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NEW CENTURY MATHS

MATHEMATICS STANDARD 2



YEAR 12

Klaas Bootsma
Sarah Hamper
Margaret Willard
Robert Yen

4TH EDITION



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Series editor: Robert Yen

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New Century Maths 12 Maths Standard 2
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PREFACE

New Century Maths 12 Mathematics Standard 2, first published in 2001, has been rewritten for the new Mathematics Standard syllabus (2017). In this 4th edition of the book, teachers will find those familiar features that have made *New Century Maths* a leading mathematics series, such as graded exercises, syllabus codes and references, Investigation, Technology, SkillCheck pre-chapter exercises, chapter summary mind maps, practice papers and a glossary/index.

Mathematics Standard 2 is designed for students who will work or study in fields that require a mathematical or statistical background. We have endeavoured to produce a practical text that captures the spirit of the course, providing relevant and meaningful examples of mathematics being used in society and industry.

The *NelsonNet* websites contain additional worksheets, video tutorials and topic tests. We wish all teachers and students using this book every success with the new course.

ABOUT THE AUTHORS

Klaas Bootsma was head teacher of mathematics at Ambarvale High School in Campbelltown and has taught at Lurnea and Grantham high schools. He was a senior HSC examiner and has worked on the HSC Advice Line. Klaas has been the lead author of *New Century Maths 9–10* for over 20 years.

Sarah Hamper teaches at Abbotsleigh School in Wahroonga and has taught at Meriden and Tara Anglican schools. Her expertise is in using modelling, problem solving and ICT in the classroom, and she has presented workshops for MANSW and nationally. Sarah is a co-author of *New Century Maths 9–10*.

Margaret Willard has extensive experience writing units of work designed for distance education and was Manager at TAFE's distance education unit, OTEN. She has served on the executive of MANSW and managed its post-secondary programs.

Robert Yen taught at Hurlstone Agricultural High School in southwest Sydney. He co-edited *Reflections*, the MANSW journal, and wrote HSC study guides for *The Sydney Morning Herald*. Robert has been writing for *New Century Maths 7–12* for over 20 years and now works for Cengage as the mathematics publisher.

CONTRIBUTING AUTHORS

Megan Boltze (Ashcroft HS) and **Deborah Smith** (Bulli HS) wrote and edited many of the NelsonNet worksheets.

John Drake, Katie Jackson and **Joanne Magner** created the video tutorials.

Trisha Goss wrote the topic tests.

Roger Walter wrote the *ExamView* questions.

Tracey MacBeth-Dunn wrote the worked solutions to all exercise sets.

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SYLLABUS REFERENCE GRID

Strand and substrand	*New Century Maths 12 Mathematics Standard 2* chapter
Algebra	
MS-A4 Types of relationships A4.1 Simultaneous linear equations A4.2 Non-linear relationships	**3** Equations and linear functions **10** Non-linear functions
Measurement	
MS-M6 Non-right-angled trigonometry	**8** The sine and cosine rules **9** Scale maps and plans
MS-M7 Rates and ratios	**2** Ratios and rates **7** Energy and sustainability **9** Scale maps and plans
Financial Mathematics	
MS-F4 Investments and loans F4.1 Investments F4.2 Depreciation and loans	**1** Interest and depreciation **4** Loans and annuities
MS-F5 Annuities	**4** Loans and annuities
Statistical Analysis	
MS-S4 Bivariate data analysis	**5** Correlation and regression
MS-S5 The normal distribution	**11** The normal distribution
Networks	
MS-N2 Network concepts N2.1 Networks N2.2 Shortest paths	**6** Networks
MS-N3 Critical path analysis	**6** Networks

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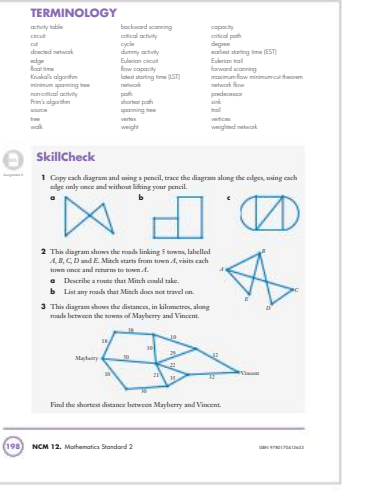
ABOUT THIS BOOK

AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins on a double-page spread showing the **Chapter contents** with syllabus codes, and a list of chapter outcomes.



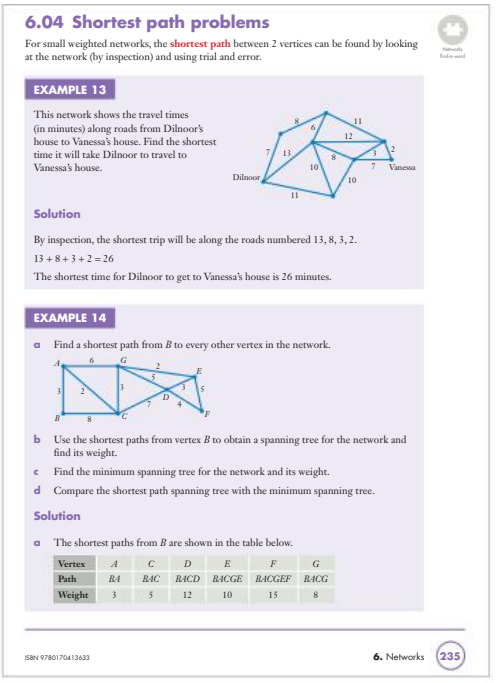
- **Terminology** previews the key words and phrases from within the chapter, while **SkillCheck** reviews important prerequisite skills and knowledge for the chapter.



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IN EACH CHAPTER

- Important facts and formulas are highlighted in a shaded box.
- Important words and phrases are printed in red and listed in the glossary at the back of the book.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.
- Worked solutions to all exercise questions are provided on the *NelsonNet* teacher website.
- **Investigations** explore the syllabus in more detail, providing ideas for modelling activities and assessment tasks.
- **Technology** promotes ICT in the classroom, using spreadsheets, graphics calculators and the Internet.
- **Did you know?** contains interesting facts and applications of the mathematics learned in the chapter.



6.04 Shortest path problems

For small weighted networks, the **shortest path** between 2 vertices can be found by looking at the network (by inspection) and using trial and error.

EXAMPLE 13

This network shows the travel times (in minutes) along roads from Dilnoor's house to Vanessa's house. Find the shortest time it will take Dilnoor to travel to Vanessa's house.

Solution

By inspection, the shortest trip will be along the roads numbered 13, 8, 3, 2.

$13 + 8 + 3 + 2 = 26$

The shortest time for Dilnoor to get to Vanessa's house is 26 minutes.

EXAMPLE 14

a Find a shortest path from B to every other vertex in the network.

b Use the shortest paths from vertex B to obtain a spanning tree for the network and find its weight.

c Find the minimum spanning tree for the network and its weight.

d Compare the shortest path spanning tree with the minimum spanning tree.

Solution

a The shortest paths from B are shown in the table below.

Vertex	A	C	D	E	F	G
Path	BA	BAC	$BACD$	$BACGE$	$BACGEF$	$BACG$
Weight	3	5	12	10	15	8

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16 If 2 people shake hands, a network diagram can be drawn where the vertices represent the people and an edge represents the handshake, as shown below.

So, for 2 people there is one handshake.

Draw a network diagram to represent each number of people shaking hands with everybody else and count:

i the number of handshakes for each person

ii the total number of handshakes.

a 3 people shake hands
b 4 people shake hands
c 5 people shake hands
d 8 people shake hands

INVESTIGATION

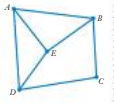
HAMILTONIAN PATHS AND CIRCUITS

1 A **Hamiltonian path** is a path that visits *every* vertex only once.

For the network shown, a Hamiltonian path is ***ABEDC*** or ***ABCDE***.

a How many Hamiltonian paths can you find for this network?

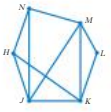
b Are all the edges of the network included in a Hamiltonian path?



2 A **Hamiltonian circuit** is a Hamiltonian path that begins and ends at the same vertex.

For the network shown, a Hamiltonian circuit is ***KHJNMLK*** or ***HKLMNJK***.

How many Hamiltonian circuits can you find in this network?



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DID YOU KNOW?

The 7 Bridges of Königsberg

In the Prussian town of Königsberg (now Kaliningrad, Russia), there were 7 bridges connecting 2 islands and the banks of the Pregel River. There was a tradition where people would walk through the town on a Sunday and it became a challenge to complete a walk crossing each bridge only once. No person was able to do it but no one could prove that it was impossible.



In 1736, the Swiss mathematician Leonhard Euler solved the problem by reducing the map to a network. He found that it was impossible to do the walk. Eventually another bridge was built. That is why a walk that crosses every bridge exactly once is called an Eulerian trail.

1 Draw a network to represent the map and explain why the original walk was impossible.

2 An eighth bridge was built. Where would the bridge have been placed so that the network could be traversed?

Exercise 6.02 Eulerian trails and circuits

Example 6

1 Which network has an Eulerian trail? Select **A**, **B**, **C** or **D**.

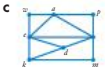
A



B



C



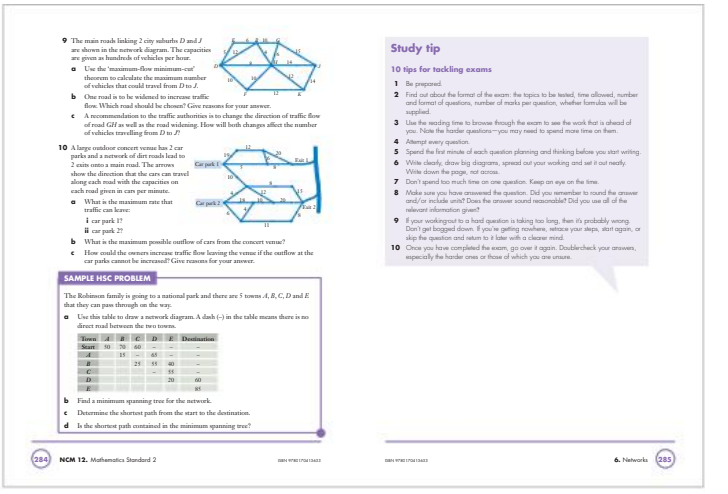
D



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AT THE END OF EACH CHAPTER

- **Sample HSC problem** demonstrates a typical HSC question using content learned from the chapter.
- **Study tip** provides useful advice for senior students in revising coursework and preparing for exams.
- **Chapter summary** concludes the chapter and includes a mind map exercise.
- **Test yourself** contains chapter revision linked to the relevant exercise set.



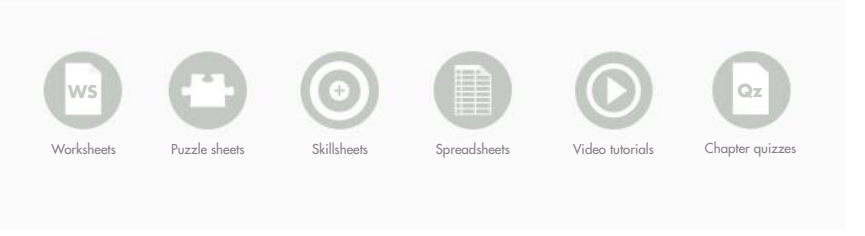
- **Practice papers** after every 3 chapters revise the skills and knowledge of those chapters.

AT THE END OF THE BOOK

- **Glossary and index** includes a comprehensive dictionary of course terminology.
- **Answers** (worked solutions are on the teacher website).

NELSONNET STUDENT WEBSITE

Margin icons link to print (PDF) and multimedia resources found on the *NelsonNet* student website, **www.nelsonnet.com.au**. These include:



- **Worksheets** and **puzzle sheets** that are write-in enabled PDFs
- **Skillsheets** of examples and exercises of prerequisite skills and knowledge
- **Video tutorials:** worked examples explained by 'flipped classroom' teachers
- **Spreadsheets:** Excel files
- **ExamView quizzes:** interactive and self-marking

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NELSONNET TEACHER WEBSITE

The *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, contains:

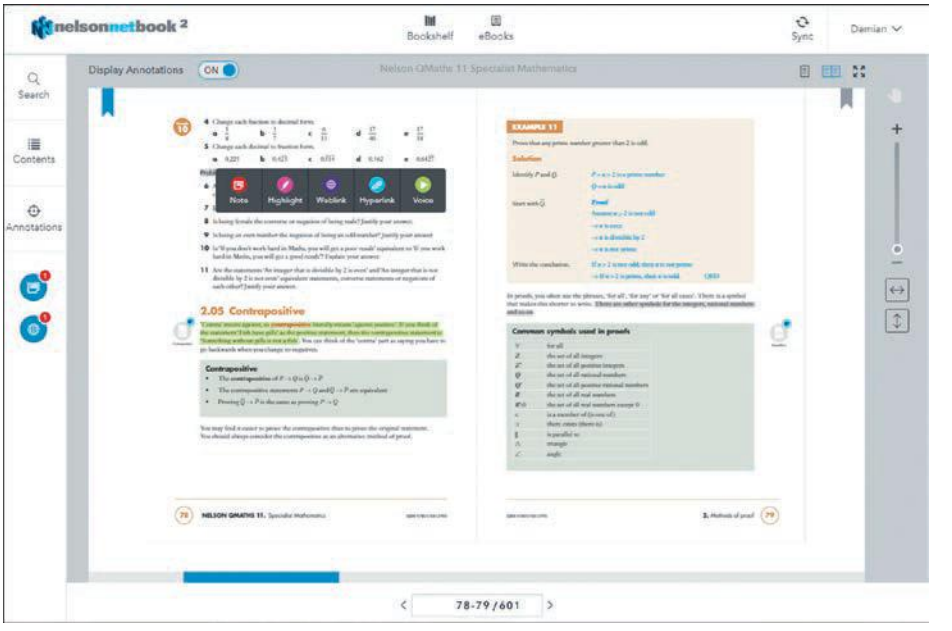
- A **teaching program**, in Microsoft Word and PDF formats
- **Topic tests**, in Microsoft Word and PDF formats
- **Worked solutions** to each exercise set
- **Chapter PDFs** of the textbook
- **ExamView** exam-writing software and questionbanks
- **Resource Finder**: search engine for *NelsonNet* resources.

Note: Complimentary access to these resources is only available to teachers who use this book as a core educational resource in their classroom. Contact your Cengage Education Consultant for information about access codes and conditions.

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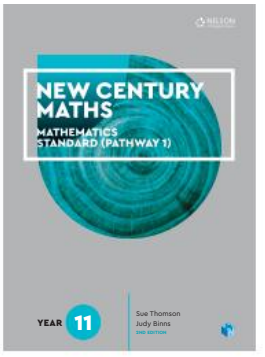
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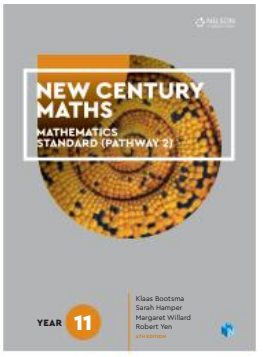
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- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students.

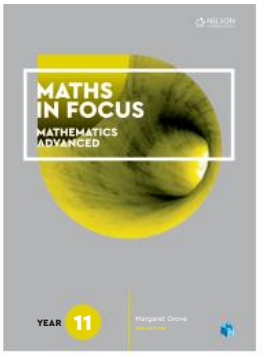


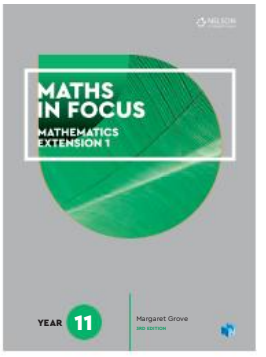
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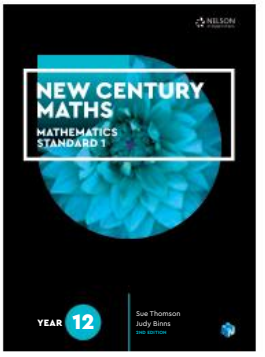
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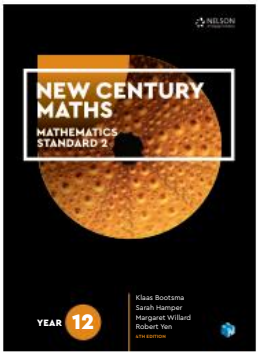


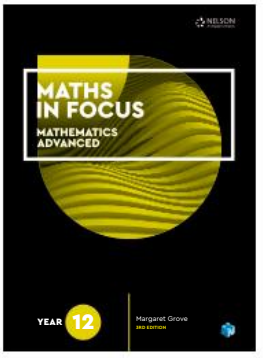


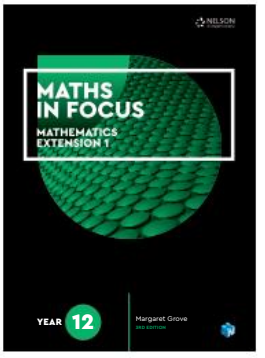


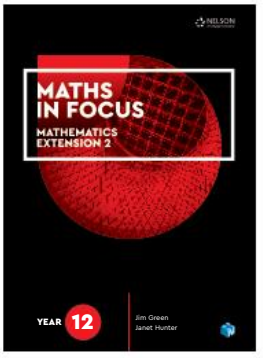












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SYMBOLS AND ABBREVIATIONS

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	parentheses, round brackets
[ ]	(square) brackets
{ }	braces
$\pm$	plus or minus
π	pi = 3.14159…
$\equiv$	is congruent to
$\|\|\|$	is similar to
$0.15\dot{2}$	the recurring decimal 0.152152 …

$^\circ$	degree
$\angle$	angle
Δ	triangle
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\therefore$	therefore
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\sqrt{\ }$	square root, radical sign
$\sqrt[3]{\ }$	cube root
$P(E)$	the probability of event E occurring
$P(\tilde{E})$	the probability of event E not occurring
LHS	left-hand side
RHS	right-hand side
%	percentage
p.a.	per annum (per year)

cos	cosine ratio
sin	sine ratio
tan	tangent ratio
$\bar{x}$	the mean
σ_n	the standard deviation
Σ	the sum of
Q_1	first quartile or lower quartile
Q_2	median (second quartile)
Q_3	third quartile or upper quartile
IQR	interquartile range
α	alpha
θ	theta
μ	micro-, mu
m	gradient

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MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study in detail the parts of a situation

apply: use knowledge or a procedure in a given situation

calculate: *See* **evaluate**

classify, **identify**: state the type, name or feature of an item or situation

comment: express an observation or opinion about a result

complete: fill in detail to make a statement, diagram or table correct or finished

compare: show how two or more things are similar or different

construct: draw an accurate diagram

convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams

decrease: make smaller

describe: state the features of a situation

estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately

evaluate, **calculate**: find the value of a numerical expression, for example, 3×8^2 or $4x + 1$ when $x = 5$

expand: remove brackets in an algebraic expression, for example, expanding $3(2y + 1)$ gives $6y + 3$

explain: describe why or how

give reasons: show the rules or thinking used when solving a problem. *See also* **justify**

graph: display on a number line, number plane or statistical graph

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: *See* **classify**

increase: make larger

interpret: find meaning in a mathematical result

justify: give reasons or evidence to support your argument or conclusion. *See also* **give reasons**

measure: determine the size of something, for example, use a ruler to determine the length of a pen

prove that: *See* **show that**

recall: remember and state

show that, **prove**: (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true

simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression

sketch: draw a rough diagram that shows the general shape or ideas, less accurate than **construct**

solve: find the value(s) of an unknown pronumeral in an equation or inequality

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state: *See* **write**

substitute: replace a variable by a number and evaluate

verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

write, **state**: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step.)

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1.



INTEREST AND DEPRECIATION

People once kept their savings under the mattress or in an old tin before they started using banks. Today's investors are more informed about financial planning and more likely to invest in the share market, a superannuation fund or property rather than leaving money in a low-interest saving account. More than half of all Australians own shares, often through a superannuation fund.

CHAPTER OUTLINE

F1.1* 1.01 Simple interest
F4.1 1.02 Compound interest
F4.1 1.03 Inflation and appreciation
F4.1 1.04 Investing in shares
F4.1 1.05 Share tables and graphs
F1.1* 1.06 Straight-line depreciation
F4.2 1.07 Declining-balance depreciation
*Year 11 revision



IN THIS CHAPTER YOU WILL:

- calculate simple interest using $I = Prn$, where P is the principal, r is the interest rate per period as a decimal and n is the number of periods
- calculate the future value (final amount), compound interest rate and present value (principal) using the formula $FV = PV(1 + r)^n$ where FV is the final value, PV is the present value, r is the interest rate per compounding period as a decimal and n is the number of compounding periods
- compare simple interest and compound interest, and compare different investment strategies
- apply the formula $FV = PV(1 + r)^n$ to problems involving inflation and appreciation
- record and graph the price of a share over time
- calculate the dividend paid on a shareholding and the dividend yield (excluding franked dividends)
- calculate the salvage value and the depreciation of an asset using the straight-line method and the declining-balance method.

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TERMINOLOGY

appreciation
compound interest
dividend
future value (*FV*)
period
quarterly
simple interest
brokerage
declining-balance method
dividend yield
inflation
present value (*PV*)
salvage value
stockbroker
compounded
depreciation
final value
market price
principal
shareholder
straight-line method

SkillCheck

WS
Assignment 1

1 Find:

a 5% of \$456 **b** 3.75% of \$78 600 **c** 4.2% of \$12 400.

2 Evaluate each expression correct to 2 decimal places.

a $3400 \times (1.015)^6$ **b** $12\,680 \times (1.004)^{12}$ **c** $760 \times (1.00056)^{25}$

3 **a** Increase \$15 600 by 8%.

b Decrease \$985 by 5%.

c Decrease \$2440 by 12%.

d Increase \$10 500 by 8% and then decrease by 10%.

4 Copy and complete each statement.

a 6.4% p.a. = _____ % per quarter **b** 4.8% p.a. = _____ % per month

c 4.5% p.a. = _____% per half year **d** 9% p.a. = _____% per month

5 If $A = P(1 + r)^n$, find:

a A if $P = 8400$, $r = 0.0125$, $n = 18$

b P if $A = 15\,600$, $r = 0.005$, $n = 36$.

6 Janelle received a 65 cent dividend for each of her 12 000 shares.

a How much did Janelle receive?

b If each share is worth \$37.50, express the 65 cent dividend as a percentage of the share value, correct to 2 decimal places.

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1.01 Simple interest

Interest is money earned on an investment, or money charged on a loan, with a bank, credit union or other financial institution. **Simple interest** (or **flat-rate interest**) is interest calculated as a percentage of the **principal** (the amount of money invested or borrowed).



Simple interest



Applications of simple interest


Simple interest riddle


What's the interest?


Simple interest

Simple interest

$$I = Prn$$

where I = interest

P = principal or initial amount

r = interest rate per **period**, expressed as a decimal

n = number of periods

EXAMPLE 1

p.a. means 'per annum' (or per year).

Dilnoor invested \$20 600 at 3.85% p.a. flat interest for 17 months.

Calculate the simple interest Dilnoor earned.

Solution

$P = \$20\,600$, $r = \dfrac{0.0385}{12}$ per month, $n = 17$ months

$I = Prn$

$= \$20\,600 \times \dfrac{0.0385}{12} \times 17$

$= \$1123.5583\ldots$

$\approx \$1123.55$

r and n are expressed in the same units (months).

Interest earned is always *rounded down* to the nearest cent.

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EXAMPLE 2

Julia invested $25 400 in a term deposit at a simple interest rate of 5.2% p.a.
How long was the term deposit if the amount of interest Julia earned was $2311.40?

Solution

$I = \$2311.40, P = \$25\,400, r = 0.052$

$I = Prn$

$\$2311.40 = \$25\,400 \times 0.052 \times n$

$\$2311.40 = \$1320.8n$

$$n = \frac{\$2311.40}{\$1320.8}$$

$= 1.75$ years

$= 1$ year 9 months

$0.75 \text{ years} = 0.75 \times 12 \text{ months} = 9 \text{ months}$

EXAMPLE 3

Tuan borrowed $11 200 over 3 years at 14% p.a. simple interest to buy a home cinema system.

a How much interest does Tuan have to pay?

b What amount does she repay altogether?

c What is Tuan's monthly payment?

Solution

a $P = \$11\,200, r = 0.14, n = 3$

$I = Prn$

$= \$11\,200 \times 0.14 \times 3$

$= \$4704$

b Amount to repay $= \$11\,200 + \$4704$ principal + interest

$= \$15\,904$

c Monthly payments $= \$15\,904 \div 36$ 3 years $= 3 \times 12 = 36$ months

$= \$441.777\ldots$

$\approx \$441.78$

Payments are always *rounded up* to the nearest cent.

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Exercise 1.01 Simple interest

1 Alana invested $12 800 at a simple interest rate of 7.2% p.a. for 3 years. Calculate the interest earned.

2 Calculate the simple interest earned from each investment. Example 1

a $4250 for 5 years at 3.65% p.a.

b $35 000 at 5.5% p.a. for 15 months

c $48 350 at 2.85% p.a. for 260 days

d $3678 for $3\frac{1}{2}$ years at 2% p.a.

3 Georgia invests $2800 for 1 year and 4 months at a flat interest rate of 4.8% p.a. What is the total value of the investment at the end of this period? Select **A**, **B**, **C** or **D**.

A $179.20 **B** $2980.62 **C** $2979.20 **D** $180.62

4 Calculate the flat-rate interest paid on each loan.

a $1450 at 3.9% p.a. for 2 years

b $1.2 million for 40 weeks at 3.5% p.a.

c $4000 at 19.95% p.a. for 37 days

d $32 700 for 5 months at 8.2% p.a.

5 For how long must $28 500 be invested at 6.4% p.a. simple interest to earn $6384? Example 2

6 For how many weeks will it take $6400 to earn $165 interest at a flat rate of 3.8% p.a.?

7 Tomasz earned $530 interest after 2 years on a term deposit of $4500. What was the simple interest rate per annum, correct to 2 decimal places?

8 What principal would earn $5678.40 interest if invested at 5% p.a. for $3\frac{1}{2}$ years?

9 Sitara earned $156.80 interest from an investment of $12 340 over 92 days. What was the simple interest rate per day, correct to 3 significant figures?

10 Liam invested $10 500 for 10 months and earned $283.76 interest. What was the simple interest rate per annum?

11 The interest on a loan over 4 years is $8260. If the simple interest rate is 12.5% p.a., calculate the amount borrowed.

12 Jackson borrowed $3600 over 2 years at 12.5% p.a. simple interest to buy a computer. Example 3

a How much interest does Jackson have to pay?

b What amount does he repay altogether?

c What is his monthly payment?

13 Layla took out a loan of $10 500 at 15.5% p.a. flat interest to buy some furniture for her apartment. The loan is to be repaid in **fortnightly** repayments over 3 years.

a How much will her furniture cost altogether?

b Calculate the fortnightly repayment.

14 Rakesh bought a new TV and borrowed $3200 to help with the purchase. If the simple interest rate is 11.25% p.a. and the **term** of the loan is 18 months, find his fortnightly repayment. Select **A**, **B**, **C** or **D**.

A $207.77 **B** $95.80 **C** $47.95 **D** $95.90

15 An amount of $22 800 was invested for 18 months with simple interest and grew to $26 083.20. Calculate the monthly interest rate.

16 Ashleigh uses her credit card to buy a notebook computer for $1399. The credit card charges 20.45% p.a. simple interest and she pays the full amount owing after 33 days. What is the amount that Ashleigh will pay?

WS Compound interest formulas

WS Compound interest

WS Compound interest table

WS Simple and compound interest

Interest calculator

Compound interest

1.02 Compound interest

Most savings accounts pay **compound interest** rather than simple interest. Compound interest is interest that is added to the principal and reinvested (**compounded**). The principal plus interest becomes the new principal on which interest is calculated. In other words, we earn 'interest on our interest' as well as on the original principal.

You should already be familiar with the compound interest **formula** $A = P(1 + r)^n$, where A is the final amount and P is the principal. In the financial world, the principal is called the **present value (*PV*)** and the final amount is called the **future value (*FV*)** or **final value**, so now we can write the following compound interest formulas.

Compound interest

$$FV = PV(1 + r)^n \quad \text{and} \quad I = FV - PV$$

where FV = future value (final amount)

Interest = future value – present value

PV = present value (principal, initial value)

n = number of **compounding periods**

r = interest rate per compounding period, expressed as a decimal

I = compound interest earned.

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EXAMPLE 4

$8500 is invested for 4 years at 7.5% p.a., compounded monthly. Calculate the interest earned.

Solution

Because interest is compounded monthly, r and n must be expressed in *months*:

$PV = \$8500,\ r = \dfrac{0.075}{12} = 0.006\ 25$ per month, $n = 4 \times 12 = 48$ months

$FV = PV(1 + r)^n$

$= \$8500(1 + 0.006\ 25)^{48}$

$= \$8500(1.006\ 25)^{48}$

$= \$11\,463.092\ldots$

$\approx \$11\,463.09$ Round down for future value

$I = FV - PV$ Interest = future value – present value

$= \$11\,463.09 - \$8500$

$= \$2963.09$

Interest earned = $2963.09

EXAMPLE 5

What present value must be invested for 8 years at 4.85% p.a. compounded **quarterly** to grow to a future value of $15 000?

Solution

$FV = \$15\,000,\ r = \dfrac{0.0485}{4} = 0.012\,125$ per quarter, $n = 8 \times 4 = 32$ quarters

$FV = PV(1 + r)^n$

$\$15\,000 = PV(1 + 0.012\ 125)^{32}$

$\$15\,000 = PV(1.012\ 125)^{32}$

$PV = \dfrac{\$15\,000}{(1.012\,125)^{32}}$

$= \$10\,199.958\ldots$

$\approx \$10\,199.96$ Round up for present value

A present value of $10 199.96 must be invested.



Compound interest rate

EXAMPLE 6

$16 000 invested for 6 years grew to a future value of $21 440. If the interest was compounded annually, find the interest rate per annum.

Solution

$PV = \$16\,000$, $FV = \$21\,440$, $n = 6$ years

$$FV = PV(1 + r)^n$$

$$\$21\,440 = \$16\,000(1 + r)^6$$

$$(1 + r)^6 = \frac{\$21\,440}{\$16\,000}$$ Divide both sides by $16 000

$$(1 + r)^6 = 1.34$$

$$1 + r = \sqrt[6]{1.34}$$ Take the 6th root of both sides by pressing SHIFT $x^{\blacksquare}$ (or 2ndF y^x) 1.34 =

$$1 + r = 1.04998\ldots.$$

$$r = 1.04998\ldots - 1$$ Subtract 1 from both sides

$$r = 0.04998\ldots$$

$$= 4.998\ldots\%$$

$$\approx 5\%$$

Exercise 1.02 Compound interest

1 Find the future value when $35 000 is invested for 5 years at 4.8% p.a. compounded yearly.

2 For each investment, calculate:

- **i** the future value (FV)
- **ii** the compound interest (I) earned.

a $18 600 at 2.99% p.a. for 6 years

b $46 000 for 8 years at 4.05% p.a. compounded monthly

c $7000 for 3 years at 6.4% p.a. compounded half-yearly

d $32 500 for 15 years at 3.5% p.a. compounded weekly

e $110 000 for 2 years at 2.25% p.a. compounded quarterly

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3 A house was purchased in Kellyville in 2005 for $890 000. Assume the value of the house has increased by 3.5% p.a. since then. Which expression gives the future value of the house in 2021? Select **A**, **B**, **C** or **D**.

A $890\,000(1 + 0.035)^{16}$ **B** $890\,000(1 + 3.5)^{16}$

C $890\,000(1 + 0.35)^{16}$ **D** $890\,000 \times 1.035 \times 16$

4 $137 000 is invested for 2 years at 5.5% p.a. compounded monthly. Calculate the interest earned.

5 What present value must be invested for 5 years at 7% p.a. compounded half-yearly to grow to a future value of $18 000?

6 Peter wants to invest some money so that it will have a future value of $38 000 in 5 years. If the interest rate is 4.5% p.a. compounded monthly, what present value should Peter invest, to the nearest dollar?

7 What present value must be invested to grow to $40 000 at:

a 3.75% p.a. for 4 years?

b 4.6% p.a. compounded quarterly for 7 years?

c 2.65% p.a. compounded monthly for 5 years?

d 7.5% p.a. compounded half-yearly for 2 years?

8 Ben's inheritance grew to a future value of $63211.20 after it had been invested for 15 years at 6.2% compounded monthly. What was the present value of his inheritance?

9 $8500 is invested for 5 years and grew to a future value of $10 095.33. If the interest is compounded annually, find the interest rate per annum.

10 Zara has $30 000 in an account which grew to a future value of $44 200 after 9 years. If the interest was compounded yearly, find the interest rate.

11 $4000 was left in a savings account for 3 years. If the future value was $4831.80, what is the interest rate p.a. if it is compounded annually. Select **A**, **B**, **C** or **D**.

A 1.06% p.a. **B** 6.5% p.a. **C** 4.02% p.a. **D** 4.5% p.a.

Example 4

12 Agata invested $15 800 in an account which grew to a future value of $18 389.31 after 4 years. If the interest was compounded monthly, find the interest rate.

Example 5

13 A principal of $25 400 is invested at 4.2% p.a. Calculate the future value of the investment after 3 years if the interest is compounded:

a yearly **b** half-yearly **c** quarterly

d monthly **e** daily.

Example 6

14 Alex won $250 000 in a lottery. She invested the winnings in a term deposit earning 5.3% p.a. interest, compounded half-yearly. How many years will it take Alex to double her money?

INVESTIGATION

DOUBLE YOUR MONEY USING THE RULE OF 72

How long will it take for money invested at a particular interest rate to double in size? We can estimate the time it takes to double our money by dividing 72 by the interest rate per compounding period.

For example:

a How long will it take a principal of $20 000 to double if interest is compounded annually at 4% p.a.?

$72 \div 4 = 18$ years

b What if the interest is 6% p.a.?

$72 \div 6 = 12$ years

1 Use the compound interest formula, $A = P(1 + r)^n$, to check that $20 000 does in fact double with the above interest rates and terms.

2 Use the 'rule of 72' to calculate how long it will take $20 000 to double if invested at an interest rate of:

i 7.5% p.a. **ii** 5%.

3 Check that your answers to Question **2** are correct using the compound interest formula.

4 Did you notice that you don't actually use '$20 000' in your calculations? That means that the time required to 'double your money' does not depend on the amount. Choose another principal amount and check that your answers to Question **2** are correct for it as well.



Compound interest graph

TECHNOLOGY

Compound interest graph

Emily invests $5000 in an account that earns 8% p.a. interest, compounded annually.

1 First, create this spreadsheet.

	A	B
1	Compound interest graph	
2	Present value	$5,000
3	Interest rate p.a.	8%
4	Number of years	15
5		
6	Number of years	Future value

2 In cell A7, enter **0** and in cell A8, enter the formula = **A7+1**.

Now **Fill down** from A8 to A22.

3 To calculate the future value in column B, in cell B7, enter the formula =**$B$2*(1+$B$3)^A7**. Now **Fill down** from cell B7 to B22.

4 Select cells A6 to B22. From the **Insert** menu, select '**Scatter with Smooth Lines and Markers**'.

What does your graph look like?

5 Change the present value (cell B2) and the interest rate (cell B3). What effect do the changes have on the graph?

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TECHNOLOGY

Simple versus compound interest

Marek wants to compare the growth of simple interest and compound interest on his investment of $10 000 at 5% p.a.

1 Create this spreadsheet, including the formulas in rows 6 and 7.

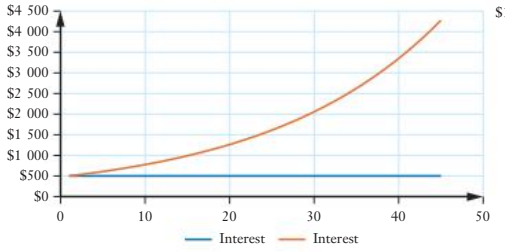
	A	B	C	D	E
1		Present value		$10 000	
2		Interest rate p.a.		0.05	
3					
4		Simple interest		Compound interest	
5	Year	Interest	Future value	Interest	Future value
6	1	=$D$1*$D$2	=$D$1+B6	=$D$1*$D$2	=$D$1+D6
7	=A6+1	=$D$1*$D$2	=C6+B7	=$D$2*E6	=E6+D7
8					

2 Fill down from row 7 to row 50 to copy the formulas down the columns.

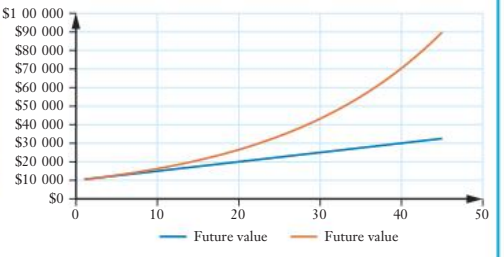
3 **a** Select columns starting with A5, B5 and D5. Insert the chart showing the line graphs for simple interest and compound interest earned per annum.

b Describe the type of graph (e.g. linear) that represents the simple interest earned and the compound interest earned.

Q 3 Simple v Compound Interest Comparison



Q 4 Simple v Compound Interest Comparison



4 Select columns starting with A5, C5 and E5. Insert the chart showing the line graphs for the growth for the simple interest and compound interest investments. What type of graph (e.g. linear) represents the value of the simple interest investment and the value of the compound interest investment?

5 **a** Modify the spreadsheet by changing the present value and/or the interest rate. What is the effect on the amount of interest earned and the final value of the investment?

b What is the effect on the graphs?

Comparing interest rates

Comparing interest rates

Simple vs compound interest: spreadsheet

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WS
Compounding periods: spreadsheet

TECHNOLOGY

Compounding periods

Create the following spreadsheet for a present value of $100 000 invested at 4% p.a. over 10 years for various compounding periods. Enter appropriate formulas to find the given values.

Let 1 year = 365 days.

	A	B	C	D	E
1	Present value	$100,000.00			
2	Interest rate	0.04			
3	Term (years)	10			
4					
5		**Interest rate per period**	**Number of periods**	**Final amount**	**Total interest earned**
6	**Simple interest**	0.04	10	$140,000.00	$40,000.00
7	**Compound interest**			**Future Value**	
8	Compounded yearly	0.04	10	148024.43	$48,024.43
9	Compounded half-yearly	0.02	20	148594.74	$48,594.74
10	Compunded quarterly	0.01	40	148886.37	$48,886.37
11	Compounded monthly	0.003333333	120	149083.27	$49,083.27
12	compounded daily	0.000109589	3650	149179.20	$49,179.20

1 By modifying the spreadsheet, show that the interest earned on $50 000 at 3.5% p.a., compounded monthly for 5 years is $9547.14.

2 Show that the difference between interest compounded yearly and interest compounded daily on a present value of $10 000 at 6.2% p.a. over 3 years is $66.33.

3 Why is it better for an investor to earn interest compounded daily rather than compounded monthly?

WS
Prices and inflation

1.03 Inflation and appreciation

Inflation

Inflation is an increase in the prices of all goods and services. The yearly percentage increase is called the **annual inflation rate** and this is usually accompanied by increases in workers' wages and salaries to 'keep up with inflation'. The annual inflation rate in Australia is generally around 2 to 3%.

Calculating the new price of goods and services after inflation is an application of increasing a quantity by a percentage repeatedly, so we can use the compound interest formula:

$$FV = PV(1 + r)^n.$$

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EXAMPLE 7

A new car costs $38 999 today. If inflation remains constant at 2.4% p.a., what will be the price of a new car in 4 years' time, rounded to the nearest dollar?

Solution

$PV = \$38\,999, r = 0.024, n = 4$

$FV = PV(1 + r)^n$

$= \$38\,999(1 + 0.024)^4$

$= \$38\,999(1.024)^4$

$= \$42\,879.853\ldots$

$\approx \$42\,879$ Round down for future value

Price in 4 years = $42 879

EXAMPLE 8

The current price of a magazine is $7.20. Calculate its price 10 years ago if the inflation rate was a constant 2.8% p.a. during this time. Round your answer up to the nearest 5 cents.

Solution

We need to work backwards and use $FV = \$7.20, r = 0.028, n = 10$.

$FV = PV(1 + r)^n$

$\$7.20 = PV(1 + 0.028)^{10}$

$= PV(1.028)^{10}$

$$PV = \frac{\$7.20}{(1.028)^{10}}$$

$= \$5.4626\ldots$

$\approx \$5.50$ Round up for present value

Price 4 years ago = $5.50

Appreciation

Appreciation is the increase in value of an item (such as artwork, gold, a prestige car, land or a house) over time. It is the opposite of **depreciation**, which is a decrease in value over time of items such as cars, furniture and electrical appliances.

Calculating the value of items after appreciation is another application of repeated percentage increase, so we can use the compound interest formula again.

EXAMPLE 9

Jackie bought a block of land for $542 500 and it appreciated at a rate of 6.5% p.a.

Calculate its value 3 years later, to the nearest $100.

Solution

$PV = \$542\,500, r = 0.065, n = 3$

$FV = PV(1 + r)^n$

$= \$542\,500\ (1.065)^3$

$= \$655\,312.671\ldots$

$\approx \$655\,300$

Value after 3 years $\approx \$655\,300$

Exercise 1.03 Inflation and appreciation

Example 7

1 A car costs $25 800 today. If inflation remains at a constant 2.6% p.a. for the next 6 years, what will be the price of the car in 6 years' time? Select **A**, **B**, **C** or **D**.

A $30 000 **B** $31 095 **C** $30 200 **D** $30 100

Example 8

2 A textbook today costs $85.00. What was its price 8 years ago if the inflation rate over this period was 3% p.a.?

3 The cost of a meat pie today is $3.50. What was its price 40 years ago if the average inflation rate over this period was 3.18% p.a.?

Example 9

4 A house cost $580 000 and appreciated in value at a rate of 7% p.a. What is the value of the house 10 years after it was purchased? Select **A**, **B**, **C** or **D**.

A $1 140 000 **B** $1 140 900 **C** $1 140 950 **D** $1 500 000

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5 A loaf of bread currently costs $4.00. Calculate its value in 3 years if the inflation rate is 2.4% p.a.

6 If gold appreciates at 6.2% p.a. and its current value is $1121 per ounce, calculate the value of gold per ounce in 5 years' time.

7 A 22-day cruise costs $23 295. Calculate its cost, to the nearest dollar, in 2 years' time if the inflation rate is 4.2% p.a.

8 A house increased in value this year from $585 300 to $670 000. What was the rate of appreciation per annum, as a percentage to one decimal place?

9 A pair of jeans costs $115 today. What was the value of the jeans 10 years ago if the annual inflation rate was constant at 2.48% p.a.?

10 A painting has a current value of $15 600. What was its value 15 years ago, to the nearest dollar, if it appreciated by 6.3% p.a.?

11 Rabiba's wages increase according to the increase in the consumer price index. If she earns $1182.40 per week and the increase in the consumer price index for the year was 2.8% calculate:

a Rabiba's new weekly wage

b the amount by which her annual income has increased.

12 **a** Name 3 items that appreciate over time.

b Name 3 items that depreciate over time.

13 Antara has a diamond ring worth $5700. What will be its value after 12 years if it appreciates by 8% annually?

14 Joshua works for an IT company and his salary increases with the annual inflation rate of 3.1%. Calculate his salary in 4 years' time if it is currently $82 000.

15 A vintage car has a current value of $75 000. If the car was bought 6 years ago, and the car appreciated at 5% p.a., what was the price paid for the car? Answer to the nearest $100.



iStock.com/Anton_Sokolov

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16 How much will $100 be worth by today's standards in 5 years' time if the inflation rate is 2.7% p.a.? (*Hint*: Final value = $100.)

17 Calculate correct to the nearest dollar the cost of a new $1499 washing machine in 3 years if the inflation rate is 2.9% annually.

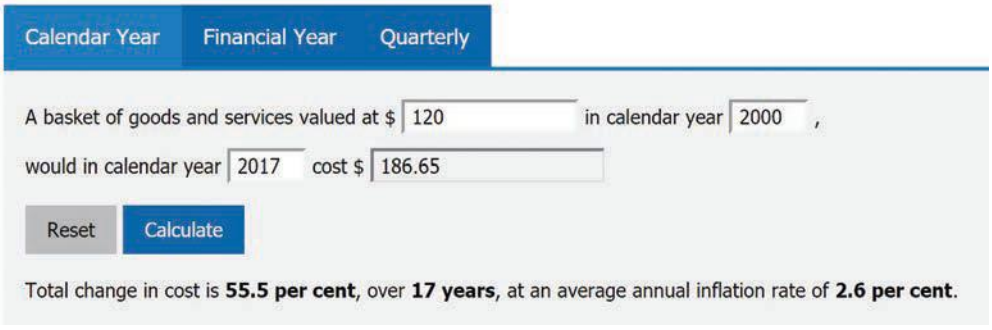
18 Ramiya's salary increased from $76 380 to $87 150 over 3 years. If the increase was due to inflation, find the annual rate of inflation over the 3 years.

19 A family spends $480 on groceries every week. If inflation remains steady at 2.3% p.a., how much would they pay for the groceries in 5 years' time?

TECHNOLOGY

Inflation online calculator

The **Reserve Bank of Australia** website has an online calculator to find an average annual inflation rate.



Source: Reserve Bank of Australia

Experiment with the inflation calculator to find how much a car would have cost in 1990 if its price today is $34 500.

DID YOU KNOW?

The Consumer Price Index

The Consumer Price Index (CPI) measures the prices paid by Australian households for goods and services consumed, and is based on the average price of a fixed basket of goods and services bought in the eight state/territory capital cities. The CPI in 2017 was around 110, compared to 100 in the base year 2011–12.

This table shows the percentage change in prices from March 2016 to March 2017.

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Goods and services	Percentage change
Food and non-alcoholic beverages	1.8
Alcohol and tobacco	6.1
Clothing and footwear	0.3
Housing	2.5
Furnishings, household equipment and services	–0.1
Health	3.8
Transport	3.8
Communication	4.8
Recreation and culture	–0.2
Education	3.3
Insurance and financial services	27
All groups	**1.6**

Source: Australian Bureau of Statistics

The percentage change in the CPI is a measure of inflation.

$$\text{Inflation rate} = \frac{\text{New CPI} - \text{Old CPI}}{\text{Old CPI}} \times 100\%$$

a **If a weekly train ticket cost $38.50 in March 2016, how much did it cost in March 2017?**

b **Find the CPI for this year and for the same time last year, and hence calculate the current inflation rate.**

1.04 Investing in shares

WS Shares and dividends

Another way of investing money is to buy **shares** in a company, such as Woolworths, BHP, National Australia Bank (NAB) or Qantas. The **market price** of shares changes daily due to many factors, such as demand for the shares, the state of the company and the economic situation at the time.

Dividend and dividend yield

When we own shares in a company we are **shareholders** and earn a **dividend** (a type of interest) on our investment. The dividend can be collected by the investor or can be reinvested in more shares to earn a higher dividend next time. The **dividend yield** is the dividend as a percentage of the market price of the share (similar to an interest rate).

Dividend yield

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price per share}} \times 100\%$$

EXAMPLE 10

Ilhea had 2000 CBA shares valued at $162 800 and received a dividend of $1.99 per share. What was the dividend yield, correct to 2 decimal places?

Solution

Market price of each share = $162 800 ÷ 2000 = $81.40

Dividend per share = $1.99

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price per share}} \times 100\%$$

$$= \frac{\$1.99}{\$81.40} \times 100\%$$

$$= 2.4447\ldots\%$$

$$= 2.44\%$$

Purchasing shares

A **stockbroker** buys and sells shares at the current market price, charging a commission called the **brokerage**. This cost should be taken into account when calculating our earnings.

EXAMPLE 11

Kalan bought 500 shares in Woolworths at $24.60 per share with a dividend yield of 4.2%. One year later, he sold the parcel of shares for $31.80 per share. His stockbroker charges brokerage of 1.8% for buying and 1% for selling a parcel of shares.

a What was the total cost to Kalan of purchasing his shares?

b Calculate Kalan's total earnings over the year, after costs.

Solution

a Purchase price = 500 × $24.60

= $12 300

Brokerage for buying = 1.8% × $12 300

= $221.40

Total cost = $12 300 + $221.40

= $12 521.40

The total cost of the shares to Kalan was $12 521.40.

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b Dividend earned = 4.2% × $12 300

= $516.60

Selling price = 500 × $31.80

= $15 900

Brokerage for selling = 1% × $15 900

= $159

Total earnings = selling price + dividend – brokerage – total cost

= $15 900 + $516.60 – $159 – $12 521.40

= $3736.20

Kalan's total earnings were $3736.20.

Exercise 1.04 Investing in shares

1 Kellie bought 200 shares at $5.31 each and received a dividend of 14c per share. What is the dividend yield? Select **A**, **B**, **C** or **D**. Example 10

A 2.64% **B** $28 **C** 0.026% **D** $2.60

2 Adrian wants to purchase 2500 ANZ Bank shares. The market price of the shares is $28.75.

a Calculate the total cost of the shares.

b Adrian's stockbroker charges a commission of 2% of the cost of shares.
Calculate the total brokerage.

c A dividend of 80c was paid when the market price of the share was $27.30.
Calculate the total dividend earned and the dividend yield, correct to 2 decimal places.

3 Terry bought 4000 shares for $8920, with a dividend yield of 12%. Calculate:

a the market price of one share

b the total dividend earned.

4 Eliza bought 200 shares in Fairfax for $286. Calculate:

a the market price of one share

b the dividend per share, to the nearest 0.1 cent, if the dividend yield is 6.2%.

5 Vicky bought 40 000 shares in Myer at $0.82 each. For buying or selling, her stockbroker charged 2% brokerage for the first $15 000 of shares and 1.4% thereafter. Example 11

a Calculate the total purchase cost of Vicky's shares.

b If Vicky collects a dividend yield of 2.6% and then sells her shares for $1.13 each, calculate her total earnings after costs.

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6 Michaela bought 8200 shares in Nine Entertainment at $1.37 each.

a Calculate the value of her shares.

b Additional costs were:

- Broker's service fee: $5.50
- Brokerage: 2.5% of the share value up to $5000
  2% of the next $10 000
  1.5% of the next $35 000
  1% of the remainder.

Calculate the total cost of buying the shares.

c Calculate Michaela's dividend if the company pays a dividend of 8.5c per share.

d Calculate the dividend yield, correct to 2 decimal places.

7 Village Roadshow shares have a market value of $4.07 with a dividend yield of 3.44%.

a What is the total market value of 3000 shares?

b What is the dividend per share?

8 Karl's share portfolio is described in the table below.

Karl's portfolio			
Number of shares	**Company**	**Market value**	**Dividend**
2200	BHP	$25.10	2.9%
700	Tabcorp Holdings	$4.22	8.7%
1500	QBE Insurance	$12.23	4.4%
1800	Coca-Cola Amatil	$8.68	5.2%

a Calculate the total value of Karl's shares.

b Calculate the total dividend earned.

9 Boun owns 10 500 AGL Energy shares. The dividend per share is 41c and the market price is $24.75.

a What dividend will Boun receive?

b Calculate the dividend yield, correct to one decimal place.

c If Boun reinvested his dividend, how many extra shares could he purchase?

10 The Macquarie Group Ltd shares have a market value of $87.13. If the dividend is 3.21% of the market value, calculate the dividend earned from 5700 Macquarie Group shares.

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11 Paula bought 12 500 shares in Qantas Air at $5.49 per share, with 1.7% brokerage.

- **a** Calculate the cost of her shares.
- **b** If the company pays a dividend of 8c per share, calculate Paula's dividend.
- **c** When the market price of the shares is $5.87, calculate:
  - **i** the value of Paula's shares
  - **ii** the dividend yield.

12 Charissa bought 2500 shares in Domino's Pizza at $51.20 each with a dividend yield of 1.9%. For buying shares, brokerage is 2.3% and for selling shares, brokerage is 1%.

- **a** Calculate the cost of purchasing the shares.
- **b** Charissa sells her shares 1 year later at $55.30. Find her total profit after costs.

1.05 Share tables and graphs


The share market game

Share tables

Daily information about share prices can be found in the finance section of a newspaper, listed in a table similar to the one below. All decimal values refer to dollars and cents. The table below is an example and does not contain factual information on the status of the company shares.

Company	Last sale	+ or – (cents)	No. sold (100s)	52 week	
				high	low
AGL Energy	25.05	+25	10 043	28.47	16.51
BHP	24.83	–27	73 267	27.95	18.88
Harvey Norman	4.15	+2	29 598	5.58	3.54
Medibank Private	2.81	+6	61 968	3.14	2.37
Myer	.816	–1	33 692	1.48	0.816
Seven Group	10.72	+1	2168	12.04	6.43
Viva Energy	2.24	–	7498	2.65	2.02
Westfield	7.93	+5	25 905	11.15	7.64

Last sale The market price of the shares at close of day

+ or – The change in price from the previous trading day

No. sold (100s) The number of shares sold (in 100s)

52 week high/low The maximum and minimum price in the past 52 weeks

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EXAMPLE 12

Use the share table on the previous page to find:

a the market price of Viva Energy shares

b the share that was 27c lower than the day before

c the price of Westfield shares the day before

d the shares that had the lowest price in the past 52 weeks

e the share that sold the least on this day.

Solution

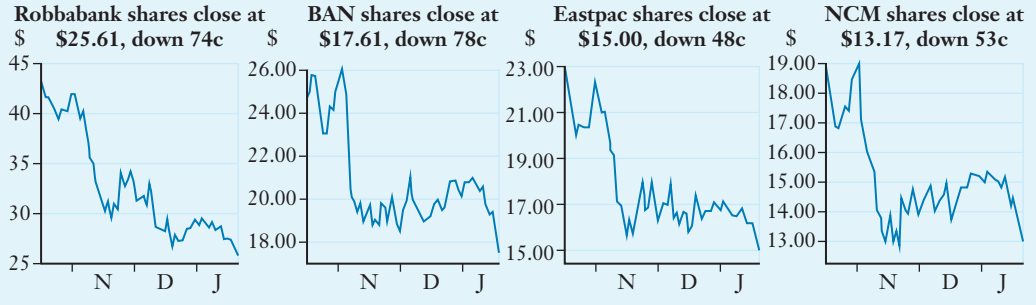
a	$2.24	Last sale
b	BHP was 27c lower on this day than on the day before.	See + or – column
c	$7.93 – 5c = $7.88	Market price was 5c higher than the day before
d	Myer shares at 81.6c per share	52 week low
e	The Seven Group had the least number sold with 216 800 shares.	See No. sold column

Share graphs

Graphs are often used to chart the changing prices of shares over a period of time.

EXAMPLE 13

These graphs show the movement in the share prices of four banks from mid-October to mid-January.



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a What is the highest price paid for BAN shares?

b What is the lowest price reached by Robbabank shares?

c What was the range of prices of NCM shares, to the nearest dollar?

d Describe the performance of Robbabank shares over the period.

e What are two similarities between the four graphs?

f Predict what is likely to happen to the share prices of the four banks in February. Justify your answer.

Solution

a $26.00

b $25.61 From the title of the graph

c The range of prices of the NCM shares is: $19.00 – $12.00 = $7.00

d Apart from a couple of small rises, the Robbabank share price continued its downward trend.

e Some similarities are:

- the share prices for all banks peaked around the beginning of November
- the prices dropped rapidly in the first half of November
- the lowest share price for all banks was in mid-January.

f It looks like the prices may fall even further.

Exercise 1.05 Share tables and graphs

Use the share table on page 23 to answer Questions **1** to **8**.

1 What was the market value of AGL Energy shares on the previous day? Select **A**, **B**, **C** or **D**.

A $25.30 **B** $25.35 **C** $24.85 **D** $24.80

2 What was the range in Westfield share prices over the year? Select **A**, **B**, **C** or **D**.

A 5 cents **B** $3.51 **C** $3.22 **D** 29 cents

Example 12

3 For the Medibank Private shares:

- **a** what is the market price?
- **b** what was the market price the previous day?
- **c** what was the lowest price in the past 52 weeks?
- **d** how many shares were sold on this day?

4 **a** Which share in the share table showed the smallest change in price from the previous day?

- **b** What was the change in price referred to in part **a**?

5 **a** Which share in the share table showed the greatest change in price from the previous day?

- **b** What was the change in price referred to in part **a**?

6 **a** Which share had the highest market price?

- **b** Which share had the lowest market price?

7 Which share had the most number sold on this day?

8 For Harvey Norman shares:

- **a** what was the lowest price paid in the past 52 weeks?
- **b** what was the highest price in the past 24 hours?

9 Use the share table below to answer the following questions.

Stock name	Code	Last sale	Change (cents)	Year	
				high	low
Caltex	CTX	31.65	–38	35.41	27.55
JB Hi-Fi	JBH	25.59	–29	31.20	21.20
NewsCorp	NWS	17.95	+35	19.96	14.97
Origin Energy	ORG	7.21	+64	8.04	4.69
Westfarmers	WES	41.47	–72	46.06	39.52

- **a** What is the market price of NewsCorp shares?
- **b** Which share had the least change from the previous day?
- **c** What is the code used for?
- **d** Which share had the greatest range for the year?
- **e** What was the price paid for a Westfarmers share the day before?
- **f** By how much did the market price of Origin Energy shares increase from the day before?
- **g** What was the percentage increase in Origin Energy shares from the previous day (correct to one decimal place)?

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10 For the graphs of the bank share prices in Example 13 on page 24, find:

Example 13

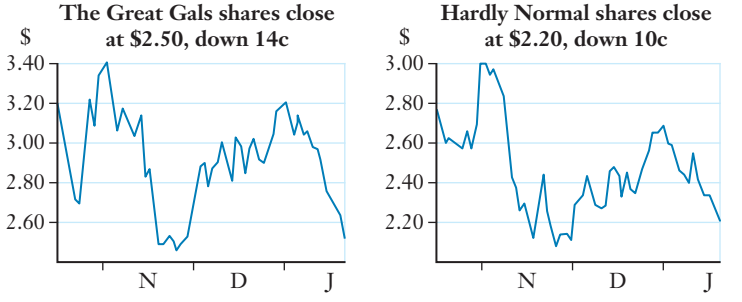
- **a** the market price of NCM in mid-November
- **b** the lowest Eastpac price over the period
- **c** the number of months covered by the period
- **d** the number of times the price of NCM shares rose above $15.00
- **e** the price of BAN shares in mid-December
- **f** an estimate of the price of BAN shares on 1 January.

11 The market value of a share was noted every Friday for 6 weeks. The results are shown in the table.

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
$11.25	$14.50	$18.60	$12.36	$13.20	$10.40

- **a** What was the range of share prices?
- **b** Draw a line graph showing the share price in the 6-week period.
- **c** Describe the trend over the 6 weeks.
- **d** Is it possible to predict what will happen in the next 6 weeks?

12 These graphs show the performance of the shares of two retailers over a 3-month period.



- **a** What was the highest price for The Great Gals over this period?
- **b** What was the lowest price for Hardly Normal over this period?
- **c** How many times was the price for The Great Gals shares $2.60?
- **d** What was the range in price for Hardly Normal shares?
- **e** Describe the performance of The Great Gals shares in this 3-month period.
- **f** Estimate what the Hardly Normal share prices were on these days.
  - **i** 1 December **ii** 31 October
- **g** Would January be a good time to buy shares in Hardly Normal? Why?
- **h** If an investor bought shares in The Great Gals around 4 November, would you suggest selling them in mid-January? Why?

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INVESTIGATION

AUSTRALIAN SECURITIES EXCHANGE (ASX)

The Australian Stock Exchange was established in 1987 in Sydney, after the amalgamation of the six state stock exchanges, and is now one of the world's top ten exchanges. Since 2006, it has been known as the Australian Securities Exchange (ASX).

More than 1200 Australian companies and 600 overseas companies are listed on the ASX. The All Ordinaries Index (abbreviated to All Ords) is a statistic that measures the value of 500 of our largest companies. The S&P/ASX 200 is a newer index that summarises the value of 200 shares.

1 Visit the ASX website and play the Schools Sharemarket Game.

2 Find the name of eight companies listed on the ASX and list the market value of their shares and the dividend yield. Compile a list of values, over a week (or longer), then graph the results and discuss your findings. Decide the company in which you would invest based on their dividend yields.

3 Visit the stock exchange in Sydney, if possible, to see how share prices are displayed.

4 Find out which country's stock exchanges are associated with these indices.

a Dow Jones **b** Nikkei

c FT100 **d** Hang Seng

WS Straight-line depreciation formula practice

WS Motorbike depreciation

Straight-line depreciation

1.06 Straight-line depreciation

Depreciation is the loss in value of an **asset** over time. For example, a new car loses around 30% to 40% of its value in its first year. There are two methods for depreciating an item:

- **straight-line method**, where the item's value decreases by the same amount each period
- **declining-balance method**, where the item's value decreases by the same percentage each period.

The formula for the straight-line method of depreciation is a **linear function** (of the form $y = mx + c$) and its graph is a straight line.

Straight-line depreciation

$$S = V_0 - Dn$$

where S = **salvage** (current) **value** of the asset after n periods

V_0 = initial value of the asset

D = amount of depreciation per period

n = number of periods

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EXAMPLE 14

Computer equipment is purchased for $28 000 and depreciates by $4500 each year. Use the straight-line method of depreciation to find:

a the value of the equipment after 3 years

b when, to the nearest month, the equipment will be worth $11 000

c when, to the nearest month, the equipment will have no value.

Solution

a $V_0 = \$28\,000$, $D = \$4500$, $n = 3$, $S = ?$

$S = V_0 - Dn$

$= \$28\,000 - \$4500 \times 3$

$= \$14\,500$

After 3 years the equipment is worth $14 500

b $S = \$11\,000$, $V_0 = \$28\,000$, $D = \$4500$

$S = V_0 - Dn$

$\$11\,000 = \$28\,000 - \$4500n$

$\$4500n = \$17\,000$

$$n = \frac{\$17\,000}{\$4500}$$

$= 3.777\ldots$ years

$= 3$ years $9.333\ldots$ months $\quad 0.777\ldots \times 12$ months $= 9.333\ldots$ months

$= 3$ years 10 months $\quad$ Rounding up

The equipment will be worth $11 000 after 3 years 10 months.

c $S = \$0, V_0 = \$28\,000, D = \$4500$

$$S = V_0 - Dn$$

$$\$0 = \$28\,000 - \$4500n$$

$$\$4500n = \$28\,000$$

$$n = \frac{\$28\,000}{\$4500}$$

$$= 6.222\ldots \text{ years}$$

$$= 6 \text{ years } 2.666\ldots \text{ months} \qquad 0.222\ldots \times 12 \text{ months} = 2.666\ldots \text{ months}$$

$$= 6 \text{ years } 3 \text{ months} \qquad \text{Rounding up}$$

The equipment will have no value after 6 years 3 months.

EXAMPLE 15

A photocopier has an initial value of \$13 200 and depreciates by \$2200 per year. The depreciation schedule shows the accumulated depreciation and value of the photocopier.

Period	Accumulated depreciation (\$)	Value of copier at end of year (\$)
0	0	13 200
1	2200	11 000
2	4400	8800
3	6600	6600
4	8800	4400
5	11 000	2200
6	13 200	0

a What is the value of the photocopier at the end of the 4th year?

b What is the total depreciation at the end of the 5th year?

c Draw a graph showing the value of the photocopier with years on the horizontal axis.

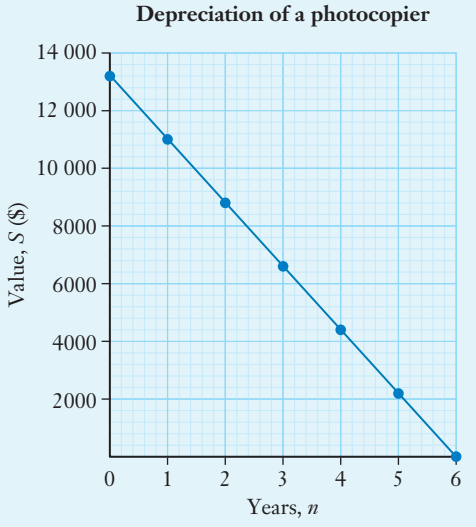
d Write down the equation of the graph.

Solution

a \$4400 **b** \$11 000

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c



d Equation of the line is $S = V_0 - Dn$
where $V_0 = \$13\,200$, $D = \$2200$
Equation is $S = 13\,200 - 2200n$.

Exercise 1.06 Straight-line depreciation

Example 14

1 A computer was purchased for $5200 and depreciates by $1100 per year. What is its salvage value after 4 years? Select **A**, **B**, **C** or **D**.

A $4100 **B** $4400 **C** $1000 **D** $800

2 Emma's Electrics purchased an industrial drill for $15 600. Using the straight-line method of depreciation, it was expected to be worth half of this amount after 3 years.

a What is the amount of depreciation of the drill per year?

b What was the drill's value after 4 years?

3 Toby bought a new tractor for his farm for $87 500 and it depreciated to $58 700 after 4 years.

a What was the annual depreciation using the straight-line method?

b What was the value of the tractor after 7 years?



iStock.com/oticki

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4 A wood lathe was purchased for $1350 and depreciated to $450 at the end of 5 years.

a Use the straight-line method to find the yearly depreciation.

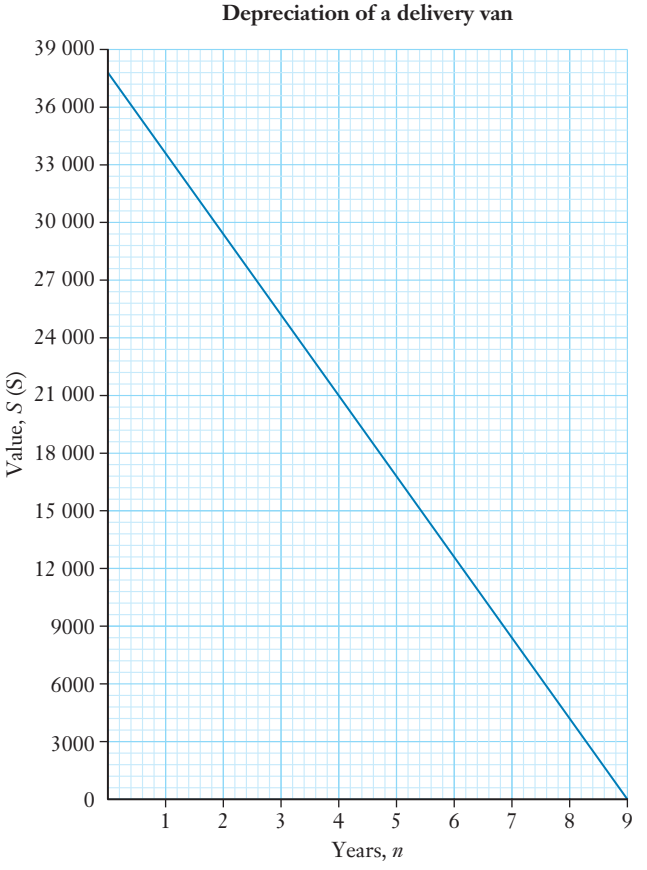
b What was the value of the lathe after 3 years?

5 Tara and Nick bought a new oven costing $8350 for their take-away food business. Using the straight-line method of depreciation, its value was $4240 after 3 years.

a What is the amount of depreciation of the oven per year?

b When will the oven have no value?

6 The graph shows the depreciation of a newsagent's delivery van over its useful life.



a What is the purchase price?

b What is the yearly depreciation of the delivery van?

c What is its salvage value after 3 years?

d What is the accumulated depreciation at the end of the 5th year?

e What is the equation of the graph?

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Example 15

7 This is the depreciation schedule for a ride-on lawn mower.

Year	Accumulated depreciation ($)	Value at end of year ($)
0	0	5200
1	840	4360
2	1680	3520
3	2520	2680
4	3360	1840



iStock.com/IStockphoto_RAW

a What is the purchase price?

b Find the depreciation in the first 2 years.

c What is the salvage value after 3 years?

d Draw a graph showing the depreciation of the lawn mower with years on the horizontal axis and value on the vertical axis.

e Write down the equation of the graph.

8 Furniture for the reception area at a doctor's surgery was bought for $16 000 and depreciates by $3100 per year.

a Copy and complete this depreciation schedule for the furniture.

Year	Accumulated depreciation ($)	Value of end of period ($)
0	0	16 000
1		
2		
3		
4		
5		

b What is the value of the furniture at the end of the 4th year?

c What is the total depreciation at the end of the 2nd year?

d Draw a graph showing the depreciation of the furniture.

e When will the furniture have no value?

f Write down the equation of the graph.

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9 The graph shows the decreasing value of a vending machine.

a What is its purchase price?

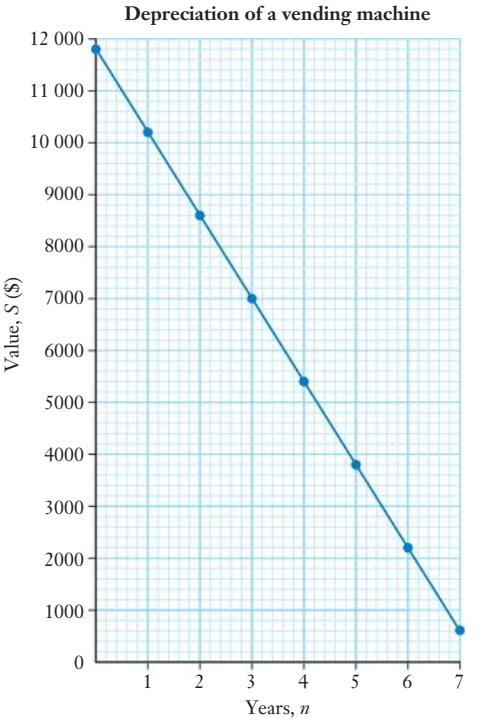
b How much depreciation has occurred after 7 years?

c After how many years is the machine worth:

i $7000? **ii** $3000?

d Find the annual depreciation amount.

e What is the equation of the line?



1.07 Declining-balance depreciation

Declining-balance depreciation formula practice

Declining-balance depreciation: spreadsheet

Declining-balance depreciation

Declining-balance depreciation

Many items do not depreciate by the same amount each year. A motor vehicle generally depreciates more in the first few years of its life. The **declining-balance method of depreciation** takes this into account and reduces the value of the asset each period by a constant **percentage** rather than a constant amount. This percentage is called the **declining balance rate**.

EXAMPLE 16

A Ferrari is purchased for $179 000 and depreciates 15% p.a.

a Calculate its depreciation in the 1st year.

b What is the value of the Ferrari at the end of the 1st year?

c What is the value at the end of the 3rd year, correct to the nearest dollar?

d Construct a table showing the depreciation and value for the first 5 years.

e Graph the depreciating value of the Ferrari over 5 years.

Solution

a Depreciation in first year $= 15\% \times \$179\,000$

$= \$26\,850$

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b Value at end of first year = $179 000 – $26 850

= $152 150

Or:

Value at end of first year = $179 000 × 85%

= $179 000 × 0.85

= $152 150

c Value at end of second year = value at end of first year × 85%

= ($179 000 × 0.85) × 0.85

= $179 000 × $(0.85)^2$

= $129 327.50

Value at end of third year = value at end of second year × 85%

= $179 000 × $(0.85)^3$

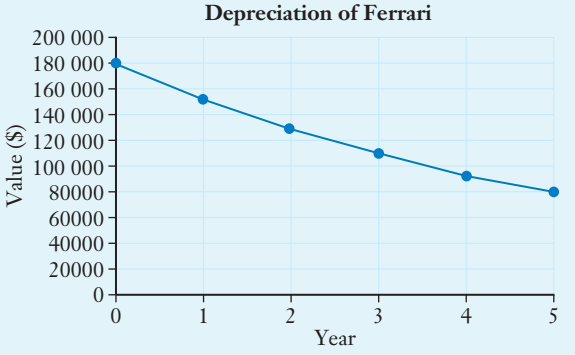
= $109 928.375

≈ $109 928 Rounded to the nearest dollar

d

Year	Depreciation ($)	Value at end of year ($)
0	0.00	179 000.00
1	26 850.00	152 150.00
2	22 822.50	129 327.50
3	19 399.13	109 928.38
4	16 489.26	93 439.12
5	14 015.87	79 423.25

e



Declining-balance depreciation

$$S = V_0(1 - r)^n$$

where S = salvage (current) value of the asset after n periods

V_0 = initial value of the asset

r = depreciation (declining balance) rate per period, expressed as a decimal

n = number of periods.

$S = V_0(1 - r)^n$ is like the compound interest formula $FV = PV(1 + r)^n$ with a *negative* interest rate. If you purchase an item for $\$V_0$ and it depreciates at a rate of r per period for n periods, its value **decreases** to $\$S$ according to the formula $S = V_0(1 - r)^n$.

EXAMPLE 17

Kevin bought a camper trailer for \$34 500 with an expected depreciation rate of 18% p.a.

a Use the declining-balance formula to find its value, correct to the nearest dollar:

i after 1 year **ii** after 5 years.

b What is the total depreciation in the first 5 years?

c After how many years, to one decimal place, will the camper trailer be worth \$1000?

Solution

a $S = ?$, $V_0 = \$34\,500$, $r = 0.18$

i When $n = 1$

$$S = V_0(1 - r)^n$$
$$= \$34\,500(1 - 0.18)^1$$
$$= \$34\,500 \times 0.82$$
$$= \$28\,290$$

Value after 1 year is \$28 290.

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ii When $n = 5$

$S = V_0(1 - r)^n$

$= \$34\,500(1 - 0.18)^5$

$= \$34\,500\ (0.82)^5$

$= \$12\,790.52459$

$\approx \$12\,791$ Rounded to the nearest dollar

Value after 5 years is \$12 791.

b Total depreciation in first 5 years = purchase price – value after 5 years

$= \$34\,500 - \$12\,791$

$= \$21\,709$

c Want $S = 1000$

Use a guess-and-check method.

Try $n = 17$:

$S = \$34\,500(0.82)^{17} = \$1182.1010...$ Too big

Try $n = 18$:

$S = \$34\,500(0.82)^{18} = \$969.3228...$ Too small

Try $n = 17.7$: $S = \$1028.7841...$

Try $n = 17.8$: $S = \$1008.5691...$

Try $n = 17.9$: $S = \$988.7512...$

The camper trailer will be worth less than \$1000 after 17.9 years.

Exercise 1.07 Declining-balance depreciation

Example 16

1 Daksh installs new machinery costing \$115 000 in his factory and it depreciates at 20% p.a.

- **a** Calculate the depreciation in the 1st year.
- **b** What is the value of the machinery at the end of the 1st year?
- **c** What is the value at the end of the 4th year?
- **d** Construct a table showing the depreciation and value for the first 6 years.
- **e** Graph the depreciating value of the machinery over 6 years.

Example 17

2 A motorcycle costs $18 340 and depreciates at 18% p.a. What is its value 3 years after purchase? Select **A**, **B**, **C** or **D**.

A $10 112 **B** $9903 **C** $15 038 **D** $11 200

3 A hospital bought new computers for its nurses' station costing $18 000. If depreciation is 32% p.a., find:

a the value of the computers after 2 years

b the depreciation in the 3rd year.

4 Quong bought a new car costing $55 000, which he is going to use for work. He is allowed to depreciate the car at 45% p.a.

a What is the depreciation in the 1st year?

b What is the value of the car after 2 years?

c When does the car have a value less than $8000?

5 The value of office furniture at B&B Accounting is shown by the graph rounded to the nearest $500. The furniture depreciates at 22% p.a.

a How much did the furniture cost?

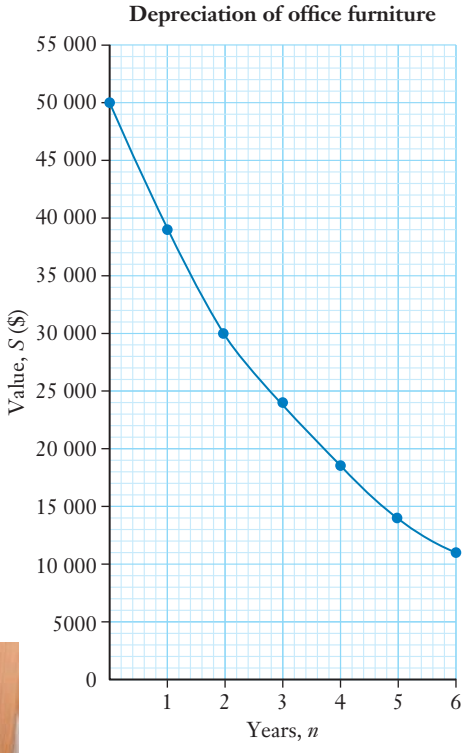
b What is its value after:

i 3 years? **ii** 6 years?

c What is the depreciation in the:

i 2nd year? **ii** 4th year?

d Use the declining-balance formula to find the value of the furniture after 8 years.





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6 Baldin Enterprises purchased machinery for $110 000. Using a declining-balance rate of 30% p.a., find the following values (to the nearest dollar).

a the value of the machinery after:

i 3 years **ii** 8 years.

b the depreciation in the:

i 3rd year **ii** 8th year.

7 The table shows the value of some sawmill equipment over 5 years.

Age of equipment (years)	Value of equipment ($)
0	180 000
1	135 000
2	101 250
3	75 937
4	56 953
5	42 715

a What is the equipment worth after 3 years?

b What is the rate of depreciation of the sawmill equipment?

c Use a guess-and-check method to find in what year its value will fall below $20 000.

d Plot the values and draw the depreciation graph for the sawmill equipment over 5 years.

e What is the vertical intercept and what does it represent?

8 Robyn's Removals bought a new van and after 3 years it was valued at $44 000. The depreciation is 24% p.a.

a How much did Robyn pay for the van, to the nearest dollar?

b What is the expected value of the van after 5 years?

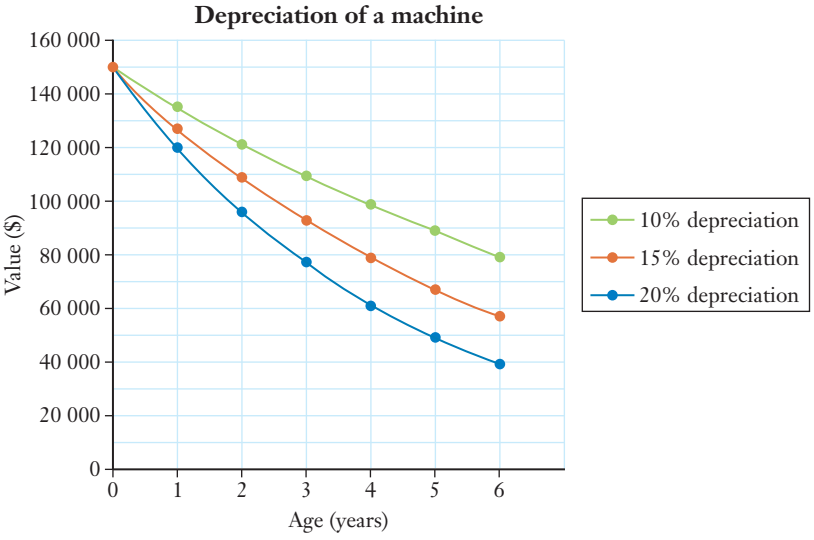


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9 Doug's Diner purchased a refrigerator and after 2 years it was valued at $12 000. What was the price of the fridge if it depreciated at 45% p.a.?

10 This graph shows the decreasing value of a machine at different annual rates of depreciation.



a What is the initial cost of the machine?

b What is the value (to the nearest $1000) of the machine after 4 years at:

i 10% p.a. depreciation? **ii** 20% p.a. depreciation?

c What is the total depreciation in the first 2 years at:

i 15% p.a. depreciation? **ii** 20% p.a. depreciation?

d Use the declining-balance formula and a rate of 15% p.a. to calculate the value of the machine when it is 10 years old. Answer to the nearest dollar.

11 Leo has spent $27 000 on equipment to set up his painting business. The equipment depreciates at 18% p.a.

a Find the value of the equipment after 4 years.

b By how much had the equipment depreciated after 2 years?

c Use a guess-and-check method to find how long it will take for the equipment to have a value less than $8000.

d Find the value of Leo's equipment after 6 years as a percentage of its original value, correct to one decimal place.

12 Kayla bought a computer for $6500. For tax purposes, she is allowed to claim depreciation at 40% p.a. How much can Kayla claim:

a at the end of the 1st year?

b in the 3rd year?

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TECHNOLOGY

Declining-balance depreciation

A spreadsheet can be used to find the value of an item and the amount of depreciation using the declining-balance method.

1 **a** Open a new spreadsheet and enter the data as shown for the depreciation of an item costing $50 000 which depreciates at 30% p.a.

	A	B	C
1	**Purchase price ($)**	50000	
2	**Depreciation rate (%)**	30	
3			
4	**Year**	**Value at end of year ($)**	**Amount of depreciation ($)**
5	0		
6			
7			

b In cell A6, enter =**A5+1** and fill down from A6 to row 15.

c In cell B5, enter =**B1** and in cell C5, enter 0.

d In cell B6, enter =**$B$1*(1-$B$2/100)^A6** and in cell C6, enter =**B5-B6**. Then Fill down from B6 and C6 down to row 15.

e Highlight cells A4 to B15. From the **Insert** menu, select '**Scatter with Smooth lines and Markers**'.

Describe the type of graph shown.

2 Use your spreadsheet to answer each question.

a What is the value of the item after 5 years?

b When does the value fall below $1000?

c What is the depreciation in the 3rd year?

d Fill down your spreadsheet to determine when the value is $0.

e Change the purchase cost to $15 000 and rate of depreciation to 22% p.a. and answer parts **a**–**d**.

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SAMPLE HSC PROBLEM

Brittany invests $15 000 in an account that earns 4.5% p.a. interest, compounded annually.

a How much interest will the account earn in 4 years?

b How much more interest would the account earn if the interest was compounded monthly?

c Explain why more interest is earned when the interest is compounded monthly.

Study tip

Switch off your phone and devices

It is very easy to achieve nothing in 3 hours of 'study time' if much of it is spent on tidying up your desk, chatting with friends on the phone or social media, having excessive fridge and TV breaks, or playing games on the computer. If you are serious about studying, you need to program into your study routine blocks of time when you cannot be interrupted by phone calls, messages or friendly visits. This is especially important when you have a big task at hand, such as preparing for an exam or completing a major assignment. Establish an arrangement with your family so that they know not to disturb you during your study time.

Put your phone and other digital devices away. Don't study in front of the TV or computer. Time passes very quickly if you do. Study *or* watch TV *or* use the computer, but don't try to multi-task. You can listen to music or have the radio on as long as it doesn't affect your concentration. As for those fun things you want to do? Do them during a study break as a reward for completing a significant amount of work.

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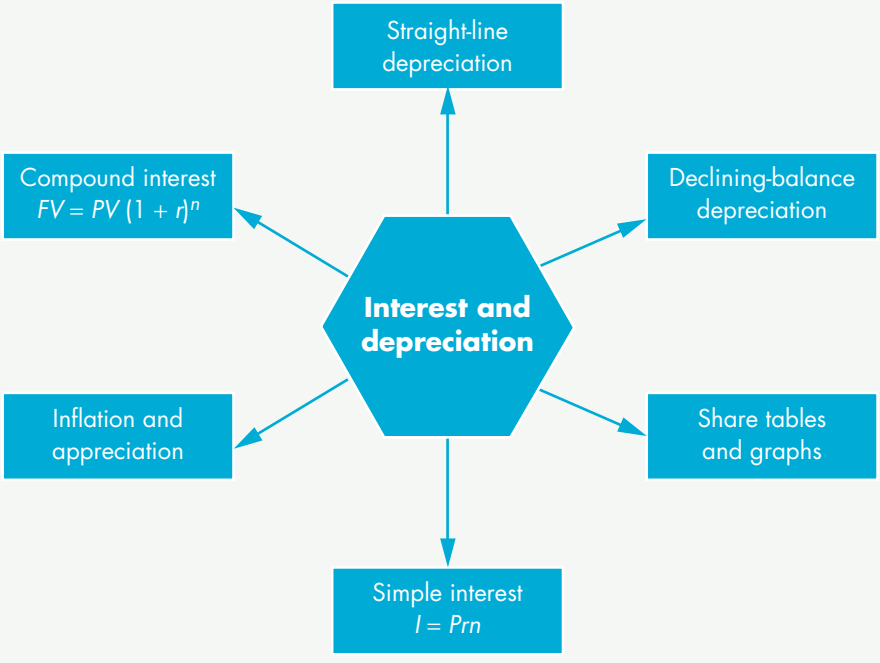
1. CHAPTER SUMMARY

Interest and depreciation crossword

Investing money crossword

This chapter, Interest and depreciation, looked at the mathematics of investing and depreciation in different ways. You applied the simple and compound interest formulas in a variety of situations, such as investments, appreciation and inflation. You investigated the share market, including share prices displayed in tables and graphs, and learnt new terminology such as market price, brokerage and dividend yield. You calculated the depreciation of assets using the straight-line and declining-balance methods.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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1. TEST YOURSELF

Exercise 1.01

1 Calculate the simple interest earned for:

a $5200 invested at 2.4% p.a. for 18 months

b $8500 invested for 5 years at 3.6% p.a

c $24 200 invested at $3\frac{1}{4}$ % p.a. for 300 days.

Exercise 1.01

2 For how long must $12 000 be invested at 5.5% p.a. simple interest to earn $2970?

Exercise 1.01

3 Nathan earned $825 interest after 3 years on a term deposit of $10 000. What was the annual flat rate of interest?

Exercise 1.02

4 $28 000 is invested for 3 years at 4.5% p.a. compounded annually. What is the future value of this investment? Select **A**, **B**, **C** or **D**.

A $3780 **B** $31 780 **C** $31 952.65 **D** $3952.65

Exercise 1.02

5 For each investment, calculate the future value and the compound interest earned.

a $8000 at 2.99% p.a. for 6 years compounded monthly

b $9500 for 2 years at 4.05% p.a. compounded daily

c $14 000 for 18 months at 6.4% p.a. compounded fortnightly

d $32 500 for 15 years at 3.5% p.a. compounded half-yearly

Exercise 1.02

6 What present value must be invested for $2\frac{1}{2}$ years at 4.25% p.a. compounded monthly to grow to a future value of $16 000?

Exercise 1.02

7 An amount of $2000 was left in an account for 7 years and earned $561.82 interest. Calculate the interest rate if the interest was compounded annually.

