JACARANDA

MATHS QUEST 9

AUSTRALIAN CURRICULUM | THIRD EDITION

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AUSTRALIAN CURRICULUM | THIRD EDITION

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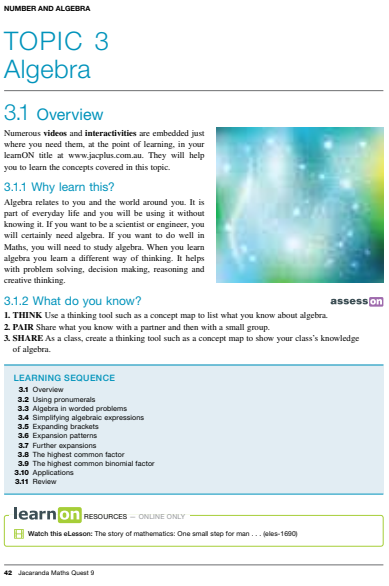
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ABOUT THIS RESOURCE

Jacaranda Maths Quest 9 Australian Curriculum Third Edition has been completely revised to help teachers and students navigate the Australian Curriculum Mathematics syllabus. The suite of resources in the Maths Quest series is designed to enrich the learning experience and improve learning outcomes for all students.

Maths Quest is designed to cater for students of all abilities: no student is left behind and none is held back. Maths Quest is written with the specific purpose of helping students deeply understand mathematical concepts. The content is organised around a number of features, in both print and online through Jacaranda's *learnON* platform, to allow for seamless sequencing through material to scaffold every student's learning.


NUMBER AND ALGEBRA

TOPIC 3
Algebra

3.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

3.1.1 Why learn this?

Algebra relates to you and the world around you. It is part of everyday life and you will be using it without knowing it. If you want to be a scientist or engineer, you will certainly need algebra. If you want to do well in Maths, you will need to study algebra. When you learn algebra you learn a different way of thinking. It helps with problem solving, decision making, reasoning and creative thinking.

3.1.2 What do you know?

assessON

1. THINK Use a thinking tool such as a concept map to list what you know about algebra.
2. PAIR Share what you know with a partner and then with a small group.
3. SHARE As a class, create a thinking tool such as a concept map to show your class's knowledge of algebra.

LEARNING SEQUENCE

3.1 Overview
3.2 Using pronumerals
3.3 Algebra in worded problems
3.4 Simplifying algebraic expressions
3.5 Expanding brackets
3.6 Expansion patterns
3.7 Further expansions
3.8 The highest common factor
3.9 The highest common binomial factor
3.10 Applications
3.11 Review

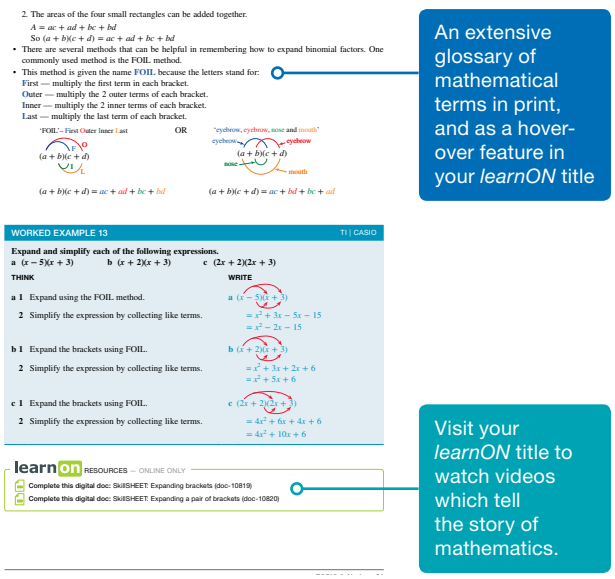
learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: One small step for man . . . (eles-1690)

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Topic introductions put the topic into a real-world context.

The learning sequence at a glance


2. The areas of the four small rectangles can be added together.
$A = ac + ad + bc + bd$
So $(a + b)(c + d) = ac + ad + bc + bd$

- There are several methods that can be helpful in remembering how to expand binomial factors. One commonly used method is the FOIL method.
- This method is given the name **FOIL** because the letters stand for:
  First — multiply the first term in each bracket.
  Outer — multiply the 2 outer terms of each bracket.
  Inner — multiply the 2 inner terms of each bracket.
  Last — multiply the last term of each bracket.

'FOIL'- First Outer Inner Last OR 'eyebrow, eyebrow, nose and mouth'

$(a + b)(c + d) = ac + ad + bc + bd$ $(a + b)(c + d) = ac + bd + bc + ad$

WORKED EXAMPLE 13 TI | CASIO

Expand and simplify each of the following expressions.
a $(x - 5)(x + 3)$ **b** $(x + 2)(x + 3)$ **c** $(2x + 2)(2x + 3)$

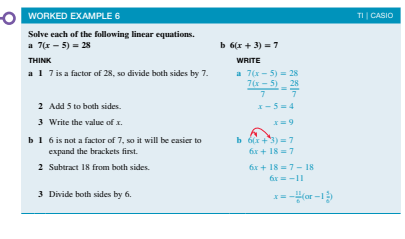
THINK	WRITE
a 1 Expand using the FOIL method.	a $(x - 5)(x + 3)$
2 Simplify the expression by collecting like terms.	$= x^2 + 3x - 5x - 15$ $= x^2 - 2x - 15$
b 1 Expand the brackets using FOIL.	b $(x + 2)(x + 3)$
2 Simplify the expression by collecting like terms.	$= x^2 + 3x + 2x + 6$ $= x^2 + 5x + 6$
c 1 Expand the brackets using FOIL.	c $(2x + 2)(2x + 3)$
2 Simplify the expression by collecting like terms.	$= 4x^2 + 6x + 4x + 6$ $= 4x^2 + 10x + 6$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Expanding brackets (doc-10819)
Complete this digital doc: SkillSHEET: Expanding a pair of brackets (doc-10820)

An extensive glossary of mathematical terms in print, and as a hover-over feature in your *learnON* title

Visit your *learnON* title to watch videos which tell the story of mathematics.


WORKED EXAMPLE 6 TI | CASIO

Solve each of the following linear equations.
a $7(x - 5) = 28$ **b** $6(x + 3) = 7$

THINK	WRITE
a 1 7 is a factor of 28, so divide both sides by 7.	a $\frac{7(x-5)}{7} = \frac{28}{7}$
2 Add 5 to both sides.	$x - 5 = 4$
3 Write the value of x.	$x = 9$
b 1 6 is not a factor of 7, so it will be easier to expand the brackets first.	b $6(x + 3) = 7$ $6x + 18 = 7$
2 Subtract 18 from both sides.	$6x + 18 = 7 - 18$ $6x = -11$
3 Divide both sides by 6.	$x = -\frac{11}{6}$ (or $-1\frac{5}{6}$)

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Expanding brackets (doc-10827)

Exercise 4.3 Solving linear equations with brackets assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–h, 3a–f, 4a–f, 5, 6, 8, 10	Questions: 1d–i, 2d–i, 3d–i, 4d–i, 5, 7–11	Questions: 1g–i, 2g–l, 3g–l, 4g–l, 5, 7–12

Individual pathway interactivity: int-4490 learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Solve each of the following linear equations.
   a. $5(x - 2) = 20$ **b.** $4(x + 5) = 8$ **c.** $6(x + 3) = 18$ **d.** $5(x - 41) = 75$
   e. $8(x + 2) = 24$ **f.** $3(x + 5) = 15$ **g.** $5(x + 4) = 15$ **h.** $3(x - 2) = -12$
   i. $7(x - 6) = 0$ **j.** $-6(x - 2) = 12$ **k.** $4(x + 2) = 4.8$ **l.** $16(x - 3) = 48$
2. WE2 Solve each of the following equations.
   a. $6(b - 1) = 1$ **b.** $2(m - 3) = 3$ **c.** $2(a + 5) = 7$ **d.** $3(m + 2) = 2$
   e. $5(p - 2) = -7$ **f.** $6(m - 4) = -8$ **g.** $-10(a + 1) = 5$ **h.** $-12(p - 2) = 6$
   i. $-9(a - 3) = -3$ **j.** $-2(m + 3) = -1$ **k.** $3(2a + 1) = 2$ **l.** $4(3m + 2) = 5$

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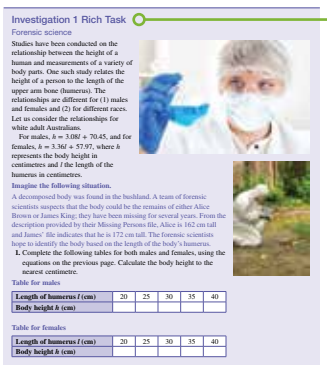
Fully worked examples throughout the text demonstrate key concepts.

Your FREE online *learnON* resources contain hundreds of videos, interactivities and traditional WorkSHEETs and SkillSHEETs to support and enhance learning.

Carefully graded questions cater for all abilities. Question types are classified according to strands of the Australian Curriculum.

Individual pathway interactivities in each sub-topic ensure consolidation of learning for every skill level.

Fully worked solutions to every question are provided online, and answers are provided at the end of each print topic.


Investigation 1 Rich Task

Forensic science

Studies have been conducted on the relationship between the height of a human and measurements of a variety of body parts. One such study relates the height of a person to the length of the upper arm bone (humerus). The relationships are different for (1) males and females and (2) for different races. Let us consider the relationships for white adult Australians.

For males, $h = 3.08l + 70.45$, and for females, $h = 3.36l + 57.97$, where h represents the body height in centimetres and l the length of the humerus in centimetres.

Imagine the following situation.

A decomposed body was found in the bushland. A team of forensic scientists suspects that the body could be the remains of either Alice Brown or James King; they have been missing for several years. From the description provided by their Missing Persons file, Alice is 162 cm tall and James' file indicates that he is 172 cm tall. The forensic scientists hope to identify the body based on the length of the body's humerus.

1. Complete the following tables for both males and females, using the equations on the previous page. Calculate the body height to the nearest centimetre.

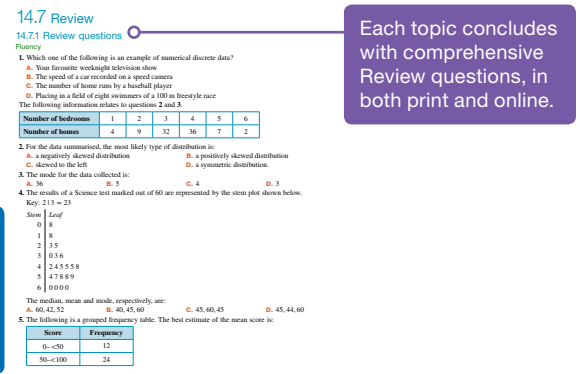
Table for males

Length of humerus l (cm)	20	25	30	35	40
Body height h (cm)					

Table for females

Length of humerus l (cm)	20	25	30	35	40
Body height h (cm)					

Engaging Investigations at the end of each topic to deepen conceptual understanding


14.7 Review

14.7.1 Review questions

Fluency

1. Which one of the following is an example of numerical discrete data?
   A. Your favourite weeknight television show
   B. The speed of a car recorded on a speed camera
   C. The number of home runs by a baseball player
   D. Placing in a field of eight swimmers of a 100 m freestyle race

The following information relates to questions **2** and **3**.

Number of bedrooms	1	2	3	4	5	6
Number of homes	4	9	32	36	7	2

2. For the data summarised, the most likely type of distribution is:
   A. a negatively skewed distribution **B.** a positively skewed distribution
   C. skewed to the left **D.** a symmetric distribution.
3. The mode for the data collected is:
   A. 36 **B.** 5 **C.** 4 **D.** 3
4. The results of a Science test marked out of 60 are represented by the stem plot shown below.
   Key: 2 | 3 = 23

Stem	Leaf
0	8
1	8
2	3 5
3	0 3 6
4	2 4 5 5 5 8
5	4 7 8 8 9
6	0 0 0 0

The median, mean and mode, respectively, are:
A. 60, 42, 52 **B.** 40, 45, 60 **C.** 45, 60, 45 **D.** 45, 44, 60

5. The following is a grouped frequency table. The best estimate of the mean score is:

Score	Frequency
0–<50	12
50–<100	24

Each topic concludes with comprehensive Review questions, in both print and online.

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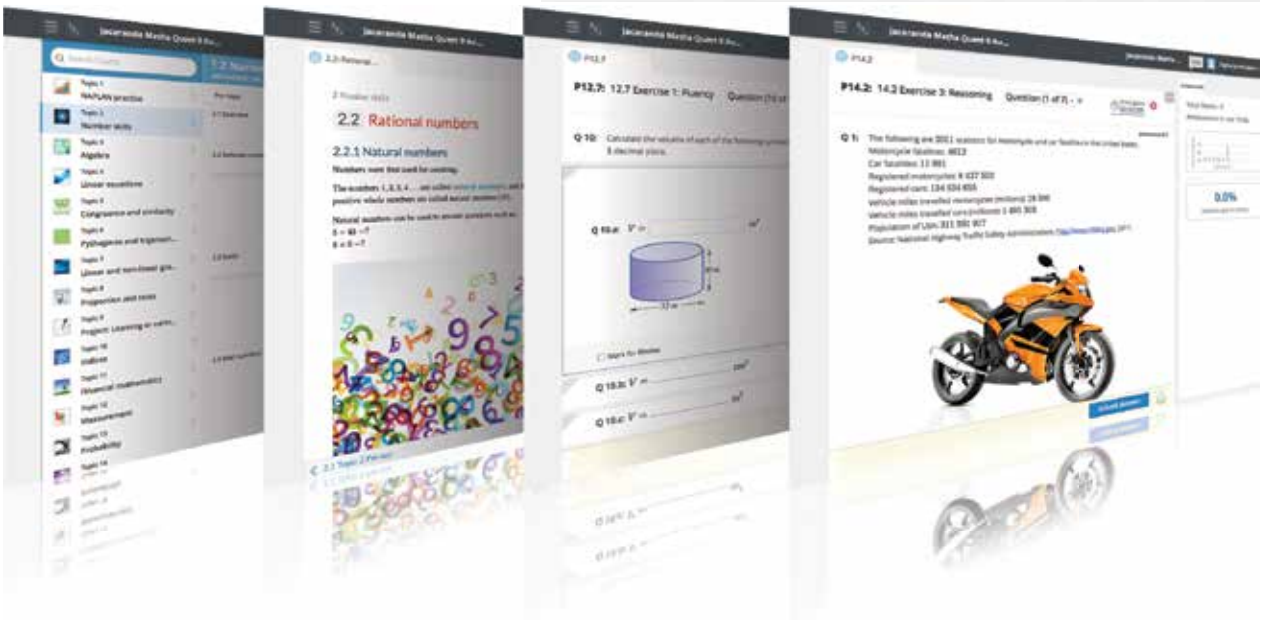
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Maths Quest contains a free activation code for *learnON* (please see instructions on the inside front cover), so students and teachers can take advantage of the benefits of both print and digital, and see how *learnON* enhances their digital learning and teaching journey.

learnON includes:

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- Collaboration in real time through class discussions
- Comprehensive summaries for each topic
- Code puzzles and dynamic interactivities help students engage with and work through challenging concepts
- Formative and summative assessments
- And much more …





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TOPIC 1
Numeracy 1

1.1 Overview



Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

1.1.1 Why learn this?

Our lives are interwoven with mathematics. Counting, measuring and pattern-making are all part of everyday life. We use numbers to mark significant events (such as birthdays) and for identification (such as passports and credit cards). We use numbers to describe ourselves (for example our height and weight). Shopping involves understanding numbers, and tallying scores in sports requires a comparison of numbers. You may not realise just how much you rely on numbers.

1.1.2 What do you know?

assessON

1. **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner, then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

LEARNING SEQUENCE

1.1 Overview
1.2 Set A
1.3 Set B

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: NAPLAN: Strategies and tips (eles-1688)

These questions provide the opportunity for students to use their numeracy skills in everyday situations.

1.2 Set A

assessON

1.2.1 Calculator allowed

1. Which of the following is the longest time?
A. 95 000 seconds **B.** 22 hours **C.** 1800 minutes **D.** 1 day

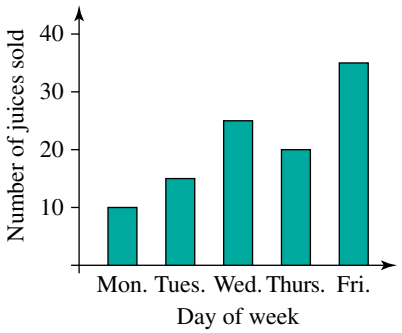
2. Anne bought 20 pens and Jo bought 30 pens. Each pen costs the same amount, and the pens cost $75 altogether. How much did Jo pay for her pens?

3. When a ball is dropped it bounces on the ground. The maximum height reached after each bounce is half the distance the ball drops. If a ball is dropped from a ladder 6 m above the ground, what is the total distance travelled by the time the ball hits the ground the third time?
A. 9 m **B.** 18 m **C.** 10.5 m **D.** 15 m

4. Last year at Fern Hill High School, there were 985 students. The student population has increased by 20% this year. How many students are at Fern Hill High School this year?
A. 1000 **B.** 1182 **C.** 1218 **D.** 1320

5. The length of a rectangle is 9 cm and its perimeter is 30 cm. What is the area of the rectangle?

6. The graph shows the number of juices sold each day of a particular week at a school. Calculate the mean daily number of juices sold at the school that week.



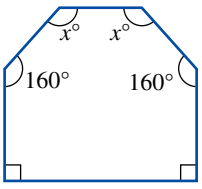
7. $(10^2 + 12 \div 3) \div (-8) = ?$
A. 13 **B.** 14
C. -13 **D.** -14

8. $\frac{3}{8}$ expressed as a percentage is:
A. 30.8% **B.** 13%
C. 37.5% **D.** 0.375%

9. Lena wants to make 2 chocolate cakes. The recipe for a chocolate cake is:
$1\frac{1}{2}$ cups self-raising flour
$\frac{1}{2}$ cup sugar
2 tablespoons cocoa
1 egg
125 grams butter.
How much of each ingredient will she need to use?
___ cups self-raising flour
___ cup sugar
___ tablespoons cocoa
___ eggs
___ grams butter



10. In the diagram shown, what is the value of x?
A. 110° **B.** 105° **C.** 120° **D.** 100°



11. Express in binary form.
113_{ten}

12. Jen bought a car on sale for 15% off the original price, which was $12 500. How much did she pay for the car?
A. $1875 **B.** $8505 **C.** $10 625 **D.** $10 510

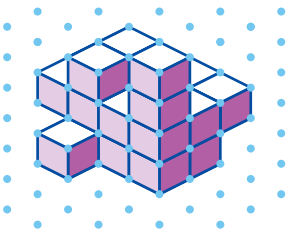
13. Connor climbs to the summit of a mountain 4250 m above sea level. Inga is scuba diving at the bottom of the ocean. The ocean floor is 27 metres below sea level. What is the vertical distance between Connor and Inga?



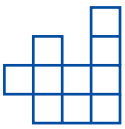
14. The ratio of cordial to water in Shinji's favourite drink is 2 : 5. How much water does he need to add to 80 mL of cordial to make his favourite drink?

A. 100 mL **B.** 200 mL **C.** 300 mL **D.** 400 mL

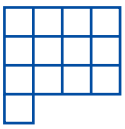
15. Which of the following represents the right side view of the 3-dimensional figure shown?



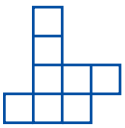
A.



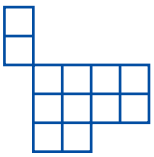
B.



C.

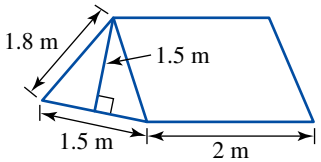


D.



16. A rectangular paddock is surrounded by a fence. The perimeter of the paddock is 870 m. If the length of the paddock is 65 m longer than its width, calculate the dimensions of the paddock.

17. Calculate the area of material required to make the tent shown, including the floor.



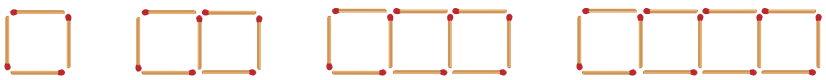
A. 12 m^2 **B.** 12.5 m^2

C. 12.45 m^2 **D.** 24.45 m^2

18. Write the rule for the following pattern made from sticks.

Let n = number of sticks.

Let s = number of squares.

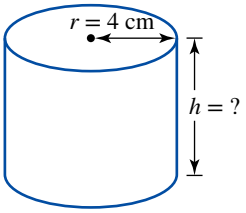


A. $n = 2s + 2$ **B.** $n = 3s + 1$ **C.** $n = 4s$ **D.** $n = s + 3$

19. If 1 teaspoon = 5 mL and 1 tablespoon = 20 mL, how many teaspoons are there in a tablespoon?

A. 3 **B.** 4 **C.** 5 **D.** 2

20. The total surface area (TSA) of a cylinder can be found using the formula $TSA = 2\pi r(r + h)$ where r is the radius and h is the height of the cylinder. Find h when $r = 4$ cm and the TSA is 377 cm^2.



21. A car travelled at 60 kilometres per hour for 2 hours and 15 minutes. How far did it travel?

A. 165 km **B.** 120 km **C.** 75 km **D.** 135 km

22. The table below shows the times for 4 students to run 100 m.

Student	Mitch	Josh	Ben	Liam
Time (s)	14.2	15.4	14.8	14.4

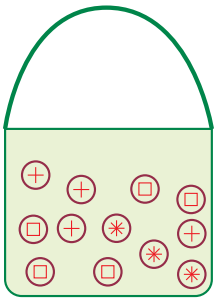
What is the average (mean) time the 4 students ran 100 m?

23. Which of the following is the smallest number?

A. $\sqrt[3]{6}$ **B.** 0.18 **C.** 0.5^2 **D.** $\frac{3}{8}$

24. Complete the following to make this equation true.
$13 \times (15 - 1) = 180 + ?$

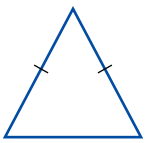
25. The diagram shows a bag of tokens. Aneko chooses a token from the bag without looking. What is the chance of Aneko choosing a star token?



A. $\frac{3}{10}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{2}$

26. Which of the following diagrams shows an equilateral triangle?

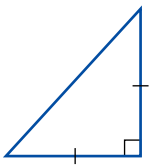
A.



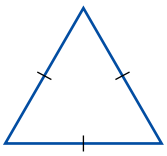
B.



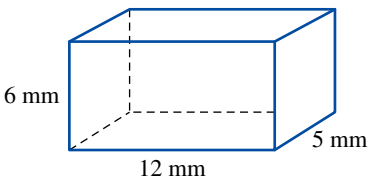
C.



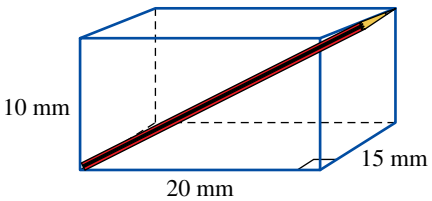
D.



27. What is the volume of this rectangular prism?



28. What is the longest pencil that can fit inside the box shown? (Round to 1 decimal place.)



A. 26.9 mm **B.** 25 mm
C. 22.4 mm **D.** 24.6 mm

29. Which coordinates will the graph of $y = -2x + 1$ pass through?

A. (5, 11) **B.** (5, −9)
C. (1, 3) **D.** (1, −3)

30. If $a = -3$, then $3a^3$ equals:

A. 27 **B.** −27 **C.** 81 **D.** −81

1.3 Set B

assessON

1.3.1 Non-calculator

1. How many faces does a rectangular prism have?

A. 4 **B.** 6
C. 8 **D.** 12

2. Oranges sell for $2.50 per kilogram. What is the total cost of 3 kg of oranges?

A. $2.50 **B.** $5.00
C. $4.50 **D.** $7.50



3. A fair, six-sided die is rolled. What is the chance of it landing on a 2?

A. $\frac{1}{6}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{2}$ **D.** 2

4. $7 + (2 \times 5) - 9 = ?$

A. 36 **B.** 17 **C.** 8 **D.** 7

5. Evaluate 1.3×0.6.

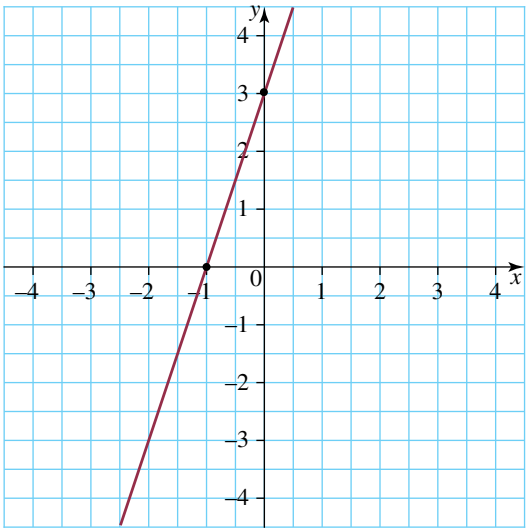
6. What is the equation of the graph shown?

A. $y = x + 3$

B. $y = x - 3$

C. $y = 3x + 3$

D. $y = 3x - 3$



7. Draw a linear graph using the table of values below.

x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4

8. Sivan recorded the colour of cars travelling along her street between 9 am and 10 am.

Colour	Blue	White	Red	Silver	Black	Gold
Number	1	6	2	5	3	3

What was the percentage of white cars?

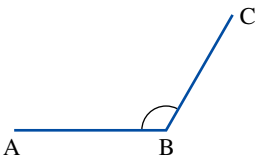
9. What type of angle is this?

A. Acute

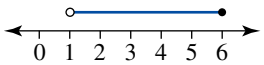
B. Obtuse

C. Reflex

D. Straight



10. What is the inequation that represents the interval shown on the number line?



A. $1 \leq x \leq 6$ **B.** $1 \leq x < 6$ **C.** $1 \geq x \geq 6$ **D.** $1 < x \leq 6$

11. A store is having a 25% off sale. What would be the reduced price of an item that was originally $120?



A. $40

B. $30

C. $100

D. $90

12. Two friends share 24 lollies. One friend gets twice as many as the other. How many lollies do they each receive?

A. 10 and 14 **B.** 8 and 16

C. 6 and 18 **D.** 4 and 20

13. Calculate the product of $\frac{3}{4}$ and $\frac{5}{6}$.

A. $1\frac{7}{12}$ **B.** $\frac{9}{10}$ **C.** $\frac{19}{12}$ **D.** $\frac{5}{8}$

14. The sum of 3 consecutive odd numbers is 21. Which equation best represents this situation?

A. $x + 3 = 21$ **B.** $3x = 21$ **C.** $3x + 6 = 21$ **D.** $6x + 3 = 21$

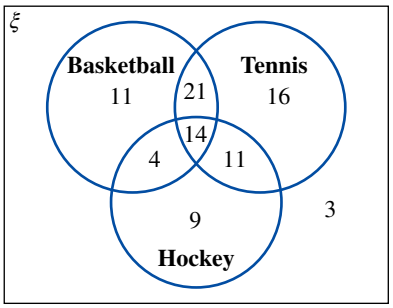
15. Find the mean and the median of the data below.

2, 1, 5, 5, 4, 0, 2, 5, 2, 6

A. Mean = 3, median = 2 **B.** Mean = 3.2, median = 4

C. Mean = 3, median = 3 **D.** Mean = 3.2, median = 3

16. The Venn diagram shows the number of students in Year 9 who play basketball, tennis and hockey. Find $n(\text{tennis} \cap \text{hockey})'$.



A. 61 **B.** 78

C. 70 **D.** 64

17. Evaluate $3^2 + 2^3 + 3^3$.

A. 40 **B.** 44

C. 36 **D.** 38

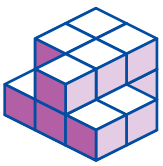
18. What is the missing number?

$5000 + 700 + ? + 6 = 5746$

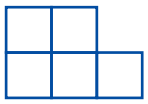
19. A piece of string 30 cm long forms a rectangle. If the length of the rectangle formed is 10 cm, what is its width?

A. 5 cm **B.** 10 cm **C.** 15 cm **D.** 20 cm

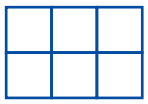
20. Which of the following is the top view of the 3-dimensional figure shown?



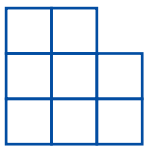
A.



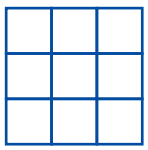
B.



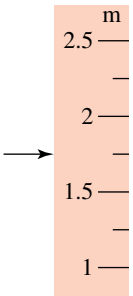
C.



D.



21. This ruler is used to measure the height of a teacher. How tall is the teacher?



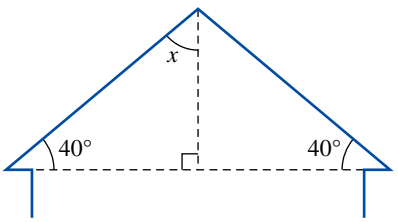
A. 1.6 m

B. 1.75 m

C. 1.075 m

D. 175 m

22. A roof makes an angle of 40° with the ceiling. What is the value of x?



A. 50°

B. 40°

C. 45°

D. 90°

23. The stem-and-leaf plot shows the number of goals scored in each netball match over a season. The range is:

Stem	*Leaf*
1	4 9
2	5 7 9
3	1 2 4 8
4	0 4
5	4 7
6	2

A. 48 **B.** 50

C. 14 **D.** 42

24. A circular spinner with 5 equal-sized sectors is labelled from 1 to 5. The spinner is spun. What is the probability of it landing on an odd number?

A. 3 **B.** $\frac{1}{2}$

C. $\frac{3}{5}$ **D.** $\frac{1}{3}$

25. Write an expression to represent the following:
p is added to the square of q and then the sum is divided by y.

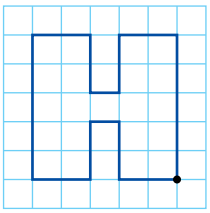
26. If one hour of parking costs $4.50, how much will it cost to park from 9 am until 2 pm?

A. $45 **B.** $22.50 **C.** $20 **D.** $16.50

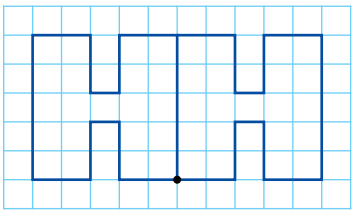
27. A map has a scale showing that 1 cm on the map represents an actual distance of 2 km. Write the appropriate scale ratio for this map.

A. 1 : 2 **B.** 1 : 200 **C.** 1 : 2000 **D.** 1 : 200 000

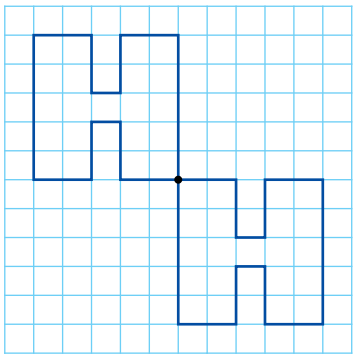
28. The shape shown is rotated 90° in a clockwise direction around the black dot. Which of the following choices represents the result?



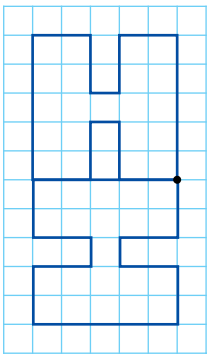
A.



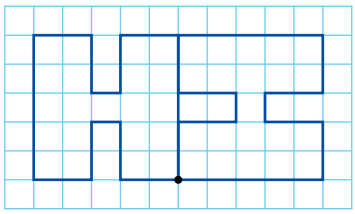
B.



C.

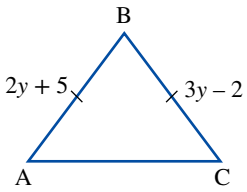


D.



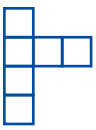
29. ABC is an isosceles triangle. What is the value of y?

A. 5 **B.** 7
C. 9 **D.** 11

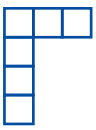


30. Which of the following is the net of a cube?

A.



B.



C.



D.



Answers

Topic 1 Numeracy 1

Exercise 1.2 Set A

1.2.1 Calculator allowed

1. C **2.** $45 **3.** D **4.** B

5. 54 cm^2 **6.** 21 **7.** C **8.** C

9. 3, 1, 4, 2, 250 **10.** A **11.** 1110001_2 **12.** C

13. 4277 m **14.** B **15.** C **16.** $l = 250$ m $w = 185$ m

17. C **18.** B **19.** B **20.** $h \approx 11$ cm

21. D **22.** 14.7 s **23.** B **24.** 2

25. C **26.** D **27.** 360 mm^3 **28.** A

29. B **30.** D

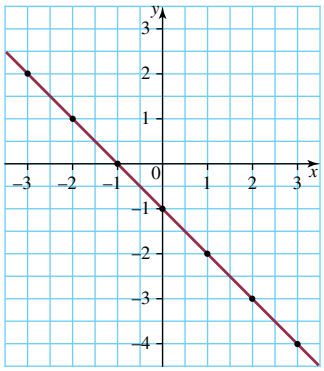
Exercise 1.3 Set B

1.3.1 Non-calculator

1. B **2.** D **3.** A **4.** C

5. 0.78 **6.** C

7.



8. 30% **9.** B **10.** D **11.** D

12. B **13.** D **14.** C **15.** D

16. D **17.** B **18.** 40 **19.** A

20. C **21.** B **22.** A **23.** A

24. C **25.** $\frac{q^2 + p}{y}$ **26.** B **27.** D

28. D **29.** B **30.** D

TOPIC 2
Number skills

2.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



2.1.1 Why learn this?

Mathematics has been a part of every civilisation throughout history. From the most primitive times people have needed numbers for counting and calculating. Our modern world is linked by computers, which rely heavily on numbers to store, find and track information.

2.1.2 What do you know?

assessON

1. **THINK** List what you know about numbers. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner, then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of real numbers.

LEARNING SEQUENCE

2.1 Overview
2.2 Rational numbers
2.3 Surds
2.4 Real numbers
2.5 Scientific notation
2.6 Review

2.2 Rational numbers

2.2.1 Natural numbers

- Numbers were first used for counting.
- The numbers $1, 2, 3, 4 \ldots$ are called **natural numbers**, and the set of natural numbers is called N. That is, positive whole numbers are called natural numbers (N).
- Natural numbers can be used to answer questions such as:
  $5 + 43 = ?$
  $8 \times 6 = ?$

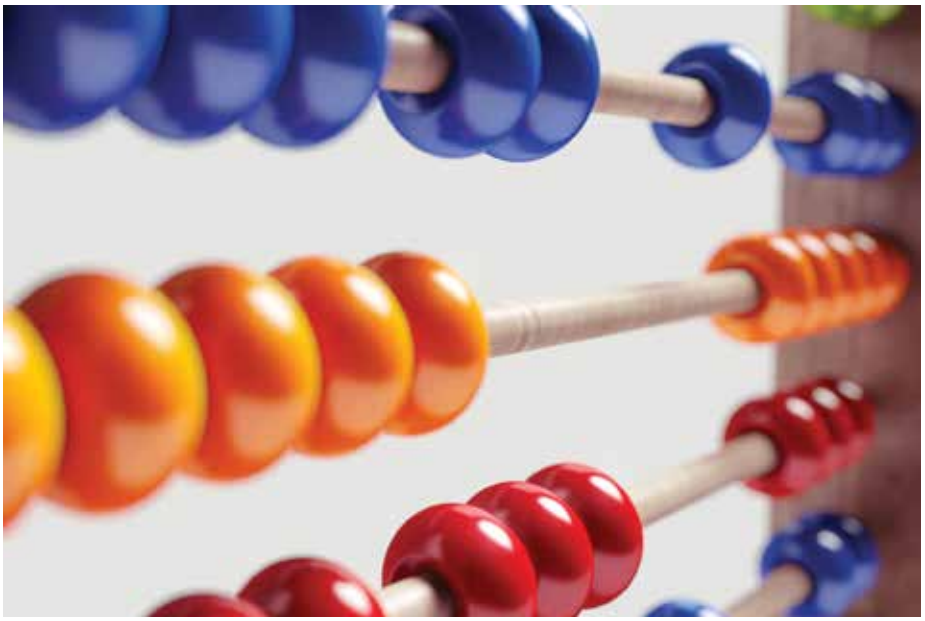


2.2.2 Integers

- What is the value of $3 - 8$?
- The answer to this question is not a natural number, so zero and the negative numbers were defined and a name was given to this new list of numbers.
- The numbers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$ are called whole numbers or **integers**, and the set of integers is called Z.
- Can you suggest any reasons why we need negative numbers?

2.2.3 Rational numbers

- What is the value of $4 \div 7$?
  The answer to this question is not an integer, but a fraction (or ratio) written as $\frac{4}{7}$.
- Fractions such as $\frac{4}{7}$ or $\frac{-11}{3}$ in which the denominator and numerator are both integers are called **rational numbers**. The set of rational numbers is called Q.
- Can you suggest any reasons why we need fractions?



- *Note:* The **quotient** $9 \div 0$ has no answer; therefore, numbers such as $\frac{9}{0}$ do not exist. They are said to be 'undefined'. Because they 'do not exist', they are not rational numbers.

WORKED EXAMPLE 1

By writing each of the following in fraction form, show that the numbers are rational.
a 7 **b** -11 **c** 0 **d** $-4\frac{2}{5}$ **e** 1.2

THINK	WRITE
Each number must be written as a fraction using integers.	
a The number 7 has to be written in fraction form. To write a number as a fraction, it must be written with a numerator and denominator. For a whole number, the denominator is 1.	**a** $7 = \frac{7}{1}$ is rational.
b The number -11 has to be written in fraction form. To write a number as a fraction, it must be written with a numerator and denominator. For a whole number, the denominator is 1.	**b** $-11 = \frac{-11}{1}$ is rational.
c The number 0 has to be written in fraction form. To write a number as a fraction, it must be written with a numerator and denominator. For a whole number, the denominator is 1.	**c** $0 = \frac{0}{1}$ is rational.
d Change $-4\frac{2}{5}$ to an improper fraction.	**d** $-4\frac{2}{5} = \frac{-22}{5}$ is rational.
e The number 1.2 can be expressed as $1 + 0.2$. This can then be expressed as $1 + \frac{2}{10}$. Write this as an improper fraction.	**e** $1.2 = \frac{12}{10}$ is rational.

2.2.4 Rational numbers written as decimals

- When a rational number is written as a decimal there are two possibilities.

  1. The decimal terminates, e.g. $\frac{5}{4} = 1.25$.

  2. The decimal repeats, or recurs, e.g. $\frac{7}{6} = 1.1666\ldots$

  For $1.1666\ldots$ the 6 is a repeating digit. This number, $\frac{7}{6}$, is called a **recurring decimal**, and it can also be written as $1.1\dot{6}$.

  $1.\dot{6}1\dot{5}$ or $1.\overline{615}$ means $1.615\,615\,615\ldots$ (The digits 615 in the decimal repeat.)

  $1.6\dot{1}\dot{5}$ or $1.6\overline{15}$ means $1.615\,151\,515\ldots$ (The digits 15 in the decimal repeat.)

  $1.61\dot{5}$ means $1.615555\ldots$ (Only the digit 5 in the decimal repeats.)

WORKED EXAMPLE 2

Write the first 8 digits of each of the following recurring decimals.

a $3.\dot{0}\dot{2}$ **b** $47.\dot{1}$ **c** $11.5\dot{4}\dot{9}$

THINK	WRITE
a In $3.\dot{0}\dot{2}$, 02 recurs.	**a** 3.0202020
b In $47.\dot{1}$, 1 recurs.	**b** 47.111111
c In $11.5\dot{4}\dot{9}$, 49 recurs.	**c** 11.549494

WORKED EXAMPLE 3

Write each fraction as a recurring decimal using dot notation. (Use a calculator for the division.)

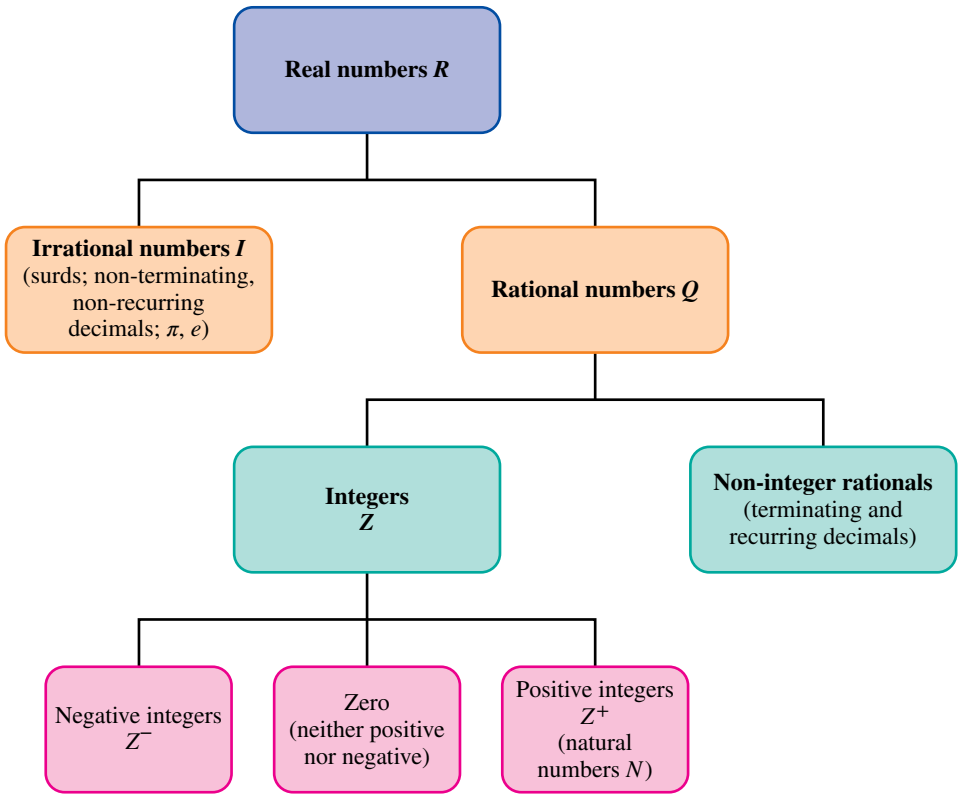
a $\frac{5}{6}$ **b** $\frac{57}{99}$ **c** $\frac{25}{11}$ **d** $\frac{4}{7}$

THINK	WRITE
a $5 \div 6 = 0.8333333$ 3 recurs — the dot goes above the 3.	**a** $\frac{5}{6} = 0.8\dot{3}$
b $57 \div 99 = 0.5757575757$ 57 recurs — dots go above the 5 and the 7.	**b** $\frac{57}{99} = 0.\dot{5}\dot{7}$
c $25 \div 11 = 2.27272727$ 27 recurs — dots go above the 2 and the 7.	**c** $\frac{25}{11} = 2.\dot{2}\dot{7}$
d $4 \div 7 = 0.571428571$ It looks as though 571428 will recur — dots go above the 5 and the 8.	**d** $\frac{4}{7} = 0.\dot{5}71428\dot{8}$

2.2.5 Is every decimal a rational number?

- 1.237 is rational, because it is a **terminating decimal**. It is easy to show that $1.237 = \frac{1237}{1000}$.
- $0.\dot{8}\dot{6}$, or $0.868\,686\ldots$, is rational because it is a recurring decimal. It can be shown that $0.\dot{8}\dot{6} = \frac{86}{99}$.
- Decimals that do not terminate and also do not recur are not rational. They cannot be written as integer fractions or ratios, and are called **irrational numbers**.

2.2.6 The number system



Note: A real number is any number that lies on the number line. Further explanation can be found in the next section.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Operations with directed numbers (doc-6100)

Exercise 2.2 Rational numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–l, 2a–f, 3a–e, 4–7	Questions: 1d–l, 2c–j, 3c–g, 4–11	Questions: 1e–l, 2f–j, 3e–j, 4–11

Individual pathway interactivity: int-4476

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Show that the following numbers are rational by writing each of them in fraction form.

a. 15 **b.** -8 **c.** $2\frac{2}{3}$ **d.** $-5\frac{1}{8}$

e. $\sqrt{16}$ **f.** $7\frac{3}{10}$ **g.** 0.002 **h.** 87.2

i. 0 **j.** 1.56 **k.** 3.612 **l.** -0.08

2. WE2 Write the first 8 digits of each of the following recurring decimals.

a. $0.\dot{5}$ **b.** $0.\dot{5}\dot{1}$

c. $0.5\dot{1}$ **d.** $6.0\dot{3}\dot{1}$

e. $5.\dot{1}8\dot{3}$ **f.** $-7.02\dot{4}$

g. $8.\dot{9}12\dot{4}$ **h.** $5.12\dot{3}\dot{4}$

i. $5.\dot{1}23\dot{4}$ **j.** $3.0\dot{0}\dot{2}$

3. WE3 Write the following fractions as recurring decimals using dot notation. (Use a calculator.)

a. $\frac{5}{9}$ **b.** $\frac{3}{11}$

c. $2\frac{2}{11}$ **d.** $\frac{3}{7}$

e. $\frac{-173}{99}$ **f.** $\frac{73}{990}$

g. $-3\frac{5}{6}$ **h.** $\frac{7}{15}$

i. $\frac{46}{99}$ **j.** $\frac{46}{990}$

Understanding

4. From the following list of numbers:

$-3, \frac{-3}{7}, 0, 2.3, 2.\dot{3}, 2\frac{3}{5}, 15$

a. write down the natural numbers

b. write down the integers

c. write down the rational numbers.

5. Write these numbers in order from smallest to largest.

$2.\dot{1}, 2.\dot{1}\dot{2}, 2.\dot{1}2\dot{1}, 2.1\dot{2}\dot{1}, 2.1\dot{2}$

Reasoning

6. Explain why all integers are rational numbers.

7. Are all fractions in which both the numerator and denominator are integers rational? Explain why or why not.

Problem solving

8. a. Using the fraction $\frac{a}{b}$, where a and b are natural numbers, write 3 recurring decimals in fractional form using the smallest natural numbers possible.

b. What was the largest natural number you used?

9. Write 2 fractions that have the following number of repeating digits in their decimal forms.

a. 1 repeating digit **b.** 2 repeating digits

c. 3 repeating digits **d.** 4 repeating digits

10. If a $2 coin weighs 6 g, a $1 coin weighs 9 g, a 50c coin weighs 15 g, a 20c coin weighs 12 g, a 10c coin weighs 5 g and a 5c coin weighs 3 g, explain what the maximum value of the coins would be if a bundle of them weighed exactly 10 kg.

11. A Year 9 student consumes 2 bottles of water every day, except on every 5th day when she only has 1. Calculate her *annual* bottled water consumption.

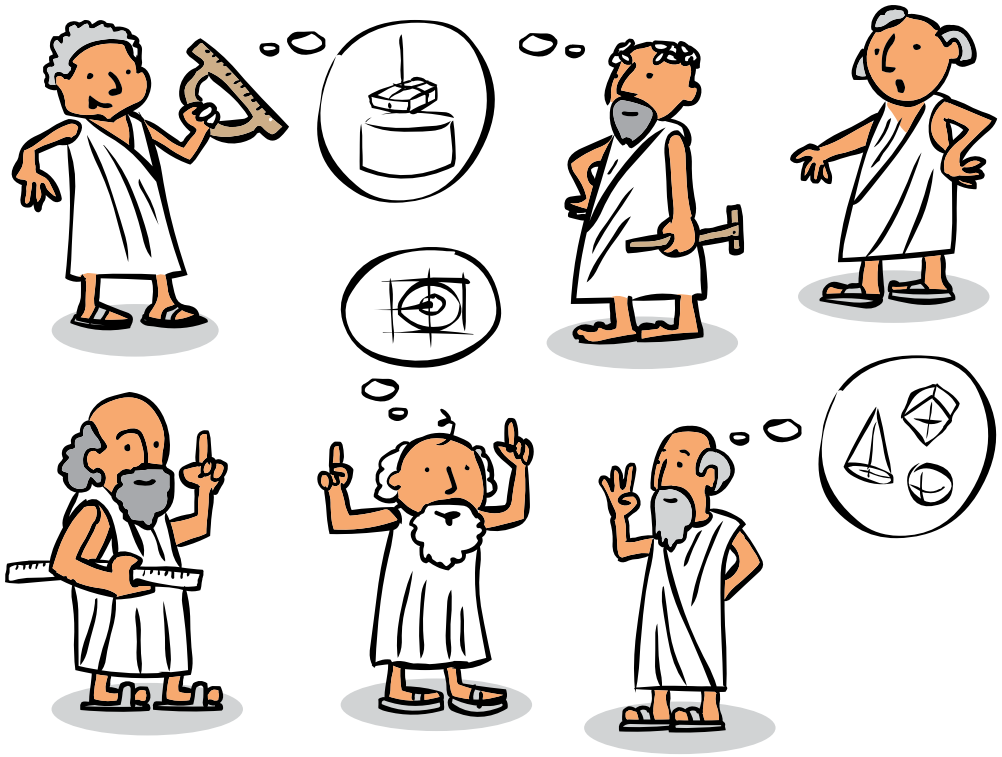
Reflection

How many recurring digits will there be for $\frac{1}{13}$?

2.3 Surds

2.3.1 Rational and irrational numbers

- Ancient mathematicians were shocked to find that rational numbers could not be used to label every point on the number line. In other words they discovered lengths that could not be expressed as fractions. These numbers are called irrational numbers.



- A number is irrational if it is not rational; that is, if it cannot be written as a fraction, nor as a terminating or recurring decimal.
- Irrational numbers are denoted by the letter I.

2.3.2 Roots

- The nth root of any positive number can be found. That is, $\sqrt[n]{b} = a$ or $a^n = b$.
  For example,
  $\sqrt{25} = 5$ because $25 = 5 \times 5$.
  Note: The 2 is usually not written in square roots, for example $\sqrt{25} = \sqrt[2]{25}$.
  $\sqrt[3]{64} = 4$ because $64 = 4 \times 4 \times 4$.

WORKED EXAMPLE 4 — TI | CASIO

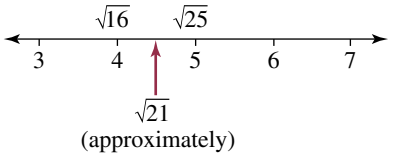
Evaluate each of the following.

a $\sqrt{16}$ **b** $\sqrt[3]{27}$ **c** $\sqrt[5]{32}$

THINK	WRITE
a Determine the number that when multiplied by itself gives $16 : 16 = 4 \times 4$.	**a** $\sqrt{16} = 4$
b Determine the number that when multiplied by itself three times gives $27 : 27 = 3 \times 3 \times 3$.	**b** $\sqrt[3]{27} = 3$
c Determine the number that when multiplied by itself five times gives $32 : 32 = 2 \times 2 \times 2 \times 2 \times 2$.	**c** $\sqrt[5]{32} = 2$

2.3.3 Surds

- When the square root of a number is an irrational number, it is called a **surd**. For example, $\sqrt{10}$ cannot be written as a fraction, nor as a recurring or terminating decimal. It is therefore irrational and is called a surd.
  $\sqrt{10} \approx 3.162\,277\,660\,17\ldots$
- The value of a surd can be approximated using a number line.
  For example, $\sqrt{21}$ will lie between 4 and 5, because $\sqrt{16} = 4$ and $\sqrt{25} = 5$. This can be shown on a number line.



WORKED EXAMPLE 5

Place $\sqrt{34}$ on a number line.

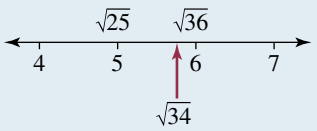
THINK

1 The square number that is smaller than 34 is 25. The square number that is larger than 34 is 36. Write the numbers just below and just above $\sqrt{34}$.

2 Draw a number line to show the approximate position of $\sqrt{34}$.

WRITE

$\sqrt{34}$ lies between $\sqrt{25}$ and $\sqrt{36}$.



- *Note:* Negative numbers do not have square roots within the set of real numbers. However, in the 18th century mathematicians described the square root of a negative number as an 'imaginary number'. Imaginary numbers and real numbers together form the set of complex numbers. Imaginary numbers have many uses in higher level mathematics used in science and engineering, but they are beyond the scope of this current course.

WORKED EXAMPLE 6

Which of the following are surds?

a $\sqrt{0}$ **b** $\sqrt{20}$ **c** $-\sqrt{9}$ **d** $\sqrt[3]{6}$

THINK

a $\sqrt{0} = 0$. This is a rational number and therefore not a surd.

b $\sqrt{20} \approx 4.472\,135\,955\ldots$ It is an irrational number and therefore a surd.

c $-\sqrt{9} = -3$. This is a rational number and therefore not a surd.

d $\sqrt[3]{6} \approx 1.817\,120\,592\,83\ldots$ It is an irrational number and therefore a surd.

WRITE

a $\sqrt{0} = 0$, which is not a surd.

b $\sqrt{20}$ is a surd.

c $-\sqrt{9} = -3$, which is not a surd.

d $\sqrt[3]{6}$ is a surd.

2.3.4 Multiplying and dividing surds

- Consider that $3 = \sqrt{9}$, $2 = \sqrt{4}$, and $6 = \sqrt{36}$.
  The multiplication $3 \times 2 = 6$ could be written as
  $\sqrt{9} \times \sqrt{4} = \sqrt{36}$.
- In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.
  For example, $\sqrt{7} \times \sqrt{3} = \sqrt{21}$.
- Similarly, $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.
  For example, $\sqrt{18} \div \sqrt{3} = \sqrt{6}$.

WORKED EXAMPLE 7

Evaluate the following, leaving your answer in surd form.
a $\sqrt{7} \times \sqrt{2}$ **b** $5 \times \sqrt{3}$ **c** $\sqrt{5} \times \sqrt{5}$ **d** $-2\sqrt{3} \times 4\sqrt{5}$

THINK	WRITE
a Apply the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.	**a** $\sqrt{7} \times \sqrt{2} = \sqrt{14}$
b Only $\sqrt{3}$ is a surd. It is multiplied by 5, which is not a surd.	**b** $5 \times \sqrt{3} = 5\sqrt{3}$
c Apply the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.	**c** $\sqrt{5} \times \sqrt{5} = \sqrt{25}$ $= 5$
d Multiply the whole numbers by each other. Multiply the surds by each other.	**d** $-2\sqrt{3} \times 4\sqrt{5} = -2 \times 4 \times \sqrt{3} \times \sqrt{5}$ $= -8 \times \sqrt{15}$. $= -8\sqrt{15}$.

WORKED EXAMPLE 8

Evaluate the following, leaving your answer in surd form.
a $\frac{\sqrt{10}}{\sqrt{5}}$ **b** $\sqrt{\frac{10}{5}}$ **c** $\frac{-6\sqrt{8}}{4\sqrt{4}}$ **d** $\frac{\sqrt{20}}{\sqrt{5}}$ **e** $\frac{5}{\sqrt{5}}$

THINK	WRITE
a Apply the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.	**a** $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}}$ $= \sqrt{2}$
b Simplify the fraction first.	**b** $\sqrt{\frac{10}{5}} = \sqrt{2}$
c Simplify the whole numbers. Then apply the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.	**c** $\frac{-6\sqrt{8}}{4\sqrt{4}} = \frac{-3\sqrt{2}}{2}$
d Apply the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.	**d** $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4}$ $= 2$
e Rewrite the numerator as the product of two surds and then simplify.	**e** $\frac{5}{\sqrt{5}} = \frac{\sqrt{5} \times \sqrt{5}}{\sqrt{5}}$ $= \sqrt{5}$

2.3.5 Simplifying surds

- Just as a rational number can be written many different ways (e.g. $\frac{1}{2} = \frac{5}{10} = \frac{7}{14}$), so can a surd, and it is expected that surds should normally be written in simplest form.
- A surd is in simplest form when the number inside the radical sign has the smallest possible value.
- Note that $\sqrt{24}$ can be factorised several ways. For example:
  $\sqrt{24} = \sqrt{2} \times \sqrt{12}$
  $\sqrt{24} = \sqrt{3} \times \sqrt{8}$
  $\sqrt{24} = \sqrt{4} \times \sqrt{6}$
  In the last case $\sqrt{4} = 2$
  $\sqrt{24} = 2 \times \sqrt{6}$
  $= 2\sqrt{6}$
  $2\sqrt{6}$ is equal to $\sqrt{24}$, and $2\sqrt{6}$ is written in simplest form.
- To simplify a surd you must find a factor that is also a perfect square: 4, 9, 16, 25, 36, 49 and so on.

WORKED EXAMPLE 9 — TI | CASIO

Simplify the following surds.

a $\sqrt{18}$ **b** $6\sqrt{20}$

THINK	WRITE
a 1 Rewrite 18 as the product of two numbers, one of which is square.	**a** $\sqrt{18} = \sqrt{9} \times \sqrt{2}$
2 Simplify.	$= 3 \times \sqrt{2}$ $= 3\sqrt{2}$
b 1 Rewrite 20 as the product of two numbers, one of which is square.	**b** $6\sqrt{20} = 6 \times \sqrt{4} \times \sqrt{5}$
2 Simplify.	$= 6 \times 2 \times \sqrt{5}$ $= 12\sqrt{5}$

- The surd $\sqrt{22}$ cannot be simplified because 22 has no perfect square factors.
- Surds can be simplified in more than one step.

$$\begin{aligned}\sqrt{72} &= \sqrt{4} \times \sqrt{18} \\ &= 2\sqrt{18} \\ &= 2 \times \sqrt{9} \times \sqrt{2} \\ &= 2 \times 3\sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

2.3.6 Entire surds

- The surd $\sqrt{45}$, when simplified, is written as $3\sqrt{5}$.
  The surd $\sqrt{45}$ is called an entire surd, because it is written entirely inside the radical sign, whereas $3\sqrt{5}$ is not.
- Writing a surd as an entire surd reverses the process of simplification.

WORKED EXAMPLE 10

Write $3\sqrt{7}$ as an entire surd.

THINK	WRITE
1 In order to place the 3 inside the radical sign it has to be written as $\sqrt{9}$.	$3\sqrt{7} = \sqrt{9} \times \sqrt{7}$
2 Apply the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.	$= \sqrt{63}$

WORKED EXAMPLE 11

Which number is larger, $3\sqrt{5}$ or $5\sqrt{3}$?

THINK	WRITE
1 Write $3\sqrt{5}$ as an entire surd.	$3\sqrt{5} = \sqrt{9} \times \sqrt{5}$ $= \sqrt{45}$
2 Write $5\sqrt{3}$ as an entire surd.	$5\sqrt{3} = \sqrt{25} \times \sqrt{3}$ $= \sqrt{75}$
3 Compare the values.	$\sqrt{75} > \sqrt{45}$
4 Write your answer.	$5\sqrt{3}$ is the larger number.

2.3.7 Addition and subtraction of surds

- Surds can be added or subtracted if they are like terms.

WORKED EXAMPLE 12

TI | CASIO

Simplify each of the following.

a $6\sqrt{3} + 2\sqrt{3} + 4\sqrt{5} - 5\sqrt{5}$ **b** $3\sqrt{2} - 5 + 4\sqrt{2} + 9$

THINK	WRITE
a Collect the $\sqrt{3}$s and the $\sqrt{5}$s.	**a** $6\sqrt{3} + 2\sqrt{3} + 4\sqrt{5} - 5\sqrt{5}$ $= 8\sqrt{3} - \sqrt{5}$
b Collect the like terms and simplify.	**b** $3\sqrt{2} - 5 + 4\sqrt{2} + 9$ $= 3\sqrt{2} + 4\sqrt{2} - 5 + 9$ $= 7\sqrt{2} + 4$

- Surds should be simplified before adding or subtracting like terms.

WORKED EXAMPLE 13

TI | CASIO

Simplify $5\sqrt{75} - 6\sqrt{12} + \sqrt{8} - 4\sqrt{3}$.

THINK	WRITE
1 Simplify $5\sqrt{75}$.	$5\sqrt{75} = 5 \times \sqrt{25} \times \sqrt{3}$ $= 5 \times 5 \times \sqrt{3}$ $= 25\sqrt{3}$

2 Simplify $6\sqrt{12}$.

$$\begin{aligned} 6\sqrt{12} &= 6 \times \sqrt{4} \times \sqrt{3} \\ &= 6 \times 2 \times \sqrt{3} \\ &= 12\sqrt{3} \end{aligned}$$

3 Simplify $\sqrt{8}$.

$$\begin{aligned} \sqrt{8} &= \sqrt{4} \times \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

4 Rewrite the original expression and simplify by adding like terms.

$$\begin{aligned} &5\sqrt{75} - 6\sqrt{12} + \sqrt{8} - 4\sqrt{3} \\ &= 25\sqrt{3} - 12\sqrt{3} + 2\sqrt{2} - 4\sqrt{3} \\ &= 9\sqrt{3} + 2\sqrt{2} \end{aligned}$$

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Try out this interactivity: Balance surds (int-2762)

Complete this digital doc: SkillSHEET: Calculating the square root of a number (doc-6101)

Complete this digital doc: SkillSHEET: Using a calculator to evaluate numbers in index form (doc-6102)

Complete this digital doc: SkillSHEET: Simplifying surds (doc-10813)

Complete this digital doc: WorkSHEET: Number skills (doc-6119)

Exercise 2.3 Surds

assessON

Individual pathways

PRACTISE
Questions:
1–9, 11a–f, 12a–f, 14

CONSOLIDATE
Questions:
1–10, 11e–j, 12e–h, 13–15

MASTER
Questions:
1k–n, 2e–l, 3i–p, 4m–p, 5l–p, 6–10, 11g–l, 12e–j, 13, 15–21

Individual pathway interactivity: int-4477

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Fluency

1. WE4 Write down the square roots of each of the following.

a. 1 **b.** 4 **c.** 0 **d.** $\frac{1}{9}$ **e.** $1\frac{9}{16}$

f. 0.16 **g.** 400 **h.** 10 000 **i.** $\frac{4}{25}$ **j.** 1.44

k. 20.25 **l.** 1 000 000 **m.** 0.0009 **n.** 256

2. WE5 Write down the value of each of the following.

a. $\sqrt{81}$ **b.** $-\sqrt{81}$ **c.** $\sqrt{121}$ **d.** $-\sqrt{441}$

e. $\sqrt[3]{8}$ **f.** $\sqrt[3]{64}$ **g.** $\sqrt[3]{343}$ **h.** $\sqrt[4]{81}$

i. $\sqrt[5]{1024}$ **j.** $\sqrt[3]{125}$ **k.** $-\sqrt{49}$ **l.** $\sqrt[3]{-27}$

3. WE6 Which of the following are surds?

a. $\sqrt[3]{0}$ **b.** $\sqrt{10}$ **c.** $-\sqrt{36}$ **d.** $\sqrt[3]{9}$

e. $-\sqrt[3]{216}$ **f.** $\sqrt[3]{-216}$ **g.** $-\sqrt{2}$ **h.** $1+\sqrt{2}$

i. 1.32 **j.** $1.\dot{3}\dot{2}$ **k.** $\sqrt[4]{64}$ **l.** $1.752\,16$

m. $\sqrt{7}+\sqrt{2}$ **n.** $\sqrt{9}+\sqrt{49}$ **o.** $\sqrt{4}-\sqrt{3}$ **p.** $\sqrt{3^2}$

4. WE7 Simplify each of the following.

a. $\sqrt{3}\times\sqrt{7}$ **b.** $-\sqrt{3}\times\sqrt{7}$ **c.** $2\times\sqrt{6}$ **d.** $2\times 3\sqrt{7}$

e. $2\sqrt{7}\times 5\sqrt{2}$ **f.** $-3\sqrt{2}\times -5\sqrt{5}$ **g.** $3\sqrt{7}\times 4$ **h.** $\sqrt{7}\times 9$

i. $-2\sqrt{5}\times -11\sqrt{2}$ **j.** $2\sqrt{3}\times 11$ **k.** $\sqrt{3}\times\sqrt{3}$ **l.** $2\sqrt{3}\times 4\sqrt{3}$

m. $\sqrt{6}\times\sqrt{6}$ **n.** $\sqrt{11}\times\sqrt{11}$ **o.** $\sqrt{51}\times\sqrt{51}$ **p.** $\sqrt{15}\times 2\sqrt{15}$

5. WE8 Simplify each of the following.

a. $\sqrt{\dfrac{12}{4}}$ **b.** $-\sqrt{\dfrac{10}{5}}$ **c.** $\dfrac{\sqrt{18}}{\sqrt{3}}$ **d.** $\dfrac{-\sqrt{15}}{-\sqrt{3}}$

e. $\dfrac{15\sqrt{6}}{5\sqrt{2}}$ **f.** $\dfrac{15\sqrt{6}}{10}$ **g.** $\dfrac{15\sqrt{6}}{\sqrt{3}}$ **h.** $\dfrac{5}{3}\sqrt{\dfrac{15}{3}}$

i. $\dfrac{-10\sqrt{10}}{5\sqrt{2}}$ **j.** $\dfrac{\sqrt{9}}{\sqrt{3}}$ **k.** $\dfrac{3}{\sqrt{3}}$ **l.** $\dfrac{7}{\sqrt{7}}$

m. $\dfrac{35}{\sqrt{35}}$ **n.** $\dfrac{\sqrt{28}}{\sqrt{7}}$ **o.** $\dfrac{-\sqrt{18}}{\sqrt{2}}$ **p.** $\dfrac{\sqrt{45}}{\sqrt{5}}$

6. WE9 Simplify each of the following.

a. $\sqrt{20}$ **b.** $\sqrt{8}$ **c.** $\sqrt{18}$ **d.** $\sqrt{49}$

e. $\sqrt{30}$ **f.** $\sqrt{50}$ **g.** $\sqrt{28}$ **h.** $\sqrt{108}$

i. $\sqrt{288}$ **j.** $\sqrt{48}$ **k.** $\sqrt{500}$ **l.** $\sqrt{162}$

7. WE9 Simplify each of the following.

a. $2\sqrt{8}$ **b.** $5\sqrt{27}$ **c.** $6\sqrt{64}$ **d.** $7\sqrt{50}$ **e.** $10\sqrt{24}$

f. $5\sqrt{12}$ **g.** $4\sqrt{42}$ **h.** $12\sqrt{72}$ **i.** $9\sqrt{45}$ **j.** $12\sqrt{242}$

8. WE10 Write each of the following in the form $\sqrt{a}$; that is, as an entire surd.

a. $2\sqrt{3}$ **b.** $5\sqrt{7}$ **c.** $6\sqrt{3}$ **d.** $4\sqrt{5}$ **e.** $8\sqrt{6}$

f. $3\sqrt{10}$ **g.** $4\sqrt{2}$ **h.** $12\sqrt{5}$ **i.** $10\sqrt{6}$ **j.** $13\sqrt{2}$

9. MC **a.** $\sqrt{1000}$ is equal to:

A. 31.6228 **B.** $50\sqrt{2}$ **C.** $50\sqrt{10}$ **D.** $10\sqrt{10}$

b. $\sqrt{80}$ in simplest form is equal to:

A. $4\sqrt{5}$ **B.** $2\sqrt{20}$ **C.** $8\sqrt{10}$ **D.** $5\sqrt{16}$

c. Which of the following surds is in simplest form?

A. $\sqrt{60}$ **B.** $\sqrt{147}$ **C.** $\sqrt{105}$ **D.** $\sqrt{117}$

d. Which of the following surds is not in simplest form?

A. $\sqrt{102}$ **B.** $\sqrt{110}$ **C.** $\sqrt{116}$ **D.** $\sqrt{118}$

e. $6\sqrt{5}$ is equal to:

A. $\sqrt{900}$ **B.** $\sqrt{30}$ **C.** $\sqrt{150}$ **D.** $\sqrt{180}$

f. Which one of the following is not equal to the rest?

A. $\sqrt{128}$ **B.** $2\sqrt{32}$ **C.** $8\sqrt{2}$ **D.** $64\sqrt{2}$

g. Which one of the following is not equal to the rest?

A. $4\sqrt{4}$ **B.** $2\sqrt{16}$ **C.** 8 **D.** 16

h. $5\sqrt{48}$ is equal to:

A. $80\sqrt{3}$ **B.** $20\sqrt{3}$ **C.** $9\sqrt{3}$ **D.** $21\sqrt{3}$

10. Challenge: Reduce each of the following to simplest form.

a. $\sqrt{675}$ **b.** $\sqrt{1805}$ **c.** $\sqrt{1792}$ **d.** $\sqrt{578}$

e. $\sqrt{a^2c}$ **f.** $\sqrt{bd^4}$ **g.** $\sqrt{h^2jk^2}$ **h.** $\sqrt{f^3}$

11. WE12 Simplify each of the following.

a. $6\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$

b. $4\sqrt{5} - 6\sqrt{5} - 2\sqrt{5}$

c. $-3\sqrt{3} - 7\sqrt{3} + 4\sqrt{3}$

d. $-9\sqrt{6} + 6\sqrt{6} + 3\sqrt{6}$

e. $10\sqrt{11} - 6\sqrt{11} + \sqrt{11}$

f. $\sqrt{7} + \sqrt{7}$

g. $4\sqrt{2} + 6\sqrt{2} + 5\sqrt{3} + 2\sqrt{3}$

h. $10\sqrt{5} - 2\sqrt{5} + 8\sqrt{6} - 7\sqrt{6}$

i. $5\sqrt{10} + 2\sqrt{3} + 3\sqrt{10} + 5\sqrt{3}$

j. $12\sqrt{2} - 3\sqrt{5} + 4\sqrt{2} - 8\sqrt{5}$

k. $6\sqrt{6} + \sqrt{2} - 4\sqrt{6} - \sqrt{2}$

l. $16\sqrt{5} + 8 + 7 - 11\sqrt{5}$

12. WE13 Simplify each of the following.

a. $\sqrt{8} + \sqrt{18} - \sqrt{32}$

b. $\sqrt{45} - \sqrt{80} + \sqrt{5}$

c. $-\sqrt{12} + \sqrt{75} - \sqrt{192}$

d. $\sqrt{7} + \sqrt{28} - \sqrt{343}$

e. $\sqrt{24} + \sqrt{180} + \sqrt{54}$

f. $\sqrt{12} + \sqrt{20} - \sqrt{125}$

g. $2\sqrt{24} + 3\sqrt{20} - 7\sqrt{8}$

h. $3\sqrt{45} + 2\sqrt{12} + 5\sqrt{80} + 3\sqrt{108}$

i. $6\sqrt{44} + 4\sqrt{120} - \sqrt{99} - 3\sqrt{270}$

j. $2\sqrt{32} - 5\sqrt{45} - 4\sqrt{180} + 10\sqrt{8}$

13. MC **a.** $\sqrt{2} + 6\sqrt{3} - 5\sqrt{2} - 4\sqrt{3}$ is equal to:

A. $-5\sqrt{2} + 2\sqrt{3}$ **B.** $-3\sqrt{2} + 23$ **C.** $6\sqrt{2} + 2\sqrt{3}$ **D.** $-4\sqrt{2} + 2\sqrt{3}$

b. $6 - 5\sqrt{6} + 4\sqrt{6} - 8$ is equal to:

A. $-2 - \sqrt{6}$ **B.** $14 - \sqrt{6}$ **C.** $-2 + \sqrt{6}$ **D.** $-2 - 9\sqrt{6}$

c. $4\sqrt{8} - 6\sqrt{12} - 7\sqrt{18} + 2\sqrt{27}$ is equal to:

A. $-7\sqrt{5}$ **B.** $29\sqrt{2} - 18\sqrt{3}$ **C.** $-13\sqrt{2} - 6\sqrt{3}$ **D.** $-13\sqrt{2} + 6\sqrt{3}$

d. $2\sqrt{20} + 5\sqrt{24} - \sqrt{54} + 5\sqrt{45}$ is equal to:

A. $19\sqrt{5} + 7\sqrt{6}$ **B.** $9\sqrt{5} - 7\sqrt{6}$ **C.** $-11\sqrt{5} + 7\sqrt{6}$ **D.** $-11\sqrt{5} - 7\sqrt{6}$

Understanding

14. WE11 Which number is larger?

a. $\sqrt{10}$ or $2\sqrt{3}$ **b.** $3\sqrt{5}$ or $5\sqrt{2}$ **c.** $10\sqrt{2}$ or $4\sqrt{5}$ **d.** $2\sqrt{10}$ or $\sqrt{20}$

15. Write these numbers from smallest to largest.

a. $6\sqrt{2}$, 8, $2\sqrt{7}$, $3\sqrt{6}$, $4\sqrt{2}$, $\sqrt{60}$

b. $\sqrt{6}$, $2\sqrt{2}$, $\sqrt{2}$, 3, $\sqrt{3}$, 2, $2\sqrt{3}$

Reasoning

16. A man wants to divide his vegetable garden into 10 equal squares, each of area exactly 2 m^2.

a. What will be the exact side length of each square? Explain your reasoning.

b. What will the side length of each square be, correct to 2 decimal places?

c. The man wishes to group the squares next to each other so as to have the smallest perimeter for his vegetable garden.

i. How should he arrange the squares?

ii. Explain why the exact perimeter of the vegetable patch is $14\sqrt{2}$ m.

iii. What will be the perimeter correct to 2 decimal places?

17. Explain why $\sqrt{a^3b^2}$ can be simplified to $ab\sqrt{a}$.

Problem solving

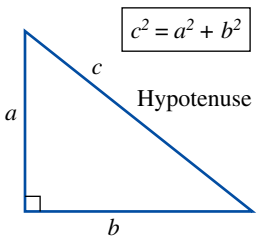
18. Kyle wanted a basketball court in his backyard; however, he could not fit a full-size court in his yard. He did get a rectangular court laid with a width of $6\sqrt{2}$ m and length of $3\sqrt{10}$ m. Calculate the area of the basketball court and represent it in its simplest surd form.

19. A netball team went on a pre-season training run. They completed 10 laps of a triangular course with side lengths $(200\sqrt{3} + 50)$ m, $(50\sqrt{2} + 75\sqrt{3})$ m and $(125\sqrt{2} - 18)$ m. What was the distance they ran in simplest surd form?



20. The area of a square is x cm^2. Would the side length of the square be a rational number? Explain your answer.

21. To calculate the length of the hypotenuse of a right-angled triangle, use the formula $c^2 = a^2 + b^2$.



a. Calculate the length of the hypotenuse for triangles with other side lengths of:

i. $\sqrt{5}, \sqrt{8}$ **ii.** $\sqrt{2}, \sqrt{7}$ **iii.** $\sqrt{15}, \sqrt{23}$.

b. Describe any pattern in your answers to part **a**.

c. State the length of the hypotenuse (without calculations) for triangles with side lengths of:

i. $\sqrt{1000}, \sqrt{500}$ **ii.** $\sqrt{423}, \sqrt{33}$ **iii.** $\sqrt{124}, \sqrt{63}$.

d. Your friend wrote down the following explanation.

$$\sqrt{b} + \sqrt{c} = \sqrt{a}$$

Explain why this is not correct and where your friend made an error.

Reflection

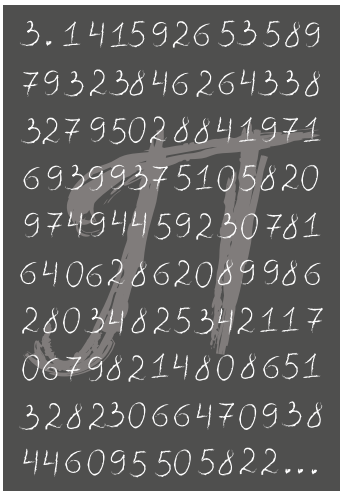
Are all square root numbers surds?

CHALLENGE 2.1

The integer 5 has a square of 25. If we add 1 to 5, we get the integer 6. The square of 6 is 36. If we increase each digit (2 and 5) in 25 by 1, we get the number 36. Therefore, the square of 6 (5 increased by 1) is the number formed by adding 1 to each digit in the square of 5. Find a pair of 2-digit numbers with the same property.

2.4 Real numbers

2.4.1 Real numbers

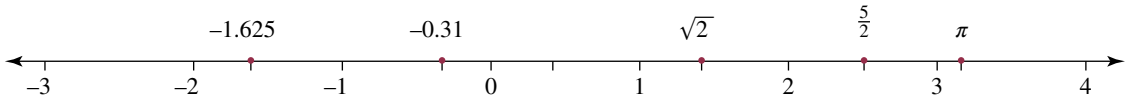


- A **real number** is any number that lies on the number line.
- Together, the rational numbers and the irrational numbers make up the set of real numbers.

  Therefore every point on the number line represents some number, rational or irrational.

2.4.2 The irrational number π

- Surds are not the only irrational numbers. Most irrational numbers cannot be written as surds. A few of these numbers are so important to mathematics and science that they are given special names.
- The most well known of these numbers is π **(pi)**.
- π is irrational, therefore it cannot be written as a fraction. If you tried to write π as a decimal, you would be writing forever, as the digits never recur and the decimal does not terminate. In the 20th century, computers were used to find the value of π to 1 trillion decimal places.
- The value of π is very close to (but not equal to) 3.14 or $\frac{22}{7}$. Most calculators store an approximate value for π.



2.4.3 Rounding decimals

- Because numbers such as $\sqrt{3}$ or π cannot be written exactly as decimals, approximate values are often used. These values can be found using a calculator and then rounded off to the desired level of accuracy.
- When we round numbers, we write them to a certain number of decimal places or significant figures.

2.4.4 Rounding to decimal places

- The rules for rounding to a certain number of decimal places are as follows.
  For example, let us round 3.456734 correct to 3 decimal places.
  - Identify the rounding digit in the third decimal place. In this case it is 6.

    3.456734
  - Look at the digit that comes after the rounding digit.

    3.456734

    If this digit is 0, 1, 2, 3 or 4, leave the rounding digit as it is.
    If this digit is 5, 6, 7, 8 or 9, then increase the rounding digit by 1.
    In this case the next digit is 7, so the rounding digit is increased by 1, from 6 to 7.
  - Leave out all digits that come after the rounding digit.
    So 3.456754 $\approx$ 3.457 correct to 3 decimal places (3 d.p.).

WORKED EXAMPLE 14

Write these numbers correct to 3 decimal places (3 d.p.).

a $\sqrt{3}$ **b** π **c** $5.1\dot{9}$ **d** $\frac{2}{3}$ **e** 7.123456

THINK	WRITE
a $\sqrt{3} = 1.7320\ldots$ The rounding digit is 2. The next digit is 0, so leave 2 as is.	**a** $\sqrt{3} \approx 1.73$
b $\pi = 3.1415\ldots$ The rounding digit is 1. The next digit is 5, so add 1.	**b** $\pi \approx 3.142$
c $5.1\dot{9} = 5.1999\ldots$ The rounding digit is 9. The next digit is 9, so add 1.	**c** $5.1\dot{9} \approx 5.200$
d $\frac{2}{3} = 0.6666\ldots$ The rounding digit is 6. The next digit is 6, so add 1.	**d** $\frac{2}{3} \approx 0.667$
e 7.123 456. The rounding digit is 3. The next digit is 4, so leave 3 as is.	**e** $7.1234 \approx 7.123$

2.4.5 Rounding to significant figures

- Another method of rounding decimals is to write them correct to a certain number of significant figures. In a decimal number the significant figures start with the first non-zero digit.
- Consider the approximate value of $\sqrt{2}$.
- $\sqrt{2} \approx 1.414$. This approximation is written correct to 3 decimal places, but it has 4 significant figures. When counting significant figures, the non-zero digits before the decimal point are included.
- The decimal 0.0302 is written to 4 decimal places, but it has only 3 significant figures. This is because significant figures start with the first non-zero digit. In this case the first significant figure is 3, and the significant figures are 302.

WORKED EXAMPLE 15

How many significant figures are there in each of the following numbers?
a 25 **b 0.04** **c 3.02** **d 0.100**

THINK		WRITE
a 25:	The first significant figure is 2.	**a** 2 significant figures
b 0.04:	The first significant figure is 4.	**b** 1 significant figure
c 3.02:	The first significant figure is 3.	**c** 3 significant figures
d 0.100:	The first significant figure is 1.	**d** 3 significant figures

WORKED EXAMPLE 16

Round these numbers correct to 5 significant figures.
a π **b** $\sqrt{200}$ **c** $0.\dot{0}\dot{3}$ **d 2530.166**

THINK	WRITE
a $\pi \approx 3.141\,59\ldots$ The first significant figure is 3. Write 4 more digits.	**a** 3.1416
b $\sqrt{200} \approx 14.1421\ldots$ The first significant figure is 1. Write 4 more digits.	**b** 14.142
c $0.\dot{0}\dot{3} = 0.030\,303\,0\ldots$ The first significant figure is 3. Write 4 more digits.	**c** 0.030 303
d 2530.16 . . . The first significant figure is 2. Write 4 more digits.	**d** 2530.2

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Complete this digital doc: SkillSHEET: Rounding to a given number of significant figures (doc-10814)

Exercise 2.4 Real numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4a–k, 5–11	Questions: 1–3, 4e–p, 5–13	Questions: 1–3, 4i–t, 5–13

Individual pathway interactivity: int-4478

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE14 Write each of the following correct to 3 decimal places.

a. $\frac{\pi}{2}$	**b.** $\sqrt{5}$	**c.** $\sqrt{15}$	**d.** 5.12×3.21	**e.** $5.\dot{1}$
f. $5.1\dot{5}$	**g.** $5.\dot{1}5$	**h.** e	**i.** 11.72^2	**j.** $\frac{3}{7}$
k. $\frac{1}{13}$	**l.** $2\frac{3}{7}$	**m.** 0.999 999	**n.** 6.581 29	**o.** 4.000 01
p. $2.79 \div 11$	**q.** 0.0254	**r.** 0.000 913 6	**s.** 5.000 01	**t.** 2342.156

2. Write the value of π correct to 4, 5, 6 and 7 decimal places.

3. WE15 How many significant figures are there in each of the following numbers?

a. 36	**b.** 207	**c.** 1631	**d.** 5.04
e. 176.2	**f.** 95.00	**g.** 0.21	**h.** 0.01
i. 0.000 316	**j.** 0.1007	**k.** 0.010	**l.** 0.0512

4. WE16 Write each number correct to 5 significant figures.

a. $\frac{\pi}{2}$	**b.** $\sqrt{5}$	**c.** $\sqrt{15}$	**d.** 5.12×3.21	**e.** $5.\dot{1}$
f. $5.1\dot{5}$	**g.** $5.\dot{1}\dot{5}$	**h.** e	**i.** 11.72^2	**j.** $\frac{3}{7}$
k. $\frac{1}{13}$	**l.** $2\frac{3}{7}$	**m.** 0.999 999	**n.** 6.581 29	**o.** 4.000 01
p. $2.79 \div 11$	**q.** 0.0254	**r.** 0.000 913 6	**s.** 5.000 01	**t.** 2342.156

5. List the following numbers in order from smallest to largest.

a. $\sqrt{7}$, 3.5, $\sqrt{18}$, 4, $\sqrt{15}$

b. 3.14, π, $\frac{22}{7}$, $\sqrt{10}$, $3.\dot{1}\dot{4}$

Understanding

6. Explain whether each statement is true or false.

a. Every surd is a rational number.

b. Every surd is an irrational number.

c. Every irrational number is a surd.

d. Every surd is a real number.

7. Explain whether each statement is true or false.

a. π is a rational number.

b. π is an irrational number.

c. π is a surd.

d. π is a real number.

8. Explain whether each statement is true or false.

a. $1.\dot{3}\dot{1}$ is a rational number.

b. $1.\dot{3}\dot{1}$ is an irrational number.

c. $1.\dot{3}\dot{1}$ is a surd.

d. $1.\dot{3}\dot{1}$ is a real number.

Reasoning

9. Explain why $\sqrt{25}$ and $-\sqrt{25}$ are defined, but $\sqrt{-25}$ is not.

10. Explain why the number 12.9954123 when rounded to 2 decimal places becomes 13.00.

Problem solving

11. The area of a circle is calculated using the formula $A = \pi \times r^2$, where r is the radius of the circle. Pi (π) is sometimes rounded to 2 decimal places (3.14). A particular circle has a radius of 7 cm.

a. Use $\pi = 3.14$ to calculate the area of the circle to 2 decimal places.

b. Use the π key on your calculator to calculate the area of the circle to 4 decimal places.

c. Round your answer for part **b** to 2 decimal places.

d. Are your answers for parts **a** and **c** different? Why or why not?

12. The volume of a sphere (a ball shape) is calculated using the formula $V = \frac{4}{3} \times \pi \times r^3$, where r is the radius of the sphere. A beach ball with a radius of 25 cm is bouncing around the crowd at the MCG during the Boxing Day Test.



a. Calculate the volume of the beach ball to 4 decimal places.

b. When the volume is calculated to 4 decimal places, how many significant figures does it have?

c. Is the calculated volume a rational number? Why or why not?

13. In a large sample of written English prose there are about 7 vowels to every 11 consonants. The letter *e* accounts for about one-third of the occurrence of vowels. How many times would you expect the letter *e* to occur in a passage of 100000 letters? Round your answer to the nearest 100.

Reflection

Is there an equal number of rational numbers and irrational numbers?

2.5 Scientific notation

2.5.1 Scientific notation

- Did you know that Jupiter is approximately 778547200 km from the sun? Astronomers and other scientists frequently use large numbers that are difficult to read and manipulate. To make them easier to use and compare, they can be written in a special way called **scientific notation** (or **standard form**).
- A number written in scientific notation looks like this:



5.316×10^2

A number between 1 and 10 $\times$ A power of 10

WORKED EXAMPLE 17

Here are some numbers written in scientific notation. Write them in decimal notation.

a 7.136×10^2 **b** 5.017×10^5 **c** 8×10^6

THINK	WRITE
a Move the decimal point 2 places to the right, since $10^2 = 100$.	**a** $7.136 \times 10^2 = 713.6$
b Move the decimal point 5 places to the right, since $10^5 = 100000$.	**b** $5.017 \times 10^5 = 501700$
c Move the decimal point 6 places to the right, since $10^6 = 1000000$.	**c** $8 \times 10^6 = 8000000$

2.5.2 Using a calculator

- Numbers written in scientific notation can be entered into your calculator using a special button. Find out how this works on your calculator.
- Some calculators have unusual ways of writing in scientific notation. For example, some show 5.71×10^4 as 5.71E4. In writing you should always show this as 5.71×10^4.

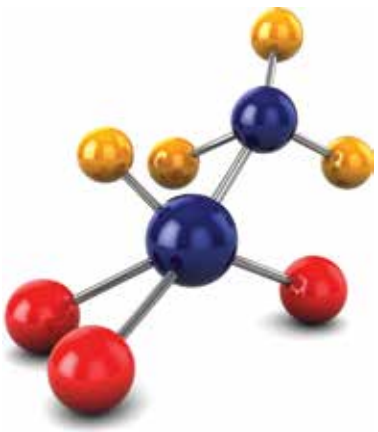
WORKED EXAMPLE 18

a Write each of these numbers in scientific notation.
b Round each answer to 3 significant figures.
i 827.2 ii 53 681 iii 51 900 000 000

THINK	WRITE
a i The first part needs to be a number between 1 and 10, so 8.272. The decimal point must be moved 2 places ($\times 10^2$).	**a i** $827.2 = 8.272 \times 10^2$
ii The first part needs to be a number between 1 and 10, so 5.3681. The decimal point must be moved 4 places ($\times 10^4$).	**ii** $53\,681 = 5.3681 \times 10^4$
iii The first part needs to be a number between 1 and 10, so 5.190. The decimal point must be moved 10 places ($\times 10^{10}$).	**iii** $51\,900\,000\,000 = 5.190 \times 10^{10}$
b i $8.272 \approx 8.27$	**b i** 8.27×10^2
ii $5.368 \approx 5.37$	**ii** 5.37×10^4
iii $5.190 \approx 5.19$	**iii** 5.19×10^{10}

2.5.3 Small numbers in scientific notation

- Very small numbers, such as the length or the mass of a molecule, can also be expressed in scientific notation.
  $0.0412 = 4.12 \div 10^2$
  $= 4.12 \times 10^{-2}$
  It will be shown later that $10^{-2} = \frac{1}{10^2}$.
- Just as
  $6.285 \times 10^2 = 628.5$
  (the decimal point is moved 2 places to the right),
  $6.285 \times 10^{-2} = 0.062\,85$
  (the decimal point is moved 2 places to the left).
  Similarly,
  $6.285 \times 10^{-1} = 0.6285$
  and
  $6.285 \times 10^{-3} = 0.006\,285$.



Very small and very large time scales

- The following units can be used to measure times at very small and very large time scales.

Unit	Scientific notation (in seconds)
Picosecond	1×10^{-12} s
Nanosecond	1×10^{-9} s
Microsecond	1×10^{-6} s
Millisecond	1×10^{-3} s
Kilosecond	1×10^{3} s
Megasecond	1×10^{6} s
Gigasecond	1×10^{9} s

WORKED EXAMPLE 19

Write these numbers in decimal notation.
a 9.12×10^{-1} **b** 7.385×10^{-2} **c** 6.32×10^{-7}

THINK	WRITE
a Move the decimal point 1 place to the left.	**a** $9.12 \times 10^{-1} = 0.912$
b Move the decimal point 2 places to the left. (Insert a zero.)	**b** $7.385 \times 10^{-2} = 0.07385$
c Move the decimal point 7 places to the left. (Insert zeros.)	**c** $6.32 \times 10^{-7} = 0.000000632$

WORKED EXAMPLE 20

Write these numbers in scientific notation.
a 0.0051 **b** 0.1321 **c** 0.0000000007

THINK	WRITE
a The first part needs to be a number between 1 and 10, so 5.1. The decimal point must be moved 3 places ($\times 10^{-3}$).	**a** $0.0051 = 5.1 \times 10^{-3}$
b The first part needs to be a number between 1 and 10, so 1.321. The decimal point must be moved 1 place ($\times 10^{-1}$).	**b** $0.1321 = 1.321 \times 10^{-1}$
c The first part needs to be a number between 1 and 10, so 7.0. The decimal point must be moved 10 places ($\times 10^{-10}$).	**c** $0.0000000007 = 7.0 \times 10^{-10}$

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Complete this digital doc: SkillSHEET: Multiplying and dividing by powers of 10 (doc-10815)

Complete this digital doc: WorkSHEET: Scientific notation (doc-10816)

Exercise 2.5 Scientific notation

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 11, 13–15	Questions: 1–8, 10, 12–15, 18	Questions: 1–19

Individual pathway interactivity: int-4479

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Fluency

1. WE17 Write these numbers in decimal notation.

a. 6.14×10^2	**b.** 6.14×10^3	**c.** 6.14×10^4	**d.** 3.518×10^2
e. 1×10^9	**f.** 3.92673×10^2	**g.** 5.911×10^2	**h.** 5.1×10^3
i. 7.34×10^5	**j.** 7.1414×10^6	**k.** 3.51×10	**l.** 8.05×10^4

2. WE18a Write these numbers in scientific notation.

a. 5000	**b.** 431	**c.** 38	**d.** 350000
e. 72.5	**f.** 725	**g.** 7250	**h.** 725000000

3. WE18b Write these numbers in scientific notation, correct to 4 significant figures.

a. 43.792	**b.** 5317	**c.** 258.95	**d.** 110.11
e. 1632000	**f.** 1 million	**g.** 123456789	**h.** 249.9

4. WE19 Write these numbers in decimal notation.

a. 2×10^{-1}	**b.** 4×10^{-3}	**c.** 7×10^{-4}	**d.** 3×10^{-2}
e. 8.273×10^{-2}	**f.** 7.295×10^{-2}	**g.** 2.9142×10^{-3}	**h.** 3.753×10^{-5}
i. 5.29×10^{-4}	**j.** 3.3333×10^{-5}	**k.** 2.625×10^{-9}	**l.** 1.273×10^{-15}

5. WE20 Write these numbers in scientific notation.

a. 0.7	**b.** 0.005	**c.** 0.0000003	**d.** 0.00000000001
e. 0.231	**f.** 0.00362	**g.** 0.000731	**h.** 0.063

6. Write these numbers in scientific notation, correct to 3 significant figures.

a. 0.006731	**b.** 0.14257	**c.** 0.0000683	**d.** 0.000000005512
e. 0.0509	**f.** 0.01246	**g.** 0.000731	**h.** 0.063

Understanding

7. Write these numbers in ascending order.

a. 8.31×10^2, 3.27×10^3, 9.718×10^2, 5.27×10^2

b. 7.95×10^2, 4.09×10^2, 7.943×10^2, 4.37×10^2

c. 5.31×10^{-2}, 9.29×10^{-3}, 5.251×10^{-2}, 5.27×10^{-1}

d. 8.31×10^2, 3.27×10^3, 7.13×10^{-2}, 2.7×10^{-3}

8. One carbon atom weighs 1.994×10^{-23} g.

a. Write the weight as a decimal.

b. How much will 1 million carbon atoms weigh?

c. How many carbon atoms are there in 10 g of carbon? Give your answer correct to 4 significant figures.





9. The distance from Earth to the Moon is approximately 3.844×10^5 km. If you could drive there without breaking the speed limit (100 km/h), how long would it take? What is this in days (correct to 2 decimal places)?

10. The Earth weighs 5.97×10^{24} kg, but the Sun weighs 1.99×10^{30} kg. How many Earths would it take to balance the Sun's weight? Give your answer correct to 2 decimal places.

11. Inside the nucleus of an atom, a proton weighs 1.6726×10^{-27} kg and a neutron weighs 1.6726×10^{-27} kg. Which one is heavier and by how much?

12. The Earth's orbit has a radius of 7.5×10^7 km; the orbit of Venus is equal to 5.4×10^7 km. How far apart are the planets when:

a. they are closest to each other

b. they are farthest apart from each other?

Reasoning

13. a. State the power(s) of 10 that you believe equations **i** and **ii** will have when solved.

i. $5.36 \times 10^7 + 2.95 \times 10^3$ **ii.** $5.36 \times 10^7 - 2.95 \times 10^3$

b. Evaluate equations **i** and **ii**, correct to 3 significant figures.

c. Were your answers to part **a** correct? Why or why not?

14. Explain why $2.39 \times 10^{-3} + 8.75 \times 10^{-7} = 2.39 \times 10^{-3}$, correct to 3 significant figures.

Problem solving

15. Write the following numbers in ascending order.
14%, 0.753, $\frac{5}{8}$, $\sqrt{2}$, 0.5^2, $\sqrt[3]{2}$, 1.5×10^{-2}

16. Distance is equal to speed multiplied by time. If we travelled at 100 km/h it would take us approximately 0.44 years to reach the Moon, 89.6 years to reach Mars, 1460 years to reach Saturn and 6590 years to reach Pluto.

a. Assuming that there are 365 days in a year, calculate the distance (as a basic numeral) between Earth and:

i. the Moon **ii.** Mars **iii.** Saturn **iv.** Pluto.

b. Write your answers to part **a** correct to 3 significant figures.

c. Write your answers to part **a** using scientific notation correct to 3 significant figures.

17. A light-year is the distance that light travels in one year. Light travels at approximately 300 000 km/s.

a. i. Calculate the number of seconds in a year (assuming 1 year = 365 days).

ii. Write your answer to part **i** using scientific notation.

b. Calculate the distance travelled by light in one year. Express your answer:

i. as a basic numeral

ii. using scientific notation.

c. The closest 'star' to Earth (other than our sun) is the star system Alpha Centauri, which is 4.3 light-years away.

i. How far is this in kilometres, correct to 4 significant figures?

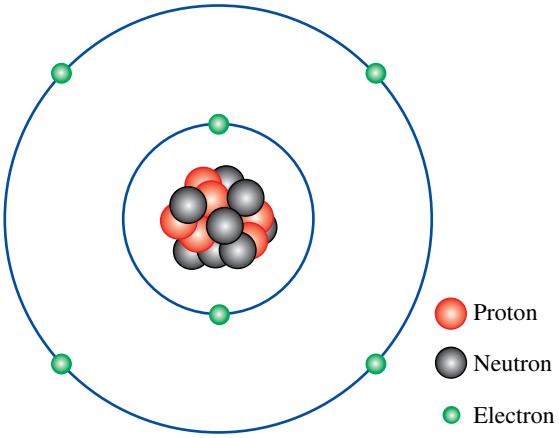
ii. Travelling at 100 km/h, how long in years would it take to reach Alpha Centauri?



18. Scientists used Earth's gravitational pull on nearby celestial bodies (such as the Moon) to calculate the mass of the Earth at approximately 5.972 sextillion metric tonnes.
a. Write 5.972 sextillion using scientific notation.
b. How many significant figures does this number have?



19. An atom consists of smaller particles called protons, neutrons and electrons. Electrons have a mass of $9.109\,381\,88 \times 10^{-31}$ kilograms, correct to 9 significant figures.
a. Write the mass of an electron correct to 5 significant figures.
b. Protons and neutrons are the same size. They are both 1836 times the size of an electron. Use the mass of an electron (correct to 9 significant figures) and your calculator to find the mass of a proton correct to 5 significant figures.
c. Use the mass of an electron correct to 3 significant figures to calculate the mass of a proton correct to 5 significant figures.
d. Why is it important to work with the original amounts and then round to the specified number of significant figures at the end of a calculation?



Reflection

What is the advantage of converting numbers into scientific notation?

CHALLENGE 2.2

The diameter of the Sun is approximately 8.656×10^5 miles. Since there are 1.61 km in a mile, convert this to kilometres using scientific notation and give your answer correct to 4 significant figures. If the Sun's diameter is 109 times that of Earth, what is Earth's diameter? Give your answer correct to 3 significant figures.

2.6 Review

2.6.1 Review questions

Fluency

1. Which of the given numbers, $\sqrt{\frac{6}{12}}$, $\sqrt{0.81}$, 5, -3.26, $0.5\dot{}$, $\frac{\pi}{5}$, $\sqrt{\frac{3}{12}}$ are rational?

A. $\sqrt{0.81}$, 5, -3.26, $0.5\dot{}$ and $\sqrt{\frac{3}{12}}$
B. $\sqrt{\frac{6}{12}}$ and $\frac{\pi}{5}$
C. $\sqrt{\frac{6}{12}}$, $\sqrt{0.81}$ and $\sqrt{\frac{3}{12}}$
D. 5, -3.26, and $\sqrt{\frac{6}{12}}$

2. For each of the following, state whether the number is rational or irrational and give the reason for your answer.
a. $\sqrt{12}$ **b.** $\sqrt{121}$ **c.** $\frac{2}{9}$ **d.** $0.6\dot{}$ **e.** $\sqrt[3]{0.08}$

3. Which of the numbers of the given set, $\{3\sqrt{2},\ 5\sqrt{7},\ 9\sqrt{4},\ 6\sqrt{10},\ 7\sqrt{12},\ 12\sqrt{64}\}$, are surds?

A. $9\sqrt{4}$, $12\sqrt{64}$
B. $3\sqrt{2}$ and $7\sqrt{12}$ only
C. $3\sqrt{2}$, $5\sqrt{7}$ and $6\sqrt{10}$ only
D. $3\sqrt{2}$, $5\sqrt{7}$, $6\sqrt{10}$ and $7\sqrt{12}$

4. Which of the following are not irrational numbers?

a. $\sqrt{7}$ **b.** $\sqrt{8}$ **c.** $\sqrt{81}$ **d.** $\sqrt{361}$

5. The number 0.000 67 written using scientific notation is:

A. 67×10^{-5} **B.** 0.67×10^{-3} **C.** 6.7×10^{-4} **D.** 6.7×10^{-5}

6. The expression $\sqrt{250}$ may be simplified to:

A. $25\sqrt{10}$ **B.** $5\sqrt{10}$ **C.** $10\sqrt{5}$ **D.** $5\sqrt{50}$

Problem solving

7. If your body produces about 2.0×10^{11} red blood cells each day, how many red blood cells has your body produced after one week?

8. If your body produces about 1.0×10^{10} white blood cells each day, how many white blood cells does your body produce each hour?

9. If the planet Earth is approximately 1.496×10^{8} km from the sun and the planet Mercury is about 5.8×10^{7} km from the sun, what is the closest distance from Mercury to Earth?

10. Determine the closest distance between Mars and Saturn if Mars is 2.279×10^{8} km from the sun and Saturn is 1.472×10^{9} km from the sun.

11. Consider the numbers 6×10^{9}, 3.4×10^{7} and 1.5×10^{9}.

a. Place the numbers in order from smallest to largest.

b. If the population of Oceania is 3.4×10^{7} and the world population is 6×10^{9}, what percentage is the Oceania population of the world population? Give your answer to 2 decimal places.

c. What must you multiply 1.5×10^{9} by to get 6×10^{9}?

d. Assume the surface area of the Earth is approximately $1.5 \times 10^{9}\,\text{km}^2$ and the world population is about 6×10^{9}. If we divided all the Earth up into equal portions, how much would each person get?

12. A newly discovered super colony of bees contains 2.05×10^{8} bees. If 0.4% of these were estimated to be queen bees, how many queen bees live in the super colony?

13. The area of Victoria is $2.27 \times 10^{5}\,\text{km}^2$. If the total area of Australia is $7.69 \times 10^{6}\,\text{km}^2$, what percentage of Australia does the state of Victoria occupy? Write your answer correct to two decimal places.

14. Third generation fibre optics that can carry up to 10 trillion (1×10^{12}) bits per second down a single strand of fibre have been recently tested. Currently the amount that one fibre can carry is tripling every 6 months and is predicted to do so for the next 20 years. In how many years will it take for the amount of bits per second being carried down a single strand of fibre exceed 6×10^{16} bits per second. Explain, in detail, your solution process.

15. Mach speed refers to the speed of sound, where Mach 1 is the speed of sound at about 343 metres per second (m/s). Mach 2 is double the speed of sound; Mach 3 is triple the speed of sound and so on. The Earth has a circumference of about 4.0×10^{4} km.

a. The fastest aircraft recorded travelled at 11 270 kilometres per hour (km/h). Express this as a Mach speed correct to 1 decimal place.

b. Create and complete a table for the four different speeds with columns 'Mach number', 'Speed in m/s' and 'Speed in km/h'.

c. Calculate how long it would take something travelling at Mach 1, 2 and 3 to circle the Earth. Give your answer in hours correct to 2 decimal places.

d. How long does it take the world's fastest aircraft to circle the Earth? Give your answer in hours correct to 2 decimal places.

e. Calculate how many times the world's fastest aircraft could circle the Earth in the time it takes an aircraft going at Mach 1, 2 and 3 to circle the Earth once. Give your answers correct to 2 decimal places.

f. Explain the relationship between your answers in the previous question and their corresponding Mach value.

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Try out this interactivity: Crossword: Topic 2 (int-0698)

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Complete this digital doc: Concept map: Topic 2 (int-10793)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

integers
irrational numbers
natural numbers
numerator
radical sign
rational numbers
recurs
recurring decimal
scientific notation
square root
surd

assessON

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

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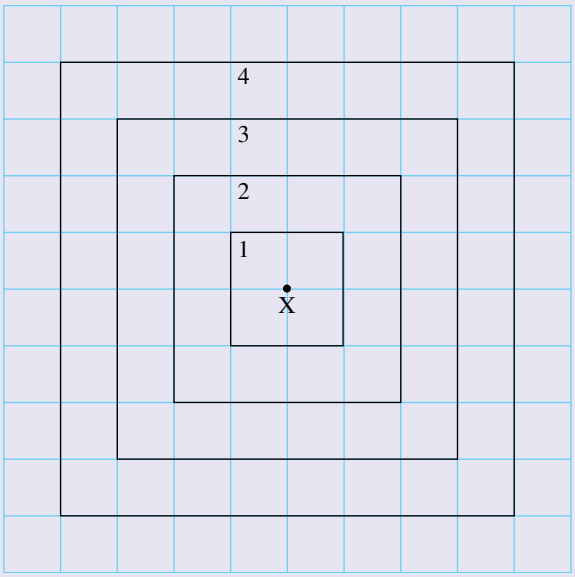


Investigation | Rich task

Concentric squares

Consider a set of squares drawn around a central point on a grid as shown in this diagram.

These squares can be regarded as concentric squares, because the central point of each square is the point X. The diagram shows four labelled squares drawn on one-centimetre grid paper.



1. Use the diagram above to complete the following table, leaving your answers in simplest surd form, if necessary. The first square has been completed for you.

Square	1	2	3	4
Side length (cm)	2			
Diagonal length (cm)	$2\sqrt{2}$			
Perimeter (cm)	8			
Area (cm^2)	4			

Observe the patterns in the table and answer questions 2 and 3.

2. What would be the side length of the tenth square of this pattern?

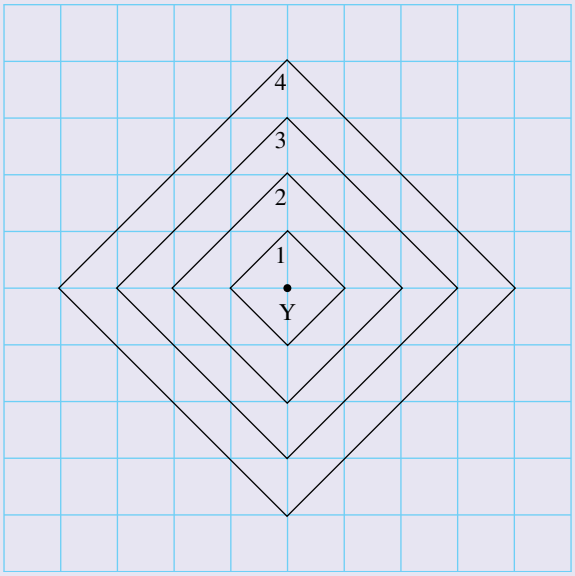
3. What would be the length of the diagonal of this tenth square?

Consider, now, a different arrangement of the squares on one-centimetre grid paper, as shown in the diagram.

These squares all still have a central point, labelled Y.

4. Use the diagram to complete the following table for these squares, leaving your answers in simplest surd form if necessary.

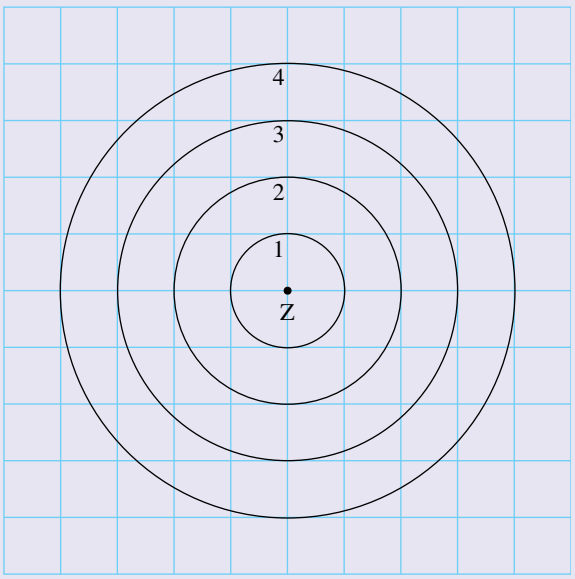
Square	1	2	3	4
Side length (cm)	$\sqrt{2}$			
Diagonal length (cm)	2			
Perimeter (cm)	$4\sqrt{2}$			
Area (cm^2)	2			



5. What would be the side length of the tenth square in this pattern of squares?
6. What would be the length of the diagonal of the tenth square?

Use the two diagrams above to answer questions 7 to 11.

7. In which arrangement does the square have the greater side length?
8. Which arrangement shows the squares with the greater diagonal length?
9. Compare the area of a square in the first diagram with the area of the corresponding square in the second diagram.
10. Compare the perimeter of corresponding squares in the two diagrams.
11. Examine the increase in area from one square to the next in the set. What increase in area would you expect from square 7 to square 8 in each of the patterns?



Consider a set of concentric circles around a centre Z, drawn on one-centimetre grid paper.

12. Investigate the change in circumference of the circles, moving from one circle to the next (smallest to largest). Write a general formula for this increase in circumference in terms of π.
13. Write a general formula in terms of π and r that can be used to calculate the increase in area from one circle to the next (smallest to largest).

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Complete this digital doc: Code puzzle: When 3 people fell in the water, why did only 2 of them get their hair wet? (doc-15889)

Answers

Topic 2 Number skills

Exercise 2.2 Rational numbers

1. a. $\frac{15}{1}$ **b.** $\frac{-8}{1}$ **c.** $\frac{8}{3}$ **d.** $\frac{-41}{8}$
e. $\frac{4}{1}$ **f.** $\frac{73}{10}$ **g.** $\frac{2}{1000}$ **h.** $\frac{872}{10}$
i. $\frac{0}{1}$ **j.** $\frac{156}{100}$ **k.** $\frac{3612}{1000}$ **l.** $\frac{-8}{100}$

2. a. 0.55555555 **b.** 0.51515151 **c.** 0.51111111 **d.** 6.0313131
e. 5.18318331 **f.** -7.02444444 **g.** 8.91249121 **h.** 5.12343431
i. 5.12341231 **j.** 3.00202021

3. a. $0.\dot{5}$ **b.** $0.\dot{2}\dot{7}$ **c.** $2.\dot{1}\dot{8}$ **d.** $0.\dot{4}28571\dot{4}$
e. $-1.\dot{7}\dot{4}$ **f.** $0.\dot{7}\dot{3}$ **g.** $-3.8\dot{3}$ **h.** $0.4\dot{6}$
i. $0.\dot{4}\dot{6}$ **j.** $0.0\dot{4}\dot{6}$

4. a. 15 **b.** -3, 0, 15 **c.** -3, $\frac{-3}{7}$, 0, 2.3, $2.\dot{3}$, $2\frac{3}{5}$, 15

5. $2.\dot{1}$, $2.\dot{1}2\dot{1}$, $2.1\dot{2}\dot{1}$, $2.\dot{1}\dot{2}$, $2.1\dot{2}$

6. Every integer a can be written as $\frac{a}{1}$ and is therefore rational.

7. Yes, because the division of an integer by another integer will always result in a rational number.

8. a. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$ **b.** 6

9. Answers will vary. Example answers are shown.
a. $\frac{1}{9}$ **b.** $\frac{10}{99}$ **c.** $\frac{100}{999}$ **d.** $\frac{1000}{9999}$

10. $3330.20

11. 657

Exercise 2.3 Surds

1. a. 1 **b.** 2 **c.** 0 **d.** $\frac{1}{3}$
e. $\frac{5}{4}$ **f.** 0.4 **g.** 20 **h.** 100
i. $\frac{2}{5}$ **j.** 1.2 **k.** 4.5 **l.** 1000
m. 0.03 **n.** 16

2. a. 9 **b.** -9 **c.** 11 **d.** -21
e. 2 **f.** 4 **g.** 7 **h.** 3
i. 4 **j.** 5 **k.** -7 **l.** -3

3. **b**, **d**, **g**, **h**, **k**, **m**, **o**

4. a. $\sqrt{21}$ **b.** $-\sqrt{21}$ **c.** $2\sqrt{6}$ **d.** $6\sqrt{7}$
e. $10\sqrt{14}$ **f.** $15\sqrt{10}$ **g.** $12\sqrt{7}$ **h.** $9\sqrt{7}$
i. $22\sqrt{10}$ **j.** $22\sqrt{3}$ **k.** 3 **l.** 24
m. 6 **n.** 11 **o.** 51 **p.** 30

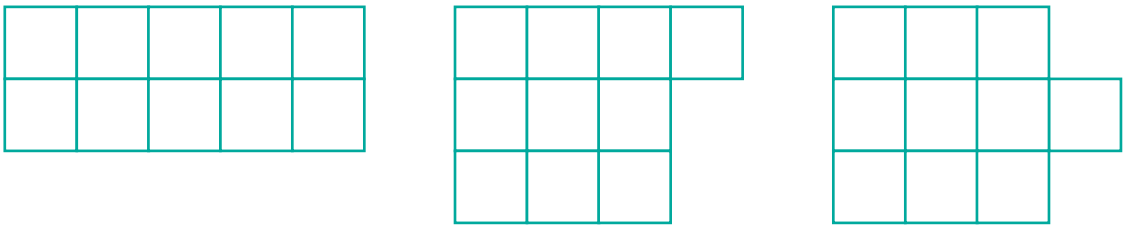
5. a. $\sqrt{3}$ **b.** $-\sqrt{2}$ **c.** $\sqrt{6}$ **d.** $\sqrt{5}$
e. $3\sqrt{3}$ **f.** $\frac{3\sqrt{6}}{2}$ **g.** $15\sqrt{2}$ **h.** $\frac{5\sqrt{5}}{3}$
i. $-2\sqrt{5}$ **j.** $\sqrt{3}$ **k.** $\sqrt{3}$ **l.** $\sqrt{7}$
m. $\sqrt{35}$ **n.** 2 **o.** -3 **p.** 3

6. a. $2\sqrt{5}$ **b.** $2\sqrt{2}$ **c.** $3\sqrt{2}$ **d.** 7
e. $\sqrt{30}$ **f.** $5\sqrt{2}$ **g.** $2\sqrt{7}$ **h.** $6\sqrt{3}$
i. $12\sqrt{2}$ **j.** $4\sqrt{3}$ **k.** $10\sqrt{5}$ **l.** $9\sqrt{2}$

7. a. $4\sqrt{2}$ **b.** $15\sqrt{3}$ **c.** 48 **d.** $35\sqrt{2}$
e. $20\sqrt{6}$ **f.** $10\sqrt{3}$ **g.** $4\sqrt{42}$ **h.** $72\sqrt{2}$
i. $27\sqrt{5}$ **j.** $132\sqrt{2}$

8. a. $\sqrt{12}$ **b.** $\sqrt{175}$ **c.** $\sqrt{108}$ **d.** $\sqrt{80}$
e. $\sqrt{384}$ **f.** $\sqrt{90}$ **g.** $\sqrt{32}$ **h.** $\sqrt{720}$
i. $\sqrt{600}$ **j.** $\sqrt{338}$

9. a. D **b.** A **c.** C **d.** C
e. D **f.** D **g.** D **h.** B
10. a. $15\sqrt{3}$ **b.** $19\sqrt{5}$ **c.** $16\sqrt{7}$ **d.** $17\sqrt{2}$
e. $a\sqrt{c}$ **f.** $d^2\sqrt{b}$ **g.** $hk\sqrt{j}$ **h.** $f\sqrt{f}$
11. a. $2\sqrt{2}$ **b.** $-4\sqrt{5}$ **c.** $-6\sqrt{3}$ **d.** 0
e. $5\sqrt{11}$ **f.** $2\sqrt{7}$ **g.** $10\sqrt{2} + 7\sqrt{3}$ **h.** $8\sqrt{5} + \sqrt{6}$
i. $8\sqrt{10} + 7\sqrt{3}$ **j.** $16\sqrt{2} - 11\sqrt{5}$ **k.** $2\sqrt{6}$ **l.** $5\sqrt{5} + 15$
12. a. $\sqrt{2}$ **b.** 0 **c.** $-5\sqrt{3}$ **d.** $-4\sqrt{7}$
e. $5\sqrt{6} + 6\sqrt{5}$ **f.** $2\sqrt{3} - 3\sqrt{5}$ **g.** $4\sqrt{6} + 6\sqrt{5} - 14\sqrt{2}$ **h.** $29\sqrt{5} + 22\sqrt{3}$
i. $9\sqrt{11} - \sqrt{30}$ **j.** $28\sqrt{2} - 39\sqrt{5}$
13. a. D **b.** A **c.** C **d.** A
14. a. $2\sqrt{3}$ **b.** $5\sqrt{2}$ **c.** $10\sqrt{2}$ **d.** $2\sqrt{10}$
15. a. $2\sqrt{7}$, $4\sqrt{2}$, $3\sqrt{6}$, $\sqrt{60}$, 8, $6\sqrt{2}$ **b.** $\sqrt{2}$, $\sqrt{3}$, 2, $\sqrt{6}$, $2\sqrt{2}$, 3, $2\sqrt{3}$
16. a. $\sqrt{2}$ m **b.** 1.41 m
c. i. Any of these:



ii. $14\sqrt{2}$ m
iii. 19.80 m

17. $\sqrt{a^3b^2} = \sqrt{a^3} \times \sqrt{b^2}$
$= \sqrt{a^2} \times \sqrt{a} \times \sqrt{b^2}$
$= a \times \sqrt{a} \times b$
$= ab\sqrt{a}$

18. $36\sqrt{5}\,m^2$
19. $(1750\sqrt{2} + 2750\sqrt{3} + 320)$ m
20. The side length will be rational if x is a perfect square; otherwise, the side length will be a surd.
21. a. i. $\sqrt{13}$ **ii.** 3 **iii.** $\sqrt{38}$
b. For side lengths $\sqrt{a}$ and $\sqrt{b}$, the length of the hypotenuse is $\sqrt{a+b}$.
c. i. $\sqrt{1500} = 10\sqrt{15}$ **ii.** $\sqrt{456} = 2\sqrt{144}$ **iii.** $\sqrt{187}$
d. The answer should be $\sqrt{b+a} = \sqrt{c}$.

Challenge 2.1

$45^2 = 2025$; $56^2 = 3136$

Exercise 2.4 Real numbers

1. a. 1.571 **b.** 2.236 **c.** 3.873 **d.** 16.435
e. 5.111 **f.** 5.156 **g.** 5.152 **h.** 2.718
i. 137.358 **j.** 0.429 **k.** 0.077 **l.** 2.429
m. 1.000 **n.** 6.581 **o.** 4.000 **p.** 0.254
q. 0.025 **r.** 0.001 **s.** 5.000 **t.** 2342.156
2. 3.1416, 3.14159, 3.141593, 3.1415927
3. a. 2 **b.** 3 **c.** 4 **d.** 3
e. 4 **f.** 4 **g.** 2 **h.** 1
i. 3 **j.** 4 **k.** 2 **l.** 3
4. a. 1.5708 **b.** 2.2361 **c.** 3.8730 **d.** 16.435
e. 5.1111 **f.** 5.1556 **g.** 5.1515 **h.** 2.7183
i. 137.36 **j.** 0.42857 **k.** 0.076923 **l.** 2.4286

m. 1.0000 **n.** 6.5813 **o.** 4.0000 **p.** 0.25364

q. 0.025400 **r.** 0.00091360 **s.** 5.0000 **t.** 2342.2

5. a. $\sqrt{7}$, 3.5, $\sqrt{15}$, 4, $\sqrt{18}$ **b.** 3.14, $3.\dot{1}\dot{4}$, π, $\frac{22}{7}$, $\sqrt{10}$

6. a. F **b.** T **c.** F **d.** T

7. a. F **b.** T **c.** F **d.** T

8. a. T **b.** F **c.** F **d.** T

9. $\sqrt{25}$ means the positive square root of 25, i.e. 5, because $5^2 = 25$. Similarly, $-\sqrt{25}$ means the negative square root of 25, which is -5 because $(-5)^2 = 25$. But -25, like all other negative numbers, has no real square root, so $\sqrt{-25}$ is 'undefined'.

10. The 5th digit (5) causes the 4th digit to round up from 9 to 10, which causes the 3rd digit to round up from 9 to 10, which causes the 2nd digit to round up from 2 to 3.

11. a. 153.86 cm^2 **b.** 153.9380 cm^2 **c.** 153.94 cm^2

d. Yes, because 3.14 is used as an estimate and is not the value of π as used on the calculator.

12. a. 65449.8470 cm^3 **b.** 9

c. Yes, any number with a finite number of decimal places is a rational number.

13. 13000

Exercise 2.5 Scientific notation

1. a. 614 **b.** 6140 **c.** 61400 **d.** 351.8

e. 1000000000 **f.** 392.673 **g.** 591.1 **h.** 5100

i. 734000 **j.** 7141400 **k.** 35.1 **l.** 80500

2. a. 5.00×10^3 **b.** 4.31×10^2 **c.** 3.8×10^1 **d.** 3.5×10^5

e. 7.25×10^1 **f.** 7.25×10^2 **g.** 7.25×10^3 **h.** 7.25×10^8

3. a. 4.379×10^1 **b.** 5.317×10^3 **c.** 2.590×10^2 **d.** 1.101×10^2

e. 1.632×10^6 **f.** 1.000×10^6 **g.** 1.235×10^8 **h.** 2.499×10^2

4. a. 0.2 **b.** 0.004 **c.** 0.0007 **d.** 0.03

e. 0.08273 **f.** 0.07295 **g.** 0.0029142 **h.** 0.00003753

i. 0.000529 **j.** 0.00003333 **k.** 0.000000002625 **l.** 0.0000000000000001273

5. a. 7×10^{-1} **b.** 5×10^{-3} **c.** 3×10^{-7} **d.** 1×10^{-11}

e. 2.31×10^{-1} **f.** 3.62×10^{-3} **g.** 7.31×10^{-4} **h.** 6.3×10^{-2}

6. a. 6.73×10^{-3} **b.** 1.43×10^{-1} **c.** 6.83×10^{-5} **d.** 5.12×10^{-9}

e. 5.09×10^{-2} **f.** 1.25×10^{-2} **g.** 7.31×10^{-4} **h.** 6.30×10^{-2}

7. a. 5.27×10^2, 8.31×10^2, 9.718×10^2, 3.27×10^3 **b.** 4.09×10^2, 4.37×10^2, 7.943×10^2, 7.95×10^2

c. 9.29×10^{-3}, 5.251×10^{-2}, 5.31×10^{-2}, 5.27×10^{-1} **d.** 2.7×10^{-3}, 7.13×10^{-2}, 8.31×10^2, 3.27×10^3

8. a. 0.00000000000000000000001994 g

b. 1.994×10^{-17} g

c. 5.015×10^{23} atoms

9. 3844 hours $\approx$ 160.17 days

10. 333333.33

11. The neutron is heavier by 2.3×10^{-30} kg.

12. a. 2.1×10^7 **b.** 1.29×10^8

13. a. Check with your teacher.

b. i. 5.36×10^7 **ii.** 5.36×10^7

c. The answers are the same because 2.95×10^3 is very small compared with 5.36×10^7.

14. 8.75×10^{-7} is such a small amount (0.000000875) that when it is added to 2.39×10^{-3}, it doesn't affect the value when given to 3 significant figures.

15. 1.5×10^{-2}, 14%, 0.5^2, $\frac{5}{8}$, 0.753, $3\sqrt{2}$, $\sqrt{2}$

16. a. i. 385440 km **ii.** 78489600 km **iii.** 1278960000 km **iv.** 5772840000 km

b. i. 385000 km **ii.** 78500000 km **iii.** 1280000000 km **iv.** 5770000000 km

c. i. 3.85×10^5 km **ii.** 7.85×10^7 km **iii.** 1.28×10^9 km **iv.** 5.77×10^9 km

17. a. i. 31 536 000 s **ii.** 3.1536×10^7 s

b. i. 9460 800 000 000 km **ii.** 9.4608×10^{12} km

c. i. 4.068×10^{13} km **ii.** 46 440 000 years

18. a. 5.972×10^{21} **b.** 4

19. a. 9.1094×10^{-31} **b.** 1.6725×10^{-27} **c.** 1.6726×10^{-27}

d. It is important to work with the original amounts and leave rounding until the end of a calculation so that the answer is accurate.

Challenge 2.2

1.394×10^6 km; 12 800 or 1.28×10^4 km

2.6 Review

1. A

2. a. Irrational, since equal to non-recurring and non-terminating decimal

b. Rational, since can be expressed as a whole number

c. Rational, since given in a rational form

d. Rational, since it is a recurring decimal

e. Irrational, since equal to non-recurring and non-terminating decimal

3. d **4.** c, d **5.** c **6.** b

7. 1.4×10^{12} **8.** 4.167×10^8 **9.** 9.16×10^7 km **10.** 1.2441×10^9 km

11. a. $3.4 \times 10^7, 1.5 \times 10^9, 6 \times 10^9$ **b.** 0.57%

c. 4 **d.** 0.25 km^2

12. 820 000 queen bees

13. 2.95%

14. During the 5th year or 5.5 years

15. a. Mach 9.1

b.

Mach number	Speed m/s	Speed km/h
1	343	1234.8
2	686	2469.6
3	1029	3704.4
9.1	3130.6	11 270

c. Travelling at Mach 1 would take 32.39 hours.

Travelling at Mach 2 would take 16.20 hours.

Travelling at Mach 3 would take 10.79 hours.

d. 3.55 hours

e. The fastest aircraft would travel the Earth 9.12 times in the time an aircraft travelling at Mach 1 would circle the Earth once.

The fastest aircraft would travel the Earth 4.56 times in the time an aircraft travelling at Mach 2 would circle the Earth once.

The fastest aircraft would travel the Earth 3.04 times in the time an aircraft travelling at Mach 3 would circle the Earth once.

f. (Number of times to circle Earth) × (Mach value) is approximately equal to 9.1.

Investigation — Rich task

1.

Square	1	2	3	4
Side length (cm)	2	**4**	**6**	**8**
Diagonal length (cm)	$2\sqrt{2}$	$\mathbf{4\sqrt{2}}$	$\mathbf{6\sqrt{2}}$	$\mathbf{8\sqrt{2}}$
Perimeter (cm)	8	**16**	**24**	**32**
Area (cm^2)	4	**16**	**36**	**64**

2. 20 cm

3. $20\sqrt{2}$ cm

4.

Square	1	2	3	4
Side length (cm)	$\sqrt{2}$	$\mathbf{2\sqrt{2}}$	$\mathbf{3\sqrt{2}}$	$\mathbf{4\sqrt{2}}$
Diagonal length (cm)	2	**4**	**6**	**8**
Perimeter (cm)	$4\sqrt{2}$	$\mathbf{8\sqrt{2}}$	$\mathbf{12\sqrt{2}}$	$\mathbf{16\sqrt{2}}$
Area (cm^2)	2	**8**	**18**	**32**

5. $10\sqrt{2}$ cm

6. 20 cm

7. First diagram

8. First diagram

9. The squares in the first diagram are twice the area of the corresponding square in second diagram.

10. The squares in the first diagram have a perimeter that is $\sqrt{2}$ times the perimeter of the corresponding square in the second diagram.

11. An increase of 60 cm^2 in the first diagram and an increase of 30 cm^2 for the second diagram

12. 2π

13. $\pi(2r + 1)$

TOPIC 3 Algebra

3.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



3.1.1 Why learn this?

Algebra relates to you and the world around you. It is part of everyday life and you will be using it without knowing it. If you want to be a scientist or engineer, you will certainly need algebra. If you want to do well in Maths, you will need to study algebra. When you learn algebra you learn a different way of thinking. It helps with problem solving, decision making, reasoning and creative thinking.

3.1.2 What do you know?

assessON

1. **THINK** Use a thinking tool such as a concept map to list what you know about algebra.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a concept map to show your class's knowledge of algebra.

LEARNING SEQUENCE

3.1 Overview
3.2 Using pronumerals
3.3 Algebra in worded problems
3.4 Simplifying algebraic expressions
3.5 Expanding brackets
3.6 Expansion patterns
3.7 Further expansions
3.8 The highest common factor
3.9 The highest common binomial factor
3.10 Applications
3.11 Review

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Watch this eLesson: The story of mathematics: One small step for man . . . (eles-1690)

3.2 Using pronumerals

- A **pronumeral** is a letter or symbol which stands for a number. Algebra is the branch of mathematics where we use and manipulate pronumerals.

3.2.1 Expressions

- In algebra an **expression** is a group of **terms** in which each term is separated by a + sign or a − sign.
- Here are some important things to note about the expression $5x - 3y + 2$:
  - The expression has 3 terms and is called a **trinomial**.
  - The pronumerals x and y may take different values and are called **variables**.
  - The term 2 has only one value and is called a **constant**.
  - The **coefficient** (number in front) of x is 5, and the coefficient of y is -3.
- An expression with 2 terms is called a binomial, and an expression with 1 term is called a monomial. A polynomial is an expression with many terms.

WORKED EXAMPLE 1

For the expression $6x - 3xy + z + 2 + x^2z + \frac{y^2}{7}$, determine:

a the number of terms
b the coefficient of the second term
c the coefficient of the last term
d the constant term
e the term with the smallest coefficient
f the coefficient of x^2z.

THINK

a Count the number of terms.

b The second term is $-3xy$. The number part is the coefficient.

c The last term is $\frac{y^2}{7}$. This can be rewritten as $\frac{1}{7} \times y^2$.

d The constant is the term with no pronumeral.

e Identify the smallest coefficient and write the whole term to which it belongs.

f x^2z can be written as $1x^2z$.

WRITE

a There are 6 terms.

b The coefficient of the second term is -3.

c The coefficient of the last term is $\frac{1}{7}$.

d The constant term is 2.

e The term with the smallest coefficient is $-3xy$.

f The coefficient of x^2z is 1.

3.2.2 Substitution

- We can evaluate (find the value of) an algebraic expression if we replace the pronumerals with their known values.
- This process is called substitution.
- Consider the expression $4x + 3y$. If we substitute the values $x = 2$ and $y = 5$, then the expression takes the value

$$\begin{aligned} 4 \times 2 + 3 \times 5 &= 8 + 15 \\ &= 23 \end{aligned}$$

- Rather than showing the multiplication signs, it is common in mathematics to write the substituted values in brackets.
  We would write the example above as:

$$\begin{aligned} 4x + 3y &= 4(2) + 3(5) \\ &= 8 + 15 \\ &= 23 \end{aligned}$$

WORKED EXAMPLE 2

Evaluate the following terms for the values $x = 4$ and $x = -4$.
a $3x^2$ **b** $-x^2$ **c** $\sqrt{5 - x}$

THINK	WRITE
a 1 Substitute 4 for x.	**a** If $x = 4$: $3x^2 = 3(4)^2$
2 Only the x term is squared.	$= 3 \times 16$
3 Evaluate.	$= 48$
4 Repeat for $x = -4$.	If $x = -4$: $3x^2 = 3(-4)^2$
	$= 3 \times 16$
	$= 48$
b 1 Substitute 4 for x.	**b** If $x = 4$: $-x^2 = -(4)^2$
2 Evaluate.	$= -16$
3 Repeat for $x = -4$.	If $x = -4$: $-x^2 = -(-4)^2$
	$= -16$
c 1 Substitute 4 for x.	**c** If $x = 4$: $\sqrt{5 - x} = \sqrt{5 - 4}$
2 Evaluate.	$= \sqrt{1}$
	$= 1$
3 Repeat for $x = -4$.	If $x = -4$: $\sqrt{5 - x} = \sqrt{5 - (-4)}$
	$= \sqrt{5 + 4}$
	$= \sqrt{9}$
	$= 3$

WORKED EXAMPLE 3

TI | CASIO

If $x = 3$ and $y = -2$, evaluate the following expressions.
a $3x + 2y$ **b** $5xy - 3x + 1$ **c** $2x^2 + y^2$

THINK	WRITE
a 1 Write the expression.	**a** $3x + 2y$
2 Substitute $x = 3$ and $y = -2$.	$= 3(3) + 2(-2)$
3 Evaluate.	$= 9 - 4$
	$= 5$
b 1 Write the expression.	**b** $5xy - 3x + 1$
2 Substitute $x = 3$ and $y = -2$.	$= 5(3)(-2) - 3(3) + 1$
3 Evaluate.	$= -30 - 9 + 1$
	$= -38$
c 1 Write the expression.	**c** $2x^2 + y^2$
2 Substitute $x = 3$ and $y = -2$.	$= 2(3)^2 + (-2)^2$
3 Evaluate.	$= 18 + 4$
	$= 22$

3.2.3 Substitution into formulas

- A **formula** is a mathematical rule, usually written using pronumerals.
- We know that the formula for the area of a rectangle is given by:

$$A = lw$$

where A represents the area of the rectangle, l represents the length and w represents the width.

- If a rectangular kitchen tile has length $l = 20$ cm and width $w = 15$ cm, we can substitute these values into the formula to find its area.

$$\begin{aligned} A &= lw \\ &= 20 \times 15 \\ &= 300 \text{ cm}^2 \end{aligned}$$

- If we are given the area of the tile to be 400 cm^2 and the width to be 55 cm, then we can calculate its length.

$$\begin{aligned} A &= lw \\ 400 &= l \times 55 \\ l &= \frac{400}{55} \\ &= \frac{80}{55} \\ &\approx 7.3 \text{ cm (to 1 d.p.)} \end{aligned}$$



WORKED EXAMPLE 4

The formula for the voltage in an electrical circuit can be found using the formula known as Ohm's Law:

$$V = IR$$

where I = current in amperes
R = resistance in ohms
V = voltage in volts.

a Calculate V when:
i $I = 2$ amperes, $R = 10$ ohms
ii $I = 20$ amperes, $R = 10$ ohms.

b Calculate I when $V = 300$ volts and $R = 600$ ohms.

THINK	WRITE
a i 1 Write the formula.	**a i** $V = IR$
2 Substitute $I = 2$ and $R = 10$.	$= (2)(10)$
3 Evaluate and express the answer in the correct units.	$= 20$ The voltage is 20 volts.
ii 1 Write the formula.	**ii** $V = IR$
2 Substitute $I = 20$ and $R = 10$.	$= (20)(10)$
3 Evaluate and express the answer in the correct units.	$= 200$ The voltage is 200 volts.
b 1 Write the formula.	**b** $V = IR$
2 Substitute $V = 300$ and $R = 600$.	$300 = I(600)$

3 Evaluate and express the answer in the correct units.

$$I = \frac{300}{600}$$
$$= 0.5$$

The current is 0.5 amperes

Note: Even if we know nothing about volts, amperes and ohms, we can still do the calculation.

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Complete this digital doc: SkillSHEET: Alternative expressions used for the four operations (doc-6122)

Exercise 3.2 Using pronumerals

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a–h, 3a–f, 4–6, 8, 9a–g, 10–12, 20	Questions: 1, 2e–l, 3c–h, 4–8, 9d–k, 13–16, 18, 20	Questions: 1, 2i–o, 3e–j, 4–6, 8, 9j–o, 15–20

Individual pathway interactivity: int-4480

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. Find the coefficient of each of the following terms.

a. $3x$ **b.** $7a$ **c.** $-2m$ **d.** $-8q$ **e.** w

f. $-n$ **g.** $\frac{x}{3}$ **h.** $\frac{y}{2}$ **i.** $-\frac{at}{4}$ **j.** $-\frac{r^2}{9}$

2. State the coefficient of the x term in each of the following expressions.

a. $6x - 3y$ **b.** $5 + 7x$ **c.** $5x^2 + 3x - 2$ **d.** $-7x^2 - 2x + 4$

e. $3x - 2x^2$ **f.** $-9x^2 - 2x$ **g.** $5x^2 + 3 - 7x$ **h.** $-11x + 5 - 2x^2$

i. $2ax^3 - \frac{x}{2} + x^2$ **j.** $1 - \frac{2x}{3} - bx^2$ **k.** $\frac{x^2}{6} + \frac{5}{7} + \frac{x}{4}$ **l.** $x^3 + x + 4$

m. $-3x - 4bx + 6$ **n.** $4cx^2 - 2x + 4ax$ **o.** $2x^2 - 5$

3. WE1 For each expression below, write:

i. the number of terms **ii.** the coefficient of the first term

iii. the constant term **iv.** the term with the smallest coefficient.

a. $5x^2 + 7x + 8$ **b.** $-9m^2 + 8m - 6$ **c.** $5x^2y - 7x^2 + 8xy + 5$

d. $9ab^2 - 8a - 9b^2 + 4$ **e.** $11p^2q^2 - 4 + 5p - 7q - p^2$ **f.** $-9p + 5 - 7q^2 + 5p^2q + q$

g. $4a - 2 + 9a^2b^2 - 3ac$ **h.** $5s + s^2t + 9 + 12t - 3u$ **i.** $-m + 8 + 5n^2m + m^2 + 2n$

j. $7c^2d + 5d^2 + 14 - 3cd^2 - 2e$

4. WE2 Evaluate the following expressions for $x = 1$, $x = 0$ and $x = -2$.

a. $5x + 2$ **b.** $x^2 + 5x + 2$ **c.** $x(x + 2)$

d. $x^2 + x - 2$ **e.** $\sqrt{x^2}$ **f.** $\frac{2}{x}$

5. MC **a.** If $x = -3$, then the value of $-5x - 3$ is:

A. -18 **B.** 12 **C.** 30 **D.** 18

b. An expression that is a trinomial is:

A. $3x^2$ **B.** $x + 5 + 2$ **C.** $x^2 - 7$ **D.** $x^2 + x + 2$

c. The expression $\sqrt{(x - 6)}$ cannot be evaluated if x is equal to:

A. 2 **B.** 6 **C.** 7 **D.** 10

d. In the expression $5x^2 - 3xy + 0.5x - 0.3y - 5$, the smallest coefficient is:

A. 5 **B.** -3 **C.** 0.5 **D.** -0.3

6. WE3 Find the value of the following expressions if $x = 2$, $y = -1$ and $z = 3$.

a. $2x$ **b.** $3xy$ **c.** $2y^2z$

d. $\frac{14}{x}$ **e.** $6(2x + 3y - z)$ **f.** $x^2 - y^2 + xyz$

7. If $x = 4$ and $y = -3$, evaluate the following expressions.

a. $4x + 3y$ **b.** $3xy - 2x + 4$ **c.** $x^2 - y^2$

Understanding

8. WE4 The change in the voltage in an electrical circuit can be found using the formula known as Ohm's Law: $V = IR$, where I = current in amperes, R = resistance in ohms and V = voltage in volts.

a. Calculate V when:

i. $I = 4$, $R = 8$ **ii.** $I = 25$, $R = 10$.

b. Calculate R when:

i. $V = 100$, $I = 25$ **ii.** $V = 90$, $I = 30$.

9. Evaluate each of the following by substituting the given values into each formula.

a. If $A = bh$, find A when $b = 5$ and $h = 3$.

b. If $d = \frac{m}{v}$, find d when $m = 30$ and $v = 3$.

c. If $A = \frac{1}{2}xy$, find A when $x = 18$ and $y = 2$.

d. If $A = \frac{1}{2}(a + b)h$, find A when $h = 10$, $a = 7$ and $b = 2$.

e. If $V = \frac{AH}{3}$, find V when $A = 9$ and $H = 10$.

f. If $v = u + at$, find v when $u = 4$, $a = 3.2$ and $t = 2.1$.

g. If $t = a + (n - 1)d$, find t when $a = 3$, $n = 10$ and $d = 2$.

h. If $A = \frac{1}{2}(x + y)h$, find A when $x = 5$, $y = 9$ and $h = 3.2$.

i. If $A = 2b^2$, find A when $b = 5$.

j. If $y = 5x^2 - 9$, find y when $x = 6$.

k. If $y = x^2 - 2x + 4$, find y when $x = 2$.

l. If $a = -3b^2 + 5b - 2$, find a when $b = 4$.

m. If $s = ut + \frac{1}{2}at^2$, find s when $u = 0.8$, $t = 5$ and $a = 2.3$.

n. If $F = \frac{mp}{r^2}$, find F correct to 2 decimal places, when $m = 6.9$, $p = 8$ and $r = 1.2$.

o. If $C = \pi d$, find C correct to 2 decimal places if $d = 11$.

Reasoning

10. The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the length of the base and h is the perpendicular height of the triangle.

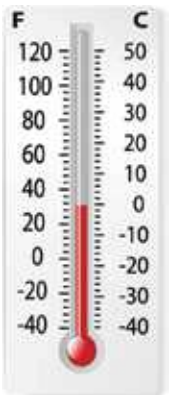
a. Show that the area is 12 cm^2 when $b = 6$ cm and $h = 4$ cm.

b. What is h if $A = 24$ cm^2 and $b = 4$ cm?

11. The formula to convert degrees Fahrenheit (F) to degrees Celsius (C) is $C = \frac{5}{9}(F - 32)$.

a. Find C when $F = 59$.

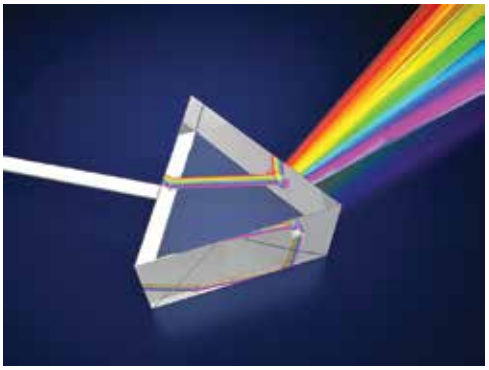
b. Show that when Celsius (C) is 15, Fahrenheit (F) is 59.



12. The length of the hypotenuse of a right-angled triangle (c) can be found using the formula, $c = \sqrt{a^2 + b^2}$, where a and b are the lengths of the other two sides.

a. Find c when $a = 3$ and $b = 4$.

b. Show that when $a = 5$ and $c = 13$, $b = 12$.



13. If the volume of a prism (V) is given by the formula $V = AH$, where A is the area of the cross-section and H is the height of the prism, determine:

a. V when $A = 7$ cm^2 and $H = 9$ cm

b. H when $V = 120$ cm^3 and $A = 30$ cm^2.

14. Using $E = F + V - 2$ where F is the number of faces on a prism, E is the number of edges and V is the number of vertices, calculate:

a. E if $F = 5$ and $V = 7$

b. F if $E = 10$ and $V = 2$.

15. The kinetic energy (E) of an object is found by using the formula $E = \frac{1}{2}mv^2$ where m is the mass and v is the velocity of the object.

a. Determine E when $m = 3$ and $v = 3.6$.

b. Show that when $E = 25$ and $v = 5$, $m = 2$.

16. The volume of a cylinder (v) is given by $v = \pi r^2 h$, where r is the radius in centimetres and h is the height of the cylinder in centimetres.

a. Determine v correct to 2 decimal places if $r = 7$ and $h = 3$.

b. Determine h correct to 2 decimal places if $v = 120$ and $r = 2$.

17. The surface area of a cylinder (S) is given by $S = 2\pi r(r + h)$ where r is the radius of the circular end and h is the height of the cylinder.

a. Calculate S (to 2 decimal places) if $r = 14$ and $h = 5$.

b. Show that for a cylinder of surface area 240 units^2 and radius 5 units, the height is 2.64 units, correct to 2 decimal places.

Problem solving

18. a. In a magic square, every row, column and diagonal adds to the same number. This number is called the magic number for that square. Complete the unfinished magic square by first finding an algebraic expression for the magic number.

	$2d+2g$	
	$d+g$	
	0	$d+2g$

b. Make three different magic squares by substituting different values for g and d in the completed magic square from part **a**.

c. Calculate the magic number for each magic square.

19. A flat, rectangular board is built by gluing together a number of square pieces of the same size. The board is m squares wide and n squares long. In terms of m and n, write expressions for:

a. the total number of squares

b. the number of completely surrounded squares

c. the perimeter of the figure, given that each square is 1 unit by 1 unit.

20. Which one of the following eight expressions is not equivalent to the others?

$x - y + z$ $\quad$ $z + x - y$

$z - y + x$ $\quad$ $x - (y - z)$

$z - (y - x)$ $\quad$ $y - (z - x)$

$-y + x + z$ $\quad$ $x + z - y$

Reflection

Which letters (pronumerals) should you avoid using when writing algebraic expressions?

3.3 Algebra in worded problems

3.3.1 Algebra in worded problems

- An important skill in algebra is to be able to convert worded expressions and sentences into algebraic expressions.
- The first step in converting a worded statement into an algebraic expression is to assign pronumerals to the unknown quantities.

WORKED EXAMPLE 5

Write an algebraic expression for each of the following, choosing an appropriate pronumeral if necessary.

a The number that is 6 more than Ben's age
b The total value of some $10 notes
c The total cost of 8 adults' and 3 children's train tickets
d The product of a and w
e w less than a

THINK	WRITE
a 1 Ben's age is the unknown quantity. *Note:* You must not say 'Let Ben = b'; b is a number.	**a** Let Ben's age = b years.
2 Write an expression for Ben's age plus 6.	$b + 6$
b 1 The number of notes is the unknown quantity.	**b** Let n = the number of $10 notes.
2 The value is 10 times the number of notes.	Total value = $10n$ dollars
c 1 The cost of tickets is the unknown quantity.	**c** Let an adult's ticket cost a dollars and a child's ticket cost c dollars. Total cost = $8a + 3c$ dollars.
2 The adults' tickets cost $8 \times a$ and the children's tickets cost $3 \times c$.	
d 'Product' means to multiply.	**d** $a \times w$
e w is subtracted from a.	**e** $a - w$

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Complete this digital doc: SkillSHEET: Algebraic expressions (doc-6123)
Complete this digital doc: SkillSHEET: Substitution into algebraic expressions (doc-6124)

Exercise 3.3 Algebra in worded problems

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7, 12, 14	Questions: 3–8, 10, 12, 14, 15	Questions: 4–11, 13, 15, 16

Individual pathway interactivity: int-4481

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. Jacqueline studies 5 more subjects than Helena. How many subjects does Jacqueline study if:
- **a.** Helena studies 6 subjects?
- **b.** Helena studies x subjects?
- **c.** Helena studies y subjects?



2. Dianne and Angela walk home from school together. Dianne's home is 2 km further from school than Angela's home. How far does Dianne walk if Angela's home is:
- **a.** 1.5 km from school
- **b.** x km from school?

3. Lisa watched television for 2.5 hours today. How many hours will she watch tomorrow if she watches:
- **a.** 1.5 hours more than she watched today
- **b.** t hours more than she watched today
- **c.** y hours fewer than she watched today?

Understanding

4. WE5 Write an algebraic expression for each of the following questions.
- **a.** If it takes 10 minutes to iron a single shirt, how long would it take to iron all of Anthony's shirts?
- **b.** Ross has 30 dollars more than Nick. If Nick has N dollars, how much money does Ross have?
- **c.** In a game of Aussie rules, Luciano kicked 4 more goals than he kicked behinds.
  - **i.** How many behinds did Luciano score if g is the number of goals kicked?
  - **ii.** How many points did Luciano score?

  (*Remember:* 1 goal scores 6 points, 1 behind scores 1 point.)

5. Jeff and Chris play Aussie Rules football for opposing teams, and Jeff's team won when the two teams played one another.



- **a.** How many points did Jeff's team score if they kicked:
  - **i.** 14 goals and 10 behinds
  - **ii.** x goals and y behinds?
- **b.** How many points did Chris's team score if his team kicked:
  - **i.** 10 goals and 6 behinds
  - **ii.** p goals and q behinds?
- **c.** How many points did Jeff's team win by if:
  - **i.** Chris's team scored 10 goals and 6 behinds, and Jeff's team scored 14 goals and 10 behinds?
  - **ii.** Chris's team scored p goals and q behinds, and Jeff's team scored x goals and y behinds?

6. Yvonne's mother gives her x dollars for each school subject she passes. If she passes y subjects, how much money does she receive?

7. Brian buys a bag containing x Smarties.



- **a.** If he divides them equally among n people, how many does each person receive?
- **b.** If he keeps half the Smarties for himself and divides the remaining Smarties equally among n people, how many does each person receive?

8. A piece of licorice is 30 cm long.

- **a.** If David cuts d cm off, how much licorice remains?
- **b.** If David cuts off $\frac{1}{4}$ of the remaining licorice, how much licorice has been cut off?
- **c.** How much licorice remains now?

Reasoning

9. One-quarter of a class of x students plays tennis on the weekend. One-sixth of the class plays tennis and also swims on the weekend.

- **a.** Write an expression to represent the number of students playing tennis on the weekend.
- **b.** Write an expression to represent the number of students playing tennis and also swimming on the weekend.
- **c.** Show that the number of students playing only tennis on the weekend is $\frac{x}{12}$.

10. During a 24-hour period, Vanessa uses her computer for c hours. Her brother Darren uses it for $\frac{1}{7}$ of the remaining time.

- **a.** For how long does Darren use the computer?
- **b.** Show that the total number of hours that Vanessa and Darren use the computer during a 24-hour period can be expressed as $\frac{6c + 24}{7}$.

11. Marty had a birthday party last weekend and invited n friends. The table on next page indicates the number of friends at Marty's party at certain times during the evening. Everybody arrived by 8.30 pm and had all left the party by 11 pm.

a. Show that 1 person arrived between 7.00 pm and 7.30 pm.
b. Between which times were the most friends present at the party?
c. How many friends were invited but did not arrive?
d. Prove that 24 friends were invited in total.
e. Between which times did the most friends arrive?

Time	Number of friends
7.00 pm	$n - 24$
7.30 pm	$n - 23$
8.00 pm	$n - 8$
8.30 pm	$n - 5$
9.00 pm	$n - 5$
9.30 pm	$n - 7$
10.00 pm	$n - 12$
10.30 pm	$n - 18$
11.00 pm	$n - 24$

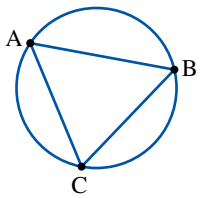
Problem solving

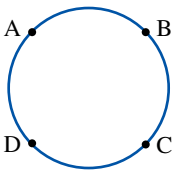
12. A child builds a pyramid as shown.



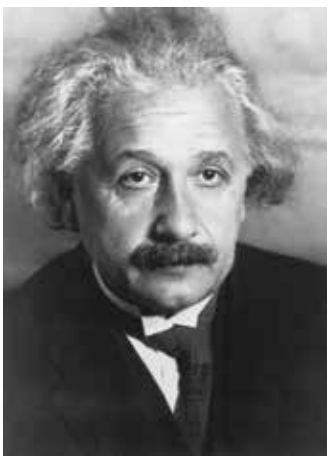
a. Find a rule which gives the number of blocks on the bottom layer, b, for a tower that is h blocks high.
b. If the child wishes the tower to be 10 blocks high, how many blocks should she begin with on the bottom layer?

13. If a circle has three equally spaced dots (A, B and C) on its circumference, only 1 triangle can be formed by joining the dots. Investigate to determine the number of triangles that could be formed by 4, 5, . . . equally spaced dots on the circumference.





From your investigation, can you see a pattern in the number of triangles (t) formed for a circle with d equally spaced dots on its circumference? Is it possible to express this as a formula?



14. A shop owner is ordering vases from a stock catalogue. Red vases cost \$4.20 and clear vases cost \$5.70. If 25 vases cost \$120, how many of each type of vase did she buy?

15. a. Investigate what the variables in the formula $E = mc^2$ mean.
b. Prepare a presentation about Albert Einstein and present this to the class.

16. a. Transpose the equation $T = \frac{x - 3}{x - 1}$ to make x the subject.
b. What is the value of T when $x = 1$? Explain.
c. What is the value of x when $T = 1$? Explain.

Reflection

Why is it important to define pronumerals or variables in terms of what they represent?

CHALLENGE 3.1

A father is 4 times the age of his son. In 4 years time, the father will be 3 times the age of his son. How old is the father now?

3.4 Simplifying algebraic expressions

- When we simplify an expression, we write it with the smallest number of terms, without using $\times$ or $\div$ signs.

3.4.1 Addition and subtraction of like terms

- **Like terms** have identical pronumeral parts.
  For example: $3x$ and x are like terms, but $3x^2$ and x are not.
  $5a^2b$ and $-2ba^2$ are like terms, but $5a^2b$ and $-2ab^2$ are not.
  $6x + 5x$ can be simplified (to $11x$), but $6x + 5$ cannot.
- When there is more than one pronumeral in a term, it is preferable to write them in alphabetical order. For example, $3xy$ is preferable to $3yx$, although these are identical terms. This makes it easy to recognise like terms.

WORKED EXAMPLE 6

Simplify the following expressions.

a $x + 2x + 3x - 5$ **b** $9a^2b + 5ba^2 + ab^2$ **c** $-12 - 4c^2 + 10 + 5c^2$

THINK	WRITE
a 1 Write the expression.	**a** $x + 2x + 3x - 5$
2 Collect the like terms: x, $2x$, $3x$.	$= 6x - 5$
b 1 Write the expression.	**b** $9a^2b + 5ba^2 + ab^2$
2 Collect the like terms: $9a^2b$, $5ba^2$.	$= 14a^2b + ab^2$
c 1 Write the expression.	**c** $-12 - 4c^2 + 10 + 5c^2$
2 Collect the like terms.	$= -12 + 10 - 4c^2 + 5c^2$
3 Simplify.	$= -2 + c^2$ or $c^2 - 2$

3.4.2 Multiplication and division

Multiplication

- Any two algebraic terms can be multiplied together.
- Terms can be multiplied together in any order, so it is easier to deal with the coefficients first and then each pronumeral in turn.

$$\begin{aligned} 3xy \times 5y &= 3 \times x \times y \times 5 \times y \\ &= 3 \times 5 \times x \times y \times y \\ &= 15 \times x \times y^2 \\ &= 15xy^2 \end{aligned}$$

WORKED EXAMPLE 7

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Simplify the following terms.

a $4a \times 2b \times a$ **b** $7ax \times -6bx \times -2abx$

THINK	WRITE
a 1 Write the term.	**a** $4a \times 2b \times a$
2 Rearrange, writing the coefficients first.	$= 4 \times 2 \times a \times a \times b$
3 Multiply the coefficients and pronumerals separately.	$= 8a^2b$
b 1 Write the term.	**b** $7ax \times -6bx \times -2abx$
2 Rearrange, writing the coefficients first.	$= 7 \times -6 \times -2 \times a \times a \times b \times b \times x \times x \times x$
3 Multiply the coefficients and pronumeral separately.	$= 84a^2b^2x^3$

3.4.3 Division

- Division is performed by writing the terms in fraction form and then cancelling to simplify.

For example, $10x \div 5x = \frac{10x}{5x}$

$= \frac{{}^{2}\cancel{10}\cancel{x}}{{}_{1}\cancel{5}\cancel{x}}$

$= 2$

WORKED EXAMPLE 8

Simplify the following terms.

a $\frac{12xy}{4xz}$ **b** $-8ab^2 \div -16a^2b$

THINK	WRITE
a 1 Write the term.	**a** $\frac{12xy}{4xz}$
2 Cancel the common factors.	$= \frac{{}^{3}\cancel{12} \times \cancel{x} \times y}{{}_{1}\cancel{4} \times \cancel{x} \times z}$
3 Write the answer.	$= \frac{3y}{z}$
b 1 Write the term as a fraction.	**b** $\frac{-8ab^2}{-16a^2b}$
2 Cancel the common factors.	$= \frac{{}^{1}\cancel{-8} \times {}^{1}\cancel{a} \times b\cancel{^2}\cancel{b}}{{}_{2}\cancel{-16} \times {}_{a}\cancel{a^2} \times \cancel{b}_{1}}$
3 Simplify and write the answer.	$= \frac{1}{2} \times \frac{1}{a} \times \frac{b}{1}$
	$= \frac{b}{2a}$

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Complete this digital doc: SkillSHEET: Like terms (doc-6125)
Complete this digital doc: SkillSHEET: Collecting like terms (doc-6126)
Complete this digital doc: SkillSHEET: Multiplying algebraic terms (doc-6127)
Complete this digital doc: SkillSHEET: Dividing algebraic terms (doc-6128)
Complete this digital doc: SkillSHEET: Adding and subtracting integers (doc-10817)
Complete this digital doc: SkillSHEET: Multiplying and dividing integers (doc-10818)
Complete this digital doc: WorkSHEET: Introductory algebra (doc-6138)

Exercise 3.4 Simplifying algebraic expressions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a–l, 3, 4a–i, 5a–l, 6, 7, 10, 12	Questions: 1, 2g–r, 3, 4g–o, 5d–o, 6–13	Questions: 1, 2p–z, 3, 4k–r, 5h–r, 6, 8–11, 13–15

Individual pathway interactivity: int-4482

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. For each of the following terms, select those terms listed in brackets that are like terms.

a. $6ab$ $(7a, 8b, 9ab, -ab, 4a^2b^2)$
b. $-x$ $(3xy, -xy, 4x, 4y, -yx)$
c. $3az$ $(3ay, -3za, -az, 3z^2a, 3a^2z)$
d. x^2 $(2x, 2x^2, 2x^3, -2x, -x^2)$
e. $-2x^2y$ $(xy, -2xy, -2xy^2, -2x^2y, -2x^2y^2)$
f. $3x^2y^5$ $(3xy, 3x^5y^2, 3x^4y^3, -x^2y^5, -3x^2y^5)$
g. $5x^2p^3w^5$ $(-5x^3w^5p^3, p^3x^2w^5, 5xp^3w^5, -5x^2p^3w^5, w^5p^2x^3)$
h. $-x^2y^5z^4$ $(-xy^5, -y^2z^5x^4, -x + y + z, 4y^5z^4x^2, -2x^2z^4y^5)$

2. WE6 Simplify the following expressions.

a. $5x + 2x$
b. $3y + 8y$
c. $7m + 12m$
d. $13q - 2q$
e. $17r - 9r$
f. $-x + 4x$
g. $5a + 2a + a$
h. $9y + 2y - 3y$
i. $7x - 2x + 8x$
j. $14p - 3p + 5p$
k. $2q^2 + 7q^2$
l. $5x^2 - 2x^2$
m. $6x^2 + 2x^2 - 3y$
n. $3m^2 + 2n - m^2$
o. $9x^2 + x - 2x^2$
p. $9h^2 - 2h + 3h + 9$
q. $-2g^2 - 4g + 5g - 12$
r. $-5m^2 + 5m - 4m + 15$
s. $12a^2 + 3b + 4b^2 - 2b$
t. $6m + 2n^2 - 3m + 5n^2$
u. $3xy + 2y^2 + 9yx$
v. $3ab + 3a^2b + 2a^2b - ab$
w. $9x^2y - 3xy + 7yx^2$
x. $4m^2n + 3n - 3m^2n + 8n$
y. $-3x^2 - 4yx^2 - 4x^2 + 6x^2y$
z. $4 - 2a^2b - ba^2 + 5b - 9a^2$

3. MC Simplify the following expressions.

a. $18p - 19p$

A. p **B.** $-p$ **C.** p^2 **D.** -1

b. $5x^2 - 8x + 6x - 9$

A. $3x - 9$ **B.** $3x^2 - 9$ **C.** $5x^2 + 2x - 9$ **D.** $5x^2 - 2x - 9$

c. $12a - a + 15b - 14b$

A. $11a + b$ **B.** 12 **C.** $11a - b$ **D.** $13a + b$

d. $-7m^2n + 5m^2 + 3 - m^2 + 2m^2n$

A. $-9m^2n + 4m^2 + 3$ **B.** $-9m^2n + 8$ **C.** $-5m^2n - 4m^2 + 3$ **D.** $-5m^2n + 4m^2 + 3$

4. WE7 Simplify the following.

a. $3m \times 2n$ **b.** $4x \times 5y$ **c.** $2p \times 4q$ **d.** $5x \times -2y$ **e.** $3y \times -4x$

f. $-3m \times -5n$ **g.** $5a \times 2a$ **h.** $4y \times 5y$ **i.** $5p \times p$ **j.** $m \times 7m$

k. $3mn \times 2p$ **l.** $-6ab \times b$ **m.** $-5m \times -2mn$ **n.** $-6a \times 3ab$ **o.** $-3xy \times -5xy \times 2x$

p. $4pq \times -p \times 3q^2$ **q.** $4c \times -7cd \times 2c$ **r.** $-3a^2 \times -5ab^3 \times 2ab^4$

5. WE8 Simplify the following.

a. $\frac{6x}{2}$ **b.** $\frac{9m}{3}$ **c.** $\frac{12y}{6}$ **d.** $\frac{8m}{2}$ **e.** $12m \div 3$

f. $14x \div 7$ **g.** $-21x \div 3$ **h.** $-32m \div 8$ **i.** $\frac{4m}{8}$ **j.** $\frac{6x}{18}$

k. $\frac{8mn}{18n}$ **l.** $\frac{16xy}{12y}$ **m.** $\frac{2}{10m}$ **n.** $\frac{6ab}{12a^2b}$ **o.** $\frac{28xyz}{14x}$

p. $\frac{70ab^2}{4b}$ **q.** $\frac{2x^2yz}{8xz}$ **r.** $-7xy^2z^2 \div 11xyz$

6. Simplify the following.

a. $5x \times 4y \times 2xy$ **b.** $7xy \times 4ax \times 2y$ **c.** $x \times 4xy \times 3yx$

d. $\frac{6x^2y}{12y^2}$ **e.** $\frac{-15x^2ab}{12b^2x^2}$ **f.** $\frac{2p^3q^2}{p^3q^2}$

g. $-4a \times -5ab^2 \times 2a$ **h.** $-a \times 4ab \times 2ba \times b$ **i.** $2a \times 2a \times 2a \times 2a$

Understanding

7. Jim buys m pens at p cents each and n books at q dollars each.

a. How much does he spend in:

i. dollars **ii.** cents?

b. What is his change from $20?

8. At a local discount clothing store 4 shirts and 3 pairs of trousers cost $138. If a pair of trousers cost 2.5 times as much as a shirt, determine the cost of each.

Reasoning

9. Class 9A were given an algebra test. One of the questions is shown below.

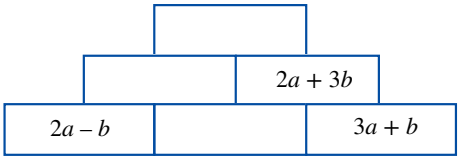
Simplify the following expression: $\frac{3ab}{2} \times \frac{4ac}{6b} \times 7c$.

Sean, who is a student in class 9A, wrote his answer as $\frac{12aabc}{12b} \times 7c$. Explain why Sean's answer is incorrect, and write the correct answer.

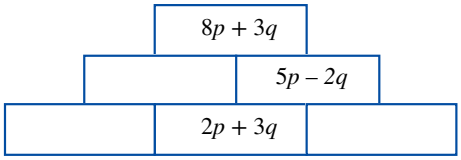
Problem solving

10. a. Fill in the empty bricks in the following pyramids. The expression in each brick is obtained by adding the expressions in the two bricks below it.

i.

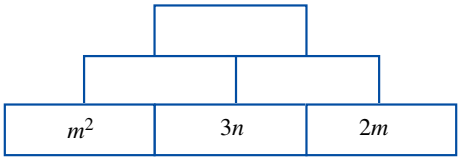


ii.

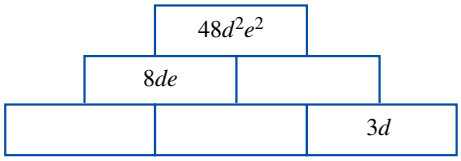


b. Fill in the empty bricks in the following pyramids. The expression in each brick is obtained by multiplying the expressions in the two bricks below it.

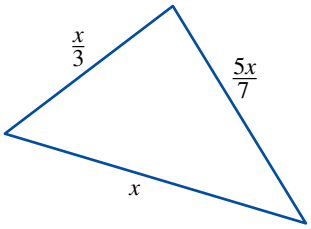
i.



ii.

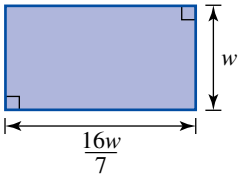


11. For the triangle shown:



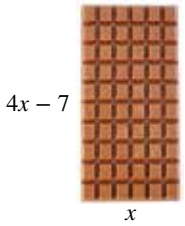
- **a.** write an expression for the perimeter of the triangle
- **b.** show that the perimeter can be simplified to $\frac{43}{21}x$
- **c.** if x is 2 metres, calculate the cost (to the nearest dollar) to frame the triangle using timber which costs \$4.50 per metre.

12. For the rectangle shown:



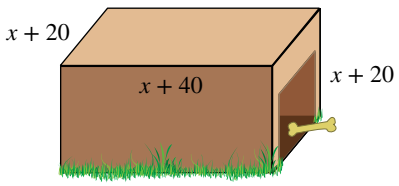
- **a.** write an expansion for the perimeter of the rectangle
- **b.** show that the perimeter can be simplified to $6\frac{4}{7}w$
- **c.** if $w = 7$ metres, calculate the cost (to the nearest cent) to create a wire frame for the rectangle if the wire frame costs \$1.57 per metre.

13. A rectangular chocolate block has dimensions x and $(4x - 7)$ as shown.



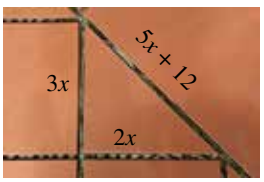
- **a.** Write an expression for the perimeter (P) in the form $P = __x + __(4x - 7)$.
- **b.** Expand and simplify this equation.
- **c.** Calculate the perimeter of the chocolate block when $x = 3$.
- **d.** Explain why x cannot equal 1.

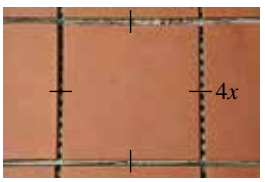
14. A dog house in the shape of a rectangular prism is to be reinforced with steel edging along its outer edges including the base. The cost of the steel edging is \$1.50 per metre. All measurements are in centimetres.



- **a.** Write an expression for the total length of the straight edges of the frame.
- **b.** Expand the expression.
- **c.** Calculate the cost of steel needed when $x = 80$ cm.

15. A tile manufacturer produces tiles that have the following side lengths. All measurements are in centimetres.





- **a.** Write an expression for the perimeter of each shape.
- **b.** Calculate the value of x for which the perimeter of the triangular tile is the same as the perimeter of the square tile.

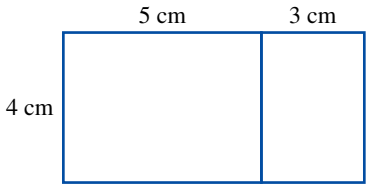
Reflection

Is the expression *ab* the same as *ba*? Explain.

3.5 Expanding brackets

3.5.1 The Distributive Law

- What is the area of the large rectangle drawn below? There are two ways of finding out.



1. The rectangle can be treated as one large shape.

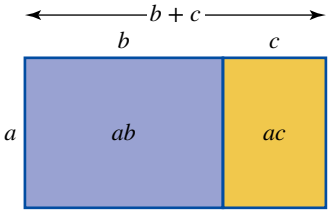
   The length of the rectangle is 4 cm, and its width is $5 + 3$ cm.

   $A = l \times w$
   $= 4(5 + 3)$
   $= 4 \times 8$
   $= 32 \text{ cm}^2$

2. The areas of the two small rectangles can be added together.

   $A = 4 \times 5 + 4 \times 3$
   $= 20 + 12$
   $= 32 \text{ cm}^2$

- The length of this rectangle is a, and its width is $b + c$.



- The area can be found in two different ways.

1. $A = l \times w$
   $= a(b + c)$

   This expression is **factorised** because it is one number (or factor) multiplied by another. The two factors are a and $b + c$.

2. The areas of the two small rectangles can be added together.

   $A = ab + ac$

   This expression is described as expanded or written without brackets.

- $a(b + c) = ab + ac$ is called the **Distributive Law**.

WORKED EXAMPLE 9

Use two different methods to find the value of the following.
a $7(5 + 15)$ **b** $10(9 - 1)$

THINK	WRITE
a 1 Method 1: Work out the brackets first, then evaluate.	**a** $7(5 + 15)$ $= 7 \times 20$ $= 140$
2 Method 2: Expand the brackets first, then evaluate.	$7(5 + 15)$ $= 7 \times 5 + 7 \times 15$ $= 35 + 105$ $= 140$

b 1 Method 1: Work out the brackets first, then evaluate.

b $10(9 - 1)$
$= 10 \times 8$
$= 80$

2 Method 2: Expand the brackets first, then evaluate.

$10(9 - 1)$
$= 10 \times 9 + 10 \times -1$
$= 90 - 10$
$= 80$

- A good way to expand brackets is to draw arrows as you work out each multiplication.

$x(x - y) = x^2 \ldots$
$x(x - y) = x^2 - xy$

WORKED EXAMPLE 10

Expand the following.
a $5(x + 3)$ **b** $-4y(2x - w)$

THINK

a 1 Draw arrows to assist in expanding.

2 Simplify.

b 1 Draw arrows to assist in expanding.

2 Simplify.

WRITE

a $5(x + 3)$
$= 4 \times x + 5 \times 3$
$= 5x + 15$

b $-4y(2x - w)$
$= -4y \times 2x - 4y \times -w$
$= -8xy + 4wy$

3.5.2 Expanding and collecting like terms

- With more complicated expansions, like terms may need to be collected after the expansion of the brackets.
  First expand the brackets, then collect the like terms.

WORKED EXAMPLE 11

TI | CASIO

Expand and simplify by collecting like terms.
a $4(x - 4) + 5$ **b** $x(y - 2) + 5x$
c $-x(y - z) + 5x$ **d** $7x - 6(y - 2x)$

THINK

a 1 Expand the brackets.

2 Simplify.

b 1 Expand the brackets.

2 Simplify.

WRITE

a $4(x - 4) + 5$
$= 4x - 16 + 5$
$= 4x - 11$

b $x(y - 2) + 5x$
$= xy - 2x + 5x$
$= xy + 3x$

c 1 Expand the brackets.	**c** $-x(y - z) + 5x$
2 Simplify. (There are no like terms.)	$= -xy + xz + 5x$
d 1 Expand the brackets.	**d** $7x - 6(y - 2x)$
2 Simplify.	$= 7x - 6y + 12x$ $= 19x - 6y$

3.5.3 Expanding two brackets

- When simplifying an expression containing two (or more) brackets, the steps are the same. Expand each bracket, and then simplify by collecting like terms.

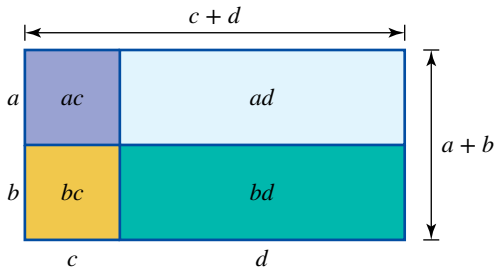
WORKED EXAMPLE 12

Expand and simplify the following expressions.
a $5(x + 2y) + 6(x - 3y)$
b $5x(y - 2) - y(x + 3)$

THINK	WRITE
a 1 Expand each bracket.	**a** $5(x + 2y) + 6(x - 3y)$
2 Simplify.	$= 5x + 10y + 6x - 18y$ $= 11x - 8y$
b 1 Expand each bracket.	**b** $5x(y - 2) - y(x + 3)$
2 Simplify.	$= 5xy - 10x - xy - 3y$ $= 4xy - 10x - 3y$

3.5.4 Expanding binomial factors

- Remember, a **binomial** is an expression containing two terms, such as $x + 3$ or $2y - z^2$.
  In this section two binomials are multiplied together.
- The rectangle below has length $a + b$ and width $c + d$.

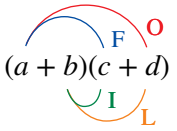


- There are two ways of finding the area of the large rectangle.
  1. $A = L \times W$
     $= (a + b) \times (c + d)$
     $= (a + b)(c + d)$
  This is a factorised expression, where the two factors are $(a + b)$ and $(c + d)$.

2. The areas of the four small rectangles can be added together.
   $A = ac + ad + bc + bd$
   So $(a + b)(c + d) = ac + ad + bc + bd$

- There are several methods that can be helpful in remembering how to expand binomial factors. One commonly used method is the FOIL method.
- This method is given the name **FOIL** because the letters stand for:
  First — multiply the first term in each bracket.
  Outer — multiply the 2 outer terms of each bracket.
  Inner — multiply the 2 inner terms of each bracket.
  Last — multiply the last term of each bracket.

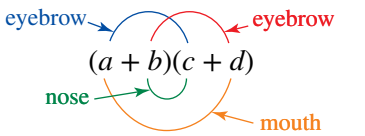
'FOIL'– First Outer Inner Last



$(a + b)(c + d) = ac + ad + bc + bd$

OR

'eyebrow, eyebrow, nose and mouth'



$(a + b)(c + d) = ac + bd + bc + ad$

WORKED EXAMPLE 13

TI | CASIO

Expand and simplify each of the following expressions.
a $(x - 5)(x + 3)$ **b** $(x + 2)(x + 3)$ **c** $(2x + 2)(2x + 3)$

THINK	WRITE
a 1 Expand using the FOIL method.	**a** $(x - 5)(x + 3)$
2 Simplify the expression by collecting like terms.	$= x^2 + 3x - 5x - 15$ $= x^2 - 2x - 15$
b 1 Expand the brackets using FOIL.	**b** $(x + 2)(x + 3)$
2 Simplify the expression by collecting like terms.	$= x^2 + 3x + 2x + 6$ $= x^2 + 5x + 6$
c 1 Expand the brackets using FOIL.	**c** $(2x + 2)(2x + 3)$
2 Simplify the expression by collecting like terms.	$= 4x^2 + 6x + 4x + 6$ $= 4x^2 + 10x + 6$

learnon RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Expanding brackets (doc-10819)
Complete this digital doc: SkillSHEET: Expanding a pair of brackets (doc-10820)

Exercise 3.5 Expanding brackets

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a–d, 3a–d, 4a, c, m, o, 5a, c, g, k, m, 6a, e, i, m, o, 7, 8a, d, g, j, m, 9, 10a, d, g, j, m, 11, 12, 18, 19	Questions: 1, 2c–f, 3c–f, 4b, f–j, 5i–p, 6b, d, j, l, p, 7, 8b, e, h, k, n, 9d–f, 10b, e, h, k, n, 13, 14, 18, 19, 22	Questions: 1, 2e–h, 3e–h, 4i–p, 5i–p, 6i–p, 7c, 8c, f, i, l, o, 9g–i, 10c, f, i, l, o, 13–17, 20–22

Individual pathway interactivity: int-4483

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE9 Use two different methods to find the value of these expressions.

a. $8(10 - 2)$ **b.** $11(99 + 1)$ **c.** $-5(3 + 1)$ **d.** $7(100 - 1)$

2. WE10 Expand the following expressions. For the first two examples, draw a diagram to represent the expression.

a. $3(x + 2)$ **b.** $4(x + 3)$ **c.** $4(x + 1)$ **d.** $7(x - 1)$

e. $-3(p - 2)$ **f.** $-(x - 1)$ **g.** $3(2b - 4)$ **h.** $8(3m - 2)$

3. WE10 Expand each of the following. For the first two examples, draw a diagram to represent the expression.

a. $x(x + 2)$ **b.** $a(a + 5)$ **c.** $x(4 + x)$ **d.** $m(7 - m)$

e. $2x(y + 2)$ **f.** $-3y(x + 4)$ **g.** $-b(3 - a)$ **h.** $-6a(5 - 3a)$

4. WE11 Expand and simplify by collecting like terms.

a. $2(p - 3) + 4$ **b.** $5(x - 5) + 8$

c. $-7(p + 2) - 3$ **d.** $-4(3p - 1) - 1$

e. $6x(x - 3) - 2x$ **f.** $2m(m + 5) - 3m$

g. $3x(p + 2) - 5$ **h.** $4y(y - 1) + 7$

i. $-4p(p - 2) + 5p$ **j.** $5(x - 2y) - 3y - x$

k. $2m(m - 5) + 2m - 4$ **l.** $-3p(p - 2q) + 4pq - 1$

m. $-7a(5 - 2b) + 5a - 4ab$ **n.** $4c(2d - 3c) - cd - 5c$

o. $6p + 3 - 4(2p + 5)$ **p.** $5 - 9m + 2(3m - 1)$

5. WE12 Expand and simplify the following expressions.

a. $2(x + 2y) + 3(2x - y)$ **b.** $4(2p + 3q) + 2(p - 2q)$

c. $7(2a + 3b) + 4(a + 2b)$ **d.** $5(3c + 4d) + 2(2c + d)$

e. $-4(m + 2n) + 3(2m - n)$ **f.** $-3(2x + y) + 4(3x - 2y)$

g. $-2(3x + 2y) + 3(5x + 3y)$ **h.** $-5(4p + 2q) + 2(3p + q)$

i. $6(a - 2b) - 5(2a - 3b)$ **j.** $5(2x - y) - 2(3x - 2y)$

k. $4(2p - 4q) - 3(p - 2q)$ **l.** $2(c - 3d) - 5(2c - 3d)$

m. $7(2x - 3y) - (x - 2y)$ **n.** $-5(p - 2q) - (2p - q)$

o. $-3(a - 2b) - (2a + 3b)$ **p.** $4(3c + d) - (4c + 3d)$

6. WE12 Expand and simplify the following expressions.

a. $a(b + 2) + b(a - 3)$ **b.** $x(y + 4) + y(x - 2)$

c. $c(d - 2) + c(d + 5)$ **d.** $p(q - 5) + p(q + 3)$

e. $3c(d - 2) + c(2d - 5)$ **f.** $7a(b - 3) - b(2a + 3)$

g. $2m(n + 3) - m(2n + 1)$ **h.** $4c(d - 5) + 2c(d - 8)$

i. $3m(2m + 4) - 2(3m + 5)$ **j.** $5c(2d - 1) - (3c + cd)$

k. $-3a(5a + b) + 2b(b - 3a)$ **l.** $-4c(2c - 6d) + d(3d - 2c)$

m. $6m(2m - 3) - (2m + 4)$ **n.** $2p(p - 4) + 3(5p - 2)$

o. $7x(5 - x) + 6(x - 1)$ **p.** $-2y(5y - 1) - 4(2y + 3)$

7. MC **a.** What is the equivalent of $3(a + 2b) + 2(2a - b)$?

A. $5a + 6b$ **B.** $7a + 4b$ **C.** $5(3a + b)$ **D.** $7a + 8b$

b. What is the equivalent of $-3(x - 2y) - (x - 5y)$?

A. $-4x + 11y$ **B.** $-4x - 11y$ **C.** $4x + 11y$ **D.** $4x + 7y$

c. What is the equivalent of $2m(n + 4) + m(3n - 2)$?

A. $3m + 4n - 8$ **B.** $5mn + 4m$ **C.** $5mn + 10m$ **D.** $5mn + 6m$

8. WE13 Expand and simplify each of the following expressions.

a. $(a + 2)(a + 3)$ **b.** $(x + 4)(x + 3)$ **c.** $(y + 3)(y + 2)$

d. $(m + 4)(m + 5)$ **e.** $(b + 2)(b + 1)$ **f.** $(p + 1)(p + 4)$

g. $(a - 2)(a + 3)$ **h.** $(x - 4)(x + 5)$ **i.** $(m + 3)(m - 4)$

j. $(y + 5)(y - 3)$ **k.** $(y - 6)(y + 2)$ **l.** $(x - 3)(x + 1)$

m. $(x - 3)(x - 4)$ **n.** $(p - 2)(p - 3)$ **o.** $(x - 3)(x - 1)$

9. Expand and simplify each of the following.

a. $(2a + 3)(a + 2)$ **b.** $(3m + 1)(m + 2)$ **c.** $(6x + 4)(x + 1)$

d. $(c - 6)(4c - 7)$ **e.** $(7 - 2t)(5 - t)$ **f.** $(1 - x)(9 - 2x)$

g. $(2 + 3t)(5 - 2t)$ **h.** $(7 - 5x)(2 - 3x)$ **i.** $(5x - 2)(5x - 2)$

10. Expand and simplify each of the following.

a. $(x + y)(z + 1)$ **b.** $(p + q)(r + 3)$ **c.** $(2x + y)(z + 4)$

d. $(3p + q)(r + 1)$ **e.** $(a + 2b)(a + b)$ **f.** $(2c + d)(c - 3d)$

g. $(x + y)(2x - 3y)$ **h.** $(4p - 3q)(p + q)$ **i.** $(3y + z)(x + z)$

j. $(a + 2b)(b + c)$ **k.** $(3p - 2q)(1 - 3r)$ **l.** $(7c - 2d)(d - 5)$

m. $(4x - y)(3x - y)$ **n.** $(p - q)(2p - r)$ **o.** $(5 - 2j)(3k + 1)$

11. MC **a.** The equivalent of $(x + 7)(x - 2)$ is:

A. $x^2 + 5x - 14$ **B.** $2x + 5$

C. $x^2 - 5x - 14$ **D.** $x^2 + 5x + 14$

b. What is the equivalent of $(4 - y)(7 + y)$?

A. $28 - y^2$ **B.** $28 - 3y + y^2$

C. $28 - 3y - y^2$ **D.** $11 - 2y$

c. The equivalent of $(2p + 1)(p - 5)$ is:

A. $2p^2 - 5$ **B.** $2p^2 - 11p - 5$

C. $2p^2 - 9p - 5$ **D.** $2p^2 - 6p - 5$

Understanding

12. Expand the following expressions using the FOIL method, then simplify.

a. $(x + 3)(x - 3)$ **b.** $(x + 5)(x - 5)$ **c.** $(x + 7)(x - 7)$

d. $(x - 1)(x + 1)$ **e.** $(x - 2)(x + 2)$ **f.** $(2x - 1)(2x + 1)$

Can you see a pattern? If so, explain.

13. Expand the following expressions using the FOIL method, then simplify.

a. $(x + 1)(x + 1)$ **b.** $(x + 2)(x + 2)$ **c.** $(x + 8)(x + 8)$

d. $(x - 3)(x - 3)$ **e.** $(x - 5)(x - 5)$ **f.** $(x - 9)(x - 9)$

Can you see a pattern? If so, explain.

14. Simplify the following expressions.

a. $2.1x(3x + 4.7y) - 3.1y(1.4x + y)$

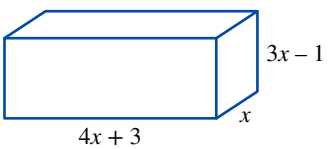
b. $(2.1x - 3.2y)(2.1x + 3.2y)$

c. $(3.4x + 5.1y)^2$

15. For the box shown at right, find in expanded form:

a. the total surface area

b. the volume.



Reasoning

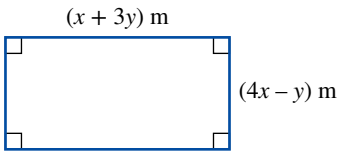
16. For each of the following shapes:

i. write down the area in factor form

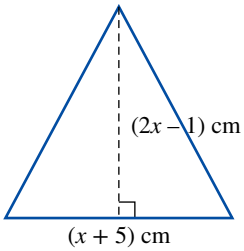
ii. expand and simplify the expression

iii. discuss any limitations on the value of x.

a.



b.



17. Show that

$(a - x)(a + x) - 2(a - x)(a - x) - 2x(a - x)$

$= -(a - x)^2$.

Problem solving

18. Three students gave the following incorrect answers when expanding $-5(3x - 20)$.

i. $-5x - 20$

ii. $-8x + 25$

iii. $15x - 100$

a. Explain the errors made by each student.

b. What is the correct answer?

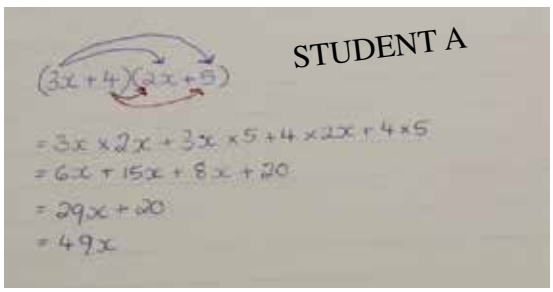
19. In a test, a student expanded brackets and obtained the following answers. Identify and correct the student's errors and write the correct expansions.

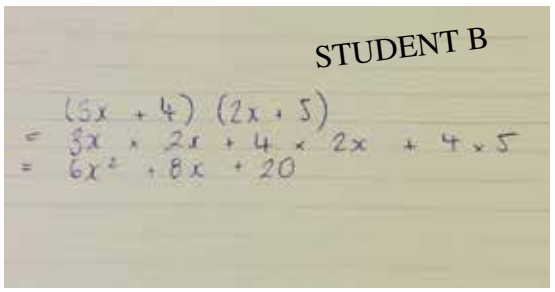
a. $-2(a - 5) = -2a - 10$

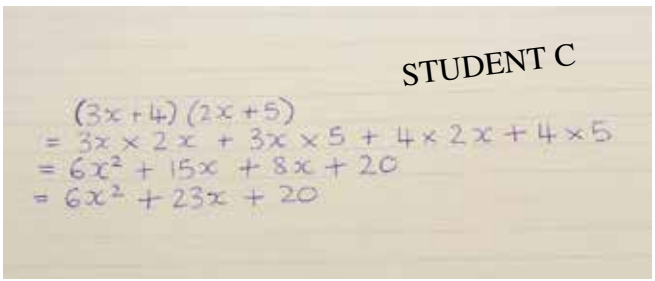
b. $2b(3b - 1) = 6b^2 - 1$

c. $-2(c - 4) = 2c + 8$

20. Shown below are three students' attempts at expanding $(3x + 4)(2x + 5)$.







a. Which student's work was correct?

b. Copy each of the incorrect answers into your workbook and correct the mistakes in each as though you were the teacher of these students.

21. Three incorrect expansions of $(x + 8)(x - 3)$ are shown below. For each incorrect expansion, explain what mistake has been made.

a. $x^2 + 11x + 24$ **b.** $x^2 + 5x + 24$ **c.** $x^2 - 3x$

22. Rectangular floor mats have an area of $(x^2 + 2x - 15)$ cm^2.

a. The length of the mat is $(x + 5)$ cm. Find an expression for the width of the mat.

b. If the length of the mat is 70 cm, what is the width?

c. If the width of the mat is 1 m, what is the length?

Reflection

Explain why, when expanded, $(x + y)(2x + y)$ gives the same result as $(2x + y)(x + y)$.

3.6 Expansion patterns

3.6.1 Difference of two squares

- Look at the expansion of $(x + 3)(x - 3)$.

$$\begin{aligned}(x + 3)(x - 3) &= x^2 - 3x + 3x - 9\\ &= x^2 - 9\end{aligned}$$

Compare it with

$$\begin{aligned}(x + 5)(x - 5) &= x^2 - 5x + 5x - 25\\ &= x^2 - 25\end{aligned}$$

In both cases the middle terms cancel each other out, leaving just two terms. There seems to be a pattern, so we can take the short cut:

$$\begin{aligned}(x + 10)(x - 10) &= x^2 - 10^2\\ &= x^2 - 100\end{aligned}$$

The terms x^2 and 100 are both perfect squares, so an expression such as $x^2 - 100$ is called a **difference of two squares**. (Remember, difference means subtraction.) It is important to recognise this type of expression and remember its name.

> Difference of two squares: $(a + b)(a - b) = a^2 - b^2$

Does this rule work if $a = 7$ and $b = 3$? Let's check.

$$\begin{aligned}(7 + 3)(7 - 3) &= 10 \times 4\\ &= 40\\ 7^2 - 3^2 &= 49 - 9\\ &= 40\end{aligned}$$

The rule works. Try some more examples yourself.

WORKED EXAMPLE 14

Use the difference of two squares rule to expand and simplify each of the following.

a $(x + 8)(x - 8)$ **b** $(6 - x)(6 + x)$

c $(2x - 3)(2x + 3)$ **d** $(3x + 5)(5 - 3x)$

THINK	WRITE
a 1 Write the factorised expression.	**a** $(x + 8)(x - 8)$
2 This expression is in the form $(a + b)(a - b)$, so apply the rule $a^2 - b^2$.	$= x^2 - 8^2$ $= x^2 - 64$

b 1	Write the factorised expression.	**b** $(6 - x)(6 + x)$
2	The difference of two squares rule can be used. Expand using the formula.	$= 6^2 - x^2$ $= 36 - x^2$
	Note: $36 - x^2$ is not the same as $x^2 - 36$.	
c 1	Write the factorised expression.	**c** $(2x - 3)(2x + 3)$
2	The difference of two squares rule can be used. Expand using the formula.	$= (2x)^2 - 3^2$ $= 4x^2 - 9$
d 1	Write the factorised expression.	**d** $(3x + 5)(5 - 3x)$
2	The difference of two squares rule can be used by rearranging the terms, since $3x + 5 = 5 + 3x$. Expand using the formula.	$(5 + 3x)(5 - 3x)$ $= 5^2 - (3x)^2$ $= 25 - 9x^2$

3.6.2 The perfect square of a binomial

- A **perfect square** is the square of a whole number. For example, 1, 4, 9, 16 . . . and so on are all perfect squares since $1 = 1 \times 1 = 1^2$, $4 = 2 \times 2 = 2^2$, $9 = 3 \times 3 = 3^2$, $16 = 4 \times 4 = 4^2$ and so on.
- Similarly, $(x + 3)^2$ is a perfect square since it is equivalent to $(x + 3)(x + 3)$, or $x + 3$ multiplied by itself. It is called the 'perfect square of a binomial.'
- Consider a picture of $(x + 3)^2$. What shape is it?

	x	3
x	x^2	$3x$
3	$3x$	9

The area is given by

$$x^2 + 3x + 3x + 9 = x^2 + 6x + 9$$

The large square is made up of two smaller squares and two rectangles, and because the rectangles are the same size ($x \times 3$), there is a short cut to the answer.
The first term is x^2,
the third term is 3^2 and
the middle term is $x \times 3 \times 2$.
Once again there is a pattern to remember. It is called the 'square of a binomial'.

> Square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

So $(x + 5)^2 = x^2 + 2 \times x \times 5 + 5^2$
$= x^2 + 10x + 25$

- Does this rule work if $a = 7$ and $b = 3$? Let's check.

$(7 + 3)^2 = 10^2$
$= 100$
$(7 + 3)^2 = 7^2 + 2 \times 7 \times 3 + 3^2$
$= 49 + 42 + 9$
$= 100$
The rule works.

- Try to remember this rule in words:
  'The square of a binomial equals the square of the first term plus the square of the second term plus twice the product of the two terms.'
- Note that $(a - b)^2 = a^2 - 2ab + b^2$
  since $a^2 = a^2$
  $(-b)^2 = b^2$
  and $2 \times a \times -b = -2ab$.

Square of a binomial: $(a \pm b)^2 = a^2 \pm 2ab + b^2$

WORKED EXAMPLE 15

Use $(a \pm b)^2 = a^2 \pm 2ab \pm b^2$ to expand and simplify the following.
a $(x + 1)(x + 1)$ **b** $(x - 2)^2$
c $(2x + 5)^2$ **d** $(4x - 5y)^2$

THINK	WRITE
a 1 This is the square of a binomial.	**a** $(x + 1)(x + 1)$
2 Apply the formula for perfect squares: $(a + b)^2 = a^2 + 2ab + b^2$.	$= x^2 + 2 \times x \times 1 + 1^2$ $= x^2 + 2x + 1$
b 1 This is the square of a binomial.	**b** $(x - 2)^2$
2 Apply the formula for perfect squares: $(a - b)^2 = a^2 - 2ab + b^2$.	$= (x - 2)(x - 2)$ $= x^2 - 2 \times x \times 2 \times 2^2$ $= x^2 - 4x + 4$
c 1 This is the square of a binomial.	**c** $(2x + 5)^2$
2 Apply the formula for perfect squares: $(a + b)^2 = a^2 + 2ab + b^2$.	$= (2x)^2 + 2 \times 2x \times 5 + 5^2$ $= 4x^2 + 20x + 25$
d 1 This is the square of a binomial.	**d** $(4x - 5y)^2$
2 Apply the formula for perfect squares: $(a - b)^2 = a^2 - 2ab + b^2$.	$= (4x)^2 - 2 \times 4x \times 5y + (5y)^2$ $= 16x^2 - 40xy + 25y^2$

learnon RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Recognising expansion patterns (doc-10821)

Exercise 3.6 Expansion patterns

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10	Questions: 1–12	Questions: 1–13

Individual pathway interactivity: int-4484

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Fluency

1. WE14 Use the difference of two squares rule to expand and simplify each of the following.

a. $(x + 2)(x - 2)$ **b.** $(y + 3)(y - 3)$ **c.** $(m + 5)(m - 5)$
d. $(a + 7)(a - 7)$ **e.** $(x + 6)(x - 6)$ **f.** $(p - 12)(p + 12)$
g. $(a + 10)(a - 10)$ **h.** $(m - 11)(m + 11)$

2. WE14 Use the difference of two squares rule to expand and simplify each of the following.

a. $(2x + 3)(2x - 3)$ **b.** $(3y - 1)(3y + 1)$ **c.** $(5d - 2)(5d + 2)$
d. $(7c + 3)(7c - 3)$ **e.** $(2 + 3p)(2 - 3p)$ **f.** $(d - 9x)(d + 9x)$
g. $(5 - 12a)(5 + 12a)$ **h.** $(3x + 10y)(3x - 10y)$ **i.** $(2b - 5c)(2b + 5c)$
j. $(10 - 2x)(2x + 10)$

3. WE15 Use the square of a binomial rule to expand and simplify each of the following.

a. $(x + 2)(x + 2)$ **b.** $(a + 3)(a + 3)$ **c.** $(b + 7)(b + 7)$ **d.** $(c + 9)(c + 9)$
e. $(m + 12)^2$ **f.** $(n + 10)^2$ **g.** $(x - 6)^2$ **h.** $(y - 5)^2$
i. $(9 - c)^2$ **j.** $(8 + e)^2$ **k.** $2(x + y)^2$ **l.** $(u - v)^2$

4. WE15 Use the perfect squares rule to expand and simplify each of the following.

a. $(2a + 3)^2$ **b.** $(3x + 1)^2$ **c.** $(2m - 5)^2$ **d.** $(4x - 3)^2$
e. $(5a - 1)^2$ **f.** $(7p + 4)^2$ **g.** $(9x + 2)^2$ **h.** $(4c - 6)^2$
i. $(3 + 2a)^2$ **j.** $(5 + 3p)^2$ **k.** $(2 - 5x)^2$ **l.** $(7 - 3a)^2$
m. $(9x - 4y)^2$ **n.** $(8x - 3y)^2$

Understanding

5. A square has a perimeter of $4x + 12$. What is its area?

6. Francis has fenced off a square in her paddock for spring lambs. The area of the paddock is $9x^2 + 6x + 1$ m^2.
By using pattern recognition, find the side length, in terms of x, of the paddock.

Reasoning

7. Show that $a^2 - b^2 = (a + b)(a - b)$ is true for each of the following.

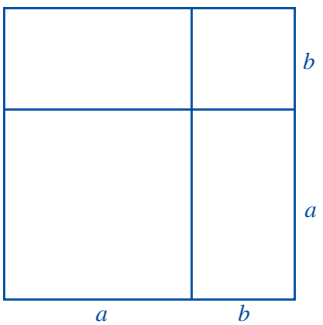
a. $a = 5, b = 4$ **b.** $a = 9, b = 1$
c. $a = 2, b = 7$ **d.** $a = -10, b = -3$

8. Anne has a square bedroom. Her sister Beth has a room that is 1 m shorter in length than Anne's room, but 1 m wider.

a. Show that Anne has the larger bedroom.
b. How much bigger is Anne's bedroom compared to Beth's bedroom?

Problem solving

9. A large square has been subdivided into two squares and two rectangles.



a. Write formulas for the areas of these four pieces, using the dimensions a and b marked on the diagram.
b. Write an equation that states that the area of the large square is equal to the combined area of its four pieces. Do you recognise this equation?

10. Expand each of the following pairs of expressions.

a. **i.** $(x - 3)^2$ and $(3 - x)^2$
ii. $(x - 15)^2$ and $(15 - x)^2$
iii. $(3x - 7)^2$ and $(7 - 3x)^2$
b. What do you notice about the answers to the pairs of expansions above?
c. Explain how this is possible.

11. Expand each of the following pairs of expressions.

a. **i.** $(x - 4)(x + 4)$ and $(4 - x)(4 + x)$

ii. $(x - 11)(x + 11)$ and $(11 - x)(11 + x)$

iii. $(2x - 9)(2x + 9)$ and $(9 - 2x)(9 + 2x)$

b. What do you notice about the answers to the pairs of expansions above?

c. Explain how this is possible.

12. a. Expand $(10k + 5)^2$.

b. Show that $(10k + 5)^2 = 100k(k + 1) + 25$

c. Using part **b**, evaluate 25^2 and 85^2.

13. The expansion of perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

and $(a - b)^2 = a^2 - 2ab + b^2$

can be used to simplify some arithmetic calculations. For example:

$$\begin{aligned} 97^2 &= (100 - 3)^2 \\ &= 100^2 - 2 \times 100 \times 3 + 3^2 \\ &= 9409 \end{aligned}$$

Use this method to calculate the following.

a. 103^2 **b.** 62^2 **c.** 997^2 **d.** 1012^2 **e.** 53^2 **f.** 98^2

Reflection

How could you represent $(x - 3)^2$ on a diagram?

CHALLENGE 3.2

Complete this algebraic magic square, so that all rows, columns and diagonals add up to the same amount.

$-3(x - 1)^2$		
	$3x$	
$(x + 1)^2$		$3(x^2 + 1)$

3.7 Further expansions

3.7.1 Expanding several sets of brackets

- We need to take care with signs when one expression is subtracted from another. For example,

$$\begin{aligned} &x^2 + x + 2 - (x - 2)(x + 2) \\ &= x^2 + x + 2 - (x^2 - 4) && \text{(Remember — all of } (x - 2)(x + 2) \text{ is being subtracted.)} \\ &= x^2 + x + 2 - x^2 + 4 \\ &= x + 6 \end{aligned}$$

WORKED EXAMPLE 16

Expand and simplify each of the following expressions.

a $(x + 3)(x + 4) + 4(x - 2)$

b $(x - 2)(x + 3) - (x - 1)(x + 2)$

THINK	WRITE
a 1 Expand each set of brackets.	**a** $(x + 3)(x + 4) + 4(x - 2)$
2 Simplify by collecting like terms.	$= x^2 + 4x + 3x + 12 + 4x - 8$ $= x^2 + 11x + 4$
b 1 Expand and simplify each pair of brackets. Keep the second expression in a bracket.	**b** $(x - 2)(x + 3) - (x - 1)(x + 2)$ $= x^2 + 3x - 2x - 6 - (x^2 + 2x - x - 2)$
2 Subtract all of the second result from the first result. Remember that $-(x^2 + x - 2) = -1(x^2 + x - 2)$.	$= x^2 + x - 6 - (x^2 + x - 2)$ $= x^2 + x - 6 - x^2 - x + 2$
3 Simplify by collecting like terms.	$= -4$

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Try out this interactivity: Expanding brackets (int-2763)

Exercise 3.7 Further expansions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–16, 31, 34	Questions: 10–22, 32–34	Questions: 18–35

Individual pathway interactivity: int-4485

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Fluency

WE16 Expand and simplify each of the following expressions.

1. $(x + 3)(x + 5) + (x + 2)(x + 3)$
2. $(x + 4)(x + 2) + (x + 3)(x + 4)$
3. $(x + 5)(x + 4) + (x + 3)(x + 2)$
4. $(x + 1)(x + 3) + (x + 2)(x + 4)$
5. $(p - 3)(p + 5) + (p + 1)(p - 6)$
6. $(a + 4)(a - 2) + (a - 3)(a - 4)$
7. $(p - 2)(p + 2) + (p + 4)(p - 5)$
8. $(x - 4)(x + 4) + (x - 1)(x + 20)$
9. $(y - 1)(y + 3) + (y - 2)(y + 2)$
10. $(d + 7)(d + 1) + (d + 3)(d - 3)$
11. $(x + 2)(x + 3) + (x - 4)(x - 1)$
12. $(y + 6)(y - 1) + (y - 2)(y - 3)$
13. $(x + 2)^2 + (x - 5)(x - 3)$
14. $(y - 1)^2 + (y + 2)(y - 4)$
15. $(p + 2)(p + 7) + (p - 3)^2$
16. $(m - 6)(m - 1) + (m + 5)^2$
17. $(x + 3)(x + 5) - (x + 2)(x + 5)$
18. $(x + 5)(x + 2) - (x + 1)(x + 2)$
19. $(x + 3)(x + 2) - (x + 4)(x + 3)$
20. $(m - 2)(m + 3) - (m + 2)(m - 4)$
21. $(b + 4)(b - 6) - (b - 1)(b + 2)$
22. $(y - 2)(y - 5) - (y + 2)(y + 6)$
23. $(p - 1)(p + 4) - (p - 2)(p - 3)$
24. $(x + 7)(x + 2) - (x - 3)(x - 4)$
25. $(c - 2)(c - 1) - (c + 6)(c + 7)$
26. $(f - 7)(f + 2) - (f + 4)(f + 5)$
27. $(m + 3)^2 - (m + 4)(m - 2)$
28. $(a - 6)^2 - (a - 2)(a - 3)$
29. $(p - 3)(p + 1) - (p + 2)^2$
30. $(x + 5)(x - 4) - (x - 1)^2$

Reasoning

31. Show that $(p - 1)(p + 2) + (p - 3)(p + 1) = 2p^2 - p - 5$.

32. Show that $(x + 2)(x - 3) - (x + 1)^2 = -3x - 7$.

Problem solving

33. a. Show that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.

b. Using part **a**, write $(2^2 + 1^2)(3^2 + 4^2)$ as the sum of two squares and evaluate.

34. a. Expand $(x^2 + x - 1)^2$.

b. Show that $(x^2 + x - 1)^2 = (x - 1)x(x + 1)(x + 2) + 1$.

c. i. Evaluate $4 \times 3 \times 2 \times 1 + 1$.

ii. What is the value of x if $4 \times 3 \times 2 \times 1 + 1 = (x - 1)x(x + 1)(x + 2) + 1$?

35. a. Expand $(a + b)(d + e)$.

b. Expand $(a + b + c)(d + e + f)$. Draw a diagram to illustrate your answer.

Reflection

On a diagram how would you show $(m + 2)(m + 3) - (m + 2)(m + 4)$?

3.8 The highest common factor

3.8.1 Factorising expressions

- **Factorising** a number or an expression means writing it as a product. For example, 35 is factorised by writing it as 7×5, the product of 7 and 5.
  The factors of 35 are 7 and 5:
  $$35 \div 7 = 5$$
  $$35 \div 5 = 7$$
- $5(x + 2)$ is also a factorised expression. (Remember that it means $5 \times (x + 2)$.)
  The factors of $5(x + 2)$ are 5 and $(x + 2)$:
  $$5(x + 2) \div 5 = (x + 2)$$
  $$5(x + 2) \div (x + 2) = 5$$
- In section 3D, factorised expressions were expanded as
  $$5(x + 2) = 5x + 10$$
  This section examines the opposite process. We will start with expanded expressions and try to factorise them.

WORKED EXAMPLE 17

Complete the following factorisations.

a $15x = 5x \times \ldots$ **b** $15x^2 = 3x \times \ldots$ **c** $-18ab^2 = 6a \times \ldots$

THINK	WRITE
a Divide $15x$ by $5x$.	**a** $\frac{15x}{5x} = 3$ So $15x = 5x \times 3$.
b Divide $15x^2$ by $3x$.	**b** $\frac{15x^2}{3x} = 5x$ So $15x^2 = 3x \times 5x$.
c Divide $-18ab^2$ by $6a$.	**c** $\frac{-18ab^2}{6a} = -3b^2$ So $-18ab^2 = 6a \times -3b^2$.

- The **highest common factor (HCF)** of two or more numbers is the largest factor that divides into all the numbers with no remainder. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. The factors of 16 are 1, 2, 4, 8 and 16. The HCF of 12 and 16 is 4.
- The simplest way to factorise an expression is to take out the highest common factor or HCF.

WORKED EXAMPLE 18

Determine the highest common factor for each of the following pairs of terms.
a $25a^2b$ **and** $10ab$ **b** $3xy$ **and** $-3xz$

THINK	WRITE
a 1 Write $25a^2b$ in expanded form.	**a** $25a^2b = 5 \times 5 \times a \times a \times b$
2 Write $10ab$ in expanded form.	$10ab = 2 \times 5 \times a \times b$
3 Write the HCF.	HCF $= 5ab$
b 1 Write $3xy$ in expanded form.	**b** $3xy = 3 \times x \times y$
2 Write $-3xz$ in expanded form.	$-3xz = -1 \times 3 \times x \times z$
3 Write the HCF.	HCF $= 3x$

3.8.2 Factorising expressions by finding the highest common factor

- To factorise an expression such as $4xy + 12x$, find the highest common factor of both terms.
  The HCF for $4xy$ and $12x$ is $4x$, so $4xy + 12x = 4x \times (\ldots)$ where the brackets contain a binomial.

  $\frac{4xy}{4x} = y$ and $\frac{12x}{4x} = 3$

  So $4xy + 12x = 4x \times (y + 3)$
  $= 4x(y + 3)$
  The answer can be checked by expanding the brackets.

WORKED EXAMPLE 19

TI | CASIO

Factorise each expression by first finding the HCF.
a $5x + 15y$ **b** $-14xy - 7y$ **c** $6x^2y + 9xy^2$

THINK	WRITE
a 1 HCF $= 5$. Write 5 multiplied by brackets.	**a** $5x + 15y = 5(\quad)$
2 Divide each term by 5 to get the binomial. $\frac{5x}{5} = x$, $\frac{15y}{5} = 3y$	$= 5(x + 3y)$
3 Check the answer by expanding.	$5(x + 3y) = 5x + 15y$ (correct)
b 1 HCF $= 7y$ or $-7y$, but $-7y$ works neatly.	**b** $-14xy - 7y$
2 $\frac{-14xy}{-7y} = 2x$, $\frac{-7y}{-7y} = 1$	$= -7y(2x + 1)$
3 Check the answer by expanding.	$-7y(2x + 1) = -14xy - 7y$ (correct)

c 1 HCF = $3xy$

2 $\frac{6x^2y}{3xy} = 2x$, $\frac{9xy^2}{3xy} = 3y$

3 Check the answer by expanding.

c $6x^2y + 9xy^2$

$= 3xy(2x + 3y)$

$3xy(2x + 3y) = 6x^2y + 9xy^2$ (correct)

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Complete this digital doc: SkillSHEET: Finding the highest common factor (doc-10822)

Complete this digital doc: SkillSHEET: Factorising by finding the HCF (doc-10823)

Exercise 3.8 The highest common factor

assessON

Individual pathways

PRACTISE
Questions:
1a–f, 2a–i, 3, 4a–i, 5a–i, 6a–i, 7a–i, 8a–i, 9

CONSOLIDATE
Questions:
1d–i, 2e–k, 3, 4g–n, 5g–n, 6g–o, 7g–o, 8g–o, 9, 11

MASTER
Questions:
1g–l, 2i–p, 3, 4i–r, 5i–o, 6i–r, 7j–o, 8j–o, 9–12

Individual pathway interactivity: int-4486

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Fluency

1. WE17 Complete each of the following factorisations by writing in the missing factor.

a. $8a = 4 \times \ldots$ **b.** $8a = 2a \times \ldots$ **c.** $12x^2 = 4x \times \ldots$
d. $-12x^2 = 3x^2 \times \ldots$ **e.** $3x^2 = x \times \ldots$ **f.** $15a^2b = ab \times \ldots$
g. $12x = -4 \times \ldots$ **h.** $10mn = 10n \times \ldots$ **i.** $10mn = -10 \times \ldots$
j. $a^2b^2 = ab \times \ldots$ **k.** $30x^2 = 10x \times \ldots$ **l.** $-15mn^2 = -3m \times \ldots$

2. WE18 Find the highest common factor (HCF) of each of the following.

a. 4 and 12 **b.** 6 and 15 **c.** 10 and 25 **d.** 24 and 32
e. 12, 15 and 21 **f.** 25, 50 and 200 **g.** 17 and 23 **h.** $6a$ and $12ab$
i. $14xy$ and $21xz$ **j.** $60pq$ and $30q$ **k.** $50cde$ and $70fgh$ **l.** $6x^2$ and $15x$
m. $6a$ and $9c$ **n.** $5ab$ and 25 **o.** $3x^2y$ and $4x^2z$ **p.** $4k$ and 6

3. MC What is $5m$ the highest common factor of?

A. $2m$ and $5m$ **B.** $5m$ and m **C.** $25mn$ and $15lm$ **D.** $20m$ and $40m$

4. WE19 Factorise each of the following expressions.

a. $4x + 12y$ **b.** $5m + 15n$ **c.** $7a + 14b$
d. $7m - 21n$ **e.** $-8a - 24b$ **f.** $8x - 4y$
g. $-12p - 2q$ **h.** $6p + 12pq + 18q$ **i.** $32x + 8y + 16z$
j. $16m - 4n + 24p$ **k.** $72x - 8y + 64pq$ **l.** $15x^2 - 3y$
m. $5p^2 - 20q$ **n.** $5x + 5$ **o.** $56q + 8p^2$
p. $7p - 42x^2y$ **q.** $16p^2 + 20q + 4$ **r.** $12 + 36a^2b - 24b^2$

5. Factorise each expression.

a. $9a + 21b$ **b.** $4c + 18d^2$ **c.** $12p^2 + 20q^2$

d. $35 - 14m^2n$ **e.** $25y^2 - 15x$ **f.** $16a^2 + 20b$

g. $42m^2 + 12n$ **h.** $63p^2 + 81 - 27y$ **i.** $121a^2 - 55b + 110c$

j. $10 - 22x^2y^3 + 14xy$ **k.** $18a^2bc - 27ab - 90c$ **l.** $144p + 36q^2 - 84pq$

m. $63a^2b^2 - 49 + 56ab^2$ **n.** $22 + 99p^3q^2 - 44p^2r$ **o.** $36 - 24ab^2 + 18b^2c$

6. Factorise the following expressions.

a. $-x + 5$ **b.** $-a + 7$ **c.** $-b + 9$

d. $-2m-6$ **e.** $-6p - 12$ **f.** $-4a - 8$

g. $-3n^2 + 15m$ **h.** $-7x^2y^2 + 21$ **i.** $-7y^2 - 49z$

j. $-12p^2 - 18q$ **k.** $-63m + 56$ **l.** $-12m^3 - 50x^3$

m. $-9a^2b + 30$ **n.** $-15p - 12q$ **o.** $-18x^2 + 4y^2$

p. $-3ab + 18m - 21$ **q.** $-10 - 25p^2 - 45q$ **r.** $-90m^2 + 27n + 54p^3$

7. Factorise each of the following expressions.

a. $a^2 + 5a$ **b.** $m^2 + 3m$ **c.** $x^2 - 6x$

d. $14q - q^2$ **e.** $18m + 5m^2$ **f.** $6p + 7p^2$

g. $7n^2 - 2n$ **h.** $a^2 - ab + 5a$ **i.** $7p - p^2q + pq$

j. $xy + 9y - 3y^2$ **k.** $5c + 3c^2d - cd$ **l.** $3ab + a^2b + 4ab^2$

m. $2x^2y + xy + 5xy^2$ **n.** $5p^2q^2 - 4pq + 3p^2q$ **o.** $6x^2y^2 - 5xy + x^2y$

8. Factorise each of the following expressions.

a. $5x^2 + 15x$ **b.** $10y^2 + 2y$ **c.** $12p^2 + 4p$

d. $24m^2 - 6m$ **e.** $32a^2 - 4a$ **f.** $-2m^2 + 8m$

g. $-5x^2 + 25x$ **h.** $-7y^2 + 14y$ **i.** $-3a^2 + 9a$

j. $-12p^2 - 2p$ **k.** $-15b^2 - 5b$ **l.** $-26y^2 - 13y$

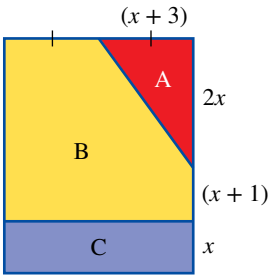
m. $4m - 18m^2$ **n.** $-6t + 36t^2$ **o.** $-8p - 24p^2$

Understanding

9. A large billboard display is in the shape of a rectangle as shown at right. There are 3 regions (A, B, C) with dimensions in terms of x as shown.

a. Determine the total area of the rectangle. Give your answer in factorised form.

b. Determine the area of each region in simplest form.



Reasoning

10. On her recent Algebra test, Marcia wrote down her answer of $(a + b)(a - b)^2$ to the question 'Using factorisation, simplify the following expression $a^2(a - b) - b^2(a - b)$'. If Marcia used difference of two squares in her solution, explain the steps she took to get her answer.

Problem solving

11. It has been said that, for any two numbers, the product of their LCM and HCF is the same as the product of the two numbers themselves.
Show whether this is true.

12. a. Factorise $36x^2 - 100y^2$ by first taking out the common factor and then using DOTS.

b. Factorise $36x^2 - 100y^2$ by first using DOTS and then taking out the common factor.

c. Do you get the same answers to parts **a** and **b**?

Reflection

How do you find the factors of terms within algebraic expressions?

3.9 The highest common binomial factor

3.9.1 The common binomial factor

- When factorising, always look for the highest common factor(s) first.
- Common factors can be expressions.
- Consider the following expression: $5(x + y) + 6b(x + y)$.
  The factors of $5(x + y)$ are 5 and $x + y$, and the factors of $6b(x + y)$ are $6b$ and $x + y$.
  So $x + y$ is a common factor.

$$\frac{5(x + y)}{x + y} = 5 \quad \text{and} \quad \frac{6b(x + y)}{x + y} = 6b$$

$$5(x + y) + 6b(x + y) = (x + y)(5 + 6b)$$

WORKED EXAMPLE 20

Factorise each of the following expressions.
a $5(x + y) + 6b(x + y)$ **b** $2b(a - 3b) - (a - 3b)$
In both these expressions it can be seen that the HCF is a binomial factor.

THINK	WRITE
a 1 The HCF $= (x + y)$.	**a** $5(x + y) + 6b(x + y)$
2 $\frac{5(x + y)}{x + y} = 5$, $\frac{6b(x + y)}{x + y} = 6b$	$= (x + y)(5 + 6b)$
b 1 The HCF $= (a - 3b)$.	**b** $2b(a - 3b) - (a - 3b)$
2 $\frac{2b(a - 3b)}{a - 3b} = 2b$, $\frac{-1(a - 3b)}{a - 3b} = -1$	$= 2b(a - 3b) - 1(a - 3b)$ $= (a - 3b)(2b - 1)$

3.9.2 Factorising by grouping terms

- If an algebraic expression has 4 terms and no common factor in all the terms, it may be possible to group the terms in pairs and find a common factor in each pair.

WORKED EXAMPLE 21

TI | CASIO

Factorise each of the following expressions by grouping the terms in pairs.
a $5a + 10b + ac + 2bc$ **b** $x - 3y + ax - 3ay$ **c** $5p + 6q + 15pq + 2$

THINK	WRITE
a 1 $5a + 10b = 5(a + 2b)$ $ac + 2bc = c(a + 2b)$	**a** $5a + 10b + ac + 2bc$
2 $a + 2b$ is a common factor.	$= 5(a + 2b) + c(a + 2b)$ $= (a + 2b)(5 + c)$
b 1 $x - 3y = 1(x - 3y)$, $ax - 3ay = a(x - 3y)$	**b** $x - 3y + ax - 3ay$
2 Take out the common factor $x - 3y$.	$= 1(x - 3y) + a(x - 3y)$ $= (x - 3y)(1 + a)$

c 1 There are no simple common factors.
Write the terms in a different order.

c $5p + 6q + 15pq + 2$

2 $5p + 15pq = 5p(1 + 3q)$
$6q + 2 = 2(3q + 1)$

$= 5p + 15pq + 6q + 2$

3 $1 + 3q = 3q + 1$

$= 5p(1 + 3q) + 2(3q + 1)$
$= 5p(1 + 3q) + 2(1 + 3q)$

4 Factorise by taking out a binomial common factor.

$= (1 + 3q)(5p + 2)$

- The answers can be checked by expanding the brackets.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: WorkSHEET: Factorising (doc-10824)

Exercise 3.9 The highest common binomial factor

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 5	Questions: 1–5	Questions: 1–7

Individual pathway interactivity: int-4487

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE20 Factorise each of the following expressions.

a. $2(a + b) + 3c(a + b)$
b. $4(m + n) + p(m + n)$
c. $7x(2m + 1) - y(2m + 1)$
d. $4a(3b + 2) - b(3b + 2)$
e. $z(x + 2y) - 3(x + 2y)$
f. $12p(6 - q) - 5(6 - q)$
g. $3p^2(x - y) + 2q(x - y)$
h. $4a^2(b - 3) + 3b(b - 3)$
i. $p^2(q + 2p) - 5(q + 2p)$
j. $6(5m + 1) + n^2(5m + 1)$

2. WE21 Factorise each of the following expressions by grouping the terms in pairs.

a. $xy + 2x + 2y + 4$
b. $ab + 3a + 3b + 9$
c. $xy - 4y + 3x - 12$
d. $2xy + x + 6y + 3$
e. $3ab + a + 12b + 4$
f. $ab - 2a + 5b - 10$
g. $m - 2n + am - 2an$
h. $5 + 3p + 15a + 9ap$
i. $15mn - 5n - 6m + 2$
j. $10pq - q - 20p + 2$
k. $6x - 2 - 3xy + y$
l. $16p - 4 - 12pq + 3q$
m. $10xy + 5x - 4y - 2$
n. $6ab + 9b - 4a - 6$
o. $5ab - 10ac - 3b + 6c$
p. $4x + 12y - xz - 3yz$
q. $5pr + 10qr - 3p - 6q$
r. $ac - 5bc - 2a + 10b$

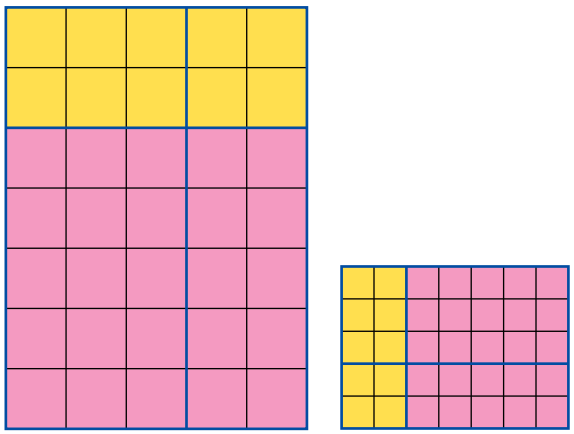
Reasoning

3. Using the method of rectangles to expand, show how $a(m + n) + 3(m + n)$ equals $(a + 3)(m + n)$.

Problem solving

4. a. Expand the expression $(3x + 2)(2x - 5)$.

b. Diagrams I and II below represent the expansion of $(3x + 2)(2x - 5)$. Examine the diagrams and explain how they work to expand the expression.



I II

c. For the products below, use grid paper to construct and appropriately colour representations for their expansions. Use your diagrams to write the expanded form of each.

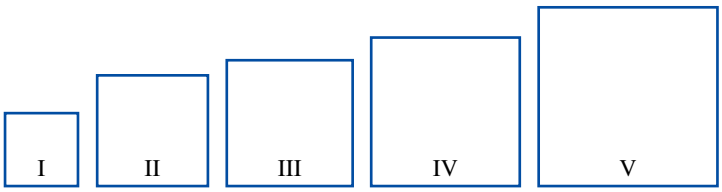
i. $(2x - 3)(x - 4)$ **ii.** $(x + 5)(-2x + 1)$

d. Expand each product in part **c** and compare with your diagrams.

5. Five squares of increasing size and five area formulas are given below.

a. Use factorisation to find the side length that correlates to each area formula.

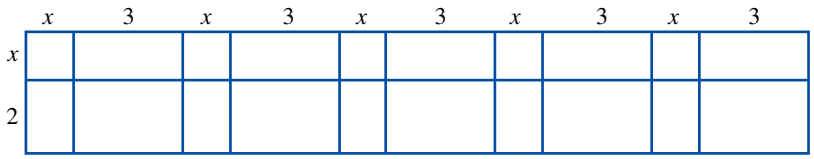
b. Using the area given and side lengths found, match the squares below with the appropriate algebraic expression for their area.



$A_a = x^2 + 6x + 9$
$A_b = x^2 + 10x + 25$
$A_c = x^2 + 16x + 64$
$A_d = x^2 - 6x + 9$
$A_e = x^2 + 12x + 36$

c. If $x = 5$ cm, use the formula given to calculate the area of each square.

6. a. Write out the product $5(x + 2)(x + 3)$ and show that it also corresponds to the diagram below.



b. Explain why $5(x + 2)(x + 3)$ is equivalent to $(5x + 10)(x + 3)$. Use bracket expansion and a labelled diagram to support your answer.

c. Explain why $5(x + 2)(x + 3)$ is equivalent to $(x + 2)(5x + 15)$. Use bracket expansion and a labelled diagram to support your answer.

7. The area formulas below relate to either squares or rectangles.

i. $9s^2 + 48s + 64$ **ii.** $25s^2 - 4$ **iii.** $s^2 + 4s + 3$ **iv.** $4s^2 - 28s - 32$

a. Without completing any algebraic operations, examine the formulas and determine which belong to squares and which to rectangles. Explain your answer.

b. Factorise each formula and classify it as a square or rectangle. Check this against your answer to part **a**.

Reflection

How do you factorise expressions with 4 terms?

3.10 Applications

3.10.1 Converting worded problems

- A very important skill is to be able to convert a worded or real-life problem into an algebra sentence or expression.

WORKED EXAMPLE 22 — TI | CASIO

A rectangular swimming pool measures 30 m by 20 m. A path around the edge of the pool is x m wide on each side.



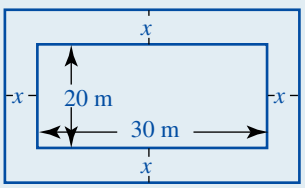
a Determine the area of the pool.
b Write an expression for the area of the pool plus the path.
c Write an expression for the area of the path.
d If the path is 1.5 m wide, calculate the area of the path.

THINK

a 1 Construct a drawing of the pool.

WRITE/DRAW

a



2 Calculate the area of the pool.

Area = length × width
= 20 × 30
= 600 m^2

b 1 Write expressions for the total length and width.	**b** Length $= 30 + x + x$ $= 30 + 2x$ Width $= 20 + x + x$ $= 20 + 2x$
2 Area = length × width	Area = length × width $= (30 + 2x)(20 + 2x)$ $= 600 + 60x + 40x + 4x^2$ $= (600 + 100x + 4x^2)$ m^2
c Find an expression for the area of the path by subtracting the area of the pool from the total area.	**c** Area of path = total area − area of pool $= 600 + 100x + 4x^2 - 600$ $= (100x + 4x^2)$ m^2
d Substitute 1.5 for x in the expression found for the area of the path.	**d** When $x = 1.5$, Area of path $= 100(1.5) + 4(1.5)^2$ $= 159$ m^2

- The algebraic expression found in part **c** of Worked example 22 allows us to calculate the area of the path for any given width, x.

WORKED EXAMPLE 23

Suppose that the page of a typical textbook is 24 cm high by 16 cm wide. The page has margins at the top and bottom of x cm and on the left and right of y cm.

a Write an expression for the height of the page that is inside the margins.

b Write an expression for the width of the page that is inside the margins.

c Write an expression for the area of the page that is inside the margins.

d Show that if the margins on the left and right are doubled, then the area available to be printed is reduced by $(48y - 4xy)$ cm^2.

e If the margins at the top and the bottom of the page are 1.5 cm and the margins on the left and right of the page are 1 cm, calculate the size of the area that is available to be printed.

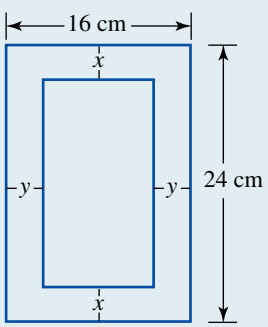


THINK

a 1 Construct a drawing, showing the key dimensions.

WRITE/DRAW

a



2 The real height (24 cm) is effectively reduced by x cm at the top and x cm at the bottom.

$$\begin{aligned}\text{Height} &= 24 - x - x \\ &= (24 - 2x) \text{ cm}\end{aligned}$$

b The width (16 cm) is reduced by y cm on the left and y cm on the right.

b
$$\begin{aligned}\text{Width} &= 16 - y - y \\ &= (16 - 2y) \text{ cm}\end{aligned}$$

c The area of the page is the product of the width and height.

c
$$\begin{aligned}\text{Area}_1 &= (24 - 2x)(16 - 2y) \\ &= (384 - 48y - 32x + 4xy) \text{ cm}^2\end{aligned}$$

d 1 If the left and right margins are doubled, they become $2y$ and $2y$, respectively. Determine the new expression for the width of the page.

d
$$\begin{aligned}\text{Width} &= 16 - 2y - 2y \\ &= (16 - 4y) \text{ cm}\end{aligned}$$

2 Determine the new expression for the reduced area.

$$\begin{aligned}\text{Area}_2 &= (24 - 2x)(16 - 4y) \\ &= 24 \times 16 + 24 \times -4y \\ &\quad - 2x \times 16 - 2x \times -2y \\ &= (384 - 96y - 32x + 8xy) \text{ cm}^2\end{aligned}$$

3 Determine the difference in area by subtracting the reduced area from the area obtained in part **c**.

$$\begin{aligned}\text{Difference in area} &= \text{Area}_1 - \text{Area}_2 \\ &= 384 - 48y - 32x + 4xy \\ &\quad - (384 - 96y - 32x + 8xy) \\ &= 384 - 48y - 32x + 4xy \\ &\quad - 384 + 96y + 32x - 8xy \\ &= 48y - 4xy\end{aligned}$$

So the amount by which the area is reduced is $(48y - 4xy)$ cm^2.

e Using the area found in **c** above, substitute the values for x and y, where $x = 1.5$ and $y = 1$.

e
$$\begin{aligned}\text{Area}_1 &= (384 - 48y - 32x + 4xy) \text{ cm}^2 \\ &= 384 - 48(1) - 32(1.5) + 4(1.5)(1) \\ &= 294 \text{ cm}^2\end{aligned}$$

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Exercise 3.10 Applications

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 8, 13, 14	Questions: 1–3, 5, 7–10, 13, 14	Questions: 1–3, 6, 7, 9–15

Individual pathway interactivity: int-4488

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. Answer the following for each shape.

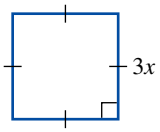
i. Determine an expression for the perimeter.

ii. Determine the perimeter when $x = 5$.

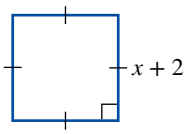
iii. Determine an expression for the area and simplify by expanding if necessary.

iv. Determine the area when $x = 5$.

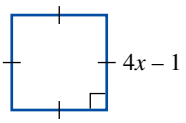
a.



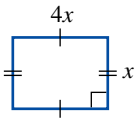
b.



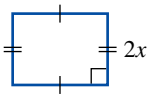
c.



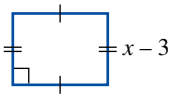
d.



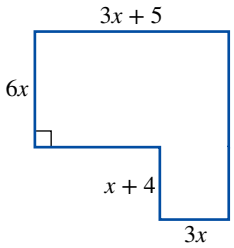
e.



f.



g.



Understanding

2. WE22 A rectangular swimming pool measures 50 m by 25 m. A path around the edge of the pool is x m wide on each side.

a. Determine the area of the pool.

b. Write an expression for the area of the pool plus the path.

c. Write an expression for the area of the path.

d. If the path is 2.3 m wide, calculate the area of the path.

e. If the area of the path is 200 m^2, write an equation that could be solved to find the width of the path.



3. WE23 The page of a book is 20 cm high by 15 cm wide. The page has margins at the top and bottom of x cm and on the left and right of y cm.

a. Write an expression for the height of the page that is inside the margins.

b. Write an expression for the width of the page that is inside the margins.

c. Write an expression for the area of the page that is inside the margins.

d. Show that if the margins on the left and right are doubled, then the area available to be printed on is reduced by $(40y - 4xy)$ cm.



4. A rectangular book cover is 8 cm long and 5 cm wide.

a. Calculate the area of the book cover.

b. i. If the length of the book cover is increased by v cm, write an expression for its new length.

ii. If the width of the book cover is increased by v cm, write an expression for its new width.

iii. Write an expression for the new area of the book cover and expand.

iv. Calculate the area of the book cover if $v = 2$ cm.

c. i. If the length of the book cover is decreased by d cm, write an expression for its new length.

ii. If the width of the book cover is decreased by d cm, write an expression for its new width.

iii. Write an expression for the new area of the rectangle and expand.
iv. Calculate the area of the book cover if $d = 2$ cm.

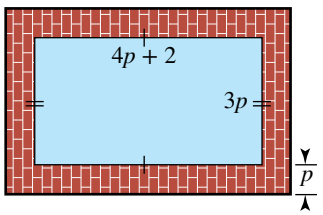
d. i. If the length of the book cover is made x times as long, write an expression for its new length.
ii. If the width of the book cover is increased by x cm, write an expression for its new width.
iii. Write an expression for the new area of the book cover and expand.
iv. Calculate the area of the book cover if $x = 5$ cm.

5. A square has dimensions of $5x$ m.
a. Write an expression for its perimeter.
b. Write an expression for its area.
c. i. If its length is decreased by 2 m, write an expression for its new length.
ii. If its width is decreased by 3 m, write an expression for its new width.
iii. Write an expression for its new area and expand.
iv. Calculate its area when $x = 6$ m.

6. A rectangular sign has a length of $2x$ cm and a width of x cm.
a. Write an expression for its perimeter.
b. Write an expression for its area.
c. i. If its length is increased by y cm, write an expression for its new length.
ii. If its width is decreased by y cm, write an expression for its new width.
iii. Write an expression for its new area and expand.
iv. Calculate its area when $x = 4$ cm and $y = 3$ cm using your expression.

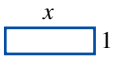
7. A square has a side length of x cm.
a. Write an expression for its perimeter.
b. Write an expression for its area.
c. i. If its side length is increased by y cm, write an expression for its new side length.
ii. Write an expression for its new perimeter and expand.
iii. Calculate the perimeter when $x = 5$ cm and $y = 9$ cm.
iv. Write an expression for its new area and expand.
d. Calculate the area when $x = 3.2$ cm and $y = 4.6$ cm.

8. A swimming pool with length $(4p + 2)$ m and width $3p$ m is surrounded by a path of width p m. Obtain the following in expanded form.
a. An expression for the perimeter of the pool
b. An expression for the area of the pool
c. An expression for the length of the pool and path
d. An expression for the width of the pool and path
e. An expression for the perimeter of the pool and path
f. An expression for the area of the pool and path
g. An expression for the area of the path
h. The area of the path when $p = 2$ m

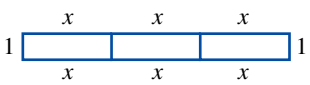


Reasoning

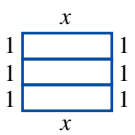
9. The figure below right is a rectangle. For the purpose of this activity we are going to call this rectangle an 'algebra rectangle'.
The length of each algebra rectangle is x cm and the width is 1 cm.
These algebra rectangles are put together to form larger rectangles in one of two ways.



Long algebra rectangle (3-long)



Tall algebra rectangle (3-tall)



Let's find the perimeter of each algebra rectangle.

Perimeter of 3-long algebra rectangle; $P = (x + x + x + x + x + x) + (1 + 1)$
$= 6x + 2$

Perimeter of 3-tall algebra rectangle; $P = (x + x) + (1 + 1 + 1 + 1 + 1 + 1)$
$= 2x + 6$

a. Find the perimeter of each of the algebra rectangles and put your results into the table below.

Type of algebra rectangle	Perimeter	Type of algebra rectangle	Perimeter
1-long		1-tall	
2-long		2-tall	
3-long	$6x + 2$	3-tall	$2x + 6$
4-long		4-tall	
5-long		5-tall	

b. Can you see a pattern? What would be the perimeter of a 20-long algebra rectangle and a 20-tall algebra rectangle?

c. For what values of x will the tall rectangle have a larger perimeter than the long rectangle?

10. The Body Mass Index (B) is used as an indicator of whether or not a person is in a healthy weight range for their height. It can be found using the formula $B = \frac{m}{h^2}$, where m is the person's mass in kilograms and h is the person's height in metres. Use the formula to answer the following questions, correct to 1 decimal place.

a. Calculate Richard's Body Mass Index if he weighs 85 kg and is 1.75 m tall.

b. A person is considered to be in a healthy weight range if his or her Body Mass Index lies in the range of 21 to 25 inclusive. Comment on Richard's weight for a person of his height.

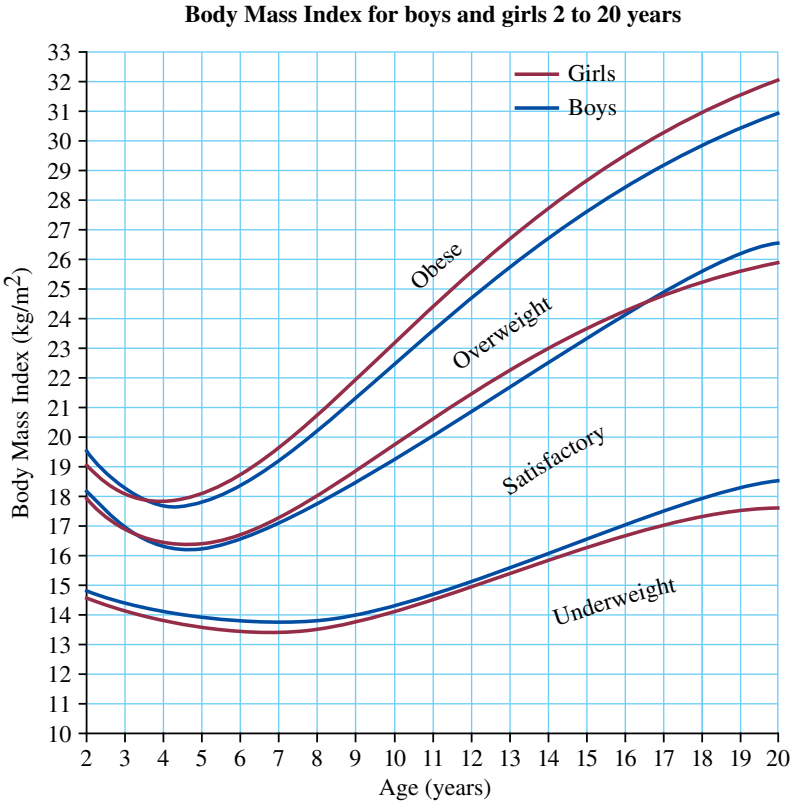
c. Calculate the Body Mass Index, correct to 1 decimal place, for each of the following people:

i. Judy, who is 1.65 m tall and has a mass of 52 kg

ii. Karen, who is 1.78 m tall and has a mass of 79 kg

iii. Manuel, who is 1.72 m tall and has a mass of 65 kg.

d. The Body Mass Index (BMI) range for children changes until adulthood. These graphs show the Body Mass Index for boys and girls aged 2 to 20 years. Study the graphs carefully and decide on possible age ranges (between 2 and 20) for Judy, Karen and Manuel if their BMI was in the Satisfactory range. State the age ranges in half-year intervals.



11. Can you evaluate 997^2 without a calculator and in less than 90 seconds? We would be able to evaluate this using long multiplication, but it would take a fair amount of time and effort. Mathematicians are always looking for quick and simple ways of solving problems. What if we consider the expanding formula that produces the difference of two squares?

$(a + b)(a - b) = a^2 - b^2$

Adding b^2 to both sides gives $(a + b)(a - b) + b^2 = a^2 - b^2 + b^2$.

Simplifying and swapping sides gives $a^2 = (a + b)(a - b) + b^2$.

We can use this new formula and the fact that multiplying by 1000 is an easy operation to evaluate 997^2.

a. If $a = 997$, what should we make the value of b so that $(a + b)$ equals 1000?

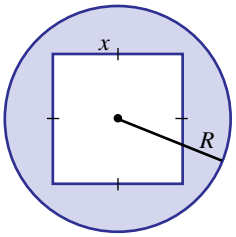
b. Substitute these a and b values into the formula to evaluate 997^2.

c. Try this method to evaluate the following.

i. 995^2 **ii.** 990^2

Problem solving

12. A new game has been created by students for the school fair. To win the game you need to hit the target with 5 darts in the shaded region.



a. Write an expression for the area of the game board (the shaded region).

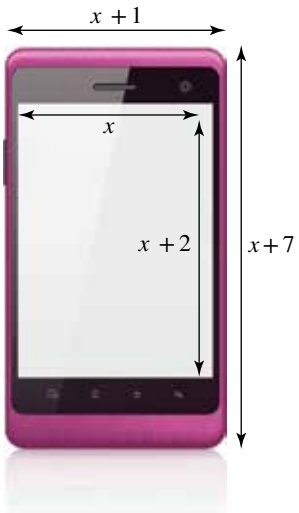
b. If $R = 7.5$ cm and $x = 4$ cm, find the area of the shaded region.

c. Show that $R = \sqrt{\frac{A + x^2}{\pi}}$ by transposing the formula found in part **a**.

d. If $A = 80$ cm^2 and $x = 3$ cm, calculate the value of R.

e. The students found that the best size for the game board is when $R = 10$ and $x = 5$. What is the percentage of the total board that is shaded?

13. A picture of a mobile phone is shown at right (dimensions are in cm).



a. Use the information in the picture to write an expression for the area of:

i. the viewing screen

ii. the front face of the phone.

b. A friend has a phone that is 4 cm longer but 1 cm narrower.

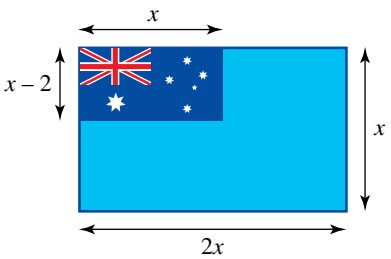
Write expressions, in expanded form, for:

i. the length and width of your friend's phone

ii. the area of your friend's phone.

c. If the phone pictured is 5 cm wide, use your answer to part **b** to find the area of the front face of your friend's phone.

14. A proposed new flag for Australian schools will have the Australian flag in the top left-hand corner. The dimensions given are in metres.



a. Write an expression, in factorised form, for the area of:

i. the Australian flag

ii. the proposed flag.

b. Use the answers from part **a** to write the area of the Australian flag as a *fraction* of the school flag and then simplify this fraction.

c. Use the fraction from part **b** to express the area of the Australian flag as a *percentage* of the proposed school flag.

d. Using the formula for the percentage of area taken up by the Australian flag, find the percentages for the following school flag widths.

i. 4 m

ii. 4.5 m

iii. 4.8 m

e. If the percentage of the school flag taken up by the Australian flag measures the importance a school places on Australia, what can be said for the three flags?

15. Cubic expressions are the expanded form of expressions with three linear factors. The expansion process for three linear factors is called trinomial expansion.

a. Explain how the area model can be altered to show that binomial expansion can also be altered to become a model for trinomial expansion.

b. Investigate the powers in cubic expressions by expanding the following expressions.

i. $(3x + 2)(x + 1)(x + 1)$

ii. $(x + 5)(2x - 2)(4x - 8)$

iii. $(3 - x)(x + 8)(5 - x)$

c. Describe the patterns in the powers in a cubic expression.

Reflection

How can you use algebraic skills in real-life situations?

