(x + 5) < 10(x + 6)$ **f** $3(3x - 1) \leq -2(x - 4)$

g $4(x + 3) \geq 2(5x - 3)$ **h** $2(2 - x) > 3(1 - 2x)$ **i** $5(3 - 2x) \leq -4(2x - 7)$

14 Solve each of these inequalities.

a $-3 \leq 2x - 5 \leq 7$ **b** $15 < \frac{5(1 - 2x)}{3} < 25$

Online resources:

- Interactive skillsheet: Representing simple linear inequalities on the Cartesian plane
- Interactive skillsheet: Solving simple linear inequalities
- Investigation: Sign and direction of inequalities
- Quick quiz: 6.9

Lesson 6.10

Review: Equations, inequalities and linear relationships

Module summary

Equations

equals sign

$2x + 1 = 5$

left-hand side of equation (LHS) — right-hand side of equation (RHS)

- A solution to an equation makes the equation a true statement.
- To solve by inspection, identify the correct value of the pronumeral.

Solving simple quadratic equations

- A simple quadratic equation is of the form $ax^2 = c$ and can be solved using inverse operations.

$ax^2 = c \quad (\div a)$

$x^2 = \frac{c}{a}$ (take the square root on both sides)

$x = \pm\sqrt{\frac{c}{a}}$

- If $\frac{c}{a} > 0$, there are 2 real solutions.
- If $\frac{c}{a} = 0$, there is one real solution ($x = 0$).
- If $\frac{c}{a} < 0$, there are no real solutions.

Finding linear equations

All linear equations can be written in the form

$$y = mx + c$$

- m is equal to the change in value of y as x increases by 1.
- c is equal to the value of y when $x = 0$.

Solving linear inequalities

$-2 > 5x + 3 \quad (-3)$

$-5 > 5x \quad (\div 5)$

$-1 > x$

$x < -1$

Check by substituting a value less than –1.

Solving an equation using inverse operations

1. Identify the operation(s) acting on the pronumeral.
2. Identify the inverse operations.
3. Apply the inverse operations to both sides of the equation.

$x + 1 = 5$

$x + 1 - 1 = 5 - 1$ ← inverse operations

$x = 4$ ← solution

Solving equations with the unknown on both sides

1. Use inverse operations to remove the pronumeral term from one side.
2. Solve the equation using inverse operations.

$4x - 2 = 3x + 1$

$4x - 3x - 2 = 3x - 3x + 1$

$x - 2 = 1$

$x - 2 + 2 = 1 + 2$

$x = 3$

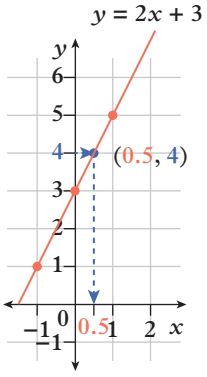
Solving linear equations using graphs

1. Make the side of the equation containing the unknown equal y.
2. Plot the graph of the equation from step 1.
3. Locate the value from the other side of the equation on the y-axis, and read off the corresponding value of x.

x	–2	–1	0	1	2
y	–1	1	3	5	7

+1 +1 +1 +1 (across the x row) +2 +2 +2 +2 (across the y row)

→ $y = 2x + 3$

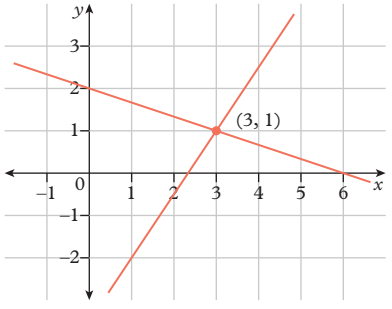


$2x + 3 = 4$

$x = 0.5$

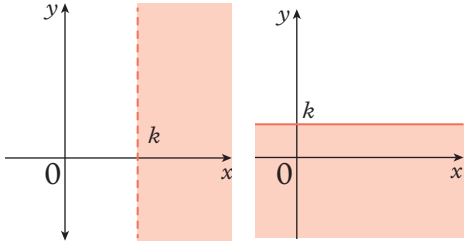
Intersecting lines

- Points of intersection can be determined graphically by plotting the equations on the same set of axes.
- Points of intersection can also be determined using a table of values.
- Intersecting lines can be used to solve equations with the unknown on both sides.



Representing linear inequalities on the Cartesian plane

- A solid boundary line indicates that the points on the line are included (≤ or ≥)
- A dotted boundary line indicates that the points on the line are not included (< or >)



$x > k$ $\quad$ $y \leq k$

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Review questions 6.10A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- Cartesian plane
- coordinates
- equation
- equivalent equations
- graph
- inequality
- inverse operations
- linear graph
- linear relationship
- origin
- plot
- point of intersection
- quadratic equation
- solution
- solve by inspection
- x-axis
- x-coordinate
- y-axis
- y-coordinate
- y-intercept

1 On the Cartesian plane, label the x-axis, y-axis and the origin.

2 Fill in the blanks using words from the key terms list above.

a The _____ of a point is the vertical distance from the origin.

b The _____ of a point is the horizontal distance from the origin.

3 Identify the key terms being referenced in each of these definitions.

a mentally identifying the number which when substituted makes an equation a true statement

b the point at which two lines meet

c the point at which a graph crosses the y-axis

4 Using an example, provide a definition in your own words for the following key terms.

a inverse operations **b** linear relationship **c** quadratic equation

5 What is the difference between an equation and an inequality?

Review questions 6.10B: Multiple choice

6.1 **1** The solution to $x + 5 = 12$ is:

A $x = 17$ **B** $x = 12$ **C** $x = 7$ **D** $x = 5$

6.1 **2** Which equation has solution $y = -4$?

A $y + 4 = 0$ **B** $2y - 3 = 4$ **C** $3y - 12 = 0$ **D** $4 - y = 0$

6.1 **3** A dance class has double the number of students that it had last week. Four of the students have to leave early, leaving 28 students in the class.

Which of the following equations can be used to determine how many students were in the class last week?

A $2s + 4 = 28$ **B** $2s - 4 = 28$ **C** $\frac{s}{2} + 4 = 28$ **D** $\frac{s}{2} - 4 = 28$

6.2 **4** Which equivalent equation is formed when 5 is subtracted from both sides of $2x - 6 = 14$?

A $2x - 1 = 14$ **B** $2x - 6 = 9$ **C** $2x - 11 = 9$ **D** $2x - 1 = 9$

6.2 **5** Which of the following operation(s) should be applied to both sides of the equation to solve $\frac{3x}{4} = 5$?

A subtract 5

B divide by $\frac{4}{3}$

C divide by 4, then multiply by 3

D multiply by 4, then divide by 3

6.3 **6** What is the solution to $3x + 7 = 2x + 9$?

A $x = 16$ **B** $x = -2$ **C** $x = \frac{16}{5}$ **D** $x = 2$

6.3 **7** The value of x in the equation $8x + 15 = 3 - x$ is:

A $x = -\frac{4}{3}$ B $x = 2$ **C** $x = -2$ **D** $x = \frac{4}{3}$

6.4 **8** What kind of solutions exist to the equation $x^2 + 9 = 0$?

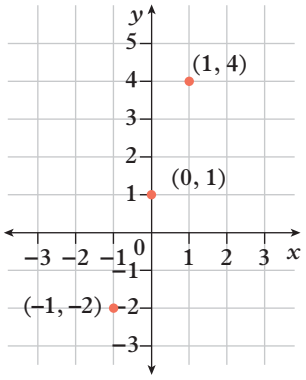
A no solutions
B one integer solution
C two integer solutions
D two non-integer solutions

6.4 **9** What are the solution(s) for x in the equation $v = 2x^2 + a$ where $v = 90$ and $a = 40$?

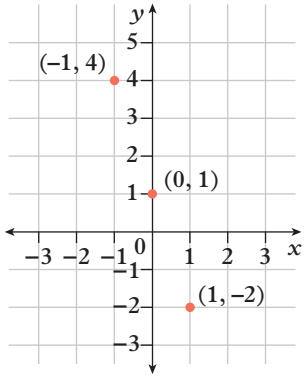
A 5 **B** ± 10 **C** ± 5 **D** ± 25

6.5 **10** Which of the following sets of points satisfy the equation of $y = 1 - 3x$?

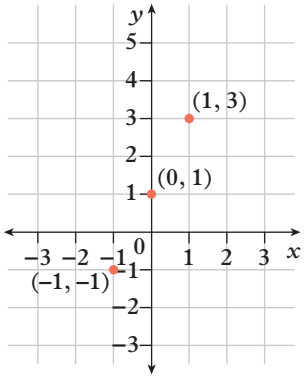
A



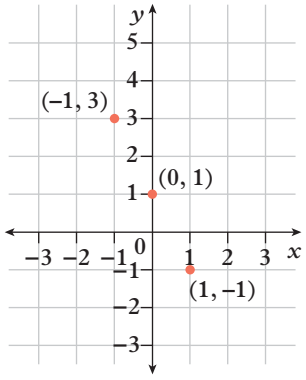
B



C



D



6.6 **11** Consider the set of related equations below.

$\frac{x}{2} + 5 = 7$ $\frac{x}{2} + 5 = 0$ $\frac{x}{2} + 5 = -1$

The equation of the graph that can be used to solve this set of equations is:

A $\frac{x}{2} + 5 = 7$ **B** $\frac{x}{2} + 5 = 0$ **C** $\frac{x}{2} + 5 = -1$ **D** $y = \frac{x}{2} + 5$

6.7 **12** What is the point of intersection for the lines $y = x$ and $y = -2x + 3$?

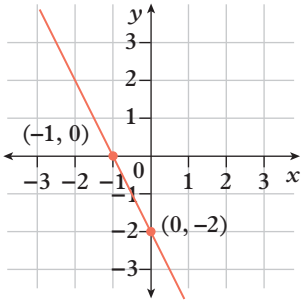
A (1, 1) **B** (1, −1) **C** (3, 3) **D** (3, 1)

6.8 **13** The equation for the table of values below is:

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

A $y = x + 1$ **B** $y = 2x - 1$ **C** $y = 2x + 1$ **D** $y = -2x - 1$

6.8 **14** The equation of this graph is:



A $y = -2x - 2$
B $y = 2x - 2$
C $y = -2x + 2$
D $y = -x - 2$

6.9 **15** What is the solution to $2x - 1 < 5$?

A $x > 3$ **B** $x < 3$ **C** $x > 2$ **D** $x < 2$

6.9 **16** Which of the following values of x satisfies the inequality $60 < 2x + 12$?

A 36 **B** 24 **C** 12 **D** 6

Review questions 6.10C: Short answer

6.1 **1** Use substitution to determine whether $x = -3$ is the solution to the following equations.

a $x + 11 = 8$ **b** $5x = -15$ **c** $4x + 12 = 0$ **d** $2x - 3 = 9$

6.1 **2** Solve the following equations by inspection.

a $a + 7 = 16$ **b** $b - 6 = 8$ **c** $4c = 12$ **d** $\frac{d - 7}{4} = 2$

6.2 **3** Use inverse operations to solve the equations.

a $5x - 2 = 13$ **b** $-\frac{1}{3}x + 8 = 11$ **c** $\frac{4c}{7} = 8$

d $4(3d + 5) = 44$ **e** $\frac{4x}{5} + 12 = 13$ **f** $\frac{6 - 8x}{7} = 10$

6.2 **4** A bookstore owner is shipping small books and large books in a box. Small books weigh half as much as large books. There are 12 of each type of book in the box.

a If a small book weighs s kg, write an equation to represent this situation given that a box weighs 36 kg.

b Solve the equation and find the weight of a large and small book.

6.3 **5** Solve the following equations algebraically.

a $5x + 6 = 3x - 4$ **b** $8x + 3 = 6x + 7$

c $-4x - 15 = 6x + 15$ **d** $x + 3 = -2 - x$

6.3 **6** Suzi's age in 11 years will be equal to five times her age five years ago. Write an equation and solve it to find how old Suzi is now.

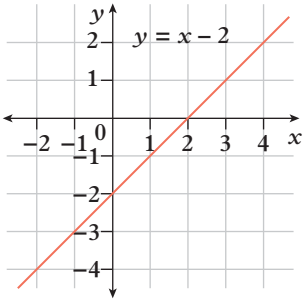
6.4 **7** Solve the following equations for x.

a $x^2 = 36$ **b** $3x^2 = 108$ **c** $x^2 + 5 = 5$

6.5 **8** Plot a graph of each of the following relationships by first completing a table of values for integer values of x from -3 to 3.

a $y = x - 4$ **b** $y = 4x + 1$ **c** $y = 3 - x$ **d** $y = 6 - 3x$

6.6 **9** Refer to the graph of $y = x - 2$ on the right to solve each equation.



a $x - 2 = -4$ **b** $x - 2 = -1$ **c** $x - 2 = -2$

6.7 **10** Solve each of the equations in question **5** graphically.

6.8 **11** Write an equation for each of the tables below.

a

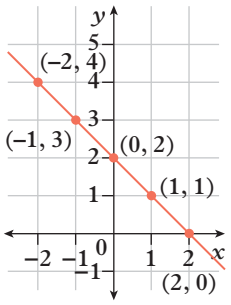
x	-2	-1	0	1	2
y	-13	-9	-5	-1	3

b

x	-2	-1	0	1	2
y	2	1	0	-1	-2

6.8 **12** Determine the equations of the graphs below.

a



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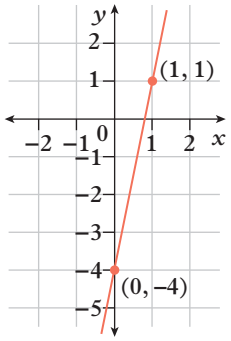
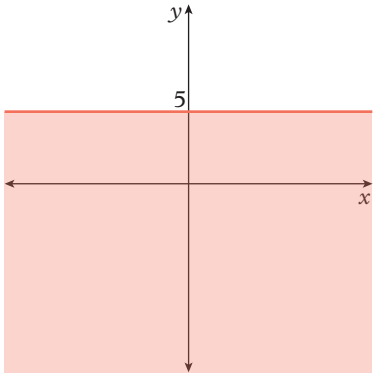


6.9 **13** Solve the following inequalities and represent the result on the Cartesian plane.

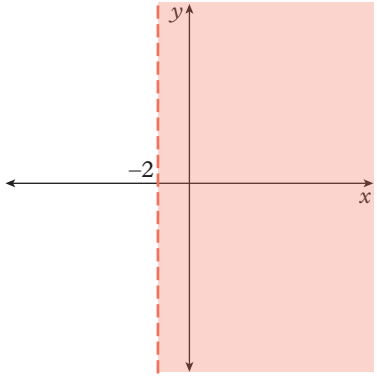
a $4x - 3 > 5$ **b** $7x + 1 \leq 7$ **c** $9 \geq 5x + 10$

6.9 **14** Write the inequality represented on each of these graphs.

a



b



Review questions 6.10D: Mathematical modelling

1 Joss is the manager of a bookstore. He uses Crate Couriers to deliver his orders. Crate Couriers charge Joss $5 per book plus $3 flat fee per delivery.

a Write an equation to find the total cost, C, of delivering x books.

b How much does it cost Joss to have 10 books delivered to a customer?

c How many books can Joss have delivered for $38?

d Complete the table below.

Number of books	0	1	2	3	4	5
Cost of delivery						

e Plot a graph of the cost of delivery vs the number of books for Crate Couriers.

f A new courier approaches Joss. Crown Couriers do not have a flat fee but they charge $6 per book delivery. At first Joss thinks Crown will be too expensive but he decides he should investigate further.

i Fill in the table for Crown Couriers below.

Number of books	0	1	2	3	4	5
Cost of delivery						

ii On the same graph as part **e**, plot the graph of cost of delivery vs number of books for Crown Couriers.

iii After looking at the two graphs, Joss can see that the situation is not so simple. Explain when Joss should use Crate Couriers and when he should use Crown Couriers.

2 A suburban party venue costs \$150 to hire, plus \$20 per person for games and food.

a If p people attend the party, write an equation for the total cost, C, of the venue hire, games and food.

b Kacie's mother gives her \$450 for venue hire, games and food. Substitute this value into the equation to determine how many people Kacie can have at her party.

c Kacie finds another party venue in the city that offers a similar service. Their costs consist of a hire fee of \$250 plus \$15 per person for games and food.

i Write a second equation for the total cost, C, of venue hire, games and food for a party attended by p people at the second venue.

ii Complete the tables below for each venue.

Suburban venue:

p	0	10	20	30	40	50
C						

City venue:

p	0	10	20	30	40	50
C						

iii Use the tables of values to plot a graph of each relationship on the same set of axes. Label each graph.

iv Do these graphs show a linear relationship? Why or why not?

v Use the graphs to find the cost of inviting 25 people to a party at each venue.

vi How many people can Kacie invite to a party at each venue, with a budget of \$450?

d Although the city venue has a higher hire fee, if Kacie invites more than a certain number of people, the venue will be cheaper than the suburban venue.

Use your graph to determine which venue is cheaper if Kacie invites:

i 10 people **ii** 50 people **iii** 20 people.

3 Sometimes collected data can be represented as a table of values and then graphed on the Cartesian plane.

a Plot the points in the following table of values on the Cartesian plane.

x	0	1	1	2	3	4	4	5	5	6	7	8
y	0	1	2	3	3	4	5	6	7	8	9	10

b Does the table of values in part **a** correspond to a linear relationship? Why or why not?

c A *line of best fit* is a line which best approximates data plotted on the Cartesian plane.

Graph the following linear relationships on the same Cartesian plane as in part **a**.

i $y = x$ **ii** $y = 2x$

d Give a reason why each of the linear relationships in part **c** are not lines of best fit for the table of values in part **a**.

e The linear relationship $y = \frac{5}{4}x$ is a *line of good fit* for the table of values in part **a**. To be a line of good fit, there should be the same number of points above and below the line, and ideally, some points on the line.

i Plot the points in the table of values and the line $y = \frac{5}{4}x$ on a new set of axes.

ii Explain why $y = \frac{5}{4}x$ is a better line of good fit than the lines in part **c**.

4 A high-capacity train has c carriages and 200 people can safely ride in each carriage. Each train has at most seven carriages.

a Write an inequality to represent the total number of passengers, P, who can ride the train at any point in terms of c.

b What is the minimum number of carriages required to safely carry 780 passengers?

c What is the maximum number of people who can safely ride the train?

d On some days, buses replace trains due to construction work on the train lines. One bus can carry 67 passengers. One day, a train with the maximum possible number of passengers arrives at a station after which all passengers must switch to a bus to continue their journey.

i What is the minimum number of buses required if the number of passengers from part **c** intend to ride the bus?

ii The transport company can provide one bus every five minutes. The first bus is waiting when the passengers arrive, and the next bus arrives five minutes later. How long does it take for all passengers to board a bus?

iii Explain why your answers to parts **b** and **d i** must be integers.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Verify solutions using substitution ☐ Solve simple linear equations by inspection ☐ Write equations to represent worded problems	☐ Go back to Lesson 6.1 Equations
☐ Solve linear equations using inverse operations	☐ Go back to Lesson 6.2 Solving linear equations
☐ Solve linear equations with the unknown on both sides	☐ Go back to Lesson 6.3 Solving equations with the unknown on both sides
☐ Explain why there are zero, one or two solutions to simple quadratic equations ☐ Solve simple quadratic equations ☐ Solve simple quadratic equations after substituting known values into a formula	☐ Go back to Lesson 6.4 Solving simple quadratic equations
☐ Use equations to complete tables of values ☐ Plot points relationships from tables of values and equations ☐ Differentiate between linear and non-linear relationships	☐ Go back to Lesson 6.5 Plotting linear relationships
☐ Solve linear equations using a graph	☐ Go back to Lesson 6.6 Solving linear equations using graphs
☐ Identify the intersection point of two lines graphically ☐ Identify the intersection point of two lines using a table of values ☐ Solve equations with the unknown on both sides using a graph	☐ Go back to Lesson 6.7 Intersecting lines
☐ Find the linear equation for a table of values ☐ Find the linear equation for a graph when the coordinates are known	☐ Go back to Lesson 6.8 Finding linear equations
☐ Solve linear inequalities using inverse operations ☐ Graph the solutions of one-variable linear inequalities	☐ Go back to Lesson 6.9 Solving linear inequalities

Computational Thinking

Curriculum link

Use algorithms and related testing procedures to identify and correct errors (VC2M8A04)

Inter-year links

Year 7	Classifying polygons
Year 9	Triangles
Year 10	Solving non-linear equations with digital tools
Year 10	Graph algorithms

Binary numbers

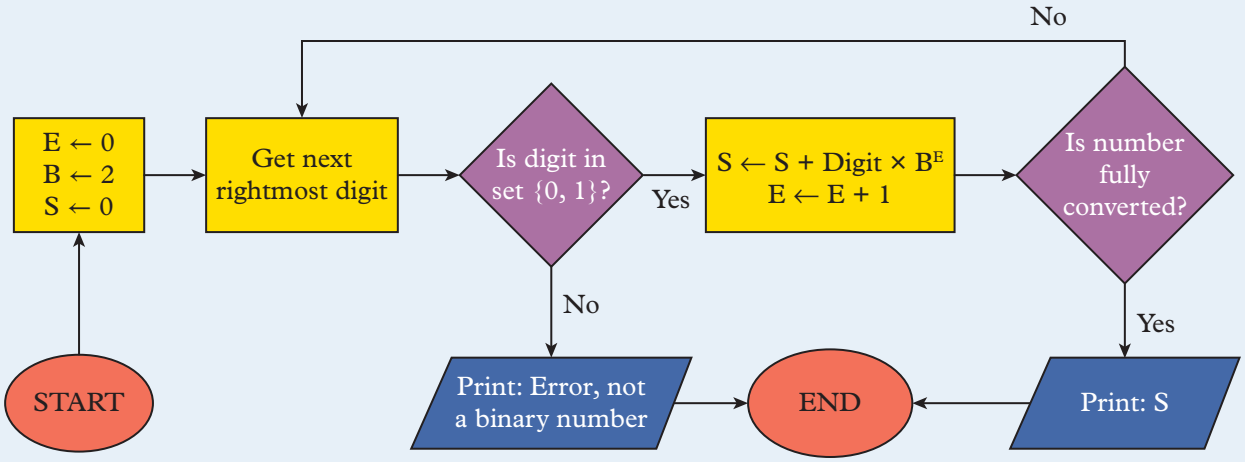
The decimal number system, also known as base-10, uses ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Decimal numbers are defined using positional notation that corresponds to powers of 10. This is 5127 in base-10:

$5127 = 5 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 7 \times 10^0$

The binary number system, also known as base-2, uses two digits: 0 and 1. Binary numbers are defined using positional notation that corresponds to powers of 2. Binary numbers are identified with a subscript of 2 after the number. The binary number 1101_2, when converted to a decimal number, is 13.

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13$

The flow chart below shows how to convert a binary number into a decimal number. The flow chart uses the variables Exponent (E), Base (B) and Sum (S) to keep track of the solution in the Sum variable, which changes value as the decimal number is calculated. A left-facing arrow, ←, is used to update the value of a variable.



Check your understanding

1 Answer the following questions.

a Use the flow chart to convert the binary number 1101_2 into a decimal number.

b What happens if 170 is the input to the flow chart?

c Where is the loop in the flow chart? What makes the loop terminate?

Task 1

Operations can be performed on binary numbers. Research how to add binary numbers using the binary addition algorithm. Perform some additions with numbers of your choice, and show the calculations are correct by converting both the numbers and results to decimal numbers. Compare addition of binary numbers with addition of decimal numbers.

Finding and correcting errors

Algorithms are useful when they always provide the correct output. When following a flow chart, there are two main types of error: user errors and algorithm errors. A *user error* occurs when the person following the flow chart makes a mistake, while an *algorithm error* means that the steps in the flow chart, even when followed correctly, may produce the incorrect result.

One method to check for an error is to perform what is known as a 'desk check'. This involves going through the algorithm, step-by-step, and keeping track of the values of any variables at each point. Repeating this process with more than one input can make it easier to identify the type of error and how to correct the error.

Check your understanding

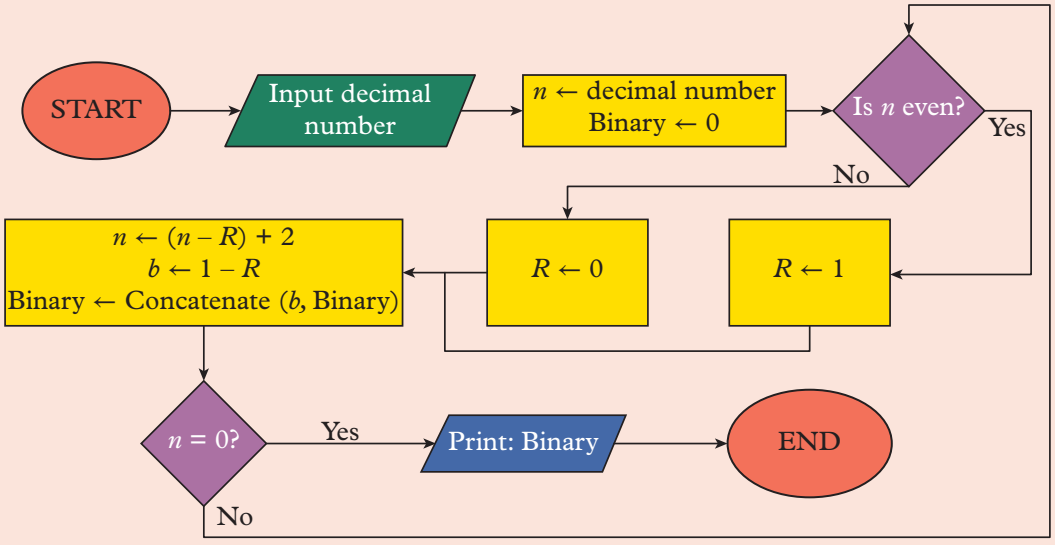
2 Answer the following questions.

a Explain the difference between a user error and an algorithm error. Provide an example of each type of error.

b Perform a desk check on the binary-to-decimal flow chart with the input 11_2.

Task 2

The flow chart below can be used to convert a decimal number to a binary number. It uses the operation 'Concatenate$(a, b) = ab$' to write two variables or numbers next to each other. For example, Concatenate$(11, 00) = 1100$. The flow chart contains several errors.



1 Perform a desk check using the input 43. Compare the output of the flow chart with the correct binary conversion of 43.

2 Identify and correct the errors in the flow chart.

Extension

The quinary number system, also known as base-5, uses five digits: 0, 1, 2, 3 and 4.

The quinary number system is used by the indigenous Gumatj people of Australia, who are part of the Yolgnu language group in the Northern Territory. Quinary numbers are identified with a subscript 5 after the number, as shown below:

$431_5 = 4 \times 5^2 + 3 \times 5^1 + 1 \times 5^0 = 116$

Create a flow chart to convert a decimal number to a quinary number. Perform desk checks on your flow chart to identify any errors. Update your flow chart based on your results.

7

Angles, triangles and Pythagoras' theorem

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Substitution
- ✔ Solving equations using inverse operations
- ✔ Solving simple quadratic equations

Curriculum links

- Use Pythagoras' theorem to solve problems involving the side lengths of right-angled triangles (VC2M8M06)

Materials

- ✔ Compass
- ✔ Ruler
- ✔ Protractor

Lesson 7.1

Angles

Learning intentions

By the end of this lesson you will be able to ...

✔ find unknown angles in diagrams.

Inter-year links

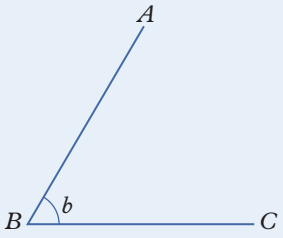
Support Understanding angles

Year 7 7.2 Angles at a point

Types of angles

Key content video
Angles at a point

- Angles can be named using either a pronumeral, for example b, or an angle symbol followed by capital letters which represent the vertices of the angle, for example $\angle ABC$.
- Angles are classified according to their size:
  - → **acute**: between 0° and 90°
  - → **right**: 90°
  - → **obtuse**: between 90° and 180°
  - → **straight**: 180°
  - → **reflex**: between 180° and 360°
  - → **revolution**: 360°.



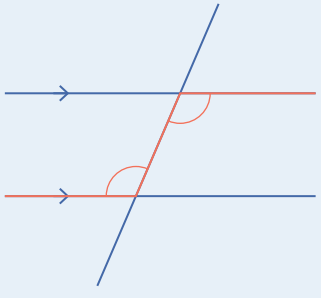
Angle properties

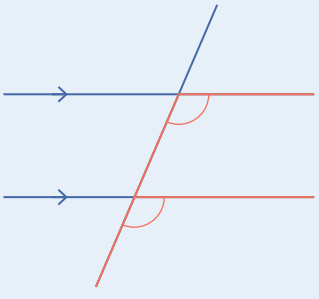
- Angles at a point that form a straight line add to 180°.
- Angles at a point that form a revolution add to 360°.
- Two angles that add to 90° are called **complementary angles**.
- Two angles that add to 180° are called **supplementary angles**.
- Angles formed when two lines intersect form **vertically opposite angles** which are equal in size.

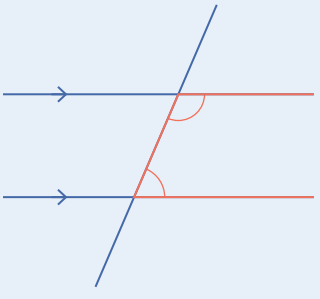
Angles and parallel lines

Key content video
Angles and parallel lines

- When two or more lines have arrowheads on them, this means that they are parallel. Multiple arrowheads may be used if there are more than one pair of parallel lines in a diagram.
- A **transversal** is a straight line that cuts across two or more other lines.
- When two or more **parallel lines** are intersected by a transversal:
  - → **Alternate angles** are equal.
  - → **Corresponding angles** are equal.
  - → **Co-interior angles** are supplementary.





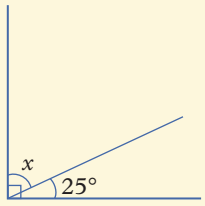


- Angles formed by a transversal can be used to determine if two lines are parallel. If any of the relationships listed above are true, then the lines crossed by a transversal are parallel.

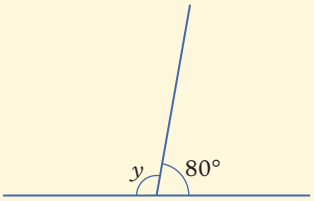
Worked example 7.1A Using complementary and supplementary angles

Find the size of each labelled unknown angle.

a



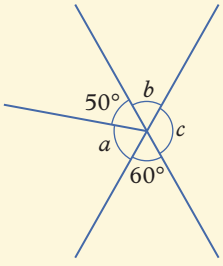
b



THINK	WRITE
a **1** Identify the relationship between the unknown and known angles. x and 25° are complementary angles.	**a** $x = 90° - 25°$
2 Calculate the size of the unknown angle. Complementary angles add to 90°, so $x = 90° - 25°$.	$= 65°$
b **1** Identify the relationship between the unknown and known angles. y and 80° are supplementary angles.	**b** $y = 180° - 80°$
2 Calculate the size of the unknown angle. Supplementary angles add to 180°, so $y = 180° - 80°$.	$= 100°$

Worked example 7.1B Using angles at a point

Find the size of each labelled unknown angle.



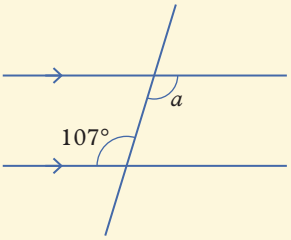
THINK	WRITE
1 Identify any relationships between unknown and known angles. b is vertically opposite to 60° and vertically opposite angles are equal in size.	$b = 60°$
2 c and 60° are supplementary angles, so $c = 180° - 60°$.	$c = 180° - 60°$ $= 120°$
3 50°, a and 60° form a straight angle, so $a = 180° - 50° - 60°$.	$a = 180° - (50° + 60°)$ $= 70°$

Worked example 7.1C Finding unknown angles on parallel lines

Consider the diagram on the right.

a Identify the relationship between the unknown and known angles.

b Find the size of angle a.



THINK

a a and 107° are the internal angles of a reverse 'Z' shape, so they are alternate angles.

b Alternate angles formed by parallel lines are equal in size.

WRITE

a a and 107° are alternate angles.

b $a = 107°$

Helpful hints

- ✔ Unless stated, you will not need to measure any of the angles in a diagram. All the information required to answer questions will be provided.
- ✔ Don't assume that two given lines are parallel if they look like they are parallel in a diagram! Pay attention to whether lines have parallel marks or if you are given that information in a question.

Learning pathways

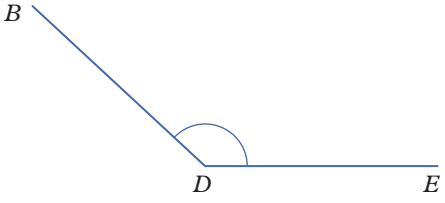
1–9, 11–13	1, 2(a, d, e), 3–5, 6(e–i), 7–9, 12–15, 17(a)	1(d–f), 3–5, 6(f–i), 7(d–f), 8, 10–12, 16–18

ANS p569

Exercise 7.1A: Understanding and fluency

1 Name each angle using the vertices and state what type of angle it is.

a



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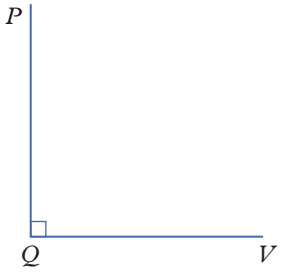

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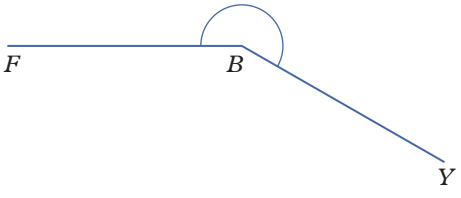
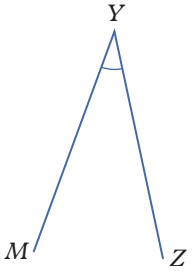


d



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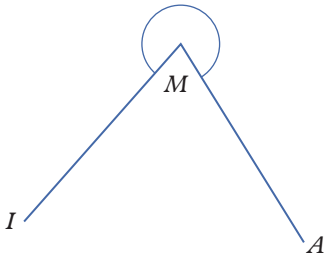

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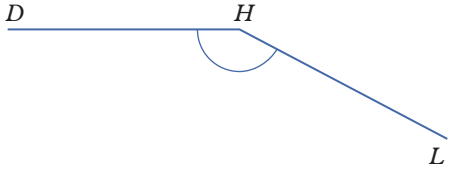
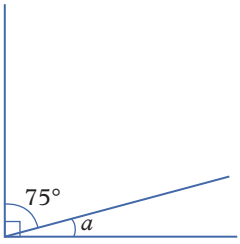


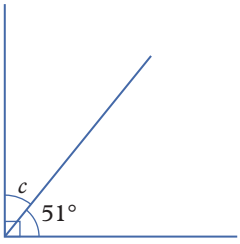
WE 7.1A **2** Find the size of each labelled unknown angle.

a



b

c



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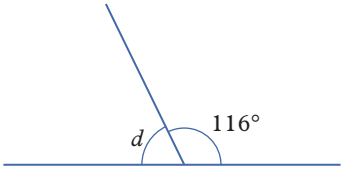

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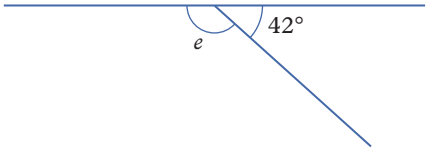

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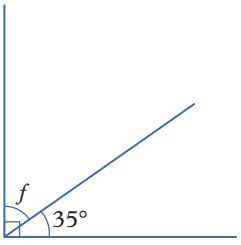
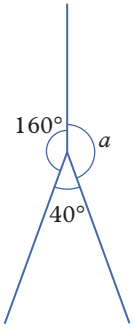


WE 7.1B **3** Find the size of each labelled unknown angle.

a



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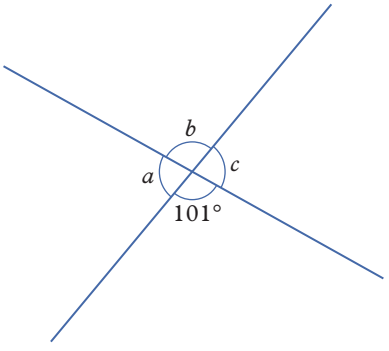
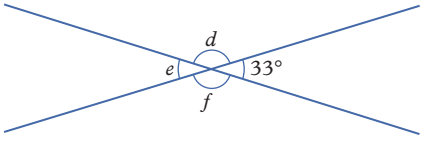
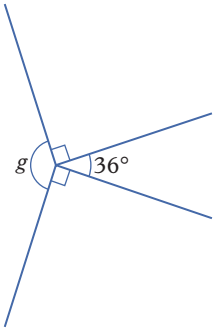


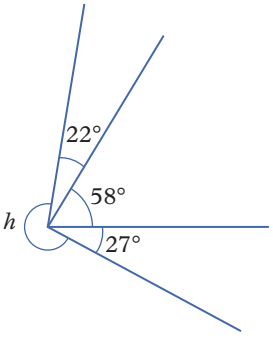
c



d



e



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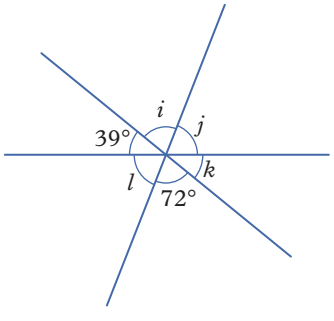
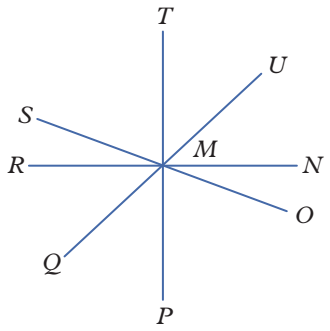


4 Identify the relationship (complementary, supplementary or vertically opposite) between each pair of angles in the following diagram.



a $\angle UMT$ and $\angle NMU$

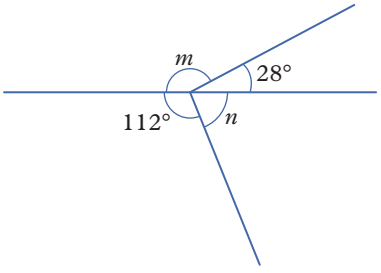
b $\angle OMT$ and $\angle PMO$

c $\angle SMR$ and $\angle NMO$

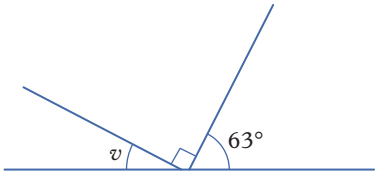
d $\angle TMQ$ and $\angle PMQ$

5 Find the value of each pronumeral in these diagrams.

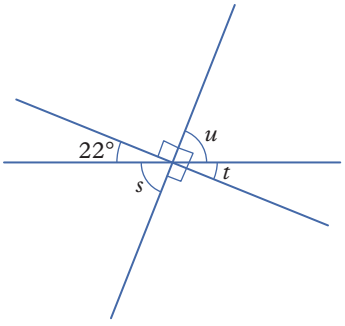
a



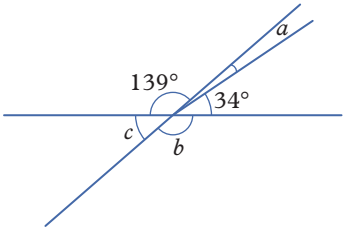
b



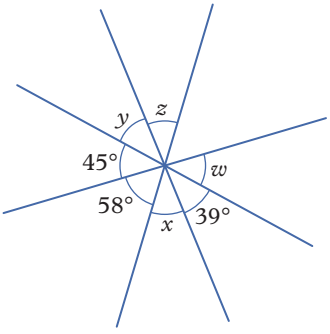
c



d



e



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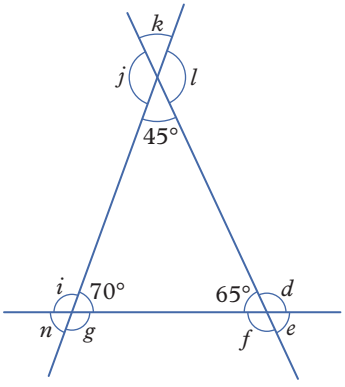
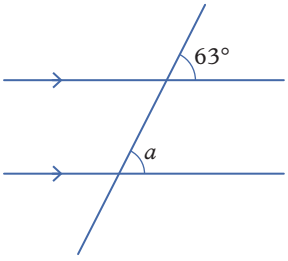
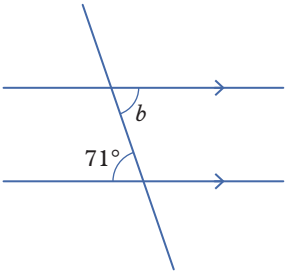


WE 7.1C **6** For each of the following diagrams, identify the relationship between the unknown and known values, and find the size of the labelled unknown angle.

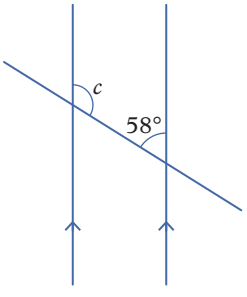
a



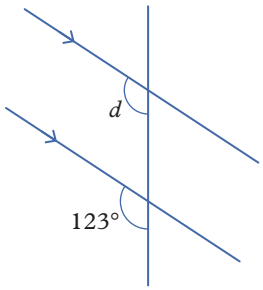
b



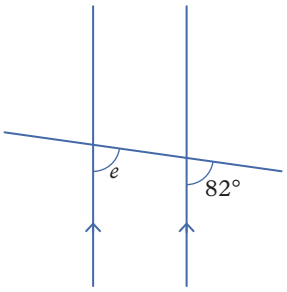
c



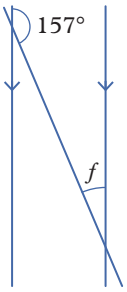
d



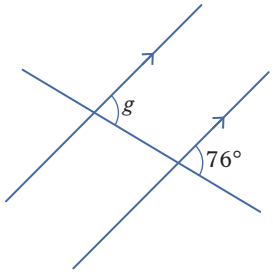
e



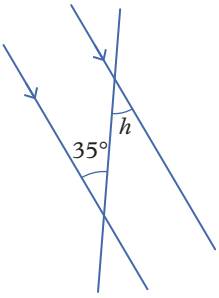
f



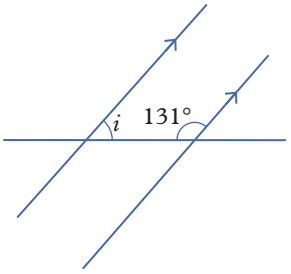
g



h

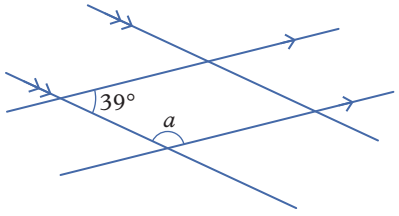


i

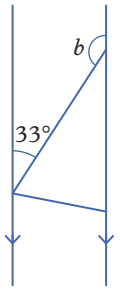


7 Find the size of each labelled unknown angle. In each case, state whether the angle is alternate, corresponding or co-interior to the given angle(s).

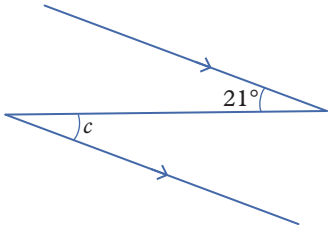
a



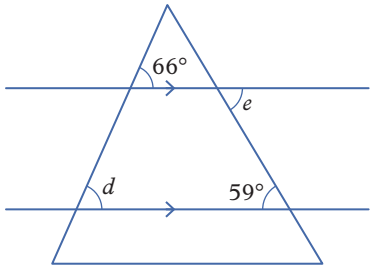
b



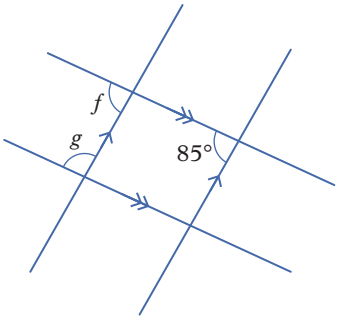
c



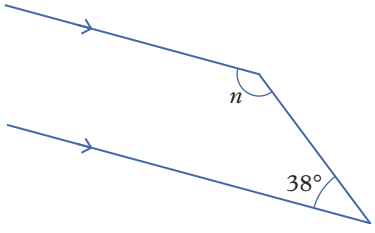
d



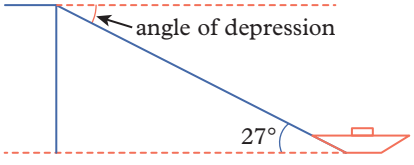
e



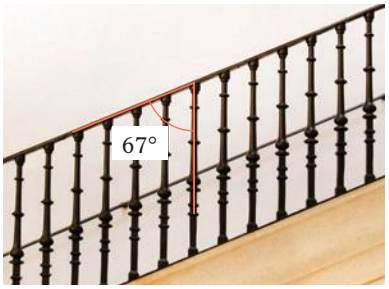
f



8 The angle of elevation (from a boat to a cliff top) is 27°. What is the angle of depression from the cliff top to the boat? Justify your answer.

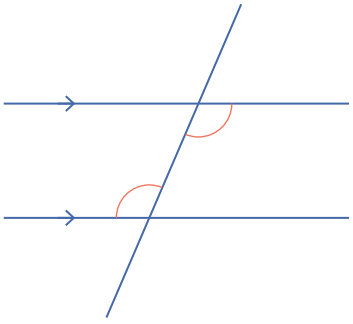


Exercise 7.1B: Problem solving and reasoning

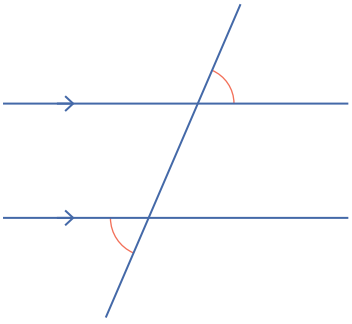


9 The Australian standard for staircases prefers that the angle that stairs make with the horizontal be between 30° and 38°, and states that it must be between 20° and 45°. The handrail is parallel to the stairs and all vertical support posts are parallel to each other. If one of the angles between the handrail and the support posts is 67°, determine if this staircase meets the Australian standard and if it is in the preferred range.

10 Alternate angles usually refer to the angles on opposite sides of the transversal between two lines; however, the angles on the outside of the lines on opposite sides of the transversal are also alternate angles. To distinguish between the two, we can call the ones between the lines alternate interior angles and the ones outside of the lines alternate exterior angles. Explain why alternate exterior angles will be equal for parallel lines.
Note: You can assume the term 'alternate angles' always refers to alternate interior angles, unless specified otherwise.



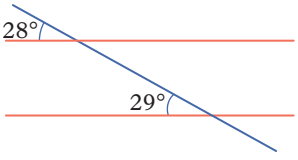
alternate interior angles



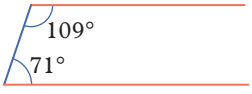
alternate exterior angles

11 Use your knowledge of co-interior, alternate and corresponding angles to check whether the pairs of red lines are parallel or not. Give a reason why each pair is or is not parallel.

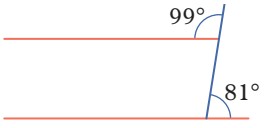
a



b



c

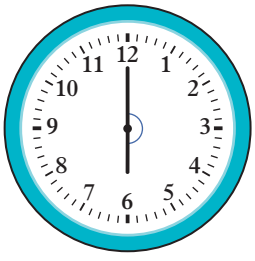


12 An understanding of angles around a point can help you to draw analogue clocks accurately.

a How many degrees are in a circle?

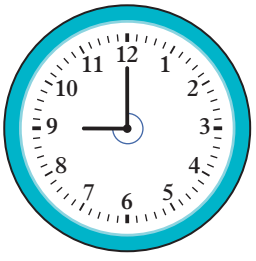
b Find the size of the angle indicated on each clock.

i



ii

iii



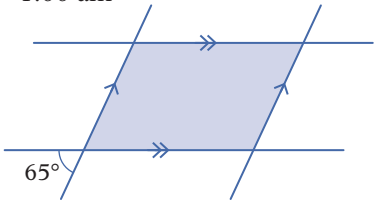
c What size angle would be between the hands of a clock displaying these times? Start from 12 o'clock and move clockwise to the other hand.

i 4:00 pm **ii** 11:00 am **iii** 8:00 pm **iv** 1:00 am

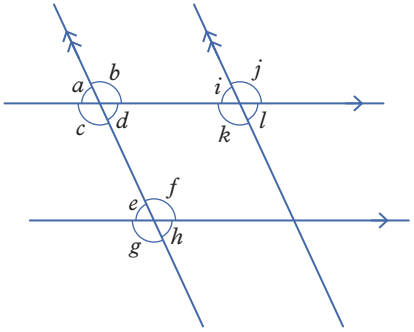
13 Consider the diagram on the right.

a What shape is formed by the two pairs of parallel lines?

b Use your knowledge of angles and parallel lines to state the size of all the internal angles of the shape.

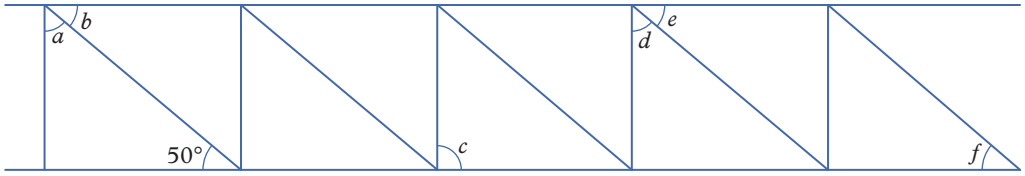


14 In the diagram on the right, identify all pairs of:



- **a** alternate angles
- **b** corresponding angles
- **c** co-interior angles.

15 The diagram below shows a building's structural support beams. If you know that the main two horizontal beams are at right angles to the vertical beams, and the slanted beams are all parallel to each other, use the information provided to find the size of each angle labelled with a pronumeral, giving reasons for your answers.



16 When a pair of parallel lines are cut by a transversal a number of angles are formed.

- **a** If a pair of parallel lines is cut by a single transversal, explain how knowing the size of one angle in the diagram means that you can find the size of every other angle.
- **b** If a pair of parallel lines is cut by a number of transversals that are parallel to each other, explain how knowing the size of one angle in the diagram still means that you can find the size of every other angle.

Exercise 7.1C: Challenge

17 The two hands on an analogue clock form a pair of angles.

- **a** What is the size of the acute angle between two consecutive hour marks on a clock face?
- **b** What is the size of:
  - **i** half of this angle
  - **ii** a quarter of this angle
  - **iii** three quarters of this angle?
- **c** Use your answers to parts **a** and **b** to find the size of the angle indicated on these clocks.

i

ii

iii

18 Find the size of the smaller angle between the two hands of a clock displaying these times.

a 10:20 **b** 6:55 **c** 4:35 **d** 2:27 **e** 11:09

Use these hints to help you.

- How many minute marks are displayed on a clock?
- What is the size of the angle between two of the minute marks?
- What is $\frac{1}{60}$ of the size of the angle between two of the minute marks?
- How far does the small hand move every time the big hand moves 1 minute?

Online resources:

- **Interactive skillsheet** Classifying angles
- **Interactive skillsheet** Angles at a point
- **Interactive skillsheet** Angles and parallel lines
- **Worksheet** Identifying and drawing angles
- **Investigation** Angles and parallel lines using technology
- **Quick quiz** 7.1

Lesson 7.2

Triangles

Learning intentions

By the end of this lesson you will be able to ...

✔ classify triangles and calculate unknown angles in triangles.

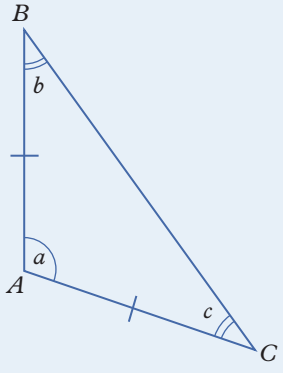
Inter-year links

Year 7 7.4 Triangles

Triangle properties

Key content video
Classifying triangles

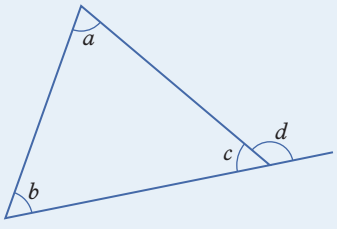
- A **triangle** is a two-dimensional (2D) shape with three straight sides.
- The **interior angles** of a triangle add to 180°.
- Triangles can be classified according to their side lengths and interior angles.
  - → An **equilateral triangle** has three sides of equal length and three interior angles of 60°.
  - → An **isosceles triangle** has two sides of equal length and two interior angles of equal size. The equal angles are opposite the sides of equal length.
  - → A **scalene triangle** has three sides of different length and three different interior angles.
  - → An **acute-angled triangle** has three interior acute angles (less than 90°).
  - → A **right-angled triangle** has one interior right angle (90°) and two interior acute angles (less than 90°).
  - → An **obtuse-angled triangle** has one interior obtuse angle (more than 90°) and two interior acute angles (less than 90°).
- When drawing geometric shapes, sides of equal length are marked using hatch marks or dashes, and angles of equal size are marked using the same number of arcs.
  For example, in this triangle, sides AB and AC are of equal length and angles b and c are equal in size.



Exterior angles

Key content video
Exterior angle of a triangle

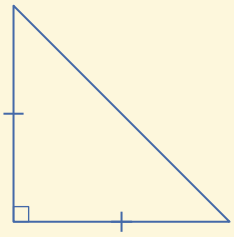
- An **exterior angle** is an angle between a side of a shape and a line extended from the adjacent side.
- The exterior angle of a triangle is supplementary to the adjacent interior angle; therefore, it is also equal to the sum of the two opposite interior angles.



$a + b + c = 180°$
$c + d = 180°$
$d = a + b$

Worked example 7.2A Classifying triangles

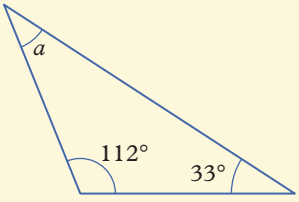
Classify the following triangle according to its sides and angles.



THINK	WRITE
1 Determine how many sides of the triangle are equal in length. The triangle has two sides equal in length, so it is an isosceles triangle. 2 Look at the angles in the triangle. The triangle has a right angle, so it is a right-angled triangle.	This is a right-angled isosceles triangle.

Worked example 7.2B Finding an unknown angle in a triangle

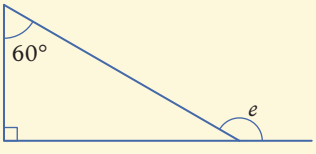
Find the size of angle a.



THINK	WRITE
The interior angles in a triangle add to 180°. Subtract each of the known angles from 180° to calculate the third angle, a.	$a = 180° - 112° - 33°$ $= 35°$

Worked example 7.2C Finding the exterior angle of a triangle

Find the size of angle e.



THINK	WRITE
The exterior angle of a triangle is equal to the sum of the two opposite interior angles.	$e = 60° + 90°$ $= 150°$

Helpful hints

- ✔ Don't guess that a triangle is equilateral or isosceles based on how it looks! This should only be determined once you know the side lengths and/or angles of a triangle.
- ✔ Every triangle with exactly two equal angles will have two sides of equal length opposite those angles and is an isosceles triangle.

Learning pathways

1–6, 7(a, b, d), 8, 9	2–6, 7(b, c, e), 10–13	2(d, f), 3(e, f), 4, 5, 7(e, f), 10, 11, 13, 14

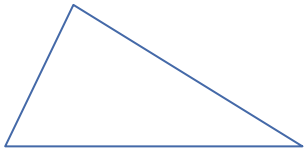
ANS p570

Exercise 7.2A: Understanding and fluency

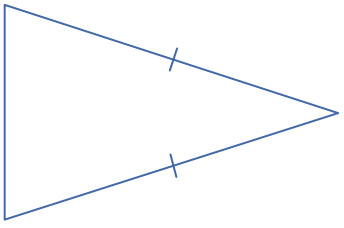
WE 7.2A **1** Classify each triangle according to its sides and angles.

a

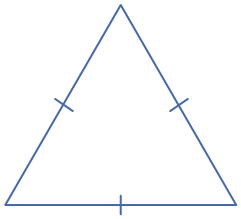
b



c



d



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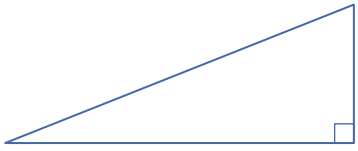

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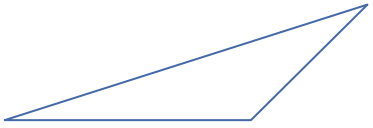
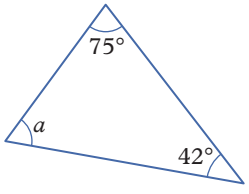
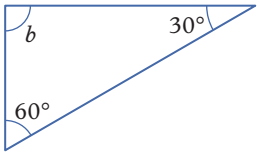


WE 7.2B **2** Find the size of each unknown angle.

a



b



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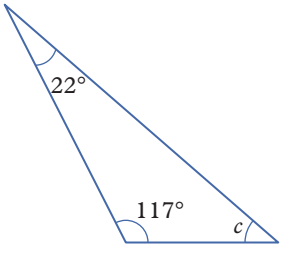

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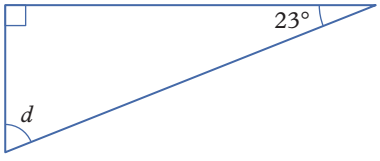

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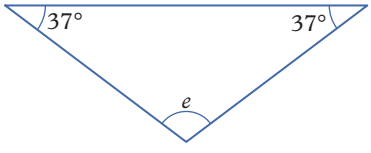

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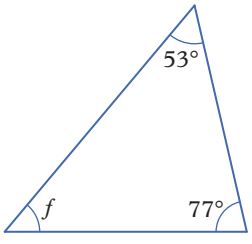
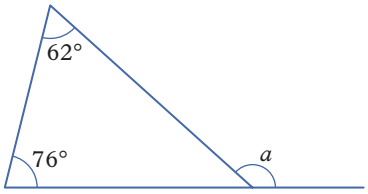


WE 7.2C **3** Find the size of each labelled unknown angle.

a



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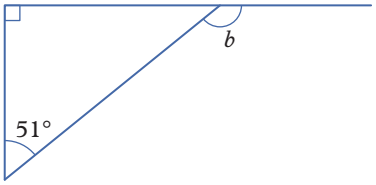

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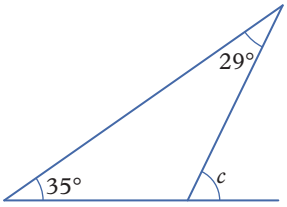

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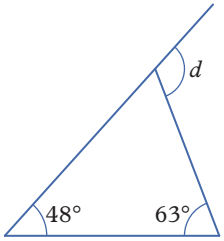

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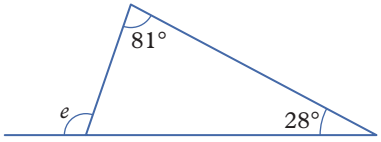

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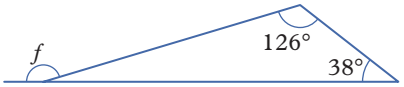
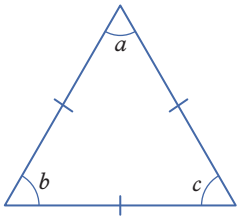


4 Use your knowledge of triangle properties to find the size of each labelled unknown angle.

a



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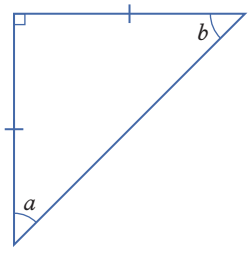

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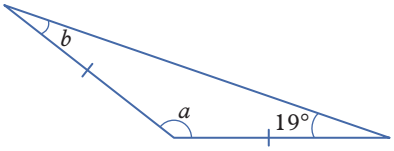

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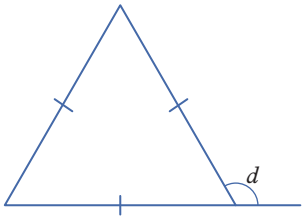

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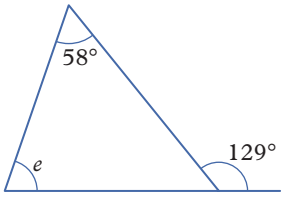

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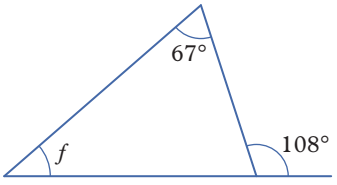
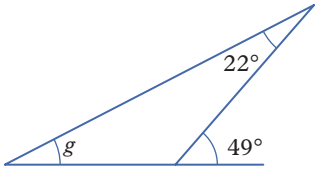


g



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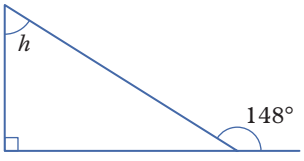
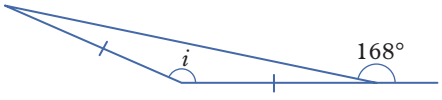


i



5 Classify each of the triangles from question **4** according to its sides and angles.

Exercise 7.2B: Problem solving and reasoning

6 Answer true or false to the following statements, giving a reason for your answers.

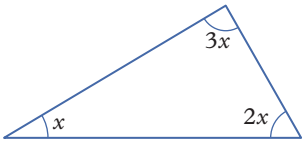
a An equilateral triangle can have a right angle.

b An isosceles triangle can have an obtuse angle.

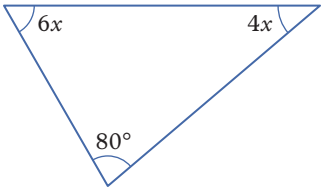
c A scalene triangle can have a right angle.

7 Use your knowledge of triangle properties and algebra to find the value of x in each triangle.

a



b



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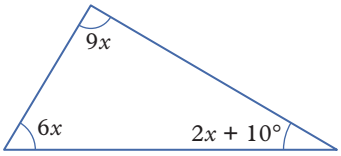
d



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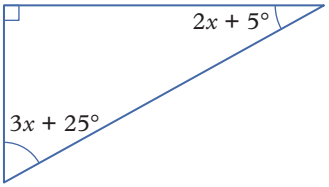

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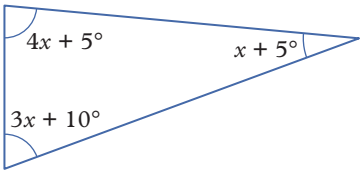
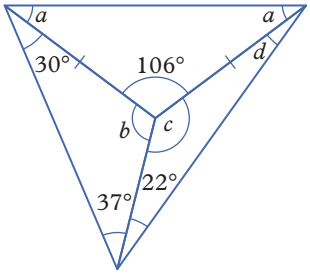


8 Rebecca measures the angles in a triangle and finds they are in a ratio of 1 : 2 : 3. That is, angle b is twice as big as angle a and angle c is three times as big as angle a.

a Find the size of each angle in this triangle.

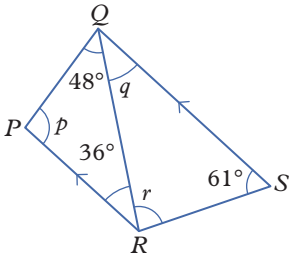
b Classify this triangle according to its sides and angles, justifying your answer.

9 Determine the values of the pronumerals a, b, c and d.



10 Consider the following triangle.

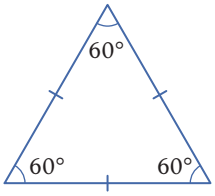
a Determine the values of the pronumerals p, q and r.



b What angle must $\angle RSQ$ be changed to in order to make PQ and RS parallel?

c Which other angle will change if PQ and RS become parallel by changing $\angle RSQ$, and to what size?

11 Consider this equilateral triangle.



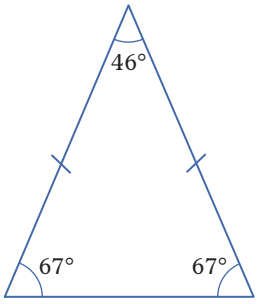
a Draw a copy of the triangle and:

i extend each side to create an exterior angle at each vertex

ii find the size of each of the exterior angles and the sum of the exterior angles.

Repeat **i** and **ii** for these two triangles.

b



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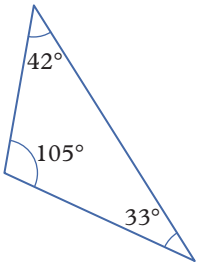


d What can you say about the sum of the exterior angles of a triangle?

12 A classmate was given a number of triangles to construct but is having trouble drawing them. For each set of instructions, draw a diagram of the triangle or provide a reason why it cannot be drawn.

a a triangle with internal angles of 90°, 50° and 60°

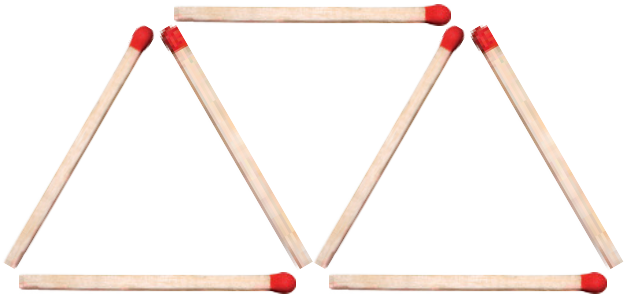
b a triangle with two internal angles of 40°

c a triangle with side lengths of 10 cm, 3 cm and 5 cm

d a triangle with two obtuse angles

Exercise 7.2C: Challenge

13 You can construct equilateral triangles using matches, as shown here.



a How many matches were used to construct these three triangles?

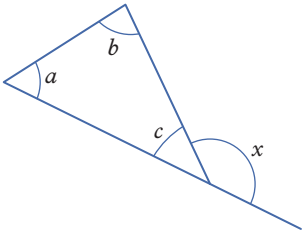
b Use:

i nine matches to construct four equilateral triangles

ii nine matches to construct five equilateral triangles

iii six matches to construct four equilateral triangles.

14 Use algebra to show that $x = a + b$.



Online resources:

Interactive skillsheet Triangle properties

Worksheet Exterior angles

Investigation Stained glass triangles

Quick quiz 7.2

Lesson 7.3

Pythagoras' theorem

Learning intentions

By the end of this lesson you will be able to ...

✔ use Pythagoras' theorem to determine whether a triangle is right-angled, or to find the length of the hypotenuse.

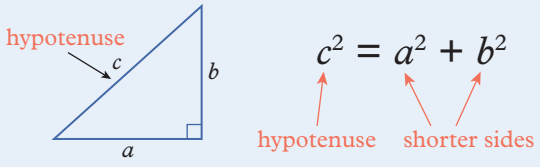
Inter-year links

Year 7	1.7 Exponents and square roots
Year 9	5.1 Pythagoras' theorem
Year 10	7.1 Pythagoras' theorem

Key content video
Pythagoras' theorem

Pythagoras' theorem for right-angled triangles

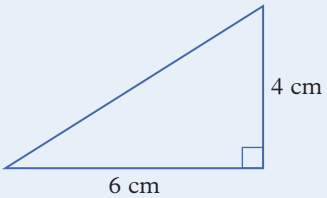
- Pythagoras of Samos was an ancient Greek philosopher and mathematician who lived from approximately 570 to 495 BCE. He has been credited with many mathematical and scientific discoveries, although the origins of his most famous theorem are disputed to this day.
- The **hypotenuse** is the longest side of a right-angled triangle. It is the side directly opposite the right angle.
- **Pythagoras' theorem** states the relationship between the three side lengths of a right-angled triangle. That is, the sum of the squares of the shorter sides is equal to the square of the hypotenuse. The relationship for the triangle shown is:



- A triangle is right-angled if it has side lengths that satisfy Pythagoras' theorem.
- Any set of three positive whole numbers that satisfy Pythagoras' theorem is called a **Pythagorean triple** (or Pythagorean triad).
  For example, 3, 4, 5 is a Pythagorean triple as $5^2 = 3^2 + 4^2$.

Finding the length of the hypotenuse

- To find the length of the hypotenuse using Pythagoras' theorem:
  1 State Pythagoras' theorem and define a, b and c.
  2 Substitute in the values for a, b and c and simplify the equation.
  3 Solve for c by finding the square root of both sides of the equation using inverse operations.
- The result for the length of the hypotenuse can be written as an exact value (whole number, terminating decimal or in surd form) or approximated to a specific number of decimal places.
  For example, the side length of the hypotenuse in this triangle can be written as $\sqrt{52}$ cm or 7.21 cm correct to two decimal places.

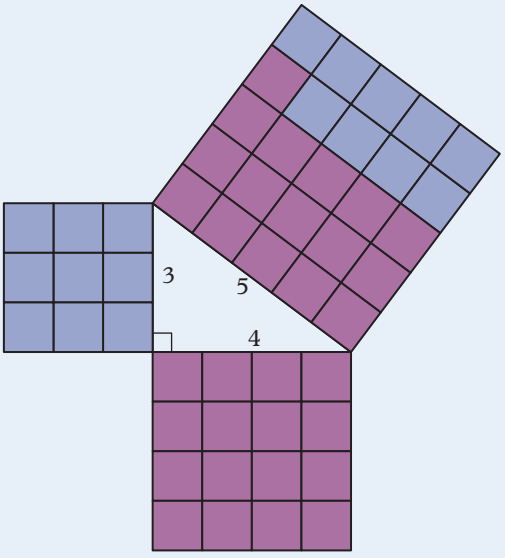


$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 6^2 + 4^2 \\ &= 36 + 16 \\ &= 52 \\ c &= \sqrt{52} \text{ cm} \\ &\approx 7.21 \text{ cm} \end{aligned}$$

→ Unless the question states otherwise, the hypotenuse should be written as an exact value.

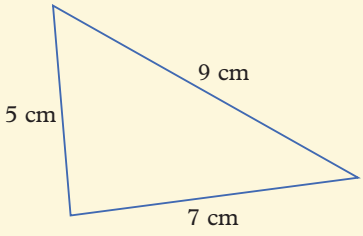
Proofs of Pythagoras' theorem

- Mathematical proofs demonstrate that a mathematical statement or theorem is true using logic and reasoning.
- There are over 300 different proofs of Pythagoras' theorem, with the earliest of these proofs dating back thousands of years.
  - → Two proofs of Pythagoras' theorem are demonstrated in question **18** of the following exercise. An investigation exploring further proofs of Pythagoras' theorem can be found on Oxford Digital.



Worked example 7.3A Using Pythagoras' theorem to determine whether a triangle is right-angled

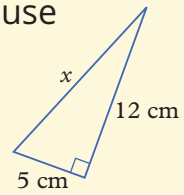
Use Pythagoras' theorem to determine whether this is a right-angled triangle.



THINK	WRITE
1 Identify the length of the longest side and define as c.	Let $c = 9$ cm.
2 Calculate the value of c^2.	$c^2 = 9^2$ $= 81$
3 Identify the lengths of the two shorter sides and define as a and b.	Let $a = 5$ cm and $b = 7$ cm.
4 Calculate the value of $a^2 + b^2$.	$a^2 + b^2 = 5^2 + 7^2$ $= 25 + 49$ $= 74$
5 Compare c^2 to $a^2 + b^2$. If the two values are equal, then the triangle is right-angled. If the two values are not equal, then it is not a right-angled triangle.	$c^2 \neq a^2 + b^2$ The triangle is not a right-angled triangle.

Worked example 7.3B Calculating the length of the hypotenuse (whole number)

Use Pythagoras' theorem to calculate the length of the hypotenuse of the triangle on the right.



THINK

1. The triangle is right-angled, so state Pythagoras' theorem and define *a*, *b* and *c*.
2. Substitute in the values for *a*, *b* and *c*, and simplify the equation.
3. Solve for *x* by finding the square root of both sides of the equation using inverse operations.

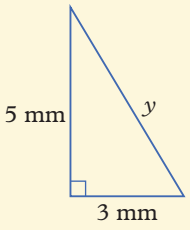
WRITE

$c^2 = a^2 + b^2$, where $a = 5$ cm, $b = 12$ cm and $c = x$.

$x^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
$x = \sqrt{169}$
$= 13$ cm

Worked example 7.3C Calculating the length of the hypotenuse (decimal value)

Calculate the length of the hypotenuse of the triangle on the right correct to two decimal places.



THINK

1. The triangle is right-angled, so state Pythagoras' theorem and define *a*, *b* and *c*.
2. Substitute in the values for *a*, *b* and *c*, and simplify the equation.
3. Solve for *y* by finding the square root of both sides of the equation.
4. Use a calculator to find the approximate length. Round to two decimal places. Note: $\sqrt{34}$ mm is the exact length.

WRITE

$c^2 = a^2 + b^2$, where $a = 3$ mm, $b = 5$ mm and $c = y$.

$y^2 = 3^2 + 5^2$
$= 9 + 25$
$= 34$

$y = \sqrt{34}$

≈ 5.83 mm

Helpful hints

- ✔ Make sure you correctly identify the hypotenuse. It is the longest side and is opposite the right angle. It may not be obvious, so always look at the numbers carefully.
- ✔ It doesn't matter which shorter sides you assign to be *a* or *b*, but the hypotenuse must always be *c*!

Learning pathways

1–7, 9, 11, 12(a, b, f), 16	2, 3, 5, 7, 10, 12(c, e, f), 13, 14, 16, 17(a, b), 19	2, 6–8, 10, 13–16, 17(c, d), 18

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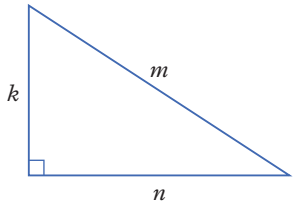
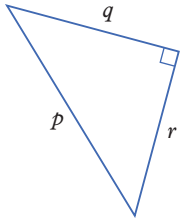
Exercise 7.3A: Understanding and fluency

1 Identify the pronumeral shown on the hypotenuse in each triangle.

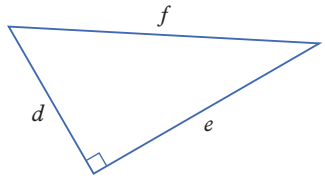
a



b

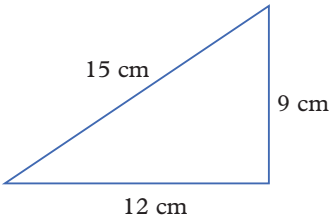


c



WE 7.3A **2** Use Pythagoras' theorem to determine whether these are right-angled triangles.

a



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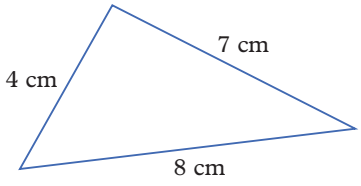

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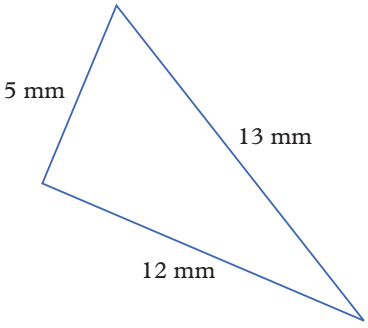

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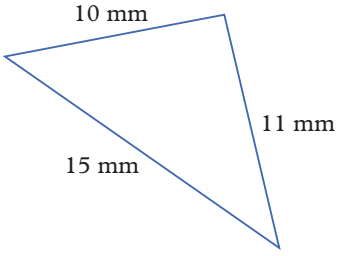
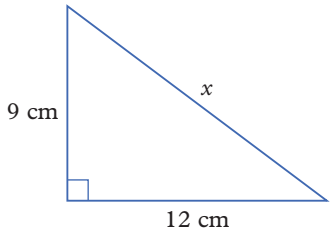


3 Determine whether each triangle with the given side lengths is right-angled or not.
Hint: The longest side in a right-angled triangle is the hypotenuse.

a 6 mm, 8 mm, 10 mm
b 7 cm, 24 cm, 25 cm
c 8 cm, 10 cm, 14 cm
d 11 mm, 15 mm, 19 mm
e 20 cm, 21 cm, 29 cm
f 10 mm, 18 mm, 20 mm

WE 7.3B **4** Use Pythagoras' theorem to calculate the length of the hypotenuse in each triangle.

a



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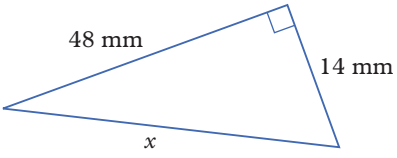

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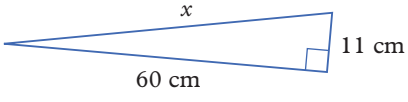

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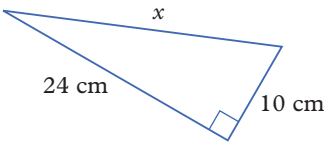
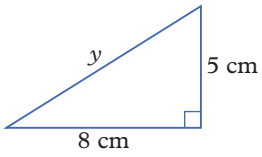


WE 7.3C **5** Calculate the length of the hypotenuse in each triangle correct to two decimal places.

a



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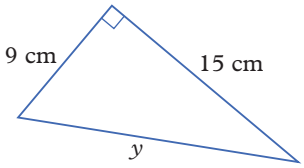
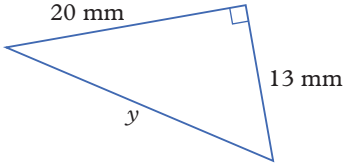
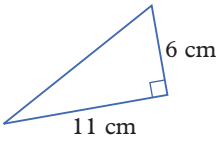


c



6 Find the unknown side length in each right-angled triangle. Give your answers **i** as exact values and **ii** correct to two decimal places.

a



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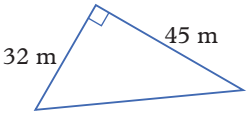

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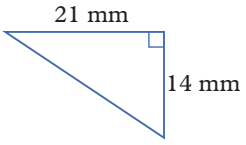
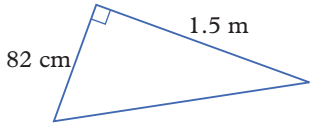


7 Find the length of the hypotenuse in each right-angled triangle correct to the nearest centimetre.
Hint: Convert all length measurements to centimetres first.

a



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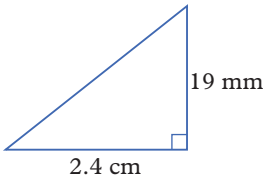

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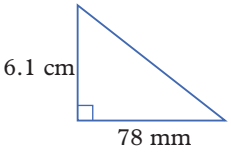


Exercise 7.3B: Problem solving and reasoning

8 Both multiplication and addition are commutative. That is, when numbers or variables are multiplied or added, the order doesn't matter as the result will be the same.
For example: $2 \times 3 = 3 \times 2$ and $2 + 3 = 3 + 2$
In Pythagoras' theorem, a and b refer to the lengths of the two shorter sides of a right-angled triangle. Does it matter which of the shorter sides is labelled a and which is labelled b? Explain by referring to commutative properties.

9 Pythagorean triples can be generated using formulas. One method for finding Pythagorean triples uses three formulas and two known values, m and n where $m > n$, to generate the shorter side lengths a and b, and the hypotenuse c. The formulas are:

- $a = m^2 - n^2$
- $b = 2mn$
- $c = m^2 + n^2$.

Use the formulas to complete the table below to generate Pythagorean triples. The first row has been completed for you.

Values		**Pythagorean triple**		
m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	$2^2 - 1^2 = 3$	$2 \times 2 \times 1 = 4$	$2^2 + 1^2 = 5$
3	1			
3	2			
4	1			
4	2			
4	3			
5	1			
5	2			
5	3			
5	4			
6	1			

10 Pythagorean triples form the lengths of right-angled triangles. Some triples are multiples of other triples and they form similar triangles (triangles with the same shape but different sizes).

For example, (6, 8, 10) and (9, 12, 15) are multiples of (3, 4, 5).

A Pythagorean triple that does not have a common factor between the three sides is called a primitive Pythagorean triple. For example, 3, 4 and 5.

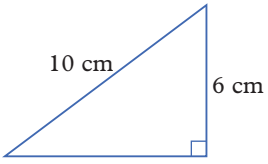
a List the primitive Pythagorean triples from question **9**.

b The Pythagoreans triples below are all multiples of a primitive Pythagorean triple. State the primitive Pythagorean triple in each case.

i (50, 120, 130) **ii** (24, 45, 51) **iii** (27, 36, 45)

iv (36, 160, 164) **v** (390, 432, 582) **vi** (231, 2420, 2431)

11 Consider Triangle A.



Triangle A

a Use Pythagoras' theorem to confirm that Triangle A is right-angled.

b A new triangle, Triangle B, is formed by doubling the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle B?

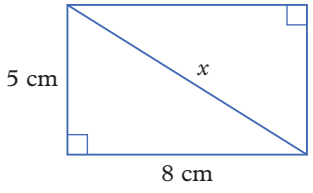
c Triangle C is formed by tripling the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle C?

d Triangle D is formed by halving the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle D?

e Will Pythagoras' theorem still work if the side lengths of Triangle A are multiplied by 10?

12 Use Pythagoras' theorem to calculate the unknown length indicated by the pronumeral in each diagram correct to two decimal places.

a



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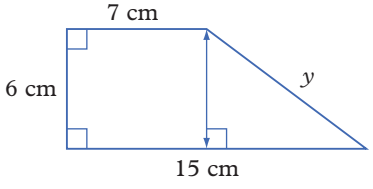

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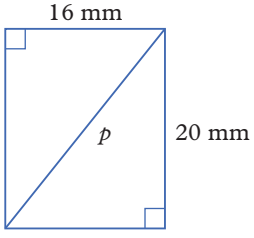

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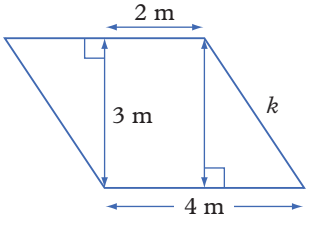
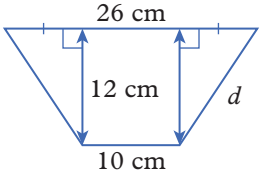
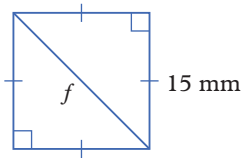


e

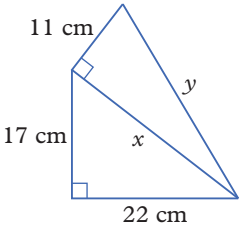


f



13 Find the lengths indicated by x and y in each diagram, correct to two decimal places.

a



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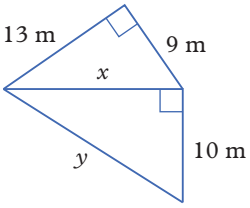

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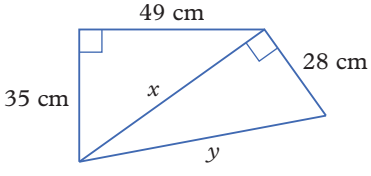
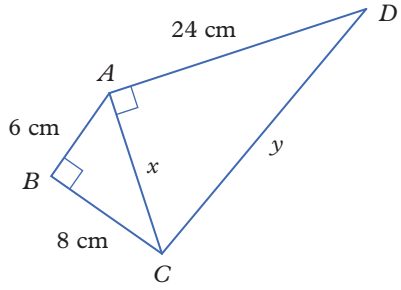


14 The square root of a positive number is sometimes given with a ± sign, for example $\sqrt{9} = \pm 3$. This is because the square of the negative value is positive: $(-3)^2 = 9$.
Explain why we only need to consider the positive solution of a square root when using Pythagoras' theorem.

15 Consider the shape *ABCD* shown on the right.

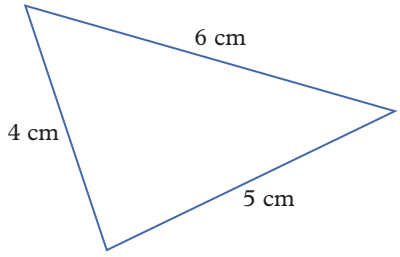


a How many right-angled triangles can you see? Name them.

b Calculate the length of *AC*. Which side lengths did you use in your calculation?

c Calculate the length of *CD*. Explain how you were able to do this.

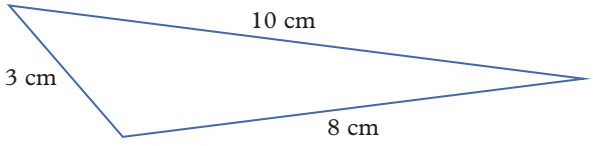
16 Consider this acute-angled triangle.



a Calculate the square of the side lengths of this triangle.

b Complete the following sentence.
In an acute-angled triangle, the square of the longest side is _____ the sum of the squares of the two other sides.

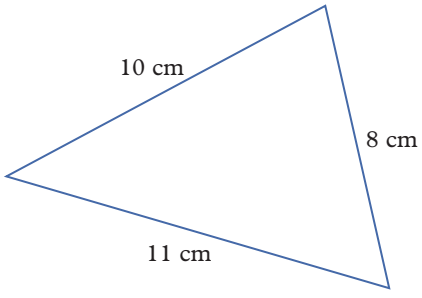
c Calculate the square of the side lengths of this obtuse-angled triangle.



d Complete the following sentence.
In an obtuse-angled triangle, the square of the longest side is _____ the sum of the squares of the two other sides.

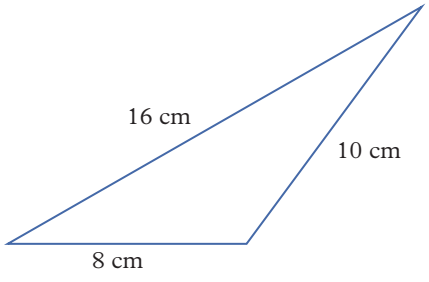
e Confirm that your sentences work for the following triangles.

i



Acute-angled triangle

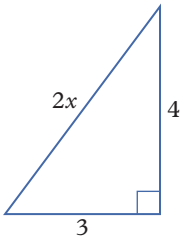
ii



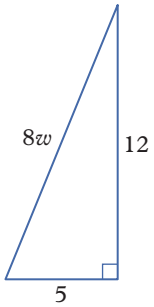
Obtuse-angled triangle

17 Determine the value of the pronumeral in each of the following. Write your answer as a fraction where appropriate.

a



b



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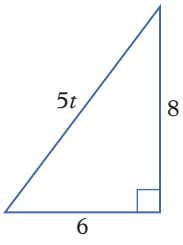

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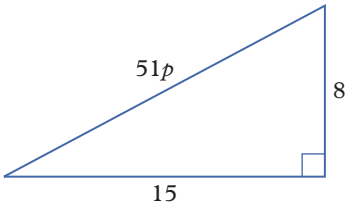


Exercise 7.3C: Challenge

18 Pythagoras' theorem is one of the most thoroughly proved theorems with hundreds of unique proofs (mathematical explanations). One proof, which we will look at, is known as 'Proof by rearrangement'. Consider Diagram 1 and Diagram 2. Both diagrams have the same four right-angled triangles with shorter side lengths of a and b, and a hypotenuse with length c.

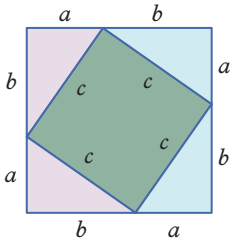


Diagram 1

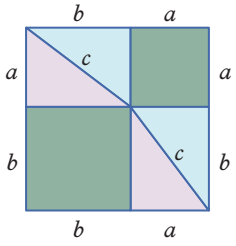


Diagram 2

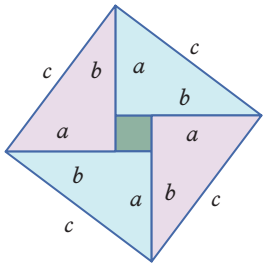


Diagram 3

a Explain why the two squares have the same area.

b Using your ideas from part **a**, explain how this pair of diagrams shows that $c^2 = a^2 + b^2$.

c We can prove this algebraically by writing an equation equating the areas of each square.

i Write an equation for the total area in each diagram by adding the area of their smaller sections.

ii The area in both diagrams is the same, so we can equate the equations from part **i.** Show that this equation is equivalent to $c^2 = a^2 + b^2$.

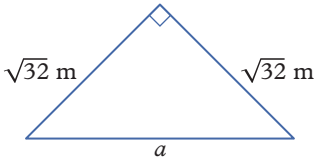
d Another proof for Pythagoras' theorem can be shown using Diagram 3.

i Using this diagram, explain why $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$ by considering the area of each part.

ii Show that the equation from part **i** is equivalent to $c^2 = a^2 + b^2$.

19 Squaring and taking the square root of a number are inverse operations, so $(\sqrt{a})^2 = a$. Use this information to determine the values of the pronumerals in these diagrams, giving your answers correct to two decimal places where required.

a



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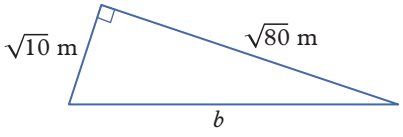

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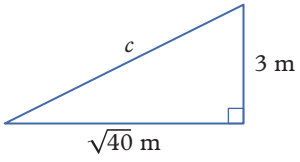

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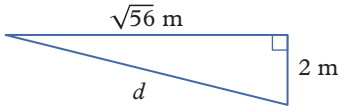

Online resources:

- **Interactive skillsheet** Pythagoras' theorem
- **Worksheet** Pythagoras' theorem
- **Desmos activity** Exploring length with geoboards
- **Quick quiz** 7.3
- **Interactive skillsheet** Finding the length of the hypotenuse
- **Investigation** Proving Pythagoras' theorem

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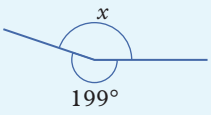
Checkpoint

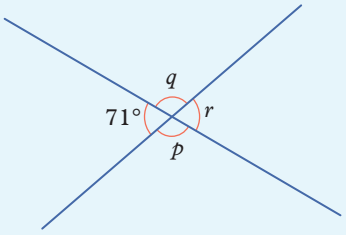
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

7.1 **1** Determine the value of each of the pronumerals.

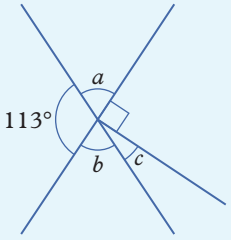
a



b

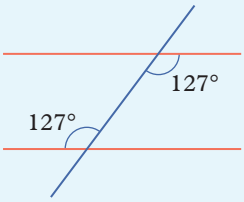


c



7.1 **2** Determine if the red lines are parallel or not parallel. State why or why not.

a



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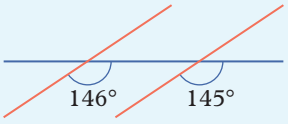

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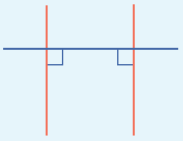
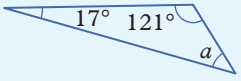


7.2 **3** Determine the size of the missing angles and classify the type of triangle according to its sides and angles.

a



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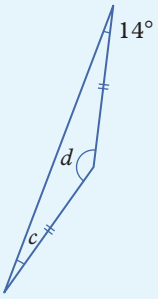
c



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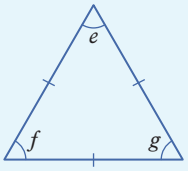
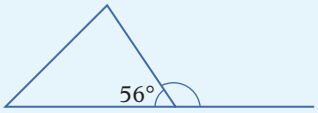


7.2 **4** Determine the size of the exterior angle marked.

a



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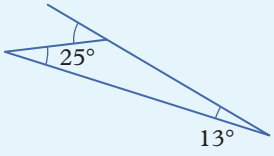

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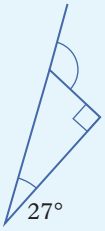

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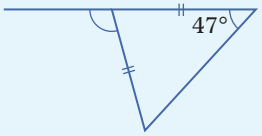
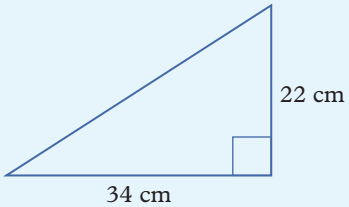


7.3 **5** Determine whether the following triples could be side lengths of a right-angled triangle.

a (21, 28, 35) **b** (24, 121, 123) **c** (38, 56, 68)

7.3 **6** Determine the length of the missing side, correct to two decimal places.

a



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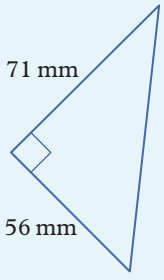

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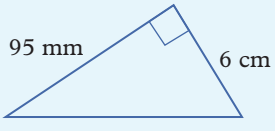


Lesson 7.4

Finding the length of a shorter side

Learning intentions

By the end of this lesson you will be able to ...

✔ use Pythagoras' theorem to find the length of a shorter side.

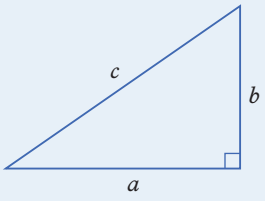
Inter-year links

Year 7	6.6 Solving equations using inverse operations
Year 9	5.1 Pythagoras' theorem
Year 10	7.1 Pythagoras' theorem

Finding the length of a shorter side

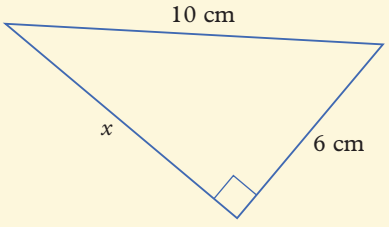
Key content video
Finding the length of a shorter side

- Pythagoras' theorem states that in a right-angled triangle with side lengths a, b and c, where c is the length of the hypotenuse, $c^2 = a^2 + b^2$.
- To find the length of one of the shorter sides using Pythagoras' theorem:
  1. State Pythagoras' theorem and define a, b and c.
  2. Substitute the values for a, b and c, and simplify the equation.
  3. Solve for a or b by finding the square root of both sides of the equation using inverse operations. Write the solution including the correct unit for the side length.



Worked example 7.4A Calculating the length of a shorter side (whole number)

Use Pythagoras' theorem to calculate the unknown side length in this triangle.



THINK

1. The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.
2. Substitute the values for a, b and c, and simplify the equation.
3. Solve for x by first subtracting 36 from both sides of the equation and then finding the square root.

WRITE

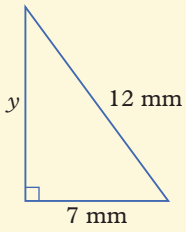
$c^2 = a^2 + b^2$, where $a = x$, $b = 6$ cm and $c = 10$ cm.

$10^2 = x^2 + 6^2$

$100 = x^2 + 36$
$64 = x^2$
$\sqrt{64} = x$
$8 = x$
$x = 8$ cm

Worked example 7.4B Calculating the length of a shorter side (decimal value)

Calculate the unknown side length in this triangle, correct to two decimal places.



THINK

1. The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.
2. Substitute the values for a, b and c, and simplify the equation.
3. Solve for y by first subtracting 49 from both sides of the equation and then finding the square root.
4. Use a calculator to find the approximate length correct to two decimal places. (Note that $\sqrt{95}$ mm is the exact length.)

WRITE

$c^2 = a^2 + b^2$, where $a = 7$ mm, $b = y$ and $c = 12$ mm.

$12^2 = 7^2 + y^2$

$144 = 49 + y^2$

$95 = y^2$

$\sqrt{95} = y$

$y \approx 9.75$ mm

Helpful hints

- ✔ A common mistake when using Pythagoras' theorem occurs in the second-last step. When looking at $y^2 = 95$, instead of taking the square root of both sides, some students will halve the result giving 47.5. The 2 is an exponent, not a factor!
- ✔ Remember that c represents the length of the hypotenuse in $c^2 = a^2 + b^2$, not the unknown side. So when finding the length of a shorter side, substitute the pronumeral in for a or b.
- ✔ The solution of an equation $x^2 = 64$ has two solutions, $x = \pm 8$, but you only need to write the positive solution if x represents a length, as lengths can't be negative.

Learning pathways

1, 2–4(1st column), 5, 6, 7(a, b), 8	2–4(2nd column), 5, 6(2nd column), 8, 12, 13(a, b)	4, 5, 6(2nd column), 8–10, 11(b), 12, 13(c, d), 14

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Exercise 7.4A: Understanding and fluency

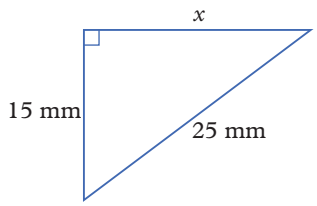
1 Solve the following equations for x. Write your answers correct to two decimal places.

a $x^2 + 25 = 36$ **b** $x^2 + 7^2 = 10^2$ **c** $16 + x^2 = 64$

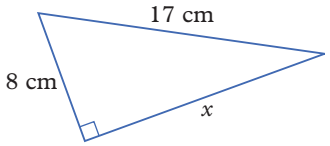
d $81 = x^2 + 9$ **e** $12^2 = 5^2 + x^2$ **f** $36 = x^2 + x^2$

WE 7.4A **2** Use Pythagoras' theorem to calculate the unknown side length in each triangle.

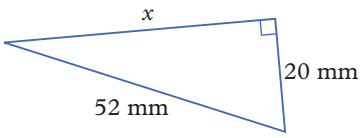
a



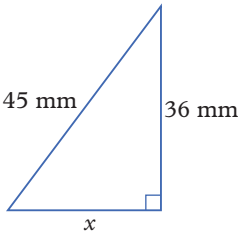
b



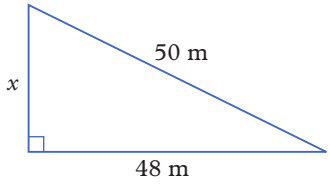
c



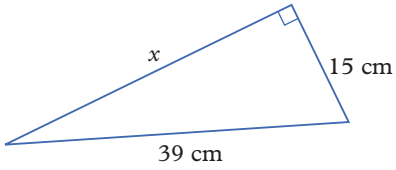
d



e

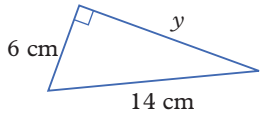


f

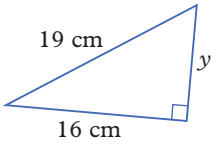


WE 7.4B **3** Calculate the unknown side length in each triangle, correct to two decimal places.

a



b



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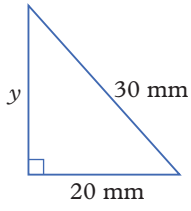

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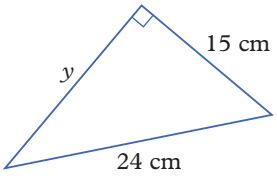

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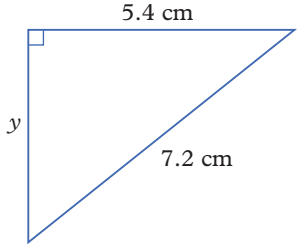

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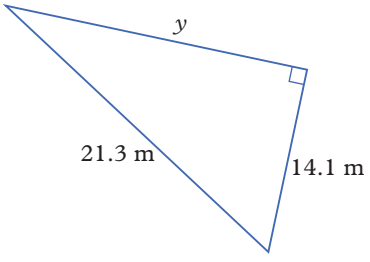
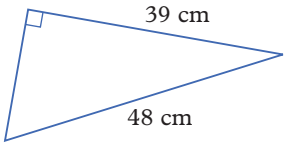


4 Find the unknown side length in each right-angled triangle. Give your answers **i** as exact values and **ii** correct to two decimal places.

a



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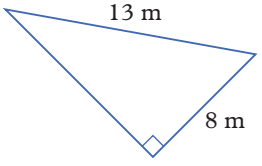

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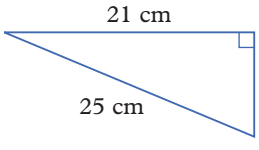
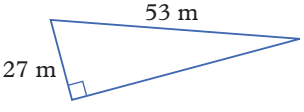
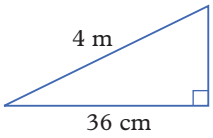


d



5 Find the unknown length in each right-angled triangle correct to the nearest centimetre.
Hint: Convert all length measurements to centimetres first.

a



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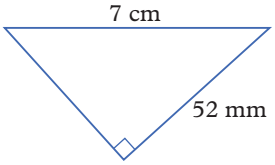

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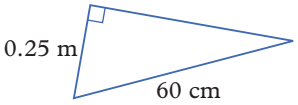

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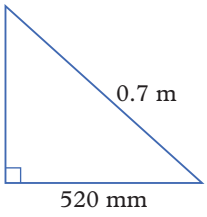
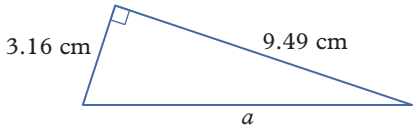


6 Calculate the unknown side length in each right-angled triangle correct to two decimal places.

a



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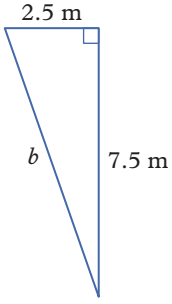

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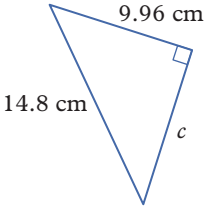

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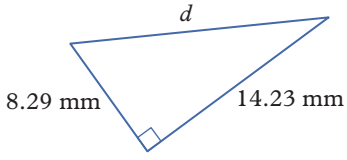

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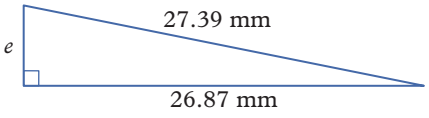

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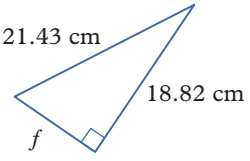
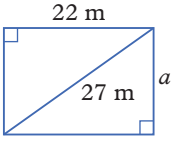


Exercise 7.4B: Problem solving and reasoning

7 Use Pythagoras' theorem to calculate the unknown lengths indicated by the pronumerals in each diagram correct to two decimal places.

a



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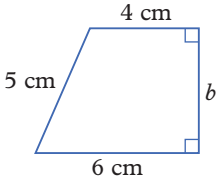

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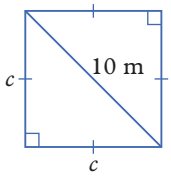
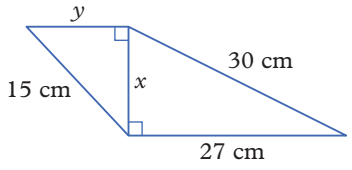


8 Find the lengths labelled x and y in each diagram correct to two decimal places.

a



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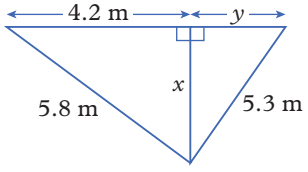

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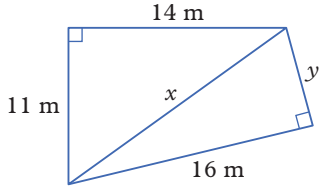

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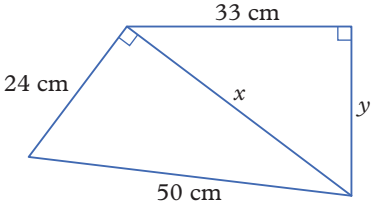
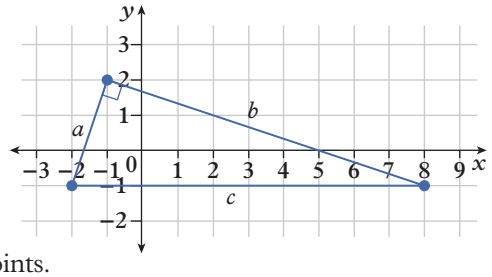
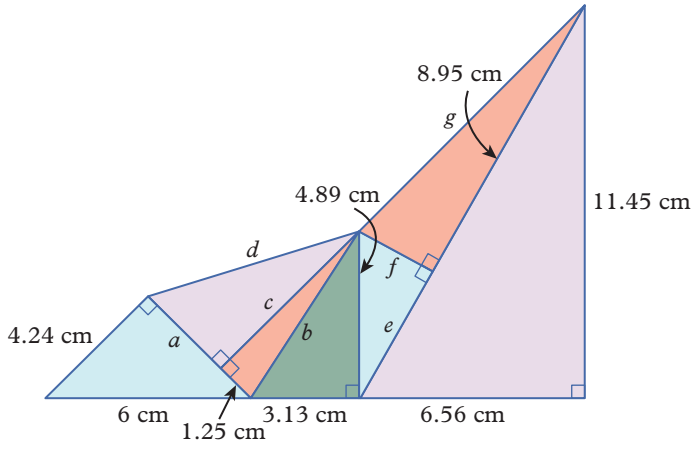


9 Consider the triangle shown on the Cartesian plane.



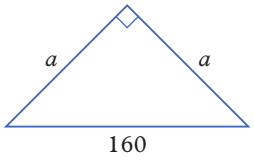
- **a** State the length of c.
- **b** Determine the square of the lengths of a and b.
- **c** Show that the triangle is right-angled.
- **d** Determine the midpoint of each side length. Write your answers using decimals where necessary.
- **e** Determine the squares of the lengths between each pair of midpoints.
- **f** Determine whether the triangle formed by the midpoints is right-angled or not.

10 Determine the value of all pronumerals in the following diagram, a–g correct to two decimal places.

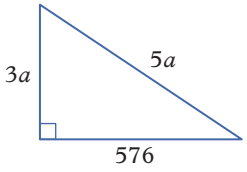


11 Determine the value of the pronumerals correct to two decimal places where appropriate.

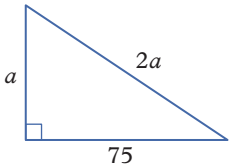
a



b



c



12 Joey is determining the unknown side length of a right-angled triangle with the two side lengths 7 cm and 15 cm. Joey writes down all possible calculations for the third length.

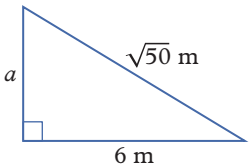
i $\sqrt{7^2 + 15^2}$ **ii** $\sqrt{15^2 + 7^2}$ **iii** $\sqrt{7^2 - 15^2}$ **iv** $\sqrt{15^2 - 7^2}$

- **a** If the missing side length is the hypotenuse, which calculation(s) could be used?
- **b** If the missing side length is not the hypotenuse, which calculation(s) could be used?
- **c** Is there a calculation listed that does not result in the side length of a right-angled triangle? Explain why or why not.

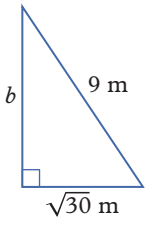
Exercise 7.4C: Challenge

13 Determine the values of the pronumerals in these diagrams, giving your answers correct to two decimal places where required.

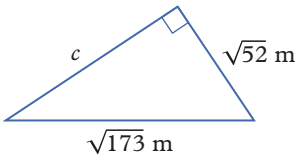
a



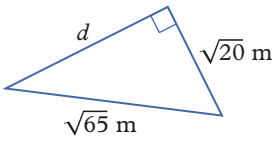
b



c



d



14 A square is drawn on each side of a right-angled isosceles triangle. The sum of the areas of the squares is 60 cm^2. What is the area of the triangle?

Online resources:

Interactive skillsheet
Finding the length of a shorter side

Quick quiz
7.4

Lesson 7.5

Applications of Pythagoras' theorem

Learning intentions

By the end of this lesson you will be able to ...

✔ use Pythagoras' theorem to solve a range of different problems.

Inter-year links

Year 9 5.1 Pythagoras' theorem

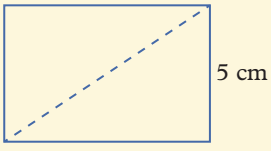
Year 10 7.1 Pythagoras' theorem

Applications of Pythagoras' theorem

- Pythagoras' theorem allows you to calculate the third side length of any right-angled triangle, providing the other two side lengths are known. $a^2 + b^2 = c^2$
- Right angles are widely used in many industries, such as architecture, engineering and construction. Right angles can also be found in nature; for example, a tree which stands vertically will form a right angle with the ground.
  → When a right angle is formed, a right-angled triangle can be drawn, and unknown distances can be calculated.

Worked example 7.5A Using Pythagoras' theorem to calculate an unknown length

Calculate the length of the diagonal of this rectangle. Give your answer as an exact value.



THINK

1 The shape is a rectangle, so the corners are right angles and the diagonal will be the hypotenuse of the triangle including the two side lengths. State Pythagoras' theorem and define a, b and c.

2 Substitute the values for a and b into the equation for Pythagoras' theorem. Let c be the unknown side and solve the equation for c.

WRITE

$c^2 = a^2 + b^2$, where $a = 5$ cm and $b = 7$ cm.

$c^2 = 5^2 + 7^2$
$= 25 + 49$
$= 74$
$c = \sqrt{74}$

Helpful hints

✔ If you are not given a diagram in a question which requires Pythagoras' theorem, draw a diagram as your first step to ensure you correctly identify the hypotenuse and the two shorter sides.

Learning pathways

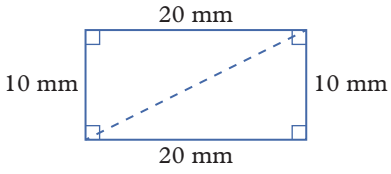
1–2(a–c), 5(a–c), 6–9, 11, 13	1–2(d–f), 5(d–f), 7, 10, 11, 13, 14, 16	3, 4, 5(d–f), 7, 9, 12, 15–20

ANS p573

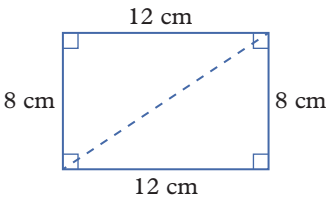
Exercise 7.5A: Understanding and fluency

WE 7.5A **1** Calculate the length of the diagonal in each of these rectangles. Give your answers in exact form.

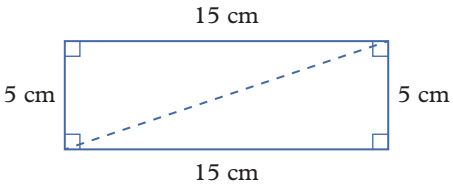
a



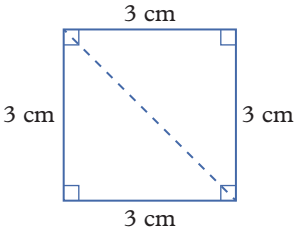
b



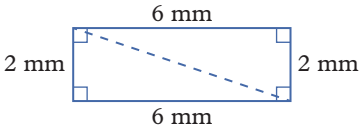
c



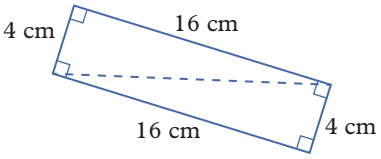
d



e



f



2 Rewrite the lengths of the diagonals of the rectangles in question **1** correct to one decimal place.

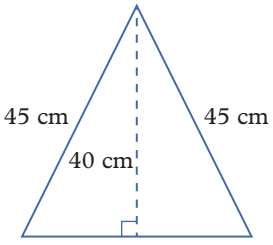
3 Calculate the length of the diagonal of a square with each of these side lengths. Give your answers as exact values in simplest form.

a 5 cm **b** 20 mm **c** 8 cm **d** 25 cm

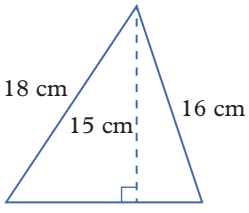
4 Rewrite the lengths of the diagonals of the squares in question **3**, correct to one decimal place.

5 Calculate the length of the unknown side for each of these triangles, using the given height of each triangle to help you. Give your answers as approximate values correct to one decimal place.

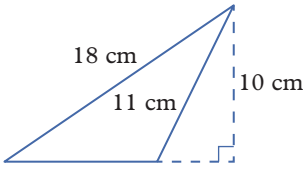
a



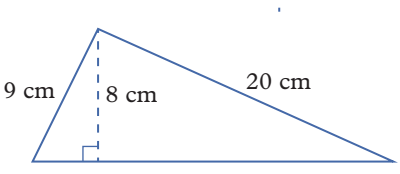
b



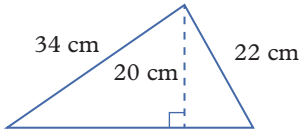
c



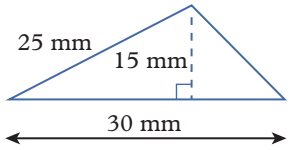
d



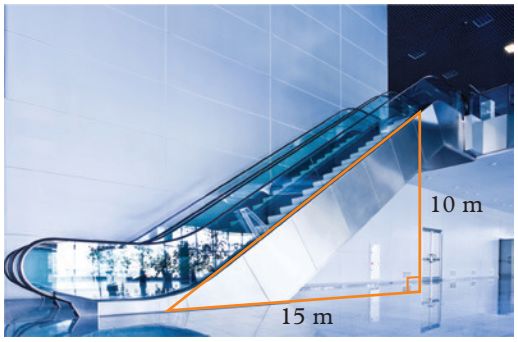
e



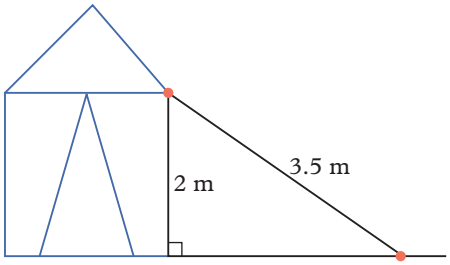
f



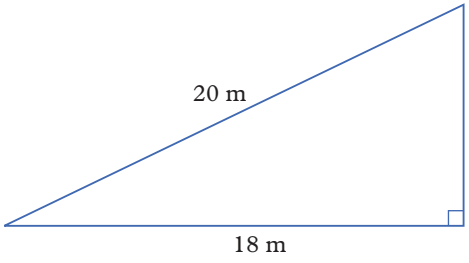
6 An escalator rises a vertical distance of 10 m for a horizontal distance of 15 m. What distance would you travel from the bottom to the top of the escalator? Give your answer correct to two decimal places.



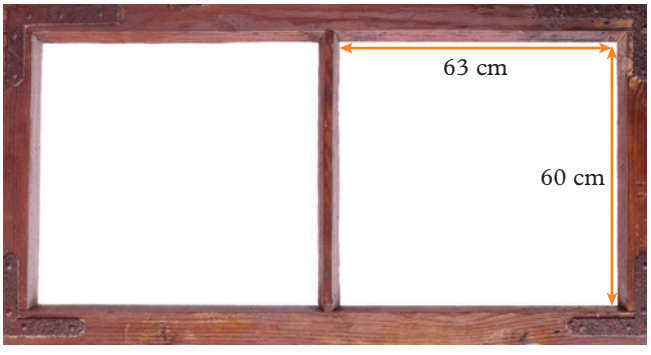
7 A guy rope on a tent is 3.5 m long and is attached to the tent at a height of 2 m. How far from the tent does the guy rope reach the ground? Give your answer correct to two decimal places.



8 A BMX rider uses the ramp shown in this diagram to perform stunts. How high is the ramp? Give your answer correct to two decimal places.



9 The timber frames in a window should meet at right angles. The diagonal length of the glass in one section of the window below is measured to be 87 cm. Use Pythagoras' theorem to check whether this window frame has been made correctly.



Exercise 7.5B: Problem solving and reasoning

10 A rectangular picture frame measures 70 cm by 24 cm. The picture framer checks that the frame is 'square' (right-angled) by measuring the distance from the top left corner to the bottom right corner of the frame. The measured distance is 73 cm.

a Draw a diagram of the frame and label it with all the known information.

b Use Pythagoras' theorem to check whether the frame is 'square'. If it is not, should the diagonal distance be longer or shorter?

c Suggest a length for the diagonal distance.

11 Josh has an extension ladder that is 9.8 m long. He needs to clean the windows on the outside of a building. The lower part of the windows are 9.8 m from the base of the building, and the windows are 1.2 m tall. Unfortunately, bushes prevent Josh from placing his ladder any closer to the base of the building than 3 m. Josh is 1.7 m tall.

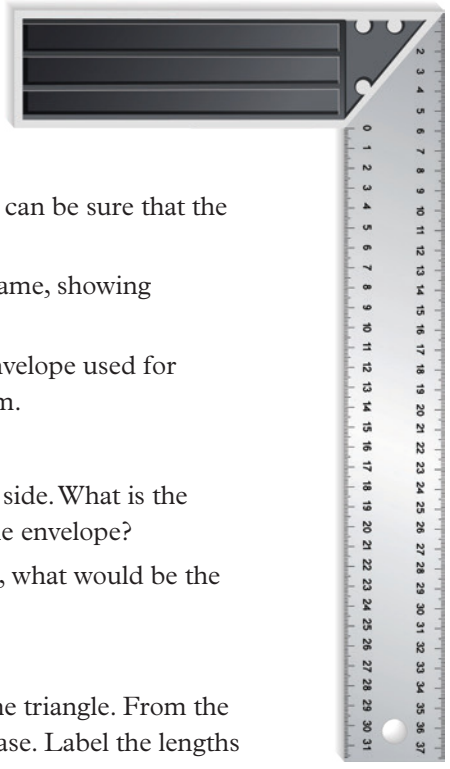
a Draw a diagram to show the situation.

b Calculate the height the ladder reaches up the building correct to one decimal place.

c Explain whether Josh will be able to clean the windows.

12 A cubby house is built on two levels in a tree. The lower level is 10 m above the ground, and the upper level is 5 m directly above the first level. The ladder from the ground to the lower level is 12 m long. Sam wants to construct another ladder to go from the ground, 1 m further along the ground from the base of the tree than the first ladder, to the top level. How long should he make this ladder?

13 A squaring tool is used when making picture frames, to ensure that the corners of the frames are true right angles.



Imagine you are constructing a square picture frame with side lengths of 54 cm.

a One of the diagonals of the frame measures 78 cm. Explain how you can be sure that the frame is not truly square.

b The other diagonal measures 76 cm. Draw a labelled sketch of the frame, showing its true shape.

14 A sheet of writing paper is 30 cm long and has a width of 21 cm. The envelope used for this writing paper has a length of 22 cm and a diagonal length of 24.6 cm.

a What is the width of the envelope?

b The writing paper is folded into equal-sized sections along its longer side. What is the minimum number of folds necessary to ensure the paper will fit in the envelope?

c If the paper was folded along its shorter side as well as its longer side, what would be the minimum number of folds needed for it to fit the envelope?

15 An equilateral triangle has side lengths of 8 cm.

a Draw a sketch of the triangle with two vertices forming the base of the triangle. From the top vertex, draw a line that is perpendicular (at right angles) to the base. Label the lengths of all sides of the figure.

b Use Pythagoras' theorem to calculate the height of the triangle. Give your answer as:

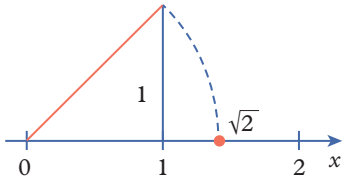
i an exact value

ii an approximate value, correct to one decimal place.

c Derive the formula for the area of any equilateral triangle with side lengths of x.

16 Builders in ancient Egypt used a piece of rope with 12 equally spaced knots as a tool. How did this show that a corner was 'square'? Explain using a diagram.

17 Pythagoras' theorem can be used to locate the position of surds on a number line. For example, to locate the position of $\sqrt{2}$, draw a perpendicular line to the number line at 1, with a height of 1 unit. This creates an isosceles right-angled triangle with a hypotenuse of length $\sqrt{2}$. Use a compass with one end placed at 0 on the number line and the other end placed at the end of the hypotenuse, and draw an arc to the number line to locate the position of $\sqrt{2}$.

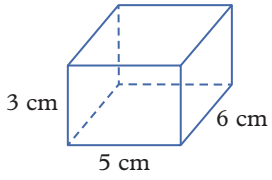


Following this technique, locate the positions of the following surds on a number line.

a $\sqrt{5}$ **b** $\sqrt{10}$

Exercise 7.5C: Challenge

18 Pythagoras' theorem can also be used to find unknown lengths within 3D objects. Consider the following rectangular prism.

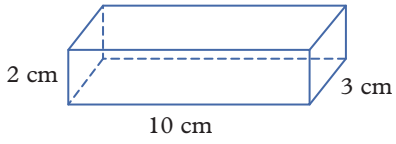


a Use Pythagoras' theorem to determine the length of the diagonal on the base of the prism. Give your answer as an exact value.

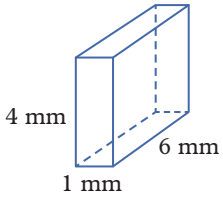
b Use your answer to part **a** and the height of the prism to determine the length of the diagonal within the prism correct to two decimal places.

19 Use the technique from question **18** to find the lengths of the diagonals within these rectangular prisms correct to two decimal places.

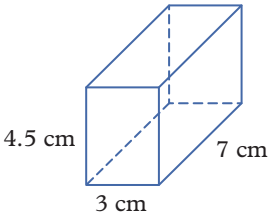
a



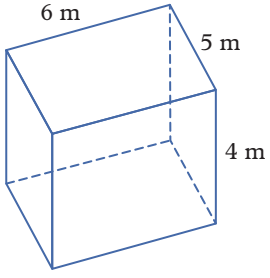
b



c



d



20 Use the technique from question **18** to write a formula which gives the length of the diagonal within a rectangular prism with side lengths of a, b and c.

Online resources:

Quick quiz
7.5

Lesson 7.6

Review: Angles, triangles and Pythagoras' theorem

Module summary

Types of angles

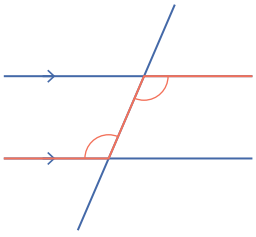
- Acute: between 0° and 90°
- Right: 90°
- Obtuse: between 90° and 180°
- Straight: 180°
- Reflex: between 180° and 360°
- Revolution: 360°

Angle properties

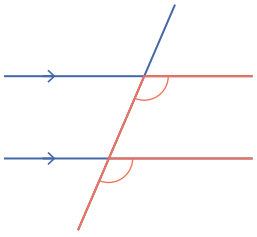
- Angles on a straight line: add to 180°
- Angles at a point: add to 360°
- Complementary angles: add to 90°
- Supplementary angles: add to 180°
- Vertically opposite angles: equal in size

Angles and parallel lines

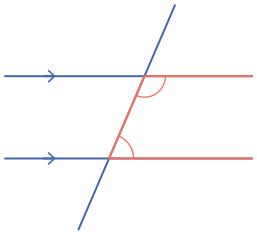
Alternate angles are equal.



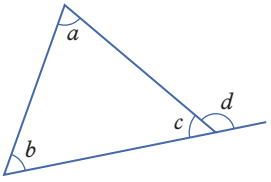
Corresponding angles are equal.



Co-interior angles are supplementary.

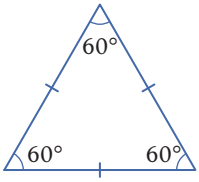


Exterior angle of a triangle

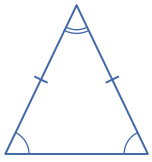


$a + b + c = 180°$
$c + d = 180°$
$d = a + b$

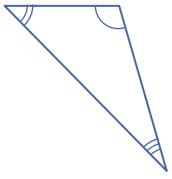
Classifying triangles by their sides



Equilateral



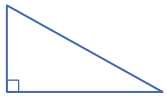
Isosceles



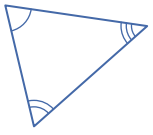
Scalene

Classifying triangles by their angles

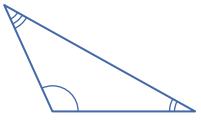
$a + b + c = 180°$



Right-angled

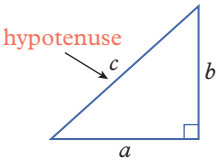


Acute-angled



Obtuse-angled

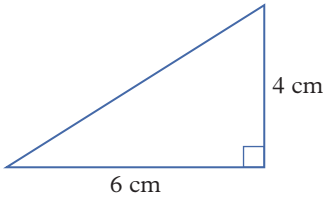
Pythagoras' theorem



$c^2 = a^2 + b^2$

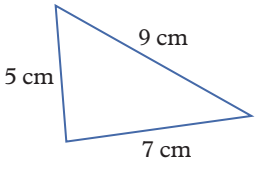
hypotenuse, shorter sides

Length of the hypotenuse



$c^2 = a^2 + b^2$
$= 6^2 + 4^2$
$= 36 + 16$
$= 52$
$c = \sqrt{52}$ cm
≈ 7.21 cm

Determining if a triangle is right-angled



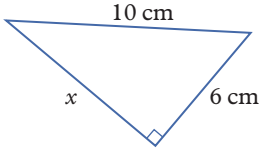
$c^2 = 9^2$
$= 81$

$a^2 + b^2 = 5^2 + 7^2$
$= 25 + 49$
$= 74$

$c^2 \neq a^2 + b^2$

The triangle is not a right-angled triangle.

Length of a shorter side



$c^2 = a^2 + b^2$
$10^2 = x^2 + 6^2$
$100 = x^2 + 36$
$64 = x^2$
$\sqrt{64} = x$
$x = 8$ cm

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Review questions 7.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- acute angle
- acute-angled triangle
- alternate angles
- co-interior angles
- complementary angles
- corresponding angles
- equilateral triangle
- exterior angle
- hypotenuse
- interior angle
- isosceles triangle
- obtuse angle
- obtuse-angled triangle
- parallel lines
- Pythagoras' theorem
- Pythagorean triple
- reflex angle
- revolution
- right angle
- right-angled triangle
- scalene triangle
- straight angle
- supplementary angles
- transversal
- vertically opposite angles

1 Which type of triangle has exactly two interior angles of equal size?

2 Which type of angles formed by a transversal are sometimes known as 'F angles'?

3 What angle type is between 90° and 180°?

4 Identify the key terms being referenced in each of these definitions.

a a straight line that cuts across two or more other lines

b a triangle with all three interior angles less than 90°

5 Using an example, provide a definition in your own words for the following key terms.

a Pythagorean triple

b complementary angles

6 Complete the following sentences using words from the key terms list.

a The _____ is the longest side of a right-angled triangle.

b An _____ is the angle between the side of a shape and a line extended from the adjacent side.

Review questions 7.6B: Multiple choice

7.1 **1** Which statement is *not* correct?

A Vertically opposite angles are equal in size.

B The sum of two obtuse angles will always be less than a revolution.

C Complementary angles add to 180°.

D Two supplementary angles make a straight angle.

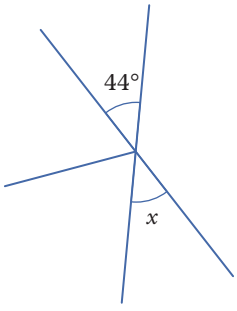
7.1 **2** What is the size of angle x?

A 136°

B 22°

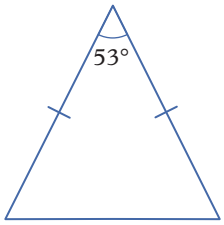
C 88°

D 44°



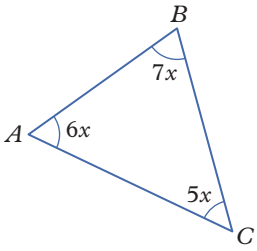
7.2 **3** What is the classification of this triangle?

A acute-angled equilateral triangle
B acute-angled isosceles triangle
C obtuse-angled isosceles triangle
D obtuse-angled scalene triangle



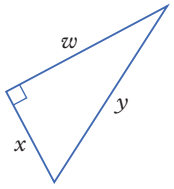
7.2 **4** What size is angle $\angle BAC$?

A 6°
B 10°
C 18°
D 60°



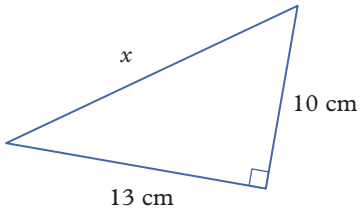
7.3 **5** Using Pythagoras' theorem, which statement shows the relationship between the side lengths of this triangle?

A $w^2 = x^2 + y^2$
B $x^2 = w^2 + y^2$
C $y^2 = x^2 + w^2$
D $y^2 = x^2 - w^2$



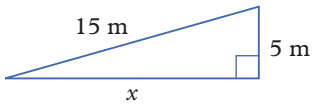
7.3 **6** What is the value of the unknown side length, x, correct to one decimal place?

A 4.8 cm
B 8.3 cm
C 16.0 cm
D 16.4 cm



7.4 **7** The unknown side length, x, in this triangle, correct to the nearest m, is:

A 3 m
B 10 m
C 14 m
D 20 m



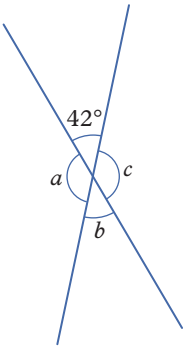
7.5 **8** Hitesh puts a 3-metre ladder against a wall. If the bottom of the ladder is 80 cm away from the base of the wall, how high up the wall is the top of the ladder?

A 2.89 m **B** 2.20 cm **C** 1.95 m **D** 3.10 m

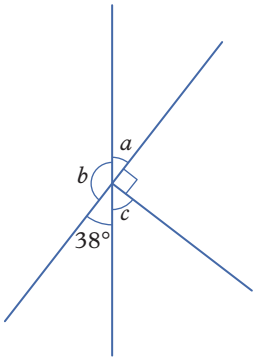
Review questions 7.6C: Short answer

7.1 **1** Without using a protractor, find the size of each unknown angle.

a

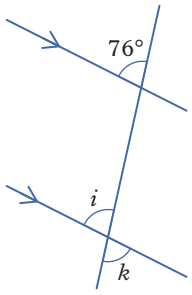


b



7.1 **2** Find the size of the unknown pronumerals.

a



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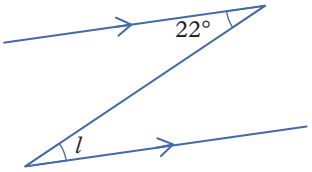
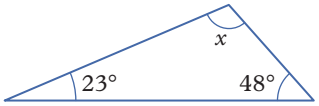


7.2 **3** Find the size of angles x and y.

a



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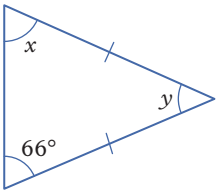
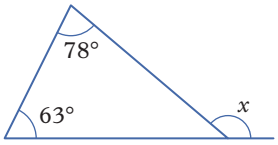
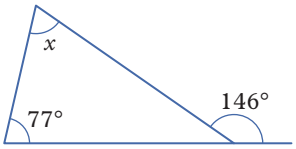


c



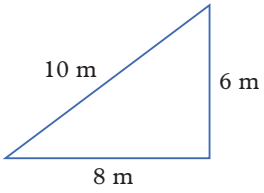
d



7.2 **4** For each triangle shown in question **3**, identify whether it is isosceles or scalene, and whether it is acute-angled or obtuse-angled.

7.3 **5** Use Pythagoras' theorem to decide whether these are right-angled triangles.

a



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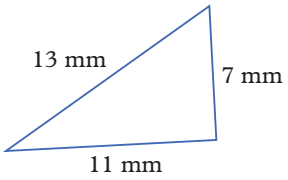
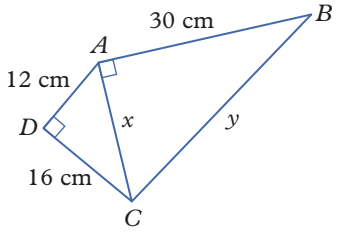
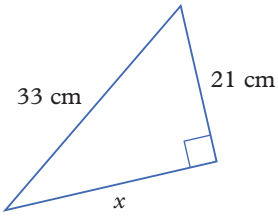


7.3 **6** Find the length of the sides represented by x and y in this diagram, correct to two decimal places.

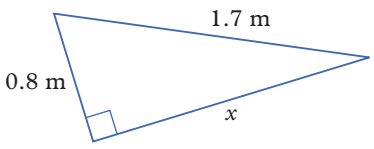


7.4 **7** Calculate the side length x in each triangle, correct to two decimal places.

a



b



7.5 **8** The size of a television screen is usually given in inches (″), where the given distance is the diagonal of the screen. Answer the following questions correct to one decimal place.

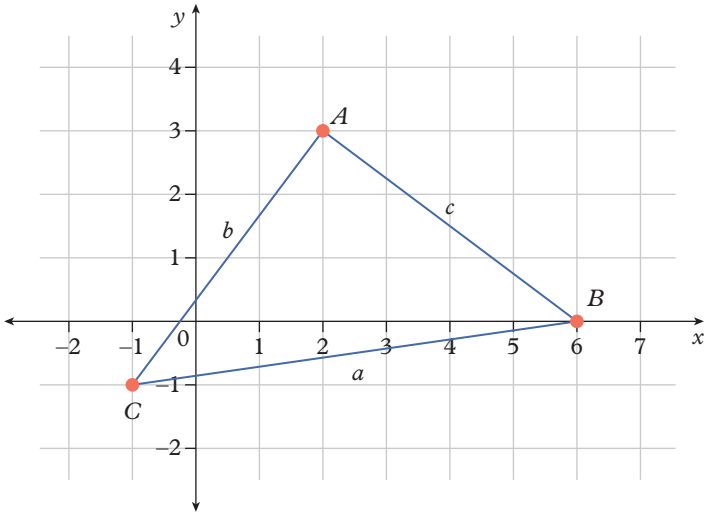
a What is the height of a 50″ television screen with a width of 43.6″?

b What is the width of a 55″ television screen with a height of 27″?

c What is the size of a television screen with a width of 65.4″ and a height of 36.8″?

Review questions 7.6D: Mathematical modelling

Consider the triangle shown on this Cartesian plane.



a State the coordinates of points A, B and C.

b Calculate the length of the line segments a, b and c, giving your answers as exact values.

c Given that $\angle BAC$ is a right angle, classify the triangle according to its sides and angles.

d In an isosceles triangle, if the unequal side is the base, the height of the triangle is the distance from the centre of the base to the opposite vertex.

i Determine the height of the triangle when BC is considered the base of the triangle.

ii Point A is moved 3 units up and 2 units left. What are the new coordinates of point A?

iii Classify the new triangle according to its sides and angles.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Calculate and use complementary and supplementary angles ☐ Calculate and use angles at a point ☐ Calculate and use angles formed by a transversal	☐ Go back to Lesson 7.1 Angles
☐ Classify triangles ☐ Calculate unknown interior and exterior angles of triangles	☐ Go back to Lesson 7.2 Triangles
☐ Use Pythagoras' theorem to determine whether a triangle is right-angled ☐ Use Pythagoras' theorem to find the length of the hypotenuse in a right-angled triangle	☐ Go back to Lesson 7.3 Pythagoras' theorem
☐ Use Pythagoras' theorem to find the length of a shorter side in a right-angled triangle	☐ Go back to Lesson 7.4 Finding the length of a shorter side
☐ Use Pythagoras' theorem to solve a range of different problems	☐ Go back to Lesson 7.5 Applications of Pythagoras' theorem

8

Units of measurement

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Substitution
- ✔ Multiplying and dividing by powers of 10
- ✔ The Cartesian plane
- ✔ Understanding time

Curriculum links

- Solve problems involving the area and perimeter of irregular and composite shapes using appropriate units (VC2M8M01)
- Solve problems involving the volume and capacity of right prisms using appropriate units (VC2M8M02)
- Solve problems involving the circumference and area of a circle using formulas and appropriate units (VC2M8M03)
- Solve problems involving time and duration, including using 12- and 24-hour time across multiple time zones (VC2M8M04)
- Describe in different ways the position and location of three-dimensional objects in 3 dimensions, including using a three-dimensional Cartesian coordinate system with the use of dynamic geometry software or other digital tools (VC2M8SP03)

Materials

- ✔ Calculator

Lesson 8.1

Length and perimeter

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the perimeter of two-dimensional shapes.

Inter-year links

Support Length and perimeter

Year 7 9.1 Length and perimeter

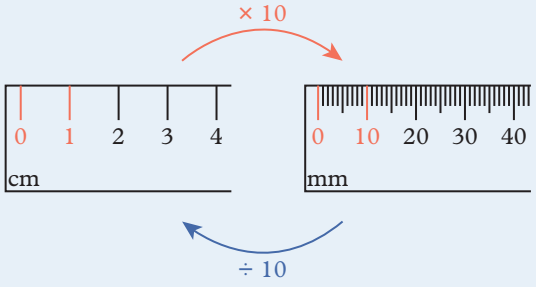
Units of length

Key content video
Length and perimeter

- To specify a **length**, you need a numerical value as well as a unit of length (for example, 5 km).

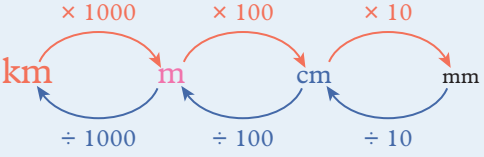
Metric units of length	
Name	**Abbreviation**
Millimetre	mm
Centimetre	cm
Metre	m
Kilometre	km

- To convert between units of length, multiply or divide by the appropriate **conversion factor**.
  For example, a length of 1 cm is the same as 10 mm; therefore, the conversion factor when converting from cm to mm is 10.



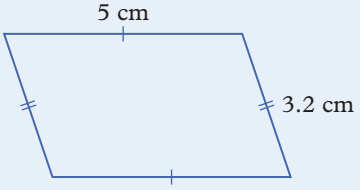
Conversion factors

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m



Perimeter

- The **perimeter** of a shape is the length of the shape's boundary.
- To calculate the perimeter, add the lengths of each side together.
  For example, the perimeter of the parallelogram is 5 cm + 3.2 cm + 5 cm + 3.2 cm = 16.4 cm.
  → All measurements must be in the same unit before being added together.
  → Sides of equal length are marked using hatch marks.