Exercise 1.03

8 A $5000 diamond ring appreciates in value at 2.5% p.a. Calculate its value in 25 years. Answer to the nearest dollar.

Exercise 1.03

9 The price of an ice block today is $3.55. What was its price 30 years ago if the average annual inflation rate over this time was 2.4% p.a.?

Exercise 1.03

10 A vintage car appreciated in value by 2% p.a. over the last 10 years. If the car is now priced at $24 000, what was its value 10 years ago?

Exercise 1.04

11 Emma has 3000 QBE shares which had a market value of $11.84 each. The dividend yield was 3.2%. Calculate the total dividend earned.

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12 Maddison bought 4000 shares at $3.80 with a dividend yield of 6.5%. Brokerage costs are 1.8% of the purchase price for both buying and selling shares.

Exercise 1.04

a Calculate the total costs of purchasing the shares.

b One year later, Maddison sold her shares for $4.80 each after collecting her dividends. Calculate her total earnings for the year, after costs.

13 Woolworths shares have a market value of $26.70 and pay a dividend of 33 cents per share.

Exercise 1.04

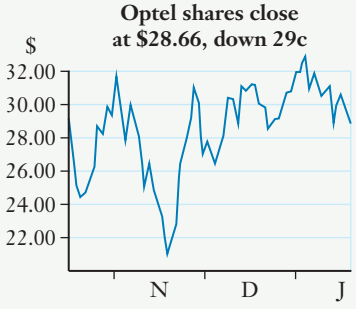
a If Belinda owned 15 000 shares, what is the total market value of her shares?

b Calculate Belinda's dividend.

c What is the dividend yield, correct to 2 decimal places?

14 The graph shows the performance of Optel shares.

Exercise 1.05



a How long is the period covered by this graph?

b What was the lowest share price over this period, and in which month did it occur?

c Describe the performance of the Optel shares over this period.

d How many times did the price reach $26?

e Estimate the value of the shares on 22 December.

f Would 20 October be a good time to buy or sell? Give reasons for your answer.

15 Use the share table below to answer the following questions.

Exercise 1.05

Stock name	Code	Last sale	Percentage change	Year high	Year low
AMP	AMP	5.34	–0.56	5.96	4.39
Bega	BGA	6.87	–0.58	7.06	3.82
Dulux Group	DLX	6.60	–1.79	7.32	5.74
Rio Tinto	RIO	64.30	–1.26	69.85	46.50
SkyNetTV	SKT	3.23	–	4.91	3.08

a What is the market price of a Rio Tinto share?

b Which share had the least change from the previous day?

c Which shares had the smallest range of prices for the year?

d What was the price paid for a Dulux Group share the day before?

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Exercise 1.06

16 Priyanka bought a new computer for $7600 and after 4 years it depreciated to $1525. If the straight-line method of depreciation was used, calculate the yearly depreciation.

Exercise 1.06

17 A furniture making company bought a new saw for $3480. After 3 years it had depreciated to half its value. What was the value after 5 years?

Exercise 1.06

18 A builder bought a new vehicle for $54 000. After how many years does the value of the vehicle fall below $8000 if it depreciated by $12 000 each year?

Exercise 1.07

19 A restaurant bought a new oven for its kitchen for $25 000. What is its value after 3 years if it depreciated by 28% p.a.? Select **A**, **B**, **C** or **D**.

A $548 **B** $15 669 **C** $12 301 **D** $9331

Exercise 1.07

20 A new van costs $44 600, including registration and insurance, with an annual depreciation rate of 35% p.a.

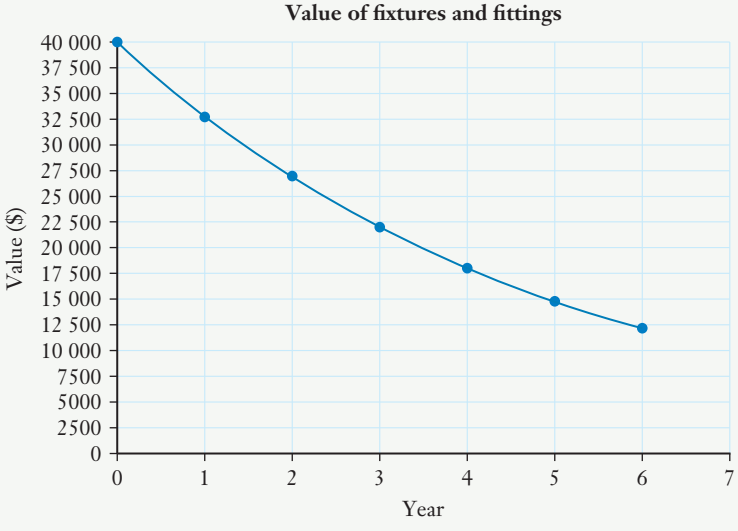
a How much will it be worth after the 1st year?

b Find the amount of the depreciation in the 3rd year.

c After how many years will the van have a value less than $6000?

Exercise 1.07

21 The graph shows the value of the new fixtures and fittings for a newsagency, which depreciate at 18% p.a.



a How much did the new fixtures and fittings cost?

b Use the graph to estimate the value of the fixtures and fittings after:

i 2 years **ii** 5 years.

c What is the depreciation in the 3rd year?

d Use the declining-balance formula to find the value of the fixtures and fittings after 8 years.

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22 Prints Charming purchased a new printer for $30 000. The company want to compare 2 methods of depreciation:

A: straight-line with $7000 depreciation per annum

B: declining-balance with 30% depreciation per annum.

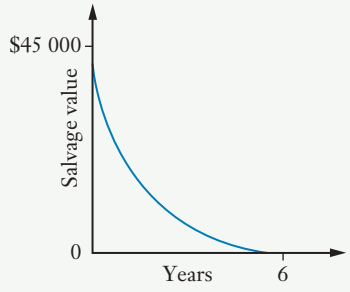
Which method gives the greater depreciation over 3 years? Give reasons.

Exercise 1.07

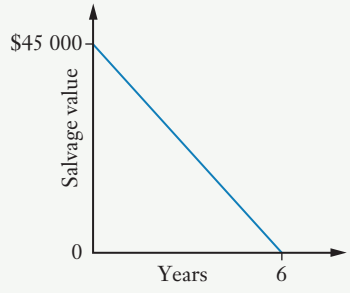
23 A boat was bought for $45 000. Using the declining-balance method of depreciation, which graph best repesents the salvage value of the boat after 6 years? Select **A**, **B**, **C** or **D**.

Exercise 1.07

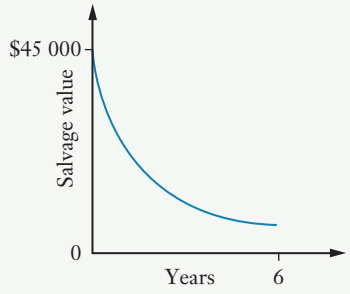
A



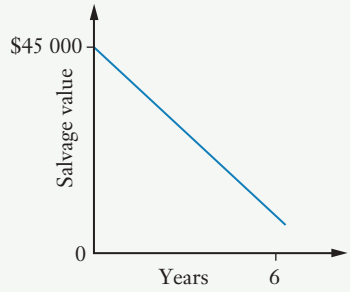
B



C



D



Qz
Chapter quiz

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MEASUREMENT

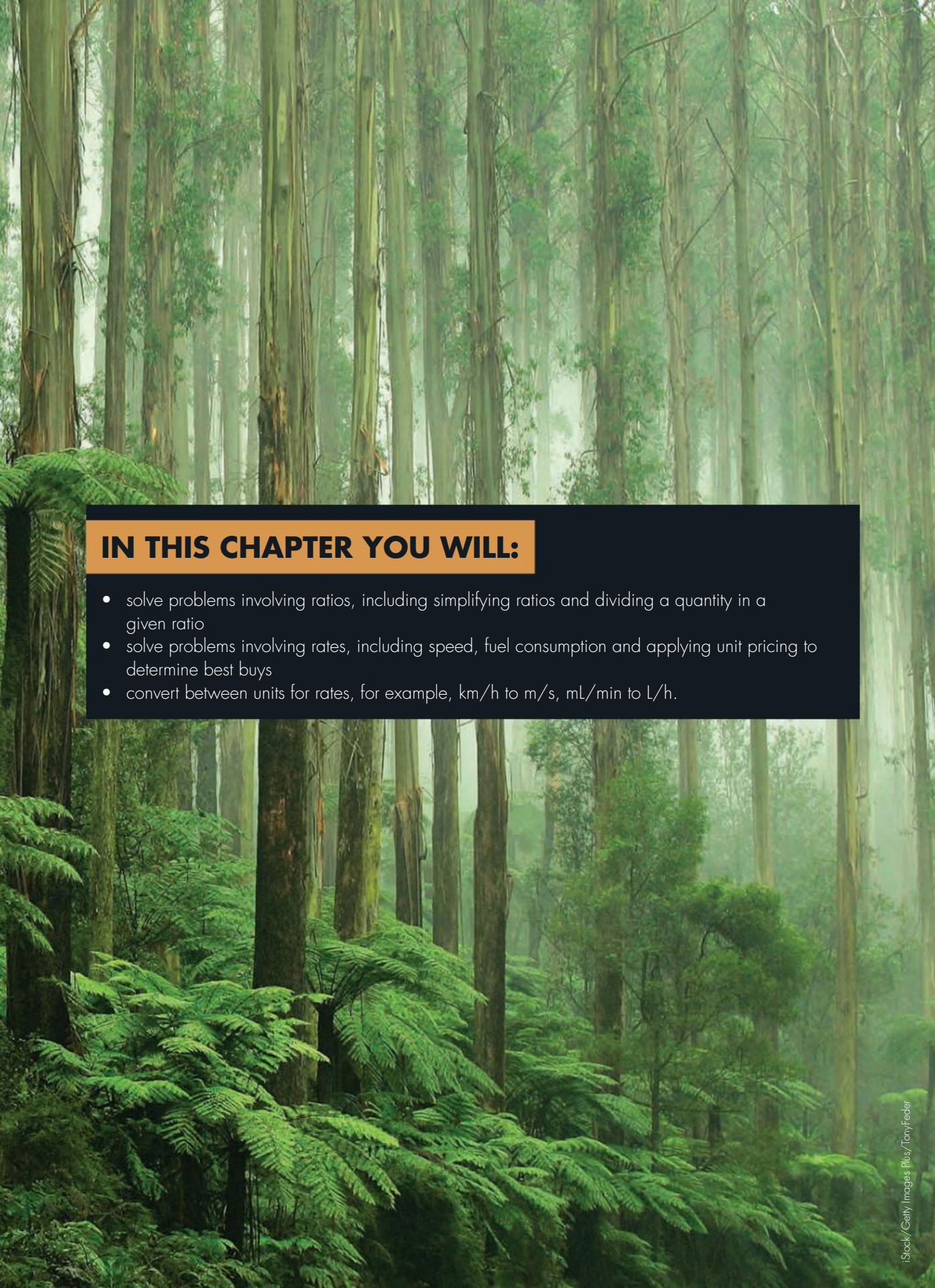
2.

RATIOS AND RATES

Despite greater awareness and concern for the natural environment, unsustainable logging of tropical rainforests continues worldwide. When thinning a forest, trees designated for removal are marked with paint. Suppose the ratio of trees to be felled to those to be left in a particular forest is 2:3 and this forest contains 32 000 trees. After felling, however, it was noted that 5% of the trees to remain had fallen accidently. How many trees were actually felled?

CHAPTER OUTLINE

M7 2.01 Ratio problems
M7 2.02 Dividing a quantity in a given ratio
M7 2.03 Rate problems
M7 2.04 Unit pricing
M7 2.05 Speed and fuel consumption
M7 2.06 Converting rates



IN THIS CHAPTER YOU WILL:

- solve problems involving ratios, including simplifying ratios and dividing a quantity in a given ratio
- solve problems involving rates, including speed, fuel consumption and applying unit pricing to determine best buys
- convert between units for rates, for example, km/h to m/s, mL/min to L/h.

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TERMINOLOGY

best buy
ratio
fuel consumption
speed
heart rate
unit price
rate
unitary method

SkillCheck

Assignment 2

Simplifying fractions

Ratios

1 Simplify each fraction.

a $\frac{14}{6}$ **b** $\frac{12}{48}$ **c** $\frac{25}{60}$

2 **a** If bananas cost \$5.10 per kilogram, how much would 250 grams cost?

b If potatoes cost \$4.15 for 5 kg, how much would 3 kg cost?

c If tomatoes cost \$2.50 for 200 g, how much would 1.5 kg cost?

3 Oliver and Mia bought an apartment for \$750 000. If Oliver paid $\frac{1}{3}$, how much did Mia pay?

4 Tom's mother said she would pay for $\frac{3}{4}$ of the cost of his first car and he had to pay the rest from his savings. If Tom had saved \$3200, what is the maximum amount the car can cost?

5 Copy and complete each statement.

a 2 kg = …… g **b** \$4.57 = …. c **c** 60 km = …… m

d 25 m = …... km **e** 3 h 16 min = …... min **f** 720 s = …… min

Ratio match-up puzzle

2.01 Ratio problems

A **ratio** compares two or more parts or shares of quantities of the same type. The numbers in a ratio are called its **terms** and a **simplified ratio** has terms that have no common factors.

EXAMPLE 1

Simplify each ratio.

a 8 : 12 : 28 **b** $\frac{2}{9} : \frac{5}{12}$ **c** 1 h 30 min : 35 min

Solution

a $8 : 12 : 28 = \frac{8}{4} : \frac{12}{4} : \frac{28}{4}$

$= 2 : 3 : 7$

Dividing each term by the highest common factor, 4

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b $\frac{2}{9} : \frac{5}{12} = \frac{2}{9} \times 36 : \frac{5}{12} \times 36$
$= 8 : 15$

Multiplying each term by 36 (the lowest common multiple of 9 and 12) to make them whole

c 1 h 30 min : 35 min = 90 min : 35 min

Converting to the same units (minutes)

$= \frac{90}{5} : \frac{35}{5}$
$= 18 : 7$

Dividing each term by 5

EXAMPLE 2

Petro and Nicki invested money in a business in the ratio 3 : 5.
If Nicki invested $465 000, find:

a the total cost of the business

b the amount Petro invested.

Solution

a Petro's share : Nicki's share = 3 : 5

5 parts (Nicki) = $465 000

1 part = $465 000 ÷ 5 — Dividing by 5 to find one part
= $93 000

Total cost of business = 8 × $93 000 — The total number of shares is 3 + 5 = 8
= $744 000

b Petro's share (3 parts) = 3 × $93 000 — From above
= $279 000

Check: $279 000 + $465 000 = $744 000 — The two shares add to equal the whole

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Exercise 2.01 Ratio problems

Example 1

1 Simplify each ratio.

a	5 : 15	**b**	4 : 2 : 10	**c**	14 : 14	**d**	105 : 75
e	17 : 68	**f**	12 : 36 : 9	**g**	0.04 : 0.4	**h**	$\frac{4}{5} : \frac{2}{3}$
i	$\frac{3}{8} : 1\frac{1}{4}$	**j**	250 mL : 2 L	**k**	3 days : 2 weeks	**l**	\$4.20 : 95 c
m	45 cm : 3 m	**n**	\$2000 : \$1 000 000			**o**	$\frac{1}{2}$ cup : $2\frac{1}{2}$ cups
p	1 fortnight : 3 weeks : 1 year			**q**	2 ha : 100 m^2	**r**	350 mL : 2.4 L

2 Concrete is made up of sand, cement and lime in the ratio 2.5 : 0.5 : 0.25. Express this ratio in simplest form.

3 What is the simplified form of 27 : 99? Select **A**, **B**, **C** or **D**.

A	9 : 33	**B**	11 : 3	**C**	3 : 11	**D**	1 : 3.6

4 What is the simplified form of 0.5 kL : 1.4 L? Select **A**, **B**, **C** or **D**.

A	500 : 1.4	**B**	1 : 2.8	**C**	7 : 2500	**D**	2500 : 7

5 15 out of 130 female smokers are likely to develop lung cancer. Express this statistic as a simplified ratio.

6 Half of Robert's class walk to school, $\frac{1}{3}$ take the bus and the rest ride a bike. Find the ratio of:

- **a** bike riders to walkers
- **b** bus travellers to the whole class
- **c** walkers to bus travellers.

7 What is the ratio of a square's side length to its perimeter?

Example 2

8 Brodie cuts a length of ribbon in 3 pieces in the ratio 1 : 3 : 5. If the largest piece is 75 cm long:

- **a** how long are the other 2 pieces?
- **b** how long was the entire length of ribbon?

9 At Point Perpendicular High School, the ratio of boys to girls is 23 : 27. If there are 529 boys, what is the total school population?

10 A large jar of jellybeans contains only yellow, white and green jellybeans, in the ratio 12 : 11 : 22. If there are 600 g of yellow jellybeans, what is the mass of:

- **a** the white jellybeans?
- **b** all of the jellybeans?

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11 The ratio of teachers to students in Westvale TAFE college is 1 : 15. If Westvale TAFE has 1200 students, how many teachers does it have?

12 Lawnmower fuel is made up of 25 parts petrol to 1 part oil. How much oil needs to be added to 3 L of petrol to make the fuel? Select **A**, **B**, **C** or **D**.

A 25 mL **B** 80 mL **C** 120 mL **D** 200 mL

13 Jesse's soccer team won 8 games for every 5 games they lost. If the team lost 15 games, how many games did they win?

14 The ratio of fiction to non-fiction books in the library is 3 : 7. If there are 13 980 fiction books, how many books are in the library?

15 A concrete mix contains cement, sand and gravel in the ratio 1 : 4 : 6.

a How many shovels of sand are needed in a mix containing 3 shovels of cement?

b If 2.2 t of gravel is used to make some concrete mix, how many kilograms of sand is used?



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16 Ivy made chocolate crackles containing copha, coconut, rice bubbles and cocoa in the ratio 2 : 2 : 5 : 1. If she used 400 g of copha, find the mass of the:

a cocoa **b** rice bubbles **c** chocolate crackles.

17 **a** Joshua is making a fruit cordial drink for a party, mixing 2 parts of concentrate to 13 parts of water. How much concentrate does Joshua need to add to 2.6 L of water?

b If Lecki makes a similar drink to Joshua, with the same ratio of concentrate to water, how much water does she need to add to 600 mL of concentrate?

18 Thelma and Louise share a company in the ratio 7 : 8. If Louise invests $384 000, how much does Thelma invest?

ISBN 9780170413633

WS Ratio problems

WS Calculating quantities in ratios

WS Ratios in other contexts

2.02 Dividing a quantity in a given ratio

Dividing a quantity in a given ratio

1 Find the total number of parts (by adding).

2 Find the size of *one* part (by dividing).

3 Find the size of each term of the ratio (by multiplying).

Check that your answers add up to the original quantity.

EXAMPLE 3

Irena's estate of $3 600 000 was divided between her children, Max, Rose and Reggie in the ratio 3 : 2 : 1. How much did each receive?

Solution

Total number of parts = 3 + 2 + 1 = 6

1 part = $3 600 000 ÷ 6

= $600 000

Max's share (3 parts) = 3 × $600 000

= $1 800 000

Rose's share (2 parts) = 2 × $600 000

= $1 200 000

Reggie's share (1 part) = $600 000

Check: $1 800 000 + $1 200 000 + $600 000 = $3 600 000

Exercise 2.02 Dividing a quantity in a given ratio

1 A cash prize of $1200 is divided between Vila and Kim in the ratio 2 : 3. What is Vila's share? Select **A**, **B**, **C** or **D**.

A $240 **B** $480 **C** $600 **D** $400

2 Divide each quantity in the given ratio.

a $31 500 in the ratio 4 : 5

b 3750 kg in the ratio 13 : 2

c 34 000 m^2 in 6 : 7 (nearest m^2)

d $2\frac{1}{2}$ cups in the ratio 1 : 4

e 7500 L in the ratio 2 : 3

f 10 ha in the ratio 5 : 3

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Example 3

3 The ratio of boys to girls at Nelson Lakes College is 4 : 5. How many girls attend the school if the total enrolment is 846 students?

4 An alloy contains copper and nickel in the ratio 7 : 4. How much of each metal (correct to the nearest kilogram) is in 500 kg of the alloy?

5 KJ and Brad own a business with shares in the ratio 5 : 3. How should they share a profit of $53 600?

6 Jeremy won $200 000 in Lotto and divided it between himself and his 3 grandchildren in the ratio 3 : 2 : 2 : 1. How much did each person receive?

7 Two-stroke fuel for an outboard motor is made up of oil and petrol in the ratio 1 : 49. How much oil and petrol are needed to make 20 L of fuel?

8 550 kg of concrete is made by mixing cement, sand and aggregate in the ratio 1 : 4 : 6. How many kilograms of each component are needed?

9 Punch is made up of fruit juice, lemonade and ginger ale in the ratio 2 : 2 : 1. How many millilitres of each ingredient are needed to make 6 L of punch?

10 Jas, Lisa and Ally share the weekly rent of a house in the ratio 3 : 2 : 1. If the rent is $660, how much does each person pay?

11 Baby formula is made up of 1 tablespoon of powdered formula to 14 tablespoons of water. How many millilitres of water are needed to make up 300 mL of formula?

12 A weed spray is made up of wetting agent and a herbicide in the ratio 1 : 100. How much wetting agent (to the nearest millilitre) do you need to make 85 L of weed spray?

13 Tim pays 30% of his weekly income of $1080 in tax. The remaining income is divided into savings and living expenses in the ratio 2 : 5.

- **a** How much does Tim save?
- **b** How much does Tim use for living expenses?

14 Tasha's daily diet consists of protein, fat and carbohydrate in the ratio 3 : 0.5 : 9.5.

- **a** Simplify this ratio.
- **b** How much protein does Tasha have in one day if she consumes 3.5 kg of food? Answer to the nearest gram.
- **c** How much would Tasha eat in a day if her diet contained 20 g of fat?
- **d** How much carbohydrate is in Tasha's 507 g bowl of pasta if it contains the same ratio of components as her whole diet?

ISBN 9780170413633

WS Rate problems

WS Rate skills

WS Medicine dosage

2.03 Rate problems

A **rate** is a measurement that compares 2 different quantities. For example, a **heart rate** is measured in beats per minute, written as beats/min.

EXAMPLE 4

a A hose delivers 3840 litres of water in one hour. What is the flow rate in litres per minute?

b The cost of 35 L of petrol is $44.45. Express this cost as a rate in cents per litre.

Solution

a $$\text{Flow rate} = \frac{3840 \text{ L}}{1 \text{ h}}$$

$$= \frac{3840 \text{ L}}{60 \text{ min}}$$ Converting 1 h to 60 min

$$= 64 \text{ L/min}$$

b $$\text{Petrol cost} = \frac{\$44.45}{35 \text{ L}}$$

$$= \frac{4445 \text{ c}}{35 \text{ L}}$$ Converting $44.45 to 4445 c

$$= 127 \text{ c/L}$$

EXAMPLE 5

Rod's heart beats 72 times per minute.

a How long will Rod's heart take to beat 2052 times?

b How many times will Rod's heart beat in one hour?

Solution

a Time taken for 2052 beats $= 2052 \div 72$

$= 28.5$ min

b Beats in one hour $= 60 \times 72$ 60 min = 1 hour

$= 4320$

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Rate problems are solved by either multiplying or dividing. One simple rule is to write the units of the rate as a fraction.


Rate problems

In the heartbeat problem from Example **5**:

$$\text{beats/minute} = \frac{\text{beats}}{\text{minutes}}$$

- To find the number of minutes, we **divide** by the rate per minute: $2052 \div 72$.
- To find the number of beats, we **multiply** by the rate per minute: 60×72.

Exercise 2.03 Rate problems

1 What units are used to express each rate?

a the cost of a mobile phone call
b a wage rate
c typing speed
d the postage rate for a parcel
e the cost of potatoes
f the speed of an athlete

2 Name a rate that uses each pair of units.

a persons/km^2
b cents/word
c litres/100 km
d megabytes/second
e births/1000 persons
f \$/m^2

3 A tree grew 31.2 cm in height in a quarter of a year. Calculate its growth rate in centimetres/week.

Example 4

4 Steve scored 42 runs in 8 overs of a cricket match. Calculate his run rate in runs per over.

5 A town's population grew by 16 500 over 6 years. What was its growth rate in persons/year?

6 Steak costs \$4.55 for 160 g. Calculate the cost as a rate in \$/kg.

7 A computer downloaded a 6.5 megabyte file in 5 seconds. Calculate the download rate in MB/s.

8 Australia's area is 7 682 300 km^2 and its population is 24 641 662. Calculate its population density in persons/km^2, correct to 2 decimal places.

9 A bus travels at a speed of 96 km/h.

Example 5

a How far will the bus travel in $3\frac{1}{2}$ hours?
b How long will the bus take to travel 480 km?

10 Jake types 64 words per minute. How long will it take him to type a 1500-word essay? Give your answer to the nearest minute.

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11 A breakfast cereal contains 3 mg of fat in every 60 g serve.

a Express the fat content as a rate in mg/g.

b How much fat is in a 100 g serving?

12 Ian's swimming pool is being filled at a rate of 50 L/min. How long will it take to fill if it has a capacity of 38 kL? Give your answer in hours and minutes.

13 A cricket team had a run rate of 4.9 runs/over. How many runs did the team score in 30 overs?

14 While travelling in the USA, Mickey exchanged A$750 for US$472.50.

a How much was A$1 worth in US dollars?

b Convert US$200 to Australian dollars, correct to the nearest cent.

15 The recommended daily dose for a child's medicine is 80 mg for every kilogram of bodyweight. Which of the following is Elizabeth's daily dose in grams, if she weighs 30 kg? Select **A**, **B**, **C** or **D**.

A 2.4 **B** 3.75 **C** 2400 **D** 375

16 A tap is dripping water at a rate of 40 drops per minute. Each drop contains 1.3 mL of water. How many litres will drip from the tap in a day?

17 A lawn fertiliser is spread at the rate of 5 kg per 100 m^2 of lawn.

a How much fertiliser is needed for a rectangular lawn measuring 11 m by 7.5 m? Give your answer to the nearest 0.1 kg.

b How many times can the whole lawn described in part **a** be treated if a 25 kg bag of fertiliser is purchased?

18 Australia's birth rate last year was 13.8 per 1000 and its death rate was 7.6 per 1000. If Australia's population was 24 641 662 at the start of the year, how many people (correct to the nearest whole number):

a were born during the year?

b died during the year?

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2.04 Unit pricing

Best buys puzzle

Have you been overwhelmed by the different sizes and prices of the same item at a supermarket? Is the large box of cereal cheaper per volume than the small box? Should you buy a 20-pack of bottled water to save money or a 4-pack to save space?

When shopping, an important skill is comparing the prices of different brands or sizes of items and calculating which is the **best buy** ('best value for money'). This can be done by comparing the **unit price** (price of one item or unit) of each brand or size and choosing the cheapest one. A unit could be one gram or one millilitre.



If you look carefully at price tags on supermarket shelves, you will notice unit prices displayed in small print under the main prices.

Unit price

Unit price = price ÷ number of items or units

Unit pricing

EXAMPLE 6

The same cooking oil is available in 2 sizes of bottle:

- 500 mL for $4.75
- 750 mL for $6.45

a Calculate the unit price for each bottle.

b Which size of bottle is the better value?

Solution

a Calculate the unit price (say, per 100 mL) for each bottle.

500 mL bottle: $4.75 ÷ 5 = $0.95 per 100 mL

750 mL bottle: $6.45 ÷ 7.5 = $0.86 per 100 mL

b The 750 mL bottle is the better buy. It has the lower unit price.

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EXAMPLE 7

Which size of fruit juice is the best buy?

- Small pack: \$1.80 for 280 mL
- Bottle: \$3.85 for 600 mL
- Large carton: \$8.55 for 1.75 L

Calculating the unit price (say, per mL):

Small pack: \$1.80 ÷ 280 = \$0.006 42… per mL

Bottle: \$3.85 ÷ 600 = \$0.006 41… per mL

Large carton: \$8.55 ÷ 1750 = \$0.004 88… per mL 1.75 L = 1750 mL

The large carton is the best buy. It has the lowest unit price.

Note: Buying the largest or cheapest item may not always be the 'best buy'. For example, buying a 10 kg bag of potatoes if you will not be able to eat them all before they go bad is not a sensible choice.

Exercise 2.04 Unit pricing

Examples 6,7

1 A 1 kg tub of ice cream costs \$7.99, while a 300 g carton of the same ice cream is \$2.30.

a Which is the better buy?

b Give one reason why this might be so.

2 Which size packet of biscuits is the best buy?

- Regular: 600 g for \$2.40
- Family: 1 kg for \$4.65
- Snack: 325 g for \$1.40

3 Nami wants to carpet her lounge room (4 m by 5m) and gets two different companies to quote for the same carpet. Which is the better buy: **A** or **B**?

A \$4130 for the room **B** \$185 per m^2

4 Which size of aluminium foil is the best value for money? Select **A**, **B**, **C** or **D**.

A \$4.95 for 20 m **B** \$7.35 for 30 m

C \$1.35 for 5 m **D** \$3.75 for 15 m

5 Batteries are sold in a pack of 4 for \$10.20 or in a pack of 10 for \$25.99. Which is more economical?

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6 Should you buy a 250 g tube of your favourite toothpaste for $7.64 or a 400 g tube for $11.25?

7 Peanut butter is sold in 380 g or 150 g jars at $5.72 and $2.63 respectively. Which is better value?

8 Which brand of rice is the best buy? Select **A**, **B**, **C** or **D**.

A Super Grain: 400 g for $1.75

B Wonder Bowl: 600 g for $2.61

C Hearty White: 1 kg for $4.43

D The Rice Stuff: 20 kg for $84.70

9 For each item, find which size has the best value for money.

a



b



c



d



10 Potatoes cost $15 for a 12.5 kg bag at a local store and $5.40 for a 5 kg bag at a supermarket. Where should you buy the potatoes for better value for money?

11 Shampoo is on sale at two stores. Warmies is offering 3 for 2 on 500 mL bottles and Coolies has 300 mL bottles on offer at 'buy one, get one free'. Where should you buy your shampoo?

12 You can buy 3 1-litre bottles of soft drink for $6.70 or 24 cans (each 330 mL) for $13.60. Which is better value?

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13 Find the cheapest brand for each item.

		Sols	Baldy	GIA	Bullworths
a	Salt	500 g for $1.89	200 g for 76 c	100 g for 39 c	1 kg for $3.74
b	Juice	250 mL for $1	50 mL for 17 c	1 L for $2	200 mL for 70 c
c	Bananas	750 g for $5.20	400 g for $2.40	250 g for $1.70	2 kg for $12.20
d	Aluminium foil	20 m for $3.29	5 m for $0.80	30 m for $4.90	18 m for $2.85

14 Teneille's netball team need new T-shirts, which cost $27.50 each. With Supplier A she can pay for 2 and get 1 free, with Supplier B she gets a 20% discount on every shirt if she buys more than 20, and Supplier C gives 3 free shirts for every 5 sold. Teneille needs 26 shirts. Who has the best deal?

15 Choose 2 products your family uses frequently and compare 3 different sizes of each of the products to determine the best buy.

16 You use a supermarket website to compare brands of toothpaste. Brand X has a unit price of 24 c/g and Brand Y has a unit price of 26 c/g. Give 2 reasons why you would buy Brand Y even though it is more expensive.

17 Karen comes home with a 10 kg bag of apples that were discounted and were a good buy. Zoe usually buys a 2 kg bag each week even though the unit price is slightly higher. Give one reason why Karen's purchase may not be the better buy.

DID YOU KNOW?

Unit pricing

In 2009, the Australian Government introduced the national unit pricing scheme, which made it compulsory for supermarkets to display unit prices when selling certain grocery items. Unit pricing enables consumers to compare the price of a range of similar items by using a common measure:

- volume: per 100 millilitres
- weight: per 100 grams
- length: per metre
- area: per square metre
- for 40 or fewer items: per item
- for more than 40 items: per 100 items

The unit price must be displayed in dollars and whole cents.

For example:





Check the unit price of each size of mushroom soup, and decide on the better buy.

ISBN 9780170413633

2.05 Speed and fuel consumption

Speed

Speed is the rate at which something is moving or changing.

Average speed

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$S = \frac{D}{T}$$

This formula can also be written as $D = ST$ or $T = \frac{D}{S}$.

Speed is usually measured in metres/second (m/s) or kilometres per hour (km/h).
At sea, speed is measured in knots (nautical miles/hour).

EXAMPLE 8

Belinda is driving from Melbourne to Sydney, a distance of 877 km, at an average speed of 94 km/h. How long, to the nearest minute, will it take her to reach Sydney?

Solution

$S = 94$ km/h; $D = 877$ km

$S = \frac{D}{T}$ or $T = \frac{D}{S}$

$94 = \frac{877}{T}$

$T = \frac{877}{94}$

$= 9.3297\ldots$ h

≈ 9 h 19 min 47.23 s — By pressing ° ′ ″ or D°M'S on the calculator

≈ 9 h 20 min — Rounded up because 47.23 s $\geq$ 30 s

It will take Belinda 9 h 20 min to reach Sydney.

WS Speed problems

WS Distance, speed and time

WS Speed formula practice

WS Racing rates

WS Speed stories

Speed, distance and time

ISBN 9780170413633

Fuel consumption

WS
How much petrol?

Fuel consumption

Fuel consumption is the rate at which a vehicle uses fuel, measured in litres per 100 km (L/100 km). For example, a large car may use 12.3 L of fuel to travel 100 km whereas a small car may use only 5.8 L. The small car has a better fuel consumption and is more fuel efficient. The table shows the fuel consumption for some of Australia's best-selling cars.

Vehicle	Fuel consumption (L/100km)		
	Average	City	Highway
Ford Focus	6.2	8.7	4.8
Ford Ranger diesel	6.6	7.8	5.8
Holden Commodore	9.0	12.5	7.0
Holden Cruze	7.0	9.5	5.5
Hyundai i30	7.3	10.3	5.5
Mazda 2	5.4	7.2	4.4
Mazda 3	5.8	7.6	4.8
Mitsubishi Lancer	10.1	13.9	8.0
Nissan Navara	9.7	13.0	7.7
Nissan X-trail	8.1	11.0	6.4
Toyota Camry Hybrid	5.2	5.7	4.9
Toyota Corolla	4.1	3.9	4.1
Toyota Hilux	11.1	15.5	9.4
Toyota Hilux diesel	9.0	10.9	7.8
Toyota Prado	11.6	15.5	9.4
Toyota Yaris	5.8	7.1	5.1
Volkswagen Golf	4.9	5.8	4.5

Source: www.greenvehicleguide.gov.au (c) Copyright 2017, Commonwealth of Australia

The fuel consumption of a vehicle depends on several factors. You use more fuel when:

- stop–start driving in traffic (urban cycle)
- driving on unsealed roads
- using a poorly-tuned engine
- driving with hard acceleration or hard braking
- using under-inflated tyres
- towing a trailer or caravan
- using the air conditioner.

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EXAMPLE 9

On a full tank of 42 L of petrol, Ali's car can travel a distance of 560 km.

a Calculate the fuel consumption for Ali's car.

b How far can Ali's car travel on 12 L of petrol?

c How much petrol will it use for a trip of 1530 km?

Solution

a Fuel consumption $= \dfrac{42 \text{ L}}{560 \text{ km}}$

$= \dfrac{42 \text{ L}}{5.6 \times 100 \text{ km}}$ Units are L/100 km, so write 560 as 5.6×100

$= \dfrac{(42 \div 5.6) \text{ L}}{100 \text{ km}}$

$= 7.5$ L/100 km

b 7.5 L is used for 100 km

1 L is used to go $\dfrac{100}{7.5}$ km Using the unitary method: find 1 L first.

12 L is used to go $12 \times \dfrac{100}{7.5} = 160$ km

Ali's car can travel 160 km on 12 L of petrol.

c Fuel for 100 km = 7.5 L

Fuel for 1 km $= \dfrac{7.5}{100}$ L Using the unitary method: find 1 km first.

Fuel for 1530 km $= 1530 \times \dfrac{7.5}{100}$

$= 114.75$ L

114.75 L of petrol is used for a trip of 1530 km.

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Exercise 2.05 Speed and fuel consumption

Examples 8

1 Cathy drove at an average speed of 57 km/h. At this rate, find how long in hours and minutes it will take her to drive:

a 507 km **b** 160 km **c** 440 km.

2 Liam ran a 36 km road race in $2\frac{1}{4}$ hours.

Examples 9

a What was his average speed in km/h?

b How far would he run in 3 h 10 min at this rate?

c How many hours and minutes would it take him to run 22 km?

3 Jess drove 93 km from Wollongong to Nowra in 1 h 30 min. What was her average speed in km/h? Select **A**, **B**, **C** or **D**.

A 46.5 **B** 31 **C** 62 **D** 60

4 If a ship travels 311.85 nautical miles between 8:00 am and 6:30 pm, what is its average speed in knots (nautical miles per hour)?

5 Use the table of fuel consumption on page 64 to answer each question, using the 'Average' rate. Answer correct to one decimal place where necessary.

a The Ford Focus has a 55 L fuel tank. How far can it travel on a full tank?

b If the Toyota Camry Hybrid travelled 732 km, how much fuel did it use?

c How far can a Mitsubishi Lancer travel on 40 L of fuel if driving:

i in the city? **ii** on the highway?

d Generally for all cars, is it more economical to drive in the city or on the highway? Why?

e Julie has a Volkswagen Golf and made 6 trips of 5.4 km, 11.3 km, 8.5 km, 5.1 km, 32.6 km and 67.4 km in one week. How much fuel did she use?

6 A spacecraft travels at 12 km/s.

a How far will it travel in one hour?

b How many days (correct to 2 decimal places) will it take to travel from Earth to Mars, a distance of 78 300 000 km?

7 A motorcycle has a fuel consumption of 3.2 L/100 km. How far can it travel on 1 L of petrol? Select **A**, **B**, **C** or **D**.

A 32 km **B** 16.1 km **C** 31.25 km **D** 3.2 km

8 The fuel consumption of larger vehicles is greater than that of smaller vehicles. Why do you think this is so?

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9 A Honda Civic hybrid has a fuel consumption of 4.6 L/100 km. How many litres of fuel will it use to travel 480 km?

10 A Ford Falcon ute consumes 11.7 L of petrol for every 100 km travelled. How far will it travel on a full petrol tank of 64 L? Give your answer to the nearest kilometre.

11 A Mazda car has a fuel consumption of 11.5 L/100 km.

a How much fuel (to the nearest litre) does the Mazda use to travel 980 km?

b How far (to the nearest kilometre) could the Mazda travel on a 55 L tank of fuel?

12 John cycled 24 km to Seth's place at an average speed of 16 km/h. Seth was not home, so John cycled back at 12 km/h. Find:

a the total time for the round trip

b John's average speed, to the nearest km/h, for the round trip.

13 The fastest land animal is the cheetah, which can run at 96 km/h. How far would it run in one minute at this speed?

14 Jack and Jill are going to a beach 5 km away. Jill will ride her bike at 12 km/h while Jack will jog at 8 km/h. How much head start should Jill give Jack so that they arrive at the beach at the same time?

15 A Kia Sorrento has a fuel consumption of 10.9 L/100 km. Petrol costs $1.49/L.

a How much petrol can be bought for $40? Give your answer correct to the nearest 0.1 litre.

b How far can the car travel on $40 worth of petrol? Give your answer correct to the nearest kilometre.

16 A plane flies from Sydney to Perth, a distance of 4127 km, at a speed of 940 km/h. How long will it take to get there?

17 A satellite orbits the Earth at a constant height of 750 km.

a Given that the Earth's radius is 6400 km, use the formula $C = 2\pi r$ to calculate the length of the satellite's circular orbit, correct to 2 decimal places.

b If the satellite is travelling at a speed of 25 000 km/h, find how long it will take to make one orbit, in hours and minutes.

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Converting rates

Converting rates

More rates

2.06 Converting rates

EXAMPLE 10

Convert:

a 125 L/h to L/day **b** 24 c/min to $/h **c** 80 km/h to m/s

Solution

a 125 L/h = (125 × 24) L/day — Converting from 'per hour' to 'per day'

= 3000 L/day

b 24c/min = $0.24/min — Converting cents to dollars

= ($0.24 × 60)/h — Converting minutes to hours

= $14.40/h

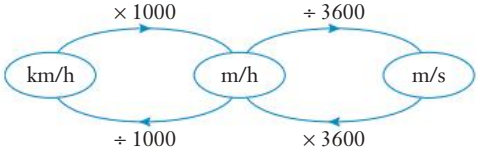
c 80 km/h = (80 × 1000) m/h — Converting kilometres to metres

= 80 000 m/h

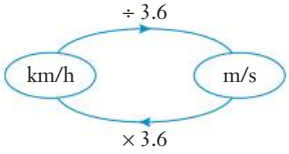
= (80 000 ÷ 3600) m/s — Converting from 'per hour' to 'per second'

$= 22\frac{2}{9}$ m/s (or $22.\dot{2}$ m/s)

For km/h ↔ m/s conversions, you may find this diagram useful.



Or you may prefer to combine the two operations into the single operation of dividing or multiplying by 3.6.



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Exercise 2.06 Converting rates

Example 10

1 Convert:

a $46 212/year to $/month
b 75 km/h to km/min
c 5 L/h to L/day
d 3 m/s to m/min
e 0.8 cm/min to cm/h
f 78 words/min to words/s
g 8.2 L/100 km to L/km
h 1750 mL/s to mL/min.

2 Convert:

a 24 c/min to $/hour
b 8 m/s to km/h
c 15 mL/s to L/h
d $10.80/h to c/min
e 60 km/h to m/s
f 3 t/day to kg/min.

3 The world's fastest cricket bowler is Pakistan's Shoaib Akhtar. In 2003, he bowled a ball 20.12 m in 0.45 seconds. Calculate the speed of the ball he bowled, to the nearest km/h.

4 The speed of sound, called Mach 1, is 1224 km/h. Which of the following is this speed in m/s? Select **A**, **B**, **C** or **D**.

A 34 **B** 44 **C** 340 **D** 441

5 It takes 20 seconds to fill a 10 L bucket with water. What is the rate for filling the bucket, in litres per hour?

6 Jamaican athlete Usain Bolt is the world's fastest man, running 100 metres in 9.58 seconds in 2009. What was his speed in km/h, correct to one decimal place?

7 The lift in Sydney Tower travels 298 m in 42 seconds. What is its average speed in km/h, correct to 2 decimal places?



iStock/Getty Images Plus/Frogman1484

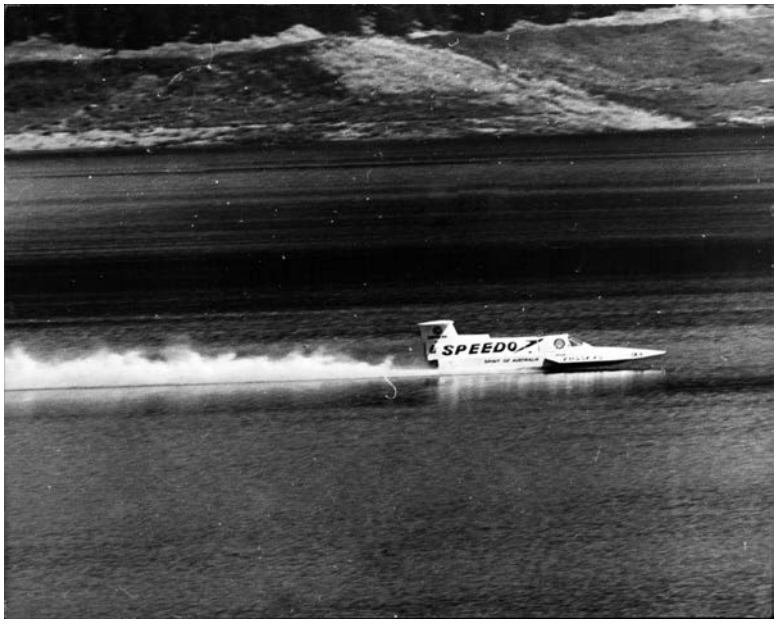
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8 A world record for water speed, 250.7 knots (nautical miles per hour), was achieved in 1977 by Ken Warby in his hydroplane *Spirit of Australia* at Blowering Lake Dam in Tumut, NSW.

a Convert 250.7 knots to km/h (correct to one decimal place) given that 1 nautical mile = 1.852 km.

b At that speed, how far (to the nearest metre) did the hydroplane travel in:

i one minute? **ii** one second?



Alamy Stock Photo/KEYSTONE Pictures USA

9 A car is travelling at 75 km/h. How far, to the nearest 0.1 metre, will it travel in the 5 seconds it takes you to cross the road?

10 If the reaction time of a car driver is 0.9 seconds, how far will the car travel in this time if its speed is:

a 60 km/h? **b** 84 km/h? **c** 100 km/h?

SAMPLE HSC PROBLEM

Police investigating an accident found that a car travelled 20.7 metres in 1.2 seconds.

a How many **significant figures** are there in the distance measurement of 20.7 metres?

b Calculate, correct to 3 significant figures, the average speed of the car:

i in m/s **ii** in km/h.

c How far could this car travel in 3.5 s at the same speed?

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Study tip

Learn from your mistakes

Any man can make mistakes, but only an idiot persists in his error.

Cicero (106–43 BCE), Roman statesman



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Life is a series of successes and failures. While the successes can be enjoyable and uplifting, there are many lessons to be learned from the failures. If you perform badly in an exam, learn from the experience. You can't change the past but you can use its mistakes to improve your future.

Don't throw away your old exams. Use them to find out where and how you went wrong. Identify your areas of weakness and make an effort to study them in more detail. Work on your exam technique and try redoing exams when you study.

Your teachers are a valuable resource. They know the HSC course best as well as your strengths and weaknesses. Also use your friends and classmates. Study in groups if you find it easier. Ask an older student for help.

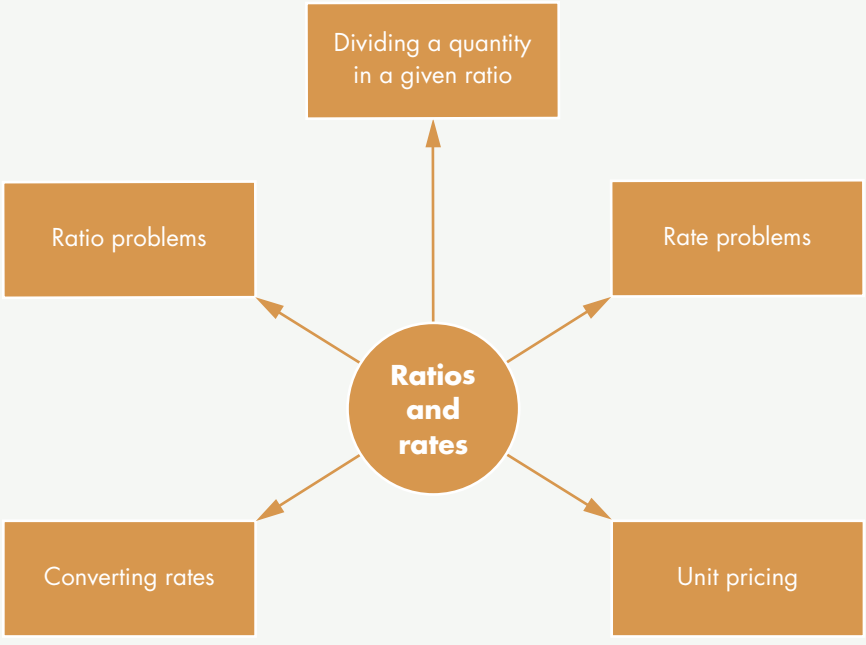
Use the library, Maths faculty or a bookstore to find study guides, summaries and past exams. Visit websites such as NESA, HSC revision and the Mathematical Association of NSW.

ISBN 9780170413633

2. CHAPTER SUMMARY

This chapter, Ratios and rates, covered practical problems involving ratios and rates, including calculating unit prices (to determine best buys), speed and fuel consumption.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



ISBN 9780170413633

2. TEST YOURSELF

1 Simplify each ratio.

a 15 : 60 **b** $1\frac{1}{2} : 3\frac{1}{4}$ **c** 5 min : 100 s **d** 50 kg : 1 t

Exercise 2.01

2 A milkshake contains 5 parts milk, 1 part chocolate flavouring and 2 parts ice cream.

a How much chocolate flavouring is there in 520 mL of milkshake?

b How much milkshake would contain 260 mL of milk?

Exercise 2.02

3 Divide $44 350 between Daniel and Anastasia in the ratio 3 : 2.

Exercise 2.02

4 A car's rental rate is $78 per day. What is the cost of hiring it for a 2-week holiday?

Exercise 2.03

5 A hose delivers water at a rate of 34 litres per minute. How many hours and minutes will it take to fill a backyard pool with 40 kL of water?

Exercise 2.03

6 Which block of land is the best buy?

A 72 ha for $398 000

B 45 ha for $215 000

C 150 ha for $804 000

Exercise 2.04

7 Sam cycled 71 km at an average speed of 17 km/h. How long was his journey in hours and minutes?

Exercise 2.05

8 A Toyota Hilux can travel 594 km on 63 L of diesel.

a Calculate its fuel consumption in L/100 km, correct to one decimal place.

b How many litres does the Hilux require to travel 250 km?

Exercise 2.05

9 An Internet kiosk charges users at a rate of $7.50 per half-hour. Convert this to a rate of cents per minute.

Exercise 2.06

10 Brazilian swimmer Cesar Cielo became the world's fastest swimmer when he swam 50 metres in 20.91 seconds in 2009.

a Calculate, correct to 2 decimal places, his average speed:

i in m/s **ii** in km/h.

b If Cesar could maintain this speed, how long (to the nearest minute) would it take him to swim 1 km?

Exercise 2.06

Qz Chapter quiz

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ALGEBRA

3.

EQUATIONS AND LINEAR FUNCTIONS

Many relationships in real life, where one quantity or variable directly affects another, can be described by a table of values, a graph or an algebraic formula. For example, the temperature of air varies with the height above sea level and the time between seeing lightning and hearing thunder varies directly with the distance from the storm. When the relationships are modelled using a linear function, the gradient and y-intercept have special significance.

CHAPTER OUTLINE

A1* 3.01 Solving equations
A1* 3.02 Formulas and equations
A1* 3.03 Changing the subject of a formula
A1* 3.04 Linear functions
A1* 3.05 Direct linear variation
A4.1 3.06 Intersection of lines

*Year 11 revision



IN THIS CHAPTER YOU WILL:

- solve linear equations, including after substituting into a formula
- change the subject of a linear formula
- graph linear functions of the form $y = mx + c$
- use linear functions to model practical situations and interpret the meaning of the gradient and y-intercept of the line in context
- graph and solve problems involving direct linear variation $y = kx$
- solve simultaneous equations graphically by identifying points of intersection
- solve problems involving simultaneous equations, including finding break-even points in cost vs revenue problems.

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TERMINOLOGY

break-even point
constant term
equation
gradient
point of intersection
subject of a formula
y-intercept
coefficient
dependent variable
formula
independent variable
proportional to
varies
constant of variation
direct linear variation
function
linear function
simultaneous equations
vertical intercept

Assignment 3

Graphing linear equations

Gradient gag

Algebra review

SkillCheck

1 Simplify each expression.

a $4m + 7 - 3m + 9$ **b** $6n \times 5mn$ **c** $\frac{12hp}{20p}$ **d** $2x + 5a - 6a + x$

2 Expand each expression.

a $3(2r - 5)$ **b** $-6(y + 7)$ **c** $-2(8 - k)$

3 Evaluate $c^2 + c - 2$ if:

a $c = 7$ **b** $c = -3$ **c** $c = 10$

4 Solve each equation.

a $3x + 17 = 38$ **b** $8d - 4 = 3d - 9$ **c** $20 - 2w = -16$

5 For the linear function $y = 3x + 1$, what is:

a the dependent variable? **b** the gradient?

c the *y*-intercept? **d** the value of x when $y = -14$?

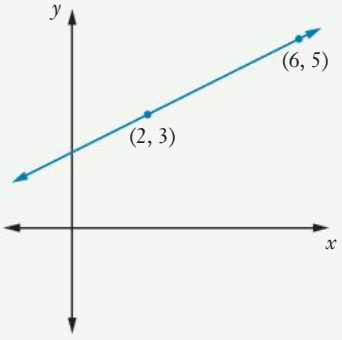
6 Graph $y = 3x + 1$ and $y = 5 - x$ on the same number plane and write the coordinates of their point of intersection.

7 Find the linear function for this table of values.

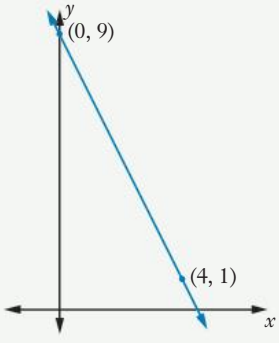
x	1	2	3	4	5
y	−2	1	4	7	10

8 Find the gradient and equation of each line below.

a



b



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3.01 Solving equations

Equations code puzzle

Equations riddle

An **equation** contains an algebraic expression and an equals (=) sign, for example, $2x + 7$ is an **expression**, while $2x + 7 = -15$ is an equation. An equation is **solved** when a value of the **variable** (for example, x) is found that makes the equation true.

Solving an equation

1 Keep the equation balanced by performing the same operation on both sides.

2 Aim to have the variable (for example, x) on one side of the equation and a number on the other side, for example, $x = 6$.

EXAMPLE 1

Solve each equation.

a $2x + 7 = -15$ **b** $\frac{b}{2} - 6 = 1$ **c** $\frac{h+10}{4} = 3$

Solution

a
$$2x + 7 = -15$$
$$2x + 7 - 7 = -15 - 7$$ Subtract 7 from both sides
$$2x = -22$$
$$\frac{2x}{2} = \frac{-22}{2}$$ Divide both sides (each term) by 2
$$x = -11$$

b
$$\frac{b}{2} - 6 = 1$$
$$\frac{b}{2} - 6 + 6 = 1 + 6$$ Add 6 to both sides
$$\frac{b}{2} = 7$$
$$\frac{b}{2} \times 2 = 7 \times 2$$ Multiply both sides (each term) by 2
$$b = 14$$

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c $\dfrac{h+10}{4} = 3$

$\dfrac{h+10}{4} \times 4 = 3 \times 4$ Multiply both sides by 4

$h + 10 = 12$

$h + 10 - 10 = 12 - 10$ Subtract 10 from both sides

$h = 2$

EXAMPLE 2

Solve each equation.

a $5y + 5 = 2y + 23$ **b** $\dfrac{2m-7}{3} = -9$ **c** $7(1 - 2t) = 28$

Solution

a $5y + 5 = 2y + 23$

$5y + 5 - 2y = 2y + 23 - 2y$ Subtract $2y$ from both sides so all the y's are on LHS

$3y + 5 = 23$

$3y + 5 - 5 = 23 - 5$ Subtract 5 from both sides

$3y = 18$

$\dfrac{3y}{3} = \dfrac{18}{3}$ Divide both sides (each term) by 3

$y = 6$

b $\dfrac{2m-7}{3} = -9$

$\dfrac{2m-7}{3} \times 3 = -9 \times 3$ Multiply both sides by 3

$2m - 7 = -27$

$2m - 7 + 7 = -27 + 7$ Add 7 to both sides

$2m = -20$

$\dfrac{2m}{2} = \dfrac{-20}{2}$ Divide both sides (each term) by 2

$m = -10$

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c $7(1-2t)=28$

$7-14t=28$ Expand LHS first

$-14t=21$ Subtract 7 from both sides

$t=\frac{21}{-14}$ Divide both sides by (–14)

$t=-1\frac{1}{2}$

Exercise 3.01 Solving equations

1 Which of the following is the solution to the equation $7+5w=4$? Select **A**, **B**, **C** or **D**. Example 1

A $w=-\frac{3}{5}$ **B** $w=-1\frac{2}{3}$ **C** $w=2.2$ **D** $w=6$

2 In which line was an error made in solving this equation? Select **A**, **B**, **C** or **D**.

$8c+16=c-12$ Line 1

$9c+16=-12$ Line 2

$9c=-28$ Line 3

$c=\frac{-28}{9}$ Line 4

$c=-3\frac{1}{9}$ Line 5

A Line 2 **B** Line 3 **C** Line 4 **D** Line 5

3 Solve each equation. Example 1

a $3d-2=16$ **b** $2p+9=14$ **c** $4u+11=25$

d $3+5a=-12$ **e** $12b+19=1+3b$ **f** $3-2a=-6$

g $3m=m-10$ **h** $\frac{3h}{4}=9$ **i** $\frac{r-1}{6}=2$

j $-\frac{2x}{5}=8$ **k** $\frac{y+2}{-3}=1$ **l** $11-4n=15$

m $5y+6=4y+11$ **n** $\frac{2c}{5}=3$ **o** $\frac{z}{3}-11=9$

4 Solve each equation. Example 2

a $\frac{4h+7}{9}=2$ **b** $\frac{2r-3}{5}=-4$ **c** $-4(2-3y)=0$

d $3k+8=2(k-10)$ **e** $\frac{w}{3}-8=6$ **f** $6(4+2k)=-6$

g $\frac{3m+2}{5}=10$ **h** $\frac{9-3x}{3}=4$ **i** $-5(4-2c)=-10$

j $13y-9=46-2y$ **k** $5(d+3)=4(d+9)$ **l** $\frac{5a+8}{6}=-4$

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5 Which of the following is the solution to the equation $\frac{7-3p}{5} = -9$? Select **A**, **B**, **C** or **D**.

A $p = \frac{7}{3}$ **B** $p = 7$ **C** $p = \frac{52}{3}$ **D** $p = \frac{38}{3}$

3.02 Formulas and equations

Sometimes, after values are substituted into a formula, the result is an equation that must be solved.

EXAMPLE 3

The mean, M, of 3 numbers, x, y and z is given by the formula $M = \frac{x+y+z}{3}$.

If 3 numbers have a mean of 22 and 2 of the numbers are 25 and 26, what is the third number?

Solution

Here $M = 22, x = 25, y = 26$.

$$M = \frac{x+y+z}{3}$$

$$22 = \frac{25+26+z}{3}$$

$$22 = \frac{51+z}{3}$$

$$\frac{51+z}{3} = 22$$

$$\frac{51+z}{3} \times 3 = 22 \times 3$$

$$51 + z = 66$$

$$51 + z - 51 = 66 - 51$$

$$z = 15$$

The third number is 15.

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Exercise 3.02 Formulas and equations

Example 3

1 The formula for converting Fahrenheit (°F) temperatures to Celsius (°C) is $C = \frac{5}{9}(F - 32)$. Convert 38°C to °F.

2 The number of days fresh milk will keep if stored at T°C is $D = \frac{6}{T + 1}$. If a carton of fresh milk lasted 4 days, at what temperature was it stored?

3 The volume of a cylinder is $V = \pi r^2 h$ where r is the base radius and h is the height. If a cylinder has a volume of 904.78 cm^3 and a base radius of 8 cm, find its height correct to one decimal place.

4 The formula for converting speeds from k km/h to M m/s is $M = \frac{5k}{18}$. Convert 12 m/s to km/h.

5 The distance, d km, that an observer can see to the horizon from a height of h m is $d = 8\sqrt{\frac{h}{5}}$. Which of the following heights allows you to see the horizon at a distance of 100 km? Select **A**, **B**, **C** or **D**.

A 358 m **B** 390 m **C** 625 m **D** 781 m

6 The simple interest earned when \$$P$ is invested at a rate of r per period for n periods is $I = Prn$ where r is a decimal. For how long must a principal of \$2300 be invested at 6.5% p.a. to earn simple interest of \$1196?

7 The average speed, S, of a moving object is $S = \frac{D}{T}$ where D is the distance travelled and T is the time taken.

a Find the distance travelled by a car in 3 hours if its average speed is 90.2 km/h.

b Find the time taken (to the nearest minute) for a cyclist to travel 250 km if his average speed is 17.5 km/h.

8 Young's rule for calculating the medicine dosage for infants is $D = \frac{yA}{y + 12}$, where y is the age of the infant in years and A is the adult dosage. How old is Libby if her medicine dosage is 0.6 mL and the adult dosage is 15 mL?

9 The formula for converting Australian dollars (\$A) to New Zealand dollars (\$NZ) is NZ = 1.2362A. Convert the following \$NZ amounts to \$A, correct to the nearest cent.

a \$NZ20 **b** \$NZ38 **c** \$NZ100

10 The body mass index (BMI) of an adult is $B = \frac{m}{h^2}$ where m is the mass in kilograms and h is the height in metres. If Kate is 1.76 m tall and has a BMI of 24.2, calculate her mass to the nearest kg.

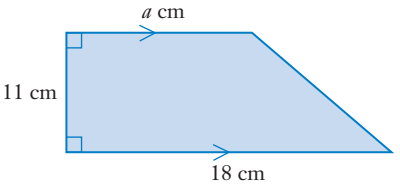
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11 The blood alcohol content (BAC) of a person who consumes alcoholic drinks is BAC = $0.0012na$ where n is the number of drinks taken and a is the amount of alcohol in each drink. Mark drank four glasses of wine, each containing 150 mL, and registered a BAC level of 0.0792.

a How many millilitres of alcohol were present in each glass?

b What percentage of the wine was alcohol?

12 The area of a trapezium is given by the formula $A = \frac{h}{2}(a + b)$ where a and b are the lengths of the parallel sides and h is the distance between them. If the area of the trapezium is 220 cm^2, find the length of the side marked a.



13 The volume (in cubic centimetres) of wood in a tree is $V = 0.4724d^2h + 9.86$ where d is the diameter of the trunk and h is the height of the lowest branch. All dimensions are in centimetres. Calculate, correct to the nearest cm, the height of the lowest branch in a tree whose trunk diameter is 55 cm and whose volume is 400 000 cm^3.

14 The air temperature, T°C, outside an aeroplane is given by the formula $T = 15 - 0.006h$ where h is the height of the plane in metres above sea level. Calculate, correct to the nearest metre, the height of a plane if the outside temperature is –39°C.

15 The density ρ kg/m^3 of a substance can be calculated using the formula $\rho = \frac{m}{V}$, where m is the mass (in kg) and V is the volume (m^3). Calculate correct to 2 significant figures the volume of a cork block if its density is 240 kg/m^3 and its mass is 20 kg.

16 A formula used to find the total resistance in an electrical circuit is $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2}$. Find the value of R_1 if R_{total} is 15 and R_2 is 20.

17 Newton's second law of motion is $a = \frac{F}{m}$, where a is the acceleration in m/s^2, F is the force in newtons (N) and m is the mass in kg. What is the mass of an object (correct to 2 significant figures) if a force of 40 N results in an acceleration of 18 m/s^2?

18 The surface area of a rectangular prism of length l, width w and height h is

$$S = 2lw + 2lh + 2wh.$$

If a rectangular prism with surface area 132 cm^2 has length 7 cm and width 3 cm, find its height.

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DID YOU KNOW?

The tallest building in the world

In 2010, the Burj Khalifa Tower in Dubai, in the United Arab Emirates, became the world's tallest building at 830 m high with a record-breaking 163 floors.

Some other tall landmarks are:

Shanghai Tower, Shanghai	632 m
Abraj Al-Bait Clock Tower, Mecca	601 m
CN Tower, Toronto	553 m
One World Trade Centre, New York	546 m
Petronas Towers, Kuala Lumpur	452 m
Eiffel Tower, Paris	324 m
Q1 Tower, Surfers Paradise	322 m
Sydney Tower	309 m



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Use the formula $d = 8\sqrt{\frac{h}{5}}$ to calculate the distance that can be seen to the horizon from the top of the Burj Khalifa Tower.

3.03 Changing the subject of a formula

To change the subject of a formula to another variable, use the same rules as for solving an equation. In the formula $y = mx + c$, y is called the **subject of the formula** because it is on the LHS. The answer is not a number but an algebraic expression (another formula).

WS Subject of a formula

Changing the subject of a formula

EXAMPLE 4

The formula for a straight line with gradient m and y-intercept c is $y = mx + c$. Change the subject of this formula to:

a c **b** m.

Solution

a $y = mx + c$

$mx + c = y$ Swap sides so new subject c appears on LHS

$c = y - mx$ Subtract mx from both sides

b $y = mx + c$

$mx + c = y$ Swap sides so new subject m appears on LHS

$mx = y - c$ Subtract c from both sides

$m = \frac{y - c}{x}$ Divide both sides by x

EXAMPLE 5

The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base and h is the height. Make h the subject of the formula.

Solution

$V = \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 h = V$ Swap sides so new subject h appears on LHS

$\pi r^2 h = 3V$ Multiply both sides by 3 to remove fraction

$h = \frac{3V}{\pi r^2}$ Divide both sides by πr^2

Exercise 3.03 Changing the subject of a formula

Example 4

1 Make x the subject of each formula.

a $y = 3x - 5$ **b** $T = 2x + 11$ **c** $d = \frac{x + 2}{8}$

d $p = 22 - x$ **e** $k = 2x + r$ **f** $C = n + \frac{x}{3}$

g $S = 12xy$ **h** $V = \frac{x - 5}{6}$ **i** $z = 12y - ax$

2 The formula relating distance, D, speed, S and time, T, is $D = ST$. Change the subject of the formula to:

a S **b** T.

3 The angle sum of a shape with n sides is $A°$, where $A = 180(n - 2)$.

a Use the formula to find the angle sum of a decagon (10 sides).

b Make n the subject of the formula.

c If the angle sum of a polygon is 2340°, how many sides does it have?

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4 The volume of a pyramid has the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height. Which of the following is the correct formula for A? Select **A**, **B**, **C** or **D**.

A $A = \frac{Vh}{3}$ **B** $A = \frac{3}{Vh}$ **C** $A = 3Vh$ **D** $A = \frac{3V}{h}$

5 The area of a trapezium has the formula $A = \frac{h}{2}(a+b)$. Change the subject of the formula to:

a h **b** a.

6 Change the subject of each formula to the variable shown in brackets.

Example 5

a $C = 2\pi r$ $[r]$ **b** $y = mx + c$ $[x]$

c $v^2 = u^2 + 2as$ $[a]$ **d** $z = \frac{x-m}{s}$ $[s]$

e $E = mc^2$ $[m]$ **f** $K = \frac{1}{2}mv^2$ $[m]$

g $10d = y^2 - b$ $[b]$ **h** $x = 6y - 16$ $[y]$

i $A = \frac{1}{2}bh$ $[b]$ **j** $N = \frac{m+d}{w}$ $[m]$

k $V = IR - E$ $[E]$ **l** $e = ir + \frac{Q}{C}$ $[C]$

m $v = \frac{m^2 r}{t}$ $[r]$ **n** $s = ut + \frac{1}{2}at^2$ $[u]$

7 The body–mass index of an adult is given by the formula $B = \frac{m}{h^2}$, where m is the adult's mass in kilograms and h is their height in metres. Change the subject of the formula to m and hence find, correct to 2 decimal places, the mass of a person with a body mass index of 28.5 and a height of 1.74 m.

8 The volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base and h is the perpendicular height. Show that this formula can be rewritten as $h = \frac{3V}{\pi r^2}$ and hence find (to the nearest centimetre) the height of a cone with volume 4251.66 cm^3 and base radius 16 cm.

9 Make F the subject of the formula $C = \frac{5}{9}(F - 32)$ and hence find F when:

a $C = 42$ **b** $C = -10$.

10 Make P the subject of the formula $M = \frac{10}{\pi hP}$. Find the value of P, correct to 2 significant figures, given that $M = 11.2$ and $h = 1.41$.

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INVESTIGATION

GETTING THE RIGHT FORMULA

Match each formula to its correct description. (Some descriptions are used twice.)

Formulas

1 $I = Prn$

2 $c^2 = a^2 + b^2$

3 $S = \dfrac{D}{T}$

4 $\bar{x} = \dfrac{\Sigma x}{n}$

5 $P(\bar{E}) = 1 - P(E)$

6 $A = \pi r^2$

7 $C = 2\pi r$

8 $y = kx$

9 $y = mx + c$

10 $A = \dfrac{h}{2}(a + b)$

11 $V = Ah$

12 $FV = PV(1 + r)^n$

13 $A = \dfrac{h}{2}(d_f + d_l)$

14 $\text{IQR} = Q_3 - Q_1$

15 $A = \dfrac{1}{2}xy$

16 $A = s^2$

17 $C = \pi d$

18 $S = V_0(1 - r)^n$

19 $A = lw$

20 $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

21 $S = V_0 - Dn$

22 $V = \pi r^2 h$

23 $A = \dfrac{1}{2}bh$

24 $P(E) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

25 $\dfrac{\text{absolute error}}{\text{measurement}} \times 100\%$

26 $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\text{change in } y}{\text{change in } x}$

Descriptions

A Area of a trapezium

B Area of a circle

C Probability of a complementary event

D Mean of a data set

E Volume of a prism

F Volume of a cylinder

G Pythagoras' theorem

H Circumference of a circle

I Probability of an event

J Interquartile range of a data set

K Linear variation

L Area of a square

M Area of a rectangle

N Speed

O Area of a triangle

P Gradient of a line

Q Simple interest

R Trapezoidal rule

S Declining-balance depreciation

T Area of a rhombus

U Straight-line depreciation

V Equation of a line

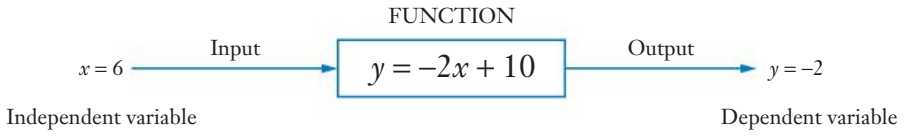
W Percentage error

X Future value of an investment after compound interest

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3.04 Linear functions

A **function** is a relationship between two variables, like a 'number machine' that converts an input value, x, into an output value, y.

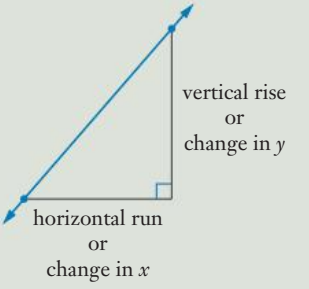


$y = -2x + 10$ is a **linear function** because its graph is a straight line. y is the **dependent variable** because the value of y depends on the value of x, while x is the **independent variable**.

Linear functions

- A linear function has the form $y = mx + c$ and its graph is a straight line.
- The **gradient**, m, is the rate of change of y relative to x:

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)} = \frac{\text{change in } y}{\text{change in } x}$$



- The **y-intercept** or **vertical intercept**, c, is the value of y when $x = 0$.

In the linear function $y = -2x + 10$, the number in front of x, **-2**, is called the **coefficient** of x, while the number on its own without an x, **10**, is called the **constant term**. For the graph of $y = -2x + 10$, -2 is the **gradient** and 10 is the **y-intercept**.

EXAMPLE 6

a Complete this table of values for the linear function $y = 2x - 1$, then graph $y = 2x - 1$ on a number plane.

x	-2	-1	0	1	2
y					

b Find the gradient and y-intercept of the line.

c How are the gradient and y-intercept of the line related to the equation $y = 2x - 1$?

Graphing linear functions
Graphing linear functions
$y = mx + c$
Graphing $y = mx + c$
Gradient and y-intercept of a line
Equation of a line
Finding the equation of a line
Gradient and y-intercept
Graphing lines: graphics calculator

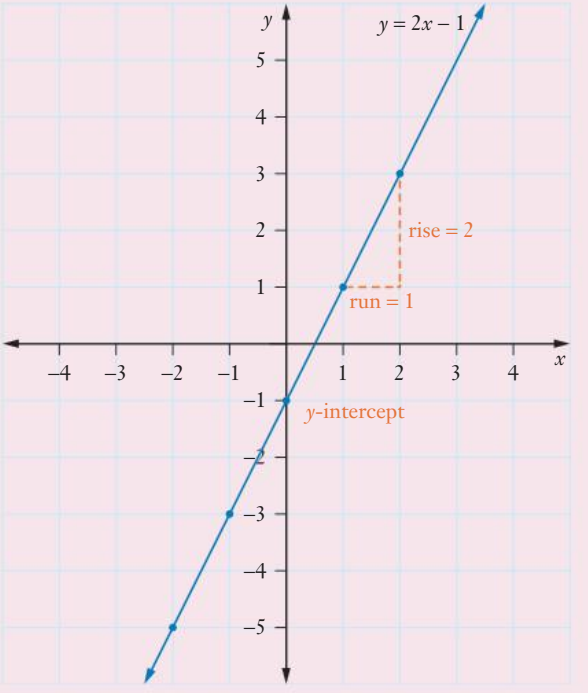
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Solution

a Substitute x-values into $y = 2x - 1$ to find y:

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

Graph the points (−2, − 5), (− 1, − 3), (0, − 1), (1, 1), (2, 3):



b From the graph, gradient $(m) = \dfrac{\text{rise}}{\text{run}} = \dfrac{2}{1} = 2$, and the y-intercept is −1.

Note also that in the table of values, as x increases by 1, y increases by 2 (the gradient) and when $x = 0$, $y = -1$ (the y-intercept).

c The gradient, 2, is the coefficient of x in the equation $y = 2x - 1$ and the y-intercept, −1, is the constant term.

EXAMPLE 7

What is the linear function for this table of values?

x	−4	−1	5	10
y	−23	−8	22	47

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Solution

The equation has the form $y = mx + c$. Select any two points to calculate the gradient, say (–1, –8) and (–4, –23):

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{-8 - -23}{-1 - -4}$$

Using the points

$$= \frac{15}{3}$$

$$= 5$$

$\therefore$ The equation has the form $y = 5x + c$.

$\therefore$ is the mathematical symbol for 'therefore'

To find c, substitute any point, say (10, 47):

$47 = 5(10) + c$

$47 = 50 + c$

$-3 = c$

$c = -3$

$\therefore$ The equation is $y = 5x - 3$.

EXAMPLE 8

A traffic researcher found a linear relationship between the toll charge and the number of vehicles using a motorway each day.

Toll, \$$t$	2.5	3.3	4	5.1	6
No. of vehicles, v	6500	6020	5600	4940	4400



a What is the independent variable?

b Find the linear function of the form $v = mt + c$.

c What is the vertical intercept and what does it represent?

d Use the function to predict the daily number of vehicles using the motorway when the toll is \$4.80.

e What will happen to the number of vehicles using the motorway if the toll increases by \$1.50?

Solution

a The independent variable is t, the toll. *v* depends on *t*

b First, find m:

$$m = \frac{\text{change in } v}{\text{change in } t}$$

$$= \frac{4400 - 5600}{6 - 4}$$ Using points (4, 5600) and (6, 4400)

$$= \frac{-1200}{2}$$

$$= -600$$

$\therefore v = -600t + c$

Second, find c:

$6500 = -600(2.5) + c$ Using point (2.5, 6500)

$= -1500 + c$

$c = 8000$

$\therefore v = -600t + 8000$

c The vertical intercept is 8000. This represents the number of vehicles using the motorway if $t = 0$, that is, if there is no toll.

d Substitute $t = 4.8$ into $v = -600t + 8000$:

$$v = -600(4.8) + 8000$$

$$= 5120$$

5120 vehicles will use the tollway when the toll is \$4.80.

e The gradient $m = -600$ means that for every \$1 increase in the toll, the number of vehicles using the motorway will decrease by 600. So if the toll increases by \$1.50:

Number of vehicles will decrease by $1.5 \times 600 = 900$.

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Alternative method:

Suppose the toll increases by \$1.50 from \$4 to \$5.50

When $t = 4$, $v = 5600$ (from table)

When $t = 5.5$, $v = -600(5.5) + 8000$

$= 4700$

Number of vehicles will decrease by $5600 - 4700 = 900$.

Exercise 3.04 Linear functions

1 Which of these is a linear function? Select **A**, **B**, **C** or **D**.

A $y = -\frac{1}{x}$ **B** $y = 12 - x$ **C** $y = 2x^2 + 9$ **D** $y = 2^x$

Example 6

2 **a** For the linear function $y = 3x + 2$, what is:

i the independent variable? **ii** the dependent variable?

b Graph $y = 3x + 2$ on a number plane after completing this table of values:

x	-2	-1	0	1	2
y					

WS A page of number planes

c Find the gradient and y-intercept of the line.

d How are the gradient and y-intercept of the line related to the equation $y = 3x + 2$?

WS Number plane grid paper

3 **a** Graph $y = 4 - 3x$ on a number plane after completing this table of values:

x	-2	-1	0	1	2
y					

b Find the gradient and y-intercept of the line.

c Explain how the gradient and y-intercept are related to the equation of the line.

4 For each linear function, write down its gradient and y-intercept.

a $y = -3x + 7$ **b** $y = 5 - x$ **c** $y = \frac{1}{2}x - 3$

5 Graph each linear function shown in Question **4** above on a separate number plane.

6 Which is the linear function for this table of values? Select **A**, **B**, **C** or **D**.

Example 7

x	4	7	12	20
y	10	31	66	122

A $y = \frac{1}{7}x - 18$ **B** $y = \frac{1}{7}x + 10$ **C** $y = 7x - 1$ **D** $y = 7x - 18$

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7 Find the linear function for each table of values.

a

k	6	10	13	15
C	152	120	96	80

b

t	1	3	6	14
v	9	21	39	87

c

n	3	8	20	35
p	141	126	90	45

d

x	2	5	11	18
t	–2	13	43	78

Example 8

8 This table shows the quarterly cost of water usage for different volumes of water.

Volume, V (kL)	16	25	48	60	81	99
Cost, C (\$)	168.24	183	220.72	240.40	274.84	304.36

a What is the independent variable and what does it represent?

b Find a linear function for C in terms of V.

c What does the gradient represent?

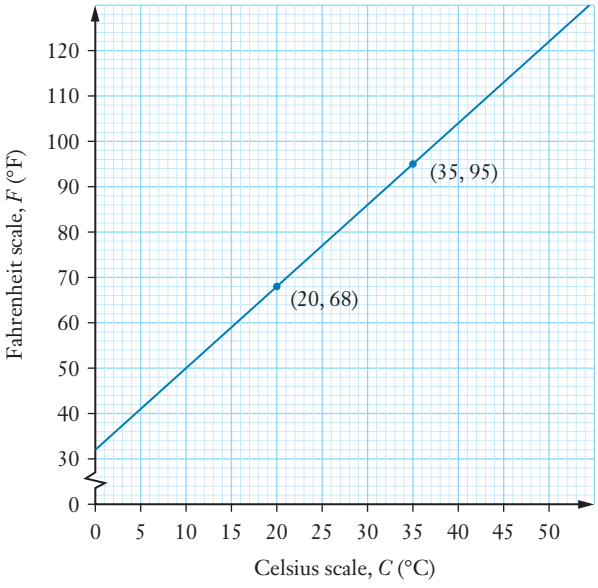
d What does the vertical intercept represent?

e What is the cost of using 54 kL of water?

f Find the volume of water usage that costs \$265.

9 This conversion graph shows the linear relationship between the Celsius and Fahrenheit scales for measuring temperature.

Celsius–Fahrenheit temperature conversion graph



a Find a formula for F in terms of C.

b What is the independent variable?

c What is the vertical intercept and what does it represent?

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d Use the formula to convert:

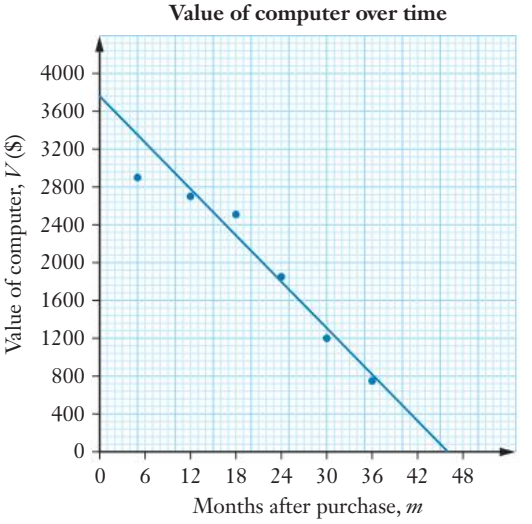
i 12°C to °F **ii** 86°F to °C.

e Use the graph to convert:

i 12°C to °F **ii** 86°F to °C.

f If C increases by 5°, what does F increase by?

10 Lizzie recorded the value of her computer every 6 months. She graphed this data and drew a 'line of fit' as shown below to estimate the values.



a Find the equation of Lizzie's line of fit.

b Use the equation to calculate the original value of her computer.

c What is the gradient of the line and what does it represent?

d Calculate the value of her computer after:

i 20 months **ii** $3\frac{1}{2}$ years.

e When will the value of the computer be zero? Answer in years and months.

f What are the limitations of this model?

11 A shoe's length (in inches) has a linear relationship with its shoe size.

Shoe size, s	2	5	7	8	12
Length, L inches	9	10	$10\frac{2}{3}$	11	$12\frac{1}{3}$

a What is the independent variable?

b Find the linear function for L in terms of s.

c What is the length of a shoe of size $7\frac{1}{2}$?

d What size is a shoe of length 13 inches?

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12 At a factory, the weekly cost, $\$C$, of manufacturing notebook computers is shown in the table, where n is the number of computers produced.

No. of computers, n	50	85	120	136	157
Weekly cost, $\$C$	21 900	31 350	40 800	45 120	50 790

a Find the linear function for C.

b What is the cost of producing 200 notebook computers?

c How many computers can be produced for $60 000?

d The factory made 44 more computers this week than last week. By how much did the cost increase from last week?

3.05 Direct linear variation

Direct linear variation

If y **varies** as x, or y is directly **proportional to** x, then $y = kx$, where k is a constant. k is called the **constant of variation** or **constant of proportionality**.

A direct linear relationship exists between x and y. If x increases (or decreases), y increases (or decreases). If x is doubled (or halved), y is doubled (or halved).

EXAMPLE 9

On a road trip, the total distance travelled, D km, varies directly as the time, t hours.

Time, t (hours)	0	2	3	5	8
Distance travelled, D (km)	0	160	240	400	640

a Graph the relationship between D and t.

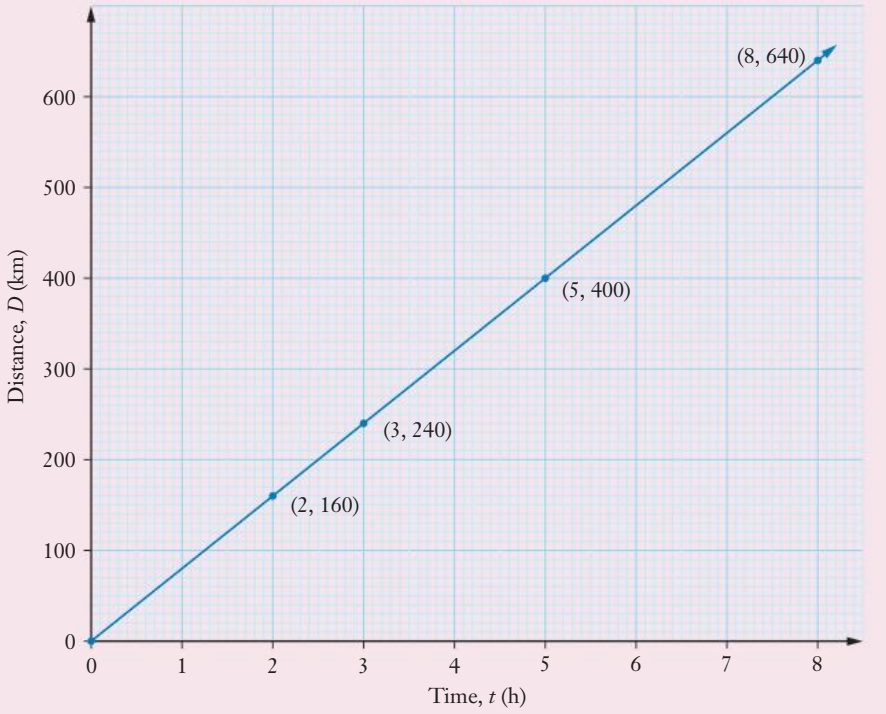
b Find the equation for D in terms of t.

c Find the time it takes to travel 550 km, in hours and minutes, correct to the nearest minute.

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Solution

a



b As D varies directly as t, the variation equation is of the form $D = kt$.

To find k, substitute a point from the table, say (3, 240):

$$D = kt$$

$$240 = k\,(3)$$

$$k = \frac{240}{3}$$

$$= 80$$

$\therefore\ D = 80t$.

c When $D = 550$:

$$550 = 80t$$

$$T = \frac{550}{80}$$

$$= 6.875 \text{ h}$$

$$= 6 \text{ h } 52 \text{ min } 30 \text{ s}$$

$$\approx 6 \text{ h } 53 \text{ min}$$

The time it takes to travel 550 km is 6 hours 53 minutes.

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Direct linear variation

EXAMPLE 10

The mass, M (in grams), of petrol varies as its volume, V (in litres). The mass of 60 L of petrol is 42 kg.

a Find the variation equation relating M to V.

b What is the mass of 35 L of petrol?

c What is the volume (correct to the nearest litre) of petrol that has a mass of 80 kg?

Solution

a As M varies directly as V, the variation equation is $M = kV$.

Substitute $M = 42$ and $V = 60$ to find k.

$$42 = k(60)$$

$$k = \frac{42}{60}$$

$$= 0.7$$

$$\therefore M = 0.7V$$

b When $V = 35$:

$$M = 0.7(35)$$

$$= 24.5$$

35 L of petrol has a mass of 24.5 kg.

c When $M = 80$:

$$80 = 0.7\,V$$

$$V = \frac{80}{0.7}$$

$$= 114.2857\ldots$$

$$\approx 114$$

80 kg of petrol has a volume of 114 L.

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Solving a linear variation problem

1 Identify the 2 variables (say x and y) and form a variation equation $y = kx$.

2 Substitute values for x and y to find k, the constant of variation.

3 Rewrite $y = kx$ using the value of k.

4 Substitute a value for x or y into $y = kx$ to solve the problem.

Exercise 3.05 Direct linear variation

Example 9

1 This table shows the weights of astronauts on Earth and on the Moon.

Weight on Earth, E (kg)	55	60	65	72	75	85
Weight on Moon, M (kg)	9.1	9.9	10.7	11.9	12.4	14.1

a Graph the relationship between M and E.

b What is the vertical intercept?

c Find the gradient of the line.

d Find the equation for M in terms of E.

e If Sally weighs 8.6 kg on the Moon, what is her weight on Earth?