3.11 Review

3.11.1 Review questions

Fluency

1. For the expression $-5xy^2 + 2y + 8y^2 + 6$, the coefficient of the second term is:

A. -5 **B.** 2 **C.** 8 **D.** 6

2. Jodie had \$55 in her purse. She spent \$$x$. The amount left is:

A. $x - 55$ **B.** $x + 55$ **C.** $55x$ **D.** $55 - x$

3. What is $\frac{3x}{5} - \frac{x}{4}$ simplified?

A. $\frac{7x}{20}$ **B.** $\frac{2x}{20}$ **C.** $\frac{2x}{1}$ **D.** $\frac{4x}{20}$

4. What does $\frac{3}{p} \div \frac{6}{p}$ equal?

A. 2 **B.** $\frac{1}{2}$ **C.** $12p$ **D.** $12p^2$

5. The volume V of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere. The volume, if $r = 3$, is:

A. 25.12 **B.** 38.37 **C.** 64.05 **D.** 113.10

6. What is the equivalent of $6 - 4x(x + 2) + 3x$?

A. $6 - 4x^2 - 5x$ **B.** $6 + 4x^2 + 5x$ **C.** $6 - 4x^2 + 5x$ **D.** $6 + 4x^2 + 5x$

7. The equivalent of $(3 - a)(3 + a)$ is:

A. $9 + a^2$ **B.** $9 - a^2$ **C.** $3 + a^2$ **D.** $3 - a^2$

8. The equivalent of $(2y + 5)^2$ is:

A. $4y^2 + 20y + 25$ **B.** $4y^2 + 10y + 25$ **C.** $2y^2 + 20y + 25$ **D.** $2y^2 + 20y + 5$

9. When factorised, $6(a + 2b) - x(a + 2b)$ equals:

A. $6 - x(a + 2b)$ **B.** $(6 - x)(a + 2b)$ **C.** $6(a + 2b - x)$ **D.** $(6 + x)(a - 2b)$

10. a. For the expression $-8xy^2 + 2x + 8y^2 - 5$:

i. state the number of terms **ii.** state the coefficient of the first term

iii. state the constant term **iv.** state the term with the smallest coefficient.

b. Write expressions for the following, where x and y represent numbers:

i. a number 8 more than y

ii. the difference between x and y

iii. the sum of x and y

iv. 7 times the product of x and y

v. 2 times x is subtracted from 5 times y.

11. a. Leo receives x dollars for each car he washes. If he washes y cars, how much money does he earn?

b. A piece of rope is 24 m long.

i. If George cuts k m off, how much is left?

ii. After George has cut k m off, he divides the rest into three pieces of equal length. How long is each piece?

12. Simplify the following expressions by collecting like terms.

a. $8p + 9p$ **b.** $5y^2 + 2y - 4y^2$ **c.** $9s^2t - 12s^2t$

d. $11c^2d - 2cd + 5dc^2$ **e.** $n^2 - p^2q - 3p^2q + 6$ **f.** $8ab + 2a^2b^2 - 5a^2b^2 + 7ab$

13. Simplify the following expressions.

a. $6a \times 2b$ **b.** $2ab \times b$ **c.** $2xy \times 4yx$ **d.** $\frac{4x}{12}$ **e.** $18 \div 4b$

14. Expand these expressions.

a. $5(x + 3)$ **b.** $-(y + 5)$ **c.** $-x(3 - 2x)$ **d.** $-4x(2m + 1)$

15. Expand and simplify by collecting like terms.

a. $3(x - 2) + 9$ **b.** $-2(5m - 1) - 3$ **c.** $4m(m - 3) + 3m - 5$ **d.** $7p - 2 - (3p + 4)$

16. Expand and simplify the following expressions.

a. $3(a + 2b) + 2(3a + b)$ **b.** $-4(2x + 3y) + 3(x - 2y)$

c. $2m(n + 6) - m(3n + 1)$ **d.** $-2x(3 - 2x) - (4x - 3)$

17. Expand and simplify these expressions.

a. $(x + 4)(x + 5)$ **b.** $(m - 2)(m + 1)$ **c.** $(3m - 2)(m - 5)$ **d.** $(2a + b)(a - 3b)$

18. Expand and simplify these expressions.

a. $(x + 4)(x - 4)$ **b.** $(9 - m)(9 + m)$ **c.** $(x + y)(x - y)$ **d.** $(1 - 2a)(1 + 2a)$

19. Expand and simplify these expressions.

a. $(x + 5)^2$ **b.** $(m - 3)^2$ **c.** $(4x + 1)^2$ **d.** $(2 - 3y)^2$

20. Expand and simplify these expressions.

a. $(x + 2)(x + 1) + (x + 3)(x + 2)$ **b.** $(m + 7)(m - 2) + (m + 3)^2$

c. $(x + 6)(x + 2) - (x + 3)(x - 1)$ **d.** $(b - 7)^2 - (b - 3)(b - 4)$

21. Expand and simplify the following.

a. $(x + 2)^2 + (x + 3)^2$ **b.** $(x - 2)^2 - (x - 3)^2$ **c.** $(x + 4)^2 - (x - 4)^2$

22. If $y = 5x^2 + 2x - 1$, find y when:

a. $x = 2$ **b.** $x = 5$.

23. The volume (V) of each of the following paint tins is given by the formula, $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Find V (to 2 decimal places).

a. $r = 7$ cm and $h = 2$ cm **b.** $r = 9.3$ cm and $h = 19.8$ cm.

24. Factorise each of the following by finding common factors.

a. $6x + 12$ **b.** $6x^2 + 12x^2y$ **c.** $8a^2 - 4b$

d. $16x^2 - 24xy$ **e.** $-2x - 4$ **f.** $b^2 - 3b + 4bc$

25. Simplify each of the following using common factor techniques.

a. $5(x + y) - 4a(x + y)$ **b.** $7a(a + 5c) - 6c(b + 5c)$ **c.** $15x(d + 2e) + 25xy(d + 2e)$

d. $2x + 2y + ax + ay$ **e.** $6xy + 4x - 6y - 4$ **f.** $pq - r + p - rq$

Problem solving

26. A rectangular rug has a length of $3x$ cm and a width of x cm.

a. Write an expression for its perimeter.

b. Write an expression for its area.

c. **i.** If its side length is increased by y cm, write an expression for its new side length.

ii. Write an expression for its new perimeter and expand.

iii. Calculate the perimeter when $x = 90$ cm and $y = 30$ cm.

iv. Write an expression for its new area and expand.

v. Calculate the area when $x = 90$ cm and $y = 30$ cm.

27. A rectangular garden bed has a length of 15 m and a width of 8 m. It is surrounded by a path of width p m.

a. Write down the total area of the garden bed and path in factorised form.

b. Expand the expression you found in **a**.

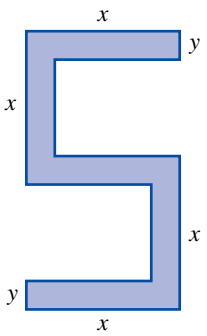
c. Find the area of the path in terms of p.

d. Write an equation that could be solved to find the width of the path if the area of the path is 200 m^2.

28. Find the circumference of a circle whose area is $\pi\left(\frac{x}{y}\right)^2$ cm^2. Give your answer in terms of π.

29. This large sign appears in a parking lot at the entrance to car park 5. It has a uniform width, with the dimensions as shown.

Write an algebraic expression for the area of the front of the sign.



30. At the end of a lesson using boxes of Smarties, the remaining Smarties are shared equally amongst the members of each group. In one group there are 6 boxes of Smarties left over and 42 Smarties that are no longer in boxes.

a. Write an expression for the total number of Smarties left over. (*Hint:* Let $n =$ the number of Smarties in a box.)

b. Write the expression from part **a** in factorised form.

c. If there are 3 people in the group, write an expression for the number of Smarties each person receives.

d. If there are x people in the group, write an expression for the number of Smarties each person receives.

31. When entering numbers into an electronic device, or even writing numbers down, errors frequently occur. A common type of error is a transposition error, which occurs when two digits are written in the reverse order. Take the number 2869, for example. With this type of error, it could be written as 8269, 2689 or 2896. A common rule for checking these errors is as follows.

If the difference between the correct number and the recorded number is a multiple of 9, a transposition error has occurred.

We can use algebraic expressions to check this rule. Let the digit in the thousands position be represented by a, the digit in the hundreds position by b, the digit in the tens position by c and the digit in the ones position by d. So the real number can be represented as $1000a + 100b + 10c + d$.

a. If the digits in the ones position and the tens position were written in the reverse order, the number would be $1000a + 100b + 10d + c$. The difference between the correct number and the incorrect one would then be:

$1000a + 100b + 10c + d - (1000a + 100b + 10d + c)$

i. Simplify this expression.

ii. Is the expression a multiple of 9? Explain.

b. If a transposition error had occurred in the tens and hundreds position, the incorrect number would be $1000a + 100c + 10b + d$. Perform the procedure shown in question **31a** to determine whether the difference between the correct number and the incorrect one is a multiple of 9.

c. Consider, lastly, a transposition error in the thousands and hundreds positions. Is the difference between the two numbers a multiple of 9?

d. Comment on the checking rule for transposition errors.

e. The price marked on the tag of a CD in a music store is x dollars and y cents. Its value is less than $100. The cashier makes a mistake and enters a value of y dollars and x cents into the register. Show whether the rule for checking transposition errors applies in this case.

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Complete this digital doc: Concept map: Topic 3 (doc-10794)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

binomial
coefficient
common factor
constant
difference of two squares
distributive law
evaluate
expand
expression
factorise
FOIL
highest common factor
pronumeral
simplify
substitution
terms
trinomial
variable

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Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

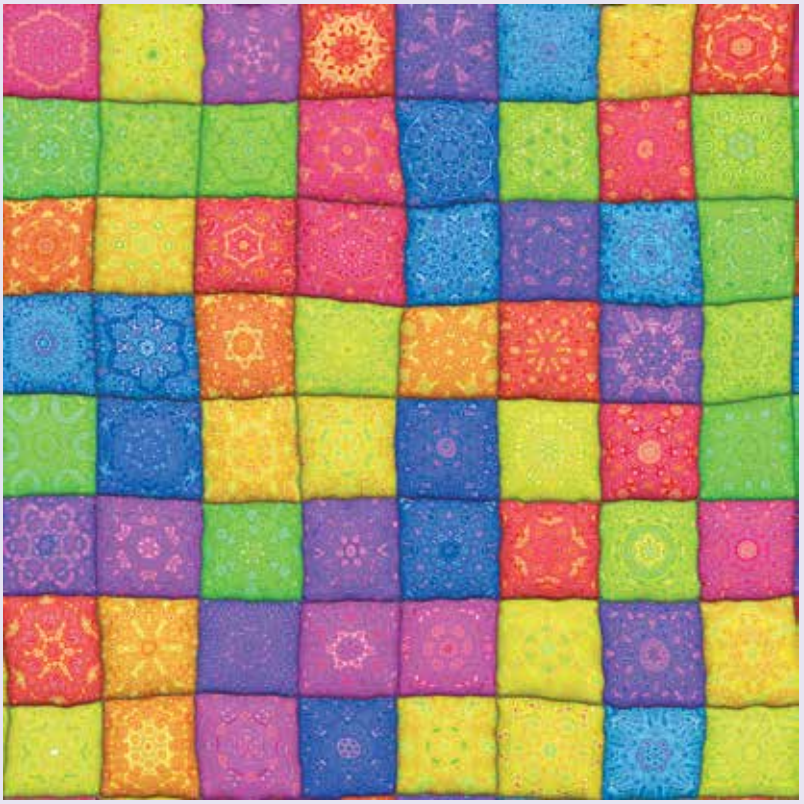
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

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Investigation | Rich task

Quilt squares



Quilts are decorative blankets that are also used to keep us warm. For many generations, people all over the world have been interested in quilt making, which involves stitching pieces of fabric together and inserting stuffing between the sewn fabric. The fabric can be arranged and sewn in a variety of ways to create attractive geometric designs. Quilts can also be used as decorative objects.

Medini is designing a special quilt. She is sewing pieces of different coloured fabrics together to make a block, and then copying the block and sewing the blocks together repeatedly.

A scaled diagram of the basic block is shown below right. The letters indicate the colours of the fabric that make up the block: yellow, black and white. The yellow and white pieces are square; the black pieces are rectangular. The blocks are sewn together in rows and columns.

y	b	y
b	w	b
y	b	y

Diagram of basic block used in the quilt

Trace or copy the basic block at right onto a sheet of paper. Repeat this process until you have nine blocks. Colour in each section and cut out all nine blocks.

1. Place all nine blocks together to form a 3 × 3 square. In the space provided below, draw and colour a scaled diagram of your result.
2. Describe the feature created by arranging the blocks in the manner described in question **1**. Observe the shapes created by the different colours.

Medini sold her design to a company that now manufactures quilts made from 100 of these blocks. Each quilt covers an area of 1.44 m^2. Each row and column has the same number of blocks. Answer the following, ignoring any seam allowances.

3. Calculate the side length of each square block.

4. If the entire quilt has an area of 1.44 m^2, what is the area of each block?

5. Determine the dimensions of the yellow, black and white pieces of fabric of each block.

6. Calculate the area of the yellow, black and white pieces of fabric of each block.

7. Determine the total area of each of the three different colours required to construct this quilt.

8. Due to popular demand, the company that manufactures these quilts now makes them in different sizes. All that the customer is required to do is to specify the approximate quilt area, the three colours required and the number of blocks in the quilt (they must conform to a square or rectangular shape). Derive a general formula that would allow the company to quickly determine the areas of the three coloured fabrics in each block. Give an example and a diagram on a separate sheet of paper to illustrate your formula.



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Complete this digital doc: Code puzzle: Why does a giraffe have a long neck? (doc-15891)

Answers

Topic 3 Algebra

Exercise 3.2 Using pronumerals

1. a. 3 **b.** 7 **c.** -2 **d.** -8 **e.** 1
f. -1 **g.** $\frac{1}{3}$ **h.** $\frac{1}{2}$ **i.** $-\frac{1}{4}$ **j.** $-\frac{1}{9}$

2. a. 6 **b.** 7 **c.** 3 **d.** -2 **e.** 3
f. -2 **g.** -7 **h.** -11 **i.** $\frac{-1}{2}$ **j.** $\frac{-2}{3}$
k. $\frac{1}{4}$ **l.** 1 **m.** -3 **n.** -2 **o.** 0

3. a. i. 3 **ii.** 5 **iii.** 8 **iv.** $5x^2$
b. i. 3 **ii.** -9 **iii.** -6 **iv.** $-9m^2$
c. i. 4 **ii.** 5 **iii.** 5 **iv.** $-7x^2$
d. i. 4 **ii.** 9 **iii.** 4 **iv.** $-9b^2$
e. i. 5 **ii.** 11 **iii.** -4 **iv.** $-7q$
f. i. 5 **ii.** -9 **iii.** 5 **iv.** $-9p$
g. i. 4 **ii.** 4 **iii.** -2 **iv.** $-3ac$
h. i. 5 **ii.** 5 **iii.** 9 **iv.** $-3u$
i. i. 5 **ii.** -1 **iii.** 8 **iv.** $-m$
j. i. 5 **ii.** 7 **iii.** 14 **iv.** $-3cd^2$

4. a. 7, 2, -8 **b.** 8, 2, -4 **c.** 3, 0, 0
d. 0, -2, 0 **e.** 1, 0, 2 **f.** 2, cannot be done, -1

5. a. B **b.** D **c.** A **d.** B

6. a. 4 **b.** -6 **c.** 6 **d.** 7 **e.** -12 **f.** -3

7. a. 7 **b.** -40 **c.** 7

8. a. i. 32 **ii.** 250
b. i. 4 **ii.** 3

9. a. 15 **b.** 10 **c.** 18 **d.** 45 **e.** 30
f. 10.72 **g.** 21 **h.** 22.4 **i.** 50 **j.** 171
k. 4 **l.** -30 **m.** 32.75 **n.** 38.33 **o.** 34.56

10. a. 12 cm^2 **b.** 12 cm

11. a. 15 **b.** Answers will vary.

12. a. 5 **b.** Answers will vary.

13. a. 63 cm^3 **b.** 4 cm

14. a. 10 **b.** 10

15. a. 19.44 **b.** Answers will vary.

16. a. 461.81 **b.** 9.55

17. a. 1671.33 **b.** 2.64

18. a.

d	$2d + 2g$	g
$2g$	$d + g$	$2d$
$2d + g$	0	$d + 2g$

Magic number = $3d + 3g$

b, **c** Answers will vary.

19. a. mn **b.** $(m - 2)(n - 2)$ **c.** $(2m + 2n)$ units

20. $y - (z - x)$

Exercise 3.3 Algebra in worded problems

1. a. 11 **b.** $x + 5$ **c.** $y + 5$

2. a. 3.5 km **b.** $(x + 2)$ km

3. a. 4 **b.** $2.5 + t$ **c.** $2.5 - y$

4. a. $10n$, where $n =$ number of shirts

b. $N + 30$, where $N =$ the number of Nick's dollars

c. i. $g - 4$ **ii.** $7g - 4$

5. a. i. 94 **ii.** $6x + y$

b. i. 66 **ii.** $6p + q$

c. i. 28 **ii.** $6x + y - 6p - q$

6. xy dollars

7. a. $\frac{x}{n}$ **b.** $\frac{x}{2n}$

8. a. $(30 - d)$ cm **b.** $\frac{30 - d}{4}$ cm **c.** $\frac{3(30 - d)}{4}$ cm

9. a. $\frac{x}{4}$ **b.** $\frac{x}{6}$ **c.** $\frac{x}{4} - \frac{x}{6} = \frac{x}{12}$

10. a. $\frac{24 - c}{7}$ hours **b.** $(c + \frac{24 - c}{7}) = \frac{6c + 24}{7}$

11. a. 1 **b.** 8.30 pm and 9.00 pm **c.** 5

d. 24 **e.** 7.30 pm and 8.00 pm

12. a. $b = 2h - 1$ **b.** 19 blocks

13. 4 dots = 4 triangles

5 dots = 10 triangles

6 dots = 20 triangles

7 dots = 35 triangles

$t = \frac{d!}{6(d - 3)!}$ where $d!$ is the product of all positive integers less than or equal to d.

14. 15 red vases and 10 clear vases

15. Answers will vary.

16. a. $x = \frac{T - 3}{T - 1}$

b, **c**. Undefined; answers will vary.

Challenge 3.1

The father is 32 years old.

Exercise 3.4 Simplifying algebraic expressions

1. a. $9ab, -ab$ **b.** $4x$ **c.** $-3za, -az$ **d.** $2x^2, -x^2$

e. $-2x^2y$ **f.** $-x^2y^5, -3x^2y^5$ **g.** $p^3x^2w^5, -5x^2p^3w^5$ **h.** $4y^5z^4x^2, -2x^2z^4y^5$

2. a. $7x$ **b.** $11y$ **c.** $19m$ **d.** $11q$

e. $8r$ **f.** $3x$ **g.** $8a$ **h.** $8y$

i. $13x$ **j.** $16p$ **k.** $9q^2$ **l.** $3x^2$

m. $8x^2 - 3y$ **n.** $2m^2 + 2n$ **o.** $7x^2 + x$ **p.** $9h^2 + h + 9$

q. $-2g^2 + g - 12$ **r.** $-5m^2 + m + 15$ **s.** $12a^2 + b + 4b^2$ **t.** $3m + 7n^2$

u. $12xy + 2y^2$ **v.** $2ab + 5a^2b$ **w.** $16x^2y - 3xy$ **x.** $m^2n + 11n$

y. $-7x^2 + 2x^2y$ **z.** $-3a^2b - 9a^2 + 5b + 4$

3. a. B **b.** D **c.** A **d.** D

4. a. $6mn$ **b.** $20xy$ **c.** $8pq$ **d.** $-10xy$

e. $-12xy$ **f.** $15mn$ **g.** $10a^2$ **h.** $20y^2$

i. $5p^2$ **k.** $6mnp$ **m.** $10m^2n$ **o.** $30x^3y^2$

j. $7m^2$ **l.** $-6ab^2$ **n.** $-18a^2b$ **p.** $-12p^2q^3$

q. $-56c^3d$ **r.** $30a^4b^7$

5. a. $3x$ **b.** $3m$ **c.** $2y$ **d.** $4m$

e. $4m$ **f.** $2x$ **g.** $-7x$ **h.** $-4m$

i. $\frac{m}{2}$ **j.** $\frac{x}{3}$ **k.** $\frac{4m}{9}$ **l.** $\frac{4x}{3}$

m. $\frac{1}{5m}$ **n.** $\frac{1}{2a}$ **o.** $2yz$ **p.** $\frac{35ab}{2}$

q. $\frac{xy}{4}$ **r.** $\frac{-7yz}{11}$

6. a. $40x^2y^2$ **b.** $56ax^2y^2$ **c.** $12x^3y^2$

d. $\frac{x^2}{2y}$ **e.** $\frac{-5a}{4b}$ **f.** 2

g. $40a^3b^2$ **h.** $-8a^3b^3$ **i.** $16a^4$

7. a. i. $(0.01mp + nq)$ dollars **ii.** $(mp + 100nq)$ cents

b. $20 - (0.01mp + nq)$ dollars

8. Shirt = \$12 each

Trousers = \$30 each

9. Explanations will vary. The correct answer is $7a^2c^2$.

10. a. i.

	$3a + 4b$	
$a + b$		$2a + 3b$
$2a - b$	**$-a + 2b$**	$3a + b$

ii.

	$8p + 3q$	
$3p + 5q$		$5p - 2q$
$p + 2q$	$2p + 3q$	**$3p - 5q$**

b. i.

	$18m^3n^2$	
$3m^2n$		**$6mn$**
m^2	$3n$	$2m$

ii.

	$48d^2e^2$	
$8de$		**$6de$**
$4d$	**$2e$**	$3d$

11. a. $P = \frac{x}{3} + x + \frac{5x}{7}$ **b.** Answers will vary. **c.** \$18

12. a. $P = 2w + \frac{32w}{7}$ **b.** Answers will vary. **c.** \$72.22

13. a. $P = 2x + 2(4x - 7)$ **b.** $P = 10x - 14$ **c.** 16

d. If $x = 1$ the rectangle will have a negative value perimeter, which is not possible.

14. a. $l = 4(x + 20) + 4(x + 40) + 4(x + 20)$ **b.** $l = 12x + 320$ **c.** \$19.20

15. a. $P_1 = 10x + 12$ cm

$P_2 = 16x$ cm

b. $x = 2$

Exercise 3.5 Expanding brackets

1. a. 64 **b.** 1100 **c.** -20 **d.** 693

2. a. $3x + 6$

3	$3x$	6
	x	2

b. $4x + 12$

4	$4x$	12
	x	3

c. $4x + 4$ **d.** $7x - 7$ **e.** $-3p + 6$

f. $-x + 1$ **g.** $6b - 12$ **h.** $24m - 16$

3. a. $x^2 + 2x$

x	x^2	$2x$
	x	2

b. $a^2 + 5a$

a	a^2	$5a$
	a	5

c. $4x + x^2$ **d.** $7m - m^2$ **e.** $2xy + 4x$

f. $-3xy - 12y$ **g.** $-3b + ab$ **h.** $-30a + 18a^2$

4. a. $2p - 2$ **b.** $5x - 17$ **c.** $-7p - 17$ **d.** $-12p + 3$

e. $6x^2 - 20x$ **f.** $2m^2 + 7m$ **g.** $3px + 6x - 5$ **h.** $4y^2 - 4y + 7$

i. $-4p^2 + 13p$ **j.** $4x - 13y$ **k.** $2m^2 - 8m - 4$ **l.** $-3p^2 + 10pq - 1$

m. $-30a + 10ab$ **n.** $7cd - 12c^2 - 5c$ **o.** $-2p - 17$ **p.** $3 - 3m$

5. a. $8x + y$ **b.** $10p + 8q$ **c.** $18a + 29b$ **d.** $19c + 22d$

e. $2m - 11n$ **f.** $6x - 11y$ **g.** $9x + 5y$ **h.** $-14p - 8q$

i. $-4a + 3b$ **j.** $4x - y$ **k.** $5p - 10q$ **l.** $-8c + 9d$

m. $13x - 19y$ **n.** $-7p + 11q$ **o.** $-5a + 3b$ **p.** $8c + d$

6. a. $2ab + 2a - 3b$ **b.** $2xy + 4x - 2y$ **c.** $2cd + 3c$ **d.** $2pq - 2p$

e. $5cd - 11c$ **f.** $5ab - 21a - 3b$ **g.** $5m$ **h.** $6cd - 36c$

i. $6m^2 + 6m - 10$ **j.** $9cd - 8c$ **k.** $-15a^2 + 2b^2 - 9ab$ **l.** $-8c^2 + 3d^2 + 22cd$

m. $12m^2 - 20m - 4$ **n.** $2p^2 + 7p - 6$ **o.** $-7x^2 + 41x - 6$ **p.** $-10y^2 - 6y - 12$

7. a. B **b.** A **c.** D

8. a. $a^2 + 5a + 6$ **b.** $x^2 + 7x + 12$ **c.** $y^2 + 5y + 6$ **d.** $m^2 + 9m + 20$

e. $b^2 + 3b + 2$ **f.** $p^2 + 5p + 4$ **g.** $a^2 + a - 6$ **h.** $x^2 + x - 20$

i. $m^2 - m - 12$ **j.** $y^2 + 2y - 15$ **k.** $y^2 - 4y - 12$ **l.** $x^2 - 2x - 3$

m. $x^2 - 7x + 12$ **n.** $p^2 - 5p + 6$ **o.** $x^2 - 4x + 3$

9. a. $2a^2 + 7a + 6$ **b.** $3m^2 + 7m + 2$ **c.** $6x^2 + 10x + 4$

d. $4c^2 - 31c + 42$ **e.** $2t^2 - 17t + 35$ **f.** $2x^2 - 11x + 9$

g. $-6t^2 + 11t + 10$ **h.** $15x^2 - 31x + 14$ **i.** $25x^2 - 20x + 4$

10. a. $xz + x + yz + y$ **b.** $pr + 3p + qr + 3q$ **c.** $2xz + 8x + yz + 4y$ **d.** $3pr + 3p + qr + q$

e. $a^2 + 3ab + 2b^2$ **f.** $2c^2 - 5cd - 3d^2$ **g.** $2x^2 - xy - 3y^2$ **h.** $4p^2 + pq - 3q^2$

i. $3xy + 3yz + xz + z^2$ **j.** $ab + ac + 2b^2 + 2bc$ **k.** $3p - 9pr - 2q + 6qr$ **l.** $7cd - 35c - 2d^2 + 10d$

m. $12x^2 - 7xy + y^2$ **n.** $2p^2 - pr - 2pq + qr$ **o.** $15k + 5 - 6jk - 2j$

11. a. A **b.** C **c.** C

12. a. $x^2 - 9$ **b.** $x^2 - 25$ **c.** $x^2 - 49$

d. $x^2 - 1$ **e.** $x^2 - 4$ **f.** $4x^2 - 1$

13. a. $x^2 + 2x + 1$ **b.** $x^2 + 4x + 4$ **c.** $x^2 + 16x + 64$

d. $x^2 - 6x + 9$ **e.** $x^2 - 10x + 25$ **f.** $x^2 - 18x + 81$

14. a. $6.3x^2 + 5.53xy - 3.1y^2$ **b.** $4.41x^2 - 10.24y^2$ **c.** $11.56x^2 + 34.68xy + 26.01y^2$

15. a. Surface area $= 38x^2 + 14x - 6$ **b.** Volume $= 12x^3 + 5x^2 - 3x$

16. a. i. $(x + 3y)(4x - y)$ **ii.** $4x^2 + 11xy - 3y^2$ **iii.** $x > -3y, x > \frac{y}{4}$

b. i. $\frac{(2x - 1)(x + 5)}{2}$ **ii.** $\frac{2x^2 + 9x - 5}{2}$ **iii.** $x > \frac{1}{2}$

17. Answers may vary.

18. a. i. Student did not multiply both terms.

ii. Student used addition instead of multiplication.

iii. Student did not change negative and positive signs.

b. $100 - 15x$

19. a. $-2(a - 5) = 2a - 10$ is incorrect because the student did not change the multiplied negative signs for -2×-5 to a positive sign. The correct answer is $-2a + 10$.

b. $2b(3b - 1) = 6b^2 - 1$ is incorrect because the student did not multiply -1 and $2b$ together. The correct answer is $6b^2 - 2b$.

c. $-2(c - 4) = 2c + 8$ is incorrect because the student has left out the negative sign when multiplying -2 and c. The correct answer is $-2c + 8$.

20. a. Student C

b. Corrections to the students' answers are shown in bold.
Student A:
$(3x + 4)(2x + 5)$
$= 3x \times 2x + 3x \times 5 + 4 \times 2x + 4 \times 5$
$= \mathbf{6x^2} + 23x + 20$
(Also, $29x + 20$ does not equal $49x$.)
Student B:
$(3x + 4)(2x + 5)$
$= 3x \times 2x \mathbf{+ 3x \times 5} + 4 \times 2x + 4 \times 5$
$= 6x^2 + 15x + 8x + 20$
$= 6x^2 + 23x + 20$

21. a. Negative sign ignored **b.** Negative sign ignored **c.** Distributive law not used

22. a. $w = \dfrac{x^2 + 2x - 15}{x + 5}$ **b.** $w = 62$ cm **c.** $l = 108$ cm

Exercise 3.6 Expansion patterns

1. a. $x^2 - 4$ **b.** $y^2 - 9$ **c.** $m^2 - 25$ **d.** $a^2 - 49$
e. $x^2 - 36$ **f.** $p^2 - 144$ **g.** $a^2 - 100$ **h.** $m^2 - 121$

2. a. $4x^2 - 9$ **b.** $9y^2 - 1$ **c.** $25d^2 - 4$ **d.** $49c^2 - 9$
e. $4 - 9p^2$ **f.** $d^2 - 81x^2$ **g.** $25 - 144a^2$ **h.** $9x^2 - 100y^2$
i. $4b^2 - 25c^2$ **j.** $100 - 4x^2$

3. a. $x^2 + 4x + 4$ **b.** $a^2 + 6a + 9$ **c.** $b^2 + 14b + 49$ **d.** $c^2 + 18c + 81$
e. $m^2 + 24m + 144$ **f.** $n^2 + 20n + 100$ **g.** $x^2 - 12x + 36$ **h.** $y^2 - 10y + 25$
i. $81 - 18c + c^2$ **j.** $64 + 16e + e^2$ **k.** $2x^2 + 4xy + 2y^2$ **l.** $u^2 - 2uv + v^2$

4. a. $4a^2 + 12a + 9$ **b.** $9x^2 + 6x + 1$ **c.** $4m^2 - 20m + 25$ **d.** $16x^2 - 24x + 9$
e. $25a^2 - 10a + 1$ **f.** $49p^2 + 56p + 16$ **g.** $81x^2 + 36x + 4$ **h.** $16c^2 - 48c + 36$
i. $9 + 12a + 4a^2$ **j.** $25 + 30p + 9p^2$ **k.** $4 - 20x + 25x^2$ **l.** $49 - 42a + 9a^2$
m. $81x^2 - 72xy + 16y^2$ **n.** $64x^2 - 48xy + 9y^2$

5. $(x^2 + 6x + 9)$ units^2

6. $(3x + 1)$ m

7. Answers will vary.

8. Anne's bedroom is larger by 1 m^2.

9. a. $a^2 + ab + ab + b^2$

b. $A = (a + b) \times (a + b)$
$= (a + b)^2$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$
Perfect squares

10. a. i. $x^2 - 6x + 9$ and $9 - 6x + x^2$

ii. $x^2 - 30x + 225$ and $225 - 30x + x^2$

iii. $9x^2 - 42x + 49$ and $49 - 42x + 9x^2$

b. The answers to the pairs of expansions are the same.

c. This is possible because when a negative number is multiplied by itself, it becomes positive; and when expanding a perfect square, where the two expressions are the same, the negative signs cancel out and result in the same answer.

11. a. i. $x^2 - 16$ and $16 - x^2$

ii. $x^2 - 121$ and $121 - x^2$

iii. $4x^2 - 81$ and $81 - 4x^2$

b. The answers to the pairs of expansions are the same, except that the negative and positive signs are reversed.

c. This is possible because when a negative number is multiplied by a positive number, it becomes negative; and when expanding a DOTS, where the two expressions have different signs, the signs will be reversed.

12. a. $(10k + 5)^2 = 100k^2 + 100k + 25$

b. $(10k + 5)^2 = 100 \times k \times k + 100 \times k + 25$
$= 100k(k + 1) + 25$

c. $25^2 = (10 \times 2 + 5)^2$

Let $k = 2$.

$25^2 = 100k(k + 1) + 25$
$= 100 \times 2 \times (2 + 1) + 25$
$= 625$

$85^2 = (10 \times 8 + 5)^2$

Let $k = 8$.

$85^2 = 100k(k + 1) + 25$
$= 100 \times 8 \times (8 + 1) + 25$
$= 7225$

13. a. 10 609 **b.** 3844 **c.** 994 009 **d.** 1 024 144 **e.** 2809 **f.** 9604

Challenge 3.2

$-3(x-1)^2$	$4x^2 - x + 4$	$-x^2 + 4x - 1$
$2x^2 + x + 2$	$3x$	$-2x^2 + 5x - 2$
$(x+1)^2$	$-4x^2 + 7x - 4$	$3(x^2+1)$

Exercise 3.7 Further expansions

1. $2x^2 + 13x + 21$ **2.** $2x^2 + 13x + 20$ **3.** $2x^2 + 14x + 26$ **4.** $2x^2 + 10x + 11$

5. $2p^2 - 3p - 21$ **6.** $2a^2 - 5a + 4$ **7.** $2p^2 - p - 24$ **8.** $2x^2 + 19x - 36$

9. $2y^2 + 2y - 7$ **10.** $2d^2 + 8d - 2$ **11.** $2x^2 + 10$ **12.** $2y^2$

13. $2x^2 - 4x + 19$ **14.** $2y^2 - 4y - 7$ **15.** $2p^2 + 3p + 23$ **16.** $2m^2 + 3m + 31$

17. $x + 5$ **18.** $4x + 8$ **19.** $-2x - 6$ **20.** $3m + 2$

21. $-3b - 22$ **22.** $-15y - 2$ **23.** $8p - 10$ **24.** $16x + 2$

25. $-16c - 40$ **26.** $-14f - 34$ **27.** $4m + 17$ **28.** $-7a + 30$

29. $-6p - 7$ **30.** $3x - 21$

31. $(p-1)(p+2) + (p-3)(p+1) = (p^2 + p - 2) + (p^2 - 2p - 3)$
$= p^2 + p^2 + p - 2p - 2 - 3$
$= 2p^2 - p - 5$

32. $(x+2)(x-3) - (x+1)^2 = (x^2 - x - 6) - (x^2 + 2x + 1)$
$= -3x - 7$

33. a. $(a^2 + b^2)(c^2 + d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$
$= (a^2c^2 + b^2d^2) + (a^2d^2 + b^2c^2)$
$= ((ac)^2 - 2abcd + (bd)^2) + ((ad)^2 + 2abcd + (bc)^2)$
$= (ac - bd)^2 + (ad + bc)^2$

b. $a = 2,\ b = 1,\ c = 3,\ d = 4$
$(2^2 + 1^2)(3^2 + 4^2) = (2 \times 3 - 1 \times 4)^2 + (2 \times 4 - 1 \times 3)^2$
$= 4 + 121$
$= 125$

34. a. $(x^2 + x - 1)^2 = (x^2 + x - 1)(x^2 + x - 1)$
$= x^4 + x^3 - x^2 + x^3 + x^2 - x - x^2 - x + 1$
$= x^4 + 2x^3 - x^2 - 2x + 1$

b. $x^4 + 2x^3 - x^2 - 2x + 1$
$= x(x^3 + 2x^2 - x - 2) + 1$
$= x(x+1)(x+2)(x-1) + 1$

c. i. $4 \times 3 \times 2 \times 1 + 1 = 25$ **ii.** $x = 2$

35. a. $(a+b)(d+e) = ad + ae + bd + be$

b. $(a+b+c)(d+e+f) = ad + ae + bd + be + cd + ce + af + bf + cf$

	a	b	c
d	ad	bd	cd
e	ae	be	ce
f	af	bf	cf

Exercise 3.8 The highest common factor

1. a. $2a$ **b.** 4 **c.** $3x$ **d.** -4

e. $3x$ **f.** $15a$ **g.** $-3x$ **h.** m

i. $-mn$ **j.** ab **k.** $3x$ **l.** $5n^2$

2. a. 4 **b.** 3 **c.** 5 **d.** 8

e. 3 **f.** 25 **g.** 1 **h.** $6a$

i. $7x$ **j.** $30q$ **k.** 10 **l.** $3x$

m. 3 **n.** 5 **o.** x^2 **p.** 2

3. C

4. a. $4(x + 3y)$ **b.** $5(m + 3n)$ **c.** $7(a + 2b)$ **d.** $7(m - 3n)$

e. $-8(a + 3b)$ **f.** $4(2x - y)$ **g.** $-2(6p + q)$ **h.** $6(p + 2pq + 3q)$

i. $8(4x + y + 2z)$ **j.** $4(4m - n + 6p)$ **k.** $8(9x - y + 8pq)$ **l.** $3(5x^2 - y)$

m. $5(p^2 - 4q)$ **n.** $5(x + 1)$ **o.** $8(7q + p^2)$ **p.** $7(p - 6x^2y)$

q. $4(4p^2 + 5q + 1)$ **r.** $12(1 + 3a^2b - 2b^2)$

5. a. $3(3a + 7b)$ **b.** $2(2c + 9d^2)$ **c.** $4(3p^2 + 5q^2)$ **d.** $7(5 - 2m^2n)$

e. $5(5y^2 - 3x)$ **f.** $4(4a^2 + 5b)$ **g.** $6(7m^2 + 2n)$ **h.** $9(7p^2 + 9 - 3y)$

i. $11(11a^2 - 5b + 10c)$ **j.** $2(5 - 11x^2y^3 + 7xy)$ **k.** $9(2a^2bc - 3ab - 10c)$ **l.** $12(12p + 3q^2 - 7pq)$

m. $7(9a^2b^2 - 7 + 8ab^2)$ **n.** $11(2 + 9p^3q^2 - 4p^2r)$ **o.** $6(6 - 4ab^2 + 3b^2c)$

6. a. $-(x - 5)$ **b.** $-(a - 7)$ **c.** $-(b - 9)$ **d.** $-2(m + 3)$

e. $-6(p + 2)$ **f.** $-4(a + 2)$ **g.** $-3(n^2 - 5m)$ **h.** $-7(x^2y^2 - 3)$

i. $-7(y^2 + 7z)$ **j.** $-6(2p^2 + 3q)$ **k.** $-7(9m - 8)$ **l.** $-2(6m^3 + 25x^3)$

m. $-3(3a^2b - 10)$ **n.** $-3(5p + 4q)$ **o.** $-2(9x^2 - 2y^2)$ **p.** $-3(ab - 6m + 7)$

q. $-5(2 + 5p^2 + 9q)$ **r.** $-9(10m^2 - 3n - 6p^3)$

7. a. $a(a + 5)$ **b.** $m(m + 3)$ **c.** $x(x - 6)$ **d.** $q(14 - q)$

e. $m(18 + 5m)$ **f.** $p(6 + 7p)$ **g.** $n(7n - 2)$ **h.** $a(a - b + 5)$

i. $p(7 - pq + q)$ **j.** $y(x + 9 - 3y)$ **k.** $c(5 + 3cd - d)$ **l.** $ab(3 + a + 4b)$

m. $xy(2x + 1 + 5y)$ **n.** $pq(5pq - 4 + 3p)$ **o.** $xy(6xy - 5 + x)$

8. a. $5x(x + 3)$ **b.** $2y(5y + 1)$ **c.** $4p(3p + 1)$ **d.** $6m(4m - 1)$

e. $4a(8a - 1)$ **f.** $-2m(m - 4)$ **g.** $-5x(x - 5)$ **h.** $-7y(y - 2)$

i. $-3a(a - 3)$ **j.** $-2p(6p + 1)$ **k.** $-5b(3b + 1)$ **l.** $-13y(2y + 1)$

m. $2m(2 - 9m)$ **n.** $-6t(1 - 6t)$ **o.** $-8p(1 + 3p)$

9. a. $2(x + 3)(4x + 1)$

b. $A = x(x + 3)$

$C = 2(x + 3)x$

$B = 5x^2 + 17x + 6$

10. Answers will vary.

11. True

12. a. $4(3x - 5y)(3x + 5y)$ **b.** $4(3x - 5y)(3x + 5y)$ **c.** Yes, the answers are the same.

Exercise 3.9 The highest common binomial factor

1. a. $(a + b)(2 + 3c)$ **b.** $(m + n)(4 + p)$ **c.** $(2m + 1)(7x - y)$ **d.** $(3b + 2)(4a - b)$

e. $(x + 2y)(z - 3)$ **f.** $(6 - q)(12p - 5)$ **g.** $(x - y)(3p^2 + 2q)$ **h.** $(b - 3)(4a^2 + 3b)$

i. $(q + 2p)(p^2 - 5)$ **j.** $(5m + 1)(6 + n^2)$

2. a. $(y + 2)(x + 2)$ **b.** $(b + 3)(a + 3)$ **c.** $(x - 4)(y + 3)$ **d.** $(2y + 1)(x + 3)$

e. $(3b + 1)(a + 4)$ **f.** $(b - 2)(a + 5)$ **g.** $(m - 2n)(1 + a)$ **h.** $(5 + 3p)(1 + 3a)$

i. $(3m - 1)(5n - 2)$ **j.** $(10p - 1)(q - 2)$ **k.** $(3x - 1)(2 - y)$ **l.** $(4p - 1)(4 - 3q)$

m. $(2y + 1)(5x - 2)$ **n.** $(2a + 3)(3b - 2)$ **o.** $(b - 2c)(5a - 3)$ **p.** $(x + 3y)(4 - z)$

q. $(p + 2q)(5r - 3)$ **r.** $(a - 5b)(c - 2)$

3. Answers will vary.

4. a. $6x^2 - 11x - 10$ **b.** Answers will vary. **c.** Answers will vary.

d. i. $2x^2 - 11x + 12$ **ii.** $-2x^2 - 9x + 5$

5. a. $\text{Side length}_a = x + 3$

$\text{Side length}_b = x + 5$

$\text{Side length}_c = x + 8$

$\text{Side length}_d = x - 3$

$\text{Side length}_e = x + 6$

b. $A_a = \text{II} = (x + 3)^2$

$A_b = \text{III} = (x + 5)^2$

$A_c = \text{V} = (x + 8)^2$

$A_d = \text{I} = (x - 3)^2$

$A_e = \text{IV} = (x + 6)^2$

c. A_a = II = 64 cm^2

A_b = III = 100 cm^2

A_c = V = 169 cm^2

A_d = I = 4 cm^2

A_e = IV = 121 cm^2

6. a. $5(x + 2)(x + 3) = 5(x^2 + 5x + 6) = 5x^2 + 15x + 30$

	x	3	x	3	x	3	x	3	x	3
x	x^2	$3x$	x^2	$3x$	x^2	$3x$	x^2	$3x$	x^2	$3x$
2	$2x$	6	$2x$	6	$2x$	6	$2x$	6	$2x$	6

b. $5(x + 2)(x + 3) = (5 \times x + 2 \times 5)(x + 3) = (5x + 10)(x + 3)$

	x	2	x	2	x	2	x	2	x	2
x	x^2	$2x$	x^2	$2x$	x^2	$2x$	x^2	$2x$	x^2	$2x$
3	$3x$	6	$3x$	6	$3x$	6	$3x$	6	$3x$	6

c. $5(x + 2)(x + 3) = (5 \times x + 3 \times 5)(x + 2) = (5x + 15)(x + 2)$

	$5x$	15
x	$5x^2$	$15x$
2	$10x$	30

7. a. i. Square, because it is a perfect square.

ii. Rectangle, because it is a DOTS.

iii. Rectangle, because it is a trinomial.

iv. Rectangle, because it is a trinomial.

b. i. $(3s + 8)^2$ **ii.** $(5s + 2)(5s - 2)$ **iii.** $(s + 1)(s + 3)$ **iv.** $4(s^2 - 7s - 8)$

Exercise 3.10 Applications

1. a. i. $12x$ **ii.** 60 **iii.** $9x^2$ **iv.** 225

b. i. $4x + 8$ **ii.** 28 **iii.** $x^2 + 4x + 4$ **iv.** 49

c. i. $16x - 4$ **ii.** 76 **iii.** $16x^2 - 8x + 1$ **iv.** 361

d. i. $10x$ **ii.** 50 **iii.** $4x^2$ **iv.** 100

e. i. $10x + 2$ **ii.** 52 **iii.** $6x^2 + 2x$ **iv.** 160

f. i. $12x - 2$ **ii.** 58 **iii.** $5x^2 - 13x - 6$ **iv.** 54

g. i. $20x + 18$ **ii.** 118 **iii.** $21x^2 + 42x$ **iv.** 735

2. a. 1250 m^2 **b.** $(1250 + 150x + 4x^2)$ m^2 **c.** $(150x + 4x^2)$ m^2

d. 366.16 m^2 **e.** $150x + 4x^2 = 200$

3. a. $(20 - 2x)$ cm **b.** $(15 - 2y)$ cm

c. $(300 - 40y - 30x + 4xy)$ cm^2 **d.** Check with your teacher.

4. a. 40 cm^2

b. i. $(8 + v)$ cm **ii.** $(5 + v)$ cm **iii.** $(v^2 + 13v + 40)$ cm^2 **iv.** 70 cm^2

c. i. $(8 - d)$ cm **ii.** $(5 - d)$ cm **iii.** $(40 - 13d + d^2)$ cm^2 **iv.** 18 cm^2

d. i. $8x$ cm **ii.** $(5 + x)$ cm **iii.** $(8x^2 + 40x)$ cm^2 **iv.** 400 cm^2

5. a. $20x$ m **b.** $25x^2$ m^2

c. i. $(5x - 2)$ m **ii.** $(5x - 3)$ m **iii.** $(25x^2 - 25x + 6)$ m^2 **iv.** 756 m^2

6. a. $6x$ cm **b.** $2x^2$ cm^2

c. i. $(2x + y)$ cm **ii.** $(x - y)$ cm **iii.** $(2x^2 - xy - y^2)$ cm^2 **iv.** 11 cm^2

7. a. $4x$ cm **b.** x^2 cm^2

c. i. $(x + y)$ cm **ii.** $(4x + 4y)$ cm **iii.** 56 cm **iv.** $(x^2 + 2xy + y^2)$ cm^2

d. 60.84 cm^2

8. a. $(14p + 4)$ m **b.** $(12p^2 + 6p)$ m^2 **c.** $(6p + 2)$ m **d.** $5p$ m

e. $(22p + 4)$ m **f.** $(30p^2 + 10p)$ m^2 **g.** $(18p^2 + 4p)$ m^2 **h.** 80 m^2

9. a.

Type of 'algebra' rectangle	Perimeter	Type of 'algebra' rectangle	Perimeter
1-long	$2x + 2$	1-tall	$2x + 2$
2-long	$4x + 2$	2-tall	$2x + 4$
3-long	$6x + 2$	3-tall	$2x + 6$
4-long	$8x + 2$	4-tall	$2x + 8$
5-long	$10x + 2$	5-tall	$2x + 10$

b. Perimeter: long $40x + 2$, tall $2x + 40$

c. $0 < x < 1$

10. a. 27.8

b. Richard is not within the healthy weight range.

c. i. 19.1 **ii.** 24.9 **iii.** 22.0

d. Judy: 9 to 20 years of age; Karen: 17.5 to 20 years of age; Manuel: 13.5 to 20 years of age

11. a. 3

b. 994 009

c. i. 990 025 **ii.** 980 100

12. a. $A = \pi R^2 - x^2$ **b.** $A = 160.71$ cm^2 **c.** Answers will vary.

d. $R = 5.32$ cm **e.** 92.0%

13. a. i. $A_{\text{screen}} = x(x + 2) = x^2 + 2x$

ii. $A_{\text{phone 1}} = (x + 1)(x + 7) = x^2 + 8x + 7$

b. i. $l = x + 11,\ w = x$

ii. $A_{\text{phone 2}} = x(x + 11) = x^2 + 11x$

c. $x = 4$ cm, $A_{\text{phone 2}} = 60$ cm^2

14. a. i. $x(x - 2)$ m^2 **ii.** $x(2x)$ m^2

b. $\dfrac{x - 2}{2x}$

c. $\dfrac{50(x - 2)}{x}\%$

d. i. 25% **ii.** 27.8% **iii.** 29.17%

e. The third flag places the most importance on Australia.

15. a. Answers will vary.

b. i. $-3x^3 - 2x^2 + 3x + 2$ **ii.** $8x^3 + 16x^2 - 104x + 80$ **iii.** $x^3 - 49x + 120$

c. In a cubic expression there are four terms with descending powers of x and ascending values of its pronumeral. For example, $(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$, where the powers of x are descending and the values of a are ascending.

3.11 Review

1. B **2.** D **3.** A **4.** B **5.** D

6. A **7.** B **8.** A **9.** B

10. a. i. 4 **ii.** -8 **iii.** -5 **iv.** $-8xy^2$

b. i. $y + 8$ **ii.** $x - y$ **iii.** $x + y$ **iv.** $7xy$ **v.** $5y - 2x$

11. a. xy dollars **b. i.** $(24 - k)$m **ii.** $\dfrac{24 - k}{3}$ m

12. a. $17p$ **b.** $y^2 + 2y$ **c.** $-3s^2t$ **d.** $16c^2d - 2cd$

e. $n^2 - 4p^2q + 6$ **f.** $15ab - 3a^2b^2$

13. a. $12ab$ **b.** $2ab^2$ **c.** $8x^2y^2$ **d.** $\frac{x}{3}$ **e.** $\frac{9}{2b}$

14. a. $5x + 15$ **b.** $-y - 5$ **c.** $-3x + 2x^2$ **d.** $-8m^2 - 4m$

15. a. $3x + 3$ **b.** $-10m - 1$ **c.** $4m^2 - 9m - 5$ **d.** $4p - 6$

16. a. $9a + 8b$ **b.** $-5x - 18y$ **c.** $-mn + 11m$ **d.** $4x^2 - 10x + 3$

17. a. $x^2 + 9x + 20$ **b.** $m^2 - m - 2$ **c.** $3m^2 - 17m + 10$ **d.** $2a^2 - 5ab - 3b^2$

18. a. $x^2 - 16$ **b.** $81 - m^2$ **c.** $x^2 - y^2$ **d.** $1 - 4a^2$

19. a. $x^2 + 10x + 25$ **b.** $m^2 - 6m + 9$ **c.** $16x^2 + 8x + 1$ **d.** $4 - 12y + 9y^2$

20. a. $2x^2 + 8x + 8$ **b.** $2m^2 + 11m - 5$ **c.** $6x + 15$ **d.** $-7b + 37$

21. a. $2x^2 + 10x + 13$ **b.** $2x - 5$ **c.** $16x$

22. a. 23 **b.** 134

23. a. 307.88 cm^3 **b.** 5379.98 cm^3

24. a. $6(x + 2)$ **b.** $6x^2(1 + 2xy)$ **c.** $4(2a^2 - b)$

d. $8x(2x - 3y)$ **e.** $-2(x + 2)$ **f.** $b(b - 3 + 4c)$

25. a. $(5 - 4a)(x + y)$ **b.** $(7a - 6c)(b + 5c)$ **c.** $5x(3 + 5y)(d + 2e)$ **d.** $(2 + a)(x + y)$

e. $2(x - 1)(3y + 2)$ **f.** $(p - r)(q + 1)$

26. a. $8x$ cm

b. $3x^2$ cm^2

c. i. $(3x + y)$ cm **ii.** $(8x + 2y)$ cm **iii.** 780 cm **iv.** $(3x^2 + xy)$ cm^2 **v.** 27 000 cm^2

27. a. $(8 + 2p)(15 + 2p)$ m^2 **b.** $(4p^2 + 46p + 120)$ m^2 **c.** $(4p^2 + 46p)$ m^2 **d.** $4p^2 + 46p = 200$

28. $\frac{2\pi x}{y}$ cm

29. $5xy - 4y^2$

30. a. $6n + 42$ **b.** $6(n + 7)$ **c.** $2(n + 7)$ **d.** $\frac{6(n + 7)}{x}$

31. a. i. $9(c - d)$

ii. Yes, this is a multiple of 9 as the number that multiplies the brackets is 9.

b. $90(b - c)$, 90 is a multiple of 9 so the difference between the correct and incorrect one is a multiple of 9.

c. $90(a - b)$, again 900 is a multiple of 9.

d. If two adjacent digits are transposed, the difference between the correct number and the transposed number is a multiple of 9.

e. Let the correct price of the CD be $\$ab.cd$. This can be written as $10a + b + 0.1c + 0.01d$.

The cashier enters $\$cd.ab$ as the price. This can be written as $10c + d + 0.1a + 0.01b$.

The difference is $10a + b + 0.1c + 0.01d - (10c + d + 0.1a + 0.01b)$.

This is equal to $9.9a + 0.99b - 9.9c - 0.99d$, which is clearly a multiple of 9.

So, the rule for checking transposition errors also applies in this case.

Investigation — Rich task

1. Solution not required

2. Answers will vary.

3. 0.12 m

4. 0.0144 m^2

5. Yellow: 0.03 m × 0.03 m

Black : 0.03 m × 0.06 m

White : 0.06 m × 0.06 m

6. Yellow : 0.0009 m^2

Black : 0.0018 m^2

White : 0.0036 m^2

7. Yellow : 0.0036 m^2

Black : 0.0072 m^2

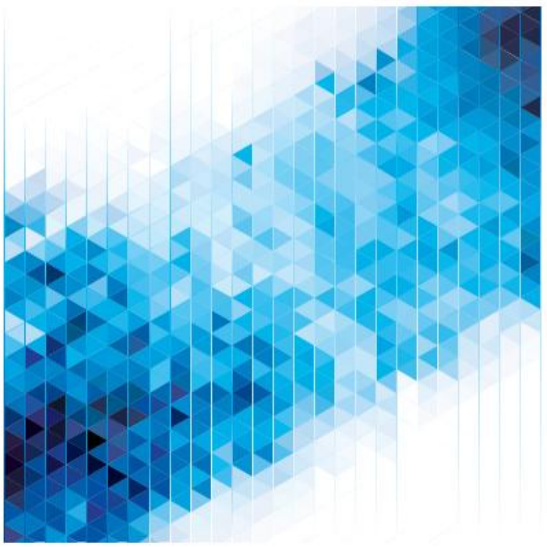
White : 0.0036 m^2

8. Answers will vary.

TOPIC 4
Linear equations

4.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



4.1.1 Why learn this?

Looking for patterns in numbers, relationships and measurements helps us to understand the world around us. A mathematical model is a mathematical representation of a situation. If we can see a pattern in a table of values or a graph that shows ordered pairs following an approximately straight line, the model is called a linear model.

4.1.2 What do you know?

assessON

1. **THINK** List what you know about linear equations. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of linear equations.

LEARNING SEQUENCE

4.1 Overview
4.2 Solving linear equations
4.3 Solving linear equations with brackets
4.4 Solving linear equations with pronumerals on both sides
4.5 Solving problems with linear equations
4.6 Rearranging formulas
4.7 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: The mighty Roman armies (eles-1691)

4.2 Solving linear equations

4.2.1 What is a linear equation?

- An **equation** is a mathematical statement that contains an equals sign (=).
- For an equation, the expression on the left-hand side of the equals sign has the same value as the expression on the right-hand side.
- Solving a **linear equation** means finding a value for the pronumeral that makes the statement true.
- 'Doing the same thing' to both sides of the equation ensures that the two expressions remain equal.

WORKED EXAMPLE 1

For each of the following equations, determine whether $x = 10$ is a solution.

a $\frac{x+2}{3} = 6$ **b** $2x + 3 = 3x - 7$ **c** $x^2 - 2x = 9x - 10$

THINK	WRITE
a 1 Substitute 10 for x in the left-hand side of the equation.	**a** LHS $= \frac{x+2}{3}$ $= \frac{10+2}{3}$ $= \frac{12}{3}$ $= 4$
2 Write the right-hand side.	RHS $= 6$
3 Is the equation true? That is, does the left-hand side equal the right-hand side?	LHS $\neq$ RHS
4 State whether $x = 10$ is a solution.	$x = 10$ is not a solution.
b 1 Substitute 10 for x in the left-hand side.	**b** LHS $= 2x + 3$ $= 2(10) + 3$ $= 23$
2 Substitute 10 for x in the right-hand side.	RHS $= 3x - 7$ $= 3(10) - 7$ $= 23$
3 Is the equation true?	LHS = RHS
4 State whether $x = 10$ is a solution.	$x = 10$ is a solution.
c 1 Substitute 10 for x in the left-hand side.	**c** LHS $= x^2 - 2x$ $= 10^2 - 2(10)$ $= 100 - 20$ $= 80$
2 Substitute 10 for x in the right-hand side.	RHS $= 9x - 10$ $= 9(10) - 10$ $= 90 - 10$ $= 80$
3 Is the equation true?	LHS = RHS
4 State whether $x = 10$ is a solution.	$x = 10$ is a solution.

4.2.2 Solving one-step equations

- If one operation has been performed on a pronumeral, it is known as a **one-step equation**.
- Simple equations can be solved by performing the **inverse operation**.
- The inverse operation has the effect of undoing the original operation.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

WORKED EXAMPLE 2

TI | CASIO

Solve each of the following linear equations.

a $x - 79 = 153$ **b** $x + 46 = 82$ **c** $6x = 100$ **d** $\frac{x}{7} = 19$

THINK	WRITE
a 1 79 is subtracted from x to give 153.	**a** $x - 79 = 153$
2 Apply the inverse operation by adding 79 to both sides of the equation.	$x = 153 + 79$
3 Write the value of x.	$x = 232$
b 1 46 is added to x to give 82.	**b** $x + 46 = 82$
2 Apply the inverse operation by subtracting 46 from both sides of the equation.	$x = 82 - 46$
3 Write the value of x.	$x = 36$
c 1 6 is multiplied by x to give 100.	**c** $6x = 100$
2 Perform the inverse operation by dividing both sides of the equation by 6.	$x = \frac{100}{6}$
3 Write the value of x.	$x = 16\frac{2}{3}$
d 1 x is divided by 7 to give 19.	**d** $\frac{x}{7} = 19$
2 Perform the inverse operation by multiplying both sides of the equation by 7.	$x = 19 \times 7$
3 Write the value of x.	$= 33$

Note: In each case the result can be checked by substituting the value obtained for x back into the original equation and confirming that it will make the equation a true statement.

4.2.3 Solving two-step equations

- If two operations have been performed on the pronumeral, it is known as a **two-step equation**.
- To solve two-step equations, determine the order in which the operations were performed.
- Perform inverse operations in the reverse order to both sides of the equation.
- Each inverse operation must be performed one step at a time.
- This principle will apply to any equation with two or more steps, as shown in the examples that follow.

WORKED EXAMPLE 3

Solve the following linear equations.

a $2y + 4 = 12$ **b** $6 - x = 8$ **c** $\frac{x}{3} - 4 = 2$ **d** $\frac{3x}{5} = 6$

THINK	WRITE
a 1 First subtract 4 from both sides.	**a** $2y + 4 = 12$ $2y + 4 - 4 = 12 - 4$
2 Divide both sides by 2.	$2y = 8$ $\frac{2y}{2} = \frac{8}{2}$
3 Write the value of y.	$y = 4$
b 1 $6 - x$ is the same as $-x + 6$. Rewrite the equation.	**b** $6 - x = 8$ $-x + 6 = 8$
2 Subtract 6 from both sides.	$-x + 6 - 6 = 8 - 6$
3 Divide both sides by -1.	$-x = 2$ $\frac{-x}{-1} = \frac{2}{-1}$
4 Write the value of x.	$x = -2$
c 1 Add 4 to both sides.	**c** $\frac{x}{3} - 4 = 2$ $\frac{x}{3} - 4 + 4 = 2 + 4$ $\frac{x}{3} = 6$
2 Multiply both sides by 3.	$\frac{x}{3} \times 3 = 6 \times 3$
3 Write the value of x.	$x = 18$
d 1 Multiply both sides by 5.	**d** $\frac{3x}{5} = 6$ $\frac{3x}{5} \times 5 = 6 \times 5$ $3x = 30$
2 Divide both sides by 3.	$\frac{3x}{3} = \frac{30}{3}$
3 Write the value of x.	$x = 10$

WORKED EXAMPLE 4 TI | CASIO

Solve the following linear equations.

a $\frac{x + 1}{2} = 11$ **b** $\frac{7 - x}{5} = -6.3$

THINK	WRITE
a 1 All of $x + 1$ has been divided by 2.	**a** $\frac{x+1}{2} = 11$
2 Multiply both sides by 2.	$\frac{x+1}{2} \times 2 = 11 \times 2$ $x + 1 = 22$
3 Subtract 1 from both sides.	$x = 21$
b 1 All of $7 - x$ has been divided by 5.	**b** $\frac{7-x}{5} = -6.3$
2 Multiply both sides by 5.	$\frac{7-x}{5} \times 5 = -6.3 \times 5$
3 $7 - x$ is the same as $-x + 7$.	$7 - x = -31.5$
4 Subtract 7 from both sides.	$7 - x - 7 = -31.5 - 7$ $-x = -38.5$
5 Divide both sides by -1.	$x = 38.5$

4.2.4 Algebraic fractions with the pronumeral in the denominator

- If a pronumeral is in the denominator, there is an extra step involved in finding the solution. Consider the following example:

$$\frac{4}{x} = \frac{3}{2}$$

In order to solve this equation, we first multiply both sides of the equations by x.

$$\frac{4}{x} \times x = \frac{3}{2} \times x$$
$$4 = \frac{3x}{2}$$
$$\text{or } \frac{3x}{2} = 4$$

The pronumeral is now in the numerator, and the equation is easy to solve.

$$\frac{3x}{2} = 4$$
$$3x = 8$$
$$x = \frac{8}{3}$$

WORKED EXAMPLE 5

Solve each of the following linear equations.

a $\frac{3}{a} = \frac{4}{5}$ **b** $\frac{5}{b} = -2$

THINK	WRITE
a 1 Multiply both sides by a.	**a** $\frac{3}{a} = \frac{4}{5}$ $3 = \frac{4a}{5}$
2 Multiply both sides by 5.	$15 = 4a$

3 Divide both sides by 4.

$$a = \frac{15}{4}$$

or $a = 3\frac{3}{4}$

b 1 Write the equation.

b $\frac{5}{b} = -2$

2 Multiply both sides by b.

$5 = -2b$

3 Divide both sides of the equation by -2.

$\frac{5}{-2} = b$

$b = -2\frac{1}{2}$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Solving one-step equations (doc-6150)

Complete this digital doc: SkillSHEET: Checking solutions to equations (doc-6151)

Complete this digital doc: SkillSHEET: Solving equations (doc-6152)

Complete this digital doc: WorkSHEET: Solving linear equations (doc-6156)

Exercise 4.2 Solving linear equations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–l, 3a–h, 4, 5, 6a–f, 7a–f, 8a–f, 9a–f, 10, 11, 12, 17	Questions: 1d–i, 2g–r, 3d–i, 4, 5, 6d–i, 7d–i, 8d–i, 9d–l, 10–13, 17–19	Questions: 1g–l, 2i–u, 3g–l, 4, 5, 6g–l, 7g–l, 8g–l, 9g–l, 10–12, 14–20

Individual pathway interactivity: int-4489

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 For each of the following equations, determine whether $x = 6$ is a solution.

a. $x + 3 = 7$ **b.** $2x - 5 = 7$ **c.** $x^2 - 2 = 38$ **d.** $\frac{6}{x} + x = 7$

e. $\frac{2(x+1)}{7} = 2$ **f.** $3 - x = 9$ **g.** $x^2 + 3x = 39$ **h.** $3(x + 2) = 5(x - 4)$

i. $x^2 + 2x = 9x - 6$ **j.** $x^2 = (x + 1)^2 - 14$ **k.** $(x - 1)^2 = 4x + 1$ **l.** $5x + 2 = x^2 + 4$

2. WE2 Solve each of the following linear equations. Check your answers by substitution.

a. $x - 43 = 167$ **b.** $x - 17 = 35$ **c.** $x + 286 = 516$ **d.** $58 + x = 81$

e. $x - 78 = 64$ **f.** $209 - x = 305$ **g.** $5x = 185$ **h.** $60x = 1200$

i. $5x = 250$ **j.** $\frac{x}{23} = 6$ **k.** $\frac{x}{17} = 26$ **l.** $\frac{x}{9} = 27$

m. $y - 16 = -31$ **n.** $5.5 + y = 7.3$ **o.** $y - 7.3 = 5.5$ **p.** $6y = 14$

q. $0.2y = 4.8$ **r.** $0.9y = -0.05$ **s.** $\frac{y}{5} = 4.3$ **t.** $\frac{y}{7.5} = 23$

u. $\frac{y}{8} = -1.04$

3. WE3a Solve each of the following linear equations.

a. $2y - 3 = 7$ **b.** $2y + 7 = 3$ **c.** $5y - 1 = 0$ **d.** $6y + 2 = 8$

e. $7 + 3y = 10$ **f.** $8 + 2y = 12$ **g.** $15 = 3y - 1$ **h.** $-6 = 3y - 1$

i. $6y - 7 = 140$ **j.** $4.5y + 2.3 = 7.7$ **k.** $0.4y - 2.7 = 6.2$ **l.** $600y - 240 = 143$

4. WE3b Solve each of the following linear equations.

a. $3 - 2x = 1$ **b.** $-3x - 1 = 5$ **c.** $-4x - 7 = -19$ **d.** $1 - 3x = 19$

e. $-5 - 7x = 2$ **f.** $-8 - 2x = -9$ **g.** $9 - 6x = -1$ **h.** $-5x - 4.2 = 7.4$

i. $2 = 11 - 3x$ **j.** $-3 = -6x - 8$ **k.** $-1 = 4 - 4x$ **l.** $35 - 13x = -5$

5. Solve each of the following linear equations.

a. $7 - x = 8$ **b.** $8 - x = 7$ **c.** $5 - x = 5$ **d.** $5 - x = 0$

e. $15.3 = 6.7 - x$ **f.** $5.1 = 4.2 - x$ **g.** $9 - x = 0.1$ **h.** $140 - x = 121$

i. $-30 - x = -4$ **j.** $-5 = -6 - x$ **k.** $-x + 1 = 2$ **l.** $-2x - 1 = 0$

6. WE3c, d Solve each of the following linear equations.

a. $\frac{x}{4} + 1 = 3$ **b.** $\frac{x}{3} - 2 = -1$ **c.** $\frac{x}{8} = \frac{1}{2}$ **d.** $-\frac{x}{3} = 5$

e. $5 - \frac{x}{2} = -8$ **f.** $4 - \frac{x}{6} = 11$ **g.** $\frac{2x}{3} = 6$ **h.** $\frac{5x}{2} = -3$

i. $-\frac{3x}{4} = -7$ **j.** $-\frac{8x}{3} = 6$ **k.** $\frac{2x}{7} = -2$ **l.** $-\frac{3x}{10} = -\frac{1}{5}$

7. WE4 Solve each of the following linear equations.

a. $\frac{z-1}{3} = 5$ **b.** $\frac{z+1}{4} = 8$ **c.** $\frac{z-4}{2} = -4$ **d.** $\frac{6-z}{7} = 0$

e. $\frac{3-z}{2} = 6$ **f.** $\frac{-z-50}{22} = -2$ **g.** $\frac{z-4.4}{2.1} = -3$ **h.** $\frac{z+2}{7.4} = 1.2$

i. $\frac{140-z}{150} = 0$ **j.** $\frac{-z-0.4}{2} = -0.5$ **k.** $\frac{z-6}{9} = -4.6$ **l.** $\frac{z+65}{73} = 1$

8. Solve each of the following linear equations.

a. $\frac{5x+1}{3} = 2$ **b.** $\frac{2x-5}{7} = 3$ **c.** $\frac{3x+4}{2} = -1$ **d.** $\frac{4x-13}{9} = -5$

e. $\frac{4-3x}{2} = 8$ **f.** $\frac{1-2x}{6} = -10$ **g.** $\frac{-5x-3}{9} = 3$ **h.** $\frac{-10x-4}{3} = 1$

i. $\frac{4x+2.6}{5} = 8.8$ **j.** $\frac{5x-0.7}{-0.3} = -3.1$ **k.** $\frac{1-0.5x}{4} = -2.5$ **l.** $\frac{-3x-8}{14} = \frac{1}{2}$

9. WE5 Solve each of the following linear equations.

a. $\frac{2}{x} = \frac{1}{2}$ **b.** $\frac{3}{x} = 7$ **c.** $\frac{-4}{x} = \frac{7}{2}$ **d.** $\frac{5}{x} = \frac{-3}{4}$

e. $\frac{0.4}{x} = \frac{9}{2}$ **f.** $\frac{8}{x} = 1$ **g.** $\frac{-4}{x} = \frac{2}{3}$ **h.** $\frac{-6}{x} = \frac{-4}{5}$

i. $\frac{1.7}{x} = \frac{1}{3}$ **j.** $\frac{6}{x} = -1$ **k.** $\frac{4}{x} = \frac{-15}{22}$ **l.** $\frac{50}{x} = \frac{-35}{43}$

10. MC **a.** The solution to the equation $82 - x = 44$ is:

A. $x = 126$ **B.** $x = -126$ **C.** $x = 122$ **D.** $x = 38$

b. What is the solution to the equation $5x - 12 = -62$?

A. $x = -14.8$ **B.** $x = 14.8$ **C.** $x = 10$ **D.** $x = -10$

c. What is the solution to the equation $\frac{x-1}{2} = 5.3$?

A. $x = 9.6$ **B.** $x = 10.6$ **C.** $x = 11.6$ **D.** $x = 2$

11. Solve each of the following linear equations.

a. $3a + 7 = 4$ **b.** $5 - b = -5$ **c.** $4c - 4.4 = 44$

d. $\frac{d - 4}{67} = 0$ **e.** $5 - 3e = -10$ **f.** $\frac{2f}{3} = 8$

g. $100 = 6g + 4.2$ **h.** $\frac{h + 2}{6} = 5.5$ **i.** $452i - 124 = -98$

j. $\frac{6j - 1}{17} = 0$ **k.** $\frac{12 - k}{5} = 4$ **l.** $\frac{l - 5.2}{3.4} = 1.5$

Understanding

12. Write the following worded statements as a mathematical sentence and then solve for the unknown.

a. Seven is added to the product of x and 3, which gives the result of 4.

b. Four is divided by x and this result is equivalent to $\frac{2}{3}$.

c. Three is subtracted from x and this result is divided by 12 to give 25.

13. Driving lessons are usually quite expensive but a discount of $15 per lesson is given if a family member belongs to the automobile club. If 10 lessons cost $760 (after the discount), find the cost of each lesson before the discount.

14. Anton lives in Australia and his pen pal, Utan, lives in USA. Anton's home town of Horsham experienced one of the hottest days on record with a temperature of 46.7 °C. Utan said that his home town had experienced a day hotter than that, with the temperature reaching 113 °F. The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. Was he correct?

Reasoning

15. Santo solved the linear equation $9 = 5 - x$. His second step was to divide both sides by -1. Trudy, his mathematics buddy, said that she multiplied both sides by -1. Explain why they are both correct.

16. Find the mistake in the following working and explain what is wrong.

$$\frac{x}{5} - 1 = 2$$
$$x - 1 = 10$$
$$x = 11$$

Problem solving

17. Sweet-tooth Sammy goes to the corner store and buys an equal number of 25-cent and 30-cent lollies for a total of $16.50. How many lollies did he buy?

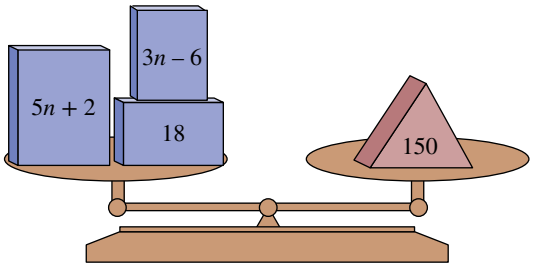
18. In a cannery, cans are filled by two machines that together produce 16000 cans during an 8-hour shift. If the newer machine produces 340 more cans per hour than the older machine, how many cans does each machine produce in an 8-hour shift?

19. General admission to an exhibition is $55 for an adult ticket, $27 for a child and $130 for a family of two adults and two children.

a. How much is saved by buying a family ticket instead of buying two adult and two child tickets?

b. Is it worthwhile buying a family ticket if the family has only one child?

20. A teacher comes across a clue shown below in a cryptic mathematics cross-number. What is the value of n that the teacher is looking for?



CHALLENGE 4.1

The value of the expression $\frac{12}{x-4}$ is an integer. What are the possible values for x, given that x is also an integer?

Reflection

How are linear equations defined?

4.3 Solving linear equations with brackets

4.3.1 Linear equations with brackets

- Consider the equation $3(x + 5) = 18$.
  There are two good methods for solving this equation.

Method 1:
First divide both sides by 3.

$$\frac{3(x+5)}{3} = \frac{18}{3}$$
$$x + 5 = 6$$
$$x = 1$$

Method 2:
First expand the brackets.

$$3(x + 5) = 18$$
$$3x + 15 = 18$$
$$3x = 3$$
$$x = 1$$

In this case, method 1 works well because 3 divides exactly into 18.
Now try the equation $7(x + 2) = 10$.

Method 1:
First divide both sides by 7.

$$\frac{7(x+2)}{7} = \frac{10}{7}$$
$$x + 2 = \frac{10}{7}$$
$$x = -\frac{4}{7}$$

Method 2:
First expand the brackets.

$$7(x + 2) = 10$$
$$7x + 14 = 10$$
$$7x = -4$$
$$x = \frac{-4}{7}$$

In this case, method 2 works well because it avoids fraction addition or subtraction.
Try both methods and choose the one that works best for each question.

WORKED EXAMPLE 6

TI | CASIO

Solve each of the following linear equations.

a $7(x - 5) = 28$ **b** $6(x + 3) = 7$

THINK	WRITE
a 1 7 is a factor of 28, so divide both sides by 7.	**a** $7(x - 5) = 28$ $\frac{7(x - 5)}{7} = \frac{28}{7}$
2 Add 5 to both sides.	$x - 5 = 4$
3 Write the value of x.	$x = 9$
b 1 6 is not a factor of 7, so it will be easier to expand the brackets first.	**b** $6(x + 3) = 7$ $6x + 18 = 7$
2 Subtract 18 from both sides.	$6x + 18 = 7 - 18$ $6x = -11$
3 Divide both sides by 6.	$x = -\frac{11}{6} (\text{or } -1\frac{5}{6})$

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Complete this digital doc: SkillSHEET: Expanding brackets (doc-10827)

Exercise 4.3 Solving linear equations with brackets

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–h, 3a–f, 4a–f, 5, 6, 8, 10	Questions: 1d–i, 2d–i, 3d–i, 4d–i, 5, 7–11	Questions: 1g–l, 2g–l, 3g–l, 4g–l, 5, 7–12

Individual pathway interactivity: int-4490

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Fluency

1. WE6 Solve each of the following linear equations.

a. $5(x - 2) = 20$	**b.** $4(x + 5) = 8$	**c.** $6(x + 3) = 18$	**d.** $5(x - 41) = 75$
e. $8(x + 2) = 24$	**f.** $3(x + 5) = 15$	**g.** $5(x + 4) = 15$	**h.** $3(x - 2) = -12$
i. $7(x - 6) = 0$	**j.** $-6(x - 2) = 12$	**k.** $4(x + 2) = 4.8$	**l.** $16(x - 3) = 48$

2. WE6 Solve each of the following equations.

a. $6(b - 1) = 1$	**b.** $2(m - 3) = 3$	**c.** $2(a + 5) = 7$	**d.** $3(m + 2) = 2$
e. $5(p - 2) = -7$	**f.** $6(m - 4) = -8$	**g.** $-10(a + 1) = 5$	**h.** $-12(p - 2) = 6$
i. $-9(a - 3) = -3$	**j.** $-2(m + 3) = -1$	**k.** $3(2a + 1) = 2$	**l.** $4(3m + 2) = 5$

3. Solve each of the following equations.

a. $9(x - 7) = 82$	**b.** $2(x + 5) = 14$	**c.** $7(a - 1) = 28$	**d.** $4(b - 6) = 4$
e. $3(y - 7) = 0$	**f.** $-3(x + 1) = 7$	**g.** $-6(m + 1) = -30$	**h.** $-4(y + 2) = -12$
i. $-3(a - 6) = 3$	**j.** $-2(p + 9) = -14$	**k.** $3(2m - 7) = -3$	**l.** $2(4p + 5) = 18$

4. Solve the following linear equations. Round the answers to 3 decimal places where appropriate.

a. $2(y + 4) = -7$	**b.** $0.3(y + 8) = 1$	**c.** $4(y + 19) = -29$	**d.** $7(y - 5) = 25$
e. $6(y + 3.4) = 3$	**f.** $7(y - 2) = 8.7$	**g.** $1.5(y + 3) = 10$	**h.** $2.4(y - 2) = 1.8$
i. $1.7(y + 2.2) = 7.1$	**j.** $-7(y + 2) = 0$	**k.** $-6(y + 5) = -11$	**l.** $-5(y - 2.3) = 1.6$

5. MC **a.** The best first step in solving the equation $7(x - 6) = 23$ would be to:

A. add 6 to both sides **B.** subtract 7 from both sides
C. divide both sides by 23 **D.** expand the brackets

b. The solution to the equation $84(x - 21) = 782$ is closest to:

A. $x = 9.31$ **B.** $x = 9.56$ **C.** $x = 30.31$ **D.** $x = -11.69$

Understanding

6. In 1974 a mother was 6 times as old as her daughter. If the mother turned 50 in the year 2000, in what year was the mother double her daughter's age?

7. New edging is to be placed around a rectangular children's playground. The width of the playground is x m and the length is 7 metres longer than the width.

a. Write down an expression for the perimeter of the playground. Write your answer in factorised form.

b. If the amount of edging required is 54 m, determine the dimensions of the playground.

Reasoning

8. Juanita is solving the following equation: $2(x - 8) = 10$. She performs the following operations to both sides of the equation in order: $+8$, $\div 2$. Explain why Juanita will not find the correct value of x using her order of inverse operations, then solve the equation.

9. As your first step to solve the equation $3(2x - 7) = 18$, you are given three options:

- Expand the brackets on the left-hand side.
- Add 7 to both sides.
- Divide both sides by 3.

Which of the options is your least preferred and why?

Problem solving

10. Five times the sum of 4 and a number is equal to 35. What is the number?

11. Kyle earns \$55 more than Noah each week, but Callum earns three times as much as Kyle. If Callum earns \$270 a week, how much do Kyle and Noah earn each week?

12. A school wishes to hire a bus to travel to a football game. The bus will take 28 passengers, and the school will contribute \$48 towards the cost of the trip. The price of each ticket is \$10. If the hiring of the bus is \$300 + 10% of the cost of all the tickets, what should be the cost per person?

Reflection

Explain the two possible methods for solving equations in factorised form.

4.4 Solving linear equations with pronumerals on both sides

4.4.1 Linear equations with pronumerals on both sides

- When solving equations, it is important to remember that whatever we do to one side of an equation we must do to the other.
- If the pronumeral occurs on both sides of the equation, first remove it from one side, as shown in the example on the following page.

WORKED EXAMPLE 7

TI | CASIO

Solve each of the following linear equations.

a $5y = 3y + 3$ **b** $7x + 5 = 2 - 4x$

c $3(x + 1) = 14 - 2x$ **d** $2(x + 3) = 3(x + 7)$

THINK	WRITE
a 1 $3y$ is smaller than $5y$. Subtract $3y$ from both sides.	**a** $5y = 3y + 3$ $5y - 3y = 3y + 3 - 3y$ $2y = 3$
2 Divide both sides by 2.	$y = \frac{3}{2}$ (or $1\frac{1}{2}$)
b 1 $-4x$ is smaller than $7x$. Add $4x$ to both sides.	**b** $7x + 5 = 2 - 4x$ $7x + 5 + 4x = 2 - 4x + 4x$ $11x + 5 = 2$
2 Subtract 5 from both sides.	$11x + 5 - 5 = 2 - 5$ $11x = -3$
3 Divide both sides by 11.	$x = \frac{-3}{11}$
c 1 Expand the bracket.	**c** $3(x + 1) = 14 - 2x$ $3x + 3 = 14 - 2x$
2 $-2x$ is smaller than $3x$. Add $2x$ to both sides.	$3x + 3 + 2x = 14 - 2x + 2x$ $5x + 3 = 14$
3 Subtract 3 from both sides.	$5x + 3 - 3 = 14 - 3$ $5x = 11$
4 Divide both sides by 5.	$x = \frac{11}{5}$
d 1 Expand the brackets.	**d** $2(x + 3) = 3(x + 7)$ $2x + 6 = 3x + 21$
2 $2x$ is smaller than $3x$. Subtract $2x$ from both sides.	$2x + 6 - 2x = 3x + 21 - 2x$ $6 = x + 21$
3 Subtract 21 from both sides.	$6 - 21 = x + 21 - 21$ $-15 = x$
4 Write the answer with the pronumeral written on the left-hand side.	$x = -15$

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Complete this digital doc: SkillSHEET: Simplifying like terms (doc-10828)

Exercise 4.4 Solving linear equations with pronumerals on both sides

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2, 3a–f, 4, 5, 6a–f, 7, 8, 11	Questions: 1d–i, 2, 3d–i, 4, 5, 6d–i, 7–12	Questions: 1g–l, 2, 3g–l, 4, 5, 6g–l, 7–14

Individual pathway interactivity: int-4491

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Fluency

1. WE7a Solve each of the following linear equations.

a. $5y = 3y - 2$ **b.** $6y = -y + 7$ **c.** $10y = 5y - 15$ **d.** $25 + 2y = -3y$

e. $8y = 7y - 45$ **f.** $15y - 8 = -12y$ **g.** $7y = -3y - 20$ **h.** $23y = 13y + 200$

i. $5y - 3 = 2y$ **j.** $6 - 2y = -7y$ **k.** $24 - y = 5y$ **l.** $6y = 5y - 2$

2. MC **a.** To solve the equation $3x + 5 = -4 - 2x$, the first step is to:

A. add $3x$ to both sides **B.** add 5 to both sides

C. add $2x$ to both sides **D.** subtract $2x$ from both sides

b. To solve the equation $6x - 4 = 4x + 5$, the first step is to:

A. subtract $4x$ from both sides **B.** add $4x$ to both sides

C. subtract 4 from both sides **D.** add 5 to both sides

3. WE7b Solve each of the following linear equations.

a. $2x + 3 = 8 - 3x$ **b.** $4x + 11 = 1 - x$ **c.** $x - 3 = 6 - 2x$ **d.** $4x - 5 = 2x + 3$

e. $3x - 2 = 2x + 7$ **f.** $7x + 1 = 4x + 10$ **g.** $5x + 3 = x - 5$ **h.** $6x + 2 = 3x + 14$

i. $2x - 5 = x - 9$ **j.** $10x - 1 = -2x + 5$ **k.** $7x + 2 = -5x + 2$ **l.** $15x + 3 = 7x - 3$

4. Solve each of the following linear equations.

a. $x - 4 = 3x + 8$ **b.** $3x + 12 = 4x + 5$ **c.** $2x + 9 = 7x - 1$

d. $-2x + 7 = 4x + 19$ **e.** $-3x + 2 = -2x - 11$ **f.** $11 - 6x = 18 - 5x$

g. $6 - 9x = 4 + 3x$ **h.** $x - 3 = 18x - 1$ **i.** $5x + 13 = 15x + 3$

5. MC **a.** The solution to $5x + 2 = 2x + 23$ is:

A. $x = 3$ **B.** $x = -3$ **C.** $x = 5$ **D.** $x = 7$

b. The solution to $3x - 4 = 11 - 2x$ is:

A. $x = 15$ **B.** $x = 7$ **C.** $x = 3$ **D.** $x = 5$

6. WE7c, d Solve each of the following.

a. $5(x - 2) = 2x + 5$ **b.** $7(x + 1) = x - 11$ **c.** $2(x - 8) = 4x$

d. $3(x + 5) = x$ **e.** $6(x - 3) = 14 - 2x$ **f.** $9x - 4 = 2(3 - x)$

g. $4(x + 3) = 3(x - 2)$ **h.** $5(x - 1) = 2(x + 3)$ **i.** $8(x - 4) = 5(x - 6)$

j. $3(x + 6) = 4(2 - x)$ **k.** $2(x - 12) = 3(x - 8)$ **l.** $4(x + 11) = 2(x + 7)$

Understanding

7. Aamir's teacher gave him an algebra problem and told him to solve it.

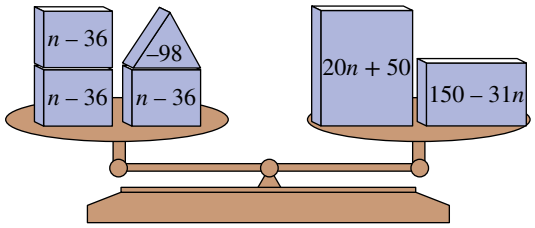
$3x + 7 = x^2 + k = 7x + 15$

Can you help him find the value of k?

8. A classroom contained an equal number of boys and girls. Six girls left to play hockey, leaving twice as many boys as girls in the classroom. What was the original number of students present?

Reasoning

9. Express the following information as an equation, then show that $n = 29$ is the solution.



10. Explain what the difficulty is when trying to solve the equation $4(3x - 5) = 6(4x + 2)$ without expanding the brackets first.

Problem solving

11. This year Tom is 4 times as old as his daughter, while in 5 years' time he will be only 3 times as old as his daughter. Find the ages of Tom and his daughter now.

12. If you multiply an unknown number by 6 and then add 5, the result is 7 less than the unknown number plus 1 multiplied by 3. Find the unknown number.

13. You are investigating getting a business card printed for your new game store. A local printing company charges $250 for the cardboard used and an hourly rate for labour of $40.

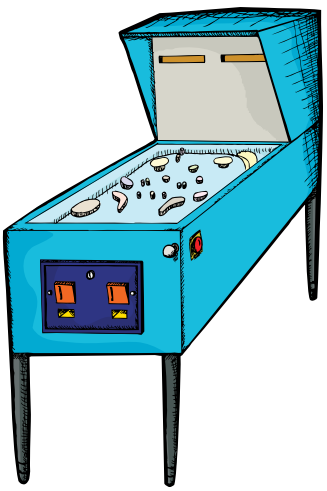


a. If h is the number of hours of labour required to print the cards, construct an equation for the cost of the cards, C.

b. You have budgeted $1000 for the printing job. How many hours of labour can you afford? Give your answer to the nearest minute.

c. The company estimates that it can print 1000 cards per hour of labour. How many cards will you get printed with your current budget?

d. An alternative to printing is photocopying. The company charges 15 cents per side for the first 10 000 cards and then 10 cents per side for the remaining cards. Which is the cheaper option for 18 750 single-sided cards and by how much?



14. A local pinball arcade offers its regular customers the following deal. For a monthly fee of $40 players get 25 $2 pinball games. Additional games cost $2 each. After a player has played 50 games in a month, all further games are $1.

a. If Tom has $105 to spend in a month, how many games can he play if he takes up the special deal?

b. How much did Tom save by taking up the special deal, compared to playing the same number of games at $2 a game?

Reflection

Draw a diagram that could represent $2x + 4 = 3x + 1$.

4.5 Solving problems with linear equations

4.5.1 Converting worded sentences to algebraic equations

- An important skill in mathematics is the ability to translate written problems into **algebraic equations** in order to solve problems.

WORKED EXAMPLE 8

Write linear equations for each of the following statements, using x to represent the unknown. (Do not attempt to solve the equations.)

a When 6 is subtracted from a certain number, the result is 15.

b Three more than seven times a certain number is zero.

c When 2 is divided by a certain number, the answer is 4 more than the number.

THINK	WRITE
a 1 Let x be the number.	**a** x = unknown number
2 Write x and subtract 6. This expression equals 15.	$x - 6 = 15$
b 1 Let x be the number.	**b** x = unknown number
2 7 times the number is $7x$. Three more than $7x$ equals $7x + 3$. This expression equals 0.	$7x + 3 = 0$
c 1 Let x be the number.	**c** x = unknown number
2 Write the term for 2 divided by a certain number.	$\frac{2}{x}$
Write the expression for 4 more than the number.	$x + 4$
3 Write the equation.	$\frac{2}{x} = x + 4$

WORKED EXAMPLE 9

In a basketball game, Hao scored 5 more points than Seve. If they scored a total of 27 points between them, how many points did each of them score?



THINK	WRITE
1 Define a pronumeral.	Let Seve's score be x.
2 Hao scored 5 more than Seve.	Hao's score is $x + 5$.
3 Between them they scored a total of 27 points.	$x + (x + 5) = 27$
4 Solve the equation.	$2x + 5 = 27$ $2x = 22$ $x = 11$
5 Since $x = 11$, this is Seve's score. Write Hao's score.	Hao's score $= x + 5$ $= 11 + 5$ $= 16$
6 Write the answer in words.	Seve scored 11 points and Hao scored 16 points.

WORKED EXAMPLE 10

Taxi charges are $3.60 plus $1.38 per kilometre for any trip in Melbourne. If Elena's taxi fare was $38.10, how far did she travel?



THINK	WRITE
1 The distance travelled by Elena has to be found. Define the pronumeral.	Let x = distance travelled (in kilometres).
2 It costs 1.38 to travel 1 kilometre, so the cost to travel x kilometres $= 1.38x$. The fixed cost is $3.60. Write an expression for the total cost.	Total cost $= 3.60 + 1.38x$
3 Let the total cost = 38.10.	$3.60 + 1.38x = 38.10$
4 Solve the equation.	$1.38x = 34.50$ $x = \frac{34.50}{1.38}$ $= 25$
5 State the solution in words.	Elena travelled 25 kilometres.

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Complete this digital doc: SkillSHEET: Writing equations from worded statements (doc-10826)

Exercise 4.5 Solving problems with linear equations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 7, 9, 11–14	Questions: 1–5, 7–10, 12–15	Questions: 1–16

Individual pathway interactivity: int-4492

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Fluency

1. WE8 Write linear equations for each of the following statements, using x to represent the unknown. (Do not attempt to solve the equations.)

a. When 3 is added to a certain number, the answer is 5.

b. Subtracting 9 from a certain number gives a result of 7.

c. Seven times a certain number is 24.

d. A certain number divided by 5 gives a result of 11.

e. Dividing a certain number by 2 equals -9.

f. Three subtracted from five times a certain number gives a result of -7.

g. When a certain number is subtracted from 14 and this result is then multiplied by 2, the result is -3.

h. When 5 is added to three times a certain number, the answer is 8.

i. When 12 is subtracted from two times a certain number, the result is 15.

j. The sum of 3 times a certain number and 4 is divided by 2, which gives a result of 5.

2. MC Which equation matches the following statement?

a. A certain number, when divided by 2, gives a result of -12.

A. $x = \frac{-12}{2}$ **B.** $2x = -12$ **C.** $\frac{x}{2} = -12$ **D.** $\frac{x}{12} = -2$

b. Dividing 7 times a certain number by -4 equals 9.

A. $\frac{x}{-4} = 9$ **B.** $\frac{-4x}{7} = 9$ **C.** $\frac{7+x}{-4} = 9$ **D.** $\frac{7x}{-4} = 9$

c. Subtracting twice a certain number from 8 gives 12.

A. $2x - 8 = 12$ **B.** $8 - 2x = 12$ **C.** $2 - 8x = 12$ **D.** $8 - (x + 2) = 12$

d. When 15 is added to a quarter of a number, the answer is 10.

A. $15 + 4x = 10$ **B.** $10 = \frac{x}{4} + 15$ **C.** $\frac{x+15}{4} = 10$ **D.** $15 + \frac{4}{x} = 10$

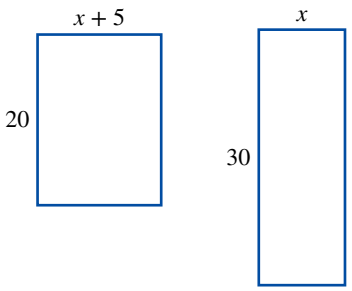
Understanding

3. When a certain number is added to 3 and the result is multiplied by 4, the answer is the same as when the same number is added to 4 and the result is multiplied by 3. Find the number.

4. WE9 John is three times as old as his son Jack, and the sum of their ages is 48. How old is John?

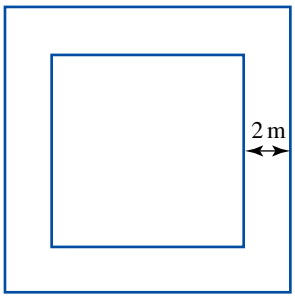
5. In one afternoon's shopping Seedevi spent half as much money as Georgia, but $6 more than Amy. If the three of them spent a total of $258, how much did Seedevi spend?

6. These rectangular blocks of land have the same area. Find the dimensions of each block, and the area.



Reasoning

7. A square pool is surrounded by a paved area that is 2 metres wide. If the area of the paving is $72\,m^2$, what is the length of the pool?





8. Maria is paid $11.50 per hour, plus $7 for each jacket that she sews. If she earned $176 for one 8-hour shift, how many jackets did she sew?

9. Mai hired a car for a fee of $120 plus $30 per day. Casey's rate for his car hire was $180 plus $26 per day. If their final cost and rental period were the same, how long was the rental period?

10. WE10 The cost of producing music CDs is quoted as $1200 plus $0.95 per disk. If Maya's recording studio has a budget of $2100, how many CDs can she have made?



11. Joseph wishes to have some flyers delivered for his grocery business. Post Quick quotes a price of $200 plus 50 cents per flyer, while Fast Box quotes $100 plus 80 cents per flyer.

a. If Joseph needs to order 1000 flyers, which distributor would be cheaper to use?

b. For what number of fliers will the cost be the same for either distributor?

Problem solving

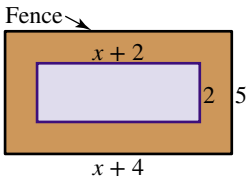
12. A number is multiplied by 8 and 16 is then subtracted. The result is the same as 4 times the original number minus 8. What is the number?

13. Carmel sells three different types of healthy drinks; herbal, vegetable and citrus fizz. One hour she sells 4 herbal, 3 vegetable and 6 citrus fizz for $60.50. The next hour she sells 2 herbal, 4 vegetable and 3 citrus fizz. The third hour she sells 1 herbal, 2 vegetable and 4 citrus fizz. The total amount in cash sales for the three hours is $136.50. Carmel made $7 less in the third hour than she did in the second hour of sales.

Determine her sales in the fourth hour, if Carmel sells 2 herbal, 3 vegetable and 4 citrus fizz.



14. A rectangular swimming pool is surrounded by a path which is enclosed by a pool fence. All measurements are in metres and are not to scale in the diagram shown.



a. Write an expression for the area of the entire fenced-off section.

b. Write an expression for the area of the path surrounding the pool.

c. If the area of the path surrounding the pool is 34 m^2, find the dimensions of the swimming pool.

d. What fraction of the fenced-off area is taken up by the pool?

Reflection

Why is it important to define the pronumeral used when forming a linear equation to solve a problem?

4.6 Rearranging formulas

4.6.1 Rearranging (transposing) formulas

- Formulas are generally written in terms of two or more pronumerals or variables.
- One pronumeral is usually written on one side of the equal sign.
- When rearranging formulas, use the same methods as for solving linear equations (use inverse operations in reverse order).

  The difference between rearranging formulas and solving linear equations is that rearranging formulas does not require a value for the pronumeral(s) to be found.
- The **subject** of the formula is the pronumeral or variable that is written by itself. It is usually written on the left-hand side of the equation.
- A formula is simply an equation that is used for some specific purpose. By now you will be familiar with many mathematical or scientific formulas.

  For example, $C = 2\pi r$ relates the circumference of a circle to its radius. When the formula is shown in this order, C is called the subject of the formula. The formula can be **transposed** (rearranged) to make r the subject.

$$C = 2\pi r \qquad \text{Divide both sides by } 2\pi.$$
$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$
$$\frac{C}{2\pi} = r$$
$$\text{or } r = \frac{C}{2\pi} \qquad \text{Now } r \text{ is the subject.}$$

WORKED EXAMPLE 11

TI | CASIO

Rearrange each formula to make x the subject.

a $y = kx + m$ **b** $6(y + 1) = 7(x - 2)$

THINK

a 1 Subtract m from both sides.

2 Divide both sides by k.

3 Rewrite the equation so that x is on the left-hand side.

b 1 Expand the brackets.

2 Add 14 to both sides.

3 Divide both sides by 7.

4 Rewrite the equation so that x is on the left-hand side.

WRITE

a
$$y = kx + m$$
$$y - m = kx$$
$$\frac{y - m}{k} = \frac{kx}{k}$$
$$\frac{y - m}{k} = x$$
$$x = \frac{y - m}{k}$$

b
$$6(y + 1) = 7(x - 2)$$
$$6y + 6 = 7x - 14$$
$$6y + 20 = 7x$$
$$\frac{6y + 20}{7} = x$$
$$x = \frac{6y + 20}{7}$$

WORKED EXAMPLE 12

For each of the following make the variable shown in brackets the subject of the formula.

a $g = 6d - 3$ (d) **b** $a = \frac{v - u}{t}$ (v)

THINK

a 1 Add 3 to both sides.

2 Divide both sides by 6.

3 Rewrite the equation so that d is on the left-hand side.

b 1 Multiply both sides by t.

2 Add u to both sides.

3 Rewrite the equation so that v is on the left-hand side.

WRITE

a
$$g = 6d - 3$$
$$g + 3 = 6d$$
$$\frac{g + 3}{6} = d$$
$$d = \frac{g + 3}{6}$$

b
$$a = \frac{v - u}{t}$$
$$at = v - u$$
$$at + u = v$$
$$v = at + u$$

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Watch this eLesson: Formulas in the real world (eles-0113)

Exercise 4.6 Rearranging formulas

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3, 6	Questions: 1e–h, 2e–h, 3–6, 8	Questions: 1g–l, 2g–n, 3–10

Individual pathway interactivity: int-4493

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE11 Rearrange each formula to make x the subject.

a. $y = ax$ **b.** $y = ax + b$ **c.** $y = 2ax - b$

d. $y + 4 = 2x - 3$ **e.** $6(y + 2) = 5(4 - x)$ **f.** $x(y - 2) = 1$

g. $x(y - 2) = y + 1$ **h.** $5x - 4y = 1$ **i.** $6(x + 2) = 5(x - y)$

j. $7(x - a) = 6x + 5a$ **k.** $5(a - 2x) = 9(x + 1)$ **l.** $8(9x - 2) + 3 = 7(2a - 3x)$

2. WE12 For each of the following, make the variable shown in brackets the subject of the formula.

a. $g = 4P - 3 \quad (P)$ **b.** $f = \frac{9c}{5} \quad (c)$ **c.** $f = \frac{9c}{5} + 32 \quad (c)$

d. $V = IR \quad (I)$ **e.** $v = u + at \quad (t)$ **f.** $d = b^2 - 4ac \quad (c)$

g. $m = \frac{y - k}{h} \quad (y)$ **h.** $m = \frac{y - a}{x - b} \quad (y)$ **i.** $m = \frac{y - a}{x - b} \quad (a)$

j. $m = \frac{y - a}{x - b} \quad (x)$ **k.** $C = \frac{2\pi}{r} \quad (r)$ **l.** $f = ax + by \quad (x)$

m. $s = ut + \frac{1}{2}at^2 \quad (a)$ **n.** $F = \frac{GMm}{r^2} \quad (G)$

Understanding

3. The cost to rent a car is given by the formula $C = 50d + 0.2k$, where d = the number of days rented and k = the number of kilometres driven. Lin has $300 to spend on car rental for her 4-day holiday. How far can she travel on this holiday?



4. A cyclist pumps up a bike tyre that has a slow leak. The volume of air (in cm^3) after t minutes is given by the formula:

$$V = 24000 - 300t$$



a. What is the volume of air in the tyre when it is first filled?
b. Write an equation and solve it to work out how long it takes the tyre to go completely flat.

Reasoning

5. The total surface area of a cylinder is given by the formula $T = 2\pi r^2 + 2\pi rh$, where r = radius and h = height. A car manufacturer wants the engine's cylinders to have a radius of 4 cm and a total surface area of 400 cm^2. Show that the height of the cylinder is approximately 11.92 cm, correct to 2 decimal places. (*Hint:* Express h in terms of T and r.)

6. If $B = 3x - 6xy$, write x as the subject. Explain the process by showing all working.

Problem solving

7. Use algebra to show that $\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$ can also be written as $u = \frac{fv}{v + f}$.

8. Consider the formula $d = \sqrt{b^2 - 4ac}$.
Rearrange the formula to make a the subject.

9. Find values for a and b, such that:
$$\frac{4}{x+1} - \frac{3}{x+2} = \frac{ax + b}{(x + 1)(x + 2)}$$

Reflection

How does rearranging formulas differ to solving linear equations?

CHALLENGE 4.2

The volume, V, of a sphere can be calculated using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. What is the radius of a spherical ball that has the capacity to hold 5 litres of water?

4.7 Review

4.7.1 Review questions

Fluency

1. The linear equation represented by the sentence 'When a certain number is multiplied by 3, the result is 5 times the certain number plus 7' is:

A. $3x + 7 = 5x$ **B.** $5(x + 7) = 3x$ **C.** $5x + 7 = 3x$ **D.** $5x = 3x + 7$

2. The solution to the equation $\frac{x}{3} = 5$ is:

A. $x = -15$ **B.** $x = 15$ **C.** $x = 1\frac{2}{3}$ **D.** $x = 3$

3. What is the solution to the equation $7 = 21 + x$?

A. $x = 28$ **B.** $x = -28$ **C.** $x = -14$ **D.** $x = 14$

4. What is the solution to the equation $5x + 3 = 37$?

A. $x = 8$ **B.** $x = -8$ **C.** $x = 6.8$ **D.** $x = 106$

5. The solution to the equation $8 - 2x = 22$ is:

A. $x = 11$ **B.** $x = 15$ **C.** $x = -15$ **D.** $x = -7$

6. The solution to the equation $4x + 3 = 7x - 33$ is:

A. $x = -12$ **B.** $x = 12$ **C.** $x = \frac{36}{11}$ **D.** $x = \frac{30}{11}$

7. The solution to the equation $7(x - 15) = 28$ is:

A. $x = 11$ **B.** $x = 19$ **C.** $x = 20$ **D.** $x = 6.14$

8. When rearranging $y = ax + b$ in terms of x, we obtain:

A. $x = \frac{y - a}{b}$ **B.** $x = \frac{y - b}{a}$ **C.** $x = \frac{b - y}{a}$ **D.** $x = \frac{y + b}{a}$

9. Which of the following are linear equations?

a. $5x + y^2 = 0$ **b.** $2x + 3 = x - 2$ **c.** $\frac{x}{2} = 3$

d. $x^2 = 1$ **e.** $\frac{1}{x} + 1 = 3x$ **f.** $8 = 5x - 2$

g. $5(x + 2) = 0$ **h.** $x^2 + y = -9$ **i.** $r = 7 - 5(4 - r)$

10. Solve each of the following linear equations.

a. $3a = 8.4$ **b.** $a + 2.3 = 1.7$ **c.** $\frac{b}{21} = -0.12$

d. $b - 1.45 = 1.65$ **e.** $b + 3.45 = 0$ **f.** $7.53b = 5.64$

11. Solve each of the following linear equations.

a. $\frac{2x - 3}{7} = 5$ **b.** $\frac{5 - x}{2} = -4$ **c.** $\frac{-3x - 4}{5} = 3$

d. $\frac{6}{x} = 5$ **e.** $\frac{4}{x} = \frac{3}{5}$ **f.** $\frac{x + 1.7}{2.3} = -4.1$

12. Solve each of the following linear equations.

a. $5(x - 2) = 6$ **b.** $7(x + 3) = 40$ **c.** $4(5 - x) = 15$

d. $6(2x + 3) = 1$ **e.** $4(x + 5) = 2x - 5$ **f.** $3(x - 2) = 7(x + 4)$

13. Liz has a packet of 45 Easter eggs. She saves 21 to eat tomorrow but rations the remainder so that she can eat 8 eggs each hour.

a. Write a linear equation in terms of the number of hours, h, to represent this situation.

b. Work out how many hours it will take to eat today's share.

14. Solve each of the following linear equations.

a. $11x = 15x - 2$ **b.** $3x + 4 = 16 - x$ **c.** $5x + 2 = 3x + 8$

d. $8x - 9 = 7x - 4$ **e.** $2x + 5 = 8x - 7$ **f.** $3 - 4x = 6 - x$

15. Translate these sentences into algebraic equations.

Use x for the certain number.

a. Twice a certain number is equal to 3 minus that certain number.

b. When 8 is added to 3 times a certain number, the result is 19.

c. Multiplying a certain number by 6 equals 4.

d. Dividing 10 by a certain number is one more than dividing that number by 6.

e. Multiply a certain number by 2, then add 5. Multiply this result by 7. This expression equals 0.

f. Twice the distance travelled is 100 metres more than the distance travelled plus 50 metres.

16. Samuel decides to go on a holiday. He travels a certain distance on the first day, twice that distance on the second day, three times that distance on the third day and four times that distance on the fourth day. If his total journey is 2000 km, how far did he travel on the third day?

17. For each of the following, make the variable shown in brackets the subject of the formula.

a. $y = 6x - 4 \quad (x)$ **b.** $y = mx + c \quad (x)$ **c.** $q = 2(P - 1) + 2r \quad (P)$

d. $P = 2l + 2w \quad (w)$ **e.** $v = u + at \quad (a)$ **f.** $s = \left(\frac{u + v}{2}\right)t \quad (t)$

g. $v^2 = u^2 + 2as \quad (a)$ **h.** $2A = h(a + b) \quad (b)$

Problem solving

18. John is comparing two car rental companies, Golden Ace Rental Company and Silver Diamond Rental Company. Golden Ace Rental Company charges a flat rate of $38 per day and $0.20 per kilometre. The Silver Diamond Rental Company charges a flat rate of $30 per day plus $0.32 per kilometre. John plans to rent a car for three days.

a. Write an algebraic equation for the cost of renting a car for three days from the Golden Ace Rental Company in terms of the number of kilometres travelled, k.

b. Write an algebraic equation for the cost of renting a car for three days from the Silver Diamond Rental Company in terms of the number of kilometres travelled, k.

c. How many kilometres would John have to travel so that the cost of hiring from each company is the same?

19. Frederika has $24 000 saved for a holiday and a new stereo. Her travel expenses are $5400 and her daily expenses are $260.

a. Write down an equation for the cost of her holiday if she stays for d days.

Upon her return from holidays, Frederika wants to purchase a new stereo system that will cost her $2500.

b. How many days can she spend on her holiday if she wishes to purchase a new stereo upon her return?

20. A maker of an orange drink purchases her raw materials from two sources. The first source provides liquid with 6% orange juice, while the second source provides liquid with 3% orange juice. She wishes to make 1 litre of drink with 5% orange juice. Let x = amount of liquid (in litres) purchased from the first source.

a. Write an expression for the amount of orange juice from the first supplier, given that x is the amount of liquid.

b. Write an expression for the amount of liquid from the second supplier, given that x is the amount of liquid used from the first supplier.

c. Write an expression for the amount of orange juice from the second supplier.

d. Write an equation for the total amount of orange juice in the mixture of the 2 supplies, given that 1 litre of drink is mixed to contain 5% orange juice.

e. How much of the first supplier's liquid should she use?

21. Rachel, the bushwalker, goes on a 4-day journey. She travels a certain distance on the first day, half that distance on the second day, a third that distance on the third day and a fourth of that distance on the fourth day. If the total journey is 50 km, how far did she walk on the first day?

22. Svetlana, another bushwalker goes on a 5-day journey, using the same pattern as Rachel in the previous question (a certain amount, then half that amount, then one third, one fourth and one fifth). If her journey is also 50 km, how far did she travel on the first day?

23. Nile.com, the Internet bookstore, advertises its shipping cost to Australia as a flat rate of $20 for up to 10 books; while Sheds & Meager, their competitor, offers a rate of $12 plus $1.60 per book.
For how many books (6, 7, 8, 9 or 10) is Nile.com's cost a better deal?

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Complete this digital doc: Concept map: Topic 4 (doc-10795)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algebraic equation
algebraic fraction
alternative
decomposed
define
expand
expression
fixed
forensic science
formula
inverse operation
justify
linear equation
one-step equation
solution
solve
subject
two-step equation

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Investigation 1 Rich Task

Forensic science



Studies have been conducted on the relationship between the height of a human and measurements of a variety of body parts. One such study relates the height of a person to the length of the upper arm bone (humerus). The relationships are different for (1) males and females and (2) for different races. Let us consider the relationships for white adult Australians.

For males, $h = 3.08l + 70.45$, and for females, $h = 3.36l + 57.97$, where h represents the body height in centimetres and l the length of the humerus in centimetres.



Imagine the following situation.

A decomposed body was found in the bushland. A team of forensic scientists suspects that the body could be the remains of either Alice Brown or James King; they have been missing for several years. From the description provided by their Missing Persons file, Alice is 162 cm tall and James' file indicates that he is 172 cm tall. The forensic scientists hope to identify the body based on the length of the body's humerus.

1. Complete the following tables for both males and females, using the equations on the previous page. Calculate the body height to the nearest centimetre.

Table for males

Length of humerus *l* (cm)	20	25	30	35	40
Body height *h* (cm)					

Table for females

Length of humerus *l* (cm)	20	25	30	35	40
Body height *h* (cm)					

2. On a piece of graph paper, draw the first quadrant of a Cartesian plane. Since the length of the humerus is the independent variable, place it on the x-axis. Place the dependent variable, body height, on the y-axis.

3. Plot the points from the two tables representing both male and female bodies from question **1** onto the set of axes drawn in question **2**. Join the points with straight lines, using different colours to represent males and females.



4. Describe the shape of the two graphs.
5. Measure the length of your humerus. Use your graph to predict your height. How accurate is the measurement?
6. The two lines of your graph will intersect if extended. At what point does this occur? Comment on this value.

The forensic scientists measured the length of the humerus of the bone remains and found it to be 33 cm.

7. Using methods covered in this activity, identify the body, justifying your decision with mathematical evidence.

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Complete this digital doc: Code puzzle: The driest place (doc-15893)

Answers

Topic 4 Linear equations

Exercise 4.2 Solving linear equations

1. a. No **b.** Yes **c.** No **d.** Yes **e.** Yes **f.** No
g. No **h.** No **i.** Yes **j.** No **k.** Yes **l.** No

2. a. $x = 210$ **b.** $x = 52$ **c.** $x = 230$ **d.** $x = 23$ **e.** $x = 142$ **f.** $x = -96$
g. $x = 37$ **h.** $x = 20$ **i.** $x = 50$ **j.** $x = 138$ **k.** $x = 442$ **l.** $x = 243$
m. $y = -15$ **n.** $y = 1.8$ **o.** $y = 12.8$ **p.** $y = 2\frac{1}{3}$ **q.** $y = 24$ **r.** $y = -\frac{1}{18}$
s. $y = 21.5$ **t.** $y = 172.5$ **u.** $y = -8.32$

3. a. $y = 5$ **b.** $y = -2$ **c.** $y = 0.2$ **d.** $y = 1$ **e.** $y = 1$ **f.** $y = 2$
g. $y = 5\frac{1}{3}$ **h.** $y = -1\frac{2}{3}$ **i.** $y = 24.5$ **j.** $y = 1.2$ **k.** $y = 22.25$ **l.** $y = \frac{383}{600}$

4. a. $x = 1$ **b.** $x = -2$ **c.** $x = 3$ **d.** $x = -6$ **e.** $x = -1$ **f.** $x = \frac{1}{2}$
g. $x = 1\frac{2}{3}$ **h.** $x = -2.32$ **i.** $x = 3$ **j.** $x = -\frac{5}{6}$ **k.** $x = 1\frac{1}{4}$ **l.** $x = 3\frac{1}{13}$

5. a. $x = -1$ **b.** $x = 1$ **c.** $x = 0$ **d.** $x = 5$ **e.** $x = -8.6$ **f.** $x = -0.9$
g. $x = 8.9$ **h.** $x = 19$ **i.** $x = -26$ **j.** $x = -1$ **k.** $x = -1$ **l.** $x = -\frac{1}{2}$

6. a. $x = 8$ **b.** $x = 3$ **c.** $x = 4$ **d.** $x = -15$ **e.** $x = 26$ **f.** $x = -42$
g. $x = 9$ **h.** $x = -1\frac{1}{5}$ **i.** $x = 9\frac{1}{3}$ **j.** $x = -2\frac{1}{4}$ **k.** $x = -7$ **l.** $x = \frac{2}{3}$

7. a. $z = 16$ **b.** $z = 31$ **c.** $z = -4$ **d.** $z = 6$ **e.** $z = -9$ **f.** $z = -6$
g. $z = -1.9$ **h.** $z = 6.88$ **i.** $z = 140$ **j.** $z = 0.6$ **k.** $z = -35.4$ **l.** $z = 8$

8. a. $x = 1$ **b.** $x = 13$ **c.** $x = -2$ **d.** $x = -8$ **e.** $x = -4$ **f.** $x = 30\frac{1}{2}$
g. $x = -6$ **h.** $x = -\frac{7}{10}$ **i.** $x = 10.35$ **j.** $x = 0.326$ **k.** $x = 22$ **l.** $x = -5$

9. a. $x = 4$ **b.** $x = \frac{3}{7}$ **c.** $x = -1\frac{1}{7}$ **d.** $x = -6\frac{2}{3}$ **e.** $x = \frac{4}{45}$ **f.** $x = 8$
g. $x = -6$ **h.** $x = 7.5$ **i.** $x = 5.1$ **j.** $x = -6$ **k.** $x = -5\frac{13}{15}$ **l.** $x = -61\frac{3}{7}$

10. a. D **b.** D **c.** C

11. a. $a = -1$ **b.** $b = 10$ **c.** $c = 12.1$ **d.** $d = 4$ **e.** $e = 5$ **f.** $f = 12$
g. $g = 15\frac{29}{30}$ **h.** $h = 31$ **i.** $i = \frac{13}{226}$ **j.** $j = \frac{1}{6}$ **k.** $k = -8$ **l.** $l = 10.3$

12. a. -1 **b.** 6 **c.** 303

13. \$91

14. No. 46.7 °C $\approx$ 116.1 °F.

15. Answers will vary.

16. The mistake is in the second line: the -1 should have been multiplied by 5.

17. 60 lollies

18. Old machine: 6640 cans; new machine: 9360 cans

19. a. \$34 **b.** Yes, a saving of \$7

20. 17

Challenge 4.1

$x = -8, -2, 0, 1, 2, 3, 5, 6, 7, 8, 10, 16$

Exercise 4.3 Solving linear equations with brackets

1. a. $x = 6$ **b.** $x = -3$ **c.** $x = 0$ **d.** $x = 56$ **e.** $x = 1$ **f.** $x = 0$
g. $x = -1$ **h.** $x = -2$ **i.** $x = 6$ **j.** $x = 0$ **k.** $x = -0.8$ **l.** $x = 6$

2. a. $b = 1\frac{1}{6}$ **b.** $m = 4\frac{1}{2}$ **c.** $a = -1\frac{1}{2}$ **d.** $m = -1\frac{1}{3}$ **e.** $p = \frac{3}{5}$ **f.** $m = 2\frac{2}{3}$

g. $a = -1\frac{1}{2}$ **h.** $p = 1\frac{1}{2}$ **i.** $a = 3\frac{1}{3}$ **j.** $m = -2\frac{1}{2}$ **k.** $a = -\frac{1}{6}$ **l.** $m = -\frac{1}{4}$

3. a. $x = 16\frac{1}{9}$ **b.** $x = 2$ **c.** $a = 5$ **d.** $b = 7$ **e.** $y = 7$ **f.** $x = -3\frac{1}{3}$

g. $m = 4$ **h.** $y = 1$ **i.** $a = 5$ **j.** $p = -2$ **k.** $m = 3$ **l.** $p = 1$

4. a. $y = -7.5$ **b.** $y = -4.667$ **c.** $y = -26.25$ **d.** $y = 8.571$ **e.** $y = -2.9$ **f.** $y = 3.243$

g. $y = 3.667$ **h.** $y = 2.75$ **i.** $y = 1.976$ **j.** $y = -2$ **k.** $y = -3.167$ **l.** $y = 1.98$

5. a. D **b.** C

6. 1990

7. a. $[2(2x + 7)]$ m **b.** Width 10 m, length 17 m

8. Answers will vary; $x = 3$.

9. Adding 7 to both sides is the least preferred option, as it does not resolve the subtraction of 7 within the brackets.

10. 3

11. Kyle: \$90, Noah: \$35

12. \$20

Exercise 4.4 Solving linear equations with pronumerals on both sides

1. a. $y = -1$ **b.** $y = 1$ **c.** $y = -3$ **d.** $y = -5$ **e.** $y = -45$ **f.** $y = \frac{8}{27}$

g. $y = -2$ **h.** $y = 20$ **i.** $y = 1$ **j.** $y = -1\frac{1}{5}$ **k.** $y = 4$ **l.** $y = -2$

2. a. C **b.** A

3. a. $x = 1$ **b.** $x = -2$ **c.** $x = 3$ **d.** $x = 4$ **e.** $x = 9$ **f.** $x = 3$

g. $x = -2$ **h.** $x = 4$ **i.** $x = -4$ **j.** $x = \frac{1}{2}$ **k.** $x = 0$ **l.** $x = -\frac{3}{4}$

4. a. $x = -6$ **b.** $x = 7$ **c.** $x = 2$ **d.** $x = -2$ **e.** $x = 13$

f. $x = -7$ **g.** $x = \frac{1}{6}$ **h.** $x = -\frac{2}{17}$ **i.** $x = 1$

5. a. D **b.** C

6. a. $x = 5$ **b.** $x = -3$ **c.** $x = -8$ **d.** $x = -7\frac{1}{2}$ **e.** $x = 4$ **f.** $x = \frac{10}{11}$

g. $x = -18$ **h.** $x = 3\frac{2}{3}$ **i.** $x = \frac{2}{3}$ **j.** $x = -1\frac{3}{7}$ **k.** $x = 0$ **l.** $x = -15$

7. -3

8. 24

9. $3(n - 36) - 98 = -11n + 200$

10. You cannot easily divide the left-hand side by 6 or the right-hand side by 4.

11. Daughter = 10 years, Tom = 40 years

12. The unknown number is -3.

13. a. $C = 40h + 250$ **b.** 18 hours, 45 minutes

c. 18750 **d.** The printing is cheaper by \$1375.

14. a. 65 games **b.** \$25

Exercise 4.5 Solving problems with linear equations

1. a. $x + 3 = 5$ **b.** $x - 9 = 7$ **c.** $7x = 24$ **d.** $\frac{x}{5} = 11$ **e.** $\frac{x}{2} = -9$

f. $5x - 3 = -7$ **g.** $2(14 - x) = -3$ **h.** $3x + 5 = 8$ **i.** $2x - 12 = 15$ **j.** $\frac{3x + 4}{2} = 5$

2. a. C **b.** D **c.** B **d.** B

3. 0

4. 36 years

5. \$66

6. 20×15; 30×10; Area = 300

7. 7 m

8. 12 jackets

9. 15 days

10. 947 CDs

11. a. Post Quick (cost $= \$700$)

b. The cost is nearly the same for 333 flyers ($366.50 and $366.40).

12. 2

13. $42.50

14. a. $A_{\text{fenced}} = (5x + 20)\ \text{m}^2$ **b.** $A_{\text{path}} = (3x + 16)\ \text{m}^2$ **c.** $l = 8$ m, $w = 2$ m **d.** $\frac{8}{25}$

Exercise 4.6 Rearranging formulas

1. a. $x = \frac{y}{a}$ **b.** $x = \frac{y - b}{a}$ **c.** $x = \frac{y + b}{2a}$ **d.** $x = \frac{y + 7}{2}$ **e.** $x = \frac{8 - 6y}{5}$

f. $x = \frac{1}{y - 2}$ **g.** $x = \frac{y + 1}{y - 2}$ **h.** $x = \frac{4y + 1}{5}$ **i.** $x = -5y - 12$ **j.** $x = 12a$

k. $x = \frac{5a - 9}{19}$ **l.** $x = \frac{14a + 13}{93}$

2. a. $P = \frac{g + 3}{4}$ **b.** $c = \frac{5f}{9}$ **c.** $c = \frac{5(f - 32)}{9}$ **d.** $I = \frac{V}{R}$ **e.** $t = \frac{v - u}{a}$

f. $c = \frac{b^2 - d}{4a}$ **g.** $y = hm + k$ **h.** $y = m(x - b) + a$ **i.** $a = y - m(x - b)$ **j.** $x = \frac{y - a + mb}{m}$

k. $r = \frac{2\pi}{C}$ **l.** $x = \frac{f - by}{a}$ **m.** $a = \frac{2(s - ut)}{t^2}$ **n.** $G = \frac{Fr^2}{Mm}$

3. 500 km

4. a. 24 000 cm^3

b. $t = 80$ min $= 1$ h 20 min

5. Answers will vary.

6. $\frac{B}{3(1 - 2y)} = x$

7. Answers will vary.

8. $a = \frac{b^2 - d^2}{4c}$

9. $a = 1$ and $b = 5$

Challenge 4.2

$r = 10.608$ cm

4.7 Review

1. C **2.** B **3.** C **4.** C **5.** D **6.** B **7.** B **8.** B

9. b, c, f, g, i

10. a. $a = 2.8$ **b.** $a = -0.6$ **c.** $b = -2.52$ **d.** $b = 3.1$ **e.** $b = -3.45$ **f.** $b = 0.749$

11. a. $x = 19$ **b.** $x = 13$ **c.** $x = -6\frac{1}{3}$ **d.** $x = -1\frac{1}{5}$ **e.** $x = 6\frac{2}{3}$ **f.** $x = -11.13$

12. a. $x = 3\frac{1}{5}$ **b.** $x = 2\frac{5}{7}$ **c.** $x = 1\frac{1}{4}$ **d.** $x = -1\frac{5}{12}$ **e.** $x = -12\frac{1}{2}$ **f.** $x = -8\frac{1}{2}$

13. a. $8h + 21 = 42$ **b.** 3 hours

14. a. $x = \frac{1}{2}$ **b.** $x = 3$ **c.** $x = 3$ **d.** $x = 5$ **e.** $x = 2$ **f.** $x = -1$

15. a. $2x = 3 - x$ **b.** $3x + 8 = 19$ **c.** $6x = 4$ **d.** $\frac{10}{x} - 1 = \frac{x}{6}$

e. $7(2x + 5) = 0$ **f.** $2x - 100 = x + 50$

16. 600 km

17. a. $x = \frac{y + 4}{6}$ **b.** $x = \frac{y - c}{m}$ **c.** $P = \frac{q - 2r}{2} + 1$ **d.** $w = \frac{P - 2l}{2}$ **e.** $a = \frac{v - u}{t}$ **f.** $t = \frac{2s}{u + v}$

g. $a = \frac{v^2 - u^2}{2s}$ **h.** $b = \frac{2A - ah}{h}$

18. a. $C_G = 114 + 0.20k$ **b.** $C_S = 90 + 0.32k$ **c.** 200 km

19. a. $5400 + 260d = C_H$ **b.** 61 days

20. a. $0.06x$ **b.** $(1 - x)$ **c.** $0.03(1 - x)$

d. $0.06x + 0.03(1 - x) = 0.05$ **e.** 0.667 or 66.7%

21. 24 km

22. $21\frac{123}{137} \approx 21.9$ km

23. 6, 7, 8, 9 or 10 books

Investigation — Rich task

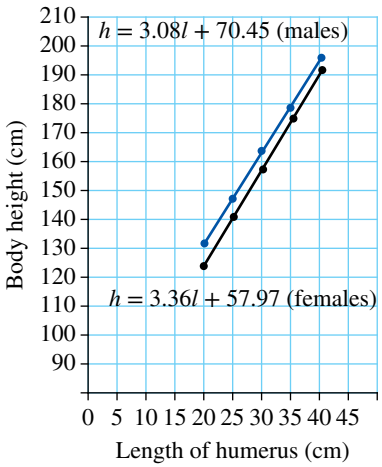
1. Table for males

Length of humerus *l* (cm)	20	25	30	35	40
Body height *h* (cm)	132	147	163	178	194

Table for females

Length of humerus *l* (cm)	20	25	30	35	40
Body height *h* (cm)	125	142	159	176	192

2 and **3**



4. Linear

5. Answers will vary.

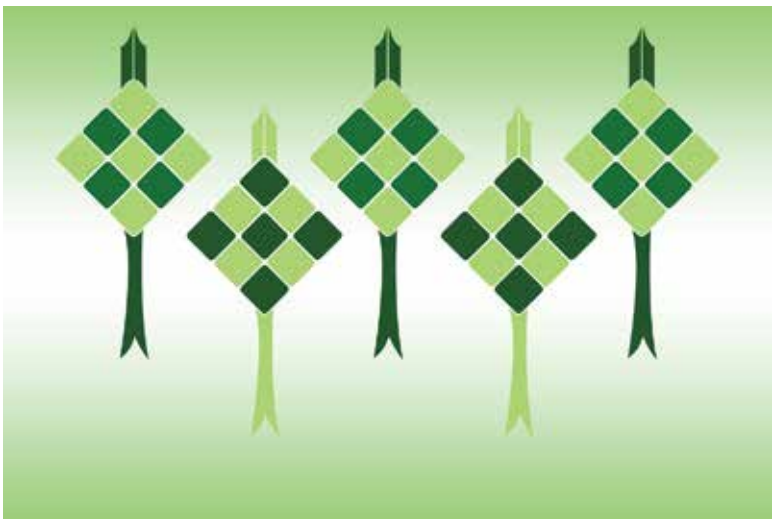
6. (44.6, 207.8)

7. James King

TOPIC 5
Congruence and similarity

5.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



5.1.1 Why learn this?

Geometry is a study of points, lines and angles and how they combine to make different shapes. Similarity and congruence between two figures are important concepts in geometry. Recognising and using congruent and similar shapes can make calculations and design more effective.

5.1.2 What do you know?

assessON

1. **THINK** List what you know about congruent and similar figures. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of congruent and similar figures.

LEARNING SEQUENCE

5.1 Overview
5.2 Ratio and scale
5.3 Congruent figures
5.4 Similar figures
5.5 Area and volume of similar figures
5.6 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: Mathematics in art (eles-1692)

5.2 Ratio and scale

5.2.1 Ratio

- **Ratios** are used to compare quantities of the same kind, measured in the same unit.
- The ratio '1 is to 4' can be written in two ways: as 1 : 4 or as $\frac{1}{4}$.
- The order of the numbers in a ratio is important.

WORKED EXAMPLE 1

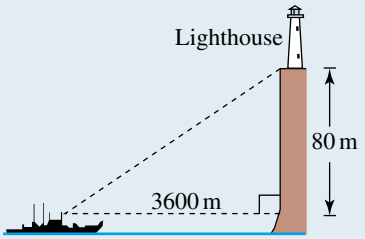
A lighthouse is positioned on an 80 m high cliff. A ship at sea is 3600 m from the base of the cliff.

a Write the following ratios in simplest form.

i Height of the cliff to the distance of the ship from shore

ii Distance of the ship from shore to the height of the cliff

b Compare the distance of the ship from shore with the height of the cliff.



THINK

a i 1 The height and distance are in the same unit (m). Write the height first.

2 Simplify the ratio by dividing both terms by the highest common factor (80).

ii 1 Write the distance from the ship to the shore first.

2 Simplify.

Note: Do not write $\frac{45}{1}$ as 45, because a ratio is a comparison of two numbers.

b 1 Write the ratio 'distance of the ship from shore to the height of the cliff'.

2 Write the answer.

WRITE

a i Height of the cliff : distance of ship from shore

$= 80 : 3600$

$= 1 : 45$

ii Distance of ship from shore : height of cliff

$= \frac{3600}{80}$

$= \frac{45}{1}$

b 45 : 1

The distance of the ship from shore is 45 times the height of the cliff.

- In simplest form, a ratio is written using the smallest whole numbers possible.

WORKED EXAMPLE 2

Express each of the following ratios in simplest form.

a 24 : 8 **b 3.6 : 8.4** **c $1\frac{4}{9} : 1\frac{2}{3}$**

THINK

a Divide both terms by the highest common factor (8).

WRITE

a 24 : 8

$= 3 : 1$

b 1 Multiply both terms by 10 to obtain whole numbers.

2 Divide both terms by the highest common factor, (12).

c 1 Change both mixed numbers into improper fractions.

2 Multiply both terms by the lowest common denominator (9) to obtain whole numbers.

b $3.6 : 8.4$
$= 36 : 84$
$= 3 : 7$

c $1\frac{4}{9} : 1\frac{2}{3}$
$= \frac{13}{9} : \frac{5}{3}$
$= 13 : 15$

- A **proportion** is a statement that indicates that two ratios are equal. A proportion can be written in two ways: $4 : 7 = x : 15$ or $\frac{2}{3} = \frac{11.5}{x}$.

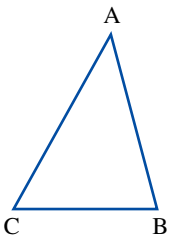
WORKED EXAMPLE 3

Find the value of x in the proportion $4 : 9 = 7 : x$.

THINK	WRITE
1 Write the ratios as equal fractions.	$\frac{4}{9} = \frac{7}{x}$
2 Multiply both sides by x.	$\frac{4x}{9} = 7$
3 Solve the equation.	$4x = 63$
	$x = 15.75$

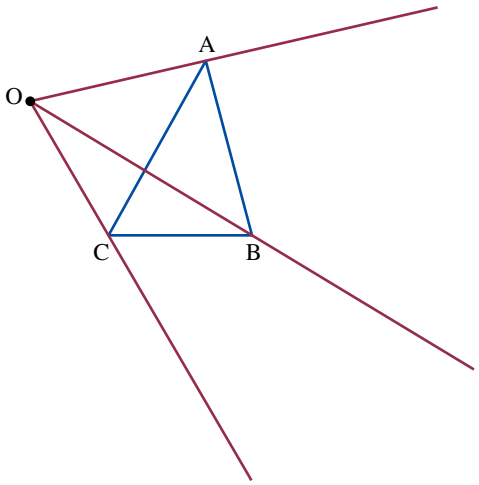
5.2.2 Scale

- Ratios are used when producing scale drawings or maps.
- Consider the case where we want to enlarge triangle ABC (called the **object**) by a **scale factor** of 2, that is, to make it twice its size. Here is one method that we can use.



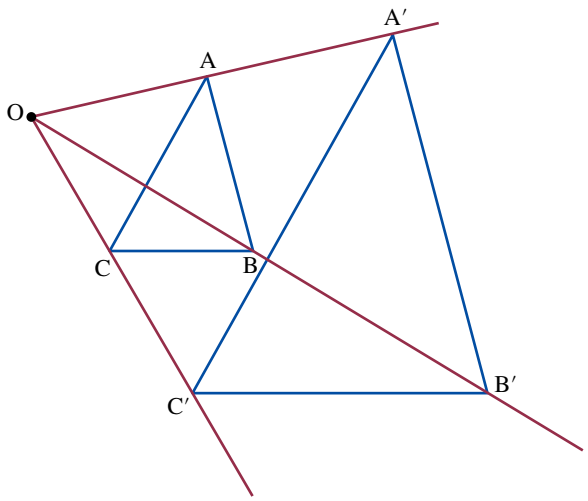


1. Mark a point O somewhere outside the triangle and draw the lines OA, OB and OC as shown.



2. Measure the length of OA and mark in the point A′ (called the **image** of A) so that the distance OA′ is twice that of OA.

3. In the same way, mark in points B′ and C′. (OB′ = 2 × OB, and OC′ = 2 × OC.)



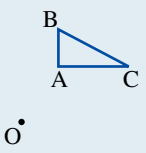
4. Joining A′B′C′ gives a triangle that has side lengths double those of ΔABC. ΔA′B′C′ is called the image of ΔABC.

- By definition:

$$\text{scale factor} = \frac{\text{image length}}{\text{object length}}$$

WORKED EXAMPLE 4

Enlarge triangle ABC by a scale factor of 3, with the centre of enlargement at point O.



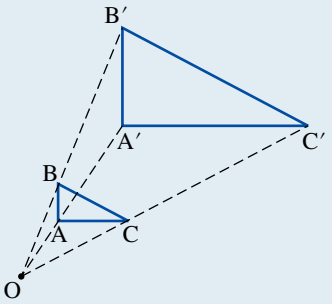
THINK

1 Join each vertex of the triangle to the centre of enlargement O with straight lines and extend them.

2 Locate points A′, B′ and C′ along the lines, OA′= 3OA, OB′= 3OB and OC′= 3OC.

3 Join points A′, B′ and C′ to complete the image.

DRAW



- Enlargements have the following properties.
  - The corresponding side lengths of the enlarged figure are changed in a fixed ratio (that is, the same ratio).
  - The corresponding angles are the same.
  - A scale factor greater than 1 produces an enlarged figure.
  - If the scale factor is a positive number less than 1, the image is smaller than the object (reduction has taken place).

WORKED EXAMPLE 5

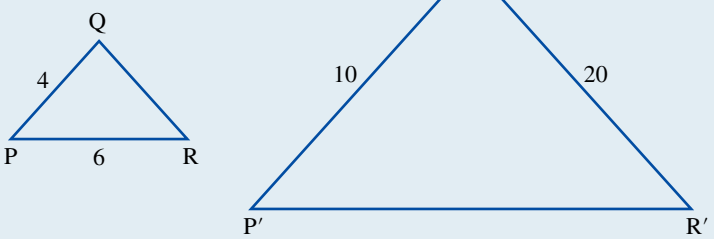
A triangle PQR has been enlarged to triangle P′Q′R′. PQ = 4 cm, PR = 6 cm, P′Q′ = 10 cm and Q′R′ = 20 cm. Calculate:
a the scale factor for the enlargement
b the length of P′R′
c the length of QR.

THINK

a 1 Draw a diagram.

WRITE/DRAW

a



2 Find two corresponding sides. P′Q′ corresponds to PQ.

$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$
$$= \frac{P'Q'}{PQ}$$
$$= \frac{10}{4}$$
$$= 2.5$$

b 1 Apply the scale factor.
$P'R' = 2.5 \times PR$

b $P'R' = 2.5 \times PR$
$= 2.5 \times 6$
$= 15$

2 Write the answer.

$P'R'$ is 15 cm long.

c 1 Apply the scale factor.
$Q'R' = 2.5 \times QR$

c $\frac{Q'R'}{QR} = \frac{20 \text{ cm}}{x \text{ cm}} = 2.5$
$Q'R' = 2.5 \times QR$
$20 = 2.5 \times QR$
$QR = \frac{20}{2.5}$
$= 8$

2 Write the answer.

QR is 8 cm long.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Simplifying fractions (doc-6190)

Complete this digital doc: SkillSHEET: Simplifying ratios (doc-6191)

Complete this digital doc: SkillSHEET: Finding and converting to the lowest common denominator (doc-6192)

Complete this digital doc: SkillSHEET: Solving equations of the type $a = \frac{x}{b}$ to find x (doc-6193)

Complete this digital doc: SkillSHEET: Solving equations of the type $a = \frac{b}{x}$ to find x (doc-6194)

Complete this digital doc: WorkSHEET: Ratio and scale (doc-6198)

Exercise 5.2 Ratio and scale

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–f, 4a–f, 5a–e, 6a–f, 7–16, 19	Questions: 1, 2, 3d–i, 4e–j, 5b–g, 6e–j, 7–14, 16, 17, 20–22	Questions: 1, 2, 3f–j, 4g–l, 5f–i, 6g–l, 7–14, 17, 18, 20, 23, 24

Individual pathway interactivity: int-4494

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

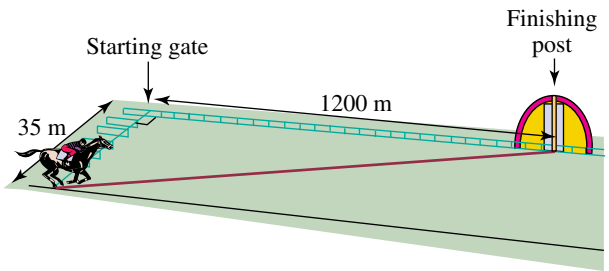
1. WE1 This horse track is 1200 m long and 35 m wide.

a. Write the following ratios in simplest form.

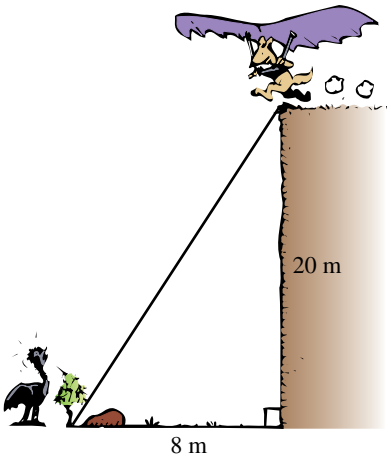
i. Track length to track width

ii. Track width to track length

b. Compare the distance of the length of the track with the width of the track.



2. A dingo perched on top of a cliff spots an emu on the ground below.



a. Write the following ratios in simplest form.
  i. Cliff height to distance from cliff base to emu
  ii. Distance of emu from cliff base cliff height

b. Compare the height of the cliff with the ground distance from the base of the cliff to the emu.

3. WE2a Express each of the following ratios in the simplest form.

a. 12 : 18 **b.** 8 : 56
c. 9 : 27 **d.** 14 : 35
e. 88 : 66 **f.** 16 : 60
g. 200 : 155 **h.** 144 : 44
i. 32 : 100 **j.** 800 : 264

4. WE2b Express each of the following ratios in simplest form.

a. 1.2 : 0.2 **b.** 3.9 : 4.5 **c.** 9.6 : 2.4 **d.** 18 : 3.6
e. 1.8 : 3.6 **f.** 4.4 : 0.66 **g.** 0.9 : 5.4 **h.** 0.35 : 0.21
i. 6 : 1.2 **j.** 12.1 : 5.5 **k.** 8.6 : 4 **l.** 0.07 : 14

5. WE2c Write each of the following ratios in the simplest form.

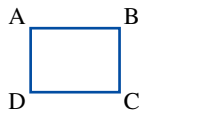
a. $1\frac{1}{2} : 2$ **b.** $2 : 1\frac{3}{4}$ **c.** $1\frac{1}{3} : 2$ **d.** $1\frac{2}{5} : 1\frac{1}{4}$ **e.** $3 : \frac{2}{3}$
f. $\frac{4}{7} : 2$ **g.** $5 : 1\frac{1}{2}$ **h.** $2\frac{3}{4} : 1\frac{1}{3}$ **i.** $3\frac{5}{6} : 2\frac{1}{2}$ **j.** $1\frac{3}{5} : 6\frac{4}{8}$

6. WE3 Find the value of the pronumeral in each of the following proportions.

a. $a : 15 = 3 : 5$ **b.** $b : 18 = 4 : 3$ **c.** $24 : c = 3 : 4$
d. $21 : d = 49 : 4$ **e.** $e : 33 = 5 : 44$ **f.** $6 : f = 5 : 12$
g. $3 : 4 = g : 5$ **h.** $9 : 8 = 5 : h$ **i.** $11 : 3 = i : 8$
j. $7 : 20 = 3 : j$ **k.** $15 : 13 = 12 : k$ **l.** $3 : 4 = l : 15$

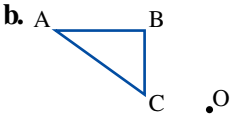
7. WE4 Enlarge each of the following figures by the given scale factor and the centre of enlargement marked O. Show the image of each figure.

a.



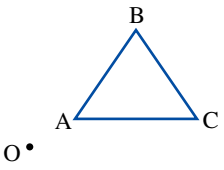
SF = 2

b.



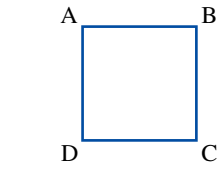
SF = 3

c.



SF = 1.5

d.

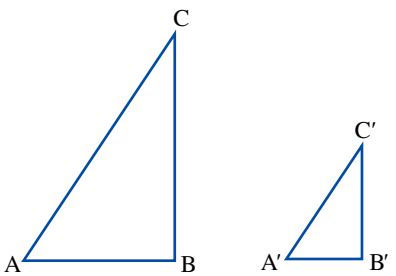


SF = $\frac{1}{2}$

8. WE5 A quadrilateral ABCD is enlarged toA′B′C′D′. AB = 7 cm, AD = 4 cm, A′B′ = 21 cm, B′C′ = 10.5 cm.
Find:

a. the scale factor for enlargement
b. A′D′
c. BC.

9. ΔABC is scaled down to ΔA′B′C′. By measuring the side lengths, determine the scale factor.



Understanding



10. The estimated volume of the Earth's salt water is 1 285 600 000 cubic kilometres. The estimated volume of fresh water is about 35 000 000 cubic kilometres.

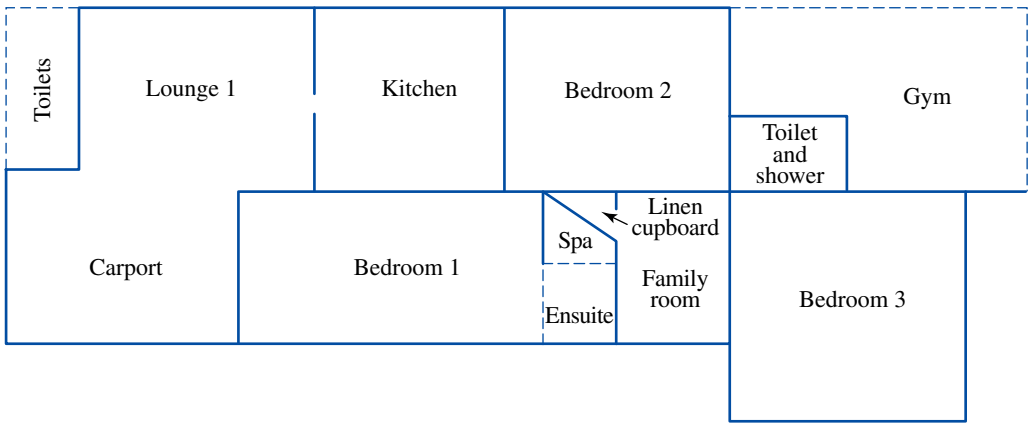
a. What is the ratio of fresh water to salt water (in simplest form)?

b. Find the value of x, to the nearest whole number, when the ratio found in **a** is expressed in the form $1 : x$.

11. Super strength glue comes in two tubes which contain Part A and Part B pastes. These pastes have to be mixed in the ratio 1 : 4 for maximum strength. How many mL of Part A would be needed for 10 mL of Part B?

12. A recipe states that butter and flour must be combined in the ratio 2 : 7. How many grams of butter would be necessary for 3.5 kg of flour?

13. The diagram below shows the ground plan of a house. Bedroom 1 is 8 m × 4 m.



a. Using the dimensions given for bedroom 1, find the scale factor when the actual house (object) is built from the plan (image).

b. Give an estimate of the dimensions of:

i. bedroom 3 **ii.** the kitchen.

Reasoning

14. Pure gold is classed as 24-carat gold. This is too soft to use as jewellery, so it is combined with other metals to form an alloy. 18-carat gold contains gold and other metals in the ratio 18 : 6. The composition of 18-carat rose gold is 75% gold, 22.25% copper and 2.75% silver.

a. Show the mass of silver in a 2.5-gram rose gold bracelet is 0.07 g.

b. Give the composition of a rose gold bracelet which has 0.5 g of copper.

15. The angles of a triangle are in the ratio 3 : 4 : 5. Show the sizes of the three angles are 45°, 60° and 75°.

16. The dimensions of a rectangular box are in the ratio 2 : 3 : 5 and its volume is 21 870 cm^2. Show the dimensions of the box are 18, 27 and 45 cm.

17. Tyler, Dylan and Aaron invested money in the ratio 11 : 9 : 4. If the profits are shared in the ratio 17 : 13 : 6, comment if this is fair for each person. Explain.



18. Five pens cost the same as 2 pens and 6 pencils and the same as 6 sharpeners and a pencil. Show a relationship between the cost of each item.

Problem solving



19. Gordon, a tourist at Kakadu National Park, takes a picture of a two-metre crocodile beside a cliff. When he develops his pictures, the two-metre crocodile is 2.5 cm long and the cliff is 8.5 cm high. What was the actual height of the cliff in cm?

20. Find the ratio of $y : z$ if $2x = 3y$ and $3x = 4z$.

21. The ratio of boys to girls among the students who signed up for a basketball competition is 4 : 3. If 3 boys drop out of the competition and 4 girls join, there will be the same number of boys and girls. How many students have signed up for the basketball competition?

22. Two quantities P and Q are in the ratio 2 : 3. If P is reduced by 1, the ratio is $\frac{1}{2}$. Find the values of P and Q.

23. In the group of students who voted in a Year 9 school leader election, the ratio of girls to boys is 2 : 3. If 10 more girls and 5 more boys had voted, the ratio would have been 3 : 4. How many students voted altogether?

24. Two cylinders are such that the ratio of their base radii is 2 : 1 and the ratio of their heights is 3 : 1. Find the ratio of their respective volumes.

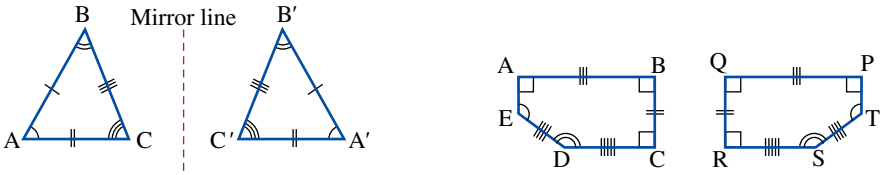
Reflection

It is possible to have a negative scale factor. How would you interpret it?

5.3 Congruent figures

5.3.1 Congruent figures

- **Congruent figures** are identical figures; that is, they have exactly the same shape and size.
- They can be superimposed exactly on top of each other, using reflection, rotation and translation.

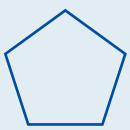


- The symbol for congruence is ≅. This is read as ‘is congruent to’.
- For the diagrams shown above, ABC ≅ A′B′C′ and ABCDE ≅ PQRST.
- When writing congruence statements, the vertices of the figures are named in corresponding order.

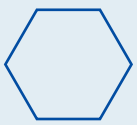
WORKED EXAMPLE 6

Select a pair of congruent shapes from the following set.

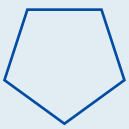
a



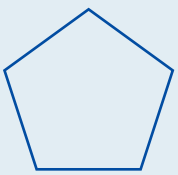
b



c



d



THINK

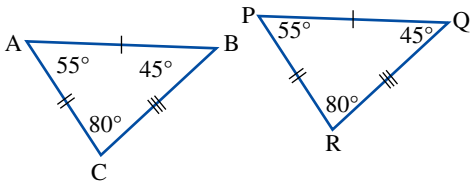
Figures **a** and **c** are identical in shape and size; they just have different orientation.

WRITE

Shape **a** ≅ shape **c**

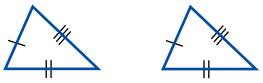
5.3.2 Testing triangles for congruence

- It is not necessary to know that all three sides and all three angles of one triangle are equal to the corresponding sides and angles of another triangle to ensure that the two triangles are congruent. There are certain minimum conditions that will guarantee that this is so.



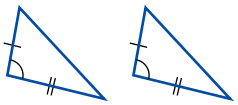
5.3.3 Side-side-side condition of congruence (SSS)

- If two triangles have equal corresponding sides, the angles opposite these corresponding sides will also be equal in size. This means that these two triangles are congruent.
- This is the side-side-side (SSS) condition of congruence.



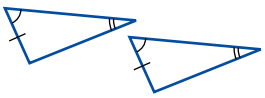
5.3.4 Side-angle-side condition of congruence (SAS)

- In this situation, two pairs of corresponding sides are equal. If the angles between these sides are equal, then the triangles are congruent.
- This is the side-angle-side (SAS) condition of congruence.



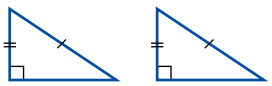
5.3.5 Angle-side-angle condition of congruence (ASA)

- Two pairs of corresponding angles are equal in these triangles. (The third pair of angles will also be equal.)
- If one pair of corresponding sides is equal, then the triangles are congruent.
- This is the angle-side-angle (ASA) condition of congruence.

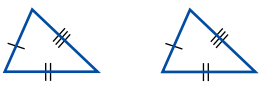
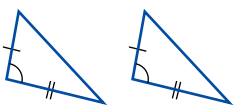
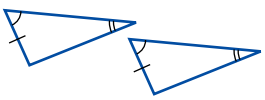
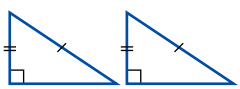


5.3.6 Right angle-hypotenuse-side condition of congruence (RHS)

- In a right-angled triangle, if the hypotenuse and one other side are equal, then the triangles are congruent.
- This is the right angle-hypotenuse-side (RHS) condition of congruence.

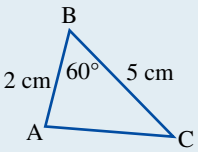


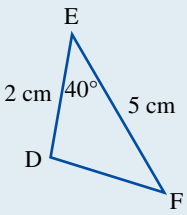
5.3.7 Summary of congruence tests

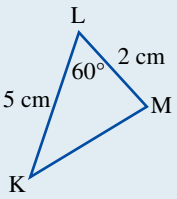
Test	Description	Abbreviation
	All corresponding sides are equal in length.	SSS (side–side–side)
	Two corresponding sides are equal in length and the included angles are equal in size.	SAS (side–angle–side)
	Two angles are equal in size and there is one pair of corresponding sides of equal length.	ASA (angle–side–angle)
	In a right-angled triangle, the hypotenuse and one pair of corresponding sides are equal in length.	RHS (right angle–hypotenuseside)

WORKED EXAMPLE 7

Which of the following triangles are congruent? Give reasons for your answer.







THINK

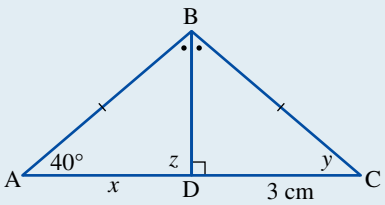
In all three triangles two given sides are of equal length (2 cm and 5 cm). Triangles ABC and KLM also have the included angle of equal size (60°). B corresponds to L, and A corresponds to M.

WRITE

$\Delta ABC \cong \Delta MLK$ (SAS)

WORKED EXAMPLE 8

Given that $\Delta ABD \cong \Delta CBD$, find the values of the pronumerals in the figure at right.



THINK

1 In congruent triangles corresponding sides are equal in length. Side AD (marked x) corresponds to side CD.

2 Since triangles are congruent, corresponding angles are equal.

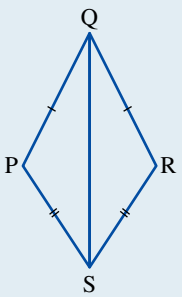
WRITE

$\Delta ABD \cong \Delta CBD$
$AD = CD$
$x = 3\,\text{cm}$

$\angle A = \angle C$
$y = 40°$
$\angle BDA = \angle BDC$
$z = 90°$

WORKED EXAMPLE 9

Prove that ΔPQS is congruent to ΔRQS.



THINK

1 Study the diagram and state which sides and/or angles are equal.

2 This fits the SSS test and proves congruence.

WRITE

$QP = QR$ (given)
$PS = RS$ (given)
QS is common.

$\Delta PQS \cong \Delta RQS$ (SSS)

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Naming angles (doc-6195)

Complete this digital doc: SkillSHEET: Complementary and supplementary angles (doc-6196)

Complete this digital doc: SkillSHEET: Angles in a triangle (doc-6197)

Complete this digital doc: WorkSHEET: Congruent figures (doc-6201)

Exercise 5.3 Congruent figures

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10, 11	Questions: 1–8, 10–13	Questions: 1–15

Individual pathway interactivity: int-4495

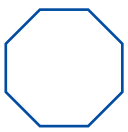
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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

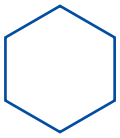
Fluency

1. WE6 Select a pair of congruent shapes from the figures in each part of the following question.

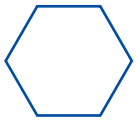
a. i.



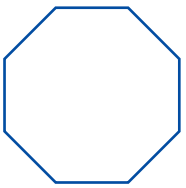
ii.



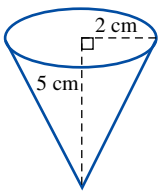
iii.



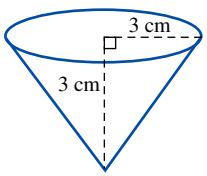
iv.



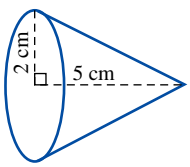
b. i.



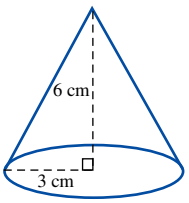
ii.



iii.



iv.



c. i.



ii.



iii.

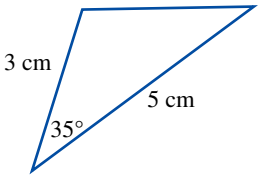


iv.

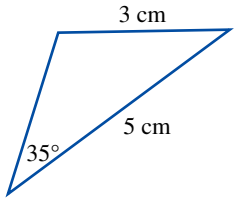


Understanding

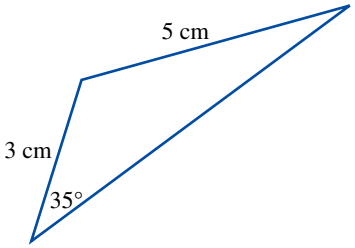
2. MC Which of the following is congruent to the triangle shown at right?



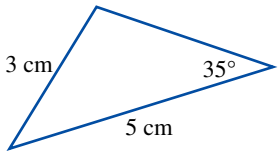
A.



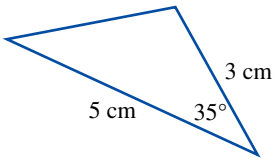
B.



C.

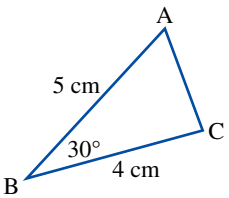


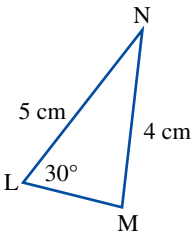
D.

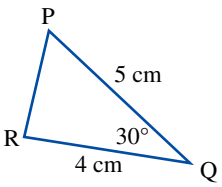


3. WE7 In each part of the question, which of the triangles are congruent? Give a reason for your answer.

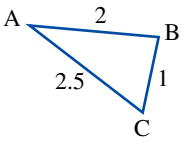
a.

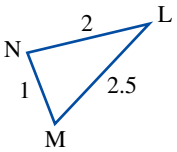


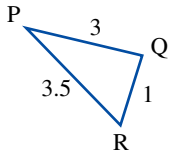




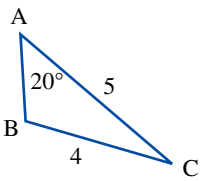
b.

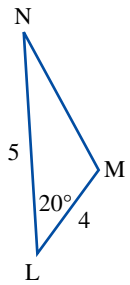


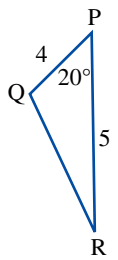




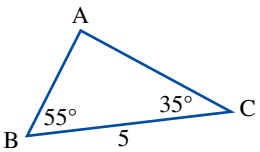
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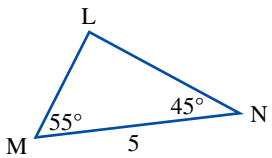


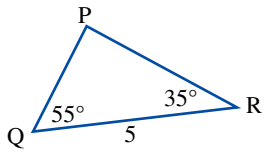




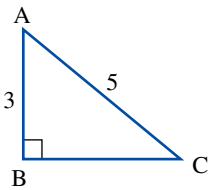
d.

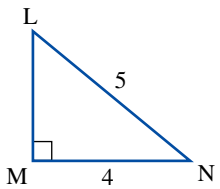


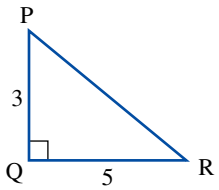




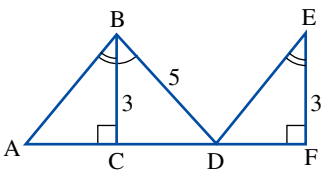
e.





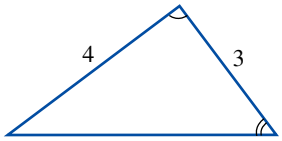


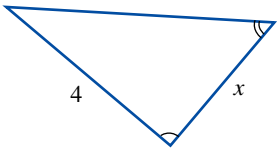
f.



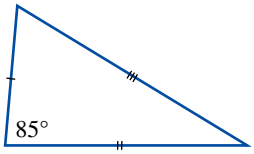
4. WE8 Find the value of the pronumeral in each of the following pairs of congruent triangles. All side lengths are in centimetres.

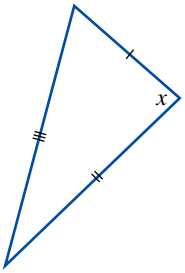
a.



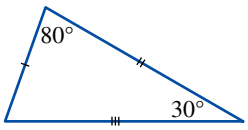


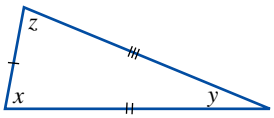
b.



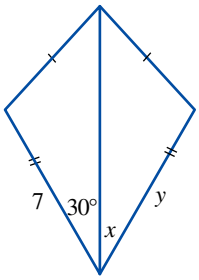


c.

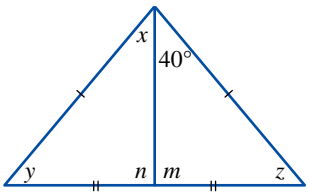




d.

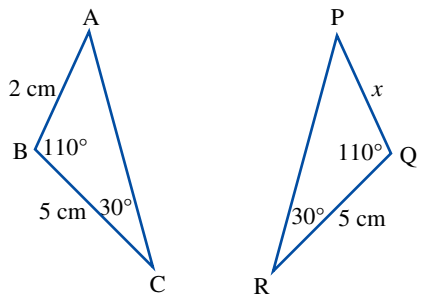


e.

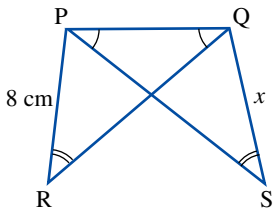


5. Find the length of the side marked with the pronumeral using congruent triangles.

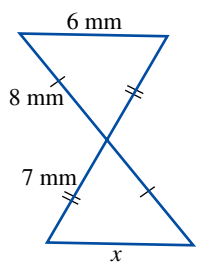
a.



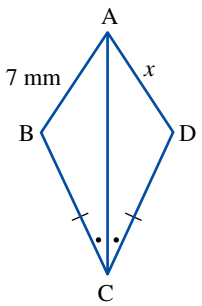
b.



c.



d.

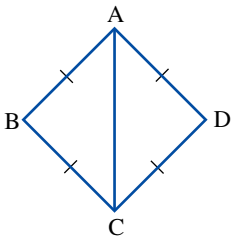


6. Do congruent figures have the same area? Explain.

Reasoning

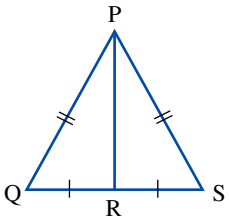
7. WE9 For each of the following, prove that:

a.



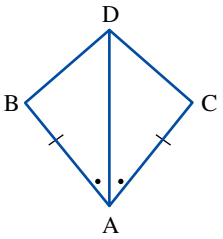
$\Delta ABC \cong \Delta ADC$

b.



$\Delta PQR \cong \Delta PSR$

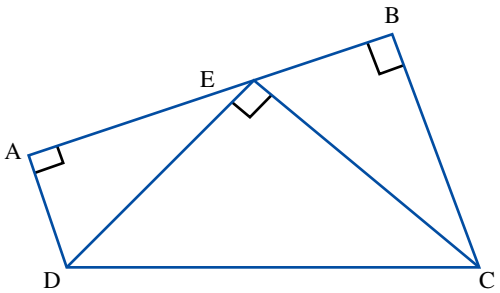
c.



$\Delta DBA \cong \Delta DCA$

8. Give an example to show that triangles with two angles of equal size and a pair of non-corresponding sides of equal length may not be congruent.

9. ABCD is a trapezium with both AD and BC perpendicular to AB. If a right-angled triangle DEC is constructed with an angle $\angle ECD$ equal to 45°, prove that $\Delta EDA \cong \Delta ECB$.



Problem solving

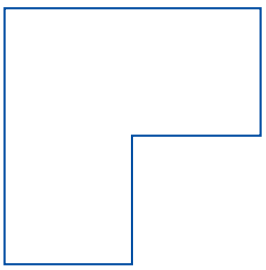
10. If two congruent triangles have a right angle, is the reason always 'right angle, hypotenuse and corresponding side'? Justify your answer.

11. A teacher asked his class to each draw a triangle that has side lengths of 5 cm and 4 cm, and an angle of 45 degrees that is not formed at the point joining the 5 cm and 4 cm side. Would the triangles drawn by every member of the class be congruent? Explain why.

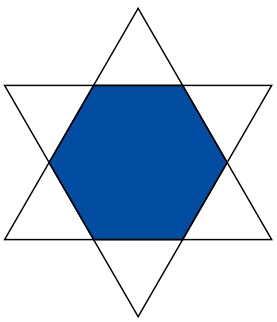
12. Make 5 congruent triangles from 9 matchsticks.

13. Make 7 congruent triangles from 9 matchsticks.

14. Show how the figure shown at right can be cut into four congruent pieces.



15. Find the ratio of the outer (unshaded) area to the inner (shaded) area of this six-pointed star.

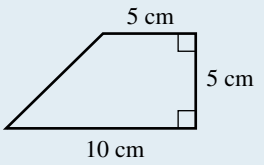


Reflection

What is the easiest way to determine if two figures are congruent?

CHALLENGE 5.1

On a piece of paper, draw the trapezium as per the dimensions given to the diagram at right. Divide your trapezium into four congruent trapeziums that are similar in shape to the original trapezium.



5.4 Similar figures

5.4.1 Similar figures



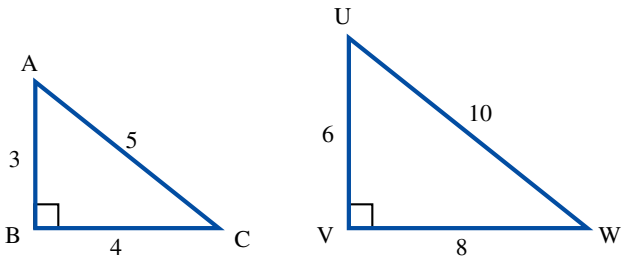
- **Similar figures** have identical shape but different size. The corresponding angles in similar figures are equal in size and the corresponding sides are in the same ratio, given by the scale factor.
- The symbol used to denote similarity is ~, which is read as 'is similar to'.
- Similar figures can be obtained as a result of enlargement or reduction.
- If an enlargement (or a reduction) takes place, the original figure can be called the object and the enlarged (or reduced) figure called the image.
- It can also be said that the object *maps to* the image.
- For any two similar figures, the scale factor can be obtained using the following formula:

$$\text{scale factor} = \frac{\text{length of the image}}{\text{length of the object}}$$

Note: The size of the scale factor indicates whether the original object has been enlarged or reduced.

- If the scale factor is greater than 1, an enlargement has occurred.
- If the scale factor is positive but less than 1, a reduction has occurred.

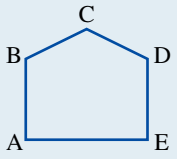
Consider the pair of similar triangles below.



- Triangle UVW is similar to triangle ABC.
  That is, $\Delta UVW \sim \Delta ABC$.
- The corresponding angles of the two triangles are equal in size:
  $\angle CAB = \angle WUV$, $\angle ABC = \angle UVW$ and $\angle ACB = \angle UWV$.
- The corresponding sides of the two triangles are in the same ratio.
  $\frac{UV}{AB} = \frac{VW}{BC} = \frac{UW}{AC} = 2$; that is, the side lengths of ΔUVW are twice as long as the corresponding sides in ΔABC.
- The scale factor is 2.
- The original figure, ΔABC, can be called the object, while ΔUVW, obtained as the result of enlargement, is the image.
- It can be said that ΔABC *maps to* ΔUVW.

WORKED EXAMPLE 10

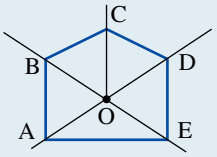
Enlarge the shape at right by a factor of 2.



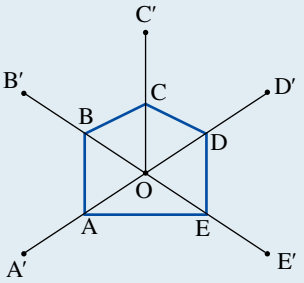
THINK

DRAW

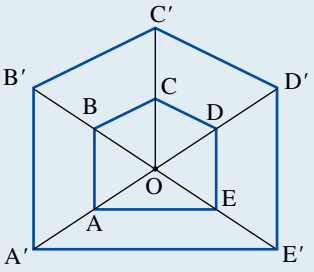
1 Select a point, O, somewhere inside the given shape and join it with straight-line segments to each vertex. Extend the lines beyond the shape.



2 Measure the distance OA and mark in the point A′ so that $OA' = 2 \times OA$. Repeat this for the other vertices.



3 Join the image vertices A′B′C′D′E′ with straight lines.



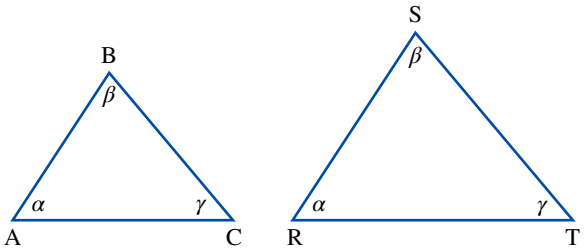
5.4.2 Testing triangles for similarity

- As with congruent triangles, it is not necessary to know that all pairs of corresponding sides are in the same ratio and that all corresponding angles are equal to ensure that two triangles are similar. There are certain minimum conditions which will guarantee that this is so.



5.4.3 Angle-angle-angle condition of similarity (AAA)

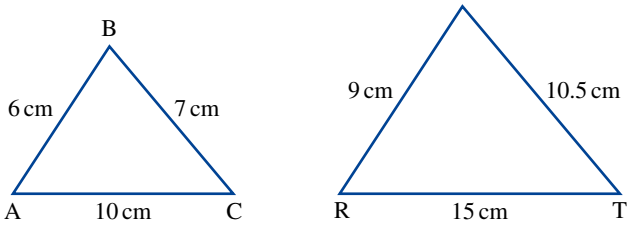
- If the angles of one triangle are the same as the angles of a second triangle, then the triangles are similar.



- This is the angle-angle-angle (AAA) condition for similarity.
- From the diagram above, $\Delta ABC \sim \Delta RST$ (AAA).

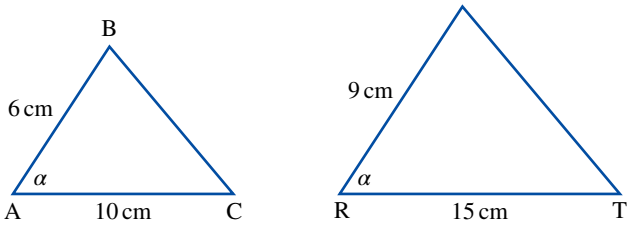
5.4.4 Side-side-side condition for similarity (SSS)

- In the diagram at right, the ratio of pairs of corresponding sides is constant.
  That is, $\frac{9}{6} = \frac{15}{10} = \frac{10.5}{7} = 1.5$.
  This is enough to show that the triangles are similar.
- This is the side-side-side (SSS) condition for similarity.
- In this case, $\Delta ABC \sim \Delta RST$ (SSS).



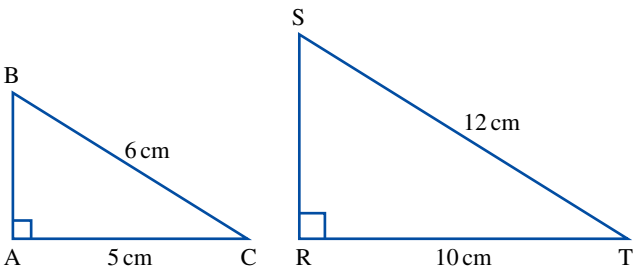
5.4.5 Side-angle-side condition for similarity (SAS)

- In the diagram at right, two pairs of sides are in the same ratio; that is, $\frac{9}{6} = \frac{15}{10} = 1.5$, and the included angles are equal as well.
  This is enough to show that the triangles are similar.
- This is the side-angle-side (SAS) condition for similarity.
  In this case $\Delta ABC \sim \Delta RST$ (SAS).



5.4.6 Right angle-hypotenuse-side condition for similarity (RHS)

- With right-angled triangles, a special condition can apply.
- If the hypotenuse and one other pair of sides are in the same ratio (e.g. in the diagram at right, $\frac{12}{6} = \frac{10}{5}$), then the triangles are similar.
- This is the right angle-hypotenuse-side (RHS) condition for similarity.
  In this case $\Delta ABC \sim \Delta RST$ (RHS).



5.4.7 Summary of similarity tests

- Triangles can be checked for similarity using one of the tests described in the table below.

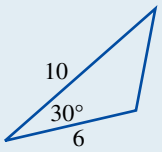
Test description	Abbreviation
All corresponding angles are equal in size	AAA (angle–angle–angle)
All corresponding sides are in the same ratio	SSS (side–side–side)
Two pairs of corresponding sides are in the same ratio and the included angles are equal in size	SAS (side–angle–side)
In right-angled triangles, the hypotenuses and one other pair of sides are in the same ratio.	RHS (right angle–hypotenuse–side)

- *Note:* When using the AAA test, it is sufficient to show that two pairs of corresponding angles are equal. Since the sum of the interior angles in any triangle is 180°, the third pair of angles will automatically be equal.

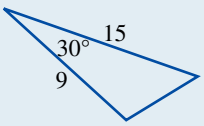
WORKED EXAMPLE 11

Find a pair of similar triangles from those shown. Give a reason for your answer.

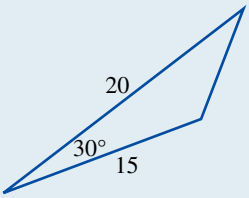
a



b



c



THINK

1 In each triangle we know the size of two sides and the included angle, so the SAS test can be applied. Since all included angles are equal (30°), we need to find ratios of corresponding sides, taking two triangles at a time.

2 Write the answer.

WRITE

For triangles **a** and **b**:

$\frac{15}{10} = \frac{9}{6} = 1.5$

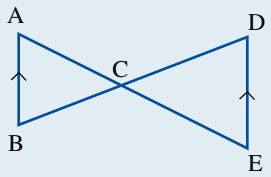
For triangles **a** and **c**:

$\frac{20}{10} = 2, \quad \frac{15}{6} = 2.5$

Triangle **a** ~ triangle **b** (SAS)

WORKED EXAMPLE 12

Prove that ΔABC is similar to ΔEDC.



THINK

1 AB is parallel to DE. ∠ABC and ∠EDC are alternate angles.

2 ∠BAC and ∠DEC are alternate angles.

3 The third pair of angles must be equal.

4 This proves that the triangles are similar.

WRITE

∠ABC = ∠EDC (alternate angles)

∠BAC = ∠DEC (alternate angles)

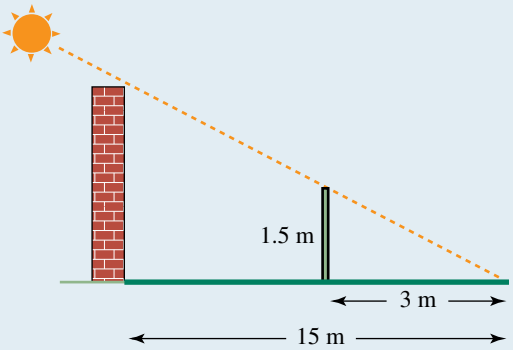
∠BCA = ∠DCE (vertically opposite angles)

ΔABC ~ ΔEDC (AAA)

- The ratio of the corresponding sides in similar figures can be used to calculate missing side lengths or angles in these figures.

WORKED EXAMPLE 13

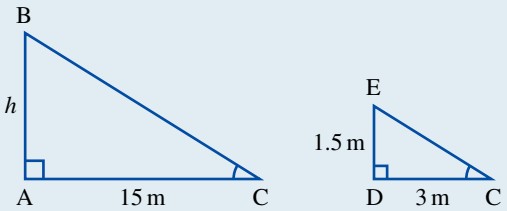
A 1.5-metre pole casts a shadow 3 metres long, as shown. Find the height of a building that casts a shadow 15 metres long at the same time of the day.



THINK

1 Represent the given information on a diagram. $\angle BAC = \angle EDC$; $\angle BCA = \angle ECD$

WRITE/DRAW



2 Triangles ABC and DEC are similar. Therefore, the ratios of corresponding sides are the same. Write the ratios.

$\Delta ABC \sim \Delta DEC$ (AAA)

$$\frac{h}{1.5} = \frac{15}{3}$$

3 Solve the equation for h.

$$h = \frac{15 \times 1.5}{3}$$
$$= 7.5$$

4 Write the answer in words, including units.

The height of the building is 7.5 m.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: WorkSHEET: Similar figures (doc-6202)

Exercise 5.4 Similar figures

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 5, 8, 11–14, 17, 18	Questions: 1–4, 6, 9, 11–14, 16, 17, 19, 21, 22	Questions: 1–4, 7, 10–16, 18, 20, 21, 23, 24

Individual pathway interactivity: int-4496

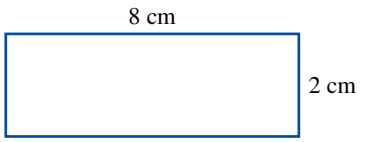
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

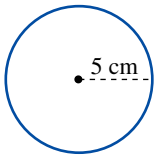
Fluency

1. WE10 Enlarge (or reduce) the following shapes by the scale factor given.

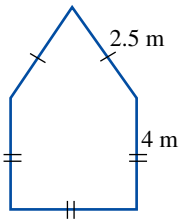
a. Scale factor = 3



b. Scale factor = 2

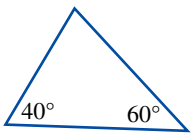


c. Scale factor = 0.5

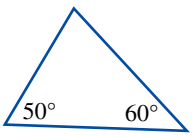


2. WE11 Find a pair of similar triangles among those shown in each part. Give a reason for your answer.

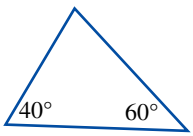
a. i.



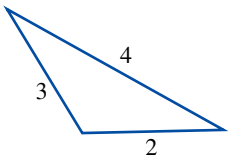
ii.



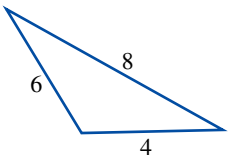
iii.



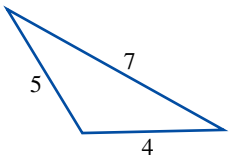
b. i.



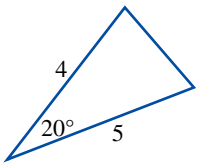
ii.



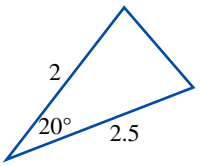
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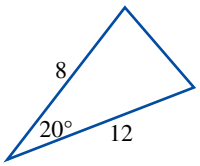
c. i.



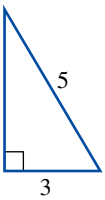
ii.



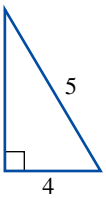
iii.



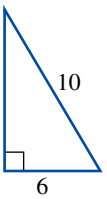
d. i.



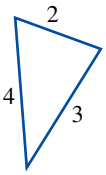
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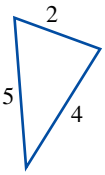
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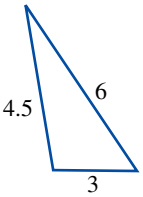
e. i.



ii.



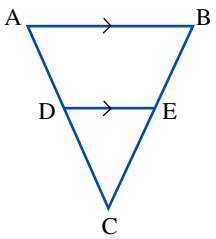
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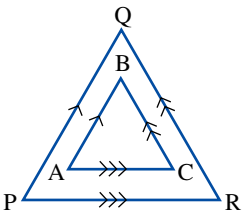
Understanding

3. Name two similar triangles in each of the following figures, ensuring that vertices are listed in the correct order.

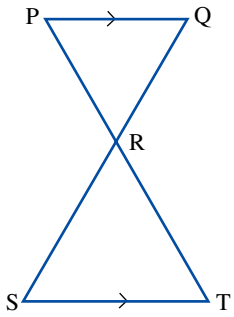
a.



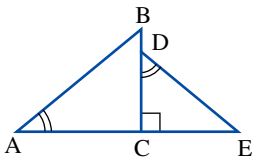
b.



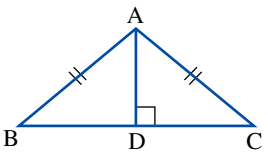
c.



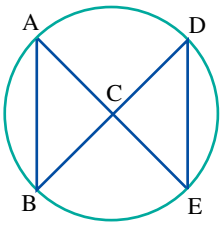
d.



e.



4. In the diagram at right C is the centre of the circle. Complete this statement: ΔABC is similar to . . .



5. ABCDEF is a regular hexagon, similar to PQRSTU.

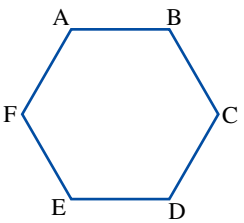
a. What is the length of:

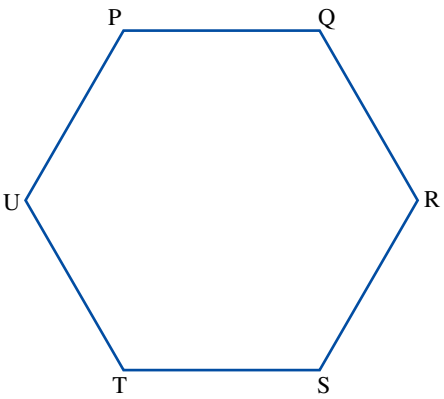
i. AB

ii. RS?

b. What is the scale factor for enlargement?

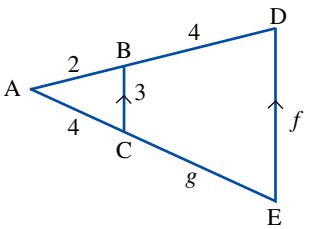
Note: You need to measure the lengths.





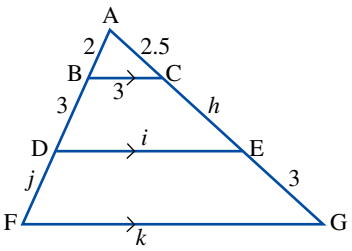
6. a. Complete this statement: $\frac{AB}{AD} = \frac{BC}{\quad} = \frac{\quad}{AE}$.

b. Find the value of the pronumerals.

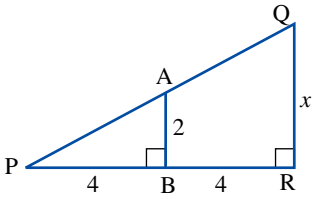


7. a. Find the values of h and i.

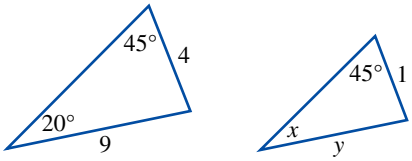
b. Find the values of j and k.



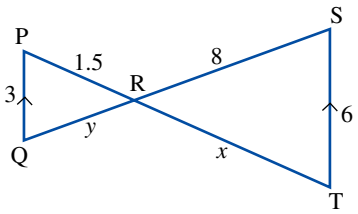
8. Find the value of the pronumeral in the diagram below.



9. If the triangles shown below are similar, find the values of x and y.



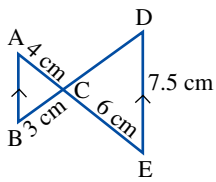
10. Find the values of x and y in the diagram below.



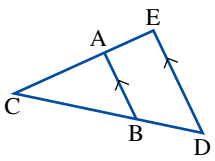
Reasoning

11. WE12 Prove that ΔABC is similar to ΔEDC in each of the following.

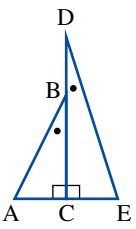
a.



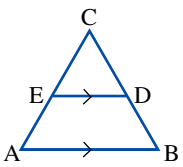
b.



c.

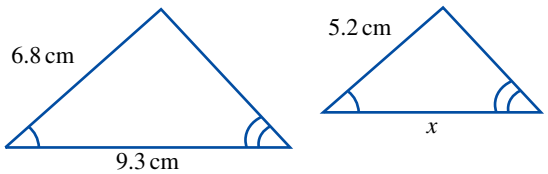


d.

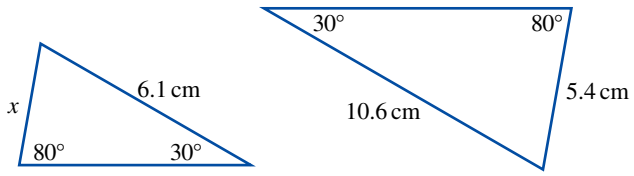


12. Find the value of each pronumeral in these triangles. Show how you arrived at your answers.

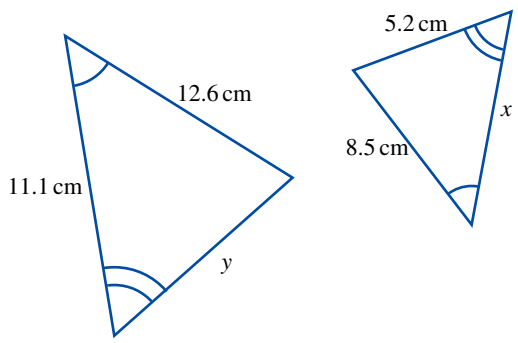
a.



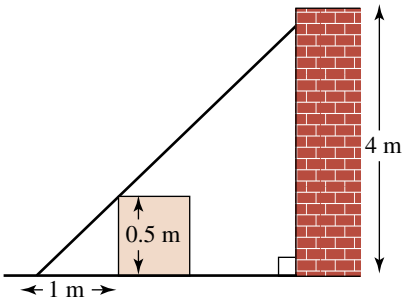
b.



c.



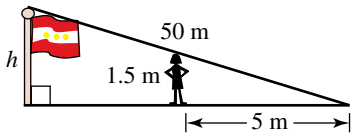
13. WE13 A ladder just touches a bench and leans on a wall 4 metres above the ground, as shown. If the bench is 50 centimetres high and is 1 metre from the base of the ladder, show that the base of the ladder is 8 metres from the wall.



14. Natalie, whose height is 1.5 metres, casts a shadow 2 metres long at a certain time of the day. If Alex is 1.8 metres tall, show that his shadow would be 2.4 m long.



15. A string 50 metres long is pegged to the ground and tied to the top of a flagpole. It just touches the head of Maureen, who is 5 metres away from the point where the string is held to the ground. If Maureen is 1.5 metres tall, show that the height, h, of the flagpole is 14.37 m.



16. Using diagrams or otherwise, explain whether the following statements are true or false.

a. All equilateral triangles are similar.

b. All isosceles triangles are similar.

c. All right-angled triangles are similar.

d. All right-angled isosceles triangles are similar.

Problem solving

17. Penny and Paul play tennis at night under floodlights. When Penny stands 2.5 m from the base of the floodlight, her shadow is 60 cm long.

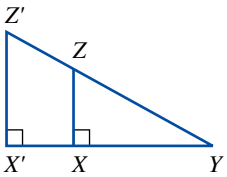
a. If Penny is 1.3 m tall, how high is the floodlight in metres, correct to 2 decimal places?

b. If Paul, who is 1.6 m tall, stands in the same place, how long will his shadow be in cm?



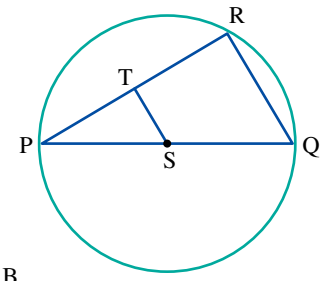
18. To determine the height of a flagpole, Jenna and Mia decided to measure the shadow cast by the flagpole. They place a 1 m ruler at a distance of 3 m from the base of the flagpole and measure the shadows that both the ruler and flagpole cast. Both shadows finished at the same point. After measuring the shadow of the flagpole, Jenna and Mia calculate that the height of the flagpole is 5 m. Determine the length of the shadow cast by the flagpole, in metres, as measured by Jenna and Mia.

19.



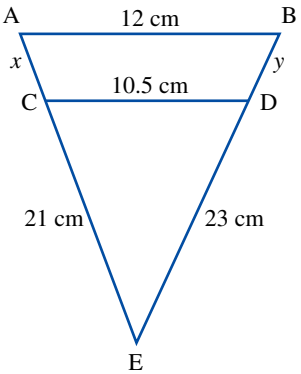
Use the diagram above to find the value of a if $XZ = 8$ cm, $X'Z' = 12$ cm, $X'X = a$ cm and $XY = (a + 1)$ cm.

20. PQ is a diameter of this circle with a centre at S. R is any point on the circumference. T is the midpoint of PR.



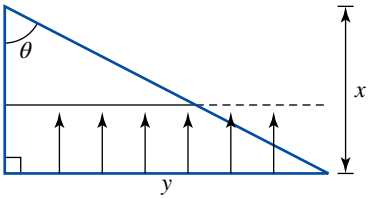
a. Write down everything you know about this figure.
b. Explain why ΔPTS is similar to ΔPRQ.
c. Find the length of TS if RQ is 8 cm.
d. Find the length of every other side also given that PT is 3 cm and angle PRQ is a right angle.

21. AB and CD are parallel lines in the figure at right.

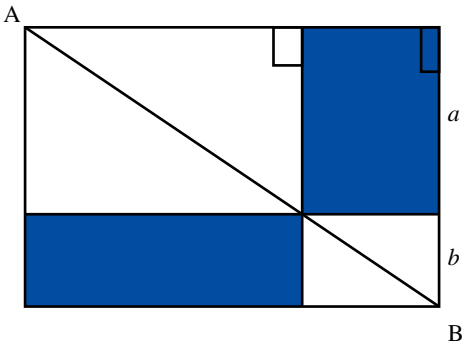


a. State the similar triangles.
b. Calculate the values of x and y.

22. For the diagram given, show that if the base of the triangle is raised to half the height of the triangle, the length of the base of the newly formed triangle will be half of its original length.



23. AB is a straight line. The fraction of the large rectangle that is shaded is $\frac{12}{25}$. Find the ratio $a : b$.



Reflection

Do similar objects have the same perimeters?

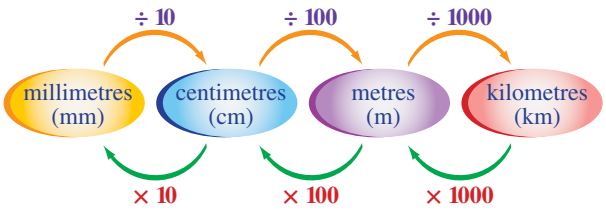
CHALLENGE 5.2

A tree that is 3.5 m tall casts a 7.5 m shadow at a certain time of the day. At that time of day, Ruby's shadow is 3 m long. How tall is Ruby?

5.5 Area and volume of similar figures

5.5.1 Units of length

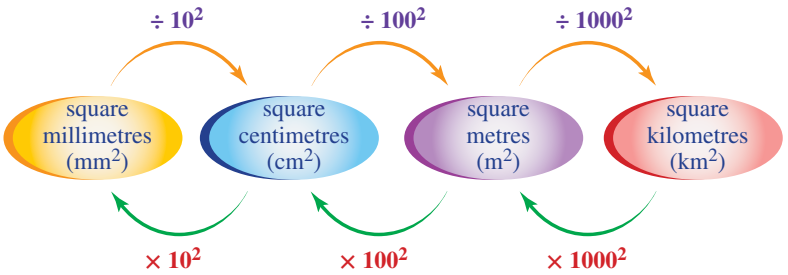
- Metric units of length include millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- To convert between the units of length, we use the conversion chart shown below.



- When converting from a large unit to a smaller unit, multiply by the conversion factor; when converting from a smaller unit to a larger unit, divide by the conversion factor.

5.5.2 Units of area

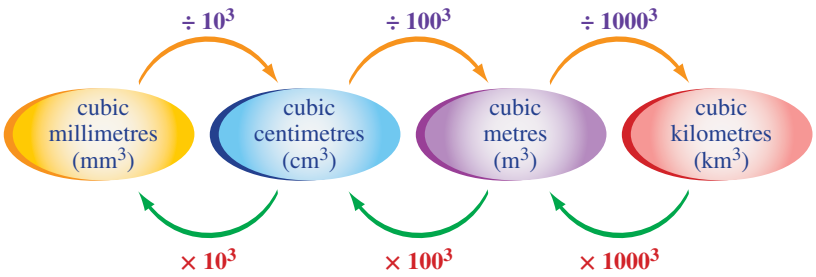
- Area is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2).
- Area units can be converted using the chart below.



- Area units are the squares of the corresponding length units.

5.5.3 Units of volume

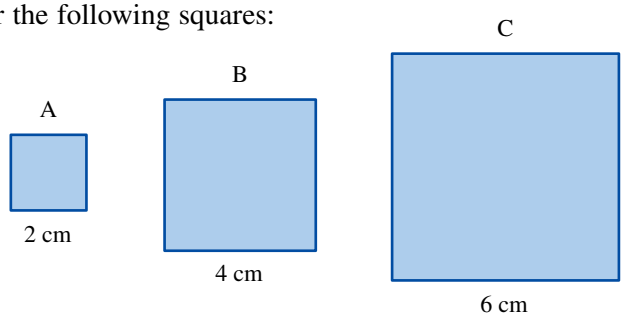
- Volume is measured in cubic units such as cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3).
- Volume units can be converted using the chart shown below.



- Volume units are the cubes of the corresponding length units.

5.5.4 Area and surface area of similar figures

- If the side lengths in any figure are increased by a scale factor of n, then the area of similar figures increases by a scale factor of n^2.
  For example, consider the following squares:



Area $= 2 \times 2$
$= 4 \text{ cm}^2$

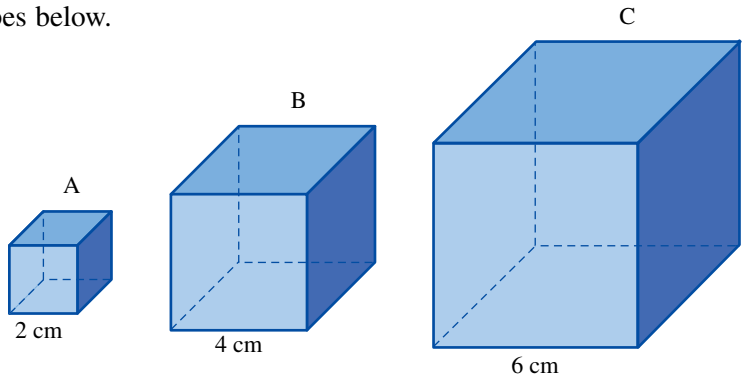
Area $= 4 \times 4$
$= 16 \text{ cm}^2$

Area $= 6 \times 6$
$= 36 \text{ cm}^2$

- The scale factors for the side lengths and the scale factors for the areas are calculated below.

Squares	Scale factor for side length	Scale factor for area
A and B	$\frac{4}{2} = 2$	$\frac{16}{4} = 4 = 2^2$
A and C	$\frac{6}{2} = 3$	$\frac{36}{4} = 9 = 3^2$
B and C	$\frac{6}{4} = \frac{3}{2}$	$\frac{36}{16} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

- If the side lengths in any figure are increased by a scale factor of n, then the surface area of similar figures increases by a scale factor of n^2.
  Consider the cubes below.



Surface Area $= 6 \times 4$
$= 24 \text{ cm}^2$

Surface Area $= 6 \times 16$
$= 96 \text{ cm}^2$

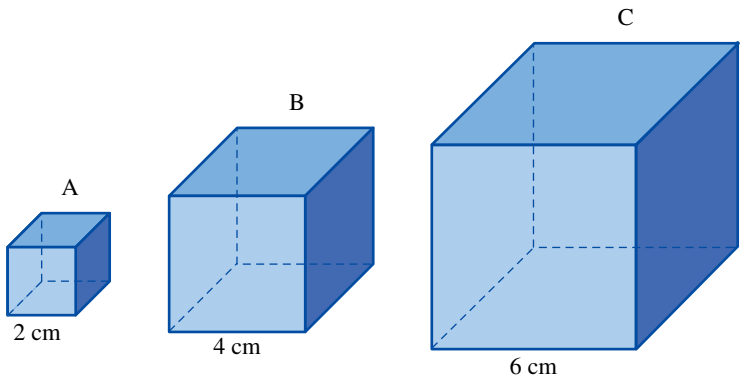
Surface Area $= 6 \times 36$
$= 216 \text{ cm}^2$

The scale factors for the side lengths and the scale factors for the surface areas are calculated below.

Cubes	Scale factor for side length	Scale factor for surface area
A and B	$\frac{4}{2} = 2$	$\frac{96}{24} = 4 = 2^2$
A and C	$\frac{6}{2} = 3$	$\frac{216}{24} = 9 = 3^2$
B and C	$\frac{6}{4} = \frac{3}{2}$	$\frac{216}{96} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

5.5.5 Volume of similar figures

- If the side lengths in any solid are increased by a scale factor of n, then the volume of similar solids increases by a scale factor of n^3.
- Once again consider the cubes shown earlier. The scale factors for the side lengths and the scale factors for the volumes are calculated below.



Volume $= 2 \times 2 \times 2$
$= 8 \text{ cm}^3$

Volume $= 4 \times 4 \times 4$
$= 64 \text{ cm}^3$

Volume $= 6 \times 6 \times 6$
$= 216 \text{ cm}^3$

Cubes	Scale factor for side length	Scale factor for volume
A and B	$\frac{4}{2} = 2$	$\frac{64}{8} = 8 = 2^3$
A and C	$\frac{6}{2} = 3$	$\frac{216}{8} = 27 = 3^3$
B and C	$\frac{6}{4} = \frac{3}{2}$	$\frac{216}{64} = \frac{27}{8} = \left(\frac{3}{2}\right)^3$

WORKED EXAMPLE 14

The side lengths of a box have been increased by a factor of 3.
a Find the surface area of the new box if the original surface area is 94 cm^2.
b Find the volume of the new box if the original volume is 60 cm^3.

THINK

a 1 State the scale factor for side length used to produce the new box.

2 The scale factor for surface area is the square of the scale factor for length.

3 Calculate the surface area of the new box.

WRITE

a Scale factor for side length = 3

Scale factor for surface area
$= 3^2$
$= 9$

Surface area of new box
$= 94 \times 9$
$= 846 \text{ cm}^2$

b 1 The scale factor for volume is the cube of the scale factor for length.

b Scale factor for volume
$= 3^3$
$= 27$

2 Calculate the volume of the new box.

Volume of new box
$= 60 \times 27$
$= 1620 \text{ cm}^3$

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Try out this interactivity: Similar figures (int-2768)

Exercise 5.5 Area and volume of similar figures

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 4, 6, 8, 13	Questions: 1–4, 6, 8, 13–16	Questions: 1, 3, 5–18

Individual pathway interactivity: int-4497

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE14a The side lengths of the following shapes have all been increased by a factor of 3. Copy and complete the following table.

	Original surface area	Enlarged surface area
a.	100 cm^2	
b.	7.5 cm^2	
c.	95 mm^2	
d.		918 cm^2
e.		45 m^2
f.		225 mm^2

2. WE14b The side lengths of the following shapes have all been increased by a factor of 3. Copy and complete the following table.

	Original volume	Enlarged volume
a.	200 cm^3	
b.	12.5 cm^3	
c.	67 mm^3	
d.		2700 cm^3
e.		67.5 m^3
f.		27 mm^3

3. A rectangular box has a surface area of 96 cm^2 and volume of 36 cm^3. Find the volume and surface area of a similar box that has side lengths double the size of the original.

Understanding

4. The area of a bathroom on a house plan is 5 cm^2. Find the area of the bathroom if the map has a scale of 1 : 100.

5. The area of a kitchen is 25 m^2.
- **a.** Change 25 m^2 to cm^2.
- **b.** Find the area of the kitchen on a plan if the scale of the plan is 1 : 120. (Give your answer correct to 1 decimal place.)

6. The volume of a swimming pool from its construction plan is calculated to be 20 cm^3. Find the volume of the pool if the plan has a scale of 1 : 75.

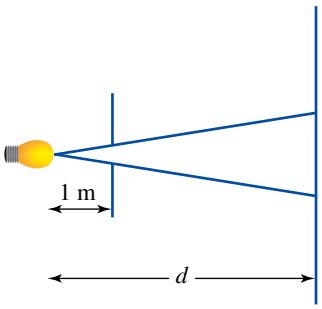
7. The total surface area of the wings on a 747 aircraft is 120 m^3.
- **a.** Change 120 m^2 to cm^2.
- **b.** Find the total surface area of the wings on a scale model built using the scale 1 : 80.



Reasoning

8. A cube has a surface area of 253.5 cm^2. (Give answers correct to 1 decimal place where appropriate.)
- **a.** Show that the side length of the cube is 6.5 cm.
- **b.** Show that the volume of the cube is 274.625 cm^3.
- **c.** Find the volume of a similar cube that has side lengths twice as long.
- **d.** Find the volume of a similar cube that has side lengths half as long.
- **e.** Find the surface area of a similar cube that has side lengths one third as long.

9. In the diagram below a light is shining through a hole, resulting in a circular bright spot with a radius of 5 cm on the screen. The hole is 10 mm wide. If the light is 1 m behind the hole, show that the light is 10 m from the screen.



10. A triangle ABC maps to triangle A′B′C′ under an enlargement, AB = 7 cm, AC = 5 cm, A′B′ = 21 cm, B′C′ = 30 cm.
- **a.** Show that the scale factor for enlargement is 3.
- **b.** Find BC.
- **c.** Find A′C′.
- **d.** If the area of ΔABC is 9 cm^2, show that the area of ΔA′B′C′ is 81 cm^2.

11. Two rectangles are similar. If the width of one rectangle is twice of width of the other, prove that the ratio of their areas is 4 : 1.

12. A pentagon has an area of 20 cm^2. If all the side lengths are doubled, show that the area of the enlarged pentagon is 80 cm^2.

Problem solving

13. **a.** Calculate the areas of squares with sides 2 cm, 5 cm, 10 cm and 20 cm.
b. State in words how the ratio of the areas is related to the ratio of the side lengths.

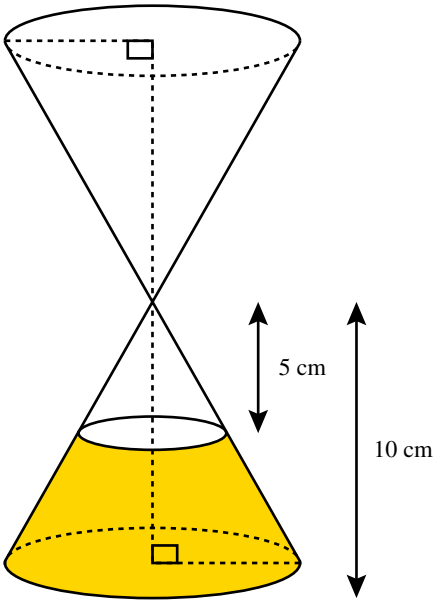
14. The areas of two similar trapeziums are 9 and 25. What is the ratio of a pair of corresponding side lengths?

15. Two cones are similar. The ratio of volumes is 27 : 64. Find the ratio of:
a. the perpendicular heights
b. the areas of the bases.

16. Rectangle A has the dimensions 5 by 4, rectangle B has the dimensions 4 by 3 and rectangle C has the dimensions 3 by 2.4.
a. Which rectangles are similar? Explain.
b. Find the area scale factor for the similar rectangles.

17. A balloon in the shape of a sphere has an initial volume of $840\,cm^3$ and is increased to a volume of $430080\,cm^3$. What is the increase in the radius of the balloon?

18. Part of an egg timer in the shape of a cone has sand poured into it as shown in the diagram. Find the ratio of the volume of sand in the cone to the volume of empty space in the bottom half of the egg timer.



Reflection

Why does a side length scale factor of 5 result in a volume that is increased by a scale factor of 125?

5.6 Review

5.6.1 Review questions

Fluency

1. Express each of the following in simplest form.
a. 8 : 16 **b.** 24 : 16 **c.** 27 : 18 **d.** 56 : 80 **e.** 8 : 20 **f.** 49 : 35

2. There are 9 girls and 17 boys in a Year 9 maths class. The ratio of boys to girls is:
A. 9 : 17 **B.** 17 : 9 **C.** 17 : 26 **D.** 9 : 26

3. Jan raised $15 for a charity fundraising event, while her friend Lara raised $25. Determine the ratio of:
a. the amount Jan raised to the amount Lara raised
b. the amount Jan raised to the total amount raised by the pair
c. the amount the pair raised to the amount Lara raised.

4. A pigeon breeder has 45 pigeons. These include 15 white, 21 speckled pigeons, and the rest grey. Determine the ratio of:
a. white to grey pigeons
b. grey to speckled pigeons
c. white pigeons to the total number of pigeons.

5. Express each of the following ratios in simplest form.
a. $4\frac{1}{2} : 1$ **b.** $6\frac{1}{8} : 9$ **c.** $7\frac{1}{4} : 3\frac{1}{2}$ **d.** 8.4 : 7.2 **e.** 0.2 : 2.48 **f.** 6.6 : 0.22

6. Jack has completed 2.5 km of a 4.5 km race. The ratio of distance completed to remaining distance is:
A. 9 : 5 **B.** 4 : 5 **C.** 4 : 9 **D.** 5 : 4

7. If $x : 16 = 5 : 4$, the value of x is:
A. 1 **B.** 4 **C.** 16 **D.** 20

8. If $40 : 9 = 800 : y$, the value of y is:

A. 20 **B.** 40 **C.** 180 **D.** 450

9. Determine the value of the pronumeral in each of the following.

a. $a : 15 = 2 : 5$ **b.** $b : 20 : 5 : 8$ **c.** $9 : 10 = 12 : c$

d. $11 : 9 = d : 5$ **e.** $7 : e = 4 : 5$ **f.** $3 : 4 = 8 : f$

10. $\Delta PQR \sim \Delta DEF$. Determine the length of the missing side in each of the following combinations.

a. PQ = 10 cm DE = 5 cm PR = 6 cm Find DF

b. PQ = 4 cm DE = 12 cm QR = 5 cm Find EF

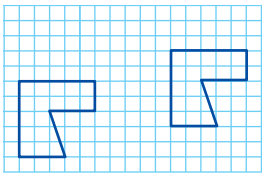
c. DE = 4 cm PQ = 6 cm EF = 8 cm Find QR

d. DF = 5 cm PR = 8 cm DE = 6 cm Find PQ

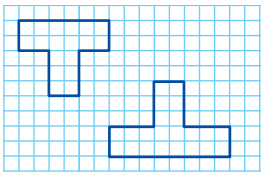
e. QR = 16 cm EF = 6 cm PQ = 12 cm Find DE

11. Which of the following pairs of shapes are congruent?

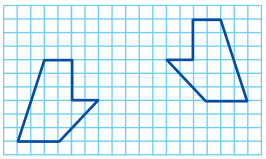
a.



b.

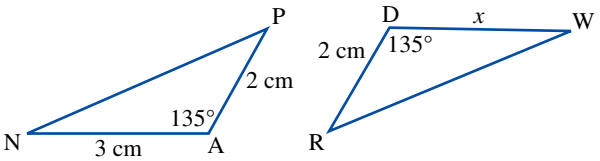


c.

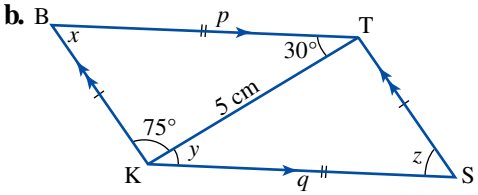


12. Name the congruent triangles in these figures and find the value of the pronumerals in each case.

a.

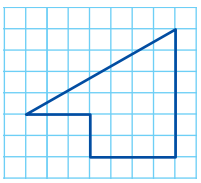


b.



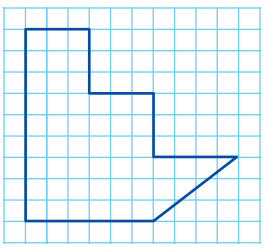
13. Copy each of the following shapes and enlarge (or reduce) them by the given factor.

a.



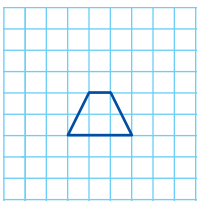
Enlarge by a factor of 2.

b.



Reduce by a factor of 3.

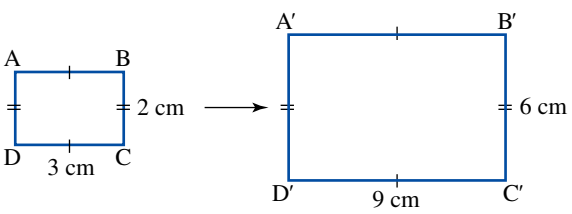
c.



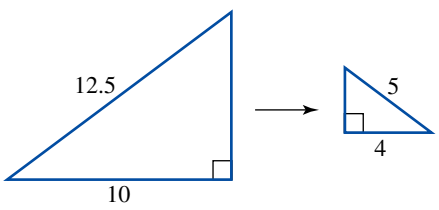
Enlarge by a factor of 4.

14. Determine the enlargement factors that have been used on the following shapes.

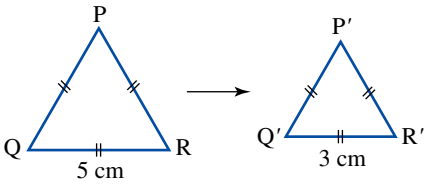
a.



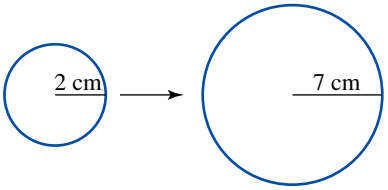
b.



c.

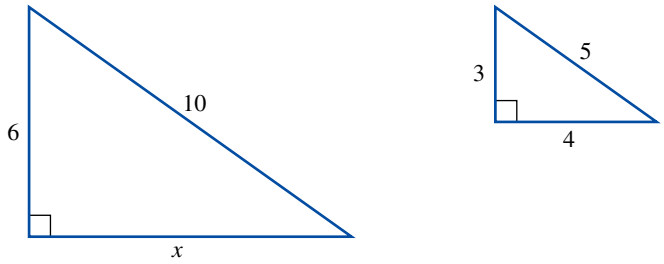


d.

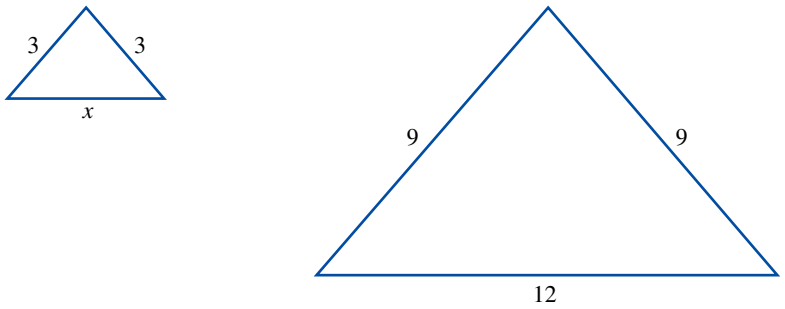


15. Each of the diagrams below shows a pair of similar triangles. Find the value of x in each case.

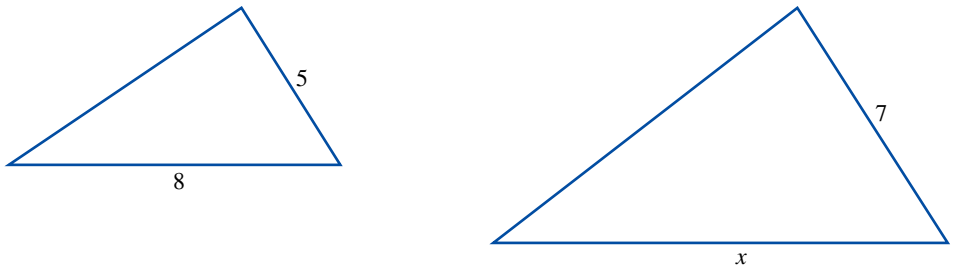
a.



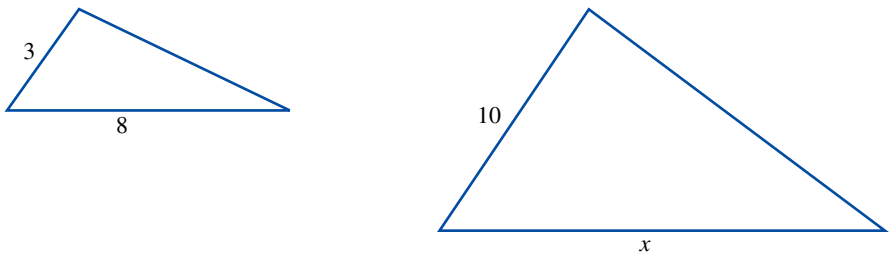
b.



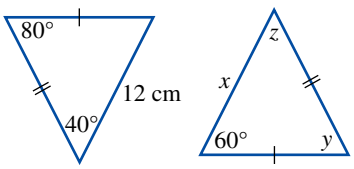
c.



d.



16. Find the value of the pronumerals in the pair of congruent triangles shown.

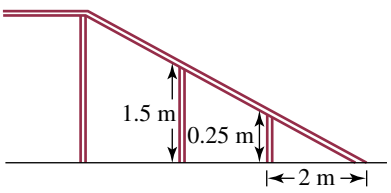


17. The area of a family room is 16 m^2 and the length of the room is 6.4 m. Find the area of the room on a plan that uses a scale of 1 : 20.

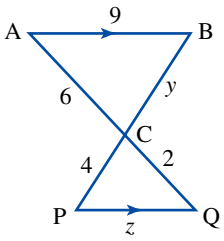
Problem solving

18. The diagram at right shows a ramp made by Josef for his automotive class. The first post has a height of 0.25 m and is placed 2 m from the end of the ramp.

If the second post is 1.5 m high, how far should it be placed from the first post?



19. a. Write a formal proof for a similarity relationship between two triangles in this figure. Give a reason for the similarity.

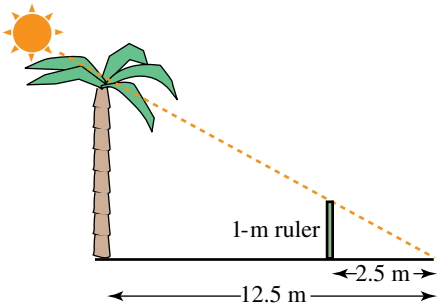


b. Determine the values of the pronumerals.

20. *Slocum* is a yacht whose length is 12 m and beam (width) is 2.50 m. A model is constructed for a maritime museum. If the length and width of the model are one fifth of the original length and width, how do their volumes compare?

21. Prior to the start of a yacht race, the judging official must certify that all of the sails are the same. Without unrigging the triangular sails from their masts, which congruency rule can the official most efficiently use to determine if the sails on each of the boats are exactly the same? Explain why this is the most appropriate rule.

22. Rachel is given a 1-m ruler and asked to estimate the height of a palm tree. She places the ruler vertically so that the ruler's shadow ends at exactly the same point as that of the palm tree. The ruler's shadow is 2.5 m long, and the palm shadow is 12.5 m long.



Rachel performed some calculations using similar triangles and calculated the height of the palm tree to be 4 m. Her friend Sandy said that her calculations were incorrect, and that the answer should be 5 m.

a. Which is the correct answer?

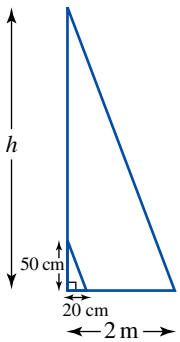
b. Explain the error one of the girls made in her calculations.

23. Calculate the height of the top of the ladder in the photo at right using similar triangles. Give your answer correct to 1 decimal place.

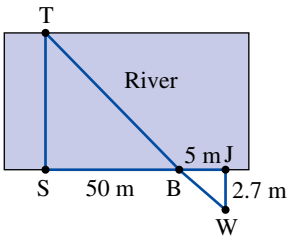


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24. A flag stick casts a shadow 2 m long. If a 50 cm ruler is placed in the same upright position and it casts a shadow 20 cm long, what is the height of the flag stick?



25. A student (S) uses a tape measure, backpack (B), water bottle (W), tree (T) and jacket (J) to help calculate the width of a river.



a. Copy and complete the following:

i. ∠____ and ∠____ are both right angles____ .

ii. ∠*SBT* and ∠____ are equal because____ .

iii. ∠*STB* and ∠____ are equal because____ .

b. Rewrite the following statement, selecting the correct alternative from within the brackets.
$\angle STB$ and $\angle JWB$ are (similar/congruent) (SSS/SAS/ASA/AAA/RHS).

c. Copy and complete the following equation.

$$\frac{\overline{ST}}{\overline{JW}} = \square$$

d. Use your statement from **c** to calculate the width of the river.

learnON RESOURCES — ONLINE ONLY

- **Try out this interactivity:** Word search: Topic 5 (int-2692)
- **Try out this interactivity:** Crossword: Topic 5 (int-2693)
- **Try out this interactivity:** Sudoku: Topic 5 (int-3205)
- **Complete this digital doc:** Concept map: Topic 5 (doc-10796)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

alternate angles
congruent figures
corresponding sides
enlargement
image
object
proportion
ratio
reduction
reflection
rotation
scale factor
similar figures
translation
vertices

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Investigation | Rich task

What's this object?



When using geometrical tools to construct shapes, we have to ensure that the measurements taken are precise. Small errors in each step of the measuring process can result in the creation of an incorrect shape. The object that you have to create for this task is one that has generated a great deal of interest over the years. It is made by combining three congruent shapes. Instructions to make the congruent shape are as follows.

Part 1 The congruent shape

1. Using a ruler, protractor, pencil and a pair of compasses, follow the instructions below to construct the first part of the object. Draw the shape in your workbook.
- Measure a horizontal line AB that is 9.5 cm long. To ensure that there is enough space for the entire object, draw the line close to the bottom of the space.
- At point B, construct an angle of 120° above the line AB. Extend the line to point C, making BC 2 cm long.
- At point C, construct an angle of 60° on the same side of the line BC as point A. Measure the line CD to be 7.5 cm long.
- At point D and above the line DC, construct an angle of 60°. Measure the length DE to be 3.5 cm.
- On the line DE, at point E, construct an angle of 120° above the line DE. Let the measurement from E to F be 2 cm. Join point F to point A.

2. What is the length of the line joining point A to point F?

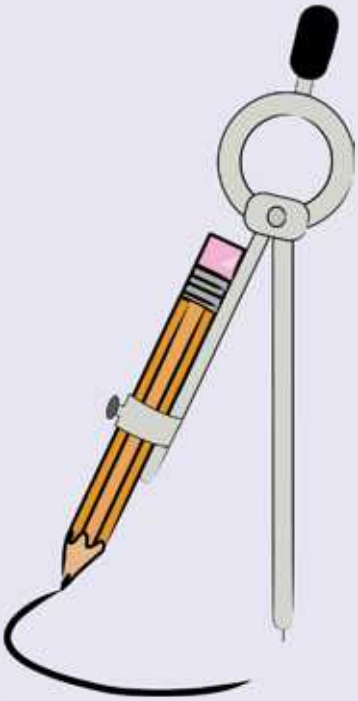
3. What do you notice about the size of the angles FAB and AFE?

4. Shade the shape with any colour you wish.

Part 2 The object

You have now constructed the shape that is to be used three times to make the final shape. To do this, follow the instructions below.
- **Trace the shape onto a piece of tracing paper twice and cut around the edges. Label the shapes with the letters used in its construction.**
- **Place the line AF of your trace against the line CD of your drawn shape. Reproduce this shape on your object. Shade this section using a different colour.**
- **Place the line DC of your second trace against the line FA of your drawn shape. Reproduce this shape on your object. Again, shade this section with a different colour.**

5. Describe the object you have created.
6. Using the internet, library or other references, investigate other impossible objects drawn as two-dimensional shapes. Recreate them on a separate sheet of paper and include reasons why they are termed impossible.



learnon RESOURCES — ONLINE ONLY

Complete this digital doc: Code puzzle: Why does a giraffe have a long neck? (doc-15895)

Answers

Topic 5 Congruence and similarity

Exercise 5.2 Ratio and scale

1. a. i. 240 : 7 **ii.** 7 : 240

b. The track is $34\frac{2}{7}$ times as long as it is wide.

2. a. i. 5 : 2 **ii.** 2 : 5

b. The cliff is 2.5 times as high as the distance from the base of the cliff to the emu.

3. a. 2 : 3 **b.** 1 : 7 **c.** 1 : 3 **d.** 2 : 5

e. 4 : 3 **f.** 4 : 15 **g.** 40 : 31 **h.** 36 : 11

i. 8 : 25 **j.** 100 : 33

4. a. 6 : 1 **b.** 13 : 15 **c.** 4 : 1 **d.** 5 : 1

e. 1 : 2 **f.** 20 : 3 **g.** 1 : 6 **h.** 5 : 3

i. 5 : 1 **j.** 11 : 5 **k.** 43 : 20 **l.** 1 : 200

5. a. 3 : 4 **b.** 8 : 7 **c.** 2 : 3 **d.** 28 : 25

e. 9 : 2 **f.** 2 : 7 **g.** 10 : 3 **h.** 33 : 16

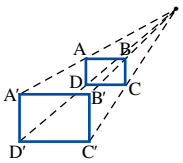
i. 23 : 15 **j.** 16 : 65

6. a. $a = 9$ **b.** $b = 24$ **c.** $c = 32$ **d.** $d = 1\frac{5}{7}$

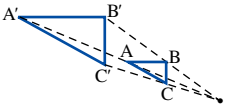
e. $e = 3\frac{3}{4}$ **f.** $f = 14\frac{2}{5}$ **g.** $g = 3\frac{3}{4}$ **h.** $h = 4\frac{4}{9}$

i. $i = 29\frac{1}{3}$ **j.** $j = 8\frac{4}{7}$ **k.** $k = 10\frac{2}{5}$ **l.** $l = 11\frac{1}{4}$

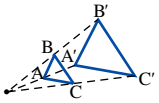
7. a.



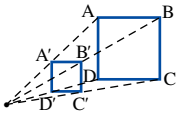
b.



c.



d.



8. a. 3 **b.** 12 cm **c.** 3.5 cm

9. $\frac{1}{2}$

10. a. 175 : 6428 **b.** 37

11. 2.5 mL

12. 1000 g

13. a. 200

b. i. 6 m × 6 m **ii.** 5 m × 5 m

14. a. Check with your teacher. **b.** 1.69 g gold, 0.5 g copper, 0.06 g silver

15. $3 + 4 + 5 = 12$
$180 \div 12 = 15$
$3 \times 15 = 45$; $4 \times 15 = 60$; $5 \times 15 = 75$
The three angles are 45°, 60° and 75°.

16. $2k \times 3k \times 5k = 30k^3$
$30k^3 = 21\,870$
$k^3 = 729$
$k = 9$
Substituting k into the ratio $(2k : 3k : 5k)$, the dimensions are 18 cm, 27 cm and 45 cm.

17. The profits aren't shared in a fair ratio. Tyler gets more profit than his share and Dylan gets less profit than his share. Only Aaron gets the correct share of the profit.

18. A pen costs twice as much as a pencil.

2 sharpeners cost the same as 3 pencils.

4 sharpeners cost the same as 3 pens.

Pen : pencil : sharpener = 1 : 2 : 3

19. 680 cm

20. $y : z = 9 : 8$

21. 49 students

22. $P = 4$, $Q = 6$

23. 125 students

24. 12 : 1

Exercise 5.3 Congruent figures

1. a. **ii** and **iii** **b.** **i** and **iii** **c.** **i** and **iv**

2. D

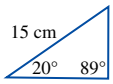
3. a. ΔABC and ΔPQR, SAS **b.** ΔABC and ΔLNM, SSS **c.** ΔLMN and ΔPQR, SAS
d. ΔABC and ΔPQR, ASA **e.** ΔABC and ΔLMN, RHS **f.** ΔABC and ΔDEF, ASA

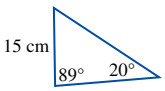
4. a. $x = 3$ cm **b.** $x = 85°$ **c.** $x = 80°$, $y = 30°$, $z = 70°$
d. $x = 30°$, $y = 7$ **e.** $x = 40°$, $y = 50°$, $z = 50°$, $n = 90°$, $m = 90°$

5. a. 2 cm **b.** 8 cm **c.** 6 mm **d.** 7 mm

6. Yes, because they are identical.

7. a. SSS **b.** SSS **c.** SAS

8. 

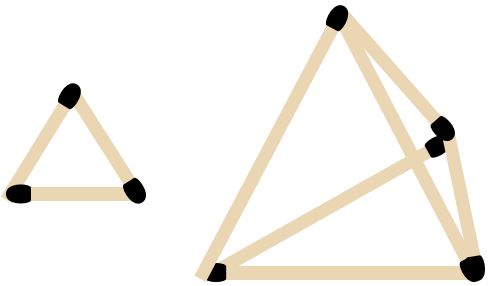
and


9. Check with your teacher.

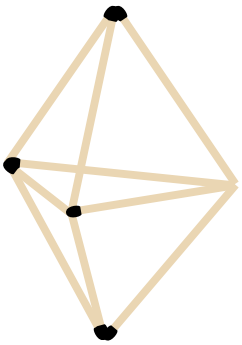
10. No; it could be ASA.

11. Because the angle is not between the two given sides, the general shape of the triangle is not set; therefore, many shapes are possible.

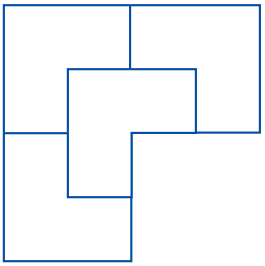
12. This can be done with an equilateral triangle and a regular tetrahedron.



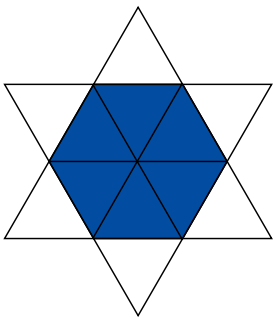
13. This can be done with a double regular tetrahedron.



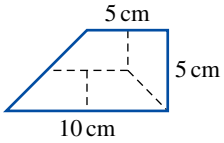
14. Each piece is similar to the original shape.



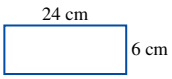
15. 1 : 1



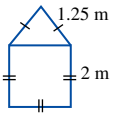
Challenge 5.1



Exercise 5.4 Similar figures

1. a.


b.


c.


2. a. **i** and **iii**, AAA **b.** **i** and **ii**, SSS **c.** **i** and **ii**, SAS
d. **i** and **iii**, RHS **e.** **i** and **iii**, SSS

3. a. Triangles ABC and DEC **b.** Triangles PQR and ABC **c.** Triangles PQR and TSR
d. Triangles ABC and DEC **e.** Triangles ADB and ADC

4. a. ΔEDC

5. Answers may vary due to inconsistencies in measurement.
a. i. 1.3 cm **ii.** 2.6 cm
b. 2

6. a. $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ **b.** $f = 9$, $g = 8$

7. a. $h = 3.75$, $i = 7.5$ **b.** $j = 2.4$, $k = 11.1$

8. $x = 4$

9. $x = 20°$, $y = 2\frac{1}{4}$

10. $x = 3$, $y = 4$

11. Check with your teacher.

12. a. $x = 7.1$ **b.** $x = 3.1$ **c.** $x = 7.5$, $y = 7.7$

13. Answers will vary.

14. Answers will vary.

15. Answers will vary.

16. a. T **b.** F **c.** F **d.** T

17. a. 6.72 m **b.** 78 cm

18. 3.75 m

19. $a = 1$ cm

20. a. Answers will vary. **b.** SAS **c.** 4 cm **d.** PR 6 cm, PS 5 cm and PQ 10 cm

21. a. ΔEDC and ΔEBA **b.** $x = 3$ cm, $y \approx 3.29$ cm

22. The triangles are similar (AAA); $l = \frac{y}{2}$

23. 2 : 3 or 3 : 2

Challenge 5.2

Ruby is 1.4 m tall.

Exercise 5.5 Area and volume of similar figures

1. a. 900 cm^2 **b.** 67.5 cm^2 **c.** 855 mm^2 **d.** 102 cm^2 **e.** 5 m^2 **f.** 25 mm^2

2. a. 5400 cm^3 **b.** 337.5 cm^3 **c.** 1809 mm^3 **d.** 100 cm^3 **e.** 2.5 m^3 **f.** 1 mm^3

3. SA = 384 cm^2, V = 288 cm^3

4. 50000 cm^2

5. a. 250000 cm^2 **b.** 17.4 cm^2

6. 8437500 cm^3

7. a. 1200000 cm^2 **b.** 187.5 cm^2

8. a. Answers will vary. **b.** Answers will vary. **c.** 2197 cm^3

d. 34.3 cm^3 **e.** 28.2 cm^2

9. Answers will vary.

10. a. Answers will vary. **b.** 10 cm **c.** 15 cm **d.** Answers will vary.

11. Answers will vary.

12. Answers will vary.

13. a. 4 cm^2, 25 cm^2, 100 cm^2, 400 cm^2

b. The ratio of the areas is equal to the square of the ratio of the side lengths.

14. 3 : 5

15. a. 3 : 4 **b.** 9 : 16

16. a. A and C are similar rectangles by the ratio 5 : 3 or scale factor $\frac{5}{3}$.

b. $\frac{25}{9}$

17. The new radius is 8 times the old radius.

18. The ratio is 7 : 1.

5.6 Review

1. a. 1 : 2 **b.** 3 : 2 **c.** 3 : 2 **d.** 7 : 10 **e.** 2 : 5 **f.** 7 : 5

2. B

3. a. 3 : 5 **b.** 3 : 8 **c.** 8 : 5

4. a. 5 : 3 **b.** 3 : 7 **c.** 1 : 3

5. a. 9 : 2 **b.** 49 : 72 **c.** 29 : 14 **d.** 7 : 6 **e.** 5 : 62 **f.** 30 : 1

6. D **7.** D **8.** C

9. a. $a = 6$ **b.** $b = 12.5$ **c.** $c = 13\frac{1}{3}$ **d.** $d = 6\frac{1}{9}$ **e.** $e = 8\frac{3}{4}$ **f.** $f = 10\frac{2}{3}$

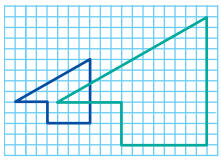
10. a. 3 cm **b.** 15 cm **c.** 12 cm **d.** 9.6 cm **e.** 4.5 cm

11. a. Congruent **b.** Not congruent **c.** Congruent

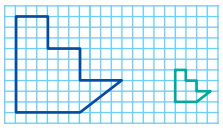
12. a. $\Delta ANP \cong \Delta DWR, x = 3$ cm

b. $\Delta TKB \cong \Delta KTS$ or $\Delta TKB \cong \Delta KST, x = 75°, y = 30°, z = 75°, p = 5$ cm, $q = 5$ cm

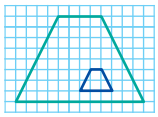
13. a.



b.



c.



14. a. 3 **b.** 0.4 **c.** $\frac{3}{5}$ **d.** 3.5

15. a. $x = 8$ **b.** $x = 4$ **c.** $x = 11\frac{1}{5}$ **d.** $x = 26\frac{2}{3}$

16. $x = 12$ cm, $y = 80°, z = 40°$

17. 400 cm^2

18. a. 10 m

19. a. $\Delta ABC \sim \Delta QPC$, AAA **b.** $y = 12, z = 3$

20. The model is $\frac{1}{125}$th the volume of the yacht.

21. The easiest way to check for congruency is to measure what is within reach. This would be the bottom of each sail and the angles at the bottom of each sail. Hence ASA would be the most appropriate rule to use.

22. a. 5 m

b. Sandy had the correct answer. Rachel used the distance of 10 m (from the ruler to the tree) in her calculations instead of 12.5 m (the whole length of the tree's shadow).

23. 2.1 m

24. 5 m

25. a. i. $\angle TSB$ and $\angle WJB$ are both right angles.

ii. $\angle SBT$ and $\angle JBW$ are equal because they are vertically opposite.

iii. $\angle STB$ and $\angle JWB$ are equal because they are alternate.

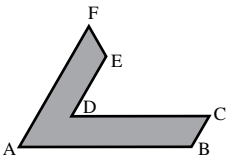
b. ΔSBT and ΔBJW are similar (AAA).

c. $\frac{\overline{ST}}{\overline{JW}} = \frac{50}{5}$

d. 27 m

Investigation — Rich task

1.



2. 7.5 cm

3. $\angle FAB = 60°$, $\angle AFE = 60°$

4. and **5.** The impossible triangle

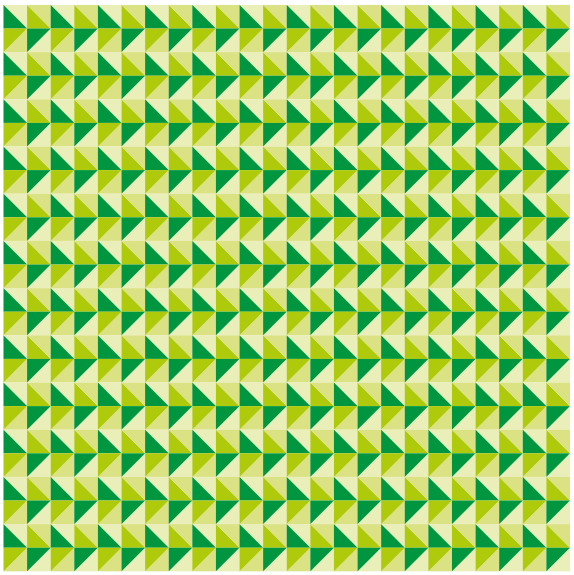


6. Teacher to check.

TOPIC 6
Pythagoras and trigonometry

6.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



6.1.1 Why learn this?

Pythagoras was a great mathematician and philosopher who lived in the 6th century BCE. He is best known for the theorem that bears his name. It concerns the relationship between the lengths of the sides in a right-angled triangle. Geometry and trigonometry are branches of mathematics where Pythagoras' theorem is still widely applied. Trigonometry is a branch of mathematics that allows us to relate the side lengths of triangles to angles. Combining trigonometry with Pythagoras' theorem allows us to solve many problems involving triangles.

6.1.2 What do you know?

assessON

1. **THINK** List what you know about trigonometry. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of trigonometry.

LEARNING SEQUENCE

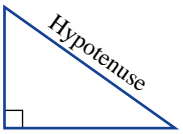
6.1 Overview
6.2 Pythagoras' theorem
6.3 Applications of Pythagoras' theorem
6.4 What is trigonometry?
6.5 Calculating unknown side lengths
6.6 Calculating unknown angles
6.7 Angles of elevation and depression
6.8 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: Secret society (eles-1693)

6.2 Pythagoras' theorem

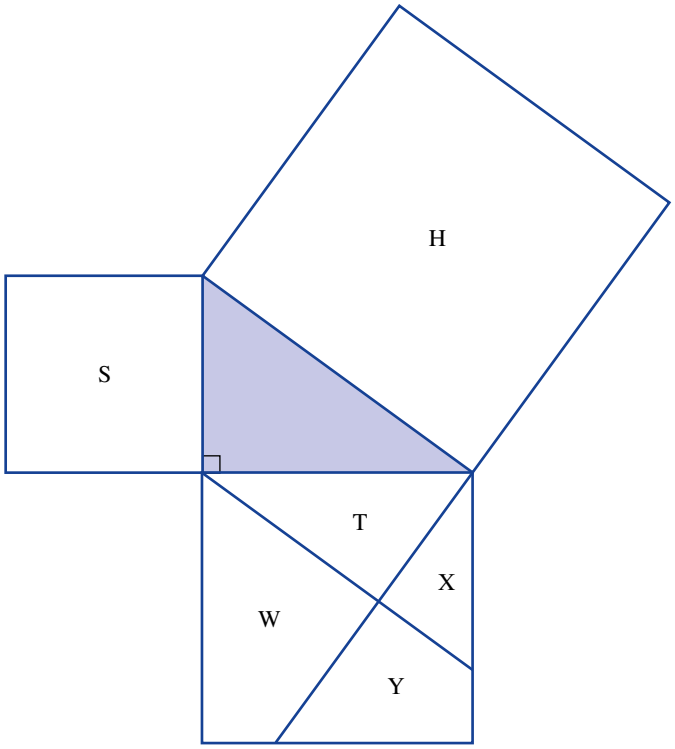
6.2.1 Right-angled triangles



- Triangles can be classified according to their side lengths or their angles.
- Equilateral, isosceles and scalene triangles are classified by the length of their sides.
- A **right-angled triangle** is classified by the type of angle it contains.
- The longest side of a right-angled triangle is opposite the right angle and is called the **hypotenuse**.
- In all triangles the longest side is opposite the largest angle.
- Pythagoras (580–501 BC) was a Greek mathematician who explored the relationship between the lengths of the sides of right-angled triangles.
- The relationship he described, and has been credited with discovering over 2500 years ago, is known as **Pythagoras' theorem**.

Activity

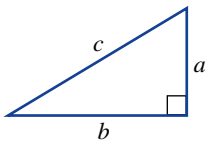
1. The diagram at right shows a right-angled triangle with squares drawn on each of its sides. Copy the figure onto a piece of paper.
2. Cut around the perimeter of the figure, then cut off the two smaller squares from the base and height of the right-angled triangle. Cut the larger of these two squares into the four pieces shown.
3. You should now have a right-angled triangle with a square H attached to its hypotenuse and five smaller pieces (S, T, W, X and Y). Rearrange these five pieces to fit exactly in the area H.
4. Since the squares from the two shorter sides of the right-angled triangle fit exactly into the square on the hypotenuse, what can you conclude?



6.2.2 Pythagoras' theorem

- Pythagoras investigated the relationship between the lengths of the sides of a right-angled triangle. This is now known as Pythagoras' theorem.

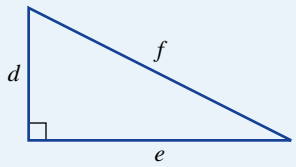
- The theorem states that:
  In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is:
  $$c^2 = a^2 + b^2$$
  where a and b are the two shorter sides and c is the hypotenuse.
- If the lengths of two sides of a right-angled triangle are given, then Pythagoras' theorem enables the length of the unknown side to be found.



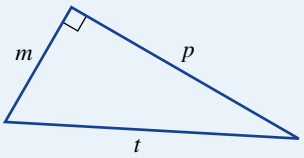
WORKED EXAMPLE 1

For the right-angled triangles shown below:
i state which side is the hypotenuse
ii write Pythagoras' theorem.

a



b

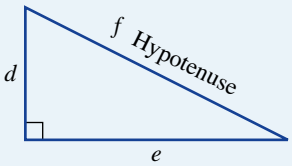


THINK

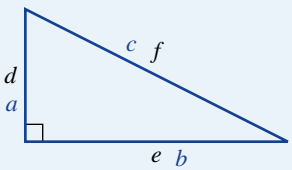
a i The hypotenuse is always opposite the right angle.

WRITE/DRAW

Side f is opposite the right angle. Therefore, side f is the hypotenuse.

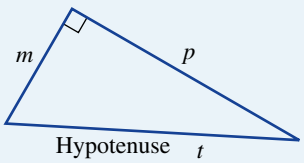


ii If the triangle is labelled as usual with a, b and c, as shown in blue, Pythagoras' theorem can be written and then the letters can be replaced with their new values (names).



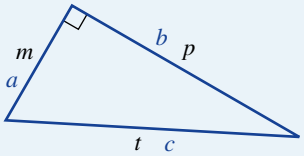
$c = f; a = d; b = e$
$c^2 = a^2 + b^2$
$f^2 = d^2 + e^2$

b i The hypotenuse is opposite the right angle.



Side t is opposite the right angle. Therefore, side t is the hypotenuse.

ii If the triangle is labelled as usual with a, b and c, as shown in blue, Pythagoras' theorem can be written and then the letters can be replaced with their new values (names).



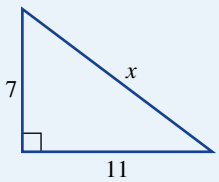
$c = t; b = p; a = m$
$c^2 = a^2 + b^2$
$t^2 = m^2 + p^2$

6.2.3 Finding the hypotenuse

- To calculate the length of the hypotenuse given the length of the two other shorter sides, a and b, substitute the values of a and b into the rule $c^2 = a^2 + b^2$ and solve for c.
- Remember that the hypotenuse is the longest side, so the value of c must be *greater than* that of either a or b.

WORKED EXAMPLE 2

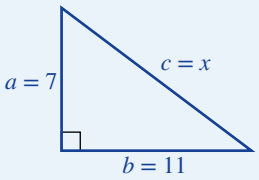
For the triangle at right, calculate the length of the hypotenuse, x, correct to 1 decimal place.



THINK

1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

WRITE/DRAW



2 Write Pythagoras' theorem.

$c^2 = a^2 + b^2$

3 Substitute the values of a, b and c into this rule and simplify.

$x^2 = 7^2 + 11^2$
$= 49 + 121$
$= 170$

4 Calculate x by taking the square root of 170. Round the answer correct to 1 decimal place.

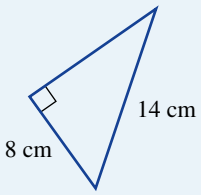
$x = \sqrt{170}$
$x = 13.0$

6.2.4 Finding the length of a shorter side

- When given both the hypotenuse and one other shorter side, either a or b, rearrange Pythagoras' theorem to find the value of the unknown shorter side, e.g., $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.
- Remember that the value of the unknown shorter side, either a or b, should be *less than* the value of the hypotenuse.

WORKED EXAMPLE 3

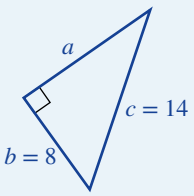
Calculate the length, correct to 1 decimal place, of the unmarked side of the triangle at right.



THINK

1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

WRITE/DRAW



2 Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
3 Substitute the values of b and c into this rule and simplify.	$a^2 = 14^2 - 8^2$ $= 196 - 64$ $= 132$
4 Find a by taking the square root of 132. Round to 1 decimal place.	$a = \sqrt{132}$ $= 11.5$ cm

6.2.5 Practical problems

- When using Pythagoras' theorem to solve practical problems, draw a right-angled triangle to represent the problem.
  - Identify the unknown variable by reading what the question is asking.
  - Identify the known values and substitute these into Pythagoras' theorem.
  - Solve for the unknown value.
  - Use the result to write the answer as a complete sentence.