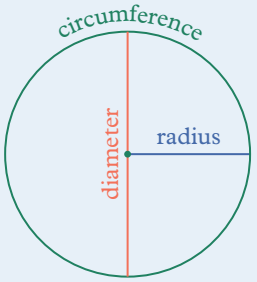
Circumference

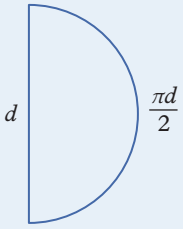
Key content video
Circumference of a circle

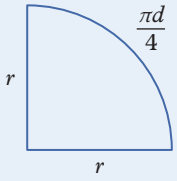
- The perimeter of a circle is called its **circumference**; this is found by multiplying the diameter by π. The irrational number π (**pi**) is a non-recurring decimal approximately equal to 3.14.
  - → The **diameter** (d) is the length of any line segment going from one point on the circumference to another point on the circumference, passing through the centre of the circle.
  - → The **radius** (r) is the length of any line segment going from the centre of the circle to a point on the circumference. The radius is half the length of the diameter.
- The length of the curved section of a semicircle is half of the circumference of a circle with the same diameter.
  - → The perimeter of a semicircle also includes the straight edge, which is the length of the diameter.
- The length of the curved section of a quarter circle is a quarter of the circumference of a circle with the same diameter.
  - → The perimeter of a quarter circle also includes two straight edges, both of which are the length of the radius.

$C = \pi d$

$C = 2\pi r$







Worked example 8.1A Converting between units of length

Convert 1.23 m into millimetres.

THINK

1 Convert from metres to centimetres.
To convert from a larger unit to a smaller unit, multiply by the conversion factor.

2 Convert from centimetres to millimetres.

WRITE

1 m = 100 cm

1.23 m = 1.23 × 100 cm
= 123 cm

1 cm = 10 mm

123 cm = 123 × 10 mm
= 1230 mm

Worked example 8.1B Calculate the circumference of a circle

Calculate the circumference of a circle with diameter 8 m, correct to two decimal places.

THINK

1 Identify which formula to use.

2 Substitute the measurement for the diameter.

3 Calculate using π on your calculator.

4 Round to two decimal places and include the appropriate unit.

WRITE

$C = \pi d$

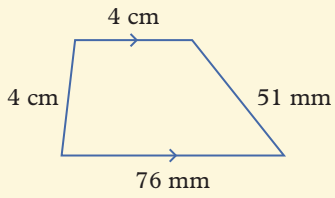
$= \pi \times 8$

$= 25.132\,741\,2 \ldots$

≈ 25.13 m

Worked example 8.1C Calculating perimeter

Calculate the perimeter of the following shape in centimetres.



THINK

1 Convert all measurements into the unit specified by the question.

2 Write out the sum of the side lengths. There are four sides so there should be four lengths in the sum.

3 Add all the lengths. Include the appropriate unit in the answer.

WRITE

51 mm = 51 ÷ 10 cm
= 5.1 cm

76 mm = 76 ÷ 10 cm
= 7.6 cm

perimeter = 4 cm + 51 mm + 76 mm + 4 cm
= 4 cm + 5.1 cm + 7.6 cm + 4 cm

= 20.7 cm

Helpful hints

- ✔ Once the units are the same, you can leave them out of calculations, but you must remember to include them in your final answer!
- ✔ Always estimate your answer first. If your calculated answer is significantly different than your estimate, check your working to ensure mistakes haven't been made.
- ✔ When converting from a larger unit to a smaller unit (e.g. m to cm) you multiply, and when converting from a smaller unit to a larger unit (e.g. m to km) you divide.
- ✔ Unless instructed otherwise, use π on your calculator instead of an approximation such as 3.14. You should only round your answers at the final step of the calculations.

Learning pathways

1, 2, 3–5(a–e), 6, 8(a–e), 9–10(1st column), 13, 15, 18, 20, 22	1(b, c, e, g), 2(b, d, f, h), 3–5(b, d, f), 8(b, d, f), 9–10(c–e), 12, 16, 17, 19, 21, 24, 25	1–2(d, f, h), 3–5(d–f), 7, 8(d–f), 9–10(e–f), 14, 16, 19, 23, 25–27

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Exercise 8.1A: Understanding and fluency

WE 8.1A **1** Convert these lengths to the units shown in brackets.

a 58 m (cm) **b** 2.7 km (m) **c** 521 cm (m) **d** 398 m (km)

e 1.2 cm (mm) **f** 3987 m (km) **g** 423 mm (cm) **h** 7 mm (cm)

2 Convert these lengths to the units shown in brackets.

a 5.34 m (mm) **b** 3.95 km (cm) **c** 235 000 mm (m) **d** 145 000 000 mm (km)

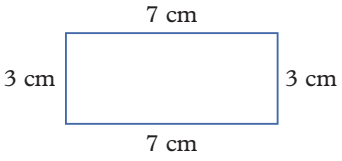
e 2368 cm (km) **f** 287 mm (m) **g** 1.934 km (cm) **h** 0.91 km (mm)

WE 8.1B **3** Find the circumference, correct to two decimal places, of a circle with:

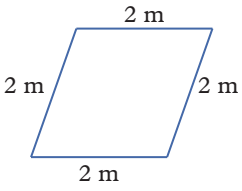
a diameter 1.2 m
b radius 2.5 cm
c diameter 46 mm
d radius 0.8 cm
e radius 91 cm
f radius 1.3 mm
g diameter 1.75 m
h diameter 2.6 cm
i diameter 82 km.

4 Calculate the perimeter of these quadrilaterals.

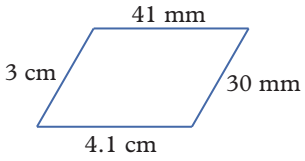
a



b



c



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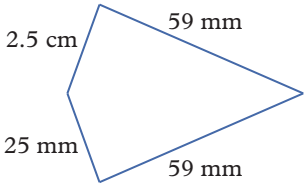

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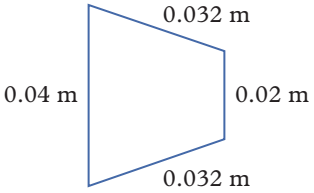

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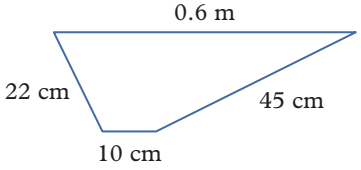
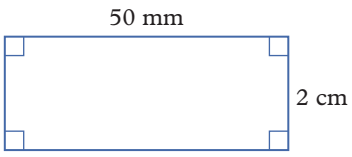
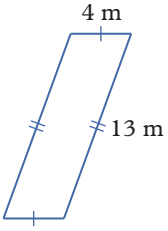


5 Calculate the perimeter of these quadrilaterals.

a



b



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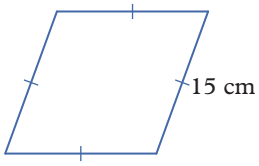

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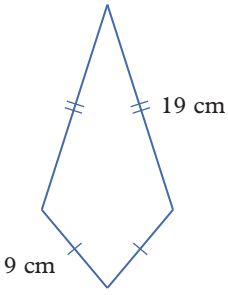
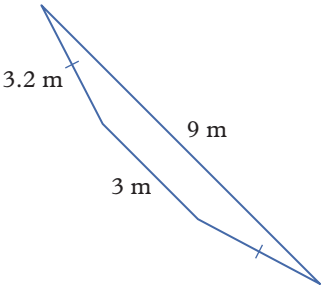


e



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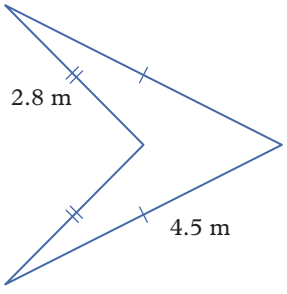
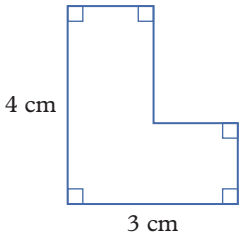


6 Calculate the perimeter of:

a a square with side length 4 cm
b a rhombus with base 15 mm
c a rectangle with length 10 mm and width 15 mm
d a parallelogram with side lengths 18 cm and 12 cm.

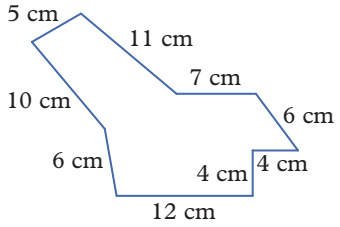
7 Calculate the perimeter of:

a an equilateral triangle with side length 2.5 cm
b a regular hexagon with side length 2.5 m
c an L-shaped hexagon with longer sides 3 cm and 4 cm
d a regular octagon (8-sided shape) with side length 7.7 cm.

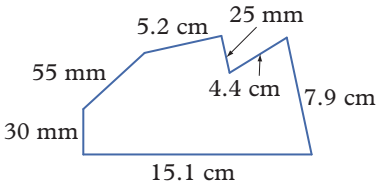


WE 8.1C **8** Calculate the perimeter of these shapes.

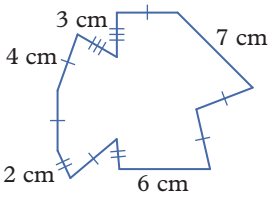
a



b



c



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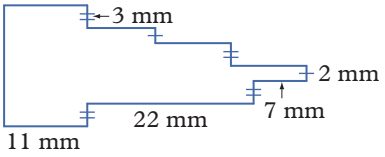

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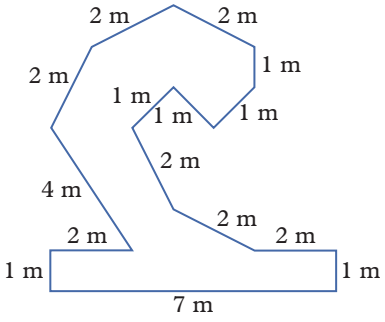

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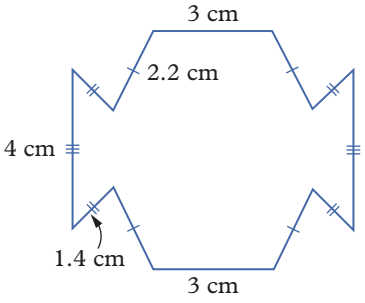
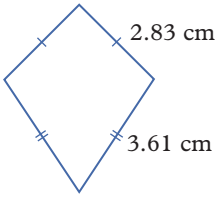


9 Calculate the perimeter of the following shapes correct to two decimal places.

a



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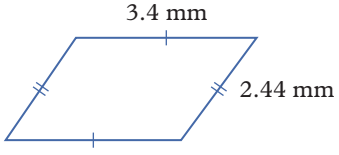

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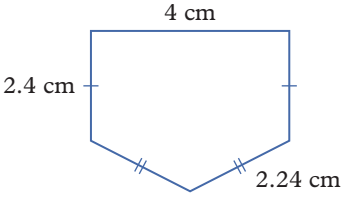
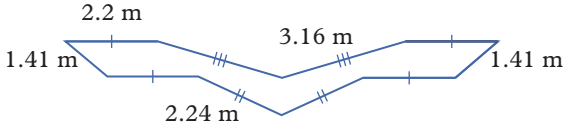
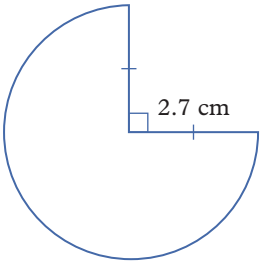


d



e



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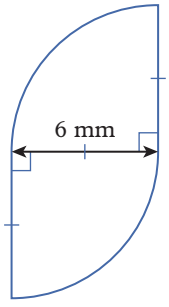
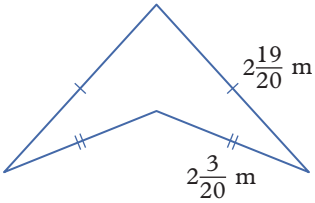
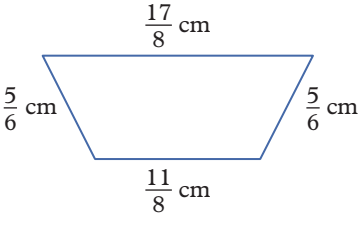


10 Calculate the perimeter of the following shapes. Leave your answers as exact values.

a



b



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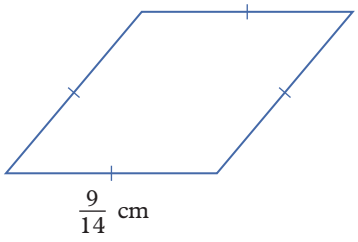

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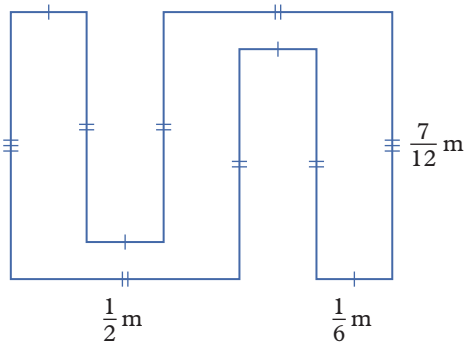

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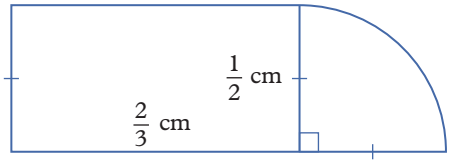

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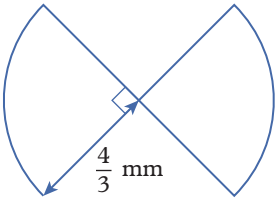
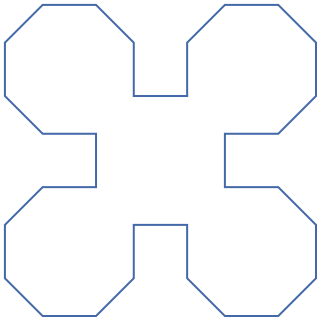


Exercise 8.1B: Problem solving and reasoning

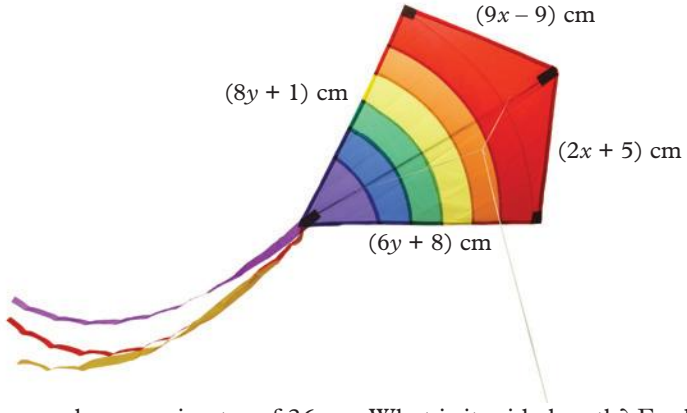
11 Trish makes earrings. One design is a rectangular jewel with wire wrapped around its perimeter twice. If the jewel is 4.5 cm long and 1.5 cm wide, how much wire (in metres) is used for 20 pairs of earrings?

12 A road has solid white lines painted on each side, and a dashed line down the middle. If every 10 m section of the road has eight 650 mm long dashes down its centre, calculate the total length of painted line required (in kilometres) for a 10 km road.

13 The perimeter of each of the congruent regular octagons that make up this shape is 48 cm. Determine the perimeter of this shape, justifying your answer.



14 Determine the values of x and y and hence calculate the perimeter, P, of this kite.



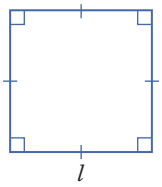
15 A square has a perimeter of 36 cm. What is its side length? Explain your answer.

16 A new television screen has a perimeter of 6.1 m. If it has a length of 1.9 m, find its width.

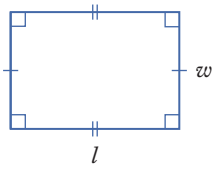
17 A circular flower bed has a circumference of 4.71 m. What is the diameter correct to two decimal places?

18 Write an expression for the perimeter of the following quadrilaterals.

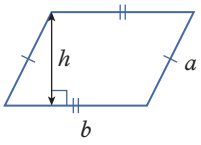
a



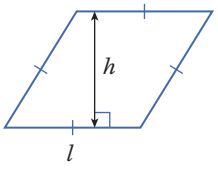
b



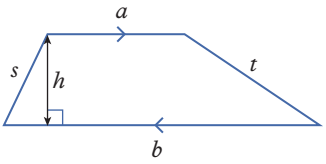
c



d



e



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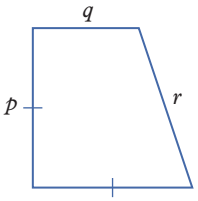
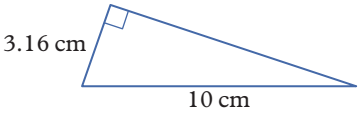
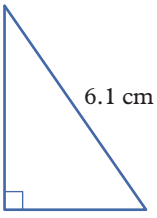


19 Use Pythagoras' theorem (covered in Module 7) to help calculate the perimeter of these right-angled triangles. Give your answers correct to two decimal places.

a



b



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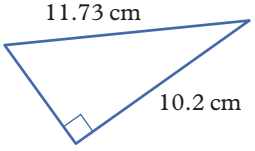
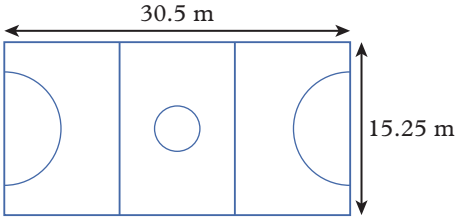


20 To warm up before a netball match, Jordan has the option of either running six laps around the netball court or doing four shuttle runs.

a How far is one lap around the netball court?

b How far is six laps around the netball court?

c A shuttle run involves starting at the left of the court and running hard to the first third-line then back to the start, running to the second third-line and back to the start, and finally running up to the end of the court and back.



i How far is the first part of the shuttle run (to the first third-line and back)? Give your answer correct to two decimal places.

ii What distance does a single shuttle run cover?

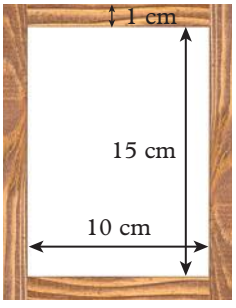
iii How far is four shuttle runs? Explain how you calculated the answer.

iv Which warm-up option is shorter?

21 Alice wants to make some photo frames for a friend. She buys some wood to fit around the outside of a standard-sized photo.

a What is the perimeter of the photo that fits in this frame?

b Alice wants the frame to be 1 cm wide, as shown in the image.



i Why is the length of wood needed greater than the perimeter of the photo?

Hint: Are the corner parts of the frame included in the photo perimeter?

ii What length of wood is needed?

iii What is the perimeter around the photo frame? Why is this different from part **c**?

c Investigate and explain how the results to parts **b i** and **b ii** would change if:

i the frame was 2 cm wide

ii the frame was 3 cm wide

iii the frame was for a 15 cm × 20 cm photo

iv the frame was for a 20 cm × 25 cm photo.

22 Alice packs the photo frames from question **21** into a gift box with the dimensions shown in the photo and wants to tie a ribbon around the present. Assuming that she will need a minimum of 60 cm of ribbon for the bow, at least how much ribbon will she need to buy in order to wrap the present as shown?

23 A rectangle has a perimeter of 30 m. List all the possible whole number dimensions for the rectangle.

24 A rectangular pool is 12.5 m long and 4.7 m wide.

a What length of paving is required around the pool's edge if the pavers used are 50 cm wide?

b Jordan incorrectly measured the length of the pool as 13.4 m and the width as 5.2 m. What is the size of the error in each measurement?

c The percentage error of a measurement can be found by calculating the size of the error as a percentage of the actual measurement. Find the percentage error for Jordan's measurements giving your answers correct to one decimal place.

d Which measurement has a greater:

i error **ii** percentage error?

e Calculate the length of paving required if Jordan's measurements are used.

f Find the percentage error in the length of paving if using Jordan's measurements. Give your answer correct to one decimal place.

25 Jackson takes on a scrapbooking project. He decides to cover the internal perimeter of each page in ribbon that is 2 cm wide. If there are 56 square pages that measure 25 cm in the book, find the length of ribbon (in metres) required to place ribbon inside the perimeter of every page. Justify your answer.

Exercise 8.1C: Challenge

26 A rectangular vegetable garden currently has a perimeter of 28 m. A fence is to be placed 1 m from the edge, around the vegetable garden, to allow space for more vegetables.

a Write down a possible set of whole number dimensions for the vegetable garden.

b What length of fencing is required?

c What length of fencing would be required if the distance between the fencing and the edge of the garden was:

i 2 m **ii** 3 m?

d Can you see a pattern or rule that you could follow in order to find the length of fencing required, no matter how close or far from the edge it is placed?

e Repeat parts **a**–**d** using a different set of dimensions. Do your findings match the rule you found in part **d**? Explain your answer.

27 An inch is 2.54 cm.

a A foot is equal to 12 inches. What is the conversion factor from feet to centimetres?

b A yard is equal to 3 feet. What is the conversion factor from yards to metres?

c One mile is equal to 1760 yards. What is the conversion factor from miles to kilometres?

d The official distance for a marathon is 26 miles and 385 yards. What is this distance in kilometres?

Online resources:

Interactive skillsheet Converting between units of length

Interactive skillsheet Perimeter

Interactive skillsheet Circumference

Worksheet Length conversions

Investigation In the extreme

Quick quiz 8.1

Lesson 8.2

Area of rectangles and triangles

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the area of rectangles and triangles.

Inter-year links

Support	Understanding area
Year 7	9.3 Area of a rectangle
Year 7	9.5 Area of a triangle
Year 9	5.2 Area of composite shapes
Year 10	8.1 Area review

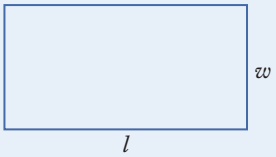
Area

Key content video
Area of a triangle

Key content video
Area of a rectangle

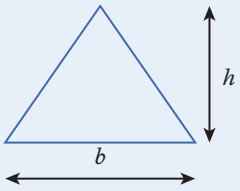
- The **area** of a 2D shape is the space occupied by that shape.
- To find the area of a **rectangle**, multiply its length (l) by its width (w).
- To find the area of a triangle, multiply its base (b) by its height (h) then divide by 2.
  → The **base** length and height must be perpendicular (at a right angle) to each other.

Rectangle



$A = lw$

Triangle



$A = \frac{1}{2}bh$

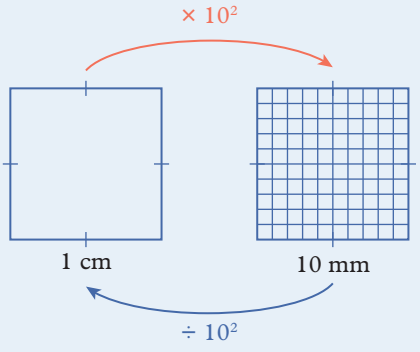
- Measurements must be in the same unit before they are substituted into area formulas.

Metric units of area	
Name	**Abbreviation**
Square millimetre	mm^2
Square centimetre	cm^2
Square metre	m^2
Square kilometre	km^2

Converting between units of area

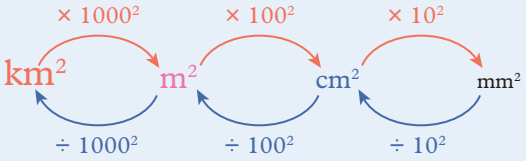
Key content video
Converting between units of area

- To convert between square units of area, multiply or divide by the square of the conversion factor between the lengths.
  For example, a square centimetre (1 cm × 1 cm = 1 cm^2) is equivalent to 100 square millimetres (10 mm × 10 mm = 10^2 mm^2 = 100 mm^2). That means we can multiply the number of cm^2 by 10^2 to determine the number of mm^2.
  Likewise we can divide the number of mm^2 by 10^2 to determine the number of cm^2, since each mm^2 is $\frac{1}{100}$ th of a cm^2.



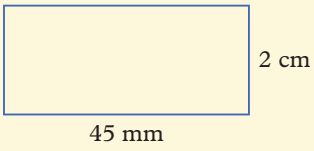
Conversion factors

$1\ cm^2 = 10^2\ mm^2$
$= 100\ mm^2$
$1\ m^2 = 100^2\ cm^2$
$= 10\ 000\ cm^2$
$1\ km^2 = 1000^2\ m^2$
$= 1\ 000\ 000\ m^2$



Worked example 8.2A Calculating the area of a rectangle

Calculate the area of this rectangle in cm^2.



THINK

1. Identify the measurements for l and w and convert the length into the unit specified by the question.
2. Substitute the values for l and w into the formula for the area of a rectangle.
3. Calculate the area of the rectangle. Remember to include the appropriate unit in the answer.

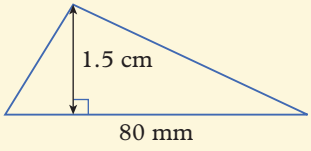
WRITE

$l = 45\ mm$
$= 4.5\ cm$
$w = 2\ cm$

$A = lw$
$= 4.5 \times 2$
$= 9\ cm^2$

Worked example 8.2B Calculating the area of a triangle

Calculate the area of this triangle in mm^2.



THINK

1. Identify the measurements for b and h and convert the height into the units specified by the question.
2. Substitute the values for b and h into the formula for the area of a triangle.
3. Calculate the area of the triangle. Remember to include the appropriate unit in the answer.

WRITE

$b = 80\ mm$
$h = 1.5\ cm$
$= 15\ mm$

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 80 \times 15$
$= 600\ mm^2$

Worked example 8.2C Converting to a smaller area unit

Convert 3.8 cm^2 into mm^2.

THINK	WRITE
1 Identify the appropriate conversion factor. To convert from a larger unit to a smaller unit, multiply by the conversion factor.	$1\ cm^2 = 10^2\ mm^2$ $= 100\ mm^2$
2 Complete the calculation.	$3.8\ cm^2 = 3.8 \times 100$ $= 380\ mm^2$

Worked example 8.2D Converting to a larger area unit

Convert 120 000 mm^2 into m^2.

THINK	WRITE
1 First convert from mm^2 to cm^2. To convert from a smaller unit to a larger unit, divide by the conversion factor.	$1\ cm^2 = 10^2\ mm^2$ $= 100\ mm^2$ $1\ m^2 = 100^2\ cm^2$ $= 10\,000\ cm^2$ $120\,000\ mm^2 = 120\,000 \div 100$ $= 1200\ cm^2$
2 Complete the calculation by converting from cm^2 to m^2.	$1200\ cm^2 = 1200 \div 10\,000$ $= 0.12\ m^2$

Helpful hints

- ✔ When multiplying by a power of ten, move the decimal point to the *right*.

  $5.23 \times 10 = 52.3$
  $5.23 \times 100 = 523.$
  $5.23 \times 1000 = 5230.$

- ✔ When dividing by a power of ten, move the decimal point to the *left*.

  $5.23 \div 10 = 0.523$
  $5.23 \div 100 = 0.0523$
  $5.23 \div 1000 = 0.005\,23$

- ✔ Remember that larger units take up more area for each square unit, so there will be less of them than for smaller units. Likewise, smaller units take up less area for each square unit, so there will be more of them than for larger units.

Learning pathways

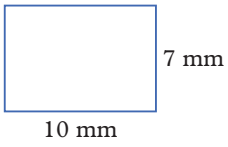
1, 2, 3(a–e), 4–7, 8–10, 11(a, b), 14, 18, 20	1, 2, 3(c–f), 4(b, c, f), 5–7, 10–12, 15, 19, 21, 22(a, b)	3(e–h), 4–7, 11–13, 16, 17, 21, 22

ANS p575

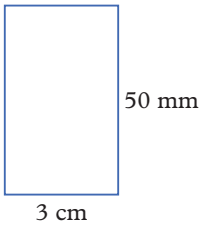
Exercise 8.2A: Understanding and fluency

WE 8.2A **1** Calculate the area of each rectangle. Where more than one unit is used, give your answer in terms of the smaller unit.

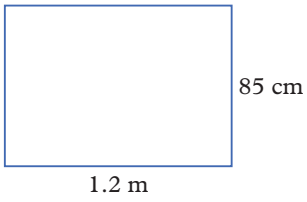
a



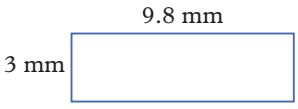
b



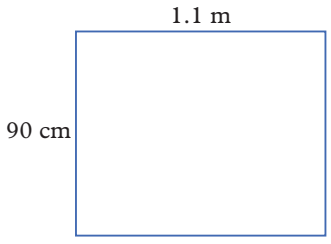
c



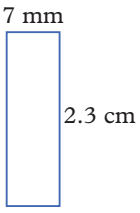
d



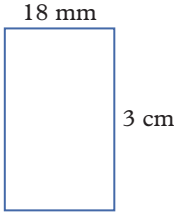
e



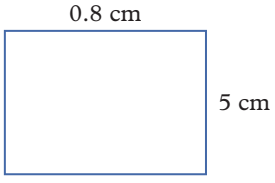
f



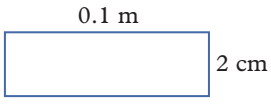
g



h

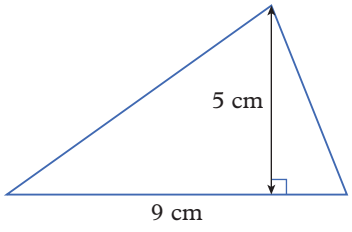


i

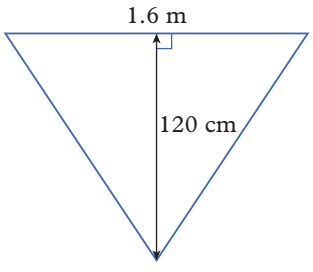


WE 8.2B **2** Calculate the area of each triangle. Where more than one unit is used, give your answer in terms of the larger unit.

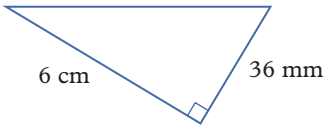
a



b



c



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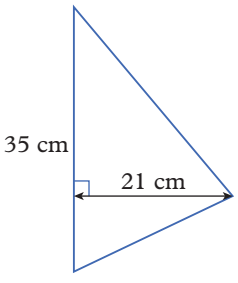
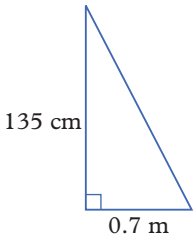
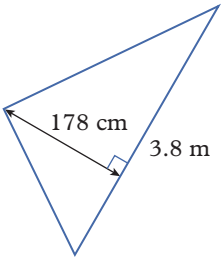


e



f



3 Calculate the area of each shape in the unit specified in brackets.

a a rectangle with length 35 mm, width 4 cm (mm^2)

b a rectangle with length 12 mm, width 17 cm (cm^2)

c a triangle with base 15 mm, height 2 cm (mm^2)

d a square with length 7.2 mm (mm^2)

e a triangle with base 25 cm, height 10 mm (cm^2)

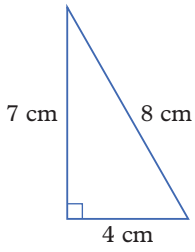
f a triangle with base 8 cm, height 42 mm (cm^2)

g a square with base 19 cm (cm^2)

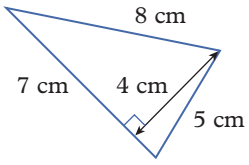
h a rectangle with length 4.2 cm, width 11 mm (mm^2)

4 Calculate the area of each shape.

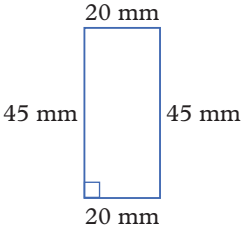
a



b



c



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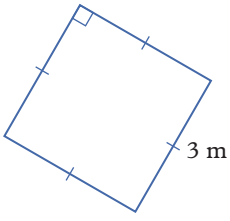

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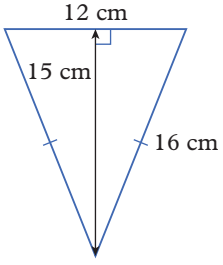

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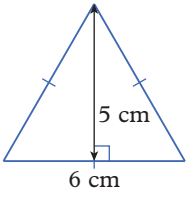


WE 8.2C **5** Convert each area measurement.

a 3 m^2 into cm^2 **b** 10 cm^2 into mm^2 **c** 6.5 m^2 into cm^2

d 19.7 cm^2 into mm^2 **e** 1.6 km^2 into m^2 **f** 0.75 km^2 into m^2

WE 8.2D **6** Convert each area measurement.

a 25 000 cm^2 into m^2 **b** 750 000 mm^2 into cm^2 **c** 400 mm^2 into cm^2

d 125 000 m^2 into km^2 **e** 675 cm^2 into m^2 **f** 500 000 000 m^2 into km^2

7 Convert each area measurement.

a 150 000 mm^2 into m^2 **b** 0.002 km^2 into cm^2

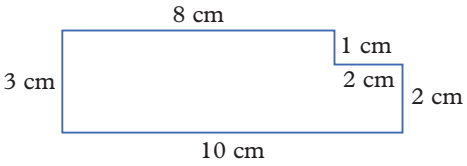
c 1.5 m^2 into mm^2 **d** 379 000 cm^2 into km^2

Exercise 8.2B: Problem solving and reasoning

8 Estimate how much material is needed to make the multi-coloured umbrella shown by approximating each coloured panel as a triangle.

9 A classmate attempted question **8** and after using 68.5 cm as the height of each triangle, obtained an answer of 8768 cm^2. Explain where they went wrong.

10 A composite shape is a shape that can be split up into two or more basic shapes. For example, the composite shape on the right can be split up into rectangles as shown in Figures A and B.



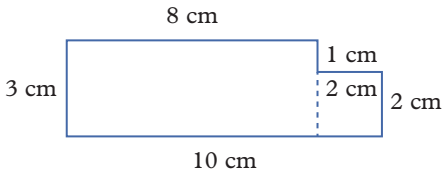


Figure A

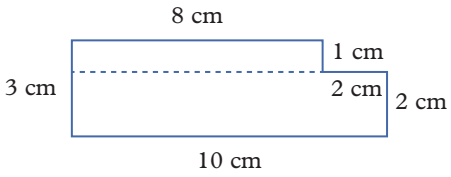


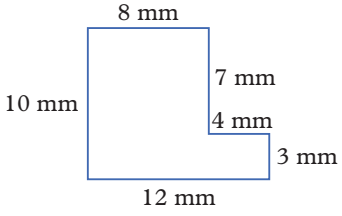
Figure B

a Find the area of each rectangle in Figure A and hence the area of the entire shape.

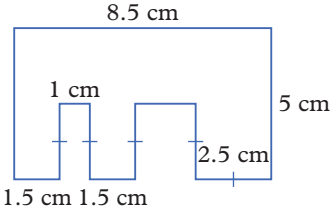
b Find the area of each rectangle in Figure B and hence the area of the entire shape.

11 Find the area of each of these composite shapes. Hint: For some parts it may be easier to subtract shapes.

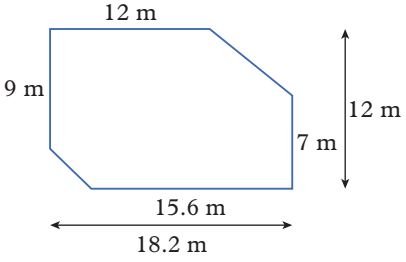
a



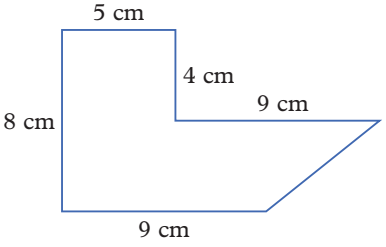
b



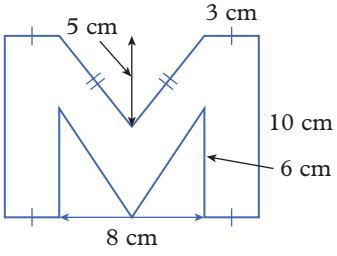
c



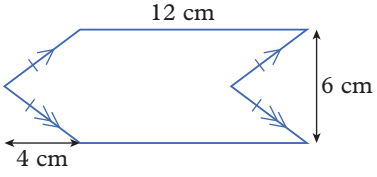
d



e

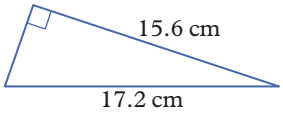


f

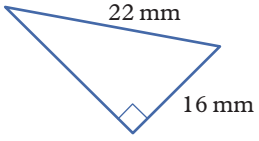


12 Use Pythagoras' theorem to calculate the area of these right-angled triangles. Give your answers correct to two decimal places.

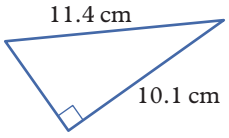
a



b



c



13 Which unit of area (mm^2, cm^2, m^2 or km^2) would be best for measuring the area of these? Justify your answers.

a basketball court **b** wristwatch face **c** national park

d five-dollar note **e** surface of a skateboard ramp **f** family-size pizza

14 Alex was completing a worksheet on conversions and provided the following answer to a question:

$40\,000\ cm^2 = (40\,000 \div 100)\ m^2$
$= 400\ m^2$

Explain where Alex made a mistake and provide the correct answer.

15 Ahmed needed to convert 11 cm^2 into mm^2. He said that because there are 10 mm in 1 cm, then there are 110 mm^2 in 11 cm^2. Explain where he went wrong and provide the correct answer.

16 A hectare is 10 000 m^2.

a How many hectares are in 1 km^2?

b There are many other units of area that are still used today that are not metric, such as the acre. A hectare is roughly equal to 2.5 acres. Approximately how many square metres are there in an acre?

17 Aaron harvests wheat on his farm and gets a yield of about 3.2 tonnes (3200 kg) per hectare.

a Write this in:

i kilograms per hectare

ii kilograms per square metre

iii grams per square metre

iv grams per square centimetre.

b Which unit do you think makes the most sense to use? Explain your answer.

c If Aaron is paid 35 cents per kilogram of wheat, find how much money he will earn if he harvests 3000 hectares of wheat.

18 A rectangle has an area of 20 cm^2 and a length of 4 cm. Find its width and explain why there is only one rectangle that you can draw that fits this description.

19 A rectangle has a perimeter of 28 cm. If the dimensions are whole numbers, find three possible areas this rectangle could have.

20 A square has an area of 64 mm^2. Sketch this square, stating its dimensions, its perimeter and why there is only one square that matches this description.

21 A wooden frame is to go around a rectangular mirror with an area of 3600 cm^2.

a If the mirror has a length of 90 cm, what is its width?

b Calculate the area of the wooden surface if the frame is to be 5 cm wide? Hint: Draw a picture to help you with your calculations.

Exercise 8.2C: Challenge

22 Mohammed is making a rectangular vegetable garden and has 24 m of fencing to put around the garden.

a List three possible sets of dimensions and corresponding areas for the garden.

b What is the maximum area that you can find using 24 m of fencing? What shape is this?

c Mohammed realises he can enclose a bigger area if he puts the garden against the shed wall and only has to fence three sides of the garden.

i List another three possible sets of dimensions and corresponding areas for the veggie garden if one side is against the shed wall.

ii What is the maximum area that you can find now, using 24 m of fencing for three sides? What shape is this? How does this relate to your answer to part **b**?

Online resources:

- Interactive skillsheet: Area of a rectangle
- Interactive skillsheet: Converting between units of area
- Investigation: Units of area and accuracy
- Quick quiz: 8.2
- Interactive skillsheet: Area of a triangle
- Worksheet: Area conversions

Lesson 8.3

Area of quadrilaterals

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the area of quadrilaterals.

Inter-year links

Support	Understanding area
Year 7	9.4 Area of a parallelogram
Year 9	5.2 Area of composite shapes
Year 10	8.1 Area review

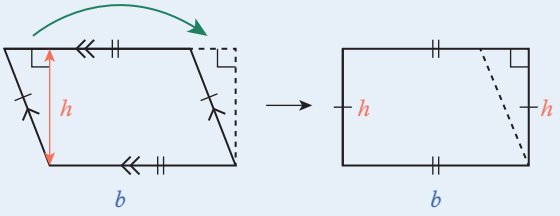
Area of quadrilaterals

Key content video
Area of quadrilaterals

- Every **parallelogram** can be rearranged into a rectangle with the same base length and height; therefore, the area of a parallelogram can be calculated using the formula $A = bh$, where b is the base length and h is the perpendicular height.

$$A = bh$$

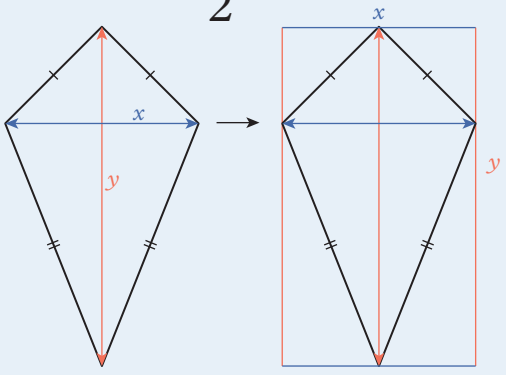
Parallelogram



- The area of a **kite** is exactly half the area of the rectangle that encloses it. Therefore, the area of a kite can be calculated using the formula $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the kite.

$$A = \frac{1}{2}xy$$

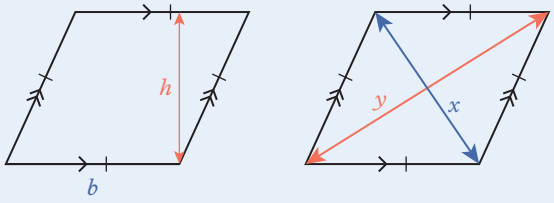
Kite



- As every **rhombus** is both a kite and a parallelogram, the area of a rhombus can be calculated using either the formula for the area of a parallelogram, $A = bh$, or the formula for the area of a kite, $A = \frac{1}{2}xy$, depending on the information provided.

$$A = bh \qquad A = \frac{1}{2}xy$$

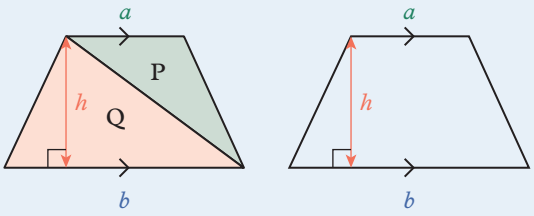
Rhombus



- Every **trapezium** can be split into two triangles, both with the same perpendicular height. The area of triangle P is $\frac{1}{2}ah$ and the area of triangle Q is $\frac{1}{2}bh$; therefore, the area of the trapezium is $A = \frac{1}{2}ah + \frac{1}{2}bh$, or more concisely, $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height.

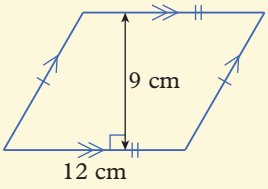
Trapezium

$$A = \frac{1}{2}(a + b)h$$



Worked example 8.3A Calculating the area of a parallelogram

Calculate the area of this parallelogram.



THINK

1. Identify the measurements for b and h.
2. Substitute the values for b and h into the formula for the area of a parallelogram.
3. Calculate the result. Remember to include the appropriate unit in the answer.

WRITE

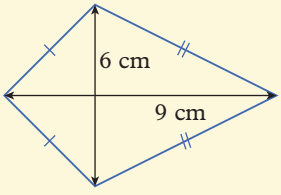
$A = bh$

$= 12 \times 9$

$= 108 \text{ cm}^2$

Worked example 8.3B Calculating the area of a kite

Calculate the area of this kite.



THINK

1. Identify the measurements for x and y.
2. Substitute the values for x and y into the formula for the area of a kite.
3. Calculate the result. Remember to include the appropriate unit in the answer.

WRITE

$x = 6$ cm, $y = 9$ cm

$A = \frac{1}{2}xy$

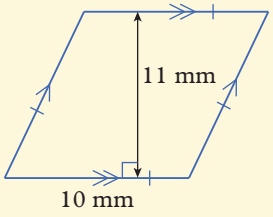
$= \frac{1}{2} \times 6 \times 9$

$= 27 \text{ cm}^2$

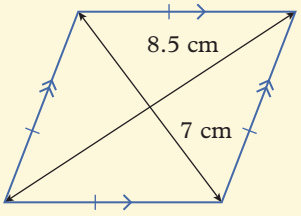
Worked example 8.3C Calculating the area of a rhombus

Calculate the area of these rhombuses.

a



b



THINK

a
1. Determine which formula to use. b and h are given, so use the formula for the area of a parallelogram.
2. Identify the given measurements.
3. Substitute the values for b and h into the formula for the area of a parallelogram.
4. Calculate the result and include the appropriate unit.

b
1. Determine which formula to use. x and y are given, so use the formula for the area of a kite.
2. Identify the given measurements.
3. Substitute the values for x and y into the formula for the area of a kite.
4. Calculate the result and include the appropriate unit.

WRITE

a $b = 10$ mm, $h = 11$ mm

$A = bh$

$b = 10$ mm, $h = 11$ mm

$A = 10 \times 11$

$= 110$ mm^2

b $A = \frac{1}{2}xy$

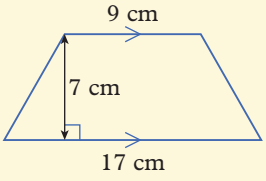
$x = 7$ cm, $y = 8.5$ cm

$A = \frac{1}{2} \times 7 \times 8.5$

$= 29.75$ cm^2

Worked example 8.3D Calculating the area of a trapezium

Calculate the area of this trapezium.



THINK

1. Identify the given measurements.
2. Substitute the values for a, b and h into the formula for the area of a trapezium.
3. Calculate the result and include the appropriate unit.

WRITE

$a = 9$ cm, $b = 17$ cm, $h = 7$ cm

$A = \frac{1}{2}(a + b)h$

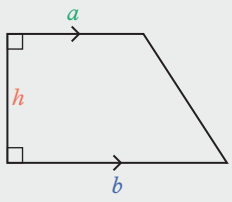
$= \frac{1}{2} \times (9 + 17) \times 7$

$= \frac{1}{2} \times 26 \times 7$

$= 91$ cm^2

Helpful hints

- ✔ All area formulas must include the multiplication of two lengths to create an area.
- ✔ In the formulas for the area of a parallelogram and trapezium, the height must be the perpendicular distance between parallel sides.
- ✔ In a right-angled trapezium, the height will be the length of the side joining the right angles.
- ✔ In the formula for the area of a kite, it doesn't matter which diagonal length you label x or y, since $xy = yx$! Likewise, in the formula for the area of a trapezium, it doesn't matter which of the parallel sides you label a or b, since $a + b = b + a$.



Learning pathways

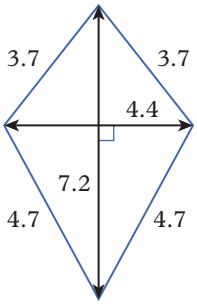
1–8, 10, 11, 13, 14, 17, 18 | 2–5(b, c), 6–8, 11–13, 15, 16, 19, 20 | 6–8, 12, 13, 16, 19–21

ANS p576

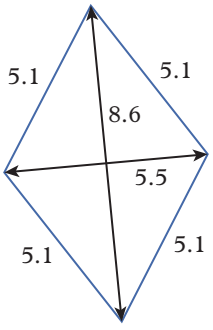
Exercise 8.3A: Understanding and fluency

1 Which of the lengths in the following shapes are needed to determine the area?

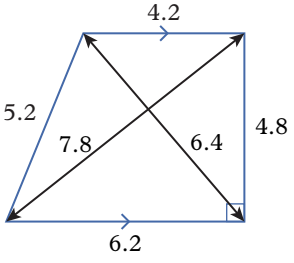
a



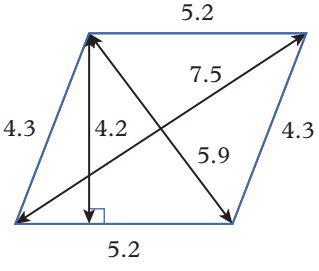
b



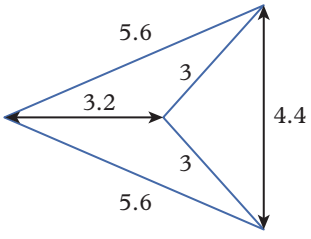
c



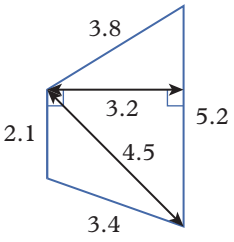
d



e

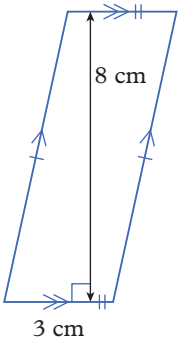


f

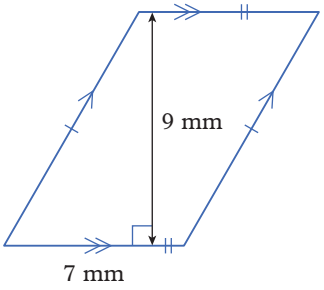


WE 8.3A **2** Calculate the area of each parallelogram.

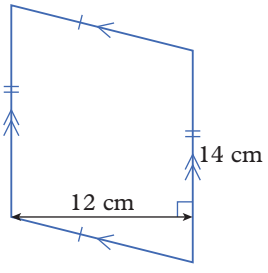
a



b

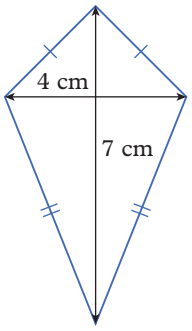


c

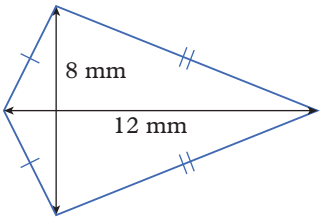


WE 8.3B **3** Calculate the area of each kite.

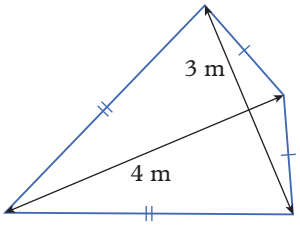
a



b

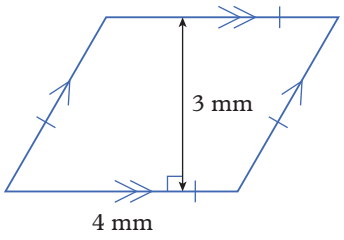


c

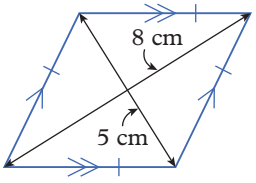


WE 8.3C **4** Calculate the area of each rhombus.

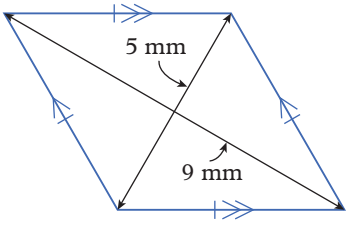
a



b

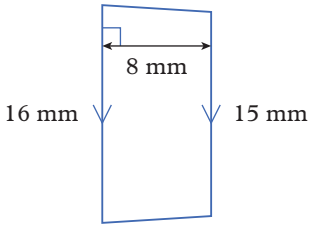


c

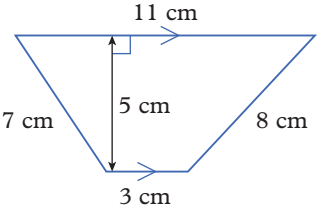


WE 8.3D **5** Calculate the area of each trapezium.

a



b



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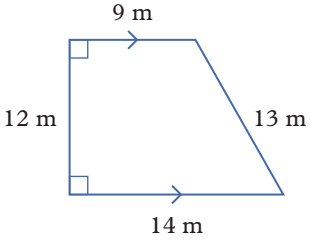


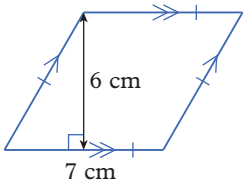
6 For each shape:

i identify if it is a parallelogram, kite, rhombus or trapezium

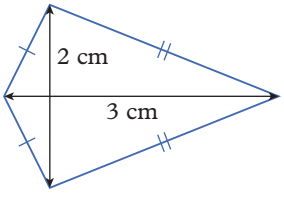
ii write the formula to find its area

iii calculate its area.

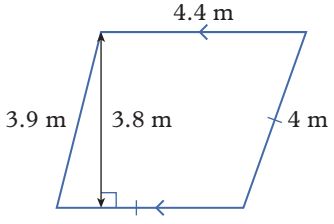
a



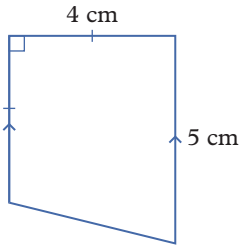
b



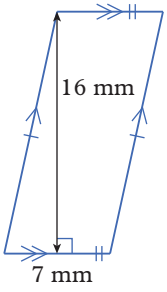
c



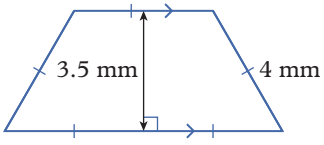
d



e

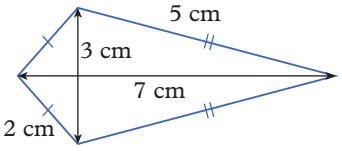


f

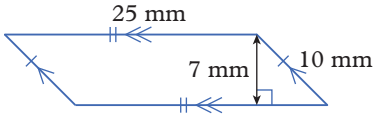


7 Calculate the area of each quadrilateral.

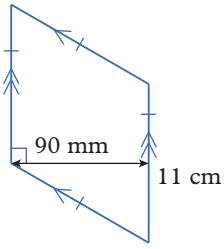
a



b



c



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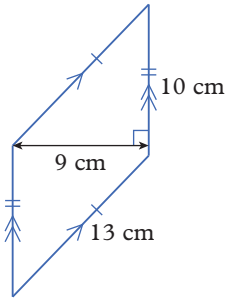
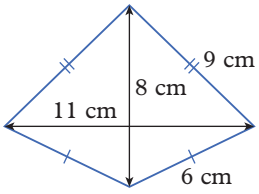


e



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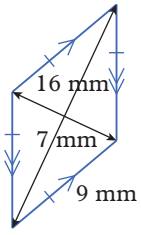
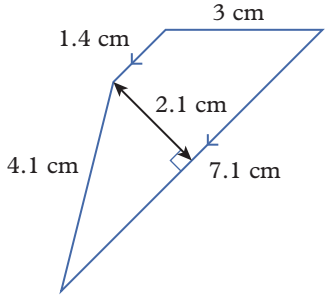
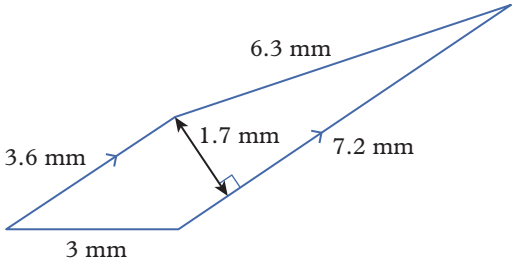


8 Evaluate the areas of the following shapes.

a



b



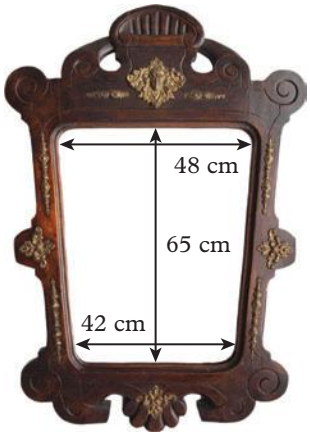
Exercise 8.3B: Problem solving and reasoning

9 Amanda wants to make some kites. She decides to make two types, a large blue kite and a smaller red kite.

a The blue kite is 95 cm long and 65 cm wide. What is its area?

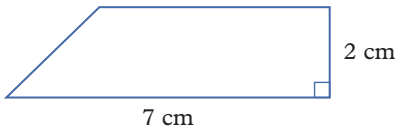
b The red kite has an area of 400 cm^2. If the kite is 40 cm long, how wide must it be?

10 Find the cost of fitting the frame in the image with a mirror if your local hardware store charges \$0.25 per cm^2 for mirror glass.

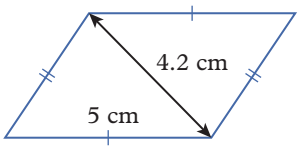


11 If no other side lengths are determined, is there enough information to calculate the area of the following shapes? Give reasons with your answers.

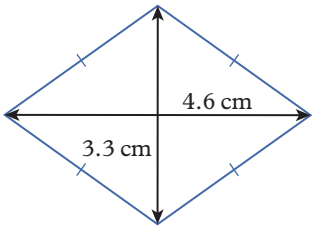
a



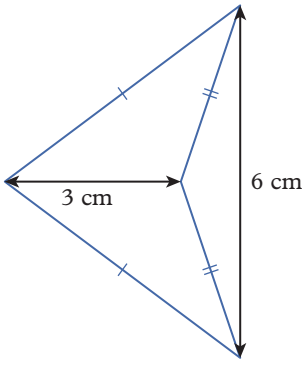
b



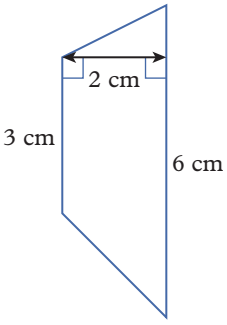
c



d



e



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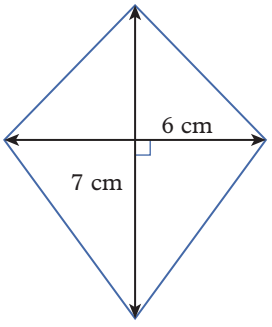
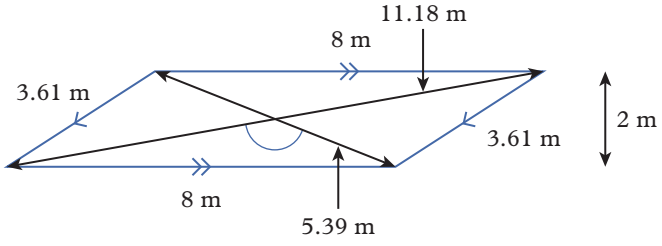


12 Some quadrilaterals can be named in multiple ways; for example, every square is also a rectangle. For each of the following quadrilaterals:

i state if the shape is a parallelogram, kite, rhombus and/or trapezium

ii determine the area of the quadrilateral, correct to one decimal place where necessary.

a



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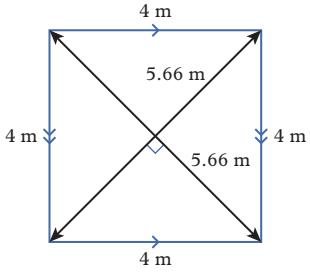

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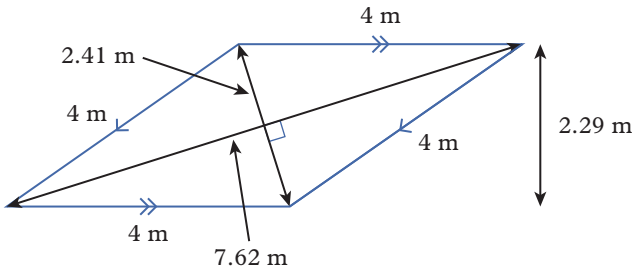

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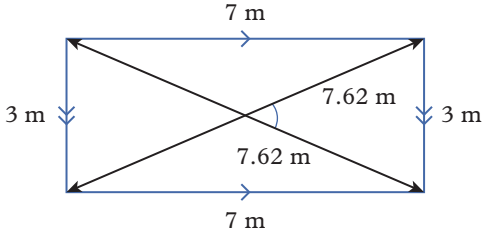
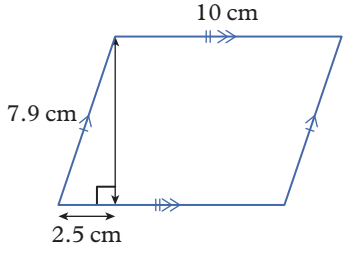
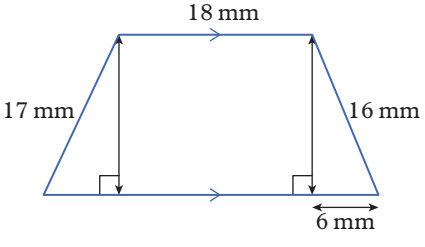


13 Use Pythagoras' theorem to help calculate the area of these right-angled triangles. Give your answers correct to two decimal places.

a



b



14 The Dockland is a famous building in Hamburg, Germany, as shown in the following photograph.

a What shape is the wall of windows shown in the photograph?

b If the wall of windows is 21 m tall and the base is 86 m long, what is the area of this entire section?

c Is every window equal in area? Explain your answer.

d Which area formulas would you need to use to calculate the areas of the individual windows?

15 Use the information provided to determine the missing lengths.

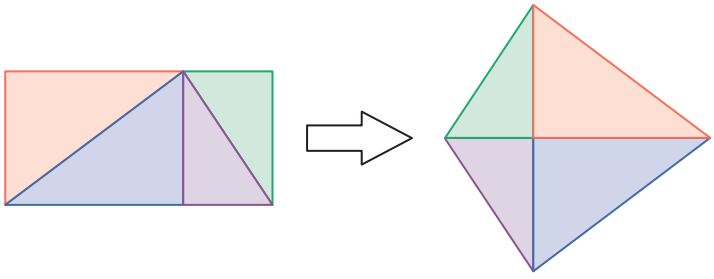
a The area of a kite is 72 cm^2 and the length of one of the diagonals is 16 cm. Determine the length of the other diagonal.

b The area of a rhombus is 17 cm^2 and the base length is 4 cm. Determine the height of the rhombus.

c The area of a trapezium is 22.5 cm^2 and the sum of the lengths of the parallel sides is 9 cm. Determine the height of the trapezium.

d The area of a trapezium is 48 mm^2, the height is 3 mm, and the length of one of the parallel sides is 1.3 cm. Determine the length of the other parallel side.

16 A rectangle has an area of 48 cm^2. The rectangle is cut into two pairs of congruent triangles which are rearranged to form a kite. If the lengths of the diagonals of the kite are integers, what possible dimensions (lengths of the diagonals) can the kite have?



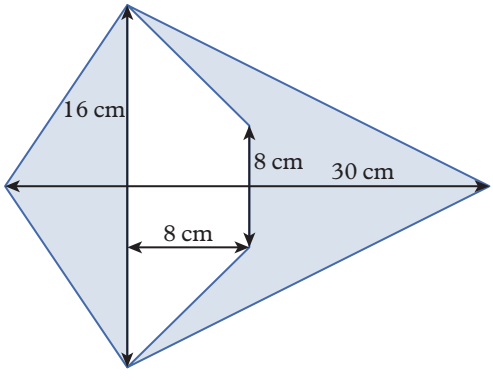
17 Consider that a square has four sides, all with the same side length.

a The perimeter of a square is 36 cm. Determine the area of the square.

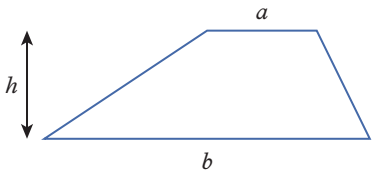
b The area of a square is 36 cm^2. Determine the perimeter of the square.

c Determine the side length of a square (in cm) such that the value of the area and perimeter are the same.

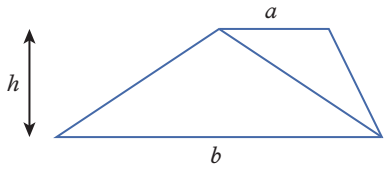
18 Calculate the shaded area in the following shape.



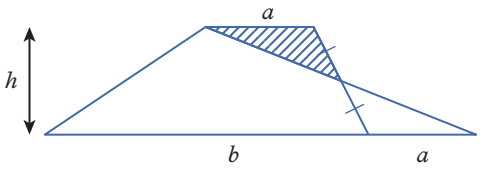
19 The formula for the area of a trapezium can be derived in multiple ways. Describe how each diagram has been constructed from the trapezium shown and how it could be used to derive the formula for the area of the trapezium.



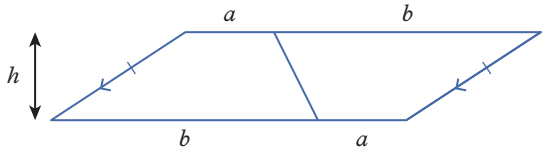
a



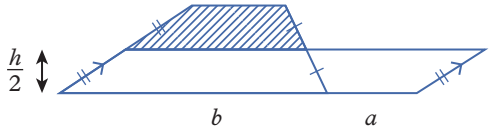
b



c



d



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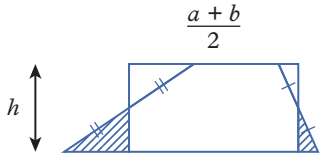
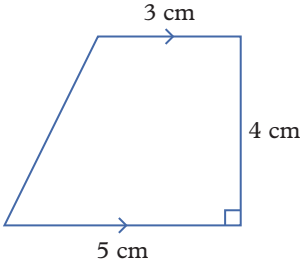


Exercise 8.3C: Challenge

20 Consider the trapezium shown. The shorter parallel side is increased by 150%, the longer parallel side is increased by 20%, and the height is decreased by 75%.



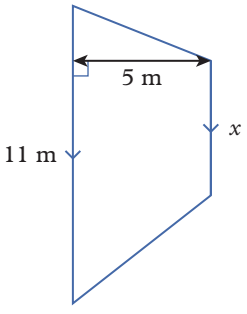
a Determine the area of:

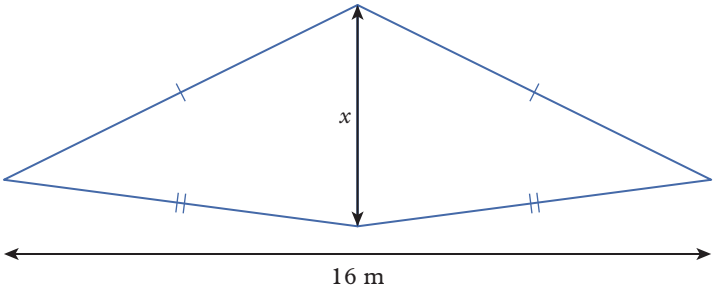
i the original trapezium

ii the altered trapezium.

b Determine the percentage change of the area, stating whether it is an increase or decrease.

21 Determine the value of x such that these two quadrilaterals have the same area. State this area.





Online resources:

Interactive skillsheet — Area of quadrilaterals

Investigation — Land to value

Quick quiz — 8.3

Lesson 8.4

Area of a circle

Learning intentions

By the end of this lesson you will be able to ...

✔ use the formula for the area of a circle to find the area of a circle, or its radius.

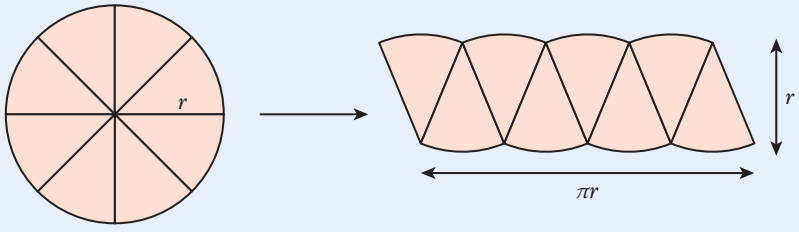
Inter-year links

Support	Understanding area
Year 7	9.2 Circumference of a circle
Year 9	5.4 Surface area of cylinders
Year 10	8.2 Surface area review

Area of a circle

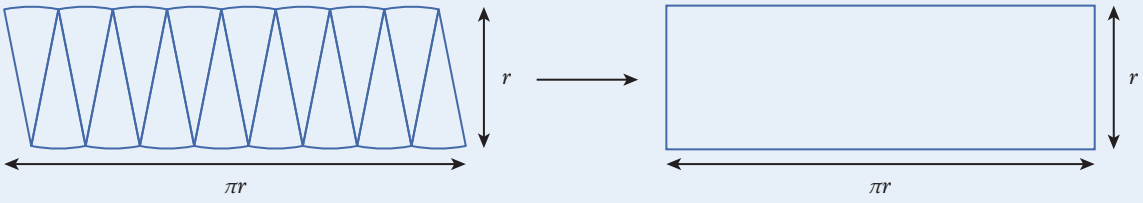
Key content video
Area of a circle

- By dividing a circle into equal parts and rearranging these parts as shown, we can find the area of a circle.

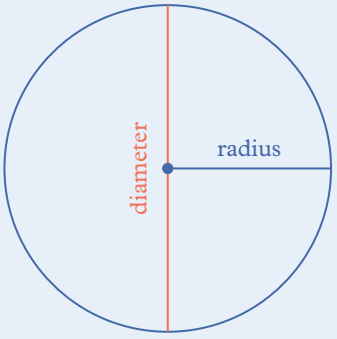


→ The base of the rearranged shape is approximately equal to half of the circumference (πr).

- As the number of parts increases, the resulting figure becomes rectangular in shape.



- Therefore, the formula for the area of a circle with radius r is



$$A = \pi r^2$$

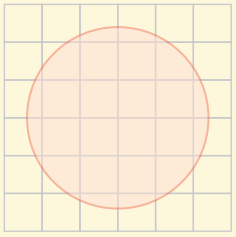
→ If you are given the diameter, find the radius before using the formula.

→ This formula can be rearranged to find the radius of a circle, given the area.

$$\pi r^2 = A \quad \rightarrow \quad r^2 = \frac{A}{\pi} \quad \rightarrow \quad r = \sqrt{\frac{A}{\pi}}$$

Worked example 8.4A Estimating the area of a circle

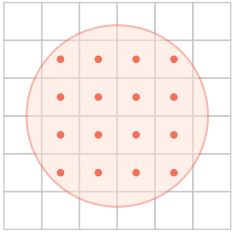
Estimate the area of this circle shown on 1 cm grid paper.



THINK

1. Count the number of squares that are completely enclosed by the shape.
2. Count the number of squares that are at least half covered by the shape. We'll include these squares in our final tally and exclude any squares that are less than half covered.
3. Give your answer for the area. To show your answer is an estimate, use the symbol ≈, which means 'is approximately equal to'.

WRITE

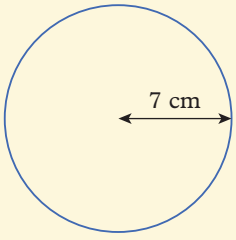


Area ≈ 16 cm^2

Worked example 8.4B Calculating the area of a circle

Calculate the area of these circles correct to two decimal places.

a



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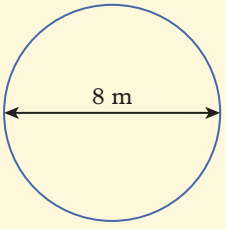


THINK

a
1. Identify the value of the radius, r.
2. Substitute the value for r into the formula for the area of a circle.
3. Calculate the result using π on your calculator.
4. Round to two decimal places and include the appropriate unit.

b
1. Identify the value of the radius, r. Divide the diameter by 2 to find the radius.
2. Substitute the value for r into the formula for the area of a circle.
3. Calculate the result using π on your calculator.
4. Round to two decimal places and include the appropriate unit.

WRITE

a $r = 7$ cm

$A = \pi r^2$

$= \pi \times 7^2$

$= 153.938...$

≈ 153.94 cm^2

b $r = d \div 2$

$= 8 \div 2$

$= 4$ m

$A = \pi r^2$

$= \pi \times 4^2$

$= 50.265...$

≈ 50.27 m^2

Worked example 8.4C Calculating the radius of a circle, given the area

Find the radius of a circle which has an area of 184 cm^2, correct to two decimal places.

THINK

1 Identify both the given value and the required value.

2 Substitute the value for A into the formula for the area of a circle.

3 Rearrange the formula to calculate the value of the radius. Remember to follow the order of operations.

4 Round to two decimal places and include the appropriate unit.

WRITE

$A = 184$ cm^2, $r = ?$

$A = \pi r^2$

$184 = \pi r^2$

$\frac{184}{\pi} = r^2$

$\sqrt{\frac{184}{\pi}} = r$

$r = 7.653...$

$r \approx 7.65$ cm

Helpful hints

✔ The formulas for the circumference of a circle ($2\pi r$) and the area of a circle (πr^2) are similar. To tell them apart, look at where the 2 is: when the 2 is an exponent, it's an area formula; when the 2 is a coefficient, it's a circumference formula.

Learning pathways

1–8, 10, 15, 16, 18(a, b, e), 19(a, b)	1, 3–7, 9, 11, 12, 14, 17, 18–19(a, c, e), 22	1(c), 3(d–f), 4–7, 11–14, 18(e, f), 19(d–f), 20–23

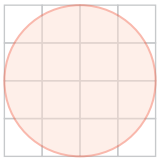
ANS p577

Exercise 8.4A: Understanding and fluency

For all relevant questions in this lesson, use π on your calculator and unless stated, otherwise round answers to two decimal places.

WE 8.4A **1** Estimate the area of these circles shown on 1 cm grid paper.

a



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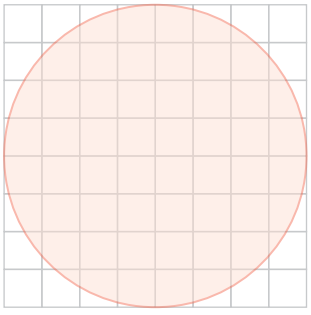

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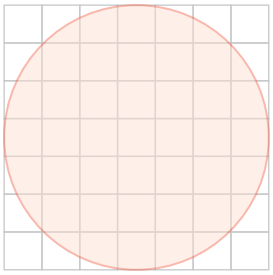


2 For each of the circles in question **1**:

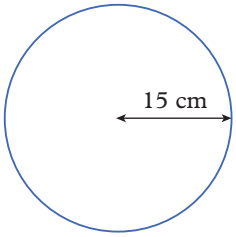
i give the diameter in cm

ii give the radius in cm

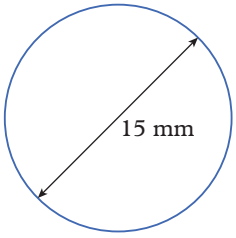
iii divide the estimated area by the square of the radius.

WE 8.4B **3** Calculate the area of each circle.

a



b



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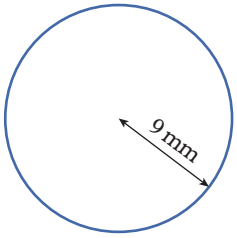

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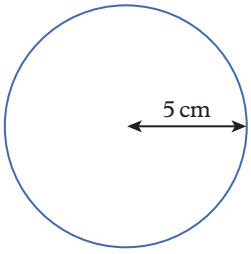

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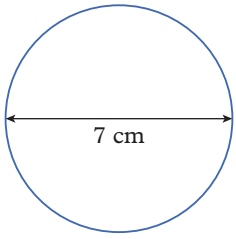

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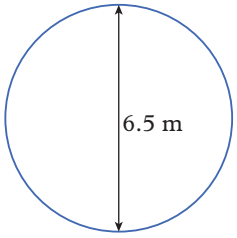
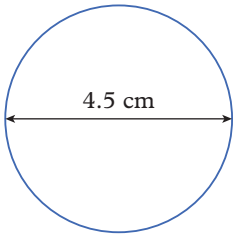


4 Calculate the exact area of each circle by leaving π in the answer.

a



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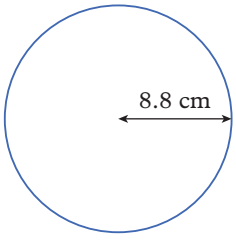

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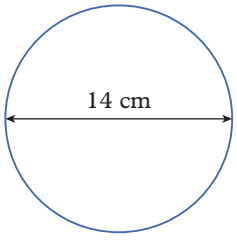


5 Calculate the area and perimeter of a circle with:

a radius 4.4 cm **b** diameter 7.5 m **c** diameter 15 mm **d** radius 29 mm.

WE 8.4C **6** Find the radius of a circle which has an area of:

a 12 cm^2 **b** 25 mm^2 **c** 100 cm^2 **d** 68 cm^2.

7 Find the diameter of a circle which has an area of:

a 80 cm^2 **b** 49 cm^2 **c** 20 mm^2 **d** 135 cm^2.

Exercise 8.4B: Problem solving and reasoning

8 Anika wants to make a circular placemat which has a diameter of 55 cm. What is the area of the placemat?

9 What is the total area covered by a circular sprinkler which has a range of 7.5 m?

10 A pizzeria sells two sizes of pizza: medium with a diameter of 12 inches and large with a diameter of 18 inches. Explain why you get more pizza by ordering one large rather than two medium pizzas.

11 The area of a circle is most often found using its radius, but it can also be found using its diameter or its circumference.

a The area of a circle can be found with $A = \frac{1}{4}\pi d^2$, where d is the diameter of the circle. Show that $\frac{1}{4}\pi d^2 = \pi r^2$.

b The area of a circle can be found with $A = \frac{1}{2}Cr$, where C is the circumference of the circle and r is the radius. Show that $\frac{1}{2}Cr = \pi r^2$.

12 Use $A = \pi r^2$, $A = \frac{1}{4}\pi d^2$ or $A = \frac{1}{2}Cr$ to solve the following problems.

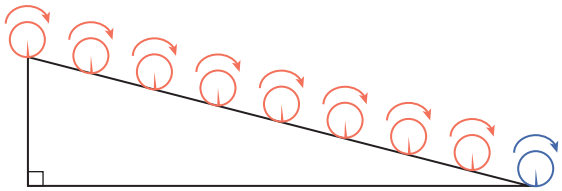
a Find A when $d = 8$ m. **b** Find A when $r = 3$ cm.

c Find A when $C = 40$ mm. **d** Find A when $C = 50$ cm.

e Find d when $A = 125$ cm^2. **f** Find C when $A = 90$ mm^2.

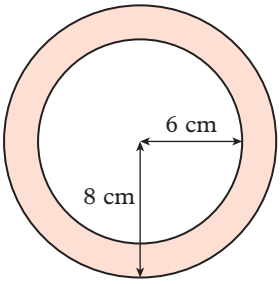
g Find C when $A = 20$ cm^2.

13 A circular object with an area of 58 cm^2 rolls down an inclined ramp after exactly 8 revolutions. How long is the ramp correct to the nearest integer?



14 You can subtract the area of one circle from another to find the area of a ring. Another name for a ring is an annulus. Find the area for each annulus.

a



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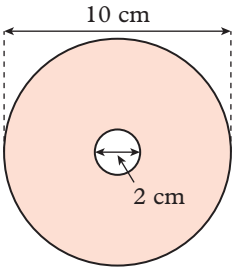

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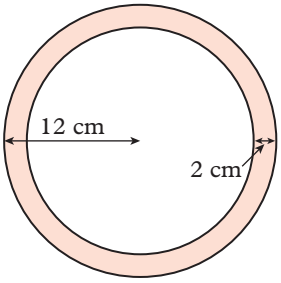
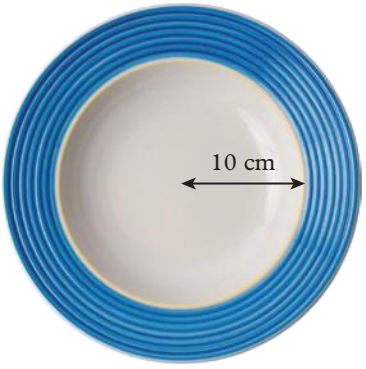


15 What is the largest area of pancake that could be cooked in a frying pan with a base diameter of 24 cm?

16 Adrian wants to cook a pancake using the frying pan from question **15** and place it on this plate.



a What is the maximum area a pancake could have if it had to fit on the white section of this plate and the white section has a radius of 10 cm?

b To make a pancake the size you found in part **a**, what distance should be between the edges of the pancake and the side of the frying pan?

c What is the entire area of the plate if it has a diameter of 30 cm?

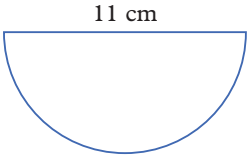
d What is the area of the painted section of the plate?

17 What is the total area that this button covers if it has a diameter of 20 mm and each hole has a radius of 1.5 mm?

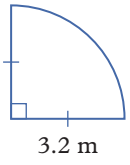
18 A semicircle is exactly half of a circle, and a quadrant is exactly a quarter of a circle. To find the area of semicircles and quadrants, find the area of the full circle and then divide by 2 or 4, respectively.

Find the area of these sections of circles.

a



b



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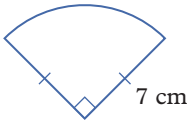

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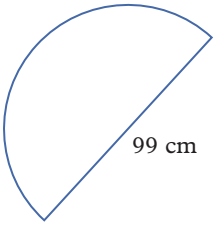

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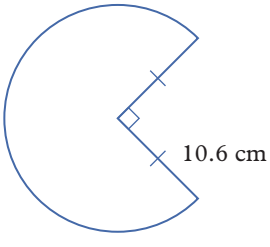

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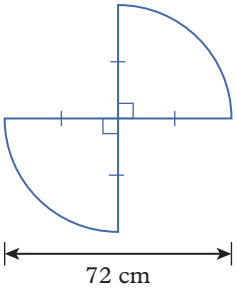
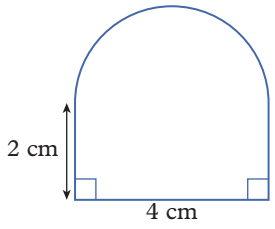


19 Use your understanding of the area of a circle to find the area of each composite shape.

a



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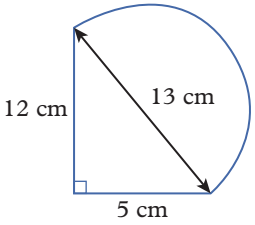

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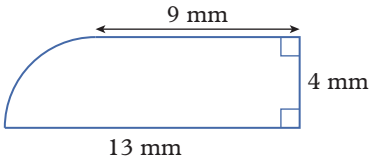

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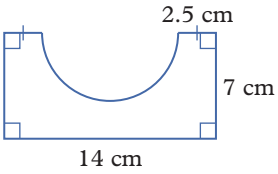

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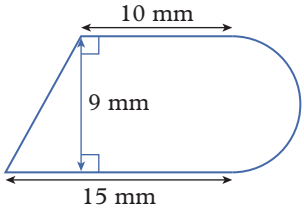

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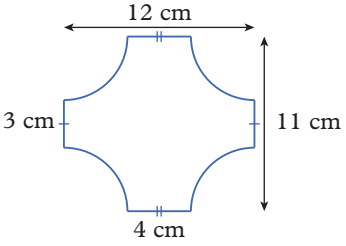


20 Explain why you can only find the exact length of the radius of a circle if the area is given with π still included in the value. Support your answer by finding the radius of a circle with an area of:

a 16 mm^2 **b** 16π mm^2.

21 The formulas for the area or circumference of a circle can be rearranged to determine other values.

a A circle has an area of 49π cm^2. Find its:

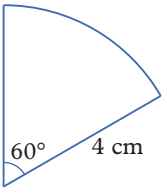
i diameter **ii** circumference (keeping π in the answer).

b A circle has a circumference of 8π m. Find its:

i radius **ii** area (keeping π in the answer).

Exercise 8.4C: Challenge

22 A sector is a portion of a circle formed by two radii and part of the circumference (an arc). You can use knowledge of angles to calculate the area of a sector. Look at the sector shown here.



a Write the angle for this sector as a fraction of the degrees in a circle.

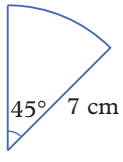
b What is the radius of this sector?

c What would be the area of the circle if it was complete? Give your answer correct to two decimal places.

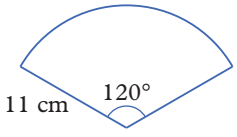
d Use your answers to parts **a** and **c** to find the area of the sector correct to two decimal places.

23 Calculate the area of each sector.

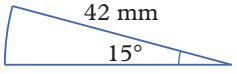
a



b



c



Online resources:

- **Interactive skillsheet** Area of a circle
- **Worksheet** Area of a circle
- **Investigation** Using arrays and rectangles to approximate areas
- **Quick quiz** 8.4

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Checkpoint

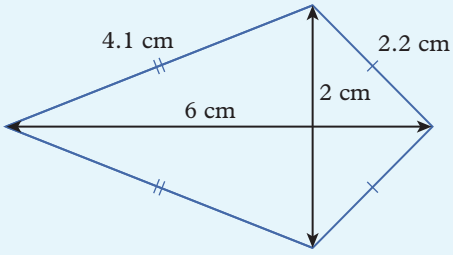
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

8.1 **1** Convert the following.

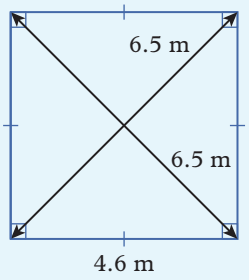
a 45 cm to m **b** 5.9 km to m **c** 4.1 m to mm **d** 0.27 km to cm

8.1 **2** Calculate the perimeter of the following shapes.

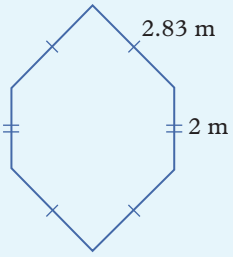
a



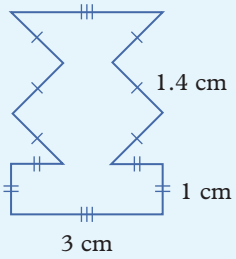
b



c

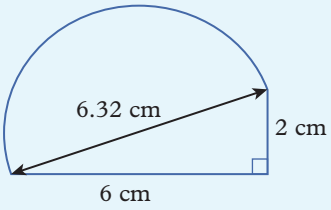


d

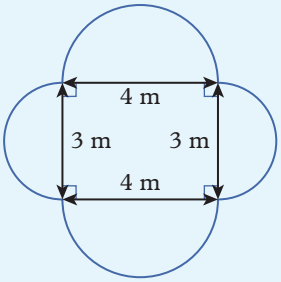


8.1 **3** Calculate the perimeter of the following composite shapes correct to two decimal places.

a



b



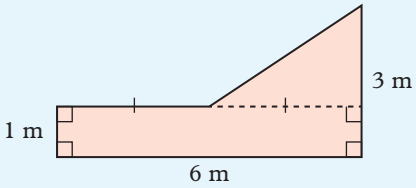
8.1 **4** Calculate the circumference of the following circles. Give your answers correct to two decimal places.

a circle with diameter 31 m **b** circle with radius 7 cm

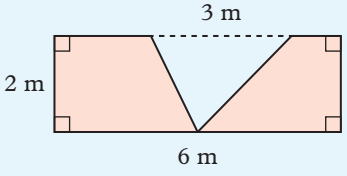
8.2 **5** The area of a rectangle with a width of 4 m is 28.8 m^2. Determine the length of the rectangle.

8.2 **6** Calculate the shaded area of the following composite shapes.

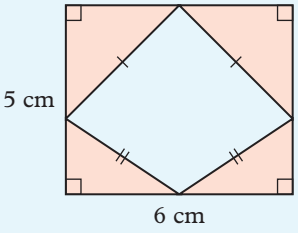
a



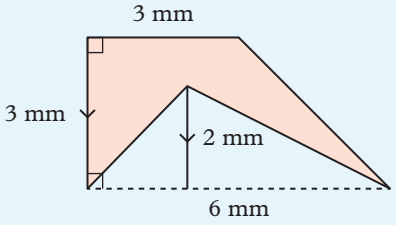
b



c



d



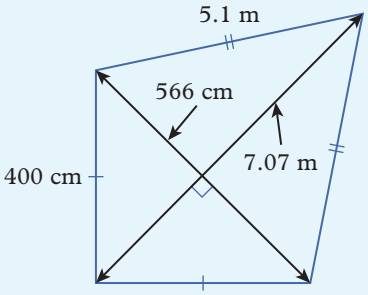
8.3 **7** Use the given information to find the missing length.

a The area of a kite is 31.5 cm^2. If one of the diagonals is 7 cm long, determine the length of the other diagonal.

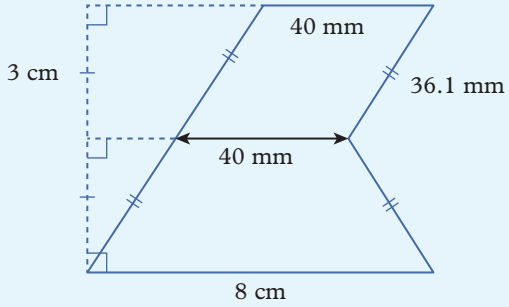
b The area of a trapezium is 37.8 cm^2. If one of the parallel sides is 9.5 cm long and the perpendicular height is 6 cm, determine the length of the other parallel side.

8.3 **8** Calculate the area of the following shapes.

a

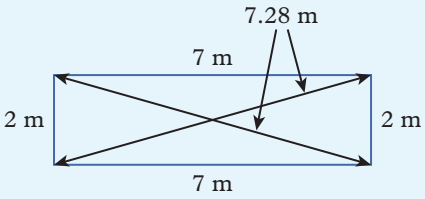


b

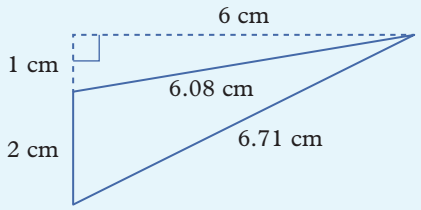


8.3 **9** Calculate the area of the following shapes, giving your answers correct to two decimal places where necessary.

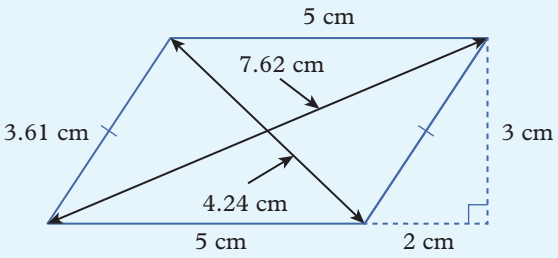
a



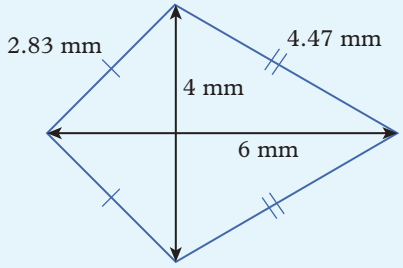
b



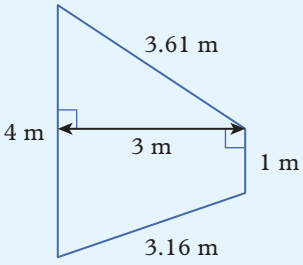
c



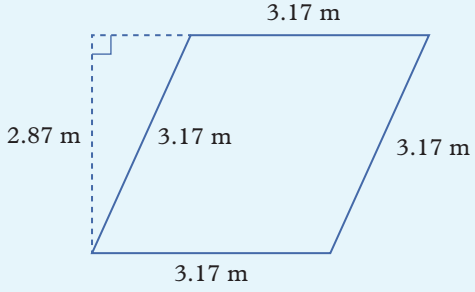
d



e

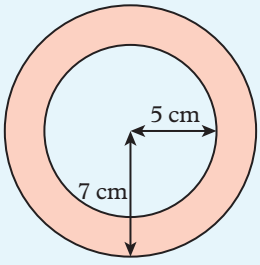


f

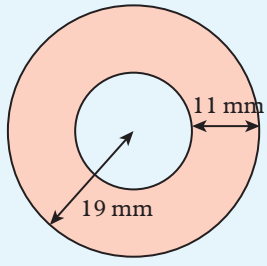


8.4 **10** Calculate the shaded area of the following shapes, giving your answers correct to two decimal places.

a



b



8.4 **11** Calculate the area of each shape in question **3**. Give your answers correct to two decimal places.

Lesson 8.5

Volume and capacity

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the volume and capacity of prisms.

Inter-year links

Support	Volume and capacity
Year 7	9.6 Volume of right prisms
Year 9	5.5 Volume of prisms and cylinders
Year 10	8.3 Volume

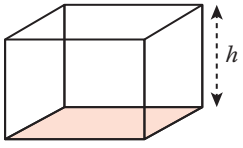
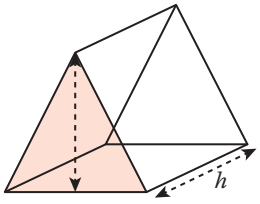
Volume of prisms

Key content video
Volume and capacity

- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.

Metric units of volume	
Name	**Abbreviation**
Cubic millimetre	mm^3
Cubic centimetre	cm^3
Cubic metre	m^3

- A **prism** is a 3D object with a consistent cross-section parallel to the base of the prism. The cross-section has the same shape as the base.
  → The shape of the cross-section of a prism is a **polygon**.
- The volume (V) of any prism can be found by multiplying the area of the base (A) by the height (h); $V = Ah$.

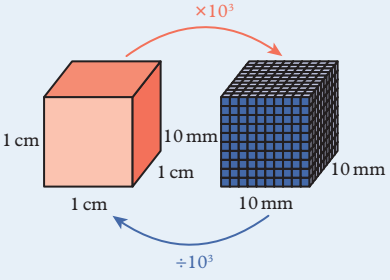
Shape	Diagram	Formula
Rectangular prism		V = (area of rectangle) × h
Triangular prism		V = (area of triangle) × h

- The base and height of a right prism are perpendicular (at right angles) to each other.

Converting between units of volume

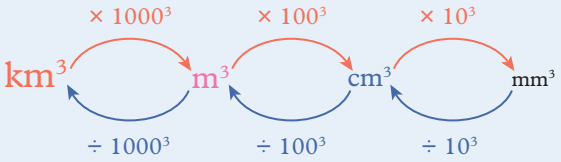
Key content video
Converting between units of volume

- To convert between units of volume, multiply or divide by the cube of the conversion factor.
  For example, a cubic centimetre (1 cm × 1 cm × 1 cm = 1 cm^3) is equivalent to 1000 cubic millimetres (10 mm × 10 mm × 10 mm = 10^3 mm^3 = 1000 mm^3). That means we can multiply the number of cm^3 by 10^3 to determine the number of mm^3. Likewise we can divide the number of mm^3 by 10^3 to determine the number of cm^3, since each mm^3 is $\frac{1}{1000}$ of a cm^3.



Conversion factors

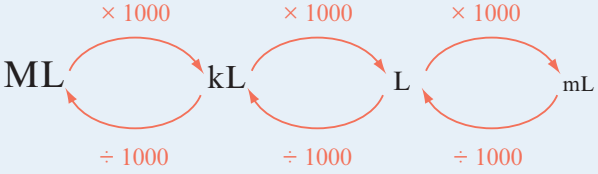
$1\ cm^3 = 10^3\ mm^3$
$= 1000\ mm^3$
$1\ m^3 = 100^3\ cm^3$
$= 1\ 000\ 000\ cm^3$
$1\ km^3 = 1000^3\ m^3$
$= 1\ 000\ 000\ 000\ m^3$



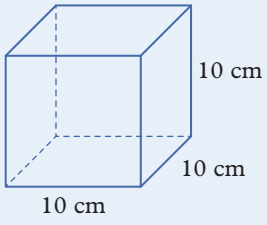
Capacity

- The **capacity** of a 3D object is a measure of how much that object can hold.

Metric units of capacity	
Name	**Abbreviation**
Millilitre	mL
Litre	L
Kilolitre	kL
Megalitre	ML



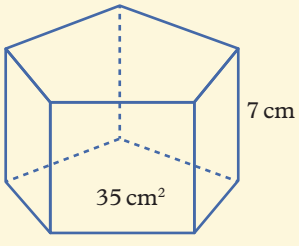
→ A container with an internal volume of $1\ cm^3$ holds 1 mL of liquid.
→ A container with an internal volume of $1000\ cm^3$ holds 1 L of liquid.
→ A container with an internal volume of $1\ m^3$ holds 1 kL of liquid.



Volume = $1000\ cm^3$
Capacity = 1 L

Worked example 8.5A Calculating the volume of a prism

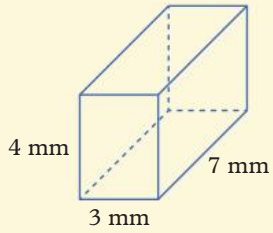
Calculate the volume of this prism.



THINK	WRITE
1 Identify the values for A and h.	$A = 35\ cm^2$, $h = 7\ cm$
2 Substitute the values for A and h into the formula for the area of a prism.	$V = Ah$ $= 35 \times 7$
3 Calculate the volume of the prism. Remember to include the appropriate unit in the answer.	$= 245\ cm^3$

Worked example 8.5B Calculating the volume of a rectangular prism

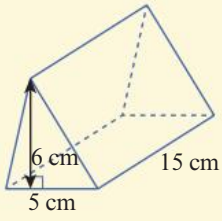
Calculate the volume of this rectangular prism.



THINK	WRITE
1 Identify the base of the prism. In this example we will designate the 3 mm by 4 mm side as the base.	$l = 4$ mm, $w = 4$ mm
2 Use the formula for the area of a rectangle, $A = lw$, to calculate the area of the base, A.	$A = lw$ $= 4 \times 3$ $= 42$ mm^2
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the volume of a prism.	$h = 7$ mm $V = Ah$ $= 12 \times 7$
4 Calculate the volume of the rectangular prism. Remember to include the appropriate unit in the answer.	$= 84$ mm^3

Worked example 8.5C Calculating the volume of a triangular prism

Calculate the volume of this triangular prism.



THINK	WRITE
1 Identify the base of the prism. In this example the base is the triangular face. As we will be dealing with two heights, we will refer to them as h_1 and h_2.	$b = 5$ cm, $h_1 = 6$ cm
2 Calculate the area of the base, A. In this example use the formula for the area of a triangle, $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh_1$ $= \frac{1}{2} \times 5 \times 6$ $= 15$ cm^2
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the area of a prism.	$h_2 = 15$ cm $V = Ah_2$ $= 15 \times 15$
4 Calculate the volume of the triangular prism. Remember to include the appropriate unit in the answer.	$= 225$ cm^3

Worked example 8.5D Converting units of volume

a Convert 2 m^3 into cm^3.

b Convert 216 mm^3 into cm^3.

THINK

a **1** Identify the appropriate conversion factor. To convert from a larger unit to a smaller unit, multiply by the conversion factor.

2 Complete the calculation.

b **1** Identify the appropriate conversion factor. To convert from a smaller unit to a larger unit, divide by the conversion factor.

2 Complete the calculation.

WRITE

a $1\ m^3 = 100^3\ cm^3$
$= 1\,000\,000\ cm^3$

$2\ m^3 = 2 \times 1\,000\,000$
$= 2\,000\,000\ cm^3$

b $1\ cm^3 = 10^3\ mm^3$
$= 1000\ mm^3$

$216\ mm^3 = 216 \div 1000$
$= 0.216\ cm^3$

Worked example 8.5E Converting units of capacity

a Convert 3 L into mL.

b Convert 500 L into kL.

THINK

a **1** Identify the appropriate conversion factor. To convert from a larger to a smaller unit, multiply by the conversion factor.

2 Complete the calculation.

b **1** Identify the appropriate conversion factor. To convert from a smaller to a larger unit, divide by the conversion factor.

2 Complete the calculation.

WRITE

a $1\ L = 1000\ mL$
$3\ L = 3 \times 1000$
$= 3000\ mL$

b $1\ kL = 1000\ L$
$500\ L = 500 \div 1000$
$= 0.5\ kL$

Helpful hints

- ✔ For rectangular prisms, any of the sides can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.
- ✔ Recall the following formulas for area:
  - → Rectangle: $A = lw$
  - → Triangle: $A = \frac{1}{2}bh$
  - → Parallelogram: $A = bh$
  - → Kite: $A = \frac{1}{2}xy$
  - → Rhombus: $A = bh$ or $A = \frac{1}{2}xy$
  - → Trapezium: $A = \frac{1}{2}(a + b)h$
- ✔ Volume and capacity are not the same! Volume is a measure of how much space an object occupies (expressed in mm^3, cm^3 or m^3), whereas capacity is a measure of how much an object can hold (expressed in mL, L, kL or ML).

Learning pathways

1–4, 5(1st column), 6, 7, 9–12, 14, 17	1(c, d), 2–4(c), 5–7(a–c),8, 11, 13–18	1–6(2nd column), 7–10, 16, 18, 19

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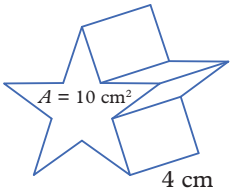
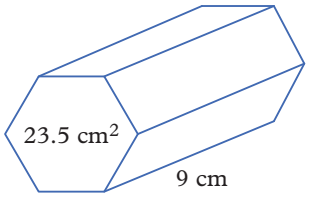
Exercise 8.5A: Understanding and fluency

WE 8.5A **1** Calculate the volume of each prism. The base area is shown.

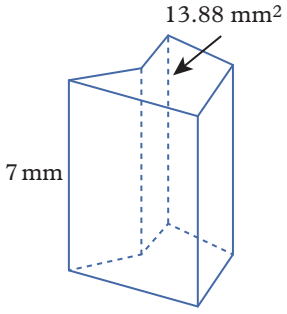
a



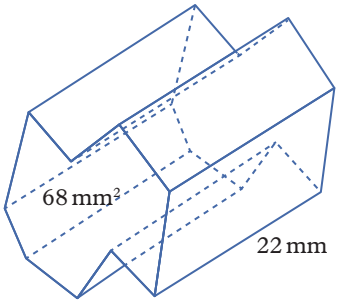
b



c

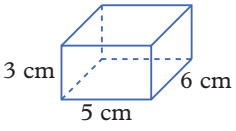


d

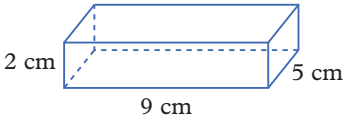


WE 8.5B **2** Calculate the volume of each rectangular prism.

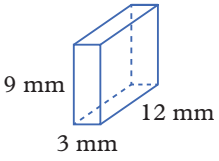
a



b

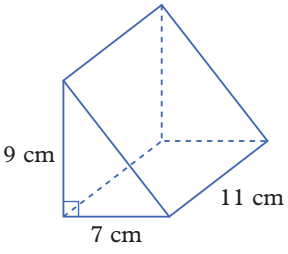


c

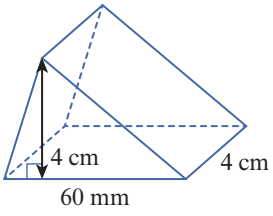


WE 8.5C **3** Calculate the volume of each triangular prism.

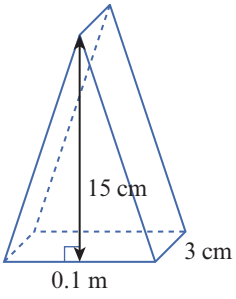
a



b

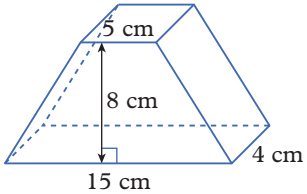


c

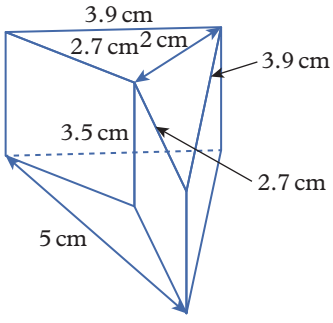


4 Use the formula $V = Ah$ to calculate the volume of each object.

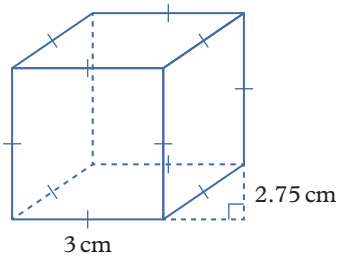
a



b



c



WE 8.5D **5** Convert each measurement.

a 9 cm^3 into mm^3 **b** 18 m^3 into cm^3 **c** 16 000 mm^3 into cm^3

d 1 750 000 m^3 into km^3 **e** 4.2 km^3 into m^3 **f** 4 000 000 cm^3 into m^3

g 4.56 m^3 into cm^3 **h** 845 000 cm^3 into m^3 **i** 0.15 km^3 into m^3

j 6.1 cm^3 into mm^3 **k** 28 mm^3 into cm^3 **l** 1 700 000 000 000 m^3 into km^3

WE 8.5E **6** Use the conversion factors to complete these statements.

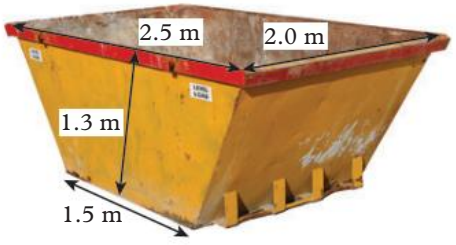
a 8 L = _____ mL **b** 15 kL = _____ L **c** 70 mL = _____ L

d 6.4 ML = _____ kL **e** 2500 L = _____ kL **f** 3 kL = _____ mL

7 Complete the table by working out the missing value for each container.

	Inside volume of container	Capacity of container
a	14 cm^3	_____ mL
b	_____ cm^3	3 L
c	5.8 m^3	_____ kL
d	750 cm^3	_____ L
e	_____ m^3	1000 kL
f	_____ m^3	6.4 ML

8 A skip has the dimensions shown. Calculate its internal volume in cubic metres.



Exercise 8.5B: Problem solving and reasoning

9 A tissue box is 21 cm long, 11.4 cm wide and 12.3 cm tall.

a What is its volume?

b If each tissue laid flat in the box is 1 mm high, how many tissues would it fit?

10 A rectangular prism has a square base with sides that measure 5 cm. If the prism has a volume of 100 cm^3, what is its height?

11 The volume of a rectangular prism can also be given by using the formula $V = lwh$, where l, w and h represent the length, width and height, respectively. Explain why it doesn't matter which of the measurements you designate as the length, width or height.

12 Priya was completing some volume conversions and wrote the following answers.

a

$7\ m^3 = (7 \times 100^3)\ cm^3$
$= 2100\ cm^3$

b

$3000\ mm^3 = (3000 \div 100^3)\ cm^3$
$= 30\ cm^3$

i Explain the mistake Priya made in each instance.

ii Provide the correct working for the conversion.

13 Consider the perfume bottle on the right.

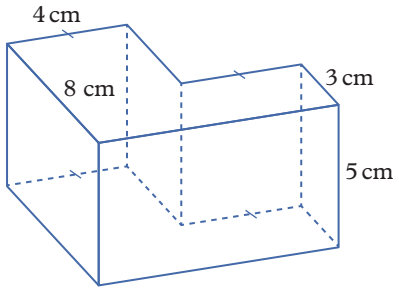
a Calculate the volume of this perfume bottle in cubic centimetres.

b Convert the volume from part **a** into cubic millimetres.

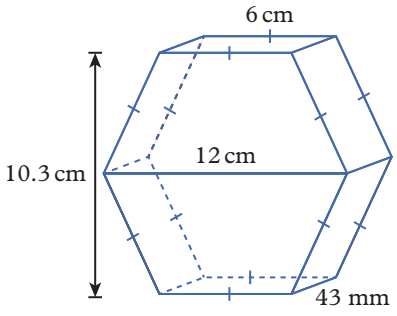
c Another way to do this is to first convert all the dimensions into millimetres and then find the volume. Show how doing the conversion first gives you the same answer as in part **b**.

14 Calculate the volume of these hexagonal prisms.

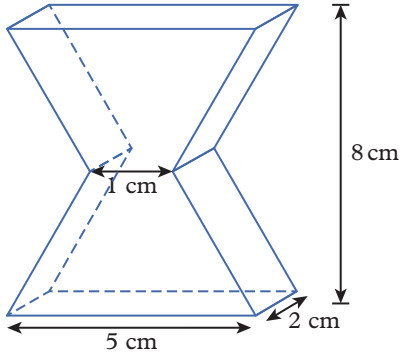
a



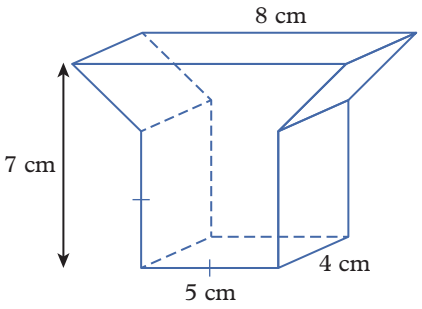
b



c



d

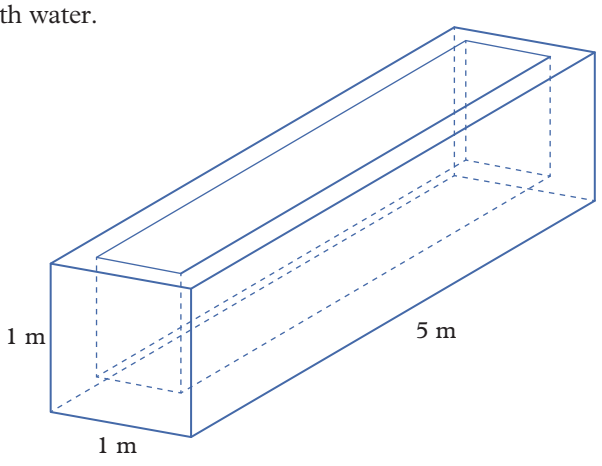


15 A concrete water trough is designed as follows. A 1 m × 5 m × 1 m concrete block is formed around a 80 cm × 480 cm × 90 cm mould to create a cavity to fill with water.

a Determine the maximum capacity of the trough in litres.

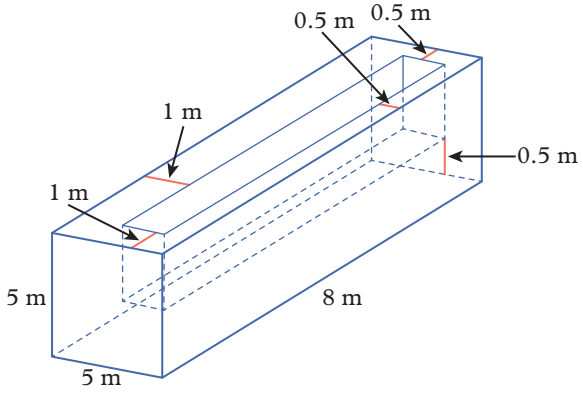
b Determine the total volume of trough and water when the trough is full, in cubic centimetres.

c Determine the volume of concrete used to create the trough in cubic centimetres.

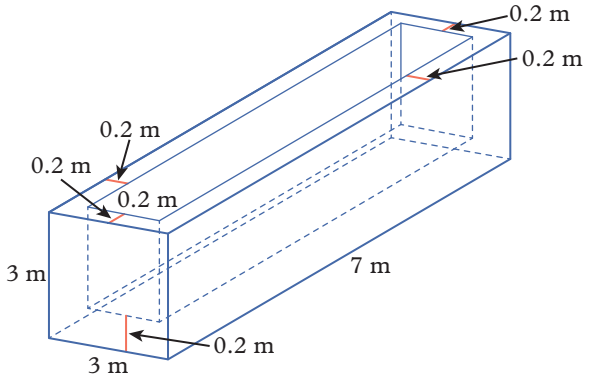


16 Calculate the capacity of each of the following troughs in litres.

a



b



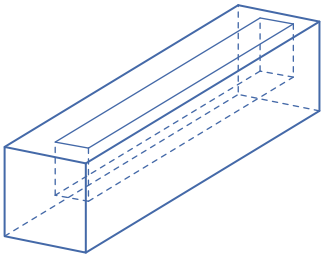
Exercise 8.5C: Challenge

17 Consider the following three troughs made from the same sized block.

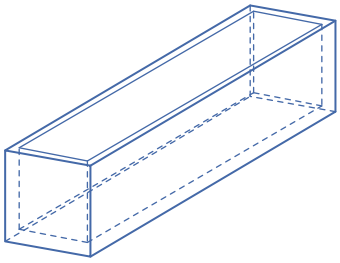
a Put these troughs in ascending order according to their capacity.

b How does the volume of the troughs relate to the capacity of the troughs?

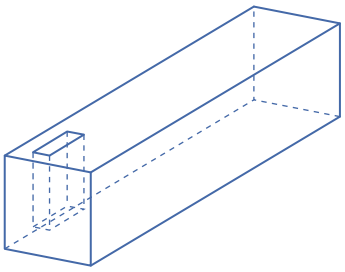
A



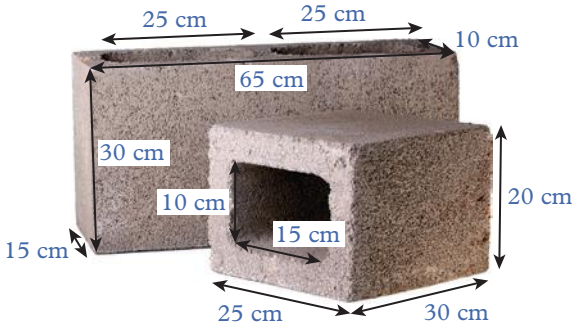
B



C



18 Find the volume of concrete needed to make these two blocks.

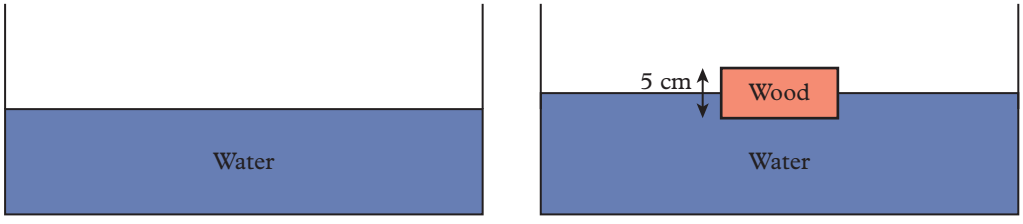


19 Archimedes knows that when an object is submerged under water, the water level will rise as the water becomes *displaced* by the object. He also knows that the volume of the submerged object is equal to the volume of the displaced water.

Archimedes has a rectangular glass container with base dimensions 30 cm × 40 cm. He fills it with 12 L of water and finds that the water level is exactly halfway up the total height of the glass container.

a Determine the height of Archimedes' container.

b Archimedes then floats a rectangular wooden block on the surface of the water. The wooden block has dimensions 10 cm × 8 cm × 5 cm. He measures $\frac{3}{5}$ of the wooden block is submerged under water.



i Determine the volume of the displaced water by first finding the volume of the object that is submerged.

ii Find the new water level by considering the relationship: volume of water in the container = base of the tank × water level − volume of the block submerged in water.

iii Determine the difference in the water level in cm.

iv Archimedes keeps adding water to the container until the top of the wooden block is at the same level as the edge of the container, what's the water level now?

v How much water has been added?

vi Archimedes added the water at a constant rate of 5 L/minute. How long did it take for the surface of the block to be at the same level as the edge of the container?

Online resources:

- **Interactive skillsheet** Volume of prisms
- **Interactive skillsheet** Converting between units of volume
- **Worksheet** Volume conversions
- **Investigation** A bit of trash
- **Desmos activity** Cylinders
- **Quick quiz** 8.5

Lesson 8.6

Three-dimensional space

Learning intentions

By the end of this lesson you will be able to...

✔ work with coordinates in three-dimensional space.

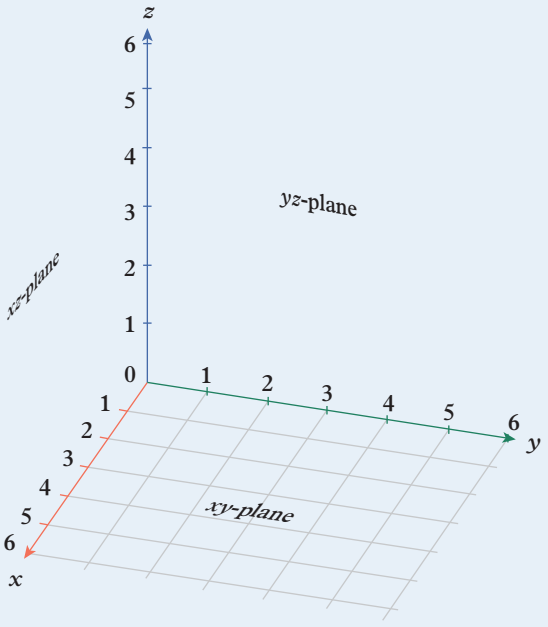
Inter-year links

Year 7 5.4 The Cartesian plane

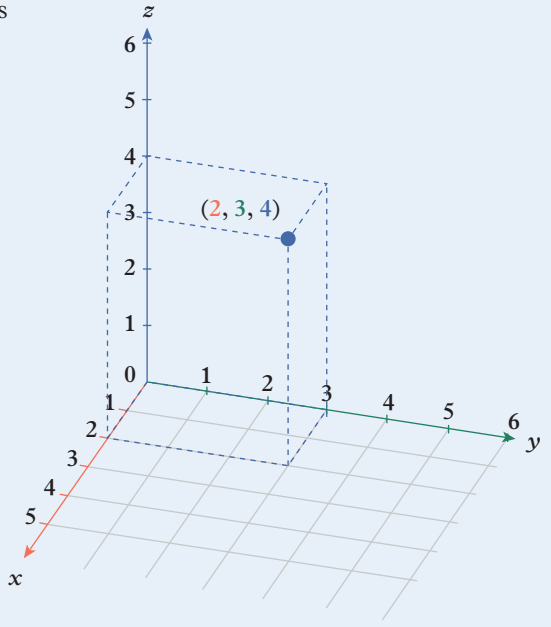
Three-dimensional coordinates

Key content video
Three-dimensional space

- A third axis, the ***z*-axis**, can be added perpendicular to the Cartesian plane (the *xy*-plane) to allow for coordinates in three-dimensional space to be displayed.
  - → In this text, we will display three-dimensional space with the *x*-axis pointing towards the observer.
  - → You may also come across other views, with either the *y*- or *z*-axis pointing towards the observer.
- When displaying three-dimensional space on a two-dimensional page, we can use cues to ensure points appear where they are intended, such as perspective (closer looks larger, further away looks smaller) and dashed line segments.
- To ensure all three axes are clearly visible, we show the space at an angle that skews the *xy*-plane, making the right angle between the *x*- and *y*-axes not appear as a right angle.
  - → Similarly, the right angles between the *x*- and *z*-axes (the *xz*-plane) as well as the *y*- and *z*-axes (the *yz*-plane) do not appear as right angles.



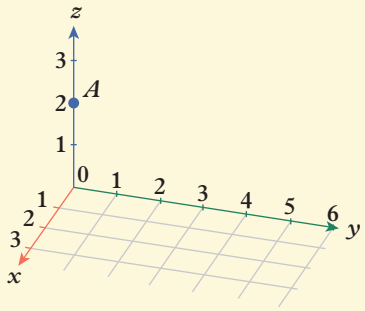
- **Three-dimensional coordinates** consist of three values which specify a location in three-dimensional space.
- The coordinates of a point in three dimensions are written (x, y, z), where:
  - → the *x*-coordinate represents how far forward (positive) or backward (negative) the point is from the origin (0, 0, 0)
  - → the *y*-coordinate represents how far right (positive) or left (negative) the point is from the origin (0, 0, 0)
  - → the ***z*-coordinate** represents how far up (positive) or down (negative) the point is from the origin (0, 0, 0).
- We can give the coordinates of the vertices of two-dimensional shapes and three-dimensional objects in three-dimensional space that can be joined up to create the shape or object.



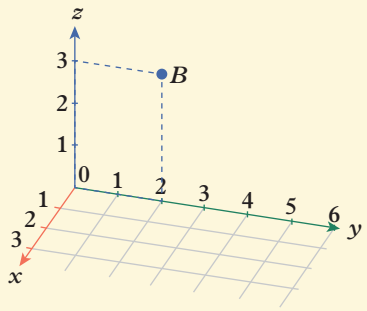
Worked example 8.6A Writing coordinates of points in three-dimensional space

Write the coordinates of the points shown below.

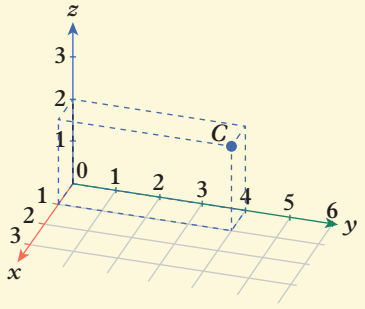
a



b



c



THINK	WRITE
a 1 The point is on the z-axis, so the x- and y-coordinates are both 0.	a The x-coordinate is 0. The y-coordinate is 0.
2 Identify the z-coordinate by reading how far up or down the point is from the origin.	The z-coordinate is 2.
3 Write the coordinate as (x, y, z).	$A(0, 0, 2)$
b 1 The point is on the yz-plane, so the x-coordinate is 0.	**b** The x-coordinate is 0.
2 Identify the y-coordinate by reading how far right or left the point is from the origin.	The y-coordinate is 2.
3 Identify the z-coordinate by reading how far up or down the point is from the origin.	The z-coordinate is 3.
4 Write the coordinate as (x, y, z).	$B(0, 2, 3)$
c 1 Identify the x-coordinate by reading how far forward or back the point is from the origin.	**c** The x-coordinate is 1.
2 Identify the y-coordinate by reading how far right or left the point is from the origin.	The y-coordinate is 4.
3 Identify the z-coordinate by reading how far up or down the point is from the origin.	The z-coordinate is 2.
4 Write the coordinate as (x, y, z).	$C(1, 4, 2)$

Worked example 8.6B Plotting points in three-dimensional space

Plot each of the following points on its own diagram. Clearly show the x-, y- and z-coordinates using dashed lines.

a $A(4, 5, 2)$ **b** $B(0, 4, 3)$

THINK

a **1** Mark the x-, y- and z-coordinates on their respective axes.

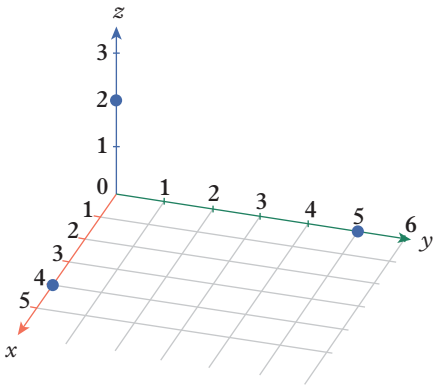
2 Mark the point $(4, 5, 0)$ on the xy-plane by using the corresponding x- and y-coordinates. Add dashed line segments from the x-axis and the y-axis to indicate the position of the point on the xy-plane.

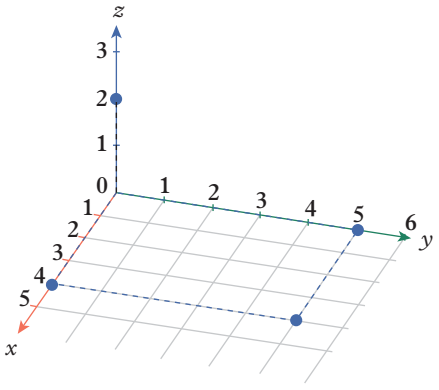
3 Move the points on the xy-plane, marked in steps 1 and 2, two units up in the z-direction, as the z-coordinate of A is 2. Add dashed line segments to connect corresponding points.

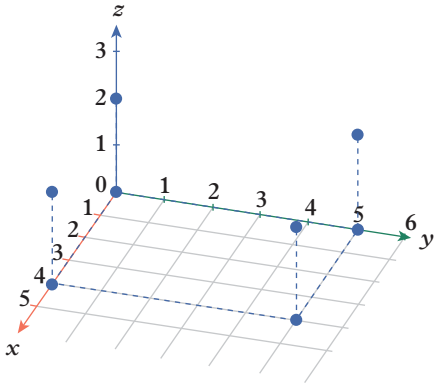
4 Add dashed line segments between the points to indicate the position of the point $A(4, 5, 2)$ in three-dimensional space.

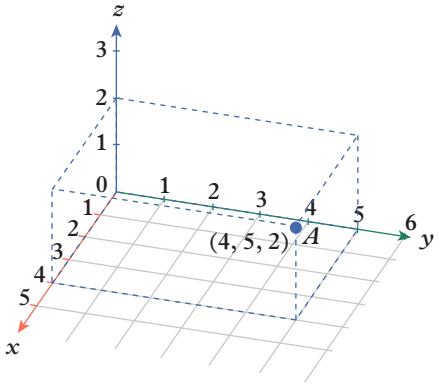
WRITE

a









THINK

b **1** Mark the x-, y- and z-coordinates on their respective axes.

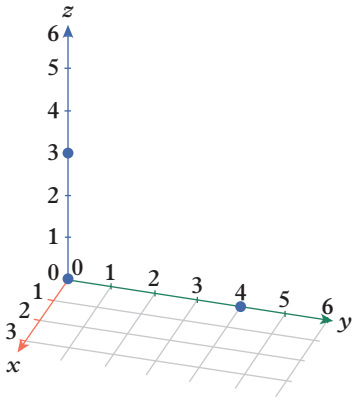
2 Mark the point $(0, 4, 0)$ on the xy-plane by using the corresponding x- and y-coordinates. The x-coordinate is 0, so the position of $(0, 4, 0)$ is on the y-axis.

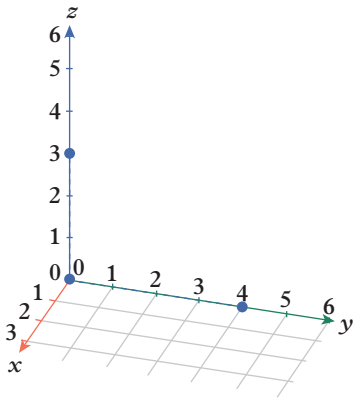
3 Move the points on the xy-plane, marked in steps 1 and 2, three units up in the z-direction, as the z-coordinate of B is 3. Add dashed line segments to connect corresponding points.

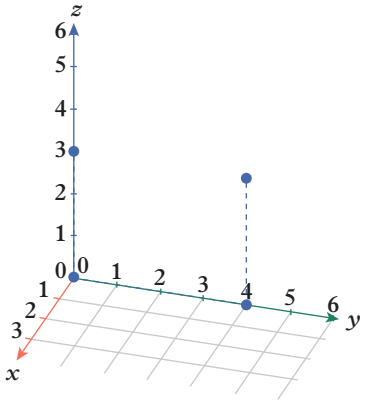
4 Add dashed line segments between the points to indicate the position of the point $B(0, 4, 3)$ in three-dimensional space.

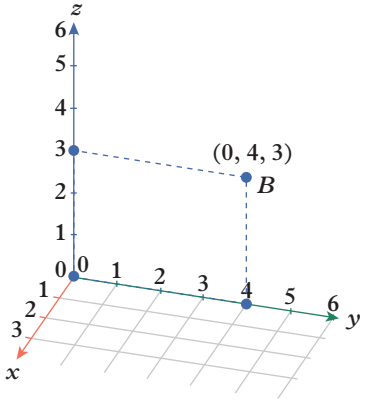
WRITE

b









Worked example 8.6C Drawing shapes and objects in three-dimensional space

Draw the following shape and object by first plotting the vertices in three-dimensional space.

a triangle with vertices (3, 1, 2), (1, 4, 3) and (5, 4, 5)

b rectangular prism with vertices (0, 0, 0), (3, 0, 0), (0, 4, 0), (3, 4, 0), (0, 0, 2), (3, 0, 2), (3, 4, 2) and (0, 4, 2)

THINK

a **1** Plot the points that are the vertices of the shape.

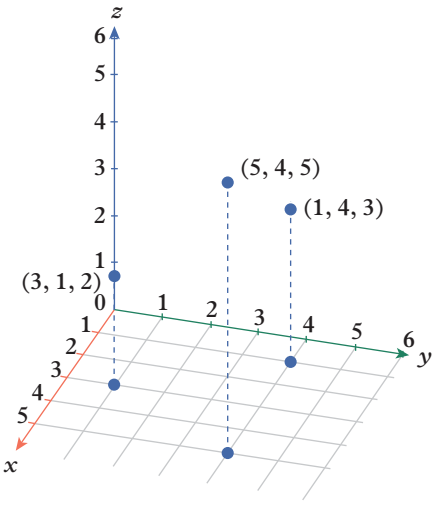
2 Join the vertices up to form the correct shape.

b **1** Plot the points that are the vertices of the object.

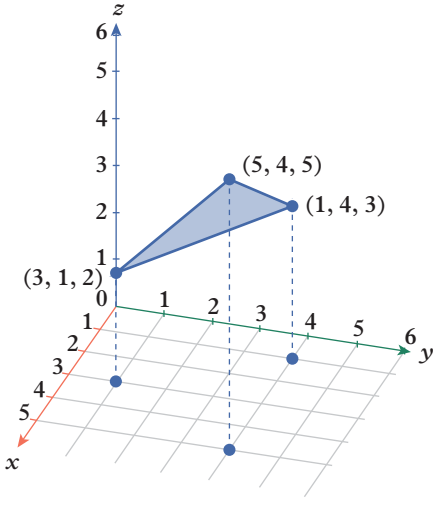
2 Join the vertices up to form the correct object. Use dashed line segments for hidden edges.

WRITE

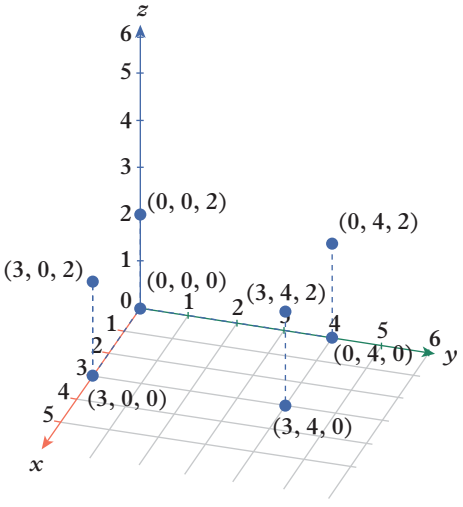
a **1**



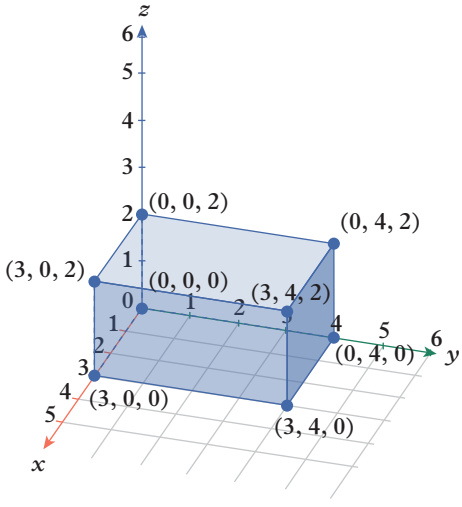
2



b **1**



2



Helpful hints

- ✔ Different points in three-dimensional space can look the same when drawn on a two-dimensional page. Clarify the position of your points by using dashed line segments.
- ✔ Use a ruler to make sure your points are the correct distance from each axis and parallel to the appropriate axis by sliding the ruler from the axis to the correct position without rotating it.

Learning pathways

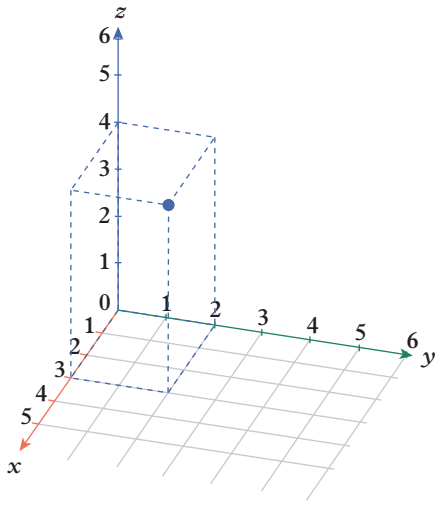
1, 2, 3–4(a), 5, 7–9(a, b)	1–3, 4–5(b–d), 6, 7(c–e), 8–10, 12	1, 2(d–f), 3–4(c), 5, 6, 7(d–f), 8–9(c, d), 10–12

ANS p578

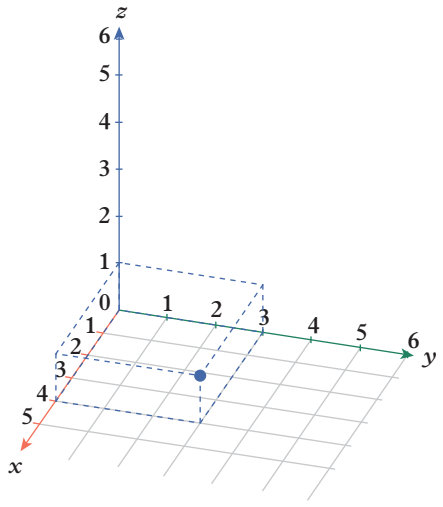
Exercise 8.6A: Understanding and fluency

WE 8.6A **1** Write the coordinates of each of the points shown below.

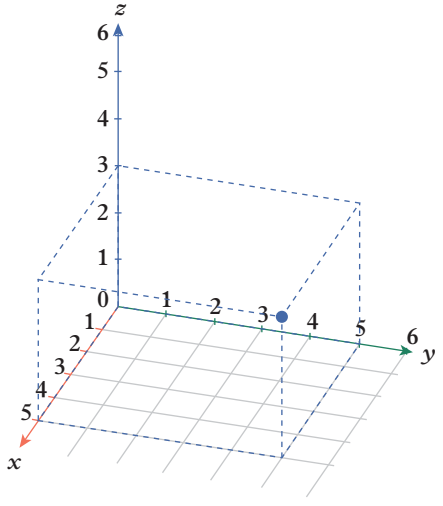
a



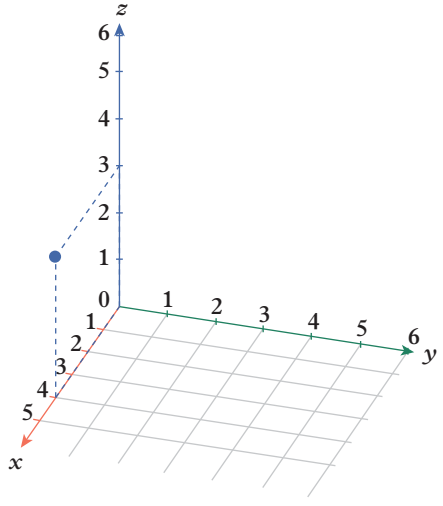
b



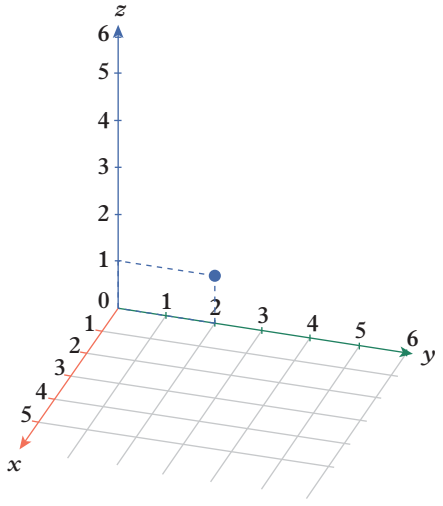
c



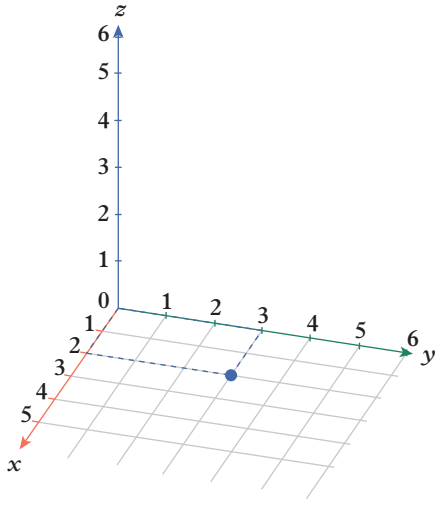
d



e



f



WE 8.6B **2** Plot each of the following points on its own diagram. Clearly show the x-, y- and z-coordinates using dashed lines.

a (3, 4, 2) **b** (2, 1, 5) **c** (5, 1, 4) **d** (4, 3, 0) **e** (3, 0, 0) **f** (0, 4, 5)

WE 8.6C **3** Draw the following shapes and objects by first plotting the vertices in three-dimensional space.

a triangle with vertices (4, 5, 5), (4, 3, 4) and (1, 1, 4)

b rectangular prism with vertices (0, 0, 0), (5, 0, 0), (0, 2, 0), (5, 2, 0), (0, 0, 4), (5, 0, 4), (0, 2, 4) and (5, 2, 4)

c rectangular prism with vertices (2, 1, 0), (3, 1, 0), (2, 5, 0), (3, 5, 0), (2, 1, 3), (3, 1, 3), (2, 5, 3) and (3, 5, 3)

4 A translation is a movement of a point, figure or shape in a given direction. Determine the coordinates of the images of the points after the listed translations.

a (1, 2, 5) is translated 5 units in the positive x-direction, 3 units in the positive y-direction, and 2 units in the negative z-direction.

b (6, 0, 5) is translated 4 units in the negative x-direction, 7 units in the positive y-direction, and 4 units in the positive z-direction.

c (12, 8, 3) is translated 8 units in the negative x-direction, 4 units in the negative y-direction, and 5 units in the positive z-direction.

d (0, 10, 14) is translated 4 units in the positive x-direction, 6 units in the negative y-direction, and 6 units in the negative z-direction.