2 This table shows the fuel used when travelling certain distances by a small car.

Fuel used, f (L)	3	12	15	18	30	33
Distance travelled, D (km)	50	200	250	300	500	550

a Graph the relationship between D and f.

b Find the equation for D in terms of f.

c Find the amount of fuel used when travelling a distance of 680 km.

3 The mass, M (g) of a piece of steel is given for different volumes, V (cm^3) in the table below.

Volume, V (cm^3)	3	8	11	25	35	60
Mass, M (g)	23.4	62.4	85.8	195	273	468

a Find the equation for M in terms of V.

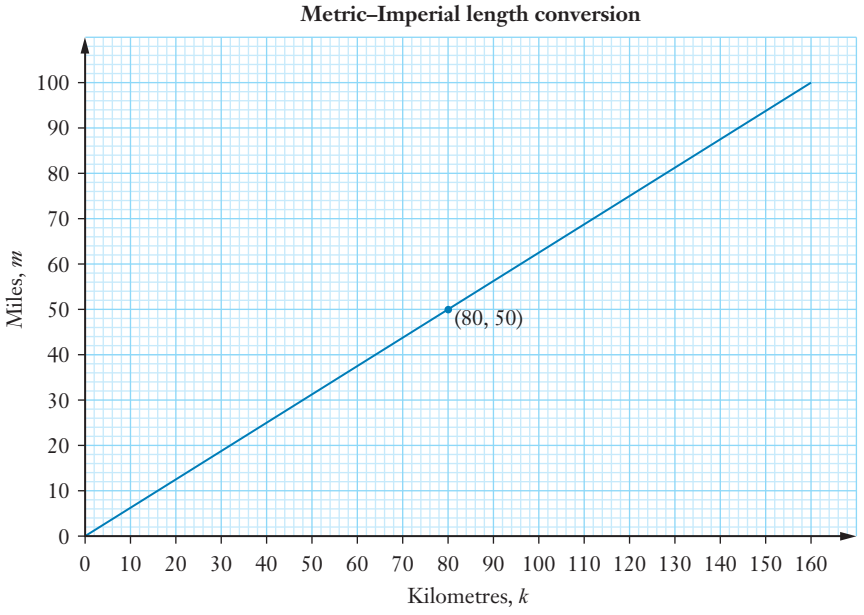
b Find the mass a piece of steel if it has a volume of 180 cm^3.

c Convert 1 m^3 to cubic centimetres (cm^3).

d If a piece of steel has a volume 1 m^3, calculate its mass in kilograms.

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4 This graph converts kilometres (a metric unit of length) to miles (an Imperial unit of length).

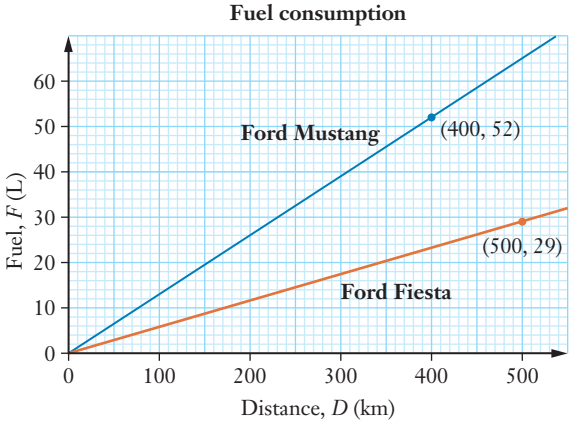


a What is the equation for m in terms of k?

b Use the equation to convert 115 km to miles.

c Convert 160 miles to kilometres.

5 This graph shows the fuel consumption of 2 models of Ford cars.



a Find the equation for F in terms of D for each car.

b Calculate how much more fuel is used by a Ford Mustang than a Ford Fiesta over a distance of:

i 100 km

ii 280 km.

c How much further can a Ford Fiesta be driven than a Ford Mustang on 50 L of fuel?

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Example 10

6 The download time, T seconds, of a computer file is directly proportional to the size of the file, S MB. On Jai's computer, a file of 1800 MB requires 36 seconds to download.

a Find the variation equation relating T to S.

b How long it will take Jai's computer to download a 3000 MB file?

c If a file took 80 seconds to download, what was its size?

7 During a thunderstorm, the time between seeing lightning and hearing thunder varies as the distance of the observer from the centre of the storm. If thunder is heard 8 seconds after the lightning for a storm that is 42 km away, calculate the distance of a storm when thunder is heard 12 seconds after the lightning. Select **A**, **B**, **C** or **D**.

A 2 km **B** 60 km **C** 63 km **D** 180 km

8 The increase in pressure experienced by a scuba diver is directly proportional to her depth under the water. The increase in pressure at depth 40 m is 235 kPa (kilopascals).

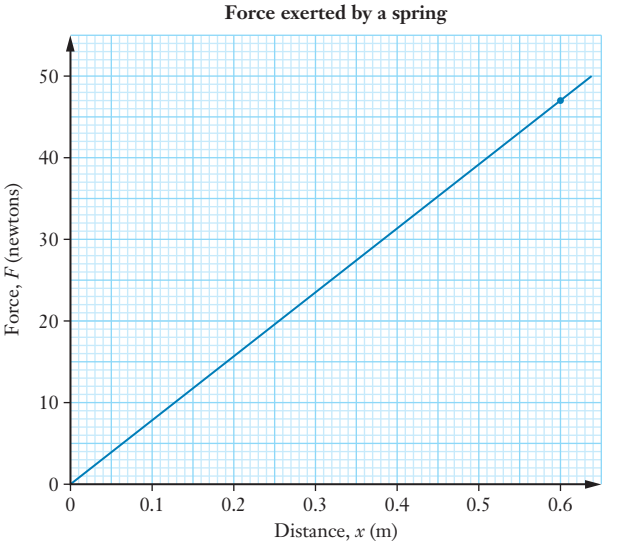
a Find the variation equation for the pressure in terms of the depth.

b What increase in pressure is experienced at a depth of 60 m?

c At what depth, to the nearest metre, is the increase in pressure 830 kPa?

9 The cost of running a car varies directly with the distance travelled. The cost is $355.20 for a distance of 500 km. If the car travels 15 000 km in one year, what is the cost of running the car for a year?

10 This graph shows the force, F (in newtons, N), exerted by a spring when it is stretched to a distance x (in metres), by a mass of 2 kg.



a Find the equation for F in terms of x.

b Calculate how much force is required to stretch the spring:

i 0.35 m **ii** 0.52 m.

c A force of 30 N is applied to the spring. How far is the spring stretched? Answer to the nearest 0.01 m.

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11 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, which of the following is the amount of time it would take for the temperature to drop 10°C? Select **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

12 At 3:30 p.m. one day, the shadows of different objects were measured, and they were found to be directly proportional to their heights. A tree of height 250 cm had a shadow of 325 cm.

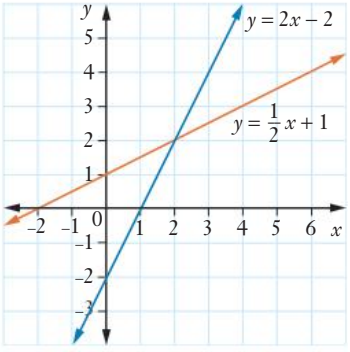
a Calculate the length of the shadow of a lamp post of height 320 cm.

b What is the height of a letterbox, correct to the nearest centimetre, if its shadow is 111 cm long?

WS
Intersection of lines

3.06 Intersection of lines

A pair of equations, such as $y = \frac{1}{2}x + 1$ and $y = 2x - 2$, can be solved together to find the values of x and y that satisfy *both* equations. Because they are solved at the same time, they are called **simultaneous equations**.



If simultaneous equations are graphed on the number plane, their solution is the coordinates of the point where their graphs intersect. For example, the graphs of $y = \frac{1}{2}x + 1$ and $y = 2x - 2$ cross at the point (2, 2), so the solution to the simultaneous equations is $x = 2, y = 2$.

Simultaneous equations can be used to solve problems involving **linear modelling**.

EXAMPLE 11

Ally is organising an outdoor morning tea to raise money for charity. The cost of hiring a marquee is $116 and the catering is $6.20 per person. To cover these costs, Ally is charging each person $12. The cost and revenue functions are represented by the formulas:

Cost: $C = 6.2n + 116$

Revenue: $R = 12n$

where C and R are in dollars and n is the number of people.

a Graph both functions on the same set of axes for $n = 0$ to 50.

b Use the graph to find the cost (to the nearest $10) of the morning tea for:

i 12 people **ii** 30 people.

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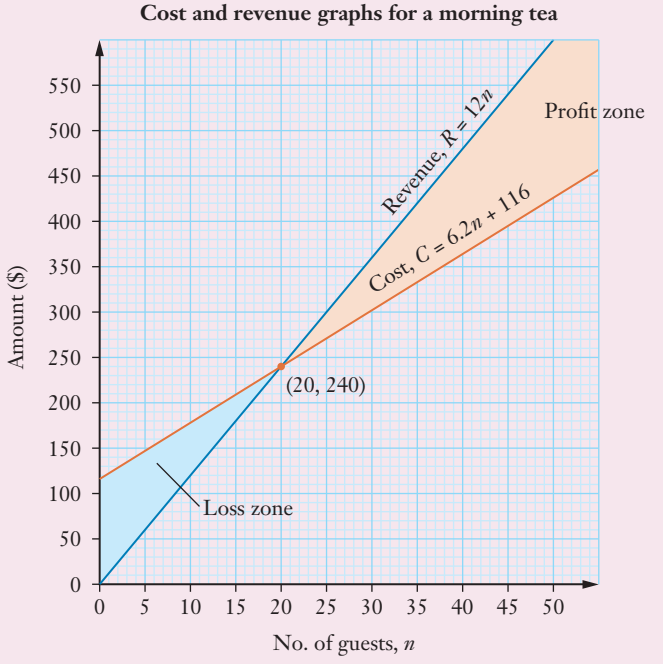
c Why would it be unwise to run a morning tea for 12 people?

d For what value of n does the cost equal the revenue?

e Why is the **point of intersection** of the two lines called the '**break-even point**'? What happens for values of n above this point?

Solution

a



b From the graph:

i \$190 **ii** \$300.

c The revenue would be less than the cost, so Ally would lose money.

d $n = 20$ $(C = R = \$240)$

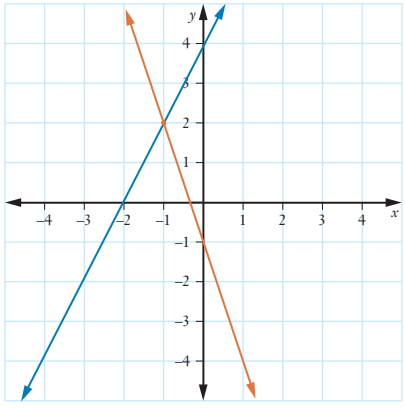
e When $n = 20$, the revenue covers the cost exactly, so Ally breaks even. For values of n above 20, the revenue is more than the cost so Ally would make a profit.

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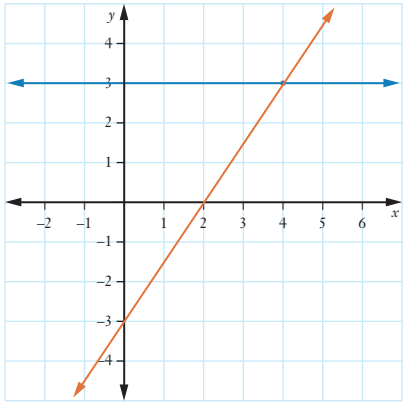
Exercise 3.06 Intersection of lines

1 For each graph, find the equations of both lines and the coordinates of their point of intersection.

a



b



2 Solve this pair of simultaneous equations graphically: $y = 2x - 1$ and $y = -x + 5$.

Example 11

3 At CopyCat Express, the charge for photocopying is 13 cents per sheet. Klaas is investigating the purchase of his own photocopier for $1200, which brings the cost down to 5 cents per sheet.

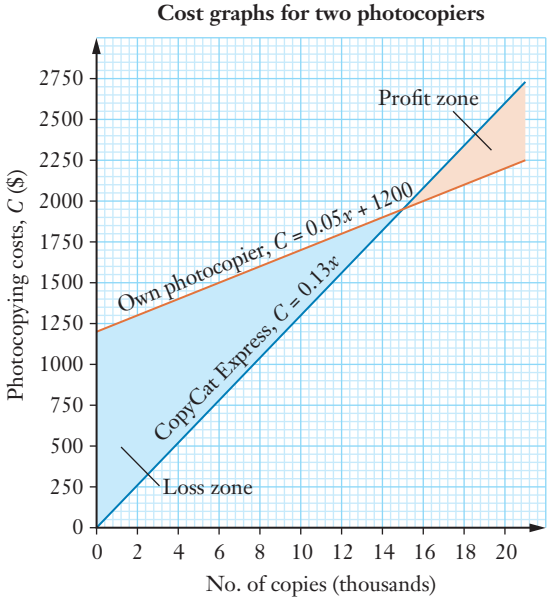
The 2 cost functions are:

CopyCat Express: $C = 0.13x$

Own photocopier: $C = 0.05x + 1200$

where C is in dollars and x represents the number of copies made.

The graph of the functions is shown.



a Use the graph to determine which option is cheaper for making 5000 copies.

b Use the formulas to find the cost of making 5000 copies for each option.

c Why is the 'Own photocopier' option more expensive for smaller values of x?

d What is the value of x at the break-even point?

e What happens when x exceeds this break-even value?

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4 Dave claims that a quick 'rule-of-thumb' for converting Celsius temperatures to Fahrenheit is 'double it and add 30'.

a Graph Dave's rule and the actual conversion formula for values of C from 0 to 100:

Dave's rule: $F = 2C + 30$

Actual conversion formula: $F = \frac{9}{5}C + 32$

where C represents the temperature in °C and F represents the temperature in °F.

b For what value of C is Dave's rule exactly equal to the actual conversion formula?

c For what values of C does Dave's rule give answers that are too high?

d For what values of C is the difference between the two formulas not more than 5°F?

e Is Dave's 'rule-of-thumb' a good one? Justify your answer.

5 Harry and Meghan are newlyweds and are deciding whether to take out private health insurance. If you are a member of a private health fund, it will pay 85% of the cost of your medical consulting fees. Annual membership of the private fund Harry and Meghan are considering costs $680 per year.

a If an average medical consulting fee is $32, write an equation for the total cost per year, C, paid for medical consultations. Let n represent the number of medical consultations. Assume Harry and Meghan do NOT take out private health insurance.

b If Harry and Meghan do take out private health insurance, show that the equation for the total cost per year, C, in terms of the number of medical consultations, n, is given by $C = 680 + 4.8n$.

c Graph on the same axes your equation from part **a** and the equation $C = 680 + 4.8n$, for values of n from 0 to 1000.

d Briefly describe what the point of intersection represents.

6 Jenni is comparing two Internet cafés. QuickByte has a monthly membership fee of $8 and charges 60 cents per hour of Internet use. Game Hunters charges $1 per hour of Internet use but has no monthly access fee. The 2 costs are represented by the formulas:

QuickByte: $C = 0.6t + 8$

Game Hunters: $C = t$

where C is in dollars and t is the number of hours of Internet use.

a Graph both formulas for values of t from 0 to 50 and find the break-even point.

b If Jenni uses an Internet café for 24 hours per month, which café is better for her? Why?

c Calculate the cost of 24 hours' Internet use for each café.

d Compare the advantages of each café, taking the break-even point into account.

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7 The trip charges for two taxi companies are shown below:

Yellow Taxis: $C = 0.9k + 4.6$ ($4.60 plus 90 c/km)

Purple Cabs: $C = 1.2k + 2.5$ ($2.50 plus $1.20/km)

where C is in dollars and k represents kilometres travelled.

a Graph both formulas for values of k from 0 to 11 and find their point of intersection.

b What does the point of intersection represent?

c Write two advantages of using each taxi company.

d Jake travels to and from work each day by taxi, a distance of 11 km each way. Determine which is the better taxi for him to use, and calculate the saving he would make using this one over 5 days.



Graphing lines: graphics calculator

TECHNOLOGY

GRAPHING LINEAR FUNCTIONS

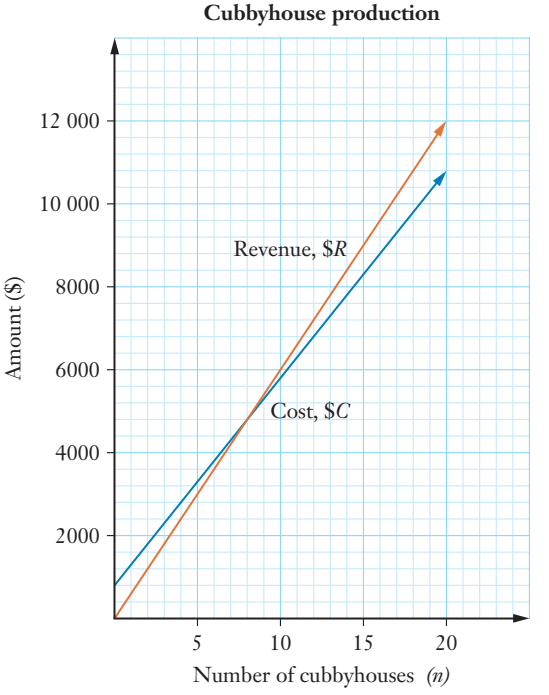
Linear functions can be graphed using graphing software. Learn to:

- graph a line by entering its equation (**graph** function)
- graph a line by entering the coordinates of two points on the line (**plot** function)
- use the **trace** and **zoom** functions to find points on a line, including the x- and y-intercepts
- solve a pair of simultaneous linear equations graphically using the **trace** and **zoom** functions to locate the point of intersection.

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SAMPLE HSC PROBLEM

A company makes children's cubbyhouses. The graph shows the cost of making n cubbyhouses in a given month and the revenue from selling n cubbyhouses.



a What is the equation of the cost function, C?

b How much does each cubbyhouse sell for?

c Hence, what is the equation of the revenue function, R?

d How many cubbyhouses need to be sold in a month to break even?

e Calculate the amount of profit made if 15 cubbyhouses are sold this month.

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Study tip

Multiple-choice questions

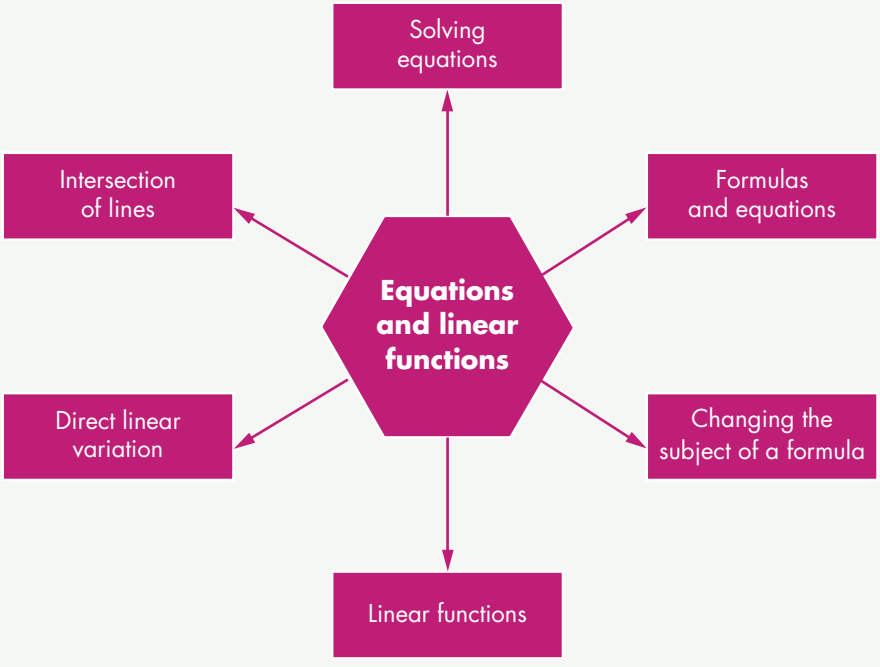
- Multiple-choice questions test precise knowledge—one or two of the answers are usually close, but there is only one correct answer.
- Multiple-choice questions are not necessarily easy, even if they are usually worth only one mark each—do not rush through them.
- There is no easy way of choosing the correct answer—you will need to carefully work out the answer and check that it matches one of the answers given.
- You don't have to **show** working, but you will need to **do** working to find the correct answer.
- If you are not sure of the answer, make an educated guess.
- Often it is possible to eliminate one or two of the options and narrow your choice.
- Do not leave a multiple-choice question blank—even if you guess, you have a one-in-four chance of achieving the correct answer.
- Don't cross out or change an answer impulsively—you may have been right the first time!

ISBN 9780170413633

3. CHAPTER SUMMARY

This chapter, Equations and linear functions, revised and extended algebra concepts introduced in Year 11. Solving equations, using formulas, changing the subject of a formula, linear functions and finding the intersection of lines by graphing simultaneous equations were covered, so be sure to include them in your topic summary. By now you should be competent in solving a variety of equations, graphing linear functions and locating points of intersection, and solving problems involving linear modelling and direct linear variation.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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3. TEST YOURSELF

Exercise 3.01

1 Solve each equation.

a $4k - 7 = 3$ **b** $-5(y - 8) = 5$ **c** $\frac{3m}{5} = 6$

d $10k - 17 = 4k + 16$ **e** $-3(1 - 2y) = 9$ **f** $2(4x - 11) = 5x$

g $\frac{2(2p+3)}{3} = -10$ **h** $\frac{3k-2}{5} = 4$ **i** $16d + 5 = 3d - 14$

Exercise 3.02

2 Eve's height, h cm, over her first 20 years can be modelled by the formula $h = 5.5a + 68$, where a is her age in years. At what age (to the nearest year) did Eve's height reach 155 cm?

Exercise 3.03

3 Change the subject of each formula to the variable indicated in brackets.

a $P = 110 + \frac{y}{2}$ (y) **b** $V = \frac{1}{2}hd^2 + 9.9$ (h) **c** $V = IR - E$ (I)

Exercise 3.04

4 The number of bacteria in a sample of milk is checked at the start of each day.

Day, d	0	1	2	3	4	5
Bacteria per mL, B	3000	15 727	28 454	41 181	53 908	66 635

a Find the linear function for B in terms of d.

b If this linear function was graphed, what would be the gradient of the line and what does it represent?

c Calculate the amount of bacteria per mL on the seventh day.

d Milk is unfit for use when the number of bacteria exceeds 100 000/mL. At the start of which day will this occur?

Exercise 3.05

5 The fuel consumption, F L/100 km, of a certain make of car, varies as the mass, M kg, of the car. One such car with a mass of 1200 kg has a fuel consumption of 13.2 L/100 km.

a What is the fuel consumption of a car that weighs 870 kg?

b What is the mass, to the nearest kilogram, of a car that has a fuel consumption of 11.9 L/100 km?

Exercise 3.06

6 The cost of manufacturing laptop computers is $C = 270p + 8400$ where p is the number of computers made and C is the cost in dollars. The revenue from selling these computers is $R = 410p$ where R is the revenue (sales) in dollars.

a Graph both functions on the same number plane for values of p from 0 to 100.

b What are the values of p, C and R at the break-even point?

c Calculate the cost and revenue from producing 50 laptop computers.

d What would happen if the company made 50 laptop computers?

e What is the vertical intercept of the cost graph and what does it represent?

f How much does one laptop computer sell for?

Qz Chapter quiz

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Practice paper 1



WS
Mathematics Standard 2 Reference sheet

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Question 1

Exercise 3.01

Which equation has $x = 4$ as the solution?

A $x + 4 = 0$
B $4 - 8x = 0$
C $2x = 8$
D $\frac{x}{2} = 8$

Question 2

Exercise 1.01

Meg borrowed \$7500 for 3 years at 11.5% p.a. flat interest to buy a jet ski. What amount does she repay altogether?

A \$2587.50
B \$2250
C \$9750
D \$10 087.50

Question 3

Exercise 2.01

Which ratio is the simplified form of 2.5 L : 14 mL?

A 1250 : 7
B 28 : 5
C 125 : 7
D 5 : 28

Question 4

Exercise 2.02

Sarah won \$420 000 in Lotto and divided it between herself and her 2 children in the ratio 3 : 2 : 2. How much did Sarah receive?

A \$180 000
B \$140 000
C \$120 000
D \$60 000

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Exercise 3.01

Question 5

A car was purchased new for \$57 000. By applying the declining-balance method of depreciation, the current value, S, of the car, at the end of n years, is given by the formula

$$S = 57\,000 \times (0.825)^n.$$

To the nearest dollar, what was the loss in the car's value during the 3rd year?

A \$9975 **B** \$6789

C \$32 006 **D** \$24 994

Exercise 3.01

Question 6

Solve $5 - 2x = -6$.

A $x = -5\frac{1}{2}$ **B** $x = 5\frac{1}{2}$

C $x = -\frac{1}{2}$ **D** $x = \frac{1}{2}$

Exercise 2.06

Question 7

If a cruise ship travels at an average speed of 22 knots between 7:00 a.m. and 2:30 p.m., how far (in nautical miles) does it travel altogether?

A 209 **B** 187

C 165 **D** 231

Exercise 2.07

Question 8

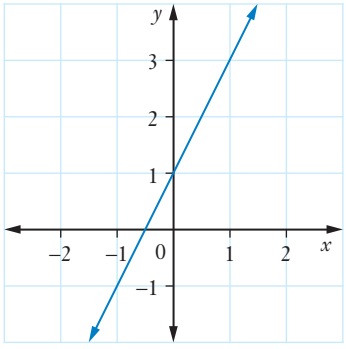
What is the equation of this line?

A $y = -3x + 1$

B $y = 3x + 1$

C $y = -2x + 1$

D $y = 2x + 1$



Exercise 1.02

Question 9

\$8600 is invested at 5% p.a. compounded monthly for 3 years. Calculate the compound interest earned.

A \$1355.58 **B** \$1388.66

C \$8707.95 **D** \$9955.58

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Question 10

Exercise 3.04

Determine the linear function for this table of values.

x	11	20	32	44	60
y	27	45	69	93	125

A $y = 3x - 6$

B $y = 2x + 5$

C $y = 2x + 11$

D $y = 3x - 15$

Section 2

3 questions: 10 marks each

Question 11 (10 marks)

a Solve each equation.

i $5a - 6 = 14$ **2 marks**

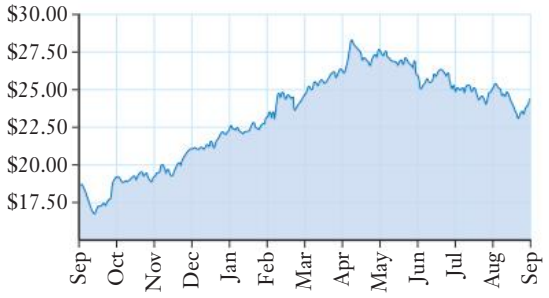
Exercise 3.01

ii $\frac{4m}{3} = 8 + m$ **2 marks**

iii $3(1 - 4x) = 15$ **2 marks**

b This graph shows the share prices of an energy company over a 12-month period, starting in September.

Exercise 1.05



i What was the highest price paid for these shares? **1 mark**

ii Describe the performance of these energy company shares over the 12-month period. **1 mark**

c Jack charges \$570 to trim trees for 3 hours, while Liam charges \$290 to trim trees for $1\frac{1}{2}$ hours. Who has the cheaper rate per hour? **2 marks**

Exercise 3.09

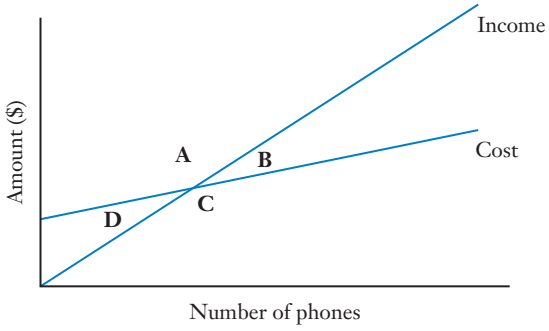
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Question 12 (10 marks)

Exercise 3.06

a A company manufactures phones. The company's income equation and cost equation are shown on the graph.

Which region of the graph (**A, B, C** or **D**) represents the profit zone? **1 mark**



Exercise 3.03

b Given the formula $S = 700(1 - r)$:

i rearrange it to make r the subject **2 marks**

ii find r, correct to 2 significant figures, given $S = 450$. **1 mark**

Exercise 2.02

c Jackie and Kim invested in a catering business in the ratio 2 : 3. If Kim invested $375 000, find:

i the total cost of the catering business **2 marks**

ii the amount Jackie invested. **1 mark**

Exercise 1.01

d Chris borrowed $12 000 to buy a car using a flat rate loan at 8.75% p.a. interest over 4 years.

i Calculate the total interest paid. **1 mark**

ii Calculate Chris' monthly repayment, to the nearest cent. **2 marks**

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Question 13 (10 marks)

a David tried to solve this equation and made errors in Line 2, Line 4 and Line 5.

Exercise 3.01

$2(A + 3) - 4(2A - 5) = 10$ … Line 1

$2A + 6 - 8A - 5 = 10$ … Line 2

$1 - 6A = 10$ … Line 3

$6A = 9$ … Line 4

$A = \frac{2}{3}$ … Line 5

i Identify the 3 errors in this solution. **1 mark**

ii Copy the equation in Line 1 and solve it correctly. **3 marks**

b The cost of manufacturing yo-yos is given by the function $C = 3n + 90$ where n is the number of yo-yos made and C is the cost in dollars. The revenue from selling yo-yos is given by the function $R = 6n$ where n is the number of yo-yos sold and R is the sales total in dollars.

Exercise 3.06

i Graph both functions on the same set of axes for values of n from 0 to 50. **1 mark**

ii Write the coordinates of the break-even point. **1 mark**

iii Calculate the cost of producing 25 yo-yos. **1 mark**

iv State whether the point where $n = 25$ is above or below the break-even point and describe what would happen if the factory produced 25 yo-yos. **1 mark**

c A patient receives her medication through an IV drip at a rate of 16 drops/minute. There are 18 drops/millilitre and the patient requires 400 mL of liquid. How long (in hours and minutes) will it take to administer her medication? **2 marks**

Exercise 2.06

END OF EXAMINATION

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4.

LOANS AND ANNUITIES

Since the Global Financial Crisis (GFC) of 2008, ideas and expectations about superannuation and saving for the future have changed significantly. Given that a boy and girl born today in Australia can expect to live to 80.3 and 84.4 years respectively, it is understandable that annuities have gained popularity over traditional term deposits as people are living longer. Annuities are considered to be low-risk and involve investing money to provide a series of guaranteed, regular payments over a set number of years (or even for life).

CHAPTER OUTLINE

F4.2 4.01 Reducing balance loans
F4.2 4.02 Credit cards
F5 4.03 Annuities
F5 4.04 Loan repayment tables
F5 4.05 Repaying a home loan

IN THIS CHAPTER YOU WILL:

- examine the progress of a reducing balance loan after each repayment
- calculate the repayments, duration, interest and total amount paid in a reducing balance loan
- calculate the costs and conditions involved in credit card purchases, interest and fees
- solve problems involving annuities, including calculating the duration and future value
- use a future value table to calculate the future value of an annuity (*FVA*), the interest earned and the contribution per period
- use a loan repayment table to calculate repayments, total amount and interest paid for a reducing balance loan
- calculate the fees and charges associated with having a home loan
- compare different types of home loans using graphs and calculations, including different payment options.



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TERMINOLOGY

annual fee
annuity
balance owing
cash advance
compound interest
contribution
credit card
deposit
fee
flat-rate loan
future value
future value of an annuity (*FVA*)
interest-free period
loan repayment table
minimum payment
mortgage
personal loan
principal
reducing balance loan
repayment
statement period
transactions

WS
Assignment 4

SkillCheck

1 How many:

a fortnights in 18 months?

b months in 25 years?

c weeks in 20 years?

d quarters in $6\frac{1}{2}$ years?

2 Complete each blank, correct to 4 decimal places where necessary.

a 12% p.a. = _____% per quarter

b 9% p.a. = _____% per month

c 14.3% p.a. = _____% per day

d 6.75 % p.a. = _____% per day

3 Evaluate each expression, correct to 2 decimal places.

a $1500 \times \dfrac{0.034}{365} \times 21$

b $76\,240 \times \dfrac{0.43}{100} \times 155$

4 Copy and complete each statement.

a Payments of \$1200 per fortnight = \$ _____ per year

b \$895 payments per week = \$ _____ per year

c Paying \$3450 per month = \$ _____ over 5 years

5 Evaluate $250\,000 + 250\,000 \times \dfrac{0.43}{100} - 1680$.

6 If $P = 870\,000$, $I = 870\,000 \times 0.00375$ and $R = \$5624$, evaluate $P + I - R$.

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4.01 Reducing balance loans



Reducing balance loan grids

A **reducing balance loan** is calculated on the **balance owing** on the loan, not on the original **principal** borrowed as with a **flat-rate loan**. You pay less interest on a reducing balance loan because the interest decreases as the balance owing decreases.

You can save interest and pay off a reducing balance loan more quickly by:

- making **repayments** more frequently, for example, weekly or **fortnightly** rather than monthly
- making extra payments of any size from time to time, for example, if you receive a bonus or pay rise at work
- increasing the size of each repayment (even by just a few dollars).


Reducing balance loans

EXAMPLE 1

Claire borrowed $595 000 to buy a studio apartment at 6.25% p.a. reducible interest. She made monthly payments of $3925. The table shows the progress of the loan for the first 2 months.

Reducing balance loan for Claire's studio apartment				
Amount borrowed	$595 000			
Interest rate (% p.a.)	6.25			
Monthly repayment (R)	$3925			
Month (n)	**Principal (P)**	**Interest (I)**	**Amount owing before repayment ($P + I$)**	**Balance ($P + I - R$)**
1	$595 000	$3098.96	$598 098.96	$594 173.96
2	$594 173.96	$3094.66	$597 268.62	$593 343.62

a Show how the interest of $3098.96 for the 1st month was calculated.

b Add rows showing the progress of the loan for the next 3 months.

c Find the principal at the start of the 6th month.

d How much has Claire paid off the principal after 5 months?

e Calculate how much interest was paid in the first 5 months.

Solution

a Amount owing (principal) at the start of the 1st month = $595 000

$$\text{Interest for 1st month} = \$595\,000 \times \frac{0.0625}{12}$$

$$= \$3098.9583...$$

$$= \$3098.96$$

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b

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment ($P + I$)	Balance ($P + I - R$)
1	\$595 000	\$3098.96	\$598 098.96	\$594 173.96
2	\$594 173.96	\$3094.66	\$597 268.62	\$593 343.62
3	\$593 343.62	\$3090.33	\$596 433.95	\$592 508.95
4	\$592 508.95	\$3085.98	\$595 594.93	\$591 669.93
5	\$591 669.93	\$3081.61	\$594 751.54	\$590 826.54
		Total \$15 451.54		

c Principal at start of 6th month is \$590 826.54 — Same as the balance at the end of the 5th month

d Amount paid off principal after 5 months = \$595 000 – \$590 826.54
= \$4173.46

e Interest paid in first 5 months = \$15 451.54 — Total of the Interest column

Exercise 4.01 Reducing balance loans

Example 1

1 From the table in Example **1** above, what was the amount that had been paid off the principal at the end of the 4th month? Select **A**, **B**, **C** or **D**.

A \$3085.98 **B** \$592 508.95 **C** \$3330.07 **D** \$591 669.93

2 a Add four rows to the table in Example **1** to show the progress of the loan for months 6 to 9. (The 5th month details are shown to start you off.)

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment ($P + I$)	Balance ($P + I - R$)
5	\$591 669.93	\$3081.61	\$594 751.54	\$590 826.54
6				
7				
8				
9				

b What is the principal at the start of the 9th month?

c What amount had been paid off the principal at the end of the 9th month?

d What total interest was charged for the first 9 months?

3 Stephanie borrows \$1 370 000 to buy a house. Interest is charged at 5.6% per annum, compounded monthly. How much does she owe at the end of the 1st month, after she has made an \$8495 repayment? Select **A**, **B**, **C** or **D**.

A \$1 355 112 **B** \$1 362 899 **C** \$1 363 567 **D** \$1 367 898

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4 For a reducing balance loan of $780 000 at 6% p.a. over 12 years, with fortnightly repayments of $3805.82, find:

a the total amount to repay

b the total interest payable.

5 Zoe borrowed $875 000 to purchase her first home. The bank lent her the money at 7.2% p.a. reducible interest and fortnightly repayments of $3444.65.

Fortnights (n)	Principal (P)	Interest (I)	Amount owing ($P + I$)	Balance ($P + I - R$)
1	$875 000	$2423.08	$877 423.08	$873 978.43
2	$873 978.43	$2420.25	$876 398.68	$872 954.03
3				
4				
5				
6				

a Copy and complete the table above showing the progress of Zoe's loan for the first 6 fortnights.

b How much had she paid off the principal after six repayments?

c How much interest did she pay in the first 12 weeks?

6 Nick borrows $36 200 for his upcoming wedding, taking out a personal loan that has the following conditions.

- Repayments can be made weekly, fortnightly or monthly.
- Extra repayments can be made at no extra cost.
- The loan approval fee is $230 (to be paid upfront when loan is established).
- The loan administration fee is $30 (charged after each 3 months of the loan).
- The interest rate is 13.6% p.a.

His monthly repayments (R) are $835 over 5 years.

a Calculate the monthly interest rate as a decimal, correct to 6 decimal places.

b The table below shows the progress of Nick's loan. Use the exact monthly interest rate to show that the interest paid in the 2nd month is $405.45.

Month	P	I	$P + I$	$P + I - R$
1	$36 200	$410.27	$36 610.27	$35 775.27
2	$35 775.27			
3				

Do not use rounded values for the interest rate in your calculations.

c Copy and complete this table.

d After 3 months Nick is charged a fee. What is the fee and how much will it be?

e How much in interest charges will Nick have paid after 3 months?

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7 Kim and Richard borrow $27 800 to finance an overseas holiday. They will repay their loan over 5 years at a rate of 12.5% p.a., with fortnightly repayments of $313.

a Calculate the fortnightly interest rate as a decimal, correct to 5 decimal places.

b Copy and complete the table below for the first 2 fortnights' repayments.

Fortnight	P	I	$P + I$	$P + I - R$
1				
2				

c How much will Kim and Richard pay in interest after 2 repayments have been made?

d How much is still owing on the loan after 2 fortnights?

WS
Reducing balance loan: spreadsheet

Reducible loans

TECHNOLOGY

Reducing balance loans on a spreadsheet

The spreadsheet shows the progress of a $400 000 loan at 4.8% p.a. reducible interest and monthly repayment $2300. Enter the labels, values and formulas shown. **Fill Down** for columns C, D and E until the amount owing in column E is negative, at row 304.

	A	B	C	D	E
1	Principal (P)	400000			
2	Interest rate (% p.a.)	4.8			
3	Repayment per period (R)	2300			
4	No. periods per year	12			
5					
6	Period no.	P	I	P+I	P+I-R
7	1	=B1	=B7*$B$2/100/$B$4	=B7+C7	=D7-$B$3
8	=A7+1				
9	=A8+1				
10					

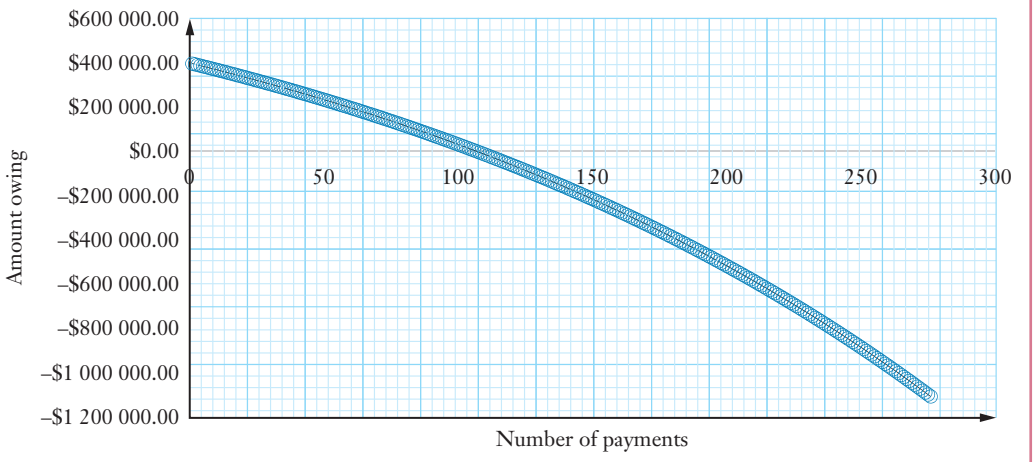
The details for the **first 3 months** and the **final month** of the loan are shown.

6	Period no.	P	I	P+I	P+I-R
7	1	$400,000.00	$1,600.00	$401,600.00	$399,300.00
8	2	$399,300.00	$1,597.20	$400,897.20	$398,597.20
9	3	$398,597.20	$1,594.39	$400,191.59	$397,891.59
10					
304	298	$2,268.92	$9.08	$2,277.99	-$22.01

From the **Insert** menu, select **Scatter with Lines and Markers** to draw a line graph showing the progress of the loan.

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1 Adjust your spreadsheet to show these loan details. (Each change is based on the original details as shown above.)



a The complete term of this loan is 23 years 0 months.

b The final repayment needed to reduce the balance to zero (and pay off the loan) is $2747.95.

c If the monthly repayment is doubled to $4600, the loan will be repaid in 9 years 0 months.

2 **a** Convert a monthly repayment of $2300 to a fortnightly repayment.

b Find the interest saved by making fortnightly repayments instead of monthly repayments.

3 Use a spreadsheet (as demonstrated in this activity) to find:

a the term for a $500 000 home loan at 5.4% pa reducible interest and monthly repayments of $3041

b the total interest on a travel loan of $40 000 at 8.49% pa reducible interest and fortnightly repayments of $410

c the final payment on a home improvement loan of $350 000 at 5.49% p.a. reducible interest and monthly repayments of $2415.

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WS
Credit card statement

WS
Credit card calculations

WS
Investigating credit cards

4.02 Credit cards

Using a **credit card** is like taking out a short-term loan. You use the card to purchase items and pay for them later when you receive a monthly statement. The interest charged can be flat or compound and is charged daily on each item from the date of purchase.

- Credit cards are more convenient than carrying cash, and are useful for purchasing items online, by phone or by post.
- A credit limit, such as $5000, is set by the card issuer, which is the maximum amount of money the cardholder is approved to spend using the credit card account.
- A monthly statement provides a record of spending.
- Many cards have an **interest-free period** of 55 days to repay the amount owing in full, otherwise interest is calculated from the date of each purchase.
- The interest-free period does not apply to **cash advances** (withdrawals) made on the card, which are charged a higher interest rate than purchases.
- Many offer reward schemes such as frequent flyer points, gift cards or cash-back rewards.
- Credit cards with high credit limits can be both an advantage and a disadvantage.
- Balance transfer credit cards need to be considered carefully as they may contain extra costs.





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EXAMPLE 2

Mitchell's credit card has a flat interest rate of 14% p.a. and no interest-free period. He uses the card to make the following purchases for the period 1 August to 31 August:

2 August	Phone recharge	$85
16 August	Jeans	$129
29 August	Movies	$22

If Mitchell pays his account in full on 3 September, how much does he pay?

Solution

The interest for each purchase is calculated separately.

Daily interest rate $= \frac{0.14}{365}$ 1 year = 365 days

Purchase amount	No. of days interest*	Interest
$85	$31 - 2 + 3 = 32$	$\$85 \times \frac{0.14}{365} \times 32 = \$1.04328\ldots$
$129	$31 - 16 + 3 = 18$	$\$129 \times \frac{0.14}{365} \times 18 = \$0.89063\ldots$
$22	$31 - 29 + 3 = 5$	$\$22 \times \frac{0.14}{365} \times 5 = \$0.04219\ldots$
Total = $236		**Total = $1.9761... ≈ $1.98**

* from date of purchase to 3 September

Do not round interest values until you calculate the total.

Total purchases = $236

Total interest = $1.98

Total payment = $236 + $1.98 = $237.98

When calculating the number of days interest in the above example, we counted the number of days from the date of purchase to the date of payment (3 September). However, some credit card accounts also **include** the date of payment in the count, charging an extra day's interest, as seen in the next example.

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EXAMPLE 3

Kylie's credit card has an interest-free period of 55 days, with interest on purchases compounded daily if the amount owing is not paid in full by the due date.
***Note*: For this card, interest is charged *up to and including* the date of payment.**

FIRSTBANK

Statement Period	14 December 2020 to 13 January 2021
Account No	2483 0222 1223 6517
Credit Limit	$5000
Available Credit	$3824.38

Ms Kylie Spender
16 Dollar Lane
Decimal Point 2178

Account Summary

Balance from previous statement	**$816.97**
Payment and other credits	$500.00CR
Purchases, cash advances	$791.00
Interest and other charges	$67.65
Closing Balance	**$1175.62**
Minimum payment required	$60.00
Due date	7 February 2021

Your Transaction Record

Date	Reference	Details	Amount
15 Dec 2020	3146532	Payment-Thank you	$500.00CR
15 Dec 2020		Interest	$8.65
18 Dec 2020	4367549	'Fresh at the Bay' Restaurant	$85.60
21 Dec 2020	3020542	ATM Withdrawal	$100.00
21 Dec 2020	3020542	ATM Fee	$4.00
31 Dec 2020	5746380	Ballet tickets	$265.50
2 Jan 2021		Annual Fee	$55.00
7 Jan 2021	2345698	Fabulous Fashions	$256.50
12 Jan 2021	6348921	Sam's Hair and Beauty	$83.40

Annual compound interest rate 22.484%
Daily compound interest rate 0.0616%

a Show how the closing balance of $1175.62 was calculated.

b What is the credit limit on this card?

c Calculate the following amounts for this period.

i cash advances **ii** purchases **iii** interest and other charges

d How much was paid by Kylie since the previous statement and on what date?

e When does the interest-free period start and end?

f What interest will be charged if this account is paid in full on 6 February?

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Solution

a Closing balance = \$816.97 − \$500 + \$791 + \$67.65

= \$1175.62

b \$5000

c **i** Cash advances = \$100 — ATM withdrawals

ii Purchases = \$791 − \$100 = \$691

iii Interest and other charges = \$67.65

d \$500 on 15 December 2020

e From 14 December 2020 to 7 February 2021 — The start of the statement period until the due date

f As 6 February falls within the interest-free period, interest is only charged on the cash advance, from 21 December to 6 February.

Number of days interest = (31 − 21 + 1) + 31 + 6 = 48 — +1 to include the date of purchase

Using the compound interest formula on $P = 100$, $r = 0.000\,616$, $n = 48$

$A = \$100(1 + 0.000\,616)^{48}$

$= 103.0000\ldots$

Interest = \$103.0000… − \$100

= \$3.0000…

= \$3.00

Exercise 4.02 Credit cards

1 Roberto is charged 0.04% compound interest per day on his credit card purchase of \$1200. Find the amount of interest owing after 30 days. Select **A**, **B**, **C** or **D**.

A \$14.40 **B** \$14.48 **C** \$4.80 **D** \$12.00

2 Karin has a credit card that charges simple interest of 22.5% p.a. and has no interest-free period. She bought shoes for \$235 on 21 December and paid her account on 29 December. Find the amount she paid. Select **A**, **B**, **C** or **D**.

A \$235 **B** \$287.88 **C** \$236.45 **D** \$1.45

3 Jad has a credit card with no interest-free period and a simple interest rate of 0.0453% per day. The purchases below were made in October. If Jad pays his account in full on 1 November, how much does he pay?

Example 2

4 October	Swim trunks	\$110
18 October	Swim goggles	\$25.00
25 October	Wetsuit	\$649.99

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4 NCM Bank credit cards have no interest-free period. For the credit card statement below, find for the due date:

a the number of days interest charged on the football tickets

b the total amount due.

Ms Adele Atkins **Due date: 1 April**		**NCM BANK** **Credit Card statement**	
Previous balance $876.15	**Payment** $876.15	**Total purchases**	**Interest** .....
Date	**Purchases**	**Amount**	
15 March	Coffee maker	$865.95	
20 March	Football tickets	$239.37	
28 March	Car tyres	$952.00	
Annual simple interest rate: 17.49%			

Example 3

5 A credit card has an interest-free period of 55 days. One monthly statement is shown. Interest is charged up to and including the date of payment.

Statement period: **1 June 2021 to 30 June 2021** Mr C Evans 1 Mountain Avenue Tournay 2126	**Account number** **Balance from previous statement** Payment and other credits Purchases Interest and other charges **Closing balance**	543210 $3256.43 $3000.00 $2195.48 $3.84 $2455.75
Annual compound interest rate 18.25% **Daily compound interest rate** 0.05%	**Minimum payment required** **Due date**	**$212** **25 July 2021**
Date	**Purchases**	**Amount**
15 June 2021	Payment – thank you	$3000.00 CR
15 June 2021	Interest	$3.84
18 June 2021	Bike repairs	$1346.58
27 June 2021	Cycling outfit	$665.50
27 June 2021	Bicycle helmet	$185.60

a Show how the closing balance of $2455.75 was calculated.

b How much was paid by Mr Evans and on what date?

c When does the interest-free period start and end?

d What interest will be charged on the 3 purchases if this account is paid in full on:

i 20 July? **ii** 30 July?

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6 This is the July statement for Claire's credit card account.

STATEMENT

PRL BANK

Ms C. Howe

Statement date: 28 August 2021 Credit limit: $5000

Previous balance	Payments	Purchases	Interest
$305.60	$305.60	$886.50	?
Date	**Purchases**	**Amount**	**Closing balance**
25 July	Cengage Car Insurance	$886.50	?

Annual percentage rate: 21% Daily percentage rate: 0.0575%

Note: Interest is charged on amounts from (and including) the purchase date up to (and including) the statement date (shown at the top of the statement).

Minimum payment due: $50 or 5% of closing balance, whichever is larger.

What is the minimum payment due on this account?

A $16.43 **B** $40.83 **C** $45.22 **D** $50

7 Sarah bought a dining table and chairs for $2600 on 11 July using a credit card. Simple interest was charged at a rate of 20.35% per annum for purchases on the credit card. No other purchases were made on this credit card account. There was no interest-free period. The period over which interest was charged included the purchase date and the payment date. What amount was paid, when Sarah paid the account in full on 19 August?

A $2642.04 **B** $2643.49 **C** $2656.53 **D** $2657.98

8 Hassan has a credit card with these specifications.

- A flat rate of interest is charged at a rate of 20% p.a. on the daily balance from the date of purchase up to and including the date of payment.
- There is no interest-free period.

Hassan's credit card transactions for the month of August are shown below.

Date	Description	Debit ($)	Credit ($)	Balance ($)
03/08/2020	Opening balance			0
07/08/2020	Talk Net Inc	286		286
10/08/2020	Sleepy Head Beds	1835		2121
28/08/2020	Petrol station	76		2197
31/08/2020	Interest charges			

On 31 August, Hassan paid this account in full, including the interest charges. What amount did he pay?

INVESTIGATION

COMPARING CREDIT CARDS

The best credit card is the one that matches your spending and repayment habits. If you plan to use the card for small or regular purchases and pay the total amount each month, choose a card with an interest-free period and a low annual fee. If, however, you plan to use the card to purchase many items and only pay off part of the debt each month, a low interest rate is better.

The rates, fees and interest-free days of 5 credit cards are shown in the table.

Card	Interest rate p.a. on purchases	Interest rate p.a. on cash advances	Annual fee	Interest-free days
ANZ First Visa	12.49%	21.74%	$58	55
Coles Low rate MasterCard	12.99%	19.99%	$49	62
NAB Low Rate Card	13.99%	21.74%	$59	55
St George Vertigo Visa	13.24%	21.49%	$0 (in first year) $55 thereafter	55
Westpac Altitude	0 % (for first 6 months) 20.24 % thereafter	20.74%	$100	45

Source: Canstar, July 2017

Select the best credit card for each person below, giving reasons for your choice.

a Cody uses his credit card frequently but always pays off the total amount due by the due date.

b Erin likes to make a lot of purchases with her credit card, and only makes the minimum payment each month.

c Mikhail often uses his credit card to make cash advances.

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4.03 Annuities

An **annuity** is a financial plan involving regular equal **contributions** (payments) to an account earning compound interest. Each contribution is made at the *end* of each compounding period. **Superannuation** is an example of an annuity, in which part of your pay is deposited regularly into a fund to provide future finance for your retirement years.

WS Annuity grids

WS Annuity: spreadsheet

WS Annuities: graphics calculator

EXAMPLE 4

The table shows the progress of an annuity over 5 years with a contribution of \$600 per year made at the *end* of each year and an interest rate of 3% p.a.

Year	Balance at start of year (P)	Interest (I)	Contribution at end of year (a)	Balance at end of year ($P + I + a$)
1	\$0	\$0	\$600	\$600
2	\$600	\$18	\$600	\$1218
3	\$1218	\$36.54	\$600	**A**
4	**B**	**C**	\$600	\$2510.18
5	\$2510.18	\$75.31	\$600	\$3185.49

a Find, to the nearest cent, the value of **A**, **B** and **C**, showing calculations.

b What is the balance at the end of the sixth year?

c If the contribution made at the end of each year, is doubled to \$1200, what is the balance at the end of the fourth year?

Solution

a Balance at end of 3rd year: **A** = \$1218 + \$36.54 + \$600 = \$1854.54

Balance at start of 4th year: **B** = \$1854.54 (same as **A**)

Interest earned in 4th year: **C** = $0.03 \times \$1854.54 = \$55.64$

Check: Balance at end of 4th year = \$1854.54 + \$55.64 + \$600 = \$2510.18

b Balance at end of 6th year = $\$3185.49 + 0.03 \times \$3185.49 + \$600 = \$3881.05$

c Balance at end of 4th year when contribution is \$600 = \$2510.18 (from table)

Balance at end of 4th year when contribution is \$1200 = $\$2510.18 \times 2 = \$5020.36$

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Future value table

An annuity earns more interest and grows faster than simply investing a principal into an account, because you keep making regular, equal payments into the account.

We know that **future value** means the final amount of an investment.

The **future value of an annuity** (*FVA*) is the total value of the investment at the end of the term, after the final contribution is made. It is the sum of all contributions plus interest.

The table below gives the future value (in dollars) of an annuity with a contribution of $1 per period. The values are rounded to 4 decimal places.

Future value interest factors (Future value of an annuity with a contribution of $1 at the end of each period)										
Period	Interest rate per period									
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0025	2.0050	2.0075	2.0100	2.0150	2.0200	2.0250	2.0300	2.0400	2.0500
3	3.0075	3.0150	3.0226	3.0301	3.0452	3.0604	3.0756	3.0909	3.1216	3.1525
4	4.0150	4.0301	4.0452	4.0604	4.0909	4.1216	4.1525	4.1836	4.2465	4.3101
5	5.0251	5.0503	5.0756	5.1010	5.1523	5.2040	5.2563	5.3091	5.4163	5.5256
6	6.0376	6.0755	6.1136	6.1520	6.2296	6.3081	6.3877	6.4684	6.6330	6.8019
7	7.0527	7.1059	7.1595	7.2135	7.3230	7.4343	7.5474	7.6625	7.8983	8.1420
8	8.0704	8.1414	8.2132	8.2857	8.4328	8.5830	8.7361	8.8923	9.2142	9.5491
9	9.0905	9.1821	9.2748	9.3685	9.5593	9.7546	9.9545	10.1591	10.5828	11.0266
10	10.1133	10.2280	10.3443	10.4622	10.7027	10.9497	11.2034	11.4639	12.0061	12.5779
11	11.1385	11.2792	11.4219	11.5668	11.8633	12.1687	12.4835	12.8078	13.4864	14.2068
12	12.1664	12.3356	12.5076	12.6825	13.0412	13.4121	13.7956	14.1920	15.0258	15.9171

EXAMPLE 5

Dominic opens an account that earns interest of 3% p.a. compounded monthly. He contributes $100 at the end of each month for six months.

a Use the table on above to calculate the future value of Dominic's annuity.

b How much interest will he earn on his investment?

Solution

a Interest rate per month = 3% ÷ 12 = 0.25%

No. of months = 6

Future value of $1 per month for 6 months = $6.0376 From table

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Future value of \$100 per month for 6 months = \$6.0376 × 100

= \$603.76

b Interest earned = \$603.76 – 6 × \$100

= \$3.76

Future value – total contributions

EXAMPLE 6



Find the contribution per period of an annuity that will give a future value of:

a \$15 000 if the annuity earns 2.5% p.a. for 3 years

b \$2600 if the annuity earns 10% p.a. compounded half-yearly for 4 years.

Solution

a FV = \$15 000, Interest rate = 2.5% p.a., No. of periods = 3 years

Future value of \$1 per year at 2.5% p.a. for 3 years = \$3.0756 — From table

Let \$$a$ be the contribution amount that will give a future value of \$15 000.

Future value of \$$a$ per year at 2.5% p.a. for 3 years = \$3.0756 × a = \$15 000

So a = \$15 000 ÷ \$3.0756

= 4877.097…

≈ 4877.10 — Round up for contribution

The contribution will be \$4877.10 per year.

b FV = \$2600, Interest rate = 10% ÷ 2 = 5% (per half-year),
No. of periods = 4 × 2 = 8 half-years

As with the compound interest formula, the interest rate and number of periods must be expressed with the same time units.

Future value of \$$a$ per half-year at 5% per half-year for 8 half-years = \$9.5491 × a = \$2600. — From table

So a = \$2600 ÷ \$9.5491

= 272.276…

≈ 272.28

The contribution will be \$272.28 per half-year.

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Exercise 4.03 Annuities

Example 4

1 The table shows the progress of an annuity with a contribution of $5000 per year made at the end of each year for 5 years.

Year	Balance at start of year (P)	Interest (I)	Contribution (a)	Balance at end of year ($P + I + a$)
1	$0	$0	$5 000	$5 000
2	$5 000	$400	$5 000	$10 400
3	$10 400	$832	$5 000	$16 232
4	$16 232	**X**	$5 000	$22 530.56
5				

a What is the annual interest rate?

b How much interest (**X**) was earned in the fourth year?

c Complete the table to find the amount of the annuity at the end of 5 years.

Example 5

2 Use the table to find the future value of an annuity (*FVA*) with a contribution of:

a $8000 per year for 5 years at 3% p.a.

b $600 per quarter for 9 months at 8% p.a. compounded quarterly

c $5400 per half-year for 5 years at 6% p.a. compounded half-yearly

d $900 per month for 10 months at 12% p.a. compounded monthly.

3 Cesar's Truck Repairs has a plant replacement fund and deposits $5000 on 30 June and 31 December each year. If the account earns interest of 8% p.a. compounded half-yearly, how much will be in the account at the end of 5 years?

4 Tom deposits $750 at the end of each month for 12 months into an account earning 6% p.a. compounded monthly. Immediately after making the last payment, he withdraws all his money to buy a boat that costs $9300. Does he have enough in his account to buy the boat? Show all working to justify your answer.

Example 6

5 Find the contribution per period (a) of an annuity whose future value (FV) is:

a $10 000, at 4% p.a. for 3 years

b $10 000, at 4% p.a. compounded quarterly for 3 years

c $26 500, at 5% p.a. compounded half-yearly for 6 years

d $9800, at 0.75% per month for 3 months.

6 Katarina would like to buy a house in 4.5 years and needs a deposit of $40 000. What contribution should she make every 6 months into an account earning interest of 6% p.a. compounded half-yearly to reach this amount?

7 Asher deposits an amount at the end of each year into a superannuation fund paying 8% p.a. compounded half-yearly. If he plans to have $400 000 after 6 years, what is his regular deposit?

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8 Carol and Bill start a trust fund for their granddaughter, Phoebe, on her 8th birthday to finance her university education. What amount should they deposit each birthday in an account earning 5% p.a. so that she will have $50 000 on her 18th birthday?

9 Selina invested $7600 at 8% p.a. compounded quarterly for three years.

a What is the final amount of her investment? Answer to the nearest dollar.

b Would she have achieved a better financial result if she had chosen an annuity of $760 per quarter for three years at the same interest rate? Justify your answer.

10 The shop owners in a small shopping centre estimate that $50 000 will be needed in 2 years' time to repaint and retile the common areas. They plan to make regular payments to a sinking fund earning 4% p.a. compounded quarterly.

a How much must be deposited in the sinking fund at the end of each quarter?

b How much interest will the sinking fund earn?

c How much would be in the sinking fund if they deposited $4000 per quarter for 2 years at the same interest rate?

11 Brianna is saving to go on holidays. She wants $6000 in 12 months so she can go on a cruise. How much must she invest at the end of each quarter at 6% p.a. compounded quarterly to achieve this?

12 The Downhams plan to buy a property in 3 years' time and will need a deposit of $60 000. If they deposit $4300 at the end of each quarter into an account earning 12% p.a. compounded quarterly, will they have the deposit in 3 years' time?

13 Azaa needs to save $16 000 for a holiday overseas in 3 years' time.

a Using the formula $PV = \frac{FV}{(1+r)^n}$, what single sum should Azaa pay into an annuity now, investing at a compound interest rate of 1.5% p.a., compounded twice a year, to ensure she can afford her holiday? Answer to the nearest dollar.

b If Azaa makes quarterly contributions of $1300 over 3 years in an account earning 3% p.a. compounded quarterly, use the future value table for an annuity on page 130 to determine (to the nearest dollar) if Azaa has enough money to take her overseas holiday. Justify your answer.

14 **a** Hadeya and Wyn invest $5000 every quarter for 3 years, invested at 8% p.a. compounded quarterly. Using the future value table on page 130, what is the future value of their annuity? Answer to the nearest cent.

b Using the formula $PV = \frac{FV}{(1+r)^n}$, what single sum could Hadeya and Wyn invest for 4 years at 3% p.a. compounded quarterly, to reach the future value of the annuity determined in part **a**?

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15 Taj wants to save $500 000 to provide him with a regularly monthly pension in his retirement.

a If Taj wants to retire in 5 years' time, use the future value table for an annuity on page 130 to determine (to the nearest dollar) the single sum he would need to invest now, at an interest rate of 3% p.a., to ensure he can afford to retire comfortably.

b If Taj had contributed $9000 every quarter, invested in an annuity over 10 years, at 6% p.a., compounded quarterly, would he have been able to accrue the $500 000 required for his retirement? Use this future value table of interest factors for an annuity in your calculations.

Period	1.5%	3%	4.5%	6%
5	5.152	5.309	5.471	5.637
10	10.703	11.464	12.288	13.181
20	23.124	26.870	31.311	36.786
40	54.268	75.401	107.030	154.762

INVESTIGATION

A FORMULA FOR FUTURE VALUE OF AN ANNUITY

The values in the future value table are calculated using a special formula for the future value of an annuity:

$$FVA = a\left[\frac{(1+r)^n - 1}{r}\right]$$

where a = contribution per period, paid at the end of the period

r = interest rate per compounding period, expressed as a decimal

n = number of compounding periods.

This formula is NOT part of the Mathematics Standard 2 course and does not have to be learned. Note that there are some similarities between this formula and the compound interest formula.

1 Owen and Emily open an account earning interest of 5% p.a. They deposit $10 000 at the end of each year for 4 years.

a Use the future value table to show that the future value of their annuity is $43 101.

b Use the FVA formula to show that the future value of their annuity is $43 101.

c Show that they will earn $3101 in interest.

2 According to the future value table, the future value of an annuity with a contribution of $1 per period, at an interest rate of 3% per period over 7 years, is $7.6625. Use the FVA formula to show that this value is correct.

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3 A **sinking fund** is an annuity where the future value has been decided and you pay regular contributions into the fund to reach this target. For example, in an apartment block, each unit owner pays into a sinking fund so money is available for maintenance and repairs.

The owners of a block of units estimate that $30 000 will be needed in 4 years' time to repaint the building. They open a sinking fund with interest of 6% p.a. compounded quarterly. Use the FVA formula to show that $1672.95 must be deposited each quarter into the sinking fund to reach this target.

DID YOU KNOW?

Who wants to be a millionaire?

Did you know that if you open an account at age 20 and save $100 per week at an interest rate of 10% p.a. compounded weekly, you will be a millionaire just after your fiftieth birthday?

How long will it take you to become a millionaire if you start saving $100 per week now at the *current* interest rate?



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4.04 Loan repayment tables

WS
Loan repayments table

Published **loan repayment tables** enable repayments for reducing balance loans to be easily calculated.

EXAMPLE 7

This table gives the monthly repayments for a reducing balance loan of $1000 for various terms and interest rates.

Monthly repayments per $1000 borrowed						
Interest rate (% p.a.)	Term (years)					
	5	10	15	20	25	30
4.5	$18.64	$10.36	$7.65	$6.33	$5.56	$5.07
5.0	$18.87	$10.61	$7.91	$6.60	$5.85	$5.37
5.5	$19.10	$10.85	$8.17	$6.88	$6.14	$5.68
6.0	$19.33	$11.10	$8.44	$7.16	$6.44	$6.00
7.0	$19.80	$11.61	$8.99	$7.75	$7.07	$6.65
8.0	$20.28	$12.13	$9.56	$8.36	$7.72	$7.34

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The Turners borrowed $570 000 at 5% p.a. reducible interest for a term of 20 years. Use the table to find:

a their monthly repayment

b the total amount repaid

c the interest paid

d how much more interest they would pay if the loan was over 25 years instead of 20 years.

Solution

a Term = 20 years, interest rate = 5% p.a.

Monthly repayment for $1000 = $6.60 — From table

Monthly repayment for $570 000 = $6.60 × 570

= $3762

b Total amount repaid = $3762 × 12 × 20 — Monthly repayments over 20 years

= $ 902 880

c Interest = $902 880 – $570 000 — Total amount repaid – principal

= $332 880

d Term = 25 years, interest rate = 5% p.a.

Monthly repayment for $1000 = $5.85

Monthly repayment for $570 000 = $5.85 × 570

= $3334.50

Total amount repaid = $3334.50 × 12 × 25 — Monthly repayments over 25 years

= $1 000 350

Interest paid = $1 000 350 – $570 000 — Total amount repaid – principal

= $430 350

Difference in interest paid = $430 350 – $332 880

= $97 470

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Exercise 4.04 Loan repayment tables

Use the table from Example **7** on page 135 to answer Questions **1**, **2** and **3**.

Example 7

1 Find the monthly repayment on a $450 000 loan at 6% p.a. for 15 years.
Select **A**, **B**, **C** or **D**.

A $4995 **B** $3798 **C** $4045.50 **D** $3222

2 Find the interest paid on a $500 000 loan at 5.5% p.a. over 25 years.
Select **A**, **B**, **C** or **D**.

A $421 000 **B** $377 500 **C** $921 000 **D** $532 000

3 For each reducing **balance** loan shown below, calculate:

i the monthly repayment
ii the total amount repaid
iii the total interest paid.

a $380 000 at 7% over 15 years **b** $925 000 at 4.5% over 25 years
c $1 230 000 at 5.5% over 20 years **d** $697 000 at 8% over 10 years.

4 This table gives the monthly loan repayments required to pay off various loans in 10 years.

Monthly loan repayments: 10-year term				
Principal borrowed	**Interest rate**			
	4% p.a.	**5% p.a.**	**6% p.a.**	**7% p.a.**
$20 000	$202.49	$212.13	$222.04	$232.22
$50 000	$506.23	$530.33	$555.10	$580.54
$80 000	$809.96	$848.52	$888.16	$928.87
$100 000	$1012.45	$1060.66	$1110.21	$1161.08

For each loan with 10-year terms, calculate the amount of interest paid.

a $50 000 at 6% p.a. **b** $100 000 at 4% p.a.
c $20 000 at 5% p.a. **d** $80 000 at 7% p.a.

5 **a** If the interest rate from Question **4c** dropped by 1%, how much less would the monthly repayment be?

b How much more interest would you pay if the interest rate from Question **4b** increased by 2%?

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6 The table below shows monthly repayments for loans over terms from 15 to 20 years.

Monthly loan repayments						
Principal	Term of loan (years)					
	15	16	17	18	19	20
$50 000	$492.37	$477.25	$464.15	$452.73	$442.73	$433.92
$60 000	$590.85	$572.70	$556.98	$543.28	$531.27	$520.70
$70 000	$689.32	$668.15	$649.81	$633.83	$619.82	$607.48
$80 000	$787.80	$763.60	$742.64	$724.37	$708.36	$694.26
$90 000	$886.27	$859.05	$835.47	$814.92	$796.91	$781.05
$100 000	$984.74	$954.50	$928.30	$905.46	$885.45	$867.83

a How much would you repay per year on a loan of:

i $60 000 for 19 years?

ii $80 000 for 17 years?

b Give **two** ways of finding the monthly repayment for a loan of $180 000 for 16 years.

c What is the interest saved by borrowing $300 000 over 15 years instead of 20 years?

7 The table shows the monthly and fortnightly repayments for some reducing balance loans over different terms.

	Repayment amounts					
	1 year		18 months		2 years	
Loan amount	monthly	fortnightly	monthly	fortnightly	monthly	fortnightly
$10 000	$885	$408	$606	$280	$467	$215
$20 000	$1770	$815	$1213	$559	$935	$430
$30 000	$2655	$1223	$1819	$838	$1402	$646
$40 000	$3540	$1630	$1869	$1117	$2425	$861

a What is the total amount to repay if you have a loan of $30 000 over 2 years with monthly repayments?

b How much interest do you save if you choose fortnightly repayments for the loan in part **a**?

c Jaryd took a $40 000 loan over 1 year, while Maia took a $40 000 loan over 2 years. If each made monthly repayments, what was the difference in the total amount repaid?

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INVESTIGATION

SUPERANNUATION PAYMENTS

With superannuation, you make regular contributions to an annuity during your working life to provide funds for your retirement. When you retire, you can receive your superannuation as a lump sum or reinvest it into another annuity that pays you in regular instalments.

Suppose you want to retire at age 70. How much superannuation will you receive upon retirement if your average annual income is $85 000? Find an online superannuation calculator, enter 18 as your age, 9.5% as the employer contribution and $0 as your super balance. Alter the 'Super balance' and 'additional contributions' to see the effect on the expected super balance at retirement.

Hannah was born in 1971. She has a super balance of $150 000 and currently contributes $50 per week to a superannuation fund. She intends retiring at age 67. Find her estimated super balance at retirement.

4.05 Repaying a home loan

EXAMPLE 8

Tanya applied for a $545 000 home loan (a **mortgage**) to purchase a city apartment. There are a number of additional costs associated with taking out the loan.

Registration of mortgage	$140
Registration of transfer	$272
Home loan establishment fee	$600
Rate lock (fixed interest rate) fee	$750
Lender mortgage insurance	$7920
Late payment charge	$20 per late payment
Loan service fee	$10 per month

This fee makes the interest rate on Tanya's loan stay the same, unlike a variable interest rate where the rate can change often.

a Tanya must also pay stamp duty (state government tax) of $20 015. If she is not locking her interest rate, what is the total amount in charges she must pay upfront?

b Hence calculate, correct to one decimal place, Tanya's upfront charges as a percentage of her home loan.

c After paying the upfront charges, how much did Tanya pay in fees in the first year if she did not make any late repayments?

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Solution

a Upfront charges = \$20 015 + \$140 + \$272 + \$600 + 7920

= \$28 947

b $\frac{\$28\,947}{\$545\,000} \times 100\% = 5.3113...$

$\approx 5.3\%$

c Loan service fee = \$10 × 12

= \$120

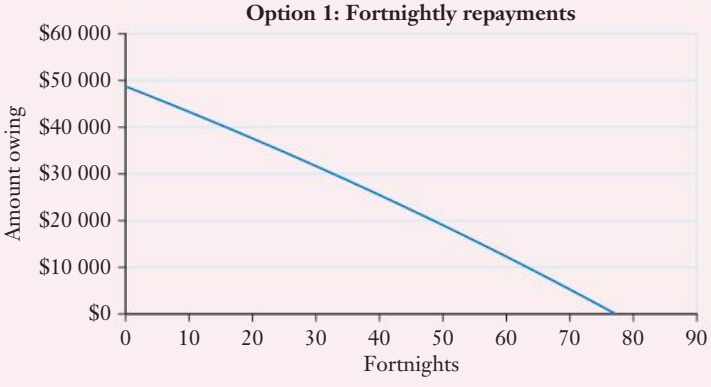
EXAMPLE 9

Jennifer takes out a personal loan to install a swimming pool costing \$48 700 at her house. She has 2 options:

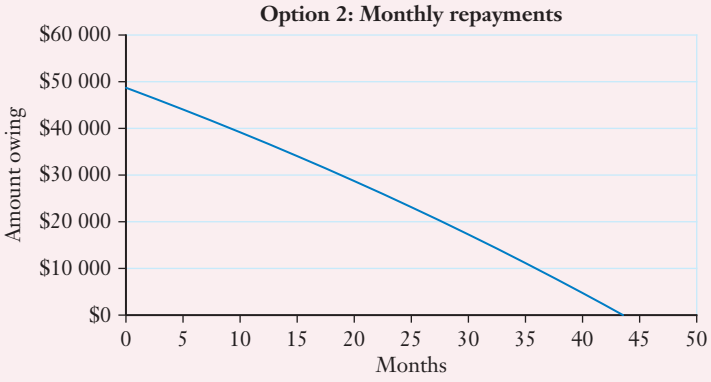
Option 1: Fortnightly repayments of \$741.11 at 10.99% p.a. reducible interest

Option 2: Monthly repayments of \$1360.70, at 10.99% p.a. reducible interest

The 2 graphs below show the progress of the loan under each option.



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a Approximately how many years and months will it take Jennifer to repay the loan if she makes her repayments:

i fortnightly? **ii** monthly?

b How much time would she save paying off the loan fortnightly instead of monthly?

c Calculate the total of Jennifer's repayments if she pays them:

i fortnightly **ii** monthly.

d How much interest would she save paying off the loan fortnightly instead of monthly?

Solution

a Reading from the graphs:

i Approximately 78 fortnights $= 78 \times 2$ weeks

$= 156$ weeks

$= 156 \div 52$ years

$= 3$ years

ii Approximately 44 months $= 44 \div 12$ years

$= 3.6666\ldots$ years

$= 3$ years $+ 0.6666\ldots \times 12$ months

$= 3$ years 8 months

b Time saved paying fortnightly $= 3$ years 8 months $- 3$ years

$= 8$ months

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c **i** Total fortnightly repayments = $\$741.11 \times 78$

$= \$57\,806.58$

ii Total monthly repayments = $\$1360.70 \times 44$

$= \$59\,870.80$

The difference in interest paid is the same as the difference in total repayments.

d Interest saved paying fortnightly = $\$59\,870.80 - \$57\,806.58$

$= \$2064.22$

Exercise 4.05 Repaying a home loan

Questions **1** to **3** refer to this table of NCM Bank loan interest rates and fees.

Home loan	Car loan	Personal loan
Interest rate: 5.20% p.a. Registration of mortgage: $140 Registration of transfer: $272 Loan establishment fee: $600 Rate lock fee: $750 (for loans ≤ $ 1 000 000) or 0.15% of the total loan amount Late payment charge: $20 Loan service fee: $10 per month	Interest rate: 8.55% p.a. Loan establishment fee: $350 Early termination fee: $50 + $\frac{\text{months remaining on loan}}{\text{total months of loan}} \times \$680$ Late payment charge: $20	Interest rate: 12.99% p.a. Loan approval fee: $150 Rate lock fee: $750 Late payment charge: $20 Loan service fee: $30 per quarter

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Example 8

1 Alia borrowed $9000 to buy a second-hand car.

a What interest rate was she charged?

b What does she pay to establish the loan?

c What will she be charged if she terminates the loan early?

2 Vaesna establishes a home loan of $500 000.

a What interest rate is he charged?

b What is the total amount in charges Vaesna must pay upfront?

c He made late home loan repayments 2 months in a row. How much did his late payment and service charges amount to?

3 Elizabeth borrows $4700 to fill her new studio apartment with furniture and electrical goods.

a What interest rate is she charged?

b How much did she pay when her loan was approved?

c In the first year of her loan she made one late payment and had to pay administration charges. How much did Elizabeth pay?

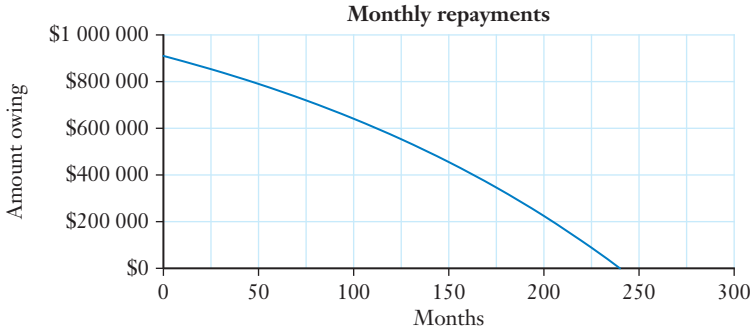


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Example 9

4 This graph shows a home loan of $910 000 repaid with monthly repayments.



a Estimate from the graph how many years and months it took to pay off this loan.

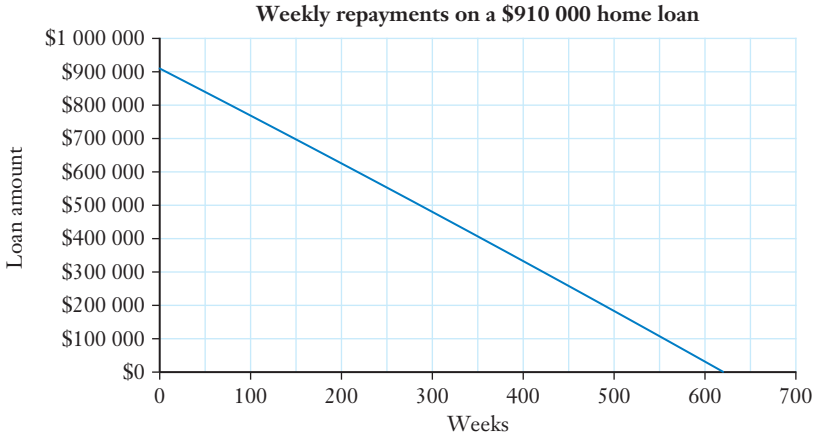
b Estimate how many years and months it took for the home loan to reduce to $600 000.

c How much money was still owing after 15 years?

d If the loan was repaid with monthly repayments of $6132, how much was repaid in total?

e Calculate how much interest was paid.

f The graph below shows the same loan of $910 000 repaid with weekly repayments. Estimate how many years and months it took to pay off this loan.



g Estimate how many years and months it took for the home loan to reduce to $700 000.

h How much of the loan was still owing after 5 years?

i If the loan was repaid with weekly repayments of $1533, how much was repaid in total?

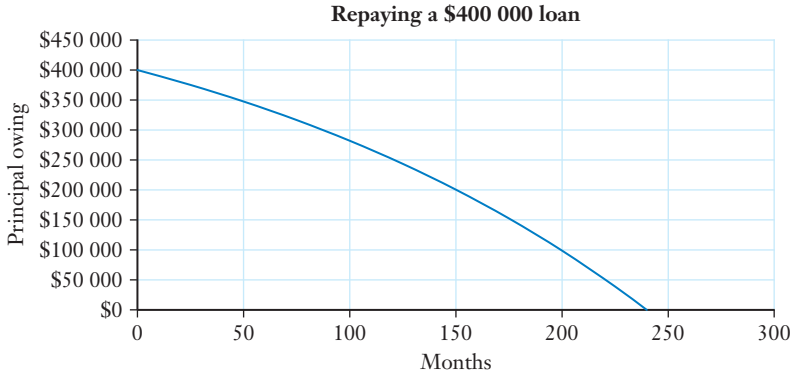
j Calculate how much interest was paid.

k How much time would be saved by paying off the loan weekly instead of monthly?

l How much interest would be saved by paying off the loan weekly instead of monthly?

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5 The graph shows the progress of a $400 000 loan over a term of 20 years.



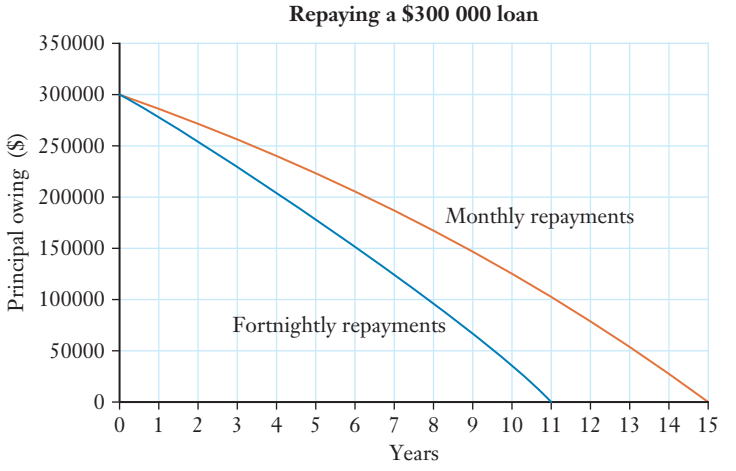
a Find, to the nearest $10 000, the principal owing after:

i 5 years **ii** 15 years.

b In what year was:

i a principal of $300 000 owing? **ii** the principal half-paid off?

6 The graph shows the progress of a $300 000 loan at 4.9% p.a. reducible interest over 15 years for monthly and fortnightly repayments.



a How much sooner can you repay the loan by making fortnightly repayments?

b After how many years was a principal of $100 000 owing if repayments were made:

i monthly? **ii** fortnightly?

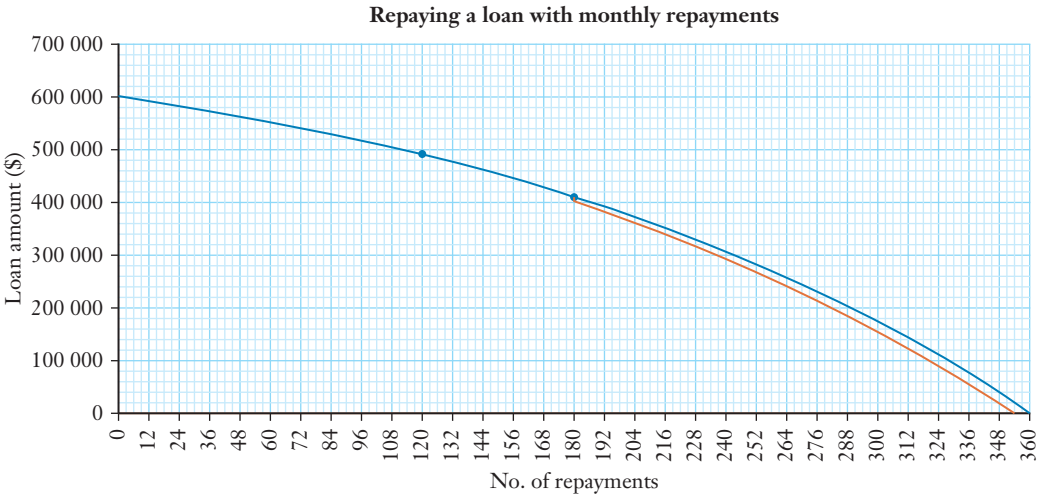
c How much of the amount borrowed was still owing after:

i 5 years with fortnightly repayments?

ii 10 years with monthly repayments?

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7 The blue graph below shows the progress of the Hayes family's home loan, at monthly repayments of $3295.



a What amount did the Hayes family borrow?

b How much will they owe after 120 months?

c How many years will it take to pay off the loan?

d In total, how much will they repay?

e Calculate how much interest they will pay.

f After 15 years (180 months) into the loan, the family made an extra payment of $10 000 off the loan. It changed the length of the loan, as shown by the orange graph and the 'dip' at 180 months. How many years and months will it take to pay off the loan now?

g How much will the Hayes family repay now, including the $10 000?

h How much interest will they pay now?

i How much time was saved on the loan after paying the extra $10 000?

j How much interest was saved on the loan after paying the extra $10 000?

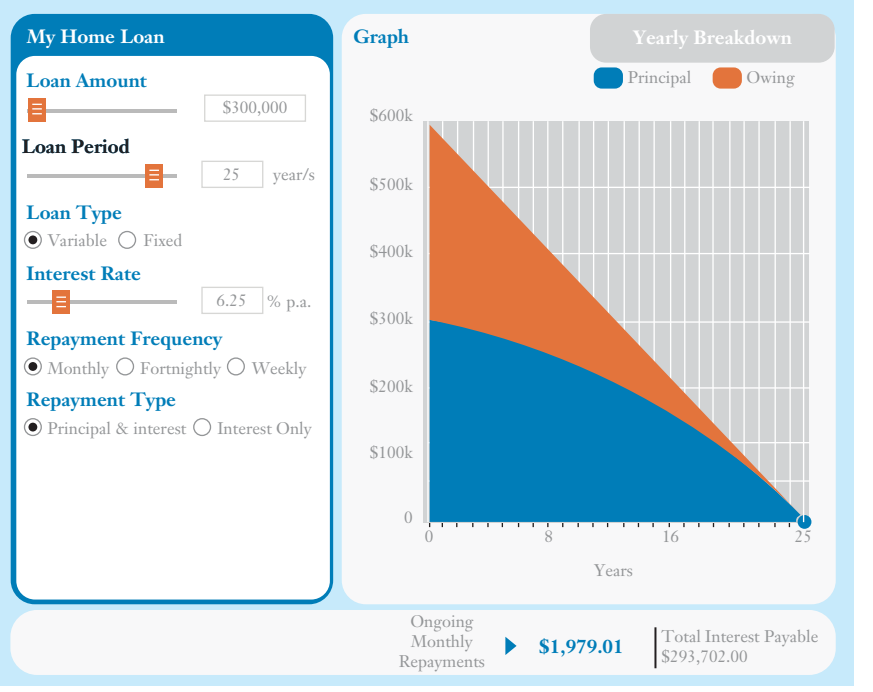
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INVESTIGATION

CHANGING VARIABLES ON HOME LOANS

Use the Internet to find an online loan calculator to see the effect of:

- higher or lower interest rates on total interest paid
- repaying fortnightly or weekly rather than monthly
- changing the term of the loan on the size of the repayments
- increasing the size of the repayment on the time for the loan to be repaid.



Reproduced with permission from St George Bank

The graph above shows the progress of a reducing balance loan over time, with total amount owing (principal + interest) shown in orange and the amount of principal owing shown in blue.