WORKED EXAMPLE 4

Calculate the value of the missing side length. Give your answer correct to 1 decimal place.



THINK	WRITE
1 • Identify the length of the hypotenuse and the two shorter sides. • Check that the units for all measurements are the same. Convert 84 mm to centimetres by dividing by 10.	$c = 12$ cm $a = 84$ mm $= 8.4$ cm Let $x =$ the length of the missing side. $b = x$
2 Substitute the values into the equation and simplify.	$c^2 = a^2 + b^2$ $12^2 = (8.4)^2 + x^2$ $144 = 70.56 + x^2$
3 Solve the equation for x by: • subtracting 70.56 from both sides, as shown in red • taking the square root of both sides.	$144 - 70.56 = 70.56 - 70.56 + x^2$ $73.44 = x^2$ $\sqrt{73.44} = \sqrt{x^2}$ $\pm 8.5697 \approx x$
4 Answer the question. • Possible values for x are 8.5697 and -8.5697. It is not possible to have a length of -8.5697. • Round the answer to 1 decimal place. • Include the appropriate units in the answer. The calculations were carried out in centimetres. The answer will be in centimetres.	$x \approx 8.5697$ is the only valid solution as x is the length of a side in a triangle. It is not possible to have a side with a negative length. Therefore, the missing side length is 8.6 cm (correct to 1 decimal place).

6.2.6 Pythagorean triads

- A **Pythagorean triad** is a group of any three whole numbers that satisfy Pythagoras' theorem. For example, {5, 12, 13} and {7, 24, 25} are Pythagorean triads.
  $13^2 = 5^2 + 12^2$; $25^2 = 24^2 + 7^2$
- Pythagorean triads are useful when solving problems using Pythagoras' theorem. If two known side lengths in a triangle belong to a triad, the length of the third side can be stated without performing any calculations.
- Some well known Pythagorean triads are: {3, 4, 5}; {5, 12, 13}; {8, 15, 17}; {7, 24, 25}.

WORKED EXAMPLE 5

Determine whether the following sets of numbers are Pythagorean triads.
a **{9, 10, 14}** **b** **{33, 65, 56}**

THINK

a 1 Pythagorean triads satisfy Pythagoras' theorem. Substitute the values into the equation $c^2 = a^2 + b^2$ and determine whether the equation is true. Remember, c is the longest side.

2 State your conclusion.

b 1 Pythagorean triads satisfy Pythagoras' theorem. Substitute the values into the equation $c^2 = a^2 + b^2$ and determine whether the equation is true. Remember, c is the longest side.

2 State your conclusion.

WRITE

a $c^2 = a^2 + b^2$
LHS $= c^2$ RHS $= a^2 + b^2$
$= 14^2$ $= 9^2 + 10^2$
$= 196$ $= 81 + 100$
$= 181$
Since LHS $\neq$ RHS, the set {9, 10, 14} is not a Pythagorean triad.

b $c^2 = a^2 + b^2$
LHS $= 65^2$ RHS $= 33^2 + 56^2$
$= 4225$ $= 1089 + 3136$
$= 4225$
Since LHS = RHS, the set {33, 65, 56} is a Pythagorean triad.

- If each term in a triad is multiplied by the same number, the result is also a triad. For example, if we multiply each number in {5, 12, 13} by 2, the result {10, 24, 26} is also a triad.
- Builders and gardeners use multiples of the Pythagorean triad {3, 4, 5} to ensure that walls and floors are right angles.



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Try out this interactivity: Pythagorean triples (int-2765)

Complete this digital doc: SkillSHEET: Rearranging formulas (doc-11429)

Complete this digital doc: SkillSHEET: Converting units of length (doc-11430)

Complete this digital doc: SkillSHEET: Rounding to a given number of decimal places (doc-11428)

Exercise 6.2 Pythagoras' theorem

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 13, 16	Questions: 1–14, 16	Questions: 1–17

Individual pathway interactivity: int-4472

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

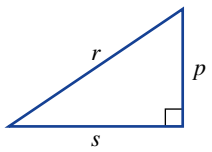
Fluency

1. WE1 For the right-angled triangles shown below:

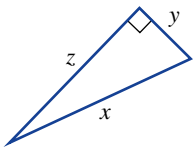
i. state which side is the hypotenuse

ii. write Pythagoras' theorem.

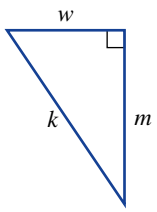
a.



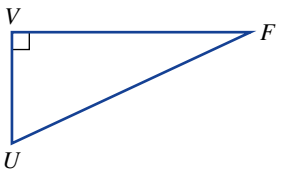
b.



c.

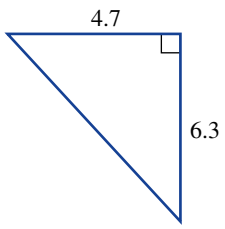


d.

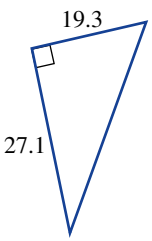


2. WE2 For each of the following triangles, calculate the length of the hypotenuse, giving answers correct to 2 decimal places:

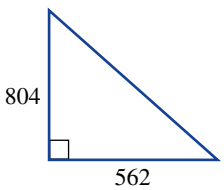
a.



b.

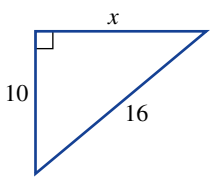


c.

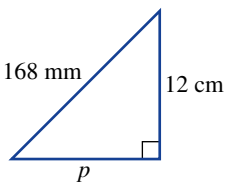


3. WE3 Calculate the value of the pronumeral in each of the following triangles. Give your answers correct to 2 decimal places.

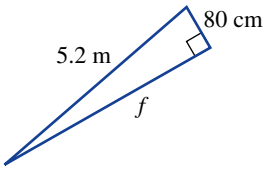
a.



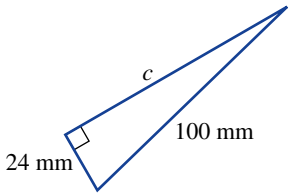
b.



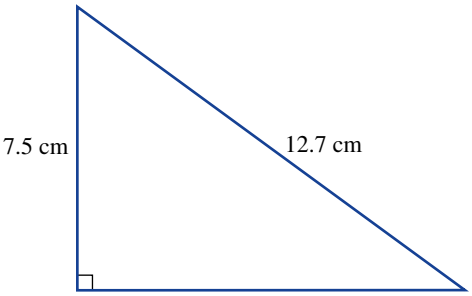
c.



d.



4. WE4 Calculate the value of the missing side length. Give your answer correct to 1 decimal place.



5. WE5 Determine whether the following sets of numbers are Pythagorean triads.

a. {2, 5, 6} **b.** {7, 10, 12} **c.** {18, 24, 30}
d. {72, 78, 30} **e.** {13, 8, 15} **f.** {50, 40, 30}

Understanding

6. In a right-angled triangle, the two shortest sides are 4.2 cm and 3.8 cm.

a. Draw a sketch of the triangle.

b. Calculate the length of the hypotenuse correct to 2 decimal places.

7. A right-angled triangle has a hypotenuse of 124 mm and another side of 8.5 cm.

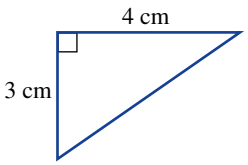
a. Draw a sketch of the triangle.

b. Calculate the length of the third side. Give your answer in millimetres correct to 2 decimal places.

8. MC Which of the following sets is formed from the triad {21, 20, 29}?

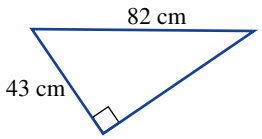
A. {95, 100, 125} **B.** {105, 145, 100} **C.** {84, 80, 87}
D. {105, 80, 87} **E.** {215, 205, 295}

9. MC What is the length of the hypotenuse in the triangle at right?



A. 25 cm **B.** 50 cm **C.** 50 mm
D. 500 mm **E.** 2500 mm

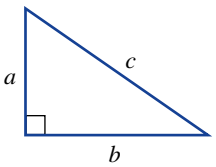
10. MC What is the length of the third side in this triangle?



A. 48.75 cm **B.** 0.698 m **C.** 0.926 m
D. 92.6 cm **E.** 69.8 mm

11. A ladder that is 7 metres long leans up against a vertical wall. The top of the ladder reaches 6.5 m up the wall. How far from the wall is the foot of the ladder, correct to 2 decimal places?

12. Two sides of a right-angled triangle are given. Find the third side in the units specified. The diagram at right shows how each triangle is to be labelled. Give your answers correct to 2 decimal places.



Remember: c is always the hypotenuse.

a. $a = 37$ cm, $c = 180$ cm; find b in cm.

b. $a = 856$ mm, $b = 1200$ mm; find c in cm.

c. $b = 4950\ m$, $c = 5.6$ km; find a in km.

d. $a = 125\,600$ mm, $c = 450$ m; find b in m.

Reasoning

13. An art student is trying to hang her newest painting on an existing hook in a wall. She leans a 1.2-m ladder against the wall so that the distance between the foot of the ladder and the wall is 80 cm.

a. Draw a sketch showing the ladder leaning on the wall.

b. How far up the wall does the ladder reach, correct to 2 decimal places?

c. The student climbs the ladder to check whether she can reach the hook from the step at the very top of the ladder. Once she extends her arm, the distance from her feet to her fingertips is 1.7 m. If the hook is 2.5 m above the floor, will the student reach it from the top step?

14. A rectangular park is 260 m by 480 m. Danny usually trains by running 5 circuits around the edge of the park. After heavy rain, two adjacent sides are too muddy to run along, so he runs a triangular path along the other two sides and the diagonal. Danny does 5 circuits of this path for training. Show that Danny runs about 970 metres less than his usual training session.

15. An adventure water park has hired Sally to build part of a ramp for a new water slide. She builds a ramp that is 12 m long and rises to a height of 250 cm. To meet the regulations, the ramp must have a gradient between 0.1 and 0.25. Show that the ramp Sally has built is within the regulations.

Problem solving

16. a. The smallest number of four Pythagorean triads is given below. Find the middle number and, hence, find the third number.

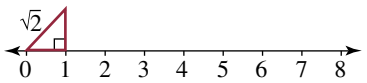
i. 9 **ii.** 11

iii. 13 **iv.** 29

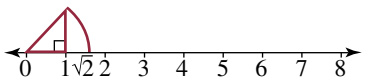
b. What do you notice about the triads formed in part **a**?

17. We know that it is possible to find the exact square root of some numbers but not others. For example, we can find $\sqrt{4}$ exactly but not $\sqrt{3}$ or $\sqrt{5}$. Our calculators can find decimal approximations of these, but because they cannot be found exactly they are called irrational numbers. There is a method, however, of showing their exact location on a number line.

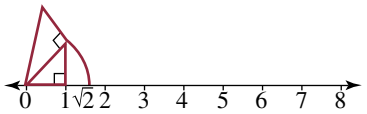
a. Using graph paper, draw a right-angled triangle with two equal sides of length 1 cm as shown below.



b. Using Pythagoras' theorem, we know that the longest side of this triangle is $\sqrt{2}$ units. Place the compass point at zero and make an arc that will show the location of $\sqrt{2}$ on the number line.



c. Draw another right-angled triangle using the longest side of the first triangle as one side, and make the other side 1 cm in length.



d. The longest side of this triangle will have a length of $\sqrt{3}$ units. Draw an arc to find the location of $\sqrt{3}$ on the number line.

e. Repeat steps **c** and **d** to draw triangles that will have sides of length $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ units and so on.

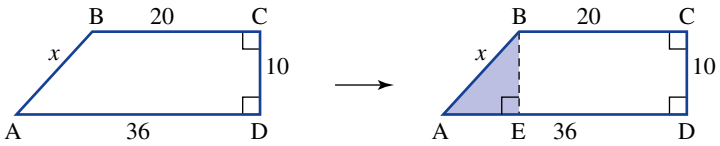
Reflection

What is the quickest way to identify whether a triangle is right-angled or not?

6.3 Applications of Pythagoras' theorem

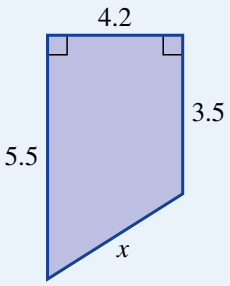
6.3.1 Using composite shapes to solve problems

- Dividing a composite shape into simpler shapes creates shapes that have known properties. For example, to calculate the value of x in the trapezium shown, a vertical line can be added to create a right-angled triangle and a rectangle. The length of x can be found using Pythagoras' theorem.



WORKED EXAMPLE 6

Calculate the value of x in the diagram below, correct to 1 decimal place.



THINK

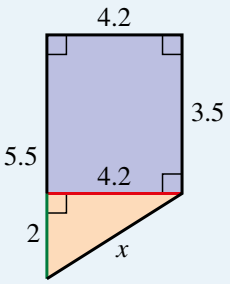
1 Divide the shape into smaller shapes that create a right-angled triangle, as shown in red.

2 x is the hypotenuse of a right-angled triangle. To use Pythagoras' theorem, the length of the side shown in green must be known. This length can be calculated as the difference between the long and short vertical edges of the trapezium: $5.5 - 3.5 = 2$.

3 Substitute the values of the side lengths into Pythagoras' theorem.

4 Answer the question.
- Possible values for x are 4.651 88 and −4.651 88. It is not possible for x to have a length of negative value.
- Round the answer to 1 decimal place.
- Include units if appropriate. There are no units.

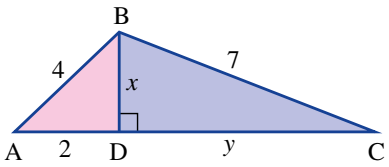
WRITE/DRAW



$c^2 = a^2 + b^2$
$x^2 = 2^2 + (4.2)^2$
$x^2 = 4 + 17.64$
$x^2 = 21.64$
$\sqrt{x^2} = \sqrt{21.64}$
$x \approx \pm 4.651\,88$

$x \approx 4.651\,88$ is the only valid solution, as x is the side length of a triangle.
Therefore $x = 4.7$ (correct to 1 decimal place).

- To find the value of y in the irregular triangle shown, the triangle can be split into two right-angled triangles: ΔABD (pink) and ΔBDC (purple). There is enough information to calculate the missing side length from ΔABD. This newly calculated length can be used to find the value of y.



WORKED EXAMPLE 7

a Calculate the perpendicular height of the isosceles triangle whose equal sides are each 15 mm long and whose third side is 18 mm long.
b Calculate the area of the triangle.

THINK

a 1 Draw the triangle and label all side lengths as described.

2 Draw an additional line to represent the height of the triangle, as shown in red, and label it appropriately. Since the triangle is an isosceles triangle, h bisects the base of 18 mm, as shown in green, and creates 2 right-angled triangles.

3 Focus on one right-angled triangle containing the height.

4 Substitute the values from this right-angled triangle into Pythagoras' theorem to calculate the value of h.

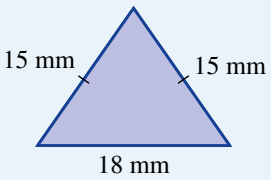
5 Answer the question.
- Possible values for h are 12 and −12. It is not possible for h to have a length of negative value.
- Include units if appropriate.

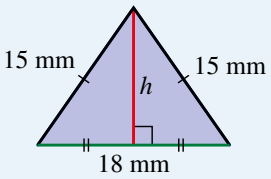
b 1 Write the formula for the area of a triangle. In this case the base is 18 mm and the height is 12 mm.

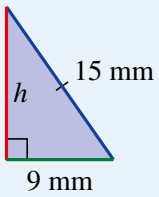
2 Answer the question. Include units if appropriate.

WRITE/DRAW

a







$$\begin{aligned} c^2 &= a^2 + b^2 \\ 15^2 &= h^2 + 9^2 \\ 225 &= h^2 + 81 \\ 225 - 81 &= h^2 + 81 - 81 \\ 144 &= h^2 \\ \sqrt{144} &= \sqrt{h^2} \\ \pm 12 &= h \end{aligned}$$

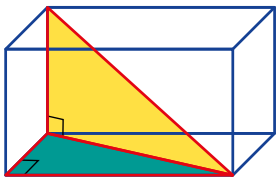
$h = 12$ is the only valid solution, since h represents the height of a triangle.
The height of the triangle is 12 mm.

b $A_\Delta = \frac{1}{2}bh$
$= \frac{1}{2} \times 18 \times 12$
$= 108$

The area of the triangle is 108 mm^2.

6.3.2 Pythagoras' theorem in 3-D

- Pythagoras' theorem can also be used to solve problems in three dimensions.
- In 3-D problems, often more than one right-angled triangle exists.
- It is often helpful to redraw sections of the 3-D diagram in 2 dimensions, allowing the right angles to be seen with greater clarity.



WORKED EXAMPLE 8

Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions 1 m × 1.5 m × 0.5 m.

THINK

1 Draw a diagram of a rectangular box with a rod in it, labelling the dimensions.

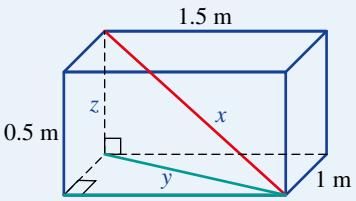
2 • Draw in a right-angled triangle that has the metal rod as one of the sides, as shown in red.
The length of y in this right-angled triangle is not known.
• Draw in another right-angled triangle, as shown in green, to calculate the length of y.

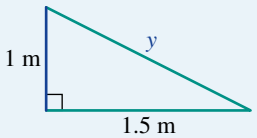
3 • Calculate the length of y using Pythagoras' theorem.
• Calculate the exact value of y.

4 Draw a right-angled triangle containing the rod and use Pythagoras' theorem to calculate the length of the rod (x).
Note: z = height of the crate = 0.5 m

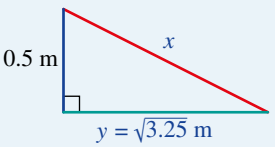
5 Answer the question.

WRITE/DRAW





$c^2 = a^2 + b^2$
$y^2 = 1.5^2 + 1^2$
$y^2 = 3.25$
$y = \sqrt{3.25}$



$c^2 = a^2 + b^2$
$x^2 = (\sqrt{3.25})^2 + 0.5^2$
$x^2 = 3.25 + 0.25$
$x^2 = 3.5$
$x = \sqrt{3.5}$
≈ 1.87

The maximum length of the metal rod is 1.87 m (correct to 2 decimal places).

Exercise 6.3 Applications of pythagoras' theorem

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 12, 14, 16, 19	Questions: 1–12, 14–16, 19, 20	Questions: 1–20

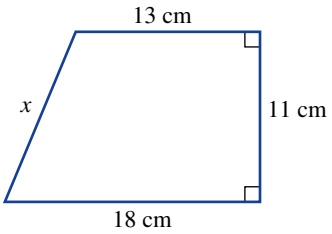
Individual pathway interactivity: int-4475

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

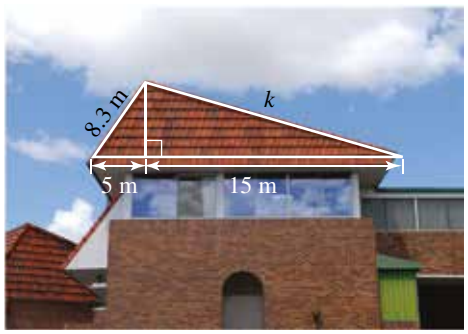
Fluency

1. WE6 Calculate the length of the side x in the figure below, correct to 2 decimal places.

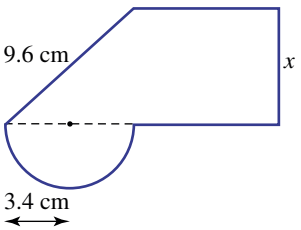


2. Calculate the values of the pronumerals in each of the following. Give your answers correct to 2 decimal places.

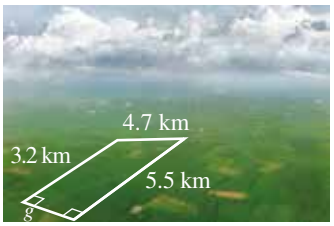
a.



b.

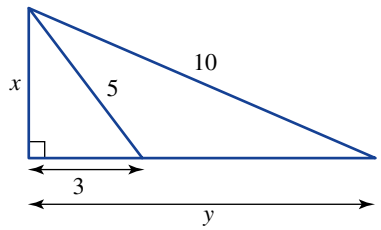


c.

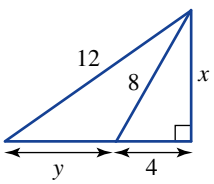


3. For each of the following diagrams, calculate the lengths of the sides marked x and y. Give your answer correct to 2 decimal places where necessary.

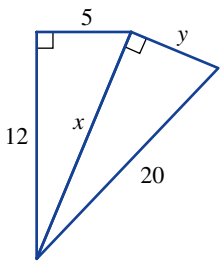
a.



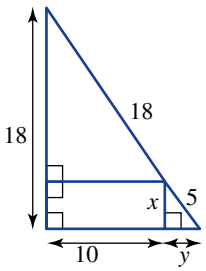
b.



c.

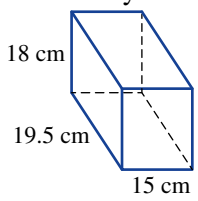


d.

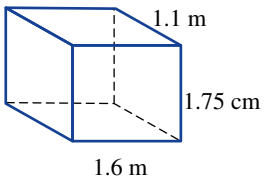


4. WE8 Calculate the length of the longest metal rod that can fit diagonally into each of the boxes shown below. Give your answers correct to 2 decimal place.

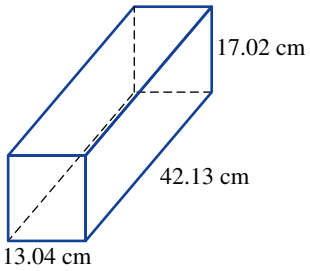
a.



b.



c.



Understanding

5. WE7 The height of an isosceles triangle is 3.4 mm and its equal sides are twice as long.
- **a.** Sketch the triangle, showing all the information given.
- **b.** Calculate the length of the third side to 2 decimal places.
- **c.** Calculate the area of the triangle to 2 decimal places.

6. The side length of an equilateral triangle is 1 m. Calculate:
- **a.** the height of the triangle in metres to 2 decimal places
- **b.** the area of the triangle in m^2 to 3 decimal places.

7. MC The longest length object that can fit into a box with dimensions 42 cm × 60 cm × 13 cm is:

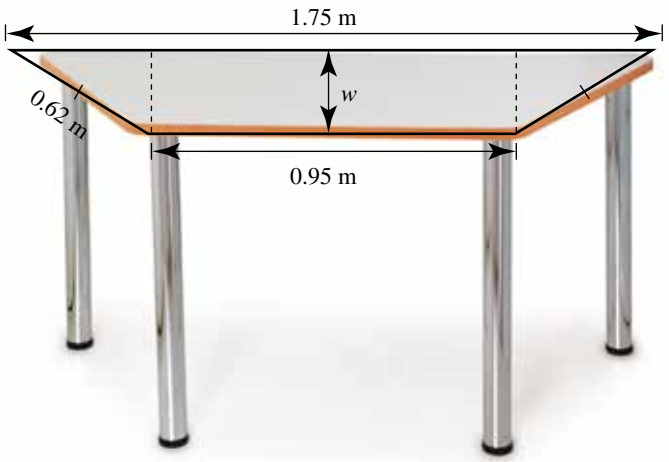
A. 74.5 cm **B.** 60 cm **C.** 73.2 cm **D.** 5533 cm **E.** 74 cm

8. A friend wants to pack an umbrella into her suitcase. If the suitcase measures 89 cm × 21 cm × 44 cm, will her 1-m umbrella fit in? Give the length (correct to 3 decimal places) of the longest object that will fit in the suitcase.

9. A friend is packing his lunch into his lunchbox and wants to fit in a 22-cm-long straw. If the lunchbox is 15 cm × 5 cm × 8 cm, will it fit in? What is the length (correct to 2 decimal places) of the longest object that will fit into the lunchbox?

10. A cylindrical pipe has a length of 2.4 m and an internal diameter of 30 cm. What is the largest object that could fit in the pipe? Give your answer correct to the nearest cm.

11. A classroom contains 20 identical desks like the one shown at right.
- **a.** Calculate the width of each desktop (labelled w) correct to 2 decimal places.
- **b.** Calculate the area of each desktop correct to 2 decimal places.
- **c.** If the top surface of each desk is to be painted, what is the total area that needs to be painted correct to 2 decimal places?
- **d.** What is the cost of the paint needed to finish the job if all desks are to be given two coats of fresh paint and paint is sold in 1-litre containers that cost $29.95 each and give coverage of 12 m^2 per litre?

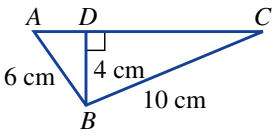


12. A hobby knife has a blade in the shape of a right-angled trapezium with the sloping edge 2 cm long and parallel sides 32 mm and 48 mm long. Calculate the width of the blade and, hence, the area.

13. A cylindrical pen holder has an internal height of 16 cm. If the diameter of the pen holder is 8.5 cm and the width of the pen holder is 2 mm, what is the largest pen that could fit completely inside the holder? Give your answer correct to the nearest mm.

Reasoning

14. Explain in your own words how a 2-D right-angled triangle can be seen in a 3-D figure.



15. Consider the figure shown at right.

a. In which order will you need to calculate the lengths of the following sides: AD, AC and DC?

b. Calculate the lengths of the sides in part **a** to 2 decimal places.

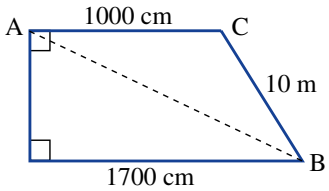
c. Is the triangle ABC right-angled? Use calculations to justify your answer.



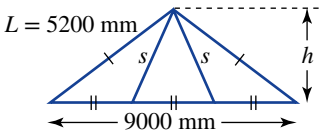
16. Katie goes on a hike and walks 2.5 km north, then 3.1 km east. She then walks 1 km north and 2 km west. She then walks in a straight line back to her starting point. Show that she walks a total distance of 12.27 km.

17. **a.** Show that the distance AB in the plan of the paddock below is 18.44 metres.

b. Prove that the angle ∠ABC is not a right angle.



18. The diagram below shows the cross-section through a roof.

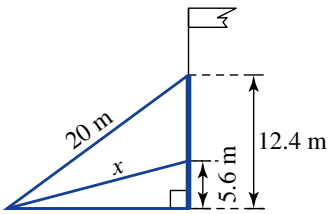


a. Calculate the height of the roof, h, to the nearest millimetre.

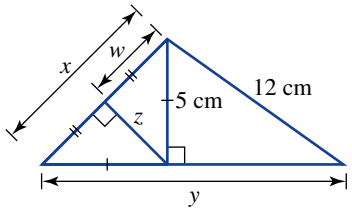
b. The longer supports, L, are 5200 mm long. Show that the shorter supports, s, are 2193 mm shorter than the longer supports.

Problem solving

19. A flagpole is attached to the ground by two wires as shown in the diagram at right. Use the information from the diagram to calculate the length of the lower wire, x, to 1 decimal place.



20. Calculate the values of the pronumerals w, x, y and z in the diagram below.



Reflection

Which simple shapes should you try to divide composite shapes up into?

6.4 What is trigonometry?

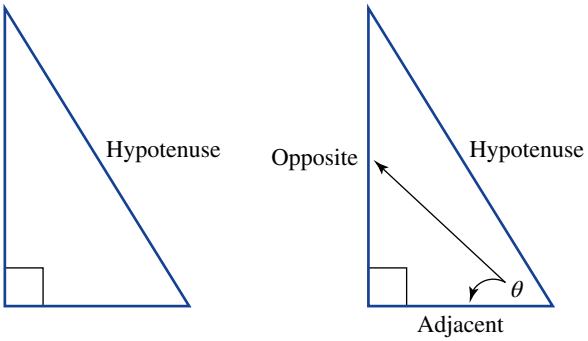


- The word **trigonometry** is derived from the Greek words *trigonon* (triangle) and *metron* (measurement). Thus, it literally means 'to measure a triangle'.
- Trigonometry deals with the relationship between the sides and the angles of a triangle.
- Modern day uses of trigonometry include surveying land, architecture, measuring distances and determining heights of inaccessible objects.
- In this section relationships between the sides and angles of a right-angled triangle will be explored.

6.4.1 Naming the sides of a right-angled triangle

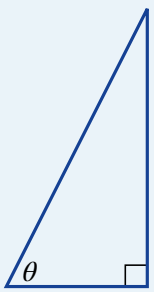
- The longest side of a right-angled triangle (the side opposite the right angle) is called the hypotenuse.
- In order to name the remaining two sides another angle, called the 'reference angle', must be added to the diagram.

  The side that is across from the reference angle, θ, is called the **opposite side**, and the remaining side (the side next to the reference angle) is called the **adjacent side**.
- *Note:* If there is no reference angle marked, only the hypotenuse can be named.



WORKED EXAMPLE 9

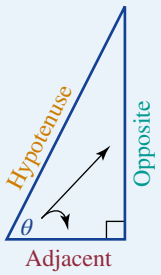
Label the sides of the right-angled triangle shown using the words hypotenuse, adjacent and opposite.



THINK

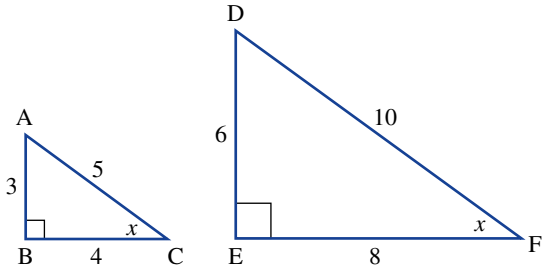
1. The hypotenuse is opposite the right angle.
2. Label the side next to angle θ as 'adjacent' and the side opposite angle θ as 'opposite'.

WRITE/DRAW



6.4.2 Similar right-angled triangles

- Consider the two right-angled triangles shown below. The second triangle (ΔDEF) is an enlargement of the first, using a scale factor of 2. Therefore, the triangles are similar (ΔABC ~ ΔDEF), and ∠BCA = ∠EFD = x.



- For ΔABC, $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{3}{4}$, and for ΔDEF, $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{6}{8} = \frac{3}{4}$.
- Now complete the table below.

	ΔABC	**ΔDEF**
$\frac{\textbf{Opposite side}}{\textbf{hypotenuse}}$		
$\frac{\textbf{Adjacent side}}{\textbf{hypotenuse}}$		

In similar right-angled triangles, the ratios of corresponding sides are equal.

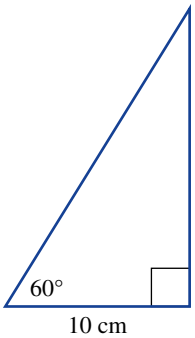
6.4.3 Experiment

A. Using a protractor and ruler, carefully measure and draw a right-angled triangle of base 10 cm and angle of 60° as shown in the diagram.

Measure the length of the other two sides to the nearest mm, and mark these lengths on the diagram as well.

Use your measurements to calculate these ratios correct to 2 decimal places:

$\frac{\text{opposite}}{\text{adjacent}} =$, $\frac{\text{opposite}}{\text{hypotenuse}} =$, $\frac{\text{adjacent}}{\text{hypotenuse}} =$



B. Draw another triangle, similar to the one in part A (all angles the same), making the base length anything that you choose, and measuring the length of all the sides. Once again, calculate the three ratios correct to 2 decimal places:

$\frac{\text{opposite}}{\text{adjacent}} =$, $\frac{\text{opposite}}{\text{hypotenuse}} =$, $\frac{\text{adjacent}}{\text{hypotenuse}} =$

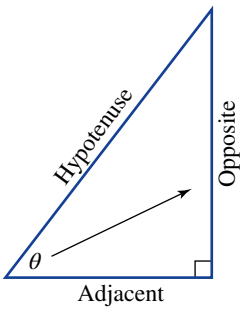
C. Compare your results with the rest of the class. What conclusions can you draw?

6.4.4 Trigonometric ratios

- Trigonometry is based upon the ratios between pairs of side lengths, and each one is given a special name as follows.
  In any right-angled triangle:

$$\textbf{sine}(\theta) = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

$$\textbf{cosine}(\theta) = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$



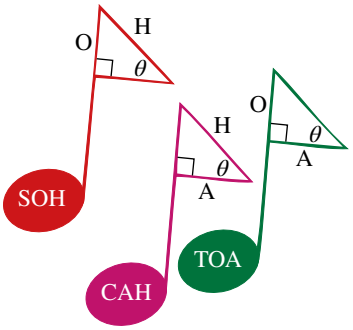
$$\textbf{tangent}(\theta) = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

- These rules are abbreviated to:

$$\sin(\theta) = \frac{\text{O}}{\text{H}}, \cos(\theta) = \frac{\text{A}}{\text{H}} \text{ and } \tan(\theta) = \frac{\text{O}}{\text{A}}.$$

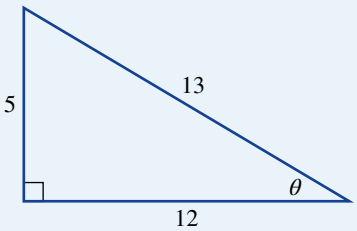
- The following mnemonic can be used to help remember the trigonometric ratios.

SOH – CAH – TOA



WORKED EXAMPLE 10

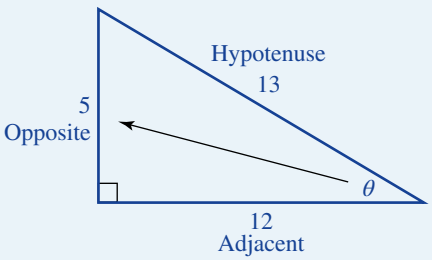
For this triangle, write the equations for the sine, cosine and tangent ratios of the given angle.



THINK

1 Label the sides of the triangle.

WRITE/DRAW



2 Write the trigonometric ratios.

$$\sin(\theta) = \frac{\text{O}}{\text{H}}, \cos(\theta) = \frac{\text{A}}{\text{H}}, \tan(\theta) = \frac{\text{O}}{\text{A}}$$

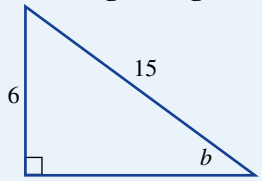
3 Substitute the values of A, O and H into each formula.

$$\sin(\theta) = \frac{5}{13}, \cos(\theta) = \frac{12}{13}, \tan(\theta) = \frac{5}{12}$$

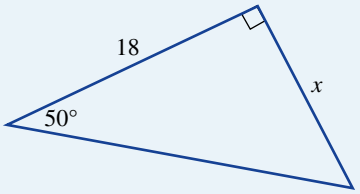
WORKED EXAMPLE 11

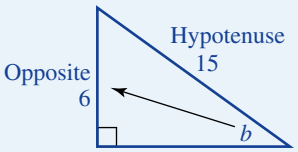
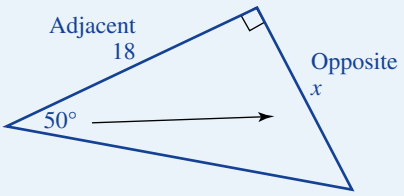
Write the trigonometric ratio that relates the two given sides and the reference angle in each of the following triangles.

a.



b.



THINK	WRITE/DRAW
a 1 Label the given sides.	**a** 
2 We are given O and H. These are used in SOH. Write the ratio.	$\sin(\theta) = \frac{\text{O}}{\text{H}}$
3 Substitute the values of the pronumerals into the ratio.	$\sin(b) = \frac{6}{15}$
4 Simplify the fraction.	$\sin(b) = \frac{2}{5}$
b 1 Label the given sides.	**b** 
2 We are given A and O. These are used in TOA. Write the ratio.	$\tan(\theta) = \frac{\text{O}}{\text{A}}$
3 Substitute the values of the angle and the pronumerals into the ratio.	$\tan(50°) = \frac{x}{18}$

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Investigation: Trigonometric ratios (int-0744)

Complete this digital doc: SkillSHEET: Rounding to a given number of decimal places (doc-10830)

Complete this digital doc: SkillSHEET: Measuring angles with a protractor (doc-10831)

Exercise 6.4 What is trigonometry?

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8	Questions: 1–11	Questions: 1–12

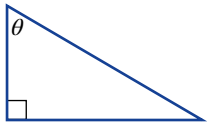
Individual pathway interactivity: int-4498

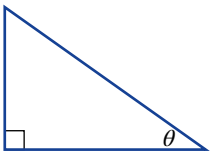
learnON ONLINE ONLY

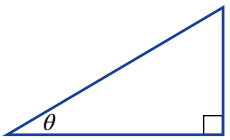
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

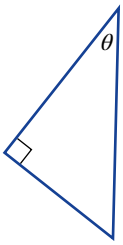
Fluency

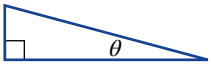
1. WE9 Label the sides of the following right-angled triangles using the words hypotenuse, adjacent and opposite.

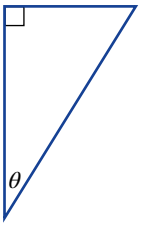
a.


b.


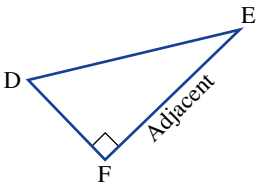
c.


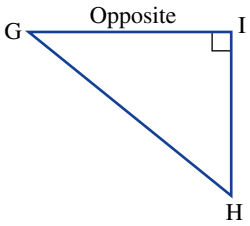
d.


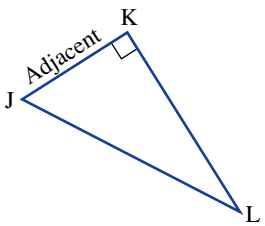
e.


f.


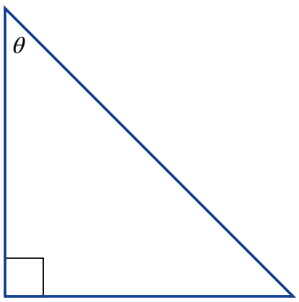
2. Label the hypotenuse, adjacent and opposite sides, and reference angle θ, where appropriate, in each of the following right-angled triangles.

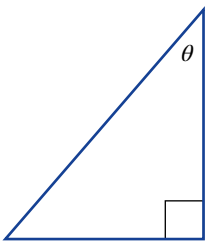
a.


b.


c.


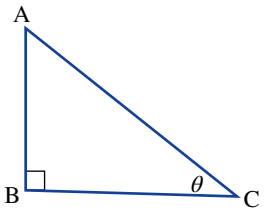
3. For each triangle below, carefully measure θ correct to the nearest degree, then carefully measure each side correct to the nearest mm. Use this information to copy and complete the table below.

a.


b.


	θ	O	A	H	sin(θ)	cos(θ)	tan(θ)
a							
b							

4. MC Which alternative correctly names the sides and angle of the triangle below?

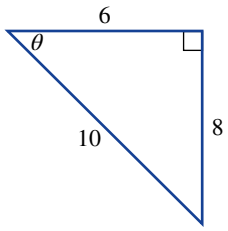


A. $\angle C = \theta$, AB = adjacent side, AC = hypotenuse, AC = opposite side
B. $\angle C = \theta$, AB = opposite side, AC = hypotenuse, AC = adjacent side
C. $\angle A = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side
D. $\angle C = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side

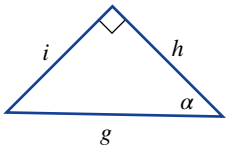
5. WE10 For each of the following triangles, write the expressions for ratios of each of the given angles:

i. sine **ii.** cosine **iii.** tangent.

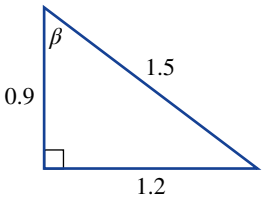
a.



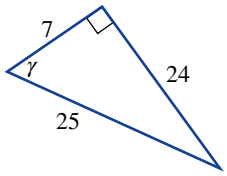
b.



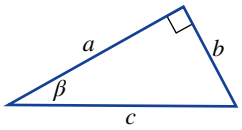
c.



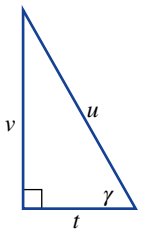
d.



e.

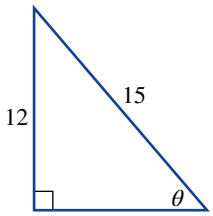


f.

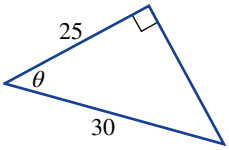


6. WE11 Write the trigonometric ratio that relates the two given sides and the reference angle in each of the following triangles.

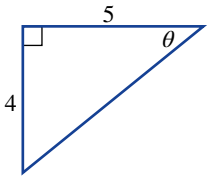
a.



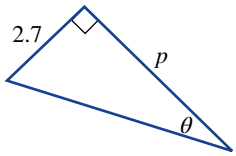
b.



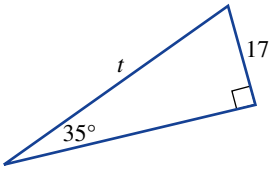
c.



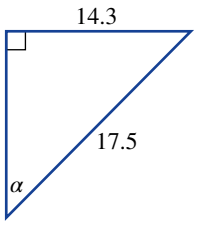
d.



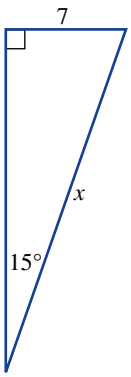
e.



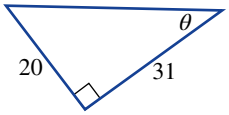
f.



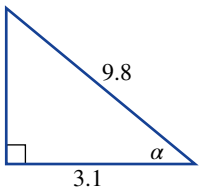
g.



h.

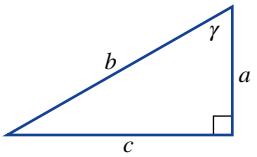


i.



Understanding

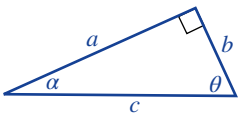
7. MC **a.** What is the correct trigonometric ratio for the triangle shown at right?



A. $\tan(\gamma) = \frac{a}{c}$ **B.** $\sin(\gamma) = \frac{c}{a}$

C. $\cos(\gamma) = \frac{c}{b}$ **D.** $\sin(\gamma) = \frac{c}{b}$

b. Which trigonometric ratio for the triangle shown at right is incorrect?

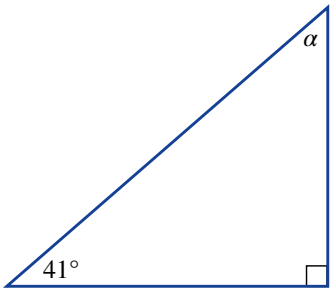


A. $\sin(\alpha) = \frac{b}{c}$ **B.** $\sin(\alpha) = \frac{a}{c}$

C. $\cos(\alpha) = \frac{a}{c}$ **D.** $\tan(\alpha) = \frac{b}{a}$

Reasoning

8. Consider the right-angled triangle shown at right.



a. Label each of the sides using the letters O, A, H with respect to the 41° angle.

b. Measure the side lengths (to the nearest millimetre).

c. Determine the value of each trigonometric ratio. (Where applicable, answers should be given correct to 2 decimal places.)

i. sin(41°) **ii.** cos(41°) **iii.** tan(41°)

d. What is the value of the unknown angle, α?

e. Determine the value of each of these trigonometric ratios, correct to 2 decimal places.

i. sin(α) **ii.** cos(α) **iii.** tan(α)

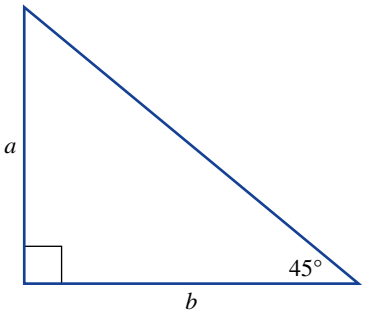
(*Hint:* First re-label the sides of the triangle with respect to angle α.)

f. What do you notice about the relationship between sin(41°) and cos(α)?

g. What do you notice about the relationship between sin(α) and cos(41°)?

h. Make a general statement about the two angles.

9. Given the triangle shown at right:



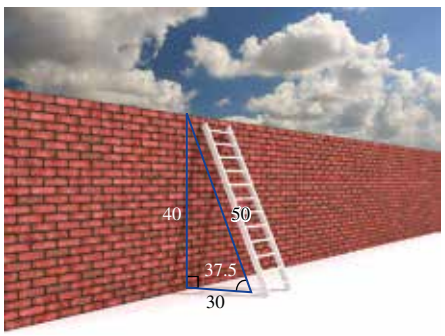
a. why does $a = b$?

b. what would the value of tan(45°) be?

Problem solving

10. If a right-angled triangle has side lengths m, $(m + n)$ and $(m - n)$, which one of the lengths is the hypotenuse? Explain your reasoning.

11. A ladder leans on a wall as shown. Use the information from the diagram to answer the following questions. In relation to the angle given, what part of the image represents:



a. the adjacent side

b. the hypotenuse

c. the opposite side?

12. Use sketches of right-angled triangles to investigate the following.

a. As the acute angle increases in size, what happens to the ratio of the length of the opposite side to the length of the hypotenuse in any right-angled triangle?

b. As the acute angle increases in size, what happens to the other two ratios (i.e. the ratio of the length of the adjacent side to the length of the hypotenuse and that of the opposite side to the adjacent)?

c. What is the largest possible value for:

i. sin(θ) **ii.** cos(θ) **iii.** tan(θ)?

Reflection

Why does sin(30°) = cos(60°)?

6.5 Calculating unknown side lengths

6.5.1 Values of trigonometric ratios

- The values of trigonometric ratios can be found using a calculator.
- Each calculator has several modes. For the following calculations, your calculator must be in degree mode.

WORKED EXAMPLE 12 — TI | CASIO

Evaluate each of the following, giving answers correct to 4 decimal places.
a $\sin(53°)$ **b** $\cos(31°)$ **c** $\tan(79°)$

THINK

a 1 Set the calculator to degree mode. Write the first 5 decimal places.

2 Round correct to 4 decimal places.

b 1 Write the first 5 decimal places.

2 Round correct to 4 decimal places.

c 1 Write the first 5 decimal places.

2 Round correct to 4 decimal places.

WRITE

a $\sin(53°) = 0.79863$

≈ 0.7986

b $\cos(31°) = 0.85716$

≈ 0.8572

c $\tan(79°) = 5.14455$

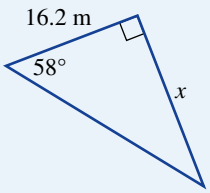
≈ 5.1446

6.5.2 Finding side lengths

If a reference angle and any side length of a right-angled triangle are known, it is possible to find the other sides using trigonometry.

WORKED EXAMPLE 13 — TI | CASIO

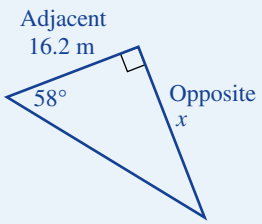
Use the appropriate trigonometric ratio to find the length of the unknown side in the triangle shown. Give your answer correct to 2 decimal places.



THINK

1 Label the given sides.

WRITE/DRAW



2 These sides are used in TOA. Write the ratio.

$$\tan(\theta) = \frac{\text{O}}{\text{A}}$$

3 Substitute the values of θ, O and A into the tangent ratio.

$$\tan(58°) = \frac{x}{16.2}$$

4 Solve the equation for x.

$$16.2 \times \tan(58°) = x$$
$$x = 16.2\tan(58°)$$

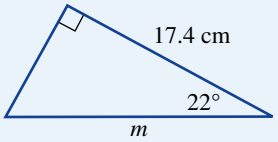
5 Calculate the value of x to 3 decimal places, then round the answer to 2 decimal places.

$$x = 25.925$$
$$x \approx 25.93 \text{ m}$$

WORKED EXAMPLE 14

TI | CASIO

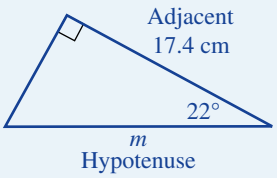
Find the length of the side marked *m* in the triangle at right. Give your answer correct to 2 decimal places.



THINK

1 Label the given sides.

WRITE/DRAW



2 These sides are used in CAH. Write the ratio.

$$\cos(\theta) = \frac{\text{A}}{\text{H}}$$

3 Substitute the values of θ, A and H into the cosine ratio.

$$\cos(22°) = \frac{17.4}{m}$$

4 Solve for *m*:
- Multiply both sides by *m*.
- Divide both sides by cos(22°).

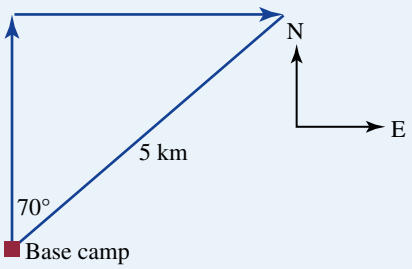
$$m\cos(22°) = 17.4$$
$$m = \frac{17.4}{\cos(22°)}$$
$$m = 18.766$$

5 Calculate the value of *m* to 3 decimal places, then round the answer to 2 decimal places.

$$m \approx 18.77 \text{ cm}$$

WORKED EXAMPLE 15

Benjamin set out on a bushwalking expedition. Using a compass, he set off on a course N 70°E (or 070°T) and travelled a distance of 5 km from his base camp.





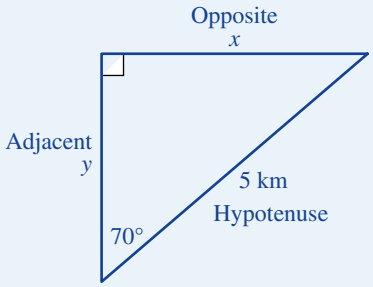
a How far east has he travelled?
b How far north has he travelled from the base camp?
Give answers correct to 2 decimal places.

THINK

a 1 Label the easterly distance *x*.
Label the northerly distance *y*.
Label the sides of the triangle:
Hypotenuse, Opposite, Adjacent.

WRITE/DRAW

a



2 To calculate the value of x, use the sides of the triangle: $x = \text{O}, 5 = \text{H}$.
These are used in SOH. Write the ratio.

$$\sin(\theta) = \frac{\text{O}}{\text{H}}$$

3 Substitute the values of the angle and the pronumerals into the sine ratio.

$$\sin(70°) = \frac{x}{5}$$

4 Make x the subject of the equation.

$$x = 5\sin(70°)$$

5 Evaluate x to 3 decimal places, using a calculator.

$$= 4.698$$

6 Round to 2 decimal places.

$$\approx 4.70 \text{ km}$$

7 Answer the question in sentence form.

Benjamin has travelled 4.70 km east of the base camp.

b 1 To calculate the value of y, use the sides: $y = \text{A}, 5 = \text{H}$.
These are used in CAH. Write the ratio.

b $\cos(\theta) = \frac{\text{A}}{\text{H}}$

2 Substitute the values of the angle and the pronumerals into the cosine ratio.

$$\cos(70°) = \frac{y}{5}$$

3 Make y the subject of the equation.

$$y = 5\cos(70°)$$

4 Evaluate y using a calculator.

$$= 1.710$$

5 Round the answer to 2 decimal places.

$$\approx 1.71 \text{ km}$$

6 Answer the question in sentence form.

Benjamin has travelled 1.71 km north of the base camp.

learnon RESOURCES — ONLINE ONLY

Watch this eLesson: Using an inclinometer (eles-0116)

Complete this digital doc: SkillSHEET: Solving equations of the type $a = \frac{x}{b}$ to find x (doc-10832)

Complete this digital doc: SkillSHEET: Solving equations of the type $a = \frac{b}{x}$ to find x (doc-10833)

Complete this digital doc: WorkSHEET: Trigonometry (doc-10835)

Exercise 6.5 Calculating unknown side lengths

assessON

Individual pathways

PRACTISE
Questions:
1–3, 4a–c, 5a–c, 6a–f, 7–9, 11, 12

CONSOLIDATE
Questions:
1–3, 4b–d, 5b–d, 6d–i, 7–10, 12–15

MASTER
Questions:
1–3, 4d–f, 5d–f, 6g–i, 7–10, 11, 13–17

Individual pathway interactivity: int-4499

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE12 **a.** Evaluate the following correct to 4 decimal places.

i. sin(55°) **ii.** sin(11.6°)

b. Copy and complete the table below. Use your calculator to find each value of sin(θ) correct to 2 decimal places.

θ	0°	15°	30°	45°	60°	75°	90°
sin(θ)							

c. Summarise the trend in these values.

2. a. Evaluate the following correct to 4 decimal places.

i. cos(38°) **ii.** cos(53.71°)

b. Copy and complete the table below. Use your calculator to find each value of cos(θ) correct to 2 decimal places.

θ	0°	15°	30°	45°	60°	75°	90°
cos(θ)							

c. Summarise the trend in these values.

3. a. Evaluate the following correct to 4 decimal places.

i. tan(18°) **ii.** tan(51.9°)

b. Copy and complete the table below. Use your calculator to find each value of tan(θ) correct to 2 decimal places.

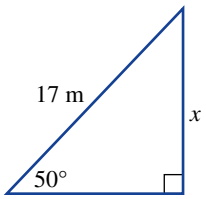
θ	0°	15°	30°	45°	60°	75°	90°
tan(θ)							

c. Find the value of tan(89°) and tan(89.9°).

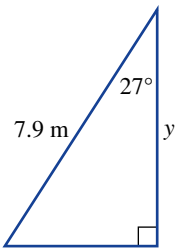
d. What do you notice about these results?

4. WE13 Use the appropriate trigonometric ratios to find the length of the unknown side in each of the triangles shown. Give the answers correct to 2 decimal places.

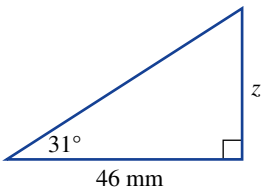
a.



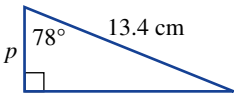
b.



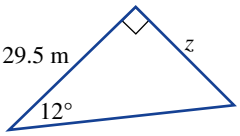
c.



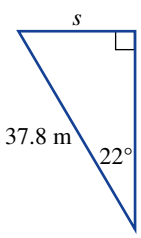
d.



e.

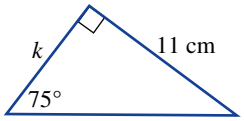


f.

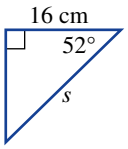


5. WE14 Use the appropriate trigonometric ratio to find the length of the unknown side in each of the triangles shown. Give the answers correct to 2 decimal places.

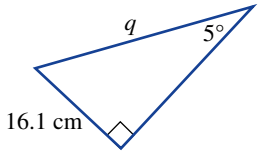
a.



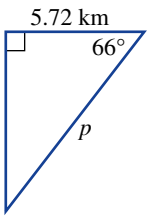
b.



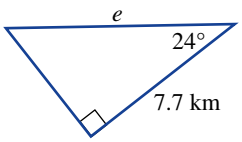
c.



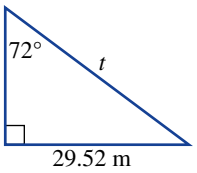
d.



e.

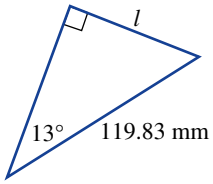


f.

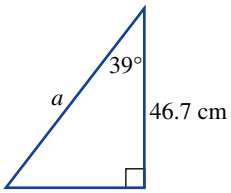


6. Find the length of the unknown side in each of the following triangles, correct to 2 decimal places. (*Note:* In some cases the unknown will be in the numerator and in other cases it will be in the denominator.)

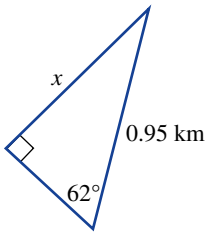
a.



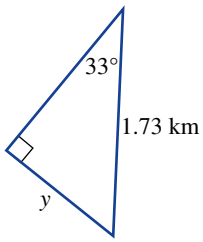
b.



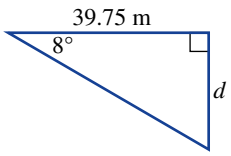
c.



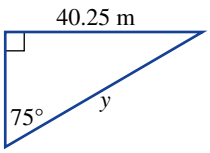
d.



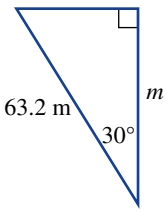
e.



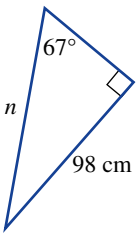
f.



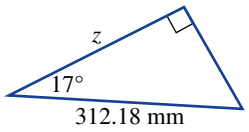
g.



h.

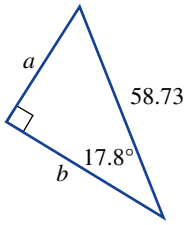


i.

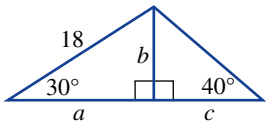


7. Find the lengths of the unknown sides in the triangles shown, correct to 2 decimal places.

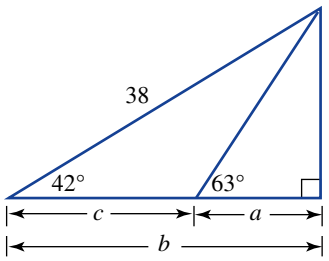
a.



b.

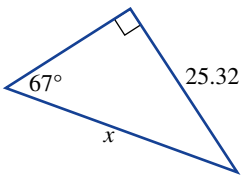


c.



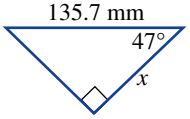
Understanding

8. MC **a.** The value of x correct to 2 decimal places is:



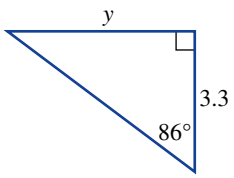
- **A.** 59.65
- **B.** 23.31
- **C.** 64.80
- **D.** 27.51

b. The value of x correct to 2 decimal places is:



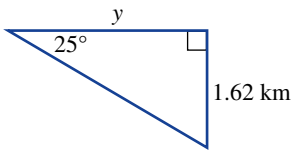
- **A.** 99.24 mm
- **B.** 92.55 mm
- **C.** 185.55 mm
- **D.** 198.97 mm

c. The value of y correct to 2 decimal places is:



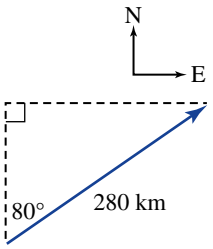
A. 47.19
B. 7.94
C. 1.37
D. 0.23

d. The value of y correct to 2 decimal places is:



A. 0.76 km
B. 1.79 km
C. 3.83 km
D. 3.47 km

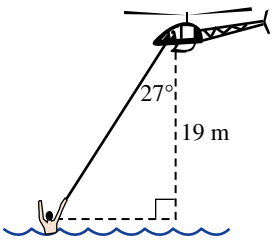
9. **WE15** A ship that was to travel due north veered off course and travelled N 80°E (or 080°T) for a distance of 280 km, as shown in the diagram.



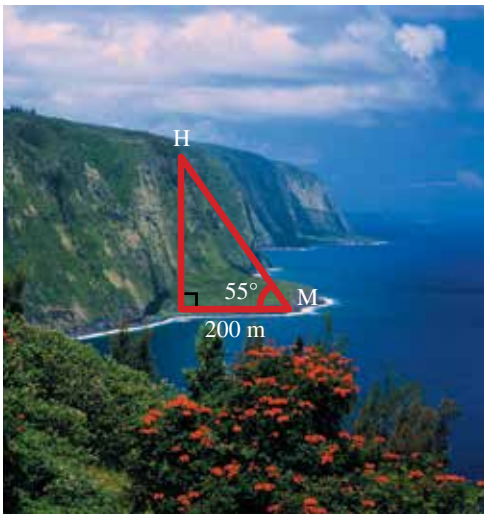
a. How far east had the ship travelled?
b. How far north had the ship travelled?



10. A rescue helicopter spots a missing surfer drifting out to sea on his damaged board. The helicopter descends vertically to a height of 19 m above sea level and drops down an emergency rope, which the surfer grips. Due to the wind the rope swings at an angle of 27° to the vertical, as shown in the diagram. What is the length of the rope?

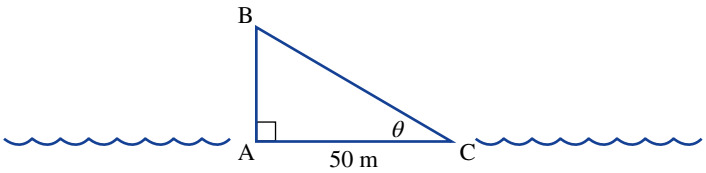


11. Walking along the coastline, Michelle (M) looks up through an angle of 55° and sees her friend Helen (H) on top of the cliff at the lookout point. How high is the cliff if Michelle is 200 m from its base? (Assume both girls are the same height.)



Reasoning

12. One method for determining the distance across a body of water is shown in the diagram below.



The required distance is AB. A surveyor moves at right angles 50 m to point C and uses a tool called a transit to measure the angle θ ($\angle$ACB).

a. If $\theta = 12.3°$, show that the length AB is 10.90 m.

b. Show that a value of $\theta = 63.44°$ gives a length of AB = 100 m.

c. Find a rule that can be used to calculate the length AC.

13. Using a diagram, explain why $\sin(70°) = \cos(20°)$ and $\cos(70°) = \sin(20°)$. In general, $\sin(\theta)$ will be equal to which cosine?

Problem solving

14. Calculate the value of the pronumeral in each of the following triangles.

a.



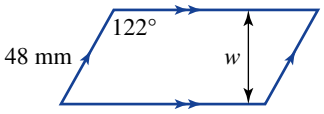
b.



c.



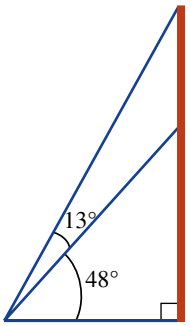
15. A tile is in the shape of a parallelogram with measurements as shown. Calculate the width of the tile, w, to the nearest mm.



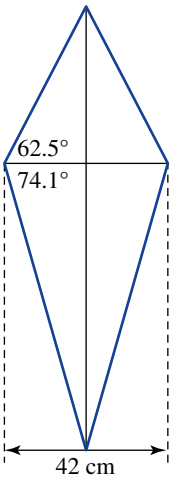
16. A pole is supported by two wires as shown. If the length of the lower wire is 4.3 m, calculate to 1 decimal place:

a. the length of the top wire

b. the height of the pole.



17. The frame of a kite is built from 6 wooden rods as shown. Calculate the total length of wood used to make the frame of the kite to the nearest metre.



Reflection

What does sin(60°) actually mean?

CHALLENGE 6.1

A track in a national park has four legs from its starting point to its return. The first leg travels due east. The second, third and fourth legs follow paths southwest, due north and finally due east back to the starting point. If the distances of the first two legs are 1.5 km and 3.5 km respectively, what distances are the third and fourth legs?

6.6 Calculating unknown angles

6.6.1 Inverse trigonometric ratios

- We have seen that $\sin(30°) = 0.5$; therefore, 30° is the inverse sine of 0.5. This is written as $\sin^{-1}(0.5) = 30°$.
- The expression $\sin^{-1}(x)$ is read as 'the inverse sine of x'. The expression $\cos^{-1}(x)$ is read as 'the inverse cosine of x'.
- The expression $\tan^{-1}(x)$ is read as 'the inverse tangent of x'.

Experiment

1. Use your calculator to find sin(30°), then find the inverse sine of the answer. Choose another angle and do the same thing.
2. Now find cos(30°) and then find the inverse cosine ($\cos^{-1}$) of the answer. Choose another angle and do the same thing.
3. Lastly, find tan(45°) and then find the inverse tangent ($\tan^{-1}$) of the answer. Try this with other angles.

- The fact that sin and $\sin^{-1}$ cancel each other out is useful in solving equations such as:

$$\begin{aligned}
\sin(\theta) &= 0.3 \quad \text{(Take the inverse sine of both sides.)} \\
\sin^{-1}(\sin(\theta)) &= \sin^{-1}(0.3) \\
\theta &= \sin^{-1}(0.3) \\
\sin^{-1}(x) &= 15° \quad \text{(Take the sine of both sides.)} \\
\sin(\sin^{-1}(x)) &= \sin(15°) \\
x &= \sin(15°)
\end{aligned}$$

Similarly, $\cos(\theta) = 0.522$ means that $\theta = \cos^{-1}(0.522)$
and $\tan(\theta) = 1.25$ means that $\theta = \tan^{-1}(1.25)$.

WORKED EXAMPLE 16 — TI | CASIO

Evaluate $\cos^{-1}(0.3678)$, correct to the nearest degree.

THINK	WRITE
1 Set your calculator to degree mode.	$\cos^{-1}(0.3678) = 68.4$
2 Round the answer to the nearest whole number and include the degree symbol.	$\approx 68°$

WORKED EXAMPLE 17

Determine the size of angle θ in each of the following. Give answers correct to the nearest degree.
a $\sin(\theta) = 0.6543$ **b** $\tan(\theta) = 1.745$

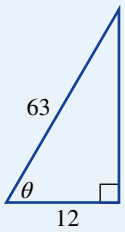
THINK	WRITE
a 1 θ is the inverse sine of 0.6543.	**a** $\sin(\theta) = 0.6543$ $\theta = \sin^{-1}(0.6543)$
2 Calculate and record the answer.	$= 40.8$
3 Round the answer to the nearest degree.	$\approx 41°$
b 1 θ is the inverse tangent of 1.745.	**b** $\tan(\theta) = 1.745$ $\theta = \tan^{-1}(1.745)$
2 Use the inverse tangent function on a calculator. Record the number shown.	$= 60.18$
3 Round the answer to the nearest degree.	$\approx 60°$

6.6.2 Finding the angle when 2 sides are known

If the lengths of any 2 sides of a right-angled triangle are known, it is possible to find an angle using inverse sine, inverse cosine or inverse tangent.

WORKED EXAMPLE 18

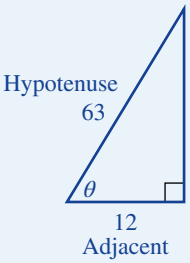
Determine the value of θ in the triangle below. Give your answer correct to the nearest degree.



THINK

1 Label the given sides. These are used in CAH. Write the ratio.

WRITE/DRAW



$$\cos(\theta) = \frac{\text{A}}{\text{H}}$$

2 Substitute the given values into the cosine ratio.

$$\cos(\theta) = \frac{12}{63}$$

3 θ is the inverse cosine of $\frac{12}{63}$.

$$\theta = \cos^{-1}\left(\frac{12}{63}\right)$$

4 Evaluate.

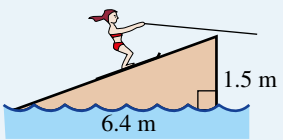
$$= 79.0$$

5 Round the answer to the nearest degree.

$$\approx 79°$$

WORKED EXAMPLE 19

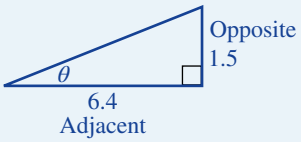
Roberta enjoys water skiing and is about to try a new ramp on the Hawkesbury River. The inclined ramp rises 1.5 m above the water level and spans a horizontal distance of 6.4 m. What is the magnitude (size) of the angle that the ramp makes with the water? Give the answer correct to the nearest degree.



THINK

1 Draw a simple diagram, showing the known lengths and the angle to be found.

WRITE/DRAW



2 Label the given sides. These are used in TOA. Write the ratio.

$$\tan(\theta) = \frac{\text{O}}{\text{A}}$$

3 Substitute the values of the pronumerals into the tangent ratio.

$$\tan(\theta) = \frac{1.5}{6.4}$$

4 θ is the inverse inverse tangent of $\frac{1.5}{6.4}$.	$\theta = \tan^{-1}\left(\frac{1.5}{6.4}\right)$
5 Evaluate.	$= 13.19$
6 Round the answer to the nearest degree.	$\approx 13°$
7 Write the answer in words.	The ramp makes an angle of 13° with the water.

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Complete this digital doc: SkillSHEET: Rounding angles to the nearest degree (doc-10836)

Exercise 6.6 Calculating unknown angles

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–c, 2, 3a–f, 4–10	Questions: 1d–f, 2, 3d–h, 4–11	Questions: 1g–i, 2, 3e–i, 4–13

Individual pathway interactivity: int-4500 learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE16 Evaluate each of the following, correct to the nearest degree.

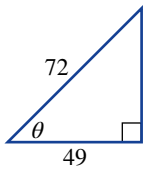
a. $\sin^{-1}(0.6294)$ **b.** $\cos^{-1}(0.3110)$ **c.** $\tan^{-1}(0.7409)$
d. $\tan^{-1}(1.3061)$ **e.** $\sin^{-1}(0.9357)$ **f.** $\cos^{-1}(0.3275)$
g. $\cos^{-1}(0.1928)$ **h.** $\tan^{-1}(4.1966)$ **i.** $\sin^{-1}(0.2554)$

2. WE17 Determine the size of the angle in each of the following. Give answers correct to the nearest degree.

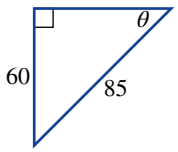
a. $\sin(\theta) = 0.3214$ **b.** $\sin(\theta) = 0.6752$ **c.** $\sin(\beta) = 0.8235$
d. $\cos(\beta) = 0.9351$ **e.** $\cos(\alpha) = 0.6529$ **f.** $\cos(\alpha) = 0.1722$
g. $\tan(\theta) = 0.7065$ **h.** $\tan(a) = 1$ **i.** $\tan(b) = 0.876$
j. $\sin(c) = 0.3936$ **k.** $\cos(\theta) = 0.5241$ **l.** $\tan(\alpha) = 5.6214$

3. WE18 Determine the value of θ in each of the following triangles. Give answers correct to the nearest degree.

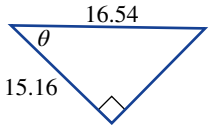
a.



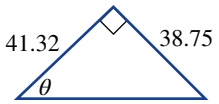
b.



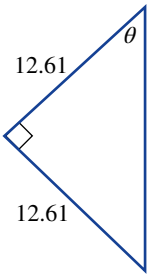
c.



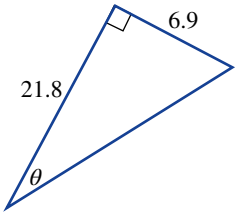
d.



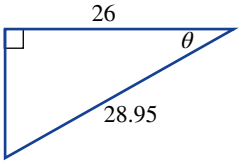
e.



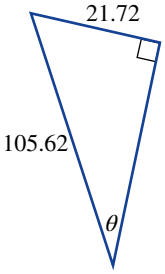
f.



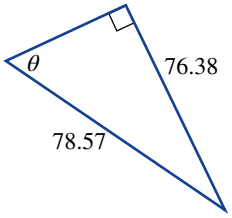
g.



h.



i.



4. MC **a.** If $\cos(\theta) = 0.8752$, the value of θ correct to 2 decimal places is:

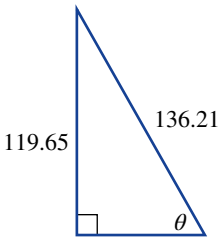
A. 61.07° **B.** 41.19° **C.** 25.84° **D.** 28.93°

b. If $\sin(\theta) = 0.5530$, the value of θ correct to 2 decimal places is:

A. 56.43° **B.** 33.57° **C.** 28.94° **D.** 36.87°

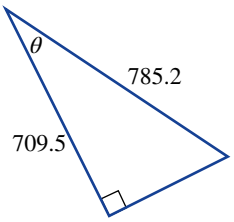
c. The value of θ in the triangle shown, correct to 2 decimal places, is:

A. 41.30°
B. 28.55°
C. 48.70°
D. 61.45°



d. The value of θ in the triangle shown, correct to 2 decimal places, is:

A. 42.10°
B. 64.63°
C. 25.37°
D. 47.90°



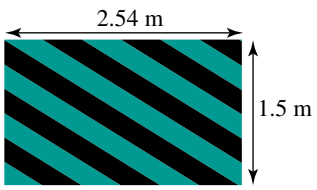
5. a. Copy and fill in the table below.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y = \cos^{-1}(x)$	90°					60°					0°

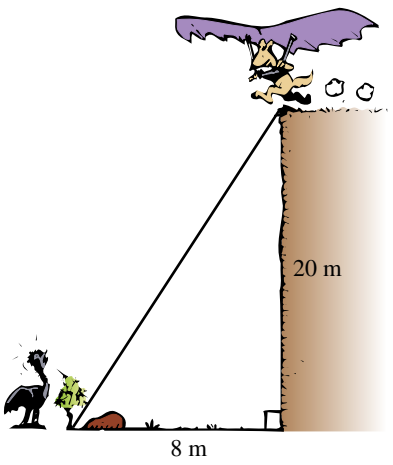
b. Plot the above table on graph paper or with a spreadsheet or suitable calculator.

Understanding

6. A piece of fabric measuring 2.54 m by 1.5 m has a design consisting of parallel diagonal stripes. What angle does each diagonal make with the length of the fabric? Give your answer correct to 2 decimal places.



7. WE19 Danny Dingo is perched on top of a cliff 20 m high watching an emu feeding 8 m from the base of the cliff. Danny has purchased a flying contraption, which he hopes will help him capture the emu. At what angle to the cliff must he swoop to catch his prey? Give your answer correct to 2 decimal places.



Reasoning

8. Jenny and Les are camping with friends Mark and Susie. Both couples have a 2-m-high tent. The top of each 2-m tent pole is to be tied with a piece of rope that will be used to keep the pole upright. So that the rope doesn't trip a passerby, Jenny and Les decide that the angle between the rope and the ground should be 60°. Answer the following questions, correct to 2 decimal places.

a. Find the length of the rope needed from the top of the tent pole to the ground to support their tent pole.

b. Further down the camping ground, Mark and Susie also set up their tent. However, they want to use a piece of rope that they know is in the range of 2 to 3 metres in length.

i. Explain why the rope will have to be greater than 2 metres in length.

ii. Show that the minimum angle the rope will make with the ground will be 41.8°.

9. Safety guidelines for wheelchair access ramps used to state that the gradient had to be in the ratio 1 : 20.

a. Using this ratio, show that the angle that the ramp had to make with the horizontal is closest to 3°.

b. New regulations have changed the ratio of the gradient, so the angle the ramp must make with the horizontal is now closest to 6°. Explain why, using this angle size, the new ratio could be 1 to 9.5.

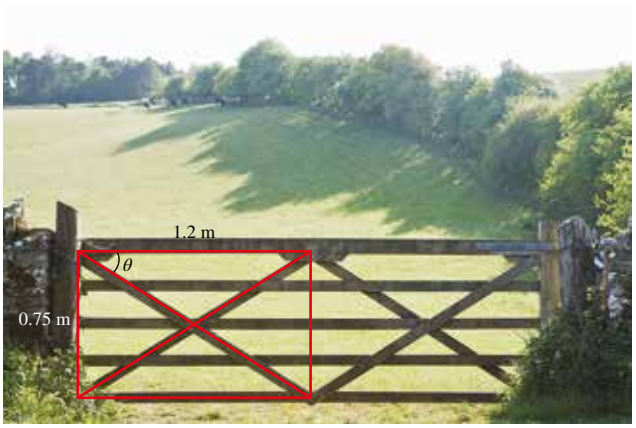
Problem solving

10. Calculate the value of the pronumeral in each of the following to 2 decimal places.

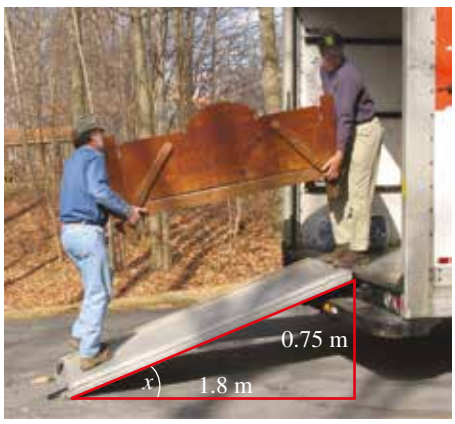
a.



b.



c.



11. A family is building a patio extension to their house. One section of the patio will have a gable roof. A similar structure is pictured with the planned post heights and span shown. To allow more light in, the family wants the peak (highest point) of the gable roof to be at least 5 m above deck level. According to building regulations, the slope of the roof (i.e. the angle that the sloping edge makes with the horizontal) must be 22°.

a. Use trigonometry to calculate whether the roof would be high enough if the angle was 22°.
b. Use trigonometry to calculate the size of the obtuse angle formed at the peak of the roof.



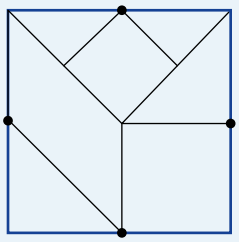
12. Use the formulas $\sin(\theta) = \frac{o}{h}$ and $\cos(\theta) = \frac{a}{h}$ to prove that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$.

Reflection

Why does cos(0°) = 1?

CHALLENGE 6.2

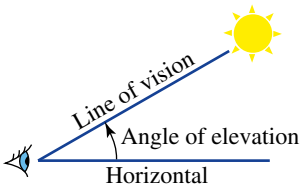
1 A square-based prism has a height twice its base length. What angle does the diagonal of the prism make with the diagonal of the base?
2 Seven smaller shapes are created inside a square with side length 10 cm as shown in the diagram. If the dots represent the midpoints of the square's sides, find the dimensions of each shape.



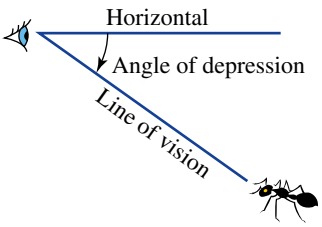
6.7 Angles of elevation and depression

6.7.1 Angles of elevation and depression

- When looking up towards an object, an **angle of elevation** is the angle between the horizontal line and the line of vision.



- When looking down at an object, an **angle of depression** is the angle between the horizontal line and the line of vision.
- Angles of elevation and depression are measured from horizontal lines.



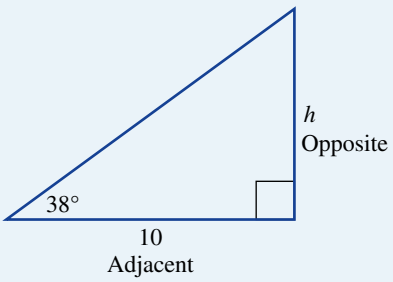
WORKED EXAMPLE 20

At a point 10 m from the base of a tree, the angle of elevation of the treetop is 38°. How tall is the tree to the nearest centimetre?

THINK

1 Draw a simple diagram. The angle of elevation is 38° from the horizontal.

WRITE/DRAW



2 Label the given sides of the triangle. These sides are used in TOA. Write the ratio.

$$\tan(38°) = \frac{h}{10}$$

3 Multiply both sides by 10.

$$10\tan(38°) = h$$

4 Calculate correct to 3 decimal places.

$$h = 7.812$$

5 Round to 2 decimal places.

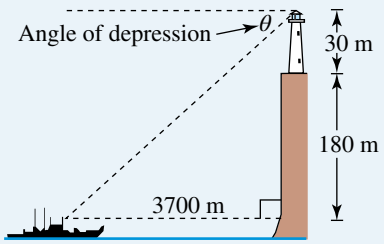
$$\approx 7.81$$

6 Write the answer in words.

The tree is 7.81 m tall.

WORKED EXAMPLE 21

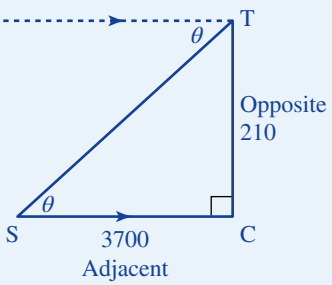
A lighthouse, 30 m tall, is built on top of a cliff that is 180 m high. Find the angle of depression (θ) of a ship from the top of the lighthouse if the ship is 3700 m from the bottom of the cliff.



THINK

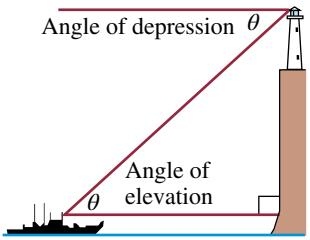
1 Draw a simple diagram to represent the situation. The height of the triangle is 180 + 30 = 210 m. Draw a horizontal line from the top of the triangle and mark the angle of depression, θ. Also mark the alternate angle.

WRITE/DRAW

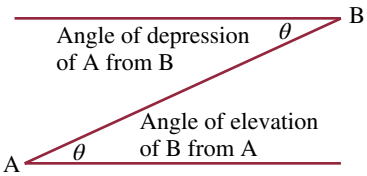


2 Label the triangle. These sides are used in TOA. Write the ratio.	$\tan(\theta) = \frac{\text{O}}{\text{A}}$
3 Substitute the given values into the ratio.	$\tan(\theta) = \frac{210}{3700}$
4 θ is the inverse tangent of $\frac{210}{3700}$.	$\theta = \tan^{-1}\left(\frac{210}{3700}\right)$
5 Evaluate.	$= 3.2$
6 Round the answer to the nearest degree.	$\approx 3°$
7 Write the answer in words.	The angle of depression of the ship from the top of the lighthouse is 3°.

- *Note:* In Worked example 21, the angle of depression from the top of the lighthouse to the ship is equal to the angle of elevation from the ship to the top of the lighthouse. This is because the angle of depression and the angle of elevation are alternate (or 'Z') angles.



- This can be generalised as follows:
  For any two objects, A and B, the angle of elevation of B, as seen from A, is equal to the angle of depression of A, as seen from B.



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Complete this digital doc: SkillSHEET: Drawing a diagram from given directions (doc-10837)

Complete this digital doc: WorkSHEET: Trigonometry using elevation and depression (doc-10838)

Exercise 6.7 Angles of elevation and depression

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 10, 12	Questions: 1–6, 8, 11–14	Questions: 1–3, 6, 7, 9–16

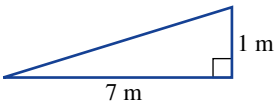
Individual pathway interactivity: int-4501

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Fluency

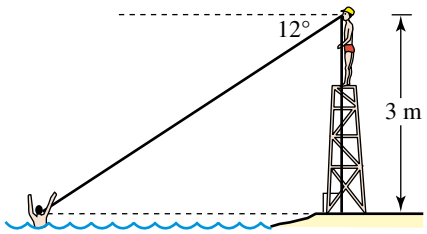
1. WE20 Building specifications require the angle of elevation of any ramp constructed for public use to be less than 3°.



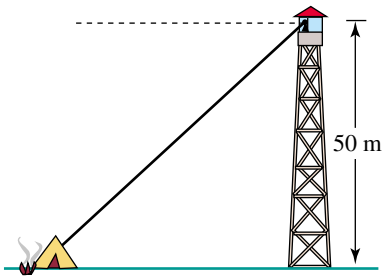


Ramps being constructed at a new shopping centre are each made in the ratio 7 m horizontal length to 1 m vertical height. Find the angle of elevation of these ramps and, hence, decide whether they meet building specifications.

2. A lifesaver standing on his tower 3 m above the ground spots a swimmer experiencing difficulty. The angle of depression of the swimmer from the lifesaver is 12°. How far is the swimmer from the lifesaver's tower? (Give your answer correct to 2 decimal places.)



3. From the top of a lookout 50 m above the ground, the angle of depression of a camp site that is level with the base of the lookout is 37°. How far is the camp site from the base of the lookout?

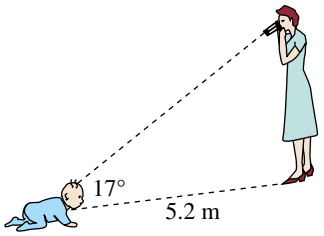


Understanding

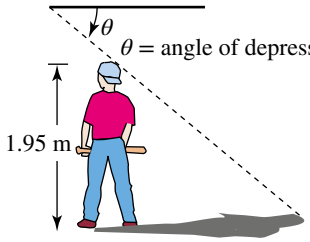
4. From a rescue helicopter 80 m above the ocean, the angles of depression of two shipwreck survivors are 40° and 60° respectively. If the two sailors and the helicopter are in line with each other:
- **a.** draw a labelled diagram to represent the situation
- **b.** calculate the distance between the two sailors, to the nearest metre.

5. The angle of elevation of the top of a tree from a point on the ground, 60 m from the tree, is 35°.
- **a.** Draw a labelled diagram to represent the situation.
- **b.** Find the height of the tree to the nearest metre.

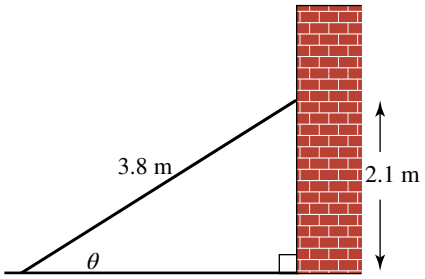
6. Miriam, an avid camerawoman from Perth, wants to record her daughter Alexandra's first attempts at crawling. As Alexandra lies on the floor and looks up at her mother, the angle of elevation is 17°. If Alexandra is 5.2 m away from her mother, how tall is Miriam? Give your answer correct to 1 decimal place.



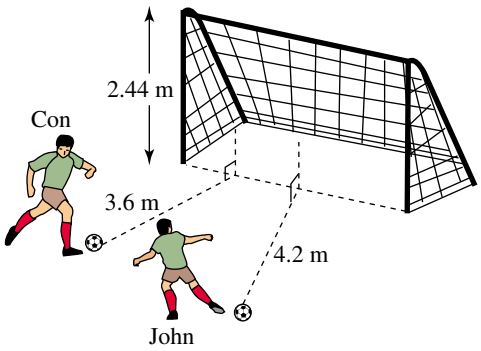
7. **WE21** Stan, who is 1.95 m tall, measures the length of the shadow he casts along the ground as 0.98 m. Find the angle of depression of the sun's rays to the nearest degree.



8. What angle does a 3.8-m ladder make with the ground if it reaches 2.1 m up the wall? How far is the foot of the ladder from the wall? (Give your answers to the nearest degree and the nearest metre.)

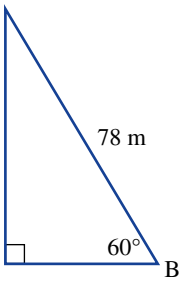


9. Con and John are practising shots on goal. Con is 3.6 m away from the goal and John is 4.2 m away, as shown in the diagram. If the height of the goal post is 2.44 m, what is the maximum angle of elevation, to the nearest degree, that each can kick the ball in order to score a goal?

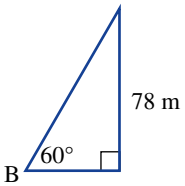


10. **MC** The angle of elevation of the top of a lighthouse tower 78 m tall, from a point B on the same level as the base of the tower, is 60°. The correct diagram for this information is:

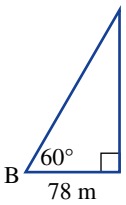
A.



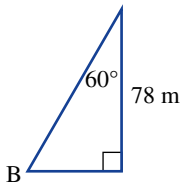
B.



C.



D.



Reasoning

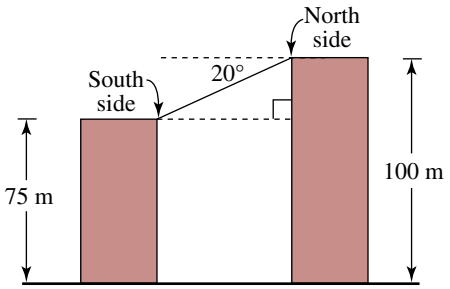
11. Lifesaver Sami spots some dolphins playing near a marker at sea directly in front of him. He is sitting in a tower that is situated 10 m from the water's edge and he is 4 m above sea level. The marker is 20 m from the water's edge.

a. Draw a diagram to represent this information.

b. Show that the angle of depression of Sami's view of the dolphins, correct to 1 decimal place, is 7.6°.

c. As the dolphins swim towards Sami, would the angle of depression increase or decrease? Justify your answer in terms of the tangent ratios.

12. Two buildings are 100 m and 75 m high. From the top of the north side of the taller building, the angle of depression to the top of the south side of the smaller building is 20°, as shown at right. Show that the horizontal distance between the north side of the taller building and the south side of the smaller building is closest to 69 metres.



Problem solving

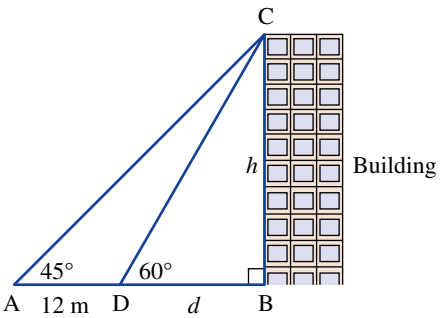
13. Rouka was hiking in the mountains when she spotted an eagle sitting up in a tree. The angle of elevation of her view of the eagle was 35°. She then walked 20 metres towards the tree and her angle of elevation was 50°. The height of the eagle from the ground was 35.5 metres.

a. Draw a labelled diagram to represent this information.

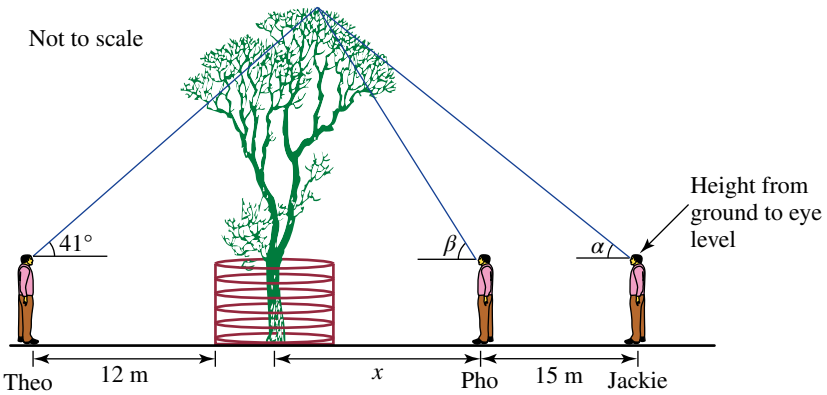
b. Determine how tall Rouka is, if her eyes are 9 cm from the top of her head. Write your answer in metres, correct to the nearest centimetre.

14. A lookout in a lighthouse tower can see two ships approaching the coast. Their angles of depression are 25° and 30°. If the ships are 100 m apart, show that the height of the lighthouse, to the nearest metre, is 242 metres.

15. At a certain distance away, the angle of elevation to the top of a building is 60°. From 12 m further back, the angle of elevation is 45° as shown in the diagram at right. Show that the height of the building is 28.4 metres.



16. A tall gum tree stands in a courtyard in the middle of some office buildings. Three Year 9 students, Jackie, Pho and Theo, measure the angle of elevation from three different positions. They are unable to measure the distance to the base of the tree because of the steel tree guard around the base. The diagram below shows the angles of elevation and the distances measured.



a. Show that $x = \dfrac{15 \tan \alpha}{\tan \beta - tan\ \alpha}$, where x is the distance, in metres, from the base of the tree to Pho's position.

b. The girls estimate the tree to be 15 m taller than them. Pho measured the angle of elevation to be 72°. What should Jackie have measured her angle of elevation to be, if these measurements are assumed to be correct? Write your answer to the nearest degree.

c. Theo did some calculations and determined that the tree was only about 10.4 m taller than them. Jackie claims that Theo's calculation of 10.4 m is incorrect.

i. Is Jackie's claim correct? Show how Theo calculated a height of 10.4 m.

ii. If the height of the tree was actually 15 metres above the height of the students, determine the horizontal distance Theo should have used in his calculations. Write your answer to the nearest centimetre.

Reflection

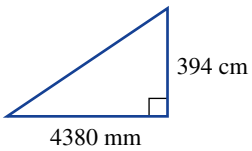
Why does the angle of elevation have the same value as the angle of depression?

6.8 Review

6.8.1 Review questions

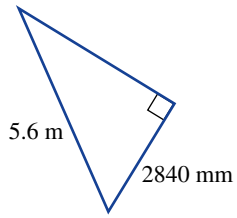
Fluency

1. What is the length of the third side in this triangle?



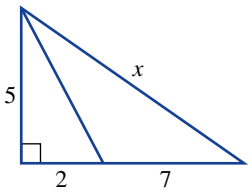
A. 34.71 m
B. 2.96 m
C. 5.89 m
D. 1722 cm

2. The most accurate measure for the length of the third side in this triangle is



A. 4.83 m.
B. 23.3 cm.
C. 3.94 m.
D. 4826 mm.

3. What is the value of x in this figure?

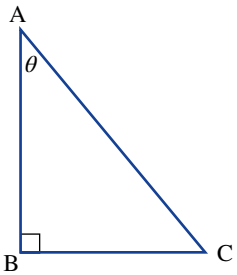


A. 5.4
B. 7.5
C. 10.1
D. 10.3

4. Which of the following triples is not a Pythagorean triple?

A. 3, 4, 5
B. 6, 8, 10
C. 5, 12, 13
D. 2, 3, 4

5. Which of the following correctly names the sides and angle of the triangle shown?

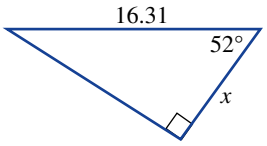


A. $\angle C = \theta$, AB = adjacent side, AC = hypotenuse, BC = opposite side
B. $\angle C = \theta$, AB = opposite side, AC = hypotenuse, AC = adjacent side
C. $\angle A = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side
D. $\angle A = \theta$, AB = adjacent side, AC = hypotenuse, BC = opposite side

6. Which one of the following statements is correct?

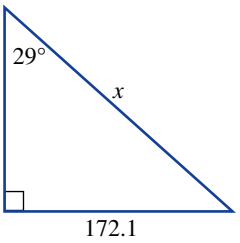
A. $\sin(60°) = \cos(60°)$
B. $\cos(25°) = \cos(65°)$
C. $\cos(60°) = \sin(30°)$
D. $\sin(70°) = \cos(70°)$

7. What is the value of x in the triangle shown, correct to 2 decimal places?



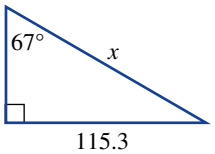
A. 26.49
B. 10.04
C. 12.85
D. 20.70

8. Which one of the following could be used to find the value of x in the triangle shown?



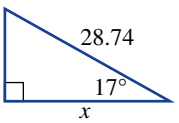
A. $x = \dfrac{172.1}{\cos(29°)}$
B. $x = \dfrac{172.1}{\sin(29°)}$
C. $x = 172.1 \times \sin(29°)$
D. $x = 172.1 \times \cos(29°)$

9. Which of the following could be used to find the value of x in the triangle shown?



A. $x = \dfrac{115.3}{\sin(23°)}$
B. $x = \dfrac{115.3}{\cos(67°)}$
C. $x = 115.3° \times \sin(67°)$
D. $x = \dfrac{115.3}{\cos(23°)}$

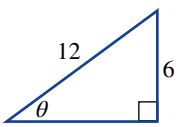
10. Which of the following could be used to find the value of x in the triangle shown?



A. $x = \dfrac{28.3}{\cos(17°)}$
B. $x = 28.74 \times \sin(17°)$
C. $x = 28.74 \times \cos(17°)$
D. $x = 28.74 \times \cos(73°)$

11. The value of $\tan^{-1}(1.8931)$ is closest to:
A. 62°
B. 0.0331°
C. 1.08°
D. 69°

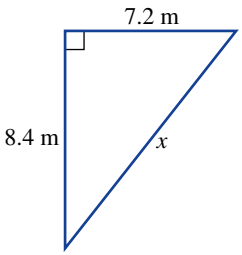
12. What is the value of θ in the triangle shown, correct to 2 decimal places?



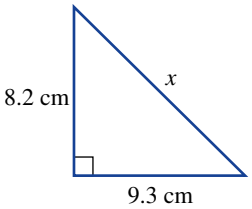
A. 40.89°
B. 60.00°
C. 35.27°
D. 30.00°

13. Calculate x to 2 decimal places.

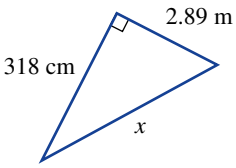
a.



b.

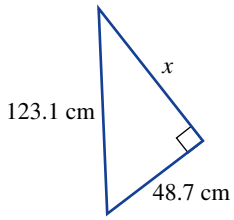


c.

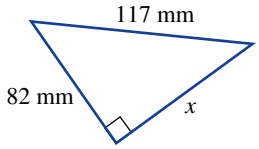


14. Calculate x, correct to 2 decimal places.

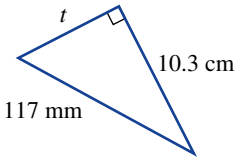
a.



b.



c.

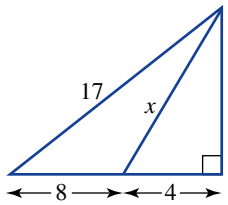


15. The top of a kitchen table measures 160 cm by 90 cm. A beetle walks diagonally across the table. How far does it walk correct to 2 decimal places?

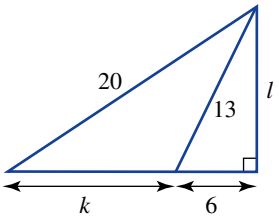
16. A broomstick leans against a wall. The stick is 1.5 m long and reaches 1.2 m up the wall. How far from the base of the wall is the bottom of the broom?

17. Calculate the unknown values in the figures below. Give your answers correct to 2 decimal places.

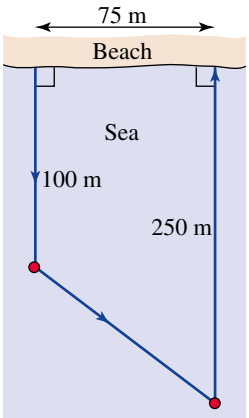
a.



b.



18. An ironwoman race involves 3 swim legs and a beach run back to the start as shown in the figure. What is the total distance covered in the race?



19. Which of the following are Pythagorean triples?

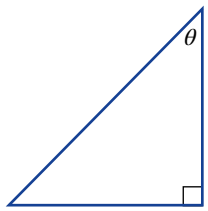
a. 15, 36, 39

b. 50, 51, 10

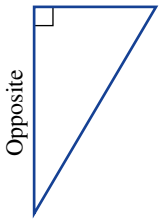
c. 50, 48, 14

20. Label the unlabelled sides of the following right-angled triangles using the words hypotenuse, adjacent and opposite, and the symbol, θ, where appropriate.

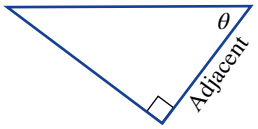
a.



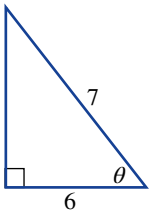
b.

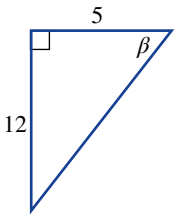


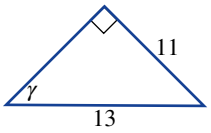
c.



21. Write trigonometric ratios that connect the lengths of the given sides and the size of the given angle in each of the following triangles. Use your ratio to then calculate the size of the angle.

a.


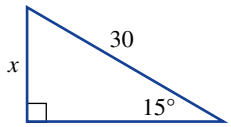
b.


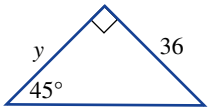
c.


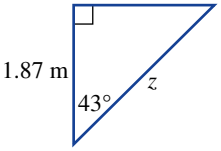
22. Use a calculator to find the value of the following trigonometric ratios, correct to 4 decimal places.

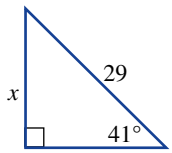
a. sin(54°) **b.** cos(39°) **c.** tan(12°)

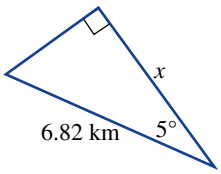
23. Find the values of the pronumerals in each of the following triangles. Give answers correct to 2 decimal places.

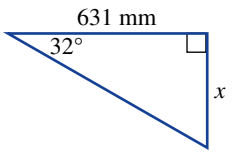
a.


b.


c.


d.


e.


f.


24. Evaluate each of the following, correct to the nearest degree.

a. $\sin^{-1}(0.1572)$ **b.** $\cos^{-1}(0.8361)$ **c.** $\tan^{-1}(0.5237)$

25. Find the size of the angle in each of the following. Give answers correct to the nearest degree.

a. $\sin(\theta) = 0.5321$ **b.** $\cos(\theta) = 0.7071$ **c.** $\tan(\theta) = 0.8235$

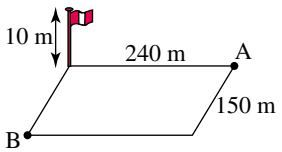
d. $\sin(\alpha) = 0.7891$ **e.** $\cos(\alpha) = 0.3729$ **f.** $\tan(\alpha) = 0.5774$

g. $\sin(\beta) = 0.8660$ **h.** $\cos(\beta) = 0.5050$ **i.** $\tan(\beta) = 8.3791$

26. At a certain time of the day a 6.7-m tall tree casts a shadow 1.87 m long. Find the angle of depression of the rays of the sun at that time. Round to 2 decimal places.

Problem solving

27. A 10-m high flagpole is in the corner of a rectangular park that measures 240 m by 150 m.



a. Find, correct to 2 decimal places:

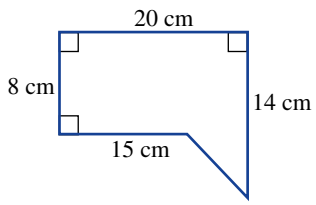
i. the length of the diagonal of the park

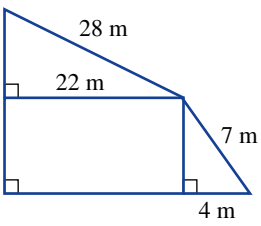
ii. the distance from A to the top of the pole

iii. the distance from B to the top of the pole.

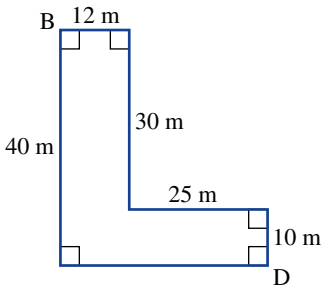
b. A bird flies from the top of the pole to the centre of the park. How far does it fly?

28. Find the perimeters of the shapes below.

a.


b.


29. Calculate the length of the shortest distance between points B and D, inside the figure.



30. Two towers are 30 m apart. From the top of tower A, the angle of depression of the base of tower B is 60°, and the angle of depression of the top of tower B is 30°. What is the height of tower B? Round to the nearest metre.

31. Determine the angles of a triangle whose sides are given by the Pythagorean triple 3, 4, 5.

32. A stack of chairs reaches 2 m in height. They must fit into a doorway which is 1.8 m high. The janitor can safely lean them to an angle of 25° to the vertical. Can the janitor move the stack through the doorway?

33. What is the side-length of the largest square that can be drawn within a circle of radius r? Give your answer as a surd.

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Try out this interactivity: Word search: Topic 6 (int-0889)

Try out this interactivity: Crossword: Topic 6 (int-0703)

Try out this interactivity: Sudoku: Topic 6 (int-3206)

Complete this digital doc: Concept map: Topic 6 (doc-10797)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

angle of depression
angle of elevation
adjacent
cosine ratio
hypotenuse
inverse
opposite
Pythagoras' theorem
Pythagorean triad
right-angled triangle
sine ratio
tangent ratio
trigonometric inverses
trigonometric ratios

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Investigation | Rich task

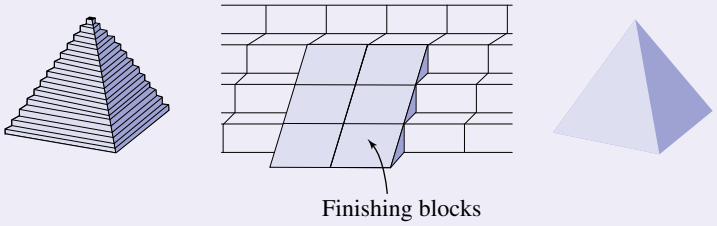
The Great Pyramid of Giza



The Great Pyramid of Giza was built over four and a half thousand years ago. It was constructed using approximately 2 300 000 rectangular granite blocks and took over 20 years to complete. When built, its dimensions measured 230 m at the base and its vertical height was 146.5 m.

1. Each side of the pyramid has a triangular face. Use the dimensions given and Pythagoras' theorem to calculate the height of each triangle. Give your answer correct to 2 decimal places.

2. Special finishing blocks were added to the ends of each row of the pyramid to give each triangular face a smooth and flat finish. Calculate the area of each face of the pyramid.



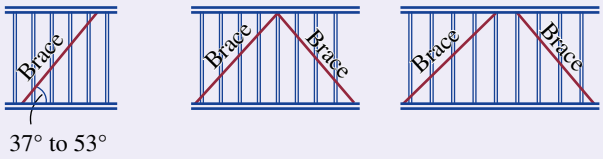
3. The edge of the pyramid joins two faces from the ground to the tip of the pyramid. Use Pythagoras' theorem to calculate the length of the edge.

Wall braces

In the building industry, wall frames are strengthened with the use of braces. These braces run between the top and bottom horizontal sections of the frame.



Industry standards stipulate that the acute angle the brace makes with the horizontal sections lies in the range 37° to 53°. Sometimes, more than one brace may be required if the frame is a long one.



1. Cut thin strips of cardboard and arrange them in the shape of a rectangle to represent a rectangular frame. Pin the corners to hold them together. Notice that the frame moves out of shape easily. Attach a brace according to the angle stipulation of the building industry. Write a brief comment to describe what effect the brace had on the frame.
2. Investigate what happens to the length of the brace required as the acute angle with the base increases from 37° to 53°.
3. Use your finding from question **2** to state which angle requires the shortest brace and which angle requires the longest brace.

Most contemporary houses are constructed with a ceiling height of 2.4 metres; that is, the height of the walls from the floor to the ceiling. Use this fact to assist in your calculations for the following questions.

4. Assume you have a section of a wall that is 3.5 metres long. What would be the length of the longest brace possible? Draw a diagram and show your working to support your answer in the space below.
5. What would be the minimum wall length in which two braces were required? Show your working, along with a diagram, in the space provided.
6. Some older houses have ceilings over 2.4 metres. Repeat questions **4** and **5** for a frame with a height of 3 metres. Draw diagrams and show your workings to support your answers in the space below.
7. Take the measurements of a wall without windows in your school or at home. Draw a scale drawing of the frame on a separate sheet of paper and show the positions in which a brace or braces might lie. Calculate the length and angle of each brace.

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Complete this digital doc: Code puzzle: What does it mean? (doc-15897)

Answers

Topic 6 Pythagoras and trigometry

Exercise 6.2 Pythagoras' theorem

1. a. i. r **ii.** $r^2 = p^2 + s^2$

b. i. x **ii.** $x^2 = y^2 + z^2$

c. i. k **ii.** $k^2 = m^2 + w^2$

d. i. FU **ii.** $(FU)^2 = (VU)^2 + (VF)^2$

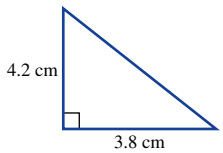
2. a. 7.86 **b.** 33.27 **c.** 980.95

3. a. $x = 12.49$ **b.** $p = 11.76$ cm **c.** $f = 5.14$ m **d.** $c = 97.08$ mm

4. 10.2 cm

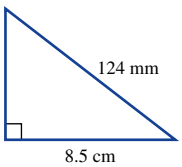
5. a. No **b.** No **c.** Yes **d.** Yes **e.** No **f.** Yes

6. a.



b. 5.66 cm

7. a.



b. 9.28 mm

8. B

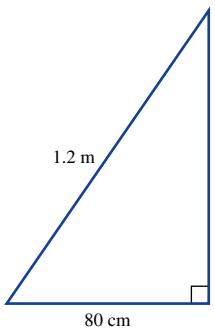
9. C

10. B

11. 2.60 m

12. a. 176.16 cm **b.** 147.40 cm **c.** 2.62 km **d.** 432.12 m

13. a.



b. 89.44 cm

c. Yes, she will reach the hook from the top step.

14. Answers will vary.

15. The horizontal distance is 11.74 m, so the gradient is 0.21, which is within the limits.

16. a. i. {9, 40, 41} **ii.** {11, 60, 61} **iii.** {13, 84, 85} **iv.** {29, 420, 421}

b. The middle number and the large number are one number apart.

17. Check with your teacher.

Exercise 6.3 Applications of Pythagoras' theorem

1. a. 12.08 cm

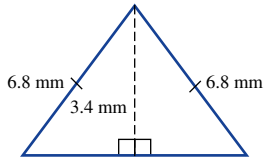
2. a. $k = 16.40$ m **b.** $x = 6.78$ cm **c.** $g = 4.10$ km

3. a. $x = 4, y = 9.17$ **b.** $x = 6.93, y = 5.80$

c. $x = 13, y = 15.20$ **d.** $x = 2.52, y = 4.32$

4. a. 30.48 cm **b.** 2.61 cm **c.** 47.27 cm

5. a.



b. 11.78 mm **c.** 20.02 mm^2

6. a. 0.87 m

b. 0.433 m^2

7. E

8. Yes, 1.015 m

9. No, 17.72 cm

10. 2.42 m

11. a. $w = 0.47$ m **b.** 0.64 m^2 **c.** 12.79 m^2 **d.** \$89.85

12. 12 mm; 480 mm^2

13. 17.9 cm

14. Even though a problem may be represented in 3-D, right-angled triangles in 2-D can often be found within the problem. This can be done by drawing a cross-section of the shape or by looking at individual faces of the shape.

15. a. AD, DC, AC

b. AD = 4.47 cm, DC = 9.17 cm, AC = 13.64 cm

c. The triangle ABC is not right-angled because $(AB)^2 + (BC)^2 \neq (AC)^2$.

16. Answers will vary.

17. Answers will vary.

18. a. 2606 mm

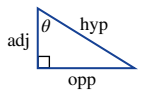
b. Answers will vary.

19. 16.7 m

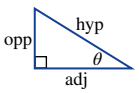
20. $w = 3.536$ m, $x = 7.071$ cm, $y = 15.909$ cm, $z = 3.536$ cm

Exercise 6.4 What is trigonometry?

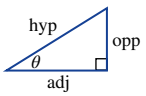
1. a.



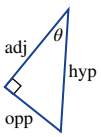
b.



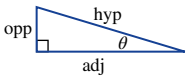
c.



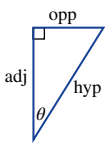
d.



e.



f.



2. a. DE = hyp DF = opp $\angle E = \theta$ **b.** GH = hyp IH = adj $\angle H = \theta$ **c.** JL = hyp KL = opp $\angle J = \theta$

3.

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
a	45°	0.71	0.71	1.00
b	35°	0.57	0.82	0.70

4. D

5. a. i. $\sin(\theta) = \frac{4}{5}$ **ii.** $\cos(\theta) = \frac{3}{5}$ **iii.** $\tan(\theta) = \frac{4}{3}$

b. i. $\sin(\alpha) = \frac{i}{g}$ **ii.** $\cos(\alpha) = \frac{h}{g}$ **iii.** $\tan(\alpha) = \frac{i}{h}$

c. i. $\sin(\beta) = 0.8$ **ii.** $\cos(\beta) = 0.6$ **iii.** $\tan(\beta) = 1.3$

d. i. $\sin(\gamma) = \frac{24}{25}$ **ii.** $\cos(\gamma) = \frac{7}{25}$ **iii.** $\tan(\gamma) = \frac{24}{7}$

e. i. $\sin(\beta) = \frac{b}{c}$ **ii.** $\cos(\beta) = \frac{a}{c}$ **iii.** $\tan(\beta) = \frac{b}{a}$

f. i. $\sin(\gamma) = \frac{v}{u}$ **ii.** $\cos(\gamma) = \frac{t}{u}$ **iii.** $\tan(\gamma) = \frac{v}{t}$

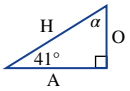
6. a. $\sin(\theta) = \frac{12}{15} = \frac{4}{5}$ **b.** $\cos(\theta) = \frac{25}{30} = \frac{5}{6}$ **c.** $\tan(\theta) = \frac{4}{5}$

d. $\tan(\theta) = \frac{2.7}{p}$ **e.** $\sin(35°) = \frac{17}{t}$ **f.** $\sin(\alpha) = \frac{14.3}{17.5}$

g. $\sin(15°) = \frac{7}{x}$ **h.** $\tan(\theta) = \frac{20}{31}$ **i.** $\cos(\alpha) = \frac{3.1}{9.8}$

7. a. D **b.** B

8. a.



b. O = 34 mm, A = 39 mm, H = 52 mm

c. i. $\sin(41°) = 0.66$ **ii.** $\cos(41°) = 0.75$ **iii.** $\tan(41°) = 0.87$

d. $\alpha = 49°$

e. i. $\sin(49°) = 0.75$ **ii.** $\cos(49°) = 0.66$ **iii.** $\tan(49°) = 1.15$

f. They are equal.

g. They are equal.

h. The sine of an angle is equal to the cosine of its complement.

9. a. The missing angle is also 45°, so the triangle is an isosceles triangle, therefore $a = b$.

b. 1

10. Provided n is a positive value, $(m + n)$ would be the hypotenuse, as it has a greater value than both m and $(m–n)$.

11. a. Ground **b.** Ladder **c.** Brick wall

12. a. The ratio of the length of the opposite side to the length of the hypotenuse will increase.

b. The ratio of the length of the adjacent side will decrease, and the ratio of the opposite side to the adjacent will increase.

c. i. 1 **ii.** 1 **iii.** ∞

Exercise 6.5 Calculating unknown side lengths

1. a. i. 0.8192 **ii.** 0.2011

b.

θ	0°	15°	30°	45°	60°	75°	90°
$\sin(\theta)$	0	0.26	0.50	0.71	0.87	0.97	1.00

c. As θ increases, so does $\sin(\theta)$, starting at 0 and increasing to 1.

2. a. i. 0.7880 **ii.** 0.5919

b.

θ	0°	15°	30°	45°	60°	75°	90°
$\cos(\theta)$	1.00	0.97	0.87	0.71	0.50	0.26	0

c. As θ increases, $\cos(\theta)$ decreases, starting at 1 and decreasing to 0.

3. a. i. 0.3249 **ii.** 1.2753

b.

θ	0°	15°	30°	45°	60°	75°	90°
$\tan(\theta)$	0	0.27	0.58	1.00	1.73	3.73	Undefined

c. $\tan(89°) = 57.29$, $\tan(89.9°) = 572.96$

d. As θ increases, $\tan(\theta)$ increases, starting at 0 and becoming very large. There is no value for tan (90°).

4. a. 13.02 m **b.** 7.04 m **c.** 27.64 mm **d.** 2.79 cm **e.** 6.27 m **f.** 14.16 m

5. a. 2.95 cm **b.** 25.99 cm **c.** 184.73 cm **d.** 14.06 km **e.** 8.43 km **f.** 31.04 m

6. a. 26.96 mm **b.** 60.09 cm **c.** 0.84 km **d.** 0.94 km **e.** 5.59 m **f.** 41.67 m

g. 54.73 m **h.** 106.46 cm **i.** 298.54 mm

7. a. $a = 17.95$, $b = 55.92$

b. $a = 15.59$, $b = 9.00$, $c = 10.73$

c. $a = 12.96$, $b = 28.24$, $c = 15.28$

8. a. D **b.** B **c.** A **d.** D

9. a. 275.75 km **b.** 48.62 km

10. 21.32 m

11. 285.63 m

12. a, b Answers will vary.

c. $AC = \dfrac{AB}{\tan(\theta)}$

13. Answers will vary.

14. a. $x = 12.87$ m **b.** $h = 3.00$ m **c.** $x = 2.60$ m

15. $w = 41$ mm

16. a. 5.9 m **b.** 5.2 m

17. 4 m

Challenge 6.1

2.47 km; 0.97 km

Exercise 6.6 Calculating unknown angles

1. a. 39° **b.** 72° **c.** 37° **d.** 53° **e.** 69° **f.** 71°
g. 79° **h.** 77° **i.** 15°

2. a. 19° **b.** 42° **c.** 55° **d.** 21° **e.** 49° **f.** 80°
g. 35° **h.** 45° **i.** 41° **j.** 23° **k.** 58° **l.** 80°

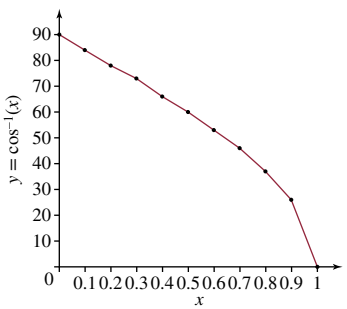
3. a. 47° **b.** 45° **c.** 24° **d.** 43° **e.** 45° **f.** 18°
g. 26° **h.** 12° **i.** 76°

4. a. D **b.** B **c.** D **d.** C

5. a.

x	$y = \cos^{-1}(x)$
0.0	90°
0.1	84°
0.2	78°
0.3	73°
0.4	66°
0.5	60°
0.6	53°
0.7	46°
0.8	37°
0.9	26°
1.0	0°

b.



6. 30.56°

7. 21.80°

8. a. 2.31 m

b. Answers will vary.

9. Answers will vary.

10. a. $\theta = 26°$ **b.** $\theta = 32.01°$ **c.** $x = 6.41°$

11. a. The roof would not be high enough. **b.** 136°

12. Answers will vary.

Challenge 6.2

13. 54.74°

14. Large square: 5 cm × 5 cm

Large triangles: 5 cm × 5 cm

Small square: $\frac{5\sqrt{2}}{2}$ cm × $\frac{5\sqrt{2}}{2}$ cm

Small triangles: $\frac{5\sqrt{2}}{2}$ cm × $\frac{5\sqrt{2}}{2}$ cm

Parallelogram: $5\sqrt{2}$ cm × 5 cm

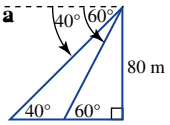
Exercise 6.7 Angles of elevation and depression

1. 8.13°. No, the ramps do not meet specifications.

2. 14.11 m

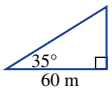
3. 66.35 m

4. a.



b. 49 m

5. a.



b. 42 m

6. 1.6 m

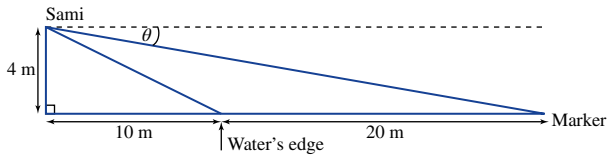
7. 63°

8. 34°, 3 m

9. Con: 34°, John: 30°

10. B

11. a.



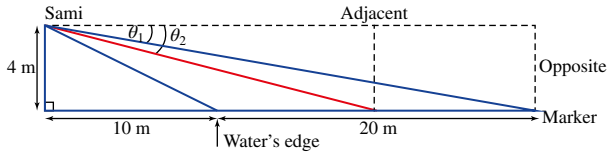
b. $\tan(\theta) = \frac{4}{30}$

$\theta = \tan^{-1}\left(\frac{4}{30}\right)$

$\theta \approx 7.595$

$\approx 7.6^\circ$

c.



As the dolphins swim towards Sami, the adjacent length decreases and the opposite remains unchanged.

$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$

Therefore, θ will increase as the adjacent length decreases.

If the dolphins are at the water's edge,

$\tan(\theta) = 10$

$\theta = \tan^{-1}\left(\frac{4}{10}\right)$

$\theta \approx 21.801$

$\approx 21.8^\circ$

12. Answers will vary.

13. a. Answers will vary. **b.** 1.64 m

14. Answers will vary.

15. Answers will vary.

16. a. Answers will vary.

b. 37°

c. i. Yes

ii. 17.26 m

6.8 Review

1. C **2.** D **3.** D **4.** D **5.** D **6.** C

7. B **8.** B **9.** D **10.** C **11.** A **12.** D

13. a. 11.06 m

b. 12.40 cm

c. 429.70 cm or 4.30 m

14. a. 113.06 cm

b. 83.46 mm

c. 55.50 mm or 5.55 cm

15. 183.58 cm

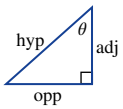
16. 0.9 m

17. a. $x = 12.69$ **b.** $l = 11.53$, $k = 10.34$

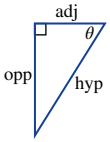
18. 593 m

19. a. True **b.** False **c.** True

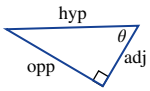
20. a.



b.



c.



21. a. $\cos(\theta) = \frac{6}{7}$, $\theta = 31°$ **b.** $\tan(\beta) = \frac{12}{5}$, $\beta = 67°$ **c.** $\sin(\gamma) = \frac{11}{13}$, $\gamma = 58°$

22. a. 0.8090 **b.** 0.7771 **c.** 0.2126

23. a. 7.76 **b.** 36.00 **c.** 2.56 m

d. 19.03 **e.** 6.79 km **f.** 394.29 mm

24. a. 9° **b.** 33° **c.** 28°

25. a. 32° **b.** 45° **c.** 39°

d. 52° **e.** 68° **f.** 30°

g. 60° **h.** 60° **i.** 83°

26. 74.41°

27. a. i. 283.02 m

ii. 240.21 m

iii. 150.33 m

b. 141.86 m

28. a. 64.81 cm

b. 84.06 m

29. 59.24 m

30. 35 m

31. 90°, 53° and 37°

32. No. The stack of chairs must lean 25.84° to fit through the doorway, which is more than the safe lean angle of 25°.

33. $\frac{2r}{\sqrt{2}} = \sqrt{2}r$

Investigation — Rich task

The Great Pyramid of Giza

1. 186.25 m **2.** 21 418.75 m^2 **3.** 218.89 m

Wall braces

1. Answers will vary.

2. Answers will vary.

3. 53° requires the shortest brace and 37° requires the longest brace.

4. 4.24 m

5. 3.62 m

6. 4.61 m; 4.52 m

7. Answers will vary.

learnON

Project: Learning or earning?



Scenario

Year 9 Students at Progressive High School have seen the paper *Are young people learning or earning*? produced by the Australian Bureau of Statistics. They start the week discussing their thoughts and then decide to do their own research. Simon decides that he will use this research to demonstrate to his parents that life is very different to when they were his age and that he is capable of undertaking a part-time job and keeping up with his studies.

Task

For this project, you will need to read the ABS article, summarise it, and then design a survey to research the topic. Your findings will reflect the work–study balance of the students in your school and demographic. After completing your survey, you will produce a presentation for your parents outlining the ABS article and detailing your research and the conclusions you have drawn.

Process

Open the ProjectsPLUS application for this chapter in the Resources tab.
Watch the introductory video lesson, and read the articles provided. Answer the questions provided in the Are Young People Learning or Earning file in the Media Centre. Summarise the thoughts of the article in 200 words or less.

- Use the **Wordle** weblink in the Resources tab and select the **create** tab. Copy the summary of your article into the text box. Keep selecting the **randomise** button until you are happy with the result. Print your final choice. Take a screenprint of your final choice. Take it into **Paint** for use as a slide in your presentation. Save your Paint file.
- Use **Surveymonkey** to survey 100 students at your school to determine who has part-time jobs and how long they work at these each week. You are able to ask only 10 questions per survey. Record your 10 questions in Word. When planning your survey think carefully about the types of questions you want to ask. For example, do you want to know why they have a part-time job?



Your 100 students need to represent the school as a population. How will you make your sample representative of this? Will it be a random sample or a stratified sample? Explain. What about the gender balance? Justify your decision. How will you notify the chosen students that they need to do the survey? What instructions will you give to the students completing your survey? How long will they have to complete it? What will you do to ensure they have all completed it? Complete the survey table provided in the Media Centre. Type your instructions to each person completing the survey. Copy this into the Survey instructions template in the Media Centre.

- **Analysis.** Record the results from your survey in a frequency distribution table. Use the Results table provided in the Media Centre. Write a paragraph summarising your findings. Include mean, median, mode and range for each year group. What percentages of the students have part-time jobs? Is there a trend as the students get older (are more or less students working)? Include a statement as to why you still want to get a part-time job. Represent your findings in a frequency histogram and frequency polygon. Use the Excel template for your results and include your graphs on that sheet. If you were to do this research again, is there anything that you would change? Why? Are there any better resources for your research? What is the mean, median and mode and range of your results?
- Open the **Suburb statistics labour force by age** weblink. Type in your postcode. Follow the prompts to download the labour force statistics by age, and by sex for your postcode, and save. If you live in an urban area repeat for a rural region, if you live in a rural region, repeat for an urban area. Save the Excel files columns for the rural and urban postcodes. Use Excel to create a column graph with a series of columns. See Sample spreadsheet file. Include the graph in your Prezi file with an explanation of what you did.
- Download the Prezi sample and the Prezi planning template to help you prepare your presentation. Your Media Centre also includes images that can help to liven up your presentation. As you arrange your images on your Prezi page make them form a large circle so that they flow smoothly when they are linked and presented.
- Use the Prezi template to develop your presentation. Remember that you are trying to convince your parents that you should be able to undertake a part-time job. Make sure you include all the results of your research, and that your presentation will grab their attention. To include tables in Prezi you need to take them into paint and save the file as a jpeg in order to upload them. Use Word to type up your dialogue to your parents when you present your case (200–500 words).

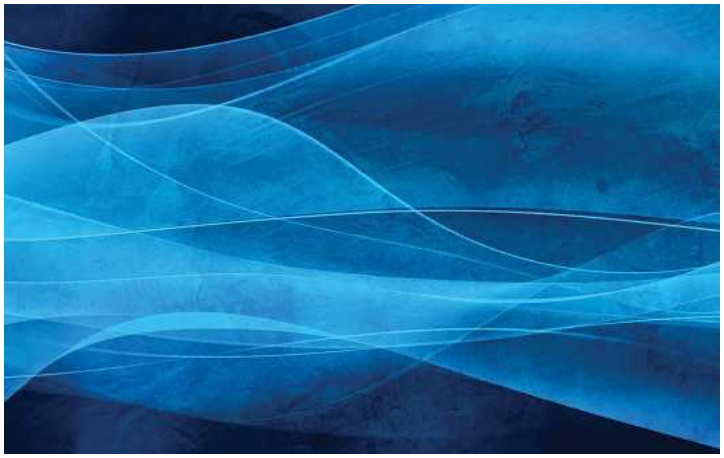


Suggested Software

- ProjectsPLUS
- Microsoft Word
- PowerPoint, Prezi, Keynote or other presentation software
- Microsoft Excel
- Surveymonkey
- Wordle

TOPIC 7
Linear and non-linear graphs

7.1 Overview



Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

7.1.1 Why learn this?

It is very common that one quantity depends on another quantity, and we can often use mathematical modelling to gain an understanding of the situation. If we know the equation for a function, we can draw a graph and use the graph to analyse, interpret and explain the relationship between the variables, and to make predictions about the future. Science, engineering, health and finance all rely heavily on using mathematical equations to model real-life situations and solve problems. Slope, gradient and distance between points are important concepts to understand.

7.1.2 What do you know?

assessON

1. **LIST** what you know about linear graphs. Use a thinking tool such as a concept map to show your list.
2. **SHARE** what you know with a partner and then with a small group.
3. **AS** a class, create a thinking tool such as a large concept map to show your class's knowledge of linear graphs.

LEARNING SEQUENCE

7.1 Overview
7.2 Plotting linear graphs
7.3 The equation of a straight line
7.4 Sketching linear graphs
7.5 Technology and linear graphs
7.6 Determining linear rules
7.7 Practical applications of linear graphs
7.8 Midpoint of a line segment and distance between two points
7.9 Non-linear relations (parabolas, hyperbolas, circles)
7.10 Review

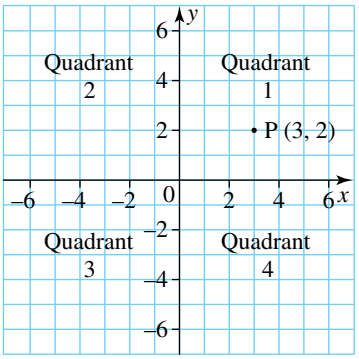
learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: How fast can humans go? (eles-1694)

7.2 Plotting linear graphs

7.2.1 The Cartesian plane

- The **Cartesian plane** is divided into 4 regions (quadrants) by the x- and y-axes, as shown at right.
- Every point in the plane is described exactly by a pair of coordinates (x, y). The point P (3, 2) is marked on the diagram.



7.2.2 Plotting from a rule

- A graph can be drawn by plotting a series of points on a Cartesian plane. To do this requires:
  1 a set of x-values
  2 a rule.

WORKED EXAMPLE 1

TI | CASIO

Plot the graph specified by the rule $y = x + 2$ for the x-values $-3, -2, -1, 0, 1, 2, 3$.

THINK

1 Draw a table and write in the required x-values.

WRITE/DRAW

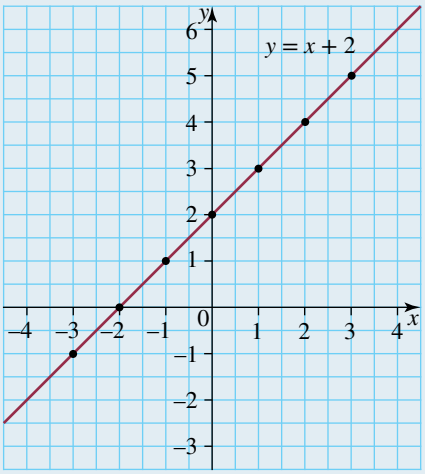
x	-3	-2	-1	0	1	2	3
y							

2 Substitute each x-value into the rule $y = x + 2$ to obtain the corresponding y-value.
When $x = -3$, $y = -3 + 2 = -1$.
When $x = -2$, $y = -2 + 2 = 0$ etc.
Write the y-values into the table.

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

3 Plot the points from the table: $(-3, 1)$ etc.

4 Join the points with a straight line and label the graph with its equation, $y = x + 2$.



- A straight line graph is called a **linear graph** and its rule is called a linear relation. The rule for a linear graph can always be written in the form $y = mx + c$, for example $y = 4x - 5$ or $y = x + 1.2$.

- If a graph is linear, then a minimum of two points need be plotted to locate the straight line. It is sensible to choose points that are some distance apart and to use a third point to check an error has not been made.

WORKED EXAMPLE 2

Plot two points and hence draw the linear graph $y = 2x - 1$.

THINK

1 Choose any two x-values, for example $x = -2$ and $x = 3$.

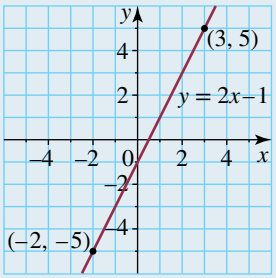
2 Calculate y by substituting each x-value into $y = 2x - 1$.
$y = 2 \times -2 - 1 = -5$
$y = 2 \times 3 - 1 = 5$

3 Plot the points $(-2, -5)$ and $(3, 5)$.

4 Draw a line through the points and add a label.

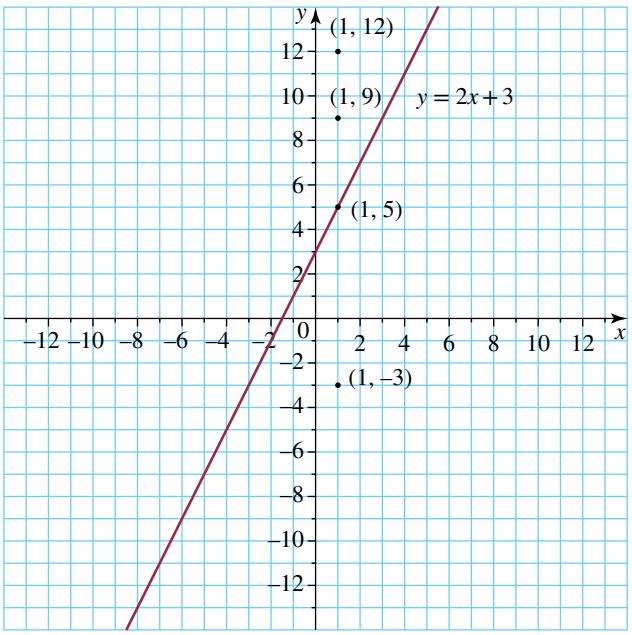
WRITE/DRAW

x	-2	3
y	-5	5



7.2.3 Points on a line

- Consider the line that has the rule $y = 2x + 3$ as shown in the graph.
  If $x = 1$, then $y = 2(1) + 3$
  $= 5$
  So the point $(1, 5)$ lies on the line $y = 2x + 3$.
- The points $(1, 0)$, $(1, -3)$, $(1, 9)$, $(1, 12)$ … are not on the line, but lie above or below it.



WORKED EXAMPLE 3

Does the point (2, 4) lie on the line given by:
a $y = 3x - 2$ **b** $x + y = 5$?

THINK	WRITE
a 1 Substitute $x = 2$ into the equation $y = 3x - 2$ and find y.	**a** $y = 3x - 2$ $x = 2$: $y = 3(2) - 2$ $= 6 - 2$ $= 4$
2 When $x = 2$, $y = 4$, so the point (2, 4) lies on the line. Write the answer.	The point (2, 4) lies on the line $y = 3x - 2$.
b 1 Substitute $x = 2$ into the equation $x + y = 5$ and find y.	**b** $x + y = 5$ $x = 2$: $2 + y = 5$ $y = 3$
2 The point (2, 3) lies on the line, but the point (2, 4) does not. Write the answer.	The point (2, 4) does not lie on the line $x + y = 5$.

learnON RESOURCES — ONLINE ONLY

- **Try out this interactivity:** Drawing a graph (int-1020)
- **Complete this digital doc:** SkillSHEET: Plotting coordinate points (doc-6161)
- **Complete this digital doc:** SkillSHEET: Substituting into a rule (doc-6162)
- **Complete this digital doc:** SkillSHEET: Completing a table of values (doc-6163)
- **Complete this digital doc:** SkillSHEET: Plotting a line from a table of values (doc-6164)

Exercise 7.2 Plotting linear graphs

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12	Questions: 1–10, 12, 13	Questions: 1–14

Individual pathway interactivity: int-4502

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. MC **a.** The point with coordinates (−2, 3) is:

A. in quadrant 1 **B.** in quadrant 2
C. in quadrant 3 **D.** in quadrant 4

b. The point with coordinates $(-1, -5)$ is:
- **A.** in the first quadrant
- **B.** in the second quadrant
- **C.** in the third quadrant
- **D.** in the fourth quadrant

c. The point with coordinates $(0, -2)$ is:
- **A.** in the third quadrant
- **B.** in the fourth quadrant
- **C.** on the x-axis
- **D.** on the y-axis

2. WE1 For each of the following rules, complete the table below and plot the linear graph.

x	-3	-2	-1	0	1	2	3
y							

a. $y = x$ **b.** $y = 2x + 2$
c. $y = 3x - 1$ **d.** $y = -2x$

Understanding

3. WE2 By first plotting 2 points, draw the linear graph given by each of the following.

a. $y = -x$ **b.** $y = \frac{1}{2}x + 4$
c. $y = -2x + 3$ **d.** $y = x - 3$

4. WE3 Do these points lie on the graph of $y = 2x - 5$?

a. $(3, 1)$ **b.** $(-1, 3)$
c. $(0, 5)$ **d.** $(5, 5)$

5. Does the given point lie on the given line?

a. $y = -x - 7$, $(1, -8)$ **b.** $y = 3x + 5$, $(0, 5)$
c. $y = x + 6$, $(-1, 5)$ **d.** $y = 5 - x$, $(8, 3)$
e. $y = -2x + 11$, $(5, -1)$ **f.** $y = x - 4$, $(-4, 0)$
g. $y = 7x - 11$, $(1, -4)$ **h.** $2x + y = 10$, $(3, 4)$

6. MC The line that passes through the point $(2, -1)$ is:

A. $y = -2x + 5$ **B.** $y = 2x - 1$
C. $y = -2x + 1$ **D.** $x + y = 1$

7. Match each point with a line passing through that point.

a. $(1, 1)$ **b.** $(1, 3)$
c. $(1, 6)$ **d.** $(1, -4)$
A. $x + y = 4$ **B.** $2x - y = 1$
C. $y = 3x - 7$ **D.** $y = 7 - x$

Reasoning

8. The line through $(1, 3)$ and $(0, 4)$ passes through every quadrant except one. Which one? Explain your answer.

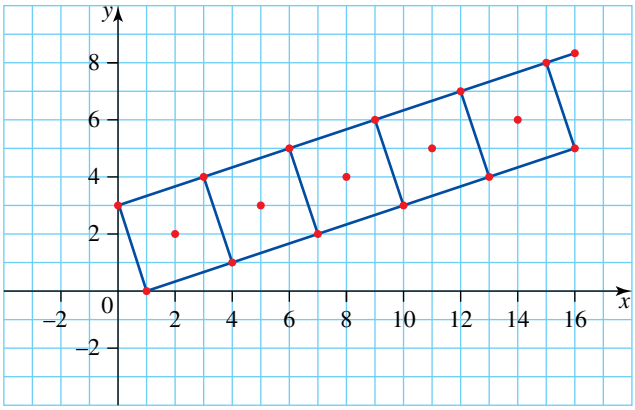
9. a. Which quadrant(s) does the line $y = x + 1$ pass through?
b. Show that the point $(1, 3)$ does not lie on the line $y = x + 1$.

10. Explain the process of how to check whether a point lies on a given line.

11. Using the coordinates $(-1, -3)$, $(0, -1)$ and $(2, 3)$, show that a rule for the linear graph is $y = 2x - 1$.

Problem solving

12. Consider this pattern of squares on the grid shown.



What would be the coordinates of the centre of the 20th square?

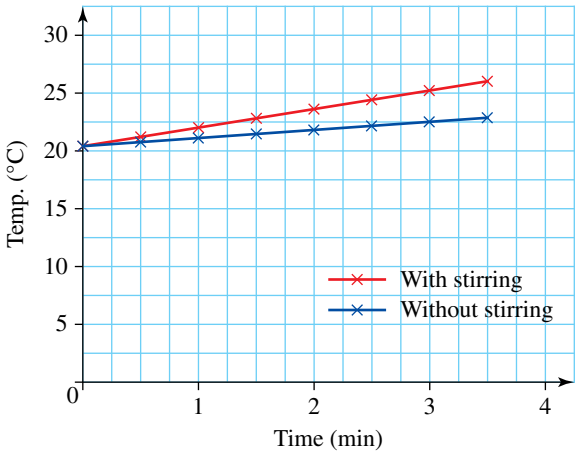
13. It is known that the mass of a certain kind of genetically modified tomato increases linearly over time. The following results were recorded.



Time, *t* (weeks)	1	4	6	9	16
Mass, *m* (grams)	6	21	31	46	81

a. Plot the above points on a Cartesian plane.
b. Determine the rule connecting mass with time.
c. Show that the mass after 20 weeks is 101 grams.

14. As a particular chemical reaction proceeds, the temperature increases at a constant rate. The graph at right represents the same chemical reaction with and without stirring. How does stirring affect the reaction?



Reflection

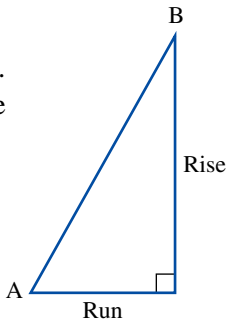
In linear equations, what does the coefficient of *x* determine?

7.3 The equation of a straight line

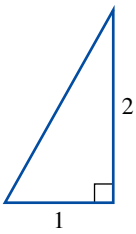
- A straight line goes on forever, and has constant steepness or **gradient**.



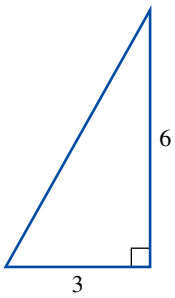
7.3.1 The gradient (m)



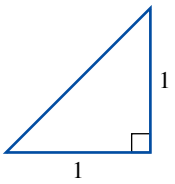
- The gradient of an interval (portion of a line) is equal to the gradient of the entire line.
- The gradient of an interval AB is defined as the distance up (rise) divided by the distance across (run), and is usually given the symbol m.
- So $m = \dfrac{\textbf{rise}}{\textbf{run}}$.
- Compare these intervals and their gradients.



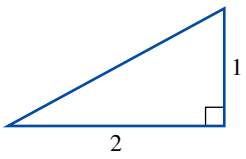
$m = 2$



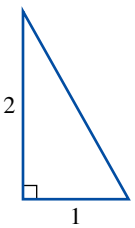
$m = 2$



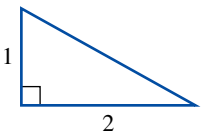
$m = 1$



$m = \frac{1}{2}$



$m = -2$



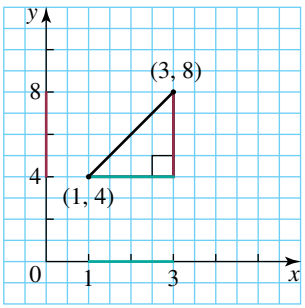
$m = -\frac{1}{2}$

- Note that if the line is sloping downwards (from left to right), the gradient has a negative value.

7.3.2 Finding the gradient of a line passing through two points

- Suppose a line passes through the points (1, 4) and (3, 8), as shown in the graph below.
- By completing a right-angled triangle, it can be seen that the rise = 8 − 4 (the difference in y-values), and the run = 3 − 1 = 2 (the difference in x-values). So

$$m = \frac{8-4}{3-1} = \frac{4}{2} = 2$$



- In general, if the line passes through the points (x_1, y_1) and (x_2, y_2), then

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

WORKED EXAMPLE 4

Find the gradient of the line passing through the points (−2, 5) and (1, 14).

THINK	WRITE
1 Let the two points be (x_1, y_1) and (x_2, y_2).	$(-2, 5) = (x_1, y_1), (1, 14) = (x_2, y_2)$
2 Write the formula for gradient.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

3 Substitute the coordinates of the given points into the formula and evaluate.

$$= \frac{14 - 5}{1 - -2}$$
$$m = \frac{9}{1 + 2}$$
$$m = \frac{9}{3}$$
$$= 3$$

4 Write the answer.

The gradient of the line passing through $(-2, 5)$ and $(1, 14)$ is 3.

Note: Let $(x_1, y_1) = (1, 14)$ and $(x_2, y_2) = (-2, 5)$.

The calculation becomes $m = \frac{y_2 - y_1}{x_2 - x_1}$

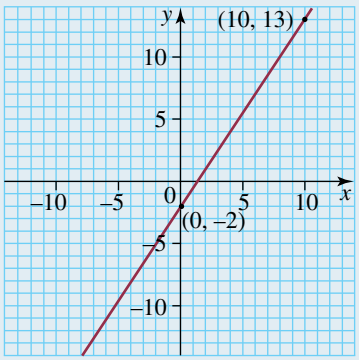
$$= \frac{5 - 14}{-2 - 1}$$
$$= \frac{-9}{-3}$$
$$= 3$$

The result is the same.

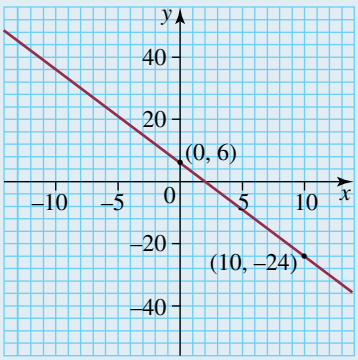
WORKED EXAMPLE 5

Calculate the gradients of the lines shown.

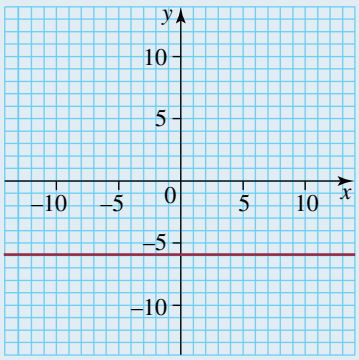
a



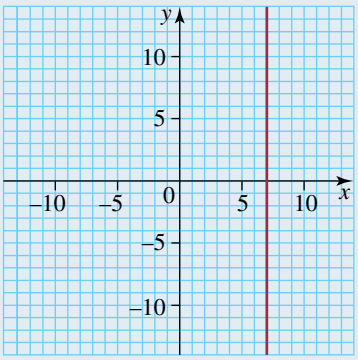
b



c



d



THINK

a 1 Write down two points that lie on the line.

WRITE

a Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (10, 13)$.
Rise $= y_2 - y_1 = 13 - -2 = 15$
Run $= x_2 - x_1 = 10 - 0 = 10$

2 Calculate the gradient by finding the ratio $\frac{\text{rise}}{\text{run}}$.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{15}{10}$$
$$= \frac{3}{2} \text{ or } 1.5$$

b 1 Write down two points that lie on the line.

b Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (10, -24)$.
Rise $= y_2 - y_1 = -24 - 6 = -30$
Run $= x_2 - x_1 = 10 - 0 = 10$

2 Calculate the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-30}{10}$$
$$= -3$$

c 1 Write down two points that lie on the line.

c Let $(x_1, y_1) = (5, -6)$ and $(x_2, y_2) = (10, -6)$.

2 There is no rise between the two points.

Rise $= y_2 - y_1$
$= -6 - -6 = 0$
Run $= x_2 - x_1$
$= 10 - 5 = 5$

3 Calculate the gradient.
Note that the gradient of a horizontal line is always zero. The line has no slope.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{0}{5}$$
$$= 0$$

d 1 Write down two points that lie on the line.

d Let $(x_1, y_1) = (7, 10)$ and $(x_2, y_2) = (7, -3)$.

2 The vertical distance between the selected points is 13 units. There is no run between the two points.

Rise $= y_2 - y_1 = -3 - 10 = 13$
Run $= x_2 - x_1 = 7 - 7 = 0$

3 Calculate the gradient.
Note: The gradient of a vertical line is always undefined.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{13}{0} \text{ undefined}$$

7.3.3 Finding the gradient of a straight line from its rule

- When an equation is written in the form $y = mx + c$, m is the value of the gradient.
  For example, consider the line with equation $y = 3x + 1$. The gradient is 3.
- To confirm this, find the gradient using the formula.
  Two points that lie on the line $y = 3x + 1$ are (0, 1) and (5, 16).

$$\text{Gradient} = \frac{16 - 1}{5 - 0}$$
$$= \frac{15}{5}$$
$$= 3$$

WORKED EXAMPLE 6

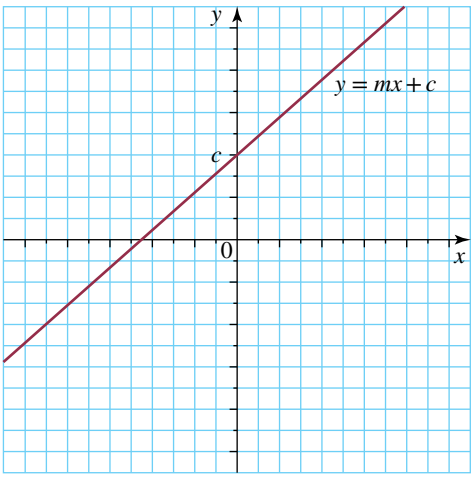
Find the gradients of the straight lines whose rules are given.

a $y = -2x + 3$ **b** $2y - 3x = 6$ **c** $y = 4$

THINK	WRITE
a The equation is the form $y = mx + c$, so the gradient is the coefficient of x.	**a** $y = -2x + 3$ $m = -2$
b 1 First rearrange the given rule so that it is in the form $y = mx + c$. (Add $3x$ to both sides, then divide both sides by 2.)	**b** $2y - 3x = 6$ $2y = 6 + 3x$ $y = \frac{6}{2} + \frac{3}{2}x$ $y = \frac{3}{2}x + 3$
2 Write the value of the gradient.	$m = \frac{3}{2}$
c 1 Rewrite the equation in the form $y = mx + c$.	**c** $y = 4$
2 Write the value of the gradient.	$m = 0$

7.3.4 The *y*-intercept

- For the line given by $y = mx + c$, when $x = 0$, $y = c$.
- The line passes through the point $(0, c)$. This is the point where the graph cuts the y-axis.
- The point where the graph cuts the y-axis is called the **y-intercept**.
- In this case the y-intercept is $(0, c)$, often simply called c.
- The y-intercept of any line is easily found by substituting 0 for x and calculating the y-value.
- $y = mx + c$ is called the 'gradient–intercept form' of the equation of a line, because it plainly displays the gradient (m) and the y-intercept (c).



WORKED EXAMPLE 7

TI | CASIO

Find the y-intercepts of the lines whose linear rules are given, and hence state the coordinates of the y-intercept.

a $y = -4x + 7$ **b** $5y + 2x = 10$ **c** $y = 2x$ **d** $y = -8$

THINK	WRITE
a The rule is in the gradient–intercept form, $y = mx + c$. The y-intercept is the value of c. State the coordinates.	**a** $y = -4x + 7$ $c = 7$ y-intercept: $(0, 7)$
b 1 To find the y-intercept, substitute $x = 0$ into the equation.	**b** $5y + 2x = 10$ $5y + 2(0) = 10$
2 Solve for y.	$5y = 10$ $y = 2$
3 Write the coordinates of the y-intercept.	y-intercept: $(0, 2)$

c The rule is in the gradient–intercept form, $y = mx + c$. The y-intercept is the value of c. State the coordinates.

c $y = 2x$
$c = 0$
y-intercept: $(0, 0)$

d The rule is in the form $y = mx + c$. State the coordinates.

d $y = -8$
$y = 0x - 8$
$c = -8$
y-intercept: $(0, -8)$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Transposing a linear equation to general form (doc-6165)
Complete this digital doc: SkillSHEET: Finding the gradient given two points (doc-10839)
Complete this digital doc: SkillSHEET: Measuring the rise and the run (doc-10840)
Complete this digital doc: SkillSHEET: Finding the gradient of a line from its equation (doc-10841)
Complete this digital doc: WorkSHEET: Linear graphs (doc-6170)

Exercise 7.3 The equation of a straight line

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–e, 3a–f, 4, 5a–f, 6–11, 15–16	Questions: 1d–i, 2c–f, 3e–j, 4, 5c–j, 6, 7b–e, 8, 9–12, 15–17	Questions: 1g–l, 2e–i, 3g–l, 4, 5f–l, 6, 7c–e, 8–19

Individual pathway interactivity: int-4503

learnON ONLINE ONLY

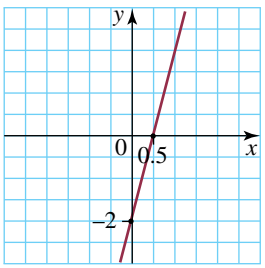
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

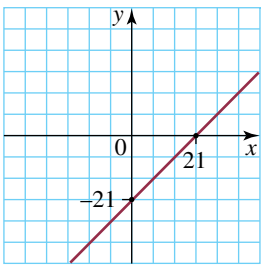
Fluency

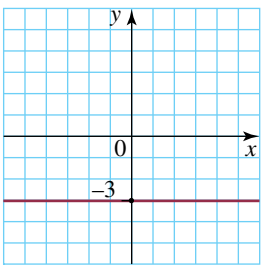
1. WE4 Find the gradients of the lines passing through the following pairs of points.

a. (2, 10) and (4, 22)
b. (1, −2) and (3, −10)
c. (−3, 0) and (7, 0)
d. (−4, −7) and (1, −1)
e. (0, 4) and (4, −4.8)
f. (−2, 122) and (1, −13)
g. (2, 3) and (17, 3)
h. (−2, 2) and (2, 2.4)
i. (1, −5) and (5, −15.4)
j. (−12, −7) and (8.4, −7)
k. (−2, −17.7) and (0, 0.3)
l. (−3, 3.4) and (5, 2.6)

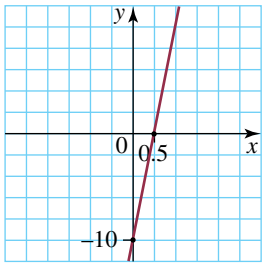
2. WE5 Calculate the gradients of the lines shown.

a.


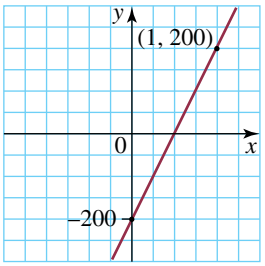
b.


c.


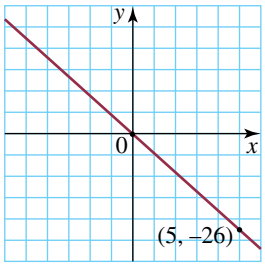
d.



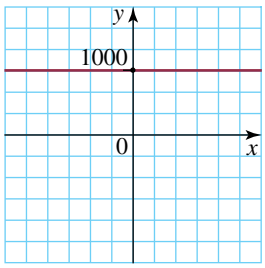
e.



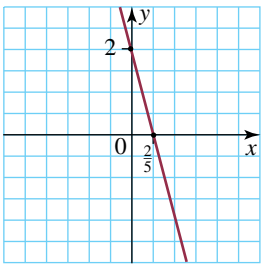
f.



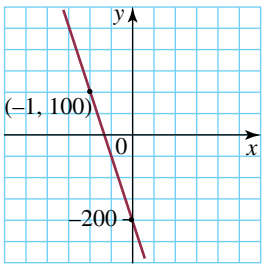
g.



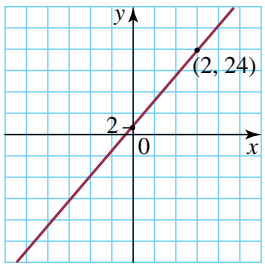
h.



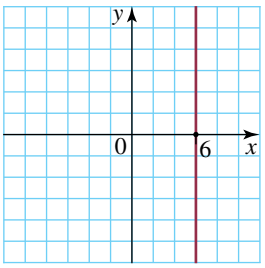
i.



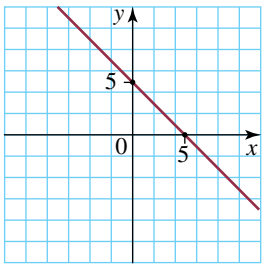
j.



k.



l.



3. WE6 Find the gradients of the straight lines whose rules are given below.

a. $y = 5x + 23$ **b.** $y = 54 - 3x$ **c.** $y = 3(x - 2)$

d. $y = 70 - 2x$ **e.** $y = \frac{1}{2}(x + 2)$ **f.** $y = \frac{x}{2} + 5.2$

g. $y = 100 - x$ **h.** $y = 100$ **i.** $y = -87 + 23x$

j. $2y - 4x - 5 = 0$ **k.** $2y + 4x = -5$ **l.** $6y - 21x = -19$

4. WE7 Find the y-intercepts of the lines whose linear rules are given in question **3**.

5. Write each equation in gradient–intercept form, then find the gradient and y-intercept of the lines defined by the following rules.

a. $y = 4x + 8$ **b.** $y = -4x + 8$ **c.** $y = -2x + 7$

d. $y = 12x$ **e.** $y = 0.5x + 2.5$ **f.** $y = -40x + 83$

g. $y + 4x = -18$ **h.** $5y - 6x = -18$ **i.** $y = \frac{x}{2} + \frac{3}{2}$

j. $15y - x = 0$ **k.** $3y - 9x = 15$ **l.** $8x - 2y = 16$

Understanding

6. MC Which of the following statements about linear graphs is *false*?

A. A gradient of zero means the graph is a horizontal line.

B. Gradient can be any real number.

C. A linear graph can have two y-intercepts.

D. In the form $y = mx + c$, the y-intercept equals c.

7. In the form $y = mx + c$, the y-intercept is c and the gradient is m. Find a formula for the x-intercept in terms of m and c. (*Hint:* The x-intercept is the point where the graph crosses the x-axis. At such a point $y = 0$.)

8. Find the coordinates of the y-intercepts of the lines with the following rules.

a. $y = -6x - 10$ **b.** $3y + 3x = -12$

c. $7x - 5y + 15 = 0$ **d.** $y = 7$

e. $x = 9$

Reasoning

9. Explain why the gradient of a vertical line is undefined.

10. Explain why the gradient of a horizontal line is zero.

11. Show the gradient of the line passing through the points (a, b) and (c, d) is $\frac{d - b}{c - a}$.

12. a. Sketch a line for the rule $y = -x + b$, if the value of b can be any value.

b. Explain the effect of the change in b for the line $y = -x + b$.

13. Find how many points (x, y) satisfy the equation $x + y = 25$, assuming that the values of both x and y must be:

a. equal

b. positive integers.

14. Water is leaking out of a water tank such that the amount of water remaining in the tank is given by the formula $V = 3000 - 48t$, where V is the volume of water in the tank in litres and t is the time it takes to drain in minutes.



a. How much water was in the tank before it started leaking?

b. How much water is left in the tank after leaking for 20 minutes?

c. Show that if the tank is checked after 65 minutes there will be no water left.

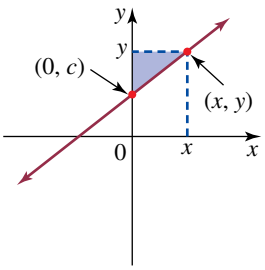
d. Use this information to plot the graph of V against t and state the intercepts.

e. In the context of this problem, explain what the gradient of -48 means.

Problem solving

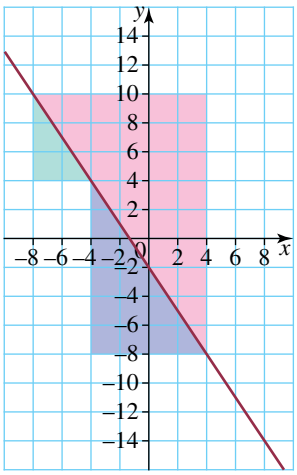
15. When using the gradient to draw a line, does it matter if you rise before you run or run before you rise? Explain.

16. a. Using the graph at right, write a general formula for the gradient m in terms of x, y and c.



b. Transpose your formula to make y the subject. What do you notice?

17. Three right-angled triangles have been superimposed on the graph below.

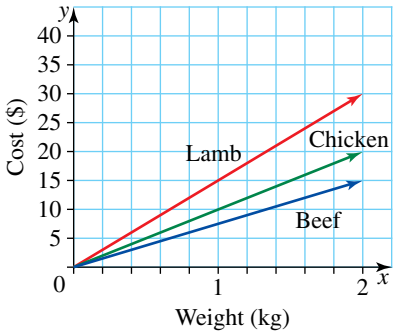


a. Use each of these to determine the gradient of the line.
b. Does it matter which points are chosen to determine the gradient of a line? Explain.
c. Describe the shape of the graph.

18. Lilly has entered the competition for the Readers' Award which awards prizes for the students who read the most books from 1 February to 31 May. First prize is awarded to a student who has read more than 50 books over the four months, second prize is awarded to a student who has read from 30 to 50 books and third prize is awarded to a student who has read between 15 and 30 books. At the beginning of the nth week, Lilly had read 5 books.

Lilly then read an average of b books each week until the end of the competition and qualified for second prize. Explain why the least value of b must be 1.56.

19. The price per kilogram for 3 different types of meat is illustrated in the graph.





a. Calculate the gradient (using units) for each graph.
b. What is the cost of 1 kg of each type of meat?
c. What is the cost of purchasing:
i. 1 kg of lamb
ii. 0.5 kg of chicken
iii. 2 kg of beef?
d. What is the total cost of the order in part **c**?
e. Copy and complete the table below to confirm your answer from part **d**.

Meat type	Cost per kilogram ($/kg)	Weight required (kg)	Cost = $/kg × kg
Lamb		1	
Chicken		0.5	
Beef		2	
		Total cost	

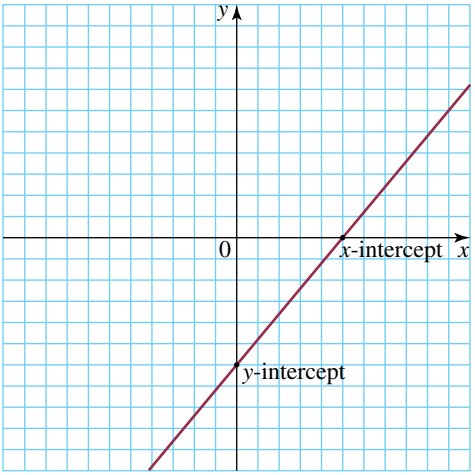
Reflection

Why is the y-intercept of a graph found by substituting $x = 0$ into the equation?

7.4 Sketching linear graphs

7.4.1 The *x*- and *y*-intercept method

- To use this method, the *x*-intercept (where the line crosses the *x*-axis and $y = 0$) and the *y*-intercept (where the line crosses the *y*-axis and $x = 0$) must be known.
- The line is drawn by locating each intercept, then drawing a straight line through those points.
- If both intercepts are at the origin, another point is needed to sketch the line.



WORKED EXAMPLE 8

Using the *x*- and *y*-intercept method, sketch the graphs of:

a $2y + 3x = 6$ **b** $y = \frac{4}{5}x + 5$ **c** $y = 2x$.

THINK

a 1 Write the rule.

2 To find the *y*-intercept, let $x = 0$.
Write the coordinates of the *y*-intercept.

3 To find the *x*-intercept, let $y = 0$.
Write the coordinates of the *x*-intercept.

4 Plot and label the *x*- and *y*-intercepts on a set of axes and rule a straight line through them. Label the graph.

b 1 Write the rule.

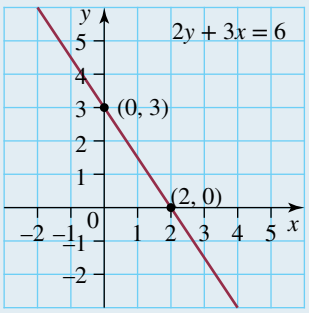
2 The rule is in the form $y = mx + c$, so the *y*-intercept is the value of *c*.

WRITE/DRAW

a $2y + 3x = 6$

$x = 0$: $2y + 3 \times 0 = 6$
$2y = 6$
$y = 3$
y-intercept: (0, 3)

$y = 0$: $2 \times 0 + 3x = 6$
$3x = 6$
$x = 2$
x-intercept: (2, 0)



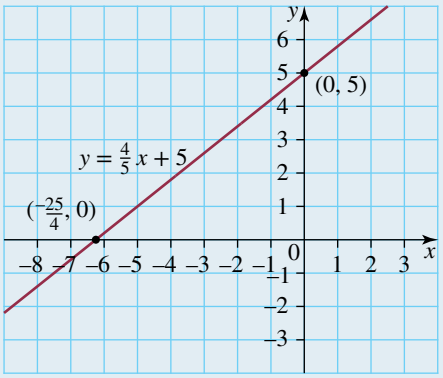
b $y = \frac{4}{5}x + 5$

$c = 5$
y-intercept: (0, 5)

3 To find the x-intercept, let $y = 0$. Write the coordinates of the x-intercept.

$$y = \frac{4}{5}x + 5$$
$y = 0$: $0 = \frac{4}{5}x + 5$
$$-5 = \frac{4}{5}x$$
$$x = -\frac{25}{4} \left(= -6\frac{1}{4}\right)$$
x-intercept: $\left(-\frac{25}{4}, 0\right)$

4 Plot and label the intercepts on a set of axes and rule a straight line through them. Label the graph.



c 1 Write the rule.

c $y = 2x$

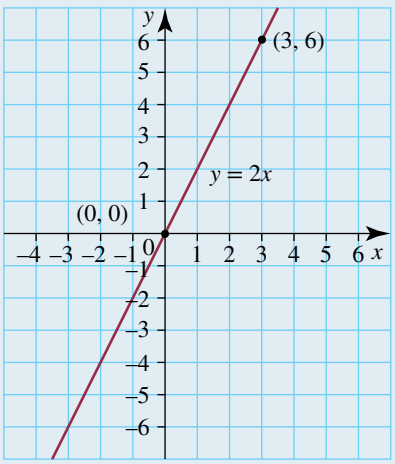
2 To find the y-intercept, let $x = 0$.
Write the coordinates of the y-intercept.

$x = 0$: $y = 2 \times 0$
$= 0$
y-intercept: $(0, 0)$

3 The x- and y-intercepts are the same point, $(0, 0)$, so one more point is required.
Choose any value for x, such as $x = 3$.
Substitute and write the coordinates of the point.

$x = 3$: $y = 2 \times 3$
$= 6$
Another point: $(3, 6)$

4 Plot the points, then rule and label the graph.
Label the graph.

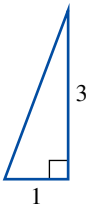


7.4.2 The gradient–intercept method

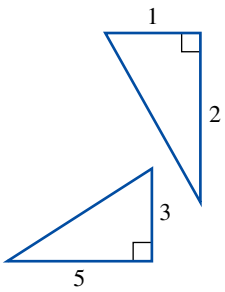
- To use this method, the gradient and the y-intercept must be known.
- The line is drawn by plotting the y-intercept, then drawing a line with the correct gradient through that point.

Note:

– A line interval of gradient 3 $\left(=\frac{3}{1}\right)$ can be drawn with a rise of 3 and a run of 1.
– Similarly, a line interval with a gradient of -2 $\left(=\frac{-2}{1}\right)$ can be shown as an interval sloping downwards.



– A line interval with a gradient of $\frac{3}{5}$ can be shown with rise = 3 and run = 5.



WORKED EXAMPLE 9

Using the gradient–intercept method, sketch the graphs of:

a $y = \frac{3}{4}x + 2$
b $4x + 2y = 3$.

THINK

a 1 From the equation, the y-intercept is 2.
Plot the point (0, 2).

2 From the equation, the gradient is $\frac{3}{4}$, so $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$.
From (0, 2), run 4 units and rise 3 units.
Mark the point P (4, 5).

3 Draw a line through (0, 2) and P (4, 5).
Label the graph.

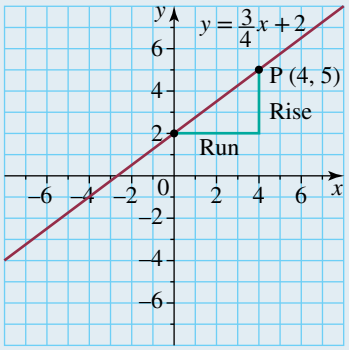
b 1 Write the rule in gradient–intercept form: $y = mx + c$.
From the equation, $m = -2$, $c = \frac{3}{2}$.
Plot the point $(0, \frac{3}{2})$.

2 The gradient is -2, so $\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$.
From $(0, \frac{3}{2})$, run 1 units and rise -2 units (i.e. go down 2 units). Mark the point P$(1, -\frac{1}{2})$.

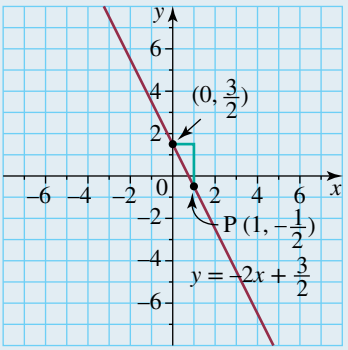
3 Draw a line through $(0, \frac{3}{2})$ and P$(1, -\frac{1}{2})$. Label the graph.

WRITE/DRAW

a



b $4x + 2y = 3$
$2y = 3 - 4x$
$y = \frac{3}{2} - 2x$
$y = -2x + \frac{3}{2}$

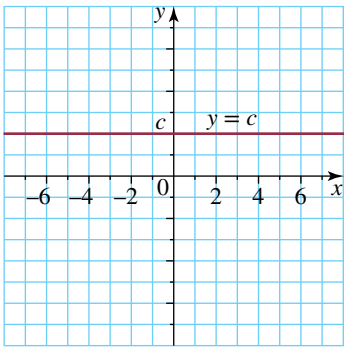


7.4.3 Vertical and horizontal lines

$y = c$

- $y = c$ is the same as $y = 0x + c$.
- This is a line with gradient 0 and y-intercept c.
- As a fraction, $0 = \frac{0}{3}, \frac{0}{4}$ and so on; therefore, a line with gradient of 0 has a rise of 0 and a run of any length except 0. This is a horizontal line.
- Using a table to find points on the line $y = c$ gives:

x	-2	0	2	4
y	c	c	c	c



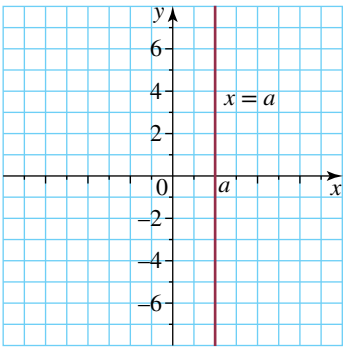
$x = a$

- This equation implies that $x = a$, no matter what value y may take.
- A table of values looks like this:

x	a	a	a	a
y	-2	0	2	4

Plotting these points gives a vertical line, as shown at right.

- The run of the graph is 0, so using the formula $m = \frac{\text{rise}}{\text{run}}$ involves dividing by zero, which cannot be done. The gradient is said to be **undefined**.



WORKED EXAMPLE 10

a On a pair of axes, sketch the graphs of:
i $x = -3$
ii $y = 4$.
b Label the point of intersection of the two lines.

THINK

a i 1 The line $x = -3$ is in the form $x = a$. This is a vertical line.

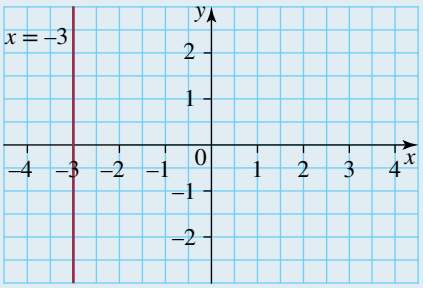
2 Rule the vertical line where $x = -3$. Label the graph.

ii 1 The line $y = 4$ is in the form $y = c$. This is a horizontal line.

2 Rule the horizontal line where $y = 4$. Label the graph.

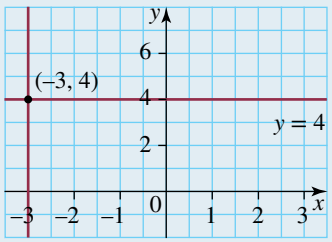
WRITE/DRAW

a



b The lines intersect at $(-3, 4)$.

b



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Complete this digital doc: SkillSHEET: Graphing linear equations using intercepts (doc-10842)

Complete this digital doc: SkillSHEET: Solving linear equations that arise when finding intercepts (doc-10843)

Exercise 7.4 Sketching linear graphs

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9, 13	Questions: 1–11, 13, 14	Questions: 1–15

Individual pathway interactivity: int-4504

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Fluency

1. WE8 Sketch the graphs of the following by finding the x- and y-intercepts.

a. $5y - 4x = 20$ **b.** $y = x + 2$ **c.** $y = -3x + 6$ **d.** $3y + 4x = -12$

e. $y = 2x - 4$ **f.** $x - y = 5$ **g.** $x + y = 4$ **h.** $2y + 7x - 8 = 0$

2. WE9 Sketch the graphs of the following using the gradient–intercept method.

a. $y = x - 7$ **b.** $y = 2x + 1$ **c.** $y = 2x + 2$ **d.** $y = -2x + 2$

e. $y = \frac{1}{2}x - 1$ **f.** $y = 4 - x$ **g.** $y = \frac{5}{4}x + 5$ **h.** $y = -x - 10$

3. WE10 Sketch the graphs of the following.

a. $y = 4$ **b.** $y = -3$ **c.** $y = -12.5$ **d.** $y = \frac{4}{5}$

4. Sketch the graphs of the following.

a. $x = 2$ **b.** $x = -6$ **c.** $x = -2.5$ **d.** $x = \frac{3}{4}$

5. Sketch the graphs of the following.

a. $y = 3x$ **b.** $y = -2x$ **c.** $y = \frac{3}{4}x$ **d.** $y = -\frac{1}{3}x$

6. MC **a.** Which of the following statements about the rule $y = 4$ is not true?

A. The gradient $m = 0$.

B. The y-intercept is at $(0, 4)$.

C. The graph is parallel to the x-axis.

D. The point $(4, 2)$ lies on this graph.

b. Which of the following statements is not true about the rule $y = -\frac{3}{5}x$?

A. The graph passes through the origin.

B. The gradient $m = -\frac{3}{5}$.

C. The x-intercept is at $x = 0$.

D. The graph can be sketched using the x- and the y-intercept method.

Understanding

7. a. $2x + 5y = 20$ is a linear equation in the form $ax + by = c$.
- **i.** Rearrange this equation into the form $y = mx + c$.
- **ii.** What is the gradient?
- **iii.** State the x- and y-intercepts.
- **iv.** Sketch this straight line.

b. If the x-intercept of a straight line is -3 and the y-intercept is 5:
- **i.** state the gradient
- **ii.** draw the graph
- **iii.** write the equation in the form $y = mx + c$ and the form $ax + by = c$.

Reasoning

8. Consider the relationship $4x - 3y = 24$.
- **a.** Rewrite this relationship, making y the subject.
- **b.** Show that the x- and y-intercepts are (6, 0) and (0, -8) respectively.
- **c.** Sketch a graph of this relationship.

9. a. Rewrite the relationship $ax + by = c$, making y the subject.
- **b.** If a, b and c are positive integer values, explain how the gradient is negative.

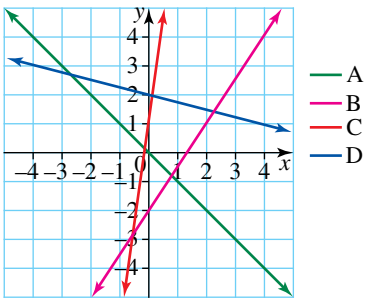
10. Josie accidently spilled a drink on her work. Part of her calculations were smudged. The line $y = \frac{1}{2}x + \frac{3}{4}$ was written in the form $ax + 4y = 3$. Show that the value of $a = -2$.

11. Explain why the descriptions 'right 3 up 2', 'right 6 up 4', 'left 3 down 2', 'right $\frac{3}{2}$ up 1' and 'left 1 down $\frac{2}{3}$' all describe the same gradient for a straight line.

12. Find the slope of the line containing the points (3, 5) and (6, 11). Find coordinates for another point that lies on the same line. Explain the method used to find the coordinate and gradient.

Problem solving

13. a. Match the descriptions given below with their corresponding line.
- **i.** Straight line with a y-intercept of (0, 1) and a positive gradient
- **ii.** Straight line with a gradient of $1\frac{1}{2}$
- **iii.** Straight line with a gradient of -1



b. Write a description for the unmatched graph.

14. a. Sketch the linear equation $y = -\frac{5}{7}x - \frac{3}{4}$:
- **i.** using the y-intercept and the gradient
- **ii.** using the x- and y-intercepts
- **iii.** using two other points.

b. Compare and contrast the methods and generate a list of advantages and disadvantages for each method. Which method do you think is best? Why?

15. Consider these two linear graphs.

$$y - ax = b \text{ and } y - cx = d.$$

Show that if these two graphs intersect where both x and y are positive, then $a > c$ when $d > b$.

Reflection

Why are gradients of vertical lines undefined?

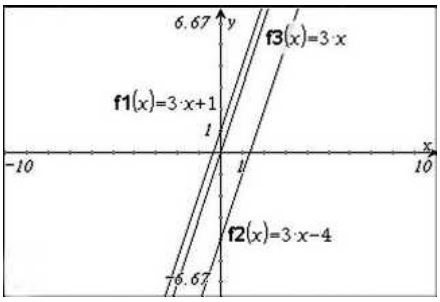
7.5 Technology and linear graphs

7.5.1 Graphing with technology

- There are many digital technologies that can be used to graph linear relationships. These include but are not limited to graphing calculators.
- Digital technologies can be very useful when you want to draw multiple graphs in order to investigate important features.

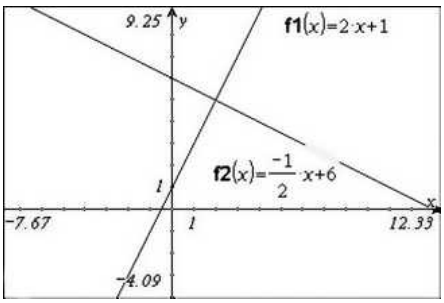
7.5.2 Parallel lines

- Lines with the same gradient are called **parallel lines**.
  That is, $m_1 = m_2$.
  For example, $y = 3x + 1$, $y = 3x - 4$ and $y = 3x$ are all parallel lines, because $m = 3$.



7.5.3 Perpendicular lines

- Lines that meet at right angles are called **perpendicular lines**.
- The product of the gradients of two perpendicular lines is equal to -1.
  That is, $m_1 \times m_2 = -1$.
  For example, $y = 2x + 1$ and $y = -\frac{1}{2}x + 6$ are perpendicular, because $2 \times -\frac{1}{2} = -1$.



Exercise 7.5 Technology and linear graphs

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 15	Questions: 1–12, 15–16	Questions: 1–14, 15–17

Individual pathway interactivity: int-4505

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Understanding

Use technology wherever possible to answer the following questions.

1. On the same Cartesian plane, sketch the following graphs.
a. $y = x$ **b.** $y = 2x$ **c.** $y = 3x$
The steepness of a graph is called the gradient of the line.
i. What happens to the steepness of the graph as the coefficient of x increases in value?
ii. Where does each graph cut the x-axis? (That is, what is the x-intercept?)
iii. Where does each graph cut the y-axis? (That is, what is the y-intercept?)

2. On another Cartesian plane, sketch the following graphs.
a. $y = -x$ **b.** $y = -2x$ **c.** $y = -3x$
i. What happens to the steepness of the graph as the magnitude of the coefficient of x decreases in value? (That is, the coefficient becomes more negative.)
ii. Where does each graph cut the x-axis? (That is, what is the x-intercept?)
iii. Where does each graph cut the y-axis? (That is, what is the y-intercept?)

3. Copy the following sentences, deleting the incorrect response.
a. If the coefficient of x is **(positive/negative)**, then the graph will have an upward slope to the right. That is, the gradient of the graph is **(positive/negative)**.

b. If the coefficient of x is negative, then the graph will have a **(downward/upward)** slope to the right. That is, the gradient of the graph is **(positive/negative)**.

c. The bigger the magnitude of the coefficient of x (more positive or more negative), the **(bigger/smaller)** the steepness of the graph.

d. If there is no constant term in the equation, the graph **(will/will not)** pass through the origin.

4. On another Cartesian plane, sketch the following graphs.

a. $y = x$ **b.** $y = x + 2$ **c.** $y = x - 2$

i. Is the coefficient of x the same for each graph? If so, what is it?

ii. Does the steepness (or gradient) of each graph differ?

iii. Where does each graph cut the x-axis? (That is, what is the x-intercept?)

iv. Where does each graph cut the y-axis? (That is, what is the y-intercept?)

5. On another Cartesian plane, sketch the following graphs.

a. $y = -x$ **b.** $y = -x + 2$ **c.** $y = -x - 2$

i. Is the coefficient of x the same for each graph? If so, what is it?

ii. Does the steepness (or gradient) of each graph differ?

iii. Where does each graph cut the x-axis? (That is, what is the x-intercept?)

iv. Where does each graph cut the y-axis? (That is, what is the y-intercept?)

6. Copy the following sentences, deleting the incorrect response.

i. For a given set of linear graphs, if the coefficient of x is the **(same/different)**, the graphs will be parallel.

ii. The constant term in the equation is the **(*y*-intercept/*x*-intercept)** or where the graph cuts the **(*y*-axis/*x*-axis)**.

iii. The **(*y*-intercept/*x*-intercept)** can be found by substituting $x = 0$ into the equation.

iv. The **(*y*-intercept/*x*-intercept)** can be found by substituting $y = 0$ into the equation.

7. On another Cartesian plane, sketch the following graphs.

a. $y = x + 5$ **b.** $y = -x + 5$ **c.** $y = 3x + 5$ **d.** $y = -\frac{2}{5}x + 5$

i. Is the coefficient of x the same for each graph?

ii. Does the steepness of each graph differ?

iii. Write down the gradient of each linear graph.

iv. Where does each graph cut the x-axis? (That is, what is the x-intercept?)

v. Where does each graph cut the y-axis? (That is, what is the y-intercept?)

8. Copy the following sentences, deleting the incorrect response.

One of the general forms of the equation of a linear graph is $y = mx + c$, where m is the **(steepness/*x*-coordinate)** of the graph. We call the steepness of the graph the gradient. The value of c is the **(*x*-coordinate/*y*-coordinate)** where the graph cuts the **(*x*-axis/*y*-axis)**. All linear graphs with the **(same/different)** gradient are **(parallel/perpendicular)**. All linear graphs that have the same y-intercept pass through **(the same/different)** point on the y-axis.

9. For each of the following lines, write:

i. the gradient **ii.** the y-intercept.

a. $y = 2x$ **b.** $y = x + 1$ **c.** $y = -3x + 5$ **d.** $y = \frac{2}{3}x - 7$

Reasoning

10. Using a CAS calculator, graph the lines $y = 3(x - 1) + 5$, $y = 2(x - 1) + 5$ and $y = -\frac{1}{2}(x - 1) + 5$. What do they all have in common? Explain how they differ from each other.

11. Show on a CAS calculator how $y = \frac{1}{2}x - \frac{5}{3}$ can be written as $6y = 3x - 10$.

12. A telephone company charges \$2.20 for international calls of 1 minute or less and \$0.55 for each additional minute. Using a CAS calculator, graph the cost for calls that last whole numbers of minutes. Explain all the important values needed to sketch the graph.

13. Shirly walks dogs after school for extra pocket money. She finds that she can use the equation $P = -15 + 10N$ to calculate her profit (in dollars) each week.
- **a.** Explain the real-world meaning of the numbers -15 and 10 and the variable N.
- **b.** Explain what the equation means.
- **c.** Using a CAS calculator, sketch the equation.

14. Graph $y = 0.2x + 3.71$ on a CAS calculator. Explain how to use the calculator to find an approximate value when $x = 70.3$.

Problem solving

15. Plot the points (6, 3.5) and $(-1, -10.5)$ using a CAS calculator and:
- **a.** find the equation of the line
- **b.** sketch the graph, showing x- and y-intercepts
- **c.** find the value of y when $x = 8$
- **d.** find the value of x when $y = 12$.

16. A school investigating the price of a site licence for their computer network found that it would cost \$1750 for 30 computers and \$2500 for 60 computers.
- **a.** Using a CAS calculator, find a linear equation that represents the cost of a site licence in terms of the number of computers in the school.
- **b.** What is the y-intercept of the linear equation and how does it relate to the cost of a site licence?
- **c.** How much would it cost for 200 computers?
- **d.** How many computers could you connect for \$3000?



17. Dylan starts his exercise routine by jogging to the gym, which burns 325 calories. He then pedals a stationary bike burning 3.8 calories a minute.
- **a.** Graph the data using a CAS calculator.
- **b.** After 15 minutes of pedalling, how many calories has Dylan burned?
- **c.** How long did it take for Dylan to burn a total of 450 calories?

Reflection

If two lines look like they intersect at right angles, can you assume that they are perpendicular?

7.6 Determining linear rules

7.6.1 Finding the equation of a straight line given the gradient and the *y*-intercept

- For a linear graph, if the gradient (m) is known and the y-intercept (c) is known, then the equation ($y = mx + c$) can be determined.

WORKED EXAMPLE 11 TI | CASIO

Determine the rule of the line whose gradient is -2 and y-intercept is 3.

THINK	WRITE
1 Write the equation of a straight line.	$y = mx + c$
2 Substitute the values $m = -2$, $c = 3$.	$m = -2, c = 3$ $y = -2x + 3$

7.6.2 Finding the rule given the gradient and the coordinates of one point

- If the gradient (m) and any point are known, then the y-intercept can be calculated.

WORKED EXAMPLE 12

Determine the rule of a straight line that goes through the point (1, −3), if its gradient is −2.

THINK	WRITE
1 Write the equation of a straight line.	$y = mx + c$
2 Substitute the value $m = -2$.	$y = -2x + c$
3 Since the line passes through the point (1, −3), substitute $x = 1$ and $y = -3$ into $y = -2x + c$ to find the value of c.	When $x = 1$, $y = -3$. $-3 = -2 \times 1 + c$
4 Solve for c.	$-3 = -2 + c$ $-3 + 2 = c$ $c = -1$
5 Write the rule.	$y = -2x - 1$

7.6.3 Finding the rule given the coordinates of two points

- If two points on the line are known, then the gradient (m) can be calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, or $m = \frac{\text{rise}}{\text{run}}$.
- Using the gradient and one of the points, the equation can be found as above.

WORKED EXAMPLE 13

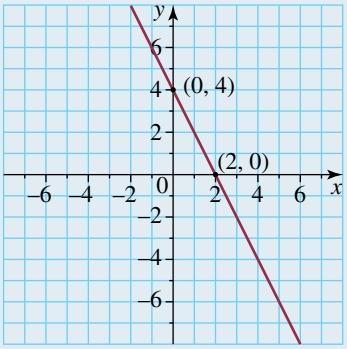
TI | CASIO

Find the rule of the straight line passing through the points (−1, 6) and (3, −2).

THINK	WRITE
1 Write the equation of a straight line.	$y = mx + c$
2 Write the formula for finding the gradient, m.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
3 Let $(x_1, y_1) = (-1, 6)$ and $(x_2, x_2) = (3, -2)$. Substitute the values into the formula and determine the value of m.	$m = \frac{-2 - 6}{3 - -1}$ $= \frac{-2 - 6}{3 + 1}$ $= -\frac{8}{4}$ $= -2$
4 Substitute the value of m into the equation.	$y = -2x + c$
5 Select either of the two points, say (3, −2), and substitute into $y = -2x + c$.	Point (3, −2): $-2 = -2(3) + c$ $-2 = -6 + c$
6 Solve for c.	$c = 4$
7 Write the rule using the values $c = 4$, $m = -2$.	$y = -2x + 4$

WORKED EXAMPLE 14

Find the equation of the linear graph shown.



THINK	WRITE
1 Read the important information from the graph. The y-intercept is 4 and the x-intercept is 2. Write the coordinates of each point.	The graph passes through $(0, 4)$ and $(2, 0)$.
2 Find the gradient using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (0, 4)$.	$m = \frac{4 - 0}{0 - 2}$ $= \frac{4}{-2}$ $= -2$ $y = -2x + c$
3 From the graph, the y-intercept is 4.	$c = 4$
4 Write the equation.	$y = -2x + 4$

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Exercise 7.6 Determining linear rules

assessON

Individual pathways

PRACTISE
Questions:
1a–c, 2, 3a–d, 4, 5a–f, 6a–d, 7a–c, 8a–c, 9–13, 16, 17

CONSOLIDATE
Questions:
1c–e, 2, 3e–h, 4, 5d–i, 6b–e, 7c–e, 8c–e, 9–14, 16–18

MASTER
Questions:
1d–f, 2, 3g–j, 4, 5g–l, 6d–f, 7d–f, 8d–f, 9, 10, 12–15, 16–20

Individual pathway interactivity: int-4506

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Fluency

1. WE11 Determine the rules of the straight lines with the gradients and y-intercepts given below.

a. Gradient = 4, y-intercept = 2
b. Gradient = -4, y-intercept = 1
c. Gradient = 4, y-intercept = 8
d. Gradient = 6, y-intercept = 7
e. Gradient = -2.5, y-intercept = 6
f. Gradient = 45, y-intercept = 135

2. Find the rule for each straight line passing through the origin and with the gradient given.

a. Gradient = -2
b. Gradient of 4
c. Gradient = 10.5
d. Gradient of -20
e. Gradient = 1.07
f. Gradient of 32

3. WE12 Determine the rule for each straight line with the gradient and passing through the point given below.

a. Gradient = 1, point = (3, 5)
b. Gradient = -1, point = (3, 5)
c. Gradient = -4, point = $(-3, 4)$
d. Gradient = 2, point = (5, -3)
e. Gradient = -5, point = (13, 5)
f. Gradient = 2, point = (10, -3)
g. Gradient = -6, point = (2, -1)
h. Gradient = -1, point = $(-2, 0.5)$
i. Gradient = 6, point = $(-6, -6)$
j. Gradient = -3.5, point = (3, 5)
k. Gradient = 1.2, point = $(2.4, -1.2)$
l. Gradient = 0.2, point = $(1.3, -1.5)$

4. Determine the rules when:

a. gradient = -4, x-intercept = -6
b. gradient = 2, x-intercept = 3
c. gradient = -2, x-intercept = 2
d. gradient = 5, x-intercept = -7
e. gradient = 1.5, x-intercept = 2.5
f. gradient = 0.4, x-intercept = 2.4.

5. WE13 Find the rule for each straight line passing through the given points.

a. $(-6, 11)$ and (6, 23)
b. (1, 2) and $(-5, 8)$
c. (4, 11) and (6, 11)
d. (3, 6.5) and (6.5, 10)
e. (1.5, 2) and (6, -2.5)
f. $(-7, 3)$ and (2, 4)
g. (25, -60) and (10, 30)
h. (5, 100) and (25, 500)
i. (1, 3) and (3, 1)
j. (2, 5) and $(-2, 6)$
k. (9, -2) and (2, -4)
l. (1, 4) and $(-0.5, 3)$

6. Determine the rules for the linear graphs that have the following x- and y intercepts.

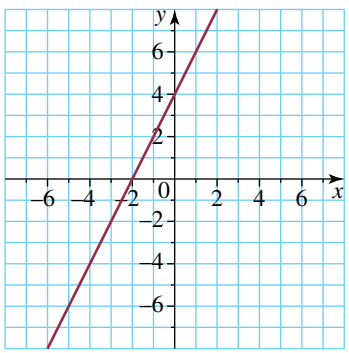
a. x-intercept = -3, y-intercept = 3
b. x-intercept = 4, y-intercept = 5
c. x-intercept = 1, y-intercept = 6
d. x-intercept = -40, y-intercept = 35
e. x-intercept = -8, y-intercept = 8
f. x-intercept = 3, y-intercept = 6
g. x-intercept = -7, y-intercept = -3
h. x-intercept = -200, y-intercept = 50

7. Find the rule for each straight line passing through the origin and:

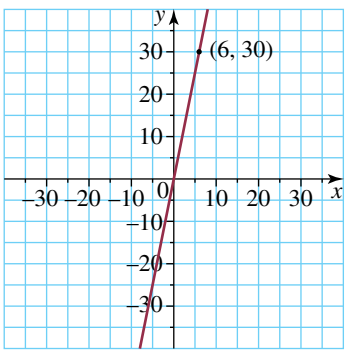
a. the point (4, 7)
b. the point (5, 5)
c. the point $(-4, 8)$
d. the point $(-1.2, 3.6)$
e. the point $(-22, 48)$
f. the point $(-105, 35)$.

8. WE14 Find the equation of the line shown on each of the following graphs.

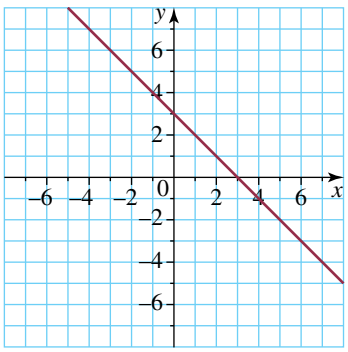
a.



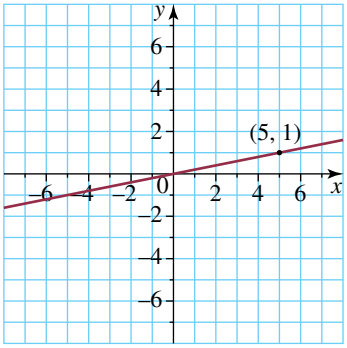
b.



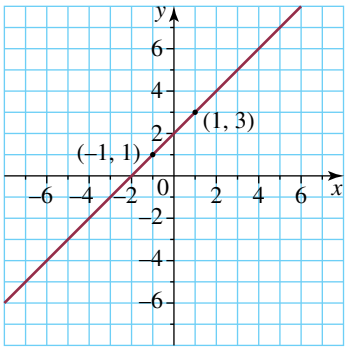
c.



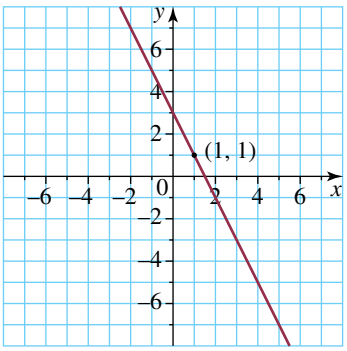
d.



e.



f.



9. MC **a.** The gradient of the straight line that passes through (3, 5) and (5, 3) is:

A. -2 **B.** -1 **C.** 0 **D.** 1

b. A straight line with an x-intercept of 10 and a y-intercept of 20 has a gradient of:

A. -2 **B.** -1 **C.** -0.5 **D.** 0

c. The rule $2y - 3x = 20$ has an x-intercept at:

A. $-\frac{3}{2}$ **B.** $-\frac{2}{3}$ **C.** 0 **D.** None of these

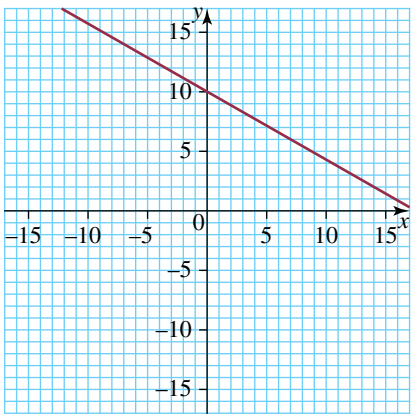
Understanding

10. Given that the x-intercept of a straight line graph is $(-5, 0)$ and the y-intercept is $(0, -12)$:

a. determine the equation of the straight line

b. find the value of y when $x = 19.3$.

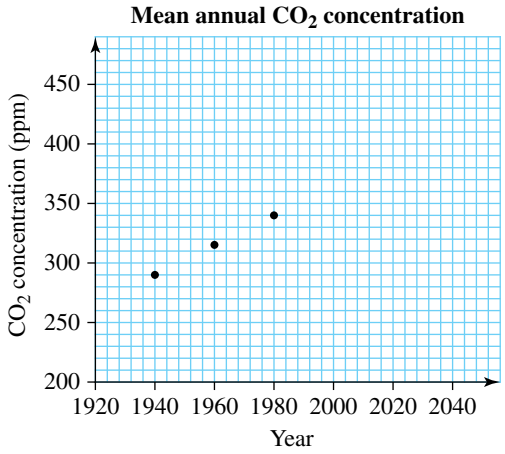
11. a. Determine the equation of the straight line shown in red in the graph below. Use the fact that when $x = 5$, $y = 7$.



b. Determine the x-intercept.

Reasoning

12. The graph below shows the carbon dioxide (CO_2) concentration in the atmosphere, measured in parts per million (ppm).



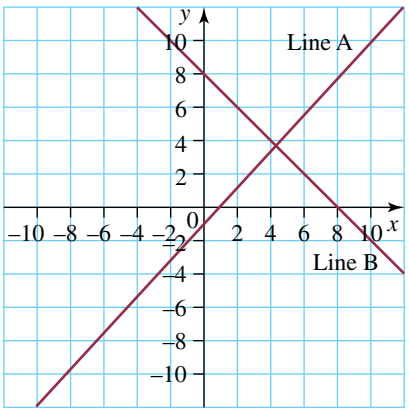
a. If the trend follows a linear pattern, find the equation for the line.

b. Explain why c cannot be read directly from the graph.

c. What might be the concentration of CO_2 in 2020? What is the assumption that is made when finding this value?

13. Show that the equation for the line that passes through the point (3, 6) parallel to the line through the points (0, −7) and (4, −15) is $y = -2x + 12$.

14. a. Determine the equations for line A and line B as shown.

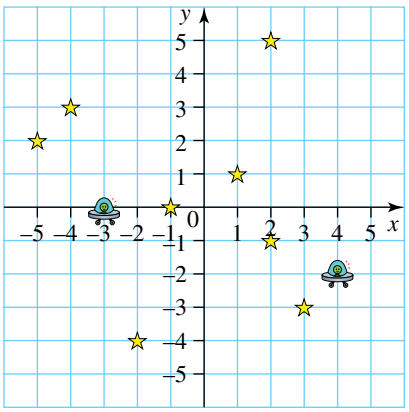


b. Write the point of intersection between line A and line B and mark it on the Cartesian plane.

c. Show that the equation of the line that is perpendicular to line B and passes through the point (−4, 6) is $y = x + 10$.

Problem solving

15. A student plays the game Space Galaxy. She has the Stars and Spaceships on her screen as shown at right.



a. Copy the diagram into your workbook. On the diagram, draw a straight line that will hit 3 stars.

b. What is the equation of the straight line that will hit 3 stars?

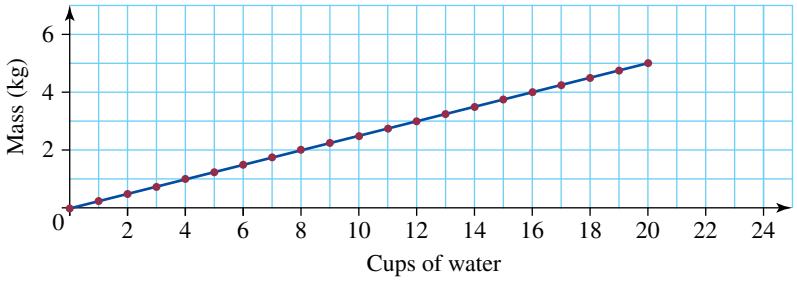
c. The student types in the equation $y = \frac{1}{2}x + \frac{1}{2}$ and manages to hit 2 stars. Draw the straight line on the screen.

d. If the student was to type in the equation from part **b** and the equation from part **c**, what is the coordinate of the star that both lines would hit?

e. If a student types $y = 2$, how many stars will they hit?

f. What is another equation of a straight line that will hit 2 stars?

16. The graph shown describes the mass in kilograms of metric cups of water.

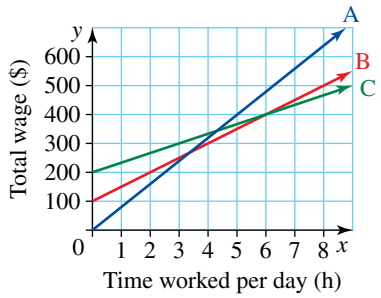


Write a rule to describe the mass of water relative to the number of cups.

17. The temperature of water in a kettle is 15°C before the temperature increases at a constant rate for 20 seconds to reach boiling point (100°C). A classmate argues that $T = 5t + 15$ describes the water temperature, citing the starting temperature of 15°C and that to reach 100°C in 20 seconds an increase of 5°C for every second is required. Explain why the equation is incorrect and devise another equation that correctly describes the temperature of the water.

18. A father wants to administer Children's Panadol® to his child. The recommended dosage is a range, 7.5–9 mL for an average weight of 12–14 kg. The child weighs 12.8 kg. The father uses a linear relationship to calculate an exact dosage. What dosage does the father calculate?

19. The graph shows the wages earned in three different remote locations.



a. What is the set allowance for each location?

b. Determine the hourly rates for each location.

c. Using your answers from parts **a** and **b**, determine linear equations that describe the wages at each location.

d. Match each working lifestyle below to the most appropriate location.

i. Working lifestyle 1: Earn the most money possible while working at most 4 hours in a day

ii. Working lifestyle 2: Earn the most money possible while working an 8-hour day

e. If a person works on average 8 hours a day, what is the advantage of location C?

f. How much money is earned at each location for a:

i. 2-hour day

ii. 6-hour day?

Reflection

Why are only 2 points needed to find the rule for the line that passes through the points?

CHALLENGE 7.1

A certain linear pattern has its Cartesian coordinates in fraction form as (x, y), $\left(\frac{1}{6}, -\frac{1}{4}\right), \left(\frac{1}{3}, -\frac{1}{6}\right), \left(\frac{1}{2}, -\frac{1}{12}\right), \left(\frac{2}{3}, 0\right)$. Find the rule that satisfies these Cartesian coordinates.

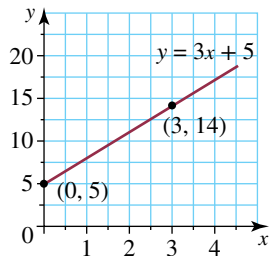
7.7 Practical applications of linear graphs

7.7.1 Finding a linear rule from a table of values

- If two **variables** are linked by a linear rule, then as one variable increases, the other increases (or decreases) at a steady rate.
- Here is an example of a linear relationship.

x	0	1	2	3
y	5	8	11	14

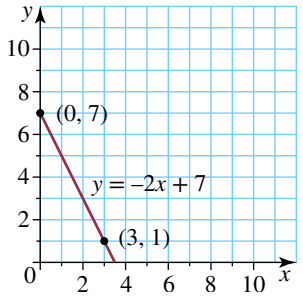
Each time x increases by 1, y increases by 3.
The linear rule connecting x and y is $y = 3x + 5$.



- Consider this relationship.

x	0	1	2	3
y	7	5	3	1

Each time x increases by 1, y decreases by 2.
The linear rule in this case is $y = -2x + 7$.



WORKED EXAMPLE 15

Find the rule connecting x and y in each of the following value tables.

a

x	0	1	2	3
y	−3	2	7	12

b

x	3	4	5	6
y	12	11	10	9

THINK

a 1 y increases at a steady rate, so this is a linear relationship. Write the rule.

2 Find m: y increases by 5 each time x increases by 1. Write the value of the gradient.

3 From the table, when $x = 0$, $y = -3$. Write the value of the y-intercept.

4 Write the rule.

b 1 y decreases at a steady rate, so this is a linear relationship.

2 y decreases by 1 each time x increases by 1. Write the value of the gradient.

3 Find c: $y = 12$ when $x = 3$.
To find the y-intercept, substitute the x- and y-values of one of the points, and solve for c.

4 Write the rule.

WRITE

a $y = mx + c$

$m = 5$

$c = -3$

$y = 5x - 3$

b $y = mx + c$

$m = -1$

$y = -x + c$
(3, 12): $12 = -(3) + c$
$12 = -3 + c$
$c = 15$

$y = -x + 15$

7.7.2 Modelling linear relationships

- Relationships between real-life variables are often modelled (described) by a mathematical equation. In other words, an equation or formula is used to link the two variables.
- For example,
  $A = l^2$ represents the relationship between the area and the side length of a square;
  $C = \pi d$ represents the relationship between the circumference and the diameter of a circle.
- If one variable changes at a constant rate compared to the other, then the two variables have a linear relationship.
- Many relationships are linear, which means they can be modelled by the equation $y = mx + c$.

WORKED EXAMPLE 16

An online bookstore sells a certain book for $21 and charges $10 for the delivery of any number of books.

a Find the rule connecting the cost ($C) with the number of books delivered (n).

b Use the rule to find the cost of delivering 35 books.

c How many books can be delivered for $1000?

THINK	WRITE
a 1 Set up a table. Cost for 1 book $= 21 + 10$ Cost for 2 books $= 2(21) + 10$ $= 52$	**a**

n	1	2	3
C	31	52	73

2 The cost rises steadily, so there is a linear relationship. Write the rule.	$C = mn + c$
3 To find the gradient, use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ with the points (1, 31) and (2, 52).	$m = \frac{52 - 31}{2 - 1}$ $= 21$ $C = 21n + c$
4 To find the value of c, substitute $C = 31$ and $n = 1$.	(1, 31): $31 = 21(1) + c$ $c = 10$
5 Write the rule.	$C = 21n + 10$
b 1 Substitute $n = 35$ and find C.	**b** $C = 21(35) + 10$ $= 735 + 10$ $= 745$
2 Write the answer.	The cost including delivery for 35 books is $745.
c 1 Substitute $C = 1000$ and find n.	**c** $1000 = 21n + 10$ $21n = 990$ $n = 47.14$
2 You cannot buy 47.14 books, so round down. Write the answer.	For $1000, 47 books can be bought and delivered.

- In Worked example 16, compare the rule $C = 21n + 10$ with the original question. It is clear that the $21n$ refers to the cost of the books (a variable amount, depending on the number of books) and that 10 refers to the fixed (constant) delivery charge.

- In this case C is called the **dependent variable**, because it depends on the number of books (n).
- The variable n is called the **independent variable**.

Exercise 7.7 Practical applications of linear graphs

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–d, 2–4, 7, 8, 10, 11	Questions: 1e–h, 2, 4, 5, 7, 9–12, 14	Questions: 1g–i, 2, 4, 6, 8–15

Individual pathway interactivity: int-4507

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE15 Find the linear rule linking the variables in each of the following tables.

a.

x	0	1	2	3
y	−5	1	7	13

b.

x	0	1	2	3
y	8	5	2	−1

c.

x	0	1	2	3
y	4	6	8	10

d.

x	0	1	2	3
y	1.1	2.0	2.9	3.8

e.

x	2	3	4	5
y	7	10	13	16

f.

x	5	6	7	8
y	12	11	10	9

g.

n	0	1	2	3
C	5	9	13	17

h.

t	0	1	2	3
n	0	5	10	15

i.

t	3	4	5	6
v	18	15	12	9

j.

d	1	2	3	4
C	11	14	17	20

2. MC Greg and Tanya hire a car. They are charged a fixed fee of $150 for hiring the car and then $25 per day. They hire the car for d days. Which one of the following could describe the rule that describes the number of days the car is hired and the total cost, C, they are charged?

A. $C = 25d$ **B.** $C = 150d$ **C.** $C = 175d$
D. $C = 25d + 150$ **E.** $C = 150d + 25$

Understanding

3. WE16 Nathan's bank balance has increased in a linear manner since he started his part-time job. If after 20 weeks of work his bank balance was at $560 and after 21 weeks of work it was at $585 find:

a. the rule that relates the size of his bank balance, A, and the time (in weeks) worked, t
b. the amount in his account after 200 weeks
c. the initial amount.



4. The cost of producing a shoe increases as the size of the shoe increases. It costs $5.30 to produce a size 6 shoe, and $6.40 to produce a size 8 shoe. Assuming that a linear relationship exists:
- **a.** determine the rule relating cost (C) to shoe size (s).
- **b.** how much does it cost to produce a size 12 shoe?

5. The number of publications in a library (N) increased steadily with time (t). After 10 years there were 7200 publications in the library, and after 12 years there were 8000 publications.
- **a.** Determine the rule predicting the number of publications in the library.
- **b.** How many publications were there after 5.5 years?
- **c.** How many publications will there be after 25 years?



6. The Robinsons' water tank sprang a leak and has been losing water at a steady rate. Four days after the leak occurred, the tank contained 552 L of water, and ten days later it held only 312 L.
- **a.** Find the rule linking the amount of water in the tank (w) and the number of days (t) since the leak occurred.
- **b.** How much water was in the tank initially?
- **c.** If water loss continues at the same rate, when will the tank be empty?

Reasoning

7. A software company claims that its staff can fix 22 bugs per month. They are working to fix a program that initially had 164 bugs.
- **a.** Determine the linear rule connecting the number of bugs left, N, and the time in months, t, from the beginning of eradication.
- **b.** How many bugs will there be left after 2 months?
- **c.** After how many months will there be 54 bugs left?
- **d.** How long will it take this company to eradicate all bugs? Justify your answer.

8. A skyscraper can be built at a rate of 4.5 storeys per month.
- **a.** How many storeys will be built after 6 months? Justify your answer.
- **b.** How many storeys will be built after 24 months? Justify your answer.



9. The cost of a taxi ride is $3.50 flag fall plus $2.14 for each kilometre travelled.
- **a.** Determine the linear rule connecting the cost, C, and the distance travelled, d.
- **b.** How much will an 11.5 km trip cost?
- **c.** How much will a 23.1 km trip cost? Justify your answer.

10. After 11 pm the taxi company charges $3.50 flag fall plus $2.57 for each kilometre travelled.
- **a.** Determine the linear rule connecting the cost, C, and the distance travelled, d.
- **b.** How much will an 11.5 km trip cost with this taxi?
- **c.** If I have $22 in my pocket, how far can I travel, correct to 1 decimal place? Justify your answer.

11. A certain kind of eucalyptus tree grows at a linear rate for its first 2 years of growth. If the growth rate is 5 cm per month, show that the tree will be 1.07 m tall after 21.4 months.

12. The pressure inside a boiler increases steadily as the temperature increases. For each 1°C, the pressure increases by 10 units, and at a temperature of 100 °C the pressure is 1200 units. If the maximum pressure allowed is 2000 units, show that the temperature cannot exceed 180 °C.

13. Michael produces and sells prints of his art at a local gallery. For each print run his profit (P) is given by the equation $P = 200n - 800$, where n is the number of prints sold.

- **a.** Sketch the graph of this rule.
- **b.** What is the y-intercept? What does this mean in this example?
- **c.** What is the x-intercept? What does this mean in this example?
- **d.** What is the gradient of the graph? What does this mean in this example?



Problem solving

14. Cheng lives in Australia and he is going on holiday to Japan. One yen (¥) buys 0.0127 Australian dollars (A$).

- **a.** Write an equation that converts Australian dollars to Japanese yen, where A represents amount of Australian dollars and Y represents amount of yen.
- **b.** Using the equation from part **a**, how many yen (¥) will Cheng receive if he has A$2500?
- **c.** There is a commission to be paid on exchanging currency. Cheng needs to pay 2.8% for each Australian dollar that is exchanged into yen. Write down an equation that calculates the total amount of yen Cheng will receive. Write your equation in terms of Y_T, total amount of yen, and Australian dollars.

15. Sam and Cody need to make a journey to the other branch of their store across town. The traffic is very busy at this time of the day so Sam catches the train that travels halfway and then walks the rest of the way. Cody travels by bike the whole way. The bike path travels along the train line and then along the roadway to the other branch of their store. The bike's speed was twice walking speed and the train's speed was four times the bike's speed. Who arrives at the destination first?

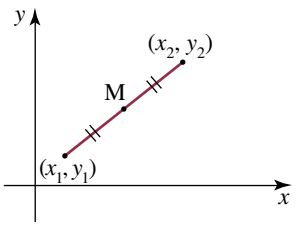
Reflection

How are the dependent and independent variables determined?

7.8 Midpoint of a line segment and distance between two points

7.8.1 Finding the midpoint of a line segment

- The x- and y-coordinates of the **midpoint** are half-way between those of the end points of a line segment.
- The coordinates of the midpoint, M, of a line can be found by averaging the x- and y-coordinates of the end points.



- The coordinates of the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) are:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

WORKED EXAMPLE 17

TI | CASIO

Find the midpoint of the segment joining (5, 9) and (−3, 11).

THINK	WRITE
1 Average the x-values: $\frac{x_1 + x_2}{2}$.	$x = \frac{5 - 3}{2}$ $= \frac{2}{2}$ $= 1$
2 Average the y-values: $\frac{y_1 + y_2}{2}$.	$y = \frac{9 + 11}{2}$ $= \frac{20}{2}$ $= 10$
3 Write the answer.	The midpoint is (1, 10).

WORKED EXAMPLE 18

TI | CASIO

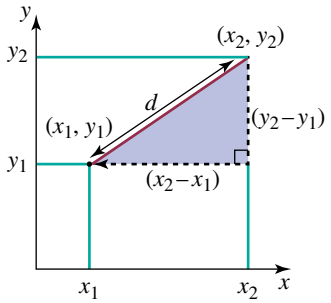
M (7, 2) is the midpoint of the line segment AB. If the coordinates of A are (1, −4), find the coordinates of B.

THINK	WRITE
1 Let B have the coordinates (x, y).	A (1, −4), B (x, y), M (7, 2)
2 The midpoint is (7, 2), so the average of the x-values is 7. Solve for x.	$\frac{1 + x}{2} = 7$ $1 + x = 14$ $x = 13$
3 The average of the y-values is 2. Solve for y.	$\frac{-4 + y}{2} = 2$ $-4 + y = 4$ $y = 8$
4 Write the answer.	Hence, the coordinates of point B are (13, 8).

7.8.2 The distance between two points

- The distance between two points on the Cartesian plane is calculated using Pythagoras' theorem applied to a right-angled triangle.
- The distance between the points (x_1, y_1) and (x_2, y_2) is calculated using the formula:

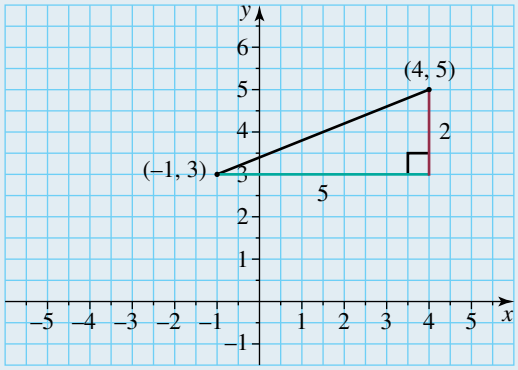
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



WORKED EXAMPLE 19

TI | CASIO

Find the distance between the points (−1, 3) and (4, 5):
a exactly **b correct to 3 decimal places.**

THINK	WRITE/DRAW
a 1 Draw a diagram showing the right-angled triangle (optional).	**a** 
2 Write the formula for the distance between two points.	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3 Let $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, 5)$. Substitute the x- and y-values into the equation.	$d = \sqrt{(4 - -1)^2 + (5 - 3)^2}$
4 Simplify.	$d = \sqrt{5^2 + 2^2}$ $d = \sqrt{25 + 4}$ $d = \sqrt{29}$
b 1 Write $\sqrt{29}$ as a decimal to 4 decimal places.	**b** $\sqrt{29} = 5.3851$
2 Write the answer correct to 3 decimal places.	$d \approx 5.385$

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Distance between two points (int-2766)

Exercise 7.8 Midpoint of a line segment and distance between two points

assessON

Individual pathways

PRACTISE
Questions:
1a–f, 2–5, 6a–f, 7, 8, 11–13

CONSOLIDATE
Questions:
1c–h, 2–5, 6c–h, 7, 8, 11–14, 16

MASTER
Questions:
1e–j, 2–5, 6e–j, 7–9, 11–17

Individual pathway interactivity: int-4508

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE17 Find the midpoint of the segment joining each of the following pairs of points.

a. (1, 3) and (3, 5)
b. (6, 4) and (4, −2)
c. (2, 3) and (12, 1)
d. (6, 3) and (10, 15)
e. (4, 2) and (−4, 8)
f. (0, −5) and (−2, 9)
g. (8, 2) and (−18, −6)
h. (−3, −5) and (7, 11)
i. (−8, −3) and (8, 27)
j. (−4, 1) and (−2, 13)

2. Find the midpoint of the segment joining each of the following pairs of points.

a. (7, −2) and (−4, 13)
b. (0, 22) and (−6, −29)
c. (−15, 8) and (−4, 11)
d. (−3, 40) and (0, −27)

3. WE18 Find the value of a in each question below so that the point M is the midpoint of the segment joining points A and B.

a. A $(-2, a)$, B $(-6, 5)$, M $(-4, 5)$

b. A $(a, 0)$, B $(7, 3)$, M $(8, \frac{3}{2})$

c. A $(3, 3)$, B $(4, a)$, M $(3\frac{1}{2}, -6\frac{1}{2})$

d. A $(-4, 4)$, B $(a, 0)$, M $(-2, 2)$

4. M is the midpoint of the line interval AB. Find the coordinates of B if:

a. A = (0, 0) and M = (2, 3)
b. A = (2, 3) and M = (0, 0)
c. A = (−3, 2) and M = (4, 2)
d. A = (3, −1) and M = (−2, −2).

5. Find the equation of a line that has a gradient of 5, and passes through the midpoint of the segment joining (−1, −7) and (3, 3).

6. WE19 Find the distance between each of the following pairs of points.

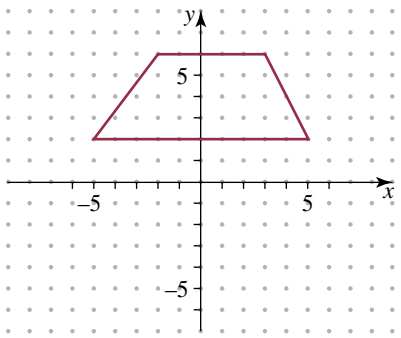
a. (4, 5) and (1, 1)
b. (7, 14) and (15, 8)
c. (2, 4) and (2, 3)
d. (12, 8) and (10, 8)
e. (14, 9) and (2, 14)
f. (5, −13) and (−3, −7)
g. (−14, −9) and (−10, −6)
h. (0, 1) and (−15, 9)
i. (−4, −8) and (1, 4)
j. (12, 9) and (−4, −3)

7. Calculate the distance between the pairs of points below, correct to 3 decimal places.

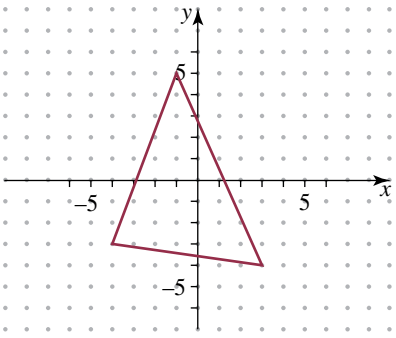
a. (−14, 10) and (−8, 14)
b. (6, −7) and (13, 6)
c. (−11, 1) and (2, 2)
d. (9, 0) and (5, −8)
e. (2, −7) and (−2, 12)
f. (9, 4) and (−10, 0)

8. Find the perimeter of each figure below, giving your answers correct to 3 decimal places.

a.

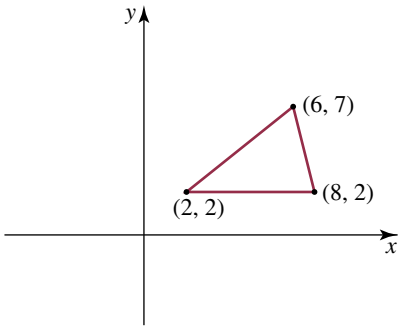


b.

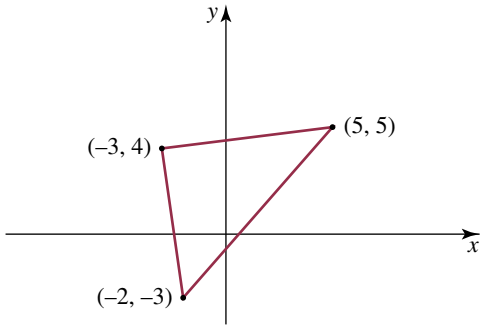


9. Find the perimeter of each triangle, giving your answers correct to 3 decimal places.

a.

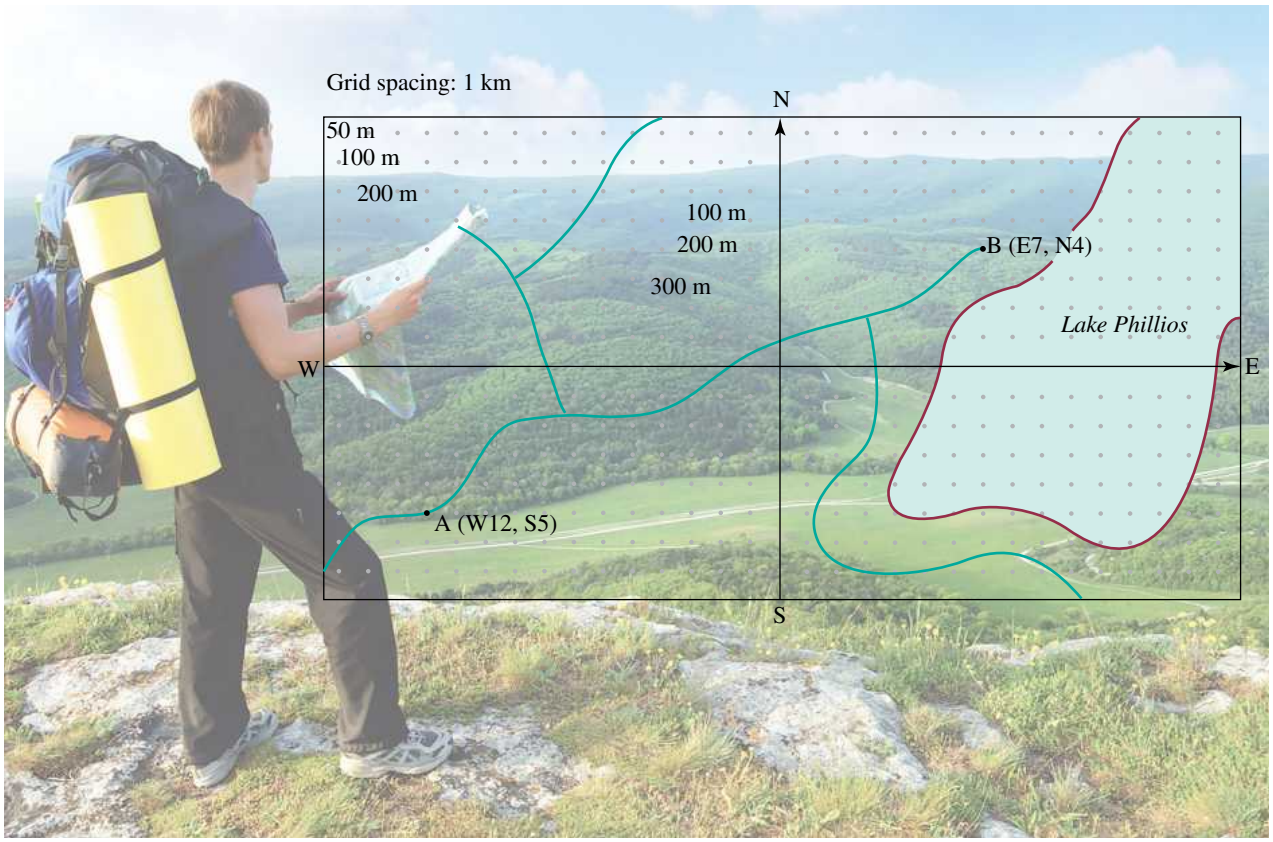


b.



Understanding

10. Two hikers are about to hike from A to B (shown on the map below). How far is it from A to B in a straight line?



Reasoning

11. a. Plot the following points on a Cartesian plane: A (−1, −4), B (2, 3), C (−3, 8) and D (4, −5).

b. Show that the midpoint of the interval AC is (−2, 2).

c. Find the exact distance between the points A and C.

d. If B is the midpoint of an interval CM, find the point M.

e. Show that the gradient of the line segment AB is $\frac{7}{3}$.

f. Find the equation of the line that passes through the points B and D.

12. The point M (−2, −4) is the midpoint of the interval AB. Show that the point B is (−9, −2), given A is (5, −6).

13. Show that the distance between the points A (2, 2) and B (6, −1) is 5.

14. Show that the point B (6, −10) is equidistant from the points A (15, 3) and C (−7, −1).
15. A triangle has vertices A (−4, 1), B (2, 3) and C (0, −3). Show that ΔABC is isosceles.

Problem solving

16. Calculate the gradient of the line through the points $(-1, 3)$ and $(3 + 4t, 5 + 2t)$.
17. A map of a town drawn on a Cartesian plane shows the main street extending from (−4, 5) to (0, −7). Five street lights are positioned in the street. There is one at either end, and three spaced evenly down the street. Give the position of the five lights in the street.

Reflection

If the two end points of a line segment both have negative *x*- and *y*-coordinates, the distance formula still produces a positive answer. Why is this so?

CHALLENGE 7.2

The distance from the origin to the *y*-intercept of a linear graph is twice the distance from the origin to the *x*-intercept. The area of the triangle formed by the line and the axes is 2.25 units^2. The line has a negative gradient and a negative *y*-intercept. Determine the gradient and *y*-intercept of the line.

7.9 Non-linear relations (parabolas, hyperbolas, circles)

7.9.1 Non-linear relationships

- There are many examples of non-linear relationships in mathematics. Some of them are the parabola, the hyperbola and the circle.
  Note: This section introduces these non-linear relationships; parabolas will be explored in more depth in Topic 17.

7.9.2 The parabola

- The **parabola** is a curve that is found in many phenomena such as those shown in the images at right.
- These images reveal a number of common features of parabolic shapes.
  – They are all symmetrical. Specifically, a line of symmetry can be drawn in the middle of the parabola, such that each half is an exact reflection of the other.
  – They have either a maximum (highest) point or a minimum (lowest point) — both are known as turning points.
  – All parabolas have the same basic shape; however, they can be wider or narrower depending on the equation.
- The most basic parabolic graph is produced by the equation:
$$y = x^2$$
- The graph of $y = x^2$ can be plotted using a table of values as shown in Worked example 20.



WORKED EXAMPLE 20

Plot the graph of $y = x^2$ for values of x from -3 to 3. State the equation of the axis of symmetry and the coordinates of the turning point.

THINK

1 Write the equation.

2 Produce a table of values using x-values from -3 to 3.

3 Draw a set of clearly labelled axes, plot the points and join them with a smooth curve. The scale on the y-axis would be from -2 to 10 and from -4 to 4 on the x-axis.

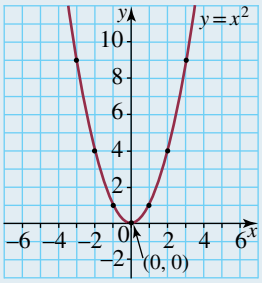
4 Write the equation of the line that divides the parabola exactly in half.

5 Write the coordinates of the turning point.

WRITE/DRAW

$y = x^2$

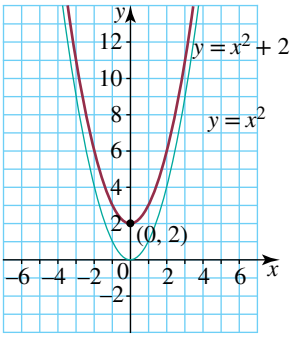
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



The equation of the axis of symmetry is $x = 0$.

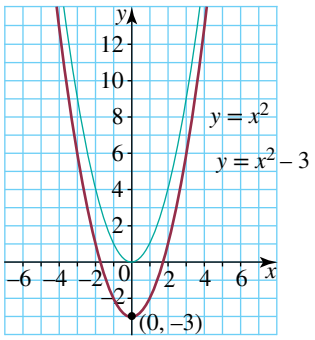
The turning point is (0, 0).

- A parabola can be sketched using its key features, such as its general shape, its axis of symmetry and its turning point or **vertex**.
- A parabola can be seen to undergo transformations by making changes to the equation $y = x^2$.
  Some transformations include:
  – vertical translations
  – horizontal translations
  – reflections.



7.9.3 Vertical translation

- Compare the graph of $y = x^2 + 2$ with that of $y = x^2$.
  – The whole graph has been moved or translated 2 units upwards.
  – The turning point has become (0, 2).
  – The y-coordinate of the turning point has increased by 2 units to show that the graph has been moved up 2 units.
- Compare the graph of $y = x^2 - 3$ with that of $y = x^2$.
  – The whole graph has been moved or translated 3 units downwards.
  – The turning point has become (0, -3).
  – The y-coordinate of the turning point has decreased by 3 units to show that the graph has been moved down 3 units.



WORKED EXAMPLE 21

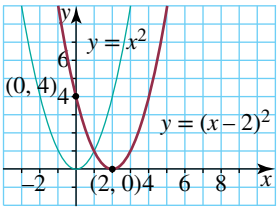
TI | CASIO

State the vertical translation (when compared with the graph of $y = x^2$) and the coordinates of the turning point for the graphs of each of the following equations.

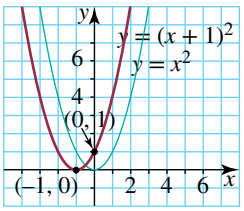
a $y = x^2 + 5$ **b** $y = x^2 - 4$

THINK	WRITE
a 1 Write the equation.	**a** $y = x^2 + 5$
2 +5 means the graph is translated upwards 5 units.	Vertical translation of 5 units up
3 Translate the turning point of $y = x^2$, which is (0, 0). The x-coordinate of the turning point remains 0, and the y-coordinate has 5 added to it.	The turning point becomes (0, 5).
b 1 Write the equation.	**b** $y = x^2 - 4$
2 −4 means the graph is translated downwards 4 units.	Vertical translation of 4 units down
3 Translate the turning point of $y = x^2$, which is (0, 0). The x-coordinate of the turning point remains 0, and the y-coordinate has 4 subtracted from it.	The turning point becomes (0, −4).

7.9.4 Horizontal translation



- Compare the graph of $y = (x - 2)^2$ with that of $y = x^2$.
  - The whole graph has been moved or translated 2 units to the right.
  - The turning point has become (2, 0).
  - The x-coordinate of the turning point has increased by 2 units to show that the graph has been moved 2 units to the right.
- Compare the graph of $y = (x + 1)^2$ with that of $y = x^2$.
  - The whole graph has been moved or translated 1 unit left.
  - The turning point has become (−1, 0).
  - The x-coordinate of the turning point has decreased by 1 unit to show that the graph has been moved left 1 unit.



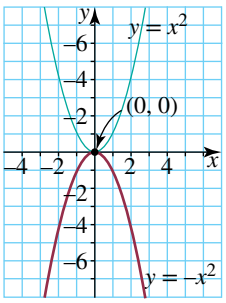
WORKED EXAMPLE 22 TI | CASIO

State the horizontal translation (when compared to the graph of $y = x^2$) and the coordinates of the turning point for the graphs of each of the following equations.
a $y = (x - 3)^2$ **b** $y = (x + 2)^2$

THINK	WRITE
a 1 Write the equation.	**a** $y = (x - 3)^2$
2 −3 means the graph is translated to the right 3 units.	Horizontal translation of 3 units to the right
3 Translate the turning point of $y = x^2$, which is (0, 0). The y-coordinate of the turning point remains 0, and the x-coordinate has 3 added to it.	The turning point becomes (3, 0).
b 1 Write the equation.	**b** $y = (x + 2)^2$
2 +2 means the graph is translated to the left 2 units.	Horizontal translation of 2 units to the left
3 Translate the turning point of $y = x^2$, which is (0, 0). The y-coordinate of the turning point remains 0, and the x-coordinate has 2 subtracted from it.	The turning point becomes (−2, 0).

7.9.5 Reflection

- Compare the graph of $y = -x^2$ with that of $y = x^2$.
  - In each case the axis of symmetry is the line $x = 0$ and the turning point is (0, 0).
  - The only difference between the equations is the $-$ sign in $y = -x^2$. The difference between the graphs is that $y = x^2$ 'sits' on the x-axis and $y = -x^2$ 'hangs' from the x-axis. (One is a reflection or mirror image of the other).
  - $y = x^2$ has a minimum turning point and $y = -x^2$ has a maximum turning point.



- Any quadratic graph where x^2 is positive has a ∪ shape and is said to be concave up. Conversely, if x^2 is negative the graph has a ∪ shape and is said to be concave down.

7.9.6 The hyperbola

- A hyperbola is a non-linear graph whose equation is $y = \frac{1}{x}$ or $xy = 1$.

WORKED EXAMPLE 23 TI | CASIO

Complete the table of values below and use it to plot the graph of $y = \frac{1}{x}$.

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	2	3
y								

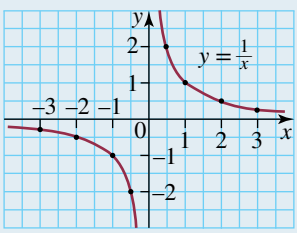
THINK

1 Substitute each x-value into the function $y = \frac{1}{x}$ to obtain the corresponding y-value.

WRITE/DRAW

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	Undefined	2	1	$\frac{1}{2}$	$\frac{1}{3}$

2 Draw a set of axes and plot the points from the table. Join them with a smooth curve.



- The graph in Worked example 23 has several important features.
  - There is no function value (y-value) when $x = 0$. At this point the hyperbola is undefined. When this occurs, the line that the graph approaches ($x = 0$) is called a vertical **asymptote**.
  - As x becomes larger and larger, the graph gets very close to but will never touch the x-axis. The same is true as x becomes smaller and smaller. The hyperbola also has a horizontal asymptote at $y = 0$.
  - The hyperbola has two separate branches. It cannot be drawn without lifting your pen from the page and is an example of a *discontinuous* graph.
  - Graphs of the form $y = \frac{k}{x}$ are the same basic shape as $y = \frac{1}{x}$, but they are wider or narrower depending on the value of k.
  - Hyperbolas can be transformed in the same way as parabolas; this will be covered in later years.

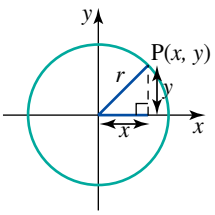
7.9.7 The circle

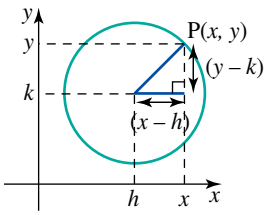
- A circle is the path traced out by a point at a constant distance (the radius) from a fixed point (the centre).
- Consider the circles shown at right. The first circle has its centre at the origin and radius r.
  - Let P (x, y) be a point on the circle.
  - By Pythagoras' theorem, $x^2 + y^2 = r^2$.
  - This relationship is true for all points, P, on the circle.
  - The equation of a circle with centre $(0, 0)$ and radius r is:

$$x^2 + y^2 = r^2$$

  - If the circle is translated h units to the right, parallel to the x-axis, and k units upwards, parallel to the y-axis, then the equation of a circle with centre (h, k) and radius r is:

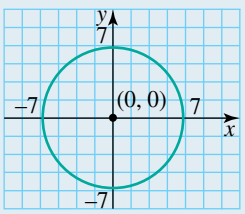
$$(x - h)^2 + (y - k)^2 = r^2$$





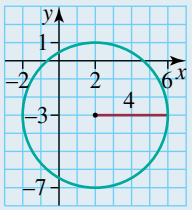
WORKED EXAMPLE 24 TI | CASIO

Sketch the graph of $x^2 + y^2 = 49$, stating the centre and radius.

THINK	WRITE/DRAW
1 Write the equation.	$x^2 + y^2 = 49$
2 State the coordinates of the centre.	Centre $(0, 0)$
3 Find the length of the radius by taking the square root of both sides. (Ignore the negative results.)	$r^2 = 49$ $r = 7$ Radius = 7 units
4 Sketch the graph.	

WORKED EXAMPLE 25

Sketch the graph of $(x-2)^2+(y+3)^2=16$, clearly showing the centre and radius.

THINK	WRITE/DRAW
1 Express the equation in general form.	$(x-h)^2+(y-k)^2=r^2$ $(x-2)^2+(y+3)^2=4^2$
2 State the coordinates of the centre.	Centre $(2, -3)$
3 State the length of the radius.	$r^2=16$ $r=4$ Radius = 4 units
4 Sketch the graph.	

Exercise 7.9 Non-linear relations (parabolas, hyperbolas, circles)

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–12, 16	Questions: 1–13, 16–17	Questions: 1–18

Individual pathway interactivity: int-4509

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE21 State the vertical translation and the coordinates of the turning point for the graph of each of the following equations.

a. $y=x^2+3$ **b.** $y=x^2-1$ **c.** $y=x^2-7$ **d.** $y=x^2+\frac{1}{4}$

2. WE22 State the horizontal translation and the coordinates of the turning point for the graph of each of the following equations.

a. $y=(x-1)^2$ **b.** $y=(x-2)^2$ **c.** $y=(x+10)^2$ **d.** $y=(x+4)^2$

3. For each of the following graphs, give the coordinates of the turning point and state whether it is a maximum or a minimum.

a. $y=-x^2+1$ **b.** $y=x^2-3$ **c.** $y=-(x+2)^2$

4. WE23 Complete the table of values below and use it to plot the graph of $y=\frac{10}{x}$.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y											

5. a. Plot the graph of each hyperbola.

b. Write the equation of each asymptote.

i. $y=\frac{5}{x}$ **ii.** $y=\frac{20}{x}$ **iii.** $y=\frac{100}{x}$

6. On the same set of axes, draw the graphs of $y = \frac{2}{x}$, $y = \frac{3}{x}$ and $y = \frac{4}{x}$.

7. Use your answer to question **6** to describe the effect of increasing the value of k on the graph of $y = \frac{k}{x}$.

8. WE24 Sketch the graphs of the following, stating the centre and radius of each.

a. $x^2 + y^2 = 49$ **b.** $x^2 + y^2 = 4^2$ **c.** $x^2 + y^2 = 36$

d. $x^2 + y^2 = 81$ **e.** $x^2 + y^2 = 25$ **f.** $x^2 + y^2 = 100$

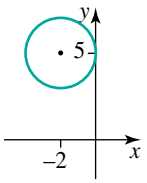
9. WE25 Sketch the graphs of the following, clearly showing the centre and the radius.

a. $(x - 1)^2 + (y - 2)^2 = 5^2$ **b.** $(x + 2)^2 + (y + 3)^2 = 6^2$ **c.** $(x + 3)^2 + (y - 1)^2 = 49$

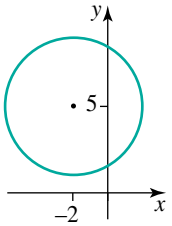
d. $(x - 4)^2 + (y + 5)^2 = 64$ **e.** $x^2 + (y + 3)^2 = 4$ **f.** $(x - 5)^2 + y^2 = 100$

10. MC The graph of $(x - 2)^2 + (y + 5)^2 = 4$ is:

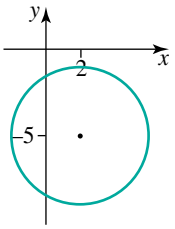
A.



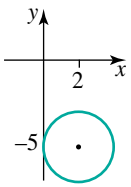
B.



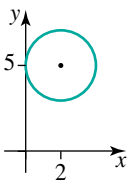
C.



D.



E.



Reasoning

11. Show that $y = x^2$ is the same as $y = (-x)^2$.

12. Explain the similarities and differences between $y = \frac{1}{x}$ and $y = -\frac{1}{x}$.

13. Show that the turning point for $y = (x + 3)^2 - 1$ is $(-3, -1)$. Explain whether this is a maximum or minimum turning point.

14. Show the equation of the circle that has centre (2, 1) and passes through the point (6, 1) is $(x - 2)^2 + (y - 1)^2 = 16$.

15. Which point on the circle $x^2 + y^2 - 12x - 4y = 9$ is closest to the origin? Explain.

Problem solving

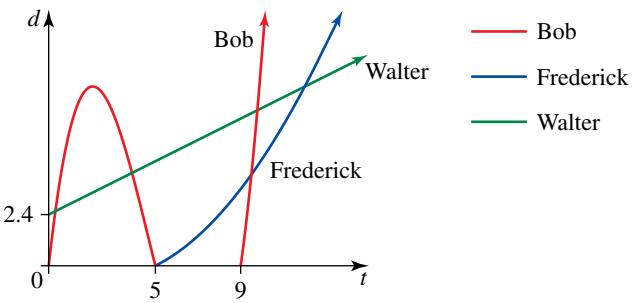
16. Three kindergarten children, Walter, Frederick and Bob, ran a 20-metre race. Each child ran according to the following equations, where d is the distance, in metres, from the starting line, and t is the time, in seconds.

Walter: $d = 2.4 + 0.5t$

Frederick: $d = 0.1t(t - 5)$

Bob: $d = 0.2t(t - 5)(t - 9)$

The following sketch shows their running paths.



a. Describe the race style of Bob.
b. How long did it take Frederick to start running?
c. At the start of the race, how far was Walter from the starting line?
d. If the winner of the 20-metre race won the race in 10.66 seconds, determine whether Frederick or Bob won the race. Justify your answer using calculations.