5 Describe the location in three-dimensional space of points with the following characteristics.

a an x- and y-coordinate of 0

b a y- and z-coordinate of 0

c an x- and z-coordinate of 0

d an x-, y-, and z-coordinate of 0

e a z-coordinate of 0

f an x-coordinate of 0

g a y-coordinate of 0

6 Multistorey car parks use a three-dimensional coordinate system to identify individual car parks. An example of this is 'floor, row, position'. For example, a car on the third floor, second row, six parking spaces along is written as 326.

a Write the coordinates of the following cars in the car park.

i fifth floor, seventh row, eight parking spaces along

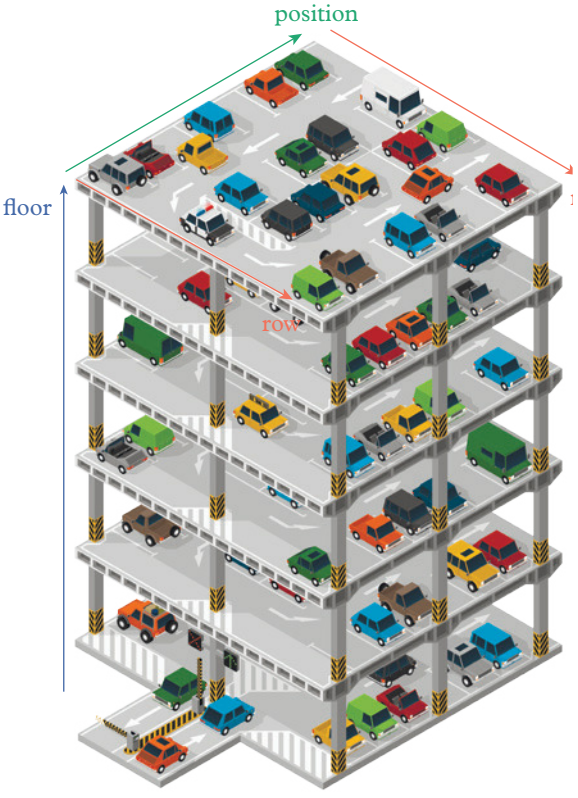
ii seventh floor, second row, one parking space along

iii second floor, eighth row, five parking spaces along

iv first floor, seventh row, nine parking spaces along

v ninth floor, seventh row, five parking spaces along

vi ground floor, fourth row, three parking spaces along



b Describe the position of the following cars in the car park.

i 512 **ii** 491 **iii** 149

iv 725 **v** 028 **vi** 294

c Describe how the following people can get from where they are to where they want to go. Describe movement between floors using 'up' and/or 'down', between rows using 'left' and/or 'right', and between positions using 'forwards' and/or 'backwards'.

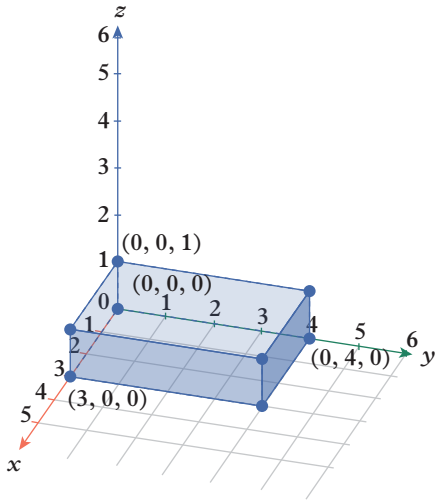
i Jacinta enters the car park from the connected shopping centre next to 408 and they want to return to their car that is parked at 591.

ii Lucinda parked their car at 789 and they want to go to the entrance of the shopping centre next to 203.

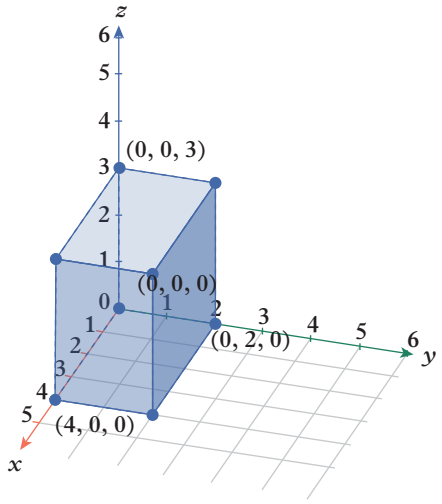
Exercise 8.6B: Problem solving and reasoning

7 Write down the coordinates of the remaining unlabelled vertices of the following prisms.

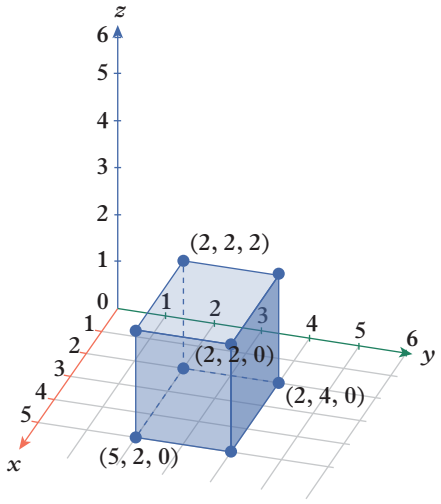
a



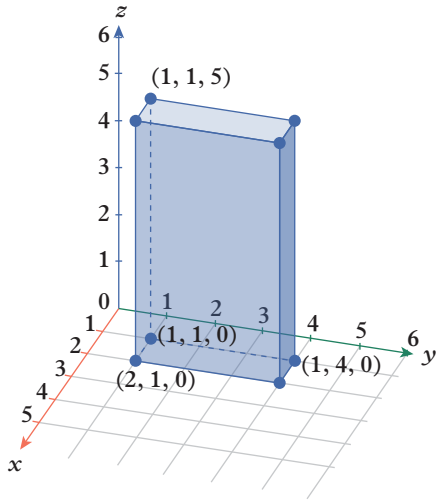
b



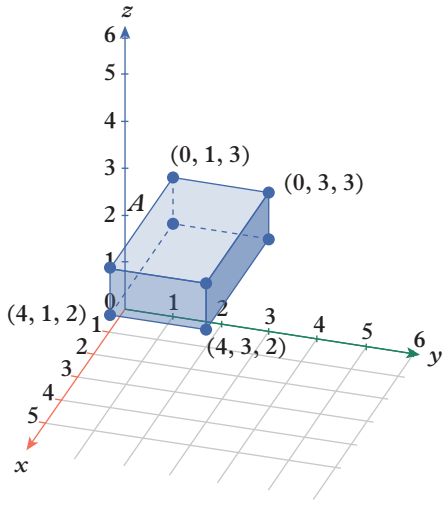
c



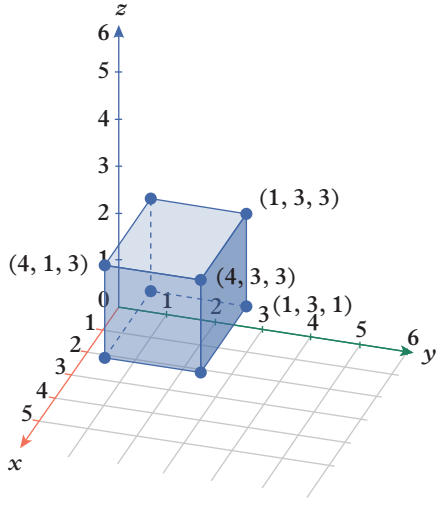
d



e



f



8 Write down the coordinates of the remaining four vertices of the following rectangular prisms.

a (0, 0, 0), (0, 2, 0), (4, 0, 0), (0, 0, 4)

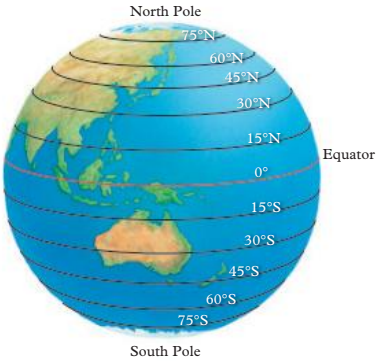
b (0, 0, 0), (0, 4, 2), (5, 4, 0), (5, 4, 2)

c (2, 2, 0), (5, 2, 0), (2, 3, 0), (2, 2, 1)

d (3, 1, 0), (5, 1, 0), (3, 3, 0), (3, 1, 3)

9 Calculate the volume of the prisms in question **8**, if each unit is equivalent to 1 cm.

10 Spherical coordinates are another three-dimensional coordinate system. A common example of spherical coordinates is coordinates on Earth, written as (latitude, longitude, altitude). Latitude measures the angle, in degrees, north or south from the equator. Longitude measures the angle, in degrees, east or west from the Prime Meridian. Altitude measures how far, in metres, the position is above sea level, where a negative altitude references a point below sea level. For example, a point that is 30° north of the equator, 45° east of the Prime Meridian and 50 m above sea level is written as (30° N, 45° E, 50 m).



a Write the coordinates of the following landmarks using their latitude, longitude and altitude.

i The Eiffel Tower is 48.8584° north of the equator, 2.2945° east of the Prime Meridian and 35 m above sea level.

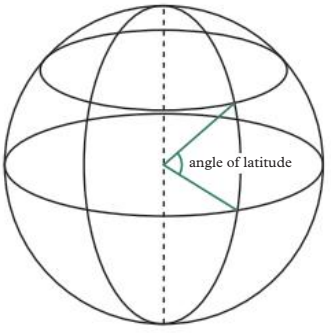
ii The Statue of Liberty is 40.6892° north of the equator, 74.0445° west of the Prime Meridian and 7.2 m above sea level.

iii Uluru is 25.3444° south of the equator, 131.0369° east of the Prime Meridian and 863 m above sea level.

iv Machu Picchu is 13.1631° south of the equator, 72.5450° west of the Prime Meridian and 2430 m above sea level.

v Mt. Fuji is 35.3606° north of the equator, 138.7274° east of the Prime Meridian and 3776 m above sea level.

vi The Eureka Skydeck is 37.8213° south of the equator, 144.9647° east of the Prime Meridian and 300 m above sea level.



b Describe the position of the following points in terms of their latitude, longitude and altitude.

i (60° N, 171° E, 709 m)

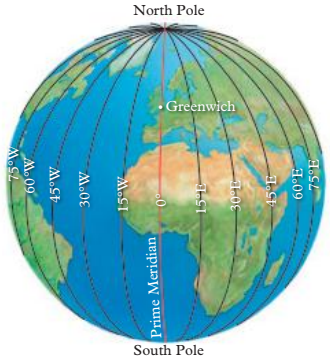
ii (25° N, 128° W, 545 m)

iii (5° S, 47° W, 135 m)

iv (79° S, 120° E, 642 m)

v (16° N, 63° W, 97 m)

vi (34° S, 152° E, 13 m)



Exercise 8.6C: Challenge

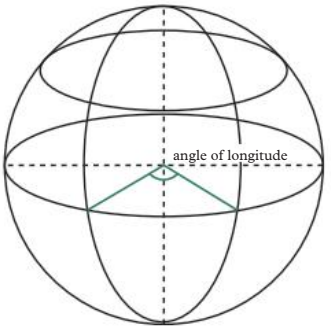
11 Find the coordinates of the images after the given transformations.

a (15° N, 124° E, 25 m) is rotated 30° north and 60° west, and translated 130 m up.

b (28° S, 48° W, 40 m) is rotated 30° north and 60° west, and translated 12 m down.

c (78° N, 4° W, 100 m) is rotated 11° north and 140° east, and translated 55 m up.

d (41° N, 19° W, 320 m) is rotated 120° south and 120° east, and translated 231 m down.



12 Pythagoras' theorem can be used to find lengths in three-dimensional space.

a Determine the distance of the point (3, 4, 0) from the origin. Hint: Make a right-angled triangle where the hypotenuse joins the origin and the point (3, 4, 0).

b Determine the distance of the point (3, 4, 12) from the origin.

Online resources:

- **Interactive skillsheet** Three-dimensional coordinates
- **BLM** 3D coordinate system
- **Investigation** Virtual maps versus street views
- **Quick quiz** 8.6

Lesson 8.7

Time

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the duration between two given times, and use time zones.

Inter-year links

Support Understanding time

Units of time

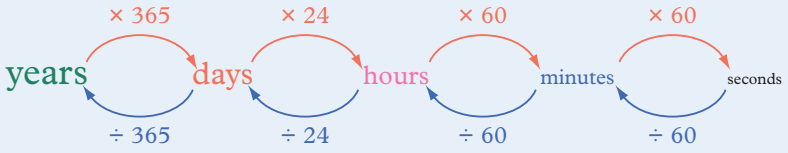
Key content video
Understanding time

- To convert between units of time, multiply or divide by the appropriate conversion factor.

  For example, 5 hours = 5 × 60
  = 300 minutes

Conversion factors

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days



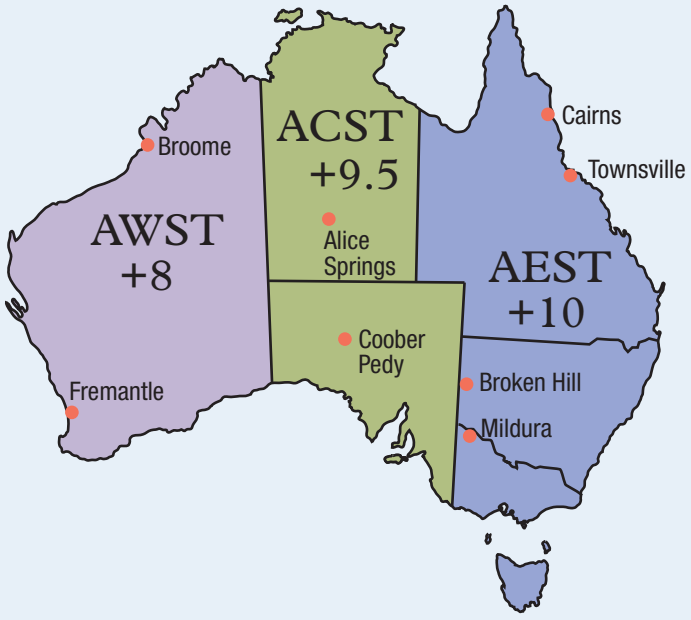
12- and 24-hour time

- In **12-hour time**, am and pm are used to describe whether a time is before or after midday. For example, 7 am represents 7 o'clock in the morning.
- In **24-hour time**, the time is given as a four-digit number. Each day starts at 0000 (midnight) and finishes at 2359.
  - → Times after midday have 12 hours added to them. For example, 2 pm is 1400 hours.
- A **duration** is the length between two given times.

Time zones

- The Earth is divided into 24 major **time zones**.
- A time zone map shows how many hours ahead or behind a time zone is from **Coordinated Universal Time (UTC)**, which is the time in Greenwich (in the UK).

  For example, UTC+10 means that the time is 10 hours ahead of UTC time.
- Australia covers three time zones:
  - → Australian Western Standard Time (AWST) is UTC+8.
  - → Australian Central Standard Time (ACST) is UTC+9.5.
  - → Australian Eastern Standard Time (AEST) is UTC+10.



Worked example 8.7A Converting between units of time

Convert 2 hours into seconds.

THINK

1. Convert from hours to minutes. To convert from a larger unit to a smaller unit, multiply by the conversion factor.
2. Convert from minutes to seconds.

WRITE

1 hour = 60 minutes

2 hours = 2 × 60
= 120 minutes

1 minute = 60 seconds

120 minutes = 120 × 60
= 7200 seconds

Worked example 8.7B Converting between 12- and 24-hour time

Convert:

a 9:20 pm into 24-hour time

b 1345 into 12-hour time.

THINK

a As the time is after midday (pm), add 1200 to this value to express the time in 24-hour time.

b As the given time is greater than 1200, subtract 1200 from 1345 and include pm to express the time in 12-hour time.

WRITE

a 9:20 pm = 0920 + 1200
= 2120

b 1345 = 1345 − 1200
= 1:45 pm

Worked example 8.7C Calculating durations of elapsed time

A car leaves home at 11:07 am and arrives at its destination at 4:12 pm. How long did it take to get there?

THINK

1. Count the number of minutes from the departure time to the next whole hour.
2. Count the number of hours from the time at the end of the previous step to the whole hour of the arrival time.
3. Count the number of minutes from the time at the end of the previous step to the arrival time.
4. Calculate the total duration by adding the individual time intervals. Remember that 60 minutes = 1 hour.

WRITE

Time from 11:07 am to 12:00 pm = 53 minutes

Time from 12:00 pm to 4:00 pm = 4 hours

Time from 4:00 pm to 4:12 pm = 12 minutes

53 mins + 4 hrs + 12 mins = 4 hrs + 65 mins
= 5 hrs and 5 mins

Worked example 8.7D Calculating times in different locations

The time is 8:35 am in Perth (AWST). Calculate the time in Brisbane (AEST).

THINK	WRITE
1 Calculate the difference between the two time zones. Eastern Standard Time (AEST) is UTC+10 and Western Standard Time (AWST) is UTC+8, so AEST is 2 hours ahead of AWST.	AEST is 2 hours ahead of AWST.
2 As AEST is 2 hours ahead of AWST, add 2 hours to the time in Perth to determine the time in Brisbane.	8:35 am + 2 hours = 10:35 am The time in Brisbane is 10:35 am.

Helpful hints

- ✔ All times expressed in 24-hour time must be given as four-digit numerals. For example, 7:35 am is 0735, not 735!
- ✔ When calculating the duration of elapsed time remember that there are only 60 minutes in 1 hour; for example, 73 minutes = 1 hour and 13 minutes.

Learning pathways

1–7, 8(1st, 2nd columns), 9(a, c, e), 10–12, 14, 17	1–7, 8(2nd, 3rd columns), 9(b, d, f), 10, 13, 15, 19(a–d)	4–7, 8(3rd column), 9(b, d, f), 16, 18–20

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Exercise 8.7A: Understanding and fluency

WE 8.7A

1 Convert each of these into minutes.

a 3 hours **b** 5 hours **c** 10 hours
d 120 seconds **e** 600 seconds **f** 6000 seconds

2 Convert each of these.

a 10 minutes into seconds **b** 5 minutes into seconds **c** 60 minutes into seconds
d 120 minutes into hours **e** 1800 minutes into hours **f** 240 minutes into hours

3 Convert each of these.

a 30 minutes into hours **b** 45 minutes into hours **c** 20 minutes into hours
d 15 seconds into minutes **e** 6 seconds into minutes **f** 48 seconds into minutes

4 Convert each of these.

a 3 hours into seconds **b** 10 hours into seconds **c** 6 hours into seconds
d 1.5 hours into seconds **e** 14 400 seconds into hours **f** 9000 seconds into hours

5 How many:

a minutes in a day **b** hours in a week **c** seconds in a day
d hours in a leap year **e** seconds in a week **f** hours in June?

WE 8.7B **6** Convert these times into 24-hour time.

a 2:30 am **b** 4:15 pm **c** 1:48 pm **d** 11:11 pm

7 Convert these times into 12-hour time.

a 1925 **b** 1200 **c** 2218 **d** 0958

WE 8.7C **8** Calculate the duration between each pair of times.

a 4:00 pm and 9:00 pm **b** 8:00 am and 11:00 pm **c** 11:30 pm and 2:30 am

d 0900 and 1400 **e** 1330 and 2230 **f** 1600 and 2130

g 5:27 pm and 7:45 pm **h** 12:52 pm and 6:05 pm **i** 6:43 pm and 2:37 am

j 0004 and 1931 **k** 1919 and 2222 **l** 0636 and 1421

WE 8.7D **9** Find the time in the given time zone.

a 3:45 pm (UTC+8) in UTC+10 **b** 3:45 pm (UTC+10) in UTC+8

c 11:05 am (UTC+9.5) in UTC+10 **d** 11:05 am (UTC+9.5) in UTC+8

e 2345 (UTC+10) in UTC+9.5 **f** 2345 (UTC+8) in UTC+9.5

10 The time is given in one of Australia's time zones. Determine what time it is in the other places.

a The time is 1:49 pm AEST. What is the time in:

i Broome, WA **ii** Alice Springs, NT **iii** Cairns, QLD?

b The time is 2:28 am ACST. What is the time in:

i Coober Pedy, SA **ii** Broken Hill, NSW **iii** Fremantle, WA?

c The time is 1417 AWST. What is the time in:

i Geelong, VIC **ii** Perth, WA **iii** Adelaide, SA?

d The time is 0040 AEST. What is the time in

i Darwin, NT **ii** Townsville, QLD **iii** Geraldton, WA?

11 Consider that there are 60 minutes in an hour.

a Convert 15 minutes into a fraction of an hour.

b Write this fraction as a decimal.

c Explain why 1.25 hours is not equivalent to 1 hour and 25 minutes.

Exercise 8.7B: Problem solving and reasoning

12 A family takes a road trip from Melbourne to Cairns. Their trip is plotted in the table below, showing when they leave, arrive and when they stop for lunch.

	Day 1	**Day 2**	**Day 3**	**Day 4**
Departure time	8:30 am	8:00 am	8:15 am	8:20 am
Start lunch	12:30 pm	12:30 pm	12:55 pm	1:05 pm
Finish lunch	1:00 pm	1:00 pm	1:40 pm	1:30 pm
Arrival time	7:30 pm	6:00 pm	7:45 pm	7:15 pm

a How long did they take from departure to arrival on their first day?

b How long was their lunch break on the first day?

c Use your answers to parts **a** and **b** to find how long they drove for on the first day.

d Find how long they drove for on the second, third and fourth days.

e How many hours driving did they do for the whole trip?

13 Mistakes can easily be made when calculating durations of time.

i Determine the error each person made when calculating the following durations.

ii Determine the correct duration.

a 2:45 pm to 6:25 pm

2:45 + 00:15 = 2:00
2:00 + 00:25 = 2:25
15 + 25 = 40 min
2:25 + 4:00 = 6:25
4 h 40 min

b 2:45 am to 6:25 pm

2:45 + 00:15 = 3:00
3:00 + 00:25 = 3:25
15 + 25 = 40 min
3:25 + 3:00 = 6:25
3 h 40 min

c 2:45 pm to 6:25 am

2:45 + 00:15 = 3:00
3:00 + 00:25 = 3:25
15 + 25 = 40 min
3:25 pm + 16:00 = 6:25 am
16 h 40 min

14 Andrew was studying for an upcoming test. He spent 85 minutes making flash cards, had a 10 minute break, spent 30 minutes quizzing himself using the flash cards, had a 30 minute lunch break, spent another 30 minutes quizzing himself using the flash cards, then sat a practice test for 50 minutes. If it was 11:10 am when he started, at what time did he finish the practice test?

15 Jaime has a bike with an odometer on it that tells him how many metres he has travelled. When he leaves for a bike ride along the Toogoom Trail, the odometer is at 1391 and the time is 1104. Upon arriving at Toogoom, the odometer reads 5403 and the time is 1334.

a How far did Jaime ride?

b How long did it take him to get to Toogoom?

c What is Jaime's average speed in m/s correct to two decimal places?

16 Harry and family are driving from Cairns, QLD, to Eucla, WA, overnight. They leave Cairns at 8 pm and Harry falls asleep during the drive. He is awoken by his parents when they arrive at the hotel in Eucla. He sees the clock in the lobby of the hotel that says it is 10 am. Harry assumes it took them 14 hours to get there.

a Why is Harry wrong?

b How long did it actually take Harry and family to get to the hotel?

c If the car travelled at an average speed of 80 km/h over the whole trip, approximately how many kilometres did they drive?

d Harry checks the route on his phone and sees it was 1490 km. Calculate the error between the approximation in part **c** and the actual distance.

e Calculate the percentage error of the approximation by dividing the error from part **d** by the actual distance and multiplying the result by 100%. Give your answer correct to the nearest whole number.

17 Use the provided time zone map to decide which place is not in the same time zone as the others.

a **A** Coral Sea Islands **B** Papua New Guinea
C Guam **D** Solomon Islands

b **A** Jakarta, Indonesia **B** Makassar, Indonesia
C Manila, Philippines **D** Shanghai, China

18 Calculate the duration between the following times. Write all answers in hours and minutes.

a 2:45 pm today and 11:10 am tomorrow

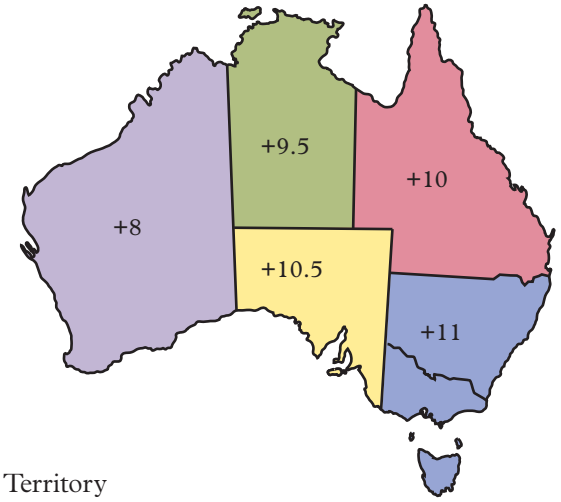
b 10:15 am on 22/08 and 11:00 am on 24/08

c 3:30 pm on 19/10 and 9:00 am on 22/10

d 1114 on 14/11/2022 and 1530 on 14/11/2023. Hint: 2023 is not a leap year.

Exercise 8.7C: Challenge

19 Daylight saving time is when people put their clocks forward 1 hour on the first Sunday in October to have an 'extra' hour of daylight in the evenings. The clocks then move backwards 1 hour on the first Sunday in April. In Australia, the Australian Capital Territory, New South Wales, South Australia, Tasmania and Victoria observe daylight saving time. The Northern Territory, Queensland and Western Australia do not. This diagram shows the different time zones in summer when daylight saving time is observed.



a In summer, how many hours is:

i Victoria ahead of Queensland

ii South Australia ahead of Western Australia

iii the Australian Capital Territory ahead of the Northern Territory

iv Tasmania ahead of Western Australia?

b In winter, if it is 6 pm in South Australia, what time is it in:

i Western Australia **ii** the Northern Territory **iii** Victoria **iv** Queensland?

c Repeat part **b**, but for summer.

d What is special about the time difference between South Australia and Queensland in summer and winter?

e Find the time in Darwin when it is:

i 4 pm on 27 January in Melbourne

ii 3 am on 16 July in Cairns

iii midday on 4 December in Perth

iv 9:30 am on 25 December in Sydney

v 5:45 pm on 30 June in Hobart

vi 11 am on 30 November in Adelaide.

20 Zoë and her friend Wes want to take a holiday in Thailand (UTC+7) during July.

a Zoë's flight leaves Melbourne (UTC+10) at 1:00 pm and arrives in Thailand at 7:00 pm. How long will the flight be?

b Wes has to take a different flight. His flight leaves at 1445. If his flight takes the same time, what time will he land in Thailand?

c On the way home they plan to stop in Bali (UTC+8). If they leave Thailand at 1000 and arrive in Bali at 1530, how long is this flight?

d What time will Zoë and Wes land at home if they leave Bali at 1330 and their flight is 8 hours long?

Online resources:

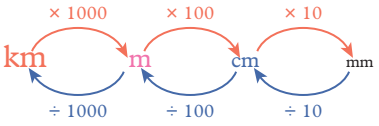
- Interactive skillsheet: Converting between units of time
- Interactive skillsheet: Time zones
- Worksheet: Converting units of time
- Quick quiz: 8.7
- Interactive skillsheet: Durations of time
- Investigation: Daylight saving time
- Worksheet: Differences in time
- Desmos activity: Talking time
- Worksheet: Using a time zone map

Lesson 8.8

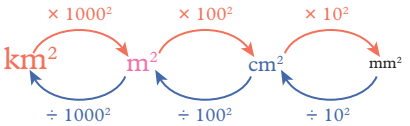
Review: Units of measurement

Module summary

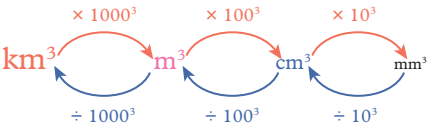
Units of length



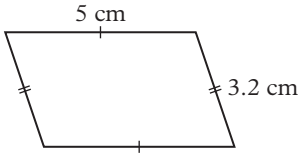
Units of area



Units of volume

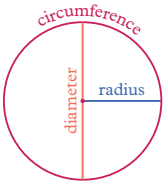


Perimeter



5 + 3.2 + 5 + 3.2 = 16.4 cm

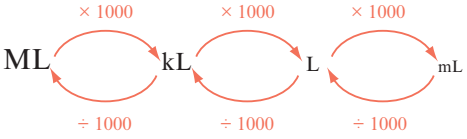
Circumference and area of a circle



$C = \pi d$ or
$C = 2\pi r$

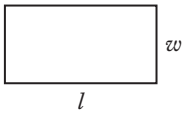
$A = \pi r^2$

Units of capacity



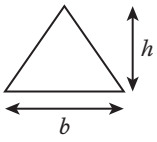
Area

Rectangle



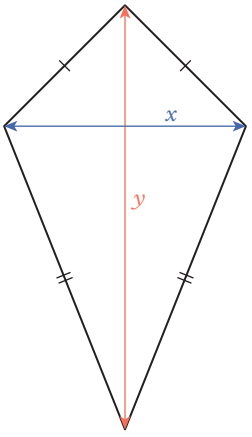
$A = lw$

Triangle



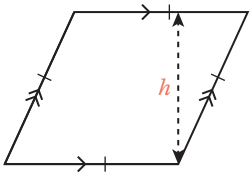
$A = \frac{1}{2}bh$

Kite

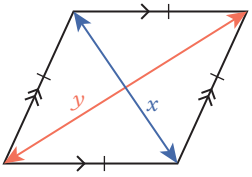


$A = \frac{1}{2}xy$

Rhombus

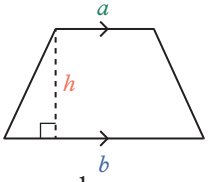


$A = bh$



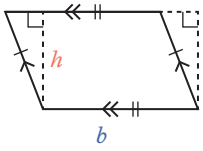
$A = \frac{1}{2}xy$

Trapezium



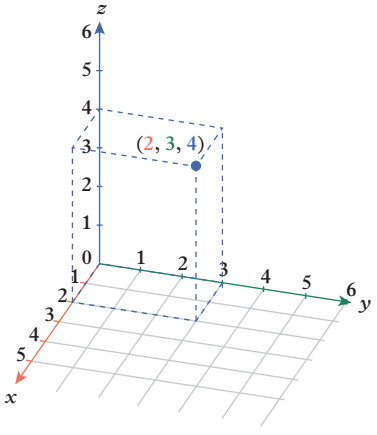
$A = \frac{1}{2}(a + b)h$

Parallelogram



$A = bh$

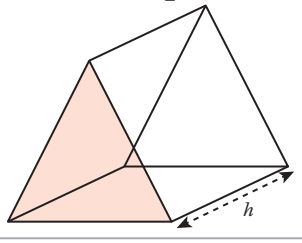
3D coordinates



Units of time

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days

Volume of prisms



$V = Ah$

where A is the area of the base and h is the height

12- and 24-hour time

7 am = 0700

2 pm = 1400

Time zones

Australian Western Standard Time (AWST) is UTC+8.

Australian Central Standard Time (ACST) is UTC+9.5.

Australian Eastern Standard Time (AEST) is UTC+10.

ANS p581

Review questions 8.8A: Mathematical literacy review

Review quiz
Take the module review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- 12-hour time
- 24-hour time
- area
- base
- capacity
- circumference
- conversion factor
- Coordinated Universal Time (UTC)
- diameter
- duration
- kite
- length
- parallelogram
- perimeter
- polygon
- prism
- radius
- rectangle
- rectangular prism
- rhombus
- three-dimensional coordinates
- time zones
- trapezium
- triangle
- triangular prism
- volume
- x-axis
- x-coordinate
- y-axis
- y-coordinate
- z-axis
- z-coordinate

1 Which axis needs to be added to the xy-plane to allow for coordinates in three-dimensions to be displayed?

2 How are the radius and circumference of a circle related?

3 Explain how to convert from 24-hour time to 12-hour time.

4 Identify the key terms being referenced in each of these definitions.

a The space occupied by a 2D shape.

b A 3D object with a consistent cross-section, in the shape of a polygon, parallel to the base.

5 Using an example, provide a definition in your own words for the following key terms.

a perimeter **b** duration

6 Complete the following sentences using words from the key terms list.

a The volume of any _____ can be found by multiplying the area of the _____ by the height.

b To find the area of a _____, multiple its base by its _____ then divide by 2.

Review questions 8.8B: Multiple choice

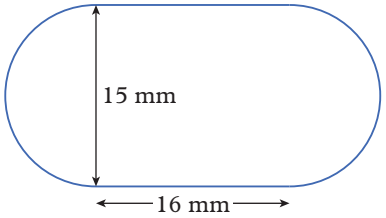
8.1 **1** What is 7.84 m equal to?

A 0.784 km **B** 7840 mm **C** 78.4 cm **D** 0.0784 km

8.1 **2** A circle has a diameter of 6 cm. What is its radius?

A 12 cm **B** 6π cm **C** 36π cm **D** 3 cm

8.1 **3** Which of these is closest to the total perimeter of this shape?



A 79.1 mm **B** 47 mm **C** 62 mm **D** 63.1 mm

8.2 **4** A triangle has a base length of 10 cm and an area of 20 cm^2. What is its height?

A 1 cm **B** 2 cm **C** 4 cm **D** 200 cm

8.2 **5** 2000 m^2 is not equal to which of the following?

A 40 m × 50 m **B** 2 000 000 000 mm^2 **C** 20 000 000 cm^2 **D** 2 km^2

8.3 **6** A rhombus has diagonals of lengths 5 cm and 6 cm. What is its area?

A 30 cm^2 **B** 60 cm^2

C 56 cm^2 **D** 15 cm^2

8.4 **7** Which of these is closest to the area of the shape in question **3**?

A 946.9 mm^2 **B** 754 mm^2

C 480 mm^2 **D** 416.7 mm^2

8.5 **8** What is the volume of an irregular prism with base area 2.5 cm^2 and length 10 mm?

A 250 mm^3 **B** 25 mm^3

C 2.5 cm^3 **D** 25 cm^3

8.5 **9** What is 3 cm^3 equal to?

A 30 mm^3 **B** 300 mm^3

C 300 m^3 **D** 3000 mm^3

8.5 **10** Which pair of volume and capacity measurements are equivalent?

A 40 mL and 4 cm^3 **B** 1 m^3 and 10 L

C 250 mL and 2.5 L **D** 500 cm^3 and 0.5 L

8.6 **11** Which of the following points is on the y-axis?

A (2, 0, 6) **B** (0, 4, 0)

C (2, 4, 0) **D** (0, 0, 2)

8.7 **12** What is the time duration between 1156 and 2014?

A 7 hours, 18 minutes **B** 8 hours, 42 minutes

C 8 hours, 18 minutes **D** 9 hours, 18 minutes

8.7 **13** If the time is 4:45 pm in Melbourne (UTC+10), what is the time in Perth (UTC+8)?

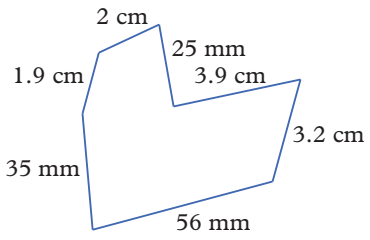
A 2:45 pm **B** 4:15 pm

C 4:45 pm **D** 6:45 pm

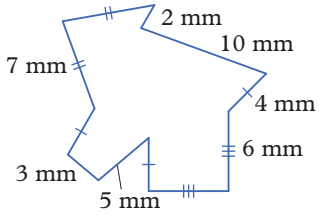
Review questions 8.8C: Short answer

8.1 **1** Calculate the perimeter of each shape.

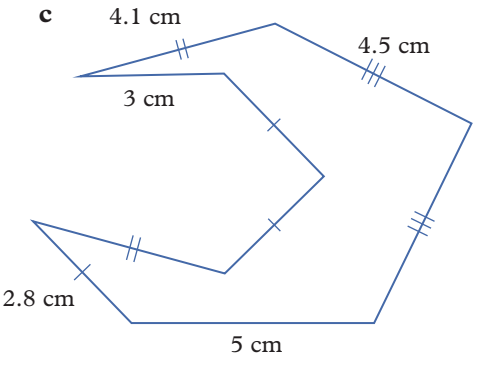
a



b

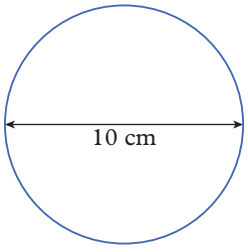


c

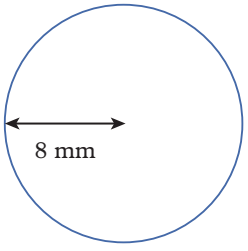


8.1 **2** Calculate the circumference of each circle correct to two decimal places.

a

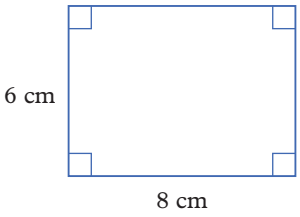


b



8.2 **3** Calculate the area of each shape.

a



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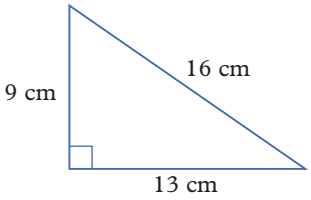
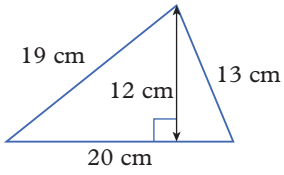
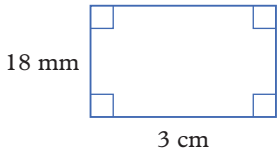


c



d

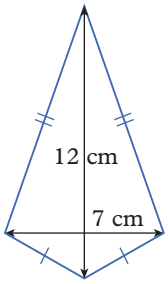


8.2 **4** Convert each area measurement.

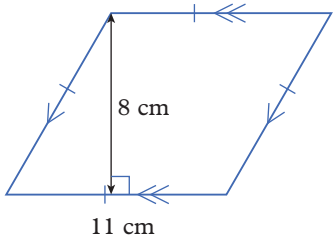
a 5 m^2 into cm^2 **b** 50 mm^2 into cm^2

8.3 **5** Calculate the area of each shape.

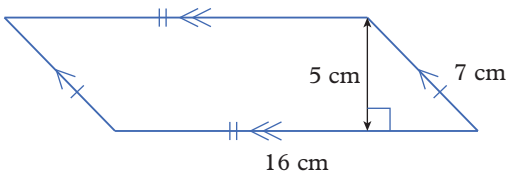
a



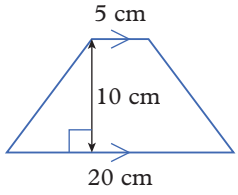
b



c



d



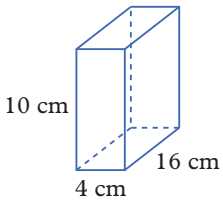
8.4 **6** Calculate the area of each circle in question **2**. Give your answers correct to two decimal places.

8.5 **7** Convert each measurement.

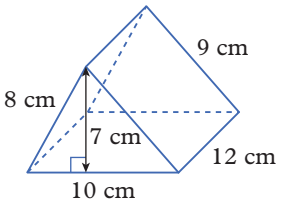
a 3.5 cm^3 into mm^3 **b** 400 cm^3 into m^3 **c** 35 000 mL to L **d** 0.124 L to mL

8.5 **8** Calculate the volume of each object.

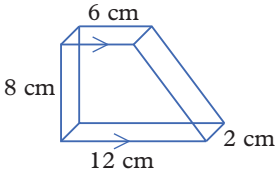
a



b



c



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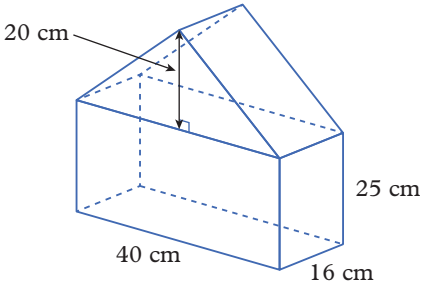


8.6 **9** Plot each of the following points on its own diagram. Clearly show the x-, y- and z-coordinates using dashed lines.

a (3, 4, 2) **b** (0, 1, 5) **c** (3, 0, 4) **d** (4, 3, 0)

8.7 **10** Calculate the duration between the following times.

a 3:45 pm to 7:30 pm
b 10:05 pm to 6:37 am
c 0712 to 1546
d 2055 to 0322

8.7 **11** Convert the following times.

a 7:23 am UTC+10 to UTC+8
b 2:49 pm UTC+9.5 to UTC+10
c 9:10 am AWST to AEST
d 1:20 pm AEST to ACST

Review questions 8.8D: Mathematical modelling

1 A friend wants to have a ball pit at their birthday party. They have set aside a rectangular space in their backyard that is 3 m long and 2 m wide.

a What area would this cover?

b How much fencing would be needed to go around the ball pit?

c Your friend can only buy the wood that they need for the fencing in planks that are 20 cm wide and 1 m tall. How many wood planks are needed for the fencing?

d Your friend wants to paint the wooden fencing inside and out with two coats of paint. What is the area to be painted? Ignore the thickness of the wood.

e They also want to paint a diamond on the front fence of the ball pit. From point to point (that is, its diagonals) it measures 55 cm vertically and 30 cm horizontally.

What is the area of this diamond?

f Around the diamond will be six circles representing the coloured balls in the ball pit. Each circle has a diameter of 6.6 cm.

i What is the circumference of each circle correct to two decimal places?

ii Find the total area to be painted for the six circles correct to two decimal places.

iii What is the volume of the pit?

iv If each ball to go into the pit has a volume of 150 cm^3, how many could theoretically fit into the pit? Be careful with unit conversions!

v Your friend finds that only about 25 000 balls fit into the pit. What explanation can you offer for this difference?

2 Oscar is packing for a weekend trip to Western Australia. It will take him 15 minutes to pack, after which he will get a taxi to the airport, which is a 40-minute drive away from his home.

a It takes 7 minutes for the taxi to arrive after Oscar has finished packing. If Oscar starts packing at 2:35 pm, at what time does he arrive at the airport?

b Oscar's airplane is scheduled to leave at 17:05 and boarding commences twenty minutes before take-off. If it takes Oscar 12 minutes to check in his bags and another 5 minutes to walk to his gate, how long does Oscar have at the gate before boarding commences?

c The flight to Perth takes 4 hours and 10 minutes. At what time, in Melbourne time, does Oscar's plane land in Perth? Give your answer in 12-hour time.

d Oscar lives in Melbourne, which is in the UTC+10 time zone, whereas Perth is in the UTC+8 time zone.

i At what time, in Perth time, does Oscar's plane land in Perth?

ii It takes 18 minutes for Oscar to collect his bags, 3 minutes to get into a taxi, and 17 minutes for the taxi to drive to Oscar's hotel. At what time does Oscar arrive at his hotel (in Perth time)?

iii Oscar wants to call his wife at 8:00 am Melbourne time the following morning. If he wants to wake up 15 minutes before the phone call, at what time does he need to wake up (in Perth time)?

3 A swimming pool is in the shape of a kite with diagonal lengths of 12 metres and 10 metres.

a What is the area of the surface of the swimming pool?

b The swimming pool has a consistent depth of 2 metres. What is the internal volume of the swimming pool?

c What is the capacity of the swimming pool in litres?

d How much water is required to fill the swimming pool to a depth of 1.6 metres?

e If a hose can fill the swimming pool at a rate of 40 litres per minute, how many hours would it take to fill the swimming pool to a depth of 1.6 metres?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert between units of length ☐ Find the perimeter of two-dimensional shapes ☐ Find the circumference of a circle	☐ Go back to Lesson 8.1 Length and perimeter
☐ Convert between units of area ☐ Calculate the area of rectangles and triangles	☐ Go back to Lesson 8.2 Area of rectangles and triangles
☐ Calculate the area of parallelograms, kites, rhombuses and trapeziums	☐ Go back to Lesson 8.3 Area of quadrilaterals
☐ Calculate the area of a circle ☐ Calculate the radius, diameter or circumference of a circle, given its area	☐ Go back to Lesson 8.4 Area of a circle
☐ Calculate the volume and capacity of right prisms ☐ Convert between units of volume and capacity	☐ Go back to Lesson 8.5 Volume and capacity
☐ Identify the coordinates of a point in three dimensional space ☐ Plot a point in three dimensional space using its coordinates ☐ Work with shapes and objects whose vertices are given as coordinates in three dimensional space	☐ Go back to Lesson 8.6 Three-dimensional space
☐ Convert between units of time ☐ Calculate the duration between two given times ☐ Use time zones to calculate the time in different locations	☐ Go back to Lesson 8.7 Time

9

Congruence and similarity

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ The Cartesian plane
- ✔ Triangle properties
- ✔ Equivalent fractions

Curriculum links

- Identify the conditions for congruence and similarity of triangles and explain the conditions for other sets of common shapes to be congruent or similar, including those formed by transformations (VC2M8SP01)
- Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems explaining reasoning (VC2M8SP02)

Materials

- ✔ Compass
- ✔ Ruler
- ✔ Protractor

Lesson 9.1

Transformations

Learning intentions

By the end of this lesson you will be able to ...

✔ describe and perform transformations on the Cartesian plane.

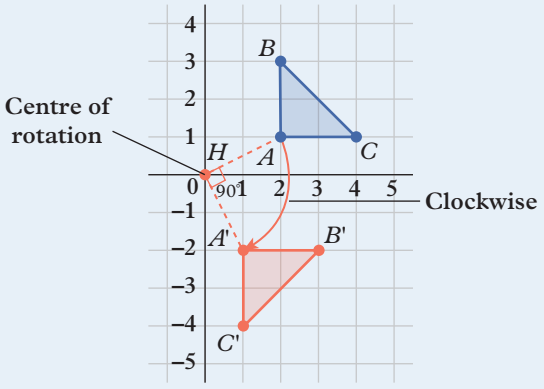
Inter-year links

Support	Transformations
Year 7	8.3 Translations
Year 7	8.4 Reflections and rotations

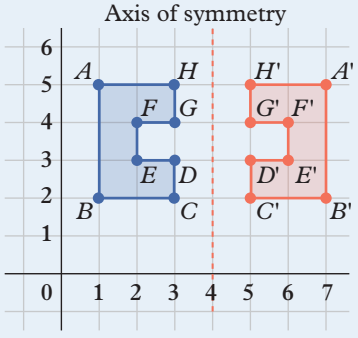
Transformations

Key content video
Transformations

- **Rotations**, **reflections** and **translations** are **isometric transformations**. Isometric transformations do not change the shape or size of a shape or object.
  → The angle and side measurements of a shape do not change after an isometric transformation.
- A transformed shape or object (also called an **image**) should always be given a name that indicates that it is a transformation; for example, the image of A is A′.
- A rotation turns a shape about a fixed point. A rotation requires:
  1 the centre of rotation
  2 the angle of rotation
  3 the direction of rotation.
  → When performing a rotation, first identify a single point to concentrate on. This is called the **point of focus**.

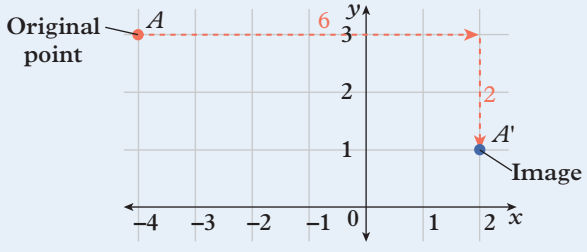


- A reflection is performed in an axis of symmetry. Each point in the image must be the same distance from the axis of symmetry as the corresponding original point.



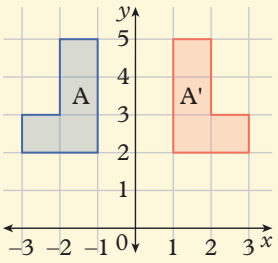
- A translation is when a shape or object is moved without turning or changing size.
  → Translations are recorded by counting how many units the shape slides up or down and left or right.

Translation 6 units right and 2 units down



Worked example 9.1A Describing transformations

Provide a full description of the transformation shown. Include the type of transformation as well as a description of the specific movement.



THINK

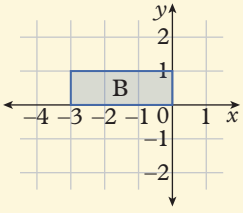
1. Determine if the shape has been translated, rotated or reflected.
2. Determine which line the shape has been reflected in.
3. Write your answer.

WRITE

The shape has been reflected in the y-axis.

Worked example 9.1B Performing a transformation

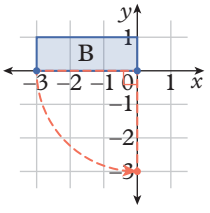
Rotate this shape 90° anti-clockwise about the origin.

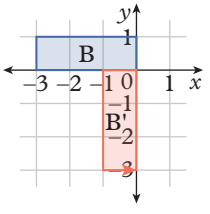


THINK

1. Select a point of focus on the shape. Since the given point of rotation is on the shape, it is easier to choose a point of focus that has an edge in common with it. Rotate this edge 90° anti-clockwise around the point of rotation.
2. Draw in the remainder of the image. The rotated shape must be the same size as the original shape.
3. Name the final image.

WRITE





Helpful hints

- ✔ When transforming a shape, every point on the original shape must be transformed.
- ✔ Start by transforming a single point on a shape, rather than trying to transform the whole shape in one go.

Learning pathways

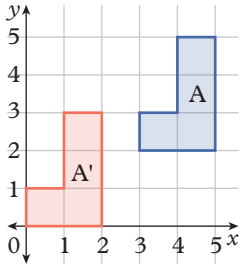
1–4, 8, 9, 10(a, b), 11	1, 3–10	1, 3, 5, 6, 9, 10, 12, 13

ANS p582

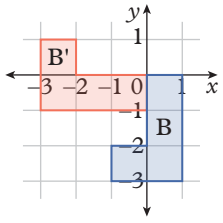
Exercise 9.1A: Understanding and fluency

WE 9.1A **1** Provide a full description of each of the transformations shown. Include the type of transformation as well as a description of the specific movement.

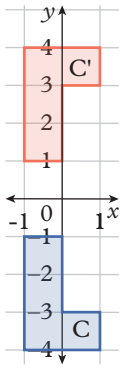
a



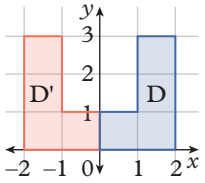
b



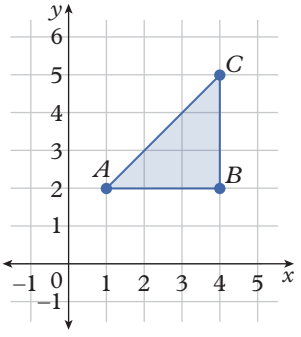
c



d



WE 9.1B **2** Transform the triangle shown on the right using the descriptions provided. Start from the original triangle each time.



a Translate 3 units left and 2 units up.

b Rotate 90° clockwise about the origin.

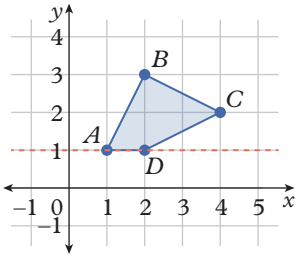
c Reflect in the y-axis.

d Rotate 180° clockwise about the origin.

e Translate 1 unit right and 4 units up.

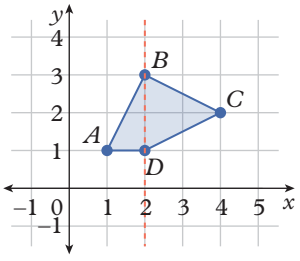
3 Transform the quadrilateral shown below using the provided descriptions. Draw each image on its own Cartesian plane.

a



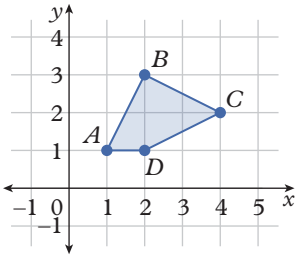
Reflect in the line AD.

b



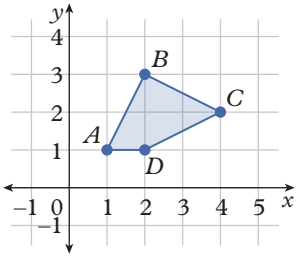
Reflect in the line BD.

c



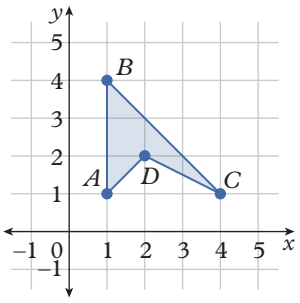
Translate 5 units left and 5 units down

d



Rotate 90° clockwise about the origin.

4 Transform the quadrilateral shown on the right using the provided descriptions. Start from the original quadrilateral each time. Draw each image on its own Cartesian plane.



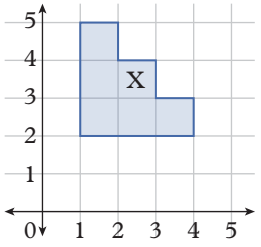
a Rotate 90° clockwise about the origin, then translate 2 units left and 3 units down.

b Translate 2 units left and 3 units down, then rotate 90° clockwise about the origin.

c Reflect in the y-axis, then rotate 90° anti-clockwise about the origin.

d Rotate 90° anti-clockwise about the origin, then reflect in the y-axis.

5 Perform each transformation on this shape. Start from the original shape each time.



a Reflect in the x-axis.

b Translate 4 units left and 3 units up.

c Reflect in the y-axis.

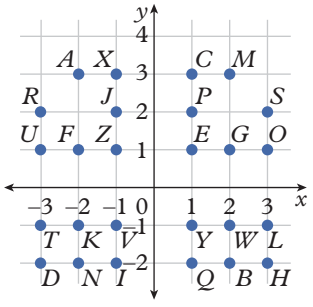
d Rotate 90° clockwise about the origin.

e Translate 1 unit right and 2 units down.

f Rotate 90° anti-clockwise about the origin.

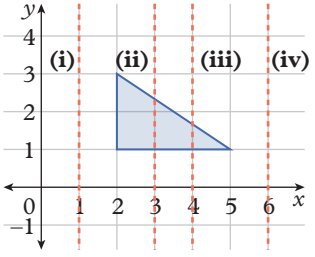
Exercise 9.1B: Problem solving and reasoning

6 Begin at point (2, 3). Apply the following transformations in order from start to finish and write down the letter at each new point to reveal a hidden phrase.



→ Reflect in the y-axis
→ Rotate 90° anti-clockwise about the origin
→ Translate 4 units right and 3 units up
→ Rotate 90° degrees clockwise about the origin
→ Translate 2 units right and 2 units up
→ Reflect in the y-axis
→ Rotate 180° anti-clockwise about the origin
→ Reflect in the x-axis
→ Rotate 360° clockwise about the origin
→ Translate 5 units left and 2 units down

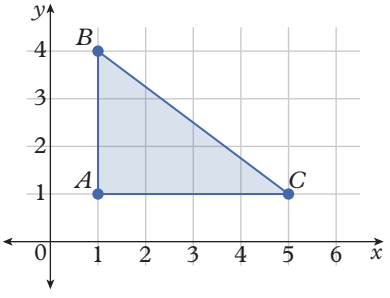
7 Consider the diagram on the right.



a Copy the triangle and reflect it in each of the four mirror lines shown. For each reflection, start from the original image.

b What do you notice about the difference between images created by a mirror line that is on one side of the object as opposed to images created by a mirror line that cuts through the object?

8 Consider the following right-angled triangle.



a If each grid square represents 1 cm^2, what is the base length (AC) and height (AB) of the triangle?

b Use Pythagoras' theorem (from Module 7) to calculate the length of the hypotenuse (BC) of the triangle.

c Reflect the triangle in the x-axis.

d What is the corresponding base length, height and hypotenuse of the reflected triangle?

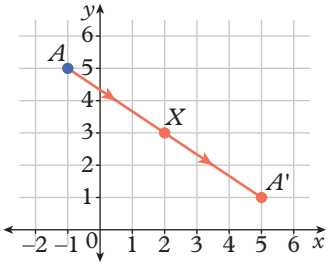
e Considering your answers to parts **a**, **b** and **d**, how does a reflection affect the side measurements of a shape?

9 Answer true or false for the following statements.

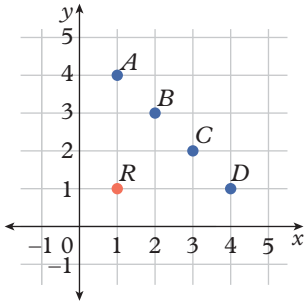
a When a shape undergoes an isometric transformation, the angle and side measurements of the shape stay the same.

b When a shape undergoes a translation, the orientation of the shape does not change.

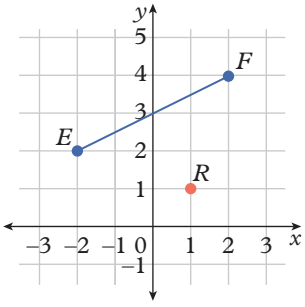
10 To reflect in a point, X, draw a straight line from the point you are reflecting, A, to the point you are reflecting in. Extend this line by the same distance to locate the position of the image, A'.



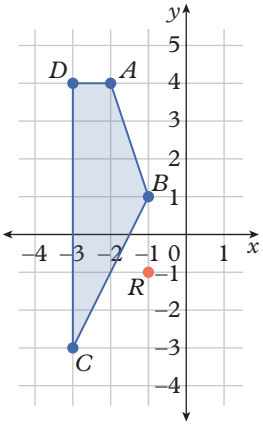
a Reflect the points A, B, C and D in the point R.



b Reflect the line segment $\overline{EF}$ in the point R.



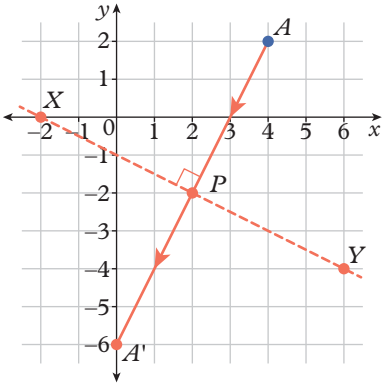
c Reflect the quadrilateral $ABCD$ in the point R.



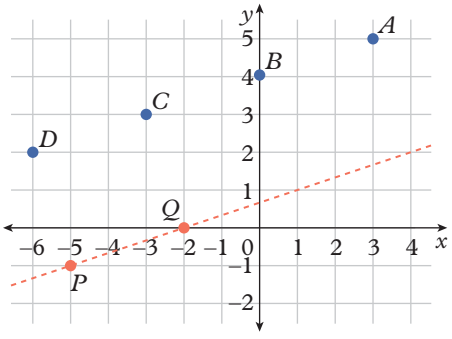
11 To reflect a point A in a line $\overline{XY}$, you need to identify the point, P, on $\overline{XY}$ that is closest to A.

Use a ruler to locate the point P on $\overline{XY}$, and draw a straight line from point P to A. The lines $\overline{AP}$ and $\overline{XY}$ will be perpendicular (at right angles) to each other.

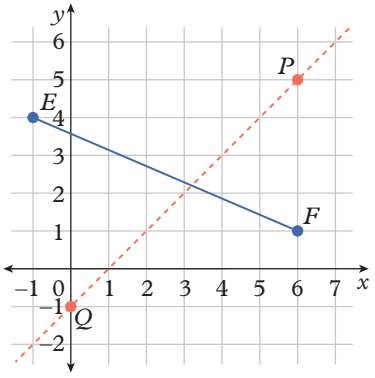
Finally, extend $\overline{AP}$ by the same distance on the other side of $\overline{XY}$ to locate the position of the image A'.



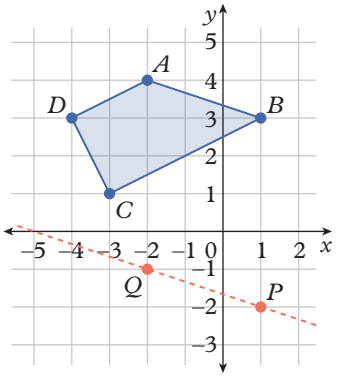
a Reflect points A, B, C and D in the line $\overline{PQ}$.



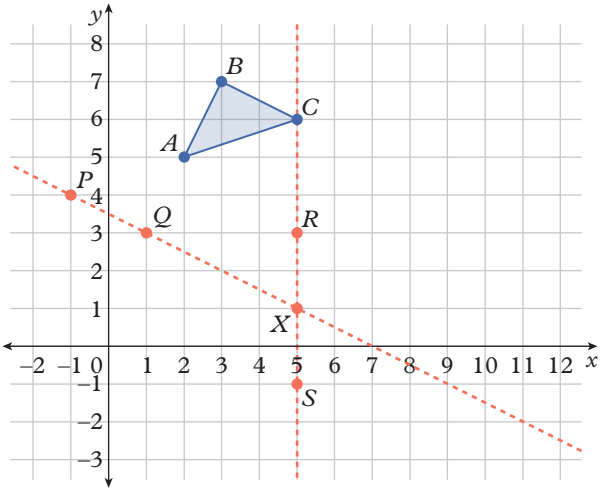
b Reflect the line segment $\overline{EF}$ in the line $\overline{PQ}$.



c Reflect the quadrilateral $ABCD$ in the line $\overline{PQ}$.



12 Consider $\triangle ABC$ and the two lines $\overline{PQ}$ and $\overline{RS}$.



a Reflect $\triangle ABC$ in the line $\overline{PQ}$.

b Reflect the image of $\triangle ABC$ from part **a** in the line $\overline{RS}$.

c Measure the following angles correct to the nearest degree.

i $\angle AXA''$

ii $\angle BXB''$

iii $\angle CXC''$

d Measure the following lengths correct to one decimal place.

i AX and XA''

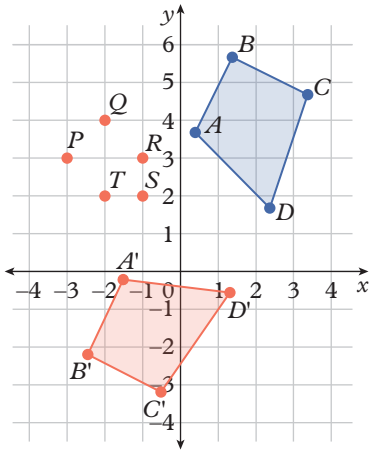
ii BX and BX''

iii CX and XC''

e Describe the single transformation from $\triangle ABC$ to $\triangle A''B''C''$.

Exercise 9.1C: Challenge

13 Identify the two points that lie on the axis of symmetry that the quadrilateral $ABCD$ was reflected in.



Online resources:

- Interactive skillsheet: Translations
- Interactive skillsheet: Reflections
- Interactive skillsheet: Rotations
- Worksheet: Transforming shapes
- Investigation: Treasure hunt
- Quick quiz: 9.1

Lesson 9.2

Congruence

Learning intentions

By the end of this lesson you will be able to ...

✔ identify congruent shapes, and use properties of congruent shapes.

Inter-year links

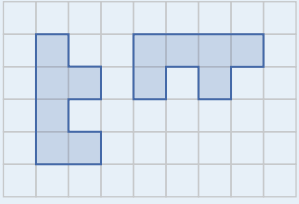
Support 2D shapes

Year 7 7.6 Polygons

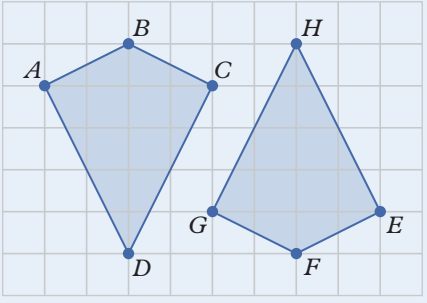
Congruence

Key content video
Congruence

- Two figures are **congruent** if they have the same shape and size, or if one shape is the mirror image of the other.
  - → Congruent figures can be in any position or orientation.
  - → Corresponding side lengths and angles are equal in congruent figures.
  - → For example, these shapes are congruent even though they are not in the same orientation.

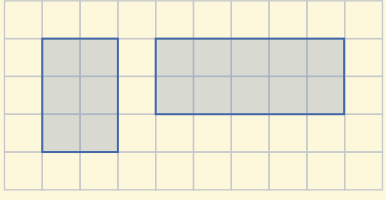


- Isometric transformations (rotations, reflections and translations) produce congruent figures.
- The symbol ≅ is used to indicate congruence. When making a congruence statement, matching vertices must be put in the same order.
  - → For example, $ABCD \cong EFGH$.
  - → This means A and E are matching vertices, B and F are matching vertices, and so on.



Worked example 9.2A Determining if two shapes are congruent

Determine if these two shapes are congruent.



THINK

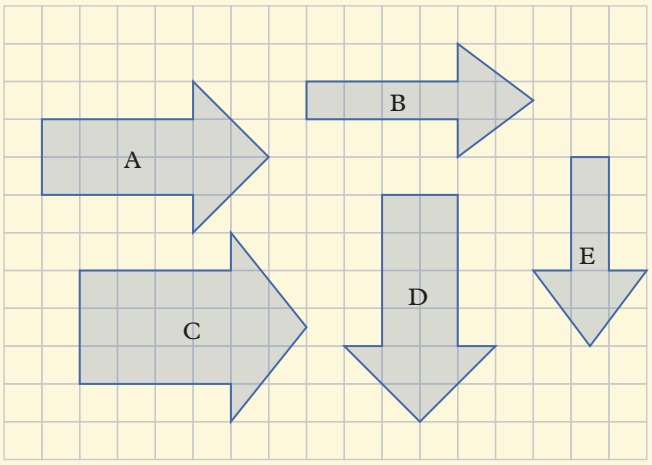
1 Determine if the basic shapes are the same.
2 Check if the corresponding sides are equal.
3 Write your answer.

WRITE

The shapes are not congruent because they do not have equal corresponding sides.

Worked example 9.2B Identifying congruent pairs

Identify any pairs of congruent shapes in the diagram on the right.



THINK

1. Compare the length and width of all the shapes.
2. C is wider than the other shapes, so it does not have a congruent pair.
3. E is shorter than the other shapes, so it does not have a congruent pair.
4. B is thinner than both A and D, so it does not have a congruent pair.
5. Shapes A and D are the same shape and the corresponding sides are equal in length, so they are a congruent pair.

WRITE

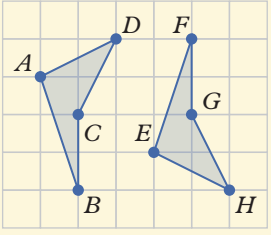
Shapes A and D are congruent.

Worked example 9.2C Matching sides of congruent pairs

Consider the pair of congruent quadrilaterals on the right.

a Match the corresponding sides for this pair of congruent quadrilaterals.

b Write a congruence statement for this pair of congruent quadrilaterals.



THINK

a 1. Pick a side from the first shape that has a different length from all other sides. For example, side AB is the only side that goes down by 3 units and across by 1 unit. Find the corresponding side on the second shape (EF).

2. Pick an adjacent side to that chosen in step 1; for example, BC. Find the corresponding side on the second shape (FG).

3. Continue picking adjacent sides on the first shape and finding the corresponding sides on the second shape until all sides have been matched.

b Use the congruence symbol ($\cong$) to write a congruence statement, ensuring that the matching vertices are put in the same order for each quadrilateral.

WRITE

a AB and EF, BC and FG, CD and GH, DA and HE

b $ABCD \cong EFGH$

Helpful hints

✔ Remember that orientation does not matter when determining congruence, but the size of the shapes does matter!

Learning pathways

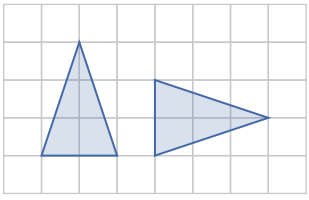
1–5, 6(a, b), 7, 8	1–4, 6, 7, 9, 11	2, 3(c, d), 4(b, d), 6(c, d), 8–12

ANS p585

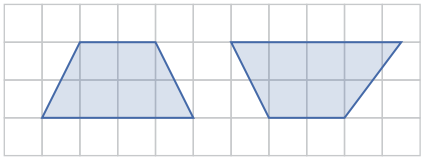
Exercise 9.2A: Understanding and fluency

WE 9.2A **1** Decide whether each pair of shapes is congruent.

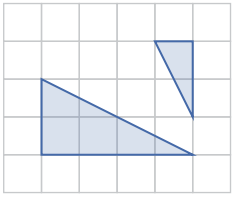
a



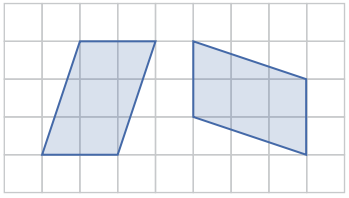
b



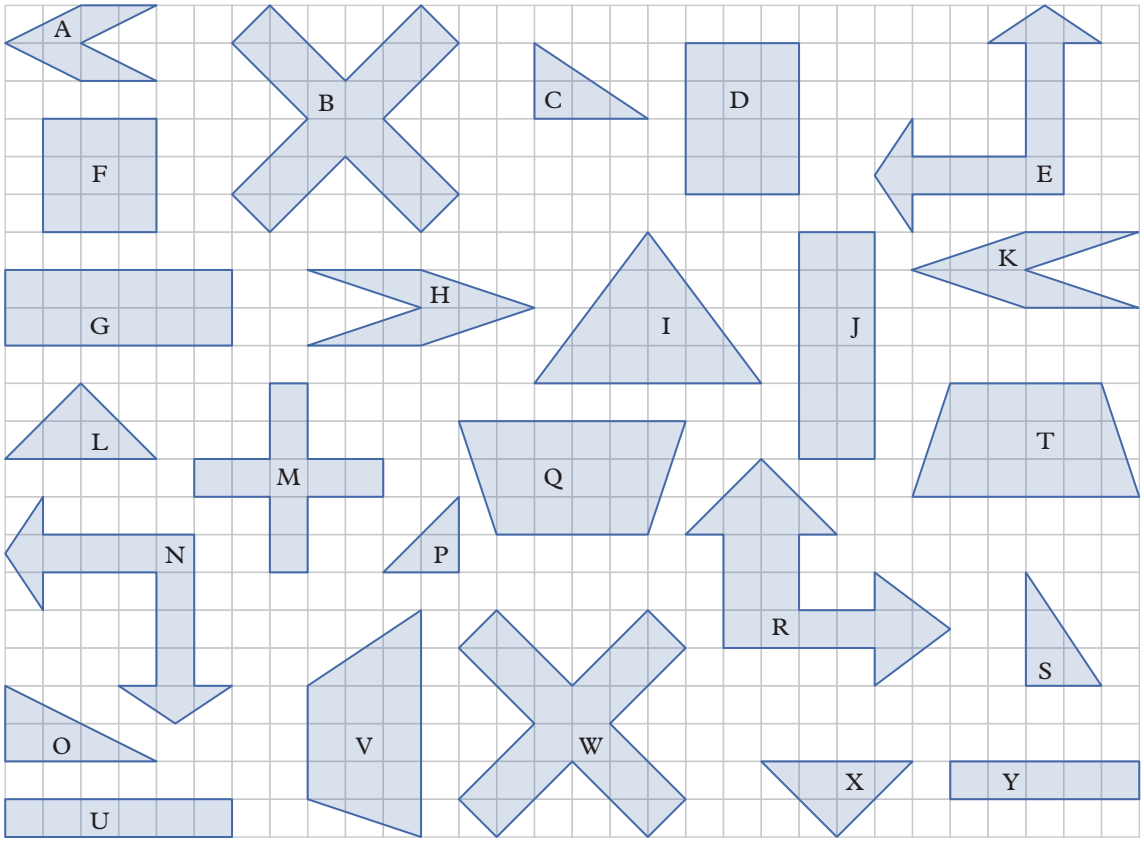
c



d



WE 9.2B **2** Identify any pairs of shapes that are congruent in the following diagram.

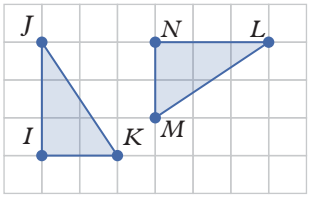


WE 9.2C **3** Consider each pair of congruent shapes below.

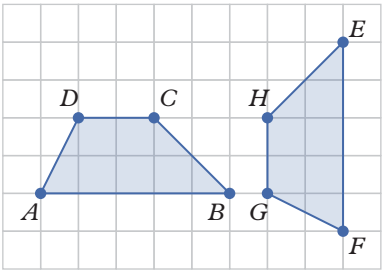
i Match the corresponding sides for each pair of congruent shapes.

ii Write a congruence statement for each pair of congruent shapes.

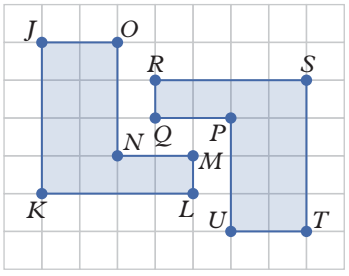
a



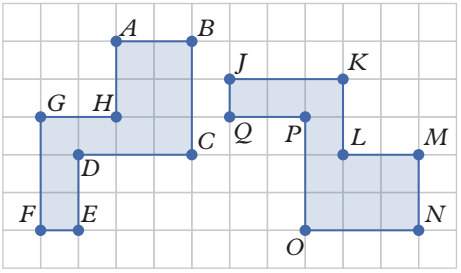
b



c

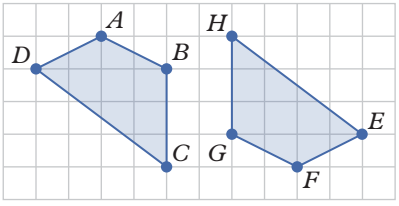


d

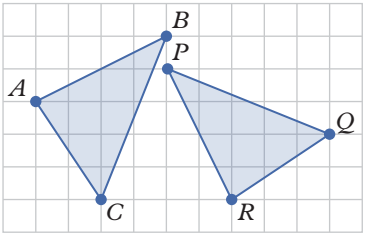


4 Match the corresponding vertices of each pair of congruent shapes.

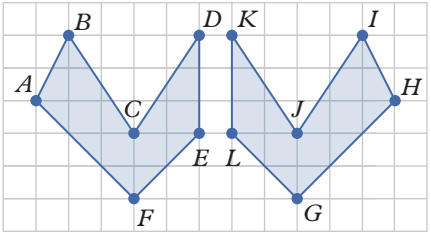
a



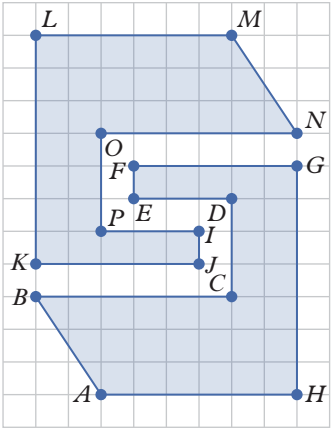
b



c



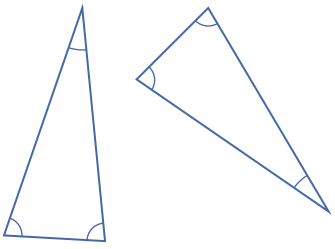
d



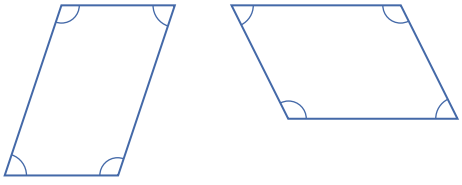
Exercise 9.2B: Problem solving and reasoning

5 For each pair of figures, use a ruler and protractor to measure all sides and angles and state if the figures are congruent.

a

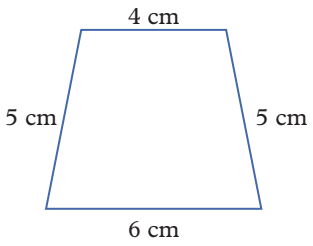


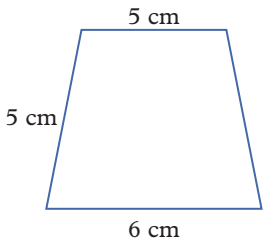
b



6 Determine if each pair of figures is congruent, justifying your decision.

a





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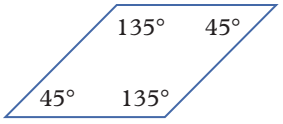

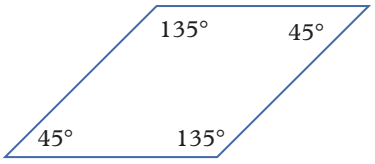

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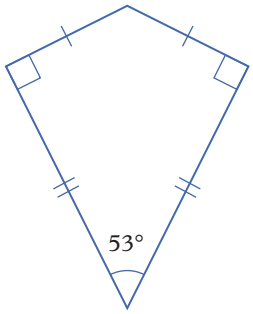

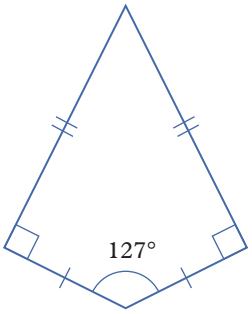

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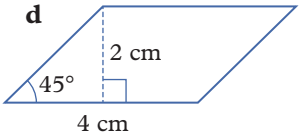
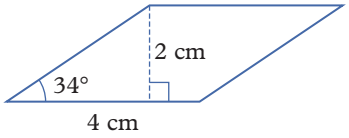
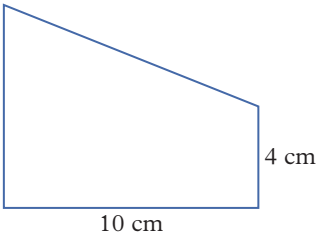
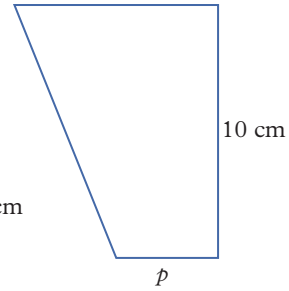




7 Find the value of the pronumerals, given that these pairs of shapes are congruent.

a





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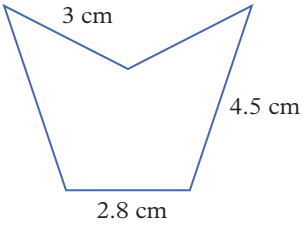
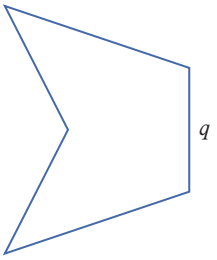
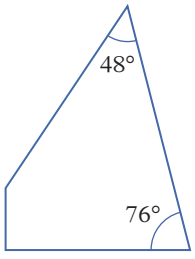
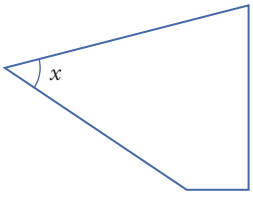




8 Find the value of the pronumerals, given that these pairs of shapes are congruent.

a





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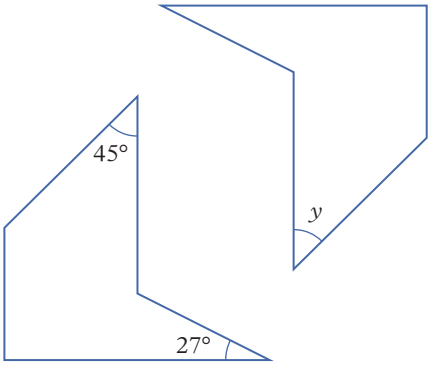


9 Answer true or false to the following statements, giving reasons for your answers.

- **a** All squares with the same side length are congruent with each other.
- **b** All rectangles with the same side lengths are congruent with each other.
- **c** All triangles with the same base length and height are congruent with each other.
- **d** All parallelograms with the same base length and height are congruent with each other.
- **e** All circles with the same radii are congruent with each other.

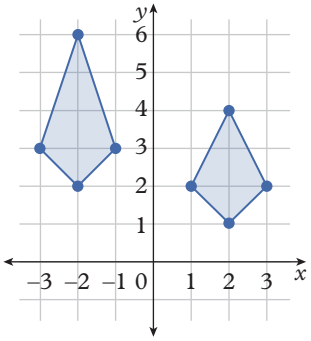
10 By considering the corresponding side lengths and angles, explain why a shape and its image are congruent after a translation, reflection or rotation.

11 For each of the following pairs of shapes:

i identify a single transformation that can be applied to the left shape to check if the two shapes are congruent or not congruent

ii state whether the shapes are congruent or not congruent.

a



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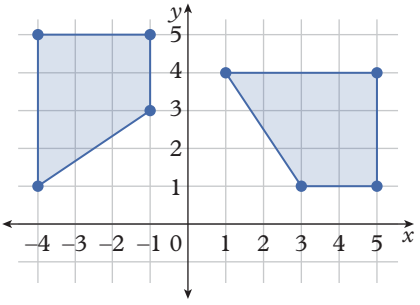

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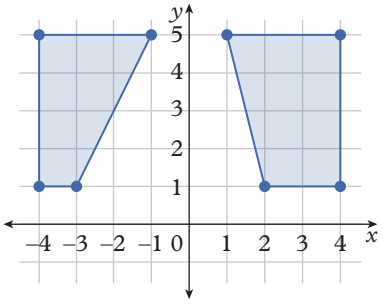

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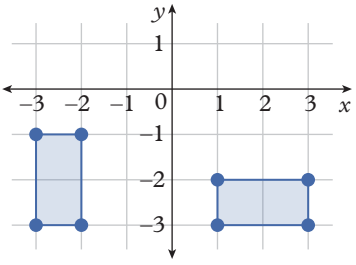
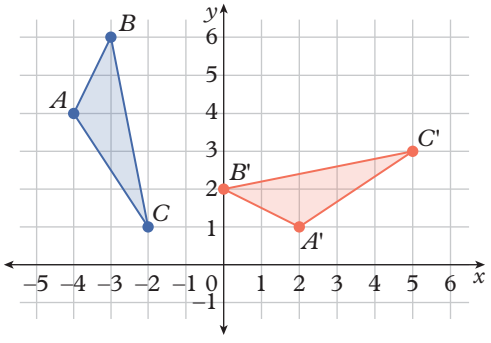


Exercise 9.2C: Challenge

12 Describe a series of transformations that can be applied to the left shape to show the two shapes are congruent.

a



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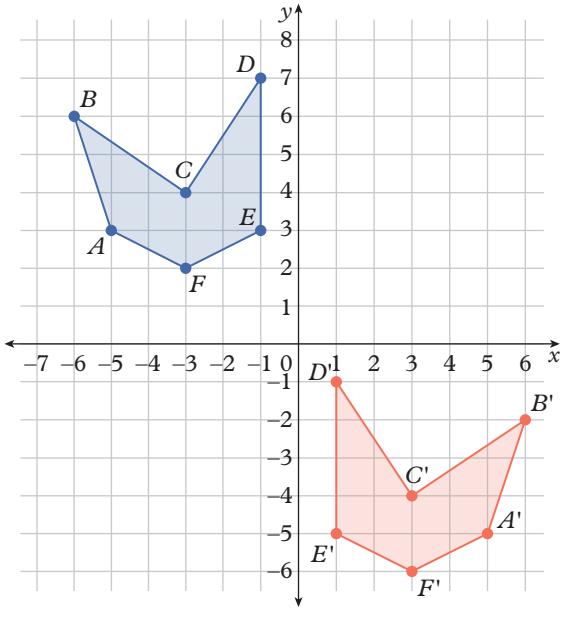


Online resources:

- **Interactive skillsheet** — Congruence
- **Worksheet** — Congruent figures and transformations
- **Investigation** — Tiling patterns in art and design
- **Quick quiz** — 9.2

Lesson 9.3

Congruent triangles

Learning intentions

By the end of this lesson you will be able to ...

✔ use congruence tests to determine if two triangles are congruent.

Inter-year links

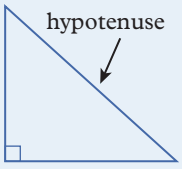
Year 7 7.4 Triangles

Year 10 6.3 Congruence and similarity

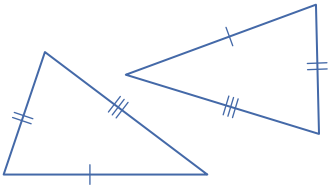
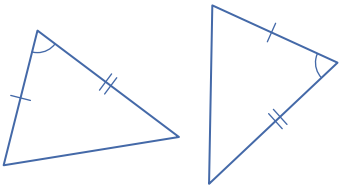
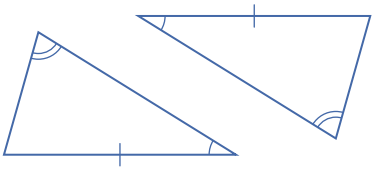
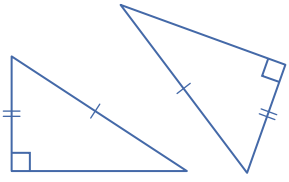
Congruent triangles

Key content video
Congruent triangles

- Two triangles are congruent if all their sides and angles are equal.
- The symbol Δ can be used instead of writing 'triangle'; for example, ΔABC.
- An **included angle** is an angle between two sides.
- The hypotenuse of a right-angled triangle is the side that is opposite the right angle.



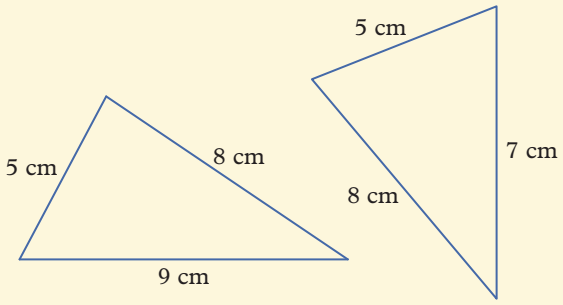
- If a pair of triangles pass any of the following tests, they are congruent.

Test	Name	Meaning	Example
SSS	side–side–side	All three side lengths are equal.	
SAS	side–angle–side	Two side lengths and the included angle (between these sides) are equal.	
AAS (or ASA or SAA)	angle–angle–side	Two angles and a corresponding side length are equal.	
RHS	right angle–hypotenuse–side	The hypotenuse and another side length of two right-angled triangles are equal.	

- In the AAS test, any corresponding side length is sufficient to prove congruence. This test may also be known as ASA or SAA.
- Other specifications, such as AAA and SSA, are not sufficient to prove congruence.

Worked example 9.3A Using the SSS congruence test

Use the SSS congruence test to determine if this pair of triangles are congruent.



THINK

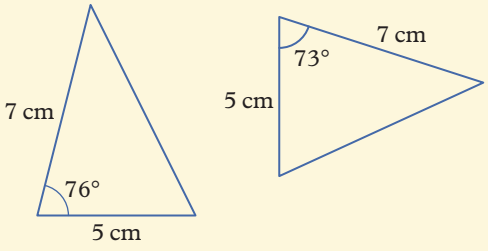
1. The triangles are congruent if all three side lengths are equal.
2. Go through each of the side lengths on the first shape and check whether there is a matching side length on the second triangle. In the second triangle there is not a matching side length for the 9 cm side in the first triangle.
3. As the test fails, the triangles are not congruent.

WRITE

The two triangles are not congruent.

Worked example 9.3B Using the SAS congruence test

Use the SAS congruence test to determine if this pair of triangles are congruent.



THINK

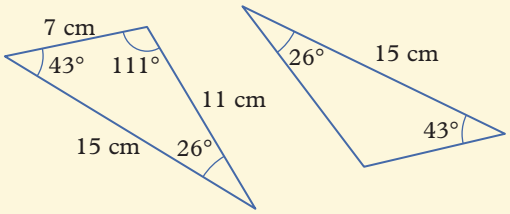
1. The triangles are congruent if two side lengths and the included angle are equal.
2. Check that there are two matching side lengths on each triangle.
3. Look at the angles between the two matching sides. The included angle in the second triangle is not equal to the included angle in the first triangle.
4. As the test fails, the triangles are not congruent.

WRITE

The two triangles are not congruent.

Worked example 9.3C Using the AAS congruence test

Use the AAS congruence test to determine if this pair of triangles are congruent.



THINK

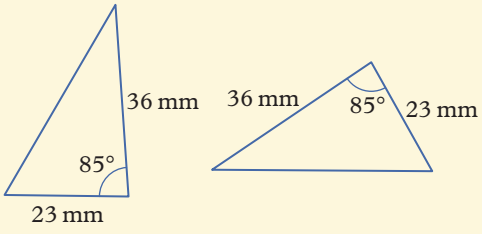
1. The triangles are congruent if two angles and a corresponding side length are equal.
2. Check that there are two pairs of equal angles.
3. Identify the corresponding sides of each triangle by using the matching angles from step **2**.
4. As the corresponding side lengths are equal the test passes, and the triangles are congruent.

WRITE

The two triangles are congruent.

Worked example 9.3D Determining if two triangles are congruent

Determine if this pair of triangles is congruent, giving a reason for your answer.



THINK

1. Identify what information is provided in each triangle. Both triangles contain two equal sides and an included angle, so we can use the SAS test.
2. Check whether the side lengths and the included angles are equal or not.
3. Provide the answer, giving a reason with your answer.

WRITE

The two triangles are congruent as they pass the SAS congruence test.

Helpful hints

- ✔ When using the SAS test, make sure the provided angle is between the corresponding sides!
- ✔ When using any congruence test, make sure that the matching side lengths are corresponding sides!
- ✔ When there is a pair of right-angled triangles, if the information for the RHS test is not provided, remember to check if there is sufficient information for either the SAS or AAS test.

Learning pathways

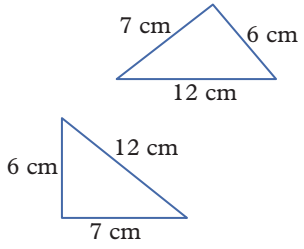
1–3, 5–7, 9, 10	1–4, 5(c, d), 6, 8, 10–12	1–3, 5(c, d), 6, 10–14

ANS p585

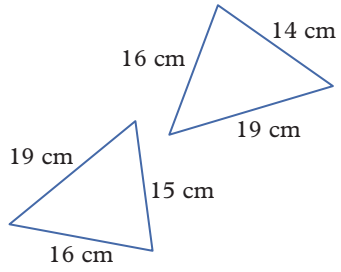
Exercise 9.3A: Understanding and fluency

WE 9.3A **1** Use the SSS congruence test to determine if these pairs of triangles are congruent.

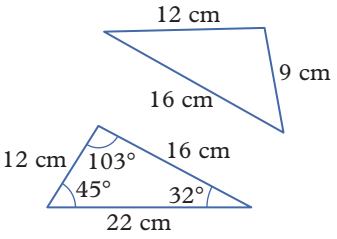
a



b

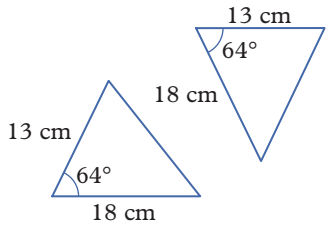


c



WE 9.3B **2** Use the SAS congruence test to determine if these pairs of triangles are congruent.

a



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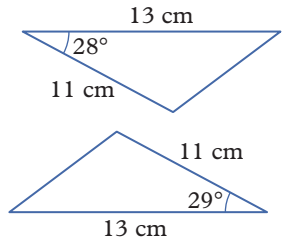

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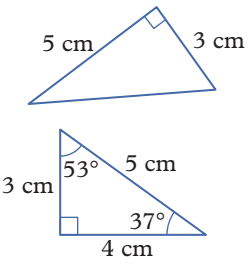
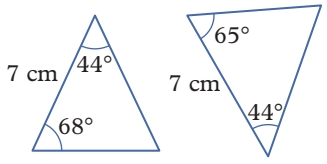


WE 9.3C **3** Use the AAS congruence test to determine if these pairs of triangles are congruent.

a



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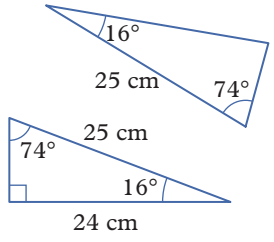

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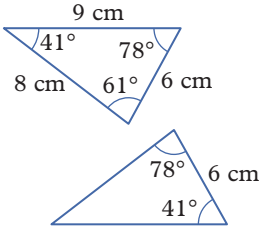
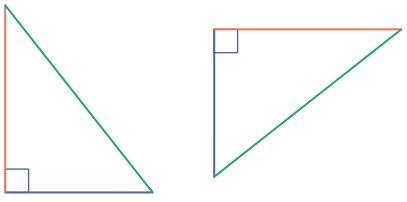


4 The RHS congruence test only applies to right-angled triangles. In every right-angled triangle the longest side (known as the hypotenuse) appears opposite the right angle.



a If both sets of shorter sides in a pair of right-angled triangles were equal, what congruence test would demonstrate that the two triangles are congruent? Explain your answer.

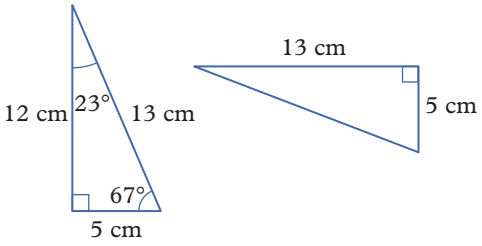
b When a triangle is right-angled and you know the length of two sides, the length of the third side can be calculated using Pythagoras' theorem.

i If you calculated the length of the third side, which other test could be used to check for congruence? Hint: You would know the lengths of all the sides.

ii How does this show that RHS is a test for congruence?

5 Use the RHS congruence test to determine if these pairs of triangles are congruent.

a



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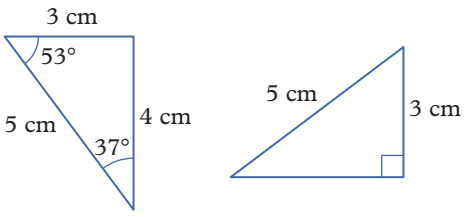

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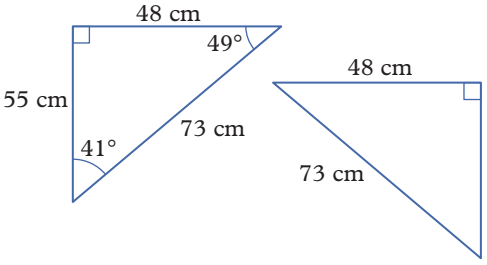

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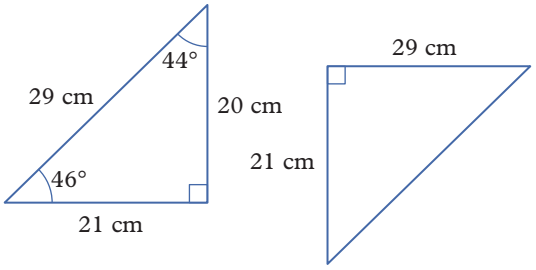
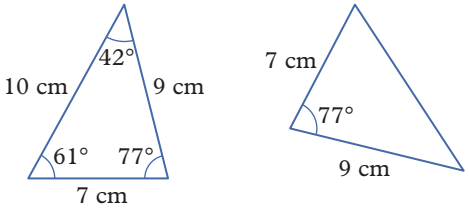


WE 9.3D **6** Determine if these pairs of triangles are congruent, giving reasons for your answers.

a



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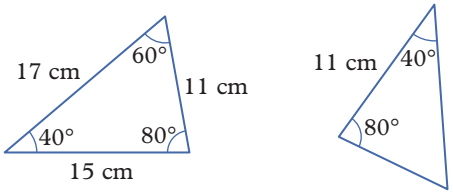

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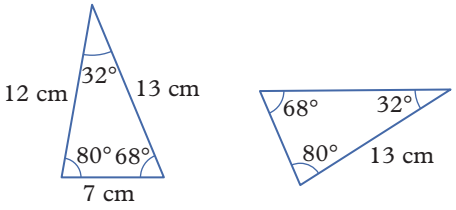

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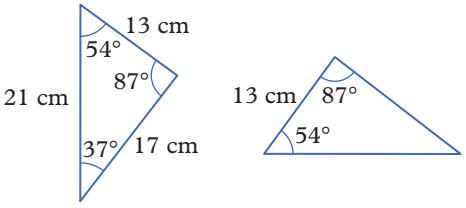

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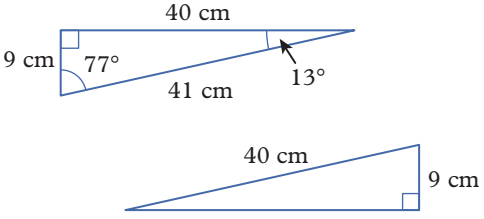
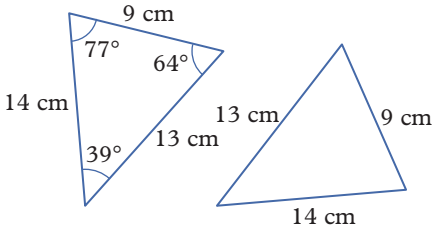


f

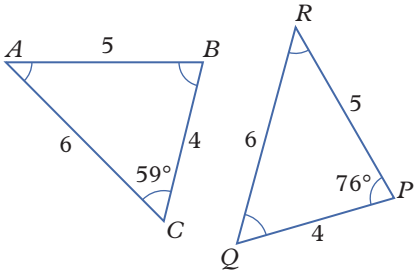


Exercise 9.3B: Problem solving and reasoning

7 Consider the pair of congruent triangles below.

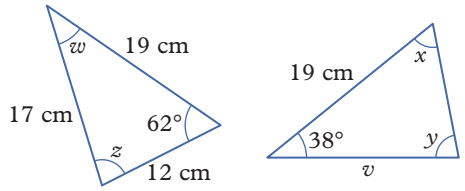
a State the congruence test that can be used to explain why this pair of triangles is congruent.

b Determine the size of the angles $\angle ABC$ and $\angle BAC$.

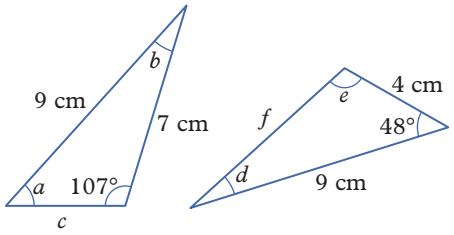


8 Each of these pairs of triangles are congruent.

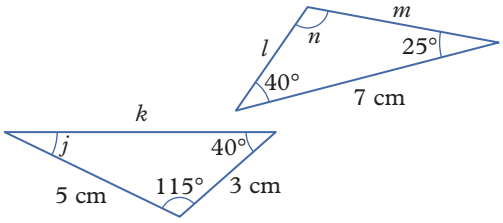
a



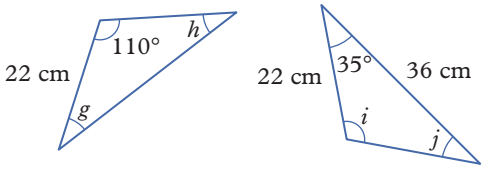
b



c



d

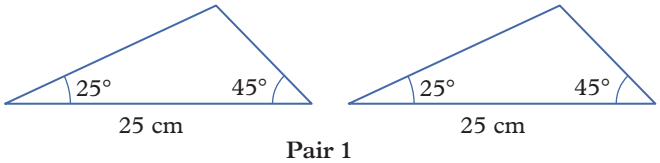


i Find the unknown side lengths and angles in each triangle.

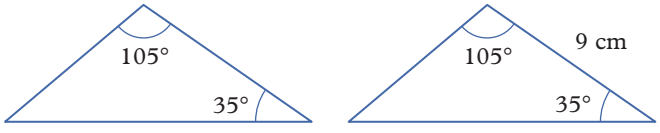
ii State the congruence tests and triangle properties that you used to solve each problem.

9 ASA (angle–side–angle) is another test for congruence. ASA refers to two matching pairs of angles and the pair of sides in between.

a Explain why Pair 1 passes the ASA congruence test but Pair 2 does not.



Pair 1



Pair 2

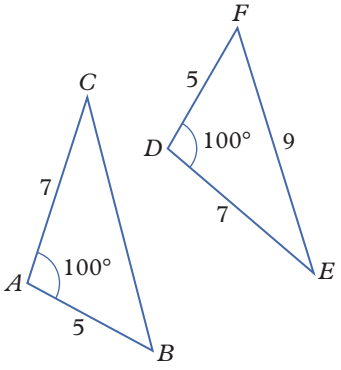
b Can you explain how, if a pair of triangles passes the ASA congruence test, they will also pass the AAS congruence test? Hint: Find the third angle of the triangles.

10 For each of the following:

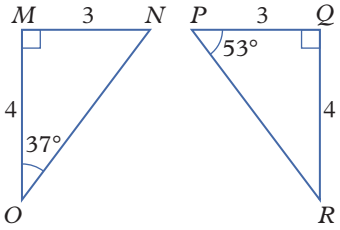
i explain why the stated congruence test is or is not able to show the triangles are congruent

ii if the test cannot show the triangles are congruent, state a test that can be used.

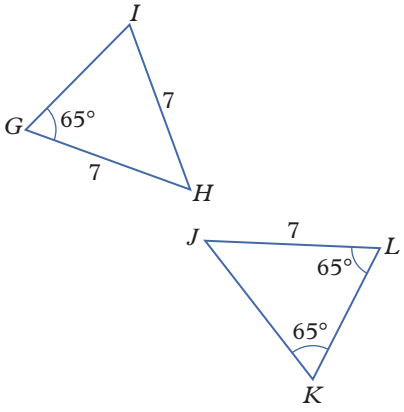
a SSS



b RHS



c AAS

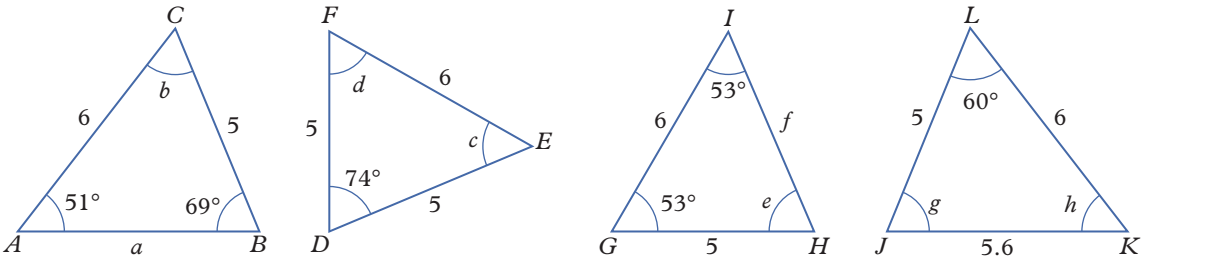


11 For each group of triangles,

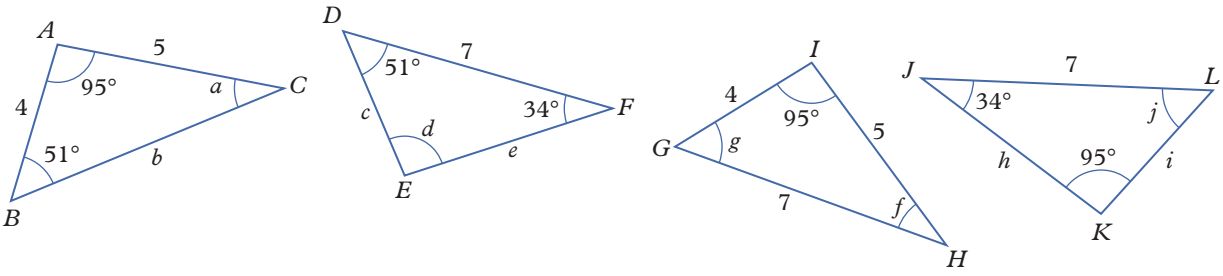
i write a congruence statement detailing which triangles are congruent

ii find the values of the pronumerals.

a



b

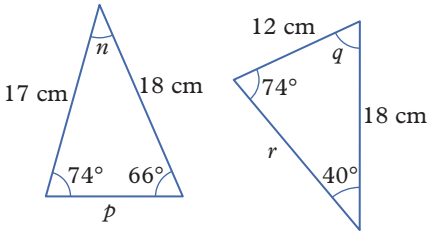


12 For each pair of triangles below:

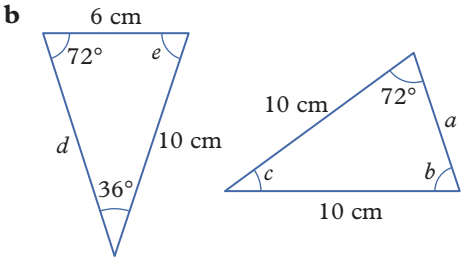
i determine if they are congruent, stating which congruence test you used

ii find the value of the pronumerals if the pair is congruent.

a

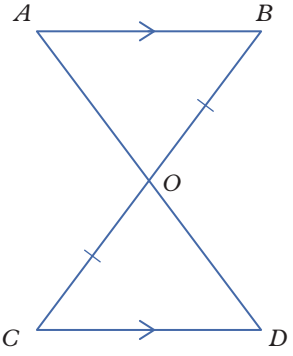


b



Exercise 9.3C: Challenge

13 Consider the figure on the right, where $\overline{AB}$ is parallel to $\overline{CD}$ and $\overline{OB} = \overline{OC}$.



a By using properties of angles on parallel lines, which pairs of angles can be stated to be equal?

b By using properties of angles at a point, which pair of angles can be stated to be equal?

c Using your answers to parts **a** and **b**, as well as the fact that $\overline{OB} = \overline{OC}$, which congruence test can be used to show that the two triangles shown in the diagram are congruent?

d Write a congruence statement for the pair of congruent triangles.

14 Up to 12 values can be given for a pair of congruent triangles (six angles and six side lengths, three each per triangle). SSS, SAS, AAS and RHS all require at least six values to be given. What is the largest number of values that can be given for two congruent triangles where someone would still not be certain the two are congruent, without determining other lengths or angles? Provide an example pair of triangles using hatch marks and arcs to label equal lengths and angles.

Online resources:

- **Interactive skillsheet** Congruent triangles
- **Worksheet** Conditions for congruence
- **Investigation** Congruence in construction
- **Quick quiz** 9.3

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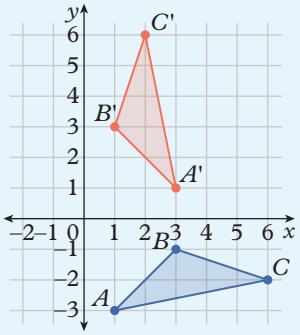
Checkpoint

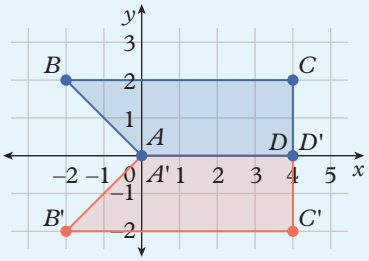
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

9.1 **1** Provide a full description of each of the transformations shown below.

a

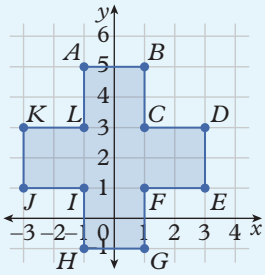


b

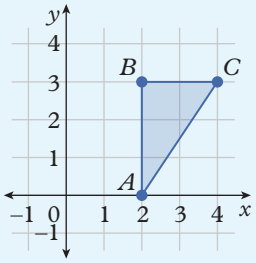


9.1 **2** Draw the images of the following shapes after the given transformations are applied.

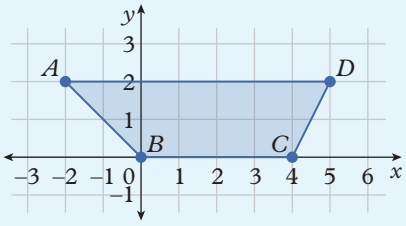
a Translate 5 units down and 2 units right.



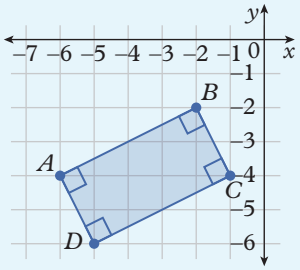
b Rotate 90° clockwise about the origin.



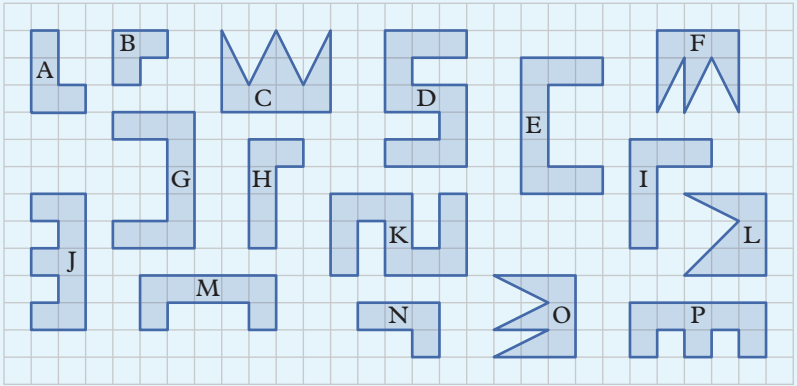
c Reflect in the y-axis.



d Rotate 90° clockwise about the origin then translate 6 units right.

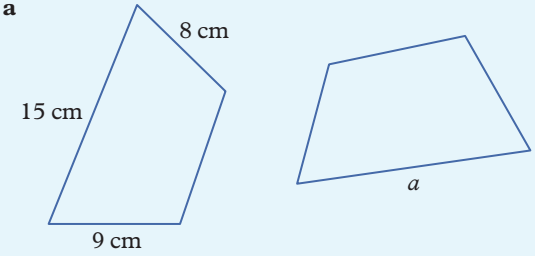


9.2 **3** Identify any pairs of shapes that are congruent in the following diagram.

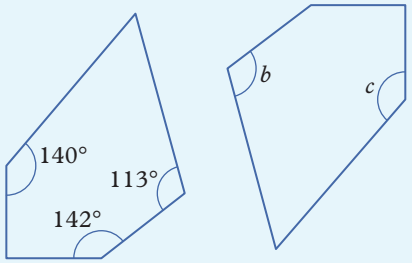


9.2 **4** Find the values of the pronumerals, given that these pairs of shapes are congruent.

a

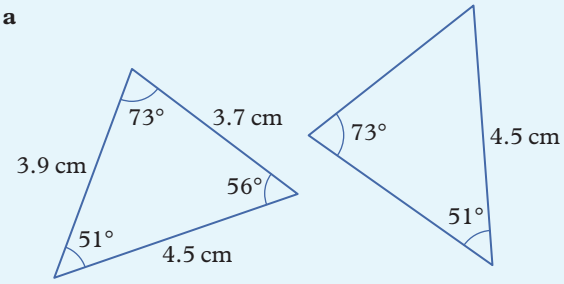


b

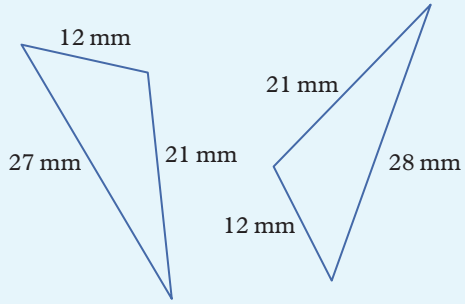


9.3 **5** Determine if these pairs of triangles are congruent, giving reasons for your answers.

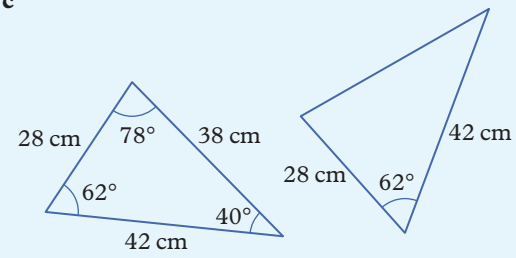
a



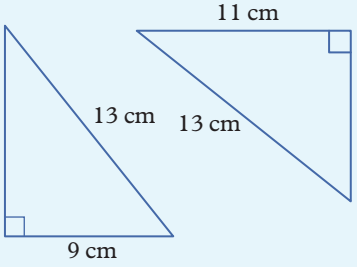
b



c

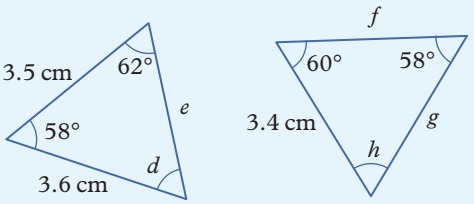


d

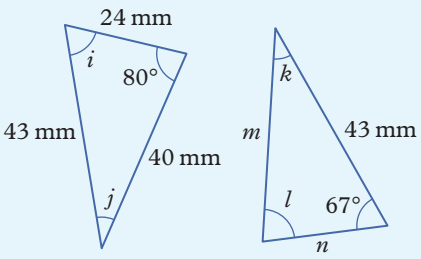


9.3 **6** Each of these pairs of triangles are congruent. Find the unknown side lengths and angles in each triangle.

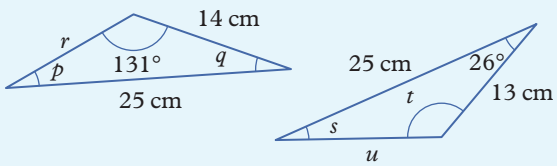
a



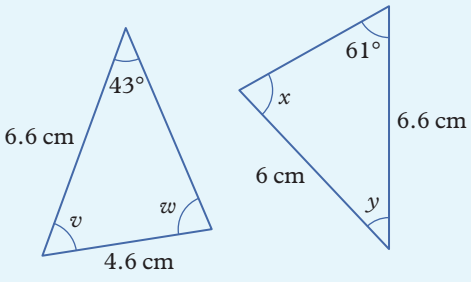
b



c



d



Lesson 9.4

Dilations and similar figures

Learning intentions

By the end of this lesson you will be able to ...

✔ find unknown lengths in similar figures, and use the scale factor to perform dilations.

Inter-year links

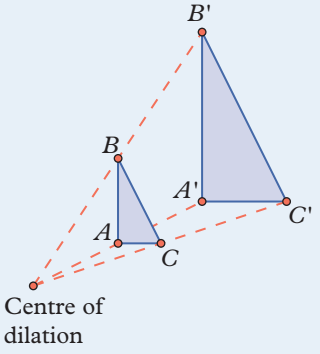
Year 7 3.2 Equivalent fractions

Year 9 6.2 Similar figures

Key content video
Dilations and similar figures

Dilations

- A **dilation** is a transformation that changes the size of an object. For a dilation, the shape and orientation of the image remains the same as the original shape.
  - → The angle measurements of a shape do not change after a dilation.
- A dilation can be either an **enlargement** or a **reduction**.
  - → An enlargement creates an image that is larger than the original shape.
  - → A reduction creates an image that is smaller than the original shape.
- A **centre of dilation** and **scale factor** need to be specified to perform a dilation.



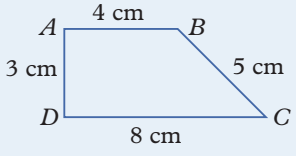
- A scale factor can be found by using the formula:

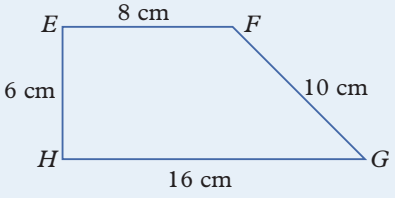
$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

  - → If the scale factor is between 0 and 1, the image is a *reduction*.
  - → If the scale factor is greater than 1, the image is an *enlargement*.
- The side length of a dilation can be found by multiplying the original side length by the scale factor.
  - → For example, when dilating a shape with a side AB:
    Length of $A'B'$ = length of AB × scale factor.

Similar figures

- Two figures are said to be **similar** if they are the same shape. Similar figures do not need to be the same size.
  - → For example, the shapes $ABCD$ and $EFGH$ are similar figures; $EFGH$ is an image of $ABCD$ after a dilation of scale factor 2.

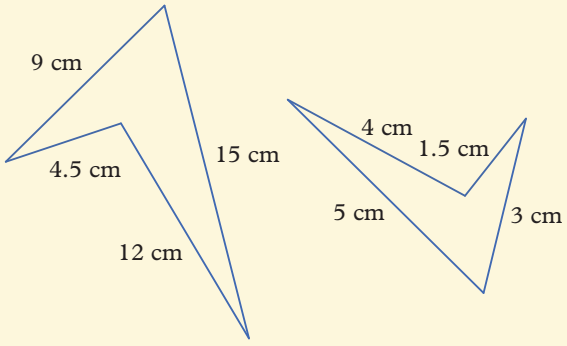




- If two figures are similar, all corresponding angles are equal in size and all corresponding sides are in the same ratio. This ratio is called the ratio of similarity.
- The symbol ~ can be used to indicate similarity. When making a similarity statement, corresponding vertices must be put in the same order.
  → For example, $ABCD \sim EFGH$.
- If two figures are known to be similar, an unknown side length can be found by using the scale factor.
- If the scale factor between two similar figures is 1, then the two figures are congruent.
  → Congruence is a special case of similarity.

Worked example 9.4A Finding the scale factor

Find the scale factor for this pair of similar figures if the figure on the left is the original shape.



THINK

1 Identify two corresponding sides. The longest side of the original figure is 15 cm and the longest side of the image is 5 cm.

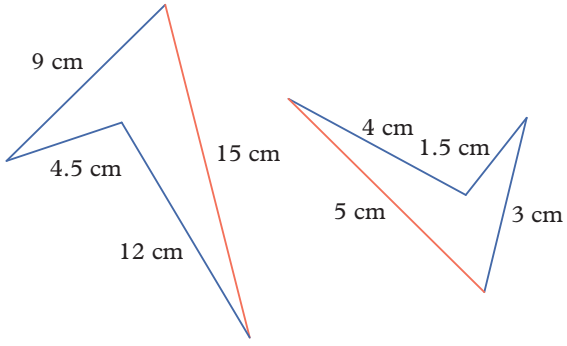
2 Substitute these lengths into the formula:

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

3 Match the remaining corresponding sides and check that each pair of sides is in the same ratio.

4 Check that the scale factor seems reasonable (between 0 and 1 means a reduction).

WRITE



$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{5 \text{ cm}}{15 \text{ cm}}$$
$$= \frac{1}{3}$$

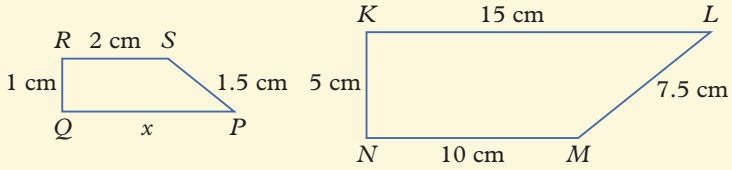
Check each corresponding pair of sides:

$$\text{Scale factor} = \frac{3}{9} = \frac{1.5}{4.5} = \frac{4}{12} = \frac{1}{3}$$

The scale factor of $\frac{1}{3}$ is a reduction, as required.

Worked example 9.4B Finding an unknown side length in similar figures

Find the value of x, given that these two figures are similar and the figure on the left is the original shape.



THINK

1. Identify two corresponding sides and calculate the scale factor. Use corresponding vertices to write the side names in the same order. As R corresponds to N and S corresponds to M, then side RS corresponds to side NM (not MN).
2. Find a pair of corresponding sides that involve x.
3. Substitute in the known values and solve for x.

WRITE

Sides RS and NM are corresponding.

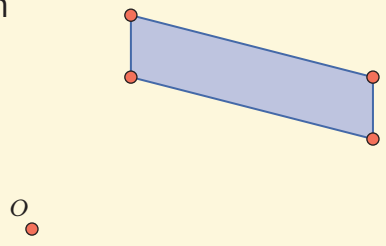
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{10 \text{ cm}}{2 \text{ cm}}$$
$$= 5$$

Side QP has length x. Sides QP and KL are corresponding.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$5 = \frac{15 \text{ cm}}{x}$$
$$5x = 15 \text{ cm}$$
$$x = \frac{15 \text{ cm}}{5}$$
$$x = 3 \text{ cm}$$

Worked example 9.4C Performing a dilation

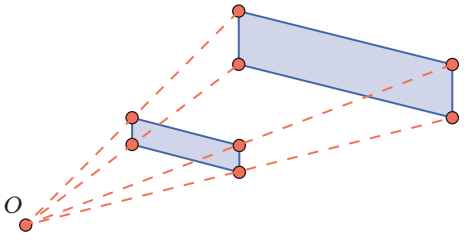
Dilate this figure by a scale factor of $\frac{1}{2}$ using the centre of dilation, O.



THINK

1. Use dashed lines to connect the centre of dilation with each of the vertices on the original shape.
2. The scale factor is $\frac{1}{2}$ so measure half the distance from the centre of dilation to each of the vertices on the original shape and mark these points. These are the corresponding vertices of the image.
3. Join the vertices of the image together to complete the dilation.

WRITE



Helpful hints

✔ The image from a dilation will always be in the same orientation as the original shape; however, similar shapes do not need to be in the same orientation.

Learning pathways

1–5, 6(a–c), 7, 10–13 | 2–5, 6(d–f), 11–14, 18 | 3–5, 6(e, f), 8–10, 13–18

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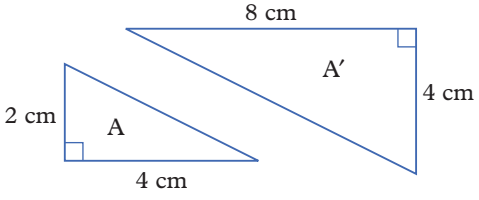
Exercise 9.4A: Understanding and fluency

1 Decide whether each scale factor produces an enlargement or a reduction.

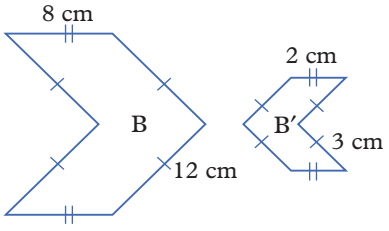
a 4 **b** $\frac{1}{3}$ **c** 2 **d** 6 **e** $\frac{1}{10}$ **f** $\frac{1}{5}$

WE 9.4A **2** Find the scale factor for each pair of similar figures.

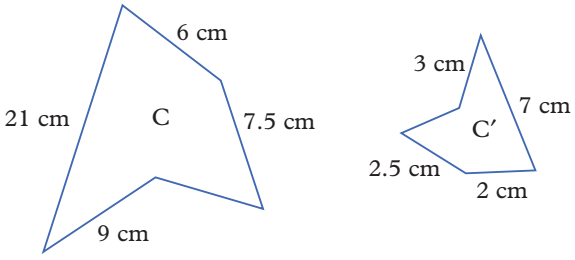
a



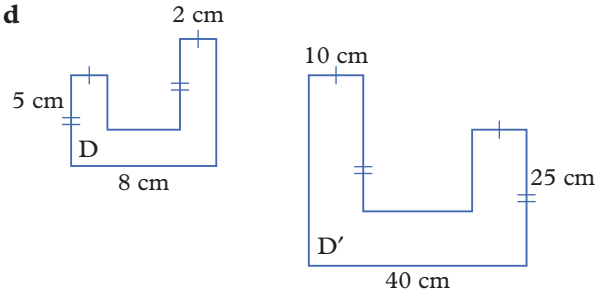
b



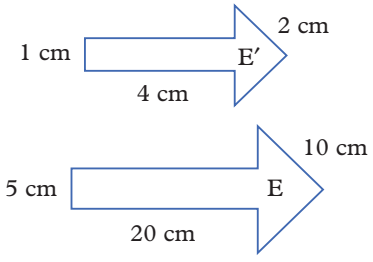
c



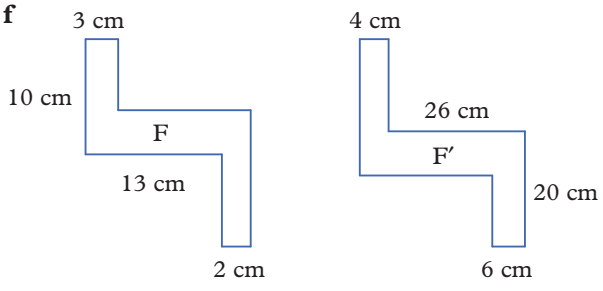
d



e

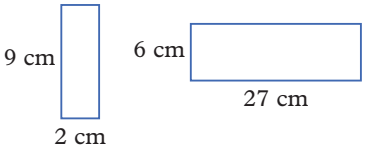


f

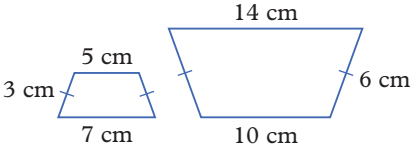


3 Find the scale factor for the similar figures if the figure on the left of each pair is the original shape.

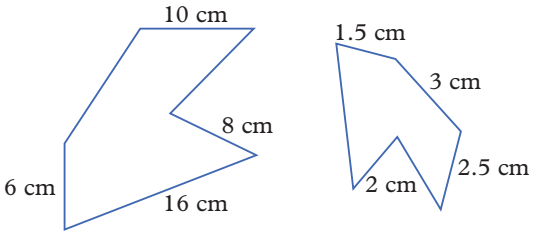
a



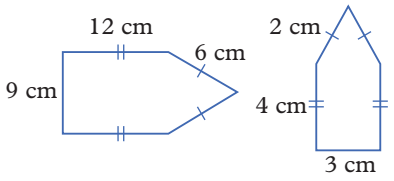
b



c

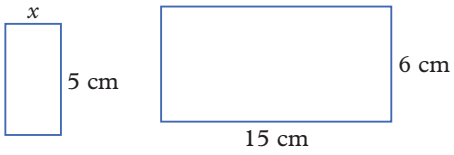


d

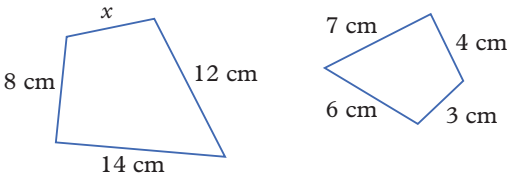


WE 9.4B **4** Find the value of x, given that these pairs of shapes are similar.

a



b



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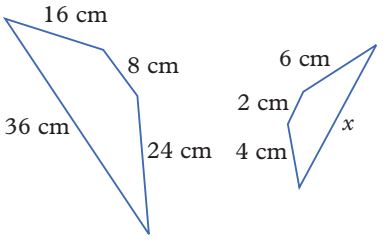
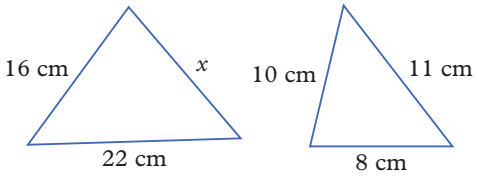
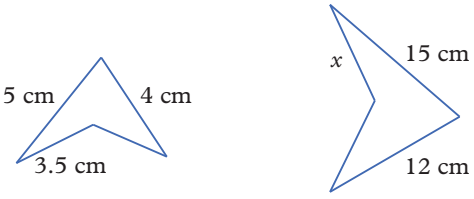


d



e



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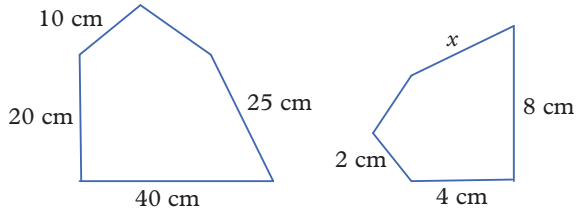
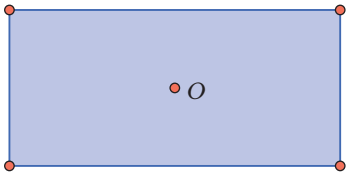


WE 9.4C **5** Use the worksheet available in your online resources to dilate each figure using the centre of dilation, O, and a scale factor of:

i 3

ii $\frac{1}{3}$

a



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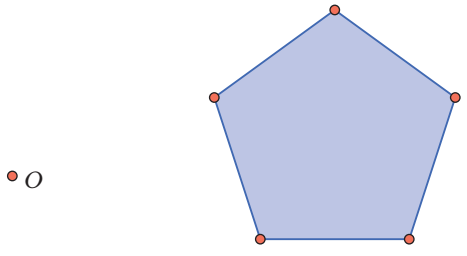
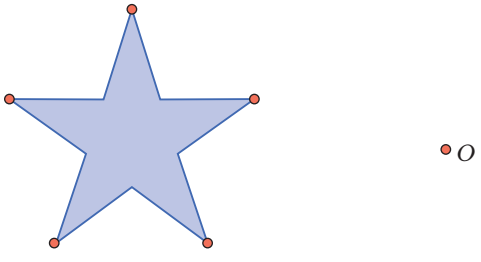
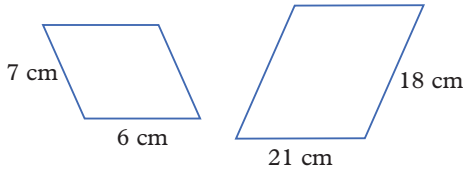


c

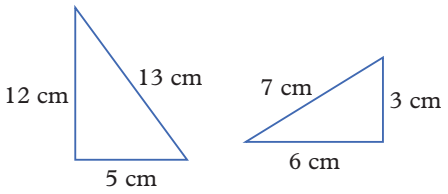


6 Determine whether each pair of shapes is similar by comparing the corresponding side lengths of the shapes.

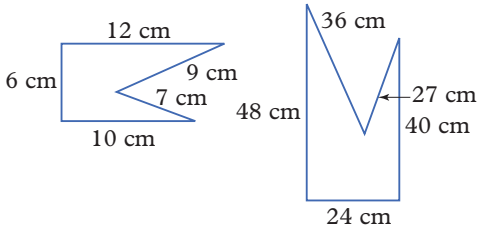
a



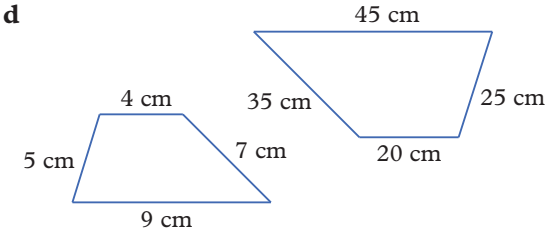
b



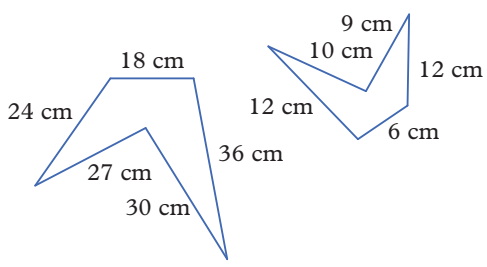
c



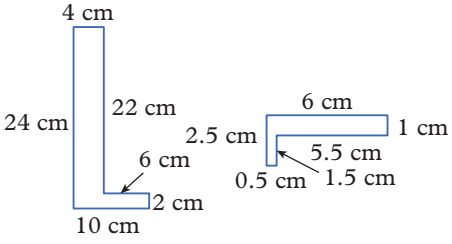
d



e



f



Exercise 9.4B: Problem solving and reasoning

7 A microscope with a scale factor of 20 000 was used to view a bacterial cell, which appeared 5.2 cm long. What was the actual length of the bacterial cell?

8 Explain why a circle is similar to all other circles, why a square is similar to all other squares, but a rectangle is not similar to all other rectangles.

9 Finn was explaining a dilation to a classmate and said that a dilation of $\frac{2}{3}$ was the same as dilating by a factor of 2 and then dilating by a factor of $\frac{1}{3}$. Can you explain why he is correct?

10 Explain the difference between congruent figures and similar figures.

11 Explain why isometric transformations produce congruent figures whereas dilations produce similar figures.

12 A standard print size for photographs is 10 cm × 15 cm. Alice wanted to enlarge a photograph to place it onto a canvas measuring 40 cm × 50 cm. Are the photograph and its corresponding canvas art similar figures?

13 A ratio can be used instead of a fraction to describe a dilation. For example, a figure that is 10 times as small as the original has a scale factor of $\frac{1}{10}$, which is the ratio 1 : 10.

a How would a scale factor of $\frac{1}{2}$ be written as a ratio?

b Explain the relationship between the fraction and the ratio. Hint: What parts of the ratio relate to the numerator and denominator?

c Consider an enlargement with a scale factor of 2. What is 2 written as a fraction?

d Use your answer to part **c** to write a scale factor of 2 as a ratio.

e Write these ratios as scale factors.

i 1 : 4 **ii** 1 : 100 **iii** 1 : 2500 **iv** 3 : 4 **v** 2 : 1 **vi** 7 : 2

f Explain how you can tell if a similar figure is an enlargement or reduction by the values in the ratio.

14 Timothy got a model plane for his birthday, which had a scale factor of 1 : 100.

a What does this scale factor mean?

b If the real-life plane has a length of 34 m, what is the length of the model plane?

c If the model plane has a wingspan of 27 cm, what is the wingspan of the real-life plane?

15 Marc has a hiking map with a ratio scale of 1 : 75 000.

a Marc planned to walk the length of a 15 km track. What distance would this be on the map?

b At lunchtime, he had walked a distance shown as 7 cm on the map. How far did he have left to walk on the track?

16 Elliot drew a regular hexagon with side lengths of 3 cm.

a What is the difference between a regular and an irregular hexagon?

b If Elliot drew another regular hexagon with different side lengths, explain how you know without measuring anything that the two hexagons must be similar.

Exercise 9.4C: Challenge

17 Emily bought Timothy a model plane to go into a special glass case that allows for a maximum height of 15 cm.

a If the plane was 30 m tall in real life, what is the maximum scale factor the model plane could be to fit into the glass case?

b When the plane arrives, Emily finds that the plane actually measures 16 cm in height.

What scale factor is the model plane?

c Assuming that all dimensions must stay in the same ratio, it is important that the plane fits into the glass case.

i What scale factor would the model need to be dilated by to fit into the space? Hint: Which measurement is the original and which is the image?

ii What scale factor would the space need to be dilated by to fit the current model?

iii How are the answers to parts **i** and **ii** different and how are they similar?

18 This table shows the dimensions of standard paper sizes, rounded to the nearest millimetre. Use the table to decide whether any or all of the following sizes are similar figures. Compare:

a sizes within the A group (e.g. A2, A4 and A7)

b sizes within the B group (e.g. B0, B3 and B6)

c sizes within the C group (e.g. C1, C5 and C8)

d sizes between different groups (e.g. A0, B0 and C0).

	A sizes (mm)	B sizes (mm)	C sizes (mm)
0	841 × 1189	1000 × 1414	917 × 1297
1	594 × 841	707 × 1000	648 × 917
2	420 × 594	500 × 707	458 × 648
3	297 × 420	353 × 500	324 × 458
4	210 × 297	250 × 353	229 × 324
5	148 × 210	176 × 250	162 × 229
6	105 × 148	125 × 176	114 × 162
7	74 × 105	88 × 125	81 × 114
8	52 × 74	62 × 88	57 × 81
9	37 × 52	44 × 62	40 × 57
10	26 × 37	31 × 44	28 × 40

Online resources:

Interactive skillsheet
Similar figures

Worksheet
Dilations

Desmos activity
Sketchy Dilations

Quick quiz
9.4

Lesson 9.5

Similar triangles

Learning intentions

By the end of this lesson you will be able to ...

✔ use similarity tests to determine if two triangles are similar.

Inter-year links

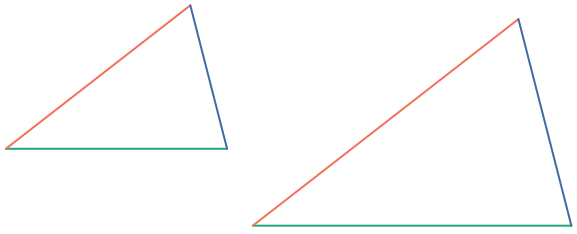
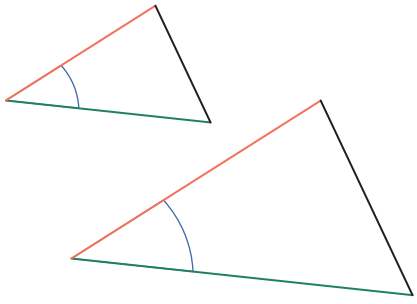
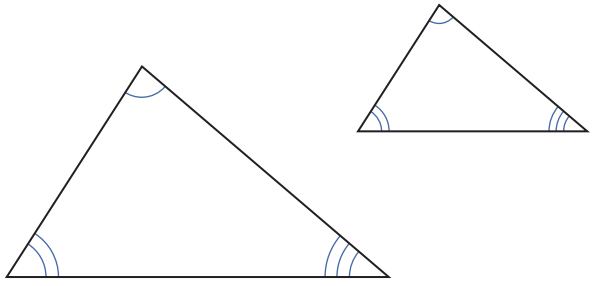
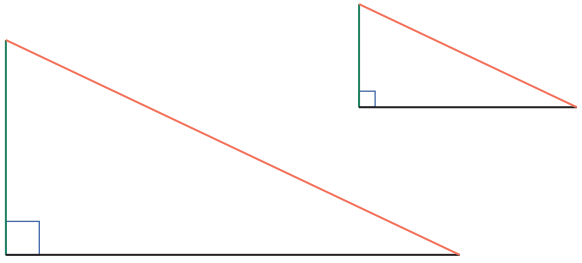
Year 7 7.4 Triangles

Year 10 6.3 Congruence and similarity

Similar triangles

Key content video
Similar triangles

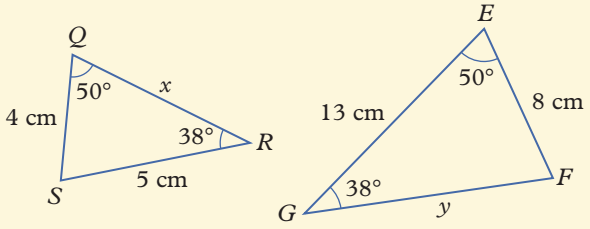
- Two triangles are similar if:
  - → corresponding pairs of angles are equal
  - → corresponding pairs of side lengths are in the same ratio.
- If a pair of triangles pass any of the following tests, they are similar.

Test	Meaning	Example
SSS	All three pairs of corresponding side lengths are in the same ratio.	
SAS	Two corresponding pairs of side lengths are in the same ratio and the included angles are equal.	
AAA	All three pairs of angles are equal.	
RHS	The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle.	

- In the SAS similarity test the matching angles must be between the corresponding pairs of side lengths to demonstrate similarity.
- Other specifications, such as SSA and AAS, are not sufficient to demonstrate similarity.
- The similarity tests are the minimum set of conditions required to demonstrate similarity. In some questions more information may be provided; however, if any of the tests are passed then a pair of triangles will be similar.
- If a pair of triangles are congruent then they will also be similar (with a scale factor of 1).

Worked example 9.5A Finding unknown side lengths in similar triangles

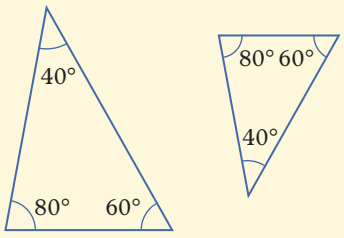
Find the value of the unknown side lengths in this pair of similar triangles.