Suppose you borrowed $300 000 to buy an apartment at 6.25% p.a. reducible interest to be repaid monthly over 25 years. Use your loan calculator to answer these questions.

a What is the size of each monthly repayment?

b How does increasing the monthly payment by $50 affect the term of the loan?

c How much sooner would you repay the loan if you made repayments fortnightly instead of monthly?

d How much sooner would you repay the loan if you made an extra payment of $10 000 after 10 years?

e What amount did you pay in stamp duty? (Use a stamp duty calculator.)

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SAMPLE HSC PROBLEM

This table is used to calculate monthly loan repayments on reducing balance loans at a lending institution.

Monthly repayments per $1000 borrowed					
Interest rate (% p.a.)	Term (years)				
	5	10	15	20	25
5.0	$18.87	$10.61	$7.91	$6.60	$5.85
5.5	$19.10	$10.85	$8.17	$6.88	$6.14
6.0	$19.33	$11.10	$8.44	$7.16	$6.44
6.5	$19.57	$11.35	$8.71	$7.46	$6.75

James borrowed $820 000 at 5.5% p.a. for 20 years.

a What is his monthly loan repayment?

b What total amount does he repay?

c How much interest does he pay?

d If James paid off this loan over 25 years at the same interest rate, how much *more* would he pay in interest?

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Study tip

Finding the key of a topic and using mnemonics

When studying a novel in English class, you look for the underlying theme of the book. When studying a mathematics topic, you can also look for '**the key**', the main idea that threads through the topic, the reason for learning it. Once you understand and appreciate the point of a topic, a lot of its concepts and ideas will fall into place.

Analyse the content of each topic/chapter for meaning and relevance. What is the key idea or theme? How and where will you use the mathematics? Use each chapter's introductory outline and review pages to help you. Ask yourself and your teacher questions about the topic. Have an opinion. Sound informed.

A **mnemonic** (pronounced 'ne-monic' with a silent first m) is a memory aid—a saying, rhyme or picture to help you remember a rule or list of ideas. You probably already know some mnemonics for spelling ('i before e except after c') and the number of days in each month ('Thirty days has September …').

To memorise a complex list or rule, make up a saying or picture that is easy to remember. For example, the initials for the definitions of basic trigonometry SOH CAH TOA can be remembered using the saying 'Some Old Hens Can Always Hide Their Old Age' or 'Sun Over Head Caused A Huge Tan On Arms'. The gradient formula $m = \frac{\text{rise}}{\text{run}}$ can be learned by picturing yourself at sun*rise* waking up to go for a *run*.

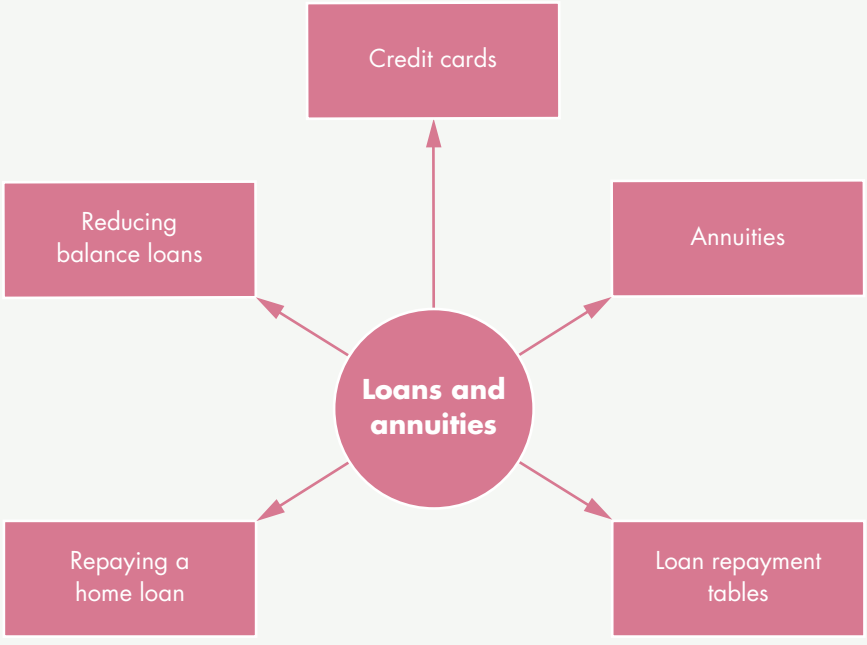
Be creative in your use of mnemonics. In fact, the sillier the phrase or picture, the better the mnemonic because absurd words and images are easier to recall.

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4. CHAPTER SUMMARY

This chapter, Loans and annuities, examined the financial mathematics of credit, loans and annuities. You should be able to calculate the interest and repayments involved in flat-rate loans, payments, reducing balance loans and credit cards, and use tables and graphs to track the progress of a reducing balance loan. You used tables to calculate future values of annuities, and loan repayments. You should understand the financial terminology of this topic, and be able to make informed decisions on different loan and investment options.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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4. TEST YOURSELF

1 Thomas borrowed $18 000 for a new bathroom at 9% p.a. reducible interest. He made monthly payments of $420.

Exercise 4.01

a Draw up a table showing the progress of the loan in the first 4 months, with columns as shown below, where $R = \$420$.

Month	P	I	$P + I$	$P + I - R$
1	$18 000			
2				
3				
4				

b How much had Thomas paid off the principal after his fourth payment?

c How much interest did he pay in the first four months?

d Calculate the total of the repayments made in the first four months.

2 Beatrice owns a credit card that has no annual fees and charges a flat rate of 19.65% p.a. interest on all purchases made. If interest is charged from the day of purchase, what is the interest charged on $2135 for 21 days? Select **A**, **B**, **C** or **D**.

Exercise 4.02

A $419.53 **B** $412.65 **C** $24.14 **D** $19.98

3 Minjee has a credit card with no interest-free period. Her monthly statement for October is shown.

Exercise 4.02

Ms Minjee Lee Payment due: 1 November 2021		**GOLF BANK** **Credit Card statement**	
Previous balance $1325.68	**Payment** $1325.68	**Total purchases**	**Interest** .....
Date	**Purchases**	**Amount**	
15 October	Golf coaching	$325.50	
20 October	Golf shoes	$295.00	
28 October	Massage	$137.00	
Annual simple interest rate: 19.78% Daily percentage rate: 0.0542%			

If Minjee pays her account in full on 5 November, how much interest does she pay?

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Use the future value of an annuity (FVA) table on page 130 to answer Questions **4** and **5**.

Exercise 4.03

4 Tahlia puts $5000 in an account at the start of each year for 10 years.

a What is the future value of her annuity if the interest rate is 2.5% p.a?

b How much interest will she earn?

Exercise 4.03

5 Irven contributes $16 000 per quarter for one year into an annuity that earns interest at 3% p.a. compounded quarterly. Calculate the future value of Irven's annuity.

Exercise 4.04

6 This table gives the monthly repayments of a car loan for different loan amounts and terms.

Monthly car loan repayments				
Loan amount	Term of loan (months)			
	12	24	36	48
$10 000	$870	$453	$314	$244
$12 000	$1043	$542	$375	$292
$14 000	$1216	$632	$437	$340
$16 000	$1390	$721	$498	$388
$18 000	$1562	$811	$561	$436
$20 000	$1735	$901	$622	$484

a Consider a car loan of $16 000 over 3 years. Use the table to calculate:

i the monthly repayment

ii the total amount repaid

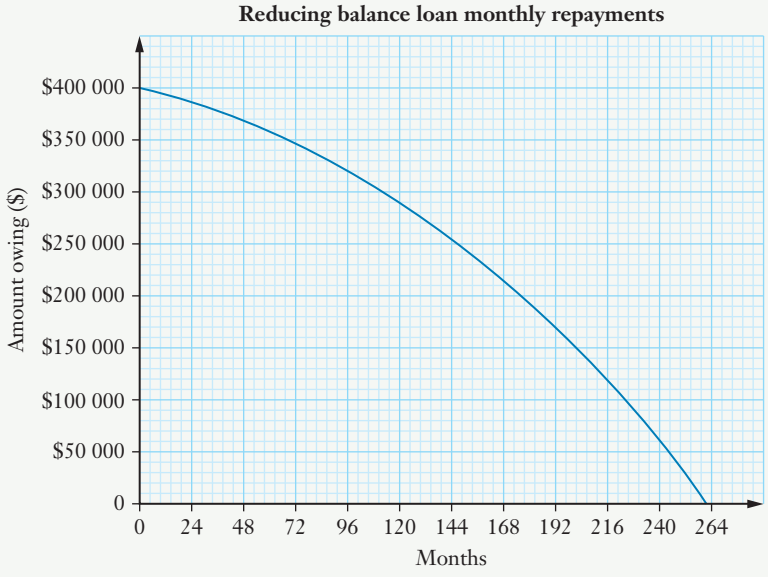
iii the total interest paid.

b If the $16 000 car loan in part **a** is paid off over 24 months instead, how much money would be saved?

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7 The graph below shows the progress of Ante's home loan, in which he made monthly repayments of $3000.

Exercise 4.05



a How much money did Ante borrow?

b What was the amount owing after 10 years?

c How many years and months did it take Ante to pay off the loan?

d What was the time taken for the loan to be half-paid?

Qz Chapter quiz

5.

CORRELATION AND REGRESSION

Usain Bolt has been called 'the fastest man alive' for holding world and Olympic records in the 100 m and 200 m sprints. Contributing to his success is his height, 195 cm, and his average stride length, 2.439 metres. This means that he only takes 41 steps in a 100 m race, which is 3 or 4 fewer steps than his rivals.

CHAPTER OUTLINE

S4 5.01 Scatterplots
S4 5.02 Correlation
S4 5.03 Line of best fit
S4 5.04 Least-squares regression line
S4 5.05 Life expectancy



IN THIS CHAPTER YOU WILL:

- use scatterplots to graph 2 sets of data and identify patterns (linear versus non-linear), associations and correlations between the bivariate data
- describe the direction (positive/negative) and strength (strong, moderate, weak) of linear patterns
- calculate the correlation coefficient of bivariate data using digital technology, and interpret its size and sign
- draw a line of best fit by eye and find its equation
- fit a least-squares regression line to bivariate data using digital technology
- interpret the gradient and vertical intercept of a line of best fit
- use a line of best fit to make predictions by interpolation or extrapolation
- use scatterplots and correlation to investigate, analyse and describe associations within bivariate data, such as biometric and life expectancy data.

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TERMINOLOGY

biometric data
bivariate data
causality
correlation
correlation coefficient (r)
dependent variable
extrapolation
independent variable
interpolation
least-squares regression line
life expectancy
linear
line of best fit
predict
regression line
scatterplot
trendline

WS
Assignment 5

SkillCheck

1 If $y = 0.34x + 45.8$, find:

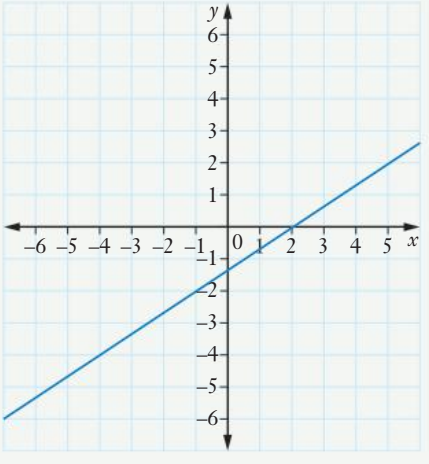
a y when $x = 18.7$ **b** x when $y = 204.3$.

2 Find the gradient of the line passing through the points:

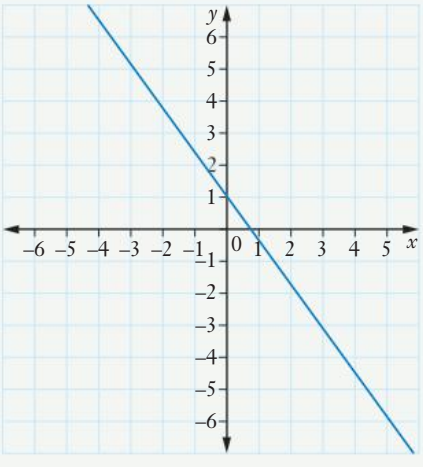
a (4, 8) and (9, 2) **b** (3.7, 22.6) and (8.5, 7.2).

3 Find the equation of each line.

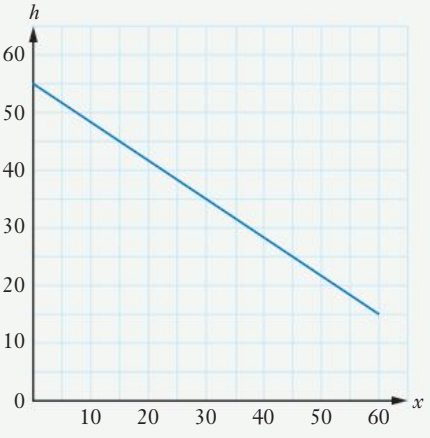
a



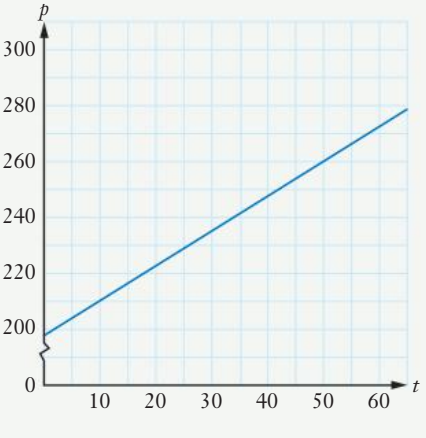
b



c



d



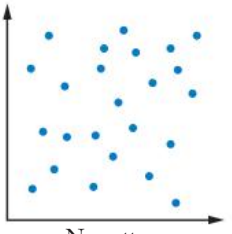
ISBN 9780170413633

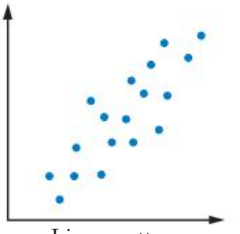
5.01 Scatterplots

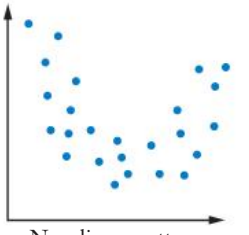
A **scatterplot** is a graph of points on the number plane. Each point represents the value of 2 different variables, so scatterplots represent **bivariate data** (data relating to 2 variables). The resulting graph may show a pattern that suggests that an association or a relationship exists between the 2 variables.

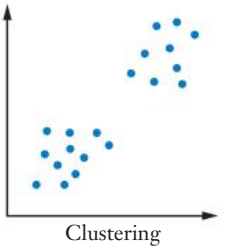
WS
A page of scatterplots

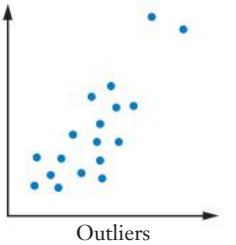
WS
Body measurements

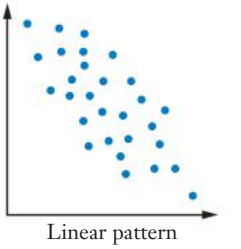

No pattern


Linear pattern

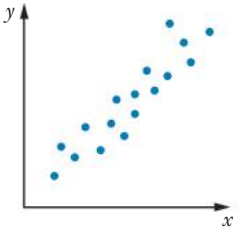

Non-linear pattern


Clustering

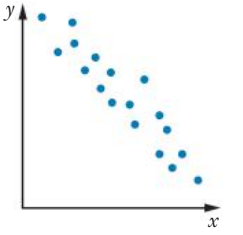

Outliers


Linear pattern

The type of **linear** pattern will indicate the strength and direction of the association.

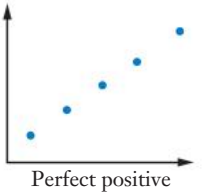


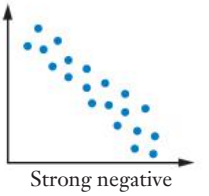
The 2 variables have a **positive association** if y increases as x increases.

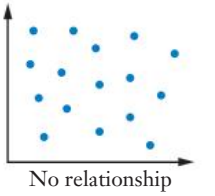


The 2 variables have a **negative association** if y decreases as x increases.

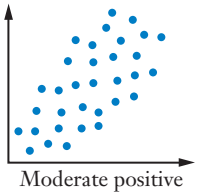
The closer the dots are to a line, the stronger the association or relationship.


Perfect positive

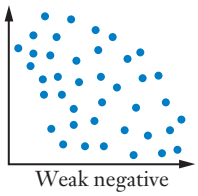

Strong negative


No relationship

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Moderate positive


Perfect negative


Weak negative

Data obtained by measuring physical characteristics, such as height, arm span and bone length, are called **biometric data**. Body measurements usually have a moderate to strong linear relationship.

EXAMPLE 1

The heights and arm spans of 10 male swimmers are shown in the table.

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Plot the data on a scatterplot.

b Describe the pattern and state whether the association between a male swimmer's height and arm span is positive, negative or zero.

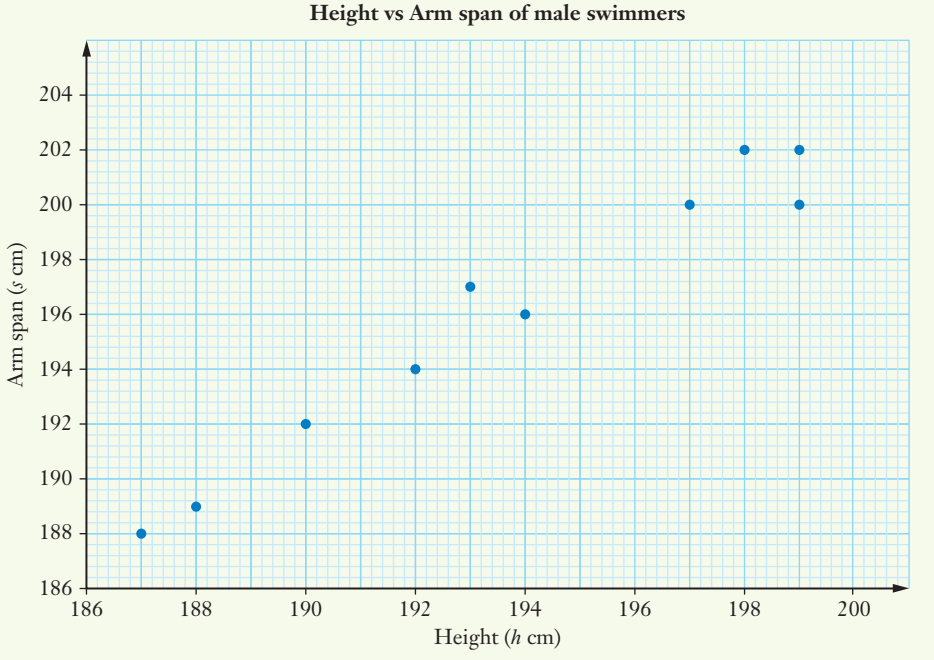
c Is the relationship strong, moderate or weak?

d Comment on the statement 'For a male swimmer, arm span and height are approximately the same'.

Note that for scatterplots, you do not need to start both axes at 0. Choose an appropriate start and end value for each axis to reflect the values of the given data.

Solution

a



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b The pattern is linear. The relationship is positive. As height increases, arm span increases.

c The relationship is strong as the points are close to a line.

d True. According to the data (table and graph), arm span and height are approximately the same for a male swimmer.

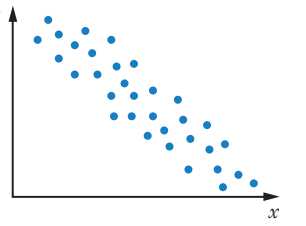
Exercise 5.01 Scatterplots

In this exercise, draw scatterplots on graph paper or using a spreadsheet.

1 a What type of association is shown on the scatterplot?

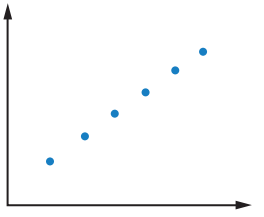
b What is the missing word from this sentence?

'As the value of x increases, the value of y _____.'

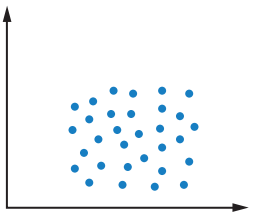


2 For each scatterplot, state whether a linear relationship exists and if it does, whether it is positive or negative, and whether it is strong, moderate or weak.

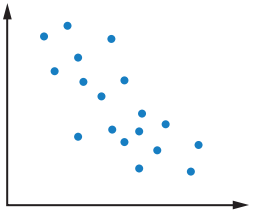
a



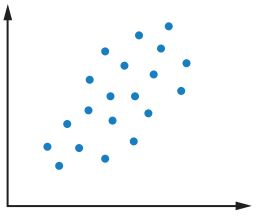
b



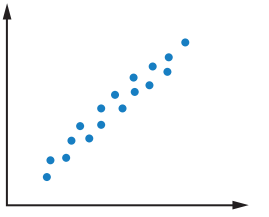
c



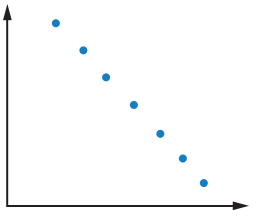
d



e



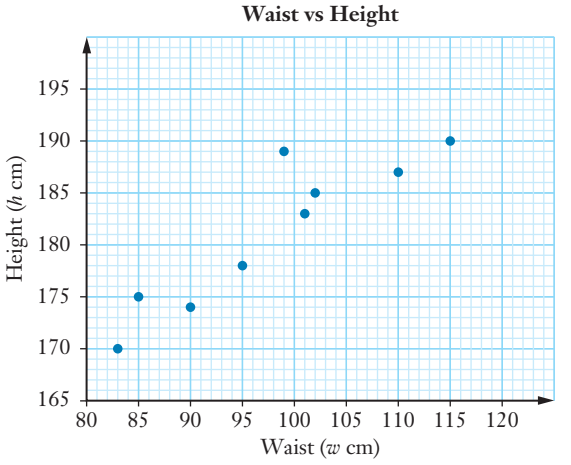
f



3 Which scatterplot in Question **2** could represent the relationship between height and stride length (the biggest step they can take from a standing position measured between toe positions) of people?

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4 The waist lengths and heights of 10 people were measured and the results graphed on a scatterplot.



a What is the waist circumference of the person who is 189 cm tall?

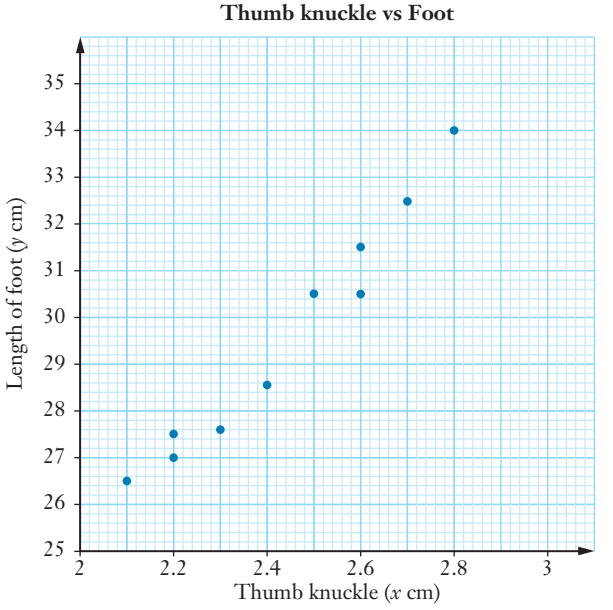
b How tall is the person with a waist circumference of 90 cm?

c Is the relationship between waist circumference and height linear?

d What is the strength of the relationship? Justify your answer.

e Consider the statement: 'A person's waist circumference is approximately half of their height'. Does the data support this statement? Justify your answer.

5 Zach measured the width of his thumb knuckle and the length of his foot, and exclaimed to Jacinta, '12 of my thumb knuckles equal the length of my foot!' They test this theory by taking their own measurements plus those of 8 friends and plotted the results.



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a Describe the pattern of the data.

b Is the association between thumb knuckle width and foot length positive or negative?

c Is the association strong, moderate or weak?

d Do you agree with Zach's theory? Justify your answer.

6 The heights and weights of 12 students are shown in the table.

Example 1

Height, *H* cm	150	151	156	163	178	157	160	165	175	153	162	168
Weight, *W* kg	54	55	61	62	76	64	60	65	84	60	55	70

a Plot the data on a scatterplot.

b Describe the pattern of the data.

c Describe the direction and strength of the association between student height and weight.

7 This is a table of typical body measurements, correct to the nearest centimetre, for girls aged 4 to 16 years.

Age (years)	4	6	8	10	12	14	16
Height	104	116	128	140	152	164	176
Chest circumference	58	60	64	68	76	84	92
Waist	54	55	58	61	65	69	73
Hip	61	64	69	74	82	90	98
Neck circumference	28	29	30	31	33	35	37
Outside leg (waist height)	59	68	77	86	94	102	110

a Plot the data for height and outside leg on a scatterplot.

b Describe the strength of the association between height and outside leg.

c Plot the data for height and hip circumference on a scatterplot.

d Describe the strength of the association between height and hip circumference.

e Plot the data for waist and neck circumference on a scatterplot.

f Describe the strength of the association between waist and neck circumference.

g Jenny measured her waist and neck and said 'my waist is double my neck measurement'. Does the data reflect this statement?

8 A paleontologist measured the thigh bone (femur) and upper arm bone (humerus) of some fossils. The results are shown in the table.

Femur (*f* cm)	60	73	56	50	67	62	72	59
Humerus (*h* cm)	70	78	59	40	70	61	71	58

a Plot the data on a scatterplot.

b Describe the direction and strength of the association between the lengths of the two bones.

c Do you think there is a linear association between femur and humerus lengths in humans? Justify your answer.

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INVESTIGATION

ANCIENT BODY MEASUREMENTS

In ancient and medieval times, people used parts of the body to measure lengths.

A **palm** or **hand breadth** was the width of your four fingers pressed together and is about 7.5 cm.

A **handspan** was the distance between thumb and little finger when the fingers are spread and is about 22.5cm.

A **foot** was the length of King Henry I's foot, defined as 12 inches or about 30.5 cm. An average person's foot today is 23.5cm.

A **cubit** was the length of the forearm, from elbow to tip of middle finger. Since people from ancient cultures differed in size, the cubit ranged from 44 to 52 cm. Today, the average adult cubit is 46 cm.

1 Show that:

a 1 cubit = 6 palms **b** 1 cubit = 2 spans **c** 1 span = 3 palms.

2 Noah's Ark was described to be 300 cubits long, 50 cubits wide and 30 cubits high. Convert these measurements to:

a spans **b** palms **c** feet **d** metres.

3 Collect measurements for the cubit, foot, palm and span from 10 to 12 people.

a Find the average for each measurement and compare with the values given above.

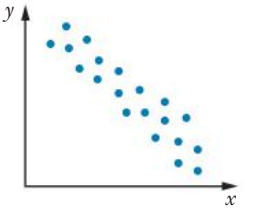
b Select any two measurements, graph them on a scatterplot and determine whether there is a relationship between them.

WS
Correlations matching game

5.02 Correlation

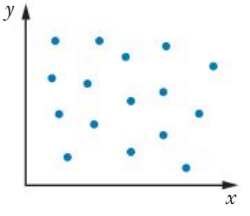
If 2 variables are plotted on a scatterplot and the points have a linear association, we say that the 2 variables are **correlated**. **Correlation** is a measure of the strength of the linear association or relationship between the 2 variables.

A



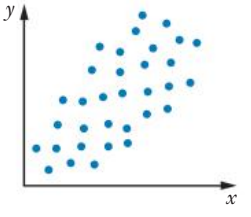
strong negative correlation

B



no correlation

C



moderate positive correlation

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Correlation and causality

A correlation between 2 variables does not necessarily mean that one variable *causes* the other to occur (**causality**). For example, a person's height and foot size have a high correlation (as height increases, foot size increases) but a change in height does not necessarily cause a change in foot size. External factors, such as age and growth rate, are usually involved in strong but non-causal relationships.

EXAMPLE 2

Each pair of variables has a strong correlation. Determine whether each pair has a causal relationship, and for those that do not, suggest an external factor that may affect the correlation.

a height and weight of a person

b length of right arm and length of left arm

c price of petrol and amount of petrol sold at a petrol station

d number of hot dogs and drinks sold at a football game

e amount of road traffic and driving time to school/work

Solution

a Not causal: affected by such factors as age, fat and muscle distribution

b Not causal: affected by age and growth rate

c Causal: the higher the price, the lower the sales (negative correlation)

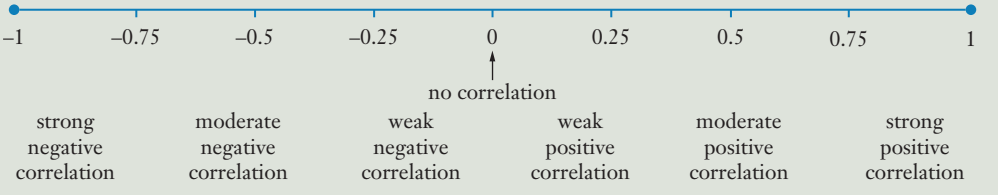
d Not causal: affected by number of people at the game

e Causal: the greater the traffic, the longer the driving time

Pearson's correlation coefficient

Pearson's correlation coefficient

The **correlation coefficient**, r, is a number between -1 and $+1$ that measures the correlation between 2 variables. This scale shows the meaning for the different values of r.

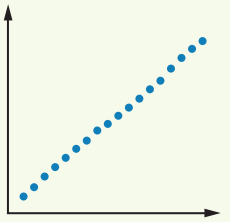


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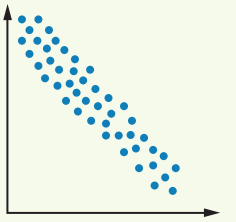
EXAMPLE 3

Match each scatterplot to a correlation coefficient below and describe the correlation.

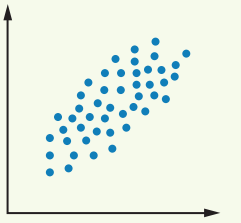
a



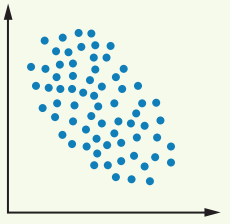
b



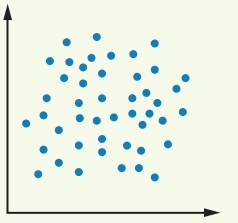
c



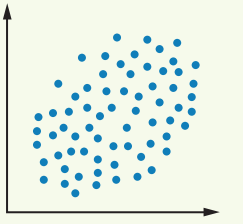
d



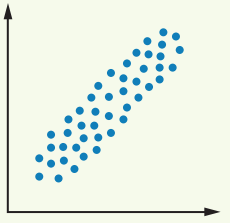
e



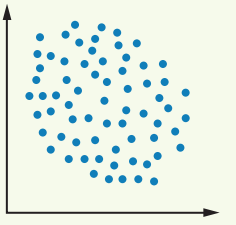
f



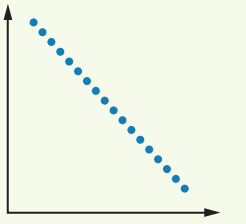
g



h



i



$r = -1$ $r = -0.5$ $r = -0.2$ $r = 0$ $r = 0.3$

$r = 0.7$ $r = 0.8$ $r = -0.9$ $r = 1$

Solution

a $r = 1$ perfect positive correlation

b $r = -0.9$ strong negative correlation

c $r = 0.7$ moderate positive correlation

d $r = -0.5$ moderate negative correlation

e $r = 0$ zero correlation

f $r = 0.3$ weak to moderate positive correlation

g $r = 0.8$ strong positive correlation

h $r = -0.2$ weak negative correlation

i $r = -1$ perfect negative correlation

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Calculating Pearson's correlation coefficient

Pearson's correlation coefficient, r, can be calculated using a scientific calculator, graphics calculator, online calculator or spreadsheet. As with standard deviation, there is a complex formula for calculating the correlation coefficient but you are not required to use it.

EXAMPLE 4

The heights and arm spans of 10 male swimmers are shown in the table (from Example **1**).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

Find the correlation coefficient, r, correct to 2 decimal places.

Solution

For bivariate data, follow the instructions for your calculator as shown in the tables below for a correlation coefficient of $r \approx 0.98$.

Operation	**Casio scientific**	**Sharp scientific**
Start statistics mode.	MODE STAT A + BX	MODE STAT LINE
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table Enter in X column 190 = 192 =, etc. enter in Y column 192 = 194 =, etc. AC to leave table	190 2ndF STO 192 M+ 192 2ndF STO 194 M+ etc.
Calculate the correlation coefficient ($r = 0.9805...$)	SHIFT 1 Reg r =	ALPHA r =

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Operation	Casio graphics	Texas Instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and CLEAR, then 2nd 0 and scroll down to select DiagnosticOn ENTER ENTER.
Clear statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2 column	SHIFT Edit … with cursor on L1 CLEAR ENTER Repeat for L2.
Enter x-scores in List 1.	190 EXE 192 EXE, etc.	190 CLEAR 192 ENTER, etc.
Enter y-scores in List 2.	192 EXE 194 EXE, etc.	192 ENTER 194 ENTER, etc.
Calculate the correlation coefficient (r = 0.9805 ...)	CALC REG X to see 'LinearReg' and the value of r.	SHIFT CALC LinReg(ax+b) ENTER to find value of r.

On a spreadsheet, enter the data into two rows as shown below, then enter the formula = PEARSON(B1:K1, B2:K2) into any blank cell to calculate the correlation coefficient $r = 0.9805....$

	A	B	C	D	E	F	G	H	I	J	K
1	**Height (cm)**	190	192	199	188	197	187	194	198	193	199
2	**Arm span (cm)**	192	194	202	189	200	188	196	202	197	200

The formal name for this linear correlation coefficient is **Pearson product-moment correlation coefficient**.

Exercise 5.02 Correlation

1 Describe each term in your own words.

a bivariate data **b** correlation **c** correlation coefficient

2 Which pair of variables below have no correlation? Select **A**, **B**, **C** or **D**.

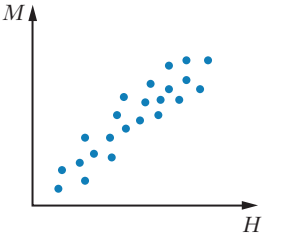
A car engine size and towing capacity

B height and ability to touch the ceiling

C number of pies eaten for lunch and handspan

D age and number of doctor visits

3 Which phrase describes the correlation between variables H and M on the scatterplot? Select **A**, **B**, **C** or **D**.



A moderate positive correlation

B weak negative correlation

C strong positive correlation

D moderate negative correlation

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4 Does each pair of variables have a positive, negative or zero correlation?

- **a** number of alcoholic drinks consumed and reaction speed
- **b** number of cigarettes smoked and probability of lung cancer
- **c** price of a packet of biscuits and the change from $10
- **d** number of road accidents and number of ambulances on the road
- **e** size of feet and swimming speed
- **f** length of femur (thigh bone) and length of humerus (upper arm bone)
- **g** tennis ability and time spent practising tennis shots
- **h** time spent awake and time spent asleep

5 Each pair of variables has a correlation. Which ones have a causal relationship?

Example 2

- **a** abdominal circumference and head circumference of a foetus
- **b** height and arm span of nurses
- **c** shoe size and age of children
- **d** number of patients in a hospital emergency room and waiting time
- **e** number of computers and number of TVs in a home
- **f** home loan interest rate and size of monthly repayments
- **g** driving speed of a vehicle and time taken to arrive home
- **h** spelling test mark and English test mark
- **i** driving speed of a vehicle and amount of fuel used
- **j** temperature and ice cream sales

6 For each correlation coefficient value, draw a scatterplot that illustrates this correlation.

Example 3

a 0.2 **b** 0.9 **c** –0.7
d 0.5 **e** –0.3 **f** 0

7 The systolic value of Victor's blood pressure was measured every 5 years.

Example 4

Age (A years)	15	20	25	30	35	40	45	50	55	60
Blood pressure (B mmHg)	117	120	121	122	123	125	127	129	131	134

Note: mmHg means millimetres of mercury.

- **a** Draw a scatterplot of the data.
- **b** Describe any correlation between the 2 variables.
- **c** Write an estimate for the correlation coefficient.
- **d** Calculate the correlation coefficient, r, correct to 2 decimal places.
- **e** How close was your estimate to the actual value?

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8 A local gym recorded the age and pulse rate of 10 people before exercise.

Age (A years)	45	20	15	62	35	49	18	38	55	24
Pulse rate (B beats/min)	71	79	83	70	68	67	86	72	68	86

a Draw a scatterplot of the data.

b Describe any correlation between the 2 variables.

c Write an estimate for the correlation coefficient.

d Calculate Pearson's correlation coefficient, r, correct to 2 decimal places.

9 The maximum monthly temperatures and monthly rainfall for Sydney last year are shown in the table.

Temperature (°C)	27.4	28.2	26.8	24.7	22.9	19	18.5	19.1	21.4	23.9	26.1	28.1
Rainfall (mm)	249.8	25.8	193.2	155	7.2	305	104.6	151.4	70	31.4	27.2	65

a Find, correct to 3 significant figures, Pearson's correlation coefficient.

b Describe the correlation between temperature and rainfall.

10 a i Find the correlation for the data in the table below.

Age (years)	14	18	15	13	16	16	17	11	13	11	12	14
Height	162	180	182	163	171	175	187	146	151	134	153	166

ii Describe the correlation between age and height.

b i Find the correlation for the data in the table below.

Height	162	180	182	163	171	175	187	146	151	134	153	166
Age (years)	14	18	15	13	16	16	17	11	13	11	12	14

ii Is the correlation between height and age different from the correlation between age and height?

11 Calculate, correct to 3 significant figures, the correlation coefficient for each set of bivariate data.

a

Waist (w cm)	54	55	58	61	65	69	73
Neck circumference (n cm)	28	29	30	31	33	35	37

b

Height (h cm)	155	168	162	175	163	178	180	165
Weight (w kg)	50	73	55	84	62	76	80	59

c

Femur (f cm)	60	73	56	50	67	62	72	59
Humerus (h cm)	70	78	59	40	70	61	71	58

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INVESTIGATION

CORRELATION OF BODY MEASUREMENTS

Does a person's height correlate with their other body measurements? Make the following measurements for 10 to 15 people.

- height
- arm span (distance between the middle fingertips when arms are outstretched)
- head circumference (measured at the widest point)
- handspan (distance between thumb and little finger when hand is outstretched)
- wrist circumference (at point where the hand bends)
- femur length (hip to knee)

1 Construct a scatterplot comparing height with arm span, head circumference, handspan, wrist circumference or femur length.

2 Are the 2 measurements correlated?

3 Is there a strong or weak correlation?

4 Are there any outliers?

5 Calculate Pearson's correlation coefficient, correct to 4 decimal places.

6 Do you think that your results are typical of the Australian adult population?

7 What conclusions can you draw about the relationships between the 2 measurements? Write a paragraph about your findings.

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WS
Height vs shoe size

WS
Lines of fit

5.03 Line of best fit

If 2 variables x and y show a strong linear relationship when graphed on a scatterplot, the linear relationship can be modelled by fitting or drawing an appropriate **line of best fit** and finding its equation $y = mx + c$. This can then be used to describe and quantify associations between the variables. The **independent variable** (in this case, x) is shown on the horizontal axis, while the **dependent variable** (in this case, y) is shown on the vertical axis.

A line of best fit can be drawn on graph paper but it is easier to graph it using technology such as a spreadsheet, dynamic geometry or graphing software.

Line of best fit

A line of best fit:

- represents most or all of the points as closely as possible
- goes through as many points as possible
- has roughly the same number of points above and below it
- is drawn so that the distances of points from the line are as small as possible.

A line of best fit can be used to **predict** what might happen:

- between the points on a scatterplot, within the range of data (called **interpolation**)
- beyond the points on the scatterplot, outside the range of data (called **extrapolation**).

EXAMPLE 5

The heights and arm spans of 10 male swimmers are shown in the table (from Example **1**).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Graph the data on a scatterplot and draw a line of best fit to represent all data points.

b Find the equation of the line of best fit.

c Use the graph and the equation to estimate the arm span of a male swimmer who is 191 cm tall.

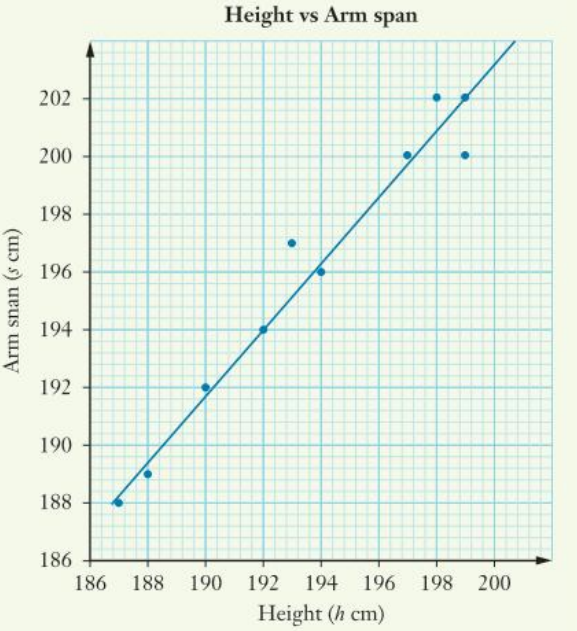
d Use the graph and the equation to estimate the height of a male swimmer with an arm span of 195 cm.

e Is it possible to predict the arm span of a female swimmer who is 165 cm tall?

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Solution

a Use a transparent ruler to draw a line of best fit. Points not on the line should be roughly equally spaced from the line.



Your line of best fit may differ from the one shown here but should look similar.

b The equation of the line is of the form $y = mx + c$.

To find the gradient m, choose 2 points on the line, say (192, 194) and (199, 202).

$$m = \frac{202 - 194}{199 - 192}$$
$$= \frac{8}{7}$$
$$= 1.1428...$$
$$\approx 1.14 \quad \text{(to 2 decimal places)}$$

$s = 1.14h + c$

To find the vertical intercept, c, substitute one point on the line, say (199, 202) again.

$$202 = 1.14(199) + c$$
$$202 = 226.86 + c$$
$$202 - 226.86 = c$$
$$c = -24.86$$

$s = 1.14h - 24.86$

c Reading from the graph, the arm span of a male swimmer who is 191 cm tall is 193 cm.

Substituting $h = 188$ into $s = 1.14h - 24.86$:

$$s = 1.14(191) - 24.86$$
$$= 192.88$$
$$\approx 193$$

The arm span of a male swimmer who is 188 cm tall is 189 cm.

d Reading from the graph, the height of a male swimmer with an arm span of 195 cm is 193 cm.

Substituting $s = 195$ into $s = 1.14h - 24.86$:

$$195 = 1.14h - 24.86$$

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$$195 + 24.86 = 1.14h$$

$$1.14h = 219.86$$

$$h = \frac{219.8}{1.14}$$

$$= 192.8596\ldots$$

$$\approx 193$$

The height of a male swimmer with an arm span of 195 cm is 193 cm.

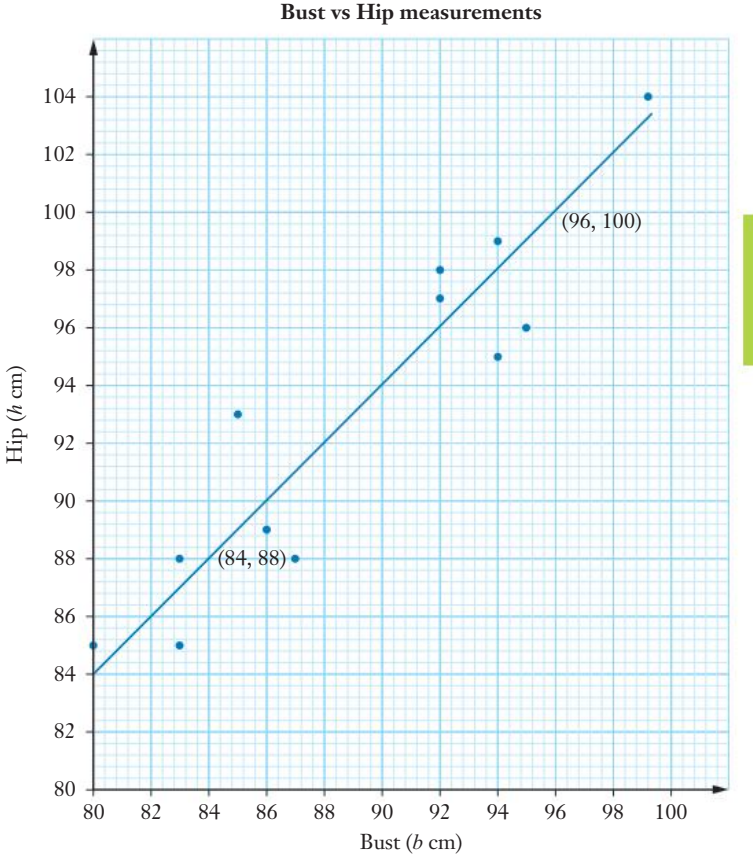
e No, you cannot predict the arm span of a female swimmer who is 165 cm tall. The data is about male swimmers only. Also, the value of 165 cm is well outside the limits of the data here.

Exercise 5.03 Line of best fit

Example 5

1 The bust and hip measurements of 12 women were graphed on a scatterplot. A line of best fit was then drawn by eye.

Bust (b cm)	83	95	99	85	94	80	87	94	92	86	83	92
Hip (h cm)	88	96	104	93	95	85	88	99	97	89	85	98



This line of best fit does not pass through any points on the scatterplot but it does go through (84, 88) and (96, 100).

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a Find the equation of the line of best fit.

b Use the equation to predict the hip size of a woman with a bust size of 90 cm.

c Use the graph to estimate the bust size for a woman with a hip size of 100 cm.

d Is it possible to predict the hip size of a woman with a 110 cm bust? Why?

2 The wrists and sleeve lengths of 12 children were taken.

Wrist (w cm)	16	18	16	16	18	17	17	16	15.5	18.5	17	16.5	16	16	18
Sleeve length (s cm)	58	60	59	59	61	60	59.5	58	57.5	61.5	58.5	59	57.5	60	60.5

a Draw a scatterplot to represent the data.

b Draw a line of best fit by eye.

c Use your graph to estimate:

i the wrist measurement of a student with a 60 cm sleeve length

ii the sleeve length of a student with a 15.6 cm wrist.

d Show that the equation of your line of best fit is close to $s = w + 40$.

e Use the equation of your line of best fit to predict:

i the wrist measurement of a student with a 58.5 cm sleeve length

ii the sleeve length of a student with a 17.5 cm wrist.

3 The ice cream sales and temperature were recorded by a shop close to a beach over 10 consecutive days.

Temperature (°C)	24	23	26	30	33	31	26	25	36	32
Ice cream sales ($)	812	830	1015	1305	1535	1260	1060	1020	1700	1470

a Draw a scatterplot to represent the data.

b Draw a line of best fit.

c Find the equation of the line of best fit.

d Use the graph to estimate the sales of ice cream for a temperature of:

i 28°C **ii** 34°C.

e Is it possible to predict ice cream sales for a temperature of:

i 20°C? **ii** 40°C?

4 Adam's weight at different ages is listed in the table below.

Age, a (years)	2	5	8	10	13	15
Weight, w (kg)	12	18	24	30	43	53

a Graph this data on a number plane and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the graph and the equation to estimate, to the nearest kilogram, Adam's weight at age 18.

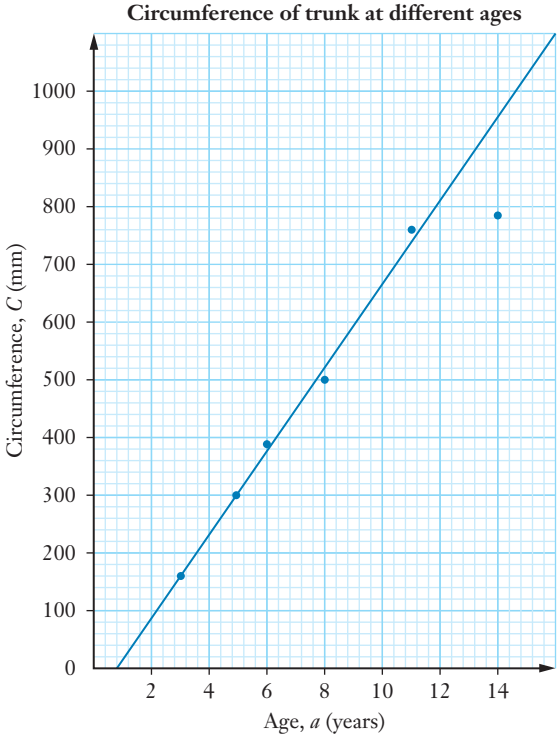
d Use the equation to find the age (correct to one decimal place) when Adam would reach a weight of 40 kg.

e What is a limitation of this model?

5 The age of a large tree can be predicted by measuring the circumference of its trunk. The following data was collected.

Age, a (years)	3	5	6	8	11	14
Circumference, C (mm)	155	306	392	502	763	790

This data is graphed below and a line of best fit drawn. Note that the point (14, 790) was treated as an outlier when graphing the line.



a Find the equation of the line of best fit.

b What is the gradient and what does it represent?

c Use the graph and the equation you found in part **a** to find the circumference of the tree at age:

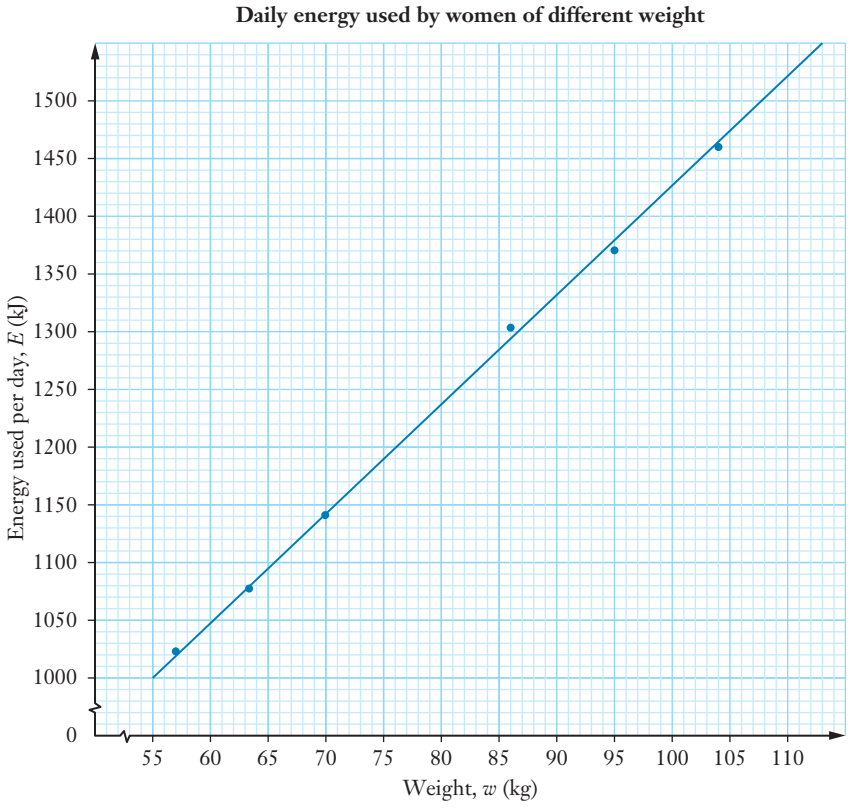
i 10 years

ii 20 years.

d Use the graph and equation to find the age of the tree when its circumference is 650 mm. Answer to the nearest year.

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6 A dietician measured the amount of energy, E kilojoules, used per day by women of different weights, w kg, and graphed the following data.



a Find the equation of the line of best fit.

b How much energy is used each day by a 75 kg woman?

c What is the weight of a woman who uses 1290 kJ of energy each day? Answer to the nearest kilogram.

d This linear model is only accurate for weights of over 50 kg. Why do you think this is so?

7 The population figures for Australia from 1980 to 2015 are given in the table below.

Year	1980	1985	1990	1995	2000	2005	2010	2015
Population (millions)	14.7	15.8	17.1	18.0	19.0	20.2	22.0	23.9

a What is the independent variable?

b Graph the data on a scatterplot and draw a line of best fit.

c Find the equation of the line of best fit.

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d Use your equation to find the population in:

i 1970 **ii** 2008.

e Use your equation to estimate the Australia's population in:

i 2020 **ii** 2030.

f What are the limitations of your model?

8 Thirteen men had their height and weight measured.

Height (cm)	195	184	168	173	178	170	187	180	185	180	175	182	175
Weight (kg)	84	74	60	70	62	68	64	72	110	70	60	75	75

a Graph the data on a scatterplot and draw a line of best fit.

b Find the equation of the line of best fit.

c Use your equation to find the weight of a man who has a height of:

i 172 cm **ii** 190 cm.

d Use your equation to find the height of a man who has a weight of:

i 80 kg **ii** 100 kg.

e What is a limitation of this model?

WS
Least-squares regression line

5.04 Least-squares regression line

A line of best fit drawn on a scatterplot is called a **regression line**. The equation of this line represents all data points. Regress mean to 'go back', so the points on the scatterplot are 'returning' to the regression line.

Least-squares regression line

The **least-squares regression line** (or **least-squares line of best fit**) is the line of best fit that best represents all data points. Its gradient and y-intercept can be calculated using formulas involving Pearson's correlation coefficient r and the means and standard deviations of the x- and y-values (see below), but you are not required to use it as the regression line can be found using a scientific calculator, graphics calculator, online calculator or spreadsheet.

The equation of the least-squares line of best fit is $y = mx + c$
where

$$m = r \times \frac{\text{standard deviation of } y\text{-scores}}{\text{standard deviation of } x\text{-scores}} \left(\text{or } \frac{r\sigma_y}{\sigma_x} \right)$$

$$c = \text{mean of } y\text{-scores} - m \times \text{mean of } x\text{-scores} = \bar{y} - m\bar{x}$$

If you measure the distance of each point from the line and then square them and add them (for example, $0.8^2 + 0.1^2 + 0^2 + \ldots$), the sum of the squared distances will be as small as possible, hence the name 'least-squares'.

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The next example demonstrates the formula for the least-squares regression line, but you do not have to learn it. However, the instructions following it demonstrate the use of technology in finding this line.

EXAMPLE 6

The heights and arm spans of 10 male swimmers were found to have a Pearson's correlation coefficient of 0.98 (from Example **4**).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Calculate the mean and standard deviation for height and arm span values. Write the standard deviation values, correct to 2 decimal places.

b Find the equation of the least-squares regression line.

c Graph the least-squares regression line.

d Use the graph and the equation to estimate to the nearest centimetre:

i the arm span of a male swimmer 193 cm tall

ii the height of a male swimmer with an arm span of 202 cm.

e Write a sentence stating any similarities or differences between the line of best fit drawn by eye in Example **5** and the least-squares regression line.

Solution

a

	Mean	**Standard deviation**
Height	$\bar{x} = 193.7$	$\sigma_x = 4.24$
Arm span	$\bar{y} = 196$	$\sigma_y = 4.88$

b
$$m = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$
$$= 0.98 \times \frac{4.88}{4.24}$$
$$= 1.1279...$$
$$\approx 1.13$$
$$c = \bar{y} - m\bar{x}$$
$$= 196 - 1.13 \times 193.7$$
$$= -22.88$$

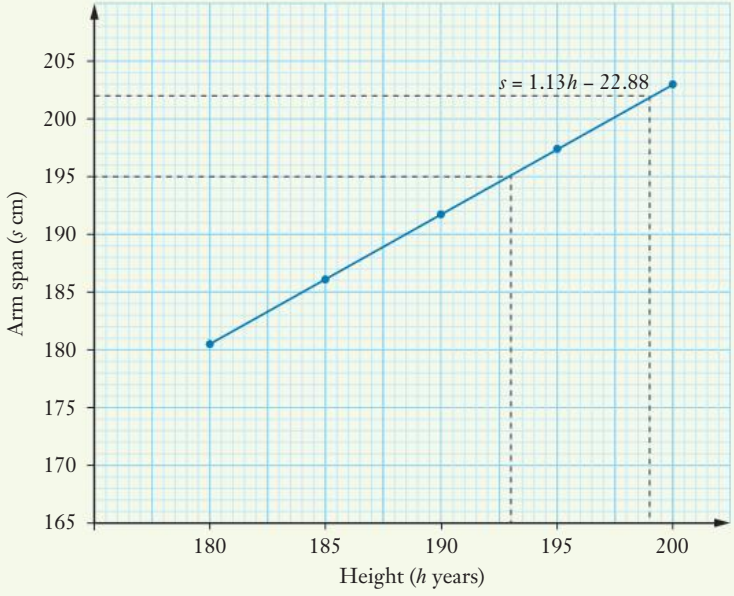
This formula is not part of the course but it is shown here to compare against the calculator methods that follow.

Equation of least-squares regression line is: $s = 1.13h - 22.88$

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c Complete a table of values for $s = 1.13h - 22.88$ and graph the line.

h	180	185	190	195	200
s	180.52	186.17	191.82	197.47	203.12



d **i** From the graph, the arm span of a male swimmer 193 cm tall is 195 cm.

From the equation, when $h = 193$,

$$s = 1.13(193) - 22.88 = 195.21$$
$$\approx 195$$

The arm span of a male swimmer 193 cm tall is 195 cm.

ii From the graph, the height of a male swimmer with an arm span of 202 cm is 199 cm.

From the equation, when $s = 202$,

$$202 = 1.13h - 22.88$$
$$202 + 22.88 = 1.13h$$
$$1.13h = 224.88$$
$$h = \frac{224.88}{1.13}$$
$$= 199.0088...$$
$$\approx 199$$

The height of a male swimmer with an arm span of 202 cm is 199 cm.

e The equation of the least-squares line of best fit ($s = 1.13h - 22.88$) is slightly different from the line of best fit drawn by eye ($s = 1.14h - 24.86$) in Example **5** but the values are close.

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Least-squares regression line on a calculator

The equation of the least-squares regression line is more efficiently found on a scientific or graphics calculator, and the line graphed on a graphics calculator or spreadsheet. See Example **4** on pages 165–166 for the calculator steps involved for the line in Example **5**, but change the last step to the following:

Operation	Casio scientific	Sharp scientific
On these calculators, the gradient is b and the y-intercept is a ($b = 1.127..., a = -22.329...$)	SHIFT 1 Reg b = SHIFT 1 Reg a =	ALPHA b = ALPHA a =

Operation	Casio graphics	Texas Instruments graphics
On these calculators, the gradient is a and the y-intercept is b ($a = 1.127..., b = -22.329...$)	CALC REG X to see 'LinearReg' and the values of a and b	STAT CALC LinReg(ax+b) ENTER to find values of a and b.
To graph the least-squares line	EXIT EXIT EXIT GRPH SET and set StatGraph1 Graph Type to Scatter EXE GRPH1 and CALC X DRAW	STAT PLOT Plot 3 On selects the scatterplot STAT CALC LinReg(ax+b) ENTER GRAPH

The calculator's equation for the least-squares regression line is $s = 1.127h - 22.329$, which is more exact than the equation $s = 1.13h - 22.88$ we found in Example **5** because we did some rounding in our calculations.

See the worksheet 'Linear regression: graphics calculator' for more details.

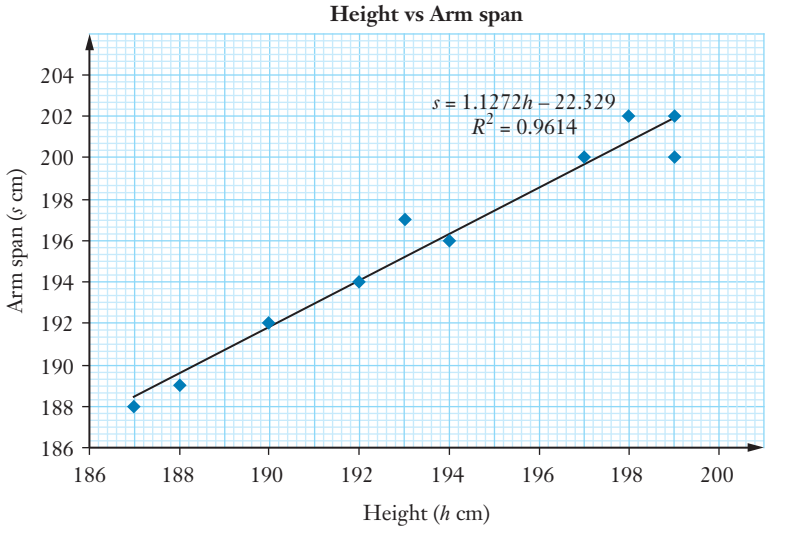
WS
Least-squares regression line graphics calculator

Least-squares regression line (trendline) on a spreadsheet

Follow these steps to graph a least-squares regression line on a spreadsheet.

1. Enter data into the spreadsheet.
2. Draw a scatterplot, add a title to the graph and the name of each axis.
3. From the toolbar, select **Add Chart Element, Trendline** and More trendline options.
4. Under **Trendline options**, tick **Linear**, **Display equation on chart** and **Display R-squared value on chart** (for Pearson's correlation coefficient, $r = \sqrt{R^2}$).

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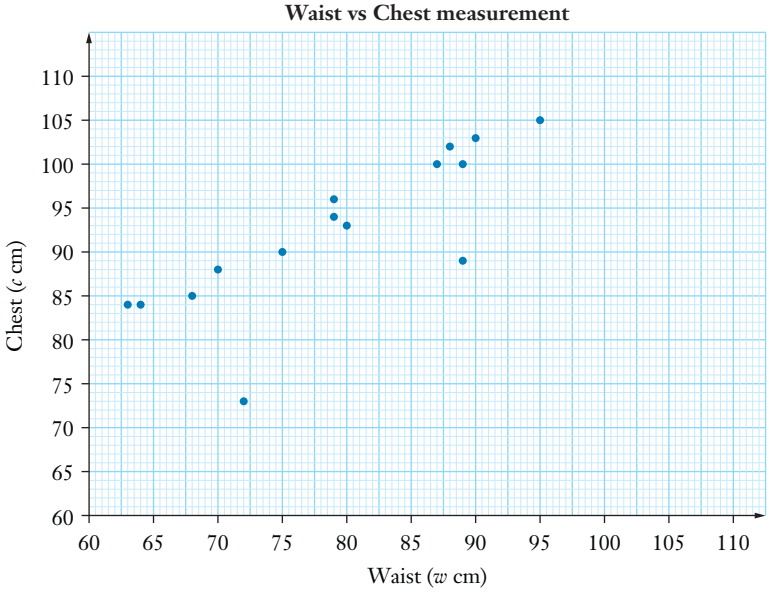
$R^2 = 0.9614$ so the correlation coefficient $r = \sqrt{0.9614} = 0.9805... \approx 0.98$

Exercise 5.04 Least-squares regression line

Example 6

1 A scatterplot comparing waist and chest measurement of 15 people is drawn.

Waist (w cm)	64	80	90	79	75	68	88	63	89	72	79	87	70	95	89
Chest (c cm)	84	93	103	94	90	85	102	84	100	73	96	100	88	105	89



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a Calculate, to 2 decimal places:

- **i** Pearson's correlation coefficient, r
- **ii** the mean and standard deviation of the waist data
- **iii** the mean and standard deviation of the chest data.

b Use the formulas to find the equation of the least-squares regression line.

c What are the coordinates of the 2 outliers?

2 Twelve students measured their height and stride.

Height (h cm)	165	140	180	176	160	164	178	170	148	157	150	162
Length of stride (s cm)	105	85	111	104	98	95	108	102	92	97	88	100

a Calculate the correlation coefficient, r, for the data (correct to 3 decimal places).

b Calculate, correct to 2 decimal places:

- **i** the mean and standard deviation for the height values
- **ii** the mean and standard deviation for the stride values.

c Find the equation of the least-squares regression line.

d Graph this line.

e Use the graph to find:

- **i** the length of stride of a student who is 167 cm tall
- **ii** the height of a student with a stride length of 90 cm.

f Use this equation to find:

- **i** the length of stride of a student who is 167 cm tall
- **ii** the height of a student with a stride length of 90 cm.

3 The table below shows the number of chirps made by crickets at different temperatures.

Temperature, t (°C)	31	22	34	29	27	24	20	27	20	28	26	28	27	28	24
Chirps, C	20	16	20	18	17	16	15	17	15	16	15	17	16	17	14

a Calculate the correlation coefficient, r, for the data, correct to 3 decimal places.

b Use a calculator or spreadsheet to find the equation of the least-squares regression line.

c

- **i** Draw a scatterplot to represent the data in the table and on this scatterplot, draw the graph of the equation of the least-squares regression line.
- **ii** Describe how close the points of the scatterplot are to the least-squares regression line.

d Use the graph and the equation to find:

- **i** the number of chirps at a temperature of 30°C
- **ii** the temperature when there are 10 chirps.

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4 The height and weight of 10 elite sportswomen are shown in the table.

Height, H (cm)	196	190	178	185	174	193	169	174	181	178
Weight, W (kg)	79	75	69	65	62	96	63	70	67	68

a Use a calculator or spreadsheet to find the equation of the least-squares regression line.

b Use the equation to find:

i the weight of a sportswoman who is 200 cm tall

ii the height when the weight is 80 kg.

5 **a** Use a graphics calculator or spreadsheet to graph the wrist and sleeve length data in the table below on a scatterplot.

Wrist (w cm)	16	18	16	16	18	17	17	16	15.5	18.5	17	16.5	16	16	18
Sleeve length (s cm)	58	60	59	59	61	60	59.5	58	57.5	61.5	58.5	59	57.5	60	60.5

b Use the calculator or spreadsheet to graph the least-squares regression line.

c What is the equation of this line?

d What is the correlation for this data? Is it a strong, moderate or weak correlation?

e Compare the least-squares regression line with the line of best fit drawn in Exercise **5.03**, Question **2**. Describe any similarities and differences between them.

6 The weight and body mass index (BMI) of 12 sportsmen are shown in the table.

Weight, w (cm)	85	88	92	78	72	67	61	62	84	90	94	57
BMI, b	24.1	23.8	23.0	22.4	21.1	21.8	20.2	20.1	23.3	23.4	27.4	20.2

a What is the correlation coefficient for this data, correct to 3 decimal places? Is it a strong, moderate or weak correlation?

b Use a calculator or spreadsheet to find the equation of the least-squares regression line.

c Use the equation to find:

i the weight of a sportsman who has a BMI of 25

ii the BMI when the weight is 80 kg.

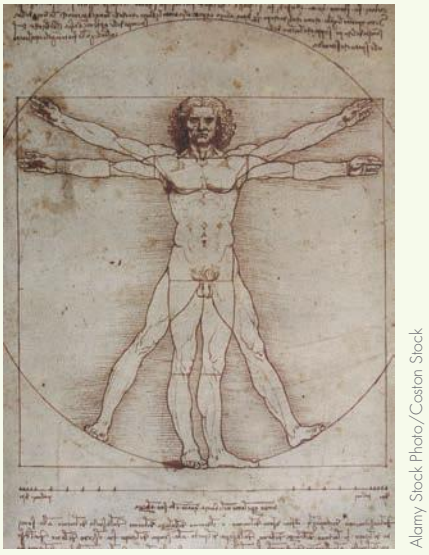
d Explain why you would not use this equation to find the BMI for a sportsman who weighed 50 kg.

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DID YOU KNOW?

Vitruvian man

The Italian artist and inventor Leonardo da Vinci lived from 1452 to 1519 during the Renaissance period. As well as painting the *Mona Lisa*, his great interest in anatomy and the work of the Roman architect Vitruvius led him to draw the *Vitruvian Man* in 1487, an ink sketch of a male figure whose outstretched limbs touch the circumference of a circle and the edges of a square, and whose navel is at the centre of the circle.



Alamy Stock Photo/Costan Stock

Da Vinci believed that each section of the human body is a percentage of the whole.

- height to head length = 8 : 1
- height to elbow–armpit distance = 8 : 1
- height to elbow–hand tip distance = 5 : 1
- height to arm span = 1 : 1
- height to handspan = 10 : 1
- height to the maximum width of the shoulders = 4 : 1
- height to length of foot = 7 : 1
- face length to ear length = 3 : 1.

Choose 2 of these ratios and collect measurements to decide if the Vitruvian theory is still true today.

5.05 Life expectancy

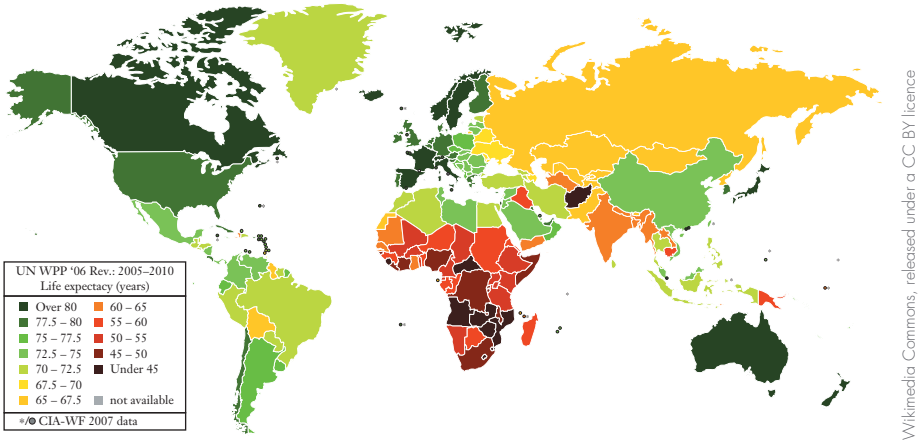
Life expectancy is the average number of years of **life remaining** to a person at a specified age.

For example:

- a boy aged 15 years with a life expectancy of 65.1 years is most likely to die at age 80.1 years
- a woman aged 85 years with a life expectancy of 6.9 years is most likely to die at age 91.9 years.

The population of a country with a good healthcare system and an abundance of food and shelter is likely to have a higher life expectancy. The world map on the next page shows the life expectancy at birth in all countries. Australia is one of the few countries shown in dark green with a life expectancy at birth of 80 years and more, whereas the African countries shown in red have a life expectancy of less than 60 years.

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TECHNOLOGY

Life expectancy calculators

1 Search the Internet for a life expectancy calculator like the one below. It calculates life expectancy based on age and country. It does not account for sex, smoking status, height/weight, education, exercise, medical conditions, diseases or injuries.

Calculate your Life Expectancy

Select Your Age* 17

Select Your Country* Australia

You are expected to live up to the age of 81.72. You have another 65 years to enjoy life. So live happily.

2 Search on the Internet for a life expectancy calculator that requires you to answer more questions relating to your lifestyle and family so that it determines a more accurate life expectancy value. Compare the answer given by this calculator to the answer given by the calculator you found in Question **1**.

3 A calculator that shows how many years cigarette smoking can take out of your lifespan can be found at the **UW Health** website **(University of Wisconsin-Madison Health)**, search for **How does smoking affect your lifespan?**

EXAMPLE 7

This table shows the number of infant deaths per 1000 live births in Australia from 2003 to 2015.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Deaths per 1000	4.8	4.7	4.9	4.7	4.1	4.1	4.2	4.1	3.8	3.3	3.6	3.4	3.2

Source:ABS

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a What percentage of babies died at birth in Australia in 2005?

b Use a graphics calculator or spreadsheet to construct a least-squares line of best fit for this data and find its equation in the form $y = mx + c$.

c Explain the meaning of the gradient of the line.

d Calculate, correct to 2 decimal places, the correlation coefficient for this data and describe the correlation.

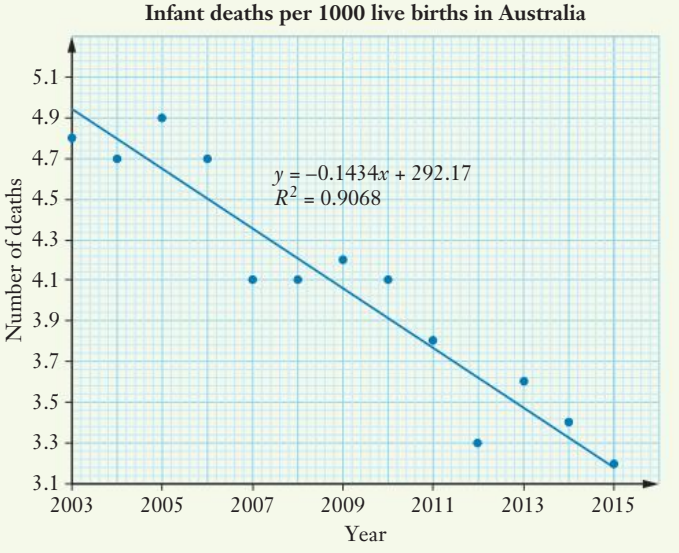
e Do you think the mortality rate will continue to decline? Justify your answer.

Solution

a $\text{Mortality rate in 2005} = \dfrac{4.9}{1000} \times 100\%$

$= 0.49\%$

b The following trendline was created using a spreadsheet.



The least-squares line of best fit has the equation $y = -0.1434x + 292.17$.

c The gradient –0.1434 is the rate at which the mortality rate is decreasing each year.

d As $R^2 = 0.9068$, the correlation coefficient is

$r = \sqrt{0.9068}$

$= 0.95226\ldots$

$= 0.95$

But the gradient and graph indicate that the correlation is negative, so $r = -0.95$.

The correlation is negative and strong.

e Yes, the mortality rate will continue to decline. Because of advancements in healthcare and medicine, infants in Australia are less likely to die during childbirth.

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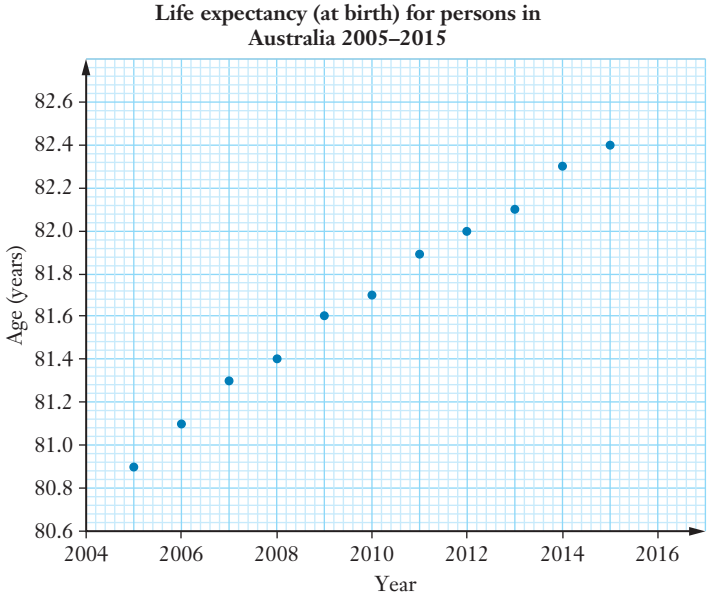
Exercise 5.05 Life expectancy

Example 7

1 The scatterplot below shows the life expectancy at birth in Australia from 2005 to 2015. It has been drawn using the table of data shown here.

Year, *Y*	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Life expectancy, *L* years	80.9	81.1	81.3	81.4	81.6	81.7	81.9	82	82.1	82.3	82.4

Source: ABS



a Describe the correlation of this data.

b Estimate the correlation coefficient.

c Use a graphics calculator or spreadsheet to create a least-squares regression line for this data, and find its equation.

d Calculate the correlation coefficient, correct to 2 decimal places.

e Explain the meaning of the value obtained for the gradient of the regression line.

2 A group of smokers and non-smokers have their ages and life expectancies recorded in the table.

Age (*A* years)	20	40	25	65	55	30	20	45	35	50	25	30	55	65	60
Life expectancy (*E* years)	48	24	32	6	8	40	52	9	25	10	44	29	19	1	1

a Draw a scatterplot for this data comparing age with life expectancy.

b If the smokers in the group have a lower life expectancy than the non-smokers, what is the most likely age at death for:

i the 30-year-old smoker?

ii the 55-year-old non-smoker?

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c What is the difference in life expectancy between the two 25-year-olds?

d Describe the correlation between age and life expectancy for this data.

e Calculate, correct to 2 decimal places, the correlation coefficient.

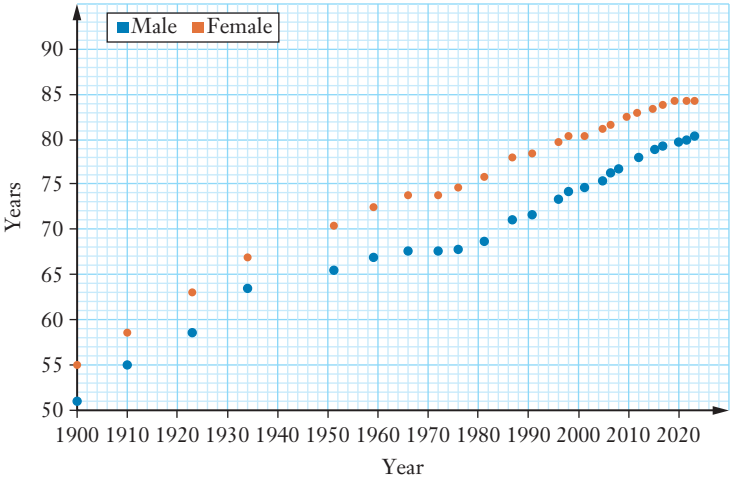
f Draw a line of best fit by eye and show that its equation is close to $E = -A + 65$.

g Use the equation of the line to estimate the life expectancy for a 45-year-old and explain why this value differs from the one in the table.

h Use appropriate technology to find the equation of the least-squares regression line for this data and hence find the most likely age of death for a person who is 18 years old now.

3 This scatterplot shows the life expectancy at birth for boys and girls in Australia from 1900 to 2015.

Life expectancy at birth in Australia from 1900 to 2015



a Write one observation about the life expectancy for:

i females **ii** males.

b When was the difference between the life expectancies for males and females:

i the least? **ii** the most?

c Describe the pattern of the life expectancies between 1900 and 2015.

d Copy the scatterplot and draw a line of best fit by eye for the female data.

e What is your equation of your line of best fit?

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4 This table shows the ages and life expectancies for 12 Indigenous Australians.

Age	69	65	70	68	77	62	77	75	68	64	65	72
Life expectancy	4	3	5	5	2	7	1	1	4	6	10	3

a Use a graphics calculator or spreadsheet to draw a scatterplot for this data.

b Use the scatterplot to describe the correlation of this data.

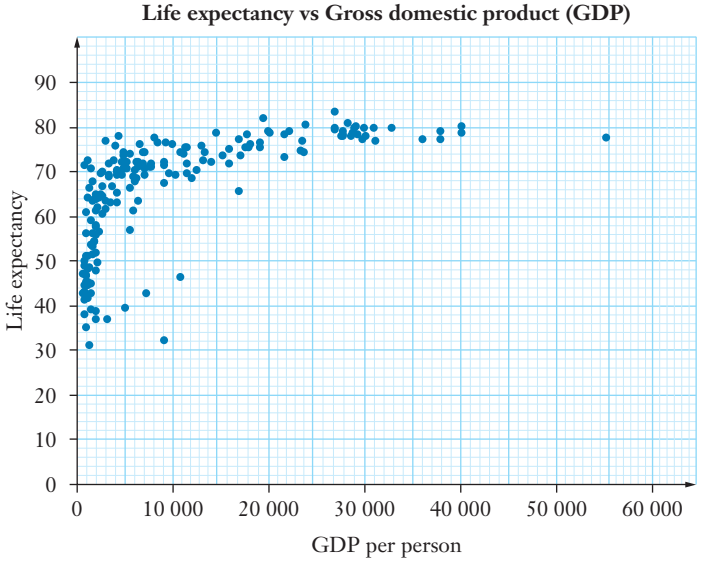
c Use a graphics calculator or spreadsheet to draw a least-squares regression line and find its equation.

d Calculate, to 2 decimal places, the correlation coefficient.

e Use the equation of the least-squares regression line to estimate the life expectancy of:

i Meg, who is 65 **ii** Joe, who is 71.

5 GDP is the total value of all goods and services produced by a country in a given year and is a good indicator of a country's standard of living, which correlates with life expectancy. This scatterplot shows the life expectancy for countries of the world compared to its gross domestic product (GDP) per person.



Source: CIA Factbook

a Describe the life expectancy data when the GDP per person is between $0 and $10 000.

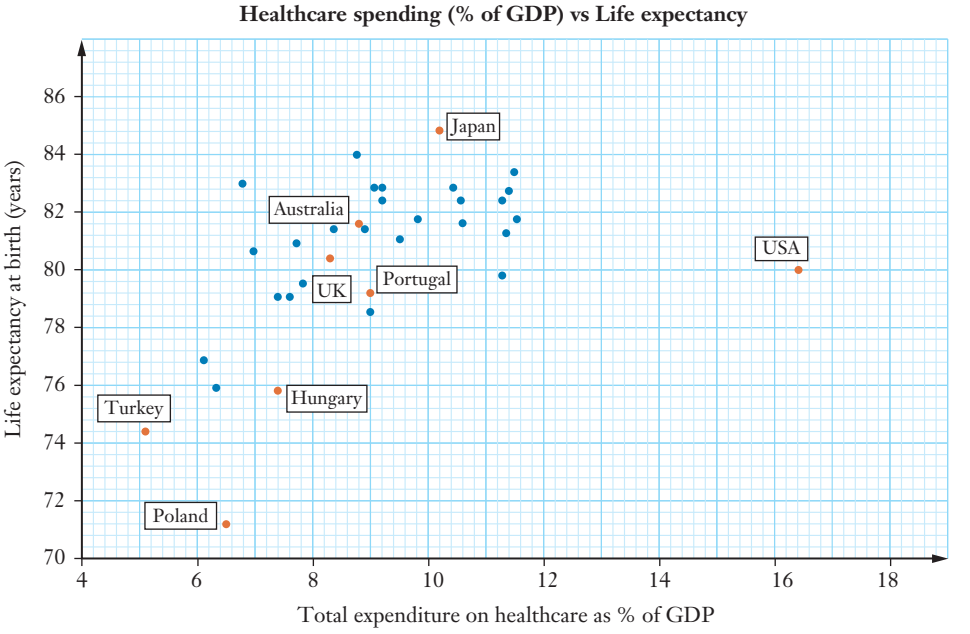
b Explain what happens as the GDP per person increases above $10 000.

c One country produces about $55 000 GDP per person. What is the life expectancy in this country?

d It is thought that above $25 000 GDP per person, life expectancy ceases to increase. Do you agree with this statement? Justify your answer.

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6 This scatterplot compares a country's percentage of GDP spent on healthcare with its life expectancy.



Source: OECD Health Statistics 2015

a What percentage of the GDP is spent on healthcare in:

i Hungary? **ii** Australia? **iii** Portugal?

b What is the life expectancy in:

i Japan? **ii** UK? **iii** Poland?

c Which country spends the highest percentage of their GDP on healthcare and what is this percentage?

d Which country spends the lowest percentage of their GDP on healthcare and what is this percentage?

e Which country spends approximately 8.5% of their GDP on healthcare?

f Compare the spending on healthcare and the life expectancy in Hungary, UK and Japan.

g Is there a correlation between life expectancy and healthcare expenditure? Why?

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SAMPLE HSC PROBLEM

The waist and hip sizes of 15 people were measured and recorded in this table.

Waist (w cm)	65	80	90	78	75	68	84	63	89	72	79	87	74	95	88
Hip (h cm)	88	100	108	102	95	88	102	89	102	78	101	105	95	109	101

a Calculate, correct to 2 decimal places, Pearson's correlation coefficient for this data.

b Find the equation of the least-squares regression line for this data.

c Use the equation to estimate (to the nearest centimetre):

i the hip of a person who has a waist of 70 cm

ii the waist of a person whose hip measurement is 100 cm.

Study tip

Practise with past exams

Why is practising with past exams such an effective study technique? Because it allows you to become familiar with the format, style and level of difficulty expected in an HSC exam, as well as the common topic areas tested. Knowing what to expect helps alleviate exam anxiety. Remember, Mathematics is the only subject in which the exam questions are fairly predictable. The people who write the exams are not going to ask many unusual questions. By the time you have worked through many past exams, this year's HSC paper won't seem that much different.

Don't throw away your old exams. Use them to identify your mistakes and weak areas for further study. Revising topics and then working on mixed questions is a great way to study maths. You might like to complete a past HSC exam under timed conditions to improve your exam technique.

You can find past HSC exam papers with solutions, practice exams and revision guides from websites, bookstores, the library, your maths faculty or the Mathematical Association of NSW (MANSW). This textbook also contains chapter tests and practice papers, with further quizzes and topic tests on the NelsonNet student and teacher websites respectively.

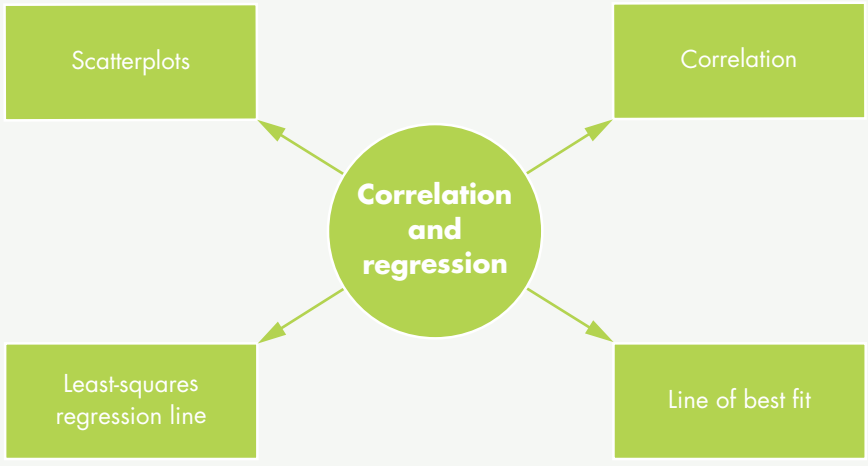
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5. CHAPTER SUMMARY

This chapter, Correlation and regression, examined scatterplots, correlation, lines of best fit and least-squares regression lines and used them to analyse bivariate data in practical contexts such as life expectancy. Be sure that you understand how to use a calculator to find Pearson's correlation coefficient, r, and the equation of the least-squares regression line. You should also know how to draw lines of best fit by hand and using technology, interpret correlation and the strength of association between 2 variables (bivariate data).

Fitting the data find-a-word

Make a summary of this topic. Use the outline at the start of this chapter as a guide.
An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes, and reminders to complete it. Gain a 'whole picture' view of the topic and identify any weak areas.