17. A circle with centre (0, 2) passes through the point (3, 6). What are the coordinates of the points where the circle crosses the y-axis?

18. Consider the relationship $\frac{1}{y} = \frac{1}{x} + 1$. It appears that both variables are being raised to the power of 1. The relationship can be transposed as shown below.

$$\frac{1}{y} = \frac{1}{x} + 1$$
$$\frac{1}{y} = \frac{1}{x} + \frac{x}{x}$$
$$\frac{1}{y} = \frac{1+x}{x}$$
$$y = \frac{x}{1+x}$$

a. Generate a table of values for x versus y using the transposed relationship.
b. Plot a graph to show the points contained in the table of values.
c. Use your plot to confirm whether this relationship is linear.

Reflection

How can you tell whether the equation of a non-linear relationship represents a parabola, a hyperbola or a circle?

7.10 Review

7.10.1 Review questions

Fluency

1. For the rule $y = 3x - 1$, what is y when $x = 2$?
A. -1 **B.** 1 **C.** 2 **D.** 5

2. What is the gradient of the linear rule $y = 4 - 6x$?
A. 6 **B.** -6 **C.** 4 **D.** -4

3. The graph with the rule $2y - x + 6 = 0$ has an x-intercept of:
A. -2 **B.** 0 **C.** 2 **D.** 6

4. The graph with the rule $2y - x + 6 = 0$ has a y-intercept of:
A. -6 **B.** -3 **C.** 0 **D.** 3

5. Consider a linear graph that goes through the points $(6, -1)$ and $(0, 5)$. The gradient of this line is:
A. 5 **B.** -5 **C.** 1 **D.** -1

6. A straight line passes through the points (2, 1) and (5, 4). Its rule is:
A. $y = x - 1$ **B.** $y = x + 1$ **C.** $y = 2x$ **D.** $4y = 5x$

7. The rule for a line whose gradient is -4 and y-intercept = 8 is:
A. $y = -4x + 32$ **B.** $y = -4x + 8$ **C.** $y = 4x - 32$ **D.** $y = 4x - 8$

8. Which of the following linear rules will *not* intersect with the straight line defined by $y = 3x$?
A. $y = 3x + 2$ **B.** $y = -3x + 1$ **C.** $y = -3x + 2$ **D.** $y = -2x + 1$

9. If $y = 2x + 1$, then a point that could not be on the line is:
A. (3, 7) **B.** $(-3, -5)$ **C.** (0, 1) **D.** $(-3, 0)$

10. The solution to $y = 3x + 1$ and $y = -3x + 1$ is:

A. $(0, 1)$ **B.** $(1, 0)$ **C.** $\left(0, -\frac{1}{3}\right)$ **D.** $\left(-\frac{1}{3}, 0\right)$

11. Write down **i** the gradient and **ii** the y-intercept of the following linear graphs.

a. $y = 8x - 3$ **b.** $y = 5 - 9x$ **c.** $2x + y - 6 = 0$ **d.** $4x - 2y = 0$ **e.** $y = \dfrac{2x - 1}{3}$

12. Which of the following lines are parallel to the line with the equation $y = 6 - x$?

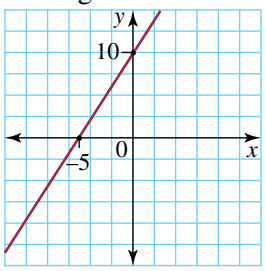
a. $y + x = 4$ **b.** $y = 13 - x$ **c.** $2y - 2x = 1$ **d.** $x + 2y - 4 = 0$

13. Determine which of the following rules will yield a linear graph.

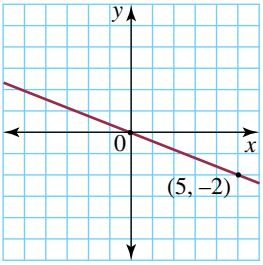
a. $3y = -5x - \frac{1}{2}$ **b.** $y = 3x^2 - 2\frac{1}{2}$ **c.** $3x + 4y + 6 = 0$ **d.** $x + y - 2xy = 0$ **e.** $2y = 4^2x + 9^2$

14. Find the gradient of the lines shown.

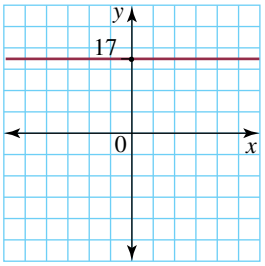
a.



b.



c.



15. Find the gradient of the line passing through the following pairs of points:

a. $(2, -3)$ and $(4, 1)$

b. $(0, -5)$ and $(4, 0)$.

16. For each of the following rules, state the gradient and the y-intercept.

a. $y = -3x + 7$ **b.** $2y - 3x = 6$ **c.** $y = -\frac{2}{5}x$ **d.** $y = 4$

17. For the following rules, use the gradient–intercept method to sketch linear graphs.

a. $y = -x + 5$ **b.** $y = 4x - 2.5$ **c.** $y = \frac{2}{3}x - 1$ **d.** $y = 3 - \frac{5}{4}x$

18. For the following rules, use the x- and y-intercept method to sketch linear graphs.

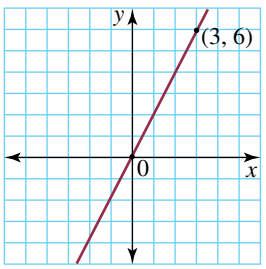
a. $y = -6x + 25$ **b.** $y = 20x + 45$ **c.** $2y + x = -5$ **d.** $4y + x - 2.5 = 0$

19. For the following rules, use an appropriate method to sketch linear graphs.

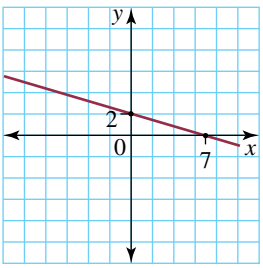
a. $y = -3x$ **b.** $y = \frac{1}{4}x$ **c.** $y = -2$ **d.** $x = 3$

20. Find the rule of the lines shown.

a.



b.



21. Determine the linear rules given the following pieces of information.

a. Gradient $= 2$, y-intercept $= -7$

b. Gradient $= 2$, x-intercept $= 7$

c. Gradient $= 2$, passing through $(7, 9)$

d. Gradient $= -5$, passing through the origin

e. y-intercept $= -2$, passing through $(1, -3)$

f. Passing through $(1, 5)$ and $(5, -6)$

g. x-intercept $= 3$, y-intercept $= -3$

h. y-intercept $= 5$, passing through $(-4, 13)$

22. Find the midpoint of the line interval joining the points $(-2, 3)$ and $(4, -1)$.

23. Find the distance between the points $(1, 1)$ and $(4, 5)$.

24. Sketch each of the following, comparing it with the graph of $y = x^2$.

a. $y = x^2 - 3$ **b.** $y = (x + 2)^2$

25. a. Plot the graph of $y = \frac{4}{x}$.

b. What sort of graph is this?

26. Sketch the following circle. Clearly show the centre and radius. $x^2 + y^2 = 16$

Problem solving

27. Louise owes her friend Sula $400 and agrees to pay her back $15 per week.

a. State a linear rule that demonstrates this reducing debt schedule and sketch the graph.

b. How many weeks does it take her to repay the debt?

c. How much does she owe after 15 weeks?

d. After how many repayments does she owe $85?

28. A bushwalker is 40 km from his base camp when he decides to head back. If he is able to walk 3.5 km each hour:

a. determine the linear rule that describes this situation and sketch its graph.

b. how long, correct to 1 decimal place, will it take him to reach base camp?

c. how far will he have walked in 6.5 hours?

29. Sue is writing test questions. She has already written 25 questions and can write a further 5 questions per hour.

a. Represent this information as a linear equation where t hours is the time spent writing test questions and n is the number of questions written.

b. Predict the total number of questions written after a further 8 hours assuming the same linear rule.

c. How long, to the nearest minute, will it take Sue to have 53 written questions?

30. Catherine earns a daily rate of $200 for working in her mother's store. She receives $5 for each necklace that she sells.

a. Write an equation to show how much money (m) Catherine earned for the day after selling (n) necklaces.

b. Graph the equation that you created in part **a**, showing the two intercepts.

c. Which part of the line applies to her earnings? Explain.

d. Which part of the line does not apply to her earnings? Explain.

31. Calculate the gradient of the line passing through the points $(2, 3)$ and $(6 + 4t, 5 + 2t)$. Write your answer in simplest form.

32. What is the point on the line $y = 2x + 7$ that is also 5 units above the x-axis ?

33. An experiment was conducted, and data collected for two variables p and t .

p	$-\frac{1}{2}$		$\frac{1}{2}$	3.6
t	$2\frac{1}{4}$	1.75		−5.95

It is known that the relationship between p and t is a linear one. What are the two missing values?

34. The distance from the origin to the y-intercept of a linear graph is three times the distance from the origin to the x-intercept. The area of the triangle formed by the line and the axes is 3.375 units^2. The line has a negative gradient and a negative y-intercept. What is the equation of the line?

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

asymptote
axis of symmetry
Cartesian plane
circle
concave down
concave up
coordinates
dependent variable
discontinuous graph
fixed
formula
gradient
horizontal
hyperbola
independent variable
interval
line segment
linear graph
linear relation
midpoint
origin
parabola
parallel
perpendicular
quadrant
translation
turning point
undefined
variable amount
vertex
vertical
***x*-intercept**
***y*-intercept**

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Try out this interactivity: Crossword: Topic 7 (int-2688)

Try out this interactivity: Sudoku: Topic 7 (int-3207)

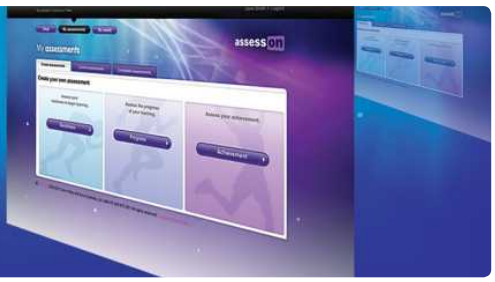
Complete this digital doc: Concept map: Topic 7 (doc-10798)

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Investigation | Rich task

Path of a billiard ball

The path of a billiard ball can be mapped by using mathematics. The billiard table can be represented by a rectangle, the ball by a point and its path by line segments. In this investigation, we will look at the trajectory of a single ball, unobstructed by other balls. A billiard table has a pocket at each of its corners and one in the middle of each of its long sides.

Consider the path of a ball that is hit on its side, from the lower left-hand corner of the table, so that it travels at a 45° angle from the corner of the table. Assume that the ball continues to move, rebounding from the sides and stopping only when it comes to a pocket. Diagrams of the table drawn on grid paper are shown below. Each grid square has a side length of 0.25 metres.

Diagram A

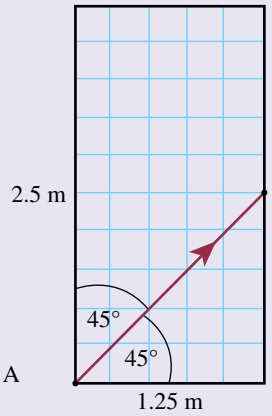


Diagram B

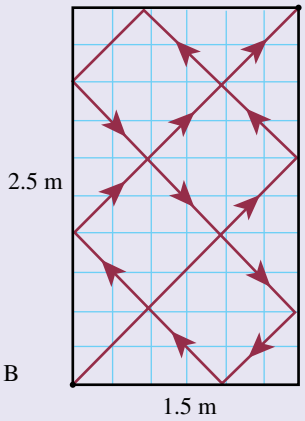
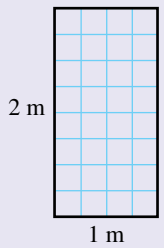


Diagram A shows the trajectory of a ball that has been hit on its side at a 45° angle, from the lower left-hand corner of a table that is 2.5 m long and 1.25 m wide. Note that, because the ball has been hit on its side at a 45° angle, it travels diagonally through each square in its path, from corner to corner. Diagram B shows the trajectory of a ball on a 2.5 m by 1.25 m table.

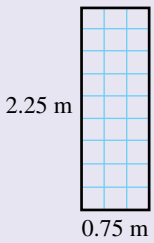
1. With reference to Diagram B, how many times does the ball rebound off the sides before going into a pocket?

2. Eight tables of different sizes are drawn on grid paper in the following diagrams. For each table, determine the trajectory of a ball hit at 45° on its side, from the lower left-hand corner of the table. Draw the path each ball travels until it reaches a pocket.

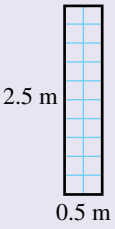
a.



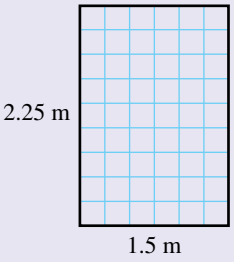
b.



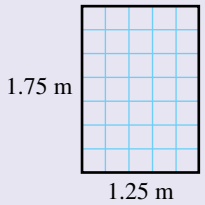
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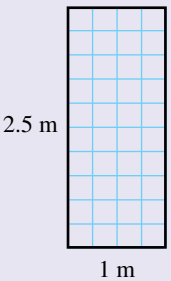
d.



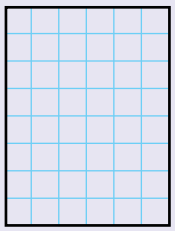
e.



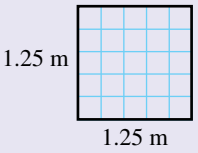
f.



g.



h.



3. Which tables show the ball travelling through the simplest path? What is special about the shape of the tables?

4. Which table shows the ball travelling through the most complicated path? What is special about the path? Draw another table (and path of the ball) with the same feature.

5. Which tables show the ball travelling through a path that does not cross itself? Draw another table (and path of the ball) with the same feature.

6. Consider the variety of table shapes. Will a ball hit on its side from the lower left-hand corner of a table at 45° always end up in a pocket (assuming it does not run out of energy)?

Simplify matters a little and consider a billiard table with no pockets in the middle of the long sides. Look, in a systematic way, for patterns for tables whose dimensions are related in a special way.

7. On paper, draw a series of billiard tables of length 3 m. Increase the width from an initial value of 0.25 m in increments of 0.25 m. Investigate the destination pocket of a ball hit from the lower left-hand corner. Complete the table below.

	Length of table (m)	Width of table (m)	Destination pocket
a	3	0.25	
b	3	0.5	
c	3	0.75	
d	3	1	
e	3	1.25	
f	3	1.5	
g	3	1.75	
h	3	2	

8. How can you predict (without drawing a diagram) the destination pocket of a ball hit from the lower left-hand corner of a table that is 3 m long? Provide an illustration with your answer to verify your prediction.

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Complete this digital doc: Code puzzle: Christmas day, 1859 (doc-15899)

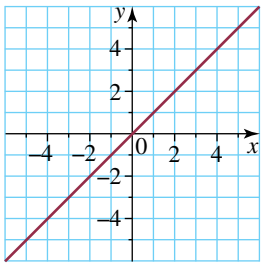
Answers

Topic 7 Linear and non-linear graphs

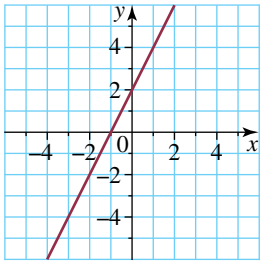
Exercise 7.2 Plotting linear graphs

1. a. B **b.** C **c.** D

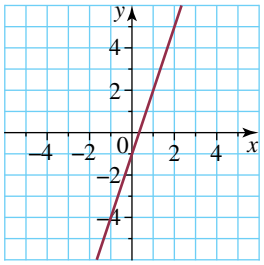
2. a. $y = x$



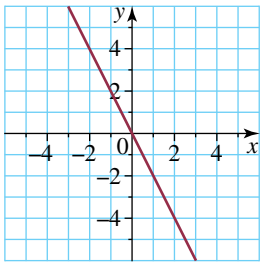
b. $y = 2x + 2$



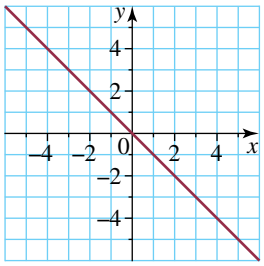
c. $y = 3x - 1$



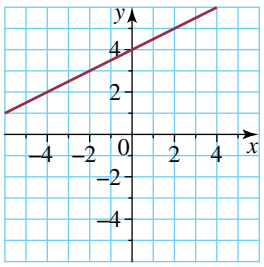
d. $y = -2x$



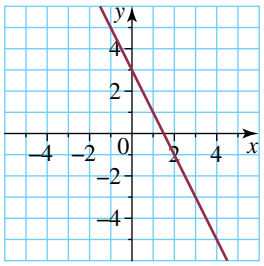
3. a. $y = -x$



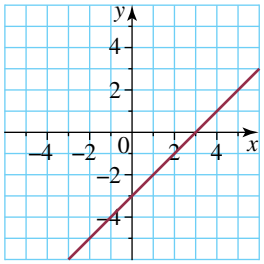
b. $y = \frac{1}{2}x + 4$



c. $y = -2x + 3$



d. $y = x - 3$



4. a. Yes **b.** No **c.** No **d.** Yes

5. a. Yes **b.** Yes **c.** Yes **d.** No
e. No **f.** No **g.** Yes **h.** Yes

6. D

7. a. B **b.** A **c.** D **d.** C

8. Third quadrant. By plotting the points on the Cartesian plane and joining them to make a line it can be seen that it does not pass through the third quadrant.

9. a. The first, second and third quadrants.

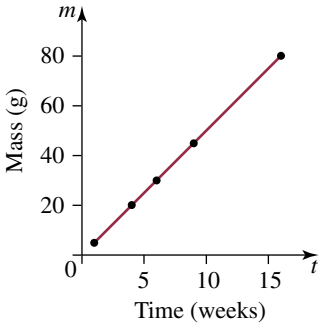
b. Answers will vary. Substitute point (1, 3) into line equation. $3 \neq 1 + 1$

10. Answers will vary. Substitute the point into the line equation. If the LHS equals the RHS of the equation, then the point lies on the line. Alternatively, graph the line and check the point on the Cartesian plane.

11. Answers will vary. Plot the points on the Cartesian plane and draw a line through the points. Find the equation of the line. Check all points lie on the line by substituting into the line equation.

12. (59, 21)

13. a.



b. $m = 5t + 1$

c. Answers will vary.

14. Stirring increases the rate of reaction.

Exercise 7.3 The equation of a straight line

1. a. 6 **b.** -4 **c.** 0 **d.** $\frac{6}{5}$
e. -2.2 **f.** -45 **g.** 0 **h.** 0.1
i. -2.6 **j.** 0 **k.** 9 **l.** -0.1

2. a. 4 **b.** 1 **c.** 0 **d.** 20
e. 400 **f.** -5.2 **g.** 0 **h.** -5
i. -300 **j.** 11 **k.** Undefined **l.** -1

3. a. 5 **b.** -3 **c.** 3 **d.** -2
e. 0.5 **f.** 0.5 **g.** -1 **h.** 0
i. 23 **j.** 2 **k.** -2 **l.** $\frac{7}{2}$ or 3.5

4. a. 23 **b.** 54 **c.** -6 **d.** 70
e. 1 **f.** 5.2 **g.** 100 **h.** 100
i. -87 **j.** $\frac{5}{2}$ **k.** $\frac{-5}{2}$ **l.** $\frac{-19}{6}$

5. a. Gradient = 4, y-intercept = 8 **b.** Gradient = -4, y-intercept = 8
c. Gradient = -2, y-intercept = 7 **d.** Gradient = 12, y-intercept = 0
e. Gradient = 0.5, y-intercept = 2.5 **f.** Gradient = -40, y-intercept = 83
g. Gradient = -4, y-intercept = -18 **h.** Gradient = 1.2, y-intercept = -3.6
i. Gradient = 0.5, y-intercept = 1.5 **j.** Gradient = $\frac{1}{15}$, y-intercept = 0
k. Gradient = 3, y-intercept = 5 **l.** Gradient = 4, y-intercept = -8

6. C

7. x-inercept = $\frac{-c}{m}$

8. a. (0, -10) **b.** (0, -4) **c.** (0, 3)
d. (0, 7) **e.** No y-intercept

9. Answers will vary. There is a rise and no run; $\frac{\text{rise}}{0}$ = undefined.

10. Answers will vary. There is no rise and a run; $\frac{0}{\text{run}} = 0$

11. Answers will vary.

12. a. Answers will vary.

b. The change in b changes the y-intercept and moves the line along the y-axis.

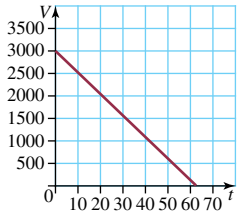
13. a. 1 point (12.5, 12.5) **b.** 26 points

14. a. 3000 L

b. 2040 L

c. Answers will vary.

d.



Intercepts: (0, 3000) and (62.5, 0)

e. Water is leaking out at a rate of 48 litres per minute.

15. It does not matter if you rise before you run or run before you rise, as long as you take into account whether the rise or run is negative.

16. a. $m = \frac{y - c}{x}$ **b.** $y = mx + c$

17. a. $m = -\frac{3}{2}$

b. It does not matter which points are chosen to determine the gradient of the graph because the gradient will always remain the same.

c. Straight line with a y-intercept of $(0, -2)$ and a slope of $-\frac{3}{2}$

18. Answers will vary.

19. a. Lamb: $m = \$15$/kg
Chicken: $m = \$10$/kg
Beef: $m = \$7.50$/kg

b. Lamb: \$15
Chicken: \$10
Beef: \$7.50

c. i. \$15 **ii.** \$5 **iii.** \$15

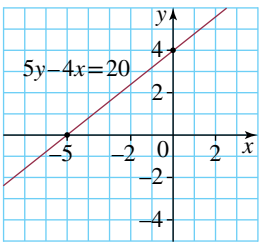
d. \$35

e.

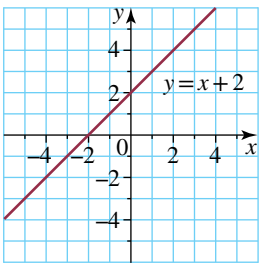
Meat type	Cost per kilogram (\$/kg)	Weight required (kg)	Cost = \$/kg × kg
Lamb	15	1	15
Chicken	10	0.5	5
Beef	7.50	2	15
		Total cost	35

Exercise 7.4 Sketching linear graphs

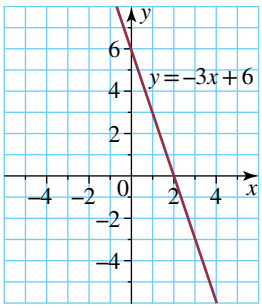
1. a.



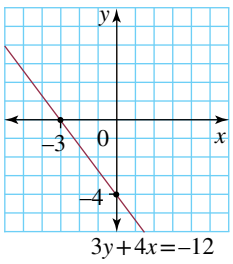
b. $y = x + 2$



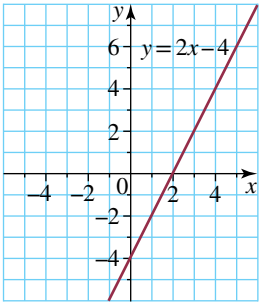
c. $y = -3x + 6$



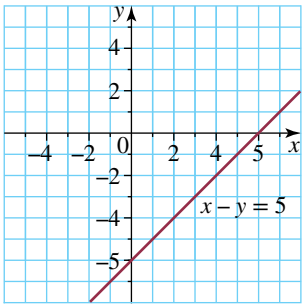
d.



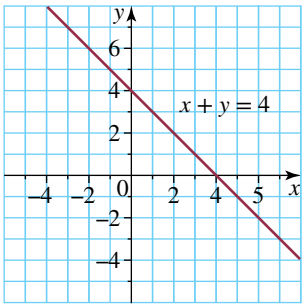
e. $y = 2x - 4$



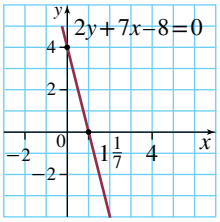
f. $x - y = 5$



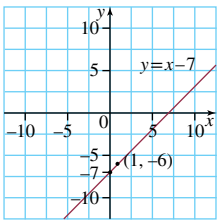
g. $x + y = 4$



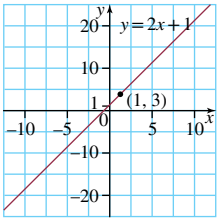
h.



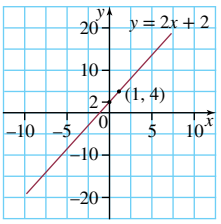
2. a.



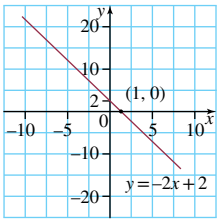
b.



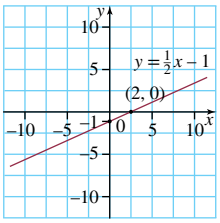
c.



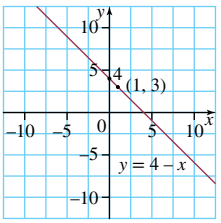
d.



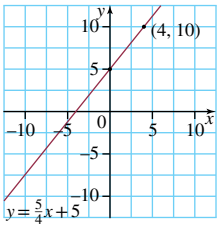
e.



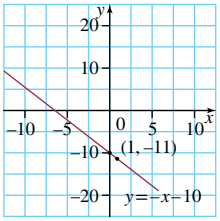
f.



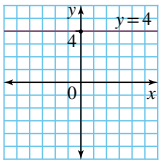
g.



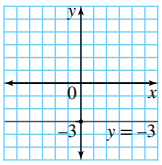
h.



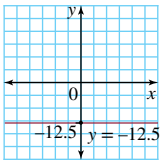
3. a.



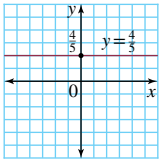
b.



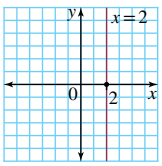
c.



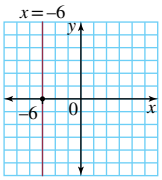
d.



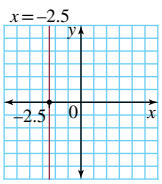
4. a.



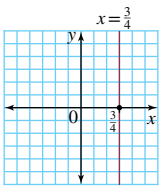
b.



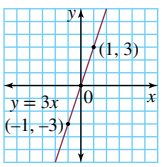
c.



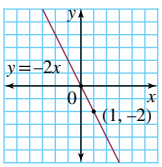
d.



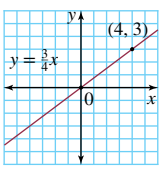
5. a.



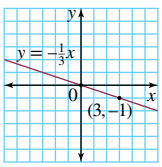
b.



c.



d.



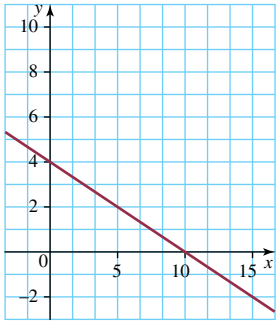
6. a. D **b.** D

7. a. **i.** $y = -\frac{2x}{5} + 4$

ii. $-\frac{2}{5}$

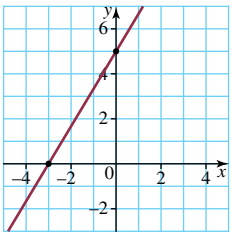
iii. The x-intercept is 10, the y-intercept is 4.

iv.



b. i. $m = \frac{\text{rise}}{\text{run}} = \frac{5}{3}$

ii.



iii. $y = \frac{5x}{3} + 5$

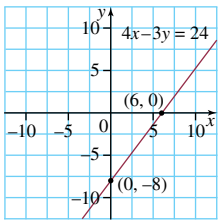
$-5x + 3y = 15$

8. a. $y = \frac{4}{3}x - 8$

b. $y = 0, 4x = 24, x = 6$

$x = 0, -3y = 24, y = -8$

c.



9. a. $y = \frac{-ax + c}{b}$

b. Answers will vary. The gradient is $\frac{-a}{b}$; substituting positive values always results in a negative gradient.

10. Answers will vary; a possible answer is shown.

$4y = \frac{4}{2}x + 3$

$4y = 2x + 3$

$-2x + 4y = 3$

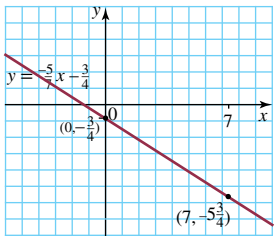
11. Answers will vary. All descriptions use the idea that a gradient is equal to $\frac{\text{rise}}{\text{run}}$, which equals $\frac{2}{3}$ in all of these cases.

12. Answers will vary. The gradient is 2. The equation of the line is $y = 2x - 1$. One point is (1, 1).

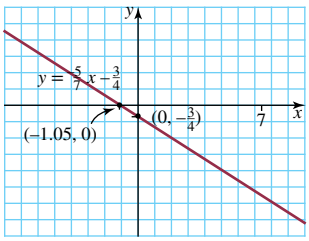
13. a. i. C **ii.** B **iii.** A

b. Straight line y-intercept of (0, 2) with a slope of $-\frac{1}{4}$

14. a. i.



ii.



iii. Answers will vary.

b. Answers will vary.

15. Answers will vary.

Exercise 7.5 Technology and linear graphs

1. **i.** As the size of the coefficient increases, the steepness of the graph increases.

ii. Each graph cuts the x-axis at $(0, 0)$.

iii. Each graph cuts the y-axis at $(0, 0)$.

2. **i.** As the magnitude of the coefficient decreases, the steepness of the graph increases.

ii. Each graph cuts the x-axis at $(0, 0)$.

iii. Each graph cuts the y-axis at $(0, 0)$.

3. **a.** positive, positive **b.** downward, negative **c.** bigger **d.** will

4. **i.** Yes, 1 **ii.** No, the lines are parallel.

iii. **a.** $(0, 0)$, i.e. $x = 0$ **b.** $(-2, 0)$, i.e. $x = -2$ **c.** $(2, 0)$, i.e. $x = 2$

iv. **a.** $(0, 0)$, i.e. $y = 0$ **b.** $(0, 2)$, i.e. $y = 2$ **c.** $(0, -2)$, i.e. $y = -2$

5. **i.** Yes, -1 **ii.** No, the lines are parallel.

iii. **a.** $(0, 0)$, i.e. $x = 0$ **b.** $(2, 0)$, i.e. $x = 2$ **c.** $(-2, 0)$, i.e. $x = -2$

iv. **a.** $(0, 0)$, i.e. $y = 0$ **b.** $(0, 2)$, i.e. $y = 2$ **c.** $(0, -2)$, i.e. $y = -2$

6. **i.** same **ii.** y-intercept, y-axis **iii.** y-intercept **iv.** x-intercept

7. **i.** No **ii.** Yes

iii. **a.** 1 **b.** -1 **c.** 3 **d.** $-\frac{2}{5}$

iv. **a.** $(-5, 0)$, i.e. $x = -5$ **b.** $(5, 0)$, i.e. $x = 5$ **c.** $(-\frac{5}{3}, 0)$, i.e. $x = -\frac{5}{3}$ **d.** $(\frac{25}{2}, 0)$, i.e. $x = \frac{25}{2}$

v. **a.** $(0, 5)$, i.e. $y = 5$ **b.** $(0, 5)$, i.e. $y = 5$ **c.** $(0, 5)$, i.e. $y = 5$ **d.** $(0, 5)$, i.e. $y = 5$

8. steepness, y-coordinate, y-axis, same, parallel, the same

9. **a.** **i.** 2 **ii.** $(0, 0)$, i.e. $y = 0$

b. **i.** 1 **ii.** $(0, 1)$, i.e. $y = 1$

c. **i.** -3 **ii.** $(0, 5)$, i.e. $y = 5$

d. **i.** $\frac{2}{3}$ **ii.** $(0, -7)$, i.e. $y = -7$

10. Answers will vary. They all have in common the point $(1, 5)$. Their gradients are all different.

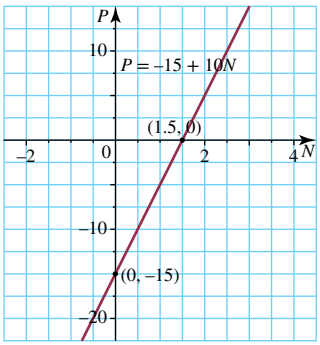
11. Answers will vary. Use solve on CAS.

12. Answers will vary but should show the y-intercept $(0, 2.2)$, point $(1, 2.75)$, and gradient 0.55.

13. **a.** N is the number of dogs walked; $-\$15$ is Shirly's starting cost and out-of-pocket expense before she walks a dog, and she earns \$10 for every dog walked.

b. She needs to walk at least 2 dogs a week before she can make a profit.

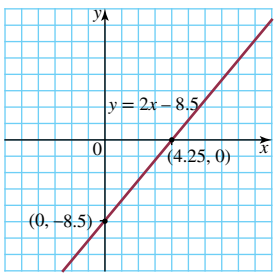
c.



14. Answers will vary.

15. **a.** $y = 2x - 8.5$

b.



c. $y = 7.5$

d. $x = 10.25$

16. a. $C = 25n + 1000$

b. (0, 1000). This is the initial cost of the site licence.

c. $6000

d. 80

17. a. The method will vary: possible methods include plotting the points (0, 325) and (1, 328.8) or using the equation $c = 3.8m + 325$.

b. 382 calories

c. 32.9 minutes

Exercise 7.6 Determining linear rules

1. a. $y = 4x + 2$ **b.** $y = -4x + 1$ **c.** $y = 4x + 8$

d. $y = 6x + 7$ **e.** $y = -2.5x + 6$ **f.** $y = 45x + 135$

2. a. $y = -2x$ **b.** $y = 4x$ **c.** $y = 10.5x$

d. $y = -20x$ **e.** $y = 1.07x$ **f.** $y = 32x$

3. a. $y = x + 2$ **b.** $y = -x + 8$ **c.** $y = -4x - 8$ **d.** $y = 2x - 13$

e. $y = -5x + 70$ **f.** $y = 2x - 23$ **g.** $y = -6x + 11$ **h.** $y = -x - 1.5$

i. $y = 6x + 30$ **j.** $y = -3.5x + 15.5$ **k.** $y = 1.2x - 4.08$ **l.** $y = 0.2x - 1.76$

4. a. $y = -4x - 24$ **b.** $y = 2x - 6$ **c.** $y = -2x + 4$

d. $y = 5x + 35$ **e.** $y = 1.5x - 3.75$ **f.** $y = 0.4x - 0.96$

5. a. $y = x + 17$ **b.** $y = -x + 3$ **c.** $y = 11$

d. $y = x + 3.5$ **e.** $y = -x + 3.5$ **f.** $y = \frac{1}{9}x + \frac{34}{9}$ (or $9y = x + 34$)

g. $y = -6x + 90$ **h.** $y = 20x$ **i.** $y = -x + 4$

j. $y = -0.25x + 5.5$ **k.** $y = \frac{2}{7}x - \frac{32}{7}$ (or $7y = 2x - 32$) **l.** $y = \frac{2}{3}x + \frac{10}{3}$ (or $3y = 2x + 10$)

6. a. $y = x + 3$ **b.** $y = \frac{-5}{4}x + 5$ **c.** $y = -6x + 6$ **d.** $y = \frac{7}{8}x + 35$

e. $y = x + 8$ **f.** $y = -2x + 6$ **g.** $y = \frac{-3}{7}x - 3$ **h.** $y = \frac{1}{4}x + 50$

7. a. $y = \frac{7}{4}x$ **b.** $y = x$ **c.** $y = -2x$

d. $y = -3x$ **e.** $y = \frac{-24}{11}x$ **f.** $y = \frac{-x}{3}$

8. a. $y = 2x + 4$ **b.** $y = 5x$ **c.** $y = -x + 3$

d. $y = \frac{1}{5}x$ **e.** $y = x + 2$ **f.** $y = -2x + 3$

9. a. B **b.** A **c.** D

10. a. $y = -2.4x - 12$ **b.** $y = -58.32$

11. a. $y = -0.6x + 10$ **b.** $(16\frac{2}{3}, 0)$

12. a. The equation is $y = \frac{5}{4}x - 2135$.

b. Values of c cannot be read directly from the graph because the graph doesn't contain the origin, and since c is found at $x = 0$, we need to use the equation.

c. In 2020, the concentration of CO_2 will be 390 ppm. The assumption is that the concentration of CO_2 will continue to follow this linear pattern.

13. $m = \dfrac{-15 + 7}{4 - 0} = -2$

$y = -2x + c$

$6 = -6 + c$

$c = 12$

The equation is $y = -2x + 12$.

14. a. Line A: $y = x - 1$ Line B: $y = -x + 8$

b. (4, 3)

c. Answers will vary. Possible methods include graphing on a Cartesian plane or using algebra.

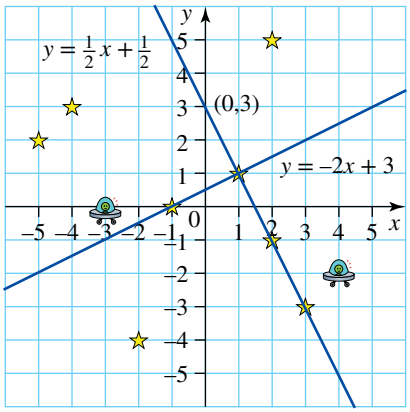
Line B: $y = -x + 8$, $m = -1$, $m_{\perp} = 1$

$y = x + c$

$6 = -4 + c$

$c = 10$

15. a, c



b. $y = -2x + 3$

d. (1, 1)

e. 1 star

f. Answers will vary but could include the following.

$y = -x - 1$, $x = 2$

16. $m = \frac{1}{4}C$

17. Answers will vary but should include the correct equation: $T = 4.25t + 15$.

18. 8.1 mL

19. a. A: \$0 B: \$100 C: \$200

b. A: \$80 B: \$50 C: \$33.33

c. A: $w = 80h$ B: $w = 50h + 100$ C: $w = 33.\dot{3}h + 200$

d. i. C **ii.** A

e. The advantage of location C is that it has the highest minimum pay, so you would be guaranteed at least \$200 per day, regardless of how many hours you work.

f. i. A: \$160 B: \$200 C: \$266.66

ii. A: \$480 B: \$400 C: \$400

Challenge 7.1

$y = \frac{1}{2}x - \frac{1}{3}$

Exercise 7.7 Practical applications of linear graphs

1. a. $y = 6x - 5$ **b.** $y = -3x + 8$ **c.** $y = 2x + 4$ **d.** $y = 0.9x + 1.1$

e. $y = 3x + 1$ **f.** $y = -x + 17$ **g.** $C = 4n + 5$ **h.** $n = 5t$

i. $v = -3t + 27$ **j.** $C = 3d + 8$

2. D

3. a. $A = 25t + 60$ **b.** \$5060 **c.** \$60

4. a. $C = 0.55s + 2.0$ **b.** \$8.60

5. a. $N = 400t + 3200$ **b.** 5400 **c.** 13 200

6. a. $W = -40t + 712$ **b.** 712 L **c.** 18 days

7. a. $N = -22t + 164$ **b.** 120 **c.** 5 **d.** 7.5 months

8. a. 27 **b.** 108

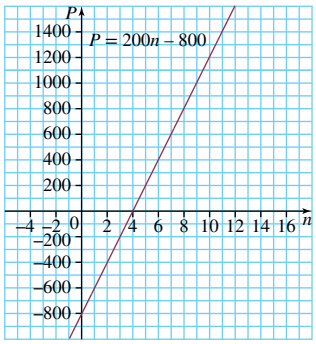
9. a. $C = 2.14d + 3.50$ **b.** \$28.11 **c.** \$52.93

10. a. $C = 2.57d + 3.50$ **b.** \$33.06 **c.** 7.20 km

11. $5 \times 21.4 = 107\,\text{cm} = 1.07\,\text{m}$

12. Answers will vary. ($P = 10t + 200$)

13. a.



b. (0, −800), i.e. $y = -800$; fixed costs are $800

c. (4, 0), i.e. $x = 4$; break-even amount

d. 200; sale price per print that contributes to profit

14. a. $Y = 78.7A$ **b.** ¥196 750 **c.** $Y_T = 76.5A$

15. Cody arrives first.

Exercise 7.8 Midpoint of a line segment and distance between two points

1. a. (2, 4) **b.** (5, 1) **c.** (7, 2) **d.** (8, 9) **e.** (0, 5)
f. (−1, 2) **g.** (−5, − 2) **h.** (2, 3) **i.** (0, 12) **j.** (−3, 7)

2. a. $(1\frac{1}{2}, 5\frac{1}{2})$ **b.** $(-3, -3\frac{1}{2})$ **c.** $(-9\frac{1}{2}, 9\frac{1}{2})$ **d.** $(-1\frac{1}{2}, 6\frac{1}{2})$

3. a. 5 **b.** 9 **c.** −16 **d.** 0

4. a. (4, 6) **b.** (−2, −3) **c.** (11, 2) **d.** (−7, −3)

5. $y = 5x - 7$

6. a. 5 **b.** 10 **c.** 1 **d.** 2 **e.** 13
f. 10 **g.** 5 **h.** 17 **i.** 13 **j.** 20

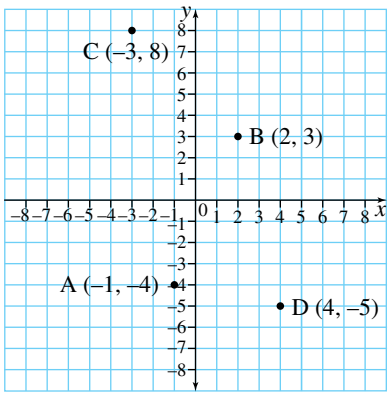
7. a. 7.211 **b.** 14.765 **c.** 13.038
d. 8.944 **e.** 19.416 **f.** 19.416

8. a. 24.472 **b.** 25.464

9. a. 17.788 **b.** 25.763

10. 21.024 km

11. a.



b. $M = \left(\frac{-1 + -3}{2}, \frac{-4 + 8}{2}\right)$
$= (-2, 2)$

c. $2\sqrt{37}$

d. (7, −2)

e. $\frac{3 - (-4)}{2 - (-1)} = \frac{7}{3}$

f. $y = -4x + 11$

12. $-2 = \dfrac{x+5}{2} \Rightarrow x = -9$

$-4 = \dfrac{-6+y}{2} \Rightarrow y = -2$

The required point is $(-9, -2)$.

13. $d = \sqrt{(6-2)^2 + (-1-2)^2)}$

$= \sqrt{16+9}$

$= 5$

14. $d_{AB} = \sqrt{(3-(-10))^2 + (15-6)^2}$

$= \sqrt{169+81}$

$= \sqrt{250}$

$= 5\sqrt{10}$

$d_{BC} = \sqrt{(-1-(-10))^2 + (-7-6)^2}$

$= \sqrt{81+169}$

$= \sqrt{250}$

$= 5\sqrt{10}$

$d_{AB} = d_{BC}$

Therefore, B is equidistant from A and C.

15. Answers will vary; a sample answer is shown.

$d_{AB} = \sqrt{(3-1)^2 + (2+4)^2}$

$= \sqrt{4+36}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$d_{BC} = \sqrt{(3+3)^2 + (0+2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$d_{AC} = \sqrt{(-3-1)^2 + (0+4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Side length AB is equal to side length BC but not equal to side length AC. Therefore, ΔABC is an isosceles triangle.

16. $\frac{1}{2}$

17. $(-4, 5), (-3, 2), (-2, -1), (-1, -4), (0, -7)$

Challenge 7.2

$m = -2, c = -3$

Exercise 7.9 Non-linear relations (parabolas, hyperbolas, circles)

1. a. Vertical 3 up, TP (0, 3)

b. Vertical 1 down, TP (0, −1)

c. Vertical 7 down, TP (0, −7)

d. Vertical $\frac{1}{4}$ up, TP $(0, \frac{1}{4})$

2. a. Horizontal 1 right, (1, 0)

b. Horizontal 2 right, (2, 0)

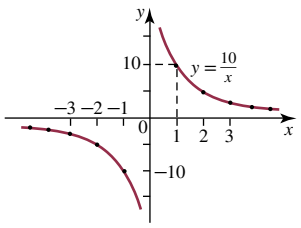
c. Horizontal 10 left, (−10, 0)

d. Horizontal 4 left, (−4, 0)

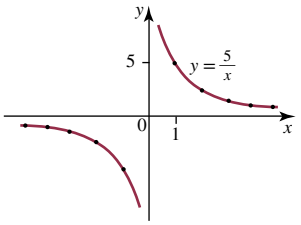
3. a. (0, 1), max **b.** (0, −3), min **c.** (−2, 0), max

4.

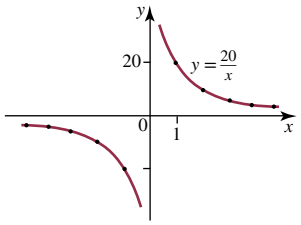
x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	−2	−2.5	−3.3	−5	−10	Undefined	10	5	3.3	2.5	2



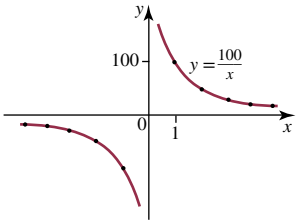
5. a. i.



ii.

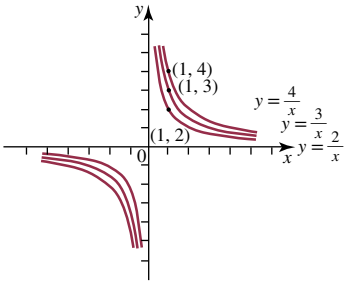


iii.



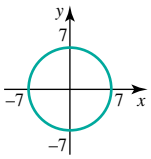
b. i. $x = 0,\ y = 0$ **ii.** $x = 0,\ y = 0$ **iii.** $x = 0,\ y = 0$

6.



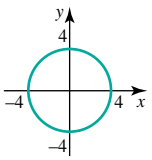
7. It increases the y-values by a factor of k and hence dilates the curve by a factor of k. As k increases, the graph is further from the origin.

8. a.



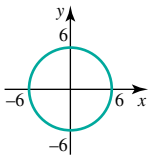
Centre (0, 0), radius 7

b.



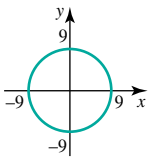
Centre (0, 0), radius 4

c.



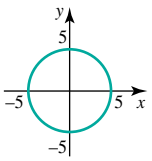
Centre (0, 0), radius 6

d.



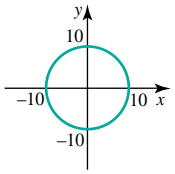
Centre (0, 0), radius 9

e.



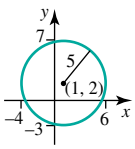
Centre (0, 0), radius 5

f.

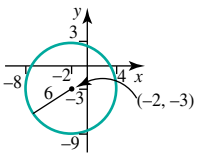


Centre (0, 0), radius 10

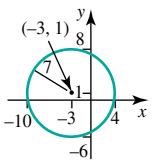
9. a.



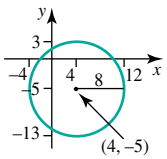
b.



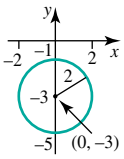
c.



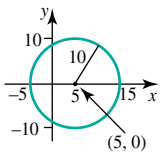
d.



e.



f.



10. D

11. Answers will vary. Possible methods include using a table of values or graphing both equations on a Cartesian plane.

12. Similarity: they both have the same asymptotes — $x = 0$ and $y = 0$.

Difference: one is a reflection of the other about the y-axis.

13. Answers will vary; use algebra or graph the equation. The turning point is a minimum.

14. $r = \sqrt{(6-2)^2 + (1-1)^2}$

$= \sqrt{16}$

$= 4$

The centre is (2, 1). The equation is $(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-2)^2 + (y-1)^2 = 16$

15. $x^2 + y^2 - 12x - 4y = 9$

$(x - 6)^2 + (y - 2)^2 = 49$

From the centre (6, 2) to the origin, the distance is $\sqrt{40} \approx 6.32$.

The equation of the line from the centre to the origin is $y = \frac{1}{3}x$.

Solve simultaneously: $y = \frac{1}{3}x$ and $(x - 6)^2 + (y - 2)^2 = 49$.

The coordinates of the point closest to the origin are $(-0.64, -0.21)$.

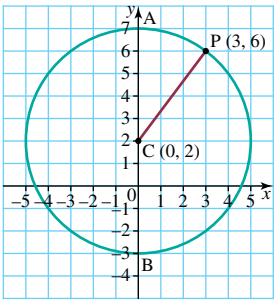
16. a. Bob started well and raced ahead of Frederick and Walter. He then turned around and went back to the starting line. He stayed at the starting line for an amount of time before turning around and sprinting to the finishing line, passing Frederick and Walter.

b. From the graph, Frederick took 5 seconds to begin running.

c. From the graph, at $t = 0$, Walter is 2.4 metres from the starting line.

d. Bob won the race.

17.

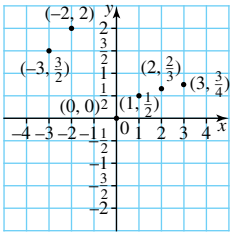


At (0, 7) and (0, −3)

18. a.

x	−3	−2	−1	0	1	2	3
y	$\frac{3}{2}$	2	Undefined	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

b.



c. Not linear

7.10 Review

1. D **2.** B **3.** D **4.** B **5.** D

6. A **7.** B **8.** A **9.** D **10.** A

11. a. i. 8 **ii.** −3

b. i. −9 **ii.** 5

c. i. −2 **ii.** 6

d. i. 2 **ii.** 0

e. i. $\frac{2}{3}$ **ii.** $-\frac{1}{3}$

12. a, b

13. a, c, e

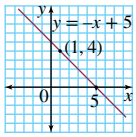
14. a. 2 **b.** $-\frac{2}{5}$ **c.** 0

15. a. 2 **b.** $\frac{5}{4}$

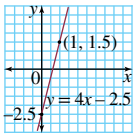
16. a. $m = -3, c = 7$ **b.** $m = \frac{3}{2}, c = 3$

c. $m = -\frac{2}{5}, c = 0$ **d.** $m = 0, c = 4$

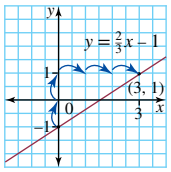
17. a.



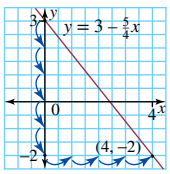
b.



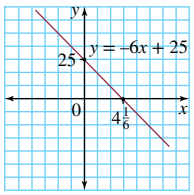
c.



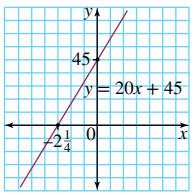
d.



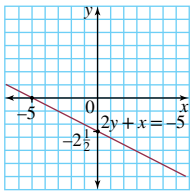
18. a.



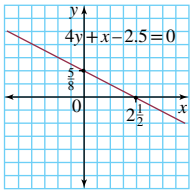
b.



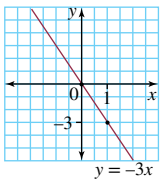
c.



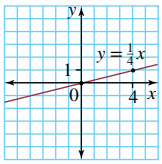
d.



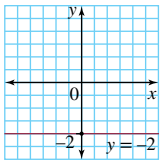
19. a.



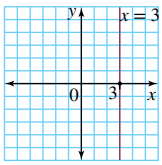
b.



c.



d.



20. a. $y = 2x$ **b.** $y = 2 - \frac{2}{7}x$

21. a. $y = 2x - 7$ **b.** $y = 2x - 14$

c. $y = 2x - 5$ **d.** $y = -5x$

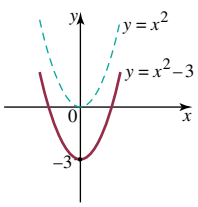
e. $y = -x - 2$ **f.** $y = -2.75x + 7.75$

g. $y = x - 3$ **h.** $y = -2x + 5$

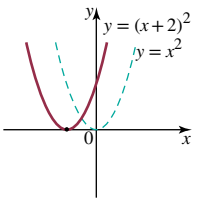
22. (1.1)

23. 5 units

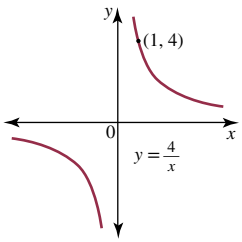
24. a. Vertical translation 3 units down; TP = (0, −3)



b. Horizontal translation 2 units to the left; TP = (2, 0)

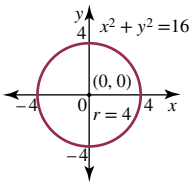


25. a.

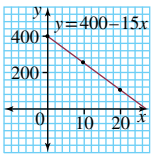


b. Hyperbola

26.



27. a. $y = 400 - 15x$

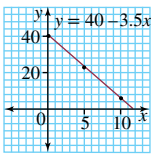


b. 26.7 or 27 weeks

c. $175

d. 21 repayments

28. a. $y = 40 - 3.5x$



b. 11.4 hours

c. 22.75 km

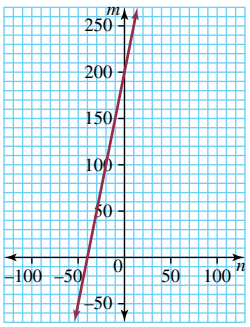
29. a. $n = 5t + 25$

b. 65

c. 336 minutes

30. a. $m = 5n + 200$

b.



c. Everything to the right of the vertical axis and including the vertical axis applies to her earnings because she can only sell 0, 1, 2, etc. necklaces.

d. Everything to the left of the vertical axis, because she cannot sell a negative number of necklaces and because she cannot make less than her base salary of $200.

31. $\frac{1}{2}$

32. $(-1, 5)$

33. $p = -0.25, t = \frac{1}{4}$

34. $y = -3x - 4.5$

Investigation — Rich task

1. 6

2. Teacher to check

3. **a** and **h**

4. **e**

5. **a**, **b**, **c** and **h**

6. Yes

7.

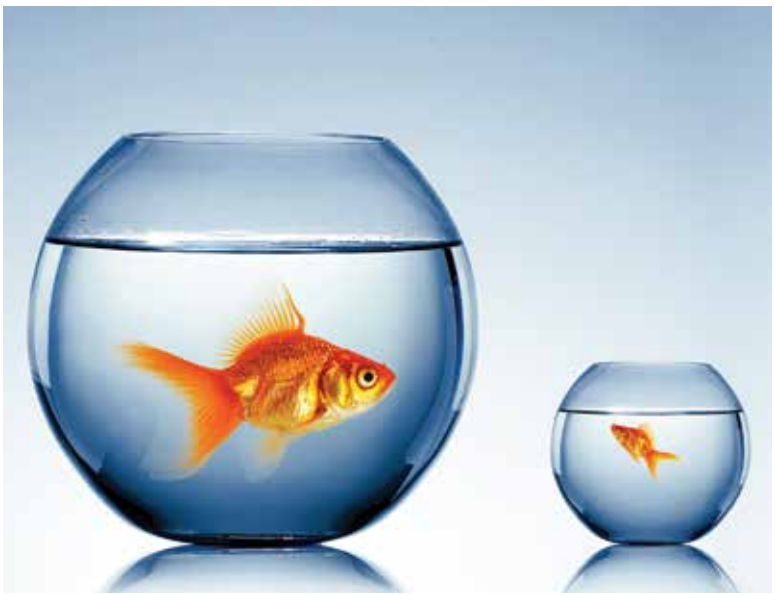
Length of table (m)	Width of table (m)	Destination pocket
3	0.25	Far left
3	0.5	Far left
3	0.75	Far left
3	1	Far right
3	1.25	Far left
3	1.5	Far left
3	1.75	Far right
3	2	Close right

8. Answers will vary.

TOPIC 8
Proportion and rates

8.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



8.1.1 Why learn this?

Proportion and rates are often used to compare quantities. Kilometres per hour, prices per kilogram, dollars per litre, and pay per hour or per week are all examples of rates. Dividing quantities according to a particular rule requires an understanding of ratios and rates.

8.1.2 What do you know?

assessON

1. **THINK** List what you know about proportion and rates. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then a small group.
3. **SHARE** As a class, use a thinking tool such as a large concept map to show your class's knowledge of proportion and rates.

LEARNING SEQUENCE

8.1 Overview
8.2 Direct proportion
8.3 Direct proportion and ratio
8.4 Inverse proportion
8.5 Introduction to rates
8.6 Constant and variable rates
8.7 Rates of change
8.8 Review

learnON RESOURCES — ONLINE ONLY

Watch this video: The story of mathematics: The Golden Ratio (eles-1695)

8.2 Direct proportion

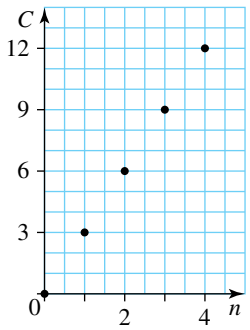
8.2.1 Direct proportion

- Suppose that ice-creams cost $3 each and that you are to buy some for your friends. There is a relationship between the cost of the ice-creams (C) and the number of the ice-creams that you buy (n).



The relationship can be illustrated in a table or a graph.

n	0	1	2	3	4
C ($)	0	3	6	9	12



- This relationship has some important characteristics:
  - As n increases, so does C.
  - When $n = 0$, $C = 0$.
  - The graph of the relationship is a straight line passing through the origin.

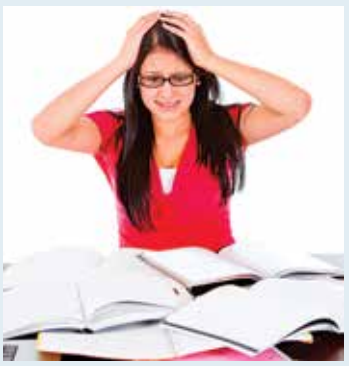
  When all of these characteristics apply, the relationship is called **direct proportion**.
- We say that 'C is directly proportional to n' or 'C varies directly as n.' This is written as $C \propto n$.

WORKED EXAMPLE 1

Does direct proportion exist between these variables?

a The height of a stack of photocopy paper (h) and the number of sheets (n) in the stack

b Your Mathematics mark (m) and the number of hours of Maths homework you have completed (n)



THINK	WRITE
a When n increases, so does h. When $n = 0$, $h = 0$. If graphed, the relationship would be linear.	**a** $h \propto n$
b As n increases, so does m. When $n = 0$, I may get a low mark at least, so $n \neq 0$.	**b** m is not directly proportional to n.

WORKED EXAMPLE 2

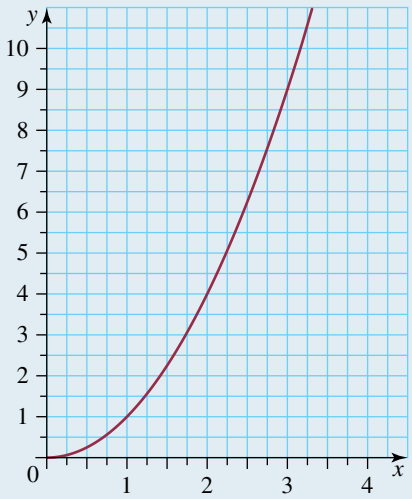
Do the following relationships show direct proportion?

a

t	0	1	2	3	4
y	0	1	3	7	15

b

n	1	2	3	4
c	5	10	15	20



THINK	WRITE
a From the table, when t increases, so does y. When $t = 0$, $y = 0$. The t-values increase by a constant amount but the y-values do not, so the relationship is not linear. When t is doubled, y is not.	**a** y is not directly proportional to t.
b From the table, as n increases, so does m. Extending the pattern gives $n = 0$, $C = 0$. The n-values and C-values increase by constant amounts, so the relationship is linear.	**b** $C \propto n$
c When x increases, so does y. When $x = 0$, $y = 0$. The graph is not a straight line.	**c** y is not directly proportional to x.

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Try out this interactivity: Direct proportion (int-2767)

Complete this digital doc: SkillSHEET: Measuring the rise and the run (doc-6174)

Exercise 8.2 Direct proportion

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2	Questions: 1, 2, 4, 5	Questions: 1–7

Individual pathway interactivity: int-4510

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency



1. WE1 For each of the following pairs of variables, state whether direct proportion exists. If it does not exist, give a reason.

a. The distance (d) travelled in a car travelling at 60 km/h and the time taken (t)

b. The speed of a swimmer (s) and the time the swimmer takes to complete one lap of the pool (t)

c. The cost of a bus ticket (c) and the distance travelled (d)

d. The perimeter (p) of a square and the side length (l)

e. The area of a square (A) and the side length (l)

f. The total cost (C) of buying n boxes of pencils

g. The weight of an object in kilograms (k) and the weight in pounds (p)

h. The distance (d) travelled in a taxi and the cost (c)

i. A person's height (h) and their age (a)

Understanding

2. WE2 For each of the following, determine whether direct proportion exists between the variables. If it does not, give a reason.

a.

x	0	1	2	3	4
y	0	1	3	8	15

b.

a	0	1	2	3	6
M	0	8	16	24	48

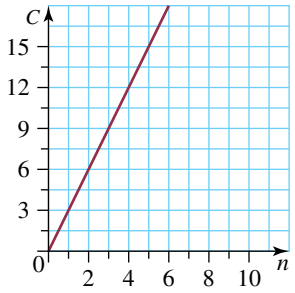
c.

t	0	1	2	4	8
d	0	3	6	9	12

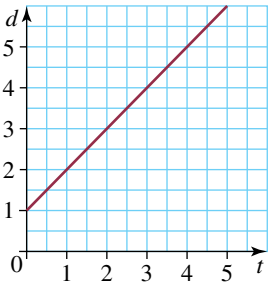
d.

n	0	1	2	3	4
C	10	20	30	40	50

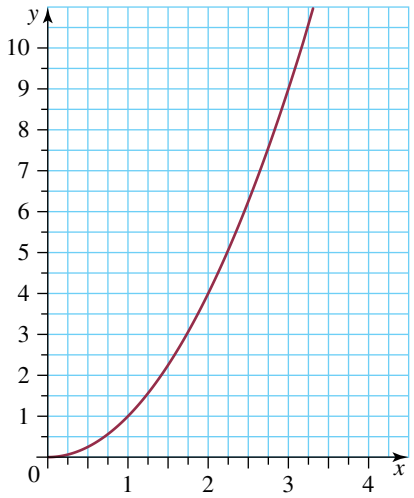
e.



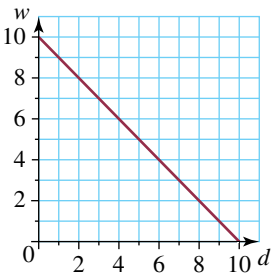
f.



g.



h.



3. List five pairs of real-life variables that exhibit direct proportion.

Reasoning

4. Which point must always exist in a table of values if the two variables exhibit direct proportionality?

5. If direct proportion exists between two variables, m and n, fill out the table and explain your reasoning.

m	0	2	5
n	0		20

Problem solving

6. Mobile phone calls are charged at 17 cents per 30 seconds

a. Does direct proportion exist between the cost of a phone bill and the number of 30-second time periods?

b. If a call went for 7.5 minutes, how much would the call cost?

7. Bruce is building a pergola and needs to buy treated pine timber. He wants 4.2-metre and 5.4-metre lengths of timber. If it costs $23.10 for the 4.2-metre length and $29.16 for the 5.4-metre length, does direct proportion exist between the cost of the timber and the length of the timber per metre? Explain.

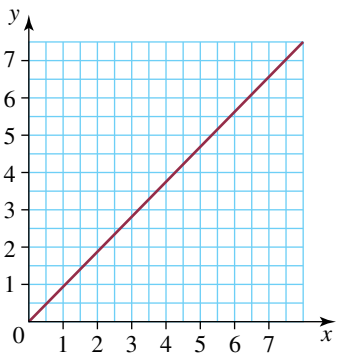
Reflection

How do you know when two quantities are directly proportional?

8.3 Direct proportion and ratio

8.3.1 Constant of proportionality

- A direct proportion relationship ($y \propto x$) is shown by the graph at right. Because the graph passes through the origin, the y-intercept is equal to zero, and the graph has the equation $y = kx$, where k is the gradient of the graph.
- In the equation $y = kx$:
  - k is a constant, and is called the **constant of proportionality** (or the constant of variation)
  - $y \propto x$ is equivalent to the equation $y = kx$
  - y is called the dependent variable and is normally placed in the bottom row of a table
  - x is called the independent variable and is normally placed in the top row of a table.



WORKED EXAMPLE 3

Given that $y \propto x$, and $y = 12$ when $x = 3$, find the constant of proportionality and state the rule linking y and x.

THINK	WRITE
1 $y \propto x$, so write the linear rule.	$y = kx$
2 Substitute $y = 12$, $x = 3$ into $y = kx$.	$12 = 3k$
3 Find the constant of proportionality by solving for k.	$k = 4$
4 State the rule.	$y = 4x$

WORKED EXAMPLE 4

The weight (W) of \$1 coins in a bag varies directly as the number of coins (n). Twenty coins weigh 180 g.
a Find the relationship between W and n.
b How much will 57 coins weigh?
c How many coins weigh 252 g?



THINK	WRITE
Summarise the information given in a table.	$W = kn$

n	20	57	
W	180		252

a 1 Substitute $n = 20$, $W = 180$ into $W = kn$.	**a**	$180 = 20k$
2 Solve for k.		$k = 9$

3	State the relationship between W and n.	$W = 9n$
b 1	State the rule.	**b** $W = 9n$
2	Substitute 57 for n to find W.	$W = 9 \times 57$ $= 513$
3	State the answer.	Fifty-seven coins will weigh 513 g.
c 1	State the rule.	**c** $W = 9n$
2	Substitute 252 for W.	$252 = 9n$
3	Solve for n.	$n = \frac{252}{9}$
4	State the answer.	$= 28$ Twenty-eight coins weigh 252 g.

8.3.2 Ratio

- If $y \propto x$, then $y = kx$, where k is constant.
  Transposing this formula gives $\frac{y}{x} = k$.
  In other words, for any pair of values (x, y) the ratio $\frac{y}{x}$ is constant and equals the constant of proportionality.
- For example, this table shows that $v \propto t$.

t	1	2	3	4
v	5	10	15	20

It is clear that $\frac{20}{4} = \frac{15}{3} = \frac{10}{2} = \frac{5}{1} = 5$.

WORKED EXAMPLE 5

Sharon works part time and is paid at a fixed rate per hour. If she earns $135 for 6 hours work, how much will she earn for 11 hours?

THINK	WRITE
1 Sharon's payment is directly proportional to the number of hours worked. Write the rule.	$P = kn$
2 Summarise the information given. The value of x is to be found.	n: 6, 11 P: 135, x
3 Since $P = kn$, $\frac{P}{n}$ is constant. Solve for x.	$\frac{135}{6} = \frac{x}{11}$ $x = \frac{135 \times 11}{6}$ $x = 247.5$
4 State the answer.	Sharon earns $247.50 for working 11 hours.

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Complete this digital doc: SkillSHEET: Rounding to a given number of decimal places (doc-6175)

Complete this digital doc: WorkSHEET Direct variation (doc-6181)

Exercise 8.3 Direct proportion and ratio

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 9–11, 14	Questions: 1–5, 7, 9, 10, 12, 14, 15	Questions: 1–5, 8–10, 12–17

Individual pathway interactivity: int-4511

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE3 If a is directly proportional to b, and $a = 30$ when $b = 5$, find the constant of proportionality and state the rule linking a and b.

2. If $a \propto b$, and $a = 2.5$ when $b = 5$, find the rule linking a and b.

3. If $C \propto t$, and $C = 100$ when $t = 8$, find the rule linking C and t.

4. If $v \propto t$, and $t = 20$ when $v = 10$, find the rule linking v and t.

5. If $F \propto a$, and $a = 40$ when $F = 100$, find the rule linking a and F.

Understanding

6. WE4 Springs are often used to weigh objects, because the extension of a spring (E) is directly proportional to the weight (W) of the object hanging from the spring. A 4-kg load stretches a spring by 2.5 cm.

a. Find the relationship between E and W.

b. What load will stretch the spring by 12 cm?

c. How much will 7 kg extend the spring?



7. Han finds that 40 shelled almonds weigh 52 g.

a. Find the relationship between the weight (W) and the number of almonds (n).

b. How many almonds would there be in a 500-g bag?

c. How much would 250 almonds weigh?

8. Petra knows that her bicycle wheel turns 40 times when she travels 100 m.

a. Find the relationship between the distance travelled (d) and the number of turns of the wheel (n).

b. How far does she go if her wheel turns 807 times?

c. How many times does her wheel turn if she travels 5 km?



9. Fiona, who operates a plant nursery, uses large quantities of potting mix. Last week she used 96 kg of potting mix to place 800 seedlings in medium-sized pots.

a. Find the relationship between the mass of potting mix (M) and the number of seedlings (n).
b. How many seedlings can she pot with her remaining 54 kg of potting mix?
c. How much potting mix will she need to pot 3000 more seedlings?

10. WE5 Tamara is paid at a fixed rate per hour. If she earns $136 for 5 hours work, how much will she earn for working 8 hours?

11. It costs $158 to buy 40 packets of cards. How much will 55 packets cost?

12. If 2.5 L of lawn fertiliser will cover an area of 150 m^2, how much fertiliser is needed to cover an area of 800 m^2?

Reasoning

13. Paul paid $68.13 for 45 L of fuel. At the same rate, how much would he pay for 70 L? Justify your answer.

14. Rose gold is an alloy of gold and copper that is used to make high-quality musical instruments. If it takes 45 g of gold to produce 60 g of rose gold, how much gold would be needed to make 500 g of rose gold? Justify your answer.



Problem solving

15. If Noah takes a group of friends to the movies for his birthday and it would cost $62.50 for five tickets, how much would it cost if there were 12 people (including Noah) in the group?

16. Sharyn enjoys quality chocolate, so she makes a trip to her favourite chocolate shop. She is able to select her favourite chocolates for $7.50 per 150 grams. Since Sharyn loves her chocolate, she decides to purchase 675 grams. How much did she spend?

17. Anthony drives to Mildura, covering an average of 75 kilometres in 45 minutes. He has to travel 610 kilometres to get to Mildura.
a. Find the relationship between the distance travelled in kilometres, D, and his driving time in hours, T.
b. How long will it take Anthony to complete his trip?
c. If he stops at Bendigo, 220 kilometres from his starting position, how long does it take him to reach Bendigo?
d. How much longer will it take him to arrive at Mildura after leaving Bendigo?

Reflection

What is ratio?

CHALLENGE 8.1

The volume of a bird's egg can be determined by the formula $V = kl^3$, where V is the volume in cm^3, l is the length of the egg in cm and k is a constant. A typical ostrich egg is 15 cm long and has a volume of 7425 cm^3. What is the volume of a typical 5 cm long chicken egg?

8.4 Inverse proportion

8.4.1 Inverse proportion

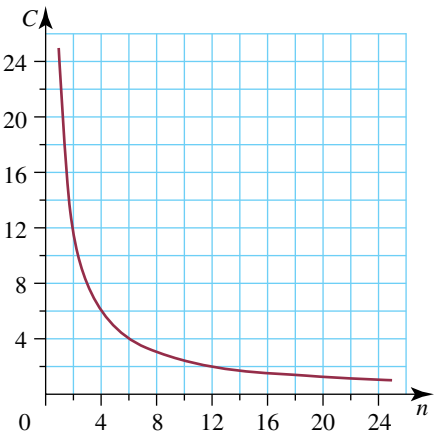


- If 24 sweets are shared between 4 children, then each child will receive 6 sweets. If the sweets are shared by 3 children, then each will receive 8 sweets.
- The relationship between the number of children (C) and the number sweets for each child (n) can be given in a table.

C	1	2	3	4	6	8	12
n	24	12	8	6	4	3	2

- As the number of children (C) increases, the number of sweets for each child (n) decreases. This is an example of inverse proportion or inverse variation.
- We say that 'n is inversely proportional to C' or 'n varies inversely as C'.

  This is written as $C \propto \frac{1}{n}$, or $C = \frac{k}{n}$, where k is a constant (the constant of proportionality). This formula can be rearranged to $Cn = k$. Note that multiplying any pair of values in the table (3×8, 12×2) gives the same result.



- The relationship has some important characteristics:
  - As C increases, n decreases, and vice versa.
  - The graph of the relationship is a **hyperbola**.

WORKED EXAMPLE 6

y is inversely proportional to x and $y = 10$ when $x = 2$.
a Calculate the constant of proportionality, k, and hence the rule relating x and y.
b Plot a graph of the relationship between x and y, for values of x from 2 to 10.

THINK

a 1 Write the relationship between the variables.

2 Rewrite as an equation using k, the constant of proportionality.

3 Substitute $y = 10$, $x = 2$ into $y = \frac{k}{x}$ and solve for k.

4 Write the rule by substituting $k = 20$ into $y = \frac{k}{x}$.

b 1 Use the rule $y = \frac{20}{x}$ to set up a table of values for x and y, taking values for x which are positive factors of k so that only whole number values of y are obtained.

For example, $x = 4$, $y = \frac{20}{4} = 5$.

WRITE/DRAW

a $y \propto \frac{1}{x}$

$y = \frac{k}{x}$

$10 = \frac{k}{2}$

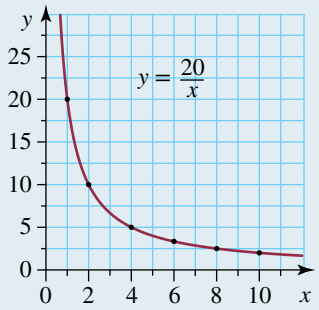
$k = 20$

$y = \frac{20}{x}$

b

x	2	4	6	8	10
y	10	5	3.3	2.5	2

2 Plot the points on a clearly labelled set of axes and join the points with a smooth curve. Label the graph.



WORKED EXAMPLE 7

When a wire is connected to a power source, the amount of electrical current (I) passing through the wire is inversely proportional to the resistance (R) of the wire. If a current of 0.2 amperes flows through a wire of resistance 60 ohms:

a find the constant of proportionality
b determine the rule relating R and I
c find the resistance if the current equals 5 amperes
d find the current that will flow through a wire of resistance 20 ohms.



THINK

Summarise the information in a table.

$I \propto \frac{1}{R}$

Write the rule.

a 1 Substitute $R = 60$, $I = 0.2$ into $I = \frac{k}{R}$.

2 Solve for k.

b Write the rule using $k = 12$.

c 1 Substitute $I = 5$ into $I = \frac{12}{R}$.

2 Solve for R.

3 Write the answer.

d 1 Substitute $R = 20$ into $I = \frac{12}{R}$.

2 Write the answer.

WRITE

R	60		20
I	0.2	5	

$I = \frac{k}{R}$

a $0.2 = \frac{k}{60}$

$0.2 \times 60 = k$

$k = 12$

b $I = \frac{12}{R}$

c $5 = \frac{12}{R}$

$5R = 12$

$R = \frac{12}{5}$

$= 2.4$

The resistance equals 2.4 ohms.

d $I = \frac{12}{20}$

$= 0.6$

The current will be 0.6 amperes.

Exercise 8.4 Inverse proportion

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10	Questions: 1–13	Questions: 1–13

Individual pathway interactivity: int-4512

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Fluency

1. Decide whether inverse proportion exists between each pair of variables. If it does exist, write an equation to describe the relationship.
- **a.** The speed of a car (s) and the time (t) it takes to complete one lap of a race circuit
- **b.** The amount of money (D) that I have and the number (n) of cards that I can buy
- **c.** The time (t) that it takes to make a pair of jeans and the number of pairs (p) that can be made in one day
- **d.** The price (P) of petrol and the amount (L) that can be bought for $80
- **e.** The price (P) of petrol and the cost (C) of buying 80 L
- **f.** The number of questions (n) in a test and the amount of time (t) available to answer each one

2. List three examples of inverse proportion.

3. WE6 y varies inversely as x and $y = 100$ when $x = 10$.
- **a.** Calculate the constant of proportionality, k, and hence the rule relating x and y.
- **b.** Plot a graph of the relationship between x and y, for values of x that are positive factors of k less than 21.

4. p is inversely proportional to q and $p = 12$ when $q = 4$.
- **a.** Calculate the constant of proportionality, k, and hence the rule relating p and q.
- **b.** Plot a graph of the relationship between q and p, for values of q that are positive factors of k less than 11.

5. y varies inversely as x and $y = 42$ when $x = 1$.
- **a.** Calculate the constant of proportionality, k, and hence the rule relating x and y.
- **b.** Plot a graph of the relationship between x and y, for values of x from 1 to 10.

Understanding



6. WE7 When a constant force is applied to an object, its acceleration is inversely proportional to its mass. When the acceleration of an object is 40 m/s^2, the corresponding mass is 100 kg.
- **a.** Calculate the constant of proportionality.
- **b.** Determine the rule relating mass and acceleration.
- **c.** Determine the acceleration of a 200-kg object.
- **d.** Determine the acceleration of a 1000-kg object.

7. The number of colouring pencils sold is inversely proportional to the price of each pencil.
Two thousand pencils are sold when the price is $0.25 each.
- **a.** Calculate the constant of proportionality.
- **b.** Determine the number of pencils that could be sold for $0.20 each.
- **c.** Determine the number of pencils that could be sold for $0.50 each.

8. The time taken to complete a journey is inversely proportional to the speed travelled. A trip is completed in 4.5 hours travelling at 75 km per hour.
- **a.** Calculate the constant of proportionality.
- **b.** Determine how long, to the nearest minute, the trip would take if the speed was 85 km per hour.
- **c.** Determine the speed required to complete the journey in 3.5 hours, correct to 1 decimal place.
- **d.** Determine the distance travelled in each case.

9. The cost per person travelling in a charter plane is inversely proportional to the number of people in the charter group. It costs $350 per person when 50 people are travelling.
- **a.** Calculate the constant of variation.
- **b.** Determine the cost per person, to the nearest cent, if there are 75 people travelling.
- **c.** Determine how many people are required to reduce the cost to $250 per person.
- **d.** Determine the total cost of hiring the charter plane.

Reasoning

10. The electrical current in a wire is inversely proportional to the resistance of the wire to that current. There is a current of 10 amperes when the resistance of the wire is 20 ohms.
- **a.** Calculate the constant of proportionality.
- **b.** Determine the current possible when the resistance is 200 ohms.
- **c.** Determine the resistance of the wire when the current is 15 amperes.
- **d.** Justify your answer to parts **b** and **c** using a graph.

11. Two equations relating the time of a trip, T, and the speed at which they travel, S, are given. For both cases the time is inversely proportional to the speed: $T_1 = \frac{5}{S_1}$ and $T_2 = \frac{7}{S_2}$. Explain what impact the different constants of proportionality have on the time of the trip.

Problem solving



12. The time it takes to pick a field of strawberries is inversely proportional to the number of pickers. It takes 2 people 5 hours to pick all of the strawberries in a field.
- **a.** Calculate the constant of proportionality.
- **b.** Determine the rule relating time (T) and the number of pickers (P).
- **c.** Determine the time spent if there are 6 pickers.

13. For a constant distance covered by a sprinter, the sprinter's speed is inversely proportional to their time. If a sprinter runs at a speed of 10.4 m/s, the corresponding time is 9.62 seconds.
- **a.** Calculate the constant of variation.
- **b.** Determine the rule relating speed (V) and time (T).
- **c.** Determine the time, correct to 2 decimal places, if they ran at a speed of 10.44 m/s.
- **d.** Determine the time, correct to 2 decimal places, if they ran at a speed of 6.67 m/s.

Reflection

Explain what is meant by inverse proportion.

8.5 Introduction to rates

- The word **rate** occurs commonly in news reports and conversation.
  'Home ownership rates are falling.'
  'People work at different rates.'
  'What is the current rate of inflation?'
  'Do you offer a student rate?'
  'The crime rate seems to be increasing.'
- Rate is a word often used when referring to a specific ratio.



8.5.1 Average speed

- If you travel 160 km in 4 hours, your average speed is 40 km per hour.
  Speed is an example of a rate, and its unit of measurement (kilometres per hour) contains a formula.
  The word 'per' can be replaced with 'divided by', so speed $= \frac{\text{distance (in km)}}{\text{time (in hours)}}$.
- It is important to note the units involved. For example, an athlete's speed is often measured in metres per second (m/s) rather than km/h.

8.5.2 Rates

- In general, to find a rate (or special ratio), one quantity is divided by another.

WORKED EXAMPLE 8

Calculate the rates suggested by these statements.
a A shearer shears 1110 sheep in 5 days.
b Eight litres of fuel costs $12.56
c A cricket team scored 152 runs in 20 overs.

THINK

a The rate suggested is 'sheep per day'.
Write the ratio (rate).

b The rate suggested is 'dollars per litre'.
Write the ratio (rate).

c The rate suggested is 'runs per over'.
Write the ratio (rate).

WRITE

a Rate $= \frac{\text{number of sheep}}{\text{number of days}}$
$= \frac{1110}{5}$
$= 222$ sheep/day

b Rate $= \frac{\text{number of dollars}}{\text{number of litres}}$
$= \frac{12.56}{8}$
$= \$1.57$ per litre

c Rate $= \frac{\text{number of runs}}{\text{number of overs}}$
$= \frac{152}{20}$
$= 7.6$ runs per over

WORKED EXAMPLE 9

The concentration of a solution is measured in g/L (grams per litre). What is the concentration of the solution when 10 g of salt is dissolved in 750 mL of water?

THINK

Concentration is measured in g/L, which means that concentration = number of grams (mass) ÷ number of litres (volume). Write the ratio (rate).

WRITE

$$\text{Concentration} = \frac{\text{mass}}{\text{volume}}$$
$$= \frac{10}{0.75}$$
$$= 13.3 \text{ g/L}$$

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Complete this digital doc: WorkSHEET: Rates (doc-6182)

Exercise 8.5 Introduction to rates

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5	Questions: 1–11	Questions: 1–11

Individual pathway interactivity: int-4513

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Fluency

1. Hayden drove from Hay to Bee. He covered a total distance of 96 km and took 1.5 hours for the trip. What was Hayden's average speed for the journey?
2. Find the average speed in km/h for each of the following.
   - **a.** 90 km in 45 min
   - **b.** 5500 km in 3 h 15 min (correct to 1 decimal place)
3. WE8 Calculate the rates suggested by these statements, giving your answers to 2 decimal places where necessary.
   - **a.** It costs $736 for 8 theatre tickets.
   - **b.** Penelope decorated 72 small cakes in 3 hours.
   - **c.** Usain Bolt has a 100 m world sprint record of 9.58 seconds.
   - **d.** It takes 30 hours to fill the swimming pool to a depth of 90 cm.
   - **e.** Peter received $260 for 15 hours work.
   - **f.** Yan received $300 for assembling 6 air conditioners.

Understanding

4. A metal bolt of volume 25 cm^3 has a mass of 100 g. Find its density (mass per unit of volume).
5. WE9 One hundred and twenty grams of sugar is dissolved in 200 mL of water. What is the concentration of this solution in g/L? Explain your answer.

Reasoning



6. In the race between the tortoise and the hare, the hare ran at 72 km per hour while the giant tortoise moved at 240 cm per minute. Compare and explain the difference in the speed of the two animals.

7. The average speed of a car is determined by the distance of the journey and the time the journey takes. Explain the two ways in which the speed can be increased.

Problem solving

8. In the 2000 Sydney Olympics, Cathy Freeman won gold in the 400-metre race. Her time was 49.11 seconds. In the 2008 Beijing Olympics, Usain Bolt set a new world record for the men's 100-metre race. His time was 9.69 seconds. Calculate the average speed of the winner for each race in kilometres per hour.

9. A school had 300 students in 2013 and 450 students in 2015. What was the average rate of growth in students per year?



10. Mt Feathertop is Victoria's second highest peak. To walk to the top involves an increase in height of 1500 m over a horizontal distance of 10 km (10 000 m). Calculate the average gradient of the track.

11. Beaches are sometimes unfit for swimming if heavy rain has washed pollution into the water. A beach is declared unsafe for swimming if the concentration of bacteria is more than 5000 organisms per litre. A sample of 20 millilitres was tested and found to contain 55 organisms. Calculate the concentration in the sample (in organisms/litre) and state whether or not the beach should be closed.

Reflection

What is an example of a rate that is not listed in this section?

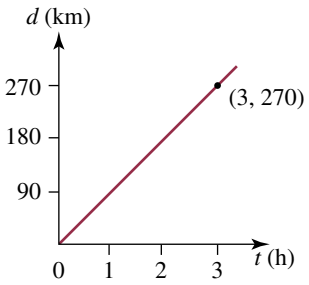
8.6 Constant and variable rates

8.6.1 Constant rates

- Consider a car travelling along a highway at a constant speed (rate) of 90 km/h. After 1 hour it will have travelled 90 km, as shown in the table below.

Time (h)	0	1	2	3
Distance (km)	0	90	180	270

- The distance–time graph is shown at right.



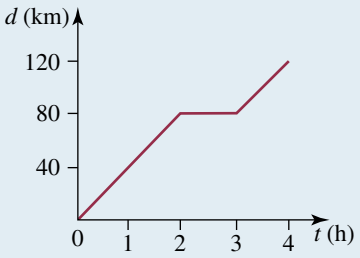
The gradient of the graph is $\frac{270 - 0}{3 - 0} = 90$.

The equation of the graph is $d = 90t$, and the gradient is equal to the speed, or rate of progress.

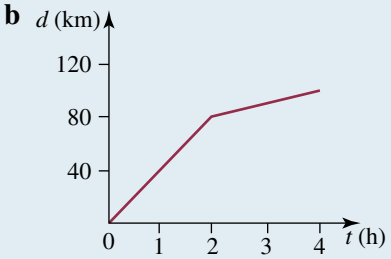
WORKED EXAMPLE 10

Each diagram below illustrates the distance travelled by a car over time. Describe the journey, including the speed of the car.

a



b



THINK

a There are three distinct sections.

1 In the first 2 hours, the car travels 80 km: $\frac{80}{2} = 40$ km/h.

2 In the next hour, the car does not move.

3 In the fourth hour, the car travels 40 km.

WRITE

a

The car travels at a speed of 40 km/h for 2 hours, and then stops for 1 hour. After this it travels for 1 hour at 40 km/h.

b There are two distinct sections.

1 In the first 2 hours, the car travels 80 km: $\frac{80}{2} = 40$ km/h.

2 In the next 2 hours, the car travels 20 km: $\frac{20}{2} = 10$ km/h.

b

The car travels at a speed of 40 km/h for 2 hours, and then travels at 10 km/h for a further 2 hours.

WORKED EXAMPLE 11

Draw a distance–time graph to illustrate the following journey.
A cyclist travels for 1 hour at a constant speed of 10 km/h, and then stops for a 30-minute break before riding a further 6 km for half an hour at a constant speed.

THINK

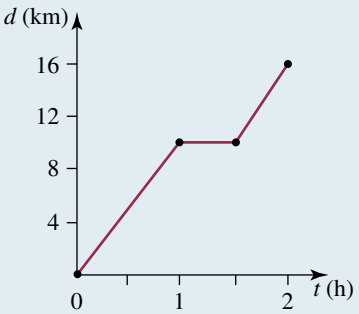
There are three phases to the journey.
The graph starts at (0, 0).

1 In the first hour, the cyclist travels 10 km. Draw a line segment from (0, 0) to (1, 10).

2 For the next half-hour, the cyclist is stationary, so draw a horizontal line segment from (1, 10) to (1.5, 10).

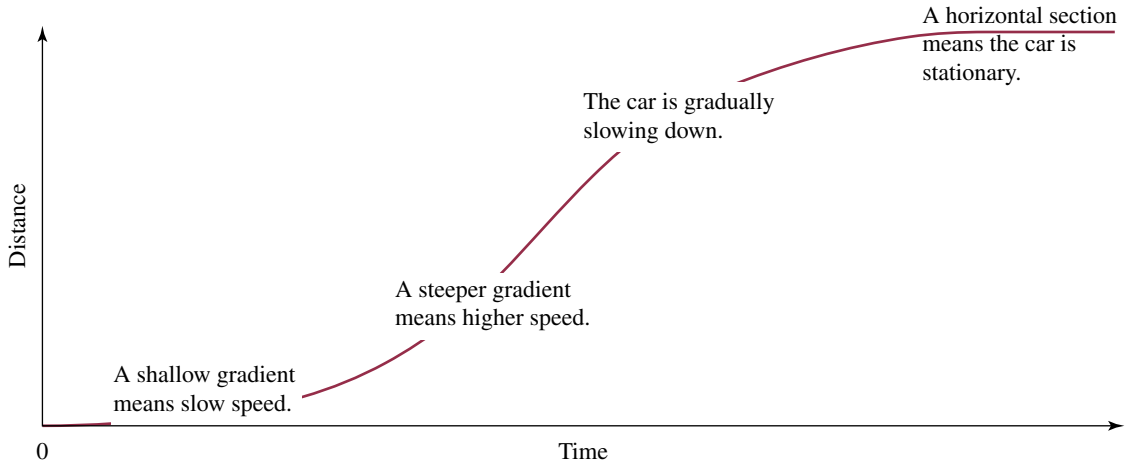
3 In the next half-hour, the cyclist travels 6 km. Draw a line segment from (1.5, 10) to (2, 16).

DRAW



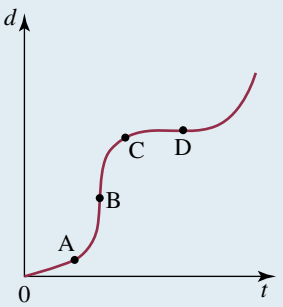
8.6.2 Variable rates

- In reality, a car tends not to travel at constant speed. It starts from rest and gradually picks up speed.
- When the speed is low, the distance–time graph will have a small gradient, and when the speed is high, the gradient will be steep.



WORKED EXAMPLE 12

The diagram at right illustrates the distance travelled by a car over time. Describe what is happening, in terms of speed, at each of the marked points.



THINK	WRITE
From the graph:	
At A the gradient is small, but becoming steeper.	At A the car is travelling slowly but accelerating.
At B the gradient is at its steepest and is not changing.	At B the car is at its greatest speed.
At C the graph is becoming flatter.	At C the car is slowing down.
At D the graph is horizontal.	At D the car is stationary.

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Complete this digital doc: SkillSHEET: Equivalent rates (doc-10845)

Exercise 8.6 Constant and variable rates

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7	Questions: 1–13	Questions: 1–10, 13

Individual pathway interactivity: int-4514

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Fluency

1. Two friends take part in a 24-kilometre mini-marathon. They run at constant speed. Ali takes 2 hours and Beth takes 3 hours to complete the journey.

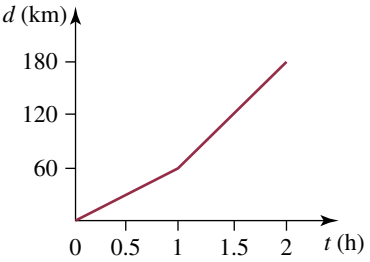
a. On the same diagram, draw a distance–time graph for each runner.

b. i. What is the equation for each graph?

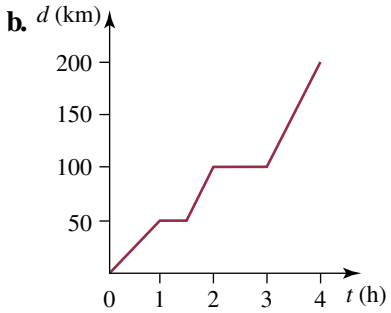
ii. What is the difference between the two graphs?

2. WE10 Each diagram below illustrates the distance travelled by a car over time. Describe the journey, including the speed of the car.

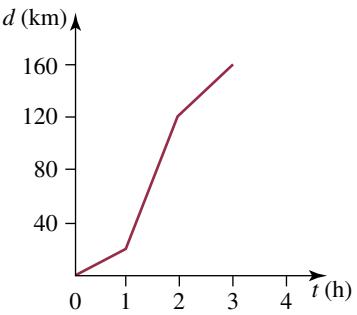
a.



b.



c.

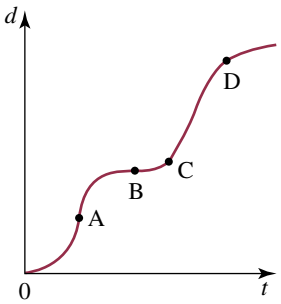


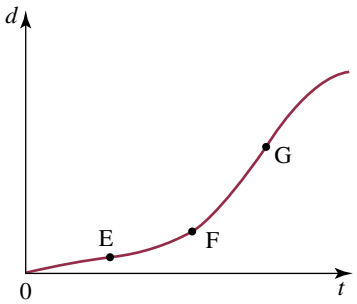
3. WE11 Draw a distance–time graph to illustrate each of the following journeys.

a. A cyclist rides at 40 km/h for 30 minutes, stops for a 30-minute break, and then travels another 30 km at a speed of 15 km/h.

b. Zelko jogs at a speed of 10 km/h for one hour, and then at half the speed for another hour.

4. WE12 The diagrams below illustrate the distances travelled by two cars over time.





a. Describe what is happening in terms of speed at each of the marked points.
b. For each diagram:
i. at which point is the speed the greatest?
ii. at which point is the speed the least?
iii. at which point is the car stationary?

Understanding

5. The table below shows the distance travelled, D, as a person runs for R minutes.



***R* (minutes)**	10	20	50
***D* (km)**	2	4	10

a. Find the rate in km/minute between the time 10 minutes and 20 minutes.
b. Find the rate in km/minute between the distance 4 km and 10 km.
c. Is the person's speed constant? Why?

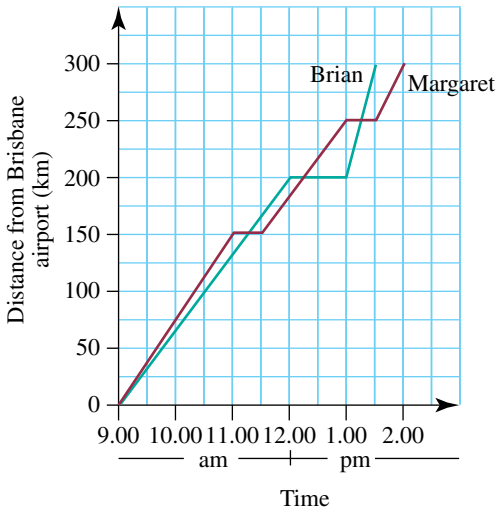
6. The table below shows the water used, W, after the start of the shower, where T is the time after the shower was started.

***T* (minutes)**	0	1	2	4
***W* (litres)**	0	20	30	100

a. Find the rate of water usage in L/minute for the 4 stages of the shower.
b. Was the rate of water usage constant?

Reasoning

7. Margaret and Brian left Brisbane airport at 9.00 am. They travelled separately but on the same road and in the same direction. Their journeys are represented by the travel graph below.



a. At what distance from the airport did their paths cross?
b. How far apart were they at 1.00 pm?
c. For how long did each person stop on the way?
d. What was the total time spent driving and the total distance for each person?
e. Calculate the average speed while driving for each person.

8. Hannah rode her bike along the bay one morning. She left home at 7.30 am and covered 12 km in the first hour. She felt tired and rested for half an hour. After resting she completed another 8 km in the next hour to reach her destination.

a. How long did Hannah take for the entire journey?

b. What is the total distance for which she actually rode her bike?

c. Draw a travel graph for Hannah's journey.



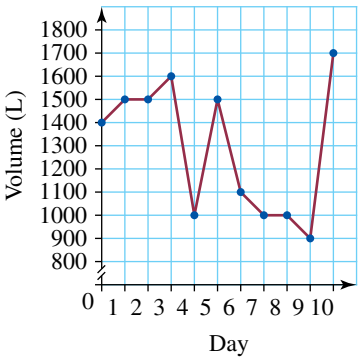
9. Rebecca and Joanne set off at the same time to jog 12 kilometres. Joanne ran the entire journey at constant speed and finished at the same time as Rebecca. Rebecca set out at 12 km/h, stopping after 30 minutes to let Joanne catch up, then she ran at a steady rate to complete the distance in 2 hours.

a. Show the progress of the two runners on a distance–time graph.

b. How long did Rebecca wait for Joanna to catch up?

Problem solving

10. Use the graph showing the volume of water in a rainwater tank to answer the following questions.



a. During which day(s) was the rate of change positive?

b. During which day(s) was the rate of change negative?

c. During which day(s) was the rate of change zero?

d. On which day did the volume of water increase at the fastest rate?

e. On which day did the volume of water decrease at the fastest rate?

11. An internet service provider charges $30 per month plus $0.10 per megabyte downloaded. The table of monthly cost versus download amount is shown.

Download (MB)	0	100	200	300	400	500
Cost ($)	30	40	50	60	70	80

a. By how much does the cost increase when the download amount increases from:

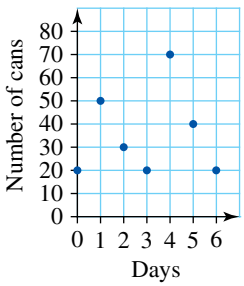
i. 0 to 100 MB

ii. 100 to 200 MB

iii. 200 to 300 MB?

b. Is the cost increasing at a constant rate?

12. The graph shown at right shows the number of soft-drink cans in a vending machine at the end of each day.

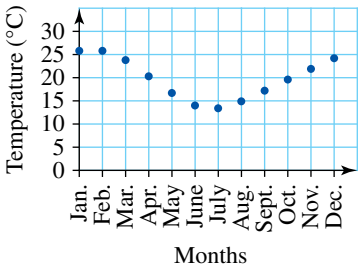


a. By how much did the number of soft-drink cans change in the first day?

b. By how much did the number of soft-drink cans change in the fifth day?

c. Is the number of soft-drink cans changing at a constant or variable rate?

13. The table below and graph at right show Melbourne's average daily maximum temperature over the year.



	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Mean maximum (°C)	25.8	25.8	23.8	20.3	16.7	14.0	13.4	14.9	17.2	19.6	21.9	24.2

Source: Weatherzone, www.weatherzone.com.au.

a. What is the average maximum temperature in:
  i. February
  ii. June?

b. What is the change in temperature from:
  i. January to August
  ii. November to December?

c. Is the temperature changing at a constant rate?

Reflection

How can you tell the difference between constant and variable rates?

CHALLENGE 8.2

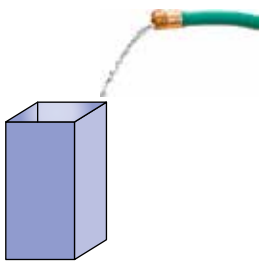
An early method of catching speeding motorists was the amphometer. It consisted of two airtight tubes placed across the road at a separation of 15 m. The tubes were connected to a timing device that detected the change in pressure as a car crossed them. In a 60 km/h zone, what was the minimum time for the car to travel across the device without being booked for speeding?

8.7 Rates of change

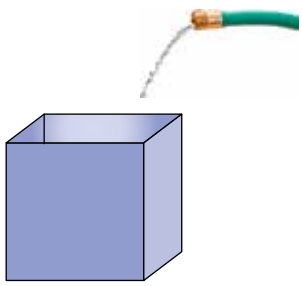
8.7.1 Rates of change

- Imagine that water is flowing out of a hose at a steady rate and will be used to fill several containers.

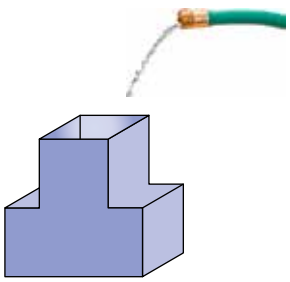
A



B



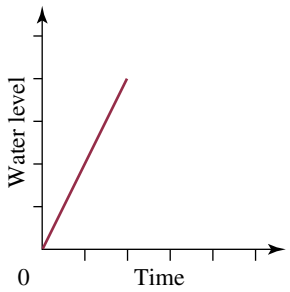
C



Because they are of different widths, they will fill at different rates. The narrow container will fill at a faster rate than the other two.

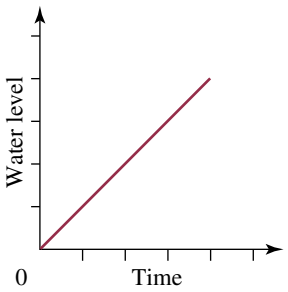
- Consider a graph of the water level against time for the three containers.

Container A is narrow, so the water level will rise quickly.



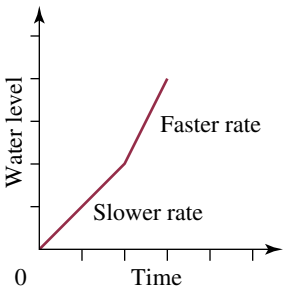
The water level changes at a constant rate.

Container B is wide and will fill at a slower rate.



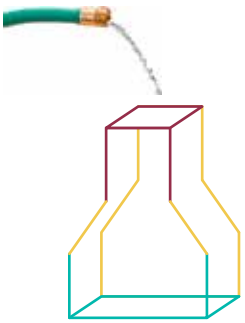
The water level changes at a constant rate.

Container C is wide at the bottom, so the water will rise slowly at first, then quickly when it reaches the narrow part.

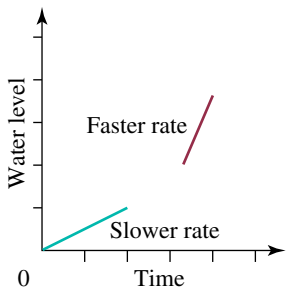


The water level changes at two different but constant rates.

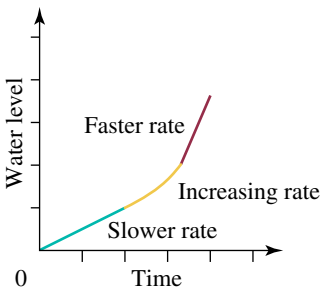
- Here is a more complex container, with three distinct sections.



The bottom section fills slowly and the top section fills quickly.



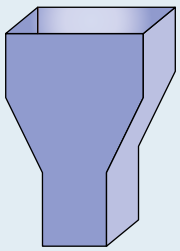
In between, the rate changes steadily from slow to fast. In this section, the rate is increasing.



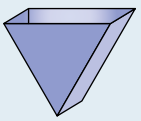
WORKED EXAMPLE 13

Each of these containers is being filled with water at a steady rate. For each container, sketch a graph of the water level against time.

a



b

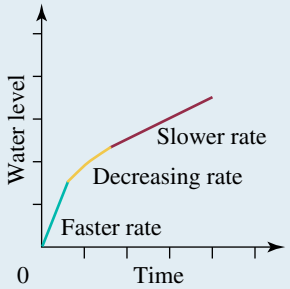


THINK

a The container has three distinct sections.
The bottom section will fill quickly.
The top section will fill slowly.
In the middle section, the rate will increase gradually from slow to quick.

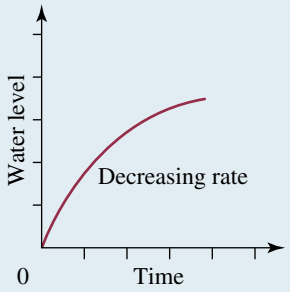
DRAW

a



b At the bottom of the container the water will rise rapidly, slowing down as the container becomes wider.

b



learnON RESOURCES — ONINE ONLY

Complete this digital doc: SkillSHEET: Completing a table of values (doc-10846)

Complete this digital doc: SkillSHEET: Plotting from a table of values (doc-10847)

Complete this digital doc: WorkSHEET: Problems using rates (doc-6187)

Exercise 8.7 Rates of change

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6	Questions: 1–8	Questions: 1–9

Individual pathway interactivity: int-4515

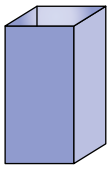
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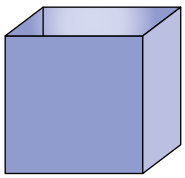
Fluency

1. These containers are filled with water at a steady rate. Match each container with the appropriate graph.

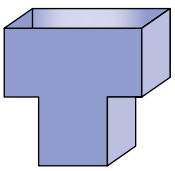
a.



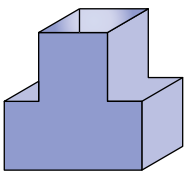
b.



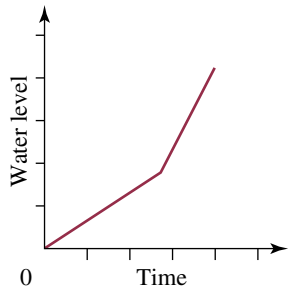
c.



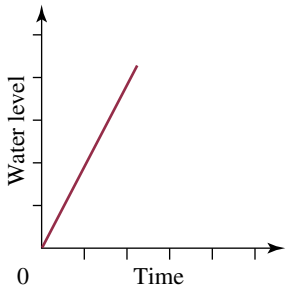
d.



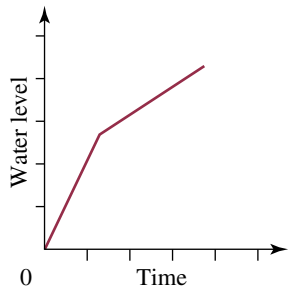
A.



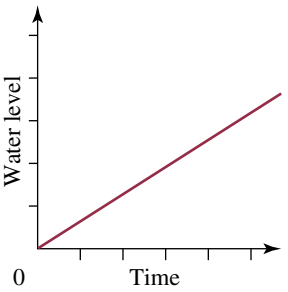
B.



C.

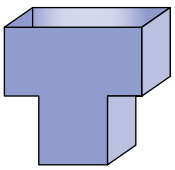


D.

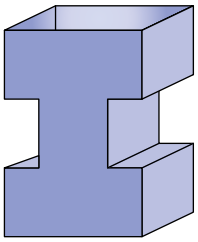


2. These containers are being filled with water at a steady rate. Sketch a graph of the water level against time.

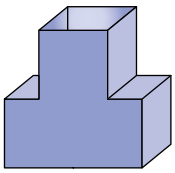
a.



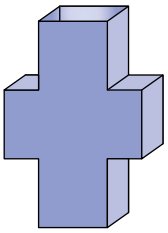
b.



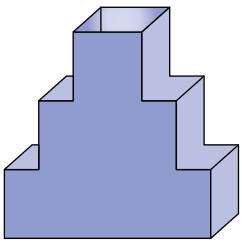
c.



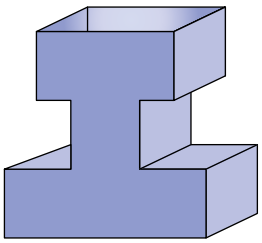
d.



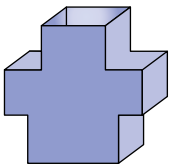
e.



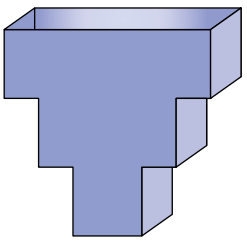
f.



g.



h.

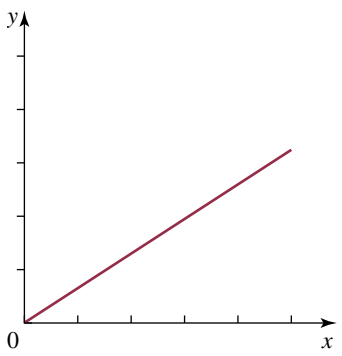


Understanding

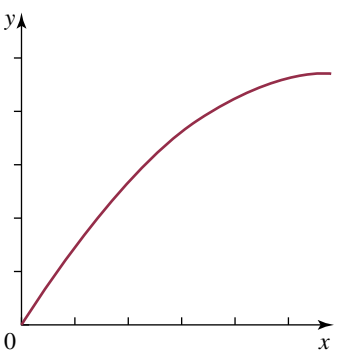
3. Which of these graphs show:

a. a steady rate **b.** an increasing rate **c.** a decreasing rate?

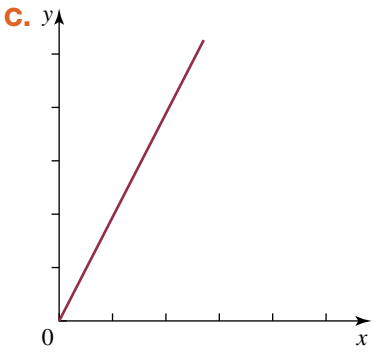
A.



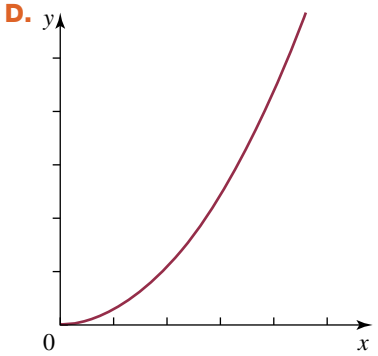
B.



C.



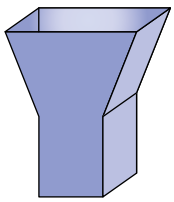
D.



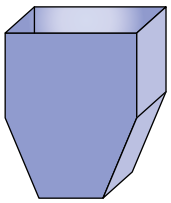
Reasoning

4. WE13 These containers are being filled with water at a steady rate. For each container, sketch a graph of water level against time and explain your reasons.

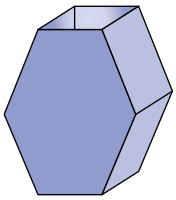
a.



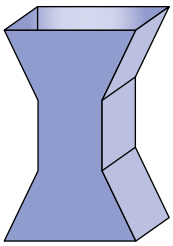
b.



c.

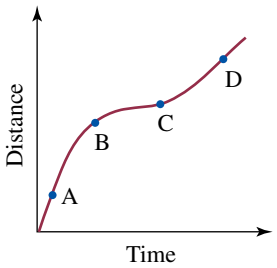


d.



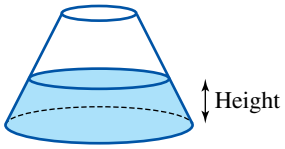
5. The distance versus time graph for a runner is shown at right. Four times during the run are marked as A, B, C and D on the graph. State, with reasoning, at which of the points the runner is:

a. travelling fastest
b. travelling slowest
c. slowing down
d. speeding up
e. travelling at constant speed.

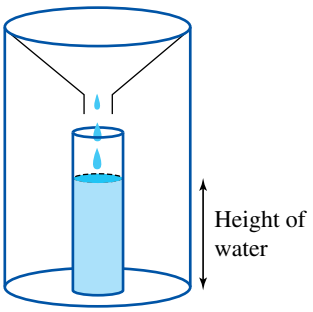


Problem solving

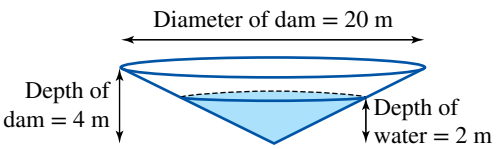
6. Water is poured at a constant rate into the container shown. Sketch the graph of height of water against time.



7. A rain gauge is made up of an inner cylinder and an outer cylinder, as shown in the diagram at right. When the inside cylinder fills and overflows, then the outside cylinder fills, so there are two scales on the device. The inner cylinder measures smaller quantities accurately and the outer cylinder measures larger quantities. For a day when the rain falls steadily, sketch a graph of the height of the water versus time.



8. A dam can be thought of as an inverted cone with a large radius and small height, as shown in the diagram at right. If the depth of water is half of the depth of the dam, at what percentage of its capacity is the dam? The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



Reflection

How can you tell from a graph whether the rate of change is constant or variable?

8.8 Review

8.8.1 Review questions

Fluency

1. t is directly proportional to s and the gradient $m = 1.5$. When $s = 2$, t is equal to:
A. 1.5 **B.** 0.66 **C.** 0.75 **D.** 3

2. y is directly proportional to x and $y = 450$ when $x = 15$. The rule relating x and y is:
A. $y = 0.0333x$ **B.** $y = 30x$ **C.** $y = 60x$ **D.** $y = 6750x$

3. If $y \propto x$ and $y = 10$ when $x = 50$, the constant of proportionality is:
A. 10 **B.** 5 **C.** 1 **D.** 0.2

4. If $y \propto x$ and $y = 10$ when $x = 50$, the value of x when $y = 12$ is:
A. 6 **B.** 60 **C.** 40 **D.** 2.4

5. If y is inversely proportional to x, then which of the following statements is true?
A. $x + y$ is a constant value **B.** $y \div x$ is a constant value
C. $y \times x$ is a constant value **D.** $y - x$ is a constant value

6. The number of calculators a company sells is inversely proportional to the selling price. If a company can sell 1000 calculators when the price is $22, how many could they sell if they reduced the price to $16?
A. 2000 **B.** 727 **C.** 6000 **D.** 1375

7. Maya is sharing her collection of 60 fuchsia plants among three members of the family in the ratio of 2 : 3 : 5. The difference between the largest and smallest share of fuchsias is:
A. 12 **B.** 18 **C.** 36 **D.** 42

8. A speed of 60 km/h is equivalent to approximately:
A. 17 m/min **B.** 1 km/s **C.** 1 m/s **D.** 17 m/s

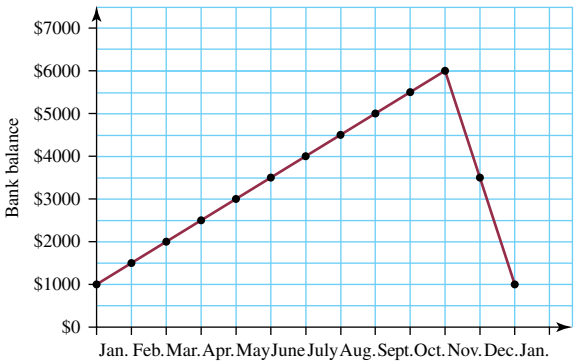
9. A metal part has a density of 37 g/mm^3. If its volume is 6 mm^3, it has a mass of:
A. 6.1 g **B.** 6.1 kg **C.** 222 kg **D.** 222 g

10. Which of the following is not a rate?
A. 50 km/h **B.** 70 beats/min **C.** 40 kg **D.** Gradient

11. How long does it take to travel 240 km at an average of 60 km/hour?
A. 0.25 hours **B.** 240 minutes **C.** 180 hours **D.** 14 400 hours

12. A plane takes 2 hours to travel 1600 km. How long does it take to travel 1000 km?
A. 150 minutes **B.** 48 minutes **C.** 2 hours **D.** 75 minutes

13. The graph at right is of Sandy's bank balance. How many months during the year has she been saving?
A. 2 months **B.** 5 months
C. 10 months **D.** 12 months



14. If $w \propto v$, and $w = 7.5$ when $v = 5$, calculate k, the constant of proportionality.

15. The gear ratio for front and back sprockets of a bicycle is 10 : 3. If the front (large) sprocket has 40 teeth, how many teeth does the back sprocket have?

16. Calculate the missing quantities in the table below.

Mass	Volume	Density
500 g	20 cm^3	
1500 g		50 g/cm^3
	120 cm^3	17 g/cm^3

17. a. A 2000-litre water tank takes 2 days to fill. Express this rate in litres per hour with 2 decimal places.

b. How far do you have to travel vertically to travel 600 metres horizontally if the gradient of a track is 0.3?

c. How long does it take for a 60-watt (60 joules/second) light globe to use 100 kilojoules?

d. Find the cost of 2.3 m^3 of sand at \$40 per m^3.

18. The graph below shows the partial variation between the height of a child over time. What is the rate of change of height with respect to age in cm/year?



Problem solving

19. The authors of a Physics textbook are going to share royalties from sales of the book in the ratio proportional to the number of chapters each has written. Miss Alan wrote 4 chapters, Mr Bradley wrote 3 chapters, Mrs Cato wrote 7 chapters and Ms Dawn wrote 6 chapters. If the expected amount to be shared is \$28 000, how much money will each author get?

20. Lisa drove to the city from her school. She covered a distance of 180 km in 2 hours.

a. What is Lisa's average speed?

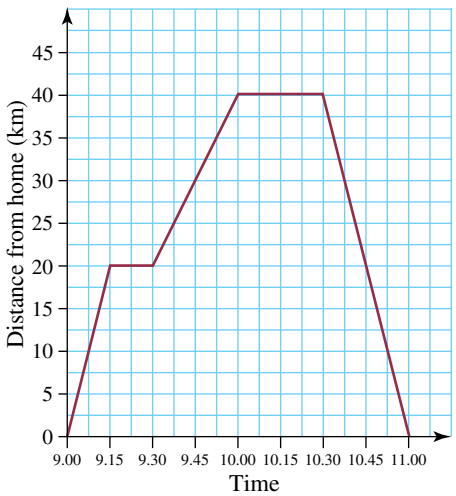
b. Lisa travelled back at an average speed of 60 km/h. How long did she take?

21. Seventy grams of ammonium sulfate crystals are dissolved in 0.5 L of water.

a. What is the concentration of the solution in g/mL?

b. Another 500 mL of water is added. What is the concentration of the solution now?

22. Karina left home at 9.00 am. She spent some time at a friend's house then travelled to the airport to pick up her sister. She then travelled straight back home. Her journey is shown by the travel graph at right.



a. How far from Karina's house is her friend's house?

b. How much time did Karina spend at her friend's place?

c. How far is the airport from Karina's house?

d. How much time did Karina spend at the airport?

e. How much time did Karina take to drive home?

f. Find the average speed of Karina's journey:
- **i.** from her home to the friend's place
- **ii.** from her friend's place to the airport
- **iii.** from the airport to her home.

23. A skyscraper can be built at a rate of 4.5 storeys per month.
- **a.** How many storeys will be built after 6 months?
- **b.** How many storeys will be built after 24 months?

24. A certain kind of eucalyptus tree grows at a linear rate for its first 2 years of growth. If the growth rate is 5 cm per month, how long will it take to grow to be 1.07 m tall?

25. The pressure inside a boiler increases as the temperature increases. For each 1 °C, the pressure increases by 10 units. At a temperature of 100 °C the pressure is 600 units. If the boiler can withstand a pressure of 2000 units, at what temperature does this occur?

26. Hector has a part-time job as a waiter at a local café and is paid $8.50 per hour. Complete the table of values relating the amount of money received to the number of hours worked.

Number of hours	0	2	4	6	8	10
Pay						

27. A fun park charges a $10 entry fee and an additional $3 per ride. Complete the following table of values relating the total cost to the number of rides.

Rides	0	2	4	6	8	10
Cost						

28. Speed is a measure of the distance covered in a period of time. Distance is a length measurement and the units can be kilometres, metres, centimetres, millimetres or other smaller metric units. The length measurements could also be in imperial units, such as miles, yards, feet or inches. Time can be measured in years, days, hours, minutes or seconds, just to name a few. This means that speed can be quoted in units such as kilometres per hour, metres per minute, miles per hour, feet per second and the like. In Australia, we use the metric measurement of length (kilometres, metres, centimetres, millimetres) and this is the case in most parts of the world. However, the United States uses imperial units of length (miles, yards, feet, inches) while the United Kingdom uses a combination of imperial and metric. How can you compare, for instance, the speed of a car travelling at 100 km/h in Australia with one travelling at 100 miles/h (or mph) in the United States?

To help compare speed in different units, a length conversion chart is useful. Time is measured in the same units throughout the world, so a time conversion chart is not necessary.

Length conversion chart

Imperial unit	**Conversion factor**	**Metric unit**
inches (ins)	25.4	mm
feet (ft)	30.5	cm
yards (yds)	0.915	m
miles	1.61	km

To convert an imperial length unit into its equivalent metric unit, multiply by the conversion factor. Divide by the conversion factor when converting from metric to imperial units.

a. Use the table to complete each of the following.
- **i.** 12 ins = _____ mm
- **ii.** 3 ft = _____ mm
- **iii.** 1 m = = _____ yd
- **iv.** 1 km = _____ miles

b. Which is the faster car — the one travelling at 100 km/h, or the one travelling at 100 miles/h?

29. The speed limit of 60 km/h in Australia would be equivalent to a speed limit of ____ miles/h in USA. Give your answer correct to 3 decimal places.

30. The first supersonic land speed record for wheeled vehicles on ground was set by Andy Green of the United States in 1997. He reached a speed of 763.035 miles/h. How fast is this in km/h? Give your answer correct to 3 decimal places.

31. A commercial aircraft covers a distance of 1700 km in 2 hours 5 minutes. How many metres does the aircraft travel each second?

32. A launched rocket covers a distance of 17 miles in 10 seconds. Calculate its speed in km/h.

33. Here are some record speeds for moving objects.

Motorcycle	149 m/sec
Train	302 miles/h
Human skiing	244 km/h
Bullet from 38-calibre revolver	4000 ft/sec

Convert these speeds to the same unit. Then place them in order from fastest to slowest.

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- **Try out this interactivity:** Crossword: Topic 8 (int-2690)
- **Try out this interactivity:** Sudoku: Topic 8 (int-3208)
- **Complete this digital doc:** Concept map: Topic 8 (doc-10799)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

average rate
constant rate
constant of proportionality
dependent variable
direct proportion
gradient
independent variable
inverse proportion
origin
stationary
variable rate
varies directly as speed

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Fastest speeds





To calculate the average speed of a journey, we can use the speed formula:

$$\text{speed} = \frac{\text{distance}}{\text{time}}.$$

Speed is a *rate*, because it compares two quantities of different units. Speed can be expressed in units such as km/h, km/min and m/s. The units of the quantities substituted into the numerator and denominator determine the final units of speed. In order to compare the speeds of different events, it is useful to convert them to the same units.



In 2008, the summer and winter Olympics were held. At the summer Olympics, sprinter Usain Bolt ran 100 m in 9.69 seconds. At the winter Olympics, cross-country skier Petter Northug covered 50 km in 2 hours and 5 minutes, and speed skater Mika Poutala covered 0.5 km in 34.86 seconds. Which competitor was the fastest? In order to answer this question, we need to determine the

speed of each competitor. Since the information is quoted in a variety of units, we need to decide on a common unit for speed.

1. Calculate the speed of each athlete in m/s, correct to 1 decimal place.

2. Which athlete was the fastest?

Consider the speed of objects in the world around you. This task requires you to order the following objects from fastest to slowest, assuming that each object is travelling at its fastest speed possible:

electric car, submarine, diesel train, car ferry, skateboard, bowled cricket ball, windsurfer, served tennis ball, solar-powered car, motorcycle, aircraft carrier, Jaguar motorcar, helicopter, airliner, rocket-powered car, the Concorde supersonic airliner.

3. From your personal understanding and experience, order the above list from fastest to slowest.

We can make a more informed judgement if we have facts available about the movement of each of these objects. Consider the following facts:

When travelling at its fastest speed, the Concorde can cover a distance of about 7000 km in 3 hours. The fastest airliner can cover a distance of 5174 km in 2 hours and a helicopter 600 km in 1.5 hours. It would take 9.5 hours for a rocket-powered car to travel 9652 km, 3.25 hours for an electric car to travel 1280 km, 1.5 hours for a Jaguar car to travel 525 km and (on a sunny day) a solar-powered car can travel 39 km in half an hour. In 15 minutes, a car ferry can travel 26.75 km, an aircraft carrier 14 km and a submarine 18.5 km. It takes the fastest diesel train 0.42 hours to travel 100 km and a motorcyclist just 12 minutes to travel the same distance. In perfect weather conditions, a skateboarder can travel 30 km in 20 minutes and a windsurfer can travel 42 km in 30 minutes. A bowled cricket ball can travel 20 m in about 0.45 seconds while a served tennis ball can travel 25 m in about 0.4 seconds.

4. Using the information above, decide on a common unit of speed and determine the speed of each of the objects.

5. Order the above objects from fastest to slowest.

6. Compare the order from question **5** with the list you made in question **3**.

7. Conduct an experiment with your classmates and record times that objects take to cover a certain distance. For example, record the time it takes to run 100 m or throw a ball 20 m. Compare the speeds with the speeds of the objects calculated during this investigation.

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Complete this digital doc: Code puzzle: Who created the fictional character Sherlock Holmes? (doc-15901)

Answers

Topic 8 Proportion and rates

Exercise 8.2 Direct proportion

1. a. Yes

b. No; as speed increases, time decreases.

c. No; doubling distance doesn't double cost.

d. Yes

e. No; doubling the side length doesn't double the area.

f. Yes

g. Yes

h. No; doubling distance doesn't double cost (due to the initial fee).

i. No; doubling age doesn't double height.

2. a. No; as x doubles, y does not.

b. Yes

c. No; as t doubles, d does not.

d. No; when $n = 0$, d does not.

e. Yes

f. No; when $t = 0$, d does not.

g. No; as x doubles, y does not.

h. No; when $d = 0$, w does not.

3. Answers will vary.

4. The point (0, 0) must always exist in a table of values of two variables exhibiting direct proportionality.

5. The missing value is 8. The relationship between n and m can be calculated by the values 5 and 20 in the table ($n = 4m$).

6. a. Yes, direct proportion does exist.

b. \$2.55

7. Direct proportion does not exist. The price per metre for the 4.2 − metre length is \$5.50, and the price per metre for the 5.4 − metre length is \$5.40.

Exercise 8.3 Direct proportion and ratio

1. $k = 6$, $a = 6b$

2. $a = 0.5b$

3. $C = 12.5t$

4. $v = 0.5t$

5. $F = 2.5a$

6. a. $E = 0.625W$ **b.** 19.2 kg **c.** 4.375 cm

7. a. $W = 1.3n$ **b.** $\approx$ 385 almonds **c.** 325 g

8. a. $d = 2.5n$ **b.** 2017.5 km **c.** 2000 turns

9. a. $M = 0.12n$ **b.** 450 seedlings **c.** 360 kg

10. \$217.60 **11.** \$217.25 **12.** 13.33 L

13. \$105.98 **14.** 375 g **15.** \$150

16. \$33.75

17. a. $D = 100T$

b. 6 hours 6 minutes

c. 2 hours 12 minutes

d. 3 hours 54 minutes

Challenge 8.1

275 cm^3

Exercise 8.4 Inverse proportion

1. a. $s = \frac{k}{t}$ **b.** No

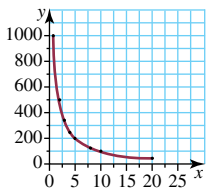
c. $t = \frac{k}{p}$ or $p = \frac{k}{t}$ **d.** $L = \frac{k}{p}$ or $p = \frac{k}{L}$

e. No **f.** $t = \frac{k}{n}$ or $n = \frac{k}{t}$

2. Answers will vary.

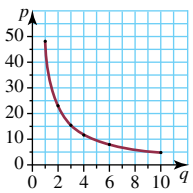
3. a. $k = 1000$, $y = \frac{1000}{x}$

b.



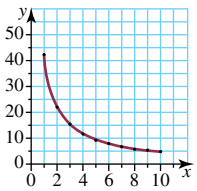
4. a. $k = 48$, $p = \frac{48}{q}$

b.



5. a. $k = 42$, $y = \frac{42}{x}$

b.



6. a. 4000 **b.** $a = \frac{4000}{m}$ **c.** 20 m/s^2 **d.** 4 m/s^2

7. a. 500 **b.** 2500 pencils **c.** 1000 pencils

8. a. 337.5 **b.** 3.97 hours = 3 h 58 min **c.** 96.4 km/h **d.** 337.5 km

9. a. 17 500 **b.** $233.33 **c.** 70 people **d.** $17 500

10. a. 200 **b.** 1 ampere **c.** $13.\dot{3}$ ohms **d.** Check with your teacher.

11. The constant of proportionality represents the distance of the trip; therefore, when this value is smaller, the time taken to complete the trip at the same speed is also smaller.

12. a. 10 **b.** $T = \frac{10}{P}$ **c.** 1 hour 40 minutes

13. a. 100.048 **b.** $T = \frac{100.048}{V}$ **c.** 9.58 seconds **d.** 15.00 seconds

Exercise 8.5 Introduction to rates

1. 64 km/h

2. a. 120 km/h **b.** 1692.3 km/h

3. a. $92 per ticket **b.** 24 cakes per hour **c.** 10.44 m/s

d. 3 cm/h **e.** $17.33/h **f.** $50 per air conditioner

4. 4 g/cm^3 **5.** 600 g/L

6. The hare runs at 120 000 cm per minute; the tortoise runs at 0.144 km per hour. The hare runs 500 times faster than the tortoise.

7. Either the distance of the journey decreases and the time remains constant, or the distance of the journey remains constant and the time decreases.

8. Cathy Freeman: 29.32 km/h; Usain Bolt: 37.15 km/h

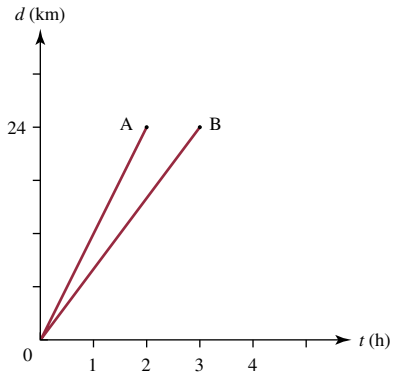
9. 75 students/year

10. 0.15

11. 2750 organisms/litre. The beach should not be closed.

Exercise 8.6 Constant and variable rates

1. a.



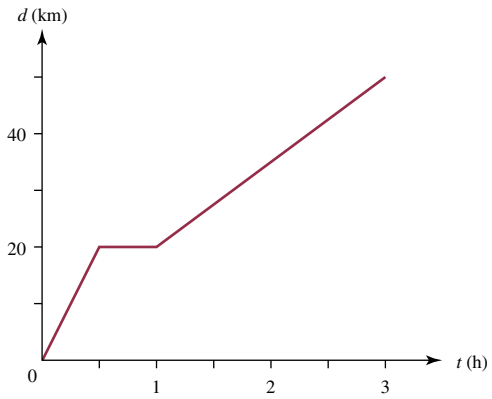
b. i. A: $d = 12t$ B: $d = 8t$ **ii.** A has a steeper gradient than B.

2. a. 1 hour at 60 km/h, then 1 hour at 120 km/h

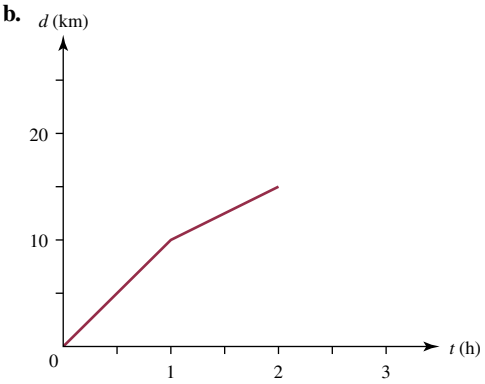
b. 1 hour at 50 km/h, a 30-minute stop, 30 minutes at 100 km/h, a 1-hour stop, then 1 hour at 100 km/h

c. 1 hour at 20 km/h, 1 hour at 100 km/h, 1 hour at 40 km/h

3. a.



b.



4. a. A The car is moving with steady speed.

B The car is momentarily stationary.

C The speed is increasing.

D The car is slowing down.

E The car is moving at a slow steady speed.

F The speed is increasing.

G The car is moving at a faster steady speed.

b. i. A, G **ii.** B, E **iii.** B

5. a. 0.2 km/min **b.** 0.2 km/min **c.** Yes, both rates are the same.

6. a. 20, 10, 35, 0 L/min **b.** No

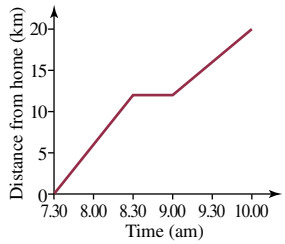
7. a. 150 km, 200 km and 250 km **b.** 50 km **c.** Both stop for 1 hour.

d. Brian — 300 km, 3.5 h; Margaret — 300 km, 4 h

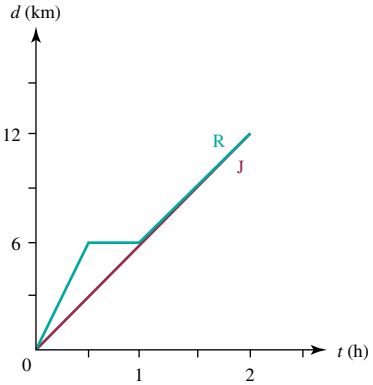
e. Brian — 85.7 km/h; Margaret — 75 km/h

8. a. 2.5 hours **b.** 20 km

c.



9. a.



b. 30 minutes

10. a. 1, 3, 5, 10 **b.** 4, 6, 7, 9 **c.** 2, 8 **d.** 10 **e.** 4

11. a. i. $10 **ii.** $10 **iii.** $10

b. Yes

12. a. 30 **b.** 30 **c.** Variable

12. a. i. 25.8 °C **ii.** 14.0 °C

b. i. −10.9 °C **ii.** 2.3 °C

c. No

Challenge 8.2

0.9 seconds

Exercise 8.7 Rates of change

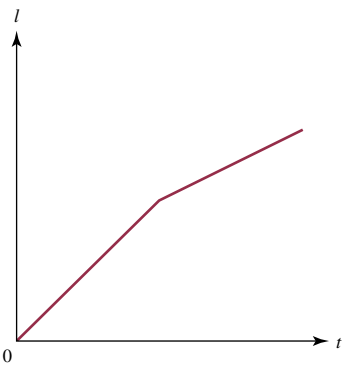
1. a. B

b. D

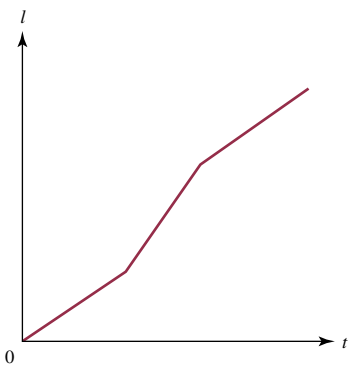
c. C

d. A

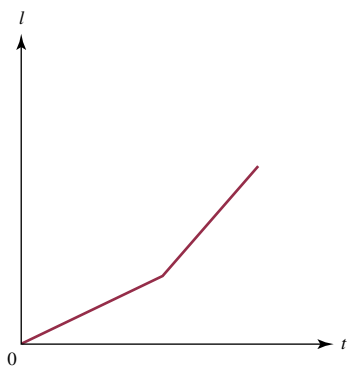
2. a.



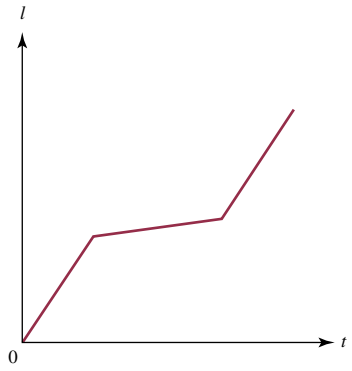
b.



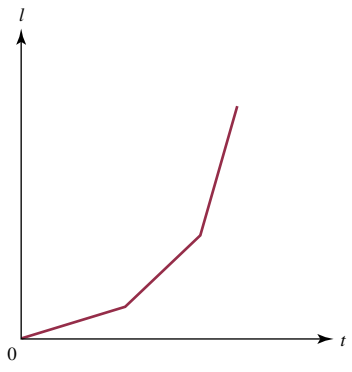
c.



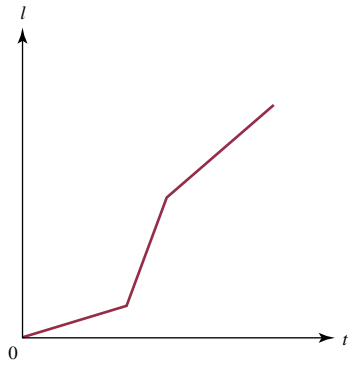
d.



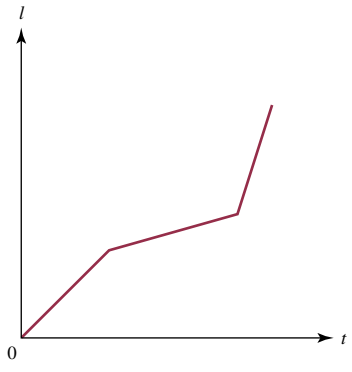
e.



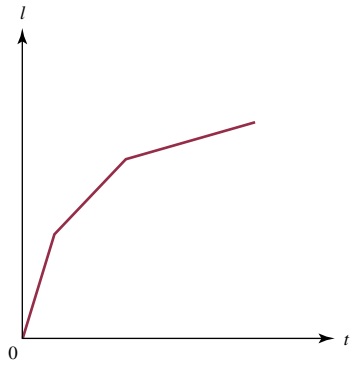
f.



g.

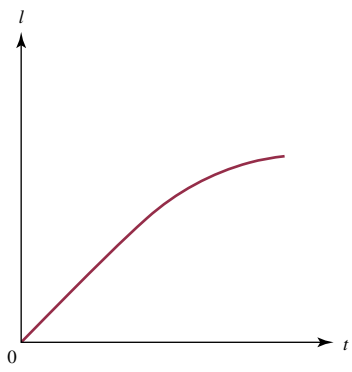


h.

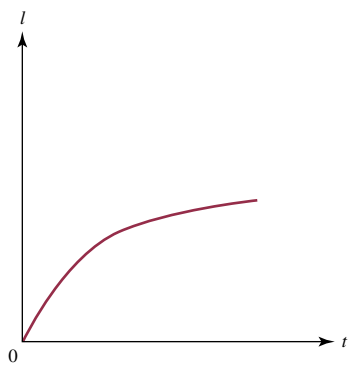


3. a. A, C **b.** D **c.** B

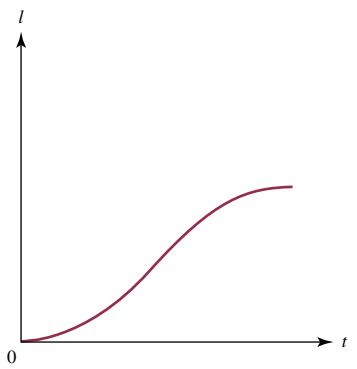
4. a.



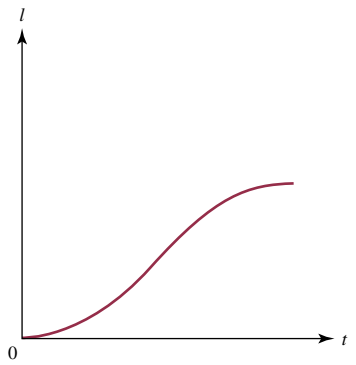
b.



c.



d.



5. a. Point A — the gradient is greatest.

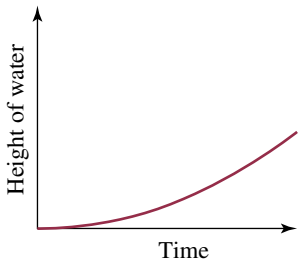
b. Point C — the gradient is least.

c. Point B — the gradient is decreasing.

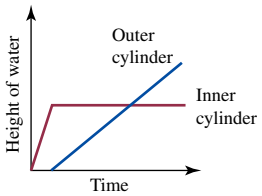
d. Point C — the gradient is increasing.

e. Points A and D — the gradient is constant.

6.



7.



8. 12.5%

8.8 Review

1. D **2.** B **3.** D **4.** B **5.** C

6. D **7.** B **8.** D **9.** D **10.** C

11. B **12.** D **13.** C **14.** 1.5 **15.** 12 teeth

16.

Mass	Volume	Density
500 g	20 cm^3	25 g/cm^3
1500 g	30 cm^3	50 g/cm^3
2040 g	120 cm^3	17 g/cm^3

17. a. 41.67 L/h **b.** 180 m **c.** 28 minutes **d.** $92

18. 18 cm/year

19. Miss Alan $5600, Mr Bradley $4200, Mrs Cato $9800, Ms Dawn $8400

20. a. 90 km/h **b.** 3 h

21. a. 0.14 g/mL **b.** 0.07 g/mL

22. a. 20 km **b.** 15 min **c.** 40 km **d.** 30 min **e.** 30 min

f. i. 80 km/h **ii.** 40 km/h **iii.** 80 km/h

23. a. 27 **b.** 108

24. 21.4 months

25. 240 °C

26.

Number of hours	0	2	4	6	8	10
Pay	$0	$17	$34	$51	$68	$85

27.

Rides	0	2	4	6	8	10
Cost	$10	$16	$22	$28	$34	$40

28. a. i. 304.8 mm **ii.** 915 mm **iii.** 1.09 yd **iv.** 0.62 miles

b. The one travelling at 100 miles/h

29. 37.267 miles/h

30. 1228.486 km/h

31. $226\frac{2}{3}$ m/sec

32. 9853.2 km/h

33. The conversions may vary but the order from fastest to slowest is:

Bullet from 38-calibre revolver

Motor cycle

Train

Human skiing

Investigation — Rich task

1. Usain Bolt = 10.3 m/s

Petter Northug = 6.7 m/s

Mika Poutala = 14.3 m/s

2. Mika Poutala

3. Answers will vary.

4. Concorde: 2333.3 km/h

Airliner: 2587 km/h

Helicopter: 400 km/h

Rocket-powered car: 1016 km/h

Electric car: 393.8 km/h

Jaguar car: 350 km/h

Solar-powered car: 78 km/h

Car ferry: 107 km/h

Aircraft carrier: 56 km/h

Submarine: 74 km/h

Diesel train: 238.1 km/h

Motorcyclist: 500 km/h

Skateboarder: 90 km/h

Windsurfer: 84 km/h

Bowled cricket ball: 160 km/h

Served tennis ball: 225 km/h

5. Airliner, Concorde, rocket-powered car, motorcyclist, helicopter, electric car, Jaguar car, diesel train, served tennis ball, bowled cricket ball, oar ferry, skateboarder, windsurfer, solar-powered car, submarine, aircraft carrier

6. Answers will vary.

7. Answers will vary.

TOPIC 9
Numeracy 2

9.1 Overview



Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

9.1.1 Why learn this?

Our lives are interwoven with mathematics. Counting, measuring and pattern-making are all part of everyday life. We use numbers to mark significant events (such as birthdays) and for identification (such as passports and credit cards). We use numbers to describe ourselves (for example our height and weight). Shopping involves understanding numbers, and tallying scores in sports requires a comparison of numbers. You may not realise just how much you rely on numbers.

9.1.2 What do you know?

assessON

1. **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner, then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

LEARNING SEQUENCE

9.1 Overview
9.2 Set C
9.3 Set D

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: NAPLAN: Strategies and tips (eles-1688)

9.2 Set C

assessON

9.2.1 Calculator allowed

1. The cube root of 64 is between:

A. 1^2 and 1^3 **B.** 1^2 and 3^2

C. 2^3 and 4^2 **D.** 3^2 and 4^2

2. A javelin is thrown 3 times. The mean distance of the 3 throws is 55 m. The fourth throw reaches 63 m. What is the mean distance after 4 throws?



A. 52 m **B.** 57 m

C. 55 m **D.** 60 m

3. The monthly cost (C) of renting a mobile phone is given by the equation $C = 20 + 0.2x$, where x is the call time in minutes. If February's bill was $120, the call time was:

A. 300 minutes **B.** 100 minutes

C. 500 minutes **D.** 450 minutes

4. A music store has two promotions running at the same time:

- Buy 2 CDs for $26.95 each and get the third free.
- 30% off all CDs which normally retail at $24.95 each.

If Emma wants to buy three CDs, what is the best price she can get?

A. $53.90 **B.** $52.40

C. $51.90 **D.** $54.50

5. A number is multiplied by 3, then 12 is subtracted. The final answer is the square of 6. What is the original number?

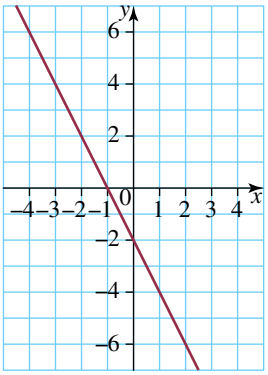
6. A painter quotes $448 to paint the walls of a bedroom with the following dimensions:

length = 4 m

width = 3 m

height = 3.2 m.

How much extra will he charge to paint the ceiling as well (at the same original cost per square metre)?



A. $100

B. $110

C. $120

D. $150

7. Solve the equation for q.

$$\frac{3 - 4q}{2} = \frac{1}{4}$$

8. What is the equation of the graph shown?

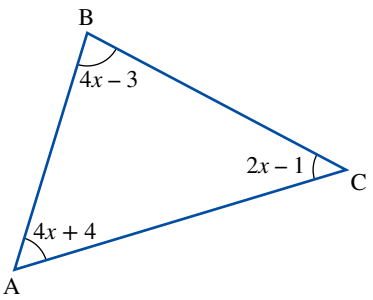
A. $y = -2x - 2$

B. $y = 2x - 2$

C. $y = -2x + 2$

D. $y = 2x + 2$

9. For the triangle shown, x equals:



A. 16

B. 17

C. 18

D. 19

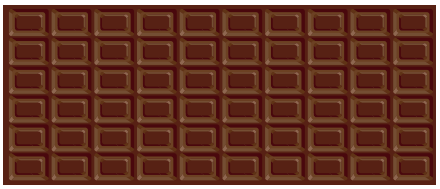
10. The scale 200 cm ⇔ 4 m simplifies to the ratio:

A. 1 : 2 **B.** 1 : 4 **C.** 2 : 4 **D.** 50 : 1

11. A house is bought for $450 000 and sold for $525 000. Express the profit as a percentage of the cost price. (Round to 2 decimal places.)

A. 14.29% **B.** 16.67% **C.** 85.71% **D.** 16.7%

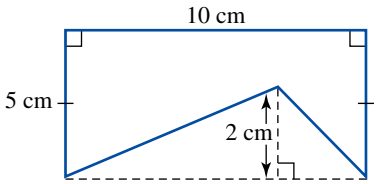
12. A 300-g block of chocolate is to be broken into its individual squares. What is the mass of 4 squares of chocolate?



A. 10 g **B.** 16 g

C. 5 g **D.** 20 g

13. The area of the composite shape shown is:



A. 30 cm^2

B. 40 cm^2

C. 50 cm^2

D. 45 cm^2

14. What is the next term in the following pattern?

1, 4, 9, 16, . . .

A. 20 **B.** 25

C. 29 **D.** 36

15. A cyclist travels 15 km in 30 minutes. The speed (in km/h) she is travelling is:



A. 40 km/h **B.** 60 km/h

C. 80 km/h **D.** 30 km/h

16. The intersection point for $y = 2x + 4$ and $x + y = 1$ is:

A. (−2, 1) **B.** (−1, −2)

C. (−1, 2) **D.** (2, 1)

17. The factorised form of $2x^2y^3 - 6xy^2$ is:

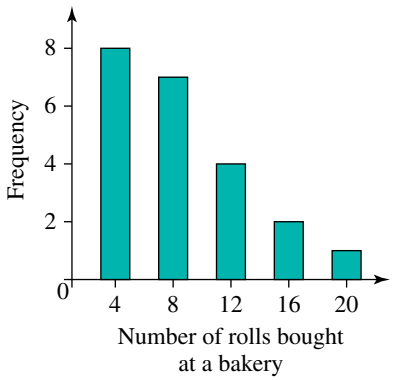
A. $2xy^2(xy - 3)$

B. $2xy(x^2y - 3)$

C. $2xy(xy - 6)$

D. $2x^2y(xy - 3)$

18. The graph shows the frequency of the number of bread rolls bought at a bakery. What is the total number of rolls bought?



A. 200

B. 188

C. 150

D. 168

19. What are the mean and median of the data shown in the table?

Score (x)	Frequency
1	1
2	2
3	3
4	4

A. The mean is 3 and median is 4. **B.** The mean is 3 and median is 3.

C. The mean is 4 and median is 3. **D.** The mean is 4 and median is 4.

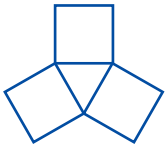
20. Which number is exactly halfway between 3.25 and 4.75?

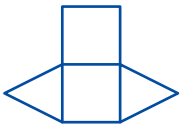
A. 3.95 **B.** 4.15 **C.** 4.25 **D.** 4

21. Write a fraction equivalent to 0.75.

22. Which diagram is the net of a square-based pyramid?

A. 

B. 

C. 

D. 

23. The table summarises how much time Lian spent on her Maths project. What was the average amount of time each day Lian worked on her project?

A. 25 minutes

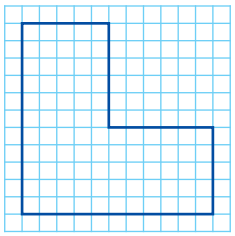
B. 30 minutes

C. 43 minutes

D. 1 hour

Time spent on project	
Day	**Time**
Monday	30 minutes
Tuesday	15 minutes
Wednesday	$1\frac{1}{4}$ hours
Thursday	50 minutes
Friday	45 minutes

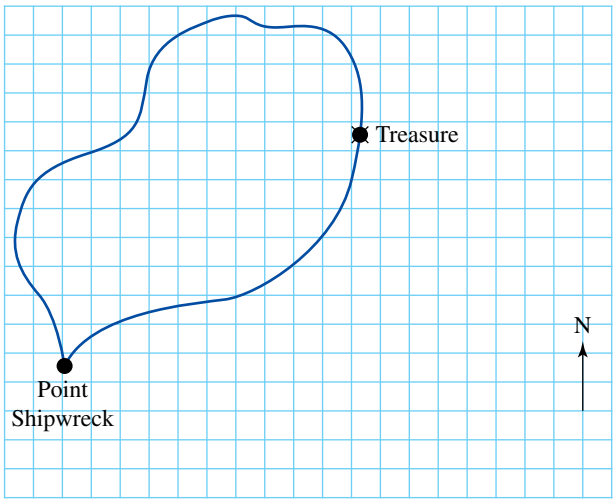
24. Draw a line of symmetry through the diagram.



25. If $x = -2$, what is the value of $\dfrac{2x}{3x - 2}$?

A. $\frac{1}{2}$ **B.** $\frac{1}{4}$ **C.** $\frac{2}{3}$ **D.** $\frac{1}{3}$

26. A map of Fern Island is shown below.



What direction is the treasure from Point Shipwreck?

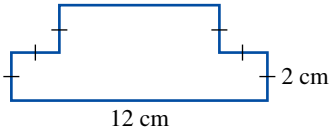
27. The shape at right is to be enlarged by a factor of 3. The new area will be:

A. 3 times the original area

B. 6 times the original area

C. 9 times the original area

D. 12 times the original area



28. Complete the table of values for the rule $y = 3x^2$.

x	−3	−2	−1	0	1	2	3
y							

29. The population of a city is expected to increase by 2.5% each year for the next 5 years. If this city's current population is 300 000, what is it expected to be in 5 years' time?

A. 339 422 **B.** 755 000 **C.** 3 750 345 **D.** 355 406

30. Find the area of material (m^2) needed to make a circular cushion with a diameter of 110 cm.
Note: Two pieces of material are cut out and sewn together to make the cushion.

A. 2.5 m^2 **B.** 2.0 m^2 **C.** 1.9 m^2 **D.** 1.5 m^2

9.3 Set D

assessON

9.3.1 Non-calculator

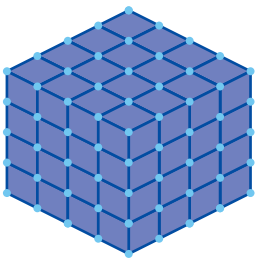
1. What is the equivalent fraction to $3\frac{2}{5}$?

A. $\frac{17}{3}$ **B.** $\frac{11}{5}$ **C.** $\frac{17}{5}$ **D.** $\frac{11}{2}$

2. The value of $\left[\left(\frac{1}{2}\right)^2 + 2\frac{1}{4}\right] \div \frac{5}{7}$ is:

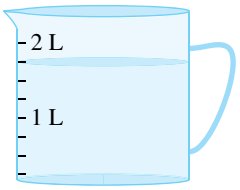
A. $\frac{25}{14}$ **B.** $3\frac{1}{2}$ **C.** $4\frac{1}{2}$ **D.** $2\frac{1}{3}$

3. A large cube, shown at right, is painted blue. How many cubes, inside this large cube, have no painted faces?



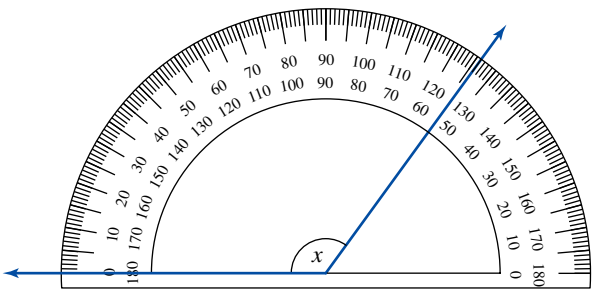
A. 8
B. 16
C. 36
D. 64

4. A glass of water (250 mL) is poured from the jug. How much water is left in the jug?



A. 1.75 L
B. 1.5 L
C. 1.25 L
D. 1 L

5. Measure the size of the unknown angle, x, using the protractor.



A. 125° **B.** 66° **C.** 54° **D.** 126°

6. 14.076 − 9.25 equals:

A. 4.826 **B.** 4.51 **C.** 4.926 **D.** 0.482

7. Jake recorded his classmates' preferred ice-cream flavours in the table below.

Ice-cream flavour	Number of students
Chocolate	14
Vanilla	3
Strawberry	7
Choc-mint	6

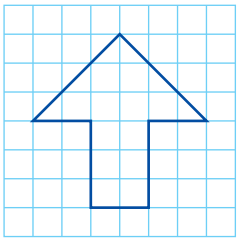
What percentage of students prefers choc-mint flavoured ice-cream?

A. 60% **B.** 20% **C.** 6% **D.** 25%

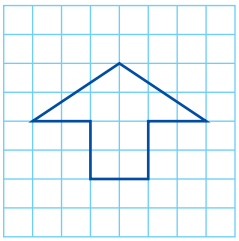
8. A car travels at 75 kilometres per hour. How far will it travel in 4 hours?

A. 300 km **B.** 150 km **C.** 100 km **D.** 250 km

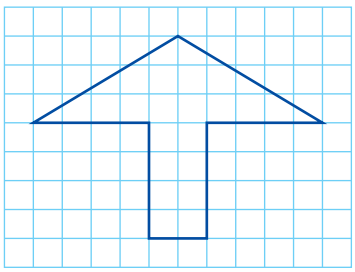
9. Which of the following diagrams shows the arrow below multiplied by a scale factor of $\frac{1}{2}$?



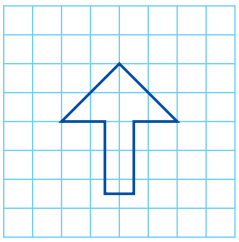
A.



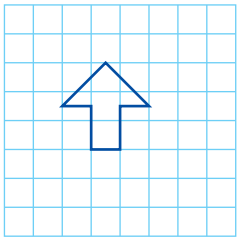
B.



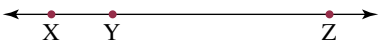
C.



D.

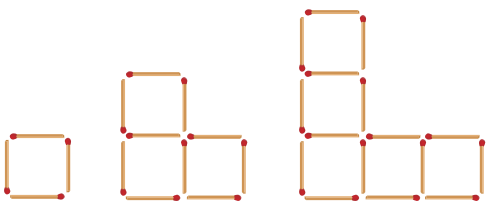


10. $\overline{XZ}$ is 32 cm. $\overline{XY}$ is a quarter of the distance of $\overline{XZ}$. Find $\overline{YZ}$.



A. 8 cm **B.** 16 cm **C.** 20 cm **D.** 24 cm

11. How many sticks would be needed to make the 5th shape in this pattern?

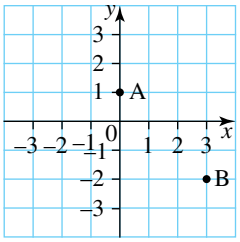


A. 10
B. 16
C. 28
D. 34

12. To convert from degrees Fahrenheit (°F) to degrees Celsius (°C), the following formula is used:
$C = \frac{5}{9}(F - 32)$.
What is the Fahrenheit temperature equivalent to 35 °C?

A. 86 °F **B.** 95 °F **C.** 70 °F **D.** 100 °F

13. The two points A and B are on which line?



A. $y = 1 - x$
B. $y = x + 1$
C. $y = x - 1$
D. $y = 3 - x$

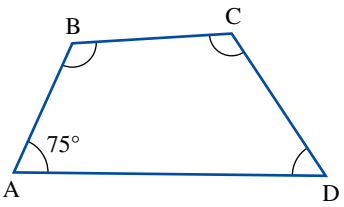
14. Solve the equation $\frac{4m}{5} + 7 = 11$.

A. $m = 5$ **B.** $m = 4$
C. $m = -5$ **D.** $m = 2$

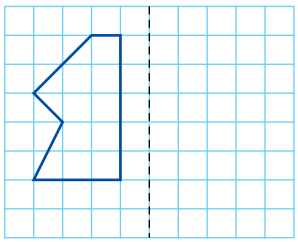
15. A football team played 22 games during a season. The team won 4 more games than it lost. How many games did the team win?

A. 9 **B.** 11 **C.** 13 **D.** 14

16. ∠BAD and ∠BCD are supplementary angles. ∠ABC is 130°. Find the size of ∠ADC.



17. Draw the reflection of the shape shown below.



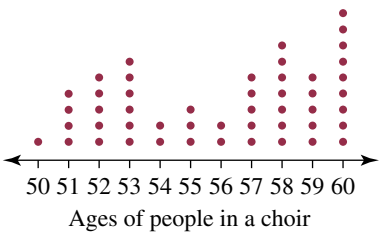
18. If $p = 3$ and $q = -5$, evaluate $q^2 - 2p$.

A. −31 **B.** −19 **C.** 21 **D.** 19

19. The area of a circle is $A = \pi r^2$. Which value best estimates the area of a circle with a radius of 5 cm?

A. 75 cm^2 **B.** 15 cm^2 **C.** 50 cm^2 **D.** 7.5 cm^2

20. This dot plot shows the ages of people in a choir. What is the most common age of the choir members?



A. 50 **B.** 58
C. 59 **D.** 60

21. What is the probability of getting Tails when a fair coin is tossed?

A. 0 **B.** $\frac{1}{2}$
C. 1 **D.** $\frac{1}{3}$

22. Insert '<' or '>' to make this statement true.
4^3 __ 2^5

23. The following scores were recorded in a test: $\frac{18}{20}$ and 88%. Which is the higher score?

24. The cost of two different books is in the ratio 4 : 5. If the more expensive book costs $25, what does the cheaper book cost?



A. $5 **B.** $10
C. $15 **D.** $20

25. Calculate the value of 3 m 75 cm + 5 m 36 cm.

A. 8 m 95 cm **B.** 9 m 11 cm
C. 9 m 20 cm **D.** 8 m 11 cm

26. There are 60 squares in a chocolate block. If Stef eats $\frac{2}{3}$ of the block, how many squares are left?

A. 30 **B.** 25 **C.** 20 **D.** 15

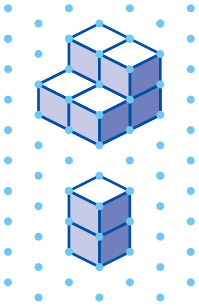
27. The length of a rectangular sporting field is four times its width. If the perimeter of the field is 200 m, find the length and width.

28. A bag of marbles contains 22 red, 20 yellow, 15 green and 18 blue marbles. Darcy chooses one marble without looking. What is the chance it will be green?

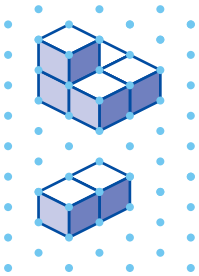
A. $\frac{1}{3}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{5}$ **D.** $\frac{1}{6}$

29. Which pair of shapes fits together to make a cube?

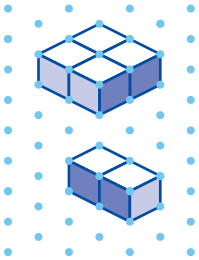
A.



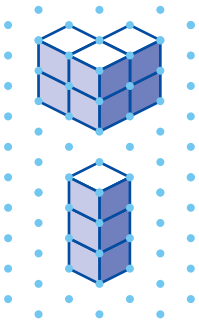
B.



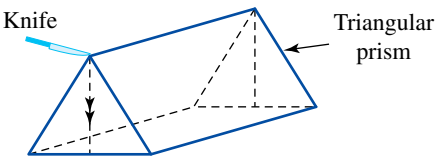
C.



D.



30. What shape is revealed when a triangular prism is cut vertically?



A. Triangle **B.** Square **C.** Rectangle **D.** Trapezium

Answers

Topic 9 Numeracy 2

9.2 Set C

9.2.1 Calculator allowed

1. B **2.** B **3.** C **4.** B

5. 16 **6.** C **7.** $q = \frac{5}{8}$ **8.** A

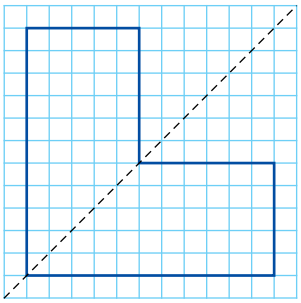
9. C **10.** A **11.** B **12.** D

13. B **14.** B **15.** D **16.** C

17. A **18.** B **19.** B **20.** D

21. $\frac{3}{4}$ **22.** D **23.** C

24.



25. A **26.** North-east **27.** C

28. 27, 12, 3, 0, 3, 12, 27 **29.** A **30.** C

9.3 Set D

9.3.1 Non-calculator

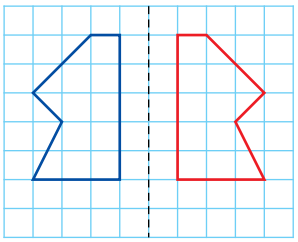
1. C **2.** B **3.** A **4.** B

5. D **6.** A **7.** B **8.** A

9. D **10.** D **11.** C **12.** B

13. A **14.** A **15.** C **16.** 50°

17.



18. D **19.** A **20.** D **21.** B

22. > **23.** $\frac{18}{20}$ **24.** D **25.** B

26. C **27.** $l = 80$ m, $w = 20$ m **28.** C **29.** A

30. C

TOPIC 10 Indices

10.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.



10.1.1 Why learn this?

Indices (the plural of index) give us a way of abbreviating multiplication, division and so on. They are most useful when working with very large or very small numbers. For calculations involving such numbers, we can use indices to simplify the process.

10.1.2 What do you know?

assessON

1. **THINK** List what you know about indices. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of indices.

LEARNING SEQUENCE

10.1 Overview
10.2 Review of index laws
10.3 Raising a power to another power
10.4 Negative indices
10.5 Square roots and cube roots
10.6 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: The population boom (eles-1697)

10.2 Review of index laws

10.2.1 Index notation

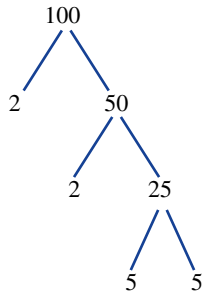
- The product of factors can be written in a shorter form called **index notation**.

Index, exponent

Base → $6^4 = 6 \times 6 \times 6 \times 6$ ← Factor form

$= 1296$

- Any composite number can be written as a product of powers of prime factors using a factor tree, or by other methods, such as repeated division.



$100 = 2 \times 2 \times 5 \times 5$

$= 2^2 \times 5^2$

WORKED EXAMPLE 1

TI | CASIO

Express 360 as a product of powers of prime factors using index notation.

THINK	WRITE
1 Express 360 as a product of a factor pair.	$360 = 6 \times 60$
2 Further factorise 6 and 60.	$= 2 \times 3 \times 4 \times 15$
3 Further factorise 4 and 15.	$= 2 \times 3 \times 2 \times 2 \times 3 \times 5$
4 There are no more composite numbers.	$= 2 \times 2 \times 2 \times 3 \times 3 \times 5$
5 Write the answer using index notation. *Note:* The factors are generally expressed with bases in ascending order.	$360 = 2^3 \times 3^2 \times 5$

10.2.2 Multiplication using indices

- The **First Index Law states:** $a^m \times a^n = a^{m+n}$.
  That is, when multiplying terms with the same bases, add the indices.

WORKED EXAMPLE 2

Simplify $5e^{10} \times 2e^3$.

THINK

1 The order is not important when multiplying, so place the coefficients first.

2 Simplify by multiplying the coefficients and applying the First Index Law (add the indices).

WRITE

$5e^{10} \times 2e^3$
$= 5 \times 2 \times e^{10} \times e^3$
$= 10e^{13}$

- When more than one base is involved, apply the First Index Law to each base separately.

WORKED EXAMPLE 3

TI | CASIO

Simplify $7m^3 \times 3n^5 \times 2m^8n^4$.

THINK

1 The order is not important when multiplying, so place the coefficients first and group the same pronumerals together.

2 Simplify by multiplying the coefficients and applying the First Index Law (add the indices).

WRITE

$7m^3 \times 3n^5 \times 2m^8n^4$
$= 7 \times 3 \times 2 \times m^3 \times m^8 \times n^5 \times n^4$
$= 42m^{11}n^9$

10.2.3 Division using indices

- The **Second Index Law states:** $a^m \div a^n = a^{m-n}$.
  That is, when dividing terms with the same bases, subtract the indices.

WORKED EXAMPLE 4

TI | CASIO

Simplify $\dfrac{25v^6 \times 8w^9}{10v^4 \times 4w^5}$.

THINK

1 Simplify the numerator and the denominator by multiplying the coefficients.

2 Simplify further by dividing the coefficients and applying the Second Index Law (subtract the indices).

WRITE

$\dfrac{25v^6 \times 8w^9}{10v^4 \times 4w^5}$
$= \dfrac{200v^6w^9}{40v^4w^5}$
$= \dfrac{{}^{5}\cancel{200}}{{}_{1}\cancel{40}} \times \dfrac{v^6}{v^4} \times \dfrac{w^9}{w^5}$
$= 5v^2w^4$

- When the coefficients do not divide evenly, simplify by cancelling.

WORKED EXAMPLE 5

Simplify $\dfrac{7t^3 \times 4t^8}{12t^4}$.

THINK	WRITE
1 Simplify the numerator by multiplying the coefficients.	$\dfrac{7t^3 \times 4t^8}{12t^4}$ $= \dfrac{28t^{11}}{12t^4}$
2 Simplify the fraction by dividing the coefficients by the highest common factor. Then apply the Second Index Law.	$= \dfrac{28}{12} \times \dfrac{t^{11}}{t^4}$ $= \dfrac{7t^7}{3}$

10.2.4 Zero index

- Any number divided by itself (except zero) is equal to 1.
  Therefore, $\dfrac{10}{10} = \dfrac{2.14}{2.14} = \dfrac{\pi}{\pi} = \dfrac{592^3}{592^3} = 1$.
- Similarly, $\dfrac{x^3}{x^3} = 1$. But using the Second Index Law, $\dfrac{x^3}{x^3} = x^0$. It follows that $x^0 = 1$.
- In the same way, $\dfrac{n^{10}}{n^{10}} = 1$, and $\dfrac{n^{10}}{n^{10}} = n^0$, so $n^0 = 1$.
- In general, any number (except zero) to the power zero is equal to 1.
- This is the **Third Index Law: $a^0 = 1$, where $a \neq 0$**.

WORKED EXAMPLE 6

Evaluate the following.
a t^0 **b** $(xy)^0$ **c** 17^0 **d** $5x^0$ **e** $(5x)^0 + 2$ **f** $5^0 + 3^0$

THINK	WRITE
a Apply the Third Index Law.	**a** $t^0 = 1$
b Apply the Third Index Law.	**b** $(xy)^0 = 1$
c Apply the Third Index Law.	**c** $17^0 = 1$
d Apply the Third Index Law.	**d** $5x^0 = 5 \times x^0$ $= 5 \times 1$ $= 5$
e Apply the Third Index Law.	**e** $(5x)^0 + 2 = 1 + 2$ $= 3$
f Apply the Third Index Law.	**f** $5^0 + 3^0 = 1 + 1$ $= 2$

WORKED EXAMPLE 7

Simplify $\dfrac{9g^7 \times 4g^4}{6g^3 \times 2g^8}$.

THINK

1 Simplify the numerator and the denominator by applying the First Index Law.

2 Simplify the fraction further by applying the Second Index Law.

3 Simplify by applying the Third Index Law.

WRITE

$$\frac{9g^7 \times 4g^4}{6g^3 \times 2g^8}$$

$$= \frac{36g^{11}}{12g^{11}}$$

$$= \frac{{}^{3}\cancel{36}g^{11}}{{}_{1}\cancel{12}g^{11}}$$

$$= 3g^0$$
$$= 3 \times 1$$
$$= 3$$

10.2.5 Cancelling fractions

- Consider the fraction $\dfrac{x^3}{x^7}$. This fraction can be cancelled by dividing the denominator and the numerator by the highest common factor (HCF), x^3, so $\dfrac{x^3}{x^7} = \dfrac{1}{x^4}$.

  Note: $\dfrac{x^3}{x^7} = x^{-4}$ by applying the Second Index Law. We will study negative indices in a later section.

WORKED EXAMPLE 8

Simplify these fractions by cancelling.

a $\dfrac{x^5}{x^7}$ **b** $\dfrac{6x}{12x^8}$ **c** $\dfrac{30x^5y^6}{10x^7y^3}$

THINK

a Divide the numerator and denominator by the HCF, x^5.

b Divide the numerator and denominator by the HCF, $6x$.

c Divide the numerator and denominator by the HCF, $10x^5y^3$.

WRITE

a $\dfrac{x^5}{x^7} = \dfrac{1}{x^2}$

b $\dfrac{6x}{12x^8} = \dfrac{6}{12} \times \dfrac{x}{x^8}$

$= \dfrac{1}{2} \times \dfrac{1}{x^7}$

$= \dfrac{1}{2x^7}$

c $\dfrac{30x^5y^6}{10x^7y^3} = \dfrac{30}{10} \times \dfrac{x^5}{x^7} \times \dfrac{y^6}{y^3}$

$= \dfrac{3}{1} \times \dfrac{1}{x^2} \times \dfrac{y^3}{1}$

$= \dfrac{3y^3}{x^2}$

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Try out this interactivity: Index laws (int-2769)

Complete this digital doc: SkillSHEET: Index form (doc-6225)

Complete this digital doc: SkillSHEET: Using a calculator to evaluate numbers in index form (doc-6226)

Exercise 10.2 Review of index laws

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 5a–e, 6, 7a–e, 8–11, 13–18	Questions: 1–3, 4a–c, 5d–g, 6, 7d–g, 8–19	Questions: 1, 2, 3e–j, 4d–f, 5f–i, 6, 7f–j, 8–20

Individual pathway interactivity: int-4516

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Express each of the following as a product of powers of prime factors using index notation.

a. 12 **b.** 72 **c.** 75

d. 240 **e.** 640 **f.** 9800

2. WE2 Simplify each of the following.

a. $4p^7 \times 5p^4$ **b.** $2x^2 \times 3x^6$ **c.** $8y^6 \times 7y^4$

d. $3p \times 7p^7$ **e.** $12t^3 \times t^2 \times 7t$ **f.** $6q^2 \times q^5 \times 5q^8$

3. WE3 Simplify each of the following.

a. $2a^2 \times 3a^4 \times e^3 \times e^4$ **b.** $4p^3 \times 2h^7 \times h^5 \times p^3$

c. $2m^3 \times 5m^2 \times 8m^4$ **d.** $2gh \times 3g^2h^5$

e. $5p^4q^2 \times 6p^2q^7$ **f.** $8u^3w \times 3uw^2 \times 2u^5w^4$

g. $9y^8d \times y^5d^3 \times 3y^4d^7$ **h.** $7b^3c^2 \times 2b^6c^4 \times 3b^5c^3$

i. $4r^2s^2 \times 3r^6s^{12} \times 2r^8s^4$ **j.** $10h^{10}v^2 \times 2h^8v^6 \times 3h^{20}v^{12}$

4. WE4 Simplify each of the following.

a. $\frac{15p^{12}}{5p^8}$ **b.** $\frac{18r^6}{3r^2}$ **c.** $\frac{45a^5}{5a^2}$

d. $\frac{60b^7}{20b}$ **e.** $\frac{100r^{10}}{5r^6}$ **f.** $\frac{9q^2}{q}$

5. WE5 Simplify each of the following.

a. $\frac{8p^6 \times 3p^4}{16p^5}$ **b.** $\frac{12b^5 \times 4b^2}{18b^2}$ **c.** $\frac{25m^{12} \times 4n^7}{15m^2 \times 8n}$

d. $\frac{27x^9y^3}{12xy^2}$ **e.** $\frac{16h^7k^4}{12h^6k}$ **f.** $\frac{12j^8 \times 6f^5}{8j^3 \times 3f^2}$

g. $\frac{8p^3 \times 7r^2 \times 2s}{6p \times 14r}$ **h.** $\frac{27a^9 \times 18b^5 \times 4c^2}{18a^4 \times 12b^2 \times 2c}$ **i.** $\frac{81f^{15} \times 25g^{12} \times 16h^{34}}{27f^9 \times 15g^{10} \times 12h^{30}}$

6. WE6 Evaluate the following.

a. m^0 **b.** $6m^0$ **c.** $(6m)^0$ **d.** $(ab)^0$

e. $5(ab)^0$ **f.** w^0x^0 **g.** 85^0 **h.** $85^0 + 15^0$

i. $x^0 + 1$ **j.** $5x^0 - 2$ **k.** $\frac{x^0}{y^0}$ **l.** $x^0 + y^0$

m. $x^0 - y^0$ **n.** $3x^0 + 11$ **o.** $3a^0 + 3b^0$ **p.** $3(a^0 + b^0)$

7. WE7 Simplify each of the following.

a. $\frac{2a^3 \times 6a^2}{12a^5}$ **b.** $\frac{3c^6 \times 6c^3}{9c^9}$ **c.** $\frac{5b^7 \times 10b^5}{25b^{12}}$ **d.** $\frac{8f^3 \times 3f^7}{4f^5 \times 3f^5}$

e. $\frac{9k^{12} \times 4k^{10}}{18k^4 \times k^{18}}$ **f.** $\frac{2h^4 \times 5k^2}{20h^2 \times k^2}$ **g.** $\frac{p^3 \times q^4}{5p^3}$ **h.** $\frac{m^7 \times n^3}{5m^3 \times m^4}$

i. $\frac{8u^9 \times v^2}{2u^5 \times 4u^4}$ **j.** $\frac{9x^6 \times 2y^{12}}{3y^{10} \times 3y^2}$

Understanding

8. WE8 Simplify the following by cancelling.

a. $\frac{x^7}{x^{10}}$ **b.** $\frac{m}{m^9}$ **c.** $\frac{m^3}{4m^9}$ **d.** $\frac{12x^6}{6x^8}$

e. $\frac{12x^8}{6x^6}$ **f.** $\frac{24t^{10}}{t^4}$ **g.** $\frac{5y^5}{10y^{10}}$ **h.** $\frac{35x^2y^{10}}{20x^7y^7}$

i. $\frac{12m^2n^4}{30m^5n^8}$ **j.** $\frac{16m^5n^{10}}{8m^5n^{12}}$ **k.** $\frac{20x^4y^5}{10x^5y^4}$ **l.** $\frac{a^2b^4c^6}{a^6b^4c^2}$

9. Find the value of each of the following expressions if $a = 3$.

a. $2a$ **b.** a^2 **c.** $2a^2$

d. $a^2 + 2$ **e.** $a^2 + 2a$

Reasoning

10. Explain why x^2 and $2x$ are not the same number. Include an example to illustrate your reasoning.

11. MC **a.** $12a^8b^2c^4(de)^0f$ when simplified is equal to:

A. $12a^8b^2c^4$ **B.** $12a^8b^2c^4f$ **C.** $12a^8b^2f$ **D.** $12a^8b^2$

b. $\left(\frac{6}{11}a^2b^7\right)^0 \times -(3a^2b^{11})^0 + 7a^0b$ when simplified is equal to:

A. $7b$ **B.** $1 + 7b$ **C.** $-1 + 7ab$ **D.** $-1 + 7b$

c. You are told that there is an error in the statement $3p^7q^3r^5s^6 = 3p^7s^6$. To make the statement correct, what should the left-hand side be?

A. $(3p^7q^3r^5s^6)^0$ **B.** $(3p^7)^0q^3r^5s^6$ **C.** $3p^7(q^3r^5s^6)^0$ **D.** $3p^7(q^3r^5)^0s^6$

d. You are told that there is an error in the statement $\frac{8f^6g^7h^3}{6f^4g^2h} = \frac{8f^2}{g^2}$. To make the statement correct, what should the left-hand side be?

A. $\frac{8f^6(g^7h^3)^0}{(6)^0f^4g^2(h)^0}$ **B.** $\frac{8(f^6g^7h^3)^0}{(6f^4g^2h)^0}$ **C.** $\frac{8(f^6g^7)^0h^3}{(6f^4)^0g^2h}$ **D.** $\frac{8f^6g^7h^3}{(6f^4g^2h)^0}$

e. What does $\frac{6k^7m^2n^8}{4k^7(m^6n)^0}$ equal?

A. $\frac{6}{4}$ **B.** $\frac{3}{2}$ **C.** $\frac{3n^8}{2}$ **D.** $\frac{3m^2n^8}{2}$

12. Explain why $5x^5 \times 3x^3$ is not equal to $15x^{15}$.

13. A multiple choice question requires a student to multiply 5^6 by 5^3. The student is having trouble deciding which of these four answers is correct: 5^{18}, 5^9, 25^{18} or 25^9.
a. Which is the correct answer?
b. Explain your answer by using another example to explain the First Index Law.

14. A multiple choice question requires a student to divide 5^{24} by 5^8. The student is having trouble deciding which of these four answers is correct: 5^{16}, 5^3, 1^{16} or 1^3.
a. Which is the correct answer?
b. Explain your answer by using another example to explain the Second Index Law.

15. a. What is the value of $\frac{5^7}{5^7}$?
b. What is the value of any number divided by itself?
c. Applying the Second Index Law dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what index?
d. Explain the Third Index Law using an example.

Problem solving

16. a. For $x^2\, x^{\Delta} = x^{16}$ to be an identity, what number must replace the triangle?
b. For $x^{\Delta}\, x^{\bigcirc}\, x^{\Diamond} = x^{12}$ to be an identity, there are 55 ways of assigning positive whole numbers to the triangle, circle, and diamond. Give at least four of these.

17. a. Can you find a pattern in the units digit for powers of 3?
b. The units digit of 3^6 is 9. What is the units digit of 3^{2001}?

18. a. Can you find a pattern in the units digit for powers of 4?
b. What is the units digit of 4^{105}?

19. a. Investigate the patterns in the units digit for powers of 2 to 9.
b. Predict the units digit for:
i. 2^{35} **ii.** 3^{16} **iii.** 8^{51}.

20. Write $4^{n+1} + 4^{n+1}$ as a single power of 2.

Reflection

How do the index laws aid calculations?

CHALLENGE 10.1

Find the value of each of the following.
a. $2^2 - 1^2$ **b.** $3^2 - 2^2$ **c.** $4^2 - 3^2$
Use your results to investigate a pattern and use it to determine the answer to $87^2 - 86^2$ without actually performing the calculation.

10.3 Raising a power to another power

10.3.1 More Index Laws

- $$\begin{aligned}(7^2)^3 &= 7^2 \times 7^2 \times 7^2 \\ &= 7^{2+2+2} \text{ (using the First Index Law)} \\ &= 7^{2\times 3} \\ &= 7^6\end{aligned}$$
- The indices are multiplied when a power is raised to another power. This is the **Fourth Index Law:** $(a^m)^n = a^{m\times n}$.
- The Fifth and Sixth Index Laws are extensions of the Fourth Index Law.

$$\textbf{Fifth Index Law: } (a \times b)^m = a^m \times b^m.$$

$$\textbf{Sixth Index Law: } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.$$

WORKED EXAMPLE 9

Simplify the following.
a $(7^4)^8$ **b** $(3a^2b^5)^3$

THINK	WRITE
a Simplify by applying the Fourth Index Law (multiply the indices).	**a** $(7^4)^8$ $= 7^{4 \times 8}$ $= 7^{32}$
b 1 Write the expression.	**b** $(3^1a^2b^5)^3$
2 Simplify by applying the Fifth Index Law for each term inside the brackets (multiply the indices).	$= 3^{1 \times 3}a^{2 \times 3}b^{5 \times 3}$ $= 3^3a^6b^{15}$
3 Write the answer.	$= 27a^6b^{15}$

WORKED EXAMPLE 10

Simplify $(2b^5)^2 \times (5b)^3$.

THINK	WRITE
1 Write the expression, including all indices.	$(2^1b^5)^2 \times (5^1b^1)^3$
2 Simplify by applying the Fifth Index Law.	$= 2^2b^{10} \times 5^3b^3$
3 Simplify further by applying the First Index Law.	$= 4 \times 125 \times b^{10} \times b^3$ $= 500b^{13}$

WORKED EXAMPLE 11 TI | CASIO

Simplify $\left(\frac{2a^5}{d^2}\right)^3$.

THINK	WRITE
1 Write the expression, including all indices.	$\left(\frac{2^1a^5}{d^2}\right)^3$
2 Simplify by applying the Sixth Index Law for each term inside the brackets.	$= \frac{2^3a^{15}}{d^6}$
3 Write the answer.	$= \frac{8a^{15}}{d^6}$

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Exercise 10.3 Raising a power to another power

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–d, 4–12, 14, 15	Questions: 1d–i, 2d–i, 3b–e, 4–12, 14–18	Questions: 1g–i, 2g–i, 3e–h, 4–18

Individual pathway interactivity: int-4517

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE9 Simplify each of the following.

a. $(e^2)^3$ **b.** $(f^8)^{10}$ **c.** $(p^{25})^4$

d. $(r^{12})^{12}$ **e.** $(a^2b^3)^4$ **f.** $(pq^3)^5$

g. $(g^3h^2)^{10}$ **h.** $(3w^9q^2)^4$ **i.** $(7e^5r^2q^4)^2$

2. WE10 Simplify each of the following.

a. $(p^4)^2 \times (q^3)^2$ **b.** $(r^5)^3 \times (w^3)^3$ **c.** $(b^5)^2 \times (n^3)^6$

d. $(j^6)^3 \times (g^4)^3$ **e.** $(q^2)^2 \times (r^4)^5$ **f.** $(h^3)^8 \times (j^2)^8$

g. $(f^4)^4 \times (a^7)^3$ **h.** $(t^5)^2 \times (u^4)^2$ **i.** $(i^3)^5 \times (j^2)^6$

3. WE11 Simplify each of the following.

a. $\left(\frac{3b^4}{d^3}\right)^2$ **b.** $\left(\frac{5h^{10}}{2j^2}\right)^2$ **c.** $\left(\frac{2k^5}{3t^8}\right)^3$ **d.** $\left(\frac{7p^9}{8q^{22}}\right)^2$

e. $\left(\frac{5y^7}{3z^{13}}\right)^3$ **f.** $\left(\frac{4a^3}{7c^5}\right)^4$ **g.** $\left(\frac{-4k^2}{7m^6}\right)^3$ **h.** $\left(\frac{-2g^7}{3h^{11}}\right)^4$

Understanding

4. Simplify each of the following.

a. $(2^3)^4 \times (2^4)^2$ **b.** $(t^7)^3 \times (t^3)^4$ **c.** $(a^4)^0 \times (a^3)^7$

d. $(b^6)^2 \times (b^4)^3$ **e.** $(e^7)^8 \times (e^5)^2$ **f.** $(g^7)^3 \times (g^9)^2$

g. $(3a^2)^4 \times (2a^6)^2$ **h.** $(2d^7)^3 \times (3d^2)^3$ **i.** $(10r^{12})^4 \times (2r^3)^2$

5. MC What does $(p^7)^2 \div p^2$ equal?

A. p^7 **B.** p^{12} **C.** p^{16} **D.** $p^{4.5}$

6. MC What does $\frac{(w^5)^2 \times (p^7)^3}{(w^2)^2 \times (p^3)^5}$ equal?

A. w^2p^6 **B.** $(wp)^6$ **C.** $w^{14}p^{36}$ **D.** w^2p^2

7. MC What does $(r^6)^3 \div (r^4)^2$ equal?

A. r^3 **B.** r^4 **C.** r^8 **D.** r^{10}

8. Simplify each of the following.

a. $(a^3)^4 \div (a^2)^3$ **b.** $(m^8)^2 \div (m^3)^4$ **c.** $(n^5)^3 \div (n^6)^2$

d. $(b^4)^5 \div (b^6)^2$ **e.** $(f^7)^3 \div (f^2)^2$ **f.** $(g^8)^2 \div (g^5)^2$

g. $(p^9)^3 \div (p^6)^3$ **h.** $(y^4)^4 \div (y^7)^2$ **i.** $\frac{(c^6)^5}{(c^5)^2}$

j. $\frac{(f^5)^3}{(f^2)^4}$ **k.** $\frac{(k^3)^{10}}{(k^2)^8}$ **l.** $\frac{(p^{12})^3}{(p^{10})^2}$

Reasoning

9. a. Simplify each of the following.

i. $(-1)^{10}$ **ii.** $(-1)^{7}$ **iii.** $(-1)^{15}$ **iv.** $(-1)^{6}$

b. Write a general rule for the result obtained when -1 is raised to a positive power. Justify your solution.

10. a. Replace the triangle with the correct index for $4^7 \times 4^7 \times 4^7 \times 4^7 \times 4^7 = (4^7)^{\Delta}$.

b. The expression $(p^5)^6$ means to write p^5 as a factor how many times?

c. If you rewrote the expression from part **b** without any exponents, as $p \times p \times p \ldots$, how many factors would you need?

d. Explain the Fourth Index Law.

11. A multiple choice question requires a student to calculate $(5^4)^3$. The student is having trouble deciding which of these three answers is correct: 5^{64}, 5^{12} or 5^7.

a. Which is the correct answer?

b. Explain your answer by using another example to explain the Fourth Index Law.

12. Jo and Danni are having an algebra argument. Jo is sure that $-x^2$ is equivalent to $(-x)^2$, but Danni thinks otherwise. Explain who is correct and justify your answer.

13. a. Without using your calculator, simplify each side to the same base and solve each of the following equations.

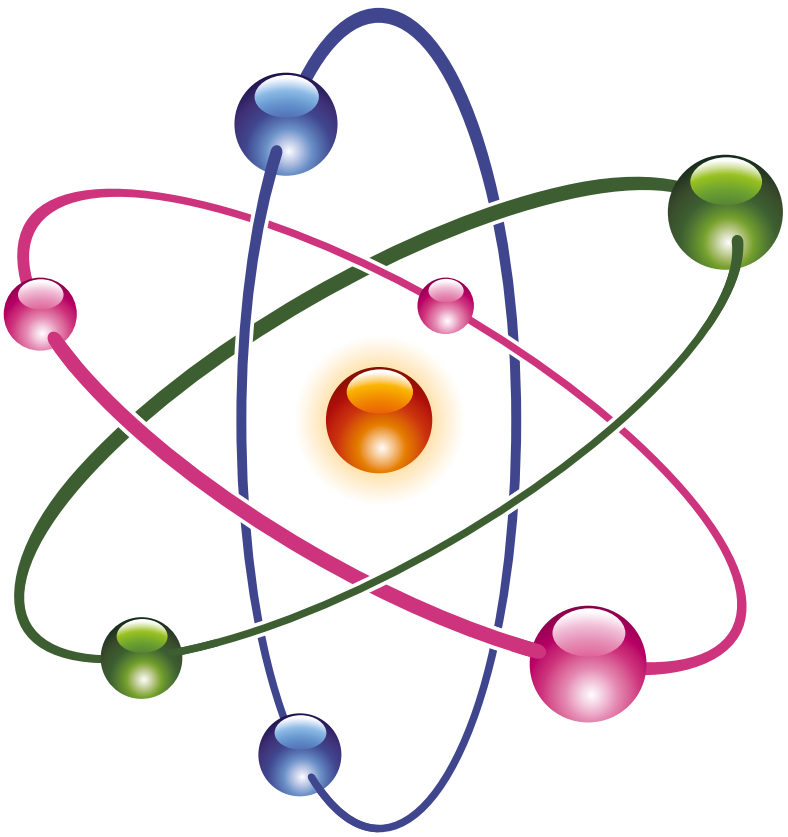
i. $8^x = 32$ **ii.** $27^x = 243$ **iii.** $1000^x = 100000$

b. Explain why all three equations have the same solution.

Problem solving

14. Consider the expression 4^{3^2}. Explain how you could get two different answers.

15. The diameter of a typical atom is so small that it would take about 10^8 of them, arranged in a line, to reach just one centimetre. Estimate how many atoms are contained in a cubic centimetre. Write this number as a power of 10.



16. Writing a base as a power itself can be used to simplify an expression.
Copy and complete the following calculations.

a. $16^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} =$ ..... **b.** $343^{\frac{2}{3}} = (7^3)^{\frac{2}{3}} =$ .....

17. Simplify the following using index laws.

a. $8^{\frac{1}{3}}$ **b.** $27^{\frac{4}{3}}$ **c.** $125^{-\frac{2}{3}}$ **d.** $512^{\frac{2}{9}}$

e. $16^{-\frac{1}{2}}$ **f.** $4^{-\frac{1}{2}}$ **g.** $32^{-\frac{1}{5}}$ **h.** $49^{-\frac{1}{2}}$

18. a. Use the index laws to simplify the following.

i. $(3^2)^{\frac{1}{2}}$ **ii.** $(4^2)^{\frac{1}{2}}$ **iii.** $(8^2)^{\frac{1}{2}}$ **iv.** $(11^2)^{\frac{1}{2}}$

b. Use your answers from part **a** to calculate the value of the following.

i. $9^{\frac{1}{2}}$ **ii.** $16^{\frac{1}{2}}$ **iii.** $64^{\frac{1}{2}}$ **iv.** $121^{\frac{1}{2}}$

c. Use your answers to parts **a** and **b** to write a sentence describing what raising a number to a power of one-half does.

Reflection

What difference, if any, is there between the operation of the index laws on numeric terms compared with similar operations on algebraic terms?

10.4 Negative indices

- As previously stated, $\frac{x^4}{x^6} = \frac{1}{x^2}$ if the numerator and denominator are both divided by the highest common factor, x^4.

  However, $\frac{x^4}{x^6} = x^{4-6} = x^{-2}$ if the Second Index Law is applied.

  It follows that $\boldsymbol{a^{-n} = \frac{1}{a^n}}$.

WORKED EXAMPLE 12

Evaluate the following.

a 5^{-2} **b** 7^{-1} **c** $\left(\frac{3}{5}\right)^{-1}$

THINK

a 1 Apply the rule $a^{-n} = \frac{1}{a^n}$.

2 Simplify.

b Apply the rule $a^{-n} = \frac{1}{a^n}$.

c 1 Apply the Sixth Index Law, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

2 Apply the rule $a^{-n} = \frac{1}{a^n}$ to the numerator and denominator.

3 Simplify and write the answer.

WRITE

a $5^{-2} = \frac{1}{5^2}$

$= \frac{1}{25}$

b $7^{-1} = \frac{1}{7^1}$

$= \frac{1}{7}$

c $\left(\frac{3^1}{5^1}\right)^{-1} = \frac{3^{-1}}{5^{-1}}$

$= \frac{1}{3} \div \frac{1}{5}$

$= \frac{1}{3} \times \frac{5}{1}$

$= \frac{5}{3}$

WORKED EXAMPLE 13

Write the following with positive indices.

a x^{-3} **b** $5x^{-6}$ **c** $\dfrac{x^{-3}}{y^{-2}}$

THINK	WRITE
a Apply the rule $a^{-n} = \dfrac{1}{a^n}$.	**a** $x^{-3} = \dfrac{1}{x^3}$
b 1 Write in expanded form and apply the rule $a^{-n} = \dfrac{1}{a^n}$.	**b** $5x^{-6} = 5 \times x^{-6}$ $= 5 \times \dfrac{1}{x^6}$
2 Simplify.	$= \dfrac{5}{x^6}$
c 1 Write the fraction using division.	**c** $\dfrac{x^{-3}}{y^{-2}} = x^{-3} \div y^{-2}$
2 Apply the rule $a^{-n} = \dfrac{1}{a^n}$.	$= \dfrac{1}{x^3} \div \dfrac{1}{y^2}$
3 Simplify.	$= \dfrac{1}{x^3} \times \dfrac{y^2}{1}$ $= \dfrac{y^2}{x^3}$

WORKED EXAMPLE 14

Simplify the following expressions, writing your answers with positive indices.

a $x^3 \times x^{-8}$

b $3x^{-2}y^{-3} \times 5xy^{-4}$

THINK	WRITE
a 1 Apply the First Index Law, $a^n \times a^m = a^{m+n}$.	**a** $x^3 \times x^{-8} = x^{3+-8}$ $= x^{-5}$
2 Write the answer with a positive index.	$= \dfrac{1}{x^5}$
b 1 Write in expanded form and apply the First Index Law.	**b** $3x^{-2}y^{-3} \times 5xy^{-4} = 3 \times 5 \times x^{-2} \times x^1 \times y^{-3} \times y^{-4}$
2 Apply the rule $a^{-n} = \dfrac{1}{a^n}$.	$= 15x^{-1}y^{-7}$ $= \dfrac{15}{1} \times \dfrac{1}{x} \times \dfrac{1}{y^7}$
3 Simplify.	$= \dfrac{15}{xy^7}$

WORKED EXAMPLE 15

Simplify the following expressions, writing your answers with positive indices.

a $\dfrac{t^2}{t^{-5}}$ **b** $\dfrac{15m^{-5}}{10m^{-2}}$

THINK

a Apply the Second Index Law, $\dfrac{a^n}{a^m} = a^{n-m}$.

b 1 Apply the Second Index Law and simplify.

2 Write the answer with positive indices.

WRITE

a $\dfrac{t^2}{t^{-5}} = t^{2-(-5)}$
$= t^{2+5}$
$= t^7$

b $\dfrac{15m^{-5}}{10m^{-2}} = \dfrac{15}{10} \times \dfrac{m^{-5}}{m^{-2}}$
$= \dfrac{3}{2} \times m^{-5-(-2)}$
$= \dfrac{3}{2} \times m^{-3}$
$= \dfrac{3}{2} \times \dfrac{1}{m^3}$
$= \dfrac{3}{2m^3}$

Exercise 10.4 Negative indices

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 13, 14	Questions: 1–11, 13–16	Questions: 1–17

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Fluency

1. Copy and complete the patterns below.

a. $3^5 = 243$
$3^4 = 81$
$3^3 = 27$
$3^2 =$
$3^1 =$
$3^0 =$
$3^{-1} = \dfrac{1}{3}$
$3^{-2} = \dfrac{1}{9}$
$3^{-3} =$
$3^{-4} =$
$3^{-5} =$

b. $5^4 = 625$
$5^3 =$
$5^2 =$
$5^1 =$
$5^0 =$
$5^{-1} =$
$5^{-2} =$
$5^{-3} =$
$5^{-4} =$

c. $10^4 = 10\,000$
$10^3 =$
$10^2 =$
$10^1 =$
$10^0 =$
$10^{-1} =$
$10^{-2} =$
$10^{-3} =$
$10^{-4} =$

2. WE12 Evaluate each of the following expressions.

a. 2^{-5} **b.** 3^{-3} **c.** 4^{-1} **d.** 10^{-2}

e. 5^{-3} **f.** $\left(\frac{1}{7}\right)^{-1}$ **g.** $\left(\frac{3}{4}\right)^{-1}$ **h.** $\left(\frac{3}{4}\right)^{-2}$

i. $\left(\frac{1}{3}\right)^{-3}$ **j.** $\left(\frac{3}{2}\right)^{-1}$ **k.** $\left(2\frac{1}{4}\right)^{-2}$ **l.** $\left(\frac{2}{7}\right)^{-2}$

3. WE13 Write each expression with positive indices.

a. x^{-4} **b.** y^{-5} **c.** z^{-1} **d.** a^2b^{-3}

e. $7m^{-2}$ **f.** $m^{-2}n^{-3}$ **g.** $(m^2n^3)^{-1}$ **h.** $\dfrac{x^2}{y^{-2}}$

i. $\dfrac{5}{x^{-3}}$ **j.** $\dfrac{x^{-2}}{w^{-5}}$ **k.** $\dfrac{1}{x^{-2}y^{-2}}$ **l.** $a^2b^{-3}cd^{-4}$

m. $\dfrac{a^2b^{-2}}{c^2d^{-3}}$ **n.** $10x^{-2}y$ **o.** $3^{-1}x$ **p.** $\dfrac{m^{-3}}{x^2}$

Understanding

4. WE14 Simplify the following expressions, writing your answers with positive indices.

a. $a^3 \times a^{-8}$ **b.** $m^7 \times m^{-2}$

c. $m^{-3} \times m^{-4}$ **d.** $2x^{-2} \times 7x$

e. $x^5 \times x^{-8}$ **f.** $3x^2y^{-4} \times 2x^{-7}y$

g. $10x^5 \times 5x^{-2}$ **h.** $x^5 \times x^{-5}$

i. $10a^2 \times 5a^{-7}$ **j.** $10a^{10} \times a^{-6}$

k. $16w^2 \times -2w^{-5}$ **l.** $4m^{-2} \times 4m^{-2}$

m. $(3m^2n^{-4})^3$ **n.** $(a^2b^5)^{-3}$

o. $(a^{-1}b^{-3})^{-2}$ **p.** $(5a^{-1})^2$

5. WE15 Simplify the following expressions, writing your answers with positive indices.

a. $\dfrac{x^3}{x^8}$ **b.** $\dfrac{x^{-3}}{x^8}$ **c.** $\dfrac{x^3}{x^{-8}}$ **d.** $\dfrac{x^{-3}}{x^{-8}}$

e. $\dfrac{10a^4}{5a^5}$ **f.** $\dfrac{6a^2c^5}{a^4c}$ **g.** $10a^2 \div 5a^8$ **h.** $5m^7 \div m^8$

i. $\dfrac{a^5b^6}{a^5b^7}$ **j.** $\dfrac{a^2b^8}{a^5b^{10}}$ **k.** $\dfrac{a^{-3}bc^3}{abc}$ **l.** $\dfrac{4^{-2}ab}{a^2b}$

m. $\dfrac{m^{-3} \times m^{-5}}{m^{-5}}$ **n.** $\dfrac{2t^2 \times 3t^{-5}}{4t^6}$ **o.** $\dfrac{t^3 \times t^{-5}}{t^{-2} \times t^{-3}}$ **p.** $\dfrac{(m^2n^{-3})^{-1}}{(m^{-2}n^3)^2}$

6. Write the following numbers as powers of 2.

a. 1 **b.** 8 **c.** 32

d. 64 **e.** $\frac{1}{8}$ **f.** $\frac{1}{32}$

7. Write the following numbers as powers of 4.

a. 1 **b.** 4 **c.** 64

d. $\frac{1}{4}$ **e.** $\frac{1}{16}$ **f.** $\frac{1}{64}$

8. Write the following numbers as powers of 10.

a. 1 **b.** 10 **c.** 10000

d. 0.1 **e.** 0.01 **f.** 0.00001

Reasoning

9. a. The result of dividing 3^7 by 3^3 is 3^4. What is the result of dividing 3^3 by 3^7?
b. Explain what it means to have a negative index.
c. Explain how you write a negative index as a positive index.

10. Indices are encountered in science, where they help to deal with very small and large numbers. The diameter of a proton is 0.000 000 000 000 3 cm. Explain why it is logical to express this number in scientific notation as 3×10^{-13}.

11. a. When asked to find an expression that is equivalent to $x^3 + x^{-3}$, a student responded x^0. Is this answer correct? Explain why or why not.
b. When asked to find an expression that is equivalent to $(x^{-1} + y^{-1})^{-2}$, a student responded $x^2 + y^2$. Is this answer correct? Explain why or why not.

12. a. When asked to find an expression that is equivalent to $x^8 – x^{-5}$, a student responded x^3. Is this answer correct? Explain why or why not.
b. Another student said that $\frac{x^2}{x^8 - x^5}$ is equivalent to $\frac{1}{x^6} - \frac{1}{x^3}$. Is this answer correct? Explain why or why not.

Problem solving

13. What is the value of n in the following expressions?
a. $4793 = 4.793 \times 10^n$
b. $0.631 = 6.31 \times 10^n$
c. $134 = 1.34 \times 10^n$
d. $0.000\,56 = 5.6 \times 10^n$

14. Write the following numbers as basic numerals.
a. 4.8×10^{-2}
b. 7.6×10^3
c. 2.9×10^{-4}
d. 8.1×10^0

15. Find a half of 2^{20}.

16. Find one-third of 3^{21}.

17. Simplify the following expressions.
a. $(2^{-1} + 3^{-1})^{-1}$
b. $\frac{3^{400}}{6^{200}}$
c. $\sqrt{16x^{16}}$

Reflection

What strategy will you use to remember the index laws?

10.5 Square roots and cube roots

10.5.1 Square root

- The symbol $\sqrt{\ }$ means square root — a number that multiplies by itself to give the original number.
- Each number actually has a positive and negative square root. For example, $(2)^2 = 4$ and $(–2)^2 = 4$. Therefore the square root 4 is +2 or –2. For this chapter, assume $\sqrt{\ }$ is positive unless otherwise indicated.
- The square root is the inverse of squaring (power 2).
- For this reason, a square root is equivalent to an index of $\frac{1}{2}$.
- In general, $\sqrt{a} = a^{\frac{1}{2}}$.

WORKED EXAMPLE 16

TI | CASIO

Evaluate $\sqrt{16p^2}$.

THINK	WRITE
1 We need to obtain the square root of both 16 and p^2.	$\sqrt{16p^2} = \sqrt{16} \times \sqrt{p^2}$
2 Which number is multiplied by itself to give 16? It is 4. Replace the square root sign with a power of $\frac{1}{2}$.	$= 4 \times (p^2)^{\frac{1}{2}}$
3 Use the Fourth Index Law.	$= 4 \times p^{2 \times \frac{1}{2}}$
	$= 4 \times p^1$
4 Simplify.	$= 4p$

10.5.2 Cube root

- The symbol $\sqrt[3]{\ }$ means cube root — a number that multiplies by itself three times to give the original number.
- The cube root is the inverse of cubing (power 3).
- For this reason, a square root is equivalent to an index of $\frac{1}{3}$.
- In general, $\sqrt[3]{a} = a^{\frac{1}{3}}$.

WORKED EXAMPLE 17

TI | CASIO

Evaluate $\sqrt[3]{8j^6}$.

THINK	WRITE
1 We need to obtain the cube root of both 8 and j^6.	$\sqrt[3]{8j^6} = \sqrt[3]{8} \times \sqrt[3]{j^6}$
2 Which number, written 3 times and multiplied gives 8? It is 2. Replace the cube root sign with a power of $\frac{1}{3}$.	$= 2 \times (j^6)^{\frac{1}{3}}$
3 Use the Fourth Index Law.	$= 2 \times j^{6 \times \frac{1}{3}}$
	$= 2 \times j^2$
4 Simplify.	$= 2j^2$

- In general terms, $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.

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Exercise 10.5 Square roots and cube roots

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10, 11	Questions: 1–8, 10–12	Questions: 1–13

Individual pathway interactivity: int-4519

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Fluency

1. Write the following in surd form.

a. $x^{\frac{1}{2}}$ **b.** $y^{\frac{1}{5}}$ **c.** $z^{\frac{1}{4}}$ **d.** $(2w)^{\frac{1}{3}}$ **e.** $7^{\frac{1}{2}}$

2. Write the following in index form.

a. $\sqrt{15}$ **b.** $\sqrt{m}$ **c.** $\sqrt[3]{t}$ **d.** $\sqrt[3]{w^2}$ **e.** $\sqrt[5]{n}$

3. WE16 Evaluate the following.

a. $49^{\frac{1}{2}}$ **b.** $4^{\frac{1}{2}}$ **c.** $27^{\frac{1}{3}}$ **d.** $125^{\frac{1}{3}}$

e. $1000^{\frac{1}{3}}$ **f.** $64^{\frac{1}{2}}$ **g.** $64^{\frac{1}{3}}$ **h.** $128^{\frac{1}{7}}$

i. $243^{\frac{1}{5}}$ **j.** $1\,000\,000^{\frac{1}{2}}$ **k.** $1\,000\,000^{\frac{1}{3}}$ **l.** $(27^{\frac{1}{3}})^2$

Understanding

4. WE17 Simplify the following expressions.

a. $\sqrt{m^2}$ **b.** $\sqrt[3]{b^3}$ **c.** $\sqrt{36t^4}$ **d.** $\sqrt[3]{m^3n^6}$

e. $\sqrt[3]{125t^6}$ **f.** $\sqrt[5]{x^5y^{10}}$ **g.** $\sqrt[4]{a^8m^{40}}$ **h.** $\sqrt[3]{216y^6}$

i. $\sqrt[3]{64x^6y^6}$ **j.** $\sqrt{25a^2b^4c^6}$ **k.** $\sqrt[7]{b^{49}}$ **l.** $\sqrt[3]{b^3} \times \sqrt{b^4}$

5. MC **a.** What does $\sqrt[3]{8000m^6n^3p^3q^6}$ equal?

A. $2666.6m^2npq^2$ **B.** $20m^2npq^2$ **C.** $20m^3n^0p^0q^3$ **D.** $7997m^2npq^2$

b. What does $\sqrt[3]{3375a^9b^6c^3}$ equal?

A. $1125a^3b^2c$ **B.** $1125a^6b^3c^0$ **C.** $1123a^6b^3$ **D.** $15a^3b^2c$

c. What does $\sqrt[3]{15\,625f^3g^6h^9}$ equal?

A. $25fg^2h^3$ **B.** $25f^0g^3h^6$ **C.** $25g^3h^6$ **D.** $5208.3fg^2h^3$

Reasoning

6. a. Using the First Index Law, explain how $3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3$.

b. What is another way that $3^{\frac{1}{2}}$ can be written?

c. Find $\sqrt{3} \times \sqrt{3}$.

d. How can $\sqrt[n]{a}$ be written in index form?

e. Without a calculator, solve:

i. $8^{\frac{1}{3}}$ **ii.** $32^{\frac{2}{5}}$.

7. a. Explain why calculating $z^{2.5}$ is a square root problem.

b. Is $z^{0.3}$ a cube root problem? Justify your reasoning.

8. Mark and Christina are having an algebra argument. Mark is sure that $\sqrt{x^2}$ is equivalent to x, but Christina thinks otherwise. Who is correct? Explain how you would resolve this disagreement.

9. Verify that $(-8)^{\frac{1}{3}}$ can be evaluated and explain why $(-8)^{\frac{1}{4}}$ cannot be evaluated.

Problem solving

10. If $n^{\frac{3}{4}} = \frac{8}{27}$, what is the value of n?

11. The mathematician Augustus de Morgan enjoyed telling his friends that he was x years old in the year x^2. Find the year of Augustus de Morgan's birth, given that he died in 1871.

12. a. Investigate Johannes Kepler.

b. Kepler's Third Law describes the relationship between the distance of planets from the Sun and their orbital periods. It is represented by the equation $d^{\frac{1}{2}} = t^{\frac{1}{3}}$. Solve for:

i. d in terms of t **ii.** t in terms of d.

13. An unknown number is multiplied by 4 and then has five subtracted from it. It is now equal to the square root of the original unknown number squared.

a. Is this a linear algebra problem? Justify your answer.

b. How many solutions are possible? Explain why.

c. Find all possible values for the number.

Reflection

How would $\sqrt[n]{a^b}$ be written in index form?

CHALLENGE 10.2

The pronumeral i is used to represent $\sqrt{-1}$. This is the basis for imaginary numbers. It follows that the value of i^2 is -1. What is the value of i^{40}?

10.6 Review

10.6.1 Review questions

Fluency

1. State the base for each of the following.

a. 4^8 **b.** 5^9 **c.** a^4 **d.** x^7

2. State the power or index for each of the following.

a. 10^3 **b.** 6^{12} **c.** g^{89} **d.** 4^0

3. Write each of the following as a basic numeral.

a. 5^2 **b.** 4^2 **c.** 2^5 **d.** 10^3 **e.** 3^4 **f.** 12^8

4. Express each of the following as a product of powers of primes.

a. 100 **b.** 121 **c.** 104 **d.** 225 **e.** 588 **f.** 10080

5. Simplify each of the following.

a. $b^7 \times b^3$ **b.** $m^9 \times m^2$ **c.** $k^3 \times k^5$ **d.** $f^2 \times f^8 \times f^4$

e. $h^4 \times h^5 \times h$ **f.** $2q^5 \times 3q^2 \times q^{10}$ **g.** $5w^3 \times 7w^{12} \times w^{14}$ **h.** $2e^2p^3 \times 6e^3p^5$

i. $5a^2b^4 \times 3a^8b^5 \times 7a^6b^8$

6. Simplify each of the following.

a. $a^5 \div a^2$ **b.** $t^5 \div t$ **c.** $r^{19} \div r^{12}$ **d.** $p^8 \div p^5$

e. $\frac{f^{17}}{f^{12}}$ **f.** $\frac{y^{100}}{y^{10}}$ **g.** $\frac{m^{24}}{m^{14}}$ **h.** $\frac{g^4 \times g^5}{g^2}$

i. $\frac{x^6 \times x^2 \times x}{x^8}$ **j.** $\frac{d^6 \times d^7 \times d^2}{d^8}$ **k.** $\frac{t^7 \times t \times t^3}{t^2 \times t^4}$ **l.** $\frac{p^5 \times p^3 \times p \times p^4}{p^2 \times p^4 \times p^2}$

m. $\frac{16k^{13}}{21} \div \frac{8k^9}{42}$ **n.** $\frac{22b^{15}}{c} \div \frac{2b^8}{c^6}$ **o.** $\frac{9d^8}{16e^{10}} \div \frac{2d^{10}}{e^{16}}$

7. Simplify the following.

a. 5^0 **b.** 12^0 **c.** 345^0 **d.** q^0

e. r^0 **f.** ab^0 **g.** $3w^0$ **h.** $5q^0 - 2q^0$

i. $100s^0 + 99t^0$ **j.** a^7b^0 **k.** $v^{10}\,w^0$ **l.** prt^0

m. $a^9b^4c^0$ **n.** $j^8k^0m^3$ **o.** $4e^2f^0 - 36(a^2b^3)^0$ **p.** $-8(18x^2y^4z^6)^0$

q. $15 - 12x\left(\frac{3x}{8}\right)^0$ **r.** $-4p^0 \times 6(q^2r^3)^0 \div 8(-12q^2)^0$ **s.** $3(6w^0)^2 \div 2(5w^5)^0$

8. Raise each of the following to the given power.

a. $(b^4)^2$ **b.** $(a^8)^3$ **c.** $(k^7)^{10}$ **d.** $(j^{100})^2$

e. $(a^5b^2)^3$ **f.** $(m^7n^{12})^2$ **g.** $(st^6)^3$ **h.** $(qp^{30})^{10}$

9. What does $\left(\frac{4b^4}{d^2}\right)^3$ equal?

A. $\frac{4b^3}{d^3}$ **B.** $\frac{12b^{12}}{d^6}$ **C.** $\frac{64b^{12}}{d^6}$ **D.** $\frac{64b^7}{d^5}$

10. Write each of the following with positive indices.

a. a^{-1} **b.** k^{-4} **c.** $4m^3 \div 2m^7$ **d.** $7x^3y^{-4} \times 6x^{-3}y^{-1}$

11. Write each of the following using a negative index.

a. $\frac{1}{x}$ **b.** $\frac{2}{y^4}$ **c.** $z \div z^4$ **d.** $45p^2q^{-4} \times 3p^{-5}q$

12. Simplify each of the following.

a. $\sqrt{100}$ **b.** $\sqrt{36}$ **c.** $\sqrt{a^2}$ **d.** $\sqrt{b^2}$

e. $\sqrt{49f^4}$ **f.** $\sqrt[3]{27}$ **g.** $\sqrt[3]{1000}$ **h.** $\sqrt[3]{x^3}$

i. $\sqrt[3]{8d^3}$ **j.** $\sqrt[3]{64f^6g^3}$

Problem solving

The chessboard problem

Legend has it that an ancient Chinese king challenged the people around him to invent a game that would keep him amused for hours on end and would challenge him mentally. Days later the king was presented with the game of chess.

The king thought that this was a marvellous game and wanted to reward the inventor. The inventor said, *'I do not ask much for my king's pleasure. All I ask for is one grain of rice for the first square, two grains of rice for the second square, four grains of rice for the third square and so on.'*

The king was amazed that one could ask for so little for such a wonderful game. He sent his mathematician away to calculate exactly what the inventor was to be paid.

The mathematician came back and said that the inventor had asked for more than all the rice in the Kingdom of China!



Questions

13. The inventor asked for one grain of rice for the first square, two for the second, four for the third and so on. For the first row of the board, write down the number of grains of rice for each square.

14. We can write 4 as 2×2, 8 as $2 \times 2 \times 2$ and so on. Write each of the amounts of grain for the first row, in index form.

15. For each square on the board, write the number of grains of rice to be paid, in index notation.

16. Complete the table below for the first row.

Square number	Grains paid	Total paid
1	1	1
2	2	3
3	4	7
4	8	
5		
6		
7		
8		

17. Can you see a pattern between the numbers in the total paid column and in the grains paid column?
18. Calculate the total number of rice grains to be paid.

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- **Try out this interactivity:** Word search: Topic 10 (int-2696)
- **Try out this interactivity:** Crossword: Topic 10 (int-2697)
- **Try out this interactivity:** Sudoku: Topic 10 (int-3209)
- **Complete this digital doc:** Concept map: Topic 10 (doc-10800)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

coefficients
composite number
cube root
exponent
factor form
fractional index
index
index laws
index notation
indices
negative index
prime factors
square root
surd form
zero index

assessON
Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement. assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.
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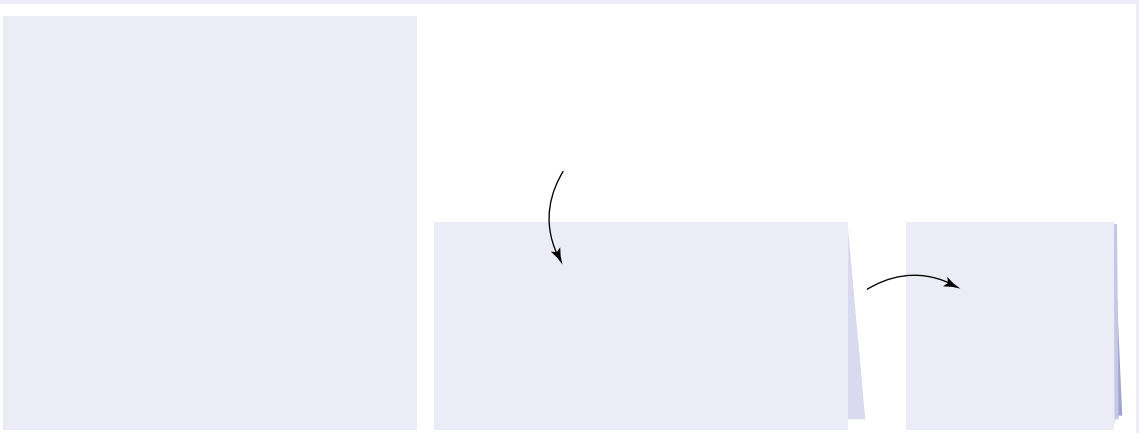


Investigation | Rich task

Paper folds



If you take a piece of paper and fold it a few times, you will soon notice that the area of the upper surface of the paper decreases with each fold. In this task, consider the relationships between the number of folds, the resulting thickness of paper and the upper surface area after each fold has been made.



One square piece of paper | One fold, two sheets thick | Two folds, four sheets thick

Take a square piece of paper with a side length of 8 cm. The upper surface of this paper has an area of 64 cm^2. Fold the paper in half. The paper now shows an area of 32 cm^2, has 1 fold and is 2 sheets thick. Make another fold. The diagram above displays the changes that take place for 2 folds.

Continue with the folding process for up to 5 folds. The thickness of the paper and the surface area of the upper face change with each fold.

1. Write the dimensions of each upper surface after each fold.

2. Calculate the area (in cm^2) of each upper surface after each fold.

3. Complete the following table to show the change in the upper surface area and the thickness after each fold.

Number of folds	0	1	2	3	4	5
Thickness of paper	1	2				
Area of upper surface after each fold (cm^2)	64	32				

4. Study the values recorded in the table in question **3**. Explain whether there is a linear relationship between the number of folds and the thickness of the paper, or between the number of folds and the area after each fold.

Let f represent the number of folds, t represent the thickness of the paper after each fold and a represent the area of the upper face after each fold. A relationship between the pronumerals may be more obvious if the values in the table are presented in a different form.

5. Complete the table below, presenting your values in index form with a base of 2.

Number of folds (f)	0	1	2	3	4	5
Thickness of paper (t)	2^0	2^1				
Area of upper surface after each fold (cm^2) (a)	2^6	2^5				

6. Consider the values in the table above to write a relationship between the following pronumerals.
- t and f
- a and t
- a and f

7. What difference, if any, would it make to these relationships if the original paper size had been a square with side length of 16 cm? Draw a table to show the change in area of each face and the thickness of the paper with each fold. Write formulas to describe these relationships.

8. Investigate these relationship with squares of different side lengths. Describe whether the relationship between the three features studied during this task can always be represented in index form.



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Complete this digital doc: Code puzzle: What is the name given to a boat with one sculler and two oarsmen? (doc-15903)

Answers

Topic 10 Indices

Exercise 10.2 Review of index laws

1. a. $2^2 \times 3$ **b.** $2^3 \times 3^2$ **c.** 3×5^2 **d.** $2^4 \times 3 \times 5$ **e.** $2^7 \times 5$ **f.** $2^3 \times 5^2 \times 7^2$

2. a. $20p^{11}$ **b.** $6x^8$ **c.** $56y^{10}$ **d.** $21p^8$ **e.** $84t^6$ **f.** $30q^{15}$

3. a. $6a^6e^7$ **b.** $8p^6h^{12}$ **c.** $80m^9$ **d.** $6g^3h^6$ **e.** $30p^6q^9$ **f.** $48u^9w^7$
g. $27d^{11}y^{17}$ **h.** $42b^{14}c^9$ **i.** $24r^{16}s^{18}$ **j.** $60h^{38}v^{20}$

4. a. $3p^4$ **b.** $6r^4$ **c.** $9a^3$ **d.** $3b^6$ **e.** $20r^4$ **f.** $9q$

5. a. $\frac{3p^5}{2}$ **b.** $\frac{8b^5}{3}$ **c.** $\frac{5m^{10}n^6}{6}$ **d.** $\frac{9x^8y}{4}$ **e.** $\frac{4hk^3}{3}$ **f.** $3f^5f^3$
g. $\frac{4p^2rs}{3}$ **h.** $\frac{9a^5b^3c}{2}$ **i.** $\frac{20f^6g^2h^4}{3}$

6. a. 1 **b.** 6 **c.** 1 **d.** 1 **e.** 5 **f.** 1
g. 1 **h.** 2 **i.** 2 **j.** 3 **k.** 1 **l.** 2
m. 0 **n.** 14 **o.** 6 **p.** 6

7. a. 1 **b.** 2 **c.** 2 **d.** 2 **e.** 2 **f.** $\frac{h^2}{2}$
g. $\frac{q^4}{5}$ **h.** $\frac{n^3}{5}$ **i.** v^2 **j.** $2x^6$

8. a. $\frac{1}{x^3}$ **b.** $\frac{1}{m^8}$ **c.** $\frac{1}{4m^6}$ **d.** $\frac{2}{x^2}$ **e.** $2x^2$ **f.** $24t^6$
g. $\frac{1}{2y^5}$ **h.** $\frac{7y^3}{4x^5}$ **i.** $\frac{2}{5m^3n^4}$ **j.** $\frac{2}{n^2}$ **k.** $\frac{2y}{x}$ **l.** $\frac{c^4}{a^4}$

9. a. 6 **b.** 9 **c.** 18 **d.** 11 **e.** 15

10. Answers will vary.

11. a. B **b.** D **c.** D **d.** A **e.** D

12. Answers will vary.

13. a. 5^9 **b.** Answers will vary.

14. a. 5^{16} **b.** Answers will vary.

15. a. 1 **b.** 1 **c.** Zero **d.** Answers will vary.

16. a. $\Delta = 14$

b. Answers will vary, but $\Delta + O + \Diamond$ must sum to 12. Possible answers include: $\Delta = 3$, $O = 2$, $\Diamond = 7$; $\Delta = 1$, $O = 3$, $\Diamond = 8$; $\Delta = 4$, $O = 4$, $\Diamond = 4$; $\Delta = 5$, $O = 1$, $\Diamond = 6$.

17. a. The repeating pattern is 1, 3, 9, 7.

b. 3

18. a. The repeating pattern is 4, 6.

b. 4

19. a. Answers will vary.

b. i. 8 **ii.** 1 **iii.** 2

20. 2^{2n+3}

Challenge 10.1

a. 3 **b.** 5 **c.** 7

$87^2 - 86^2 = 173$

Exercise 10.3 Raising a power to another power

1. a. e^6 **b.** f^{80} **c.** p^{100} **d.** r^{144} **e.** a^8b^{12}
f. p^5q^{15} **g.** $g^{30}h^{20}$ **h.** $81w^{36}q^8$ **i.** $49e^{10}r^4q^8$

2. a. p^8q^6 **b.** $r^{15}w^9$ **c.** $b^{10}n^{18}$ **d.** $j^{18}g^{12}$ **e.** q^4r^{20}
f. $h^{24}j^{16}$ **g.** $a^{21}f^{16}$ **h.** $t^{10}u^8$ **i.** $i^{15}j^{12}$

3. a. $\frac{9b^8}{d^6}$ **b.** $\frac{25h^{20}}{4j^4}$ **c.** $\frac{8k^{15}}{27t^{24}}$ **d.** $\frac{49p^{18}}{64q^{44}}$ **e.** $\frac{125y^{21}}{27z^{39}}$

f. $\frac{256a^{12}}{2401c^{20}}$ **g.** $\frac{-64k^6}{343m^{18}}$ **h.** $\frac{16g^{28}}{81h^{44}}$

4. a. 2^{20} **b.** t^{33} **c.** a^{21} **d.** b^{24} **e.** e^{66}

f. g^{39} **g.** $324a^{20}$ **h.** $216d^{27}$ **i.** $40\,000r^{54}$

5. B

6. B

7. D

8. a. a^6 **b.** m^4 **c.** n^3 **d.** b^8 **e.** f^{17} **f.** g^6

g. p^9 **h.** y^2 **i.** c^{20} **j.** f^7 **k.** k^{14} **l.** p^{16}

9. a. i. 1 **ii.** -1 **iii.** -1 **iv.** 1

b. $(-1)^{\text{even}} = 1(-1)^{\text{odd}} = -1$

10. a. 5 **b.** 6 **c.** 30 **d.** Answers will vary.

11. a. 5^{12} **b.** Answers will vary.

12. Danni is correct. Explanations will vary but should involve $(-x)(-x) = (-x)^2 = x^2$ and $-x^2 = -1 \times x^2 = -x^2$.

13. a. i. $x = \frac{5}{3}$ **ii.** $x = \frac{5}{3}$ **iii.** $x = \frac{5}{3}$

b. When equating the powers, $3x = 5$.

14. Answers will vary. Possible answers are 4096 and 262 144.

15. $10^8 \times 10^8 \times 10^8 = (10^8)^3$atoms

16. a. 4^1 **b.** 7^2

17. a. 2^1 **b.** 3^4 **c.** $\frac{1}{5^2}$ **d.** 2^2

e. $\frac{1}{2^2}$ **f.** $\frac{1}{2}$ **g.** $\frac{1}{2}$ **h.** $\frac{1}{7}$

18. a. i. 3 **ii.** 4 **iii.** 8 **iv.** 11

b. i. 3 **ii.** 4 **iii.** 8 **iv.** 11

c. Raising a number to a power of one-half is the same as finding the square root of that number.

Exercise 10.4 Negative indices

1. a. $3^5 = 243$, $3^4 = 81$, $3^3 = 27$, $3^2 = 9$, $3^1 = 3$, $3^0 = 1$, $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$, $3^{-3} = \frac{1}{27}$, $3^{-4} = \frac{1}{81}$, $3^{-5} = \frac{1}{243}$

b. $5^4 = 625$, $5^3 = 125$, $5^2 = 25$, $5^1 = 5$, $5^0 = 1$, $5^{-1} = \frac{1}{5}$, $5^{-2} = \frac{1}{25}$, $5^{-3} = \frac{1}{125}$, $5^{-4} = \frac{1}{625}$

c. $10^4 = 10\,000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$, $10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{100}$, $10^{-3} = \frac{1}{1000}$, $10^{-4} = \frac{1}{10\,000}$

2. a. $\frac{1}{32}$ **b.** $\frac{1}{27}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{100}$ **e.** $\frac{1}{125}$ **f.** 7

g. $\frac{4}{3}$ **h.** $\frac{16}{9}$ **i.** 27 **j.** $\frac{2}{3}$ **k.** $\frac{16}{81}$ **l.** $\frac{49}{4}$

3. a. $\frac{1}{x^4}$ **b.** $\frac{1}{y^5}$ **c.** $\frac{1}{z}$ **d.** $\frac{a^2}{b^3}$ **e.** $\frac{7}{m^2}$ **f.** $\frac{1}{m^2n^3}$

g. $\frac{1}{m^2n^3}$ **h.** x^2y^2 **i.** $5x^3$ **j.** $\frac{w^5}{x^2}$ **k.** x^2y^2 **l.** $\frac{a^2c}{b^3d^4}$

m. $\frac{a^2d^3}{b^2c^2}$ **n.** $\frac{10y}{x^2}$ **o.** $\frac{x}{3}$ **p.** $\frac{1}{m^3x^2}$

4. a. $\frac{1}{a^5}$ **b.** m^5 **c.** $\frac{1}{m^7}$ **d.** $\frac{14}{x}$ **e.** $\frac{1}{x^3}$ **f.** $\frac{6}{x^5y^3}$

g. $50x^3$ **h.** 1 **i.** $\frac{50}{a^5}$ **j.** $10a^4$ **k.** $\frac{-32}{w^3}$ **l.** $\frac{16}{m^4}$

m. $\frac{27m^6}{n^{12}}$ **n.** $\frac{1}{a^6b^{15}}$ **o.** a^2b^6 **p.** $\frac{25}{a^2}$

5. a. $\frac{1}{x^5}$ **b.** $\frac{1}{x^{11}}$ **c.** x^{11} **d.** x^5 **e.** $\frac{2}{a}$ **f.** $\frac{6c^4}{a^2}$

g. $\frac{2}{a^6}$ **h.** $\frac{5}{m}$ **i.** $\frac{1}{b}$ **j.** $\frac{1}{a^3b^2}$ **k.** $\frac{c^2}{a^4}$ **l.** $\frac{1}{16a}$

m. $\frac{1}{m^3}$ **n.** $\frac{3}{2t^9}$ **o.** t^3 **p.** $\frac{m^2}{n^3}$

6. a. 2^0 **b.** 2^3 **c.** 2^5 **d.** 2^6 **e.** 2^{-3} **f.** 2^{-5}

7. a. 4^0 **b.** 4^1 **c.** 4^3 **d.** 4^{-1} **e.** 4^{-2} **f.** 4^{-3}

8. a. 10^0 **b.** 10^1 **c.** 10^4 **d.** 10^{-1} **e.** 10^{-2} **f.** 10^{-5}

9. a. $3^{-4} = \frac{1}{3^4}$

b. Answers will vary but should convey that if dividing, the power in the numerator is lower than that in the denominator.

c. Answers will vary.

10. Answers will vary but should convey that the negative 13 means the decimal point is moved 13 places to the left of 3. Using scientific notation allows the number to be expressed more concisely.

11. a. No. The equivalent expression with positive indices is $\frac{x^6 + 1}{x^3}$.

b. No. The equivalent expression with positive indices is $\frac{(xy)^2}{(x + y)^2}$.

12. a. No. The equivalent expression with positive indices is $\frac{x^{13} - 1}{x^5}$.

b. No. The correct equivalent expression is $\frac{1}{x^6 - x^3}$.

13. a. 3 **b.** –1 **c.** 2 **d.** –4

14. a. 0.048 **b.** 7600 **c.** 0.00029 **d.** 8.1

15. 2^{19}

16. 3^{20}

17. a. $\frac{6}{5}$ **b.** $\left(\frac{3}{2}\right)^{200}$ **c.** $4x^8$

Exercise 10.5 Square roots and cube roots

1. a. $\sqrt{x}$ **b.** $\sqrt[5]{y}$ **c.** $\sqrt[4]{z}$ **d.** $\sqrt[3]{2w}$ **e.** $\sqrt{7}$

2. a. $15^{\frac{1}{2}}$ **b.** $m^{\frac{1}{2}}$ **c.** $t^{\frac{1}{3}}$ **d.** $(w^2)^{\frac{1}{3}}$ **e.** $n^{\frac{1}{5}}$

3. a. 7 **b.** 2 **c.** 3 **d.** 5 **e.** 10 **f.** 8

g. 4 **h.** 2 **i.** 3 **j.** 1000 **k.** 100 **l.** 9

4. a. m **b.** b **c.** $6t^2$ **d.** mn^2 **e.** $5t^2$ **f.** xy^2

g. a^2m^{10} **h.** $6y^2$ **i.** $4x^2y^2$ **j.** $5ab^2c^3$ **k.** b^7 **l.** b^3

5. a. B **b.** D **c.** A

6. a. $3^{\frac{1}{2}+\frac{1}{2}} = 3^1$ **b.** $\sqrt{3}$ **c.** 3 **d.** $a^{\frac{1}{n}}$

e. i. 2 **ii.** 4

7. a. $z^{2.5} = z^{\frac{5}{2}} = (z^5)^{\frac{1}{2}} = \sqrt{z^5}$

b. No, it is the tenth root: $z^{0.3} = z^{\frac{3}{10}} = (z^3)^{\frac{1}{10}} = \sqrt[10]{z^3}$.

8. Mark is correct: $\sqrt{x^2} = x^{\frac{1}{2}\times 2} = x^1 = x$; x can be a positive or negative number.

9. $(-2^3)^{\frac{1}{3}} = -2$; answers will vary but should include that we cannot take the fourth root of a negative number.

10. $\frac{16}{81}$

11. $\sqrt{1871} \approx 43.25$
$42^2 = 1764$
$43^2 = 1849$
He was 43 years old in 1849. Therefore, he was born in $1849 - 43 = 1806$.

12. a. Answers will vary.

b. i. $d = t^{\frac{2}{3}}$ **ii.** $t = d^{\frac{3}{2}}$

13. a. No, since it has x^2 and $\sqrt{\ }$.

b. 2, because the square root of a number has a positive and a negative answer

c. $\frac{5}{3}$, 1

Challenge 10.2

$i^{40} = (i^2)^{20} = (-1)^{20} = 1$

10.6 Review

1. a. 4 **b.** 5 **c.** a **d.** x

2. a. 3 **b.** 12 **c.** 89 **d.** 0

3. a. 25 **b.** 16 **c.** 32 **d.** 1000
e. 81 **f.** 429 981 696

4. a. $2^2 \times 5^2$ **b.** 11^2 **c.** $2^3 \times 13$ **d.** $3^2 \times 5^2$
e. $2^2 \times 3 \times 7^2$ **f.** $2^5 \times 3^2 \times 5 \times 7$

5. a. b^{10} **b.** m^{11} **c.** k^8 **d.** f^{14} **e.** h^{10} **f.** $6q^{17}$
g. $35w^{29}$ **h.** $12e^5p^8$ **i.** $105a^{16}b^{17}$

6. a. a^3 **b.** t^4 **c.** r^7 **d.** p^3 **e.** f^5 **f.** y^{90}
g. m^{10} **h.** g^7 **i.** x **j.** d^7 **k.** t^5 **l.** p^5
m. $4k^4$ **n.** $11b^7c^5$ **o.** $\dfrac{9e^6}{32d^2}$

7. a. 1 **b.** 1 **c.** 1 **d.** 1 **e.** 1
f. a **g.** 3 **h.** 3 **i.** 199 **j.** a^7
k. v^{10} **l.** pr **m.** a^9b^4 **n.** j^8m^3 **o.** $4e^2 - 36$
p. -8 **q.** $15 - 12x$ **r.** -3 **s.** 54

8. a. b^8 **b.** a^{24} **c.** k^{70} **d.** j^{200}
e. $a^{15}b^6$ **f.** $m^{14}n^{24}$ **g.** s^3t^{18} **h.** $q^{10}p^{300}$

9. C

10. a. $\dfrac{1}{a}$ **b.** $\dfrac{1}{k^4}$ **c.** $\dfrac{2}{m^4}$ **d.** $\dfrac{42}{y^5}$

11. a. x^{-1} **b.** $2y^{-4}$ **c.** z^{-3} **d.** $135p^{-3}q^{-3}$

12. a. 10 **b.** 6 **c.** a **d.** b **e.** $7f^2$ **f.** 3
g. 10 **h.** x **i.** $2d$ **j.** $4f^2g$

13.

1	2	4	8	16	32	64	128

14.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7

15.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7
2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}
2^{24}	2^{25}	2^{26}	2^{27}	2^{28}	2^{29}	2^{30}	2^{31}
2^{32}	2^{33}	2^{34}	2^{35}	2^{36}	2^{37}	2^{38}	2^{39}
2^{40}	2^{41}	2^{42}	2^{43}	2^{44}	2^{45}	2^{46}	2^{47}
2^{48}	2^{49}	2^{50}	2^{51}	2^{52}	2^{53}	2^{54}	2^{55}
2^{56}	2^{57}	2^{58}	2^{59}	2^{60}	2^{61}	2^{62}	2^{63}

16.

Square number	Grains paid	Total paid
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

17. The total paid is 1 less than twice the number of grains paid.
Total paid = 2(Grains paid) − 1

18. $2(2^{63}) - 1 = 18\,446\,744\,073\,709\,551\,615$ or $1.844\,674\,407 \times 10^{19}$

Investigation — Rich task

1. Fold 1, 8 cm × 4 cm; fold 2, 4 cm × 4 cm; fold 4, 2 cm × 2 cm; fold 5, 2 cm × 1 cm

2. Fold 1, 32 cm^2; fold 2, 16 cm^2; fold 4, 4 cm^2; fold 5, 2 cm^2

3.

Number of folds	0	1	2	3	4	5
Thickness of paper	1	2	4	8	16	32
Area of surface after each fold (cm^2)	64	32	16	8	4	2

4. There is no linear relationship.

5.

Number of folds (f)	0	1	2	3	4	5
Thickness of paper (t)	2^0	2^1	2^2	2^3	2^4	2^5
Area of surface after each fold (cm^2) (a)	2^6	2^5	2^4	2^3	2^2	2^1

6. $t = 2^f$, $at = 2^6$, $a = 2^{6-f}$

7. $t = 2^f$, $at = 2^8$, $a = 2^{8-f}$

8. Check with your teacher.

TOPIC 11
Financial mathematics

11.1 Overview



Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

11.1.1 Why learn this?

Unfortunately for most of us, money is not in endless supply. If we monitor our income and expenses we can make our money go further. Understanding budgets and investments can help us to keep track of our money and reach our financial goals.

11.1.2 What do you know?

assessON

1. **THINK** List what you know about financial mathematics. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of financial mathematics.

LEARNING SEQUENCE

11.1 Overview
11.2 Salaries and wages
11.3 Special rates
11.4 Piecework
11.5 Commission and royalties
11.6 Loadings and bonuses
11.7 Taxation and net earnings
11.8 Simple interest
11.9 Compound interest
11.10 Review

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Watch this eLesson: The story of mathematics: The high life (eles-1698)

11.2 Salaries and wages

- Employees may be paid for their work in a variety of ways. Most employees receive either a **wage** or a **salary**.

11.2.1 Salaries

- A salary is a fixed annual (yearly) amount, usually paid fortnightly or monthly. A person who receives a salary is paid to do a job, regardless of the number of hours worked.

WORKED EXAMPLE 1

Susan has an annual salary of $63 048.92. How much is she paid:
a weekly **b fortnightly** **c monthly?**

THINK	WRITE
a 1 Annual means per year, so divide the salary by 52 because there are 52 weeks in a year.	**a** $63\,048.92 \div 52$ ≈ 1212.48
2 Write the answer in a sentence.	Susan's weekly salary is $1212.48.
b 1 There are 26 fortnights in a year, so divide the salary by 26.	**b** $63\,048.92 \div 26$ ≈ 2424.96
2 Write the answer in a sentence.	Susan's fortnightly salary is $2424.96.
c 1 There are 12 months in a year, so divide the salary by 12.	**c** Monthly salary $= 63\,048.92 \div 12$ ≈ 5254.08
2 Write the answer in a sentence.	Susan's monthly salary is $5254.08.

11.2.2 Wages

- A wage is based on a fixed rate per hour. Hours outside the normal work period are paid at a higher rate.

WORKED EXAMPLE 2

Frisco has casual work at a fast-food store. He is paid $12.27 per hour Monday to Saturday and $24.54 per hour on Sunday. Calculate his wage for a week in which he worked from 5.00 pm to 10 pm on Friday and from 6 pm to 9.00 pm on Sunday.

THINK	WRITE
1 Work out the number of hours Frisco worked each day. He worked 5 hours on Friday and 3 hours on Sunday.	Friday: $5 \times 12.27 = 61.35$ Sunday: $3 \times 24.54 = 73.62$
2 Find the total amount earned.	$61.35 + 73.62 = 134.97$
3 Write the answer in a sentence.	Frisco's wage was $134.97.

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Complete this digital doc: SkillSHEET: Converting units of time (doc-10849)

Complete this digital doc: SkillSHEET: Multiplying and dividing a quantity (money) by a whole number (doc-10850)

Complete this digital doc: SkillSHEET: Multiplying and dividing a quantity (money) by a fraction (doc-10851)

Complete this digital doc: SkillSHEET: Increasing a quantity by a percentage (doc-10852)

Complete this digital doc: SkillSHEET: Adding periods of time (doc-10853)

Exercise 11.2 Salaries and wages

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 11, 13	Questions: 1–6, 7, 9, 10, 12, 14, 15, 17	Questions: 1–6, 9, 12–20

Individual pathway interactivity: int-4520

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Fluency

1. WE1 Johann has an annual salary of $57 482. How much is he paid:
a. weekly **b.** fortnightly **c.** monthly?

2. Anna earns $62 300 per annum. How much does she earn:
a. weekly **b.** fortnightly **c.** monthly?

3. Find the annual salary of workers with the following weekly incomes.
a. $368 **b.** $892.50 **c.** $1320.85

4. How much is earned per annum by those paid fortnightly salaries of:
a. $995 **b.** $1622.46 **c.** $3865.31?

5. Which of each pair is on the higher salary?
a. $3890 per month or $45 700 per annum **b.** $3200.58 per fortnight or $6700 per month

6. Find the hourly rate for these workers.
a. Rahni earns $98.75 for 5 hours.
b. Francisco is paid $54.75 for $4\frac{1}{2}$ hours.
c. Nhan earns $977.74 for a 38-hour week.
d. Jessica works $7\frac{1}{2}$ hours a day for 5 days to earn $1464.75.



Understanding

7. Henry is a second-year apprentice motor mechanic. He receives the award wage of $12.08 per hour. Jenny, a fourth-year apprentice, earns $17.65 per hour.
a. How much does Henry earn in a 38-hour week?
b. How much more does Jenny earn in the same period of time?