THINK	WRITE
1 Identify the equal corresponding angles and then match the corresponding sides opposite these angles. Sides QS and EF are both opposite the 38° angle. Sides RS and GF are both opposite the 50° angle.	
2 Use the corresponding pairs of sides to write a similarity statement for the pair of triangles.	$QSR \sim EFG$
3 Using the pair of corresponding sides that have known measurements (QS and EF), find the scale factor between $\triangle QSR$ and $\triangle EFG$. Consider $\triangle QSR$ as the original triangle and $\triangle EFG$ as the image.	Corresponding pairs: $QS = 4$ cm and $EF = 8$ cm $\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$ $= \frac{8}{4}$ $= 2$
4 Use the scale factor formula to find y in $\triangle EFG$.	Corresponding pairs: $RS = 5$ cm and $GF = y$ $\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$ $2 = \frac{y}{5}$ $y = 2 \times 5$ $y = 10$ cm
5 Use the scale factor formula to find x in $\triangle QSR$.	Corresponding pairs: $QR = x$ and $EG = 13$ cm $\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$ $2 = \frac{13}{x}$ $x = \frac{13}{2}$ $x = 6.5$ cm

Worked example 9.5B Determining which similarity test to use

Which test could you use to check if this pair of triangles are similar?



THINK

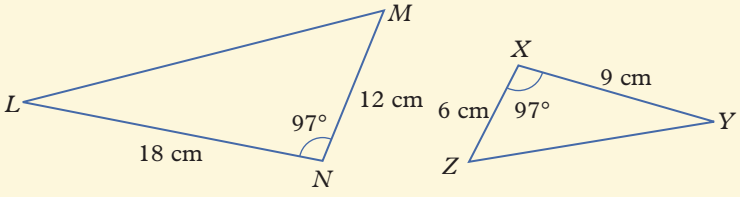
Look at the information provided in each triangle. All angles are given in both triangles.

WRITE

The AAA similarity test could be used to check whether these triangles are similar.

Worked example 9.5C Determining if two triangles are similar

Determine whether this pair of triangles are similar, stating the similarity test that was used.



THINK

1. Both triangles show two sides and an included angle, so we can use the SAS similarity test to determine whether this pair of triangles are similar.
2. Calculate the scale factor between the two shorter sides (NM and XZ) and the two longer sides (NL and XY).
3. Compare the scale factors to determine whether the SAS test passes.
4. Write your final answer.

WRITE

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{XZ}{NM}$$
$$= \frac{6\text{ cm}}{12\text{ cm}}$$
$$= \frac{1}{2}$$

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{XY}{NL}$$
$$= \frac{9\text{ cm}}{18\text{ cm}}$$
$$= \frac{1}{2}$$

The two triangles are similar as they pass the SAS similarity test.

Helpful hints

✔ Make sure that you identify the corresponding sides and angles correctly. This is a key skill. Remember that the given triangles may not be in the same orientation.

Learning pathways

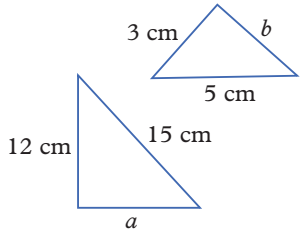
1, 2, 4–6, 9, 10, 12, 13	1, 3–5, 7, 8, 11, 12, 14, 15	1, 4, 5, 7, 10, 12, 13, 15–18

ANS p588

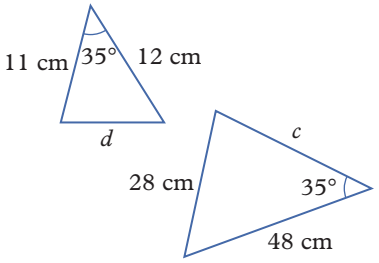
Exercise 9.5A: Understanding and fluency

WE 9.5A **1** Find the values of the unknown side lengths in each pair of similar triangles.

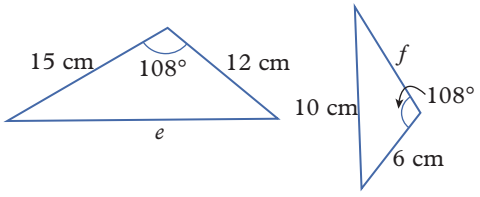
a



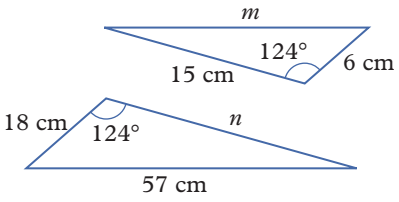
b



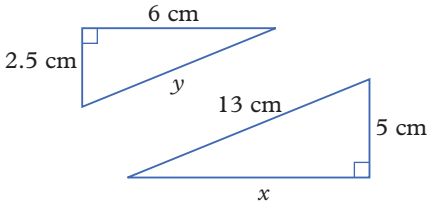
c



d



e



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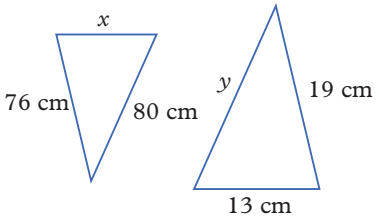
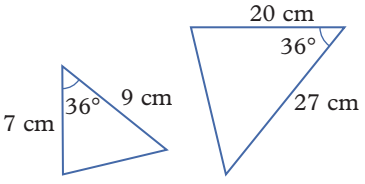
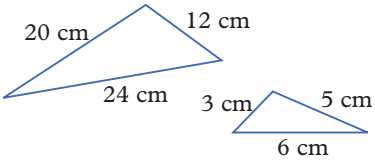


WE 9.5B **2** State the test you could use to check whether each pair of triangles is similar.

a



b



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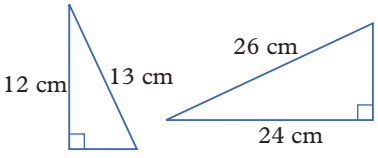

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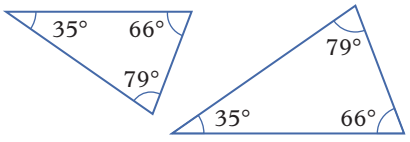
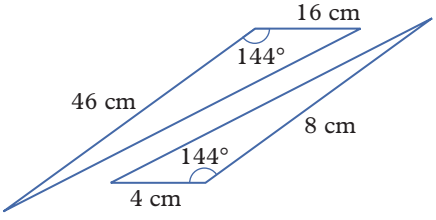
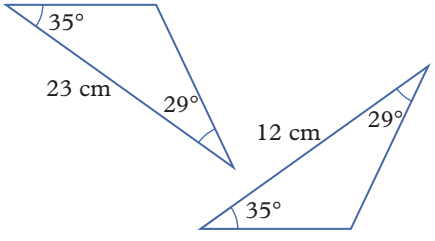


e

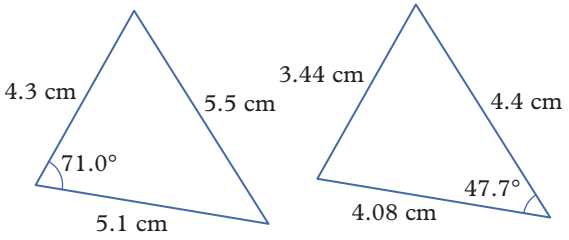


f

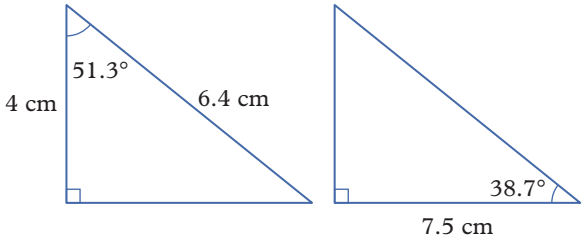


3 Which test(s) could you use to check that these triangles are similar?

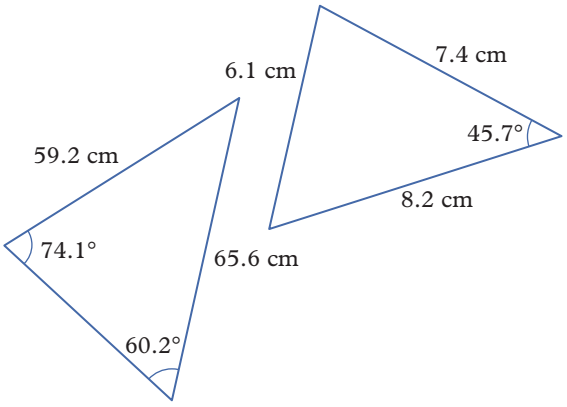
a



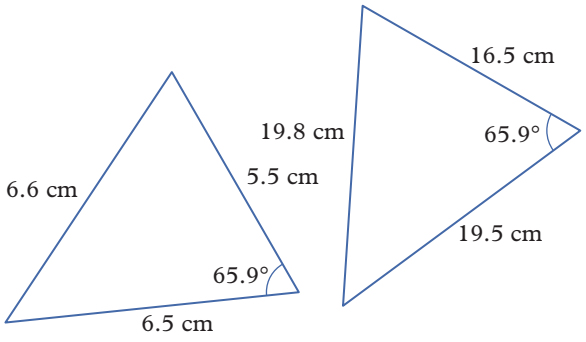
b



c

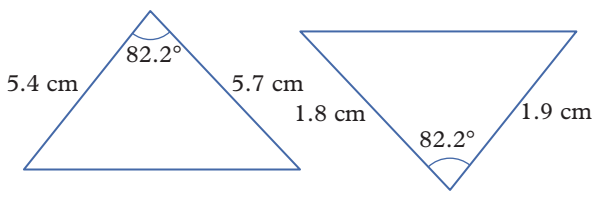


d

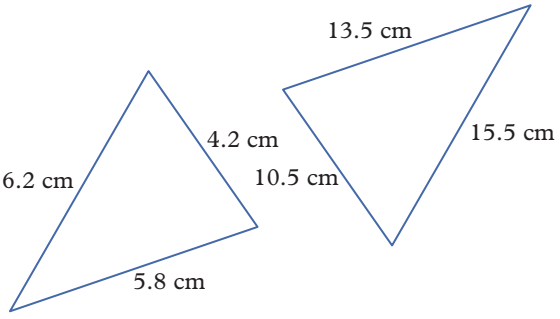


WE 9.5C **4** Determine if each pair of triangles are similar. State the similarity test used in each instance.

a



b



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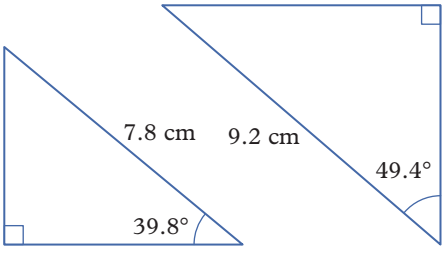

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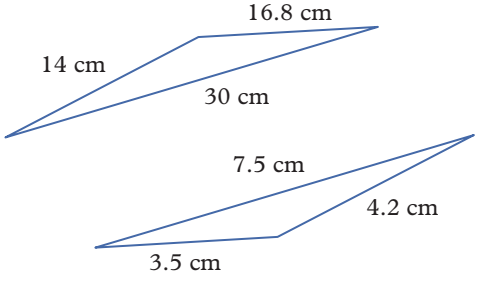

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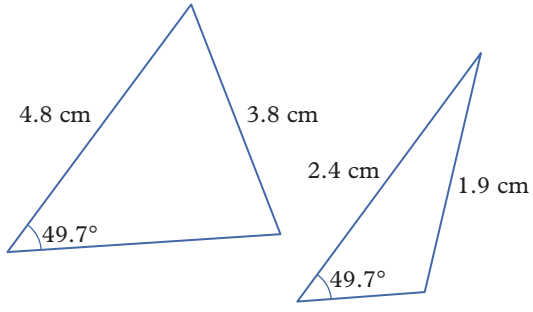

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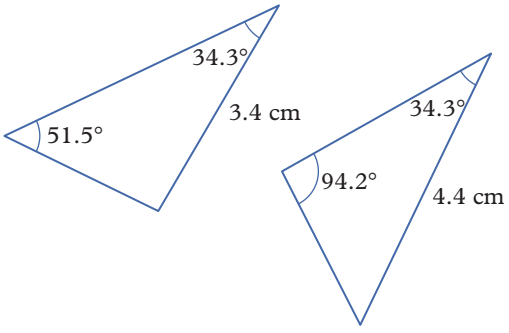
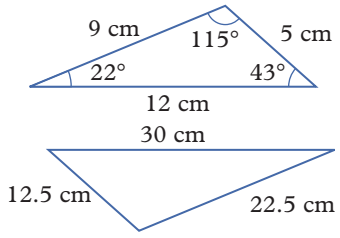


5 Determine whether each pair of triangles are similar.

a



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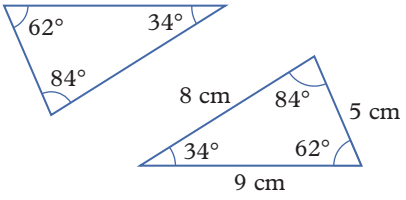

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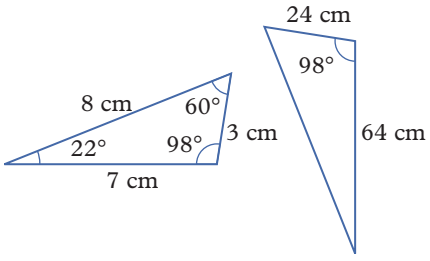

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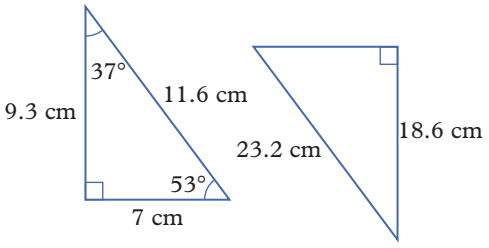

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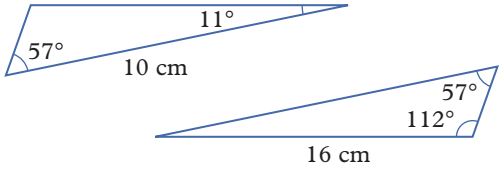

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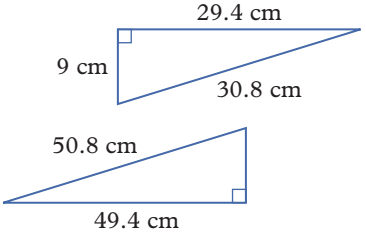


Exercise 9.5B: Problem solving and reasoning

6 Determine if the pairs of triangles from question **2** are similar.

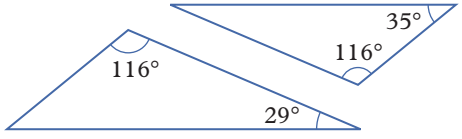
7 Consider the four similar-triangle tests. What conditions need to be added to each of the tests for the similar triangles to be congruent?

8 Why are all equilateral triangles similar to each other?

9 Explain why only two angles need to be checked when using AAA as a test for similarity.

10 Use your understanding of triangle properties to determine whether these pairs of triangles are similar.

a



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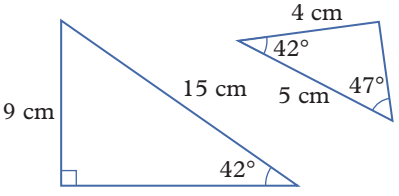

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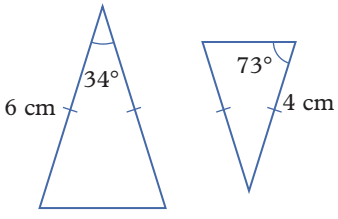

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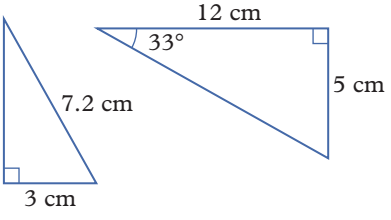
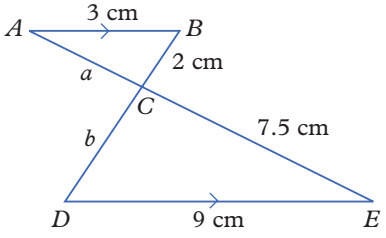


11 Consider the diagram on the right.

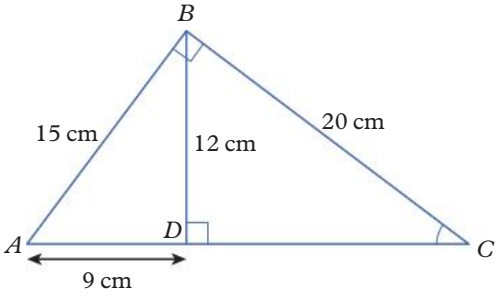
a Using an understanding of angles and parallel lines, explain why the pair of triangles in the diagram are similar.

b If triangle *ABC* is the original figure, find the scale factor between the two triangles.

c Find the length of the unknown sides.



12 Consider the following diagram.



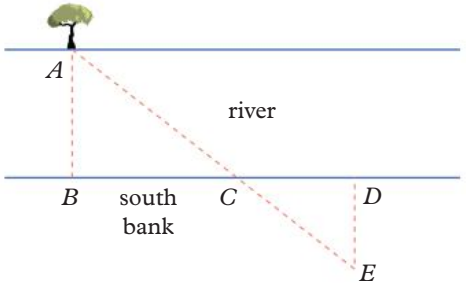
a Which similarity test can be used to confirm that $\triangle ADB \sim \triangle BDC$?

b Using the ratio of similar sides, what is the side length CD?

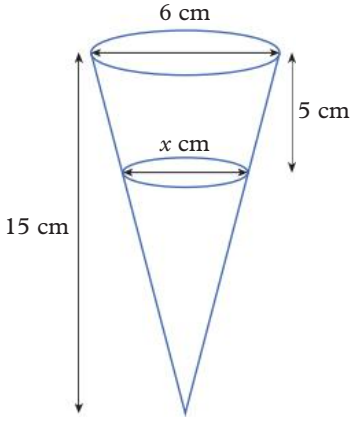
c Confirm your answer to part **b** by using Pythagoras' theorem.

d Explain why $\triangle ABC$ is also similar to $\triangle ADB$ and $\triangle BDC$.

13 In order to find the width of a river, Ahmed places three markers along its southern bank. Marker B is directly opposite a tree at position A. Marker C is 50 m east of marker B and 40 m west of marker D. Ahmed then stands at point E, 30 m south of marker D and he can see marker C in a direct line with the tree at point A. Draw the diagram and mark in these measurements, and then use similar triangles to find the width of the river.



14 An ice-cream cone is 15 cm high with an opening of diameter 6 cm. If chocolate coats the inside of the cone from the bottom to 5 cm below its top, find the largest diameter of the chocolate coating inside the cone.



15 Draw diagrams to help you solve these similar triangle problems.

a Scott has a red triangular flag that is 10 cm high and 15 cm long. He wants to create a larger version from 2 m of red material. If he uses its full length, what height would this larger flag be?

b Christy bought a new netball ring but she wasn't sure that it was the official height (3.05 m). She measured her shadow as 117 cm and the netball ring's shadow as 195 cm. If Christy is 1.8 m tall, is the netball ring the official height?

c Oscar wants to know the height of a flagpole. A guide wire positioned 3.5 m from its base reached to exactly half its height. Oscar stood exactly halfway between the flagpole and where the guide wire attached to the ground and the guide wire touched the top of his head. If Oscar is 1.75 m tall, how tall is the flagpole?

16 Sean and Tania want to know whose house is taller. One morning, Sean measures his shadow as 1.9 m and the shadow of his house as 3.42 m. Later that day, Tania measures her shadow as 2.3 m and the shadow of her house as 3.8 m. If Sean is 172 cm tall and Tania is 165 cm tall, whose house is taller?

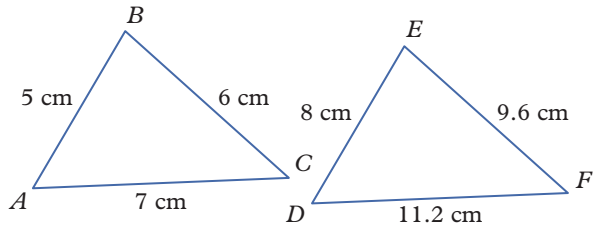
17 Explain why you cannot just compare the shadow lengths of the houses in question **16** to decide which house is taller.

Exercise 9.5C: Challenge

18 The process of showing that two triangles are similar can be formalised and generalised further. We want to prove beyond a shadow of a doubt that a given fact is true by writing up bulletproof reasoning behind every step we take. We call these geometric proofs.

Follow the worked examples and then complete the proofs.

Example: Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{8\text{ cm}}{5\text{ cm}} = 1.6$

$\frac{EF}{BC} = \frac{9.6\text{ cm}}{6\text{ cm}} = 1.6$

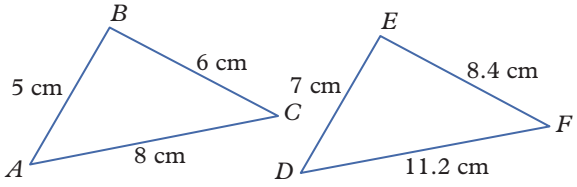
$\frac{FD}{CA} = \frac{11.2\text{ cm}}{7\text{ cm}} = 1.6$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

Corresponding sides are in the same ratio.

$\therefore \triangle ABC \sim \triangle DEF$ by SSS

a Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{\square}{\square} =$

$\frac{EF}{BC} = \frac{\square}{\square} =$

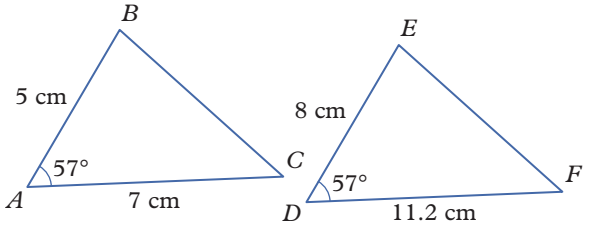
$\frac{FD}{CA} = \frac{\square}{\square} =$

$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

Corresponding sides are in the same ratio.

$\therefore \triangle$_____ $\sim \triangle$____ by ____

Example: Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{8\text{ cm}}{5\text{ cm}} = 1.6$

$\frac{FD}{CA} = \frac{11.2\text{ cm}}{7\text{ cm}} = 1.6$

$\angle EDF = 57°$

$\angle BAC = 57°$

$\frac{DE}{AB} = \frac{FD}{CA}$

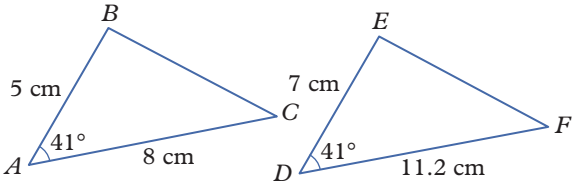
Corresponding sides are in the same ratio.

$\angle EDF = \angle BAC$

Corresponding included angles are equal.

$\therefore \triangle ABC \sim \triangle DEF$ by SAS

b Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{\square}{\square} =$

$\frac{FD}{CA} = \frac{\square}{\square} =$

$\angle EDF =$ ____°

$\angle BAC =$ ____°

$\frac{\square}{\square} = \frac{\square}{\square}$

Corresponding sides are in the same ratio.

$\angle$____ $= \angle$_____

Corresponding included angles are equal.

$\therefore \triangle$_____ $\sim \triangle$____ by ____

Online resources:

- **Interactive skillsheet** — Similar triangles
- **Investigation** — Using mirrors to calculate length
- **Quick quiz** — 9.5

Lesson 9.6

Quadrilaterals

Learning intentions

By the end of this lesson you will be able to ...

✔ classify quadrilaterals, and find unknown angles in quadrilaterals.

Inter-year links

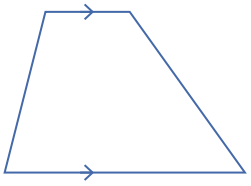
Year 7 7.5 Quadrilaterals

Year 10 6.4 Proofs and quadrilaterals

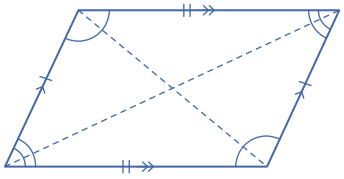
Properties of quadrilaterals

Key content video
Classifying quadrilaterals

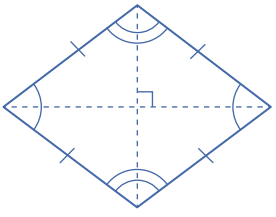
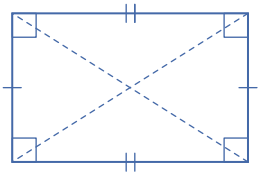
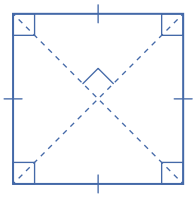
- A **quadrilateral** is a two-dimensional (2D) shape with four straight sides.
- The interior angles in a quadrilateral sum to 360°.
- Quadrilaterals can be classified by their side lengths and interior angles.

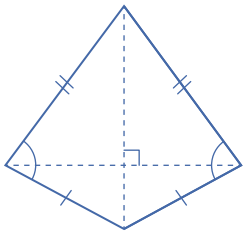
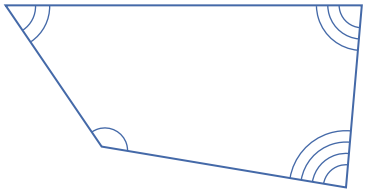
Name and definition	Properties	Example
Trapezium A quadrilateral with at least one pair of parallel sides	• two pairs of adjacent supplementary angles	

A parallelogram is a special type of trapezium.

Parallelogram A quadrilateral with two pairs of parallel sides	• two pairs of opposite angles that are equal in size • opposite sides are equal in length • diagonals bisect each other (divide each other into two equal parts)	

Three special types of parallelogram are the rhombus, rectangle and square.

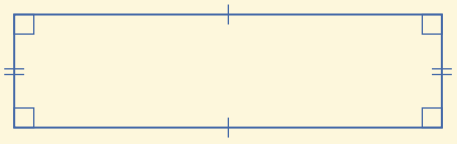
Rhombus A quadrilateral with four sides of equal length	• diagonals are perpendicular • diagonals bisect the vertex angles	
Rectangle A quadrilateral with four right angles	• diagonals are equal in length	
Square A quadrilateral with four sides of equal length and four right angles	• diagonals are equal in length • diagonals are perpendicular • diagonals bisect the vertex angles	

Name and definition	Properties	Example
Kite A quadrilateral with two pairs of adjacent sides that are equal in length	• no parallel sides • one pair of opposite angles that are equal in size • diagonals are perpendicular • one diagonal bisects the other diagonal • one diagonal bisects a pair of vertex angles	
Irregular quadrilateral A quadrilateral with no special properties	• four different internal angles • no parallel sides	

- The definition of a quadrilateral is also a property of that quadrilateral.

Worked example 9.6A Classifying quadrilaterals

Classify this quadrilateral.



THINK

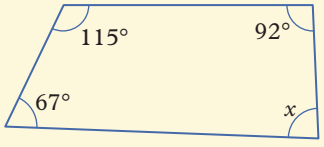
Examine the properties of this quadrilateral. The quadrilateral has opposite sides that are equal in length and it has four right angles, so it is a rectangle.

WRITE

The quadrilateral is a rectangle.

Worked example 9.6B Finding an internal angle in a quadrilateral

Find the size of angle x.



THINK

The internal angles in a quadrilateral add to 360°. Subtract each of the known angles from 360° to determine the size of the fourth angle, x.

WRITE

$x = 360° - 115° - 92° - 67°$
$= 86°$

Helpful hints

- ✔ The diagonals of a non-square rectangle are not perpendicular (at right angles to each other)!
- ✔ As rhombuses, rectangles and squares are all parallelograms, they also have the properties of parallelograms.
- ✔ Every square is also a rhombus and a rectangle, as well as being a parallelogram.

Learning pathways

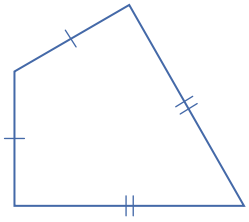
1–4, 5(a–c), 7, 8, 10, 11, 14	1–4, 5(d–f), 6, 7, 9, 12, 14–16	3–5, 8, 13–19

ANS p589

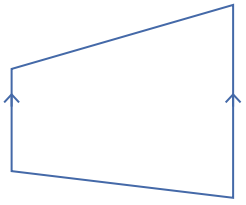
Exercise 9.6A: Understanding and fluency

WE 9.6A **1** Classify each quadrilateral. Give reasons with your answers.

a



b



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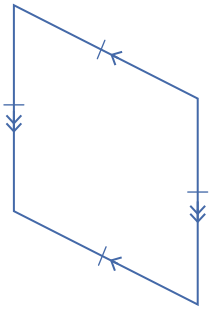

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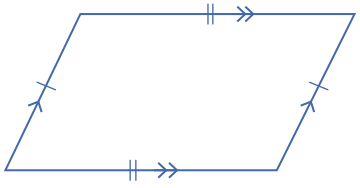

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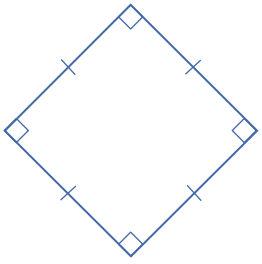

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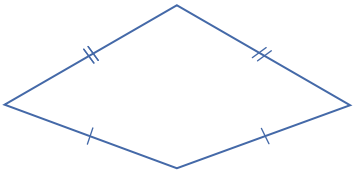
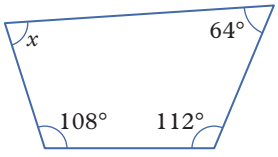


WE 9.6B **2** Find the size of angle x in each quadrilateral.

a



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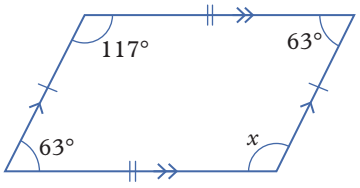

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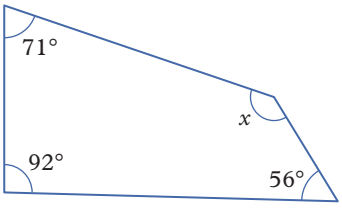

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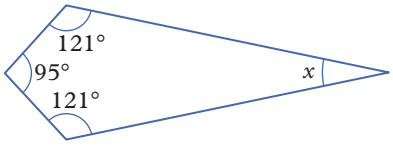

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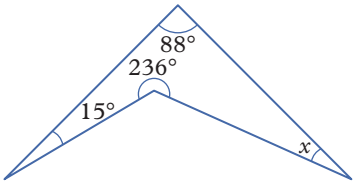

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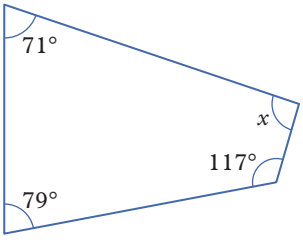
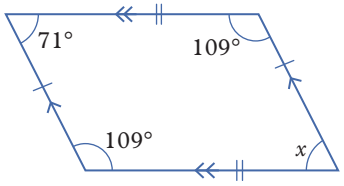
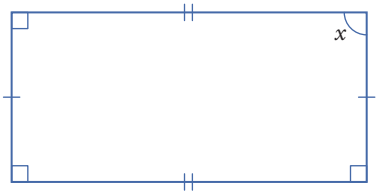


3 Classify each quadrilateral and find the size of angle x.

a



b



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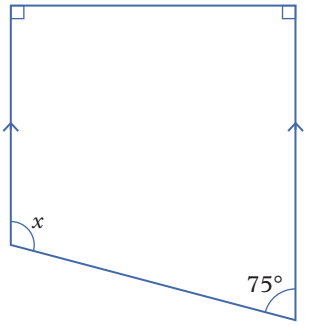

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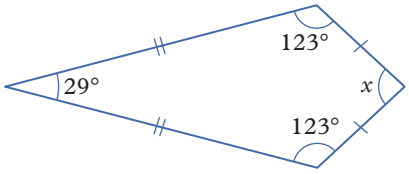

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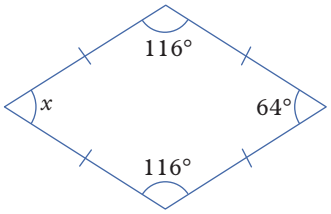

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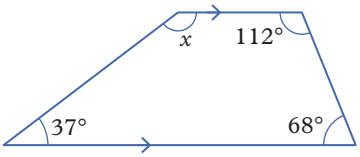
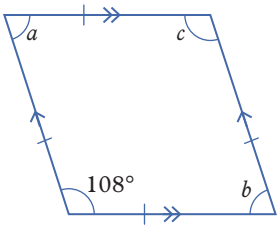


4 Use your knowledge of quadrilaterals to:

i name each figure

ii find the size of the unknown angles.

a



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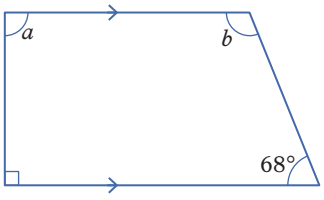

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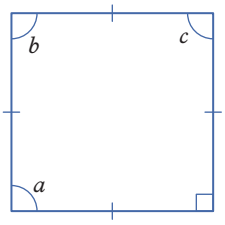

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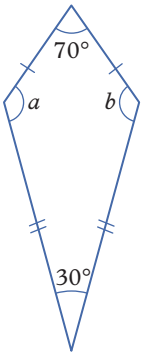

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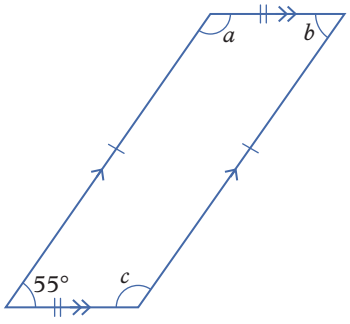

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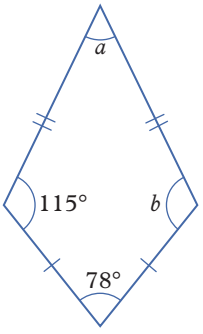
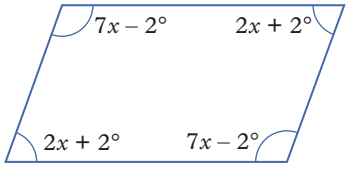
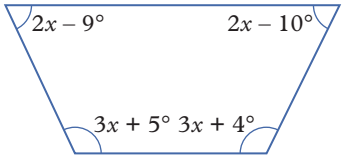


5 Find the value of x in each quadrilateral.

a



b



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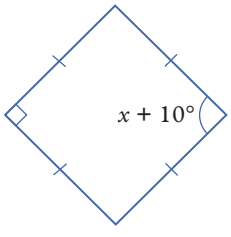

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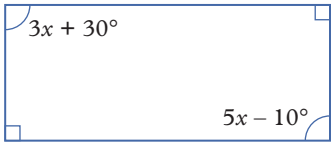

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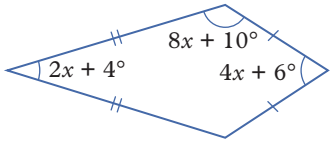

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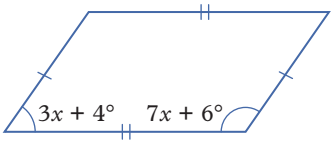
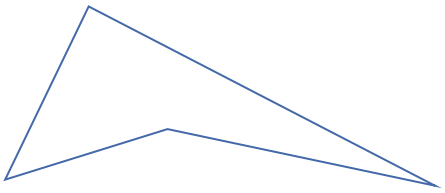


Exercise 9.6B: Problem solving and reasoning

6 An irregular polygon can be convex or concave. A convex polygon has internal angles all smaller than 180°. A concave polygon has at least one reflex angle (greater than 180°).



a Explain why a concave quadrilateral can have only one internal reflex angle.

b Find the size of the reflex angle in a quadrilateral if the other three angles measure 20°, 20° and 50°.

7 Decide if each statement is true or false. If it is false, give a reason why.

a A square is a type of parallelogram.

b A kite has a pair of parallel sides.

c A parallelogram is a type of rectangle.

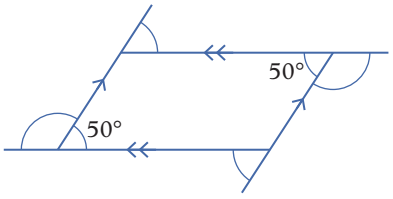
d A trapezium has exactly one pair of parallel sides.

e A rhombus always has four equal sides and four equal angles.

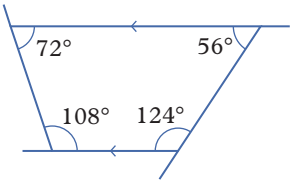
f A rectangle always has four equal sides and four equal angles.

8 Exterior angles can be found for quadrilaterals.
Consider the following quadrilaterals.

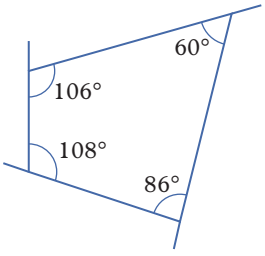
Quadrilateral 1:



Quadrilateral 2:



Quadrilateral 3:



a For Quadrilateral 1, state the:

i size of each exterior angle

ii sum of the exterior angles.

b For Quadrilateral 2, state the:

i size of each exterior angle

ii sum of the exterior angles.

c For Quadrilateral 3, state the:

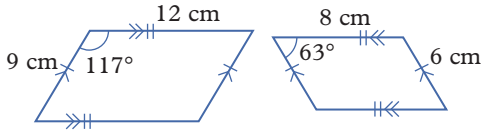
i size of each exterior angle

ii sum of the exterior angles.

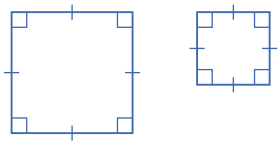
d What can you say about the sum of the exterior angles of a quadrilateral?

9 Use your understanding of quadrilateral properties to determine whether each pair of shapes is similar.

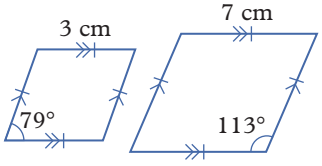
a



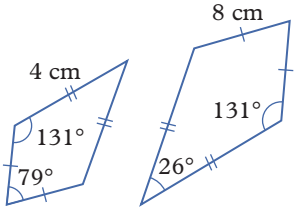
b



c

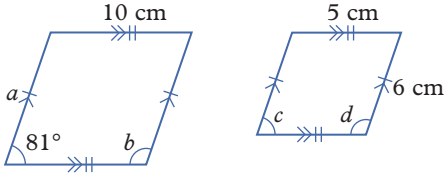


d

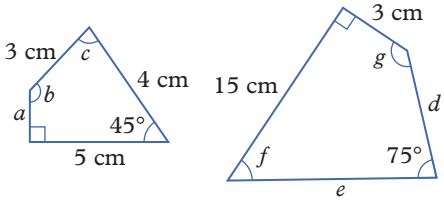


10 Use your understanding of quadrilateral properties to find the value of the pronumerals in each pair of similar figures.

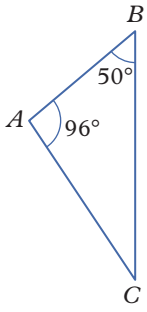
a



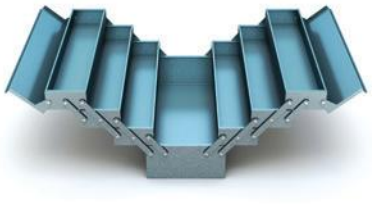
b



11 The following triangle is reflected in the line BC to create a kite. What are the four interior angles of the kite?

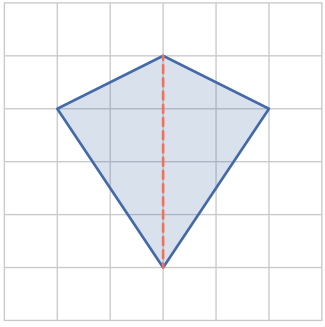


12 Many toolboxes are designed so that the trays of the boxes stay horizontal as they open. Use the properties of parallelograms to explain how these toolboxes work.

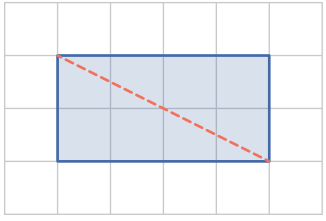


13 The diagonals of some quadrilaterals are axes of symmetry; that is, one half of the quadrilateral can be reflected in the diagonal to give the other half of the quadrilateral. Reflect the left-hand half of each of these quadrilaterals in the red diagonal to determine whether it is an axis of symmetry.
Hint: If you are unsure how to reflect in a line, review **Lesson 9.1** question **6**.

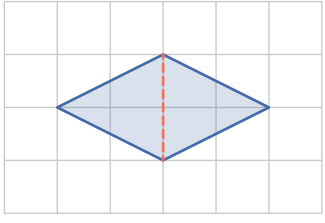
a kite



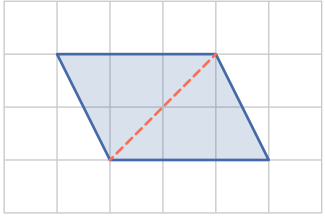
b rectangle



c rhombus



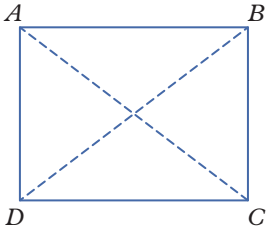
d parallelogram



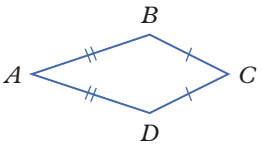
14 Consider the rectangle *ABCD*.

a Explain why $\triangle ABD \cong BAC$, stating the congruence test you used.

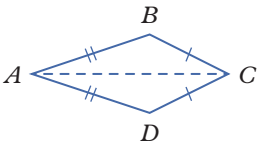
b Hence, explain why the diagonals of a rectangle are equal in length.



15 Consider the kite *ABCD* shown with equal side lengths marked. We can establish other properties of kites by using congruent triangles. Copy and complete the explanations for each part.



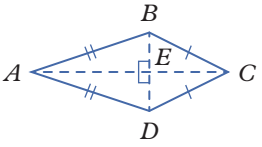
a Add the diagonal *AC* to the kite to cut it into two triangles.
Explain why $\triangle ABC \cong \triangle ADC$ by stating the three pairs of congruent sides.
△___ ≅ △____ by _____ as __ ≅ __, __ ≅ __, and __ is common to both triangles.



b Explain why $\angle ABC = \angle ADC$, $\angle BCA = \angle DCA$, and $\angle BAC = \angle DAC$.
∠___ = ∠____, ∠___ = ∠____, and ∠___ = ∠___ because they are _____ angles in _____ triangles.

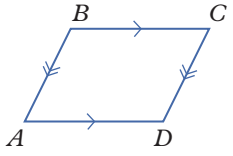
c Use the congruent angles in part **b** to explain why diagonal *AC* bisects (cuts in half) $\angle BAD$ and $\angle BCD$.
$\angle BAD$ = ∠___ + ∠____.
Since $\angle BAC = \angle DAC$, $\angle BAD$ = 2∠___ and so ∠___ has been bisected into two equal angles by diagonal __.
Likewise, $\angle BCD$ = ∠___ + ∠___.
Since $\angle BCA = \angle DCA$, $\angle BCD$ = 2∠___ and so ∠___ has been bisected into two equal angles by diagonal __.

d Add the perpendicular height to $\triangle ABC$ and $\triangle ADC$ if the base is *AC* for both triangles. Explain why the two perpendicular heights, *BE* and *DE*, form the diagonal *BD*.
Since △___ ≅ △___, the perpendicular heights __ and __ will be in the same relative position in the triangles, at point _ to matching vertices _ and _.
These two line segments align to form the diagonal __.



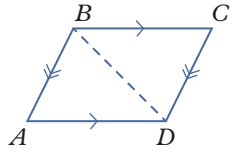
e Explain why *BE* = *DE* and why diagonal *AC* bisects diagonal *BD* at a right angle.
Since △___ ≅ △___, __ and __ are _____ lengths in _____ triangles, which are therefore _____.
∠___ and ∠____ are right angles since __ and __ are perpendicular heights to __ on which point *E* lies.
Since __ intersects __ at point *E*, diagonal __ has _____ diagonal __ at a _____ angle.

16 Consider the parallelogram *ABCD* shown with parallel sides as marked. We can establish other properties of a parallelogram by using congruent triangles. Copy and complete the explanations for each part.



a Determine which pairs of angles in *ABCD* are equal.
∠___ = ∠___, and ∠____ = ∠___

b Add diagonal *BD* to the parallelogram. Explain why $\angle BDA = \angle DBC$ and $\angle DBA = \angle BDC$.
__ and __ are parallel and __ is a transversal.
∠___ = ∠___ as they are _____ angles on _____ lines.
Likewise, __ and __ are parallel and __ is a _____ so ∠___ = ∠___ as they are _____ angles on _____ lines.



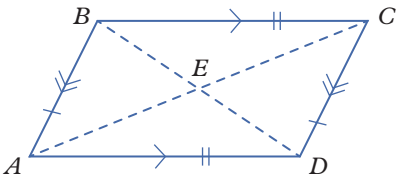
c Explain why it does not matter which diagonal (*AC* or *BD*) was added to find two congruent triangles.
Either diagonal will act as a _____ to both pairs of _____ sides, so the _____ angles formed by the _____ will be congruent.

d Explain why $\triangle ABD \cong \triangle CDB$.
△___ ≅ △___ by ___ as ∠____ = ∠____, ∠____ = ∠____, and __ is common to both triangles.

e Use the fact that $\triangle ABD \cong \triangle CDB$ to explain why *AB* = *CD* and *AD* = *CB*.
△___ ≅ △___, where side _____ corresponds to side _____ and side _____ corresponds to side _____, so ___ = ___ and ___ = ___.

17 In question **16**, we showed that for the parallelogram $ABCD$, given the parallel sides shown:

- $\angle BDA = \angle DBC$
- $\angle DBA = \angle BDC$
- $AB = CD$
- $AD = CB$.



Copy and complete the explanations for each part.

a Add diagonal AC to the parallelogram. Explain why $\angle AED = \angle BEC$ and $\angle BEA = \angle CED$.

$\angle$___ = $\angle$___ as they are _____ angles.
Likewise, $\angle$___ = $\angle$___ as they are _____ angles.

b State which two pairs of triangles are congruent and why they are congruent.

Δ___ $\cong$ Δ___ by ___ as $\angle$____ = $\angle$____, $\angle$____ = $\angle$____, and __ = ___.
Δ___ $\cong$ Δ___ by ___ as $\angle$____ = $\angle$____, $\angle$____ = $\angle$____, and __ = ___.

c Explain why $BE = DE$ and $AE = CE$.

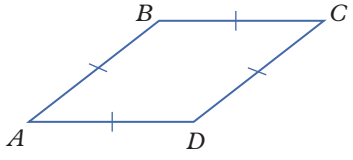
__ = __ and __ = __ as they are _____ lengths in _____ triangles, which are _____.

d Explain why the diagonals AC and BD bisect each other.

Diagonal __ = __ + __, and since __ = __, diagonal __ = 2__ and so has been _____ into two equal line segments by diagonal __.
Likewise, diagonal __ = __ + __, and since __ = __, diagonal __ = 2__ and so has been _____ into _____ by diagonal __.
Therefore, _____.

Exercise 9.6C: Challenge

18 Consider the rhombus $ABCD$ with equal sides marked.



a Add diagonal AC and explain why $\Delta ABC \cong \Delta ADC$.

b State what type of triangles ΔABC and ΔADC are. Explain how you know.

c Explain why $\angle BAC = \angle BCA$ and $\angle DAC = \angle DCA$.

d Explain why $\angle ABC = \angle ADC$, $\angle BCA = \angle DCA$, and $\angle BAC = \angle DAC$.

e Explain why the diagonal AC bisects $\angle BAD$ and $\angle BCD$.

f Explain why a similar process to parts **a–e** can be used to show that the diagonal BD bisects $\angle ABC$ and $\angle ADC$.

19 Each part below gives a clue about a different quadrilateral. Use the clues to:

i find the size of each angle **ii** name the quadrilateral.

a All angles are equal and all sides are equal.

b Two pairs of angles are equal and one pair of angles is twice the size of the other pair. Two pairs of side lengths are equal and one pair is twice the length of the other pair.

Online resources:

Interactive skillsheet
Quadrilateral properties

Investigation
Stained glass quadrilaterals

Quick quiz
9.6

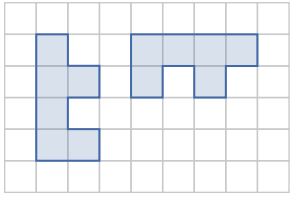
Lesson 9.7

Review: Congruence and similarity

Module summary

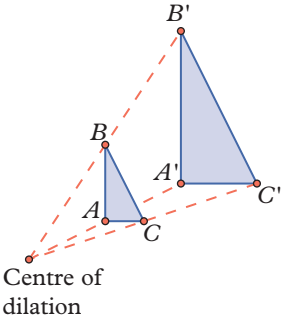
Congruent figures

- Same shape and size. Isometric translations produce congruent figures.



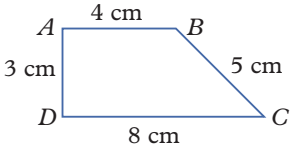
Dilations

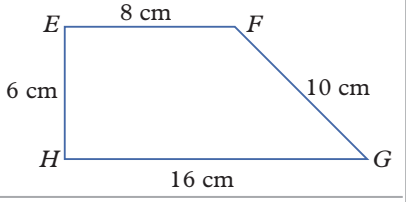
- An enlargement creates an image that is larger than the original shape.
- A reduction creates an image that is smaller than the original shape.
- A centre of dilation and scale factor are needed to perform a dilation.
- scale factor = $\frac{\text{image length}}{\text{original length}}$
- Length of $A'B'$ = length of AB × scale factor



Similar figures

- Same shape but do not need to be the same size.
- All corresponding angles are equal in size.
- All corresponding sides are in the same ratio.





Congruent triangles

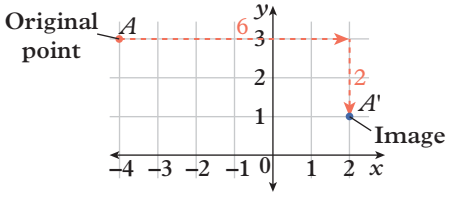
- SSS (side–side–side): all three side lengths are equal.
- SAS (side–angle–side): two side lengths and the included angle are equal.
- AAS (angle–angle–side): two angles and a corresponding side are equal.
- RHS (right angle–hypotenuse–side): the hypotenuse and another side of a right-angled triangle are equal.

Similar triangles

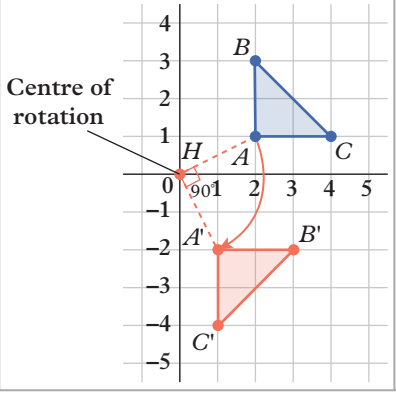
- SSS (side–side–side): all three pairs of corresponding side lengths are in the same ratio.
- SAS (side–angle–side): two corresponding pairs of side lengths are in the same ratio and the included angles are equal.
- AAA (angle–angle–angle): all three pairs of corresponding angles are equal.
- RHS (right angle–hypotenuse–side): the hypotenuses are in the same ratio as another set of sides in a right-angled triangle.

Translations

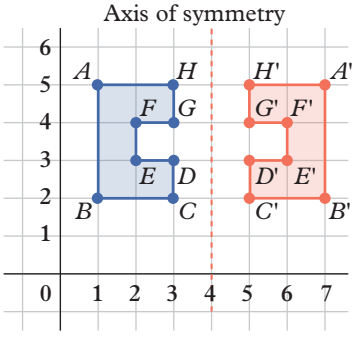
Translation 6 units right and 2 units down



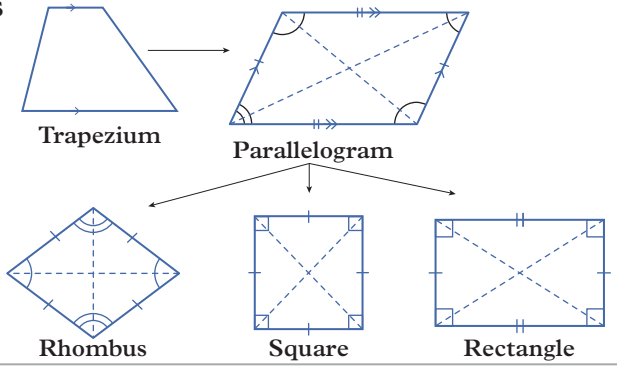
Rotations



Reflections



Quadrilaterals



ANS p590

Review questions 9.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- centre of dilation
- congruent
- dilation
- enlargement
- image
- included angle
- irregular quadrilateral
- isometric transformation
- kite
- parallelogram
- point of focus
- quadrilateral
- rectangle
- reduction
- reflection
- rhombus
- rotation
- scale factor
- similar
- square
- translation
- trapezium

1 Name three types of isometric transformations.

2 Name the three special types of parallelograms.

3 What three things does a rotation require?

4 Identify the key terms being referenced in each of these definitions.

a a quadrilateral with four sides of equal length

b a transformed shape or object

5 Using an example, provide a definition in your own words for the following key terms.

a included angle

b trapezium

6 Complete the following sentences using words from the key terms list.

a A dilation can be either an _____or a _____.

b Two figures are _____ if they have the same shape and size, or if one shape is the mirror image of the other.

Review questions 9.7B: Multiple choice

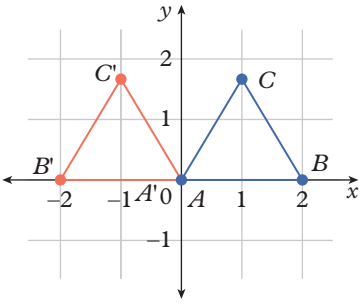
9.1 **1** Which of the following transformations would *not* give the image of the equilateral triangle shown?

A Translate 2 units left.

B Translate 2 units right.

C Rotate 120° anti-clockwise about the origin.

D Rotate 240° clockwise about the origin.

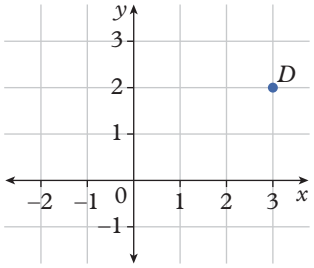


9.1 **2** The point (1, −2) on the Cartesian plane results in which image point after a 90° anti-clockwise rotation about the origin?

A (1, 2) **B** (2, 1) **C** (2, −1) **D** (−1, −2)

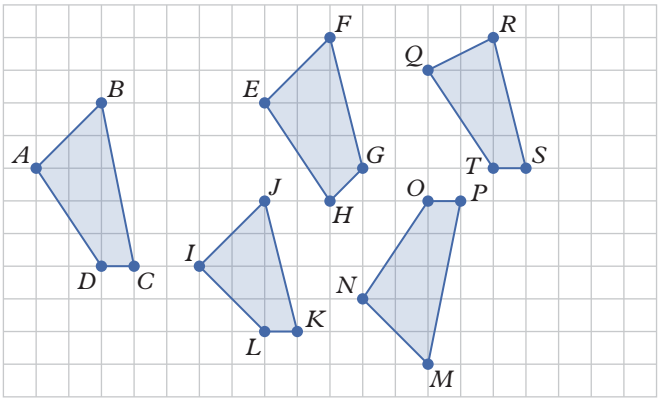
9.1 **3** What are the coordinates of the point D if it is translated by 2 units left and 1 unit down, and then reflected in the x-axis?

A $(1, -1)$
B $(-3, 1)$
C $(-1, 1)$
D $(1, -3)$



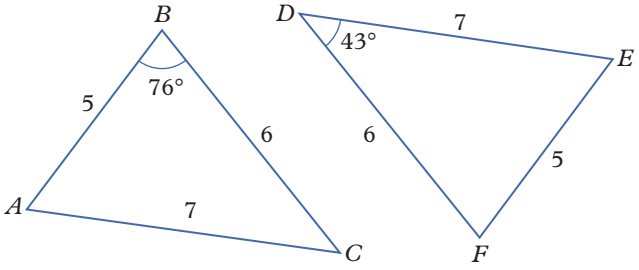
9.2 **4** Which of the following is congruent to $ABCD$?

A $EFGH$
B $IJKL$
C $NMPO$
D $QRST$



9.3 **5** Using the triangles shown, determine the size of $\angle BAC$.

A 76°
B 43°
C 14°
D 61°

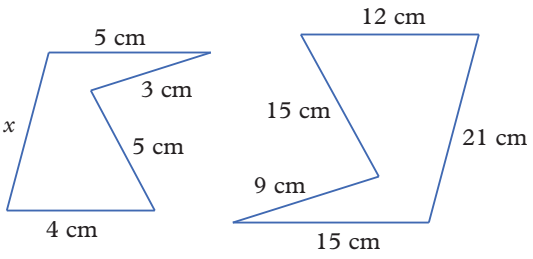


9.4 **6** Which scale factor will not result in an enlargement?

A $\frac{1}{3}$ **B** $\frac{5}{4}$ **C** 2 **D** $\frac{10}{3}$

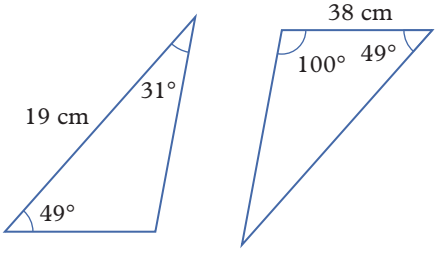
9.4 **7** Find the value of x for these similar figures.

A 21 cm
B 63 cm
C 7 cm
D 8.75 cm



9.5 **8** Which test would you use to determine whether these two triangles are similar?

A AAA
B SSS
C SAS
D AAS



9.6 **9** Which statement is true?

A A rhombus is a rectangle.

B A parallelogram is a square.

C A square is a rhombus.

D A trapezium cannot have a right angle.

10 A quadrilateral ABCD exists such that AC and BD intersect at a 90° angle. Which of the following shapes could ABCD not be classified as?

A kite

B rhombus

C square

D rectangle

Review questions 9.7C: Short answer

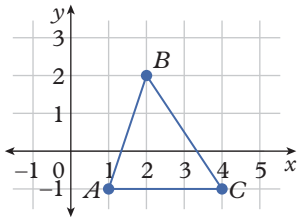
9.1 **1** Transform the triangle shown below right using the descriptions provided. Start from the original triangle each time.

a Translate 2 units right and 3 units down.

b Reflect in the x-axis.

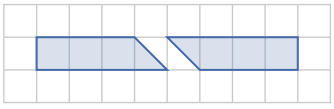
c Rotate 180° clockwise about the origin, then translate 3 units up.

d Translate 2 units right and 1 unit down, then reflect in the y-axis.

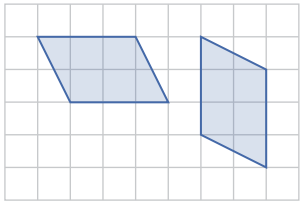


9.2 **2** Decide whether each pair of shapes is congruent.

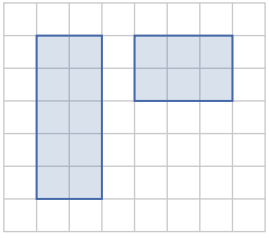
a



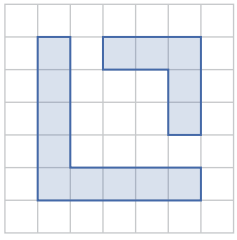
b



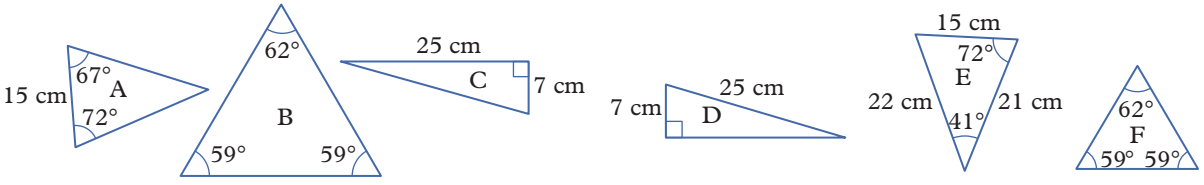
c



d



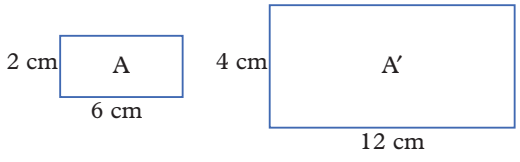
9.3 **3** Identify any pairs of congruent triangles.



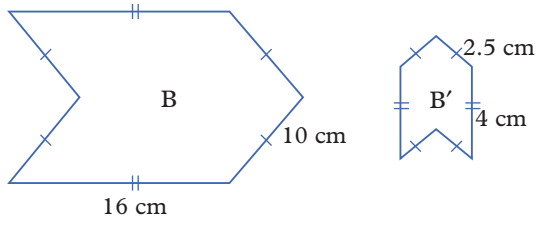
9.3 **4** Explain your answers to question **3** using one of the congruence tests (SSS, SAS, AAS, RHS).

9.4 **5** Describe each transformation as a reduction or an enlargement and state the scale factor.

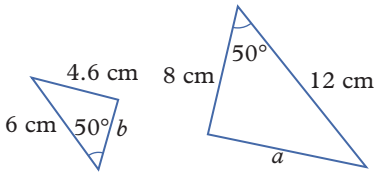
a



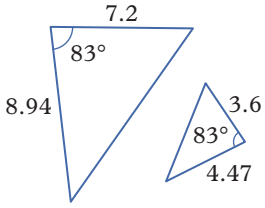
b



9.5 **6** Given that these two triangles are similar, find the value of the unknown side lengths.

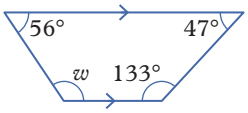


9.5 **7** Determine whether these two triangles are similar. If similar, state the similarity test you used, or explain why they are not similar.

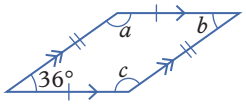


9.6 **8** Find the size of the unknown angles in these quadrilaterals.

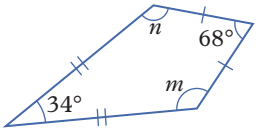
a



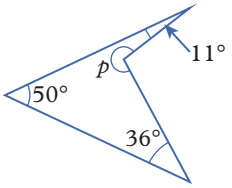
b



c



d

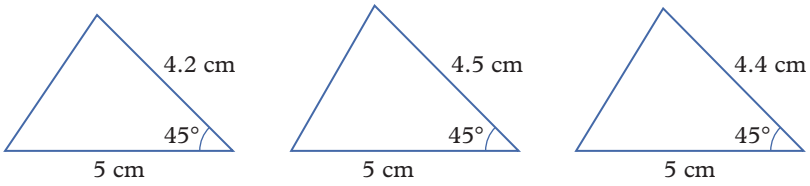


9.6 **9** Name the quadrilaterals shown in question **8**.

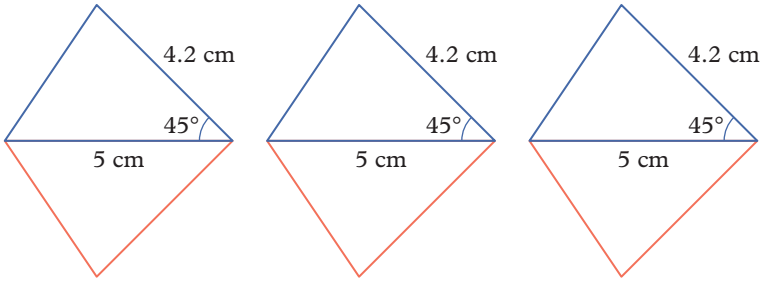
Review questions 9.7D: Mathematical modelling

1 Jess is constructing congruent kites to use for an art project.

a She starts by constructing three triangles using the lengths 5 cm and 4.2 cm and the included angle 45°. Jess thinks her compass may have slipped when constructing the triangles and so decides to measure her lengths and angles. Explain why her triangles are not congruent.



b Jess reconstructs her incorrectly drawn triangles so that they are all congruent. Her plan is to draw the reflection of these triangles below the 5 cm length to create her kites. Explain why these shapes will be kites referring to congruent lengths and triangles.

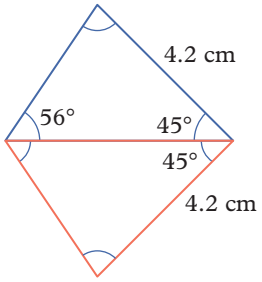


c Construct another kite for Jess using two congruent triangles with the same 5 cm length, 4.2 cm lengths, and included angles of 45°.

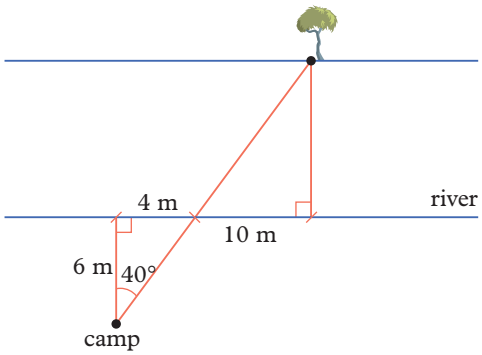
d Jess measures the other base angle of her original triangle to find it is 56°. She says it is now possible to determine the size of all the interior angles in her kite.

i Explain why the size of all the other angles can be determined.

ii Determine the size of the four interior angles of the kite.



2 While preparing to study a species of gum tree on a field trip, Stephanie set up a camp 6 m from a river's edge. From the camp, she measures the angle to a gum tree she wishes to get to and records various measurements, as shown in the diagram.



a Considering both the triangles shown in Stephanie's diagram, what are the angle sizes in each triangle?

b Are the two triangles congruent or only similar? Provide a reason to support your answer.

c What is the scale factor linking the triangle attached to the camp site to the triangle attached to the gum tree?

d Briefly explain how the triangles can be used to determine the width of the river.

e Calculate the width of the river.

f Explain how Pythagoras' theorem can be used to calculate the straight line distance between the camp and the gum tree.

g Calculate the straight line distance between the camp and the gum tree correct to two decimal places.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Describe transformations ☐ Perform transformations	☐ Go back to Lesson 9.1 Transformations
☐ Determine if shapes are congruent ☐ Match vertices and sides of congruent pairs	☐ Go back to Lesson 9.2 Congruence
☐ Use congruence tests to determine if two triangles are congruent	☐ Go back to Lesson 9.3 Congruent triangles
☐ Determine scale factors ☐ Find unknown lengths in similar figures ☐ Perform dilations	☐ Go back to Lesson 9.4 Dilations and similar figures
☐ Find unknown angles and lengths in similar triangles ☐ Determine whether two triangles are similar	☐ Go back to Lesson 9.5 Similar triangles
☐ Classify quadrilaterals ☐ Find missing angles in quadrilaterals ☐ Use congruent triangles to establish properties of quadrilaterals	☐ Go back to Lesson 9.6 Quadrilaterals

Computational Thinking

Curriculum link

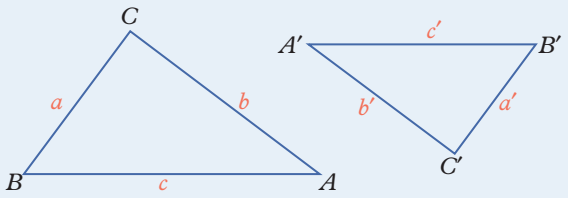
Design and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works (VC2M8SP04)

Inter-year links

Year 7	Classifying polygons
Year 9	Triangles
Year 10	Solving non-linear equations with digital tools
Year 10	Graph algorithms

Congruent and similar triangles

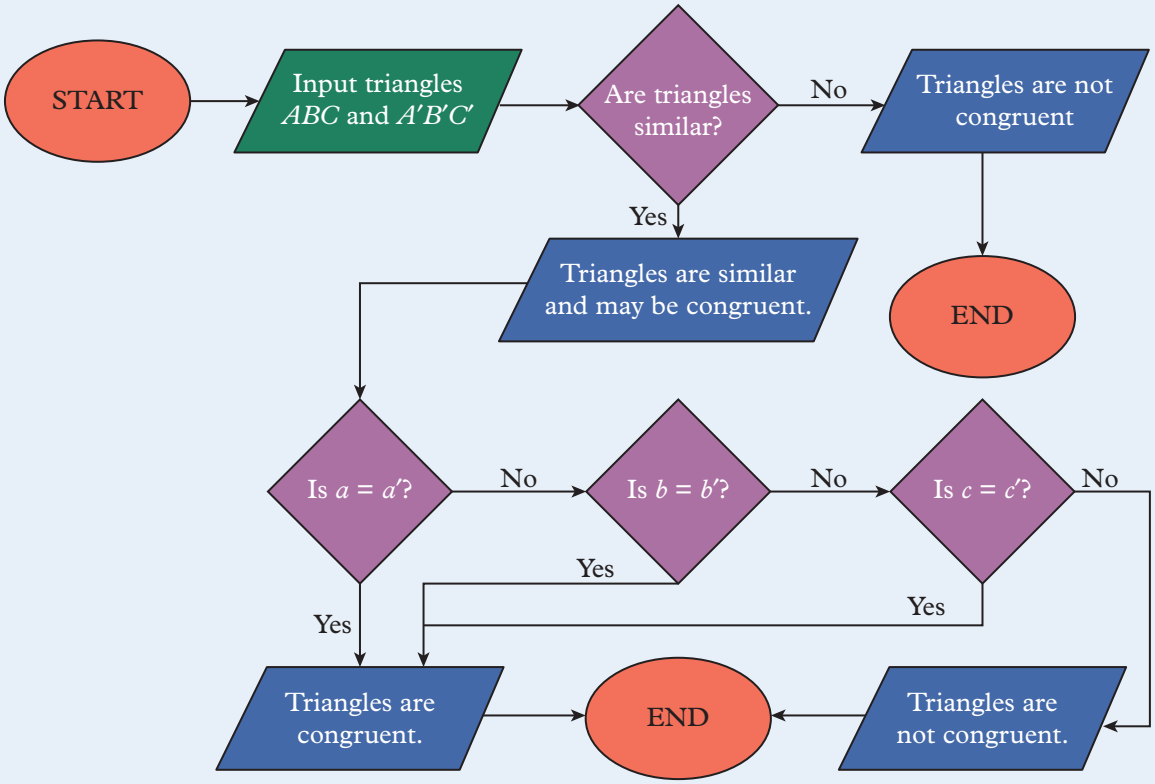
One way to label a triangle is to call the vertices A, B and C and the sides a, b and c, where angle A is opposite side a, angle B is opposite side b and angle C is opposite side c. These labels can be chosen so that $A \leq B \leq C$ and, as a result, have $a \leq b \leq c$. You may assume that you can always correctly label a triangle using this convention.



The two triangles above, $\triangle ABC$ and $\triangle A'B'C'$, are both labelled using this convention.

An algorithm to determine whether two triangles are congruent or similar is useful if it can take any two triangles as input. This means that the algorithm can be used regardless of the information you have about the side lengths and angles. If not enough is known about the triangles, the algorithm *should* report that not enough information is known to determine congruency or similarity.

The following flowchart can be used to test if $\triangle ABC$ and $\triangle A'B'C'$ (labelled using the above convention) are congruent. At the second, third and fourth decision points, the 'No' path should be followed if any of the required variables are unknown.

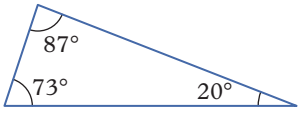


Check your understanding

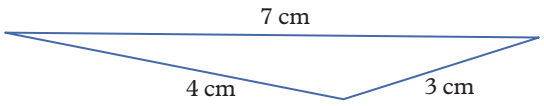
1 Answer the following questions.

a Label the vertices and sides of the triangles below using the convention previously introduced.

i



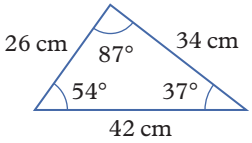
ii

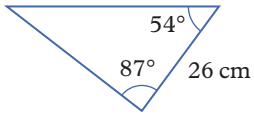


b Which congruence test is applied by the flow chart?

c Why can the test from part **b** always be applied to similar triangles?

d Use the flow chart to determine whether the following triangles are similar. Write the sequence of decisions as you go through the steps in the flow chart.





e Given a pair of triangles, should the output of the flow chart always be 'congruent' or 'not congruent'? Is there an output missing from the flow chart?

Task

1 Design a flow chart for each of the four similarity tests. Assume for an input pair of triangles that you know whether it is suitable to apply a particular test. You should still test all possible angles and side ratios.

2 Test your flow charts using pairs of triangles from **Lesson 9.5** as well as others of your choice. Describe any flaws in your flow charts and create updated versions based on your results.

3 Design a flow chart which takes a pair of triangles as input (labelled using the same convention as before), and then outputs which similarity test you should apply, or whether there is not enough information to test for similarity.

Extension

Pseudocode is another way to represent an algorithm without using a particular coding language. Specific terms and formatting using indentations are used to represent the steps in the algorithm.

The algorithms discussed in this lesson all require the ability to write angles and side lengths in ascending order. The pseudocode on the right represents an algorithm which can be used to sort three values into ascending order.

Line number	Pseudocode
1	**input** three numbers a, b, c
2	**if** $a > b$ **then**
3	$d \leftarrow a$
4	$a \leftarrow b$
5	$b \leftarrow d$
6	**end if**
7	**if** $b > c$ **then**
8	$d \leftarrow b$
9	$b \leftarrow c$
10	$c \leftarrow d$
11	**end if**
12	**if** $a > b$ **then**
13	$d \leftarrow a$
14	$a \leftarrow b$
15	$b \leftarrow d$
16	**end if**
17	**print** a, b, c

1 Input the following lists of numbers into the sorting algorithm. Keep track of the values of all variables using a desk check.

a 1, 2, 3 **b** 2, 1, 3 **c** 1, 3, 2

d 2, 3, 1 **e** 3, 1, 2 **f** 3, 2, 1

2 What is the purpose of variable d?

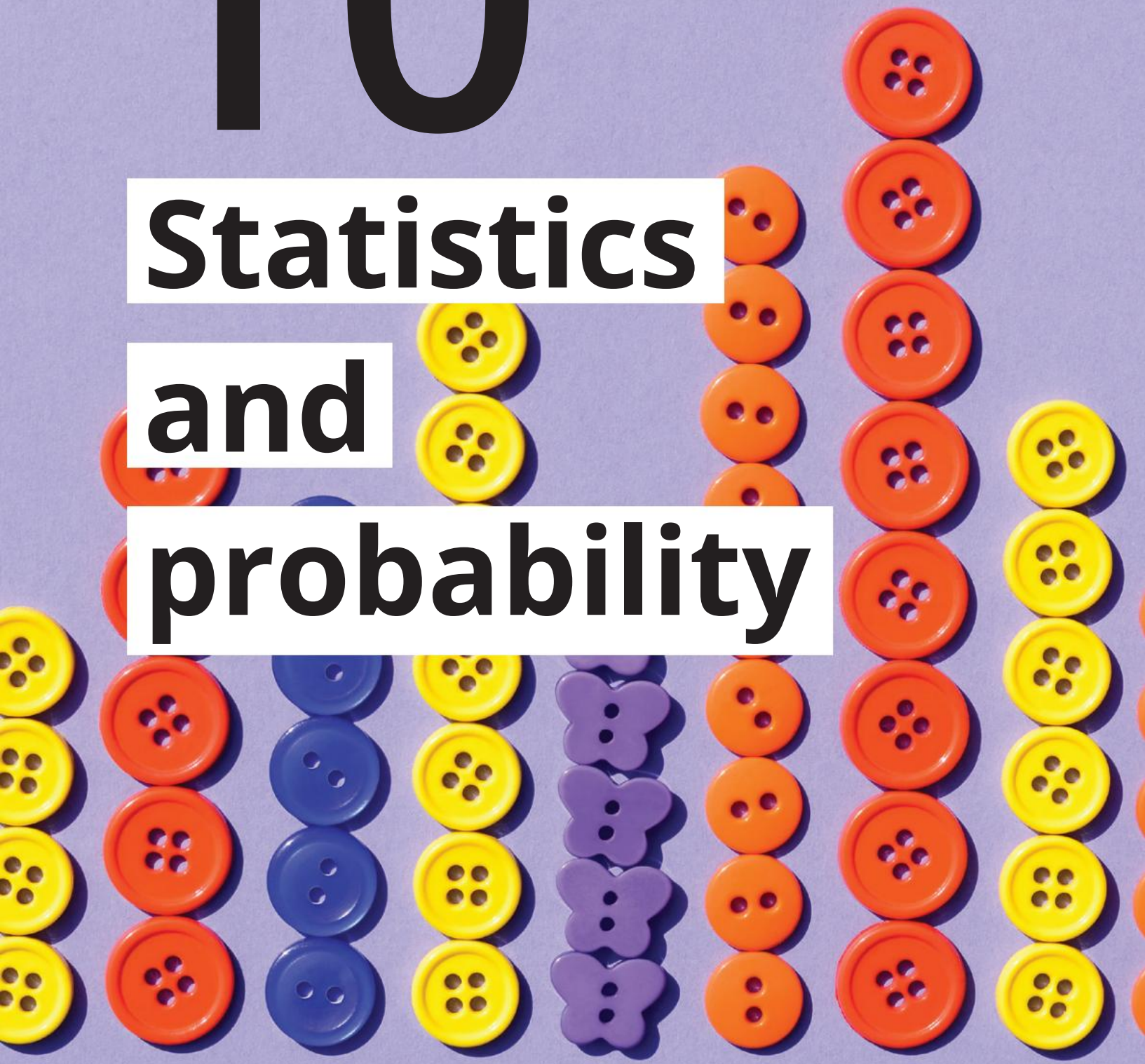
3 Which lists of numbers from question **1** were the most complicated to sort using this algorithm? Explain.

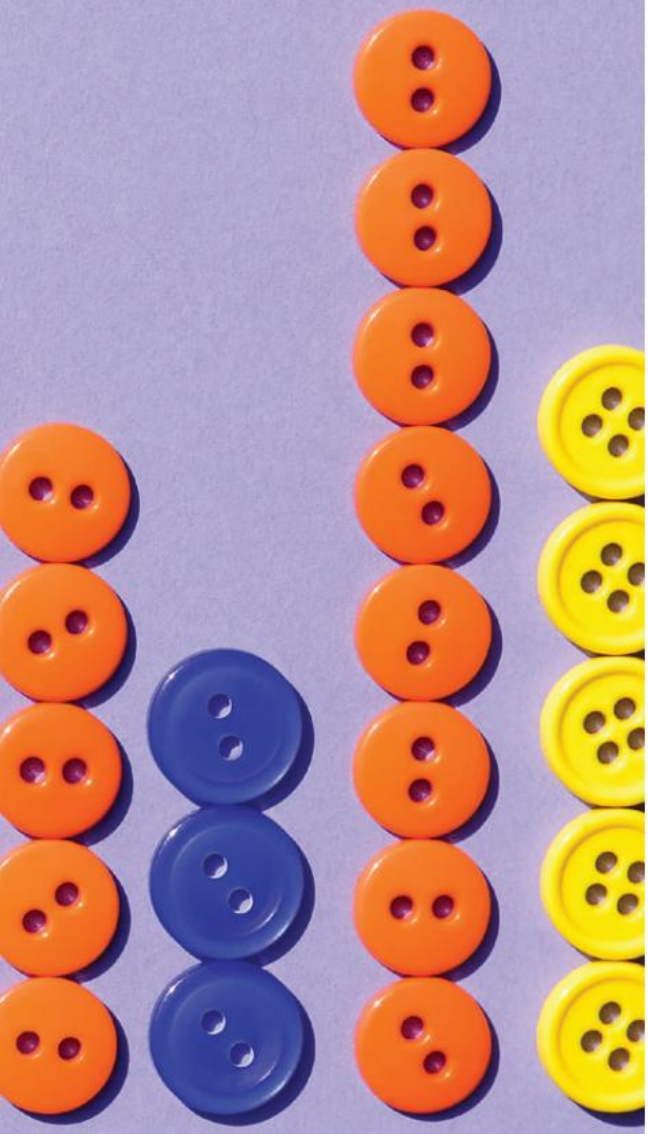
4 Write a pseudocode algorithm which can be used to sort four numbers into ascending order.

5 Research another sorting algorithm and compare it to this one. Describe their similarities and differences, and whether it uses any additional features.

10

Statistics and probability





Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Fractions, decimals and percentages
- ✔ Adding and subtracting fractions
- ✔ Equivalent fractions

Curriculum links

- Distinguish between a population and a sample, and investigate techniques for data collection... (VC2M8ST01)
- Analyse and report on the distribution of data from primary and secondary sources... (VC2M8ST02)
- Compare variations in distributions and proportions obtained from random samples of the same size... (VC2M8ST03)
- Plan and conduct statistical investigations involving samples of a population... (VC2M8ST04)
- Recognise that complementary events have a combined probability of one; use this relationship to calculate probabilities in applied contexts (VC2M8P01)
- Determine all possible outcome combinations for 2 events, using two-way tables, tree diagrams and Venn diagrams... (VC2M8P02)
- Conduct repeated chance experiments and simulations, using digital tools to determine probabilities for compound events... (VC2M8P03)

Lesson 10.1

Collecting data and sampling methods

Learning intentions

By the end of this lesson you will be able to ...

✔ determine whether a sample is biased, and identify different sampling methods.

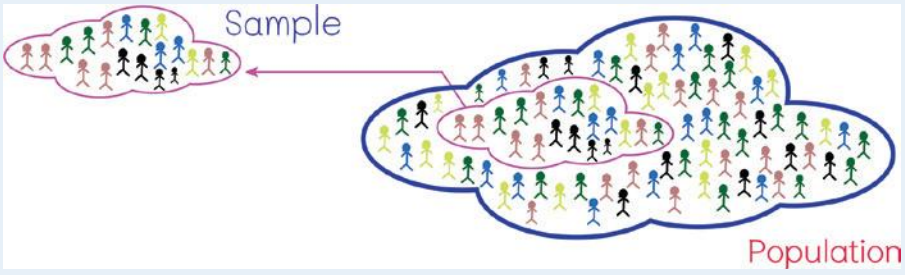
Inter-year links

Support	Collecting data
Year 9	7.1 Surveys and sampling methods
Year 10	10.9 Analysing statistical reports

Key content video
Collecting data

Collecting data

- When collecting data, the **population** refers to all potential pieces of data under consideration.
- A **census** is a survey of an entire population.
  → Taking a census can be impractical, expensive and time consuming.
- A **sample** is a selection of data that is part of the population.
  → Samples can be used to estimate characteristics of the population.
- When taking a sample, every piece of data should have an equal probability of being selected. Otherwise, the sample will be **biased**.
  → A biased sample may be systematically different from the population. For example, a biased sample may favour one outcome over others.
- A fair (unbiased) sample is likely to represent the characteristics of the population.
  → Fair samples must be of a reasonable size to accurately reflect the population.



- Collecting data by observation is when a researcher observes events as they naturally occur. For example, counting the number of different types of vehicles passing through an intersection.
  → Environmental conditions may bias data collected by observation. For example, trucks may make up a higher proportion of vehicles on the road at nighttime when compared to daytime.
- Data collected by the researcher is known as primary data. If the data being studied was collected by someone else, it is called secondary data.

Surveys

- A **survey** is a question, or a list of questions, designed to extract specific information about a chosen topic.
- When designing a survey, the goal should be clear. Think about what you are researching, and how the survey questions relate to your goal.
- Survey questions can be either closed or open-ended. Closed questions give a limited number of responses to choose from, whereas open-ended questions allow the respondent to choose their own response.
  → Some closed questions may only contain two options (e.g. 'Yes' or 'No'), whereas others may contain a list of options (e.g. 'What is your favourite colour from these options?').

- Survey questions should be neutral and not lead the respondent towards a particular answer.
  → A leading question, such as 'I think the new office design is fantastic. What do you think?', may lead to biased responses which do not accurately reflect the views of the population.

Sampling methods


Key content video
Sampling methods

- **Simple random sampling** techniques ensure that every potential piece of data has an equal chance of being selected.
  → **Systematic sampling** selects data points at fixed intervals (for example, every 10th person). The starting point should be random.
  → **Stratified sampling** divides the population into separate categories (for example, age) and takes a random sample from each category. The size of each sample is proportional to how large the category is.
- **Convenience sampling** is when a sample that is easily accessible is selected. For example, selecting students from your class to complete a survey.
- **Judgemental sampling** is when a sample is selected by the researcher. For example, a researcher may ask students to take part in a study.
- **Quota sampling** divides the population into certain categories (like stratified sampling) and takes a specified number from each category. The size of each sample is chosen by the researcher, meaning that potential pieces of data may have different chances of being selected.
- **Cluster sampling** divides the population into groups and then a random sample of the groups (or clusters) is selected. The population within a cluster should be diverse but the composition of all clusters should be similar (for example, all clusters are pairs consisting of one male and one female).

Worked example 10.1A Census or sample?

Zoe wants to find out what the favourite local park of the residents of her town is. She asks everybody in her school what their favourite park in town is. Is this a census or a sample?

THINK

1 Identify the population. The population is Zoe's town.
2 Identify who is surveyed. Zoe's school has been surveyed.
3 Determine whether the survey taken is of the whole population (census) or if it is a sample.

WRITE

This is a sample.

Worked example 10.1B Determining whether a sample is biased

Mohamed surveyed his friends and family to find out the most popular radio station in their hometown. Decide whether Mohamed's sample is fair or biased, giving reasons.

THINK

Determine whether the sample is a random sample of the population, or if it is biased and potentially unrepresentative of the population.

WRITE

Mohamed's friends and family may share common interests and opinions that do not truly reflect the population, so this is a biased sample.

Worked example 10.1C Identifying the sampling method

Maria selects a sample of 10 people to ask which brand of shampoo they use. She selects people for her sample according to their hair colour, in proportion to how common that hair colour is in the population. Determine the sampling method Maria has used.

THINK

1. Determine whether this is a simple random sample, a convenience sample, a judgemental sample, a cluster sample or a quota sample. Every person has an equal chance of being selected, so this is a simple random sample.
2. Determine whether this is a systematic sample or a stratified sample. The population has been divided into categories with an equal proportion taken from each category, so this is a stratified sample.

WRITE

This is a stratified sample.

Worked example 10.1D Writing meaningful survey questions

Alec is investigating the favourite sport of students at his school. He asks 50 students 'Do you prefer the AFL or NRL?' Will his question lead to biased responses? If so, how could he improve his survey question?

THINK

1. This question only asks about two sports, without considering the possibility that other sports may be the favourite of some students at Alec's school. Therefore, this question will lead to biased answers.
2. Consider how Alec could ask a question that includes other potential responses.

WRITE

This question will lead to biased answers.

Alec could ask the students to pick their favourite sport from a list of options.

Helpful hints

- ✔ Handpicking a sample will not provide a random sample of a population, even if the person picking believes that their sample is representative of the population.
- ✔ The only way to take an unbiased sample is to take a random sample from the whole population, for example by using systematic sampling or stratified sampling.
- ✔ A census will always be fair as it covers the whole population.

Learning pathways

1–7, 9, 10	1–2(a, c, d), 3–6, 8, 11–13, 15, 17	1–2(c, d), 4(d), 5, 8, 10, 12, 14–18

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Exercise 10.1A: Understanding and fluency

1 Identify the population in each situation.

- **a** Surveying students at a local school to find what the town's Year 8 students think of a local issue
- **b** Surveying people at a supermarket to find national attitudes to war
- **c** Surveying every thousandth person from a Western Australian phone book to find the favourite movie in Western Australia
- **d** Surveying people randomly to find a town's attitudes to a new mall being built

WE 10.1A **2** Decide whether a census or a sample has been taken in each of these situations.

- **a** Nikita questions every person in her class to find out the favourite chocolate of the entire class.
- **b** Jacqui questions every person in her year level to find out the favourite film for the school.
- **c** Travis asks everyone he knows how many pets they have in order to predict the average number of pets in any household in the state.
- **d** Aaron asks everybody in his family what their favourite dessert is to decide what to cook for a family dinner.

3 Decide whether a census or a sample would be most appropriate for gathering information in each situation.

- **a** Deciding who will be the next school captain
- **b** Getting feedback on the latest version of Windows
- **c** Finding which store in your town (or suburb) sells PlayStations at the cheapest price
- **d** Finding the average number of blue M&Ms in a packet

WE 10.1B **4** For the following situations decide whether the survey is fair or biased, giving reasons.

- **a** Dane asks every 100th person from the local phone book if they prefer skiing or snowboarding to find out the town's favourite winter sport.
- **b** Jacey asks everybody in the school what they think of the uniform to see how popular it is with the students.
- **c** Ryan asks everybody in a casino what they think about laws to restrict certain forms of gambling to find out the common opinion in Australia.
- **d** Tracey asks every 10th person in Woolworths which supermarket they prefer in order to find the town's favourite.

WE 10.1C **5** Determine the sampling method used in each of these situations.

- **a** Jamie asks every fifth person in a store their opinion on a new magazine.
- **b** Helen selects a panel that consists of three girls and two boys out of a group of 15 girls and 10 boys.
- **c** Xavier calls every phone number in the phone book ending in a randomly chosen digit to complete a survey.
- **d** Mia decides to survey 20 students from public schools and 30 students from private schools.
- **e** Devon chooses 20 people on the oval at lunchtime to ask if they enjoy sport.
- **f** Rita posts a survey she needs filled in for class to her Instagram story for her followers to complete.
- **g** Sara measures the diameter of all the roses in her family's well-maintained garden to determine the average diameter of any rose.

WE 10.1D **6** Identify if the following survey questions will lead to biased responses. If so, suggest how the survey questions could be improved.

	Research topic	Question
a	Finding the most popular animal	Do you like cats or dogs?
b	Researching the most popular holiday film this year	Do you like horror movies?
c	Investigating the most popular chocolate bar	What is your favourite chocolate bar?
d	Finding the most common hobby amongst Year 8 students	What do you think the most common hobby might be?
e	Researching the most popular holiday destinations	Where are three places you would like to go to in the holidays?
f	Investigating voting intentions	Who do you think will win the election?

Exercise 10.1B: Problem solving and reasoning

7 Lizzie wants to conduct a survey looking at the attitudes of people in town towards graffiti.

a What is the population?

b Somebody suggests Lizzie ask random people that she sees at the local mall. What potential problems are there with this method?

c Somebody else suggests that Lizzie ask every fifth person who walks past the local post office. What kind of sampling is this?

d What potential problems are there with this second method?

e Give an example of how Lizzie might collect data using stratified sampling.

8 Write two different questions that provide fair answers if you were to investigate:

a favourite soft drinks

b number of pets in a household

c internet usage per week

d student opinions about a new school rule.

9 A survey is to be done on a new school uniform. Provide a method of sampling the students at the school that uses:

a systematic sampling

b stratified sampling

c quota sampling

d cluster sampling.

10 For each survey:

i identify the population

ii provide an example of how you might collect a fair sample of the population.

a Investigating the top five favourite celebrities of Year 8s at your school

b Investigating a town's opinion on a new national law

c Investigating the average number of people in an Australian family

11 The Victorian Government wants to determine the view of state residents on their new policies. Instead of surveying the population, they choose to use cluster sampling.

a What is an advantage of using cluster sampling instead of surveying the entire population?

b The government decides to survey the residents of three randomly chosen local government areas in Victoria. Why might the results of the survey not accurately represent the views of the whole population?

12 A new video game for a popular franchise was announced with a video of the gameplay on social media. To determine the fanbase's reaction to the announcement, the developers checked the top 20 posts with the hashtag they included in the announcement and found 18 negative posts and 2 positive posts.

a What type of sample did the developers use?

b Is this sample fair or biased? Explain.

c In the lead up to the launch of the game, the developers checked 20 reviews of the game. They had paid 5 of these reviewers and provided review copies to 10 others. They found the majority of the reviews were positive.

i What type of sample did the developers use?

ii Is this sample fair or biased? Explain.

13 Judgemental selection for a sample involves deliberately choosing people or objects for your sample.

a Why might deliberately selecting people or objects give a better sample than a randomly selected sample?

b Why might deliberately selecting people or objects give a worse sample than a randomly selected sample?

14 Sally wants to find out what people in town think of a local issue. She makes a post on her social media, linking to an online survey she has created. How might this method of surveying be biased?

15 Nikki researches the favourite flavour of gelato at her local store by observing what flavours are purchased over a 2-hour period on a sunny Sunday afternoon. How might environmental conditions bias the data that Nikki collects?

16 All the gamers at a school are asked to identify the main equipment they use to play video games. Marci samples this population to determine the most popular type of device/console that gamers at her school use. The number of students Marci interviewed is also shown in the table. Determine if Marci's sample is perfectly stratified, approximately stratified, or not stratified. Explain why.

Equipment used to play video games	Population (percentage)	Marci's sample (frequency)
PlayStation	23%	13
Xbox	17%	8
Nintendo	27%	17
Mobile	15%	7
PC	18%	9

Exercise 10.1C: Challenge

17 A class of 25 students state whether they prefer half-hour- or full-hour-long episodes for TV shows. Their preferences are: full, half, half, full, full, half, half, half, half, half, half, half, half, half, full, full, half, half, full, full, full, half, half, full, half.

a Select every second response, starting from the second response, and determine the percentage who prefer half-hour episodes, correct to the nearest percent.

b What type of sample is this?

c Calculate the percentage of the population who prefer half-hour episodes.

d Calculate the percentage error between the sample percentage and the population percentage, correct to the nearest tenth of a percent.

18 A new vaccine has been developed and is ready for human trials in a sample of the population. The developers of the vaccine are considering how large the sample should be.

a Give a possible consequence of the sample being too small.

b Give a possible consequence of the sample being too large.

Online resources:

Interactive skillsheet
Sampling methods

Investigation
First day disasters

Quick quiz
10.1

Lesson 10.2

Summary statistics

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate summary statistics from raw data and frequency tables.

Inter-year links

Support	Understanding data
Year 7	10.1 Summary statistics
Year 9	7.4 Summary statistics from tables and displays
Year 10	10.1 Five-number summary and interquartile range

Summary statistics

Key content video
Calculating summary statistics

- The **mean** is a numerical average of the values in a data set.
  To find the mean, add all the numbers together, then divide by the number of values in the set.

$$\text{mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$

- The **median** is the middle of a data set when the numbers are listed in order.
  The median divides a data set in half.
  → If there are n values in a data set, the median is the $\frac{(n+1)}{2}$ th value.
  For example, if there are 11 values, the median is the 6th value.
  If there are 16 values, the median is the 8.5th number, meaning it is halfway between the 8th and 9th values.

2, 3, 4, 5, 6
median = 4

1, 2, 3, 4, 5, 6
median $= \frac{3+4}{2} = 3.5$

- The **mode** is the most common value in a data set. There can be more than one mode in a data set.
- The **range** is the difference between the largest (maximum) and smallest (minimum) values in a data set.
  range = maximum value − minimum value

Outliers

- An **outlier** is an extreme data point that is much higher or lower than the other values in a data set.

Worked example 10.2A Calculating summary statistics

Find the **a** mean, **b** median, **c** mode and **d** range for this data set: 8, 4, 1, 2, 9, 2, 4, 13, 2.

THINK

a Add the numbers together and divide by the total number of values.

b List the numbers in the data set in ascending order and find the middle value.

c List the numbers in the data set in ascending order and identify the number which occurs most often.

d Identify the minimum and maximum values of the data set. Subtract the minimum value from the maximum value.

WRITE

a $\text{mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$
$= \frac{8+4+1+2+9+2+4+13+2}{9} = \frac{45}{9} = 5$

b 1, 2, 2, 2, (4), 4, 8, 9, 13
The median is 4.

c 1, 2, 2, 2, 4, 4, 8, 9, 13
The mode is 2.

d range = max − min
= 13 − 1
= 12

Worked example 10.2B Summary statistics from a frequency table

For the data shown in the frequency table on the right, find:

a the mean

b the median

c the mode

d the range.

Score (x)	Frequency (f)
0	4
1	6
2	8
3	7
Total	**25**

THINK

a Add a 'score × frequency' column to the table and complete it.
The total of this column is the sum of all data points, whereas the total of the frequency column is the total number of data points.

b Add a cumulative frequency column to the table and complete it.
Add 1 to the number of values and divide by 2 to find the position of the median. Then identify the row which contains the median score by using the cumulative frequency column.

c Identify the score with the highest frequency.

d Identify the minimum and maximum values in the score column of the data set. Subtract the minimum value from the maximum value.

WRITE

Score (x)	Frequency (f)	Score × frequency	Cumulative frequency
0	4	$0 \times 4 = 0$	4
1	6	$1 \times 6 = 6$	$4 + 6 = 10$
2	8	$2 \times 8 = 16$	$10 + 8 = 18$
3	7	$3 \times 7 = 21$	$18 + 7 = 25$
Total	**25**	**43**	

a $\text{mean} = \dfrac{\text{sum of all data points}}{\text{total number of data points}}$

$= \dfrac{43}{25}$

$= 1.72$

b $\dfrac{(25+1)}{2} = 13$, so the median is the 13th score.

Therefore the median is 2.

c The mode is 2.

d range = max − min

$= 3 - 0$

$= 3$

Helpful hints

- ✔ If every value in a data set is different then there is no mode, but if at least one value is repeated, then there is a mode! Remember that there can be more than one mode.
- ✔ Remember that the median is the middle number of an ordered data set, not the middle of the range.
- ✔ If a data set has an odd number of values then the median is one of the values; otherwise, the median may not appear in the data set.
- ✔ As with the median, the mean may not appear in a data set.

Learning pathways

 1–4, 5(a–c), 6–9, 11, 14

 3, 4, 5(d–f), 6, 8, 10, 12, 16, 17(a)

 4, 5(d–f), 8, 11, 13, 15–18

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Exercise 10.2A: Understanding and fluency

WE 10.2A **1** Consider this data set: 5, 8, 7, 11, 4, 9, 5. Write the numbers in order. Then for this data, find:

a the mean **b** the median **c** the mode **d** the range.

2 Find the mean (to one decimal place), median, mode and range for each set of data.

a 11, 19, 18, 12, 12 **b** 2, 8, 4, 9, 7, 6, 2, 4, 1 **c** 8, 7, 3, 5, 7, 7, 4, 8, 6, 0, 9, 7

3 Monica collected this data on the heights (in centimetres) of her classmates.

161 152 148 167 162 149 159 155 161 145 155 156

For this data, find:

a the mean (to two decimal places) **b** the median **c** the mode **d** the range.

4 Mario collected data on the heights (in centimetres) of his classmates.

154 157 158 161 162 198 168 147 152 152 153 150

For this data, find:

a the mean (to two decimal places) **b** the median **c** the mode **d** the range.

WE 10.2B **5** For the data shown in each frequency table, find:

i the mean (to two decimal places) **ii** the median **iii** the mode **iv** the range.

a

Score (x)	Freq (f)
1	9
2	11
3	6
4	4
Total	**30**

b

Score (x)	Freq (f)
2	9
3	6
4	5
5	4
Total	**24**

c

Score (x)	Freq (f)
10	4
20	8
30	6
40	2
Total	**20**

d

Score (x)	Freq (f)
1	4
2	8
3	6
4	2
5	5
Total	**25**

e

Score (x)	Freq (f)
13	2
14	4
15	1
16	2
17	1
Total	**10**

f

Score (x)	Freq (f)
0	8
10	12
20	20
30	6
40	4
Total	**50**

6 A Year 8 class were asked how many online quizzes they did last week. The results were:

0 1 6 3 5 1 2 0 0 2 1 3
2 1 0 0 1 1 2 18 2 3 5 3

a Create a frequency table to represent the data.

b Find the **i** mean (to one decimal place), **ii** median and **iii** mode.

c Find the range.

d Identify the outlier.

7 Calculate summary statistics for the data shown in this table. Give your answers to one decimal place where necessary.

Score (x)	Frequency (f)
11	4
12	8
13	16
14	17
15	13
16	4
17	2
Total	**64**

8 Calculate summary statistics for the data shown in this table.

Score (x)	Frequency (f)
–0.3	4
–0.2	7
–0.1	7
0	2
0.1	6
0.2	3
0.3	1
Total	**30**

Exercise 10.2B: Problem solving and reasoning

9 The weights (to the nearest kilogram) of school bags of a Year 8 class were recorded for an investigation.

2 6 11 4 2 4 8 5
9 3 4 2 3 5 6 5
2 4 3 5 5 7 10 10

a Arrange the data into a frequency table.

b Calculate the four summary statistics for this data. Round your answers to one decimal place where necessary.

c Use your answers to part **b** to write two sentences about the weights of these school bags.

10 Monique collected this data on the number of instant messages people in her class sent per day.

6 39 15 26 18 37 48 14 88 9 22 74
42 22 56 33 16 28 31 48 52 17 21 55

By calculating the summary statistics for this data, write two sentences about Monique's data.

11 The weights of 10 oranges are recorded to the nearest gram.

110 125 118 136 104 142 123 202 130 128

a Identify the outlier in this data set.

b Calculate the value of the mean **i** with and **ii** without the outlier included in the data set.

c Calculate the value of the range **i** with and **ii** without the outlier included in the data set.

d Discuss how the outlier affects the value of the mean and the range.

12 The mean is often given as a value to represent a data set; however, the presence of an outlier can significantly affect the mean.

a Explain why an outlier can affect the mean, using the comparison of the heights in Monica's and Mario's classes from questions **3** and **4** to help you.

b Would you use mean or median to describe 'average' height in Mario's class? Explain your answer.

c Identify the outlier in Mario's class.

d Recalculate the mean for Mario's class without the outlier. How does this compare to the median?

e What is the range for Mario's class without the outlier? Compare to your answer for question **4d**.

13 A green bag of 10 grocery items weighs 3 kg.

a What is the average (mean) weight of the grocery items?

b A bag of chicken fillets is removed from the green bag and the average weight changes to 200 g. What was the weight of the chicken fillets?

14 For each of the following, a mistake has been made in determining the summary statistic.

i Determine the error made for each statistic.

ii Determine the difference between the actual statistic and the incorrect statistic. Calculate each difference as 'actual statistic – incorrect statistic'.

a Calculate the median of this list of data: 8, 3, 5, 1, 2, 7, 5.

8, 3, 5, 1, 2, 7, 5

The number in the middle of the list is 1, so the median is 1.

b Calculate the range of this list of data: 8, 3, 5, 1, 2, 7, 5.

8, 3, 5, 1, 2, 7, 5

The first and last numbers of the list are 8 and 5, so the range is $5 - 8 = -3$.

c Calculate the mean from this table.

Score (x)	Frequency (f)
1	3
2	4
4	7
7	2

$\text{mean} = \dfrac{1 + 2 + 4 + 7}{4} = 3.5$

The mean is the sum of the scores (1, 2, 4, 7) divided by the number of scores, 4, which is 3.5.

d Calculate the range from this table.

Score (x)	Frequency (f)
1	24
2	15
3	52
4	17

The largest value in the table is 52 and the smallest is 1, so the range is $52 - 1 = 51$.

e Calculate the median from this table.

Score (x)	Frequency (f)
4	1
6	0
8	4
10	12
12	13

The median is the middle number, and the number in the middle of the score column is 8.

15 Consider this set of 15 values: $10, 12, 12, 13, 13, 13, 14, 15, 15, 15, 16, 16, 17, 19, x$, where x is the final number in the set but has not been determined yet. The value of x is not necessarily the largest value in the set.

a Complete the table below, where in each row x is the value given in the left-hand cell.

x	Mean (correct to two decimal places)	Median	Mode(s)	Range
13	14.20	14		9
8	13.87		13, 15	11
1		14		
−3	13.13			
15	14.33			9
25			13, 15	

b Which statistics changed the most as the value of x changed? Which changed the least?

c For what value(s) of x did the median change from 14? Explain why this happened.

d For what value(s) of x did the range change from 9? Explain why this happened.

e For what value(s) of x did the mode change from both 13 and 15? Explain why this happened.

f For what other values of x will the mode change to being a different value from 13 and 15?

g Complete these sentences.

i As the value of x decreases, the value of the mean _____.

ii As the value of x increases, the value of the mean _____.

h If $x = 100$, how do you expect the statistics will change? Calculate the mean, median, mode and range to check your expectation.

16 Jacinta is looking to buy a laptop and has narrowed her choices down to two. To help her make her decision, she looks at the customer ratings for both laptops on the website she will purchase from.

Laptop A	
Rating	Frequency
★★★★★	10
★★★★	1
★★★	0
★★	1
★	10

Laptop B	
Rating	Frequency
★★★★★	1
★★★★	3
★★★	10
★★	3
★	1

a Calculate the mean, median and range for both laptops' ratings.

b Give reasons, related to the ratings, why Jacinta may want to and may not want to buy Laptop A.

c Give reasons, related to the ratings, why Jacinta may want to and may not want to buy Laptop B.

Exercise 10.2C: Challenge

17 Find the solution to the following problems.

a A set of 4 numbers has a mean, median and mode of 7, and a range of 4. Write down the four numbers.

b A set of 5 numbers has a median of 10 and modes of 3 and 12. Calculate the mean and range of the data.

18 Consider the set of data 20, 24, 29, 34, 40, 55, x, y, where x and y are not necessarily the largest values in the set. Determine the values of x and y such that the mean = 37.125, the median = 31.5, and the range = 50.

Online resources:

Interactive skillsheet The mean

Interactive skillsheet The median

Interactive skillsheet The mode

Interactive skillsheet The range

Investigation Pool usage

Quick quiz 10.2

Lesson 10.3

Analysing data

Learning intentions

By the end of this lesson you will be able to ...

✔ identify summary statistics, and identify whether a sample can predict population parameters.

Inter-year links

Year 7	10.2 Describing data with dot plots
Year 9	7.5 Describing data
Year 10	10.3 Distributions of data

Using samples to make predictions

Key content video
Analysing data

- Samples can be used to predict **population parameters** (characteristics of a population). To be confident of these predictions, a sample must be:
  - → gathered using a fair and unbiased technique
  - → of a sufficient size.
- Random samples drawn from the same population will vary from each other. For this reason, predictions made from a random sample are unlikely to exactly match the population parameters.
  - → When a population is very small (less than 50) a census should be taken, if possible. This means the results will be perfectly accurate, rather than having to rely on making predictions.
- The larger the sample size, the more likely it is that the sample will be representative of the population. Therefore, predictions made from large samples are more reliable than predictions made from small samples.
  - → There is no minimum sample size, however a sample size of less than 50 may be too small to make accurate predictions.
- Outliers will have a bigger effect on predictions of population parameters when the sample size is small.
- Biased samples should not be used to predict population parameters.

Describing data

- Summary statistics can be used to describe data.
  - → The mean, median and mode are **measures of centre** as they each identify a central position in a data set.
  - → The range is a **measure of spread** as it provides an indication of how spread out the data is.
- When deciding which measure of centre should be used to describe data, the presence of outliers should be taken into consideration.
  - → Outliers can skew the value of the mean but will not impact the median or mode.
  - → A small sample size with the mean being significantly higher or lower than the median may indicate an outlier skewing the value of the mean.
- When describing a population or sample, people use words such as:
  - → 'average', which usually refers to the mean, but sometimes median
  - → 'most common', 'most popular' and 'most often', which refer to the mode
  - → 'middle' and 'centre', which refer to the median
  - → 'spread' or 'span', which usually refer to the range.

Worked example 10.3A Identifying summary statistics

Identify which summary statistics are being used in these sentences.

a The most popular team was Hawthorn.

b The levels of a chemical in the water varied by up to 20 g.

THINK

a The term 'most popular' refers to the mode.

b The word 'varied' refers to the spread of the data, which is the range.

WRITE

a mode

b range

Worked example 10.3B Identifying whether a sample can predict population parameters

The table below shows the levels a group of Year 8 students reached when playing a computer game on their first attempt.

Given this data, could you reasonably predict the population parameters? Explain your answer.

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Year 8 students	Draw names out of a hat	60 students from Year 8	4.4	4	4	8

THINK

1. Look at the sample size. Is it of a sufficient size?
2. Look at the sampling technique. Was the sample gathered using a fair and unbiased technique?
3. Determine if you would be confident enough to use these results to predict the population parameters.

WRITE

You can be confident in using this sample to predict the population parameters as the sample size is sufficient and the sampling technique is random.

Helpful hints

✔ Outliers have more of an impact on the mean for small samples than they do for large samples! For example, in a sample of the heights of 10 students, a very tall student would significantly affect the mean, whereas in a sample of 100 students, a very tall student would not have much impact on the mean.

Learning pathways

 1–7, 9  1, 3–6, 8–10  1, 3, 6, 8–11

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Exercise 10.3A: Understanding and fluency

WE 10.3A **1** Identify which summary statistics are being used in these sentences.

a The average height for a 14 year old is 162 cm.
b The temperature covered a span of 10°C.
c The most popular music in the class was hip-hop.
d The times varied by up to 40 minutes.
e The train is most often 2 minutes late.
f Andrew is in the middle of his height range.

2 Tanya wants to investigate the most popular sports team in South Australia. Which samples do you think would better reflect the population? Explain your answers.

a A sample of 20 people or a sample of 100 people
b A sample of 619 people or a sample of 500 people
c A sample of 50 randomly selected people or a sample of 200 students
d A sample of 100 people in Australia or a sample of 50 people in South Australia

WE 10.3B **3** Given the data in these tables, state whether you could reasonably predict the population parameters. Explain your answers.

	Population	Sampling technique	Sample	Mean	Median	Mode	Range
a	Your school	Asking your friends	50 of your friends	8	7	8	3
b	Australia	Stratified sampling	1000 people in Canberra	16	16	14	20
c	Victoria	Systematic sampling	30 people in Victoria	8	3	3	26
d	Audience at a circus (200 people)	Stratified sampling	100 people at the circus	4	3	5	10
e	Your suburb	Systematic sampling	500 people in your suburb	9	8	9	3

4 Laani reads that there are 2.5 people in an average Australian household. She decides to conduct a survey to see if her results match. She asks everybody that she knows at school and displays her results in this table.

Number of people in household	2	3	4	5	6	7	8
Frequency	6	19	38	20	14	2	1

a Find the mean, median and mode for this data.
b What is the population of the survey? What is the sample?
c How many people has Laani surveyed?
d Why might this sample not truly reflect the population?

5 Laani repeats the survey from question **4**, but this time she uses a random sampling technique of the population. She surveys 20 people and finds an average of 2.9 people per household.

a How is this different from her original sample?
b Why might you still not be confident that this sample reflects the population? Hint: How might an outlier affect this sample?
c In the end Laani surveys 200 people, finding an average of 2.6 people per household.
i Why might Laani be more confident of her results with a larger sample?
ii As sample size increases, you can see that the sample mean more closely resembles the population mean. Do you think this would be the same for other summary statistics? Explain your answer.

6 Three different samples are taken from a population. The results are summarised below.

Sample: 20 people	
Category	**Frequency**
A	5
B	2
C	7
D	4
E	2

Sample: 50 people	
Category	**Frequency**
A	14
B	4
C	19
D	7
E	6

Sample: 200 people	
Category	**Frequency**
A	52
B	23
C	84
D	24
E	17

a The percentage frequency is the frequency divided by the size of the sample and is then written as a percentage. Convert each table to percentage frequency.

b Which category(s) is most likely to be the mode for the population? Explain by referring to the sample percentage frequencies.

c Which category(s) is most likely to be the least common for the population? Explain by referring to the sample percentage frequencies.

Exercise 10.3B: Problem solving and reasoning

7 Jade conducted an online survey about Australians' favourite season of the year. She received the following results.

Favourite season	Frequency
Spring	51
Summer	78
Autumn	43
Winter	28

a What proportion of people preferred each of the seasons in Jade's survey?

b Karl conducted a similar online survey and received the following results.

Favourite season	Frequency
Spring	99
Summer	134
Autumn	94
Winter	73

What proportion of people preferred each of the seasons in Karl's survey?

c Explain why there is a variation in the proportions from the two surveys.

d Assuming each survey was collected randomly, how might you report on the favourite seasons of Australians?

8 Chris wants to find the average number of mobile phones in Australian households. He decides to take a sample of 50 people. The results are shown in the frequency table below.

a Find the mean, median and mode for this data.

b What might you need to consider before using this data to predict the population average?

c Use either the sample mean or median to estimate the average number of mobile phones in Australian households. Explain your decision.

Number of mobile phones	0	1	2	3	4	5	6
Frequency	1	1	11	8	16	9	4

9 Lauren wants to know the opinion of people at her school on reality TV. She takes five samples of people and tabulates the percentages.

Opinion	20 random Year 8s	20 random students	4 random students from each year level	20 random teachers	15 random students and 5 random teachers
Love it	10%	15%	25%	40%	25%
Like it	15%	20%	12.5%	25%	15%
No opinion	30%	15%	12.5%	15%	20%
Don't like	40%	30%	37.5%	15%	15%
Despise it	5%	20%	12.5%	5%	25%

a How many Year 8s in the first sample like reality TV?

b Using the percentages in the table, what is the mean percentage of people who love reality TV at Lauren's school?

c Explain why the percentages varied so much from sample to sample.

d Based on your answer from part **c**, do these samples give a clear picture of the opinions on reality TV at Lauren's school?

10 Suzie takes a sample of 100 Year 8s at her school, calculates the mean height to be 1.52 m and determines there are 85% who are right-handed. Suzie claims that the average height of all Year 8s in Australia is 1.52 m and 85% of Year 8s in Australia are right-handed.

a Explain why Suzie does not have enough information to make that claim.

b What claim can Suzie make given her sample?

Exercise 10.3C: Challenge

11 The State Theatre has 2000 seats. Over four consecutive shows, all the seats were filled. The country of origin of every attendee was noted. The data for the four shows is given in the following table.

	Thursday night	Friday night	Saturday night	Sunday night
Australia	1119	1340	1461	900
China	131	98	79	116
UK	273	245	182	320
USA	370	215	170	553
Other	107	102	108	111
Total	**2000**	**2000**	**2000**	**2000**

a Find the average number of attendees from each country (including other) over the four nights.

b Historical data can be used to make predictions about future events. The theatre manager uses the data above to predict the country of origin of theatre attendees the following week on any night.

i Use your answers from part **a** to find the average proportion of attendees from each country (including 'other') written as a percentage.

ii If only 1200 seats are available in the theatre due to required maintenance, how many people from each country would the theatre manager expect to be in attendance? Round your answers to the nearest integer.

c The theatre manager now wants to only consider the Sunday night shows. Should the theatre manager continue using the answers to part **b** to make predictions? If not, what could they do instead?

d How could the theatre manager improve their predictions for parts **b** to **c**?

Online resources:

Worksheet
Connecting samples and populations

Investigation
Ice cream flavours

Quick quiz
10.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

10.1 **1** Identify whether the following situations use a sample or a census.

a Checking what every third person leaving a shop bought to determine the most popular item that shop sells
b Walking the entire length of a path to determine if there are any problems to fix along the path
c Interviewing all Year 8 students at a school about the best place to have lunch at their school

10.1 **2** Identify whether the following questions contain bias.

a Asking 'What is your opinion of the Prime Minister? Please rate on a scale from 1 to 10, where 1 is dreadful and 10 is amazing' to determine random people's opinions of the Prime Minister
b Asking 'Are you smart?' to determine if someone is intelligent
c Asking 'What is your favourite chocolate, and why is it Crunchie?' to determine your classmates' favourite type of chocolate

10.1 **3** Identify the sampling method in each of the following situations.

a Ringing every 5th number on the school's phone directory to survey families at the school
b Using the most recent Australian census to determine the number of random people from different nationalities to interview
c Posting a survey to a group page on Facebook for people in the group to answer if they want to

10.2 **4** Calculate the **i** mean, **ii** median, **iii** mode and **iv** range of the following data sets.

a 17, 20, 6, 11, 20, 14, 20, 14, 18, 7
b 3.3, 5, 1.5, 0.3, 3.2, 3.3, 0.3, 2.9, 4.1, 2.8

10.2 **5** The mean for a set of data is 30 and the median for the same set is 18. Which statistic should be used for the average? Explain why.

10.3 **6** Can these sample statistics be used to predict population parameters? Explain why or why not.

	Population	Sampling technique	Sample	Mean	Median	Mode	Range
a	Students at your school (1200 students)	Randomly selected from a list of all students	120 students	4	5	4	10
b	Students at your school (1200 students)	Randomly selected from a list of all students	16 students	6	6	7	5
c	Students at your school (1200 students)	Online survey attached to school newsletter	75 students	10	12	9	14
d	Students at your school (1200 students)	All students who had been given a detention this month	50 students	17	20	18	21

10.3 **7** Thirty people entering a shopping centre are asked if they are going to Woolworths. The responses are:

Yes, Yes, Yes, Yes, No, No, No, Yes, No, No, Yes, Yes, Yes, No, No,
No, No, Yes, Yes, Yes, Yes, Yes, Yes, No, No, No, Yes, Yes, No, No.

a Select every second response, starting from the second response, and determine the percentage who are planning to go to Woolworths, correct to the nearest percent.
b Select every third response, starting from the third response, and determine the percentage who are planning to go to Woolworths.
c Explain why the two samples have different percentages despite being taken from the same population.

Statistical Investigations

The statistical investigation process

Statistics can be used to answer a wide range of real-life questions. By collecting and analysing data, we can draw evidence-based conclusions, allowing us to make informed decisions.

A statistical investigation involves the following steps:

1. Identifying a question to investigate
2. Collecting data
3. Analysing the data
4. Reporting on the findings.

Curriculum link

Plan and conduct statistical investigations involving samples of a population; use ethical and fair methods to make inferences about the population and report findings, acknowledging uncertainty. (VC2M8ST04)

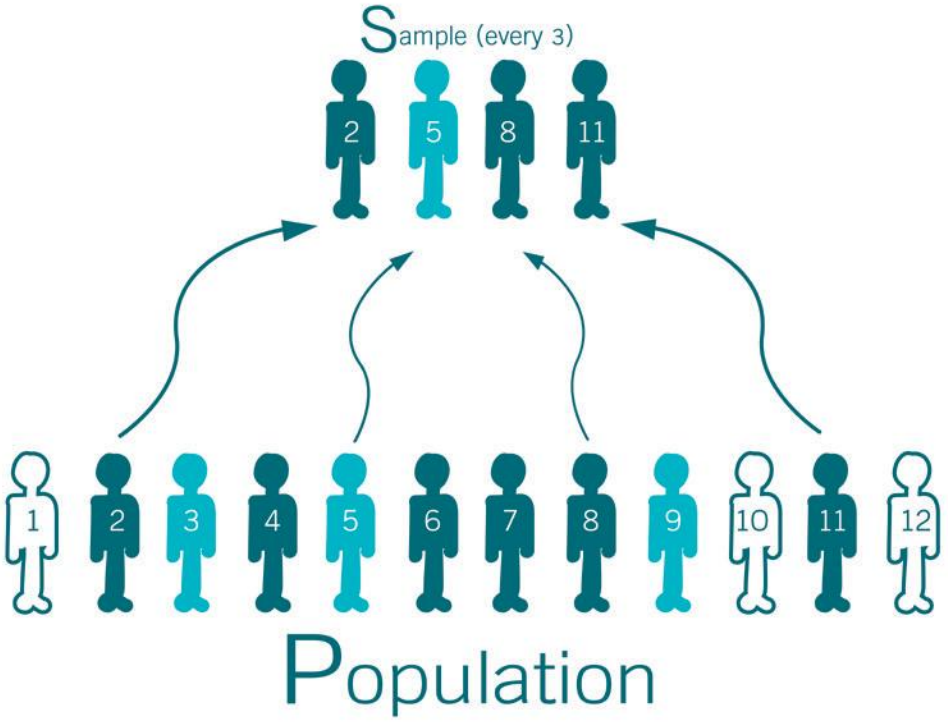
Populations and samples

Statistical investigations can be used to research issues and answer questions about populations.

Recall that:

- the population refers to all potential pieces of data under consideration
- a census is a survey of an entire population
- a sample is a selection of the data that is part of the population.

Often it is not possible or practical to take a census and so a sample is used to make inferences about the population. Ensuring that samples are taken randomly (that is, every person or item in the population has a fair chance of being sampled) is important to give the best chance that the sample will accurately represent the population. However, taking a random sample is not always possible and so other sampling methods may be used.



Investigation

Choose one of the following questions to ask your classmates. Record the results so they can be analysed later.

- How many pets do you have?
- What is your height (in cm)?
- How many siblings do you have?
- How many hours did you sleep last night?

Investigation questions

1 If you take all the Year 8 students at your school as the population (and your school has multiple Year 8 classes) then your sample will not be a random sample. Which sampling method did you use?

2 If you sampled a different class, would the data be the same or different? Explain your answer.

3 How confident can you be that your sample accurately represents the population (all Year 8 students at your school)?

Analysis

1 How many people are in your sample?

2 Analyse the data you collected by calculating each of the following summary statistics.

a Mean

b Median

c Mode

d Range

3 Are there any outliers in your sample?

4 Write a short report on your findings from this investigation.

Investigate Further

Perform another investigation using one of the other investigation questions or using a question you come up with yourself. If you come up with your own question, make sure you will receive numerical data. For example, 'How many minutes did it take you to travel to school today?' will result in numerical data, whereas 'What method of transport did you use to travel to school today?' will result in categorical data, for which you will not be able to calculate summary statistics.

Lesson 10.4

Theoretical probability

Learning intentions

By the end of this lesson you will be able to ...

✔ use the language of probability, calculate theoretical events, and identify complementary events.

Inter-year links

Support	Understanding probability
Year 7	10.5 Theoretical probability
Year 9	8.1 Two-step chance experiments
Year 10	11.1 Theoretical probability

Theoretical probability

Key content video
Calculating theoretical probability

- The **theoretical probability** of an **event** describes the chance that the event will occur. All probabilities have a value in the range of 0 (impossible) to 1 (certain).
- The **sample space** of an **experiment** is a set of the possible outcomes in the experiment.
  - → An event is one or more outcomes from the sample space of an experiment.
  - → The probabilities of all outcomes in a sample space add up to 1.
- A **favourable outcome** is an outcome that you are interested in.
- If all outcomes in the sample space are equally likely to occur, the theoretical probability of an event is calculated by:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes in event}}{\text{total number of possible outcomes}}$$

  - → If outcomes within the sample space are not equally likely to occur, you can write them as a list of outcomes that reflect their chance of occurring.

    For example, for selecting a letter at random from the word ENERGY, the list can be written as E, N, E, R, G, Y, where each outcome is equally likely to occur.

Describing events

Key content video
Describing events in probability

- 'at least' means equal to or above the given value. For example, 'rolling at least 3' means 3, 4, 5, …
- 'at most' means equal to or below the given value. For example, 'rolling at most 5' means 5, 4, 3, …
- 'more than' means above the given value. For example, 'more than 3' means 4, 5, 6, …
- 'fewer than' means below the given value. For example, 'fewer than 5' means 4, 3, 2, …
- 'and' means both outcomes must be met. For example, 'selecting a red card and a king' means selecting a red king.
- 'or' means either of the given outcomes.
  - → 'or' can be inclusive, meaning if both outcomes are met the outcome is favourable. For example, 'rolling a prime number or more than 4' when rolling a standard six-sided die means 2, 3, 5 or 6.
  - → 'or' can also be exclusive, meaning if both outcomes are met the outcome is not favourable. For example, 'rolling a prime number or more than 4 but not both' when rolling a standard six-sided die means 2, 3 or 6 (5 is not included).

Complementary events

Key content video
Complementary events

- The complement of event A is the event that A does not occur. For example, the complement of the event 'rolling a 1 on a standard six-sided die' is 'not rolling a 1 on a standard six-sided die.'
  - → If the probability of event A is Pr(A), then the probability of the complement of event A is Pr(A').

- The probabilities of **complementary events** always add to 1. This means that $\Pr(A') = 1 - \Pr(A)$.

Mutually exclusive events

- Two events are **mutually exclusive** if they cannot occur at the same time. For example, it is not possible for a coin to land on both heads and tails.
- If two events are mutually exclusive, then the probability of them both occurring at the same time is 0. That is, if A and B are mutually exclusive, then $\Pr(A \text{ and } B) = 0$.

Worked example 10.4A Interpreting the language of probability

Identify the favourable outcomes in each of the following situations.

a Rolling at least 5 on a standard six-sided die

b Landing on a 1 or a prime number when this spinner is spun

c Landing on a multiple of 2 or a multiple of 3, but not both, when this spinner is spun

d Getting a multiple of 2 and a multiple of 3 when randomly picking a number from 1–12

THINK

a 'at least' means any outcome equal to or above the value is favourable.

b 'or' means any section which satisfies either condition is favourable.

c 'not both' means that any section which satisfies both conditions is not favourable.

d 'and' means that both conditions must be met for an outcome to be favourable.

WRITE

a $\{5, 6\}$

b $\{1, 2, 3, 5, 7\}$

c $\{2, 3, 4, 8\}$

d $\{6, 12\}$

Worked example 10.4B Calculating theoretical probability

The letters from the word FRANCE are placed in a bag and a letter is selected at random from the bag.

a State the number of possible outcomes.

b What is the probability of selecting a C?

c What is the probability of selecting a vowel?

THINK

a Count the total number of outcomes, with each letter counting as a separate outcome.

b Count the number of favourable outcomes and substitute into the formula with the total number of outcomes.

c Count the number of favourable outcomes (A and E) and substitute into the formula with the total number of outcomes.

WRITE

a total number of outcomes = 6

b $\Pr(C) = \frac{1}{6}$

c $\Pr(\text{vowel}) = \frac{2}{6}$
$= \frac{1}{3}$

Worked example 10.4C Using complementary events

A standard six-sided die is rolled and the number on the upper face is noted.

a List the sample space.

b What is the probability of 'rolling at most 2'?

c What is the complementary event to 'rolling at most 2'?

d What is the probability of this complementary event in part **c**?

THINK	WRITE
a List all the possible outcomes within curly brackets.	**a** $\{1, 2, 3, 4, 5, 6\}$
b The favourable outcomes are rolling a 1 and rolling a 2.	**b** $\Pr(\text{at most } 2) = \frac{2}{6}$ $= \frac{1}{3}$
c The complement to 'at most' a given number is 'more than' that number.	**c** More than 2
d Complementary events add to 1. Subtract the original probability from 1 to calculate the complementary probability.	**d** $\Pr(\text{more than } 2) = 1 - \frac{1}{3}$ $= \frac{2}{3}$

Helpful hints

✔ When using 'at least', 'at most', 'more than' or 'fewer than', make sure you identify whether or not the given value is included in the favourable outcome.

Learning pathways

1, 2, 4–9, 11, 14–17	1, 3–7, 9–13, 16, 17, 20–22, 24(a, b)	1(c–f), 5, 6, 7(c–f), 11, 13, 14, 17–20, 22–24

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Exercise 10.4A: Understanding and fluency

WE 10.4A **1** A 10-sided die numbered from 1 to 10 is rolled. Identify the favourable outcomes in each of the following situations.

a Getting at least a 5

b Getting at most a 5

c Getting an odd number and a prime number

d Getting an odd number and not a prime number

e Getting an odd number or a prime number or both

f Getting an odd number or a prime number, but not both

WE 10.4B **2** A letter is selected at random from the word LEMON. State:

a the number of possible outcomes

b the probability of selecting an L

c the probability of selecting a vowel.

3 Calculate the theoretical probability of:

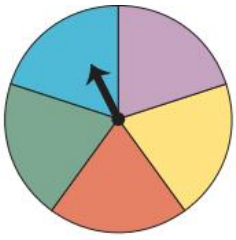
a rolling a standard six-sided die and obtaining a 6

b selecting the letter Z from the word CRAZY

c selecting a blue marble from a bag containing three red, two blue, one yellow and four green marbles

d randomly guessing the correct answer to a multiple-choice question with options A–E.

4 Consider this spinner if spun once.



a State the total number of possible outcomes.

b What is the probability of landing on blue?

c What is the probability of landing on yellow or green?

d What is the probability of landing on anything except red?

5 Calculate the theoretical probability of:

a selecting a consonant from the word CRAZY

b selecting a green or yellow marble from a bag containing four blue, three yellow, two red and six green marbles

c rolling a standard six-sided die and obtaining a number greater than 4

d selecting a C, E or S from the 34-letter word SUPERCALIFRAGILISTICEXPIALIDOCIOUS

e rolling a standard six-sided die and obtaining any number except 6.

6 Given that P(A') = 1 − P(A), find Pr(A') if Pr(A) is equal to:

a 0.4 **b** $\frac{3}{4}$ **c** 0.1 **d** $\frac{1}{2}$ **e** $\frac{1}{8}$ **f** 0.7 **g** $\frac{2}{3}$ **h** 0.25

7 A 10-sided die numbered from 1 to 10 is rolled. State, in words (without writing 'not event') and as a set of values, the complementary event of each of the following.

a Getting at least a 5

b Getting at most a 5

c Getting an odd number and a prime number

d Getting an odd number and not a prime number

e Getting an odd number or a prime number or both

f Getting an odd number or a prime number, but not both

WE 10.4C **8** A standard six-sided die is rolled. State:

a the sample space

b the probability of rolling at least a 5

c the complementary event to rolling at least a 5

d the probability of the complementary event.

9 A letter is selected at random from the word EXCITEMENT.

a What is the probability of selecting a consonant?

b What is the complementary event to selecting a consonant?

c What is the probability of the complementary event?

10 In the image shown there are 5 blue, 5 red, 4 yellow and 1 pink button. A button is selected at random from the collection. Assume that every button has an equally likely chance of being selected.

a How many buttons are there?

b What is the probability of selecting a blue button?

c State the complementary event to selecting a blue button and find its probability.

d What is the probability of selecting a white button?

e What is the probability of selecting a button with four holes?

f State the complementary event to part **e** and find its probability.

11 A number of households were surveyed about the number of eggs they used in total on Sunday morning. The data collected is summarised in the table below.

Number of eggs	0	1	2	3	4	5	6	7	8
Frequency	5	2	4	3	5	2	6	3	4

Calculate the probability that a household selected at random will have used:

a at least one egg **b** at most four eggs **c** at least four eggs **d** an even number of eggs.

e Calculate the probability of the complementary event of part **a**. State how many eggs were used in these households.

f Are parts **b** and **c** complementary events? Explain why or why not. If they are not complementary, describe the event that is complementary to part **b** and to part **c**.

Exercise 10.4B: Problem solving and reasoning

12 Explain why, when calculating theoretical probability, the outcomes listed in the sample space must be equally likely to occur for the number of elements to be used as the total number of possible outcomes.

13 A standard deck of cards has four suits with 13 cards each: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king.

a How many cards are there in total?

b What is the probability of selecting an ace?

c What is the probability of the complementary event to 'selecting an ace'?

d What is the probability of selecting a picture card (jack, queen or king)?

e Are you more likely to select a queen of hearts or an ace? Explain.

f What is the probability of selecting a picture card that is not a heart?

g What is the probability of the complementary event to part **f**?

h What is the probability of selecting an even number from the deck?

14 Consider the phrase YEAR EIGHT IS BETTER THAN YEAR SEVEN.

a How many letters are there in total? **b** What is the probability of selecting an E?

c What is the probability of the complementary event to part **b**?

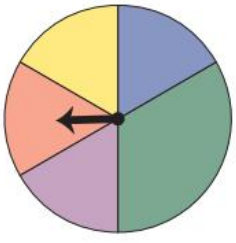
d What is the probability of selecting an A or a B? **e** What is the complementary event to part **d**?

f Now consider the letters as vowels and consonants.

i How many vowels are in the phrase? **ii** State the probability of selecting a vowel.

iii State the complement of this event and its probability.

15 Consider this spinner, where the green section is twice the size of every other section. If it is spun once:



a What is the probability that you will land on red or blue?

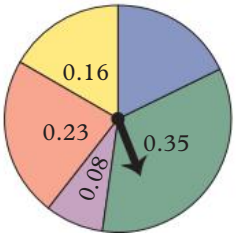
b Explain why the answer to part **a** is not $\frac{2}{5}$, even though there are five different colours.

c What is the probability that you will land on yellow?

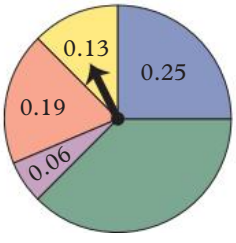
d State the complementary event to 'landing on yellow' and state its probability.

16 The numbers on these spinners represent the probabilities of landing on each labelled sector. Calculate the probability of landing on the unlabelled sector in each spinner.

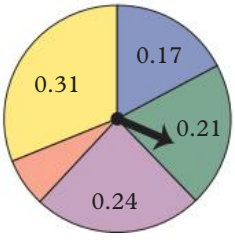
a



b



c



17 When rolling a standard six-sided die, explain why the outcomes 'rolling an odd number' and 'rolling a multiple of 2' are mutually exclusive events.

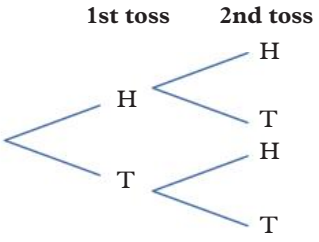
18 A spinner has $\Pr(\text{blue}) = \frac{1}{2}$, $\Pr(\text{red}) = \frac{1}{4}$, $\Pr(\text{green}) = \frac{1}{8}$ and the remaining portion is yellow. State the probability of spinning yellow.

19 At a sporting match there are 200 spectators. Ninety-five people are there to support Team A, eighty-six people are there to support Team B and the remainder don't have a preference.

a Find the probability that a randomly chosen spectator doesn't have a preference as to who wins.

b If you are there to support Team B, what is the probability that a person who sits next to you will also support Team B?

20 Some experiments consist of more than one stage. In these experiments, a tree diagram can be used to display the outcomes of the experiment. In a tree diagram, the outcomes of each stage are recorded at the end of each set of branches. Write the sample space for the experiment detailed in the tree diagram on the right, where a coin was tossed twice, and the results were recorded.



21 Maynard flipped two coins and got two tails. He said that the complementary event of 'flipping two tails' is 'flipping two heads'. Can you explain where he may have gone wrong?

22 A packet of M&Ms contains 55 individual pieces. You count out 13 blue, 7 brown, 9 green, 11 orange, 7 red and the rest yellow.

a What is the probability of selecting a yellow M&M?

b What is the probability of selecting a blue or green M&M?

c Imagine you select a yellow M&M.

i How many M&Ms are left in the packet now?

ii How many of these are yellow?

iii What is the probability of selecting a yellow M&M from the packet now?

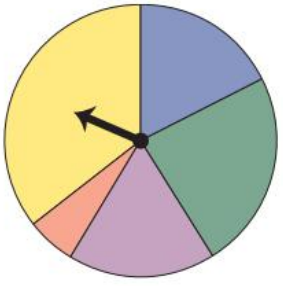
d Imagine that the second M&M you select is blue.

i How many M&Ms are left in the packet now?

ii How many of these are blue?

iii What is the probability of selecting a blue or green M&M now?

Exercise 10.4C: Challenge



23 Use a protractor and your knowledge of angles in a circle to find the probability of spinning the spinner shown on the right and landing in each of the sectors.

24 A bag contains a number of marbles coloured blue, yellow, green and red. You have a $\frac{1}{4}$ chance of selecting a blue marble, $\frac{1}{5}$ chance of selecting green and $\frac{3}{5}$ chance of not selecting yellow.

a What is the probability of selecting a yellow marble?

b What is the probability of selecting a red marble?

c Hence find the smallest number of marbles that could be in the bag.

Online resources:

- Interactive skillsheet: The language of probability
- Interactive skillsheet: Calculating theoretical probability
- Interactive skillsheet: Complementary events
- Investigation: Scrabble probabilities
- Quick quiz: 10.4

Lesson 10.5

Two-way tables

Learning intentions

By the end of this lesson you will be able to ...

✔ create, complete and use two-way tables.

Inter-year links

Year 9 8.5 Two-way tables

Year 10 11.3 Two-way tables and Venn diagrams

Two-way tables

Key content video
Understanding two-way tables

Key content video
Creating and completing two-way tables

- **Two-way tables** display frequencies for two different categories or attributes of data. For example, this two-way table shows whether students at two different schools play musical instruments.

	School A	School B	Total
Musical instrument	10	4	14
No musical instrument	3	13	16
Total	13	17	30

→ The values in the total column are calculated by summing the values in each row and column respectively. The total in the bottom right-hand corner is also equal to the size of the data set.

- Two-way tables can also be used to calculate probabilities. For example, the **relative frequency** that someone at school A plays a musical instrument is $\frac{10}{13}$, as there are 13 students from school A and 10 of them play a musical instrument. This means that if one of these students from school A is selected at random then the probability they play a musical instrument is $\frac{10}{13}$.

Worked example 10.5A Understanding two-way tables

Consider this two-way table, which shows whether students in two classes prefer cereal or toast for breakfast.

	Class A	Class B	Total
Cereal	4	9	13
Toast	7	5	12
Total	11	14	25

a How many students were surveyed?

b How many students in class A prefer toast for breakfast?

c How many students prefer cereal for breakfast?

THINK

a The total number of people surveyed is displayed in the bottom right-hand corner.

b Identify the cell that represents both Class A and toast.

c Identify the cell that shows the total of the cereal row.

WRITE

a 25

b 7

c 13

Worked example 10.5B Creating and completing a two-way table

50 students from Year 7 and Year 8 were surveyed on whether they took public or private transport to school. 19 of the Year 7 students took private transport and 13 of 27 Year 8 students took public transport.

a Create a two-way table and input the data that is given in this question.

b Complete the two-way table.

THINK

a 1 Draw the table with one of the categories (year level) placed in the rows and the other category (type of transport) placed in the columns. Then add the third row and column to write the totals.

2 Insert the information from the question into the relevant cells in the table.

b 1 The numbers in each row and column add to give the value in the total cells, so wherever there are two out of three cells in a row or column, the third can be found using addition or subtraction.