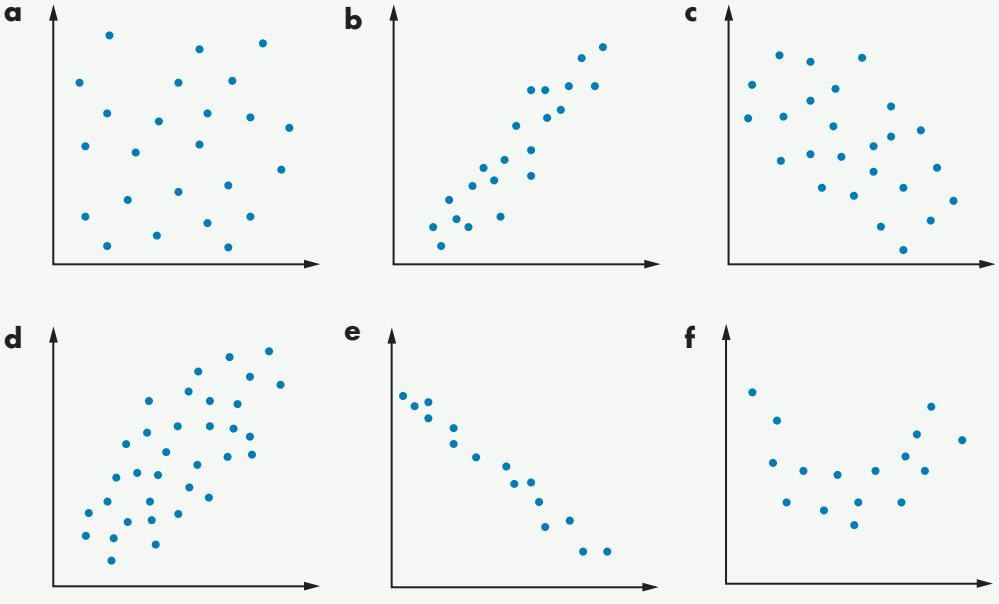


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5. TEST YOURSELF

Exercise 5.01

1 For each scatterplot, state whether a linear relationship or association exists and if it does, whether it is positive or negative and whether it is strong, moderate or weak.



Exercise 5.01

2 The heights and arm spans of 12 boys and 12 girls were measured in centimetres.

Height	145	151	154	158	160	161	163	165	169	170	172	180
Boy arm span	146	150	153	158	159	160	164	163	170	167	170	178
Girl arm span	142	131	153	159	157	159	160	162	167	166	168	176

a Using graph paper or appropriate technology, draw a scatterplot comparing the height of boys and their arm spans.

b Draw a similar scatterplot comparing the height of girls and their arm spans.

c Describe the relationship between the boys' heights and arm spans.

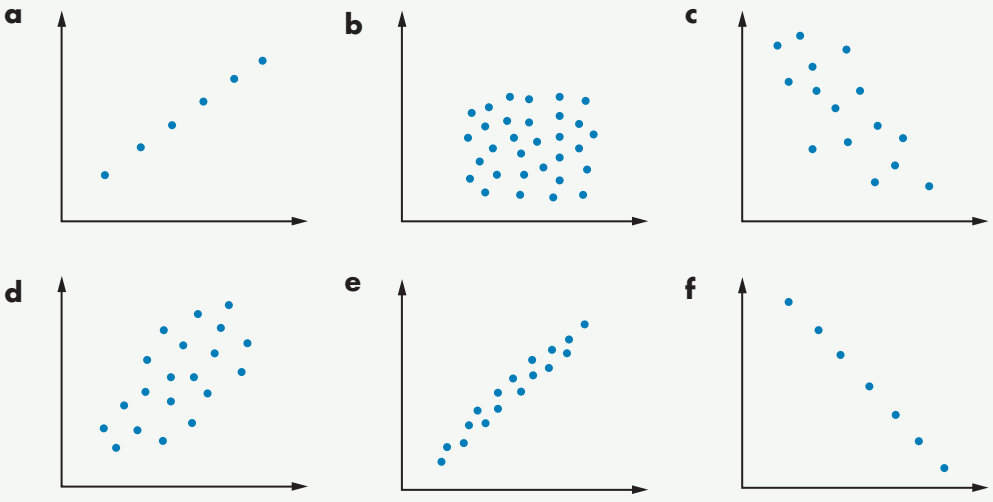
d Is the relationship between height and arm span for girls stronger or weaker than that for boys? Justify your answer.

e There is an outlier in the girls' data. What is it and what does it represent?

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3 For each scatterplot, describe the correlation between the 2 variables and write an estimate for the correlation coefficient.

Exercise 5.02



4 A local gym recorded the age and resting pulse rate of 10 people before exercise.

Exercise 5.02

Age	45	20	15	62	35	49	18	38	55	24
Pulse rate	71	79	83	70	68	67	86	72	68	86

a Draw a scatterplot of the data.

b Describe any correlation between the 2 variables.

c Write an estimate for the correlation coefficient.

d Calculate, correct to 2 decimal places, the correlation coefficient.

e How close was your estimate to the actual value?

5 Adriana is investigating the effect of price on the demand of a tablet device.

Exercise 5.03

After surveying a large sample of consumers for market research, she recorded the results shown in this table.

Price, $\$P$	80	140	190	250	330	400
Demand, D%	90	75	57	44	19	8

a Graph this data on a scatterplot and construct a line of best fit.

b Find the equation of the line.

c Use the graph and the equation to determine the demand for the computer if it is priced at $270. Give your answer to the nearest percentage.

d Use the graph to estimate the price at which the demand will be zero. Give your answer to the nearest $5.

e What is the vertical intercept? Why is the linear model inappropriate for this value?

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Exercise 5.03

6 Jane measured the height and weight of 10 people.

Height, h (cm)	150	175	193	180	165	154	170	159	171	185
Weight, w (kg)	52	70	86	70	60	71	58	62	72	80

a What is the independent variable?

b Plot the points on a scatterplot and construct a line of best fit.

c Find the equation of the line of best fit.

d Use your equation to find the height of a person who weighs:

i 65 kg **ii** 75 kg.

e Can your linear model be used to find the weight of a person who height of 200 cm?

Exercise 5.04

7 The table lists the population of a town from 1965 to 2015.

Year	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015
Population (000's)	32	34	37	42	40	45	50	51	53	55	59

a Draw a scatterplot of the data.

b What is Pearson's correlation coefficient of this data, correct to 3 decimal paces?

c Find the equation of the least-squares regression line.

d Graph the equation of the least-squares regression line on your scatterplot.

e Use the graph and the equation to estimate (to the nearest thousand):

i the population in 2008 **ii** the population in 2020.

Exercise 5.04

8 The heights and shoe sizes of a group of Year 12 students were measured.

Height, H (cm)	174	170	178	179	182	173	176	175	180	177
Shoe size, S	10	7.5	12	11	11.5	9.5	9.5	10.5	11	10

a Use a graphics calculator or spreadsheet to graph height and shoe size on a scatterplot.

b Use the calculator or spreadsheet to graph the least-squares regression line.

c What is the equation of this line?

d What is the correlation coefficient for this data? Is it a strong, moderate or weak correlation?

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Exercise 5.05

9 The table shows the life expectancy at birth for males and females in NSW from 2001 to 2015.

Year, y	2001	2003	2005	2007	2009	2011	2013	2015
Male, M	76.9	77.7	78.5	79.1	79.5	79.8	80.0	80.4
Female, F	82.4	82.9	83.3	83.8	84.3	84.2	84.3	84.6

a Draw a scatterplot showing the life expectancy at birth for males and females. Use different colours for the male and female data.

b Use a graphics calculator or spreadsheet to create a least-squares regression line for males and for females. Find the equation for male and female life expectancy.

c Calculate the correlation coefficient for male and female life expectancy, correct to 2 decimal places.

d Briefly summarise your results, referring to your graphs, equations and the correlation coefficients.

Qz Chapter quiz

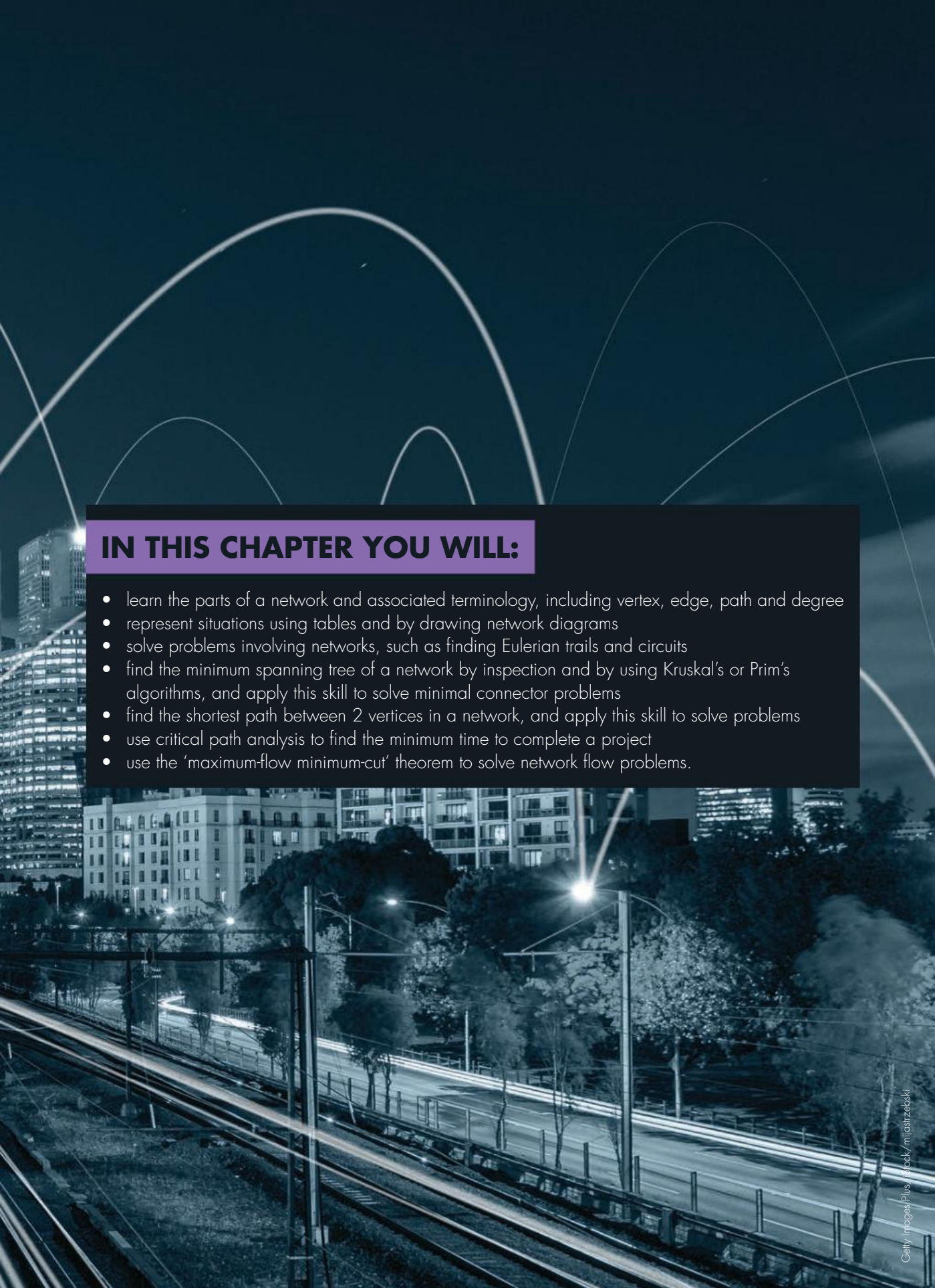
6.

NETWORKS

A **network** is a system or structure of things that are related. Bus, rail and air routes are examples of networks. Mathematicians study networks and use graph theory to try to improve them: faster process times, shorter distances, lower costs. Networks are used to represent computer systems, electrical and plumbing systems for housing, GPS road navigation, garbage bin collection routes and social media connections such as Facebook.

CHAPTER OUTLINE

N2 6.01 Networks
N2 6.02 Eulerian trails and circuits
N2 6.03 Minimum spanning trees
N2 6.04 Shortest path problems
N3 6.05 Activity tables and forward scanning
N3 6.06 Backward scanning and critical path analysis
N3 6.07 Network flow problems
N3 6.08 The 'maximum-flow minimum-cut' theorem



IN THIS CHAPTER YOU WILL:

- learn the parts of a network and associated terminology, including vertex, edge, path and degree
- represent situations using tables and by drawing network diagrams
- solve problems involving networks, such as finding Eulerian trails and circuits
- find the minimum spanning tree of a network by inspection and by using Kruskal's or Prim's algorithms, and apply this skill to solve minimal connector problems
- find the shortest path between 2 vertices in a network, and apply this skill to solve problems
- use critical path analysis to find the minimum time to complete a project
- use the 'maximum-flow minimum-cut' theorem to solve network flow problems.

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TERMINOLOGY

activity table
backward scanning
capacity
circuit
critical activity
critical path
cut
cycle
degree
directed network
dummy activity
earliest starting time (EST)
edge
Eulerian circuit
Eulerian trail
float time
flow capacity
forward scanning
Kruskal's algorithm
latest starting time (LST)
maximum-flow minimum-cut theorem
minimum spanning tree
network
network flow
non-critical activity
path
predecessor
Prim's algorithm
shortest path
sink
source
spanning tree
trail
tree
vertex
vertices
walk
weight
weighted network

WS
Assignment 6

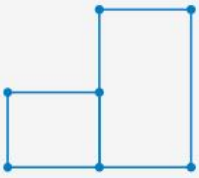
SkillCheck

1 Copy each diagram and using a pencil, trace the diagram along the edges, using each edge only once and without lifting your pencil.

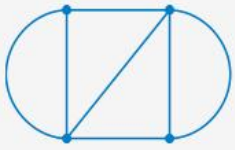
a



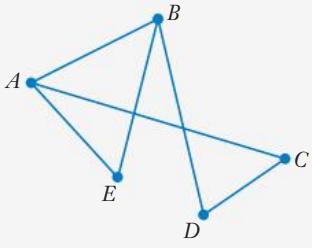
b



c



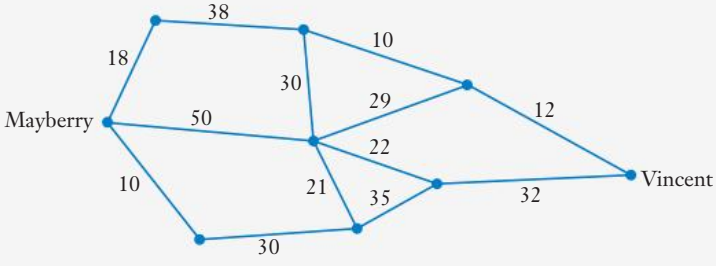
2 This diagram shows the roads linking 5 towns, labelled A, B, C, D and E. Mitch starts from town A, visits each town once and returns to town A.



a Describe a route that Mitch could take.

b List any roads that Mitch does not travel on.

3 This diagram shows the distances, in kilometres, along roads between the towns of Mayberry and Vincent.



Find the shortest distance between Mayberry and Vincent.

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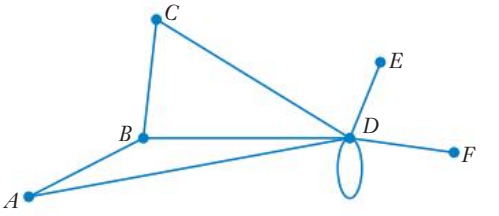
6.01 Networks

WS
Networks terminology

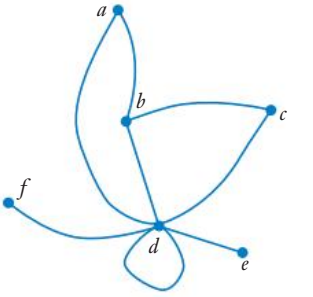
A **network** is a collection of objects connected to each other in some way.
A **graph** is a diagram of a network and contains points called **vertices** (or **nodes**), which are joined by lines called **edges** (or **arcs**). A **loop** is an edge that connects a vertex to itself.

This network has 6 vertices, labelled *A* to *F*.

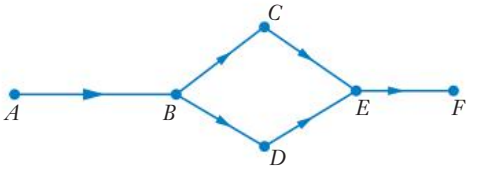
It has 8 edges, including one loop at *D*.



The same network can also be drawn with curved lines and with the vertices in different positions and labelled by lower-case letters, as long as it shows the same links.

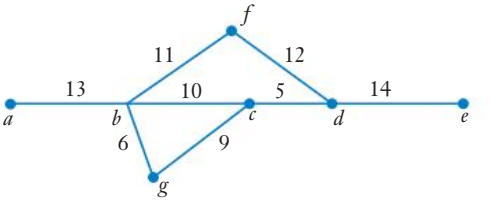


A **directed network** has arrows on its edges that indicate direction.



A **weighted network** has numbers on its edges called **weights**. The numbers could represent distances, times, costs or other quantities.

The **weight** of a network is the sum of the weights of its edges.



EXAMPLE 1

For the weighted network *abcdefg* shown above, find:

a the weight of the network

b the shortest path from *a* to *c*

c the longest path from *a* to *c* without using an edge more than once.

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Solution

a Weight of network = 13 + 10 + 5 + 14 + 11 + 12 + 6 + 9
= 80

b Shortest path from a to c = 13 + 10 = 23 — The path *abc*

c Longest path = 13 + 11 + 12 + 5 = 41 — The path *abfdc*

Degree of a vertex

The **degree** (or **order**) of a vertex is the number of edges connected to it. An **even vertex** has an even number of edges connected to it. An **odd vertex** has an odd number of edges.

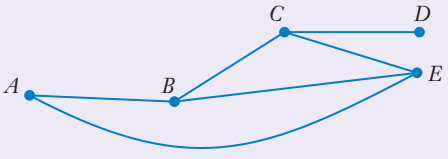
The **sum of the degrees** of all vertices in a network is equal to **twice the number of edges**, because each edge belongs to 2 vertices. This means that the sum of degrees in a network is always even.

EXAMPLE 2

a Find the degree of each vertex in this network.

b How many vertices are even?

c Show that the sum of the degrees is equal to twice the number of edges.



Solution

a

Vertex	*A*	*B*	*C*	*D*	*E*
Degree	2	3	3	1	3

b There is one even vertex. — *A* has degree 2, which is even.

c Number of edges = 6

Sum of degrees of vertices = 12, which is twice the number of edges.

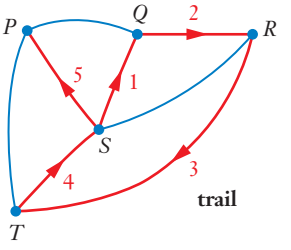
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Walks, trails, paths and cycles

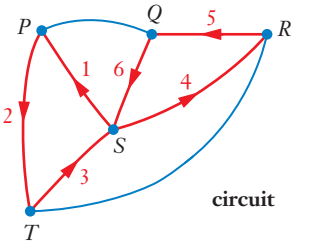
A **walk** is any route along the edges of a network, which may include repeated vertices or edges. A walk is **closed** if it starts and ends at the same vertex.

A **trail** is a walk with no repeated edges. A **circuit** is a closed trail.

In the networks below, *SQRTSP* is a **trail** (each edge used once only) and *SPTSRQS* is a **circuit** (returns to *S*), where the numbers show the order. Note however that *S* is used more than once in both cases.



trail

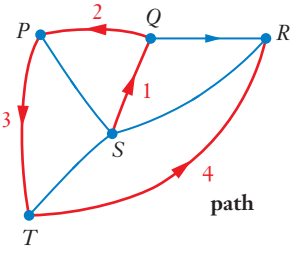


circuit

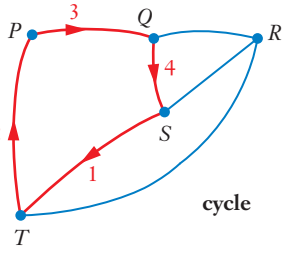
Think of a jogger running along a trail or circuit and not repeating an edge.

A **path** is a walk with no repeated vertices (or edges). A **cycle** is a closed path.

In the networks below, *SQPTR* is a **path** (each vertex used once only) and *STPQS* is a **cycle** (returns to *S*).



path

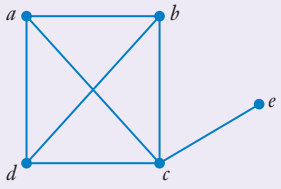


cycle

Think of a cyclist riding along a path or cycle and not repeating a vertex.

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EXAMPLE 3



a State whether each walk is a trail or path.

i *abcad* **ii** *abdce* **iii** *adbca*

b Which of the walks in part **a** is a cycle (a closed path)?

c Write down a closed walk that is not a cycle.

d Write down 2 cycles that use the edge *ab*.

Solution

a **i** *abcad* is a trail as no edges are repeated but the vertex *a* is repeated.

ii *abdce* is a path since no vertices (and edges) are repeated.

iii *adbca* is a path since no vertices (and edges) are repeated.

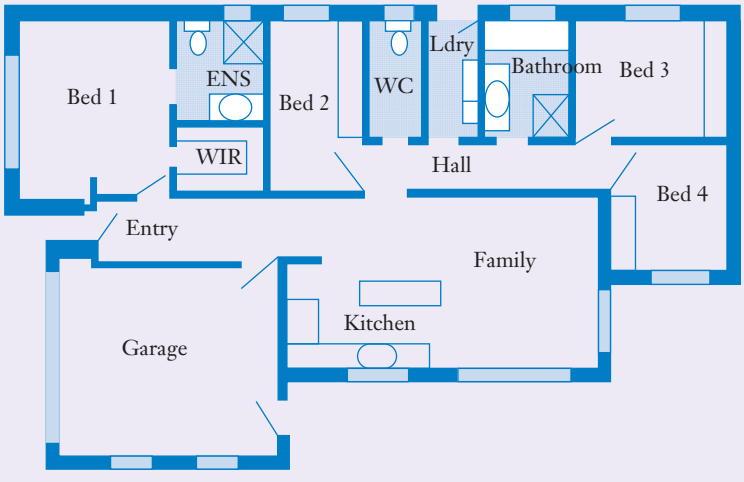
b *adbca* is a cycle.

c *adbcdb* is a path which is not a cycle (the edge *db* is used twice).

d *abdca* and *cabdc* are cycles that use the edge *ab*.

EXAMPLE 4

Model this house plan as a network, showing doors as edges and the rooms (including entry and hall) as vertices.

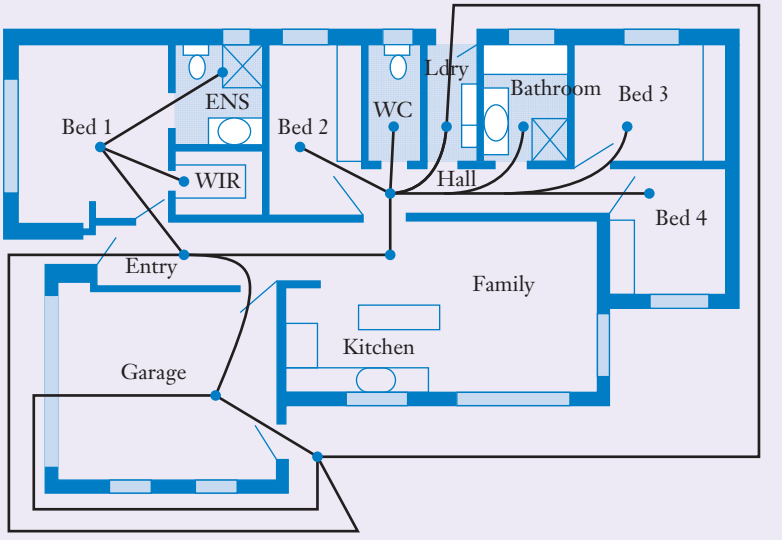


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Solution

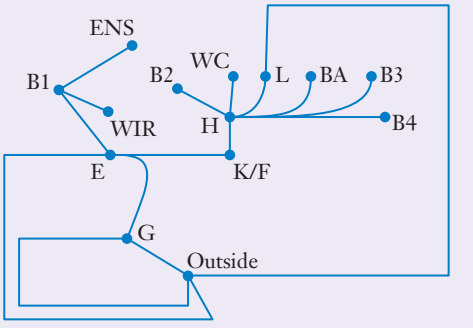
Put a dot over each room (including the entry and hall). Also put one dot outside.

Then connect the dots through the doorways.

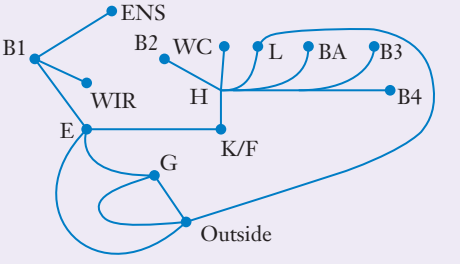


Redraw the network without the plan.

Show connections clearly and label each vertex.



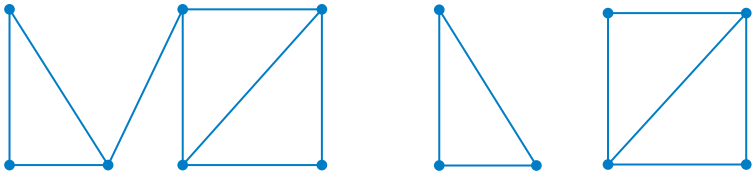
The network could also then be drawn as shown.



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Connected networks

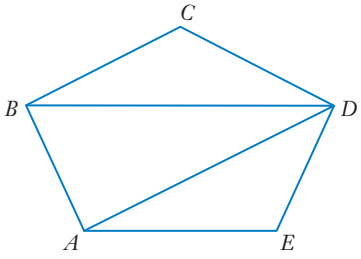
A network is **connected** if there is a path between any 2 vertices on the network.



Connected **Not connected**

Networks and tables

Networks can also be represented using a **matrix** or table of numbers. In the matrix and table below, a 1 means the 2 vertices are joined by an edge and 0 (or '–') means the 2 vertices are not joined by an edge.



diagram/graph

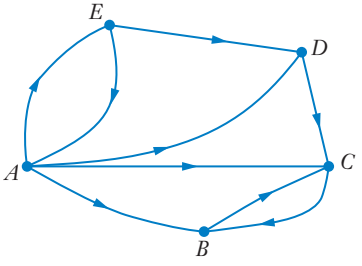
	A	*B*	*C*	*D*	*E*
A	0	1	0	1	1
B	1	0	1	1	0
C	0	1	0	1	0
D	1	1	1	0	1
E	1	0	0	1	0

matrix

	A	*B*	*C*	*D*	*E*
A	0	1	0	1	1
B	1	0	1	1	0
C	0	1	0	1	0
D	1	1	1	0	1
E	1	0	0	1	0

table

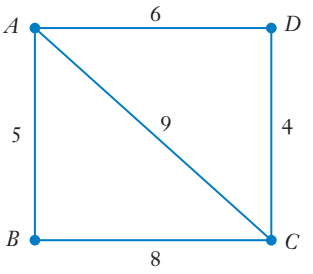
Directed networks can be shown in a table (or matrix).



		To:				
		A	*B*	*C*	*D*	*E*
From:	***A***	0	1	1	1	1
	B	0	0	1	0	0
	C	0	1	0	0	0
	D	0	0	1	0	0
	E	1	0	0	1	0

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Weighted networks can be shown in a table (or matrix).



network

	A	*B*	*C*	*D*
A	0	5	9	6
B	5	0	8	0
C	9	8	0	4
D	6	0	4	0

table

	A	*B*	*C*	*D*
A	0	5	9	6
B	5	0	8	0
C	9	8	0	4
D	6	0	4	0

matrix

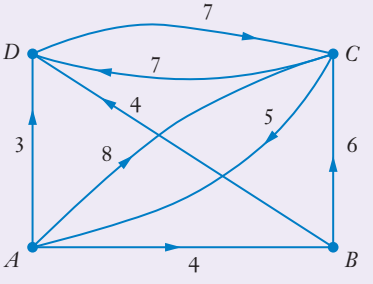
EXAMPLE 5

Draw the directed weighted network represented by this table.

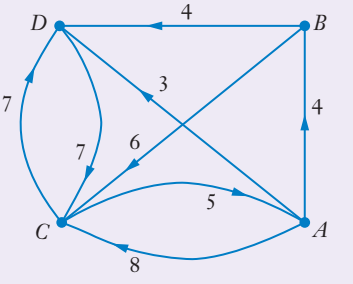
		To:			
		A	*B*	*C*	*D*
From:	***A***	–	4	8	3
	B	–	–	6	4
	C	5	–	–	7
	D	–	–	7	–

Solution

Two possible network diagrams are:



or

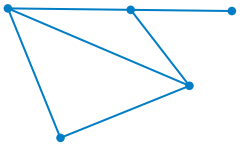


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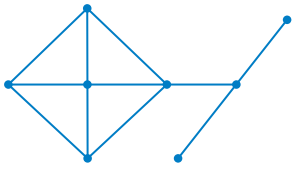
Exercise 6.01 Networks

1 For each network, count the number of: **i** vertices **ii** edges.

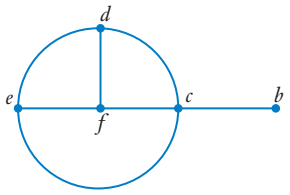
a



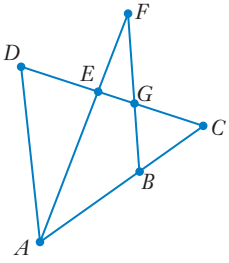
b



c



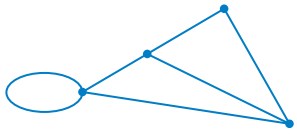
d



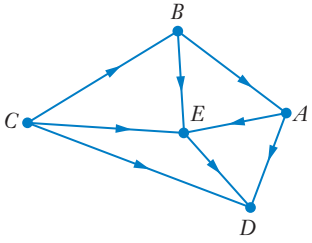
e



f



2 This directed network shows water pipes connecting facilities in a camping ground. Between which 2 facilities can water flow? Select **A**, **B**, **C** or **D**.



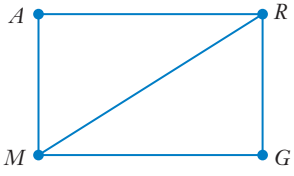
A *A* to *C*

B *B* to *D*

C *B* to *C*

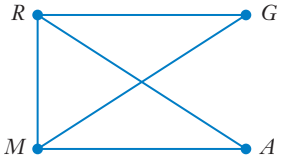
D *A* to *B*

3 This network diagram shows Facebook connections between Alex (*A*), Mariah (*M*), Georgia (*G*) and Rabibah (*R*). An edge represents a friendship between 2 people.

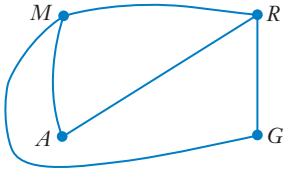


Which network diagram does *not* show the same information?

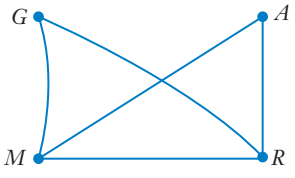
A



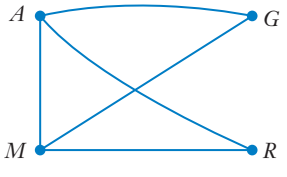
B



C

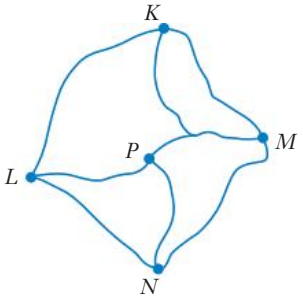


D

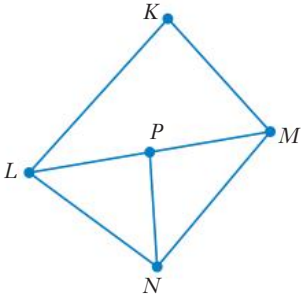


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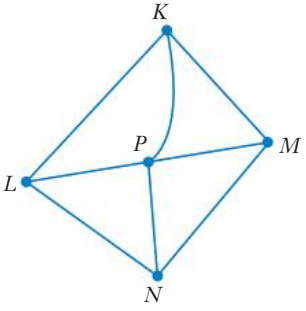
4 Which network diagram represents this map showing the roads linking 4 country towns?



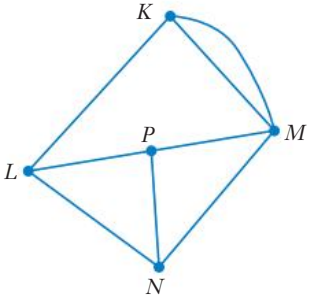
A



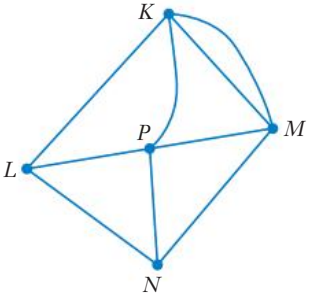
B



C



D



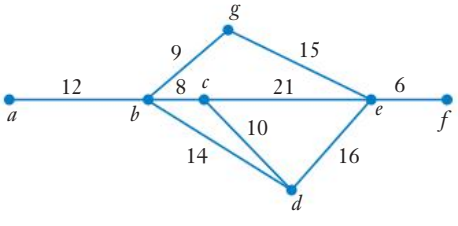
5 For each network, find:

- **i** the weight of the network
- **ii** the shortest path from a to f
- **iii** the longest path from a to f without using an edge more than once.

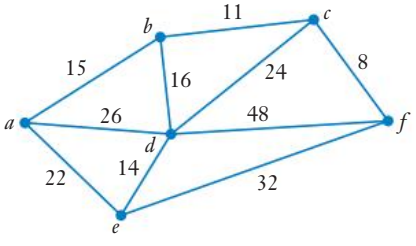
Example 1

WS Network diagrams

a



b



The worksheet 'Network diagrams' contains the diagrams from Questions 5, 6 and 13 for writing upon.

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Example 2

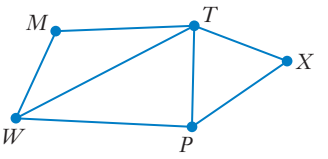
6 For each network:

i find the degree of each vertex

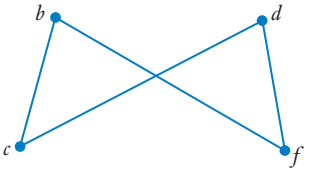
ii count how many vertices are even

iii show that the sum of the degrees is equal to twice the number of edges.

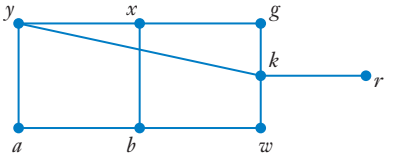
a



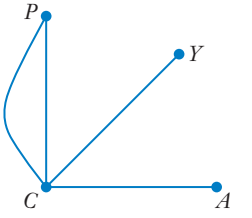
b



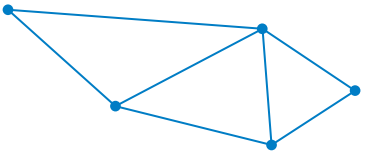
c



d



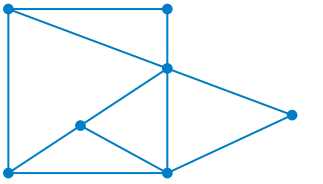
7 How many odd vertices has this network?



A 1 **B** 2

C 3 **D** 4

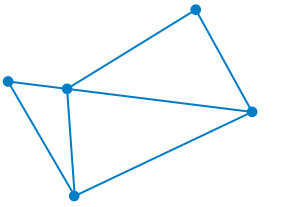
8 How many even vertices has this network?



A 2 **B** 3

C 4 **D** 5

9 What is the sum of the degrees of all vertices in this network?



A 10 **B** 12

C 14 **D** 16

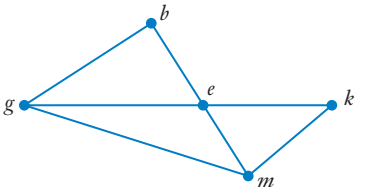
Example 3

10 **a** State whether each walk is a trail or path.

i *gbemke* **ii** *gmkeb*

iii *begmk* **iv** *ebgmke*

v *gbekmeg*



b Which of the walks in part **a** is:

i a circuit (a closed trail)? **ii** a cycle (a closed path)?

c Write down a closed walk that is not a cycle.

d Write down 2 cycles that use the edge *gm*.

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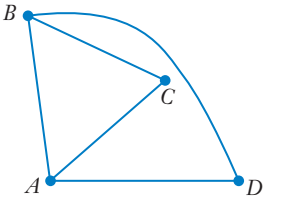
11 Which of the following walks is a cycle?

A *ABCD*

B *BCDAC*

C *CADBAC*

D *BCADB*



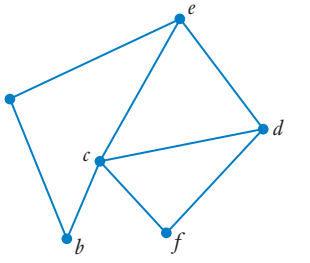
12 Which of the following walks is a trail?

A *abcedcf*

B *bcedcb*

C *abcdae*

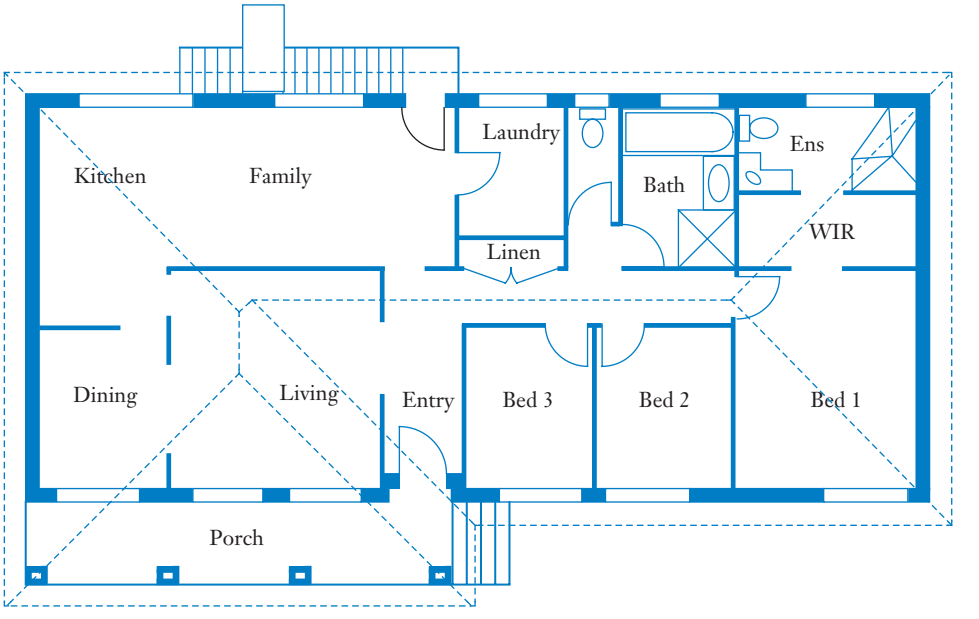
D *bcfdcba*



13 Model each house plan as a network, showing doors as edges and the rooms (including entry and hall) as vertices.

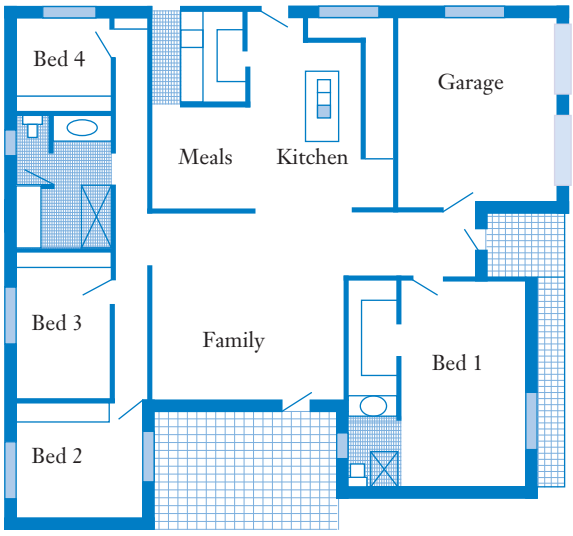
Example 4

a



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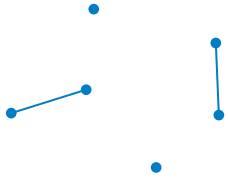
b



14 On the Sydney rail network, trains travelling from Campbelltown to Central go along 2 possible routes. They both stop at Glenfield, and then one branch passes through Liverpool, Lidcombe, Redfern and then Central, while the other branch passes through East Hills, Wolli Creek, International Airport and Domestic Airport, before reaching Central. From Central, all trains go around the City Circle, stopping at Town Hall, Wynyard, Circular Quay, St James, Museum and returning to Central.

Draw a network diagram that represents these routes.

15 What is the smallest number of edges required to make this diagram a connected network?



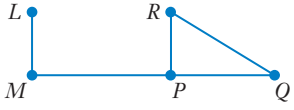
A 1

B 2

C 3

D 4

16 Five towns L, M, P, Q and R are linked by roads as shown by the network diagram, table and matrix.



	L	M	P	Q	R
L	0	1	0	0	0
M	1	0	1	0	0
P	0	a	0	1	1
Q	0	0	1	0	1
R	0	b	1	1	0

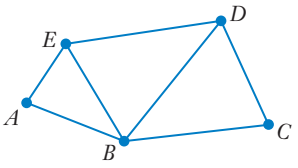
$$\begin{array}{c} \\ L \\ M \\ P \\ Q \\ R \end{array}\begin{array}{c} \begin{array}{ccccc} L & M & P & Q & R \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & a & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & b & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a What is the meaning of a 0 in the table or matrix?

b Write the values of a and b.

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17 Copy and complete the table below for this network.



	A	*B*	*C*	*D*	*E*
A	0	1			
B			1		
C					
D					
E					

18 Draw the network that is represented by the table and matrix.

a

	A	*B*	*C*	*D*
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0

b

	A	*B*	*C*	*D*	*E*
A	0	1	0	1	0
B	1	0	1	1	0
C	0	1	0	1	0
D	1	1	1	0	1
E	0	0	0	1	0

19 Draw the directed weighted network that is represented by the following.

Example 5

a

		To:				
		J	*K*	*L*	*M*	*N*
From:	***J***	–	–	–	–	–
	K	5	–	5	–	4
	L	–	4	–	8	4
	M	–	–	–	–	6
	N	–	–	6	–	–

b

		To:			
		A	*B*	*C*	*D*
From:	***A***	–	6	10	3
	B	4	–	7	6
	C	–	5	–	–
	D	–	–	5	–

20 The table shows the road distances (in km) between 9 towns in northern NSW.

	A	*B*	*C*	*GI*	*Gr*	*T*	*U*	*W*	*Y*
A	–	–	–	98	–	–	172	–	–
B	–	–	64	–	131	–	–	–	–
C	–	64	–	–	100	126	–	–	–
GI	98	–	–	–	164	93	–	129	–
Gr	–	131	100	164	–	–	110	–	–
T	–	–	126	93	–	–	–	–	193
U	172	–	–	–	110	–	–	–	–
W	–	–	–	129	–	–	–	–	85
Y	–	–	–	–	–	193	–	85	–

A – Armidale
B – Ballina
C – Casino
GI – Glen Innes
Gr – Grafton
T – Tenterfield
U – Uranga
W – Warialda
Y – Yetman

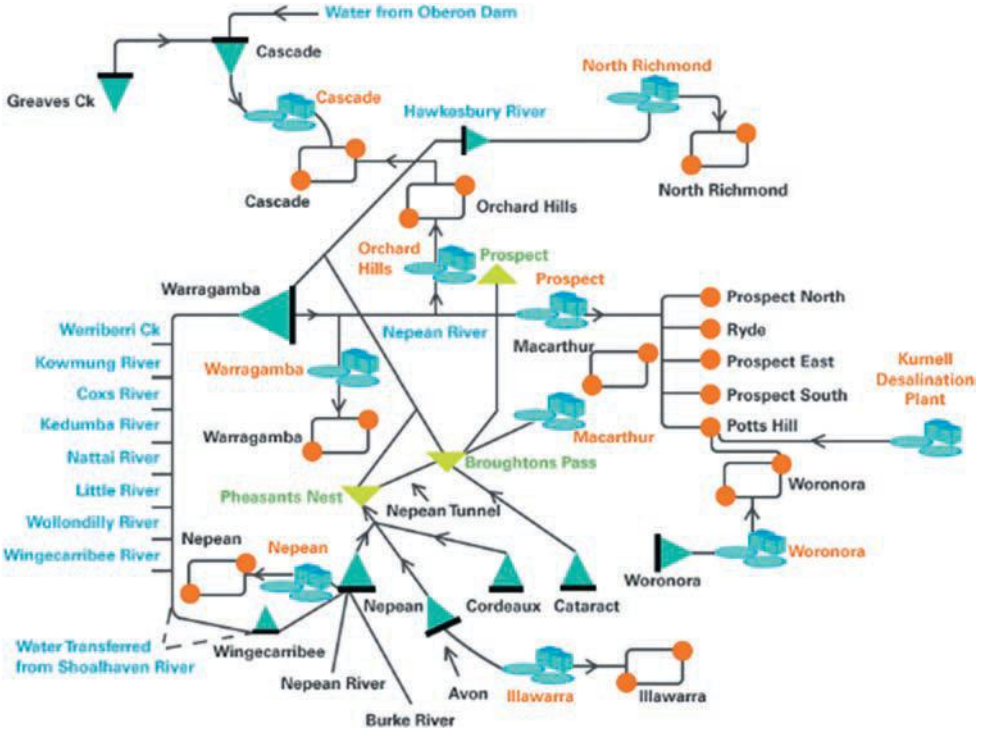
Draw a network diagram to represent the information in the table.

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INVESTIGATION

USING NETWORKS

1 This network diagram shows the Sydney water supply.



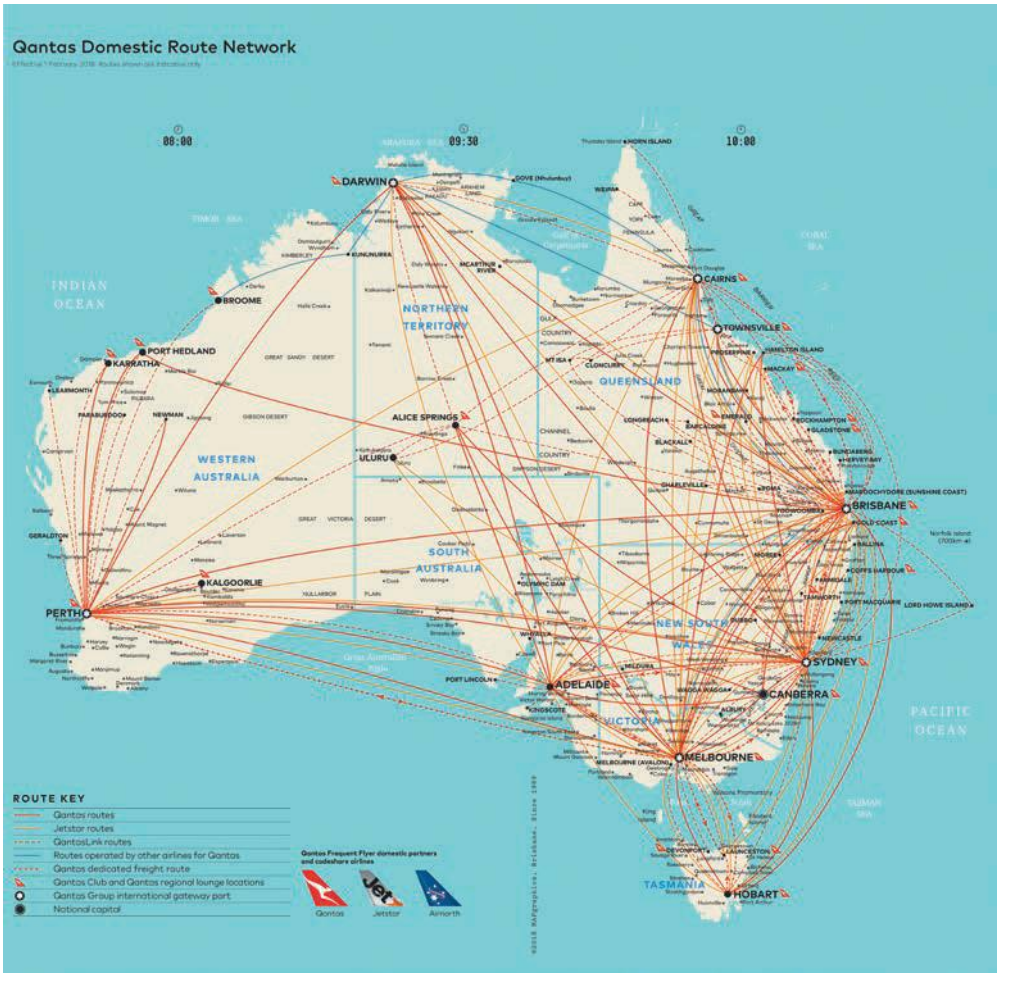
Courtesy of Sydney Water

- Go to the **Sydney Water** website and access the water network.
- Obtain a copy of the water network diagram and show how water is supplied to:

a Sydney **b** Blacktown.

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2 The network below shows the domestic air routes for Qantas.



Reproduced with permission from Qantas Airways Limited

a From which state and territory capital cities can you fly directly to every other capital city?

b From which capital cities can you fly directly to:

i Uluru?

ii Lord Howe Island?

iii Broome?

iv Cairns?

3 Network diagrams can also be used to represent social networks such as Facebook.

With a group of about 10 people in your class or Year 12, draw a diagram showing contacts through a social media platform such as Facebook and Instagram.

4 What is meant by the idea of '**six degrees of separation**'? Write a brief report on your findings and decide whether this could apply to you and any randomly selected student at your school.

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6.02 Eulerian trails and circuits

Eulerian trails

An **Eulerian trail** in a network is a walk that uses *every* edge in the network only once.

For an Eulerian trail to exist, the network must have exactly 2 odd vertices.

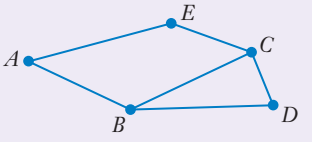
Eulerian trails must start at one odd vertex and finish at the other odd vertex.

Eulerian is pronounced 'Oiler-ian', and is named after the Swiss mathematician and engineer, Leonhard Euler (1707–1783).

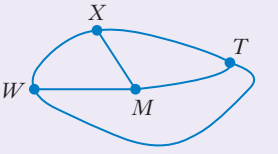
EXAMPLE 6

Does each network have an Eulerian trail?

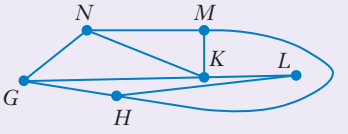
a



b



c



Solution

a The network has 2 odd vertices at B and C so it has an Eulerian trail, such as $CDBCEAB$.

b The network has all (4) odd vertices (degree 3) so it does not have an Eulerian trail.

c The network has 4 odd vertices (degree 3) so it does not have an Eulerian trail.

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Eulerian circuits

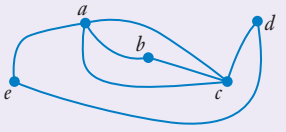
An **Eulerian circuit** in a network is a walk that uses *every* edge in the network only once and which **starts and ends** at the same vertex.

For an Eulerian circuit to exist, the vertices in the network must all be even.

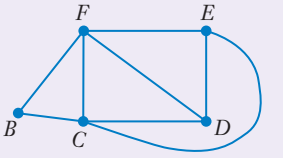
EXAMPLE 7

Does each network have an Eulerian circuit?

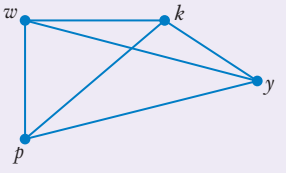
a



b



c



Solution

a The network contains an Eulerian circuit since all vertices are even (vertices b, d and e have degree 2, vertices a and c have degree 4). One possible Eulerian circuit is *abcdeaca*.

b Vertices D and E are odd (degree 3) so the network does not contain an Eulerian circuit.

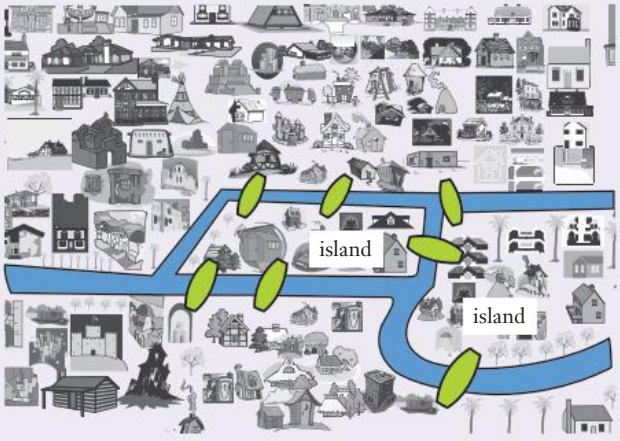
c All vertices have degree 3, so the network does not contain an Eulerian circuit.

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DID YOU KNOW?

The 7 Bridges of Königsberg

In the Prussian town of Königsberg (now Kaliningrad, Russia), there were 7 bridges connecting 2 islands and the banks of the Pregel River. There was a tradition where people would walk through the town on a Sunday and it became a challenge to complete a walk crossing each bridge only once. No person was able to do it but no one could prove that it was impossible.



In 1736, the Swiss mathematician Leonhard Euler solved the problem by reducing the map to a network. He found that it was impossible to do the walk. Eventually another bridge was built. That is why a walk that crosses every bridge exactly once is called an Eulerian trail.

1 **Draw a network to represent the map and explain why the original walk was impossible.**

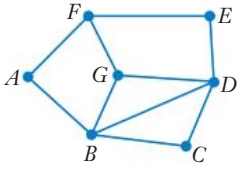
2 **An eighth bridge was built. Where would the bridge have been placed so that the network could be traversed?**

Exercise 6.02 Eulerian trails and circuits

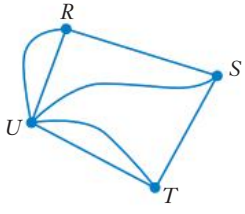
Example 6

1 Which network has an Eulerian trail? Select **A**, **B**, **C** or **D**.

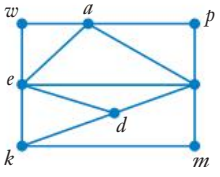
A



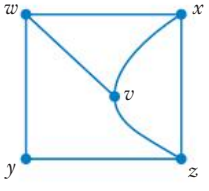
B



C



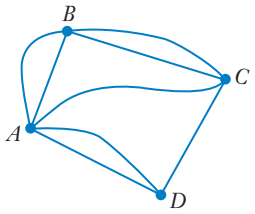
D



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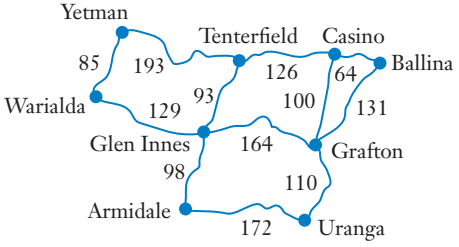
2 **a** Explain why this graph has an Eulerian trail.

b Copy the graph and show the trail.



3 The map shows the major roads between towns in northern NSW. Does the road network contain an Eulerian trail?

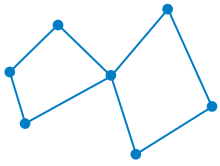
Give reasons.



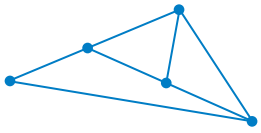
4 Which network contains an Eulerian circuit? Select **A**, **B**, **C** or **D**.

Example 7

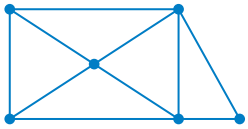
A



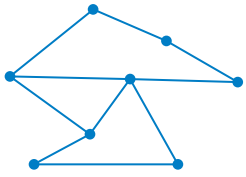
B



C



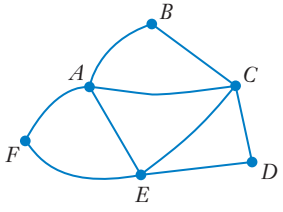
D



5 **a** Explain why the graph on the right has an Eulerian circuit.

b Write down a circuit.

c Copy the graph and show your circuit.



6 **a** Janet has to make deliveries to each place shown on the map. Can Janet make the deliveries using each road only once?

Give reasons.

b Is it possible for Janet to start and finish from the same place?

Give reasons.



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7 For each description of a network, decide whether it contains an Eulerian trail, an Eulerian circuit or neither.

- **a** A walk uses each edge of the network only once.
- **b** A walk starts and ends at the same vertex in the network.
- **c** There are no vertices of odd degree.
- **d** There are 2 vertices of odd degree.
- **e** There are more odd vertices than even vertices.
- **f** There is one even vertex and 2 odd vertices.
- **g** There are 3 even vertices and 2 odd vertices.
- **h** There are 5 odd vertices.
- **i** There are 6 even vertices and 6 odd vertices.
- **j** There are 5 even vertices and no odd vertices.

8 The blue diagram below is a route network map for a bus company in Blacktown.

- **a** Explain why the network map is an Eulerian trail but not an Eulerian circuit.
- **b** Draw a 'simplified' network diagram of the bus route.

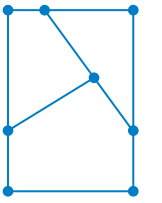


Reproduced with permission from Busways Group Pty Ltd

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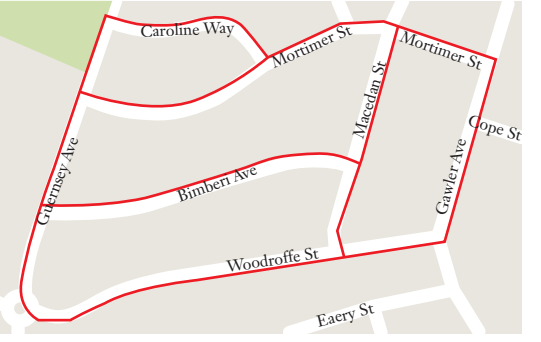
9 a The streets marked red on the map can be redrawn as a network diagram as shown.





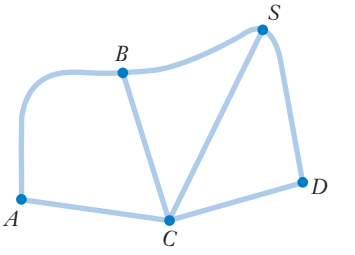
Is it possible to deliver leaflets to every home by walking along each street only once? Give reasons.

b Redraw the roads shown on the map as a network and determine if Naheed can drive along each street only once.



10 A line-marking truck is painting lines down the centre of roads shown in the network.

They start marking the lines from the intersection at S.



a Is it possible for the lines to be marked without going down a street twice?

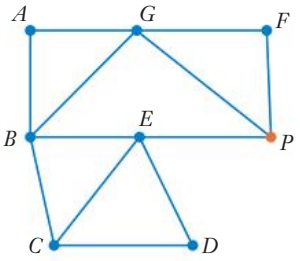
b Explain why it is not possible for the truck to start and finish at S.

c An extra road between S and B is added to the network. Show that it is now possible to start and finish at S and write a route that could be used.

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WS
Eulerian trails and circuits

11 The street map for the delivery of local newspapers is drawn as a network. The route has to be organised so that the delivery person starts at *P* and does not drive along the same road twice.



a Explain why the delivery cannot start and finish at *P*.

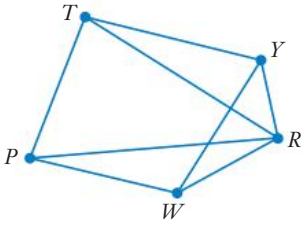
b A road is added to the network so that the delivery can start and finish at *P*. Which 2 vertices would the road join?

c Find a suitable route for the newspaper delivery.

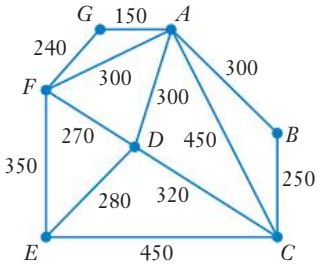
The worksheet 'Eulerian trails and circuits' contains the diagrams from Questions 11 to 15 for writing upon.

12 Crystal has to visit each city once on this network diagram, starting and ending at *T*.

Find possible routes Crystal could take.



13 This network represents a large park with walking paths. The vertices are rest areas and the numbers are the lengths of the paths in metres.

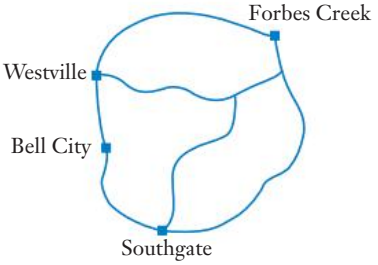


a Find the sum of the degrees of the vertices in the network diagram.

b Saieesha goes to the park and decides to walk along each path. Where should she start her walk?

c How far will Saieesha walk?

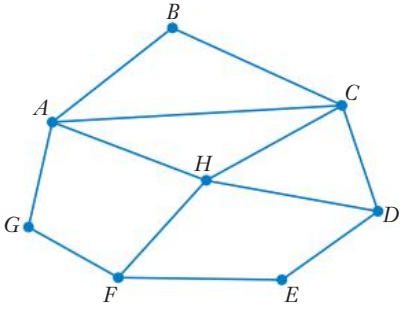
14 How many different ways can a person travel from Forbes Creek to Bell City without travelling along any road more than once? Select **A**, **B**, **C** or **D**.



A 5 **B** 6

C 7 **D** 8

15 Klaasville Tourist Centre publishes a map showing 8 'must-see' destinations for tourists, with the easiest routes shown to go from one destination to another.



a **i** Charissa and her family want to visit each destination, but use each road only once. They start from *D*. At which destination will they finish their trip?

ii How many destinations will Charissa and her family see exactly twice, no matter which route they take?

b The Roberts family decides they only wish to visit each destination once, starting at *B*. Write down the order in which they will see all the tourist destinations.

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16 If 2 people shake hands, a network diagram can be drawn where the vertices represent the people and an edge represents the handshake, as shown below.



Shutterstock.com/Hanna Kuprevich

So, for 2 people there is one handshake.

Draw a network diagram to represent each number of people shaking hands with everybody else and count:

i the number of handshakes for each person

ii the total number of handshakes.

a 3 people shake hands

b 4 people shake hands

c 5 people shake hands

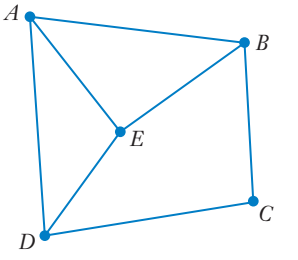
d 8 people shake hands

INVESTIGATION

HAMILTONIAN PATHS AND CIRCUITS

1 A **Hamiltonian path** is a path that visits *every* vertex only once.

For the network shown, a Hamiltonian path is ***ABEDC*** or ***ABCDE***.



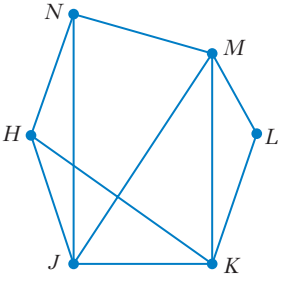
a How many Hamiltonian paths can you find for this network?

b Are all the edges of the network included in a Hamiltonian path?

2 A **Hamiltonian circuit** is a Hamiltonian path that begins and ends at the same vertex.

For the network shown, a Hamiltonian circuit is ***KHJNMLK*** or ***HKLMNJK***.

How many Hamiltonian circuits can you find in this network?

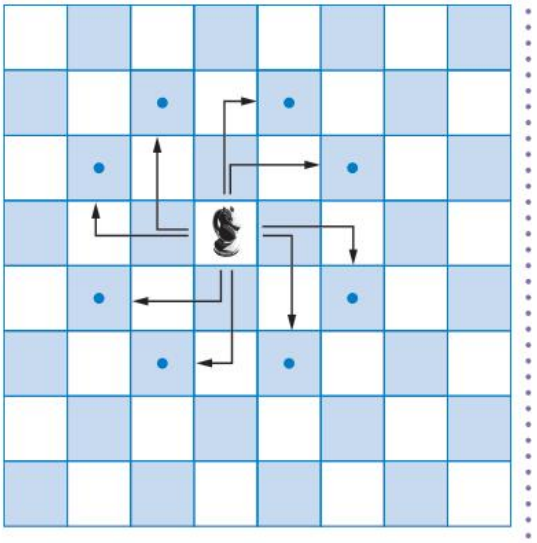


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3 An application of Hamiltonian paths is finding a knight's tour (on a chess board), where the knight moves to every square on a chess board only once.

The possible moves that a knight can move from one space to another are shown.

How many knight's tours can be found? (You may be surprised!)



4 The **Travelling Salesman Problem** is another example of applying Hamiltonian paths and circuits. Use the Internet to find what this problem is and determine a method to solve it.

WS
Minimum spanning trees

6.03 Minimum spanning trees

A **tree** is a network in which any 2 vertices are connected by exactly one path. This means that a tree is connected and has no cycles.

Here are some examples of trees.



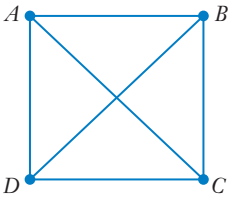
These are not trees. The left one is not connected and the right one has a cycle.



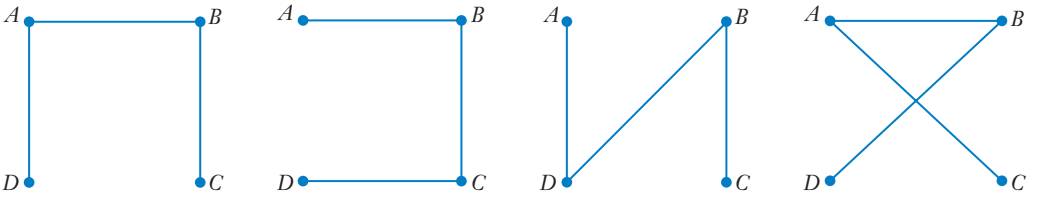
A **spanning tree** in a network is a tree in the network that connects all of the vertices. A network can have many different spanning trees.

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For example, for this network:



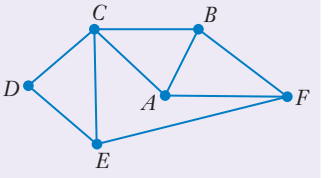
some of the spanning trees are:



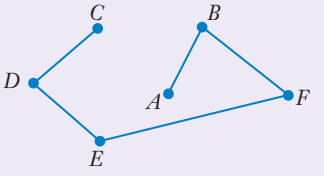
Note that each spanning tree connects the 4 vertices with 3 edges. Generally, if there are n vertices in the network, then there will be $(n - 1)$ edges in a spanning tree. Each edge can be paired with a vertex, except for one vertex.

EXAMPLE 8

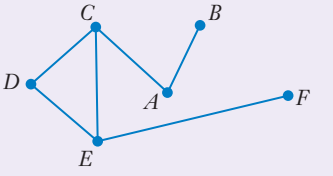
Determine whether each graph is a spanning tree of the network shown on the right.



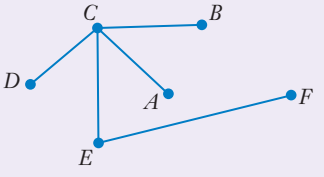
a



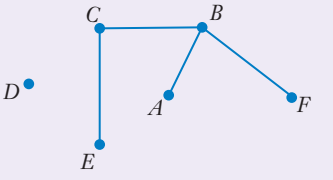
b



c



d



Solution

a This is a spanning tree as it connects all the vertices and has no cycles.

b This is not a spanning tree as there is a cycle ($CDEC$).

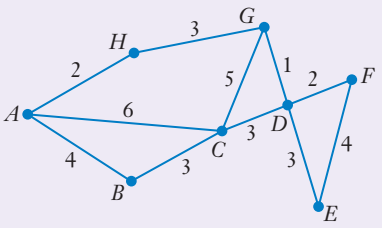
c This is a spanning tree.

d This is not a spanning tree because it does not connect vertex D.

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EXAMPLE 9

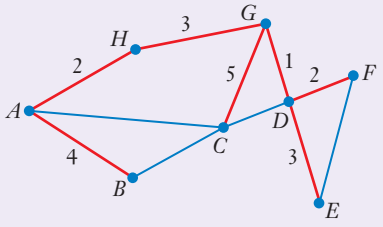
This network diagram represents 8 computers that are to be connected with optical fibre cables. The numbers show the costs (in $100s) of the cables.



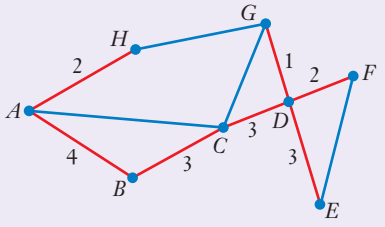
Draw 2 spanning trees for the network and determine which is the cheaper option.

Solution

Two possible spanning trees are:



Weight = 4 + 2 + 3 + 5 + 1 + 2 + 3
= 20
Cost = $2000



Weight = 2 + 4 + 3 + 3 + 3 + 1 + 2
= 18
Cost = $1800
This option is cheaper.

A **minimum spanning tree** in a weighted network is the spanning tree with the smallest weight. It is possible to find minimum spanning trees by trial and error but this can be time-consuming. Instead, we use two methods, **Kruskal's algorithm** or **Prim's algorithm**, where an algorithm is a process of precise steps, often repeated.

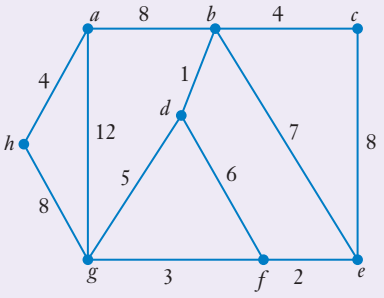
Kruskal's algorithm for minimum spanning trees

1. Sort the edges in increasing order of weight.
2. Make a diagram of the network with all vertices, but no edges.
3. Choose the edge with smallest weight.
4. Then choose the next edge with smallest weight.
5. Continue this process, adding the next edge, making sure no cycles are introduced, until all vertices are connected.

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EXAMPLE 10

Use Kruskal's algorithm to find a minimum spanning tree for the network shown.

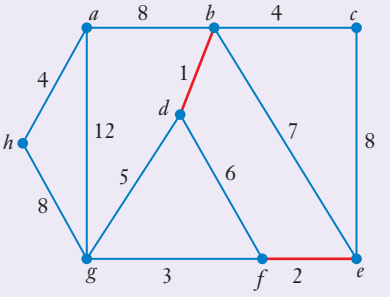


Solution

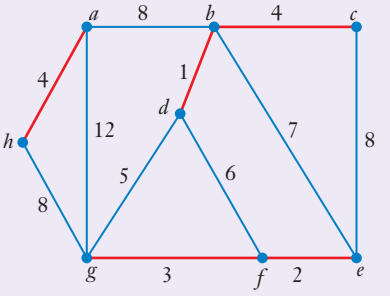
1 Sort the edges in increasing order.

Edge	Weight	
bd	1	✓
ef	2	✓
fg	3	✓
ah	4	✓
bc	4	✓
dg	5	✓
df	6	×
be	7	×
ab	8	✓
hg	8	
ce	8	
ag	12	

2 Choose the edge with the smallest weight, *bd*, then the edge with the next smallest edge, *ef*. Tick off each edge from the table as you select them.



3 Continue the process of selecting the next smallest edge from the table, choosing edge *fg*, then edge *ah*, followed by edge *bc*.

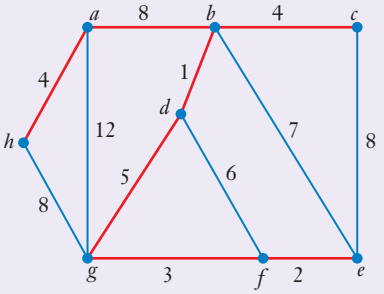


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4 Choose *dg*, but **not** *df* as it will make a cycle. Put a cross next to it in the table. For the same reason, don't choose *be*. Choose *ab* next.

All vertices are now connected (8 vertices connected by 7 edges) so the minimum spanning tree is completed.

Weight of minimum spanning tree
$= 1 + 2 + 3 + 4 + 4 + 5 + 8$
$= 27$

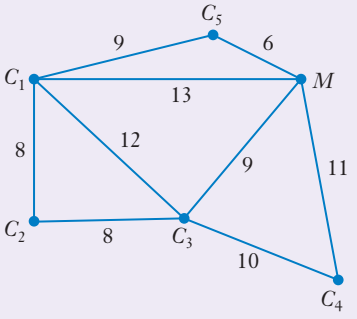


Note: This is not the only answer. A network can have more than one minimum spanning tree, but they will all have the same minimum weight.

EXAMPLE 11

William needed to connect electricity to 5 cabins at a camping site. The weighted graph shows the distances between the cabins (in metres) and the location of the electricity source (*M*).

Use Kruskal's algorithm to find the least amount of cable needed to connect electricity to all sites.

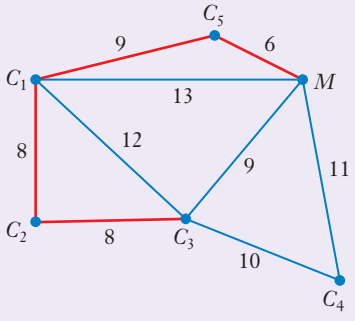


Solution

1

Edge	Weight	
MC_5	6	✓
C_1C_2	8	✓
C_2C_3	8	✓
C_1C_5	9	✓
MC_3	9	×
C_3C_4	10	✓
MC_4	11	
C_1C_3	12	
MC_1	13	

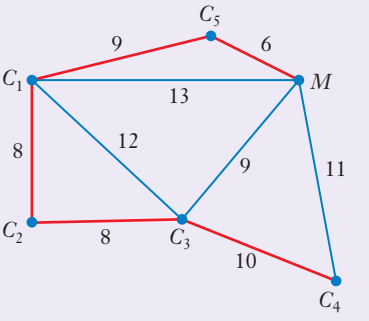
2 Choose the first 4 edges from the table, starting with MC_5.



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3 Edge MC_3 is not chosen, since it will make a cycle. C_3C_4 completes the minimum spanning tree.

Least amount of cable required
$= 6 + 9 + 8 + 8 + 10$
$= 41$ m



Prim's algorithm for minimum spanning trees

Prim's algorithm

While Kruskal's algorithm looks at the edges, Prim's algorithm focuses on the vertices.

1 Choose any vertex within the network.

2 Choose the edge with the least weight connecting this vertex to another vertex.

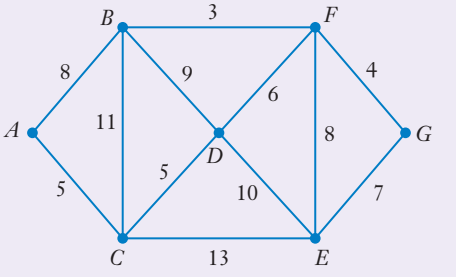
3 Look at all edges connecting with these 2 vertices and select the next edge with least weight to add to the tree.

4 Look at all the vertices in your tree and choose the next edge with least weight to add to the tree.

5 Repeat step 4 until all vertices are connected.

EXAMPLE 12

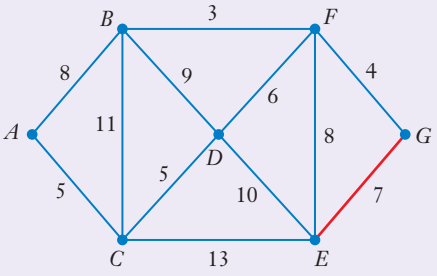
Use Prim's algorithm to find a minimum spanning tree for the network shown.



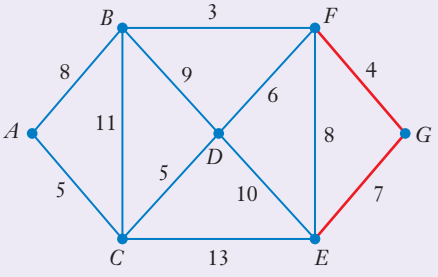
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Solution

1 Start with any vertex, say *E*.
From *E*, the smallest edge is *EG* (7).

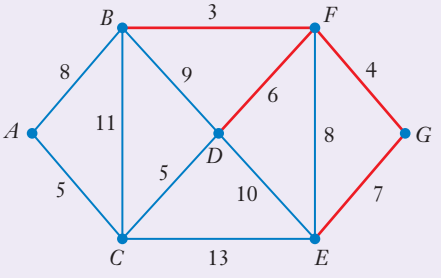


2 The next smallest edge from *E* or *G* is *GF* (4).

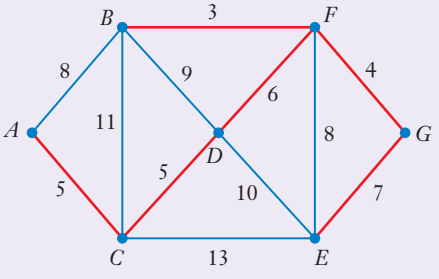


3 The next smallest edge from *F* or *E* is *FB* (3), which is added to the tree.

Then the the next smallest edge from *B*, *E* or *F* is *FD* (6).



4 The next smallest edge to connect to the tree is *DC* (5), then edge *CA* (5).



The minimum spanning tree is complete (7 vertices, 6 edges).

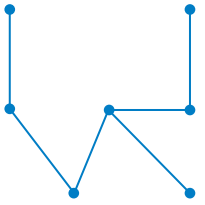
Weight of minimum spanning tree = 7 + 4 + 3 + 6 + 5 + 5 = 30.

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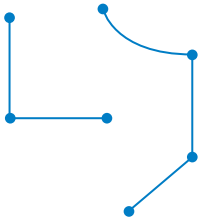
Exercise 6.03 Minimum spanning trees

1 For each network, state whether or not it is a tree. Give reasons.

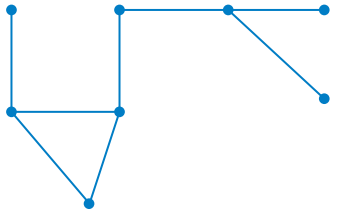
a



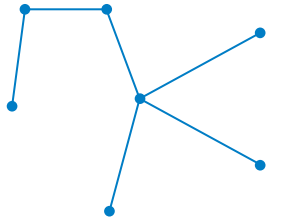
b



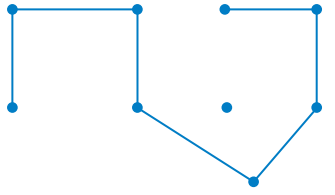
c



d



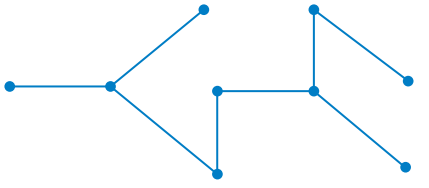
e



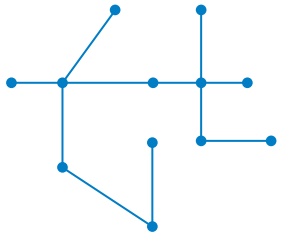
2 For each tree, count:

i the number of vertices **ii** the number of edges.

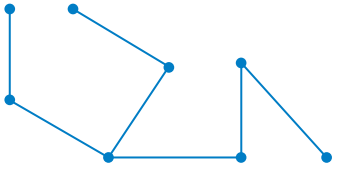
a



b



c



3 Which statement is true about trees? Select **A**, **B**, **C** or **D**.

A The number of vertices is equal to the number of edges.

B A tree with n vertices has $(n - 1)$ edges.

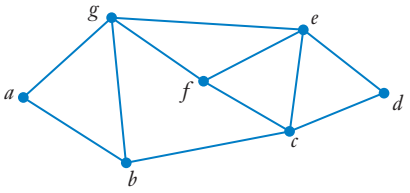
C A tree with n vertices has $(n + 1)$ edges.

D A tree with n vertices has $2n$ edges.

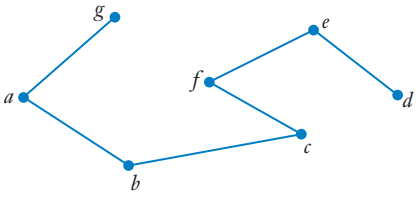
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Example 8

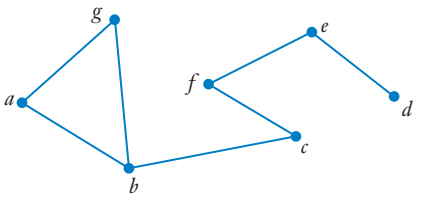
4 Determine which of the following is a spanning tree of the network shown.



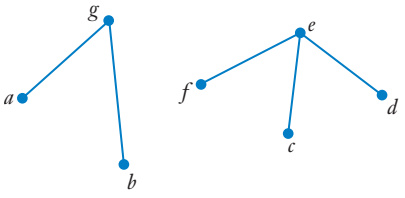
a



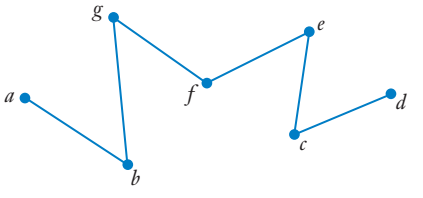
b



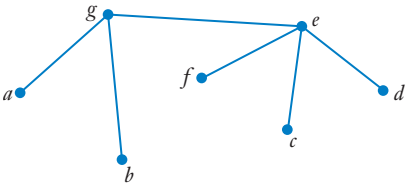
c



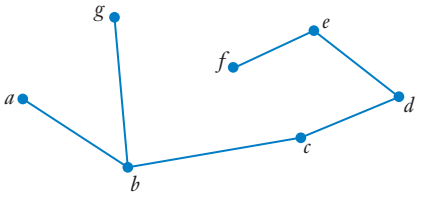
d



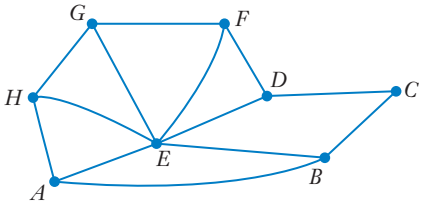
e



f



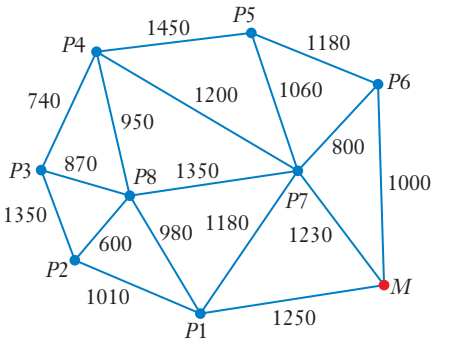
5 Find 3 spanning trees for the network below.



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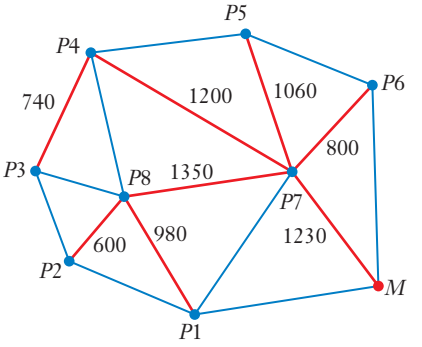
6 Consider this network which shows the cost of laying power cables from a mains power supply (M) to different areas of a camping and caravan park.

Example 9

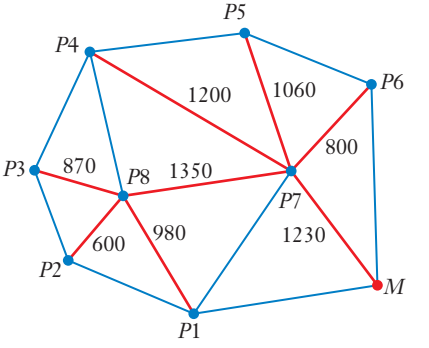


a Spanning trees for the network are shown below. Find the weight of each tree.

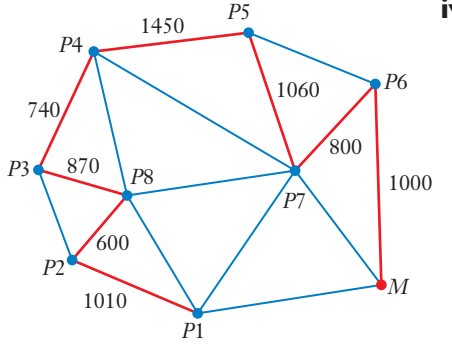
i



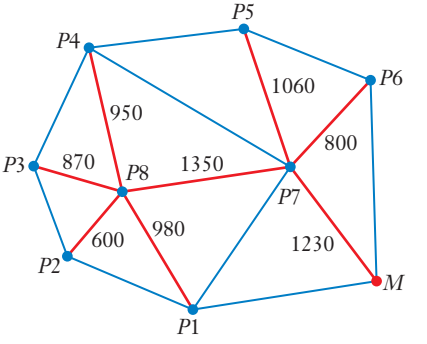
ii



iii



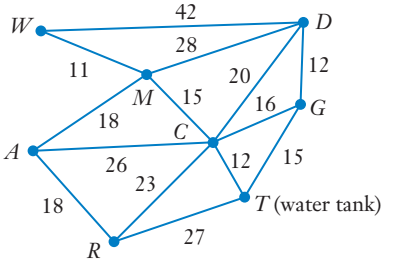
iv



b Which spanning tree is the best option? Is this tree the minimum spanning tree?

7 The network diagram shows the distances between different points in a garden. Pipes will need to be laid so the garden can be watered from a water tank at T.

Draw spanning trees for the network and by finding their weight, determine the minimum length of water pipe required to supply water to all parts of the garden.



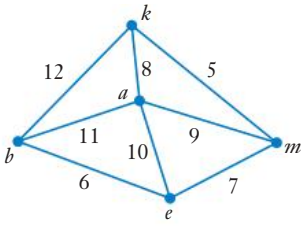
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Example 10

8 **a** List the edges of this network in increasing order.

b Use Kruskal's algorithm to find the minimum spanning tree.

c What is the weight of the minimum spanning tree?