8. WE2 Juan has casual work for which he is paid $13.17 per hour Monday to Saturday and $26.34 per hour on Sundays. Calculate his total pay for a week in which he worked from 11 am to 5 pm on Thursday and from 2.00 pm to 7.00 pm on Sunday.

9. Mimi worked the following hours in one week.

Wednesday	5.00 pm to 9.00 pm
Thursday	6.00 pm to 9.00 pm
Friday	7.00 pm to 11.00 pm

If her pay is $21.79 per hour up to 9.00 pm and $32.69 per hour after that, what is her total pay?

10. Who earns more money each week: Rhonda, who receives $38.55 an hour for 38 hours work, or Rob, who receives $41.87 an hour for 36 hours work?

11. Glenn is a chef and receives $1076.92 for a week in which he works 35 hours. What is his hourly rate of pay?

12. Julie is considering two job offers for work as a receptionist. Job A pays $878.56 for a 38-hour working week. Job B pays $812.16 for a 36-hour working week. Which job has the higher hourly rate of pay?



13. Russell and Gabrielle go to work in different department stores. Russell is paid $981.77 per week. Gabrielle is paid $26.36 per hour. How many hours must Gabrielle work to earn more money than Russell?

14. Calculate what pay each of the following salary earners will receive for each of the periods specified.

a. Annual salary $83 500, paid each week

b. Annual salary $72 509, paid each fortnight

c. Annual salary $57 200, paid each week

d. Annual salary $105 240, paid each month

Reasoning

15. MC When Jack was successful in getting a job as a trainee journalist, he was offered the following choice of four salary packages. Which should Jack choose? Show your working.

A. $456 per week

B. $915 per fortnight

C. $1980 per calendar month

D. $23 700 per year

16. In his job as a bookkeeper, Minh works 38 hours per week and is paid $32.26 per hour. Michelle, who works 38 hours per week in a similar job, is paid a salary of $55 280 per year. Who has the higher paying job? Show your working.

Problem solving

17. A lawyer is offered a job with a salary of $74 000 per year, or $40 per hour. Assuming that they work 80 hours every fortnight, which is the greater pay?

18. Over the last four weeks, a woman has worked 35, 36, 34 and 41 hours. If she earns $24.45 per hour, how much did she earn for each of the two fortnights?

19. An employee brags that he works a 40-hour week (8 hours a day, Monday–Friday) and earns $62 000 p.a.

a. What is this as an hourly rate?

b. If the employee works on average an extra half an hour a day Monday–Friday and then another 4 hours over the weekend (for the same annual salary), how is his hourly rate affected?

c. If the employee was earning the hourly rate for which he bragged about and was being paid for every hour worked, what would be his potential earnings for the year?

Reflection

What would be your preferred method of being paid and why?

CHALLENGE 11.1

Mark saves $10 per week. Phil saves 5 cents in the first week, 10 cents the second week and doubles the amount each week? How many weeks will it take for Phil to have more savings than Mark?

11.3 Special rates

11.3.1 Special rates

- A normal Australian working week is 38 hours. Wage earners who work extra hours are 'working overtime'.
- **Overtime** is paid when a wage earner works more than the regular hours each week. When an employee works overtime a higher rate is paid. This higher rate of pay is called a penalty rate. The rate is normally calculated at either time and a half, which means that the person is paid $1\frac{1}{2}$ times the normal rate of pay, or double time, which means that the person is paid twice the normal rate of pay, or double time and a half, which means that the person is paid $2\frac{1}{2}$ times the normal rate of pay.
- A person may also be paid these overtime rates for working at unfavourable times, such as at night or during weekends.
- To calculate the hourly rate earned when working overtime, we multiply the normal hourly rate by the overtime factor, which is $1\frac{1}{2}$ for overtime, 2 for double time and $2\frac{1}{2}$ for double time and a half.

WORKED EXAMPLE 3

Ursula works as a waitress and earns $23.30 per hour. Last week she received the normal rate for 30 hours of work as well as time and a half for 3 hours of overtime and double time for 5 hours of work on Sunday. What was her total wage?



THINK	WRITE
1 Calculate Ursula's normal pay.	Normal pay: $30 \times 23.30 = 699.00$
2 Calculate Ursula's pay for 3 hours at time and a half.	Overtime: $3 \times 1.5 \times 23.30 = 104.85$
3 Calculate Ursula's pay for 5 hours at double time.	Sunday: $5 \times 2 \times 23.30 = 233.00$
4 Find the total amount.	Total = 1036.85
5 Write the answer in a sentence.	Ursula's total wage was $1036.85.

11.3.2 Time sheets and pay slips

- Employers often use records called *time sheets* to monitor the number of hours worked by each employee.
- Details of the hours worked and the rate of pay are given to each employee on a pay slip, which they receive with their wages.

WORKED EXAMPLE 4

Fiona works in a department store, and in the week before Christmas she works overtime. Her time sheet is shown below. Fill in the details on her pay slip.

	Start	Finish	Normal hours	O'time 1.5
M	9.00	15.00	6	
T	9.00	17.00	8	
W	9.00	17.00	8	
T	9.00	19.00	8	2
F	9.00	19.00	8	2
S				

Pay slip for: Fiona BLACK	Week ending December 21
Normal hours	
Normal rate	$17.95
Overtime hours	
Overtime rate	
Total wage	

THINK

1. Calculate the number of normal hours worked.
2. Calculate the number of overtime hours worked.
3. Calculate the overtime rate.
4. Calculate the total pay by multiplying the number of normal hours by the normal rate and adding the overtime amount, calculated by multiplying the number of overtime hours by the overtime rate.
5. Fill in the amounts on the pay slip.

WRITE

Normal hours: $6 + 8 + 8 + 8 + 8 = 38$

Overtime hours: $2 + 2 = 4$

Overtime rate $= 1.5 \times 17.95$
$= 26.93$

Total pay $= 38 \times 17.95 + 4 \times 26.93$
$= 789.82$

Pay slip for: Fiona BLACK	Week ending December 21
Normal hours	38
Normal rate	$17.95
Overtime hours	4
Overtime rate	$26.93
Total wage	$789.82

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Complete this digital doc: SkillSHEET: Multiplying a quantity (money) by a decimal (doc-10854)

Exercise 11.3 Special rates

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 6, 9, 10, 13–15	Questions: 1–5, 7, 9, 10, 13–16, 18	Questions: 1–5, 8, 9, 11–21

Individual pathway interactivity: int-4521

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Fluency

1. Calculate the following special rates:
- **a.** time-and-a-half when the hourly rate is $15.96
- **b.** double time when the hourly rate is $23.90
- **c.** double time-and-a-half when the hourly rate is $17.40.

2. Calculate the following total weekly wages:
- **a.** 38 hours at $22.10 per hour, plus 2 hours at time-and-a-half
- **b.** 40 hours at $17.85 per hour, plus 3 hours at time-and-a-half
- **c.** 37 hours at $18.32 per hour, plus 3 hours at time-and-a-half and 2 hours at double time.

3. Julio is paid $956.08 for a regular 38-hour week.
- **a.** What is his hourly rate of pay?
- **b.** How much is he paid for 3 hours of overtime at time-and-a-half rates?
- **c.** What is his wage for a week in which he works 41 hours?

4. WE3 Geoff is a waiter in a restaurant and works 8 hours most days. Calculate what he earns for 8 hours work on the following days:
- **a.** a Monday, when he receives his standard rate of $21.30 per hour
- **b.** a Sunday, when he is paid double time
- **c.** a public holiday, when he is paid double time-and-a-half.

5. Albert is paid $870.58 for a 38-hour week. What was his total wage for a week in which he worked 5 extra hours on a public holiday with a double-time-and-a-half penalty rate?

Understanding

6. Jeleesa (aged 16) works at a supermarket on Thursday nights and weekends. The award rate for a 16-year-old is $7.55 per hour. Calculate what she would earn for:
- **a.** 4 hours work on Thursday night
- **b.** 6 hours work on Saturday
- **c.** 4 hours work on Sunday at double time
- **d.** the total of the three days.



7. Jacob works in a pizza shop and is paid $13.17 per hour.
- **a.** Jacob is paid double time-and-a-half for public holiday work. What does he earn per hour on public holidays? (Answer to the nearest cent.)
- **b.** What is Jacob's pay for a public holiday where he works 6 hours?

8. If Bronte earns $7.80 on normal time, how much does she receive per hour:
- **a.** at time and a half
- **b.** at double time
- **c.** at double time and a half?

9. Copy and complete the following time sheet. Calculate the number of hours Susan worked this week.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	9.00 am	5.00 pm	
Tuesday	Normal	9.00 am	5.00 pm	
Wednesday	Normal	9.00 am	5.00 pm	
Thursday	Normal	9.00 am	5.00 pm	
Friday	Normal	9.00 am	3.00 pm	

10. WE4 Copy and complete Susan's pay slip for this week.

Pay slip for: Susan WHITE	Week ending 17 August
Normal hours	
Normal pay rate	$25.60
Overtime hours	0
Overtime pay rate	$38.40
Total pay	

11. Below is a time sheet for Jason, who works in a department store. Copy and complete the table.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	9.00 am	5.00 pm	
Tuesday	Normal	9.00 am	5.00 pm	
Wednesday	Normal	—		
Thursday	Normal	1.00 pm	9.00 pm	
Friday	Normal	—		
Saturday	Time and a half	8.00 am	12.00 pm	

12. Copy and complete the pay slip for Jason for the week described in question **11**.

Pay slip for: Jason RUDD	Week ending 21 December
Normal hours	
Normal pay rate	$10.90
Overtime hours	
Overtime pay rate	
Total pay	

13. Brett does shift work. Copy and complete his time sheet.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	7.00 am	3.00 pm	
Tuesday	Normal	7.00 am	3.00 pm	
Wednesday				
Thursday				
Friday	Normal	11.00 pm	7.00 am	
Saturday	Time and a half	11.00 pm	7.00 am	
Sunday	Double time	11.00 pm	7.00 am	

14. Copy and complete Brett's pay slip for the week.

Pay slip for: Brett SIMPSON	**Week ending 15 September**
Normal hours	
Normal pay rate	$16.80
Time-and-a-half hours	
Time-and-a-half pay rate	
Double time hours	
Double time pay rate	
Gross pay	

Reasoning

15. Calculate the following total weekly wages:

a. 38 hours at $18.40 per hour, plus 2 hours at time and a half

b. 32 hours at $23.70 per hour plus 6 hours on a Sunday at double time

c. 38 hours at $26.42 per hour, plus 2 hours overtime and 4 hours on a public holiday that incurred the maximum penalty rate.

16. Ruby earns $979.64 for her normal 38-hour week, but last week she also worked 6 hours overtime at time-and-a-half rates.

a. Calculate how much extra she earned and give a possible reason for her getting time-and-a-half rates.

b. What was Ruby's total wage?

17. MC A standard working week is 38 hours and a worker puts in 3 hours overtime at time-and-a-half and 2 hours at double time. To how many hours at the standard rate is her total work time equivalent?

A. 43 **B.** $46\frac{1}{2}$ **C.** $44\frac{1}{2}$ **D.** 45

Problem solving

18. Glen works 32 hours per week at $22/h and is paid overtime for any time worked over the 32 hours per week. In one week Glen worked 42 hours and was paid $814. Overtime is paid at 1.5 times the standard wage. Was Glen paid the correct amount? (yes or no). If no, then provide the correct amount.

19. Joshua's basic wage is $22 per hour. His overtime during the week is paid at time and a half. Over the weekend he is paid double time. Calculate his gross wage in a week when he works his basic 40 hours, together with 1 hour overtime on Monday, 2 hours overtime on Wednesday and 4 hours overtime on Saturday.

20. The table below shows the pay sheet for a small company. If a person works up to 36 hours, the regular pay is $14.50 per hour.
For hours over 36 and up to 40, the overtime is time and a half.
For hours over 40, the overtime is double time.
Complete the table below.

	Hours worked	**Regular pay**	**Overtime pay**	**Total pay**
a	32			
b	38.5			
c	40.5			
d	47.2			

21. Vicki is a supervisor at a local factory. Each fortnight she calculates the wages of the employees. Overtime is paid to any employee who works more than 35 hours each week. The overtime rate is $1\frac{1}{2}$ times the hourly rate. The table below shows the number of hours worked and the hourly rates for three employees for one fortnight.

Employee	Hours worked	Hourly rate
Stewart	72	$12.75
Helen	56	$19.80
Amber	x	$21.50

a. Determine the total amount, in dollars, in wages for Helen and Stewart. Write your answer to the nearest cent.

b. Amber worked for x hours including some overtime. Her fortnightly wage was $1988.75.

i. Determine the number of hours she worked.

ii. Was it possible for Amber to earn this amount if she did not do any overtime?

c. Tax is charged at 45 cents in each dollar earned. Determine the amount of tax, in dollars, Amber pays for the fortnight. Write your answer correct to the nearest cent.

Reflection

In what situations would being paid according to time sheets be preferable to receiving a wage or salary?

11.4 Piecework

11.4.1 Piecework

- **Piecework** is a system of payment by which a worker is paid a fixed amount for each job or task they complete.

WORKED EXAMPLE 5

Mitchell has a job washing cars in a car yard. He is paid $5.20 per car washed. Calculate the amount Mitchell earns in an afternoon when he washes 24 cars.

THINK	WRITE
1 Multiply the number of cars Mitchell washes by the amount paid for each car.	Amount earned $= 24 \times 5.20$ $= 124.80$
2 Write the answer in a sentence.	Mitchell earns $124.80.

- A person may also be paid on a sliding scale where the pay rate increases as the number of completed tasks increases.

WORKED EXAMPLE 6

Angelica is a machinist in a clothing factory. Each week she is paid $4.28 per garment for the first 180 garments, and $5.35 per garment thereafter. What will she be paid if she produces 223 garments?



THINK	WRITE
1 Calculate the number of 'extra' garments Angelica makes.	Extra garments $= 223 - 180$ $= 43$
2 Calculate her total payment by adding the payment she receives for the first 180 garments to the payment she receives for the extra garments.	Payment $= 180 \times 4.28 + 43 \times 5.35$ $= 1000.45$
3 Write the answer in a sentence.	Angelica earns a total payment of $1000.45.

- In some cases, piecework is paid for multiple rather than single units. For example, for letterbox deliveries you may be paid per 1000 deliveries made.

WORKED EXAMPLE 7

Holly is delivering brochures to letterboxes in her local area. She is paid $43.00 per 1000 brochures delivered. Calculate the amount Holly will earn for a delivery of 3500 brochures.

THINK	WRITE
1 Calculate the number of thousands of brochures Holly will deliver.	$3500 \div 1000 = 3.5$ So Holly will deliver 3.5 thousand brochures.
2 Multiply the number of thousands of brochures delivered by 43 to calculate what Holly will earn.	Holly's pay $= 3.5 \times 43.00$ $= 150.50$
3 Write the answer in a sentence.	Holly will earn $150.50.

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Exercise 11.4 Piecework

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7	Questions: 1–5, 6, 7, 9	Questions: 1, 2, 4, 6–11

Individual pathway interactivity: int-4522

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Fluency

1. WE5 Hitani is paid 65 cents for each teacup she decorates. How much is she paid for decorating 150 teacups?

2. WE6 Jack makes leather belts. The piece rate is $1.25 each for the first 50 belts and $1.50 thereafter. What is his income for a day in which he produces 68 belts?

3. A production-line worker is paid $1.50 for each of the first 75 toasters assembled, then $1.80 per toaster thereafter. How much does she earn on a day in which she assembles 110 toasters?

4. WE7 Rudolf earns $42.50 per 1000 leaflets delivered to letterboxes. Calculate what Rudolf will earn for a week in which he delivers 7500 leaflets.

Understanding

5. Dimitri earns $7.20 for each box of fruit picked.

a. How much does he make for picking 20 boxes?

b. How many boxes must he pick to earn at least $200?

c. If he takes 4 hours to pick 12 boxes, what is his hourly rate of pay?

6. Pauline uses her home computer for word processing under contract to an agency. She is paid $3 per page for the first 50 pages, $4 per page from 51 to 100 pages, and $5 per page thereafter. Calculate her total pay for a period in which she prepares:

a. 48 pages **b.** 67 pages **c.** 123 pages.

Reasoning

7. Rani delivers bills to letterboxes and is paid $43 per thousand.

a. How much does she earn for delivering 2500 items?

b. How many thousand must she deliver to earn $1000?

c. If she takes 6 hours to deliver each thousand on average, what is her hourly rate of pay?



8. Georgio delivers pizzas. He is paid $3 per delivery from 5 pm to 9 pm and $4 per delivery after 9 pm.

a. How much does he earn on a night in which he makes 12 deliveries by 9 pm and 4 deliveries between 9 pm and 10.30 pm?

b. What are his average earnings per hour if he has worked from 5 pm to 10.30 pm?

Problem solving

9. A shoemaker is paid $5.95 for each pair of running shoes he can make.

a. If the shoemaker made 235 pairs of shoes last week, what was the amount paid?

b. The shoemaker is offered a bonus of 5% if he can make more than 250 pairs of shoes in a week. If he makes 251 pairs, what is the total amount earned, including the bonus?

10. A secretarial assistant gets paid $12 per page that she types. If she manages to type more than 20 pages in a day, she gets a 10% bonus. If a typist typed 32 pages on Tuesday, how much did she earn?

11. There are both fixed and variable costs associated with some products. Consider the cost of importing a radio from China and selling it in Australia. The costs are:

- import of product $12.50 per unit
- transportation costs $400 per 1000 units
- warehouse rental space $1 per unit per month
- advertising costs $2000 per month (fixed cost).

a. If this company imports and sells 500 units per month, what is the total cost per month?

b. At 500 units per month and a selling price of $25.00, what is the total profit per month?

Reflection

What are the advantages and disadvantages of being paid by piecework?

11.5 Commission and royalties

11.5.1 Commission and royalties

- **Commission** is a method of payment used mainly for salespeople. The commission paid is usually calculated as a percentage of the value of goods sold.
- A **royalty** is a payment made to a person who owns a copyright. For example, a musician who writes a piece of music is paid a royalty on CD and online sales. An author who writes a book is also paid a royalty based on the number of books sold. Royalties are calculated as a percentage of sales.

WORKED EXAMPLE 8

Mohamad is a songwriter who is paid a royalty of 12% on all sales of his music. Calculate the royalty that Mohamad earns if a song he writes sells CDs to the value of $150 000.



THINK	WRITE
1 Find the royalty by calculating 12% of $150 000.	Royalty = 12% of 150 000 = 0.12 × 150 000 = 18 000
2 Write the answer in a sentence.	Mohamad earns $18 000 in royalties.

- Sometimes a salesman is paid a small wage, called a retainer, plus a percentage of the value of the goods sold.

WORKED EXAMPLE 9

Gemma, a car salesperson is paid a retainer of $350 per week, plus a commission of 8% of the profits made by the company on cars that she sells.
a How much does Gemma earn in a week when no sales are made?
b How much does she earn in a week when $5000 profit was generated by her sales?

THINK	WRITE
a If no sales are made, only the retainer is paid.	**a** Gemma earns $350.
b 1 Find the commission paid by calculating 8% of $5000.	**b** Commission = 8% of 5000 = 0.08 × 5000 = $400
2 Find the total amount paid by adding the retainer and the commission.	Total earnings = 350 + 400 = $750
3 Write the answer in a sentence.	Gemma earns $750.

- Sometimes the commission is broken into several parts with differing rates.

WORKED EXAMPLE 10

A real estate agency receives 2% commission on the first $300 000 of a sale and 3% on the remainder. How much commission is received on the sale of a $380 000 property?

THINK	WRITE
1 Calculate the difference between $380 000 and $300 000.	380 000 − 300 000 = 80 000
2 Calculate 2% of $300 000.	2% of 300 000 = 6000
3 Calculate 3% of $80 000.	3% of 80 000 = 2400
4 Calculate the total commission by adding the commission earned on $300 000 and the commission earned on $80 000.	6000 + 2400 = 8400
5 Write the answer in a sentence.	The commission received is $8400.

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Complete this digital doc: SkillSHEET: Converting a percentage into a decimal (doc-10856)
Complete this digital doc: SkillSHEET: Finding a percentage of a quantity (money) (doc-10857)
Complete this digital doc: Spreadsheet: Converting percentages to fractions or decimals (doc-10905)
Complete this digital doc: Spreadsheet: Finding a percentage of an amount (doc-10906)

Exercise 11.5 Commission and royalties

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7, 11	Questions: 1–4, 6, 7, 10, 11, 13	Questions: 1, 3, 4, 5c, 6d, 7, 8, 9b, 10b, 11–16

Individual pathway interactivity: int-4523

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Fluency

1. WE8 Danyang is a writer who is paid a royalty of 10% on all sales. Calculate the royalty she earns in a year if a book she writes sells copies to the value of $30 000.

2. A home-improvements company pays commission at the rate of 16% on all sales. What would a person earn who had sales to the value of:
a. $8000 **b.** $6972.50?

3. Linda is a car salesperson who is paid a 1.5% commission on her sales. Calculate the amount of money Linda earns in a week where her sales total $95 000.

4. WE9 Gordon is paid a retainer of $200 per week plus a commission of 6% of the profits made by the company on the goods that he sells.
a. How much does Gordon earn in a week when no sales are made?
b. How much does Gordon earn in a week during which a $70000 profit was generated by his sales?

5. Alfonso gets a retainer of $235 per week plus a commission of $5\frac{1}{2}$% on sales. What are his total earnings in a week in which his sales are:
a. $1000 **b.** $4500 **c.** $17384?

Understanding



6. Bryce is an author. His publisher pays him a fixed allowance of $500 per month, plus $4\frac{1}{2}$% royalty on sales. What would be his income for a month in which his book sales totalled:
a. $0 **b.** $2000
c. $15000 **d.** $23750?

7. WE10 A real estate agency receives 2% commission on the first $250000 of a sale and 4% on the rest. How much commission is received on the sale of a $370000 property?

8. At a second real estate agency, the commission rate is 5% on the first $180000 of sale price and 2% on the remainder. Find the commission on the sale of the $370000 property.

9. Ingrid's real estate agency pays her 1% commission on the first $500000 of sale price, then 4% thereafter. How much commission would she receive on the sale of a property worth:
a. $480000 **b.** $510000 **c.** $735000?

10. Yanu works for a boat broker who pays him 6% of the first $50000 of the sale price, then $3\frac{3}{4}$% on the rest. Calculate the commission he receives on the following sales.
a. $40000 **b.** $70000 **c.** $395000

Reasoning

11. Veronica earns $400 per week plus 4% on sales, whereas Francis earns 6% commission only.
a. How much does each earn on sales of $8400?
b. What level of sales would yield each the same income?

12. Wolfgang, a car salesman, is paid a weekly retainer of $550, plus 10% of the dealer's profit on each vehicle. Find his total income for weeks in which the dealer's profits on vehicles he sold were:
a. $3500 **b.** $5980 **c.** $7036.00

13. Using the commission table for house sales below, calculate the commission on each of the following sales.

Sale price	Commission	Plus
Between $0 and $80000	2% of sale price	0
Between $80001 and $140000	1.5% of amount over $80000	$1600 (2% of $80000)
$140000 and over	1.1% of amount over $140000	$2500 (2% of $80000 + 1.5% of $60000)

a. $76000 **b.** $122500 **c.** $145000 **d.** $600000

Problem solving

14. Mr Hartney is a used car salesman. He receives a basic monthly salary of $2400 together with 5% commission on all sales. Although his sales for the month amounted to $48300 he also had deductions for insurance ($12.80), association fees ($25.70) and income tax ($1100). Calculate the amount, in dollars, he took home that month.

15. A rock musician makes a royalty on all record sales according to the following formula.

Sales from	Sales to	Royalty rate
0	$100 000	3%
$100 001	$500 000	3.5% on amount over $100 000
$500 001	1 million	4% on amount over $500 000
1 million	and above	5% on amount over 1 million

Calculate the royalties for the following years:

a. 2007 — sales = $456 000
b. 2008 — sales = $1 234 500
c. 2009 — sales = $986 400
d. 2010 — sales = $2 656 000.

16. Four years ago Inka became an employee of TrakRight Tourism where her starting annual salary was $55 600. After her first year, she received a 2% pay rise. The next year she received a 3% pay rise. Last year she received an x% pay rise. If her annual salary is now $61 042, determine the value of x, correct to one decimal place.

Reflection

What are the major advantages and disadvantages of each method of getting paid?

11.6 Loadings and bonuses

11.6.1 Loadings

- If a wage or salary earner has to work in difficult or hazardous conditions, then the worker may be granted an extra payment or **loading**.
- Most workers are granted a 'holiday loading'. For a 4-week period each year they are paid an extra 17.5% of their usual wage.



WORKED EXAMPLE 11

Rohan works as an electrician and receives $38.20 per hour for a 36-hour working week. If Rohan works 'at heights' he receives $2.50 per hour height loading. Calculate Rohan's wage in a week where he works 15 hours at heights.

THINK	WRITE
1 Calculate Rohan's normal weekly wage.	Normal wage $= 36 \times 38.20$ $= \$1375.20$
2 Calculate Rohan's loading for the time he worked at heights.	Loading $= 15 \times 2.5$ $= \$37.50$
3 Calculate Rohan's total wage.	Total wage $= 1375.20 + 37.50$ $= \$1412.70$

WORKED EXAMPLE 12

Jelena works as a hairdresser and is paid a normal rate of $19.70 per hour for a 38-hour working week.

a Calculate Jelena's normal weekly wage.

b For her 4 weeks annual leave, Jelena is paid a loading of 17.5%. Calculate the amount that Jelena receives in holiday loading.

c Calculate the total amount that Jelena receives for her 4 weeks annual leave.

THINK	WRITE
a Calculate Jelena's normal wage by multiplying the hours worked by the hourly rate.	**a** Normal wage $= 38 \times 19.70$ $= \$748.60$
b 1 Find 17.5% of Jelena's normal wage.	**b** 17.5% of $\$748.60 = \$131.01$
2 Multiply this amount by 4 to find the holiday loading.	Holiday loading $= 4 \times 131.01$ $= \$524.04$
c Find the total amount received by multiplying Jelena's normal weekly pay by 4 and adding the holiday loading.	**c** Holiday pay $= 4 \times 748.60 + 524.04$ $= \$3518.44$

11.6.2 Bonuses

- Many people who are employed in managerial positions receive a bonus if the company achieves certain performance targets. The bonus may be a percentage of their annual salary or a percentage of the company's profits.

WORKED EXAMPLE 13

Brooke is the Chief Executive Officer of a fashion company on a salary of $240 000 per year. Brooke will receive a bonus of 1% of her salary for every percentage point that she increases the company profit. If the company profit grows from $3.1 million to $4.4 million in one year, calculate the amount of Brooke's bonus.

THINK	WRITE
1 Calculate the increase in profit.	Increase in profit $= \$4.4\text{ m} - \$3.1\text{ m}$ $= \$1.3\text{ m}$
2 Express the increase in profit as a percentage.	Percentage increase $= \frac{1.3}{3.1} \times 100\%$ $= 41.9\%$
3 Calculate this percentage of Brooke's annual salary.	Bonus $= 41.9\%$ of $\$240\,000$ $= \$100\,560$
4 Write the answer in a sentence.	Brooke's bonus is $\$100\,560$.

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Complete this digital doc: SkillSHEET: Expressing one quantity as a percentage of another (doc-10858)

Exercise 11.6 Loadings and bonuses

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 3, 4, 7, 9, 10	Questions: 1, 2, 4, 5, 8–13	Questions: 1, 2, 4, 6–15

Individual pathway interactivity: int-4524

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Fluency

1. WE11 Rashid works as an electrician and receives $35.40 per hour for a 35-hour working week. If Rashid works at heights he receives a height loading of $0.32 per hour. Calculate Rashid's wage in a week where he works 18 hours at heights.

2. Patrick is a railway linesman. If he works in wet weather he is paid a loading of 43 cents per hour. If he normally works a 38-hour working week at $21.02 per hour and 16 hours are spent working in wet weather, find Patrick's pay for the week.



3. Saci is an industrial cleaner and is paid at the rate of $19.82 per hour. If Saci works in a confined space, she is paid a loading of $0.58 per hour. Calculate Saci's pay for a week in which she works 38 hours where 19 of those hours were in a confined space.

4. WE12 Jordan works as the manager of a supermarket and is paid a normal rate of $37.60 per hour for a 38-hour working week.
- **a.** Calculate Jordan's normal weekly wage.
- **b.** For her 4 weeks annual leave, Jordan is paid a loading of 17.5%. Calculate the amount that Jordan receives in holiday loading.
- **c.** Calculate the total amount that Jordan receives for her 4 weeks annual leave.

5. Charlie earns $22.80 per hour for a 38-hour week.
- **a.** Calculate the amount Charlie will earn in a normal working week.
- **b.** Calculate the total amount Charlie will receive for his 4 weeks annual leave if he receives a 17.5% holiday loading.

6. Liam is paid $15.95 per hour for a 36-hour working week.
- **a.** Calculate Liam's weekly wage.
- **b.** Liam takes one week's holiday for which he is given a 17.5% loading. Calculate the holiday loading.

7. Karen receives an annual salary of $63 212.
- **a.** What is her fortnightly pay?
- **b.** What is she paid for her annual 4-week holiday, for which she receives an extra 17.5% loading?

Understanding



8. Brian earns $956.46 for a standard 38-hour week and a $27.53 per week allowance for working on scaffolding. Calculate his total pay for a week in which he works on scaffolding and does 4 hours overtime at time-and-a-half.

9. WE13 Eric is a director of a mining company on a salary of $380 000 per year. Eric is told that at the end of the year he will receive a bonus of 1% of his salary for every percentage point that he increases the company profit. If the company profit grows from $4.9 m to $6.4 m in one year, calculate the amount of Eric's bonus.

Reasoning

10. Sally is the manager of a small bakery that employs 12 people. As an incentive to her workers she agrees to pay 15% of the business's profits in Christmas bonuses for her employees. The business makes a profit of $400 000 during the year.

a. Find the total amount that Sally pays in bonuses.

b. If the bonus is shared equally what amount does each employee receive as a Christmas bonus?

c. If one employee earns $42 000 per year, calculate the Christmas bonus as a percentage of annual earnings, correct to 2 decimal places. Explain your answer.

11. Shane, the director of an exercise company, earns a salary of $275 000 a year. Shane gets paid incentives if he is able to increase the company's profit. He gets:

- 5% if he increases the profit by 0.1–10%.
- 7.5% if he increases the profit by 10.1–20%.
- 10% if he increases the profit by more than 20%.

If the company's profit grows from $1.2 million to $1.4 million in a year:

a. explain what percentage incentive Shane will get and why

b. calculate his salary for the year.

Problem solving

12. Kevin owns a sports store and has 7 staff working for him. He offers each of them a 5.5% end-of-year bonus on any profits over $100 000. This year the store made a profit of $275 000.

a. Find the amount each employee earned in bonuses.

b. What is the cost to Kevin in total bonuses for the year?

c. If one employee earned $64 625 including bonuses for the year, what was their base salary p.a.?

13. Jimmy is a high-rise window cleaner. He gets paid $15 per window for the first five levels. For the next 15 levels he gets an extra 15% per window, and above this he gets 20% extra as danger money. How much does Jimmy earn for cleaning:

a. a total of 20 windows on levels 3 to 4

b. 10 windows on levels 4 and a total of 27 windows on levels 10 to 13

c. a total of 30 windows on levels 11 to 14 and a total of 30 windows on levels 21 to 25?



14. Denise works for a real estate agent. She receives a basic wage of $250 per week plus commission on sales. The rate of commission is variable. For houses up to $300 000, the commission is 0.5%. For houses over $300 000, the commission is an additional 0.25% on the amount over $300 000. How much pay did she receive in the week she sold a house for:
a. $280 000 **b.** $428 000?

15. When Jack goes on holidays, he is paid $17\frac{1}{2}$% holiday loading in addition to his normal pay. When he went on 2 weeks' leave, his holiday pay was $1504. What is his normal weekly pay?

Reflection

How are bonuses used to encourage workers?

11.7 Taxation and net earnings

11.7.1 Income tax

- In Australia, people who earn more than $18 200 in a financial year must pay a percentage of their earnings as **income tax**.
- The rates of taxation for Australian residents for 2014–15 are shown in the table below.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000



The above rates do not include the Medicare levy of 2.0%.

WORKED EXAMPLE 14

Find the amount of tax paid on an annual income of:
a $22 000 **b $92 000.**

THINK	WRITE
a 1 $22 000 is in the $18 201 to $37 000 bracket.	**a**
2 The tax payable is 19c (0.19) for every dollar over $18 200.	
3 Calculate the amount over $18 200 by subtracting $18 200 from $22 000.	$22 000 − $18 200 = $3800
4 Apply the rule '19c for every dollar over $18 200'.	Tax payable = 0.19 × 3800 = 722
5 Write the answer in a sentence.	The tax payable on $22 000 is $722.
b 1 $92 000 is in the $80 001 to $180 000 bracket.	**b**
2 Calculate the amount over $80 000 by subtracting $80 001 from $92 000.	$92 000 − $$80 000 = $12 000

3 Apply the rule '$17547 plus 37c for each $1 over $80000'.

$$\text{Income tax} = 17547 + 0.37 \times 12000$$
$$= 21987$$

4 Write the answer in a sentence.

The tax payable on $92000 is $21987.

11.7.2 Medicare levy

- Medicare is the scheme that gives Australian residents access to health care.
- Most taxpayers pay 2.0% of their taxable income to pay for this scheme. This is called the Medicare levy.
- People who have private medical insurance can reclaim some of this money.

11.7.3 Pay As You Go (PAYG) taxation

- When you receive a pay cheque, some of the money has been taken out by the employer to cover your income tax and Medicare levy. This is called 'pay as you go' (PAYG) taxation.
- The initial amount, before tax is taken out, is called your **gross salary** and the amount that you actually receive is called your **net salary**.
- The amount of money to be deducted by the employer each week is published by the Australian Tax Office, as shown in the following table.

PAYG TABLE: Weekly tax withheld ($)					
Gross wage	**With tax-free threshold**	**Gross wage**	**With tax-free threshold**	**Gross wage**	**With tax-free threshold**
450	24	950	165	1450	339
500	38	1000	183	1500	356
550	48	1050	200	1550	374
600	59	1100	217	1600	393
650	69	1150	235	1650	413
700	80	1200	252	1700	432
750	96	1250	270	1750	452
800	113	1300	287	1800	471
850	130	1350	304	1850	491
900	148	1400	321	1900	510

Note: Most Australian citizens qualify for the tax-free threshold. For the purposes of this section, apply the tax-free threshold values.

11.7.4 Deductions

- Often other sums of money, such as union fees and private health insurance, are deducted from gross pay.



11.7.5 Family Tax Benefit

- When a family has young or dependent children, the government may pay an allowance called the 'Family Tax Benefit', which is added to a person's gross salary.

WORKED EXAMPLE 15

Fiona has a gross wage of $900 per week.

a Use the PAYG table to find the amount of tax that should be deducted.

b What percentage of her gross pay is deducted?

c If Fiona receives $98 in family allowance but has deductions of $71 (superannuation) and $5.50 (union fee), what is her net pay?



THINK	WRITE
a From the table, PAYG tax payable on a gross wage of $900 per week is $148.	**a** $148
b Find 148 as a percentage of 900.	**b** $\frac{148}{900} \times 100 = 16.44\%$ deducted
c 1 Fiona receives $98 in family allowance. Add this to her gross weekly wage to find her total income.	**c** Total income $= 900 + 98$ $= \$998$
2 Calculate her total deductions.	Total deductions $= 148 + 71 + 5.50$ $= \$224.50$
3 Calculate her net pay by subtracting her total deductions from her total income.	Net pay $= 998 - 224.50$ $= \$773.50$

Exercise 11.7 Taxation and net earnings

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10	Questions: 1–11	Questions: 1–11

Individual pathway interactivity: int-4525

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Fluency

1. WE14 Find the amount of tax paid on an annual income of:

a. $15 000 **b.** $22 000 **c.** $44 000 **d.** $88 000.

2. WE15 In the PAYG a tax table, look up the amount of tax that must be deducted from the following weekly earnings and find this as a percentage of the gross pay.

a. $650 **b.** $1100 **c.** $1550

3. For each of the following, calculate the net pay.
- **a.** Gross pay $450.00, tax $24.00 and union fees $4.75
- **b.** Gross pay $550.00, tax $48.00, private health insurance $25.85 and superannuation $53.80
- **c.** Gross pay $850.00, tax $130.00, loan repayment $160.00 and insurance payment $45.40

4. Calculate the net annual salary of a person who has a gross annual salary of $57 200 with deductions of $17 264 for tax, annual union fees of $262.75, social club payments of $104.00, and a family allowance of $4392.20.

5. Sergio works as a security guard and receives gross pay of $950.00 each week. His tax totals $165 per week. If his other deductions are $60.10 for superannuation and $5.05 for union fees, what is his net pay?

Understanding

6. Lieng works as an interior decorator and earns $1350 per week.
- **a.** How much tax should be deducted from her pay each week?
- **b.** What percentage of her gross pay is her tax?
- **c.** If Lieng also has deductions of $105 for superannuation, $5.20 for union fees, and $4.00 for a social club, what is her net weekly pay?

Reasoning

7. Yelena works as a chef and is paid $22.86 per hour and works a 35-hour week.
- **a.** Calculate Yelena's gross weekly earnings.
- **b.** How much tax should be deducted from Yelena's pay?
- **c.** What percentage of her gross pay (correct to 2 decimal places) is deducted in tax?
- **d.** If Yelena also has deductions of $56.20 for superannuation and $22.50 for her health insurance, and gets $60.00 taken out to pay off her car loan, what is her net pay?
- **e.** What percentage of her gross pay is her net pay? Give your answer correct to 2 decimal places.



8. Debbie earns $72 000 per year.
- **a.** Explain why she takes home only $57 053.
- **b.** Give reasons why this figure could possibly be different again.

Problem solving

9. Jacko works at an IT firm and earns $1725 a week.
- **a.** How much does he earn a year, gross?
- **b.** How much tax will he need to pay per year?
- **c.** If he has no deductions, how much will he need to pay for the Medicare levy?

10. Tamara works as a swimming instructor and earns $21.50 per hour when working a 38-hour week.
- **a.** Using the PAYG table, find the amount of tax that should be deducted from Tamara's salary per week, correct to the nearest dollar.
- **b.** What percentage of her gross salary is deducted? Give your answer to one decimal place.
- **c.** If Tamara receives $82 per week in family allowance but pays $50 per week towards her superannuation, what is her net weekly pay?

Reflection

What strategies would you use to remember how to calculate income tax?

11.8 Simple interest

11.8.1 Simple interest

- **Interest** is the fee charged for the use of someone else's money. It is normally a percentage of the amount borrowed.
- Lenders or investors receive interest from banks for lending them money.
- Borrowers pay interest to banks and other financial institutions.
- **Simple interest** or 'flat rate' interest can be calculated using a simple formula: $I = PRN$
  where I = the amount of interest to be paid
  P = the principal, which is the amount of money borrowed
  R = the interest rate, usually given as a percentage
  N = the number of times that the interest must be paid.
- The abbreviation p.a. stands for 'per annum', which means 'each year'. For example, an interest rate of 5% p.a. for 4 years means that $R = 5\%$ (or 0.05) and $N = 4$.

WORKED EXAMPLE 16

Zac borrows $3000 for 2 years at 9% p.a. simple interest.
a How much interest is he charged? **b What total amount must he repay?**

THINK	WRITE
a 1 Write the simple interest formula, and the known values of the variables.	**a** $I = PRN$, $P = 3000$, $R = 9\% = 0.09$, $N = 2$
2 Substitute the values into the formula to find I.	$I = 3000 \times 0.09 \times 2$ $= \$540$
3 Write the answer in a sentence.	Zac is charged $540 interest.
b 1 Repayment = amount borrowed + interest.	**b** $3000 + 540 = 3540$
2 Write the answer in a sentence.	Zac must repay $3540 in total.

- Care needs to be taken with examples where the term of the investment is given in months or even in days. In these examples, the period of the investment needs to be expressed in years.
- The simple interest formula can also be used to find the principal, interest rate or the term of the investment by substituting the known values into the formula, and solving the resulting equation.

WORKED EXAMPLE 17

Anthony invested $1000 at a simple interest rate of 4.6% p.a. For how long must he invest it in order to earn at least $100 in interest?

THINK	WRITE
1 Write the formula and the known values of the variables.	$I = PRN$, where $I = 100, P = 1000,$ $R = 4.6\% = 0.046$
2 Substitute the given values into the formula.	$100 = 1000 \times 0.046 \times N$
3 Solve the equation.	$100 = 46N$ $N = \frac{100}{46}$ ≈ 2.1739

4 Change the decimal part of the years into months.	$0.1739 \times 12 = 2.09$ months
5 Write the answer in a sentence using years and months.	Anthony must invest for 2 years and 2 months.

- The same method is used when R or P are to be found.

WORKED EXAMPLE 18

The Smiths need to buy a new refrigerator at a cost of $1679. They will pay a deposit of $200 and borrow the balance at an interest rate of 19.5% p.a. The loan will be paid off with 24 equal monthly payments.

a How much money do the Smiths need to borrow?
b What is the term of the loan?
c How much interest will they pay?
d What will be the total cost of the refrigerator?
e How much is each payment?



THINK	WRITE
a 1 Subtract the deposit from the cost to find the amount still owing.	**a** $1679 - 200 = 1479$
2 Write the answer in a sentence.	They must borrow $1479.
b The term is 24 months as this is the length of time between borrowing and paying back. The interest rate is per year.	**b** 24 months is 2 years. The term is 2 years.
c 1 Identify the principal (P), interest rate (R) and time period (N), and use the formula.	**c** $P = 1479$, $R = \frac{19.5}{100} - 0.195$, $N = 2$ $I = PRN$ $= 1479 \times 0.195 \times 2$ $= 576.81$
2 Write the answer in a sentence.	The interest will be $576.81.
d Add the interest to the initial cost.	**d** $1679.00 + 576.81 = 2255.81$ The total cost will be $2255.81.
e 1 Subtract the deposit from the total cost to find the amount to be repaid.	**e** $2255.81 - 200 = 2055.81$
2 Divide the total payment into 24 equal payments.	$2055.81 \div 24 = 85.66$
3 Write the answer in a sentence.	Each payment will be $85.66.

- Spreadsheets are often used to make simple interest calculations easier.

11.8.2 Developing a simple interest spreadsheet

- The spreadsheet below calculates the total amount of simple interest for a given number of years.

	A	B	C	D	E	F
1						
2		Principal	1000			
3		Interest rate (per year)	5			
4		Time (years)	6			
5						
6		*Year*	*Principal*	*Interest*	*New value*	
7		1	1000	50	1050	
8		2	1000	50	1100	
9		3	1000	50	1150	
10		4	1000	50	1200	
11		5	1000	50	1250	
12		6	1000	50	1300	
13						

- Inputs (yellow cells)
  - Cell D2: the amount of principal. Above, the principal is $1000.
  - Cell D3: the interest rate, as a percentage. Above, the interest rate is 5%.
  - Cell D4: the term. Above, the term is 6 years.
- Outputs (Row 7 and beyond)
  - Column B: shows the years: 1, 2, 3, … 6
  - Column C: shows the principal each year. Set C7 = D2 and fill down.
  - Column D: shows the interest calculation. Set D7 = C7*D3/100 and fill down.
  - Cell E7: Shows the new value after year 1. Set E7 = C7 + D7.
  - Cell E8: Shows the new value after year 2. Set E8 = E7 + D8 and fill down.
- For time periods greater than 6 years, highlight Row 12's cells and fill down.

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Complete this digital doc: WorkSHEET: Simple interest (doc-6246)

Exercise 11.8 Simple interest

assessON

Individual pathways

PRACTISE
Questions:
1, 3–5, 7, 8, 10, 11, 14

CONSOLIDATE
Questions:
1, 2, 6–10, 12, 14–16

MASTER
Questions:
1, 3, 5, 7, 9–18

Individual pathway interactivity: int-4526

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Fluency

1. WE16 Monique borrows $5000 for 3 years at 8% per annum simple interest.
   a. How much interest is she charged?
   b. What total amount must she repay?

2. Calculate the simple interest earned on an investment of $15000 at 5.2% p.a. over 30 months.

3. For each loan in the table, calculate:
   i. the simple interest **ii.** the amount repaid.

	Principal ($)	Interest rate per annum	Time
a	1000	5%	2 years
b	4000	16%	3 years
c	8000	4.5%	48 months
d	2700	3.9%	2 years 6 months
e	15678	9.2%	42 months

4. Find the *final value* of each of the following investments.
   a. $3000 for 2 years at 5% p.a.
   b. $5000 for 3 years at 4.3% p.a.

5. Hasim borrows $14950 to buy a used car. The bank charges a 9.8% p.a. flat rate of interest over 60 months.
   a. What total amount must he repay?
   b. How much is each equal monthly repayment?

6. Carla borrows $5200 for an overseas trip at 8.9% p.a. simple interest over 30 months. If repayment is made in equal monthly instalments, how much is each instalment?



7. WE17 Michael invested $2000 at a simple interest rate of 4% p.a. For how long must he invest it in order to earn $200 in interest?

8. If Jodie can invest her money at 8% p.a., how much does she need to invest to earn $2000 in 2 years?

9. If the simple interest charged on a loan of $9800 over 3 years is $2352, what percentage rate of interest was charged?

10. Find the missing quantity in each row of the table.

	Principal	Rate of interest p.a.	Time	Interest earned
a	$2000	6%		$240.00
b	$3760	5.8%		$545.20
c		7%	3 years	$126.00
d		4.9%	1 year 9 months	$385.88
e	$10000		$1\frac{1}{2}$ years	$1200.00
f	$8500		42 months	$1041.25

Understanding



11. WE18 Mika is buying a used car priced at $19 998. He has a deposit of $3000 and will pay the balance in equal monthly payments over 4 years. The simple interest rate will be 12.9% p.a.
- **a.** How much money is he borrowing?
- **b.** How much interest will he pay?
- **c.** What will be the total cost of the car?
- **d.** How many payments will he make?
- **e.** How much is each payment?

12. A new sound system costs $3500, but it can be purchased for no deposit, followed by 48 equal monthly payments, at a simple interest rate of 16.2% p.a.
- **a.** What will be the total cost of the sound system?
- **b.** Under a 'no deposit, no payment for 2 years' scheme, 48 payments are still required, but the first payment isn't made for two years. (This will stretch the loan over 6 years.) How much will the system cost using this scheme?
- **c.** What will be the monthly payment under each of the schemes above?

Reasoning

13. A $269 000 business is purchased on $89 000 deposit with the balance payable over 5 years at 8.95% p.a. flat rate.
- **a.** How much money is borrowed to purchase this business?
- **b.** How much interest is charged?
- **c.** What total amount must be repaid?
- **d.** Find the size of each of the equal monthly repayments, and explain two ways how these payments could be reduced.

14. If a bank offers interest on its savings account of 4.2% p.a. and the investment is invested for 9 months, explain why 4.2 is not substituted into the simple interest formula as the interest rate.

Problem solving

15. A Year 9 girl is paid $79.50 in interest for an original investment of $500 for 3 years. What is the annual interest rate?

16. A loan is an investment in reverse; you *borrow* money from a bank and are *charged* interest. The value of a loan becomes its total cost.

A worker wishes to borrow $10 000 from a bank, which charges 11.5% interest per year. If the loan is over 2 years:
- **a.** calculate the total interest paid
- **b.** calculate the total cost of the loan.

17. For the following question assume that the interest charged on a home loan is simple interest.
- **a.** Tex and Molly purchase their first home and arrange for a home loan of $375 000. Their home loan interest rate rises 0.25% per annum within the first 6 months of the loan. Determine the monthly increase, in dollars, of their repayments.
- **b.** Brad and Angel's interest on their home loan is also increased by 0.25% per annum. Their monthly repayments increase by $60. Determine the amount of their loan, in dollars.

18.
- **a.** Theresa invests $4500 at 5.72% per annum that attracts simple interest for 6 months. Show that at the end of 6 months she should expect to have $4628.70.
- **b.** Barry has $6273 in his bank account at a simple interest rate of 4.86% per annum. After 39 days he calculates that he will have $6305.57 in his account. Did Barry calculate his interest correctly? Justify your answer by showing your calculations.
- **c.** Juanita receives $10 984 for the sale of her car. She invests x% of $10 984 in an account at 6.68% per annum simple interest for $1\frac{1}{2}$ years. She spends the remainder of the money from the sale of her car. At the end of the investment she has exactly enough money to purchase a car for $11 002. Find the value of x, correct to 2 decimal places.

Reflection

How does interest affect the way we live?

11.9 Compound interest

11.9.1 Compound interest

- Consider $1000 invested for 3 years at 10% p.a. simple interest.
- Each year the value of the investment increases by $100, reaching a total value of $1300.
- The simple interest process can be summarised in the following table.

	Principal	Interest	Total value
Year 1	$1000	$100	$1100
Year 2	$1000	$100	$1200
Year 3	$1000	$100	$1300

Total interest = $300

- Under the system called **compound interest**, the interest is added to the principal at the end of each year; in other words, it is *compounded* annually.
- The compound interest process can be summarised in this table.

	Principal	Interest	Total value
Year 1	$1000	$100	$1100
Year 2	$1100	$110	$1210
Year 3	$1210	$121	$1331

Total interest = $331

- The principal grows each year and so does the interest.
- Over many years, the difference between simple interest and compound interest can become enormous.

WORKED EXAMPLE 19

Complete the table to find the interest paid when $5000 is invested at 11% p.a. compounded annually for 3 years.

	Principal	Interest	Total value
Year 1	$5000		
Year 2			
Year 3			

Total interest =

THINK	WRITE
1 Interest for year 1 = 11% of $5000	$11\% = \frac{11}{100} = 0.11$
Find the principal for year 2 by adding the interest to the year 1 principal.	$I = 0.11 \times 5000 = 550$ $5000 + 550 = 5550$
2 Interest for year 2 = 11% of $5550	$0.11 \times 5550 = 610.50$
Find the total value at the end of year 2. This is the principal for year 3.	$5550 + 610.50 = 6160.50$

3 Interest for year 3 = 11% of $6160.50

$0.11 \times 6160.50 = 677.66$
$6160.50 + 677.66 = 6838.16$

4 Calculate the interest earned over 3 years by subtracting the year 1 principal from the final amount.

$6838.16 - 5000 = 1838.16$

	Principal	Interest	Total value
Year 1	$5000	$550	$5550
Year 2	$5550	$610.50	$6160.50
Year 3	$6160.50	$677.66	$6838.16

Total interest = $1838.16

- There is a quicker way of finding the total value of the investment. Look again at Worked example 19.

  The investment grows by 11% each year, so its value at the end of the year is $111\%\left(\frac{111}{100} = 1.11\right)$ of its value at the start of the year.

  111% of 5000
  $= 1.11 \times 5000$
  $= 5550$
- This process is repeated each year for 3 years.

  $5000 \xrightarrow{\times 1.11} 5550 \xrightarrow{\times 1.11} 6160.50 \xrightarrow{\times 1.11} 6838.16$
- After 3 years the value of the investment is $6838.16.

WORKED EXAMPLE 20

Complete the table to find the value, after 4 years, of an investment of $2000 compounded annually at 8% p.a.

Year	Start of year	End of year
Year 1	$2000	
Year 2		
Year 3		
Year 4		

THINK

1 Interest is compounded at 8%, so at the end of the first year the value is 108% of the initial value.

2 For the value at the end of year 2, calculate 108% of the amount accumulated in year 1, so find 108% of 2160.

3 For the value at the end of year 3, calculate 108% of the amount accumulated in year 2, so find 108% of 2332.80.

4 For the value at the end of year 4, calculate 108% of the amount accumulated in year 3, so find 108% of 2519.424.

5 Complete the table.

WRITE

$108\% = \frac{108}{100} = 1.08$
$1.08 \times 2000 = 2160$

$1.08 \times 2160 = 2332.80$

$1.08 \times 2332.80 = 2519.424$

$1.08 \times 2519.424 = 2720.98$

Year	Start of year	End of year
Year 1	$2000	$2160
Year 2	$2160	$2332.80
Year 3	$2332.80	$2519.42
Year 4	$2519.42	$2720.98

- The repeated multiplication above can be developed into a formula for compound interest.
- In Worked example 20 the principal ($2000) was multiplied by 108% four times (because there were 4 years). The final amount, A, was given by

$$A = 2000 \times 108\% \times 108\% \times 108\% \times 108\%$$
$$= 2000(108\%)^4$$
$$= 2000(1 + 8\%)^4$$

In general, $A = P(1 + R)^n$

where A = the final value of the investment
P = the principal
R = the interest rate
n = the number of investment periods.

WORKED EXAMPLE 21

a Find the final value of $40 000 invested at 7.5% p.a. compounding annually for 8 years.
b How much interest is earned by the investment?

THINK	WRITE
a 1 Write the compound interest formula.	**a** $A = P(1 + R)^n$
2 Write the values of P, R (converting the percentage to a decimal) and n.	$P = 40\,000,\ R = 0.075,\ n = 8$
3 Substitute the values into the formula and calculate.	$A = 40\,000(1.075)^8$ $= 71\,339.11$
4 Write the answer in a sentence.	The value of the investment is $71 339.11.
b Subtract the initial principal from the final value of the investment to find the interest earned.	**b** $71\,339.11 - 40\,000 = 31\,339.11$ The interest earned was $31 339.11.

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Complete this digital doc: Spreadsheet: Simple and compound interest (doc-10907)

Exercise 11.9 Compound interest

Individual pathways

assessON

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 7, 9, 11	Questions: 1–3, 5–7, 9–12	Questions: 1–4, 7–14

Individual pathway interactivity: int-4527

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Fluency

1. WE19 Complete the tables to find the interest paid when:

a. $1000 is invested at 12% p.a. compounded annually for 3 years

	Principal	Interest	Total value
Year 1	$1000		
Year 2			
Year 3			

Total interest =

b. $100 000 is invested at 9% p.a. compounded annually for 4 years.

	Principal	Interest	Total value
Year 1	$100 000		
Year 2			
Year 3			
Year 4			

Total interest =

2. WE20 Complete the tables to find the final value of each investment.

a. $5000 invested at 12% p.a. compounded annually for 3 years

	Start of year	End of year
Year 1	$5000	
Year 2		
Year 3		

b. $200 000 invested at 7% p.a. compounded annually for 3 years

	Start of year	End of year
Year 1	$200 000	
Year 2		
Year 3		

c. $100 000 invested at 8.5% p.a. compounded annually for 5 years

	Start of year	End of year
Year 1	$100 000	
Year 2		
Year 3		
Year 4		
Year 5		

d. $12 000 invested at 15% p.a. compounded annually for 4 years

	Start of year	End of year
Year 1	$12 000	
Year 2		
Year 3		
Year 4		

3. WE21 For each of the following investments, use the compound interest formula to find:
i. the total value **ii.** the amount of interest paid.
a. $8000 is invested for 8 years at 15% p.a. interest compounding annually.
b. $50000 is invested for 4 years at 6% p.a. interest compounding annually.
c. $72000 is invested for 3 years at 7.8% p.a. interest compounding annually.
d. $150000 is invested for 7 years at 6.3% p.a. interest compounding annually.
e. $3500 is invested for 20 years at 15% p.a. interest compounding annually.
f. $21000 is invested for 10 years at 9.2% p.a. interest compounding annually.

4. Peter invests $5000 for 3 years at 6% p.a. simple interest, and Maria invests the same amount for 3 years at 5.8% p.a. compounding annually.
a. Calculate the value of Peter's investment on maturity.
b. Calculate the value of Maria's investment on maturity.
c. Explain why Maria's investment is worth more, although she received a lower interest rate.

5. Gianni invests $8000 at 15% p.a. compounded annually, and Dylan invests $8000 at 15% p.a. flat rate. How much more than Dylan's investment will Gianni's investment be worth after:
a. 1 year **b.** 2 years
c. 5 years **d.** 10 years?

6. When her granddaughter was born, Barbara invested $100 at the rate of 7% p.a. compounding annually. She plans to give it to her granddaughter on her eighteenth birthday. What will the amount be?



Understanding

7. Mai's investment account has compounded at a steady 9% for the last 10 years. If it is now worth $68000, how much was it worth:
a. last year **b.** ten years ago?

8. Chris and Jenny each invested $10000. Chris invested at 6.5% p.a. compounding annually, and Jenny took a flat rate of interest. After 5 years, their investments had equal value.
a. Find the value of Chris's investment after 5 years.
b. Find Jenny's interest rate.
c. Find the value of each investment after 6 years.

Reasoning

9. Two investment options are available to invest $3000.
A Invest for 5 years at 5% p.a. compounding monthly.
B Invest for 5 years at 5% p.a. compounding weekly.
Explain which option you would you choose and why.

10. There are 3 factors that affect the value of a compound interest investment: the principal, the interest rate and the length of the investment.
a. Let the interest rate be 10% p.a. and the length of the investment be 2 years. Calculate the value of an investment of:
i. $1000 **ii.** $2000 **iii.** $4000.
b. Comment on the effect of increasing the principal on the value of the investment.
c. Let the principal be $1000 and the interest rate be 10% p.a. Calculate the value of an investment of:
i. 2 years **ii.** 4 years **iii.** 8 years.
d. Comment on the effect of increasing the length of the investment on the value of the investment.

e. Let the principal be $1000 and the length of the investment be 5 years. Calculate the value of an investment of:
i. 6% interest p.a. **ii.** 8% interest p.a. **iii.** 10% interest p.a.
f. Comment on the effect of increasing the interest rate on the value of the investment.

Problem solving

11. Calculate the value of each of the following investments if the principal is $1000.
a. Interest rate = 8% p.a., compounding period = 1 year, time = 2 years
b. Interest rate = 8% p.a., compounding period = 6 months, time = 2 years
c. Interest rate = 8% p.a., compounding period = 3 months, time = 2 years

12. A bank offers a term deposit for 3 years at an interest rate of 8% p.a. with a compounding period of 6 months. What would be the end value of a $5000 investment under these conditions?

13. A building society offers term deposits at 9%, compounded annually. A credit union offers term deposits at 10% but with simple interest only.
a. After 2 years, which has the larger value?
b. After 3 years, which has the larger value?
c. How many years before the compound interest offer has the greater value?

14. One aspect of compound interest is of great importance to investors: how long does it take to double my money? Consider a principal of $100 and an annual interest rate of 10% (annual compounding).
a. How long does it take for this investment to be worth $200?
b. How long would it take for the investment to be worth $400 (a second doubling)?

Reflection

Is compound interest 'fairer' than simple interest?

CHALLENGE 11.2

Which would be better, and by what percentage — a wage rise of 20% or two successive wage rises of 10%?

11.10 Review

11.10.1 Review questions

Fluency

1. Jane earns an annual salary of $45 650. Calculate her fortnightly pay.

2. Express $638.96 per week as an annual salary.

3. Frank works as a casual shop assistant and is paid $8.20 an hour from 3.30 pm to 5.30 pm, Monday to Friday, and $9.50 an hour from 7 am to 12 noon on Saturday. What is his total pay for the week?

4. Below are the pay details for 4 people. Who receives the most money?
a. Billy receives $18.50 per hour for a 40-hour working week.
b. Jasmine is on an annual salary of $38 400.
c. Mladin receives $1476.90 per fortnight.
d. Thuy receives $3205 per month.

5. Daniel earns $10 an hour for a regular 38-hour week. If he works overtime, he is paid double time. How much would he earn if he worked 42 hours in one week?

6. Bjorn earns $468.75 per week award wage for a 38-hour week.
a. What is his standard hourly rate?
b. If he is paid time-and-a-half for normal overtime, what is his pay for a week in which he works 41 hours?

7. Xana makes gift cards as a hobby and is paid 55 cents for each one.
   a. How much would she earn if she made 50 cards?
   b. How many cards would she need to make to earn $44?
8. Kim sells cakes to the local shop. If the shop pays for the ingredients, she is paid $3.20 for each cake that she makes and 50 cents for each slice. If Kim makes 5 cakes and 15 slices one week, how much does she earn?
9. A salesperson earns a $250 per week retainer, plus 2% commission on the first $10 000 sale and 1.5% on the remainder. What is the total income for a week in which sales to the value of $18 000 were made?
10. Jennifer is to start a new job selling mobile phones. She is paid commission only at the rate of 17.5% of sales. What value of sales must she make in order to receive commission of $600 in one week?

Problem solving

11. Phillipa is paid an annual salary of $48 800.
   a. Calculate Phillipa's gross weekly salary.
   b. Calculate the total amount Phillipa will receive for her 4 weeks annual leave if she is paid a 17.5% holiday loading.
12. The annual cost of operating the rail system in Sydney is $300m. The Chief Executive Officer of the rail network is promised a bonus of 0.15% of any saving that she can make to the running of the system. Calculate the bonus paid if the cost is reduced to $275m.
13. Geoff works as a waiter and is paid $12.50 per hour. If he works 40 hours per week, calculate:
   a. Geoff's gross pay
   b. the tax that Geoff should pay
   c. the percentage of gross pay that Geoff pays in tax
   d. Geoff's net pay if he also has $12.50 deducted for superannuation, $5.35 for union fees and $100.00 for a loan repayment.
14. a. Find the simple interest on an investment of $4000 for 9 months at 4.9% per annum.
   b. To what amount will the investment grow by the end of its term?
15. Calculate the monthly instalment needed to repay a loan of $12 500 over 40 months at 9.75% p.a. flat interest rate.
16. A simple interest loan of $5000 over 4 years incurred interest of $2300. What interest rate was charged?
17. Daniela is to invest $16 000 for 2 years at 9% p.a. with interest compounded annually. Calculate:
   a. the value of the investment
   b. the amount of interest earned.
18. Find the amount of interest paid on $40 000 for 5 years at 6% p.a. with interest compounded six monthly.

Non-calculator questions

19. Calculate 3.5% of 900.
20. Rearrange the formula $I = PRN$ to make N the subject.
21. Milos works in a supermarket and earns $11.70 per hour. If Milos works Saturday he is paid at time-and-a-half. State the hourly rate for which Milos works on Saturday.
22. Frank invests $1000 at 5% p.a. for three years with interest compounded annually. Which of the following calculations will correctly find the value of Frank's investment?
   A. $\$1000 \times 5 \times 3$
   B. $\$1000 \times 1.05 \times 3$
   C. $\$1000 \times 1.05 \times 1.05 \times 1.05$
   D. $\$1000 \times 1.05 \times 1.05 \times 1.05 - \$1000$

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Try out this interactivity: Sudoku: Topic 11 (int-3210)

Complete this digital doc: Concept map: Topic 11 (doc-10801)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

$(1 + R)$
bonus
commission
compound interest
gross
initial
investment
loading
net
overtime
p.a.
percentage rate
piecework
principal
retainer
royalty
salary
simple interest
time period
time sheet
wage

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Australian currency



Australia's coins have distinctive features and our notes are unique in colour and texture. Since decimal currency was introduced in Australia in 1966, our notes and coins have undergone many changes. Only our five-, ten- and twenty-cent coins are still minted as they were back then. The one- and two-cent coins are no longer in circulation, the fifty-cent coin is a different shape, the one- and two-dollar notes have been replaced by coins, and our notes have changed from paper to a special type of plastic.

Coins have two sides: an obverse side and a reverse side. The obverse side of all Australian coins depicts our reigning monarch, Queen Elizabeth II, and the year in which the coin was minted. The reverse side depicts a typical Australian feature and sometimes a special commemorative event.

Note: Answer the following questions on a separate sheet of paper.

1. What is depicted on the reverse side of each Australian coin?

The table below includes information on Australia's current coins in circulation. Use the table to answer questions 2 to 4.

Coin	Diameter (mm)	Mass (g)	Composition
Five-cent	19.41	2.83	75% copper, 25% nickel
Ten-cent	23.60	5.65	75% copper, 25% nickel
Twenty-cent	28.52	11.30	75% copper, 25% nickel
Fifty-cent	31.51	15.55	75% copper, 25% nickel
One-dollar	25.00	9.00	92% copper, 6% aluminium, 2% nickel
Two-dollar	20.50	6.60	92% copper, 6% aluminium, 2% nickel

2. What are the metal compositions of each of the coins?

3. Which is the heaviest coin and which is the lightest? List the coins in order from lightest to heaviest.

4. Which has the smaller diameter — the five-cent coin or the two-dollar coin? Indicate the difference in size.

The table below displays information on Australia's current notes in circulation. The column on the far right compares the average life of the previously used paper notes with that of the current plastic notes. Use the table to answer questions 5 to 9.

Note	Date of issue	Size (mm)	Average life of notes (months)	
			Plastic	Paper
Five-dollar	07/07/1992 24/04/1995 01/01/2001	130 × 65	40	6
Ten-dollar	01/11/1993	137 × 65	40	8
Twenty-dollar	31/10/1994	144 × 65	50	10
Fifty-dollar	04/10/1995	151 × 65	About 100	24
One-hundred-dollar	15/05/1996	158 × 65	About 450	104

5. What denomination notes are available in our Australian currency?
6. On what date was Australia's first plastic note issued and what was the denomination of the note?
7. Suggest a reason for the three issue dates for the five-dollar note.
8. Why do you think each note is of a different size?
9. The table clearly shows that the plastic notes last about five times as long as the paper notes we once used. Why do you think the fifty-dollar and one-hundred-dollar notes last longer than the five- and ten-dollar notes?

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Complete this digital doc: Code puzzle: What are the smallest and largest muscles in the human body? (doc-11905)

Answers

Topic 11 Financial mathematics

Exercise 11.2 Salaries and wages

1. a. $1105.42 **b.** $2210.85 **c.** $4790.17

2. a. $1198.08 **b.** $2396.15 **c.** $5191.67

3. a. $19 136 **b.** $46 410 **c.** $68 684.20

4. a. $25 870 **b.** $42 183.96 **c.** $100 498.06

5. a. $3890 per month **b.** $3200.68 per fortnight

6. a. $19.75/h **b.** $12.17/h **c.** $25.73/h **d.** $39.06/h

7. a. $459.04 **b.** $670.70

8. $210.72

9. $261.49

10. Rob earns more.

11. $30.77/h

12. Job A

13. 38 hours

14. a. $1605.77 **b.** $2788.81 **c.** $1100 **d.** $8770

15. B

16. Minh

17. $40 per hour

18. $1735.95, $1833.75

19. a. $29.81 **b.** $25.64 **c.** $72 080.58

Challenge 11.1

12 weeks

Exercise 11.3 Special rates

1. a. $23.94 **b.** $47.80 **c.** $43.50

2. a. $906.10 **b.** $794.33 **c.** $833.56

3. a. $25.16 **b.** $113.22 **c.** $1069.30

4. a. $170.40 **b.** $340.80 **c.** $426.00

5. $1156.96

6. a. $30.20 **b.** $45.30 **c.** $60.40 **d.** $135.90

7. a. $32.93 **b.** $197.55

8. a. $11.70 **b.** $15.60 **c.** $19.50

9. 8, 8, 8, 8, 6

10.

Pay slip for Susan White	Week ending 17 August
Normal hours	38
Normal pay rate	$25.60
Overtime hours	0
Overtime pay rate	$38.40
Total pay	$972.80

11. 8, 8, 8, 4

12.

Pay slip for Jason Rudd	Week ending 21 December
Normal hours	24
Normal pay rate	$10.90
Overtime hours	4
Overtime pay rate	$16.35
Total pay	$327.00

13. 8, 8, 8, 8, 8

14.

Pay slip for Brett Simpson	Week ending 17 August
Normal hours	24
Normal pay rate	$16.80
Time-and-a half hours	8
Time-and-a half pay rate	$25.20
Double time hours	8
Double time pay rate	$33.60
Total pay	$873.60

15. a. $754.40 **b.** $1042.80 **c.** $1347.42

16. a. $232.02 **b.** $1211.66

17. B

18. No. Glen should have been paid $1034.

19. $1155

20.

	Hours worked	Regular pay	Overtime pay	Total pay
a	32	$464	$0	$464
b	38.5	$522	$54.38	$576.38
c	40.5	$522	$101.50	$623.50
d	47.2	$522	$295.80	$817.80

21. a. Stewart $930.75, Helen $1108.80

b. i. 85 hours

ii. No, because she would have worked 92.5 hours, which is 22.5 hours over the required 70 hours per fortnight.

c. $894.94

Exercise 11.4 Piecework

1. $97.50

2. $89.50

3. $175.50

4. $318.75

5. a. $144.00 **b.** 28 **c.** $21.60

6. a. $144 **b.** $218 **c.** $465

7. a. $107.50 **b.** 24 **c.** $7.17

8. a. $52 **b.** $9.45

9. a. $1398.25 **b.** $1568.12

10. $422.40

11. a. $8950 **b.** $3550

Exercise 11.5 Commission and royalties

1. $3000

2. a. $1280 **b.** $1115.60

3. $1425

4. a. $200 **b.** $4400

5. a. $290 **b.** $482.50 **c.** $1191.12

6. a. $500 **b.** $590 **c.** $1175 **d.** $1568.75

7. $9800

8. $12 800

9. a. $4800 **b.** $5400 **c.** $14 400

10. a. $2400 **b.** $3750 **c.** $15 937.50

11. a. Veronica earns $736; Francis earns $504.

b. $20 000

12. a. $900 **b.** $1148 **c.** $1253.60

13. a. $1520 **b.** $2237.50 **c.** $2555 **d.** $7560

14. $3676.50

15. a. $15 460 **b.** $48 725 **c.** $36 456 **d.** $119 800

16. 4.5%

Exercise 11.6 Loadings and bonuses

1. $1244.76

2. $805.64

3. $764.18

4. a. $1428.80 **b.** $1000.16 **c.** $6715.36

5. a. $866.40 **b.** $4072.08

6. a. $574.20 **b.** $100.49

7. a. $2431.23 **b.** $5713.39

8. $1135.01

9. $116 326.53

10. a. $60 000 **b.** $5000 **c.** 11.90%

11. a. Shane receives a 7.5% incentive as the company's profit has grown by 16.7% (which is between 10.1% and 20%).

b. $295 625

12. a. $9625 **b.** $67 375 **c.** $55 000

13. a. $300 **b.** $615.75 **c.** $1057.50

14. a. $1650 **b.** $2070

15. $640

Exercise 11.7 Taxation and net earnings

1. a. 0 **b.** $722 **c.** $5847 **d.** $20 507

2. a. $69, 10.62% **b.** $217, 19.73% **c.** $374, 24.13%

3. a. $421.25 **b.** $422.35 **c.** $514.60

4. $43 961.45

5. $719.85

6. a. $297 **b.** 22% **c.** $938.80

7. a. $800.10 **b.** $113 **c.** 14.12%

d. $548.40 **e.** 68.54%

8. a. Debbie is taxed $14 947 on the $72 000 she earns.

b. There may be other deductions on Debbie's net pay, such as superannuation, union fees, private health insurance and the Medicare levy.

9. a. $89 700 **b.** $21 136 **c.** $1794

10. a. $113 **b.** 13.8% **c.** $736

Exercise 11.8 Simple interest

1. a. $1200 **b.** $6200

2. $1950

3. a. i. $100 **ii.** $1100

b. i. $1920 **ii.** $5920

c. i. $1440 **ii.** $9440

d. i. $263.25 **ii.** $2963.25

e. i. $5048.32 **ii.** $20 726.32

4. a. $3300 **b.** $5645

5. a. $22 275.50 **b.** $371.26

6. $211.90

7. 2.5 years

8. $12 500

9. 8%

10. a. 2 years **b.** 2.5 years **c.** $600
d. $4500.06 **e.** 8% **f.** 3.5%

11. a. $16 998 **b.** $8770.97 **c.** $28 768.97
d. 48 **e.** $536.85

12. a. $5768 **b.** $6902 **c.** $120.17, $143.79

13. a. $180 000 **b.** $80 550 **c.** $260 550 **d.** $4342.50

14. The interest rate in the simple interest formula needs to be converted from a percentage into a decimal: 4.2% = 0.042.

15. 5.3%

16. a. $2300 **b.** $12 300

17. a. $78.13 **b.** $288 000

18. a. Answers will vary. **b.** Yes **b.** 91.04

Exercise 11.9 Compound interest

1. a.

	Principal	Interest	Total value
Year 1	$1000	$120	$1120
Year 2	$1120	$134.40	$1254.40
Year 3	$1254.40	$150.53	$1404.93

Total interest = $404.93

b.

	Principal	Interest	Total value
Year 1	$100 000	$9000	$109 000
Year 2	$109 000	$9810	$118 810
Year 3	$118 810	$10 692.90	$129 502.90
Year 4	$129 502.90	$11 655.26	$141 158.16

Total interest = $41 158.16

2. a.

	Start of year	End of year
Year 1	$5000	$5600
Year 2	$5600	$6272
Year 3	$6272	$7024.64

b.

	Start of year	End of year
Year 1	$200 000	$214 000
Year 2	$214 000	$228 980
Year 3	$228 980	$245 008.60

c.

	Start of year	End of year
Year 1	$100 000	$108 500
Year 2	$108 500	$117 722.50
Year 3	$117 722.50	$127 728.91
Year 4	$127 728.91	$138 585.87
Year 5	$138 585.87	$150 365.67

d.

	Start of year	End of year
Year 1	$12 000	$13 800
Year 2	$13 800	$15 870
Year 3	$15 870	$18 250.50
Year 4	$18 250.50	$20 988.08

3. a. i. $24 472.18 **ii.** $16 472.18
b. i. $63 123.85 **ii.** $13 123.85
c. i. $90 196.31 **ii.** $18 196.31
d. i. $230 050.99 **ii.** $80 050.99
e. i. $57 282.88 **ii.** $53 782.88
f. i. $50 634.40 **ii.** $29 634.40

4. a. $5900 **b.** $5921.44
c. Each year, Maria's principal increases.

5. a. 0 **b.** $180 **c.** $2090.86 **d.** $12 364.46

6. $337.99

7. a. $62 385.32 **b.** $28 723.93

8. a. $13 700.87 **b.** 7.4%
c. Chris, $14 591.42; Jenny, $14 440

9. Option B would be the best choice, as the shorter the time between the compounding periods, the greater the interest paid.

10. a. i. $1210 **ii.** $2420 **iii.** $4840

b. Increasing the principal will increase the value of the investment because it will have a higher value of interest.

c. i. $1210 **ii.** $1464.10 **iii.** $2143.59

d. Increasing the length of the investment will increase the value of the investment because it will have a higher value of interest.

e. i. $1338.23 **ii.** $1469.33 **iii.** $1610.51

f. Increasing the interest rate will increase the value of the investment because it will have a higher value of interest.

11. a. $1166.40 **b.** $1169.86 **c.** $1171.66

12. $6326.60

13. a. Simple interest **b.** Simple interest **c.** 4 years

14. a. 7.27 years **b.** 14.55 years

Challenge 11.2

Two successive wage rises of 10%

11.10 Review

1. $1755.77 **2.** $33 225.92 **3.** $129.50

4. a

5. $460

6. a. $12.34 **b.** $524.28

7. a. $27.50 **b.** 80 cards

8. $23.50 **9.** $570 **10.** $3428.57

11. a. $938.46 **b.** $4410.76

12. $37 500

13. a. $500 **b.** $35.00 **c.** 7% **d.** $347.15

14. a. $147 **b.** $4147

15. $414.06

16. 11.5%

17. a. $19 009.60 **b.** $3009.60

18. $13 756.66 **19.** 31.5 **20.** $N = \frac{I}{PR}$ **21.** $17.55

22. C

Investigation — Rich task

1. Five-cent coin: echidna

Ten-cent coin: lyrebird

Twenty-cent coin: platypus

Fifty-cent coin: coat of arms

One-dollar coin: five kangaroos

Two-dollar coin: Aboriginal elder Gwoya Jungarai

2. Refer to the table.

3. Five-cent, ten-cent, two-dollar, one-dollar, twenty-cent, fifty-cent coin

4. The five-cent coin has a smaller diameter; 1.09 mm.

5. $5, $10, $20, $50, $100

6. 7 July 1992; $5

7. Answers will vary.

8. The different sizes allow blind people to tell the difference between each note.

9. The fifty-dollar and one-hundred-dollar notes are used less frequently.

TOPIC 12 Measurement

12.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



12.1.1 Why learn this?

Measurement skills are required in many real-world situations. Measurements may be in one, two or three dimensions. We use measurements to describe such things as length, temperature, time, area and capacity. Measurement concepts have significant connections to other areas of mathematics, such as geometry and statistics.

12.1.2 What do you know?

assessON

1. **THINK** List what you know about measurement. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of measurement.

LEARNING SEQUENCE

12.1 Overview
12.2 Measurement
12.3 Area
12.4 Area and perimeter of a sector
12.5 Surface area of rectangular and triangular prisms
12.6 Surface area of a cylinder
12.7 Volume of prisms and cylinders
12.8 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of the mathematics: Measurement matters (eles-1699)

12.2 Measurement

12.2.1 Timescales

- Scientists sometimes need to work with very large or very small timescales.
- Very large or very small numbers are best written in scientific notation (as described in Topic 2).

WORKED EXAMPLE 1

Alpha Centauri is the closest star system to Earth at a distance of 4.3 light-years. Light travels at 300 000 km/s. Correct to 4 significant figures:
a write the speed of light in scientific notation
b determine the distance travelled by light in 1 minute
c determine the distance travelled by light in 1 day
d determine the length of a light-year in kilometres
e determine the distance, in km, of Alpha Centauri from Earth.

THINK

a Light travels at 300 000 km/s.

b Light travels at the rate of 3.0×10^5 km per second.
1 minute = 60 seconds, so multiply the answer to part **a** by 60.

c Light travels at the rate of 1.8×10^7 km per minute.
1 hour = 60 minutes, 1 day = 24 hours, so multiply the answer to part **b** by 60 and by 24.

d Light travels at the rate of 2.592×10^{10} km per day. 1 year = 365.25 days, so multiply the answer to part **c** by 365.25.

e 1 Alpha Centauri is 4.3 light-years from Earth. Multiply the previous answer by 4.3.

2 Express the answer in scientific notation.

WRITE

a $300\,000 = 3.0 \times 10^5$ km/s

b $3.0 \times 10^5 \times 60 = 180 \times 10^5$
$= 1.8 \times 10^7$ km/min

c $1.8 \times 10^7 \times 60 \times 24 = 2592 \times 10^7$
$= 2.592 \times 10^{10}$ km/day

d $2.592 \times 10^{10} \times 365.25 = 946.728 \times 10^{10}$
$\approx 9.467 \times 10^{12}$ km/year
A light-year is 9.467×10^{12} km.

e $9.467\,28 \times 10^{12} \times 4.3 = 40.7093 \times 10^{12}$
$\approx 4.070 \times 10^{13}$ km

Alpha Centauri is $4.070\,93 \times 10^{13}$ km from Earth.

12.2.2 Small timescales

- Slow-motion cameras have made it possible to watch events in ultra slow motion, often 1000 times slower than normal. For example, a 1-second recording could take 15 minutes to watch at ultra slow speed.
- There are many examples of slow-motion recordings on the internet. Some useful weblinks have been provided in the Resources tab.



WORKED EXAMPLE 2

Mary bought a new camera that could record at the rate of 16 000 frames per second.
a Write this rate in scientific notation.
b She records a balloon bursting that takes 2 seconds. How many frames is this?
c She replays the video and slows it down so that it takes 15 minutes to play. How many frames per second is this?

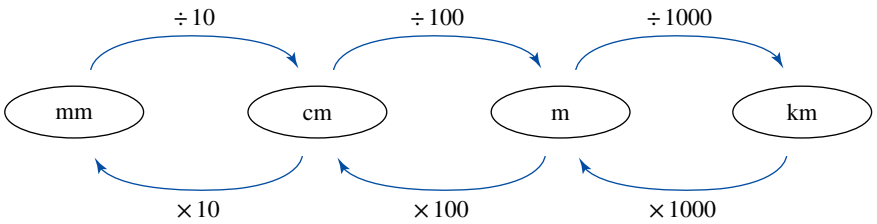
THINK	WRITE
a Write the speed in the form $a \times 10^n$.	**a** $16\,000 = 1.6 \times 10^4$ frames/s
b Multiply the answer to part **a** by 2.	**b** $1.6 \times 10^4 \times 2 = 3.2 \times 10^4$ frames
c 1 Calculate the number of seconds in 15 minutes.	**c** $15 \times 60 = 900$ seconds
2 Frames per second $= \frac{\text{number of frames}}{\text{number of seconds}}$.	$\frac{3.2 \times 10^4}{900} = 0.003\,556 \times 10^4$ $= 3.556 \times 10$ $= 36.56$
3 Write the answer in words.	The rate is 35.56 frames per second.

12.2.3 Length and perimeter

- In the **metric system**, units of length are based on the metre. The following units are commonly used.

millimetre	(mm)	one-thousandth of a metre
centimetre	(cm)	one-hundredth of a metre
metre	(m)	one metre
kilometre	(km)	one thousand metres

- The chart below is useful when converting from one unit of length to another.



For example, $36\,000$ mm $= 36\,000 \div 10 \div 100$ m
$= 36$ m

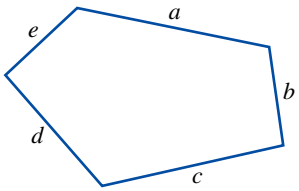
WORKED EXAMPLE 3

Convert the following lengths into cm.
a 37 mm **b 2.54 km**

THINK	WRITE
a There will be fewer cm, so divide by 10.	**a** $37 \div 10 = 3.7$ cm
b There will be more cm, so multiply. km → m: × 1000 m → cm: × 100	**b** $2.54 \times 1000 = 2540$ m $2540 \times 100 = 254\,000$ cm

12.2.4 Perimeter

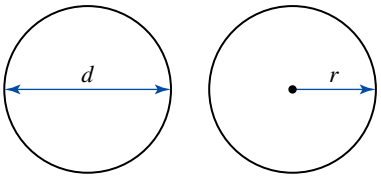
- The **perimeter** of a plane (flat) figure is the distance around the outside of the figure.
- If the figure has straight edges, then the perimeter can be found by simply adding all the side lengths.
- Ensure that all lengths are in the same unit.



Perimeter $= a + b + c + d + e$

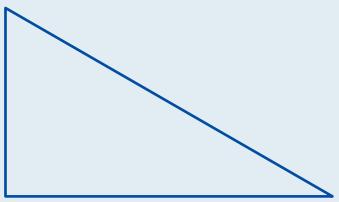
12.2.5 Circumference

- **Circumference** is a special name given to the perimeter of a circle.
- Circumference is calculated using the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius of the circle.
- Use your calculator's value for π unless otherwise directed.



WORKED EXAMPLE 4

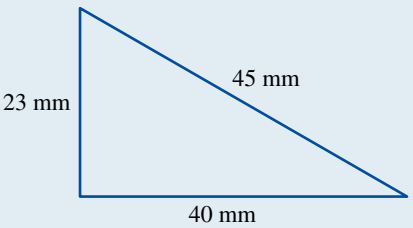
Find the perimeter of this figure, in millimetres.



THINK

1 Measure each side to the nearest mm.

WRITE/DRAW

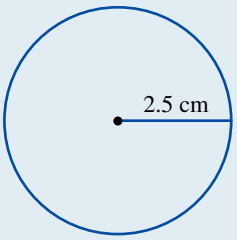


2 Add the side lengths together.

$P = 45 + 40 + 23$
$= 118$ mm

WORKED EXAMPLE 5 TI | CASIO

Determine the circumference of the circle at right. Give your answer correct to 2 decimal places.



THINK

1 The radius is known, so apply the formula $C = 2\pi r$.

2 Substitute $r = 2.5$ into $C = 2\pi r$.

3 Calculate the circumference to 3 decimal places and then round correct to 2 decimal places.

WRITE

$C = 2\pi r$

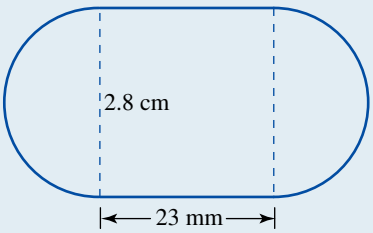
$C = 2 \times \pi \times 2.5$

$C \approx 15.707$
$C \approx 15.71$ cm

- Sometimes a standard formula will only form part of the calculation of perimeter.

WORKED EXAMPLE 6

Determine the perimeter of the shape shown. Give your answer in cm correct to 2 decimal places.



THINK	WRITE
1 There are two straight sections and two semicircles. The two semicircles make up a full circle, the diameter of which is known, so apply the formula $C = \pi d$.	$C = \pi d$
2 Substitute $d = 2.8$ into $C = \pi d$.	$= \pi \times 2.8$ ≈ 8.796 cm
3 Convert 23 mm to cm (23 mm = 2.3 cm). The perimeter is the sum of all the outside lengths.	$P = 8.796 + 2 \times 2.3$ $= 13.396$
4 Round to 2 decimal places.	≈ 13.40 cm

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Converting units of length (doc-6294)

Complete this digital doc: SkillSHEET: Substitution into perimeter formulas (doc-10872)

Complete this digital doc: SkillSHEET: Perimeter of squares, rectangles, triangles and circles (doc-10873)

Exercise 12.2 Measurement

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4 (left), 5, 6, 7, 8a–c, 9a–c, 10, 11, 13a–e, 14, 17, 18, 21, 22

CONSOLIDATE
Questions:
1, 2, 3, 4 (right), 6, 8b–d, 9b–d, 10, 12, 13d–h, 15, 17, 19, 21–23

MASTER
Questions:
1, 2, 3, 4 (right), 6, 8d–f, 9d–f, 10, 12, 13g–i, 16–24

Individual pathway interactivity: int-4528

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 NGC 6782 is a relatively nearby galaxy, residing about 183 million light-years from Earth. Given that light travels at approximately 300 000 km/s:

a. write the speed of light in scientific notation

b. determine the distance travelled by light in 1 hour

c. determine the distance travelled by light in 1 week

d. determine the length of a light-year in kilometres

e. determine the distance, in km, between NGC 6782 and Earth.

2. WE2 John bought a new camera that could record at the rate of 12 000 frames per second.

a. Write this rate in scientific notation.

b. He records his friend surfing a wave that takes 3 minutes. How many frames is this?

c. He replays the video and slows it down so that it takes 25 minutes to play. How many frames per second is this?

3. Pick ten different objects in the room. Estimate their length, then measure them with a ruler or tape measure. Fill in the table.

Object	Estimated length	Actual length in cm	Actual length in m

4. WE3 Fill in the gaps for each of the following.

a. 5 cm = _____ mm

b. 1.52 m = _____ cm

c. 12.5 mm = _____ m

d. 0.0322 m = _____ mm

e. 6.57 m = _____ km

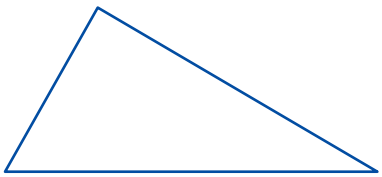
f. 64 cm = _____ km

g. 0.000 014 35 km = _____ mm

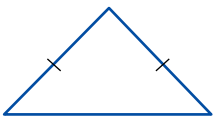
h. 18.35 cm = _____ km

5. WE4 Find the perimeter of the following figures, in millimetres, after measuring the sides.

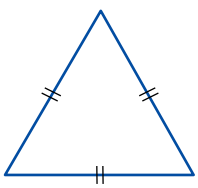
a.



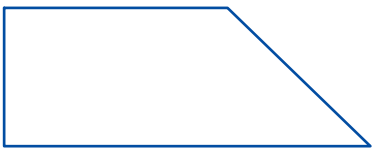
b.



c.

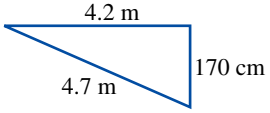


d.

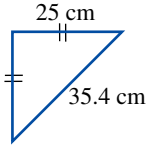


6. Find the perimeter of each of the following figures. Give your answers in centimetres (cm).

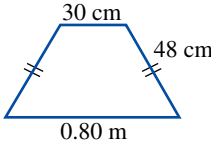
a.



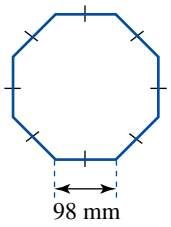
b.



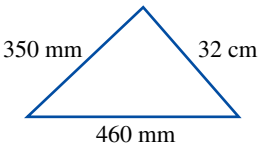
c.



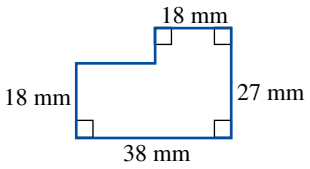
d.



e.

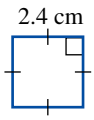


f.

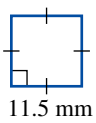


7. Find the perimeter of each of the squares below.

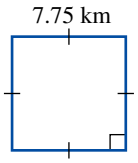
a.



b.

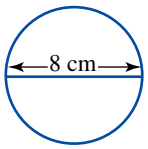


c.

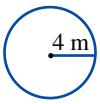


8. WE5 Find the circumference of the circles below. Give your answer correct to 2 decimal places.

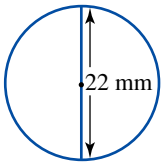
a.



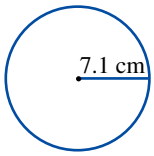
b.



c.



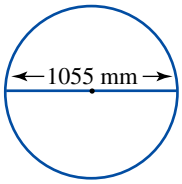
d.



e.

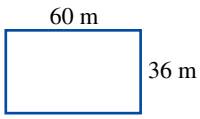


f.

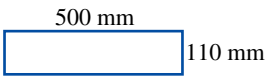


9. Find the perimeter of the rectangles below.

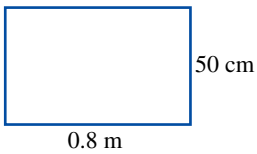
a.



b.



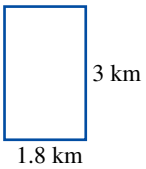
c.



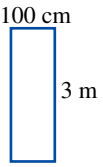
d.



e.



f.



10. MC A circle has a radius of 34 cm. Its circumference, to the nearest centimetre, is:

A. 102 cm **B.** 214 cm **C.** 107 cm **D.** 3630 cm

Understanding

11. Timber is sold in standard lengths, which increase in 300-mm intervals from the smallest available length of 900 mm. (The next two standard lengths available are therefore 1200 mm and 1500 mm.)

a. Write the next four standard lengths (after 1500 mm) in mm, cm and m.

b. How many pieces of length 600 mm could be cut from a 2.4-m length of timber?

c. If I need to cut eight pieces of timber, each 41 cm long, what is the smallest standard length I should buy?
Note: Ignore any timber lost due to the cuts.



12. The world's longest bridge is the Akashi–Kaikyo Bridge, which links the islands of Honshu and Shikoku in Japan. Its central span covers 1.990 km.

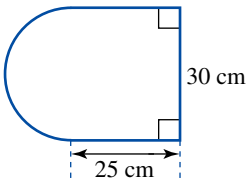


a. How long is the central span, in metres?

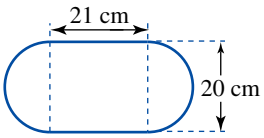
b. How much longer is the span of the Akashi–Kaikyo Bridge than the Sydney Harbour Bridge, which spans 1149 m?

13. WE6 Find the perimeter of the shapes shown below. Give your answer correct to 2 decimal places.

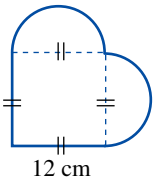
a.



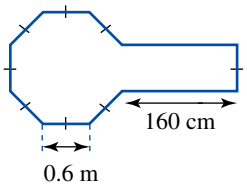
b.



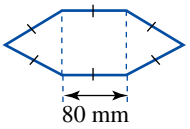
c.



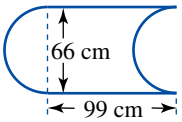
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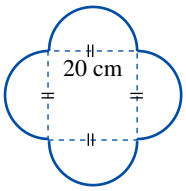
e.



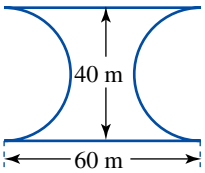
f.



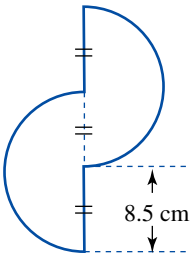
g.



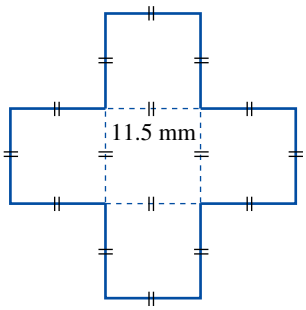
h.



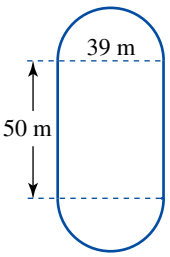
i.



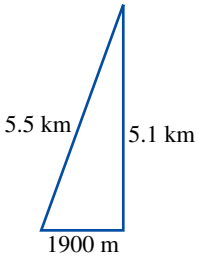
j.



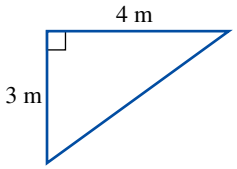
14. Find the perimeter of the racetrack shown in the plan below.



15. Yacht races are often run over a triangular course as shown at right. What distance would the yachts cover if they completed 3 laps of the shown course?



16. Use Pythagoras' theorem to find the length of the missing side and, hence, find the perimeter of the triangular frame shown at right.



Reasoning

17. The Hubble Space Telescope is over 13 m in length. It orbits the Earth at a height of 559 km, where it can take extremely sharp images outside the distortion of the Earth's atmosphere.



a. If the radius of the Earth is 6371 km, show that the distance travelled by the Hubble Space Telescope in one orbit, to the nearest km, is 43 542 km.

b. If the telescope completes one orbit in 96 minutes, show that its speed is approximately 7559 m/s.

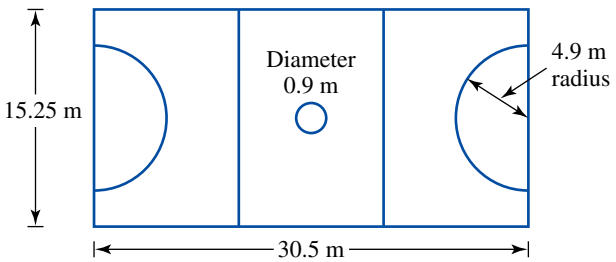
18. A bullet can travel in air at 500 m/s.

a. Show how the bullet travels 50 000 cm in 1 second.

b. How long does it take for the bullet to travel 1 centimetre?

c. If a super-slow-motion camera can take 100 000 pictures each second, how many shots would be taken by this camera to show the bullet travelling 1 cm?

19. Edward is repainting all the lines of a netball court at the local sports stadium. The dimensions of the netball court are shown below.



a. Calculate the total length of lines that need to be repainted.
Edward starts painting at 8 pm when the centre is closing, and it takes him $1\frac{1}{2}$ minutes on average to paint each metre of line.

b. Show that it will take him 233 minutes to complete the job.

20. The radius of the Earth is accepted to be roughly 6400 km.

a. How far, to the nearest km, do you travel in one complete rotation of the Earth?

b. As the Earth spins on its axis once every 24 hours, what speed are you moving at?

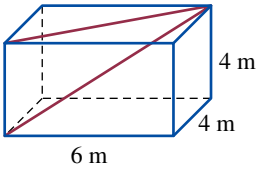
c. If the Earth is 150 000 000 km from the sun, and it takes 365.25 days to circle around the sun, show that the speed of the Earth's orbit around the sun is 107 515 km/h. Give your answer to the nearest whole number.

Problem solving

21. One-fifth of an 80-cm length of jewellery wire is cut off. A further 22-cm length is then removed. Is there enough wire remaining to make a 40-cm necklace?

22. You are a cook in a restaurant where the clock has just broken. You have a four-minute timer, a seven-minute timer and a pot of boiling water. A very famous fussy food critic enters the restaurant and orders a pasta dish. You remember from a TV show on which she appeared that she likes her pasta cooked for nine minutes exactly. How will you measure nine minutes using the timers?

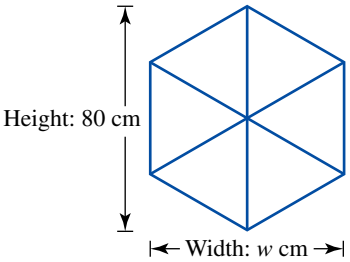
23. A spider is sitting in one top corner of a room that has dimensions 6 m by 4 m by 4 m. It needs to get to the corner of the floor that is diagonally opposite. The spider must crawl along the ceiling, then down a wall, until it reaches its destination.



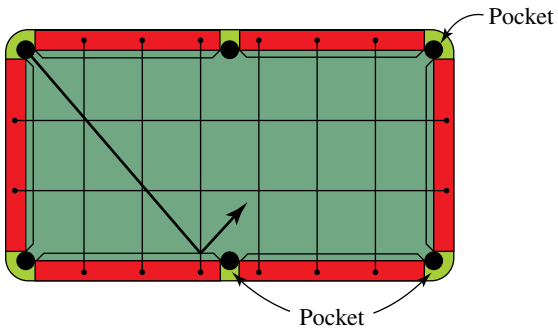
a. If the spider crawls first to the diagonally opposite corner of the ceiling, then down the wall to its destination, what distance would it crawl?

b. What is the shortest distance from the top back corner to the lower left corner?

24. A church needs to repair one of its regular hexagonal shaped stained glass windows. Use the information given in the diagram to find the width of the window.



25. Imagine this is an electronic game of billiards in a games arcade.



The ball is ejected from the top left-hand corner at 45° to the side of the table. It follows the path as indicated, always rebounding at the same angle, until it reaches the pocket in the lower right-hand corner. The grid squares are 5 cm square. What distance does the ball travel on its journey to the pocket? Give an exact answer.

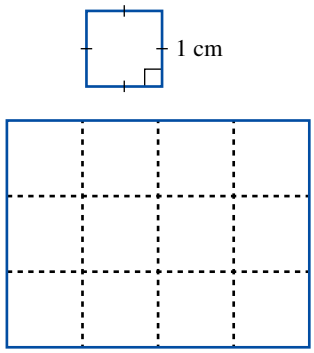
Reflection

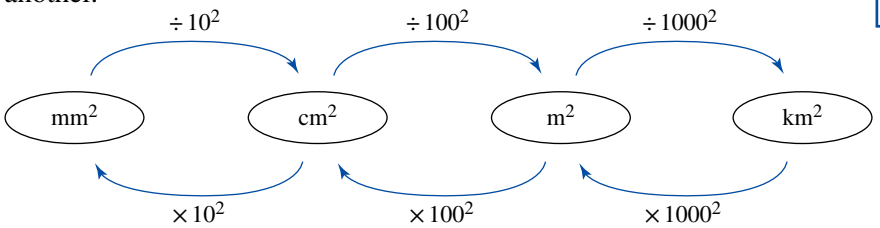
What are some ways to remember how to convert between the various metric units?

12.3 Area

12.3.1 Area

- The diagram at right shows a square of side length 1 cm.
  By definition it has an area of 1 cm^2 (1 square centimetre).
  Note: This is a 'square centimetre', not a 'centimetre squared'.
- **Area** tells us how many squares it takes to cover a figure, so the area of the rectangle at right is 12 cm^2.
- Area is commonly measured in square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), or square kilometres (km^2).
- The chart below is useful when converting from one unit of area to another.





For example, $54\ km^2 = 54 \times 1000^2 \times 100^2$
$= 540\,000\,000\,000\ cm^2$

- Another common unit is the **hectare** (ha), a 100 m × 100 m square equal to 10 000 m^2, which is used to measure small areas of land.

$$1\ ha = 10\,000\ m^2$$

WORKED EXAMPLE 7

Convert 1.3 km^2 into:
a square metres **b hectares.**

THINK

a There will be more m^2, so multiply by 1000^2.

b Divide the result of part **a** by 10 000, as 1 ha = 10 000 m^2.

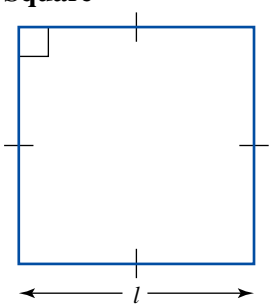
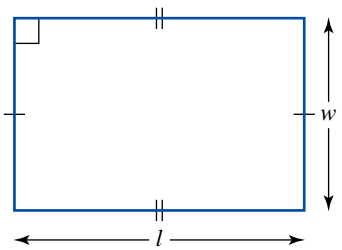
WRITE

a $1.3 \times 1000^2 = 1\,300\,000\ m^2$

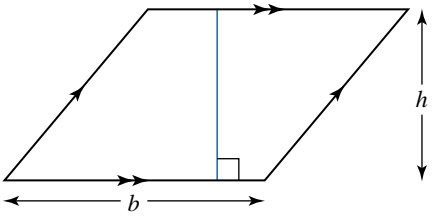
b $1\,300\,000 \div 10\,000 = 130$ ha

12.3.2 Using formulas to calculate area

- There are many useful formulas to find the area of simple shapes. Some common ones are summarised here.

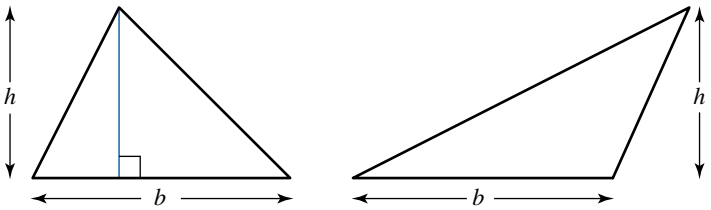
Square	**Rectangle**
	
$A = l^2$	$A = lw$

Parallelogram



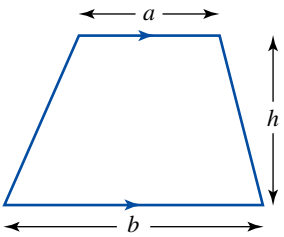
$A = bh$

Triangle



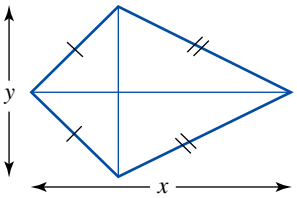
$A = \frac{1}{2}bh$

Trapezium



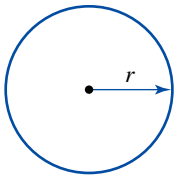
$A = \frac{1}{2}(a + b)h$

Kite



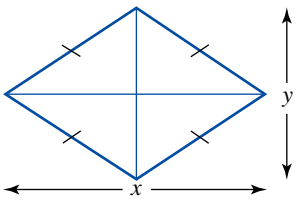
$A = \frac{1}{2}xy$

Circle



$A = \pi r^2$

Rhombus

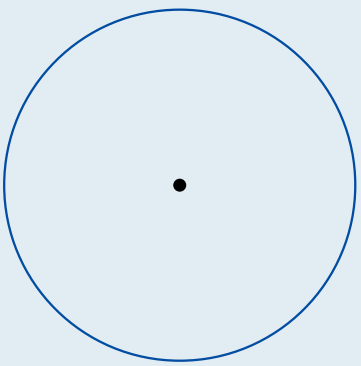


$A = \frac{1}{2}xy$

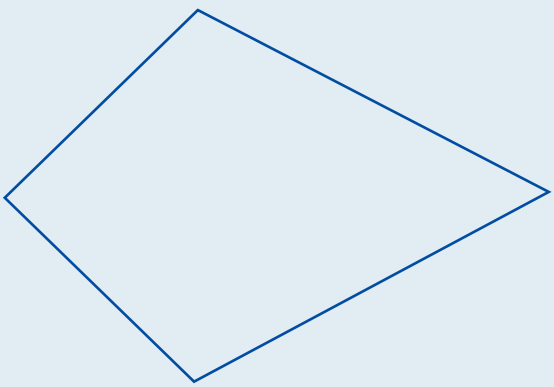
WORKED EXAMPLE 8

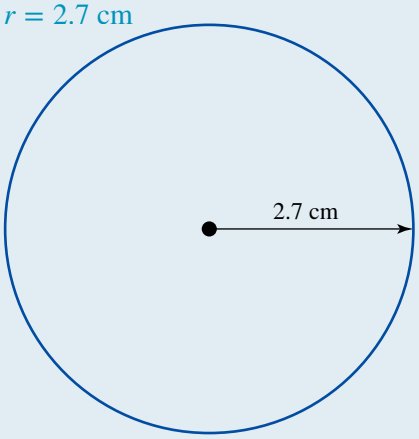
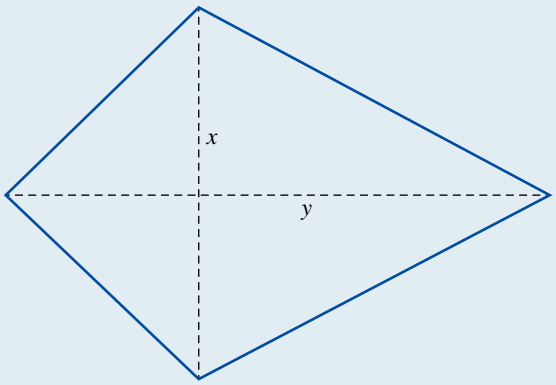
By making the appropriate measurements, calculate the area of each of the following figures in cm^2, correct to 1 decimal place.

a



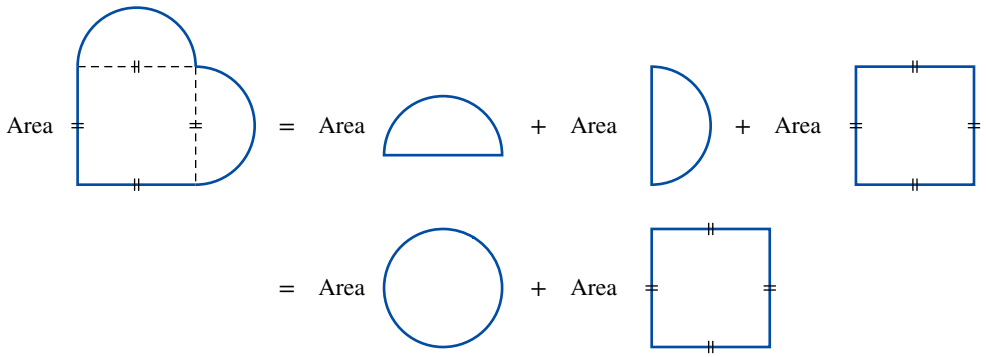
b

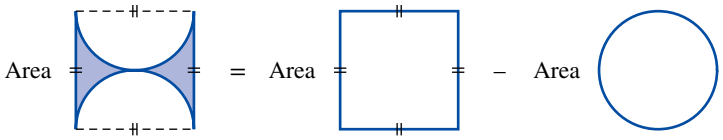


THINK	WRITE/DRAW
a 1 The figure is a circle. Use a ruler to measure the radius.	**a** $r = 2.7$ cm 
2 Apply the formula for area of a circle: $A = \pi r^2$.	$A = \pi r^2$ $= \pi \times 2.7^2$ ≈ 22.90
3 Round the answer to 1 decimal place.	$A \approx 22.9$ cm^2
b 1 The figure is a kite. Measure the diagonals.	**b** 
	$x \approx 4.9$ cm, $y \approx 7.2$ cm
2 Apply the formula for area of a kite: $A = \frac{1}{2}xy$.	$A = \frac{1}{2}xy$ $= \frac{1}{2} \times 4.9 \times 7.2$ $= 17.64$
3 Round the answer to 1 decimal place.	$A \approx 17.6$ cm^2

12.3.3 Composite shapes

- A composite shape is made up of smaller, simpler shapes. Here are two examples.

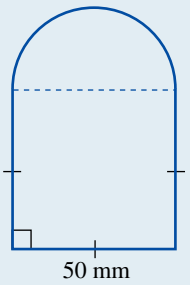


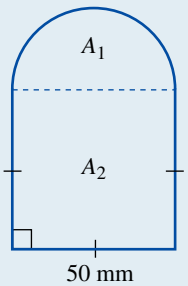


- Observe that in the second example the two semicircles are subtracted from the square to obtain the shaded area on the left.

WORKED EXAMPLE 9

Calculate the area of the figure shown, giving your answer correct to 1 decimal place.



THINK	WRITE/DRAW
1 Draw a diagram, divided into basic shapes.	
2 A_1 is a semicircle of radius 25 mm. The area of a semicircle is half the area of a complete circle.	$A_1 = \frac{1}{2}\pi r^2$
3 Substitute $r = 25$ into the formula and evaluate, correct to 4 decimal places.	$A_1 = \frac{1}{2} \times \pi \times 25^2$ $\approx 981.7477 \text{ mm}^2$
4 A_2 is a square of side length 50 mm. Write the formula.	$A_2 = l^2$
5 Substitute $l = 50$ into the formula and evaluate A_2.	$= 50^2$ $= 2500 \text{ mm}^2$
6 Sum to find the total area.	Total area $= A_1 + A_2$ $= 981.7477 + 2500$ $= 3481.7477 \text{ mm}^2$
7 Round the final answer correct to 1 decimal place.	$\approx 3481.7 \text{ mm}^2$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Substitution into area formulas (doc-10874)

Complete this digital doc: SkillSHEET: Area of squares, rectangles, triangles and circles (doc-10875)

Complete this digital doc: WorkSHEET: Length and area (doc-6303)

Exercise 12.3 Area

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–k, 4, 5a–e, 6, 7, 9a–b, 10, 11, 12, 15, 18, 21–23	Questions: 1, 2, 3c–k, 5d–i, 7, 9b–c, 10–13, 16–19, 21–23, 25	Questions: 1, 2, 3d–o, 5f–j, 8, 9c–d, 10, 11, 13, 14, 16–25

Individual pathway interactivity: int-4529

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. MC To convert an area measurement from square kilometres to square metres:

A. divide by 1000 **B.** multiply by 1000

C. divide by 1 000 000 **D.** multiply by 1 000 000

2. WE7 Fill in the gaps:

a. 13 400 m^2 = _____ km^2 **b.** 0.04 cm^2 = _____ mm^2

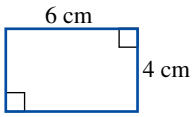
c. 3 500 000 cm^2 = _____ m^2 **d.** 0.005 m^2 = _____ cm^2

e. 0.043 km^2 = _____ m^2 **f.** 200 mm^2 = _____ cm^2

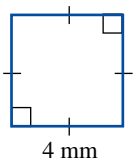
g. 1.41 km^2 = _____ ha **h.** 3800 m^2 = _____ ha

3. WE8 Calculate the area of each of the following shapes. (Where appropriate, give your answer correct to 2 decimal places.)

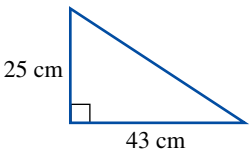
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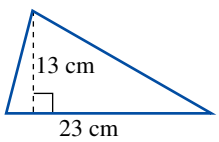
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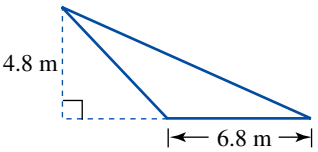
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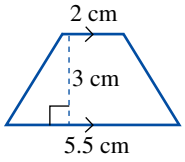
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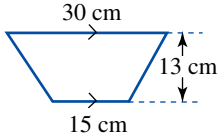
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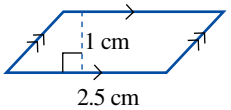
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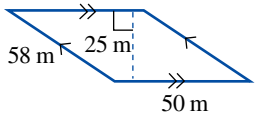
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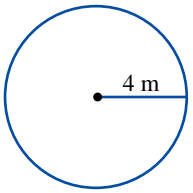
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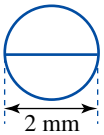
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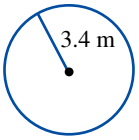
j.



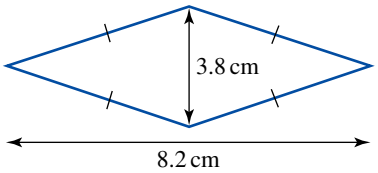
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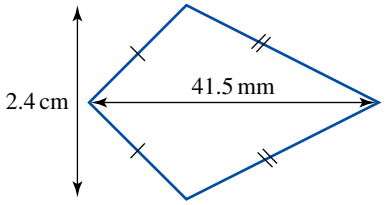
l.



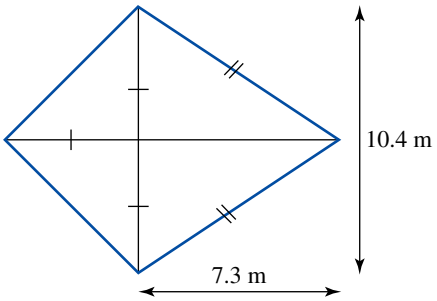
m.



n.



o.

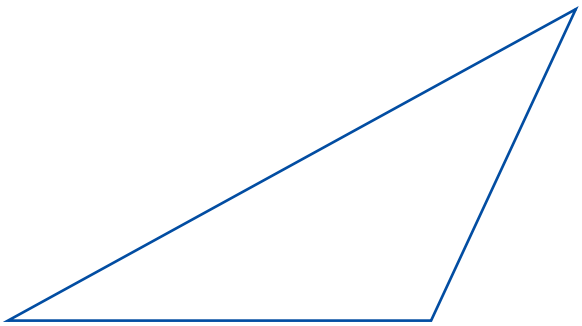


4. **WE8** By making appropriate measurements, calculate the area of each of the following figures in cm^2, correct to 1 decimal place.

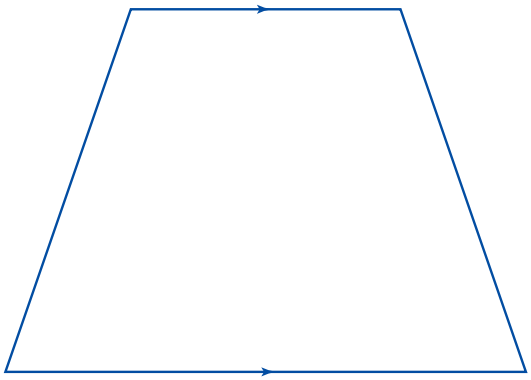
a.



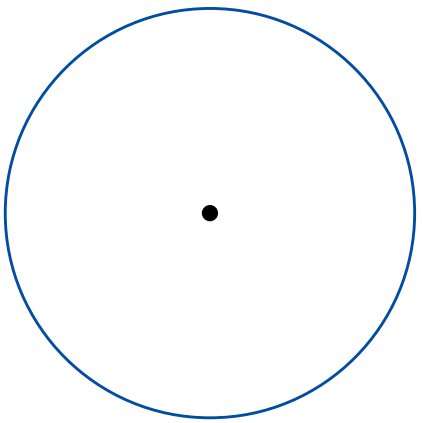
b.



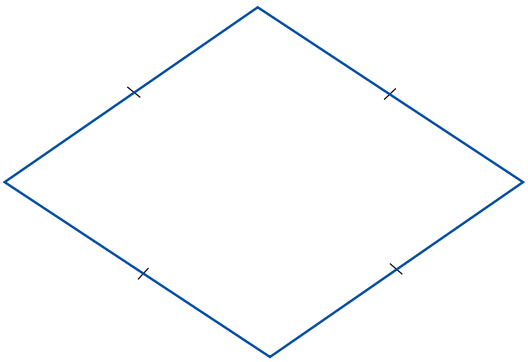
c.



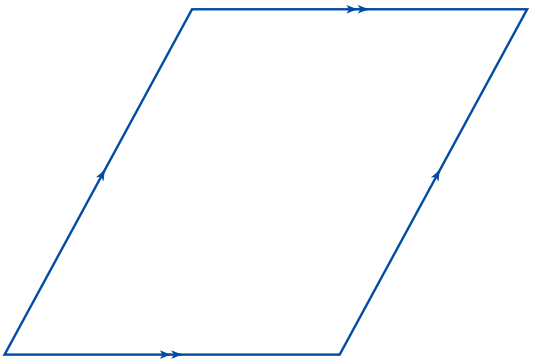
d.



e.

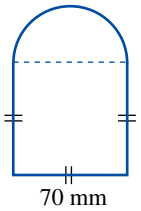


f.

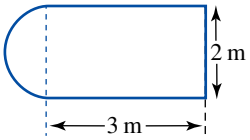


5. WE9 Calculate the areas of the composite shapes shown. Where appropriate, express your answers correct to 1 decimal place.

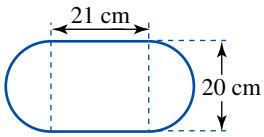
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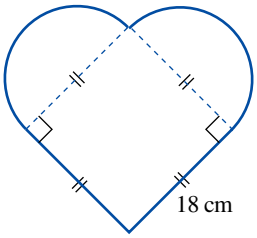
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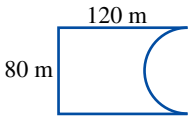
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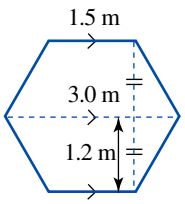
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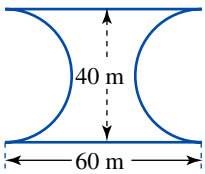
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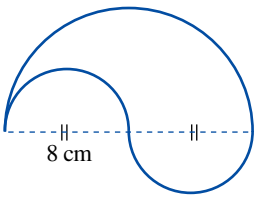
f.



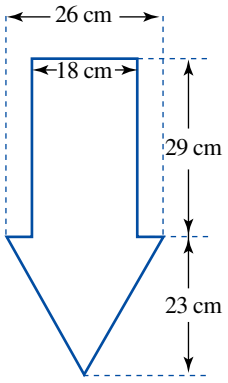
g.



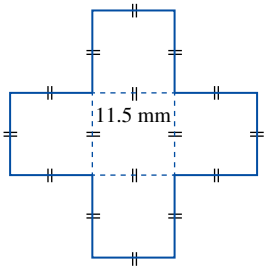
h.



i.

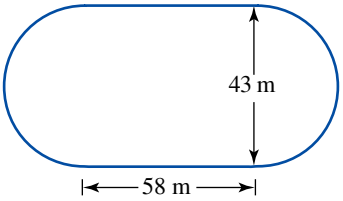


j.



Understanding

6. What would be the cost of covering the sportsground shown in the figure at right with turf if the turf costs $7.50 per square metre?



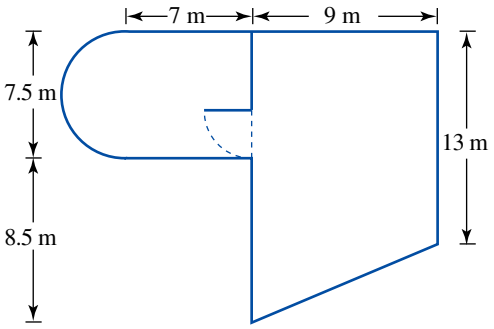
7. The Murray–Darling River Basin is Australia's largest catchment. Irrigation of farms in the Murray–Darling Basin has caused soil degradation due to rising salt levels. Studies indicate that about 500 000 hectares of the basin could be affected in the next 50 years.

a. Convert the possible affected area to square kilometres. (1 km^2 = 100 hectares.)

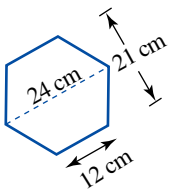
b. The total area of the Murray–Darling Basin is about 1 million square kilometres, about one-seventh of the continent. What percentage of this total area may be affected by salinity?



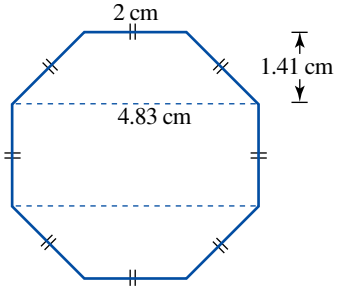
8. The plan at right shows two rooms, which are to be refloored. Calculate the cost if the flooring costs $45 per square metre. Allow 10% more for wastage and round to the nearest $10.



9. Calculate the area of the regular hexagon shown at right by dividing it into two trapeziums.

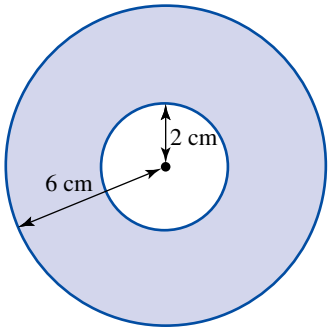


10. Calculate the area of the regular octagon by dividing it into two trapeziums and a rectangle, as shown in the figure.

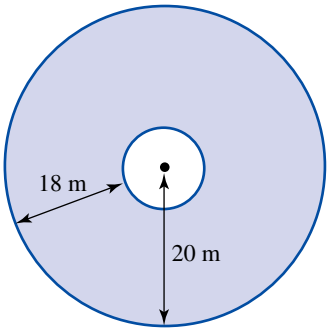


11. An annulus is a shape formed by two concentric circles (two circles with a common centre). Calculate the area of each of the annuli shown below by subtracting the area of the smaller circle from the area of the larger circle. Give answers correct to 2 decimal places.

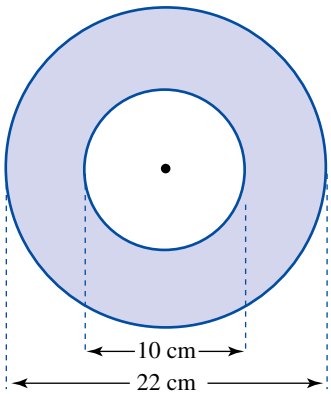
a.



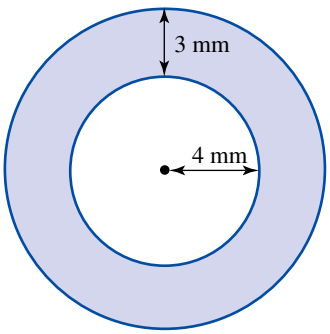
b.



c.



d.



12. MC A pizza has a diameter of 30 cm. If your sister eats one-quarter, what is the remaining area of the pizza?

A. 168.8 cm^2 **B.** 530.1 cm^2 **C.** 706.9 cm^2 **D.** 176.7 cm^2

13. A circle has an area of 4500 cm^2. Calculate its diameter to the nearest mm.

Reasoning

14. A sheet of paper measures 29.5 cm by 21.0 cm.

a. What is the area of the sheet of paper?

b. What is the radius of the largest circle that can be drawn on this sheet?

c. What is the area of this circle?

d. If the interior of the circle is shaded red, show that 56% of the paper is red.

15. a. Find the area of a square with side lengths 40 cm.

b. If the midpoints of each side of the previous square are joined by straight lines to make another square, find the area of the smaller square.

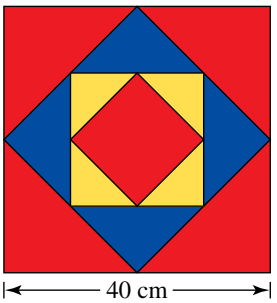
c. Now the midpoints of the previous square are also joined with straight lines to make another square. Find the area of this even smaller square.

d. This process is repeated again to make an even smaller square. What is the area of this smallest square?

e. What pattern do you observe? Justify your answer.

f. What percentage of the original square's area does the smallest square take up?

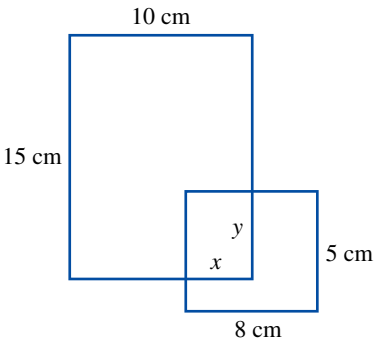
g. Show that the area of the combined figure that is coloured red is 1000 cm^2.



16. A chessboard is made up of 8 rows and 8 columns of squares. Each *little* square is 42 cm^2 in area. Show that the shortest distance from the upper right corner to the lower left corner of the chessboard is 73.32 cm.



17. Two rectangles of sides 15 cm by 10 cm and 8 cm by 5 cm overlap as shown. Show that the difference in area between the two non-overlapping sections of the rectangles is 110 cm^2.



18. The area of a square is x cm^2. Would the side length of the square be a rational number? Explain your answer.

19. Show that the triangle with the largest area for a given perimeter is an equilateral triangle.

20. Show that a square of perimeter $4x + 20$ has an area of $x^2 + 10x + 25$.

Problem solving

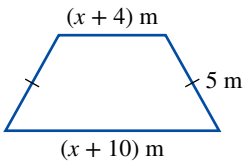
21. The area of a children's square playground is 50 m^2.

a. What is the exact length of the playground?

b. Pine logs 3 m long are to be laid around the playground. How many logs will need to be bought? (The logs can be cut into smaller pieces if required.)



22. A sandpit is designed in the shape of a trapezium, with the dimensions shown. If the area of the sandpit is 14 m^2, what will be its perimeter?

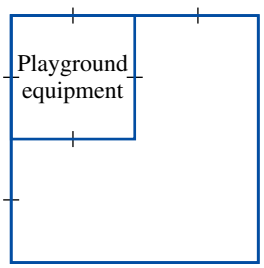


23. The area of a room must be determined so that floor tiles can be laid. The room measures 2.31 m by 4.48 m. This was rounded off to 2 m by 4 m to calculate the area.

a. What problems might arise?

b. If each tile is 0.5 m by 0.5 m, how many tiles are actually required?

24. The playground equipment is half the length and half the width of the square kindergarten yard it is in.



a. What fraction of the kindergarten yard is occupied by the play equipment?

b. During a working bee, the playground equipment area is extended 2 m in length and 1 m in width. If x represents the length of the kindergarten yard, write an expression for the area of the play equipment.

c. Write an expression for the area of the kindergarten yard *not* taken up by the playground equipment.

d. The kindergarten yard that is not taken up by the playground equipment is divided into 3 equal-sized sections:
- a grassed area
- a sandpit
- a concrete area.

i. Write an expression for the area of the concrete area.

ii. The children usually spend their time on the play equipment or in the sandpit. Write a simplified expression for the area of the yard where the children usually play.

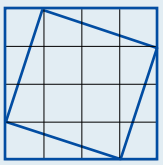
25. A rectangular classroom has a perimeter of 28 m and its length is 4 m shorter than its width. What is the area of the classroom?

Reflection

If you know the conversion factor between two units of length, for example mm and m, how can you quickly work out the conversion factor between the corresponding units of area?

CHALLENGE 12.1

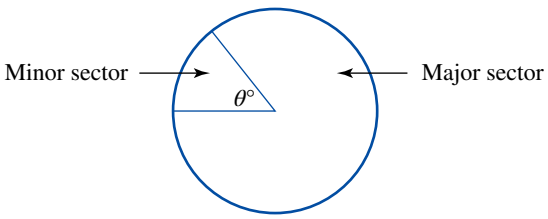
The diagram shows one smaller square drawn inside a larger square on grid paper. Express the area of the smaller square as a fraction of the larger square.



12.4 Area and perimeter of a sector

12.4.1 Sectors

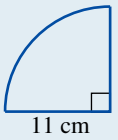
- If you draw two **radii** inside a circle, they divide the circle into two regions called **sectors**.
- A circle, like a pizza, can be cut into many sectors.
- Two important sectors that have special names are the **semicircle** (half circle) and the **quadrant** (quarter circle).





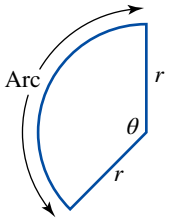
WORKED EXAMPLE 10

Calculate the area enclosed by the figure at right, correct to 1 decimal place.



THINK	WRITE
1 The figure is a quadrant or quarter-circle. Write the formula for its area.	$A = \frac{1}{4} \times \pi r^2$
2 Substitute $r = 11$ into the formula.	$A = \frac{1}{4} \times \pi \times 11^2$
3 Evaluate and round the answer correct to 1 decimal place. Include the units.	≈ 95.03 $\approx 95.0 \text{ cm}^2$

12.4.2 The formula for the area of a sector

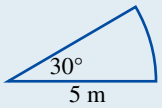


- Usually sectors are specified by the angle (θ) between the two radii.
  For example, in a quadrant, $\theta = 90°$, so a quadrant is $\frac{90}{360}$ or $\frac{1}{4}$ of a circle.
- For any value of θ, the area of the sector is given by

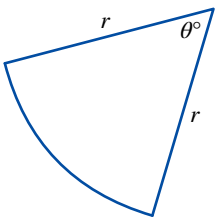
$$\text{Area}_{\text{sector}} = \frac{\theta}{360} \times \pi r^2$$

WORKED EXAMPLE 11

Calculate the area of the sector shown, correct to 1 decimal place.



THINK	WRITE
1 This sector is $\frac{30}{360}$ of a circle. Write the formula for its area.	$A = \frac{30}{360} \pi r^2$
2 Substitute $r = 5$ into the formula.	$A = \frac{30}{360} \times \pi \times 5^2$ ≈ 6.54
3 Evaluate and round the answer correct to 1 decimal place. Include the units.	$\approx 6.5 \text{ m}^2$

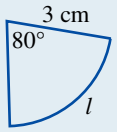


- The perimeter of a sector consists of 2 radii and a curved section, which is the arc of a circle.
- The length of the arc will be $\frac{\theta}{360}$ of the circumference of the circle.
- If the circumference of the circle = C, then the perimeter of the sector, P, will be given by

$$P = 2r + \frac{\theta}{360} \times C$$
$$= 2r + \frac{\theta}{360} \times 2\pi r$$

WORKED EXAMPLE 12

Calculate the perimeter of the sector shown, correct to 1 decimal place.



THINK	WRITE
1 The sector is $\frac{80}{360}$ of a circle. Write the formula for the length of the curved side.	$l = \frac{80}{360} \times 2\pi r$
2 Substitute $r = 3$ and evaluate l. Don't round off until the end.	$l = \frac{80}{360} \times 2 \times \pi \times 3$ ≈ 4.189 cm
3 Add all the sides together to calculate the perimeter.	$P = 4.189 + 3 + 3$ $= 10.189$ cm
4 Round the answer to 1 decimal place.	≈ 10.2 cm

Exercise 12.4 Area and perimeter of a sector

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–12	Questions: 1–12, 15, 16	Questions: 1d, 2c, 3–8, 10–17

Individual pathway interactivity: int-4530

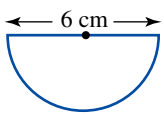
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

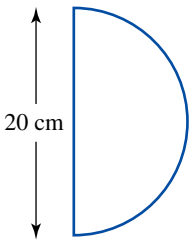
Fluency

1. Calculate the area of the semicircles below, correct to 2 decimal places.

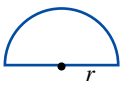
a.



b.

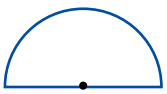


c.



$r = 4.2$ cm

d.

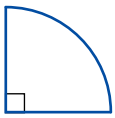


$D = 24$ mm

2. WE10 For each of the quadrants shown, calculate to 1 decimal place:

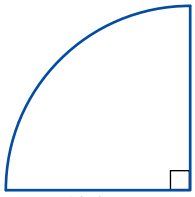
i. the perimeter **ii.** the area enclosed.

a.



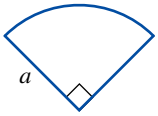
4 cm

b.



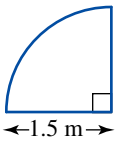
12.2 cm

c.



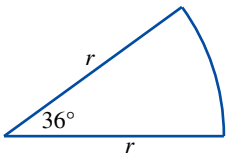
$a = 11.4$ m

d.



3. MC Which is the correct formula for calculating the area of this sector?

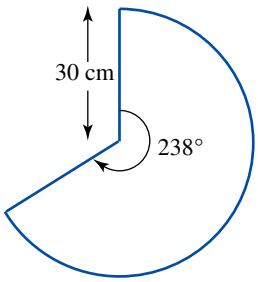
A. $A = \frac{1}{4}\pi r^2$ **B.** $A = \frac{3}{4}\pi r^2$ **C.** $A = \frac{1}{2}\pi r^2$ **D.** $A = \frac{1}{10}\pi r^2$



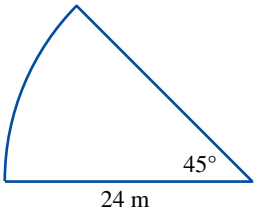
4. WE11,12 For each of the sectors shown, calculate to 1 decimal place:

i. the perimeter **ii.** the area.

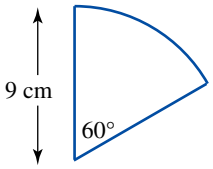
a.



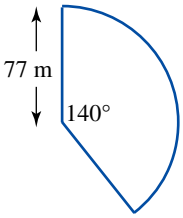
b.



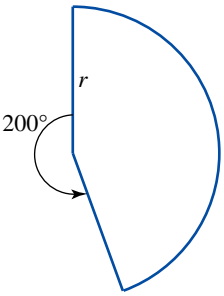
c.



d.

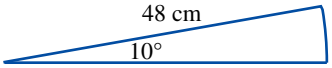


e.

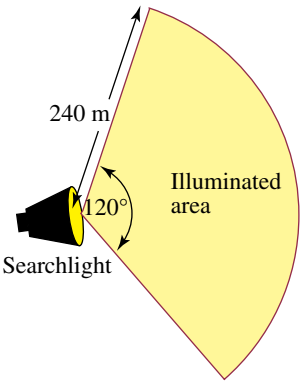


$r = 74$ cm

f.

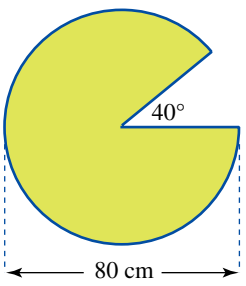


5. A searchlight lights up the ground to a distance of 240 m. What area does the searchlight illuminate if it can swing through an angle of 120°, as shown in the diagram below? (Give your answer correct to 1 decimal place.)

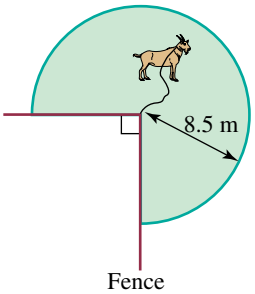


Understanding

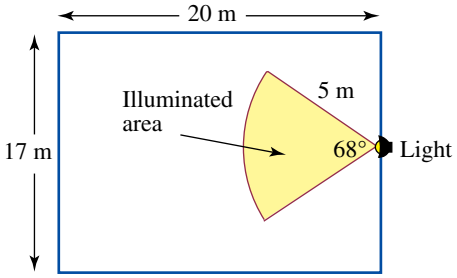
6. Calculate the perimeter, correct to 1 decimal place, of the figure at right.



7. A goat is tethered by an 8.5 m rope, to the outside of a corner post in a paddock, as shown in the diagram below. Calculate the area of grass (shaded) on which the goat is able to graze. (Give your answer correct to 1 decimal place.)



8. A beam of light is projected onto a theatre stage as shown in the diagram.



a. Calculate the illuminated area (correct to 1 decimal place) by finding the area of the sector.

b. Calculate the percentage of the total stage area that is illuminated by the light beam.

9. MC A sector has an angle of 80° and a radius of 8 cm; another sector has an angle of 160° and a radius of 4 cm. The ratio of the area of the first sector to the area of the second sector is:

A. 1 : 2 **B.** 2 : 1 **C.** 1 : 1 **D.** 1 : 4

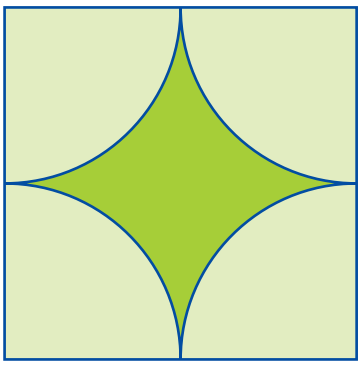
10. Calculate the radius of the following sectors, correct to 1 decimal place.

a. Area = 100 m^2, angle = 13°

b. Area = 100 m^2, curved arc = 12 m

c. Perimeter = 100 m, angle = 11°

11. Four baseball fields are to be constructed inside a rectangular piece of land. Each field is in the shape of a sector of a circle, as shown in light green. The radius of each sector is 80 m.



a. Calculate the area of one baseball field, correct to the nearest whole number.

b. What percentage, correct to 1 decimal place, of the total area is occupied by the four fields?

c. The cost of the land is $24 000 per hectare. What is the total purchase price of the land?

Reasoning

12. Answer the following questions. Where appropriate, give all answers to the nearest whole number.

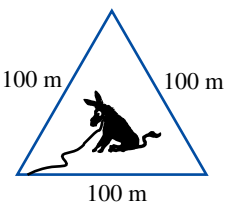
a. A donkey inside a square enclosure is tethered to a post at one of the corners. Show that the length of the rope required so that the donkey eats only half of the grass in the enclosure is 120 m.



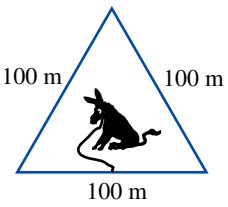
b. Suppose two donkeys are tethered at opposite corners of the square region shown at right. How long should the rope be so that the donkeys together can graze half of the area?

c. This time four donkeys are tethered, one at each corner of the square region. How long should the rope be so that all the donkeys can graze only half of the area?

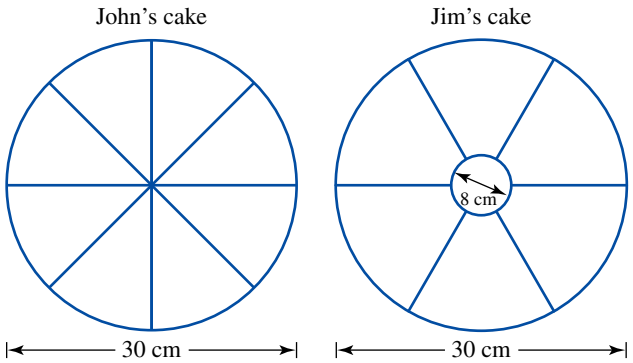
d. Another donkey is tethered to a post inside an enclosure in the shape of an equilateral triangle. The post is at one of the vertices. Show that a rope of length 64 m is required so that the donkey eats only half of the grass in the enclosure.



e. This time the donkey is tethered halfway along one side of the equilateral triangular region shown at right. How long should the rope be so that the donkey can graze half of the area?



13. John and Jim are twins, and on their birthday they have identical birthday cakes, each cake of diameter 30 cm. Grandma Maureen cuts John's cake into 8 equal sectors. Grandma Mary cuts Jim's cake with a circle in the centre and then 6 equal portions from the rest.



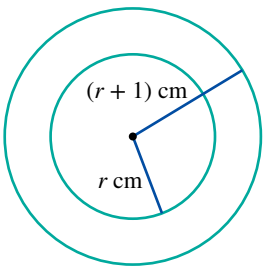
a. Show that each sector of John's cake makes an angle of 45° at the centre of the cake.
b. What area of cake, correct to 1 decimal place, is a slice of John's cake?
c. What area of cake, correct to 1 decimal place, is the small central circular part of Jim's cake?
d. What area of cake, correct to 1 decimal place, is a larger portion of Jim's cake?
e. If each boy eats one slice of the largest part of their own cake, does John eat the most cake? Justify your answer.



14. A lighthouse has a light beam in the shape of a sector of a circle that rotates at 10 revolutions per minute and covers an angle of 40°. A person stands 200 m from the lighthouse and observes the beam. Show that the time between the end of one flash and the start of the next is approximately 5.33 seconds.

Problem solving

15. A metal washer (shown at right) has an inner radius of r cm and an outer radius of $(r + 1)$ cm.



a. State, in terms of r, the area of the circular piece of metal that was cut out of the washer.
b. State, in terms of r, the area of the larger circle.
c. Show that the area of the metal washer in terms of r is $\pi(2r + 1)$ cm^2.
d. If r is 2 cm, what is the exact area of the washer?
e. If the area of the washer is 15π cm^2, show that the radius would be 7 cm.

16. The area of a sector of a circle is π cm^2, and the length of its arc is 2 cm. What is the radius of the circle (in terms of π)?

17. An arbelos is a shape enclosed by three semicircles.
The word, in Greek, means 'shoemaker's knife' as it resembles the blade of a knife used by cobblers. Investigate to determine a relationship between the lengths of the three semicircular arcs.



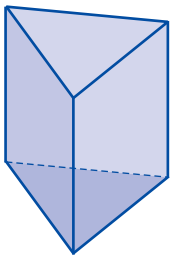
Reflection

What is the relationship between the curved arc of a sector and the area of the sector?

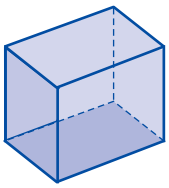
12.5 Surface area of rectangular and triangular prisms

12.5.1 Prisms

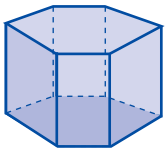
- A **prism** is a solid object with a uniform (unchanging) **cross-section** and all sides flat.
- Here are some examples of prisms.



Triangular prism

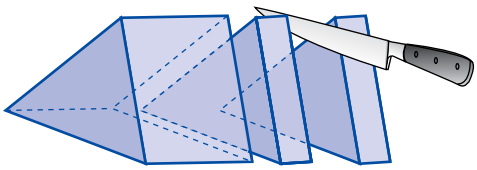


Rectangular prism (cuboid)



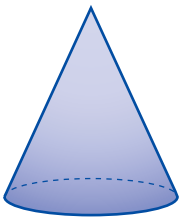
Hexagonal prism

- A prism can be sliced (cross-sectioned) in such a way that each 'slice' has an identical base.

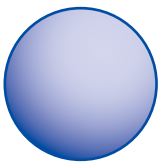


'Slicing' a prism into pieces produces congruent cross-sections.

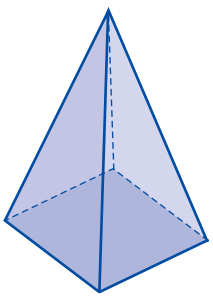
- The following objects are not prisms because they do not have uniform cross-sections.



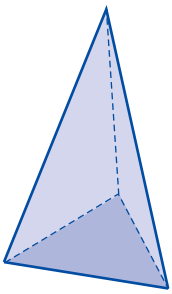
Cone



Sphere



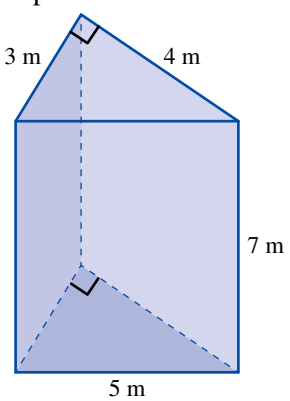
Square pyramid

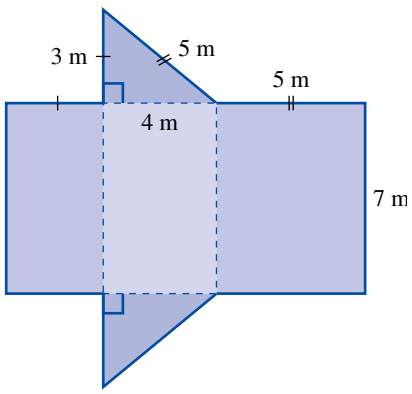


Triangular pyramid

12.5.2 Surface area of a prism

- Consider the **triangular prism** shown below.
- It has 2 bases, which are right-angled triangles, and 3 rectangular sides. In all, there are 5 faces, and the net of the prism is drawn below.

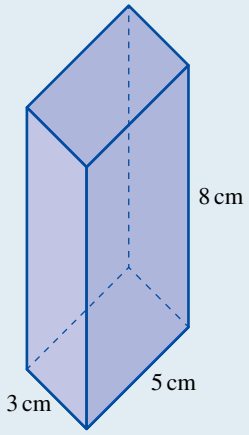




- The area of the net is the same as the total surface area (SA) of the prism.

WORKED EXAMPLE 13

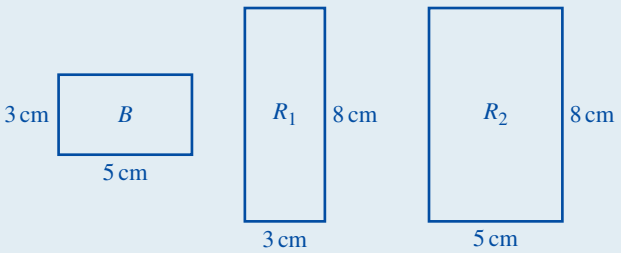
Find the surface area of the rectangular prism (cuboid) shown.



THINK

1 There are 6 faces: 2 rectangular bases and 4 rectangular sides. Draw diagrams for each pair of faces and label each region.

WRITE/DRAW



2 Find the area of each rectangle by applying the formula $A = lw$.

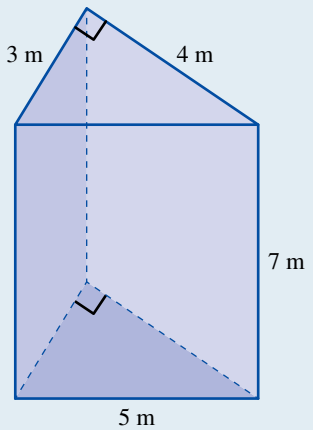
$B = 3 \times 5 \quad R_1 = 3 \times 8 \quad R_2 = 5 \times 8$
$= 15 \qquad = 24 \qquad = 40$

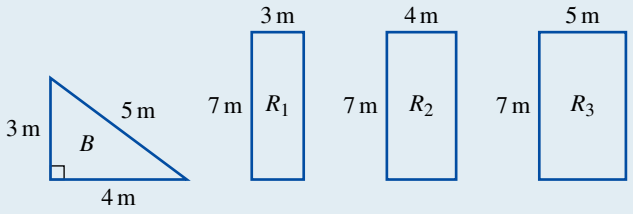
3 The total surface area is the sum of the area of 2 of each shape. Write the answer.

$\text{SA} = 2B + 2 \times R_1 + 2 \times R_2$
$= 30 + 48 + 80$
$= 158 \text{ cm}^2$

WORKED EXAMPLE 14

Find the surface area of the right-angled triangular prism shown.



THINK	WRITE/DRAW
1 There are 5 faces: 2 triangles and 3 rectangles. Draw diagrams for each face and label each region.	
2 Find the area of the triangular base by applying the formula $A = \frac{1}{2}bh$.	$B = \frac{1}{2} \times 3 \times 4$ $= 6 \text{ m}^2$
3 Find the area of each rectangular face by applying the formula $A = lw$.	$R_1 = 3 \times 7 = 21$ $R_2 = 4 \times 7 = 28$ $R_3 = 5 \times 7 = 35$
4 The total surface area is the sum of all the areas of the faces, including 2 bases.	$SA = 2B + R_1 + R_2 + R_3$ $= 12 + 21 + 28 + 35$ $= 96 \text{ m}^2$

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Surface area of prisms (int-2771)

Complete this digital doc: SkillSHEET: Surface area of cubes and rectangular prisms (doc-10876)

Complete this digital doc: SkillSHEET: Surface area of triangular prisms (doc-10877)

Complete this digital doc: WorkSHEET: Surface area (doc-6304)

Exercise 12.5 Surface area of rectangular and triangular prisms

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–c, 2–15, 17	Questions: 1b–d, 2b, 3b, 4–17	Questions: 1d–f, 2c, 3c, 4–18

Individual pathway interactivity: int-4531

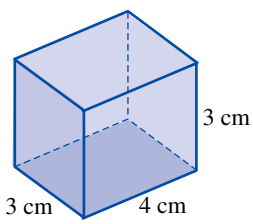
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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

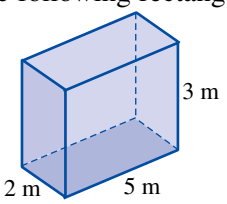
Fluency

1. WE13 Find the surface area (SA) of the following rectangular prisms (cuboids).

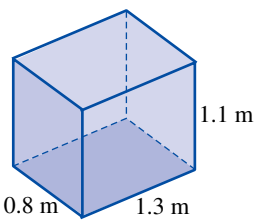
a.



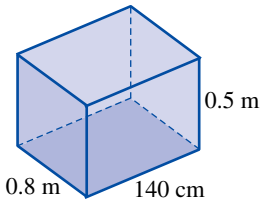
b.



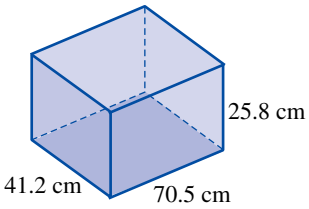
c.



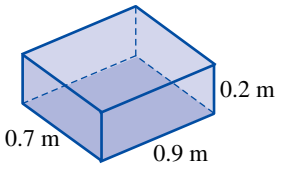
d.



e.

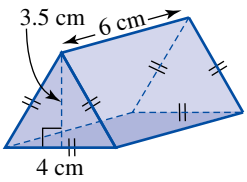


f.

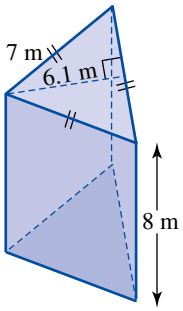


2. **WE14** Find the surface area (SA) of each of the triangular prisms below.

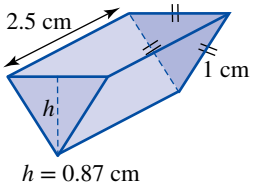
a.



b.

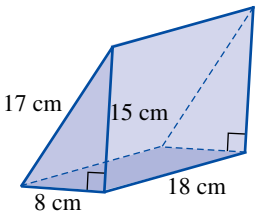


c.

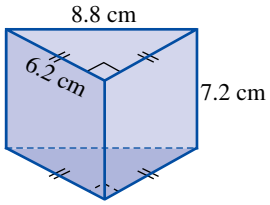


3. Find the surface area (SA) of each of the triangular prisms below.

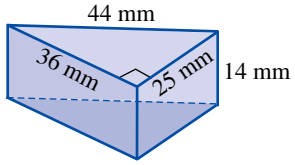
a.



b.



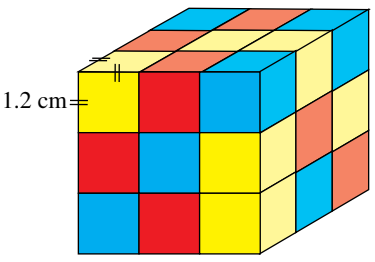
c.



4. Maya is planning to buy and paint the outside surface of one of these shipping containers. How many cans of paint should Maya buy, if the base of the container is not painted, and each can of paint covers about 40 m^2?

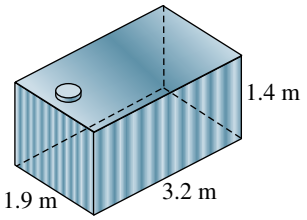


5. The aim of the Rubik's cube puzzle is to make each face of the cube one colour. Find the surface area of the Rubik's cube if each small coloured square is 1.2 cm in length. Assume that there are no gaps between the squares.



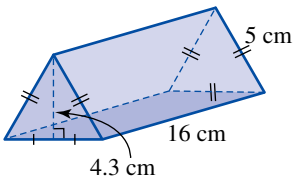
6. How many square metres of iron sheet are needed to construct the water tank shown?

7. What is the surface area of the tank in the previous question if no top is made?

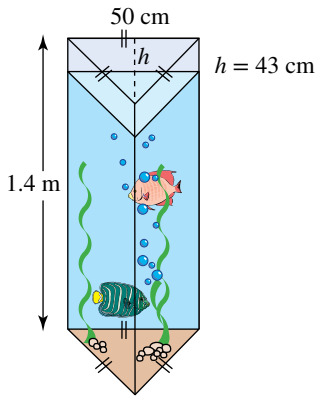


Understanding

8. What area of cardboard would be needed to construct a box to pack this prism, assuming that no overlap occurs?



9. An aquarium is a triangular prism with the dimensions shown. The top of the tank is open. What area of glass was required to construct the tank? Give your answer correct to 2 decimal places.

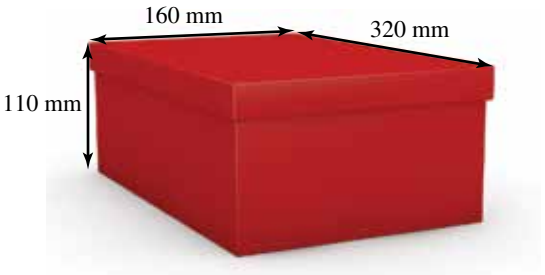


10. A tent is constructed as shown. What area of canvas is needed to make the tent, if a floor is included?



11. How many square centimetres of cardboard are needed to construct the shoebox at right, assuming no overlap and ignoring the overlap on the top? Draw a sketch of a net that could be used to make the box.

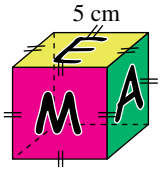
12. Find the surface area of a square-based prism of height 4 cm, given that the side length of its base is 6 cm.



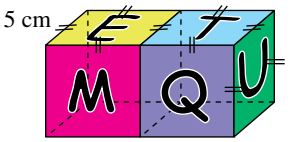
13. A prism has an equilateral triangular base with a perimeter of 12 cm. If the length of the prism is 24 cm, determine the total surface area of the prism. (*Hint:* What is the area of 1 triangle?)

Reasoning

14. a. Find the surface area of the toy block shown.



b. If two of the blocks are placed together as shown, what is the surface area of the prism which is formed?



c. What is the surface area of the prism formed by three blocks?

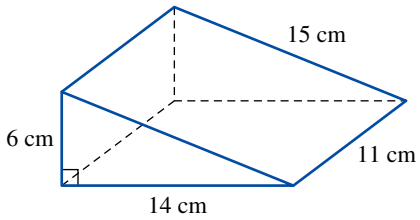
d. Use the pattern to determine the surface area of a prism formed by eight blocks arranged in a line. Explain your reasoning.

15. A cube has a side length of 2 cm. Show that the least surface area of a solid formed by joining eight such cubes is 96 cm^2.

Problem solving

16. Ken wants to paint his son's bedroom walls blue and the ceiling white. The room measures 3 m by 4 m with a ceiling height of 2.6 m. There is one 1 m by 2 m door and one 1.8 m by 0.9 m window. Each surface takes two coats of paint and 1 L of paint covers 16 m^2 on the walls and 12 m^2 on the ceiling. Cans of wall paint cost $33.95 for 1 L, $63.90 for 4 L, $147 for 10 L and $174 for 15 L. Ceiling paint costs $24 for 1 L and $60 for 4 L. What is the least it would cost Ken to paint the room?

17. A wedge in the shape of a triangular prism, as drawn below, is to be painted.



a. Draw a net of the wedge so that it is easier to calculate the area to be painted.

b. What area is to be painted? (Do not include the base.)

18. A swimming pool has a length of 50 m and a width of 28 m. The shallow end of the pool has a depth of 0.80 m, which increases steadily to 3.8 m at the deep end.

a. Calculate how much paint would be needed to paint the floor of the pool.

b. If the pool is to be filled to the top, how much water will be needed?

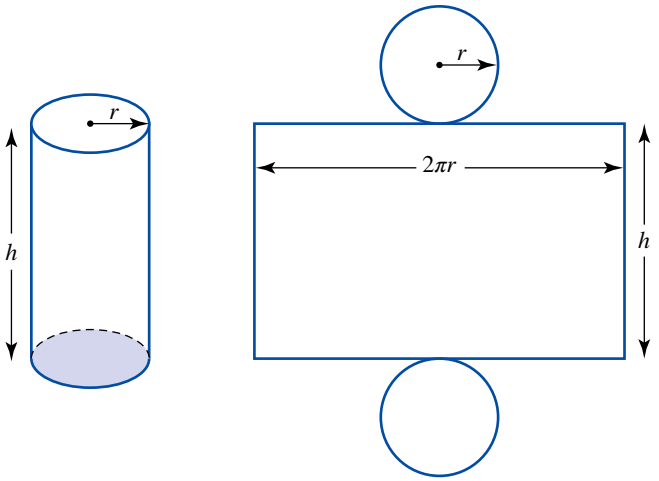
Reflection

What is the quickest method of calculating the total surface area of a six-sided box?

12.6 Surface area of a cylinder

12.6.1 Surface area of a cylinder

- A **cylinder** is a solid object with two identical flat circular ends and one curved side. It has uniform cross-section.
- The net of a cylinder has two circular bases and one rectangular face. The rectangular face is the curved surface of the cylinder.

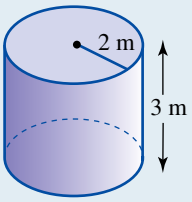


- Because the rectangle wraps around the circular base, the width of the rectangle is the same as the circumference of the circle. Therefore, the width is equal to $2\pi r$.
- The area of each base is πr^2, and the area of the rectangle is $2\pi rh$. Because there are two bases, the surface area of the cylinder is given by:

$$\begin{aligned} \text{SA} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h) \end{aligned}$$

WORKED EXAMPLE 15

a **Use the formula $A = 2\pi rh$ to calculate the area of the curved surface of the cylinder, correct to 1 decimal place.**
b **Use the formula $\text{SA} = 2\pi rh + 2\pi r^2$ to calculate the surface area of the cylinder, correct to 1 decimal place.**



THINK	WRITE
a 1 Write the formula for the curved surface area.	**a** $A = 2\pi rh$
2 Identify the values of the pronumerals.	$r = 2,\ h = 3$
3 Substitute $r = 2$ and $h = 3$.	$A = 2 \times \pi \times 2 \times 3$
4 Evaluate, round to 1 decimal place and include units.	≈ 37.69 $\approx 37.7 \text{ m}^2$
b 1 Write the formula for the surface area of a cylinder.	**b** $\text{SA} = 2\pi rh + 2\pi r^2$
2 Identify the values of the pronumerals.	$r = 2,\ h = 3$
3 Substitute $r = 2$ and $h = 3$.	$\text{SA} = (2 \times \pi \times 2 \times 3) + (2 \times \pi \times 2^2)$
4 Evaluate, round to 1 decimal place and include units.	≈ 62.83 $\approx 62.8 \text{ m}^2$

Exercise 12.6 Surface area of a cylinder

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 9, 11, 13	Questions: 1–7, 10, 12–14, 16	Questions: 1–6, 8–16

Individual pathway interactivity: int-4532

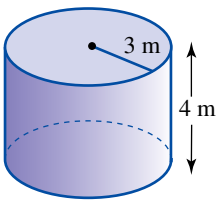
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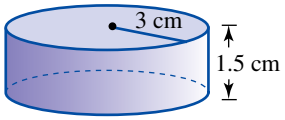
Fluency

1. WE15a Use the formula $A = 2\pi rh$ to find the area of the curved surface of each of the cylinders below. (Express your answers correct to 1 decimal place.)

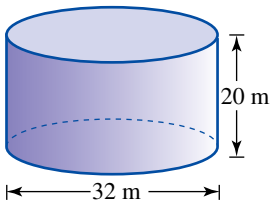
a.



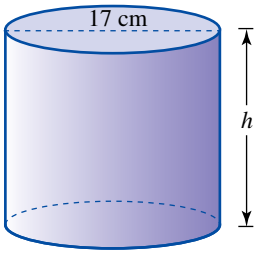
b.



c.

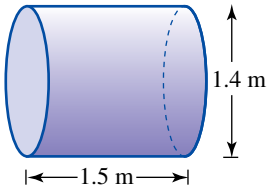


d.

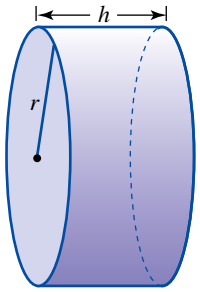


$h = 21$ cm

e.



f.



$r = 2.4$ m
$h = 1.7$ m

2. WE15b Use the formula $SA = 2\pi rh + 2\pi r^2$ to find the total surface area of each of the cylinders in question **1**.

3. A can containing an energy drink has a height of 130 mm and a radius of 24 mm. Draw the net of the can. (*Hint:* Plan ahead.)

4. A can of asparagus spears is 137 mm tall and has a diameter of 66 mm, a can of tomatoes is 102 mm tall and has a diameter of 71 mm, and a can of beetroot is 47 mm tall with a diameter of 84 mm.

a. Which can has the largest surface area? Which can has the smallest surface area?

b. What is the difference between the largest and smallest surface areas, correct to the nearest cm^2?

5. A cylinder has a radius of 15 cm and a height of 45 mm. Determine its surface area correct to 1 decimal place.

6. If the radius of a cylinder is twice its height, write a formula for the surface area in terms of its height only.



Understanding

7. A cylinder has a surface area of 2000 cm^2 and a radius of 8 cm. Determine the cylinder's height correct to 2 decimal places.

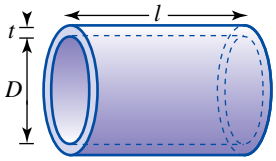
8. A 13 m-high storage tank was constructed from stainless steel (including the lid and the base). The diameter is 3 metres as shown.



a. What is the surface area of the tank? Give your answer correct to 2 decimal places.

b. How much did the steel cost for the side of the tank if it comes in sheets 1 m wide that cost $60 a metre?

9. The concrete pipe shown in the diagram has the following measurements: $t = 30$ mm, $D = 18$ cm, $l = 27$ cm.



Find the following areas. Give your answers correct to 2 decimal places.

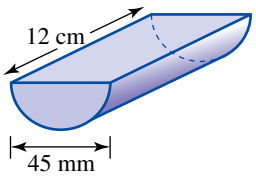
a. Calculate the outer curved surface area.

b. Calculate the inner curved surface area.

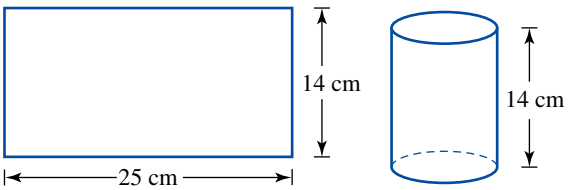
c. Calculate the total surface area of both ends.

d. Hence calculate the surface area for the entire shape.

10. Wooden mouldings are made by cutting cylindrical dowels in half as shown at right. Calculate the surface area of the moulding.



11. Kiara has a rectangular sheet of cardboard with dimensions 25 cm by 14 cm. She rolls the cardboard to form a cylinder so that the shorter side, 14 cm, is its height, and glues the edges together with a 1-cm overlap.



a. What is the radius of the circle Kiara needs to construct to put at the top of her cylinder? Give your answer correct to 2 decimal places.

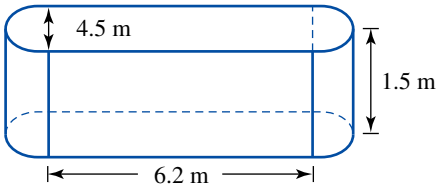
b. What is the total surface area of her cylinder if she also makes the top and bottom of her cylinder out of cardboard? Give your answer correct to 1 decimal place.

Reasoning

12. Cylinder A has a 10% greater radius and a 10% greater height compared with Cylinder B. Show that the ratio of their surface areas is 121 : 100.

Problem solving

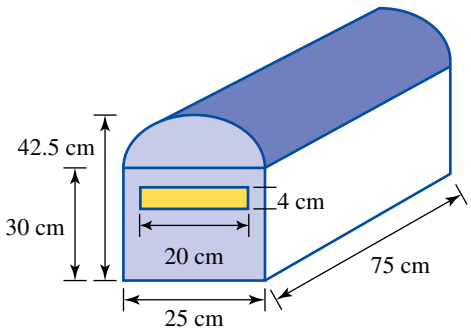
13. An above-ground swimming pool has the following shape, with semicircular ends.



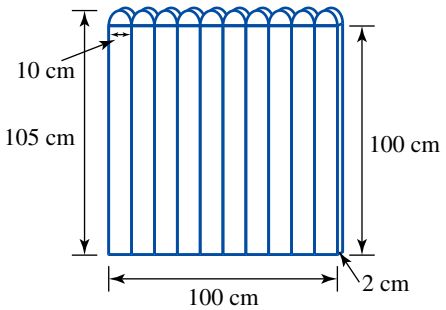
How much plastic would be needed to line the base and sides of the pool?

14. A over-sized solid wooden die is constructed for a children's playground. The side dimensions of the die are 50 cm. The number on each side of the die will be represented by cylindrical holes which will be drilled out of each side. Each hole will have a diameter of 10 cm and depth of 2 cm. All surfaces on the die will be painted (including the die holes). Show that the total area required to be painted is 1.63 m^2.

15. The following letterbox is to be spray-painted on the outside. What is the total area to be spray-painted? Assume that the end of the letterbox is a semicircle above a rectangle. The letter slot is open and does not require painting.



16. A timber fence is designed as shown below. How many square metres of paint are required to completely paint the fence front, back, sides and top with 2 coats of paint? Assume each paling is 2 cm in thickness and that the top of each paling is a semicircle.



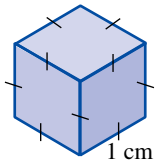
Reflection

Devise an easy way to remember the formula for the surface area of a cylinder.

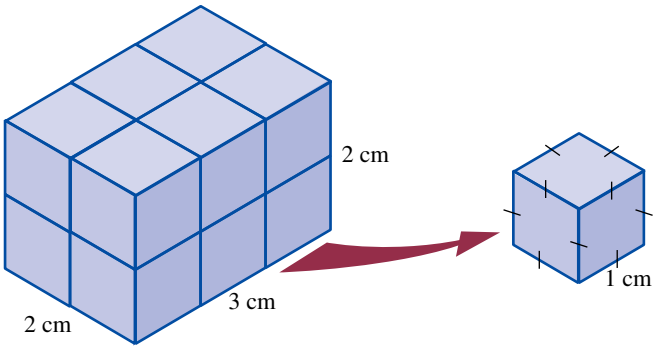
12.7 Volume of prisms and cylinders

12.7.1 Volume

- The diagram at right shows a cube of side length 1 cm. By definition the cube has a **volume** of 1 cm^3 (1 cubic centimetre). *Note:* This is a 'cubic centimetre', not a 'centimetre cubed'.
- The volume of a solid is the amount of space it fills or occupies.



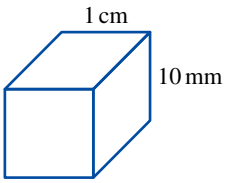
- The volume of some solids can be found by dividing them into cubes with 1-cm sides.



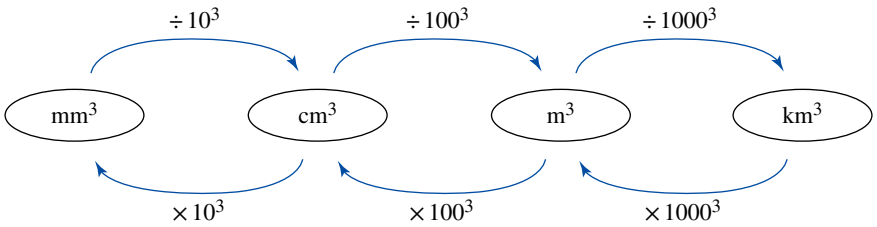
This solid fills the same amount of space as 12 cubic centimetres.
The volume of this cuboid is 12 cm^3.

- Volume is commonly measured in cubic millimetres (mm^3), cubic centimetres (cm^3), cubic metres (m^3) or cubic kilometres (km^3).
- The volume of the cube at right is equal to 1 cm^3 or 1000 mm^3, so

$1\ cm^3 = 1000\ mm^3$,
or $1\ cm^3 = 10^3\ mm^3$.
Similarly $1\ m^3 = 100^3\ cm^3$
and $1\ km^3 = 1000^3\ m^3$.



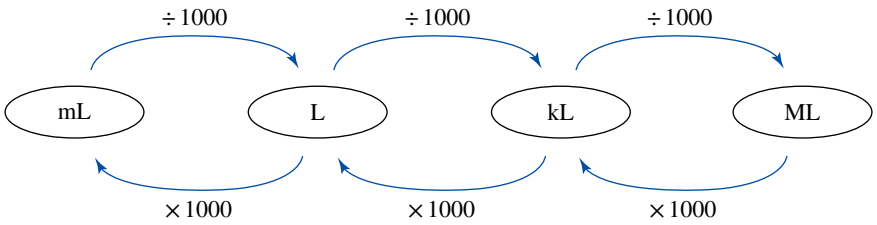
- The chart below is useful when converting from one unit of volume to another.



For example, $3\ m^3 = 3 \times 100^3 \times 10^3\ mm^3$
$= 3000000000\ mm^3$.

12.7.2 Capacity

- **Capacity** is a term usually applied to the measurement of liquids and containers.
- The capacity of a container is the volume of liquid that it can hold.
- The standard measurement for capacity is the litre (L).
  Other common units are the millilitre (mL), kilolitre (kL), and megalitre (ML), where
  1 L = 1000 mL
  1 kL = 1000 L
  1 ML = 1 000 000 L.



- The units of capacity and volume are related as follows:
  $1\,cm^3 = 1$ mL and
  $1\,m^3 = 1000$ L.

WORKED EXAMPLE 16

Convert:
a **13.2 L into cm^3** **b** **3.1 m^3 into litres**
c **0.13 cm^3 into mm^3** **d** **3.8 kL into m^3.**

THINK

a 1 1 L = 1000 mL, so multiply by 1000.

2 1 mL = 1 cm^3. Convert to cm^3.

b $1\,m^3 = 1000$ L, so multiply by 1000.

c There will be more mm^3, so multiply by 10^3.

d 1 kL = 1 m^3. Convert to m^3.

WRITE

a $13.2 \times 1000 = 13\,200$ mL
$= 13\,200\ cm^3$

b $3.1 \times 1000 = 3100$ L

c $0.13 \times 1000 = 130\ mm^3$

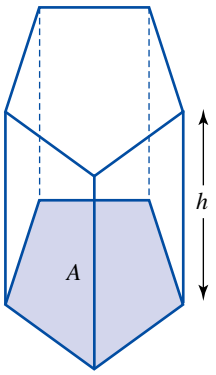
d $3.8\ kL = 3.8\ m^3$

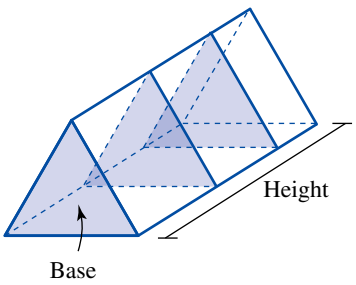
12.7.3 Volume of a prism

- The volume of a prism can be found by multiplying its cross-sectional area (A) by its height (h).

$$\textbf{Volume} = A \times h$$

- The cross-section (A) of a prism is often referred to as the base, even if it is not at the bottom of the prism.
- The height (h) is always measured perpendicular to the base, as shown in the diagram below.

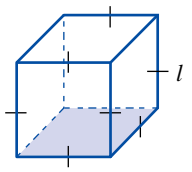




12.7.4 Volume of a cube or a cuboid

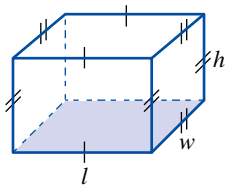
- A specific formula can be developed for the volumes of cubes and cuboids.

Cube



Volume = base area × height
$= l^2 \times l$
$= l^3$

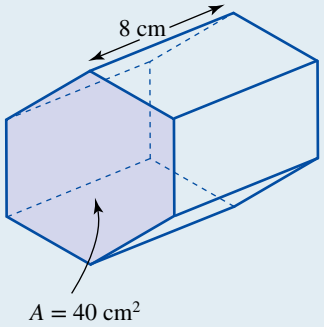
Cuboid (rectangular prism)



Volume = base area × height
$= lw \times h$
$= lwh$

WORKED EXAMPLE 17

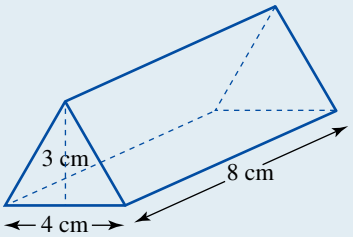
Calculate the volume of the hexagonal prism.



THINK	WRITE
1 Write the formula for the volume of a prism.	$V = Ah$
2 Identify the values of the pronumerals.	$A = 40,\ h = 8$
3 Substitute $A = 40$ and $h = 8$ into the formula and evaluate. Include the units.	$V = 40 \times 8$ $= 320$ cm^3

WORKED EXAMPLE 18

Calculate the volume of the prism.

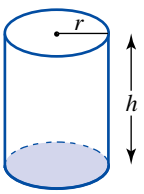


THINK	WRITE
1 The base of the prism is a triangle. Write the formula for the area of the triangle.	$A = \frac{1}{2}bh$
2 Substitute $b = 4$, $h = 3$ into the formula and evaluate.	$= \frac{1}{2} \times 4 \times 3$ $= 6$ cm^3
3 Write the formula for volume of a prism.	$V = A \times h$
4 State the values of A and h.	$A = 6,\ h = 8$
5 Substitute $A = 6$ and $h = 8$ into the formula and evaluate. Include the units.	$V = 6 \times 8$ $= 48$ cm^3

12.7.5 Volume of a cylinder

- A cylinder has a circular base and uniform cross-section.
- A formula for the volume of a cylinder is shown.

Cylinder



Volume = base area × height
= area of circle × height
= $\pi r^2 \times h$
= $\pi r^2 h$

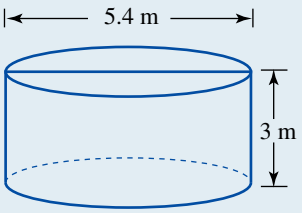
WORKED EXAMPLE 19

Calculate the capacity, in litres, of a cylindrical water tank that has a diameter of 5.4 m and a height of 3 m. (Give your answer correct to the nearest litre.)

THINK

1 Draw a labelled diagram of the tank.

WRITE/DRAW



2 The base is a circle, so $A = \pi r^2$. Write the formula for the volume of a cylinder.

$V = \pi r^2 h$

3 Write the values of r and h.

$r = 2.7,\ h = 3$

4 Substitute $r = 2.7,\ h = 3$ into the formula and find the volume.

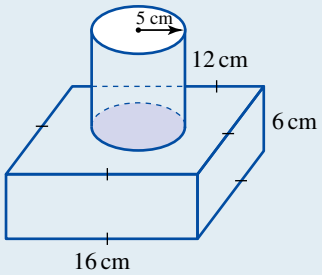
$V = \pi \times 2.7^2 \times 3$
$\approx 68.706\,631\ \text{m}^3$

5 Convert this volume to litres (multiply by 1000).

$V = 68.706\,631 \times 1000$
$= 68\,706.631$
$\approx 68\,707\ \text{L}$

WORKED EXAMPLE 20

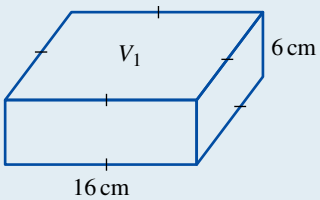
Find the total volume of this solid.

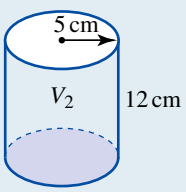


THINK

1 The solid is made from two objects. Draw and label each object.
Let V_1 = volume of the square prism.
Let V_2 = volume of the cylinder.

WRITE/DRAW





2 Find V_1 (square prism).

$V_1 = Ah$
$= 16^2 \times 6$
$= 1536\ \text{cm}^3$

3 Find V_2 (cylinder).

$$V_2 = \pi r^2 h$$
$$= \pi \times 5^2 \times 12$$
$$\approx 942.478 \text{ cm}^3$$

4 To find the total volume, add the volumes found above.

$$V = V_1 + V_2$$
$$= 1536 + 942.478$$
$$= 2478.478$$
$$\approx 2478 \text{ cm}^3$$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Volume of cubes and rectangular prisms (doc-10878)

Complete this digital doc: SkillSHEET: Volume of triangular prisms (doc-10879)

Complete this digital doc: SkillSHEET: Volume of cylinders (doc-10880)

Complete this digital doc: WorkSHEET: Volume (doc-6305)

Exercise 12.7 Volume of prisms and cylinders

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–e, 4a, 5a, b, 6a, b, 7, 9a, b, 10, 13, 14, 20–22	Questions: 1, 3b–f, 4b, 5b, 6b–e, 8, 9c, d, 11, 13–16, 20–23	Questions: 1, 3e–h, 4c, 5c, d, 6d–f, 8, 9e, f, 12–24

Individual pathway interactivity: int-4533

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE16 **a.** Convert the following units into mL.

i. 325 cm^3 **ii.** 2.6 m^3 **iii.** 5.1 L **iv.** 0.63 kL

b. Convert the following units into cm^3.

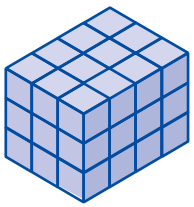
i. 5.8 mL **ii.** 6.1 L **iii.** 3.2 m^3 **iv.** 59.3 mm^3

c. Convert the following units into kL.

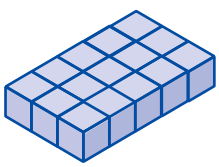
i. 358 L **ii.** 55.8 m^3 **iii.** 8752 L **iv.** 5.3 ML

2. Calculate the volumes of the cuboids below. Assume that each small cube has sides of 1 cm.

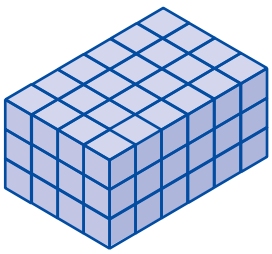
a.



b.

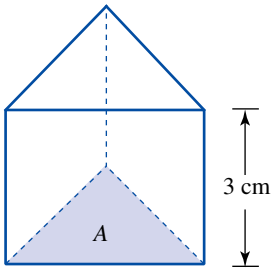


c.



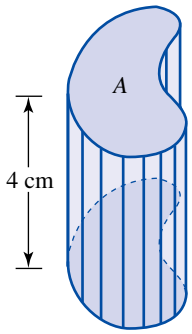
3. WE17 Calculate the volumes of these objects.

a.



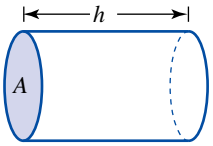
$A = 4\ \text{cm}^2$

b.



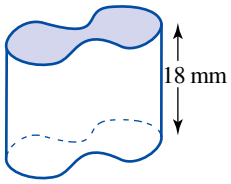
$A = 17\ \text{cm}^2$

c.



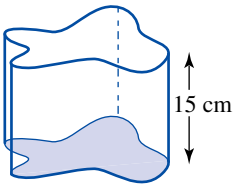
$A = 3.2\ \text{m}^2$
$h = 3.0\ \text{m}$

d.



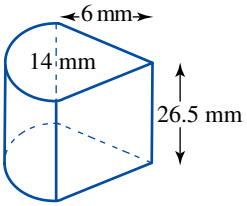
Base area $= 35\ \text{mm}^2$

e.

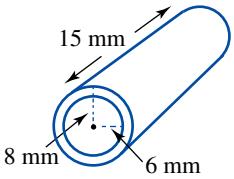


Base area $= 28\ \text{cm}^2$

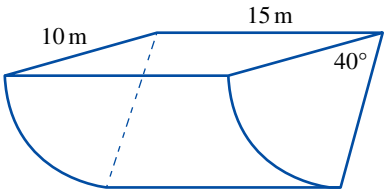
f.



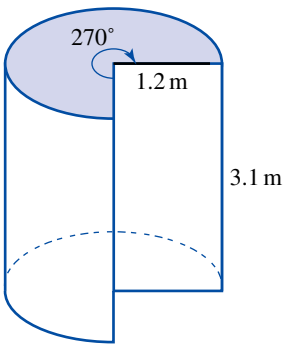
g.



h.

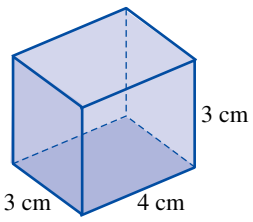


i.

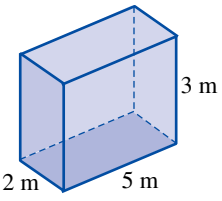


4. Calculate the volume of these rectangular prisms.

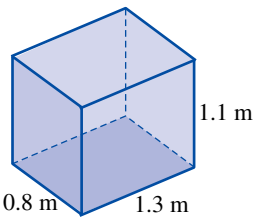
a.



b.

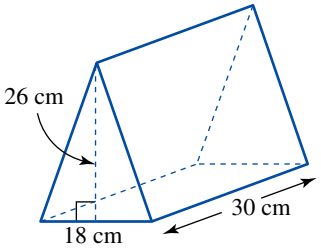


c.

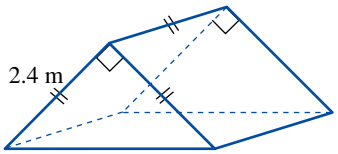


5. WE18 Calculate the volume of the prisms shown below. Give your answers correct to 2 decimal places.

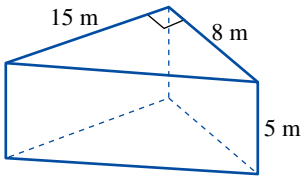
a.



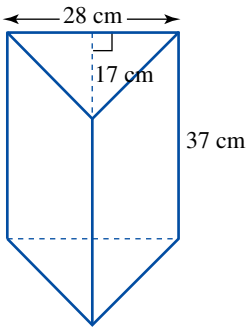
b.



c.

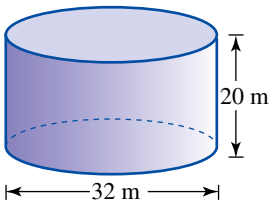


d.

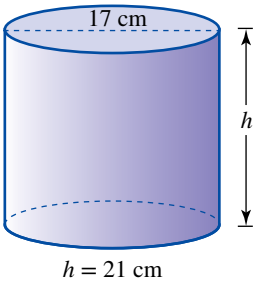


6. Calculate the volume of the following cylinders. Give your answers correct to 1 decimal place.

a.

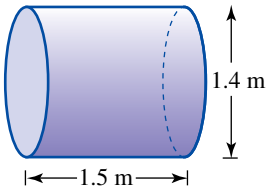


b.

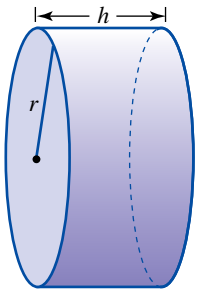


$h = 21$ cm

c.

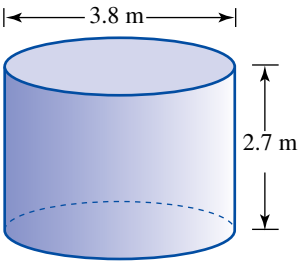


d.

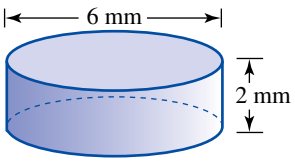


$r = 2.4$ m
$h = 1.7$ m

e.



f.



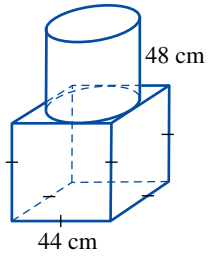
7. WE19 Calculate the volume of water, in litres (L), that can fill a cylindrical water tank that has a diameter of 3.2 m and a height of 1.8 m.

8. What is the capacity in litres of the Esky shown below?

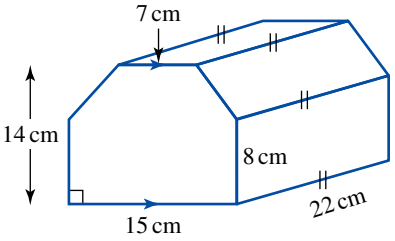


9. WE20 Find the volume of each solid to the nearest cm^3.

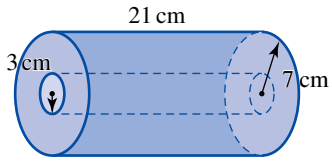
a.



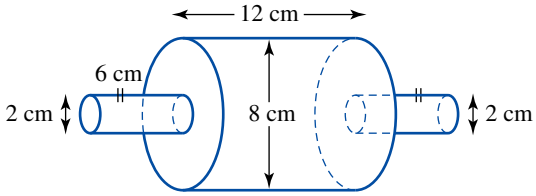
b.



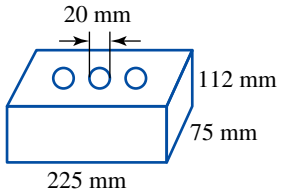
c.



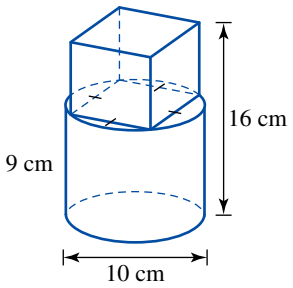
d.



e.



f.



Understanding

10. Calculate the capacity, in litres, of the cylindrical storage tank shown. Give your answer correct to 1 decimal place.



11. What is the capacity (in mL) of this cylindrical coffee plunger when it is filled to the level shown? Give your answer correct to 1 decimal place.



12. Until its closure in 2001, Fresh Kills on Staten Island outside New York City was one of the world's biggest landfill garbage dumps. (New Yorkers throw out about 100 000 tonnes of refuse weekly.) Calculate the approximate volume (m^3) of the Fresh Kills landfill if it covers an area of 1215 hectares and is about 240 m high. (*Note:* 1 hectare = 10 000 m^2.)

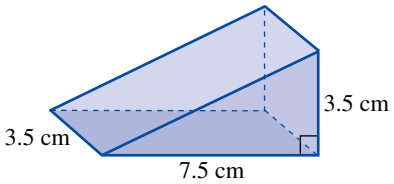


13. Sudhira is installing a rectangular pond in a garden. The pond is 1.5 m wide, 2.2 m long and has a uniform depth of 1.5 m.

a. Calculate the volume of soil (m^3) that Sudhira must remove to make the hole before installing the pond.

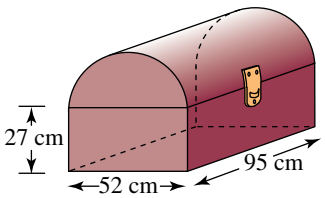
b. What is the capacity of the pond in litres? (Ignore the thickness of the walls in this calculation.)

14. a. Calculate the volume of plastic needed to make the door wedge shown. Give your answer correct to 2 decimal places.



b. The wedges can be packed snugly into cartons 45 cm × 70 cm × 35 cm. How many wedges fit into each carton?

15. Calculate the internal volume (in cm^3) of the wooden chest shown correct to 2 decimal places. (Ignore the thickness of the walls.)



16. An internal combustion engine consists of 4 cylinders. In each cylinder a piston moves up and down. The diameter of each cylinder is called the bore and the height that the piston moves up and down within the cylinder is called the stroke (stroke = height).

a. If the bore of a cylinder is 84 mm and the stroke is 72 mm, calculate the volume (in litres) of 4 such cylinders correct to 2 decimal places.

b. When an engine gets old, the cylinders have to be 're-bored', that is, the bore is increased by a small amount (and new pistons put in them). If the re-boring increases the diameter by 1.1 mm, what is the increase (in litres) of the volume of the 4 cylinders? Give your answer correct to 3 decimal places.

17. a. Calculate the volume, in m^3, of the refrigerator shown.

b. What is the capacity of the refrigerator if the walls are all 5 cm thick?

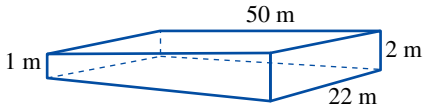


Reasoning



18. A cylindrical glass is designed to hold 1.25 L. Show that its height is 132 mm if it has a diameter of 110 mm.

19. Mark is responsible for the maintenance of the Olympic (50 m) pool at an aquatic centre. The figure below shows the dimensions of an Olympic pool.



a. What is the shape of the pool?

b. Draw the cross-section of the prism and calculate its area.

c. Show that the capacity of the pool is 1 650 000 L.

d. Mark needs to replace the water in the pool every 6 months. If the pool is drained at 45 000 L per hour and refilled at 35 000 L per hour, how long will it take to:

i. drain **ii.** refill (in hours and minutes)?

20. Use examples to show that the volume of a cone is equal to $\frac{1}{3}$ the volume of the cylinder it can be taken from.

21. Use examples to show that the volume of a pyramid is equal to $\frac{1}{3}$ the volume of the rectangular prism it can be taken from.

Problem solving

22. A cylindrical container of water has a diameter of 16 cm and is 40 cm tall. How many full cylindrical glasses can be filled from the container if the glasses have a diameter of 6 cm and are 12 cm high?

23. a. A square sheet of metal with dimensions 15 cm by 15 cm has a 1-cm square cut out of each corner. The remainder of the square is folded to form an open box. Calculate the volume of the box.

b. Write a general formula for calculating the volume of a box created from a metal sheet of any size with any size square cut out of the corners.

Reflection

Why is a cylinder often classified with prisms?

CHALLENGE 12.2

A foam cube has a side length of 10 cm. What increase in side length would be necessary to double the volume of the cube?

12.8 Review

12.8.1 Review questions

Fluency

1. Which of the following is true?

A. 5 cm is 100 times as big as 5 mm.

B. 5 metres is 100 times as big as 5 cm.

C. 5 km is 1000 times as small as 5 metres.

D. 5 mm is 100 times as small as 5 metres.

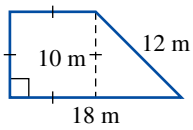
2. The circumference of a circle with a diameter of 12.25 cm is:

A. 471.44 cm **B.** 384.85 mm

C. 76.97 cm **D.** 117.86 cm

3. The area of the following shape is:

A. 216 m^2 **B.** 140 m^2 **C.** 150 m^2 **D.** 90 m^2

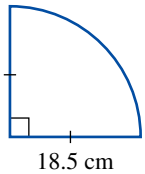


4. The area of a circle with diameter 7.5 cm is:

A. 176.7 cm^2 **B.** 47.1 cm^2 **C.** 23.6 cm^2 **D.** 44.2 cm^2

5. The perimeter of the shape shown is:

A. 1075.2 cm **B.** 55.5 cm **C.** 153.2 cm **D.** 66.1 cm



6. The surface area and volume of a cube with side length 7 m are respectively:

A. 294 m^2, 343 m^3 **B.** 49 m^2, 343 m^3 **C.** 147 m^2, 49 m^3 **D.** 28 m^2, 84 m^3

7. The surface area of a rectangular box with dimensions 7 m, 3 m, 2 m is:

A. 41 m^2 **B.** 42 m^2 **C.** 72 m^2 **D.** 82 m^2

8. The surface area and volume of a cylinder with radius 35 cm and height 40 cm are:

A. 16 493.36 cm^2 and 153 938 cm^3 **B.** 8796.5 cm^2 and 11 246.5 cm^3

C. 153 938 cm^2 and 11 246.5 cm^3 **D.** 8796.5 cm^2 and 153 938 cm^3

9. Fill in the gaps by converting the units of length.

a. 26 mm = _____ cm **b.** 1385 mm = _____ cm

c. 1.63 cm = _____ mm **d.** 1.5 km = _____ m

e. 0.077 km = _____ m **f.** 2850 m = _____ km

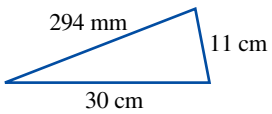
10. Calculate the cost of 1.785 km of cable, if the cable costs $4.20 per metre.

11. Calculate the circumference of circles with the following dimensions (correct to 1 decimal place).

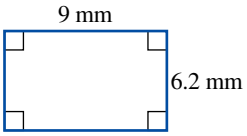
a. radius 4 cm **b.** radius 5.6 m **c.** diameter 12 cm

12. Calculate the perimeter of the following shapes (correct to 1 decimal place).

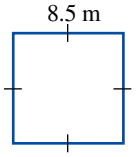
a.



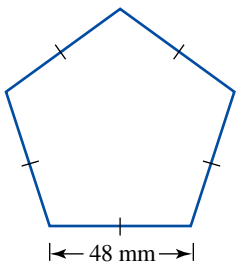
b.



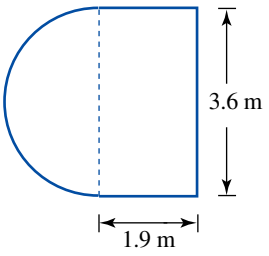
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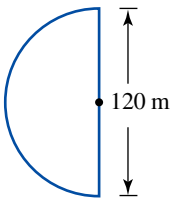
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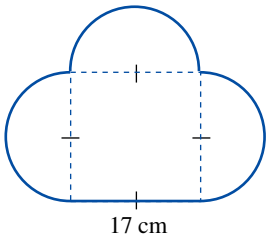
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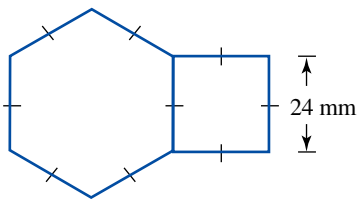
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g.

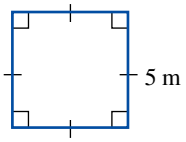


h.

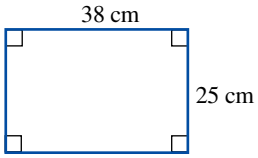


13. Calculate the area of the following shapes.

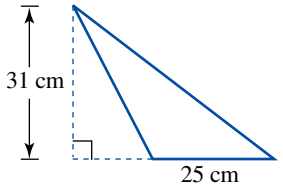
a.



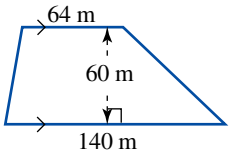
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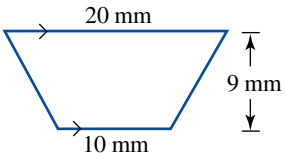
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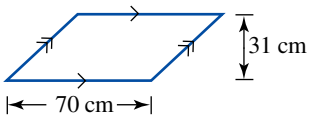
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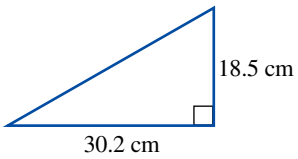
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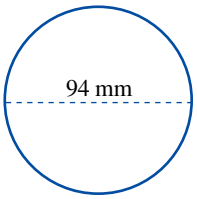
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g.

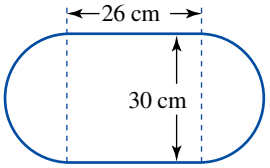


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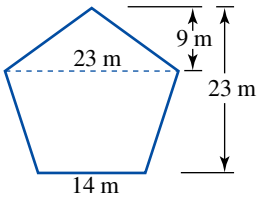


14. Calculate the area of the following 2-dimensional shapes by dividing them into simpler shapes. (Where necessary, express your answer correct to 1 decimal place.)

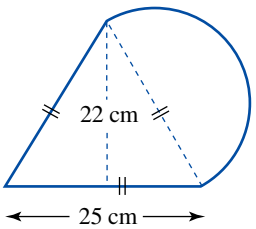
a.



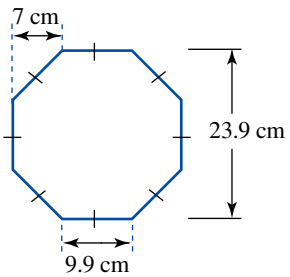
b.



c.

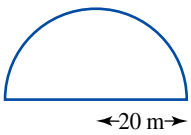


d.

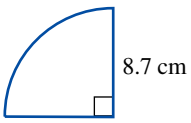


15. Calculate the area of the sectors below (correct to 2 decimal places).

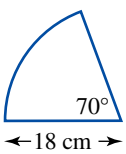
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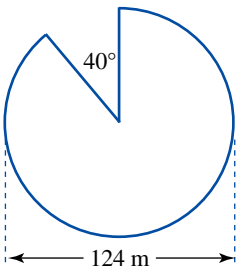
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c.

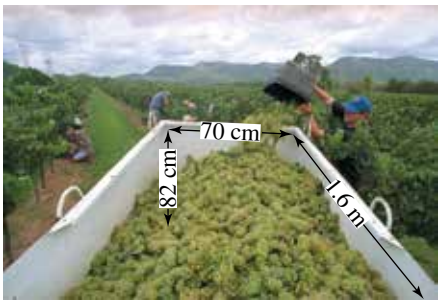


d.



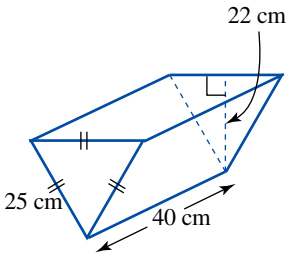
16. Calculate the perimeter of the sectors in question **15**, correct to 2 decimal places.

17. Calculate the inner surface area of the grape collecting vat using the dimensions for length, width and depth shown.



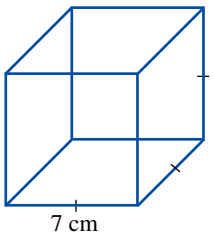
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18. Calculate the surface area of the triangular prism below.

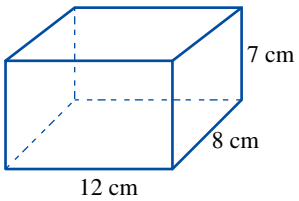


19. Calculate the volume of each of the following, giving your answers to 2 decimal places where necessary.

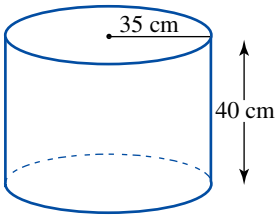
a.



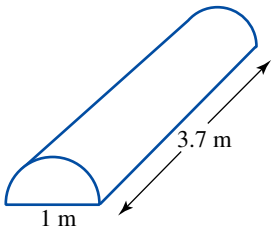
b.



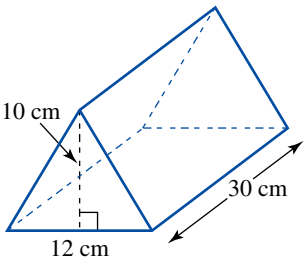
c.



d.



e.



Problem solving

1. Calculate the cost of painting an outer surface (including the lid) of a cylindrical water tank, which has a radius of 2.2 m and a height of 1.6 m, if the paint costs $1.90 per square metre.

2. A harvester travels at 11 km/h. The comb (harvesting section) is 8.7 m wide. How many hectares per hour can be harvested with this comb?

3. A block of chocolate is in the shape of a triangular prism. The face of the prism is an equilateral triangle of 5-cm sides, and the length is 22 cm.

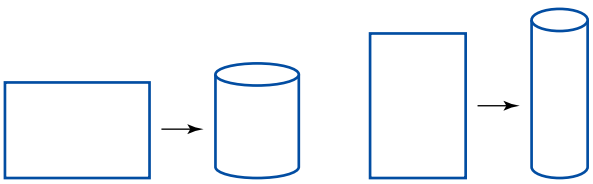
a. Determine the area of one of the triangular faces.

b. Calculate the total surface area of the block.

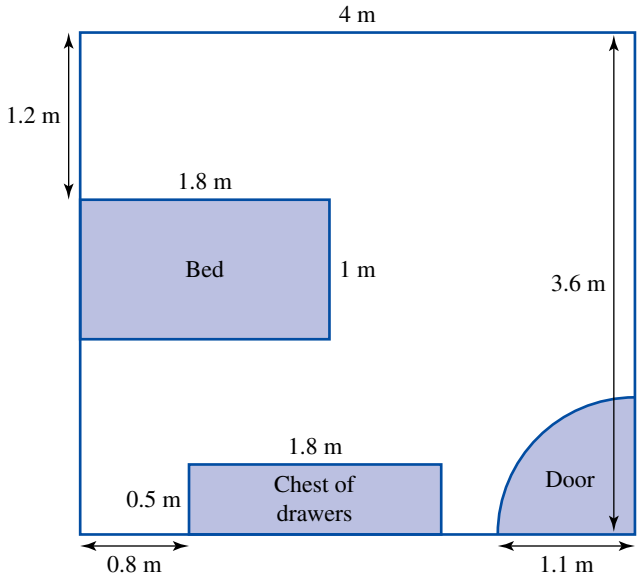
c. Calculate the volume of the block.

d. If it cost $0.025 per cm^3 to produce the block, what is the total production cost?

4. An A4 sheet of paper has dimensions 210 mm × 297 mm and can be rolled two different ways (by rotating the paper) to make baseless cylinders as illustrated at right.
Which cylinder has the greater volume?



5. Nina decides to invite some friends for a sleepover. She plans to have her guests sleep on inflatable mattresses with dimensions 60 cm by 170 cm. A plan of Nina's bedroom is shown below.



a. What area of floor space is available?

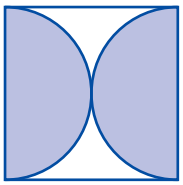
b. The area of an inflatable mattress is 1.02 m^2. Nina's best friend suggests that by dividing the available floor space by the area of an inflatable mattress the number of mattresses that can fit into the bedroom can be calculated. Is the friend correct?

c. How many self-inflatable mattresses can fit in the room as it is?

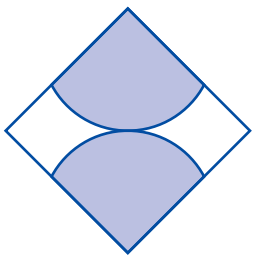
d. How would this change if the bed could be moved within the bedroom?

6. These congruent squares have shaded parts of circles inside them.

i.



ii.



Compare the shaded area in figure **i** with that in figure **ii**.

7. An isosceles triangle has its base on the side of a square. The areas of the triangle and the square are equal. What is the length of one of the equal sides of the isosceles triangle in terms of the length of one side of the square? Give an exact answer.

8. The widespread manufacture of soft drinks in cans only really started in the 1960s. Today cans of soft drink are very common. They can be purchased in a variety of sizes. You can also purchase them as a single can or bulk-buy them in larger quantities. Does the arrangement of the cans in a multi-pack affect the amount of packaging used to wrap them?

For this investigation we will look at the packaging of 375-mL cans. This can has a radius of approximately 3.2 cm and a height of approximately 12 cm. We will consider packaging these cans in cardboard in the shape of a prism, ignoring any overlap for glueing.

a. A 6-can pack could be packed in a single row of cans.

i. Draw a diagram to show what this pack would look like.

ii. Calculate the amount of cardboard required for the package.

b. The cans could also be packaged in a 3 × 2 arrangement.

i. Show the shape of this pack.

ii. Calculate the amount of cardboard required for this pack.

c. Which of the two 6-can pack arrangements is the more economical in terms of its packaging?
d. These cans can be packaged as a single layer of 4×3. How much cardboard would this require?
e. They could also have a two-level arrangement of 3×2 each layer. What packaging would this require?
f. It is also possible to have a three-level arrangement of 2×2 each layer. How much packaging would be needed in this case?
g. Comment on the best way to package a 12-can pack so that it uses the minimum amount of wrapping. Provide diagrams and mathematical evidence to support your conclusion.

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Try out this interactivity: Word search: Topic 12 (int-2708)
Try out this interactivity: Crossword: Topic 12 (int-2709)
Try out this interactivity: Sudoku: Topic 12 (int-3211)
Complete this digital doc: Concept map: Topic 12 (doc-10802)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

3-dimensional
arc
area
capacity
circle
circumference
composite shape
cross-section
cube
cuboid
cylinder
hectare
length
net
plane figure
perimeter
prism
quadrant
radii
scientific notation
sector
semicircle
total surface area
volume

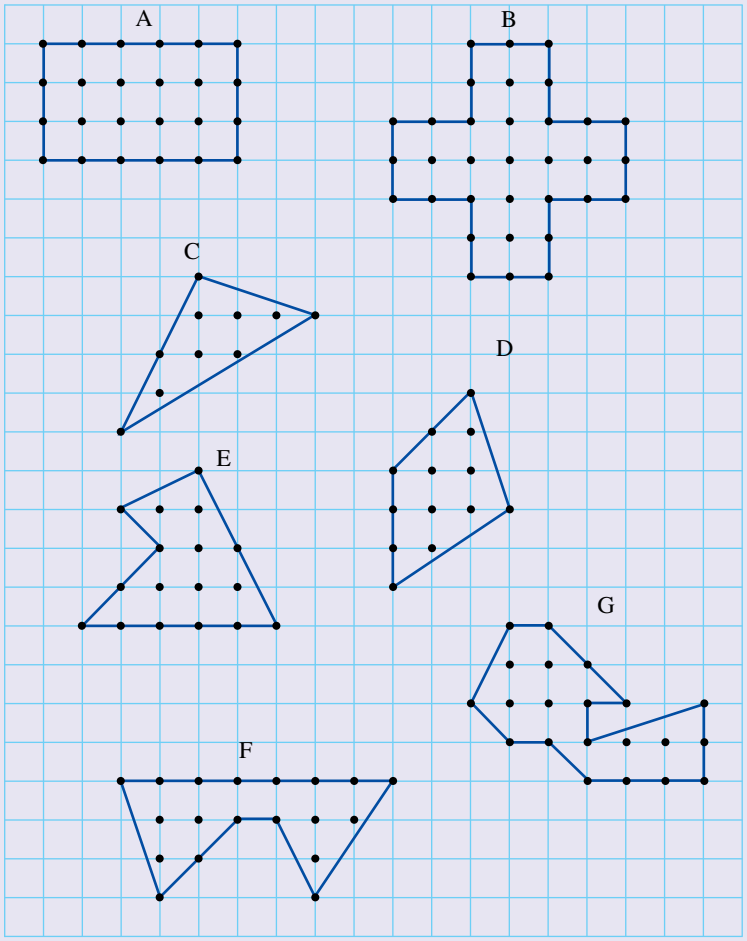
assessON
Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.
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Investigation | Rich task

Areas of polygons

The area of plane figures can be found using a formula. For example, if the figure is a rectangle, its area is found by multiplying its length by its width. If the figure is a triangle, the area is found by multiplying half the base length by its perpendicular height. If the figures are complex, break them down into simple shapes and find the area of these shapes. The area of a complex shape is the sum of the area of its simple shapes. Drawing polygons onto grid paper is one method that can be used to determine their area.

Consider the following seven polygons drawn on 1-cm grid paper.



1. By counting the squares or half-squares, determine the area of each polygon in cm^2.
In some cases, it may be necessary to divide some sections into half-rectangles in order to determine the exact area.

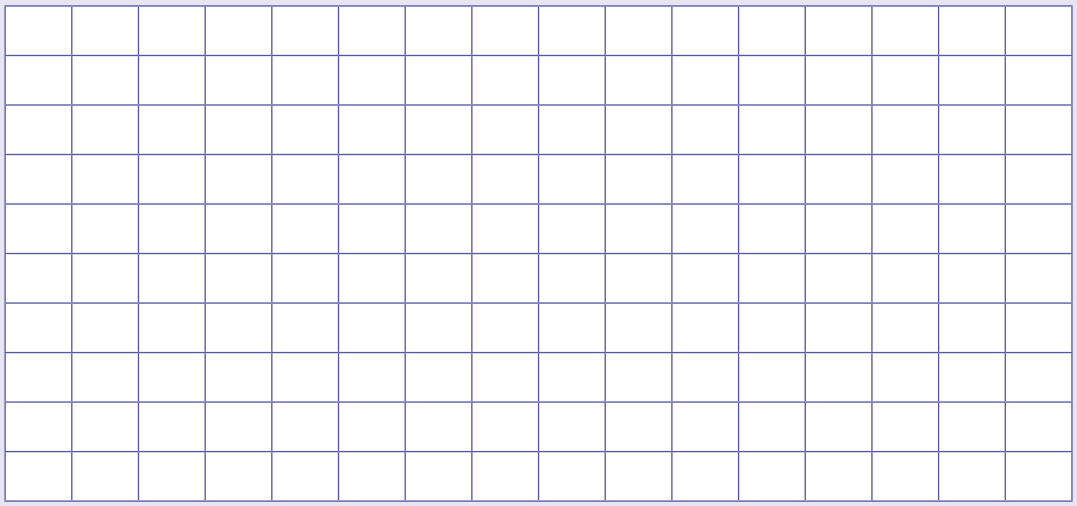
Formulas define a relationship between dimensions of figures. In order to search for a formula to find the area of polygons drawn on the grid paper, consider the next question.

2. For each of the polygons provided above, complete the table on top of the next page. Count the number of dots on the perimeter of each polygon and count the number of dots that are within the perimeter of each polygon.

Polygon	Dots on perimeter (b)	Dots within perimeter (i)	Area of polygon (A)
A			
B			
C			
D			
E			
F			
G			

3. Choose a pronumeral to represent the headings in the table. Investigate and determine a relationship between the area of each polygon and the dots on and within the perimeter. Test that the relationship determined works for each polygon. Write the relationship as a formula.

4. Draw some polygons on the grid paper provided below. Use your formula to determine the area of each shape. Confirm that your formula works by counting the squares in each polygon. The results of both methods (formula and method) should be the same.



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Complete this digital doc: Code puzzle: When is a wall absolutely plumb? (doc-15907)

Answers

Topic 12 Measurement

Exercise 12.2 Measurement

Note: **Answers may vary due to screen resolution.**

1. a. 300 000=3.0 × 10^5 km/s **b.** 1.08 × 10^9 km

c. 1.8144 × 10^{11} km **d.** 9.434 88 × 10^{12} km

e. NGC 6782 is 1.726 583 04 × 10^{21} km from Earth.

2. a. 1.2 × 10^4 **b.** 2.16 × 10^6 frames **c.** 1440 frames per second

3. Answers will vary.

4. a. 50 mm **b.** 152 cm **c.** 0.0125 m **d.** 32.2 mm

e. 0.006 57 km **f.** 0.000 64 km **g.** 14.35 mm **h.** 0.000 183 5 km

5. a. 117 mm **b.** 67 mm **c.** 75 mm **d.** 122 mm

6. a. 1060 cm **b.** 85.4 cm **c.** 206 cm

d. 78.4 cm **e.** 113 cm **f.** 13 cm

7. a. 9.6 cm **b.** 46 mm **c.** 31 km

8. a. 25.13 cm **b.** 25.13 m **c.** 69.12 mm

d. 44.61 cm **e.** 19 741.77 km **f.** 3314.38 mm

9. a. 192 m **b.** 1220 mm **c.** 260 cm

d. 74 mm **e.** 9.6 km **f.** 8 m

10. B

11. a. 1800 mm, 2100 mm, 2400 mm, 2700 mm, 180 cm, 210 cm, 240 cm, 270 cm 1.8 m, 2.1 m, 2.4 m, 2.7 m

b. 4 **c.** 3300 mm

12. a. 1990 m **b.** 841 m

13. a. 127.12 cm **b.** 104.83 cm **c.** 61.70 cm **d.** 8 m

e. 480 mm **f.** 405.35 cm **g.** 125.66 cm **h.** 245.66 m

i. 70.41 cm **j.** 138 mm

14. 222.5 m **15.** 37.5 km **16.** 12 m

17. Answers will vary.

18. a. Answers will vary. **b.** 0.000 02 sec **c.** 2 pictures

19. a. 155.62 m **b.** Answers will vary.

20. a. 40 212 km **b.** 1676 km/h **c.** Answers will vary.

21. Yes, 42 cm of wire remains.

22. To start, flip both timers over and put the pasta in the water.

When the four-minute timer runs out, flip it back over immediately. (Total time: 4 minutes)

When the seven-minute timer runs out, flip that back over immediately too. (Total time: 7 minutes)

The four-minute timer will run out again. Flip the *seven*-minute timer back over. (Total time: 8 minutes)

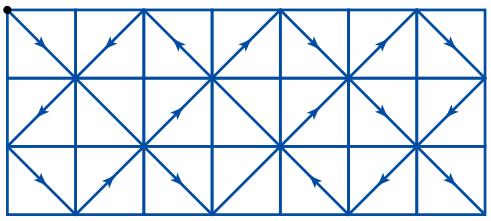
The seven-minute timer had only been running for a minute, so it will only run for a minute more before running out. (Total time: 9 minutes)

There are other ways.

23. a. $(2\sqrt{13} + 4)$ m **b.** $2\sqrt{17}$ m

24. 69.3 cm

25. This is the journey of the ball.



The ball travels along the diagonal of each of the squares. The length of each diagonal is $5\sqrt{2}$ cm. Total length travelled $= 21 \times 5\sqrt{2}$ cm $= 105\sqrt{2}$ cm

Exercise 12.3 Area

Note: **Answers may vary due to screen resolution.**

1. D

2. a. 13 400 m^2 = 0.0134 km^2 **b.** 0.04 cm^2 = 4 mm^2
c. 3 500 000 cm^2 = 350 m^2 **d.** 0.005 m^2 = 50 cm^2
e. 0.043 km^2 = 43 000 m^2 **f.** 200 mm^2 = 2 cm^2
g. 1.41 km^2 = 141 ha **h.** 3800 m^2 = 0.38 ha

3. a. 24 cm^2 **b.** 16 mm^2 **c.** 537.5 cm^2
d. 149.5 cm^2 **e.** 16.32 m^2 **f.** 11.25 cm^2
g. 292.5 cm^2 **h.** 2.5 cm^2 **i.** 1250 m^2
j. 50.27 m^2 **k.** 3.14 mm^2 **l.** 36.32 m^2
m. 15.58 cm^2 **n.** 4.98 cm^2 **o.** 65 m^2

4. a. 17.5 cm^2 **b.** 11.5 cm^2 **c.** 30.7 cm^2
d. 19.6 cm^2 **e.** 19.4 cm^2 **f.** 25.0 cm^2

5. a. 6824.2 mm^2 **b.** 7.6 m^2 **c.** 734.2 cm^2 **d.** 578.5 cm^2
e. 7086.7 m^2 **f.** 5.4 m^2 **g.** 1143.4 m^2 **h.** 100.5 cm^2
i. 821 cm^2 **j.** 661.3 mm^2

6. $29 596.51

7. a. 5000 km^2 **b.** 0.5%

8. $10 150 **9.** 378 cm^2 **10.** $\approx$ 19.3 cm^2

11. a. 100.53 cm^2 **b.** 1244.07 m^2 **c.** 301.59 cm^2 **d.** 103.67 mm^2

12. B **13.** 75.7 cm or 757 mm

14. a. 619.5 cm^2 **b.** 10.5 cm **c.** 346.36 cm^2 **d.** Answers will vary.

15. a. 1600 cm^2 **b.** 800 cm^2 **c.** 400 cm^2
d. 200 cm^2 **e.** The area halves each time. **f.** 12.5%
g. Answers will vary.

16. Answers will vary.

17. Answers will vary.

18. Answers will vary. The side length may be a rational number or a surd. It will be rational if x is a perfect square.

19. Answers will vary.

20. Check with your teacher.

21. a. $\sqrt{50} = 5\sqrt{2}$ m **b.** 10 logs

22. 17 m

23. a. Answers will vary. **b.** 45

24. a. $\frac{1}{4}$ **b.** $\frac{1}{4}(x^2 + 6x + 8)$ **c.** $\frac{1}{4}(3x^2 - 6x - 8)$
d. i. $\frac{1}{12}(3x^2 - 6x - 8)$ **ii.** $\frac{1}{6}(3x^2 + 6x + 8)$

25. 45 m^2

Challenge 12.1

$\frac{5}{8}$

Exercise 12.4 Area and perimeter of a sector

1. a. 14.14 cm^2 **b.** 157.08 cm^2
c. 27.71 cm^2 **d.** 226.19 mm^2

2. a. i. 14.3 cm **ii.** 12.6 cm^2
b. i. 43.6 cm **ii.** 116.9 cm^2
c. i. 40.7 m **ii.** 102.1 m^2
d. i. 5.4 m **ii.** 1.8 m^2

3. D

4. a. i. 184.6 cm **ii.** 1869.2 cm^2

b. i. 66.8 m **ii.** 226.2 m^2

c. i. 27.4 cm **ii.** 42.4 cm^2

d. i. 342.1 m **ii.** 7243.6 m^2

e. i. 354.6 cm **ii.** 7645.9 cm^2

f. i. 104.4 cm **ii.** 201.1 cm^2

5. 60 318.6 m^2

6. 303.4 cm

7. 170.2 m^2

8. a. 14.8 m^2 **b.** 4.4%

9. B

10. a. 29.7 m **b.** 16.7 m **c.** 45.6 m

11. a. 5027 m^2 **b.** 78.5% **c.** \$61 440

12. a. Answers will vary. **b.** 85 m each **c.** 60 m each

d. Answers will vary. **e.** 37 m

13. a. Answers will vary. **b.** 88.4 cm^2 **c.** 50.3 cm^2

d. 109.4 cm^2 **e.** Jim

14. Answers will vary.

15. a. πr^2 cm^2 **b.** $\pi(r + 1)^2$ cm^2 **c.** Answers will vary.

d. 5π cm^2 **e.** Answers will vary.

16. Radius $= \pi$ cm

17. The length of the large arc equals the sum of the lengths of the two smaller arcs.

Exercise 12.5 Surface area of rectangular and triangular prisms

1. a. 66 cm^2 **b.** 62 m^2 **c.** 6.7 m^2

d. 4.44 m^2 **e.** 11 572.92 cm^2 **f.** 1.9 m^2

2. a. 86 cm^2 **b.** 210.7 m^2 **c.** 8.37 cm^2

3. a. 840 cm^2 **b.** 191.08 cm^2 **c.** 2370 mm^2

4. 2 cans of paint **5.** 77.76 cm^2 **6.** 26.44 m^2

7. 20.36 m^2 **8.** 261.5 cm^2 **9.** 2.21 m^2

10. 15.2 m^2 **11.** 2080 cm^2 **12.** 168 cm^2

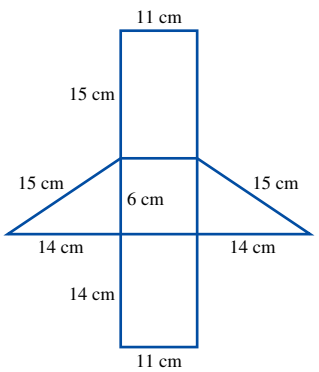
13. 301.86 cm^2

14. a. 150 cm^2 **b.** 250 cm^2 **c.** 350 cm^2 **d.** 850 cm^2

15. Answers will vary. The solid formed is a cube with side length 4 cm.

16. \$145.85 (2 L of ceiling paint, 1 L + 4 L for walls)

17. a.



b. 315 cm^2

18. a. 1402.52 m^2

b. 3220 m^3

Exercise 12.6 Surface area of a cylinder

1. a. 75.4 m^2 **b.** 28.3 cm^2 **c.** 2010.6 m^2
d. 1121.5 cm^2 **e.** 6.6 m^2 **f.** 25.6 m^2

2. a. 131.9 m^2 **b.** 84.8 cm^2 **c.** 3619.1 m^2
d. 1575.5 cm^2 **e.** 9.7 m^2 **f.** 61.8 m^2

3. Check with your teacher.

4. a. Asparagus is largest, beetroot is smallest. **b.** 118 cm^2

5. $\approx$ 1837.8 cm^2

6. $12\pi h^2$

7. $\approx$ 31.79 cm

8. a. $\approx$ 136.66 m^2 **b.** $7351.33

9. a. $\approx$ 2035.75 cm^2 **b.** $\approx$ 1526.81 cm^2
c. $\approx$ 395.84 cm^2 **d.** $\approx$ 3958.40 cm^2

10. 154.73 cm^2

11. a. 3.82 cm **b.** 427.7 cm^2

12. Answers will vary.

13. 83.6 m^2

14. The area to be painted is 1.63 m^2.

15. 11 231.12 cm^2

16. 4.30 m^2

Exercise 12.7 Volume of prisms and cylinders

1. a. i. 325 mL **ii.** 2 600 000 mL **iii.** 5100 mL **iv.** 630 000 mL
b. i. 5.8 cm^3 **ii.** 6100 cm^3 **iii.** 3 200 000 cm^3 **iv.** 0.0593 cm^3
c. i. 0.358 kL **ii.** 55.8 kL **iii.** 8.752 kL **iv.** 5300 kL

2. a. 36 cm^3 **b.** 15 cm^3 **c.** 72 cm^3

3. a. 12 cm^3 **b.** 68 cm^3 **c.** 9.6 m^3
d. 630 mm^3 **e.** 420 cm^3 **f.** $\approx$ 3152.7 mm^3
g. $\approx$ 1319.5 mm^3 **h.** $\approx$ 523.6 m^3 **i.** $\approx$ 10.5 m^3

4. a. 36 cm^3 **b.** 30 m^3 **c.** 1.144 m^3

5. a. 7020 cm^3 **b.** 6.91 m^3 **c.** 300 m^3 **d.** 8806 cm^3

6. a. 16 085.0 m^3 **b.** 4766.6 cm^3 **c.** 2.3 m^3
d. 30.8 m^3 **e.** 30.6 m^3 **f.** 56.5 mm^3

7. 14 476.5 L

8. 176.4 L

9. a. 158 169 cm^3 **b.** 4092 cm^3 **c.** 2639 cm^3
d. 641 cm^3 **e.** 1784 cm^3 **f.** 1057 cm^3

10. 75 322.8 L

11. 1145.1 mL

12. 2.916×10^9 m^3

13. a. 4.95 m^3 **b.** 4950 L

14. a. 45.94 cm^3 **b.** 2400 (to the nearest whole number)

15. 234 256.54 cm^3

16. a. 1.60 L **b.** 0.042 L

17. a. 1.1475 m^3 **b.** 840 L

18. Answers will vary.

19. a. Prism **b.** 75 m^2 **c.** Answers will vary.
d. i. 36 h 40 min **ii.** 47 h 9 min

19. Answers will vary.

20. Answers will vary.

21. 23

22. a. 169 cm^3 **b.** $V = x(l-2x)(w-2x)$

Challenge 12.2

Increase side length to 12.6 cm.

12.8 Review

1. B **2.** B **3.** B **4.** D

5. D **6.** A **7.** D **8.** A

9. a. 26 mm = 2.6 cm **b.** 1385 mm = 138.5 cm **c.** 1.63 cm = 16.3 mm

d. 1.5 km = 1500 m **e.** 0.077 km = 77 m **f.** 2850 m = 2.85 km

10. $7497

11. a. 25.1 cm **b.** 35.2 m **c.** 37.7 cm

12. a. 70.4 cm **b.** 30.4 mm **c.** 34 m **d.** 240 mm or 24 cm

e. 13.1 m **f.** 308.5 m **g.** 97.1 cm **h.** 192 mm or 19.2 cm

13. a. 25 m^2 **b.** 950 cm^2 **c.** 387.5 cm^2 **d.** 6120 m^2

e. 135 mm^2 **f.** 2170 cm^2 **g.** 279.4 cm^2 **h.** 6939.8 mm^2

14. a. 1486.9 cm^2 **b.** 362.5 m^2 **c.** 520.4 cm^2 **d.** 473.2 cm^2

15. a. 628.32 m^2 **b.** 59.45 cm^2 **c.** 197.92 cm^2 **d.** 10 734.47 m^2

16. a. 102.83 m **b.** 31.07 cm **c.** 57.99 cm **d.** 470.27 m

17. 48 920 cm^2 or 4.892 m^2

18. 3550 cm^2

19. a. 343 cm^3 **b.** 672 cm^3 **c.** 153 938.04 cm^3

d. 1.45 m^3 **e.** 1800 cm^3

20. $99.80

21. 9.57 hectares

22. a. 10.83 cm^2 **b.** 351.65 cm^2 **c.** 238.26 cm^3 **d.** $5.96

23. The cylinder of height 210 mm has the greater volume.

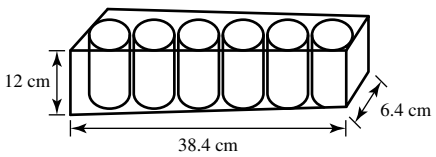
24. a. 10.75 m^2 **b.** No

c. 8 **d.** 9 mattresses could fit in the room.

25. The shaded areas are the same area.

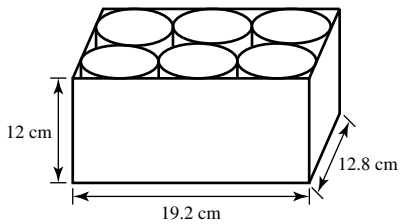
26. $\frac{\sqrt{17}}{2}$ times the side length of the square

27. a. i.



ii. 1566.72 cm^2

b. i.



ii. 1259.52 cm^2

c. 3 × 2 arrangement

d. 2058.24 cm^2

e. 2027.52 cm^2

f. Yes, 2170.88 cm^2

g. The smallest amount of packaging required is 3 × 2, with two levels of packaging.

Investigation — Rich task

1. **A:** 15 cm^2 **B:** 20 cm^2 **C:** 6 cm^2 **D:** 8.5 cm^2 **E:** 10.5 cm^2 **F:** 11.5 cm^2 **G:** 12.5 cm^2

2.

Polygon	Dots on perimeter (b)	Dots within perimeter (i)	Area of polygon (A)
A	16	8	15
B	24	9	20
C	4	6	6
D	7	6	8.5
E	11	6	10.5
F	13	6	11.5
G	15	6	12.5

3. Answers will vary.

4. Answers will vary.

TOPIC 13 Probability

13.1 Overview



Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

13.1.1 Why learn this?

Probability lies at the heart of nature. Think about all the events that had to happen for you to be born, for example . . . the odds are extraordinary.

Probability is that part of mathematics that gives meaning to the idea of uncertainty, of not fully knowing or understanding the occurrence of some event. We often hear that there is a good chance of rain, people bet with different odds that a favourite horse will win at Caulfield, and so on. In each case, we are making a guess as to what will be the outcome of some event.

It is important to learn about probability so that you can understand that chance is involved in many decisions that you will have to make in your life and in everyday events.

13.1.2 What do you know?

assessON

1. **THINK** List what you know about probability. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge about probability.

LEARNING SEQUENCE

13.1 Overview
13.2 Theoretical probability
13.3 Experimental probability
13.4 Venn diagrams and two-way tables
13.5 Two-step experiments
13.6 Mutually exclusive and independent events
13.7 Conditional probability
13.8 Review

learnON RESOURCES — ONLINE ONLY

Watch this video: The story of mathematics: What are the chances? (eles-1700)

13.2 Theoretical probability

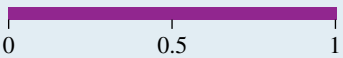
13.2.1 The language of probability

- The **probability** of an event is a measure of the likelihood that the event will take place.
- If an event is certain to occur, then it has a probability of 1.
- If an event is impossible, then it has a probability of 0.
- The probability of any other event taking place is given by a number between 0 and 1.
- The higher the probability, the more likely it is for the event to occur.
- Descriptive words such as 'impossible', 'unlikely', 'likely' and 'certain' are commonly used when referring to the chance of an event occurring. Some of these are shown on the **probability scale** below.

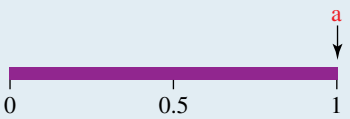
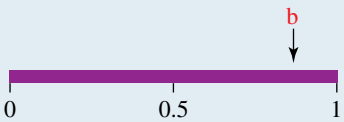
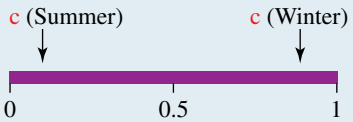


WORKED EXAMPLE 1

On the probability scale given at right, insert each of the following events at appropriate points.



a You will sleep tonight.
b You will come to school the next Monday during a school term.
c It will snow in Victoria this year.

THINK	WRITE/DRAW
a 1 Carefully read the given statement and label its position on the probability scale.	**a** 
2 Provide reasoning.	Under normal circumstances, I will certainly sleep tonight.
b 1 Carefully read the given statement and label its position on the probability scale.	**b** 
2 Provide reasoning.	It is very likely but not certain that I will come to school on a Monday during term. Circumstances such as illness or public holidays may prevent me from coming to school on a specific Monday during a school term.
c 1 Carefully read the given statement and label its position on the probability scale.	**c** 

2 Provide reasoning.

It is highly likely but not certain that it will snow in Victoria during winter. The chance of snow falling in Victoria in summer is highly unlikely but not impossible.

13.2.2 Key terms

- The study of probability uses many special terms that must be clearly understood. Here is an explanation of some of the more common terms.

  Chance experiment: A chance experiment is a process, such as rolling a die, that can be repeated many times.

  Trial: A trial is one performance of an experiment to get a result. For example, each roll of the die is called a trial.

  Outcome: The outcome is the result obtained when the experiment is conducted. For example, when a normal six-sided die is rolled the outcome can be 1, 2, 3, 4, 5 or 6.

  Sample space: The set of all possible outcomes is called the sample space and is given the symbol ξ. For the example of rolling a die, $\xi = \{1, 2, 3, 4, 5, 6\}$.

  Event: An event is the favourable outcome of a trial and is often represented by a capital letter. For example, when a die is rolled, A could be the event of getting an even number; $A = \{2, 4, 6\}$.

  Favourable outcome: A favourable outcome for an event is any outcome that belongs to the event. For event A above (rolling an even number), the favourable outcomes are 2, 4 and 6.

WORKED EXAMPLE 2

For the chance experiment of rolling a die:

a list the sample space

b list the events:

i rolling a 4 **ii rolling an even number**

iii rolling at least 5 **iv rolling at most 2**

c list the favourable outcomes for:

i {4, 5, 6} **ii not rolling 5**

iii rolling 3 or 4 **iv rolling 3 and 4.**



THINK	WRITE
a The outcomes are the numbers 1 to 6.	**a** $\xi = \{1, 2, 3, 4, 5, 6\}$
b i This describes only 1 outcome.	**b i** $\{4\}$
ii The possible even numbers are 2, 4 and 6.	**ii** $\{2, 4, 6\}$
iii 'At least 5' means 5 is the smallest.	**iii** $\{5, 6\}$
iv 'At most 2' means 2 is the largest.	**iv** $\{1, 2\}$
c i The outcomes are shown inside the brackets.	**c i** 4, 5, 6
ii 'Not 5' means everything except 5.	**ii** 1, 2, 3, 4, 6
iii The event is $\{3, 4\}$.	**iii** 3, 4
iv There is no number that is both 3 and 4.	**iv** There are no favourable outcomes.

13.2.3 Theoretical probability

- When a coin is tossed, there are two possible outcomes, Heads or Tails. That is, $\xi = \{H, T\}$.
- In ideal circumstances, the two outcomes have the same likelihood of occurring, so they are allocated the same probability.

  For example, $P(\text{Heads}) = \frac{1}{2}$ (This says the probability of Heads $= \frac{1}{2}$.)

  and $P(\text{Tails}) = \frac{1}{2}$.
- The total of the probabilities equals 1, as there are no other possible outcomes.
- In general, if all outcomes are **equally likely** to occur (ideal circumstances), then the probability of event A occurring is given by

  $$P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}.$$



WORKED EXAMPLE 3

A die is rolled and the number uppermost is noted. Determine the probability of each of the following events.

a $A = \{1\}$
b $B = \{\text{odd numbers}\}$
c $C = \{4 \text{ or } 6\}$

THINK	WRITE
There are 6 possible outcomes.	
a A has 1 favourable outcome.	**a** $P(A) = \frac{1}{6}$
b B has 3 favourable outcomes: 1, 3 and 5.	**b** $P(B) = \frac{3}{6}$ $= \frac{1}{2}$
c C has 2 favourable outcomes.	**c** $P(C) = \frac{2}{6}$ $= \frac{1}{3}$

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- **Complete this digital doc:** SkillSHEET: Probability scale (doc-6307)
- **Complete this digital doc:** SkillSHEET: Understanding a deck of playing cards (doc-6308)
- **Complete this digital doc:** SkillSHEET: Listing the sample space (doc-6309)
- **Complete this digital doc:** SkillSHEET: Theoretical probability (doc-6310)
- **Try out this interactivity:** Random number generator (int-0089)

Exercise 13.2 Theoretical probability

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–16	Questions: 1–18	Questions: 1–19

Individual pathway interactivity: int-4534

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency



1. WE1 On the given probability scale, insert each of the following events at appropriate points.
- **a.** The school will have a lunch break on Friday.
- **b.** Australia will have a swimming team in the Commonwealth Games.
- **c.** Australia will host two consecutive Olympic Games.
- **d.** At least one student in a particular class will obtain an A for Mathematics.
- **e.** Mathematics will be taught in secondary schools.
- **f.** In the future most cars will run without LPG or petrol.
- **g.** Winter will be cold.
- **h.** Bean seeds, when sown, will germinate.

2. Indicate the chance of each event listed in question **1** using one of the following terms: certain, likely, unlikely, impossible.

3. WE2a For each chance experiment below, list the sample space.
- **a.** Rolling a die
- **b.** Tossing a coin
- **c.** Testing a light bulb to see whether it is defective or not
- **d.** Choosing a card from a normal deck and noting its colour
- **e.** Choosing a card from a normal deck and noting its suit



4. WE2b A normal 6-sided die is rolled. List each of the following events.
- **a.** Rolling a number less than or equal to 3
- **b.** Rolling an odd number
- **c.** Rolling an even number or 1
- **d.** Not rolling a 1 or 2
- **e.** Rolling at most a 4
- **f.** Rolling at least a 5

5. WE2c A normal 6-sided die is rolled. List the favourable outcomes for each of the following events.
- **a.** $A = \{3, 5\}$
- **b.** $B = \{1, 2\}$
- **c.** C = 'rolling a number greater than 5'
- **d.** D = 'not rolling a 3 or a 4'
- **e.** E = 'rolling an odd number or a 2'
- **f.** F = 'rolling an odd number and a 2'
- **g.** G = 'rolling an odd number and a 3'

6. A card is selected from a normal deck of 52 cards and its suit is noted.

a. List the sample space.

b. List each of the following events.

i. 'Drawing a black card'

ii. 'Drawing a red card'

iii. 'Not drawing a heart'

iv. 'Drawing a black or a red card'

7. How many outcomes are there for:

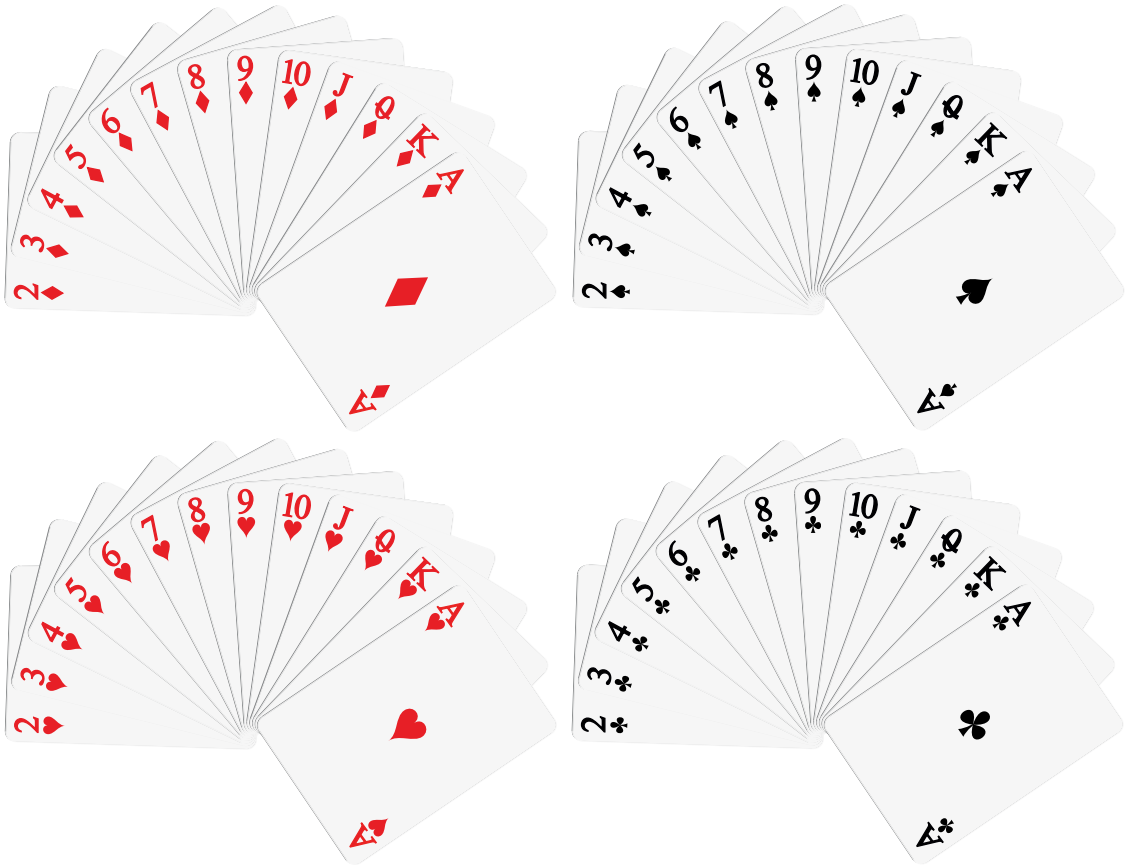
a. rolling a die

b. tossing a coin

c. drawing a card from a standard deck

d. drawing a card and noting its suit

e. noting the remainder when a number is divided by 5?



8. A card is drawn at random from a standard deck of 52 cards.

Note: 'At random' means that every card has the same chance of being selected.

Find the probability of selecting:

a. an ace **b.** a king **c.** the 2 of spades **d.** a diamond.

9. WE3 A card is drawn at random from a deck of 52. Find the probability of each event below.

a. A = {5 of clubs}

b. B = {black card}

c. C = {5 of clubs or queen of diamonds}

d. D = {hearts}

e. E = {hearts or clubs}

f. F = {hearts and 5}

g. G = {hearts or 5}

h. H = {aces or kings}

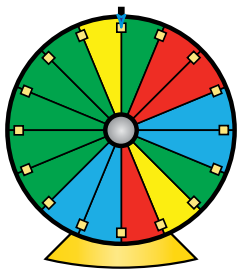
i. I = {aces and kings}

j. J = {not a 7}

10. A letter is chosen at random from the letters in the word PROBABILITY. What is the probability that the letter is:

a. B **b.** not B **c.** a vowel **d.** not a vowel?

11. The following coloured spinner is spun and the colour is noted. What is the probability of each of the events given below?



a. $A = \{\text{blue}\}$

b. $B = \{\text{yellow}\}$

c. $C = \{\text{yellow or red}\}$

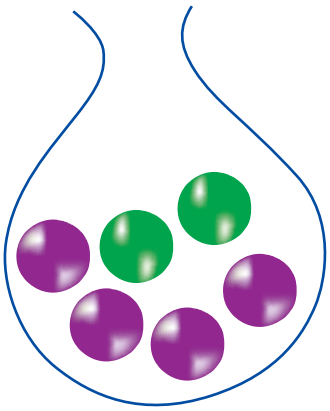
d. $D = \{\text{yellow and red}\}$

e. $E = \{\text{not blue}\}$

Understanding

12. a. A bag contains 4 purple balls and 2 green balls. If a ball is drawn at random, then what is the probability that it will be:

i. purple **ii.** green?



b. Design an experiment like the one in part **a** but where the probability of drawing a purple ball is 3 times that of drawing a green ball.

13. Design spinners (see question **11**) using red, white and blue sections so that:

a. each colour has the same probability of being spun

b. red is twice as likely to be spun as either of the other 2 colours

c. red is twice as likely to be spun as white and 3 times as likely to be spun as blue.

Reasoning

14. Do you think that the probability of tossing Heads is the same as the probability of tossing Tails if your friend tosses the coin? What are some reasons that it might not be?