2 The total column contains two cells with information. Identify the value of the missing cell (A).

3 The Year 8 row has two cells with information. Identify the value of the missing cell (B).

4 The Year 7 row now has two cells with information. Identify the value of the missing cell (C).

5 The private transport column now has two cells with information. Identify the value of the missing cell (D).

6 Identify the value of the missing cell in the public transport column (E).

7 Complete the table by writing the values of A, B, C, D, E in the table.

WRITE

a

	Public transport	Private transport	Total
Year 7			
Year 8			
Total			

	Public transport	Private transport	Total
Year 7		19	
Year 8	13		27
Total			50

b

	Public transport	Private transport	Total
Year 7	C	19	A
Year 8	13	B	27
Total	E	D	50

A: $50 - 27 = 23$

B: $27 - 13 = 14$

C: $23 - 19 = 4$

D: $19 + 14 = 33$

E: $4 + 13 = 17$

	Public transport	Private transport	Total
Year 7	4	19	23
Year 8	13	14	27
Total	17	33	50

Worked example 10.5C Using two-way tables to calculate probability

Consider the two-way table on the right. What is the probability that a person chosen randomly from this group:

a is from Year 8 and prefers rap

b prefers pop

c is from Year 10 or prefers rap, but not both?

	Year 8	Year 10	Total
Pop	12	9	21
Rap	13	16	29
Total	25	25	50

THINK

a Identify the cell which represents both Year 8 and rap. The value in this cell is the number of favourable outcomes. The total number of possible outcomes is the number in the bottom right-hand corner.

b Identify the row which represents pop. The total at the end of this row is the number of favourable outcomes.

c Identify the cells which represent Year 10 or rap. Add the values in these cells, not including the cell which represents Year 10 and rap. Simplify the result.

WRITE

a Number of favourable outcomes = 13

$\Pr(\text{Year 8 and prefers rap}) = \frac{13}{50}$

b Number of favourable outcomes = 21

$\Pr(\text{prefers pop}) = \frac{21}{50}$

c Number of favourable outcomes = 9 + 13
= 22

$\Pr(\text{Year 10 or prefers rap, but not both}) = \frac{22}{50}$
$= \frac{11}{25}$

Helpful hints

- ✔ Calculate all missing values in a two-way table before identifying which values are being asked for!
- ✔ When filling in a two-way table, check your values by making sure the sum of each row and column equals the values in the total row and column.
- ✔ When creating a two-way table each category must cover all the data. Using complementary events can help!

Learning pathways

1–3, 5–8, 10, 11, 13, 14	2, 4–9, 12, 14, 16, 17	4, 5, 8, 11, 12, 14–19

ANS p596

Exercise 10.5A: Understanding and fluency

WE 10.5A **1** Consider the two-way table on the right.

	Year 7	Year 8	Total
Play sport	11	17	28
Don't play sport	12	10	22
Total	23	27	50

a What are the four different categories?

b How many students were surveyed?

c How many Year 8 students play sport?

d How many Year 7 students are there?

e What does the number 12 in the table represent?

f How many students don't play sport?

2 Consider this two-way table which shows whether children and adults prefer books or movies.

	Books	Movies	Total
Children	12	39	51
Adults	29	20	49
Total	41	59	100

a What are the four different categories?

b How many people were surveyed?

c How many adults prefer books?

d How many children were surveyed?

e How many children prefer movies?

f How many people prefer books?

WE 10.5B **3** 50 students from Year 7 and Year 8 were surveyed on whether they prefer action or comedy movies. 23 of the Year 7 students preferred comedy movies and 10 of 18 Year 8 students preferred action movies.

a Create a two-way table to represent this data.

b Use the given information to complete the two-way table.

4 100 people from two Melbourne suburbs were surveyed on whether they prefer cats or dogs. 48 of 59 people from Heidelberg prefer dogs and 19 people from Preston prefer cats.

a Create a two-way table to represent this data.

b Use the given information to complete the two-way table.

5 Complete these two-way tables.

a

	Weekday	Weekend	Total
Volleyball	14		31
Tennis			36
Total		48	

b

	Year 8	Year 9	Total
Spring	11		26
Summer		19	
Total	29		

6 Use the objects in this photo to complete the two-way table below.

	Lollies	Not lollies	Total
Red			
Not red			
Total			

WE 10.5C **7** Consider this two-way table which shows whether Year 8 and Year 12 students prefer TikTok or Instagram.

	Year 8	Year 12	Total
TikTok	27	43	70
Instagram	20	10	30
Total	47	53	100

What is the probability a person chosen randomly from the group:

a is a Year 12 student and prefers TikTok

b prefers Instagram

c is a Year 8 student or prefers TikTok, or both?

8 Consider this two-way table which shows whether action or comedy films were first released at the cinema or on a streaming service.

	Cinema	Streaming	Total
Action	5	6	11
Comedy	19	10	29
Total	24	16	40

a How many films were recorded in total?

b How many action films were released on streaming services?

c How many cinema releases were comedies?

d What is the probability of a film chosen randomly from the group being:

i a streaming action release

ii a comedy cinema release

iii released at the cinema

iv a comedy or released on streaming, but not both?

9 Consider the two-way table shown in question **1**. What is the probability of a person chosen randomly being:

a a Year 8 student who plays sport
b a Year 7 student who doesn't play sport
c a Year 7 student
d a student who plays sport
e a student who is in Year 7 or who plays sport, but not both
f a student who is in Year 8 or who doesn't play sport, or both?

Exercise 10.5B: Problem solving and reasoning

10 A group of 100 people were surveyed about whether they prefer to put Vegemite or jam on their toast. In total, 64 people said they prefer Vegemite. Of the 64 people who prefer Vegemite, 40 said they prefer to have it on white bread. This is exactly the same as the total number of people who prefer non-white bread.

a Construct a two-way table to represent the survey data.
b Find the probability that a person chosen randomly from the group prefers jam on non-white bread.

11 A group of 150 people were surveyed about their opinions on video games and board games. 60 people like video games and board games, 105 people like either video games or board games or both, and 5 more people like board games but not video games than people who like video games but not board games.

a Construct a two-way table to show the survey data.
b Find the probability that a randomly selected person from the survey likes:
i neither video games nor board games
ii video games or board games but not both
iii video games regardless of their opinion of board games
iv board games but do not like video games.

12 A survey asked people about their phone company. 40% of the people surveyed were from country regions. Telstra was the preferred company for only 35% of those surveyed. Of the people surveyed, 55% lived in the city and did not use Telstra as their phone provider.

	Telstra	Other	Total
Country			0.4
City		0.55	
Total			1

a Copy and complete the two-way table.
b If 200 people were surveyed, redraw the table to represent the numbers of people surveyed.
c A person is chosen randomly from the people who use Telstra. What is the probability that they are from the:
i country
ii city?

13 Two classes of students were surveyed on their chocolate preference. In a Year 8 class of 24 students, 16 preferred M&Ms over Smarties. Of the 18 people who preferred Smarties, 10 came from the Year 7 class.

	Year 7	Year 8	Total
M&Ms		16	
Smarties	10		18
Total		24	50

a Copy and complete the two-way table.
b What is the probability that a person chosen randomly from this group:
i prefers Smarties over M&Ms
ii is in Year 7
iii is in Year 8 and prefers M&Ms?
c A person is chosen randomly from the Year 8 class. What is the probability that he or she prefers M&Ms?
d Explain how part **c** is different from part **b iii**.

14 Two-way tables don't always have exactly two columns and two rows. This table, which shows the preferred sports of people in northern and southern Australia, has three columns.

	Soccer	AFL	Rugby	Total
Northern	10	12	28	50
Southern	17	30	3	50
Total	27	42	31	100

a What is the probability that a person chosen randomly from the group:
i likes AFL best
ii likes rugby but is not from northern Australia
iii likes soccer or rugby
iv is from northern Australia and likes soccer?
b What can you say about the difference in sport preference between northern and southern Australia?

15 A group of 200 students were surveyed about the number of siblings they have and if they have pets. Of the 120 students who have pets, 18 have no siblings, 53 have one sibling, 37 have two siblings, and the rest have three or more siblings. In total, 24 students have no siblings, 65 students have one sibling, 48 students have two siblings, and the rest have three or more siblings.

a Create a two-way table to represent this information.

b Find the probability that a student chosen at random from this group:

i has pets and at least one sibling **ii** has at most two siblings and has no pets

iii has three or more siblings or has pets but not both.

16 Consider this two-way table which shows the hair and eye colours of a random sample of 100 Australians.

	Blond hair	Brown hair	Total
Blue eyes	16		
Brown eyes		31	
Total		61	100

a Copy and complete the table.

b Find the probability that a person chosen randomly from the group:

i has blond hair **ii** has blue eyes **iii** has blond hair and blue eyes.

c A person is randomly chosen from the group. You know that they have blond hair.

i How many people have blond hair? **ii** How many people with blond hair have blue eyes?

iii What is the probability that somebody with blond hair has blue eyes?

iv Explain why this is different from part **b iii**.

d In probability, the word 'given' is often used when we wish to only consider a subset within the sample space. For example, part **c iii** could be asked as 'What is the probability somebody has blue eyes, given they have blond hair?' This tells you to look at the blond group as the total.

To answer the question 'What is the probability that somebody has blue eyes, given that they have brown hair?', you can follow these steps.

i The overall group that you are looking at is people with brown hair. How many people are in this group?

ii The specific group that you want is people with brown hair and blue eyes. How many people are in this group?

iii Use your answers to parts **i** and **ii** to find the probability that somebody has blue eyes, given that they have brown hair.

17 Consider the table in question **14**. A person is randomly chosen from the group.

a Find the probability that the person likes AFL, given that they are from:

i southern Australia **ii** northern Australia.

b Find the probability that the person is from northern Australia, given that they like:

i rugby **ii** AFL.

Exercise 10.5C: Challenge

18 Use the following information to construct a two-way frequency table for the events A, B and their complements. 'A' and 'not A' should be the columns, whereas 'B' and 'not B' should be the rows.

- The probability of A is $\frac{3}{10}$.
- The probability of A and B is $\frac{1}{6}$.
- The probability of A or B or both is $\frac{2}{3}$.

The numbers in your two-way table must be the smallest possible integers for the given information.

19 As you have seen, not every value needs to be known to complete a two-way table. How many cells with values need to be provided to complete a three row by three column two-way table? What restrictions are there on where these numbers need to go?

Online resources:

Interactive skillsheet
Two-way tables

Investigation
Pizza menus

Quick quiz
10.5

Lesson 10.6

Venn diagrams

Learning intentions

By the end of this lesson you will be able to ...

✔ create, complete and use Venn diagrams.

Inter-year links

Year 9 8.6 Venn diagrams

Year 10 11.3 Two-way tables and Venn diagrams

Venn diagrams

Key content video
Understanding Venn diagrams

- A **Venn diagram** is used to display the relationship between different categories or attributes of data.
  - → Venn diagrams are named after John Venn, who conceived of them in 1880, so the 'V' in 'Venn' should always be capitalised.
  - → The information in a two-way table can be displayed in a Venn diagram.
- In a Venn diagram each circle represents a different category, and the rectangle containing the circles represents the entire data set.
- Each of the regions in a Venn diagram represents a different section of the data. If there are two categories, A and B, in a data set then:

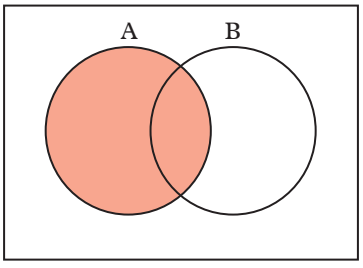
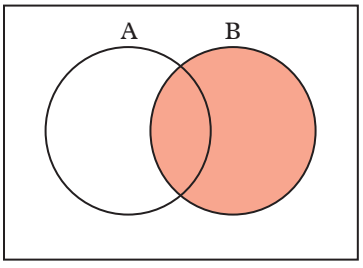
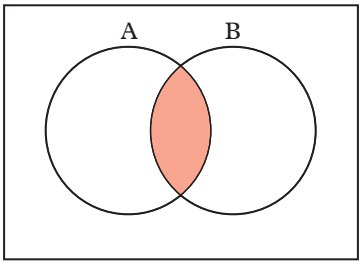
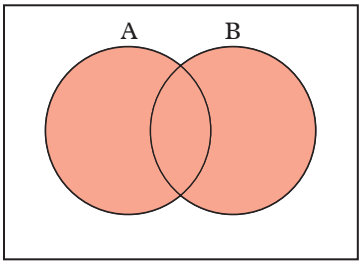
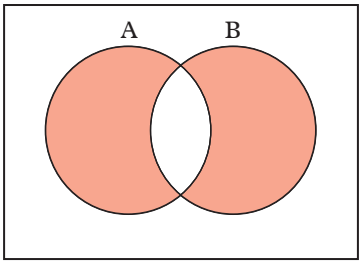
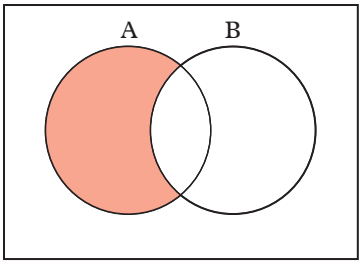
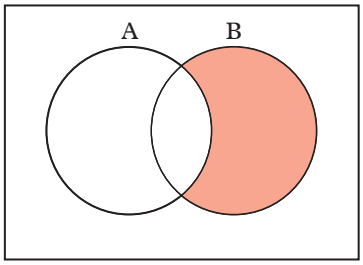
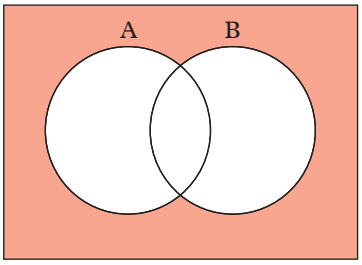
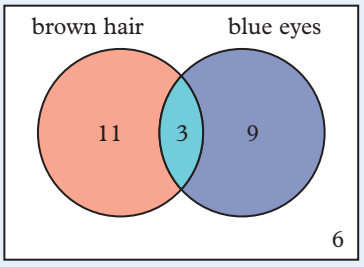
Diagram	What the shaded region represents	Diagram	What the shaded region represents
	A		B
	A and B		A or B
	A or B but not both		A and not B

Diagram	What the shaded region represents	Diagram	What the shaded region represents
	B and not A		Neither A nor B

- Venn diagrams can be labelled by either putting the frequency of items in each section, or by putting the individual items in each section.
  For example, in the following Venn diagram representing whether people have brown hair and blue eyes:

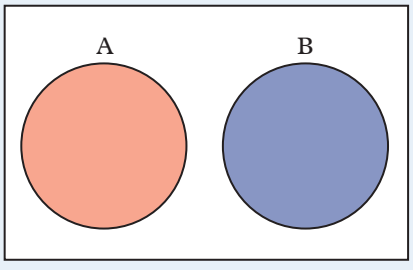
Key content video
Creating Venn diagrams



→ 11 people have brown hair without blue eyes
→ 3 people have brown hair and blue eyes
→ 9 people have blue eyes without brown hair
→ 6 people have neither brown hair nor blue eyes.

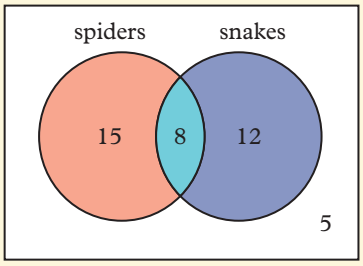
Mutually exclusive categories

- If two categories are mutually exclusive, meaning that there are no data points that belong in both categories, then the circles representing the categories can be drawn with no overlap. For example, categories A and B are mutually exclusive in the following Venn diagram.



Worked example 10.6A Understanding Venn diagrams

This Venn diagram shows whether people are afraid of spiders and snakes.



a How many people are afraid of both snakes and spiders?

b How many people are afraid of spiders?

c How many people are not afraid of either?

d How many people were surveyed?

THINK

a Identify the required region(s). Those afraid of both snakes and spiders are the number in the middle section, where the two circles overlap.

b Identify the required region(s). Those afraid of spiders are the numbers in the left-hand circle. Add together the numbers within this circle.

c Identify the required region(s). Those who are not afraid of either are the number outside the two circles.

d Add all the numbers within the Venn diagram.

WRITE

a 8 people are afraid of both snakes and spiders.

b $15 + 8 = 23$

23 people are afraid of spiders.

c There are five people who are not afraid of either snakes or spiders.

d $15 + 8 + 12 + 5 = 40$

40 people were surveyed.

Worked example 10.6B Using Venn diagrams to calculate probability

Consider the Venn diagram shown in **Worked example 10.6A**. What is the probability that a person chosen at random from the group is afraid of:

a both snakes and spiders

b snakes

c spiders only

d snakes or spiders, but not both?

THINK

a Identify the region which represents both snakes and spiders. The value in this region is the number of favourable outcomes. Sum all the numbers in the Venn diagram to get the total number of outcomes.

b Identify the region which represents snakes. Add the values in this region to get the number of favourable outcomes.

c Identify the regions which represents spiders only. The value in this region is the number of favourable outcomes.

WRITE

a Number of favourable outcomes = 8

Total number of outcomes $= 15 + 8 + 12 + 5$
$= 40$

$\Pr(\text{snakes and spiders}) = \frac{8}{40}$
$= \frac{1}{5}$

b Number of favourable outcomes $= 8 + 12$
$= 20$

$\Pr(\text{snakes}) = \frac{20}{40}$
$= \frac{1}{2}$

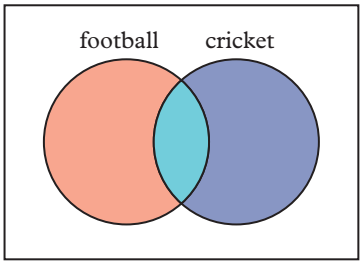
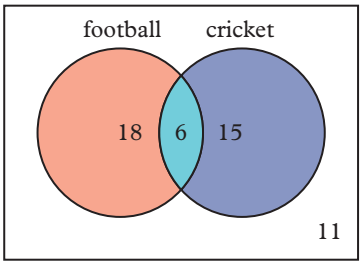
c Number of favourable outcomes = 15

$\Pr(\text{spiders only}) = \frac{15}{40}$
$= \frac{3}{8}$

THINK	WRITE
d Identify the regions which represent snakes or spiders. Add the values in these regions, not including the regions which represent snakes and spiders.	**d** Number of favourable outcomes = 15 + 12 = 27 Pr(snakes or spiders, but not both) = $\frac{27}{40}$

Worked example 10.6C Creating a Venn diagram

50 people were surveyed about whether they play football or cricket. 24 people in total play football, 21 people in total play cricket, and 11 people play neither sport.
Draw a Venn diagram to represent this situation.

THINK	WRITE
1 Draw two circles, one representing football and the other representing cricket. Draw a rectangle around the two diagrams.	
2 Enter the value representing the people who play neither sport (11) outside the two circles.	
3 Determine the number of people who play either sport by subtracting the number of people who play neither (11) from the total number of people (50).	either sport = 50 − 11 = 39
4 Add the number of people who play football (24) to the number of people who play cricket (21). Subtract the number of people who play either sport (39) from this value to find the number of people who play both sports. Enter this value in the region where the circles overlap.	both sports = 24 + 21 − 39 = 6
5 Determine the number of people who play football but not cricket by subtracting the number who play both sports (6) from the number who play football (24). Enter this value in the appropriate region of the diagram.	football but not cricket = 24 − 6 = 18
6 Determine the number of people who play cricket but not football by subtracting the number who play both sports (6) from the number who play cricket (21). Enter this value in the appropriate region of the diagram.	cricket but not football = 21 − 6 = 15
	

Helpful hints

- ✔ Don't forget to put the frequency which belongs to none of the categories outside the circles!
- ✔ When creating a Venn diagram, a common mistake is to forget that the total frequency for each category is the sum of the sections that circle has been split up into. For example, if A has 12 data points with 3 also belonging to B, then the number in the intersection of A and B will be 3, and the number in the rest of the A circle will be 9.

Learning pathways

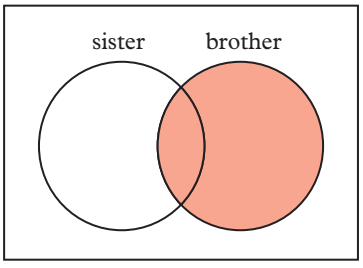
1–3, 5, 6, 8–12, 14, 15(a, b)	1, 3, 4, 6, 7, 9–11, 13–15, 17	1, 5, 6, 10, 11, 14–18

ANS p598

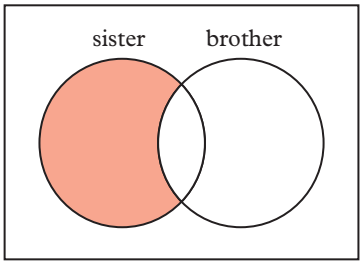
Exercise 10.6A: Understanding and fluency

1 These Venn diagrams represent whether people have brothers or sisters. Write what the shaded section in each diagram represents.

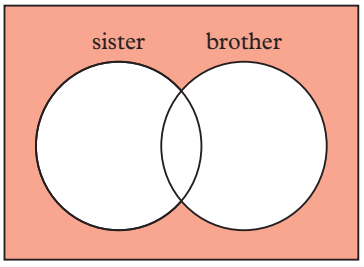
a



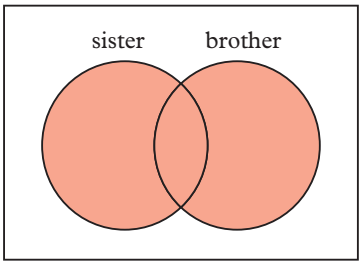
b



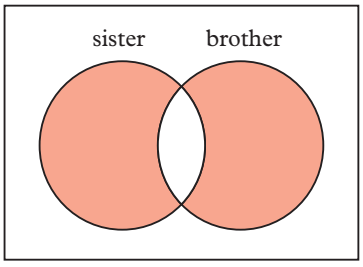
c



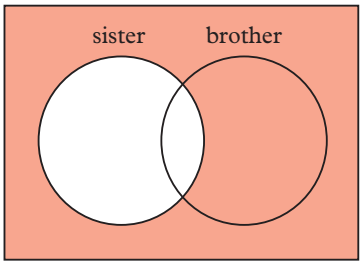
d



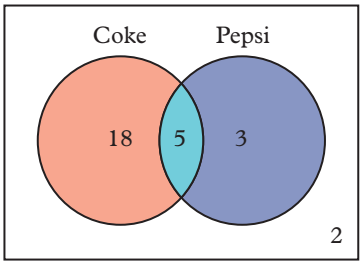
e



f



WE 10.6A **2** Consider this Venn diagram which shows the cola soft drink preferences of a group of people.



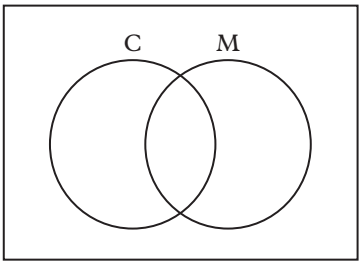
a How many people like only Coke?

b How many people like Coke?

c How many people like both Coke and Pepsi?

d How many people like neither?

3 This Venn diagram represents whether people support Collingwood (AFL) or Melbourne Storm (NRL). Section C represents those who support Collingwood and section M represents those who support Melbourne Storm.



Shade the correct section of the diagram for each of these groups.

a Collingwood supporters who also support Melbourne Storm

b Collingwood supporters who don't support Melbourne Storm

c All people who support Melbourne Storm

d People who don't support either Collingwood or Melbourne Storm

e People who support Collingwood or Melbourne Storm, but not both

f Melbourne Storm supporters who don't support Collingwood

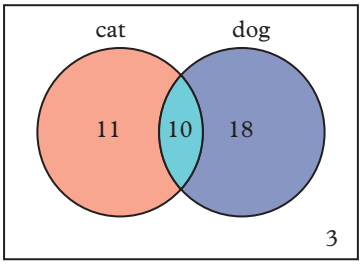
4 Consider the Venn diagram on the right.

a How many people own a cat and a dog?

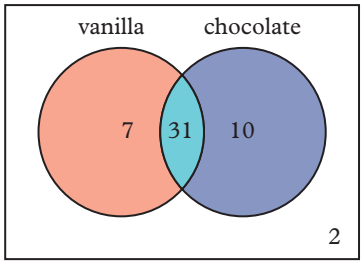
b How many people own a cat?

c How many people do not own either a cat or a dog?

d How many people were surveyed?



5 Consider this Venn diagram which shows whether people like a new chocolate and vanilla frosting.



a How many people:

i like both chocolate and vanilla

ii like chocolate only

iii like vanilla

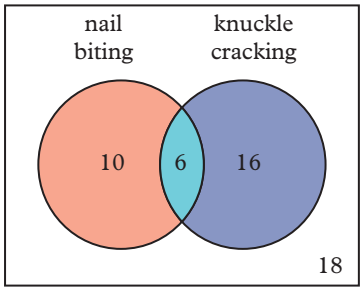
iv do not like either

v like chocolate or vanilla

vi do not like chocolate?

b How many people were surveyed?

WE 10.6B **6** Consider this Venn diagram, showing the incidence of two bad habits.



What is the probability that a person chosen at random from the group:

a bites their nails and cracks their knuckles

b cracks their knuckles but doesn't bite their nails

c bites their nails

d bites their nails or cracks their knuckles

e doesn't do either?

7 Consider the Venn diagram shown in question **4**. What is the probability that a person chosen at random from the group owns:

a both a cat and a dog

b a dog but not a cat

c a cat

d neither a dog nor a cat

e a cat or a dog, but not both?

8 Consider the Venn diagram shown in question **5**. What is the probability that a person chosen at random from the group likes:

a vanilla but not chocolate

b chocolate

c vanilla and chocolate

d vanilla or chocolate?

WE 10.6C **9** In a group of 40 people, 26 people like KFC and 30 people like McDonald's. There are four people who don't like either. Draw a Venn diagram to represent this situation.

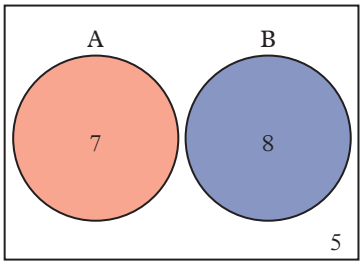
10 The photo on the right contains the following pieces of fruit:

1 banana, 4 oranges, 3 nectarines, 1 pineapple, 1 apple, 2 lemons, 2 kiwi fruits

Construct a Venn diagram using the objects in this photo, using the categories 'yellow' and 'round fruit'. Add your numbers together when you finish to check that you have completed it correctly.

Exercise 10.6B: Problem solving and reasoning

11 Use the Venn diagram shown below to calculate the probability of randomly selecting:



a A

b A but not B

c B

d B but not A

e A and B

f A or B or both

g A or B but not both

h neither A nor B

i A and B or neither A nor B.

12 A survey found that $\frac{9}{20}$ people go for daily walks, $\frac{7}{10}$ of people binge watch TV, and $\frac{1}{5}$ of people do both.

a If 100 people were surveyed, how many would you expect to:

i go for daily walks but not binge watch TV

ii binge watch TV but not go for daily walks

iii either go for daily walks or binge watch TV or both

iv neither go for daily walks nor binge watch TV?

b If 60 people were surveyed, how many:

i go for daily walks or binge watch TV or both

ii neither go for daily walks nor binge watch TV?

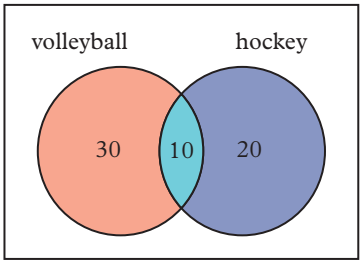
c What fraction of people surveyed:

i both go for daily walks and binge watch TV, or neither go for daily walks nor binge watch TV

ii either go for daily walks or binge watch TV but not both?

13 Clyde has created a Venn diagram to represent the following information:

30 people surveyed want to play volleyball, 20 people surveyed want to play hockey, and 10 people surveyed want to play both.

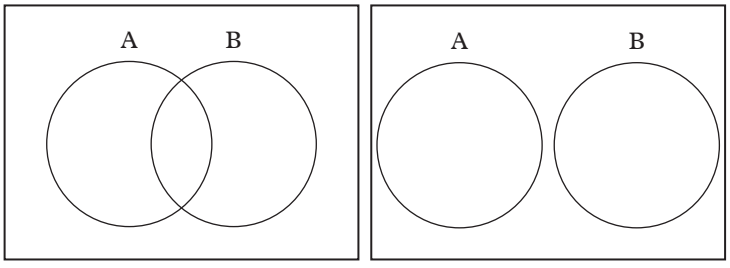


Clyde is unsure if his Venn diagram is correct as the wording of the number of people did not clearly state if the numbers represented the people who only voted for volleyball, hockey or both, or if the people who voted both were included in the people who only voted for volleyball or hockey.

a If 50 people were surveyed, Clyde's Venn diagram cannot be correct. Explain why and create the correct Venn diagram.

b If 60 people were surveyed, can Clyde's Venn diagram be correct? Explain why or why not.

14 Consider the following Venn diagrams.



a Shade the regions of both Venn diagrams that represent each of the following.

i A and B

ii A and not B

iii not A and B

iv not A and not B

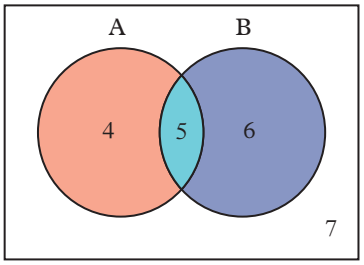
b What do you notice about the regions you shaded in part **a**?

c Use your observation in part **b** to write a rule about the sum of the probabilities of the four events listed in part **a** for the first Venn diagram. Hint: What must the probabilities add up to?

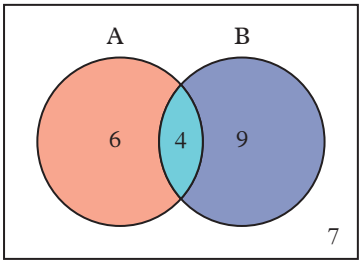
d We say that the events in the second Venn diagram are 'mutually exclusive'. How does your rule from part **c** change when A and B are mutually exclusive events?

15 Two-way tables and Venn diagrams display much of the same information. Consider how this two-way table and this Venn diagram are based on the same information.

	A	Not A	Total
B	5	6	11
Not B	4	7	11
Total	9	13	22



a Draw a two-way table that represents the information provided in this Venn diagram.

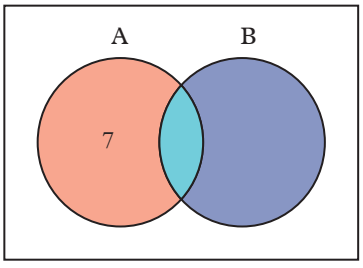


b Draw a Venn diagram that represents the information provided in this two-way table.

	A	Not A	Total
B	10	12	22
Not B	15	8	23
Total	25	20	45

c Complete the two-way table and Venn diagram shown so that they represent the same set of information.

	A	Not A	Total
B	5		
Not B		11	
Total			50



d For the information in the two-way tables and Venn diagrams in parts **a**, **b** and **c**, calculate the relative frequency of choosing the following out of the universal set.

i A and B

ii A and not B

iii A or B or both

iv A or B but not both

v not B

16 Consider this Venn diagram. If A is the set of people who like white chocolate and B is the set of people who like dark chocolate, find the probability that a person chosen randomly from the group:

a likes white chocolate

b likes white chocolate but not dark chocolate

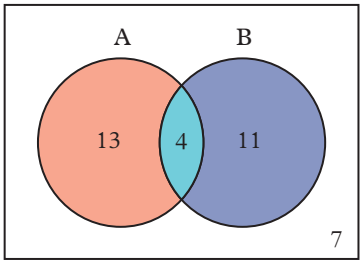
c likes both white and dark chocolate

d likes white chocolate, given that they like dark chocolate

e likes dark chocolate, given that they like white chocolate

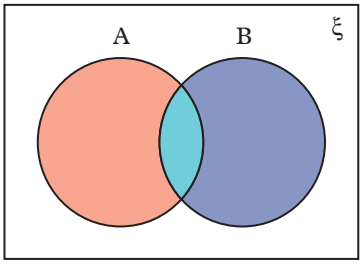
f likes white chocolate or dark chocolate but not both

g does not like dark chocolate.



Exercise 10.6C: Challenge

17 Set notation can be used to describe categories in Venn diagrams. This Venn diagram shows sets A and B. Copy the diagram five times.



a On the first diagram, shade the overlapping section of A and B. In set notation, this is the intersection of sets A and B. Label this $A \cap B$.

b On the second diagram, shade inside the circles. In set notation, this is the union of sets A and B. Label this $A \cup B$.

c On the third diagram, shade everything that does not belong to set A; that is, everything outside this circle. This is the complement of set A. Label this A'.

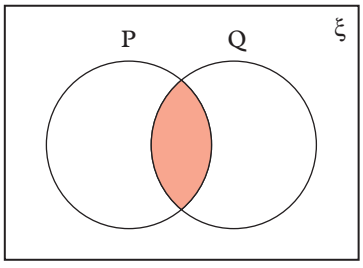
d Everything inside the rectangle is said to belong to the universal set. Label this with ξ. Shade everything inside the rectangle to represent ξ on the fourth diagram.

e Use what you have learnt to shade B' on the fifth diagram.

18 The following diagrams show sets P and Q.

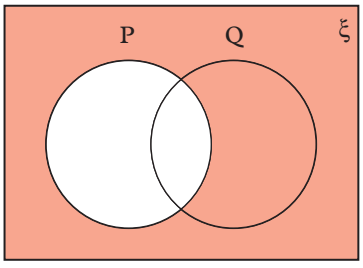
Identify what each of the shaded regions represent.

a



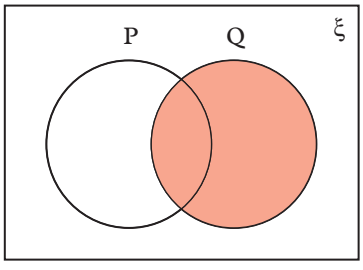
A Q **B** $P \cap Q$
C $P \cup Q$ **D** P'

b



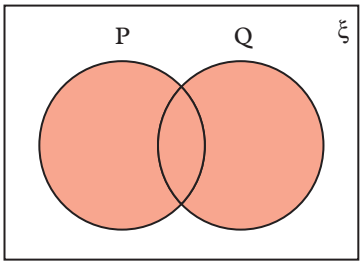
A Q **B** $P \cap Q$
C $P \cup Q$ **D** P'

c



A Q **B** $P \cap Q$
C $P \cup Q$ **D** P'

d



A Q **B** $P \cap Q$
C $P \cup Q$ **D** P'

Online resources:

Interactive skillsheet
Venn diagrams

Investigation
Venn diagrams and social networks

Quick quiz
10.6

Simulations

Monte Carlo experiments

Curriculum link

Conduct repeated chance experiments and simulations, using digital tools to determine probabilities for compound events, and describe results. (VC2M8P03)

The relative frequency of an event occurring can be determined by an experiment involving a number of trials.

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

The Law of Large Numbers states that the more times a random chance experiment is performed, the more likely it is that the relative frequency of an event will approach the theoretical probability of that event.

A Monte Carlo experiment is an investigative technique involving simulations of random chance experiments. This technique is often used when it is too difficult to calculate the exact probability of an event; however, it can also be used to confirm theoretical results. Monte Carlo experiments involving a large number of trials can be trusted to be accurate due to the Law of Large Numbers.

Simulation: Finding the difference between two dice

Many games involve rolling two standard six-sided dice and adding the two scores together. The probabilities of each sum are well known, but it is not as common to investigate the difference between the two numbers.

The difference between two numbers is defined as the greater value minus the lower value, meaning that the difference is always positive or 0. For example, the difference between 3 and 5 is 2 and the difference between 5 and 3 is also 2.

Instructions

For this simulations we will use a spreadsheet program to simulate rolls of two six-sided dice and to calculate the difference between the two numbers on the dice.

Open up a new Microsoft Excel document, or otherwise a similar spreadsheet program.

Code	Explanation
1 In cell A1, type '=RANDBETWEEN(1,6)' and press Enter. Type the same formula into cell B1.	This randomly generates a whole number from 1 to 6 with equal chance, just like rolling a die generates a whole number from 1 to 6 with equal chance.
2 In cell C1 type '=MAX(A1,B1)-MIN(A1,B1)'.	This subtracts the smaller number from the greater number, which calculates the difference between the two numbers.
3 Select the cells A1, B1 and C1 together, then drag down from the bottom right of C1 down to cell C100.	This will simulate 100 trials of the experiment, storing all the differences in column C.

4 In column E, type in the numbers 0 to 5. This means that 0 should be in E1, 1 should be in E2, and so on until 5 is in E6.	Column E is the list of possible differences. The COUNTIF formula counts the number of times each of the possible differences occurs. Your spreadsheet should look similar to the image below.
5 In cell F1 type in '=COUNTIF(C:C,E1)'.	
6 Drag down the cell F1 down to F6.	

	A	B	C	D	E	F
1	6	2	4		0	18
2	3	1	2		1	25
3	5	3	2		2	22
4	3	4	1		3	14
5	5	6	1		4	18
6	1	3	2		5	3
7	4	1	3			
8	3	1	2			
9	3	1	2			
10	3	2	1			
11	3	5	2			
12	2	1	1			
13	5	1	4			
14	1	6	5			
15	4	4	0			

Simulation questions

1 Explain why the possible values of the difference are 0, 1, 2, 3, 4 and 5.

2 In your simulation, which difference occurred the most times? Which difference occurred the least times?

You may notice that every time you execute a formula, the whole spreadsheet refreshes. This can also be done manually by pressing F9. Refreshing the spreadsheet means all the formulas are executed again, and so all the randomly generated numbers are re-generated.

3 Repeatedly press F9 while focusing on the numbers in column F. What do you notice? Which difference seems to be the most likely? Which difference seems to be the least likely?

Since there are only 100 trials, the relative frequencies may not match the theoretical probabilities closely. Highlight cells A1, B1 and C1 and this time drag the bottom right corner down to C1000. This will simulate 1000 trials. Scroll back to the top to view the counts.

4 Based on these 1000 trials, which difference occurred the most times? Which difference occurred the least times?

A difference of 5 is least likely, since it can only occur when the two numbers on the dice are 1 and 6.

5 What is the theoretical probability of rolling two dice and the difference being 5? Write your answer correct to two decimal places and compare your answer to the relative frequency in your simulation of 1000 trials.

Extension

Calculate, correct to two decimal places, the theoretical probabilities of each difference (0, 1, 2, 3, 4, 5). Then, compare these probabilities to the relative frequencies of each difference in a simulation with a large number of trials (1000 trials or more).

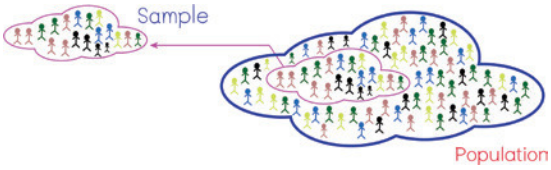
Due to the Law of Large Numbers, the relative frequencies should be very close to the theoretical probabilities (assuming you calculated them correctly). Write a short report on your Monte Carlo experiment, discussing how well the results match the theoretical probabilities you calculated.

Lesson 10.7

Review: Statistics and probability

Module summary

Collecting data



- Fair samples of a reasonable size are likely to accurately represent the characteristics of the population.

Using samples to make predictions

- Random samples should be:
  - → fair and unbiased
  - → of a sufficient size.
- The larger the sample size, the more likely it is that the sample will be representative of the population.

Sampling methods

- Simple random sampling techniques: every potential piece of data has an equal chance of being selected
  - → Systematic sampling: selecting data at fixed intervals
  - → Stratified sampling: dividing the population into categories and taking a random sample from each category
- Non-random sampling techniques: different pieces of data have an unequal chance of being selected
  - → Convenience sampling: taking a sample that is easily accessible
  - → Judgemental sampling: a sample is selected by a researcher
  - → Quota sampling: dividing the population into categories and taking a specified number from each category
  - → Cluster sampling: dividing the population into similar but diverse groups and taking a random sample of the groups

Theoretical probability

- Assuming all outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes in event}}{\text{total number of possible outcomes}}$$

- The complement of an event is the probability that the event does not occur.
- The probabilities of complementary events add to 1.

Summary statistics

- mean = $\frac{\text{sum of all data points}}{\text{total number of data points}}$
- median = middle value of an ordered data set
- mode = most common value in a data set
- range = maximum value – minimum value
- outlier = extreme data point much higher or lower than the other values

Describing data

- A small sample size with the mean being significantly different to the median may indicate the presence of an outlier.
- 'average' usually refers to the mean.
- 'most common', 'most popular' and 'most often' refer to the mode.
- 'middle' and 'centre' usually refer to the median.
- 'spread' or 'span' usually refer to the range.

Two-way tables

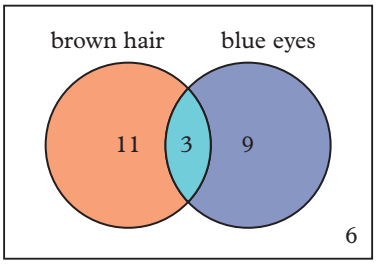
	School A	School B	Total
Musical instrument	10	4	14
No musical instrument	3	13	16
Total	13	17	30

- 10 students at school A play a musical instrument.
- 17 students from school B are in the data set.

Describing events

- 'at least' – equal to or above the given value
- 'at most' – equal to or below the given value
- 'more than' – above the given value
- 'fewer than' – below the given value
- 'and' – both outcomes must be met
- 'or' – either of the given outcomes

Venn diagrams



- 3 students have brown hair and blue eyes.
- 9 students have blue eyes but not brown hair.
- 6 students have neither brown hair nor blue eyes.
- 11 students have brown hair but not blue eyes.
- 29 students were surveyed.

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Review questions 10.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- biased
- census
- cluster sampling
- complementary events
- convenience sample
- event
- experiment
- favourable outcome
- judgemental sample
- maximum
- mean
- measure of centre
- measure of spread
- median
- minimum
- mode
- outlier
- population
- population parameters
- probability
- quota sampling
- range
- relative frequency
- sample
- sample space
- simple random sampling
- stratified sampling
- summary statistics
- survey
- systematic sampling
- two-way table
- Venn diagram

1 Which type of sampling is when a sample is selected by the researcher?

2 Which summary statistic is a numerical average of the values in a data set?

3 If all outcomes in a sample space are equally likely to occur, explain how the theoretical probability of an event can be calculated.

4 What are the three measures of centre in the key word list?

5 Identify the key terms being referenced in each of these definitions.

a the difference between the largest and smallest values in a data set
b all potential pieces of data under consideration

6 Using an example, provide a definition in your own words for the following key terms.

a outlier **b** Venn diagram

7 Complete the following sentences using words from the key term list.

a A _____ is a selection of data that is part of the _____.
b An _____ is one or more outcomes from the _____ of an experiment.

Review questions 10.7B: Multiple choice

10.1 **1** Which of these is stratified sampling?

A Selecting every third person who walks past
B Drawing names out of a hat
C Surveying 15 people in a class of 25
D Surveying 3 boys and 2 girls out of a group of 12 boys and 8 girls

10.2 **2** For the data set 1, 5, 4, 3, 2, 5, 5, 3, 4, 8, the mean is:

A 3 **B** 4 **C** 5 **D** 7

10.2 **3** For the data set in question **2**, the median is:

A 3 **B** 4 **C** 5 **D** 7

10.3 **4** Belinda wants to investigate her local town's opinion on a current issue. Which of these is the most appropriate method of taking a sample to use?

A Asking everybody in the entire town (population 15 000)

B Surveying every fourth person at the local mall until 200 are surveyed

C Asking everybody she knows

D Surveying every 10th person in the local phone book until 100 are surveyed

10.4 **5** What is the complementary probability to rolling a 1 or 2 on a standard die?

A $\frac{1}{6}$ **B** $\frac{2}{6}$ **C** $\frac{3}{6}$ **D** $\frac{4}{6}$

10.5 **6** Consider this two-way table.

	Brown eyes	Not brown eyes	Total
Blond	3	12	15
Not blond	19	16	35
Total	22	28	50

What is the probability of a person chosen randomly from the group having brown eyes?

A $\frac{11}{25}$ **B** $\frac{3}{22}$ **C** $\frac{19}{50}$ **D** $\frac{14}{25}$

10.5 **7** This two-way table shows the ice cream preferences of Year 7 and Year 8 students out of chocolate or vanilla. How many Year 8 students prefer chocolate? Complete the two-way table to find the answer.

	Chocolate	Vanilla	Total
Year 7		65	170
Year 8			
Total		110	350

A 105 **B** 135 **C** 45 **D** 145

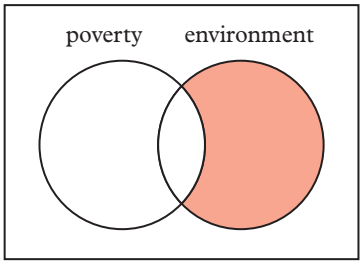
10.6 **8** A group of students were asked what they believed the two biggest issues facing the world in the 20th century are. What does the shaded section represent?

A the environment

B poverty and the environment

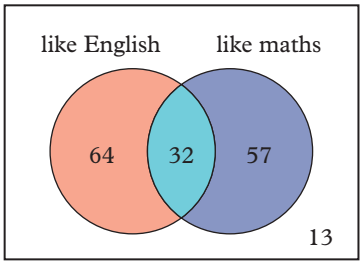
C not the environment

D the environment only



10.6 **9** The Year 8 students at a school were asked whether they like English and maths. The results are displayed in this Venn diagram. How many students don't like English?

A 13 **B** 96 **C** 70 **D** 57



Review questions 10.7C: Short answer

10.1 **1** For the following surveys, identify:

i if it is a census or sample

ii if the sampling technique is fair or biased, providing reasons. If it is biased, provide a fair alternative.

a Liana asks her class members who their favourite celebrity is to find the most popular celebrity in the state.

b Joe asks every fifth person on the school enrolment list how many people are in their family to find the average number of people in a family at his school.

c Gerard asks everybody in his class if they like chocolate to find out the most popular snack in his class.

10.1 **2** Write a fair question that could be used on a survey on each of the following topics.

a The most popular AFL team in Victoria

b The most common way that students travel to your school

c The preferred flavour in a Neapolitan tub of ice cream (strawberry, vanilla, chocolate) of Year 8 students at your school

10.2 **3** Calculate summary statistics (mean, median, mode and range) for each of these data sets.

a 1, 4, 3, 2, 5, 2, 4, 1, 1, 6, 4 **b** 11, 18, 21, 17, 13, 11, 16, 2, 9, 11 **c** 2, 2, 2, 2, 29, 3

10.2 **4** Consider this frequency table.

Score	Frequency
12	3
13	6
14	8
15	2
16	0
17	0
18	1

Calculate summary statistics for this data.

10.2 **5** Recalculate the summary statistics in the frequency table from question **4** without including the 18. Which summary statistics change, and which stay the same?

10.3 **6** Explain why the following sampling methods should *not* be used to find the most popular teacher in school.

a Asking everybody in Year 8

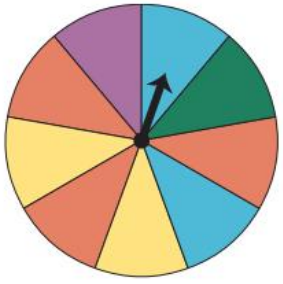
b Asking 200 people randomly from the phone book

c Asking one person from each year level

10.3 **7** Explain whether you could reasonably predict the population parameters from this table.

Population	Tasmania
Sampling technique	Stratified sampling
Sample	50 people from Tasmania
Mean	7
Median	10
Mode	11
Range	26

10.4 **8** Consider the spinner on the right.



a What is the theoretical probability of spinning blue?

b What is the theoretical probability of spinning green or red?

c Find the theoretical probability of the complementary event to part **b**.

10.4 **9** Consider the following heights of 10 students in a class:

159 cm, 181 cm, 159 cm, 161 cm, 168 cm,
147 cm, 166 cm, 167 cm, 158 cm, 155 cm.

Find the probability that a randomly selected student from the class:

a is at least 166 cm tall

b is at most 166 cm tall

c is less than the mean height.

10.5 **10** Consider this two-way table.

	Like Vegemite	Don't like Vegemite	Total
Like Milo	97	56	153
Don't like Milo	18	29	47
Total	115	85	200

What is the probability that a person chosen randomly from the group:

a likes Vegemite

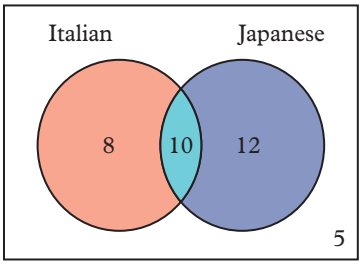
b likes Vegemite but not Milo

c likes Milo but not Vegemite

d doesn't like Milo

e likes Milo or Vegemite, but not both?

10.6 **11** Consider the Venn diagram on the right.



a How many people were surveyed?

b How many people like Italian food?

c What is the probability that a person chosen randomly from the group:

i likes both food types

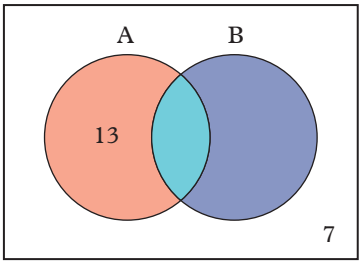
ii likes Italian food only

iii does not like Japanese food

iv likes Japanese or Italian food?

10.6 **12** The following Venn diagram and two-way table represent the same information. Complete both diagrams.

	B	B'	Total
A	11		
A'			
Total			41



Review questions 10.7D: Mathematical modelling

1 Drew wanted to find the average age of people who went to the local beach. One Saturday he went to the beach and asked the age of every fifth person who walked past. The results are shown.

14	17	18	23	21	19	23	45	23	46	61	15	16	17	39	32
9	19	27	26	31	5	21	19	18	16	15	26	22	59	72	13
49	43	16	15	19	23	22	21	63	56	8	24	35	16	19	25

a Describe the sampling method. Is Drew taking a census or a sample?

b Calculate summary statistics (mean, median, mode and range) for the data. Round to one decimal place where necessary.

c What limitations does this sample have if you want to use it to predict the population parameters?

d Make a prediction about what you would expect to see in the population.

2 A carnival game involves selecting a counter from a bag that contains numbered counters from 1 to 10. The frequency of each counter is shown in the table below.

Number	1	2	3	4	5	6	7	8	9	10
Frequency	4	6	8	10	12	12	10	8	6	4

You must pay 6 tickets to play and you get the number of tickets on the counter as a prize.

a Calculate the probability of playing and ending up with at least one more ticket than you started with.

b An alternative game is also offered which requires paying 9 tickets instead of 6. In this game if you get a non-prime even number you get 6 tickets, if you get an odd prime number you get 10 tickets, if you get an even prime number you get 30 tickets, but if the number is not even and not prime you get no tickets.

For the alternative game:

i Construct a two-way table to show the frequency of each outcome.

ii What is the probability of receiving no tickets back?

iii What is the probability of receiving more tickets than you paid?

c The expected number of tickets you receive from playing these games can be calculated by multiplying the probability of each outcome by the number of tickets received from that outcome and adding the results together.

i Calculate the expected number of tickets received for playing the first game.

ii Calculate the expected number of tickets received for playing the alternative game.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Distinguish between a population and a sample ☐ Distinguish between sampling methods ☐ Identify sources of bias in data collection methods	☐ Go back to Lesson 10.1 Collecting data and sampling methods
☐ Calculate the mean, median, mode and range of a numerical data set ☐ Evaluate how individual values affect the mean, median and range	☐ Go back to Lesson 10.2 Summary statistics
☐ Use sample properties to predict population parameters ☐ Use summary statistics to describe key features of data	☐ Go back to Lesson 10.3 Analysing data
☐ Describe events using the language of probability ☐ Identify complementary events ☐ Calculate the probability of an event and its complement	☐ Go back to Lesson 10.4 Theoretical probability
☐ Interpret and create two-way tables ☐ Calculate probabilities using two-way tables	☐ Go back to Lesson 10.5 Two-way tables
☐ Interpret and create Venn diagrams ☐ Calculate probabilities using Venn diagrams	☐ Go back to Lesson 10.6 Venn diagrams

Semester 2 review

Short answer

ANS p600

1 Verify the solutions in the brackets for each of the following equations.

a $3x - 5 = 19$ $(x = 8)$

b $3 - 8x = -1$ $\left(y = \frac{1}{2}\right)$

c $-7x + 9 = 37$ $(x = -3)$

d $3w + 8 = 5w - 2$ $(w = 5)$

2 Solve the following equations for x.

a $5x - 3 = 47$

b $8(x + 3) = 48$

c $8x + 8 = 7x - 3$

d $4x + 7 = 9x - 2$

e $x^2 - 25 = 0$

f $4x^2 = 121$

3 Plot the following points on the Cartesian plane and state whether the plot is linear or non-linear.

a $(-2, 5), (-1, 2), (0, -1), (1, -4)$

b $(0, 1), (1, 2), (2, 4), (3, 8)$

c $(-1, -1), (0, 0), (1, 1), (2, 8)$

d $\left(\frac{1}{6}, -\frac{2}{3}\right), \left(\frac{1}{4}, -\frac{1}{2}\right), \left(\frac{1}{3}, -\frac{1}{3}\right), \left(\frac{1}{2}, 0\right)$

4 Solve the equations using the graphs below.

a Using the graph of $y = 3x + 2$:

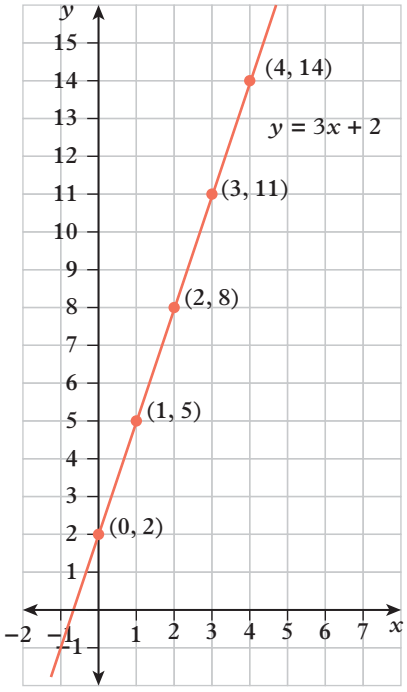
i $3x + 2 = 11$

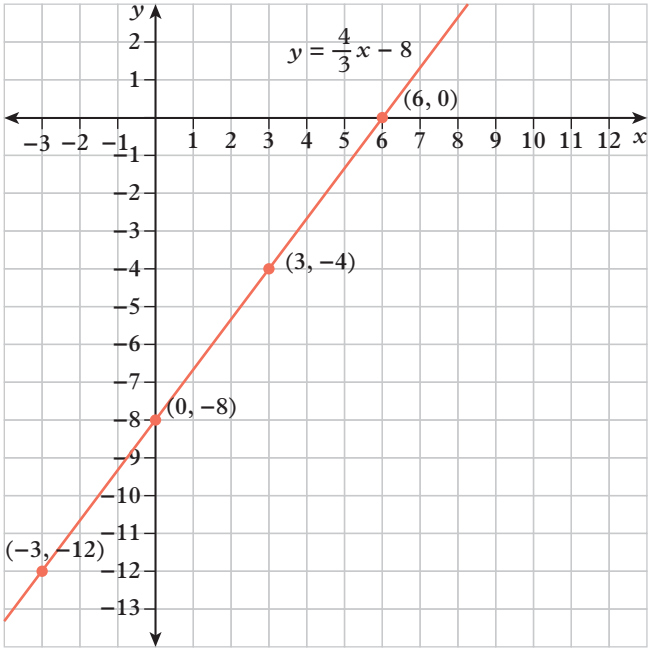
ii $3x + 2 = 2$

b Using the graph of $y = \frac{4}{3}x - 8$:

i $\frac{4}{3}x - 8 = -12$

ii $\frac{4}{3}x - 8 = 0$





5 For each equation below:

i create a table of values for the equation for integer values of x from 0 to 3

ii plot the points on the Cartesian plane and state whether the equation is linear or non-linear.

a $y = 2x + 3$

b $y = 2^x + 3$

6 Determine the equation of each of the following linear relationships.

a

x	0	1	2	3	4
y	7	5	3	1	−1

b

x	−2	−1	0	1	2
y	−5	−1	3	7	11

c

x	0	3	6	9	12
y	−2	−5	−8	−11	−14

d

x	−2	0	2	4	6
y	5	6	7	8	9

7 Solve the following equations graphically.

a $3x + 4 = 2x + 18$ **b** $4 - 2x = 5x - 3$

8 State if the following angles are complementary, supplementary or equal.

a vertically opposite angles

b the non-right angles in a right-angled triangle

c corresponding angles on parallel lines cut by a transversal

d co-interior angles on parallel lines cut by a transversal

e the exterior angle and the interior angle at the same vertex

f the included angles between unequal sides in a kite

g adjacent angles in parallelograms

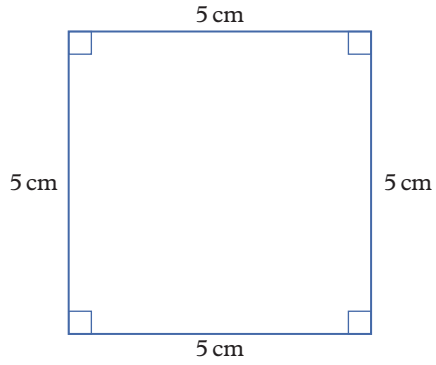
h corresponding angles in congruent shapes

9 For each of the following quadrilaterals:

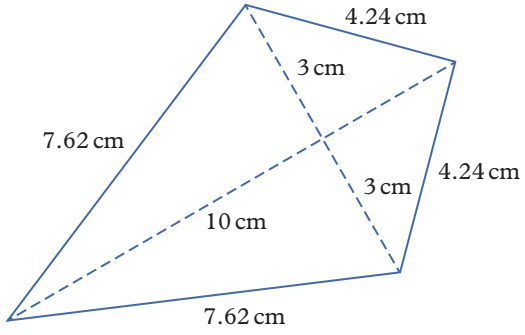
i classify the type of quadrilateral, giving the most specific name

ii determine the area.

a



b



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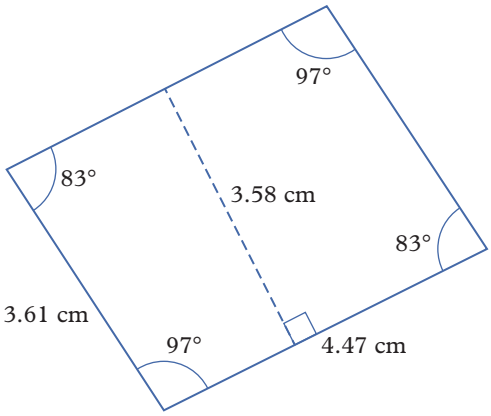
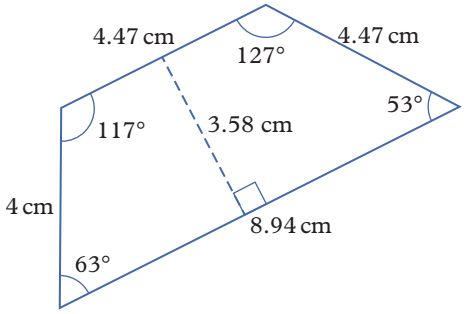
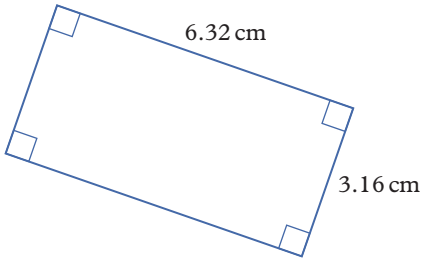


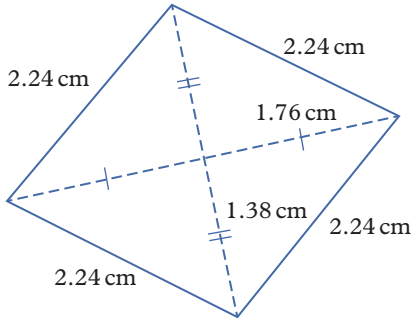
d



e

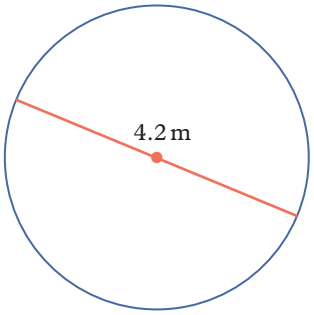


f

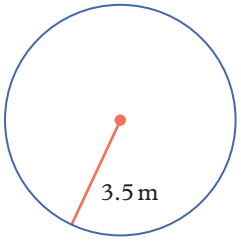


10 Determine the area and perimeter of the following shapes, correct to two decimal places.

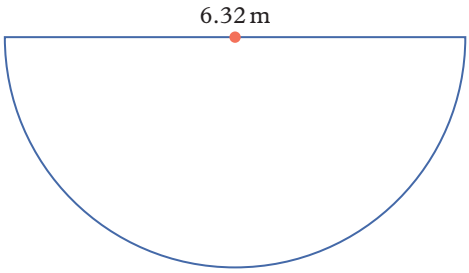
a



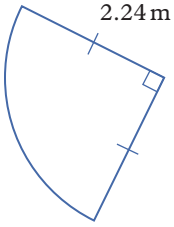
b



c



d



11 For each of the following, write and then solve an equation using the given information.

a 5 more than 3 times a number is equal to 29.

b The perimeter of a rectangle with a length of 3.4 m is 10.1 m. Find the width of the rectangle.

c The circumference of a circle is 30 cm. Find the length of the diameter of the circle, correct to two decimal places.

d The area of a triangle with a side length of 1.5 cm is 2.25 cm^2. Find the perpendicular height to the 1.5 cm side length.

e The area of a parallelogram with a base length of 7.2 cm is 24.48 cm^2. Find the perpendicular height to the base.

f The area of a circle is 30 cm^2. Find the length of the radius of the circle, correct to two decimal places.

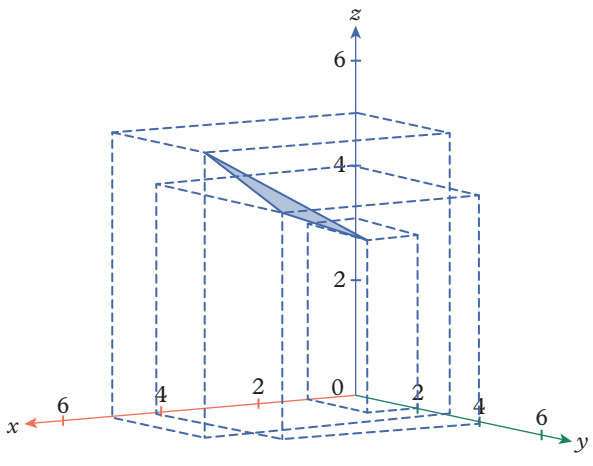
g Two of the interior angles of a triangle are 34° and 57°. Find the size of the third interior angle.

h The mean of 5, 8, 13, 2 and one other number is 6.2. Find the last number.

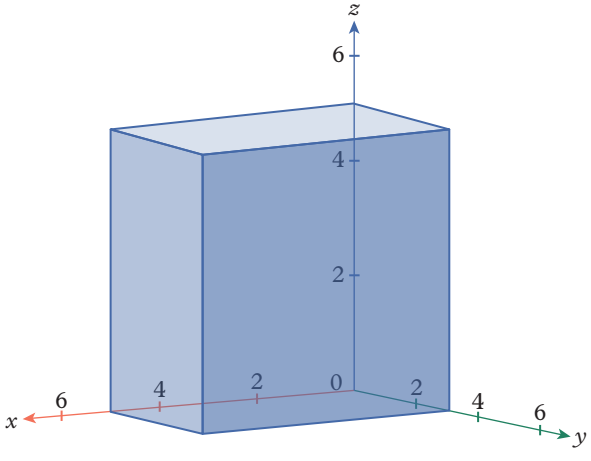
i The probability of three out of four outcomes from a sample space are 0.3, 0.12, 0.44. Find the probability of the fourth outcome.

12 List the coordinates of the vertices of the following shaded shapes and objects.

a

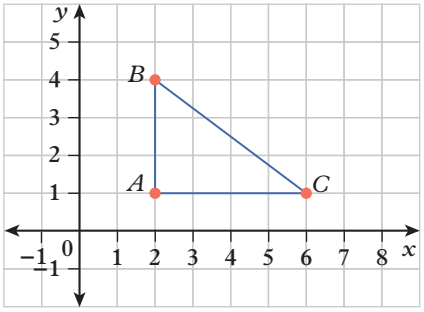


b

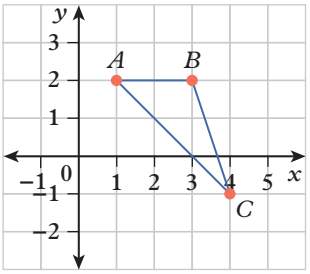


13 Apply the transformation(s) to the shapes shown.

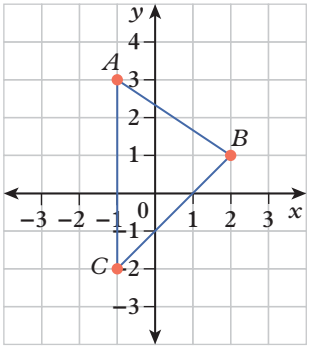
a Rotate 90° clockwise about the origin.



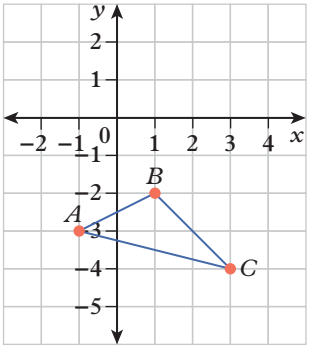
b Reflect across the x-axis.



c Translate 3 units left and 4 units up.

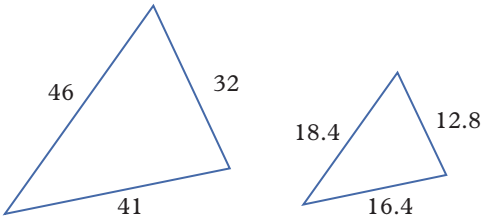


d Rotate 90° anti-clockwise about the origin, then reflect in the y-axis.

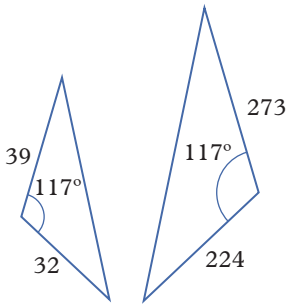


14 Determine if the following pairs of triangles are similar. If they are similar, state the test that explains why.

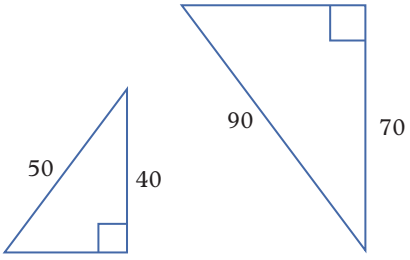
a



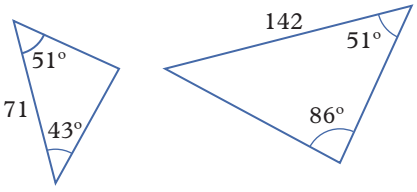
b



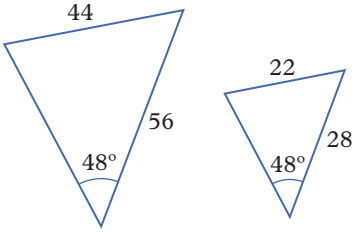
c



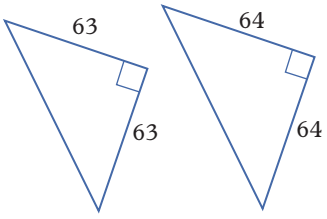
d



e

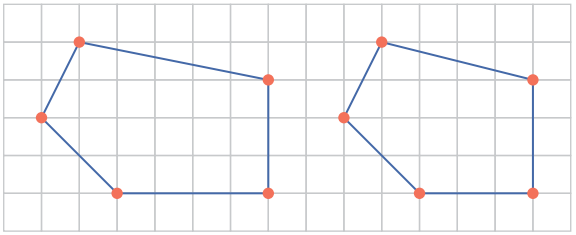


f

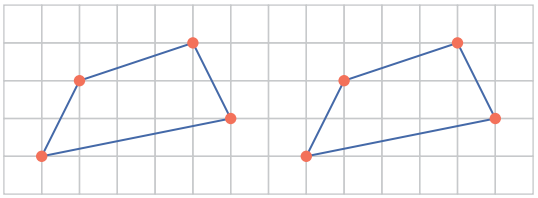


15 State if the two shapes are congruent or not.

a

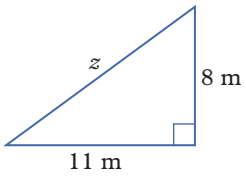


b

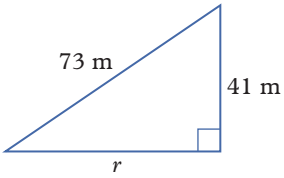


16 Determine the length of the unknown side lengths, correct to two decimal places.

a



b



17 Determine whether these side lengths can form right-angled triangles.

a 7.7 cm, 41.7 cm, 42.7 cm

b 29 mm, 128 mm, 131 mm

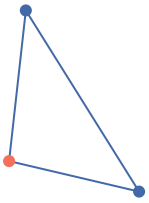
c 20 cm, 48 cm, 52 cm

d 4 m, 4.2 m, 5.8 m

18 Dilate each figure using the orange centre of dilation and the scale factor provided.

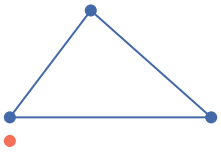
a

Scale factor: $\frac{1}{2}$



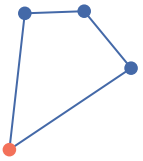
b

Scale factor: 2



c

Scale factor: 3



19 Find the probability of the following events.

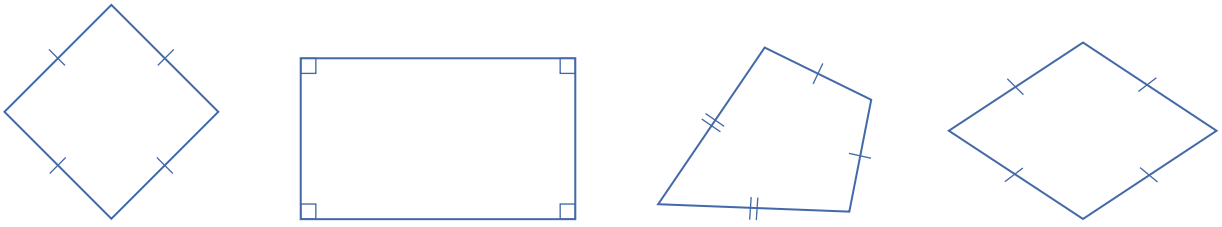
a randomly selecting a number with a recurring decimal from the following list

$\frac{2}{3}, \frac{5}{7}, \frac{3}{10}, \frac{3}{6}, \frac{7}{25}, \frac{5}{12}, \frac{11}{20}, \frac{18}{120}, \pi$

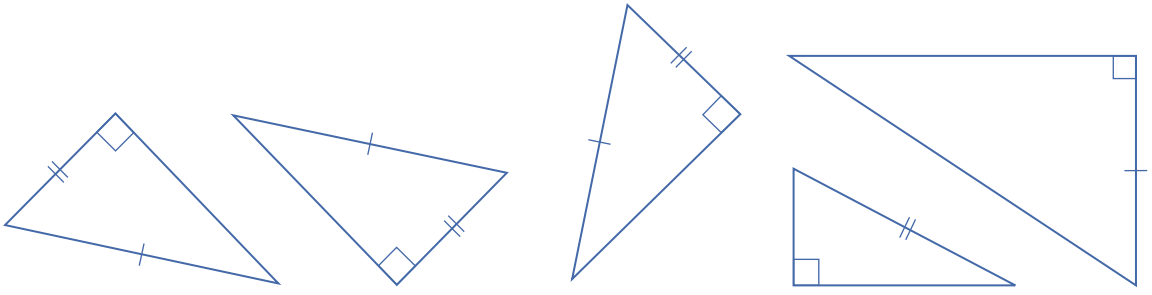
b randomly selecting an expression from the following list

$2x + 4y, -\frac{4}{3}, 8x + 1 = 15, y = 4 - 3x, 3 \times 4 + 5 = 17, 0.2z$

c randomly selecting a quadrilateral from the shapes below that has perpendicular diagonals



d randomly selecting a triangle that is congruent to at least one other triangle from the following shapes

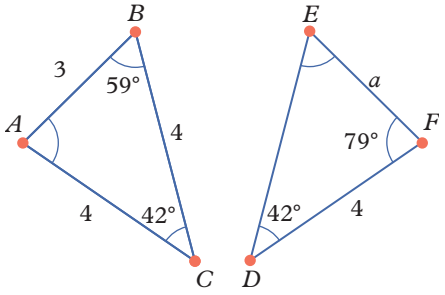


20 For each of the following pairs of triangles:

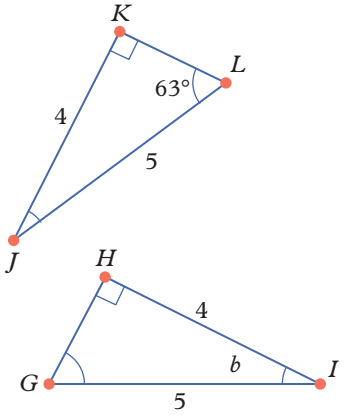
i state the test that explains why the triangles are congruent

ii find the values of the pronumeral.

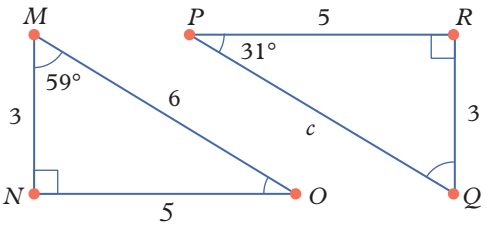
a



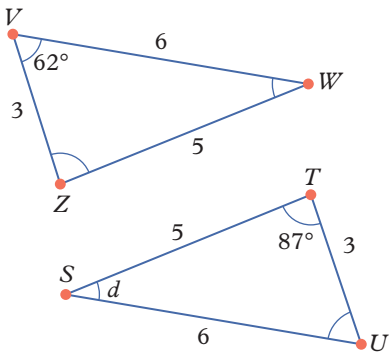
b



c

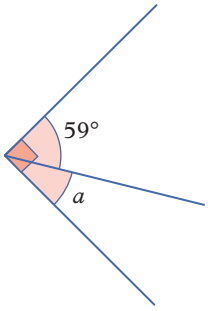


d

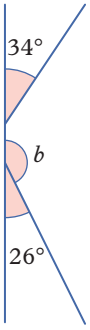


21 Find the value of the pronumerals.

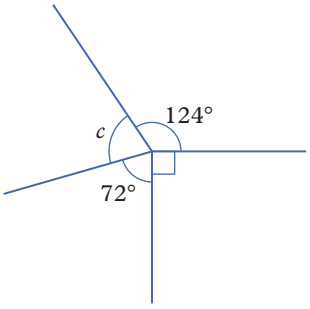
a



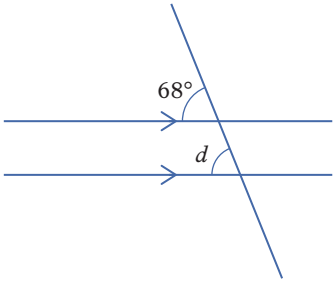
b



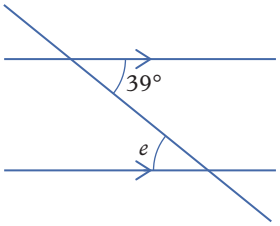
c



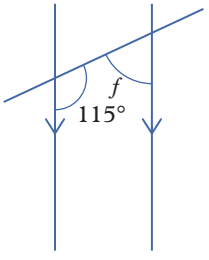
d



e



f



22 Convert each of the following into the unit given in the brackets.

a 24 mm (m)

b 5 m^3 (cm^3)

c 8001 km^2 (m^2)

d 4240 mm^3 (cm^3)

e 0.2 cm^2 (m^2)

f 30 km (cm)

g 4320 seconds (hours)

h 2.5 hours (minutes)

i 1823 (12-hour time)

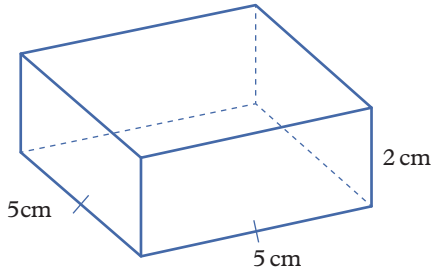
j 12:30 am (24-hour time)

k 1240 mL (L)

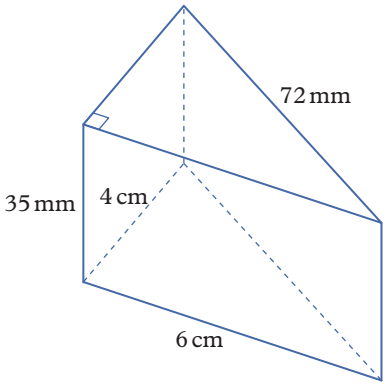
l 35L (mL)

23 Calculate the volume of the following objects.

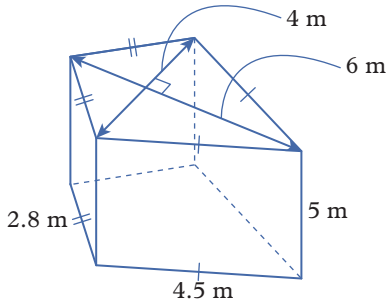
a



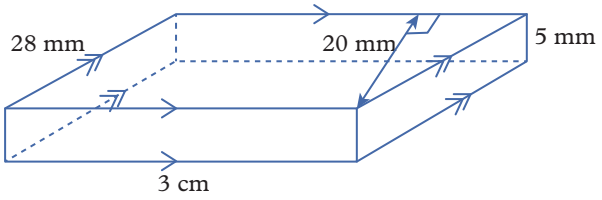
b



c



d

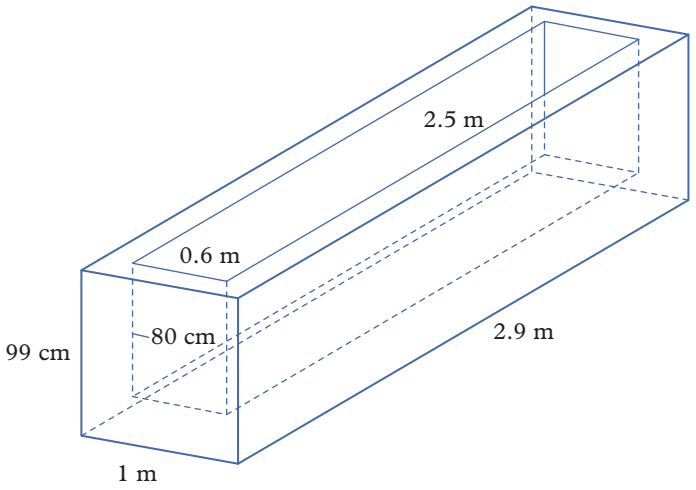


24 A rectangular prism has side lengths of 1 m, 2.9 m and 99 cm.

a Calculate the volume of the rectangular prism.

b A rectangular prism with side lengths of 0.6 m, 2.5 m and 80 cm is carved out of the original rectangular prism. Calculate the capacity of the carved out prism.

c Calculate the new volume of the object.



25 Convert the following times into the time zones in brackets.

a 2:03 pm AEST (ACST)

b 5:41 am AWST (AEST)

c 9:00 pm UTC+10 (UTC+1)

d 8:31 am UTC−2 (UTC+5)

26 Determine the length of time elapsed when these times are in the same time zone.

a 0847 to 1423

b 9:45 pm to 1:10 am

27 Determine the length of time elapsed for these pairs of times.

a 6:38 pm AEST to 6:12 pm AWST

b 0555 UTC+1 to 2320 UTC+10

28 Calculate the mean of the following data set.

Score	Frequency
3	2
5	4
7	6
9	8

29 Students were surveyed on their preferred hobbies and schedules.

a Complete the following two-way table, representing the survey results.

	Reading books	Playing board games	Playing video games	Total
Weekday	25			65
Weekend			71	135
Total		30	105	

b If a student from the survey is selected at random, find the probability that they:

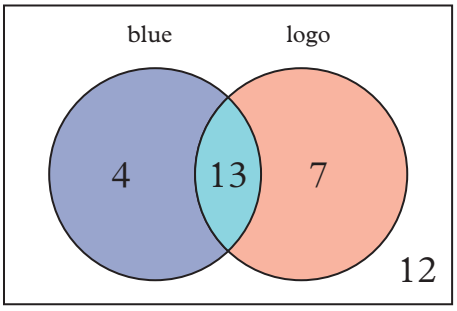
i prefer playing board games on the weekend

ii prefer playing board games or video games on a weekday

iii do not prefer reading books

iv prefer doing their hobby on the weekend, or prefer playing video games, but not both.

30 The Venn diagram on the right shows the number of counters in a bag that are blue or are marked with a logo. Calculate the relative frequency of the following.



a blue counters that are marked with a logo

b blue counters that are not marked with a logo

c blue counters or counters marked with a logo, or both

d blue counters or counters marked with a logo, but not both

e neither blue counters nor counters marked with a logo

31 For each of the following, determine:

i if it is a sample or a census

ii if it is fair or biased (if it is a sample)

iii what sampling method is being used (if it is a sample)

iv whether the sample could be used to predict population characteristics (if it is a sample).

a 1000 randomly selected students in Victoria were surveyed about their opinions of mathematics.

b A group of 10 friends throw discuses to determine the average distance they can throw a discus.

c 50 people at a shopping centre in Victoria on a weekday are selected to participate in a survey to determine the state average number of sick days people take.

32 A number of households were asked how many bathrooms they have. The data collected is shown below.

3 2 1 1 2 1 8 2 2 1.5 2 3 2.5 1 1.5 2

a Calculate the mean (to two decimal places), median, mode and range.

b Recalculate the mean, median, mode and range excluding the outlier. Which summary statistics change and which stay the same?

33 The Victorian government wants to know the approximate number of households with pet cats in the state. They need to choose a sampling method.

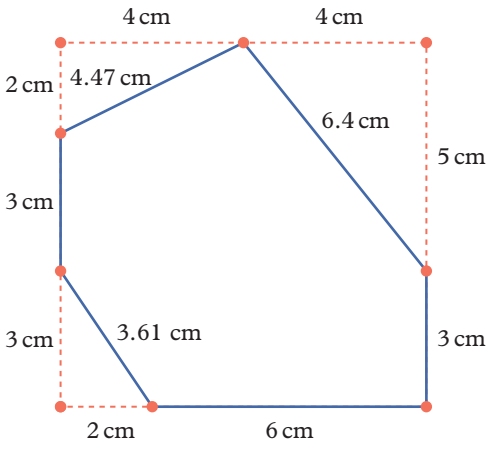
- **a** Why might they choose not to conduct a census?
- **b** Suggest one way they could use quota sampling.
- **c** They decide to use cluster sampling by taking a random sample of both rural and urban local government areas. Suggest one advantage and one disadvantage of this method.

34 Write a sentence to describe the following statistics.

- **a** survey of shoppers about how long they spent at the shops, mean = 1 h 20 m
- **b** survey about the number of board games people owned, median = 4
- **c** survey about people's favourite colour, mode = blue
- **d** sample of the length of a number of chess matches, range = 34 turns

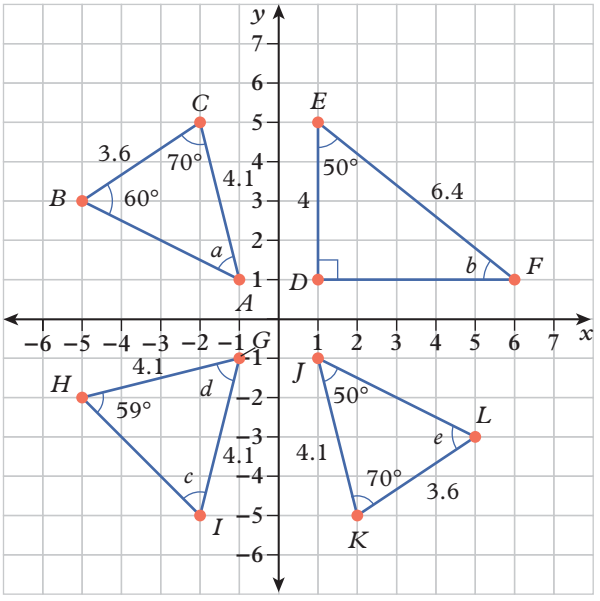
Mathematical modelling

1 Kala is constructing a prism, using cardboard, with an irregular hexagonal base. She plans to construct the base using right-angled triangles. Her plan is shown at right.



- **a** Use a ruler and protractor to construct the hexagonal base.
- **b** Calculate the area of the hexagonal base.
- **c** Kala initially wants the prism to be 8 cm tall.
  - **i** What is the total amount of cardboard Kala will need?
  - **ii** How much space will Kala's prism take up?
- **d** Kala instead decides to choose the height of the prism such that the value of its surface area and volume are equal. Let h be the height of Kala's prism.
  - **i** Write expressions, in terms of h, for the surface area and volume of the prism.
  - **ii** Determine the height of the prism, correct to two decimal places, if the value of the surface area and volume are equal.

2 Consider the triangles to the right.



- **a** Determine the value of all pronumerals.
- **b** Determine the mean, median, mode and range of the interior angles of the triangles.
- **c** What type of data are the angles in the triangles?
- **d** If one interior angle is selected at random from any of the triangles, what is the probability that it is acute?
- **e** Two of the triangles are congruent. Write the congruency statement and the test that explains why they are congruent.
- **f** What single transformation could be applied to one of the congruent triangles to create the other congruent triangle?
- **g** Which triangle has the largest area?

3 A survey is being conducted by Molly in Melbourne (UTC+10) and Lester in London (UTC+1). They want to see if people in Melbourne and London have similar or different preferences between going to the beach, camping or to the snow. Every day for four days both Molly and Lester survey people from their respective cities. The results are shown below.

Destination	Melbourne	London
Beach	21	14
Camping	17	8
Snow	12	18

M	L
38	19
25	12
37	23

M	L
34	60
21	22
25	68

M	L
14	44
12	30
9	46

a Convert the tables to percentage frequency, correct to the nearest percent.

b Write a short paragraph about whether the preferences of people in Melbourne and London are similar or different. Include references to the percentages from part **a**.

c Molly and Lester decide to organise a video call to discuss their results. Lester suggests 9 am London time. Determine the local time in Melbourne in 24-hour time for Molly.

d During their video call, they decide to check if the season influenced people's opinions by repeating the experiment in six months' time. The percentage frequencies from each day in the later surveys are shown below.

Destination	Melbourne	London
Beach	37%	32%
Camping	33%	24%
Snow	30%	44%

M	L
40%	22%
27%	25%
33%	53%

M	L
31%	30%
39%	20%
30%	50%

M	L
33%	33%
40%	22%
27%	45%

By referring to the tables, explain whether the change of season did or did not impact people's preferences.

4 Kate works at a toy store which specialises in crocheted children's toys. She earns $20 per hour. Each of her shifts lasts eight hours and she works five shifts a week.

a What is the base rate that Kate earns per week?

b Kate earns a bonus of $15 if she opens the store and a bonus of $20 if she is responsible for closing the store. She opens the store twice a week and closes the store three times a week.

How much does Kate earn per week including bonuses?

c The regional manager offers Kate a promotion where she can choose between earning an extra 15% per hour or increasing the store opening and closing bonuses by $5 each. The number of times she opens and closes the store would stay the same. Kate wants to use her knowledge of linear relationships to compare the amount she would earn in one week with each option.

i Write the equation of the linear relationship between Kate's pay, P_1, and hours worked, h, if she chooses the first option.

ii Write the equation of the linear relationship between Kate's pay, P_2, and hours worked, h, if she chooses the second option.

iii Sketch both linear relationships on the same set of axes for values of h between 0 and 40.

iv Based on your graphs, which option should Kate choose?

d Kate is offered a job by a rival toy company where she would earn $30 per hour but there would be no bonuses for opening and closing the store.

i Write the equation representing the linear relationship between Kate's potential pay, P_3, and hours worked, h.

ii Graph the linear relationship you found in part **i** on the same set of axes as the other two linear relationships for values of h between 0 and 40.

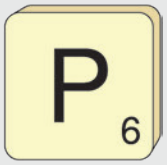
iv Assume Kate will continue to work the same number of hours each week. Will she earn more per week if she accepts the new job?

ANS p604

EXPLORATIONS 2

1 Foodle

Foodle is a word game in which players place letter tiles in a row to make the name of a food. A player's score is the sum of the numbers on the tiles that make up the word. Each letter of the alphabet scores a different number of points from 1 to 26. The letter P scores 6 points.



a The word PEA scores 10 points, PEAR scores 18 points, PEPPER scores 28 points and PAELLA scores 23 points. How many points is the letter L worth?

b The word PEACH scores 19 points and CELERY scores 27 points. How many points does CHERRY score?

c The word CARROT scores 44 points. How many points does POTATO score?

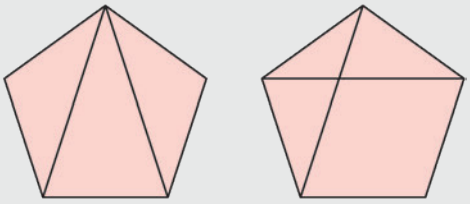
d The word GRAPE scores 22 points, GARLIC scores 31 points, ONION scores 57 points and CREAM scores 29 points. How many points does MELON score?

e Find the lowest possible score for COCONUT.

f The word OCTOPUS scores 89 points and BURGER scores 69 points. Find the lowest and highest possible scores for BRUSSELS SPROUT.

g Explain why the score for BEANS must be even.

2 Pentagon wedges



a What is the angle sum of any pentagon? Hint: What is the smallest number of triangles into which a pentagon can be divided?

b What is the angle at each corner of a regular pentagon?

c The diagram shows the two cases when a pair of diagonals is drawn on a regular pentagon. What special types of triangles and quadrilaterals are formed by the diagonals? Find the angles in each shape.

d Given an isosceles triangle, we call it *narrow* if it has the same angles as the acute-angled triangles in the diagrams above. We call it *wide* if it has the same angles as the obtuse-angled triangles.

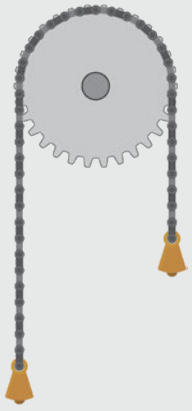
i Show how to divide a narrow triangle into two smaller triangles, one of which is narrow and the other wide.

ii Show that a regular pentagon can be divided into 33 narrow triangles and 67 wide triangles. Can you generalise this result to other pairs of numbers?

iii There are four types of isosceles triangle that can be divided into two smaller isosceles triangles. Two of these are the narrow and wide triangles described above. Find the other two types.

3 Bicycle chimes

Max pulls apart an old bicycle and builds a machine using the bicycle chain and one of the sprocket wheels. He breaks the chain, hangs it over the wheel and attaches a bell to each end, as shown. He then attaches a motor that turns the wheel clockwise for exactly one hour and then anti-clockwise for exactly one hour, both at the same constant rate.



Max positions the centre of the wheel on the wall so that at 12 noon the bell on the left hits the floor and rings. The wheel turns clockwise for one hour so that at exactly 1 pm the right bell hits the floor and rings. At 2 pm the left bell rings again, and this process continues alternately every hour.

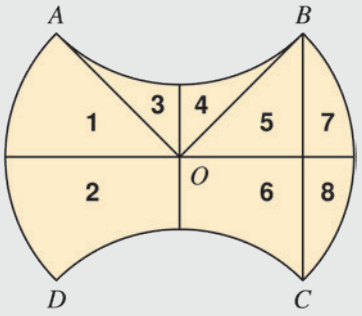
a At 2:10 pm the difference in height between the two bells is 80 cm. How far off the floor is the left bell at 3 pm?

b The diameter of the sprocket wheel is 10 cm. To two decimal places, how many revolutions does the wheel make per hour?

c As the years go by, the sprockets on the wheel wear out and the chain starts to slip. Max replaces the old wheel with a 14 cm-diameter sprocket wheel from the old bicycle. The motor still turns the wheel clockwise and anti-clockwise at the same constant rate as before. To the nearest millimetre, by how much does Max need to raise or lower the centre of the new wheel so that the bells still ring every hour?

4 Dogbones

In the dogbone shown, points A, B, C and D are the vertices of a square with centre O. The four circular arcs which join the vertices all have radius 4 units.



a Find the exact area of each numbered piece, in square units. Hint: Find the areas of shapes 1 and 5 first, then shapes 7 and 4.

b Show how to rearrange the 8 pieces to make the following shapes:

i two identical rectangles

ii a right-angled isosceles triangle

iii a symmetrical hexagon

5 Freaky frequencies

Sebastian is practising converting lists of data into frequency distribution tables. For example, starting with the list {1, 2, 2, 3, 3, 2, 4, 1}, he puts all possible scores in ascending order in the first column, namely 1, 2, 3 and 4. He then records the frequency of each score (that is, the number of times it occurs in the list) in the second column, as shown.

Score	Freq.
1	2
2	3
3	2
4	1

Sebastian notices something unusual about this example: reading the entries in the table from left to right then top to bottom produces the list {1, 2, 2, 3, 3, 2, 4, 1}, which exactly matches the one he started with! He calls such a collection of data a *freaky-frequency list*.

a Find, with reasons, the only freaky-frequency list of length 2.

b Show that there are exactly two freaky-frequency lists that involve only the numbers 1, 2, 3 and 4.

c Given any list of data which is *not* a freaky-frequency list, we can still read off the entries in its table to produce another list. For example, starting with {1}, the table's only row is $\boxed{1}\,\boxed{1}$ which produces the list {1, 1}. This in turn has only the row $\boxed{1}\,\boxed{2}$ in its table, producing the list {1, 2}. Continuing in this way, we get the following sequence of lists:

{1} → {1,1} → {1,2} → {1,1,2,1} → {1,3,2,1} → ...

i Find the 100th list in the above sequence.

ii Find the 100th list in the sequence which starts with the list {1, 2, 3, 4, 5}.

iii The fourth list in a sequence is {1, 1, 2, 3, 3, 1, 5, 1}. Find the length of the first list.

6 Robot designs

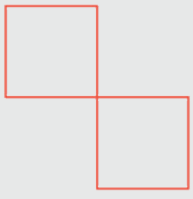
My drawing robot accepts the following commands:

- Fx : move forward x units.
- Lx : turn x degrees to the left.
- Rn [...] : repeat the instructions in the brackets n times.

For example, the instructions R3 [F30 L120] tell the robot to draw an equilateral triangle with a side length of 30 units.

I tried to program my robot to draw the three designs shown below, but each set of instructions I came up with had a single mistake. Find and correct each mistake.

a R2 [R4 [F50 L90] L90]



b R3 [R3 [F50 L120] L60]

c R6 [R6 [F50 L60] L120]

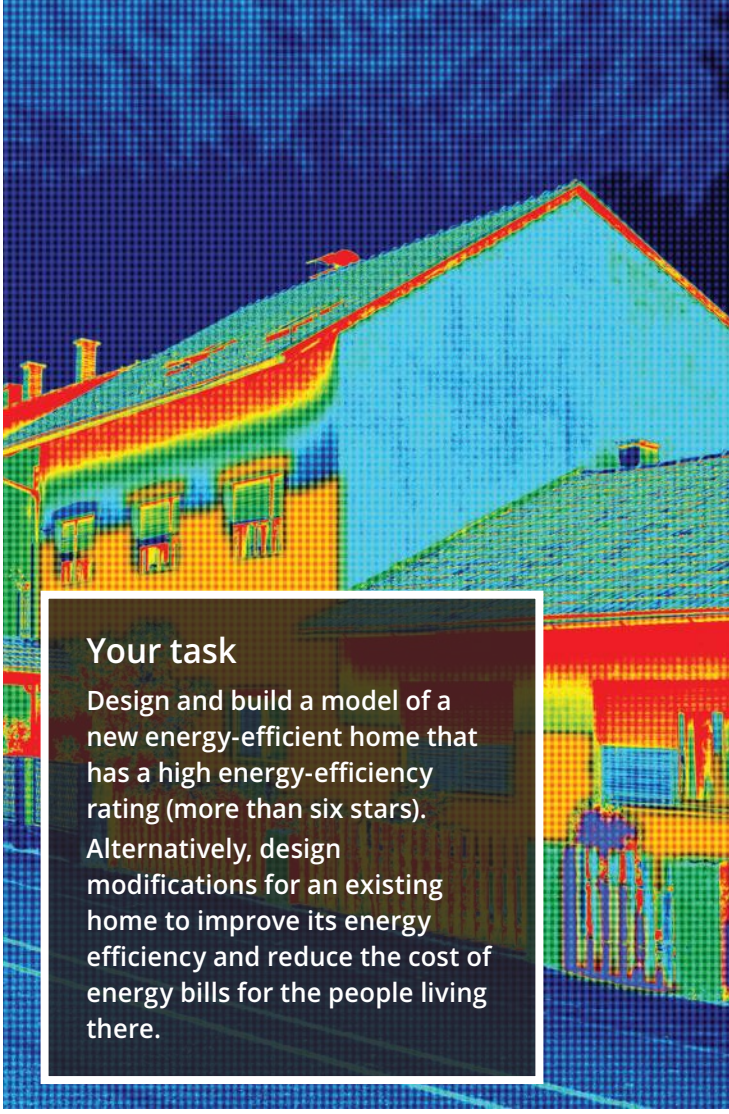
[STEAM project 1]

How can we build more energy-efficient homes so that we live more sustainably?

Have you ever considered how much energy it takes to heat or cool your home? A study by the Australian Government found that homes contribute 10 per cent of Australia's total carbon emissions. Reducing the amount of energy needed to heat or cool a home will not only save people money on their gas and electricity bills, it will make housing more sustainable.

NatHERS

The Nationwide House Energy Rating Scheme (NatHERS) is a rating system that is used across Australia to identify the energy efficiency of new homes. It gives a star rating out of ten for thermal performance (where ten stars is the most energy efficient). Computer modelling software assesses the design to estimate the amount of energy needed to heat or cool the home. The assessment is based on the local climate, the orientation of the home and materials that will be used in construction. In much of Australia, new home designs must reach a minimum of seven stars to be approved for construction. There are many ways to improve the energy efficiency of your home. For example, if you insulate the walls, floors and roof, and use awnings (covers that extend over windows or doors), you can reduce the amount of energy needed to cool the home in summer. Or in cooler climates, if you design the home so that living areas are on the northern side, you can maximise the sunlight that is available to heat the home in winter.



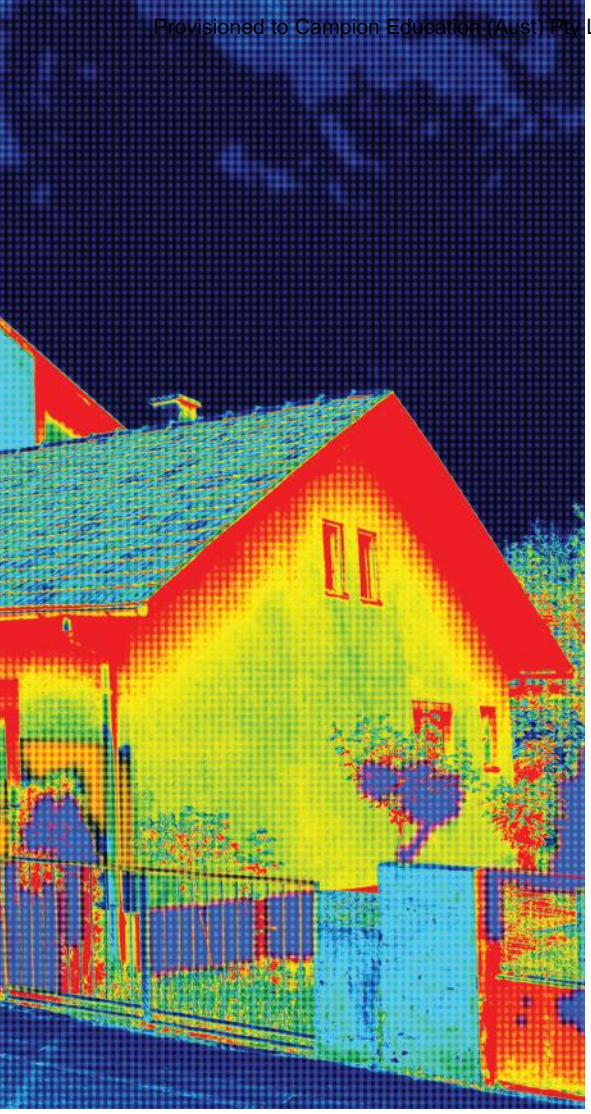
Your task

Design and build a model of a new energy-efficient home that has a high energy-efficiency rating (more than six stars). Alternatively, design modifications for an existing home to improve its energy efficiency and reduce the cost of energy bills for the people living there.

Figure 1 Infrared imaging shows warmer temperatures as red and cooler temperatures as blue. A building with insulation will lose less thermal energy to the outside environment in winter and will gain less thermal energy from the outside environment in summer.

Retrofitting homes

Not everyone wants to or can afford to build a new home. People who rent sometimes have little choice when it comes to the energy efficiency of their home, but this doesn't mean that they need to have large gas and electricity bills. There are things that they can do to reduce the movement of thermal energy in summer or winter. Covering the floor with rugs, preventing heat from moving in or out of windows, and controlling windows to manage airflow are all ways to make a home more energy-efficient and liveable.



HUMANITIES

In Geography this year, you will learn about the importance of sustainable housing in Australia's urban centres and major cities. You will explore how the forecast growth of Australia's cities could impact their liveability and the environmental issues resulting from urbanisation.

In Economics and Business, you will study the housing market, the interaction between businesses and consumers in meeting consumer demands and when the government intervenes in markets.

To complete this task successfully, you will need to study seasonal weather patterns at the location of your energy-efficient home, to improve energy use.

MATHS

In Maths this year, you will learn how to determine the area and volume of different shapes, using and converting between appropriate units. You will also learn skills for dealing with percentage changes in financial contexts. These skills will help you to quantify the costs and benefits of design features and predict the popularity of market incentives. You will perform calculations with and without digital technology.

To complete this task successfully, you will need to perform calculations for your model home, and then scale up to estimate the potential benefits of your design at a national level.

You will find help for applying these maths skills in **Lessons 3.2** 'Calculating percentages', **3.3** 'Mark-ups and discounts', **8.3** 'Area of quadrilaterals' and **8.4** 'Area of a circle' of *Oxford Maths 8 Victorian Curriculum 2E*.

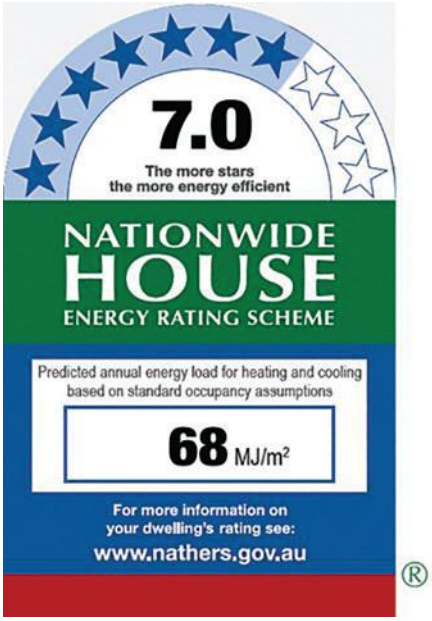


Figure 2 Energy rating schemes help consumers to understand the energy efficiency of products or appliances. The more stars, the more energy efficient the product or appliance is.

SCIENCE

In Science this year, you will learn about how energy can be transferred between objects and transformed (such as from the Sun's light energy to electrical energy). You will also examine earthquake resistant materials used in construction.

To complete this task successfully, you will need to identify how each element of a home's design can affect its heating and cooling needs, and how construction materials can affect a home's stability.

You will also need to identify the elements that can be changed in a new home design, and compare these to the elements that can be changed in an established home.

[STEAM project 1]

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for? How big is their family and home?
- What factors might affect the liveability of the home?
- What do they need? What do they not need?
- What does it feel like to face these limitations on liveability? What words would you use to describe how it would feel to face these limitations?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your home modifications, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1. Describe the orientation of the land where the home will be built or modified (include the location of sunrise and sunset, the angle of sun in the middle of summer and winter and any existing shadows).
2. Describe any existing energy-efficient features of the current home or design.
3. Describe how you will measure any improvement in energy-efficient features of the home.

Ideate

Once you know who you're designing for, and what the criteria are, it's time to get creative!

- Outline the criteria or requirements your design must fulfil (for example, improving heating or cooling of the home).
- Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Draw each individual design.

Include in the individual designs:

a labels for each part of the design

b the materials you will use for the home's construction

c a description of how the modifications will improve the energy star rating of the home

d an estimation of the cost of applying the modifications.

Present your design to your group.

Build the prototype

Choose and build two or three model homes for your group design (scale: 2 cm = 1 m).

Use the following questions as a guideline for your prototype.

- What materials will you need to build your model home?
- What skills will you need to build your model home?
- How will you test the effectiveness of your model home? (What will you compare it to?)
- How will you identify each energy-efficient feature of your model home?
- How will you collect data that supports your claims about energy efficiency?

Test

Use the scientific method to design and test the energy efficiency of your model home. You will test more than one design so that you can compare them, so you will need to control your variables between tests.

What criteria will you use to determine the success of your model home?

Conduct your tests and record your results in an appropriate table.

Communicate

Present your home design as though you are trying to get your peers to invest in it.

In your presentation, you will need to:

- construct a labelled diagram of your model home in its orientation, including the location of sunrise and sunset, the angle of sun in the middle of summer and winter, and any existing shadows
- describe the key energy-efficient features of your model home
- explain how each energy-efficient feature affects the liveability of your model home
- explain the principles that support your design (the importance of energy efficiency and how the existing landscape affected the design of the home)
- estimate the cost of implementing your design
- explain and quantify the benefits on a national scale if all new homes were to include your design features.

Online resources:

Student guidebook
This helpful booklet will guide you step-by-step through the project.

What is the design cycle?
This video will help you to better understand each phase of the design cycle.

How to manage a project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

[STEAM project 2]

How can we use technology so that the impact of natural disasters is reduced?

Your task

Develop an innovative technology or strategy that will reduce the impact of natural disasters in your community. The technology or strategy should be useful and easy to use and address one (or more) of the steps of disaster management.

Figure 1 Hundreds of people (and animals) huddled on Malua Bay beach in NSW for almost 24 hours as bushfires tore through communities in the nearby Batemans Bay area.

Across the world, climate patterns are changing and weather events are becoming more extreme. In 2020, twenty per cent of Australia's forests were burnt, a record-breaking thirty named hurricanes developed over the Atlantic Ocean and an extreme monsoon season in Asia caused flooding across a quarter of Bangladesh. Australia is particularly prone to natural disasters. Bushfires, flooding, drought periods and cyclones dramatically affect people's lives and alter natural landscapes.

Natural disasters can be devastating for communities affected by them. Recovering from natural disasters also comes with huge economic costs. This can sometimes lead to a harmful cycle in which a nation gets stuck between experiencing disaster and responding to disaster.

Disaster management

Climate action is one of the 17 Sustainable Development Goals agreed to in 2015 by world leaders. One of the identified targets in combating climate change is to strengthen the resilience and adaptive capacity of all countries to natural disasters and hazards related to climate.

It is essential that communities prepare for disasters so as to reduce their impact. The United Nations Office for Disaster Risk Reduction (UNDRR) recommends using technology as part of disaster management. For example:

- warning systems in Japan that trigger emergency brakes in bullet trains if earthquakes are detected
- systems in sub-Saharan Africa that monitor rainfall data and analyse trends to forecast and build resilience to drought
- atlases in China that record the risk of natural disasters, using location data from spatial technologies such as GPS.

After the 2019–20 Australian bushfires, it was argued that satellite technology, drones and mobile phone apps would have been extremely helpful for fighting the fires and communicating with the people affected.

Disaster management should include four steps:

- **prevention** – reducing hazards before a disaster takes place, so its impact is not as severe (e.g. building schools that are earthquake resistant)
- **preparation** – training people so they know how to act when a disaster happens (e.g. running evacuation drills)
- **response** – taking action during a disaster (e.g. emergency crews and volunteers taking on emergency operations)
- **recovery** – taking action to help people rebuild their lives (e.g. restoring services in a community).

Disaster management is an ongoing process that can occur like a cycle between the phases of prevention, preparation, response and recovery.

Figure 2 Floodwaters near Sydney

HUMANITIES

In Geography this year, you will learn about landscapes and landforms and how they can be degraded by both human and natural causes. You will study a geomorphic hazard, its impacts on a place and various ways of responding to it.

In History, you might also study how past societies have dealt with disasters and managed their responses; for example, how Rapa Nui (Easter Island) inhabitants adapted to life without trees, how Shogunate Japan developed policies to sustain its forests, or how an unstable climate (including drought and heavy monsoons) affected the Khmer Empire.

To complete this task successfully, you will need to investigate a potential natural hazard in your local area. You will research any current disaster-response plans related to managing such an event and gain an understanding of how various people and businesses in the community would be impacted.

MATHS

In Maths this year, you will consolidate and extend your skills in describing and interpreting. This will include creating and analysing plots of non-linear data, investigating the use of sampling methods and broadening your understanding of measures of spread. You will also perform calculations with percentage changes in financial contexts. You will analyse and represent data, both with and without digital technology.

To complete this task successfully, you will need to weigh up the costs of your disaster-management technology or strategy against its potential benefits, including by estimating the likelihood of natural disasters and the severity of their effects.

You will find help for applying these maths skills and statistics in **Lessons 3.2** 'Calculating percentages', **3.3** 'Mark-ups and discounts', **6.5** 'Plotting linear relationships' and **10.1** 'Collecting data and sampling methods' of *Oxford Maths 8 Victorian Curriculum 2E*.

SCIENCE

In Science this year, you will learn about how the energy of the Earth over long periods of time generates forces that can melt rocks, produce volcanoes and make diamonds. You will also learn how the kinetic energy in the air and waves can cause damage to the surrounding environments during cyclones and tsunamis. The Australian bush also contains large amounts of chemical energy that is transformed into thermal energy during the summer fire season.

To complete this task successfully, you will need to consider how energy is transferred and transformed during natural disasters. This understanding will allow you to predict and potentially reduce the impact of the disaster in your local community.

[STEAM project 2]

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What natural disaster could they face?
- How often could this disaster occur, and on what scale?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would you use to describe these feelings?
- What could the cost of such a disaster be in terms of lives lost, income lost, damage to private property and public infrastructure, and environmental impact?

To answer these questions, you may need to investigate using different resources or even conduct interviews or surveys.

Define

Before you start to design your innovative strategy or technology, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1. Define the term 'innovation'. Describe how much a strategy needs to differ from current practice to be considered innovative.
2. Describe how the natural disaster will affect the people in the community.
3. Describe how you could measure whether your solution will reduce the impact of the natural disaster.
4. Explain how you could determine what cost (price) should be acceptable for implementing your solution.

Ideate

Once you know who you're designing for and you know what the criteria are, it's time to get creative!

- Outline the criteria or requirements your designed strategy or technology must fulfil (for example, how many people will be helped, how much they will be helped, how long they will need the help).
- Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Draw each design strategy or technological idea. Label each stage of the strategy or part of the technology and explain how it will be used by the community.

Include in the individual strategy or technology:

a the timeline for the activation of the strategy or technology (i.e. how long after the disaster will the idea be ready to be used?)

b the number of people and materials that will be needed for the strategy or technology to reduce the impact of the natural disaster

c a description of how the community will benefit from the strategy or technology.

Present your strategy or technology to your group.

Build the prototype

Choose and build two or three versions of the prototype strategy or technology for your group design.

Use the following questions as a guideline for your prototype.

- What equipment do you have access to?
- What skills do you have or will you need, to make your prototype strategy or technology?
- How could you model your strategy or technology if equipment is not available?
- How could you test the effectiveness of the prototype strategy or technology with the community?

Test

Use the scientific method to design an experiment that will test the effectiveness of your prototype strategy or technology. Alternatively, conduct a survey of the community to determine their opinion of the usefulness of the prototype.

Conduct your tests or survey your community, and record your results in an appropriate table.

Consider how you could use the results of the experiment or survey to modify your design.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your strategy or technology.

In your presentation, you will need to:

- describe how the natural disaster will affect your community
- describe your prototype strategy or technology
- describe how the prototype will reduce the impact of the natural disaster on the community
- describe the materials, people and money that will be required to have the required effect on the natural disaster.

Online resources:

Student guidebook
This helpful booklet will guide you step-by-step through the project.

What is the design cycle?
This video will help you to better understand each phase in the design cycle.

How to manage your project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to define a problem
This 'how-to' video will help you to narrow your ideas down and define a specific problem.

Answers

MODULE 1 Integers

EX p6 1.1 Rounding and estimating

1 **a** 400 **b** 500 **c** 500 **d** 500
e 400 **f** 400

2 **a** 7000 **b** 6000 **c** 6000 **d** 7000
e 7000 **f** 6000

3 **a** **i** 480 **ii** 6380 **iii** 56 030
iv 738 490 **v** 8080 **vi** 904 510
b **i** 500 **ii** 6400 **iii** 56 000
iv 738 500 **v** 8100 **vi** 904 500

4 **a** **i** 36 000 **ii** 8000 **iii** 184 000
iv 51 000 **v** 6000 **vi** 1000
b **i** 36 400 **ii** 7500 **iii** 183 900
iv 50 700 **v** 6100 **vi** 800

5 **a** 80 **b** 40 **c** 80 **d** 300
e 200 **f** 600 **g** 900 **h** 2000
i 7000 **j** 20 000

6 **a** 90 **b** 100 **c** 1000
d 90 000 **e** 9000 **f** 1000

7 Sample answers:
a 37, 42, 35 **b** 692, 705, 744
c 2995, 3125, 3451 **d** 52 168, 46 854, 50 369
e 795 002, 755 036, 827 039
f 1 758 364, 2 498 567, 2 215 000

8 **a** 1600 **b** 290 **c** 600
d 20 **e** 20 000 **f** 3800
g 40 **h** 5500 **i** 2000
j 2 100 000 **k** 11 000 **l** 15 000

9 **a** 1200 **b** 50 **c** 1500
d 40 **e** 12 000 **f** 6200
g 125 **h** 50

10 **a** 10 **b** 1 800 000 **c** 6200 **d** 7000
e 560 000 000 **f** 18 000 **g** 150 **h** 19 200

11 **a** 40 000 **b** 35 800 **c** 36 000
d 35 780 **e** 40 000

12 **a** 20 000 000 km^2 **b** 17 098 200 km^2
c 17 098 000 km^2 **d** 17 100 000 km^2
e 17 100 000 km^2 **f** 17 000 000 km^2

13 **a** \$300
b overestimation

14 Tina is correct. Since the leading digit is 9 and the second digit is 6, the first digit increases by one and the digits that follow are replaced with zero. Increasing 9 by one gives 10 so there is a total of five digits in the answer.

15 **a** 30 **b** \$1800 **c** no
d 20 months

16 **a** 400 **b** \$2800
c \$2695; estimate is \$105 more

17 **a** 40
b The estimate of 40 would be lower than the actual number of boxes as the approximate price (\$50) is higher than the actual price (\$47). There would be money left over.

18 **a** Australia: 8 000 000 km^2
Indonesia: 2 000 000 km^2
b **i** 6 000 000 km^2
ii approximately 4 times the area of Indonesia
c **i** 5 787 464 km^2
ii 4.0387 times the area of Indonesia

19 **a** 48 000 **b** 52 196

20 **a** WA: 3 000 000 km^2, Qld: 2 000 000 km^2, NT: 1 000 000 km^2, SA: 1 000 000 km^2, NSW: 800 000 km^2, Vic: 200 000 km^2, Tas: 70 000 km^2, ACT: 2000 km^2
b 8 072 000 km^2
c WA: 2 530 000 km^2, Qld: 1 731 000 km^2, NT: 1 349 000 km^2, SA: 983 000 km^2, NSW: 801 000 km^2, Vic: 227 000 km^2, Tas: 68 000 km^2, ACT: 2000 km^2
d 7 691 000 km^2
e Values rounded to the nearest thousand produce an estimate much closer to the actual values.

21 **a** $\frac{8}{30} = \frac{4}{15} \approx 0.27$ km^2 per person
b New Zealand has approximately $\frac{3}{50} \approx 0.06$ km^2 per person. So, Australia has approximately 4.5 times more area per person than New Zealand.

22 Record yourself speaking for a minute and count the number of words you say. Consider how many minutes of the last hour you have spoken for. Multiplying by 12, is this a good estimate of your day?

EX p12 1.2 Adding and subtracting whole numbers

1 **a** **i** 97 **ii** 161 **iii** 979 **iv** 10897
b **i** 719 **ii** 819 **iii** 584 **iv** 3591
c **i** 123 **ii** 528 **iii** 1123 **iv** 8104
d **i** 177 **ii** 1139 **iii** 8824 **iv** 6570

2 **a** 958 **b** 692 **c** 1219 **d** 1502

3 **a** 5241 **b** 10 475 **c** 18 872 **d** 122 530

4 **a** 1223 **b** 1857 **c** 4141 **d** 4918
e 7366 **f** 20 350

5 **a** 91 852 **b** 128 077 **c** 24 921 **d** 2460

6 **a** **i** 63 **ii** 87 **iii** 94 **iv** 693
b **i** 33 **ii** 695 **iii** 256 **iv** 366
c **i** 60 **ii** 5190 **iii** 210 **iv** 112
d **i** 62 **ii** 890 **iii** 523 **iv** 1136

7 **a** 39 **b** 276 **c** 1380 **d** 3378
8 **a** 29 **b** 3489 **c** 4418 **d** 6320
9 **a** 7166 **b** 998 **c** 74 949 **d** 43 642
10 **a** 476 **b** 5074 **c** 38 150 **d** 8473
11 2514
12 **a** 1 586 191 **b** 496 059 **c** 6 530 941
13 **a** 81 593 **b** 53 248
14 **a** 1390 km **b** $1088 **c** $734 **d** $178
15 **a** 6199 **b** 4637
16 **a** 60 km **b** 125 m
17 85 pairs of shoes
18 **a** 5 **b** 2000
19 S=4, E=5, D=1, W=6, A=8, T=3, R=2, F=7, U=9, I=0
S=5, E=8, D=9, W=3, A=6, T=1, R=2, F=4, U=0, I=7
S=6, E=8, D=9, W=3, A=5, T=1, R=2, F=4, U=0, I=7
S=7, E=1, D=3, W=4, A=8, T=9, R=6, F=5, U=0, I=2
S=8, E=1, D=3, W=4, A=8, T=9, R=6, F=5, U=0, I=2
S=8, E=5, D=1, W=6, A=4, T=3, R=2, F=7, U=9, I=0

EX p18

1.3 Multiplying and dividing whole numbers

1 **a** 126 **b** 612 **c** 1421 **d** 2745
e 1666 **f** 8217 **g** 2280 **h** 779
i 6262 **j** 29 029 **k** 1067 **l** 14 985
2 **a** 408 **b** 630 **c** 444 **d** 5280
e 600 **f** 92 100 **g** 700 **h** 29 000
i 186 000
3 **a** 630 **b** 9600 **c** 7700 **d** 6300
e 320 400 **f** 8 295 000
g 24 867 000 **h** 83 700 000
4 **a** 720 000 **b** 448 000 000
c 9 600 000 **d** 375 000 000
5 **a** 44 764 **b** 24 402 **c** 105 754 **d** 147 936
e 665 640 **f** 4 903 600
6 **a** 332 820 **b** 226 044
c 316 448 **d** 1 726 202
e 1 509 024 **f** 29 792 690
7 **a** **i** 800 000 **ii** 270 000 **iii** 60 000 000
iv 640 000 000
b **i** 803 890 **ii** 287 013 **iii** 59 804 375
iv 654 565 110
8 **a** 57 remainder 7 **b** 58
c 22 remainder 6 **d** 75
e 76 remainder 5 **f** 1420 remainder 2
9 **a** 1456 **b** 1399 remainder 5
c 1403 remainder 1 **d** 16 855 remainder 1
e 6254 remainder 3 **f** 212 remainder 5
10 **a** 23 remainder 5 **b** 81 remainder 5
c 35 remainder 1 **d** 163
11 **a** 92 **b** 23
12 **a** **i** 9 **ii** 9 **b** no
c **i** 3
ii −3; subtraction does not obey the commutative law.
d **i** 18
ii 18; multiplication obeys the commutative law.
e **i** 2
ii 0.5; division does not obey the commutative law.
13 **a** **i** 18 **ii** 18 **iii** 18
b no
c Multiplication obeys the associative law, but subtraction and division do not.
For example, multiplication:
$5 \times 4 \times 2 = 4 \times 5 \times 2 = 2 \times 5 \times 4 = 40$;
subtraction: $(7 - 2) - 3 = 2$ and $7 - (2 - 3) = 8$ (does not obey the associative law);
division: $(8 \div 4) \div 2 = 1$ and $8 \div (4 \div 2) = 4$ (does not obey the associative law).
14 **a** **i** 54 + 16, since the result (70) is a multiple of 10 and easier to add
ii 188
b **i** 25 × 4, since the result (100) is easier to multiply by
ii 8700
c As order is not important, any two of the three numbers can be added or multiplied first. If this results in a number that is a multiple of 10, it can make the final operation easier.
15 **a** **i** 44 **ii** 20 + 24 = 44; the answers are the same
b **i** 48 **ii** 60 − 12 = 48; the answers are the same
c Each number inside the brackets is multiplied by the number at the front of the brackets.
d Sample answer:
$10 \times (5 + 4) = 10 \times 5 + 10 \times 4 = 50 + 40 = 90$
$10 \times (5 + 4) = 10 \times 9 = 90$
16 **a** **i** 320 **ii** 64 **iii** 384
b **i** 45 000 **ii** 90 **iii** 44 910
c Write one of the numbers to be multiplied as either a sum or difference that involves a multiple of 10 or power of 10 and a single-digit number.
17 **a** $12 **b** 3 **c** **i** $10 **ii** $400
18 **a** 65 **b** 11 **c** $195
19 **a** 402 L **b** 9 **c** $1206
20 23
21 **a** 60 g **b** 32

EX p24

1.4 Multiples, factors, exponents and roots

1 **a** 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
b 18, 36, 54, 72, 90
c 18

2 **a** 24 **b** 36 **c** 150 **d** 60
e 140 **f** 45

3 **a** 210 **b** 315 **c** 72 **d** 120
e 360 **f** 54

4 **a** 1, 3, 5, 9, 15, 45
b 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
c 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
d 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
e 1, 2, 4, 5, 10, 20, 25, 50, 100
f 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

5 **a** 18 **b** 3 **c** 6

6 **a** 14 **b** 12 **c** 9 **d** 4
e 120 **f** 25

7 **a** LCM = 360, HCF = 3
b LCM = 144, HCF = 4
c LCM = 60, HCF = 1
d LCM = 924, HCF = 1

8 **a** 9, 27 **b** 4, 8 **c** 100, 1000
d 49, 343 **e** 36, 216 **f** 144, 1728
g 64, 512 **h** 16, 64 **i** 1, 1
j 81, 729 **k** 121, 1331 **l** 196, 2744

9 **a** 3 **b** 2 **c** 5 **d** 8
e 9 **f** 6 **g** 10 **h** 7
i 8 **j** 13 **k** 14 **l** 15

10 **a** 24 **b** 33 **c** 49 000 **d** 16
e 512 **f** 100 000

11 **a** 972 **b** 36 **c** 6 **d** 32
e 256 **f** 900

12 60

13 **a** 120 **b** 8

14 90

15 24

16 8

17 Pontiac, Desiree, Sebago

18 **a** **i** 1 **ii** 121 **iii** 12 321 **iv** 1 234 321
b Sample answer: The resulting number displays digits in a pattern with 1 as the first digit, 2 as the second digit, 3 as the third digit, etc, increasing by one each time until it reaches a value that is the same as the number of digits in the base. The digits that follow then decrease by one with the number finishing at 1. For example, the base of 111^2 contains three digits so the first three digits in the number are 123, followed by 21, giving 12 321.
i 123 454 321 **ii** 12 345 654 321
iii 1 234 567 654 321 **iv** 123 456 787 654 321

19 500

EX p26

Checkpoint

1 **a** 600 **b** 900 **c** 7900 **d** 2800
e 45 600 **f** 76 600

2 **a** 1200 **b** 40 **c** 2800 **d** 30
e 15 000 **f** 9800 **g** 100 **h** 500

3 **a** 162 **b** 1577 **c** 12 158 **d** 7058
e 6717 **f** 15 368

4 **a** 18 **b** 486 **c** 89 **d** 5655
e 8369 **f** 3826

5 **a** 321 **b** 577 **c** 6471 **d** 108 206

6 **a** 33 024 **b** 64 296 **c** 354 198 **d** 768 746
e 594 336 **f** 4 207 980

7 **a** 98 remainder 3 **b** 92 remainder 2
c 1665 remainder 3 **d** 8488 remainder 6
e 6212 remainder 9 **f** 108 964 remainder 5

8 **a** 42 **b** 36 **c** 60 **d** 90
e 72 **f** 108

9 **a** 6 **b** 9 **c** 15 **d** 5
e 150 **f** 144

10 **a** 100 **b** 9 **c** 144 **d** 8
e 148 **f** 253

EX p29

1.5 Integers

1 **a** negative integers: −1, −2, −3, −4, −5
b positive integers: 1, 2, 3, 4, 5
c 0

2 **a** 9 **b** 5 **c** 1 **d** 5
e 0 **f** −4

3 **a** 5 **b** 0 **c** −2 **d** −5
e −1 **f** −9

4 **a** 0 **b** 24 **c** 14 **d** 125
e −25 **f** 0

5 **a** −70 **b** −21 **c** −68 **d** −50
e −354 **f** −450

6 **a** $-3 < 2$ **b** $-8 < -4$ **c** $0 > -1$
d $9 > -9$ **e** $-3 > -5$ **f** $-7 < -2$
g $4 > -6$ **h** $5 > -12$ **i** $6 > -4$
j $-18 < -8$ **k** $-6 > -7$ **l** $-8 < 0$

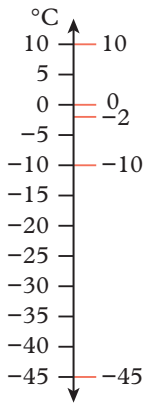
7 **a** −20, −12, −11, −7, 8, 10, 14
b −88, −48, −28, −18, −8, 8, 68
c −41, −33, −19, 0, 6, 29, 42
d −140, −126, −82, 3, 73, 104, 145

8 **a** 71, 4, −9, −10, −15, −27
b 10, 5, −1, −2, −5, −11
c 21, 5, 0, −9, −14, −19
d 71, 51, 12, −11, −62, −71, −72

9 −6, −5, −4, −3, −2, −1, 0, 1, 2, 3

10 **a** −3 **b** 2174 **c** −18 **d** −408
e 5895 **f** 500 **g** −46 **h** −40

11 a



b higher

c −45°C, −10°C, −2°C, 0°C, 10°C

d warmer

12 a +86 **b** $14 **c** −14

13 a long sighted **b** short sighted

c Elle has weaker vision.

d normal vision; that is, neither long nor short sighted

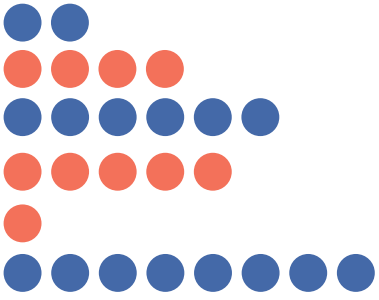
14 a i $34 **ii** $51

b i $13 **ii** $35

c Account is overdrawn; the person owes money to the bank.

15 a three

b i



ii

iii

iv

v

vi

c The reference point for opposite integers is always zero as both integers are the same number of units away from 0. Moving forward to the positive integer and then moving back as indicated by the opposite integer will always result in zero. When combined, opposite integers 'cancel' each other to result in zero. This can be modelled by equal numbers of blue and red dots.

d Two blue dots and two red dots produce two zero pairs; the remaining three red dots represent the integer −3.

e i 5 **ii** −4 **iii** −4

16 −9, −8, −7, −6, −5

EX p34

1.6 Adding and subtracting integers

1 a C **b** A **c** B

2 a −15 **b** −9 **c** 9

3 a negative **b** positive **c** negative

d zero **e** positive **f** positive

g negative **h** negative **i** negative

j positive

4 a 3 **b** 6 **c** 5 **d** −8

e −4 **f** −3 **g** −6 **h** 5

i −4 **j** −11

5 a −3 − 7 **b** 1 + 6 **c** −4 − 5

d 2 + 4 **e** 5 − 9 **f** −6 + 8

6 a 3 **b** 6 **c** −4 **d** 0

e −1 **f** 5 **g** 1 **h** −5

i 9 **j** 0

7 a

+	−5	−3	0	1	4
−3	−8	−6	−3	−2	1
−2	−7	−5	−2	−1	2
−1	−6	−4	−1	0	3
3	−2	0	3	4	7
6	1	3	6	7	10

b

+	−10	−14	−7	13	19
11	1	−3	4	24	30
14	4	0	7	27	33
−10	−20	−24	−17	3	9
−22	−32	−36	−29	−9	−3
−18	−28	−32	−25	−5	1

8 a 10 **b** −4 **c** −3

d 12 **e** −14 **f** −31

9 a 15 **b** −10 **c** −620 **d** 850

10 four levels below ground; 15 − 19 = −4

11 first level below ground; 3 − 9 + 5 = −1

12 51°C; 33 − (−18) = 51

13 a −60 **b** −60 + (+35) **c** $25

14 a A: 50, B: −2, C: 32

b i 50 − (−2) = 50 + 2

ii 32 − (−2) = 32 + 2

iii 50 − (+32) = 50 − 32

c i 52 m **ii** 34 m **iii** 18 m

d 52 m; it shows the greatest distance the bungee rope extends to before rebounding a lesser distance each time after.

15 a −900 − (+15) = −915; 915 m down

b −2500 − (−900) = −1600; 1600 m down

c 15 − (−2500) = +2515; 2515 m up

d 9 m above the sea level

16 a 45 **b** −30 **c** 1261 **d** −367

e −1020 **f** −3260

17 a 33 **b** 77 **c** −125 **d** −40

18 a $63; (−247) − (−310) = 63

b Initially, account is overdrawn by $310. During the month, $63 is added to the account and the overdrawn amount is reduced from $310 to $247.

1.7 Multiplying and dividing integers

EX p38

1 **a** 5×9 **b** $6 \times (-7)$
c -10×6 **d** $-4 \times (-3)$

2 **a** -12 **b** 35 **c** -12 **d** 27 **e** -8 **f** -24
g 72 **h** 20 **i** -44 **j** -7 **k** 8 **l** 45

3 **a** -5 **b** 4 **c** 6 **d** -7 **e** 17 **f** -8
g -7 **h** 9 **i** -6 **j** -12 **k** 10 **l** -9

4 **a** -8 **b** -12 **c** 17 **d** 15 **e** 8 **f** -3

5 **a** -35 **b** -3 **c** 7 **d** -60 **e** -4 **f** 60
g -5 **h** -36 **i** 90 **j** -1 **k** -38 **l** 45

6 **a** -3 **b** -7 **c** -9 **d** 100 **e** -70 **f** -3

7 **a**

×	−2	−1	0	+1	+2
−4	8	4	0	−4	−8
−2	4	2	0	−2	−4
0	0	0	0	0	0
+2	−4	−2	0	2	4
+4	−8	−4	0	4	8
+6	−12	−6	0	6	12

b

×	−25	−20	−10	10	20
7	−175	−140	−70	70	140
5	−125	−100	−50	50	100
0	0	0	0	0	0
−2	50	40	20	−20	−40
−4	100	80	40	−40	−80
−6	150	120	60	−60	−120

8 **a** $-1 \times (-7) = 7$ **b** $-1 \times +12 = -12$
c $-1 \times (-88) = 88$ **d** $-1 \times 25 = -25$
e $-1 \times (8 - 5) = -1 \times 3 = -3$
f $-1 \times (-3 + 2) = -1 \times (-1) = 1$

9 **a** 60 **b** -42 **c** -54 **d** -48 **e** -130
f 84

10 **a** 36 **b** -5 **c** -24 **d** -6 **e** 30
f -42

11 **a** 100 **b** 4 **c** 16
d -27 **e** -125 **f** -1000

12 **a** positive **b** negative **c** positive
d negative **e** negative **f** negative

13 **a** 10, 8, 6, 4, 2, 0, −2, −4, −6, −8, −10; the terms decrease by 2, or you get the next number by subtracting 2 (or adding −2)
b −15, −12, −9, −6, −3, 0, 3, 6, 9, 12, 15; the terms increase by 3, or you get the next number by adding 3 (or subtracting −3)
c 20, 16, 12, 8, 4, 0, −4, −8, −12, −16, −20; the terms decrease by 4, or you get the next number by subtracting 4 (or adding −4)
d −25, −20, −15, −10, −5, 0, 5, 10, 15, 20, 25; the terms increase by 5, or you get the next number by adding 5 (or subtracting −5)

14 **a** **i** $2 \times 3 = 6$ so $6 \div 2 = 3$ or $6 \div 3 = 2$
ii $2 \times (-3) = -6$ so $-6 \div 2 = -3$ or $-6 \div (-3) = 2$
iii $-2 \times 3 = -6$ so $-6 \div (-2) = 3$ or $-6 \div 3 = -2$
iv $-2 \times (-3) = 6$ so $6 \div (-2) = -3$ or $6 \div (-3) = -2$
b yes, same pattern
c **i** positive **ii** positive
iii negative **iv** negative

15 **a** **i** 0 **ii** 0 **iii** 0 **iv** 0
b The answer is always zero.
c **i** 0 **ii** 0 **iii** 0 **iv** 0
d The answer is always zero.

16 **a** −120 **b** +80 **c** 200 m **d** +110 **e** +40
f **i** 40 m **ii** 230 m
g −160 **h** −40
i Hayden. He needs to walk 20 m west.
j Natalie: 320 m; Tyler: 240 m; Hayden: 120 m; Rhys: 320 m; Sophie: 160 m; Imogen: 220 m

17 **a** $(+3)^2$, $(-3)^2$ and 3^2
b $(-3)^2 = -3 \times (-3) = +9$
while $-3^2 = -(3^2) = -(3 \times 3) = -9$
c **i** yes **ii** no **iii** no **iv** yes
d A number of the form $-x^a$ will always produce a negative result regardless of whether the power is odd or even. A negative base raised to an odd power will also produce a negative result.

18 **a** 7 and −7 **b** 9 and −9 **c** 1 and −1
d 2 and −2 **e** 8 and −8 **f** 10 and −10

19 **a** 3 **b** −5 **c** 4 **d** −1 **e** −4 **f** −10

20 **a** −6, −3 **b** 9, −5 **c** −11, 6 **d** −4, 14
e 4, 6 or −6, −4 **f** −9, −1 or 1, 9
g −16, 2 or −2, 16

21 **a** 13, 17, 29 and 53 **b** −53, −29, −17 and −13
c −49, −25, −23, −13, −7, −5, −1, 1, 5, 7, 13, 23, 25 and 49

1.8 Order of operations

EX p43

1 **a** multiplication, subtraction, addition
b subtraction (within brackets), division, addition
c square root, multiplication, addition, subtraction
d squaring, multiplication, subtraction, addition
e addition (within inner brackets), subtraction (within outer brackets), addition
f addition (within brackets), squaring, division, addition

2 **a** 23 **b** 40 **c** 75 **d** 38 **e** 59 **f** 47

3 **a** −9 **b** 0 **c** −5 **d** 7 **e** −16 **f** 14

4 **a** 2 **b** 390 **c** 22 **d** 5 **e** 123 **f** 36
g 51 **h** 555

5 **a** 6 **b** −207 **c** −4 **d** −16 **e** 25 **f** 0

6 **a** −5 **b** 23 **c** −144 **d** −100 000 **e** 18
f −36 500 **g** 6400 **h** 441

7 **a** 80 **b** 1 **c** 6 **d** 75

8 a A negative has been added to the answer.
b Addition has been calculated before the multiplication.
c Addition has been calculated before the division.
d A negative divided by a negative is a positive.
e Negative 4 to the power of 3 is −64, so the answer is also negative.
f Ignored the negative in front of the 5.

9 a 64 **b** 10 **c** 0 **d** 4 **e** −8 **f** −21

10 C

11 a $195 - 3 \times 58$
b $200 - 3 \times 60 = 20$; \$20
c \$21; the estimate is \$1 less.

12 $14 \times 7 + 11 \times 6 + 9 \times 4 + 16 \times 5 + 25 \times 1 = \$305$

13 a 13 **b** −6
c $13 - (-6)$ **d** 19 m

14 a

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Difference	6	7	2	4	5	3	1

b Monday
c i 1°C **ii** −3°C
d 4°C

EX p47

15 a deposit **b** withdrawal
c i He has money in the bank.
ii He owes the bank money.
d −\$96 **e** \$134
f −\$27, a withdrawal of \$27
g Yes, from 21 May to 24 May. A fee and/or interest is charged.

16 Negative. For the sum of two numbers to be zero, they must have different signs. As one number is positive and the other negative, the product will be negative (+ × − → −), unless they were both zero.