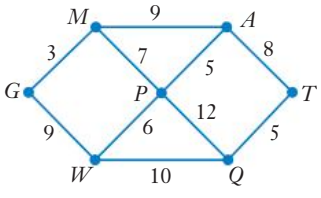
WS
Spanning tree diagrams

9 For each network below:

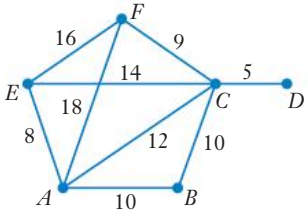
i use Kruskal's algorithm and showing the order in which the edges are selected, find the minimum spanning tree for each network and calculate its weight

ii verify that a spanning tree with n vertices has $(n - 1)$ edges.

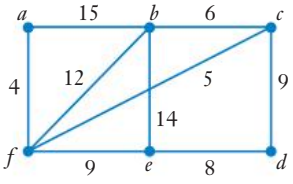
a



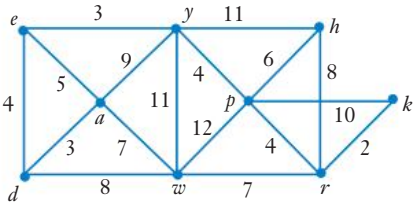
b



c



d



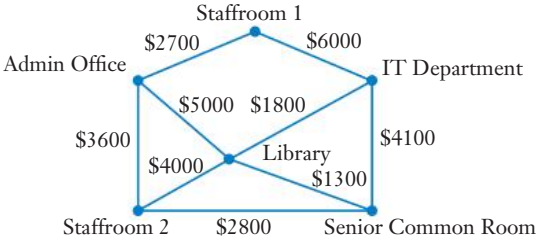
The worksheet 'Spanning tree diagrams' contains the diagrams from Questions 9, 13, 14 and 15.

Example 11

10 The cost involved in connecting a school's computer network is shown in the network diagram.

a Use Kruskal's algorithm to find the minimum spanning tree for the network.

b Calculate the minimum cost.



11 The minimum spanning tree for the network includes 2 edges with weighting a and b.

The length of the minimum spanning tree is 25.

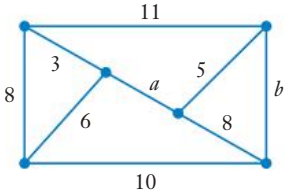
What are the values of a and b? Select **A**, **B**, **C** or **D**.

A $a = 6, b = 5$

B $a = 8, b = 4$

C $a = 9, b = 3$

D $a = 7, b = 10$



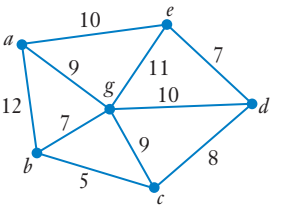
ISBN 9780170413633

Example 12

12 **a** Use Prim's algorithm to find the minimum spanning tree for this network, stating the order in which you choose the edges.

b What is the total weight of the tree?

c Is there another minimum spanning tree for this network?

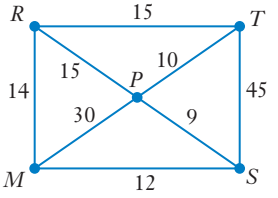


13 For each network below:

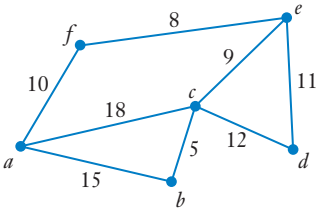
i use Prim's algorithm to find the minimum spanning tree, showing the order in which you choose the edges

ii calculate the weight of the tree.

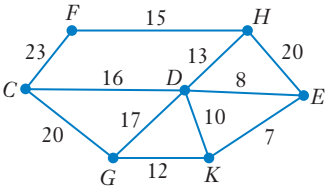
a



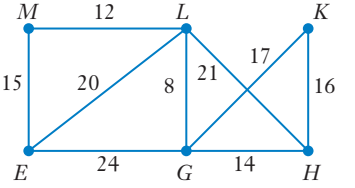
b



c



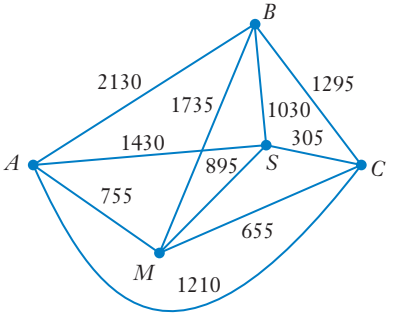
d



14 The network diagram shows the distances, in kilometres, between Adelaide (*A*), Brisbane (*B*), Canberra (*C*), Melbourne (*M*) and Sydney (*S*).

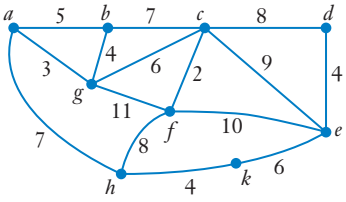
a Use Prim's algorithm to find a minimum spanning tree for the 5 cities, showing the order in which the edges are chosen.

b Calculate the length of your spanning tree.



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15 **a** The graph shows the road distances between 9 suburbs of a city, in kilometres. Find a minimum spanning tree for the road network, using:



i Kruskal's algorithm

ii Prim's algorithm.

b Are the minimum spanning trees the same?

c Find the length of the minimum spanning tree.

d Find the length of the shortest route from *a* to *d*.

16 The table shows the road distances between 6 towns in kilometres. A dash (–) indicates the towns are not directly connected.

	A					
B	22	***B***				
C	30	8	***C***			
D	15	42	–	***D***		
E	–	10	15	–	***E***	
F	40	9	12	24	–	***F***

a Draw a network diagram showing the information in the table.

b Find the minimum spanning tree for the network using:

i Kruskal's algorithm

ii Prim's algorithm.

c Find the length of the minimum spanning tree.

17 The costs of connecting 6 locations in a school with optical fibre cable are given in this table.

	B	***C***	***D***	***E***	***F***
A	7000	–	8000	–	10 000
B	–	3000	4200	1500	–
C	–	–	5100	4000	6000
D	–	–	–	6000	3600

a Draw a network diagram showing the information in the table.

b Find the minimum spanning tree for the network (showing the order in which edges are added).

c Use the minimum spanning tree to find the least cost of connecting the cable.

TECHNOLOGY

Spanning tree practice

Search for 'minimum spanning tree practice—GeoGebra' on the Internet to practise finding the minimum spanning tree for a network.

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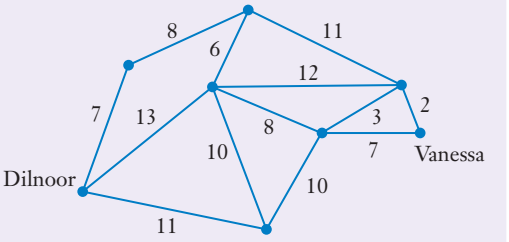
6.04 Shortest path problems

Networks find-a-word

For small weighted networks, the **shortest path** between 2 vertices can be found by looking at the network (by inspection) and using trial and error.

EXAMPLE 13

This network shows the travel times (in minutes) along roads from Dilnoor's house to Vanessa's house. Find the shortest time it will take Dilnoor to travel to Vanessa's house.



Solution

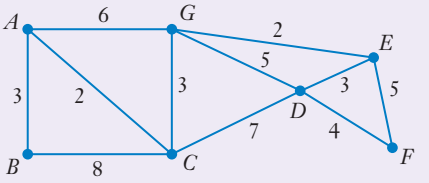
By inspection, the shortest trip will be along the roads numbered 13, 8, 3, 2.

$13 + 8 + 3 + 2 = 26$

The shortest time for Dilnoor to get to Vanessa's house is 26 minutes.

EXAMPLE 14

a Find a shortest path from B to every other vertex in the network.



b Use the shortest paths from vertex B to obtain a spanning tree for the network and find its weight.

c Find the minimum spanning tree for the network and its weight.

d Compare the shortest path spanning tree with the minimum spanning tree.

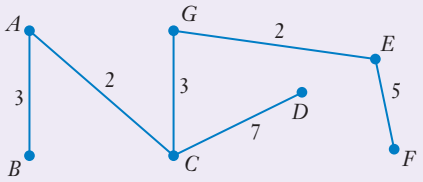
Solution

a The shortest paths from B are shown in the table below.

Vertex	A	C	D	E	F	G
Path	BA	BAC	$BACD$	$BACGE$	$BACGEF$	$BACG$
Weight	3	5	12	10	15	8

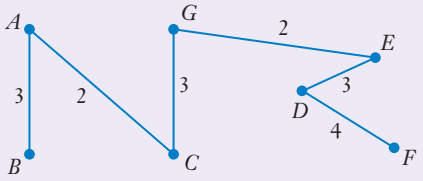
ISBN 9780170413633

b When the shortest paths from B are used, this spanning tree is obtained.



Weight of spanning tree = 3 + 2 + 3 + 2 + 5 + 7 = 22

c The minimum spanning tree is:



Weight of minimum spanning tree = 3 + 2 + 3 + 2 + 3 + 4 = 17

d The weight of the shortest path spanning tree is greater than the weight of the minimum spanning tree. Also, the shortest path between the vertices C and D and between E and F is not contained in the minimum spanning tree.

Note that the shortest path between 2 vertices is a path connecting those vertices whereas a minimum spanning tree is a tree that connects all vertices. For some networks, there may be a shortest path between 2 vertices that is not contained in any minimum spanning tree.

Finding the shortest path

1 Redraw the network with circles at each vertex, except at the starting vertex, S.

2 For all vertices one edge away from S, write down the shortest distance inside the circle.

3 For all vertices 2 edges away from S, write down the shortest distance inside the circle.

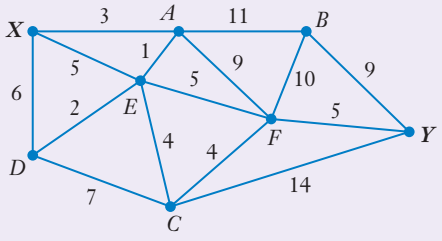
4 Continue this process until the finish vertex, F, is reached.

5 The shortest path is then identified by starting at F and moving backwards to the vertex from which the shortest distance at F was obtained, and continuing until S is reached.

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EXAMPLE 15

Find the shortest path from X to Y in this weighted network.

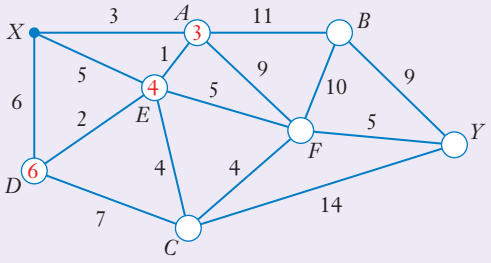


Solution

1 Redraw the network with circles at each vertex, except at the starting vertex, X.

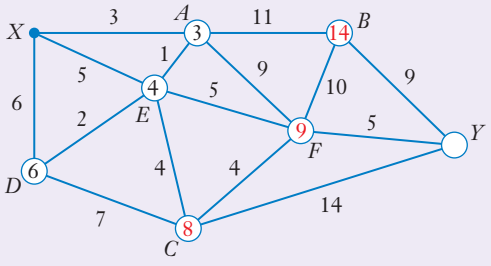
2 For all vertices **one** edge away from X, write down the shortest distance inside the circle.

The shortest distance to E is $3 + 1 = 4$ (path XAE).

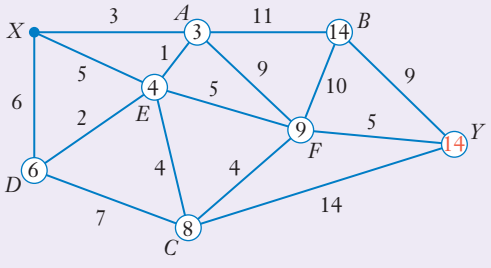


3 For all vertices **2** edges away from X, write down the shortest distance inside the circle with the help of the previous circle values.

The shortest distance to C is $4 + 4 = 8$, using the path $XAEC$.

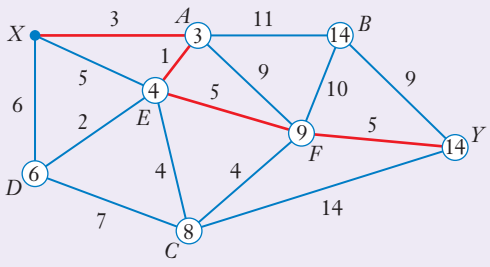


4 Continuing the process, the shortest distance from X to Y is $9 + 5 = 14$ (via F).



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5 To identify the shortest path, start at Y, move back to the vertex from which the shortest distance to X was obtained, continuing this until X is reached.



$14 - 5 = 9$

$9 - 5 = 4$

$4 - 1 = 3$

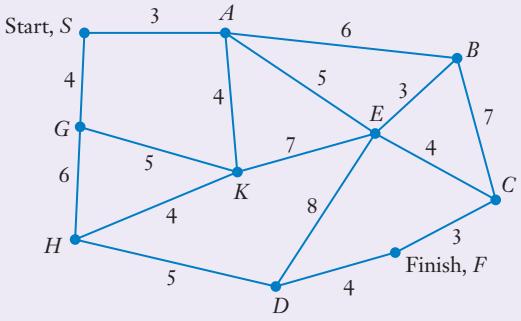
$3 - 3 = 0$

The shortest path is $XAEFY$ and has weight 14.

EXAMPLE 16

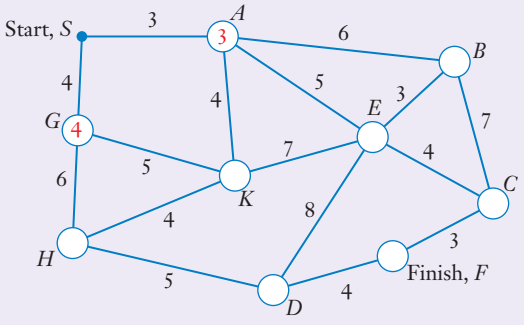
Robert wanted to run the shortest distance, starting at S, through a large nature reserve, finishing at F. The reserve is shown by the graph, where the distances are in kilometres.

Along which path did Robert run and how far did he run?



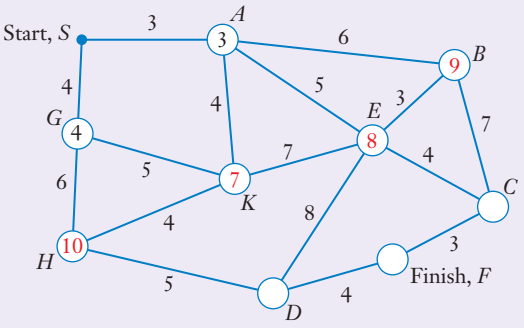
Solution

For all vertices **one** edge away from S, write down the shortest distance inside the circle.



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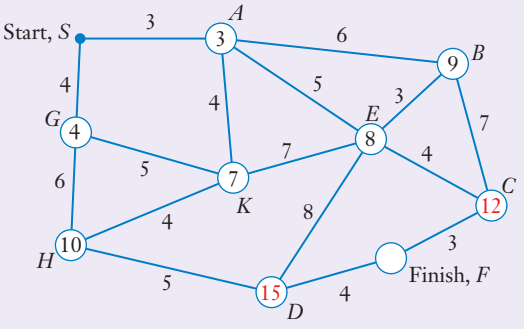
For all vertices **2** edges away from S, write down the shortest distance inside the circle.



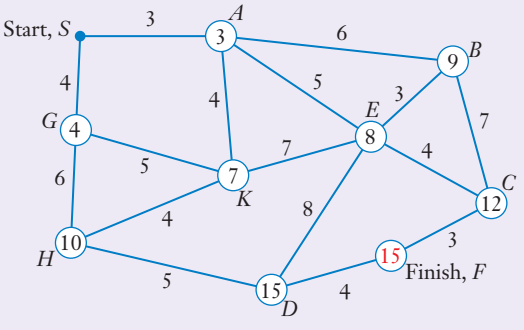
Vertices C and D are **3** edges away from S, so:

C is $8 + 4 = 12$ (via E)

D is $10 + 5 = 15$ (via H)

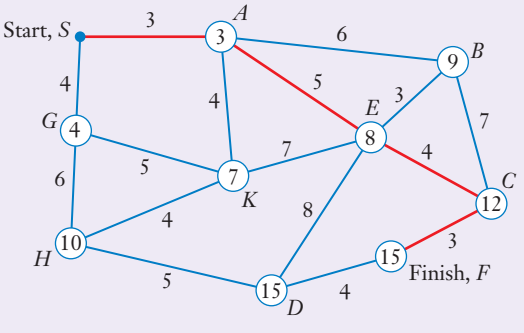


The shortest distance from S to F is then $12 + 3 = 15$ (via C).



To identify the shortest path, start at F, move backwards until S is reached.

$15 - 3 = 12$
$12 - 4 = 8$
$8 - 5 = 3$
$3 - 3 = 0$



So Robert ran the shortest path *SAECF*, which has a distance of 15 km.

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Exercise 6.04 Shortest path problems

Example 13

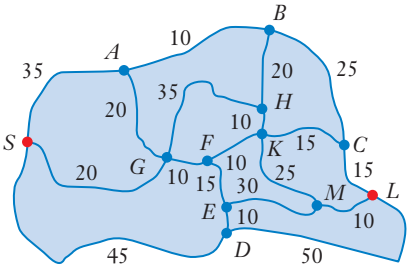
1 The network shows walking trails in a national park and the times in minutes it takes to walk the trails.

Find the shortest time it will take to walk from the start, *S*, to a lookout, *L*.

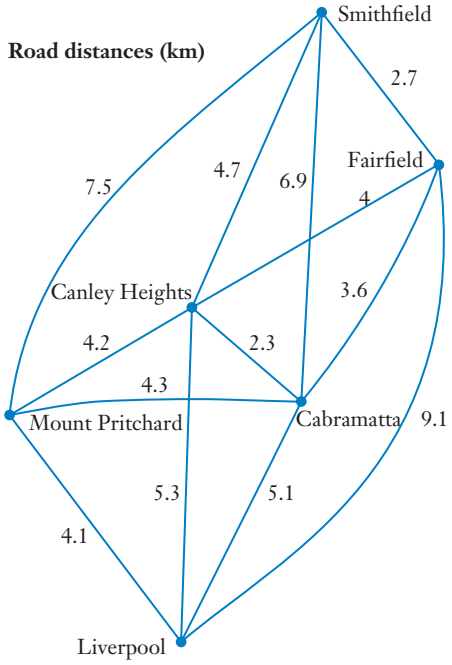
Select **A**, **B**, **C** or **D**.

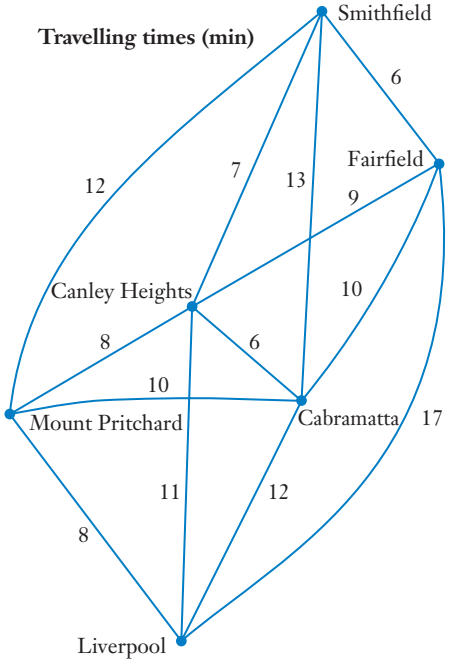
A 55 min **B** 70 min

C 85 min **D** 90 min



2 The graphs show the network of roads between 6 suburbs of south-western Sydney.





a If travelling from Mount Pritchard to Fairfield, find:

i the route that is the shortest distance

ii the route that takes the least amount of time.

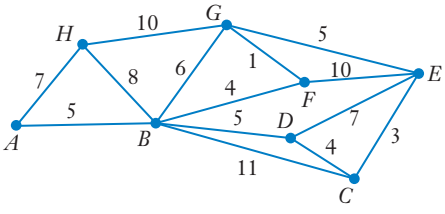
b Are the two routes the same?

Example 14

3 a Find the shortest path from *A* to every other vertex in the network.

b Use the shortest paths from part **a** to obtain a spanning tree for the network and find its weight.

c Draw a minimum spanning tree for the network.



d Compare the shortest spanning tree with the minimum spanning tree. Are they the same? Is the shortest path from *A* to *E* included in this minimum spanning tree?

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4 **a** Use the first graph from Question **2** to draw:

i the spanning tree that shows the shortest path from Liverpool to every other suburb and find its weight

ii the minimum spanning tree for the network and find its weight.

b Is the shortest path from Liverpool to Smithfield contained in the minimum spanning tree?

c Which other shortest distances from Liverpool are not contained in the minimum spanning tree?

5 **a** Use the second graph from Question **2** to draw:

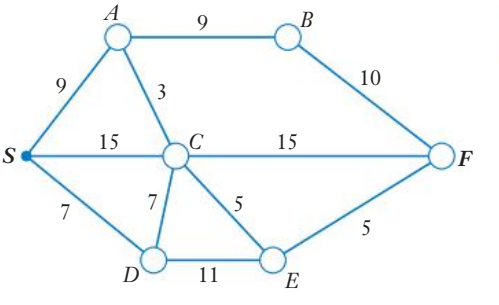
i the spanning tree that shows the shortest time to travel from Liverpool to every other suburb and find its weight

ii the minimum spanning tree for the network and find its weight.

b Is the shortest time to travel from Liverpool to Smithfield contained in the minimum spanning tree?

c Which other shortest travel times from Liverpool are not contained in the minimum spanning tree?

6 Copy this network diagram to find the shortest path from S to F, or download the worksheet 'Shortest path problems' from NelsonNet.



a For all vertices one edge away from S, write down the shortest distance inside the circle.

b Do the same for vertices 2 edges away, and continue the process until F is reached.

c Then start at F and move back to the vertex from which the shortest distance to F was obtained, continuing this until S is reached. This path will give the shortest distance.

Example 15

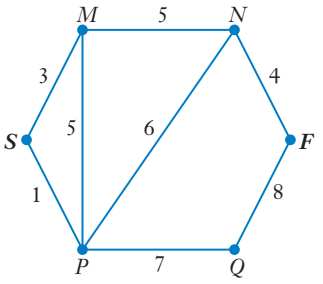
WS Shortest path problems

The worksheet 'Shortest path problems' contains the diagrams from Questions 6, 7, 9 and 10.

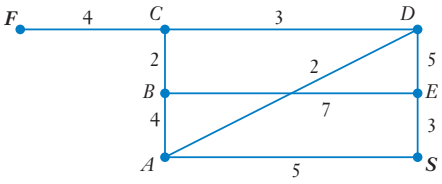
ISBN 9780170413633

7 Find the shortest path from S to F in each network.

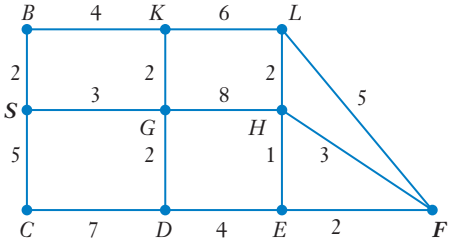
a



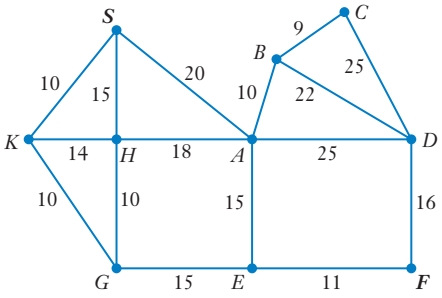
b



c

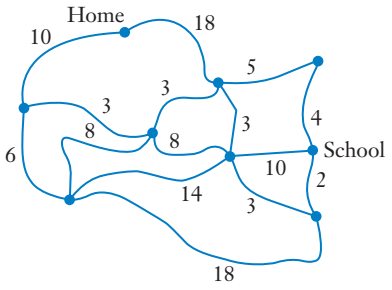


d



Example 16

8 The network shows Emily's bus travel times (in minutes) along roads from her home to school. What is the shortest time, in minutes, for Emily to travel from home to school? Select **A**, **B**, **C** or **D**.

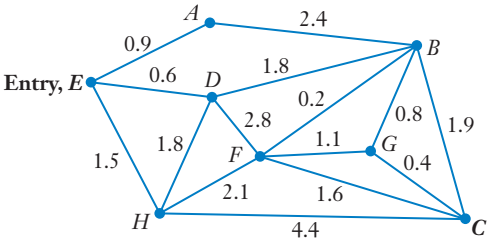


A 20 **B** 24

C 26 **D** 27

9 The network diagram shows walking trails in a nature reserve. The numbers on the edges are in kilometres.

Find the length of the shortest path from the entry, E, to the camping site, C.

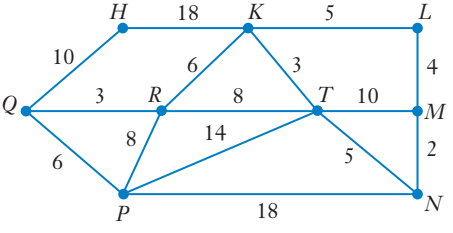


10 **a** Draw a minimum spanning tree for the network shown using either Kruskal's or Prim's algorithm and find its weight.

b Is this spanning tree unique?

c **i** Find the shortest path from Q to N.

ii Is this path contained in the minimum spanning tree?



d Find the spanning tree that gives the shortest distance from Q to every other vertex.

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11 The table shows the travelling times (in minutes) between 8 different locations in the Sydney metropolitan area, as shown on the map.



	Cron	Cam	Liv	Par	Horn	Pen	Rich
Syd	38	–	38	39	41	60	55
	Cron	–	47	59	–	–	–
		Cam	26	–	–	45	–
			Liv	31	–	42	47
				Par	34	35	40
					Horn	56	50
						Pen	22

a Draw a network diagram showing the information in the table.

b What is the length of the shortest path from Cronulla to Richmond?

c Draw a spanning tree to show the quickest travelling time from Sydney to each location and find its weight.

d **i** Draw a minimum spanning tree for the network. Write down the order in which the edges were added.

ii What is the weight of the minimum spanning tree?

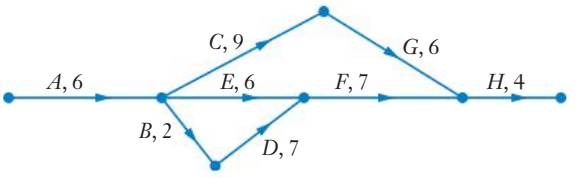
e Is the path for the quickest travelling time from Cronulla to Richmond contained in the minimum spanning tree? Give a possible reason.

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WS
Drawing directed graphs

6.05 Activity tables and forward scanning

When a large number of activities are involved in the completion of a task or project, a weighted directed network can be drawn to show the sequence of activities and the time required (in hours or days) to complete each activity, both shown on the edges of the network.



Activity tables

An **activity table** shows the order and completion time of each activity required to complete the task or project. The activity table usually includes the activity, which is coded by a letter or number, estimated time taken and predecessors. A **predecessor** is a previous activity that needs to be completed before the current activity can begin, such as putting on socks before putting on shoes.

When drawing a directed network of the activities of a task:

- Label the first and last vertices with **Start** and **Finish** respectively.
- Represent activities by arrows and start and end them at a vertex.
- Connect activities that do not have prerequisites to the start box.
- Begin activities with the same predecessor at the same vertex.
- Ensure that any 2 vertices are **not connected** by multiple edges.

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EXAMPLE 17

Each table lists the activities involved in completing a project.

Draw a directed network diagram for each project.

a

Activity	Predecessor
A	–
B	–
C	–
D	*A*
E	*B*
F	*C*
G	*E*, *F*
H	*D*
I	*G*, *H*

b

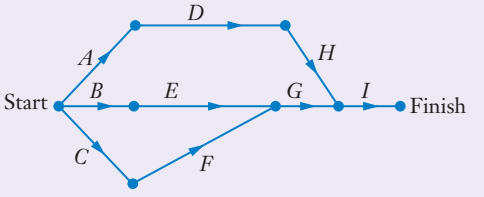
Activity	Predecessor	Time (hours)
A	–	2
B	*A*	3
C	*A*	4
D	*C*	2
E	*B*	5
F	*D*	2
G	*E*, *F*	1

c

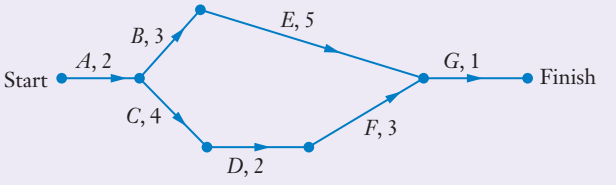
Activity	Predecessor	Time (hours)
A	–	1
B	–	2
C	*A*, *B*	3
D	*A*, *B*	2
E	*D*	2
F	*C*, *F*	1

Solution

a

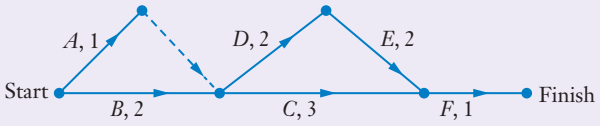


b

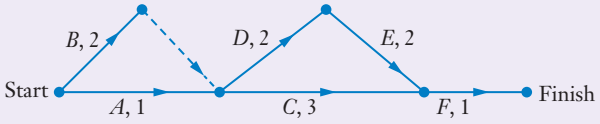


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c



or



Activities C and D are preceded by A *and* B, so a 'dummy activity' is required from A (or from B) as shown (so that 2 vertices are not connected by multiple edges).

Dummy activities

A **dummy activity** is used to eliminate repeated edges in a network. A dummy activity has zero time and is shown as a directed edge with a broken line or weight zero.

'dummy' means an artificial substitute here.

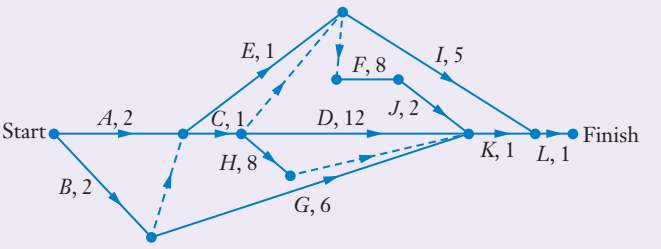
EXAMPLE 18

Draw a network diagram to show these activities involved in putting on a school musical production.

Code	Activity	Time (weeks)	Predecessor
A	Choose musical and director	2	–
B	Obtain scripts and copyright for show	2	–
C	Hold auditions and choose cast	1	A, B
D	Rehearse	12	C
E	Book venue	1	A
F	Build and paint sets	8	E
G	Obtain props	6	B
H	Make costumes	8	C
I	Advertise show and sell tickets	5	C, E
J	Set up lights and sound system	2	F
K	Hold dress rehearsal	1	D, G, H, J
L	Stage show	1	I, K

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Solution



Activity C is preceded by activities A and B, so a dummy activity is required from B.

Activity F is preceded by activity E but not C, so a dummy activity is required from E.

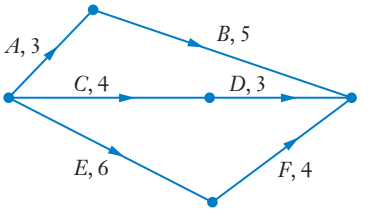
Activity I is preceded by activities E and C, so a dummy activity is required from C.

Activity K is preceded by D, G, H and J, so a dummy activity is required from activity H.

Critical paths

Critical path analysis is a step-by-step technique used to organise activities needed to complete a task or project in the shortest time possible.

Consider the simple network below, where the times for each activity are in hours.



For any task to be completed, all activities involved in that task must be completed, including those 'critical' activities that take the longest times. The **critical path** is the longest time path between the start and finish vertices in a network. The times of the activities for each path in the network above are:

'critical' means most essential.

$AB = 3 + 5 = 8$ h, $CD = 4 + 3 = 7$ h and $EF = 6 + 4 = 10$ h.

The critical path is EF (the longest time path), so the shortest time to complete all activities in the project is 10 hours.

For more complex networks, a more efficient method is used to find the critical path. It involves a process called **scheduling**, which is done using **forward** and **backward scanning** to find the **earliest** and **latest starting times** respectively for each activity in a project.

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Forward scanning

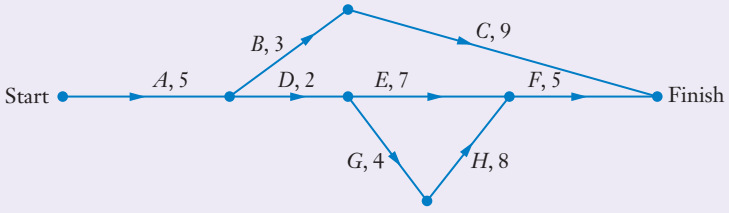
Forward scanning through a network calculates the **earliest starting time (EST)** for every activity in the network, dependent on the completion of predecessor activities, and requires these steps:

1. Redraw the network with empty boxes at each vertex.
2. At the **start** vertex, write 0 in the top box.
3. Work along each path or activity from the start (left to right), writing the total time of the path at each vertex (in the top box).
4. When 2 or more paths meet, select the highest total as the path to follow.
5. Continue until the **finish** vertex is reached.

Forward scanning

EXAMPLE 19

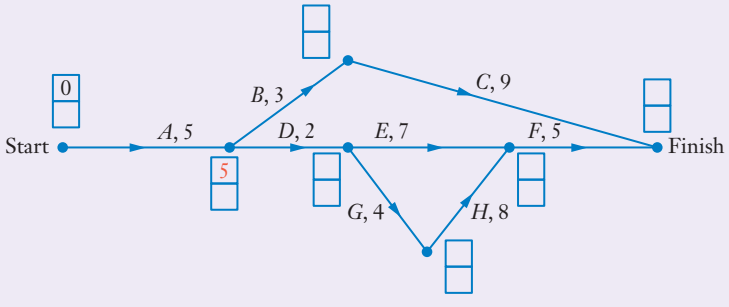
Determine the earliest start time (EST) for each activity in the network. Activity times shown are in hours.



Solution

Draw boxes above each vertex and enter 0 as the EST at the start vertex.

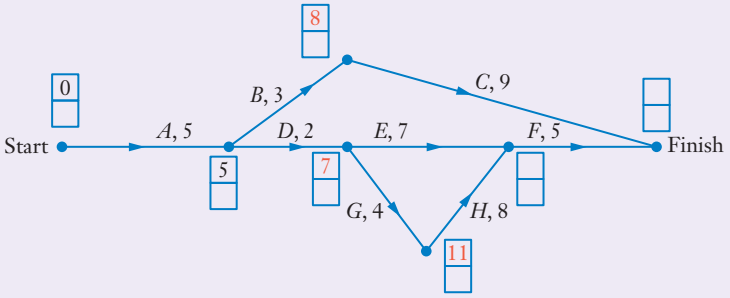
The top box at the end of activity A is for the **earliest start time** for activities B and D, so it is $0 + 5 = 5$ hours (after Start).



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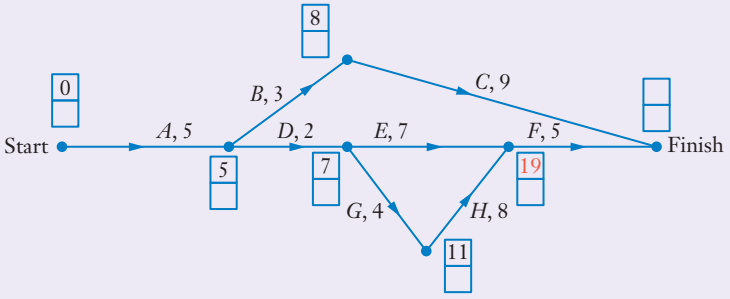
The EST for activity C is $5 + 3 = 8$ (hours after Start).

The EST for activities E and G is $5 + 2 = 7$ and the EST for activity H is $7 + 4 = 11$.



Two paths join for the start of activity F. The possible totals are $7 + 7 = 14$ or $11 + 8 = 19$.

Choose 19, the higher total, for the EST because F can't begin until *both* E and H have been completed.

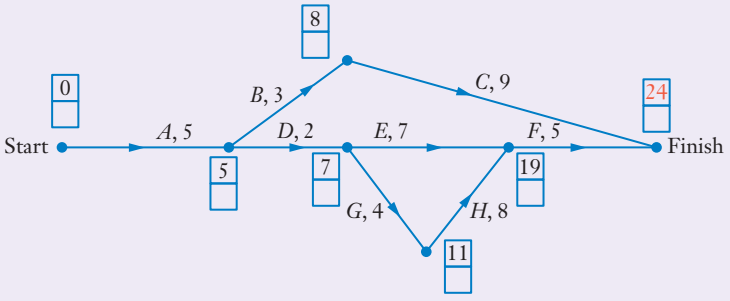


There are also 2 paths joining for the finish vertex. To calculate the EST for the finish, compare the times from activities C or F.

Activity C: $8 + 9 = 17$

Activity F: $19 + 5 = 24$

So, the final EST is the higher total, 24 hours. This means that the entire project will require 24 hours to complete.



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Exercise 6.05 Activity tables and forward scanning

Example 17

1 Draw a directed network diagram for each project.

a

Activity	Predecessor
A	–
B	–
C	*A*
D	*B*, *C*
E	*B*, *C*
F	*E*

b

Activity	Predecessor
A	–
B	–
C	*A*
D	–
E	*C*, *D*
F	*B*
G	*B*
H	*G*
J	*E*, *F*, *H*

c

Activity	Predecessor
A	–
B	–
C	*A*
D	*A*
E	*B*, *D*
F	*B*, *D*
G	*C*
H	*F*
J	*E*
K	*E*
L	*G*, *J*
M	*E*
N	*K*, *H*

d

Activity	Predecessor
A	–
B	*A*
C	*A*
D	*C*
E	*B*
F	*B*
G	*F*
H	*E*

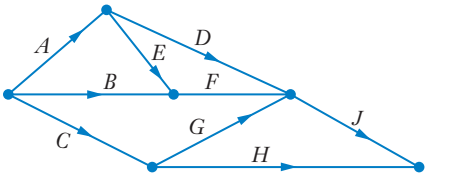
2 Which statement is **false** about the directed network? Select **A**, **B**, **C** or **D**.

A *G* is preceded by *C*.

B *D* is preceded by *A* and *E*.

C *F* is preceded by *B* and *E*.

D *J* is preceded by *D*, *F* and *G*.

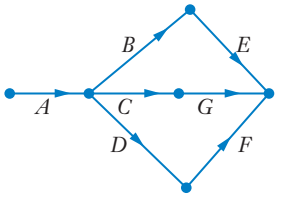


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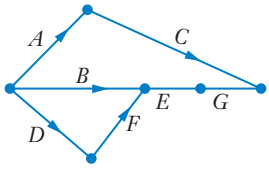
3 Which is the correct network diagram for this activity table?

Activity	Predecessor
A	–
B	–
C	*A*
D	*A*
E	*B*
F	*D*
G	*E*, *C*

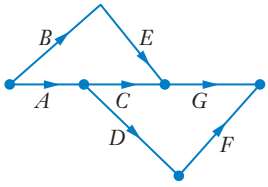
A



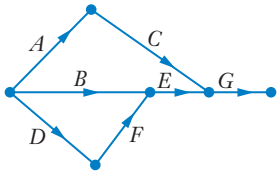
B



C



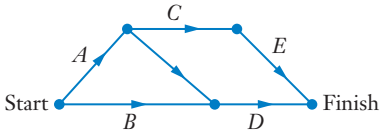
D



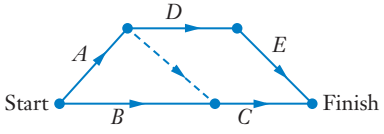
4 Which is the correct network diagram for this activity table?

Activity	Predecessor
A	–
B	–
C	*A*
D	*A*, *B*
E	*C*

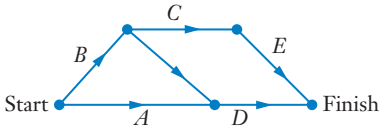
A



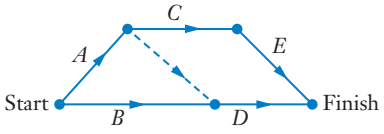
B



C

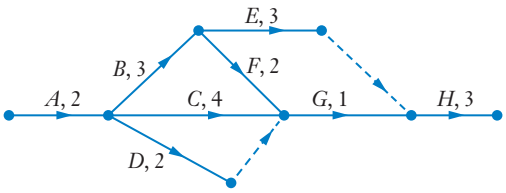


D



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5 Which of the activity tables is represented by this network diagram?



A

Activity	Duration (hours)	Predecessor
A	2	–
B	3	*A*
C	4	*A*
D	2	*A*
E	3	*A, B*
F	2	*B*
G	1	*C, F*
H	3	*E, G*

B

Activity	Duration (hours)	Predecessor
A	2	–
B	3	*A*
C	4	*A*
D	2	*A*
E	3	*B*
F	2	*B*
G	1	*C, D, F*
H	3	*E, G*

C

Activity	Duration (hours)	Predecessor
A	2	–
B	3	*A*
C	4	*A*
D	2	*A*
E	3	*B*
F	2	*B, E*
G	1	*C*
H	3	*E*

D

Activity	Duration (hours)	Predecessor
A	2	–
B	3	*A*
C	4	*A*
D	2	*A*
E	3	*B*
F	2	*B*
G	1	*C, F*
H	3	*E*

6 Draw a network diagram for each activity table, including dummy activities where required.

a

Activity	Duration (hours)	Predecessor
A	3	–
B	2	–
C	4	–
D	2	*C*
E	4	*A*
F	2	*E*
G	3	*A, B*
H	4	*D, G*
J	3	*B, F, H*

b

Activity	Duration (days)	Predecessor
A	2	–
B	3	–
C	3	*A*
D	1	*A*
E	2	*C*
F	5	*C*
G	3	*D, F*
H	2	*E*
J	2	*B*
K	1	*G, J, H*

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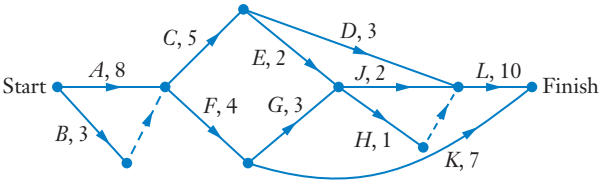
c

Activity	Duration (months)	Predecessor
A	5	–
B	2	*A*
C	3	*B, D*
D	3	–
E	5	–
F	1	*D*
G	2	*E, F*

d

Activity	Duration (months)	Predecessor
A	3	–
B	2	–
C	4	*A*
D	2	*C*
E	1	*B, D*
F	5	*A, B*

7 Draw an activity table for this network diagram.



8 Draw a network diagram for each activity table.

Example 18

a Making a cup of coffee

Code	Activity	Time (s)	Predecessor
A	Put water in kettle	10	–
B	Put coffee in cup	5	–
C	Put sugar in cup	5	–
D	Put milk in cup	10	*B, C*
E	Boil water	140	*A*
F	Pour hot water into cup	8	*D, E*
G	Stir and drink cup of coffee	300	*F*

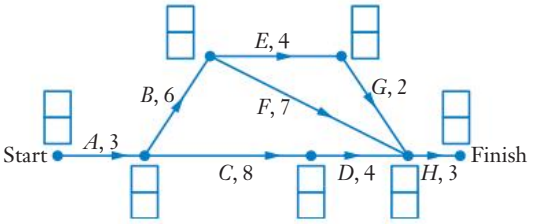
b Cooking a baked lamb dinner.

Code	Activity	Time (min)	Predecessor
A	Make mint sauce and let stand	90	–
B	Preheat oven to 180°C	5	–
C	Put lamb in oven and cook	75	*B*
D	Cut potatoes, pumpkin and carrots	5	–
E	Bake potatoes	60	*D*
F	Bake carrots and pumpkin	35	*D*
G	Put lamb and baked vegetables on plate and cover with foil	8	*C, E, F*
H	Steam beans	10	–
J	Make gravy with pan juice from lamb	7	*C*
K	Serve dinner	5	*A, G, H, J*

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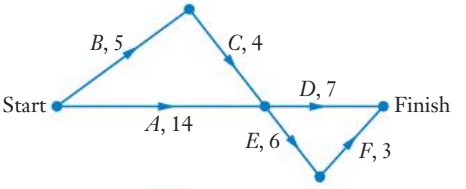
Example 19

9 Copy and complete the earliest starting times (ESTs) for the activities by forward scanning the network.

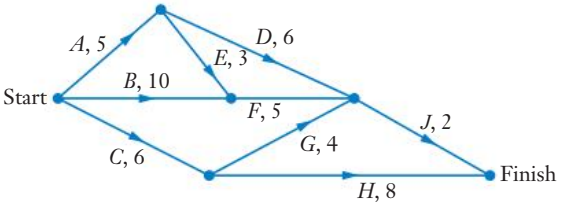


10 Determine the ESTs for the activities in each network.

a



b



WS Calculating EST and LST

WS Critical paths and activity float times

6.06 Backward scanning and critical path analysis

Backward scanning

Backward scanning is used to calculate the **latest starting time (LST)** for every activity. This is the latest time an activity can be started without affecting the completion time of the project.

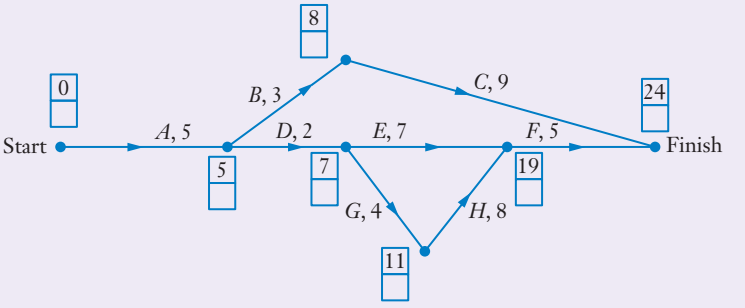
Backward scanning is done after forward scanning is completed.

1. Begin LST calculations at the **finish** vertex. At the finish, LST = EST, so write the LST in the bottom box.
2. Work backwards from right to left, subtracting the time of each activity and writing the LST in the bottom box at each vertex.
3. If there is more than one path to the vertex, take the lowest difference as the LST.
4. Continue until you reach the **start** vertex. The LST at the start vertex must be 0.

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EXAMPLE 20

Find the latest starting times (LST) for each activity in the network from the previous example, given the ESTs.



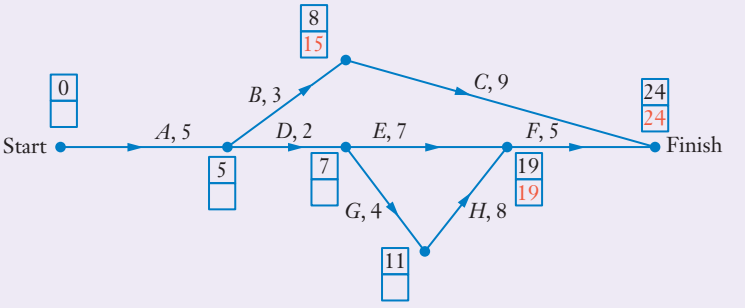
Solution

Working backwards from the right:

The LST for the finish is 24.

The LST for activity C is $24 - 9 = 15$ (hours after Start).

The LST for activity F is $24 - 5 = 19$.

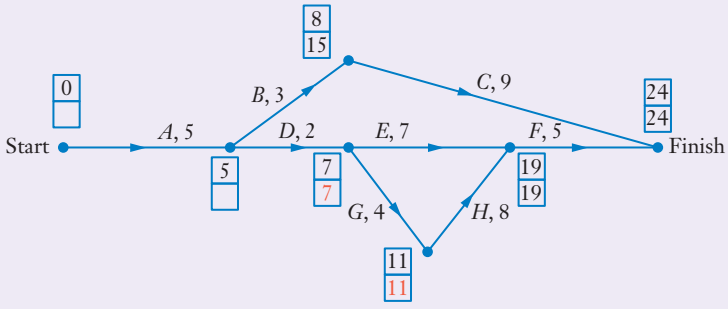


The LST for activity H is $19 - 8 = 11$.

There are 2 paths leading back to the end of activity D, so the differences are $11 - 4 = 7$ (via G) or $19 - 7 = 12$ (via E). Choose the lower difference, because enough time must be allowed to complete *both* E and G.

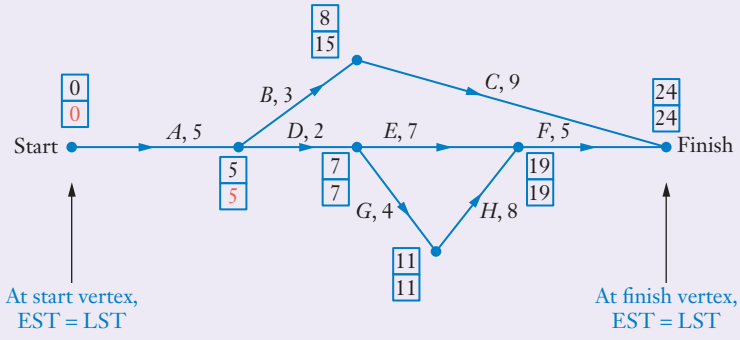
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So the LST for activities E and G is 7.

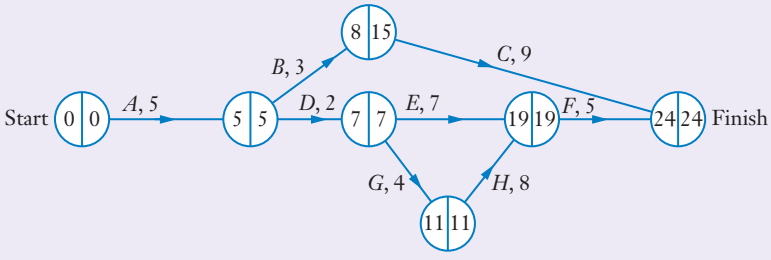


There are also 2 paths leading back to the end of activity A, so the differences are $7 - 2 = 5$ (via D) or $15 - 3 = 12$ (via B). The LST for activities B and D is the lower difference of 5.

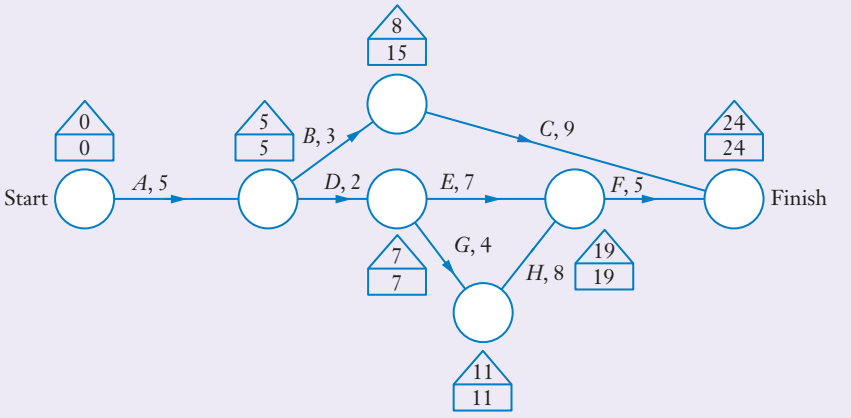
The LST for the start vertex is then $5 - 5 = 0$ (the LST for start should always be 0).



The ESTs and LSTs can also be displayed on network diagrams in circles or pentagons, as shown below and on the next page.



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Identifying the critical path

The **critical path** is the **longest time path** from start to finish in a network. It is also gives the shortest amount of time in which to complete a project. On the critical path, activities have equal EST and LST values. The critical path in the example above is *ADGHF*. Activities on the critical path, such as *F*, are called **critical activities**. Activities not on the critical path, such as *E*, are **non-critical activities**.

A critical activity must start and end on time, otherwise the project's completion will be delayed. The start and end times of a non-critical activity can be delayed without affecting the project's completion time. For example, if building a house, waiting for paint to dry could be a critical activity because it takes a long time while cleaning the paint brushes could be a non-critical activity because it doesn't take long.

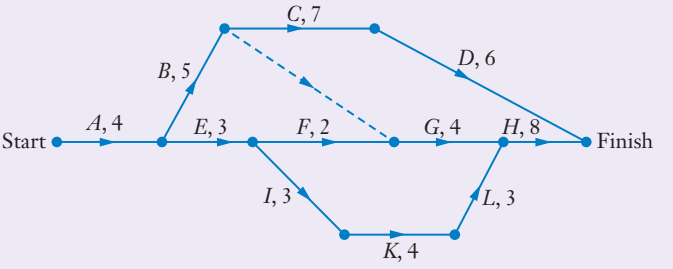
EXAMPLE 21

For the network shown, where the activity times are in hours, find:

a the earliest starting time (EST) of all activities

b the latest starting time (LST) of all activities

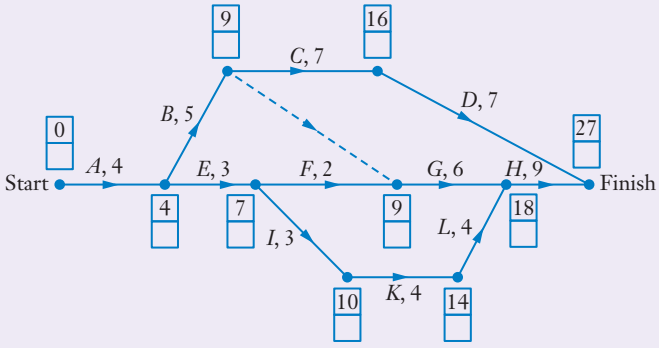
c the critical path and its length.



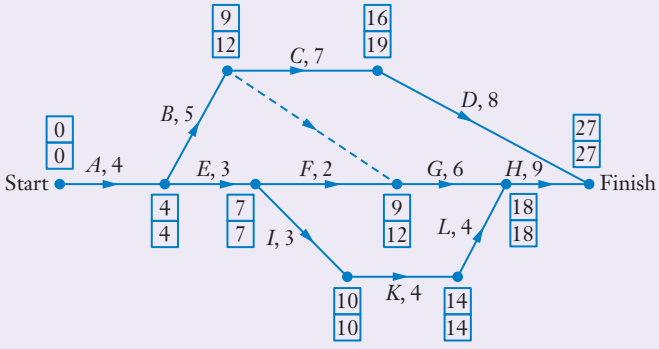
ISBN 9780170413633

Solution

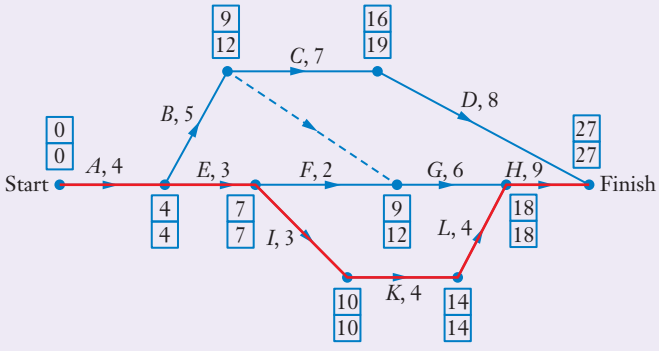
a Forward scanning to find the ESTs for each activity:



b Backward scanning to find the LSTs for each activity:



c *AEIKLH* is the critical path since the activities where EST = LST are on this path.



Length of critical path is 27 hours. The EST and LST of Finish

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Float time

The **float time** (or **slack time**) for a non-critical activity is the longest time by which the activity (such as cleaning paintbrushes) can be delayed without affecting the time to complete the project.

Float time

Float time = $LST_{next} - EST_{next}$ if it is the only path leading to the next activity

or Float time = $LST_{next} - EST$ – activity time

where EST_{next} is the EST of the next activity, which is the earliest ending time of the activity, while LST_{next} is the LST of the next activity, which is the latest ending time of the activity.

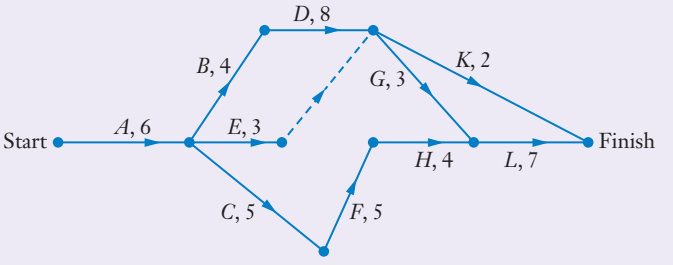
EXAMPLE 22

For each project shown, where the activity times are in days:

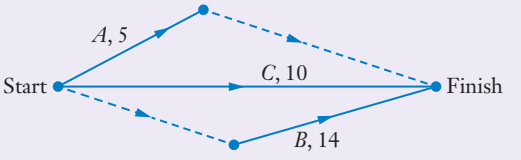
i determine the critical path

ii find the float times for the non-critical activities.

a



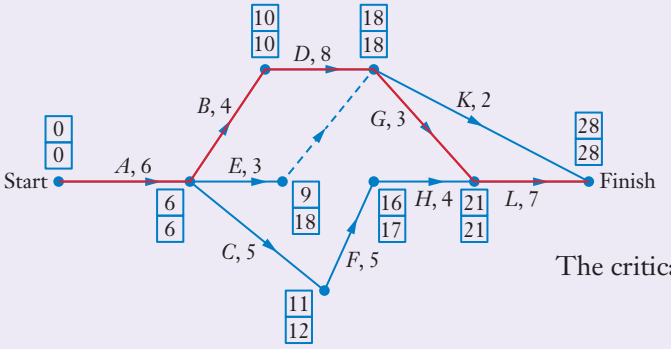
b



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Solution

a **i** The ESTs and LSTs for each activity are calculated as shown.



The critical path is *ABDGL*.

ii Non-critical activities are *C*, *E*, *F*, *H* and *K*.

From the diagram, activity *C* can end between 11 and 12 days.

Float time for $C = 12 - 11 = 1$ day $\quad LST_{next} - EST_{next}$

or Float time for $C = 12 - 6 - 5 = 1$ day $\quad LST_{next} - EST - \text{activity time}$

Float time for $E = 18 - 9 = 9$ days

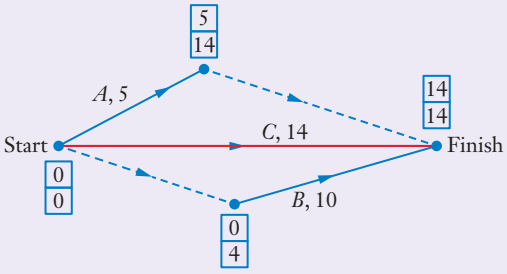
Float time for $F = 17 - 16 = 1$ day

Activities *H* and *K* have other paths leading to their next activity, so the first formula cannot be used. Activity *H*'s EST is 16, latest end time is 21, with an activity time of 4.

Float time for $H = 21 - 16 - 4 = 1$ day $\quad LST_{next} - EST - \text{activity time}$

Float time for $K = 28 - 18 - 2 = 8$ days

b **i** The ESTs and LSTs are shown below.



The critical path is *C*.

ii Non-critical activities are *A* and *B*.

Float time for $A = 14 - 5 = 9$ days $\quad LST_{next} - EST_{next}$

Float time for $B = 14 - 0 - 10 = 4$ days $\quad LST_{next} - EST - \text{activity time}$

A critical activity has no 'slack', so its float time is always 0.

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Exercise 6.06 Backward scanning and critical path analysis

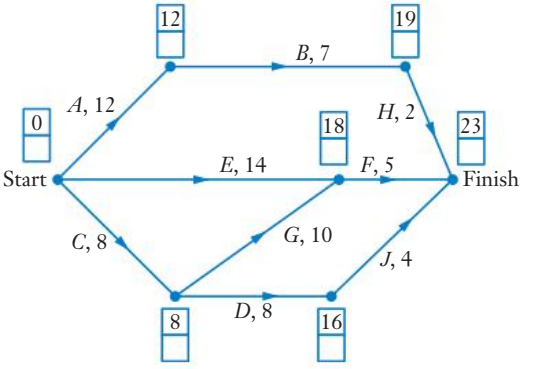
Example 20

WS Critical path analysis

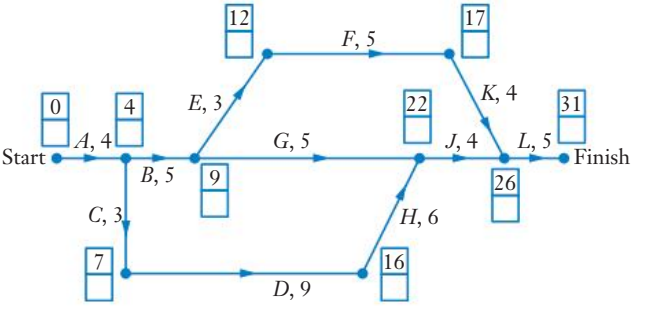
The worksheet 'Critical path analysis' contains the diagrams for Questions 1 to 7.

1 Backward scan each network to find the latest starting time (LST) for each activity.

a

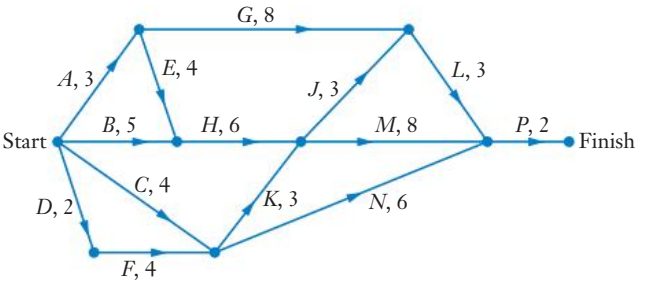


b

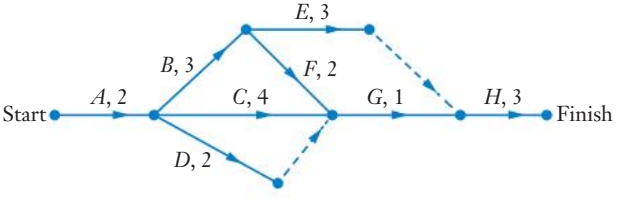


2 Determine the ESTs and LSTs for each network.

a



b



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Example 21

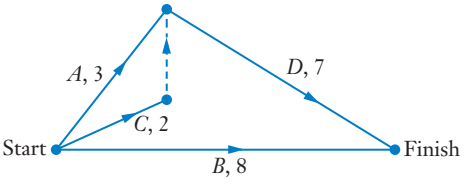
3 For each network shown, find:

i the EST

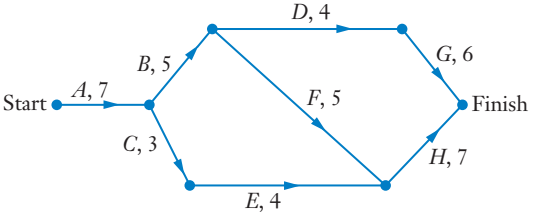
ii the LST

iii the critical path and its length.

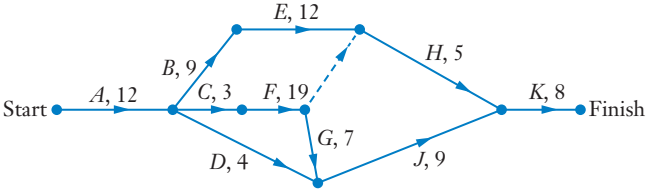
a



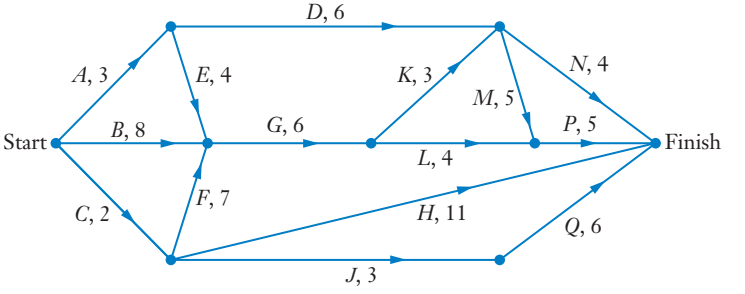
b



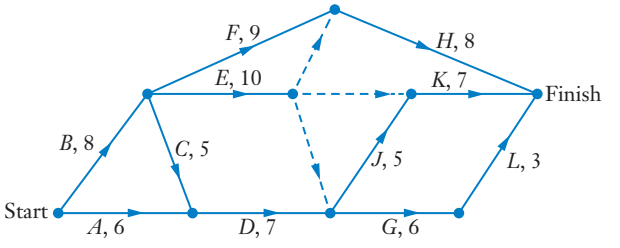
c



d



4 This network diagram shows activities that must be completed in a project.



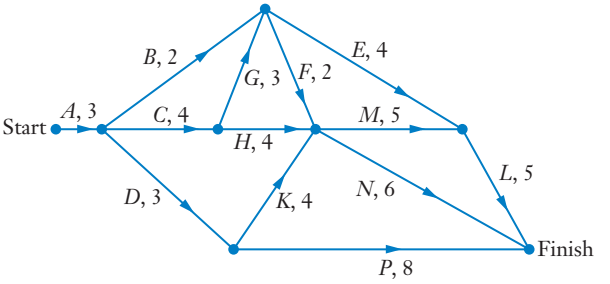
Which activities below are included in the critical path?

A *F* and *L** **B** *B* and *J** **C** *C* and *G** **D** *E* and *K*

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Example 22

5 This network shows the activities that are needed to complete a project, where the activity times are in months.



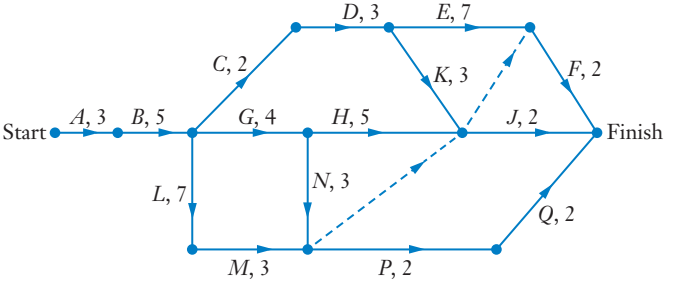
a What is the earliest start time for activity *M*?

b Find the critical path and the completion time for the project.

c Find the float time for the non-critical activity *K*. What does this float time mean?

6 Find the float times for the non-critical activities in Question **3**.

7 This network shows the activities for the completion of a project, where the time is in hours.



a Find the critical path and the minimum completion time.

b Is the critical path unique? Give reasons.

c Which activity has a float time of 3 hours?

d Which activities could be deleted without changing the completion time?

e The project time needs to be shortened by 3 hours. Which activity can be reduced to achieve this? Give reasons.

8 The maintenance of paths in a large city park will involve the completion of 8 activities. The table shows these activities and their completion times.

Activity	Duration (days)	Predecessor
A	5	–
B	6	*A*
C	5	*A*
D	8	*B*
E	12	*C*
F	4	*C*
G	6	*D, E*
H	5	*F, G*

a Draw a network diagram for the activity table and determine the EST and LST for each activity.

b Find the critical path and calculate the completion time for the maintenance of the park.

c Find the float time for the non-critical activities.

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9 The activities for building a covered area attached to the back of a house are shown in the activity table.

Code	Activity	Time (days)	Predecessor
A	Clear area	2	–
B	Obtain materials	3	–
C	Lay concrete and allow to dry	4	*A*
D	Cut timber	1	*B*
E	Put up framework for covered area	2	*C*, *D*
F	Put on roof and attach gutters	1	*E*
G	Paint timber used in covered area	3	*F*
H	Landscape around covered area	4	*F*

a Draw a network diagram for the activity table and determine the EST and LST for each activity.

b Find the critical path and calculate the completion time for the covered area.

c Find the float time for the non-critical activities.

d If the covered area needs to be finished by 23 September, when should work on the covered area begin?

10 The company has a project that requires the activities in the table below to be completed in 70 days. The critical path includes activities *B*, *D*, *E*, and *F*.

The float for *G* is 4 days and the float for *C* is 9 days.

Activity	Duration (days)	Predecessor
A	*m*	–
B	20	–
C	*n*	*A*
D	24	*A*, *B*
E	11	*D*
F	*p*	*C*, *E*
G	*r*	*D*
H	3	*G*

Draw a network diagram and find possible values for *m*, *n*, *p* and *r*.

Show all necessary working.

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11 A park is to have some new playground equipment installed. The activities and their predecessors are shown in the table.

Activity	Predecessor
A	–
B	A
C	A
D	A
E	B
F	C
G	D
H	E
J	F, G
K	J, H

a Use the information in the activity table to draw a network diagram.

b There is only one critical path for this network. How many non-critical activities are there? Give reasons for your answer.

c The table below gives the earliest start times (ESTs) and the latest start times (LSTs) for some of the activities. All times are in days.

Activity	EST	LST
B	14	15
D	14	16
K	26	26

i How long will activity A take?

ii What is the critical path for this project?

iii The minimum time to complete installing the equipment is 35 days. How many days will it take to complete activity K?

iv Activity B will take 6 days to complete. What is the maximum time to complete activities E and H?

6.07 Network flow problems

WS Network flow capacity

Another application of directed networks is managing flow of road traffic, water, mail deliveries, online purchases, concert bookings, Internet use and plane arrivals and departures at airports. These are all examples of **network flow** problems.

In a flow network, the start is called the **source** (often labelled S) and the end is called the **sink** (labelled T). The weights on the edges are called **capacities** and represent the amount of flow an edge can hold.

The **flow capacity** of a network is the total flow through the network.

The **inflow** of a vertex is the total capacity of all edges entering the vertex.

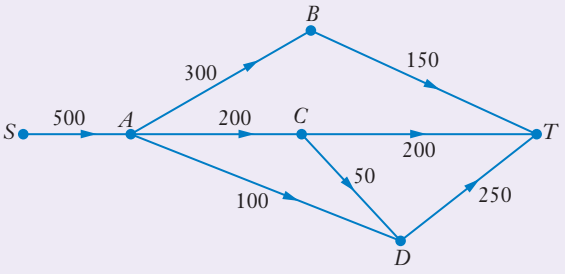
The **outflow** of a vertex is the total capacity of all edges leaving the vertex.

The **maximum outflow** of a vertex is the smaller of the inflow or outflow of the vertex.

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EXAMPLE 23

This directed network shows the flow capacities, in litres per minute, of water pipes on a property.



Find:

a the inflow capacity of vertex D

b the outflow capacity of vertex D

c the maximum outflow of vertex D.

Solution

a Inflow capacity of vertex $D = 100 + 50$
$= 150$ L/min

Add all edges entering vertex A.

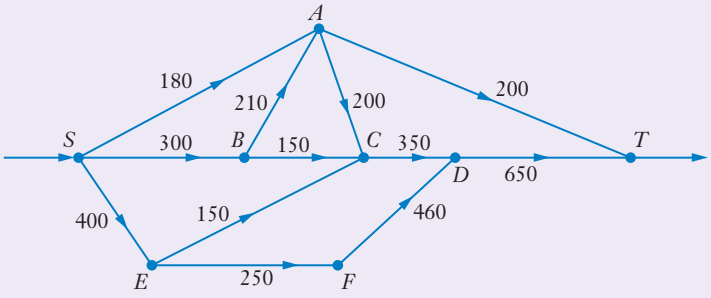
b Outflow capacity of vertex $D = 250$ L/min

c Maximum outflow of vertex $D = 150$ L/min

Although D has an outflow capacity of 250 L/min, it is not fully used because the inflow is only 150 L/min.

EXAMPLE 24

The network represents road traffic in a city, starting at S (the source) and ending at T (the sink). The capacities are the maximum number of cars per hour along each road.



a Determine the maximum flow possible from the source to the sink.

b Redraw the network showing the traffic flow along each road in order to achieve the maximum flow.

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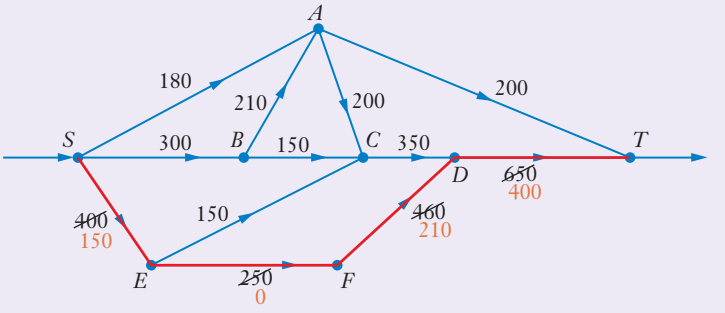
Solution

a Examine the capacity of every possible route from S to T.

Consider the route $SEFDT$ below.

The flow capacity is a maximum of 250 cars (since that is all EF can carry).

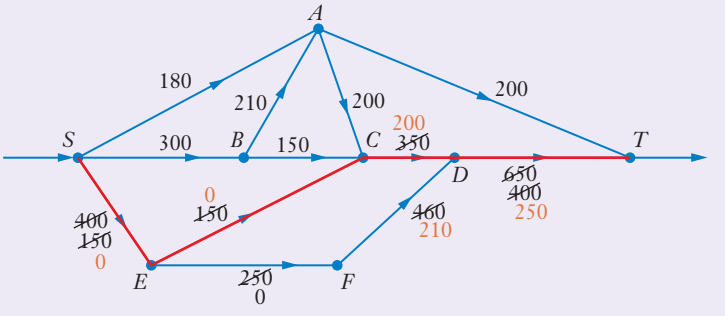
By subtracting 250 from each road, this leaves the roads involved with a carrying capacity as shown in the diagram.



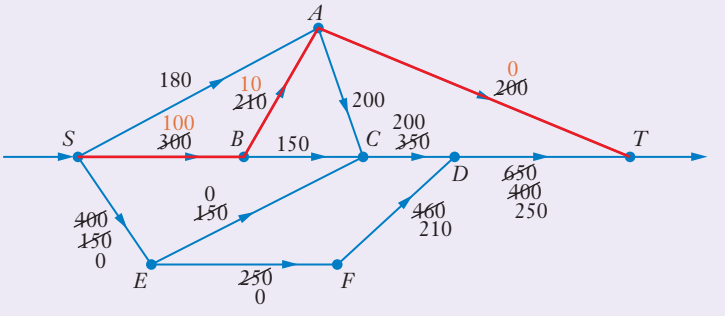
Now consider $SECDT$ below.

The flow capacity is 150 cars (the carrying capacity of SE and EC),

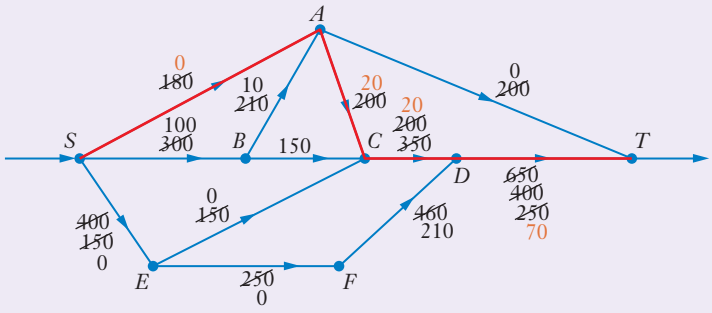
Subtract 150 from the roads involved, as shown in the diagram.



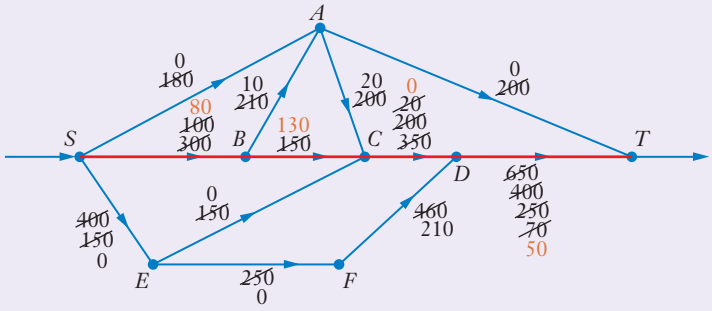
For $SBAT$, the flow capacity is 200 cars (the carrying capacity of AT).



For *SACDT*, the flow capacity is 180 cars (the capacity of *SA*).



For *SBCDT*, the capacity is 20 cars (the capacity of *CD*).



For *SAT*, there is no capacity left now.

Total flow:

SEFDT	250 cars
SECDT	150 cars
SBAT	200 cars
SACDT	180 cars
SBCDT	20 cars
	800 cars

The maximum flow from *S* to *T* for the network of roads is 800 cars/hour.

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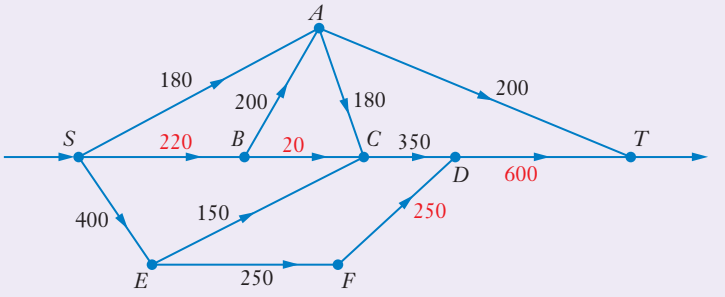
b To achieve the maximum flow, the flow along each road can be found by looking at any unused capacity for each road and subtracting it from the original capacity of the road, as shown below.

$SB = 300 - 80 = 220$

$BC = 150 - 130 = 20$

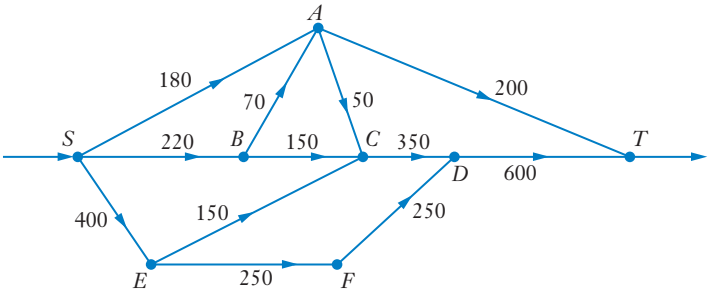
$DT = 650 - 50 = 600$

$FD = 460 - 210 = 250$



Note that in Example 24, the routes could have been chosen in a different order, resulting in a flow diagram with different values, but the maximum flow would still be 800 cars/hour.

For example, if the routes were chosen in the order *SEFDT*, *SECDT*, *SAT*, *SBAT*, *SBACDT* and *SBCDT*, the flow along each road would be:



Capacity of a cut

A method of finding the maximum flow of a network is to use 'cuts'.

A **cut** is a line drawn through the edges of a flow network that *separates* the source from the sink of the network but does not pass through any vertex. It acts like an arrangement of roadblocks that stops all flow from the source to the sink.

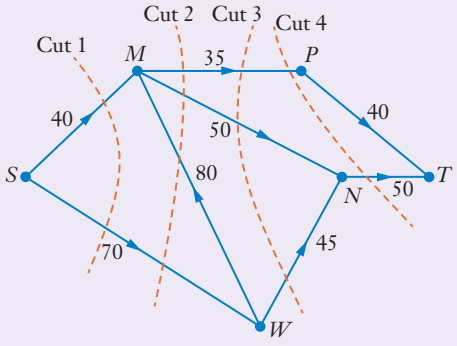
Any cut through a network blocks or stops the flow, so we can compare the effects of different cuts on the size of the flow being blocked. The **capacity of a cut** is the amount of flow blocked by the cut, found by summing the cut edges passing from source to sink.

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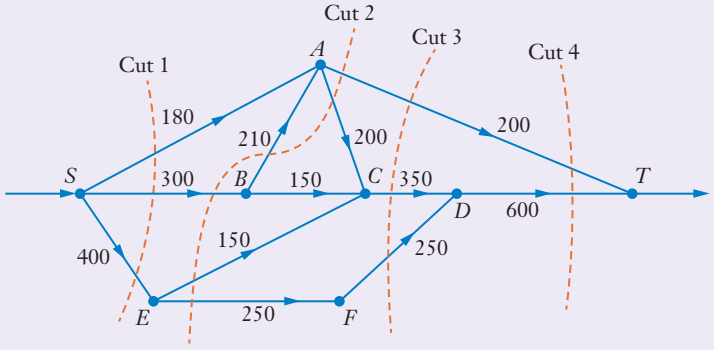
EXAMPLE 25

Calculate the capacity of each cut in each network.

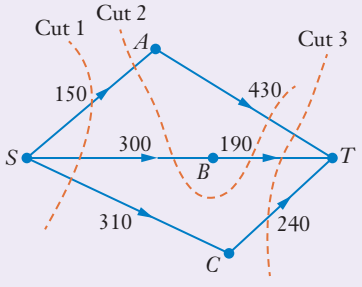
a



b



c



Solution

a The capacities of the cuts are:

cut 1 = 40 + 70 = 110

cut 2 = 35 + 50 + 70 = 155

MW is flowing in the opposite direction to the other edges cut, so its capacity is not counted.

cut 3 = 35 + 50 + 45 = 130

cut 4 = 35 + 50 = 85

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b cut 1 = 180 + 300 + 400 = 880

cut 2 = 200 + 200 + 300 + 150 + 250 = 1100

The edge *AB* is cut in the opposite direction to all the other cut edges.

cut 3 = 200 + 350 + 250 = 800

cut 4 = 200 + 600 = 800

c cut 1 = 150 + 300 + 310 = 760

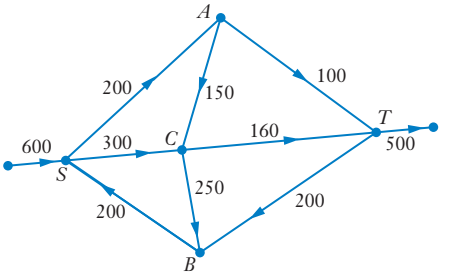
cut 2 is **not** a valid cut as it does not stop the flow from *S* to *T* via *C*.

cut 3 = 430 + 190 + 240 = 860

Exercise 6.07 Network flow problems

Example 23

1 The directed network shows the traffic capacity, in vehicles per hour, of roads in the centre of a city suburb, where the vertices represent intersections.



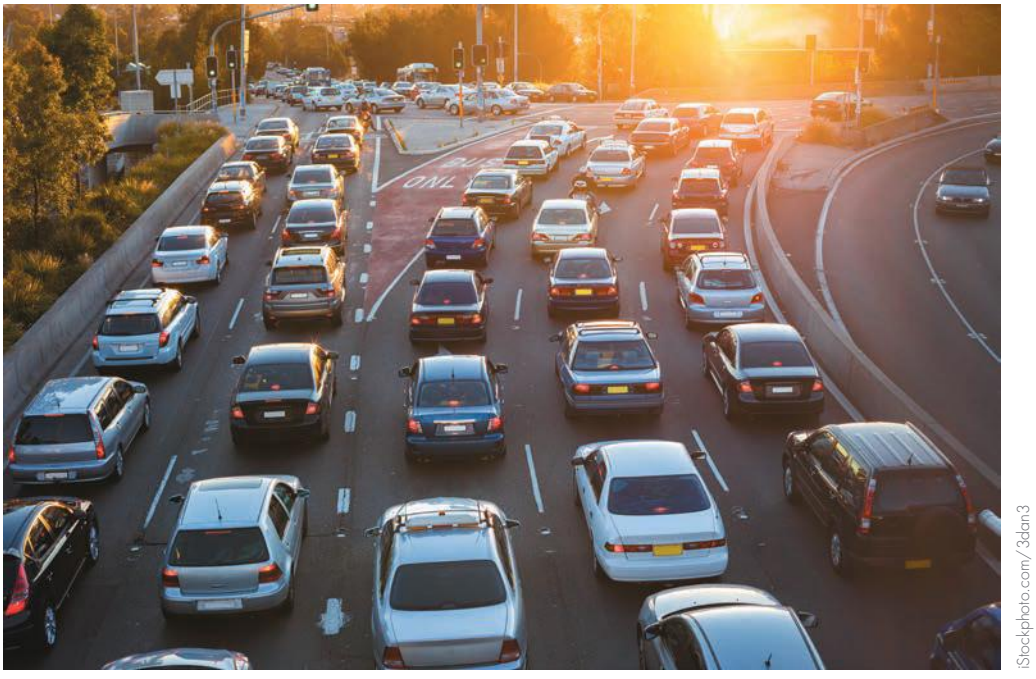
For each vertex below, calculate:

i the inflow capacity

ii the outflow capacity

iii the maximum outflow.

a *S* **b** *C* **c** *B* **d** *T*

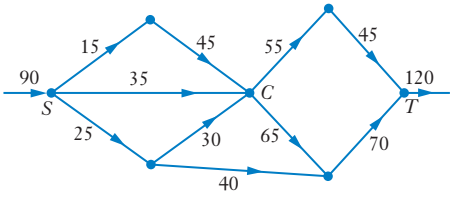


iStockphoto.com/3dan3

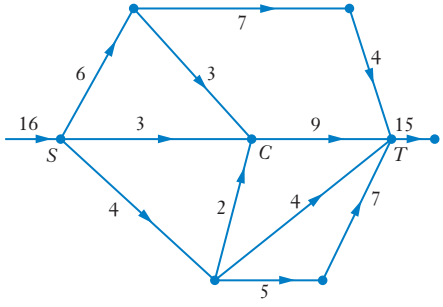
2 On these directed graphs, the values give the maximum flow between the vertices. For each graph, find the maximum outflow at:

i S **ii** C **iii** T

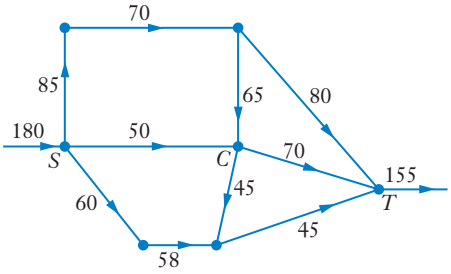
a



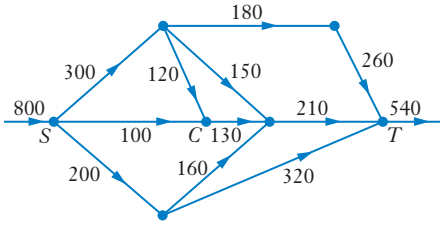
b



c

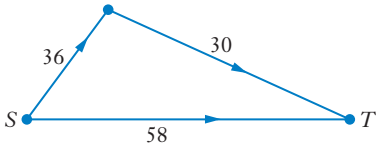


d

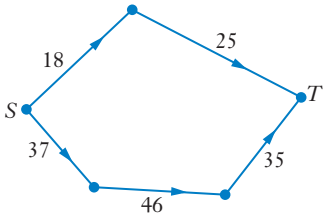


3 Determine the maximum flow possible from the source at S to the sink T.

a



b



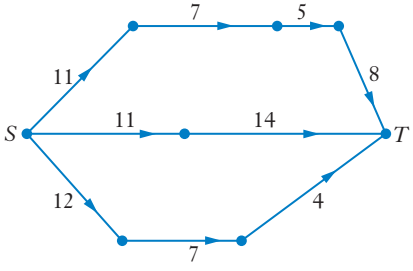
4 What is the maximum flow from the source at S to the sink at T?