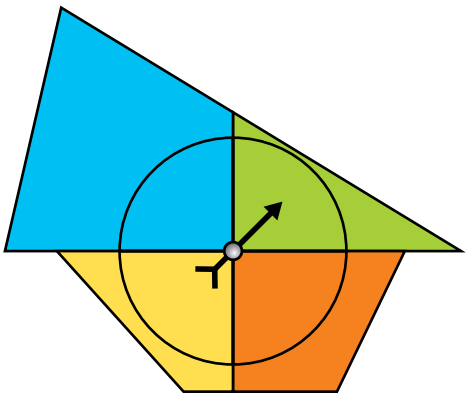
15. If the following four probabilities were given to you, which two would you say were not correct? Give reasons why.

0.725, −0.5, 0.005, 1.05

Problem solving

16. Consider this spinner.

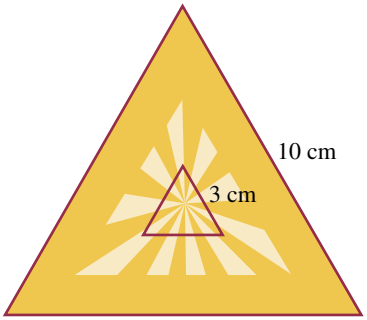
Discuss whether the spinner has equal chance of falling on each of the colours.



17. A box contains two coins. One is a double-headed coin, and the other is a normal coin with Heads on one side and Tails on the other. You draw one of the coins from a box and look at one of the sides. It is Heads. What is the probability that the other side shows Heads also?

18. 'Unders and Overs' is a game played with two normal six-sided dice. The two dice are rolled, and the numbers uppermost added to give a total. Players bet on the outcome being 'under 7', 'equal to 7' or 'over 7'. If you had to choose one of these outcomes, which would you choose? Explain why.

19. Justine and Mary have designed a new darts game for their Year 9 Fete Day. Instead of a circular dart board, their dart board is in the shape of two equilateral triangles. The inner triangle (bullseye) has a side length of 3 cm, while the outer triangle has side length 10 cm.



Given that a player's dart falls in one of the triangles, what is the probability that it lands in the bullseye? Write your answer correct to 2 decimal places.

Reflection

Write a sentence using the word 'probability' that shows its meaning.

13.3 Experimental probability

13.3.1 Relative frequency

- A die is rolled 12 times and the outcomes are recorded in the table below.

Outcome	1	2	3	4	5	6
Frequency	3	1	1	2	2	3

In this chance experiment there were 12 trials.
The table shows that the number 1 was rolled 3 times out of 12.

- So the **relative frequency** of 1 is 3 out of 12, or $\frac{3}{12} = \frac{1}{4}$.
As a decimal, the relative frequency of 1 is equal to 0.25.
- In general, the relative frequency of an outcome $= \dfrac{\text{the frequency of the outcome}}{\text{total number of trials}}$.
If the number of trials is very large, then the relative frequency of each outcome becomes very close to the theoretical probability.

WORKED EXAMPLE 4

For the chance experiment of rolling a die, the following outcomes were noted.

Outcome	1	2	3	4	5	6
Frequency	3	1	4	6	3	3



a How many trials were there?
b How many threes were rolled?
c What was the relative frequency for each number written as decimals?

THINK

a Adding the frequencies will give the number of trials.

b The frequency of 3 is 4.

c Add a relative frequency row to the table and complete it.

WRITE

a $1 + 3 + 4 + 6 + 3 + 3 = 20$ trials

b 4 threes were rolled.

c

Outcome	1	2	3	4	5	6
Frequency	3	1	4	6	3	3
Relative frequency	$\frac{3}{20} = 0.15$	$\frac{1}{20} = 0.05$	$\frac{4}{20} = 0.2$	$\frac{6}{20} = 0.3$	$\frac{3}{20} = 0.15$	$\frac{3}{20} = 0.15$

Group experiment

- Organise for the class to toss a coin at least 500 times. For example, if the class has 20 students, each one should record 25 outcomes and enter their information into a grid as shown below.

Group	H	T
A	17	8
B	15	10
C		
etc.		

Total T	Total H	Total outcomes	Relative frequency (Heads)	Relative frequency (Tails)
17	8	25	$\frac{17}{25} = 0.68$	$\frac{8}{25} = 0.32$
32	18	50	$\frac{32}{50} = 0.64$	$\frac{18}{50} = 0.36$
		75		

- Questions:
  1. In an ideal situation, what would you expect the relative frequencies to be? Has this occurred?
  2. As more information was added to the table, what happened to the relative frequencies?
  3. What do you think might happen if the experiment was continued for another 500 tosses?
- A rule called the law of large numbers indicates that as the number of trials increases, then the relative frequencies will tend to get closer to the expected value (in this case 0.5).

13.3.2 Experimental probability

- Sometimes it is not possible to calculate theoretical probabilities and in such cases experiments, sometimes called simulations, are conducted to determine the **experimental probability**.
- The relative frequency is equal to the experimental probability.

$$\textbf{Experimental probability} = \frac{\textbf{the frequency of the outcome}}{\textbf{total number of trials}}$$

For example, the spinner shown at right (made from light cardboard and a toothpick) is not symmetrical, and the probability of each outcome cannot be determined theoretically.



However, the probability of each outcome can be found by using the spinner many times and recording the outcomes. If a large number of trials is conducted, the relative frequency of each outcome will be very close to its probability.

WORKED EXAMPLE 5

The spinner shown above was spun 100 times and the following results were achieved.

Outcome	1	2	3	4
Frequency	7	26	9	58

a How many trials were there?
b What is the experimental probability of each outcome?
c What is the sum of the 4 probabilities?

THINK	**WRITE**
a Adding the frequencies will determine the number of trials.	**a** $7 + 26 + 9 + 58 = 100$ trials
b The experimental probability equals the relative frequency.	**b** $P(1) = \frac{7}{100}$ $= 0.07$ $P(2) = \frac{26}{100}$ $= 0.26$ $P(3) = \frac{9}{100}$ $= 0.09$ $P(4) = \frac{58}{100}$ $= 0.58$
c Add the probabilities (they should equal 1).	**c** $0.07 + 0.26 + 0.09 + 0.58 = 1$

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Complete this digital doc: WorkSHEET: Probability I (doc-6313)

Exercise 13.3 Experimental probability

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 6–13, 19–21, 25	Questions: 1–4, 6–11, 15, 16, 22–25	Questions: 1–3, 5–8, 12–14, 16–25

Individual pathway interactivity: int-4535

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Fluency

1. WE4 Each of the two tables below contains the results of a chance experiment (rolling a die). For each table, find:

i. the number of trials held
ii. the number of fives rolled
iii. the relative frequency for each outcome, correct to 2 decimal places
iv. the sum of the relative frequencies.

a.

Number	1	2	3	4	5	6
Frequency	3	1	5	2	4	1

b.

Number	1	2	3	4	5	6
Frequency	52	38	45	49	40	46

2. A coin is tossed in two chance experiments. The outcomes are recorded in the tables below. For each experiment, find:

i. the relative frequency of both outcomes
ii. the sum of the relative frequencies.

a.

Outcome	H	T
Frequency	22	28

b.

Outcome	H	T
Frequency	31	19

3. Construct an irregular spinner using cardboard and a toothpick. By carrying out a number of trials, estimate the probability of each outcome.

4. WE5 An unbalanced die was rolled 200 times and the following outcomes were recorded.

Number	1	2	3	4	5	6
Frequency	18	32	25	29	23	73

Using these results, find:
a. P(6)
b. P(odd number)
c. P(at most 2)
d. P(not 3).

5. A box of matches claims on its cover to contain 100 matches.
A survey of 200 boxes established the following results.

Number of matches	95	96	97	98	99	100	101	102	103	104
Frequency	1	13	14	17	27	55	30	16	13	14

If you were to purchase a box of these matches, what is the probability that:
a. the box would contain 100 matches
b. the box would contain at least 100 matches
c. the box would contain more than 100 matches
d. the box would contain no more than 100 matches?



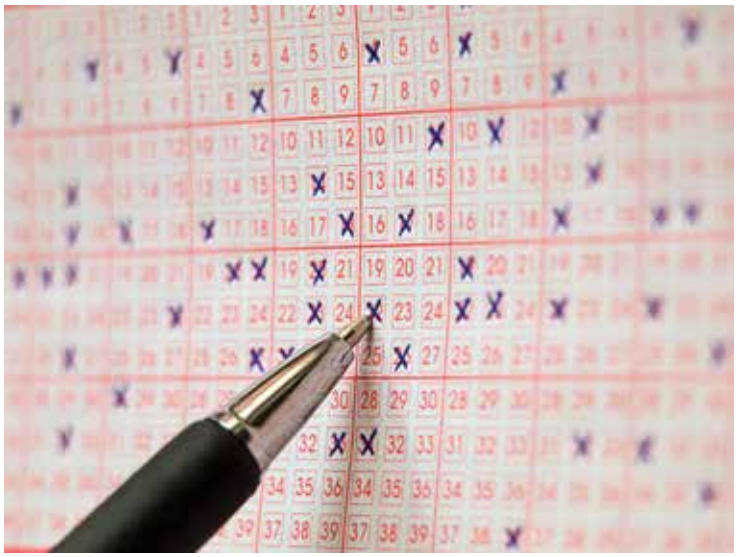
Understanding

6. Here is a series of statements based on experimental probability.
If a statement is not reasonable, give a reason why.
a. I tossed a coin 5 times and there were 4 Heads, so P(H) = 0.8.

b. Sydney Roosters have won 1064 matches out of the 2045 that they have played, so P(Sydney will win their next game) = 0.54.
c. P(The sun will rise tomorrow) = 1.
d. At a factory, a test of 10 000 light globes showed that 7 were faulty. Therefore, P(faulty light globe) = 0.0007.
e. In Sydney it rains an average of 143.7 days each year, so P(it will rain in Sydney on the 17th of next month) = 0.39.

7. At a birthday party, some cans of soft drink were put in a container of ice. There were 16 cans of Coke, 20 cans of Sprite, 13 cans of Fanta, 8 cans of Sunkist and 15 cans of Pepsi. If a can was picked at random, what is the probability that it was:
a. a can of Pepsi **b.** not a can of Fanta?

8. MC In Tattslotto, 6 numbers are drawn from the numbers 1, 2, 3, … 45. The number of different combinations of 6 numbers is 8 145 060. If you buy 1 ticket, what is the probability that you will win the draw?
A. $\frac{1}{8\,145\,060}$ **B.** $\frac{1}{45}$ **C.** $\frac{45}{8\,145\,060}$ **D.** $\frac{1}{6}$ **E.** $\frac{6}{8\,145\,060}$



9. WE8 If a fair coin is tossed 400 times, how many Tails are expected?

10. If a fair die is rolled 120 times, how many threes are expected?

11. WE9 MC A survey of high school students asked 'Should Saturday be a normal school day?' 350 students voted yes, and 450 voted no. What is the probability that a student chosen at random said no?
A. $\frac{7}{16}$ **B.** $\frac{9}{16}$ **C.** $\frac{7}{9}$ **D.** $\frac{9}{14}$ **E.** $\frac{1}{350}$

12. In a poll of 200 people, 110 supported party M, 60 supported party N and 30 were undecided. If a person is chosen at random from this group of people, what is the probability that he or she:
a. supports party M
b. supports party N
c. supports a party
d. is not sure what party to support?

13. A random number is picked from $N = \{1, 2, 3, \ldots 100\}$. What is the probability of picking a number that is:
a. a multiple of 3
b. a multiple of 4 or 5
c. a multiple of 5 and 6?

14. The numbers 3, 5 and 6 are combined to form a three-digit number such that no digit may be repeated.

a. **i.** How many numbers can be formed?

ii. List them.

b. Find P(the number is odd).

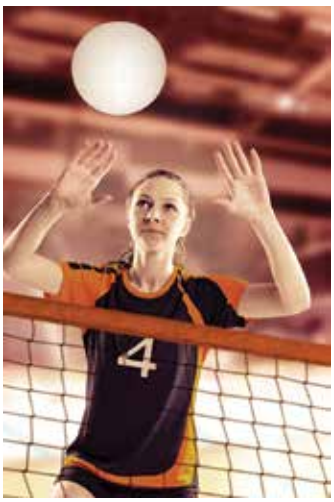
c. Find P(the number is even).

d. Find P(the number is a multiple of 5).

15. MC In a batch of batteries, 2 out of every 10 in a large sample were faulty. At this rate, how many batteries are expected to be faulty in a batch of 1500?

A. 2 **B.** 150 **C.** 200 **D.** 300 **E.** 750

16. Svetlana, Sarah, Leonie and Trang are volleyball players. The probabilities that they will score a point on serve are 0.6, 0.4, 0.3 and 0.2 respectively. How many points on serve are expected from each player if they serve 10 times each?



17. MC A survey of the favourite leisure activity of 200 Year 9 students produced the following results.

Activity	Playing sport	Fishing	Watching TV	Video games	Surfing
Number of students	58	26	28	38	50

The probability (given as a percentage) that a student selected at random from this group will have surfing as their favourite leisure activity is:

A. 50% **B.** 100% **C.** 25% **D.** 0% **E.** 29%

18. The numbers 1, 2 and 5 are combined to form a three-digit number, allowing for any digit to be repeated up to three times.

a. How many different numbers can be formed?

b. List the numbers.

c. Determine P(the number is even).

d. Determine P(the number is odd).

e. Determine P(the number is a multiple of 3).

Reasoning

19. John has a 12-sided die numbered 1 to 12 and Lisa has a 20-sided die numbered 1 to 20. They are playing a game where the first person to get the number 10 wins. They are rolling their dice individually.

a. Find P(John gets a 10). **b.** Find P(Lisa gets a 10). **c.** Is this game fair? Explain.

20. At a supermarket checkout, the scanners have temporarily broken down and the cashiers must enter in the bar codes manually. One particular cashier overcharged 7 of the last 10 customers she served by entering the incorrect bar code.

a. Based on the cashier's record, what is the probability of making a mistake with the next customer?

b. Should another customer have any objections with being served by this cashier?

c. Justify your answer to part **b**.

21. If you flip a coin 6 times, how many of the possible outcomes could include a Tail on the second toss?

Problem solving

22. In a jar, there are 600 red balls, 400 green balls, and an unknown number of yellow balls. If the probability of selecting a green ball is $\frac{1}{5}$, how many yellow balls are in the jar?

23. In another jar there are an unknown number of balls, N, with 20 of them green. The other colours contained in the jar are red, yellow and blue, with P(red or yellow) = $\frac{1}{2}$, P(red or green) = $\frac{1}{4}$ and P(blue) = $\frac{1}{3}$. Determine the number of red, yellow and blue balls in the jar.

24. The gender of babies in a set of triplets is simulated by flipping 3 coins. If a coin lands Tails up, the baby is a boy. If a coin lands Heads up, the baby is a girl. In the simulation, the trial is repeated 40 times. The following results show the number of Heads obtained in each trial:
0, 3, 2, 1, 1, 0, 1, 2, 1, 0, 1, 0, 2, 0, 1, 0, 1, 2, 3, 2, 1, 3, 0, 2, 1, 2, 0, 3, 1, 3, 0, 1, 0, 1, 3, 2, 2, 1, 2, 1.

a. Calculate the probability that exactly one of the babies in a set of triplets is female.

b. Calculate the probability that more than one of the babies in the set of triplets is female.

25. A survey of the favourite foods of Year 9 students is recorded, with the following results.

Meal	Tally
Hamburger	45
Fish and chips	31
Macaroni and cheese	30
Lamb souvlaki	25
BBQ pork ribs	21
Cornflakes	17
T-bone steak	14
Banana split	12
Corn-on-the-cob	9
Hot dogs	8
Garden salad	8
Veggie burger	7
Smoked salmon	6
Muesli	5
Fruit salad	3

a. Estimate the probability that macaroni and cheese is the favourite food of a randomly selected Year 9 student.

b. Estimate the probability that a vegetarian dish is a randomly selected student's favourite food.

c. Estimate the probability that a beef dish is a randomly selected student's favourite food.

Reflection

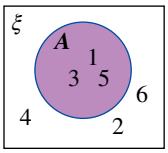
What are the most important similarities between theoretical and experimental probability calculations?

13.4 Venn diagrams and two-way tables

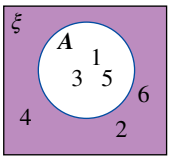
13.4.1 The complement of an event

- Suppose that a die is rolled: $\xi = \{1, 2, 3, 4, 5, 6\}$.
- If A is the event 'rolling an odd number', then $A = \{1, 3, 5\}$.

- There is another event called 'the **complement** of A', or 'not A'. This event contains all the outcomes that do not belong to A. It is given the symbol A'.
- In this case $A' = \{2, 4, 6\}$.
- A and A' can be shown on a **Venn diagram**.



A is coloured.



A' (not A) is coloured.

WORKED EXAMPLE 6

For the sample space $\xi = \{1, 2, 3, 4, 5\}$, list the complement of each of the following events.

a $A = \{\text{multiples of 3}\}$

b $B = \{\text{square numbers}\}$

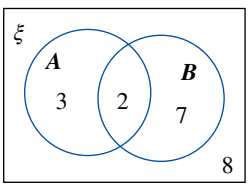
c $C = \{1, 2, 3, 5\}$

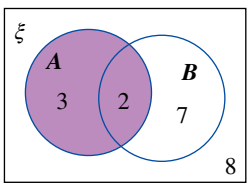
THINK	WRITE
a The only multiple of 3 in the set is 3. Therefore $A = \{3\}$. A' is every other element of the set.	**a** $A' = \{1, 2, 4, 5\}$
b The only square numbers are 1 and 4. Therefore $B = \{1, 4\}$. B' is every other element of the set.	**b** $B' = \{2, 3, 5\}$
c $C = \{1, 2, 3, 5\}$. C' is every other element of the set.	**c** $C' = \{4\}$

13.4.2 Venn diagrams and two-way tables

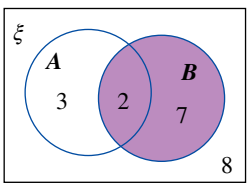
Venn diagrams

- Venn diagrams convey information in a concise manner and are often used to illustrate sample spaces and events. Here is an example.
  - In a class of 20 students, 5 study Art, 9 study Biology, and 2 students study both subjects.
  - This information is shown on the diagram below, where $A = \{\text{students who study Art}\}$ and $B = \{\text{students who study Biology}\}$.





A contains 5 students.

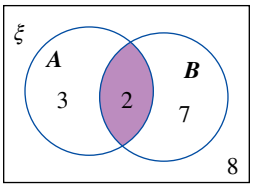


B contains 9 students.

Note: In the case shown above, 8 students in the class study neither Art nor Biology.

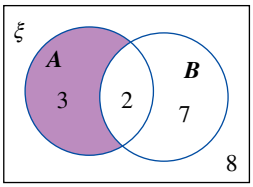
- The Venn diagram has 4 regions, each with its own name.

$A \cap B$



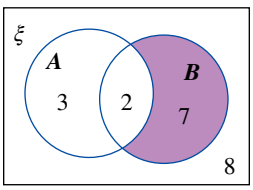
There are 2 students who study Art and Biology.
They occupy the region called 'A and B' or $A \cap B$.

$A \cap B'$



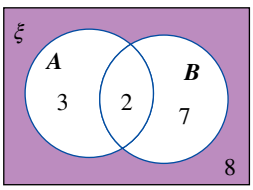
There are 3 students who study Art but not Biology.
They occupy the region called 'A and not B' or $A \cap B'$.

$A' \cap B$



There are 7 students who study Biology but not Art.
They occupy the region called 'not A and B' or $A' \cap B$.

$A' \cap B'$



The remaining 8 students study neither subject.
They occupy the region called 'not A and not B' or $A' \cap B'$.

13.4.3 Two-way tables

- The information can also be summarised in a **two-way table**.

	Biology	Not Biology	Total
Art	2	3	5
Not Art	7	8	15
Total	9	11	20

Note: Nine students in total study Biology and 11 do not. Five students in total study Art and 15 do not.

13.4.4 Number of outcomes

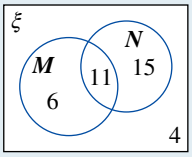
- If event A contains 7 outcomes or members, this is written as $n(A) = 7$.
- So $n(A \cap B') = 3$ means that the event 'A and not B' has 3 outcomes.

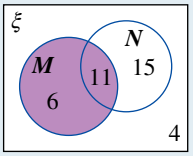
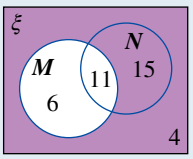
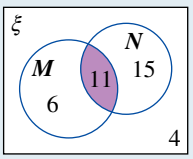
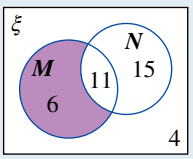
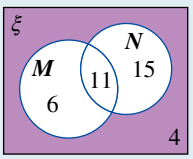
WORKED EXAMPLE 7

For the Venn diagram shown, write down the number of outcomes in each of the following.

a M **b** M' **c** $M \cap N$

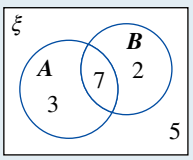
d $M \cap N'$ **e** $M' \cap N'$

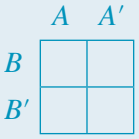
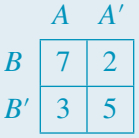


THINK	WRITE/DRAW
a Identify the regions showing M and add the outcomes.	**a**  $n(M) = 6 + 11 = 17$
b Identify the regions showing M' and add the outcomes.	**b**  $n(M') = 4 + 15 = 19$
c $M \cap N$ means 'M and N'. Identify the region.	**c**  $n(M \cap N) = 11$
d $M \cap N'$ means 'M and not N'. Identify the region.	**d**  $n(M \cap N') = 6$
e $M' \cap N'$ means 'not M and not N'. Identify the regions.	**e**  $n(M' \cap N') = 4$

WORKED EXAMPLE 8

Show the information from the Venn diagram on a two-way table.



THINK	WRITE
1 Draw a 2 × 2 table and add the labels A, A', B and B'.	
2 There are 7 elements in A and B. There are 3 elements in A and 'not B'. There are 2 elements in 'not A' and B. There are 5 elements in 'not A' and 'not B'.	

3 Add in a column and a row to show the totals.

	A	A'	Total
B	7	2	8
B'	3	5	8
Total	10	7	17

WORKED EXAMPLE 9

Show the information from the two-way table on a Venn diagram.

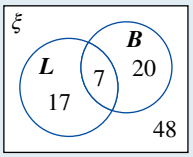


	Left-handed	Right-handed
Blue eyes	7	20
Not blue eyes	17	48

THINK

1 Draw a Venn diagram that includes a sample space and events L for left-handedness and B for blue eyes. (Right-handedness = L')

DRAW



2 $n(L \cap B) = 7$

$n(L \cap B') = 17$

$n(L' \cap B) = 20$

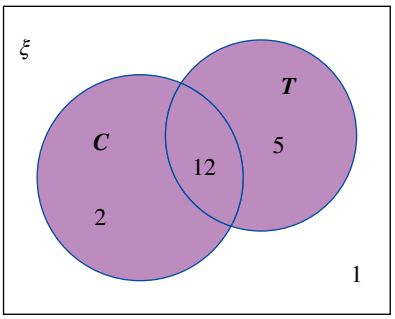
$n(L' \cap B') = 48$

13.4.5 Event *A* or *B*

- This Venn diagram illustrates the results of a survey of 20 people, showing whether they drink tea and whether they drink coffee.

  In all there are 19 people who drink tea or coffee. They are found in the shaded region of the diagram. This large group of people is written as $C \cup T$ and called 'C or T'. Note that the people who drink both tea and coffee, $C \cap T$, are included in this group. $n(C \cup T) = 19$

  A number of people drink tea or coffee, but not both. This group contains the 2 people who drink only tea and the 5 people who drink only coffee. n(people who drink tea or coffee, but not both) = 7



WORKED EXAMPLE 10

In a class of 24 students, 11 students play basketball, 7 play tennis, and 4 play both sports.

a Show the information on a Venn diagram.

b If one student is selected at random, then find the probability that:

i the student plays basketball

ii the student plays tennis or basketball

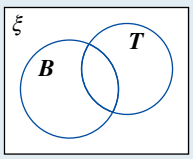
iii the student plays tennis or basketball but not both.

THINK

a 1 Draw a sample space with events B and T.

DRAW

a



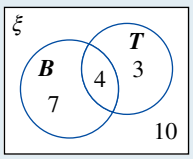
2 $n(B \cap T) = 4$

$n(B \cap T') = 11 - 4 = 7$

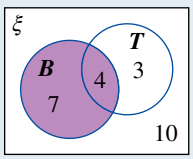
$n(T \cap B') = 7 - 4 = 3$

So far, 14 students out of 24 have been placed.

$n(B' \cap T') = 24 - 14 = 10$



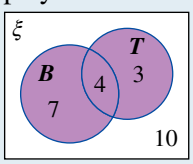
b i Identify the number of students who play basketball.



$$P(B) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

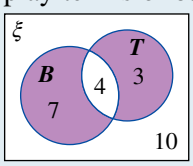
b i
$$P(B) = \frac{\text{number of students who play basketball}}{\text{total number of students}}$$
$$= \frac{n(B)}{24}$$
$$= \frac{11}{24}$$

ii Identify the number of students who play tennis or basketball.



ii
$$P(T \cup B) = \frac{n(T \cup B)}{24}$$
$$= \frac{14}{24}$$
$$= \frac{7}{12}$$

iii Identify the number of students who play tennis or basketball but not both.



iii $n(B \cap T') + n(B' \cap T) = 3 + 7$
$= 10$

P(tennis or basketball but not both)

$$= \frac{10}{24}$$
$$= \frac{5}{12}$$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Determining complementary events (doc-6311)

Complete this digital doc: SkillSHEET: Calculating the probability of a complementary event (doc-6312)

Exercise 13.4 Venn diagrams and two-way tables

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7, 9, 11, 13, 15–17	Questions: 1–4, 6, 8, 10, 13–15, 17–19	Questions: 1, 3, 4, 6, 8, 10, 12, 14, 16–20

Individual pathway interactivity: int-4536

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE6 If $\xi = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, list the complement of each of the following events.

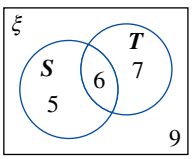
a. $A = \{\text{multiples of } 3\}$

b. $B = \{\text{numbers less than } 20\}$

c. $C = \{\text{prime numbers}\}$

d. $D = \{\text{odd numbers and numbers greater than } 16\}$

2. WE7 For the Venn diagram shown, write down the number of outcomes in:



a. ξ

b. S

c. T

d. $S \cap T$

e. $T \cap S$

f. $T \cap S'$

g. $S' \cap T'$.

3. WE8 Show the information from question **2** on a two-way table.

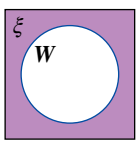
4. WE9 Show the information from this two-way table on a Venn diagram.

	S	S'
V	21	7
V'	2	10

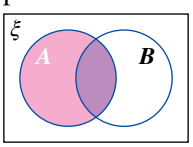
5. For each of the following Venn diagrams, use set notation to write the name of the region coloured in:

i. purple **ii.** pink.

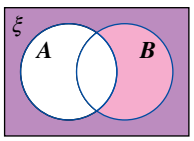
a.



b.



c.



6. The membership of a tennis club consists of 55 men and 45 women. There are 27 left-handed people, including 15 men.



a. Show the information on a two-way table.

b. Show the information on a Venn diagram.

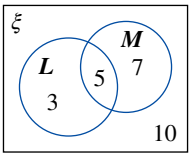
c. If one member is chosen at random, find the probability that the person is:

i. right-handed

ii. a right-handed man

iii. a left-handed woman.

7. Using the information given in the Venn diagram, if one outcome is chosen at random, find:



a. P(L) **b.** P(L') **c.** P($L \cap M$) **d.** P($L \cap M'$).

8. WE10 Using the information given in the table, if one family is chosen at random, find the probability that they own:

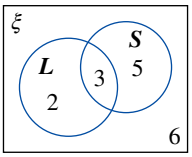
Pets owned by families

	Cat	No cat
Dog	4	11
No dog	16	9

a. a cat **b.** a cat and a dog **c.** a cat or a dog or both

d. a cat or a dog but not both **e.** neither a cat nor a dog.

9. A group of athletes was surveyed and the results were shown on a Venn diagram. S = {sprinters} and L = {long jumpers}.



a. How many athletes were included in the survey?

b. If one of the athletes is chosen at random, what is the probability that the athlete competes in:

i. long jump **ii.** long jump and sprints

iii. long jump or sprints **iv.** long jump or sprints but not both?

Understanding

10. If ξ = {children}, S = {swimmers} and R = {runners}, describe in words each of the following.

a. S' **b.** $S \cap R$

c. $R' \cap S'$ **d.** $R \cup S$

11. A group of 12 students was asked whether they liked hip hop (H) and whether they liked classical music (C). The results are shown in the table on the following page.

a. Show the results on:

i. a Venn diagram

ii. a two-way table.

b. If one student is selected at random, find:

i. P(H)

ii. P($H \cup C$)

iii. P($H \cap C$)

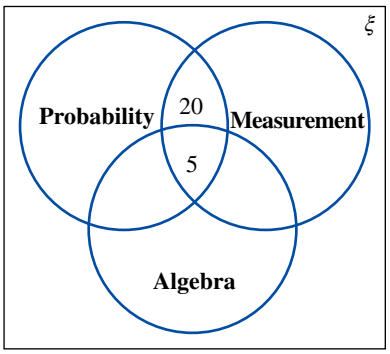
iv. P(student likes classical or hip hop but not both).

	C	H
Ali	✓	✓
Anu		
Chris		✓
George		✓
Imogen		✓
Jen	✓	✓
Luke	✓	✓
Pam	✓	
Petra		
Roger	✓	
Seedevi		✓
Tomas		

12. Place the elements of the following sets of numbers in their correct position in a single Venn diagram.
A = {prime numbers from 1 to 20}
B = {even numbers from 1 to 20}
C = {multiples of 5 from 1 to 20}
ξ = {numbers between 1 and 20 inclusive}

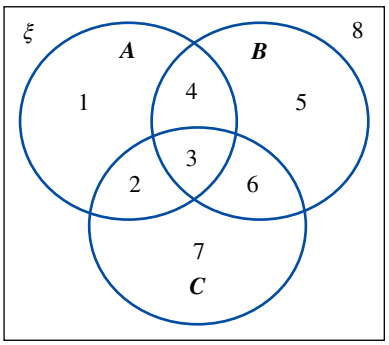
Reasoning

13. One hundred Year 9 Maths students were asked to indicate their favourite topic in mathematics. Sixty chose Probability, 50 chose Measurement and 43 chose Algebra. Some students chose two topics: 15 chose Probability and Algebra, 18 chose Measurement and Algebra, and 25 chose Probability and Measurement. Five students chose all three topics.



a. Copy and complete the Venn diagram at right.
b. How many students chose Probability only?
c. How many students chose Algebra only?
d. How many students chose Measurement only?
e. How many students chose any two of the three topics?
A student is selected at random from this group. Find the probability that this student has chosen:
f. Probability
g. Algebra
h. Algebra and Measurement
i. Algebra and Measurement but not Probability
j. all of the topics.

14. Create a Venn diagram using two circles to accurately describe the relationships between the following quadrilaterals: rectangle, square and rhombus.



15. Use the Venn diagram at right to write the numbers of the correct regions for each of the following problems.
a. $A' \cup (B' \cap C)$
b. $A \cap (B \cap C')$
c. $A' \cap (B' \cup C')$
d. $(A \cup B \cup C)'$

16. A recent survey taken at a cinema asked 90 teenagers what they thought about three different movies. In total, 47 liked 'Hairy Potter', 25 liked 'Stuporman' and 52 liked 'There's Something About Fred'.
16 liked 'Hairy Potter' only.
4 liked 'Stuporman' only.
27 liked 'There's Something About Fred' only.
There were 11 who liked all three films and 10 who liked none of them.
a. Construct a Venn diagram showing the results of the survey.
b. What is the probability that a teenager chosen at random liked 'Hairy Potter' and 'Stuporman' but not 'There's Something About Fred'?

Problem solving

17. 120 children attended a school holiday program during September. They were asked to select their favourite board game from Cluedo, Monopoly and Scrabble. They all selected at least one game, and 4 children chose all three games.
In total, 70 chose Monopoly and 55 chose Scrabble.
Some children selected exactly two games — 12 chose Cluedo and Scrabble, 15 chose Monopoly and Scrabble, and 20 chose Cluedo and Monopoly.
a. Draw a Venn diagram to represent the children's selections.
b. What is the probability that a child selected at random did not choose Cluedo as a favourite game?

18. Valleyview High School offers three sports at Year 9: baseball, volleyball and soccer. There are 65 students in Year 9.
2 have been given permission not to play sport due to injuries and medical conditions.
30 students play soccer.
9 students play both soccer and volleyball but not baseball.
9 students play both baseball and soccer (including those who do and don't play volleyball).
4 students play all three sports.
12 students play both baseball and volleyball (including those who do and don't play soccer).
The total number of players who play baseball is 1 more than the total of students who play volleyball.
a. Determine the number of students who play volleyball.
b. If a student was selected at random, what is the probability that this student plays soccer and baseball only?

19. A Venn diagram consists of overlapping ovals which are used to show the relationships between sets. Consider the numbers 156 and 520. Show how a Venn diagram could be used to determine their:
a. HCF **b.** LCM.

20. A group of 200 shoppers was asked which type of fruit they had bought in the last week. The results are shown in the table on the following page.

Fruit	Number of shoppers
Apples (A) only	45
Bananas (B) only	34
Cherries (C) only	12
A and B	32
A and C	15
B and C	26
A and B and C	11

a. Display this information in a Venn diagram.
b. Calculate $n(A \cap B' \cap C)$.
c. How many shoppers purchased apples and bananas but not cherries?
d. Calculate the relative frequency of shoppers who purchased:
i. apples
ii. bananas or cherries.
e. Estimate the probability that a shopper purchased cherries only.

Reflection

How will you remember the difference between when one event *and* another occurs and when one event *or* another occurs?

CHALLENGE 13.1

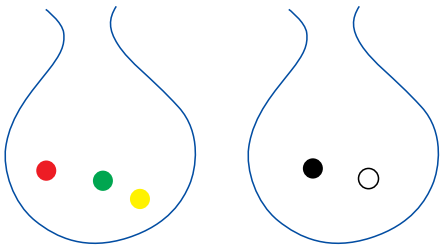
Bar codes are used to identify items. One particular system uses bars arranged in groups of five, each group containing two long bars and three short bars. In how many different ways can two long bars and three short bars be arranged in a group?



13.5 Two-step experiments

13.5.1 The sample space

- Imagine two bags (that are not transparent) that contain coloured counters. The first bag has a mixture of black and white counters, and the second bag holds red, green and yellow counters. In a probability experiment, one counter is to be selected at random from each bag and its colour noted.



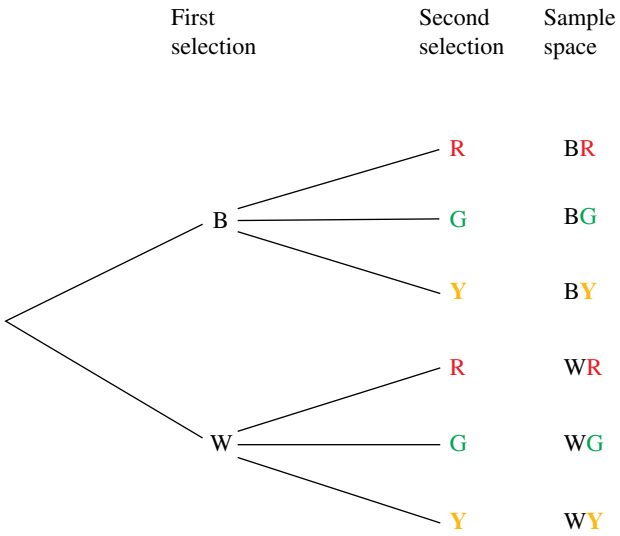
Bag 1 **Bag 2**

- The sample space for this experiment can be found using a table called an **array** that systematically displays all the outcomes.

		Bag 2		
		R	G	Y
Bag 1	B	B R	B G	B Y
	W	W R	W G	W Y

The sample space, $\xi = \{\text{BR, BG, BY, WR, WG, WY}\}$.

- The sample space can also be found using a **tree diagram**.



WORKED EXAMPLE 11

Two dice are rolled and the numbers uppermost are noted. List the sample space in an array.

a How many outcomes are there?

b How many outcomes contain at least one 5?

c What is P(at least one 5)?



THINK

Draw an array (a table) showing all the possible outcomes.

WRITE/DRAW

		Second die					
		1	2	3	4	5	6
First die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a The table shows 36 outcomes.

a There are 36 outcomes.

b Count the outcomes that contain 5. The cells are shaded in the table.

b Eleven outcomes include 5.

c There are 11 favourable outcomes and 36 in total.

c $P(\text{at least one 5}) = \frac{11}{36}$

WORKED EXAMPLE 12

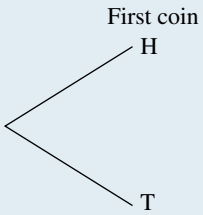
Two coins are tossed and the outcomes are noted. Show the sample space on a tree diagram.
a How many outcomes are there?
b Find the probability of tossing at least one Head.



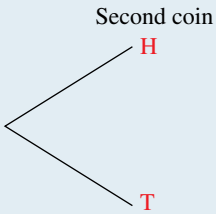
THINK

1 Draw a tree representing the outcomes for the toss of the first coin

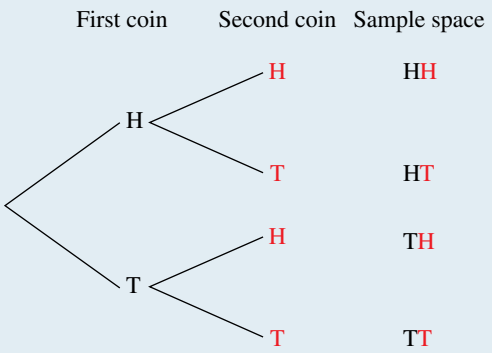
WRITE/DRAW



2 For the second coin the tree looks like this:



Add this tree to both ends of the first tree.



3 List the outcomes.

a Count the outcomes in the sample space.

b Three outcomes have at least one Head.

a There are 4 outcomes (HH, HT, TH, TT).

b P(at least one Head) $= \frac{3}{4}$

13.5.2 Two-step experiments

- When a coin is tossed, P(H) $= \frac{1}{2}$, and when a die is rolled, P(3) $= \frac{1}{6}$.
  If a coin is tossed and a die is rolled, what is the probability of getting a Head *and* a 3?
- Consider the sample space.



	1	2	3	4	5	6
H	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
T	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

There are 12 outcomes, and P(Head and 3) $= \frac{1}{12}$.

- In this case, P(Head and 3) = P(H) × P(3); that is, $\frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$.

- In general, if A is the outcome of one event and B is the outcome of a separate event, then

$$\mathbf{P}(A \cap B) = P(A) \times P(B).$$

WORKED EXAMPLE 13

In one cupboard Joe has 2 black t-shirts and 1 yellow one. In his drawer there are 3 pairs of white socks and 1 black pair. If he selects his clothes at random, what is the probability that his socks and t-shirt will be the same colour?

THINK

If they are the same colour then they must be black.

$\text{P(black t-shirt)} = \text{P}(B_t) = \frac{2}{3}$

$\text{P(black socks)} = \text{P}(B_s) = \frac{1}{4}$

WRITE

$$\begin{aligned}\text{P}(B_t \cap B_s) &= \text{P}(B_t) \times \text{P}(B_s) \\ &= \frac{2}{3} \times \frac{1}{4} \\ &= \frac{1}{6}\end{aligned}$$

13.5.3 Choosing with replacement

- Consider what happens when replacement is allowed in an experiment. Worked example 14 illustrates this situation.

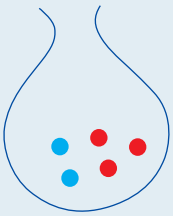
WORKED EXAMPLE 14

A bag contains 3 red and 2 blue counters. A counter is taken at random from the bag, its colour is noted, then it is returned to the bag and a second counter is chosen.

a Show the outcomes on a tree diagram.

b Find the probability of each outcome.

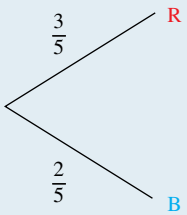
c Find the sum of the probabilities.



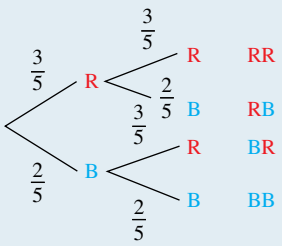
THINK

WRITE/DRAW

a 1 Draw a tree for the first trial. Write the probability on the branch.
Note: The probabilities should sum to 1.



2 For the second trial the tree is the same. Add this tree to both ends of the first tree.



3 List the outcomes.

b For both draws $P(R) = \frac{3}{5}$ and $P(B) = \frac{2}{5}$. Use the rule $P(A \cap B) = P(A) \times P(B)$ to determine the probabilities.

b
$$\begin{aligned} P(R \cap R) &= P(R) \times P(R) \\ &= \frac{3}{5} \times \frac{3}{5} \\ &= \frac{9}{25} \end{aligned}$$
$$\begin{aligned} P(R \cap B) &= P(R) \times P(B) \\ &= \frac{3}{5} \times \frac{2}{5} \\ &= \frac{6}{25} \end{aligned}$$
$$\begin{aligned} P(B \cap R) &= P(B) \times P(R) \\ &= \frac{2}{5} \times \frac{3}{5} \\ &= \frac{6}{25} \end{aligned}$$
$$\begin{aligned} P(B \cap B) &= P(B) \times P(B) \\ &= \frac{2}{5} \times \frac{2}{5} \\ &= \frac{4}{25} \end{aligned}$$

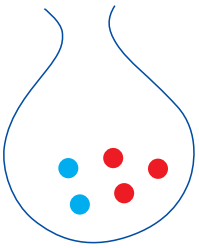
c Add the probabilities.

c $\frac{9}{25} + \frac{6}{25} + \frac{6}{25} + \frac{4}{25} = 1$

- In Worked example 14, $P(R) = \frac{3}{5}$ and $P(B) = \frac{2}{5}$ for both trials. This would not be so if a counter is selected *but not replaced.*

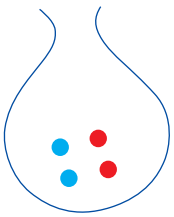
13.5.4 Choosing without replacement

- Let us consider again the situation described in Worked example 14, and consider what happens if the first marble is not replaced.
- Initially the bag contains 3 red and 2 blue counters, and either a red counter or a blue counter will be chosen.



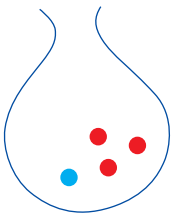
- $P(R) = \frac{3}{5}$ and $P(B) = \frac{2}{5}$.
- If the counter is not replaced, then the sample space is affected as follows:

If the first counter randomly selected is red, then the sample space for the second draw looks like this:



So $P(R) = \frac{2}{4}$ and $P(B) = \frac{2}{4}$.

If the first counter randomly selected is **blue**, then the sample space for the second draw looks like this:



So $P(R) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$.

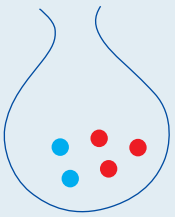
WORKED EXAMPLE 15

A bag contains 3 red and 2 blue counters. A counter is taken at random from the bag and its colour is noted, then a second counter is drawn, without replacing the first one.

a Show the outcomes on a tree diagram.

b Find the probability of each outcome.

c Find the sum of the probabilities.

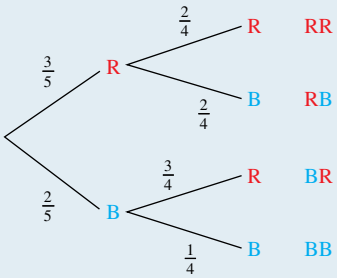


THINK

a Draw a tree diagram, listing the probabilities.

WRITE/DRAW

a



b Use the rule $P(A \cap B) = P(A) \times P(B)$ to determine the probabilities.

b
$$P(R \cap R) = P(R) \times P(R) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$
$$P(R \cap B) = P(R) \times P(B) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$
$$P(B \cap R) = P(B) \times P(R) = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$$
$$P(B \cap B) = P(B) \times P(B) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

c Add the probabilities.

c $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} + \frac{1}{10} = 1$

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Exercise 13.5 Two-step experiments

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 12	Questions: 1–15	Questions: 1–17

Individual pathway interactivity: int-4537

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Fluency

1. In her cupboard Rosa has 3 scarves (red, blue and pink) and 2 beanies (brown and purple). If she randomly chooses 1 scarf and 1 beanie, show the sample space in an array.



2. WE11 If two dice are rolled and their *sum* is noted, complete the array below to show the sample space.

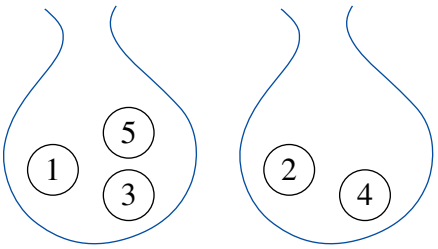
		Die 1					
		1	**2**	**3**	**4**	**5**	**6**
Die 2	**1**	2					
	2					7	
	3						
	4						
	5						
	6			9			

a. What is P(rolling a total of 5)?
b. What is P(rolling a total of 1)?
c. What is the most probable outcome?

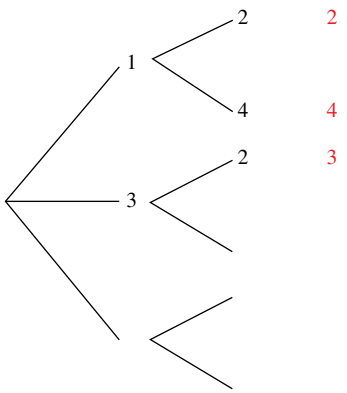
3. One box contains red and blue pencils, and a second box contains red, blue and green pencils. If one pencil is chosen at random from each box and the colours are noted, draw a tree diagram to show the sample space.



4. **WE12** A bag contains 3 discs labelled 1, 3 and 5, and another bag contains two discs, labelled 2 and 4, as shown below. A disc is taken from each bag and the larger number is recorded.



a. Complete the tree diagram below to list the sample space.



b. What is:

i. P(2) **ii.** P(1) **iii.** P(odd number)?

5. Two dice are rolled and the difference between the two numbers is found.

a. Use an array to find all the outcomes.

b. Find:

i. P(odd number) **ii.** P(0)

iii. P(a number more than 2) **iv.** P(a number no more than 2).

6. **WE13** A die is rolled twice. What is the probability of rolling:

a. a 6 on the first roll **b.** a double 6

c. an even number on both dice **d.** a total of 12?

7. A coin is tossed twice.

a. Show the outcomes on a tree diagram.

b. What is:

i. P(2 Tails) **ii.** P(at least 1 Tail)?

8. WE14 A bag contains 3 red counters and 1 blue counter. A counter is chosen at random. A second counter is drawn with replacement.

a. Show the outcomes and probabilities on a tree diagram.

b. Find the probability of choosing:

i. a red counter then a blue counter **ii.** two blue counters.

9. WE15 A bag contains 3 black balls and 2 red balls. If two balls are selected, randomly, without replacement:

a. show the outcomes and their probabilities on a tree diagram

b. find P(2 red balls).

Understanding

10. The kings and queens from a deck of cards are shuffled, then 2 cards are chosen. Find the probability that 2 kings are chosen:

a. if the first card is replaced

b. if the first card is not replaced.

11. Each week John and Paul play 2 sets of tennis against each other. They each have an equal chance of winning the first set. When John wins the first set, his probability of winning the second set rises to 0.6, but if he loses the first set, he has only a 0.3 chance of winning the second set.



a. Show the possible outcomes on a tree diagram.

b. What is:

i. P(John wins both sets)

ii. P(Paul wins both sets)

iii. P(they win 1 set each)?

Reasoning

12. A bag contains 4 red and 6 yellow balls. If the first ball drawn is yellow, explain the difference in the probability of drawing the second ball if the first ball was replaced compared to not being replaced.

13. Three dice are tossed and the total is recorded.

a. What are the smallest and largest possible totals?

b. Calculate the probabilities for all possible totals.

Problem solving

14. You draw two cards, one after the other without replacement, from a deck of 52 cards.

a. What is the probability of drawing two aces?

b. What is the probability of drawing two face cards (J, Q, K)?

c. What is the probability of getting a 'pair'? (22, 33, 44 … QQ, KK, AA)?

15. A chance experiment involves flipping a coin and rolling two dice. Determine the probability of obtaining Tails and two numbers whose sum is greater than 4.

16. In a jar there are 10 red balls and 6 green balls. Jacob takes out two balls, one at a time, without replacing them. What is the probability that both balls are the same colour?

17. In the game of 'Texas Hold'Em' poker, 5 cards are progressively placed face up in the centre of the table for all players to use. At one point in the game there are 3 face-up cards (two hearts and one diamond). You have 2 diamonds in your hand for a total of 3 diamonds. Five diamonds make a flush. Given that there are 47 cards left, what is the probability that the next two face-up cards are both diamonds?

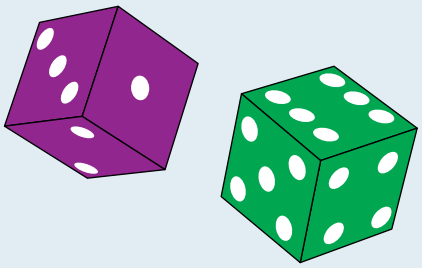


Reflection

How does replacement affect the probability of an event occurring?

CHALLENGE 13.2

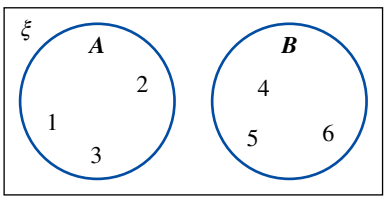
Professional gambling cheats use a variety of techniques to help their cause. One such method is to re-number the dice with only three numbers on each die. Consider two dice. The first die is numbered 1, 1, 2, 2, 3 and 3. The second die is numbered 4, 4, 5, 5, 6 and 6. When rolling two fair dice, the chance of getting a total of 7 is $\frac{1}{6}$. Would you be advantaged or disadvantaged by using the two re-numbered dice? Explain.



13.6 Mutually exclusive and independent events

13.6.1 Mutually exclusive events

- If two events cannot both occur at the same time then it is said the two events are **mutually exclusive**. For example, when rolling a die, the events 'getting a 1' and 'getting a 5' are mutually exclusive.
- If two sets are disjoint (have no elements in common), then the sets are mutually exclusive. For example, if $A = \{\text{prime numbers} > 10\}$ and $B = \{\text{even numbers}\}$, then A and B are mutually exclusive.
- If A and B are two mutually exclusive events (or sets), then $P(A \cap B) = \emptyset$.
- Consider the Venn diagram shown. Since A and B are disjoint, then A and B are mutually exclusive sets.
- If two events A and B are mutually exclusive, then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$.



13.6.2 Examples of mutually exclusive events

- Draw a card from a standard deck: the drawn card is a heart or a club.
  - Reason: it is impossible to get both a heart and a club at the same time.

- Record the time of arrival of overseas flights: a flight is late, on time or it is early.
  - Reason: it is impossible for the flight to arrive late, on time or early all at the same time.

13.6.3 Examples of non-mutually exclusive events

- Draw a card from a standard deck: the drawn card is a heart or a king.
  - Reason: it is possible to draw the king of hearts.
- Record the mode of transport of school students: count students walking or going by bus.
  - Reason: a student can walk (to the bus stop) and take a bus.

WORKED EXAMPLE 16

A card is drawn from a pack of 52 cards. What is the probability that the card is a diamond or a spade?

THINK	WRITE
1 The events are mutually exclusive because diamonds and spades cannot be drawn at the same time.	The two events are mutually exclusive as $P(A \cap B) = \varnothing$.
2 Determine the probability of drawing a diamond and the probability of drawing a spade.	Number of diamonds, $n(E_1) = 13$ Number of spades, $n(E_2) = 13$ Number of cards, $n(S) = 52$ $P(\text{diamond}) = \frac{13}{52}$ $P(\text{spade}) = \frac{13}{52}$ $= \frac{1}{4}$ $= \frac{1}{4}$
3 Write the probability.	$P(A \cup B) = P(A) + P(B)$ P(diamond or spade) $= P(\text{diamond}) + P(\text{spade})$
4 Evaluate and simplify.	$= \frac{1}{4} + \frac{1}{4}$ $= \frac{1}{2}$

13.6.4 Independent events

- Two events are considered independent if the outcome of one event is not dependent on the outcome of the other event.
- For example, if E_1 = {rolling a 4 on a first die} and E_2 = {rolling a 2 on a second die}, the outcome of event E_1 is not influenced by the outcome of event E_2, so the events are independent.

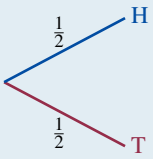
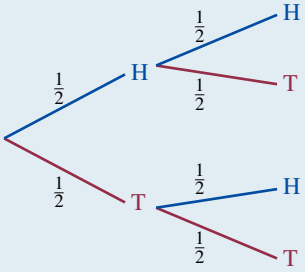
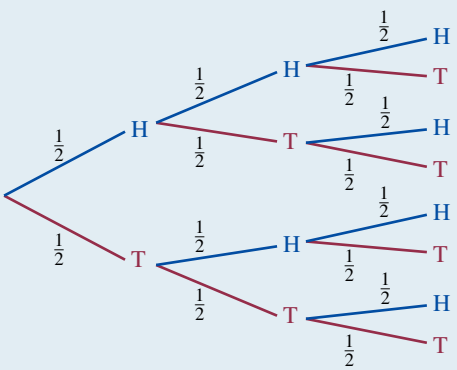
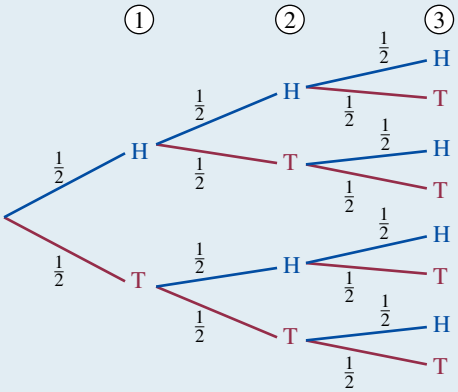
WORKED EXAMPLE 17

Three coins are flipped simultaneously. Draw a tree diagram for the experiment. Calculate the following probabilities.

a P(3 Heads)

b P(2 Heads)

c P(at least 1 Head)

THINK	WRITE/DRAW
1 Use branches to show the individual outcomes for the first part of the experiment (flipping the first coin). **2** Link each outcome of the first flip with the outcomes of the second part of the experiment (flipping the second coin). **3** Link each outcome from the second flip with the outcomes of the third part of the experiment (flipping the third coin).	First coin  First coin Second coin  First coin Second coin Third coin 
4 Determine the probability of each outcome. *Note:* The probability of each result is found by multiplying along the branches and in each case this will be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.	① ② ③  Outcomes Probability HHH $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ HHT $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ HTH $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ HTT $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ THH $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ THT $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ TTH $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ TTT $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ 1
a The probability of three heads is P(H, H, H)	**a** P(3 Heads) $= \frac{1}{8}$.

b 1 {2 Heads} has 3 satisfactory outcomes: (H, H, T), (H, T, H) and (T, H, H), which are mutually exclusive.

b $P(2\text{Heads})$
$= P(H, H, T) + P(H, T, H) + P(T, H, H)$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$= \frac{3}{8}$

2 Write your answer.

The probability of obtaining exactly 2 Heads is $\frac{3}{8}$.

c 1 At least 1 Head means any outcome that contains one or more Head. This is every outcome except three Tails. That is, it is the complementary event to obtaining 3 Tails.

c $P(\text{at least 1 Head})$
$= 1 - P(T, T, T)$
$= 1 - \frac{1}{8}$
$= \frac{7}{8}$

2 Write your answer.

The probability of obtaining at least 1 Head is $\frac{7}{8}$.

Note: The probabilities of all outcomes add to 1.

13.6.5 Dependent events

- Many real-life events have some dependence upon each other, and their probabilities are likewise affected.
- Examples include:
  - the chance of rain today and the chance of a person taking an umbrella to work
  - the chance of growing healthy vegetables and the availability of good soil
  - the chance of Victory Soccer Club winning this week and winning next week
  - drawing a card at random, not replacing it, and drawing another card.
- It is important to be able to recognise the difference between *dependent* events and *independent* events.

WORKED EXAMPLE 18

A jar contains three black marbles, five red marbles, and two white marbles. Find the probability of choosing a black marble (with replacement), then choosing another black marble.

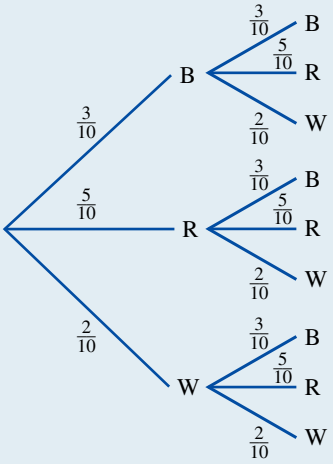
THINK

1 The events, draw 1 and draw 2, are independent because the result of the first draw is not dependent on the result of the second draw.

WRITE/DRAW

E_1 and E_2 are independent events.

2 Demonstrate using a tree diagram.



3 Determine the probability.

$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$

$P(\text{black and black}) = P(\text{black}) \times P(\text{black})$

Evaluate and simplify.

$P(\text{black and black}) = \frac{3}{10} \times \frac{3}{10}$

$= \frac{9}{100}$

- If the first marble had not been replaced in the previous worked example, the second draw would be dependent on the outcome of the first draw, and so it follows that the sample space for the second draw is different from that for the first draw.

WORKED EXAMPLE 19

Repeat Worked example 15 without replacing the first marble before the second one is drawn.

THINK

1 The words 'without replacing' indicate that the two events are dependent.
Write the sample space and state the probability of choosing a black marble on the first selection.

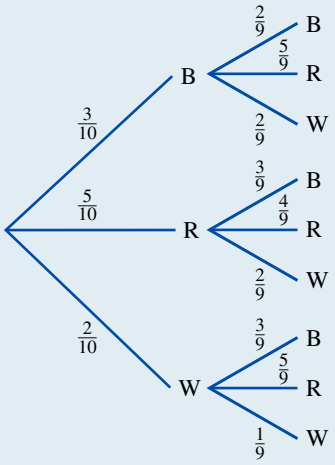
WRITE/DRAW

There are 10 marbles and 3 of these are black.
The sample space is
{B, B, B, R, R, R, R, R, W, W}.
$P(E_1) = P(\text{black}) = \frac{3}{10}$

2 Assume that a black marble was chosen in the first selection. Determine how many black ones remain, and the total number of remaining marbles.
Write the sample space and state the probability of choosing a black marble on the second selection.

A black one was chosen, leaving 2 black ones and a total of 9 marbles.
The sample space is
{B, B, R, R, R, R, R, W, W}.
$P(E_2) = P(\text{black}) = \frac{2}{9}$

3 Demonstrate using a tree diagram.



4 Multiply the probabilities.

$$\begin{aligned} P(E_1 \text{ and } E_2) &= P(E_1) \times P(E_2) \\ &= \frac{3}{10} \times \frac{2}{9} \\ &= \frac{1}{15} \end{aligned}$$

5 Answer the question.

The probability of choosing two black marbles without replacing the first marble is $\frac{1}{15}$.

Exercise 13.6 Mutually exclusive and independent events

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 5, 7, 9, 11, 13–16, 18, 20, 22, 24, 26, 29	Questions: 1, 3–6, 8, 10, 12–15, 18, 20, 24–26, 28, 29	Questions: 1, 5–8, 11, 13, 14, 16, 17, 19–23, 25, 27, 28, 30

Individual pathway interactivity: int-4538

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Fluency

1. MC If a card is drawn from a pack of 52 cards, what is the probability that the card is not a queen?

A. $\frac{4}{52}$ **B.** $\frac{4}{48}$

C. $\frac{13}{12}$ **D.** $\frac{48}{52}$

2. MC Which events are not mutually exclusive?

A. Drawing a queen and drawing a jack from 52 playing cards

B. Drawing a red card and drawing a black card from 52 playing cards

C. Drawing a vowel and drawing a consonant from cards representing the 26 letters of the alphabet

D. Obtaining a total of 8 and rolling doubles (when rolling two dice)

3. When a six-sided die is rolled 3 times, the probability of getting 3 sixes is $\frac{1}{216}$. What is the probability of not getting 3 sixes?

4. MC Eight athletes compete in a 100-m race. The probability that the athlete in lane 1 will win is $\frac{1}{5}$. What is the probability that one of the other athletes wins? (Assume that there are no dead heats.)



A. $\frac{1}{5}$ **B.** $\frac{5}{8}$

C. $\frac{8}{5}$ **D.** $\frac{4}{5}$

5. A pencil case has 4 red pens, 3 blue pens and 5 black pens. If a pen is randomly drawn from the pencil case, find:

a. P(drawing a blue pen)

b. P(not drawing a blue pen).

6. Seventy Year 9 students were surveyed. Their ages ranged from 13 years to 15 years, as shown in the table below.

Age	13	14	15	Total
Boys	10	20	9	39
Girls	7	15	9	31
Total	17	35	18	70

A student from the group is selected at random. Find:

a. P(selecting a student of the age of 13 years)

b. P(not selecting a student of the age of 13 years)

c. P(selecting a 15-year-old boy)

d. P(not selecting a 15-year-old boy).

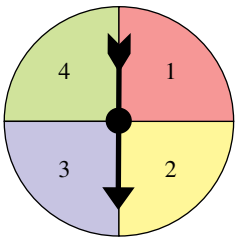


7. WE16 A card is drawn from a pack of 52 cards. What is the probability that the card is a king or an ace?

8. MC A die is rolled. Find the probability of getting an even number or a 3.

A. $\frac{3}{6}$ **B.** $\frac{4}{6}$

C. $\frac{1}{6}$ **D.** $\frac{5}{6}$



9. If you spin the following spinner, what is the probability of obtaining:

a. a 1 or a 3

b. an even number or an odd number?

10. The probabilities of Dale placing 1st, 2nd, 3rd or 4th in the local surf competition are:

1st $= \frac{1}{6}$ 2nd $= \frac{1}{5}$ 3rd $= \frac{2}{5}$ 4th $= \frac{7}{30}$.

Find the probability that Dale places:

a. 1st or 2nd **b.** 3rd or 4th **c.** 1st, 2nd or 3rd **d.** not 1st.

11. WE17 A circular spinner that is divided into two equal halves, coloured red and blue, is spun 3 times.

a. Draw a tree diagram for the experiment.

b. Calculate the following probabilities.

i. P(3 red sectors) **ii.** P(2 red sectors) **iii.** P(1 red sector)

iv. P(0 red sectors) **v.** P(at least 1 red sector) **vi.** P(at least 2 red sectors)



12. WE18 There are two yellow tickets, three green tickets, and four black tickets in a jar. Choose one ticket, replace it, then choose another ticket. Find the probability that a yellow ticket is drawn first, then a black ticket.

13. WE19 Repeat question **12** with the first ticket not being replaced before the second ticket is drawn.

14. A coin is tossed two times. Determine P(a Head and a Tail in any order).

15. A coin is tossed three times. Determine P(H, H, T) (in that order).

16. A coin and a die are tossed. What is the probability of a Heads–2 outcome?

Understanding

17. Holty is tossing two coins. He claims that flipping two Heads and flipping zero Heads are complementary events. Is he correct? Explain your answer.

18. Each of the numbers 1, 2, 3, . . . 20 is written on a card and placed in a bag. If a card is drawn from the bag, find:

a. P(drawing a multiple of 3 or a multiple of 10)

b. P(drawing an odd number or a multiple of 4)

c. P(drawing a card with a 5 or a 7)

d. P(drawing a card with a number less than 5 or more than 16).

19. From a shuffled pack of 52 cards, a card is drawn. Find:

a. P(hearts or the jack of spades)

b. P(a queen or a jack)

c. P(a 7, a queen or an ace)

d. P(neither a club nor the king of spades).

20. MC Which are not mutually exclusive?

A. Obtaining an odd number on a die and obtaining a 4 on a die

B. Obtaining a Head on a coin and obtaining a Tail on a coin

C. Obtaining a red card and obtaining a black card from a pack of 52 playing cards

D. Obtaining a diamond and obtaining a king from a pack of 52 playing cards

21. Greg has a 30% chance of scoring an A on an exam, Carly has 70% chance of scoring an A on the exam, and Chilee has a 90% chance of scoring an A on the exam. What is the probability that all three can score an A on the exam?

22. From a deck of playing cards, a card is drawn at random, noted, replaced and another card is drawn at random. Find the probability that:

a. both cards are spades

b. neither card is a spade

c. both cards are aces

d. both cards are the ace of spades

e. neither card is the ace of spades.

23. Repeat question **22** with the first drawn card not being replaced before the second card is drawn.

24. Assuming that it is equally likely that a boy or a girl will be born, answer the following.

a. Show the gender possibilities of a 3-child family on a tree diagram.

b. In how many ways is it possible to have exactly 2 boys in the family?

c. What is the probability of getting exactly 2 boys in the family?

d. Which is more likely, 3 boys or 3 girls in the family?

e. What is the probability of having at least 1 girl in the family?

Reasoning

25. Give an example of mutually exclusive events that are not complementary events using:

a. sets

b. a Venn diagram.

26. Explain why all complementary events are mutually exclusive but not all mutually exclusive events are complementary.

27. A married couple plans to have four children.

a. List the possible outcomes in terms of boys and girls.

b. What is the probability of them having exactly two boys?

c. Another couple plans to have two children. What is the probability that they have exactly one boy?

Problem solving

28. A bag contains 6 marbles, 2 of which are red, 1 is green and 3 are blue. A marble is drawn, the colour is noted, the marble is replaced and another marble is drawn.



a. Show the possible outcomes on a tree diagram.

b. List the outcomes of the event 'the first marble is red'.

c. Calculate P(the first marble is red).

d. Calculate P(2 marbles of the same colour are drawn).

29. A tetrahedron (prism with 4 identical triangular faces) is numbered 1, 1, 2, 3 on its 4 faces. It is rolled twice. The outcome is the number facing downwards.

a. Show the results on a tree diagram.

b. Are the outcomes 1, 2 and 3 equally likely?

c. Find the following probabilities:

i. P(1, 1)

ii. P(1 is first number)

iii. P(both numbers the same)

iv. P(both numbers are odd).

30. Robyn is planning to watch 3 footy games on one weekend. She has a choice of two games on Friday night: (A) Carlton vs West Coast and (B) Collingwood vs Adelaide. On Saturday, she can watch one of three games: (C) Geelong vs Brisbane, (D) Melbourne vs Fremantle and (E) North Melbourne vs Western Bulldogs. On Sunday, she also has a choice of three games: (F) St Kilda vs Sydney, (G) Essendon vs Port Adelaide and (H) Richmond vs Hawthorn. She plans to watch one game each day and will choose a game at random.



a. To determine the different combinations of games Robyn can watch, she draws a tree diagram using codes A, B, . . . H. List the sample space for Robyn's selections.

b. Robyn's favourite team is Carlton. What is the probability that one of the games Robyn watches involves Carlton?

c. Robyn has a good friend who plays for St Kilda. What is the probability that Robyn watches both the matches involving Carlton and St Kilda?

Reflection

What is the difference between independent events and mutually exclusive events?

13.7 Conditional probability

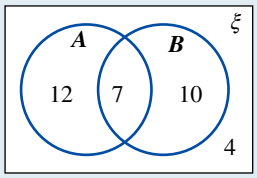
13.7.1 Conditional probability

- The probability that an event occurs given that another event has already occurred is called **conditional probability**.
- The probability that event B occurs, given that event A has already occurred is denoted by $P(B \mid A)$. The symbol 'l' stands for 'given'.
- The formula for conditional probability is:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0.$$

WORKED EXAMPLE 20

This Venn diagram below shows the results of a survey where students were asked to indicate whether they liked apples or bananas.



If one student is selected at random:
a What is the probability that the student likes bananas?
b What is the probability that the student likes bananas, given that they also like apples?
c Comment on any differences between the answers for parts a and b.

THINK	WRITE
a 1 Find the total number of students.	**a** Total number of students in survey $= 12 + 7 + 10 + 4$ $= 33$
2 Find the total number of students who like bananas.	Total number who like bananas $= 7 + 10$ $= 17$
3 Determine the probability using the correct formula.	$P(\text{bananas}) = P(B)$ $= \frac{\text{Total number who like bananas}}{\text{Total number of students}}$ $= \frac{17}{33}$
4 Write the answer.	The probability that a student likes bananas is $\frac{17}{33}$.
b 1 Determine the number of students who like apples.	**b** Number of students who like apples $= 12 + 7$ $= 19$
2 Find the probability that a student likes apples.	$P(\text{apples}) = P(A)$ $= \frac{\text{Number of students who like apples}}{\text{Total number of students}}$ $= \frac{19}{33}$

3 Note the number liking both apples and bananas. This is the overlapping region of the two sets.

Number who like both apples and bananas
$= n(A \cap B)$
$= 7$

4 Determine the probability a student likes both apples and bananas.

$$P(A \cap B) = \frac{7}{33}$$

5 Apply the formula to determine the conditional probability.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{\frac{7}{33}}{\frac{19}{33}}$$
$$= \frac{7}{19}$$

6 Write the answer.

The probability that a student likes bananas, given that they also like apples is $\frac{7}{19}$.
This answer is also supported by the figures in the Venn diagram.

c Why aren't the answers for parts **a** and **b** both the same?

c The answer for part **a** determines the proportion of students who like bananas out of the whole group of students. The part **b** answer gives the proportion of students who like bananas out of those who like apples.

Note: These probabilities could also be expressed as decimals or percentages.

- It is possible to transpose the conditional probability formula to determine $P(A \cap B)$.

$$\mathbf{P(A \cap B) = P(A) \times P(B \mid A)}$$

WORKED EXAMPLE 21

In a student survey, the probability that a student likes apples is $\frac{19}{33}$. The probability that a student likes bananas, given that they also like apples, is $\frac{7}{19}$. What is the probability that a student selected at random likes both apples and bananas?

THINK	WRITE
1 Write the given information.	$P(A) = \frac{19}{33}$ $P(B \mid A) = \frac{7}{19}$
2 Apply the rearranged conditional probability formula.	$P(A \cap B) = P(A) \times P(B \mid A)$ $= \frac{19}{33} \times \frac{7}{19}$ $= \frac{7}{33}$
3 Answer the question.	The probability that a student selected at random likes both apples and bananas is $\frac{7}{33}$.

- Conditional probability can also be determined by examining outcomes from a tree diagram.

WORKED EXAMPLE 22

Three coins are flipped simultaneously.

a Display the outcomes as a tree diagram.

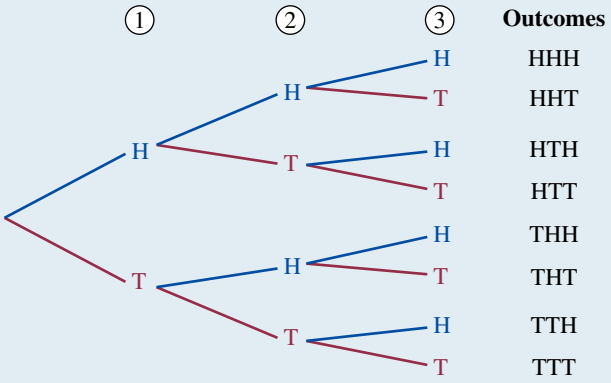
b Determine the probability that a Head will result from the third coin, given that the first two coins resulted in a Head (H) and a Tail (T).

THINK

a Draw a tree diagram to display the flipping of three coins. Write the individual outcomes.

WRITE/DRAW

a



b 1 Look for the outcomes where the first two flips resulted in a Head and a Tail.

b There are four outcomes where the first two flips are a Head and a Tail — HTH, HTT, THH and THT.

2 How many of these outcomes have a Head for the third flip?

There are two of these outcomes where the third flip resulted in a Head — HTH and THH.

3 Calculate the probability.

From four possible outcomes, two satisfy the conditions.

$$P(\text{H on third flip} \mid \text{H and T on first two flips}) = \frac{2}{4} = \frac{1}{2}$$

- A two-way table can also be used to determine conditional probability.

WORKED EXAMPLE 23

Two dice are rolled and the numbers are added.

a Show the results in a two-way table.

b Determine the probability that the sum of the two dice is 7, given that their total is greater than 6.

THINK

a Show the results of rolling two dice in a two-way table.

WRITE/DISPLAY

a

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b 1 Which outcomes have a total greater than 6?	**b** There are 36 outcomes. 21 of these have a total greater than 6 (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), (6, 2) . . . etc.
2 Which of these outcomes have a total equal to 7?	There are 6 of these outcomes that have a total of 7 – (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6).
3 Write the probability.	P(total of 7 | total greater than 6) $= \frac{6}{21} = \frac{2}{7}$

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Exercise 13.7 Conditional probability

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 5, 7, 9–11	Questions: 1–4, 6–13	Questions: 1–15

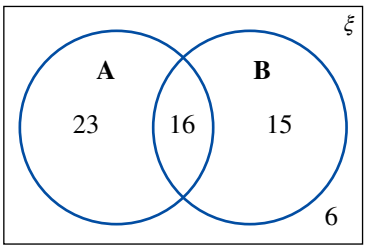
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Fluency

1. WE21 A group of motocross racers was asked to comment on which of two tracks, A or B, they used. The results were recorded in the Venn diagram below.



a. How many motocross racers were surveyed?
b. Calculate $P(A \cap B)$.
c. Calculate:
i. $P(A)$ **ii.** $P(B \mid A)$.
d. Calculate:
i. $P(B)$ **ii.** $P(A \mid B)$.

2. Consider your answers to question **1**.
a. Use your answers from part **c** to determine $P(A \cap B)$.
b. Use your answers from part **d** to determine $P(A \cap B)$.
c. Comment on your answers to parts **a** and **b** in this question.

3. A survey was conducted to determine whether a group of students preferred drink A or drink B. The results of the survey produced the following probabilities.
$P(A) = \frac{7}{10}$ and $P(B \mid A) = \frac{3}{7}$.
Determine $P(A \cap B)$.



4. WE22 Two fair coins are tossed.
a. Display the outcomes as a tree diagram.
b. Determine the probability that a Head results on the second coin, given that the first coin also resulted in a Head.

5. WE23 Two standard dice are rolled and the numbers are added together.

a. Show the results in a two-way table.

b. Determine the probability that the sum of the two dice is even, given that their total is greater than 7.

Understanding

6. A group of 40 people was surveyed regarding the types of movies, comedy or drama, that they enjoyed. The results are shown below.

28 enjoyed comedy.

17 enjoyed drama.

11 liked both comedy and drama.

6 did not like either type.



a. Draw a Venn diagram to display the results of the survey.

b. Determine the probability that a person selected at random:

i. likes comedy

ii. likes drama

iii. likes both comedy and drama

iv. likes drama, given that they also like comedy

v. likes comedy, given that they also like drama.

c. Arrange the probabilities in part **b** in order from least probable to most probable.

7. A teacher gave her class two tests. Only 25% of the class passed both tests, but 40% of the class passed the first test. What percentage of those who passed the first test also passed the second test?

Reasoning

8. If $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cup B) = 0.6$, calculate:

a. $P(A \cap B)$ **b.** $P(B \mid A)$ **c.** $P(A \mid B)$.

9. A group of 80 boys is auditioning for the school musical. They are all able to either sing, play a musical instrument, or both. Of the group, 54 can play a musical instrument and 35 are singers. What is the chance that if a randomly selected student is a singer he can also play a musical instrument?

10. A white die and a black die are rolled. The dice are 6-sided and unbiased. Consider the following events.

Event A: the white die shows a 6.

Event B: the black die shows a 2.

Event C: the sum of the two dice is 4.

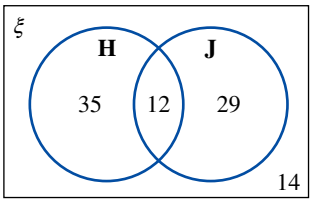
Determine the following probabilities.

a. $P(A \mid B)$ **b.** $P(B \mid A)$ **c.** $P(C \mid A)$ **d.** $P(C \mid B)$

11. A die is rolled and the probability of rolling a 6 is $\frac{1}{6}$. However, with the condition that the number rolled was an even number, its probability is $\frac{1}{3}$. Explain why the probabilities are different, using conditional probability.

Problem solving

12. A group of students was asked to nominate their favourite form of dance, hip hop (H) or jazz (J). The results are illustrated in the Venn diagram. Use the Venn diagram given to calculate the following probabilities relating to a student's favourite form of dance.



a. What is the probability that a randomly selected student likes jazz?

b. What is the probability that a randomly selected student enjoys hip hop, given that they like jazz?

13. At a school classified as a 'Music school for excellence', the probability that a student elects to study Music and Physics is 0.2. The probability that a student takes Music is 0.92. What is the probability, correct to 2 decimal places, that a student takes Physics, given that the student is taking Music?

14. A medical degree requires applicants to participate in two tests, an aptitude test and an emotional maturity test. This year, 52% passed the aptitude test and 30% passed both tests. Use the conditional probability formula to calculate the probability, correct to 2 decimal places, that a student who passed the aptitude test also passed the emotional maturity test.

15. The probability that a student is well and misses a work shift the night before an exam is 0.045, while the probability that a student misses a work shift is 0.05. What is the probability that a student is well, given they miss a work shift the night before an exam?





Reflection

How can you determine when a probability question is a conditional one?

13.8 Review

13.8.1 Review questions

Fluency

1. In a trial, it was found that a drug cures $\frac{2}{5}$ of those treated by it. If 700 sufferers are treated with the drug, how many of them are not expected to be cured?

A. 280 **B.** 420 **C.** 140 **D.** 350

2. Two dice are rolled and their numbers added. The probability that their total is odd, given that the total is less than 5 is:

A. $\frac{2}{5}$ **B.** $\frac{1}{2}$ **C.** $\frac{3}{5}$ **D.** $\frac{1}{3}$

3. If a die is rolled and a coin tossed, what is the probability of a 6–Heads result?

A. $\frac{1}{6}$ **B.** $\frac{1}{2}$ **C.** $\frac{1}{8}$ **D.** $\frac{1}{12}$

4. Which one of the following does not represent independent events?

A. Flipping a coin, then rolling a die

B. The colour of your hair, and your marks in school

C. Choosing a card from a standard deck of cards without replacing it, then choosing another card from the same deck

D. Flipping a coin ten times

5. Twelve nuts are taken from a jar containing macadamias and cashews. If 3 macadamias are obtained, the experimental probability of obtaining a cashew is:

A. $\frac{1}{12}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{3}$ **D.** $\frac{3}{4}$

6. From a normal pack of 52 playing cards, one card is randomly drawn and replaced. If this is done 208 times, the number of red or picture cards expected to turn up is:

A. 150 **B.** 130 **C.** 128 **D.** 120

7. A cubic die with faces numbered 2, 3, 4, 5, 6 and 6 is rolled. The probability of rolling an even number is:

A. $\frac{1}{3}$ **B.** $\frac{2}{3}$ **C.** $\frac{1}{6}$ **D.** $\frac{1}{2}$

8. The probability of rolling an odd number or a multiple of 2 using the die in question **7** is:

A. 1 **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{3}{4}$

The following information applies to questions **9** and **10**.

Students in a Year 9 class chose the following activities for a recreation day.

Activity	Tennis	Fishing	Golf	Bushwalking
Number of students	8	15	5	7

9. If a student is selected at random from the class, the probability that the student chose fishing is:

A. $\frac{1}{7}$ **B.** $\frac{2}{7}$ **C.** $\frac{3}{7}$ **D.** $\frac{4}{7}$

10. If a student is selected at random, the probability that the student did not choose bushwalking is:

A. $\frac{1}{35}$ **B.** $\frac{2}{5}$ **C.** $\frac{3}{5}$ **D.** $\frac{4}{5}$

11. The mass of 40 students in a Year 9 Maths class was recorded in a table.

Mass (kg)	Less than 50	50–<55	55–<60	60–<65	65 and over
Number of students	4	6	10	15	5

What is the experimental probability of selecting a student who has:

a. a mass of 55 kg or more, but less than 60 kg?

b. a mass less than 50 kg?

c. a mass of 65 kg or greater?

12. Calculate the following expected values.

a. The number of Heads in 80 tosses of a coin

b. The number of sixes in 200 rolls of a die

c. The number of hearts if a card is picked from a reshuffled pack and replaced 100 times

13. Consider a family of three children. Given that the first child is a girl, what is the probability that the other two children are both boys?

14. Insert each of the letters **a** to **d** to represent the following events at appropriate places on the given probability scale below.



a. You will go to school on Christmas Day.

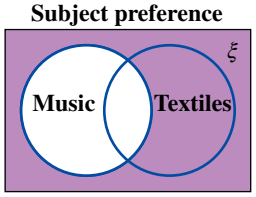
b. All Year 9 students can go to university without doing Year 10.

c. Year 9 students will study Maths.

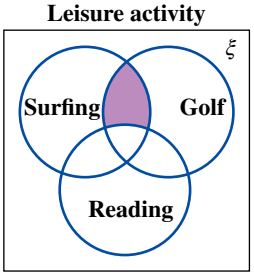
d. An Australian TV channel will telecast the news at 6.00 pm.

15. Indicate the set that each of the shaded regions represents.

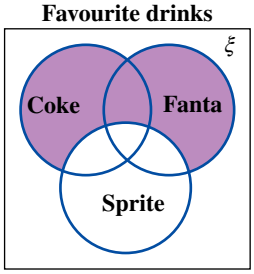
a. **Subject preference**



b. **Leisure activity**



c. **Favourite drinks**



16. A normal six-sided die is rolled. What is the probability of getting an odd number or a multiple of 4?

17. A card is drawn from a pack of 52 cards. What is the probability that the card is a heart or a club?

18. An equal number of red (R), black (B) and yellow (Y) counters makes up a total of 30 in a bag.

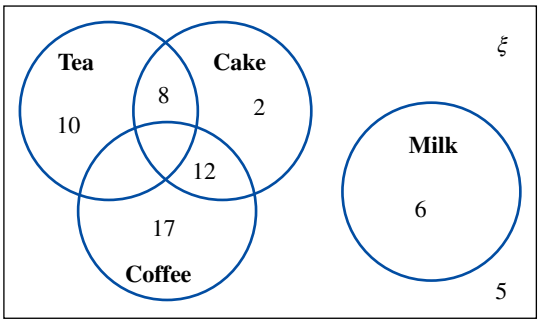
a. List the sample space for picking a counter from the bag.

b. Event A is 'the first counter is yellow' if two counters are selected. List the sample space.

c. Is 'choosing a green counter' an outcome? Explain your answer.

Problem solving

19. Teachers at a school opted for the choice of morning recess refreshments shown in the Venn diagram below.



a. How many teachers are in the set 'cake ∩ coffee'?

b. What is the total number of teachers surveyed?

c. If a teacher is selected at random, find the probability that the teacher:

i. chose tea

ii. chose coffee only

iii. chose milk

iv. did not choose tea, coffee, cake or milk

v. did not choose coffee.

20. Thirty-two students ordered fried rice, chicken wings and dim sims for lunch. Four students ordered all three, 2 ordered fried rice and chicken wings only, 3 ordered fried rice and dim sims only, and 3 ordered chicken wings and dim sims only. When the waiter organised the orders, he found that 16 students ordered fried rice and 18 students ordered dim sims.

a. Show this information on a Venn diagram.

b. How many students ordered chicken wings only?

c. If a student is picked at random, find the probability that the student has:

i. ordered chicken wings and dim sims

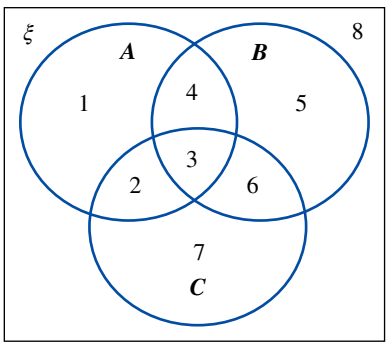
ii. ordered fried rice

iii. not ordered dim sims.

21. The following are options for dorm rooms at a university. You are required to choose one option from each of the four categories. How many different combinations of rooms are there to choose from?

- Upstairs or downstairs rooms
- Single or double rooms
- Male or female roommates
- A choice of 10 different locations on campus

22. Use the Venn diagram to calculate each of the following.



a. $A' \cup (B' \cap C')$

b. $A' \cap (B \cap C')$

c. $A \cap (B' \cup C')$

d. $(A \cup B \cup C)' \cap (A \cup B \cup C)$

23. A witness described a getaway car as having a NSW registration plate starting with TLK. The witness could not remember the three digits that followed, but recalled that all three digits were different. How many cars in NSW could have a registration plate with these letters and numbers?

24. There are 12 people trying out for a badminton team. Five of them are girls. What is the probability that a team chosen at random to play is a mixed doubles team?

25. An ace is chosen from a deck of standard cards and not replaced. A king is then chosen from the deck. What is the probability of choosing an ace and a king in this order?

26. If you randomly select one number from 1 to 500 (inclusive), what is the probability that the selected number will have at least one four in the digits?

27. If you flip a coin 8 times, how many of the possible outcomes would you expect to have a Head on the second toss?

28. Any future king is the oldest male child. The future king of Mainland has two siblings. What is the probability he has an older sister?

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- **Try out this interactivity:** Word search: Topic 13 (int-2711)
- **Try out this interactivity:** Crossword: Topic 13 (int-2712)
- **Try out this interactivity:** Sudoku: Topic 13 (int-3212)
- **Try out this interactivity:** Concept map: Topic 13 (int-13668)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

array
certain
chance
chance experiment
complementary
conditional
dependent
equally likely
event
experiment
experimental probability
favourable outcome
impossible
independent
intersection
likely
mutually exclusive
outcome
probability
random
sample space
scale
theoretical probability
tree diagram
trial
two-way table
Venn diagram

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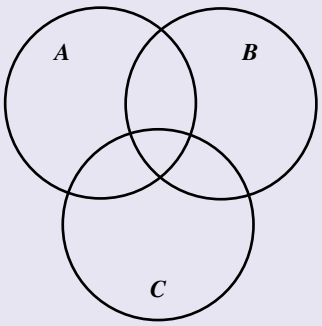
Investigation | Rich task

Sand-rings





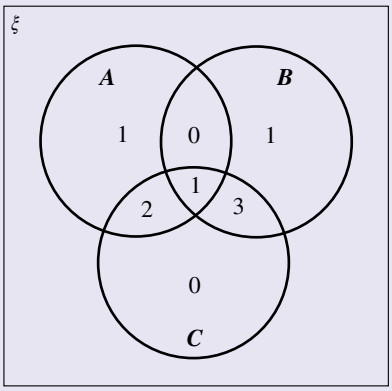
A class of students and their teacher spent a day at the beach as part of a school excursion. Part of the day was devoted to activities involving puzzles in the sand. One o the popular — and most challenging — puzzles was 'sand-rings'. Sand-rings involve drawing three rings in the sand as shown in the diagram at right.



The first sand-rings puzzle requires 8 shells to be arranged inside the circles, so that 4 shells appear inside circle A, 5 shells appear inside circle B and 6 shells appear inside circle C. The overlapping of the circles shows that the shells can be counted in 2 or 3 circles. One possible arrangement is shown below. Use this diagram to answer questions 1 to 4.

1. How many shells appear inside circle A, but not circle B?
2. How many shells appear in circles B and C, but not circle A?





A shell is selected at random from the sand.

3. What is the probability it came from circle A?
4. What is the probability it was not in circle C?
5. The class was challenged to find the rest of the arrangements of the 8 shells. (*Remember:* 4 shells need to appear in circle A, 5 in circle B and 6 in circle C.)

After completing the first puzzle, the students are given new rules. The number of shells to be arranged in the circles is reduced from 8 to 6. However, the number of shells to be in each circle remains the same; that is, 4 shells in circle A, 5 shells in circle B and 6 shells in circle C.



6. Using 6 shells, in how many ways can the shells be arranged so that there are 4, 5 and 6 shells in the three circles?
7. Explain the system or method you used to determine your answer to question **6** above. Draw diagrams in the space provided to show the different arrangements.
8. Using 7 shells, in how many ways can the shells be arranged so that there are 4, 5 and 6 shells in the three circles?
9. Again, explain the system or method you used to determine your answer to question **8**. Draw diagrams in the space provided to show the different arrangements.
10. What would be the minimum number of shells required to play sand-rings, so that there are 4, 5 and 6 shells in the three circles?
11. Modify the rules of this game so that different totals are required for the 3 circles. Challenge your classmates to find all possible solutions to your modified game.

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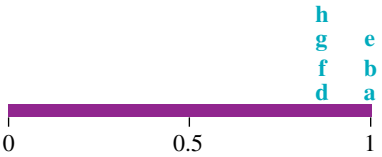
Complete this digital doc: Code puzzle: Roy G. Biv is a mnemonic (memory aid) for what purpose? (doc-15909)

Answers

Topic 13 Probability

Exercise 13.2 Theoretical probability

1.



2. **a.** Certain **b.** Certain **c.** Unlikely **d.** Likely
e. Certain **f.** Likely **g.** Likely **h.** Likely

3. **a.** {1, 2, 3, 4, 5, 6} **b.** {H, T} **c.** {defective, not defective}
d. {red, black} **e.** {hearts, clubs, diamonds, spades}

4. **a.** {1, 2, 3} **b.** {1, 3, 5} **c.** {1, 2, 4, 6} **d.** {3, 4, 5, 6}
e. {1, 2, 3, 4} **f.** {5, 6}

5. **a.** 3, 5 **b.** 1, 2 **c.** 6 **d.** 1, 2, 5, 6
e. 1, 2, 3, 5 **f.** no favourable outcomes **g.** 3

6. **a.** {hearts, clubs, diamonds, spades}
b. **i.** {clubs, spades}
ii. {hearts, diamonds}
iii. {clubs, diamonds, spades}
iv. {hearts, clubs, diamonds, spades}

7. **a.** 6 **b.** 2 **c.** 52 **d.** 4 **e.** 5

8. **a.** $\frac{1}{13}$ **b.** $\frac{1}{13}$ **c.** $\frac{1}{52}$ **d.** $\frac{1}{4}$

9. **a.** $\frac{1}{52}$ **b.** $\frac{1}{2}$ **c.** $\frac{1}{26}$ **d.** $\frac{1}{4}$
e. $\frac{1}{2}$ **f.** $\frac{1}{52}$ **g.** $\frac{4}{13}$ **h.** $\frac{2}{13}$
i. 0 **j.** $\frac{12}{13}$

10. **a.** $\frac{2}{11}$ **b.** $\frac{9}{11}$ **c.** $\frac{4}{11}$ **d.** $\frac{7}{11}$

11. **a.** $\frac{1}{4}$ **b.** $\frac{1}{8}$ **c.** $\frac{5}{16}$ **d.** 0 **e.** $\frac{3}{4}$

12. **a.** **i.** $\frac{2}{3}$ **ii.** $\frac{1}{3}$
b. Answers will vary.

13, 14 Check with your teacher.

15. Probabilities must be between 0 and 1, so -0.5 and 1.05 can't be probabilities.

16. The coloured portions outside the arc of the spinner shown are of no consequence. The four colours within the arc of the spinner are of equal area (each $\frac{1}{4}$ circle), so there is equal chance of falling on each of the colours.

17. $\frac{2}{3}$

18. There are 36 outcomes, 15 under 7, 6 equal to 7 and 15 over 7. So, you would have a greater chance of winning if you chose 'under 7' or 'over 7' rather than 'equal to 7'.

19. 0.09

Exercise 13.3 Experimental probability

1. **a.** **i.** 16 **ii.** 4

iii.

Outcome	1	2	3	4	5	6
Relative frequency	0.19	0.06	0.31	0.13	0.25	0.06

iv. 1

b. **i.** 270 **ii.** 40

iii.

Outcome	1	2	3	4	5	6
Relative frequency	0.19	0.14	0.17	0.18	0.15	0.17

iv. 1

2. a. **i.** r.f.(H) = 0.44, r.f.(T) = 0.56 **ii.** 1

b. **i.** r.f.(H) = 0.62, r.f.(T) = 0.38 **ii.** 1

3. Each answer will be different.

4. a. 0.365 **b.** 0.33 **c.** 0.25 **d.** 0.875

5. a. 0.275 **b.** 0.64 **c.** 0.365 **d.** 0.635

6. a. Not reasonable; not enough trials were held.

b. Not reasonable; the conditions are different under each trial.

c. Not reasonable; there are seasonal influences.

d. Reasonable; enough trials were performed under the same conditions.

e. Not reasonable; monthly rainfall in Sydney is not consistent throughout the year.

7. a. $\frac{5}{24}$ **b.** $\frac{59}{72}$

8. A **9.** 200 **10.** 20 **11.** B

12. a. $\frac{11}{20}$ **b.** $\frac{3}{10}$ **c.** $\frac{17}{20}$ **d.** $\frac{3}{20}$

13. a. $\frac{33}{100}$ **b.** $\frac{40}{100} = \frac{2}{5}$ **c.** $\frac{3}{100}$

14. a. **i.** 6 **ii.** {356, 365, 536, 563, 635, 653}

b. $\frac{2}{3}$ **c.** $\frac{1}{3}$ **d.** $\frac{1}{3}$

15. D

16. Svetlana 6, Sarah 4, Leonie 3, Trang 2

17. C

18. a. 27

b. {111, 112, 115, 121, 122, 125, 151, 152, 155, 211, 212, 215, 221, 222, 225, 251, 252, 255, 511, 512, 515, 521, 522, 525, 551, 552, 555}

c. $\frac{1}{3}$ **d.** $\frac{2}{3}$ **e.** $\frac{1}{3}$

19. a. $\frac{1}{12}$ **b.** $\frac{1}{20}$

c. No, because John has a higher probability of winning.

20. a. $\frac{7}{10}$ **b, c.** Yes, far too many mistakes

21. 32

22. 1000 balls

23. Red = 10, yellow = 50, blue = 40

24. a. $\frac{7}{20}$ **b.** $\frac{2}{5}$

25. a. $\frac{30}{241}$ **b.** $\frac{91}{241}$ **c.** $\frac{59}{241}$

Exercise 13.4 Venn diagrams and two-way tables

1. a. $A' = \{11, 13, 14, 16, 17, 19, 20\}$ **b.** $B' = \{20\}$

c. $C' = \{12, 14, 15, 16, 18, 20\}$ **d.** $D' = \{12, 14, 16\}$

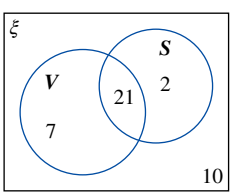
2. a. 27 **b.** 11 **c.** 13 **d.** 6

e. 6 **f.** 7 **g.** 9

3.

	T	T'
S	6	5
S'	7	9

4.



5. a. **i.** W'

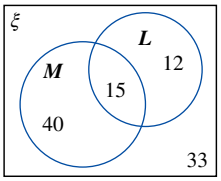
b. **i.** $A \cap B$ **ii.** $A \cap B'$

c. **i.** $A' \cap B'$ **ii.** $A' \cap B$

6. a.

	Left-handed	Right-handed
Male	15	40
Female	12	33

b.



c. **i.** $\frac{73}{100}$ **ii.** $\frac{2}{5}$ **iii.** $\frac{3}{25}$

7. a. $\frac{8}{25}$ **b.** $\frac{17}{25}$ **c.** $\frac{1}{5}$ **d.** $\frac{3}{25}$

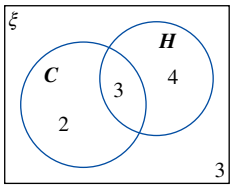
8. a. $\frac{1}{2}$ **b.** $\frac{1}{10}$ **c.** $\frac{31}{40}$ **d.** $\frac{27}{40}$ **e.** $\frac{9}{40}$

9. a. 16 **b.** **i.** $\frac{5}{16}$ **ii.** $\frac{3}{16}$ **iii.** $\frac{5}{8}$ **iv.** $\frac{7}{16}$

10. a. Children who are not swimmers **b.** Children who are swimmers and runners

c. Children who neither swim nor run **d.** Children who swim or run or both

11. a. **i.**

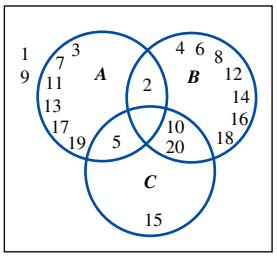


ii.

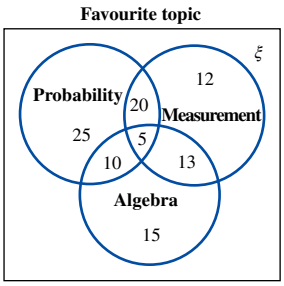
	H	H'
C	3	2
C'	4	3

b. **i.** $\frac{7}{12}$ **ii.** $\frac{3}{4}$ **iii.** $\frac{1}{4}$ **iv.** $\frac{1}{2}$

12.



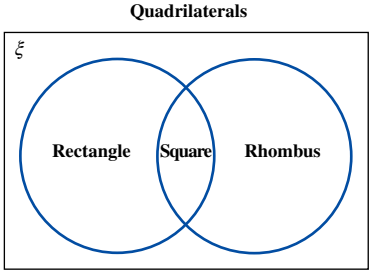
13. a.



b. 25 **c.** 15 **d.** 12 **e.** 43 **f.** $\frac{3}{5}$

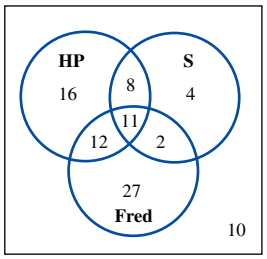
g. $\frac{43}{100}$ **h.** $\frac{9}{50}$ **i.** $\frac{13}{100}$ **j.** $\frac{1}{20}$

14.



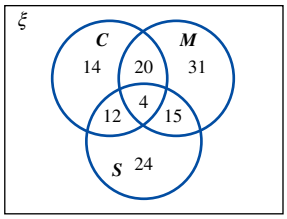
15. a. 2, 5, 6, 7, 8 **b.** 4 **c.** 5, 7, 8 **d.** 8

16. a.



b. $\frac{8}{90} = \frac{4}{45}$

17. a.

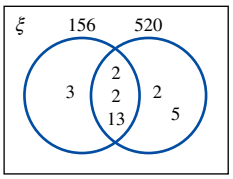


b. $\frac{7}{12}$

18. a. 31 students

b. $\frac{1}{13}$

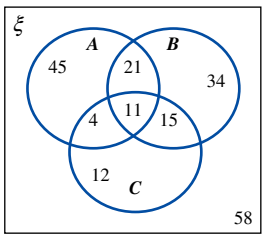
19.



a. HCF = $2 \times 2 \times 13 = 52$

b. LCM = $3 \times 2 \times 2 \times 13 \times 2 \times 5 = 1560$

20. a.



b. 4 **c.** 21 **d. i.** $\frac{81}{200}$ **ii.** $\frac{97}{200}$

e. $\frac{3}{50}$

Challenge 13.1

120

Exercise 13.5 Two-step experiments

1.

		Scarves		
		R	**Bl**	**Pi**
Beanies	**Br**	Br, R	Br, Bl	Br, Pi
	Pu	Pu, R	Pu, Bl	Pu, Pi

2.

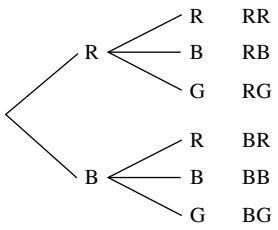
		Die 1					
		1	**2**	**3**	**4**	**5**	**6**
Die 2	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

a. $\frac{1}{9}$

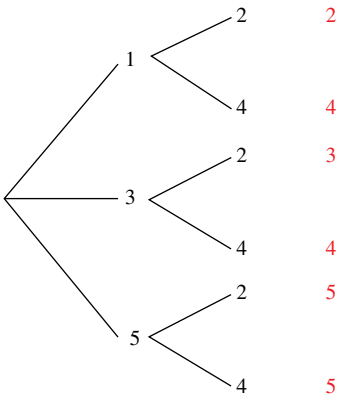
b. 0

c. A total of 7

3.



4. a.



b. i. $\frac{1}{6}$ **ii.** 0 **iii.** $\frac{1}{2}$

5. a.

		Die 1					
		1	**2**	**3**	**4**	**5**	**6**
Die 2	**1**	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

b. i. $\frac{1}{2}$ **ii.** $\frac{1}{6}$ **iii.** $\frac{1}{3}$ **iv.** $\frac{2}{3}$

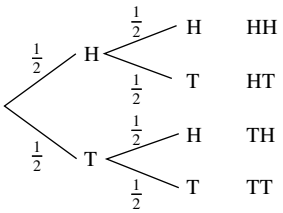
6. a. $\frac{1}{6}$

b. $\frac{1}{36}$

c. $\frac{1}{4}$

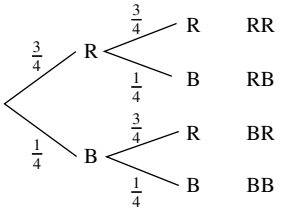
d. $\frac{1}{36}$

7. a.



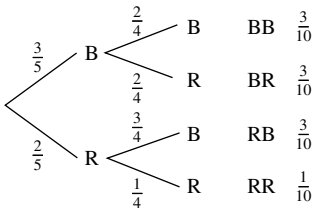
b. i. $\frac{1}{4}$ **ii.** $\frac{3}{4}$

8. a.



b. i. $\frac{3}{16}$ **ii.** $\frac{1}{16}$

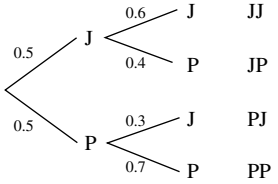
9. a.



b. $\frac{1}{10}$

10. a. $\frac{1}{4}$ **b.** $\frac{3}{14}$

11. a.



b. i. 0.3 **ii.** 0.35 **iii.** 0.35

12. If the first ball is replaced, the probability of drawing a yellow ball stays the same on the second draw, i.e. $\left(\frac{3}{5}\right)$.

If the first ball isn't replaced, the probability of drawing a yellow ball on the second draw decreases, i.e. $\left(\frac{5}{9}\right)$.

13. a. Smallest total: 3, largest total: 18

b.

Total	Probability
3	$\frac{1}{216}$
4	$\frac{3}{216}$
5	$\frac{6}{216}$
6	$\frac{10}{216}$
7	$\frac{15}{216}$
8	$\frac{21}{216}$
9	$\frac{25}{216}$
10	$\frac{27}{216}$
11	$\frac{27}{216}$
12	$\frac{25}{216}$
13	$\frac{21}{216}$
14	$\frac{15}{216}$
15	$\frac{10}{216}$
16	$\frac{6}{216}$
17	$\frac{3}{216}$
18	$\frac{1}{216}$

14. a. $\frac{1}{221}$ **b.** $\frac{11}{221}$ **c.** $\frac{1}{17}$

15. $\frac{5}{12}$ **16.** $\frac{1}{2}$ **17.** $\frac{45}{1081}$

Challenge 13.2

Advantaged. The chance of getting a total of 7 would be $\frac{1}{3}$.

Exercise 13.6 Mutually exclusive and independent events

1. D **2.** A **3.** $\frac{215}{216}$ **4.** D

5. a. $\frac{1}{4}$ **b.** $\frac{3}{4}$

6. a. $\frac{17}{70}$ **b.** $\frac{53}{70}$ **c.** $\frac{9}{70}$ **d.** $\frac{61}{70}$

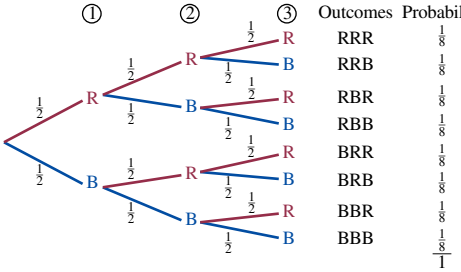
7. $\frac{2}{13}$

8. B

9. a. $\frac{1}{2}$ **b.** 1

10. a. $\frac{11}{30}$ **b.** $\frac{19}{30}$ **c.** $\frac{23}{30}$ **d.** $\frac{5}{6}$

11. a.



Outcomes	Probability
RRR	$\frac{1}{8}$
RRB	$\frac{1}{8}$
RBR	$\frac{1}{8}$
RBB	$\frac{1}{8}$
BRR	$\frac{1}{8}$
BRB	$\frac{1}{8}$
BBR	$\frac{1}{8}$
BBB	$\frac{1}{8}$
	1

b. i. $\frac{1}{8}$ **ii.** $\frac{3}{8}$ **iii.** $\frac{3}{8}$ **iv.** $\frac{1}{8}$ **v.** $\frac{7}{8}$ **vi.** $\frac{1}{2}$

12. $\frac{8}{81}$ **13.** $\frac{1}{9}$ **14.** $\frac{1}{2}$ **15.** $\frac{1}{8}$

16. $\frac{1}{12}$

17. No, because there also the possibility of 1 Head (HT or TH).

18. a. $\frac{8}{20}$ **b.** $\frac{15}{20}$ **c.** $\frac{4}{20}$ **d.** $\frac{8}{20}$

19. a. $\frac{14}{52}$ **b.** $\frac{2}{13}$ **c.** $\frac{3}{13}$ **d.** $\frac{38}{52}$

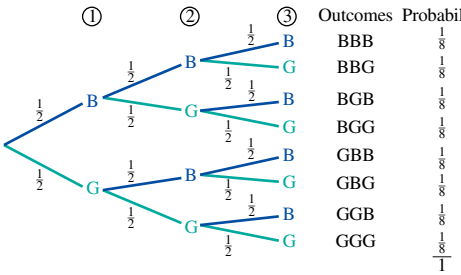
20. D

21. $\frac{189}{1000}$ or 0.189

22. a. $\frac{1}{16}$ **b.** $\frac{9}{16}$ **c.** $\frac{1}{169}$ **d.** $\frac{1}{2704}$ **e.** $\frac{2601}{2704}$

23. a. $\frac{1}{17}$ **b.** $\frac{19}{34}$ **c.** $\frac{1}{221}$ **d.** 0 **e.** $\frac{25}{26}$

24. a.



Outcomes	Probability
BBB	$\frac{1}{8}$
BBG	$\frac{1}{8}$
BGB	$\frac{1}{8}$
BGG	$\frac{1}{8}$
GBB	$\frac{1}{8}$
GBG	$\frac{1}{8}$
GGB	$\frac{1}{8}$
GGG	$\frac{1}{8}$
	1

b. 3

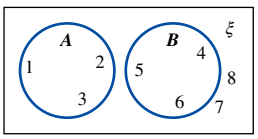
c. $\frac{3}{8}$

d. They are equally likely.

e. $\frac{7}{8}$

25. a. $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, A = $\{1, 2, 3\}$, B = $\{4, 5, 6\}$

b.

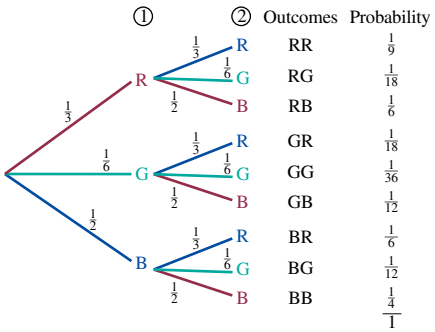


26. If two events are complementary, they cannot occur at the same time, thus their intersection is ø, the same as mutually exclusive sets. However, if events are mutually exclusive, they do not need to have a sum equal to 1.

27. a. {BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG}

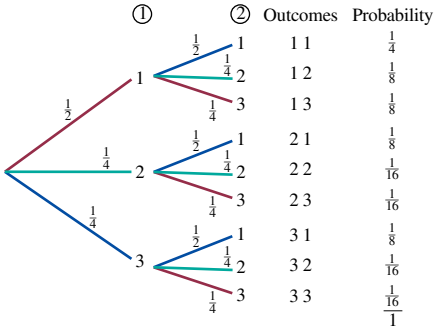
a. $\frac{3}{8}$ **b.** $\frac{1}{2}$

28. a.



b. {(R, R), (R, G), (R, B)} **c.** $\frac{1}{3}$ **d.** $\frac{7}{18}$

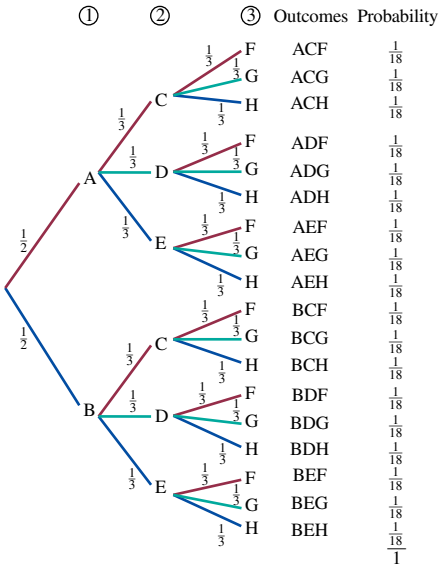
29. a.



b. No

c. i. $\frac{1}{4}$ **ii.** $\frac{1}{2}$ **iii.** $\frac{3}{8}$ **iv.** $\frac{9}{16}$

30. a.



Sample space = {ACF, ACG, . . ., BEG, BEH}

b. $\frac{1}{2}$ **c.** $\frac{1}{6}$

Exercise 13.7 Conditional probability

1. a. 60

b. $\frac{4}{15}$

c. i. $\frac{13}{20}$ **ii.** $\frac{16}{39}$

d. i. $\frac{31}{60}$ **ii.** $\frac{16}{31}$

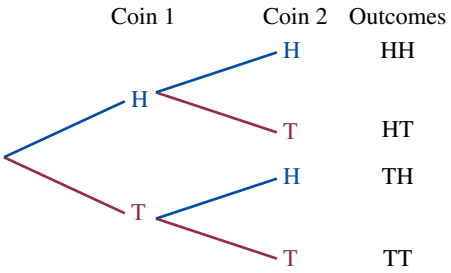
2. a. $\frac{4}{15}$

b. $\frac{4}{15}$

c. They are the same, and equal to the probability calculated in question **1** part **b**.

3. $\frac{3}{10}$

4. a.



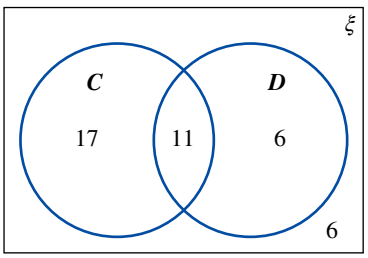
b. $\frac{1}{2}$

5. a.

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b. $\frac{3}{5}$

6. a.



b. i. $\frac{7}{10}$ **ii.** $\frac{17}{40}$ **iii.** $\frac{11}{40}$ **iv.** $\frac{11}{28}$ **v.** $\frac{11}{17}$

c. Order from least to most probable: **iii**, **iv**, **ii**, **v**, **i**.

7. 62.5%

8. a. 0.2 **b.** $\frac{2}{3}$ **c.** $\frac{2}{5}$

9. $\frac{9}{35}$

10. a. $\frac{1}{6}$ **b.** $\frac{1}{6}$ **c.** 0 **d.** $\frac{1}{6}$

11. Conditional probability reduces the sample space that the probability is calculated from. In this instance the sample space is reduced from 6 numbers (1, 2, 3, 4, 5, 6) to 3 numbers (2, 4, 6).

12. a. $P(J) = \frac{41}{90}$

b. $P(H \mid J) = \frac{12}{41}$

13. 0.22

14. 0.58

15. 0.9

13.8 Review

1. B **2.** D **3.** D **4.** C **5.** D

6. C **7.** B **8.** A **9.** C **10.** D

11. a. $\frac{1}{4}$ **b.** $\frac{1}{10}$ **c.** $\frac{1}{8}$

12. a. 40 **b.** 33 **c.** 25

13. $\frac{1}{4}$

14. b



15. a. Students who do not like music

b. People who like surfing and golf as leisure activities, but not reading

c. People who like Coke or Fanta or both but not Sprite

16. $\frac{2}{3}$

17. $\frac{1}{2}$

18. a. {R, B, Y}

b. {(YR), (YY), (YB)}

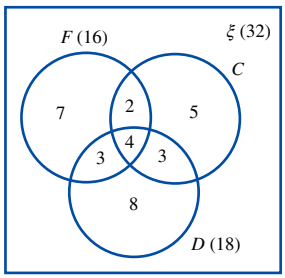
c. No, there is no green counter.

19. a. 12

b. 60

c. i. $\frac{3}{10}$ **ii.** $\frac{17}{60}$ **iii.** $\frac{1}{10}$ **iv.** $\frac{1}{12}$ **v.** $\frac{31}{60}$

20. a.



a. 5

b. i. $\frac{7}{32}$ **ii.** $\frac{1}{2}$ **iii.** $\frac{7}{16}$

21. 80

22. a. { 1, 5, 6, 7, 8 }

b. { 5 }

c. { 1, 2, 4 }

d. ø

23. 720 **24.** $\frac{35}{66}$ **25.** $\frac{4}{663}$ **26.** $\frac{176}{500}$ **27.** 128 **28.** $\frac{3}{7}$

Investigation — Rich task

1. 3 **2.** 4 **3.** $\frac{1}{2}$ **4.** $\frac{1}{4}$ **5.** 18 **6.** 2

7. Answers will vary.

8. 8

9. Answers will vary.

10. 6

11. Answers will vary.

TOPIC 14 Statistics

14.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



14.1.1 Why learn this?

Data gathered from observation and experimentation can be interpreted and analysed using statistics. Statistics allow us to understand real-world issues such as the popularity of politicians, the effectiveness of drugs and the reliability of weather predictions.

14.1.2 What do you know?

assessON

1. **THINK** List the types of data you know are collected by the Australian Bureau of Statistics (ABS) every 5 years. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as large concept map to show your class's knowledge of the types of data collected by the ABS every 5 years.

LEARNING SEQUENCE

14.1 Overview
14.2 Sampling
14.3 Collecting data
14.4 Displaying data
14.5 Measures of central tendency
14.6 Measures of spread
14.7 Review

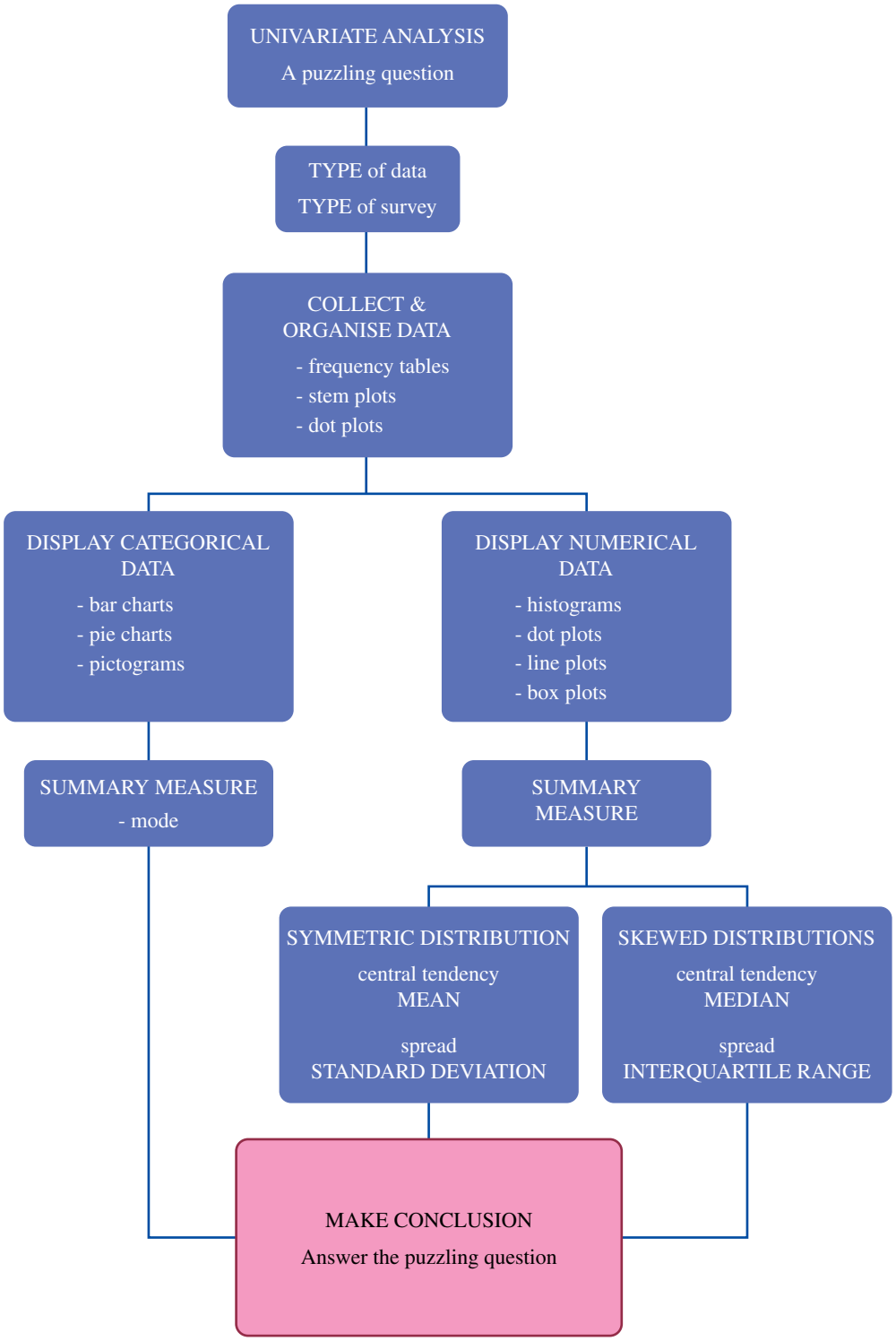
learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: The world's mega cities (eles-1701)

14.2 Sampling

14.2.1 Introduction

- **Statistics** is the collection and analysis of data in order to respond to a question. Data have to be collected, recorded, organised, presented and analysed in a manner appropriate to the question.
- The process involves many stages as shown in the flow chart below.



14.2.2 Types of data

- When performing statistical analyses, the type of data being collected is important.
- The different types of data determine the choice of displays, tables and evaluations that are best used.
- The types of data can be classified as follows.

Categorical data			**Numerical data**	
Data recorded are non-numerical and use labels or categories. For example, hair colour or year level.			Data are recorded in numerical form. For example, values can be the number of family members or the heights of students in Year 9.	
Nominal	**Ordinal**		**Discrete**	**Continuous**
Categories that are named for each of the possible responses, such as types of pets: cat, dog, bird, fish, and others.	Categories that are named with some notion of order, such as quality of work: excellent, good, satisfactory, not satisfactory, poor.		Numerical values that can only be expressed in integer form or whole numbers, such as counting the number of stamps in a stamp collection, or counting the number of goals kicked each game by a full forward.	Numerical values that can be expressed as fractions, decimals, as well as rounded off integer values, such as the measured heights of basketball players, or the change in temperature during the day.
Examples of some types of statistical tools that can be used for each type of data include:				
frequency tables, bar charts, pie charts, percentages, and mode	frequency tables, bar charts, pie charts, percentages, and mode		frequency tables, histograms, box plots, mean, median, and range	frequency tables (grouped), histograms (grouped), box plots, mean, median, range

WORKED EXAMPLE 1

Classify each of the following data as categorical data or numerical data.
a Favourite sport watched on TV
b Quantity of books carried in a school bag by 20 Year 9 students
c Cars passing your home each hour of a day
d Eye colour of 100 pre-school children

THINK

Decide how the data would initially be recorded, as either numbers or labels.

WRITE

a Favourite sport watched on TV Possible data could include football, netball, soccer or tennis. These are labels, so they are categorical data.

b Quantity of books carried in a school bag by 20 Year 9 students
The recorded data would be gathered by counting the number of books in a school bag. This is numerical data.

c Cars passing your home each hour of a day
The recorded data would be the number of cars. This is numerical data.

d Eye colour of 100 pre-school children
The records kept would be blue, brown, green or hazel. These are labels and so it is categorical data.

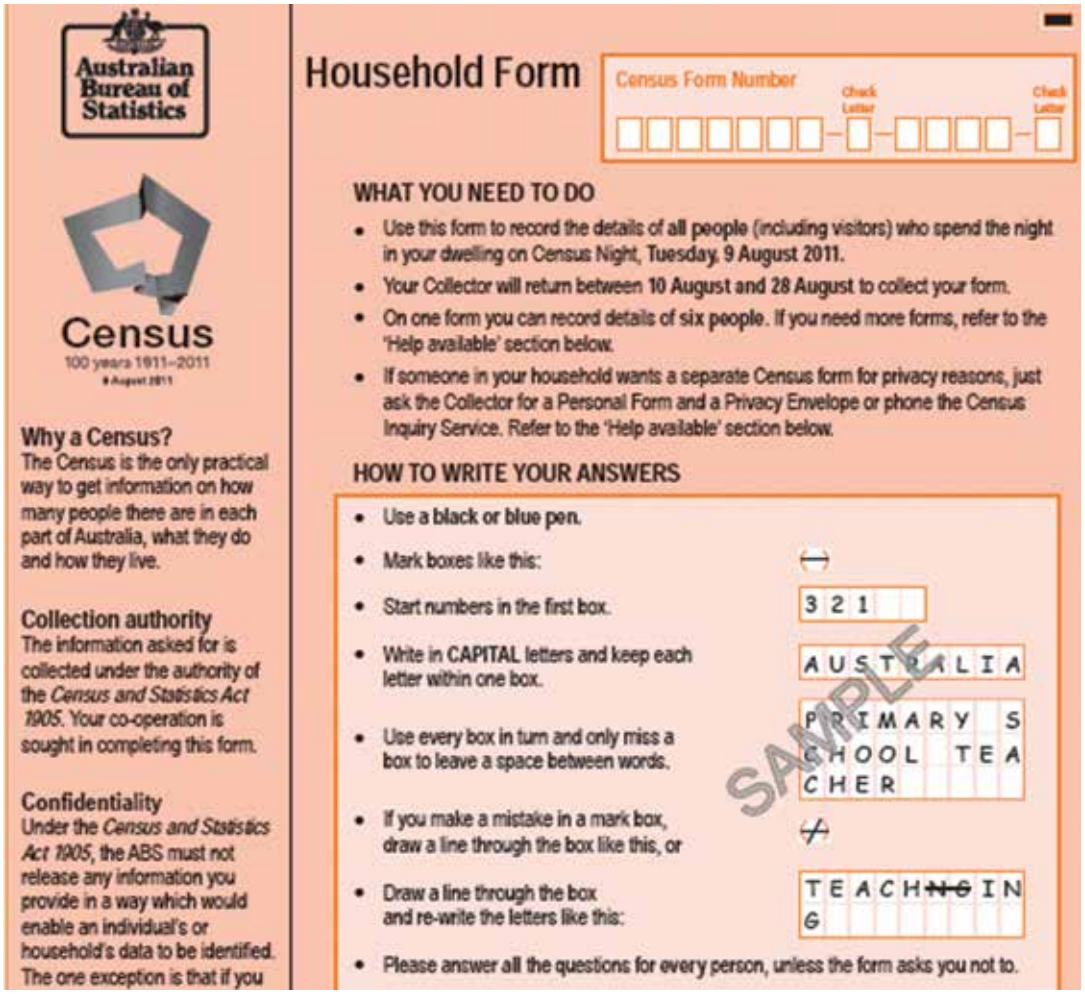
WORKED EXAMPLE 2

Classify each of the following data into the correct groups, stating if it is categorical and nominal or ordinal, or numerical and discrete or continuous.

a Incomes of the senior Geelong football players
b Type of transport used to go to work
c Attendance at a zoo each day
d Hair colour of 100 Year 9 students
e Mass of an individual Easter egg being manufactured
f A student's behavioural report

THINK	WRITE
1 For each, determine whether the data collected is a numerical value or a category.	**a** Incomes of the senior Geelong football players As there is a wide range of incomes and not accurately stated, they will be grouped in categories such as \$50 000–<\$100 000, \$100 000–<\$150 000, and so on. As these are labels and have order it is more appropriate to classify the data as categorical ordinal data.
2 If numerical data, decide if it is collected by counting (numerical discrete data) or collected by measuring (numerical continuous data).	**b** Type of transport used to go to work A worker's transport mode could include car, bus or train. As these are labels with no logical order it is classified as categorical nominal data. **c** Attendance at a zoo each day You would count the number of zoo patrons and record them as whole numbers (integer); therefore, it is classified as numerical discrete data.
3 If categorical, decide the types of labels possible, and if the labels can be ranked in a logical order (categorical ordinal data) or if they cannot be logically ranked (categorical nominal data).	**d** Hair colour of 100 Year 9 students. The students' hair colour would include blonde, brunette, redhead and so on. These are labels with no logical order so it is categorical nominal data. **e** Mass of an individual Easter egg being manufactured. You would measure the mass of each egg on a scale. The mass of each egg can then be recorded as a whole number (i.e. 55 grams), or more accurately as a decimal number (i.e. 55.23). Therefore, the data is numerical continuous data. **f** A student's behavioural report The student's behaviour could be stated as: excellent, good, satisfactory or improvement needed. These are labels that do have a logical order; therefore, they are categorical ordinal data.

14.2.3 Size of data sets


Australian Bureau of Statistics

Census
100 years 1911–2011

Why a Census?
The Census is the only practical way to get information on how many people there are in each part of Australia, what they do and how they live.

Collection authority
The information asked for is collected under the authority of the *Census and Statistics Act 1905*. Your co-operation is sought in completing this form.

Confidentiality
Under the *Census and Statistics Act 1905*, the ABS must not release any information you provide in a way which would enable an individual's or household's data to be identified. The one exception is that if you

Household Form

Census Form Number

WHAT YOU NEED TO DO
- Use this form to record the details of all people (including visitors) who spend the night in your dwelling on Census Night, Tuesday, 9 August 2011.
- Your Collector will return between 10 August and 28 August to collect your form.
- On one form you can record details of six people. If you need more forms, refer to the 'Help available' section below.
- If someone in your household wants a separate Census form for privacy reasons, just ask the Collector for a Personal Form and a Privacy Envelope or phone the Census Inquiry Service. Refer to the 'Help available' section below.

HOW TO WRITE YOUR ANSWERS
- Use a black or blue pen.
- Mark boxes like this:
- Start numbers in the first box. 3 2 1
- Write in CAPITAL letters and keep each letter within one box. AUSTRALIA
- Use every box in turn and only miss a box to leave a space between words. PRIMARY SCHOOL TEACHER
- If you make a mistake in a mark box, draw a line through the box like this, or
- Draw a line through the box and re-write the letters like this: TEACHING
- Please answer all the questions for every person, unless the form asks you not to.

SAMPLE

- The size of the data collected can be a **population** or a **sample**.
- In statistics population refers to every element or identity that relates to the investigation. A population can be as small as 10 or as large as 6 billion. It depends on what the outcome relates to. For instance, to find the average age of *Year 9 students for your school*, the population contains only *Year 9 students from your school*. If the average age of *Year 9 students in Australia* was needed, then the population includes all *Year 9 students in Australia*.
- A sample is a representative portion of the population. For example, a sample of 20 Year 9 students could be taken to represent all Year 9 students at your school.
- A single piece of data is often referred to as a score or a value.

14.2.4 Types of data collection

- There are three basic ways to collect data:
  1. **Survey** — data collected from a portion of the population or sample. This type of data collection is most commonly used in market research. It is important that surveys are correctly conducted as the conclusions drawn from the data should represent those of the population from which they were drawn.
  2. **Census** — data collected from an entire population. The most common is the one conducted by the Australian Bureau of Statistics. Every five years the entire nation takes part in a census.
  3. Experimental — data collected by measuring or counting or assigning labels.

14.2.5 Question design and data collection methods

- It is important to keep in mind that the questions in a survey or census need to be clear and unambiguous.
- Questions should be unbiased, not too personal, and able to be answered by most respondents.
- The design of the questionnaire should enable the collected data to be analysed easily.
- It is important to consider the methods used to collect data, as they can bias the results of the analysis, leading to incorrect conclusions.
- The sample for the data set can be collected by either a random or a stratified technique.
  - **Random selection** ensures all elements of a population have an equal chance of being selected.
  - **Stratified selection** ensures that groups within a population have a similar representation in a sample.

WORKED EXAMPLE 3

For the following data collection activities, define whether the amount of data collected is a sample or population and what type of data collection was used: census, survey or experimental; and random, stratified or biased.

a Sixty Year 7 students collected data on the amount they contributed to a charity, then used this data to find the average donation from Bolan College's 1523 students.

b A data set of the force, measured in newtons, needed to break wooden ice block sticks.

c The average age of the members of your own family.

THINK

Know the key terminology used by statisticians. Decide if it is a:

- sample or population
- census, survey or experimental
- random, stratified or biased.

WRITE

a It is a sample because only 60 of the Bolan College's 1523 students were surveyed. The data are biased as only Year 7 students were surveyed.

b It is a sample as there is an unknown number of wooden ice block sticks. It is impossible to collect data on the entire population. The data are experimental as the sticks are randomly supplied or collected.

c Survey of all members of a family about the family is a population.

WORKED EXAMPLE 4

A health club is planning to conduct a survey to research the needs of its members. Fifty members of the club will be asked to participate in the survey. The club has 120 male adult, 90 male adolescent, 60 female adult and 30 female adolescent members. Determine the number of members required from each group to perform a stratified sample survey that is an accurate approximation of the needs of the whole population.



THINK	WRITE
1 Determine what fraction of the population is to be used in the sample.	Total population is 120 + 90 + 60 + 30 = 300 members. Fraction of members surveyed $= \frac{50}{300} = \frac{1}{6}$
2 Use this fraction to calculate the number to be surveyed from each group within the population.	Number of male adults $= \frac{1}{6} \times \frac{120}{1} = 20$ Number of male adolescents $= \frac{1}{6} \times \frac{90}{1} = 15$ Number of female adults $= \frac{1}{6} \times \frac{60}{1} = 10$ Number of female adolescents $= \frac{1}{6} \times \frac{30}{1} = 5$
3 Summarise the sampling strata.	50 members surveyed will be: 20 male adults 15 male adolescents 10 female adults 5 female adolescents.

14.2.6 Random number generators

- Random numbers can be obtained from number generators built into most calculators and computers.
- Computer programs and the scientific calculator function RAND randomly select a number from 0 to less than 1. This value multiplied by the size of the sample (plus 1 to avoid choosing the value zero) will ensure any sample size selected is unbiased.
  Below is an example of randomly selecting a whole number (integer) from 1 to 15.

Button	Possible outcome	Example 1	Example 2	Example 3
RAND	0.000 to 0.999	0.456	0.056	0.932
×15 (multiply)	0.000 to 14.985	6.84	0.84	13.98
+1	1.000 to 15.985	7.84	1.84	14.98
INT (ignore decimal part)	1 to 15	7	1	14

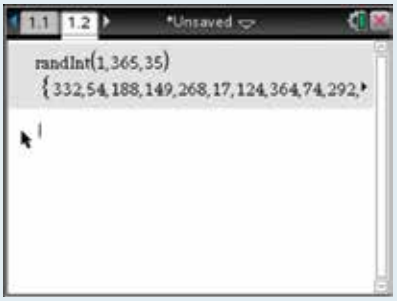
- Graphics calculators can also be used to generate random numbers.

WORKED EXAMPLE 5 — TI | CASIO

Use an appropriate random number generator to select a sample for the following surveys.
a Randomly select 35 days from a year's worth of maximum temperature.
b Randomly select 15 months from January 2006 to December 2010.

THINK	WRITE/DISPLAY
a Using a graphics calculator	**a** Julian date system
1 Use the Julian date, which associates each day with the numbers 1 to 365.	

2 Use the random number generator function as follows: randInt (first number, last number, sample size). Complete the entry line as: randInt(1,365,35).
Then press ENTER.
Note: Make the sample size slightly bigger than needed in case numbers double-up.



3 Scroll to the right to view all 35 random numbers that are in the range 1 to 365.

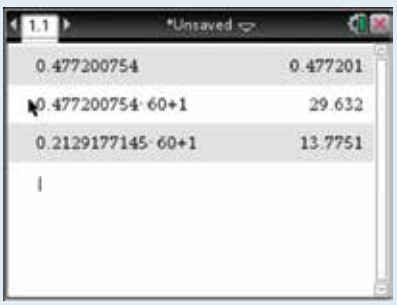
4 Use the Julian calendar to convert the number to a calendar date.

Day 332 is 28 November.
Day 54 is 23 February.
Day 188 is 7 July.

b Using a scientific calculator

Use the random number button to generate a number from 0 to less than 1, then multiply the generated number by 60. Finally add 1 to the generated number (only use the integers, ignore the decimals).
Random number = integer(random number × 60 + 1)

b Label the months numerically as:
1. January 2006
2. February 2006
. . .
60. December 2010.

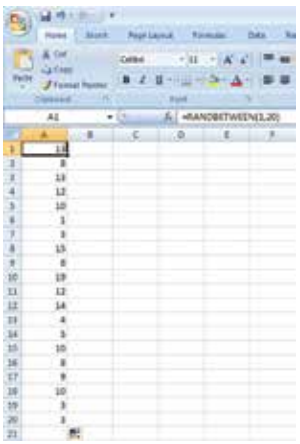


Repeat until 15 months have been selected.

29.632 becomes the 29th month — May 2008.
13.775 becomes the 13th month — January 2007.
(Repeat this 15 times ignoring any doubling up of months.)

14.2.7 Generating random numbers using an Excel spreadsheet

- The command **RANDBETWEEN** in an Excel spreadsheet generates a random number within the values that you specify.
- If your spreadsheet does not have this function, press the **Office Button** and choose **Excel options**. Choose **Add-Ins** from the menu on the right, click on **Analysis Toolpak**, and press **Go** at the bottom of the page. Tick **Analysis ToolPak**, and press **OK**.
  1. In order to enter 20 random numbers, enter the function **=RANDBETWEEN(1,20)** into cell **A1**. A random number in the range 1 to 20 will appear in cell A1.
  2. Use the **Fill Down** function to fill this formula down to cell A20. You should now have 20 random numbers as shown in the screen shot on the right.



Note that each time you perform an action on this spreadsheet, the random numbers will automatically change. In order to switch to manual mode:
– choose the **Calculator Options menu**
– select **Manual**
– press **F9** to obtain a recalculation of your random numbers.

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Random number generator (int-0089)

Exercise 14.2 Sampling

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 11, 13, 14, 15–17, 20	Questions: 1–3, 5, 7, 8, 10, 12, 14, 15–18, 20–21	Questions: 1–4, 6, 8, 10, 12, 14–22

Individual pathway interactivity: int-4540

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Classify each of the following data according to whether they are numerical or categorical data.
   - **a.** Favourite TV programs of each student in your year level
   - **b.** Shoe size of the top ten international models
   - **c.** Hours of computer games played at home each week by each student in your class
   - **d.** Birth weight of each baby in a young mothers' group
   - **e.** The favourite colours for cars sold in Canberra in the last month
2. WE2 Classify each of the following numerical data according to whether they are discrete or continuous.
   - **a.** Length of a stride of each golfer in a local golf club
   - **b.** Distance travelled on one full tank from the top ten most popular vehicles
   - **c.** Rainfall recorded for each day during spring
   - **d.** Shoe size of each student in your class
   - **e.** Membership size of the 16 AFL teams
3. Classify each of the following categorical data as nominal or ordinal.
   - **a.** Favourite magazine of each student in your year level
   - **b.** Sales ranking of the top ten magazines
   - **c.** Age group of the readers of the top ten magazines
   - **d.** Gender of 100 randomly chosen readers of each of the top ten magazines
   - **e.** Type of magazine of the top ten magazines
4. WE3 Classify each of the following data collections according to whether they are samples or populations.
   - **a.** Favourite TV programs of secondary school students by surveying students in Year 8
   - **b.** Size of shoes to be stocked in a department store determined by measuring the feet of the top ten international models

c. Number of hours computer games are played by your class at home each week by surveying each student in your class

d. Survey of first 40 customers at the opening of a new store to gauge customer satisfaction

e. The most popular duco colours for cars sold in the last month in Victoria, taken from the registration board database of all 2310 cars registered last month.



5. For those surveys that involve sampling from question **4**, classify the sample as random, stratified and/or biased.

6. WE4 Determine the number of people that need to be surveyed from each of the following groups if stratified surveying is to be conducted.

a. Survey of 40 members from a club of 1500 members, with 1200 males members and 300 female members

b. Survey of 10% of all clients from a database of 10 000 clients where an appropriate representation of the 70% female customers and 30% male customers is maintained

c. Survey of 100 students from a school with a population of 900 students. There are 90 Year 12 students, 135 Year 11 students, 153 Year 10 students, 180 Year 9 students, and 180 Year 8 students, with the remainder of the population being Year 7 students.

d. Survey 8 class members such that there is an unbiased gender representation of the 20 girls and 12 boys.

Understanding

7. WE5 Devise a strategy for randomly selecting samples for the following scenarios.

a. Select 4 AFL clubs from the 18 clubs available to represent the league at an international sporting convention.

b. Select 5 students from your class.

c. Select 50 students from your school.

d. Select 65 consecutive days from a non-leap year for analysis of the daily maximum temperatures.

8. MC A tennis club has 400 members where 280 members are male. If the club wants to conduct a sample survey of 80 members, the number of males needed for an unbiased survey is:

A. 32 **B.** 24 **C.** 40 **D.** 56

9. MC Which of the following statements is false?

A. A good sample size is 50 or more.

B. Results from samples give an estimate.

C. Results from a population give an estimate.

D. The use of random numbers or a stratified selection process ensures unbiased results.

10. MC If a random number generator is used and the first two numbers are 0.796 and 0.059, the first two random numbers, respectively, for a selected sample of size 50 are:

A. 2.95 and 39.8 **B.** 2 and 39 **C.** 4 and 41 **D.** 3 and 40

11. Randomly select 5 days of a non-leap year.

12. Use random number generators for the following unbiased selection processes. For each case show:

i. an association table

ii. the random numbers chosen

iii. how to convert the random numbers to a random choice from a population.

a. An unbiased selection of 6 days from the month of March

b. An unbiased selection of 5 students from your mathematics class

c. An unbiased selection of 30 randomly chosen student lockers numbered from 001 to 860

Reasoning

13. The following are 2011 statistics for motorcycle and car fatalities in the United States.
Motorcycle fatalities: 4612
Car fatalities: 11 981
Registered motorcycles: 8 437 502
Registered cars: 134 534 655
Vehicle miles travelled motorcycles (millions): 18 500
Vehicle miles travelled cars (millions): 1 495 303
Population of USA: 311 591 917
Source: National Highway Traffic Safety Administration, http://www.nhtsa.gov, 2011.
a. According to the data, which is the safer mode of transport, a motorcycle or car? Explain.
b. What ratios or percentages could be extracted from this data to reinforce your conclusion?
c. If you wanted to advocate riding a motorcycle, which information would you include? Which information would you omit?
d. Write a sentence that could convince your readers that a motorcycle is a safer mode of transport.
e. Write a sentence that could convince your readers that a car is a safer mode of transport.

14. A stratified random sample was taken from two high schools, a 200-student high school and a larger high school. The sample assessed how many students watched more than four hours of television per day. If the small school represented 12.5% of the total students surveyed and a total of 208 students were randomly surveyed:
a. Show that 26 students were surveyed in the small school.
b. How many students were surveyed in the large school?
c. How many students were in the large school?
d. Show that 1600 students were in both schools.

15. Explain how data are classified into different groups, giving examples for each group.

16. Identify and justify two sets of information a person might want to collect from each of the following sources.
a. A maternity hospital **b.** An airport **c.** Shoppers at a supermarket

17. Develop a question to ask about the issue 'A new Australian flag'. Decide whether a census or survey of a sample population should be conducted. Give reasons for your decision.

18. A TV station requires viewers to vote online in ranking 4 couples in a home renovating show. Is this a random sample? Is it a biased survey? Explain your reasoning.

19. Explain how a random number generator can obtain seven numbers from the number range 0 to 45.

Problem solving

20. Questions are being written for a survey about students' views on sports. Which of the following questions would you use and why?
a. What do you think about sport?
b. List these sports in order from your most favourite to least favourite: football; tennis; netball; golf.
c. How often do you watch sports on television? Once a week; once a day; once a month; once a year; never.
d. Why do you like sports?
e. How many live sporting events do you attend each year?
f. Can you play a sport?
g. Which sports do you watch on the television? Football; tennis; netball; golf; all of these; none of these.

21. Businesses collect and analyse all sorts of data.
a. What sort of data do businesses collect?
b. Who collects the data for the businesses?
c. What is the data used for?

22. A company involved in constructing the desalination plant in the Wonthaggi region wishes to gather some data on public opinion about the presence of the desalination plant in the area and its possible effects on the environment. The company has decided to conduct a survey and has asked you to plan and implement the survey and then analyse the data and interpret the results.

a. What questions could you include in the survey?
b. What are you trying to find out by asking these questions?
c. What type(s) of data will you collect?
d. How would you display this data visually?
e. What could the company use this data for?

Reflection

What is the chance any five students chosen in your class are born on the same day of the month?

14.3 Collecting data

14.3.1 Recording and organising data

- Data need to be recorded and organised so that graphs and tables can be drawn and analysis conducted. Data are usually presented in a frequency table to begin with.



WORKED EXAMPLE 6

The following is a result of a survey of 30 students showing the number of people in their family living at home. Draw a frequency table to summarise the data.
2 4 5 6 4 3 4 3 2 5 6 6 5 4 5 3 3 4 5 8 7 4 3 4 2 5 4 6 4 5

THINK

1 Identify the data as numerical discrete data.

2 There are only 7 different scores so the data can be left ungrouped.

3 Use a frequency table of two (or three) columns. The first column is for the possible scores (x) and can be labelled as 'Number of members in the family'. The last column is for the frequency (f) and can be labelled 'Number of families'. An optional column between these two can be used as a tally for very large sets of data.

4 Add up the frequency (Σf) to confirm scores from 30 students have been recorded.

WRITE/DRAW

Number of members in the family (x)	Tally	Number of families (f)
2	𝍸	3
3	𝍸	5
4	𝍸 𝍸	9
5	𝍸	7
6	𝍸	4
7	𝍸	1
8	𝍸	1
		$\Sigma f = 30$

WORKED EXAMPLE 7

A survey was conducted, asking students the type of vehicle driven by their parents. The responses were collected and recorded as shown below. Display the data as a frequency table and as a dot plot.

Sedan	**4WD**	**Sedan**	**Sedan**
4WD	**SUV**	**SUV**	**Station wagon**
SUV	**Convertible**	**Station wagon**	**4WD**
Sedan	**Sports car**	**Convertible**	**Station wagon**
Sedan	**Station wagon**	**SUV**	**Station wagon**
SUV	**SUV**	**Sedan**	**Sedan**

THINK

WRITE/DRAW

1 Identify this data as categorical ordinal data.

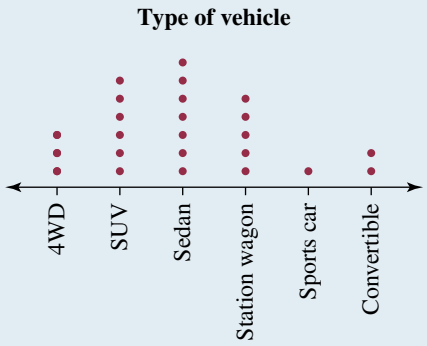
2 There are 6 different scores or categorical labels.

3 Use a frequency table of two (or three) columns. The first column is for the possible scores (x) and can be labelled as 'Type of vehicle'. The last column is for the frequency (f) and can be labelled 'Number of vehicles'. An optional middle column can be used as a tally for very large sets of data.

Type of vehicle (x)	Tally	Number of vehicles (f)
4WD	𝍸	3
SUV	𝍸 |	6
Sedan	𝍸 ||	7
Station wagon	𝍸	5
Sports car	|	1
Convertible	||	2
		$\Sigma f = 24$

4 Add up the frequency (Σf) to confirm 24 scores have been recorded.

5 Place a dot for each vehicle type in the appropriate column.



WORKED EXAMPLE 8

The golf scores for 30 golfers were recorded as follows. Summarise the data as a frequency table.
73 69 75 79 68 85 78 76 72 73 71 70 81 73 74
87 78 79 68 75 76 72 63 72 74 71 70 75 66 82



THINK

1 Identify the data as numerical discrete data.

2 There are scores from 63 to 87, so they can be grouped as 2s or 5s.

3 For this example the data will be grouped by 2s. Grouping in 2s will give $24 \div 2 = 12$ groups. The groups have been made to start with even numbers, giving us 13 groups.

4 Sum the frequencies to check all 30 scores were recorded.

WRITE/DRAW

Golf scores (x)	Tally	Number of players (f)
62–63	|	1
64–65		0
66–67	|	1
68–69	|||	3
70–71	||||	4
72–73	𝍸 |	6
74–75	𝍸	5
76–77	||	2
78–79	||||	4
80–81	|	1
82–83	|	1
84–85	|	1
86–87	|	1
		$\Sigma f = 30$

14.3.2 Stem plots

- A stem plot or **stem-and-leaf plot** is a graph that displays numerical data.
- All data values are shown in stem plots.
- Stem plots have two columns: the stem holds the group value and the leaf holds the final digit in the data value.

- The following stem plot shows the sizes of 20 students' DVD collections.

DVD collection size of 20 students

Key: 2 | 5 = 25 DVDs

Stem	*Leaf*
0	4
1	4 6
2	2 3 5 8
3	0 4 4 6 7 7 9
4	1 3 5 9
5	4 6

- From the stem plot shown, the scores collected were 4, 14, 16, 22, 23, 25, 28, 30, 34, 34, 36, 37, 37, 39, 41, 43, 45, 49, 54, 56.
- The distribution of the data is evident in a stem plot. In this example, it is a normal or symmetrical distribution.

WORKED EXAMPLE 9

The following scores represent the distance (in km) travelled by a group of people over a particular weekend. Summarise the data in a stem plot.
48 67 87 2 34 105 34 45 63 98 12 23 35 54 65 41 34 23 12 38 18 58 53 44 39 29
Comment on what the shape of the distribution tells us.

THINK

1 Identify the data as numerical discrete data. Write a title and a key.

2 There are scores from 2 to 105, so they can be grouped in 10s. The stem will hold the tens place value and leaf holds the last digit (the unit/ones place value) for each score.

3 Transfer each score by recording its last digit in the leaf alongside the row corresponding to the stem with its tens place digit.

4 Reorder each row in the leaf so the digits are in ascending order from the stem.

5 Comment on the shape of the distribution.

WRITE/DRAW

Distance travelled over one weekend

Key: 4 | 7 = 47 km

Stem	*Leaf*
0	2
1	2 2 8
2	3 3 9
3	4 4 4 5 8 9
4	1 4 5 8
5	3 4 8
6	3 5 7
7	
8	7
9	8
10	5

The distribution is not symmetrical. Most people travelled around 20 km to 50 km over the weekend, with only 3 people travelling more than 70 km.

- Sometimes the leaves in the rows of a stem plot become too long. This can be overcome by breaking the stems into smaller intervals. The second part of the interval is identified with an asterisk (*).

WORKED EXAMPLE 10

The heights of 30 students are measured and recorded as follows.
125, 143, 119, 136, 127, 131, 139, 122, 140, 118, 120, 123, 132, 134, 127, 129, 124, 131, 138, 133, 122, 128, 130, 135, 141, 139, 121, 138, 131, 126
Represent the data in a stem plot.

THINK

1 Write a title and key, then draw up the stem plot with the numbers in each row of the leaf column in ascending order from the stem.

WRITE/DRAW

Heights of 30 students (in cm)
Key: 11 | 8 = 118 cm

Stem	*Leaf*
11	8 9
12	0 1 2 2 3 4 5 6 7 7 8 9
13	0 1 1 1 2 3 4 5 6 8 8 9 9
14	0 1 3

2 The leaves of the two middle stem values are long. They would be easier to interpret with the stem broken up into smaller intervals, i.e. intervals of 5. The stem of 12 would then include the numbers from 120 to 124 inclusive, while the stem of 12* would include the numbers from 125 to 129 inclusive. Reconstruct the stem plot.

Heights of 30 students (in cm)
Key: 11* | 8 = 118 cm

Stem	*Leaf*
11*	8 9
12	0 1 2 2 3 4
12*	5 6 7 7 8 9
13	0 1 1 1 2 3 4
13*	5 6 8 8 9 9
14	0 1 3

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Presenting data in a frequency table (doc-6317)
Complete this digital doc: SkillSHEET: Arranging a set of data in ascending order (doc-10954)
Complete this digital doc: WorkSHEET: Statistics I (doc-6323)

Exercise 14.3 Collecting data

assessON

Individual pathways

PRACTISE
Questions:
1, 3–6, 8, 10, 12, 13, 17

CONSOLIDATE
Questions:
1–4, 6, 8, 10, 12–15, 17, 18

MASTER
Questions:
1, 3, 5, 7, 9, 11–18

Individual pathway interactivity: int-4541

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE6, 7 The following set of data represents the scores a class of students achieved in a multiple choice Maths test.

4, 6, 8, 3, 6, 9, 1, 3, 5, 6, 4, 7, 5, 9, 3, 2, 7, 8, 9, 6, 5, 4, 6, 5, 3, 5, 7, 6, 7, 7

a. Present the data as a dot plot.

b. Present the data in a frequency table.

2. The following set of data shows the types of pets kept by 20 people.

cat, dog, fish, cat, cat, cat, dog, dog, bird, turtle, dog, cat, snake, cat, dog, dog, frog, bird

a. Present the data as a dot plot.

b. Present the data in a frequency table.

c. Explain whether this data can be presented in a stem plot.

3. WE8, 9

State	Death by motor vehicle accidents — Males aged 15–24 years (rates per 100 000 people)			
	2008	2009	2010	2011
NSW	14	13	18	9
Vic.	19	25	14	12
Qld	25	17	26	15
SA	23	34	22	26
WA	22	24	36	27
Tas.	42	18	36	42
NT	43	60	71	45
ACT	0	11	14	21

a. Present the data shown in the table above as a frequency table by grouping the data in intervals: 0–9, 10–19, . . . etc.

b. Present the data as a stem plot.

4. A sample of students surveyed for the Australian Bureau of Statistics were asked: 'What is the colour of your eyes?' The results were as follows:

Brown	Hazel	Blue	Brown	Brown	Green	Green	Brown	Brown	Hazel
Brown	Brown	Brown	Brown	Blue	Brown	Green	Blue	Blue	Blue
Brown	Brown	Brown	Brown	Hazel	Blue	Hazel	Blue	Brown	Blue
Brown	Blue	Green	Hazel	Hazel	Hazel	Green	Blue	Blue	Brown



a. Construct a dot plot for the data.
b. Set up a frequency table for the data.
c. Comment on the proportion of people who have brown eyes.

5. Draw a stem-and-leaf plot for each of the following sets of data. Comment on each distribution.
a. 18, 22, 20, 19, 20, 21, 19, 20, 21
b. 24, 19, 31, 43, 20, 36, 26, 19, 27, 24, 31, 42, 29, 25, 38
c. 346, 353, 349, 368, 371, 336, 346, 350, 359, 362
d. 49, 43, 88, 81, 52, 67, 70, 85, 83, 44, 47, 82, 55, 77, 84, 48, 46, 83
e. 2.8, 3.6, 1.5, 1.3, 1.9, 2.4, 2.7, 3.3, 4.1, 2.9, 1.1, 4.6, 4.2, 3.9

6. WE10 Redraw the stem plots in question **5** so that the stems are intervals of 5 rather than intervals of 10.

7. The following times (to the nearest minute) were achieved by 40 students during a school outdoor running activity.

23	45	25	48	21	56	33	34	63	43	42	41	26	44
45	41	40	39	37	53	26	55	48	39	29	52	57	33
31	32	71	60	49	52	32	28	47	42	37	33		

a. Construct a stem plot with groupings of 5 minutes.
b. Construct a frequency table. (Use groupings of 5 minutes.)
c. If the top 10 runners were chosen for the representative team, what was the qualifying time as given by:
i. the stem plot **ii.** the frequency table.
d. How many times was the time of 33 minutes recorded? Explain which summary (stem plot or frequency table) was able to give this information.

8. The ages of participants in a Pump class at a gym are listed below:
17, 21, 36, 38, 23, 45, 32, 53, 18, 25, 14, 29, 42, 26, 18, 27, 37, 19, 34, 20, 35.
Display the data as a stem plot using an interval of 5.

9. MC Consider the stem plot shown.

7 | 6 = 7.6

Stem	*Leaf*
7	8
8	0 8 9
9	1 6 7 8
10	3 5 8
11	2

Which of the following data sets matches the above stem plot?
A. 78 80 88 89 91 96 97 98 103 105 108 112
B. 8 0 8 9 1 6 7 8 3 5 8 2
C. 7.8 8.0 8.8 8.9 9.1 9.6 9.7 9.8 1.03 1.05 1.08 1.12
D. 7.8 8.0 8.8 8.9 9.1 9.6 9.7 9.8 10.3 10.5 10.8 11.2

Understanding

10. The following data represent the life expectancy (in years) of Australians in 40 different age groups.

83.7	84.5	84.7	85.2	84.6	84.5	85.9	84.9	88.3	86.5
84.1	84.6	84.8	84.4	84.7	84.5	86.0	85.0	88.5	86.6
84.2	84.6	84.8	84.4	84.7	85.7	87.3	85.0	88.8	86.8
84.3	84.6	84.9	84.5	87.1	85.8	87.6	85.1	89.0	86.9

a. Present the data as a frequency table.
b. Present the data as a stem plot.
c. Comment on the distribution of life expectancy for the 40 different age groups.

11. For the median ages from 2002 to 2009 given below, choose the most appropriate technique for summarising the data. Justify your choice.

Median age of Australian females and males from 2002 to 2009

Year	Females	Males
2002	36.6	35.2
2003	36.9	35.3
2004	37.1	35.5
2005	37.3	35.7
2006	37.4	35.9
2007	37.6	36.1
2008	37.6	36.1
2009	37.7	36.1

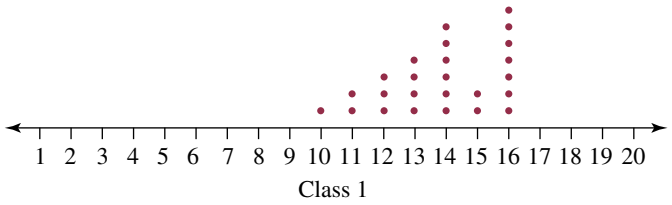
Reasoning

12. The dots below are arranged in a particular pattern, even though they may appear to be randomly coloured. Each row is generated by the row immediately above it.



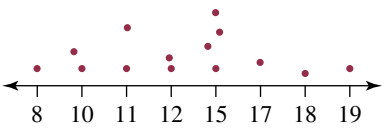
a. Can you work out the pattern and add the next 3 rows?
b. Do you think a row could ever have all red dots?
c. Do you think a row could ever have all black dots?
d. Does a row ever repeat itself?

13. The two dot plots below display the latest Maths test results for two Year 9 classes. The results show the marks out of 20.



a. How many students are in each class?
b. For each class, how many students scored 15 out of 20 for the test?
c. For each class, how many students scored more than 10 for the test?
d. Use the dot plots to describe the performance of each class on the test.

14. The following diagram is the result of a student's attempt to draw a dot plot to display the values 8, 10, 10, 11, 11, 12, 12, 15, 15, 15, 15, 17, 18, 19.



a. List the mistakes that the student made in drawing the dot plot.
b. Draw the correct dot plot.

15. a. Sometimes you are asked to represent two sets of data on separate stem-and-leaf plots. How can you represent two sets of data (keeping them separate) on a single stem-and-leaf plot? Demonstrate your method using the data sets below that show the heights (in cm) of 12 boys and 12 girls in a Year 8 class:

Boys: 154, 156, 156, 158, 159, 160, 162, 164, 168, 171, 172, 180
Girls: 145, 148, 152, 153, 155, 156, 160, 161, 161, 163, 165, 168

b. Consider the stem-and-leaf plot below. How could you make this easier to read and use?

Stem	*Leaf*
1	0 2 2 2 5 7 8 9
2	0 0 0 0 0 1 1 2 2 2 2 2 2 4 4 7 7 7 7 8 8 8 8 9 9
3	0 2 2 3 3 3 5 5 7 8 9

16. The masses of mice (measured in grams) recorded by a Year 9 Science class are shown below.

25.1, 24.8, 25.1, 27.3, 25.3, 29.5, 24.5, 26.7, 24.0, 26.3, 25.4, 26.3, 23.9, 25.8, 25.4, 24.6, 25.1, 23.9, 33.2, 24.5, 28.1, 27.3

a. Draw an ordered stem-and-leaf plot for this data.
b. Calculate the range.
c. Draw an ordered stem-and-leaf plot for the masses of the mice rounded to the nearest gram. Discuss whether a stem-and-leaf plot is suitable for the rounded data. Compare the stem-and-leaf plot with the original masses of mice. What can you conclude?

Problem solving

17. The following data shows the speeds of 30 cars recorded by a roadside camera outside a school, where the speed limit is 40 km/h.

20, 27, 30, 36, 45, 39, 15, 22, 29, 30, 30, 38, 40, 40, 40, 40, 42, 44, 20, 45, 29, 30, 34, 37, 40, 45, 60, 38, 35, 32

a. Present the data as an ordered stem-and-leaf plot.
b. Write a paragraph for the school newsletter about the speed of cars outside the school.



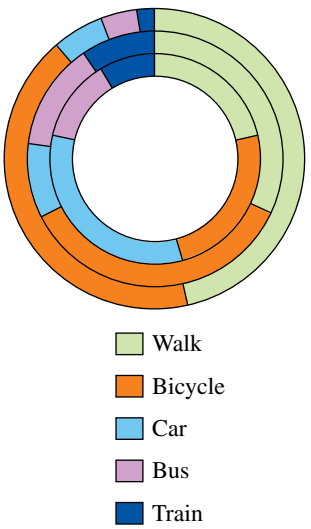
18. A donut graph can be used to compare two or more sets of data using the basic idea of a pie graph. A donut graph comparing students' methods of travel to school on Monday, Tuesday and Wednesday is shown.

a. What similarities and differences does a donut graph have to a pie graph?
b. Write a series of instructions for drawing a donut graph.
c. Trial your instructions on the following data by creating a donut graph to represent the data.

Method of travel	Monday	Tuesday	Wednesday
Walk	5	7	25
Ride	6	8	23
Car	8	2	3
Bus	3	3	2
Train	2	2	1

d. Compare your donut chart with the one shown at right.
e. What are the advantages and disadvantages of using a donut graph instead of a multiple column graph to compare two or more sets of data?

Method of travel to school
Monday (inner ring)
Tuesday (middle ring)
Wednesday (outer ring)



Reflection

What are two things that need to be considered when deciding on the most suitable method to collect data?

CHALLENGE 14.1

Analysis of written English prose reveals the proportion of vowels and consonants to be roughly the same in every large sample — approximately 7:11. The letter *e* accounts for one-third of the occurrence of vowels. In a passage consisting of 10 000 letters, approximately how many times would you expect the letter *e* to occur?

14.4 Displaying data

14.4.1 Displaying data

- Displaying data using graphs reveals many important features at a glance.
- There are many representations available to a statistician, but they must be correctly matched with the type of data. These are summarised in the table below.



	Categorical data	Numerical discrete data	Numerical continuous data
Types of graphs	Dot plots	Dot plots	Histograms
	Line plots	Line plots	Frequency polygons
	Pictographs	Histograms — ungrouped	Box plots
	Bar charts	Histograms — grouped	
	Pie charts	Frequency polygons	
		Stem plots	

14.4.2 Bar charts and pie charts

- **Bar charts** are used to display categorical data only.
- The height of each bar represents **frequency**, **relative frequency** or percentage frequency.
- The width of bars and spaces between bars need to be kept uniform.
  Note: Bars cannot touch one another.
- **Pie charts** (circle graphs or sector graphs) use pieces of the pie or sectors to represent a category.
- The size of the piece of pie is in proportion to the frequency (as a percentage) compared with the total (100%).
- The size of each sector is measured as a proportion of the 360 degrees in a circle.

WORKED EXAMPLE 11

TI | CASIO

The Kelly Clan is having a major family reunion. The age groups of the 200 family members that are attending have been recorded, as shown below.

Age group	Number
Baby	20
Toddler	24
Child	34
Teenager	36
Adult	50
Pensioner	36

Display the frequency table as a:
a percentage bar chart
b pie chart.

THINK

a 1 Add two extra columns to the frequency table to calculate the percentage frequency and the size of the sector.
For percentage use the rule
$\% = \frac{f}{\Sigma f} \times \frac{100}{1}$.
For size of sector use the rule
$\text{degrees}^\circ = \frac{f}{\Sigma f} = 360^\circ$.

2 Give the bar chart a suitable title.

3 Identify the category names and label the horizontal axis with the names of the categories of people. A space between each category is the convention for bar charts.

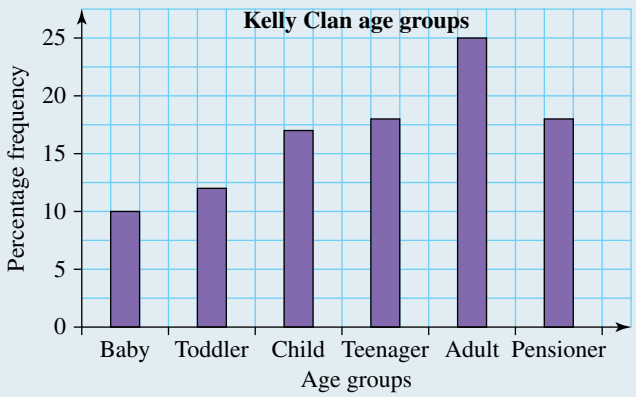
4 Label the vertical axis with a suitable scale and title.

WRITE/DRAW

a

Age group	Number (f)	Percentage frequency	Size of sector
Baby	20	$\frac{20}{200} \times \frac{100}{1} = 10\%$	$\frac{20}{200} \times 360^\circ = 36^\circ$
Toddler	24	$\frac{24}{200} \times \frac{100}{1} = 12\%$	$\frac{24}{200} \times 360^\circ = 43.2^\circ \approx 43^\circ$
Child	34	$\frac{34}{200} \times \frac{100}{1} = 17\%$	$\frac{34}{200} \times 360^\circ = 61.2^\circ \approx 61^\circ$
Teenager	36	$\frac{36}{200} \times \frac{100}{1} = 18\%$	$\frac{36}{200} \times 360^\circ = 64.8^\circ \approx 65^\circ$
Adult	50	$\frac{50}{200} \times \frac{100}{1} = 25\%$	$\frac{50}{200} \times 360^\circ = 90^\circ$
Pensioner	36	$\frac{36}{200} \times \frac{100}{1} = 18\%$	$\frac{36}{200} \times 360^\circ = 64.8^\circ \approx 65^\circ$
Σ	$\Sigma f = 200$	100%	360°

Kelly Clan age group categories include baby, toddler, child, teenager, adult and pensioner.



5 Draw the bars to the correct percentage frequency height for each category.

b 1 Use the information calculated in the extended frequency table from **a1**.

2 Give the pie chart a suitable title.

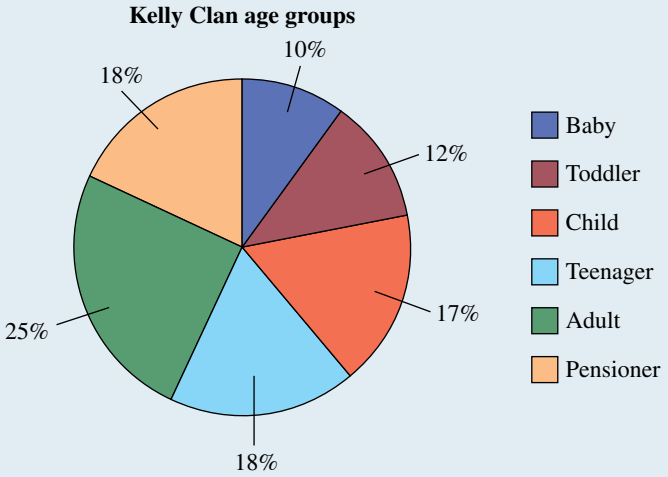
3 Create a legend with a suitable colour for each category.

4 Draw a circle.

5 Using a protractor, measure and draw the correct angle of each sector.

6 Colour the sectors using the legend.

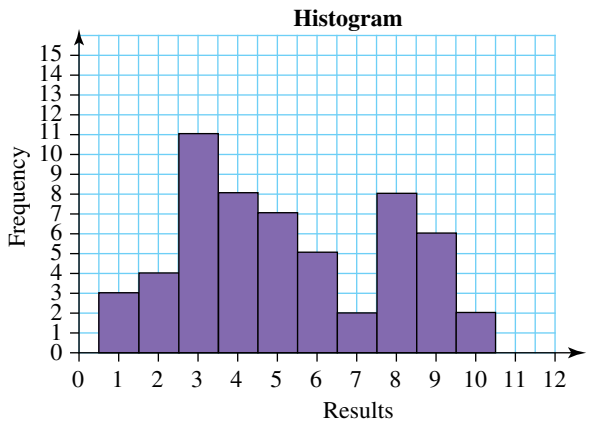
b

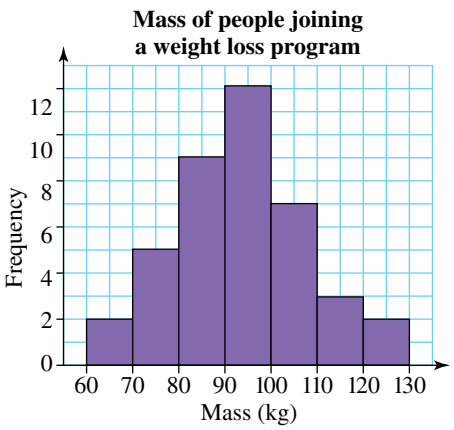


14.4.3 Histograms and frequency polygons

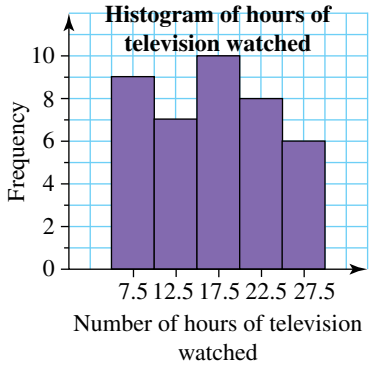
- **Histograms** are similar to bar charts except the columns have no gaps between them.
- Histograms can display display discrete and continuous numerical data.
- Histograms should include:
  - a title
  - clearly labelled axes
  - separate axes scaled evenly
  - columns of equal width, with no gaps between
  - a half-interval gap at each end of the graph.

- Each column of the histogram has the range of values shown on either edge of the column.

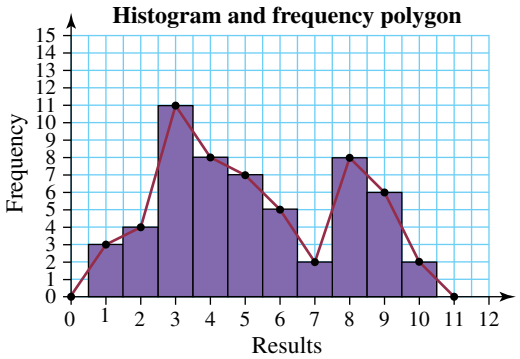




- Alternatively, the centre score (midpoint) of each class interval can be marked.
- The midpoint is calculated by taking the average of the two extreme values of that class interval.



- **Frequency polygons** are constructed by connecting the midpoints of the tops of the columns in the histograms with straight lines.
- The lines close the frequency polygon on either side of the histogram on the horizontal axis.
- The ends of the frequency polygon should be a half-interval either side of the first and last columns of the histogram.



WORKED EXAMPLE 12

A sample of 40 people was surveyed regarding the number of hours per week they spent watching television. The results are listed below.
12.9, 18.3, 9.9, 17.1, 20, 7.8, 24.2, 16.7, 9.1, 27, 7.2, 16, 26.5, 15, 7.4, 28, 11.4, 20, 9, 11, 23.7, 19.8, 29, 12.6, 19, 12.5, 16, 21, 8.3, 5, 16.4, 20.1, 17.5, 10, 24, 21, 5.9, 13.8, 29, 25

a What is the range for the data.

b Organise the data into 5 class intervals and use this to create a frequency distribution table that displays the class intervals, midpoints and frequencies.

c Construct a histogram and frequency polygon to represent the data.

THINK	WRITE/DRAW
a Determine the range of the number of hours of television watched. *Note:* The range is the difference between the largest and smallest values.	**a** Range = largest value − smallest value = 29 − 5 = 24

b 1 Determine the size of the class intervals. Class intervals have been recorded as 5–<10, 10–<15 and so on to accommodate the continuous data.

2 Rule a table with three columns, headed 'Hours of television watched' (class interval), 'Midpoint' (class centre) and 'Frequency'.
Note: The midpoint of a class interval is calculated by taking the average of the two extreme values of that class interval. For example, the midpoint of the 5–10 class interval is $\frac{5 + 10}{2} = 7.5$.

3 Tally the scores and enter the information into the frequency column.

4 Calculate the total of the frequency column.

c 1 Rule a set of axes on graph paper. Label the horizontal axis 'Number of hours of television watched' and the vertical axis 'Frequency'.

2 Leaving a $\frac{1}{2}$-unit or interval space, draw in the first column so that it starts and finishes halfway between class intervals and reaches a vertical height of 9 people.

3 Draw the columns for each of the other scores.

4 To draw the frequency polygon, mark the midpoints of the tops of the columns obtained in the histogram.

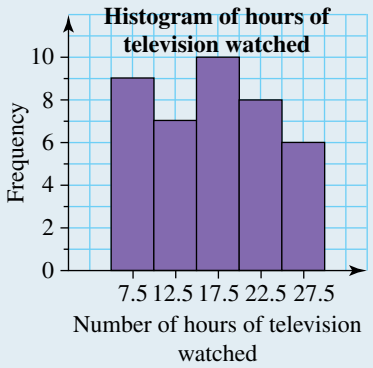
5 Join the midpoints by straight line intervals.

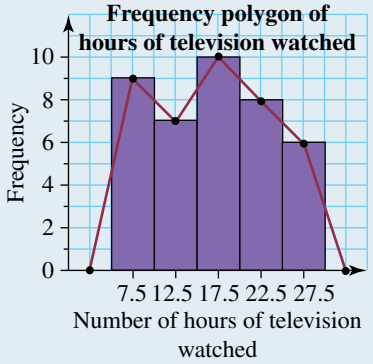
6 Close the frequency polygon by drawing lines to points placed a half-interval either side of the first and last columns of the histogram.

b Class intervals of 5 hours will create 5 groups, and cover a range of 25 numbers.

Hours of television watched	Midpoint	Frequency
5–<10	7.5	9
10–<15	12.5	7
15–<20	17.5	10
20–<25	22.5	8
25–<30	27.5	6
	Total	40

c





WORKED EXAMPLE 13

A stem plot was used to display the weight of airline luggage for flights to the Gold Coast.

a Convert the stem plot into a histogram.

b Show the frequency polygon without the histogram superimposed.

Weights of airline luggage

Key: 21 | 3 = 21.3 kg

Stem	*Leaf*
18	5 8 9
19	2 3 6 7
20	5 7 8 8 9 9
21	0 2 6 9
22	0 1 9
23	2



THINK

a Histogram

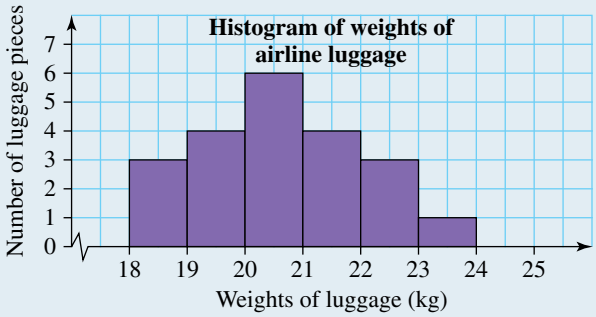
1. Identify the data as numerical continuous data with six groups or class intervals. Label the *x*-axis as 'Weight of airline luggage (kg)'. Leave a half-interval gap at the beginning and use an appropriate scale for the *x*-axis.
2. Label the *y*-axis as 'Frequency' (or 'Number of luggage pieces').
3. Draw the first column on the *x*-axis to a height of its frequency.
4. Repeat for all class intervals.

b Frequency (histogram) polygon

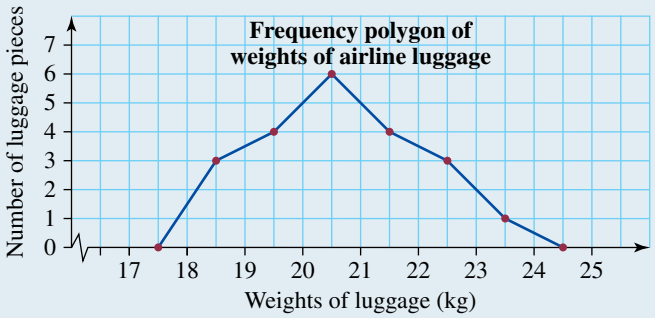
Use the same procedure as for the histogram, except the columns are replaced with lines with end points lined up with the middle of the class intervals shown on the *x*-axis and to the height of the frequency on the *y*-axis.

DRAW

a



b



14.4.4 Back-to-back stem-and-leaf plots

- It is important to be able to compare data sets when the data are related.
- When two sets of data are related, we can present them as **back-to-back stem-and-leaf plots**.

WORKED EXAMPLE 14

The ages of male and female groups using a ten-pin bowling centre are listed.
Males: 65, 15, 50, 15, 54, 16, 57, 16, 16, 21, 17, 28, 17, 27, 17, 22, 35, 18, 19, 22, 30, 34, 22, 31, 43, 23, 48, 23, 46, 25, 30, 21.
Females: 16, 60, 16, 52, 17, 38, 38, 43, 20, 17, 45, 18, 45, 36, 21, 34, 19, 32, 29, 21, 23, 32, 23, 22, 23, 31, 25, 28.
Display the data as a back-to-back stem-and-leaf plot and comment on the distribution.



THINK

1 Rule up three columns, headed Leaf (female), Stem, and Leaf (male).

2 Make a note of the smallest and largest values of both sets of data (15 and 65). List the stems in ascending order in the middle column.

WRITE

Key: 1 | 5 = 15

Leaf (female)	*Stem*	*Leaf (male)*
9 8 7 7 6 6	1	5 5 6 6 6 7 7 7 8 9
8 5 3 2 3 3 1 9 1 0	2	1 8 7 2 2 2 3 3 5 1
1 2 2 4 6 8 8	3	5 0 4 1 0
5 5 3	4	3 8 6
2	5	0 4 7
0	6	5

3 Beginning with the males, work through the given data and enter the leaf (unit component) of each value in a row beside the appropriate stem.

4 Repeat step 3 for the set of data for females.

5 Include a key to the plot that informs the reader of the meaning of each entry.

6 Redraw the stem-and-leaf plot so that the numbers in each row of the leaf columns are in ascending order.
Note: The smallest values are closest to the stem column and increase as they move away from the stem.

Key: 1 | 5 = 15

Leaf (female)	*Stem*	*Leaf (male)*
9 8 7 7 6 6	1	5 5 6 6 6 7 7 7 8 9
9 8 5 3 3 3 2 1 1 0	2	1 1 2 2 2 3 3 5 7 8
8 8 6 4 2 2 1	3	0 0 1 4 5
5 5 3	4	3 6 8
2	5	0 4 7
0	6	5

7 Comment on any interesting features.

The youngest male attending the ten-pin bowling centre is 15 and the oldest 65; the youngest and oldest females attending the ten-pin bowling centre are 16 and 60 respectively. Ten-pin bowling is most popular for men in the teens and 20s, and for females in the 20s and 30s.

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Back-to-back stem plot (int-2773)

Exercise 14.4 Displaying data

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 4–6, 9–11, 13	Questions: 1, 3, 6, 7, 9, 11, 12, 14, 17	Questions: 1, 4, 6, 8–10, 15, 16–18

Individual pathway interactivity: int-4542

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE11 The number of participants in various gym and fitness classes for the local fitness club is given as:

Aerobics — 120 members
Boxing — 15 members
Pilates — 60 members
Spin — 45 members
Aqua aerobics — 30 members
Step — 90 members

Represent this information as a:

a. bar chart

b. pie chart.

2. WE12 In a survey, 40 people were asked about the number of hours a week they spent watching television. The results are listed below.

10.3, 13, 7.1, 12.4, 16, 11, 6, 14, 6, 11.1, 5.8, 14, 12.9, 8.5, 27.5, 17.3, 13.5, 8, 14, 10.2, 13, 7.9, 15, 10.7, 16, 8.4, 18.4, 14, 21, 28, 9, 12, 11.5, 13, 9, 13, 29, 5, 24, 11

a. Are the data continuous or discrete?

b. What is the range of the data?

c. Organise the data into class intervals and create a frequency distribution table that displays the class intervals, midpoints and frequencies.

d. Construct a histogram and frequency polygon.

3. The number of phone calls made per week in a sample of 56 people is listed below.

21, 50, 8, 64, 33, 58, 35, 61, 3, 51, 5, 62, 16, 44, 56, 17, 59, 23, 34, 57, 49, 2, 24, 50, 27, 33, 55, 7, 52, 17, 54, 78, 69, 53, 2, 42, 52, 28, 67, 25, 48, 63, 12, 72, 36, 66, 15, 28, 67, 13, 23, 10, 72, 72, 89, 80

a. Organise the data into a grouped frequency distribution table using a suitable class interval.

b. Display the data as a combined histogram and frequency polygon.

4. For the following data, construct:

a. a frequency distribution table

b. a histogram and frequency polygon.

Class sizes at Jacaranda Secondary College									
38	24	20	23	27	27	22	17	30	26
25	16	29	26	15	26	19	22	13	25
21	19	23	18	30	20	23	16	24	18
12	26	22	25	14	21	25	21	31	25

5. WE13 The number of bananas sold each recess at a high school canteen for the past five weeks was recorded as shown.

4 6 3 1 3 4 5 7 2 1 6 2 3 5 7 3 4 2 2 5 8 4 2 1 4

Represent this information as:

a. a histogram

b. a frequency polygon without the histogram superimposed.

6. The distances 300 students have to travel to attend a primary school are summarised in the table below:

Distance km	Number of students
0–<2	112
2–<4	65
4–<6	56
6–<8	44
8–<10	15
10–<12	8

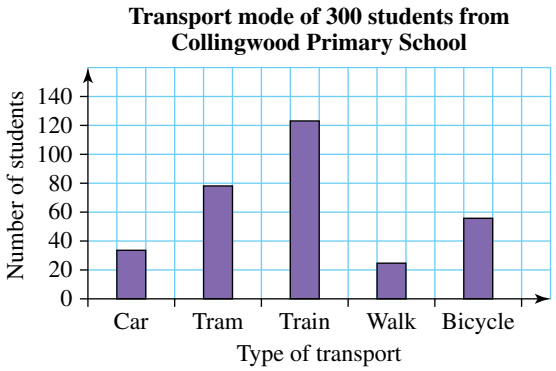
Represent this information as:

a. a histogram

b. a frequency polygon without the histogram superimposed.

7. MC From the bar chart, the number of students who catch public transport to school is:

A. 80 **B.** 100 **C.** 120 **D.** 200



Understanding

8. An investigation into transport needs for an outer suburb community recorded the number of passengers boarding a bus during each of its journeys.

12 43 76 24 46 24 21 46 54 109 87 23 78 37 22 139

65 78 89 52 23 30 54 56 32 66 49

Construct a histogram using class intervals of 20.

9. A drama teacher is organising his DVD collection into categories of 'Type of show'. He recorded the following for the first 20 shows nominated.

musical, musical, drama, pantomime, horror, musical, pantomime, action, drama, pantomime, drama, drama, musical, drama, pantomime, romance, drama, musical, romance, drama

a. Show the data set as a bar chart.

He continued on with the task, and the classification for his collection of 200 DVDs is summarised in the frequency table shown below.

Type of DVD	Number of DVDs	Size of sector for a pie chart
Musical	52	
Drama	98	
Pantomime	30	
Horror	10	
Action	5	
Romance	5	
Total	200	360°

b. Copy and complete the frequency table.

c. Construct a pie chart for the 200-DVD collection.

10. The height, in centimetres, of 30 students in Year 9 was recorded as follows.
146.5, 163, 156.1, 168, 159.4, 170.1, 152.1, 174.5, 156, 163, 157.8, 161, 178, 151, 148, 167, 162.9, 157, 166.7, 154.6, 150.3, 166, 160, 155.9, 164, 157, 171.1, 168, 158, 162

a. Is the data continuous or discrete?

b. Organise the data into 7 classes, starting at 145 cm and draw up a frequency distribution table.

c. Display the data as a histogram.

d. Is it possible to obtain the range of heights from the frequency distribution table, without any other information? Explain.

e. MC How many students stood *at least* 160 cm tall?

A. 15 **B.** 16 **C.** 14 **D.** 7

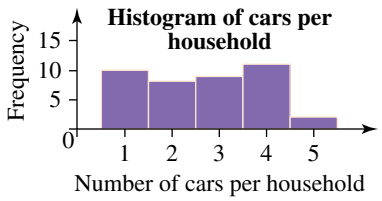
f. MC What percentage were under the minimum height of 165 cm for the basketball team?

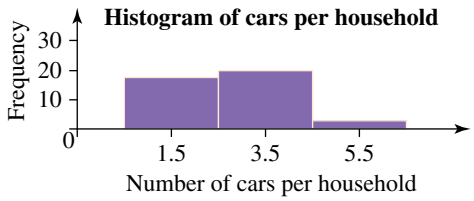
A. 21% **B.** 30% **C.** 87% **D.** 70%

g. Reorganise the data into class intervals of 4 cm; that is, 145–<148 cm.

h. Draw a new histogram and compare it to the previous one. Discuss any advantages or disadvantages of having a smaller class interval.

11. Could the following histograms be produced from the same data sets? Explain.

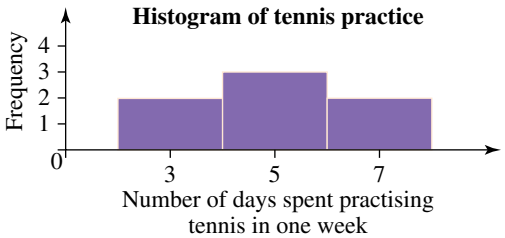




12. The following histogram illustrates the number of days that tennis team members practised in one week.

a. What does the frequency represent in this problem?

b. What is the average number of days the team members practised in one week?



Reasoning

13. WE14 The number of goals scored in football matches by Mitch and Yani were recorded as follows:

Mitch	0	3	1	0	1	2	1	0	0	1
Yani	1	2	0	1	0	1	2	2	1	1

Display the data as a back-to-back stem-and-leaf plot and comment on the distribution.

14. Answer the following questions for the back-to-back stem-and-leaf plot in question **13**.

a. How many times did each player score more than 1 goal?

b. Who scored the greatest number of goals in any one match?

c. Who scored the greatest number of goals overall?

d. Who is the more consistent performer? Explain.

15. Percentages in a mathematics exam for two classes were as follows:

9A	32	65	60	54	85	73	67	65	49	96	57	68
9B	46	74	62	78	55	73	60	75	73	77	68	81

a. Construct a back-to-back stem-and-leaf plot of the data.

b. What percentage of each group scored above 50? Give your answers to the nearest whole number.

c. Which group had more scores over 80?

d. Compare the clustering for each group.

e. Comment on extreme values.

f. Calculate the average percentage for each group.

16. MC The back-to-back stem-and-leaf plot displays the heights of a group of Year 9 students.

Key: 13 | 7 = 137 cm

Leaf (boys)	*Stem*	*Leaf (girls)*
9 8	13	7 8
9 8 8 7 6	14	3 5 6
9 8 8	15	1 2 3 7
7 6 6 5	16	3 5 6
8 7 6	17	1

a. The total number of Year 9 students is:

A. 13 **B.** 17 **C.** 30 **D.** 36

b. The tallest male and shortest female heights respectively are:

A. 186 cm and 137 cm **B.** 171 cm and 148 cm

C. 137 cm and 188 cm **D.** 178 cm and 137 cm

c. Compare the heights of boys and girls.

Problem solving

17. When reading the menu at the local Chinese restaurant, you notice that the dishes are divided into sections. The sections are labelled chicken, beef, duck, vegetarian and seafood.

a. What type of data is this?

b. What is the best way to represent this data?

18. You are working in the marketing department of a company that makes sunscreen and your boss wants to know if there are any unusual results from some market research you completed recently. You completed a survey of people who use your sunscreen, asking them how many times a week they used it. What will you say to your boss? Analyse the data set below to help you and try to explain any unusual values.

7, 2, 5, 4, 7, 5, 7, 2, 5, 4, 3, 5, 7, 7, 4, 5, 21, 5, 5, 2

Reflection

Give two reasons why stem plots are preferred by statisticians rather than frequency tables.

14.5 Measures of central tendency

14.5.1 Mean, median and mode

- The mean, median and mode are know as **measures of central tendency**.
- The **mean** is a measure of the centre of a set of data. The mean is more commonly known as the average.
  - To calculate the mean from a set of raw scores, use the formula:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}}$$

  - To calculate the mean of data organised in a frequency table, use the formula:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{\text{sum of (scores} \times \text{frequency)}}{\text{sum of frequencies}}$$

- The **median** is the middle score (symbol M or Q_2) of a set of data organised from lowest to highest.
  - The median score $= \frac{n+1}{2}$ th score, where n is the number of pieces of data or scores.
  - If n is odd, then the median is the middle score.
  - If n is even, then the median is the average of the two middle scores.
- The **mode** is the score (or scores) that occurs most frequently.

WORKED EXAMPLE 15

TI | CASIO

3 7 4 5 6 2 8 3 3 5
For the data above, calculate the:
a median **b mean** **c mode.**

THINK

a Median

1 Organise the data or scores from smallest to largest.

2 The median or middle score is found by first counting how many scores there are. If there is an even number of scores, then the median is the average of the two middle scores. Use the rule median score $= \frac{n+1}{2}$ th score where n is the number of scores.

b Mean

1 Add up all the scores.

2 Divide by the number of scores.

c The mode is the score that occurs the most frequently in the list.

WRITE

a Median

2 3 3 3 4 5 5 6 7 8

Median score $= \frac{n+1}{2}$th score

$= \frac{10+1}{2} = 5.5$th score

The median score is between the 5th and 6th scores, which is between 4 and 5.

$\therefore$ median = 4.5.

b Mean

$\text{Mean} = \bar{x}$

$= \frac{\Sigma x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}}$

$\bar{x} = \frac{3+7+4+5+6+2+8+3+3+5}{10}$

$\bar{x} = \frac{46}{10} = 4.6$

c Mode = 3

WORKED EXAMPLE 16

The combined number of goals scored in each of the 48 first round games of the 2010 World Cup is summarised in the frequency table below.

Total combined goals scored	Number of matches
0	6
1	13
2	12
3	9
4	5
5	2
6	0
7	1
Σf	48



Find the:

a median number of goals scored

b mean number of goals scored

c modal number of goals scored.

THINK

a Median

The sum of the frequencies is the number of scores, n. If there is an even number of scores, then the median is the average of two middle scores.

Find where the 24th and 25th scores lie.

b Mean

Set up a third column on the frequency table and multiply the score and its frequency.

Sum the products of xf.

Divide by the sum of the frequencies, Σf.

WRITE/DRAW

a Median

$$\text{Median score} = \frac{n+1}{2}\text{th match}$$

$$= \frac{48+1}{2} = 24.5\text{th match}$$

This score is between the 24th match and the 25th match.

The 24th score is 2 combined goals.

The 25th score is 2 combined goals.

Median = 2 combined goals

b Mean

Combined goals x	Number of matches f	$x \times f$
0	6	0
1	13	13
2	12	24
3	9	27
4	5	20
5	2	10
6	0	0
7	1	7
Σf	48	$\Sigma xf = 101$

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{\text{sum of (scores} \times \text{frequency)}}{\text{sum of frequencies}}$$
$$= \frac{101}{48}$$
$$= 2.1$$

c Mode
Look for the score with the highest frequency.

c Mode = 1 combined goal scored

- Sometimes the mean, median and mode are calculated from graphs.

WORKED EXAMPLE 17

Find the median of each data set.

a

Stem	*Leaf*
1	8 9
2	2 2 5 7 7 8
3	0 1 4 6 7
4	0 5

b



THINK

a 1 Check that the scores are arranged in ascending order.

2 Locate the position of the median using the rule $\frac{n+1}{2}$ where $n = 15$.
Note: The position of the median is $\frac{15+1}{2} = 8$ (the eighth score).

3 Answer the question.

b 1 Observe the data in the given dot plot.

2 Locate the position of the median using the rule $\frac{n+1}{2}$ where $n = 12$.
Note: The position of the median is $\frac{12+1}{2} = 6.5$; that is, between the sixth and seventh scores.

3 Obtain the average of the two middle scores.

4 Answer the question.

WRITE/DISPLAY

a The scores are arranged in ascending order.

Key: 1 | 8 = 18

Stem	*Leaf*
1	8 9
2	2 2 5 7 7 **8**
3	0 1 4 6 7
4	0 5

The median of the set of data is 28.

b The 6th score is 19.
The 7th score is 19.

$$\text{Median} = \frac{19+19}{2}$$
$$= \frac{38}{2}$$
$$= 19$$

The median of the set of data is 19.

14.5.2 Grouped data

- To calculate the mean of a grouped data set, multiply the midpoint of each class interval by the corresponding frequency.
- The median is the middle value when the values are in order.
- The mode of a grouped data set is the most common class interval, which is known as the **modal class**.

WORKED EXAMPLE 18

The actual diagonal lengths of 45 plasma and LCD televisions in a store are presented as a frequency table. Calculate the:
a median (to the nearest inch) b mean (to the nearest inch) c modal class interval.

Diagonal length (inches)	Number of TVs
10–<20	1
20–<30	3
30–<40	8
40–<50	19
50–<60	9
60–<70	3
70–<80	2
Σf	45

THINK

a 1 Median
The sum of the frequencies is the number of scores, n. If there is an odd number of scores, then the median is the middle score.

2 Find where the 23rd score lies in the table by looking at the cumulative frequencies.
<20 inches 1 TV
<30 inches 4 TVs
<40 inches 12 TVs
<50 inches 31 TVs
The 23rd score is in the 40–<50 inches group. It is the 11th score in that group of 19 televisions.

WRITE/DISPLAY

a Median

$$\text{Median score} = \frac{n+1}{2}\text{th score}$$
$$= \frac{45+1}{2} = 23\text{rd score}$$

The 23rd score is in the 40–<50 inches class.
It is the 11th score in a group of 19 scores.

$$\frac{11}{19} \times 10\,\text{inches} = 5.79\ \text{inches}$$
$$\text{Median} = 40 + 5.79\ \text{inches}$$
$$= 45.79\ \text{inches}$$
$$\approx 46\ \text{inches}$$

b 1 Mean
Set up a third and fourth column in the frequency table.

2 Record the midpoint as the middle value of each interval; e.g. $\frac{10 + 20}{2} = 15$.

3 Multiply the midpoint of each interval, x_{mid} and the interval's frequency, f.

4 Add up all the products of $x_{mid} \times f$.

5 Divide $x_{mid} \times f$ by the sum of the frequencies, Σf.

6 Write your answer.

c The modal class is the class interval with the highest frequency.

b Mean

Diagonal length (inches)	Number of TVs (f)	Class midpoint x_{mid} (inches)	Product of x_{mid} and f
10–<20	1	15	$1 \times 15 = 15$
20–<30	3	25	$3 \times 25 = 75$
30–<40	8	35	$8 \times 35 = 280$
40–<50	19	45	$19 \times 45 = 855$
50–<60	9	55	$9 \times 55 = 495$
60–<70	3	65	$3 \times 65 = 195$
70–<80	2	75	$2 \times 75 = 150$
Σf	$= 45$	$\Sigma x_{mid} \times f$	$= 2065$

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{\text{sum of (score} \times \text{frequency)}}{\text{sum of frequencies}}$$
$$= \frac{2065}{45}$$
$$= 45.89 \approx 46 \text{ inches}$$

The mean TV size is 46 inches.

c Modal class is 40 to less than 50 inches.

learnON RESOURCES — ONLINE ONLY

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Complete this digital doc: SkillSHEET: Finding the middle score (doc-10956)

Complete this digital doc: SkillSHEET: Finding the middle score for data arranged in a dot plot (doc-10957)

Complete this digital doc: SkillSHEET: Finding the most frequent score (doc-10958)

Exercise 14.5 Measures of central tendency

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3a, 4a, 5–7, 10–16, 19, 20

CONSOLIDATE
Questions:
1, 2, 3b, 4b, 5, 6, 8, 10–16, 19–21

MASTER
Questions:
1, 2, 3c, 4c, 5, 6, 9, 10, 12, 14, 16, 19–22

Individual pathway interactivity: int-4543

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Fluency

1. **WE15** Calculate the median, mean and mode for the small raw data sets given. Confirm your answers for the median and mean using a graphics calculator. Where necessary, give your answers to 2 decimal places.

a. 27 32 45 48 53 55 55 57 59 61 75 81

b. 8 8 2 8 8 8 8 8 9 9 1 9 5 9 7 10 0

2. **WE16** Calculate the median, mean (to 2 decimal places) and mode for this ungrouped frequency table.

Score	Frequency
0	2
1	5
2	12
3	10
4	7
5	2

3. **WE17a** Find the median of each of the following data sets.

a. Key: 1 | 0 = 10

Stem	*Leaf*
1	0 2
2	1 3 3 5
3	
4	4

b. Key: 10 | 0 = 100

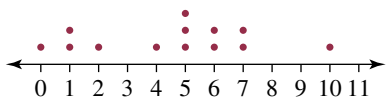
Stem	*Leaf*
10	0
11	0 2 2 2
12	0 4 6 6
13	3

c. Key: 6.1 | 8 = 6.18

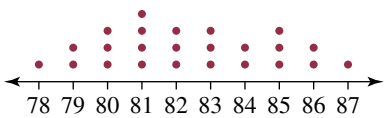
Stem	*Leaf*
6.1	8 8 9
6.2	0 5 6 8
6.3	0 1 2 4 4 4

4. **WE17b** Find the median of each of the following data sets.

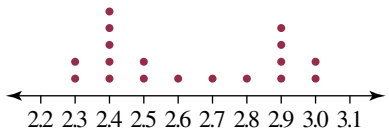
a.



b.



c.



5. **WE18** Calculate the median, mean and mode for the following grouped frequency tables. Where necessary, give your answers to 1 decimal place.

a.

Score x	Frequency f
120–129	2
130–139	8
140–149	10
150–159	5
160–169	1

b.

Score	Frequency
120–129	2
130–139	8
140–149	10
150–159	16
160–169	5
170–179	1

Hint: The tables above show grouped discrete data, where the midpoint of the first class interval is $\frac{120 + 129}{2} = 124.5$.

6. For the stem plot given below, calculate:

a. the median

b. the mean (to 2 decimal places)

c. the mode.

d. Key: 3* | 7 = 37

Stem	*Leaf*
1	3 4
1*	5 6 7 9
2	0 2 3 3 4
2*	5 5 5 6 6 7 8
3	1 2 3 4
3*	5 7
4	1

7. MC For the following set of data:

1, 5, 2, 8, 7, 3, 5, 7, 2, 8, 7, 6, 4, 5, 5

the median, mean and mode is best given by:

A. 5, 6, 5 **B.** 6, 3, 5 **C.** 5, 5, 5 **D.** 5, 5, 7

8. MC The median number of members per family surveyed and presented in this frequency table is:

Number of members per family	Frequency
3	19
4	53
5	21
6	4
7	2
8	1

A. 3 **B.** 4 **C.** 4.5 **D.** 4.6

9. MC For this grouped frequency table, the best estimate of the mean score is:

Score	Frequency
0–<50	1
50–<100	4
100–<150	10
150–<200	3
200–<250	1
250–<300	1

A. 100 **B.** 150 **C.** 125 **D.** 130

Understanding

10. The frequency table on next page is a summary of 100 long jump attempts by a group of primary school students during a recent school sports carnival. For the data given, determine the:

a. type of data **b.** median **c.** mean **d.** mode.

Where necessary, round to the nearest whole number.

Long jump distance (cm)	Number of jumps
120–<140	22
140–<160	44
160–<180	17
180–<200	14
200–<220	2
220–<240	0
240–<260	0
260–<280	1



11. The grouped frequency table below is a summary of 40 golf scores recorded at the local council's 9-hole golf course.
For the data given, determine the:
a. type of data **b.** median **c.** mean **d.** modal class.

Golf score	Number of games
30–39	4
40–49	32
50–59	2
60–69	1
70–79	1

12. A survey of the number of people living in each house on a residential street produced the following data:
2, 5, 1, 6, 2, 3, 2, 1, 4, 3, 4, 3, 1, 2, 2, 0, 2, 4.
a. Prepare a frequency distribution table with an $f \times x$ column and use it to find the average (mean) number of people per household. Give your answer to 1 decimal place.
b. Draw a dot plot of the data and use it to find the median number per household.
c. Find the modal number per household.
d. Which of the measures would be most useful to:
i. a real estate agent renting out houses
ii. a government population survey
iii. an ice-cream mobile vendor?

Reasoning

13. A small business pays these wages (in thousands of dollars) to its employees:
18, 18, 18, 18, 26, 26, 26, 35, 80 (boss).
a. MC What is the wage earned by most workers?
A. $29 400 **B.** $18 000 **C.** $29 000 **D.** $26 000
b. What is the average wage?
c. MC Find the median of the distribution.
A. $26 000 **B.** $26 500 **C.** $18 000 **D.** $18 500
d. Which measure might be used in wage negotiations by:
i. the union, representing the employees (other than the boss)
ii. the boss?
Explain each answer.

14. A class of 26 students had a median mark of 54 in Mathematics; however, no-one actually obtained this result.

a. Explain how this is possible. **b.** Explain how many must have scored below 54.

15. A soccer team had averaged 2.6 goals per match after 5 matches. After their sixth match, the average had dropped to 2.5. Explain how they scored 2 goals in that latest match.

16. A tyre manufacturer selects 48 tyres at random from the production line for testing. The total distance travelled during the safe life of each tyre is shown in the table.

Distance in km ('000)	82	78	56	52	50	46
Number of tyres	2	4	10	16	12	4

a. Calculate the mean, median and mode.

b. Which measure best describes average tyre life? Explain.

c. Recalculate the mean with the 6 longest-lasting tyres removed. By how much is it lowered? Give your answer to the nearest km.

d. If you selected a tyre at random, what tyre life has the highest probability of being selected?

e. In a production run of 10 000 tyres, how many could be expected to last for a maximum of 50 000 km?

f. As the manufacturer, for what distance would you be prepared to guarantee your tyres? Why?

17. If the mean = 11, mode = 4, and the median = 9 (and the missing values are positive integers):

a. find two values for the following set of numbers:
4, ___, 19, ____, 21, 4

b. find two different values that meet the stated criteria

c. can you substitute 9 for both missing values? Explain.

18. a. Create a data set of 8 numbers that meet the following criteria.

- Data values are positive integers
- Mean = 9.5
- Median = 7
- Mode = 6
- Maximum value in data set = 21
- Minimum value in data set = 1

b. Is there only one data set that can meet this criteria? Explain.

Problem solving

19. Calculate the mean for this data set.

Score	4	8	12	16	20	24
Frequency	$4a$	$3a$	$2a$	a	$\frac{a}{2}$	$\frac{a}{4}$

20. a. Create a set of five values with a mean of 4 and a median of 5. Demonstrate that this is true.

b. Change only one of your numbers in the set so that the mean is now 5 and the median remains 5. Demonstrate that this is true.

21. Seven students did a very difficult Science test. The test was out of 15. Six of the students got the following marks: 4, 4, 5, 5, 5, 6. Determine the mark that the 7th student obtained if:

a. the mean is the same as the median **b.** the mean is greater than the median

c. the mean is equal to the mode **d.** the mean is greater than 6.

22. On a piece of paper is a list of six 2-digit prime numbers. They have a mean and median of 39. Their mode is 31, and the smallest number is 13. What are the six numbers? Explain how you deduced your answer.

Reflection

What is the unique feature of the data collected when all three measures of central tendency are the same or very similar?

CHALLENGE 14.2

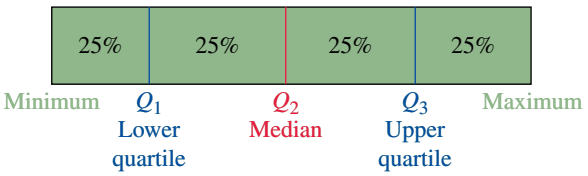
I have chosen 5 different whole numbers. Their mean is 5. The product of the 5 numbers is 945. Two of the numbers are 1 and 9. What are the remaining 3 numbers?

14.6 Measures of spread

14.6.1 Measures of spread

- **Measures of spread** are used to describe a distribution. There are three common measures of spread.
- The **range** is the difference between the highest and lowest scores.

  Range = highest score – lowest score
  $= x_H - x_L$
- The **interquartile range (IQR)** is the range of the middle 50% of all the scores in an ordered set. When the interquartile range is calculated, the data is first organised into quartiles, each containing 25% of the data. The word 'quartiles' comes from the word 'quarter'.



  Interquartile range = upper quartile − lower quartile
  This can be written as:

$$IQR = Q_{upper} - Q_{lower}$$

  or

$$IQR = Q_3 - Q_l$$

- The **standard deviation** is a unique measure for symmetrical or normal distributions that measures the average deviation of all the scores from the mean of the data set.
  – For data from a sample, use the formula

$$\text{Sample standard deviation} = s = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n - 1}}.$$

  – For data that is from a population, use the formula

$$\text{Population standard deviation} = \sigma = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n}}.$$

  Note: The standard deviation value is best calculated using a calculator with a statistical function.

WORKED EXAMPLE 19 TI | CASIO

Find the range and interquartile range of the following data set.
5 6 8 4 12 5 7 8 4 10 9 6 5

THINK	WRITE
1 Arrange data from lowest to highest score.	4 4 5 5 5 6 6 7 8 8 9 10 12
2 Calculate the range.	Range = highest score − lowest score $= x_H - x_L$ $= 12 - 4$ $= 8$

3 Find the median by using the rule: median score $= \frac{n+1}{2}$th score.	Median score $= \frac{n+1}{2}$th score $= \frac{13+1}{2} = 7$th score Median is the 7th score. $\therefore$ median $= 6$
4 Find the lower quartile (Q_{lower}) from the 6 scores in the lower half of the data set.	4 4 5 5 5 6 **6** 7 8 8 9 10 12 Q_{lower} is between 3rd and 4th scores $Q_{\text{lower}} = 5$
5 Find the upper quartile (Q_{upper}) from the 6 scores in the upper half of the data set.	Q_{upper} is between 3rd last and 4th last scores. $Q_{\text{upper}} = \frac{8+9}{2} = 8.5$
6 Subtract the lower quartile from the upper quartile to find the interquartile range.	$IQR = Q_{\text{upper}} - Q_{\text{lower}}$ $= 8.5 - 5$ $= 3.5$

WORKED EXAMPLE 20

For the stem plot shown, determine the range and the interquartile range.

Key: 4 \| 7 = 4.7 kg

Stem	*Leaf*
1	**9**
2	**2 7**
3	**0 1 3 7**
4	**2 4 5 7 8**
5	**1 2 8**
6	**0**

THINK	WRITE
1 Calculate the range by finding the difference between the lowest and highest scores.	Range = highest score − lowest score $= x_{\text{H}} - x_{\text{L}}$ $= 6.0 - 1.9$ $= 4.1$ kg
2 Count the number of scores, n.	$n = 16$
3 Find the median by using the rule median score $= \frac{n+1}{2}$th score.	Median score $= \frac{\text{n}+1}{2}$th score $= \frac{16+1}{2} = 8.5$th score The median is between the 8th and 9th scores, i.e. between 4.2 and 4.4. $\therefore$ the median $= 4.3$ kg.

4 Find the lower quartile, Q_{lower} (1st quartile) from the 8 scores in the lower half of the data set.

Stem	*Leaf*
1	9
2	2 7
3	0 1 3 7
4	2 4 5 7 8
5	1 2 8
6	0

Q_{lower} is between the 4th score (3.0) and the 5th score (3.1).

$\therefore Q_{lower} = 3.05$ kg

5 Find the upper quartile, Q_{upper} (3rd quartile) from the 8 scores in the upper half of the data set.

Q_{upper} is between the 4th last score (5.1) and the 5th last score (4.8), that is 4.95 kg.

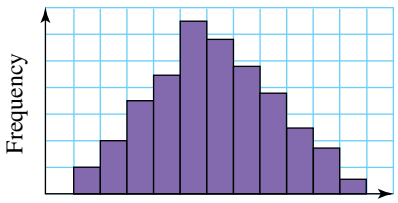
$\therefore Q_{upper} = 4.95$ kg

6 Determine the interquartile range by subtracting the lower quartile from the upper quartile.

$$\begin{aligned} IQR &= Q_{upper} - Q_{lower} \\ &= 4.95 - 3.05 \\ &= 1.9 \text{ kg} \end{aligned}$$

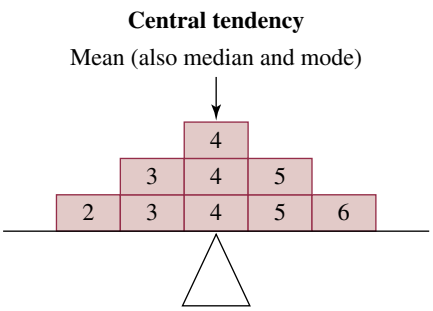
14.6.2 Types of distributions

- Distribution refers to the way data are spread out.
- Depending on how data are distributed or spread out, particular measures of centre and spread may provide different results for the data.
- **Symmetrical distributions**, also referred to as bell-shaped or normal distributions, have a clear centre of the distribution and an even spread on either side.

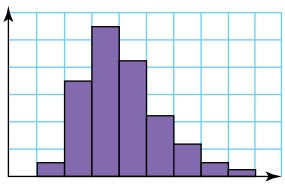


Stem	*Leaf*
0	7
1	2 3
2	2 4 5 7 9
3	0 2 3 6 8 8
4	4 7 8 9 9
5	2 7 8
6	1 3

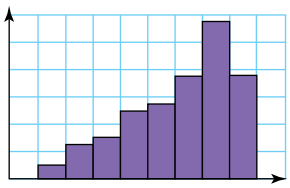
- For symmetrical distributions, all three measures of central tendency (the mean, the median and the mode) give similar results.
  - The mean is the preferred measure of central tendency.
  - Standard deviation is the preferred measure of spread.



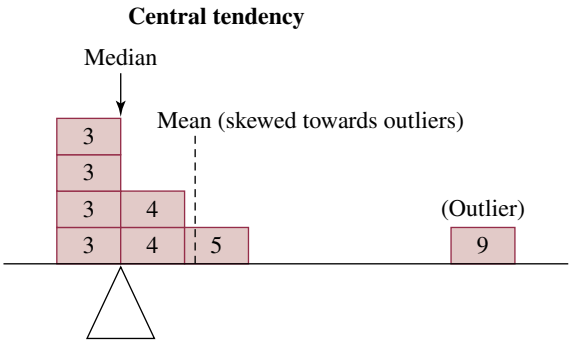
- A skewed distribution is a non-symmetrical distribution.
  - Right or **positively skewed distribution** has few high scores that are spread over a large range to the right. The most common scores are located to the left.
  - Left or **negatively skewed distribution** has few low scores that are spread over a large range to the left. The most common scores are located to the right.
  - For skewed distributions, the mean is affected by outlier(s) or unusual scores.



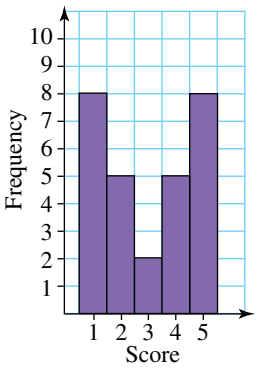
Positively skewed distribution



Negatively skewed distribution



  - The mean is shifted towards the outlier(s).
  - The median is not affected by outliers and is thus the preferred measure of central tendency.
  - Interquartile range is the preferred measure of spread.
- **Bi-modal distributions** have two peaks. When this occurs it indicates that the measurement or observation includes two different groups. An example would be measuring the heights of 100 adults. We could find two modes, one for the heights of adult males and the other mode for adult females. Such distributions would then suggest that the data analysis be modified to separate the two unique groups.



- Measures of central tendency and spread are paired with an appropriate distribution as summarised in the table below. For skewed distributions, the mean is affected by outliers or unusual scores. The median is not affected and is thus the preferred measure of central tendency. The mean is affected by outliers and is shifted towards the outlier.

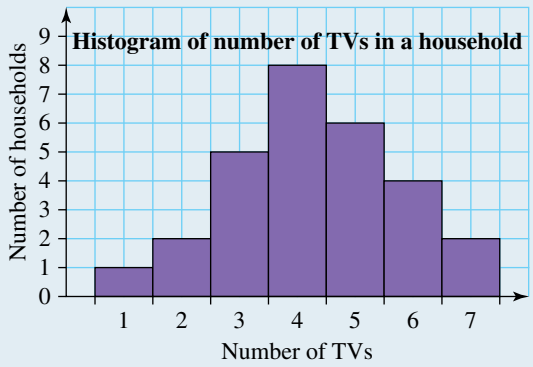
	Symmetric distribution	Skewed distribution
Central tendency	Mean	Median
Spread	Standard deviation	Interquartile range

- The mode is used primarily for describing the centre of categorical data.

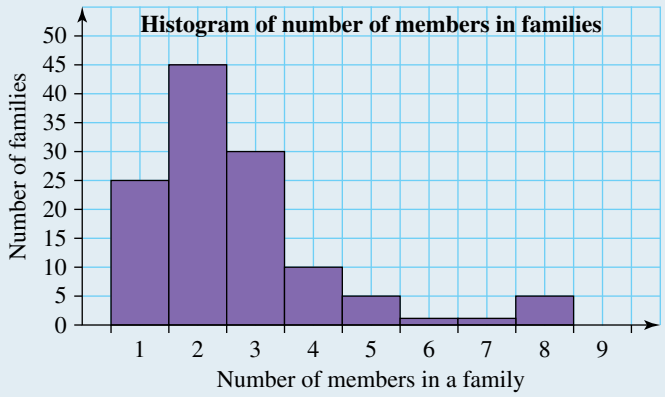
WORKED EXAMPLE 21

For each of the distributions shown, describe the type of distribution, determining its centre and spread using the most appropriate measures.

a



b



THINK

a **1** Describe the shape of the distribution.

2 Select appropriate measures of centre and spread.

3 Determine the mean and standard deviation.

4 State the answer.

b **1** Describe the shape of the distribution.

2 Select appropriate measures of centre and spread.

WRITE

a The graph is roughly symmetrical.

Measure of centre: mean
Measure of spread: standard deviation.

$$\text{Mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}}$$
$$= \frac{120}{28} = 4.3$$

$$\text{Sample standard deviation} = s = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n - 1}}$$
$$= \sqrt{\frac{59.7}{27}}$$
$$= 1.5$$

The mean number of TVs per household is 4.3 and the standard deviation is 1.5.

b The graph is positively skewed.

Measure of centre: median
Measure of spread: interquartile range.

3 Determine the median and interquartile range.	The median is the average of the 61st and 62nd scores. $\text{Median} = \frac{2+2}{2} = 2$ Q_{lower} is the 31st score. $Q_{\text{lower}} = 2$ Q_{upper} is the 92nd score. $Q_{\text{upper}} = 3$ $IQR = Q_{\text{upper}} - Q_{\text{lower}}$ $= 3 - 2 = 1$
4 State the answer.	The median number of members in a family is 2 and the interquartile range is 1.

WORKED EXAMPLE 22

Determine the best measures of central tendency and spread for the heights of 25 people shown in the stem plot at right. Give your answer to the nearest centimetre.

Key: 17 | 8 = 178 *cm*

Stem	*Leaf*
14	8
15	3 6
16	1 4 7
17	0 0 7 8 9
18	1 2 2 4 6 8 8
19	3 4 7 7
20	0 2
21	2

THINK	WRITE/DISPLAY
1 Describe the distribution.	A symmetrical or normal distribution.
2 State the most appropriate measures of centre and spread.	Measure of centre: mean Measure of spread: standard deviation.
3 Calculate the mean.	$\text{Mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}}$ $= \frac{4509}{25}$ $= 180.36$
4 Use a calculator to determine the standard deviation.	Sample standard deviation $= s = 16.29$
5 State the answer.	The mean height of the 25 people is 180 cm and the standard deviation is 16 cm.

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Exercise 14.6 Measures of spread

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–12, 16	Questions: 1, 3, 5, 6, 8, 10, 11–14, 16	Questions: 1–3, 5, 7, 9, 11–16

Individual pathway interactivity: int-4544

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE19 Find the range and interquartile range for the following raw data sets.

a. 120 124 125 135 135 137 141 145 151

b. 18 22 25 27 29 31 35 37 40 42 44

2. WE20 This stem plot gives the spread of 25 cars caught by a roadside speed camera.

Key: 8 | 2 = 82 km/h

8* | 6 = 86 km/h

Determine:

a. the range

b. the interquartile range

c. the sample standard deviation of the distribution.

Stem	*Leaf*
8	2 2 4 4 4 4
8*	5 5 6 6 7 9 9
9	0 1 1 2 4
9*	5 6 9
10	0 2
10*	
11	4

3. For the following raw data sets:

i. find the range and interquartile range

ii. use a statistical function on a calculator and find the standard deviation (to 2 decimal places).

a. 20 45 73 65 85 127 101

b. 1.22 1.28 1.48 1.52 1.62 1.72 1.85 1.95 2.03

4. For the ungrouped frequency table shown, calculate:

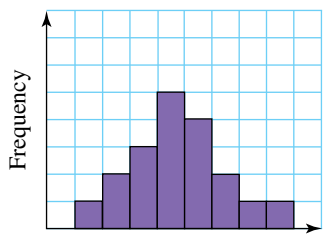
a. the spread — the range, interquartile range and sample standard deviation

b. the central tendency — the mode, the median and mean (correct to 2 decimal places).

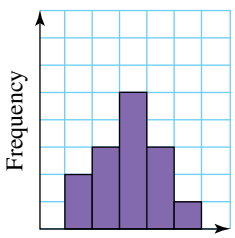
Score	Frequency
0	24
1	17
2	12
3	7
4	2
5	1

5. For each of the following histograms, describe the shape of the distribution of the data.

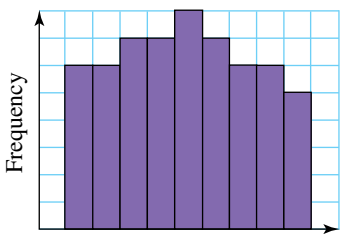
a.



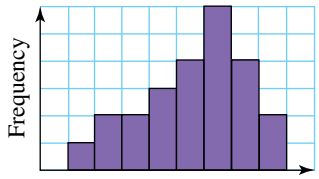
b.



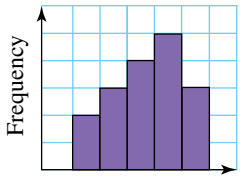
c.



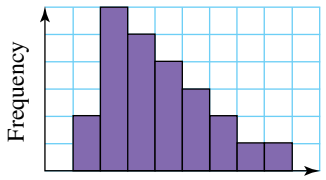
d.



e.



f.



6. For each of the following stem plots, state the type of distribution.

a. Key: 14 | 6 = 14.6

Stem	*Leaf*
12	0
13	8 9
14	0 8 9 9
15	1 5 6 6 6 7 8
16	3 5 7 8
17	0 2
18	3 5

b. Key: 14 | 6 = 146

Stem	*Leaf*
13	1 2 4 6 7 8 9
14	0 4 5 6 8 9 9
15	1 5 6 6 7 8
16	3 7 8
17	0 2
18	3

c. Key: 14 | 6 = 1.46

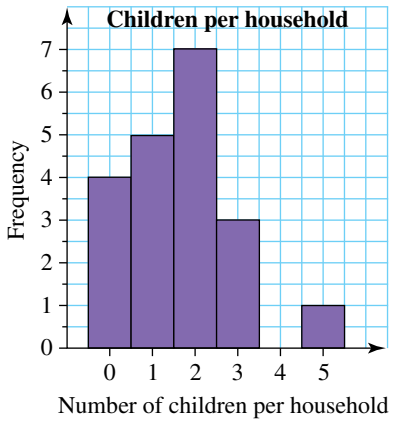
Stem	*Leaf*
12	7 8
13	6 7 8 9
14	0 9
15	1 5 6
16	3 7 8 9 9
17	0 2
18	3

d. Key: 14 | 6 = 1.46

Stem	*Leaf*
1	1
2	
3	6 9
4	0 4 9
5	1 3 5 6 8
6	2 3 7 8 9 9
7	0 4 6 7 7 8

Understanding

7. WE21 This histogram represents the results of data collected from households in a suburban street. Describe the type of distribution, determining its centre and spread using the most appropriate measure.



8. WE22 The number of customers served each hour for the past 24-hour period is summarised in the following stem plot.
Key: 2 | 4 = 24 customers

Stem	*Leaf*
0	0 0 1 4
0*	5 5 8 8 9 9 9
1	1 2 3 4 4 4
1*	5 5 5 7
2	0 2
2*	
3	4

a. State the type of data and type of distribution.
b. Choose the most appropriate measure of central tendency and spread. Give a reason for your choice.
c. Calculate the centre and spread chosen in **b**.

9. MC Which of the following statements is false?
A. Median and interquartile range are not affected by outliers.
B. Mean and standard deviation are affected by outliers.
C. Interquartile range is a good measure of spread for skewed distributions.
D. Range and interquartile range can be used on categorical data.

10. MC

Score	0	1	2	3	4	5	6	7
Frequency	3	4	5	7	10	22	34	45

For the frequency table on previous page, the most likely type of distribution is a:

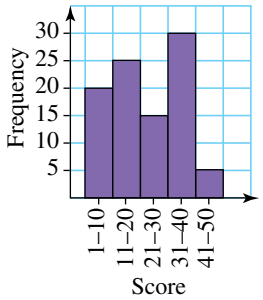
A. negatively skewed distribution

B. positively skewed distribution

C. bimodal distribution

D. normal distribution

12. Consider the distribution shown at right.



a. Are the data symmetrical?

b. What is the modal class?

c. Can the mean and median be seen from the graph and, if so, what are their values?

Reasoning

13. A Geography teacher with a class of 20 students commented that test results for his class out of a score of 50 were negatively skewed. Show and explain a data set for the possible results obtained for the 20 students.

14. Show a data set where the range and interquartile range are the same. Explain what factors cause this.

15. Explain the effect on the range when a set of scores has:

a. 5 added to each score

b. each score doubled

c. each score increased by 25%.

Problem solving

16. A symmetric distribution is one where the mean and median are equal. A uniform distribution (one where the elements are all the same) is symmetric. Give an example of a non-uniform distribution which is symmetric.

17. This back-to-back stem-and-leaf plot displays the speeds of 25 cars caught by a speed camera on a major highway on Easter Sunday and Christmas Day.

Easter Sunday		Christmas Day
8 6 3	6	2 3 6 9 9
8 7 4 1	7	1 2 2 4 5 5 7 8
6 6 5 4 2 1	8	0 0 3 4 6 7
9 8 7 7 5 2 1	9	1 2 3 3
7 7 5 2 1	10	2 4



The police commissioner was heard to comment that he was surprised to find drivers behaved better on Christmas Day than they did on Easter Sunday. His department had done an analysis of the data and found that both the mean and the interquartile range of speeds on Easter Sunday were greater than those for Christmas Day. Discuss his statement.

Reflection

A famous phrase possibly first used by Mark Twain was 'Lies, damned lies and statistics'. This phrase suggests statistics can be used to twist the truth. Suggest a golden rule to avoid statistics being used inappropriately.

14.7 Review

14.7.1 Review questions

Fluency

1. Which one of the following is an example of numerical discrete data?

A. Your favourite weeknight television show
B. The speed of a car recorded on a speed camera
C. The number of home runs by a baseball player
D. Placing in a field of eight swimmers of a 100 m freestyle race

The following information relates to questions **2** and **3**.

Number of bedrooms	1	2	3	4	5	6
Number of homes	4	9	32	36	7	2

2. For the data summarised, the most likely type of distribution is:

A. a negatively skewed distribution
B. a positively skewed distribution
C. skewed to the left
D. a symmetric distribution.

3. The mode for the data collected is:

A. 36 **B.** 5 **C.** 4 **D.** 3

4. The results of a Science test marked out of 60 are represented by the stem plot shown below.
Key: 2 | 3 = 23

Stem	*Leaf*
0	8
1	8
2	3 5
3	0 3 6
4	2 4 5 5 5 8
5	4 7 8 8 9
6	0 0 0 0

The median, mean and mode, respectively, are:

A. 60, 42, 52 **B.** 40, 45, 60 **C.** 45, 60, 45 **D.** 45, 44, 60

5. The following is a grouped frequency table. The best estimate of the mean score is:

Score	**Frequency**
0–<50	12
50–<100	24
100–<150	23
150–<200	4

A. 63 **B.** 90 **C.** 92 **D.** 100

6. The range and IQR for the following set of data is: 46, 46, 49, 53, 61, 63, 67, 72, 81, 84, 93

A. 46 and 34 **B.** 46 and 65 **C.** 47 and 63 **D.** 47 and 32

7. The sample standard deviation for the following set of data is (to 1 decimal place):
25, 27, 33, 42, 47, 54, 58, 59, 65, 66

A. 15.3 **B.** 14.5 **C.** 47.6 **D.** 26.0

8. For the following surveys or studies conducted, decide what type of data would be collected.
- **a.** The blood type of all workers at a shoe factory
- **b.** A survey of 500 Australians' views (strongly agree, agree, disagree, strongly disagree) on bringing in corporal punishment
- **c.** Mobile phones owned in the households of students in your class
- **d.** The top 20 fastest recorded times for the women's 100 metres butterfly

9. Classify each of the following data collections according to whether they are samples or populations.
- **a.** A survey of 1000 travellers on the quality of train service provided.
- **b.** A survey of all the students at a secondary college about the state government's rules on under-age alcohol laws.
- **c.** Local school's mathematics performance in this year's Year 9 cohort of all this year's students' assessed examination results.

10. Determine the number from each group to be surveyed in the following, if stratified sampling was to be conducted.
- **a.** Survey of 60 members from a tennis club of 600 members with 200 male adult members, 100 female adult members and 300 teenage members.
- **b.** Survey of 100 students from a school population of 1250 students where there are 100 Year 12s, 150 Year 11s, 250 Year 10s, 250 Year 9s, 250 Year 8s and 250 Year 7s.

11. The following set of data shows the types of mobiles owned by the first 30 shoppers surveyed.
Nokia, Samsung, LG, Nokia, Nokia, Nokia, Samsung, Samsung, Blackberry, LG, Samsung, Nokia, LG, Nokia, Samsung, Samsung, LG, Nokia, Samsung, Blackberry, Samsung, LG, Nokia, Nokia, Nokia, Nokia, Samsung, Samsung, Blackberry, LG
- **a.** Present the data as a dot plot.
- **b.** Present the data in a frequency table.

12. The following data represent the number of mp3 songs found on rented laptops. An initial analysis of 25 laptops resulted in the following data being collected.
47, 103, 45, 111, 67, 54, 59, 44, 31, 89, 34, 2, 43, 96, 70, 90, 66, 48, 67, 82, 74, 56, 35, 9, 77
- **a.** Present the data as stem plot.
- **b.** Comment on the shape of distribution.
- **c.** Present the data as a frequency table grouped in intervals of 20.
- **d.** State the type of data, type of distribution and give the most appropriate measures of central tendency and spread.

13. The following table shows the salaries for AFL players in 2004.

AFL players' salaries	Number of players
0–$100 000	146
$100 001–$200 000	188
$200 001–$300 000	107
$300 001–$400 000	57
$400 001–$500 000	24
$500 001–$600 000	12
$600 001–$700 000	4
$700 001–$800 000	0
$800 001–$900 000	4
Total number of players	542

a. Draw a histogram to display the data.
b. Describe the type of distribution the histogram displays.
c. Calculate the median salary of these football players (to the nearest thousand).
d. Find the modal salary for AFL footballers in 2004.

Problem solving

14. The workers in an office are trying to obtain a wage rise. In the previous year, the ten people who work in the office each received a 2% wage rise, while the company CEO received a 42% wage rise.
a. What was the mean wage rise received in the office last year? Give your answer to 1 decimal place.
b. What was the median percentage wage rise received?
c. What was the modal percentage wage rise received?
d. The company is trying to avoid paying the rise. Explain the statistics you think they would quote about last year's rise. Give your reason.
e. What statistics do you think the trade union would quote about wage rises? Explain why.
f. Which statistic do you think is the most 'honest' reflection of last year's wage rises? Explain why.

15. A stratified random sample was taken from two high schools; a 312-student high school, and a larger high school. The sample would assess how many students watched more than four hours of television per day. If the small school represented 12.5% of the total students surveyed, and 200 total students were randomly surveyed:
a. How many students were surveyed in the small school?
b. How many students were surveyed in the large school?
c. How many students were in the large high school?
d. How many students were in both schools?

16. If the mean = 15, mode = 8 and the median = 14 (and the missing values are positive integers):
a. find two values for the following set: 8, ___, 25, ___, 21, 8
b. find two different values that meet the stated criteria.

17. Create a data set of 9 numbers that meet the following criteria.
- Data values are positive integers
- Mean = 8
- Median = 7
- Mode = 10
- Maximum value in data set = 14
- Minimum value in data set = 3

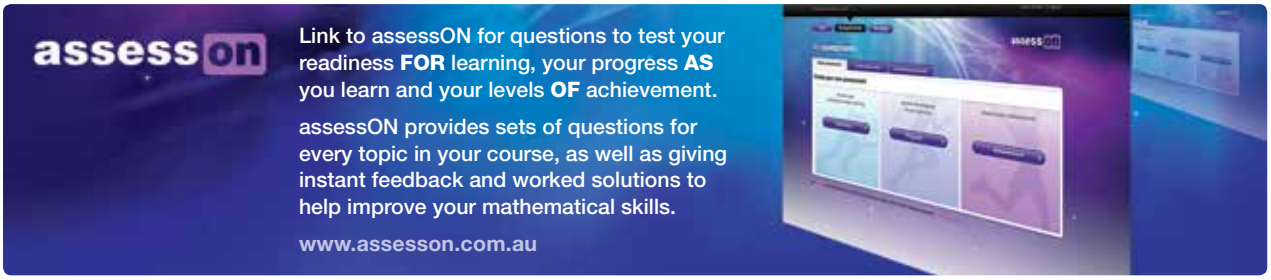
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Try out this interactivity: Word search: Topic 14 (int-2714)
Try out this interactivity: Crossword: Topic 14 (int-2715)
Try out this interactivity: Sudoku: Topic 14 (int-3213)
Complete this digital doc: Concept map: Topic 14 (doc-10804)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

categorical data
data
deviation
dot plot
frequency table
mean
median
mode
numerical data
outlier
pictogram
pie chart
skewed
statistics
stem plot
survey
symmetric
univariate

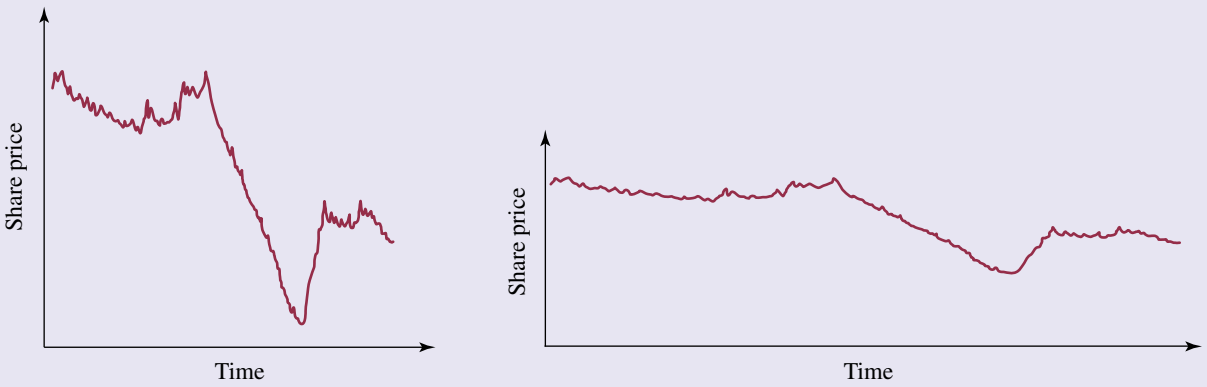


Investigation | Rich task

Graphical displays of data



Statistical data can be communicated in the form of written prose, tables and graphs. It is difficult to compare and see trends in written prose. To gain a full understanding of trends and comparisons of data presented in table form, we often need to perform additional calculations. The visual impact of graphs is immediate; however, often the impression we receive is a misleading one if we fail to look carefully at the graph. Graphs can be manipulated to produce the desired effect. Consider the two graphs below.



These graphs both display the same data (the value of shares over the same period of time), yet they look quite different. If the objective of the graph is to create the impression that the value of the shares has dropped quite dramatically, the first graph would serve this purpose. On the other hand, if we are to be convinced that the shares have not changed very much in value, the second graph reflects this point of view.

1. What technique is used to create two quite opposing impressions in the two graphs?

The following table shows the times for the men's and women's 100-m sprints in the Summer Olympic Games. A sports journalist wants to convince readers that, since 1928, the women have been more successful in reducing their time than the men.

Year	100 m men	Men's time (seconds)	100 m women	Women's time (seconds)
1928	Percy Williams, CAN	10.80	Betty Robinson, USA	12.20
1932	Eddie Tolan, USA	10.30	Stella Walsh, POL	11.90
1936	Jesse Owens, USA	10.30	Helen Stephens, USA	11.50
1948	Harrison Dillard, USA	10.30	Fanny Blankers-Koen, HOL	11.90
1952	Lindy Remigino, USA	10.40	Marjorie Jackson, AUS	11.50
1956	Bobby Morrow, USA	10.50	Betty Cuthbert, AUS	11.50
1960	Armin Hary, GER	10.20	Wilma Rudolph, USA	11.00
1964	Bob Hayes, USA	10.00	Wyomia Tyus, USA	11.40
1968	Jim Hines, USA	9.95	Wyomia Tyus, USA	11.08
1972	Valery Borzov, URS	10.14	Renate Stecher, GDR	11.07
1976	Hasely Crawford, TRI	10.06	Annegret Richter, FRG	11.08
1980	Allan Wells, GBR	10.25	Lyudmila Kondratyeva, URS	11.06
1984	Carl Lewis, USA	9.99	Evelyn Ashford, USA	10.97
1988	Carl Lewis, USA	9.92	Florence Griffith Joyner, USA	10.54
1992	Linford Christie, GBR	9.96	Gail Devers, USA	10.82
1996	Donovan Bailey, CAN	9.84	Gail Devers, USA	10.94
2000	Maurice Greene, USA	9.87	Marion Jones, USA	10.75
2004	Justin Gatlin, USA	9.85	Yuliya Nesterenko, BLR	10.93
2008	Usain Bolt, JAM	9.69	Shelley-Ann Fraser, JAM	10.78

Note: The Summer Olympic Games were not held in 1940 and 1944.

2. On a separate sheet of graph paper, draw two graphs to support the sports journalist's view.
3. Describe features of your graphs that helped you present the journalist's view effectively.
4. Imagine you are the sports journalist and you are about to publish the graphs in the local newspaper. Write a short journal article that would accompany the graphs.

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Complete this digital doc: Code puzzle: The five smallest countries in the world (in area) (doc-15911)

Answers

Topic 14 Statistics

Exercise 14.2 Sampling

1. a. Categorical **b.** Numerical **c.** Numerical **d.** Numerical **e.** Categorical

2. a. Continuous **b.** Continuous **c.** Continuous **d.** Discrete **e.** Discrete

3. a. Nominal **b.** Ordinal **c.** Ordinal **d.** Nominal **e.** Nominal

4. a. Sample **b.** Sample **c.** Population **d.** Sample **e.** Population

5. a. Biased **b.** Biased **d.** Biased

6. a. 32 males, 8 females

b. 700 females, 300 males

c. Y12s — 10, Y11s — 15, Y10s — 17, Y9s — 20, Y8s — 20, Y7s — 18

d. 5 girls, 3 boys

7. a. 1–18, randint(1,18,4) **b.** 1–x, randint(1,x,5) **c.** 1–x, randint(1,x,50) **d.** 1–365, randint(1,365,65)

8. D **9.** C **10.** D **11.** randint(1,365,7)

12. a. randint(1,31,7) **b.** randint(1,class size,6) **c.** randint(1,860,35)

13. a. Car **b–e.** Answers will vary.

14. a. Answers will vary. **b.** 182 **c.** 1400 **d.** Answers will vary.

15. Answers will vary but should include the following groups.

Categorical data can be:

- nominal — arranged in categories, for example by colour or age
- ordinal — arranged in categories that have an order; for example, karate ability ranges from white belt (beginner) to black belt (accomplished).

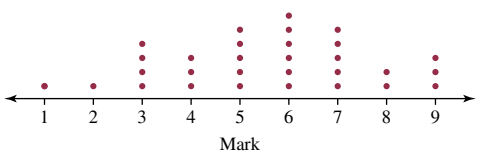
Numerical data can be:

- discrete — data can have only particular values, for example amounts of money or shoe sizes
- continuous — data can have *any* value within a range, for example the time it takes a person to run around an oval.

16–22. Answers will vary.

Exercise 14.3 Collecting data

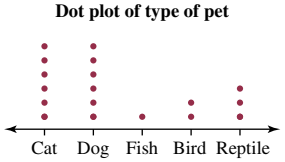
1. a.



b.

Score x	Frequency f
1	1
2	1
3	4
4	3
5	5
6	6
7	5
8	2
9	3

2. a.



b.

Type of pet x	Frequency f
Cat	6
Dog	6
Fish	1
Bird	2
Reptile	3

c. No, only numeric data can be presented in a stem plot.

3. a.

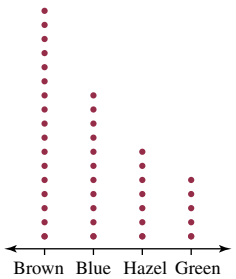
Deaths by motor vehicle accidents Rates per 100 000 people x	Frequency f
0–<10	2
10–<20	11
20–<30	10
30–<40	3
40–<50	4
50–<60	0
60–<70	1
70–<80	1
	$\Sigma f = 32$

b. Deaths by motor vehicle accidents per 100 000 people in Australian states from 2008 to 2011

Key: 7 | 3 = 73 deaths per 100 000 people

Stem	*Leaf*
0	0 9
1	1 2 3 4 4 4 5 7 8 8 9
2	1 2 2 3 4 5 5 6 6 7
3	4 6 6
4	2 2 3 5
5	
6	0
7	1

4. a. **Dot plot of eye colour of 40 people**



b.

Score x	Frequency f
Brown	17
Blue	11
Hazel	7
Green	5
$\Sigma f =$	40

c. 17 out of 40 people or 42.5% of the people have brown eyes.

5. a. Key: 1 | 8 = 18

Stem	*Leaf*
1	8 9 9
2	0 0 0 1 1 2

Not symmetrical. Most scores in low 20s.

b. Key: 1 | 9 = 19

Stem	*Leaf*
1	9 9
2	0 4 4 5 6 7 9
3	1 1 6 8
4	2 3

Not symmetrical.

c. Key: 33 | 6 = 336

Stem	*Leaf*
33	6
34	6 6 9
35	0 3 9
36	2 8
37	1

Almost symmetrical.

d. Key: 4 | 3 = 43

Stem	*Leaf*
4	3 4 6 7 8 9
5	2 5
6	7
7	0 7
8	1 2 3 3 4 5 8

Almost symmetrical.

e. Key: 1 | 1 = 1.1

Stem	*Leaf*
1	1 3 5 9
2	4 7 8 9
3	3 6 9
4	1 2 6

Not symmetrical.

6. a. Key: 1* | 8 = 18 2 | 0 = 20

Stem	*Leaf*
1*	8 9 9
2	0 0 0 1 1 2

b. Key: 1* | 9 = 19 2 | 0 = 20

Stem	*Leaf*
1*	9 9
2	0 4 4
2*	5 6 7 9
3	1 1
3*	6 8
4	2 3

c. Key: 33* | 6 = 336 35 | 0 = 350

Stem	*Leaf*
33*	6
34	
34*	6 6 9
35	0 3
35*	9
36	2
36*	8
37	1

d. Key: 4 | 3 = 43 4* | 6 = 46

Stem	*Leaf*
4	3 4
4*	6 7 8 9
5	2
5*	5
6	
6*	7
7	0
7*	7
8	1 2 3 3 4
8*	5 8

e. Key: 1 | 1 = 1.1 1* | 5 = 1.5

Stem	*Leaf*
1	1 3
1*	5 9
2	4
2*	7 8 9
3	3
3*	6 9
4	1 2
4*	6

7. a. Running times for 40 students
Key: 4 | 3 = 43 minutes, 4* | 8 = 48 minutes

Stem	*Leaf*
2	1 3
2*	5 6 6 8 9
3	1 2 2 3 3 3 4
3*	7 7 9 9
4	0 1 1 2 2 3 4
4*	5 5 7 8 8 9
5	2 2 3
5*	5 6 7
6	0 3
6*	
7	1

b.

Run times x **(minutes)**	**Number of runners** f
20–<25	2
25–<30	5
30–<35	7
35–<40	4
40–<45	7
45–<50	6
50–<55	3
55–<60	3
60–<65	2
65–<70	0
70–<75	1
$\Sigma f =$	40

c. i. 32 min

ii. 30 min to less than 35 min

d. 3, stem plot

8. Key: 1 | 4 = 14
1* | 7 = 17

Stem	*Leaf*
1	4
1*	7 8 8 9
2	0 1 3
2*	5 6 7 9
3	2 4
3*	5 6 7 8
4	2
4*	5
5	3

9. D

10. a.

Life expectancy x (years)	**Number of age groups f**
83–<84	1
84–<85	20
85–<86	7
86–<87	5
87–<88	3
88–<89	3
89–<90	1
$\Sigma f =$	40

b. Life expectancy of Australians across 40 different age groups.
Key: 86 | 1 = 86.1 years

Stem	*Leaf*
83	7
84	1 2 3 4 4 5 5 5 5 6 6 6 6 7 7 7 8 8 9 9
85	0 0 1 2 7 8 9
86	0 5 6 8 9
87	1 3 6
88	3 5 8
89	0

c. Answers will vary. Half the people will live to 84, with others living longer.