17 −6, −4, 5

18 a 10 **b** 12

19 a Sample answer:

Number	Equation
0	$4 \div 4 \times 4 - 4$
1	$4 \div 4 + 4 - 4$
2	$4 - (4 + 4) \div 4$
3	$(4 \times 4 - 4) \div 4$
4	$4 + 4 \times (4 - 4)$
5	$(4 \times 4 + 4) \div 4$
6	$(4 + 4) \div 4 + 4$
7	$4 + 4 - 4 \div 4$
8	$4 \div 4 \times 4 + 4$
9	$4 \div 4 + 4 + 4$
10	Not possible

Number	Equation
11	Not possible
12	$4 \times (4 - 4 \div 4)$
13	Not possible
14	Not possible
15	$4 \times 4 - 4 \div 4$
16	$4 \times 4 + 4 - 4$
17	$4 \times 4 + 4 \div 4$
18	Not possible
19	Not possible
20	$4 \times (4 \div 4 + 4)$

b Sample answer (considering the numbers not possible in part **a**):

Number	Equation
10	$(4 + 4 + 4) - \sqrt{4}$
11	Not possible
13	Not possible

Number	Equation
14	$4 \times 4 - 4 \div \sqrt{4}$
18	$4 \times 4 + 4 - \sqrt{4}$
19	Not possible

c Sample answer (considering the numbers not possible in part **a** or **b**):

Number	Equation
11	$(4! \times \sqrt{4} - 4) \div 4$
13	$(4! \times \sqrt{4} + 4) \div 4$
19	$4! - (4 + 4 \div 4)$

d Place brackets around each calculation and add a negative sign in front. This will keep the value of the calculation the same but give a negative result.

Review: Integers

Review questions 1.9A: Mathematical literacy review

1 compensation method
2 I (indices/exponents)
3 the whole-number result of the division
4 a integers
b estimate
c product
5 a The multiples of a whole number are the numbers produced by multiplying that number by another whole number. For example, the multiples of 5 are 5, 10, 15, 20, …
b The square root of a number is the number that when squared results in the original number. For example, the square root of 49 is 7, as $7 \times 7 = 49$.
c A list of numbers are written in ascending order when they are in order from smallest to largest. For example, the list 23, 27, 29, 35, 40 is written in ascending order.
6 a The commutative law of addition states that the order in which numbers are added does not change the result.
b The factors of a whole number are the whole numbers which divide exactly into the number.

Review questions 1.9B: Multiple choice

1 B **2** D **3** A **4** D **5** A **6** D
7 C **8** D **9** D **10** A **11** C **12** A
13 A **14** C **15** D **16** B **17** B **18** A
19 D **20** A

Review questions 1.9C: Short answer

1 a 200 **b** 70 000 **c** 4000
d 100 000
2 a 14 000 **b** 60 000 **c** 120 000 **d** 50

3 $2 296 240
4 312 398
5 168
6 11
7 18
8 **a** 216 **b** 128 **c** 64 **d** 121
9 **a** 2 **b** 9 **c** 3 **d** 5
10 **a** $-5 < 2$ **b** $-5 < -2$
c $7 > -4$ **d** $4 > -7$
11 **a** −40, −24, −22, −14, 16, 20, 24
b −35, −27, −17, 0, 9, 21, 63
c −66, −46, −26, −16, −6, 6, 86
d −409, −300, −87, 3, 45, 65, 190
12 **a** −9 **b** +9 **c** −12 **d** +19
13 $122
14 21°C
15 **a** −1 **b** −10 **c** −8 **d** −10
e 0 **f** −100
16 **a** −56 **b** +45 **c** −9 **d** +5
e 0 **f** −60 **g** −3 **h** +14
17 **a** 180 **b** −35 **c** 12 **d** −40
18 **a** −8 **b** 9 **c** −1 **d** 100
19 **a** 21 **b** 22 **c** −130 **d** −10
20 **a** −180 **b** 43 **c** −11 **d** −8

Review questions 1.9D: Mathematical modelling

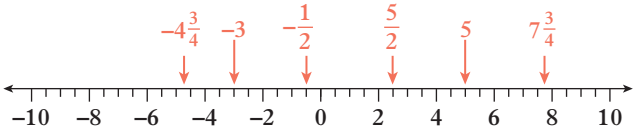
1 **a** 2700 kg, 1800 kg **b** 900 kg
c 4500 kg **d** 900 kg
e **i** 450 kg **ii** 270 kg and 180 kg
f **i** 1795 kg, 1775 kg, 1675 kg
ii The elephant should be losing 5 kg each day, but Jez's expression suggests the elephant lost 20 kg on the second day and 100 kg on the third day.
iii $2 \times 3^2 \times 10^2 - n \times 5$
2 **a** 1 568 672
b 8 000 000, 7 000 000, 5 000 000, 3 000 000, 2 000 000, 600 000, 500 000, 200 000
c **i** 1 000 000; the difference between the estimated and the actual value is 568 672 people
ii 26 300 000
3 **a** **i** Wednesday **ii** Friday
b Monday: 7°C, Tuesday: 7°C, Wednesday: 11°C, Thursday: 7°C, Friday: 4°C, Saturday: 7°C, Sunday: 6°C
c Wednesday
d average minimum temperature: –2°C, average maximum temperature: 5°C
e 7°C

MODULE 2 Fractions and decimals

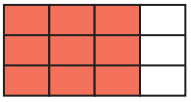
2.1 Fractions

EX p57

1 **a** $\frac{1}{-2} = -\frac{1}{2} = \frac{-1}{2}$ **b** $\frac{-10}{3} = -\frac{10}{3} = \frac{10}{-3}$
c $\frac{-5}{-6} = \frac{5}{6}$ **d** $\frac{-2}{-3} = \frac{2}{3}$
e $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$ **f** $-\frac{-3}{-7} = -\frac{3}{7} = \frac{-3}{7} = \frac{3}{-7}$
2



3 **a** $\frac{3}{4}$
b Sample answer:



c $\frac{9}{12}$
4 **a** $\frac{3}{10} = \frac{6}{20}$ **b** $\frac{2}{7} = \frac{10}{35}$ **c** $\frac{18}{33} = \frac{6}{11}$ **d** $\frac{16}{48} = \frac{2}{6}$
e $-\frac{4}{5} = -\frac{24}{30}$ **f** $\frac{-8}{36} = \frac{-2}{9}$ **g** $-1\frac{3}{8} = -\frac{55}{40}$
h $3\frac{3}{4} = \frac{75}{20}$ **i** $\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = \frac{8}{32} = \frac{20}{80} = \frac{25}{100}$
j $-\frac{3}{7} = \frac{-15}{35} = \frac{-21}{49} = \frac{-30}{70} = \frac{-300}{700} = -\frac{90}{210}$
k $1\frac{1}{5} = \frac{12}{10} = \frac{60}{50} = \frac{300}{250} = \frac{54}{45} = \frac{72}{60}$
l $-2\frac{1}{3} = -\frac{14}{6} = \frac{-35}{15} = \frac{-49}{21} = -\frac{21}{9} = -\frac{700}{300}$
5 **a** $\frac{8}{20}, \frac{22}{55}, \frac{4}{10}, \frac{10}{25}, \frac{18}{45}$ **b** $\frac{20}{15}, \frac{28}{21}, 1\frac{4}{12}$
6 **a** $\frac{3}{8} < \frac{1}{2}$ **b** $\frac{4}{7} > \frac{5}{14}$ **c** $\frac{-9}{10} < \frac{-4}{5}$
d $-\frac{5}{12} > -\frac{2}{3}$ **e** $1\frac{5}{6} > 1\frac{19}{24}$ **f** $\frac{17}{15} < \frac{7}{3}$
g $-\frac{2}{3} > -\frac{3}{4}$ **h** $\frac{5}{6} > \frac{4}{5}$ **i** $-\frac{11}{8} > -\frac{5}{3}$
j $2\frac{3}{4} < \frac{17}{6}$ **k** $-\frac{29}{3} < -9\frac{1}{2}$ **l** $3\frac{5}{12} > \frac{25}{8}$
7 **a** $\frac{7}{9}$ **b** $\frac{11}{4}$ **c** $-\frac{12}{5}$
d $-\frac{43}{6}$ **e** $3\frac{7}{11}$ **f** $-8\frac{1}{2}$
g $\frac{16}{3}$ **h** $-\frac{55}{9}$ **i** $3\frac{5}{7}$
8 **a** $\frac{4}{5}$ **b** $\frac{2}{3}$ **c** $\frac{7}{9}$
d $\frac{5}{8}$ **e** $\frac{10}{3}$ $\left(\text{or } 3\frac{1}{3}\right)$ **f** $\frac{11}{6}$ $\left(\text{or } 1\frac{5}{6}\right)$
9 **a** $\frac{2}{5}$ **b** $-\frac{3}{7}$ **c** $-\frac{2}{5}$
d $-\frac{5}{3}$ $\left(\text{or } -1\frac{2}{3}\right)$ **e** $\frac{8}{9}$ **f** $\frac{3}{7}$
g $3\frac{3}{4}$ **h** $-5\frac{5}{9}$ **i** $4\frac{3}{11}$
j $-1\frac{4}{7}$ **k** $2\frac{3}{8}$ **l** $-7\frac{5}{6}$

10 a $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{7}{5}$

b $\frac{3}{8}, \frac{9}{8}, 1\frac{1}{4}, \frac{7}{4}, 2\frac{5}{8}, 2\frac{3}{4}$

c $-\frac{9}{2}, -1\frac{9}{10}, -\frac{9}{5}, -1\frac{1}{2}, -\frac{2}{5}, -\frac{3}{10}$

11 a $\frac{19}{11}, 1\frac{5}{11}, \frac{13}{11}, \frac{9}{11}, \frac{5}{11}, \frac{3}{11}$

b $-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -1\frac{1}{4}, -\frac{4}{3}, -1\frac{1}{2}$

c $1\frac{1}{3}, \frac{5}{6}, \frac{1}{6}, -\frac{5}{12}, -\frac{11}{6}, -2\frac{1}{6}$

12 a $\frac{3}{5}$ **b** $\frac{2}{9}$ **c** $\frac{7}{24}$ **d** $\frac{3}{10}$

e $\frac{1}{6}$ **f** $\frac{2}{5}$ **g** $\frac{2}{7}$ **h** $\frac{3}{20}$

i $\frac{7}{15}$ **j** $\frac{3}{17}$ **k** $\frac{1}{30}$ **l** $\frac{2}{15}$

13 a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{5}{8}$ **d** $\frac{2}{3}$

14 $\frac{1}{4}$

15 a $\frac{3}{4}, \frac{6}{8} = \frac{3}{4}, \frac{9}{12} = \frac{3}{4}, \frac{12}{16} = \frac{3}{4}, \frac{15}{20} = \frac{3}{4}, \frac{18}{24} = \frac{3}{4}, \frac{21}{28} = \frac{3}{4}, \frac{24}{32} = \frac{3}{4}, \frac{27}{36} = \frac{3}{4}$

b All of the fractions made by dividing a multiple of 3 by the corresponding multiple of 4 will simplify to $\frac{3}{4}$ as they are equivalent fractions.

c $\frac{7}{5}, \frac{14}{10} = \frac{7}{5}, \frac{21}{15} = \frac{7}{5}, \frac{28}{20} = \frac{7}{5}, \frac{35}{25} = \frac{7}{5}, \frac{42}{30} = \frac{7}{5}, \frac{49}{35} = \frac{7}{5}, \frac{56}{40} = \frac{7}{5}, \frac{63}{45} = \frac{7}{5}$

d All of the fractions made by dividing a multiple of 7 by the corresponding multiple of 5 will simplify to $\frac{7}{5}$ as they are equivalent fractions.

16 a $\frac{4}{7}$ **b** $-\frac{7}{11}$ **c** $\frac{1}{2}$ **d** $-\frac{4}{9}$

e $\frac{1}{3}$ **f** $-\frac{1}{8}$ **g** $\frac{1}{24}$ **h** $2\frac{14}{15}$

17 The first test, the second test, the third test

18 15

19 a i Since $\frac{4}{7} > \frac{1}{2}$ and $\frac{7}{15} < \frac{1}{2}$, we can compare the fractions using the benchmark $\frac{1}{2}$.

ii $\frac{4}{7} > \frac{7}{15}$

b i Since the numerators are the same, we can compare the denominators.

ii $\frac{15}{17} < \frac{15}{4}$

c i Both fractions are one piece away from 1; we can compare the difference between the fractions and the benchmark 1.

ii $\frac{99}{100} < \frac{100}{101}$

d i Both mixed numbers have a whole number part of 1, so we only need to compare the fraction parts. Since the numerators are the same, we can compare the denominators.

ii $1\frac{1}{8} > 1\frac{1}{11}$

e i Since the numerators and denominators are quite small, finding any equivalent fractions will be simple.

ii $1\frac{3}{5} < 1\frac{2}{3}$

f i Since 10 is a multiple of 5, we can compare using an equivalent fraction of $\frac{39}{5}$ with a denominator of 10.

ii $\frac{77}{10} < \frac{39}{5}$

20 a Ash (lower value is better rate in serving)

b 10 **c** 153

21 a D

b Answers will vary but may include some of the strategies from question **19**.

EX p64

2.2 Adding and subtracting fractions

1 a The first number is written more simply as $\frac{1}{7}$ and the operations + (−) can be replaced with −.

b $-\frac{2}{7}$

2 a $\frac{13}{17}$ **b** $\frac{2}{5}$ **c** 1 **d** $\frac{1}{3}$

3 a 15 **b** 14 **c** 30 **d** 24

e 15 **f** 20 **g** 12 **h** 18

4 a $\frac{21}{22}$ **b** $\frac{7}{20}$ **c** $1\frac{17}{30}$ **d** $\frac{7}{24}$

e $\frac{1}{30}$ **f** $1\frac{7}{12}$ **g** $\frac{7}{18}$ **h** $\frac{31}{40}$

5 a $\frac{4}{7}$ **b** $-2\frac{1}{3}$ **c** $-\frac{2}{8} = -\frac{1}{4}$ **d** $-\frac{2}{5}$

e $-1\frac{2}{3}$ **f** $-1\frac{1}{4}$ **g** $\frac{2}{9}$ **h** $-1\frac{1}{10}$

6 a $-\frac{1}{5}$ **b** $-\frac{3}{7}$ **c** $-\frac{9}{11}$ **d** $-1\frac{1}{6}$

e $-\frac{3}{14}$ **f** $\frac{1}{8}$

7 a $\frac{3}{8}$ **b** $3\frac{1}{4}$ **c** $4\frac{7}{15}$ **d** $2\frac{3}{22}$

e $7\frac{5}{12}$ **f** $2\frac{7}{10}$ **g** $-\frac{2}{3}$ **h** $-5\frac{7}{10}$

8 a $-\frac{1}{3}$ **b** 0 **c** $-\frac{2}{3}$ **d** $-\frac{3}{20}$

e $\frac{7}{18}$ **f** $-2\frac{6}{7}$ **g** $1\frac{11}{24}$ **h** $-\frac{1}{20}$

9 a $\frac{1}{3}$ **b** $\frac{5}{3}$ **c** $\frac{7}{3}$ **d** $-\frac{9}{2}$

e $-\frac{10}{3}$ **f** $-\frac{7}{3}$ **g** $-\frac{1}{3}$ **h** $-\frac{1}{6}$

10 a $-\frac{7}{3}$ **b** $\frac{1}{5}$ **c** $\frac{19}{30}$ **d** $-\frac{11}{6}$

e −2 **f** $\frac{10}{3}$ **g** 3 **h** $\frac{31}{42}$

11 a i $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$

b i $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$

c Each diagonal adds to $1\frac{1}{2}$.

d They are all the same. **e** $1\frac{1}{2}$

12 a

$\frac{2}{15}$	$\frac{7}{15}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
$\frac{4}{15}$	$\frac{1}{5}$	$\frac{8}{15}$

b

$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	$\frac{7}{24}$
$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{4}$

c

$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$
0	$-\frac{5}{6}$	$\frac{1}{3}$

13 a positive **b** negative **c** positive **d** zero
e negative **f** negative **g** zero **h** negative

14 a $1\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $-\frac{1}{2}$ **d** $-1\frac{1}{2}$
e $10\frac{1}{4}$ **f** $-10\frac{1}{4}$

15 It is better to convert to improper fractions first when the fraction part of the mixed number you are subtracting is greater than the fraction part of the mixed number you are subtracting from.

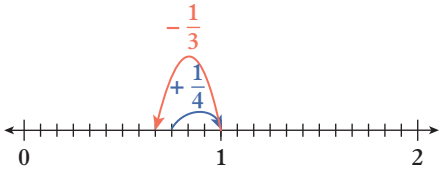
16 $\frac{1}{3} + 1\frac{3}{4} = 2\frac{1}{12}$ L; the container is not large enough.

17 $4\frac{7}{12}$ hours

18 a $\frac{3}{10}$ **b** $\frac{29}{30}$ **c** $\frac{1}{30}$

19 a B **b** D **d** A **e** C

E



c $\frac{3}{2} + \frac{1}{6} = \left(\frac{3}{2} + \frac{3}{6}\right) - \frac{2}{6} = 2 - \frac{1}{3} = 1\frac{2}{3}$

20 D

EX p70

2.3 Multiplying and dividing fractions

1 a $\frac{9}{20} \times \frac{5}{3}$ **b** $\frac{9}{20} \times \frac{15}{7}$ **c** $\frac{20}{9} \times \frac{7}{15}$ **d** $\frac{20}{9} \times \frac{15}{7}$
e $\frac{2}{9} \times \frac{4}{7}$ **f** $\frac{2}{9} \times \frac{21}{5}$ **g** $\frac{9}{2} \times \frac{5}{21}$ **h** $\frac{9}{2} \times \frac{21}{5}$

2 a $\frac{3}{20}$ **b** $\frac{6}{5}$ or $1\frac{1}{5}$ **c** $\frac{7}{10}$ **d** $\frac{12}{35}$
e $\frac{1}{50}$ **f** 9 **g** $\frac{20}{21}$ **h** 1

3 a $\frac{9}{8}$ or $1\frac{1}{8}$ **b** 4 **c** 10
d $\frac{3}{4}$ **e** $12\frac{4}{5}$ or $\frac{64}{5}$ **f** 6
g $2\frac{5}{8}$ or $\frac{21}{8}$ **h** $32\frac{1}{2}$ or $\frac{65}{2}$ **i** 144
j $60\frac{1}{5}$ or $\frac{301}{5}$ **k** $\frac{40}{23}$ **l** $\frac{125}{39}$

4 a $\frac{10}{21}$ **b** $\frac{5}{8}$ **c** $\frac{3}{8}$ **d** $4\frac{1}{2}$ or $\frac{9}{2}$
e 7 **f** $\frac{1}{7}$ **g** $1\frac{4}{5}$ or $\frac{9}{5}$ **h** 40

5 a $6\frac{3}{5}$ or $\frac{33}{5}$ **b** $\frac{9}{16}$ **c** $\frac{1}{2}$ **d** $\frac{2}{9}$
e $1\frac{1}{2}$ or $\frac{3}{2}$ **f** $\frac{2}{7}$ **g** $\frac{17}{28}$ **h** 14
i $\frac{28}{17}$ or $1\frac{11}{17}$ **j** $4\frac{5}{27}$ or $\frac{113}{27}$ **k** $\frac{3}{26}$ **l** $\frac{31}{10}$

6 a $\frac{4}{11}$ **b** $-\frac{1}{3}$ **c** $-1\frac{1}{5}$
d $\frac{108}{245}$ **e** $-\frac{7}{12}$ **f** $-\frac{5}{27}$
g -15 **h** $\frac{2}{3}$ **i** $-1\frac{1}{2}$

7 a $\frac{1}{3}$ **b** $\frac{7}{5}$ **c** $\frac{1}{6}$ **d** 2
e $-\frac{5}{12}$ **f** 6 **g** $\frac{1}{5}$ **h** $\frac{5}{2}$

8 a $\frac{8}{15}$ **b** $\frac{5}{6}$ **c** $\frac{8}{15}$ **d** $-\frac{2}{3}$
e $\frac{8}{63}$ **f** $\frac{15}{4}$ **g** $-\frac{5}{8}$ **h** $-\frac{2}{3}$

9 a $1\frac{5}{11}$ **b** $\frac{3}{2}$ or $1\frac{1}{2}$ **c** $-\frac{1}{12}$ **d** $8\frac{11}{12}$
e -1 **f** $-\frac{13}{14}$ **g** -1 **h** $\frac{1}{3}$

10 a, b i $\frac{3}{7}$ **ii** $\frac{5}{8}$ **iii** $-\frac{5}{16}$ **iv** $-\frac{5}{24}$
c Yes, the answers match. It is easier to use the method in a when the divisor is a factor of the numerator in the dividend.

11 a 24 buttons **b** 25 red buttons **c** $\frac{5}{4}$

12 a 8 **b** 29 kg

13 a i 7 hours **ii** \$36 **iii** 14 kg **iv** 32 cm
b, c i 21 hours **ii** \$84 **iii** 21 kg **vi** 40 cm
d i 27 days **ii** 8 mm

14 66

15 a i 200 g butter, 1 cup sugar, 4 large eggs, 3 cups flour, 2 tsp baking powder, 1 cup milk, $1\frac{1}{2}$ cups cocoa powder, 2 cups boiling water
ii 50 g butter, $\frac{1}{4}$ cup sugar, 1 large egg, $\frac{3}{4}$ cup flour, $\frac{1}{2}$ tsp baking powder, $\frac{1}{4}$ cup milk, $\frac{3}{8}$ cup cocoa powder, $\frac{1}{2}$ cup boiling water
iii $66\frac{2}{3}$ g butter, $\frac{1}{3}$ cup sugar, $1\frac{1}{3}$ large eggs, 1 cup flour, $\frac{2}{3}$ tsp baking powder, $\frac{1}{3}$ cup milk, $\frac{1}{2}$ cup cocoa powder, $\frac{2}{3}$ cup boiling water
b $1\frac{1}{8}$ cups icing sugar, $112\frac{1}{2}$ g butter, 90 g chocolate, $\frac{3}{8}$ cup thickened cream

16 a $\frac{8}{125}$ **b** $\frac{4}{9}$ **c** $-\frac{8}{27}$ **d** $\frac{81}{49}$ **e** $\frac{1000}{27}$

17 a $\frac{4}{5}$ **b** $\frac{7}{3}$ **c** $\frac{9}{10}$ **d** $\frac{5}{3}$ **e** $\frac{11}{6}$

18 a 36 **b** 147 **c** $42\frac{2}{3}$ **d** $36\frac{4}{5}$

19 a $18\frac{17}{18}$ **b** $78\frac{8}{15}$ **c** $39\frac{1}{5}$ **d** $18\frac{115}{168}$

20 By writing a division calculation as a multiplication by the reciprocal, we can apply the distributive law to the dividend (left fraction) by multiplying the whole part and the fraction part of the sum by the reciprocal of the divisor (right fraction). We can then confirm this is equivalent to dividing each part by the original divisor by writing the multiplications as divisions by the reciprocal.

However, we cannot apply the distributive law to the divisor as it is no longer the sum of 5 and $\frac{2}{3}$ after we get its reciprocal.

$$3\frac{9}{10} \div 5\frac{2}{3} = 3\frac{9}{10} \div \frac{17}{3} = 3\frac{9}{10} \times \frac{3}{17}$$
$$= \left(3 \times \frac{3}{17}\right) + \left(\frac{9}{10} \times \frac{3}{17}\right)$$
$$= \left(3 \div 5\frac{2}{3}\right) + \left(\frac{9}{10} \div 5\frac{2}{3}\right)$$

21 a $-\frac{5}{7}$ **b** $\frac{39}{16}$

EX p74

Checkpoint

1 a $\frac{6}{10} = \frac{48}{80}$ **b** $\frac{55}{20} = 2\frac{3}{4}$ **c** $\frac{63}{18} = \frac{28}{8}$ **d** $\frac{6}{5} = \frac{-18}{-15}$

2 a $\frac{4}{5}$ **b** $\frac{4}{3}$ **c** $8\frac{1}{4}$ **d** $-\frac{5}{9}$

3 a $\frac{2}{3} < \frac{3}{4}$ **b** $\frac{7}{9} > -\frac{8}{9}$ **c** $-\frac{17}{10} > -\frac{9}{5}$ **d** $\frac{30}{36} > \frac{36}{54}$

4 a 12 **b** 18 **c** 24 **d** 33

5 a $\frac{16}{15} = 1\frac{1}{15}$ **b** $\frac{35}{9} = 3\frac{8}{9}$

c $\frac{56}{25} = 2\frac{6}{25}$ **d** $-\frac{99}{28} = -3\frac{15}{28}$

6 a $-\frac{1}{12}$ **b** $-\frac{101}{15} = -6\frac{11}{15}$

c $\frac{43}{48}$ **d** $-\frac{25}{21} = -1\frac{4}{21}$

7 a 10 **b** $-\frac{32}{27} = -1\frac{5}{27}$

c $-2\frac{1}{5}$ **d** $\frac{40}{9} = 4\frac{4}{9}$

8 a $\frac{5}{3}$ **b** 7 **c** $\frac{1}{9}$ **d** $\frac{3}{17}$

9 a $\frac{3}{5}$ **b** -7 **c** $-\frac{16}{35}$ **d** $\frac{8}{9}$

10 a $\frac{27}{25} = 1\frac{2}{25}$ **b** $-\frac{7}{10}$ **c** $-\frac{65}{36} = -1\frac{29}{36}$ **d** $\frac{27}{175}$

11 a 65 GB **b** $7\frac{1}{2}$ L **c** 750 g **d** \$4.25

EX p78

2.4 Decimals

1

	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
a		0	.	0	0	6	5
b		0	.	6	5	0	0
c		6	.	0	0	5	
d	6	0	.	5			

2 a 5 thousandths or $\frac{5}{1000}$ **b** 5 tens or 50

c 5 ones or 5 **d** 5 tenths or $\frac{5}{10}$

e 5 ten thousandths or $\frac{5}{10\,000}$

f 5 hundredths or $\frac{5}{100}$

3 a 3 **b** 3 **c** 1 **d** 2 **e** 4 **f** 5

4 a 0.104, 0.14, 0.142, 0.2, 0.214, 0.41

b 8.072, 8.2, 8.702, 8.97, 9.782, 9.87

c –15.40, –10.45, 10.54, 15.04, 105.4, 150.4

d –800.8, –800.08, –800.07, –799.8, 799.08, 800.7

5 a 13.713, 13.7, 13.31, 13.173, 13.137, 13.07

b 0.685, 0.658, 0.6508, 0.65, 0.6085, 0.6058

c 555.55, 5.5555, 5.55000, 5.00500, –50.0505, –50.5050

d 999.90, 999.099, 100, –99.009, –99.90, –100

6 Comparing each place value from left to right, 9.46 contains 6 hundredths whereas 9.452 only contains 5 hundredths. (The first two place values are identical.)

7 a 25.68 **b** −673.82 **c** 0.78

d −101.02 **e** 8148.26 **f** −54.26

g 330.41 **h** −4.85 **i** 16.00

8 a 57.23 **b** 8.914 **c** 92.0871

d −0.757 **e** −135.2 **f** 65.071 23

g 7.510 **h** −208.0099 **i** 1.727 73

j 845.8 **k** −0.1235 **l** −0.123 456 8

9 a $1\frac{3}{100}$ **b** $2\frac{9}{10}$ **c** $3\frac{7}{1000}$ **d** $-\frac{1}{10\,000}$

e $10\frac{7}{10}$ **f** $-\frac{3}{100\,000}$ **g** $12\frac{9}{1\,000\,000}$

h $-\frac{7}{10\,000\,000}$

10 a $\frac{3}{25}$ **b** $-\frac{27}{50}$ **c** $\frac{803}{1000}$ **d** $-\frac{1}{25}$

e $\frac{1}{40}$ **f** $-\frac{3}{5000}$ **g** $-\frac{37}{50}$ **h** $\frac{63}{250}$

i $-\frac{3}{2000}$ **j** $-\frac{6}{125}$ **k** $\frac{17}{100\,000}$ **l** $-\frac{17}{20}$

11 a 0.7 **b** −0.29 **c** 0.451 **d** −0.2047

e 0.04 **f** −0.009 **g** 0.0005 **h** −0.063

12 a 0.8 **b** 0.625 **c** −0.5 **d** 1.25

e 0.125 **f** −0.4 **g** 0.35 **h** −0.36

i 0.025 **j** −0.6 **k** 0.22 **l** −0.3125

13 Rosa; the digit in the fourth decimal place is 5 so you must round up.

14 Jayden; the digit in the third decimal place is 6 so you must round up. Adding one to the digit in the second decimal place gives 10 which means you write 0 in the second decimal place and add one to the first decimal place (7 + 1 = 8).

15 Kota; Peter correctly rounded to 16 but did not write the two decimal places that were asked for, so it appears he rounded to the nearest whole number.

16 Sabine has not rounded directly to one decimal place. For this number, rounding to two decimal places meant that the digit in the hundredths place changed from 4 to 5 which results in rounding up rather than rounding down.

17 0.7 cm

18 a 1.568 m **b** 1.657 m

c 1.568 m, 1.58 m, 1.615 m, 1.63 m, 1.651 m, 1.657 m

d 1.65 m, 1.63 m, 1.57 m, 1.62 m, 1.58 m, 1.66 m

e 1.57 m, 1.58 m, 1.62 m, 1.63 m, 1.65 m, 1.66 m; the measurements are still all different and there is no change to the order.

f 1.7 m, 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m

g 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m, 1.7 m; there are only two different measurements.

19 **a** 0.02 mm **b** 0.018 mm **c** 0.0184 mm

20 **a** **i** 0.3 mm **ii** 0.28 mm **iii** 0.285 mm

b $\frac{569}{2000}$ mm

21 **a** \$42.35 **b** \$207.60 **c** \$7.25

d \$15.05 **e** \$60.70 **f** \$100.00

22 Sample answer: 0.918273645

23 **a** $4.65 \leq 4.7 < 4.75$

b $12.745 \leq 12.75 < 12.755$

EX p84 2.5 Adding and subtracting decimals

1 **a** 10.1 **b** 26.25

c 2.052 **d** 29.38

2 **a** 247.96 **b** 33.7686 **c** 690.984

d 106.5953 **e** 0.2776 **f** 119.0154

3 **a** 82.83 **b** 785.654 **c** 0.1344 **d** 26.251

4 **a** 2.7312 **b** 177.196 **c** 3794.295

d 64668.44 **e** 8999.445 **f** 1.9999

5 **a** −2.6 **b** −9 **c** 2.6

d 48.266 **e** −3.9078 **f** −495.238

6 **a** −3.7 **b** −13.26 **c** 3.7

d −61.783 **e** −273.7973 **f** −1.209

7 **a** −0.9 **b** −8.74

c −82.85 **d** −0.2488

8 **a** −7.6 **b** 5.21

c 1.1 **d** −88.189 34

9 Estimate: 12 km, exact: 12.025 km

10 **a** **i** 368 cm **ii** 382.9 cm

b Sample answer: Left hand; Lee may be right-handed and use her left hand less, so there is less wear on the nails.

c 750.9 cm

11 **a** D

b When you expand the brackets using the distributive law, the calculation becomes 7.2 − 9.4 − 2.3; however, the original calculation calls for + 2.3. A similar expression that is equivalent to the original calculation is 7.2 + (−9.4 + 2.3).

c Not when one subtraction follows another. For example, the correct answer to 10 − 5 − 3 is 2, but if the second subtraction (5 − 3) is performed first the result is 10 − 2 = 8, which is incorrect.

12 **a** −13.3168 **b** −40.911

c 0.0472 **d** −163.0612

13 **a** Yes, \$20.61 extra **b** 3 weeks

14 The sum will have at most x decimal places.

15 **a**

$$-(3.12 + 4.2) = -3.12 + (-4.2)$$
$$= -3.12 - 4.2$$

If you find the sum of 3.12 and 4.2, then multiply by negative 1, you get the same answer as −3.12 − 4.2.

b

$$-(4.2 - 3.12) = -4.2 - (-3.12)$$
$$= -4.2 + 3.12$$
$$= 3.12 + (-4.2)$$
$$= 3.12 - 4.2$$

If you subtract 3.12 from 4.2, then multiply by negative 1, you get the same answer as 3.12 − 4.2.

16 **a** 21 − 13.9 = 7.1

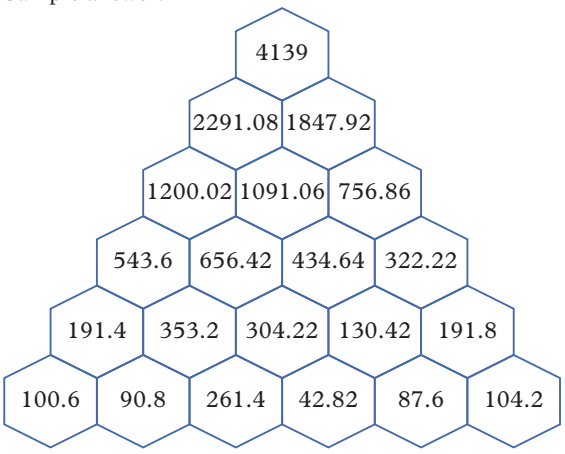
b 161.67 − 100.28 = 61.39

c 9.319 − 14.78 = −5.461

d 92.08 − 181.88 = −89.8

17 **a** 81.178° clockwise **b** 519.628°

18 Sample answer:



EX p90 2.6 Multiplying and dividing decimals

1 **a** 0.052 **b** 1040 **c** 9.21

d 0.034 **e** 0.000 57 **f** 0.045 78

2 **a** 194.8 **b** 1222.4 **c** 52 058.1

d 2.947 59

3 **a** 43.831 **b** −30.864 **c** 2281.44

d 112.5234 **e** −4.232 **f** −2.024 204

g 7.890 54 **h** 1124.4616

4 **a** 18.149 **b** −1.3448 **c** −8.3249

d 21.6985 **e** 0.591 25 **f** −0.156 75

g 0.0256 **h** −27.614 28

5 **a** 10 **b** 35.68

c To keep an equivalent division problem

d 8.92

6 **a** 72.4 ÷ 2 **b** 49.02 ÷ (−6) **c** 1976.4 ÷ 3

d −172.8 ÷ (−8) **e** 1436.8 ÷ 4 **f** −749.1 ÷ 6

g 48.72 ÷ (−15) **h** 280.8 ÷ 12 **i** −163.8 ÷ (−9)

7 **a** 36.2 **b** −8.17 **c** 658.8

d 21.6 **e** 359.2 **f** −124.85

g −3.248 **h** 23.4 **i** 18.2

8 **a** 15.5 **b** −3.15 **c** −225.6 **d** −0.5112

e 208.125 **f** 105 **g** −10.7 **h** 12

9 **a** 1.875 **b** 11.552 **c** 10.8

d 19.322 **e** −30.568 **f** −49.998 078

10 **a** \$29.40 **b** \$5.85 **c** \$3.37
d \$9.88 **e** \$4.94 **f** \$11.99

11 **a** \$65 **b** \$65.45 **c** \$34.55

12 **a** **i** \$0.60 **ii** \$2.89 **iii** \$18.95
b **i** \$4.65 **ii** \$6.85 **iii** \$13.61
c **i** \$12.95 **ii** \$1.59 **iii** \$1.95

13 **a** box: \$0.75, individual: \$0.72
b Buying individual pencils, as the cost per pencil is less

14 **a** 400 g pack: \$0.92, loose: \$0.99
b William, as the cost is cheaper per 100 g
c Someone may need more or less than the 400 g in a package.

15 **a** −0.000 544 **b** −59.094 09
c 0.0004 **d** −8000

16 **a** −0.064 **b** −0.000 000 001
c 0.0081 **d** 97.656 25

17 **a** $6.3 \times 2 = 12.6$ **b** $9.2 \times 5 = 46$
c $0.49 \times 4 = 1.96$ **d** $7.4 \times \frac{4}{5} = 5.92$
e $0.041 \times \frac{2}{5} = 0.0164$ **f** $10.21 \times \frac{4}{25} = 1.6336$

18 perimeter = 20.27 cm, area = 18.5097 cm^2

19 **a** 108 **b** $\frac{9}{70}$ **c** 27.072
d 0.0132 **e** −20.34 **f** $5\frac{1}{4}$

20 Sample answer: $5.074 - 3.96 \times 1.28 = 0.0052$

EX p95

2.7 Terminating, non-terminating and recurring decimals

1 **a** N **b** T **c** N **d** T **e** N **f** N
g N **h** T **i** N **j** T **k** N **l** T

2 **c, f, g, k**

3 Sample answer: A terminating decimal 'terminates' or stops after a number of decimal places. A non-terminating decimal has an endless (infinite) number of decimal places. A recurring decimal is a special type of non-terminating decimal in which some or all of the digits after the decimal point form a recurring (repeating) pattern.

4 **a** $0.\dot{9}$ (or $0.\overline{9}$) **b** $-4.\overline{36}$ (or $-4.\dot{3}\dot{6}$)
c $87.2\dot{5}$ **d** $3.\overline{476}$ (or $3.4\dot{7}\dot{6}$)
e $-0.738\dot{2}$ **f** $16.19\dot{6}$
g $483.\dot{8}$ **h** $-6.72\overline{91}$ (or $6.72\dot{9}\dot{1}$)
i $-0.18\overline{367}$ (or $-0.18\dot{3}6\dot{7}$)
j $1.\overline{356\,28}$ (or $1.\dot{3}56\,2\dot{8}$)
k $28.\overline{70}$ (or $28.\dot{7}\dot{0}$)
l $-9.23071\dot{4}$

5 **a** 0.111 111 111 111…
b 5.377 777 777 777…
c −14.828 282 828 282…
d −21.569 696 969 696…
e 8.104 104 104 104…
f 0.651 365 136 513…
g −232.684 444 444 444…
h 44.761 359 359 359…
i −94.570 202 020 202…
j −1.739 247 392 473…
k 11.812 111 111 111…
l −6.904 040 404 040…

6 **a** 0.7 **b** −0.5 **c** $0.\dot{3}$ **d** 1.25
e $0.\overline{18}$ **f** −0.625 **g** $2.\dot{6}$ **h** −0.75

7 **a** −1.125 **b** 1.4
c $2.8\dot{3}$ **d** $-1.\overline{142\,857}$
e 3.6 **f** $-4.\dot{1}$ **g** $1.\overline{72}$ **h** $-2.41\dot{6}$

8 **a** **i** **a, b, d, f, h** **ii** **c, e, g**
b **c, e, g** **c** No

9 **a** They can be expressed as fractions.
b Yes, because they can be expressed as fractions.

10 **a** rational **b** rational **c** rational
d irrational **e** rational **f** irrational
g rational **h** rational **i** rational
j rational **k** irrational **l** rational

11 **a** **i** $0.\dot{1}$ **ii** $0.\dot{2}$ **iii** $0.\dot{3}$ **iv** $0.\dot{4}$
b recurring decimal
c **i** $0.\dot{5}$ **ii** $0.\dot{6}$ **iii** $0.\dot{7}$ **iv** $0.\dot{8}$

12 **a** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
b **i** $0.\overline{09}$ **ii** $0.\overline{18}$ **iii** $0.\overline{27}$ **iv** $0.\overline{36}$
c The recurring digits are the multiples of 9. $\frac{9}{11} = 0.\overline{81}$

13 **a**

Unit fraction	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{11}$
Decimal	$0.\dot{1}$	0.1	0.2	0.5	$0.1\dot{6}$	$0.\overline{09}$
Denominator's prime factorisation	3^2	2×5	5	2	2×3	11
Is 2 or 5 a factor of the denominator?	No	Yes	Yes	Yes	Yes	No
Any prime factors other than 2 and/ or 5?	Yes	No	No	No	Yes	Yes
Is the decimal terminating?	No	Yes	Yes	Yes	No	No

Unit fraction	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{15}$
Decimal	0.25	0.05	$0.0\dot{6}$	0.125	$0.08\dot{3}$
Denominator's prime factorisation	2^2	$2^2 \times 5$	3×5	2^3	$2^2 \times 3$
Is 2 or 5 a factor of the denominator?	Yes	Yes	Yes	Yes	Yes
Any prime factors other than 2 and/ or 5?	No	No	Yes	No	Yes
Is the decimal terminating?	Yes	Yes	No	Yes	No

b The fraction has a terminating decimal when the only factors of the denominator are powers of 2 or 5.

c **i** terminating, 0.04

ii non-terminating, $0.041\dot{6}$

iii non-terminating, $0.0\dot{5}$

iv terminating, 0.025

v terminating, 0.008

d **i** non-terminating, $3.\overline{428\,571}$

ii terminating, 0.21875

iii terminating, 0.7

iv non-terminating, $0.\overline{153\,846}$

v terminating, 2.8

14 a 3.160 494, 3.140 845, 3.142 857, 3.141 593, 3.141 600, 3.141 057

b π will sit at 3.14

c $\frac{355}{113}$

d Sample answer: It is an irrational number (non-terminating but not recurring decimal), so it cannot be written as an exact fraction or decimal.

15 a 1 IV, 2 III, 3 V, 4 VI, 5 II, 6 I

b To find another number with the same recurring part, so that we could subtract to eliminate the decimal part.

c **i** $\frac{2}{9}$ **ii** $\frac{7}{9}$ **iii** $\frac{7}{3}$ **iv** $\frac{17}{3}$

16 a **i** 0.111 111 11… **ii** 0.010 101 01…

iii 0.001 001 001… **iv** 0.000 100 01…

b **i** $0.\dot{1}$ **ii** $0.\overline{01}$ **iii** $0.\overline{001}$

iv $0.\overline{0001}$

c **i** $0.\overline{12}$ **ii** $0.\overline{765}$ **iii** $0.\overline{0012}$ **iv** $0.\dot{6}$

d **i** $\frac{59}{99}$ **ii** $\frac{401}{999}$ **iii** $\frac{3}{9} = \frac{1}{3}$ **iv** $6\frac{23}{99}$

e **i** $\frac{59}{990}$ **ii** $\frac{401}{9990}$

iii $\frac{3}{9000} = \frac{1}{3000}$ **iv** $\frac{6}{10} + \frac{23}{990} = \frac{617}{990}$

EX p100

Review: Fractions and decimals

Review questions 2.8A: Mathematical literacy review

1 a lowest **b** lowest **c** highest

2 Sample answer: In a mixed number, the whole number part represents the total number of units or objects, while the fraction part represents a portion of a unit or object. The whole number part of the mixed number does not change during the simplifying process because it represents a quantity that cannot be further simplified and is separate from the fractional part of the mixed number.

3 Keep the first number the same (the dividend).
Change the division symbol to a multiplication symbol.
Flip the second number (the divisor) and write it as its reciprocal.

4 a True

b False. Multiplying by a fraction can make a number either larger or smaller, depending on the value of the fraction. For example, multiplying a number by $\frac{3}{2}$ makes a number larger.

c False. Non-terminating, non-repeating decimals cannot be written as fractions.

d True

5 Sample answer:

a A digit is a single numerical symbol used to represent a value; a decimal place refers to the position of a digit within a decimal number. For example, in the number 12.345, 1, 2, 3, 4 and 5 are digits and the number has three decimal places.

b A placeholder zero is a zero used to hold the place of a digit with a value of zero in a number. A trailing zero is a zero at the end of a number, after all the significant digits have been written. For example, in the number 1.030, the zero between 1 and 3 is a placeholder zero, the zero on the right of 3 is a trailing zero.

c A rational number is a number that can be expressed as a ratio of two integers, irrational numbers are non-terminating, non-repeating decimals that cannot be written as fractions.

Review questions 2.8B: Multiple choice

1 B **2** C **3** C **4** B **5** D **6** B
7 D **8** C **9** A **10** C **11** C

Review questions 2.8C: Short answer

1 a $\frac{7}{12}$ **b** $-1\frac{7}{8}$ $\left(\text{or } -\frac{15}{8}\right)$

c $-3\frac{9}{16}$ **d** $5\frac{2}{3}$

2 a $\frac{29}{35}$ **b** $\frac{13}{30}$ **c** $6\frac{1}{33}$ **d** $2\frac{29}{35}$

3 a $\frac{6}{35}$ **b** $\frac{1}{2}$ **c** $21\frac{1}{4}$ **d** $\frac{9}{4}$ or $2\frac{1}{4}$

4 a $\frac{6}{7}$ **b** $-\frac{2}{3}$ **c** $-\frac{299}{9} = -33\frac{2}{9}$ **d** $\frac{18}{35}$

5 a $-\frac{1}{4}$ **b** $\frac{1}{15}$ **c** $-\frac{6}{7}$

d $4\frac{1}{2}$ **e** $-\frac{4}{35}$ **f** $-\frac{7}{8}$

6 a 24 km/h **b** $\frac{1}{3}$ of a cup

c 70 metres **d** 3 minutes

7 a 0.0012, 0.120, 0.123, 0.2, 0.213, 0.231

b 8.16, 8.9, 8.916, 9.16, 9.6, 9.916

c $-5, -0.5, -0.05, 0.05, 0.50, 5$

d $-14.2, -12.04, -10.42, -2.41, -2.14, -0.421$

8 a 0.717, 0.71, 0.7, 0.107, 0.0701, 0.017

b 2.721, 2.72, 2.7, 2.27, 2.127, 2.1

c 90, 89, 80.9, −0.89, −0.899, −9.8

d $-0.01, -0.099, -0.1, -0.9, -0.99, -1$

9 a 13.22 **b** −0.0012 **c** 54.546 **d** −181.2

10 a 143.67 **b** 23.48

c 265.7842 **d** 989.5993

11 **a** -3.83 **b** 4.182 **c** -2.313 **d** 114.78

12 **a** 25.2512 **b** -10.9315

c -19.722 **d** $416.758\,52$

13 **a** **i** terminating **ii** non-terminating

iii terminating **iv** non-terminating

b only part **ii**

14 **a** $0.\dot{3}$ or $0.\overline{3}$ **b** $5.\overline{385}$ or $5.\dot{3}8\dot{5}$

c $-45.68\overline{57}$ or $45.68\dot{5}\dot{7}$

d $-0.092\,\overline{451}$ or $-0.092\,\dot{4}5\dot{1}$

15 **a** $-0.233\,333\,333\,333\ldots$ **b** $3.272\,727\,272\,727\ldots$

c $46.358\,787\,878\,787\ldots$

d $-17.364\,854\,854\,854\ldots$

16 **a** $0.0\dot{3}$ or $0.0\overline{3}$ **b** $-0.\dot{5}71428\dot{5}$ or $-0.\overline{571428}$

c $-9.\dot{5}$ or $-9.\overline{5}$ **d** $1.\dot{5}\dot{4}$ or $1.\overline{54}$

Review questions 2.8D: Mathematical modelling

1 **a** **i** Wednesday **ii** Friday

b 5.8°C

c Tuesday: 6.3°C, Wednesday: 8.7°C, Thursday: 5.9°C, Friday: 4.5°C, Saturday: 5.7°C, Sunday: 5.1°C

d Wednesday

e **i** −1.8°C **ii** 4.2°C

f 6.0°C

2 **a** \$39.30 **b** 13 **c** \$0.43

d **i** \$35.88 **ii** \$26.91

iii yes, because it is a 33.3% discount

iv \$1.99

3 **a** 399 students

b **i** 378 students **ii** 198 students

c **i** $\frac{67}{266}$ **ii** 3 students above one-quarter

d **i** 417 students **ii** $\frac{139}{532}$

e 1600 students

MODULE 3 Percentages, ratios and rates

EX 3.1 Percentages, fractions and decimals

p109

1 **a** 60% **b** 40% **c** 60% **d** 52%

2 **a** 40% **b** 60% **c** 40% **d** 48%

3 **a** 24% **b** 77% **c** 83% **d** 58%

e 7% **f** 46%

4 **a** $\frac{17}{100}$, 0.17 **b** $\frac{21}{100}$, 0.21

c $\frac{7}{20}$, 0.35 **d** $\frac{2}{25}$, 0.08

e $2\frac{3}{20}$, 2.15 **f** $\frac{49}{50}$, 0.98

g $1\frac{37}{50}$, 1.74 **h** $\frac{7}{25}$, 0.28

5 **a** $\frac{7}{100}$, 0.07 **b** $\frac{31}{1000}$, 0.031

c $1\frac{31}{50}$, 1.62 **d** $\frac{721}{1000}$, 0.721

e $3\frac{47}{100}$, 3.47 **f** $\frac{8109}{10\,000}$, 0.8109

g $\frac{1057}{10\,000}$, 0.1057 **h** $\frac{339}{5000}$, 0.0678

6 **a** 71%, $\frac{71}{100}$ **b** 28%, $\frac{7}{25}$

c 5%, $\frac{1}{20}$ **d** 8%, $\frac{2}{25}$

e 90%, $\frac{9}{10}$ **f** 31%, $\frac{31}{100}$

g 54%, $\frac{27}{50}$ **h** 95%, $\frac{19}{20}$

7 **a** 36.9%, $\frac{369}{1000}$ **b** 24.8%, $\frac{31}{125}$

c 1006%, $10\frac{3}{50}$ **d** 81%, $\frac{81}{100}$

e 705.7%, $7\frac{57}{1000}$ **f** 185.9%, $1\frac{859}{1000}$

g 1210%, $12\frac{1}{10}$ **h** 24.46%, $\frac{1223}{5000}$

8 **a** **i** $\frac{16}{20}$ **ii** $\frac{80}{100}$ **iii** 80%

b **i** $\frac{15}{20}$ **ii** $\frac{75}{100}$ **iii** 75%

c **i** $\frac{7}{10}$ **ii** $\frac{70}{100}$ **iii** 70%

d **i** $\frac{6}{10}$ **ii** $\frac{60}{100}$ **iii** 60%

9 **a** 0.6, 60% **b** 0.7, 70%

c 0.085, 8.5% **d** 0.92, 92%

e 1.3, 130% **f** 1.5, 150%

g 0.0275, 2.75% **h** 3.5, 350%

10 **a** 225% **b** 600.5% **c** 730% **d** 375%

e 428% **f** 580% **g** 102.4% **h** 1326%

11 **a** 62.5% or 0.625 **b** 67.5% or 0.675

c $533\frac{1}{3}$% or $5.\dot{3}$ **d** 18.75% or 0.1875

e 8.8% or 0.088 **f** $322\frac{2}{9}$% or $3.\dot{2}$

g $73\frac{1}{3}$% or $0.7\dot{3}$ **h** $142\frac{6}{7}$% or $1.\overline{428\,571}$

12 **a** $\frac{1}{400}$, 0.0025 **b** $\frac{11}{300}$, $0.03\dot{6}$

c $\frac{1}{160}$, 0.006 25 **d** $\frac{1}{40}$, 0.025

e $\frac{11}{125}$, 0.088 **f** $\frac{47}{300}$, $0.15\dot{6}$

g $\frac{8}{225}$, $0.03\dot{5}$ **h** $\frac{79}{600}$, $0.131\dot{6}$

13 **a** $0.46 < \frac{1}{2}$ **b** $\frac{3}{5} > 57\%$

c $0.95 > 71\%$ **d** $59\% > \frac{4}{7}$

e $\frac{5}{4} > 122\%$ **f** $252\% < 21.2$

g $13.4 > 1\frac{3}{4}$ **h** $1\frac{1}{6} > 206\%$

14 **a** $\frac{3}{5} = \frac{60}{100} = 60\%$ **b** $\frac{9}{20} = \frac{45}{100} = 45\%$

c $\frac{17}{25} = \frac{68}{100} = 68\%$ **d** $\frac{46}{200} = \frac{23}{100} = 23\%$

e $\frac{17}{20} = \frac{85}{100} = 85\%$ **f** $\frac{1}{5} = \frac{20}{100} = 20\%$

g $\frac{144}{600} = \frac{24}{100} = 24\%$ **h** $\frac{75}{250} = \frac{3}{10} = \frac{30}{100} = 30\%$

15 **a** 80% **b** 81% **c** 92%

d 88% **e** 71.5% **f** $66\frac{2}{3}$%

16 **a** Both are correct.

b Sample answers include: shade 30 out of 200, shade 60 out of 400.

17 As the capacity of each container is different, 50% (or 50 mL out of every 100 mL) would be a different volume of liquid for each.

18 All percentages can be converted to fractions with the numerator and denominator both being integers, therefore all percentages are rational numbers.

19 a $\frac{3}{4}$, $\frac{4}{5}$, 0.89, 1.3, $\frac{7}{5}$, 200%

b $4\frac{3}{10}$ 4.9, $\frac{22}{4}$, 560%, $\frac{13}{2}$, 7.2

c 0.28, 65%, $\frac{3}{2}$, $1\frac{4}{5}$, $\frac{16}{5}$, 340%

20 a 66% **b** $55\frac{1}{2}\%$ **c** $79\frac{2}{3}\%$ **d** 83.1%

e 24% **f** $4\frac{2}{5}\%$ **g** 74.7% **h** 20.44%

21 a $\frac{3}{13}$ **b** 0.2308 **c** 23.08%

d i 30.77% **ii** 7.69%

iii 38.46% **iv** 84.62%

22 $\frac{21}{400}$

23 Sample answer: 0, $\frac{10}{11}\%$, 0.52, $\frac{7}{11}$, 0.76, $\frac{88}{115}$, 100%

24 a 10 cm = 100 mm, so a length of 1 mm would represent 1%. For 80%, shade 80 mm (or 8 cm); for 92%, shade 92 mm (or 9.2 cm); for 25%, shade 25 mm (or 2.5 cm); for 17%, shade 17 mm (or 1.7 cm); for 43%, shade 43 mm (or 4.3 cm); for 66%, shade 66 mm (or 6.6 cm).

b Double the percentage amount. For example, 80% means 80 out of 100 or 160 out of 200. Shade 160 of the 200 squares in the grid to represent 80%.

EX p116 3.2 Calculating percentages

1 a 25% **b** 90% **c** 76% **d** 75%

e 120% **f** 125%

2 a i 80% **ii** 20%

b i 70% **ii** 30%

c i 30% **ii** 70%

d i 90% **ii** 10%

3 a i $17\frac{1}{2}\%$ **ii** 17.5%

b i $\frac{1}{5}\%$ **ii** 0.2%

c i $36\frac{4}{5}\%$ **ii** 36.8%

d i $28\frac{1}{2}\%$ **ii** 28.5%

e i $72\frac{1}{2}\%$ **ii** 72.5%

f i $587\frac{1}{2}\%$ **ii** 587.5%

4 a 23% **b** 25% **c** 35%

d 70% **e** 7.5%

5 a $1\frac{1}{4}\%$ or 1.25% **b** 10%

c 25% **d** 75%

e 10% **f** $6\frac{1}{4}\%$ or 6.25%

g 3% **h** 25%

6 a 84 **b** 756 **c** 9 **d** 1152

e 725 **f** 112.5

7 a 453.6 m **b** 135 seconds

c 231 cm **d** 100.8 L

e 76 buttons **f** 11.76 mL

8 a \$360 **b** \$1250 **c** \$100 **d** \$1575

e \$16 **f** \$576.45 **g** \$110.50 **h** \$448.50

i \$69.30 **j** \$129.15 **k** \$41.29 **l** \$3690

9 a \$4440 **b** 114 kg **c** \$1030.96

d \$786.25 **e** \$89.03 **f** 67.5 min

g 5.84 km **h** 381.84 L

10 a \$400 **b** \$850 **c** \$3200

d 150 **e** 270 **f** \$900

11 a 460 **b** 1750 L

c 105 songs **d** 9 minutes

e 66 minutes **f** 4 hours 26 minutes

12 a $\frac{89}{420}$ **b** 21.19%

c i 26.97% **ii** 74.17% **iii** 127.27%

d i Sample answer: Big Jake may be 252 cm and Thumbelina may be 53 cm.

ii Assuming the proportions of the horses remain the same, using the heights in part **d**, the percentage from part **b** changes to 21.03% (not much difference!).

13 a 40 **b** 8

c Part **b** calculates 20% of the remaining cupcakes not the original amount.

d 32

14 a i six people **ii** increase **iii** 24%

b i 35 chocolate bars **ii** decrease

iii 87.5% or $87\frac{1}{2}\%$

c i 0.5 km **ii** increase

iii $33.\dot{3}\%$ or $33\frac{1}{3}\%$

d i 15 cm **ii** increase **iii** 10%

e i 2 minutes **ii** decrease **iii** 10%

15 a Tom: 1 cm, Lisa: 1 cm

b Tom: 2.3%, Lisa: 1.2%; Tom, because his error is compared to a smaller value.

16 1.59%

17 494 wombats

18 a minimum 45 807, maximum 82 743

b 40 544 **c** 36 490

19 a 64 **b** 98 **c** 460

20 a 44% **b** 44%

c The percentage increase is the same for both shapes.

d When the side lengths of any shape are increased by 20%, the area increases by 44%.

21 a 4 kg **b** 1.2 kg **c** 24%

EX p121 3.3 Mark-ups and discounts

1 a \$36.05 **b** \$1060.50 **c** \$110.15 **d** \$480

e \$1615 **f** \$70.08 **g** \$233.28 **h** \$3337.61

2 a \$1560 **b** \$336.75 **c** \$1350

d \$260.75 **e** \$948.15 **f** \$465

g $552 **h** $787.50 **i** $323.75

3 **a** $142.10 **b** $36.25 **c** $94.25 **d** $147.90

4 **a** $1413.50 **b** $264 **c** $661.60
d $107.25 **e** $346.12 **f** $80.25

5 **a** **i** $194.40 **ii** $74.40
b **i** $778.70 **ii** $419.30
c **i** $42.42 **ii** $7.49

6 **a** $300 **b** $31.25 **c** $13.95
d $86.65 **e** $11 422.85 **f** $624.60
g $49.90

7 **a** $141.50 **b** $214.50 **c** $165.45
d $109.55

8 **a** 87.5% **b** $138.20

9 **a** **i** $440 **ii** $60
b **i** $152.58 **ii** $26.93
c **i** $229.08 **ii** $19.92
d **i** $860.11 **ii** $35.84

10 **a** $820 **b** $600 **c** $1.41 **d** $1120

11 **a** $44.60 **b** $37.91 **c** $82.51

12 **a** 210% **b** $188.80

13 **a** **i** $768.80 **ii** $148.80
b **i** $130.43 **ii** $40.48
c **i** $2284.20 **ii** $1015.20
d **i** $835.68 **ii** $385.18

14 Jane is finding 20% of the discount price, not the original price. Her calculations will give: $0.2 \times \$198 = \$39.60$
Original price = $198 + $39.60 = $237.60
Find 20% of $237.60 and see if it gives a reduced price of $198.
A 20% discount means you pay 80% or 0.8 of the original price of the bike:
$198 ÷ 0.8 = $247.50
20% of $247.50 gives a reduced price of $198.
Hence, Tim is correct.

15 $247.50

16 70% of $699 = $489.30; 80% of $699 is $559.20, then 90% of $559.20 is $503.28; the customer is correct.

17 **a** $2800 **b** $980

18 **a** **i** $5.55 **ii** $1.77 **iii** $2.85
iv $25.73 **v** $3.56 **vi** $6.56
b **i** 43.04% **ii** 42.74% **iii** 43.22%
iv 43.02% **v** 42.97% **vi** 42.92%
c When you increase the price by 40%, you find 140%, or 1.4 times the original. When you increase the price by 30% then 10%, you first find 130% of, or 1.3 times, the original price, then you find 110% of, or 1.1 times, this second price. Increasing by 30% then 10% means multiplying by 1.3, then 1.1, which is equivalent to multiplying by $1.3 \times 1.1 = 1.43 = 143\% = 100\% + 43\%$. Increasing by 30%, then 10% is the same as increasing by 43%. The percentage is not exactly 43% because of errors introduced by rounding.

19 'All stock discounted by 45%' means all stock will receive a discount of 45%. 'Stock discounted by up to 45%' means the discount can be any amount from 0% to 45%; for example, 10%, 20%, 35%, 45%.

20 **a** $1161.62 **b** 1085.10 **c** 1277.78
d It does not matter as multiplication is commutative. That is, a 45% discount then a 10% discount is the same as a 10% discount then a 45% discount:
$0.55 \times 0.90 = 0.90 \times 0.55$

3.4 Profit and loss

EX p125

1 **a** profit of $10 **b** loss of $14
c loss of $4.55 **d** loss of $154.48

2 **a** **i** loss of $60 **ii** 25%
b **i** profit of $7.30 **ii** 292%
c **i** loss of $0.40 **ii** 7.69%
d **i** loss of $5390 **ii** 21.88%
e **i** profit of $230 **ii** 191.67%
f **i** loss of $21.95 **ii** 73.29%

3 **a** **i** 33.33% **ii** 74.49%
iii 8.33% **iv** 28%
v 65.71% **vi** 274.38%
b If a loss occurred the percentages will be greater, but if a profit occurred the percentages will be less.

4 **a** 28.57% **b** 17.07% **c** 4.92% **d** 51.50%

5 **a** loss of $48 **b** 24.24%

6 **a** $252 **b** 375%

7 **a** **i** loss of $14 **ii** 29%
b **i** profit of $127.50 **ii** 113%
c **i** profit of $50.05 **ii** 139%
d **i** loss of $411 **ii** 25%
e **i** loss of $12 395 **ii** 41%
f **i** profit of $268 865 **ii** 171%

8 **a** $2.55 per kg **b** 86%

9 **a** $27.45 **b** 11%

10 **a** **i** $250 **ii** $150
b **i** $2000 **ii** $1200
c **i** $1000 **ii** $750
d **i** $213.75 **ii** $146.25
e **i** $223.84 **ii** $96
f **i** $491.53 **ii** $432.25

11 **a** No, as it makes Joseph's cars more expensive than his competitor's.
b 111% profit
c A maximum percentage increase enables Joseph to keep his selling price below the selling price of his competitor's cars while maximising the profit.

12 **a** 25% **b** 125%
c 125% = 100% (the purchase price)
+ 25% (the price increase)

13 a 20% **b** 80%

c 20% + 80% = 100%
If percentage decrease is subtracted from 100%, the remaining percentage value gives the percentage of the original price of car.

14 a 75% **b** 25%

15 a 100% increase **b** 200% increase

c 0% increase or decrease **d** 50% decrease

e 300% increase **f** 100% decrease

16

Commodity	Starting price $	Final price $	Movement %
Gold	1741.66	1732.95	↓ 0.5
Silver	34.04	33.56	↓ 1.4
Oil	102.15	102.46	↓ 0.3
Copper	3.67	3.71	↓ 1.2

17 a $1 569 600

b i $1 874 610 **ii** $33 390

c profit of $305 010 **d** 19.43%

18 a $15.30 **b** $16.83

c $16.15; less than price in part **b**, because it doesn't take into account that GST is 10% of the marked-up price, not the wholesale price.

19 a i 60% **ii** 166.67%
iii 250% **iv** 150%

b i 60% **ii** 166.67%
iii 250% **iv** 150%

c i 75% **ii** 133.33%
iii 400% **iv** 300%

d i 68.42% **ii** 146.15%
iii 316.67% **iv** 216.67%

e i 42.89% **ii** 233.17%
iii 175.09% **iv** 75.09%

f i 87.94% **ii** 113.71%
iii 829.17% **iv** 729.17%

20

Cost price	Selling price	Cost price as a percentage of selling price	Number of sales
$3.60	$12	30%	35
$5	$15	33.33%	20
$13	$21	61.90%	37
$1.10	$2.50	44%	150
$25.90	$35	74%	25
$5.44	$8	68%	125
$50.49	$99	51%	8
$0.28	$1.12	25%	271
$24.32	$32	76%	19
$3.60	$6	60%	15

Revenue	Total profit	Profit as a percentage of the revenue
$420	$294	70%
$300	$200	66.67%
$777	$296	38.10%
$375	$210	56%
$875	$227.50	26%
$1000	$320	32%
$792	$388.08	49%
$303.52	$227.64	75%
$608	$145.92	24%
$90	$36	40%

21 a To decrease by 30%, we can multiply by (100% − 30%) = 70%, which is the same percentage we would multiply by to find 70% of its value.

b To increase by 30%, we can multiply by (100% + 30%) = 130%, which is the same percentage we would multiply by to find 130% of its value.

22 a $22 000 **b** $2000 loss

23 a selling price = wholesale price × 1.8 × 1.1
= wholesale price × 1.98
He can add 98% to the wholesale price or multiply the wholesale price by 1.98.

b $1200 = wholesale price × 1.6 × 1.1
= wholesale price × 1.76
So wholesale price = $1200 ÷ 1.76 = $681.82
The single calculation is dividing by 1.76.

c $400 = original price × 1.1 × 0.75
= original price × 0.825
So original price = $400 ÷ 0.825 = $484.85
The single calculation is dividing by 0.825.

24 56.14%

EX p129

Checkpoint

1 a $\frac{1}{4}$, 0.25 **b** $\frac{19}{25}$, 0.76 **c** $\frac{1}{25}$, 0.04

d $\frac{1}{200}$, 0.005 **e** $1\frac{1}{5}$, 1.2 **f** $\frac{16}{625}$, 0.0256

2 a 63%, $\frac{63}{100}$ **b** 12%, $\frac{3}{25}$ **c** 85%, $\frac{17}{20}$

d 330%, $3\frac{3}{10}$ **e** 0.6%, $\frac{3}{500}$ **f** 750%, $7\frac{1}{2}$

3 a 0.375, 37.5% **b** 0.325, 32.5%

c $5.\dot{6}$, $566\frac{2}{3}\%$ **d** 0.3125, 31.25%

e 0.44, 44% **f** $8.1\dot{6}$, $816\frac{2}{3}\%$

4 a 20% **b** 25% **c** 12.5% **d** 12.5%

e 20% **f** $1\frac{1}{3}\%$

5 a 133.4 L **b** $1800 **c** 19.6 cm **d** 306 m

e 488 beans **f** $192.50

6 a $11 520 **b** 700 km **c** $4927.50 **d** $42.37

e 8.45 km **f** 108 minutes

7 **a** $2880 **b** $807.50 **c** $7896
d $956.25 **e** $1159.10 **f** $900

8 **a** **i** $3.20 **ii** $35.20
b **i** $8.50 **ii** $93.50
c **i** $125 **ii** $1375
d **i** $0.35 **ii** $3.85
e **i** $2.60 **ii** $28.60
f **i** $3.50 **ii** $38.50

9 **a** $450 **b** $9800 **c** $8600 **d** $8455
e $700 **f** $25 302.50

10 **a** **i** loss of $1750 **ii** 9.46%
b **i** profit of $60 **ii** 400%
c **i** profit of $88 **ii** 733.33%
d **i** loss of $1499 **ii** 37.48%

11 **a** **i** $770 **ii** $280
b **i** $495 **ii** $231
c **i** $687.50 **ii** $385
d **i** $330 **ii** $114

EX p133 3.5 Ratios

1 **a** 3 : 4 **b** 3 : 7 **c** $\frac{3}{7}$ **d** $\frac{4}{7}$

2 **a** **i** 11 : 8 : 7 **ii** 9 : 40
b **i** $\frac{13}{40}$ **ii** 40%

3 **a** 45 **b** 10 **c** 72 **d** 2
e 95 **f** 7 **g** 6 **h** 8
i 16 **j** 80 **k** 1 **l** 35
m 12 **n** 9 **o** 44 **p** 48

4 **a** 2 : 5 = 8 : 20 = 20 : 50
b 6 : 7 = 30 : 35 = 54 : 63
c 420 : 360 = 70 : 60 = 7 : 6
d 225 : 300 = 9 : 12 = 3 : 4
e 136 : 104 = 68 : 52 = 17 : 13
f 180 : 48 = 75 : 20 = 15 : 4
g 20 : 48 = 40 : 96 = 100 : 240
h 216 : 636 = 72 : 212 = 18 : 53

5 **a** Yes. The second ratio is obtained by multiplying the first ratio by 3.
b No
c Yes. The second ratio is obtained by dividing the first ratio by 2.
d Yes. The second ratio is obtained by multiplying the first ratio by 5.
e No **f** No **g** No
h Yes. The second ratio is obtained by dividing the first ratio by 3.
i No
j Yes. The second ratio is obtained by multiplying the first ratio by 4.
k No
l Yes. The second ratio is obtained by dividing the first ratio by 9.

6 **a** 1 : 12 **b** 3 : 4 **c** 53 : 18
d 2 : 5 **e** 23 : 59 **f** 6 : 7
g 1 : 3 **h** 162 : 35 **i** 2 : 3
j 8 : 9 **k** 5 : 6 **l** 33 : 14
m 85 : 16 **n** 4 : 5 (16 : 20)
o 2 : 1 **p** 6 : 1

7 **a** 20 : 33 **b** 95 : 57 **c** 113 : 167
d 256 : 175 **e** 499 : 575 **f** 23 : 2
g 4 : 13 **h** 19 : 21 **i** 42 : 117
j 16 : 39 **k** 19 : 11 **l** 23 : 7 : 3
m 15 :17 : 19 **n** 5 : 7 : 11

8 **a** 13 : 21 **b** 120 : 17 **c** 27 : 500
d 4 : 3 **e** 1700 : 793 **f** 7 : 36
g 509 : 1300 **h** 13 : 640 **i** 17 : 2340
j 5 : 6 **k** 18 : 11 **l** 1 : 4
m 5000 : 23 : 673 **n** 5631 : 1753 : 1 000 000

9 **a** 13 : 1 **b** 1 : 9 **c** 17 : 9 **d** 4 : 7

10 3 : 17

11 **a** Flour and butter because the quantities are in the same units
b Flour to butter is 300 : 60 = 5 : 1. (Butter to flour is 60 : 300 = 1 : 5.)
c For 50 pancakes (10 × 5), multiply each number in the ratio by 5 to keep in proportion. 300 : 60 = 1500 : 300

12 12

13 **a** 41 : 9 **b** $\frac{9}{50}$

14 80 : 332 (or 20 : 83)

15 For every four games she wins, she loses five; or out of every nine games she plays, she wins four and loses five.

16 **a** Measurements must be put into same units.
b Convert one to same unit as the other measurement.
c 1670 feet = (1670 × 0.3048) m ≈ 509 m; 270 : 509

17 **a** No, units are not the same.
b 57 : 2500
c More water will result in a weaker solution so increase the second value in the ratio. Less bleach will also result in a weaker solution so decrease the first value in the ratio. Both options could also apply.

18 **a** 41 : 204 **b** 41 : 163

19 **a** 5 : 2 **b** 3 : 5 **c** 3 : 2

20 **a** $\Phi = \frac{x}{y}, \Phi = \frac{x+y}{x}$
When $y = 1$, $\Phi = x$, $\Phi = \frac{x+1}{x}$
When $y = 1$, the value of the Golden Ratio $\Phi = x$, and the longer length $x = \frac{(x+1)}{x}$.
b $\Phi^2 = \Phi + 1$
c $1.618\,034^2 \simeq 2.618\,034$
$1.618\,034 + 1 = 2.618\,034$
Both sides of the equation give the same value.

21 Sample answer

1st ratio	2nd ratio
3 : 5	9 : 15
7 : 20	14 : 40
4 : 7	40 : 70
5 : 25	1 : 5
25 : 30	5 : 6
3 : 7	12 : 28
8 : 16	4 : 8
8 : 12	2 : 3
8 : 24	1 : 3
2 : 4	1 : 2

EX p138 3.6 Modelling with ratios

1 **a** 50 and 450 **b** 100 and 400
c 150 and 350 **d** 300 and 200
e 300 and 200 **f** 400 and 100

2 **a** 21 cups and 14 cups **b** 28 cups and 7 cups
c $24\frac{1}{2}$ cups and $10\frac{1}{2}$ cups
d $31\frac{1}{2}$ cups and $3\frac{1}{2}$ cups
e $10\frac{1}{2}$ cups and $17\frac{1}{2}$ cups and 7 cups
f $10\frac{1}{2}$ cups and 14 cups and $10\frac{1}{2}$ cups

3 **a** \$5520 and \$3680 **b** \$5750 and \$3450
c \$6900 and \$2300 **d** \$4600 and \$4600
e \$4200 and \$5000 **f** \$1840 and \$7360

4 **a** \$80 and \$160 **b** \$216 and \$24
c \$40 and \$200 **d** \$150 and \$90
e \$90, \$30 and \$120 **f** \$168, \$48 and \$24

5 **a** \$2.70 and \$0.90 **b** \$0.40 and \$3.20
c \$0.60 and \$0.90 and \$2.10

6 **a** $\frac{16}{45}$ and $\frac{8}{45}$ **b** $\frac{8}{25}$ and $\frac{16}{75}$
c $\frac{16}{75}$ and $\frac{8}{45}$ and $\frac{32}{225}$

7 **a** 4000 non-fiction books **b** 7500 books

8 **a** 15 tomato seedlings **b** 36 seedlings

9 72 pens

10 **a** 1.05 L **b** 8000 mL

11 **a** 21.25 L **b** 6000 mL

12 64

13 **a** 12 **b** 27

14 60°, 80°, 40°

15 **a** 27.84 cm **b** 69.6 cm

16 \$28

17 **a** 4 : 2 : 1 **b** $\frac{2}{7}$ **c** 120 biscuits

18 The ratio of the ages simplifies to 8 : 7 : 6. Adding them up and dividing \$210 by the total gives one part equal to \$10. The money divided by the girls' ages would be \$80 to Lydia, \$70 to Sylvia and \$60 to Julia. Dividing the money into thirds gives each girl \$70 each. In both cases, Sylvia gets the same amount of money.

19 **a** $\frac{3}{5}$ **b** 3 : 5 : 12
c 15% to a present, 25% to trophies, 60% to snacks
d \$30 on a present, \$50 on trophies, \$120 on snacks

20 **a** 7 : 2 : 5 **b** 4 : 1 : 2
c 16 : 5 : 6 **d** 12 : 1 : 2

21 **a** width 72 cm, height 42 cm **b** 3024 cm^2

22 5 L

23 12 : 40 : 35

24 **a** Can A has 3200 mL of white paint and 800 mL of red paint.
Can B has 3500 mL of white paint and 500 mL of red paint.
b 67 : 13 **c** $\frac{13}{80}$ **d** $83\frac{3}{4}$% or 83.75%
e To make 4 : 1 from 67 : 13, one option could be to add 1 part red and 3 parts white. To make 7 : 1 from 67 : 13, one option could be to add 24 parts white.

25 180

EX p144 3.7 Rates

1 **a** 60 km/h **b** 591 L/min
c four quarters/game **d** \$16/h
e 45 cm/year **f** five patients/h
g \$1.49/day **h** \$28.99/month
i \$5.80/box **j** 0.5 kg/week
k 290 mm/year **l** 14 km/L

2 **a** m/s **b** m/s **c** cents/apple or \$/kg
d \$/m^2 **e** L/min **f** km/h

3 **a** 50 m/min **b** 2 L/bottle
c 4 cents per second **d** 15 students/teacher
e 26 seats/row **f** 25 newspapers/bundle
g 6.8 runs/over **h** \$2.50/muffin
i 6.8 goals/game **j** 4444 tickets/min
k 3.5 goals/game **l** 5.25 m^2/L

4 **a** 14 km/L **b** \$1.99/mango
c 12 L/min **d** 15.3 km/h
e 30.18 people/km^2 **f** 18 million km/min

5 **a** 2400 words **b** 800 words **c** 45 minutes

6 **a** \$15.20 **b** \$608 **c** 96 hours

7 **a** 386 m **b** 1 389 600 m **c** $33\frac{1}{3}$ minutes

8 \$355.50

9 **a** 54.75 km/h **b** 854.35 km/h **c** 32.5 km/h
d 0.38 km/min **e** 2.10 m/s **f** 1.89 m/s

10 **a** \$0.0012/mL **b** \$25 000/kg **c** 10 m/s
d 0.18 km/h **e** 200 kg/L **f** 14.2 g/mL

11 **a** Alexis 350 m per min, Kym 340 m per min
b Alexis, as she travelled a greater distance per minute
c Speed is distance (m) per one unit of time (min) so speed unit is m per min or m/min.

12 **a** Max 5.75 runs/over, William 5.6 runs/over
b Max's team

13 **a** 56 bpm **b** 27 min

14 a 0.3125 km (312.5 m)

b i 17.1875 km **ii** 18.75 km
iii 26.5625 km **iv** 28.125 km

c i 2 min **ii** 1 h 40 min
iii 1 h 20 min **iv** 6 min 24 s

15 a 7.5 kg of potatoes ($0.95/kg compared to $0.99/kg)

b packet of 10 batteries ($0.989/battery compared to $0.9975/battery)

c 500 g tin of coffee ($0.018/g compared to $0.01993/g)

d 650 g tin of peaches ($0.0039/g compared to $0.0042/g)

16 a Plan A: 80c, Plan B: 88c; Plan A

b Plan A: $1.60, Plan B: $1.64; Plan A

c Plan A: $2.40, Plan B: $2.40; both plans are the same.

d Plan A: $3.20, Plan B: $3.16; Plan B

e Use Plan A if most calls are shorter than 5 min, use Plan B if most calls are longer than 5 min, use either plan for calls of exactly 5 min.

17 €80 = $125, Martha can't afford it.

18 a 285 g **b** 35 cm^3 **c** 8.14 g/cm^3

19 a $5 per year **b** $100 **c** $I = PrT$

20 6 minutes. When doubling the amount of window washers and the number of windows, the minutes taken remains the same.

21 260.5 m

22 a $1000 **b** $4588

c i $1888 **ii** $10 288
iii $34 988 **iv** $58 388

d They are the amount of tax paid on earnings below the corresponding row in the table.
$4288 = 0.16 × ($45 000 − $18 200)
$31 288 = $4288 + 0.3 × ($135 000 − 45 000)
$51 6380 = $31 288 + 0.37 × ($190 000 − $135 000)

EX p148

Review: Percentages, ratios and rates

Review questions 3.8A: Mathematical literacy review

1 speed

2 A ratio is a comparison between two or more quantities with the same unit, whereas a rate is a comparison between two or more quantities with different units.

3 A profit is when the selling price is greater than the cost price, and a loss is when the cost price is greater than the selling price.

4 a rate **b** cost price

5 a an amount added to the cost price in order to obtain a profit

b income generated by the sale of goods or services

6 a A ratio is written in simplest form when the only common factor between the terms in the ratio is 1.

b A percentage is a proportion of a whole that has been divided into one hundred parts.

Review questions 3.8B: Multiple choice

1 D **2** C **3** B **4** A **5** B **6** D
7 D **8** A **9** C **10** B **11** D

Review questions 3.8C: Short answer

1 a 7 **b** $\frac{7}{20}$ **c** $\frac{35}{100}$ **d** 35%

2 a 0.23 **b** 0.62 **c** 0.1652 **d** 2.916

3 a 32% **b** 762% **c** 3162.5% **d** 0.03%

4 a 75% **b** 70% **c** 85% **d** 46.875%

5 a 40% **b** 16% **c** 46.4%

6 a 34 **b** 15.5 **c** 2250 **d** 888

7 a $1045 **b** $904.32 **c** $903.60

8 a 37.5% profit **b** 40% loss
c 51.68% profit **d** 31.57% loss

9 a profit of $5.64 **b** loss of $3.75
c profit of $33.20

10 a 76% **b** 15% **c** 74%

11 a 35 : 47 **b** 7 : 31 **c** 2 : 9 **d** 9 : 25

12 a 3 : 11 **b** 16 : 27 **c** 1 : 2 : 6 **d** 1 : 5 : 7

13 a 42 **b** 16 **c** 12 **d** 35

14 a 8 **b** 17 **c** 17 **d** 31

15 a $600, $900 **b** $450, $1050
c $300, $400, $800 **d** $300, $375, $825

16 a 100 km/h **b** 3 L/s
c $17.50/h **d** 4.333 books/month

17 a 1.5 points/min **b** $1.39/L
c $0.0048/mL or 0.48 c/mL **d** $16.80/h

Review questions 3.8D: Mathematical modelling

1 a i $\frac{13}{20}$ **ii** 0.65

b $13 **c** $33 **d** $455

e i $3.30 **ii** $29.70 **iii** $9.70 **iv** 48.5%

2 a 3 : 5 **b** $65.60

c i 2.4 cubic metres **ii** $512

d i 12 **ii** 6
iii 2 Grevilleas, 3 Callistemons, and 4 Banksias

3 a 3 : 2 **b** 9 : 5 **c** 2 cups

d $1\frac{1}{5}$ cups of pecans, $1\frac{1}{5}$ cups of cashews, $\frac{4}{5}$ cup of sunflower seeds and $\frac{4}{5}$ cup of pumpkin seeds.

MODULE 4 Exponents and roots

EX p156

4.1 Exponents

1 a 5 **b** 3 **c** 5 × 5 × 5 **d** 125

2 a 2 × 2 × 2 × 2 × 2 = 32

b 4 × 4 × 4 × 4 × 4 × 4 = 4096

c 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 = 1

d 3 × 3 × 3 × 3 × 3 = 243

e 10 × 10 × 10 × 10 × 10 × 10 × 10 = 10 000 000

f 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 = 256

g 5 × 5 × 5 × 5 = 625

h 7 × 7 × 7 × 7 × 7 = 16 807

i $8 \times 8 \times 8 = 512$

j $6 \times 6 \times 6 \times 6 = 1296$

3

	Exponent form	Base	Exponent	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	81
b	2^5	2	5	$2 \times 2 \times 2 \times 2 \times 2$	32
c	8^3	8	3	$8 \times 8 \times 8$	512
d	6^5	6	5	$6 \times 6 \times 6 \times 6 \times 6$	7776

4 D

5 **a** $(-6) \times (-6) \times (-6) = -216$

b $(-7) \times (-7) \times (-7) \times (-7) = 2401$

c $(-5) \times (-5) \times (-5) \times (-5) = 625$

d $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$

e $(-4) \times (-4) \times (-4) \times (-4) \times (-4) = -1024$

f $(-10) \times (-10) \times (-10) \times (-10) \times (-10) = -100\,000$

6 **a** $0.8 \times 0.8 = 0.64$

b $3.4 \times 3.4 \times 3.4 = 39.304$

c $(-0.5) \times (-0.5) \times (-0.5) \times (-0.5) = 0.0625$

d $0.3 \times 0.3 \times 0.3 \times 0.3 = 0.0081$

e $0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.000\,01$

f $(-0.2) \times (-0.2) \times (-0.2) \times (-0.2) \times (-0.2) \times (-0.2) \times (-0.2) \times (-0.2) = -0.000\,012\,8$

7 **a** $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

b $\left(-\frac{7}{9}\right) \times \left(-\frac{7}{9}\right) = \frac{49}{81}$

c $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{625}{1296}$

d $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{81}{10\,000}$

e $\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{1}{32}$

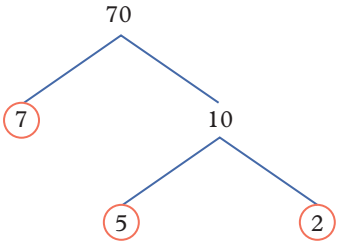
f $3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} = \frac{1\,000\,000}{729}$

8 **a** 10 556 001 **b** 1 030 301 **c** −62 748 517

d 321. 419 125 **e** 13.0321 **f** 293 162.506 24

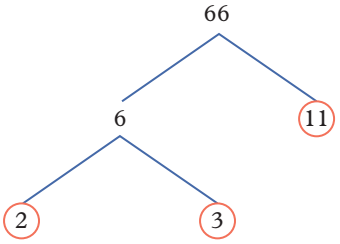
g $\frac{5832}{12\,167}$ **h** $\frac{117\,649}{1\,771\,561}$ **i** $-\frac{19\,683}{1\,953\,125}$

9 **a**



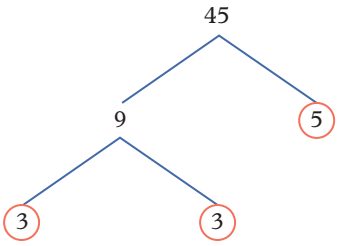
$70 = 2 \times 5 \times 7$

b



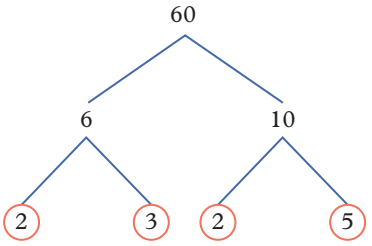
$66 = 2 \times 3 \times 11$

c



$45 = 3^2 \times 5$

d



$60 = 2^2 \times 3 \times 5$

10 **a** $75 = 3 \times 5^2$

b $48 = 2^4 \times 3$

c $90 = 2 \times 3^2 \times 5$

d $126 = 2 \times 3^2 \times 7$

e $288 = 2^5 \times 3^2$

f $1092 = 2^2 \times 3 \times 7 \times 13$

11 **a** 160 **b** 206

c 4.3601 **d** $\frac{11}{10}$ $\left(\text{or } 1\frac{1}{10}\right)$

12 **a** $(-5)^3 \times (-9)^4$ **b** $(-4)^6 \times 3^3$

c $7^2 \times (-6)^6$ **d** $(-8)^4 \times (-10)^2$

13 **a** $(-2) \times (-2) \times (-2) \times (-4) \times (-4) = -128$

b $(-5) \times (-5) \times 3 \times 3 \times 3 \times 3 = 2025$

c $(-10) \times (-10) \times (-10) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 32\,000$

d $(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-1) \times (-1) = -243$

e $(-6) \times (-6) \times (-2) \times (-2) \times (-2) = -288$

f $(-1) \times (-1) \times (-1) \times (-1) \times 3 \times 3 = 9$

14 **a** **i** −1 **ii** 1 **iii** −1

iv 1 **v** −1 **vi** 1

b those with an even exponent (**ii**, **iv**, **vi**)

c those with an odd exponent (**i**, **iii**, **v**)

d Negative numbers raised to an even power will give a positive result and negative numbers raised to an odd power will give a negative result.

e **i** −1 **ii** 1 **iii** 1

iv −1 **v** 1 **vi** −1

15 **a** negative **b** positive

c negative **d** positive

16 **a** $20^4 = 2^8 \times 5^4$, $20^5 = 2^{10} \times 5^5$

b $20^n = 2^{2n} \times 5^n$

17 **a** $2^3 \times 3 \times 7$ **b** $2 \times 3 \times 7$

c $2^2 \times 7$ **d** $2^2 \times 3 \times 5 \times 7$

18 **a** 490 **b** 140 **c** 1960 **d** 4900

19 a \$180

b i \$3, \$9, \$27, \$81, \$243, \$729

ii \$30 per week

iii $\$3^{11}$

iv \$177 147

20 a i 2000 cells **ii** 4000 cells **iii** 8000 cells

b 2000, 4000, 8000, 16 000, 32 000

c i 2 **ii** 4 **iii** 8

d 2, 4, 8, 16, 32

e $2^1, 2^2, 2^3, 2^4, 2^5$

f i 2^6 **ii** 2^{10} **iii** 2^{24}

g 16 777 216

h 16 777 216 000

21 a $144 = 2^4 \times 3^2$

b $270 = 2 \times 3^3 \times 5$

c HCF $= 2 \times 3^2 = 18$

d i 13 **ii** 12

iii 55 **iv** 27

22 a $30 = 2 \times 3 \times 5$ **b** $63 = 3^2 \times 7$

c 630

d i 1260 **ii** 288 **iii** 525 **iv** 480

23 a $150 = 2 \times 3 \times 5^2$ **b** 12

c 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150

d i 16 **ii** 10 **iii** 6 **iv** 12

24 60 days

25 $\frac{5}{4}c$

EX p161 4.2 Products and quotients of powers

1 a 3^5 **b** 4^8 **c** 6^6 **d** 5^9 **e** 8^3 **f** 2^5

2 a 243 **b** 65 536 **c** 46 656

d 1 953 125 **e** 512 **f** 32

3 a 9^3 **b** 4^1 **c** 6^3

d 4^5 **e** 10^3 **f** 6^4

4 a 729 **b** 4 **c** 216

d 1024 **e** 1000 **f** 1296

5 a $2^{11} = 2048$ **b** $5^6 = 15\,625$

c $(-3)^7 = -2187$ **d** $4^8 = 65\,536$

e $5^3 = 125$ **f** $(-3)^6 = 729$

g $9^3 = 729$ **h** $2^7 = 128$

i $(-6)^5 = -7776$ **j** $8^1 = 8$

k $10^9 = 1\,000\,000\,000$ **l** $7^2 = 49$

6 a $(1.5)^9$ **b** $(0.7)^6$ **c** $\left(\frac{1}{2}\right)^7$ **d** $\left(\frac{7}{5}\right)^{14}$

e $(-9.2)^3$ **f** $(0.63)^7$ **g** $\left(\frac{3}{4}\right)^4$ **h** $\left(-\frac{5}{2}\right)^2$

7 a 10 **b** 5 **c** 8 **d** 4 **e** 21 **f** 17

8 a $5^2 \times 5^5 \times 5^4 = 5^7 \times 5^4$
$= 5^{11}$

b $20^8 \times 20 \times 20^{10} = 20^9 \times 20^{10}$
$= 20^{19}$

c $8^{32} \div 8^7 \div 8^{10} = 8^{25} \div 8^{10}$
$= 8^{15}$

d $7^{24} \div 7^6 \div 7 = 7^{18} \div 7^1$
$= 7^{17}$

9 a 2^{12} **b** 3^6 **c** 10^{10} **d** 4^9 **e** 5^{16} **f** 9^{14}

10 a 6^2 **b** 7^4 **c** 12 **d** 8^4 **e** 5 **f** 11^5

11 a 3^2 **b** 5^3 **c** 3^7 **d** 7^5 **e** 11^4 **f** 13^6

12 10^{10} light-years

13 a Exponents have been divided instead of subtracted; $5^8 \div 5^2 = 5^6$.

b Exponents have been multiplied instead of added; $2^3 \times 2^3 = 2^6$.

c Exponents have been divided instead of subtracted; $\frac{4^7}{4^7} = 4^0$.

d The first exponent has been subtracted from the second, instead of the other way round; $9^8 \div 9^{10} = 9^{-2}$.

14 a $x^2 \times x^3$ **b** x^5 **c** 36 × 216 mm

EX p164 4.3 Raising exponents and the zero exponent

1 a 6^{20} **b** 2^{12} **c** 7^{12} **d** 11^8 **e** 8^{15} **f** 3^{14}

2 a $5^3 \times 9^3$ **b** $\frac{3^6}{4^6}$ **c** $\frac{9^4}{5^4}$

d $11^7 \times 4^7$ **e** $\frac{7^3}{10^3}$ **f** $3^9 \times 8^9$

3 a 1 **b** 1 **c** 1

4 a 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1

5 a 0.5^{12} **b** 1 **c** 3.8^{25}

d $2^{20} \times 2.5^8$ **e** 1 **f** 6.6^{18}

6 a 4^{10} cm^2 **b** 4^{15} cm^3

7 a 6 **b** 4 **c** 0 **d** 5 **e** 10 **f** 0

8 a 3 **b** 2 **c** 3 **d** 1 **e** 2 **f** 4

9 $(3^{15})^0, (3^2)^5, (3^4)^3, 3^{14}, ((3^2)^3)^3$

10 a 1

b 2^0

c Sample answer: $\frac{15^5}{15^5} = 15^{(5-5)}$
$= 15^0$
$= 1$

11 a 243, 81, 27, 9, 3

b Each term in the sequence has an exponent of one less than the previous term, so the next term is 3^0. The value of each term in the sequence is the value of the previous term in the sequence divided by 3, so the value of the next term is $3 \div 3 = 1$. Therefore, $3^0 = 1$.

12 a true **b** true **c** false **d** true

13 a $x = 4$ **b** $x = 2$

14 Sample answer: $(2^3)^6 = 2^{18}$
$= (2^2)^9$
$= 4^9$

15 $x = 2, y = 4$, or $x = 4, y = 2$

EX p166 Checkpoint

1 a $2 \times 2 \times 2 \times 2 = 16$

b $3 \times 3 \times 3 \times 3 = 81$

c $(-5) \times (-5) \times (-5) = -125$

d $(-10) \times (-10) \times (-10) \times (-10) \times (-10) \times (-10)$
$= 1\,000\,000$

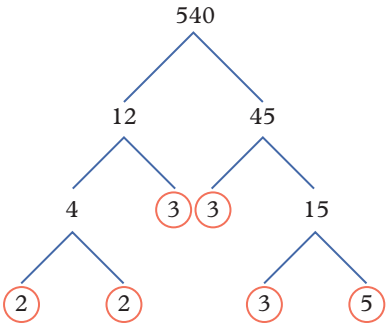
2 **a** $2.5 \times 2.5 \times 2.5 \times 2.5 = 39.0625$

b $(-0.7) \times (-0.7) \times (-0.7) \times (-0.7) \times (-0.7) = -0.168\,07$

c $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

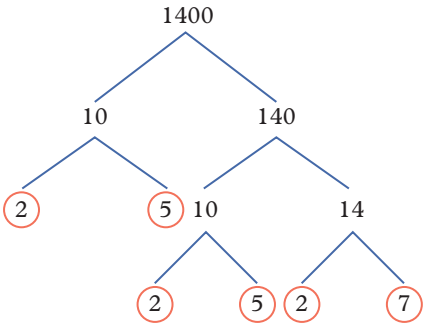
d $\left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) = \frac{1}{625}$

3 **a**



$540 = 2^2 \times 3^3 \times 5$

b



$1400 = 2^3 \times 5^2 \times 7$

4 **a** $2^3 \times 3$ **b** $2^3 \times 3^2$

c $3 \times 5 \times 13$ **d** $2 \times 5^2 \times 7$

5 **a**

Number of minutes	0	1	2	3	4	5
Size of colony	10	30	90	270	810	2430
Number of times as large as original amount	1	3	9	27	81	243

b $3^0, 3^1, 3^2, 3^3, 3^4, 3^5$

c **i** 3^{10} **ii** 3^{60} **iii** 3^{1440}

d 590 490

6 **a** 2^8 **b** 7^{20} **c** 11^6 **d** 9^{18}

7 **a** 3^4 **b** 10^{10} **c** 14^4 **d** 8^8

8 **a** 64 **b** 625

c 36 **d** 10 000 000

9 **a** 4^{18} **b** 9^6 **c** 13^{40} **d** 10^{96}

10 **a** $3^6 \times 5^6$ **b** $\frac{2^4}{7^4}$ **c** $9^5 \times 15^5$ **d** $\frac{9^{10}}{8^{10}}$

11 **a** 1 **b** 1 **c** 1 **d** 1

EX p168 4.4 Combining the exponent laws

1 **a** 3^{14} **b** 9^{41} **c** 12^{36} **d** 8^0

2 **a** 4^2 **b** 7^2 **c** 6^{11} **d** 11^6

e 5^4 **f** 8^3 **g** 2^{10} **h** 4^3

3 **a** 3^6 **b** 3^5 **c** 4^{21} **d** 2^8

4

Base 2	**Exponent form**	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	Basic numeral	16	32	64	128	256	512	1024
Base 3	**Exponent form**	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	Basic numeral	81	243	729	2187	6561	19 683	59 049

5 **a** **i** $2^5 \times 2^4$ **ii** $2^{10} \div 2^8$ **iii** $3^{10} \div 3^6$

iv $\frac{2^7 \times 2^5}{2^4}$ **v** $\frac{3^3 \times 3^5}{3^7}$ **vi** $2^6 \times 2^8 \div 2^9$

vii $\frac{2^9 \times 2^6}{2^7 \times 2^4}$ **viii** $\frac{2^7 \times 2^{10}}{2^8 \times 2^6}$

b **i** 2^9 **ii** 2^2 **iii** 3^4 **iv** 2^8

v 3^1 **vi** 2^5 **vii** 2^4 **viii** 2^3

c **i** 512 **ii** 4 **iii** 81 **iv** 256

v 3 **vi** 32 **vii** 16 **viii** 8

6 **a** 256 **b** 81 **c** 64 **d** 2187

7 **a** 72 **b** 96

c 9000 **d** 64 000 000

8 **a** 2 **b** 12 **c** 4 **d** 0

e 5 **f** 9 **g** 3 **h** 3

9 Sample answer: The number on the right-hand side of each equation is negative because the negative sign is located outside the brackets. When a negative number is raised to a power, if the exponent is even the answer will be positive, and if the exponent is odd the answer will be negative, so $(-5)^2$ is positive, but $(-5)^3$ is negative.

10 **a** true **b** true **c** false **d** false

11 **a** 33 **b** 36 **c** $\frac{9}{4096}$

12 **a** **i** $3^5 + 3^3$ **ii** 270

b **i** $5^6 + 5^4$ **ii** 16 250

c **i** $7^6 + 7^3$ **ii** 117 992

13 $x = 9$

14 384

EX p171 4.5 Roots

1 **a** 14 **b** 16 **c** 33 **d** 162 **e** 1.5

f 7.3 **g** 8.2 **h** 21.8 **i** 5.03

2 **a** not a surd **b** surd **c** not a surd

d not a surd **e** surd **f** not a surd

3 **a** 2 **b** 5 **c** 3 **d** 10

4 **a** **i** not a surd **ii** not a surd **iii** surd

iv not a surd **v** surd **vi** surd

b **iii** 4.626 065 009… **v** 2.923 012 786…

vi 0.562 341 325…

5 **a** $\frac{1}{4}$ **b** $\frac{1}{8}$ **c** $\frac{2}{5}$ **d** $\frac{3}{7}$ **e** $\frac{9}{10}$ **f** $\frac{6}{11}$

6 **a** **i** $\sqrt{31}$ **ii** 5.57

b **i** $\sqrt{318}$ **ii** 17.83

c **i** $\sqrt[3]{12}$ **ii** 2.29

d **i** $\sqrt[3]{81}$ **ii** 4.33

7 **a** 2^6 **b** 2^3 **c** 8

8 **a** The value has been divided by 2 instead of finding the square root; $\sqrt{16} = 4$.

b The square root has been found instead of the cube root; $\sqrt[3]{64} = 4$.

9 A calculator can only display a finite number of digits, 10 in this instance, so this is a rounded value, and does not show that $\sqrt{50}$ is a non-terminating, non-recurring decimal, meaning that $\sqrt{50}$ is irrational.

10 **a** 2 **b** $2\sqrt{2}$

11 **a** Sample answer: The area of a square is found by squaring the side length, so the side length can be found by taking the square root of the area.

b **i** 13 cm

ii 14.142 135 623… m

iii 16.4 km

c answer **ii** is a surd

12 **a** −3

b −2

c There are no positive or negative numbers that when squared equal −4.

13 **a** **i** 8 and 8 **ii** 18 and 18 **iii** 20 and 20

b $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

14 **a** 18 **b** 26 **c** 44 **d** 72

15 **a** **i** not a perfect square

ii perfect square; $\sqrt{289} = 17$

iii perfect square; $\sqrt{576} = 24$

iv not a perfect square

b **i** perfect cube; $\sqrt[3]{216} = 6$

ii perfect cube; $\sqrt[3]{512} = 8$

iii not a perfect cube

iv perfect cube; $\sqrt[3]{1331} = 11$

EX p174 Review: Exponents and roots

Review questions 4.6A: Mathematical literacy review

1 exponent form

2 factor trees

3 a surd

4 **a** order of operations **b** irrational number

5 **a** when dividing numbers in exponent form with the same base, subtract the second exponent from the first exponent

b a positive whole number that is divisible by at least one whole number other than itself and 1

6 **a** base

b simplification

Review questions 4.6B: Multiple choice

1 A **2** B **3** D **4** C **5** D

6 B **7** D **8** D **9** B **10** A

Review questions 4.6C: Short answer

1 **a** $3 \times 3 \times 3 = 27$

b $2 \times 2 \times 2 \times 2 = 16$

c $8 \times 8 = 64$

d $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

2 **a** $2.2 \times 2.2 \times 2.2 \times 2.2 = 23.4256$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{27}{343}$

c $(-0.9) \times (-0.9) \times (-0.9) \times (-0.9) \times (-0.9) = -0.590\,49$

d $\left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) = \frac{256}{625}$

3 **a** $45 = 3^2 \times 5$

b $124 = 2^2 \times 31$

4 **a** 3^{12} **b** 2^{10} **c** 12^4 **d** 4^{11}

5 **a** 6^6 **b** 5^0 or 1 **c** 10^7 **d** 7^1

6 **a** 9^{15} **b** $6^{10} \times 3^{10}$ **c** $\frac{3^3}{10^3}$ **d** $\frac{2^8 \times 7^8}{9^8}$

7 **a** 4 **b** 1 **c** 1 **d** 25

8 **a** $7^2 = 49$ **b** $2^5 = 32$

9 **a** **i** $\frac{5^4}{5^2}$ **ii** 5^2 **iii** 25

b **i** $\frac{3^2 \times 3^4}{3^3}$ **ii** 3^3 **iii** 27

c **i** Sample answer: $\frac{2^6 \times 2^8}{2^2 \times 2^4}$

ii 2^8 **iii** 256

d **i** Sample answer: $2^4 \times ((2^2)^2)^2$

ii 2^{12} **iii** 4096

10 **a** **i** not a surd **ii** surd

iii not a surd **iv** surd

b **ii** 26.46 **iv** 3.48

11 **a** 6 **b** 7 **c** 330 **d** 65.5

Review questions 4.6D: Mathematical modelling

1 **a**

Number of days	Mass of metal (grams)	$\frac{\text{mass of metal}}{\text{original mass of metal}}$
0	240	1
1	120	$\frac{1}{2}$
2	60	$\frac{1}{4}$
3	30	$\frac{1}{8}$

b $\left(\frac{1}{2}\right)^0, \left(\frac{1}{2}\right)^1, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3$ **c** $240 \times \left(\frac{1}{2}\right)^7$

d $\frac{15}{8}$ **e** 1.875 grams **f** 8 days

2 **a** $\frac{(2 \times 5 \times 7^2) \times (2^2 \times 3^2)^2}{(2^2 \times 3^3 \times 7) \times (3 \times 13^2)^0}$ **b** $2^3 \times 3 \times 5 \times 7$

c 840

3 **a** 0.1 mm **b** 0.2 mm **c** 3.2 mm **d** $a \times 2^n$

4 **a** $1 < 2 < 4 < 6 < 9 < 16 < 20 < 25$

b **ii**, **iv**, **vii**

c **i** 1 **ii** 1.41 **iii** 2 **iv** 2.45
v 3 **vi** 4 **vii** 4.47 **viii** 5

d $1 < 1.41 < 2 < 2.45 < 3 < 4 < 4.47 < 5$

e The bigger the number is, the bigger the square root of that number is. This allows us to compare the square root of an integer with another integer.

f **i** < **ii** > **iii** < **iv** <
v > **vi** < **vii** > **viii** <

g $\frac{1}{25} < \frac{1}{20} < \frac{1}{16} < \frac{1}{9} < \frac{1}{6} < \frac{1}{4} < \frac{1}{2} < 1$

h **ii**, **iv**, **vii**

i **i** 1 **ii** 0.71 **iii** $\frac{1}{2}$ **iv** 0.41
v $\frac{1}{3}$ **vi** $\frac{1}{4}$ **vii** 0.22 **viii** $\frac{1}{5}$

j $\frac{1}{5} < 0.22 < \frac{1}{4} < \frac{1}{3} < 0.41 < \frac{1}{2} < 0.71 < 1$

k Yes. Even with fractions, the bigger the number is, the bigger the square root of that number is. This allows us to compare the square root of a fraction with another fraction.

l **i** < **ii** < **iii** < **iv** >
v > **vi** > **vii** > **viii** >

MODULE 5 Algebra

5.1 Pronumerals

EX p183

1 **a** **i** $6a$, b and -1

ii a and b

iii The coefficient of $6a$ is 6. The coefficient of b is 1.

iv -1

b **i** $2xy$ and $-3x$

ii x and y

iii The coefficient of $2xy$ is 2. The coefficient of $-3x$ is -3.

iv No constant terms

c **i** $11f$, $-g$, $15h$ and -10

ii f, g and h

iii The coefficient of $11f$ is 11. The coefficient of $-g$ is -1. The coefficient of $15h$ is 15.

iv -10

d **i** $-\frac{9}{4}p$

ii p

iii The coefficient of $-\frac{9}{4}p$ is $-\frac{9}{4}$.

iv No constant terms

e **i** $\frac{3}{4}c$, d and $\frac{1}{3}$

ii c and d

iii The coefficient of $\frac{3}{4}c$ is $\frac{3}{4}$. The coefficient of d is 1.

iv $\frac{1}{3}$

f **i** $\frac{x}{2}$, $\frac{y}{11}$, $-\frac{7}{10}$ and $-\frac{z}{8}$

ii x, y and z

iii The coefficient of $\frac{x}{2}$ is $\frac{1}{2}$. The coefficient of $\frac{y}{11}$ is $\frac{1}{11}$. The coefficient of $-\frac{z}{8}$ is $-\frac{1}{8}$.

iv $-\frac{7}{10}$

g **i** $-\frac{m}{5}$, $-\frac{n}{7}$ and $-8p$

ii m, n and p

iii The coefficient of $-\frac{m}{5}$ is $-\frac{1}{5}$. The coefficient of $-\frac{n}{7}$ is $-\frac{1}{7}$. The coefficient of $-8p$ is -8.

iv No constant terms

h **i** $\frac{5x}{8}$, $-9y$, $\frac{z}{4}$ and $-\frac{2}{5}$

ii x, y and z

iii The coefficient of $\frac{5x}{8}$ is $\frac{5}{8}$. The coefficient of $-9y$ is -9. The coefficient of $\frac{z}{4}$ is $\frac{1}{4}$.

iv $-\frac{2}{5}$

2 **a** expression **b** equation **c** equation
d equation **e** expression **f** equation

3 **a** false, it is an equation

b true

c false, there are 2 terms

d true

e false, the coefficient of $-d$ is -1

f false, the coefficient of $\frac{b}{12}$ is $\frac{1}{12}$

g true

h true

4 **a** $a + 5$ **b** $\frac{z}{3}$ **c** $y - 23$ **d** $50m$
e $\frac{2}{5}x$ **f** $2p$ **g** $\frac{s}{2}$ **h** $\frac{3}{4}w + 5$

5 **a** $7x + 2$ **b** $\frac{x}{3} - 6$ **c** $\frac{3}{4}x - 10$
d $13 + \frac{1}{3}x$ **e** $15x - 28$ **f** $\frac{1}{2}x - 5$

6 **a** The sum of c and 8

b x less than 11

c x divided by 5

d the product of nine-tenths and q

e 5 less than the product of 3 and x

f 16 more than the product of ten-thirds and a

7 **a** $3(x + 6)$ **b** $\frac{n - 10}{7}$ **c** $\frac{3}{5}fg$ **d** $\sqrt{t} - 1$
e $\frac{b + 12}{15}$ **f** $\sqrt{z + 9}$
g $\frac{2}{3}v - 7$ **h** $3(33 + y)$

8 **a** $8(x - 3)$ **b** $6(x + 20)$ **c** $\frac{5}{6}x - 2$
d $\frac{4}{5}x + 12$ **e** $2(x + 10)$ **f** $\frac{x + 4}{12} + 16$

9 $n + 18$

10 $\frac{c}{4}$

11 $8x$

12 $y = x - 10$

13 **a** **i** $y - 6$ **ii** $y - 10$
b **i** $y + 5$ **ii** $y + 23$
c $y - x$ **d** $2y$

14 a $c + p$ **b** $p - 1$ **c** $c + p - 1$

15 a $8n$ and 12

b the delivery fee

c the price of each brush

d \$92

16 a $130t + 25$ **b** \$415

17 a $2(x + 7)$ **b** 40 chickens

18 a Naomi is correct. If one block is left over, then $x - 1$ blocks are divided between the three students.

b Sample answer: An extra block is added to the container so that the blocks can be divided equally between three students.

19 a $\frac{4x}{9} - \frac{5}{9}$ **b** $\frac{4}{9}$ **c** $-\frac{5}{9}$

20 a $0.8xy + 0.7ab$ **b** Sample answer: $67a$

c 4 t-shirts, 2 pairs of pants

EX p188 5.2 Substitution

1 a 12 **b** 2 **c** 2 **d** 10 **e** $1\frac{3}{5}$ $\left(\text{or } \frac{8}{5}\right)$

f 80 **g** 254 **h** −1 **i** 76 **j** 0

2 a 5 **b** −7 **c** 31 **d** −3 **e** 102

f 8 **g** −4 **h** −63

3 a 11 **b** 26 **c** 2 **d** −4

4 a 100 **b** 0 **c** 1 **d** $\frac{1}{3}$

5 a 30 **b** 2 **c** 8 **d** $1\frac{2}{5}$ $\left(\text{or } \frac{7}{5}\right)$

6 a 15 **b** $-\frac{3}{5}$ **c** $\frac{2}{9}$ **d** 2

7 a 13 **b** −5 **c** 7 **d** 8

8 a 4 **b** $-11\frac{3}{4}$ $\left(\text{or } -\frac{47}{4}\right)$

c −12 **d** $-9\frac{3}{4}$ $\left(\text{or } -\frac{39}{4}\right)$

9 a 10 **b** 8

c −18 **d** $1\frac{1}{2}$ $\left(\text{or } \frac{3}{2}\right)$

e 4 **f** $-10\frac{2}{3}$ $\left(\text{or } -\frac{32}{3}\right)$

g 151 **h** $3\frac{3}{5}$ $\left(\text{or } \frac{18}{5}\right)$

10 a 3 m/s **b** 94 km/h

11 a 4.9 m

b i 44.1 m **ii** 122.5 m

c increase; average speed in first second is 4.9 m/s, in first 3 seconds is $\frac{44.1}{3}$ or 14.7 m/s, in first 5 seconds is $\frac{122.5}{5}$ or 24.5 m/s.

12 a 4 cm² **b** 70 cm² **c** $6\frac{1}{4}$ cm² $\left(\text{or } \frac{25}{4} \text{ cm}^2\right)$

d i the square **ii** the triangle

13 a i Instead of substituting $x = 5$ into $2x$ and multiplying 2 by 5, x has been replaced with 5 in the expression $2x$ without multiplying.

ii 16

b i Incorrect order of operations. Multiplication has been performed before applying the exponent.

ii 9

c i 2^0 does not equal 0, $2^0 = 1$.

ii 1

d i The negative sign has been left outside the brackets. **ii** 21

14 a \$1050 **b** \$2433.31

15 a $P = 0.95(x - y) - z$ **b** \$17 540.50

EX p192 5.3 Adding and subtracting algebraic terms

1 a $20x$, $-12x$ and $3x$

$8y$, $-15y$ and y

b $5x$, $\frac{3}{7}x$, $-25x$ and $\frac{x}{7}$

$\frac{5}{7}y$ and $6y$

c $4xy$, $-yx$, $\frac{1}{8}xy$ and $-\frac{yx}{10}$

d $-a$ and $12.5a$

$5a^2$, $\frac{4}{9}a^2$, $2.3a^2$ and $-\frac{9a^2}{10}$

e $\frac{11}{18}p^3$, $5.5p^3$ and p^3

$\frac{p^2}{5}$, $-11p^2$ and $-0.05p^2$

f $\frac{9ab}{10}$, $\frac{5}{6}ab$ and $7.2ba$

$7abc$, $-cab$ and $13cab$

g $-9fg$ and $\frac{4fg}{5}$

$120f^2g$ and $-\frac{14}{3}f^2g$

$\frac{20fg^2}{3}$ and $0.33fg^2$

h $\frac{2}{9}xy^2z$, $-12y^2xz$ and $99zy^2x$

$0.095zyx$ and $18.6xyz$

2 a true

b false; the powers of the pronumerals are not the same

c true **d** true

e false; the powers of the pronumerals are not the same

f true

3 a $9x$ **b** $8kp$ **c** $-4a^2$

d $4g$ **e** $-2xy$ **f** $2g^2h$

g $\frac{1}{4}abc$ **h** $-\frac{4}{15}xyz^2$

4 a true

b false; $10k$

c false; $3p + 6q$ (does not simplify further)

d false, $6m^2 + 2n$

e false, $9ab + 3a$

f true

g true

h false; $\frac{7}{18}p^2q - \frac{5}{6}p^2$

5 a $6b + 5$ **b** $-n - 5mn$ **c** $4 - 8c^2$ **d** $gh - 3$

e $-pq - 10q^2p + 8p^2q$

f $11x^2y - 12xy^2$

g $\frac{31}{14}rst + \frac{1}{6}st$

h $-\frac{1}{2}abc - \frac{1}{2}ab - 2$

i $\frac{2}{5}g^3h + \frac{11}{6} + \frac{4g^2h}{5} - \frac{1}{15}gh$

j $\frac{9}{4}m^2n - \frac{3}{4}m^2n^4 + \frac{1}{6}$

6 a 165 **b** $16x^2 + xy$ **c** yes

d simplified expression; only had to substitute values into two terms

7 a $6a + 14b$; 72 **b** $3a + 9b$; 42

c $5ab + 3b - 2a$; 74 **d** $-2ab + a^2$; -5

e $-5b^2 + 7a^2$; 130 **f** $\frac{1}{4}a^2 - \frac{2}{3}b^2$; $\frac{1}{4}$

g $-11ab - 2ab^2$; -255 **h** $-2ab^2 - 3a^2b$; -315

i $\frac{19}{20}a^2b - \frac{19}{6}ab^2$; $-\frac{285}{4}$

8 a Tania didn't keep the minus sign with $2y$ and the addition sign with $7x$.

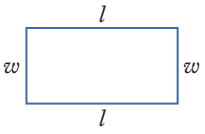
b $3x + 7x - 2y - 4y$

9 a $11x + 2y$ **b** $-\frac{7}{6}$

c $-2y$ **d** $-2xy + 8x$

e $16x^2y - 6xy^2 - 5$ **f** $-\frac{37}{20}x^2y + \frac{9}{4}xy^2$

10 a



b $l + w + l + w$ **c** $2l + 2w$

d $P = 2l + 2w$

e i 14 m **ii** 18.4 m **iii** 46 m

f Sample answer: length 8 m, width 2 m

11 a $8x + 4$ **b** $18x + 6$ **c** $36x + 4$ **d** $40x + 2$

12 a 36 cm **b** 78 cm **c** 148 cm **d** 162 cm

13 a The a has been multiplied four times instead of added four times. The expression simplifies to $4a$.

b The terms have been treated as like terms and added together instead of as unlike terms. The expression $2x + 3y$ is already in its simplest form.

c The x in each term has been multiplied together instead of treated as like terms. The expression simplifies to $13x$.

d The $-x$ term has been interpreted as dividing by x to cancel out the x in the $8x$ term. Instead, $-x$ is the same as $-1x$ so the expression simplifies to $7x$.

14 a $12(2x + 3) + 11(2x + 5)$ **b** $10(x - 3y + 4)$

c $2(x + 2) + 8(x - 2)$ **d** $\frac{13}{3x + 1} + \frac{12}{3x - 1}$

e $7\sqrt{7 - 2x} - 4\sqrt{7 + 2x}$ **f** $9(a + b) - \frac{3}{a + b} - \sqrt{a + b}$

15 a i $\frac{4}{3}$ and $\frac{4}{3}$ **ii** $-\frac{4}{3}$ and $-\frac{4}{3}$ **iii** $\frac{2}{3}$ and $\frac{8}{3}$

b i 0 and 0 **ii** 1 and -1 **iii** 1 and -1

c i 0 and 0 **ii** -1 and -1 **iii** 1 and 1

d i $-\frac{3}{5}$, $-\frac{3}{5}$, $-\frac{3}{5}$, and $\frac{3}{5}$

ii $\frac{3}{5}$, $\frac{3}{5}$, $\frac{3}{5}$, and $-\frac{3}{5}$

iii $-\frac{6}{5}$, $-\frac{6}{5}$, $-\frac{6}{5}$, and $\frac{6}{5}$

16 a The expressions are not equivalent. The expressions are equal for $x = 1$ and $x = -1$, $\frac{4}{3x}$ is dividing x and $\frac{4}{3}x$ is multiplying x, hence why the expressions are different for $x = 2$.

b The expressions are not equivalent. Despite the fact that the expressions are equal for $x = 0$, squaring a positive or negative number always results in a positive number whereas taking the negative of a squared number always results in a negative: $(-x)^2 = (-x)(-x) = xx = x^2$ but $-x^2 = -xx$.

c The expressions are equivalent. Cubing the negative of a number always results in the negative of the cube of that number: $(-x)^3 = (-x)(-x)(-x) = -xxx = -x^3$.

d The first three expressions are equivalent but $\frac{-3x}{-5}$ is not equivalent. Looking at the coefficients of the first three expressions, they all simplify to the same negative fraction. The coefficient of the final expression simplifies to a positive.

17 $11a^2b^7c^5d^5e^9f^{11}g^{14} + 9a^2b^5c^5d^7e^9f^{11}g^{14}$

18 $2.25x$ or $\frac{9}{4}x$

19 $\frac{27}{4}x$

EX p197

5.4 Multiplying algebraic terms

1 a $4x$ **b** $6k$ **c** $7d$ **d** $5m$

e p **f** $-3h$ **g** $-9a$ **h** $-10y$

i $-8c$ **j** $-e$ **k** $2f$ **l** $3m$

2 a $3ab$ **b** $4mn$ **c** $2pt$ **d** $-8cd$

e $9xy$ **f** $5fk$ **g** abc **h** ehy

i $8q$ **j** $-18m$

3 a B **b** A **c** C

4 a $5m^2$ **b** $-2a$ **c** $-xy$ **d** $2n^2$

e $-6c^2d^2$ **f** $7a^2bc$ **g** $-2s^2t^3$

h $-10f^2g^3$ **i** pq^3r^3 **j** $-11x^3y^2z^2$

5 a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

b $a \times a \times a \times a \times a \times a \times a = a^7$

c $m \times m \times m \times m \times m \times m \times m \times m = m^8$

d $x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x = x^{10}$

6 a a^5 **b** k^5 **c** c^7 **d** e^6f^5 **e** a^5d^6 **f** $m^{10}n^5$

7 a $12abc$ **b** $-a^2h^3$ **c** $21wxyz$

d $12x^4y^5$ **e** $21a^7$ **f** $-5p^{12}$

g $24n^2$ **h** $15x^8y^{10}$ **i** $-a^8b^6$

j $-18x^2yz$ **k** $\frac{1}{5}w^4x^5y^6z$ **l** $-\frac{18}{7}a^5b^7c^2$

8 a

×	$4a$	3	$6m$	$2abc$
$3d$	$12ad$	$9d$	$18dm$	$6abcd$
ab^2	$4a^2b^2$	$3ab^2$	$6ab^2m$	$2a^2b^3c$
$5mn$	$20amn$	$15mn$	$30m^2n$	$10abcmn$
$2ac$	$8a^2c$	$6ac$	$12acm$	$4a^2bc^2$

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$	$-6x^2$	$-8x^2yz$	$10xy$	$-4x^3$
xy^2	$3x^2y^2$	$4x^2y^3z$	$-5xy^3$	$2x^3y^2$
$6xy$	$18x^2y$	$24x^2y^2z$	$-30xy^2$	$12x^3y$
$-7yz$	$-21xyz$	$-28xy^2z^2$	$35y^2z$	$-14x^2yz$

9 **a** $-70bghk$ **b** $30g^{10}h^3$ **c** $-12b^9$ **d** $48m^2nq$

e $-21fg^3h^3$ **f** $-\frac{9}{40}m^{11}n^3$ **g** $24a^6b^5c$

h $10w^3x^8y^8$ **i** $-\frac{5}{2}a^8b^{13}c^7d^4$

10 **a** $2 \times 3 = 6$ **b** $8 \times -2 = -16$

c $6 \times k = 6k$ **d** $3 \times x = 3x$

e $a \times b = ab$ **f** $x \times -7y = -7xy$

g $5d \times c = 5cd$ **h** $4a \times 2b^2 = 8ab^2$

i $5n^3 \times 3mn = 15mn^4$ **j** $6e \times -2e^2f = -12e^3f$

k $5jk^2l \times 4l^2 = 20jk^2l^3$ **l** $5xy \times -xz^5 = -5x^2yz^5$

11 **a** C **b** C

c The coefficient of x is $\frac{7}{5}$; the coefficient of y is $-\frac{1}{5}$.

d C **e** D **f** A

g When using algebraic terms, placing pronumerals or a numeral and a pronumeral next to each other generally implies multiplication. However, for mixed numbers, placing a numeral in front of a fraction implies addition. Putting the algebraic and mixed numbers together leads to conflicting conventions presented simultaneously. For clarity, we do not generally use mixed numbers as coefficients.

12 **a** **i** $(3x^2)^4 = (3x^2) \times (3x^2) \times (3x^2) \times (3x^2)$

$= 3 \times 3 \times 3 \times 3 \times x^2 \times x^2 \times x^2 \times x^2$

$= 81x^8$

ii $\left(\frac{x^2}{3}\right)^4 = \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right)$

$= \frac{x^2 \times x^2 \times x^2 \times x^2}{3 \times 3 \times 3 \times 3}$

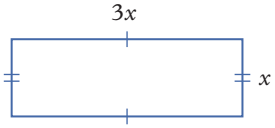
$= \frac{x^8}{81}$

b **i** $64x^{30}$ **ii** $\frac{y^{45}}{32}$

iii $64a^6b^{21}$ **iv** $\frac{625u^{24}}{16v^{20}}$

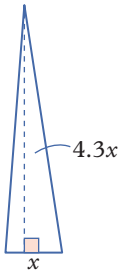
13 **a** $48xy$ **b** $12x^2y^2$ **c** $14x^2y^2$

14 **a** **i**



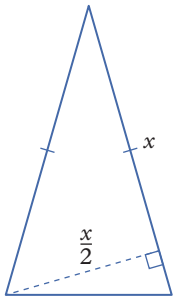
ii $3x^2$

b **i**



ii $2.15x^2$

c **i**



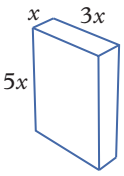
ii $\frac{x^2}{4}$

15 **a** $3x$ **b** $3x^2$ **c** 192 cm^2

d When $x = 8$, width = 8 cm and length = 24 cm.
Area = length × width = $24 \times 8 = 192$ cm^2.
This gives the same result as substituting $x = 8$ directly into $3x^2$.

e $5x$

f



g

Face	Area
top	$3x \times x = 3x^2$
bottom	$3x \times x = 3x^2$
front	$3x \times 5x = 15x^2$
back	$3x \times 5x = 15x^2$
left side	$x \times 5x = 5x^2$
right side	$x \times 5x = 5x^2$

h $3x^2 + 3x^2 + 15x^2 + 15x^2 + 5x^2 + 5x^2 = 46x^2$

i 736 cm^2

j **i** 1150 cm^2 **ii** 6624 cm^2 **iii** 11.5 cm^2

iv 202.86 cm^2

k $15x^3$

l **i** 1875 cm^3 **ii** 25 920 cm^3

iii 1.875 cm^3 **iv** 138.915 cm^3

16 a^{200} since we are multiplying 58 a's, 97 a's, and 45 a's to make a total of 200 a's multiplied together.

17 $2^{12a+5} \times 3^{13a+16}$

EX Checkpoint

p201

1 **a** **i** x and y **ii** 4 **iii** 4, 9, −4

iv −3 **v** x: 1, y: 2

b **i** m, n, and v **ii** 3 **iii** $\frac{5}{3}$, 1

iv $-\frac{9}{13}$ **v** m: 2, n: 1, v: 3

2 **a** $\frac{x}{8}$ **b** $7(a + b + c)$ **c** $y(y + 3)$

d $\sqrt{c - a}$ or $\sqrt{a - c}$

3 **a** $20g$ **b** $6g + b$ **c** $\frac{1}{3}(t + r + p)$

d $0.15b + 3$ or $\frac{3b}{20} + 3$

4 **a** 84 **b** 4 **c** 574 **d** $\frac{17}{2}$

5 **a** $y = 31$ **b** $s = 64$

6 **a** 61 **b** 20

7 **a** $7x + 4y + 10$ **b** $2ab + 5a + 10b$

c $8pqr$ **d** $30ikl + 8jkl$

8 **a** $-3x + 4x^2$ **b** $12xy^2 + 2x^2y$

9 **a** $-7.46x + 0.26y$ **b** $-\frac{15}{8}x^2$

10 **a** $63xy$ **b** $27abcd$

c $-110ghpq$ **d** $36abcuvxy$

11 **a** $36x^5$ **b** $56x^4y^4$

12 **a** $0.372x^2$ **b** $\frac{5}{2}x^{12}$ or $\frac{5x^{12}}{2}$

EX p203 5.5 Dividing algebraic terms

1 **a** 2 **b** $\frac{9}{5}$ **c** $\frac{8}{3}$

d $\frac{5}{3}$ **e** $\frac{58}{11}$ **f** $\frac{27}{4}$

2 **a** a **b** $\frac{5x}{y}$ **c** 9 **d** 12 **e** $\frac{1}{3}$

f $4a$ **g** 18 **h** $\frac{2}{3}$ **i** $\frac{p}{6}$ **j** $\frac{5}{3y}$

k $6a^4$ **l** $\frac{5}{n^4}$ **m** $\frac{2bm^3}{3}$ **n** $2t^4$ **o** $\frac{4j}{5}$

3 **a** a **b** $\frac{15x}{y}$ **c** $\frac{10}{e}$ **d** $14w^2$ **e** $\frac{mn}{3}$

f $9ab$ **g** $2c$ **h** $\frac{2b^2}{3}$ **i** $\frac{m^2np}{6}$ **j** $\frac{3wx}{2y}$

k $7a^2b$ **l** $\frac{8}{m^2n^2}$ **m** $\frac{3bcm^2}{4}$ **n** $\frac{2t^3}{s^2}$ **o** $\frac{8h^2jk}{9}$

4 **a** x **b** $2k$ **c** $9a$ **d** $9x$ **e** $\frac{n}{4}$ **f** bcd

5 **a** $-3y$ **b** $-5a^2$ **c** $\frac{3v^2}{2}$ **d** $-\frac{6r}{7p}$

6 **a** x^5 **b** m^4 **c** $6a^6$ **d** n^5 **e** 5

7 **a** $16m \div 8 = 2m$ **b** $\frac{27jk}{-9j} = -3k$

c $16x^3y^2 \div 2xy = 8x^2y$ **d** $\frac{2a^3b^2}{10a^2b^3} = \frac{a}{5b}$

8 **a** $-x^2y$ **b** $10ab^4$ **c** $5x + 27y$

d $21n + 4m - 6mn$

9 **a** $\frac{5}{36}$ **b** 64 **c** 40.5 **d** 7

10 **a** length = $2x$ cm; area = $2x^2$ cm^2

b $100x^2$ cm^2 **c** 50 pavers

d **i** 40 cm **ii** 80 cm **iii** 3200 cm^2

iv 400 cm **v** 160 000 cm^2

e 50; same

f No; the expression used to calculate the number of pavers simplifies to 50 in each case as the pronumeral x is cancelled in the fraction.

11 **a** $\frac{3}{10}$ **b** $\frac{27x^2y}{16}$ **c** $\frac{x^3y^4}{z^4}$ **d** $\frac{1}{a^3b^3c^6}$

12 **a** $2^{14} \times 13^5 \times d^{12}$ **b** $3^4 \times 7^4 \times a^9 \times b^6$

c $p^9q^7r^{15}s^6$ **d** $r^{45}s^{51}t^{78}u^{84}v^{99}$

13 **a** $17gh$ **b** $4u$

c $53r^2$ **d** $2a^3 + 8a^2$

14 $\frac{x+1}{4}$

15 **a** Sample answer: $x^9y^7 \div x^1y^2 = x^8y^5$, sum is 8 + 5 = 13

b 9 cannot be an exponent of the divisor (second term) otherwise a subtraction would result in a negative exponent. The largest exponent for the quotient that can be created using a 9 is 8 by subtracting 1. Using the remaining digits, the three largest possible sums are 15 = 8 + 7, 14 = 8 + 6 and 13 = 8 + 5. Since 9 and 8 are already in use, it is not possible to have a difference of 7 with the remaining digits: 7 = 9 – 2 = 8 – 1. Similarly, since 9, 8 and 1 are in use, it is not possible to have a difference of 6 with the remaining digits: 6 = 9 – 3 = 8 – 2 = 7 – 1. However, it is possible to have a difference of 5 as 5 = 7 – 2. If instead we started with 9 – 2 = 7, the largest sum is still 13 since 15 = 7 + 8 but 8 = 9 – 1 and 14 = 7 + 7.

EX p208 5.6 Expanding

1 **a** $3a + 6$ **b** $5k - 35$

c $10 + 2a$ **d** $8a + 8b$

e $8x + 24$ **f** $-2 + 16m$

g $35a + 21b$ **h** $48c - 30d$

i $-15m - 6n$

2 **a** $ab + 4a$ **b** $3x + wx$

c $cd - de$ **d** $2p^2 + 3p$

e $k - 4k^2$ **f** $8xy + 6wx$

g $3x - 12x^2$ **h** $\frac{2}{3}m^2 + \frac{7}{3}mn$

i $-6a^2 + 12ab$ **j** $8e^3 + 10e^2$

k $\frac{1}{2}a^4 - 15a^3b$ **l** $-11x^2y - 11y^3$

3 **a** $6x + 2y + 14$ **b** $10ab + 20d - 15$

c $14c - 7d + 35ef$ **d** $-18g + 12h - 6gh$

e $-20k - 8km + 36$ **f** $-6xy - 12 + 21w$

4 **a** Area = $3(a + 2)$

×	a	2
2	$3a$	6

Area = $3a + 6$

$3(a + 2) = 3a + 6$

b Area = $x(x + 4)$

×	x	4
x	x^2	$4x$

Area = $x^2 + 4x$

$x(x + 4) = x^2 + 4x$

5 **a** $5 \times a + 5 \times 4 + 2 \times a + 2 \times 7$

$= 5a + 20 + 2a + 14 = 7a + 34$

b $3b \times c + 3b \times 2 + 2c \times a + 2c \times 5b$

$= 3bc + 6b + 2ac + 10bc = 13bc + 6b + 2ac$

c $d \times d + d \times 8 + 6 \times d + 6 \times (-3)$

$= d^2 + 8d + 6d - 18 = d^2 + 14d - 18$

d $4k \times 2k + 4k \times 3 + (-2) \times k + (-2) \times (-5)$

$= 8k^2 + 12k - 2k + 10$

$= 8k^2 + 10k + 10$

6 **a** $7a + 27$ **b** $10b$

c $11mn + 20m + 3kn$ **d** $21x + 5xy + \frac{16}{3}y$

e $\frac{k^2}{4} + \frac{19k}{4} + 20$ **f** $c^2 - 3c + 35$

g $11rst + 5rst^2 + 36rt$ **h** $h^3 - 4h^2 - 14h$

7 $-(a + 2) = -1 \times (a + 2)$
$= -1 \times a - 1 \times 2$
$= -a - 2$

8 **a** $7p + 20$ **b** $2d - 14$

c $x^2 + 3x + 5$ **d** $k^2 + k - 1$

e $2e^2 + \frac{3}{2}$ **f** $3ab + 4a - 6$

g $-\frac{2}{3}wy - w - 2$ **h** $mnp - 4m^2p + n$

9 **a** 440 **b** $15a^2 + 50a$ **c** 440

d Sample answer: It was easier to evaluate the expression in expanded form as there are just two terms to evaluate.

10 **a** $2a^2 + 10a$; 28 **b** $30a - 6a^2$; 36

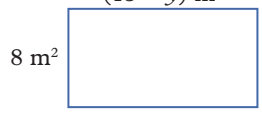
c $15a^2 - 65a$; −70 **d** $2a^2 - 3a$; 2

11 **a** $c + 3$ **b** $2(c + 3)$ **c** $2c + 6$ **d** 42

12 **a** 120 m²

b **i** $8y$ m² **ii** $(120 + 8y)$ m²

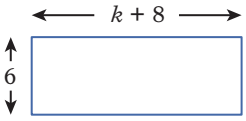
iii $(15 + y)$ m²



iv $8(15 + y)$ m²

v $(120 + 8y)$ m², the expressions are equal as they represent the same area of lawn.

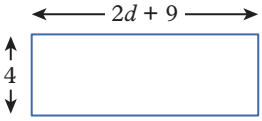
13 **a**



×	k	8
6	$6k$	48

Area $= 6(k + 8) = 6k + 48$

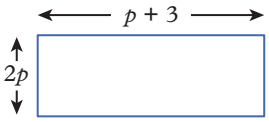
b



×	$2d$	9
4	$8d$	36

Area $= 4(2d + 9) = 8d + 36$

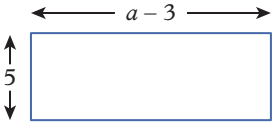
c



×	p	3
$2p$	$2p^2$	$6p$

Area $= 2p(p + 3) = 2p^2 + 6p$

14 **a**



×	a	−8
5	$5a$	−15

Area $= 5(a - 3) = 5a - 15$

b **i** The side lengths of a rectangle must be positive numbers, so a side length of −3 is impossible. Similarly, the area of a rectangle is a positive number, so there is no rectangle with area −15.

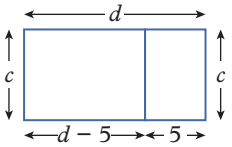
ii The area model can only be used to understand expansions involving positive numbers.

c **i** $a - 3$, 5 **ii** $5(a - 3)$

d **i** a, 5 **ii** $5a$ **iii** 3, 5 **iv** 15

v $5a - 15$

15



16 $5a(b + 3) = 5ab + 15a$, so if a and b are both positive integers, $5a \times b + 3$ is always less than $5a(b + 3)$.

17 **a** $7x^5 - 10x^4 - 21x^2$ **b** $4a^3c$

c $6m^3np^2 + 4m^2n - 6m^2np^2 + 24n$ **d** $3wy$

18 **a** −20 **b** −432 **c** −256 **d** 90

5.7 Factorising

EX p213

1 **a** C **b** D **c** B **d** A

2 **a** 4 **b** $5d$ **c** $2j$ **d** $3ab$ **e** 9

f $7y$ **g** bc **h** $2bcd$ **i** $2y$ **j** $3c$

k $6g^2$ **l** $4b$ **m** m^2p^2 **n** $5r^2s^4t$

3 **a** $2x + 6 = 2\left(\frac{2x}{2} + \frac{6}{2}\right) = 2(x + 3)$

b $8a - 36 = 4\left(\frac{8a}{4} - \frac{36}{4}\right) = 4(2a - 9)$

c $25mn + 30n = 5n\left(\frac{25mn}{5n} + \frac{30n}{5n}\right) = 5n(5m + 6)$

d $kmn - mn = mn\left(\frac{kmn}{mn} - \frac{mn}{mn}\right) = mn(k - 1)$

e $12ef - 18df = 6f\left(\frac{12ef}{6f} - \frac{18df}{6f}\right) = 6f(2e - 3d)$

f $-26ab + 24cd = -2\left(\frac{-26ab}{-2} + \frac{24cd}{-2}\right)$
$= -2(13ab - 12cd)$

g $-abc + acd = -ac\left(\frac{-abc}{-ac} + \frac{acd}{-ac}\right) = -ac(b - d)$

h $-21xyz - 28wxy = -7xy\left(\frac{-21xyz}{-7xy} - \frac{28wxy}{-7xy}\right)$
$= -7xy(3z + 4w)$

4 **a** $4(a + 2)$ **b** $7(e + 3)$ **c** $5(9 + k)$

d $3(1 + 4y)$ **e** $9(3 - p)$ **f** $6b(4a + 3)$

g $2t(6s - 7)$ **h** $5n(m + 2)$ **i** $3g(1 - 3h)$

j $2x(11y - 2z)$ **k** $3b(a + c)$ **l** $8m(2n - p)$

m $9xy(3z - 2w)$ **n** $3mn(4k + 5)$ **o** $-3fg(7e - 8h)$

5 **a** $h^2 + 7h = h\left(\frac{h^2}{h} + \frac{7h}{h}\right) = h(h + 7)$

b $10a + 5a^2 = 5a\left(\frac{10a}{5a} + \frac{5a^2}{5a}\right) = 5a(2 + a)$

c $6p^3 + 9p^5 = 3p^3\left(\frac{6p^3}{3p^3} + \frac{9p^5}{3p^3}\right) = 3p^3(2 + 3p^2)$

d $6xy + 4x^2y = 2xy\left(\frac{6xy}{2xy} + \frac{4x^2y}{2xy}\right) = 2xy(3 + 2x)$

e $mn^2 + 5mn = mn\left(\frac{mn^2}{mn} + \frac{5mn}{mn}\right) = mn(n + 5)$

f $24efg^2 + 32fg^7h^3 = 8fg^2\left(\frac{24efg^2}{8fg^2} + \frac{32fg^7h^3}{8fg^2}\right)$
$= 8fg^2(3e + 4g^5h^3)$

6 **a** $c(c + 4)$ **b** $k(6 - k)$ **c** $a(a - 2)$
d $3m^2(1 + m^2)$ **e** $p^3(10 + 11p^2)$ **f** $h(4 - 7h^3)$
g $3x^2y(2 + xy^5)$ **h** $5bc^2(bc^3 - 2)$ **i** $4mn^2(2 + 9mn)$
j $3xyz(9z - 5x)$ **k** $2a^2bc^4(7ac^6 + 8b^6)$
l $6efg^2(2e + 3f^4)$

8 **a** -5 **b** $-2p$ **c** $-3k$ **d** $-7st$
e $-4x$ **f** $-4m^2$ **g** $-xy$ **h** $-4bc^2$

9 **a** $-4(a + 3)$ **b** $-2(3 + 4k)$ **c** $-7m(1 - 2n)$
d $-5b(3 - 7a)$ **e** $-3t(s + 3r)$ **f** $-6bc(3a + 4d)$
g $-3x^{12}(4 - 7x^9)$ **h** $-h(h + 2)$
i $-8x^2y^7(2 - x^3y^3)$ **j** $-5ab^6(9 + 2a^{14}b^4c^8)$
k $-6xyz^2(7 + 6x^3y^{12})$ **l** $-abc^3(11a - 9bc^6)$

10 **a** 3 **b** bd **c** $5m$ **d** $6a$
e $4xz$ **f** $7i^2j^2k$

11 **a** $5(3p + 3q - 1)$
b $k(6j - 3i - 10l)$
c $-2np(4mq - 2jk + km)$
d $-9f(3g - h)$
e $17uv(wx + 2x - 3xz - 4)$
f $8a^7b^7c^4(7ab^5 + 4a^2c^5 - 10b^4c^3)$

12 **a** $x(x + 6)$ **b** x and $x + 6$ **c** $x + 6$
d $(x + 6) \times x = x \times (x + 6) = x \times x + x \times 6 = x^2 + 6x$
e length = 3 + 6 = 9 cm; width = 3 cm so
area = length × width = 9 × 3 = 27 cm^2
using $x^2 + 6x$, area = $3^2 + 6 \times 3 = 9 + 18 = 27$ cm^2

13 **a** **i** $32 - 24x$ **ii** $8(4 - 3x)$
b **i** $7(y + 7)$ **ii** $7y + 49$
c **i** $6b^5 - 15b^8$ **ii** $3b^3(2b^2 - 5b^5)$
d **i** $25a(a - 5b)$ **ii** $25a^2 - 125ab$

14 **a** $5x \times 3y + \frac{1}{2} \times 5x \times 2x = 15xy + 5x^2$
b $5x(3y + x)$

15 **a** **i** $6\left(\frac{x}{6} + \frac{5y}{6} + \frac{11z}{6}\right)$ **ii** $-(-x - 5y - 11z)$
iii $\frac{1}{2}(2x + 10y + 22z)$ **iv** $x\left(1 + \frac{5y}{x} + \frac{11z}{x}\right)$
v $-3x\left(-\frac{1}{3} - \frac{5y}{3x} - \frac{11z}{3x}\right)$

b **i** $6\left(\frac{x}{3} - \frac{y}{2} + z\right)$ **ii** $-(-2x + 3y - 6z)$
iii $\frac{1}{2}(4x - 6y + 12z)$ **iv** $x\left(2 - \frac{3y}{x} + \frac{6z}{x}\right)$
v $-3x\left(-\frac{2}{3} + \frac{y}{x} - \frac{2z}{x}\right)$

c **i** $6(3x^2 + 6x^3y - 9x^4z)$
ii $-(-18x^2 - 36x^3y + 54x^4z)$
iii $\frac{1}{2}(36x^2 + 72x^3y - 108x^4z)$
iv $x(18x + 36x^2y - 54x^3z)$
v $-3x(-6x - 12x^2y + 18x^3z)$

d **i** $6\left(\frac{2xy}{3} - x - \frac{5xz}{3}\right)$ **ii** $-(-4xy + 6x + 10xz)$
iii $\frac{1}{2}(8xy - 12x - 20xz)$ **iv** $x(4y - 6 - 10z)$
v $-3x\left(-\frac{4y}{3} + 2 + \frac{10z}{3}\right)$

16 **a** **i** When evaluating $\frac{12q}{-6}$, the student got $2q$, not $-2q$, and used a positive sign instead of a negative sign.
ii $-6(3p - 2q)$

b **i** The student did not take out the HCF, $8r$, as a factor so the expression is not fully factorised.
ii $8r - 16r^3 = 8r\left(\frac{8r}{8r} - \frac{16r^3}{8r}\right) = 8r(1 - 2r^2)$

c **i** The student simplified $\frac{5y^3}{5y^3}$ to 0, not 1, as the numerator and denominator are the same.
ii $20y^4 + 5y^3 = 5y^3\left(\frac{20y^4}{5y^3} + \frac{5y^3}{5y^3}\right) = 5y^3(4y + 1)$

d **i** The student took out the first term as the factor, not the HCF.
ii $6x^3y + 7xy^3 = xy\left(\frac{6x^3y}{xy} + \frac{7xy^3}{xy}\right) = xy(6x^2 + 7y^2)$

17 **a** y and $(x + 2)$
b $y(7x - 5)$ and $(x + 2)(7x - 5)$

18 $\frac{5}{8}x^{12}y^{17}(y^2 - 5x^9y^3 + 3x^5 - 2x^{20}y^{23})$

EX p218

Review: Algebra

Review questions 5.8A: Mathematical literacy review

1 Sample answer:
a Multiply the difference of a and b by 2, then subtract the result by 4
b Subtract $2b$ from a, then subtract the result by 4
c Divide the difference of a and b by 2, then subtract the result by 4
d Subtract the difference of half of b and 4 from a
e The difference between a and the square of b
f The square of the difference between a and b

2 The associative law:
$a + (b + c) = (a + b) + c$
$a \times (b \times c) = (a \times b) \times c$ (or $a(bc) = (ab)c$)
The commutative law: $a + b = b + a$
$a \times b = b \times a$ (or $ab = ba$)
The distributive law: $a(b + c) = ab + ac$

3 A term is a single number, pronumeral, or a product or quotient of numbers and pronumerals. An expression is A quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations. This means all terms are expressions but not all expressions are terms.