A 16

B 20

C 23

D 25



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Example 24

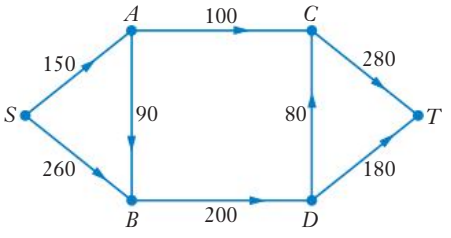
WS Network flow problems

5 For each network:

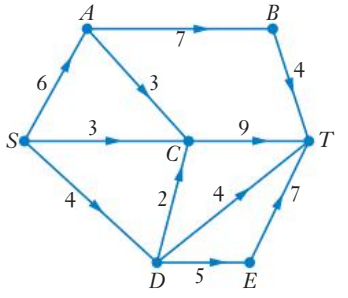
i determine the maximum flow possible from the source, S, to the sink, T (by using the inflow and outflow for the vertices along different paths)

ii redraw the network showing the flow along each edge in order to achieve the maximum flow.

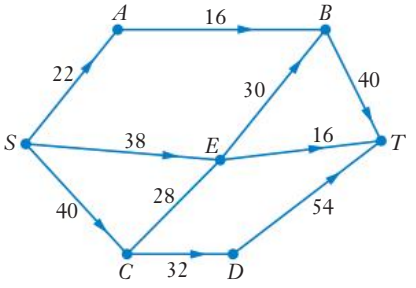
a



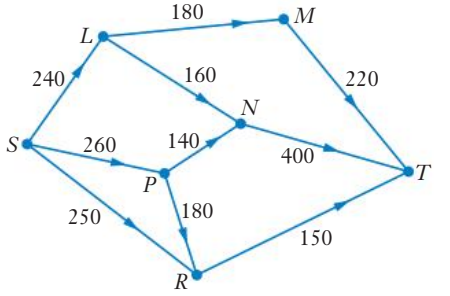
b



c

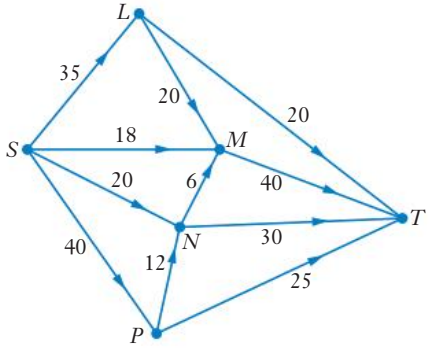


d



The worksheet 'Network flow problems' contains the diagrams from Questions 5, 6, 7 and 9 for writing upon.

6 This diagram shows water flowing from a main dam at S to 4 dams, which then supply a town at T. The edge capacities are given in thousands of litres per minute.



a Find the maximum flow through the network from the main dam to the town.

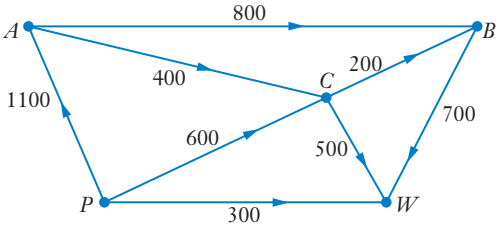
b Redraw the network showing the flow along each edge in order to achieve the maximum flow.

c What is the outflow of each dam?

d The capacity of only one edge is increased. Which one should it be? Give reasons for your answer.

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7 The network diagram shows the traffic flow, in cars per hour, along roads when travelling from a suburb at P to another suburb at W in morning peak hour.

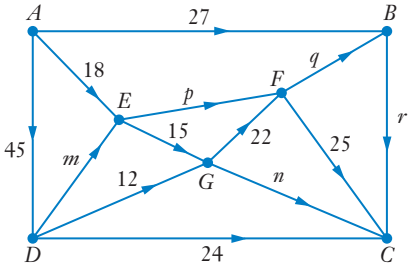


a Find the maximum flow P to W.

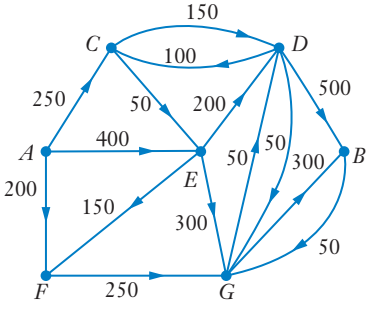
b Redraw the network showing the flow along each edge in order to achieve the maximum flow.

c One road is to be widened to increase its capacity. Which road should it be and by how much will it increase the traffic flow between the 2 suburbs? Give reasons for your answer.

8 The network shows the flow along each edge that would achieve the maximum flow from the source at A to the sink at C. Find the values of m, n, p, q and r.



9 The network diagram shows the flow capacities of traffic (in cars per hour) along roads between points A and B in a city.



a Find the maximum flow from A to B and redraw the network diagram showing the flow along each road to obtain the maximum flow.

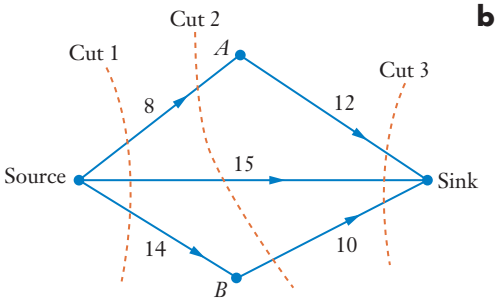
b One road is to be widened to increase traffic flow between A and B. Which road should be chosen in order to achieve the largest increase in traffic flow? Give reasons.

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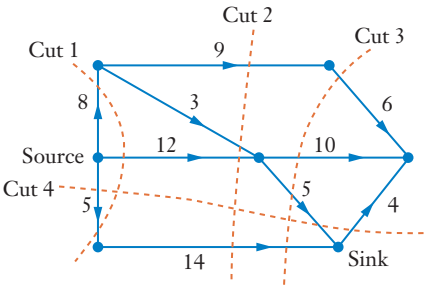
Example 25

10 Find the value of each cut in each network diagram.

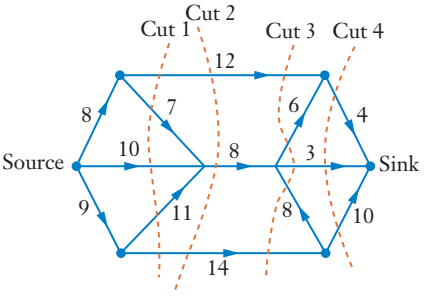
a



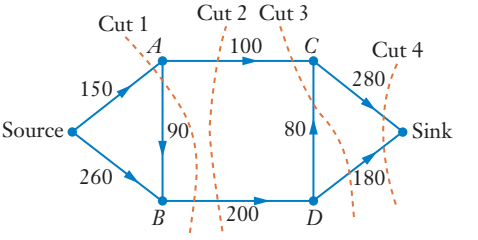
b



c



d



11 The network shows a cut that separates the source at S from the sink at T.

a Which of the following is the capacity of this cut? Select **A**, **B**, **C** or **D**.

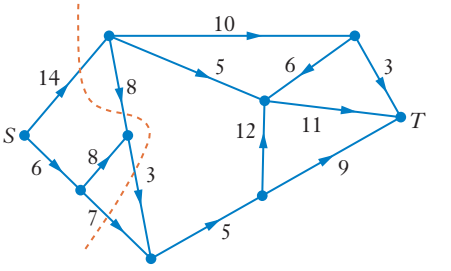
A 24 **B** 30

C 32 **D** 40

b What is the maximum flow from S to T?

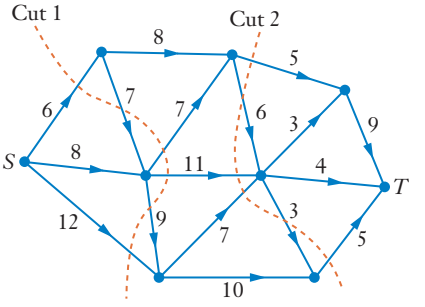
A 23 **B** 20

C 19 **D** 18

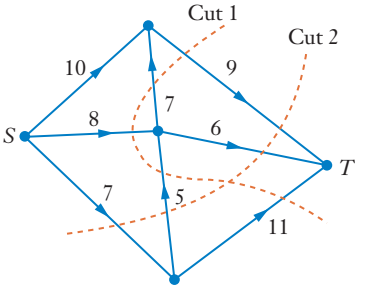


12 Find the capacity of each cut.

a



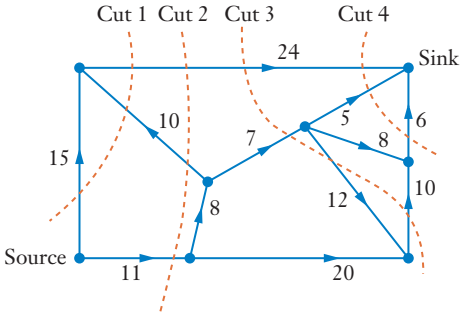
b



13 **a** In the network shown, which of the cuts is not a valid cut?

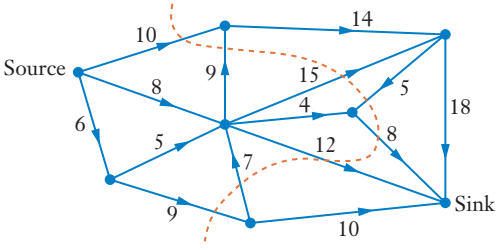
Explain why.

b Find the capacities of the other cuts.



14 Calculate the capacity of the cut shown.
Select **A**, **B**, **C** or **D**.

A 75 **B** 70
C 63 **D** 61



6.08 The 'maximum-flow minimum-cut' theorem

Any cut through a flow network blocks or contains the flow through the network, so maximum flow through a network cannot be larger than the capacity of any cut.

The 'maximum-flow minimum-cut' theorem

The **maximum flow** of a network is the **capacity of the minimum cut**.

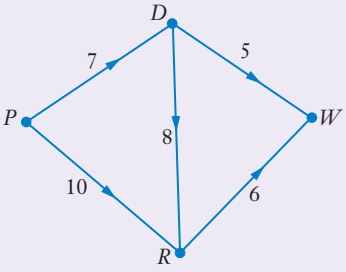
To find the maximum flow, calculate the capacity of each cut and identify the cut that has the minimum capacity.

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EXAMPLE 26

a Find the maximum flow for this network from the source P to the sink W.

b Show that the maximum flow is the same as the cut of minimum value.

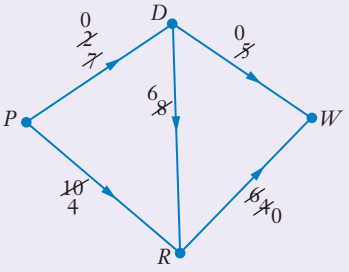


Solution

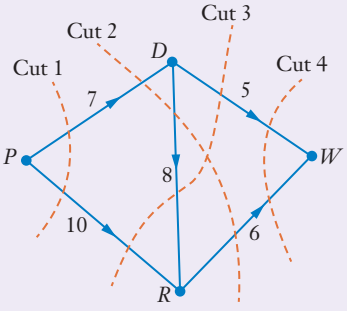
a The flows which give a maximum are:

PDW	5
$PDRW$	2
PRW	4
Total	11

The maximum flow from P to W is 11.



b The cuts that can be made to the network are shown.



The capacities of the cuts are:

cut 1 = 7 + 10 = 17

cut 2 = 7 + 6 = 13

DR is not included as it is cut in the opposite direction to the cuts *PD* and *RW*.

cut 3 = 5 + 8 + 10 = 23

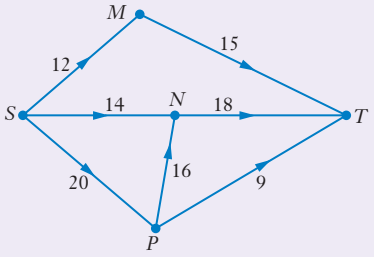
cut 4 = 5 + 6 = 11

The minimum cut is 11, which is the same as the maximum flow.

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EXAMPLE 27

This graph shows the amount of water that flows from a main dam (S) to 3 other dams which then supply a town with water (the sink, T). The capacities are given in thousands of litres per minute.



a Find the capacity of the minimum cut.

b Show that the maximum flow is equal to the minimum cut.

c What is the outflow of M?

d What is the excess flow capacity of MT?

e If the capacity of one edge is to be increased, determine which one. Give reasons.

Solution

a Cut 1 = 12 + 14 + 20 = 46

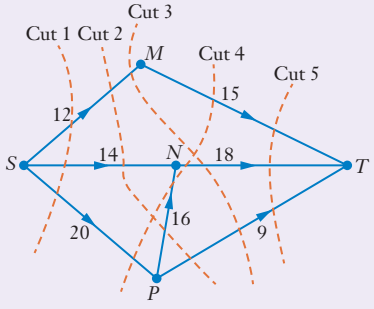
Cut 2 = 12 + 14 + 16 + 9 = 51

Cut 3 = 12 + 18 + 9 = 39

Cut 4 = 15 + 18 + 20 = 53 PN is not included.

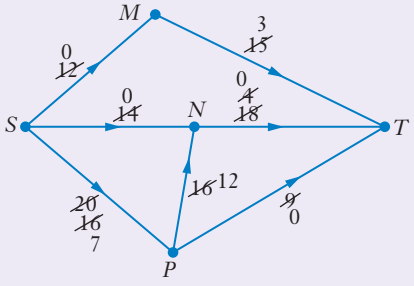
Cut 5 = 15 + 18 + 9 = 42

The minimum cut = 39



b The flows that give a maximum are:

SMT	12
SNT	14
$SPNT$	4
SPT	9
Total	39



So, the maximum flow = minimum cut = 39 000 L/min

c Possible outflow at M = 15 but inflow at M = 12 (which is less), so outflow at M = 12.

$\therefore$ Outflow of M is 12 000 L/min

d Excess flow capacity of MT is 15 000 – 12 000 = 3000 L/min.

e Maximum flow = 39 000 but outflow at S = 12 000 + 14 000 + 20 000 = 46 000 from original diagram. 46 000 – 39 000 = 7000, so the capacity of either PT or NT could be increased by 7000 L/min, but not MT because SM = 12 so increasing MT's capacity would not make a difference.

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EXAMPLE 28

a Convert the information in the table into a network diagram.

b Find the capacity of the minimum cut to determine the flow capacity of the network from A to E.

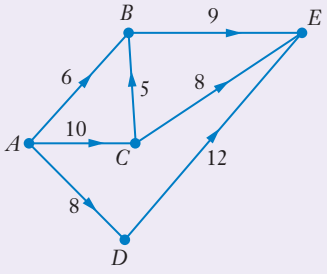
c Redraw the network showing the flow along each edge in order to achieve the maximum flow.

d Is the flow sufficient to meet demand?

From	To	Capacity
A	B	6
A	C	10
A	D	8
B	E	9
C	B	5
C	E	8
D	E	12

Solution

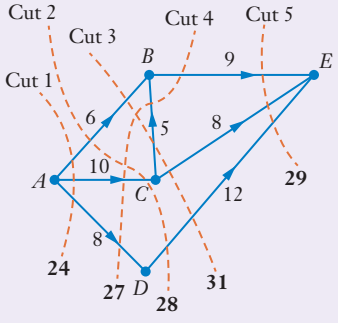
a A network diagram of the information is shown.



b The cuts and their capacities are shown.

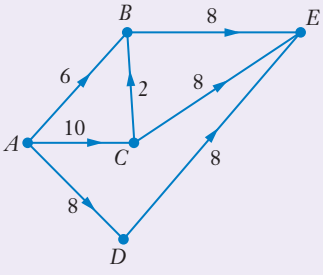
Cut 1 is the minimum cut and has a flow capacity of 24.

So the maximum flow through the network is 24.



c The flows that give a maximum flow are:

ABE	6
ACE	8
ADE	8
$ACBE$	2
Total	24



The flow along for each edge is shown (by subtracting unused capacities from edges: see Example **24b** on page 269).

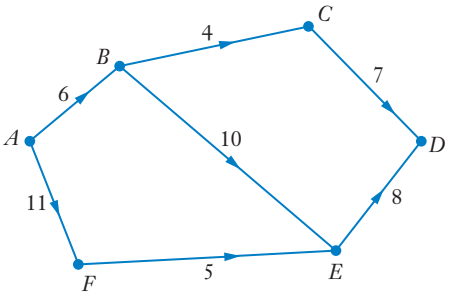
d The inflow at E could have a capacity of $9 + 8 + 12 = 29$ (see part **a**). Since the maximum flow is only 24, it is not sufficient to meet demand.

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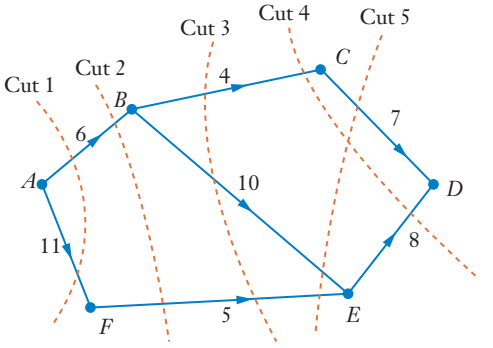
Exercise 6.08 The 'maximum-flow minimum-cut' theorem

Example 26

1 a Find the maximum flow for this network, from the source A to the sink D.

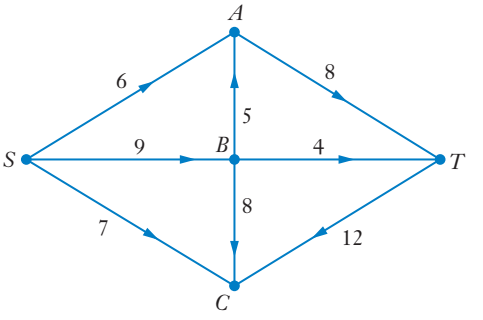


b Find the capacities of the cuts shown to determine the cut of minimum value.



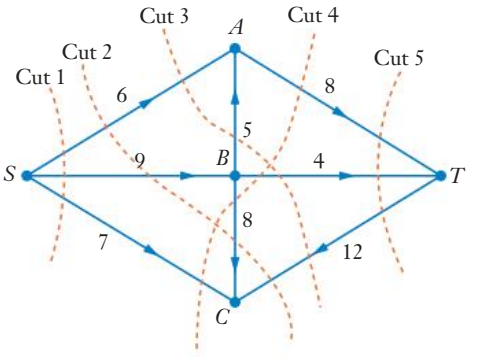
c Verify the result 'maximum flow = minimum cut' for this network.

2 a Find the maximum flow for this network, from the source at S to the sink at T.



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b Find the capacities of the cuts shown to determine the cut of minimum value.



c Which cut has a capacity equal to the maximum flow?

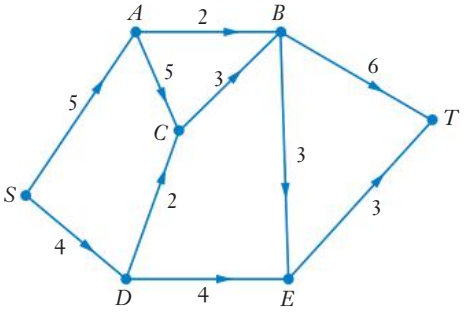
3 For each network diagram:

i find the maximum flow of the network from the source at S to the sink at T

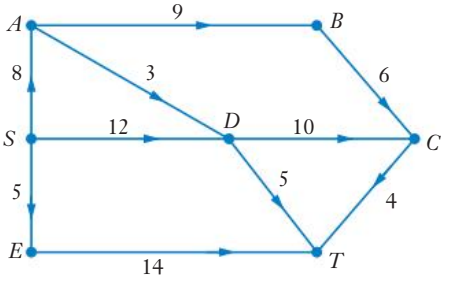
ii find the cut of minimum value and show that the maximum flow is equal to the cut of minimum value.

WS
Maximum-flow minimum-cut

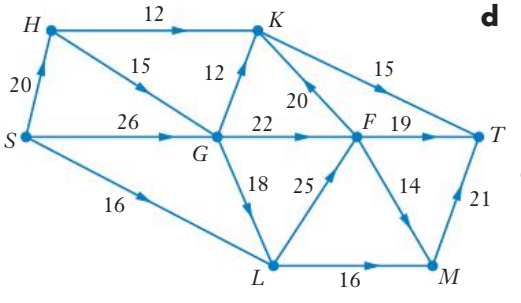
a



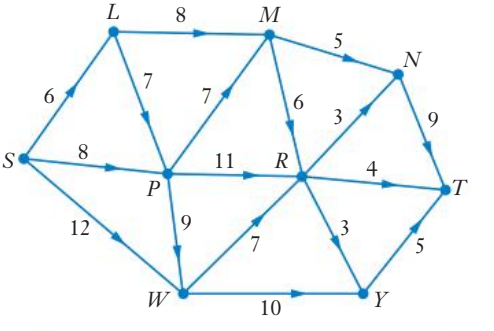
b



c



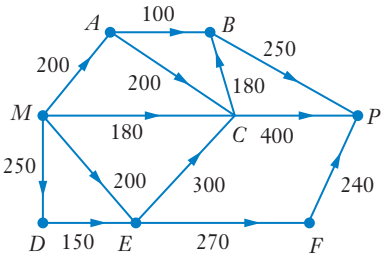
d



The worksheet 'Maximum-flow minimum-cut' contains the diagrams from Questions 3 to 7.

Example 27

4 The network graph shows the traffic flow from one city suburb, M, to another suburb, P, where the capacities are the maximum number of cars per hour.



a Find the capacity of the minimum cut.

b Show that the maximum flow is equal to the minimum cut.

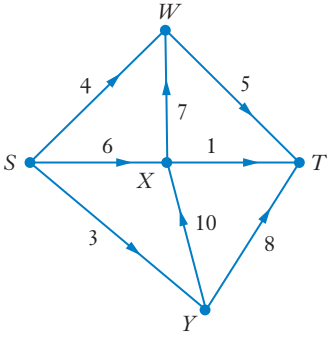
c What is the outflow of E?

d What is the excess flow capacity of EF?

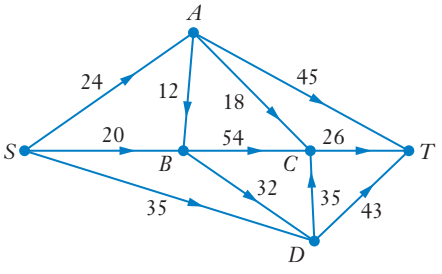
e If the capacity of one edge is to be increased, determine which one. Give reasons.

5 Determine the maximum flow from S to T of each network by drawing the line of minimum cut and finding its capacity.

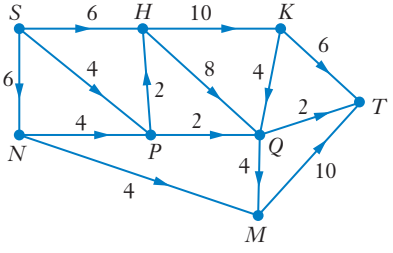
a



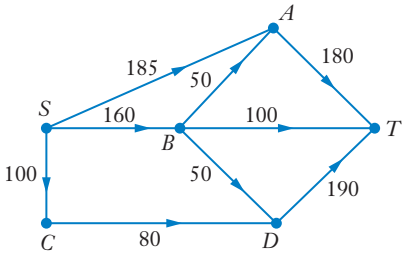
b



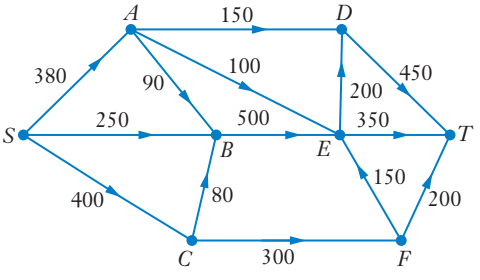
c



d



6 This diagram shows a network of pipes from an oil storage tank to a terminal from where it can be transported to other locations. The capacities are in L/min.



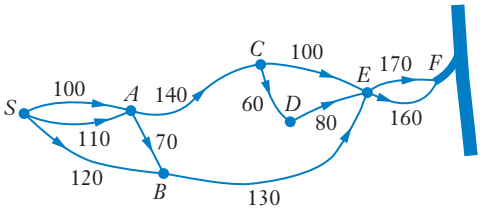
a Determine the maximum flow by finding the capacity of the minimum cut.

b Redraw the network diagram showing the flow along each pipe to obtain the maximum flow.

c The capacity of one pipe is to be increased to improve the maximum flow. Which pipe should be chosen? Give reasons.

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7 This diagram represents roads leaving an outer city suburb (S), passing through several intersections, and connecting to a freeway (F). The numbers on the edges represent the maximum number of vehicles per hour that can travel along that road.





iStockphoto.com/Peter_Virag

a Determine the maximum flow of the road network by finding the capacity of the minimum cut.

b The flow capacity from the suburb to the freeway is presently not maximised. Suggest a way that the traffic flow could be maximised with an additional road and increasing the capacity of one road. Justify your answer with a diagram and calculations.

8 The flow capacities of 2 networks are tabulated below. For each table:

Example 28

a

From	To	Capacity
A	B	90
A	C	60
B	D	70
C	D	50
C	E	80
D	E	40

b

From	To	Capacity
A	B	18
A	C	17
A	D	4
B	D	5
B	E	8
B	F	12
C	D	6
C	F	10
D	F	15
E	F	16

i convert the information presented into network diagrams, showing the direction and capacity of each edge

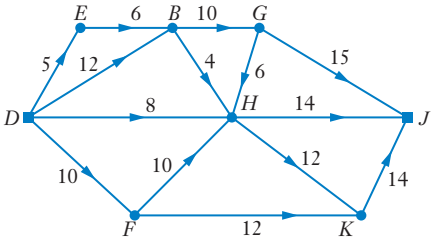
ii find the capacity of the minimum cut to determine the network's flow capacity

iii redraw each network showing the flow along each edge in order to achieve the maximum flow

iv determine if the flow is sufficient to meet demand.

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9 The main roads linking 2 city suburbs D and J are shown in the network diagram. The capacities are given as hundreds of vehicles per hour.

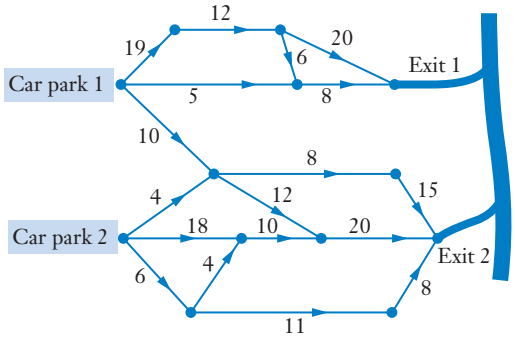


a Use the 'maximum-flow minimum-cut' theorem to calculate the maximum number of vehicles that could travel from D to J.

b One road is to be widened to increase traffic flow. Which road should be chosen? Give reasons for your answer.

c A recommendation to the traffic authorities is to change the direction of traffic flow of road GH as well as the road widening. How will both changes affect the number of vehicles travelling from D to J?

10 A large outdoor concert venue has 2 car parks and a network of dirt roads lead to 2 exits onto a main road. The arrows show the direction that the cars can travel along each road with the capacities on each road given in cars per minute.



a What is the maximum rate that traffic can leave:

i car park 1?

ii car park 2?

b What is the maximum possible outflow of cars from the concert venue?

c How could the owners increase traffic flow leaving the venue if the outflow at the car parks cannot be increased? Give reasons for your answer.

SAMPLE HSC PROBLEM

The Robinson family is going to a national park and there are 5 towns A, B, C, D and E that they can pass through on the way.

a Use this table to draw a network diagram. A dash (–) in the table means there is no direct road between the two towns.

Town	*A*	*B*	*C*	*D*	*E*	Destination
Start	50	70	60	–	–	–
A		15	–	65	–	–
B			25	55	40	–
C				–	55	–
D					20	60
E						85

b Find a minimum spanning tree for the network.

c Determine the shortest path from the start to the destination.

d Is the shortest path contained in the minimum spanning tree?

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Study tip

10 tips for tackling exams

1. Be prepared.
2. Find out about the format of the exam: the topics to be tested, time allowed, number and format of questions, number of marks per question, whether formulas will be supplied.
3. Use the reading time to browse through the exam to see the work that is ahead of you. Note the harder questions—you may need to spend more time on them.
4. Attempt every question.
5. Spend the first minute of each question planning and thinking before you start writing.
6. Write clearly, draw big diagrams, spread out your working and set it out neatly. Write down the page, not across.
7. Don't spend too much time on one question. Keep an eye on the time.
8. Make sure you have answered the question. Did you remember to round the answer and/or include units? Does the answer sound reasonable? Did you use all of the relevant information given?
9. If your working-out to a hard question is taking too long, then it's probably wrong. Don't get bogged down. If you're getting nowhere, retrace your steps, start again, or skip the question and return to it later with a clearer mind.
10. Once you have completed the exam, go over it again. Double-check your answers, especially the harder ones or those of which you are unsure.

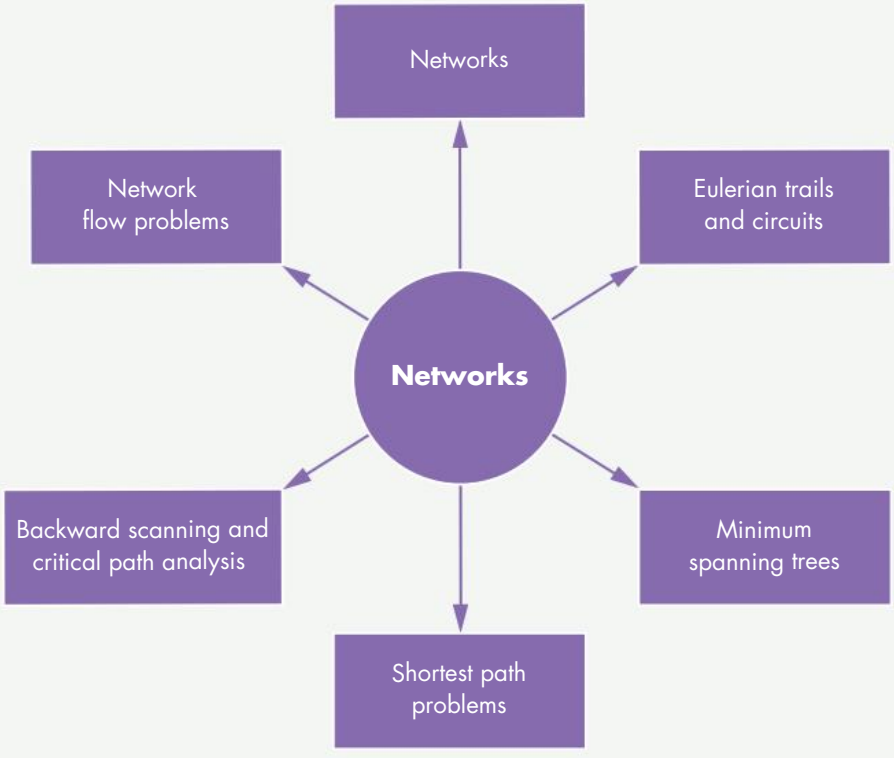
ISBN 9780170413633

6. CHAPTER SUMMARY

WS
Networks summary cards

This chapter, Networks, looked at ways of modelling situations using graphs called networks. You became familiar with network terminology such as vertex, edge, tree, weight, degree, walk, trail, path, circuit and cycle. Kruskal's and Prim's algorithms were used to find minimum spanning trees and shortest path problems were solved. You used critical path analysis to schedule activities to complete a project in the shortest possible time. Finally, you examined and solved network flow problems, using the maximum-flow minimum-cut theorem.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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6. TEST YOURSELF

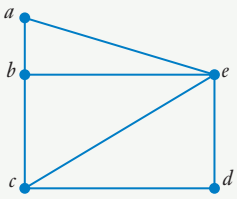
1 For each network:

Exercise 6.01

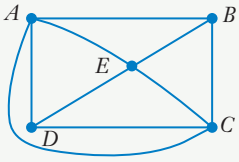
i find the degree of each vertex

ii show that the sum of the degrees of the vertices is equal to twice the number of edges.

a



b



2 State whether each walk is a path or a trail.

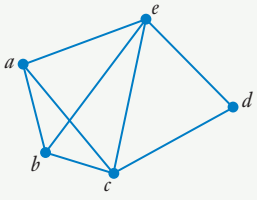
Exercise 6.01

a *abcde*

b *abcaedceb*

c *dcbaecab*

d *baedceb*



3 Which of the walks in Question **2** are:

Exercise 6.01

a closed? **b** circuits? **c** cycles?

4 Represent the information in the table as a weighted network.

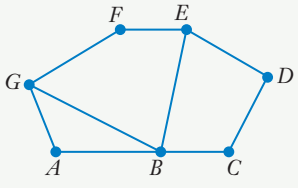
Exercise 6.01

	A	*B*	*C*	*D*	*E*	*F*
A	–	7	6	8	–	–
B	7	–	5	2	–	–
C	6	5	–	4	–	–
D	8	2	4	–	3	5
E	–	–	–	3	–	4
F	–	–	–	5	4	–

5 **a** For the network shown, write down a walk that is an Eulerian trail.

Exercise 6.02

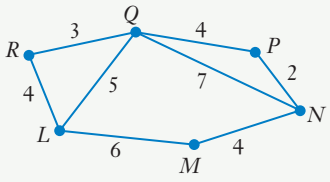
b Does this network have an Eulerian circuit?
Give reasons for your answer.



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Exercise 6.02

6 This network represents a system of roads with distances given in kilometres.



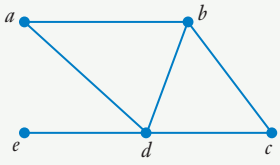
a Stephanie starts from L and rides her bike along every road in the network only once.

At which intersection did she finish?

b Stephanie now has to start and finish at R and ride along each road at least once. What is the shortest distance the cyclist will have to ride?

Exercise 6.03

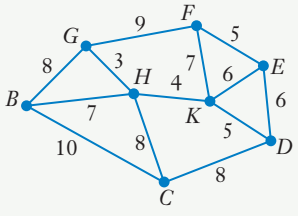
7 Draw all the spanning trees for the network below.



Exercise 6.03

8 Use Kruskal's algorithm to find a minimum spanning tree for the network below.

Show the order in which the edges for the spanning tree are selected and calculate its weight.



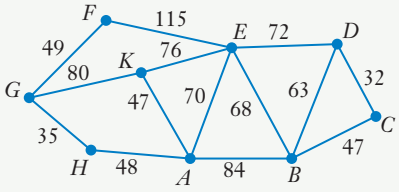
Exercise 6.03

9 Use Prim's algorithm to find the minimum spanning tree for the network in Question **8**, showing the order in which you choose the edges.

Exercise 6.03

10 **a** Use Prim's algorithm to find the minimum spanning tree, showing the order in which you choose the edges.

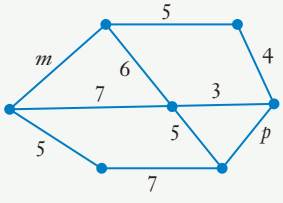
b Can you find more than one minimum spanning tree for this network?



Exercise 6.03

11 The minimum spanning tree for this network has a length of 23.

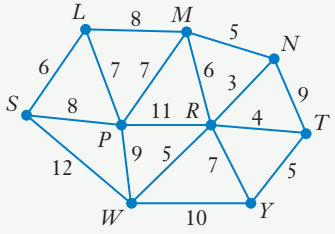
What are the values of m and p?



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12 The network diagram shows the distances between locations in a camping ground.

The distances are given in hundreds of metres.

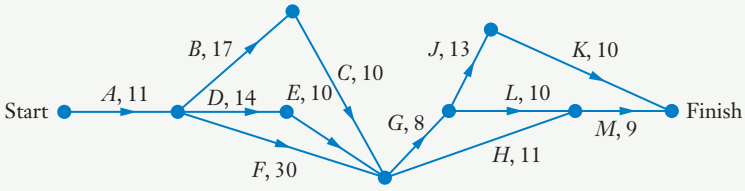


a Draw the minimum spanning tree for the network and find its length.

b Find the shortest path from S to T. Is this path contained in the minimum spanning tree?

Exercise 6.04

13 A local council has organised a project to complete maintenance on the facilities in the parks and reserves. This network diagram shows the activities and their times in days.

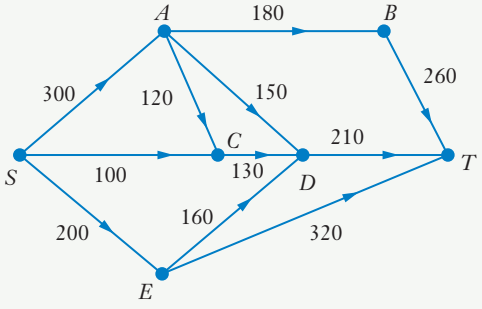


a Find the critical path by forward and backward scanning and state the minimum time to complete the project.

b Find the float times for the non-critical activities.

Exercises 6.05, 6.06

14 **a** Determine the maximum flow possible from the source at S to the sink at T by using the inflow and outflow for the vertices along different paths.



b Redraw the network showing the flow along each edge to achieve the maximum flow.

c Copy the network diagram from part **a** and by taking cuts, find the capacity of the minimum cut. Verify that this value is the same as the maximum flow.

d Is it possible to increase the maximum flow for this network? Give reasons.

Exercises 6.07, 6.08

Qz Chapter quiz

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WS
Mathematics Standard 2 Reference sheet

Practice paper 2

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Exercise 4.01

Question 1

To build a house, Ben and Anita borrow $800 000 at 5.25% p.a. over 30 years, with monthly repayments of $4417.63. What is the total amount of interest that they will pay?

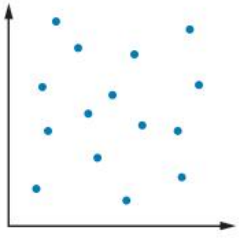
A $1 590 347 **B** $132 528.90 **C** $790 347 **D** $1 260 000

Exercise 5.02

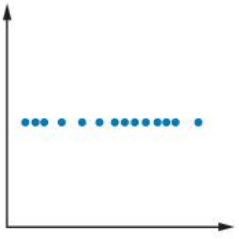
Question 2

Which scatterplot has a zero correlation coefficient?

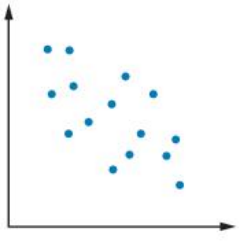
A



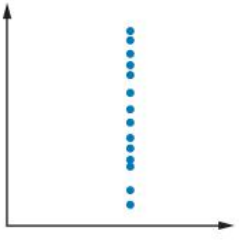
B



C



D



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Exercise 5.02

Question 3

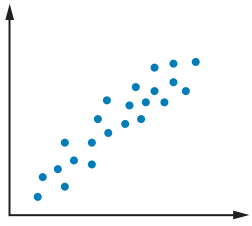
Describe the correlation between the variables shown on the scatterplot.

A Weak negative correlation

B Strong positive correlation

C Strong negative correlation

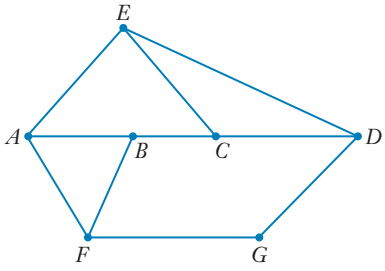
D Weak positive correlation



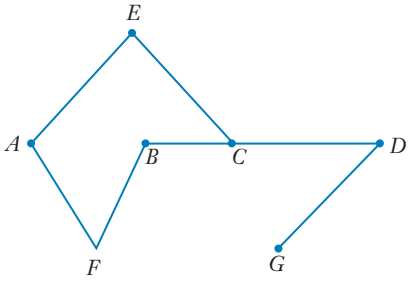
Exercise 6.03

Question 4

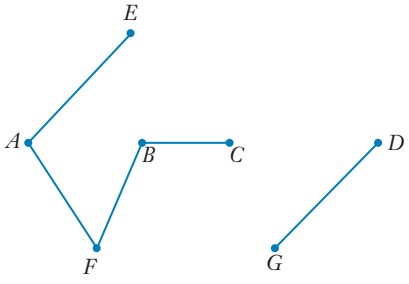
Which of the following is a spanning tree of this network?



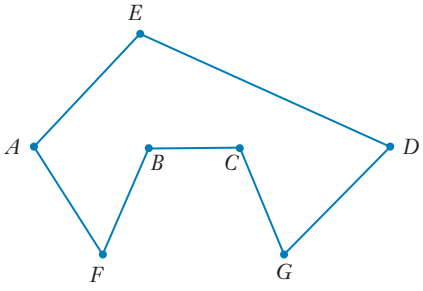
A



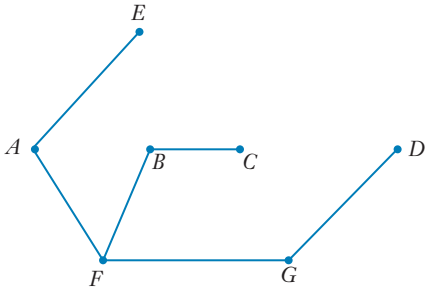
B



C



D



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Exercise 4.03

Question 5

This table shows the future value of an annuity with a contribution of $1 at the end of each period.

Period	Interest rate per period			
	3%	5%	8%	12%
5	5.3091	5.5256	5.8666	6.3528
8	8.8923	9.5491	10.6366	12.2997
10	11.4639	12.5779	14.4866	17.5487
12	14.1920	15.9171	18.9771	24.1331

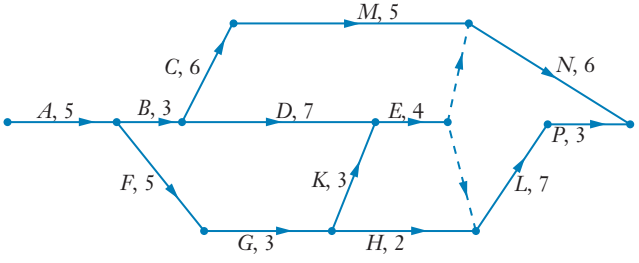
Use the table to calculate the future value of an annuity with a contribution of $8000 per year at 5% p.a. for 8 years.

A $95 491 **B** $76 392.80 **C** $46 932.80 **D** $112 000

Exercise 6.05

Question 6

The network diagram shows activities that must be completed in a project. The completion time for each activity is in hours.



Activities which are included for the critical path for this project are:

A *M* and *N* **B** *F* and *H* **C** *E* and *G* **D** *D* and *L*

Exercise 5.05

Question 7

Which of the following is the best description when comparing life expectancy and living standards?

A No correlation **B** Negative correlation

C Positive correlation **D** Constant correlation

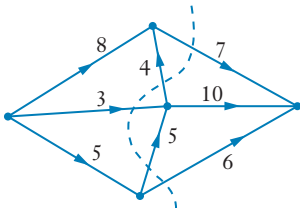
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Question 8

Exercise 6.06

What is the capacity of the cut shown in the network diagram?

A 25
B 21
C 20
D 19



Question 9

Exercise 4.02

Rafael is charged interest at a rate of 20.24% p.a. compounded daily on his credit card purchases of $550. How much interest does he pay after 26 days?

A $7.98 **B** $7.92 **C** $557.98 **D** $557.92

Question 10

Exercise 4.04

The table shows fortnightly repayments for personal loans over 5 years at a fixed interest rate.

Interest rate (% p.a.)	Loan amounts				
	$5000	$10 000	$20 000	$30 000	$50 000
4.5	$43	$86	$172	$258	$430
5	$45	$89	$178	$268	$446
8	$47	$94	$187	$281	$468
12	$51	$103	$205	$308	$513
13.5	$53	$106	$212	$319	$531

Talita borrows $30 000 over 5 years at 12% p.a. with fortnightly repayments.

How much would she repay over 5 years?

A $308 **B** $1540 **C** $36 960 **D** $40 040

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Section 2

3 questions: 12 marks each

Exercise 4.01

Question 11 (12 marks)

a Amanda borrowed $70 000 for building an extension to her house. She was charged 11% p.a. reducible interest and monthly payments of $840.

Month (n)	Principal (P)	Interest (I)	Amount owing ($P + I$)	Balance ($P + I - R$)
1	$70 000	$641.67	$70 641.67	$69 801.67
2	$69 801.67	$639.85	$70 441.52	$69 601.52
3	$69 601.52	$638.01	$70 239.53	$69 399.53
4	$69 399.53	$636.16	$70 035.69	$69 195.69
5	$69 195.69	$634.29	$69 829.98	$68 989.98
6				

i Show how the interest of $636.16 for the 4th month is calculated. **1 mark**

ii How much of the loan is left after the 4th month? **1 mark**

iii What is the total interest paid in the first 4 months? **1 mark**

iv Complete the table to determine how much will be owing at the end of the 6th month. **1 mark**

Exercise 5.03

b This table shows the age (A years) and life expectancy (E years) of a sample of 8 executives.

Age (A years)	20	25	30	20	45	35	25	30
Life expectancy (E years)	48	54	43	52	18	25	44	29

i Draw a scatterplot of this data. **1 mark**

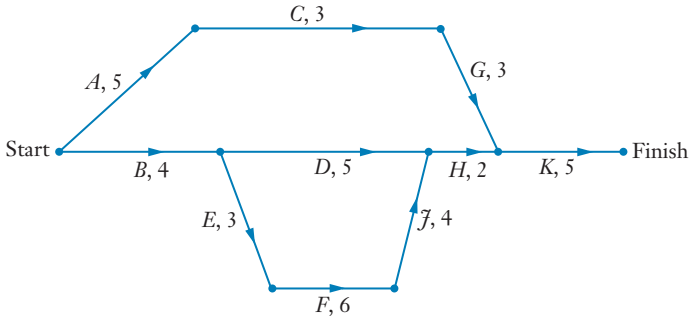
ii On your scatterplot, draw a line of best fit by eye. **1 mark**

iii Use your line of best fit to estimate the age of an executive with a life expectancy of 20 years. **1 mark**

iv Find the equation of your line of best fit. **1 mark**

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c Ten activities have been identified for renovating the canteen and changing rooms used by sporting clubs at a local sports ground. The network diagram shows the activities and the time needed to complete them in weeks.



i By finding the critical path, determine the minimum time to complete the renovations. **1 mark**

ii Determine the float time for activity *C*. **1 mark**

iii The contractors doing the renovations are able to speed up the completion of the project. Activities *A*, *B*, *D*, *F* and *G* can be reduced in time.

Which of these activities would **not** result in an earlier completion of the project? **1 mark**

iv The reduction in time of which activity would result in the earliest completion of the project. Give reasons. **1 mark**

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Question 12 (12 marks)

Exercise 6.05

a A company must complete 10 activities before the installation of new equipment can begin. These activities and their predecessors are given in the table below.

Activity	Duration	Predecessor
A	3	–
B	2	–
C	2	–
D	3	*A*
E	5	*B, D*
F	3	*C*
G	2	*E, F*
H	3	*G*
J	10	*A*
K	2	*H, J*

i Draw a network diagram for the activity table, showing the earliest starting times and the latest starting times for each activity. **3 marks**

ii What is the minimum time before the company can begin installing the new equipment? **1 mark**

Exercise 4.02

b Kaylene has a credit card account with no interest-free period and an interest rate of 20.24% p.a. compounded daily. Interest is calculated from the date of purchase up to and including the date of payment.

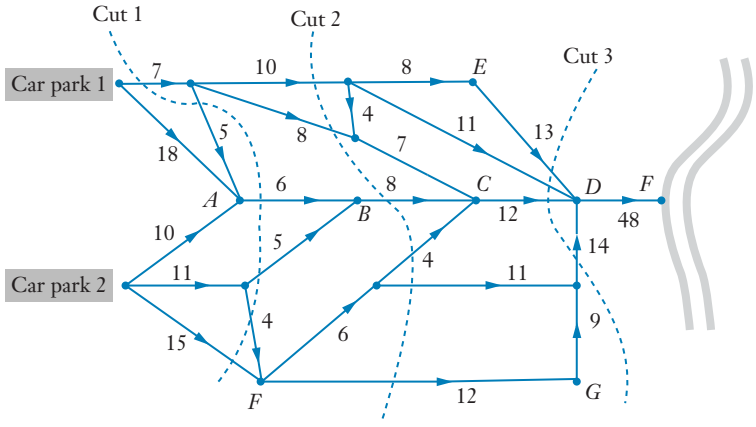
She made the following purchases for the period 1 January to 31 January.

8 Jan	Popcorn maker	$119
15 Jan	Petrol	$84.56
24 Jan	Phone bill	$70.50

If Kaylene pays her account in full on 5 February, how much does she pay? **3 marks**

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c The diagram shows the traffic flow (in cars per minute) from two car parks along a network of roads leading to a main road.



i By finding the value of the cuts shown, determine the maximum flow of cars onto the main road. **2 marks**

ii What is the outflow of cars from the car parks? **1 mark**

iii Suggest one way in which traffic flow from the car parks to the main road can be maximised. **2 marks**

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Question 13 (12 marks)

Exercise 4.03

a Kasia invested $8000 every 6 months in an account earning 8% p.a. compounded half-yearly.

i Use the future value table below to find how much Kasia will have at the end of 5 years (correct to the nearest cent). **2 marks**

Future value interest factors
(Future value of an annuity with a contribution of $1 at the end of each period)

Period	Interest rate per period								
	1%	2%	3%	4%	5%	6%	8%	10%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0800	2.1000	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2464	3.3100	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.5061	4.6410	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.8666	6.1051	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.3359	7.7156	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.9228	9.4872	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.6366	11.4356	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	12.4876	13.5795	14.7757
10	10.4622	10.9494	11.4639	12.0061	12.5779	13.1808	14.4866	15.9374	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	16.6455	18.5312	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	18.9771	21.3843	24.1331

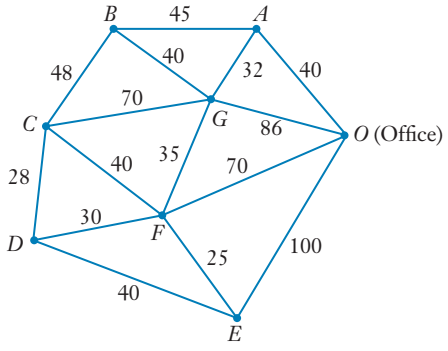
ii Use the formula $FV = PV(1 + r)^n$ to find the single sum invested now that would produce the same future value after 5 years at the same interest rate (correct to the nearest dollar). **2 marks**

iii Kasia decided that she needed $120 000 after 3 years. What amount should she invest every 3 months into an account earning 4% p.a. compounded quarterly to reach this amount (correct to the nearest dollar)? **1 mark**

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Exercise 6.04

b The diagram below shows the distances between the office and cabins in a coastal holiday resort. Pathways are to be laid between the cabins and the office.



i Find the shortest path from the office (O) to every cabin in the resort. **1 mark**

ii Use the shortest paths from part **i** to obtain a spanning tree and find its weight. **1 mark**

iii Use Kruskal's or Prim's algorithm to find a minimum spanning tree for the resort and find its weight. **1 mark**

iv If the cost of the pathways is \$140/metre, what is the lowest cost of laying the pathways? **1 mark**

Exercise 5.04

c 8 basketball players had their height and arm span measured.

Height (x cm)	175	184	200	183	172	187	190	178
Arm span (y cm)	170	185	195	182	174	188	190	177

i Calculate, correct to 4 decimal places, Pearson's correlation coefficient, r, for this data. **1 mark**

ii Find the equation of the least-squares regression line for this data. **2 marks**

END OF EXAMINATION

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MEASUREMENT

7.

ENERGY AND SUSTAINABILITY

To truly transform our economy, protect our security, and save our planet from the ravages of climate change, we need to ultimately make clean, renewable energy the profitable kind of energy.

Barack Obama, US President, 2009–2017

Sustainability is about being able to live within the limits of what the environment can provide, without impacting on and destroying our natural environment. Gradually, we are changing our mindset and lifestyle to use less energy and consider how we can create energy without polluting our world and wasting resources. The greatest energy expense in the home is heating and cooling. Did you know that an extra 1°C difference between the outside and inside temperature can add about 10% to room heating and cooling costs? Also, recycling one glass container saves enough energy to power a 100 W light globe for 4 hours or a fluorescent globe for 20 hours.

CHAPTER OUTLINE

M7	7.01	Heart rates
M1.3*	7.02	Food and energy consumption
M7	7.03	Electricity usage in the home
M7	7.04	Energy consumption and the costs of appliances
M7	7.05	Energy-efficient housing

*Year 11 revision



IN THIS CHAPTER YOU WILL:

- calculate heart rates and target heart rates
- relate units of energy to food and nutrition
- calculate electricity usage costs
- compare the energy consumption and running costs of household appliances
- investigate the requirements and features of an energy-efficient home.

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TERMINOLOGY

calorie
domestic electricity
exercise intensity
joule (J)
kilowatt-hour (kWh)
megajoule (MJ)
resting heart rate (RHR)
target heart rate (THR)
capacity
energy-efficient
gigawatt (GW)
kilojoule (kJ)
LED light
megawatt (MW)
running cost
watt (W)
consumption
energy rating label
insulation
kilowatt (kW)
maximum heart rate (MHR)
off-peak electricity
standby power



Assignment 7

SkillCheck

1 **a** During exercise an athlete has a heart rate of 55 beats per minute. Convert this rate to beats per second.

b How many times would this athlete's heart beat in half an hour?

2 The cost of 60 L of diesel is $76.20. Express this cost as a rate in cents per litre.

3 Convert these units.

a 9.2 cm to mm
b 8600 mm to metres
c 42 000 mL to litres
d 3160 L to kilolitres
e 7.5 m^3 to L
f 780 seconds to minutes

4 The quarterly electricity bill was $502.40, before **GST**. Calculate the electricity bill after 10% GST is added, correct to the nearest cent.

5 How many days from 8/04/19 to 7/07/19?

6 The natural gas account for total supply charges over the period 30/03/20 to 29/06/20 was $81.42. What was the average daily cost for the natural gas supply? Answer in cents per day.

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7.01 Heart rates

WS Heartbeat rates

WS I heart maths

To maintain your health, doing aerobic exercise increases the work of your heart and lungs, building stamina. As a result, your heart rate speeds up. To gain the maximum benefit from exercise without overworking your heart, there is a safe range for heartbeat while exercising.

EXAMPLE 1

Natalie takes her pulse during exercise and counts 30 beats in 20 seconds. Calculate her heart rate in beats per minute.

Solution

$$\text{Heart rate} = \frac{30 \text{ beats}}{20 \text{s}}$$

$$= 1.5 \text{ beats/s}$$

$$= 1.5 \times 60 \text{ beats/min} \qquad 1 \text{ minute} = 60 \text{ seconds}$$

$$= 90 \text{ beats/min}$$



Shutterstock.com/Sergey Nivens

A person's **maximum heart rate (MHR)** is the upper limit of what the cardiovascular system can handle during physical activity.

Maximum heart rate (MHR)

MHR for males = 220 – age in years

MHR for females = 226 – age in years

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EXAMPLE 2

Exercising at between 55% and 85% of your maximum heart rate for 20 to 30 minutes each day is recommended.

a Zoe is 25 years old. What is her maximum heart rate (MHR)?

b For what heart rate range should Zoe aim when exercising?

c Owen has an MHR of 189. How old is he?

d Calculate Owen's minimum recommended heart rate when exercising for 25 minutes.

Solution

a Zoe's MHR = 226 – age in years MHR for females

= 226 – 25

= 201 beats/minute

b Zoe's minimum heart rate = 55% × 201

= 110.55

≈ 111 beats/minute

Zoe's maximum heart rate = 85% × 201

= 170.85

≈ 171 beats/minute

Zoe should aim for a heart rate between 111 and 171 beats/minute.

c Owen's MHR = 220 – age in years MHR for males

189 = 220 – Owen's age

Owen's age = 220 – 189

= 31

d Owen's minimum heart rate = 55% × 189

= 103.95

≈ 104 beats/minute

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EXAMPLE 3

Gemma exercised at an intensity that made her pulse 147 beats/minute. How many times did her heart beat during her 45-minute workout?

Solution

Number of beats in 45 min $= 147 \times 45$ For beats, multiply by the rate

$= 6615$

Target heart rate



Target heart rates

Target heart rate (THR), or **training heart rate**, is the desired heart rate needed to reach the level of exertion necessary for cardiovascular (aerobic) fitness. It is specific to a person's age, gender and physical fitness. Cardiovascular fitness provides maximum health for the heart and lungs.

Exercise intensity measures how hard your body works during physical activity.

Intensity	Example
Low intensity exercise	A casual walk or easy pace bike ride
Moderate intensity exercise	Brisk walk, walk uphill or a strenuous yoga session
High intensity exercise	Circuit training, various forms of weight training, sprint in the pool

Target heart rate

$$\text{THR} = I \times (\text{MHR} - \text{RHR}) + \text{RHR}$$

where RHR = resting heart rate

MHR = maximum heart rate

I = exercise intensity as a decimal

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EXAMPLE 4

The table shows the exercise intensity (as a percentage) required during exercise for different outcomes.

Low intensity	Moderate intensity	High intensity
40–50%	50–70%	70–85%

James works out in the gym every day for cardiovascular fitness. He is 21 years old with a resting pulse of 72 beats/minute. What is his minimum and maximum target heart rate for moderate intensity exercise?

Solution

$\text{MHR} = 220 - 21$ Using the MHR formula for males

$= 199$ beats/minute

$\text{RHR} = 72$ beats/minute

$I = 0.5$ to 0.7 for moderate exercise

$\text{Minimum THR} = 0.5 \times (199 - 72) + 72$

$= 135.5$ beats/min

$\text{Maximum THR} = 0.7 \times (199 - 72) + 72$

$= 160.9$ beats/min

For moderate intensity exercise, James should aim for a target heart rate between 135.5 and 160.9 beats/minute.

Exercise 7.01 Heart rates

Example 1

1 When Jackie was training, she checked her pulse. She counted 41 beats per 20 seconds. Calculate Jackie's pulse in beats per minute.

2 David was exercising, checked his pulse and counted 28 beats in 10 seconds. How could David's pulse be calculated in beats/minute? Select **A**, **B**, **C** or **D**.

A $28 \times 10 \times 60$ **B** $28 \div 10 \times 60$ **C** $28 \div 10 \div 60$ **D** $28 \times 10 \div 60$

3 Calculate the pulse in beats/minute for these four participants in a fun run.

		Number of heart beats	Time in seconds	Pulse in beats/minute
a	Julia	66	30	
b	Rodney	30	10	
c	Amir	45	25	
d	Serena	39	15	

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4 Calculate the maximum heart rate (MHR) of each person.

Example 2

a Lauren, aged 52

b Harry, aged 34

c Steven, aged 18

5 Calculate the age of each person.

a Mark's maximum heart rate is 196 beats/minute.

b Charlotte's maximum heart rate is 221 beats/minute.

6 While we sleep, our heart rate is at its resting pulse. Adrianna sleeps for 7.5 hours each night.

a Calculate the number of minutes in 7.5 hours.

b If Adrianna is 17 years old and has a resting pulse of 56, how many times does Adrianna's heart beat while she is asleep?

7 Matt is 20 years old and his resting pulse rate is 74 beats/minute. How many times does his heart beat during a 45-minute nap?

8 Piyush exercised at a high intensity that made his pulse 147 beats/minute. How many times did his heart beat during his 20-minute workout?

Example 3

9 Raina likes to do an easy pace bike ride early in the morning. She is 45 years old with a resting pulse rate of 85 beats/minute. Using the table in Example **4** on the previous page, what is her target heart rate range for this low intensity exercise? Select **A**, **B**, **C** or **D**.

Example 4

A 157.2–175.5 **B** 152.2–166.6

C 133–152.2 **D** 123.4–133

10 Annabelle is undertaking high intensity exercise as part of her fitness regime. She is 16 years old and has a resting heart rate of 75 beats/min.

a Use the table in Example **4** on the previous page to calculate her minimum and maximum target heart rate for high intensity exercise.

b Annabelle's heart pumps 65 mL/beat. When she is resting, how many litres of blood does Annabelle's heart pump every minute?

c When she is undertaking high intensity exercise, Annabelle's pulse is 150 beats/minute and her heart pumps 24 L of blood every minute. How many millilitres of blood does Annabelle's heart pump with each beat when she is doing high intensity exercise?

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INVESTIGATION

HEART RATE

Experts advise that to exercise safely, our heart rate should be between 50% and 75% of our maximum heart rate (MHR). This is the safe range within which you should exercise without overworking your heart muscle.

Find an online heart rate calculator such as the one below from myDr.com.au and use it to calculate the maximum heart rate and target heart rate range for you and your family.

Your age: 17 years

CALCULATE

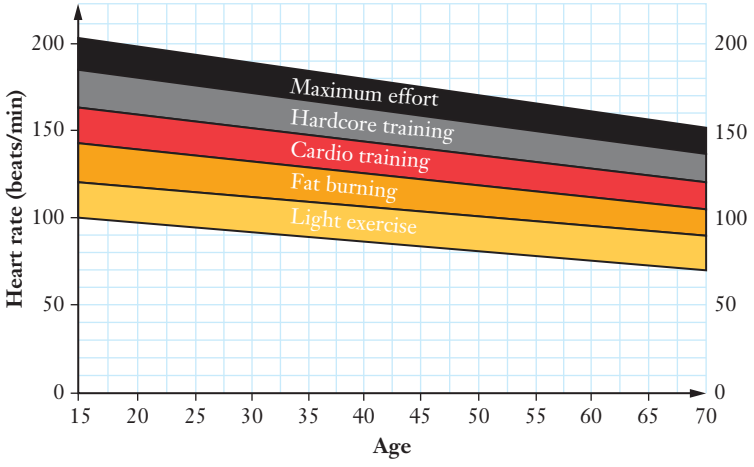
While you're exercising you should count between:

51 and **76** beats in 30 seconds

Your rate should not exceed: **102** beats

© myDr.com.au, 2001-2018.

This graph shows the range of heart rates for various ages and types of exercise.



a What is the heart rate range for a 20-year-old doing 'light activity'?

b How old is a person in an aerobics class with a heart rate between 110 and 125 beats/minute?

c Within what range of heart rates should a 17-year-old work?

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7.02 Food and energy consumption

Nutrition in food provides a source of energy to maintain life and also for growth. Good nutrition allows the body to receive the right balance of nutrients for the body to function and is important in fighting illness and disease.

Energy can be supplied to the body by taking in the right nutrients so the body can function effectively. The more active a person is, the more energy they need.

Energy intake is measured in **joules** (J) or **calories** (Cal). Calories are an older unit of energy. Sometimes on food packaging, you will see the word 'kilocalories'. When describing energy in food and nutrition, kilocalories and calories are the same thing.

Energy units

1 kilojoule (kJ) = 1000 joules (J)

1 megajoule (MJ) = 1 000 000 joules (J) = 1000 kJ

1 calorie (Cal) = 4.184 kJ

EXAMPLE 5

Convert:

a 150 J to kJ **b** 12 MJ to kJ **c** 76 Cal to kJ

d 1800 kJ to Cal **e** 0.14 MJ to Cal

Solution

a $150\text{ J} = 150 \div 1000$ $1\text{ kJ} = 1000\text{ J}$
$= 0.15\text{ kJ}$

b $12\text{ MJ} = 12 \times 1000$ $1\text{ MJ} = 1000\text{ kJ}$
$= 12\,000\text{ kJ}$

c $76\text{ Cal} = 76 \times 4.184$ $1\text{ Cal} = 4.184\text{ kJ}$
$= 317.984\text{ kJ}$

d $1800\text{ kJ} = 1800 \div 4.184$ $1\text{ Cal} = 4.184\text{ kJ}$
$= 430.210\,325$
$\approx 430.2\text{ Cal}$

e $0.14\text{ MJ} = 0.14 \times 1000$ $1\text{ MJ} = 1000\text{ kJ}$
$= 140\text{ kJ}$
$= 140 \div 4.184$ $1\text{ Cal} = 4.184\text{ kJ}$
$= 33.460\,803\,0\ldots$
$\approx 33.5\text{ Cal}$

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Exercise 7.02 Food and energy consumption

Example 5

1 Convert each unit of energy.

a 3170 kJ to J **b** 62 MJ to kJ **c** 840 Cal to kJ

d 1795 kJ to MJ **e** 350 kJ to Cal **f** 1.1 MJ to Cal

2 A 200 mL glass of apple juice contains 99 calories. How many kilojoules (kJ) is this?

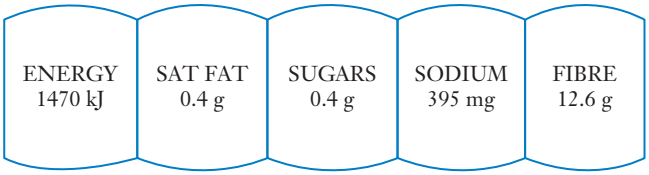
3 A box of six apricot muesli bars contains a total of 18.6 grams of fat.

a If 1 gram of fat = 170 kJ, calculate the total number of kilojoules in the box.

b How many kilojoules are there in one muesli bar?

c How many calories are there in one muesli bar?

4 Nikeeta bought a 1 kg box of breakfast cereal. The box contained this dietary intake information per 100 g.



PER 100 g

Calculate the total amount of energy in this box of breakfast cereal in:

a megajoules (MJ)

b calories.

5 This nutrition information panel is for a packet of soup.

Calculate:

a the number of kilojoules (kJ) per package represented by total fat (1 g of fat = 170 kJ)

b the number of calories in a packet containing 1044 grams in total.

Vegetable Soup



Nutrition information
Servings per package: 3
Serving size: 348 g

	Average Quantity per Serving	Average Quantity per 100 g
Energy	471 kJ (113 Cal)	135 kJ (32 Cal)
Protein	4.3 g	1.2 g
Fat, Total - Saturated	0.9 g 0.2 g	0.3 g less than 0.1 g
Carbohydrate - Sugars	18.6 g 11.3 g	5.3 g 3.2 g
Sodium	1090 mg	313 mg

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6 Jacinta bought her lunch, which consisted of the following items.

Item	Energy (in kJ)
Tuna avocado sushi roll	1272
Anzac biscuit	1000
Reduced fat milk	525

a Calculate the total energy intake for Jacinta's lunch.

b What percentage of the recommended daily intake (of 8700 kJ) does her lunch represent?

7 Daniel is making a chicken schnitzel wrap at home for his lunch. The energy content of each ingredient is shown in the table.

Food	Energy content (kJ)
1 single chicken breast	677
1 wrap	595
1 egg	300
1 tablespoon of skim milk	50
1 teaspoon of oil	185
1 cup of lettuce	33
1 tomato	97
1 cucumber	48
1 cup of breadcrumbs	1786.6
1 serve of mayonnaise	45

a Calculate the total amount of energy, in kilojoules (kJ), contained in Daniel's meal if he consumes the following:

Chicken schnitzel wraps

1 wrap	Half a cucumber
1 single chicken breast	1 serve of mayonnaise
1 egg	1 tablespoon of skim milk
Half a tomato	A quarter of a cup of breadcrumbs

b If Daniel is 55 years old, use the table below to determine what percentage of his daily energy intake this meal represents.

Estimated daily energy requirement (kilojoules) for a sedentary person (e.g. office work or study, no strenuous physical activity)		
Age (years)	**Male**	**Female**
12–15	800–9900	7400–800
16–18	10300–10900	8400–8500
19–30	10300*	8400^
31–50	9900*	8000^
51–70	9100*	7600^
Adults over 70	8300*	700^

Source: National Health and Medical Research Council
www.nutritionaustralia.org/national/resource/balancing-energy-and-out>
* estimated energy intake for male 180 cm tall, 71.3 kg
^ estimated energy intake for female 170 cm tall, 63.6 kg

8 This table shows the energy used by Alex, an 18-year-old male weighing 90 kg with a height of 186 cm, in a number of physical activities with a duration of 30 minutes.

Activity	Energy used (kJ)
Golf	834
Skateboarding	618
Basketball	741
Cycling (gentle)	741
Soccer	865
Swimming	1357
Skiing	865
Running/jogging	1551
Rock climbing	988
Touch football	988

a Name the 2 physical activities that would use the most energy.

b Calculate how long in minutes Alex would need to swim if he consumes:

i a chocolate muffin with an energy content of 2510 kJ

ii a Thai beef salad with an energy content of 1070 kJ.

c Lara is an 18-year-old female, 174 cm tall and weighing 65 kg. When she swims, she uses 1120 kJ per 30 minutes. Calculate how long it will take her to use the energy in a chocolate muffin when swimming, then do the same for a Thai beef salad, then compare those results with Alex's times.

ISBN 9780170413633

TECHNOLOGY

Compare mean daily energy of food groups for males and females

Data from the Australia Bureau of Statistics National Health Survey shows the mean daily energy content (in kJ) for both males and females, based on 4 different foods consumed by various age groups in Australia.

1 Use a spreadsheet to enter the following data, separately for males and females.

2 Draw scattergraphs with smooth lines and markers.

	Age group							
	2–3	4–8	9–13	14–18	19–30	31–50	51–70	71 and over
MALES	kJ							
Fruit products and dishes	508.1	457.7	390.3	254.1	299.0	376.1	392.6	482.9
Milk and dairy products	1211.5	1207.1	1243.3	1110.6	1086.0	905.7	872.6	937.6
Vegetable products and dishes	247.9	333.6	526.0	582.1	688.2	646.6	571.9	485.9
Snack foods (incl. chips)	93.4	305.0	298.4	258.0	194.7	145.6	72.6	41.1
	2–3	4–8	9–13	14–18	19–30	31–50	51–70	71 and over
FEMALES								
Fruit products and dishes	427.9	450	400.8	255.7	284.9	350.2	430.5	438.3
Milk and dairy products	1254.6	987.7	1161.2	1017.1	744.0	739.5	704.8	775.3
Vegetable products and dishes	268.4	321.1	432.5	615.0	643.2	562.7	496.1	479.9
Snack foods (incl. chips)	128.0	196.0	232.7	171.4	120.3	98.1	44.4	34.9

Source: Australian Bureau of Statistics

3 Use your graphs and analyse the data shown. For the 4 different food types consumed, describe any similarities/differences you can see between the male and female data regarding their consumption by referring to the energy content levels provided (in kJ).

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DID YOU KNOW?

Food labelling in Australia

In Australia there are strict rules regarding the labelling of food sold. Food labels enable consumers to be protected in regard to public health and safety, requiring the display of information such as use by dates, ingredients, allergens, instructions for storage and preparation, as well as advice and warning statements. Food Standards Australia determines what information must be shown on food labels.

Visit the Australian Government Food Standards website and choose the Consumer portal. Find information required regarding:

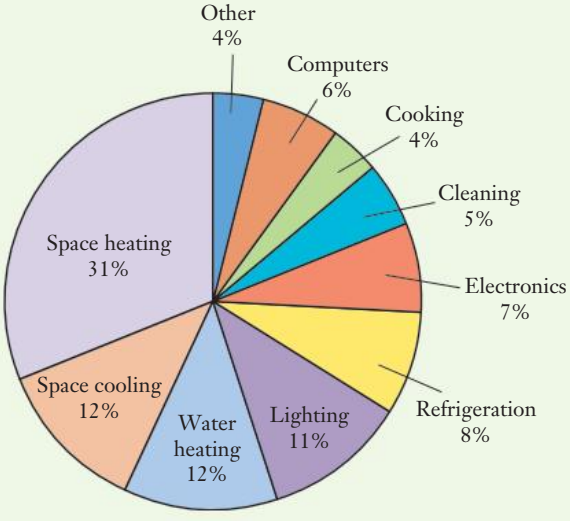
- **country of origin labelling**
- **ingredients lists and percentage labelling**
- **labelling of food additives**
- **nutrition information panels**
- **'Use by' and 'Best before' dates.**

WS
Power problems

7.03 Electricity usage in the home

EXAMPLE 6

This sector graph shows the typical residential electricity usage.



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a Calculate, to the nearest degree, the angle size of the 'Lighting' sector.

b What is typically the largest energy user in the home?

c If the cost of a household electricity bill was $860.75 for the April-June quarter, how much is typically spent on space heating?

d What could be included in 'Other'?

e If the cost of 'Other appliances' for one quarter is $65.20, what is the total electricity bill?

Solution

a Angle size of 'lighting' sector = $11\% \times 360° = 39.6° \approx 40°$

b Space heating

c Cost of space heating = $31\% \times \$860.75$

$= \$266.8325$

$\approx \$266.83$

d 'Other' may include iron, kettle, clothes dryer and other devices.

e Other appliances = $4\% = \$65.20$

$1\% = \$65.20 \div 4$ Using the unitary method

$= \$16.30$

Total electricity bill $(100\%) = \$16.30 \times 100$

$= \$1630$

Units of power

The **watt** (abbreviation **W**) is a unit of **power** and is equal to one joule of energy per second.

Unit	Relationships
microwatt (μW)	$1\ \mu W = \frac{1}{1000000}\ W = 10^{-6}\ W$
milliwatt (mW)	$1\ mW = \frac{1}{1000}\ W = 10^{-3}\ W$
watt (W)	$1\ W = 1000\ mW = 1\,000\,000\ \mu W$
kilowatt (kW)	$1\ kW = 1000\ W$
megawatt (MW)	$1\ MW = 1000\ kW = 1\,000\,000\ W$ or $10^6\ W$
gigawatt (GW)	$1\ GW = 1000\ MW = 1\,000\,000\,000\ W$ or $10^9\ W$

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EXAMPLE 7

Convert:

a 6540 W to kW **b** 82 000 MW to W **c** 165 200 GW to kW

Give answers in **scientific notation** where convenient.

Solution

a $6540\text{ W} = 6540 \div 1000\text{ kW}$ 1 kW = 1000 W

$= 6.54\text{ kW}$

b $82\,000\text{ MW} = 82\,000 \times 1\,000\,000\text{ W}$ 1 MW = 1 000 000 W

$= 82\,000\,000\,000\text{ W}$

$= 8.2 \times 10^{10}\text{ W}$

c $165\,200\text{ GW} = 165\,200 \times 1000\text{ MW}$ 1 GW = 1000 MW

$= 165\,200 \times 1000 \times 1000\text{ kW}$ 1 MW = 1000 kW

$= 165\,200\,000\,000\text{ kW}$

$= 1.652 \times 10^{11}\text{ kW}$

Units of energy usage

Electrical energy usage is measured in **watt-hours (Wh),** which is the amount of electrical energy used by a one-watt load (such as a light globe) drawing power for one hour.

The electricity usage for households is measured in **kilowatt-hours (kWh)**, which is 1000 watt-hours. The **cost** of electricity usage is given in **cents/kilowatt-hour** (c/kWh).

Domestic and off-peak electricity

Domestic rate electricity refers to electricity that is used during the day.

Off-peak rate electricity refers to electricity that is used during the late evening and early morning, such as between 11 p.m. and 7 a.m., when the demand from households and businesses is much lower. Off-peak electricity is charged at a cheaper rate than domestic electricity, and is often used to heat water in homes.

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EXAMPLE 8

Calculate the cost of electricity usage for a household where domestic usage is 954 kWh at \$0.583110/kWh and off-peak usage is 748 kWh at \$0.158620/kWh.

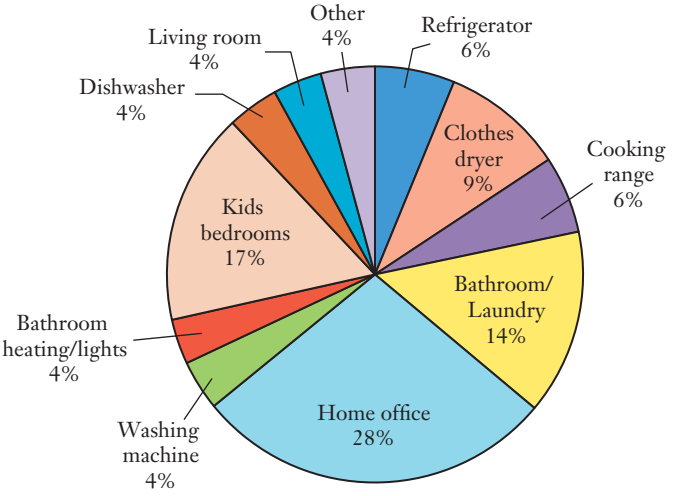
Solution

$$\begin{aligned}\text{Cost of domestic usage} &= 954 \times \$0.583\,110\\ &= \$556.28694\\ \text{Cost of off-peak usage} &= 748 \times \$0.158\,620\\ &= \$118.64776\\ \text{Total cost} &= \$556.28694 + \$118.64776\\ &= \$674.9347\\ &\approx \$674.93\end{aligned}$$

Exercise 7.03 Electricity usage in the home

Example 6

1 The sector graph shows the electricity usage for a household in the last 30 days.



a What fraction of energy use is for the refrigerator?

b What could be included in the 'Other' sector?

c If the household bill for 90 days is \$764.35, how much did 'Home office' cost?

d If dishwasher use costs \$0.67 per day, what would be the total electricity bill for 90 days?

e Calculate, to the nearest degree, the angle size of the 'Kids bedrooms' sector.

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2 The divided bar graph below shows the electricity use for an average household.



4% Cooking 7% Lighting 7% Fridges/freezers 16% Other appliances
3% Standby power 38% Heating and cooling 25% Water heating

a What fraction of electricity was used for water heating (hot water)?

b What percentage of electricity is used for cooking?

c If the average household uses 5000 kWh of electricity per year, how much electricity is for:

i lighting?

ii other appliances (which includes a swimming pool)?

Example 7

3 Express each measurement of power in scientific notation.

a 6 500 000 MW **b** 4 000 000 kW **c** 3 375 000 000 GW

4 Which of the following is 2 160 000 MW expressed in scientific notation, correct to 2 significant figures? Select **A**, **B**, **C** or **D**.

A 2 200 000 MW **B** 2.1×10^6 MW

C 2.16×10^6 MW **D** 2.2×10^6 MW

5 Convert:

a 3400 W to kW **b** 38 500 000 kW to MW **c** 3 945 000 mW to kW

6 How many kW in 7 650 000 W? Select **A**, **B**, **C** or **D**.

A 765 kW **B** 7.65 kW **C** 7650 kW **D** 76 500 kW

7 Convert 3.25 GW to:

a megawatts **b** kilowatts **c** watts.

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Example 8

8 This is an electricity bill from Lightning Energy.

Lightning Energy

Account Number: 71433973
Date Due: Thursday 16 May 2019
Amount Due: $1289.87

Ms B. SPARKS
13 AMPERE CRESCENT
LIGHTNING RIDGE, NSW 2834

ELECTRICITY ACCOUNT

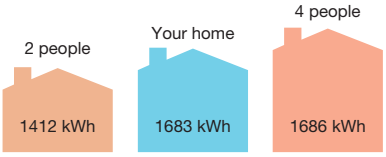
Account Summary

Description	Amount
Previous Balance	$825.79
Less payment(s) received to 08/03/19	$825.79CR
Subtotal	$0.00
Bill period: 17 Feb 2019 to 19 May 2019 (92 days)	$1289.87
Total Amount Due (Incl. GST)	**$1289.87**

How much energy are you using?

Bill period: 17 Feb to 19 May 2019 (92 days)

Compare with other homes in your area.



Average usage data supplied by Australian Energy Regulator based on homes with gas and no pool during autumn.
Visit energymadeeasy.gov.au for more information.

Snapshot.

Average daily cost:
$10.67

Average daily usage:
18.30 kWh

Same time last year:
19.72 kWh

Your electricity supply details.

Supply period: 17 February 2019 to 19 May 2019

Energy Plan: Savers

Meter No.	Read Date	Days	Start Read	End Read	kWh
0899123	19 May 2019	92	59930	60872	1683

Your next meter read is between **11 Aug 2019** and **17 Aug 2019**. Please ensure easy access to your meter on these days.

New charges and credits.

Usage and supply charges	Units	Price	Amount
Peak	1297 kWh	$0.583	$1106.34
Off Peak	386 kWh	$0.159	$61.37
Supply charge	92 days	$0.9173	$84.39
Other Charges			
Payment processing fee			$2.25
Total charges			$1254.35
Credits			
7% Pay On Time Discount			$81.74 CR
TOTAL NEW CHARGES AND CREDITS			$1172.61
Total GST			$117.26
TOTAL AMOUNT PAYABLE (incl. GST)			$1289.87

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a Is this bill paid monthly, two-monthly or quarterly?

b What was the average daily electricity usage over this period?

c Has the electricity bill increased or decreased since the last bill? By how much?

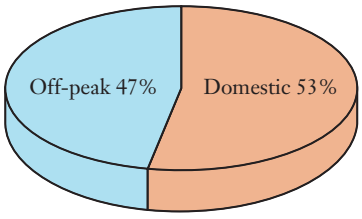
d How much electricity was used in the period covered by this bill?

e How many days of usage were covered by this bill?

f When was the electricity meter read? What was the reading?

g Look at the graphs under the heading 'Compare with other homes in your area' on the bill. This bill is for a household of 2 adults and 2 children. Do you think this house is running efficiently? Justify your answer.

9 The graph below shows the domestic and off-peak electricity usage for a Parramatta household over a 91-day period.



a If the total usage was 2472 kWh, how many kilowatt hours was off-peak usage?

b Calculate the cost of:

i off-peak usage at \$0.158620/kWh

ii domestic usage at \$0.583110/kWh.

c What was the average daily cost for domestic usage?

10 The pricing information for an electricity supplier is given below.

Tariff type	Inc GST
Off-peak (cents/kWh)	21.384
Supply charge (cents/day)	70.763
Domestic First 1000 kWh/qtr (cents/kWh)	26.906
Next 750 kWh/qtr (cents/kWh)	26.026
Remaining usage kWh/qtr (cents/kWh)	24.398

Calculate the electricity bill for a Taree household with off-peak usage of 514 kWh and domestic usage of 1986 kWh for a 90-day period.

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INVESTIGATION

RENEWABLE ENERGY

A 1-in-50-year storm caused major damage to energy infrastructure in South Australia in 2016, resulting in a statewide blackout when more than 1.5 million residents lost power. On hot days, there is not enough electricity to go around. To become self-powered, there is much discussion about **renewable energy** becoming a more sustainable, affordable and competitive means of generating energy.

1 **a** Research the following renewable energy sources:

- solar
- wind
- tidal
- batteries (such as lithium ion batteries to store energy once it is produced).

b Find out their possible uses.

c Find where each one is being productively used as a viable source of energy in Australia.

2 Research if there are any developments regarding new renewable energy applications.

7.04 Energy consumption and the costs of appliances

WS Electrical appliances

In 2017, the price of electricity in New South Wales increased by about 20%. This was equivalent to an increase of about $320 per year, per household. To reduce costs and improve efficiency you need to know how and where energy is used in the home.

EXAMPLE 9

How much energy is used (in kWh) when a 100 W light globe is on for 3 hours?

Solution

Energy used in 3 h = 100 W × 3 h

= 300 Wh

= 300 ÷ 1000 kWh 1 kWh = 1000 Wh

= 0.3 kWh

ISBN 9780170413633

WS Household energy costs

WS Energy ratings

Running costs of electrical appliances

You can calculate the **running cost** of an electrical appliance by multiplying the power used (in kWh) by the cost of electricity.

Running cost

Running cost = power used (kWh) × electricity cost ($/kWh)

There is also a daily supply charge for electricity (approximately $0.90 per day) in addition to the running cost.

EXAMPLE 10

Haresh wants to know how much it costs to charge his smartphone and his laptop each year. He uses a 12 W phone charger and it takes about 40 minutes, 4 times a week to charge his smartphone. His laptop uses 120 W/h and he charges it for 50 minutes every day. If the cost of electricity is $0.319025/kWh, calculate the cost of charging his smartphone and his laptop for one year.

Solution

$$\text{Smartphone: Power used in one year} = 12\text{ W} \times \frac{40}{60} \times 4 \times 52$$
$$= 1664\text{ Wh}$$
$$= 1.664\text{ kWh}$$
$$\text{Laptop: Power used in one year} = 120\text{ W} \times \frac{50}{60} \times 365$$
$$= 36\,500\text{ Wh}$$
$$= 36.5\text{ kWh}$$
$$\text{Total power} = 1.664 + 36.5$$
$$= 38.164\text{ kWh}$$
$$\text{Annual cost} = 38.164 \times \$0.319025$$
$$= \$12.1752701$$
$$\approx \$12.18$$

It will cost Haresh $12.18 to charge the smartphone and laptop for the year.

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Standby power

Some appliances do not need to be left on all the time. A refrigerator must stay on all of the time, but a TV has a standby mode so that it can be turned on quickly. Standby power must still be paid for, even though the appliance is actually not in use! To save on energy costs, you should switch appliances completely off when not in use by using the OFF switch or at the power point.

EXAMPLE 11

The power consumption of a microwave oven in standby mode is about 2.4W.
What is the cost of the standby mode for the microwave oven over a year if electricity is charged at the rate of 31.9025 cents/kWh?

Solution

$$\begin{aligned}\text{Power consumption} &= 2.4 \times 24 \times 365 \text{ Wh} \\ &= 21\,024 \text{ Wh} \\ &= 21.024 \text{ kWh} \\ \text{Annual cost} &= 21.024 \times \$0.319025 \\ &= \$6.707182 \\ &= \$6.71\end{aligned}$$

Exercise 7.04 Energy consumption and the costs of appliances

Example 9

1 Calculate how much energy (in kWh) is used when each appliance is used for 5 hours.

- **a** a 2000 W washing machine
- **b** a 1600 W swimming pool pump
- **c** a 70 W fan
- **d** a 900 W vacuum cleaner

2 How much energy is used over a year if it takes 45 minutes each week to iron clothes using a 2400 W iron? Select **A**, **B**, **C** or **D**.

A 5.616 kWh **B** 56.16 kWh **C** 9.36 kWh **D** 93.6 kWh

3 How much energy is used over 4 hours by an air conditioner if the power input is 0.65 kW?

Example 10

4 Piper uses a 1250 W electric lawn mower for 30 minutes each fortnight to mow her lawns.

- **a** How many kWh of energy does the lawn mower use over a year?
- **b** If electricity is charged at the rate of 32.8735 cents/kWh, calculate the cost of using the lawn mower over a year.