11. Median age of Australian females and males from 2002 to 2009

Key: 36 | 1 = 36.1 years

Stem	*Leaf*
35	2 3
35*	5 7 9
36	1 1 1
36*	6 9
37	1 3 4
37*	6 6 7

The alternative method — showing categories for 35, 36 and 37 years — is not appropriate as it only allows for 3 groups, which is below the 5 minimum groups allowed.

Score x	**Frequency f**
35–<36	5
36–<37	5
37–<38	6
$\Sigma f =$	16

12. a. 

The pattern is: two dots of the same colour produce a red dot in the row below; two dots of different colour produce a black dot in the row below. The last dot in the row is always black.

b. No, there must be a black dot at the end of each row.

c. No, as the pattern starts to repeat in Row 9 and no rows have all black dots.

d. Yes

13. a. Class 1: 25; class 2: 27

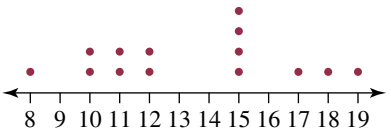
b. Class 1: 2; class 2: 3

c. Class 1: 24; class 2: 20

d. Eleven students in class 2 scored better than class 1 but 6 students scored worse than class 1. Class 2's results are more widely spread than class 1's results.

14. a. Dots are not aligned vertically. The horizontal scale is not equally spaced.

b.



15. a. Key: 15 | 4 = 154 cm

Boys *Leaf*	*Stem*	*Girls* *Leaf*
	14	4 8
9 8 6 6 4	15	2 3 5 6
8 4 2 0	16	0 1 1 3 5 8
2 1	17	
0	18	

b. The data could be grouped into smaller intervals, such as 5, instead of 10. For example, the stem value of 2 could be broken into two groups, encompassing values 20–24 and 25–29.

16. a. Key: 25 | 1 = 25.1 g

Stem	*Leaf*
23	9 9
24	0 5 5 6 8
25	1 1 1 3 4 4 8
26	3 3 7
27	3 3
28	1
29	5
30	
31	
32	
33	2

b. The range is $33.2 - 23.9 = 9.3$.

c. Answers will vary but could include that the plot for rounded values provides less detail.

Key: 2 | 4 = 24 g
2* | 6 = 26 g

Stem	*Leaf*
2	4 4
2*	5 5 5 5 5 5 5 5 5 5 6 6 6 7 7 7 8
3	0 3

17. a. Key: 3 | 2 = 32 km/h

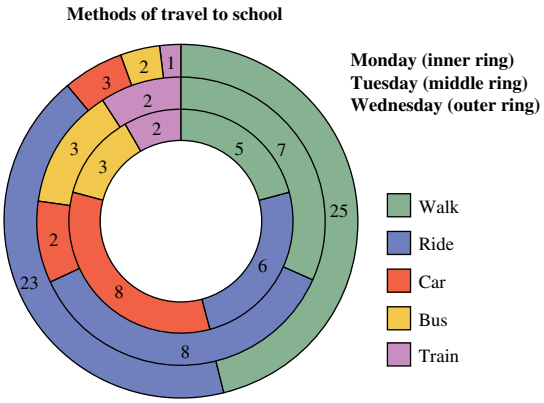
Stem	*Leaf*
1	5
2	0 0 2 7 9 9
3	0 0 0 0 2 4 5 6 7 8 8 9
4	0 0 0 0 0 2 4 5 5 5
5	
6	0

b. Answers will vary. A possible answer could include that there were a high number of speeds ending in 0 (possibly indicating a machine malfunction), and most of the speeds (24 out of 30) were within the legal speed limit.

18. a. Answers will vary.

b. Draw circles of different size with the same centre. Then use the same instructions as for creating a pie chart but mark only the lines *between* the two circles.

c.



d. They are the same graph.

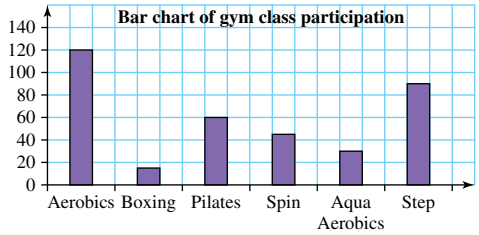
e. Answers will vary.

Challenge 14.1

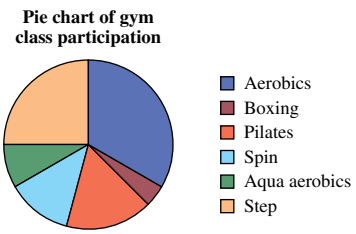
1296

Exercise 14.4 Displaying data

1. a.



b.



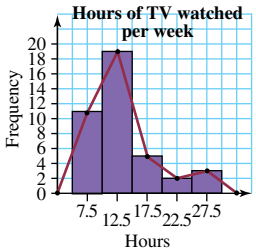
2. a. Continuous

b. 24

c.

Class interval	Midpoint (class centre)	Tally	Frequency
5–<10	7.5	𝍸 𝍸 I	11
10–<15	12.5	𝍸 𝍸 𝍸 IIII	19
15–<20	17.5	𝍸	5
20–<25	22.5	II	2
25–<30	27.5	III	3
		Total	40

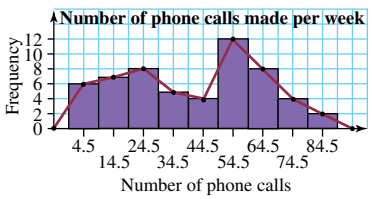
d.



3. a.

Class interval	Midpoint (class centre)	Tally	Frequency
0–9	4.5	𝍸 I	6
10–19	14.5	𝍸 II	7
20–29	24.5	𝍸 III	8
30–39	34.5	𝍸	5
40–49	44.5	IIII	4
50–59	54.5	𝍸 𝍸 II	12
60–69	64.5	𝍸 III	8
70–79	74.5	IIII	4
80–89	84.5	II	2
		Total	56

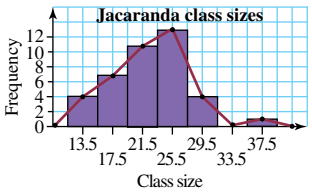
b.



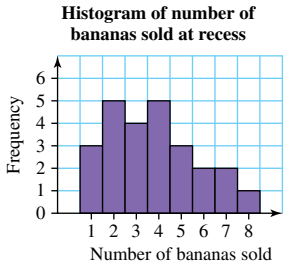
4. a.

Class interval	Midpoint (class centre)	Tally	Frequency
12–15	13.5	IIII	4
16–19	17.5	IIII II	7
20–23	21.5	IIII IIII I	11
24–27	25.5	IIII IIII III	13
28–31	29.5	IIII	4
32–35	33.5	—	0
36–39	37.5	I	1
		Total	40

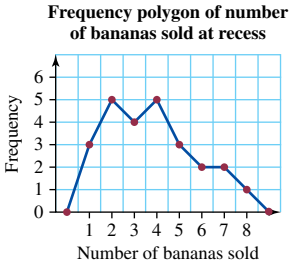
b.



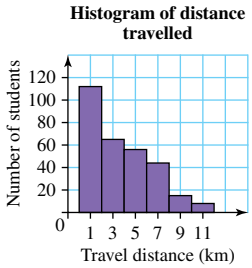
5. a.



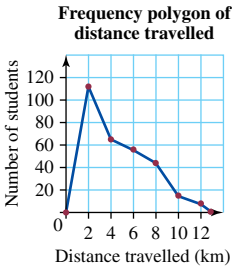
b.



6. a.

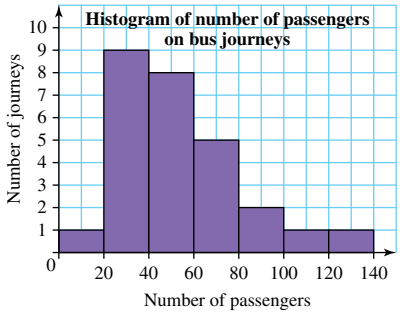


b.

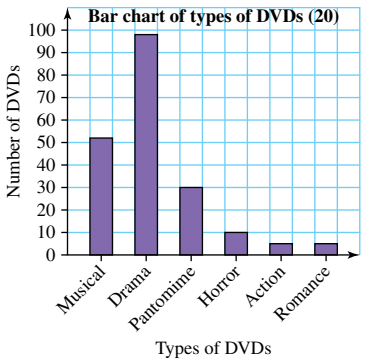


7. D

8.



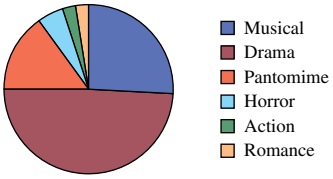
9. a.



b.

Type of DVD	Number of DVDs	Size of sector for a pie chart
Musical	52	93.6°
Drama	98	176.4°
Pantomime	30	54°
Horror	10	18°
Action	5	9°
Romance	5	9°
Total	200	360°

c. Pie chart of types of DVDs (200)

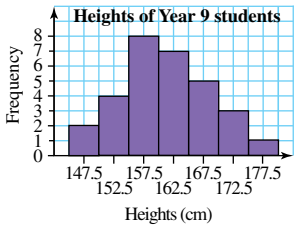


10. a. Continuous

b.

Class interval	Midpoint (class centre)	Tally	Frequency
145–<150	147.5	II	2
150–<155	152.5	IIII	4
155–<160	157.5	𝍸 III	8
160–<165	162.5	𝍸 II	7
165–<170	167.5	𝍸	5
170–<175	172.5	III	3
175–<180	177.5	I	1
		Total	30

c.



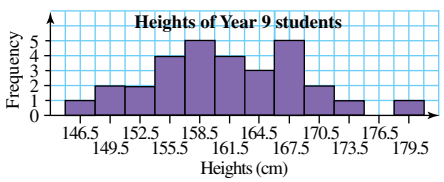
d. No. Individual scores are lost when data are grouped into class intervals.

e. B **f.** D

g.

Class interval	Midpoint (class centre)	Tally	Frequency
145–<148	146.5	I	1
148–<151	149.5	II	2
151–<154	152.5	II	2
154–<157	155.5	IIII	4
157–<160	158.5	𝍸	5
160–<163	161.5	IIII	4
163–<166	164.5	III	3
166–<169	167.5	𝍸	5
169–<172	170.5	II	2
172–<175	173.5	I	1
175–<178	176.5	—	0
178–<181	179.5	I	1
		Total	30

h.



Discuss in class.

11. Yes — change the class intervals.

12. a. Number of players **b.** 5

13. Key: 0 | 0 = 0

Leaf (*Mitch*)	*Stem*	*Leaf* (*Yani*)
3 2 1 1 1 1 0 0 0 0	0	0 0 1 1 1 1 1 2 2 2

Mitch scored between 0 and 3 goals inclusive.
Yani scored between 0 and 2 goals inclusive.

14. a. Mitch: 2, Yani: 3

b. Mitch

c. Yani: 11

d. Yani, as he scored goals in more games.

15. a. Key: 3 | 2 = 32

Leaf (*9A*)	*Stem*	*Leaf* (*9B*)
2	3	
9	4	6
7 4	5	5
8 7 5 5 0	6	0 2 8
3	7	3 3 4 5 7 8
5	8	1
6	9	

b. 9A: 83%, 9B: 92% **c.** 9A: 2 **d, e.** Discuss in class. **f.** 9A: 64.25%; 9B: 68.5%

16. a. C **b.** D

c. In general the boys are slightly taller than the girls, with the tallest 3 students all being boys.

17. a. Categorical **b.** A column graph or a pie chart

18. The average amount of times per week people used the sunscreen was 5. The minimum amount of times was 2 and the maximum amount was 21, although this value was much higher than the rest of the data. Fifty per cent of the people surveyed used the sunscreen between 4 and 7 times per week.

Exercise 14.5 Measures of central tendency

1. a. 55, 54, 55 **b.** 8, 6.88, 8

2. 2.5, 2.55, 2

3. a. 23 **b.** 116 **c.** 6.28

4. a. 5 **b.** 82 **c.** 2.55

5. a. 143.5, 142.6, 140–149 **b.** 150.9, 148.5, 150–159

6. a. 25 **b.** 25.24 **c.** 25

7. C **8.** B **9.** D

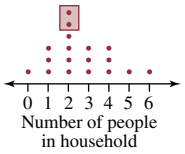
10. a. Numerical continuous **b.** 153 **c.** 157 **d.** 140–<160

11. a. Numerical discrete **b.** 45.2 **c.** 45.25 **d.** 40–49

12. a.

Score (x)	**Frequency** (f)	**Freq. × score** ($f \times x$)
0	1	0
1	3	3
2	6	12
3	3	9
4	3	12
5	1	5
6	1	6
	$n = 18$	$\Sigma fx = 47$

$\bar{x} = 2.6$

b. 

2 **c.** 2

d. i. Median **ii.** Mean **iii.** Mode

13. a. B **b.** $29 444 **c.** A **d.** Answers will vary.

14. a. The median was calculated by taking the average of the 2 middle scores.

b. 13

15. Answers will vary.

16. a. 55 250 km, 52 000 km, 52 000 km **b.** Discuss in class.

c. 51 810 km; it is reduced by 3440 km. **d.** 52 000 km

e. 3333 **f.** 50 000 km; 92% last that distance or more.

17. a, b Answers will vary; options include 7, 11 and 8, 10.

c. No; 4 is the mode.

18. a. Answers will vary. **b.** No

19. 8.93

20. Example:

a. 1, 2, 5, 6, 6

$$\frac{1+2+5+6+6}{5} = \frac{20}{5} = 4$$

b. 1, 2, 5, 6, 11

$$\frac{1+2+5+6+11}{5} = \frac{25}{5} = 5$$

21. a. 6 **b.** >6 **c.** 6 **d.** >13

22. 13, 31, 31, 47, 53 and 59

Challenge 14.2

3, 5 and 7

Exercise 14.6 Measures of spread

1. a. 31, 18.5 **b.** 26, 15

2. a. 32 km/h **b.** 10 km/h **c.** 7.5 km/h

3. a. i. 107, 56 **ii.** 35.35

b. i. 0.81, 0.52 **ii.** 0.28

4. a. 5, 2, 1.24 **b.** 0, 1, 1.19

5. a. Symmetric **b.** Symmetric **c.** Symmetric

d. Negatively skewed **e.** Negatively skewed **f.** Positively skewed

6. a. Symmetric **b.** Positively skewed **c.** Bimodal

d. Negatively skewed with an outlier

7. The data is positively skewed, with a median of 2, and an interquartile range of 1. The mean is not an appropriate measure of centre as it is affected by the outlier.

8. a. Numerical discrete data with a positively skewed distribution

b. Median and interquartile range are best measure of central tendency and spread for a skewed distribution as they are not affected by outliers.

c. Median = 11.5 customers, IQR = 8.5 customers

9. D **10.** A

11. a. No **b.** 31–40 **c.** No

12. Answers will vary.

13. Answers will vary. Possible data sets will have the highest score = Q_3 and the lowest score = Q_1.

14. a. The same **b.** Double **c.** Increase by 25%

15. For example {1, 2, 2, 3, 3, 3, 4, 4, 5}

16. Easter Sunday: mean = 87.7, IQR = 21
Christmas Day: mean = 79.9, IQR = 17.5

14.7 Review

1. C **2.** D **3.** C **4.** D

5. B **6.** D **7.** A

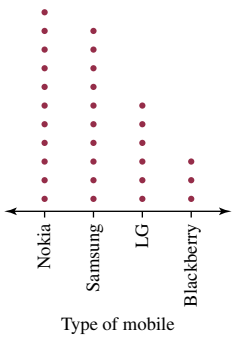
8. a. Categorical nominal **b.** Categorical ordinal **c.** Numerical discrete **d.** Numerical continuous

9. a. Sample **b.** Sample **c.** Population

10. a. 20 male adults, 10 female adults and 30 teenager members.

b. 8 Year 12s, 12 Year 11s and 20 from Years 7, 8, 9 and 10.

11. a.



b. Frequency table of mobile brands

Brand of mobile	Number of mobiles
Nokia	11
Samsung	10
LG	6
Blackberry	3

12. a. Key: 1 | 2 = 12 songs

Stem	*Leaf*
0	2 9
1	
2	
3	1 4 5
4	3 4 5 7 8
5	4 6 9
6	6 7 7
7	0 4 7
8	2 9
9	0 6
10	3
11	1

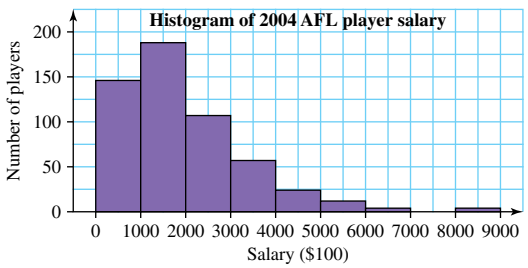
b. Positively skewed with outliers

c.

Number of songs	Number of laptops
0–19	2
20–39	3
40–59	8
60–79	6
80–99	4
100–119	2

d. Numerical discrete, symmetric distribution, mean = 60 (rounded to the nearest whole number) and standard deviation = 27.3

13. a.



b. Positively skewed

c. The median is $166 755.

d. The modal class is $100 001 to $200 000.

14. a. 5.6% **b.** 2% **c.** 2%

d. In order to give the impression that the pay rise is generous, the company would quote the mean.

e. The trade union would quote the median or the mode, because they would be arguing that the wage rise is too low.

f. The mode is probably the most 'honest' reflection of last year's wage rise, as it represents the situation of most of the workers.

15. a. 25 **b.** 175 **c.** 2184 **d.** 2496

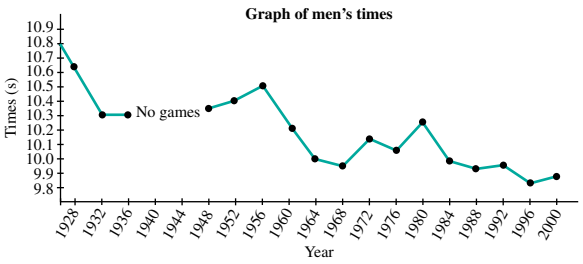
16. a. 8, 20 **b.** 9, 19

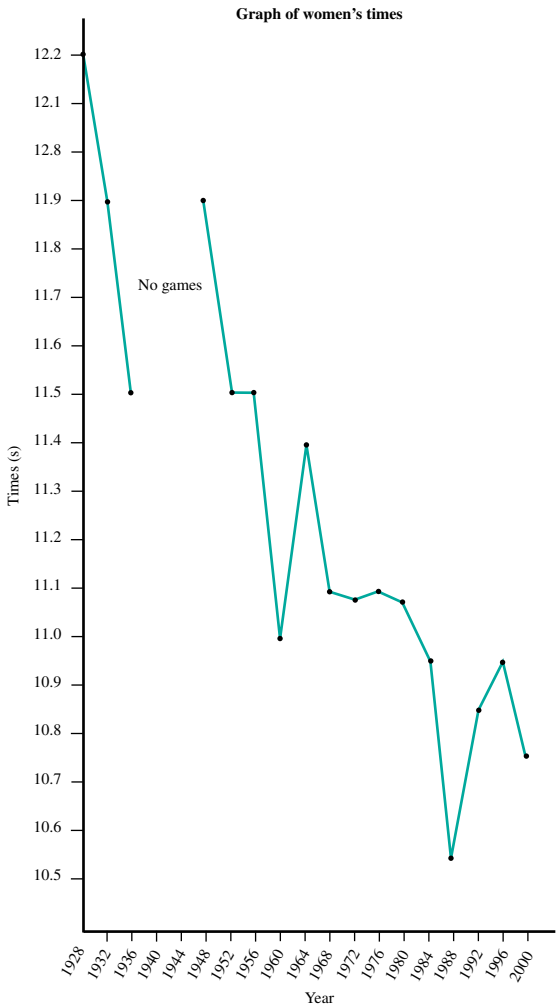
17. 3, 4, 5, 6, 7, 10, 10, 13, 14

Investigation — Rich task

1. Stretching the axes vertically gives the impression of a fall in the value of the shares. Stretching the axes horizontally gives the impression the value of the shares has not changed very much.

2.





3. For the women's graph, stretching the vertical axis and reducing the distance between the values on the horizontal axis gives the impression women's times have fallen dramatically. To show that the men's times have not fallen greatly, you can reduce the distance between the values on the vertical axis and stretch the values on the horizontal axis.

4. Teacher to check.

TOPIC 15
Numeracy 3

15.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

15.1.1 Why learn this?

Our lives are interwoven with mathematics. Counting, measuring and pattern-making are all part of everyday life. We use numbers to mark significant events (such as birthdays) and for identification (such as passports and credit cards). We use numbers to describe ourselves (for example our height and weight). Shopping involves understanding numbers, and tallying scores in sports requires a comparison of numbers. You may not realise just how much you rely on numbers.



15.1.2 What do you know?

assessON

1. **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner, then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

LEARNING SEQUENCE

15.1 Overview
15.2 Set E
15.3 Set F

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: NAPLAN: Strategies and tips (eles-1688)

15.2 Set E

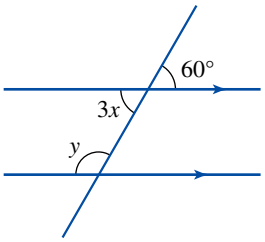
15.2.1 Calculator allowed

1. The volume of a cylindrical glass with diameter 7.2 cm and height 15.4 cm is:

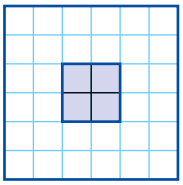
A. 2 L **B.** 627 cm^3

C. 550 cm^3 **D.** 750 mL

2. Use the diagram below to determine the values of x and y.



3. A square window is to be inserted into a square wall. The side length of the window is 70 cm. What is the area of the remaining wall?



A. 39 200 m^2

B. 3.92 m^2

C. 4.41 m^2

D. 6.8 m^2

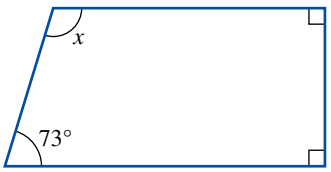
4. The formula $y = \frac{3x - 4}{5}$ rearranged to make x the subject is:

A. $x = \frac{5y - 3}{4}$ **B.** $x = \frac{4y - 5}{3}$

C. $x = \frac{3y + 4}{5}$ **D.** $x = \frac{5y + 4}{3}$

5. How many hours are there in 4 weeks and 3 days?

6. The value of x in the diagram at right is equal to:



A. 107°

B. 110°

C. 77°

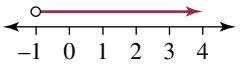
D. 90°

7. An unknown number is multiplied by 6 and then added to 4. The result is 46. Which equation best describes this?

A. $6 + 4x = 46$ **B.** $46 + 6x = 4$

C. $6x + 4 = 46$ **D.** $6(x + 4) = 46$

8. The number line shown can be described by the equation:



A. $x < -1$ **B.** $x \geq -1$

C. $x > -1$ **D.** $x < 1$

9. Arrange the following in descending order.

2.202, 0.12, $\left(\frac{1}{2}\right)^2$, $\sqrt{\frac{1}{4}}$, 0.01

10. A car uses 10 L of fuel to travel 80 km. How much fuel is needed to travel 400 km?

A. 30 L **B.** 40 L

C. 50 L **D.** 60 L

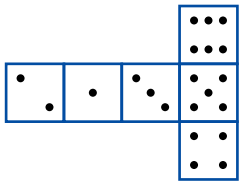
11. A length of rope is 7.5 m. It is to be cut into 2 pieces in the ratio of 2 : 3. What will be the two new lengths?

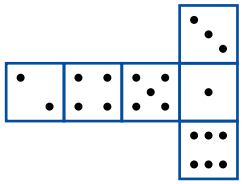


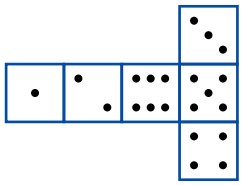
A. 2.5 m and 5 m
B. 3 m and 4.5 m
C. 3.5 m and 4 m
D. 1 m and 6.5 m

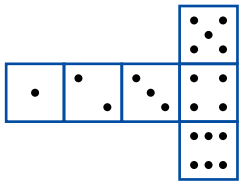
12. A 5-kg bag of potatoes cost $7.99. The cost of loose potatoes is $1.55 per kg. Is it cheaper to buy:
A. 5 kg of loose potatoes?
B. a 5-kg bag of potatoes?

13. Which diagram of the net of a cube has opposite faces that sum to 7?

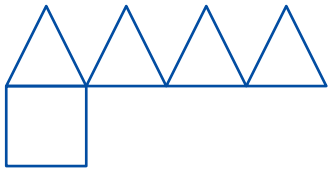
A.


B.


C.


D.


14. Which of the following shapes is represented by the net shown at right?

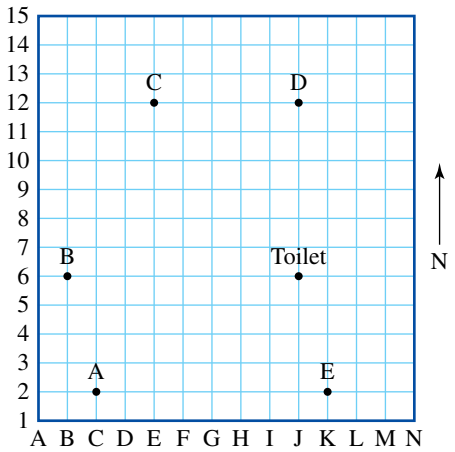


A. A triangular prism
B. A triangular pyramid
C. A square pyramid
D. A cube

15. If $A = \{\text{odd numbers between 0 and 10}\}$ and $B = \{3, 6, 9\}$ then $A \cup B$ is equal to:

A. $\{1, 3, 5, 7, 9\}$ **B.** $\{3, 9\}$ **C.** $\{2, 4, 8, 10\}$ **D.** $\{1, 3, 5, 6, 7, 9\}$

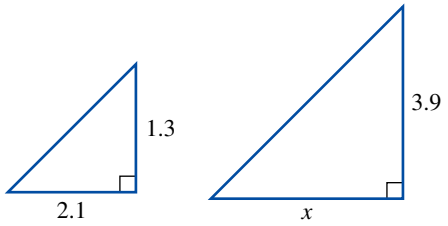
16. The following map shows campsites in a campground.



Jon is camped at grid reference C2. If he is facing north and looks due east, what is he looking at?

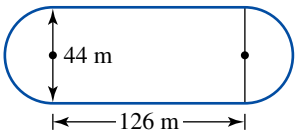
A. Toilets **B.** Campsite C **C.** Campsite A **D.** Campsite E

17. Using the similar triangles shown below, find the value of x.



A. 1.3 **B.** 2.6 **C.** 6.3 **D.** 9.3

18. The perimeter of the race track shown to the right is:



A. 390.23 m
B. 395.2 m
C. 405 m
D. 309.35 m

19. There are 1500 people in a fun run; 1125 are over 18 years of age. What percentage of people are aged 18 years and under?

A. 25% **B.** 55% **C.** 45% **D.** 35%

20. The vertices of a triangle are A (1, 1), B (3, 4) and C (4, 2). Give the vertices of the triangle if it is reflected in the y-axis.

21. The data in the table below shows the number of books read by 6 students in Year 9 during one month.

Number of books read	3, 6, 8, 4, 4, 2

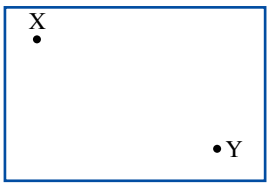
Which of the following is true?

A. Mean = median
B. Median = mode
C. Mean = mode
D. Mean < mode

22. A bike worth \$420 has been reduced by \$$x$. The new value of the bike (in dollars) is:

A. $420 - x$
B. $420x$
C. $\frac{420}{x}$
D. $420 + x$

23. X and Y are 12.5 cm apart on a map.
Calculate the actual distance between X and Y using the given key.



Key: 1 cm ⇔ 6 km

A. 12.5 km
B. 7.5 km
C. 75 km
D. 42 km

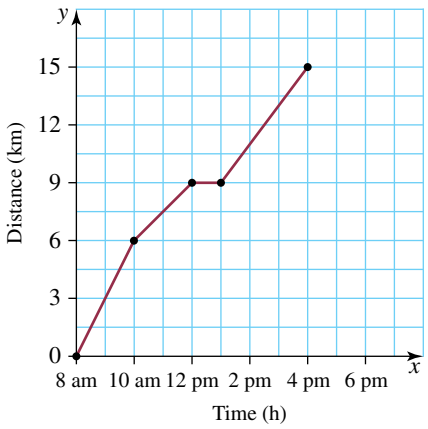
24. Pat bought a packet of 100 marbles for \$5.00. The cost of one marble is:

A. \$0.50 **B.** \$0.20 **C.** \$0.02 **D.** \$0.05

25. A bag contains 6 red, 8 green and 4 blue counters. What is the ratio of red to blue counters?

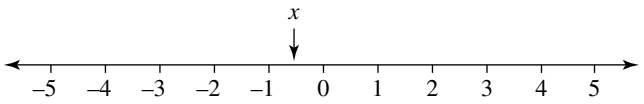
A. 3 : 2
B. 2 : 3
C. 3 : 4
D. 2 : 1

26. The graph at right shows the distance travelled by a hiker. The average speed of the hiker is closest to:



A. 15 km/h
B. 1 km/h
C. 2 km/h
D. 8 km/h

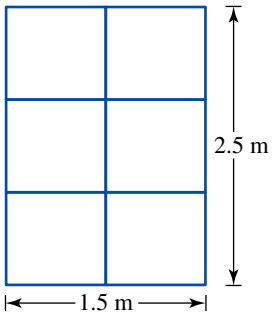
27. What is the value of x on the number line shown?



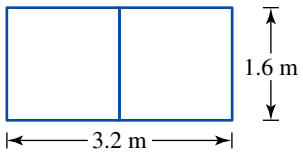
A. -1.5
B. -0.5
C. 0.5
D. 1.5

28. Which window has the largest area?

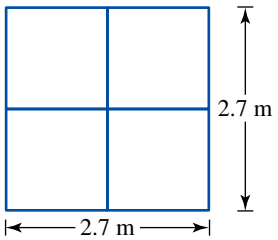
A.



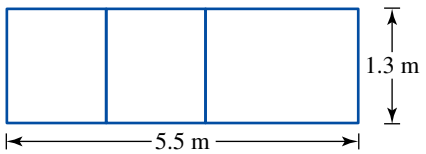
B.



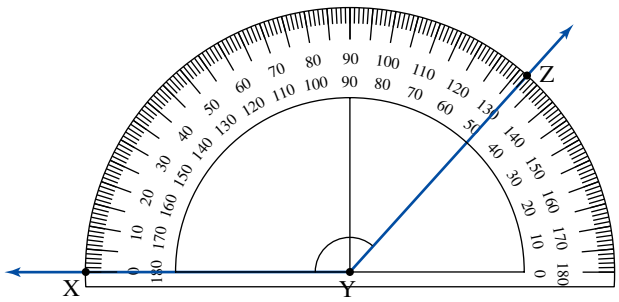
C.



D.



29. What is the size of $\angle XYZ$?



30. $24 \times ? = 16$
The missing value is:

A. $\frac{1}{3}$ **B.** $\frac{2}{3}$ **C.** $\frac{1}{2}$ **D.** $\frac{3}{4}$

15.3 Set F

15.3.1 Non-calculator

1. The sum of 11 and a number, r, is 37. What is r?

A. 27 **B.** 14 **C.** 26 **D.** 8

2. $\sqrt{150}$ is between:

A. 8 and 10 **B.** 10 and 12 **C.** 12 and 14 **D.** 14 and 16

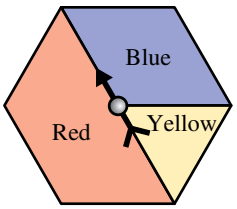
3. The arrow on the spinner is spun 50 times. Which table most likely represents the results?

A.

Colour	Number of spins
Red	25
Blue	17
Yellow	8

B.

Colour	Number of spins
Red	16
Blue	25
Yellow	9



C.

Colour	Number of spins
Red	8
Blue	15
Yellow	27

D.

Colour	Number of spins
Red	25
Blue	12
Yellow	13

4. You are goiwng to share $3\frac{1}{4}$ pizzas with your friends. How many $\frac{1}{6}$ sized slices can you cut from the pizzas?

A. 20 **B.** $19\frac{1}{2}$ **C.** 18 **D.** $16\frac{1}{2}$

5. $2 \times 2 \times 2 \times 2 \times 2$ is equal to:

A. $2^2 \times 2^2$ **B.** 8×2 **C.** 2×5 **D.** $4^2 \times 2$

6. There are 12 boys and 14 girls in a class. What fraction of the class is boys?

A. $\frac{6}{13}$ **B.** $\frac{6}{7}$ **C.** $\frac{1}{2}$ **D.** $\frac{3}{4}$

7. Of 100 000 newspapers printed, 5% are delivered to homes, with the remaining papers going to the newsagent. How many papers are delivered to the newsagent?

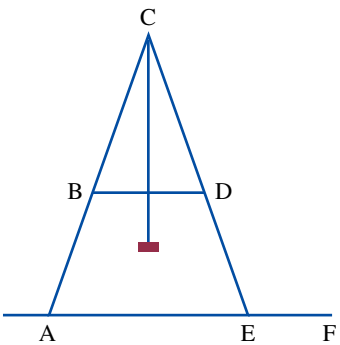
A. 5000
B. 50 000
C. 95 000
D. 75 000



8. A cricketer played 10 innings and scored the following number of runs.
45, 0, 33, 7, 11, 25, 57, 80, 33, 47
What will change if the score of 80 is removed from the set?

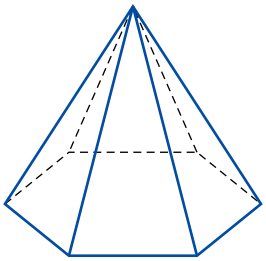
A. Mean and range
B. Mean and mode
C. Mean and median
D. Mean only

9. The side view of an A-frame swing looks like this.
$\angle BCD = 68°$.
Find $\angle BDC$ and $\angle DEF$.

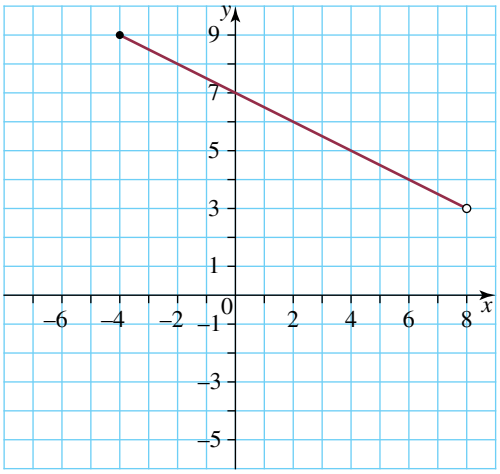


10. How many faces does this pyramid have?

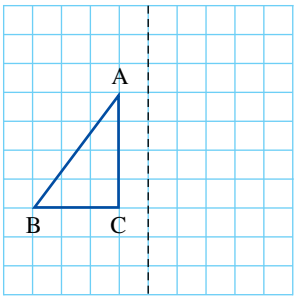
A. 4
B. 5
C. 6
D. 7



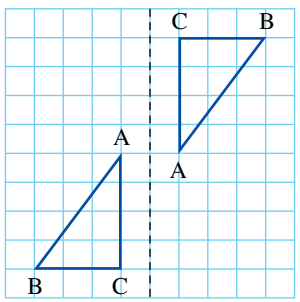
11. For the following graph, state the domain (possible x-values) and range (possible y-values).



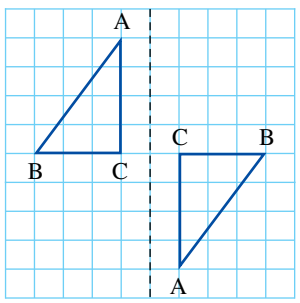
12. Which of the following represents the reflection of this triangle in the mirror shown below?



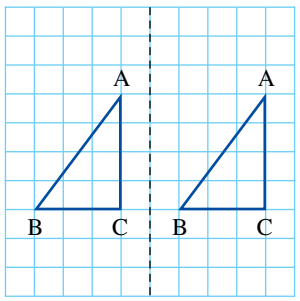
A.



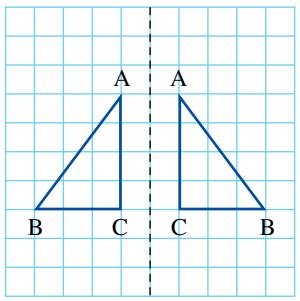
B.



C.



D.



13. Which of the following is the rearranged version of the formula $m = \frac{n - qr}{p}$?

A. $p = mn - qr$ **B.** $n = mp + qr$ **C.** $n = \frac{m}{p} + \frac{q}{r}$ **D.** $n = \frac{m + qr}{p}$

14. If a number is halved and then increased by 4, it will be equal to 2 less than the original number. What is the original number?

A. 8 **B.** 10 **C.** 12 **D.** 20

15. When $a = 1$ and $b = -1$, the value of $(a - b)^2$ is:

A. 0 **B.** 1 **C.** 2 **D.** 4

16. Which linear graph joins the points $(5, -9)$ and $(-3, 7)$?

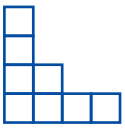
A. $y = x + 4$ **B.** $y = 2x + 1$ **C.** $y = -x - 4$ **D.** $y = -2x + 1$

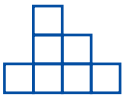
17. A T-shirt worth \$35 is discounted by 15%. Which of the following calculations gives the new price?

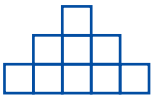
A. 35×0.85 **B.** 35×0.15 **C.** $35 + 0.15$ **D.** $35 - 15$

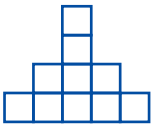
18. A fair coin is tossed twice. Write down the four possible outcomes.

19. Which of these patterns does not possess a line of symmetry?

A. 

B. 

C. 

D. 

20. Arrange the following in ascending order.

$\frac{1}{2}, 0.2^2, \sqrt{4}, \left(\frac{1}{2}\right)^2$

21. The solution to $\frac{4 - x}{3} - 5 > 3$ is:

A. $x > 20$ **B.** $x > -20$ **C.** $x < -20$ **D.** $x < 20$

22. Fill in the missing value to make the following equation true.

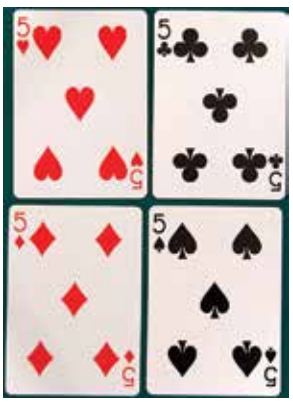
$2(2h - 1) = 3h - 2 + ?$

23. Evaluate $\sqrt{(-2)^2 + (-4)^2 + 5}$.

A. 5 **B.** 25

C. 10 **D.** 15

24. What is the probability of choosing a 5 in a regular deck of cards (no jokers)?



A. $\frac{5}{52}$

B. $\frac{1}{4}$

C. $\frac{1}{13}$

D. $\frac{1}{2}$

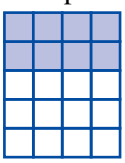
25. What percentage of this shape is shaded?

A. 8%

B. 20%

C. 50%

D. 40%



26. The probability of hiring a certain coloured bicycle is shown in the table below.

Colour	White	Red	Silver	Blue
Probability	0.55	0.25	?	0.05

What is the probability of hiring a silver bicycle if there are no colours other than the ones shown?

A. 0.15 **B.** 0.25 **C.** 0.2 **D.** 0.5

27. Two people buy a $5 ticket in a $200 000 lottery. They each contribute $2 and $3. How much would each collect if they won?

28. Andrew rides x km to school. Isabelle rides 3 km more. Together they ride 13 km. Which equation represents this situation?

A. $x - 3 = 13$ **B.** $2x + 3 = 13$ **C.** $x + 3 = 13$ **D.** $2x - 3 = 13$

29. The maximum temperature recorded for a June day was 14 °C. The minimum temperature recorded that day was −2 °C. What is the difference between the maximum and minimum temperatures recorded that day?

30. The mean, median and mode for the following set of data is:

0, 1, 1, 2, 3, 3, 3, 3

A. Mean = 2, median = 3, mode = 1

B. Mean = 2, median = 3, mode = 3

C. Mean = 2, median = 2.5, mode = 3

D. Mean = 3, median = 2.5, mode = 3

Answers

Topic 15 Numeracy 3

15.2 Set E Calculator allowed

1. B

2. $x = 20°$, $y = 120°$

3. B

4. D

5. 744 hours

6. A **7.** C **8.** C

9. 2.202, $\sqrt{\frac{1}{4}}$, $\left(\frac{1}{2}\right)^2$, 0.12, 0.01

10. C **11.** B **12.** A **13.** C

14. C **15.** D **16.** D **17.** C

18. A **19.** A

20. A′(−1, 1), C′(−4, 2), B′(−3, 4)

21. B **22.** A **23.** C **24.** D

25. A **26.** C **27.** B **28.** C

29. 132° **30.** B

15.3 Set F Non-calculator

1. C **2.** C **3.** A **4.** B

5. D **6.** A **7.** C **8.** A

9. ∠BDC = 56°
∠DEF = 124°

10. D

11. Domain: $-4 \leq x < 8$
Range: $3 < y \leq 9$

12. D **13.** B **14.** C **15.** D

16. D **17.** A

18. HH, HT, TH, TT

19. B

20. 0.2^2, $\left(\frac{1}{2}\right)^2$, $\frac{1}{2}$, $\sqrt{4}$

21. C **22.** h **23.** A **24.** C

25. D **26.** A

27. $80 000 and $120 000

28. B **29.** 16 °C **30.** C

learnon

Project: Backyard flood

Scenario



Mrs Efficiency is at home busy preparing for Christmas and needs to water the fruit trees in the back yard. When she goes to connect the hose to the tap she discovers that Mr Efficiency has taken the one she usually uses. Her only option is to use one twice the diameter. She assumes this will deliver water at twice the rate of the smaller hose and so leaves it for half her usual time of 20 minutes. She returns to a flood in the backyard.

At the evening meal she discusses the event with the family. She is really distressed because she realises that her action has led to a waste of water (a very scarce resource). Her Year 10 daughter decides to research the situation and present a summary to her mother to explain what has happened.

Task

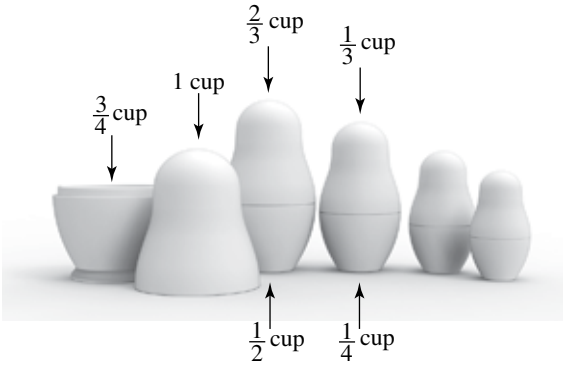
- For this project, you will look at interactive websites to summarise how doubling and halving dimensions affects length, area and volume.
- You will use Google Sketchup to design Babushka solids to be used as measures for cooking.
- You will also investigate if the weight gain of a baby fits this same model.
- You will then write up your procedure and conclusions in Word and present them as a book in Calameo. You should assume the role of Mrs Efficiency's daughter who is preparing a report for her mother. Remember to provide some advice to Mrs Efficiency regarding conservation of water.

Process

Watch the introductory video lesson and find the series of topics to help you complete your task.

- Research. Keep a journal of your discoveries as you proceed with the task. The journal entries should be short phrases to explain what you are doing/thinking, similar to using Twitter or sending a text message to a friend — do not use abbreviations for your words. Start each message with the title of the topic you are reporting on. Each person in your group should report on at least 3 different sources of information. You can view and comment on the other group members' entries and rate the comments they have made. When your research is complete, print your Research Report to hand to your teacher.
- Open the **Enlargements Teaching Tool**. Complete the table. Answer the questions provided in the Enlargements Teaching Tool file in the *Media Centre*. Summarise your discoveries in 200 words or fewer. Include screenprints of each enlargement.
- Wordle is a site that creates an image of the words in an article according to the frequency of their usage. Open the Wordle site. Select the **Create** tab. Copy the summary of your article into the text box. Keep selecting the **Randomise** button until you are happy with the result. Print your final choice. Take a screenprint of your final choice. Take it into **Paint** for use as an image in your presentation. Save your Paint file as a jpeg file.
- Open the Area Calculator site. Make sure the calculations are done in metres. For each of the shapes, choose a number to enter into the box to calculate the area. Recalculate the area for half this dimension and double it. How can this be summarised? Repeat this process for the Volume Calculator site. Summarise your findings. Include this summary in your report.

- Download **Sketchup**. Watch the introductory video to get started. Use the measuring tool to help you create a set of nesting solids to be used instead of babushka (matryoshka) measuring cups — look at the photo to see the proportions of the cups. Take a screenprint of your masterpiece to include in your report.
  Remember: 1 cup is equal to 250 mL.



- Use the Australian Icon weblink in the Resources tab and choose 3 icons to compare with the real object. Calculate the scale factor (range of numbers). Give measurements and calculations to support your conclusion. Calculate the enlargement factor of the icon from the real object. Calculate how many times larger the surface area and volume of the icon is compared with real object.
- Use Microsoft Word when preparing your presentation. Your *Media Centre* includes images that can help to liven up your presentation. As you arrange your images on your Word page make them appear as they would in the pages of a book. Each page of your Word document will be a page in your Calameo production of your report. Summarise the findings of your research. Give formulas for length, area, surface area and volume in terms of the scale factor. Discuss whether or not the weight gain of a baby fits this same model. Write a paragraph explaining the error of Mrs Efficiency's calculations, and why her backyard flooded. Assume the role of Mrs Efficiency's daughter who is presenting her research findings to her mother.



- Use Word to develop your presentation. Remember that you are trying to deliver a comprehensive summary of similarity and clarify why there was a flood in the backyard. You are also showing applications of similarity. Make sure you include all the results of your research, and that your presentation will grab the attention of those listening. Try to limit your report to either 4 or 8 pages for the most professional delivery. Use Word to type up your dialogue to present your report (200–500 words).
- Before you present your findings, use the **Calameo** weblink in the Resources tab and sign up for a free account. You will need an email address so ensure you have your teacher's or your parents' permission to do this. Take the **Guided tour** to learn how to prepare a Calameo presentation. **Create a publication** and upload your Word report. Choose the radio button for a Private Publication Mode. Click the

Start uploading button. To show your report, click the button **Read the publication** and choose full screen view before you commence your delivery. (If the pagination alters using Word — and this is a problem — delete your Calameo publication and repeat the process by uploading a pdf of your report.)

SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft Word
- Internet Explorer
- Google Sketchup
- Wordle
- Calameo
- Enlargements Teaching Tool

TOPIC 16
Quadratic algebra

16.1 Overview



Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

16.1.1 Why learn this?

Algebraic techniques help us to understand many things in the everyday world. What we learn in mathematics can be applied to many other subjects; for example, physics is often described as using mathematics as a tool for studying the world around us. The concepts studied in quadratic algebra can be applied to solving quadratic equations. Many physical phenomena can be described using quadratic equations, and in this chapter you will learn the techniques necessary to solve quadratic equations.

16.1.2 What do you know?

assessON

1. **THINK** List what you know about quadratics. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of quadratics.

LEARNING SEQUENCE

16.1 Overview
16.2 Factorisation patterns
16.3 Factorising monic quadratics
16.4 Factorising non-monic quadratics
16.5 Simplifying algebraic fractions
16.6 Quadratic equations
16.7 The Null Factor Law
16.8 Solving the quadratic equation $ax^2 + bx + c = 0$
16.9 Solving quadratic equations with two terms
16.10 Applications
16.11 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLession: The story of mathematics: Planetary orbits (eles-1702)

16.2 Factorisation patterns

16.2.1 Difference of two squares

- Recall from Topic 3 that $(a + b)(a - b) = a^2 - b^2$.
  $a^2 - b^2$ is known as the 'difference of two squares'.
- The rule can be used in reverse to factorise the difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

- Factorise by taking out any common factor first.
  For example:

$$\begin{aligned} &5x^2 - 80 \\ &= 5(x^2 - 16) \\ &= 5(x - 4)(x + 4) \end{aligned}$$

WORKED EXAMPLE 1 TI | CASIO

Factorise each of the following expressions.
a $x^2 - 9$ **b** $64m^2 - 25n^2$ **c** $(c + 7)^2 - 16$ **d** $3x^2 - 48$

THINK	WRITE
a 1 The expression is a difference of two squares.	**a** $x^2 - 9$
2 Rewrite, showing the two squares.	$= x^2 - 3^2$
3 Factorise using the formula $a^2 - b^2 = (a + b)(a - b)$ where $a = x$ and $b = 3$.	$= (x + 3)(x - 3)$
b 1 The expression is a difference of two squares.	**b** $64m^2 - 25n^2$
2 Rewrite, showing the two squares.	$= (8m)^2 - (5n)^2$
3 Factorise.	$= (8m + 5n)(8m - 5n)$
c 1 The expression is a difference of perfect squares.	**c** $(c + 7)^2 - 16$
2 Rewrite, showing the two squares.	$= (c + 7)^2 - 4^2$
3 Put $c + 7$ in each bracket and complete the factors.	$= (c + 7 + 4)(c + 7 - 4)$
4 Simplify.	$= (c + 11)(c + 3)$
d 1 There is a common factor.	**d** $3x^2 - 48$
2 Take out the common factor.	$= 3(x^2 - 16)$
3 $x^2 - 16$ is a difference of two squares.	$= 3(x^2 - 4^2)$
4 Factorise.	$= 3(x + 4)(x - 4)$

16.2.2 Perfect squares

- Recall from Topic 3 that a perfect square is an expression such as $(x + 2)^2$ or $(t - 2)^2$. The expansion of perfect squares produces the following patterns or identities.

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

- The identities can also be used in reverse to factorise the quadratic expressions.

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

- Is $x^2 + 8x + 16$ a perfect square?
  The constant term must be the square of half the coefficient of the x-term if the expression is a perfect square. Because half of 8 is 4 and $4^2 = 16$, $x^2 + 8x + 16$ is a perfect square.
  $$x^2 + 8x + 16 = (x + 4)^2$$

WORKED EXAMPLE 2

TI | CASIO

Factorise each of the following expressions.
a $x^2 - 6x + 9$ **b** $64m^2 + 80mn + 25n^2$ **c** $3x^2 - 24x + 48$

THINK	WRITE
a 1 x^2 and 9 are perfect squares, so $x^2 - 6x + 9$ might be $(x - 3)^2$. Check as follows.	**a** $x^2 - 6x + 9$
2 Halve the coefficient of the x-term and then square the result.	$\frac{-6}{2} = -3, (-3)^2 = 9$
3 The middle term fits, so factorise using the formula $a^2 - 2ab + b^2 = (a - b)^2$, where $a = x$ and $b = 3$.	$x^2 - 6x + 9 = (x - 3)^2$
b 1 $64m^2$ and $25n^2$ are perfect squares, so $64m^2 + 80mn + 25n^2$ might be $(8m + 5n)^2$.	**b** $64m^2 + 80mn + 25n^2$
2 Check the middle term.	$8m \times 5n \times 2 = 80mn$
3 The middle term fits, so write down the factors.	$64m^2 + 80mn + 25n^2 = (8m + 5n)^2$
c 1 There is a common factor.	**c** $3x^2 - 24x + 48$
2 Factorise.	$= 3(x^2 - 8x + 16)$
3 Factorise $x^2 - 8x + 16$.	$= 3(x - 4)^2$

learnon RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Factorising difference of two squares expressions (doc-10980)

Exercise 16.2 Factorisation patterns

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–e, 2a–e, 3a–f, 4a–f, 5–8, 10, 11	Questions: 1d–g, 2d–g, 3d–i, 4d–i, 5–9, 11, 12–14	Questions: 1e–i, 2e–i, 3g–l, 4g–l, 5–16

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Factorise each of the following expressions using the difference of two squares rule.

a. $x^2 - 25$ **b.** $x^2 - 81$ **c.** $a^2 - 16$
d. $25 - p^2$ **e.** $121 - a^2$ **f.** $36 - y^2$
g. $4b^2 - 25$ **h.** $9a^2 - 16$ **i.** $25d^2 - 1$

2. WE1b Factorise each of the following expressions.

a. $x^2 - y^2$ **b.** $a^2 - b^2$ **c.** $p^2 - q^2$
d. $25m^2 - n^2$ **e.** $81x^2 - y^2$ **f.** $p^2 - 36q^2$
g. $36m^2 - 25n^2$ **h.** $16q^2 - 9p^2$ **i.** $4m^2 - 49n^2$

3. WE1c Factorise each of the following expressions.

a. $(x + 9)^2 - 16$ **b.** $(p + 8)^2 - 25$ **c.** $(p - 2)^2 - q^2$
d. $(c - 6)^2 - d^2$ **e.** $(x + 7)^2 - y^2$ **f.** $(p + 5)^2 - q^2$
g. $(a + 3)^2 - b^2$ **h.** $(a - 3)^2 - 1$ **i.** $(b - 1)^2 - 36$

4. WE1d Factorise each of the following expressions completely after removing any common factors.

a. $2m^2 - 32$ **b.** $5y^2 - 45$ **c.** $6p^2 - 24$
d. $4m^2 - 100$ **e.** $288 - 2x^2$ **f.** $80 - 5a^2$
g. $100x^2 - 25y^2$ **h.** $144p^2 - 4q^2$ **i.** $32y^2 - 2x^2$
j. $ax^2 - 9a$ **k.** $8b - 2bx^2$ **l.** $3(b + 5)^2 - 48$

5. WE2 Factorise each of the following expressions by recognising the perfect square rule.

a. $x^2 + 10x + 25$ **b.** $p^2 - 24p + 144$ **c.** $n^2 + 20n + 100$
d. $x^2 + 2xy + y^2$ **e.** $u^2 - 2uv + v^2$ **f.** $64 + 16e + e^2$
g. $4m^2 - 20m + 25$ **h.** $49 + 42a + 9a^2$ **i.** $81x^2 + 72xy + 16y^2$
j. $2x^2 + 24x + 72$ **k.** $3x^2 - 24xy + 48y^2$ **l.** $18a^2 + 24ab + 8b^2$

6. a. Which of the expressions below are the difference of two squares?

b. Which of the expressions below are perfect squares?

i. $x^2 - 36$ **ii.** $x^2 + 36$ **iii.** $x^2 + 6x + 36$
iv. $x^2 - 12x + 36$ **v.** $x^2 + 12x - 36$ **vi.** $x^2 + 12x + 36$

7. MC **a.** The expression $x^2 - 121$ factorises to:

A. $(x - 11)^2$ **B.** $(x - 11)(x + 12)$
C. $(x + 11)(x - 11)$ **D.** $(x - 11)(x - 11)$

b. What does the expression $36m^2 - 12m + 1$ factorise to?

A. $(6m - 1)(6m - 1)$ **B.** $36(m + 1)(m - 1)$
C. $(6m + 1)^2$ **D.** $6(6m - 1)(6m - 1)$

c. What does the expression $16a^2 - 25b^2$ factorise to?

A. $16(a + 5b)(a - 5b)$ **B.** $4(4a - 5b)^2$
C. $(16a + 25b)(16a - 25b)$ **D.** $(4a + 5b)(4a - 5b)$

d. The expression $5c^2 - 20c + 20$ factorises to:

A. $5(c - 2)(c - 2)$ **B.** $5(c + 4)(c - 4)$
C. $5(c - 4)(c - 4)$ **D.** $(5c + 20)(5c - 20)$

e. The expression $(x + 4)^2 - 9$ factorises to:

A. $9(x + 4)(x - 4)$ **B.** $(x + 13)(x - 5)$
C. $(x + 7)(x + 1)$ **D.** $(x - 1)(x + 5)$

Understanding

8. A circular pool with a radius of r metres is surrounded by a circular path 1 m wide.

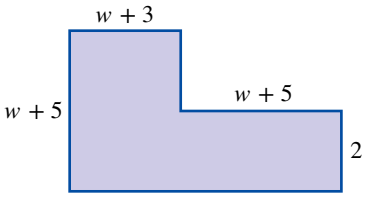
a. Find the area of the surface of the pool.

b. Find, in terms of r, the distance from the centre of the pool to the outer edge of the path.

c. Find the area of the circle that includes the path and the pool. (Don't expand the expression.)
d. Write an expression for the area of the path.
e. Simplify this expression.
f. If the pool had a radius of 5 m, what would be the area of the path to the nearest square metre?
g. If the pool had a radius of 7 m, what would be the area of the path to the nearest square metre?



9. An L-shaped piece of farm land is to be planted with wheat.



a. Write an expression for the area of the wheat in simplest form.
b. Calculate how much wheat is to be planted if w is:
i. 5 **ii.** 10 **iii.** 20.
c. Invent your own L-shaped area using the pronumeral w that has an area that can be expressed as a perfect square.

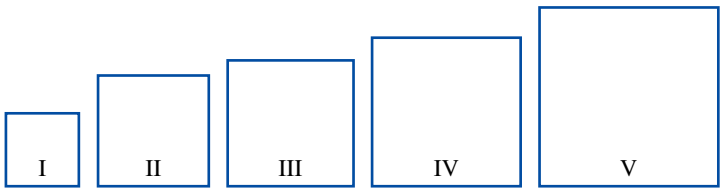
10. Factorise the following using the difference of two squares rule.
a. $x^2 - 10$ **b.** $4x^2 - 32$ **c.** $x^4 - y^2$
d. $(x + 2)^2 - (x - 3)^2$ **e.** $y^2 - 4x^4$ **f.** $10^4 - x^4$

Reasoning

11. Is $(x + 4)^2$ equal to $x^2 + 16$? Explain using a numerical example.
12. Show that $(a + b)(c + d) = (c + d)(a + b)$.
13. Show that $(a + b)(a - b) = (a - b)(a + b)$.

Problem solving

14. Five squares of increasing size and five area formulas are given below.
a. Use factorisation to find the side length that correlates to each area formula.
b. Using the area given and side lengths found, match the squares below with the appropriate algebraic expression for their area.



$A_a = x^2 + 6x + 9$
$A_b = x^2 + 10x + 25$
$A_c = x^2 + 16x + 64$
$A_d = x^2 - 6x + 9$
$A_e = x^2 + 12x + 36$

c. If $x = 5$ cm, use the formula given to calculate the area of each square.

15. The area of a rectangle is represented by $x^2 - 49$ mm^2.

a. Factorise the expression.

b. Using the factorised expression, find a possible length and width for the rectangle.

c. If $x = 25$ mm, find the dimensions for the rectangle.

d. If $x = 50$ mm, how much bigger is the area of the rectangle than in part **c**?

16. The expansion of perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

and $(a - b)^2 = a^2 - 2ab + b^2$

can be used to simplify some arithmetic calculations. For example:

$$\begin{aligned} 97^2 &= (100 - 3)^2 \\ &= 100^2 - 2 \times 100 \times 3 + 3^2 \\ &= 9409 \end{aligned}$$

Use this method to calculate the following.

a. 103^2 **b.** 62^2 **c.** 997^2

d. 1012^2 **e.** 53^2 **f.** 98^2

Reflection

What is the best way to remember all three expansion patterns from this section?

CHALLENGE 16.1

Use the difference of two squares to completely factorise $x^8 - 1$.

16.3 Factorising monic quadratics

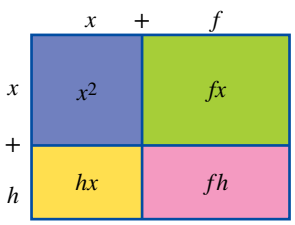
16.3.1 Quadratic trinomials

- A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where a, b and c are constants.
- If the expression contains $1x^2$, that is if $a = 1$, then it is called a monic quadratic trinomial.

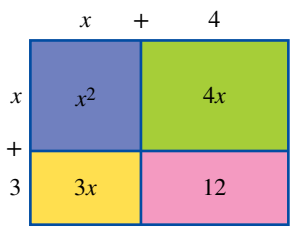
Factorising monic quadratics

- The area model of binomial expansion can be reversed to find a pattern for factorising a general quadratic expression. For example:

$$\begin{aligned} (x + f)(x + h) &= x^2 + fx + hx + fh \\ &= x^2 + (f + h)x + fh \end{aligned}$$



$$\begin{aligned} (x + 4)(x + 3) &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$



- To factorise a general quadratic, look for factors of c that sum to b.

$$x^2 + bx + c = (x + f)(x + h)$$

Factors of c that sum to b

For example, $x^2 + 7x + 12 = (x + 3)(x + 4)$.

$3 + 4 = 7$

$3 \times 4 = 12$

WORKED EXAMPLE 3

Factorise the following quadratic expressions.
a $x^2 + 5x + 6$ **b** $x^2 + 10x + 24$

THINK

a 1 The general quadratic expression has the pattern $x^2 + 5x + 6 = (x + f)(x + h)$. f and h are a factor pair of 6 that add to 5.
Calculate the sums of factor pairs of 6. The factors of 6 that add to 5 are 2 and 3, as shown in blue.

2 Substitute the values of f and h into the expression in its factorised form.

b 1 The general quadratic expression has the pattern $x^2 + 10x + 24 = (x + f)(x + h)$, where f and h are a factor pair of 24 that add to 10.
Calculate the sums of factor pairs of 24. The factors of 24 that add to 10 are 4 and 6, as shown in blue.

2 Substitute the values of f and h into the expression in its factorised form.

WRITE

a

Factors of 6	Sum of factors
1 and 6	7
2 and 3	5

$(x^2 + 5x + 6) = (x + 2)(x + 3)$

b

Factors of 24	Sum of factors
1 and 24	25
2 and 12	14
3 and 8	11
4 and 6	10

$x^2 + 10x + 24 = (x + 4)(x + 6)$

WORKED EXAMPLE 4

TI | CASIO

Factorise the following expressions.
a $x^2 - 9x + 18$ **b** $x^2 + 6x - 16$

THINK

a 1 The general quadratic expression has the pattern $x^2 - 9x + 18 = (x + f)(x + h)$, where f and h are a factor pair of 18 that add to -9.
Calculate the sums of factor pairs of 18. As shown in blue, -3 and -6 are factors of 18 that add to -9.

2 Substitute the values of f and h into the expression in its factorised form.

b 1 The general quadratic expression has the pattern $x^2 + 6x - 16 = (x + f)(x + h)$, where f and h are a factor pair of -16 that add to 6.
Calculate the sums of factor pairs of -16. As shown in blue, -2 and 8 are factors of -16 that add to 6.

2 Substitute the values of f and h into the expression in its factorised form.

WRITE

a

Factors of 18	Sum of factors
1 and 18	19
−1 and −18	−19
2 and 9	11
−2 and −9	−11
3 and 6	9
−3 and −6	−9

$x^2 - 9x + 18 = (x - 3)(x - 6)$

b

Factors of –16	Sum of factors
1 and −16	−15
−1 and 16	15
2 and −8	−6
−2 and 8	6

$x^2 - 6x + 16 = (x - 2)(x + 8)$

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Factorising quadratic equations (int-2775)

Try out this interactivity: Factorise trinomials (int-0748)

Complete this digital doc: SkillSHEET: Factorising quadratic trinomials (doc-10981)

Complete this digital doc: SkillSHEET: Finding the factor pair that adds to a given number (doc-10982)

Exercise 16.3 Factorising monic quadratics

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–k, 2, 3, 4a–i, 5, 6a–i, 7–9, 11	Questions: 1g–o, 4g–o, 5, 6g–o, 7–10, 12–14	Questions: 1m–k, 4i–o, 6i–o, 7–11, 12–14

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. Expand each of the following.

a. $(x + 4)(x + 2)$	**b.** $(x - 2)(x - 4)$	**c.** $(x - 4)(x - 5)$	**d.** $(x + 4)(x + 5)$
e. $(m + 1)(m + 5)$	**f.** $(m - 1)(m - 5)$	**g.** $(t + 8)(t + 11)$	**h.** $(t - 10)(t - 20)$
i. $(x + 2)(x - 3)$	**j.** $(x + 3)(x - 2)$	**k.** $(v + 5)(v - 8)$	**l.** $(v - 5)(v + 8)$
m. $(x + 7)(x - 2)$	**n.** $(t + 1)(t - 12)$	**o.** $(n + 15)(n - 2)$	**p.** $(a + 3)(a - 6)$
q. $(z + 4)(z + 4)$	**r.** $(z + 11)(z - 8)$	**s.** $(n - 3)(n - 7)$	**t.** $(v - 1)(v + 2)$

2. Make a complete systematic list of pairs of positive numbers that add up to:

a. 5	**b.** 6	**c.** 12	**d.** 20.

3. Make a complete systematic list of pairs of negative numbers that add up to:

a. -5	**b.** -7	**c.** -8	**d.** -11.

4. WE3 Factorise each of the following.

a. $x^2 + 4x + 3$	**b.** $x^2 - 4x + 3$	**c.** $x^2 + 12x + 11$	**d.** $x^2 - 12x + 11$
e. $a^2 + 6a + 5$	**f.** $a^2 - 6a + 5$	**g.** $x^2 - 7x + 12$	**h.** $x^2 - 7x + 10$
i. $n^2 + 8n + 16$	**j.** $n^2 + 10n + 16$	**k.** $y^2 - 12y + 27$	**l.** $x^2 - 13x + 42$
m. $t^2 - 8t + 12$	**n.** $t^2 + 11t + 18$	**o.** $u^2 + 5u + 6$	

5. Make a systematic list of 5 pairs of numbers with opposite signs whose sum is:

a. 6	**b.** -6	**c.** 3	**d.** -3
e. -8	**f.** 14	**g.** -5	**h.** 7.

6. WE4 Factorise each of the following.

a. $x^2 + 3x - 18$	**b.** $x^2 - 3x - 18$	**c.** $x^2 - 2x - 15$	**d.** $x^2 + 2x - 15$
e. $n^2 - 13n - 14$	**f.** $n^2 + 2n - 35$	**g.** $v^2 + 5v - 6$	**h.** $v^2 - 5v - 6$
i. $t^2 + 4t - 12$	**j.** $t^2 - 5t - 14$	**k.** $x^2 - x - 20$	**l.** $x^2 + x - 20$
m. $n^2 + n - 90$	**n.** $n^2 - 3n - 70$	**o.** $x^2 - 4x - 5$	

Understanding

7. Consider the quadratic trinomial $x^2 + 7x + c$ where c is a positive integer.
- **a.** Factorise the expression if $c = 6$.
- **b.** What other positive whole number values can c take if the expression is to be factorised? Factorise the expression for each of these values.

8. Consider the quadratic trinomial $x^2 + 3x + c$ where c is a negative integer.
- **a.** Factorise the expression if $c = -4$.
- **b.** Find 3 more values of c for which the expression can be factorised, and factorise each one.

9. Factorise the following trinomials, by first removing a common factor.
- **a.** $2x^2 - 10x - 28$ **b.** $4x^2 + 28x + 40$ **c.** $-2x^2 - 2x + 24$ **d.** $5x^2 - 40x + 75$

10. MC When factorised $x^2 - 3x - 18$ is equal to:
- **A.** $(x - 3)(x - 6)$ **B.** $(x - 3)(x + 6)$ **C.** $(x + 3)(x + 6)$ **D.** $(x + 3)(x - 6)$ **E.** $(x + 2)(x - 9)$

Reasoning

11. Show that $x^2 + 8x + 10$ has no factors, if only whole numbers can be used.

12. To factorise the quadratic trinomial $6x^2 + 9x + 3$, the expression is rewritten as an equivalent form consisting of four terms, $6x^2 + \square x + \square x + 3$, that are then grouped using the 'two and two' method, $6x^2 + \square x$ $+ \square x + 3$. Investigate all the possible combinations for $+\square x + \square x = 9x$ and show that there is only one combination that will factorise the trinomial appropriately.

Problem solving

13. Rectangular floor mats have an area of $x^2 + 2x - 15$.
- **a.** If the length of the mat is $(x + 5)$ cm, find an expression for the width.
- **b.** If the length of the mat is 70 cm, what is the width?
- **c.** If the width of the mat is 1 m, what is the length?

14. A particular rectangle has an area of 120 m^2.
- **a.** Generate a list of possible whole number dimensions for the rectangle.
- **b.** The area of the rectangle can also be expressed as $A = x^2 + 2x - 48$. Determine algebraic expressions for the length and the width of the rectangle in terms of x.
- **c.** Use your list of possible dimensions and your algebraic expressions for width and length to determine the dimensions of the rectangle. *Hint:* The value of x in the length and width must be the same.

Reflection

How will you memorise the steps involved in factorising quadratic trinomials when $a = 1$?

16.4 Factorising non-monic quadratics

16.4.1 Factorising quadratic trinomials where $a \neq 1$

- When a quadratic trinomial in the form $ax^2 + bx + c$ is written as $ax^2 + mx + nx + c$, where $m + n = b$, the four terms can be factorised by grouping.

$2 \times 12 = 24$
Numbers that multiply to give 24
and add to 11
therefore (8) and (3).

$2x^2 + 11x + 12 = 2x^2 + (8x) + (3x) + 12$
$= 2x^2 + 8x + 3x + 12$
$= 2x(x + 4) + 3(x + 4)$
$= 2x(x + 4) + 3(x + 4)$
$= (x + 4)(2x + 3)$

- There are many combinations of numbers that satisfy $m + n = b$, however only one particular combination can be grouped and factorised.
  For example,

$$\begin{aligned} 2x^2 + 11x + 12 &= 2x^2 + 7x + 4x - 12 \\ &= 2x^2 + 7x + 4x + 12 \\ &= x(2x + 7) + 4(x + 3) \\ &\text{cannot be factorised further} \end{aligned}$$

or

$$\begin{aligned} 2x^2 + 11x + 12 &= 2x^2 + 8x + 3x + 12 \\ &= 2x^2 + 8x + 3x + 12 \\ &= 2x(x + 4) + 3(x + 4) \\ &= (x + 4)(2x + 3) \end{aligned}$$

- In examining the general binomial expansion, a pattern emerges that can be used to help identify which combination to use for $m + n = b$.

$$\begin{aligned} (dx + e)(fx + g) &= dfx^2 + dgx + efx + eg \\ &= dfx^2 + (dg + ef)x + eg \end{aligned}$$

$$\begin{aligned} m + n &= dg + ef \\ &= b \end{aligned} \quad \text{and} \quad \begin{aligned} m \times n &= dg \times ef \\ &= dgef \\ &= dfeg \\ &= ac \end{aligned}$$

- Therefore, m and n are factors of ac that sum to b.
- To factorise a general quadratic where $a \neq 1$, look for factors of ac that sum to b. Then rewrite the quadratic trinomial with four terms that can then be grouped and factorised.

$$ax^2 + bx + c = ax^2 + mx + nx + c$$

Factors of ac that sum to b

WORKED EXAMPLE 5

TI | CASIO

Factorise $6x^2 - 11x - 10$.

THINK

1 Write the expression and look for common factors and special patterns. The expression is a general quadratic with $a = 6$, $b = -11$ and $c = -10$.

WRITE

$6x^2 - 11x - 10 = 6x^2 + -11x + -10$

2 Since $a \neq 1$, rewrite $ax^2 + bx + c$ as $ax^2 + mx + nx + c$, where m and n are factors of ac (6×-10) that sum to b (-11).
- Calculate the sums of factor pairs of -60. As shown in blue, 4 and -15 are factors of -60 that add to -11.

Factors of −60	Sum of factors
−60, 1	−59
−20, 3	−17
−30, 2	−28
15, −4	11
−15, 4	−11

3 Rewrite the quadratic expression: $ax^2 + bx + c = ax^2 + mx + nx + c$ with $m = 4$ and $n = -15$.

$6x^2 - 11x - 10 = 6x^2 + 4x + -15x - 10$

4 Factorise using the grouping method:
$6x^2 + 4x = 2x(3x + 2)$ and
$-15x - 10 = -5(3x + 2)$
Write the answer.

$6x^2 - 11x - 10 = 2x(3x + 2) + -5(3x + 2)$
$= (3x + 2)(2x - 5)$

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Complete this digital doc: WorkSHEET Quadratics I (doc-10986)

Exercise 16.4 Factorising non-monic quadratics

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–e, 2a–e, 3–8	Questions: 1d–h, 2d–h, 3–12	Questions: 1g–i, 2g–i, 3–12

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE5 Factorise each of the following quadratic trinomials.

a. $2x^2 + 10x + 12$ **b.** $3x^2 + 15x + 12$ **c.** $2x^2 - 12x + 16$
d. $3x^2 - 12x + 9$ **e.** $4x^2 - 4x - 24$ **f.** $3x^2 + 9x - 30$
g. $2x^2 + 8x - 42$ **h.** $3x^2 - 9x - 54$ **i.** $5x^2 + 20x - 60$

2. Factorise each of the following quadratic trinomials.

a. $2x^2 + 7x + 3$ **b.** $2x^2 + 7x + 6$ **c.** $3x^2 + 7x + 2$
d. $2x^2 - 7x + 3$ **e.** $3x^2 - x - 2$ **f.** $5x^2 + 3x - 2$
g. $7x^2 - 17x + 6$ **h.** $10x^2 - 11x - 6$ **i.** $2x^2 + 5x - 12$

3. a. MC The expression $3x^2 + 21x + 36$ is equal to:

A. $3(x + 6)(x + 2)$ **B.** $3(x + 4)(x + 3)$ **C.** $(3x + 9)(x + 4)$ **D.** $(3x + 2)(x + 18)$

b. The expression $2x^2 - 16x + 14$ is equal to:

A. $2(x - 1)(x - 7)$ **B.** $2(x + 1)(x + 7)$ **C.** $(2x - 7)(x - 2)$ **D.** $(2x - 2)(x - 7)$

c. The expression $12x^2 + 58x - 10$ is equal to:

A. $(12x - 2)(x - 5)$ **B.** $2(6x + 1)(x - 5)$ **C.** $2(6x + 5)(x - 1)$ **D.** $2(6x - 1)(x + 5)$

Understanding

4. Factorise the following trinominals using the cross-product method.

a. $2x^2 + 3x - 2$ **b.** $8x^2 + 2x - 3$ **c.** $6x^2 + x - 2$ **d.** $2x^2 + 9x + 10$

5. Factorise the following trinominals using *both* the grouping terms and the cross-product methods. Discuss which method is simpler to perform.

a. $3x^2 - x - 4$ **b.** $6x^2 + 5x - 21$

6. Factorise:

a. $6x^2 - 5x - 6$ **b.** $4x^2 + 13x + 10$ **c.** $4x^2 - 19x + 15$ **d.** $6x^2 + 5x - 4$.

7. Factorise $3x^2 + 15x + 12$:

a. by first taking out a common factor
b. by first separating the middle term, then grouping.

9. a. Explain why the quadratic $4x^2 - 9x - 9$ can be written as $4x^2 + 3x - 12x - 9$.
b. Factorise the quadratic $4x^2 - 9x - 9$.

Problem solving

10. a. Factorise $12x^2 - 4x - 5$.
b. Assume that the factors represent the length and width of a rectangle.
i. What is the area of the rectangle?
ii. For what value of x is the rectangle a square?
iii. For what values of x is one side twice as long as the other? Give both possible answers.

11. The area formulas below relate to either squares or rectangles.
i. $9s^2 + 48s + 64$ **ii.** $25s^2 - 4$ **iii.** $s^2 + 4s + 3$ **iv.** $4s^2 - 24s - 36$
a. Without completing any algebraic operations, examine the formulas and determine which belong to squares and which to rectangles. Explain your answer.
b. Factorise each formula and classify it as a square or rectangle. Check this against your answer to part **a**.

12. Factorise the following expressions.
a. $9b^2 + 48b + 64$ **b.** $(ac)^2 - d^2$ **c.** $a^2 + ab - 182b^2$ **d.** $b^2 - 2abc + a^2c^2$

Reflection

Which of the two methods presented do you prefer to use?

16.5 Simplifying algebraic fractions

16.5.1 Cancelling common factors

- **Algebraic fractions** are fractions that have algebraic expressions in the numerator and/or denominator, for example:

$$\frac{a}{2}, \frac{b+3}{4} \text{ and } \frac{x^2+3x}{4x+2}.$$

- Just as fractions can be simplified by cancelling common factors, so too can algebraic fractions.
- A fraction can only be simplified if the numerator and denominator are both products.
- A factor can be cancelled if it is a factor of the whole of the numerator and the whole of the denominator. For example:

$$\frac{2\cancel{a}}{\cancel{a}b} = \frac{2}{b} \qquad \frac{2\cancel{(x+3)}}{5a\cancel{(x+3)}} = \frac{2}{5a}$$

WORKED EXAMPLE 6 TI | CASIO

Simplify each of the following.

a $\dfrac{4}{x+3} \times \dfrac{x+3}{5}$ **b** $\dfrac{(z-3)(z-4)}{(z-5)(z+1)} \times \dfrac{(z+1)(z-1)}{(z+3)(z-3)}$

THINK	WRITE
a 1 Write the expression.	**a** $\dfrac{4}{x+3} \times \dfrac{x+3}{5}$
2 Cancel the common factors.	$= \dfrac{4}{1\cancel{(x+3)}} \times \dfrac{\cancel{(x+3)}^1}{5}$
3 Simplify the answer.	$= \frac{4}{5}$

b 1 Write the expression.

b $\frac{(z-3)(z-4)}{(z-5)(z+1)} \times \frac{(z+1)(z-1)}{(z+3)(z-3)}$

2 Cancel the common factors.

$= \frac{{}^{1}\cancel{(z-3)}(z-4)}{(z-5)\cancel{(z+1)}_{1}} \times \frac{{}^{1}\cancel{(z+1)}(z-1)}{(z+3)\cancel{(z-3)}_{1}}$

3 Simplify the resultant fraction.

$= \frac{(z-4)(z-1)}{(z-5)(z+3)}$

WORKED EXAMPLE 7

TI | CASIO

Simplify the following.

a $\frac{1}{x-3} \div \frac{3}{x-3}$ **b** $\frac{(z+2)(z-3)}{(z-4)} \div \frac{z-3}{z-4}$

THINK

a 1 Write the expression.

2 Change to a multiplication problem by changing the ÷ to a × and turning the second fraction upside down.

3 Cancel the common factors.

4 Simplify the fraction.

b 1 Write the expression.

2 Change to multiplication.

3 Cancel the common factors.

4 Simplify the resultant fraction.

WRITE

a $\frac{1}{x-3} \div \frac{3}{x-3}$

$= \frac{1}{x-3} \times \frac{x-3}{3}$

$= \frac{1}{{}_{1}\cancel{(x-3)}} \times \frac{\cancel{(x-3)}^{1}}{3}$

$= \frac{1}{3}$

b $\frac{(z+2)(z-3)}{(z-4)} \div \frac{z-3}{z-4}$

$= \frac{(z+2)(z-3)}{(z-4)} \times \frac{(z-4)}{(z-3)}$

$= \frac{(z+2)\cancel{(z-3)}^{1}}{{}_{1}\cancel{(z-4)}} \times \frac{\cancel{(z-4)}^{1}}{\cancel{(z-3)}_{1}}$

$= \frac{z+2}{1}$

$= z+2$

WORKED EXAMPLE 8

TI | CASIO

Simplify each of the following fractions by first factorising the numerator and then the denominator and cancelling as appropriate.

a $\frac{3x+9}{15}$ **b** $\frac{5}{10x+20}$ **c** $\frac{10x+15}{6x+9}$ **d** $\frac{x^2+4x}{x^2-5x}$

THINK

a 1 Write the fraction.

2 Factorise both the numerator and denominator.

WRITE

a $\frac{3x+9}{15}$

$= \frac{3(x+3)}{3 \times 5}$

3 Cancel the common factors.

$$= \frac{{}^{1}\cancel{3}(x+3)}{{}_{1}\cancel{3} \times 5}$$

4 Simplify the resulting fraction.

$$= \frac{x+3}{5}$$

b 1 Write the fraction.

b $\dfrac{5}{10x+20}$

2 Factorise both the numerator and denominator.

$$= \frac{5 \times 1}{10(x+2)}$$

3 Cancel the common factors.

$$= \frac{\cancel{5}^{1} \times 1}{\cancel{10}_{2}(x+2)}$$

4 Simplify the resulting fraction.

$$= \frac{1}{2(x+2)}$$

c 1 Write the fraction.

c $\dfrac{10x+15}{6x+9}$

2 Factorise both the numerator and denominator.

$$= \frac{5\cancel{(2x+3)}^{1}}{3\cancel{(2x+3)}^{1}}$$

3 Cancel the common factors.

$$= \frac{5}{3}$$

d 1 Write the fraction.

d $\dfrac{x^2+4x}{x^2-5x}$

2 Factorise both the numerator and denominator.

$$= \frac{{}^{1}\cancel{x}(x+4)}{{}^{1}\cancel{x}(x-5)}$$

4 Cancel the common factors.

$$= \frac{x+4}{x-5}$$

WORKED EXAMPLE 9

Simplify each of the following algebraic fractions.

a $\dfrac{x^2+3x-4}{x-1}$ **b** $\dfrac{x^2-7x-8}{x^2+3x+2}$ **c** $\dfrac{x^2-6x+5}{2x^2-16x+30}$

THINK **WRITE**

a 1 Write the expression.

a $\dfrac{x^2+3x-4}{x-1}$

2 Factorise the numerator and denominator and cancel the common factors.

$$= \frac{{}^{1}\cancel{(x-1)}(x+4)}{\cancel{x-1}^{1}}$$

3 Simplify.

$$= x+4$$

b 1 Write the original fraction.

b $\dfrac{x^2-7x-8}{x^2+3x+2}$

2 Factorise the numerator and denominator.

$$= \frac{{}^{1}\cancel{(x+1)}(x-8)}{{}^{1}\cancel{(x+1)}(x+2)}$$

3 Cancel any common factors and simplify.

$= \frac{x - 8}{x + 2}$

c **1** Write the original fraction.

c $\frac{x^2 - 6x + 5}{2x^2 - 16x + 30}$

2 Factorise the numerator and denominator.

$= \frac{(x - 1)(x - 5)^1}{2(x - 3)(x - 5)^1}$

3 Cancel any common factors and simplify.

$= \frac{x - 1}{2(x - 3)}$

- *Note:* A fraction such as $\frac{x - 1}{x - 3}$ cannot be simplified any further. It may be tempting to cancel the x in the numerator and denominator, but x is neither a factor of $(x - 1)$ nor $(x - 3)$, so it cannot be cancelled.

WORKED EXAMPLE 10

Simplify $\frac{2x + 4}{3} \times \frac{3x - 3}{x + 2}$ by factorising the numerator and denominator and cancelling as appropriate.

THINK

1 Write the expression.

2 Factorise each numerator and denominator and cancel common factors.

3 Simplify.

WRITE

$\frac{2x + 4}{3} \times \frac{3x - 3}{x + 2}$

$= \frac{2(x + 2)^1}{{}_1\cancel{3}} \times \frac{{}^1\cancel{3}(x - 1)}{x + 2_1}$

$= \frac{2(x - 1)}{1}$

$= 2(x - 1)$

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Complete this digital doc: SkillSHEET: Simplifying algebraic fractions (doc-10983)

Exercise 16.5 Simplifying algebraic fractions

assesson

Individual pathways

PRACTISE
Questions:
1a, c, e, k, 2a, c, e, k, 3a–h, 4a–h, 5a–h, 6a, c, e, g, 7

CONSOLIDATE
Questions:
1a, c, g, i, k, 2a–k, 3e–i, 4e–i, 5e–i, 6a–i, 7–10

MASTER
Questions:
1b–l, 2b–l, 3i–p, 4i–p, 5i–p, 6b–j, 7–10

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE6 Simplify the following.

a. $\frac{x+1}{2} \times \frac{1}{x+1}$ **b.** $\frac{y-1}{3} \times \frac{2}{y-1}$ **c.** $\frac{p-2}{4} \times \frac{4}{p-2}$

d. $\frac{b+5}{3} \times \frac{9}{b+5}$ **e.** $\frac{m+4}{m} \times \frac{m}{m+4}$ **f.** $\frac{3p}{p-6} \times \frac{p-6}{p}$

g. $\frac{y-7}{4y} \times \frac{y}{y-7}$ **h.** $\frac{ha}{a-2} \times \frac{a-2}{2a}$ **i.** $\frac{p-3}{p+3} \times \frac{5(p+3)}{p-2}$

j. $\frac{(x+2)(x-7)}{x+1} \times \frac{x+1}{x-7}$ **k.** $\frac{(b-2)(b+3)}{(b+5)(b-4)} \times \frac{b-4}{b-2}$ **l.** $\frac{x-3}{x+2} \times \frac{(x+2)(x+1)}{(x-6)(x-3)}$

2. WE7 Simplify the following.

a. $\frac{x+2}{3} \div \frac{x+2}{x}$ **b.** $\frac{m-3}{4} \div \frac{m-3}{8}$ **c.** $\frac{5}{x-3} \div \frac{10}{x-3}$

d. $\frac{12}{a+2} \div \frac{4}{a+2}$ **e.** $\frac{m}{m-6} \div \frac{m}{m-6}$ **f.** $\frac{p+2}{p-1} \div \frac{p-2}{p-1}$

g. $\frac{s-6}{3(s+1)} \div \frac{s}{s+1}$ **h.** $\frac{(a+1)(a-3)}{a+2} \div \frac{a+1}{a+2}$ **i.** $\frac{8m}{(m+2)(m-5)} \div \frac{2m}{m-5}$

j. $\frac{(a+1)(a-3)}{(a-5)(a+4)} \div \frac{a+1}{a+4}$ **k.** $\frac{m(m-2)}{(m+1)(m+5)} \div \frac{m}{m+1}$ **l.** $\frac{p}{p+7} \div \frac{p(p-4)}{(p+2)(p+7)}$

3. WE8 Simplify each of the following by first factorising the numerator and then the denominator and cancelling as appropriate.

a. $\frac{2a+2}{2}$ **b.** $\frac{3a+6}{9}$ **c.** $\frac{4a-4}{4}$ **d.** $\frac{7p-21}{7}$

e. $\frac{4x+8}{8}$ **f.** $\frac{3x-12}{6}$ **g.** $\frac{9x+54}{3}$ **h.** $\frac{12x-96}{4}$

i. $\frac{12x-24}{18}$ **j.** $\frac{9x-45}{15}$ **k.** $\frac{2}{2x+10}$ **l.** $\frac{-6}{6x-24}$

m. $\frac{2x+2}{3x+3}$ **n.** $\frac{3x-6}{4x-8}$ **o.** $\frac{5x-15}{6x-18}$ **p.** $\frac{9x+18}{3x+6}$

Understanding

4. WE8 Simplify each of the following fractions by first factorising the numerator and then the denominator and cancelling as appropriate.

a. $\frac{x^2+3x}{x}$ **b.** $\frac{p^2-5p}{p}$ **c.** $\frac{y^2-4y}{y}$ **d.** $\frac{a^2+10a}{a}$

e. $\frac{7x}{5x-x^2}$ **f.** $\frac{5b}{5b-b^2}$ **g.** $\frac{9y}{y^2-9y}$ **h.** $\frac{4m}{m^2-4m}$

i. $\frac{7x}{x-2x^2}$ **j.** $\frac{3m}{2m^2-m}$ **k.** $\frac{a-2a^2}{a^2+3a}$ **l.** $\frac{2b^2+b}{5b-8b^2}$

m. $\frac{3x^2+x}{9x-x^2}$ **n.** $\frac{7b-2b^2}{b^2+5b}$ **o.** $\frac{9m^2+18m}{12m^2-3m}$ **p.** $\frac{32a-16a^2}{24a+16a^2}$

5. WE9 Simplify each of the following algebraic fractions by first factorising the numerator, then the denominator.

a. $\frac{x^2 + 5x + 6}{x + 3}$ **b.** $\frac{x^2 + 7x + 12}{x + 4}$ **c.** $\frac{x^2 - 9x + 20}{x - 5}$ **d.** $\frac{b^2 - 8b + 7}{b - 1}$

e. $\frac{a^2 - 18a + 81}{a - 9}$ **f.** $\frac{a^2 - 22a + 121}{a - 11}$ **g.** $\frac{x^2 - 49}{x - 7}$ **h.** $\frac{m^2 - 64}{m - 8}$

i. $\frac{a^2 - 7a + 12}{a^2 - 16}$ **j.** $\frac{p^2 - 4p - 5}{p^2 - 25}$ **k.** $\frac{x^2 + 6x + 9}{x^2 + 2x - 3}$ **l.** $\frac{m^2 - 2m + 1}{m^2 + 5m - 6}$

m. $\frac{y^2 - 4y - 12}{y^2 - 36}$ **n.** $\frac{x^2 - 4x + 4}{x^2 - 4}$ **o.** $\frac{x^2 + 3x - 40}{x^2 + 6x - 16}$ **p.** $\frac{x^2 + 3x - 18}{x^2 - 6x + 9}$

6. WE10 Simplify each of the following by factorising the numerator and denominator and cancelling as appropriate.

a. $\frac{20x + 40}{2x + 8} \times \frac{x + 4}{x + 2}$ **b.** $\frac{9a + 27}{3a - 3} \times \frac{a - 1}{a + 3}$ **c.** $\frac{m + 5}{m - 2} \times \frac{8m - 16}{2m + 10}$

d. $\frac{q - 3}{q + 1} \times \frac{15q + 15}{5q - 15}$ **e.** $\frac{3x - 6}{x + 6} \times \frac{x + 6}{4x - 8}$ **f.** $\frac{3a + 6}{2a - 6} \div \frac{a + 3}{8a - 24}$

g. $\frac{10x - 5}{4x + 28} \div \frac{20x - 10}{6x + 1}$ **h.** $\frac{m^2 + 8m + 15}{m^2 + 7m + 10} \times \frac{m^2 + 6m + 8}{m^2 + 10m + 21}$

i. $\frac{x^2 + 6x + 8}{x^2 + 5x + 6} \times \frac{x^2 + 8x + 15}{x^2 + 7x + 12}$ **j.** $\frac{x^2 + x - 6}{x^2 + 5x - 14} \times \frac{x^2 + 9x + 14}{x^2 - x - 12}$

Reasoning

7. Explain why $\frac{x + 5}{x + 1}$ does not equal $\frac{5}{1}$. Illustrate your answer with an example.

8. Show that $\frac{3x + 11}{x + 2} = 3 + \frac{5}{x + 2}$.

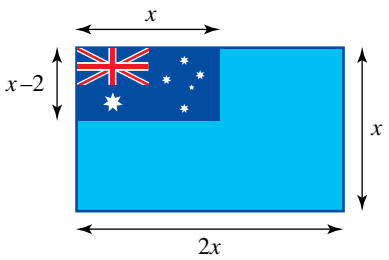
Problem solving

9. Simplify the following fractions.

a. $\frac{c^2 + 14c + 24}{c^2 - 16c - 36}$ **b.** $\frac{m^2 + 2m + 1}{m^2 + 9m + 8}$ **c.** $\frac{f^2 - 81}{f^2 - 3f - 54}$

d. $\frac{r^2 - 28r + 196}{r^2 + 28r + 196}$ **e.** $\frac{w^2 + 10w + 25}{w^2 + 25w + 100}$ **f.** $\frac{p^2 - 49}{p^2 - 64}$

10. A proposed new flag for Australian schools will have the Australian flag in the top left-hand corner. The dimensions given are in metres.



a. Write an expression, in factorised form, for the area of:
i. the Australian flag **ii.** the proposed flag.

b. Use the answers to part **a** to write the area of the Australian flag as a *fraction* of the school flag and then simplify this fraction.

c. Use the fraction from part **b** to express the area of the Australian flag as a *percentage* of the proposed school flag.

d. Using the formula for the percentage of area taken up by the Australian flag, find the percentages for the following school flag widths.
i. 4 m **ii.** 4.5 m **iii.** 4.8 m

Reflection

What is the sequence of steps involved in simplifying algebraic expressions that are being multiplied or divided?

CHALLENGE 16.2

Simplify the expression $1 + \dfrac{\dfrac{1}{x}}{1 + \dfrac{\dfrac{1}{x}}{1 + \dfrac{1}{x}}}$.

16.6 Quadratic equations

16.6.1 Quadratic equations

A **quadratic equation** is an equation of the general form $ax^2 + bx + c = 0$.
For example, $x^2 + 2x - 7 = 0$ is a quadratic equation, and so are $2x^2 = 8$ and $x^2 = 5x$.

WORKED EXAMPLE 11

Rearrange the following quadratic equations so that they are in general form and state the values of a, b and c.

a $5x^2 - 2x + 3 = 2x^2 + 4x - 12$ **b** $\dfrac{x^2}{2} - \dfrac{1}{6} = x\left(\dfrac{x}{3}\right) - 4$ **c** $x(3 - 2x) = 4(x - 6)$

THINK	WRITE
a 1 Write the equation.	**a** $5x^2 - 2x + 3 = 2x^2 + 4x - 12$
2 Subtract $2x^2$ from both sides of the equation.	$3x^2 - 2x + 3 = 4x - 12$
3 Subtract $4x$ from both sides of the equation.	$3x^2 - 6x + 3 = -12$
4 Add 12 to both sides of the equation. The equation is now in the general form $ax^2 + bx + c = 0$.	$3x^2 - 6x + 15 = 0$
5 Write the values of a, b and c.	$a = 3, b = -6, c = 15$
b 1 Write the equation.	**b** $\dfrac{x^2}{2} - \dfrac{1}{6} = x\left(\dfrac{x}{3}\right) - 4$
2 Expand the bracket. Lowest common denominator (LCD) = 6.	$\dfrac{x^2}{2} - \dfrac{1}{6} = \dfrac{x^2}{3} - 4$
3 Multiply through by 6.	$\dfrac{x^2}{2} \times \dfrac{6}{1} - \dfrac{1}{6} \times \dfrac{6}{1} = \dfrac{x^2}{3} \times \dfrac{6}{1} - 4 \times 6$
4 Simplify each fraction.	$3x^2 - 1 = 2x^2 - 24$
5 Collect like terms on the left-hand side. The equation is now in the general form $ax^2 + bx + c = 0$.	$x^2 - 1 = -24$ $x^2 + 23 = 0$
6 Write the values of a, b and c.	$a = 1, b = 0, c = 23$
c 1 Write the equation.	**c** $x(3 - 2x) = 4(x - 6)$
2 Expand the brackets.	$3x - 2x^2 = 4x - 24$
3 To collect like terms on the left-hand side of the equation, subtract $4x$ from both sides.	$-2x^2 - x = -24$

4	Add 24 to both sides.	$-2x^2 - x + 24 = 0$
5	Multiply all terms by -1 to make the x^2 term positive. The equation is now in the general form $ax^2 + bx + c = 0$.	$2x^2 + x - 24 = 0$
6	Write the values of a, b and c.	$a = 2,\ b = 1,\ c = -24$

16.6.2 Solving the equation $ax^2 + c = 0$

- Solving a quadratic equation means finding the values of x that satisfy the equation.
- Some quadratic equations have two solutions, some have only one solution, and some have no solutions.

WORKED EXAMPLE 12

TI | CASIO

Solve the following equations.

a $2x^2 - 18 = 0$ **b** $x^2 + 9 = 0$ **c** $3x^2 + 4 = 4$ **d** $3(x + 2)^2 = 27$

THINK		WRITE	
a 1	Write the equation. The aim is to make x the subject.	**a**	$2x^2 - 18 = 0$
2	Add 18 to both sides.		$2x^2 = 18$
3	Divide both sides by 2.		$x^2 = 9$
4	Take the square root of both sides. There are two square roots of 9.		$x = 3, -3$ Two solutions: $x = 3,\ x = -3$
b 1	Write the equation. The aim is to make x the subject.	**b**	$x^2 + 9 = 0$
2	Take 9 from both sides.		$x^2 = -9$
3	Take the square root of both sides.		-9 has no square root. The equation has no solution.
c 1	Write the equation. The aim is to make x the subject.	**c**	$3x^2 + 4 = 4$
2	Take 4 from both sides.		$3x^2 = 0$
3	Divide both sides by 3.		$x^2 = 0$
4	Take the square root of both sides. Zero has only one square root.		$x = 0$ One solution: $x = 0$
d 1	Write the equation. The aim is to make x the subject.	**d**	$3(x + 2)^2 = 27$
2	Divide both sides by 3.		$(x + 2)^2 = 9$
3	Take the square root of both sides. This gives two linear equations.		$x + 2 = 3$ or $x + 2 = -3$
4	Solve the two equations.		$x = 1,\ x = -5$

16.6.3 Confirming the solutions to quadratic equations

- Solutions to quadratic equations are confirmed by substituting the values into the equation.

WORKED EXAMPLE 13

Determine whether any of the following are solutions to the quadratic equation $x^2 = 4x - 3$.
a $x = 1$ **b** $x = 2$ **c** $x = 3$

THINK	WRITE
a 1 Substitute $x = 1$ into both sides of the equation.	**a** $x^2 = 4x - 3$
2 Evaluate the left-hand side.	LHS $= (1)^2$ $= 1$
3 Evaluate the right-hand side.	RHS $= 4(1) - 3$ $= 4 - 3$ $= 1$
4 Since the left-hand side equals the right-hand side, $x = 1$ is a solution. Write the answer.	$x = 1$ is a solution to the equation.
b 1 Substitute $x = 2$ into both sides of the equation.	**b** $x^2 = 4x - 3$
2 Evaluate the left-hand side.	LHS $= (2)^2$ $= 4$
3 Evaluate the right-hand side.	RHS $= 4(2) - 3$ $= 8 - 3$ $= 5$
4 The left-hand side does not equal the right-hand side, so $x = 2$ is not a solution. Write the answer.	$x = 2$ is not a solution to the equation.
c 1 Substitute $x = 3$ into both sides of the equation.	**c** $x^2 = 4x - 3$
2 Evaluate the left-hand side.	LHS $= (3)^2$ $= 9$
3 Evaluate the right-hand side.	RHS $= 4(3) - 3$ $= 12 - 3$ $= 9$
4 The left-hand side equals the right-hand side, so $x = 3$ is a solution. Write the answer.	$x = 3$ is a solution to the equation.

Exercise 16.6 Quadratic equations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 5, 7, 9, 10	Questions: 1, 3, 5, 7–12	Questions: 2, 4–12

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Fluency

1. WE11 Rearrange each of the following quadratic equations so that they are in the form $ax^2 + bx + c = 0$.

a. $3x - x^2 + 1 = 5x$ **b.** $5(x - 2) = x(4 - x)$ **c.** $x(5 - 2x) = 6(5 - x^2)$

d. $4x^2 - 5x + 2 = 6x - 4x^2$ **e.** $5(x - 12) = x(4 - 2x)$ **f.** $x^2 - 4x - 16 = 1 - 4x$

2. MC **a.** Which of the following is a quadratic equation?

A. $2x - 1 = 0$ **B.** $2^x - 1 = 0$ **C.** $x^2 - x = 1 + x^2$ **D.** $x^2 - x = 1 - x^2$

b. Which of the following is not a quadratic equation?

A. $x(x - 1) = 2x - 1$ **B.** $-3x^2 + 2x = 1$

C. $3(x + 2) + 5(x + 3) = 2(x + 1)$ **D.** $2(x - 1) + 3x = x(2x - 3)$

3. WE12a, b, c Solve the following quadratic equations.

a. $x^2 - 16 = 0$ **b.** $2x^2 + 18 = 0$ **c.** $3x^2 + 2x = x(x + 2)$

d. $2(x^2 + 7) = 16$ **e.** $x^2 + 17 = 13$ **f.** $-3x^2 + 17 = 5$

4. WE12 Solve the following quadratic equations.

a. $4(x + 2)^2 - 4 = 0$ **b.** $4(x + 2)^2 + 4 = 0$ **c.** $2(x + 3)^2 - 32 = 0$

d. $2(x - 4)^2 - 32 = 0$ **e.** $(x + 2)^2 - 9 = 0$ **f.** $-(x - 2)^2 - 9 = 0$

g. $3(x + 5)^2 - 10 = 2$ **h.** $3(x - 4)^2 + 48 = 36$ **i.** $2(x - 8)^2 + 4 = 4$

Understanding

5. WE13 Determine whether $x = 4$ is a solution to the following equations.

a. $x^2 = x + 12$ **b.** $x^2 = 3x + 1$ **c.** $x^2 = 4x$

6. Determine whether $x = -3$ is a solution to the following equations.

a. $x^2 = x + 12$ **b.** $x^2 = 3x + 1$ **c.** $x^2 = 4x$

7. Determine whether $x = 0$ is a solution to the following equations.

a. $x^2 = x + 12$ **b.** $x^2 = 3x + 1$ **c.** $x^2 = 4x$

8. Determine whether either of the following are solutions to the equation $5(x - 1)^2 + 7 = 27$.

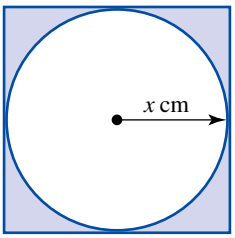
a. $x = -1$ **b.** $x = 1$

Reasoning

9. a. Show that the shaded area can be represented by $A = x^2(4 - \pi)$.

b. If the shaded area $= 10$ cm^2, find the value of x correct to 3 significant figures.

c. Why can't x be equal to -3.41?



10. Explain why $x^2 + 7x + 4 = 7x$ has no solutions.

Problem solving

11. The sum of 5 and the square of a number is 41.

a. What is the number?

b. Is there more than one possible number? If so, what is the other number and why is there more than one?

12. Two less than a number is squared and its result is tripled. The difference between this and 7 is 41.

a. Write the algebraic equation. **b.** What is the number? Give both possible answers.

16.7 The Null Factor Law

16.7.1 The Null Factor Law

- The **Null Factor Law** states that if the product of two or more factors is zero, then *at least one* of the factors must be zero.

If $a \times b = 0$
then:
$a = 0$
$b = 0$ or
$a = b = 0$.

For example, if $(x - 5)(x - 2) = 0$ then:
$(x - 5) = 0$ or $(x - 2) = 0$ or both $x - 5 = x - 2 = 0$.
If $x - 5 = 0$ If $x - 2 = 0$
$x = 5$ $x = 2$
Both $x = 5$ and $x = 2$ make the equation true.

- This can be checked by substituting the values into the original equation.
  For example:

if $x = 5$, then:
$(x - 5)(x - 2) = (5 - 5)(x - 2)$
$= 0(x - 2)$
$= 0$

if $x = 2$, then:
$(x - 5)(x - 2) = (x - 5)(2 - 2)$
$= (x - 5)0$
$= 0$

WORKED EXAMPLE 14

TI | CASIO

Solve each of the following quadratic equations.
a $(x - 2)(2x + 1) = 0$ **b** $(4 - 3x)(6 + 11x) = 0$ **c** $x(x - 3) = 0$ **d** $(x - 1)^2 = 0$

THINK	WRITE
a 1 The product of 2 factors is 0, so apply the Null Factor Law.	**a** $(x - 2)(2x + 1) = 0$
2 One of the factors must equal zero.	Either $x - 2 = 0$ or $2x + 1 = 0$.
3 Solve the equations.	$x = 0$ $2x = -1$ $= -\frac{1}{2}$
4 Write the solutions.	$x = -\frac{1}{2}, x = 2$
b 1 The product of 2 factors is 0, so apply the Null Factor Law.	**b** $(4 - 3x)(6 + 11x) = 0$
2 One of the factors must equal zero.	Either $4 - 3x = 0$ or $6 + 11x = 0$.
3 Solve the equations.	$-3x = -4$ $11x = -6$ $x = \frac{4}{3}$ $x = \frac{6}{11}$
4 Write the solutions.	$x = \frac{4}{3}, -\frac{6}{11}$
c 1 The product of 2 factors is 0, so apply the Null Factor Law.	**c** $x(x - 3) = 0$
2 One of the factors must equal zero.	Either $x = 0$ or $x - 3 = 0$.
3 Solve the equations.	$x = 3$
4 Write the solutions.	$x = 0, 3$
d 1 Write the equation.	**d** $(x - 1)^2 = 0$
2 Rewrite the squared factors as the product of two factors.	$(x - 1)(x - 1) = 0$
3 Apply the Null Factor Law.	Either $x - 1 = 0$ or $x - 1 = 0$.
4 Solve the equations. The two solutions are the same.	$x = 1$ $x = 1$
5 Write the solution.	$x = 1$

- *Note:* Quadratic equations can have a maximum of two solutions. (This is known as the **Fundamental Theorem of Algebra**.)

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Exercise 16.7 The Null Factor Law

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–i, 2a–d, 3–6	Questions: 1d–l, 2c–f, 3–7	Questions: 1h–o, 2e–h, 3–9

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE14 Solve each of the following quadratic equations.

a. $(x - 2)(x + 3) = 0$ **b.** $(2x + 4)(x - 3) = 0$ **c.** $(x + 2)(x - 3) = 0$

d. $(2x + 5)(4x + 3) = 0$ **e.** $(x + 4)(2x + 1) = 0$ **f.** $(2x - 1)(x + 30) = 0$

g. $(2x + 1)(3 - x) = 0$ **h.** $(1 - x)(3x - 1) = 0$ **i.** $x(x - 2) = 0$

j. $(x + \frac{1}{3})(2x - \frac{1}{2}) = 0$ **k.** $(5x - 1.5)(x + 2.3) = 0$ **l.** $(2x + \frac{1}{3})(2x - \frac{1}{3}) = 0$

m. $(x - 2)^2 = 0$ **n.** $x(4x - 15) = 0$ **o.** $(x + 4)^2 = 0$

2. The Null Factor Law can be extended to products of more than 2 factors. Use this to find all the solutions to the following equations.

a. $(x - 2)(x + 2)(x + 3) = 0$ **b.** $(x + 2)(x + 2)(2x - 5) = 0$

c. $(x + 2)(x + 2)(x + 4) = 0$ **d.** $x(x + 2)(3x + 12) = 0$

e. $(2x - 2.2)(x + 2.4)(x + 2.6) = 0$ **f.** $(2x + 6)(x + \frac{1}{2})(9x - 15) = 0$

g. $3(x - 3)^2 = 0$ **h.** $(x + 1)(x - 2)^2 = 0$

3. MC **a.** The solutions to $(2x - 4)(x + 7) = 0$ are:

A. $x = 4,\ x = 7$ **B.** $x = 4,\ x = -7$

C. $x = 2,\ x = 7$ **D.** $x = 2,\ x = -7$

b. The Null Factor Law cannot be applied to the equation $x(x + \frac{1}{2})(\frac{x}{2} - 1) = 1$ because:

A. there are more than 2 factors **B.** the right-hand side equals 1

C. the first factor is a simple x-term **D.** the third term has x in a fraction

Understanding

4. Rewrite the following equations so that the Null Factor Law can be used. Then solve the resulting equation.

a. $x^2 + 10x = 0$ **b.** $2x^2 - 14x = 0$ **c.** $25x^2 - 40x = 0$

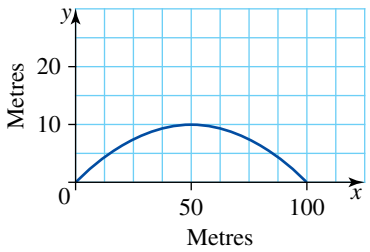
Reasoning

5. a. Explain why $x = 2$ is not a solution of the equation $(2x - 2)(x + 2) = 0$.
b. What are the solutions to the equation?

6. What is the maximum number of solutions that a quadratic such as $x^2 - x - 56 = 0$ can have? Explain why.

Problem solving

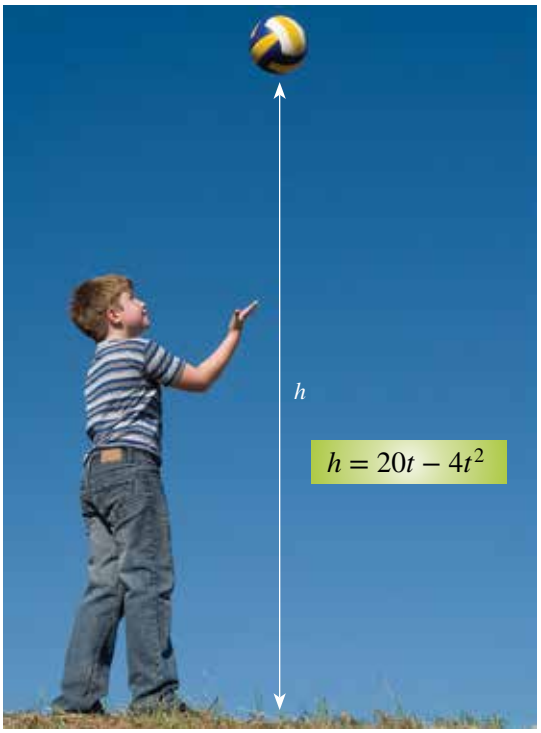
7. A bridge is constructed with a supporting structure in the shape of a parabola as shown in the following diagram. The origin (0, 0) is at the left-hand edge of the bridge, which is 100 m long.



a. What is the maximum height of the bridge support?
b. If the equation of the support is $y = ax(b - x)$, determine the values of a and b. (*Hint:* Let $y = 0$.)
c. What is the height of the support when $x = 62$?

8. Consider a ball thrown upwards so that it reaches a height of h metres after t seconds. The expression $20t - 4t^2$ represents the height of the ball, in metres, after t seconds.
a. Factorise the expression for height.
b. How high is the ball after:
i. 1 second
ii. 5 seconds?
c. How did factorising the expression make it easier to evaluate?

9. The product of 7 more than a number and 14 more than that same number is 330. What are the possible values of the number?



Reflection

When can the Null Factor Law be used, and when is it not able to be used?

16.8 Solving the quadratic equation $ax^2 + bx + c = 0$

16.8.1 Solving the quadratic equation $ax^2 + bx + c = 0$

- If $ax^2 + bx + c$ can be factorised, then the quadratic equation $ax^2 + bx + c = 0$ can be solved using the Null Factor Law.

WORKED EXAMPLE 15 TI | CASIO

Solve the following quadratic equations.
a $x^2 - 5x - 6 = 0$ **b** $x^2 + 14x = 15$ **c** $2x^2 + 7x + 3 = 0$

THINK	WRITE
a 1 Write the equation.	**a** $x^2 - 5x - 6 = 0$
2 Factorise $x^2 - 5x - 6$.	$(x - 6)(x + 1) = 0$
3 Apply the Null Factor Law.	$x - 6 = 0 \quad \vert \quad x + 1 = 0$
4 Solve the equations.	$x = 6 \quad \vert \quad x = -1$
5 Write the solutions.	$x = -1,\ x = 6$
b 1 Write the equation and transpose so that the right-hand side equals 0.	**b** $x^2 + 14x = 15$ $x^2 + 14x - 15 = 0$
2 Factorise $x^2 - 14x - 15$.	$(x + 15)(x - 1) = 0$
3 Apply the Null Factor Law.	$x + 15 = 0 \quad \vert \quad x - 1 = 0$ $x = -15 \quad \vert \quad x = 1$
4 Solve the linear equations.	
5 Write the solutions.	$x = -15,\ x = 1$
c 1 Write the equation, making sure that the right-hand side is equal to zero. Check for common factors.	**c** $2x^2 + 7x + 3 = 0$
2 Rewrite the expression on the left-hand side by breaking the middle term into two terms.	$2x^2 + 6x + x + 3 = 0$
3 Factorise by grouping terms.	$2x(x + 3) + 1(x + 3) = 0$ $(2x + 1)(x + 3) = 0$
4 Apply the Null Factor Law.	$2x + 1 = 0 \quad \vert \quad x + 3 = 0$
5 Solve the linear equations.	$x = -\frac{1}{2} \quad \vert \quad x = -3$
6 Write the solutions.	$x = -\frac{1}{2},\ x = -3$

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Exercise 16.8 Solving the quadratic equation $ax^2 + bx + c = 0$

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a, c, g, k, o, 2–5, 7	Questions: 1b, d, f, j, n, 2–5, 7–9	Questions: 1b, d, f, h, j, l, n, 2–11

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE15 Solve each of the following quadratic equations.

a. $x^2 - 6x + 8 = 0$ **b.** $x^2 + 6x + 8 = 0$ **c.** $x^2 + 6x + 5 = 0$ **d.** $x^2 + x - 6 = 0$

e. $x^2 + 2x - 15 = 0$ **f.** $x^2 + 4x + 4 = 0$ **g.** $x^2 + 2x - 24 = 0$ **h.** $x^2 - 5x - 24 = 0$

i. $x^2 - x - 12 = 0$ **j.** $x^2 + 13x + 12 = 0$ **k.** $x^2 - 10x = 11$ **l.** $x^2 + x = 20$

m. $x^2 + 29x = -100$ **n.** $x^2 - 15x = -50$ **o.** $0 = x^2 - 2x - 8$

2. Solve each of the following quadratic equations.

a. $2x^2 + 7x + 3 = 0$ **b.** $2x^2 + x - 1 = 0$ **c.** $2x^2 - x - 15 = 0$ **d.** $2x^2 + 8x + 6 = 0$

e. $3x^2 + 13x + 14 = 0$ **f.** $3x^2 + 5x + 2 = 0$ **g.** $5x^2 - 22x + 21 = 0$ **h.** $5x^2 - 17x + 6 = 0$

i. $7x^2 - 2x - 5 = 0$ **j.** $7x^2 - 33x + 20 = 0$ **k.** $4x^2 + 4x - 3 = 0$ **l.** $6x^2 + 17x + 5 = 0$

3. MC **a.** The quadratic equation $x^2 + 2x - 8 = 0$ has solutions:

A. $x = 4$ and $x = -2$ **B.** $x = -4$ and $x = 2$

C. $x = -4$ and $x = 4$ **D.** $x = -2$ and $x = 2$

b. The quadratic equation $x^2 - 7x - 8 = 0$ has solutions:

A. $x = 1$ and $x = 8$ **B.** $x = -1$ and $x = -8$

C. $x = -8$ and $x = 1$ **D.** $x = -1$ and $x = 8$

c. The quadratic equation $4a^2 + 13a + 3$ has solutions:

A. $a = 3$ and $a = 1$ **B.** $a = -3$ and $a = -1$

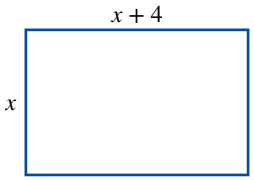
C. $a = \frac{1}{4}$ and $a = 3$ **D.** $a = -3$ and $a = -\frac{1}{4}$

Understanding

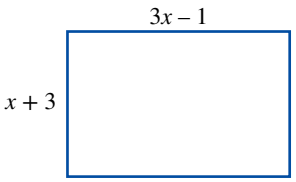
4. Solve the following for x.

$2x^2 + 4x + 5 = 3x^2 + 9x - 1$

5. The rectangle shown has an area of 45 cm^2. By solving a quadratic equation, find the dimensions of the rectangle.

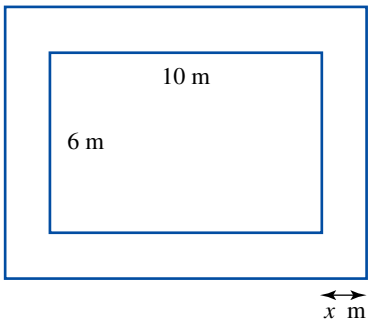


6. Find the dimensions of this rectangle, which has an area of 48 m^2.

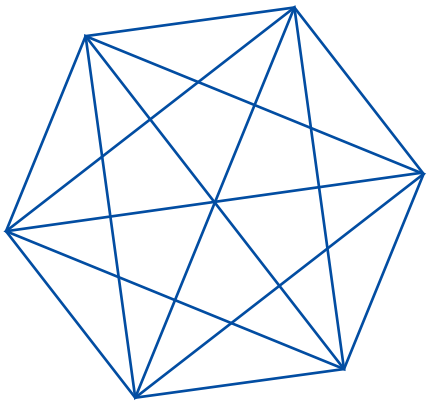


Reasoning

7. A small rectangular pool, 6 m by 10 m, is surrounded by a paved pathway of uniform width as shown. How wide is the pavement if it covers an area of 57 m^2?



8. The number of diagonals in a polygon is given by the formula $D = \frac{n}{2}(n - 3)$, where n is the number of sides in the polygon.



a. How many diagonals are there in:
i. a triangle **ii.** a square **iii.** a decagon?

b. What type of polygon has:
i. 20 diagonals **ii.** 170 diagonals?

Problem solving

9. a. Write algebraic expressions for each of the parts below. Your friend picks a number, n.
i. He adds 1 to it.
ii. He adds 7 to it.
iii. He multiplies these two new numbers together.
iv. He then divides the product in part **iii** by the product of 2 more than the original number, n, and 1 more than the original number, n.

b. The result of the division is 2. What number, n, did he begin with?

10. Calculate the value(s) of x for which the expressions $4x^2 + 13x - 16$ and $3x^2 + 8x - 2$ are equal by following the steps outlined below:
a. Equate the expressions.
b. Rewrite the resulting equation by collecting like terms so that the equation is equal to zero.
c. Factorise the quadratic equation.
d. Apply the Null Factor Law and solve.

11. A girl, her brother and her teacher are shown below. The product of the teacher's and brother's ages is 256.
a. Write an algebraic expression in terms of g to represent the teacher's age.
b. Use the algebraic expressions to write an expression for the product of the brother's and teacher's ages and expand this product.
c. Write an equation using your answer to part **b** for the product of their ages. *Hint:* An equation has an 'is equal to' part to it.
d. Rearrange your answer to part **c** by collecting like terms and making it equal to zero.
e. Find the value of g by solving the equation in part **d** using the Null Factor Law.
Note: Discard any value that does not make sense.
f. State the ages of the girl, her brother and the teacher.



16.9 Solving quadratic equations with two terms

16.9.1 Solving quadratic equations of the form $ax^2 + c = 0$

- Earlier in this topic, the equation $2x^2 - 18 = 0$ was solved using algebraic techniques. It can also be solved using the Null Factor Law.

WORKED EXAMPLE 16

Solve each of the following quadratic equations.
a $x^2 - 1 = 0$ **b** $2x^2 - 18 = 0$

THINK	WRITE
a 1 Write the equation: $x^2 - 1$ is the difference of two squares.	**a** $x^2 - 1 = 0$
2 Factorise the left-hand side using the difference of two squares rule.	$(x + 1)(x - 1) = 0$
3 Apply the Null Factor Law.	$x + 1 = 0 \quad \vert \quad x - 1 = 0$
4 Solve the equations.	$x = -1 \quad \vert \quad x = 1$
5 State the solutions.	$x = 1, x = -1$ (This can be abbreviated to $x = \pm 1$.)
b 1 Write the equation.	**b** $2x^2 - 18 = 0$
2 Take out the common factor.	$2(x^2 - 9) = 0$
3 Divide both sides of the equation by 2.	$x^2 - 9 = 0$
4 Factorise the left-hand side using the difference of two squares rule.	$(x + 3)(x - 3) = 0$
5 Apply the Null Factor Law.	$x + 3 = 0 \quad \vert \quad x - 3 = 0$
6 Solve the equations.	$x = -3 \quad \vert \quad x = 3$
7 State the solutions.	$x = \pm 3$

16.9.2 Solving quadratic equations of the form $ax^2 + bx = 0$

- The equation $ax^2 + bx = 0$ can be solved easily using the Null Factor Law.

WORKED EXAMPLE 17

TI | CASIO

Solve each of the following equations.
a $x^2 + 4x = 0$ **b** $-2x^2 - 4x = 0$

THINK	WRITE
a 1 Write the equation.	**a** $x^2 + 4x = 0$
2 Factorise by taking out a common factor of x.	$x(x + 4) = 0$
3 Apply the Null Factor Law.	$x = 0 \quad x + 4 = 0$ $x = 0 \quad x = -4$

4 Solve the equations.

5 Write the solutions. $x = -4,\ x = 0$

b 1 Write the equation. **b** $-2x^2 - 4x = 0$

2 Factorise by taking out the common factor of $-2x$. $-2x(x + 2) = 0$

3 Apply the Null Factor Law. $-2x = 0 \quad x + 2 = 0$
$x = 0 \quad x = -2$

4 Solve the equations.

5 Write the solutions. $x = -2,\ x = 0$

WORKED EXAMPLE 18

If the square of a number is multiplied by 5, the answer is 45. Find the number.

THINK	WRITE
1 Define the number.	Let x be the number.
2 Write an equation that can be used to find the number.	$5x^2 = 45$
3 Transpose to make the right-hand side equal to zero. Solve the equation.	$5x^2 - 45 = 0$ $5(x^2 - 9) = 0$ $x^2 - 9 = 0$ $x^2 - 3^2 = 0$ $(x + 3)(x - 3) = 0$ $x + 3 = 0$ or $x - 3 = 0$ $x = -3$ or $x = 3$
4 Write the answer in a sentence.	The number is either 3 or -3.

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Exercise 16.9 Solving quadratic equations with two terms

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7	Questions: 1–9, 11, 12	Questions: 1–13

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Fluency

1. WE16 Solve each of the following quadratic equations using the Null Factor Law.

a. $x^2 - 9 = 0$ **b.** $x^2 - 16 = 0$ **c.** $2x^2 - 18 = 0$ **d.** $2x^2 - 50 = 0$

e. $100 - x^2 = 0$ **f.** $49 - x^2 = 0$ **g.** $3x^2 - 27 = 0$ **h.** $5x^2 - 20 = 0$

i. $x^2 + 6 = 0$ **j.** $2x^2 + 18 = 0$ **k.** $-x^2 + 9 = 0$ **l.** $-3x^2 + 48 = 0$

m. $-4x^2 + 100 = 0$ **n.** $x^2 = 0$ **o.** $-x^2 = 0$

2. WE17 Solve each of the following equations.

a. $x^2 + 6x = 0$ **b.** $x^2 - 8x = 0$ **c.** $x^2 + 9x = 0$ **d.** $x^2 - 11x = 0$

e. $2x^2 - 12x = 0$ **f.** $2x^2 - 15x = 0$ **g.** $3x^2 - 2x = 0$ **h.** $4x^2 + 7x = 0$

i. $2x^2 - 5x = 0$ **j.** $x^2 + x = 0$ **k.** $4x^2 - x = 0$ **l.** $-x^2 - 5x = 0$

m. $-2x^2 - 24x = 0$ **n.** $-x^2 + 18x = 0$ **o.** $x^2 - 2.5x = 0$

3. MC **a.** The solutions to $4x^2 - 36 = 0$ are:

A. $x = 3$ and $x = -3$ **B.** $x = 9$ and $x = -9$

C. $x = 1$ and $x = -1$ **D.** $x = 2$ and $x = -2$

b. What are the solutions to $x^2 - 5x = 0$?

A. $x = 1$ and $x = 5$ **B.** $x = 0$ and $x = -5$

C. $x = 0$ and $x = 5$ **D.** $x = -1$ and $x = 5$

Understanding

4. WE18 If the square of a number is multiplied by 2, the answer is 32. Find the number.

5. A garden has two vegetable plots. One plot is a square; the other plot is a rectangle with one side 3 m shorter than the side of the square and the other side 4 m longer than the side of the square. Both plots have the same area. Sketch a diagram and find the dimensions of each plot.

6. The square of a number is equal to 10 times the same number. What is the number?

Reasoning

7. Explain why the equation $x^2 + 9 = 0$ cannot be solved using the Null Factor Law.

8. Solve the equation $m^2x^2 - n^2 = 0$:

a. by working backwards **b.** using the Null Factor Law.

9. a. Explain why the equation $x^2 + bx = 0$ cannot be solved by working backwards.

b. Solve the equation using the Null Factor Law.

10. A quadratic expression may be written as $ax^2 + bx + c$. Using examples, explain why b and c may take any values, but a cannot equal zero.

Problem solving

11. The square of a number tripled is equal to 24 times the same number. What is the number?

12. The sum of 6 times the square of a number and 72 times the same number is equal to zero. What is the number?

13. A rectangular horse paddock has a width of x and a length that is 30 metres longer than its width. The area of the paddock is $50x$ square metres.

a. Write the algebraic equation for the area described.

b. Factorise the equation.

c. Solve the equation.

d. Can you use all your solutions? Explain why or why not.



16.10 Applications

16.10.1 Using quadratic equations to solve problems

- There are many situations in science, engineering, economics and other fields where quadratic equations can be applied to finding solutions to problems.

WORKED EXAMPLE 19

TI | CASIO

The distance travelled by an accelerating skier is given by the formula $d = 3t + t^2$, where t is the time in seconds and d is the distance in metres. If the distance travelled was 130 m, for how long was the skier travelling?



THINK	WRITE
1 Write the equation.	$d = 3t + t^2$
2 Substitute the given value, $d = 130$, into the equation.	$130 = 3t + t^2$
3 Rearrange the equation so that it is in the form $ax^2 + bx + c = 0$.	$0 = 3t + t^2 - 130$ $t^2 + 3t - 130 = 0$
4 Factorise the left-hand side of the equation, using the factors of -130 that add up to 3.	$-130: -10 + 13 = 3$ $(t - 10)(t + 13) = 0$
5 Solve the equation by using the Null Factor Law.	Either $t - 10 = 0$ or $t + 13 = 0$ $t = 10$ $t = -13$
6 Evaluate the result.	The only feasible answer is $t = 10$ because time is always positive.
7 Answer the question in a sentence.	The skier was travelling for 10 seconds.

WORKED EXAMPLE 20

When 10 is added to the square of a positive number the result is equal to 3 times the number subtracted from twice its square. Find the number.

THINK	WRITE
1 Define the unknown quantity.	Let x be the positive number.

2 Translate each part of the sentence into an algebraic expression or term.	The square of a positive number: x^2 Add 10 to the (positive number)^2: $x^2 + 10$ Three times the number: $3x$ Three times the number is subtracted from twice its square: $2x^2 - 3x$
3 Form a quadratic equation and rearrange it so that it is in the form $ax^2 + bx + c = 0$.	$x^2 + 10 = 2x^2 - 3x$ $10 = x^2 - 3x$ $x^2 - 3x - 10 = 0$
4 Solve the equation by factorising.	$(x - 5)(x + 2) = 0$ $x - 5 = 0$ or $x + 2 = 0$ $x = 5$ $x = -2$
5 Evaluate the result.	The number is positive so $x = 5$ is the only valid solution.
6 Answer the question in a sentence.	The number is 5.

Exercise 16.10 Applications

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 3, 5, 7, 9	Questions: 1, 3–5, 7, 8, 10	Questions: 2, 4–10

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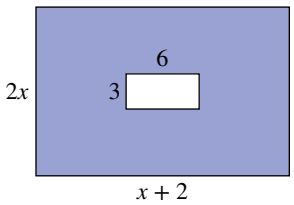
Fluency

1. The product of two consecutive numbers is 42. What are the numbers?

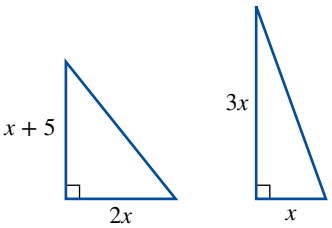
2. The product of two consecutive odd numbers is 143. What are the numbers?

Understanding

3. If the shaded area is 180 square units, find the value of x.

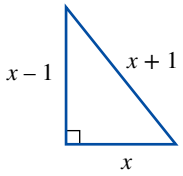


4. If these triangles have the same area, what is the value of x?

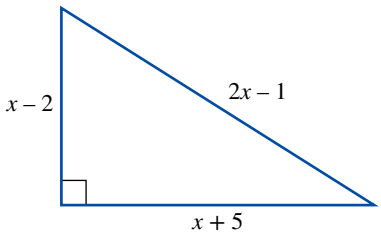


Reasoning

5. Find the side lengths of this right-angled triangle. Show your working.

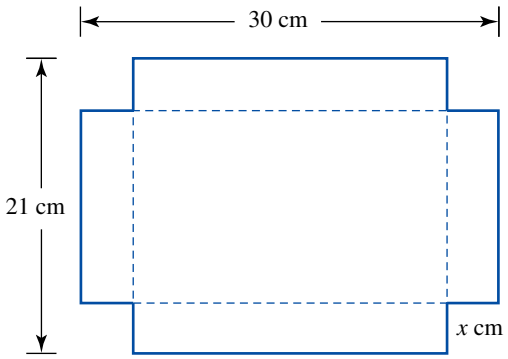


6. Find the side lengths of this right-angled triangle. Show your working.



Problem solving

7. A sheet of A4 paper is approximately 30 cm by 21 cm. Four corners are cut from a sheet as shown, and the remaining paper is folded to make a box with a base area of 286 cm^2. What are the dimensions of the box?



8. A gardener maintains a large rectangular rose garden of length 15 m and width 12 m. He plans to lay a wide pathway around the garden that will require 160 m^2 of paving. What will be the width of the pathway?



9. A cricket ball is struck high into the air. After t seconds its height, h, is given by the formula $h = 20t - 5t^2$.



a. What is its height after 1 second?

b. When will the ball strike the ground?

10. Each term in the sequence $-3, -1, 1, 3, 5 \ldots$ is 2 more than the previous term. The sum S of the first n terms in the sequence is given by the formula $S = n^2 - 4n$.

a. What is the sum of the first 4 terms?

b. How many terms are needed to make a sum of 77?

Reflection

Describe a situation that might require the application of quadratic equations.

16.11 Review

16.11.1 Review questions

Fluency

1. Which of the following is *not* a quadratic equation?

A. $x^2 - 1 = 0$ **B.** $x^2 - 1 + 2x = 0$ **C.** $x^2 - \frac{1}{2} = 0$ **D.** $x^2 - \frac{1}{2} = 0$

2. Which of the following is in general form?

A. $x^2 + 1 = 0$ **B.** $x^2 - 1 = 2$ **C.** $2x^2 = x^2 + 3$ **D.** $x^2 = x^2 + 1$

3. The Null Factor Law cannot be applied to the equation $x(x + 3)(x - 2) = 1$ because:

A. the first factor is a simple x term **B.** there are more than two factors

C. the right hand side equals 1 **D.** the second term is positive

4. If the solutions to the quadratic equation $(x - 3)(x - b) = 0$ are 3 and -5, then b is equal to:

A. 5 **B.** -5 **C.** 3 **D.** -3

5. The solutions to $3x^2 - 27 = 0$ are:

A. $x = 3$ and $x = -3$ **B.** $x = 9$ and $x = -9$ **C.** $x = 1$ and $x = -1$ **D.** $x = 2$ and $x = -2$

6. The quadratic equation $2x^2 + 7x + 3 = 0$ has solutions at:

A. $x = 3$ and $x = 1$ **B.** $x = -3$ and $x = -1$

C. $x = \frac{1}{2}$ and $x = 3$ **D.** $x = -3$ and $x = -\frac{1}{2}$

7. The quadratic equation $9(x + 9)^2 - 9 = 0$ has:

A. the solutions 9 and -9 **B.** the solutions $x = 1$ and -1

C. the solutions 8 and 10 **D.** the solutions -8 and -10

8. The expression $4a^2 - 12a + 9$ factorises to:

A. $(4a + 9)(4a + 9)$ **B.** $(4a - 9)(4a - 9)$ **C.** $(2a - 3)(2a - 3)$ **D.** $(2a - 3)(2a + 3)$

9. What does the expression $9m^2 - 16n^2$ factorise to?

A. $9(m - 4n)(m + 4n)$ **B.** $3(3m - 4n)^2$ **C.** $(9m - 16n)(9m + 16n)$ **D.** $(3m - 4n)(3m + 4n)$

10. What does the expression $5x^2 + 20x - 60$ factorise to?

A. $(5x - 12)(x + 5)$ **B.** $(5x + 15)(x - 4)$ **C.** $5(x + 6)(x - 2)$ **D.** $5(x - 6)(x + 2)$

11. What does the expression $3x^2 - 13x + 4$ factorise to?

A. $(3x - 1)(x - 4)$ **B.** $(3x - 2)(x + 2)$ **C.** $(3x - 2)^2$ **D.** $(3x - 4)(x - 1)$

12. $\dfrac{x + 3}{x + 2} \div \dfrac{(x + 3)^2}{x + 2}$ is equivalent to:

A. $\dfrac{(x + 3)^2}{(x + 2)^2}$ **B.** $\dfrac{1}{x + 3}$ **C.** $\dfrac{x + 2}{x + 3}$ **D.** $\dfrac{-1}{2(x + 2)}$

13. Find the solutions to the quadratic equation $(x - 3)(2x + 8) = 0$.

14. Determine the solutions to the equation $(4 - x)(2x - 7) = 0$.

15. Factorise each of the following using the difference of two squares rule.

a. $x^2 - 64$ **b.** $a^2 - 144$ **c.** $49b^2 - 1$

d. $4f^2 - 9g^2$ **e.** $(n + 1)^2 - m^2$ **f.** $(r - 1)^2 - 4s^2$

16. Factorise each of the following quadratic trinomials.

a. $c^2 + 5c + 4$ **b.** $p^2 + 10p - 24$ **c.** $y^2 - 10y + 24$

d. $x^2 + 3x + 2$ **e.** $m^2 - 7m + 10$ **f.** $m^2 + 24m + 44$

17. Factorise each of the following.

a. $2a^2 + 16a + 24$ **b.** $3b^2 - 24b + 36$ **c.** $4c^2 - 16c - 48$

d. $2x^2 + 3x + 1$ **e.** $3x^2 - x - 2$ **f.** $6x^2 + 13x + 6$

18. Find the solutions to the following quadratic equations.

a. $2x^2 - 98 = 0$ **b.** $5x^2 - 125 = 0$ **c.** $2x^2 + 14x = 0$

d. $x^2 - 4x - 5 = 0$ **e.** $x^2 - 11x + 10 = 0$ **f.** $2x^2 - 9x - 5 = 0$

19. Solve the following equations, identifying those with no real solutions.

a. $x^2 + 11x + 10 = 0$ **b.** $3x^2 + 6x = 0$ **c.** $-2x^2 - 1 = 0$

20. Determine the solutions to the following equations.

a. $(x + 2)^2 - 16 = 0$ **b.** $4(x - 3)^2 - 36 = 0$ **c.** $(x + 1)^2 = 25$

21. Simplify each of the following.

a. $\dfrac{(x - 3)(x + 4)}{x + 1} \times \dfrac{x + 1}{x - 3}$ **b.** $\dfrac{x(x + 5)}{x - 5} \div \dfrac{x + 5}{x(x - 5)}$

c. $\dfrac{4x - 12}{2x + 2} \times \dfrac{x + 1}{x - 3}$ **d.** $\dfrac{10x + 20}{3x - 3} \div \dfrac{5x + 10}{6x + 6}$

e. $\dfrac{x^2 + 7x + 12}{x^2 + 4x + 3} \times \dfrac{x^2 - 6x - 7}{x^2 + 2x - 8}$ **f.** $\dfrac{x^2 - 25}{x^2 + 4x - 5} \div \dfrac{x^2 - 6x + 5}{x^2 + 5x - 6}$

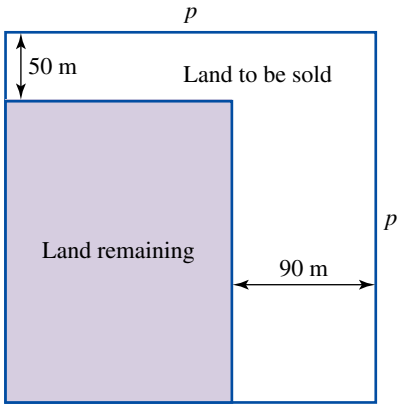
Problem solving

22. The distance travelled by a motorbike is given by the formula $d = 18t + 2t^2$, where t is the time in seconds and d is the distance in metres. How long would it take the motorbike to travel a distance of 180 m?

23. When 20 is subtracted from the square of a certain number, the result is 8 times that number. Find the number, which is negative.

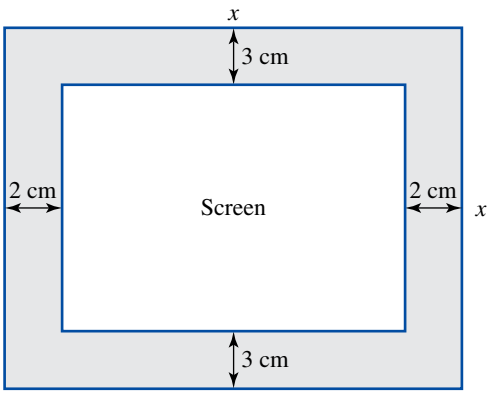
24. A ball is thrown from the balcony of an apartment building. The ball is h metres above the ground when it is a horizontal distance of x metres from the building. The path of the ball follows the rule $h = -x^2 + 3x + 28$. How far from the building will the ball land?

25. David owned land in the shape of a square with side length p.
He decided to sell part of this land by reducing it by 50 metres in one direction and 90 metres in the other.



a. Find an expression in terms of p for the area of the original land.
b. Find an expression in terms of p, in its simplest form, for the area of land David has remaining after he sells the section of land shown.
c. Find an expression in terms of p for the area of land he sold.

26. A new computer monitor is made up of a rectangular screen surrounded by a hard plastic frame in which speakers can be inserted.



a. Write expressions for the length and width of the screen in terms of x.
b. Write an expression for the area of the screen, using expanded form.
c. Find the area of the screen if $x = 30$ cm.

27. An astronaut needs to find a positive number such that twice the number plus its square gives 35.
a. Write this as an equation.
b. Solve the equation to find the answer for him.

28. A window cleaner was 3 metres off the ground, cleaning the windows of a high-rise building, when he dropped his bucket. The height, h m, of the bucket above the ground t seconds after it is dropped is given by the equation $h = 3 - t^2 - 2t$.
a. What is the value of h when the bucket hits the ground?
b. Rewrite the equation, replacing h with the value it takes when the bucket hits the ground.
c. Solve the equation.
d. Find the time it takes for the bucket to reach the ground.

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Complete this digital doc: Concept map: Topic 16 (int-10805)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algebraic fraction
common factor
constant term
denominator
difference of two squares
expand
factorise
Null Factor Law
numerator
perfect square
quadratic equation
quadratic trinomial

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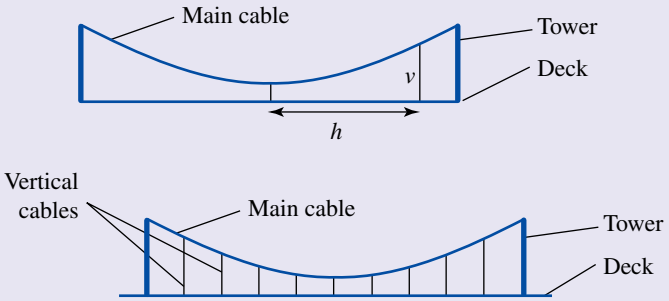
Suspension bridges



When you hold up a chain at both ends, it forms a curve. Its shape is actually a *catenary* — a trigonometric function with a shape similar to that of a quadratic equation whose shape is a parabola. When constructing a suspension bridge, the main cables are attached to a tower at both ends and the curve is a catenary. However, when vertical cables are attached from the main cable to the deck, the curve takes the shape of a parabola. A catenary curves under its own weight; the bridge's main cables are curving not just under their own weight, but also curving from holding up the weight of the deck.

There are many well-known suspension bridges around the world, for example, the Golden Gate Bridge in San Francisco.

A sketch of one side of the bridge can be represented as follows (we will consider just one side of the bridge in our calculations). Note that there are no vertical cables at the ends where the main cable is attached to the towers.



The shape of the cable can be represented by the equation $v = 0.000832\,h^2 + 3$ where v represents the vertical distance between the main cable and the deck (in metres) and h represents the horizontal distance from the centre point of the main cable (in metres). The curve is symmetrical around this central point.

Let us consider a suspension bridge with 55-m high towers at the ends and vertical cables evenly spaced every 50 m.

1. Substitute into the equation to calculate the horizontal distance of the towers from the centre of the main cable.

2. Determine the length of the deck.

3. Draw a diagram displaying the spacing of all the vertical cables on the bridge.
4. How many vertical cables are there in the structure?
5. Use the equation to calculate the length (to the nearest metre) of each of these vertical cables.
6. What is the total length of cable required for these vertical cables?



The Golden Gate Bridge consists of a main suspension span in the centre with side suspension spans at both ends.

Here are some facts about the structure of the bridge.

- Length of suspension span including main span and side spans: 1966 m
- Horizontal distance between towers: 1280 m
- Clearance above mean high water: 67 m
- Height of tower above roadway: 152 m
- The bridge has two main cables that pass over the tops of the two main towers and are secured at either end. Length of each main cable: 2332 m
- Total length of wire used in main cables and vertical cables: 129 000 km

7. Draw a sketch of the bridge showing as much detail as possible.

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Complete this digital doc: Code puzzle: Inventions and their inventors (doc-15913)

Answers

Topic 16 Quadratic algebra

Exercise 16.2 Factorisation patterns

1. a. $(x + 5)(x - 5)$ **b.** $(x + 9)(x - 9)$ **c.** $(a + 4)(a - 4)$
d. $(5 + p)(5 - p)$ **e.** $(11 + a)(11 - a)$ **f.** $(6 + y)(6 - y)$
g. $(2b + 5)(2b - 5)$ **h.** $(3a + 4)(3a - 4)$ **i.** $(5d + 1)(5d - 1)$

2. a. $(x + y)(x - y)$ **b.** $(a + b)(a - b)$ **c.** $(p + q)(p - q)$
d. $(5m + n)(5m - n)$ **e.** $(9x + y)(9x - y)$ **f.** $(p + 6q)(p - 6q)$
g. $(6m + 5n)(6m - 5n)$ **h.** $(4q + 3p)(4q - 3p)$ **i.** $(2m + 7n)(2m - 7n)$

3. a. $(x + 13)(x + 5)$ **b.** $(p + 13)(p + 3)$ **c.** $(p - 2 + q)(p - 2 - q)$
d. $(c - 6 + d)(c - 6 - d)$ **e.** $(x + 7 + y)(x + 7 - y)$ **f.** $(p + 5 + q)(p + 5 - q)$
g. $(a + 3 + b)(a + 3 - b)$ **h.** $(a - 2)(a - 4)$ **i.** $(b + 5)(b - 7)$

4. a. $2(m + 4)(m - 4)$ **b.** $5(y + 3)(y - 3)$ **c.** $6(p + 2)(p - 2)$
d. $4(m + 5)(m - 5)$ **e.** $2(12 + x)(12 - x)$ **f.** $5(4 + a)(4 - a)$
g. $25(2x + y)(2x - y)$ **h.** $4(6p + q)(6p - q)$ **i.** $2(4y + x)(4y - x)$
j. $a(x + 3)(x - 3)$ **k.** $2b(2 + x)(2 - x)$ **l.** $3(b + 9)(b + 1)$

5. a. $(x + 5)^2$ **b.** $(p - 12)^2$ **c.** $(n + 10)^2$ **d.** $(x + y)^2$
e. $(u - v)^2$ **f.** $(8 + e)^2$ **g.** $(2m - 5)^2$ **h.** $(7 + 3a)^2$
i. $(9x + 4y)^2$ **j.** $2(x + 6)^2$ **k.** $3(x - 4y)^2$ **l.** $2(3a + 2b)^2$

6. a. **i** **b.** **iv**, **vi**

7. a. C **b.** A **c.** D **d.** A **e.** C

8. a. $\pi r^2\,\text{m}^2$ **b.** $(r + 1)$ m **c.** $\pi(r + 1)^2\text{m}^2$ **d.** $(\pi(r + 1)^2 - \pi r^2)\,\text{m}^2$
e. $\pi(2r + 1)\,\text{m}^2$ **f.** $35\,\text{m}^2$ **g.** $47\,\text{m}^2$

9. a. $w^2 + 10w + 25$
b. **i.** 100 **ii.** 225 **iii.** 625
c. Check with your teacher.

10. a. $(x + \sqrt{10})(x - \sqrt{10})$ **b.** $4(x + \sqrt{8})(x - \sqrt{8})$ **c.** $(x^2 - y)(x^2 + y)$ **d.** $5(2x - 1)$
e. $(y - 2x^2)(y + 2x^2)$ **f.** $(10^2 - x^2)(10^2 + x^2) = (10 - x)(10 + x)(100 + x^2)$

11.–13. Check with your teacher.

14. a. Side length$_a$ $= x + 3$
Side length$_b$ $= x + 5$
Side length$_c$ $= x + 8$
Side length$_d$ $= x - 3$
Side length$_e$ $= x + 6$

b. $\text{A}_a = \text{II} = (x + 3)^2$
$\text{A}_b = \text{III} = (x + 5)^2$
$\text{A}_c = \text{V} = (x + 8)^2$
$\text{A}_d = \text{I} = (x - 3)^2$
$\text{A}_e = \text{IV} = (x + 6)^2$

c. $\text{A}_a = \text{II} = 64$ cm
$\text{A}_b = \text{III} = 100$ cm
$\text{A}_c = \text{V} = 169$ cm
$\text{A}_d = \text{I} = 4$ cm
$\text{A}_e = \text{IV} = 121$ cm

15. a. $(x - 7)(x + 7)$
b. Length $= x + 7; x - 7$
c. Length = 32 mm, width = 18 mm
d. Area $= 2451\,\text{mm}^2$; $1875\,\text{mm}^2$ larger

16. a. 10609 **b.** 3844 **c.** 994009 **d.** 1024144 **e.** 2809 **f.** 9604

Challenge 16.1

$(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$

Exercise 16.3 Factorising monic quadratics

1. a. $x^2 + 6x + 8$ **b.** $x^2 - 6x + 8$ **c.** $x^2 - 9x + 20$ **d.** $x^2 + 9x + 20$
e. $m^2 + 6m + 5$ **f.** $m^2 - 6m + 5$ **g.** $t^2 + 19t + 88$ **h.** $t^2 - 30t + 200$
i. $x^2 - x - 6$ **j.** $x^2 + x - 6$ **k.** $v^2 - 3v - 40$ **l.** $v^2 + 3v - 40$
m. $x^2 + 5x - 14$ **n.** $t^2 - 11t - 12$ **o.** $n^2 + 13n - 30$ **p.** $a^2 - 3a - 18$
q. $z^2 + 8z + 16$ **r.** $z^2 + 3z - 88$ **s.** $n^2 - 10n + 21$ **t.** $v^2 + v - 2$

2. a. 1, 4; 2, 3 **b.** 1, 5; 2, 4; 3, 3 **c.** 1, 11; 2, 10 … 6, 6 **d.** 1, 19; 2, 18 … 10, 10

3. a. −1, −4; −2, −3 **b.** −1, −6; −2, −5; −3, −4
c. −1, −7; −2, −6; −3, −5; −4, −4 **d.** −1, −10; −2, −9 … − 5, −6

4. a. $(x + 1)(x + 3)$ **b.** $(x - 1)(x - 3)$ **c.** $(x + 1)(x + 11)$ **d.** $(x - 1)(x - 11)$
e. $(a + 5)(a + 1)$ **f.** $(a - 5)(a - 1)$ **g.** $(x - 4)(x - 3)$ **h.** $(x - 5)(x - 2)$
i. $(n + 4)^2$ **j.** $(n + 8)(n + 2)$ **k.** $(y - 9)(y - 3)$ **l.** $(x - 7)(x - 6)$
m. $(t - 6)(t - 2)$ **n.** $(t + 9)(t + 2)$ **o.** $(u + 2)(u + 3)$

5. a. 7 and −1; 8 and −2; 9 and −3; 10 and −4; 11 and −5
b. 1 and −7; 2 and −8; 3 and −9; 3 and −10; 5 and −11
c. 4 and −1; 5 and −2; 6 and −3; 7 and −4; 8 and −5
d. 1 and −4; 2 and −5; 3 and −6; 4 and −7; 5 and −8
e. 1 and −9; 2 and −10; 3 and −11; 4 and −12; 5 and −13
f. 15 and −1; 16 and −2; 17 and −3; 18 and −4; 19 and −5
g. 1 and −6; 2 and −7; 3 and −8; 4 and −9; 5 and −10
h. 8 and −1; 9 and −2; 10 and −3; 11 and −4; 12 and −5

6. a. $(x + 6)(x - 3)$ **b.** $(x - 6)(x + 3)$ **c.** $(x - 5)(x + 3)$ **d.** $(x + 5)(x - 3)$
e. $(n - 14)(n + 1)$ **f.** $(n + 7)(n - 5)$ **g.** $(v + 6)(v - 1)$ **h.** $(v - 6)(v + 1)$
i. $(t + 6)(t - 2)$ **j.** $(t - 7)(t + 2)$ **k.** $(x - 5)(x + 4)$ **l.** $(x - 4)(x + 5)$
m. $(n + 10)(n - 9)$ **n.** $(n - 10)(n + 7)$ **o.** $(x - 5)(x + 1)$

7. a. $(x + 1)(x + 6)$ **b.** $x^2 + 7x + 10 = (x + 2)(x + 5)$, $x^2 + 7x + 12 = (x + 3)(x + 4)$

8. a. $(x - 1)(x + 4)$ **b.** Answers will vary.

9. a. $2(x + 2)(x - 7)$ **b.** $4(x + 2)(x + 5)$ **c.** $-2(x + 4)(x - 3)$ **d.** $5(x - 5)(x - 3)$

10. D

11. Answers will vary.

12. $3x + 6x = 9x$

$$\begin{aligned} 6x^2 + 3x + 6x + 3 &= 3x(2x + 1) + 3(2x + 1) \\ &= (3x + 3)(2x + 1) \\ &= 3(x + 1)(2x + 1) \end{aligned}$$

13. a. $w = \dfrac{x^2 + 2x - 15}{x + 5}$ **b.** $w = 62$ cm **c.** $l = 103$ cm

14. a. 1 × 120 m
2 × 60 m
3 × 40 m
4 × 30 m
5 × 24 m
6 × 20 m
8 × 15 m
10 × 12 m
b. $l = x + 8$, $w = x - 6$
c. 6 × 20 m

Exercise 16.4 Factorising non-monic quadratics

1. a. $2(x + 2)(x + 3)$ **b.** $3(x + 4)(x + 1)$ **c.** $2(x - 2)(x - 4)$
d. $3(x - 3)(x - 1)$ **e.** $4(x + 2)(x - 3)$ **f.** $3(x + 5)(x - 2)$
g. $2(x + 7)(x - 3)$ **h.** $3(x - 6)(x + 3)$ **i.** $5(x + 6)(x - 2)$

2. a. $(x + 3)(2x + 1)$ **b.** $(x + 2)(2x + 3)$ **c.** $(x + 2)(3x + 1)$

d. $(x - 3)(2x - 1)$ **e.** $(x - 1)(3x + 2)$ **f.** $(x + 1)(5x - 2)$

g. $(x - 2)(7x - 3)$ **h.** $(2x - 3)(5x + 2)$ **i.** $(x + 4)(2x - 3)$

3. a. B **b.** A **c.** D

4. a. $(2x - 1)(x + 2)$ **b.** $(4x + 3)(2x - 1)$ **c.** $(3x + 2)(2x - 1)$ **d.** $(2x + 5)(x + 2)$

5. a. $(3x - 4)(x + 1)$ **b.** $(2x - 3)(3x + 7)$

6. a. $(2x - 3)(3x + 2)$ **b.** $(4x + 5)(x + 2)$ **c.** $(x - 1)(4x - 15)$ **d.** $(3x + 4)(2x - 1)$

7. $3(x + 1)(x + 4)$

8. a. $(4x + 3)(x - 5)$ **b.** $(x + 3)(3x + 1)$ **c.** $(5x + 3)(2x - 3)$ **d.** $(3x - 1)(5x + 2)$

9. a. The middle term, $-9x$, can be split as $3x - 12x$. **b.** $(4x + 3)(x - 3)$

10. a. $(6x - 5)(2x + 1)$

b. i. $12x^2 - 4x - 5$ **ii.** 1.5 **iii.** 1.1 or 3.5

11. a. i. Square, because it is a perfect square.

ii. Rectangle, because it is a DOTS.

iii. Rectangle, because it is a trinomial.

iv. Rectangle, because it is a trinomial.

b. i. $(3s + 8)^2$

ii. $(5s + 2)(5s - 2)$

iii. $(s + 1)(s + 3)$

iv. $4s^2 - 6s - 9$

12. a. $(3b + 8)^2$ **b.** $(ac + d)(ac - d)$ **c.** $(a - 13b)(a + 14b)$ **d.** $(b - ac)^2$

Exercise 16.5 Simplifying algebraic fractions

1. a. $\frac{1}{2}$ **b.** $\frac{2}{3}$ **c.** 1 **d.** 3

e. 1 **f.** 3 **g.** $\frac{1}{4}$ **h.** $\frac{h}{2}$

i. $\frac{5(p - 3)}{p - 2}$ **j.** $x + 2$ **k.** $\frac{b + 3}{b + 5}$ **l.** $\frac{x + 1}{x - 6}$

2. a. $\frac{x}{3}$ **b.** 2 **c.** $\frac{1}{2}$ **d.** 3

e. 1 **f.** $\frac{p + 2}{p - 2}$ **g.** $\frac{s - 6}{3s}$ **h.** $a - 3$

i. $\frac{4}{m + 2}$ **j.** $\frac{a - 3}{a - 5}$ **k.** $\frac{m - 2}{m + 5}$ **l.** $\frac{p + 2}{p - 4}$

3. a. $a + 1$ **b.** $\frac{a + 2}{3}$ **c.** $a - 1$ **d.** $p - 3$

e. $\frac{x + 2}{2}$ **f.** $\frac{x - 4}{2}$ **g.** $3(x + 6)$ **h.** $3(x - 8)$

i. $\frac{2(x - 2)}{3}$ **j.** $\frac{3(x - 5)}{5}$ **k.** $\frac{1}{x + 5}$ **l.** $-\frac{1}{x - 4}$

m. $\frac{2}{3}$ **n.** $\frac{3}{4}$ **o.** $\frac{5}{6}$ **p.** 3

4. a. $x + 3$ **b.** $p - 5$ **c.** $y - 4$ **d.** $a + 10$

e. $\frac{7}{5 - x}$ **f.** $\frac{5}{5 - b}$ **g.** $\frac{9}{y - 9}$ **h.** $\frac{4}{m - 4}$

i. $\frac{7}{1 - 2x}$ **j.** $\frac{3}{2m - 1}$ **k.** $\frac{1 - 2a}{a + 3}$ **l.** $\frac{2b + 1}{5 - 8b}$

m. $\frac{3x + 1}{9 - x}$ **n.** $\frac{7 - 2b}{b + 5}$ **o.** $\frac{3(m + 2)}{4m - 1}$ **p.** $\frac{2(2 - a)}{3 + 2a}$

5. a. $x + 2$ **b.** $x + 3$ **c.** $x - 4$ **d.** $b - 7$

e. $a - 9$ **f.** $a - 11$ **g.** $x + 7$ **h.** $m + 8$

i. $\frac{a - 3}{a + 4}$ **j.** $\frac{p + 1}{p + 5}$ **k.** $\frac{x + 3}{x - 1}$ **l.** $\frac{m - 1}{m + 6}$

m. $\frac{y + 2}{y + 6}$ **n.** $\frac{x - 2}{x + 2}$ **o.** $\frac{x - 5}{x - 2}$ **p.** $\frac{x + 6}{x - 3}$

6. a. 10 **b.** 3 **c.** 4 **d.** 3

e. $\frac{3}{4}$ **f.** $\frac{12(a+2)}{a+3}$ **g.** $\frac{6x+1}{8(x+7)}$ **h.** $\frac{m+4}{m+7}$

i. $\frac{x+5}{x+3}$ **j.** $\frac{x+2}{x-4}$

7. Check with your teacher.

8. Check with your teacher.

9. a. $\frac{(c+12)}{(c-18)}$ **b.** $\frac{(m+1)}{(m+8)}$ **c.** $\frac{(f+9)}{(f+6)}$ **d.** Cannot be simplified

e. $\frac{(w+5)}{(w+20)}$ **f.** Cannot be simplified

10. a. i $x(x-2)$ m^2 **ii.** $x(2x)$ m^2

b. $\frac{(x-2)}{2x}$m^2

c. $\frac{50(x-2)}{x}\%$

d. i. 25% **ii.** $27.\dot{8}\%$ **iii.** $29.1\dot{7}\%$

Challenge 16.2

$\frac{x+1}{x(x+2)} + 1$

Exercise 16.6 Quadratic equations

1. a. $a = 1, b = 2, c = -1$ **b.** $a = 1, b = 1, c = -10$ **c.** $a = 4, b = 5, c = -30$

d. $a = 8, b = -11, c = 2$ **e.** $a = 2, b = 1, c = -60$ **f.** $a = 1, b = 0, c = -17$

2. a. D **b.** C

3. a. $x = 4, x = -4$ **b.** There is no solution. **c.** $x = 0$

d. $x = 1, x = -1$ **e.** There is no solution. **f.** $x = 2, x = -2$

4. a. $x = -1, x = -3$ **b.** There is no solution. **c.** $x = 1, x = -7$

d. $x = 0, x = 8$ **e.** $x = 1, x = -5$ **f.** There is no solution.

g. $x = -3, x = -7$ **h.** There is no solution. **i.** $x = 8$

5. a. Y **b.** N **c.** Y

6. a. Y **b.** N **c.** N

7. a. N **b.** N **c.** Y

8. a. Y **b.** N

9. a. Check with your teacher. **b.** $x = 3.41$ **c.** A negative number is not appropriate for a length.

10. Check with your teacher.

11. a. 6 or −6

b. There are two possible numbers, as the square root of a positive number, in this case 36, has two possible solutions, one positive and one negative.

12. a. $3(x-2)^2 - 7 = 41$

$7 - 3(x-2)^2 = 41$ has no solution.

b. −2 and 6

Exercise 16.7 The Null Factor Law

1. a. $x = -3, x = 2$ **b.** $x = -2, x = 3$ **c.** $x = -2, x = 3$

d. $x = -2\frac{1}{2}, x = -\frac{3}{4}$ **e.** $x = -4, x = -\frac{1}{2}$ **f.** $x = \frac{1}{2}, x = -30$

g. $x = -\frac{1}{2}, x = 3$ **h.** $x = 1, x = \frac{1}{3}$ **i.** $x = 0, x = 2$

j. $x = -\frac{1}{3}, x = \frac{1}{4}$ **k.** $x = -2.3, x = 0.3$ **l.** $x = -\frac{1}{6}, x = \frac{1}{6}$

m. $x = 2$ **n.** $x = 0, x = \frac{15}{4}$ **o.** $x = -4$

2. a. $x = 2, x = -2, x = -3$ **b.** $x = -2, x = 2.5$ **c.** $x = -2, x = -4$
d. $x = 0, x = -2, x = -4$ **e.** $x = 1.1, x = -2.4, x = -2.6$ **f.** $x = -3, x = -\frac{1}{2}, x = 1\frac{2}{3}$
g. $x = 3$ **h.** $x = -1, x = 2$

3. a. D **b.** B

4. a. $x(x + 10) = 0; x = 0$ or $x = -10$
b. $2x(x - 7) = 0; x = 0$ or $x = 7$
c. $5x(5x - 8) = 0; x = 0$ or $x = \frac{8}{5}$

5. a. When $x = 2$, the first bracket equals 2 and the second bracket equals 4; therefore, the product is 8.
b. -2 and 1

6. A quadratic can have a maximum of two solutions, because a quadratic can at most be factorised into two separate pairs of brackets, each of which represent one solution.

7. a. 10 metres
b. $y = \frac{1}{250}x(100 - x); a = \frac{1}{250}, b = 100$
c. 9.424 m

8. a. $h = -4t(t - 5)$
b. i. 16 m **ii.** 0 m
c. Answers will vary.

9. -29 or 8

Exercise 16.8 Solving the quadratic equation $ax^2 + bx + c = 0$

1. a. $x = 2, x = 4$ **b.** $x = -2, x = -4$ **c.** $x = -1, x = -5$ **d.** $x = 2, x = -3$
e. $x = 3, x = -5$ **f.** $x = -2$ **g.** $x = 4, x = -6$ **h.** $x = 8, x = -3$
i. $x = -3, x = 4$ **j.** $x = -12, x = -1$ **k.** $x = 11, x = -1$ **l.** $x = 4, x = -5$
m. $x = -25, x = -4$ **n.** $x = 5, x = 10$ **o.** $x = 4, x = -2$

2. a. $-3, -\frac{1}{2}$ **b.** $-1, \frac{1}{2}$ **c.** $-2\frac{1}{2}, 3$ **d.** $-3, -1$
e. $-2\frac{1}{3}, -2$ **f.** $-1, -\frac{2}{3}$ **g.** $1\frac{2}{5}, 3$ **h.** $\frac{2}{5}, 3$
i. $-\frac{5}{7}, 1$ **j.** $\frac{5}{7}, 4$ **k.** $-1\frac{1}{2}, \frac{1}{2}$ **l.** $-2\frac{1}{2}, -\frac{1}{3}$

3. a. B **b.** D **c.** D

4. $x = 1, -6$

5. 9 m by 5 m

6. 8 m by 6 m

7. 1.5 m

8. a. i. 0 **ii.** 2 **iii.** 35
b. i. Octagon **ii.** Icosahedron

9. a. i. $n + 1$ **ii.** $n + 7$ **iii.** $(n + 1)(n + 7)$ **iv.** $\frac{(n + 7)}{(n + 2)}$
b. $n = 3$

10. a. $4x^2 + 13x - 16 = 3x^2 + 8x - 2$
b. $x^2 + 5x - 14 = 0$
c. $(x - 2)(x + 7) = 0$
d. $x = 2$ or -7

11. a. $g + 18$
b. $g^2 + 12g - 108$
c. $g^2 + 12g - 108 = 256$
d. $g^2 + 12g - 364 = 0$
e. $g = 14$
f. The girl is 14 years old, her brother is 8 years old and her teacher is 32 years old.

Exercise 16.9 Solving quadratic equations with two terms

1. a. $x = -3, x = 3$ **b.** $x = -4, x = 4$ **c.** $x = -3, x = 3$ **d.** $x = -5, x = 5$
e. $x = -10, x = 10$ **f.** $x = -7, x = 7$ **g.** $x = -3, x = 3$ **h.** $x = -2, x = 2$
i. No real solutions **j.** No real solutions **k.** $x = -3, x = 3$ **l.** $x = -4, x = 4$
m. $x = -5, x = 5$ **n.** $x = 0$ **o.** $x = 0$

2. a. $x = 0, x = -6$ **b.** $x = 0, x = 8$ **c.** $x = 0, x = -9$ **d.** $x = 0, x = 11$
e. $x = 0, x = 6$ **f.** $x = 0, x = 7.5$ **g.** $x = 0, x = \frac{2}{3}$ **h.** $x = 0, x = -1\frac{3}{4}$
i. $x = 0, x = 2\frac{1}{2}$ **j.** $x = 0, x = -1$ **k.** $x = 0, x = \frac{1}{4}$ **l.** $x = 0, x = -5$
m. $x = 0, x = -12$ **n.** $x = 0, x = 18$ **o.** $x = 0, x = 2.5$

3. a. A **b.** C

4. 4 or -4

5. The square plot is 12 m × 12 m; the rectangular plot is 16 m × 9 m.

6. The number is 0 or 10.

7. $x^2 + 9$ cannot be factorised.

8. a. $x = \pm\frac{n}{m}$ **b.** $x = \pm\frac{n}{m}$

9. a. x cannot be isolated. **b.** $x = 0, -b$

10. If $a = 0$, then the expression is not quadratic.

11. 8

12. -12 or 0

13. a. $x(x + 30) = 50x$
b. $x(x - 20) = 0$
c. $x = 0$ or $x = 20$
d. No, x cannot be 0, because the width has to have a positive value.

Exercise 16.10 Applications

1. 6 and 7 or -7 and -6

2. 11 and 13 or -13 and -11

3. $x = 9$

4. $x = 10$

5. 3, 4, 5

6. 5, 12, 13

7. Length = 22 cm, width = 13 cm, height = 4 cm

8. 2.5 m

9. a. 15 m **b.** 4 seconds

10. a. 0 **b.** 11 terms

16.11 Review

1. D **2.** A **3.** C **4.** B
5. A **6.** D **7.** D **8.** C
9. D **10.** C **11.** A **12.** B

13. $x = -4, x = 3$

14. $x = 3.5, x = 4$

15. a. $(x + 8)(x - 8)$
b. $(a + 12)(a - 12)$
c. $(7b + 1)(7b - 1)$
d. $(2f + 3g)(2f - 3g)$
e. $(n + 1 + m)(n + 1 - m)$
f. $(r - 1 + 2s)(r - 1 - 2s)$

16. a. $(c + 1)(c + 4)$ **b.** $(p - 2)(p + 12)$ **c.** $(y - 6)(y - 4)$ **d.** $(x + 1)(x + 2)$

e. $(m - 2)(m - 5)$ **f.** $(m + 22)(m + 2)$

17. a. $2(a + 6)(a + 2)$ **b.** $3(b - 6)(b - 2)$ **c.** $4(c - 6)(c + 2)$ **d.** $(x + 1)(2x + 1)$

e. $(x - 1)(3x + 2)$ **f.** $(2x + 3)(3x + 2)$

18. a. $x = -7, x = 7$ **b.** $x = -5, x = 5$ **c.** $x = -7, x = 0$ **d.** $x = -1, x = 5$

e. $x = 1, x = 10$ **f.** $x = -\frac{1}{2}, x = 5$

19. a. $x = -10, x = -1$ **b.** $x = -2, x = 0$ **c.** No real solutions

20. a. $x = -6, x = 2$ **b.** $x = 0, x = 6$ **c.** $x = -6, x = 4$

21. a. $x + 4$ **b.** x^2 **c.** 2 **d.** $\frac{4(x + 1)}{x - 1}$

e. $\frac{x - 7}{x - 2}$ **f.** $\frac{x + 6}{x - 1}$

22. 6 s

23. $x = -2$

24. 7 metres

25. a. p^2 **b.** $p^2 - 140p + 4500$ **c.** $140p - 4500$

26. a. Length $= (x - 4)$ cm, width $= (x - 6)$ cm

b. $(x^2 - 10x + 24)\,\text{cm}^2$

c. $624\,\text{cm}^2$

27. a. $x^2 + 2x = 35$ **b.** 5

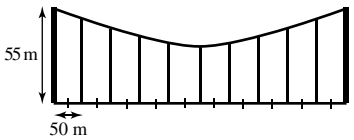
28. a. $h = 0$ **b.** $0 = 3 - t^2 - 2t$ **c.** $t = -3$ or $t = 1$ **d.** 1 s

Investigation — Rich task

1. $h = 250$ m

2. 500 m

3.

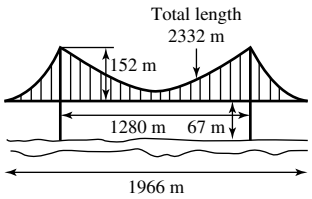


4. 18

5. 3 m; 5 m; 11 m; 22 m; 36 m

6. 305 m

7.



TOPIC 17
Quadratic functions

17.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



17.1.1 Why learn this?

A quadratic function can be used to describe a physical thing such as the arch of a bridge, the path of a batted ball, water rising from a fountain, or a whale's spout. Over the centuries the quadratic equation has played a major role in the whole of human civilisation as we know it. We would not be able to watch satellite television or engage in other modern activities without the developments that have come about through the application of quadratic functions.

17.1.2 What do you know?

assessON

1. **THINK** List what you know about quadratic graphs. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of quadratic graphs.

LEARNING SEQUENCE

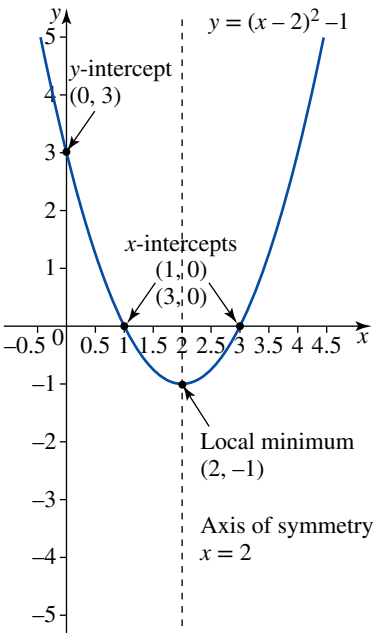
17.1 Overview
17.2 Graphs of quadratic functions
17.3 Plotting points to graph quadratic functions
17.4 Sketching parabolas of the form $y = ax^2$
17.5 Sketching parabolas of the form $y = ax^2 + c$
17.6 Sketching parabolas of the form $y = (x - h)^2$
17.7 Sketching parabolas of the form $y = (x - h)^2 + k$
17.8 Sketching parabolas of the form $y = (x + a)(x + b)$
17.9 Applications
17.10 Review

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: The story of mathematics: Catapults and projectiles (eles-1703)

17.2 Graphs of quadratic functions

17.2.1 Graphs of quadratic functions

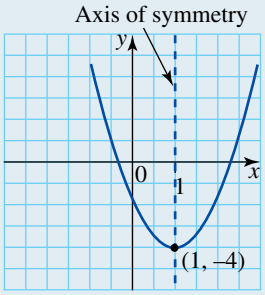


- The graph at right is a typical parabola with features as listed below.
  - The dotted line is the **axis of symmetry**; the parabola is the same on either side of this line.
  - The **turning point** is the lowest point on the graph; the point where the graph changes direction. It is also called the local minimum.
  - The parabola is upside down, or inverted, if the turning point is the highest point on the graph.
  - The x-intercept(s) is where the graph crosses (or sometimes just touches) the x-axis. Not all parabolas have x-intercepts.
  - The y-intercept is where the graph crosses the y-axis. All parabolas have one y-intercept.

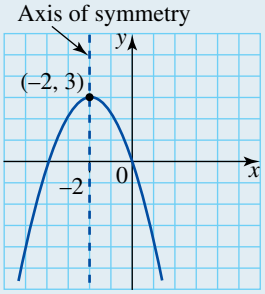
WORKED EXAMPLE 1

For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the turning point and whether it is a maximum or a minimum.

a



b



THINK

a 1 State the equation of the vertical line that cuts the parabola in half.

2 State the turning point.

3 Determine the nature of the turning point by observing whether it is the highest or lowest point of the graph.

b 1 State the equation of the vertical line that cuts the parabola in half.

2 State the turning point.

3 Determine the nature of the turning point by observing whether it is the highest or lowest point of the graph.

WRITE

a Axis of symmetry is $x = 1$.

The turning point is at $(1, -4)$.

Minimum turning point

b Axis of symmetry is $x = -2$.

The turning point is at $(-2, 3)$.

Maximum turning point

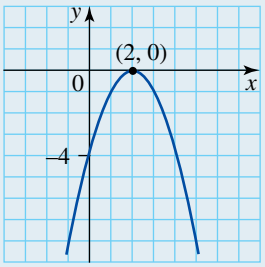
17.2.2 The *x*- and *y*-intercepts

- The ***x*-intercept** is where the graph crosses (or just touches) the *x*-axis.
- The ***y*-intercept** is where the graph crosses the *y*-axis. All parabolas have one *y*-intercept.
- When sketching a parabola, the *x*-intercepts (if any) and the *y*-intercept should always be marked on the graph, with their respective coordinates.

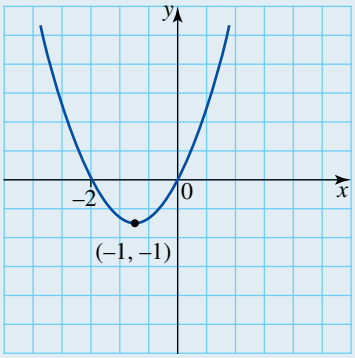
WORKED EXAMPLE 2

For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the turning point, whether the point is a maximum or a minimum, and the *x*- and *y*-intercepts.

a



b



THINK	WRITE
a 1 State the equation of the vertical line that cuts the parabola in half.	**a** Axis of symmetry is $x = 2$.
2 State the turning point and its nature; that is, determine whether it is the highest or lowest point of the graph.	Maximum turning point is at (2, 0).
3 Observe where the parabola crosses the *x*-axis. In this case, the graph touches the *x*-axis when $x = 2$, so there is only one *x*-intercept.	The *x*-intercept is 2. It occurs at the point (2, 0).
4 Observe where the parabola crosses the *y*-axis.	The *y*-intercept is -4. It occurs at the point (0, –4).
b 1 State the equation of the vertical line that cuts the parabola in half.	**b** Axis of symmetry is $x = -1$.
2 State the turning point and its nature; that is, determine whether it is the highest or lowest point of the graph.	Minimum turning point is at $(-1, -1)$.
3 Observe where the parabola crosses the *x*-axis.	The *x*-intercepts are -2 and 0. They occur at the points $(-2, 0)$ and (0, 0).
4 Observe where the parabola crosses the *y*-axis.	The *y*-intercept is 0. It occurs at the point (0, 0).

learnon RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Equation of a vertical line (doc-10989)

Exercise 17.2 Graphs of quadratic functions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 16, 17	Questions: 1a, c, e, 2a, c, e, 3, 4, 6, 7, 9–11, 13, 15–17	Questions: 1a, c, f, 2a, f, 3, 4, 7, 9–19

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

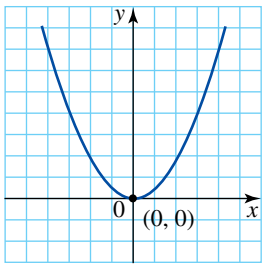
1. WE1 For each of the graphs below:

i. state the equation of the axis of symmetry

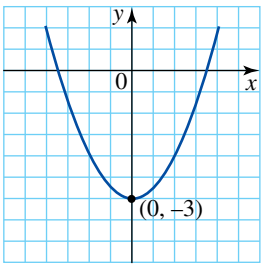
ii. give the coordinates of the turning point

iii. indicate whether it is a minimum or maximum turning point.

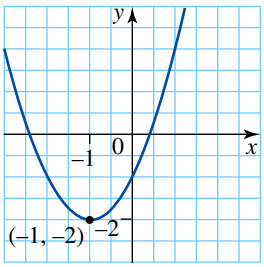
a.



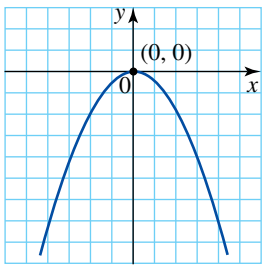
b.



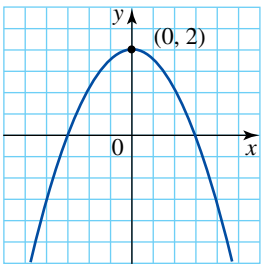
c.



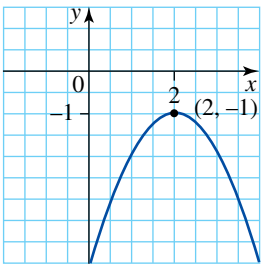
d.



e.



f.



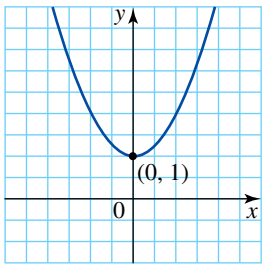
2. For each of the following graphs, state:

i. the equation of the axis of symmetry

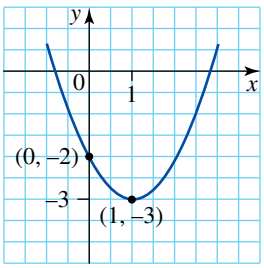
ii. the coordinates of the turning point

iii. whether the turning point is a maximum or minimum.

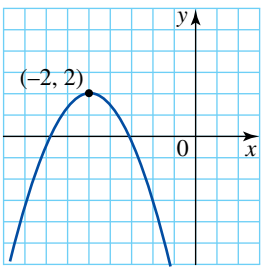
a.



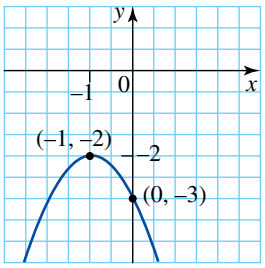
b.



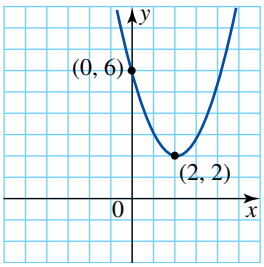
c.



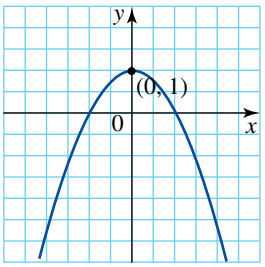
d.



e.

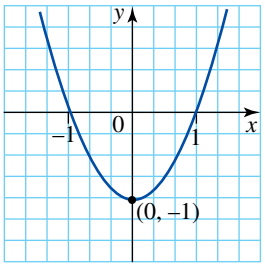


f.

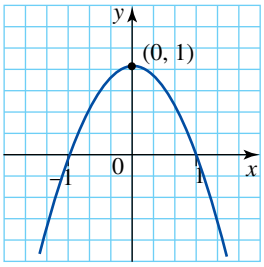


3. WE2 For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the turning point and whether it is a maximum or a minimum, and the x- and y-intercepts.

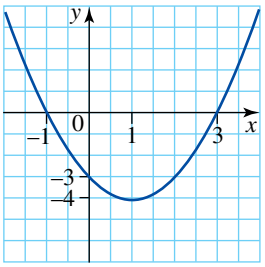
a.



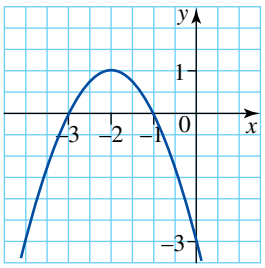
b.



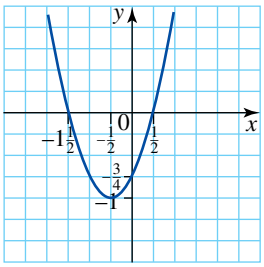
c.



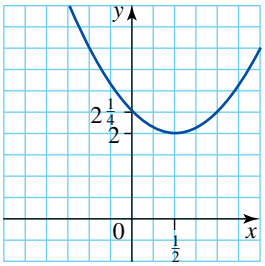
d.



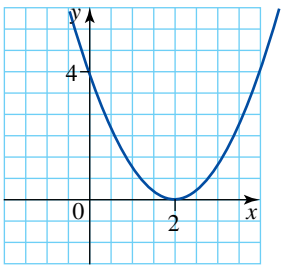
e.



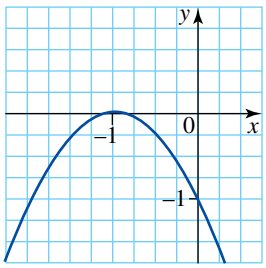
f.



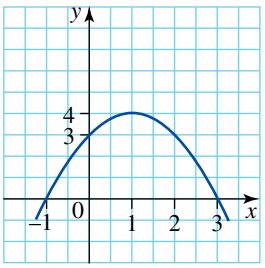
g.



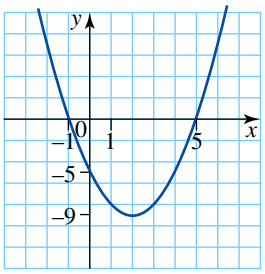
h.



i.



j.



4. MC **a.** The axis of symmetry for the graph shown at right is:

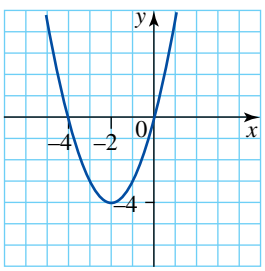
A. $x = 0$ **B.** $x = -2$

C. $x = -4$ **D.** the y-axis

b. The coordinates of the turning point for the graph are:

A. $(0, 0)$ **B.** $(-2, 4)$

C. $(-2, -4)$ **D.** $(2, -4)$



c. The y-intercept is:

A. 0 **B.** -2 **C.** -4 **D.** 2

d. The x-intercepts are:

A. 0 and 4 **B.** 0 and 2 **C.** 0 and -2 **D.** 0 and -4

Understanding

5. Consider the table of values below.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

a. Plot these points on graph paper. What shape is the graph?

b. Locate the axis of symmetry.

c. Locate the y-intercept.

d. Locate the x-intercept(s).

6. Consider the function $y = x^2 + x$. Complete this table of values for the function.

x	-6	-4	-2	0	2	4	6	8
y				0				

7. MC Which of the following rules is not a parabola?

A. $y = -2x^2$ **B.** $y = 2x^2 - x$ **C.** $y = -2 \div x^2$ **D.** $y = -2 + x^2$

8. Consider the graph of $y = -x^2$.

a. State the turning point of this graph.

b. State whether the turning point is a maximum or a minimum.

9. Given the following information, make a sketch of the graph involved.

a. Maximum turning point $= (-2, -2)$, y-intercept $= (0, -6)$

b. Minimum turning point $= (-3, -2)$, x-intercepts $(1, 0)$ and $(-7, 0)$

Reasoning

10. Consider the parabola given by the rule $y = x^2$ and the straight line given by $y = x$. Show that the two graphs meet at $(0, 0)$ and $(1, 1)$.

11. A window-cleaning company varies its charges as the square of the height of the building. Let the heights of buildings be 10, 15, 20, … 50 m. The cost of window washing for a 10-m building is \$100.

a. Determine the window washing costs for the buildings listed above. By plotting a graph, what shape is the graph?

b. What would be the cost for a 45-m building?

c. If the cost is \$729, show that the height of the building is 27 m.

12. Another window-cleaning company also varies its charges as the square of the height of the building. The cost for a 20-m building is \$600.

a. Show that the cost for a 40-m building is \$2400.

b. If the cost was \$2053.50, what was the height of the building?

13. If the axis of symmetry of a parabola is $x = -4$ and one of the x-intercepts is $(10, 0)$, show the other x-intercept is $(-18, 0)$.

14. If the x-intercepts of a parabola are $(-2, 0)$ and $(5, 0)$, show that the axis of symmetry is $x = 1.5$.

Problem solving

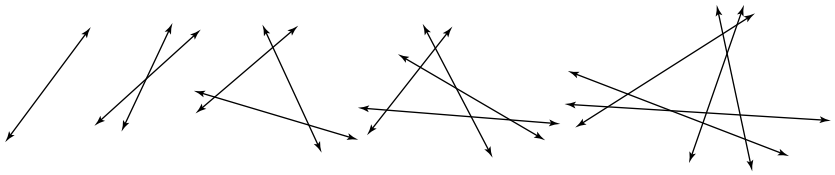
15. On a set of axes, sketch a parabola that has no x-intercepts and has an axis of symmetry $x = -2$. Can the parabola have a maximum or minimum turning point or both? Explain your reasoning.

16. Sketch a graph where the turning point and the x-intercept are the same. Suggest a possible equation.

17. Sketch a graph where the turning point and the y-intercept are the same. Suggest a possible equation.

18. a. Complete the table to show how many intersections there are for 3, 4, 5, and 6 lines.

n (number of lines)	1	2	3	4	5	6
N (number of intersections)	0	1				



b. Find the rule connecting the number of lines, n, with the number of intersections, N, and hence show that the relationship is quadratic.

c. If you could draw 100 lines, how many intersections would there be?

Reflection

What are the major features of all parabolas?

17.3 Plotting points to graph quadratic functions

17.3.1 Graphing quadratic functions

- If there is a rule connecting y and x, a table of values can be used to determine actual coordinates.
- When drawing straight line graphs, a minimum of two points is required. For parabolas there is *no minimum* number of points, but between 6 and 12 points is a reasonable number.
- The more points used, the 'smoother' the parabola will appear. The points should be joined with a smooth curve, not ruled.
- Ensure that points plotted include (or are near) the main features of the parabola, namely the axis of symmetry, the turning point and the x- and y-intercepts.

WORKED EXAMPLE 3 — TI | CASIO

Copy and complete the table of values for each of the following equations, then list the coordinates of each of the points.

a $y = x^2$

x	−3	−2	−1	0	1	2	3
$y = x^2$	9						

b $y = -x^2 + 2$

x	−3	−2	−1	0	1	2	3
$y = -x^2 + 2$							

c $y = 2x^2 - 5x + 1$

x	−2	−1	0	1	2	3	4
$y = 2x^2 - 5x + 1$							

THINK

a 1 Write the equation.

2 Substitute the x-values into the equation to obtain the corresponding y-values.

WRITE

a $y = x^2$

When $x = -3$, $y = (-3)^2 = 9$

When $x = -2$, $y = (-2)^2 = 4$

…

When $x = 3$, $y = (3)^2 = 9$

3 Complete the table of values.

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

4 List the coordinates of each of the points.

$(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)$

b 1 Write the equation.

b $y = -x^2 + 2$

2 Substitute the x-values into the equation to obtain the corresponding y-values.

When $x = -3$, $y = -(-3)^2 + 2 = -7$
When $x = -2$, $y = -(-2)^2 + 2 = -2$
…
When $x = 3$, $y = -(3)^2 + 2 = -7$

3 Complete the table of values.

x	-3	-2	-1	0	1	2	3
$y = -x^2 + 2$	-7	-2	1	2	1	-2	-7

4 List the coordinates of each of the points.

$(-3, -7), (-2, -2), (-1, 1), (0, 2), (1, 1), (2, -2), (3, -7)$

c 1 Write the equation.

c $y = 2x^2 - 5x + 1$

2 Substitute the x-values into the equation to obtain the corresponding y-values.

When $x = -2$, $y = 2(-2)^2 - 5(-2) + 1 = 19$
When $x = -1$, $y = 2(-1)^2 - 5(-1) + 1 = 8$
…
When $x = 4$, $y = 2(4)^2 - 5(4) + 1 = 13$

3 Complete the table of values.

x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x + 1$	19	8	1	-2	-1	4	13

4 List the coordinates of each of the points.

$(-2, 19), (-1, 8), (0, 1), (1, -2), (2, -1), (3, 4), (4, 13)$

- Occasionally a list of x-values will be provided and the corresponding y-values can be calculated. In the following example, the set of x-values is specified as $-4 \leq x \leq 2$.

WORKED EXAMPLE 4

TI | CASIO

Plot the graph of $y = x^2 + 2x - 3$, $-4 \leq x \leq 2$ and, hence, state:
a the equation of the axis of symmetry
b the coordinates of the turning point and whether it is a maximum or a minimum
c the x- and y-intercepts.

THINK

1 Write the equation.

2 Complete a table of values by substituting into the equation each integer value of x from -4 to 2. For example, when $x = -4$, $y = (-4)^2 + 2 \times (-4) - 3 = 5$.

3 List the coordinates of the points.

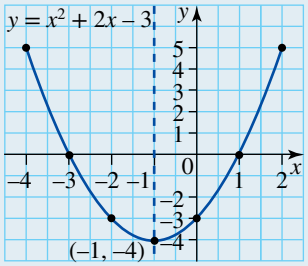
WRITE/DRAW

$y = x^2 + 2x - 3$

x	-4	-3	-2	-1	0	1	4
y	5	0	-3	-4	-3	0	5

$(-4, 5), (-3, 0), (-2, -3), (-1, -4), (0, -3), (1, 0), (2, 5)$

4 Draw and label a set of axes, plot the points listed and join the points to form a smooth curve.



a Find the equation of the line that divides the parabola exactly into two halves.

a Axis of symmetry is $x = -1$.

b Find the point where the graph turns or changes direction, and decide whether it is the highest or lowest point of the graph. State the coordinates of this point.

b Minimum turning point is at $(-1, -4)$.

c **1** State the x-coordinates of the points where the graph crosses the x-axis.

2 State the y-coordinate of the point where the graph crosses the y-axis.

c The x-intercepts are at -3 and 1. They occur at the points $(-3, 0)$ and $(1, 0)$.

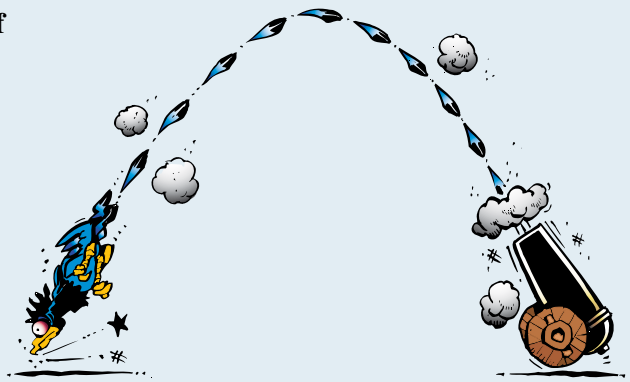
The y-intercept is at -3. It occurs at the point $(0, -3)$.

- A rule connecting x and y will be occasionally provided. From this rule, pairs of x- and y-values can be calculated.
- In the following example, the rule is given as $h = -5x^2 + 25x$.
- Graphs can be drawn using a graphics calculator, a graphing program on your computer or by hand.

WORKED EXAMPLE 5

TI | CASIO

Rudie, the cannonball chicken, was fired out of a cannon. His path could be traced by the equation $h = -5x^2 + 25x$, where h is Rudie's height, in metres, above the ground and x is the horizontal distance, in metres, from the cannon. Plot the graph for $0 \leq x \leq 5$ and use it to find the maximum height of Rudie's path.



THINK

1 Write the equation.

2 Complete a table of values by substituting into the equation each integer value of x from 0 to 5. For example, when $x = 0$, $h = -5 \times 0^2 + 25 \times 0 = 0$.

3 List the coordinates of the points.

WRITE

$h = -5x^2 + 25x$

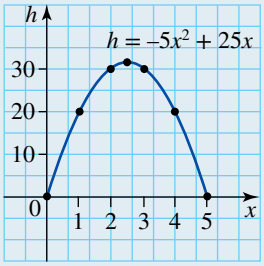
x	0	1	2	3	4	5
h	0	20	30	30	20	0

(0, 0), (1, 20), (2, 30), (3, 30), (4, 20), (5, 0)

4 As a parabola is symmetrical, the greatest value of h must be greater than 30 and occurs when x lies between the 2 and 3, so find the value of h when $x = 2.5$.

When $x = 2.5$, $h = -5 \times (2.5)^2 + 25 \times 2.5$
$= 31.25$

5 Draw and label a set of axes, plot the points from the table and join the points to form a smooth curve.



6 The maximum height is the value of h at the highest point of the graph.

$h = 31.25$

7 Answer the question in a sentence.

The maximum height of Rudie's path is 31.25 metres.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Substitution into quadratic equations (doc-10990)

Complete this digital doc: SkillSHEET: Plotting coordinate points (doc-10991)

Exercise 17.3 Plotting points to graph quadratic functions

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4a, c, e, g, 5–7, 10, 14

CONSOLIDATE
Questions:
1, 2, 3, 4b, d, f, h, 5, 6, 8, 10, 12, 14, 15

MASTER
Questions:
1, 2, 3, 4a, f, g, 5, 6, 9–17

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE3 Copy and complete the table of values for each of the following equations, then list the coordinates of each of the points.

a. $y = 2x^2$

x	−3	−2	−1	0	1	2	3
$y = 2x^2$							

b. $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
$y = x^2 - 4$							

c. $y = -x^2 + 4x + 5$

x	−2	−1	0	1	2	3	4	5	6
$y = -x^2 + 4x + 5$									

2. Using the equations in question **1**:

i. plot the points and join with a smooth curve

ii. identify the axis of symmetry and state its equation.

3. Complete the following table of values, plot the points, and then join the points with a smooth curve.

x	−2	−1	0	1	2	3	4	5
$y = -x^2 + x$								

4. WE4 Plot the graph of each of the following and, hence, state:

i. the equation of the axis of symmetry

ii. the coordinates of the turning point and whether it is a maximum or a minimum

iii. the x- and y-intercepts.

Remember: $-7 \leq x \leq 0$ means the graph is drawn from $x = -7$ to $x = 0$.

a. $y = x^2 + 8x + 15, -7 \leq x \leq 0$

b. $y = x^2 - 1, -3 \leq x \leq 3$

c. $y = x^2 - 4x, -1 \leq x \leq 5$

d. $y = x^2 - 2x + 3, -2 \leq x \leq 4$

e. $y = x^2 + 12x + 35, -9 \leq x \leq 0$

f. $y = -x^2 + 4x + 5, -2 \leq x \leq 6$

g. $y = 2x^2 - 16x - 18, -1 \leq x \leq 9$

h. $y = -x^2 - 4x - 3, -4 \leq x \leq 2$

5. Consider the equations for $-3 \leq x \leq 3$:

i. $y = x^2 + 2$ **ii.** $y = x^2 + 3$.

a. Make a table of values and plot the points on the same set of axes.

b. State the equation of the axis of symmetry for each equation.

c. State the x-intercepts for each equation.

Understanding

6. WE5 A missile was fired from a boat during a test. The missile's path could be traced by the equation $h = -\frac{1}{2}x^2 + x$, where h is the missile's height above the ground, in kilometres, and x is the horizontal distance from the boat, in kilometres.

Plot the graph for $0 \leq x \leq 2$ and use it to find the maximum height of the missile's path, in metres.

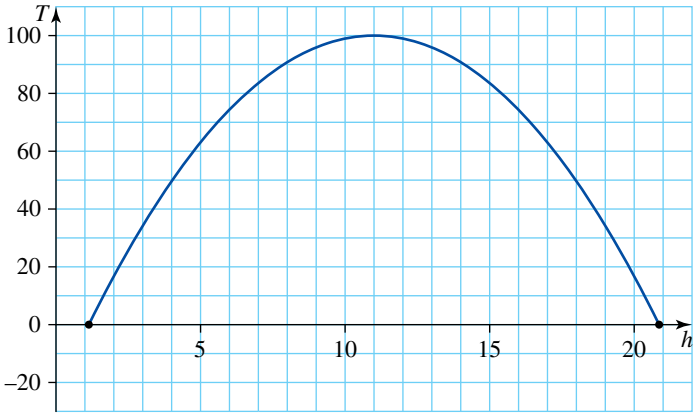
7. The speed versus distance graph of a car braking efficiently has the equation $s = \frac{v^2}{260}$, where v is the speed, in km/h, and s is the stopping distance, in metres.

a. Use graph paper to plot this graph for $0 \leq v \leq 120$ (choose an appropriate scale). From the graph find the stopping distance, to the nearest metre, for a car at:

i. 60 km/h **ii.** 100 km/h **iii.** 120 km/h.

b. What is the maximum speed a car can travel if it must stop within 25 m? Round answers to the nearest whole number. Compare your answers from the graph and formula.

8. SpaceCorp sent a lander to Mars to measure the temperature change over a period of time. The results were plotted on a set of axes shown on the following page. From the graph it can be seen that the temperature change follows the quadratic rule $T = -h^2 + 22h - 21$, where T is the temperature in degrees Celsius, and the time elapsed, h, is in hours.



a. What was the initial temperature on Mars?
b. When was the temperature measured as 0 °C?
c. When was the highest temperature recorded?
d. What was the highest temperature recorded?

9. A ball was thrown from the top of a building. Its height above the ground, h, is given by $h = -4.9t^2 + 5t + 12.5$, where h is in metres and t is the time, in seconds, since the ball was thrown.
a. What is the height of the building? (*Hint:* Where was the ball at $t = 0$?)
b. Sketch a graph of h versus t.
c. How long did it take for the ball to reach the ground?

Reasoning

10. The Grand Old Duke of York marched his men up a hill that followed the path of the equation $y = -x^2 + 16x$, where y is the vertical distance travelled and x is the horizontal distance. Both measurements are in metres. Plot the graph for $0 \leq x \leq 16$, and use it to find out whether the Duke and his men were halfway across when they were halfway up. If not, explain how far across they were on the upward journey. Round answers to 1 decimal place.

11. When a golfer hits the ball from the tee with a 7 iron, the ball follows the path of the parabola $h_1 = -0.01x^2 + 0.95x$. When hit with a 9 iron, the ball follows the path of the parabola $h_2 = -0.02x^2 + 1.6x$. In both cases, h is the height above ground, in metres, and x is the horizontal distance from the golfer, in metres. The green is 80 metres away from the tee. Plot both graphs to determine which club the golfer should use in order to land the ball on the front of the green.

12. On a basketball court, Sam threw a basketball such that it followed the path defined by the equation $y = -0.1(x - 13)(x + 1)$, where y is the height of the ball in metres and x is the horizontal distance from Sam in metres.
a. How far off the ground was the ball when Sam threw it?
b. How far from Sam did the ball bounce?
c. If Saney, another player, is 10 metres away from Sam, explain how high he will need to jump in order to catch the ball directly above his head. (Saney is 1.85 m tall and his arm length is 70 cm.)

13. In order for Ruby to participate in a trip to the snow this year, she will need to fund-raise. By selling mini Easter eggs at school for different prices each day, Ruby found that the relationship between the cost (c) of a sachet of mini Easter eggs and the total profit (P) was modelled by the equation $P = (c + 2)(6 - c)$. One day she did not sell any Easter eggs but a teacher kindly gave her $12 towards her trip.
a. Draw a graph of $P = (c + 2)(6 - c)$.
b. Explain why the graph should not appear to the left of the P axis.



c. What will the mini Easter eggs cost if the maximum profit is to be made? How much will this profit be?

d. Up to what amount could Ruby charge before she makes a loss?

Problem solving

14. a. The axis of symmetry of a parabola is $x = -4$. If one x-intercept is -10, what is the other x-intercept?

b. Suggest a possible equation for the parabola.

15. a. Compare the turning points of the parabolas $y = 2x^2 - 6x$ and $y = 2x^2 - 6x - 3$.

b. Find an equation for a quadratic curve that has the same axis of symmetry as $y = 2x^2 - 6x$ and a turning point $(1.5, -2.5)$.

16. Water comes out of a garden hose in the shape of a parabola and can be modelled by the equation $y = -0.016x^2 + 0.5x + 4.5$, where x is the horizontal distance from the hose's nozzle and y is the height of the water (both in metres). Investigate whether the water can be jetted over a 2-metre fence that is located 35.5 metres from the nozzle.



17. The data below models the equation of $y = ax^2 + bx + c$. Use the table to show that $c = 0$ and find the values of a and b.

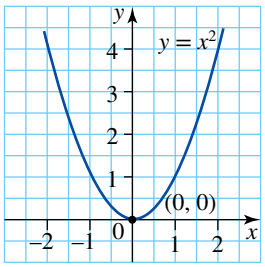
x	0	1	2	3	4
y	0	2	6	12	20

Reflection

If given a choice, what is the best way to choose the points to be plotted on a graph?

17.4 Sketching parabolas of the form y = ax2

17.4.1 The graph of the quadratic function $y = x^2$



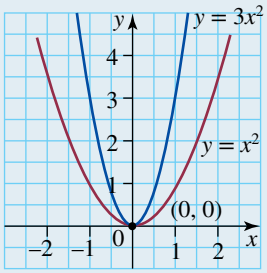
- The simplest parabola, $y = x^2$, is shown at right.
  - Both the x- and y-axes are clearly indicated, along with their scales.
  - The turning point $(0, 0)$ is indicated.
  - The x- and y-intercepts are indicated. For this graph they are all $(0, 0)$.
  - This is an example of a parabola that just touches (does not cross) the x-axis at $(0, 0)$.

17.4.2 Parabolas of the form $y = ax^2$, where $a > 0$

- A coefficient in front of the x^2 term affects the **dilation** of the graph, making it wider or narrower than the graph of $y = x^2$.
- If $a > 1$ then the graph becomes narrower, whereas if $0 < a < 1$, the graph becomes wider.
- The following features of the parabola remain *unchanged*, for $a > 0$, regardless of the value of a:
  - the axis of symmetry is $x = 0$
  - the turning point is $(0, 0)$
  - the x-intercept is $(0, 0)$
  - the y-intercept is $(0, 0)$
  - the shape of the parabola is always upright or a U shape.

WORKED EXAMPLE 6 TI | CASIO

On the same set of axes sketch the graph of $y = x^2$ and $y = 3x^2$, marking the coordinates of the turning point and the intercepts. State which graph is narrower.

THINK	WRITE/DRAW
1 Write the equation of the first graph.	$y = x^2$
2 State its axis of symmetry.	The axis of symmetry is $x = 0$.
3 State the coordinates of the turning point.	The turning point is $(0, 0)$.
4 State the intercepts.	The x-intercept is 0 and the y-intercept is 0.
5 Find the coordinates of one other point.	When $x = 1$, $y = 1$. $(1, 1)$
6 Write the equation of the second graph.	$y = 3x^2$
7 State its axis of symmetry.	The axis of symmetry is $x = 0$.
8 State the coordinates of the turning point.	The turning point is $(0, 0)$.
9 State the intercepts.	The x-intercept is 0 and the y-intercept is 0.
10 Find the coordinates of one other point.	When $x = 1$, $y = 3$. $(1, 3)$
11 Sketch the graphs, labelling the turning point.	
12 State which graph is narrower.	The graph of $y = 3x^2$ is narrower.

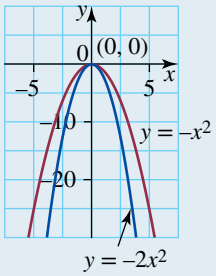
17.4.3 Parabolas of the form $y = ax^2$, where $a < 0$

- When $a < 0$, the graph is inverted; that is, it is $\cap$ shaped.
- A coefficient in front of the x^2 term affects the dilation of the graph, making it wider or narrower than the graph of $y = x^2$.
- If $-1 < a < 0$, the graph is wider than $y = -x^2$.
- If $a < -1$, the graph is narrower than $y = -x^2$.
- The following features of the parabola remain unchanged for $a < 0$, regardless of the value of a:
  - the axis of symmetry is $x = 0$
  - the turning point is $(0, 0)$
  - the x-intercept is $(0, 0)$
  - the y-intercept is $(0, 0)$
  - The shape of the parabola is always inverted or an upside-down U shape ($\cap$).

WORKED EXAMPLE 7 TI | CASIO

On the same set of axes sketch the graphs of $y = -x^2$ and $y = -2x^2$, marking the coordinates of the turning point and the intercept. State which graph is narrower.

THINK	WRITE/DRAW
1 Write the equation of the first graph.	$y = -x^2$
2 State its axis of symmetry.	The axis of symmetry is $x = 0$.

3 State the coordinates of the turning point.	The turning point is (0, 0).
4 State the intercepts.	The x-intercept is (0, 0) and the y-intercept is also (0, 0).
5 Calculate the coordinates of one other point.	When $x = 1$, $y = -1$, $(1, -1)$.
6 Write the equation of the second graph.	$y = -2x^2$
7 State its axis of symmetry.	The axis of symmetry is $x = 0$.
8 State the coordinates of the turning point.	The turning point is (0, 0).
9 State the intercepts.	The x-intercept is (0, 0) and the y-intercept is also (0, 0).
10 Calculate the coordinates of one other point.	When $x = 1$, $y = -2$, $(1, -2)$.
11 Sketch the two graphs on a single set of axes, labelling the turning point (as well as intercepts and maximum).	 The graph of $y = -2x^2$ is narrower.

Exercise 17.4 Sketching parabolas of the form $y = ax^2$

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 8, 10, 13, 16	Questions: 1–7, 9, 10, 12, 13, 16–18	Questions: 1–7, 10, 12–20

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE6 On the same set of axes, sketch the graph of $y = x^2$ and $y = 4x^2$, marking the coordinates of the turning point and the intercepts. State which graph is narrower.
2. On the same set of axes sketch the graph of $y = x^2$ and $y = \frac{1}{2}x^2$, marking the coordinates of the turning point and the intercepts. State which graph is narrower.
3. Sketch the graph of the following table. State the equation of the graph.

x	−4	−3	−2	−1	0	1	2	3	4
y	4	2.25	1	0.25	0	0.25	1	2.25	4

4. WE7 Using the same set of axes, sketch the graphs of $y = -x^2$ and $y = -0.5x^2$, marking the coordinates of the turning point and the intercepts. State which graph is narrower.

5. Using the same set of axes from question **4**, sketch the graph of $y = -5x^2$, marking the coordinates of the turning point and the intercepts. State which graph is the narrowest.

6. MC **a.** The graph of $y = -3x^2$ is:

A. wider than $y = x^2$ **B.** narrower than $y = x^2$

C. the same width as $y = x^2$ **D.** a reflection of $y = x^2$ in the x-axis

b. The graph of $y = \frac{1}{3}x^2$ is:

A. wider than $y = x^2$ **B.** narrower than $y = x^2$

C. the same width as $y = x^2$ **D.** a reflection of $y = x^2$ in the x-axis

c. The graph of $y = \frac{1}{2}x^2$ is:

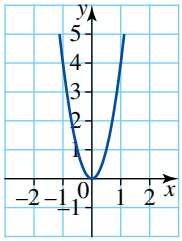
A. wider than $y = \frac{1}{4}x^2$ **B.** narrower than $y = \frac{1}{4}x^2$

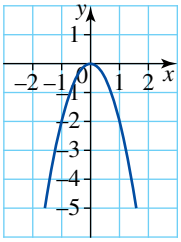
C. the same width as $y = \frac{1}{4}x^2$ **D.** a reflection of $y = \frac{1}{4}x^2$ in the x-axis

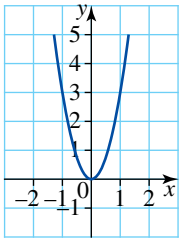
7. Match each of the following parabolas with the appropriate equation from the list.

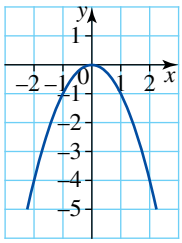
i. $y = 3x^2$ **ii.** $y = -x^2$ **iii.** $y = 4x^2$

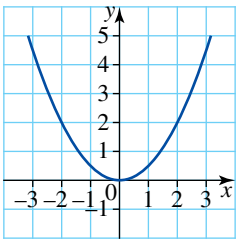
iv. $y = \frac{1}{2}x^2$ **v.** $y = -4x^2$ **vi.** $y = -2x^2$

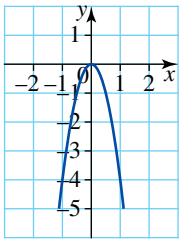
a.


b.


c.


d.


e.


f.


Understanding

8. Write an equation for a parabola that has a minimum turning point and is narrower than $y = x^2$.

9. Write an equation for a parabola that has a maximum turning point and is wider than $y = x^2$.

10. Find the equation of a quadratic relation if it has an equation of the form $y = ax^2$ and passes through:

a. $(1, 3)$ **b.** $(-1, -1)$.

11. Consider the equation $y = -3.5x^2$. Calculate the values of y when x is:

a. 10 **b.** -10 **c.** -3 **d.** 1.5 **e.** -2.2.

12. The amount of power (watts) in an electric circuit varies as the square of the current (amperes). If the power is 100 watts when the current is 2 amperes, calculate:

a. the power when the current is 4 amperes **b.** the power when the current is 5 amperes.

Reasoning

13. a. Sketch the following graphs on the same axes: $y = x^2$, $y = 2x^2$ and $y = -3x^2$. Shade the area between the two graphs above the x-axis and the area inside the graph below the x-axis. Describe the shape that has been shaded.

b. Sketch the following graphs on the same axes: $y = x^2$, $y = \frac{1}{3}x^2$ and $y = -4x^2$. Shade the area inside the graphs of $y = x^2$ and $y = -4x^2$. Also shade the area between the graph of $y = \frac{1}{3}x^2$ and the x-axis. Describe the shape that you have drawn.

14. This figure at right shows the parabolic shape of a skate ramp. The rule of the form $y = ax^2$ describes the shape of the ramp. If the top of the ramp has coordinates (3, 6) find a possible equation that describes the shape. Justify your answer.



15. The total sales of a fast food franchise varies as the square of the number of franchises in a given city. Let S be the total sales (in millions of dollars per month) and f be the number of franchises. If sales $= \$25$ million when $f = 4$, then:

a. show that the equation relating S and f is $S = 1\,562\,500f^2$

b. determine the number of franchises needed to (at least) double the sales from $\$25\,000\,000$.

Problem solving

16. Xanthe and Carly were comparing parabolas on their CAS calculators. Xanthe graphed $y = 0.001x^2$ with window settings $-1000 \leq x \leq 1000$ and $0 \leq y \leq 1000$. Carly graphed $y = x^2$ with window settings $-k \leq x \leq k$ and $0 \leq y \leq k$. Except for the scale markings, the graphs looked exactly the same. What is the value of k?

17. The parabola $y = x^2$ is rotated 90° clockwise about the origin. Find the equation.

Reflection

List the features of a parabola that remain unchanged when a changes from positive to negative in $y = ax^2$.

17.5 Sketching parabolas of the form $y = ax^2 + c$

17.5.1 Sketching parabolas of the form $y = ax^2 + c$

- Adding a constant, c, to the rule $y = ax^2$ **translates** the graph vertically.
- If $c > 0$, the value of y increases, therefore the graph translates vertically upwards.
- If $c < 0$, the value of y decreases, therefore the graph translates vertically downwards.

WORKED EXAMPLE 8 — TI | CASIO

For each part of the question, sketch the graph of $y = x^2$, then, on the same axes, sketch the given graph, clearly labelling the turning point.

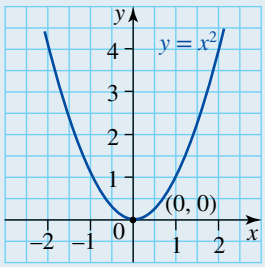
a $y = x^2 + 2$ **b** $y = -x^2 - 3$

THINK

a 1 Sketch the graph of $y = x^2$ by drawing a set of labelled axes, marking the turning point (0, 0) and noting that it is symmetrical about the y-axis.

WRITE/DRAW

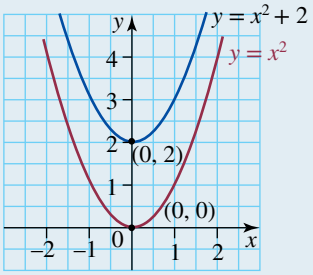
a



2 Find the turning point of $y = x^2 + 2$ by adding 2 to the y-coordinate of the turning point of $y = x^2$.

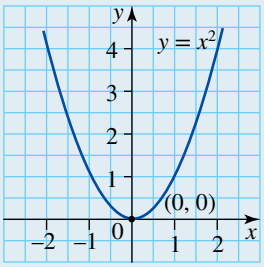
The turning point of $y = x^2 + 2$ is (0, 2).

3 Using the same axes as for the graph of $y = x^2$, sketch the graph of $y = x^2 + 2$, marking the turning point and making sure that it is the same width as the graph of $y = x^2$. (The coefficient of x^2 is the same for both graphs.)



b 1 Sketch the graph of $y = x^2$ by drawing a set of labelled axes, marking the turning point (0, 0) and noting that it is symmetrical about the y-axis.

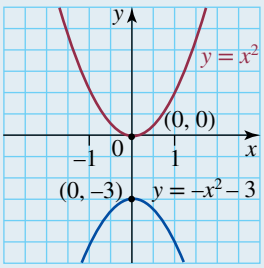
b



2 Find the turning point of $y = -x^2 - 3$ by subtracting 3 from the y-coordinate of the turning point of $y = x^2$.

The turning point of $y = -x^2 - 3$ is (0, –3).

3 Using the same axes as for the graph of $y = x^2$, sketch the graph of $y = -x^2 - 3$, marking the turning point, inverting the graph and making sure that the graph is the same width as the graph of $y = x^2$.



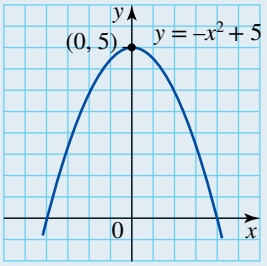
- If the graph is of the form $y = -x^2 + c$ and $c > 0$, the graph translates vertically upwards, and if $c < 0$, the graph translates vertically downwards.

WORKED EXAMPLE 9

Sketch the graph of $y = -x^2 + 5$, drawing clearly labelled axes and marking the axis of symmetry, turning point and y-intercept. State whether the turning point is a maximum or minimum.

THINK	WRITE/DRAW
1 The equation of the axis of symmetry is the same as for $y = x^2$.	The axis of symmetry is $x = 0$.
2 The turning point has been moved up c units and is (0, c).	The turning point is (0, 5).
3 The coefficient of x^2 is negative so the graph is inverted.	Maximum turning point

4 Draw clearly labelled axes, mark the turning point and draw the graph.



learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Vertical translation: y = x² + c (int-1192)

Complete this digital doc: WorkSHEET: Quadratic functions I (doc-11009)

Exercise 17.5 Sketching parabolas of the form $y = ax^2 + c$ assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 13, 15	Questions: 1–10, 12, 13, 15–17	Questions: 1–8, 10–19

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE8 For each part of the question, sketch the graph of $y = x^2$, then, on the same axes, sketch the given graph, clearly labelling the turning point.

a. $y = x^2 + 1$ **b.** $y = x^2 + 4$ **c.** $y = x^2 - 1$

d. $y = x^2 - 4$ **e.** $y = -x^2 + 1$ **f.** $y = -x^2 - 1$

2. How does a positive number at the end of the equation affect the graph?

3. How does a negative number at the end of the equation affect the graph?

4. WE9 Sketch each of the following graphs on clearly labelled axes, marking the axis of symmetry, turning point and y-intercept of each one. State whether the turning point is a maximum or a minimum.

a. $y = x^2 + 2$ **b.** $y = x^2 - 5$ **c.** $y = -x^2 + 3$

d. $y = -x^2 + 4$ **e.** $y = -x^2 - 3$ **f.** $y = x^2 - \frac{1}{2}$

5. Sketch the following graphs, indicating the turning point and estimating the x-intercepts.

a. $y = 4 - x^2$ **b.** $y = -4 - x^2$ **c.** $y = 1 - x^2$

6. a. Does the turning point change if there is a negative number in front of the x^2 term in the equation $y = x^2$?

b. How does a negative coefficient of x^2 affect the graph?

c. What is the axis of symmetry for all the graphs in this exercise?

7. MC **a.** The turning point for the graph of the equation $y = -x^2 + 8$ is:

A. (0, 0) **B.** (−1, 8) **C.** (0, 8) **D.** (0, −8)

b. The turning point of the graph of the equation $y = x^2 - 16$ is:

A. $(1, -16)$ **B.** $(-16, 1)$ **C.** $(0, 0)$ **D.** $(0, -16)$

c. The graph of $y = x^2 - 7$ moves the graph of $y = x^2$ in the following way:

A. up 1 **B.** down 1 **C.** up 7 **D.** down 7

d. The y-intercept of the graph of $y = -x^2 - 6$ is:

A. 1 **B.** -1 **C.** 6 **D.** -6

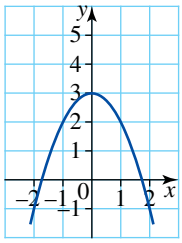
Understanding

8. Match each of the following parabolas with the appropriate equation from the list.

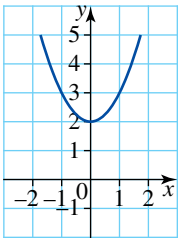
i. $y = x^2 - 3$ **ii.** $y = x^2 + 3$ **iii.** $y = 3 - x^2$

iv. $y = x^2 + 2$ **v.** $y = -x^2 + 2$ **vi.** $y = -x^2 - 2$

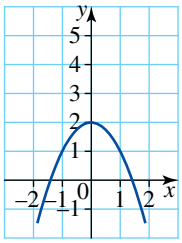
a.



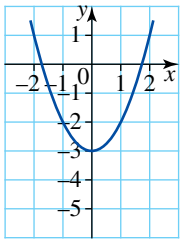
b.



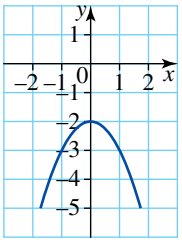
c.



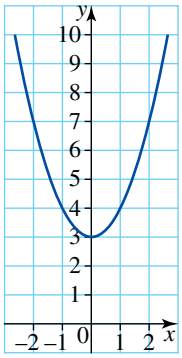
d.



e.



f.



9. The vertical cross-section through the top of the mountain called the Devil's Tower can be approximated by the graph $y = -x^2 + 5$. Sketch the graph. If the x-axis represents sea level, and both x and y are in kilometres, find the maximum height of the mountain.

10. The cross-section of a large bowl can be given by the rule $y = 3x^2 - 243$, where both x (measured across the bowl) and y (the depth of the bowl) are measured in centimetres.

a. By factorising the rule, find the points where $y = 0$. What are these points called?

b. If the bowl's rim occurs at the point where $y = 0$, find the greatest depth of the bowl.

c. What is the width of the bowl at its rim?

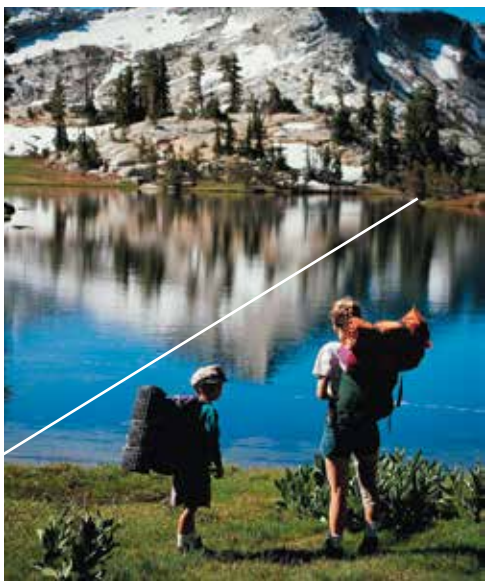
11. The photo at right shows an imaginary line drawn across the surface of a lake. A vertical cross-section of the lake is taken at the line, such that the depth of the lake can be approximated by the graph $y = x^2 - 12$, where x and y are in metres.

a. What would be a suitable 'domain' (set of possible x-values) for this graph?

b. Sketch the graph over this domain.

c. What is the greatest depth of the lake?

d. What is the width of the lake along the white line shown in the photo?



Reasoning

12. Show that the equation of the parabola that is of the form $y = x^2 + c$ and passes through:
a. $(2, 1)$ is $y = x^2 - 3$ **b.** $(-3, -1)$ is $y = x^2 - 10$.

13. A ball is thrown vertically upwards and follows a path given by $h = at^2 + c$, where h is the height of the ball in metres and t is the time of flight in seconds. If the ball reached a height of 12 m and took 2 seconds to reach this height and another 2 seconds to reach the ground, show that the rule for path of the ball is $h = -3t^2 + 12$.

14. Sketch $y = x^2 - 2$ and $y = -x^2 + 2$ on the same set of axes. Use algebra to explain where they intersect.

Problem solving

15. The path of a ball rolling off the end of a table follows a parabolic curve and can be modelled by the equation $y = ax^2 + c$. A student rolls a ball off a tabletop that is 128 cm above the floor, and the ball lands 80 cm horizontally away from the desk. If the student sets a cup 78 cm above the floor to catch the ball during its fall, where should the cup be placed?

16. On a set of axes, plot the points $(0, 2)$, $(1, 4)$, $(-3, 20)$ and $(-2, 10)$. On the same set of axes, reflect the points about the y-axis and plot the points. Join all the points together and find the equation.

17. What is the meaning of the intersection point between the graphs with equations $y = -2.5x^2 + 277.5$ and $y = -16x^2 + 555$?

18. For the equation $y = x^2 - 5$, find the exact value of the y-coordinate when the x-coordinate is $2 + \sqrt{5}$.

Reflection

Describe the effect of changing a and c, separately, in the rule $y = ax^2 + c$.

CHALLENGE 17.1

A rocket is released from a rocket blaster on the ground and travels in an upward parabolic path to a maximum height of 125 meters, before falling and landing on the ground 10 meters from where it was launched. Determine the equation of the path taken by the rocket.

17.6 Sketching parabolas of the form $y = (x - h)^2$

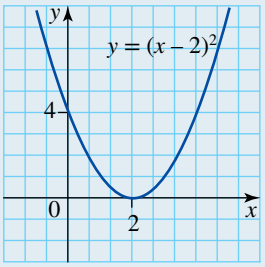
17.6.1 Sketching $y = (x - h)^2$

- Subtracting a constant value h from $y = x^2$ translates the parabola h units to the right; otherwise the parabola's shape is unchanged.
  - The equation is $y = (x - h)^2$, where h is a positive quantity.
  - The x-intercept occurs when $x = h$.
- Adding a constant value h to x in $y = x^2$ translates the parabola h units to the left; otherwise the parabola's shape is unchanged.
  - The equation is $y = (x + h)^2$, where h is a positive quantity.
  - The x-intercept occurs when $x = -h$.
- The y-intercept occurs when $x = 0$, and is always at $y = h^2$.

WORKED EXAMPLE 10 TI | CASIO

On clearly labelled axes, sketch the graph of $y = (x - 2)^2$, marking the turning point and y-intercept. State whether the turning point is a maximum or a minimum.

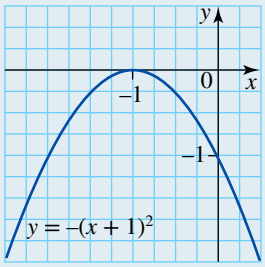
THINK	WRITE/DRAW
1 Write the equation.	$y = (x - 2)^2$

2 Find the axis of symmetry ($x = h$ where h is 2).	The axis of symmetry is $x = 2$.
3 Find the turning point, which has been moved to the right.	The turning point is (2, 0).
4 The sign in front of the bracket is positive so the parabola is upright.	Minimum turning point
5 Substitute 0 for x in the equation to find the y-intercept.	y-intercept: when $x = 0$, $y = (0 - 2)^2$ $= 4$ The y-intercept is 4.
6 Draw a clearly labelled set of axes, mark the turning point and y-intercept and draw the graph of $y = (x - 2)^2$. Note that the sign in the brackets is negative so the graph moves 2 units to the right.	

- Observe that the turning point also shifts by the same amount and direction as the x-intercept.

WORKED EXAMPLE 11

Sketch the graph of $y = -(x + 1)^2$, labelling the turning point, stating whether it is a maximum or minimum, finding the y-intercept and the equation of the axis of symmetry.

THINK	WRITE/DRAW
1 Write the equation.	$y = -(x + 1)^2$
2 Find the turning point, which has been moved to the left.	The turning point is $(-1, 0)$.
3 The sign in front of the bracket is negative so the parabola is inverted.	Maximum turning point
4 The y-intercept is where $x = 0$, so substitute 0 for x in the equation.	y-intercept: when $x = 0$, $y = -(0 + 1)^2$ $= -1$ The y-intercept is -1.
5 Find the axis of symmetry ($x = h$, where h is -1).	The axis of symmetry is $x = -1$.
6 Draw a clearly labelled set of axes, mark the turning point and y-intercept and draw the graph of $y = -(x + 1)^2$.	

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Horizontal translation: $y = (x + h)^2$ (int-1193)

Exercise 17.6 Sketching parabolas of the form $y = (x - h)^2$ assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 12, 15	Questions: 1–10, 12, 13, 15, 16	Questions: 1–8, 9a, 10a, c, 11–18

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE10 On clearly labelled axes, sketch the graphs of each of the following, marking the turning point and y-intercept. State whether the turning point is a maximum or a minimum.

a. $y = (x - 1)^2$ **b.** $y = (x - 3)^2$ **c.** $y = (x + 4)^2$

d. $y = (x + 2)^2$ **e.** $y = (x - 5)^2$ **f.** $y = (x + 6)^2$

2. State the axis of symmetry for each of the graphs in question 1.

3. How does a positive number for h in $y = (x - h)^2$ affect the graph of $y = x^2$?

4. How does a negative number for h in $y = (x - h)^2$ affect the graph of $y = x^2$?

5. WE11 On clearly labelled axes, sketch the graphs of each of the following, marking the turning point and y-intercept. State whether the turning point is a maximum or a minimum.

a. $y = -(x - 1)^2$ **b.** $y = -(x - 3)^2$ **c.** $y = -(x + 4)^2$

d. $y = -(x + 2)^2$ **e.** $y = -(x - 5)^2$ **f.** $y = -(x + 6)^2$

6. Do the answers to questions **3** and **4** change if there is a negative sign in front of the bracket? How does this negative sign affect the graph?

7. MC **a.** The axis of symmetry for the graph $y = (x + 3)^2$ is:

A. $y = 0$ **B.** $x = 3$ **C.** $x = -3$ **D.** $x = 2$

b. The turning point of the graph $y = (x + 3)^2$ is:

A. $(0, 0)$ **B.** $(1, 3)$ **C.** $(1, -3)$ **D.** $(-3, 0)$

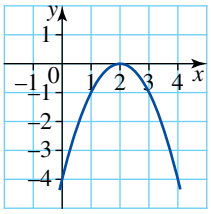
Understanding

8. Match each of the following parabolas with the appropriate equation from the list.

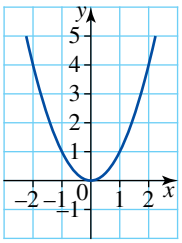
i. $y = (x - 2)^2$ **ii.** $y = x^2$ **iii.** $y = (x + 3)^2$

iv. $y = -(x + 2)^2$ **v.** $y = (x - 3)^2$ **vi.** $y = -(x - 2)^2$

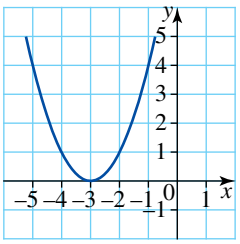
a.



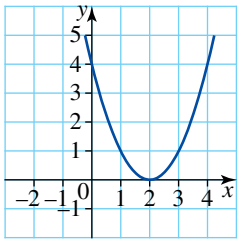
b.



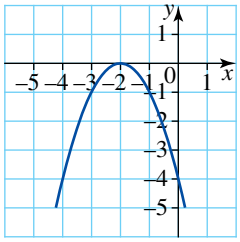
c.



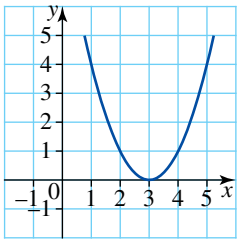
d.



e.



f.



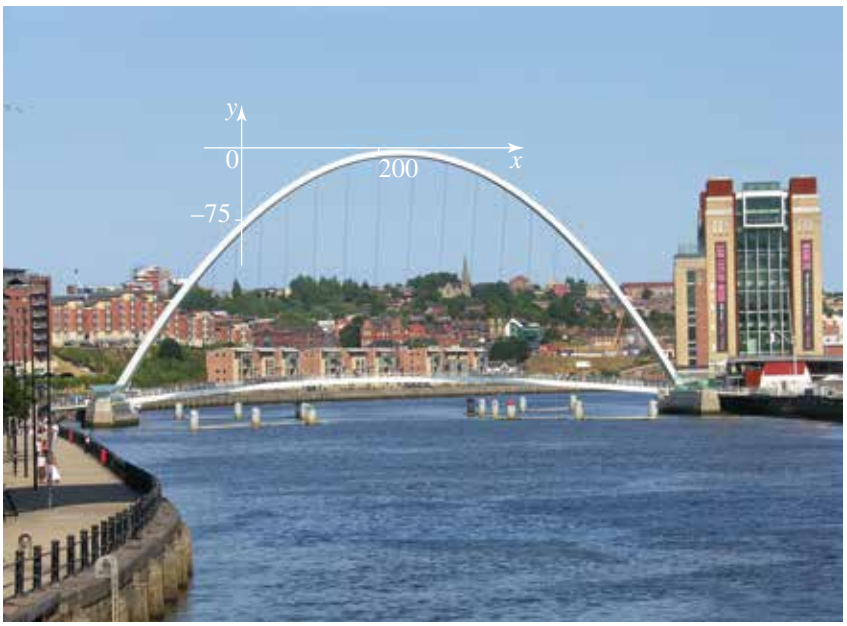
9. Find the equation of the parabola that is of the form $y = (x - h)^2$ and passes through:
a. $(3, 1)$ **b.** $(-1, 9)$.

10. State the differences, if any, between the following pairs of parabolas.
a. $y = (1 - x)^2$ and $y = (x - 1)^2$
b. $y = (x - 2)^2$ and $y = (x + 2)^2$
c. $y = -(x - 4)^2$ and $y = (4 - x)^2$

11. Write the rule for the graph in the form $y = a(x - h)^2$ such that the y-intercept is $= -12$ and the x-intercept is at $x = 2$.

Reasoning

12. The figure below shows the span of the Gateshead Millennium Bridge in England. A set of axes has been superimposed onto the photo. Use the coordinates $(200, 0)$ and $(0, -75)$ to show that a possible equation is $y = -0.001\,875\,(x - 200)^2$.



13. a. By sketching the graph, or using another method, determine the x- and y-intercepts of $y = 4(x - 3)^2$.
b. Using the result from part **a**, show that the x-intercept and y-intercept for the equation $y = a(x - h)^2$ are $(h, 0)$ and $(0, ah^2)$ respectively.

14. The amount of profit made by a new company each month can be modelled by the rule $y = a(t - h)^2$, where y is the monthly profit and t is the integer value of the month (0 = January). If the January profit was \$9000 and the March profit was \$0:
a. determine the values of a and h
b. determine the month when the profit is \$20 250
c. explain why normally three points are required to uniquely determine a parabola, but in this example only two points are required.

Problem solving

15. The graph of a quadratic function touches the x-axis and has its turning point at $(3, 0)$. Graph two parabolas that apply to this description and find equations for them. How many examples are possible? Explain.

16. Sketch the graphs of $y = (x - 2)^2$ and $y = (2 - x)^2$. What do you notice about the graphs? Explain why this is true.

17. Find the equation of a parabola that has the turning point $(-2, 0)$ and is the same size as the graph $y = 2(x - 3)^2$.

Reflection

What changes and what remains the same when a changes from a positive to a negative number in $y = a(x - h)^2$?

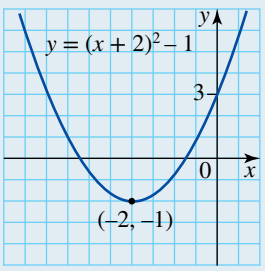
17.7 Sketching parabolas of the form $y = (x - h)^2 + k$

17.7.1 Sketching $y = (x - h)^2 + k$

- The equation $y = (x - h)^2 + k$ combines a vertical translation of k and a horizontal translation of h together.
- The equation $y = (x - h)^2 + k$ is called **turning point form** because the turning point is given by the coordinates (h, k).

WORKED EXAMPLE 12

Sketch the graph of $y = (x + 2)^2 - 1$, marking the turning point and the y-intercept, and indicate the type of turning point.

THINK	WRITE/DRAW
1 Write the equation.	$y = (x + 2)^2 - 1$
2 State the turning point. As the equation is in the form $y = (x - h)^2 + k$, the turning point is (h, k).	The turning point is $(-2, -1)$.
3 There is no sign outside the brackets, so the parabola is upright.	Minimum turning point
4 Find the y-intercept by substituting $x = 0$ into the equation.	y-intercept: when $x = 0$, $y = (0 + 2)^2 - 1$ $= 3$
5 Draw clearly labelled axes, mark the coordinates of the turning point, the y-intercept and draw a symmetrical graph.	

- Note that in the previous example the graph has shifted 2 units left ($h = 2$) and 1 unit down ($k = -1$).
- The graph has the same shape as $y = x^2$.

WORKED EXAMPLE 13

TI | CASIO

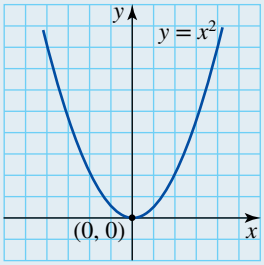
On the same set of axes, sketch the graphs of each of the following, clearly marking the coordinates of the turning point and the *y*-intercept:
a $y = x^2$ **b** $y = (x - 2)^2$ **c** $y = (x - 2)^2 + 1$.
State the changes that are made from a to b and from a to c.

THINK

a Sketch the graph of $y = x^2$, marking the coordinates of the turning point and the *y*-intercept.

WRITE/DRAW

a



b 1 Write the equation.

2 Find the coordinates of the turning point.

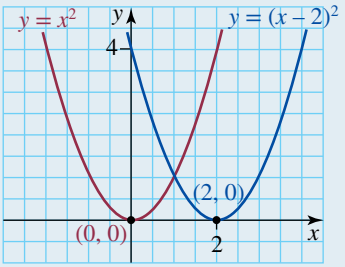
3 Find the *y*-intercept.

b $y = (x - 2)^2$

The turning point is (2, 0).

y-intercept: when $x = 0$,
$y = (0 - 2)^2$
$= 4$

4 On the same set of axes, sketch the graph of $y = (x - 2)^2$, marking the coordinates of the turning point and the *y*-intercept.



c 1 Write the equation.

2 Find the coordinates of the turning point.

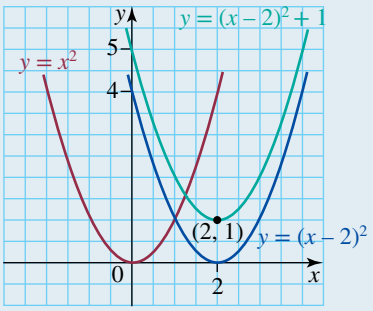
3 Find the *y*-intercept.

c $y = (x - 2)^2 + 1$

The turning point is (2, 1).

y-intercept: when $x = 0$,
$y = (0 - 2)^2 + 1$
$= 4 + 1$
$= 5$

4 On the same set of axes, sketch the graph of $y = (x - 2)^2 + 1$, marking the coordinates of the turning point and the *y*-intercept.



5 State how $y = x^2$ is changed to form $y = (x - 2)^2$.
State how $y = x^2$ is changed to form $y = (x - 2)^2 + 1$.

If $y = x^2$ is moved 2 units to the right, the resulting graph is $y = (x - 2)^2$.
If $y = x^2$ is moved 2 units to the right and 1 unit up, the resulting graph is $y = (x - 2)^2 + 1$.

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Complete this digital doc: WorkSHEET: Quadratic functions II (doc-11010)

Exercise 17.7 Sketching parabolas of the form $y = (x - h)^2 + k$

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 12, 13, 16	Questions: 1, 2, 3a, c, e, 4–8, 10–12, 14, 16–18	Questions: 1, 2, 3a, e, f, 4–10, 12–20

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE12 Sketch the graph of each of the following, marking the turning point, the type of turning point and the y-intercept.

a. $y = (x–1)^2 + 1$ **b.** $y = (x + 2)^2 - 1$ **c.** $y = (x - 3)^2 + 2$
d. $y = (x + 3)^2 - 2$ **e.** $y = -(x + 2)^2 - 1$ **f.** $y = -(x - 1)^2 + 2$

2. Explain why the equation presented in the form $y = (x - h)^2 + k$ is known as 'turning point form'.

3. WE13 Using the same set of axes, sketch each (**a–f**) of the following sets (**i**, **ii**, **iii**) of graphs, clearly marking the coordinates of the turning point and the y-intercept. State the changes that are made from **i** to **ii** and from **i** to **iii**.

	i.	**ii.**	**iii.**
a.	$y = x^2$	$y = (x - 1)^2$	$y = (x - 1)^2 + 3$
b.	$y = x^2$	$y = (x - 2)^2$	$y = (x - 2)^2 - 1$
c.	$y = x^2$	$y = (x + 3)^2$	$y = (x + 3)^2 + 2$
d.	$y = x^2$	$y = (x - 4)^2$	$y = (x - 4)^2 - 1$
e.	$y = x^2$	$y = -(x + 1)^2$	$y = -(x + 1)^2 - 2$
f.	$y = x^2$	$y = -(x - 2)^2$	$y = -(x - 2)^2 - 2$

4. MC **a.** For the graph of $y = (x + 5)^2 - 2$, the coordinates of the turning point are:
A. $(5, -2)$ **B.** $(-2, 5)$ **C.** $(-2, -5)$ **D.** $(-5, -2)$

b. For the graph of $y = -(x - 3)^2 + 7$, the axis of symmetry is:
A. $x = 3$ **B.** $x = 7$ **C.** $x = -3$ **D.** $y = 3$

c. For the graph of $y = (x - 1)^2 - 4$, the y-intercept is:
A. 1 **B.** -1 **C.** -3 **D.** 4

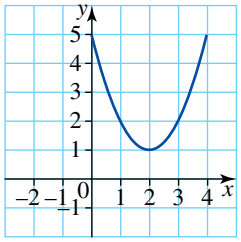
5. The graph of $y = x^2$ is translated 2 units to the left and 3 units up.
- **a.** State the equation of the translated graph.
- **b.** State the location of the turning point.

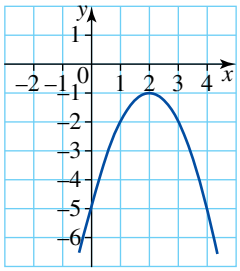
6. The turning point (maximum) of a graph is $(-3, -4)$.
- **a.** State the equation of the graph in turning point form.
- **b.** Calculate the y-intercept.

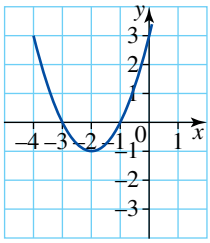
7. Sketch the graph that is the 'mirror' image (reflected vertically) of $y = (x - 2)^2 + 3$.

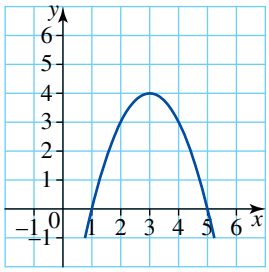
Understanding

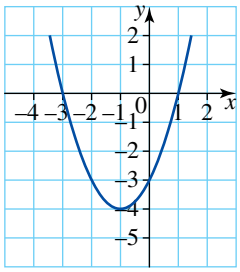
8. State the equation of each of the following given it is of the form $y = (x - h)^2 + k$ or $y = -(x - h)^2 + k$.

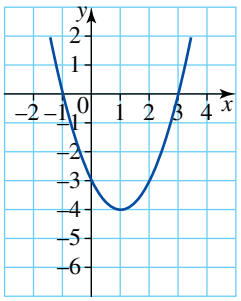
a.


b.


c.


d.


e.


f.


9. Aravind threw a ball into the air. It followed the path of a parabola defined by the equation $h = -(x - 2)^2 + 5$, where h metres represents the height of the ball above the ground x metres horizontally from Aravind's hand.
- **a.** Sketch the graph showing the path of the ball during its flight from Aravind's hand until it reaches the ground, marking on the graph the turning point and the y-intercept.
- **b.** Use the graph to find the height of the ball when it leaves Aravind's hand.
- **c.** Use the graph to find the maximum height that the ball reaches.

10. Write the equations for the following transformations on $y = x^2$.
- **a.** Reflection in the x-axis and translation of 2 units to the left
- **b.** Translation of 1 unit down and 3 units to the right
- **c.** Translation of 1 unit left and 2 units up
- **d.** Reflection in the x-axis, translation of 4 units to the right and 6 units up

11. A rocket is shot in the air from a given point $(0, 0)$. The rocket follows a parabolic trajectory. It reaches a maximum height of 400 m and lands 300 m away from the launching point.
- **a.** How far horizontally from the launching point does the rocket reach its maximum height?
- **b.** State the equation of the path of the rocket.