4 Factorising and expanding are the opposite of each other. This means that factorising and then expanding an expression gives the original expression, and that expanding and then factorising an expression gives the original expression. For example, expanding $7(a + 2)$ gives $7a + 14$, and factorising again gives $7(a + 2)$.

Review questions 5.8B: Multiple choice

1 B **2** D **3** A **4** B **5** A **6** C
7 D **8** C **9** D **10** D **11** C **12** C

Review questions 5.8C: Short answer

1 **a** $4m + 3$ **b** $\frac{5m}{2}$ **c** $\frac{m+3}{7}$
d $\frac{3}{5}(m - 6)$

2 **a** $25x$ **b** $13y$ **c** $20p$
d $25x + 13y + 20p$

3 **a** 24 **b** 80 **c** -16 **d** -7

4 **a** $v = \sqrt{u^2 + 2as}$ **b** 16 m/s

5 **a** $6a, \frac{1}{7}a, \frac{a}{10}$ and $b, -b, \frac{3}{8}b$
b $0.5x, 1.7x, 10x, \frac{5}{6}x$ and $95y, \frac{1}{2}y$
c $5de, -ed, 0.01de$ and $\frac{2ef}{15}, 12fe, ef$
d $p^2, 5.1p^2, -15p^2, \frac{3}{2}p^2$ and $2p^3, \frac{3p^3}{2}$
e $\frac{9}{10}ab, -6.5ba, -ab$ and $6.5abc, 0.9cab, \frac{11}{20}bca$
f $\frac{4}{9}f^2g$ and $\frac{gf^2}{3}$; $-23g^2f$ and $-3.14fg^2$; $-2.3g^2$; $\frac{6}{7}f^2$

6 **a** $12m + 8n$ **b** $8cd + 2c + 4$
c $-7k^2 + 5$ **d** $2ab^2 + 2a^2b$
e $-5xyz + \frac{11}{2}xy - \frac{2}{5}y$ **f** $-\frac{3}{4}pq^2 - 8q^2$

7 **a** x^{14} **b** m^2 **c** h^{11}
d $p^{12}q^6$ **e** $x^{25}y^{15}$ **f** $a^6b^{30}c^{12}$

8 **a** $-5fg$ **b** $2p^2s$ **c** $56g^{10}$
d $24ab^2c$ **e** $-6m^{12}n^{12}$ **f** $\frac{12}{5}e^2f^{10}g^{18}$

9 **a** k^4 **b** $8c^8$ **c** x^3y
d m^3 **e** $p^{11}q^2r^{18}$ **f** $ab^{12}c^{10}$

10 **a** $\frac{3a}{4}$ **b** $3c$ **c** $\frac{3ad}{5}$
d $\frac{2ehk}{5}$ **e** $\frac{4x^7y^5}{7}$ **f** 1

11 **a** $5a + 15$ **b** $28b - 4c$
c $3ac + 5cd$ **d** $-24m^3 - 6mn$

12 **a** $-12k + 31$ **b** $2y^2 - 13y - 4$

13 **a** 2 **b** $3p$ **c** $5b$
d $8st$ **e** a^2b **f** $6mn^9p$

14 **a** $2(7h - 9)$ **b** $10a(2b + c)$ **c** $4c(2c + 3)$
d $-5fg(e + h)$ **e** $5ab(3a - 8)$ **f** $6x^2y^3(3y^2 - 4wx^8)$

Review questions 5.8D: Mathematical modelling

1 **a** $yx + y(x + 2) + y(x + 4) + y(x + 6) + y(x + 8) + y(x + 10) + y(x + 12)$
b \$30.10 **c** $7xy + 42y$
d \$30.24 **e** $7y(x + 6)$
f \$30.03

2 **a** $2a^2 + 2b^2$ **b** a^2b^2
c $a^2b^2c^3$
d **i** $8a^2 + 4b^2$ **ii** $8a^2b^2$
iii $8a^2b^2c^3$

EX p222

SEMESTER 1 Review

Short Answer

1 **a** 569 000 **b** 600 000 **c** 208 492.175
d 208 492.18 **e** 0.000 05 **f** 28 180

2 **a** $\frac{7}{8} = \frac{21}{24}$ **b** $\frac{-8}{10} = \frac{-12}{15}$
c $2\frac{4}{5} = \frac{70}{25}$ **d** $\frac{-1.2}{-0.07} = \frac{1200}{70}$

3 **a** $\frac{3}{4}$ **b** $\frac{50}{3} = 16\frac{2}{3}$ **c** $7\frac{3}{4}$ **d** 7 : 6
e 3 : 2 : 6 **f** 20 : 1 **g** 1 : 6 **h** 60 km/h

4 **a** 1309 **b** -38 **c** $\frac{25}{18} = 1\frac{7}{18}$
d 21.425 **e** 24 **f** $10\frac{7}{15} = \frac{157}{15}$

5 **a** 213 **b** 7 **c** $-\frac{27}{56}$
d 38.035 **e** 72 **f** $\frac{91}{24} = 3\frac{19}{24}$

6 **a** 4410 **b** -60 **c** $\frac{5}{3} = 1\frac{2}{3}$
d 133.56 **e** 32 **f** $\frac{40}{3} = 13\frac{1}{3}$

7 **a** 643 **b** 85 **c** $-\frac{8}{15}$
d 0.07 **e** 27 **f** $\frac{49}{34} = 1\frac{15}{34}$

8 **a** 216 **b** -243
c $\frac{125}{64} = 1\frac{61}{64}$ **d** 0.0016
e $\frac{2401}{16} = 150\frac{1}{16}$ **f** 1

9 **a** 6 **b** 2
c $\frac{9}{5} = 1\frac{4}{5}$ **d** 0.7

10 **a** 5 **b** 58
c $-\frac{13}{12} = -1\frac{1}{12}$ **d** 10.432

11 **a** $3y - 5xy + 5x$ **b** $8b - 8a$
c 2^7 **d** $35x^3y^6$
e 3^8 **f** $8a^5b^4$
g 3^{30} **h** $2^6 \times 3^5$
i $2x + 4$ **j** $-6p^4$

12 a **i** 14 000 **ii** 15 505.499 72
iii 9.71%
b **i** 4500 **ii** 4245.875 477...
iii 5.99%

13 a 8.4201, 24.01, 24.1, 824.01, 842.01, 842.1
b −5.31, −5.1, −5.031, 3.15, 5.103, 5.13
c $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}, \frac{11}{12}$
d $\frac{6}{-2}, -\frac{3}{2}, -\frac{4}{3}, \frac{-3}{4}, \frac{-3}{-4}, \frac{4}{3}$

14 a **i** non-terminating **ii** recurring
b **i** non-terminating **ii** non-recurring
c **i** terminating
d **i** non-terminating **ii** recurring

15

Fraction	Decimal / recurring decimal	Percentage
$\frac{29}{20} = 1\frac{9}{20}$	1.45	145%
$\frac{7}{8}$	0.875	87.5%
$\frac{5}{2} = 2\frac{1}{2}$	2.5	250%
$\frac{21}{5}$	4.2	420%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\% = 33\frac{1}{3}\%$
$\frac{27}{25} = 1\frac{2}{25}$	1.08	108%
$\frac{4}{33}$	$0.\overline{12}$	$12.\overline{12}\% = 12\frac{4}{33}\%$

16 a $\frac{4}{5}$ **b** 40% **c** 32%
d 21 : 4 **e** 19 eggs/week **f** 85 km/h

17 a 45 **b** 44 **c** 108.9
d 64.96 **e** 96 **f** $1200 and $1500
g 21 blue **h** 0.8 kg **i** 4 km
j 8 min 20 seconds

18 a $p^2 - q^2$ **b** $3x + \frac{y}{12}$ **c** $\frac{x+6}{x-6}$ **d** $\sqrt{7k}$

19 a $32 - 24x$ **b** $-22x + 38$
c $12ab + 10b^2 + 6b$ **d** $30x^3y^2 - 45x^2y^5$

20 a $9(4x + 3)$ **b** $6(2a + 6b - 3c)$
c $\frac{5}{3}p^3(2 + 5p^4)$
d $\frac{uv}{4}\left(-\frac{3u}{2} + 5v\right) = -\frac{uv}{4}\left(\frac{3u}{2} - 5v\right)$

21 a 55c or $0.55 **b** $25
c $1.98 **d** $64
e $49.50

22 a LCM = 90, HCF = 6
b LCM = 2100, HCF = 70
c LCM = $2^4 \times 3^9 \times 5^4$, HCF = $2^3 \times 3^6 \times 5^4$
d LCM = $2^7 \times 3^8 \times 5^8 \times 7^4 \times 11^2$, HCF = $2^3 \times 7$

23 a 854 **b** $2\frac{1}{2} = \frac{5}{2}$
c $8\frac{4}{5} = \frac{44}{5}$ **d** 0.65

24 a false **b** true **c** false
d true **e** true **f** false

25 a $2^3 \times 3 \times 5$ **b** $5^3 \times 7$

Mathematical modelling

1 a lower quality flowers $0.90
medium quality flowers $2.25
higher quality flowers $3.51
b $25.29
c **i** $31.19 **ii** $13.36
d **i** $261.25 **ii** $173
e $10 cheaper

2 a **i** 372 + 654 = 1026 **ii** 1025 − 856 = 169
iii 632 × 72 = 45 504 **iv** 2760 ÷ 8 = 345
v $4^4 = 256$ **vi** $\sqrt{16} = 4$
vii $\sqrt[3]{27} = 3$ **viii** $2^3 \times 2^2 = 2^5$
ix $3^3 \div 3^3 = 1$
b **i** 9.75% **ii** 36.69% **iii** 87.5% **iv** 89.28%
v 93.75% **vi** 100% **vii** 200% **viii** 100%
ix 100%
c **i** 272 + 654 = 926 **ii** 1087 − 856 = 231
iii 79 × 72 = 5688 **iv** 296 ÷ 8 = 37
v $2^4 = 16$ **vi** $\sqrt{64} = 8$
vii $\sqrt[3]{729} = 9$ **viii** $2^4 \times 2^2 = 2^6$
ix $0^3 \div 3^3 = 0$

3 a **i** $2^8 \times 3^7 \times 5^4$ **ii** $3^8 \times 2^7 \times 7^4$
iii $3^8 \times 5^7 \times 7^4$ **iv** $a^8b^7c^4$
b **i** $\frac{2^2}{3^3 \times 5^2}$ **ii** $\frac{3^2}{2^3 \times 7^2}$
iii $\frac{3^2}{5^3 \times 7^2}$ **iv** $\frac{a^2}{b^3c^2}$
c **i** $2^3 \times 3^2 \times 5$ **ii** $3^3 \times 2^2 \times 7$
iii $3^3 \times 5^2 \times 7$ **iv** a^3b^2c
d **i** $2^5 \times 3^5 \times 5^3$ **ii** $3^5 \times 2^5 \times 7^3$
iii $3^5 \times 5^5 \times 7^3$ **iv** $a^5b^5c^3$

4 a $5(w + 3)$ **b** $5w + 15$
c 25 m²
d $w = 1$: $A = 20$ m²
$w = 2$: $A = 25$ m²
$w = 3$: $A = 30$ m²
$w = 4$: $A = 35$ m²
e **i** $w + 13$ ii $2x(w + 13)$
iii $2wx + 26x$ iv 16 m²

EXPLORATIONS 1

EX p226

1 a The sum of digits is always a multiple of 3.
b Swap the first two digits.
c The value changes by a multiple of 999 999, which is itself a multiple of 7.
d With 2 notes, put an odd digit first and an even digit second.

e For 5, 6, 8, 9, 10 and 12 notes, swap the first two digits, and for 11 notes, swap the 1st and 3rd digits (among other possibilities).

f Brooke can always win by swapping the 1st and 7th digits (among other possibilities).

2 a $7\overline{)1.^{1}0^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0}$ $0.1428571\ldots$

b $\frac{2}{7} = 0.\overline{285714}$, $\frac{3}{7} = 0.\overline{428571}$, $\frac{4}{7} = 0.\overline{571428}$, $\frac{5}{7} = 0.\overline{714285}$, $\frac{6}{7} = 0.\overline{857142}$

c $\frac{1}{14} = \frac{5}{70} = 0.0\overline{714285}$

d $\frac{1}{70}$, so $\frac{3}{14} = 0.2\overline{142857}$

e $\frac{5}{14} = 0.3\overline{571428}$, $\frac{9}{14} = 0.6\overline{428571}$, $\frac{11}{14} = 0.7\overline{857142}$ and $\frac{13}{14} = 0.9\overline{285714}$

f $\frac{2}{63} = \frac{1}{7} - \frac{1}{9} = 0.\overline{031746}$

3 a 10 hours **b** 71 hours

c 7 professionals and 19 hours

d $\frac{(x + y/2)(t - T)}{X + Y/2} = \frac{(2x + y)(t - T)}{2X + T}$

4 a 480 mL

b 600 mL, 500 mL, 400 mL, 300 mL, 200 mL, 100 mL and 0 mL

c 525 mL, 450 mL, 375 mL, 300 mL, 225 mL, 150 mL and 75 mL

5 a **i** 2^{12} **ii** 4^{6} **iii** 64^{2} **iv** 16^{3}

b $1 = 2^0 = 4^0 = 1^2 = 1^3$ and $64 = 2^6 = 4^3 = 8^2 = 4^3$

c $18^2 \times 216^3$, $3888^2 \times 6^3$, $144^2 \times 54^3$ or 486×24^3

d Only 20^1 and 24^1 cannot be written this way (1 is both a square and a cube). For any whole number n, the only power which may not be able to be written this way is n^1, since all higher powers always can.

e There are many solutions. Two are given for each part: one which can be constructed from powers of 2 and a second which is essentially a coincidence.

i $128^3 = 1024^2 + 16^5$ (and $7^3 = 10^2 + 3^5$)

ii $32^5 = 4096^2 + 256^3$ (and $8^5 = 104^2 + 28^3$)

f **i** Yes. For example, 7-24-25 is a Pythagorean triple, so $5^4 = 25^2 = 7^2 + 24^2$.

ii No (aside from $0^6 = 0^3 + 0^3$). The sixth power is also a perfect cube, but Fermat's last theorem says that no cube is the sum of two smaller cubes.

6 a 1 house point. No, adding 14 or 18 will not improve his score.

b If the first number is n, the total is $4(2n + 7)$ which is divisible by at least 2 and 4. The smallest total with only 2 house points is 44, when $n = 2$.

c If the first number ends in 1, the total is of the form $60n+21$, which is only divisible by last digits 1 and 3; similarly for the other cases. The smallest number of house points is 0 (for example, from 4, 5, 6, 7, 8, 9).

d The total is always odd, so cannot be divided by 2, 4, 6 or 8. The smallest total is 315.

e 5 consecutive numbers, with smallest total 120.

MODULE 6 Equations, inequalities and linear relationships

6.1 Equations

EX p232

1 a **i** expression **ii** equation **iii** equation **iv** expression **v** equation **vi** equation

b Equations contain an equals sign (=).

2 a yes **b** no **c** no **d** yes **e** no **f** yes

3 a no **b** no **c** yes **d** yes **e** yes **f** no

4 a yes **b** yes **c** no **d** no **e** yes **f** no **g** no **h** yes

5 a $a = 7$ **b** $b = 6$ **c** $c = 8$ **d** $d = 12$ **e** $e = 2$ **f** $f = -3$ **g** $g = 1$ **h** $h = 0$ **i** $i = -3$ **j** $j = -5$ **k** $k = 7$ **l** $l = 35$ **m** $m = 4$ **n** $n = -10$ **o** $x = -14$ **p** $p = 0$ **q** $q = 6$ **r** $r = -12$ **s** $s = -4$ **t** $t = 25$

6 D

7 C

8 A

9 C

10 a **i** $n + 4 = 11$ **ii** $n = 7$

b **i** $n - 3 = -2$ **ii** $n = 1$

c **i** $n + 15 = 21$ **iii** $n = 6$

d **i** $n - 8 = -4$ **iii** $n = 4$

e **i** $2n = 14$ **ii** $n = 7$

f **i** $\frac{n}{4} = 9$ **ii** $n = 36$

g **i** $n - 7 = -20$ **ii** $n = -13$

h **i** $11n = 22$ **ii** $n = 2$

i **i** $5n = 55$ **ii** $n = 11$

j **i** $\frac{n}{6} = -3$ **ii** $n = -18$

11 a $3x + 8 = 11$ **b** $\frac{x}{6} - 2 = 1$

c $\frac{x + 7}{2} = 5$ **d** $3(x - 9) = 3$

12 a $x + 38$ **b** $x + 38 = 62$

c $x = 24$ **d** \$24

13 a $26 + x$ **b** $26 + x = 81$

c $x = 55$ **d** 55 rolls

14 a $8x = 96$ **b** $x = 12$ **c** 12 people

15 a m = number of macadamia nuts

b $\frac{m}{4}$ **c** $\frac{m}{4} = 9$

d $m = 36$ **e** 36 macadamia nuts

16 a $8t = 24$, where t = cost of each chocolate

b \$3

17 **a** $a = 2$ **b** $b = 1$ **c** $c = 3$ **d** $d = 0$
e $e = 6$ **f** $f = 10$ **g** $g = 4$ **h** $h = -7$

18 **a** $x = 5$ **b** $x = 4$ **c** $x = -1$ **d** $x = 9$
e $x = -10$ **f** $x = 0$ **g** $x = -3$ **h** $x = -4$

19 **a** $x = 1$ **b** $x = 18$
c $x = 3$ **d** $x = 10$

20 **a** $\frac{f}{2} + 3 = 7$, where f = number of pieces of fruit initially in the bowl
b 8 pieces of fruit

21 **a** $2c + 7 = 11$, where c = number of cars in the car park at 9 am
b 2 cars

22 **a** $25c$ **b** $40a$ **c** $25c + 40a = 155$
d Substituting $a = 2$ and $c = 3$ into the equation shows Poh's calculations are correct.

23 **a** $2(x + 30) = 270$ **b** 105 minutes

24 **a** The time Arvan spends practising every day is not the same every day.
b $x + x + 2x + 2x - 20 + x = 190$ (or $7x - 20 = 190$)
c

Day	Time spent practising	Comments
Monday	30	Good start!
Tuesday	30	Same as Monday
Wednesday	60	Double Monday!
Thursday	40	20 minutes less than yesterday
Friday	30	Same as Monday
Total	190	

d The average (mean) time Arvan spends practising

25 **a** $h + \frac{h}{2} + 10 = 100$
b $h = 60$ **c** $60 + \frac{60}{2} + 10 = 100$

EX p238 6.2 Solving linear equations

1 **a** -6 **b** $\div 4$ **c** $+7$ **d** $\times 3$
e $\div (-2)$ **f** $\times (-8)$ **g** $+11$ **h** $\div (-20)$

2 **a** $x = 13$ **b** $x = 42$ **c** $x = 12$ **d** $x = 27$
e $x = 1$ **f** $x = -1$ **g** $x = -25$ **h** $x = -3$

3 **a** $+6$ **b** -9 **c** $\div 3$ **d** $\times 4$
e $+2$ **f** $\times 5$ **g** $\div 7$ **h** -1

4 **a** $-3, \div 3$ **b** $+8, \div 2$ **c** $-5, \times 2$ **d** $\div 5, -5$
e $+11, \div 5$ **f** $\times 5, \div 2$ or $\div 2, \times 5$
g $\div 9, -4$ **h** $\times 3, -4$

5 **a** $x = 12$ **b** $x = 14$ **c** $x = 7$ **d** $x = 10$
e $x = 0$ **f** $x = -22$ **g** $x = -5$ **h** $x = -3$
i $x = -2$ **j** $x = 0$ **k** $x = -3$ **l** $x = -17$

6 **a** $x = 8$ **b** $x = 1$ **c** $x = 24$ **d** $x = 6$
e $x = 18$ **f** $x = 33$ **g** $x = 20$ **h** $x = 2$
i $x = 7$ **j** $x = 6$ **k** $x = 1$ **l** $x = 13$

7 **a** $x = 1$ **b** $x = -2$ **c** $x = 4$ **d** $x = 2$
e $x = 15$ **f** $x = 2$ **g** $x = -9$ **h** $x = 10$
i $x = -22$ **j** $x = -15$ **k** $x = -1$ **l** $x = 6$

8 **a** $x = \frac{1}{5}$ **b** $x = 4\frac{1}{2}$ $\left(\text{or } x = \frac{9}{2}\right)$
c $x = 5\frac{2}{3}$ $\left(\text{or } x = \frac{17}{3}\right)$ **d** $x = \frac{4}{5}$
e $x = 4\frac{2}{5}$ $\left(\text{or } x = \frac{22}{5}\right)$
f $x = -11\frac{1}{2}$ $\left(\text{or } x = -\frac{23}{2}\right)$

9 No, as Caleb has not performed the inverse operation on both sides of the equation. He needs to add 37 to obtain the solution.

10 **a** **i, ii** $x = 2$ **iii** The answer is the same.
b **i, ii** $x = 0$ **iii** The answer is the same.
c **i, ii** $x = -1$ **iii** The answer is the same.
d **i, ii** $x = 12$ **iii** The answer is the same.

11 **a** Both students have solved the equation correctly.
b Yuhan's method: Eliminating the fraction first means you don't need to work with fractions after that step; however, you need to consistently work on both sides of the equation.
Connor's method: By completing one calculation at each step, the solution is easy to follow; however, you need to be confident working with fractions.

12 **a** $x = 14$ **b** $x = -6$ **c** $x = -10$
d $x = \frac{3}{4}$ **e** $x = 1.8$ **f** $x = 1\frac{7}{8}$ $\left(\text{or } x = \frac{15}{8}\right)$

13 **a** **i** $2(x + 4) = 26$ **ii** $x = 9$
b **i** $280 + 15x = 1600$ **ii** $x = 88$
c **i** $257 + 56x = 649$ **ii** $x = 7$
d **i** $\frac{x - 6}{2} = 11$ **ii** $x = 28$

14 $216 + \frac{t}{3} + t = 256$, $t = 30$

15 $5\left(x \times \frac{3}{4}\right) = 75$, $x = 20$

16 **a** $10x - 5\left(\frac{3}{2}x\right)$
b You do not know how many points Jenny had at the end of the week.
c The expression simplifies to $\frac{5}{2}x$, which will always be a positive value as the number of jobs completed will be positive (or zero).
d 8

17 **a** $x = 3$ **b** $x = 13$ **c** $x = 9$ **d** $x = -5$
e $x = -4$ **f** $x = 8$ **g** $x = 10$ **h** $x = 14$

18 **a** **i** $-5, \div 3, \surd$ **ii** 3
b **i** $\div 4, +2, \surd$ **ii** 4
c **i** square, $-5, \div 2$ **ii** 10

19 12 years old

20 **a** $\frac{12 + 12}{4} = \frac{24}{4}$
$= 6$
b Giovanni performed the inverse operations in the incorrect order. He subtracted 12 from each side first, before multiplying by 4, whereas he should've multiplied each side by 4 first.
c 48

EX p242

6.3 Solving equations with the unknown on both sides

1 **a** $x = 13$ **b** $x = 4$ **c** $x = 5$
d $x = 2$ **e** $x = -6$ **f** $x = 14$
g $x = 10$ **h** $x = 15$ **i** $x = 8$
j $x = -13$ **k** $x = 8$ **l** $x = 0$

2 **a** $a = 2$ **b** $b = 4$ **c** $c = -1$ **d** $d = 2$
e $e = -4$ **f** $f = 5\frac{1}{2}$ $\left(\text{or } f = \frac{11}{2}\right)$
g $g = 3\frac{1}{3}$ $\left(\text{or } g = \frac{10}{3}\right)$ **h** $h = -3$
i $i = 2\frac{1}{2}$ $\left(\text{or } i = \frac{5}{2}\right)$ **j** $j = 5$
k $k = -\frac{6}{7}$ **l** $l = -4\frac{1}{2}$ $\left(\text{or } l = -\frac{9}{2}\right)$

3 **a** $x = 4$ **b** $x = -2$
c $x = -1$ **d** $x = 3$
e $x = 1\frac{7}{8}$ $\left(\text{or } x = \frac{15}{8}\right)$ **f** $x = 1$
g $x = 6$ **h** $x = 5$
i $x = -6$ **j** $x = 4\frac{2}{3}$ $\left(\text{or } x = \frac{14}{3}\right)$
k $x = 21$ **l** $x = 4\frac{1}{2}$ $\left(\text{or } x = \frac{9}{2}\right)$

4 **a** $x = 6$ **b** $x = 4$ **c** $x = 4$
d $x = 19$ **e** $x = -\frac{1}{5}$ **f** $x = -16$
g $x = -2$ **h** $x = \frac{2}{3}$

5 **a** $6x + 4 = 4x + 2$ **b** $x = -1$

6 **a** Andy: added $2x$ to left side instead of subtracting; Kwame: did not subtract $2x$ from both sides of equation; Hayley: simplified right side incorrectly (-13, not $+13$)

b
$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= -13 \\ 4x + 11 - 11 &= -13 - 11 \\ 4x &= -24 \\ \frac{4x}{4} &= \frac{-24}{4} \\ x &= -6 \end{aligned}$$

7 **a** $p = 3$ **b** $q = 2$ **c** $r = -3$
d $s = 4$ **e** $t = -2$ **f** $u = 13$

8 **a** B **b** \$6

9 **a** D **b** 12

10 **a** $3a + 400 = 7a + 160$ **b** 60 cents

11 **a** $2a + 2$ **b** $3a - 4 = 2a + 2$
c $a = 6$; Sean's cousin is 6 years old.
d Sean is 14 years old.

12 **a** $n + 10$ **b** $3n + 4$ **c** $3n + 4 = n + 10$
d $n = 3$ **e** 3 **f** 9 **g** 13

13 **a** $6g + 27$ **b** $12g + 3$ **c** $12g + 3 = 6g + 27$
d $g = 4$; home team: 8 goals 3 behinds; away team: 4 goals 27 behinds; each team scored 51 points

14 **a** $12t + 4 = 2(4t + 8)$ or $12t + 4 = 8t + 16$
b $t = 3$
c home team: 3 tries; away team: 9 tries

15 **a** $5x - 10 = 3x + 24$ **b** $x = 17$

16 **a** $x = 29$ **b** $x = 1$ **c** $x = 6$
d $x = -7$ **e** $x = 3$ **f** $x = -2$

17 **a** Expand the brackets, apply inverse operations to get x^2 alone on one side of the equation, then take the square root of both sides.
b **i** $x = 2$ **ii** $x = 0$

EX p247

6.4 Solving simple quadratic equations

1 **a** $x = \pm 10$ **b** $y = \pm 3$ **c** $z = \pm 7$
d $a = \pm 9$ **e** $k = \pm 1$ **f** $h = 0$
g $m = \pm 17$ **h** $n = \pm 200$

2 **a** $b = \pm\frac{5}{8}$ **b** $u = \pm\frac{12}{7}$
c $d = \pm\frac{5}{4} = \pm 1\frac{1}{4}$ **d** $g = \pm\frac{11}{3} = \pm 3\frac{2}{3}$
e $w = \pm 0.6$ **f** $i = \pm 0.02$
g $x = \pm 0.25$ **h** $y = \pm 1.001$

3 **a** $x = \pm 11$ **b** $y = \pm 0.7$ **c** $z = \pm\frac{9}{4}$
d $a = \pm 9$ **e** $b = \pm 9$ **f** $c = \pm\frac{3}{4}$
g $m = \pm 8$ **h** $n = \pm 12$ **i** $p = \pm 5$
j $p = \pm\frac{1}{2}$ **k** $q = \pm 3$ **l** $r = \pm\frac{2}{3}$

4 **a** $x = \pm\sqrt{10}$ **b** $y = \pm\sqrt{91}$ **c** $z = \pm\sqrt{0.4}$
d $i = \pm\sqrt{1.6}$ **e** $j = \pm\sqrt{\frac{13}{6}}$ **f** $k = \pm\sqrt{7}$
g $m = \pm\sqrt{\frac{3}{5}}$ **h** $n = \pm\sqrt{11}$

5 **a** $x \approx \pm 3.16$ **b** $y \approx \pm 9.54$ **c** $z \approx \pm 0.63$
d $i \approx \pm 1.26$ **e** $j \approx \pm 1.47$ **f** $k \approx \pm 2.65$

6 $l = 5$ cm and $h = 10$ cm

7 **a** $A = 6l^2$ **b** $l = 6$ cm

8 **a** $v = \pm 38.28$ **b** $c = \pm 12.21$ **c** $u = \pm 41.23$
d $v = \pm 1.58$ **e** $b = \pm 9.85$

9 **a** 1, a length cannot be negative
b 1, radius is a length and cannot be negative

10 **a** $l = 3$ cm **b** $l = 9$ cm **c** $l = 14$ cm
d $l = \sqrt{20}$ cm **e** $l = \sqrt{116}$ cm **f** $l = \sqrt{200}$ cm

11 **a** $x = 8$ cm **b** $x = 14$ cm **c** $x = \sqrt{212}$ cm
d $x = 6$ cm **e** $x = 11$ cm **f** $x = \sqrt{208}$ cm

12 **a** The equation is not in the form $x^2 = c$ so it does not matter if 'the 25 is positive' yet.
b $x^2 + 25 = 0$ is equivalent to $x^2 = -25$, which has zero real solutions as no real number (positive, negative or zero) squared is negative.

13 **a** $a = \pm 5$ **b** $b = \pm\sqrt{\frac{305}{7}}$ **c** $c = \pm 4$
d $d = \pm\sqrt{75}$ **e** $e = \pm\sqrt{5}$ **f** $f = \pm 13$

14 **a** $T = 10.04$ seconds
b The period is a length of elapsed time so it cannot be negative.

15 **a** $x = -11$ or $x = 1$ **b** $x = -16$ or $x = 0$
c $x = -6$ or $x = 14$ **d** $x = 0$ or $x = 14$
e $x = 0.53$ or $x = 9.47$
f $x = -18.49$ or $x = 0.49$

16 **a** $x = -5 \pm \sqrt{35}$ **b** $x = -8 \pm \sqrt{65}$
c $x = 4 \pm \sqrt{97}$ **d** $x = 7 \pm \sqrt{51}$
e $x = 5 \pm \sqrt{21}$ **f** $x = -9 \pm \sqrt{91}$

EX p250 Checkpoint

1 **a** yes **b** yes **c** no **d** yes

2 **a** $a = 4$ **b** $p = -21$ **c** $z = -17$
d $a = 1$ **e** $j = -15$ **f** $y = 7$

3 **a** $x + 6 = 18$ **b** $x - 10 = -4$
c $4x + 9 = 33$ **d** $\frac{x + 12}{3} = 6$
e $8(x + 3) = 64$

4 **a** $x = 14$ **b** $x = 25$ **c** $x = 8$
d $x = 35$ **e** $x = -41$ **f** $x = -9$

5 **a** $x = 4$ **b** $x = 2$ **c** $x = -20$
d $x = 12$ **e** $x = 54$ **f** $x = 13$

6 **a** $x = 4$ **b** $x = 6$ **c** $x = -32$
d $x = -16$ **e** $x = -6$ **f** $x = -41$

7 **a** $x = \frac{1}{7}$ **b** $x = -\frac{9}{2}$ **c** $x = \frac{42}{5}$
d $x = \frac{6}{5}$ **e** $x = \frac{22}{3}$ **f** $x = -21$

8 **a** $x = 4$ **b** $x = 3$ **c** $x = 8$
d $x = -\frac{11}{4}$ **e** $x = 10$ **f** $x = -3$

9 **a** $x = -\frac{21}{4}$ **b** $x = -\frac{34}{7}$ **c** $x = 1$
d $x = -39$ **e** $x = \frac{41}{4}$ **f** $x = 0$

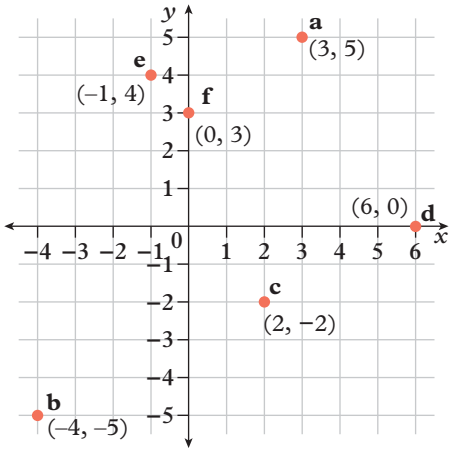
10 **a** $x = \pm 2$ **b** $x = \pm 4$ **c** $x = \pm\frac{4}{3}$

11 **a** $x = \pm 1$ **b** $x = 0$ **c** $x = \pm\frac{2}{3}$

EX p254 6.5 Plotting linear relationships

1 **a** A **b** C **c** F **d** B **e** E **f** D

2



3 **a**

x	−2	−1	0	1	2
y	−1	0	1	2	3

b

x	−3	−2	−1	0	1
y	−4	−2	0	2	4

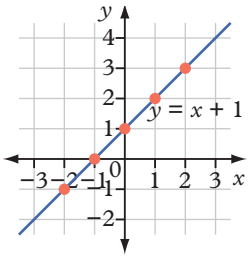
c

x	−1	0	1	2	3
y	4	3	2	1	0

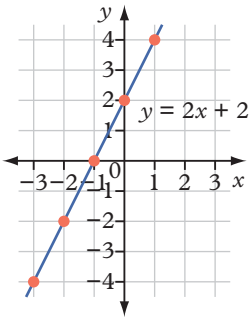
d

x	−1	0	1	2	3
y	−7	−4	−1	2	5

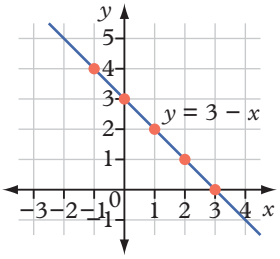
4 **a**



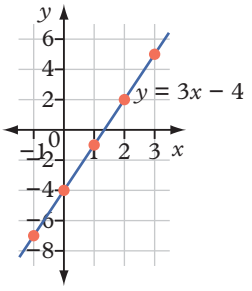
b



c

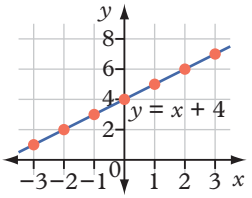


d



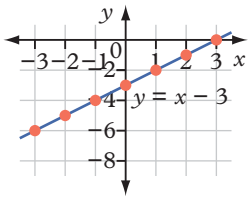
5 **a**

x	−3	−2	−1	0	1	2	3
y	1	2	3	4	5	6	7



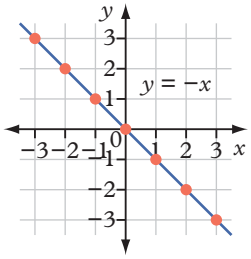
b

x	−3	−2	−1	0	1	2	3
y	−6	−5	−4	−3	−2	−1	0



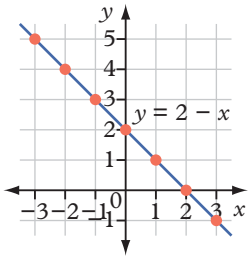
c

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3



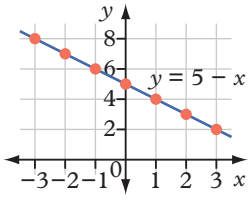
d

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1



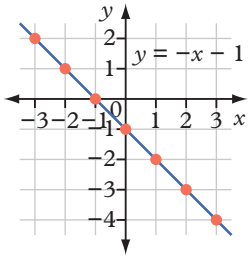
e

x	−3	−2	−1	0	1	2	3
y	8	7	6	5	4	3	2



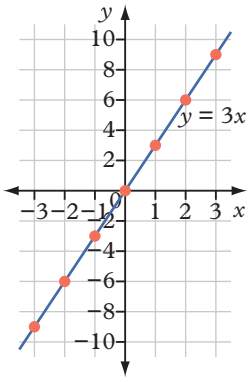
f

x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4



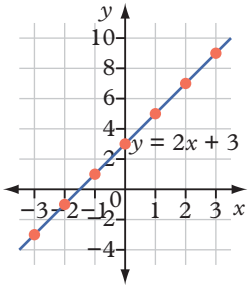
g

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9



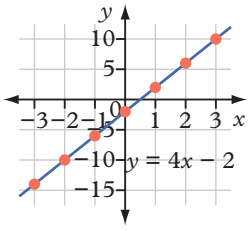
h

x	−3	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7	9



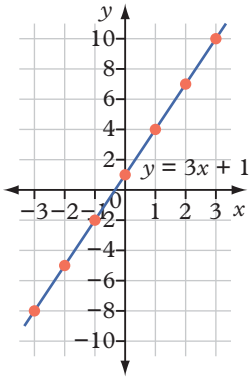
i

x	−3	−2	−1	0	1	2	3
y	−14	−10	−6	−2	2	6	10



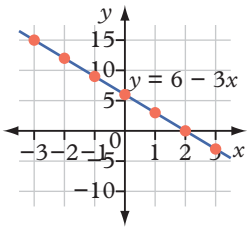
j

x	−3	−2	−1	0	1	2	3
y	−8	−5	−2	1	4	7	10



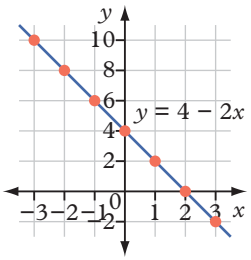
k

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9



l

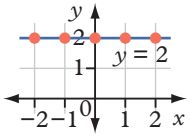
x	−3	−2	−1	0	1	2	3
y	10	8	6	4	2	0	−2



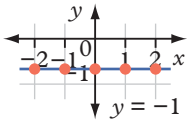
6 **a** linear **b** non-linear **c** linear

7 **a** linear **b** linear **c** non-linear

8 **a** **i**



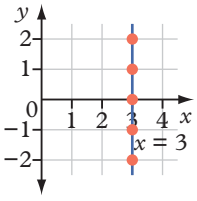
ii



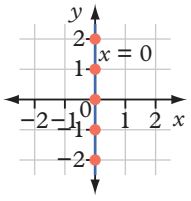
b Yes, they are straight lines.

c The horizontal line passes through the y-value described by the equation.

9 **a** **i**



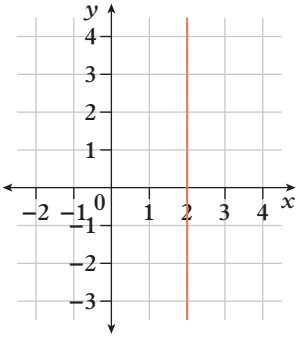
ii



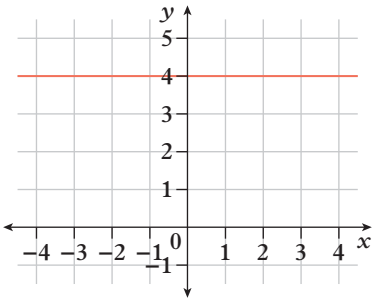
b Yes, they are straight lines.

c The vertical line passes through the x-value described by the equation.

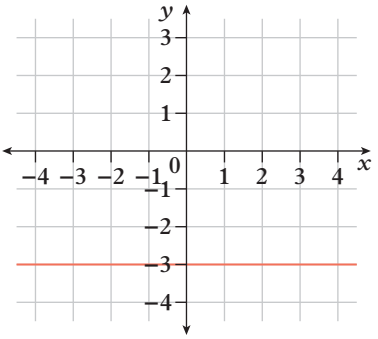
10 **a**



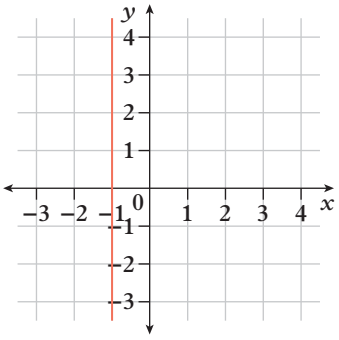
b



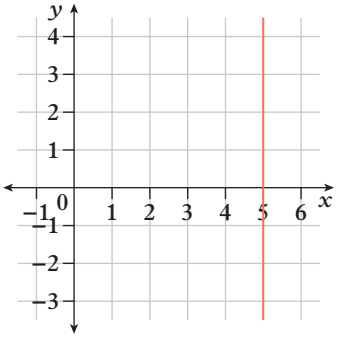
c



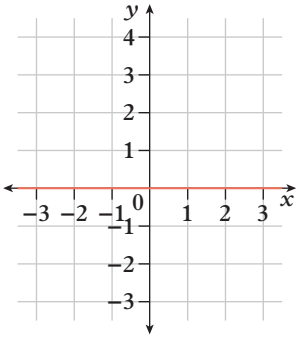
d



e



f



11 a First difference = 1, constant, linear

b First difference = –3, constant, linear

c First difference = 2, 3, 5, 7, not constant, non-linear

12 a time and height

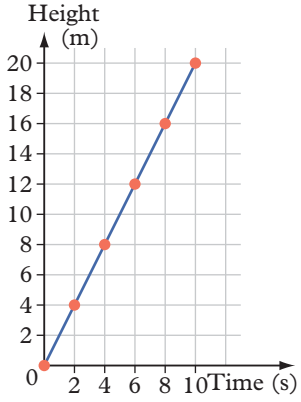
b i time; shown first in table of values

ii height

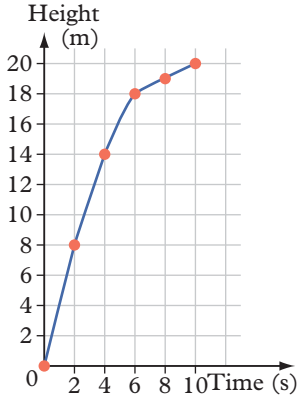
c Red balloon: (0, 0), (2, 4), (4, 8), (6, 12), (8, 16), (10, 20)

Blue balloon: (0, 0), (2, 8), (4, 14), (6, 18), (8, 19), (10, 20)

d Red balloon:



Blue balloon:



e i red balloon: 10 m; blue balloon: 16 m

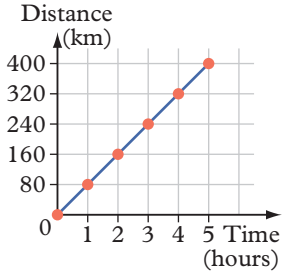
ii red balloon: 3.5 s; blue balloon: approximately 1.75 s

f the red balloon, as the points form a straight line

g the red balloon, as the behaviour is more predictable than the blue balloon

13 a independent = time, dependent = distance

b



c Yes, as the points form a straight line.

d 200 km

e $3\frac{1}{4}$ hours

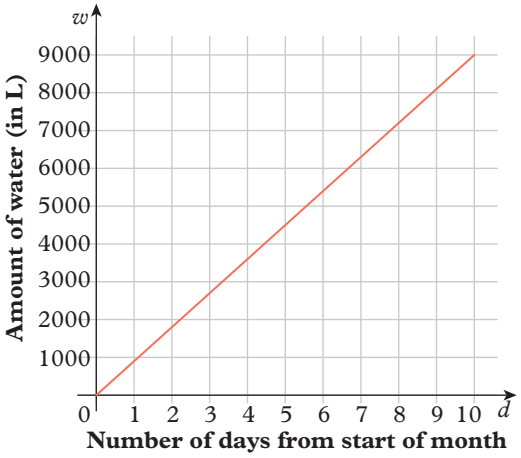
f 560 km; assume Jamie maintains the same speed

g $d = 80t$; multiply the number of hours by 80 to determine the distance Jamie travels.

h $d = 80 \times 7 = 560$

14 a independent = number of days, dependent = amount of water

b



c yes, as the points form a straight line

d 6300 L

e i the equation, as the result can be calculated exactly and because the graph doesn't cover the range needed

ii approx. 22 days $\left(22\frac{2}{9} \text{ days}\right)$

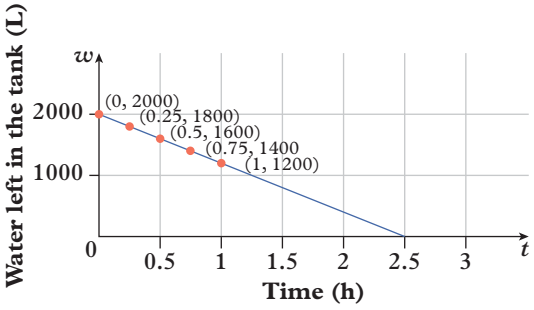
15 a 2000 L

b $w = 2000 - 800t$

c i water left (w)

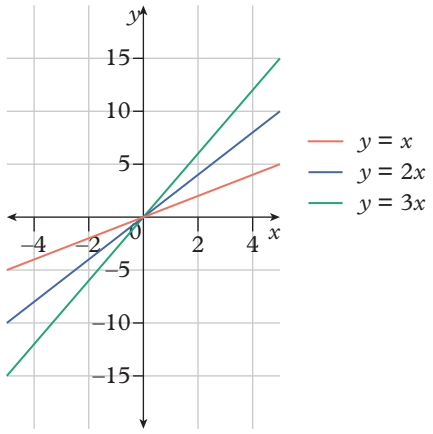
ii time passed (t)

d



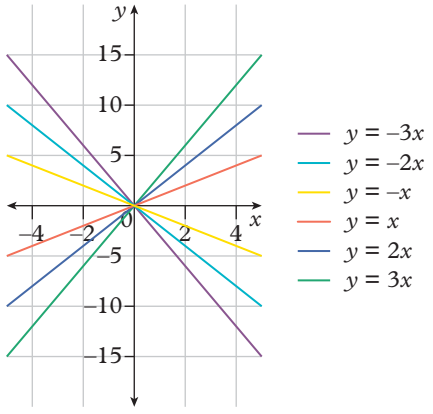
e 0 L

16 a



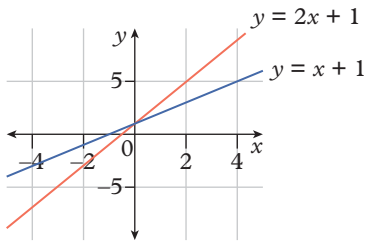
b The graphs all intercept the x- and y-axes at the same point, the origin (0, 0). This point is also the point of intersection of these graphs. The y-value of these line graphs all increase as you move from left to right. The y-values of $y = 3x$ increase more quickly than the y-values of $y = 2x$, which increase more quickly than the y-values of $y = x$.

c



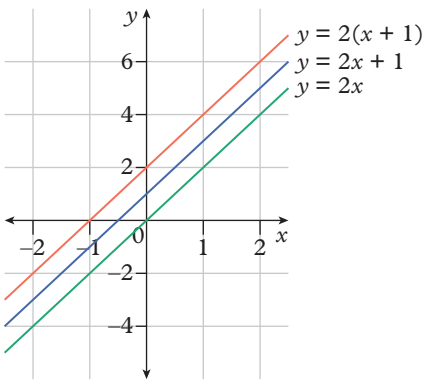
d The graphs all intercept the x- and y-axes at the same point, the origin (0, 0). This point is also the point of intersection of these graphs. Unlike the graphs drawn in part **a**, the y-values of the graphs drawn in part **c** all decrease as you move from left to right. The y-values of $y = –3x$ decrease more quickly than the y-values of $y = –2x$, which decrease more quickly than the y-value of $y = –x$.

e



The graphs both intersect the y-axis at $y = 1$, the point (0, 1). This point is also the point of intersection of these graphs. These graphs do not intersect the x-axis at the same point. The y-values of both graphs increase moving from left to right. The y-values of $y = 2x + 1$ increase more quickly.

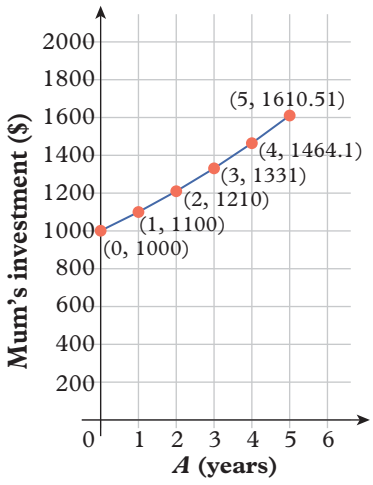
f

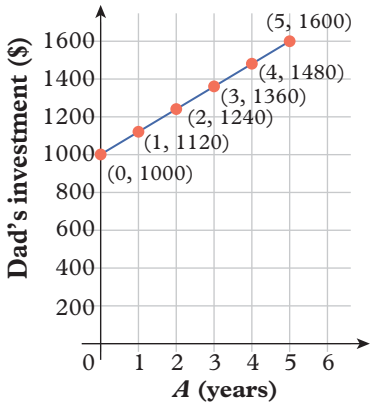


The graphs all intersect the x- and y-axes at different points. The y-values of each graph increases moving from left to right, and they all increase by 2 for each increase of 1 in the x-value. The graph $y = 2x + 1$ is the same as the graph of $y = 2x$ shifted one unit up, and the graph of $y = 2(x + 1)$ is the same as the graph of $y = 2x$ shifted two units up. The graph of $y = 2(x + 1)$ is also the same as the graph of $y = 2x$ shifted one unit left. The graph of $y = 2x + 1$ is also the same as the graph of $y = 2x$ shifted 0.5 units left.

17 a D vs A

b





Mum's investment: approx. $2600
Dad's investment: $2200

c Sample answer: Compound interest seems like the better choice. Other factors to consider are the interest rate and the terms and conditions of the investments.

EX p260 6.6 Solving linear equations using graphs

1 **a** **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$ **iv** $x = 2$

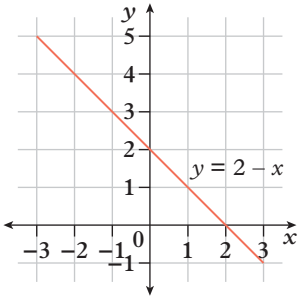
b **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$ **iv** $x = 2$

c **i** $x = 8$ **ii** $x = 4$ **iii** $x = 0$ **iv** $x = -2$

2 **a** $x = 2$ **b** $x = -1$ **c** $x = 1$ **d** $x = 0$ **e** $x = -2$ **f** $x = -3$

3 **a** $x = 2$ **b** $x = -2$ **c** $x = 0$ **d** $x = -4$ **e** $x = -3$ **f** $x = 1$

4 **a**

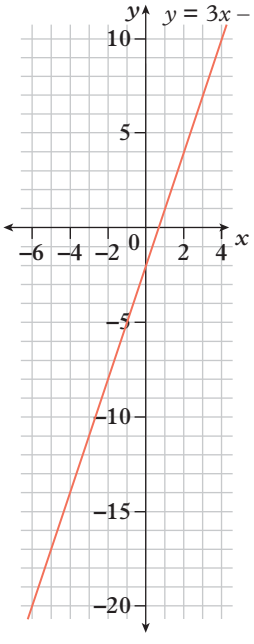


b **i** $x = 1$ **ii** $x = 3$ **iii** $x = 2$ **iv** $x = 0$ **v** $x = 0.5$ **vi** $x = 1.5$

5 **a** $y = 3x - 2$

b **i** $y = 7$ **ii** $y = -5$ **iii** $y = -20$

c

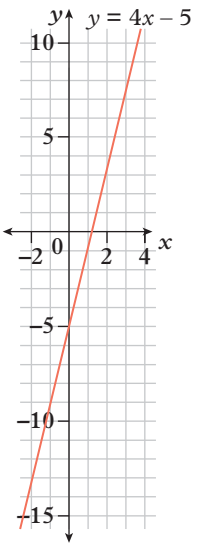


i $x = 3$ **ii** $x = -1$ **iii** $x = -6$

6 **a** $y = 4x - 5$

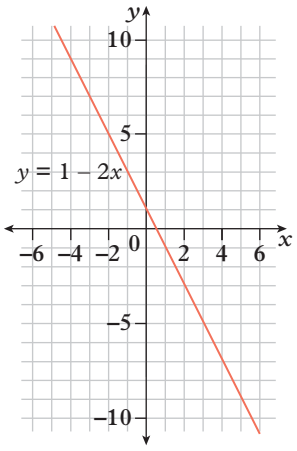
b **i** $y = 3$ **ii** $y = -13$ **iii** $y = -7$

c



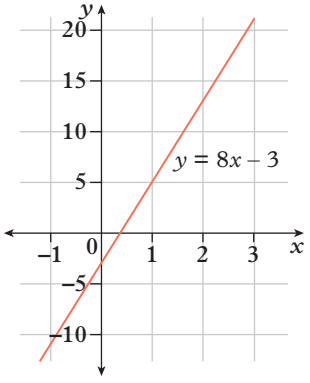
i $x = 2$ **ii** $x = -2$ **iii** $x = -0.5$

7



a $x = -1$ **b** $x = 3$ **c** $x = -3$ **d** $x = 5$ **e** $x = -4$ **f** $x = 2.5$

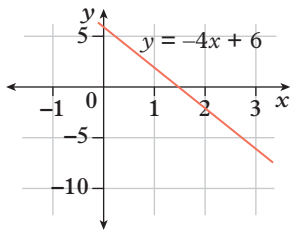
8



a $x = 1$ **b** $x = 2$ **c** $x = 3$ **d** $x = 0$ **e** $x = -1$ **f** $x = -0.5$

9 a $x = 2.5$

b



$x = 2.5$

c Sample answer: The graphical solution method can be used to solve many similar equations quickly; however, it takes longer to draw a graph and it is difficult to read non-integer values accurately from a graph.

10 a **i** $d = 50$ **ii** $d = 150$ **iii** $d = 200$

b **i** $0.64d = 50$ **ii** $d = 78.125$, \$78

c shirt:

i $0.64d = 64$ **ii** $d = 100$ **iii** \$100

shoes:

i $0.64d = 100$ **ii** $d = 156$ **iii** \$156

jeans:

i $0.64d = 42$ **ii** $d = 66$ **iii** \$66

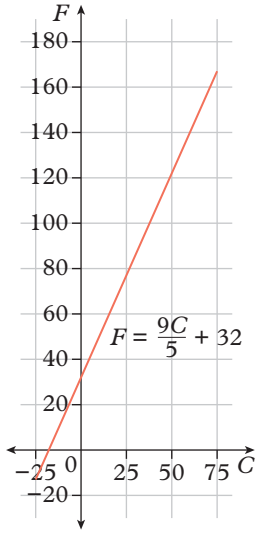
leather jacket:

i $0.64d = 140$ **ii** $d = 219$ **iii** \$219

11 a

C	−25	0	25	50	75
F	−13	32	77	122	167

b



c **i** $C \approx 4$ **ii** $C = 40$ **iii** $C = -10$

d meat:

i $\frac{9}{5}C + 32 = -5$ **ii** $C \approx -21$

dairy products and vegetables:

i $\frac{9}{5}C + 32 = 37$ **ii** $C \approx 3$

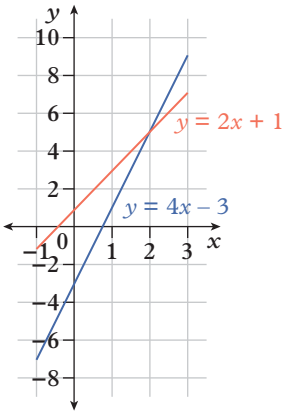
tomatoes:

i $\frac{9}{5}C + 32 = 68$ **ii** $C = 20$

iii 'Meat to be frozen should be stored in the freezer at −21°C, dairy products and vegetables stored in the fridge at about 3°C and tomatoes kept at room temperature, ideally around 20°C.'

e 60°C

12 a, b



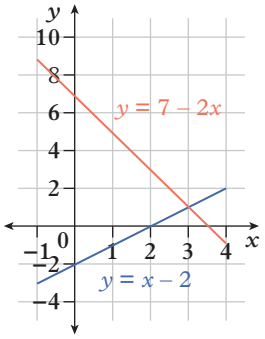
c (2, 5)

d $x = 2$

e $4x - 3 = 2x + 1$
$4(2) - 3 = 2(2) + 1$
$8 - 3 = 4 + 1$
$5 = 5$

13 a **i** $y_1 = x - 2$ and $y_2 = 7 - 2x$

ii

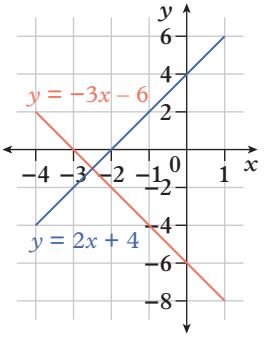


iii $x = 3$

iv $x - 2 = 7 - 2x$
$3 - 2 = 7 - 2(3)$
$1 = 7 - 6$
$1 = 1$

b **i** $y_1 = 2x + 4$ and $y_2 = -3x - 6$

ii



iii $x = -2$

iv $2x + 4 = -3x - 6$

$2(-2) + 4 = -3(-2) - 6$

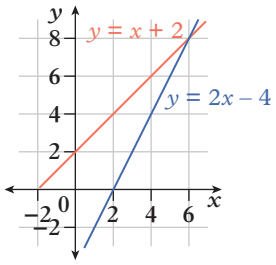
$-4 + 4 = 6 - 6$

$0 = 0$

14 a $x + 2 = 2x - 4$

b $y_1 = x + 2$ and $y_2 = 2x - 4$

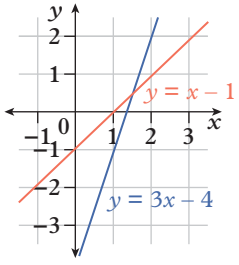
c



$x = 6$

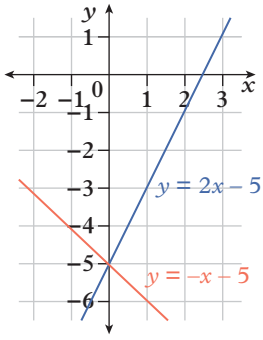
d six lamingtons

15 a



$x = 1.5$

b



$x = 0$

EX p266 6.7 Intersecting lines

1 a (2, 3) **b** (3, 2) **c** (4, −1)

d (−0.5, 1) **e** (−2, −3) **f** (−60, 80)

2 a (4, 3) **b** (0, 3) **c** (−2, 1)

d (−1, −2) **e** (−25, −2) **f** (5, 31)

3 a (1, 1)

x	−1	0	1	2	3
$y = 3x - 2$	−5	−2	1	4	7
$y = 2x - 1$	−3	−1	1	3	5

b (−2, 1)

x	−3	−2	−1	0	1
$y = 2x + 5$	−1	1	3	5	7
$y = -2x - 3$	3	1	−1	−3	−5

c (2, −2)

x	−2	−1	0	1	2
$y = 7x - 16$	−30	−23	−16	−9	−2
$y = 4x - 10$	−18	−14	−10	−6	−2

d (−1, −4)

x	−3	−2	−1	0	1
$y = -2x - 6$	0	−2	−4	−6	−8
$y = 6x + 2$	−16	−10	−4	2	8

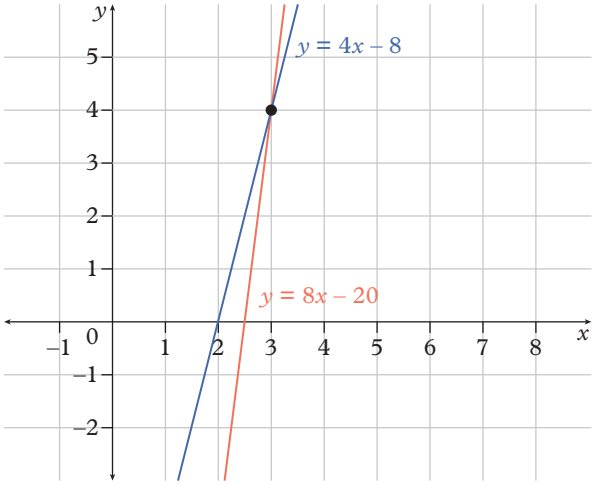
e (−26, 3)

x	−27	−26	−25	−24	−23
$y = -x - 23$	4	3	2	1	0
$y = 2x + 55$	1	3	5	7	9

f (5, −58)

x	2	3	4	5	6
$y = -3x - 43$	−49	−52	−55	−58	−61
$y = 4x - 78$	−70	−66	−62	−58	−54

4 a



b (3,4)

c $x = 3$

d i $y = 4x - 8$ $y = 8x - 20$

LHS = 4 LHS = 4

RHS = 4(3) − 8 = 4 RHS = 8(3) − 20 = 4

ii $4x - 8 = 8x - 20$

$4x + 12 = 8x$

$12 = 4x$

$3 = x$

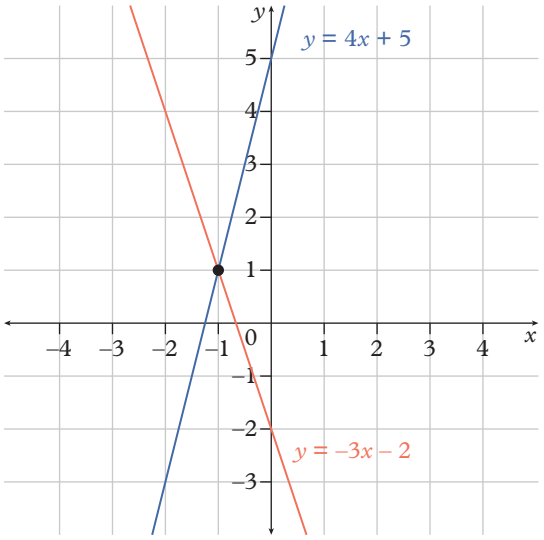
5 a $x = 2$ **b** $x = -3$ **c** $x = -2$

d $x = -4$ **e** $x = 1$ **f** $x = -6$

6 a $x = -1$ **b** $x = -5$ **c** $x = 5$

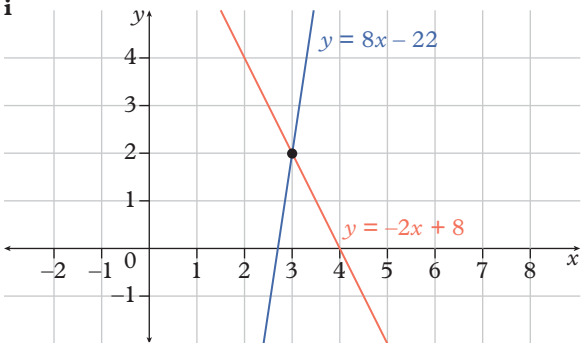
d $x = 6$ **e** $x = -1$ **f** $x = -4$

7 a i



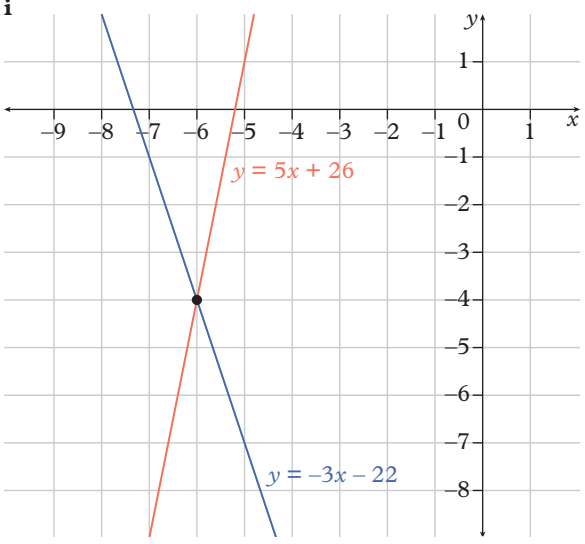
ii (−1, 1) **iii** $4x + 5 = -3x - 2$

b i



ii (3, 2) **iii** $8x - 22 = -2x + 8$

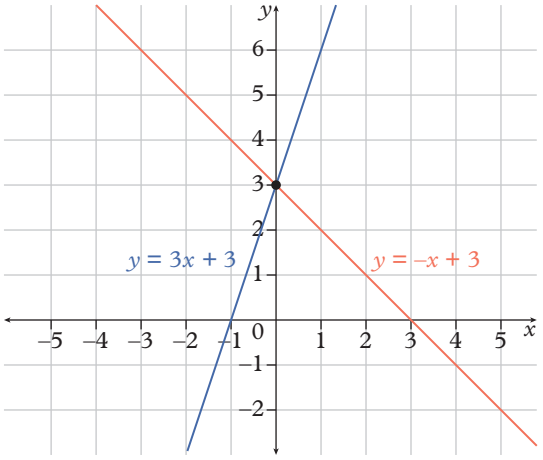
c i



ii (−6, −4) **iii** $5x + 26 = -3x - 22$

EX p272

d i



ii (0, 3) **iii** $3x + 3 = -x + 3$

8 $A(-3,3)$: $y = -2x - 3$ and $y = -x$

$B(0,2)$: $y = x + 2$ and $y = -x + 2$

$C(1,1)$: $y = x$ and $y = -x + 2$

$D(-3,-1)$: $y = 2x + 5$ and $y = x + 2$

$E(-2,1)$: $y = -2x - 3$ and $y = 2x + 5$

$F(0,0)$: $y = -x$ and $y = x$

9 a i The x-coordinate of the intersection point lies between 6 and 7.

ii The y-coordinate of the intersection point lies between 5 and 6.

b (6.2, 5.4)

x	6	6.1	6.2	6.3	6.4	6.5
$y = 2x - 7$	5	5.2	5.4	5.6	5.8	6
$y = 7x - 38$	4	4.7	5.4	6.1	6.8	7.5

10 a (7.3, −0.2) **b** (5.1, 3.5) **c** (9.1, −5.3)
d (7.8, 6) **e** (−5.5, 3) **f** (5.6, −8.6)

11 a Any pair of parallel lines do not intersect.

b The lines are parallel so the values of y change at the same rate. This means the lines will always stay the same distance (difference in y-values for each x-value) apart.

If two lines are not parallel, then there must be some point on the Cartesian plane where they intersect.

12 If two line graphs intersect in more than one point, then they are the same graph and they have the same equation.

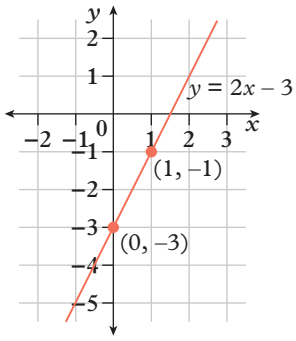
13 a (2, −1) and (5, 2) **b** (−1, −2) and (1, 2)
c (−3, 2), (0, −1) and (1, 2) **d** (−1, 3) and (2, 0)
e (−3, 0), (3, 0) and (0, −3) **f** (−1, 2)

6.8 Finding linear equations

1 a $y = 2x + 2$ **b** $y = -3x + 4$ **c** $y = 2x + 1$
d $y = -2x - 3$ **e** $y = 2x + 4$ **f** $y = -3x - 5$

2 a $y = 2x + 1$ **b** $y = -3x + 1$ **c** $y = 4x - 2$
d $y = -\frac{1}{3}x + 3$ **e** $y = -4x - 2$ **f** $y = -\frac{x}{2}$

3



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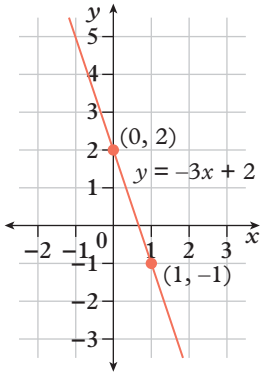


5 **a** IV **b** I **c** II **d** III

6 **a** III **b** II **c** I **d** IV

7 **a** I **b** II **c** IV **d** III

8 **a** $y = -4x + 5$ **b** $y = -3x - 5$

9 **a** $x = 4$ **b** $y = 2$ **c** $y = -3$ **d** $x = -3$

10 **a** **i** $y = 3x + 3$ **ii** $y = 3x + 1$ **iii** $y = 3x - 1$
iv $y = 3x - 3$

b **i** $y = -2x + 3$ **ii** $y = -2x + 1$ **iii** $y = -2x - 1$
iv $y = -2x - 3$

c The equations all have the same x coefficient (gradient)

d The equations all have the same x coefficient (gradient)

11 Linear graphs that have the same value for m in the equation $y = mx + c$ are parallel.

12 **a**

Rows	Tiles in the row
1	16
2	24
3	32
4	40

b $t = 8r + 8$

13 $y = -2x - 1.5$

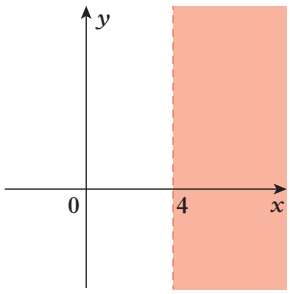
6.9 Solving linear inequalities

EX p279

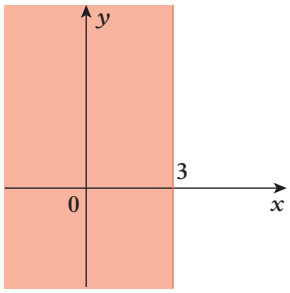
1 **a** $2 < 3$ **b** $3 > 2$ **c** $3 \geq 2$ **d** $-3 \leq -2$

2 **a** $x < 0.5$ **b** $x > \frac{4}{5}$ **c** $x \leq -12$ **d** $x \geq -1.2$
e $x > 0$ **f** $y < 13$ **g** $y > -\frac{5}{3}$ **h** $y \leq 2.13$
i $y \geq 99$

3 **a**



b



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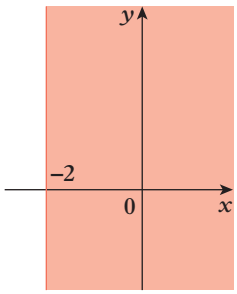
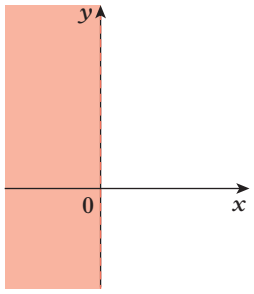
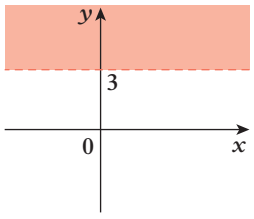


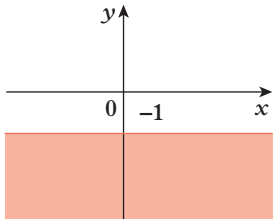
d



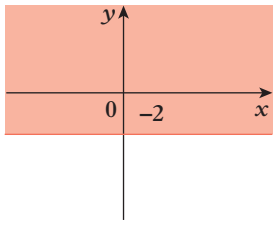
e



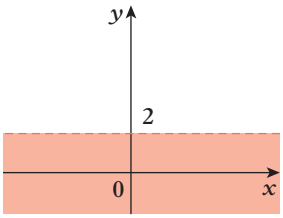
f



g



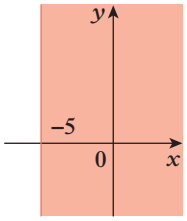
h



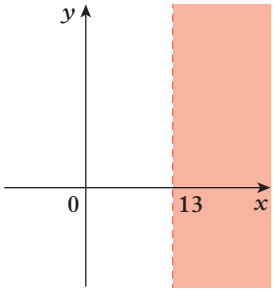
4 **a** **i** $1 < 14$ **ii** $-13 < 0$
iii $-12 < 27$ **iv** $-\frac{4}{3} < 3$
b **i** $x + 5 \geq 11$ **ii** $x - 9 \geq -3$
iii $3x \geq 18$ **iv** $\frac{x}{3} \geq 2$

5 **a** $x > 4$ **b** $x \leq 30$ **c** $x < -5$ **d** $x \leq 15$
e $x \geq 2$ **f** $x \leq 2$ **g** $x > -28$ **h** $x > -\frac{9}{10}$

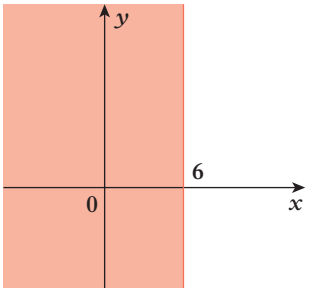
6 **a** $x \geq -5$



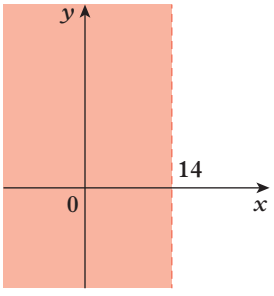
b $x > 13$



c $x \leq 6$



d $x < 14$



7 For a linear equation, there can be 0, 1 or infinitely many solutions. Consider the linear equation $ax + b = cx + d$. If $a \neq c$, then there is one solution. If $a = c$ and $b \neq d$, there are are no solutions. If $a = c$ and $b = d$, there are infinitely many solutions (since LHS = RHS for all values of x). For linear inequalities, there will always be an infinite number of solutions.

8 **a** $p \geq 650\,000$, where p is the selling price in dollars
b $h > 97$, where h is the height of a person in centimetres
c $s \leq 60$, where s is the speed in km/h

9 **a** $3p \leq 20$, where p is the cost of a pack of gum in dollars; $p \leq 6\frac{2}{3}$
b Todd could buy 1, 2, 3, 4, 5 or 6 packs of gum.

10 **a** $\frac{23x - 60}{2}$ **b** $\frac{23x - 60}{2} \geq 100$ c 12

11 **a** 25 min **b** 16 min

12 **a** $x > 4$ **b** $x \leq -2$ **c** $x \geq 3$ **d** $x > 4$
e $x < 1$ **f** $x \leq \geq -1$ **g** $x \leq 5$ **h** $x < 2$

13 **a** $x > 11$ **b** $x \leq 13$ **c** $x \geq -2$
d $x > -19$ **e** $x > -15$ **f** $x \leq 1$
g $x \leq 3$ **h** $x > -\frac{1}{4}$ **i** $x \geq -\frac{13}{2}$

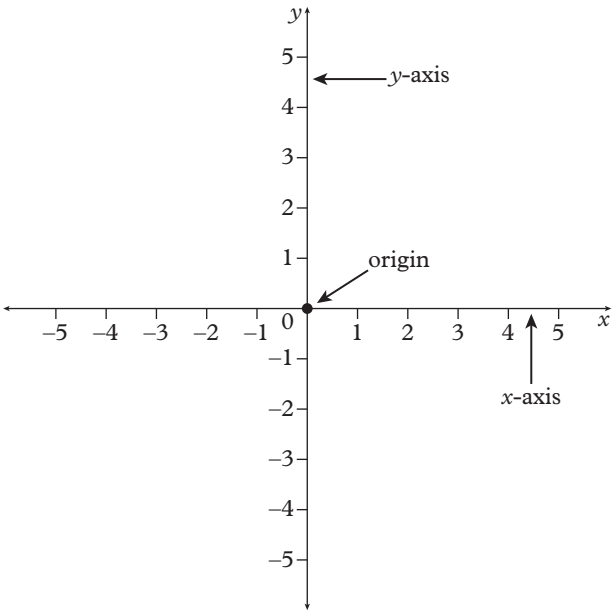
14 **a** $1 \leq x \leq 6$ **b** $-4 > x > -7$ or $-7 < x < -4$

EX p282

Review: Equations, inequalities and linear relationships

Review questions 6.10A: Mathematical literacy review

1



2 **a** The y-coordinate of a point tells you how far above or below the origin a point is located.
b The x-coordinate of a point tells you how far left or right of the origin a point is located.

3 **a** solve by inspection
b point of intersection
c y-intercept

4 a Inverse operations reverse the effect of another operation. For example, the inverse operation of adding three (+3) is subtracting three (–3).

b A linear relationship is a relationship between two variables which produces a straight line graph. For example, $y = 2x - 3$ is a linear relationship.

c A quadratic equation is an equation in which the highest exponent of any variable is 2. For example, $y = 2x^2 + x + 1$ is a quadratic equation.

5 An equation has an equals sign (=) while an inequality has a comparison symbol (>, <, ≤, ≥).

Review questions 6.10B: Multiple choice

1	C	**2**	A	**3**	B	**4**	C	**5**	D	**6**	D
7	A	**8**	A	**9**	C	**10**	B	**11**	D	**12**	A
13	B	**14**	A	**15**	B	**16**	A				

Review questions 6.10C: Short answer

1 a yes **b** yes **c** yes **d** no

2 a $a = 9$ **b** $b = 14$ **c** $c = 3$ **d** $d = 15$

3 a $x = 3$ **b** $x = -9$ **c** $c = 14$ **d** $d = 2$

e $x = 1\frac{1}{4}$ $\left(\text{or } x = \frac{5}{4}\right)$ **f** $x = -8$

4 a $12s + 24s = 36$

b $s = 1$; small book = 1 kg, large book = 2 kg

5 a $x = -5$ **b** $x = 2$ **c** $x = -3$

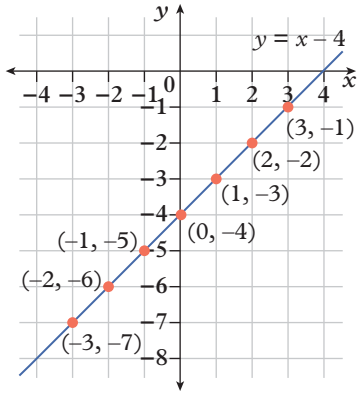
d $x = -2\frac{1}{2}$ $\left(\text{or } x = -\frac{5}{2}\right)$

6 $s + 11 = 5(s - 5)$, Suzi is 9 years old.

7 a ±6 **b** ±6 **c** 0

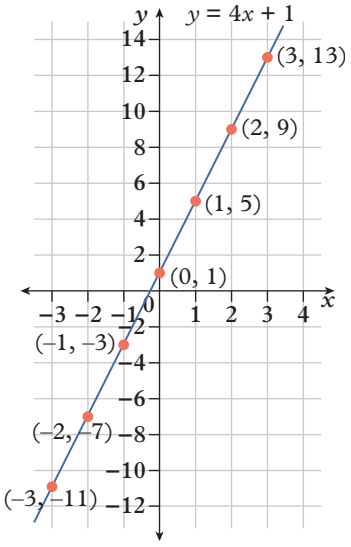
8 a

x	–3	–2	–1	0	1	2	3
y	–7	–6	–5	–4	–3	–2	–1



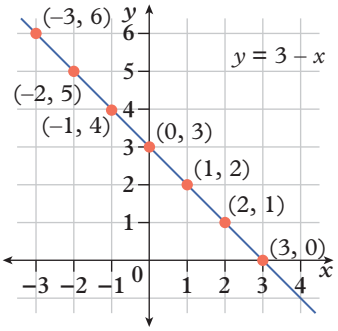
b

x	–3	–2	–1	0	1	2	3
y	–11	–7	–3	1	5	9	13



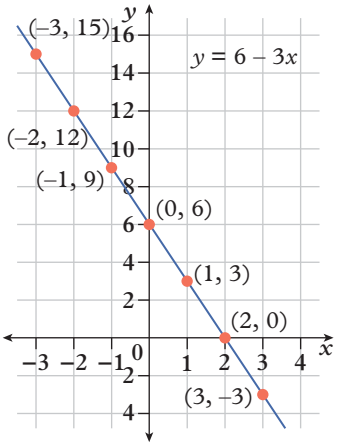
c

x	–3	–2	–1	0	1	2	3
y	6	5	4	3	2	1	0



d

x	–3	–2	–1	0	1	2	3
y	15	12	9	6	3	0	–3



9 a $x = -2$ **b** $x = 1$ **c** $x = 0$

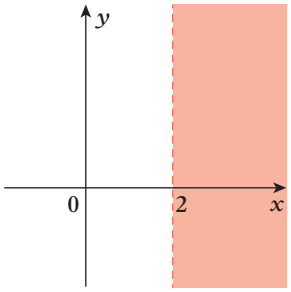
10 a $x = -5$ **b** $x = 2$ **c** $x = -3$

d $x = -2\frac{1}{2}$ $\left(\text{or } x = -\frac{5}{2}\right)$

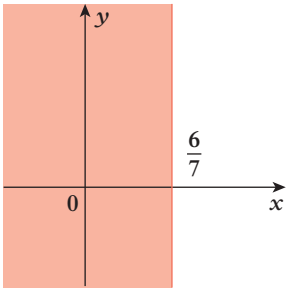
11 a $y = 4x - 5$ **b** $y = -x$

12 a $y = -x + 2$ **b** $y = 5x - 4$

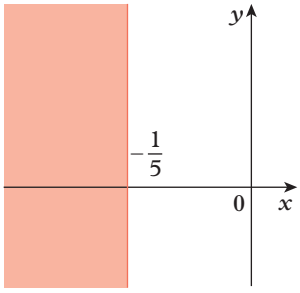
13 a $x > 2$



b $x \leq \frac{6}{7}$



c $x \leq -\frac{1}{5}$



14 a $y \leq 5$ **b** $x > -2$

Review questions 6.10D: Mathematical modelling

1 a $C = 5x + 3$

b \$53

c 7

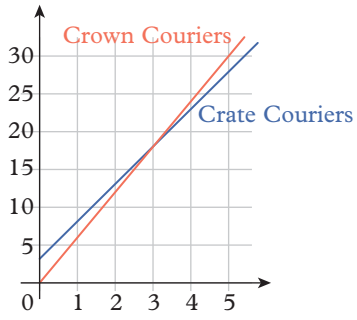
d

Number of books	0	1	2	3	4	5
Cost of delivery	\$3	\$8	\$13	\$18	\$23	\$28

e, f i

Number of books	0	1	2	3	4	5
Cost of delivery	\$0	\$6	\$12	\$18	\$24	\$30

ii



iii Joss should use Crate Couriers for delivering more than 3 books and he should use Crown Couriers for fewer than 3 books. For 3 books it makes no difference.

2 a $C = 150 + 20p$

b 15 people can attend the party.

c i $C = 250 + 15p$

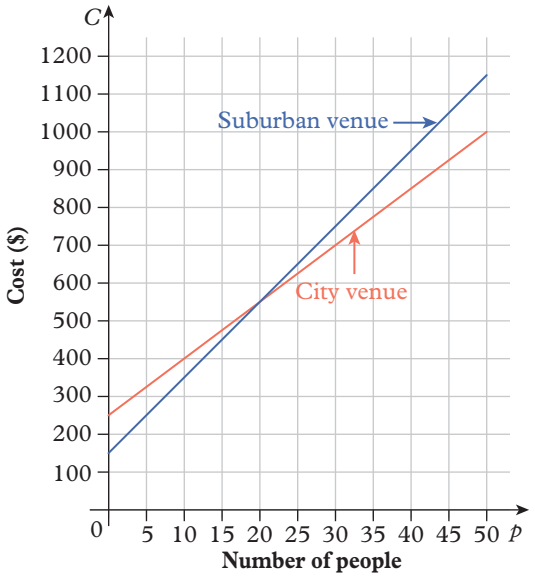
ii Suburban venue

p	0	10	20	30	40	50
C	150	350	550	750	950	1150

City venue

p	0	10	20	30	40	50
C	250	400	550	700	850	1000

iii



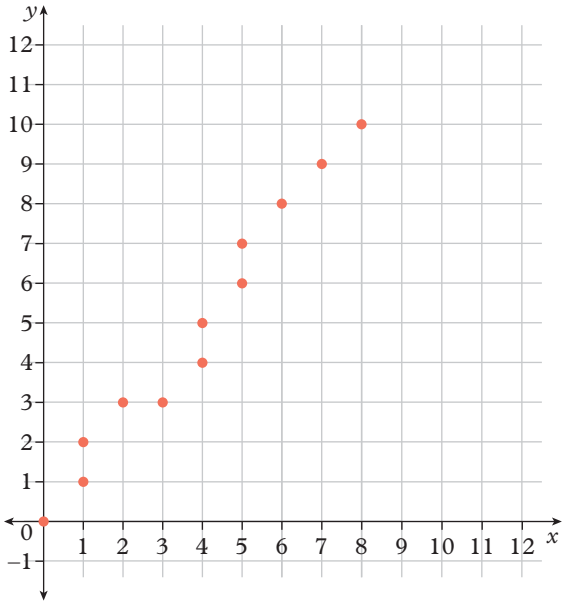
iv Yes, as the points form a straight line.

v suburban = \$650, city = \$625

vi suburban = 15, city = 13

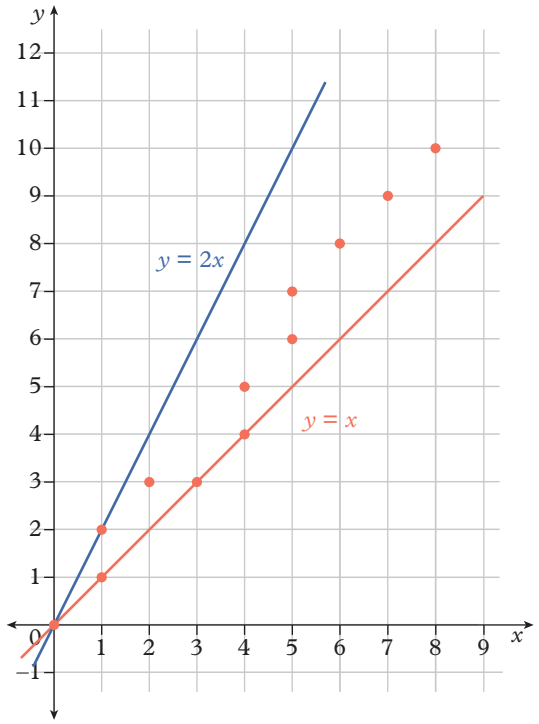
d i suburban **ii** city **iii** same cost

3 a



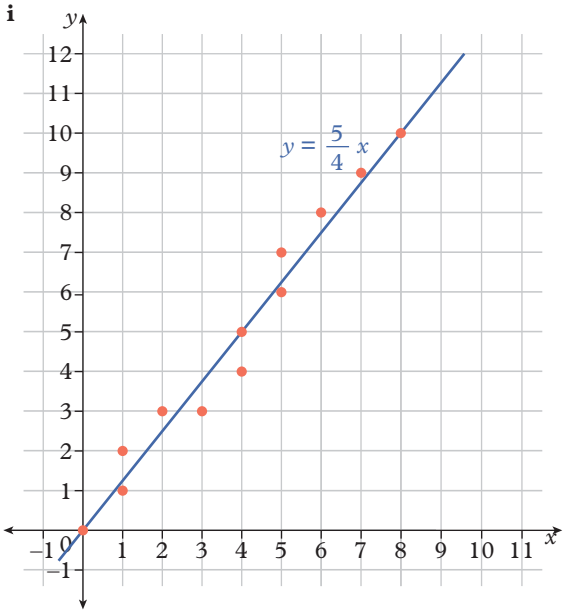
b No. The points can't be joined by a straight line.

c



d Most of the points are above the graph of $y = x$ and most of the points are below the graph of $y = 2x$.

e i



ii The graph of $y = \frac{5}{4}x$ passes through some of the points and the other points are split above and below the line.

4 a $P \leq 200c$

b 4

c 1400

d i 21

ii 100 minutes (1 hour 40 minutes)

iii The numbers of carriages and buses must be integers. You can't have just part of a bus or carriage!

EX p288 Computational thinking: Binary numbers

1 a 13

b The flowchart reports an error when it reaches the second digit (7).

c At the 'Is number fully converted?' decision point. The loop terminates and the flow chart reports the number when it is fully converted.

2 a A user error is introduced by the person following the flowchart while an algorithm error is an error in steps in the flowchart itself. Not increasing the value of S correctly would be a user error, while having a decision point of 'Is digit in set {0, 1, 2}?' would be an algorithm error.

b **1** Start

2 E = 0, B = 2, S = 0

3 Get the next rightmost digit, which is 1

4 Check if digit is in {0, 1}, then follow Yes path in flowchart

5 Update the variable S to S + 1 × 2^0, giving S = 1

6 Increase the E variable by 1 to E = 1

7 Check if the number is fully converted, then follow No path in flowchart

8 Get the next rightmost digit, which is 1

9 Check if digit is in {0, 1}, then follow Yes path in flowchart

10 Update the variable S to S + 1 × 2^1, giving S = 3

11 Increase the E variable by 1 to E = 2

12 Check if number is fully converted, then follow Yes path in flowchart

13 Print the contents of the S variable, which is 3

14 End

MODULE 7 Angles, triangles and Pythagoras' theorem

EX p294 7.1 Angles

1 a $\angle BDE$ or $\angle EDB$, obtuse angle

b $\angle PQV$ or $\angle VQP$, right angle

c $\angle FBY$ or $\angle YBF$, reflex angle

d $\angle MYZ$ or $\angle ZYM$, acute angle

e $\angle IMA$ or $\angle AMI$, reflex angle

f $\angle DHL$ or $\angle LHD$, obtuse angle

2 **a** $a = 15°$ **b** $b = 15°$ **c** $c = 39°$
d $d = 64°$ **e** $e = 138°$ **f** $f = 55°$

3 **a** $a = 160°$
b $a = 79°$, $b = 101°$, $c = 79°$
c $d = 147°$, $e = 33°$, $f = 147°$
d $g = 144°$
e $h = 253°$
f $i = 72°$, $j = 69°$, $k = 39°$, $l = 69°$

4 **a** complementary **b** supplementary
c vertically opposite **d** supplementary

5 **a** $m = 152°$, $n = 68°$
b $v = 27°$
c $s = 68°$, $t = 22°$, $u = 68°$
d $a = 7°$, $b = 139°$, $c = 41°$
e $w = 45°$, $x = 38°$, $y = 39°$, $z = 38°$
f $d = 115°$, $e = 65°$, $f = 115°$, $g = 110°$, $i = 110°$, $j = 135°$, $k = 45°$, $l = 135°$, $n = 70°$

6 **a** **i** corresponding **ii** $a = 63°$
b **i** alternate **ii** $b = 71°$
c **i** co-interior **ii** $c = 122°$
d **i** corresponding **ii** $d = 123°$
e **i** corresponding **ii** $e = 82°$
f **i** co-interior **ii** $f = 23°$
g **i** corresponding **ii** $g = 76°$
h **i** alternate **ii** $h = 35°$
i **i** co-interior **ii** $i = 49°$

7 **a** $a = 141°$, co-interior **b** $b = 147°$, co-interior
c $c = 21°$, alternate
d $d = 66°$, corresponding; $e = 59°$, alternate
e $f = 85°$, corresponding; $g = 95°$, co-interior
f $n = 142°$, co-interior

8 27°, angles are alternate and equal

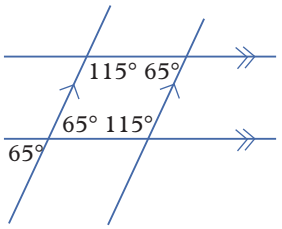
9 The stairs have an angle of 23° with respect to the horizontal and meet the standards but are not in the preferred range.

10 Alternate exterior angles are supplementary to the alternate interior angles (which are equal). The supplements of equal angles will be equal to each other, so alternate exterior angles are equal for parallel lines.

11 **a** Angles are corresponding but not equal, so the lines are not parallel.
b Angles are co-interior and supplementary, so the lines are parallel.
c 99° is supplementary to the angle alternate to 81°, so the lines are parallel.

12 **a** 360°
b **i** 180° **ii** 90° **iii** 270°
c **i** 120° **ii** 330° **iii** 240°
iv 30°

13 **a** parallelogram
b



14 **a** c and f; d and e; i and d; b and k
b a and e; b and f; c and g; d and h; j and b; i and a; l and d; k and c
c c and e; d and f; i and b; d and k

15 $a = 40°$ (angles in a triangle), $b = 50°$ (alternate angles), $c = 90°$ (right angle), $d = 40°$ (corresponding angle), $e = 50°$ (corresponding angle), $f = 50°$ (corresponding angle)

16 **a** The transversal creates two intersections. Around each intersection there will be two pairs of vertically opposite equal angles and they are related to each other by being supplementary. The pairs of angles from one intersection and the other are related and either equal or supplementary by being co-interior, corresponding or alternate. So all angles are either equal or supplementary to a given angle.
b From part **a**, the angles formed by one transversal are all related and either equal or supplementary to a given angle. For each new parallel transversal, all of its angles correspond to an angle from the previous transversal. So no matter how many parallel transversals are added, all angles are equal or supplementary to a given angle.

17 **a** 30°
b **i** 15° **ii** 7.5° **iii** 22.5°
c **i** 165° **ii** 97.5° **iii** 52.5°

18 **a** 170° **b** 122.5° **c** 72.5° **d** 88.5° **e** 79.5°

EX p302

7.2 Triangles

1 **a** obtuse-angled isosceles
b acute-angled scalene
c acute-angled isosceles
d acute-angled equilateral
e right-angled scalene
f obtuse-angled scalene

2 **a** 63° **b** 90° **c** 41°
d 67° **e** 106° **f** 50°

3 **a** 138° **b** 141° **c** 64°
d 111° **e** 109° **f** 164°

4 **a** $a = 60°$, $b = 60°$, $c = 60°$
b $a = 45°$, $b = 45°$ **c** $a = 142°$, $b = 19°$
d $d = 120°$ **e** $e = 71°$ **f** $f = 41°$
g $g = 27°$ **h** $h = 58°$ **i** $i = 156°$

5 **a** acute-angled equilateral
b right-angled isosceles
c obtuse-angled isosceles
d acute-angled equilateral
e acute-angled scalene
f acute-angled scalene
g obtuse-angled scalene
h right-angled scalene
i obtuse-angled isosceles

6 **a** False; the angles in an equilateral must be 60° (acute).
b True; the other angles in an obtuse-angled isosceles triangle will both be less than 45°.
c True; the other angles must sum to 90° and not be equal.

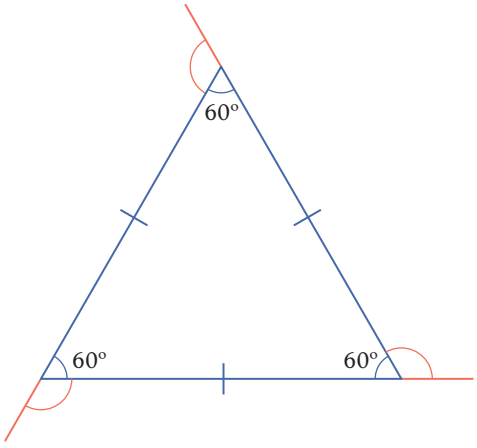
7 **a** $x = 30°$ **b** $x = 10°$ **c** $x = 10°$
d $x = 10°$ **e** $x = 12°$ **f** $x = 20°$

8 **a** $a = 30°$, $b = 60°$, $c = 90°$
b Right-angled scalene; the triangle has three different interior angles including a right angle.

9 $a = 37°$, $b = 113°$, $c = 141°$, $d = 17°$

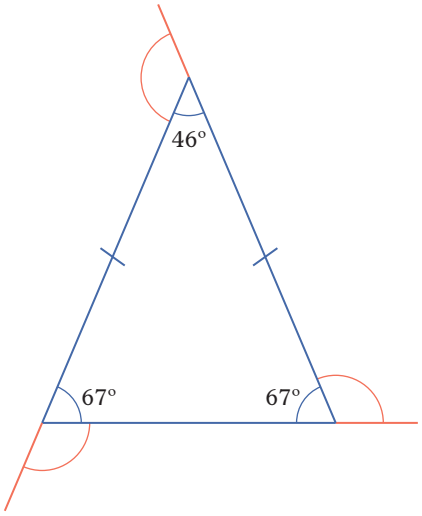
10 **a** $p = 96°$, $q = 36°$, $r = 83°$
b 96° **c** $\angle QRS = 48°$

11 **a** **i**



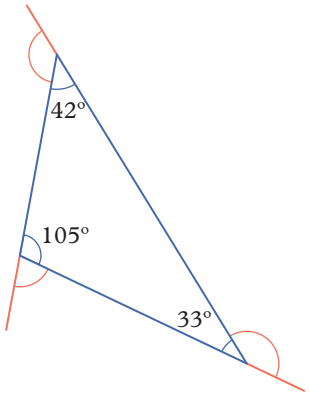
ii Each exterior angle is 120°; they add to 360°.

b **i**



ii Exterior angles are 134°, 113° and 113°; they add to 360°.

c **i**

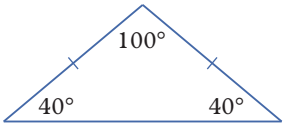


ii Exterior angles are 138°, 147° and 75°; they add to 360°.

d The exterior angles of any triangle add to 360°.

12 **a** Not possible. The angle sum of a triangle is 180°. These angles add to 200°.

b

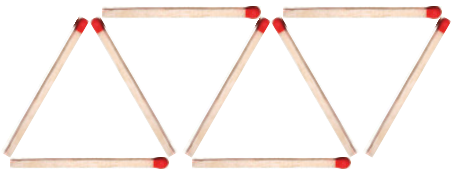


c Not possible; if the base length is 10 cm, the other two sides are not long enough to meet.

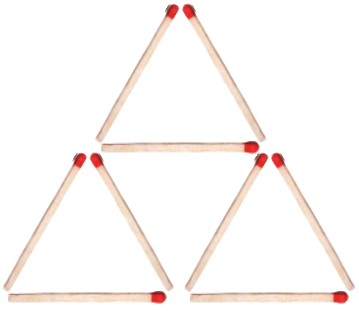
d Not possible; the angle sum of a triangle is 180°. Two obtuse angles add to more than 180°.

13 **a** seven matches

b **i**

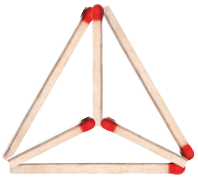


ii



(The large triangle is the fifth triangle.)

iii



(triangular pyramid)

14 $x + c = 180°$
$a + b + c = 180°$
$x + c = a + b + c$
$x = a + b$

EX p309 7.3 Pythagoras' theorem

1 **a** m **b** p **c** f

2 **a** right-angled **b** not right-angled
c right-angled **d** not right-angled

3 **a** right-angled **b** right-angled
c not right-angled **d** not right-angled
e right-angled **f** not right-angled

4 **a** 15 cm **b** 50 mm **c** 61 cm **d** 26 cm

5 **a** 9.43 cm **b** 17.49 cm **c** 23.85 mm

6 **a** **i** $\sqrt{157}$ cm **ii** 12.53 cm
b **i** $\sqrt{3049}$ m **ii** 55.22 m
c **i** $\sqrt{637}$ mm **ii** 25.24 mm

7 **a** 171 cm **b** 3 cm **c** 10 cm

8 No, because $a^2 + b^2$ is the same as $b^2 + a^2$.

9

Values		Pythagorean triad		
m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	$2^2 - 1^2 = 3$	$2 \times 2 \times 1 = 4$	$2^2 + 1^2 = 5$
3	1	8	6	10
3	2	5	12	13
4	1	15	8	17
4	2	12	16	20
4	3	7	24	25
5	1	24	10	26
5	2	21	20	29
5	3	16	30	34
5	4	9	40	41
6	1	35	12	37

10 **a** (3, 4, 5), (5, 12, 13), (15, 8, 17), (7, 24, 25), (21, 20, 29), (9, 40, 41), (35, 12, 37)
b **i** (5, 12, 13) **ii** (8, 15, 17)
iii (3, 4, 5) **iv** (9, 40, 41)
v (65, 72, 97) **vi** (21, 220, 221)

11 **a** $10^2 = 6^2 + 8^2$, $100 = 36 + 64$
b yes; $20^2 = 12^2 + 16^2$
c yes; $30^2 = 18^2 + 24^2$
d yes; $5^2 = 3^2 + 4^2$
e yes; $100^2 = 60^2 + 80^2$

12 **a** 9.43 cm **b** 10 cm **c** 25.61 mm
d 3.61 m **e** 14.42 cm **f** 21.21 mm

13 **a** $x = 27.80$ cm, $y = 29.90$ cm
b $x = 15.81$ m, $y = 18.71$ m
c $x = 60.22$ cm, $y = 66.41$ cm

14 Pythagoras' theorem calculates the length of a side, and a length is always a positive value.

15 **a** Triangle ABC and Triangle ACD
b $AC = 10$ cm; $AB = 6$ cm and $BC = 8$ cm
c $CD = 26$ cm; the hypotenuse in triangle ABC becomes one of the smaller sides in triangle ACD.

16 **a** 16, 25, 36
b In an acute triangle, the square of the longest side is less than the sum of the squares of the two other sides.
c 9, 64, 100
d In an obtuse triangle, the square of the longest side is greater than than the sum of the squares of the two other sides.
e **i** $121 < 164$ (64 + 100) **ii** $169 > 100$ (36 + 64)

17 **a** $x = \frac{5}{2}$ **b** $w = \frac{13}{8}$ **c** $t = 2$ **d** $p = \frac{1}{3}$

18 **a** Both squares have side lengths of $a + b$, so they both have an area of $(a + b)^2$.
b Both squares have the same area and include four identical triangles. So, the remaining space must be equal. The remaining area (shaded green) in Diagram 1 is a square with side lengths of c so it has an area of c^2. The remaining area of Diagram 2 is two squares, one with side length a and the other with side length b. So their areas are a^2 and b^2 respectively. Therefore, equating the remaining areas, $c^2 = a^2 + b^2$.
c **i** Diagram 1: $c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$
Diagram 2: $a^2 + b^2 + 4 \times \frac{1}{2}ab = a^2 + b^2 + 2ab$
ii $c^2 + 2ab = a^2 + b^2 + 2ab$
$c^2 = a^2 + b^2$
d **i** The total area of Diagram 3 with side lengths of c is c^2. The square is made up of four right-angled triangles with an area of $\frac{1}{2}ab$ and a square with side lengths of $b - a$ and an area of $(b - a)^2$.
ii $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$
$c^2 = b^2 - 2ab + a^2 + 2ab$
$c^2 = a^2 + b^2$

19 **a** $a = 8$ m **b** $b = 9.49$ m
c $c = 7$ m **d** $d = 7.75$ m

EX p314 Checkpoint

1 **a** $x = 161°$
b $p = 109°$, $q = 109°$, $r = 71°$
c $a = 67°$, $b = 67°$, $c = 23°$

2 **a** parallel; the alternate angles are equal
b not parallel; the corresponding angles are not equal
c parallel; the co-interior angles are supplementary

3 **a** $a = 42°$, obtuse-angled scalene
b $b = 90°$, right-angled scalene
c $c = 14°$, $d = 152°$, obtuse-angled isosceles
d $e = 60°$, $f = 60°$, $g = 60°$, acute-angled equilateral

4 **a** 124°
b 38°
c 117°
d 94°

5 **a** yes **b** no **c** no

6 **a** 40.50 cm **b** 90.43 mm
c 112.36 mm or 11.24 cm

EX p316 7.4 Finding the length of a shorter side

1 **a** $x = \pm 3.32$ **b** $x = \pm 7.14$ **c** $x = \pm 6.93$
d $x = \pm 8.49$ **e** $x = \pm 10.91$ **f** $x = \pm 4.24$

2 **a** 20 mm **b** 15 cm **c** 48 mm
d 27 mm **e** 14 m **f** 36 cm

3 **a** 12.65 cm **b** 10.25 cm **c** 22.36 mm
d 18.73 cm **e** 4.76 cm **f** 15.96 m

4 **a** **i** $\sqrt{783}$ cm **ii** 27.98 cm
b **i** $\sqrt{105}$ m **ii** 10.25 m
c **i** $\sqrt{184}$ cm **ii** 13.56 cm
d **i** $\sqrt{2080}$ m **ii** 45.61 m

5 **a** 398 cm **b** 5 cm **c** 55 cm **d** 47 cm

6 **a** 10.00 cm **b** 7.91 m **c** 10.95 cm
d 16.47 mm **e** 5.31 mm **f** 10.25 cm

7 **a** 15.65 m **b** 4.58 cm **c** 7.07 m

8 **a** $x = 13.08$ cm, $y = 7.35$ cm
b $x = 4$ m, $y = 3.48$ m
c $x = 17.80$ m, $y = 7.81$ m
d $x = 43.86$ cm, $y = 28.90$ cm

9 **a** $c = 10$
b $a^2 = 10$ and $b^2 = 90$
c A triangle is right-angled if its side lengths satisfy Pythagoras' theorem. $c^2 = 10^2 = 100$ and $a^2 + b^2 = 10 + 90 = 100$.
Therefore, the triangle is right-angled.
d $(-1.5, 0.5)$, $(3.5, 0.5)$, $(3, -1)$
e 22.5, 2.5, 25
f $c^2 = 25$ and $a^2 + b^2 = 22.5 + 2.5 = 25$. So, the triangle is right-angled.

10 $a = 3.00$ cm, $b = 5.81$ cm, $c = 5.67$ cm, $d = 6.41$ cm, $e = 4.25$ cm, $f = 2.43$ cm, $g = 9.27$ cm

11 **a** $a = 113.14$ **b** $a = 144$ **c** $a = 43.30$

12 **a** **i** and **ii** **b** **iv**
c **iii**; The calculation subtracts the square of the shorter side from the square of the hypotenuse. This is the wrong way around. The calculation is the square root of a negative number, which is not a real number as needed for the length of a right-angled triangle.

13 **a** $a = 3.74$ m
b $b = 7.14$ m
c $c = 11$ m
d $d = 6.71$ m

14 7.5 cm^2

EX p321 7.5 Applications of Pythagoras' theorem

1 **a** $\sqrt{500}$ mm **b** $\sqrt{208}$ cm **c** $\sqrt{250}$ cm
d $\sqrt{18}$ cm **e** $\sqrt{40}$ mm **f** $\sqrt{272}$ cm

2 **a** 22.4 mm **b** 14.4 cm **c** 15.8 cm
d 4.2 cm **e** 6.3 mm **f** 16.5 cm

3 **a** $\sqrt{50}$ cm **b** $\sqrt{800}$ mm
c $\sqrt{128}$ cm **d** $\sqrt{1250}$ cm

4 **a** 7.1 cm **b** 28.3 mm **c** 11.3 cm
d 35.4 cm

5 **a** 41.2 cm **b** 15.5 cm **c** 10.4 cm
d 22.5 cm **e** 36.7 cm **f** 18.0 mm

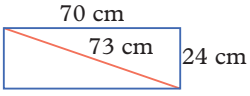
6 18.03 m

7 2.87 m

8 8.72 m

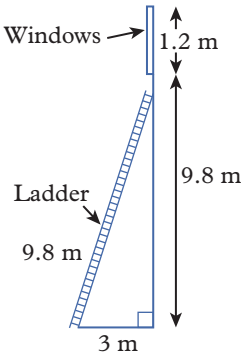
9 yes, because $60^2 + 63^2 = 87^2$

10 **a**



b The frame is not square. The diagonal distance should be longer.
c 74 cm

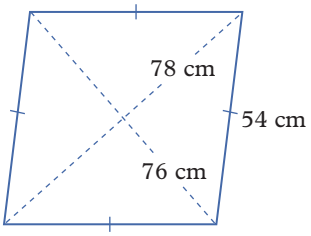
11 **a**



b 9.3 m
c The top of the window is 9.8 + 1.2 = 11 m above the ground. If Josh stands on the very top of the ladder, he is the same height above the ground as the top of the window (9.3 m + 1.7 m = 11 m). He will be able to clean the windows.

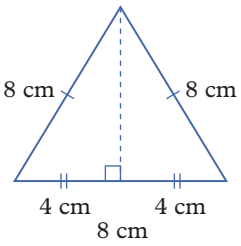
12 16.83 m

13 **a** $78^2 > 54^2 + 54^2$, so the frame is not truly square. The two sides meet at an obtuse angle.
b



14 **a** 11 cm **b** 2 folds **c** 2 folds

15 a

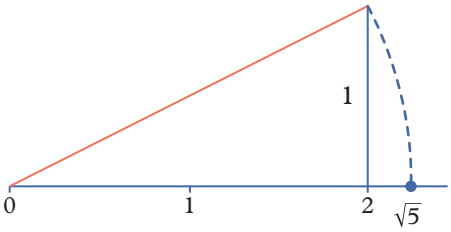


b **i** $4\sqrt{3}$ cm **ii** 6.9 cm

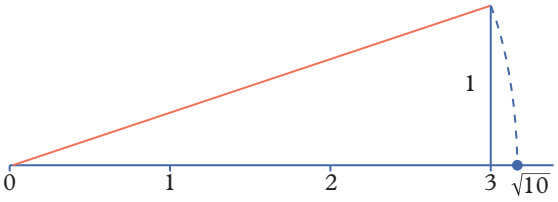
c $A = \frac{\sqrt{3}x^2}{4}$

16 The rope with 12 equally spaced knots forms a right-angled triangle with sides of 3 intervals, 4 intervals and 5 intervals. Pythagoras' theorem applies ($3^2 + 4^2 = 5^2$).

17 a



b



18 a $\sqrt{61}$ cm **b** 8.37 cm

19 a 10.63 cm **b** 7.28 mm **c** 8.85 cm **d** 8.77 cm

20 diagonal length = $\sqrt{a^2 + b^2 + c^2}$

EX p326

Review: Angles, triangles and Pythagoras' theorem

Review questions 7.6A: Mathematical literacy review

1 isosceles triangle

2 corresponding angles

3 obtuse angle

4 a transversal **b** acute-angled triangle

5 a any set of three positive whole numbers that satisfies Pythagoras' theorem

b angles that add to 90° (a right angle)

6 a The hypotenuse is the longest side of a right-angled triangle.

b An exterior angle is the angle between a side of a shape and a line extended from the adjacent side.

Review questions 7.6B: Multiple choice

1 C **2** D **3** B **4** D

5 C **6** B **7** C **8** A

Review questions 7.6C: Short answer

1 a $a = 138°, b = 42°, c = 138°$

b $a = 38°, b = 142°, c = 52°$

2 a $j = 76°, k = 76°$

b $l = 22°$

3 a $x = 109°$ **b** $x = 66°, y = 48°$

c $x = 141°$ **d** $x = 69°$

4 a obtuse-angled scalene

b acute-angled isosceles

c acute-angled scalene

d acute-angled scalene

5 a right-angled

b not right-angled

6 $x = 20$ cm, $y = 36.06$ cm

7 a 25.46 cm **b** 1.50 m

8 a 24.5" **b** 47.9" **c** 75.0"

Review questions 7.6D: Mathematical modelling

a $A(2, 3), B(6, 0), C(-1, -1)$

b $a = \sqrt{50}, b = 5, c = 5$

c The triangle is a right-angled isosceles triangle.

d **i** $\sqrt{12.5}$

ii $(0, 6)$

iii The new triangle is an acute-angled isosceles triangle.

MODULE 8 Units of measurement

8.1 Length and perimeter

EX p334

1 a 5800 cm **b** 2700 m **c** 5.21 m

d 0.398 km **e** 12 mm **f** 3.987 km

g 42.3 cm **h** 0.7 cm

2 a 5340 mm **b** 395 000 cm **c** 235 m

d 145 km **e** 0.023 68 km **f** 0.287 m

g 193 400 cm **h** 910 000 mm

3 a 3.77 m **b** 15.71 cm **c** 144.51 mm

d 5.03 cm **e** 571.77 cm **f** 8.17 mm

g 5.50 m **h** 8.17 cm **i** 257.61 km

4 a 20 cm **b** 8 m

c 142 mm (14.2 cm) **d** 168 mm (16.8 cm)

e 0.124 m **f** 137 cm (1.37 m)

5 a 14 cm (or 140 mm) **b** 34 m

c 60 cm **d** 56 cm

e 18.4 m **f** 14.6 m

6 a 16 cm **b** 60 mm **c** 50 mm **d** 60 cm

7 a 7.5 cm **b** 15 m **c** 14 cm **d** 61.6 cm

8 a 65 cm **b** 43.6 cm (436 mm)

c 47 cm **d** 112 mm

e 31 m **f** 28.4 cm

9 a 12.88 cm **b** 11.68 mm **c** 13.28 cm **d** 22.42 m

e 18.12 cm **f** 30.85 mm

10 a $10\frac{1}{5}$ m **b** $5\frac{1}{6}$ cm **c** $2\frac{4}{7}$ cm

d $4\frac{5}{6}$ m or $\frac{29}{6}$ m **e** $\frac{\pi}{4} + \frac{7}{3}$ cm **f** $\frac{16 + 4\pi}{3}$ mm

11 480 cm = 4.8 m

12 25.2 km

13 192 cm, the shape is made up of 32 edges of the octagons, each with a side length of 6 cm.

14 $x = 2$, $y = 3.5$, $P = 76$ cm

15 9 cm, the side length of a square is a quarter of the perimeter.

16 1.15 m

17 1.50 m

18 **a** $4l$ **b** $2w + 2l = 2(w + l)$
c $2a + 2b = 2(a + b)$ **d** $4l$
e $a + b + s + t$ **f** $2a + 2b = 2(a + b)$

19 **a** 22.65 cm **b** 14.60 cm **c** 27.72 cm

20 **a** 91.5 m **b** 549 m
c **i** 20.33 m **ii** 122 m
iii 488 m, four times the distance of one shuttle run (calculated in part **ii**)
iv four shuttle runs

21 **a** 50 cm
b **i** The corners of the frame are where the frame is longer than the width and length of the photo. Because of this, the sides of the frame must be longer than the sides of the photo.
ii 54 cm
iii 58 cm; the perimeter is longer because the corner lengths are counted twice, horizontally and vertically.
c **i** length of wood = 58 cm, perimeter = 66 cm
ii length of wood = 62 cm, perimeter = 74 cm
iii length of wood = 74 cm, perimeter = 78 cm
iv length of wood = 94 cm, perimeter = 98 cm

22 $20 \times 4 + 15 \times 4 + 60 = 200$ cm

23 1 cm × 14 cm, 2 cm × 13 cm, 3 cm × 12 cm, 4 cm × 11 cm, 5 cm × 10 cm, 6 cm × 9 cm, 7 cm × 8 cm

24 **a** 36.4 m
b length: 0.9 m, width 0.5 m
c length: 7.2%, width: 10.6%
d **i** length **ii** width
e 39.2 m
f 7.7%

25 51.52 m; there are 56 pages, each of which will require 92 cm of ribbon.

26 **a** Some possible answers are: 6 m × 8 m, 10 m × 4 m, 12 m × 2 m.
b 8 m longer than garden perimeter: 36 m
c **i** 44 m **ii** 52 m
d Multiply the distance by 8 and add the answer to the initial perimeter (28 m).
e The rule works for all cases. Start with the initial rectangle or the inside rectangle. Once you've found this perimeter, the outer perimeter can be found by adding 8 × the distance to the fence to the inside perimeter.

27 **a** 1 foot = 30.48 cm
b 1 yard = 0.9144 m
c 1 mile = 1.609 344 km
d 42.194 988 km

8.2 Area of rectangles and triangles

EX p343

1 **a** 70 mm² **b** 1500 mm² **c** 10 200 cm²
d 29.4 mm² **e** 9900 cm² **f** 161 mm²
g 540 mm² **h** 4 cm² **i** 20 cm²

2 **a** 22.5 cm² **b** 0.96 m² **c** 10.8 cm²
d 367.5 cm² **e** 0.4725 m² **f** 3.382 m²

3 **a** 1400 mm² **b** 20.4 cm² **c** 150 mm²
d 51.84 mm² **e** 12.5 cm² **f** 16.8 cm²
g 361 cm² **h** 462 mm²

4 **a** 14 cm² **b** 14 cm² **c** 900 mm²
d 9 m² **e** 90 cm² **f** 15 cm²

5 **a** 30 000 cm² **b** 1000 mm² **c** 65 000 cm²
d 1970 mm² **e** 1 600 000 m² **f** 750 000 m²

6 **a** 2.5 m² **b** 7500 cm² **c** 4 cm²
d 0.125 km² **e** 0.0675 m² **f** 500 km²

7 **a** 0.15 m² **b** 20 000 000 cm²
c 1 500 000 mm² **d** 0.000 037 9 km²

8 8704 cm²

9 They used the slant height (68.5 cm) as the height of each trianglular section rather than the perpendicular height (68 cm).

10 **a** 24 cm² and 4 cm²; 28 cm²
b 8 cm² and 20 cm²; 28 cm²

11 **a** 92 mm² **b** 35 cm² **c** 199 m²
d 66 cm² **e** 96 cm² **f** 72 cm²

12 **a** 56.51 cm² **b** 120.80 mm² **c** 26.70 cm²

13 The area units should give numerical values in the units, tens and hundreds which people can easily understand.
a m² **b** mm² **c** km² **d** cm² **e** m² **f** cm²

14 Alex divided by the conversion factor instead of the square of the conversion factor.

$40\,000 \text{ cm}^2 = (40\,000 \text{ cm} \div 100^2) \text{ m}^2$
$= 4 \text{ m}^2$

15 $11 \text{ cm}^2 = (11 \times 10^2) \text{ mm}^2 = 1100 \text{ mm}^2$; Ahmed forgot to square the conversion factor.

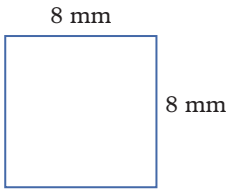
16 **a** 100 **b** 4000 m²

17 **a** **i** 3200 kg/hectare **ii** 0.32 kg/m²
iii 320 g/m² **iv** 0.032 g/cm²
b Grams per square metre will make the most sense for most people, as they are units people are more familiar with. For farmers, tonnes per hectare makes the most sense, as a farmer can have thousands of hectares of land.
c $3 360 000

18 Width is 5 cm. There is only one answer because only one number gives 20 when multiplied by 4.

19 Possible answers include: 40 cm² (10 cm × 4 cm), 48 cm² (6 cm × 8 cm), 24 cm² (12 cm × 2 cm).

20 A square has equal side lengths and its area is the square of its side length. There is only one positive number that when multiplied by itself gives 64: 8 × 8 = 64, so the dimensions must be 8 mm × 8 mm. The perimeter is 32 mm.



21 **a** 40 cm **b** 1400 cm²

22 **a** Possible answers include: 8 m × 4 m (32 m²), 7 m × 5 m (35 m²), 6 m × 6 m (36 m²), 2.4 m × 9.6 m (23.04 m²)

b 36 m²; the maximum area corresponds to a square with side lengths 6 m.

c **i** Possible answers include: 2 m × 20 m (20 m side on shed); 40 m², 4 m × 16 m (16 m side on shed); 64 m², 5 m × 14 m (14 m side on shed); 70 m².

ii 72 m² is the maximum area for a rectangle with dimensions 12 m × 6 m. Its shape is a half square. Its area is twice the area of the square in part **b**.

EX p350 8.3 Area of quadrilaterals

1 **a** 4.4 and 7.2 **b** 8.6 and 5.5 **c** 4.2, 4.8 and 6.2
d 4.2 and 5.2 **e** 3.2 and 4.4 **f** 2.1, 3.2, and 5.2

2 **a** 24 cm² **b** 63 mm² **c** 168 cm²

3 **a** 14 cm² **b** 48 mm² **c** 6 m²

4 **a** 12 mm² **b** 20 cm² **c** 22.5 mm²

5 **a** 124 mm² **b** 35 cm² **c** 138 m²

6 **a** **i** rhombus **ii** $A = bh$ **iii** 42 cm²

b **i** kite **ii** $A = \frac{1}{2}xy$ **iii** 3 cm²

c **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 15.96 m²

d **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 18 cm²

e **i** parallelogram **ii** $A = bh$ **iii** 112 mm²

f **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 21 mm²

7 **a** 10.5 cm² **b** 175 mm²
c 99 cm² (or 9900 mm²) **d** 90 cm²
e 44 cm² **f** 56 mm²

8 **a** 8.925 cm² **b** 9.18 mm²

9 **a** 3087.5 cm² **b** 20 cm

10 $731.25

11 **a** No, there is not enough information to confirm it is a trapezium, as we do not know the two sides that appear horizontal are parallel.

b No, there is enough information to confirm it is a parallelogram, but not enough information to determine its area as the perpendicular height is not known.

c Yes, there is enough information to confirm it is a kite, and the length of the perpendicular diagonals are both known.

d Yes, there is enough information to confirm it is a rhombus and both diagonal lengths are known (which are perpendicular since it is a rhombus).

e Yes, there is enough information to confirm it is a trapezium as the two vertical sides are both at right angles to the 2 cm length, which acts as a transversal indicating the vertical sides are parallel.

f No, there is not enough information to confirm it is a kite even though the diagonals are perpendicular.

12 **a** **i** parallelogram, trapezium

ii 8 × 2 = 16 m², $\frac{1}{2}(8 + 8) \times 2 = 16$ m²

b **i** parallelogram, kite, rhombus, trapezium

ii 4 × 4 = 16 m², $\frac{1}{2} \times 5.66 \times 5.66 \approx 16.0$ m²

(kite and rhombus), $\frac{1}{2}(4 + 4) \times 4 = 16$ m²

c **i** parallelogram, kite, rhombus, trapezium

ii 4 × 2.29 = 9.2 m², $\frac{1}{2} \times 2.41 \times 7.63 = 9.2$ m²

(kite and rhombus), $\frac{1}{2}(4 + 4) \times 2.29 = 9.2$ m²

d **i** parallelogram, trapezium

ii 3 × 7 = 21 m², $\frac{1}{2}(7 + 7) \times 3 = 21$ m²

13 **a** 74.94 cm² **b** 479.18 mm²

14 **a** parallelogram **b** 1806 m²

c No, as the windows on the far left or far right sides are not similar to others away from the side edges.

d For the rectangular windows, $A = lw$. For the trapezium windows, $A = \frac{1}{2}(a + b)h$. For the triangular windows, $A = \frac{1}{2}bh$.

15 **a** 9 cm **b** 4.25 cm **c** 5 cm

d 19 mm (1.9 cm)

16 1 cm × 96 cm, 2 cm × 48 cm, 3 cm × 32 cm, 4 cm × 24 cm, 6 cm × 16 cm, 8 cm × 12 cm

17 **a** 81 cm² **b** 24 cm **c** 4 cm

18 144 cm²

19 **a** The trapezium has been cut into two triangles, one with base length a, the other with base length b, and both with height h. The area of the trapezium is the sum of the area of the two triangles $\frac{1}{2}ah + \frac{1}{2}bh$, which is equal to $\frac{1}{2}(a + b)h$.

b Part of the trapezium has been cut off and rotated to form a triangle with a base length of $(a + b)$ and height h. The area of this triangle is equal to the area of the trapezium, which is $\frac{1}{2}(a + b)h$.

c The trapezium has been duplicated and rotated to form a parallelogram with a base length of $(a + b)$ and height h. The area of the trapezium is half of the area of the parallelogram, which is $\frac{1}{2}(a + b)h$.

d The top half of the trapezium has been cut off and rotated to form a parallelogram with a base length of $(a + b)$ and height $\frac{h}{2}$. The area of the trapezium is equal to the area of the parallelogram, which is $\frac{1}{2}(a + b)h$.

e Two triangles have been cut off and rotated to form a rectangle with a length of the average of the parallel sides, $\frac{a+b}{2}$, and width h. The area of the trapezium is equal to the area of the rectangle, which is $\frac{1}{2}(a + b)h$.

20 a i 16 cm^2 **ii** 6.75 cm^2

b 57.8125% decrease

21 $x = 5$ m, $A = 40$ m^2

EX p358 8.4 Area of a circle

1 a 12 cm^2 **b** 52 cm^2 **c** 37 cm^2

2 a i 4 cm **ii** 2 cm **iii** 3

b i 8 cm **ii** 4 cm **iii** 3.25

c i 7 cm **ii** 3.5 cm **iii** 3.02

3 a 706.86 cm^2 **b** 176.71 mm^2 **c** 254.47 mm^2

d 78.54 cm^2 **e** 38.48 cm^2 **f** 33.18 m^2

4 a 5.0625π cm^2 **b** 77.44π cm^2 **c** 49π cm^2

5 a area = 60.82 cm^2, perimeter = 27.65 cm

b area = 44.18 m^2, perimeter = 23.56 m

c area = 176.71 mm^2, perimeter = 47.12 mm

d area = 2642.08 mm^2, perimeter = 182.21 mm

6 a 1.95 cm **b** 2.82 mm

c 5.64 cm **d** 4.65 cm

7 a 10.09 cm **b** 7.90 cm

c 5.05 mm **d** 13.11 cm

8 2375.83 cm^2

9 176.71 m^2

10 A medium pizza has an area of 113.10 inch^2, so two medium pizzas have an area of 226.19 inch^2, whereas a large pizza has an area of 254.47 inch^2 which is greater than the area of two medium pizzas.

11 a $\frac{1}{4}\pi d^2 = \frac{1}{4}\pi(2r)^2 = \frac{1}{4}\pi \times 4r^2 = \pi r^2$

b $\frac{1}{2}Cr = \frac{1}{2}(2\pi r)r = \pi r^2$

12 a 50.27 m^2 **b** 28.27 cm^2 **c** 127.32 mm^2

d 198.94 cm^2 **e** 12.62 cm **f** 33.63 mm

g 15.85 cm

13 216 cm

14 a 87.96 cm^2 **b** 75.40 cm^2 **c** 138.23 cm^2

15 452.39 cm^2

16 a 314.16 cm^2 **b** 2 cm **c** 706.86 cm^2

d 392.70 cm^2

17 285.88 mm^2

18 a 47.52 cm^2 **b** 8.04 m^2 **c** 38.48 cm^2

d 3848.84 cm^2 **e** 264.74 mm^2 **f** 2035.75 cm^2

19 a 14.28 cm^2 **b** 96.37 cm^2 **c** 48.57 mm^2

d 66.19 cm^2 **e** 144.31 mm^2 **f** 81.73 cm^2

20 To find the length of the radius, you first need to divide A by π. Unless the value of A contains π, the result of this calculation will be irrational; therefore, the exact length of the radius cannot be found.

a $\sqrt{\frac{16}{\pi}} \approx 2.26$ mm **b** $\sqrt{16} = 4$ mm

21 a i 14 cm **ii** 14π cm

b i 4 m **ii** 16π m^2

22 a $\frac{60}{360}\left(\text{or } \frac{1}{6}\right)$ **b** 4 cm **c** 50.27 cm^2

d 8.38 cm^2

23 a 19.24 cm^2 **b** 126.71 cm^2 **c** 230.91 mm^2

EX p362 Checkpoint

1 a 0.45 m **b** 5900 m

c 4100 mm **d** 27 000 cm

2 a 12.6 cm **b** 18.4 m **c** 15.32 m **d** 18.4 cm

3 a 17.93 cm **b** 21.99 m

4 a 97.39 m **b** 43.98 cm

5 7.2 m

6 a 9 m^2 **b** 9 m^2 **c** 15 cm^2 **d** 7.5 mm^2

7 a 9 cm **b** 3.1 cm

8 a 200 081 cm^2 = 20.0081 m^2

b 3000 mm^2 = 30 cm^2

9 a 14 m^2 **b** 6 cm^2 **c** 15 cm^2 **d** 12 mm^2

e 7.5 m^2 **f** 9.10 m^2

10 a 75.40 cm^2 **b** 753.98 mm^2

11 a 37.37 cm^2 **b** 31.63 m^2

EX p368 8.5 Volume and capacity

1 a 40 cm^3 **b** 211.5 cm^3 **c** 97.16 mm^3

d 1496 mm^3

2 a 90 cm^3 **b** 90 cm^3 **c** 324 mm^3

3 a 346.5 cm^3 **b** 48 cm^3 **c** 225 cm^3

4 a 320 cm^3 **b** 17.5 cm^3 **c** 24.75 cm^3

5 a 9000 mm^3 **b** 18 000 000 cm^3

c 16 cm^3 **d** 0.00175 km^3

e 4 200 000 000 m^3 **f** 4 m^3

g 4 560 000 cm^3 **h** 0.845 m^3

i 150 000 000 m^3 **j** 6100 mm^3

k 0.028 cm^3 **l** 1700 km^3

6 a 8000 mL **b** 15 000 L **c** 0.07 L

d 6400 kL **e** 2.5 kL **f** 3 000 000 mL

7 a 14 mL **b** 3000 cm^3 **c** 5.8 kL

d 0.75 L **e** 1000 m^3 **f** 6400 m^3

8 5.2 m^3

9 a 2944.62 cm^3 **b** 123

10 4 cm

11 Multiplication is commutative, so it doesn't matter in which order you multiply the length, width and height; therefore, it doesn't matter which of the measurements you designate as the length, width or height.

12 a i Priya multiplied by the power (3) instead of mutliplying by 100 three times.

ii $7\ m^3 = (7 \times 100^3)\ cm^3 = 7\ 000\ 000\ cm^3$

b i Priya used the incorrect conversion factor.

ii $3000\ mm^3 = (3000 \div 10^3)\ cm^3 = 3\ cm^3$

13 a $82.5\ cm^3$ **b** $82\ 500\ mm^3$

c $25 \times 30 \times 110 = 82\ 500\ mm^3$

14 a $220\ cm^3$ **b** $398.61\ cm^3$

c $48\ cm^3$ **d** $152\ cm^3$

15 a 3 456 000 mL

b $5\ 000\ 000\ cm^3$

c $1\ 544\ 000\ cm^3$

16 a 102 375 L **b** 48 048 L

17 a B, A, C

b As the capacity of the troughs increases, the volume of the troughs decreases.

18 small block: $10\ 500\ cm^3$, large block: $14\ 250\ cm^3$

19 a 20 cm

b i $240\ cm^3$ **ii** 10.2 cm **iii** 0.2 cm

iv 19.2 cm **v** 10.8 L **vi** 2.16 minutes

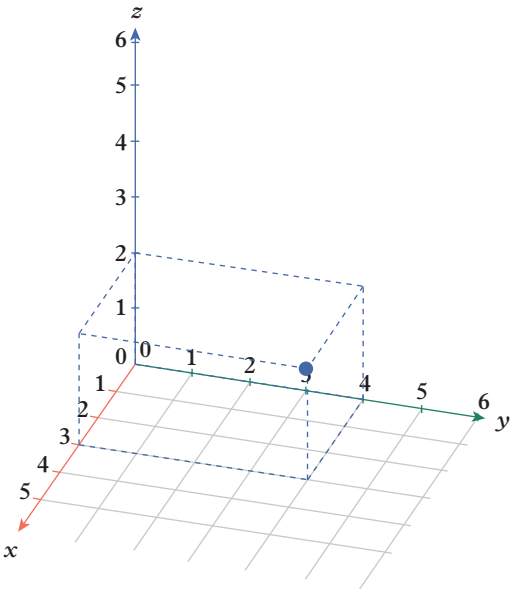
EX 8.6 Three-dimensional space

p377

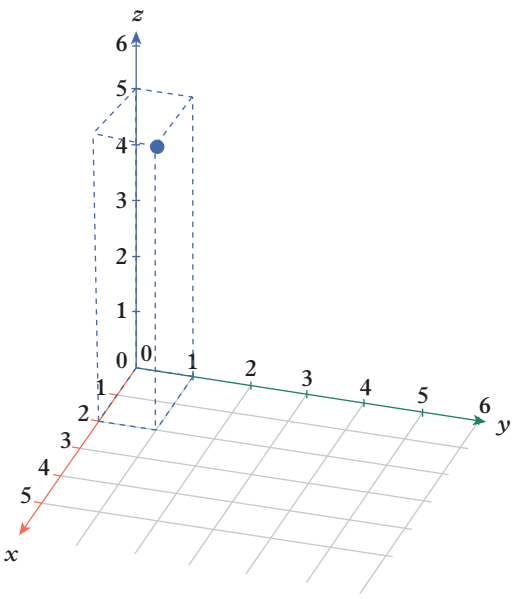
1 a (3, 2, 4) **b** (4, 3, 1) **c** (5, 5, 3)

d (4, 0, 3) **e** (0, 2, 1) **f** (2, 3, 0)

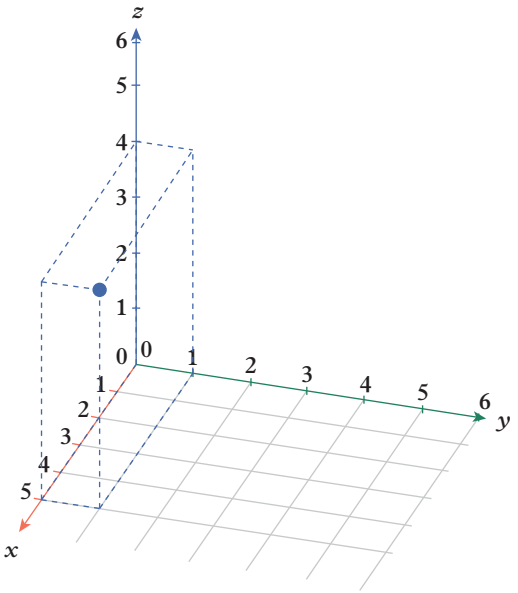
2 a



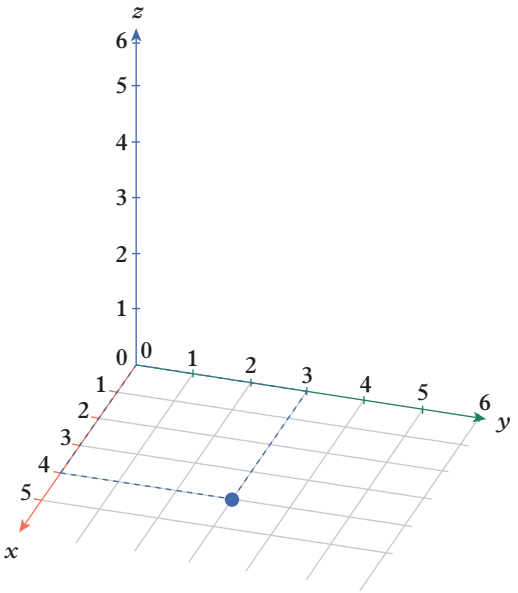
b



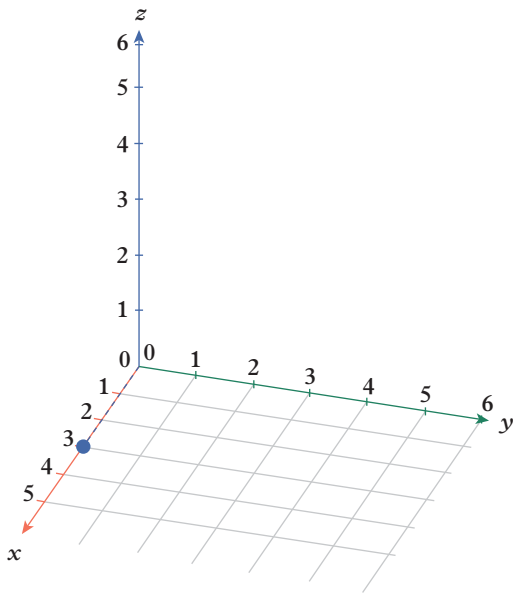
c



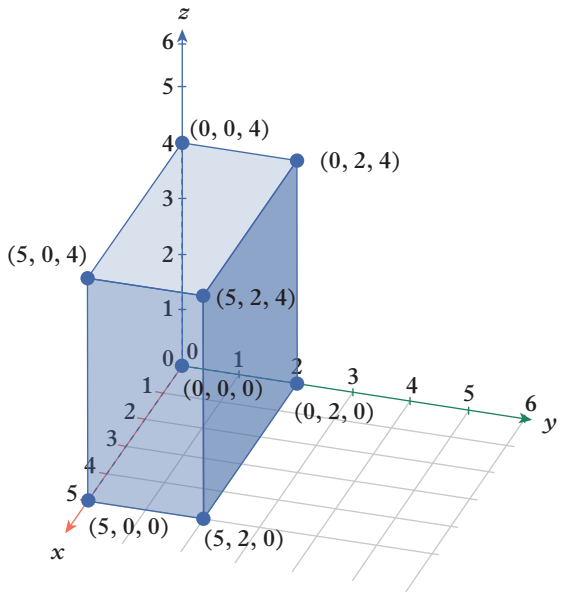
d



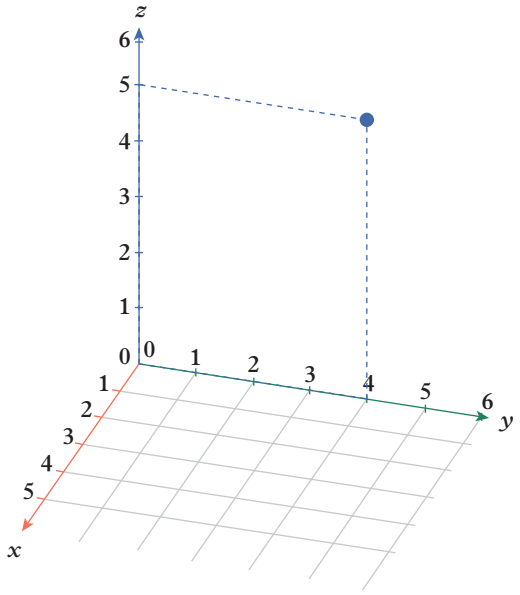
e



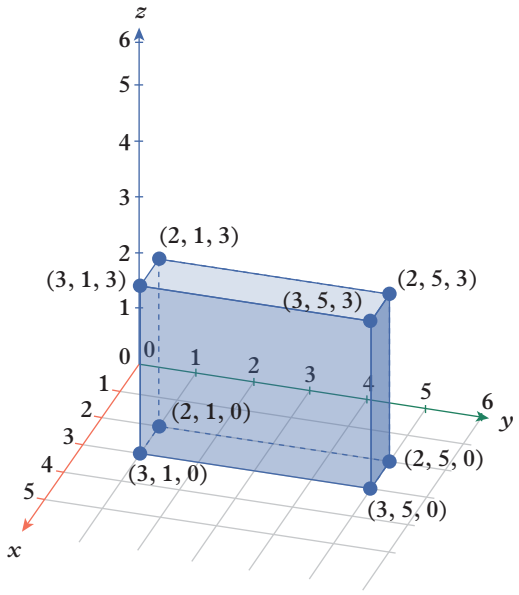
b



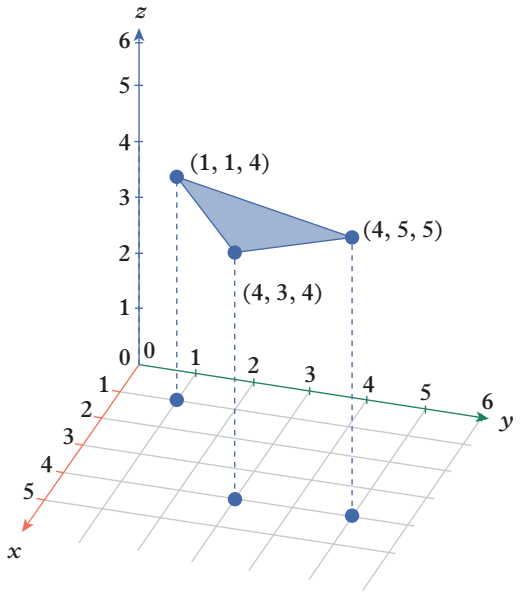
f



c



3 a



4 **a** (6, 5, 3) **b** (2, 7, 9) **c** (4, 4, 8) **d** (4, 4, 8)

5 **a** *z*-axis **b** *x*-axis **c** *y*-axis **d** origin
e *xy*-plane **f** *yz*-plane **g** *xz*-plane

6 **a** **i** 578 **ii** 721 **iii** 285
iv 179 **v** 975 **vi** 043

b **i** fifth floor, first row, two parking spaces along
ii fourth floor, ninth row, one parking space along
iii first floor, fourth row, nine parking spaces along
iv seventh floor, second row, five parking spaces along
v ground floor, second row, eight parking spaces along
vi second floor, ninth row, four parking spaces along

c **i** one floor up, nine rows right, seven spaces backwards
ii five floors down, eight rows left, six spaces backwards

7 **a** (3, 4, 0), (3, 0, 1), (0, 4, 1), (3, 4, 1)
b (4, 2, 0), (4, 0, 3), (0, 2, 3), (4, 2, 3)
c (5, 4, 0), (5, 2, 2), (2, 4, 2), (5, 4, 2)
d (2, 4, 0), (2, 1, 5), (1, 4, 5), (2, 4, 5)
e (0, 1, 2), (0, 3, 2), (4, 1, 3), (4, 3, 3)
f (1, 1, 3), (1, 1, 1), (4, 1, 1), (4, 3, 1)

8 **a** (4, 2, 0), (0, 2, 4), (4, 0, 4), (4, 2, 4)
b (0, 4, 0), (5, 0, 0), (5, 0, 2), (0, 0, 2)
c (5, 3, 0), (5, 2, 1), (2, 3, 1), (5, 3, 1)
d (5, 3, 0), (3, 3, 3), (5, 1, 3), (5, 3, 3)

9 **a** 32 cm^3 **b** 40 cm^3 **c** 3 cm^3 **d** 12 cm^3

10 **a** **i** (48.8584° N, 2.2945° E, 35 m)
ii (40.6892° N, 74.0445° W, 7.2 m)
iii (25.3444° S, 131.0369° E, 863 m)
iv (13.1631° S, 72.5450° W, 2430 m)
v (35.3606° N, 138.7274° E, 3776 m)
vi (37.8213° S, 144.9647° E, 300 m)
b **i** 60° north of the equator, 171° east of the Prime Meridian, 709 m above sea level
ii 25° north of the equator, 128° west of the Prime Meridian, 545 m above sea level
iii 5° south of the equator, 47° west of the Prime Meridian, 135 m above sea level
iv 79° south of the equator, 120° east of the Prime Meridian, 642 m above sea level
v 16° north of the equator, 63° west of the Prime Meridian, 97 m above sea level
vi 34° south of the equator, 152° east of the Prime Meridian, 13 m above sea level

11 **a** (45° N, 64° E, 155 m)
b (2° N, 108° W, 28 m)
c (89° N, 136° E, 155 m)
d (79° S, 101° E, 89 m)

12 **a** 5 units **b** 13 units

8.7 Time

EX p383

1 **a** 180 minutes **b** 300 minutes **c** 600 minutes
d 2 minutes **e** 10 minutes **f** 100 minutes

2 **a** 600 seconds **b** 300 seconds **c** 3600 seconds
d 2 hours **e** 30 hours **f** 4 hours

3 **a** 0.5 or $\frac{1}{2}$ hour **b** 0.75 or $\frac{3}{4}$ hour
c $\frac{1}{3}$ hour **d** 0.25 or $\frac{1}{4}$ minute
e 0.1 or $\frac{1}{10}$ minute **f** 0.8 or $\frac{4}{5}$ minute

4 **a** 10 800 seconds **b** 36 000 seconds
c 21 600 seconds **d** 5400 seconds
e 4 hours **f** 2.5 hours

5 **a** 1440 minutes **b** 168 hours
c 86 400 seconds **d** 8784 hours
e 604 800 seconds **f** 720 hours

6 **a** 0230 **b** 1615 **c** 1348 **d** 2311

7 **a** 7:25 pm **b** 12:00 pm **c** 10:18 pm **d** 9:58 am

8 **a** 5 hours **b** 15 hours
c 3 hours **d** 5 hours
e 9 hours **f** 5 hours 30 minutes
g 2 hours 18 minutes **h** 5 hours 13 minutes
i 7 hours 54 minutes **j** 19 hours 27 minutes
k 3 hours 3 minutes **l** 7 hours 45 minutes

9 **a** 5:45 pm **b** 1:45 pm
c 11:35 am **d** 9:35 am
e 2315 **f** 0115 (next day)

10 **a** **i** 11:49 am **ii** 1:19 pm **iii** 1:49 pm
b **i** 2:28 am **ii** 2:58 am **iii** 12:58 am
c **i** 1617 **ii** 1417 **iii** 1547
d **i** 0010 **ii** 0040 **iii** 2240

11 **a** $\frac{1}{4}$ hour **b** 0.25 hour
c 1.25 hours = 1 hour 15 minutes. Decimals can be written as fractions out of 100, but minutes are written as fractions of 60 (of an hour).

12 **a** 11 hours **b** $\frac{1}{2}$ hour **c** 10.5 hours
d 9.5 hours, 10.75 hours, 10.5 hours
e 41 hours 15 minutes

13 **a** **i** Didn't change the hour after adding 15 mins to 2:45.
ii 3 hours 40 minutes
b **i** Didn't consider am/pm difference.
ii Duration = 15 h 40 min
c **i** Miscounted from 3:25 pm to 6:25 am.
ii Duration = 15 h 40 min

14 3:05 pm

15 **a** 4012 m **b** 2 hours 30 minutes **c** 0.45 m/s

16 **a** 14 hours after 8 pm AEST is 10 am AEST, not 10 am AWST, which is the time zone the clock is likely set to.
b 16 hours **c** 1280 km
d 210 km **e** 14%

17 **a** D **b** A

18 **a** 20 hours 25 minutes **b** 48 hours 45 minutes
c 65 hours 30 minutes **d** 8764 hours 16 minutes

19 **a** **i** 1 hour **ii** 2.5 hours **iii** 1.5 hours **iv** 3 hours
b **i** 4:30 pm **ii** 6 pm **iii** 6:30 pm **iv** 6:30 pm
c **i** 3:30 pm **ii** 5 pm **iii** 6:30 pm **iv** 5:30 pm
d In winter, Queensland is ahead, but in summer, South Australia is ahead.
e **i** 2:30 pm **ii** 2:30 am **iii** 1:30 pm **iv** 8 am **v** 5:15 pm **vi** 10 am

20 **a** 9 hours **b** 2045
c 4 hours 30 minutes **d** 2330

EX p389

Review: Units of measurement

Review questions 8.8A: Mathematical literacy review

1 z-axis

2 If you multiply the radius of a circle by 2π then you get the circumference of the circle

3 If the given time is less than 1000, remove the first 0 and add 'am' to express the time in 12-hour time. If the given time is between 1000 and 1159, add 'am' to express the time in 12-hour time. If the given time is 1200 or higher, subtract 1200 and add 'pm' to express the time in 12-hour time. Remember to include a colon ':' between the hours and minutes in 12-hour time.

4 **a** area **b** prism

5 **a** A perimeter is the length of the boundary of a 2D shape. For example, the perimeter of a square with a side length of 5 cm is 20 cm.

b A duration is the length between two given times. For example, the duration between 2 pm and 4:30 pm is 2 hours and 30 minutes.

6 **a** The volume of any prism can be found by multiplying the area of the base by the height.

b To find the area of a triangle, multiple its base by its height then divide by 2.

Review questions 8.8B: Multiple choice

1 B **2** D **3** A **4** C **5** D
6 D **7** D **8** C **9** D **10** D
11 B **12** C **13** A

Review questions 8.8C: Short answer

1 **a** 22.6 cm (or 226 mm) **b** 58 mm **c** 33.6 cm

2 **a** 31.42 cm **b** 50.27 mm

3 **a** 48 cm^2 **b** 58.5 cm^2 **c** 120 cm^2
d 5.4 cm^2 (540 mm^2)

4 **a** 50 000 cm^2 **b** 0.5 cm^2

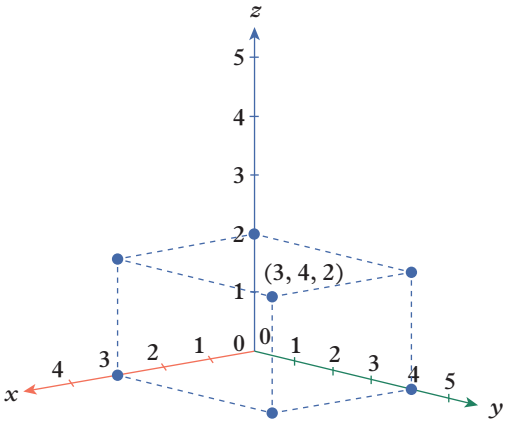
5 **a** 42 cm^2 **b** 88 cm^2 **c** 80 cm^2 **d** 125 cm^2

6 **a** 78.54 cm^2 **b** 201.06 mm^2

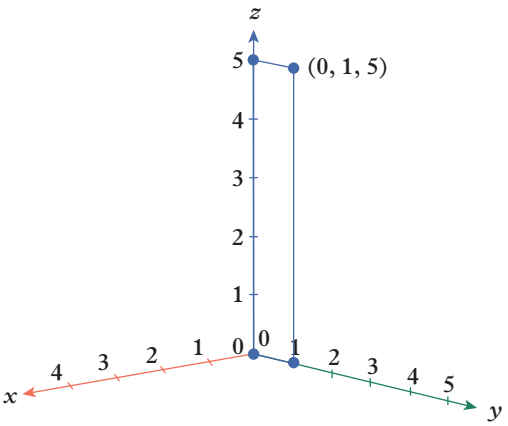
7 **a** 3500 mm^3 **b** 0.0004 m^3 **c** 35 L **d** 124 mL

8 **a** 640 cm^3 **b** 420 cm^3
c 144 cm^3 **d** 22 400 cm^3

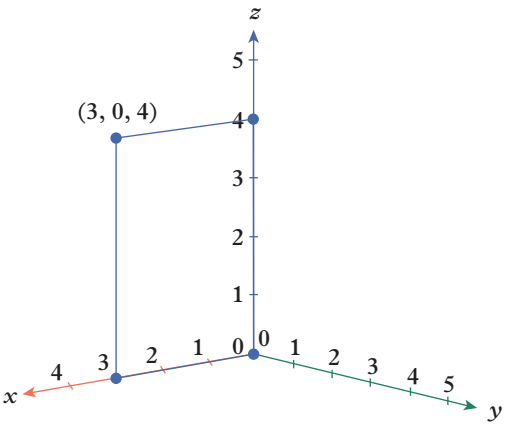
9 **a**



b



c



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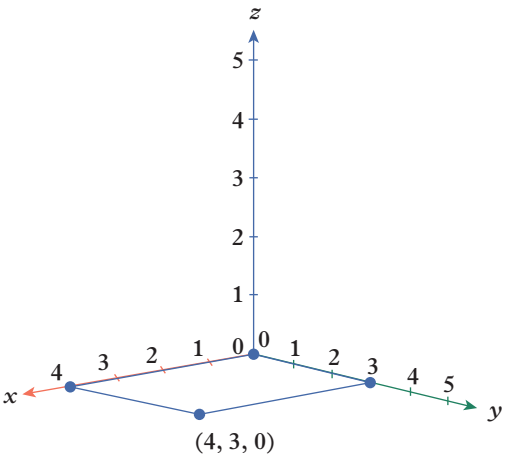


10 **a** 3 hours 45 minutes **b** 8 hours 32 minutes
c 8 hours 34 minutes **d** 6 hours 27 minutes

11 **a** 5:23 am **b** 3:19 pm
c 11:10 am **d** 12:50 pm

Review questions 8.8D: Mathematical modelling

1 **a** 6 m^2 **b** 10 m **c** 50
d 40 m^2 for two coats **e** 825 cm^2
f **i** 20.73 cm **ii** 205.27 cm^2
iii 6 m^3 **v** 40 000
v The balls will not take up the entire space as there will be space between them. The theoretical calculation does not take into account this space.

2 **a** 3:37 pm **b** 51 minutes **c** 9:15 pm
d **i** 7:15 pm **ii** 7:53 pm **iii** 5:45 am

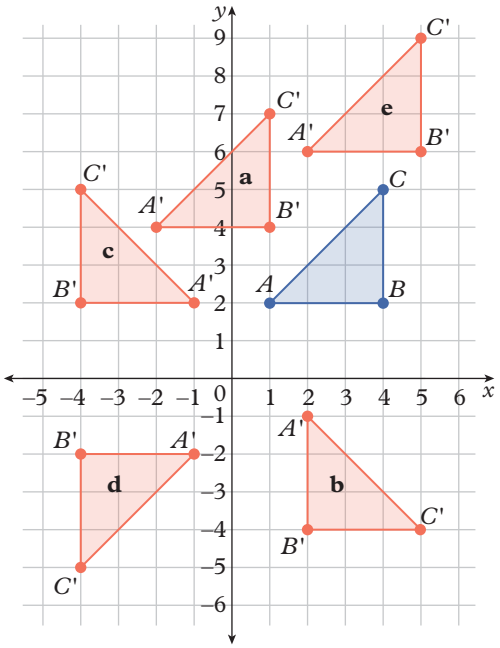
3 **a** 60 m^2 **b** 120 m^3 **c** 120 000 L
d 96 000 L **e** 40 hours

MODULE 9 Congruence and similarity

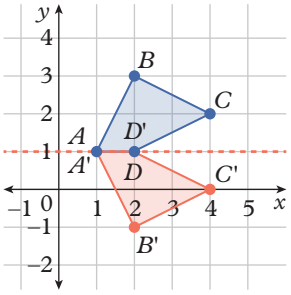
EX p398 9.1 Transformations

1 **a** translated 3 units left and 2 units down
b rotated 90° clockwise (or 270° anti-clockwise) about the origin
c reflected in the x-axis
d reflected in the y-axis

2



3 **a**



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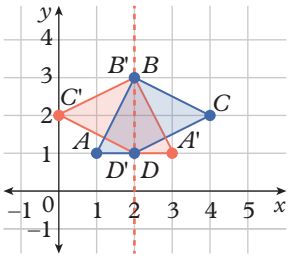

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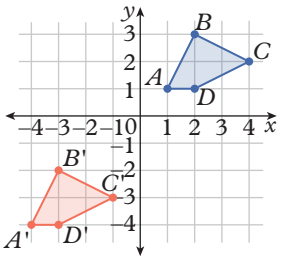

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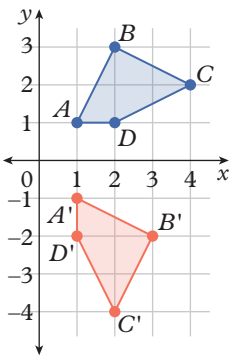
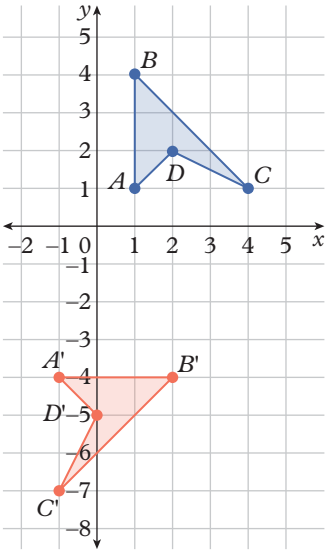


4 **a**



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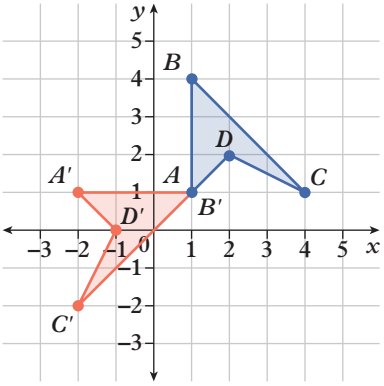

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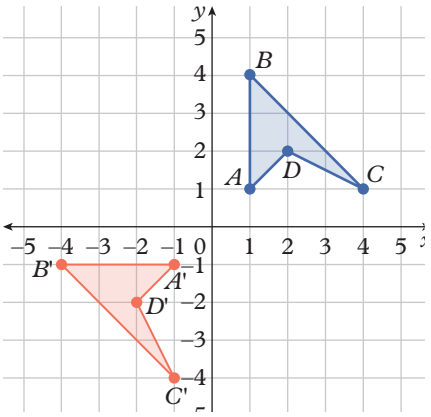

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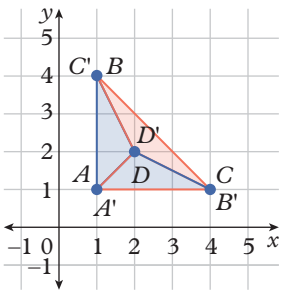
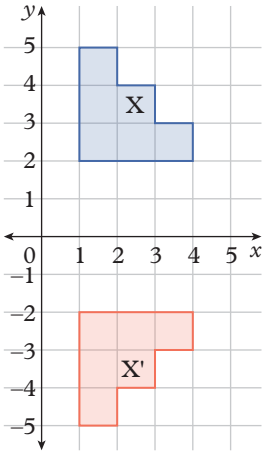


5 a



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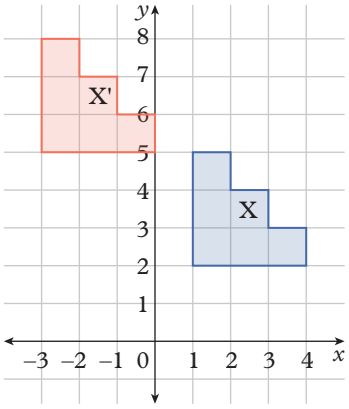

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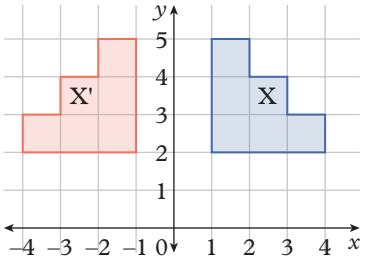

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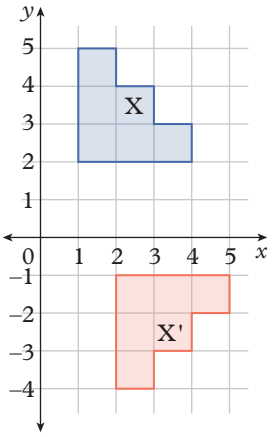

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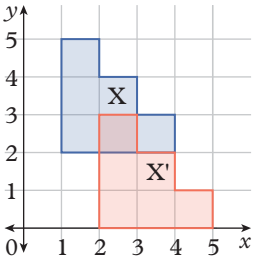

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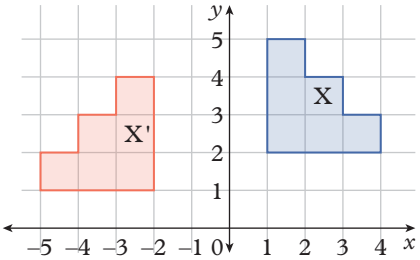
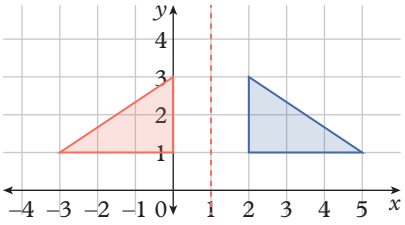


6 MADE YOU LOOK

7 a i



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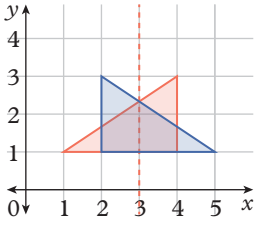

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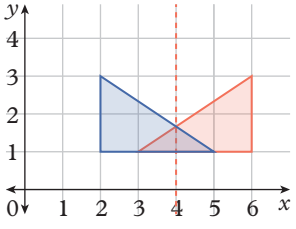

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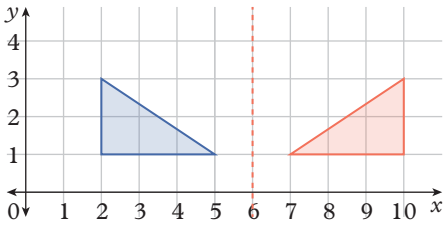
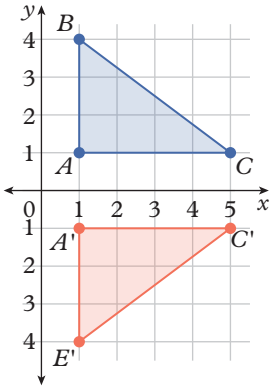


b When a mirror line is on one side of an object the original object and image do not overlap, unlike when the mirror line cuts through an object.

8 a $AC = 4$ cm, $AB = 3$ cm

b $BC = 5$ cm

c



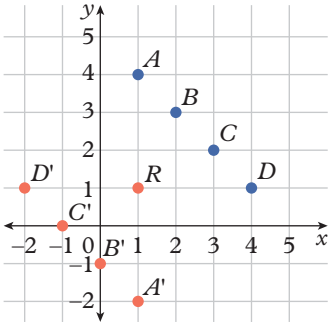
d $A'C' = 4$ cm, $A'B' = 3$ cm, $B'C' = 5$ cm

e The side measurements of a shape do not change after a reflection.

9 a True

b True

10 a



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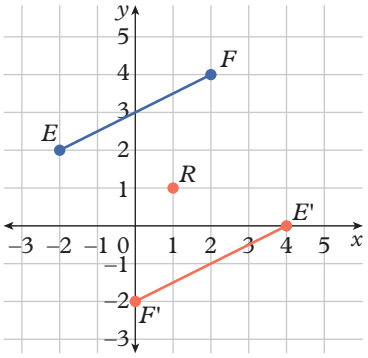

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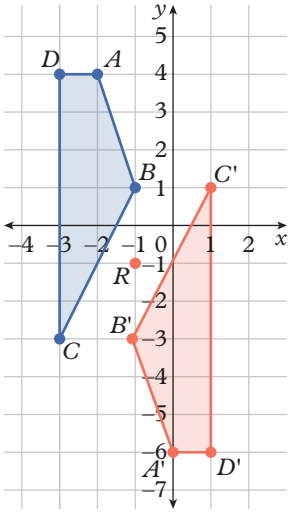
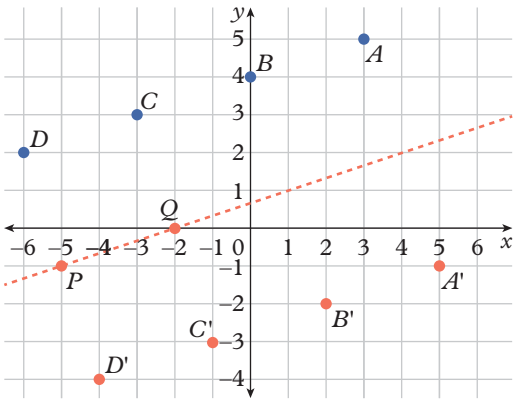


11 a



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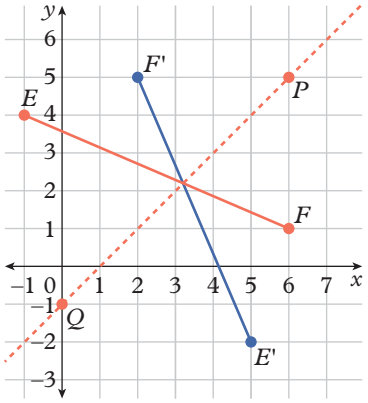

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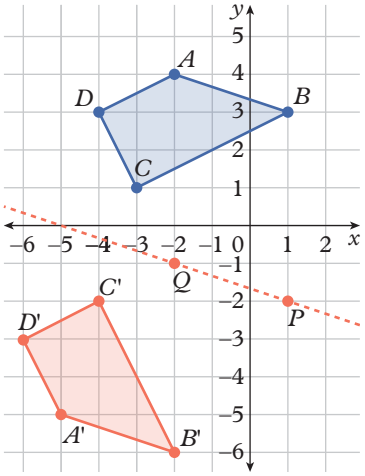
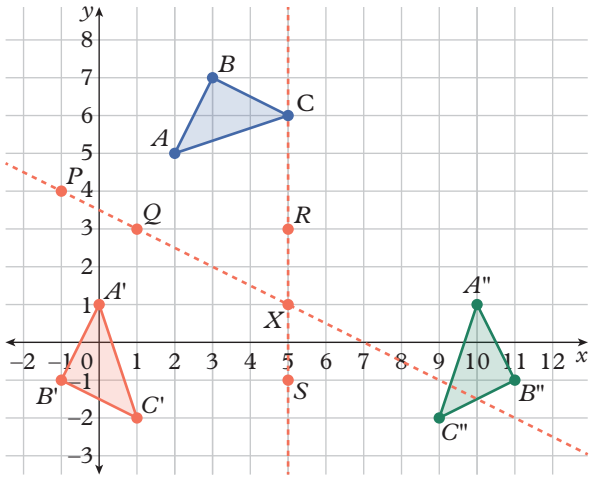


12 a, b



c **i** 127° (or 233°) **ii** 127° (or 233°)
iii 127° (or 233°)

d **i** 5.0 and 5.0 **ii** 6.3 and 6.3
iii 5.0 and 5.0

e Rotation of 127° clockwise (or 233° anticlockwise) about point X

13 P and S

9.2 Congruence

EX p404

1 **a** yes **b** no **c** no **d** yes

2 B and W, C and S, E and N, G and J, H and K, L and X, Q and T

3 **a** **i** IJ and NL, JK and LM, KI and MN
ii $IJK \cong NLM$

b **i** AD and FG, DC and GH, CB and HE, BA and EF
ii $ABCD \cong FEHG$

c **i** JK and TS, KL and SR, LM and RQ, MN and QP, NO and PU, OJ and UT
ii $JKLMNO \cong TSRQPU$

d **i** AB and MN, BC and NO, CD and OP, DE and PQ, EF and QJ, FG and JK, GH and KL, HA and LM
ii $ABCDEFGH \cong MNOPQJKL$

4 **a** A and F; B and G; C and H; D and E

b A and R; B and P; C and Q

c A and H; B and I; C and J; D and K; E and L; F and G

d A and M; B and N; C and O; D and P; E and I; F and J; G and K; H and L

5 **a** congruent **b** not congruent

6 **a** not congruent; corresponding sides not equal length

b not congruent; corresponding angles equal but corresponding sides not equal

c congruent; corresponding angles and corresponding side lengths are equal.

d not congruent; corresponding angles are not equal.

7 **a** $p = 4$ cm **b** $q = 2.8$ cm

8 **a** $x = 48°$ **b** $y = 45°$

9 **a** True; all angles in squares are 90° so the corresponding sides and angles are equal.

b True; all angles in rectangles are 90° so the corresponding sides and angles are equal.

c False; the angles in triangles with the same base length and height are not always equal.

d False; the angles in parallelograms with the same base length and height are not always equal.

e True; all circles with the same radii are the same shape and size.

10 Isometric transformations do not change the angle or side measurements of a shape, so the corresponding side lengths and angles of a shape and its image will be equal after a translation, reflection or rotation, and the shape and image will be congruent.

11 **a** **i** Translate 4 units right and 1 unit down.
ii not congruent

b **i** Rotate 90° clockwise about the origin.
ii congruent

c **i** Reflect in the y-axis.
ii not congruent

d **i** Rotate 90° anticlockwise about the origin.
ii congruent

12 **a** Rotate 90° anticlockwise about the origin, then translate 6 units right and 5 units up.

b Reflect in the y-axis, then translate 8 units down.

9.3 Congruent triangles

EX p411

1 **a** congruent **b** not congruent
c not congruent

2 **a** congruent **b** not congruent
c not congruent

3 **a** not congruent **b** congruent
c not congruent

4 **a** SAS, as the included angle in both triangles is equal (a right angle)

b **i** SSS
ii When given the hypotenuse and another side of a right-angled triangle you can always calculate the length of the third side, and SSS is a test for congruence, therefore RHS is also a test for congruence.

5 **a** not congruent **b** congruent
c congruent **d** not congruent

6 **a** congruent, SAS (7 cm, 77°, 9 cm)
b not congruent, ASA **c** not congruent, ASA
d congruent, ASA (54°, 13 cm, 87°)
e not congruent, RHS
f congruent, SSS (9 cm, 13 cm, 14 cm)

7 **a** SSS

b $\angle ABC = 76°$ and $\angle BAC = 45°$

8 a i $v = 17$ cm, $w = 38°$, $x = 62°$, $y = 80°$, $z = 80°$

ii AAS and angle sum of a triangle

b i $a = 48°$, $b = 25°$, $c = 4$ cm, $d = 25°$, $e = 107°$, $f = 7$ cm

ii AAS and angle sum of a triangle

c i $j = 25°$, $k = 7$ cm, $l = 3$ cm, $m = 5$ cm, $n = 115°$

ii angle sum of a triangle and SAS

d i $g = 35°$, $h = 35°$, $i = 110°$, $j = 35°$

ii isosceles triangle facts and SAS

9 a Pair 1 supports ASA as the corresponding sides between the angles are equal whereas the equal sides in pair 2 are not in corresponding positions.

b The third angle of the triangles can be found using the angle sum of a triangle, and once you find the third angles, you have the requirements for AAS (two angles and one corresponding side).

10 a i No, because not all three sides are known on both triangles.

ii SAS

b i No, because the length of the hypotenuse is not known.

ii SAS or AAS

c i Yes, because both are isosceles triangles, so all angles can be determined and there is a pair of congruent side lengths.

11 a i $\triangle ABC \cong \triangle JKL$ and $\triangle DEF \cong \triangle GHI$

ii $a = 5.6$, $b = 60°$, $c = 53°$, $d = 53°$, $e = 74°$, $f = 5$, $g = 69°$, $h = 51°$

b i $\triangle ABC \cong \triangle DEF \cong \triangle GHI \cong \triangle JKL$

ii $a = 34°$, $b = 7$, $c = 4$, $d = 95°$, $e = 5$, $f = 34°$, $g = 51°$, $h = 5$, $i = 4$, $j = 51°$

12 a i congruent; angle sum of a triangle is 180°, ASA

ii $n = 40°$, $p = 12$ cm, $q = 66°$, $r = 17$ cm

b i congruent; isosceles triangles, ASA

ii $a = 6$ cm, $b = 72°$, $c = 36°$, $d = 10$ cm, $e = 72°$

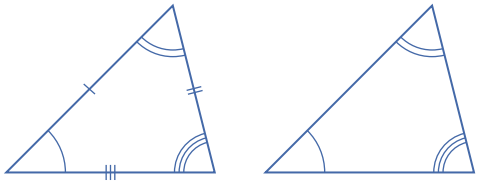
13 a $\angle ABO = \angle DCO$, $\angle BAO = \angle CDO$

b $\angle AOB = \angle COD$

c ASA

d $\triangle ABO \cong \triangle DCO$

14 9 values: all 6 angles and 3 different side lengths placed on either triangle



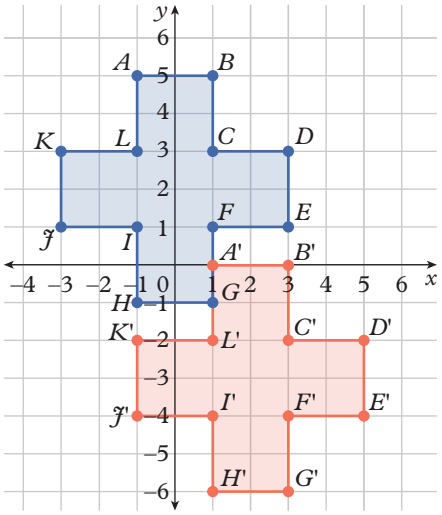
EX Checkpoint

p415

1 a rotated 90° clockwise about the origin

b reflected in the x-axis

2 a



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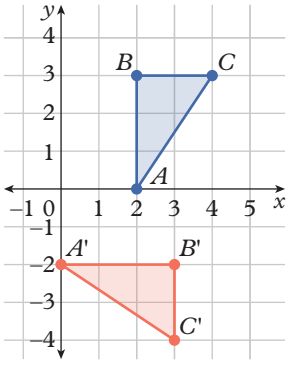

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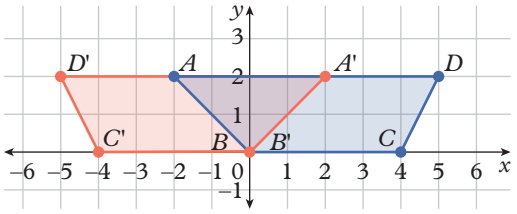

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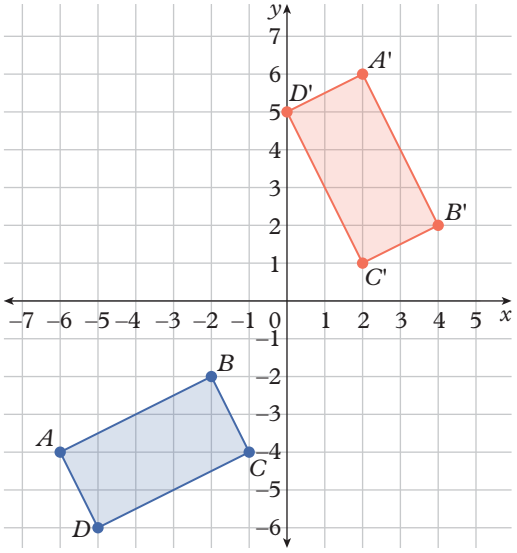


3 A and N, D and K, E and G, F and O, J and P

4 **a** $a = 15$ cm

b $b = 113°$, $c = 140°$

5 **a** congruent, AAS

b not congruent, SSS

c congruent, SAS

d not congruent, RHS

6 **a** $d = 60°$, $e = 3.4$ cm, $f = 3.6$ cm, $g = 3.5$ cm, $h = 62°$

b $i = 67°$, $j = 33°$, $k = 33°$, $l = 80°$, $m = 40$ mm, $n = 24$ mm

c $p = 26°$, $q = 23°$, $r = 13$ cm, $s = 23°$, $t = 131°$, $u = 14$ cm

d $v = 61°$, $w = 76°$, $x = 76°$, $y = 43°$

EX p420 9.4 Dilations and similar figures

1 **a** enlargement **b** reduction **c** enlargement

d enlargement **e** reduction **f** reduction

2 **a** 2 **b** $\frac{1}{4}$ **c** $\frac{1}{3}$

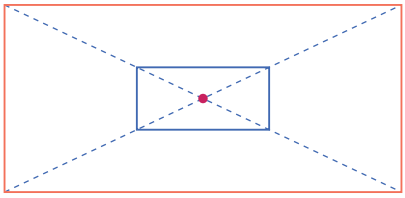
d 5 **e** $\frac{1}{5}$ **f** 2

3 **a** 3 **b** 2 **c** $\frac{1}{4}$ **d** $\frac{1}{3}$

4 **a** $x = 2$ cm **b** $x = 6$ cm **c** $x = 9$ cm

d $x = 20$ cm **e** $x = 10.5$ cm **f** $x = 5$ cm

5 **a** **i**



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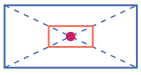
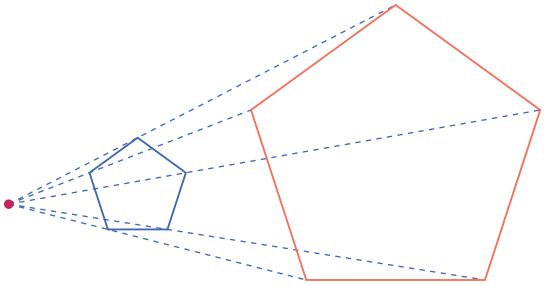


b **i**



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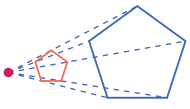
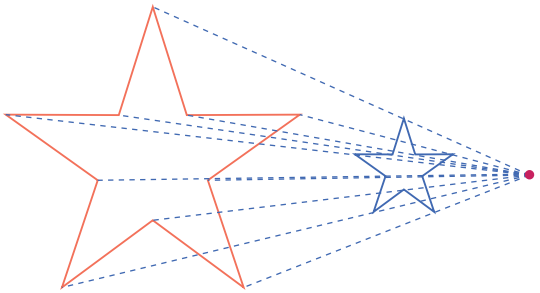


c **i**



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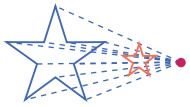


6 **a** similar **b** not similar **c** not similar

d similar **e** not similar **f** similar

7 0.0026 mm

8 All circles and squares are exactly the same shape, whereas rectangles can vary in shape.

9 Finn is correct because $\frac{2}{3}$ is the same as $2 \times \frac{1}{3}$.

10 Congruent figures are the same size and shape. Similar figures are the same shape, but not necessarily the same size.

11 With isometric transformation both the angle measurements and side lengths stay the same, so they produce congruent figures, whereas with dilations only the angle measurements stay the same, so they produce similar figures.

12 No

13 **a** 1 : 2

b The first part of the ratio is the numerator and the second part is the denominator.

c $\frac{2}{1}$ **d** 2 : 1

e **i** $\frac{1}{4}$ **ii** $\frac{1}{100}$ **iii** $\frac{1}{2500}$

iv $\frac{3}{4}$ **v** 2 **vi** $\frac{7}{2}$

f If the first part of the ratio is smaller than the second part, it is a reduction. If the first part of the ratio is larger than the second part, it is an enlargement.

14 **a** Size of plane is reduced by a scale factor of 100.

b 34 cm **c** 27 m

15 **a** 20 cm **b** 9750 m or 9.75 km

16 **a** A regular hexagon has all side lengths equal and all angles equal (120°); an irregular hexagon does not.

b Because all angles will be the same (they are for every regular hexagon) and all side lengths must be equal, any regular hexagon is similar to any other regular hexagon.

17 **a** $\frac{1}{200}$ **b** $\frac{2}{375}$

c **i** $\frac{15}{16}$ **ii** $\frac{16}{15}$

iii The numerators and denominators are swapped because in part **c** the plane is the original, but in part **d** the case is the original.

18 a Looking at all paper sizes (from A0 to A8), the sizes are almost all similar (scale factor around 0.71), though rounding limits exact dilations to even numbers and odd numbers.

b similar relationship in B-sized paper

c similar relationship in C-sized paper

d B series covers sizes that are not found in the A series, so they are not similar. The C series sizes are meant to be similar to A sizes, but there are some slight differences due to rounding errors.

9.5 Similar triangles

EX p427

1 a $a = 9$ cm, $b = 4$ cm

b $c = 44$ cm, $d = 7$ cm

c $e = 20$ cm, $f = 7.5$ cm

d $m = 19$ cm, $n = 45$ cm

e $x = 12$ cm, $y = 6.5$ cm

f $x = 52$ cm, $y = 20$ cm

2 a SAS **b** SSS **c** RHS

d AAA **e** SAS **f** AAA

3 a SSS **b** AAA **c** SAS

d SSS or SAS

4 a Yes; SAS

b No; all corresponding side pairs are not in the same ratio

c No; corresponding angles are not equal.

d Yes; SSS

e No; SSA is not a condition for similarity

f Yes; AAA

5 a similar **b** similar **c** not similar

d similar **e** similar **f** not similar

6 a not similar **b** similar **c** similar

d similar **e** not similar **f** similar

7 With all the similarity conditions, if one pair of corresponding sides is equal then the triangles will be congruent.

8 All equilateral triangles have three equal angles (60°), meeting similarity condition AAA.

9 The third angle is equal to 180° minus the sum of the other two angles. Therefore, since the two triangles have the same two angles, their third angles will also be equal.

10 a similar **b** not similar **c** similar

d not similar

11 a Using knowledge of alternate angles within parallel lines ($\angle ABC = \angle EDC$ and $\angle BAC = \angle DEC$) and vertically opposite angles ($\angle ACB = \angle ECD$), these triangles have three corresponding equal pairs of angles and hence are similar.

b scale factor = 3

c $a = 2.5$ cm, $b = 6$ cm

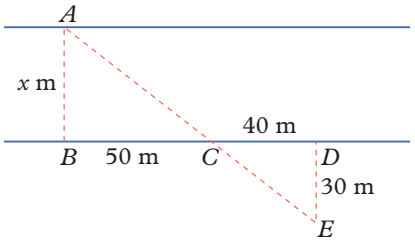
12 a RHS

b 16 cm

c $12^2 + 16^2 = 20^2$ (144 + 256 = 400)

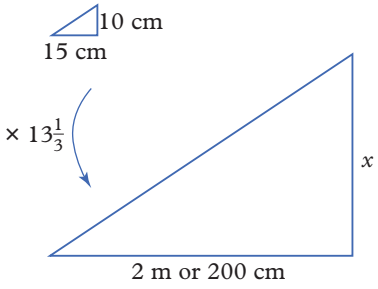
d $\triangle ABC$ can be shown to be similar to $\triangle ADB$ and $\triangle BDC$ by using any of the four similarity tests.

13 $x = 37.5$ m

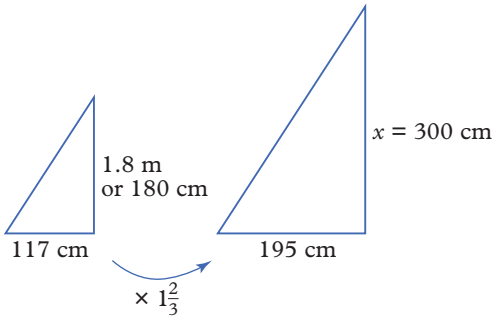


14 4 cm

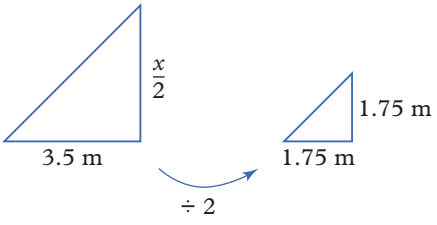
15 a 133.33 cm



b Ring is 5 cm too short, at 300 cm tall.



c 7 m



16 Sean's house is 3.1 m tall; Tania's house is 2.7 m tall. Sean's house is taller.

17 You need to have the second similar triangle to compare house heights and shadows because the measurements were taken at different times of day so the lengths of the shadows would change.

18 a $\frac{DE}{AB} = \frac{7}{5} = 1.4$

$\frac{EF}{BC} = \frac{8.4}{6} = 1.4$

$\frac{FD}{CA} = \frac{11.2}{8} = 1.4$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

Corresponding sides are in the same ratio

$\therefore \triangle ABC \sim \triangle DEF$ by SSS

b $\frac{DE}{AB} = \frac{7}{5} = 1.4$

$\frac{FD}{CA} = \frac{11.2}{8} = 1.4$

$\angle EDF = 41°$

$\angle BAC = 41°$

$\frac{DE}{AB} = \frac{FD}{CA}$

Corresponding sides are in the same ratio.

$\angle EDF = \angle BAC$

Corresponding angles are equal

$\therefore \triangle ABC \sim \triangle DEF$ by SAS

9.6 Quadrilaterals

EX p434

1 a kite; two pairs of adjacent sides equal in length

b trapezium; one pair of opposite sides that are parallel

c rhombus; all four sides equal in length and opposite sides parallel, opposite angles equal, but not 90°

d parallelogram; opposite sides are parallel and equal in length, opposite angles are equal

e square; all four sides equal in length and all angles are 90°

f kite; two pairs of adjacent sides equal in length

2 a $x = 76°$ **b** $x = 117°$ **c** $x = 141°$

d $x = 23°$ **e** $x = 21°$ **f** $x = 93°$

3 a parallelogram; $x = 71°$

b rectangle; $x = 90°$ **c** trapezium; $x = 105°$

d kite; $x = 85°$ **e** rhombus; $x = 64°$

f trapezium; $x = 143°$

4 a i rhombus

ii $a = 72°$, $b = 72°$ and $c = 108°$

b i trapezium **ii** $a = 90°$, $b = 112°$

c i square **ii** $a = 90°$, $b = 90°$, $c = 90°$

d i kite **ii** $a = 130°$, $b = 130°$

e i parallelogram **ii** $a = 125°$, $b = 55°$, $c = 125°$

f i kite **ii** $a = 52°$, $b = 115°$

5 a $x = 20°$ **b** $x = 37°$ **c** $x = 80°$

d $x = 20°$ **e** $x = 15°$ **f** $x = 17°$

6 a Having two internal reflex angles would make the angle sum exceed 360°.

b 270°

7 a True

b False. A kite has no parallel sides. It contains two pairs of adjacent sides equal in length, but not parallel.

c False. A rectangle is a type of parallelogram. The reverse is not true as a parallelogram does not necessarily have four angles equal to 90°.

d False. A trapezium has at least one pair of parallel sides. It can have two pairs of parallel sides.

e False. A rhombus has four equal sides and two pairs of angles equal in size. When all four angles are equal in size it is called a square.

f False. A rectangle has two pairs of opposite equal sides and four equal angles.

8 a i 130°, 50°, 130°, 50° **ii** 360°

b i 108°, 124°, 56°, 72° **ii** 360°

c i 74°, 120°, 94°, 72° **ii** 360°

d The exterior angles of any quadrilateral add to 360°.

9 a similar

b similar

c not similar

d not similar

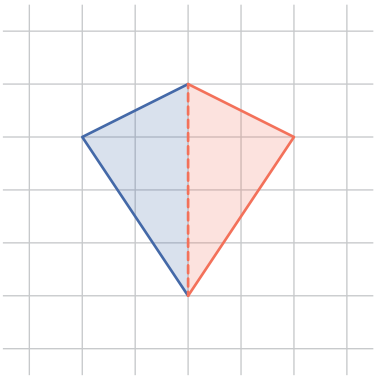
10 a $a = 12$ cm, $b = 99°$, $c = 81°$, $d = 99°$

b $a = 1$ cm, $b = 150°$, $c = 75°$, $d = 9$ cm, $e = 12$ cm, $f = 45°$, $g = 150°$

11 96°, 100°, 96°, 68°

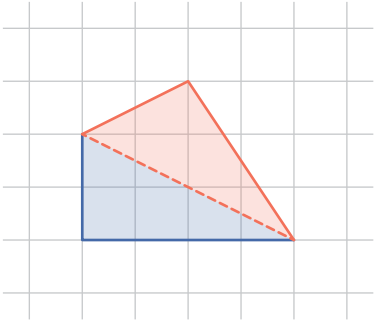
12 The hinges of the toolbox create a parallelogram with the trays being parallel sides. These sides stay parallel as the toolbox opens, meaning that the trays stay horizontal.

13 a



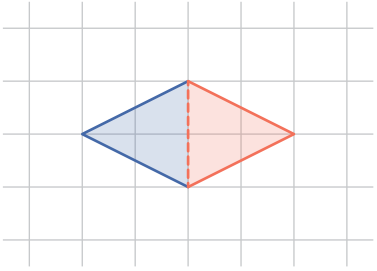
The diagonal is a line of symmetry.

b



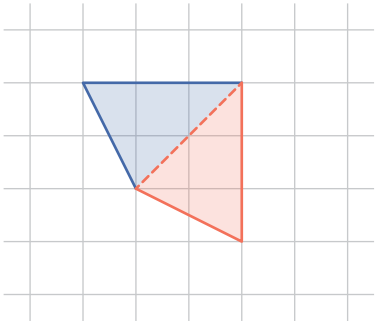
The diagonal is not a line of symmetry.

c



The diagonal is a line of symmetry.

d



The diagonal is not a line of symmetry.

14 a Sides AD and BC are equal in length, and AB is a common side to both $\triangle ABD$ and $\triangle BAC$. As both triangles have a right angle, $\triangle ABD \cong \triangle BAC$ by the SAS congruency test.

b $BD = AC$ as they are corresponding sides in congruent triangles.

15 a $\triangle ABC \cong \triangle ADC$ by SSS as $AB = AD$, $BC = DC$, and AC is common to both triangles.

b $\angle ABC = \angle ADC$, $\angle BCA = \angle DCA$, and $\angle BAC = \angle DAC$ because they are corresponding angles in congruent triangles.

c $\angle BAD = \angle BAC + \angle DAC$. Since $\angle BAC = \angle DAC$, $\angle BAD = 2\angle BAC$ and so $\angle BAD$ has been bisected into two equal angles by diagonal AC. Likewise, $\angle BCD = \angle BCA + \angle DCA$. Since $\angle BCA = \angle DCA$, $\angle BCD = 2\angle BCA$ and so $\angle BCD$ has been bisected into two equal angles by diagonal AC.

d Since $\triangle ABC \cong \triangle ADC$, the perpendicular heights BE and DE will be in the same relative position in the triangles, at point E to matching vertices B and D. These two line segments align to form the diagonal BD.

e Since $\triangle ABC \cong \triangle ADC$, BE and DE are corresponding lengths in congruent triangles, which are therefore congruent. $\angle BEA$ and $\angle DEA$ are right angles since BE and DE are perpendicular heights to AC on which point E lies. Since AC intersects BD at point E, diagonal AC has bisected diagonal BD at a right angle.

16 a $\angle BCD = \angle DAB$, and $\angle ABC = \angle CDA$

b BC and AD are parallel and BD is a transversal. $\angle BDA = \angle DBC$ as they are alternate angles on parallel lines. Likewise, AB and CD are parallel and BD is a transversal so $\angle DBA = \angle BDC$ as they are alternate angles on parallel lines.

c Either diagonal will act as a transversal to both pairs of parallel sides, so the alternate angles formed by the transversal will be congruent.

d $\triangle ABD \cong \triangle CDB$ by AAS as $\angle BDA = \angle DBC$, $\angle DBA = \angle BDC$, and BD is common to both triangles.

e $\triangle ABD \cong \triangle CDB$, where side AB corresponds to side CD and side AD corresponds to side CB, so $AB = CD$ and $AD = CB$.

17 a $\angle AED = \angle BEC$ as they are vertically opposite angles. Likewise, $\angle BEA = \angle CED$ as they are vertically opposite angles.

b $\triangle AED \cong \triangle CEB$ by AAS as $\angle AED = \angle BEC$, $\angle BDA = \angle DBC$, and $AD = CB$.
$\triangle AEB \cong \triangle CED$ by AAS as $\angle ABD = \angle CDB$, $\angle BEA = \angle CED$, and $AB = DC$.

c $BE = DE$ and $AE = CE$ as they are corresponding lengths in congruent triangles, which are congruent.

d Diagonal $BD = BE + DE$, and since $BE = DE$, diagonal $BD = 2BE$ and so has been bisected into two equal line segments by diagonal AC. Likewise, diagonal $AC = AE + CE$, and since $AE = CE$, diagonal $AC = 2AE$ and so has been bisected into two equal line segments by diagonal BD. Therefore, the diagonals AC and BD bisect each other.

18 a $\triangle ABC \cong \triangle ADC$ by SSS as $AB = AD$, $BC = DC$, and AC is common to both triangles.

b $\triangle ABC$ and $\triangle ADC$ are isosceles triangles because they each have two equal lengths, $AB = BC$ and $AD = CD$.

c $\angle BAC = \angle BCA$ and $\angle DAC = \angle DCA$ as they are angles opposite equal lengths in an isosceles triangle.

d $\angle ABC = \angle ADC$, $\angle BCA = \angle DCA$, and $\angle BAC \cong \angle DAC$ because they are corresponding angles in congruent triangles.

e $\angle BCD = \angle BCA + \angle DCA$, and since $\angle BCA = \angle DCA$, $\angle BCD = 2\angle BCA$ and so $\angle BCD$ has been bisected into two equal angles by diagonal AC. Likewise $\angle BAD = \angle BAC + \angle DAC$, and since $\angle BAC = \angle DAC$, $\angle BAD = 2\angle BAC$ and so $\angle BAD$ has been bisected into two equal angles by diagonal AC. Therefore, diagonal AC bisects $\angle BAD$ and $\angle BCD$.

f Diagonal DB forms two congruent isosceles triangles $\triangle ABD$ and $\triangle CBD$ in the same way diagonal AC did. So all subsequent steps can be repeated for $\triangle ABD$ and $\triangle CBD$ and the associated angles.

19 a i 90° **ii** square

b i 60°, 60°, 120°, 120° **ii** parallelogram

EX p441

Review: Congruence and similarity

Review questions 9.7A: Mathematical literacy review

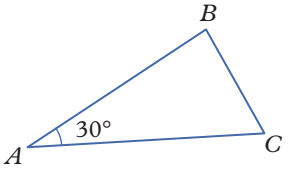
1 translations, reflections, rotations

2 rectangle, rhombus, square

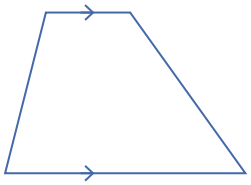
3 centre of rotation, angle of rotation, direction of rotation

4 a rhombus **b** image

5 a An included angle is the angle between two given sides. For example, the 30° angle is the included angle between AB and AC in this triangle.



b A trapezium is a quadrilateral with at least one pair of parallel sides. For example, this is a trapezium.



6 a A dilation can be either an enlargement or a reduction.

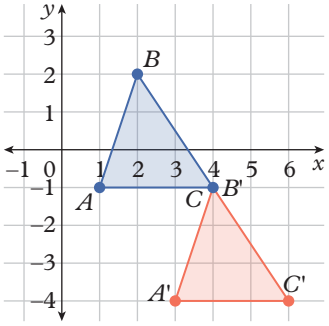
b Two figures are congruent if they have the same shape and size, or if one shape is the mirror image of the other.

Review questions 9.7B: Multiple choice

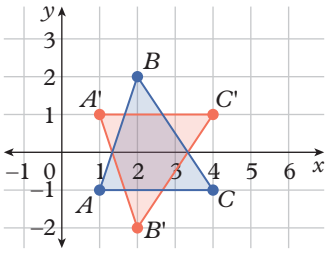
1 B **2** B **3** A **4** C **5** D
6 A **7** C **8** A **9** C **10** D

Review questions 9.7C: Short answer

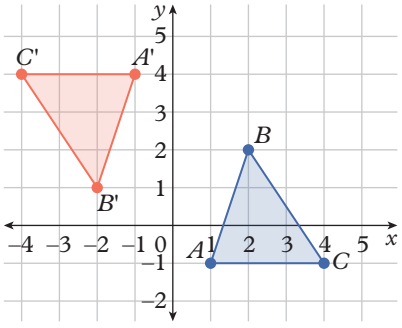
1 a



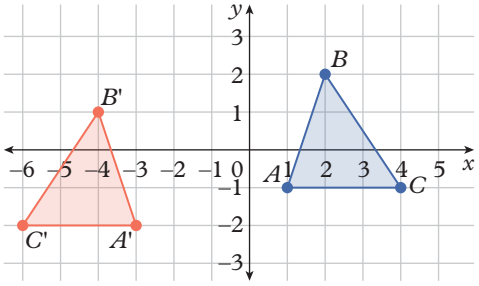
b



c



d



2 a congruent
b congruent
c not congruent
d not congruent

3 A and E

4 A and E are congruent due to the condition ASA.

5 a enlargement; scale factor = 2
b reduction; scale factor = $\frac{1}{4}$

6 $a = 9.2$ cm and $b = 4$ cm

7 Yes, by SAS

8 a $w = 124°$
b $a = 144°$, $b = 36°$, $c = 144°$
c $m = 129°$, $n = 129°$
d $p = 263°$

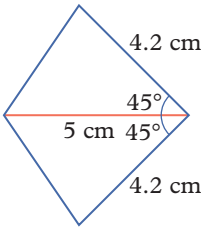
9 a trapezium **b** parallelogram
c kite **d** irregular quadrilateral

Review questions 9.7D: Mathematical modelling

1 a The triangles have an equal length and included angle but the remaining lengths are not equal, so the triangles are not congruent by SAS.

b A reflection creates a congruent image with congruent lengths and angles. The quadrilateral formed will therefore have pairs of congruent adjacent sides (the two 4.2 cm and the two lengths opposite the 45° angles). So the quadrilateral is a kite.

c



d i Given two interior angles in one triangle, the third must make the total 180°. The two triangles are congruent, so the corresponding angles must be equal.

ii 90°, 112°, 79°, 79°

2 a camp triangle: 40°, 90°, 50° gum-tree triangle: 40° (gum tree), 90°, 50°

b Corresponding angles are the same but side lengths are not; triangles are similar (AAA) but not congruent.

c 2.5

d Side length of 6 m in 'camp triangle' corresponds to the side in 'gum tree triangle' that represents width of the river. To determine width of the river, multiply 6 m by scale factor (6 × 2.5).

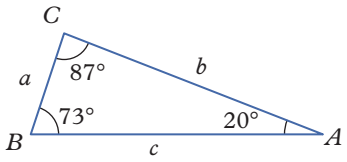
e 15 m

f We can use Pythagoras' theorem to calculate the lengths of the hypotenuses of both similar triangles and add them together to find the straight-line distance between the camp and gum tree.

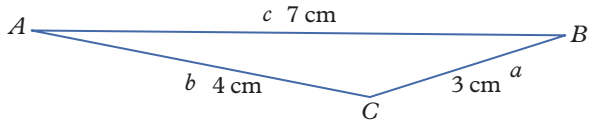
g 25.24 m

EX p446 Computational thinking: Geometric proofs

1 a i



ii



b AAS

c The flowchart first checks that the two triangles are similar, so they must satisfy AAA. We then only need to check for one pair of corresponding side lengths being equal to satisfy the AAS test.

d Label triangles as $\triangle ABC$ (left) and $\triangle A'B'C'$ (right).

1. Start
2. Input $\triangle ABC$ and $\triangle A'B'C'$.
   Variables are: $A = 87°$, $B = 54°$, $C = 37°$, $A' = 87°$, $B' = 54°$, $C' = 37°$, $a = 42$ cm, $b = 34$ cm, $c = 26$ cm, $c' = 26$ cm
3. Check if triangles are similar (Yes by AAA), then follow Yes path in flowchart
4. Report triangles are similar and may be congruent
5. Check if $a = a'$. a' is unknown so follow No path in flowchart
6. Check if $b = b'$. b' is unknown so follow No path in flowchart
7. Check if $c = c'$, then follow Yes path in flowchart
8. Report triangles are congruent
9. End

e Sometimes there is not enough information to tell whether two triangles are congruent, so a third output like 'Not enough information' should be added.

MODULE 10 Statistics and probability

EX p453 10.1 Collecting data and sampling methods

1 a all students in Year 8 in that town

b all people in the country

c everybody in Western Australia

d all people living in the town

2 a census **b** sample **c** sample **d** census

3 a census **b** sample **c** census **d** sample

4 a Biased, as the question does not allow all possible answers

b Fair, as every student has an opportunity to give their opinion

c Biased, as only getting opinions from people who visit casinos, not across the country

d Biased, as only Woolworths customers are targeted

5 a systematic **b** stratified **c** cluster **d** quota **e** judgemental **f** convenience **g** convenience

6 a Biased. Ask for favourite animals from a list of options.

b Biased. Ask to pick an option from a selection of films showing over the holidays.

c Fair

d Biased. Ask Year 8 students what their favourite hobby is.

e Fair

f Biased. Ask who you plan to vote for in the election.

7 a The people in the town

b People at the local mall may have different views on graffiti from the wider population, or Lizzie may choose to ask a certain type of person.

c Systematic sampling

d It is possible that people walking past the post office may also have different views on graffiti.

e Samples from different income brackets according to the size of each income bracket, choosing random samples from different age groups according to the size of each age group

8 Sample answers:

a What is your favourite soft drink? Which soft drink flavour do you recommend?

b How many pets do you have at home? Give the type and total number of pets at home.

c How many hours do you spend using the internet each week? How much is your weekly download?

d Are you in favour of the new school rule? What are your thoughts about the new school rule?

9 Sample answers:

a Each student is numbered from 1 to the end. Depending on the numbers required for the sample, every 20th (for example) student is selected from the list.

b A sample is selected with a fair representation of girls and boys across each year level, and in proportion to the numbers of students at each year level.

c A sample of 20 girls and 20 boys is taken from each year level (for example).

d One year level is chosen at random and all students in that year are surveyed (for example).

10 a i Year 8 students at your school

ii Sample answer: select every fifth student from the Year 8 list of students.

b i the people in the town

ii Sample answer: select every 10th (or agreed number) household from town records.

c i the Australian population

ii Sample answer: select a sample of families across Australia in proportion to the population in each state (targeting families in proportion to those living in the country as well as in cities).

11 a Cluster sampling is likely to be much cheaper than surveying the entire population (for example).

b The composition and diversity of local government areas may vary significantly depending on whether they are rural, urban, suburban or inner city (for example).

12 a Convenience sampling

b Biased. The most popular posts are likely to have more extreme views (strongly like and strongly dislike) than posts that are more neutral (like, dislike, don't care). The most popular posts are also more likely to be from accounts with a large number of followers rather than a random viewer of the announcement.

c i Judgemental

ii Biased. They paid for one-quarter of the reviews they checked, which are more likely to be positive. The reviewers that were sent review copies may be less biased, but also less critical so they receive review copies of other games. These 15 reviews make up three-quarters of the reviews the developers checked and therefore they are likely to be skewed towards being more positive.

13 a Deliberately selecting people or objects allows you to choose people or objects that fit the criteria you need for your sample, which may not be possible when only randomly selecting people or objects.

b Deliberately selecting people or objects may bias your selection if you choose people or objects that are a better fit for your desired result than a randomly selected sample of the population would be.

14 Those with a certain opinion of the local issue may be more likely to complete the survey.

15 Sample answer: Some flavours may be more popular on a sunny day, or on the weekend, or at a particular time of day. Any of these factors could bias the data that Nikki collects.

16 Marci's sample has percentages that are close to the population percentages (24%, 15%, 31%, 13%, 17%) but are not exactly the same, so her sample is approximately stratified.

17 a 67% **b** systematic

c 64% **d** 4.7% error

18 a There may not be enough people to find all side effects of the vaccine (for example).

b There may be negative side effects that affect many of the participants in the trial, which could have been avoided in a smaller trial (for example).

EX p458 10.2 Summary statistics

1 a 7 **b** 7 **c** 5 **d** 7

2 a mean = 14.4, median = 12, mode = 12, range = 8

b mean ≈ 4.8, median = 4, mode = 2 and 4, range = 8

c mean ≈ 5.9, median = 7, mode = 7, range = 9

3 a 155.83 cm **b** 155.5 cm

c 155 cm and 161 cm **d** 22 cm

4 a 159.33 cm **b** 155.5 cm

c 152 cm **d** 51 cm

5 a i 2.17 **ii** 2 **iii** 2 **iv** 3

b i 3.17 **ii** 3 **iii** 2 **iv** 3

c i 23 **ii** 20 **iii** 20 **iv** 30

d i 2.84 **ii** 3 **iii** 2 **iv** 4

e i 14.6 **ii** 14 **iii** 14 **iv** 4

f i 17.2 **ii** 20 **iii** 20 **iv** 40

6 a

Number of quizzes	Frequency (f)
0	5
1	6
2	5
3	4
4	0
5	2
6	1
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	1
Total	24

b i 2.6 **ii** 2 **iii** 1

c 18 **d** 18

7 mean ≈ 13.7, median = 14, mode = 14, range = 6

8 mean = –0.06, median = –0.1, mode = –0.2 and –0.1, range = 0.06

9 a

Weight (kg)	Frequency
2	4
3	3
4	4
5	5
6	2
7	1
8	1
9	1
10	2
11	1

b mean = 5.2, median = 5, mode = 5, range = 9

c Sample answer: The middle school bag in weight was 5 kg. The school bags varied in weight by 9 kg.

10 Sample answer: There was a difference of 82 between the student who sent the fewest instant messages and the student who sent the most instant messages. The average number of instant messages sent was 34.

11 a 202

b i 131.8 **ii** 124

c i 98 **ii** 38

d The outlier increased both the mean and the range, but it had a much larger effect on the range than the mean.

12 a A large outlier value will increase the mean and a low outlier value will decrease the mean. Both Monica and Mario's classes have the same median, but the outlier in Mario's class has increased the mean compared to Monica's class.

b The median, as it is the middle value and is not affected by the presence of outliers

c 198 cm

d Mean ≈ 155.82. There is a 3.51 cm difference in the mean when the outlier is disregarded. The result is much closer to the median value.

e 21 cm. There is a 30 cm difference in the range when the outlier is disregarded.

13 a 0.3 kg or 300 g **b** 1.2 kg or 1200 g

14 a i They did not order the data first. The median is only in the middle of the list if the data is in ascending or descending order.

ii difference = 4

b i They did not order the data first. The maximum and minimum are only at the ends of the list if the data is in ascending or descending order.

ii difference = 10

c i They did not take the frequencies into account in the numerator or denominator in the calculation of the mean.

ii difference = –0.1875

d i They looked at all numbers in the table rather than only in the score column. The range is the difference between the maximum and minimum score, not any value in the table.

ii difference = –48

e i They did not take the frequencies into account to determine the middle number, they only looked at the scores.

ii difference = 2

15 a

x	Mean (correct to two decimal places)	Median	Mode(s)	Range
13	14.20	14	13	9
8	13.87	14	13, 15	11
1	13.40	14	13, 15	18
–3	13.13	14	13, 15	22
15	14.33	15	15	9
25	15.00	15	13, 15	15

b The mean and the range changed the most. The median and the mode changed the least.

c 15 and 25. For x = 13, 8, 1, and −3 (any number 14 or less), 14 was in the 8th position and so was the median, whereas if x is 15 or more, there is one less number below 14, which now takes the 7th position, and the 8th position is now taken by 15, so 15 becomes the median. This is the case for x = 15 and x = 25. Changing x to be a number greater than 14 increases the median.

d 8, 1, −3 and 25. The values of 13 and 15 are within the range of the data from 10 to 19. All other values are outside of the range and so change the minimum or maximum value, which affects the value of the range.

e 13 and 15. When x = 13, there are four 13s, which is now the most frequent number by itself. The same thing occurs when x = 15. If x = 12 or 16, then 12 or 16 would have equal frequency with 13 and 15 and become a third mode. For any other value of x, the highest frequency numbers do not change.

f x = 12 and x = 16

g i As the value of x decreases, the value of the mean decreases.

ii As the value of x increases, the value of the mean increases.

h The mean and range will increase, the median will be 15 and the mode will be 13 and 15. Mean = 20. Range = 90.

16 a Mean = 3, median = 3, range = 4 for laptops A and B

b Laptop A has nearly half of its ratings as 5 stars. However, it also has nearly half of its ratings as 1 star. It is a love-it or hate-it laptop, so Jacinta will likely either really like Laptop A or really dislike it.

c Laptop B has most of its ratings as 3 stars and only one 1 star and 5 stars rating. So, most people find the laptop rather average, not amazing, but also not terrible. Jacinta will have a better idea how she will find the laptop.

17 a 5, 7, 7, 9 **b** mean = 8, range = 9

18 x = 25 and y = 70 or x = 70 and y = 25

10.3 Analysing data

EX p464

1 a mean **b** range **c** mode

d range **e** mode **f** median

2 a sample of 100 people, as larger is more reliable

b sample of 619 people, as larger is more reliable

c sample of 20 randomly selected people, as this sample is more likely to be representative of the population

d 50 people in South Australia, as they are all in the population

3 a The sample is probably taken from the population; however, it is not a random sample so it should not be used to predict the population parameters.

b The sample is limited to Canberra instead of the whole of Australia, so it should not be used to predict the population parameters.

c The sample of 30 is too small to allow you to accurately predict the population parameters.

d The sample is randomly taken from the population and of a sufficient size, so it can be used to predict the population parameters.

e The sampling technique is fair and the sample is of a sufficient size, so these results can be used to predict the population parameters.

4 a mean = 4.27, median = 4, mode = 4

b Population: every Australian household; sample: students Laani knows at her school.

c 100

d The people Laani knows are all part of a family. This means that she is not including the households in Australia with no school age children.

5 a Her sample is not restricted to her school or people she knows, and people are selected in a random process.

b The sample size (20) is very small. One outlier would affect the mean.

c i The larger the sample, the more reliable the result.

ii All of the summary statistics should be more reliable as the sample size increases.

6 a

Sample: 20 people	
Category	**%**
A	25
B	10
C	35
D	20
E	10

Sample: 50 people	
Category	**%**
A	28
B	8
C	38
D	14
E	12

Sample: 200 people	
Category	**%**
A	26
B	11.5
C	42
D	12
E	8.5

b Category C. It is the most common category in every sample.

c Category B or E. They are the least common categories in every sample.

7 a Spring: 25.5%, Summer: 39%, Autumn: 21.5%, Winter: 14%

b Spring: 24.75%, Summer: 33.5%, Autumn: 23.5%, Winter: 18.25%

c There is a natural variation when samples are taken from the same population.

d Sample answer: Summer appears to be Australia's favourite season, with winter the least popular.

8 a mean: 3.6 mobiles, median: 4 mobiles, mode: 4 mobiles

b Consider how the survey was conducted, who was in the sample and how large the sample is compared to the population.

c Either measure would be satisfactory for a fair survey. However, care needs to be taken in the event of outliers, which may make the median a more reliable measure.

9 a 3 Year 8s

b 23%

c The samples are from different subgroups of people within the school (students from different year levels and teachers) who may have different opinions based on their age.

d No, as the samples have widely varied percentages between categories and each sample considers different subgroups

10 a Suzie only sampled 100 Year 8s and they were all from her school. Her school may not be representative of all Year 8s in Australia.

b The average height of all Year 8s in Suzie's school is 1.52 m and 85% of Year 8s in Suzie's school are right-handed.

11

	Australia	China	UK	USA	Other
a	1205	106	255	327	107
b i	60.25%	5.3%	12.75%	16.35%	5.35%
ii	723	64	153	196	64

c No, as the proportions of attendees on the Sunday night differ greatly from the average over the four shows. They should use only the Sunday night data to make predictions.

d They could collect more data from future shows on the same four consecutive nights as the proportions may vary. The larger the data set, the more accurate the predictions should be.

EX p467 Checkpoint

1 a sample **b** census
c sample

2 a no bias **b** bias
c bias

3 a systematic **b** stratified
c convenience

4 a i 14.7 **ii** 15.5
iii 20 **iv** 14

b i 2.67 **ii** 3.05
iii 0.3 and 3.3 **iv** 4.7

5 The median. The mean is much higher than the median, so it was likely increased by the presence of an outlier, which has little to no effect on the median.

6 **a** Yes. The sample is randomly chosen and is of a suitable size.

b No. The sample size is too small.

c No. The sample is opt-in and will only favour a particular subset of students (those who read the newsletter and want to fill in the survey).

d No. The sample favours a particular subset of students (those who had detention).

7 **a** 53%

b 50%

c The percentage of people who are planning to go to Woolworths will vary from sample to sample by random chance, even when taken from the same population.

EX p472 10.4 Theoretical probability

1 **a** {5, 6, 7, 8, 9, 10} **b** {1, 2, 3, 4, 5}

c {3, 5, 7} **d** {1, 9}

e {1, 2, 3, 5, 7, 9} **f** {1, 2, 9}

2 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$

3 **a** $\frac{1}{6}$ **b** $\frac{1}{5}$ **c** $\frac{2}{10} = \frac{1}{5}$ **d** $\frac{1}{5}$

4 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{4}{5}$

5 **a** $\frac{4}{5}$ **b** $\frac{9}{15} = \frac{3}{5}$ **c** $\frac{2}{6} = \frac{1}{3}$

d $\frac{8}{34} = \frac{4}{17}$ **e** $\frac{5}{6}$

6 **a** 0.6 **b** $\frac{1}{4}$ **c** 0.9 **d** $\frac{1}{2}$

e $\frac{7}{8}$ **f** 0.3 **g** $\frac{1}{3}$ **h** 0.75

7 **a** Getting at most 4 or getting less than 5 {1, 2, 3, 4}

b Getting at least 6 or getting more than 5 {6, 7, 8, 9, 10}

c Getting an odd but not a prime, a prime but not an odd, or neither an odd nor a prime {1, 2, 4, 6, 8, 9, 10}

d Getting an odd and a prime, a prime and not an odd, or neither an odd nor a prime; alternatively, getting either an even number or a prime number {2, 3, 4, 5, 6, 7, 8, 10}

e Getting neither an odd nor a prime {4, 6, 8, 10}

f Getting an odd and a prime or neither an odd nor a prime{3, 4, 5, 6, 7, 8, 10}

8 **a** {1, 2, 3, 4, 5, 6} **b** $\frac{1}{3}$

c probability of rolling less than 5 **d** $\frac{2}{3}$

9 **a** $\frac{6}{10}$ or $\frac{3}{5}$ **b** selecting a vowel **c** $\frac{4}{10}$ or $\frac{2}{5}$

10 **a** 15 **b** $\frac{5}{15}$ or $\frac{1}{3}$

c selecting a non-blue button; $\frac{10}{15}$ or $\frac{2}{3}$

d 0 **e** $\frac{7}{15}$

f not selecting a button with four holes; $\frac{8}{15}$

11 **a** $\frac{29}{34}$ **b** $\frac{19}{34}$ **c** $\frac{10}{17}$ **d** $\frac{12}{17}$

e $\frac{5}{34}$, 0 eggs

f At most four and no less than four are not complementary as the sum of their probabilities is not equal to one (they both include four). More than four eggs (or at least five eggs) is complementary to at most four eggs. Less than four eggs is complementary to at least four eggs.

12 The sample space usually lists every different outcome rather than every possible outcome; these must be equally likely or an incorrect answer will be obtained.

13 **a** 52 **b** $\frac{1}{13}$ **c** $\frac{12}{13}$ **d** $\frac{3}{13}$

e ace; there are four aces to choose from whereas there is only one queen of hearts.

f $\frac{9}{52}$ **g** $\frac{43}{52}$ **h** $\frac{5}{13}$

14 **a** 30 **b** $\frac{7}{30}$ **c** $\frac{23}{30}$ **d** $\frac{4}{30}$ or $\frac{2}{15}$

e selecting a Y, E, R, I, G, H, T, S, H, N or V

f **i** 12 **ii** $\frac{12}{30}$ or $\frac{2}{5}$

iii selecting a consonant; $\frac{18}{30}$ or $\frac{3}{5}$

15 **a** $\frac{2}{6}$ or $\frac{1}{3}$

b The green sector is twice the size of other sectors, so the five colours are not all equally likely. It is like having six outcomes but two of these are green.

c $\frac{1}{6}$ **d** not spinning yellow; $\frac{5}{6}$

16 **a** 0.18 **b** 0.37 **c** 0.07

17 All multiples of 2 are even numbers, so it is not possible to roll an odd number and to roll a multiple of 2 in the same roll of a standard six-sided die. Therefore, 'rolling an odd number' and 'rolling a multiple of 2' are mutually exclusive events.

18 $\frac{1}{8}$

19 **a** $\frac{19}{200}$ **b** $\frac{85}{199}$

20 {HH, HT, TH, TT}

21 He forgot the two outcomes of one head, one tail.

22 **a** $\frac{8}{55}$ **b** $\frac{22}{55}$ or $\frac{2}{5}$

c **i** 54 **ii** 7 **iii** $\frac{7}{54}$

d **i** 53 **ii** 12 **iii** $\frac{21}{53}$

23 blue: $\frac{1}{6}$, green: $\frac{17}{72}$, purple: $\frac{13}{72}$, orange: $\frac{1}{18}$, yellow: $\frac{13}{36}$

24 **a** $\frac{2}{5}$ **b** $\frac{3}{20}$ **c** 20

EX p478 10.5 Two-way tables

1 **a** Year 7 students who play sport, Year 8 students who play sport, Year 7 students who don't play sport, Year 8 students who don't play sport

b 50 **c** 17 **d** 23

e Year 7 students who don't play sport

f 22

2 **a** Children who prefer books, adults who prefer books, children who prefer movies, adults who prefer movies

b 100 **c** 29 **d** 51 **e** 39 **f** 41

3 **a, b**

	Year 7	Year 8	Total
Action	9	10	19
Comedy	23	8	31
Total	32	18	50

4 **a, b**

	Cats	Dogs	Total
Heidelberg	11	48	59
Preston	19	22	41
Total	30	70	100

5 **a**

	Weekday	Weekend	Total
Volleyball	14	17	31
Tennis	5	31	36
Total	19	48	67

b

	Year 8	Year 9	Total
Spring	11	15	26
Summer	18	19	37
Total	29	34	63

6

	Lollies	Not lollies	Total
Red	5	3	8
Not red	4	2	6
Total	9	5	14

7 **a** $\frac{43}{100}$ **b** $\frac{30}{100}$ or $\frac{3}{10}$ **c** $\frac{90}{100}$ or $\frac{9}{10}$

8 **a** 40 **b** 6 **c** 19

d **i** $\frac{6}{40}$ or $\frac{3}{20}$ **ii** $\frac{19}{40}$

iii $\frac{24}{40}$ or $\frac{3}{5}$ **iv** $\frac{25}{40}$ or $\frac{5}{8}$

9 **a** $\frac{17}{50}$ **b** $\frac{12}{50}$ or $\frac{6}{25}$ **c** $\frac{23}{50}$

d $\frac{28}{50}$ or $\frac{14}{25}$ **e** $\frac{29}{50}$ **f** $\frac{39}{50}$

10 **a**

	White bread	Non-white bread	Total
Vegemite	40	24	64
Jam	20	16	36
Total	60	40	100

b probability(jam on non-white toast) $\frac{16}{100}$ or $\frac{4}{25}$

11 **a**

	Like board games	Dislike board games	Total
Like video games	60	20	80
Dislike video games	25	45	70
Total	85	65	150

b **i** $\frac{45}{150}$ or $\frac{3}{10}$ **ii** $\frac{45}{150}$ or $\frac{3}{10}$

iii $\frac{8}{15}$ **iv** $\frac{1}{6}$

12 **a**

	Telstra	Other	Total
Country	0.3	0.1	0.4
City	0.05	0.55	0.6
Total	0.35	0.65	1

b

	Telstra	Other	Total
Country	60	20	80
City	10	110	120
Total	70	130	200

c **i** $\frac{6}{7}$ **ii** $\frac{1}{7}$

13 **a**

	Year 7	Year 8	Total
M&Ms	16	16	32
Smarties	10	8	18
Total	26	24	50

b **i** $\frac{18}{50}$ or $\frac{9}{25}$ **ii** $\frac{26}{50}$ or $\frac{13}{25}$ **iii** $\frac{16}{50}$ or $\frac{8}{25}$

c $\frac{16}{24}$ or $\frac{2}{3}$

d Part **c** asked for students who preferred M&Ms from the Year 8 class only, whereas part **b** asked who is in Year 8 and prefers M&Ms as a proportion of the whole group.

14 **a** **i** $\frac{42}{100}$ or $\frac{21}{50}$ **ii** $\frac{3}{100}$

iii $\frac{58}{100}$ or $\frac{29}{50}$ **iv** $\frac{10}{100}$ or $\frac{1}{10}$

b The northern part has a preference for rugby and the southern part has a preference for AFL.

15 **a**

	0	1	2	3+	Total
Pets	18	53	37	12	120
No pets	6	12	11	51	80
Total	24	65	48	63	200

b **i** $\frac{51}{100}$ **ii** $\frac{29}{200}$ **iii** $\frac{159}{200}$

16 **a**

	Blond hair	Brown hair	Total
Blue	16	30	46
Brown	23	31	54
Total	39	61	100

b **i** $\frac{39}{100}$ **ii** $\frac{46}{100}$ or $\frac{23}{50}$ **iii** $\frac{16}{100}$ or $\frac{4}{25}$

c **i** 39 **ii** 16 **iii** $\frac{16}{39}$

iv Part **b iii** asks for the proportion of the entire group who have blond hair and blue eyes, whereas part **c iii** asks for the proportion of blonds who have blue eyes.

d **i** 61 **ii** 30 **iii** $\frac{30}{61}$

17 **a** **i** $\frac{30}{50}$ or $\frac{3}{5}$ **ii** $\frac{12}{50}$ or $\frac{6}{25}$

b **i** $\frac{28}{31}$ **ii** $\frac{12}{42}$ or $\frac{2}{7}$

18

	A	Not A	Total
B	5	11	16
Not B	4	10	14
Total	9	21	30

19 8. The numbers, by themselves, should not fill a row or column.

EX p486 10.6 Venn diagrams

1 **a** people with brothers

b people with sisters but not brothers

c people without sisters or brothers

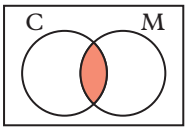
d people with sisters or brothers or both

e people with sisters or brothers but not both

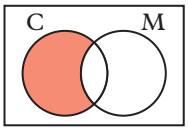
f people without sisters

2 **a** 18 **b** 23 **c** 5 **d** 2

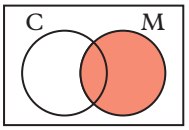
3 **a**



b



c



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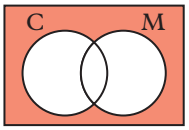
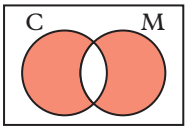
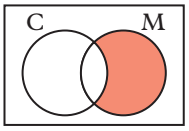


e



f



4 **a** 10 **b** 21 **c** 3 **d** 42

5 **a** **i** 31 **ii** 10 **iii** 38

iv 2 **v** 48 **vi** 9

b 50

6 **a** $\frac{6}{50}$ or $\frac{3}{25}$ **b** $\frac{16}{50}$ or $\frac{8}{25}$ **c** $\frac{16}{50}$ or $\frac{8}{25}$ **d** $\frac{32}{50}$ or $\frac{16}{25}$

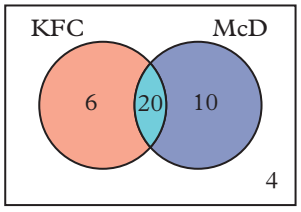
e $\frac{18}{50}$ or $\frac{9}{25}$

7 **a** $\frac{10}{42}$ or $\frac{5}{21}$ **b** $\frac{18}{42}$ or $\frac{3}{7}$ **c** $\frac{21}{42}$ or $\frac{1}{2}$ **d** $\frac{3}{42}$ or $\frac{1}{14}$

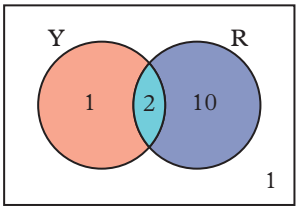
e $\frac{29}{42}$

8 **a** $\frac{7}{50}$ **b** $\frac{41}{50}$ **c** $\frac{31}{50}$ **d** $\frac{48}{50}$ or $\frac{24}{25}$

9



10



11 **a** $\frac{7}{20}$ **b** $\frac{7}{20}$ **c** $\frac{2}{5}$ **d** $\frac{2}{5}$ **e** 0

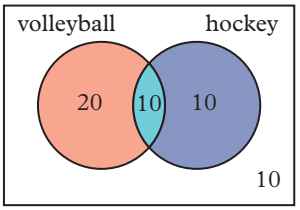
f $\frac{3}{4}$ **g** $\frac{3}{4}$ **h** $\frac{1}{4}$ **i** $\frac{1}{4}$

12 **a** **i** 25 **ii** 50 **iii** 95 **iv** 5

b **i** 57 **ii** 3

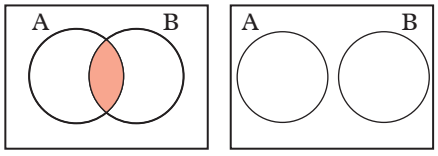
c **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

13 **a** The total of the numbers in the Venn diagram is more than 50 so it cannot represent the votes of 50 people.

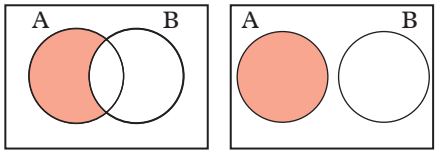


b If the 30 people who wanted to play volleyball does not include the 10 people who want to play both then it can be correct as the total of the numbers in the Venn diagram is 60. If the 30 people includes the people who want to play both then it is not correct and there would be 20 people who want to play neither volleyball nor hockey.

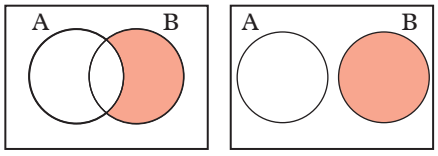
14 **a** **i**



ii



iii



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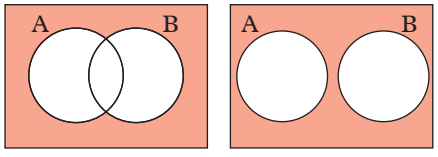


b When all of the shaded regions of each Venn diagram are put together, they cover the entire Venn diagram.

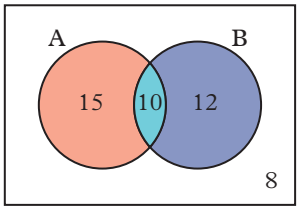
c Pr(A and B) + Pr(A and not B) + Pr(Not A and B) + Pr(Not A and Not B) = 1

d Since Pr(A and B) = 0, the rule simplifies to Pr(A and not B) + Pr(Not A and B) + Pr(Not A and Not B) = 1

15 a

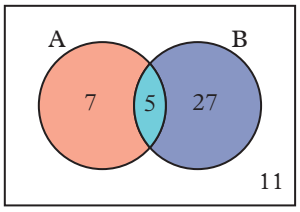
	A	Not A	Total
B	4	9	13
Not B	6	7	13
Total	10	16	26

b



c

	A	Not A	Total
B	5	27	32
Not B	7	11	18
Total	12	38	50



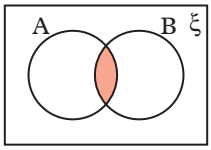
d **a** **i** $\frac{2}{13}$ **ii** $\frac{3}{13}$ **iii** $\frac{19}{26}$ **iv** $\frac{15}{26}$ **v** $\frac{1}{2}$

b **i** $\frac{2}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{37}{45}$ **iv** $\frac{3}{5}$ **v** $\frac{23}{45}$

c **i** $\frac{1}{10}$ **ii** $\frac{7}{50}$ **iii** $\frac{39}{50}$ **iv** $\frac{17}{25}$ **v** $\frac{9}{25}$

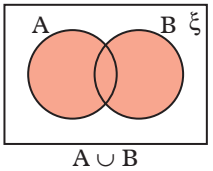
16 a $\frac{17}{35}$ **b** $\frac{13}{35}$ **c** $\frac{4}{35}$ **d** $\frac{4}{15}$ **e** $\frac{4}{17}$ **f** $\frac{24}{35}$ **g** $\frac{20}{35}$ or $\frac{4}{7}$

17 a



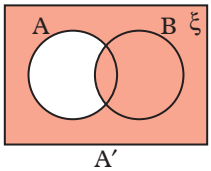
$A \cap B$

b



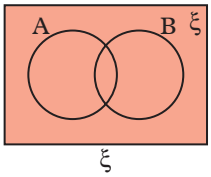
$A \cup B$

c



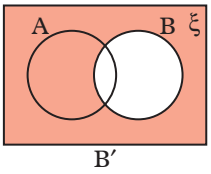
A'

d



ξ

e



B'

18 a B **b** D **c** A **d** C

EX p495

Review: Statistics and probability

Review questions 10.7A: Mathematical literacy review

1 judgemental sampling

2 mean

3 dividing the number of favourable outcomes in the event by the total number of possible outcomes

4 mean, median, mode

5 **a** range

b population

6 **a** an extreme piece of data that is much higher or lower than the rest of the data in the data set

b a display of the relationship between different categories or attributes of data, which uses circles to represent each category or attribute

7 **a** A sample is a selection of data that is part of the population.

b An event is one or more outcomes from the sample space of an experiment.

Review questions 10.7B: Multiple choice

1 D **2** B **3** B **4** D **5** D

6 A **7** B **8** D **9** C

Review questions 10.7C: Short answer

1 **a** **i** sample

ii Biased; increase the sample size and sample randomly from the state's population.

b **i** sample **ii** fair

c **i** census

ii Biased; Gerard should ask people which snack they like best.

2 **a** What is your favourite AFL team?

b How do you travel to school?

c What is your preferred ice cream flavour in a Neapolitan tub containing strawberry, vanilla and chocolate?

3 **a** mean = 3, median = 3, mode = 1 and 4, range = 5

b mean = 12.9, median = 12, mode = 11, range = 19

c mean ≈ 6.67, median = 2, mode = 2, range = 27

4 mean = 13.7, median = 14, mode = 14, range = 6

5 mean = 13.47, median = 14, mode = 14, range = 3
The mean and range change, and the median and mode stay the same.

6 **a** The sample does not include teachers who do not teach Year 8.

b The sample will almost certainly include people who do not know the teachers in school.

c The sample is too small and they may not all have experience with all the teachers in school.

7 The sampling technique appears fair, although 50 is too small a sample for the population and there appears to be at least one outlier from looking at the value of the range compared to the mean and the median. The results could not be reliably used to predict the population parameters.

8 **a** $\frac{2}{9}$ **b** $\frac{4}{9}$ **c** $\frac{5}{9}$

9 **a** $\frac{2}{5}$ **b** $\frac{7}{10}$ **c** $\frac{3}{5}$

10 **a** $\frac{115}{200}$ or $\frac{23}{40}$ **b** $\frac{18}{200}$ or $\frac{9}{100}$

c $\frac{56}{200}$ or $\frac{7}{25}$ **d** $\frac{47}{200}$ **e** $\frac{74}{200}$ or $\frac{37}{100}$

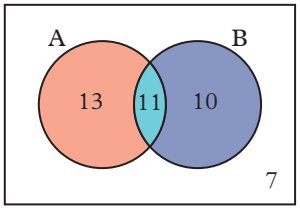
11 **a** 35 **b** 18

c **i** $\frac{10}{35}$ or $\frac{2}{7}$ **ii** $\frac{8}{35}$

iii $\frac{13}{35}$ **iv** $\frac{30}{35}$ or $\frac{6}{7}$

12

	B	B'	Total
A	11	13	24
A'	10	7	17
Total	21	20	41



Review questions 10.7D: Mathematical modelling

1 **a** systematic, sample

b mean ≈ 26.7, median = 21.5, mode = 19, range = 67

c Details such as type of beach, weather on the day, time of year or if this beach is popular for holiday makers are not known.

d The average age of people who go to the local beach is approximately 26.7 years. The ages can range from 5 to 72.

2 **a** $\frac{7}{20}$

b **i**

	Even	Not even	Total
Prime	6	30	36
Not prime	34	10	44
Total	40	40	80

ii $\frac{1}{8}$

iii $\frac{9}{20}$

c **i** 5.5

ii 8.55

EX p500

SEMESTER 2 Review

Short Answer

1 **a** solution **b** solution

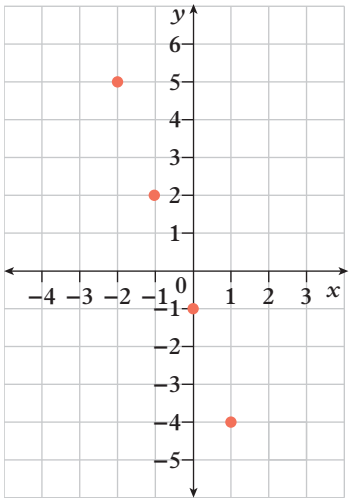
c not the solution **d** solution

2 **a** $x = 10$ **b** $x = 3$

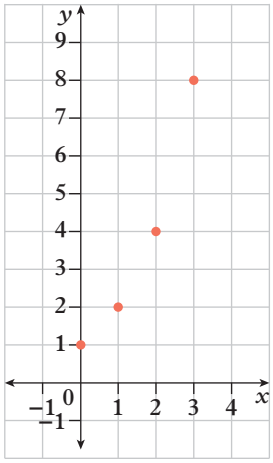
c $x = -11$ **d** $x = \frac{9}{5} = 1\frac{4}{5}$

e $x = \pm 5$ **f** $x = \pm\frac{11}{2} = \pm 5.5$

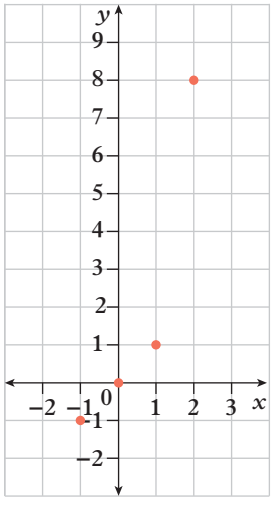
3 **a** linear



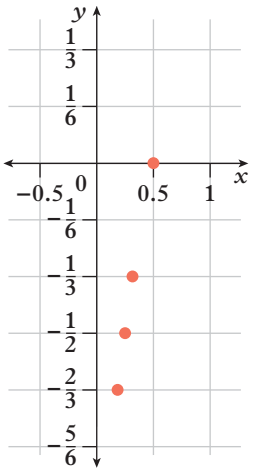
b non-linear



c non-linear



d linear



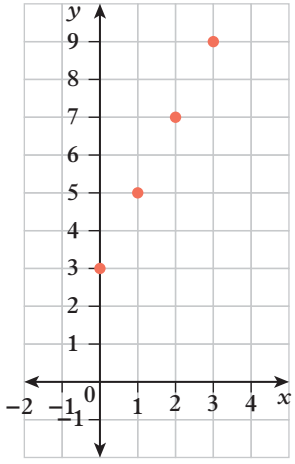
4 a i $x = 3$ **ii** $x = 0$

b i $x = -3$ **ii** $x = 6$

5 a i $x = 6$

x	0	1	2	3
y	3	5	7	9

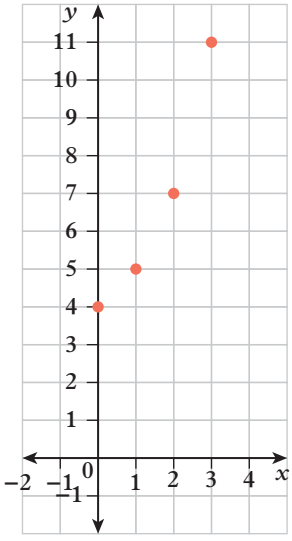
ii linear



b

x	0	1	2	3
y	4	5	7	11

ii non-linear



6 a $y = -2x + 7$

b $y = 4x + 3$

c $y = -x - 2$

d $y = \frac{1}{2}x + 6$

7 a $x = 14$

b $x = 1$

8 a equal

b complementary

c equal

d supplementary

e supplementary

f equal

g supplementary

h equal

9 a i square **ii** $A = 25$ cm^2

b i kite **ii** $A = 30$ cm^2

c i parallelogram **ii** $A = 16.0026$ cm^2

d i trapezium **ii** $A = 24.0039$ cm^2

e i rectangle **ii** $A = 19.9712$ cm^2

f i rhombus **ii** $A = 4.8576$ cm^2

10 a $A = 13.85$ m^2, $P = 13.19$ m

b $A = 38.48$ m^2, $P = 21.99$ m

c $A = 15.69$ m^2, $P = 16.25$ m

d $A = 3.94$ m^2, $P = 8.00$ m

11 a $3x + 5 = 29$
$x = 8$

b $2 \times 3.4 + 2w = 10.1$
$w = 1.65$ m

c $\pi D = 30$
$D = 9.55$ cm

d $\frac{1}{2} \times 1.5 \times h = 2.25$
$h = 3$ cm

e $7.2h = 24.48$
$h = 3.4$ cm

f $\pi r^2 = 30$
$r = 3.09$ cm

g $34 + 57 + x = 180°$
$x = 89°$

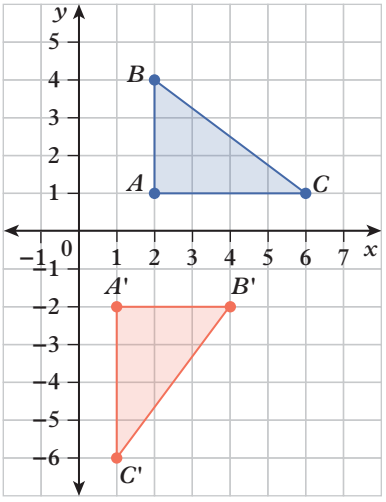
h $\frac{5 + 8 + 13 + 2 + x}{5} = 6.2$
$x = 3$

i $0.3 + 0.12 + 0.44 + x = 1$
$x = 0.14$

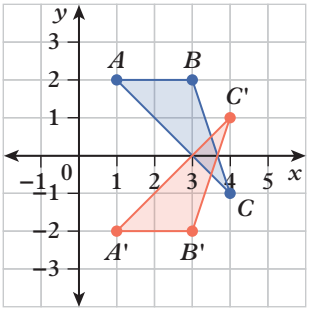
12 a (1, 2, 3), (4, 4, 4), (5, 3, 5)

b (0, 0, 0), (5, 0, 0), (5, 3, 0), (0, 3, 0), (0, 3, 5), (5, 3, 5), (5, 0, 5), (0, 0, 5)

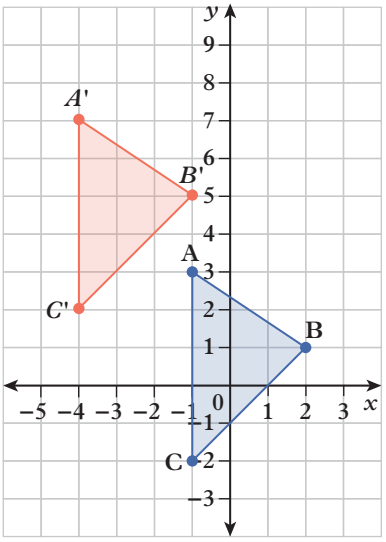
13 a



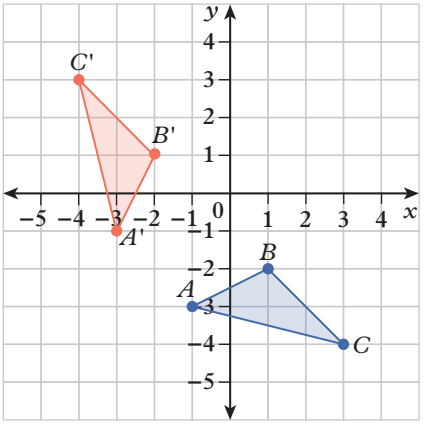
b



c



d



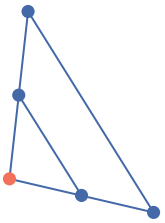
14 a yes, SSS **b** yes, SAS **c** no
d yes, AAA **e** no **f** yes, SAS

15 a not congruent **b** congruent

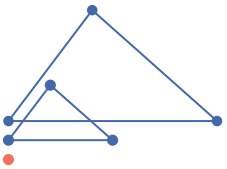
16 a $z = 13.60$ m **b** $r = 60.40$ m

17 a no **b** no **c** yes **d** yes

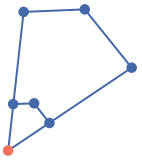
18 a



b



c



19 a $\frac{3}{9} = \frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{3}{5}$

20 a i AAS or ASA **ii** $a = 3$
b i RHS **ii** $b = 27°$
c i SAS **ii** $c = 6$
d i SSS **ii** $d = 31°$

21 a $a = 31°$ **b** $b = 154°$ **c** $c = 74°$
d $d = 68°$ **e** $e = 39°$ **f** $f = 65°$

22 a 0.024 m **b** 5 000 000 cm^3
c 8 001 000 000 m^2 **d** 4.24 cm^3
e 0.000 02 m^2 **f** 3 000 000 cm
g 1.2 hours **h** 150 minutes
i 6:23 pm **j** 0030
k 1.24 L **l** 35 000 mL

23 a 50 cm^3 **b** 42 cm^3 **c** 60 m^3 **d** 3 cm^3

24 a 2.871 m^3 or 2 871 000 cm^3
b 1 200 000 mL or 1200 L
c 1.671 m^3 or 1 671 000 cm^3

25 a 1:33 pm ACST
b 7:41 am AEST
c 12:00 pm UTC+1
d 3:31 pm UTC+5

26 a 5 h 36 m
b 3 h 25 m

27 a 1 h 34 m
b 8 h 25 m

28 mean = 7

29 a

	Reading books	Playing board games	Playing video games	Total
Weekday	25	6	34	65
Weekend	40	24	71	135
Total	65	30	105	200

b i $\frac{3}{25} = 0.12$ **ii** $\frac{1}{5} = 0.2$

iii $\frac{27}{40} = 0.675$ **iv** $\frac{49}{100} = 0.49$

30 a $\frac{13}{36}$ **b** $\frac{1}{9}$ **c** $\frac{2}{3}$ **d** $\frac{11}{36}$ **e** $\frac{1}{3}$

31 a i sample

ii fair

iii simple random sampling

iv yes

b i census

c i sample **ii** biased

iii judgemental **iv** no

32 a mean = 2.22, median = 2, mode = 2, range = 7

b mean = 1.83, median = 2, mode = 2, range = 2. The mean and range change. The mode and median stay the same.

33 a It may be too expensive (for example).

b Choose a number of both rural and urban households to survey (for example).

c This method is much cheaper than a census, but it doesn't take into account how location and climate may impact the species of pets in a household (for example).

34 a The average length of time shoppers spent at the shops was 1 h 20 m.

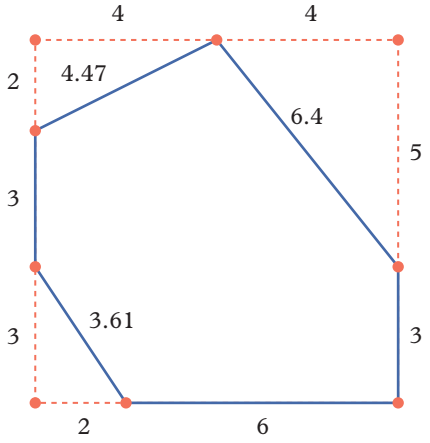
b The average number of board games people owned was 4.

c The most common favourite colour was blue.

d The length of a number of chess matches spanned 34 turns.

Mathematical modelling

1 a



b 47 cm^2

c i 305.84 cm^2 **ii** 376 cm^3

d i surface area: $26.48h + 94$, volume: $47h$

ii $h = 4.58$ cm

2 a $a = 50°$, $b = 40°$, $c = 59°$, $d = 62°$, $e = 60°$

b mean = 60°, median = 59.5°, mode = 50°, range = 50°

c continuous numerical data

d $\frac{11}{12}$

e $\triangle ABC \cong \triangle JLK$, SAS

f rotation of 180° about the origin

g $\triangle DEF$

3 a

Destination	Melbourne	London
Beach	42%	35%
Camping	34%	20%
Snow	24%	45%

M	L
38%	35%
25%	22%
37%	43%

M	L
43%	40%
26%	15%
31%	45%

M	L
40%	37%
34%	25%
26%	38%

b People in Melbourne and London have similar preferences about going to the beach, with sample percentages in the high 30s to low 40s in both cities. However, the sample percentages for camping and going to the snow vary much more significantly. Camping in Melbourne varies between 25% and 34% whereas in London it varies between 15% and 25%. Going to the snow in Melbourne varies between 24% and 37% whereas in London it varies between 38% and 45%. Going to the beach was most popular in Melbourne while going to the snow was most popular in London. The least popular activity in London is camping, while in Melbourne camping and snow were similar in preference.

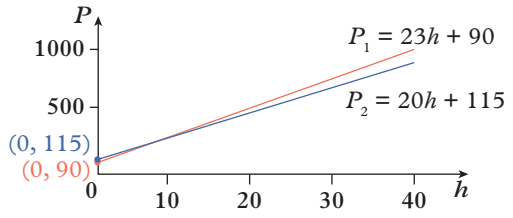
c 1800

d While some variation would still be expected if the seasons did not impact people's preferences, the percentages changed consistently across the samples. Most notably, the sample percentages for going to the snow increased in London to vary between 45% and 53% while in Melbourne they stayed between 27% and 33% (within the variation of the previous samples). The sample percentages for going to the beach in Melbourne and London dropped to vary between 22% and 40%. Camping increased in Melbourne from 27% to 40% but stayed within the same variation for London. For this time period, going to the snow became Melbourne's least popular activity, but the most popular and least popular activities in London remained the same: snow and camping, respectively.

4 a \$800 **b** \$890

c i $P_1 = 23h + 90$ **ii** $P_2 = 20h + 115$

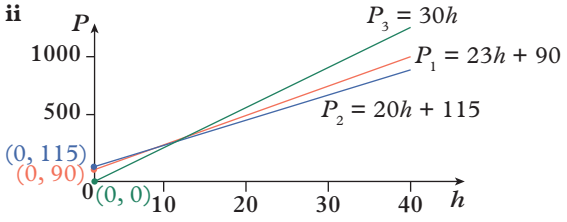
iii



iv Option 1

d i $P_3 = 30h$

ii



iii Yes

EX EXPLORATIONS 2

p510

1 a 5

b 36

c 55

d 45

e 65

f 209 and 221

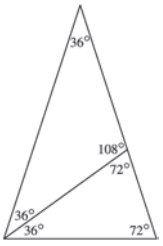
g The word BEANS scores
$b + 1 + 3 + 13 + s = b + s + 17$ points.
$s = b + 1$; therefore, $b + s$ is always odd, and $b + s + 17$ is always even.

2 a 540°

b 108°

c Isosceles triangles with apex angle 36° and base angles 72°, isosceles triangles with apex angle 108° and base angles 36°, and a rhombus with angles 72° and 108°.

d i



ii Show that a wide triangle can also be divided into one narrow and one wide triangle. It generalises to any pair of numbers as long as there is at least one narrow triangle and two wide triangles.

iii Apex angle 90° and base angles 45° (it divides into two more of the same type) and apex angle $25\frac{5}{7}°$ and base angles $77\frac{1}{7}°$ (it divides into two different types).

3 a 120 cm

b 3.82 revolutions per hour

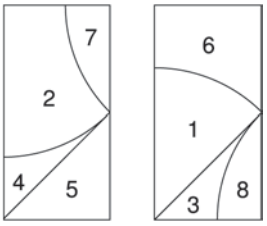
c Raise 209 mm

4 a

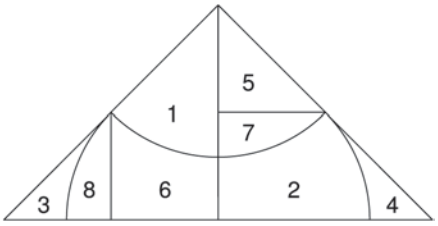
Number	1	2	3 and 4	5	6	7 and 8
Area	2π	8	$8 - 2\pi$	4	$12 - 2\pi$	$2\pi - 4$

b One possible arrangement is shown for each shape. There are many others.

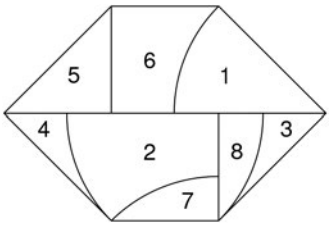
i



ii



iii



5 a {2, 2}

b The other freaky frequency list involving only the numbers 1, 2, 3 and 4 is {1, 3, 2, 1, 3, 3, 4, 1}

c i {1, 2, 2, 3, 3, 2, 4, 1}

ii {1, 4, 2, 2, 3, 2, 4, 2, 5, 1, 6, 1}

iii 9

6 Corrections are underlined:

a R2 [R4 [F50 L90] <u>L180</u>]

b <u>R6</u> [R3 [F50 L120] L60]

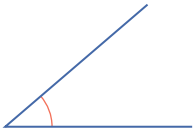
c R6 [R6 [F50 L60] <u>L60</u>]

Glossary

12-hour time
time measured over a 12-hour period from midnight and then again from midday; am and pm are used to describe whether a time is before or after midday

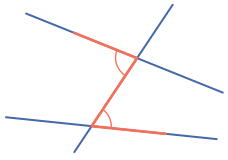
24-hour time
time measured over a 24-hour period from midnight. Usually shown with four digits to indicate the number of hours and minutes from midnight; for example, 1530 means 15 hours and 30 minutes after midnight – or 3:30 pm

acute angle
angle greater than 0° but less than 90°



acute-angled triangle
triangle with three interior acute angles (less than 90°)

alternate angles
pair of angles not adjacent to (next to) each other on opposite sides of a transversal between two lines, sometimes called 'Z angles'



area
amount of space enclosed by a two-dimensional (2D) shape. Common units are mm^2, cm^2, m^2, km^2, ha

ascending order
where a list of numbers is written from smallest to largest

associative law
regardless of how three or more numbers are grouped, when added or multiplied, the result is the same; for example,

$$(4 + 5) + 6 = 9 + 6 = 15$$
$$4 + (5 + 6) = 4 + 11 = 15$$
$$(4 \times 5) \times 6 = 20 \times 6 = 120$$
$$4 \times (5 \times 6) = 4 \times 30 = 120$$

base
1. for a value expressed in exponent form, the base is the number which is repeatedly multiplied; for example, 2^4 has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$
2. in a 2D shape, it is a nominated side of the shape. The base and height are perpendicular to each other
3. in a 3D object, it is a nominated face of the object. The base and height are perpendicular to each other

basic numeral
the numerical value of an expression in exponent form or expanded form; for example, the basic numeral of 2^4 (or $2 \times 2 \times 2 \times 2$) is 16.

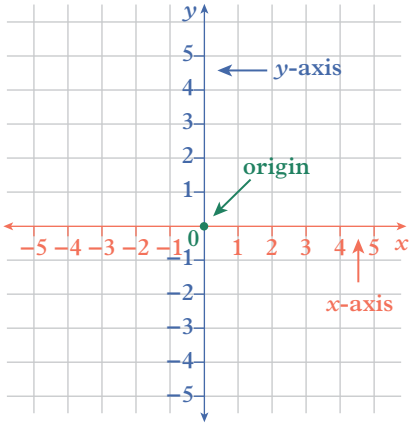
biased
a biased sample is one where the method of collecting data produces a sample that does not accurately reflect the population

BIDMAS
acronym used to remember the correct order of operations: Brackets, Indices (and roots), Division and Multiplication, and Addition and Subtraction

by-parts method
a way to add two or more numbers where the digits in each place value are added separately

capacity
amount of liquid that a three-dimensional object (container) can hold

Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point on the plane to be described. It consists of a horizontal x-axis and a vertical y-axis that intersect at right angles at a point called the origin



census
a survey of an entire population

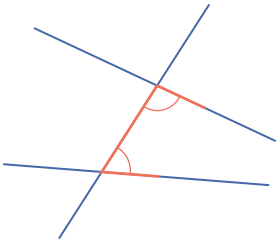
centre of dilation
the point from which a dilation is made

circumference
perimeter of a circle

cluster sampling
dividing the population into similarly diverse groups and then taking a random sample of the groups

coefficient
the number by which a pronumeral is multiplied; for example, the coefficient of $5m$ is 5 and the coefficient of $-3a$ is -3

co-interior angles
pair of angles on the same side of a transversal between two lines, sometimes called 'C angles'



common factor
factor that divides exactly into two or more whole numbers; for example, 12 and 9 have a common factor of 3

commutative law
the order in which two numbers are added or multiplied is not important; for example, $3 + 4 = 7$ and $4 + 3 = 7$; $3 \times 4 = 12$ and $4 \times 3 = 12$

compensation method
a way to add or subtract numbers whereby one number is rounded to make the calculation easier, and then the amount by which the numbers was rounded by is added to or subtracted from the result

complementary angles
angles that add to make a right angle (90°)

complementary events
a pair of events, A and A' (not A), where only one of the two events can take place at a single time, and their probabilities sum to 1

composite number
a positive whole number that is divisible by at least one whole number other than itself and 1; for example, 8 has factors of 1, 2, 4 and 8, and so is a composite number

congruent
two figures are congruent when they are identical in shape and size, or if one figure is the mirror image of the other. Congruent figures can be in any position or orientation

constant
a term without any pronumeral component; for example, the constant in the expression $3b - 4c + 10$ is 10

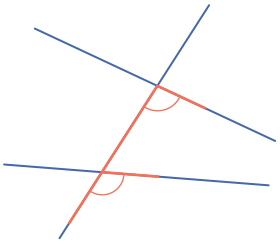
convenience sampling
when a sample that is easily accessible is selected; for example, selecting students from your class to complete a survey

conversion factor
a multiplier for converting units; for example, multiply by 10 to convert from cm to mm

Coordinated Universal Time (UTC)
time zone referenced from Greenwich in London at 0° longitude (also known as Greenwich Mean Time). Parts of the world to the east of Greenwich are ahead of UTC, and parts of the world to the west are behind UTC; for example, Sydney is in the UTC+10 time zone. This means it is 10 hours ahead of UTC

coordinates
pair of numbers that describe the position of a point on a plane or graph

corresponding angles
pair of angles on the same side of a transversal and in corresponding positions on the two lines (both above a line or both below a line), sometimes called 'F angles'



cost price (or wholesale price)
the cost paid to create or obtain a good or service before it is sold to consumers

cube
to raise to the power of three; for example, the cube of 2 is 8 because $2^3 = 8$

cube root
number that when cubed results in the original number; for example, the cube root of 8 (or $\sqrt[3]{8}$) is 2

decimal
number with a whole number component and a decimal component. The digits to the left of the decimal point indicate the whole number and the digits to the right of the decimal point indicate a value less than one; for example, 3.456

denominator
bottom number of a fraction; for example, the denominator of $\frac{2}{3}$ is 3

descending order
where a list of numbers is written from largest to smallest

diameter
line segment from one side of a circle to the other through the centre of the circle. Also defined as the length of this line segment

dilation
a transformation that produces an enlargement or reduction of the original figure

discount
an amount off the original price; the difference between the regular price and a lower price

distributive law
the same result is obtained whether you add the numbers inside a pair of brackets first before multiplying by another number or multiply each number inside the brackets by the number outside the brackets before adding, $a(b + c) = ab + ac$

dividend
number that is being divided by another number (divisor); for example, 6.84 is the dividend, 2 is the divisor and 3.42 is the quotient in $6.84 \div 2 = 3.42$

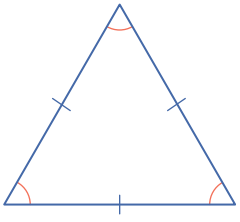
divisor
number that divides into another number (dividend); for example, in $15 \div 3 = 5$, 3 is the divisor

duration
the length between two given times

enlargement
an image produced by a dilation that is larger than the original figure

equation
a statement expressing the equality of two mathematical expressions through the use of an equals sign

equilateral triangle
triangle with all three sides equal in length. All three angles are also equal in size



equivalent equations
equations that have the same solution(s)

equivalent fractions
different fractions that have the same value. An equivalent fraction is formed by multiplying or dividing the numerator and the denominator of a fraction by the same number; for example, $\frac{24}{36} = \frac{4}{6} = \frac{2}{3}$

equivalent rate
different rates that have the same value. An equivalent rate can be found by mutiplying or dividing both quantities in a rate by the same value

equivalent ratios
different ratios that have the same value. An equivalent ratio can be produced by multiplying or dividing all numbers in a ratio by the same value

estimate
an approximation of a value or number

evaluate
to determine the value of a mathematical expression

event
a specified result that may be obtained in a probability experiment

expand
remove the brackets in an expression by using the distributive law

expanded form
an expression is written in expanded form when all multiplications are written in full, without using exponents or abbreviations

experiment
a repeatable procedure that has a clearly defined set of possible results

exponent
for a value expressed in exponent form, the exponent indicates the number of times the base is written as a repeated multiplication; for example, 2^4 has an exponent of 4 and can be written as $2 \times 2 \times 2 \times 2$. We read 2^4 as '2 to the power of 4'

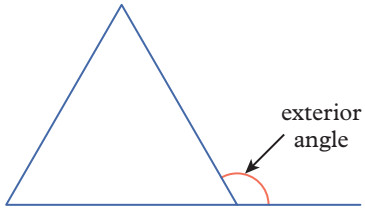
exponent form
a number or expression written using a base and an exponent; for example, 2^4 is written in exponent form

exponent laws
a set of rules that can be used to simplify expressions written in exponent form

	Exponent law
Product of powers	$a^m \times a^n = a^{m+n}$
Quotient of powers	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$
Power of a power	$(a^m)^n = a^{m \times n}$

expression
a mathematical statement containing two or more numbers or variables and at least one mathematical operation

exterior angle
an angle between any side of a shape and a line extended from the adjacent side



factor
a whole number or algebraic term that divides exactly into another whole number or algebraic term; for example, the factors of 8 are 1, 2, 4 and 8, and the factors of $2x$ are 2 and x

factorising
to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$

factor tree
tree diagram where composite numbers are broken down into factor pairs until only prime factors remain

favourable outcome
the outcome within a sample space that we are interested in

formula
relationship or rule between two or more variables, usually represented using pronumerals; for example, $s = \frac{d}{t}$ is the formula for speed equals distance divided by time

graph
a continuous line made up of infinitely many coordinate points

grouping symbols
symbols including brackets, roots and fraction lines which show where a group starts and ends, and help to establish the order to apply mathematical operations

GST (Goods and Services Tax)
introduced by the Australian Government in 1999. A 10% tax on the selling price of products or services

highest common factor (HCF)
highest or largest factor that is common to two or more given numbers or algebraic terms; for example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so the HCF of 8 and 12 is 4

hypotenuse
the side opposite the right angle of a right-angled triangle

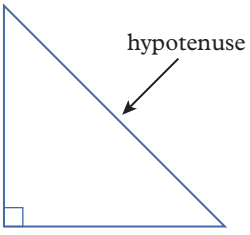


image
the result of completing a transformation on a shape or object. If the original shape is labelled A, then the image (new shape) is labelled A'

improper fraction
fraction where the numerator is greater than (or equal to) the denominator; for example, $\frac{9}{7}$ or $\frac{6}{6}$

included angle
an angle between two given sides

index
see exponent

inequality
mathematical statement that compares the values of two expressions, often using the symbols $<$, $>$, $\leq$, $\geq$ or $\neq$

integer
a whole number that is positive, negative or zero

interior angle
an angle inside a shape or object

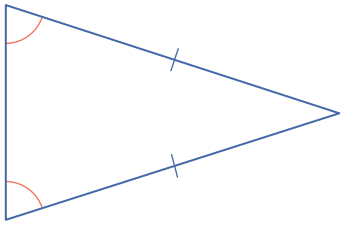
inverse operation
operation that reverses the effect of a previous operation; for example, multiplication and division are inverse operations, and addition and subtraction are inverse operations

irrational number
number that cannot be written exactly as a fraction. These are non-terminating decimals that are not recurring decimals; for example, π, $\sqrt{2}$, $\sqrt{3}$

irregular quadrilateral
quadrilateral with four different internal angles and no parallel sides

isometric transformation
a type of transformation (such as a translation, rotation or reflection) applied to a shape or object that does not change its shape or size

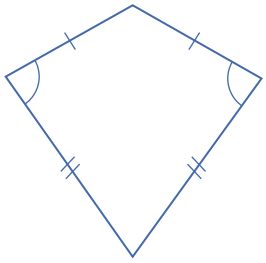
isosceles triangle
triangle with two sides equal in length. The angles opposite the equal sides are also equal



judgemental sampling
when a sample is selected by the researcher; for example, a researcher may ask students to take part in a study

jump method
a way to add or subtract numbers whereby one of the given numbers is broken down, then each part of the number is added or subtracted in stages

kite
quadrilateral with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides

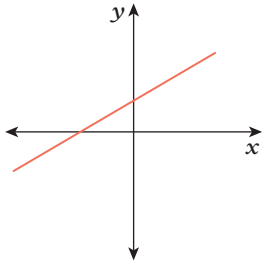


leading digit
first non-zero digit of a number; for example, the leading digit of 328 is 3

length
measurement from one end to the other of a shape or object. In a rectangle, it is the longer side measurement. Common units are mm, cm, m, km

like terms
algebraic terms that contain exactly the same pronumerals raised to the same exponents in any order

linear graph
a graph that forms a straight line on the Cartesian plane



linear relationship
relationship between two variables that produces a linear graph

loss
amount lost when the selling price is less than the cost price

lowest common denominator (LCD)
lowest common multiple of the denominators in two or more fractions; for example, $\frac{1}{3}$ and $\frac{1}{2}$ have an LCD of 6

lowest common multiple (LCM)
lowest (first) multiple that is common in the multiples list of two or more numbers; for example, multiples of 3 are 3, 6, 9, 12, 15, 18, … and multiples of 4 are 4, 8, 12, 16, 20, 24, … so the LCM of 3 and 4 is 12

magnitude
the value of a number ignoring the sign; for example, the magnitude of -3 is 3

mark-up
amount added to a cost price; for example, an original price has a 25% mark-up added to obtain the selling price

mean
a measure of the centre of a data set (numerical average) calculated by adding all of the data values and then dividing by the number of values

measure of centre
number used to represent the central point in a data set. Measures of centre include the mean, median and mode

measure of spread
number used to indicate the amount of variation in a data set. The range is a measure of spread

median
a measure of the centre of a data set. For an ordered set of data, it is the middle number if there is an odd number of values, and the average of the two middle numbers if there is an even number of values

mixed number
number made up of a whole number component and a proper fraction component; for example, $3\frac{4}{5}$

mode
a measure of the centre of a data set determined by finding the value, or values, that occurs most frequently in the data set

multiples
result of multiplying a given number by whole numbers; for example, multiples of 4 are 4, 8, 12, 16, … (4×1, 4×2, 4×3, 4×4, …)

mutually exclusive
two events are mutually exclusive if they cannot occur at the same time; for example, it is not possible for a coin to land on both heads and tails

negative integers
whole numbers that are less than zero in value and are always preceded by a negative sign, –, placed directly before the first digit

non-linear graph
graph of a non-linear relationship, which is not a straight line

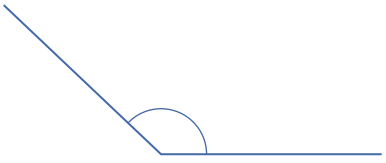
non-linear relationship
relationship between two variables which is not linear. The coordinate points describing this relationship do not lie on a straight line when plotted on the Cartesian plane

non-terminating decimal
decimals with an endless (infinite) number of decimal places; for example, 5.761 238… or 4.266 666…

n^{th} root
the n^{th} root of a number, x, is the number whose n^{th} power is x

numerator
top number of a fraction; for example, the numerator of $\frac{2}{3}$ is 2

obtuse angle
angle greater than 90° but less than 180°

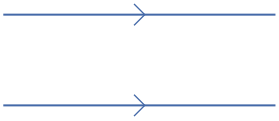


obtuse-angled triangle
triangle with one interior obtuse angle (more than 90°)

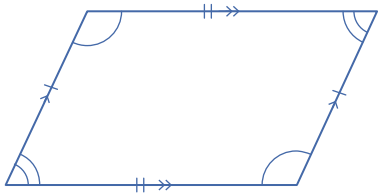
origin
point where the x- and y-axes cross on a Cartesian plane. Coordinates are (0, 0)

outlier
extreme piece of data that is much higher or lower than the rest of the data set

parallel lines
lines (or rays or segments) that are drawn side by side and are the same distance apart so they never touch are parallel. Matching arrowheads are used to indicate that lines are parallel



parallelogram
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size. Rectangles, squares and rhombuses are also classified as parallelograms



percentage
a portion of a whole divided into one hundred parts. The symbol for percentage is %

percentage decrease
occurs when a percentage amount is subtracted from the original amount

percentage error
the error of a measurement written as a percentage of the true value of the quantity

percentage increase
occurs when a percentage amount is added to the original amount

percentage loss
the amount of loss expressed as a percentage of the cost price (or selling price)

percentage profit
the amount of profit expressed as a percentage of the cost price (or selling price)

perimeter
length of the boundary of a two-dimensional (2D) shape. Common units are mm, cm, m, km

pi
irrational number (3.141 592 653 …) used in calculations involving the circumference and area of a circle. It is the ratio of the circumference of a circle to its diameter. The symbol for pi is π

plot
a Cartesian plane composed of individual coordinate points

point of focus
a point to concentrate on when reflecting or rotating a shape

point of intersection
point where two or more lines or graphs intersect on the Cartesian plane

polygon
general name for a two-dimensional shape with straight sides

population
all potential pieces of data that we are seeking information about

population parameters
the characteristics of a population, including the mean, median, mode and range

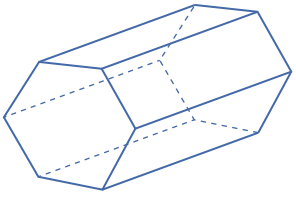
positive integers
whole numbers that are greater than zero in value and can be represented using a positive sign, +. The positive sign can also be left out

power
a number written in exponent form

prime factorisation
the unique product of the primes that make up an integer

prime number
a positive whole number greater than 1 that has exactly two factors, itself and 1; for example, the only factors of 7 are 1 and 7, so 7 is a prime number

prism
3D object with a consistent cross-section parallel to the base of the prism



product
result of a multiplication; for example, $10 \times 3 = 30$, where 30 is the product

profit
amount made when the selling price is greater than the cost price

pronumeral
letter or symbol that takes the place of a number

proper fraction
fraction where the numerator is smaller than the denominator; for example, $\frac{4}{5}$

Pythagoras' theorem
theorem that states that the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the other side lengths

Pythagorean triple
any set of three positive whole numbers that satisfy Pythagoras' theorem

quadratic equation
an equation in terms of a pronumeral (usually x) whose highest power is 2; for example, $x^2 + 3x - 2 = 0$

quadrilateral
two-dimensional shape with four straight sides

quota sampling
dividing the population into separate categories (such as gender or age) and then taking a sample from each category, with the size of each sample chosen by the researcher

quotient
result of a division; for example, $28 \div 4 = 7$ where 7 is the quotient

radical sign
the sign used to indicate a root. If a number appears at the top left of the radical sign, this shows which root is being specified; for example, $\sqrt[4]{\ }$ is specifying the fourth root

radius
line segment from the centre of a circle to the edge of the circle. Also defined as the length of this line segment; it is half the diameter of the circle

range
a measure of spread determined by finding the difference between the highest and lowest values in a data set

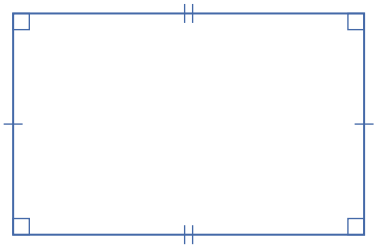
rate
a comparison between two or more different quantities. It is a measure of how one quantity changes for each increase in the unit of another quantity

ratio
a comparison of two or more quantities of the same kind; for example, 'when comparing the number of red jellybeans to blue jellybeans, the ratio is 2 : 3'

rational number
number that can be written exactly as a fraction. This includes whole numbers, fractions, terminating decimals and recurring decimals

reciprocal
the multiplicative inverse of a number which can be found by writing the number as a fraction and swapping the numerator and denominator; for example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. When a number is multiplied by its reciprocal, the result is 1

rectangle
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°. A rectangle is also classified as a parallelogram



rectangular prism
prism with three pairs of identical rectangular faces

recurring decimal
a special type of non-terminating decimal. The digits after the decimal point form a recurring or repeating pattern; for example,

$0.333\,33\ldots = 0.\dot{3}$, and

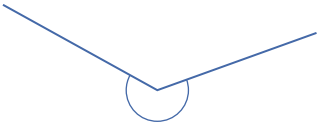
$5.476\,767\,6\ldots = 5.4\overline{76}$

reduction
an image produced by a dilation that is smaller than the original figure

reference point
a point or number used to define the location of another point relative to the reference point. This point is often zero on a number line or Cartesian plane

reflection
a transformation where a shape or object is reflected (flipped) in a mirror line to produce its exact mirror image

reflex angle
angle that is greater than 180° but less than 360°



relative frequency
the number of times a favourable outcome occurs divided by the total number of outcomes:

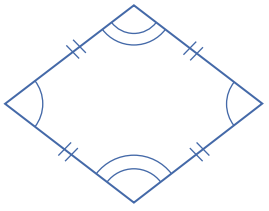
$$\text{relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

remainder
the part of the dividend that is left over when the number is not exactly divisible by the divisor; for example, 22 divided by 7 is 3 remainder 1

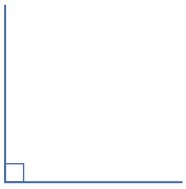
revenue
income generated by the sale of goods or services

revolution
angle that is exactly 360°

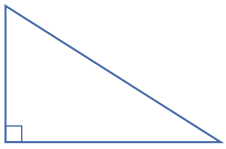
rhombus
quadrilateral with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size. A rhombus is a type of a parallelogram



right angle
angle that is exactly 90°. Often represented by a small box drawn where the lines meet



right-angled triangle
triangle where the size of one interior angle is 90°



rotation
a transformation where a shape or object is rotated (turned) by a specified amount in either a clockwise or anti-clockwise direction around a given point

round down
to find an approximation that is less than the original number

round up
to find an approximation that is greater than the original number

rounding
involves replacing the number with an approximation that is simpler and easier to use in calculations, but which makes any calculations less accurate; for example, $123\,343 \approx 120\,000$

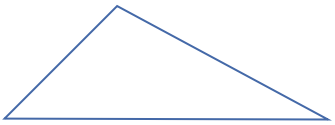
sample
a selection from the population

sample space
list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a six-sided die is {1, 2, 3, 4, 5, 6}

scale factor
the ratio between the corresponding measurements of two similar figures, calculated by dividing the image length by the original length:

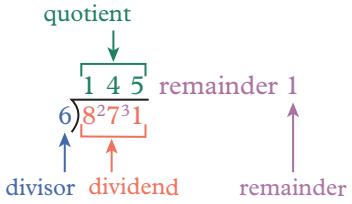
$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

scalene triangle
triangle with three sides of different lengths. All three angles are also different in size



selling price
the price that a product or service is sold at by the seller

short division
an algorithm used to simplify a division problem by breaking it down into easy steps



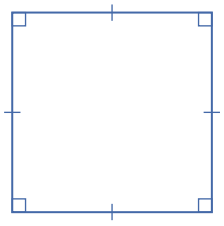
similar
two figures are similar when they have the same shape but are not necessarily the same size. Dilations produce similar figures

simple random sampling
techniques that ensure that every potential piece of data has an equal chance of being selected. Types of simple random sampling include systematic sampling and stratified sampling

solution
value of the pronumeral that makes an equation a true statement

solve by inspection
a mental strategy to solve simple equations, whereby the solution is identified without performing any calculations

square
1 quadrilateral with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°



2 to raise to the power of 2; for example, $3^2 = 9$

square root
the square root of a number is the number that when multiplied by itself produces the original number; for example, the square root of 9 (or $\sqrt{9}$) is 3

straight angle
angle that is exactly 180°

stratified sampling
dividing the population into separate categories (such as gender or age) and then taking a random sample from each category that is proportional to its size

subject
a pronumeral written by itself on one side of an equation and which does not appear on the other side of the equation; for example, y is the subject in the equation $y = mx + c$

substitution
replacement of a pronumeral with a given number; for example, substituting $x = 2$ into $3x + 4$ gives $3 \times 2 + 4$

supplementary angles
angles that add to make a straight angle (180°)

surd
a root of a number that results in an irrational solution

survey
a question, or list of questions, designed to extract specific information about a chosen topic

systematic sampling
selecting a sample at fixed intervals (such as every third person). The starting point should be random

term
a single pronumeral, single number, or the product of a number and one or more pronumerals; for example, the expression $3b - 4c + 10$ has three terms

terminating decimal
decimal that 'terminates' or stops after a set number of decimal places; for example, 0.625

theoretical probability
chance of something happening based on mathematical reasoning, shown on a scale from 0 to 1 (inclusive)

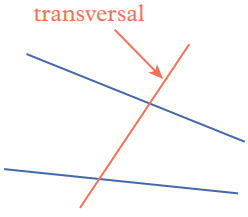
three-dimensional coordinates
three numbers that describe the position of a point in three-dimensional space

time zones
bands on the surface of the Earth where countries experience the same time. Each time zone usually spans a difference in longitude of 15°

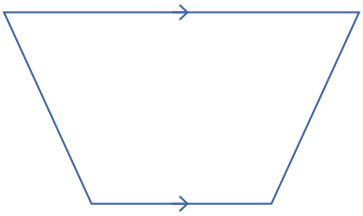
trailing zeros
zeros to the right of the last non-zero digit after the decimal point in a decimal; for example, 3.900 has two trailing zeros

translation
a transformation where a shape or object is translated (moved) in a straight line without turning or changing size

transversal
line that crosses (intersects) a pair or set of lines



trapezium
quadrilateral with one pair of opposite sides that are parallel



triangle
two-dimensional shape with three straight sides

triangular prism
3D object with five faces and a consistent cross-section in the shape of a triangle

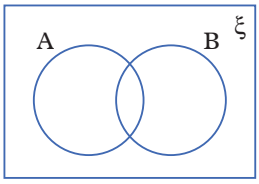
two-way table
a table that displays frequencies for two different categories or attributes of data

unitary method
a method for solving various financial problems by first calculating the value of a single unit, for example, 1% of a quantity

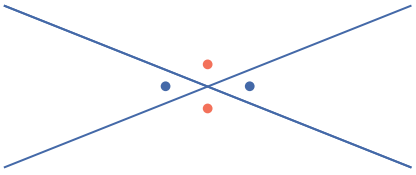
unknown number
a number that is represented using a pronumeral; for example, the x in $x + 3 = 5$

variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

Venn diagram
a display of the relationship between different categories or attributes of data. The entire data set is represented by a rectangle and each category is represented by a circle



vertically opposite angles
pair of equal angles not adjacent to (next to) each other, formed where two lines intersect



volume
amount of space that a three-dimensional (3D) object occupies

x-axis
horizontal number line that forms part of the Cartesian plane

x-coordinate
first of a pair or trio of numbers (coordinates) that describes a point's horizontal distance from the origin on the Cartesian plane

y-axis
vertical number line that forms part of the Cartesian plane

y-coordinate
second of a pair or trio of numbers (coordinates) that describes a point's vertical distance from the origin on the Cartesian plane

y-intercept
the value of the y-coordinate where a line meets the y-axis (when $x = 0$). In a linear equation of the form $y = mx + c$, the y-intercept is equal to the constant, c

z-axis
the axis perpendicular to the Cartesian plane (the xy-plane) in a three-dimensional coordinate system

z-coordinate
third of a trio of numbers (coordinates) that describes how far up or down a point is from the origin in a three-dimensional coordinate system

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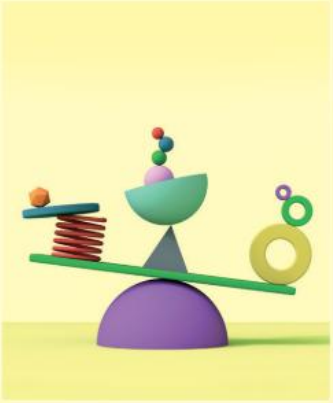

Solving equations is like balancing a pair of scales. You have to keep the equation balanced by making changes on each side of the equals sign!