12. MC A parabola has an equation of $y = x^2 - 6x + 14$. Its turning point form is:

A. $y = (x - 6)^2 + 14$ **B.** $y = (x - 14)^2 - 6$

C. $y = (x - 3)^2 + 5$ **D.** $y = (x - 5)^2 + 3$

Reasoning



13. Nikki wanted to keep a carnivorous plant, so after school she recorded the temperature on her windowsill for 8 hours every day for several months. One summer evening the temperature followed the relationship $t = (h - 5)^2 + 15$, where t is the temperature in degrees Celsius, h hours after 4 pm.

a. Find the temperature on the windowsill at 4 pm.

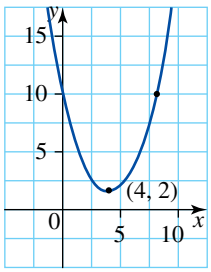
b. Show that the minimum temperature reached during the 8-hour period is 15 °C.

c. Find the number of hours it took for the windowsill to reach the minimum temperature.

d. Sketch a graph of the relationship between the temperature and the number of hours after recording began. Mark the turning point and the t-intercept on the graph.

14. A graph has an axis of symmetry at $x = 1$ and a y-intercept of -4. If the graph's minimum value is -10, determine its equation in the form $y = a(x - h)^2 + k$. Demonstrate the correctness of your result.

15. Students were asked to choose values for a, h and k and substitute them into the general equation $y = a(x - h)^2 + k$ to form a quadratic equation. The graph shown was generated using the values 0.5, 2 and 4 for the three variables. Match the numerical values with the appropriate variable. Justify your answer.



Problem solving

16. a. Describe how to transform the parabola $y = (x - 3)^2$ to obtain the parabolas $y = (x - 3)^2 - 2$ and $y = (x - 3)^2 + 1$.

b. Another parabola is created by moving $y = (x - 3)^2$ so that its turning point is $(5, -4)$. Write an equation for this parabola.

17. Find an equation for the parabola that has its turning point at $(-1, 4)$ and passes through the point $(1, 3)$.

18. Find the x-intercepts of $y = a(x - 2a)^2 - 9a^3$ in terms of a.

19. Use the three points $(-2, 1)$, $(3, 1)$ and $(0, 7)$ to determine a, h and k in the equation $y = a(x - h)^2 + k$.

20. Find the x-intercepts of $y = a(x - h)^2 - k$ in terms of a, h and k.

Reflection

What are the advantages of having the equation of a parabola in turning point form?

17.8 Sketching parabolas of the form $y = (x + a)(x + b)$

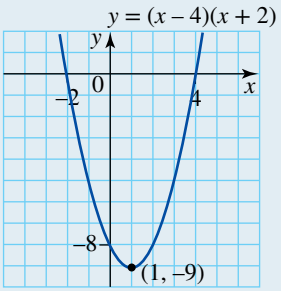
17.8.1 Sketching parabolas of the form $y = (x + a)(x + b)$

- The equation $y = (x + a)(x + b)$ form consists of a pair of linear factors $(x + a)$ and $(x + b)$ multiplied together.
- The x-intercepts are found by setting each factor to 0, namely:
  $(x + a) = 0$, or $x = -a$
  $(x + b) = 0$, or $x = -b$.
- The y-intercept can be found by letting $x = 0$ in the original equation. That is, $y = (0 + a)(0 + b)$, or $y = ab$.

- The axis of symmetry is half-way between the x-intercepts $-a$ and $-b$. That is, $x = \frac{-(a + b)}{2}$.
- Substitute the x-value of the axis of symmetry into $y = (x + a)(x + b)$ to find the y-coordinate of the turning point.

WORKED EXAMPLE 14

Sketch the graph of $y = (x - 4)(x + 2)$ by first finding the x- and y-intercepts and then the turning point.

THINK	WRITE/DRAW
1 Write the equation.	$y = (x - 4)(x + 2)$
2 Find the x-intercepts.	x-intercepts: when $y = 0$, $(x - 4)(x + 2) = 0$ $x - 4 = 0$ or $x + 2 = 0$ $x = 4$ $\quad$ $x = -2$ The x-intercepts are -2 and 4.
3 Find the y-intercept.	y-intercept: when $x = 0$, $y = (0 - 4)(0 + 2)$ $= -4 \times 2$ $= -8$ The y-intercept is -8.
4 Find the x-value of the turning point by averaging the values of the two x-intercepts.	At the turning point, $x = \frac{4 + -2}{2} = 1$
5 Find the y-value of the turning point by substituting the x-value of the turning point into the equation of the graph.	When $x = 1$, $y = (1 - 4)(1 + 2)$ $= -3 \times 3$ $= -9$
6 State the coordinates of the turning point.	The turning point is $(1, -9)$.
7 Sketch the graph.	

- Quadratic equations written in the form $y = ax^2 + bx + c$ can sometimes be factorised into intercept form.

WORKED EXAMPLE 15

TI | CASIO

Sketch the graph of $y = x^2 + 6x + 8$.

THINK	WRITE/DRAW
1 Write the equation.	$y = x^2 + 6x + 8$

2 Factorise the expression on the right-hand side of the equation, $x^2 + 6x + 8$.

$= (x + 2)(x + 4)$

3 Find the two x-intercepts.

x-intercepts: when $y = 0$,
$(x + 2)(x + 4) = 0$
$x + 2 = 0$ or $x + 4 = 0$
$x = -2$ $x = -4$

The x-intercepts are -4 and -2.

4 Find the y-intercept.

y-intercept: when $x = 0$,
$y = 0 + 0 + 8$
$= 8$

The y-intercept is 8.

5 Find the x-value of the turning point.

At the turning point, $x = \frac{-2 + -4}{2}$
$= -3$

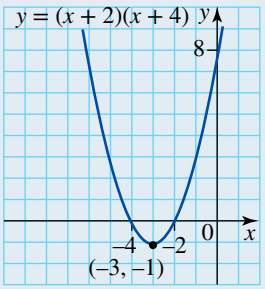
6 Find the y-value of the turning point.

When $x = -3$
$y = (-3 + 2)(-3 + 4)$
$= -1 \times 1$
$= -1$

7 State the turning point.

The turning point is $(-3, -1)$.

8 Sketch the graph.



learnon RESOURCES — ONLINE ONLY

Try out this interactivity: Sketching parabolas (int-2776)

Complete this digital doc: SkillSHEET: Solving quadratic equations of the form $(x + a)(x + b) = 0$ (doc-10992)

Complete this digital doc: SkillSHEET: Factorising quadratic trinomials of the form $ax^2 + bx + c$ where $a = 1$ (doc-11011)

Complete this digital doc: SkillSHEET: Solving quadratic trinomials of the type $ax^2 + bx + c = 0$ where $a = 1$ (doc-11012)

Complete this digital doc: WorkSHEET: Quadratic functions III (doc-11013)

Exercise 17.8 Sketching parabolas of the form $y = (x + a)(x + b)$

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 10, 12	Questions: 1a, b, f, 2b, e, h, k, 3, 4, 6, 9, 10, 12, 13	Questions: 1a, b, d, 2c, f, i, l, 3–15

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE14 Sketch the graph of each of the following by first finding the x- and y-intercepts and then the turning point.

a. $y = (x + 2)(x + 6)$ **b.** $y = (x - 3)(x - 5)$ **c.** $y = (x - 3)(x + 1)$

d. $y = (x + 4)(x - 6)$ **e.** $y = (x - 5)(x + 1)$ **f.** $y = (x + 1)(x - 2)$

2. WE15 Sketch the graph of each of the following.

a. $y = x^2 + 8x + 12$ **b.** $y = x^2 + 8x + 15$ **c.** $y = x^2 + 6x + 5$ **d.** $y = x^2 + 10x + 16$

e. $y = x^2 + 2x - 8$ **f.** $y = x^2 + 4x - 5$ **g.** $y = x^2 - 4x - 12$ **h.** $y = x^2 - 6x - 7$

i. $y = x^2 - 10x + 24$ **j.** $y = x^2 - 6x + 5$ **k.** $y = x^2 - 5x + 6$ **l.** $y = x^2 - 3x - 10$

3. Sketch the graph of a parabola whose x-intercepts are at $x = -4$ and $x = 6$ and whose y-intercept is at $y = 2$.

4. State a possible equation of a graph with an axis of symmetry at $x = 4$ and one intercept at $x = 6$.

Understanding

5. Out on the cricket field, Michael Clarke chases the ball. He picks it up, runs 4 metres towards the stumps (which are 25 metres away from where he picks up the ball), then throws the ball, which follows the path described by the quadratic equation $y = -0.1(x - m)(x - n)$, where m and n are positive integers. The ball lands 1 metre from the stumps, where another player quickly scoops it up and removes the bails. Taking the origin as the point where Michael picked up the ball initially:

a. Find how far the ball has travelled horizontally while in flight.

b. Find the values of m and n.

c. Find the x-intercepts.

d. Find the axis of symmetry.

e. Find the turning point.

f. Find the highest point reached by the cricket ball.

g. Find how far horizontally the ball has travelled when it reaches its highest point.

h. Sketch the flight of the ball, showing all of the relevant details on the graph.

6. McDonald's golden arches were designed by Jim Schindler in 1962. The two arches both approximate the shape of a parabola.

A large McDonald's sign stands on the roof of a shopping centre. The shape of one of the parabolas can be modelled by the quadratic function $y = -x^2 + 8x - 7$, where both x and y are measured in metres. The complete sign is supported by a beam underneath the arches. What is the minimum length required for this beam?



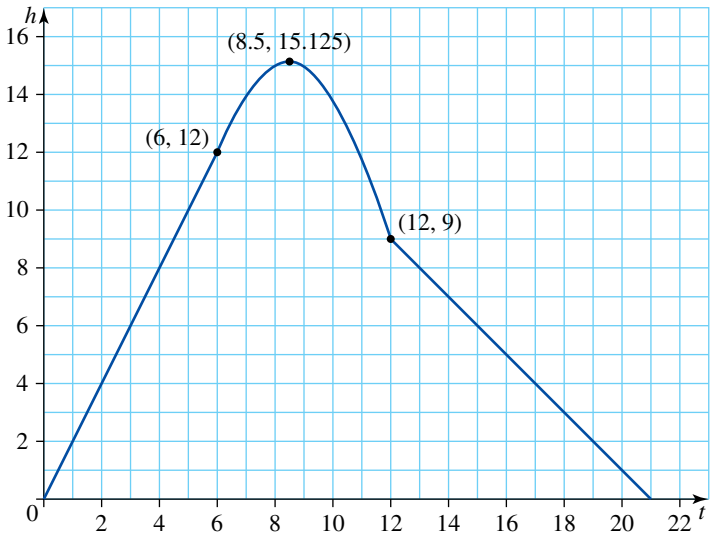
7. For the equation $y = (x - 3)(x - 9)$:
- **a.** determine the turning point
- **b.** rewrite the equation in turning point form
- **c.** expand both forms of the equation, showing that they are equivalent
- **d.** sketch the equation showing the x-intercepts and turning point.

8. The dimensions of a rectangular backyard can be given by $(x + 2)$ m and $(x - 4)$ m. Find the value of x if the yard's area is $91\,m^2$.

Reasoning

9. The height of an object, $h(t)$, thrown into the air is determined by the formula $h(t) = -8t^2 + 128t$, where t is time in seconds and h is height in metres.
- **a.** Does the graph of the formula have a maximum or a minimum turning point? Explain or show your reasoning.
- **b.** What is the maximum height of the object and at what time is this height reached?

10. Daniel is in a car on a roller-coaster ride as shown in the following graph, where the height, h, is in metres above the ground and time for the ride, t, is in minutes.
- **a.** The ride can be represented by three separate equations. Show that the first section of the ride is $h = 2t$ from $t = 0$ to $t = 6$. The middle section is a parabola and the final section is a straight line. Find the other two equations for the ride.
- **b.** What is the required domain (set of possible t-values) for each section of the ride?



Problem solving

11. On a set of axes, sketch several parabolas of the form $y = -x^2 + bx$. What do you notice about the turning points of each graph sketched?

12. Find a quadratic equation given the x-intercepts of the parabola are $-\frac{1}{2}$ and $\frac{3}{4}$. Is there more than one equation possible?

13. Find the equation of the parabola that passes through the points $(-1, -2)$, $(1, -4)$ and $(3, 10)$.

Reflection

What are the advantages of having the equation of a parabola in intercept form?

17.9 Applications

17.9.1 Applications

- Quadratic graphs and equations can be used to solve practical problems in science and engineering.
- Quadratic graphs and equations can be used to solve problems where a maximum or minimum needs to be found.
- When working with physical phenomena, ensure that the solution achieved satisfies any physical constraints of the problem. For example, if measurements of length or time are involved, there can be no negative solutions.

WORKED EXAMPLE 16

TI | CASIO

A flare is fired from a yacht in distress off the coast of Brisbane. The flare's height, h metres above the horizon t seconds after firing, is given by $h = -2t^2 + 18t + 20$.

a When will the flare fall into the ocean?

b How high is the flare after 2 seconds?

c At which other time will the flare be at the same height?

d For how long is the flare above the 'lowest visible height' of 56 m?

THINK

a Substitute $h = 0$ into the equation and solve for t using the Null Factor Law. Since t cannot equal -1 seconds, answer the question with the appropriate solution.

b Substitute $t = 2$ into the equation and solve for h. Answer the question.

c Substitute $h = 48$ into the equation, rearrange, factorise and solve for t.

d Substitute $h = 56$ into the equation, rearrange, factorise and solve for t. Answer the question.

WRITE

a $0 = -2t^2 + 18t + 20$
$= -2(t^2 - 9t - 10)$
$= -2(t - 10)(t + 1)$
$t = 10$ or -1
$t = 10$ seconds

b $h = -2(2)^2 + 18(2) + 20$
$= -8 + 36 + 20$
$= 48$
The flare is 48 metres high after 2 seconds.

c $48 = -2t^2 + 18t + 20$
$0 = -2t^2 + 18t - 28$
$= -2(t^2 - 9t + 14)$
$= -2(t - 7)(t - 2)$
$t = 7$ seconds

d $56 = -2t^2 + 18t + 20$
$= -2t^2 + 18t - 36$
$= -2(t^2 - 9t + 18)$
$= -2(t - 6)(t - 3)$
$t = 3$ seconds and 6 seconds
The flare is above 56 m for 3 seconds (between 3 and 6 seconds).

Exercise 17.9 Applications

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 9, 12	Questions: 1–4, 6, 7, 10, 12, 13	Questions: 1–4, 7–15

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. A spurt of water emerging from an outlet just below the surface of an ornamental fountain follows a parabolic path described by the equation $h = -x^2 + 8x$ where h is the height of the water and x is the horizontal distance from the outlet in metres.
- **a.** Sketch the graph of $h = -x^2 + 8x$.
- **b.** State the maximum height of the water above the surface of the fountain.

2. The position d metres below the starting point of a ball when it is dropped from a great height is given by the equation $d = 250 - 4.9t^2$. If it falls for 1 second, it drops 4.9 m, giving a value of $d = 245.1$ m.
- **a.** How far has it dropped after:
  - **i.** 2 seconds
  - **ii.** 7 seconds?
- **b.** Sketch a graph of this relation.

3. A car travels along a highway for a number of minutes according to the relationship $P = 20t^2 + 20t - 120$, where P is the distance from home in metres and t is time in minutes.
- **a.** What is the distance from home when $t = 0$?
- **b.** How long does it take the car to reach home?

Understanding

4. WE16 A rocket fired from Earth travels in a parabolic path. The equation for the path is $h = -0.05d^2 + 4d$, where h is the height in km above the surface of the earth and d is the horizontal distance travelled in km.



- **a.** Find the height of the rocket after:
  - **i.** 30 km
  - **ii.** 60 km.
- **b.** How far away does the rocket land?
- **c.** What is the maximum height of the rocket and how far did it travel before it reached this height?
- **d.** Sketch the path of the rocket.

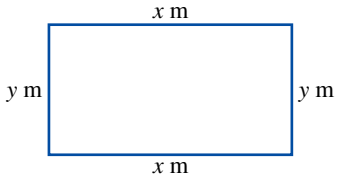
5. The height of a golf ball hit from the top of a hill is given by the quadratic rule $h = -t^2 + 5t + 14$, where h is in metres and t in seconds.



- **a.** From what height was the golf ball hit?
- **b.** What was the height of the ball after 2 seconds?
- **c.** When does the golf ball hit the ground?
- **d.** What is the maximum height the ball reaches?
- **e.** Sketch the graph of the flight of the ball.

6. Cave Ltd manufactures teddy bears. The daily profit, $\$P$, is given by the rule $P = -n^2 + 70n - 1200$, where n is the number of teddy bears produced.
- **a.** If they produce 40 bears a day, what is the profit?
- **b.** Sketch the graph of P for appropriate values of n.
- **c.** How many bears do they need to produce before they start making a profit?
- **d.** What is the maximum profit and how many teddy bears do they need to manufacture to make this amount?

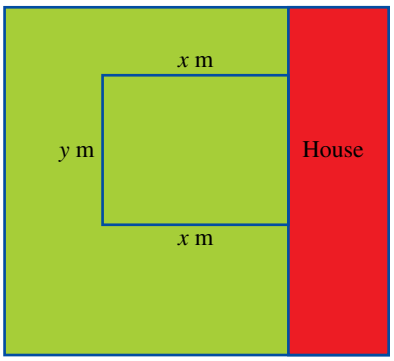
7. A school playground is to be mulched and the grounds keeper needs to mark out a rectangular perimeter of 80 m.



- **a.** Find the perimeter of the playground in terms of x and y.
- **b.** Show that $y = 40 - x$.

c. Find the area to be mulched, A, in terms of x only.
d. Sketch the graph of A against x for suitable values of x.
e. Determine the maximum area of the playground.
f. What are the dimensions of the playground for this maximum area?

8. A woman wished to build a fence around part of her backyard, as shown in the diagram. The fence will have one side abutting the wall of the house. She has enough fencing material for 55 m of fence.



a. Show that $y = 55 - 2x$.
b. Write an expression for the area enclosed by the fence in terms of x alone.
c. Determine the dimensions of the fence such that the area is a maximum and calculate that area.

Reasoning

9. A basketball thrown from the edge of the court to the goal shooter is described by the formula $h = -t^2 + 6t + 1$, where h is the height of the basketball in metres after t seconds.



a. What was the height of the ball when it was first thrown?
b. What was the height of the basketball after 2 s?
c. Show that the ball was first at a height of 6 m above the ground at 1 second.
d. During which time interval was the ball above a height of 9 m?
e. Plot the path of the basketball.
f. What was the maximum height of the basketball during its flight?
g. How long was the ball in flight if it was caught at a height of 1 m above the ground on its downward path?

10. Jack and his dog were playing outside. Jack was throwing a stick in the air for his dog to catch. The height of the stick (in metres) followed the equation $h = 20t - 5t^2$.



a. The graph of h is a parabola. Is the parabola upright or inverted?
b. Factorise the expression on the right-hand side of the equation.
c. Find the t-intercepts. These will be the two points at which the stick is on the ground, once at take-off and once at landing. How long does the stick remain in the air? (For simplicity, assume that Jack throws the stick from ground level.)
d. Justify whether the parabola has a maximum or minimum turning point, and find its coordinates.
e. What will be the maximum height reached by the stick? Explain.
f. Use the information you have found to produce a sketch of the path of the stick.

11. An engineer wishes to build a footbridge in the shape of an inverted parabola across a 40-m wide river. The bridge will be symmetrical and the greatest difference between the lowest part and highest part will be 4 m. Taking the origin as one side, show that the equation that models this footbridge is $y = -0.01x(x - 40)$. Explain what x and y are, and state the domain (the possible values that x can take).

Problem solving

12. A car engine spark plug produces a spark of electricity. The size of the spark depends on how far apart the terminals are. The percentage performance, Z, of a certain brand is thought to be $Z = -400(g - 0.5)^2 + 100$, where g is the distance between the terminals.
- **a.** Sketch the graph of Z.
- **b.** When is the performance greatest?
- **c.** From your graph, find the values of g for which the percentage performance is greater than 50%.

13. The monthly profit or loss, p (in thousands of dollars), for a new brand of soft drink is given by $p = -2(x - 7.5)^2 + 40.5$, where x is the number (integer) of months after its introduction (when $x = 0$).
- **a.** In which month was the greatest profit made?
- **b.** Between which months did the company make a profit?

14. NASA uses a parabolic flight path to simulate zero gravity and the gravity experienced on the moon. Use the internet to investigate the flight path and create a mathematical model to represent the flight path.

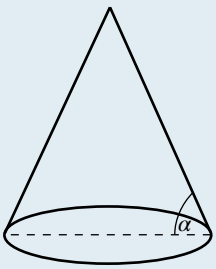
15. An arch bridge is modelled in the shape of a parabolic arch. The arch span is 50 m wide and the maximum height of the arch above water level is 5.5 m. A floating platform 35 m wide is towed under the bridge. What is the greatest height of the deck above water level if the platform is to be towed under the bridge with at least a 35 cm clearance on either side?

Reflection

What are some important things to remember when solving practical problems with quadratic graphs and equations?

CHALLENGE 17.2

Take a solid cone standing on its circular base. Let the angle that the slant height makes with the base be α. Cross-sections of the cone can reveal shapes of a parabola, an ellipse and a circle. Describe how these shapes can be obtained. Make reference to the angle α in each case.



17.10 Review

17.10.1 Review questions

Fluency

1. a. The axis of symmetry for the graph shown at left is:
A. $x = 0$ **B.** $x = 2$ **C.** $x = 4$ **D.** the y-axis

b. The coordinates of the turning point for the graph are:
A. $(0, 0)$ **B.** $(2, 4)$ **C.** $(2, -4)$ **D.** $(-2, -4)$

c. The y-intercept is:
A. 0 **B.** -2 **C.** -4 **D.** 2

d. The x-intercepts are:
A. 0 and 4 **B.** 0 and 2 **C.** 0 and -2 **D.** 0 and -4

2. a. The graph of $y = -4x^2$ is:
A. wider than $y = x^2$ **B.** narrower than $y = x^2$
C. the same width as $y = x^2$ **D.** a reflection of $y = x^2$ in the x-axis

b. The graph of $y = \frac{1}{2}x^2$ is:

A. wider than $y = x^2$ **B.** narrower than $y = x^2$

C. the same width as $y = x^2$ **D.** a reflection of $y = x^2$ in the x-axis

c. Compared to the graph of $y = x^2$, the graph of $y = -2x^2$ would be:

A. half as wide **B.** twice as wide

C. moved 2 units to the right **D.** moved 2 units up

3. a. The turning point for the graph of the equation $y = -x^2 + 6$ is:

A. $(0, 0)$ **B.** $(-1, 6)$ **C.** $(0, 6)$ **D.** $(0, -6)$

b. The turning point of the graph of the equation $y = x^2 - 12$ is:

A. $(1, -12)$ **B.** $(-12, 1)$ **C.** $(0, 12)$ **D.** $(0, -12)$

c. The graph of $y = x^2 - 4$ moves the graph of $y = x^2$ in the following way:

A. up 4 **B.** down 4 **C.** no change **D.** up 2

d. The y-intercept of the graph of $y = -x^2 - 3$ is:

A. 1 **B.** -1 **C.** 3 **D.** -3

4. a. The axis of symmetry for the graph $y = (x + 1)^2$ is:

A. $y = 0$ **B.** $x = 1$ **C.** $x = -1$ **D.** $x = 2$

b. The turning point of the graph $y = (x + 1)^2$ is:

A. $(0, 0)$ **B.** $(1, 1)$ **C.** $(1, -1)$ **D.** $(-1, 0)$

c. Compared to the graph of $y = x^2$, the graph of $y = (x - 2)^2$ would be:

A. twice as wide **B.** moved 2 units to the left

C. moved 2 units to the right **D.** moved 2 units up

5. a. For the graph of $y = (x + 6)^2 - 1$, the coordinates of the turning point are:

A. $(6, -1)$ **B.** $(-1, 6)$ **C.** $(-1, -6)$ **D.** $(-6, -1)$

b. For the graph of $y = -(x - 4)^2 + 5$, the axis of symmetry is:

A. $x = 4$ **B.** $x = 5$ **C.** $x = -4$ **D.** $y = -5$

c. For the graph of $y = (x - 2)^2 - 3$, the y-intercept is:

A. -5 **B.** 1 **C.** -4 **D.** -3

6. To change the graph of $y = x^2$ to make the graph of $y = 2(x - 1)^2 - 3$, we need to:

A. move it 1 unit to the right and 3 units down

B. make it narrower by a factor of 3, move it 1 unit to the left and 3 units down

C. make it wider by a factor of 2, move it 1 unit to the right and 3 units down

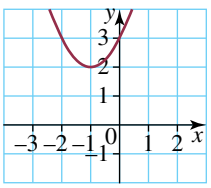
D. make it narrower by a factor of 2, move it 1 unit to the right and 3 units down

7. Human cannonball Stephanie Smith was fired from a cannon. Her flight path is a parabola. If c is her height above the net and b is her horizontal distance from the cannon, which rule could describe her flight?

A. $y = x^2 + c$ **B.** $y = bx^2 + c$

C. $y = (x - b)^2 + c$ **D.** $y = -(x - b)^2 + c$

8. The curve shown on the right could have the rule:



A. $y = -(x + 1)^2 - 2$ **B.** $y = (x + 1)^2 + 2$

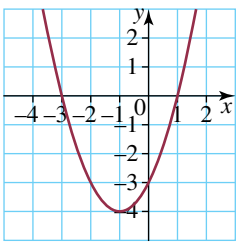
C. $y = (x + 1)^2 - 2$ **D.** $y = -(x + 2)^2 - 1$

9. If $y = x^2 - x - 6$, then the x-intercepts would be:

A. -2 and 3 **B.** -3 and 2

C. 2 and 3 **D.** -2 and -3

10. The sketch in the diagram shown on the right is a graph of:



A. $y = (x - 1)(x - 3)$

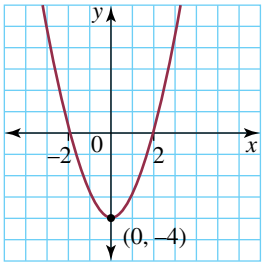
B. $y = (x + 1)(x + 3)$

C. $y = (x - 1)(x + 3)$

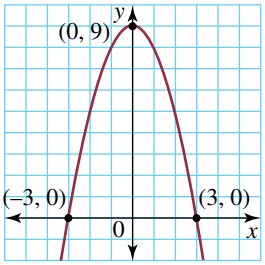
D. $y = (x + 1)(x - 3)$

11. For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the turning point, whether the point is a maximum or a minimum, and the x- and y-intercepts.

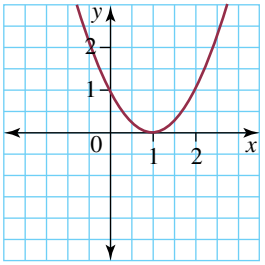
a.



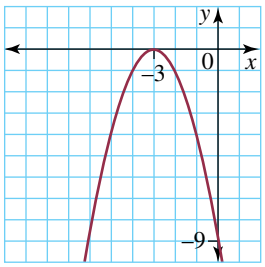
b.



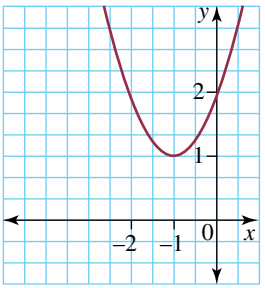
c.



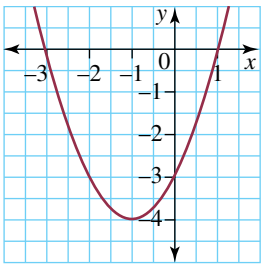
d.



e.



f.



12. Draw a table of values, use it to plot the graph of each of the following equations, and hence state the equation of the axis of symmetry, the coordinates of the turning point and whether it is a maximum or a minimum, and the x- and y-intercepts.

a. $y = x^2 - 4x, -2 \leq x \leq 6$ **b.** $y = -x^2 - 2x + 8, -5 \leq x \leq 3$

c. $y = 2x^2 - 4x + 4, -2 \leq x \leq 3$ **d.** $y = -x^2 + 6x - 5, 0 \leq x \leq 6$

13. Sketch each of the following graphs, labelling the axis of symmetry, the turning point and the intercepts, and stating the type of turning point.

a. $y = 2x^2$ **b.** $y = \frac{1}{2}x^2$ **c.** $y = -4x^2$ **d.** $y = -\frac{1}{3}x^2$

14. Sketch each of the following graphs, labelling the axis of symmetry, the turning point and the y-intercept, and stating the nature of the turning point.

a. $y = x^2 + 2$ **b.** $y = x^2 - 4$ **c.** $y = x^2 + 5$ **d.** $y = x^2 - 3$

15. Sketch each of the following graphs, labelling the turning point and the y-intercept, and stating the type of turning point.

a. $y = (x + 1)^2$ **b.** $y = -(x - 2)^2$ **c.** $y = (x - 5)^2$ **d.** $y = -(x + 2)^2$

16. Sketch each of the following graphs, labelling the turning point and the intercepts, and stating the nature of the turning point.

a. $y = (x - 1)^2 + 1$ **b.** $y = (x + 2)^2 + 3$ **c.** $y = (x - 3)^2 - 2$ **d.** $y = -(x + 1)^2 - 4$

Problem solving

17. A railway bridge has an arch below it that can be modelled by the equation $H = -(x + 1)^2 + 16$, where H is the height of the bridge, in metres. Sketch a graph of the bridge and, hence, find its maximum height.

18. Sketch the graph of each of the following by finding the x- and y-intercepts and the turning point.

a. $y = (x + 2)(x - 2)$ **b.** $y = (x + 3)(x - 7)$ **c.** $y = x^2 - 2x - 15$ **d.** $y = x^2 + 3x + 2$

19. A valley in the countryside has a river running through the centre of it. A vertical cross-section of the valley can be modelled by the equation $H = \frac{1}{10}x^2 - 6x + 100$, where H is the height above sea level, in metres, and x is the horizontal distance, in metres. If two villages are situated in the valley, one a horizontal distance of 10 metres from the river and the other 20 metres from the river, what are their heights, respectively, above sea level?

20. Children playing on a cricket pitch throw a ball that follows the path $y = \frac{1}{120}(x - 16)^2$, where y is the height, in yards, above the ground and x is the horizontal distance from the stumps at the end from which the ball is thrown. If the stumps are 22 yards away and 0.5 yards high, will the ball hit the stumps? Find out by sketching the graph. (As a cricket pitch is 22 yards long, a yard is a convenient unit to use in this example. A yard is about 91.5 cm.)

21. A toy rocket is thrown up into the air from a balcony. The path the rocket takes is given by the equation $h = 49 - (t - 2)^2$ where h is the height above the ground in metres and t is the time in seconds.

a. How high is the balcony?

b. When does the rocket land on the ground?

c. What is the highest point the rocket reaches?

d. Sketch a graph of the rockets path for $0 \leq t \leq 9$.

22. A piece of wire 1 m long is bent into the shape of a rectangle.

a. If w cm is the width and l cm is the length, write a rule connecting w and l.

b. If the area of the rectangle is A cm^2, write an equation connecting l in terms of A.

c. Sketch the graph of A versus l.

d. What will be the maximum area enclosed by the wire?

23. The organising committee of the Greensborough Tennis Club found that the profit, $\$P$, from a Saturday morning barbeque depended on the number of sausages sold, n. The estimated profit is given by $P = -n^2 + 50n - 400$.

a. What is the profit if the number of sausages sold is 35?

b. Sketch the graph of P versus n for $0 \leq n \leq 50$.

c. Find the maximum profit possible and the number of sausages that need to be sold to make this amount.

d. What would be the minimum number of sausages that would need to be sold to avoid a loss?

24. In a demonstration using the width of a street, a skateboarder performs an ollie over a distance of 2 m. He remains on the ground for $\frac{1}{4}$ m before he begins the ollie, the path of which can be described by the quadratic equation $y = -2(x - p)(x - q)$ (where all dimensions are in metres), and lands with 35 cm to spare before hitting the opposite edge of the curb. Taking the origin as the start of his run:

a. How far has he travelled horizontally?

b. Find the values of p and q.

c. State the axis of symmetry of the ollie.

d. Find the highest point reached by the skateboarder.

e. Sketch the path of the skateboarder, showing all of the relevant details on the graph.

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Try out this interactivity: Word search: Topic 17 (int-0694)

Try out this interactivity: Crossword: Topic 17 (int-0708)

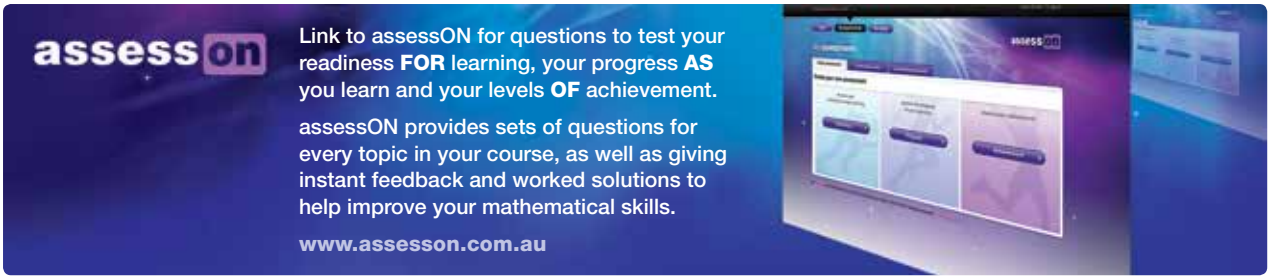
Try out this interactivity: Sudoku: Topic 17 (int-3215)

Complete this digital doc: Concept map: Topic 17 (doc-10807)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

applications
axis of symmetry
constant
cross-section
dilation
domain
horizontal translation
intercept
intercept form
inverted
maximum
minimum
modelled
parabola
parabolic shape
quadratic function
turning point
vertical translation
***x*-intercept**
***y*-intercept**



Investigation | Rich task

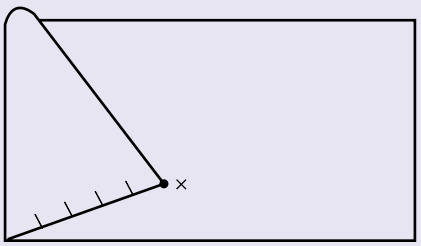
Constructing a parabola



The word *parabola* comes from the Greek language meaning 'thrown', because it is the path followed by a projectile in flight. Notice that the water streams shown in the photo above are moving in the path of a parabola.
This investigation explores the technique of folding paper to display the shape of a parabola. The instructions are given on the following page. Take care with each step to ensure your finished product is a well-constructed parabola that can be used in later parts of this investigation.

Forming a parabola by folding paper

- **Take a sheet of A4 paper. Cut it into two pieces by dividing the longer side into two. Only one of the halves is required for this investigation.**
- **Along one of the longer sides of your piece of paper, mark points that are equally spaced 1 cm apart. Start with the first point being on the very edge of the paper.**
- **Turn over the piece of paper and mark a point, X, 3 cm above the centre of the edge that has the markings on the reverse side.**
- **Fold the paper so that the first point you marked on the edge touches point X. Make a sharp crease and open the paper flat.**
- **Fold the paper again so that the second mark touches the point X. Crease and unfold again.**
- **Repeat this process until all the marks have been folded to touch point X.**
- **With the paper flat and the point X facing up, you should notice the shape of a parabola appearing in the creases.**



1. Trace the curve with a pencil.

The point X is called the *focus* of the parabola. Consider the parabola to represent a mirror. Rays of light from the focus would hit the mirror (parabola) and be reflected. The angle at which each ray hits the mirror is the same size as the angle at which it is reflected.

2. Using your curve traced from your folding activity, accurately draw a series of lines to represent rays of light from the point X to the parabola (mirror). Use a protractor to carefully measure the angle each line makes with the mirror and draw the path of these rays after reflection in the mirror.

3. Draw a diagram to describe your finding from question **2**. Provide a brief comment on your description.

4. Retrace your parabola onto another sheet of paper. Take a point other than the focus and repeat the process of reflection of rays of light from this point by the parabolic mirror.

5. Draw a diagram to describe your finding from question **4**. Provide a brief comment on your description.

6. Give examples where these systems could be used in society.



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Complete this digital doc: Code puzzle: Whatever happened to Australia's first 50-cent coin of 1966? (doc-15915)

Answers

Topic 17 Quadratic functions

Exercise 17.2 Graphs of quadratic functions

1. a. Axis of symmetry: $x = 0$, turning point $(0, 0)$, minimum

b. Axis of symmetry: $x = 0$, turning point $(0, -3)$, minimum

c. Axis of symmetry: $x = -1$, turning point $(-1, -2)$, minimum

d. Axis of symmetry: $x = 0$, turning point $(0, 0)$, maximum

e. Axis of symmetry: $x = 0$, turning point $(0, 2)$, maximum

f. Axis of symmetry: $x = 2$, turning point $(2, -1)$, maximum

2. a. $x = 0$, $(0, 1)$, minimum

b. $x = 1$, $(1, -3)$, minimum

c. $x = -2$, $(-2, 2)$, maximum

d. $x = -1$, $(-1, -2)$, maximum

e. $x = 2$, $(2, 2)$, minimum

f. $x = 0$, $(0, 1)$, maximum

3. a. $x = 0$; TP $(0, -1)$, minimum; x-intercepts are -1 and 1, y-intercept is -1.

b. $x = 0$; TP $(0, 1)$, maximum; x-intercepts are -1 and 1, y-intercept is 1.

c. $x = 1$; TP $(1, -4)$, minimum; x-intercepts are -1 and 3, y-intercept is -3.

d. $x = -2$; TP $(-2, 1)$, maximum; x-intercepts are -3 and -1, y-intercept is -3.

e. $x = -\frac{1}{2}$; TP $(-\frac{1}{2}, -1)$, minimum; x-intercepts are $-1\frac{1}{2}$ and $\frac{1}{2}$, y-intercept is $-\frac{3}{4}$.

f. $x = \frac{1}{2}$; TP $(\frac{1}{2}, 2)$, minimum; no x-intercepts, y-intercept is $2\frac{1}{4}$.

g. $x = 2$; TP $(2, 0)$, minimum; x-intercept is 2, y-intercept is 4.

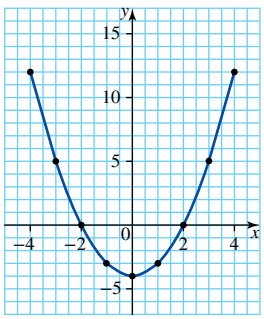
h. $x = -1$; TP $(-1, 0)$, maximum; x-intercept is -1, y-intercept is -1.

i. $x = 1$; TP $(1, 4)$, maximum; x-intercepts are -1 and 3, y-intercept is 3.

j. $x = 2$; TP $(2, -9)$, minimum; x-intercepts are -1 and 5, y-intercept is -5.

4. a. B **b.** C **c.** A **d.** D

5. a.



The shape is a parabola.

b. $x = 0$ **c.** $y = -4$ **d.** $x = -2$, $x = 2$

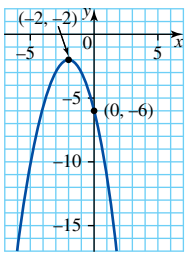
6.

x	-6	-4	-2	0	2	4	6	8
y	**30**	**12**	**2**	0	**6**	**20**	**42**	**72**

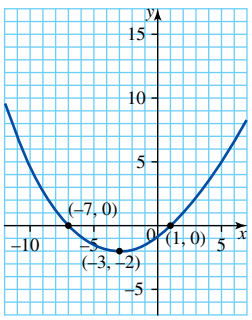
7. C

8. a. $(0, 0)$ **b.** Maximum

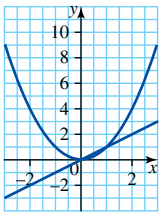
9. a.



b.



10.



By inspection or algebra the graphs meet at $(0, 0)$ and again at $(1, 1)$.

11. a. The graph has the shape of a parabola.

Height	10	15	20	25	30	35	40	45	50
Cost	100	225	400	625	900	1225	1600	2025	2500

b. \$2025

c. Answers will vary.

12. a. Answers will vary. **b.** 37 metres

13. Answers will vary.

14. Answers will vary.

15. Answers will vary. If the parabola is upright, it has a minimum turning point. If the parabola is inverted, it has a maximum turning point.

16. Answers will vary but must be of the form $y = a(x - h)^2$.

17. Answers will vary but must be of the form $y = ax^2 + c$.

18. a.

***n* (number of lines)**	1	2	3	4	5	6
***N* (number of intersections)**	0	1	3	6	10	15

b. $N = \dfrac{n(n-1)}{2}$

c. $N = \dfrac{100(100-1)}{2}$

$= 4950$ intersections

Exercise 17.3 Plotting points to graph quadratic functions

1. a. $y = 2x^2$

x	−3	−2	−1	0	1	2	3
$y = 2x^2$	18	8	2	0	2	8	18

$(-3, 18), (-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8), (3, 18)$

b. $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
$y = x^2 - 4$	5	0	−3	−4	−3	0	5

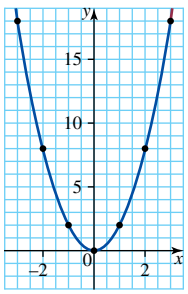
$(-3, 5), (-2, 0), (-1, -3), (0, -4), (1, -3), (2, 0), (3, 5)$

c. $y = -x^2 + 4x + 5$

x	−2	−1	0	1	2	3	4	5	6
y	−7	0	5	8	9	8	5	0	−7

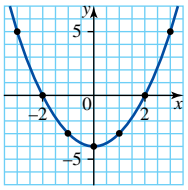
$(-2, -7), (-1, 0), (0, 5), (1, 8), (2, 9), (3, 8), (4, 5), (5, 0), (6, -7)$

2. a. i.



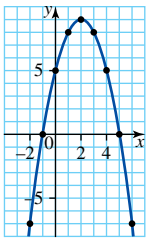
ii. $x = 0$

b. i.



ii. $x = 0$

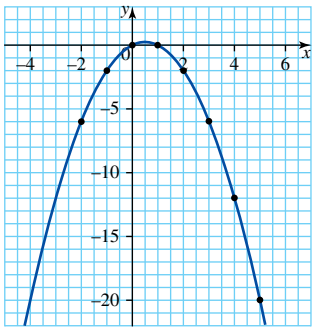
c. i.



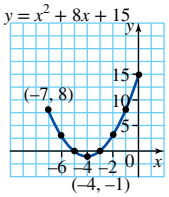
ii. $x = 2$

3.

x	-2	-1	0	1	2	3	4	5
$y = -x^2 + x$	-6	-2	0	0	-2	-6	-12	-20

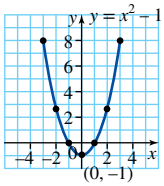


4. a. $y = x^2 + 8x + 15, -7 \leq x \leq 0$



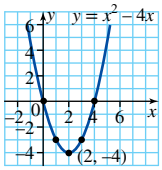
i. $x = -4$ **ii.** $(-4, -1)$, minimum **iii.** x-intercepts are -5 and -3, y-intercept is 15.

b. $y = x^2 - 1, -3 \leq x \leq 3$



i. $x = 0$ **ii.** $(0, -1)$, minimum **iii.** x-intercepts are -1 and 1, y-intercept is -1.

c. $y = x^2 - 4x, -1 \leq x \leq 5$

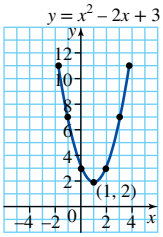


i. $x = 2$,

ii. $(2, -4)$, minimum

iii. x-intercepts are 0 and 4, y-intercept is 0.

d. $y = x^2 - 2x + 3$, $-2 \leq x \leq 4$

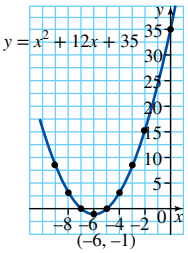


i. $x = 1$

ii. (1, 2), minimum

iii. No x-intercepts, y-intercept is 3.

e. $y = x^2 + 12x + 35$, $-9 \leq x \leq 0$

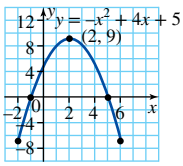


i. $x = -6$

ii. $(-6, -1)$, minimum

iii. x-intercepts are -7 and -5, y-intercept is 35.

f. $y = -x^2 + 4x + 5$, $-2 \leq x \leq 6$

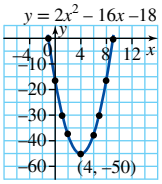


i. $x = 2$

ii. (2, 9), maximum

iii. x-intercepts are -1 and 5, y-intercept is 5.

g. $y = 2x^2 - 16x - 18$, $-1 \leq x \leq 9$

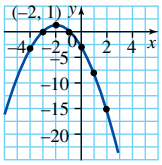


i. $x = 4$

ii. $(4, -50)$, minimum

iii. x-intercepts are -1 and 9, y-intercept is -18.

h. $y = -x^2 - 4x - 3$, $-4 \leq x \leq 2$



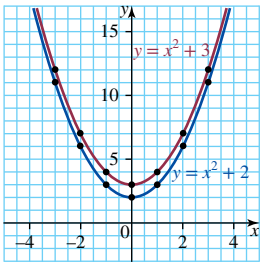
i. $x = -2$

ii. $(-2, 1)$, maximum

iii. x-intercepts are -3 and -1, y-intercept is -3.

5. a.

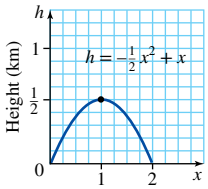
x	-3	-2	-1	0	1	2	3
$y = x^2 + 2$	11	6	3	2	3	6	11
$y = x^2 + 3$	12	7	4	3	4	7	12



b. Axis of symmetry: $x = 0$ for both equations.

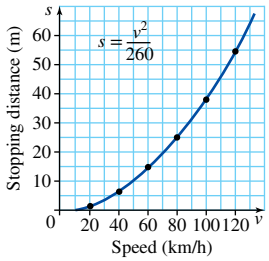
c. No x-intercepts for either equation.

6.



The maximum height is 500 m.

7. a. Stopping distances



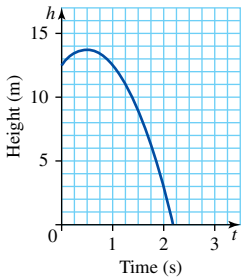
i. 14 m **ii.** 38 m **iii.** 55 m

b. 81 km/h

8. a. −21 °C **b.** 1 hour and at 21 hours **c.** $t = 11$ hours **d.** 100 °C

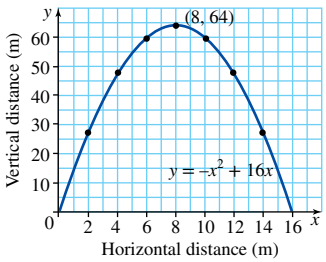
9. a. 12.5 metres

b.



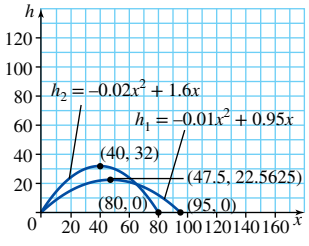
c. 2.2 seconds

10.



No. They were 2.3 metres across.

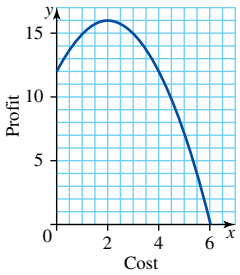
11.



The 9 iron

12. a. 1.3 m **b.** 13 m **c.** 75 cm

13. a.



b. The graph should not be to the left of the P axis because Ruby would be selling the mini Easter eggs at a negative cost there.

c. The cost of the Easter eggs should be \$2 to make a profit of \$16.

d. Ruby could charge up to \$6 before she makes a loss.

14. a. $\frac{x_2 + 10}{2} = -4$

$x_2 = 2$

b. Answers will vary but must be of the form $y = a(x - 2)(x + 10)$.

15. a. The turning point of $y = 2x^2 - 6x$ is $\left(\frac{3}{2}, \frac{-9}{2}\right)$.

The turning point of $y = 2x^2 - 6x - 3$ is $\left(\frac{3}{2}, \frac{-15}{2}\right)$.

b. Both graphs have the same x-coordinate of the turning point. $y = \left(x - \frac{3}{2}\right)^2 - 2.5$

16. $y = -0.016x^2 + 0.5x + 4.5$

$y = -0.016 \times 35.5^2 + 0.5 \times 35.5 + 4.5$

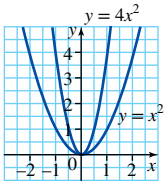
$y = 2.086$

The water just makes it over the fence.

17. $y = x^2 + x$

Exercise 17.4 Sketching parabolas of the form $y = ax^2$

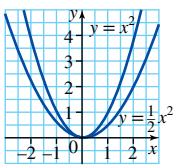
1.



$y = 4x^2$ is narrower.

Turning point for each is at $(0, 0)$ x-int. and y-int. is 0 for both.

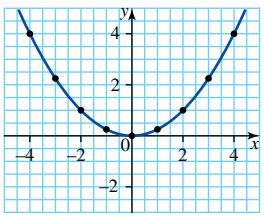
2.



$y = x^2$ is narrower.

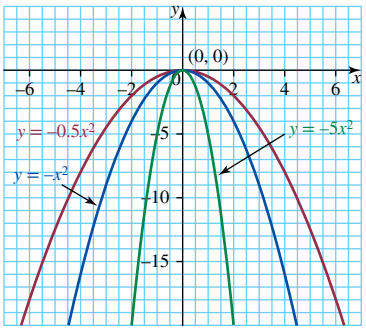
Turning point for each is at $(0, 0)$ x-int. and y-int. is 0 for both.

3.



$y = 0.25x^2$

4, 5.



$y = -x^2$ is narrower than $y = -0.5x^2$.
$y = -5x^2$ is the narrowest.

6. a. B **b.** A **c.** B

7. a. iii **b.** vi **c.** i **d.** ii **e.** iv **f.** v

8. Answers will vary; for example, $y = 2x^2$.

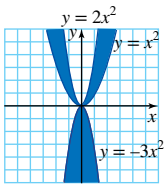
9. Answers will vary; for example, $y = -0.5x^2$.

10. a. $y = 3x^2$ **b.** $y = -x^2$

11. a. −350 **b.** −350 **c.** −31.5 **d.** −7.875 **e.** −16.94

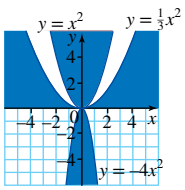
12. a. 400 watts **b.** 625 watts

13. a.



A tie

b.



A bow

14. $y = \frac{2}{3}x^2$

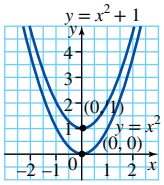
15. a. Answers will vary. **b.** 6

16. $k = 1$

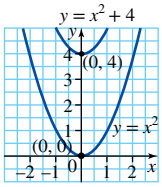
17. $x = y^2$

Exercise 17.5 Sketching parabolas of the form $y = ax^2 + c$

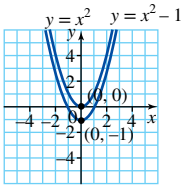
1. a.



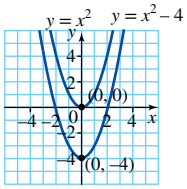
b.



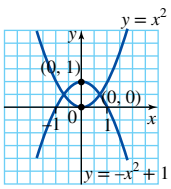
c.



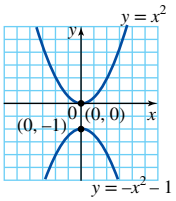
d.



e.



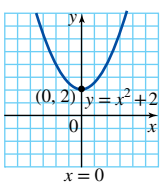
f.



2. A positive number moves the graph up.

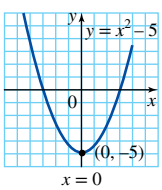
3. A negative number moves the graph down.

4. a.



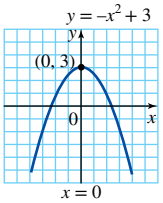
Minimum

b.



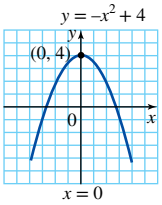
Minimum

c.



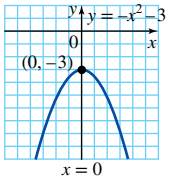
Maximum

d.



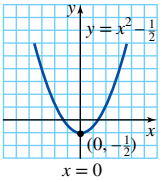
Maximum

e.



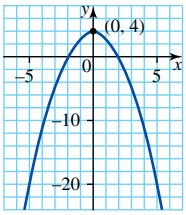
Maximum

f.

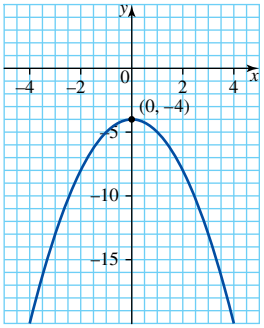


Minimum

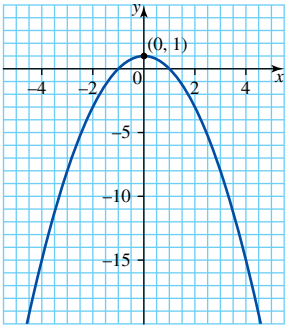
5. a. x-intercepts: $(-2, 0)$ $(2, 0)$



b. No x-intercepts



c. x-intercepts: $(-1, 0)$ $(1, 0)$



6. a. No

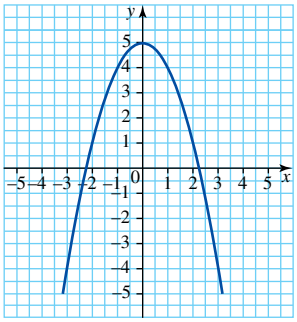
b. A negative sign inverts the graph.

c. $x = 0$ (the y-axis)

7. a. C **b.** D **c.** D **d.** D

8. a. iii **b.** iv **c.** v **d.** i **e.** vi **f.** ii

9. 5 km above sea level



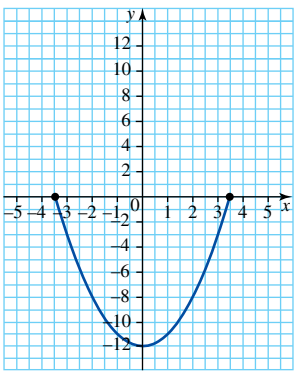
10. a. x-intercepts: $(-9, 0)$, $(9, 0)$

b. 243 cm

c. 18 cm

11. a. $-2\sqrt{3} \leq x \leq 2\sqrt{3}$

b.



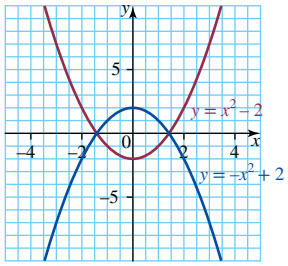
c. 12 m

d. 6.9 metres

12. a, **b** Answers will vary.

13. Answers will vary.

14. Intersections at $(-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$



15. The cup should be placed 50 cm horizontally away from the desk.

16. $y = 2x^2 + 2$

17. The parabolas intersect symmetrically at (−4.53, 226.11) and (4.53, 226.11).

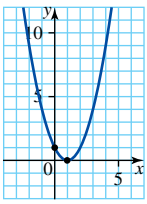
18. $y = 4 + 4\sqrt{5}$

Challenge 17.1

$y = -5x^2 + 50x$

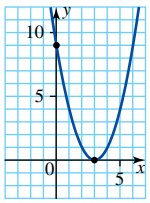
Exercise 17.6 Sketching parabolas of the form $y = (x - h)^2$

1. a. Minimum turning point (1, 0)



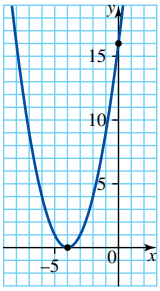
y-intercept (0, 1)

b. Minimum turning point (3, 0)



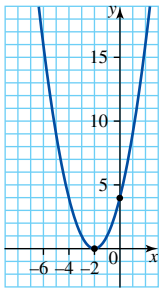
y-intercept (0, 9)

c. Minimum turning point (−4, 0)



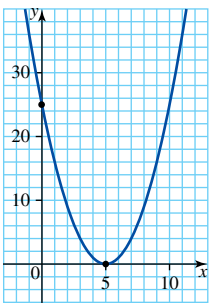
y-intercept (0, 16)

d. Minimum turning point (−2, 0)



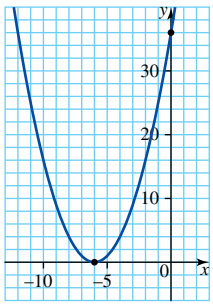
y-intercept (0, 4)

e. Minimum turning point (5, 0)



y-intercept (0, 25)

f. Minimum turning point (−6, 0)



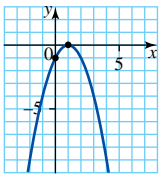
y-intercept (0, 36)

2. a. $x = 1$ **b.** $x = 3$ **c.** $x = -4$ **d.** $x = -2$ **e.** $x = 5$ **f.** $x = -6$

3. The graph is translated h units to the right.

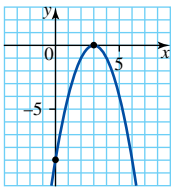
4. The graph is translated h units to the left.

5. a. Maximum turning point (1, 0)



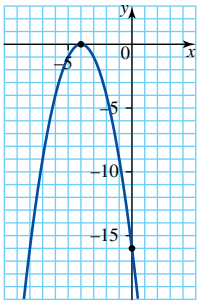
y-intercept $(0, -1)$

b. Maximum turning point (3, 0)



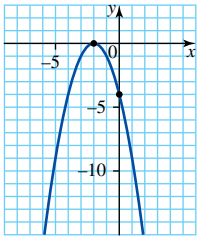
y-intercept $(0, -9)$

c. Maximum turning point $(-4, 0)$



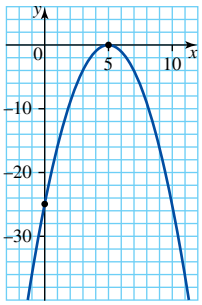
y-intercept $(0, -16)$

d. Maximum turning point $(-2, 0)$



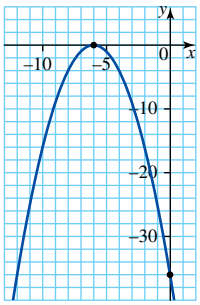
y-intercept $(0, -4)$

e. Maximum turning point (5, 0)



y-intercept $(0, -25)$

f. Maximum turning point $(-6, 0)$



y-intercept $(0, -36)$

6. No, the graph is inverted, but with the same axis of symmetry and turning point. The y-intercept changes.

7. a. C **b.** D

8. a. vi **b. ii** **c. iii** **d. i** **e. iv** **f. v**

9. a. $y = (x - 2)^2$ or $y = (x - 4)^2$ **b.** $y = (x - 2)^2$ or $y = (x + 4)^2$

10. a. No difference

b. 2nd graph is translated 4 units to the left of the 1st graph.

c. 2nd graph is 'inverted' and is the mirror image of the 1st graph in the x-axis.

11. $y = -3(x - 2)^2$

12. Answers will vary.

13. a. x-intercept (3, 0), y-intercept (0, 36) **b.** Answers will vary.

14. a. $a = 2250$, $h = 2$

b. $t = 5$ (June)

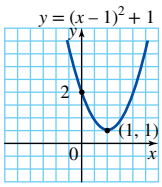
c. There are only two unknown quantities (a and h). In this case there is only one x-intercept while most parabolas have two x-intercepts.

15. Answers will vary. The graphs must have turning points at (3, 0) and be of the form $y = a(x - 3)^2$.

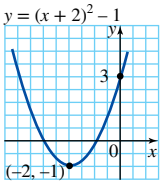
16. They are the same graph.

17. $y = 2(x + 2)^2$

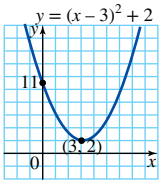
Exercise 17.7 Sketching parabolas of the form $y = (x - h)^2 + k$

1. a. 

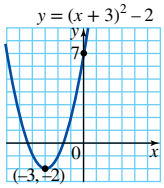
Minimum

b. 

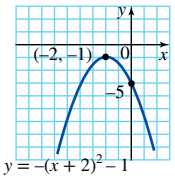
Minimum

c. 

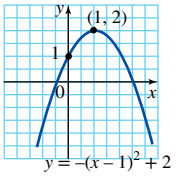
Minimum

d. 

Minimum

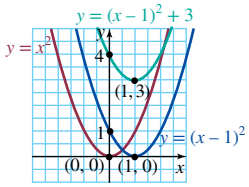
e. 

Maximum

f. 

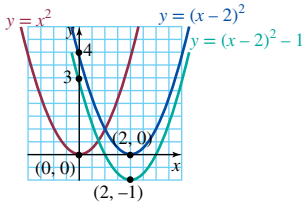
Maximum

2. It is easy to find the turning point (h, k) from the equation.

3. a. i. 

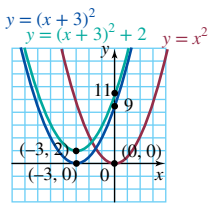
ii. 1 right

iii. 1 right, 3 up

b. i. 

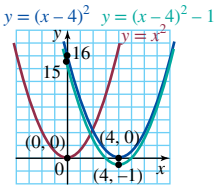
ii. 2 right

iii. 2 right, 1 down

c. i. 

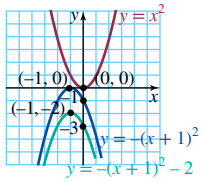
ii. 3 left

iii. 3 left, 2 up

d. i. 

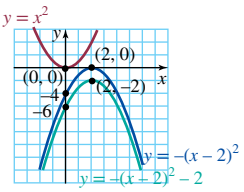
ii. 4 right

iii. 4 right, 1 down

e. i. 

ii. Reflect in the x-axis, 1 left

iii. Reflect in the x-axis, 1 left, 2 down

f. i. 

ii. Reflect in the x-axis, 2 right

iii. Reflect in the x-axis, 2 right, 2 down

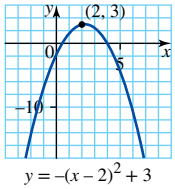
4. a. D **b.** A **c.** C

5. a. $y = (x + 2)^2 + 3$ **b.** $(-2, 3)$

6. a. Answers may vary, for example $y = (x + 3)^2 - 4$ is the simplest.

b. 5

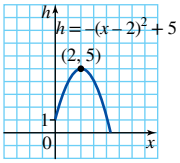
7.



8. a. $y = (x - 2)^2 + 1$ **b.** $y = -(x - 2)^2 - 1$ **c.** $y = (x + 2)^2 - 1$

d. $y = -(x - 3)^2 + 4$ **e.** $y = (x + 1)^2 - 4$ **f.** $y = (x - 1)^2 - 4$

9. a.



b. 1 metre **c.** 5 metres

10. a. $y = -(x + 2)^2$ **b.** $y = (x - 3)^2 - 1$

c. $y = (x + 1)^2 + 2$ **d.** $y = -(x - 4)^2 + 6$

11. a. 150 metres **b.** $y = \frac{-4}{225}(x - 150)^2 + 400$

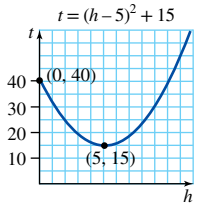
12. C

13. a. 40°C

b. Answers will vary.

c. 5 hours

d.



14. $y = 6(x - 1)^2 - 10$

Axis of symmetry $x = 1$

Therefore $h = 1$.

$k = -10$ (minimum)

y-intercept $= -4$ implies $-4 = a(0 - 1)^2 - 10$, or $a = 6$.

15. Minimum value at $y = 2$, so $k = 2$.

Axis of symmetry at $x = 4$, so $h = 4$.

Therefore $a = 0.5$.

$y = 0.5(x - 4)^2 + 2$

Check using y-intercept

$y = 0.5(-4)^2 + 2 = 10$, therefore confirmed.

16. a. $y = (x - 3)^2 - 2$ is obtained by translating $y = (x - 3)^2$ vertically 2 units down.

$y = (x - 3)^2 + 1$ is obtained by translating $y = (x - 3)^2$ vertically 1 unit up.

b. $y = (x - 5)^2 - 4$

17. $y = -\frac{1}{4}(x + 1)^2 + 4$

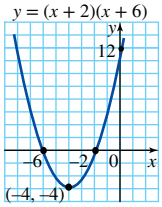
18. $x = 5a$ and $x = -a$.

19. $y = -\left(x - \frac{1}{2}\right)^2 + \frac{29}{4}$

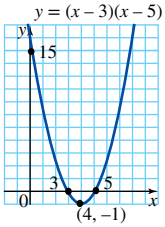
20. $x = h \pm \sqrt{\frac{k}{a}}$

Exercise 17.8 Sketching parabolas of the form $y = (x + a)(x + b)$

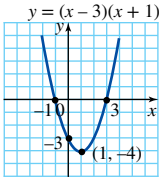
1. a. $y = (x + 2)(x + 6)$



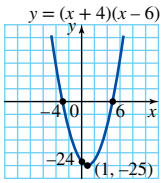
b. $y = (x - 3)(x - 5)$



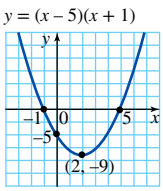
c. $y = (x - 3)(x + 1)$



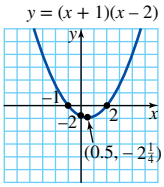
d. $y = (x + 4)(x - 6)$



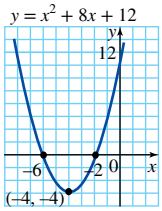
e. $y = (x - 5)(x + 1)$



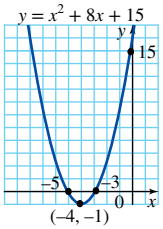
f. $y = (x + 1)(x - 2)$



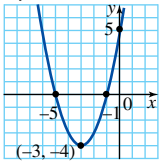
2. a. $y = x^2 + 8x + 12$



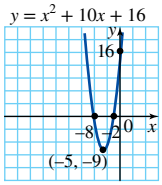
b. $y = x^2 + 8x + 15$



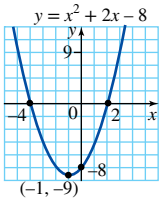
c. $y = x^2 + 6x + 5$



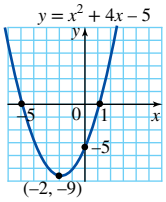
d. $y = x^2 + 10x + 16$



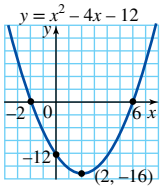
e. $y = x^2 + 2x - 8$



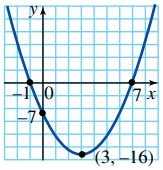
f. $y = x^2 + 4x - 5$



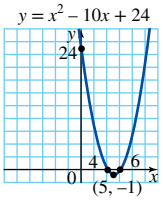
g. $y = x^2 - 4x - 12$



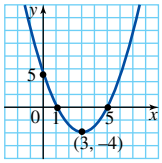
h. $y = x^2 - 6x - 7$



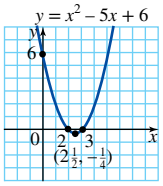
i. $y = x^2 - 10x + 24$



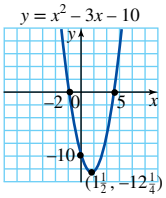
j. $y = x^2 - 6x + 5$



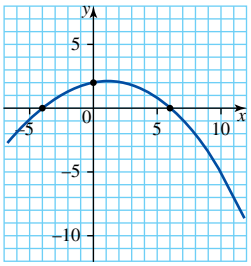
k. $y = x^2 - 5x + 6$



l. $y = x^2 - 3x - 10$



3.



4. $y = (x - 2)(x - 6)$

5. a. 20 m

b. 4, 24

c. (4, 0), (24, 0)

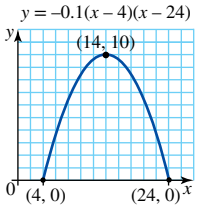
d. $x = 14$

e. (14, 10)

f. 10 m

g. 10 m

h.



6. 12 m

7. a. $(6, -9)$

b. $y = (x - 6)^2 - 9$

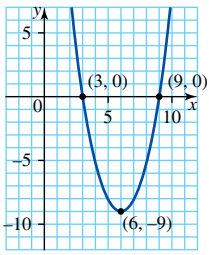
c. Intercept form expanded:

$y = x^2 - 12x + 27$

Turning point form expanded:

$y = x^2 - 12x + 36 - 9 = x^2 - 12x + 27$

d.



8. $x = 11$

9. a. Since the coefficient of t^2 is negative, the turning point is a maximum.

b. Maximum height = 512 m at $t = 8$ seconds.

10. a. From $t = 6$ to $t = 12$; $h = -0.5(t - 8.5)^2 + 15.125$

From $t = 12$ to $t = 21$; $h = -t + 21$

b. $0 \leq t \leq 6$, $6 \leq t \leq 12$, $12 \leq t \leq 21$

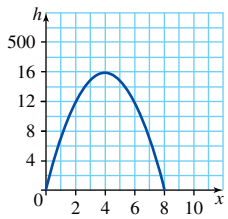
11. The general turning point is $\left(\frac{b}{2}, \frac{b^2}{4}\right)$.

12. Answers will vary but must be of the general form $y = a\left(x + \frac{1}{2}\right)\left(x - \frac{3}{4}\right)$.

13. $y = 2x^2 - x - 5$

Exercise 17.9 Applications

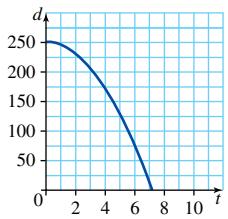
1. a.



b. 16 m

2. a. i. 19.6 m **ii.** 240.1 m

b.



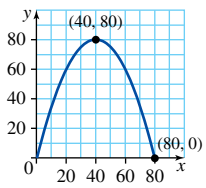
3. a. 120 metres **b.** 2 minutes

4. a. i. 75 km **ii.** 60 km

b. 80 km

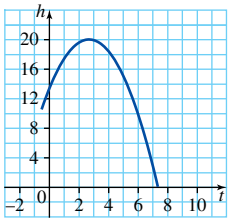
c. 80 km high after travelling 40 km horizontally

d.



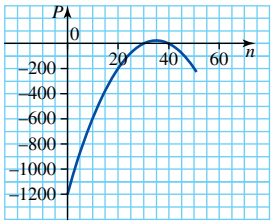
5. a. 14 m **b.** 20 m **c.** 7 s **d.** 20.25 m

e.



6. a. No profit

b.

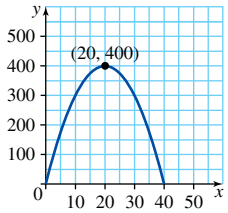


c. 30 bears

d. $25; 35 bears

7. a. $2(x + y) = 80$ **b.** Answers will vary. **c.** $A = x(40 - x)$

d.



e. 400 m^2

f. $x = 20$ m; $y = 20$ m

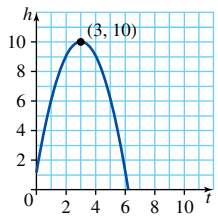
8. a. Perimeter $= 2x + y = 55$ m, so $y = 55 - 2x$

b. Area $= xy = x(55 - 2x) = 55x - 2x^2$

c. $x = 13.75$ m, $y = 27.5$ m, area $= 378.125$ m^2.

9. a. 1 m **b.** 9 m **c.** Answers will vary. **d.** From 2 to 4 seconds

e.



f. 10 m

g. 6 s

10. a. Inverted

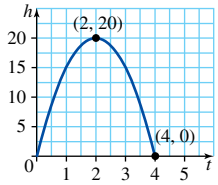
b. $h = 5t(4 - t)$

c. $t = 0$, $t = 4$; 4 seconds

d. Maximum turning point at (2, 20)

e. 20 m

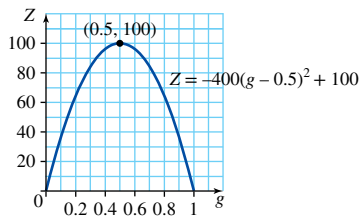
f.



11. Let x = horizontal distance, y = vertical distance across the bridge $(0 \leq x \leq 40)$

12. $y = -0.01x(x - 40)$

13. a.



b. g = 0.5

c. $0.15 \leq g \leq 0.85$

14. a. 7th to 8th month

b. 3rd and 12th months

15. Answers will vary.

16. The height of the platform deck above the water is 2.7 m.

Challenge 17.2

Answers will vary.

17.10 Review

1. a. B

b. C

c. A

d. A

2. a. B

b. A

c. A

3. a. C

b. D

c. B

d. D

4. a. C

b. D

c. C

5. a. D

b. A

c. B

6. D

7. D

8. B

9. A

10. C

11. a. $x = 0$, TP$(0, -4)$, minimum, x-intercepts are -2 and 2, y-intercept is -4.

b. $x = 0$, TP$(0, 9)$, maximum, x-intercepts are 3 and -3, y-intercept is 9.

c. $x = 1$, TP$(1, 0)$, minimum, x-intercept is 1, y-intercept is 1.

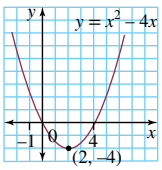
d. $x = -3$, TP$(-3, 0)$, maximum, x-intercept is -3, y-intercept is -9.

e. $x = -1$, TP$(-1, 1)$, minimum, no real x-intercepts, y-intercept is 2.

f. $x = -1$, TP$(-1, -4)$, minimum, x-intercepts are 1 and -3, y-intercept is -3.

12. a. $y = x^2 - 4x$, $-2 \leq x \leq 6$

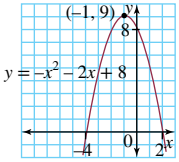
x	-2	-1	0	1	2	3	4	5	6
y	12	5	0	-3	-4	-3	0	5	12



$x = 2$, TP$(2, -4)$, minimum, x-intercepts are 0 and 4, y-intercept is 0.

b. $y = -x^2 - 2x + 8, -5 \leq x \leq 3$

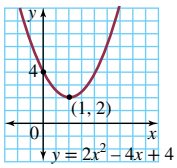
x	-5	-4	-3	-2	-1	0	1	2	3
y	-7	0	5	8	9	8	5	0	-7



$x = -1$, TP$(-1, 9)$, maximum, x-intercepts are -4 and 2, y-intercept is 8.

c. $y = 2x^2 - 4x + 4, -2 \leq x \leq 3$

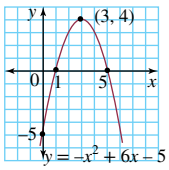
x	-2	-1	0	1	2	3
y	20	10	4	2	4	10



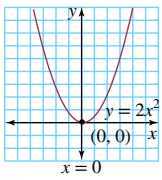
$x = 1$, TP$(1, 2)$, minimum, no x-intercepts, y-intercept is 4.

d. $y = -x^2 + 6x - 5, 0 \leq x \leq 6$

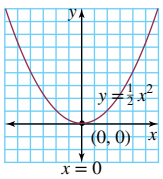
x	0	1	2	3	4	5	6
y	-5	0	3	4	3	0	-5



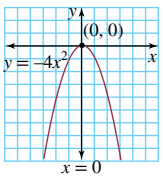
$x = 3$, TP$(3, 4)$, maximum, x-intercepts are 1 and 5, y-intercept is -5.

13. a.


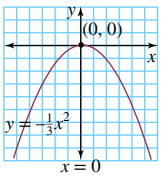
Minimum

b.


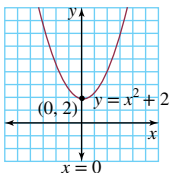
Minimum

c.


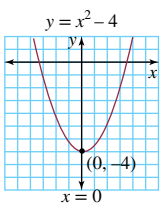
Maximum

d.


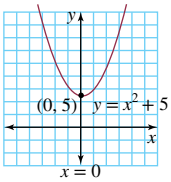
Maximum

14. a.


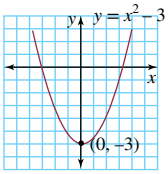
Minimum

b.


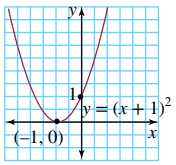
Minimum

c.


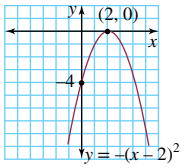
Minimum

d.


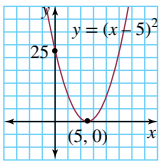
Minimum

15. a.


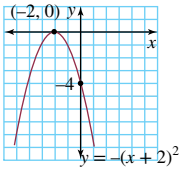
Minimum

b.


Maximum

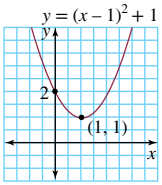
c.


Minimum

d.


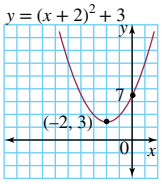
Maximum

16. a. $y = (x - 1)^2 + 1$



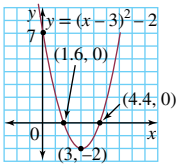
Minimum

b. $y = (x + 2)^2 + 3$



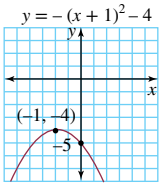
Minimum

c.



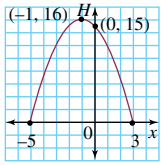
Minimum

d. $y = -(x + 1)^2 - 4$



Maximum

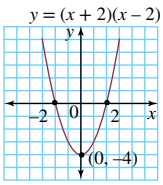
17.



$H = -(x + 1)^2 + 16$

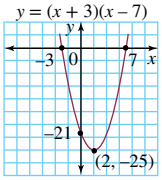
16 metres

18. a. $y = (x + 2)(x - 2)$



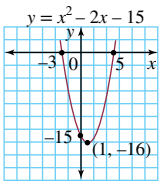
Minimum

b. $y = (x + 3)(x - 7)$



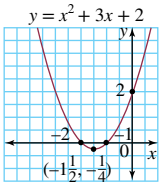
Minimum

c. $y = x^2 - 2x - 15$



Minimum

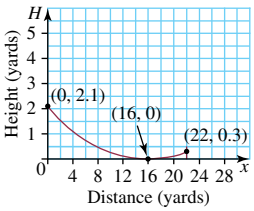
d. $y = x^2 + 3x + 2$



Minimum

19. 50 metres, 20 metres

20.



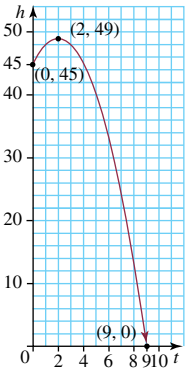
Yes

21. a. 45 m

b. 9 s

c. 49 m

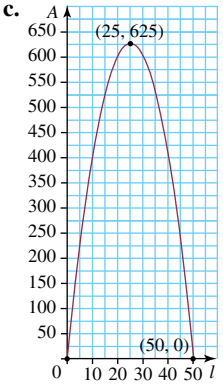
d.



22. a. $w + l = 50$

b. $A = l(50 - l)$

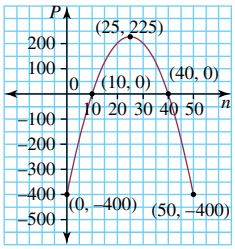
c.



d. 625 cm^2

23. a. \$125

b.



c. \$225 for 25 sausages

d. 10 or 40 sausages

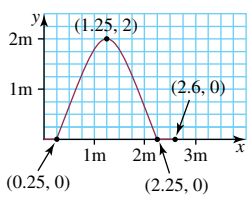
24. a. 2.6 m

b. $p = 0.25$, $q = 2.25$

c. $x = 1.25$

d. 2 m

e.



Investigation — Rich task

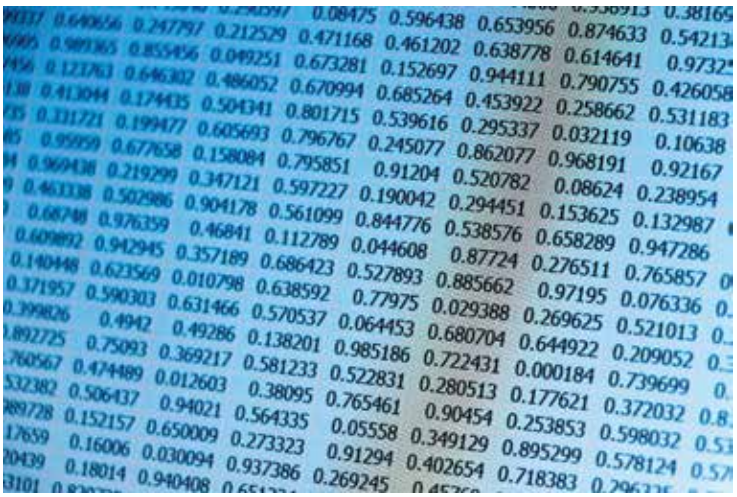
1–5. Check with your teacher.

6. Answers will vary.

TOPIC 18 Programming

18.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



18.1.1 Why learn this?

Computer programming is the process of creating instructions for a computer to follow to perform tasks.

In this topic the computer language JavaScript is used in most of the examples and questions. JavaScript has been chosen for its popularity and availability on nearly all web browsers.

A computer is often required to sort lists of data in ascending and descending order. These lists may be very long, but a program can loop through thousands of pieces of data every second. This topic will explore generating many random numbers to estimate probabilities, and will introduce how a program can process and organise large amounts of data. This topic will also explore various algorithms to sort data.

18.1.2 What do you know?

assessON

1. **THINK** Write what you know about computer programming languages. Use a thinking tool such as a concept map to display your ideas.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, combine all the ideas into one large concept map or poster.

LEARNING SEQUENCE

online only

18.1 Overview
18.2 Programs
18.3 Arrays
18.4 Loops
18.5 Set structures
18.6 Sorting algorithms
18.7 The Monte Carlo method
18.8 Review

18.2 Programs

18.2.1 Values

- Three of the most common types of values used in computer programming are:
  - **numbers** (numerical values): for example `87, -365, 1.5, -8.14213`
  - **strings** (any text surrounded in quotation marks `""`): for example `"A name", "3.14", "State: Victoria", "29#$%3", ""`
  - **Booleans** (used for logic, this type only has two possible values): either `false` or `true`.
- In JavaScript the expression `typeof value` returns either "`number`", "`string`" or "`boolean`".

WORKED EXAMPLE 1

Answer these questions for each of the following values.
i Is the value a number, a string or Boolean?
ii Apply the expression `typeof value` to each value. What is the result of the expression `typeof value`?

a `"Katherine"` **b** `3.14159265`
c `true` **d** `"false"`

THINK

a i `"Katherine"` is in quotation marks, so this value is a string.

ii 1 Apply the expression `typeof value` to the value `"Katherine"`.

2 `typeof "Katherine"` is `"string"`.

b i `3.14159265` is a numerical value and is not in quotation marks, so this value is a number.

ii 1 Apply the expression `typeof value` to the value `3.14159265`.

2 `typeof 3.14159265` is `"number"`.

c i `true` is a Boolean value.

ii 1 Apply the expression `typeof value` to the value `true`.

2 `typeof true` is `"boolean"`.

d i `"false"` is in quotation marks, so it is a string.

ii 1 Apply the expression `typeof value` to the value `"false"`.

2 `typeof "false"` is `"string"`.

WRITE

a i String

ii
```
typeof "Katherine"
```
```
"string"
```

b i Number

ii
```
typeof 3.14159265
```
```
"number"
```

c i Boolean

ii
```
typeof true
```
```
"boolean"
```

d i String

ii
```
typeof "false"
```
```
"string"
```

18.2.2 Variables

- Computer languages use memory locations to store values. These named containers are called **variables**. There are complex rules as to what is a valid variable name. For simplicity, this topic will restrict the variable names to simple rules.
- Variable names:
  – must not start with a number
  – can only contain upper and lower case letters, numbers, and the underscore character (_), and cannot contain spaces
  – cannot be JavaScript keywords. The following are JavaScript keywords which should not be used as variable names.

```
abstract, arguments, Array, boolean, break, byte, case,
catch, char, class, const, continue, Date, debugger,
default, delete, do, double, else, enum, eval, export,
extends, final, finally, float, for, function, goto,
hasOwnProperty, if, implements, import, in, Infinity,
instanceof, int, interface, isFinite, isNaN, isPrototypeOf,
length, let, long, Math, name, NaN, native, new, null,
Number, Object, package, private, protected, prototype,
public, return, short, static, String, super, switch,
synchronized, this, throw, throws, toString, transient,
true, try, typeof, undefined, valueOf, var, void, volatile,
while, with, yield, false
```

WORKED EXAMPLE 2

Each of the following is a valid variable name. True or false?

a `variableName` **b** `100_hundred` **c** `base^power`
d `number_1` **e** `while`

THINK	WRITE
a `variableName` is valid because it only uses letters.	**a** True
b `100_hundred` is invalid because it uses a number as the first character.	**b** False
c `base^power` is invalid because it uses a `^` character.	**c** False
d `number_1` is valid as it only uses letters, numbers and the `_` character. It also does not start with a number.	**d** True
e `while` is invalid because `"while"` is a JavaScript keyword.	**e** False

18.2.3 Numerical expressions

- In JavaScript, numerical expressions involving numbers, brackets, and plus and minus signs evaluate normally.
- Multiplication uses the character `*`.
- Division uses the character `/`.
- Fractions can be evaluated using the division character, but the numerator and denominator expressions must be put in brackets.

- $\frac{3+1}{31+1+32}$ `=(3+1)/(31+1+32)`
- The `%` symbol is used to find remainder after a division. For example, `76%7` evaluates to `6`, as 7 divides into 76 ten times with a 6 remainder.

WORKED EXAMPLE 3

Write the following as JavaScript expressions.

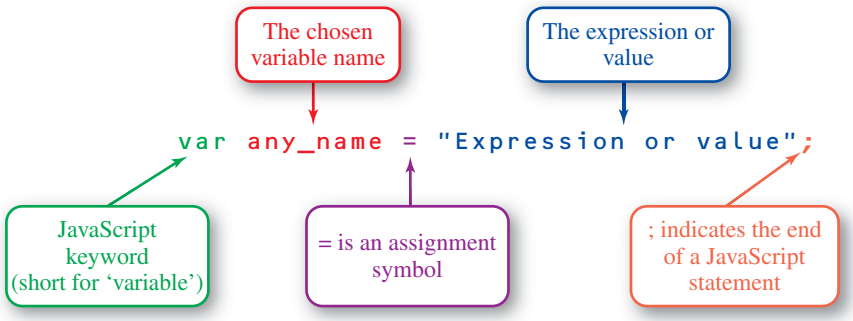
a $12 - (16 - 4)$ **b** 22×1.5 **c** $120 \div (5 \times 4 \times 3)$

d $\frac{120 - 110}{11 - 1}$ **e The remainder from the division $71 \div 8$**

THINK	WRITE
a This is a simple numerical expression with no special characters required.	**a** `12-(16-4)`
b Substitute $\times$ with `*`.	**b** `22*1.5`
c Replace the $\times$ with a `*` and the $\div$ with a `/`.	**c** `120/(5*4*3)`
d The fraction is a division. The numerator and denominator require brackets.	**d** `(120-110)/(11-1)`
e Use the modulus symbol, `%`, to find the remainder from the division $71 \div 8$.	**e** `71%8`

18.2.4 Assigning

- When assigning to a variable for the first time, the statement should begin with the JavaScript keyword `var`. JavaScript uses the following structure to assign a value or expression to a variable.



WORKED EXAMPLE 4

Write JavaScript statements to make the following variable assignments.

a Assign the value `1000` to the variable `thousand`.

b Assign `"Programming"` to the variable `topic`.

c Assign the mathematical expression $60 \times 60 \times 24 \times 7$ to the variable `seconds`.

THINK	WRITE
a 1 Identify the variable name, which in this case is `thousand`.	**a**
2 Identify the value, which in this case is `1000`.	

3 Apply the JavaScript structure to write the statement.

```
var thousand = 1000;
```

b 1 Identify the variable name, which in this case is `topic`.

b

2 Identify the value, which in this case is `"Programming"`.

3 Apply the JavaScript structure to write the statement.

```
var topic = "Programming";
```

c 1 Identify the variable name, which in this case is `seconds`.

c

2 Identify the mathematical expression, which in this case is $60 \times 60 \times 24 \times 7$.

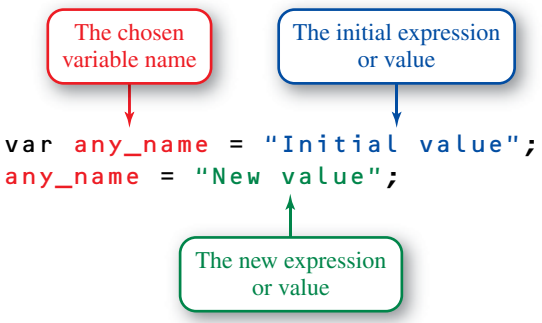
3 Convert the mathematical expression into a JavaScript expression: `60*60*24*7`.

4 Apply the JavaScript structure to write the statement.

```
var seconds = 60*60*24*7;
```

18.2.5 Reassigning

- A variable can change value and type as the program runs through a sequence of statements in order. When a variable is reassigned, the statement does not require JavaScript keyword `var`. JavaScript uses the following sequence to assign and reassign a value or expression to a variable. The value of the variable changes with each assignment.



WORKED EXAMPLE 5

Manually simulate running the following programs. For each program:
i what is the final type of the variable `a`
ii what is the final value stored in the variable `a`?

a
```
var a = 123.3;
a = "123.3";
```

b
```
var a = (2+3+15)/(20-7-3);
a = a / (40*7/140);
```

c
```
var a = "String";
var b = 122;
a = b;
b = "String";
```

THINK	WRITE
a **1** Write the initial assignment.	**a** `a = 123.3`
2 Write the reassignment.	`a = "123.3"`
i `"123.3"` is in quotation marks, so this value is a string.	**i** String
ii The last value assigned to `a` was `"123.3"`.	**ii** `"123.3"`
b **1** Write the initial assignment.	**b** `a = (2+3+15)/(20-7-3)`
2 Evaluate the expression.	`a = 2`
3 Write the reassignment.	`a = a/(40*7/140)`
4 On the right side of the assignment, substitute `2` for `a`.	`a = 2/(40*7/140)`
5 Substitute `2` for `40*7/140`.	`a = 2/2`
6 Substitute `1` for `2/2`.	`a = 1`
i `1` is a numerical value not in quotation marks, so this value is a number.	**i** Number
ii The last value assigned to `a` was `1`.	**ii** `1`
c **1** Write the initial assignment to `a`.	**c** `a = "String"`
2 Write the initial assignment to `b`.	`b = 122`
3 Write the reassignment to `a`.	`a = b`
4 Substitute `122` for `b`.	`a = 122`
5 Write the reassignment to `b`.	`b = "String"`
i `122` is a numerical value not in quotation marks, so this value is a number.	**i** Number
ii The last value assigned to `a` was `122`.	**ii** `122`

18.2.6 Comments

- Someone reading a code may know the computer language in which the code is written, but that does not necessarily mean they will be able to understand it.
- Comments are added to codes to give hints about function of the program. The comments are completely ignored by the computer, so they are only for our benefit.
- JavaScript comments start with `//`. All the text after `//` on a line is ignored by the computer. For example, consider the following line:
  `var PI = 3.1415926536; // A 10 digit approximation to π`
  The first part of the line, `var PI = 3.1415926536;`, is the actual JavaScript code to be run.

The second part (until the end of the line),
`// A 10 digit approximation to π,`
is ignored by the computer and is only there for our own reference. The text could be changed and would make no difference to the running of the program.

WORKED EXAMPLE 6

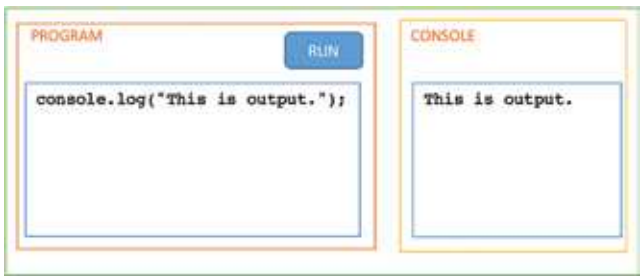
Add a comment to each statement of the form `// variable=value` where `variable` is the variable name and `value` is the evaluated value.

a `var pi = 3+1/10+3/100;`
b `var myAge = 21+22;`
c `var sum = 1+2+3+4;`

THINK	WRITE
a 1 The variable name is `pi`.	**a**
2 The calculated value is `3+1/10+4/100 = 3.14`.	
3 Append the comment.	`var pi=3+1/10+4/100; // pi=3.14`
b 1 The variable name is `myAge`.	**b**
2 The calculated value is `21+22 = 43`.	
3 Append the comment.	`var myAge=21+22; // myAge=43`
c 1 The variable name is `sum`.	**c**
2 The calculated value is `1+2+3+4 = 10`.	
3 Append the comment.	`var sum=1+2+3+4; // sum=10`

18.2.7 Output

- The **console** is a special region in a web browser for monitoring the running of JavaScript programs. Most web browsers, including Chrome, Firefox, Safari, Internet Explorer, Microsoft Edge and Opera, allow you to activate the console through the menu options.
- In order to see the result of an expression or value, the JavaScript console.log function can be used. This function outputs results to the console.



Note that the quotation marks at the start and end of strings are not shown in the output.

WORKED EXAMPLE 7

What is the output to the console after each of the following statements runs?

a `console.log(((12%5)+12)/(17%10));`

b `console.log("Some information");`

THINK	WRITE
a 1 Show the original statement.	**a** `console.log(((12%5)+12)/(17%10));`
2 `(12%5)` simplifies to `2`. `(17%10)` simplifies to `7`.	`console.log((2+12)/7);`
3 `(2+12)` simplifies to `14`.	`console.log(14/7);`
4 `14/7` simplifies to `2`.	`console.log(2);`
5 Write the output to console.	`2`
b 1 Show the original statement.	**b** `console.log("Some information");`
2 Write the output to console. Note that the quotation marks in strings are not shown in the output.	`Some information`

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Try out this interactivity: CodeBlocks Activity 1 (int-6573)

Exercise 18.2 Programs

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9, 11, 13	Questions: 1–13	Questions: 1–14

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Answer these questions for each of the following values.

i. Is the value a number, a string or Boolean?

ii. Apply the expression `typeof value` to each value. What is the result of the expression `typeof value`?

a. `-0.69314718` **b.** `false` **c.** `"true"`

d. `1000000` **e.** `true` **f.** `"Yet another string"`

g. `18` **h.** `"false"` **i.** `"False"`

h. `"-10"` **k.** `2.71828183` **l.** `"What is this?"`

m. `"1283"`

2. WE2 Each of the following is a valid variable name. True or false?

a. `4_four` **b.** `value_122` **c.** `divide/this`
d. `value_-10` **e.** `for` **f.** `someName`
g. `throw` **h.** `some_name` **i.** `21_twenty_one`
j. `multiply*this`

3. WE5 Manually simulate running the following programs. For each program:

i. what is the final type of the variable `result`
ii. what is the final value stored in the variable `result`?

a.
```
var result = "Was a string, now a number.";
result = 12
```
b.
```
var result = 12;
var x = 145;
result = x;
```
c.
```
var result = "Was a string. Now what?";
result = false
```
d.
```
var result = (3+15)/(20-2*7);
result = result / (158%5);
```
e.
```
var result = (1+2+3+4+5)/(1+1+1);
result = result / (125/(5*5));
result = true;
```
f.
```
var result = "result is a string";
var FALSE = true;
result = FALSE;
FALSE = "true";
```

4. WE7 What is the output to the console after each of the following statements runs?

a. `console.log("A simple string");`
b. `console.log("Some data");`
c. `console.log(111%(23%12));`
d. `console.log(true);`
e. `console.log("True");`
f. `console.log("12");`

5. Answer these questions for each of the following expressions and values.

i. Is the value a number, string or Boolean?
ii. Apply the expression `typeof expression` to each expression or value. What is the result of the expression `typeof expression`?

a. `10/2` **b.** `"True"`
c. `10*10*10` **d.** `99%5`

Understanding

6. WE3 Write the following as JavaScript expressions.

a. $\frac{12 - 6}{5 - 2}$
b. The remainder from the division $16 \div 5$
c. $12 + 12 - (-13 - 14 - 1)$
d. $12 \div (4 \times 3)$
e. $12 + (6 - 4 - 1)$
f. 112×2.5
g. $160 \div (4 \times 4 \times 8 \div 2)$
h. The remainder from the division $111 \div 14$
i. $24 \times 2.5 \times 2.5$

7. WE4 Write JavaScript statements to make the following variable assignments.
   a. Assign "JavaScript" to the variable `computerLanguage`.
   b. Assign the value `true` to the variable `event`.
   c. Assign the value `false` to the variable `finished`.
   d. Assign the expression `1024*1024` to the variable `megabyte`.
   e. Assign "Mathematics" to the variable `subject`.
   f. Assign the mathematical expression $60 \times 24 \times 7$ to the variable `minutes`.
   g. Assign the value `1000000` to the variable `million`.
8. WE6 Add a comment to each statement of the form `// variable=value` where `variable` is the variable name and `value` is the evaluated value
   a. `var lotsOfFactors = 2*3*5*7*11*13;`
   b. `var e = 2+7/10+1/100+8/1000+3/10000;`
   c. `var product = 1*2*3*4;`
   d. `var yourAge = 225/15;`
9. Manually simulate running the following program.

```
var final = 12;
final = final % 5;
final = 160 / final;
final = final * final;
final = final / 100;
final = final / 32;
final = final * final + final;
```

   a. What is the final type of the variable `final`?
   b. What is the final value stored in the variable `final`?
10. Manually simulate running the following program.

```
var w = "xyz";
var x = 1;
var y = 1;
var z = x+y;
y = z*(x+x+x);
x = y+z+x;
z = (y+z)*x;
x = x+z+y*x;
z = w;
```

   a. What is the final type of the variable `x`?
   b. What is the final value stored in the variable `x`?
   c. What is the final type of the variable `y`?
   d. What is the final value stored in the variable `y`?
   e. What is the final type of the variable `z`?
   f. What is the final value stored in the variable `z`?

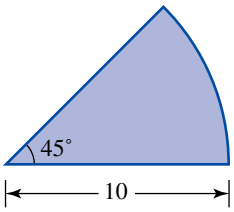
Reasoning

11. Write a program to:
   - store the value `2` in the variable `inLitres`
   - convert `inLitres` to millilitres and store into the variable `inMillilitres`.
12. Write a program to:
   - store the value `100` in the variable `crossSectionalArea`
   - store the value `12` in the variable `height`
   - calculate the volume of a prism given the `crossSectionalArea` and `height`, and store the result into the variable `prismVolume`.

Problem solving

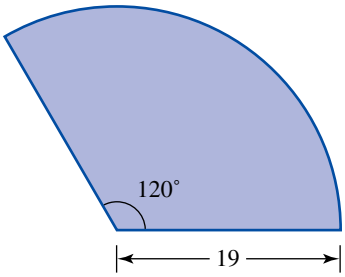
13. Write a program to calculate the `area` of a sector with `radius` equal to `10` and `angle` equal to `45` degrees.
The start of the program is shown below.

```
var radius = 10;
var angle = 45;
```



14. Write a program to calculate the `perimeter` of a sector with `radius` equal to `19` and `angle` equal to `120` degrees.
The start of the program is shown below.

```
var radius = 19;
var angle = 120;
```



Reflection

How would you explain to someone how to determine whether a variable is valid or not?

18.3 Arrays



- Computers are very good at repeating the same program many times extremely fast. However, it is redundant to apply the same program on the same data many times, so it is important to apply the program on a list of many values. This list is called an **array**.

18.3.1 List of values

- A variable can contain an array of values. An array takes the form `[value0, value1, value2, ...]` where `value0, value1, value2, ...` are the different values of the array. An array can have zero, one or more values, and the values can be any type (i.e. number, string or Boolean).
- An array with many values can be split over multiple lines. For example:

```
var array = [
value0,
value1,
value2,
...
];
```

WORKED EXAMPLE 8

Classify each of the following as an array, number, string or Boolean.

a `"Array"`
b `3.4`
c `[2,3,4]`
d `[ ]`
e `["Array"]`
f `[true,false]`
g `false`
h `[1,true,"string",[]]`

THINK	WRITE
a `"Array"` is in quotation marks, so this value is a string.	**a** String
b `3.4` is a numerical value not in quotation marks, so this value is a number.	**b** Number
c `[2,3,4]` is a list of 3 numbers, so this is an array.	**c** Array
d `[]` is an empty list, so this is an array.	**d** Array
e `["Array"]` is a list of 1 string, so this is an array.	**e** Array
f `[true,false]` is a list of 2 Booleans, so this is an array.	**f** Array
g `false` is a Boolean value.	**g** Boolean
h `[1,true,"string",[]]` is a list of 4 different types of values, so this is an array.	**h** Array

18.3.2 Testing for arrays

The function `Array.isArray(value)` can be used to test if `value` is an array. The function returns `true` if `value` is an array; otherwise, the function returns `false`.

WORKED EXAMPLE 9

Evaluate the following JavaScript expressions manually.

a `Array.isArray([])`
b `Array.isArray("[3]")`
c `Array.isArray(true)`
d `Array.isArray("array")`
e `Array.isArray([2])`
f `Array.isArray([2,"45"])`

THINK	WRITE
a 1 `[]` is an empty list, so this is an array.	**a**
2 The function `Array.isArray` returns `true`.	`true`
b 1 `"[3]"` is in quotation marks, so this value is a string, even though the contents looks like an array.	**b**
2 The function `Array.isArray` returns `false`.	`false`
c 1 `true` is a Boolean value.	**c**
2 The function `Array.isArray` returns `false`.	`false`
d 1 `"array"` is in quotation marks, so this value is a string.	**d**
2 The function `Array.isArray` returns `false`.	`false`
e 1 `[2]` is a list of 1 number, so this is an array.	**e**
2 The function `Array.isArray` returns `true`.	`true`
f 1 `[2,"45"]` is a list of 2 different types of values, so this is an array.	**f**
2 The function `Array.isArray` returns `true`.	`true`

18.3.3 Length of an array

The expression `array.length` will return the number of values in the `array`.

WORKED EXAMPLE 10

Evaluate the following JavaScript expressions manually.

a `[].length`
b `[3].length`
c `[99,"99"].length`
d `[3.1416,"pi",true,[2,3]].length`

THINK	WRITE
a `[]` is an empty list, so it has a length of `0`.	**a** `0`
b `[3]` is a list of `1` number.	**b** `1`
c `[99,"99"]` is a list of `2` different types of values.	**c** `2`
d `[3.1416,"pi",true,[2,3]]` is a list of `4` different types of values. *Note:* Do not count the number of values in the `[2,3]` array, as this is counted as one value in the longer array.	**d** `4`

18.3.4 Array indexes

- When we isolate values in a list, we usually count the item at the start of the list as the first term. JavaScript arrays are slightly different: the values inside an array are accessed using an integer **index** starting at `0`. That is, the first value in an `array` (of length `N`) is accessed with index `0`, the second value is accessed with index `1`, the third value is accessed with index `2` and the last value is accessed with index `N-1`.

`var array=`	`[`	`"first"`	`,`	`"second"`	`,`	`"third"`	`,...,`	`"last"`	`];`
Location		1st		2nd		3rd	. . .	Nth	
JavaScript index		`0`		`1`		`2`		`N-1`	

WORKED EXAMPLE 11

What is the JavaScript index of the following values in the array
`["first","second","third","fourth","second last","last"]`**?**

a `"first"` **b** `"third"`
c `"last"` **d** `"second last"`

THINK	WRITE
a `"first"` is the 1st value, so its index is `0`.	**a** `0`
b `"third"` is the 3rd value, so its index is `2`.	**b** `2`
c `"last"` is the 6th value, so its index is `5`.	**c** `5`
d `"second last"` is the 5th value, so its index is `4`.	**d** `4`

18.3.5 Accessing values in an array

- A value inside an `array` is accessed by its `index` using the expression `array[index]`. This expression returns the value with that index.

WORKED EXAMPLE 12

Given the array assignment

```
var array = ["a string",true,4,5,"false"];
```

evaluate the following JavaScript expressions manually.

a `array[1]` **b** `array[3]` **c** `array[2]`

d `array[array[2]]` **e** `[1,2,5][2]`

THINK	WRITE
a `array[1]` has an index of `1`, which is the 2nd value in the array.	**a** `true`
b `array[3]` has an index of `3`, which is the 4th value in the array.	**b** `5`
c `array[2]` has an index of `2`, which is the 3rd value in the array.	**c** `4`
d 1 `array[2]` has an index of `2`, which is the 3rd value in the array.	**d**
2 The 3rd value is `4`.	`array[2]=4`
3 Write the original expression.	`array[array[2]]`
4 Substitute `4` for `array[2]`.	`= array[4]`
5 `array[4]` has an index of `4`, which is the 5th value in the array.	`= "false"`
e 1 Indexes can be applied directly to an array. `[1,2,5][2]` has an index of `2`, which is the 3rd value in the array `[1,2,5]`.	**e**
2 The 3rd value in the array `[1,2,5]` is `5`.	`[1,2,5][2] = 5`

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Exercise 18.3 Arrays

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 11, 13	Questions: 1–9, 11–13	Questions: 1–14

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE8 Classify each of the following as an array, number, string or Boolean.
   a. `3+0.1+0.04`
   b. `[2.1,3.2,4.3,5.4,6.5,7.6]`
   c. `[true,false,true,true]`
   d. `false`
   e. `1237.3+1.2`
   f. `["a","b","c","d"]`
   g. `["Many","strings","in","an","array"]`
   h. `[1.3/2,false,"one type of many",[]]`
   i. `"Anthony"`
   j. `[]`
   k. `["A single string in an array"]`
   l. `"Is this an array, number or string?"`
2. WE9 Evaluate the following JavaScript expressions manually.
   a. `Array.isArray(["[true,false,true,true]"])`
   b. `Array.isArray([true,"true","True","TRUE"])`
   c. `Array.isArray([3.14159,2.718282])`
   d. `Array.isArray(true)`
   e. `Array.isArray([2.2,-13])`
   f. `Array.isArray([])`
   g. `Array.isArray("[2.1,3.4,5]")`
   h. `Array.isArray(false)`
   i. `Array.isArray("[true,false,true,true]")`
   j. `Array.isArray([2,"45",true,false,[1,23,4]])`
   k. `Array.isArray("Strings are like arrays")`
3. WE10 Evaluate the following JavaScript expressions manually.
   a. `[true,"true","True","TRUE"].length`
   b. `[2.718282,"e",true,[2,7,1,8,2,8,2].length].length`
   c. `[2,3,4,5].length`
   d. `[3,[2,3],4].length`
4. WE11 What is the JavaScript index of the following values in the array `[3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3]`?
   a. 2 b. 4 c. 6 d. 7 e. 8
5. WE12 Given the array assignment
   ```
   var listOfItems = [true,5,4,false,2.3,"some string",
   [2,31]];
   ```
   evaluate the following JavaScript expressions manually.
   a. `listOfItems[2]`
   b. `listOfItems[0]`
   c. `listOfItems[1+3]`
   d. `listOfItems[listOfItems[1]]`

Understanding

6. Evaluate the following JavaScript expressions manually.
   a. `[2,3,4,5][0]`
   b. `Array.isArray([3,2][1])`
   c. `[2.718282,"e",true,[2,7,1,8,2,8,2].length][1+1]`

d. `typeof Array.isArray(["[false,2,3]"])`
e. `Array.isArray([false,"false","False","FALSE"].length)`
f. `typeof Array.isArray([3,[2,4]][1])`
g. `Array.isArray([2.2,[-13]][1].length)`
h. `[Array.isArray([[].length]),2]`
i. `[3,[2,3],4][2]`

7. Manually simulate running the following program.

```
var value = 12;
value = [value,value,value*2];
value = [value.length,value[0],value[1]*value[2]];
value = value.length;
value = [value, value*value, value*value*3];
```

a. What is the final type of the variable `value`?
b. What is the final value stored in the variable `value`?

8. Manually simulate running the following program.

```
var pi = 3.14;
var pi100 = pi*100;
var d0 = pi100%10;
var d1 = ((pi100-d0)/10)%10;
var d2 = ((pi100-10*d1-d0)/100)%10;
var digits = [d2,d1,d0];
```

a. What is the final type of the variable `digits`?
b. What is the final value stored in the variable `digits`?

9. Manually simulate running the following program.

```
var values = [10,2,3,5,6];
var total = 0;
total = total + values[0];
total = total + values[1];
total = total + values[2];
total = total + values[3];
total = total + values[4];
var mean = total/values.length;
```

a. What is the final type of the variable `mean`?
b. What is the final value stored in the variable `mean`?

10. Manually simulate running the following program.

```
var numbers = [1,2,3,4,5,6];
var factorial = 1;
factorial = factorial*numbers[0];
factorial = factorial*numbers[1];
factorial = factorial*numbers[2];
factorial = factorial*numbers[3];
factorial = factorial*numbers[4];
```

a. What is the final type of the variable `factorial`?
b. What is the final value stored in the variable `factorial`?

Reasoning

11. Write a program to:
- store the values `2`, `3`, `5`, `7`, `11` and `13` in the array `primes`
- multiply all the values and store into the variable `primorial`.

12. Write a program to total all the prime numbers between 2 and 20.

Problem solving

13. Manually determine all of the possible unique combinations of rearranging the items in the array `[1,2,3]`. Store this list as an array of arrays in the variable `combinations`.

14. Let a three-item array represent three coin tosses, where one side of the coin shows `true` and the other `false`. For example, the throw below would be represented as `[true,true,false]`.



Determine all of the possible unique combinations of tossing the three coins. Store this list as an array of arrays in the variable `sampleSpace`.

Reflection

Does the length of an array always have to be a positive integer value?

18.4 Loops

18.4.1 Loops and Boolean expressions



- A similar task can be repeated many times in sequence. For example, the following program calculates 4 + 4 + 4 + 4 + 4 and outputs the sum.

```
var sum = 0;
sum = sum + 4;
sum = sum + 4;
sum = sum + 4;
sum = sum + 4;
sum = sum + 4;
console.log(sum);
```

- Loops can execute a task many times without needing multiple copies of the same code.
- Two numerical values can be compared with each other by using different combinations of symbols as shown in the following table. The comparisons return Boolean results as shown.

Expression	Description
`!x`	Opposite of `x`, where `x` is a Boolean.
`x1===x2`	`true` if `x1` equals `x2`, otherwise `false`
`x1!==x2`	`true` if `x1` does not equal `x2`, otherwise `false`
`x1>x2`	`true` if `x1` is greater than `x2`, otherwise `false`
`x1>=x2`	`true` if `x1` is greater than or equal to `x2`, otherwise `false`
`x1<x2`	`true` if `x1` is less than `x2`, otherwise `false`
`x1<=x2`	`true` if `x1` is less than or equal to `x2`, otherwise `false`

WORKED EXAMPLE 13

Evaluate the following JavaScript expressions manually.

a `!false` **b** `!true` **c** `3.24===3.42`
d `2.4!==4.2` **e** `6.4>-3.2` **f** `9.9>=9.9`
g `3.005<3.01` **h** `1.99<=1.01`

THINK	WRITE
a The opposite of `false` is `true`.	**a** `true`
b The opposite of `true` is `false`.	**b** `false`
c The statement "`3.24` equals `3.42`" is `false`.	**c** `false`
d The statement "`2.4` does not equal `4.2`" is `true`.	**d** `true`
e The statement "`6.4` is greater than `-3.2`" is `true`.	**e** `true`
f The statement "`9.9` is greater than or equal to `9.9`" is `true`.	**f** `true`
g The statement "`3.005` is less than `3.01`" is `true`.	**g** `true`
h The statement "`1.99` is less than or equal to `1.01`" is `false`.	**h** `false`

- The statement `boolean1&&boolean2` returns `true` if `boolean1` and `boolean2` are both `true`; otherwise, the statement returns `false`.

WORKED EXAMPLE 14

Evaluate the following JavaScript expressions manually.

a `false&&false` **b** `true&&false`
c `false&&true` **d** `true&&true`

THINK	WRITE
a At least one Boolean is `false`.	**a** `false`
b At least one Boolean is `false`.	**b** `false`
c At least one Boolean is `false`.	**c** `false`
d Both of the Booleans are `true`.	**d** `true`

- The statement `boolean1||boolean2` returns `true` if either `boolean1` or `boolean2` are `true`; otherwise, the statement returns `false`.

WORKED EXAMPLE 15

Evaluate the following JavaScript expressions manually.

a `false||false` **b** `true||false` **c** `false||true` **d** `true||true`

THINK	WRITE
a Both of the Booleans are `false`.	**a** `false`
b At least one Boolean is `true`.	**b** `true`
c At least one Boolean is `true`.	**c** `true`
d At least one Boolean is `true`.	**d** `true`

18.4.2 Decisions

- In JavaScript, the `if` structure is used to make a decision to run a section of code `{statement 1, statement 2, ..., statement n}` if `decision` is `true`.

```
if (decision) {
  statement 1
  statement 2
  statement n
}
```

WORKED EXAMPLE 16

Manually simulate running the following programs. For each program, what is the final value stored in `value`?

a
```
var value=15;
if (value%2>0) {
  value = value + 1;
}
```

b
```
var value=false;
if (value) {
  value = 10;
}
```

THINK	WRITE
a 1 Write the first assignment.	**a** `value=15`
2 Write the expression for the if `decision`.	`decision=(value%2)>0`
3 Substitute `15` for `value`.	`decision=(15%2)>0`
4 Substitute `1` for `15%2`.	`decision=1>0`
5 Substitute `true` for `1>0`.	`decision=true`
6 `decision` is `true`, so run the statements inside the `{}` block.	
7 The code inside the `{}` block reassigns `value`.	`value=value+1`
8 Substitute `15` for `value` on the right-hand side of the assignment.	`value=15+1`
9 Substitute `16` for `15+1`.	`value=16`
b 1 Write the first assignment.	**b** `value=false`
2 Write the expression for the if `decision`.	`decision=value`
3 Substitute `false` for `value`.	`decision=false`
4 `decision` is `false`, so ignore the statements inside the `{}` block.	
5 The value remains unchanged.	`value=false`

18.4.3 While loops

- In JavaScript, a `while` loop is used to repeat a section of code `{statement 1, statement 2, ..., statement n}` while `repeatDecision` is `true`.

```
while (repeatDecision) {
  statement 1
  statement 2
  statement n
}
```

- It is extremely important that at least one statement in the loop eventually changes the `repeatDecision` expression to `false`. If `repeatDecision` never changes to `false`, the program will never finish (without intervention). This is referred to as an **infinite loop**.

WORKED EXAMPLE 17

Each of the following causes an infinite loop. True or false?

a
```
while(true) {
  console.log("For ever");
}
```
b
```
while(false) {
  console.log("Never");
}
```
c
```
var y=1;
while(y<=10) {
  console.log(y);
  y=y+1;
}
```
d
```
var z = 1;
var y = 1;
while(z<=5) {
  console.log(y);
  y=2*y;
}
```

THINK	WRITE
a 1 `repeatDecision` is `true`.	**a**
2 `repeatDecision` never changes to `false` and causes an infinite loop.	True
b 1 `repeatDecision` is `false`.	**b**
2 `repeatDecision` starts `false`, so the loop is never executed.	False
c 1 `repeatDecision` is `y<=10`.	**c**
2 `y` is incremented each time the loop is executed with `y=y+1;`.	
3 After 10 executions, `y=11` and `y<=10` becomes `false`.	False

d 1 `repeatDecision` is `z<=5`. **d**

2 `z` is initialised at 1 and never changes inside the loop.

3 `repeatDecision` is `z<=5` and never changes to `false`, so it causes an infinite loop. True

18.4.4 Loop a number of times

The following structure is one way to repeat a section of code `{statement 1, statement 2, ..., statement n}` a fixed `number` of times. The variable `i` is an index used to track the number of times through the loop.

```
var i=1;
while (i <= number) {
  statement 1
  statement 2
  statement n
  i=i+1;
}
```

WORKED EXAMPLE 18

How many times is the loop code executed in each of the following programs?

a
```
var i=1;
while (i<=42) {
  console.log(i);
  i=i+1;
}
```

b
```
var a=1;
var i=1;
while (i<=1000) {
  a=a+i;
  i=i+1;
}
```

c
```
var j=1;
var b=1;
while (j<=5) {
  b=b*j;
  j=j+1;
}
```

THINK	WRITE
a The `number` is `42`.	**a** `42`
b The `number` is `1000`.	**b** `1000`
c 1 The change of the index from `i` to `j` is irrelevant to the loop count. Any variable can be used as an index as long as it is changed at every location in the loop structure (i.e. at initialisation, in the decision and when it is increased by 1).	**c**
2 The `number` is `5`.	`5`

- The previous program can be restructured to calculate 4 + 4 + 4 + 4 + 4 and output the total. Notice the use of comments after each line to explain what the code is doing.

```
var total = 0;                  // Initialise the running total
var i = 1;                      // i tracks number from 1 to 5
while (i <= 5) {                // Check i does not exceed 5
  total = total + 4;            // Add 4 to the running total
  i = i + 1;                    // Increment i
}
console.log(total);             // Output the total
```

WORKED EXAMPLE 19

For each of the following programs:
i how many times is the loop executed?
ii what is the final value stored in the variable a?

a
```
var a=1;
var i=1;
while (i<=4) {
  a=a*i;
  i=i+1;
}
```

b
```
var a=0;
var i=1;
while (i<=3) {
  a=a+i;
  i=i+1;
}
```

c
```
var a=0;
var i=1;
while (i<=24334) {
  a=a+7;
  i=i+1;
}
```

THINK	WRITE
a i The `number` is 4.	**a** 4
ii 1 Write the first assignment involving `a`.	a=1
2 The `statement` is `a=a*i;`.	
3 Repeat for all values of `i`.	a=a*1 a=a*2 a=a*3 a=a*4
4 Note the pattern.	a=1*1*2*3*4
5 Evaluate `a`.	a=24

b i	The `number` is `3`.	**b** `3`
ii	**1** Write the first assignment involving `a`.	`a=0`
	2 The `statement` is `a=a+i;`.	

	3 Repeat for all values of `i`.	`a=a+1`
		`a=a+2`
		`a=a+3`

	4 Note the pattern.	`a=0+1+2+3`
	5 Evaluate `a`.	`a=6`
c i	The `number` is `24334`.	**c** `24334`
ii	**1** Write the first assignment involving `a`.	`a=0`
	2 The `statement` is `a=a+7;`.	

	3 Repeat for `24334`.	`a=a+7; // i=1`
		`a=a+7; // i=2`
		`...`
		`a=a+7; // i=24334`

	4 Note the pattern.	`a=0+7+7+...+7`
	5 Note the simplification as a multiplication.	`a=7*24334`
	6 Evaluate `a`.	`a=170338`

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Try out this interactivity: CodeBlocks Activity 3 (int-6575)

Exercise 18.4 Loops

assessON

Individual pathways

PRACTISE
Questions:
1–6, 8–10, 13, 15

CONSOLIDATE
Questions:
1–11, 13–15

MASTER
Questions:
1–16,

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Fluency

1. WE13 Evaluate the following JavaScript expressions manually.

a. `16.41>-13.12`	**b.** `-999>=-999`	**c.** `4.06<4.1`
d. `!true`	**e.** `113.3===113.30`	**f.** `2.41!==412.2`
g. `101.99<=102.01`	**h.** `12===21`	**i.** `13.40===13.400`
j. `21.41!==-42212.22`	**k.** `-1.1>-1.12`	**l.** `!false`
m. `123.3===213.2`	**n.** `-4<=-4`	

2. WE14 Evaluate the following JavaScript expressions manually.

a. `false&&true`

b. `true&&false`

c. `true&&true`

d. `false&&false`

3. WE15 Evaluate the following JavaScript expressions manually.

a. `true||true`

b. `false||true`

c. `false||false`

d. `true||false`

4. WE16 Manually simulate running the following programs. For each program, what is the final value stored in `x`?

a.
```
var x=153;
if ((x%7)===6) {
  x = x*x;
}
```

b.
```
var x=false&&true;
if (x===false) {
  x = 100;
}
```

5. WE17 Each of the following causes an infinite loop. True or False?

a.
```
while(false) {
  var i = 10;
  console.log(i);
}
```

b.
```
while(true) {
  console.log("Too many");
}
```



6. WE18 How many times is the loop code executed in each of the following programs?

a.
```
var i=1;
while (i<=99) {
  console.log(2*i);
  i=i+1;
}
```

b.
```
var u=1;
var i=1;
while (i<=100) {
  u=u*i;
  i=i+1;
}
```

c.
```
var k=1;
var b=1000;
while (k<=5) {
  b=b-k;
  k=k+1;
}
```

d.
```
while(false) {
  console.log("This sentence is ignored.");
}
```

e.
```
var i=1;
while (i<=900) {
  console.log(2*i);
  i=i+1;
}
```

7. WE19 For each of the following programs:

i. how many times is the loop executed?

ii. what is the final value stored in the variable `h`?

a.
```
var h=0;
var i=1;
while (i<=14) {
  h=h+i;
  i=i+1;
}
```

b.
```
var h=1;
var k=1;
while (k<=9) {
  h=h*k;
  k=k+1;
}
```

c.
```
var h=0;
var index=1;
while (index <=5) {
  h=h*3;
  index=index+1;
}
```

Understanding

8. Consider the following program.

```
var total=0;
var i=1;
var j=1;
while (j<=5) {
  total=total+2*i;
  j=j+1;
}
```

a. What is the final value stored in the variable `total`?

b. What is the final value stored in the variable `j`?

c. What is the final value stored in the variable `i`?

9. Each of the following causes an infinite loop. True or false?

a.
```
var repeat=true;
var i=1;
while(repeat) {
  repeat = (i%31)!==0;
  i=i+1;
}
```

b.
```
var repeat=true;
var i=1;
while(repeat) {
  repeat = (i%31)!==0;
  i=(i%30)+1;
}
```

10. How many times is the loop code executed in each of the following programs? Keep in mind the loop indexes are not always increased by one each time.

a.
```
var max=1000;
var m=1;
while (m<=max) {
  console.log(m*m);
  m=m+1;
}
```

b.
```
var a=0;
var b=100;
while (a<=b) {
  a=a+2;
  b=b+1;
}
```

c.
```
var k=0;
while (k<100) {
  k=k+1;
}
```

d.
```
var j=0;
while (j<100) {
  j=j+2;
}
```

11. What is the final value stored in the variable `total` by the following program?
```
var total=0;
var x=1;
while (x<=100) {
  total = total+20;
  x=x+1;
}
```

12. What is the output of the following program?
```
var total = 0;
var a=1;
while (a<=10) {
  total = total + a*a;
  a=a+1;
}
console.log(total);
```

Reasoning

13. Write a program to multiply all of the integers between 1 and 10, and store the product into the variable `factorial_10`.

14. Write a program to add all of the integers between 1 and 1 000 000, and store the sum into the variable `total`.

Problem solving

15. Given two variables `value1` and `value2` assigned to `362` and `232` respectively, assign the first positive integer to `product` where `value1` and `value2` divide exactly. Output the value of `product` to the console. The start and end of the program is shown below. Fill in the middle to find the value `product`.

```
var value1 = 362;
var value2 = 232;
...
console.log(product);
```

16. Given an `array` of integers `[362,232,4,8,5]`, assign the first positive integer to `product` where all the values in the `array` divide into `product` exactly. Write a program to find and output the value `product`. The start and end of the program is shown below. Fill in the middle to find the value `product`.

```
var array = [362,232,4,8,5];
...
console.log(product);
```

Reflection

What is the result of an infinite loop?

18.5 Set structures

18.5.1 Array set structure

- An array can be used to store a set of values. This set structure is useful to represent a list of values where the order might be important and values might be repeated in the list.

WORKED EXAMPLE 20

For each of the following situations, represent the data as an array of values.

a Tim finished last in a race between friends. Jane was first to cross the line, closely followed by Wayne. Grace also finished the race.

b The list of prime numbers from 1 to 21

c The first 8 digits of π after the decimal place

THINK	WRITE
a The friends finished the race in the order Jane, Wayne, Grace and Tim.	**a** `["Jane","Wayne", "Grace","Tim"]`
b The primes between 1 and 21 are 2, 3, 5, 7, 11, 13, 17 and 19.	**b** `[2,3,5,7,11,13, 17,19]`
c $\pi = 3.14159265$ correct to 8 decimal places. The 8 digits after the decimal place are 1, 4, 1, 5, 9, 2, 6 and 5.	**c** `[1,4,1,5,9,2,6,5]`

18.5.2 Loop over an array

The following structure is one way to repeat a section of code

`{statement 1, statement 2, ..., statement n}`

for each `item` in an `array`. The variable `i` is the array index and is also used to track the current loop iteration.

```
var array = [...];
var i=0;
while (i < array.length) {
  var item = array[i];
  statement 1
  statement 2
  statement n
  i=i+1;
}
```

WORKED EXAMPLE 21

How many times is the loop code executed in each of the following programs?

a
```
var friends = ["Jane","Wayne","Grace","Tim"];
var places = ["1st","2nd","3rd","Last"];
var i=0;
while (i < friends.length) {
  var item = friends[i];
  var place = places[i];
  console.log(place+" : "+item);
  i=i+1;
}
```

b
```
var sum = 0;
var primes = [2,3,5,7,11,13,17,19];
var j=0;
while (j < primes.length) {
  var prime = primes[j];
  sum = sum + prime;
  j=j+1;
}
console.log(sum);
```

c
```
var pi = 3;
var place = 1;
var digits = [1,4,1,5,9,2,6,5];
var k=0;
while (k < digits.length) {
  var item = digits[k];
  place = place / 10;
  pi = pi + item * place;
  k=k+1;
}
console.log(pi);
```

THINK	WRITE
a 1 The code follows the structure to loop over the array `friends=["Jane","Wayne","Grace","Tim"]`.	**a**
2 The length of the `friends` array is 4.	4
b 1 The code follows the structure to loop over the array `primes=[2,3,5,7,11,13,17,19]`.	**b**
2 The length of the `primes` array is 8.	8
c 1 The code follows the structure to loop over the array `digits=[1,4,1,5,9,2,6,5]`.	**c**
2 The length of the `digits` array is 8.	8

18.5.3 Sum

- There are various reasons for looping over an array. One reason is to sum all the values. The following structure is one way to sum all the values in an `array`.

```
var array = [...];
var total = 0;
var i=0;
while (i < array.length) {
  var item = array [i];
  total = total + item;
  i=i+1;
}
console.log(total);
```

1. First the program initialises the running `total` with the value `0`.
2. Then the `while` loop adds each `item` to a running `total` one at a time.
3. Finally, the program outputs the `total` to the console.

WORKED EXAMPLE 22

What is the output to the console generated by the following program?

```
var sum = 0;
var primes = [2,3,5,7,11,13];
var j=0;
while (j < primes.length) {
  var item = primes[j];
  sum = sum + item;
  j=j+1;
}
console.log(sum);
```

THINK	WRITE
1 The code follows the structure to find the sum of all the items in the array `primes=[2,3,5,7,11,13]`.	
2 Write the sum of all items in the array `primes` and assign to `sum`.	`sum=2+3+5+7+11+13`
3 Evaluate the sum of items.	`sum=41`
4 Write the `total` to the console.	`41`

18.5.4 Mean

- The mean or average of a set of values is the sum of all the values divided by the number of values. Finding the `mean` from an `array` is only a little more work after calculating the `total`. Building on the sum structure, the following structure is one way to find the `mean` of the values in an `array`.

```
var array = [...];
var total = 0;
var i=0;
while (i < array.length) {
  var item = array[i];
  total = total + item;
  i=i+1;
}
var mean = total / array.length;
console.log(mean);
```

1. First the program initialises the running `total` with the value `0`.
2. The `while` loop adds each `item` to a running `total` one at a time.
3. The `mean` is calculated by dividing the `total` with the number of items.
4. Finally, the program outputs the `mean` to the console.

WORKED EXAMPLE 23

What is the output to the console generated by the following program?

```
var total = 0;
var digits = [3,1,4,1,5,9,2,6,5,4];
var k=0;
while (k < digits.length) {
  var item = digits[k];
  total = total + item;
  k=k+1;
}
var mean = total / digits.length;
console.log(mean);
```

THINK	WRITE
1 The code follows the structure to find the mean of all the items in the array `digits=[3,1,4,1,5,9,2,6,5,4]`.	
2 Write the sum of all items in the array `digits` and assign to `total`.	`total=3+1+4+1+5+9+2+6+5+4`
3 Evaluate the sum of the items.	`total=40`
4 Write the assignment to calculate the `mean`.	`mean=total/digits.length`
5 Substitute `40` for `total` and `10` for `digits.length`.	`mean=40/10`
6 Evaluate the `mean`.	`mean=4`
7 Write the `mean` to the console.	`4`

Exercise 18.5 Set structures

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 11, 13	Questions: 1–9, 11–13	Questions: 1–14

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Fluency



1. WE20 For each of the following situations, represent the data as an array of values.
   a. The days of the week starting at `"Monday"`
   b. The months of the year
   c. The lowercase letters of alphabet
   d. The number of days in each month during a leap year
2. WE21 How many times is the loop code executed in each of the following programs?

   a.
```
var e = 2;
var base = 1;
var eDigits = [7,1,8,2,8,1,8,2,8,4,5,9];
var m=0;
while (m < eDigits.length) {
  var item = eDigits[m];
  base = base / 10;
  e = e + item * base;
  m=m+1;
}
console.log(e);
```

   b.
```
var d = ["Sun","Mon","Tues","Wednes","Thurs","Fri", "Satur"];
var j=0;
while (j < d.length) {
  var start = d[j];
  console.log(start+"day");
  j=j+1;
}
```

   c.
```
var primorial = 1;
var primes = [2,3,5,7,11,13,17,19,23,29];
var j=0;
while (j < primes.length) {
  var prime = primes[j];
  primorial = primorial * prime;
  j=j+1;
}
console.log(primorial);
```

3. WE22 What is the output to the console generated by the program below to 12 decimal places?

```
var e = 2;
var base = 1;
var eDigits = [7,1,8,2,8,1,8,2,8,4,5,9];
var m=0;
while (m < eDigits.length) {
  var item = eDigits[m];
  base = base / 10;
  e = e + item * base;
  m=m+1;
}
console.log(e);
```

4. What is the output to the console generated by the program below?

```
var primorial = 1;
var primes = [2,3,5,7,11,13,17,19,23,29];
var j=0;
while (j < primes.length) {
  var prime = primes[j];
  primorial = primorial * prime;
  j=j+1;
}
console.log(primorial);
```

5. WE23 What is the output to the console generated by the program below?

```
var sum = 0;
var eDigits = [2,7,1,8,2,8,1,8,2,8,4,5,9];
var i=0;
while (i < eDigits.length) {
  var digit = eDigits[i];
  sum = sum + digit;
  i=i+1;
}
var mean = sum / eDigits.length;
console.log(mean);
```

Understanding

6. What is the output to the console generated by the program below?

```
var isPrime = true;
var testPrime=31;
var j = 2;
while (j < testPrime) {
  var notMultiple = testPrime%j !== 0;
  isPrime = isPrime && notMultiple;
  j = j+1;
}
console.log(isPrime);
```

7. What is the output to the console generated by the program below?

```
var primorial = 1;
var testPrime=2;
while (testPrime <= 30) {
```

```
  var isPrime = true;
  var j = 2;
  while (j < testPrime) {
    var notMultiple = testPrime%j !== 0;
    isPrime = isPrime && notMultiple;
    j = j+1;
  }
  if (isPrime) {
    primorial = primorial * testPrime;
  }
  testPrime = testPrime+1;
}
console.log(primorial);
```

8. Consider the program below.

```
var values = [1.2,3,5,6,2,2,1];
var result = values[0];
var i = 0;
while (i < values.length) {
  var item = values[i];
  if (item > result) {
    result = item;
  }
  i = i + 1;
}
console.log(result);
```

a. What is the output to the console generated by the program?

b. What is the purpose of the program?

9. Consider the program below.

```
var values = [2,13,2,1,1,8];
var min = values[0];
var max = values[0];
var i = 0;
while (i < values.length) {
  var item = values[i];
  if (item > max) {
    max = item;
  }
  if (item < min) {
    min = item;
  }
  i = i + 1;
}
var result = max - min;
console.log(result);
```

a. What is the output to the console generated by the program?

b. What is the purpose of the program?

10. Consider that the function `Math.sqrt(value)` calculates the square root of `value`. Also consider the program below. What is the output to the console generated by the program?

```
var scores = [10, 16, 6, 12];
var n = scores.length;
var sum = 0;
var i = 0;
while (i < n) {
  var x = scores[i];
  sum = sum + x;
  i = i + 1;
}
var mean = sum/n;
var sumSquares = 0;
var i = 0;
while (i < n) {
  var diff = scores[i]-mean;
  var diffSquare = diff*diff;
  sumSquares = sumSquares + diffSquare;
  i = i + 1;
}
var variance = sumSquares/(n-1);
var result = Math.sqrt(variance);
console.log(result);
```

Reasoning

11. Write a program to find the mean of the values `10`, `17`, `6`, `3`, `26`, `23`, `1`, `23`, `4` and `5` and store the result in the variable `mean`.

12. Write a program to count the number of primes between 1 and 1000, store the count in the variable `count`, and output `count` to the console.

Problem solving

13. Write a program to find the 1000th `prime` and output `prime` to the console.



14. Output all the solutions to the equation $a^2 + b^2 = c^2$ where a, b and c are integers and $1 \leq a \leq b \leq 30$. Use the expression `"a="+a+" b="+b+" c="+c` to output to the console.

Reflection

What does the word 'array' mean? Think about where else you have heard it used, and then look up a definition.

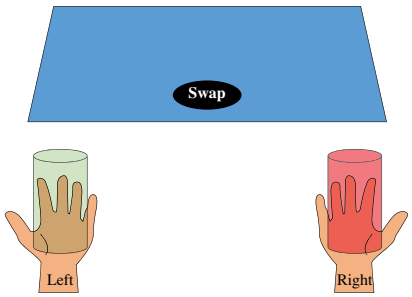
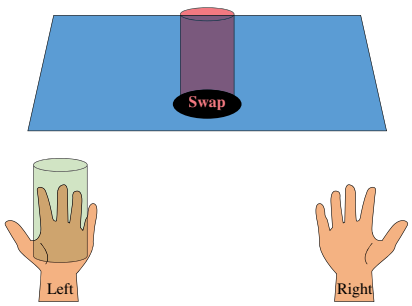
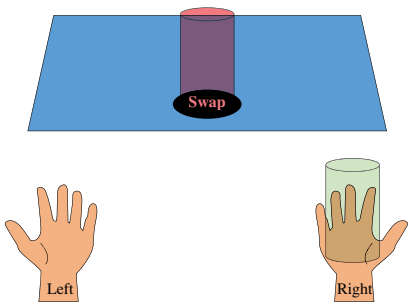
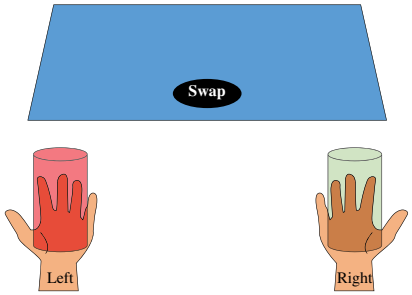
18.6 Sorting algorithms

18.6.1 Algorithms

- An **algorithm** is a step-by-step set of tasks to solve a particular problem. A program is the implementation of an algorithm.
- An array of items is easier to manage when the array is sorted. For example, finding the minimum and maximum values in a list is easier if the list is sorted so that they are located at the start and end of the list.
- In this section we will investigate various algorithms to sort an array. The sorting algorithm requires a number of smaller algorithms, which will be investigated first.

Swapping the contents of variables

- Imagine you are holding an item in each of your hands. Each hand can hold only one of each item. In order to swap the items, you need to place one item down in a temporary location.

Step	Diagram	Explanation	Swapping program
1		Start with green in the left hand and red in the right.	`var left = "green";` `var right = "red";`
2		Move the contents in the right hand to the swap location.	`var swap = right;`
3		Move the contents in the left hand to the right hand.	`right = left;`
4		Finally, move the contents in the swap location to the left hand.	`left = swap;`

WORKED EXAMPLE 24

Each of the following programs swaps the contents of `variable1` and `variable2`. True or false?

a
```
var variable1 = 12;
var variable2 = 3;
var swap = variable1;
variable1 = variable2;
variable2 = swap;
```

b
```
var variable1 = "blue";
var variable2 = "yellow";
var swap = variable1;
variable2 = variable1;
variable1 = swap;
```

c
```
var variable1 = true;
var variable2 = false;
var swap = variable2;
variable2 = variable1;
variable1 = swap;
```

THINK	WRITE
a 1 Write the first assignment.	**a** `variable1 = 12`
2 Write the second assignment.	`variable2 = 3`
3 Write the third assignment.	`swap = variable1`
4 Substitute `12` for `variable1`.	`swap = 12`
5 Write the fourth assignment.	`variable1 = variable2`
6 Substitute `3` for `variable2`.	`variable1 = 3`
7 Write the fifth assignment.	`variable2 = swap`
8 Substitute `12` for `swap`.	`variable2 = 12`
9 `variable1` and `variable2` have successfully swapped.	True
b 1 Write the first assignment.	**b** `variable1 = "blue"`
2 Write the second assignment.	`variable2 = "yellow"`
3 Write the third assignment.	`swap = variable1`
4 Substitute `"blue"` for `variable1`.	`swap = "blue"`
5 Write the fourth assignment.	`variable2 = variable1`
6 Substitute `"blue"` for `variable1`.	`variable1 = "blue"`
7 Write the fifth assignment.	`variable1 = swap`
8 Substitute `"blue"` for `swap`.	`variable2 = "blue"`
9 `variable1` and `variable2` both ended up with the same value, so they did not successfully swap.	False

c 1	Write the first assignment.	`c variable1 = true`
2	Write the second assignment.	`variable2 = false`
3	Write the third assignment.	`swap = variable2`
4	Substitute `false` for `variable2`.	`swap = false`
5	Write the fourth assignment.	`variable2 = variable1`
6	Substitute `true` for `variable1`.	`variable2 = true`
7	Write the fifth assignment.	`variable1 = swap`
8	Substitute `false` for `swap`.	`variable2 = false`
9	`variable1` and `variable2` have successfully swapped.	True

WORKED EXAMPLE 25

Complete the rest of the program to swap the contents of the two variables.

```
var containerA = "Cream";
var containerB = "Jam";
```

THINK	WRITE
1 Write the start of the program as provided.	`var containerA = "Cream";` `var containerB = "Jam";`
2 Store the contents of `containerA` in a temporary variable `swap`.	`var swap = containerA;`
3 Overwrite the contents of `containerA` with the value in `containerB`.	`containerA = containerB;`
4 Overwrite the contents of `containerB` with the contents of the temporary variable `swap`.	`containerB = swap;`

18.6.2 Swapping two array items

- To sort an array, you must be able to swap values that are out of place. The following structure is one way to `swap` the two items in an `array` with the indexes `index1` and `index2`.

```
var array = [...];
var swap = array [index1];
array[index1] = array[index2];
array[index2] = swap;
```

WORKED EXAMPLE 26

Complete the rest of the program to swap the contents of the 3rd and 6th item in the array.

```
var array = [15,11,2,4,5,9,11];
```

THINK	WRITE
1 Rewrite the start of the program.	`var array=[15,11,2,4,5,9,11];`
2 Store the array index 2 of the 3rd item in the variable `index1`.	`var index1=2;`
3 Store the array index 5 of the 6th item in the variable `index2`.	`var index2=5;`
4 Store the contents of `array[index1]` in a temporary variable `swap`.	`var swap=array[index1];`
5 Overwrite the contents of `array[index1]` with the value in `array[index2]`.	`array[index1]=array[index2];`
6 Overwrite the contents of `array[index2]` with the contents of the temporary variable `swap`.	`array[index2]=swap;`

18.6.3 Defining a function

- In more complex programs, it is useful to wrap a section of code in a JavaScript function. This gives the section of code
  `{statement 1, statement 2, ..., statement n}`
  a descriptive `name`. A function with a `name` can be given zero, one or more inputs, `x1,x2,...`.

```
function name(x1,x2,...) {
  statement 1
  statement 2
  statement n
  return output;
}
var store = name(v1,v2,...);
```

- Once a function is defined, it can be called with the statement `name(v1,v2,...);`. The value `v1` is assigned to the input `x1`, the value `v2` is assigned to the input `x2`, and so on.
- In JavaScript, a function `name` can `return` an internal value `output` to be `stored`.

WORKED EXAMPLE 27

What is the output to the console of each of the following programs?

a
```
function outputSquared(r) {
  console.log(r*r);
}
outputSquared(-11);
```

b
```
function hexagonPerimeter(side) {
  console.log(6*side);
}
hexagonPerimeter(8.5);
```

c
```
function noInputs() {
  return "No inputs";
}
console.log(noInputs());
```

THINK	WRITE
a 1 The input `r` equals `-11`.	**a** `r=-11`
2 Write the output statement.	`console.log(r*r)`
3 Substitute `-11` for `r`.	`console.log(-11*-11)`
4 Evaluate the expression.	`console.log(121)`
5 Write the output to the console.	`121`
b 1 The input `side` equals `8.5`.	**b** `side=8.5`
2 Write the output statement.	`console.log(6*side)`
3 Substitute `8.5` for `side`.	`console.log(6*8.5)`
4 Evaluate the expression.	`console.log(51)`
5 Write the output to the console.	`51`
c 1 The function has no inputs.	**c**
2 Write the return expression.	`noInputs()="No inputs"`
3 Write the output statement.	`console.log(noInputs());`
4 Substitute `"No inputs"` for `noInputs()`.	`console.log("No inputs");`
5 Write the output to the console.	`No inputs`

18.6.4 The swapping function

- A sorting algorithm requires swapping two array items. It is useful to define the swapping algorithm as a reusable function. The function `swapItems` has three inputs `array`, `index1` and `index2`, where `array` is the list of values, and `index1` and `index2` are the indexes of the items to be swapped.

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
var anArray = ["1","3","2"];
swapItems (anArray, 1, 2);
console.log(anArray);
```

- Once this function is defined, it can be called with the statement `swapItems(array,index1,index2)`.

WORKED EXAMPLE 28

Given the swapping function shown below, what is the output to the console of each of the following programs?

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
a var digits = [1,2,3,4,5,6,7,8];
  swapItems(digits, 2, 7);
  console.log(digits[2]);
b var a = ["1st","2nd","last","3rd"];
  swapItems(a, 2, 3);
  console.log(a[a.length-1]);
```

THINK	WRITE
a 1 Write the first assignment.	`a digits = [1,2,3,4,5,6,7,8]`
2 Calling `swapItems` with array `digits` and indexes `2` and `7` will swap the 3rd and 8th items.	`digits = [1,2,8,4,5,6,7,3]`
3 Write the output statement.	`console.log(digits[2])`
4 `digits[2]` is the 3rd item in the `digits` array.	
5 Substitute `8` for `digits[2]`.	`console.log(8)`
6 Write the output to the console.	`8`
b 1 Write the first assignment.	`b a = ["1st","2nd","last","3rd"];`
2 Calling `swapItems` with array `a` and indexes `2` and `3` will swap the 3rd and 4th items.	`a = ["1st","2nd","3rd","last"]`
3 Write the output statement.	`console.log(a[a.length-1]);`
4 Substitute `4` for `a.length`	`console.log(a[4-1]);`
5 Substitute `3` for `4-1`.	`console.log(a[3]);`
6 `a[3]` is the 4th item in the `a` array.	
7 Substitute `"last"` for `a[3]`.	`console.log("last")`
8 Write the output to the console.	`last`

18.6.5 Decisions

- While sorting an array, you must make many decisions to swap two values in the array if they are out of place. In JavaScript, the `if` structure can be used to make a decision to swap items if `decision` is `true`.

```
if (decision) {
  // swap
}
```

WORKED EXAMPLE 29

Manually simulate running the following programs. For each program, what is the final value stored in the array?

a
```
var primes=[2,3,5,13,11,7];
var i=0;
var j=1;
if (primes[i]>primes[j]) {
  var swap=primes[i];
  primes[i]=primes[j];
  primes[j]=swap;
}
```

b
```
var digits=[2,3,4,9,8,7];
var i=3;
var j=4;
if (digits[i]>digits[j]) {
  var swap=digits[i];
  digits[i]=digits[j];
  digits[j]=swap;
}
```

THINK	WRITE
a 1 Write the first assignment.	**a** `primes=[2,3,5,13,11,7]`
2 Write the second assignment.	`i=0`
3 Write the third assignment.	`j=1`
4 Write the expression for the if `decision`.	`decision=primes[i]>primes[j]`
5 Substitute `0` for `i` and `1` for `j`.	`decision=primes[0]>primes[1]`
6 Substitute `2` for `primes[0]` and `3` for `primes[1]`.	`decision=2>3`
7 Substitute `false` for `2>3`.	`decision=false`
8 `decision` is `false`, so ignore the statement inside the `{}` block.	
9 The array `primes` remains unchanged.	`primes=[2,3,5,13,11,7]`
b 1 Write the first assignment.	**b** `digits=[2,3,4,9,8,7]`
2 Write the second assignment.	`i=3`
3 Write the third assignment.	`j=4`
4 Write the expression for the if `decision`.	`decision=digits[i]>digits[j]`
5 Substitute `3` for `i` and `4` for `j`.	`decision=digits[3]>digits[4]`
6 Substitute `9` for `digits[3]` and `8` for `digits[4]`.	`decision=9>8`
7 Substitute `true` for `9>8`.	`decision=true`

8 `decision` is `true`, so run the statements inside the `{}` block.

9 The code inside the `{}` block swaps the `digits[3]` and `digits[4]` items.

```
digits=[2,3,4,8,9,7]
```

- In JavaScript, the `if else` structure is used execute one of the two different sections of code.
  1. `{statement 1, statement 2, ..., statement n}` if `decision` is `true`.
  2. `{statement A, statement B, ..., statement Z}` if `decision` is `false`.

```
if (decision) {
  statement 1
  statement 2
  statement n
} else {
  statement A
  statement B
  statement Z
}
```

WORKED EXAMPLE 30

Manually simulate running the following programs. For each program, what is the final value stored in the variable `mean`?

a
```
var mean=0;
var values = [4,10];
if (values.length > 1) {
  mean = (values[0]+values[1])/2;
} else {
  mean = values[0];
}
```

b
```
var mean=0;
var values = [99];
if (values.length > 1) {
  mean = (values[0]+values[1])/2;
} else {
  mean = values[0];
}
```

THINK

a 1 Write the first assignment.

2 Write the second assignment.

3 Write the `decision`.

WRITE

a
```
mean=0
values = [4,10]
decision = values.length > 1
```

4	Substitute `2` for `values.length`.	`= 2 > 1`
5	Substitute `true` for `2 > 1`.	`= true`
6	`decision` is `true`, so run the statements inside the first `{}` block	`mean = (values[0]+values[1])/2`
7	Substitute `4` for `values[0]` and `10` for `values[1]`.	`mean = (4+10)/2`
8	Substitute `7` for `(4+10)/2`.	`mean = 7`
b 1	Write the first assignment.	**b** `mean=0`
2	Write the second assignment.	`values = [99]`
3	Write the `decision`.	`decision = values.length > 1`
4	Substitute `1` for `values.length`.	`= 1 > 1`
5	Substitute `false` for `1 > 1`.	`= false`
6	`decision` is `false`, so run the statements inside the second `{}` block	`mean = values[0]`
7	Substitute `99` for `values[0]`.	`mean = 99`

18.6.6 Bubble sort

- One of the simplest sorting algorithms is the **bubble sort**. The algorithm scans an array, comparing adjacent values. If any adjacent values are out of place, they are swapped. This scanning process is repeated until no swaps are made during a complete scan.



Values					Scan	Array	Swap?	Action	After swap
9	1	5	2	3	1	`[9,1,5,2,3]`	`9>1 = true`	Swap	`[1,9,5,2,3]`
1	9	5	2	3	1	`[1,9,5,2,3]`	`9>5 = true`	Swap	`[1,5,9,2,3]`
1	5	9	2	3	1	`[1,5,9,2,3]`	`9>2 = true`	Swap	`[1,5,2,9,3]`
1	5	2	9	3	1	`[1,5,2,9,3]`	`9>3 = true`	Swap	`[1,5,2,3,9]`
1	5	2	3	9	2	`[1,5,2,3,9]`	`1>5 = false`		
1	5	2	3	9	2	`[1,5,2,3,9]`	`5>2 = true`	Swap	`[1,2,5,3,9]`
1	2	5	3	9	2	`[1,2,5,3,9]`	`5>3 = true`	Swap	`[1,2,3,5,9]`
1	2	3	5	9	2	`[1,2,3,5,9]`	`5>9 = false`		
1	2	3	5	9	3	`[1,2,3,5,9]`	`1>2 = false`		
1	2	3	5	9	3	`[1,2,3,5,9]`	`2>3 = false`		
1	2	3	5	9	3	`[1,2,3,5,9]`	`3>5 = false`		
1	2	3	5	9	3	`[1,2,3,5,9]`	`5>9 = false`		

- The function `bubbleScan` has one input called `array`, where `array` is the list of values to be scanned in sequence.
- The `bubbleScan` swaps any adjacent items not in order and stops after one pass.
- The `bubbleScan` returns `true` if any values were swapped (the array was modified) during the scan; otherwise, it returns `false`.
- The function `bubbleScan` uses the previously defined `swapItems` function.

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
function bubbleScan(array) {
  var swapped = false;
  var i=1; // start at the second item
  while (i < array.length) {
    // compare adjacent items
    if (array[i-1]>array[i]) {
      // swap if items are out of place
      swapItems(array, i-1, i);
      swapped = true;
    }
    i=i+1;
  }
  return swapped;
}
var anArray = [9,1,5,2,3];
console.log(bubbleScan(anArray));
console.log(anArray);
```

- Once the function is defined, it can be called with the statement `bubbleScan(array)`.

WORKED EXAMPLE 31

Given the function definitions shown below, what is returned from each of the following function calls?

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
function bubbleScan(array) {
  var swapped = false;
  var i=1;
  while (i < array.length) {
    if (array[i-1]>array[i]) {
      swapItems(array, i-1, i);
      swapped = true;
    }
    i=i+1;
  }
  return swapped;
}
```

a `bubbleScan([2,1])`
b `bubbleScan([1,2,3,4,5,7,6])`
c `bubbleScan([])`
d `bubbleScan([1,2,3,4,5,7])`
e `bubbleScan([99])`

THINK	WRITE
a `[2,1]` is not sorted, so `bubbleScan` returns `true`.	**a** `true`
b `[1,2,3,4,5,7,6]` is not sorted (7 and 6 are out of place), so `bubbleScan` returns `true`.	**b** `true`
c `[]` is empty but is still considered sorted, so `bubbleScan` returns `false`.	**c** `false`
d `[1,2,3,4,5,7]` is sorted, so `bubbleScan` returns `false`.	**d** `false`
e `[99]` is sorted, so `bubbleScan` returns `false`.	**e** `false`

- The function `bubbleSort` has one input `array`, where `array` is the list of values to be sorted. The function `bubbleSort` uses the previously defined `bubbleScan` and `swapItems` functions.
- The `bubbleScan` is called many times until the `array` is no longer modified.

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
function bubbleScan(array) {
  var swapped = false;
  var i=1; // start at the second item
  while (i < array.length) {
  // compare adjacent items
  if (array[i-1]>array[i]) {
    // swap if items are out of place
    swapItems(array, i-1, i);
    swapped = true;
  }
  i=i+1;
  }
  return swapped;
}
function bubbleSort(array) {
  // Scan the array until there are no more changes
  // bubbleScan(array) is now used as the decision to
  // loop.
  // bubbleScan(array) is called many times each time
  // changing the array.
  // Eventually the array becomes sorted and
  // bubbleScan(array) returns false.
  while(bubbleScan(array)) {
  // Does nothing here
  }
  return array;
}
console.log(bubbleSort([9,1,5,2,3]));
```

- Once the function is defined, it can be called with the statement `bubbleSort(array)`. The original `array` is sorted in place and returned once the `bubbleScan` detects no more changes.

WORKED EXAMPLE 32

Given the function definitions shown below, what is returned from each of the following function calls?

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
```

```
function bubbleScan(array) {
  var swapped = false;
  var i=1;
  while (i < array.length) {
    if (array[i-1]>array[i]) {
      swapItems(array, i-1, i);
      swapped = true;
    }
    i=i+1;
  }
  return swapped;
}
function bubbleSort(array) {
  while(bubbleScan(array)) {
  }
  return array;
}
```

a `bubbleSort([2,1])`
b `bubbleSort([1,2,3,4,5,7,6])`
c `bubbleSort([])`
d `bubbleSort([1,2,3,4,5,7])`
e `bubbleSort([99])`

THINK	WRITE
a `[2,1]` when sorted is `[1,2]`.	**a** `[1,2]`
b `[1,2,3,4,5,7,6]` when sorted is `[1,2,3,4,5,6,7]`.	**b** `[1,2,3,4,5,6,7]`
c `[]` is empty and already sorted.	**c** `[]`
d `[1,2,3,4,5,7]` is already sorted.	**d** `[1,2,3,4,5,7]`
e `[99]` is already sorted.	**e** `[99]`

18.6.7 Selection sort

- Another sorting algorithm is called **selection sort**. The algorithm swaps the minimum value with the item at the start of the array. This minimum value is locked in place and the process is repeated on the rest of the array. The second smallest value is now found and locked into place. The process is repeated on the remainder of the array until the entire array is sorted.

Stage	Values					Subset array	Minimum
Initial order	9	1	5	2	3	`[9,1,5,2,3]`	1
1st pass	1	9	5	2	3	`[9,5,2,3]`	2
2nd pass	1	2	5	9	3	`[3,1,2]`	3
3rd pass	1	2	3	9	5	`[2,1]`	5
4th pass	1	2	3	5	9	`[1]`	9
Sorted	1	2	3	5	9	`[]`	

18.6.8 Median

- The **median** is the middle item of a sorted array.
- If `array.length` is odd, then the median is the middle item.
  `index = (array.length-1)/2`
  For example, an odd length array `[12,15,23,86,101]` of length 5:
  ```
  index = (array.length-1)/2
  index = (5-1)/2
  index = 2
  ```

Value	12	15	23	86	101
index	0	1	index = 2	3	4

- If `array.length` is even, then the median is the average of the two middle items.
  ```
  index1 = array.length/2 - 1
  index2 = array.length/2
  ```
  For example, an even length array `[8,15,16,18,100,201]` of length 6:
  ```
  index1 = array.length/2 - 1
  index1 = 6/2 - 1
  index1 = 2

  index2 = array.length/2
  index2 = 6/2
  index2 = 3
  ```

Value	8	15	16	18	100	201
index	0	1	index1 = 2	index2 = 3	4	5

WORKED EXAMPLE 33

Given the function definition shown below, what is returned from each of the following function calls?

```
function isOdd(items) {
  return (items%2)===1;
}
function middleValue(array) {
  var middle = 0;
  if (isOdd(array.length)) {
    var index = (array.length-1)/2;
    middle = array[index];
  } else {
    var index1 = array.length/2 - 1;
    var index2 = array.length/2;
    middle = (array[index1]+array[index2])/2;
  }
  return middle;
}
```

a `isOdd(3)`
b `middleValue([3,6,9])`

THINK	WRITE
a 1 The input `items` equals `3`.	`a items = 3`
2 Write the output statement.	`isOdd(3) = (items%2)===1`
3 Substitute `3` for `value`.	`isOdd(3) = (3%2)===1`
4 Substitute `1` for `3%2`, as `2` divides into `3` with `1` remainder.	`isOdd(3) = 1===1`
5 Substitute `true` for `1===1`.	`isOdd(3) = true`
b 1 Write the function call.	`middleValue([3,6,9])`
2 Write the first assignment in the function.	`middle = 0`
3 Write the decision in the function.	`decision = isOdd(array.length)`
4 Substitute `[3,6,9]` for `array`.	`= isOdd([3,6,9].length)`
5 Substitute `3` for `[3,6,9].length`.	`= isOdd(3)`
6 Substitute `true` for `isOdd(3)`.	`= true`
7 `decision` is `true`, so run the statements inside first `{}` block.	
8 The first assignment is in the first `{}` block.	`index = (array.length-1)/2`
9 Substitute `[3,6,9]` for `array`.	`index = ([3,6,9].length-1)/2`
10 Substitute `3` for `[3,6,9].length`.	`index = (3-1)/2`
11 Substitute `1` for `(3-1)/2`.	`index = 1`
12 The second assignment is in the first `{}` block.	`middle = array[index]`
13 Substitute `1` for `index`.	`middle = array[1]`
14 Substitute `6` for `array[1]`.	`middle = 6`
15 Write the return statement.	`middleValue([3,6,9]) = middle`
16 Substitute `6` for middle.	`middleValue([3,6,9]) = 6`

Exercise 18.6 Sorting algorithms

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7–10, 12, 14	Questions: 1–10, 12, 14, 15	Questions: 1–16

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE24 Each of the following programs swap the contents of `x1` and `x2`. True or false?

   a.
   ```
   var x1 = true;
   var x2 = false;
   var a = x2;
   x2 = x1;
   x1 = a;
   ```

   b.
   ```
   var x1 = -12;
   var x2 = 32;
   var temporary = x1;
   x1 = x2;
   x2 = temporary;
   ```

   c.
   ```
   var x1 = "Monday";
   var x2 = "Tuesday";
   var temp = x1;
   x2 = x1;
   x1 = temp;
   ```

2. WE27 What is the output to the console of each the following programs?

   a.
   ```
   function alwaysDoingTheSameThing() {
     return "This text again!!!!";
   }
   console.log(alwaysDoingTheSameThing());
   ```

   b.
   ```
   function outputCubed(x) {
     var squared = x*x;
     console.log(x*squared);
   }
   outputCubed(-5);
   ```

   c.
   ```
   function octagonPerimeter(sideLength) {
     var sides = 8;
     var perimeter = sides * sideLength;
     console.log(perimeter);
   }
   octagonPerimeter(1.25);
   ```

   d.
   ```
   function regularPolygonPerimeter(sides, sideLength)
   {
     var perimeter = sides * sideLength;
     return perimeter;
   }
   console.log(regularPolygonPerimeter(200,0.1));
   ```

3. WE28 Given the swapping function shown below, what is the output to the console of each of the following programs?

   ```
   function swapItems(array, index1, index2) {
     var swap = array[index1];
     array[index1] = array[index2];
     array[index2] = swap;
   }
   ```

   a.
   ```
   var items = [true,false,"true","false",1,2];
   swapItems(items, 4, 5);
   console.log(items[5]);
   ```

b.
```
var months = [
"","January","February","March","April","September","June",
"July","August","May","October","November",
"December"
];
swapItems(months, 9, 5);
console.log(months[9]);
```

4. WE30 Manually simulate running the following programs. For each program, what is the final value stored in the variable `changing`?

a.
```
var changing=0;
var scores = [2,6,73,2];
if (scores.length === 3) {
  changing = scores[0]+scores[1]+scores[2];
} else {
  changing = scores[0];
}
```

b.
```
var changing=0;
var values = [237.2,237.1];
if (values[0]>values[1]) {
  changing = values[0];
} else {
  changing = values[1];
}
```

c.
```
var changing=0;
var values = [1,6,2];
if (values[0]>values[1]) {
  changing = values[0];
} else {
  changing = values[1];
}
if (changing < values[2]) {
  changing = values[2];
}
```

5. WE31 Given the function definitions shown below, what is returned from each of the following function calls?

```
function swapItems(array, i, j) {
  var temporary = array[j];
  array[j] = array[i];
  array[i] = temporary;
}
function scan(array) {
  var hasSwapped = false;
  var i=1;
  while (i < array.length) {
    if (array[i-1]>array[i]) {
      swapItems(array, i-1, i);
      hasSwapped = true;
    }
    i=i+1;
```

```
  }
  return hasSwapped;
}
```
a. `scan([10,200,30])`
b. `scan([1.5,2.4,3.3,4.2,5.1])`
c. `scan([-3.2])`
d. `scan([10,20,30,40,50,70,60])`
e. `scan([2.2,2.1])`
f. `scan([])`

6. WE33 Given the function definition shown below, what is returned from each of the following function calls?

```
function isEven(items) {
  return (items%2)===0;
}
function middleValue(array) {
  var middle = 0;
  if (isEven(array.length)) {
    var i1 = array.length/2 - 1;
    var i2 = array.length/2;
    middle = (array[i1]+array[i2])/2;
  } else {
    var i = (array.length-1)/2;
    middle = array[i];
  }
  return middle;
}
```
a. `isEven(3)`
b. `middleValue([3,6,9])`
c. `isEven([2,3,5,7,11,13,17,19,23,29].length)`
d. `middleValue([2,3,5,7,11,13,17,19,23,29])`

Understanding

7. WE25 Complete the rest of the program to swap the contents of the two variables.

```
var valueA = 1.414213562373095;
var valueB = 3.141592653589793;
```

8. WE26 Complete the rest of the program to swap the contents of the 3rd and 4th item in the array.

```
var array=["Mon","Tue","Thu","Wed","Fri","Sat","Sun"];
```

9. WE29 Manually simulate running the programs. What is the final value stored in the array?

a.
```
var squares=[1,4,9,16,100,25,36,49,64,81];
var i=4;
var j=9;
if (squares[i]>squares[j]) {
  var temporary=squares[i];
  squares[i]=squares[j];
  squares[j]=temporary;
}
```
b.
```
var scores=[1.2,1.3,1.6,1.5];
var a=2;
var b=3;
```

```
  if (scores[a]>scores[b]) {
    var swap=scores[a];
    scores[a]=scores[b];
    scores[b]=swap;
  }
```

10. WE32 Given the function definitions shown below, what is returned from each of the following function calls?

```
function swapItems(array, i1, i2) {
  var temp = array[i1];
  array[i1] = array[i2];
  array[i2] = temp;
}
function bubbleScan(array) {
  var swapDone = false;
  var i=1;
  while (i < array.length) {
    if (array[i-1]>array[i]) {
      swapItems(array, i-1, i);
      swapDone = true;
    }
    i=i+1;
  }
  return swapDone;
}
function sort(array) {
  while(bubbleScan(array)) {
  }
  return array;
}
```

a. `sort([10,200,30])`
b. `sort([4.2,1.5,5.1,2.4,3.3])`
c. `sort([-3.2])`
d. `sort([10,50,70,20,30,40,60])`
e. `sort([2.1,2.2])`
f. `sort([])`

11. Given the function definitions shown below, what is returned from each of the following function calls?

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
function selectionScan(startIndex, array) {
  var minIndex = startIndex;
  var i = startIndex;
  while (i < array.length) {
    if (array[i]<array[minIndex]) {
      minIndex = i;
    }
```

```
    i=i+1;
  }
  swapItems(array, startIndex, minIndex);
  return array;
}
```

a. `selectionScan(0, [1.5,2.4,3.3,4.2,0.1])`
b. `selectionScan(0, [-3.3])`
c. `selectionScan(0, [2.2,2.1])`
d. `selectionScan(0, [-233.2, -22])`
e. `selectionScan(1, [2.1,2.2])`
f. `selectionScan(0, [10,200,3])`
g. `selectionScan(1, [3,200,10])`
h. `selectionScan(2, [3,10,200])`

Reasoning

12. The `selectionScan` function scans for the minimum value on the subset of the `array`. The subset starts from the `startIndex` until the end of the `array`.

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
function selectionScan(startIndex, array) {
  var minIndex = startIndex;
  var i = startIndex;
  while (i < array.length) {
    if (array[i]<array[minIndex]) {
      minIndex = i;
    }
    i=i+1;
  }
  swapItems(array, startIndex, minIndex);
  return array;
}
```

Write a function `selectionSort` that has one input called `array`, where `array` is the list of values to be sorted using the selection sort algorithm in ascending order. The function `selectionSort` can use the previously defined `selectionScan` and `swapItems` functions.

13. Rewrite the following functions so the function `bubbleSort` now sorts the array in descending order.

```
function swapItems(array, index1, index2) {
  var swap = array[index1];
  array[index1] = array[index2];
  array[index2] = swap;
}
function bubbleScan(array) {
  var swapped = false;
  var i=1;
  while (i < array.length) {
    if (array[i-1]>array[i]) {
```

```
      swapItems(array, i-1, i);
      swapped = true;
    }
    i=i+1;
  }
  return swapped;
}
function bubbleSort(array) {
  while(bubbleScan(array)) {
  }
  return array;
}
```

Problem solving

14. Write a function `findMedian` to find and return the median of an unsorted `array`.

15. Write two functions, `findLowerQuartile` and `findUpperQuartile`, to find the 1st quartile and 3rd quartile respectively of a sorted `array`.

16. Write a function `findIQR` to find the interquartile range (IQR) of an unsorted `array`.

Reflection

Can you think of any algorithms that you use in your everyday life?

18.7 The Monte Carlo method

18.7.1 Pseudo-random numbers

- A technique called **Monte Carlo method** was developed to solve problems that are too difficult to analyse directly. The Monte Carlo method approximates the solutions using many random samples. The more random samples are generated, the higher is the accuracy of the final result.



- In order to generate random samples, a random number generator is required. Computers do not generate truly random numbers; instead, they use a mathematical algorithm to generate **pseudo-random** numbers that appear to be random. These algorithms require a **seed** value to get started. One method is to initialise the seed using the computer's internal clock. The following assignment stores the number of milliseconds since midnight on 1 January 1970.

```
var seed = new Date().getTime();
```

The seed is then updated and stored each time the pseudo-random generator is called.

WORKED EXAMPLE 34

Manually simulate running the following program. What are the final values stored in the `array`?

```
var seed = 1;
function random() {
  seed = (5*seed)%7;
  return seed;
}
var array = [random(),random(),random()];
```

THINK	WRITE
1 `random()` is called 3 times starting with the `seed` 1.	
2 Write the 1st `seed` update.	`seed = (5*seed)%7`
3 Substitute `1` for `seed` only on the right-hand side.	`seed = (5*1)%7`
4 Substitute `5` for `(5*1)%7`.	`seed = 5`
5 Write the 2nd `seed` update.	`seed = (5*seed)%7`
6 Substitute `5` for `seed` only on the right-hand side.	`seed = (5*5)%7`
7 Substitute `4` for `(5*5)%7`.	`seed = 4`
8 Write the 3rd `seed` update.	`seed = (5*4)%7`
9 Substitute `4` for `seed` only on the right-hand side.	`seed = (5*4)%7`
10 Substitute `6` for `(5*4)%7`.	`seed = 6`
11 Write the `array` assignment.	`array = [random(),random(), random()]`
12 `random()` returns a different `seed` value each time.	`= [5,4,6]`

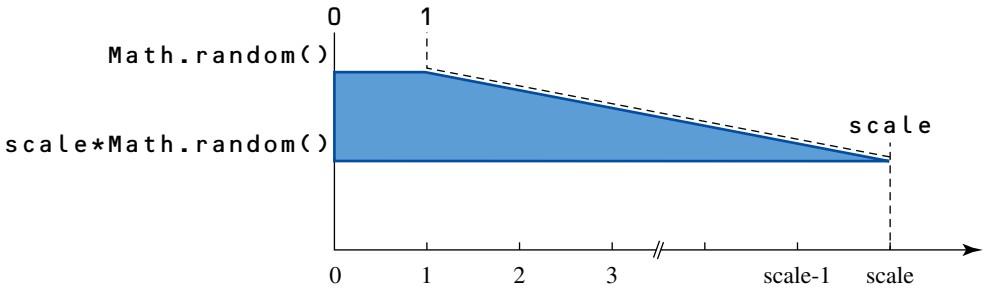
- JavaScript has a function called `Math.random` that generates pseudo-random numbers. Every time `Math.random()` is called, a different random number between `0` (inclusive) and `1` (exclusive) will be returned. The following statement stores a random number in the variable `sample`.

```
var sample = Math.random();
```

18.7.2 Scaled random numbers

- `Math.random()` returns a number between `0` (inclusive) and `1` (exclusive). Often a larger or smaller range of random numbers is required. The value returned from `Math.random()` can be multiplied by a scaling factor to adjust the range. The following statement stores a random number between `0` (inclusive) and `scale` (exclusive) in the variable `sample`.

```
var sample = scale*Math.random();
```



- The shaded region and blue lines in the diagram represent the values transitioning from the random number to the scaled random number. Numbers from `0` to `1` are scaled to numbers from `0` to `scale`. The transition is done in one step.

WORKED EXAMPLE 35

Write an expression to return a random number between 0 and:
a 2 **b** 2.5 **c** −2

THINK

a The `scale` factor is `2`.

b The `scale` factor is `2.5`.

c The `scale` factor is `-2`.

WRITE

a
```
2*Math.random()
```

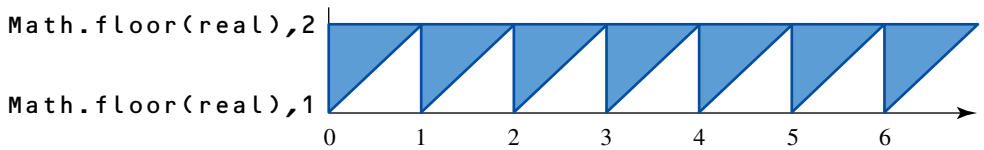
b
```
2.5*Math.random()
```

c
```
-2*Math.random()
```

18.7.3 Floor

- The expression `scale*Math.random()` returns a number between `0` and `scale`, but sometimes random integers are required. JavaScript has a function called `Math.floor` that rounds a `real` number down to the `integer` below. For example, this function will round `1.367` to `1`.

```
var integer = Math.floor(real);
```



- The shaded region and blue lines in the diagram represent the values transitioning from the real number to the integer `Math.floor(real)`. The transition is done in one step. The diagram shows how values slightly larger than `1` are reduced to `1`, values slightly larger than `2` are reduced to `2`, and so on.

WORKED EXAMPLE 36

Evaluate the following JavaScript expressions manually.
a `Math.floor(2.3)` **b** `Math.floor(501)`
c `Math.floor(-6)` **d** `Math.floor(-1.4)`
e `Math.floor(0)`

THINK

a Remove the digits after the decimal place to floor a positive number.

WRITE

a 2.3 = 2

b No change is required to floor an integer.

b `501`

c No change is required to floor an integer.

c `-6`

d Remove the digits after the decimal place and subtract `1` to floor a negative number.

d `-1.4 - 1 = -2`

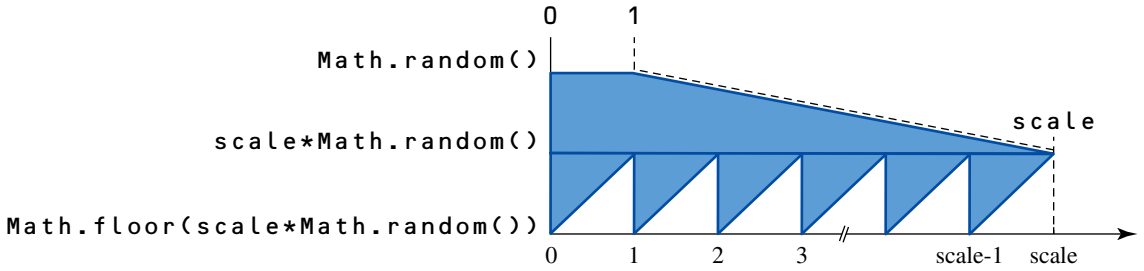
e No change is required to floor an integer.

e `0`

18.7.4 Random integers

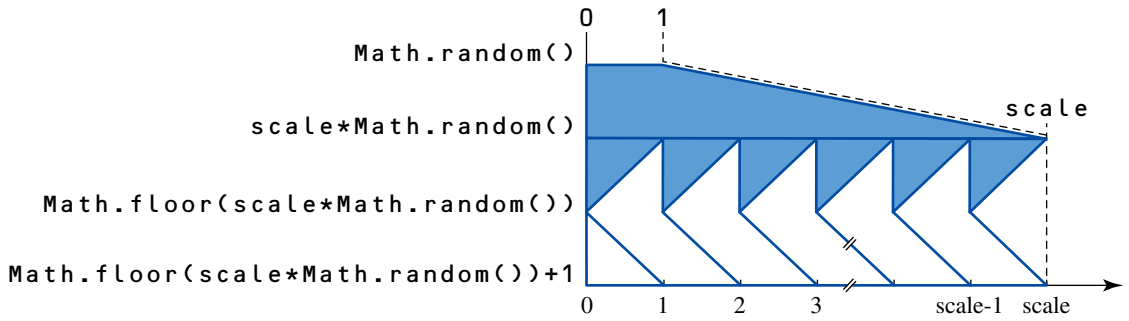
- By combining `Math.random` with `Math.floor`, we can generate a random `integer` between `0` and `scale-1` inclusive.

```
var integer = Math.floor(scale*Math.random());
```



- The shaded region and blue lines in the diagram above represent the values transitioning from `Math.random()` to `scale*Math.random()` to the random `integer`. The transition is done in two steps.
- Adding `1` to `Math.floor(scale*Math.random())` will generate a random `integer` between `1` and `scale` inclusive.

```
var integer = Math.floor(scale*Math.random()) + 1;
```



- The shaded region and blue lines in the diagram above represent the values transitioning in three steps from `Math.random()` to the random `integer`.

WORKED EXAMPLE 37

List all of the possible outcomes of the following expressions as ascending arrays of numbers.

a `Math.floor(6*Math.random())`

b `Math.floor(2*Math.random())`

c `Math.floor(4*Math.random())+1`

d `Math.floor(Math.random())`

THINK	WRITE
a 1 The `scale` factor is `6`.	**a**
2 The expression randomly returns integers between `0` and `6-1`.	`[0,1,2,3,4,5]`
b 1 The `scale` factor is `2`.	**b**
2 The expression randomly returns integers between `0` and `2-1`.	`[0,1]`
c 1 The `scale` factor is `4`.	**c**
2 The expression adds 1, so it randomly returns integers between `1` and `4`.	`[1,2,3,4]`
d 1 The `scale` factor is hidden, but it is `1`.	**d**
2 The expression randomly returns integers between `0` and `1-1`.	`[0]`

18.7.5 Random index and random integer functions

- Generating a random integer between `0` and `N-1` can be wrapped into a function `index`.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
```

- Generating a random integer between `1` and `N` can be wrapped into a function `integer`.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
```

WORKED EXAMPLE 38

Given the following function definitions, list all of the possible outcomes of the expressions below as ascending arrays of numbers.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
```

a `integer(7)` **b** `integer(3)` **c** `index(4)` **d** `index(1)`

THINK	WRITE
a The expression `integer(7)` randomly returns integers between `1` and `7`.	**a** `[1,2,3,4,5,6,7]`
b The expression `integer(3)` randomly returns integers between `1` and `3`.	**b** `[1,2,3]`
c The expression `index(4)` randomly returns integers between `0` and `4-1`.	**c** `[0,1,2,3]`
d The expression `index(1)` randomly returns integers between `0` and `1-1`.	**d** `[0]`

18.7.6 Random selection function and random samples

- Randomly selecting an `item` in an `array` can be wrapped into a function `select`. First an index `i` is randomly selected between `0` and `array.length-1`. Then the `item` is selected from the `array` using the random index `i`.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function select(array) {
  var i = index(array.length);
  var item = array[i];
  return item;
}
```

- The `generate` function returns a `random` sample. The details of how the `random` sample is generated depend on the problem. If more than one random value is required, the `random` sample can become an array of values.

```
function generate() {
  ...
  return random;
}
```

WORKED EXAMPLE 39

Given the following function definitions, define a `generate` function to return a `random` sample for each of the problems shown below.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
```

```
function integer(N) {
  return index(N)+1;
}
function select(array) {
  var i = index(array.length);
  var item = array[i];
  return item;
}
```

a Select one of the strings "1st", "2nd" or "3rd".
b Show the results from throwing two fair six-sided dice.
c Show an index between 0 and 99.
d Show an hour integer between 1 and 12 and a minute integer between 0 and 59.

THINK	WRITE
a 1 Define the function **generate**.	**a** `function generate() {`
2 Store the **list** of items.	`var list = ["1st","2nd","3rd"];`
3 **select** a **random** item.	`var random = select(list);`
4 Return the **random** item.	`return random;` `}`
b 1 Define the function **generate**.	**b** `function generate() {`
2 Randomly generate the 1st die.	`var die1 = integer(6);`
3 Randomly generate the 2nd die.	`var die2 = integer(6);`
4 Create the **random** sample array.	`var random = [die1,die2];`
5 Return the **random** item.	`return random;` `}`
c 1 Define the function **generate**.	**c** `function generate() {`
2 Randomly generate an **index** between 0 and 100−1.	`var random = index(100);`
3 Return the **random** item.	`return random;` `}`
d 1 Define the function **generate**.	**d** `function generate() {`
2 Randomly generate the 1st integer between 1 and 12.	`var hour = integer(12);`
3 Randomly generate the 2nd integer between 0 and 59.	`var minute = index(60);`
4 Create the **random** sample array.	`var random = [hour,minute];`
5 Return the **random** item.	`return random;` `}`

18.7.7 Testing a sample

- For the Monte Carlo method, each random sample needs to be tested against the event. The **isEvent** function tests the sample and returns **true** if the **test** sample is the event; otherwise, it returns **false**. The details of how the **test** sample is checked against the event depend on the problem.

```
function isEvent(test) {
  ...
  return event;
}
```

WORKED EXAMPLE 40

For each of the following situations, define the `isEvent` function to return `true` if the `test` sample is the event, and otherwise return `false`.
a The three items in an array `test` sample add to 20.
b The sample `test` is even.

THINK	WRITE
a 1 Define the function `isEvent`.	**a** `function isEvent(test) {`
2 Add the three items in the array.	`var t = test[0]+test[1]+test[2];`
3 Compare the sum with `20`.	`var event = t === 20;`
4 Return the result of the test `event`.	`return event;` `}`
b 1 Define the function `isEvent`.	**b** `function isEvent(test) {`
2 Check if `test` is even.	`var event = test%2 === 0;`
3 Return the result of the test `event`.	`return event;` `}`

18.7.8 Approximating probability

- For some events, the theoretical probability can be difficult to calculate. The Monte Carlo method can be used to simulate the event with many random samples. If a very large number of samples are used, the relative frequency of an event will approach the theoretical probability of the event.
- The follow program skeleton can be used to `estimate` the probability using a number of random `samples`.
- The `generate` function return a `random` sample. The details of how the `random` sample is generated depend on the problem.
- The `isEvent` function tests the sample and returns `true` if the `test` sample is the event; otherwise, it returns `false`. The details of how the `test` sample is checked against the event depend on the problem.

```
function generate() {
  ...
  return random;
}
function isEvent(test) {
  ...
  return event;
}
function estimate(samples) {
  var events = 0;
  var i = 1;
```

```
  while(i <= samples) {
    if (isEvent(generate())) {
      events = events + 1;
    }
    i = i + 1;
  }
  return events/samples;
}
var probability = estimate(totalSamples);
```

- Once the function is defined, it can be called with the statement `estimate(totalSamples)`.

WORKED EXAMPLE 41

Create a program to estimate the probability that the results of three fair 6-sided dice sum to 10 using a million samples. Output the estimated probability to the console.

THINK

1 Selecting a random integer between `1` and `6` will be useful for simulating each die. Define the random generating `integer` and `index` functions.

WRITE

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
```

2 Define the `generate` function to return an array of three integers, `d1`, `d2` and `d3`, each independently randomly selected between `1` and `6`.

```
function generate() {
  var d1 = integer(6);
  var d2 = integer(6);
  var d3 = integer(6);
  return [d1, d2, d3];
}
```

3 Define the `isEvent` function to test if the `sum` of three integers in an array `d` equals `10`.

```
function isEvent(d) {
  var sum = d[0]+d[1]+d[2];
  var event = sum === 10;
  return event;
}
```

4 Define the function to `estimate` the probability using a number of random `samples`. The function `estimate` calls the already defined functions `generate` and `isEvent`.

```
function estimate(samples) {
  var events = 0;
  var i = 1;
  while(i <= samples) {
    if (isEvent(generate())) {
      events = events + 1;
    }
    i = i + 1;
  }
  return events/samples;
}
```

5 Call the function `estimate` with `1000000` samples and store the result in `probability`.	`var probability=estimate (1000000);`
6 Output `probability` to the console.	`console.log(probability);`

Exercise 18.7 Monte Carlo method

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9, 11, 13	Questions: 1–13	Questions: 1–14

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE34 Manually simulate running the following program. What are the final values stored in the `array`?

```
var seed = 1;
function random() {
  seed = (13*seed)%29;
  return seed;
}
var array = [random(),random(),random(),random(),random()];
```

2. WE36 Evaluate the following JavaScript expressions manually.

a. `Math.floor(-10.42342)`
b. `Math.floor(-6)`
c. `Math.floor(-1.422)`
d. `Math.floor(1.45)`
e. `Math.floor(267)`
f. `Math.floor(999)`
g. `Math.floor(-612)`
h. `Math.floor(0)`
i. `Math.floor(3.45)`

3. WE37 List all of the possible outcomes of the following expressions as ascending arrays of numbers.

a. `Math.floor(8*Math.random())`
b. `Math.floor(9*Math.random())+1`
c. `Math.floor(13*Math.random())`
d. `Math.floor(4*Math.random())+1`

4. WE38 Given the following function definitions, list all of the possible outcomes of the expressions below as ascending arrays of numbers.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
```

a. `index(2)` **b.** `index(11)` **c.** `integer(6)`
d. `index(5)` **e.** `index(1)` **f.** `integer(0)`
g. `integer(10)` **h.** `integer(1)` **i.** `index(0)`

5. Given the following function definitions, list all of the possible outcomes of the expressions below as ascending arrays of numbers.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
```

a. `index(11)+10` **b.** `index(12)*10`
c. `integer(9)*11` **d.** `integer(5)*100 + 500`

Understanding

6. WE35 Write an expression to return a random number between `0` and:
a. `100` **b.** `0.001` **c.** `-0.1252`
d. `3.14` **e.** `-99` **f.** `10`

7. JavaScript has a function `Math.ceil(real)` to round a `real` number up to the integer above. Evaluate the following JavaScript expressions manually.
a. `Math.ceil(-120.42)` **b.** `Math.ceil(-16)`
c. `Math.ceil(16.4222)` **d.** `Math.ceil(9.4335)`
e. `Math.floor(57.4)` **f.** `Math.ceil(444)`
g. `Math.floor(-62.3)` **h.** `Math.ceil(0)`
i. `Math.floor(-16.4222)` **j.** `Math.floor(9.4335)`

8. WE39 Given the following function definitions, define a `generate` function to return a `random` sample for each of the problems shown below.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
function select(array) {
  var i = index(array.length);
  var item = array[i];
  return item;
}
```

a. A day integer between `1` and `365`
b. One of the strings `"January"`, `"February"`, `"March"`, `"April"`, `"May"`, `"June"`, `"July"`, `"August"`, `"September"`, `"October"`, `"November"` and `"December"`
c. The results from throwing one fair twenty-sided die
d. One of the strings `"Monday"`, `"Tuesday"`, `"Wednesday"`, `"Thursday"` or `"Friday"`
e. The results from throwing three fair six-sided dice
f. An index between `0` and `9`
g. An integer between `100` and `200`

h. Two independent real numbers between 0 and 1
i. One real number between 0 and 2
j. One real number between −1 and 1
k. Two independent real numbers between −1 and 1

9. WE40 For each of the following situations, define the `isEvent` function to return `true` if the `test` sample is the event, and otherwise return `false`.
a. The four items in an array `test` add to less than 10.
b. The two items in an array `test` have a difference of 5.
c. The sample `test` is odd.
d. The sum of the squares of two items in an array `test` is less than `1`.
e. At least one of the two items in an array `test` is `6`.

10. a. Define a `generate` function to return an array of two independent real numbers between `-1` and `1`.
b. Define the `isEvent` function where the input `test` is an array of two numbers. The function `isEvent` returns `true` if `test[0]*test[0]+test[1]*test[1]` is less than `1`; otherwise, it returns `false`.

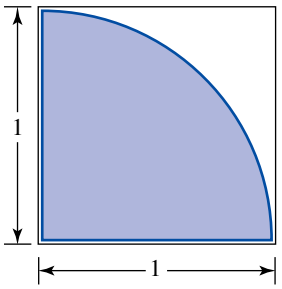
Reasoning

11. WE41 Create a program to estimate the `probability` that four fair 6-sided dice sum to 12 using a thousand samples. Output the estimated probability to the console.

12. Create a program to estimate the probability that five fair 6-sided dice form a 'straight'. A 'straight' is where the five dice can be arranged to form either `[1,2,3,4,5]` or `[2,3,4,5,6]`. Use a million samples and output the estimated probability to the console.

Problem solving

13. Create a Monte Carlo program to estimate the probability that any random point in a unit square is also inside the enclosed quarter circle that has a unit radius.



14. Create a Monte Carlo program to estimate the value of π.

Reflection

Think of some ways that random numbers can be generated if you do not have access to a computer.

18.8 Review

18.8.1 Review questions

Fluency

1. Consider the following values and expressions.
i. Is the value or expression result a number, a string or Boolean?
ii. Apply `typeof expression` to each value or expression. What is the result of `typeof expression`?

a. `-0.125` **b.** `true` **c.** `"more strings"`
d. `"FALSE"` **e.** `"False"` **f.** `"True"`
g. `100` **h.** `false` **i.** `124/4`
j. `5*5*5` **k.** `949%25`

2. Each of the following is a valid variable name. True or false?

a. `someRandomName` **b.** `41_forty_one`
c. `and&&this` **d.** `42_meaning_of_life`
e. `number_22` **f.** `remainder%this`
g. `negative_-10` **h.** `function`

3. Manually simulate running the following programs. For each program:

i. what is the final type of `variable`?
ii. what is the final value stored in `variable`?

a.
```
var variable = (1*2*3*4)/(1+1+1);
variable = true;
```

b.
```
var variable = 2;
var x = false;
variable = x;
```

c.
```
var x = 1272.2/5;
var t = 0;
var variable = -(15%4)/(230-23332*7.2);
variable = variable*x*t/23712.3;
```

d.
```
var variable = "Am I a string, a Boolean or a number?";
variable = true;
variable = 12.23;
```

e.
```
var variable = "I have changed a few times.";
variable = 2.34;
variable = true;
variable = [true,false];
variable = "String";
```

4. What is the output to the console after each of the following statements runs?

a.
```
console.log("Testing");
```

b.
```
console.log("Results");
```

c.
```
console.log(11%(121%3));
```

d.
```
console.log(false);
```

5. Classify each of the following as an array, number, string or Boolean.

a. `[3/20,true,false,"string",["not","empty"]]`
b. `"Violet"`
c. `true`
d. `[]`
e. `["One string"]`
f. `"What type is this?"`
g. `3+0.1+0.04+0.001+0.006`
h. `[-2.3,2.3,33]`
i. `[true,false]`
j. `37.3+1.232`
k. `["A","B","C"]`
l. `["Five","strings","in","this","array"]`

6. Evaluate the following JavaScript expressions manually.

a. `Array.isArray("[a,invalid,array]")`
b. `[1,10,100,1000,10000].length`
c. `Array.isArray([])`
d. `[6,[5,64,3],4,[4,5]].length`
e. `Array.isArray(true)`
f. `Array.isArray([1,2,6,24,120])`
g. `Array.isArray(false)`
h. `Array.isArray([81,"672","blah",[122,71,41]])`
i. `[2,3,4,[5,6]].length`
j. `Array.isArray("[true,false,true]")`
k. `[3.4223,"v",false,[8,2,8,2].length,"4"].length`
l. `Array.isArray(["[4,5,6,7]"])`
m. `Array.isArray([false,"false","False","FALSE"])`

7. What is the JavaScript index of each of the following values in the array shown below?

```
["a","b","c","d","e","f","g","h","i","j","k","l","m"]
```

a. `"m"` **b.** `"a"` **c.** `"g"` **d.** `"i"` **e.** `"c"`

8. Given the array assignment shown below, evaluate the following JavaScript expressions manually.

```
var items = [8,4,32,5,"False",1.45,"results",[1,2,3],"Eight"];
```

a. `items[3]` **b.** `items[3/3]`
c. `items[items[0]]` **d.** `items[0]`

9. Evaluate the following JavaScript expressions manually.

a. `-34<=-34` **b.** `1.41>-1.12` **c.** `true&&false`
d. `!true` **e.** `false&&false` **f.** `123===132`
g. `143.3===143.30` **h.** `-49>=-49` **i.** `true||true`
j. `40.08<40.1` **k.** `71.99<=72.01` **l.** `true&&true`
m. `false||true` **n.** `false||false` **o.** `4.500===4.5000`
p. `2.4221!==242.21` **q.** `false&&true` **r.** `true||false`

10. Manually simulate running the following programs. For each program, what is the final value stored in `t`?

a.
```
var t = 81;
if (t===9*9) {
  t = t/3;
}
```

b.
```
var t = false||true;
if (t) {
  t = 817;
}
if (t%2===1) {
  t = "false";
}
```

11. Each of the following causes an infinite loop. True or false?

a.
```
var double=1;
while(double===1025) {
  console.log(double);
  double=2*double;
}
```

b.
```
var j = 0;
while(false) {
  j = j + 1;
  console.log(i);
}
```

c.
```
while(true) {
  console.log("Go for
  ever!!!!");
}
```

d.
```
var triple=1;
while(triple!==243) {
  console.log(triple);
  triple=3*triple;
}
```

12. For each the following programs:

i. how many times is the loop executed?

ii. what is the final value stored in the variable `w`?

a.
```
var w = 200;
var p=1;
while (p<=9) {
  w=w-p;
  p=p+1;
}
```

b.
```
var w=0;
var i=1;
while (i<=13) {
  w=w+i;
  i=i+1;
}
```

c.
```
var w=0;
var z=1;
  while (z<=10) {
  console.log(5*z);
  z=z+1;
  w=w+2;
}
```

d.
```
var w=2048;
var i=1;
while (i<=10) {
  w=w/2;
  i=i+1;
}
```

e.
```
var y=1;
var w=1000;
while (y<=5) {
  w=w-y;
  y=y+1;
}
```

f.
```
var w = "Some string";
while(false) {
  w = "Another string";
}
```

13. How many times is the loop code executed in each of the following programs?

a.
```
var number = 0;
var digits = [0,1,2,3,4,5,6,7,8,9];
var i=0;
while (i < digits.length) {
  var item = digits[i];
  number = number*10 + item;
  i=i+1;
}
console.log(number);
```

b.
```
var weekend = ["Saturday","Sunday"];
var k=0;
while (k < weekend.length) {
  var day = weekend[k];
  console.log(day);
  k=k+1;
}
```

14. What is the output to the console generated by the program below?

```
var number = 0;
var digits = [0,1,2,3,4,5,6,7,8,9];
var i=0;
while (i < digits.length) {
  var item = digits[i];
  number = number*10 + item;
  i=i+1;
}
console.log(number);
```

15. Each of the following programs swaps the contents of `var1` and `var2`. True or false?

a.
```
var var1 = true;
var var2 = false;
var temporary = var2;
var2 = var1;
var1 = temporary;
```

b.
```
var var1 = 3.14;
var var2 = 3.1416;
var temp = var1;
var1 = var2;
var1 = temp;
```

c.
```
var var1 = "April";
var var2 = "May";
var t = var1;
var2 = var1;
var1 = t;
```

16. What is the output to the console of each of the following programs?

a.
```
function theSame() {
  return "Always this!!!";
}
console.log(theSame());
```

b.
```
function outputFloor(x) {
  var floor = Math.floor(x);
  console.log(floor);
}
outputFloor(2.34);
```

c.
```
function circleArea(radius) {
  var pi = 3.14159;
  var area = pi * radius * radius;
  console.log(area);
}
circleArea(10);
```

d.
```
function circumference(radius) {
  var pi = 3.14159;
  var perimeter = 2 * pi * radius;
  console.log(perimeter);
}
circumference(10);
```

17. Given the swapping function shown below, what is the output to the console of each of the following programs?

```
function swap(array, i, j) {
  var temporary = array[i];
  array[i] = array[j];
  array[j] = temporary;
}
```

a.
```
var array = [5,1,2,3,4,0];
swap(array, 0, 5);
console.log(array[5]);
```

b.
```
var w = ["Monday","Wednesday","Friday","Thursday","Tuesday"];
var weekdays = w;
swap(weekdays, 1, 2);
console.log(weekdays[1]);
swap(weekdays, 4, 1);
```

18. Manually simulate running the following program. What is the final value stored in the variable `final`?

```
var final=0;
var scores = [3,4];
if (scores.length === 3) {
  final = scores[0]+scores[1]+scores[2];
}
if (scores.length === 2) {
  final = scores[0]+scores[1];
}
if (scores.length === 1) {
  final = scores[0];
}
```

19. Given the function definitions shown below, what is returned from each of the following function calls?

```
function swapItems(array, i, j) {
  var temporary = array[j];
  array[j] = array[i];
  array[i] = temporary;
}
function bubbleScan(array) {
  var swapped = false;
  var k=1;
  while (k < array.length) {
    if (array[k-1]>array[k]) {
    swapItems(array, k-1, k);
    swapped = true;
    }
  k=k+1;
  }
  return swapped;
}
```

a. `bubbleScan([7,4,5,2,3,1])`
b. `bubbleScan([12.2,12.1,12.3])`
c. `bubbleScan([1,2,2,3])`
d. `bubbleScan([10,20,30,40,50,60])`
e. `bubbleScan([10,20,15])`
f. `bubbleScan([])`

20. Manually simulate running the following program. What are the final values stored in the `array`?

```
var seed = 1;
function random() {
  seed = (23*seed)%31;
  return seed;
}
var array = [random(),random(),random(),random()];
```

21. Evaluate the following JavaScript expressions manually.

a. `Math.floor(44)`
b. `Math.floor(-4.442)`
c. `Math.floor(-6)`
d. `Math.floor(13.415)`
e. `Math.floor(0)`
f. `Math.floor(-3.45)`
g. `Math.floor(3.45)`

22. Given the function definitions shown below, list all the possible outcomes of each of the following expressions as ascending arrays of numbers.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
```

a. `index(0)`
b. `integer(0)`
c. `index(11)`
d. `integer(5)`
e. `index(11)+20`
f. `integer(9)`
g. `integer(4)*10 + 60`
h. `integer(1)`
i. `integer(4)*12`
j. `Math.floor(7*Math.random())+1`
k. `index(2)`
l. `index(8)`
m. `index(8)*100`
n. `Math.floor(6*Math.random())`
o. `index(10)`

Problem solving

23. Create a program to estimate the probability that the results of five fair 6-sided dice form exactly four of a kind. Use 100 000 samples and output the estimated probability to the console.

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Complete this digital doc: How to write your own program (doc-18769)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algorithm
array
Booleans
bubble sort
console
index
infinite loop
median
Monte Carlo method
numbers
pseudo-random
seed
selection sort
strings
variables

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Answers

Topic 18 Programming

Exercise 18.2 Programs

1. a. i. Number **ii.** `"number"`
b. i. Boolean **ii.** `"boolean"`
c. i. String **ii.** `"string"`
d. i. Number **ii.** `"number"`
e. i. Boolean **ii.** `"boolean"`
f. i. String **ii.** `"string"`
g. i. Number **ii.** `"number"`
h. i. String **ii.** `"string"`
i. i. String **ii.** `"string"`
j. i. String **ii.** `"string"`
k. i. Number **ii.** `"number"`
l. i. String **ii.** `"string"`
m. i. String **ii.** `"string"`

2. a. False **b.** True **c.** False **d.** False **e.** False **f.** True
g. False **h.** True **i.** False **j.** False

3. a. i. Number **ii.** `12`
b. i. Number **ii.** `145`
c. i. Boolean **ii.** `false`
d. i. Number **ii.** `1`
e. i. Boolean **ii.** `true`
f. i. Boolean **ii.** `true`

4. a. `A simple string` **b.** `Some data` **c.** `1`
d. `true` **e.** `True` **f.** `12`

5. a. i. Number **ii.** `"number"`
b. i. String **ii.** `"string"`
c. i. Number **ii.** `"number"`
d. i. Number **ii.** `"number"`

6. a. `(12-6)/(5-2)` **b.** `16%5` **c.** `12+12-(-13-14-1)`
d. `12/(4*3)` **e.** `12+(6-4-1)` **f.** `112*2.5`
g. `160/(4*4*8/2)` **h.** `111%14` **i.** `24*2.5*2.5`

7. a.
```
var computerLanguage = "JavaScript";
```
b.
```
var event = true;
```
c.
```
var finished = false;
```
d.
```
var megabyte = 1024*1024;
```
e.
```
var subject = "Mathematics";
```
f.
```
var minutes = 60*24*7;
```
g.
```
var million = 1000000;
```

8. a.
```
var lotsOfFactors = 2*3*5*7*11*13; // lotsOfFactors=30030
```
b.
```
var e = 2+7/10+1/100+8/1000+3/10000; // e = 2.7183
```
c.
```
var product = 1*2*3*4; // product = 24
```
d.
```
var yourAge = 225/15; // yourAge = 15
```

9. a. Number **b.** `6`

10. a. Number **b.** `135` **c.** Number **d.** `6` **e.** String **f.** `"xyz"`

11.
```
var inLitres = 2;
var inMillilitres = 1000*inLitres;
```

12.
```
var crossSectionalArea = 100;
var height = 12;
var prismVolume = height*crossSectionalArea;
```

13. There are many possible programs to solve this problem; an example is shown. The value of `area` should be 39.27 (correct to 2 decimal places).
```
var radius = 10;
var angle = 45;
var pi = 3.141592653589793;
var area = pi*radius*radius * angle/360;
```

14. There are many possible programs to solve this problem; an example is shown. The value of `perimeter` should be 77.79 (correct to 2 decimal places).
```
var radius = 19;
var angle = 120;
var pi = 3.141592653589793;
var arcLength = 2*pi*radius * angle/360;
var perimeter = arcLength + radius + radius;
```

Exercise 18.3 Arrays

1. **a.** Number **b.** Array **c.** Array **d.** Boolean **e.** Number **f.** Array
g. Array **h.** Array **i.** String **j.** Array **k.** Array **l.** String

2. **a.** `true` **b.** `true` **c.** `true` **d.** `false` **e.** `true` **f.** `true`
g. `false` **h.** `false` **i.** `false` **j.** `true` **k.** `false`

3. **a.** `4` **b.** `4` **c.** `4` **d.** `3`

4. **a.** `6` **b.** `2` **c.** `7` **d.** `13` **e.** `11`

5. **a.** `4` **b.** `true` **c.** `2.3` **d.** `"some string"`

6. **a.** `2` **b.** `false` **c.** `true`
d. `"boolean"` **e.** `false` **f.** `"boolean"`
g. `false` **h.** `[true,2]` **i.** `4`

7. **a.** Array **b.** `[3,9,27]`

8. **a.** Array **b.** `[3,1,4]`

9. **a.** Number **b.** `5.2`

10. **a.** Number **b.** `120`

11. There are many possible solutions to this problem; an example is shown. The value of `primorial` should be `30030`.
```
var primes = [2,3,5,7,11,13];
var primorial = 1;
primorial = primorial*primes[0];
primorial = primorial*primes[1];
primorial = primorial*primes[2];
primorial = primorial*primes[3];
primorial = primorial*primes[4];
primorial = primorial*primes[5];
```

12. There are many possible solutions to this problem; an example is shown. The value of `total` should be `77`.
```
var primes = [2,3,5,7,11,13,17,19];
var total = 0;
total = total + primes[0];
total = total + primes[1];
total = total + primes[2];
total = total + primes[3];
total = total + primes[4];
total = total + primes[5];
total = total + primes[6];
total = total + primes[7];
```

13. There are many solutions to this problem; an example is shown. The 6 arrays could be in any order and still be correct.
```
var combinations = [
[1,2,3],
[1,3,2],
[2,1,3],
```

```
[2,3,1],
[3,1,2],
[3,2,1]
];
```

14. There are many solutions to this problem; an example is shown. The 8 arrays could be in any order and still be correct.

```
var sampleSpace = [
[false,false,false],
[false,false,true],
[false,true,false],
[false,true,true],
[true,false,false],
[true,false,true],
[true,true,false],
[true,true,true]
];
```

Exercise 18.4 Loops

1. a. `true` **b.** `true` **c.** `true` **d.** `false` **e.** `true` **f.** `true`
g. `true` **h.** `false` **i.** `true` **j.** `true` **k.** `true` **l.** `true`
m. `false` **n.** `true`

2. a. `false` **b.** `false` **c.** `true` **d.** `false`

3. a. `true` **b.** `true` **c.** `false` **d.** `true`

4. a. `23409` **b.** `100`

5. a. False **b.** True

6. a. 99 **b.** 100 **c.** 5 **d.** 0 **e.** 900

7. a. i. 14 **ii.** `105`
b. i. 9 **ii.** `362880`
c. i. 5 **ii.** `0`

8. a. `10` **b.** `6` **c.** `1`

9. a. False **b.** True

10. a. 1000 **b.** 101 **c.** 100 **d.** 50

11. `2000`

12. `385`

13. There are many programs to solve this problem; an example is shown. The value of `factorial_10` should be `3628800`.

```
var product = 1;
var a=1;
while (a<=10) {
 product = product*a;
 a=a+1;
}
var factorial_10 = product;
```

14. There are many programs to solve this problem; an example is shown. The value of `total` should be `500000500000`.

```
var sum = 0;
var a=1;
while (a<=1000000) {
 sum = sum + a;
 a=a+1;
}
var total = sum;
```

15. There are many programs to solve this problem; an example is shown. The value of `product` should be `41992`.

```
var value1 = 362;
var value2 = 232;
var product = 0;
var found = false;
while (!found) {
 product = product + 1;
 if ((product%value1 === 0) && (product%value2 === 0)) {
```

```
    found = true;
   }
  }
  console.log(product);
```

16. There are many programs to solve this problem; an example is shown. The value of `product` should be `209960`.

```
var array = [362,232,4,8,5];
var product = 0;
var found = false;
while (!found) {
 product = product + 1;
 found = true;
 var i = 0;
 while (i < array.length) {
  if (product%array[i] !== 0) {
   found = false;
  }
  i = i + 1;
 }
}
console.log(product);
```

Exercise 18.5 Set structures

1. a. `["Monday","Tuesday","Wednesday", "Thursday","Friday", "Saturday","Sunday"]`

b. `["January","February","March","April","May","June","July","August", "September","October","November","December"]`

c. `["a","b","c","d","e","f","g","h","i","j","k","l","m","n","o","p","q","r","s", "t","u","v","w","x","y","z"]`

d. `[31,29,31,30,31,30,31,31,30,31,30,31]`

2. a. `12` **b.** `7` **c.** `10`

3. `2.718281828459`

4. `6469693230`

5. `5`

6. `true`

7. `6469693230`

8. a. `6`

b. The purpose of the program is to scan the array and find the largest value. The largest value is then sent as output to the console.

9. a. `12`

b. The purpose of the program is to scan the array and find the largest and smallest value. The difference of the largest and smallest value is computed to find the range of the values. The range is then sent as output to the console.

10. `4.1633`

11. There are many possible programs to solve this problem; an example is shown. The value of `mean` should be `10`.

```
var total = 0;
var digits = [10,17,6,3,26,23,1,23,4,5];
var k=0;
while (k < digits.length) {
 var item = digits[k];
 total = total + item;
 k=k+1;
}
var mean = total / digits.length;
console.log(mean);
```

12. There are many possible programs to solve this problem; an example is shown. The value of `count` should be `168`. There are more efficient algorithms to count primes; this is a simple version that uses only the concepts shown in this topic.

```
var count = 0;
var i = 2;
while (i <= 1000) {
 var isPrime = true;
```

```
  var j = 2;
  while ((j < i) && isPrime) {
    var multiple = i%j === 0;
    isPrime = isPrime && !multiple;
    j = j+1;
  }
  if (isPrime) {
    count = count + 1;
  }
  i = i+1;
}
console.log(count);
```

13. There are many possible programs to solve this problem; an example is shown. The value of `prime` should be `7919`. There are more efficient algorithms to count primes; this is a simple version that uses only the concepts shown in this topic.

```
var count = 0;
var prime = 1;
while (count !== 1000) {
 prime = prime + 1;
 var isPrime = true;
 var j = 2;
 while ((j < prime) && isPrime) {
  var multiple = prime%j === 0;
  isPrime = isPrime && !multiple;
  j = j+1;
 }
 if (isPrime) {
  count = count + 1;
 }
}
console.log(prime);
```

14. The program should output the following list in any order.

```
a=3  b=4  c=5
a=5  b=12 c=13
a=6  b=8  c=10
a=7  b=24 c=25
a=8  b=15 c=17
a=9  b=12 c=15
a=10 b=24 c=26
a=12 b=16 c=20
a=15 b=20 c=25
a=16 b=30 c=34
a=18 b=24 c=30
a=20 b=21 c=29
a=21 b=28 c=35
```

There are many possible programs to solve this problem; an example is shown. There are more efficient algorithms to find solutions to the equation; this is a simple version that uses only the concepts shown in this topic.

```
var a = 1;
while (a <= 30) {
 var b = a;
 while (b <= 30) {
  var c = b;
  while (c <= a+b) {
   if (a*a+b*b===c*c) {
    console.log("a="+a+" b="+b+" c="+c);
   }
   c = c+1;
  }
  b = b+1;
 }
 a = a+1;
}
```

Exercise 18.6 Sorting algorithms

1. a. True **b.** True **c.** False

2. a. `This text again!!!!`

b. `-125` **c.** `10` **d.** `20`

3. a. `1` **b.** `September`

4. a. `2` **b.** `237.2` **c.** `6`

5. a. `true` **b.** `false` **c.** `false` **d.** `true` **e.** `true` **f.** `false`

6. a. `false` **b.** `6` **c.** `true` **d.** `12`

7.
```
var valueA = 1.414213562373095;
var valueB = 3.141592653589793;
var swap = valueA;
valueA = valueB;
valueB = swap;
```

8.
```
var array = ["Mon","Tue","Thu","Wed","Fri","Sat","Sun"];
var swap = array[2];
array[2] = array[3];
array[3] = swap;
```

9. a. `[1, 4, 9, 16, 81, 25, 36, 49, 64, 100]`

b. `[1.2, 1.3, 1.5, 1.6]`

10. a. `[10,30,200]` **b.** `[1.5,2.4,3.3,4.2,5.1]` **c.** `[-3.2]`

d. `[10,20,30,40,50,60,70]` **e.** `[2.1,2.2]` **f.** `[]`

11. a. `[0.1,2.4,3.3,4.2,1.5]` **b.** `[-3.3]` **c.** `[2.1,2.2]`

d. `[-233.2,-22]` **e.** `[2.1,2.2]` **f.** `[3,200,10]`

g. `[3,10,200]` **h.** `[3,10,200]`

12. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions.
```
function selectionSort(array) {
 var j = 0;
 while (j < array.length) {
  selectionScan(j, array);
  j = j+1;
 }
}
```

13. There are many possible programs to solve this problem; an example is shown. This is a simple change from the original version that only swaps the decision.
`array[i-1]>array[i]` to `array[i-1]<array[i]`.
```
function swapItems(array, index1, index2) {
 var swap = array[index1];
 array[index1] = array[index2];
 array[index2] = swap;
}
function bubbleScan(array) {
 var swapped = false;
 var i=1;
 while (i < array.length) {
  if (array[i-1]<array[i]) {
   swapItems(array, i-1, i);
   swapped = true;
  }
  i=i+1;
  }
  return swapped;
}
function bubbleSort(array) {
 while(bubbleScan(array)) {
 }
 return array;
}
```

14. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions.

```
function swapItems(array, index1, index2) {
 var swap = array[index1];
 array[index1] = array[index2];
 array[index2] = swap;
}
function bubbleScan(array) {
 var swapped = false;
 var i=1;
 while (i < array.length) {
  if (array[i-1]>array[i]) {
   swapItems(array, i-1, i);
   swapped = true;
  }
  i=i+1;
 }
 return swapped;
}
function bubbleSort(array) {
 while(bubbleScan(array)) {
 }
 return array;
}
function isOdd(items) {
 return (items%2)===1;
}
function middleValue(array) {
 var middle = 0;
 if (isOdd(array.length)) {
  var index = (array.length-1)/2;
  middle = array[index];
 } else {
  var index1 = array.length/2 - 1;
  var index2 = array.length/2;
  middle = (array[index1]+array[index2])/2;
  }
  return middle;
}
function findMedian(array) {
 bubbleSort(array);
 median = middleValue(array);
 return median;
}
```

15. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions.

```
function isOdd(items) {
 return (items%2)===1;
}
function middle(array, min, max) {
 var range = max - min;
 var middle = 0;
 if (isOdd(range)) {
  var index = min + (range-1)/2;
  middle = array[index];
 } else {
  var index1 = min + range/2 - 1;
  var index2 = min + range/2;
   middle = (array[index1]+array[index2])/2;
 }
 return middle;
}
```

```
function findLowerQuartile(array) {
 var half = 0;
 if (isOdd(array.length)) {
  half = (array.length-1)/2;
 } else {
  half = array.length/2;
 }
 var Q1 = middle(array, 0, half);
 return Q1;
}
function findUpperQuartile(array) {
 var half = 0;
 if (isOdd(array.length)) {
  half = (array.length-1)/2;
 } else {
  half = array.length/2;
 }
 var Q3 = middle(array, array.length-half, array.length);
 return Q3;
}
```

16. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions.

```
function swapItems(array, index1, index2) {
 var swap = array[index1];
 array[index1] = array[index2];
 array[index2] = swap;
}
function bubbleScan(array) {
 var swapped = false;
 var i=1;
 while (i < array.length) {
  if (array[i-1]>array[i]) {
   swapItems(array, i-1, i);
   swapped = true;
  }
  i=i+1;
 }
 return swapped;
}
function bubbleSort(array) {
 while(bubbleScan(array)) {
 }
 return array;
}
function isOdd(items) {
 return (items%2)===1;
}
function middle(array, min, max) {
 var range = max - min;
 var middle = 0;
 if (isOdd(range)) {
  var index = min + (range-1)/2;
  middle = array[index];
 } else {
  var index1 = min + range/2 - 1;
  var index2 = min + range/2;
  middle = (array[index1]+array[index2])/2;
 }
 return middle;
}
function findIQR(array) {
 bubbleSort(array);
```

```
 var half = 0;
 if (isOdd(array.length)) {
  half = (array.length-1)/2;
  } else {
  half = array.length/2;
 }
 var Q1 = middle(array, 0, half);
 var Q3 = middle(array, array.length-half, array.length);
 var IQR = Q3 - Q1;
 return IQR;
}
```

Exercise 18.7 The Monte Carlo method

1. [13,24,22,25,6]

2. **a.** -11 **b.** -6 **c.** -2 **d.** 1 **e.** 267
f. 999 **g.** -612 **h.** 0 **i.** 3

3. **a.** [0,1,2,3,4,5,6,7]
b. [1,2,3,4,5,6,7,8,9]
c. [0,1,2,3,4,5,6,7,8,9,10,11,12]
d. [1,2,3,4]

4. **a.** [0,1]
b. [0,1,2,3,4,5,6,7,8,9,10]
c. [1,2,3,4,5,6]
d. [0,1,2,3,4]
e. [0]
f. [1]
g. [1,2,3,4,5,6,7,8,9,10]
h. [1]
i. [0]

5. **a.** [10,11,12,13,14,15,16,17,18,19,20]
b. [0,10,20,30,40,50,60,70,80,90,100,110]
c. [11,22,33,44,55,66,77,88,99]
d. [600,700,800,900,1000]

6. **a.** 100*Math.random()
b. 0.001*Math.random()
c. -0.1252*Math.random()
d. 3.14*Math.random()
e. -99*Math.random()
f. 10*Math.random()

7. **a.** -120 **b.** -16 **c.** 17 **d.** 10 **e.** 57
f. 444 **g.** -63 **h.** 0 **i.** -17 **j.** 9

8. **a.**
```
function generate() {
 var random = integer(365);
 return random;
}
```
b.
```
function generate() {
 var months = [
 "January","February","March","April","May","June","July","August",
 "September","October","November","December"];
 var random = select(months)
 return random;
}
```

c.
```
function generate() {
  var random = integer(20);
  return random;
}
```

d.
```
function generate() {
  var weekdays = [
  "Monday","Tuesday","Wednesday","Thursday","Friday"];
  var random = select(weekdays)
  return random;
}
```

e.
```
function generate() {
  var d1 = integer(6);
  var d2 = integer(6);
  var d3 = integer(6);
  var random = [d1,d2,d3];
  return random;
}
```

f.
```
function generate() {
  var random = index(10);
  return random;
}
```

g.
```
function generate() {
  var random = 100+index(101);
  return random;
}
```

h.
```
function generate() {
  var real1 = Math.random();
  var real2 = Math.random();
  var random = [real1,real2];
  return random;
}
```

i.
```
function generate() {
  var random = 2*Math.random();
  return random;
}
```

j.
```
function generate() {
  var random = 2*Math.random()-1;
  return random;
}
```

k.
```
function generate() {
  var real1 = 2*Math.random()-1;
  var real2 = 2*Math.random()-1;
  var random = [real1, real2];
  return random;
}
```

9. a.
```
function isEvent(test) {
  var sum = test[0]+test[1]+test[2]+test[3];
  var event = sum < 10;
  return event;
}
```

b.
```
function isEvent(test) {
  var diff1 = test[0]-test[1];
  var diff2 = test[1]-test[0];
  var event = (diff1 === 5) || (diff2 === 5);
  return event;
}
```

c.
```
function isEvent(test) {
  var event = (test%2)===1;
  return event;
}
```

d.
```
function isEvent(test) {
   var square1 = test[0]*test[0];
   var square2 = test[1]*test[1];
   var sum = square1 + square2;
   var event = sum < 1;
   return event;
  }
```

e.
```
function isEvent(test) {
   var event = (test[0]===6) || (test[1]===6);
   return event;
  }
```

10. a.
```
function generate() {
    var real1 = 2*Math.random()-1;
    var real2 = 2*Math.random()-1;
    return [real1, real2];
  }
```

b.
```
function isEvent(test) {
   var sum = test[0]*test[0] + test[1]*test[1];
   var event = sum < 1;
   return event;
  }
```

11. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions. The `probability` should be approximately `0.095`.
```
function index(N) {
 var real = Math.random()*N;
 return Math.floor(real);
}
function integer(N) {
 return index(N)+1;
}
function generate() {
 var d1 = integer(6);
 var d2 = integer(6);
 var d3 = integer(6);
 var d4 = integer(6);
 return [d1,d2,d3,d4];
}
function isEvent(test) {
 var sum = test[0]+test[1]+test[2]+test[3];
 var event = sum === 12;
 return event;
}
function estimate(samples) {
 var events = 0;
 var i = 1;
 while(i <= samples) {
  if (isEvent(generate())) {
   events = events + 1;
  }
  i = i + 1;
 }
 return events/samples;
}
var probability = estimate(1000);
console.log(probability);
```

12. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions. The `probability` should be approximately `0.031`.
```
function index(N) {
 var real = Math.random()*N;
 return Math.floor(real);
}
```

```
function integer(N) {
 return index(N)+1;
}
function generate() {
 var d1 = integer(6);
 var d2 = integer(6);
 var d3 = integer(6);
 var d4 = integer(6);
 var d5 = integer(6);
 return [d1,d2,d3,d4,d5];
}
function swapItems(array, index1, index2) {
 var swap = array[index1];
 array[index1] = array[index2];
 array[index2] = swap;
}
function bubbleScan(array) {
 var swapped = false;
 var i=1;
 while (i < array.length) {
  if (array[i-1]>array[i]) {
   swapItems(array, i-1, i);
   swapped = true;
  }
  i=i+1;
 }
 return swapped;
}
function bubbleSort(array) {
 while(bubbleScan(array)) {
 }
 return array;
}
function isEvent(test) {
 bubbleSort(test);
 var straight1 =
 test[0]===1 &&
 test[1]===2 &&
 test[2]===3 &&
 test[3]===4 &&
 test[4]===5;
 var straight2 =
 test[0]===2 &&
 test[1]===3 &&
 test[2]===4 &&
 test[3]===5 &&
 test[4]===6;
 var event = straight1 || straight2;
 return event;
}
function estimate(samples) {
 var events = 0;
 var i = 1;
 while(i <= samples) {
  if (isEvent(generate())) {
   events = events + 1;
  }
  i = i + 1;
 }
 return events/samples;
}
var probability = estimate(1000000);
console.log(probability);
```

13. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions. The `probability` should be approximately $\frac{\pi}{4} \approx 0.785$.

```
function generate() {
 var real1 = Math.random();
 var real2 = Math.random();
 return [real1, real2];
}
function isEvent(test) {
 var sum = test[0]*test[0] + test[1]*test[1];
 var event = sum < 1;
 return event;
}
function estimate(samples) {
 var events = 0;
 var i = 1;
 while(i <= samples) {
  if (isEvent(generate())) {
   events = events + 1;
  }
  i = i + 1;
 }
 return events/samples;
}
var probability = estimate(1000000);
console.log(probability);
```

14. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions. The `probability` should be approximately $\frac{\pi}{4} \approx 0.785$. This can be multiplied by 4 to estimate $\pi \approx 3.14$.

```
function generate() {
 var real1 = Math.random();
 var real2 = Math.random();
 return [real1, real2];
}
function isEvent(test) {
 var sum = test[0]*test[0] + test[1]*test[1];
 var event = sum < 1;
 return event;
}
function estimate(samples) {
 var events = 0;
 var i = 1;
 while(i <= samples) {
  if (isEvent(generate())) {
  events = events + 1;
  }
  i = i + 1;
 }
 return events/samples;
}
var probability = estimate(1000000);
var estimatePi = probability * 4;
console.log(estimatePi);
```

18.8 Review

1. a. i. Number **ii.** `"number"`

b. i. Boolean **ii.** `"boolean"`

c. i. String **ii.** `"string"`

d. i. String **ii.** `"string"`

e. i. String **ii.** `"string"`

f. **i.** String **ii.** `"string"`
g. **i.** Number **ii.** `"number"`
h. **i.** Boolean **ii.** `"boolean"`
i. **i.** Number **ii.** `"number"`
j. **i.** Number **ii.** `"number"`
k. **i.** Number **ii.** `"number"`

2. **a.** True **b.** False **c.** False **d.** False
e. True **f.** False **g.** False **h.** False

3. **a.** **i.** Boolean **ii.** `true`
b. **i.** Boolean **ii.** `false`
c. **i.** Number **ii.** `0`
d. **i.** Number **ii.** `12.23`
e. **i.** String **ii.** `"String"`

4. **a.** `Testing` **b.** `Results` **c.** `0` **d.** `false`

5. **a.** Array **b.** String **c.** Boolean **d.** Array **e.** Array **f.** String
g. Number **h.** Array **i.** Array **j.** Number **k.** Array **l.** Array

6. **a.** `false` **b.** `5` **c.** `true` **d.** `4` **e.** `false` **f.** `true`
g. `false` **h.** `true` **i.** `4` **j.** `false` **k.** `5` **l.** `true`
m. `true`

7. **a.** `12` **b.** `0` **c.** `6` **d.** `8` **e.** `2`

8. **a.** `5` **b.** `4` **c.** `"Eight"` **d.** `8`

9. **a.** `true` **b.** `true` **c.** `false` **d.** `false` **e.** `false` **f.** `false`
g. `true` **h.** `true` **i.** `true` **j.** `true` **k.** `true` **l.** `true`
m. `true` **n.** `false` **o.** `true` **p.** `true` **q.** `false` **r.** `true`

10. **a.** `27` **b.** `"false"`

11. **a.** True **b.** False **c.** True **d.** False

12. **a.** **i.** `9` **ii.** `155`
b. **i.** `13` **ii.** `91`
c. **i.** `10` **ii.** `20`
d. **i.** `10` **ii.** `2`
e. **i.** `5` **ii.** `985`
f. **i.** `0` **ii.** `"Some string"`

13. **a.** `10` **b.** `2`

14. `123456789`

15. **a.** True **b.** True **c.** False

16. **a.** `Always this!!!` **b.** `2` **c.** `314.159` **d.** `62.8318`

17. **a.** `5` **b.** `Friday`

18. `7`

19. **a.** `true` **b.** `true` **c.** `false` **d.** `false` **e.** `true` **f.** `false`

20. `[23,2,15,4]`

21. **a.** `44` **b.** `-5` **c.** `-6` **d.** `13`
e. `0` **f.** `-4` **g.** `3`

22. **a.** `[0]`
b. `[1]`
c. `[0,1,2,3,4,5,6,7,8,9,10]`
d. `[1,2,3,4,5]`
e. `[20,21,22,23,24,25,26,27,28,29,30]`
f. `[1,2,3,4,5,6,7,8,9]`
g. `[70,80,90,100]`

h. `[1]`

i. `[12,24,36,48]`

j. `[1,2,3,4,5,6,7]`

k. `[0,1]`

l. `[0,1,2,3,4,5,6,7]`

m. `[0,100,200,300,400,500,600,700]`

n. `[0,1,2,3,4,5]`

o. `[0,1,2,3,4,5,6,7,8,9]`

23. There are many possible programs to solve this problem; an example is shown. This is a simple version that uses only the concepts shown in this topic and previously defined functions. The `probability` should be approximately `0.016`.

```
function index(N) {
  var real = Math.random()*N;
  return Math.floor(real);
}
function integer(N) {
  return index(N)+1;
}
function generate() {
  var d1 = integer(6);
  var d2 = integer(6);
  var d3 = integer(6);
  var d4 = integer(6);
  var d5 = integer(6);
  return [d1,d2,d3,d4,d5];
}
function isEvent(test) {
  var counts = [0,0,0,0,0,0];
  counts[test[0]] = counts[test[0]] + 1;
  counts[test[1]] = counts[test[1]] + 1;
  counts[test[2]] = counts[test[2]] + 1;
  counts[test[3]] = counts[test[3]] + 1;
  counts[test[4]] = counts[test[4]] + 1;
  var four1 = counts[1] === 4;
  var four2 = counts[2] === 4;
  var four3 = counts[3] === 4;
  var four4 = counts[4] === 4;
  var four5 = counts[5] === 4;
  var four6 = counts[6] === 4;
  var event = four1||four2||four3||four4||four5||four6;
  return event;
}
function estimate(samples) {
  var events = 0;
  var i = 1;
  while(i <= samples) {
    if (isEvent(generate())) {
      events = events + 1;
  }
  i = i + 1;
 }
 return events/samples;
}
var probability = estimate(100000);
console.log(probability);
```

GLOSSARY

adjacent side (trigonometry): the side next to the reference angle in a right-angled triangle
algebraic equations: equations that involve algebraic expressions
algebraic fractions: fractions that contain pronumerals (letters)
angle of depression: angle measured down from a horizontal line (through the observation point) to the line of vision
angle of elevation: angle measured up from the horizontal line (through the observation point) to the line of vision
area: amount of surface enclosed by the shape. Area is measured in square units; such as square metres, m^2, and square kilometres, km^2.
array (probability): a table that systematically displays all the outcomes of an experiment

		Bag 2		
		R	G	Y
Bag 1	B	B R	B G	B Y
	W	W R	W G	W Y

asymptote: a line that a graph approaches but never reaches
axis of symmetry: the straight line that sits midway between two halves of a symmetrical graph, or between an object and its image. An object is reflected along an axis of symmetry (mirror).
back-to-back stem-and-leaf plot: a method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem' in a stem-and-leaf plot; for example, comparing the pulse rate before and after exercise

Pulse rate

Before		After
9 8 8 8	6	
8 6 6 4 1 1 0	7	
8 8 6 2	8	6 7 8 8
6 0	9	0 2 2 4 5 8 9 9
4	10	0 4 4
0	11	8
	12	4 4
	13	
	14	6

bar chart (graph): a graph drawn in a similar way to a column graph, with horizontal bars instead of vertical columns. Categories are graphed on the vertical axis and the frequencies (numbers) on the horizontal axis.
bi-modal distributions: a distribution of data that has two modes
binomial: an expression containing two terms, for example $x + 3$ or $2y - z^2$
capacity: maximum amount of fluid contained in an object. It is usually applied to the measurement of liquids and measured in units; such as, millilitres (mL), litres (L) and kilolitres (kL).
cartesian coordinates (x, y): pairs of numbers that give the position of a point on the 2-dimensional flat plane, the Cartesian plane. The first number is the x-coordinate and the second number is the y-coordinate.
cartesian plane: an area that consists of two number lines at right angles to each other and includes all the space between the two number lines

census: process of collecting data from the entire target population

chance experiment: a process, such as rolling a die, that can be repeated many times

circumference: distance around the outside of a circle. It is given by the rule $2\pi r$ or πD, where r is the radius and D is the diameter of the circle.

coefficient: the number part of a term

commission: a method of payment in which payment is calculated as a percentage of the value of goods sold

complement (of a set): complement of a set, A, written A' is the set of elements that are in ξ but not in A

compound [interest]: interest that is paid on the sum of the principal plus preceding interest over time; $A = P(1 + R)^n$

conditional probability: where the probability of an event is conditional (depends) on another event occurring first. For two events A and B, the conditional probability of event B, given that event A occurs, is denoted by $P(B \mid A)$ and can be calculated using the formula:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0.$$

congruent figures: identical figures with exactly the same shape and size

constant: term without a pronumeral; that is, a number

constant of proportionality: the number multiplying x in the equation $y = kx$; equal to the gradient of the corresponding graph; also called the constant of variation

cosine (cos) ratio: ratio of the adjacent side to the hypotenuse in a right-angled triangle;

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ or } \cos\theta = \frac{\text{adj}}{\text{hyp}}$$

cross-section: identical 'slice' produced when cuts are made across a prism parallel to its ends

cylinder: solid with circular ends

dependent variable: y-axis (or vertical) variable

difference of two squares: an expression in which one perfect square is subtracted from another, for example $x^2 - 100$; factorised by $(a + b)(a - b) = a^2 - b^2$

direct proportion: describes a particular relationship between two variables or quantities; as one variable increases, so does the other variable

distributive law: the product of one number with the sum of two others equals the sum of the products of the first number with each of the others; for example $4(6 + 2) = 4 \times 6 + 4 \times 2$. It is also applicable to algebra; for example, $3x(x + 4) = 3x^2 + 12x$.

equally likely (probability): describes outcomes that have the same probability of occurring in an experiment

equation: a mathematical statement that shows two equal expressions

event: set of favourable outcomes in each trial of a probability experiment

expanding (algebra): multiplying everything inside the brackets by what is directly outside the brackets. Expanding is the opposite of factorising.

experimental probability: probability determined by observing an experiment and gathering data;

$$\Pr(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

expression: group of terms separated by + or − signs. Expressions do not contain equals signs.

factorising: breaking down a number or expression into smaller factors

factors: whole numbers that divide exactly into another whole number, without any remainder

favourable outcome: any outcome of a trial that belongs to the event. For example, if A is the event of getting an even number when rolling a die, the favourable outcomes are 2, 4 and 6.

fifth index law: to remove brackets containing a product, raise every part of the product to the index outside the brackets. So, $(ab)^m = a^m b^m$.

first index law: when terms with the same base are multiplied, the indices are added. So, $a^m \times a^n = a^{m+n}$.

FOIL: diagrammatic method of expanding a pair of brackets where the letters in **FOIL** represent the order of the expansion; **F**irst, **O**uter, **I**nner and **L**ast

formula: a mathematical rule, usually written using pronumerals

fourth index law: to remove brackets, multiply the indices inside the brackets by the index outside the brackets. Where no index is shown, assume that it is 1. So, $(a^m)^n = a^{mn}$.

frequency: the number of times a piece of data occurs in a set

frequency polygon: special type of line graph with the same scaled axes as the histogram. Mid-points on top of the histogram columns are joined by straight lines. The polygon is closed by drawing lines at each end down to the score (x)-axis.

fundamental theorem of algebra: Quadratic equations can have a maximum of two solutions.

gradient (slope) *m*: measure of how steep something is. The gradient of a straight line is given by $m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$ and is constant anywhere along that line.

gross salary: total salary paid before tax is deducted

hectare (ha): a unit of area equal to the space enclosed by a square with side lengths of 100 m (1 ha = 10 000 m^2). Hectares are often used to measure land area. There are 100 hectares in 1 square kilometre.

highest common factor (HCF): the largest of the set of factors common to two or more numbers. For example, the HCF of 16 and 24 is 8.

histogram: column graph with no gaps between the columns and each column 'straddles' an x-axis score, starting and finishing halfway between scores. The x-axis scale is continuous and usually a half-interval is left before the first column and after the last column.

hyperbola: the shape of the graph formed by an inverse variation relationship

hypotenuse: longest side of a right-angled triangle, always opposite the right angle

image: an enlarged or reduced copy of a shape (the original shape is called the object)

income tax: a tax paid as a percentage of income

independent variable: x-axis (or horizontal) variable

index notation: a short way of writing a number or variable when it is multiplied by itself repeatedly; for example, $6 \times 6 \times 6 \times 6 = 6^4$

integers (*Z*): positive whole numbers, negative whole numbers and zero

interest: a fee charged for the use of someone else's money, normally as a percentage of the amount borrowed

interquartile range (IQR): difference between the upper quartile, Q_U and the lower quartile, Q_L; that is, IQR = $Q_U - Q_L$. It is the range of the middle half of the data.

inverse operation: opposite operations such as + and –, or ÷ and ×

inverse proportion: variation in which increasing one variable decreases the other

irrational numbers (*I*): cannot be written as fractions. Examples of irrational numbers include surds, π and non-terminating, non-recurring decimals.

like terms: terms containing exactly the same pronumeral (letter) part. For example, $3ab$ and $7ab$ are like terms, but $5a$ is not.

linear equations: equations in which the variable has an index (power) of 1

linear graph: a relationship between two variables where a straight line is formed when the points are plotted

loading: a payment added to a wage or salary, normally as a percentage of the base wage or salary

mean: measure of the centre (average) of a set of data, given by Mean = $\frac{\text{sum of all scores}}{\text{number of scores}}$ or $\bar{x} = \frac{\Sigma x}{n}$.

When data are presented in a frequency distribution table $\bar{x} = \frac{\Sigma(f \times x)}{n}$.

measures of spread: measures of how far data values are spread from the centre or from each other; range and *interquartile range (IQR)* are both measures of spread

measure of central tendency: a single value that represents a data set; *mean*, *median* and *mode* are all measures of central tendency

median: measure of the centre (middle) of a set of data. It is the middle score for an odd number of scores arranged in numerical order (ordered). For an even number of scores, the median is the average of the two middle scores when they are ordered. Its location is determined by the rule: $\frac{n+1}{2}$.

metric system: the system of measurement based on the metre

midpoint: centre score of each class interval in grouped data

modal class: term used when analysing grouped data. It is given by the class interval with the highest frequency.

mode: measure of the centre of a set of data. It is the score that occurs most often. There may be no mode (all scores occur once), one mode or more than one mode (two or more scores occur equally frequently).

monic: a monic pronumeral or monic expression is one in which the coefficient or the leading term is 1. For example, $x^2 + 4x - 3$ is monic, but $6x^2 + 4x - 3$ is not.

mutually exclusive: cannot occur together. On a Venn diagram, two mutually exclusive events appear as disjoint sets.

natural numbers (N): positive whole numbers; 1, 2, 3, 4, . . .

negatively skewed distribution: a distribution describes a plot that has larger amounts of data as the values of the data increase

net salary: total salary received by a person after tax has been deducted from the gross salary

null factor law (NFL): used to solve quadratic equations; states that if $a \times b = 0$, then either $a = 0$ or $b = 0$ or both $a = 0$ and $b = 0$

object (drawing): the original figure

one-step equation: an equation in which one operation has been performed on the pronumeral; for example, $x - 79 = 153$, $6x = 100$

opposite side (trigonometry): the side opposite the reference angle in a right-angled triangle

outcome: result obtained from a probability experiment

overtime: the time in which a worker is working over their regular hours of employment

parabola: graph of a quadratic function

parallel lines: lines that run in the same direction and never meet

perfect square: a number or expression that is the square of another number or expression

perimeter: distance around the outside (border) of a shape

perpendicular bisector: line dividing a line segment into two equal parts and perpendicular (at right angles) to it

perpendicular lines: lines that intersect at right angles to each other

pi (π): the Greek letter π represents the ratio of the circumference of any circle to its diameter. The number π is irrational, with an approximate value of $\frac{22}{7}$, and a decimal value of $\pi = 3.141\ 59\ldots$.

piecework: a system of payment in which a worker is paid a fixed amount for each job or task they complete

pie chart (graph): a type of graph mostly used to represent categorical data. A circle is used to represent all the data, with each category being represented by a sector of the circle, whose size is proportional to the size of that category compared to the total.

population: the whole group from which a sample is drawn

positively skewed distribution: a distribution describes a plot that has larger amounts of data as the values of the data decrease

prisms: solid shapes with identical opposite ends joined by straight edges. They are 3-dimensional figures with uniform cross-sections.

probability: likelihood or chance of a particular event (result) occurring, given by:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}.$$

The probability of an event occurring ranges from 0 (impossible/will not occur) to 1 (certainty/will definitely occur) inclusive.

probability scale: a scale showing the range of possible probabilities



pronumeral: letter used in place of a number

proportion: corresponding elements are in proportion if there is a constant ratio; for example,

$\frac{\text{circumference}}{\text{diameter}} = \pi$ for all circles

pythagoras' theorem: in any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. This is often expressed as $c^2 = a^2 + b^2$.

pythagorean triad: a group of any three whole numbers that satisfy Pythagoras' theorem; for example, {5, 12, 13} and {7, 24, 25} (52 + 122 = 132).

quadrants: four regions of the Cartesian plane produced by the intersection of the x- and y-axes

quadratic equation: equation where the term with the highest power is a squared term

quadratic trinomial: algebraic expression containing three terms, where the pronumeral with the highest power is a squared term

quotient: the answer you obtain when you divide one number by another

radius [plural 'radii']: the distance from the centre of a circle to its circumference

random selection: selection of a sample in which all elements of a population have an equal chance of being selected

rate: a ratio that compares the size of one quantity with that of another quantity; a method of measuring the change in one quantity in response to another

ratio: comparison of two or more quantities of the same kind

rational numbers (Q): can be written as fractions, where the denominator is not zero

real numbers (R): set of all rational and irrational numbers

recurring decimals: decimals with one or more digits repeated continuously; for example 0.999. . . They can be expressed exactly by placing a dot or horizontal line over the repeating digits; as in $8.343434\ldots = 8.\dot{3}\dot{4}$ or $8.\overline{34}$.

relative frequency: frequency of a particular score divided by the total sum of the frequencies; given by the rule: $\text{Relative frequency of a score} = \frac{\text{frequency of the score}}{\text{total sum of frequencies}}$

right-angled triangle: a triangle with one 90° angle

royalty: a payment made to a person who owns a copyright

salary: a fixed annual amount of payment to an employee, usually paid in fortnightly or monthly instalments

sample: part of a population chosen so as to give information about the population as a whole

sample space: lists all the possible outcomes from a probability experiment; written as ξ or S, and the list is enclosed in a pair of curled brackets { }

scale factor: the factor by which an object is enlarged or reduced

scientific notation (standard form): used to express very large or very small numbers. To express a number in standard form, write it as a number between 1 and 10 multiplied by a power of 10; for example 64 350 000 can be written as 6.435×10^7 in standard form.

second index law: when terms with the same base are divided, the indices are subtracted. So $a^m \div a^n = a^{m-n}$.

sector: portion of a circle formed by two radii and the arc between them

semicircle: a half circle

similar figures: identical shapes with different sizes. The corresponding angles are equal in size and the corresponding sides are in the same ratio, called a scale factor.

simple interest: the interest accumulated when the interest payment in each period is a fixed fraction of the principal. The formula used is $I = \frac{p \times r \times T}{100}$, where I is the interest earned (in \$) when a principal of \$$P$ is invested at an interest rate of r % p.a. for a period of T years.

sine (sin) ratio: ratio of the opposite side to the hypotenuse in a right-angled triangle;

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ or $\sin\theta = \frac{\text{opp}}{\text{hyp}}$

sixth index law: to remove brackets containing a fraction, multiply the indices of both the numerator and denominator by the index outside the brackets. So, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

standard deviation: for symmetrical or normal distributions, a measure of the average deviation of all the scores from the mean of the data set. For data from a sample, use the formula Sample standard deviation, $s = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n - 1}}$. For data from a population, use the formula Population standard deviation, $\sigma = \sqrt{\frac{\Sigma(\bar{x} - x)^2}{n - 1}}$.

standard form: *see* scientific notation

statistics: the branch of mathematics that deals with the collection, organisation, display, analysis and interpretation of data, which are usually presented in numerical form

stem-and-leaf plots: display organised data. Each piece of data in a stem-and-leaf plot is made up of two components, a stem and a leaf.

stratified selection: selection of a sample so that groups within a population have a similar representation in the sample

subject: the subject of a formula is the pronumeral or variable that is written by itself. It is usually written on the left-hand side of the equation.

surds: any nth root of a number that results in an infinite non-recurring decimal pattern

survey: collection of data from a sample of a population

symmetrical: the identical size, shape and arrangement of parts of an object on opposite sides of a line or plane

symmetrical distribution: a distribution that has a clear centre and an even spread on either side; also referred to as a bell-shaped or normal distribution

tangent (tan) ratio: ratio of the opposite side to the adjacent side in a right-angled triangle;

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ or $\tan\theta = \frac{\text{opp}}{\text{adj}}$

terminating decimals: decimals with a fixed number of decimal places; for example 0.6 and 2.54

terms: parts of an expression. Terms may contain one or more pronumerals, such as $6x$ or $3xy$, or they may consist of a number only.

third index law: any term (excluding 0) with an index of 0 is equal to 1. So, $a^0 = 1$.

translated (quadratics): moving a parabola horizontally (left/right) or vertically (up/down)

transpose: rearrange a formula to make a particular pronumeral the subject

tree diagram: diagram used to list all possible outcomes of two or more events; the branches show the possible links between one outcome and the next outcome

trial: number of times a probability experiment is conducted

triangular prism: a prism with a triangular base

trigonometry: the branch of mathematics that deals with the relationship between the sides and the angles of a triangle

trinomial: an algebraic expression containing three terms

turning point: point at which the graph of a quadratic function (parabola) changes direction (either up or down)

turning point form (quadratics): the form of the equation of a graph that gives the coordinates of the turning point. For a parabola with the equation $y = (x - h)^2 + k$, the turning point is (h, k).

two-step equation: an equation in which two operations have been performed on the pronumeral; for example, $2y + 4 = 12$, $6 - x = 8$, $\frac{x}{3} - 4 = 2$

two-way table: lists all the possible outcomes of a probability experiment in a logical manner

undefined: a numerical value that cannot be calculated

variable: a letter or symbol in an equation or expression that may take many different values

venn diagrams: series of circles, representing sets; within a rectangle, representing the universal set. They show the relationships between the sets.

vertex: point where two rays or arms of an angle meet

volume: amount of space a 3-dimensional object occupies. The units used are cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).

wage: a payment to an employee based on a fixed rate per hour

***x*-intercept:** point where a graph cuts the x-axis

***y*-intercept:** point where a graph cuts the y-axis. In the equation of a straight line, $y = mx + c$, the constant term, c, represents the y-intercept of that line.

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