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5 A hair dryer of 1000 W is used by a family, on average, for 1 hour a week during the winter quarter (92 days). Calculate the cost of using the hair dryer during this quarter if electricity costs $0.319025/kWh.

6 The power input of the wash cycle of a front loader washing machine is 1600 W. The machine is used 5 times a week and it takes 36 minutes to complete a cold water wash cycle. Electricity is charged at the rate of 32.8735 cents/kWh. What is the total cost of using this washing machine for one year?

7 A house has 22 lights, which are used for an average of 5 hours per day. Electricity is charged at the rate of 31.9025 cents/kWh. Calculate the cost of lighting for a year if:

- **a** 75 W 'vintage style' light globes are used
- **b** 15 W compact fluorescent lamps are used
- **c** 12 W LED lights are used
- **d** 40 W halogen lights are used.

8 The energy consumption for a 3-star top-loader washing machine, assuming daily use, is 124 kWh/year on cold wash and 519 kWh/ year using warm wash. If electricity is charged at a rate of 31.9025 cents/kWh, calculate the difference in cost between cold and warm washing for the year.

9 The energy consumption of a small refrigerator is 365 kWh per year. Find the cost of running the small fridge for one year if electricity costs 32.8735 cents/kWh.

10

- **a** A 3-bedroom house has 18 lights. Compact fluorescent lights of 30 W are used and the lights are on for 7 hours per day. The cost of electricity is 34.5960 cents/kWh. What is the cost of lighting for one year?
- **b** If 12 W LED lights are used instead of CFLs, what is the cost of lighting for one year and how much will be saved?

11 The average lifetime of a major appliance is 10 years. Using 35.3285 cents/kWh as an estimate for the cost of electricity, calculate the lifetime running cost of electricity for each television.

- **a** Smart TV with a 4.5-star energy rating label and energy consumption 480 kWh
- **b** Smart TV with a 5-star energy rating label and energy consumption 458 kWh

Example 11

12 Find the cost of standby for these appliances for one year if electricity is charged at the rate of 31.9025 cents/kWh (assume each appliance is on 24 h per day 365 days of the year).

- **a** a washing machine in standby mode using 4 W/h
- **b** a modem on standby power using 10 W per hour

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13 Standby power can account for as much as 10% of the electricity used in a home. The average Australian home uses 6570 kWh per year.

a How much energy can be consumed in one year by appliances on standby in an Australian home?

b Calculate the cost of standby power for one year if the cost of electricity is 31.9025 cents/kWh.

TECHNOLOGY

Energy calculators

The cost of appliances, heating and cooling a home, and lighting can be calculated using online energy calculators which can be found on the Internet. Search for **Energy Use Calculator**.

1 Use the energy calculator to find the yearly cost of running appliances such as washing machines, LCD/LED display or TV screen, laptops and notebooks, refrigerators, freezers, dishwashers, recharging mobile phones and so on, as well as calculating the cost of lighting, heating and cooling a home.

2 Prepare a report, putting your information in a table so that the total energy use of your home can be calculated over a year.

3 Appliances can be compared in terms of their running costs for different numbers of years. Visit the **Energy Rating** government website to 'Compare Products', select an appliance and compare the cost of 2 or 3 different brands and usage costs over 1 year and 10 years. Record or print your results.

4 Repeat this process for at least 3 other appliances that are used in your home.

WS Energy ratings

7.05 Energy-efficient housing

WS Designing a sustainable house

An **energy-efficient** home is designed to make the most of natural heating (the sun), cooling (shade and cooling breezes) and lighting. Using 'green' design principles will reduce the need for appliances such as air conditioners and heaters and also reduce your impact in the environment. The following features should be considered when designing 'clean and green' home.

- **Orientation:** Living areas should face north to capture winter sun and the long axis of the house should run from east to west.
- **Insulation:** Roof insulation, light-coloured roofing and double-glazed windows reduce energy costs for heating and cooling.
- **Windows/skylights:** Full-length windows facing north allow winter sun to provide light and heat.
- **Ventilation:** Cross-ventilation allows air flow for cooling of the house.
- **Landscaping/shading:** Plants and trees can shade walls and windows.
- **Building materials:** Choose materials that can absorb and store heat.

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EXAMPLE 12

As a general rule, total window area should be less than 25% of the floor area of a house. Give an estimate for the maximum window area of a rectangular house with dimensions 15 m by 7.8 m.

Solution

Floor area = 15 m × 7.8 m

= 117 m^2

Window area = 25% × 117

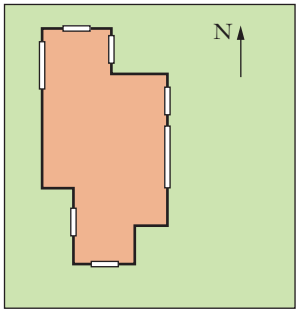
= 29.25 m^2

The maximum window area is 29.25 m^2.

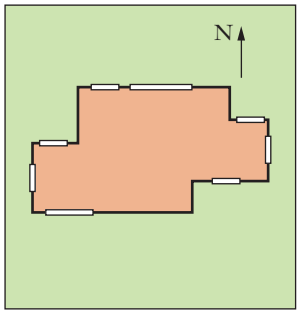
Exercise 7.05 Energy-efficient housing

1 Which site plan shows the house placed in the most energy-efficient orientation? Give reasons for your answer.

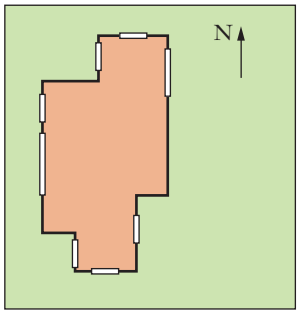
A



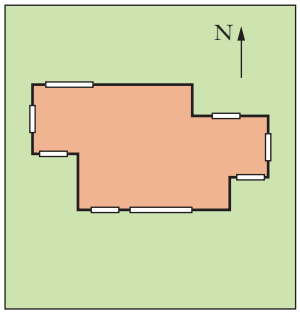
B



C



D



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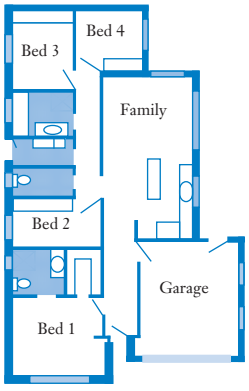
Example 12

2 **a** The plan of the house has been drawn to scale. If the length of the house is 20 m, calculate the scale used.

b Calculate the area of the house.

c The window and glazed door area is 30% of the floor area. Give an estimate for the window and glazed door area.

d Which side of the house should be facing north?



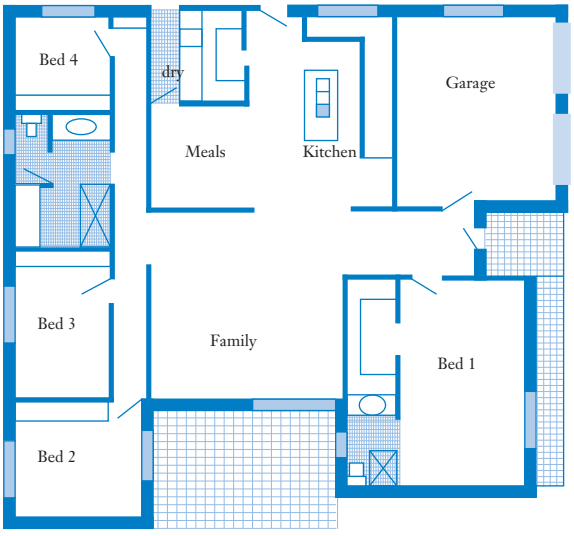
3 The house shown in the plan on the right will have the garage facing the street, which is in an easterly direction.

a In what direction will the kitchen face?

b In which direction will bedrooms 2, 3 and 4 face?

c Has the house been orientated in an energy-efficient way? Give reasons.

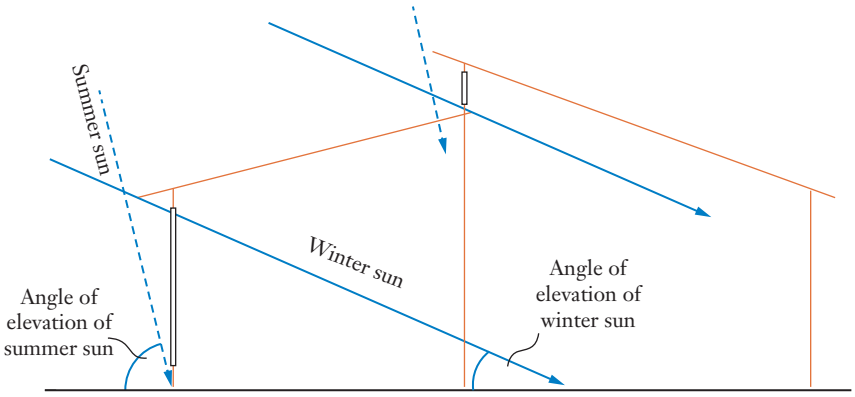
d Does this house have cross-ventilation? Give reasons.



4 What is roof insulation and how does it reduce energy costs in the home in both summer and winter?

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5 The diagram below is the side view of a house showing the angles of elevations for the winter and summer sun.



Scale 1 : 100

a Use a protractor to find the angle of elevation of:

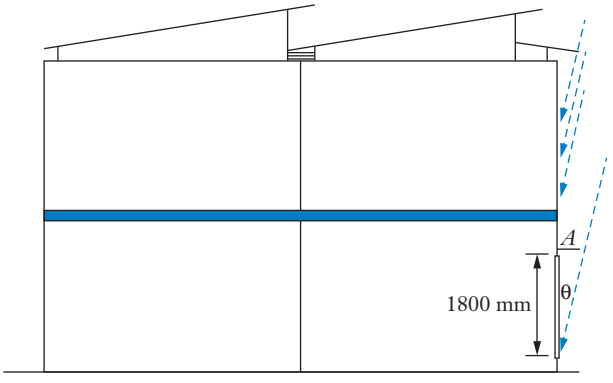
i the summer sun **ii** the winter sun.

b Will the house benefit from the position of the window and skylight? Give reasons.

6 Name two ways of using green design principles to:

a cool your home in summer **b** bring more light into your home in winter.

7 The diagram below is the side view of a two-storey house facing north, showing the angle of elevation of the summer sun. A shade awning is to be placed at A, 150 mm above the head of the window.



Scale 1 : 100

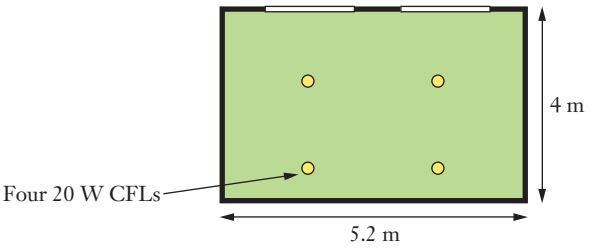
a Use the angle of elevation of the summer sun obtained in Question **5**, find θ, the angle between the window and the rays of the summer sun.

b Use trigonometry to calculate the length of the shade awning so the summer sun does not come through the downstairs window.

c The owner of the house does not want a shade awning for the upstairs window facing north. What needs to be done to reduce the amount of summer sun coming through this window?

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8 The Building Code of Australia has introduced restrictions on the amount of electrical lighting in and around the home. The lamp power density limits are 5 W/m^2 for a house and 4 W/m^2 for a balcony or verandah.



a Calculate the area of the room shown in the diagram.

b What is the lamp capacity (watts/lamp × number of lamps)?

c Calculate the lamp power density (lamp capacity ÷ area).

d Do the lights in the room comply to the limits?

9 Calculate the lamp power density for the room in Question **8** if the following lights were used instead of the CFL's and state which lights comply with the 5 W/m^2 limit:

a 2 × 100 W incandescent lights

b 3 × 50 W halogen lights

c 5 × 18 W LEDs.

10 The **Building Sustainability Index (BASIX)** was introduced by the NSW Government in 2004 to ensure that new homes are built to be more efficient in energy and water use. It assesses home design examining features such as location, size, building materials, water source and fittings, hot water, cooling and heating systems. Every development application for a new home must be submitted to the local council with a BASIX certificate. This extract of a BASIX report shows information regarding the energy commitments of a new house.

Energy commitments	Certifier check
Hot water	
The applicant must install the following hot water system in the development, or a system with a higher energy rating: solar hot water system with electric booster with a performance of 5 stars	
Cooling system	
The living areas must not incorporate any cooling system, or any ducting which is designed to accommodate a cooling system	
The bedrooms must not incorporate any cooling system, or any ducting which is designed to accommodate a cooling system	
Heating system	
The living areas must not incorporate any heating system, or any ducting that is designed to accommodate a heating system	
The bedrooms must not incorporate any heating system, or any ducting which is designed to accommodate a heating system	

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Ventilation	
The applicant must install the following exhaust systems in the development:	
At least one bathroom: individual fan, ducted to façade or roof; operation control: manual switch on/off	
Kitchen: individual fan, ducted to façade or roof; operation control: manual switch on/off	
Laundry: individual fan, ducted to façade or roof; operation control: manual switch on/off	
Artificial lighting	
The applicant must ensure that the 'primary type of artificial lighting' is fluorescent or light emitting diode (LED) lighting in each of the following rooms, and where the word 'dedicated' appears, the fittings for those lights must only be capable of accepting fluorescent or light emitting diode (LED) lamps:	
- all bathrooms/toilets	
- the laundry	
Natural lighting	
The applicant must install a window and/or skylight in the kitchen of the dwelling for natural lighting	
The applicant must install a window and/or skylight in bathrooms(s)/toilet(s) for natural lighting	

Source: NSW Government, Department of Planning and Environment
www.planningportal.nsw.gov.au/planning-tools/basix>

a What type of hot water system must be installed?

b What rooms must include an exhaust fan?

c What other design features could be used for ventilation that are not mentioned in the report?

d What type of lights are required in all bathrooms and toilets?

e What is the difference between artificial and natural lighting?

f What is a skylight and in which rooms is it appropriate to install one?

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INVESTIGATION

BUILDING AN ENERGY-EFFICIENT HOME

When building a new home, there are many things that can be done to reduce the amount of electricity used. For example, up to 35% of heat gain and loss from a house can be through a ceiling that is not insulated. Insulating the roof, walls and suspended floors will reduce energy use and cut electricity costs.

1 Investigate the following websites to find different ways a house can be made to be more energy-efficient.

- **Resources and energy NSW:** search for 'Reduce your power bills'
- Australian Government: **Your Energy Savings**
- Australian Government: **Your Home** and select 'Energy'

2 What are 'ideal' settings for heating and for cooling?

3 How can landscaping help to reduce your energy bill?

4 What type of lighting should be used in outdoor areas?

SAMPLE HSC PROBLEM

An electric kettle is rated at 2400 W and takes $2\frac{1}{2}$ minutes to boil water when full.

Charlotte boils a full kettle of water on average 5 times a day.

The cost of electricity is 34.5062 cents/kWh.

What is the cost of using the kettle for a year (to the nearest cent)?

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Study tip

Take advantage of 'down time'

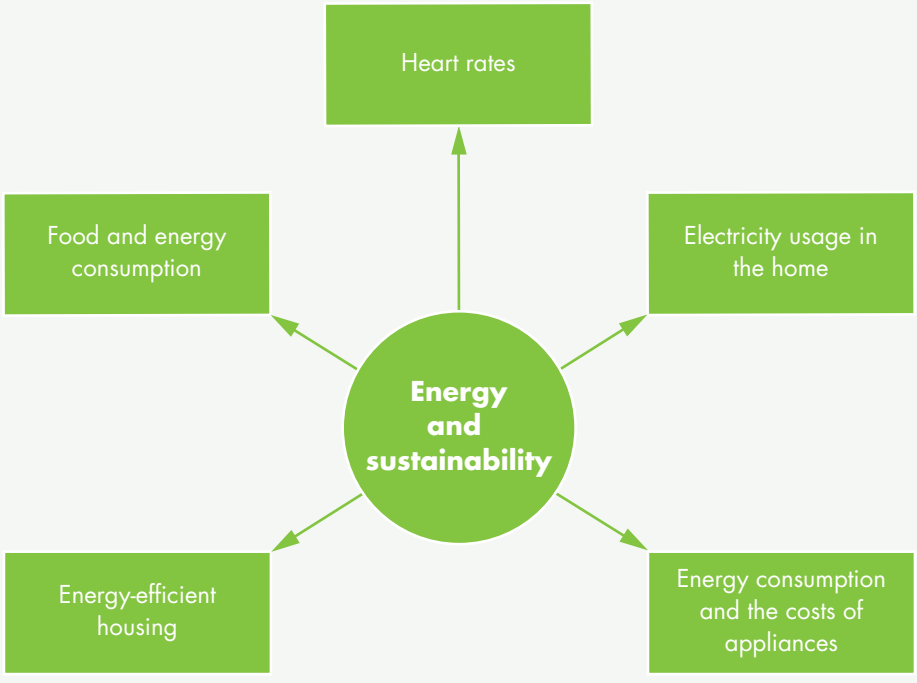
Ever read a page of text and not taken anything in? At some times during the day, such as mid-afternoon or before dinner, your mind and body will be less motivated to work. When writers have 'writer's block' and run out of ideas, they do something different to take their minds off it. Take a break from your studies. Go for a walk or jog. Get some fresh air. Play with your dog. Have a glass of water. Stretch, meditate or exercise. Alternatively, use your 'down time' to complete some task that requires less effort, such as organising your desk or folders, catching up on missed work or doing some background reading. By changing your mood and environment, you can overcome boredom and fatigue and return to your studies with a 'second wind'.

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7. CHAPTER SUMMARY

This chapter, Energy and sustainability, looked at the human body and calculations involving heart rate as well as the science of nutrition and energy in food. You should have an understanding of the units related to power and be able to interpret electricity bills. You have calculated and compared household electricity usage and costs. You have learned to calculate the costs of running different household appliances over a period of time and should be able to interpret their energy ratings and compare their costs. Finally, energy-efficient housing and the requirements of the Building Sustainability Index (BASIX) were investigated.

Make a summary of this topic. Use the outline at the start of the chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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7. TEST YOURSELF

Exercise 7.01

1 Anh exercised at a high intensity that made his pulse 148 beats/minute. How many times did his heart beat during his 25-minute workout?

Exercise 7.01

2 Sarah is 32 years old.

a What is her maximum heart rate (MHR)?

b Exercising 55–85% of your maximum heart rate for 20–30 minutes is recommended. For what heart rate range should Sarah aim when exercising?

Exercise 7.02

3 Convert:

a 5.23 kJ to J **b** 18 MJ to kJ **c** 9200 calories to kJ

d 110.5 kJ to MJ **e** 730 kJ to calories **f** 0.32 MJ to calories.

Exercise 7.02

4 A packet of biscuits contained this dietary intake (D.I.) information per 23.2 g serve (4 biscuits). There are 11 serves per packet.

biscuits 		
ENERGY	406 kJ	4.7% D.I.
PROTEIN	2.8 g	5.6% D.I.
FAT	2.6 g	3.7% D.I.
CARBS	14.1 g	4.6% D.I.
FIBRE	2.6 g	8.7% D.I.
SODIUM	125 mg	5.4% D.I.

Calculate the total amount of energy in this packet of biscuits in:

a kilojoules **b** calories.

Exercise 7.03

5 Convert:

a 15 300 kW to MW **b** 5.12 GW to kW **c** 39 000 000 mW to kW.

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6 a The pricing information for an electricity supplier is given below.

Exercise 7.03

Tariff type	Excluding GST
Off-peak (cents/kWh)	15.862
Domestic First 1750 kWh/qtr (cents/kWh)	27.906
Remaining Usage kWh/qtr (cents/kWh)	25.462
Supply charge (cents/day)	70.763

Calculate the quarterly (92 days) electricity bill (including 10% GST) for a Gosford household with off-peak usage of 1128 kWh and domestic usage of 2375 kWh.

7 The average daily household electricity consumption in Canberra is 22.7 kWh/day while in Sydney it is 20.1 kWh/day. Give one possible reason for this difference in electricity usage.

Exercise 7.03

8 In a household of 2 adults and 3 teenagers, each person uses, on average, 12 minutes of hot water per shower. This house contains a 3.6 kW hot water unit and each person showers twice per day. If electricity is charged at 31.9025 cents/kWh when the hot water unit is being used, what is the electricity cost for the hot water used by this household in one week? Select **A**, **B**, **C** or **D**.

Exercise 7.04

A $482.37 **B** $68.91 **C** $16.08 **D** $2.30

9 Claire has a 1400 W coffee machine in her kitchen which is used on average 45 minutes per day. If the cost of electricity is 31.9025 cents/kWh, how much will it cost her to use the coffee machine over one year?

Exercise 7.04

10 The energy consumption for a 4-star 8 kg top-loader washing machine is 112 kWh/year for a cold wash and 385 kWh per year for a warm wash. If electricity is charged at a rate of 34 cents/kWh, calculate how much cheaper per year it is to wash in cold water than in warm.

Exercise 7.04

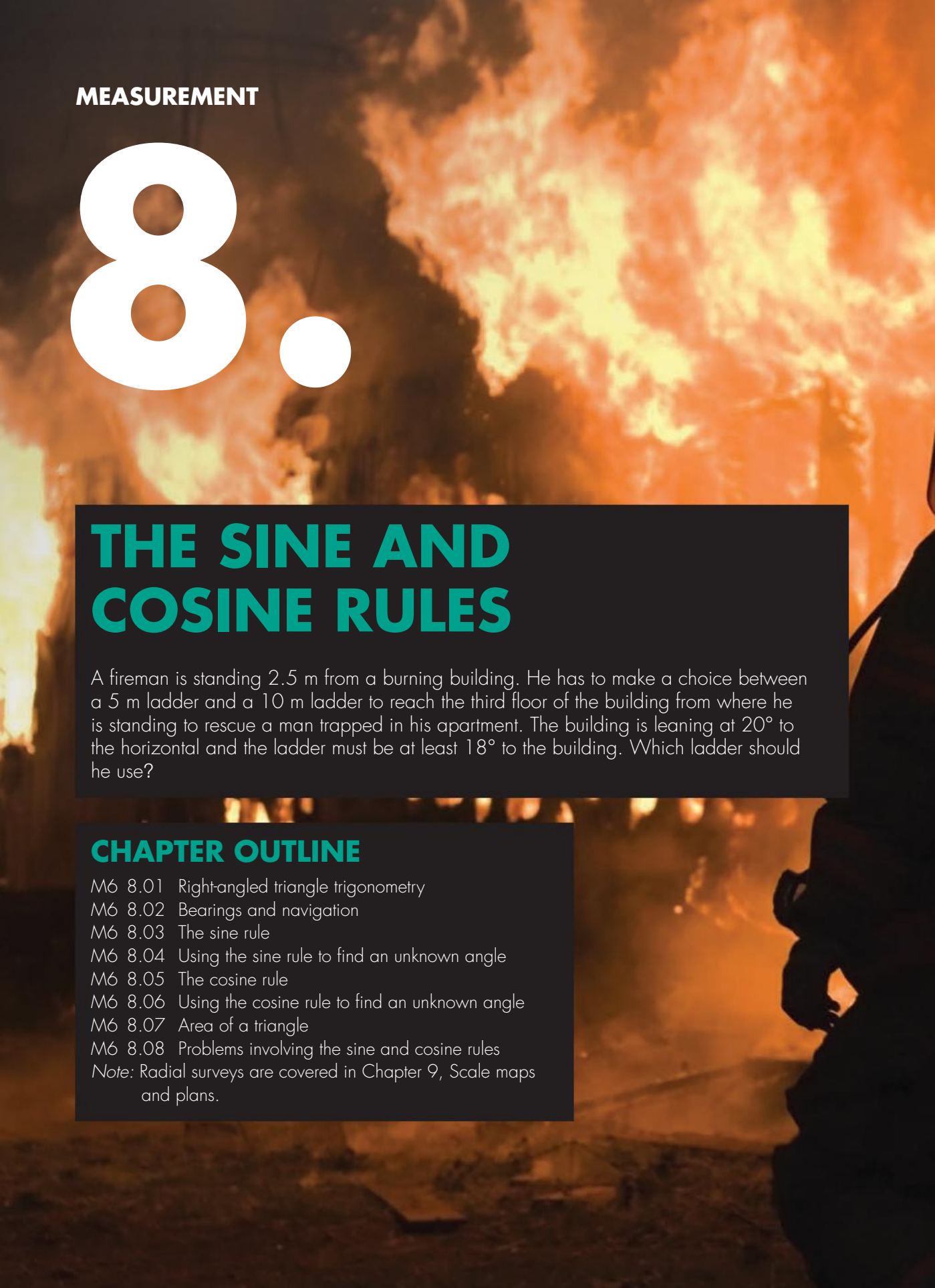
11 a Name one type of home insulation and explain how it reduces energy costs.

b Why should an energy-efficient house face north?

Exercise 7.05

Qz Chapter quiz

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MEASUREMENT

8.

THE SINE AND COSINE RULES

A fireman is standing 2.5 m from a burning building. He has to make a choice between a 5 m ladder and a 10 m ladder to reach the third floor of the building from where he is standing to rescue a man trapped in his apartment. The building is leaning at 20° to the horizontal and the ladder must be at least 18° to the building. Which ladder should he use?

CHAPTER OUTLINE

M6 8.01 Right-angled triangle trigonometry
M6 8.02 Bearings and navigation
M6 8.03 The sine rule
M6 8.04 Using the sine rule to find an unknown angle
M6 8.05 The cosine rule
M6 8.06 Using the cosine rule to find an unknown angle
M6 8.07 Area of a triangle
M6 8.08 Problems involving the sine and cosine rules

Note: Radial surveys are covered in Chapter 9, Scale maps and plans.



IN THIS CHAPTER YOU WILL:

- solve trigonometry problems for right-angled triangles, including those involving angles of elevation and depression
- solve problems involving compass bearings and true bearings
- investigate traditional navigational methods
- solve trigonometry problems for all types of triangles using the sine and cosine rules
- use the formula $A = \frac{1}{2}ab\sin C$ to find the area of a triangle.

Getty Images/TheImageArea

TERMINOLOGY

angle of depression
angle of elevation
compass bearing
cosine rule
included angle
obtuse angle
sine rule
three-figure bearing
true bearing

WS Assignment 8

WS Trigonometry calculations

SkillCheck

1 Find the value of each angle, correct to the nearest degree.

a $\cos A = \frac{1}{2}$ **b** $\sin \theta = 0.9674$ **c** $\tan X = 13.58$

d $\sin \alpha = 0.5873$ **e** $\cos \beta = -0.3465$

2 Write each angle size, correct to the nearest minute.

a 37.15° **b** 48°42′38.2″ **c** 125.68° **d** 4°31′56.7″

3 Evaluate each expression, correct to 2 decimal places.

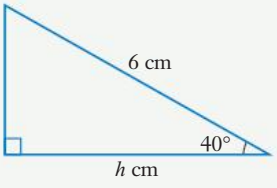
a $25 \sin 18°25'$ **b** $\frac{9}{\cos 32°}$

c $4^2 + 8^2 - 2 \times 4 \times 8 \cos 116°$ **d** $\frac{17 \sin 40°}{\sin 68°}$

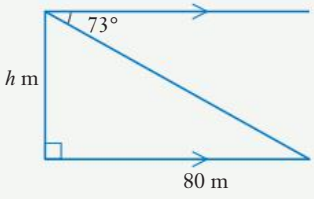
e $\frac{6^2 + 7^2 - 11^2}{2 \times 6 \times 7}$ **f** $\frac{1}{2} \times 9 \times 5 \sin 38°$

4 Find h, correct to 2 decimal places.

a

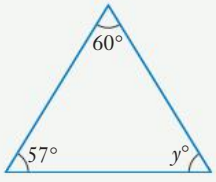


b

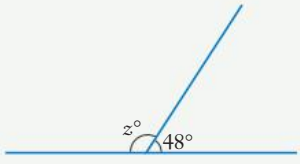


5 Find the value of each pronumeral, giving reasons.

a



b



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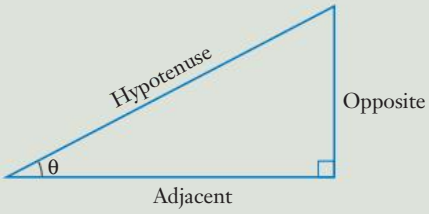
8.01 Right-angled triangle trigonometry

The trigonometric ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{(SOH)}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \text{(CAH)}$$

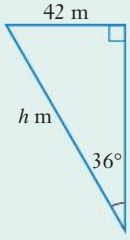
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \quad \text{(TOA)}$$



Some students memorise the initials SOH-CAH-TOA (pronounced 'so car toe-ah').

EXAMPLE 1

Find h, correct to 2 significant figures.



Solution

$$\sin 36° = \frac{42}{h}$$

$\frac{\text{opposite}}{\text{hypotenuse}}$

$$h = \frac{42}{\sin 36°}$$

← If the unknown side is in the denominator, divide to find the answer.

$$h = 71.4546\ldots$$

$$\approx 71$$

← From the diagram, 71 m seems a reasonable answer.

Make sure that your calculator is in degrees (D) mode, not radian (R) or grad (G).

Trigonometry review

Trigonometry match-up

Finding an unknown side

Finding an unknown angle

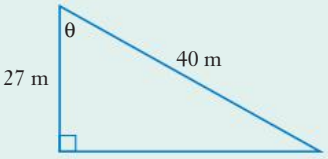
Mixed trig questions

Trigonometry problems

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EXAMPLE 2

Find θ, correct to the nearest minute.



Solution

$\cos\theta = \frac{27}{40}$

$\theta = 47.5458\ldots°$

$\approx 47°32'45.06''$

$\approx 47°33'$

$\frac{\text{adjacent}}{\text{hypotenuse}}$

SHIFT cos 27 $a^{b}/_{c}$ 40 = $°\,'\,''$ or DMS

From the diagram, 47°33′ seems a reasonable answer.

Angles of elevation and depression

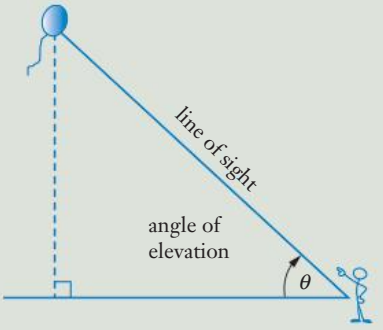
Angles of elevation and depression

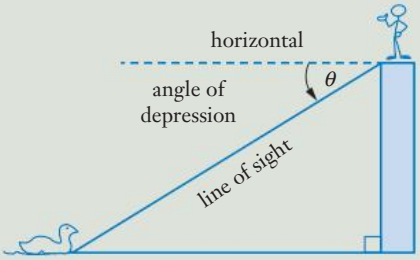
Angle of depression

Angle of depression

Angles of elevation and depression

- The **angle of elevation** is the angle *looking up*: if we feel **elevated**, things are 'looking up'.
- The **angle of depression** is the angle *looking down*: if we feel **depressed**, things are 'looking down'.





Angle of elevation and angle of depression problems usually involve the tangent (tan) ratio.

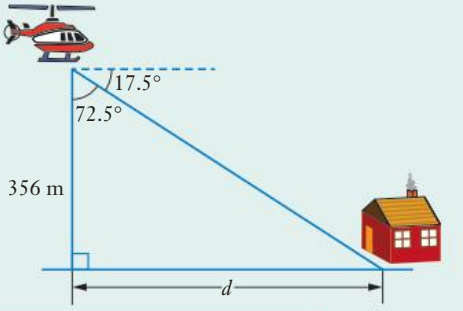
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EXAMPLE 3

The angle of depression of Katie's house from a helicopter 356 m above the ground is 17.5°. Calculate, correct to 3 significant figures, the ground distance between the helicopter and Katie's house.

Solution

Draw a diagram and let the ground distance be d. Angle of depression is 17.5°, so the adjacent angle inside the triangle is $90° - 17.5° = 72.5°$



$$\tan 72.5° = \frac{d}{356}$$

$$d = 356 \times \tan 72.5°$$

$$= 1129.0877\ldots$$

$$\approx 1130 \text{ m}$$

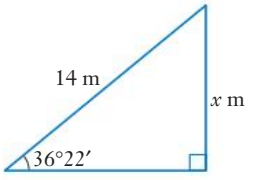
A ground distance of 1130 m looks reasonable.

Exercise 8.01 Right-angled triangle trigonometry

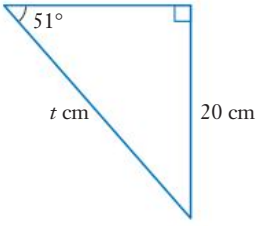
1 Find the value of the pronumeral in each triangle, correct to 2 decimal places.

Example 1

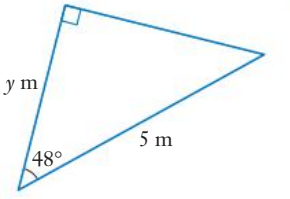
a



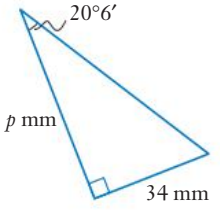
b



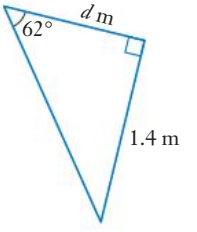
c



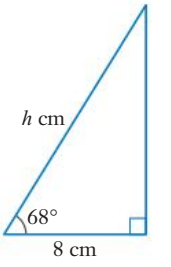
d



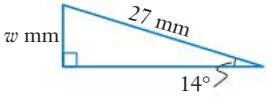
e



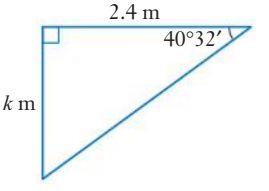
f



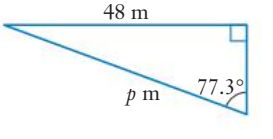
g



h



i

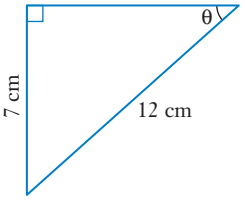


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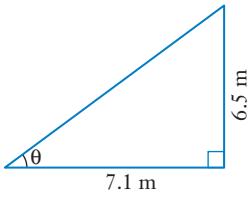
Example 2

2 Find θ, to the nearest degree.

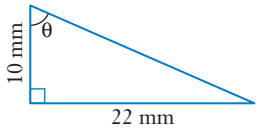
a



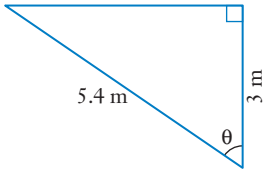
b



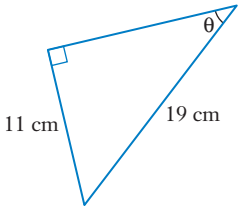
c



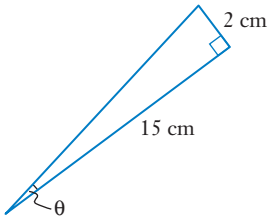
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e

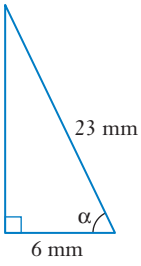


f

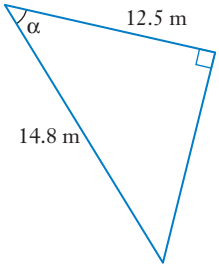


3 Find α, to the nearest minute.

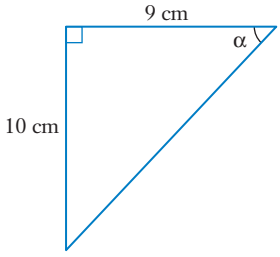
a



b



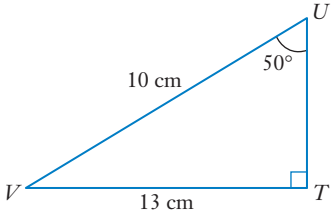
c



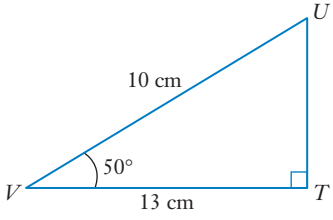
4 In $\triangle TUV$, $\angle T$ is right-angled, $\angle U = 50°$, $UV = 13$ cm and $TV = 10$ cm.

Which diagram fits this description? Select **A**, **B**, **C** or **D**.

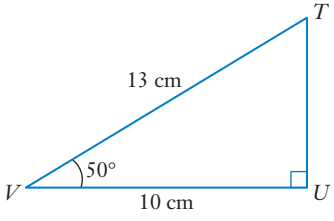
A



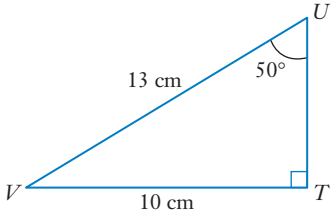
B



C



D



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5 **a** Mark the following information on a diagram.
In $\triangle PQR$, $\angle P = 90°$, $\angle R = 76° 10'$ and $PR = 4.1$ m.

b Find the size of the unknown angle.

c Find the length of the 2 unknown sides, correct to 2 decimal places.

6 A kite with a string of length 120 m is 96 m above the ground. What is the size of the angle the string makes with the horizontal? Select **A**, **B**, **C** or **D**.

A 37° **B** 39° **C** 51° **D** 53°

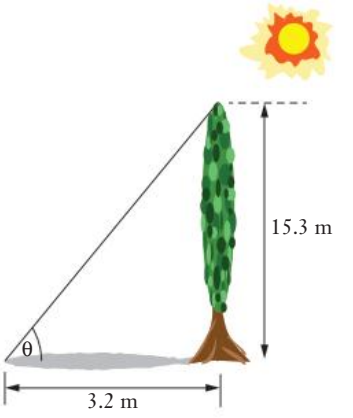
7 A 5.8 m ladder leaning against a wall makes an angle of 72° with the ground. Calculate, correct to one decimal place:

a the distance the ladder reaches up the wall

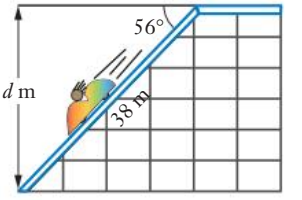
b the distance between the foot of the ladder and the base of the wall.

8 A tree of height 15.3 m casts a shadow of 3.2 m along the ground. Calculate the angle of elevation of the Sun, θ, correct to the nearest minute.

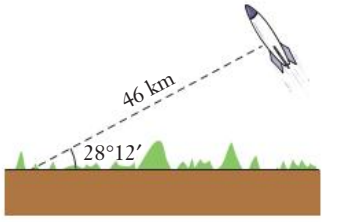
Example 3



9 A section of a rollercoaster ride has an angle of depression of 56°. If the length of the section is 38 m, what is the length of its vertical drop, correct to one decimal place?



10 A radar station measures the angle of elevation of a missile to be 28° 12′ and the line-of-sight distance to be 46 km. What is the altitude (height) of the missile, correct to one decimal place?



11 A lifeguard on top of an observation tower 12.5 m high notices a surfer in trouble at an angle of depression of 8°. How far, correct to one decimal place, is the surfer from the tower?

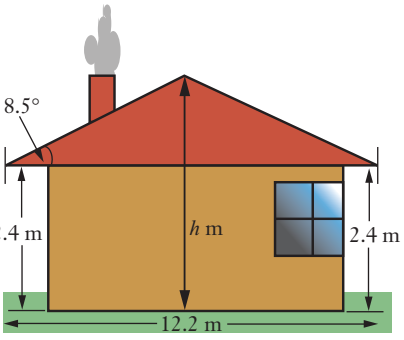
ISBN 9780170413633

12 The roof of this house has a pitch of 8.5°.

How high, h m, is the apex of the roof above the ground? Select **A**, **B**, **C** or **D**.

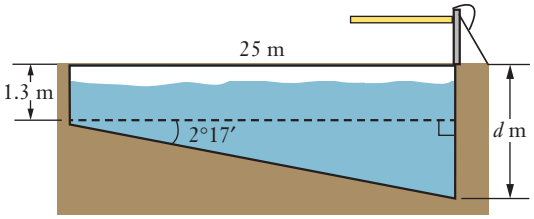
A 0.9 m **B** 3.3 m

C 3.6 m **D** 4.2 m

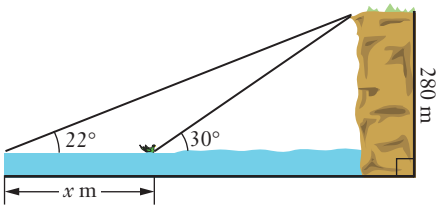


13 A boat is 420 m from the base of a cliff. The angle of depression of the boat from the top of the cliff is 28°. Find the height of the cliff, to the nearest metre.

14 The floor of a swimming pool inclines at 2° 17′ to the horizontal. The shallow end is 1.3 m deep and the pool is 25 m long. What is the depth, to the nearest 0.1 m, at the deep end?

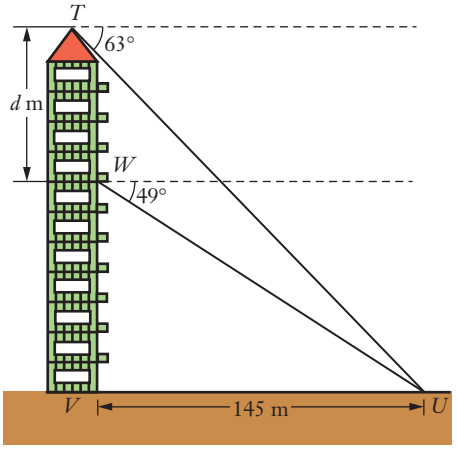


15 Jason finds that the angle of elevation of a cliff, 280 m above sea level, is 22°. After sailing towards the cliff, he finds that the angle of elevation increases to 30°. How far did Jason walk? Answer correct to the nearest metre.



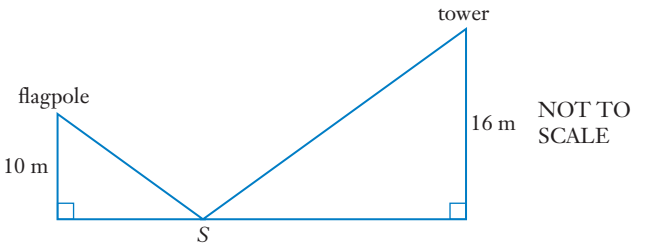
16 From the rooftop, T, of her apartment block, Rose observes the playground in the courtyard, U, at an angle of depression of 63°. From her apartment window, W, the angle of depression is 49°. The playground is 145 m from the apartment block. Show that the vertical distance between the rooftop and Rose's apartment is d metres, where:

$$d = 145(\tan 63° - \tan 49°).$$



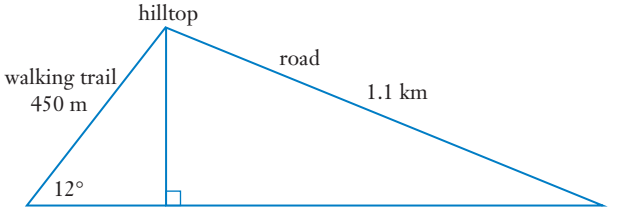
ISBN 9780170413633

17 Sophie is standing at a point, S, between a 10 m flagpole and a 16 m tower. From Sophie's position, the angle of elevation to the top of the tower is 73° and to the top of the flagpole is 34°.



Calculate the distance between the flagpole and the tower.

18 A hilltop is viewed from 2 different locations. The walking trail up the mountain is 450 m long at an angle of inclination of 12°. The bus travels on a road 1.1 km long on the opposite side of the mountain.



Calculate the angle of inclination of the road.

8.02 Bearings and navigation

Bearings use angles to show the direction of a location from a given point.

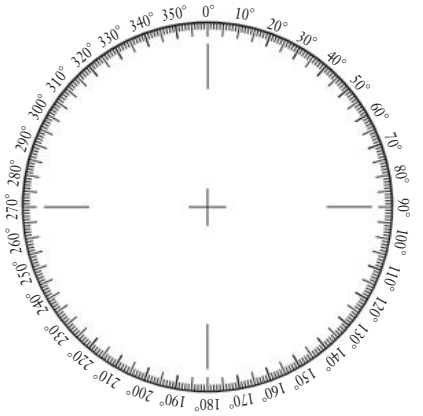


True bearings or **three-figure bearings** are angles between 000° and 360° measured in a clockwise direction from north. In the diagram, the **true bearing** of Wagga Wagga from Sydney is 249°.

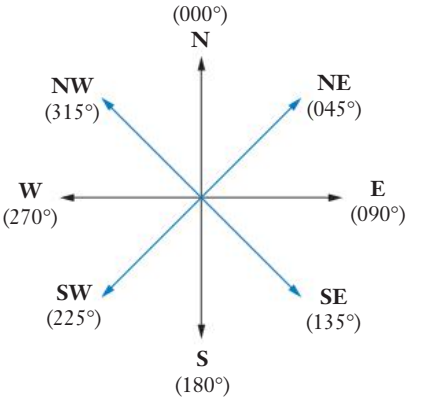
Every which way
Compass bearings
True bearings
True bearings
Bearings

ISBN 9780170413633

WS Bearings
WS A page of bearings
WS Identifying bearings
WS NSW map bearings
WS 16 points of the compass
WS Bearings match-up
WS Elevations and bearings



True bearings



Compass bearings

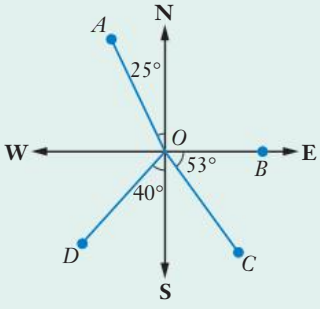
Compass bearings are angles measured from the north–south axis towards east or west. The compass bearing of Wagga Wagga from Sydney is S 69° W.

EXAMPLE 4

Write the location of points A, B, C and D from point O expressed as a:

a true bearing

b compass bearing.



Solution

a A: $360° - 25° = 335°$

B: $090°$

C: $90° + 53° = 143°$

D: $180° + 40° = 220°$

b Take the bearing from the north–south line.

A: N 25° W

B: E

C: S 37° E

D: S 40° W

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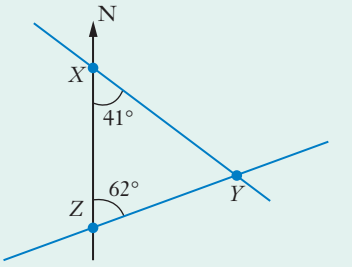
EXAMPLE 5

Find the true bearing of:

a Y from Z

b Y from X

c X from Y.

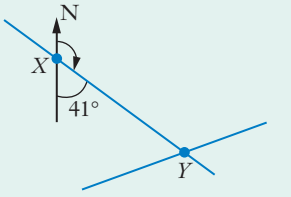


Solution

a Starting position is after the 'from', so start from Z and measure angle to Y from north. The true bearing of Y from Z is 062°.

b Starting position is X, measure angle to Y from north.

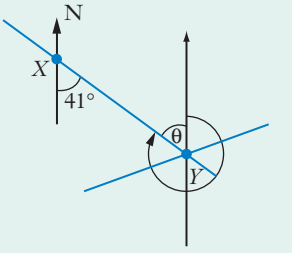
The true bearing of Y from X is $180° - 41° = 139°$.



c Starting position is Y, measure angle to X from north.

$\theta = 41°$ Alternate angles on parallel lines

The true bearing of X from Y is $360° - 41° = 319°$.



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EXAMPLE 6

A triathlete runs 10 km on a bearing of 200° from the end of a cycle leg to the finish line.

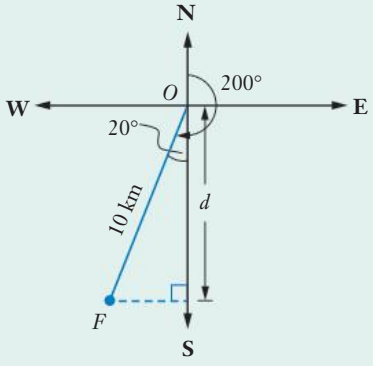
a How far south does the triathlete run? Answer to the nearest kilometre.

b What is the true bearing of the end of the cycle leg from the finish line?

c What is the compass bearing of the finish line from the end of the cycle leg?

Solution

Let O be the end of the cycle leg and the centre of the compass rose and F be the finish line.



a Let d be the distance the triathlete runs south.

Now $\angle FOS = 200° - 180° = 20°$

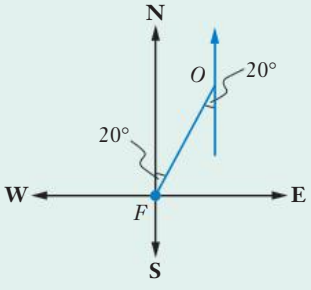
$$\cos 20° = \frac{d}{10}$$

$$d = 10 \cos 20°$$

$$= 9.3969\ldots$$

$$\approx 9 \text{ km}$$

b To find the bearing of O from F, draw another compass rose through F.



$\angle NFO = 20°$ Alternate angles on parallel lines

The bearing of the end of the cycle leg from the finish line is 020°.

Take note where the bearing is *from*: this is where you draw the centre of your compass rose and *from* where you take the bearing.

c S 20° W

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Exercise 8.02 Bearings and navigation

1 In which quadrant (NE, SE, SW, NW) does each bearing lie?

a 332° **b** 245° **c** 169°

d 007° **e** 218° **f** 095°

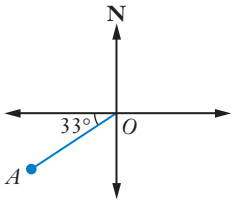
2 For each point from O, write the:

i true bearing

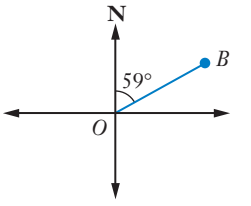
ii compass bearing.

Example 4

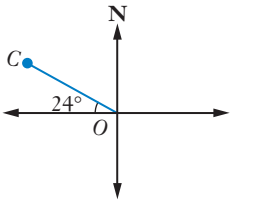
a



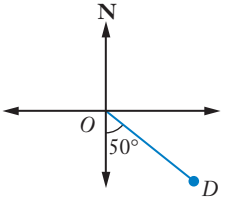
b



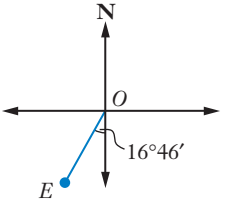
c



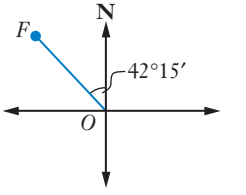
d



e



f



3 Sketch each bearing on a separate compass rose.

a 081° **b** 247° **c** 300°

d 107° **e** 180° **f** 282°

4 Write the true bearing of:

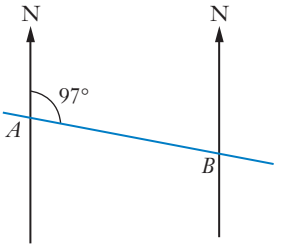
a NW **b** SW **c** NE.

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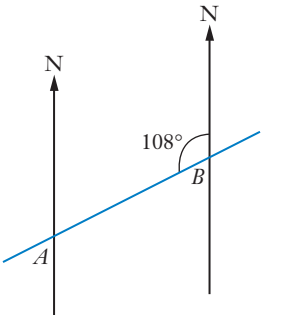
Example 5

5 Find the bearing of B from A.

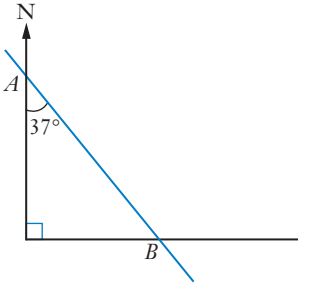
a



b

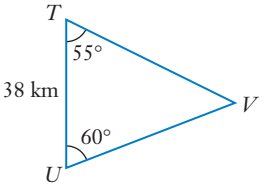


c

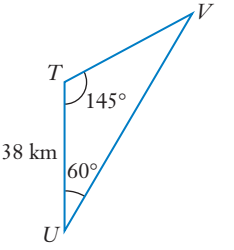


6 Tomerong (T) is 38 km due north of Uptown (U). Vincentia (V) is on a bearing of 145° from Tomerong and 060° from Uptown. Which diagram represents this? Select **A**, **B** or **C**.

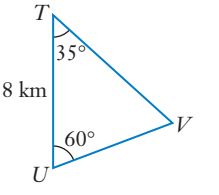
A



B



C



Example 6

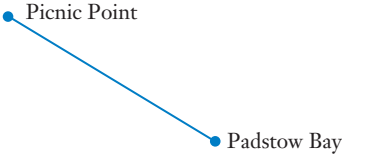
7 Turia walked for 7.2 km on a bearing of 204° from base camp.

a How far west is she from camp? Answer correct to one decimal place.

b How far south is she from camp?

c What is the bearing of the camp from her current position?

8 Picnic Point is 6.4 km west and 4.5 km north of Padstow Bay. What is the bearing of Padstow Bay from Picnic Point? Select **A**, **B**, **C** or **D**.



A 125°

B 145°

C 305°

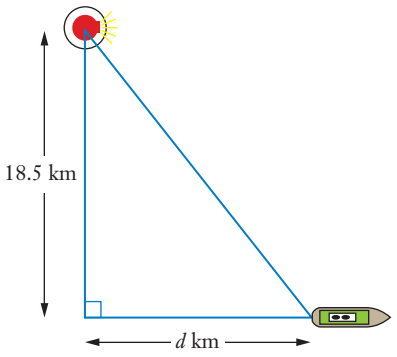
D 325°

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9 A ship's captain calculates that a lighthouse is 18.5 km due north of his ship. He sails due east for some time and calculates that the lighthouse is now on a bearing of 322° from the ship.

a Find the distance travelled by the ship, correct to one decimal place.

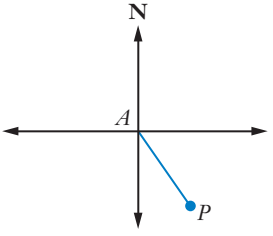
b What is the time taken to travel this distance if the ship's average speed is 48 km/h? Answer to the nearest minute.



10 A plane, P, takes off from an airport, A, at 10:15 a.m. and flies on a bearing S 30° E at a speed of 700 km/h.

a How far due south of the airport, A, is the plane at 1:45 p.m.? Answer to the nearest kilometre.

b What is the compass bearing of the airport from the plane then?



11 A hot air balloon travelled 11 km due south of Camden but drifted 1.1 km west. From Camden to the nearest degree, what was the balloon's:

a true bearing?

b compass bearing?

12 Vanda began a bushwalk and walked 3 km due west. She then ran 1.5 km due north to the lookout. What was her location from the lookout to her starting point for the bushwalk expressed as a:

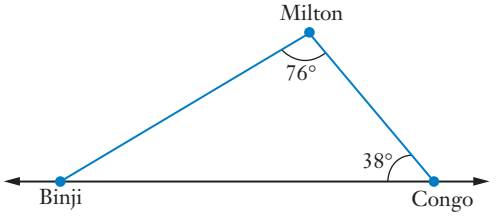
a true bearing?

b compass bearing?

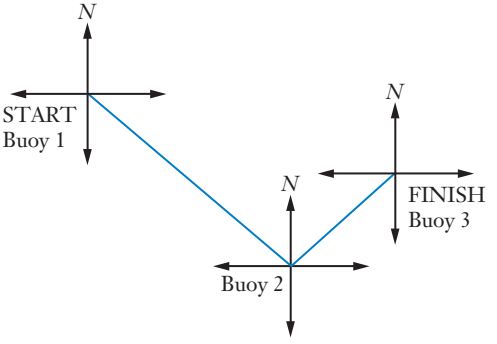
13 Three towns are shown in the diagram.

a What is the true bearing of Milton from Binji?

b What is the compass bearing of Congo from Milton?



14 Mel went in a stand-up paddle board race. She paddled from buoy 1 on a bearing of 135° for 4.2 km then turned at buoy 2 on a bearing of 045° and continued for 2.8 km to the finish line at buoy 3. What is her true bearing from the finish line (buoy 3) to the start (buoy 1)?



ISBN 9780170413633

INVESTIGATION

TRADITIONAL VERSUS MODERN NAVIGATION

Aboriginals, Torres Strait Islanders, Maoris and Polynesians made extensive return journeys to distant islands long before the invention of compasses, charts, satellite navigation and digital communication technology. They used the positions of stars (such as the Southern Cross contellation and the Sun), the shapes of clouds and the movements of birds, dolphins and ocean currents to determine where they were.

1 The early Polynesian seafarers found Easter Island in the Pacific Ocean using no instruments. They navigated by observing the ocean waves and currents as well as by reading the stars. They knew that if you can identify certain stars as they rise and set, you can find your direction. This was known as a 'star compass' and the four cardinal points (north, south, east and west) were located according to the rising and setting sun.

Investigate the Polynesian star compass and how the early navigators travelled the seas.

2 The Ramsden sextant was invented in the 18th century and is used to find the angle of a celestial body (such as the moon) above the horizon.

Investigate how a sextant works.



Getty Images/Tetra Images

3 Today, there are navigation systems that are used on land, in the sea, in the air and in space.

Investigate one or two navigational aids, such as GPS, radio, radar and satellite.



Public Domain via Wikimedia Commons

This vehicle, developed by a Stanford University team in 2005 and nicknamed 'Stanley', navigates without a driver.

ISBN 9780170413633

DID YOU KNOW?

Why 360 degrees? Why 60 minutes?

Our number, measurement and currency use a decimal or base 10 system. But in 2000 BCE, the ancient Babylonians used a base 60 system. Why did they choose 60?

- 60 is divisible by 1, 2, 3, 4, 5, 6, 10, 15, 20, 30 and 60 whereas 10 is only divisible by 1, 2, 5 and 10.
- $60 \times 6 = 360$ is the Babylonian approximation for the number of days in a year, so they made one revolution equal to 360°, meaning that the Earth travelled 1° around the Sun each day of their year.

The Babylonian influence on our measurement system has lasted more than 4000 years. Our units for measuring time and angles still follow a base 60 system:

- 1 hour = 60 minutes = 3600 seconds
- 1 month ≈ 30 days
- 1 day = 24 hours
- 1 year = 12 months

Many imperial units of measurement and currency use multiples of 6.

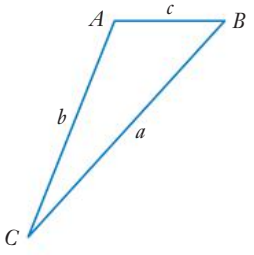
Find out how many:

a inches in a foot **b** feet in a yard **c** feet in a fathom
d items in a dozen **e** items in a gross **f** pence in a shilling.

8.03 The sine rule

WS Discovering the sine rule

The **sine rule** is a relationship between the sides and their opposite angles in a non-right triangle. Notice that in any triangle, the smallest angle is always opposite the shortest side ($\angle C$ and c in the diagram) and the largest angle is always opposite the longest side ($\angle A$ and a in the diagram aside).



If the angles of a triangle are labelled with capital letters A, B and C, then the sides opposite these angles are labelled with matching lower-case letters a, b and c.

The sine rule

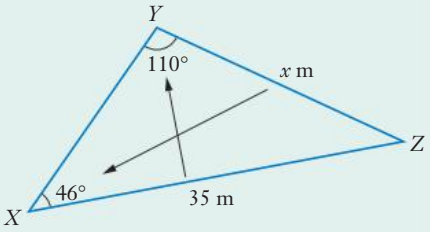
The sine rule is used with 2 sides of a triangle and the angles opposite these sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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EXAMPLE 7

Calculate x, correct to 2 significant figures.



Solution

$$\frac{x}{\sin 46°} = \frac{35}{\sin 110°}$$

$\frac{x}{\sin X} = \frac{y}{\sin Y}$

$$x = \frac{35 \sin 46°}{\sin 110°}$$

Making x the subject

$$= 26.7926\ldots$$

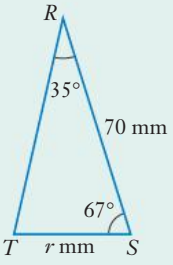
$$\approx 27$$

From the diagram, an answer of 27 m looks reasonable.

EXAMPLE 8

In $\triangle RST$, $\angle R = 35°$, $\angle S = 67°$, $t = 70$ mm.

Find r, correct to one decimal place.



Solution

$$\frac{r}{\sin 35°} = \frac{70}{\sin T°}$$

$\frac{r}{\sin R} = \frac{t}{\sin T}$

$$\frac{r}{\sin 35°} = \frac{70}{\sin 78°}$$

$\angle T = 180° - 35° - 67° = 78°$

$$r = \frac{70 \sin 35°}{\sin 78°}$$

$$= 41.0473\ldots$$

$$\approx 41.0$$

From the diagram, an answer of 41.0 mm looks reasonable.

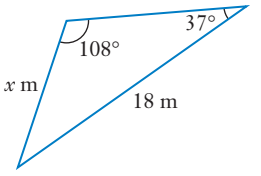
ISBN 9780170413633

Exercise 8.03 The sine rule

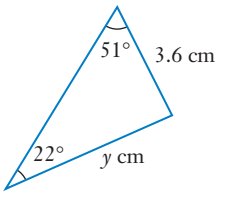
1 Find the value of the pronumeral, correct to 2 decimal places.

Example 7

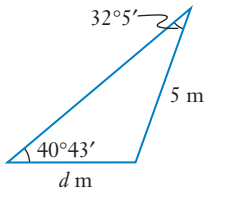
a



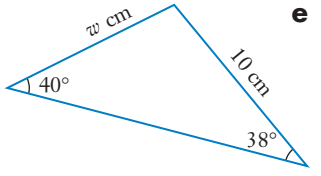
b



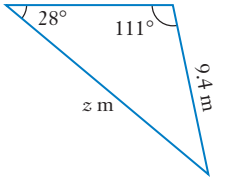
c



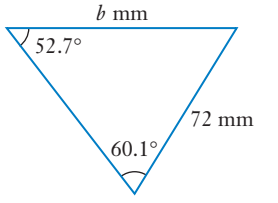
d



e



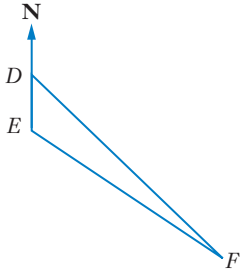
f



2 Dundee (D) is north of Engel (E). Federal (F) is 18 km on a bearing of 133° from Dundee and on a bearing of 124° from Engel. Copy and complete the diagram, then calculate, to the nearest 0.1 km, the distance between:

a Engel and Federal

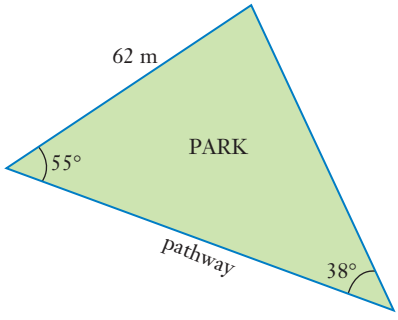
b Engel and Dundee.



3 In $\triangle MTV$, $\angle M = 71°$, $\angle T = 46°$ and $TV = 18.3$ cm. Calculate the length of MV, correct to one decimal place.

Example 8

4 A park, is to have a pathway constructed along one boundary, as shown in the diagram below. Find the length of the path, correct to the nearest metre.



5 Iftikaar sails on a bearing of 163° from A to B for 31 nautical miles. He then sails on a bearing of 195° to C, where he is directly south of A.

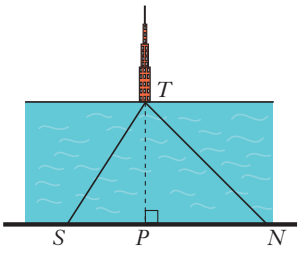
a Draw a triangle, ABC, showing this information.

b Explain why $\angle ABC = 148°$.

c Calculate how far south Iftikaar has travelled, to the nearest nautical mile.

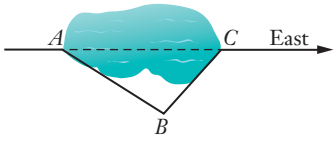
ISBN 9780170413633

6 Sarah and Natalie, standing 230 m apart on one side of a river, observe a tower on the opposite side. Given that $\angle TSP = 78°$ and $\angle TNP = 66°$, calculate to the nearest metre:



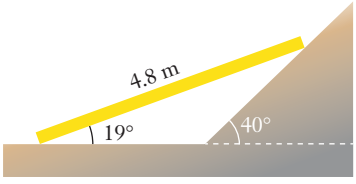
- **a** the distance ST between Sarah and the tower
- **b** the distance NT between Natalie and the tower
- **c** the width TP of the river, using either ST or NT.

7 A surveyor travelling due east needs to make a detour on a bearing of 128° to avoid a flooded area. After detouring 8 km, he turns exactly north-east and travels until he joins the original line of travel.



- **a** Copy the diagram and mark on it all the information given.
- **b** Find the size of each angle in $\triangle ABC$.
- **c** Calculate the length of AC, correct to one decimal place.

8 A 4.8 m plank is placed between the ground and a ramp that is inclined at 40° to the ground.

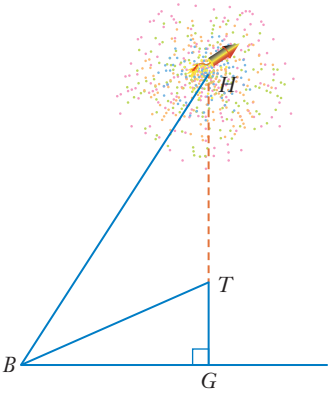


If the plank is inclined at 19° to the ground, how far up the ramp is the other end of the plank? Select **A**, **B**, **C** or **D**.

A 2.4 m **B** 4.1 m **C** 9.5 m **D** 10.6 m

9 Queenstown is 8.5 km due east of Princesville. The bearings of Royal from Princesville and Queenstown are 037° and 324° respectively. Calculate the distance between Princesville and Royal, correct to the nearest 0.1 km.

10 Boun (B) sees fireworks being launched from the top of a tower, TG, at an angle of elevation of 38°. When the fireworks are at their highest point, H, 800 m above the tower, the angle of elevation is 80°.



- **a** Copy the diagram and mark on it all the information given.
- **b** Show that $\angle BTH = 128°$.
- **c** Calculate the distance between Boun and the fireworks at H, to the nearest metre.
- **d** Calculate the distance between Boun and the top, T, of the tower, correct to the nearest metre.
- **e** Calculate the height of the tower, to the nearest metre.

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INVESTIGATION

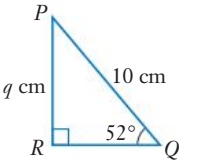
THE SINE RULE AND RIGHT-ANGLED TRIANGLES

Find q in the diagram, correct to 2 decimal places, using:

a the sine ratio

b the sine rule

Why do both methods give the same answer?



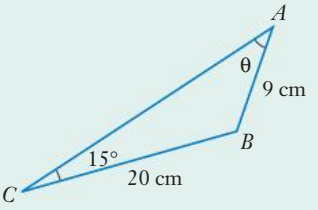
8.04 Using the sine rule to find an unknown angle

WS
Sine rule problems

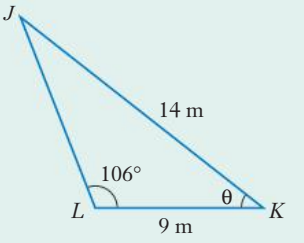
EXAMPLE 9

Find the value of θ, correct to the nearest minute.

a



b



Solution

Look for 2 sides and the angles opposite them.

a

$$\frac{20}{\sin\theta} = \frac{9}{\sin 15°} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{\sin\theta}{20} = \frac{\sin 15°}{9}$$ Turn both fractions upside-down

$$\sin\theta = \frac{20\sin 15°}{9}$$ Make sin θ the subject

$$= 0.5751\ldots$$

$$\theta = 35.1103\ldots°$$

$$= 35°6'37.36''$$

$$\approx 35°7'$$

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b The side opposite θ is not known, so calculate $\angle J$ first.

$$\frac{9}{\sin J} = \frac{14}{\sin 106°} \qquad \frac{j}{\sin J} = \frac{l}{\sin L}$$

$$\frac{\sin J}{9} = \frac{\sin 106°}{14}$$

$$\sin J = \frac{9 \sin 106°}{14}$$

$$= 0.6179\ldots$$

$$J = 38.1668\ldots°$$

$$\therefore \theta = 180° - 106° - 38.1668\ldots° \qquad \text{Angle sum of triangle}$$

$$= 35.8331\ldots°$$

$$= 35°\,49'\,59.25''$$

$$\approx 35°\,50'$$

Obtuse-angled answers

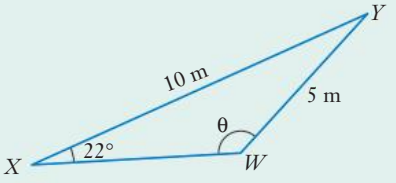
If sin θ is positive, θ could be acute *or* **obtuse** since sin θ = sin (180° − θ).

For example, sin 30° = sin 150° = 0.5. In this course, assume the acute-angled answer only, unless the obtuse-angled answer is asked for (as in the next example).

Obtuse angles using the sine rule

EXAMPLE 10

Find θ, to the nearest degree, if it is **obtuse**.



Solution

$$\frac{10}{\sin\theta} = \frac{5}{\sin 22°}$$

$$\frac{\sin\theta}{10} = \frac{\sin 22°}{5}$$

$$\sin\theta = \frac{10 \sin 22°}{5}$$

$$\sin\theta = 0.7492\ldots$$

$$\theta = 48.5222\ldots°$$

$$\approx 49°$$

But θ is obtuse, so

$$\therefore \theta \approx 180° - 49°$$

$$= 131°$$

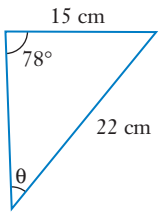
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Exercise 8.04 Using the sine rule to find an unknown angle

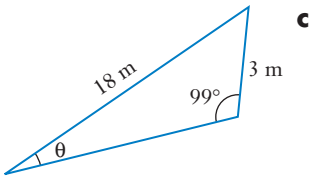
Example 9

1 Find θ in each triangle. Give your answer correct to the nearest minute.

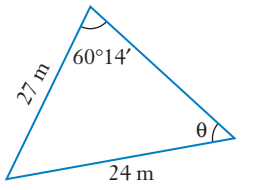
a



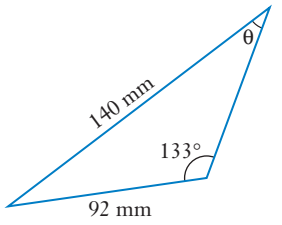
b



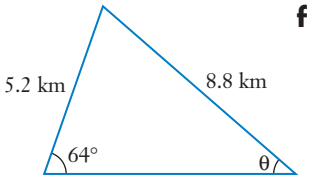
c



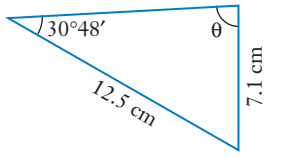
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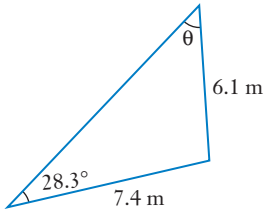
e



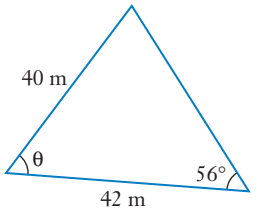
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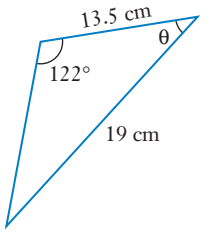
g



h



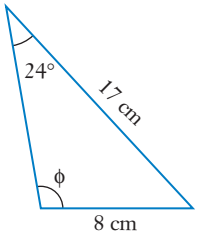
i



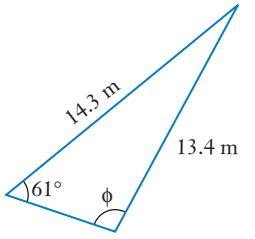
Example 10

2 Find ϕ in each triangle, correct to the nearest degree, given that ϕ is obtuse.

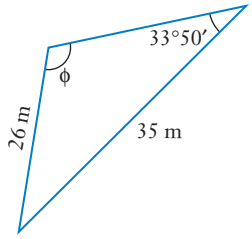
a



b



c



3 In $\triangle UWS$, $UW = 28$ mm, $WS = 40$ mm and $\angle S = 31°$. Calculate the size of $\angle U$, to the nearest minute, if it is:

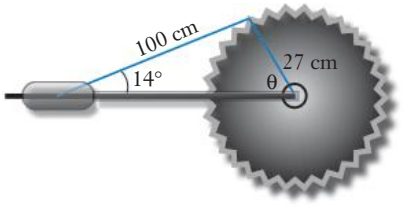
a acute **b** obtuse.

4 A surveyor positions three checkpoints M, N and P around a lake at a picnic ground. $MN = 728$ m, $MP = 638$ m and $\angle N = 57°$. Find the size of $\angle M$. Select **A**, **B**, **C** or **D**.

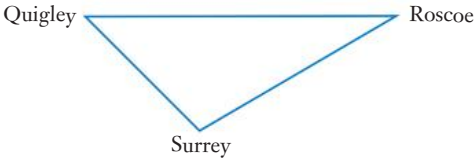
A 50° **B** 73 **C** 76° **D** 96°

ISBN 9780170413633

5 The crank and connecting rod of an engine are 27 cm and 100 cm long respectively. What angle does the crank make with the horizontal when the angle made by the connecting rod is 14°? Answer to the nearest degree.

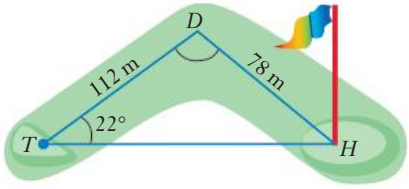


6 Surrey is 21 km on a bearing of 140° from Quigley. Roscoe is 31 km from Surrey and due east of Quigley. Find the bearing of Surrey from Roscoe to the nearest degree.



7 A bushwalker travels N 15° W from A to B for 8.5 km. She then turns and walks for 13.2 km to C, directly north of A. Show that the bearing of C from B is N 10° E.

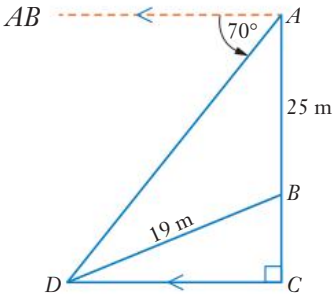
8 The 18th hole of a golf course has a tee at T, a hole at H and a dogleg at D. If $TD = 112$ m, $DH = 78$ m and $\angle HTD = 22°$, find $\angle TDH$, correct to the nearest degree.



9 James walked 2.8 km due west of home, then turned and walked on a bearing of 296° until he was 4.9 km from home. What is James's bearing from home? Select **A**, **B**, **C** or **D**.

A 011° **B** 015° **C** 281° **D** 349°

10 The angle of depression from A to D is 70°. The length of AB is 25 metres and the length of BD is 19 metres. Calculate the angle of elevation from D to B. Answer to the nearest degree.



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8.05 The cosine rule

The **cosine rule** is a relationship between the 3 sides of a triangle and one of its angles. The formula has the unknown side squared as the subject.

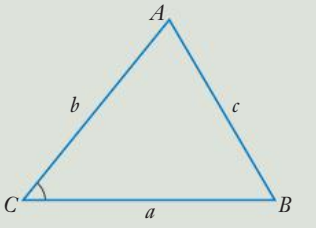
The cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

where c is the unknown side

a and b are the 2 known sides

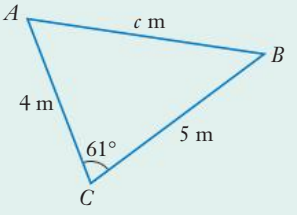
$\angle C$ is the angle opposite c.



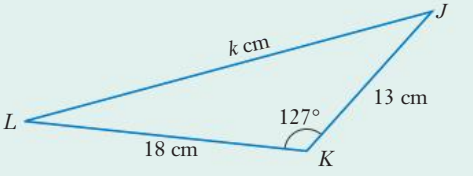
EXAMPLE 11

Find the value of the pronumeral in each triangle, correct to 2 significant figures.

a



b



Solution

a $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 5^2 + 4^2 - 2 \times 5 \times 4\cos 61°$

$c^2 = 21.6076\ldots$

$c = \sqrt{21.6076}$ Taking the square root

$= 4.6483\ldots$

≈ 4.6 ← From the diagram, a length of 4.6 m looks reasonable.

b $k^2 = 18^2 + 13^2 - 2 \times 18 \times 13\cos 127°$

$k^2 = 774.6494\ldots$

$k = \sqrt{774.6494}$

$= 27.8325\ldots$

≈ 28 ← From the diagram, a length of 28 cm looks reasonable.

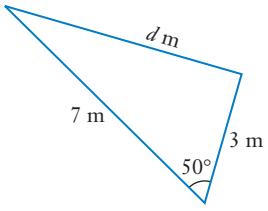
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Exercise 8.05 The cosine rule

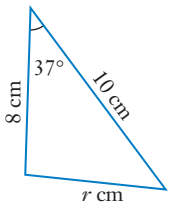
Example 11

1 Find the value of each pronumeral, correct to 2 decimal places.

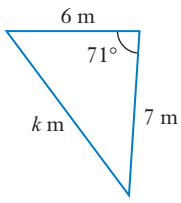
a



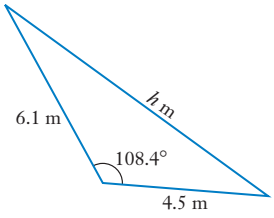
b



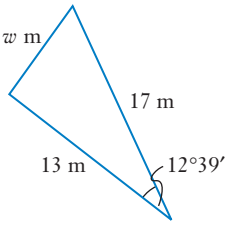
c



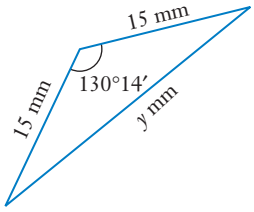
d



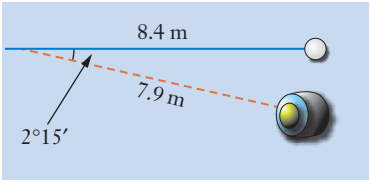
e



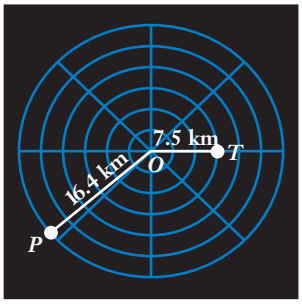
f



2 In a game of lawn bowls, Amelie is aiming to hit the jack (target ball) 8.4 m away. If she bowls 2° 15′ off-centre and her bowl travels 7.9 m, how far is her bowl from the jack? Answer correct to one decimal place.



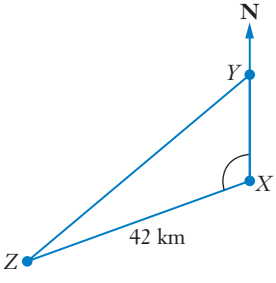
3 An air traffic controller looks on his radar screen and sees that plane P is 16.4 km from the airport, O, while plane T is 7.5 km from the airport. If he knows that the angle $\angle POT = 136°$, what is the distance between the planes? Answer correct to one decimal place.



4 The bearing of Z from X is 230° and the distance of Z from X is 42 km.

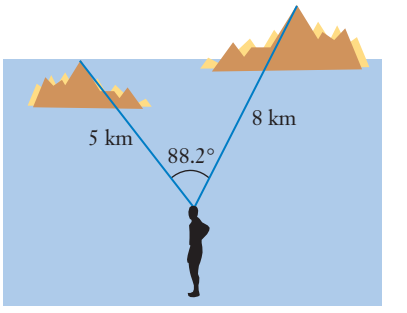
a Explain why $\angle X$ is 130°.

b If Y is due north of X and $XY = 23$ km, calculate the distance of Z from Y, correct to the nearest kilometre.

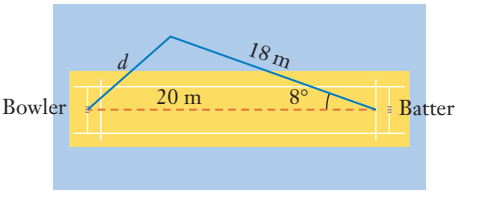


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5 Shaun observes the peaks of 2 hills from a plain. One is at a distance of 5 km and the other at a distance of 8 km. If the angle between the lines of sight is 88.2°, calculate the distance between the peaks, correct to the nearest metre.



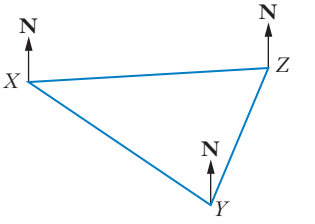
6 In a cricket match, the distance between the bowler and the batter was 20 m. During one bowl, the batter hit the ball at an angle of 8° to the line of the pitch and the bowler ran and caught the ball after it had travelled 18 m.



How far did the bowler run to catch the ball? Select **A**, **B**, **C** or **D**.

A 1.1 m **B** 2.0 m **C** 3.3 m **D** 4.0 m

7 A yacht navigates from X to Y on a bearing of 130° for 4.2 km. It then turns and navigates to Z on a bearing of 025° for 2.9 km.

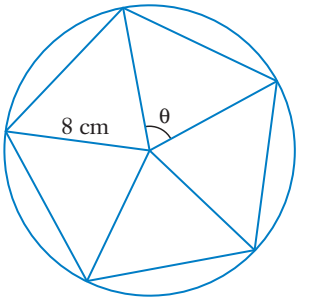


a Copy the diagram and mark the given information on it.

b Explain why $\angle XYZ = 75°$.

c Calculate the distance XZ, correct to one decimal place.

8 A regular pentagon is inscribed in a circle of radius 8 cm.

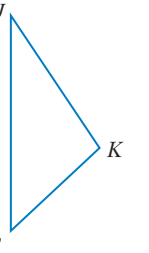


a How many degrees in a revolution?

b What is the size of angle θ?

c Calculate the perimeter of the pentagon, giving the answer correct to 2 decimal places.

9 Jonesville (J) is 84 km due north of Leeton (L). Kellyfield (K) is 60 km S 32° E of Jonesville.

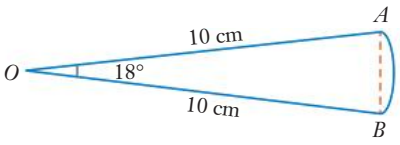


a Copy the diagram and clearly mark on it the above information.

b Calculate, to the nearest kilometre, the distance between Kellyfield and Leeton.

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10 OAB is a sector of a circle with radius 10 cm and central angle 18°.



a Use the arc length formula to calculate the length of arc AB, correct to 2 decimal places.

b Use the cosine rule to calculate the length of interval AB, correct to 2 decimal places.

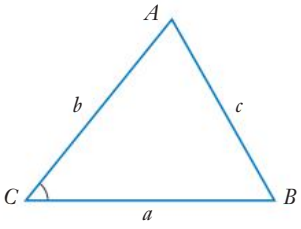
c Which is longer—the arc AB or the interval AB—and by how much?

WS
Cosine rule problems

The cosine rule for angles

8.06 Using the cosine rule to find an unknown angle

The cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$ can be used to find an unknown angle if the lengths of all 3 sides of the triangle are known. The formula can be rearranged so that $\cos C$ is the subject.



Using the cosine rule to find an unknown angle

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

where $\angle C$ is the unknown angle

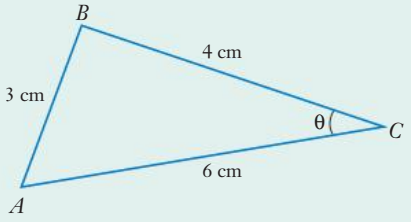
a and b are the 2 sides adjacent to $\angle C$

c is the side opposite $\angle C$.

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EXAMPLE 12

Find θ, to the nearest degree.



Solution

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos \theta = \frac{4^2 + 6^2 - 3^2}{2 \times 4 \times 6}$$

$$\cos \theta = 0.8958\ldots$$

$$\theta = 26.3843\ldots°$$

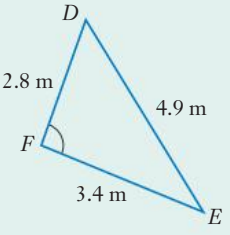
$$\approx 26°$$

Calculator sequence: SHIFT COS ANS =

EXAMPLE 13

$\triangle DEF$ has $DE = 4.9$ m, $EF = 3.4$ m and $DF = 2.8$ m.

Find $\angle F$ to the nearest minute.



Solution

$$\cos F = \frac{3.4^2 + 2.8^2 - 4.9^2}{2 \times 3.4 \times 2.8}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos F = -0.2421\ldots$$

$$F = 104.0118\ldots°$$

SHIFT COS ANS =

$$= 104° 0' 42.51''$$

$$\approx 104° 1'$$

cos F is negative so F is obtuse.

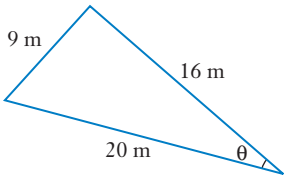
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Exercise 8.06 Using the cosine rule to find an unknown angle

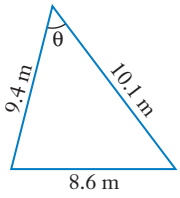
Example 12

1 Find θ, to the nearest degree, in each triangle.

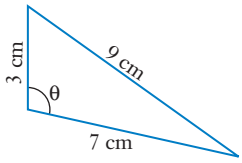
a



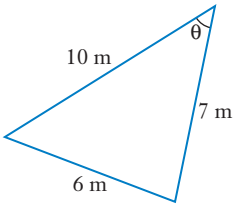
b



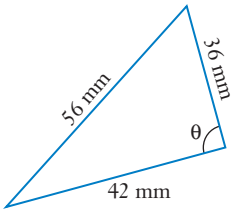
c



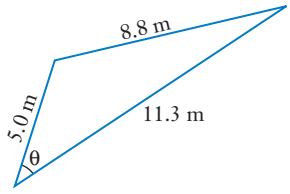
d



e



f



2 In $\triangle GST$, $GS = ST = 8.4$ cm and $GT = 12.7$ cm.

Example 13

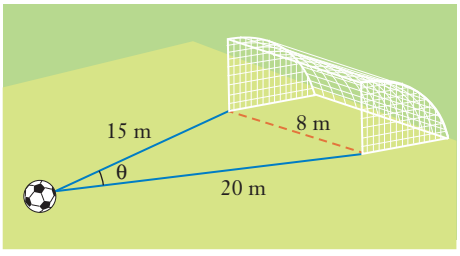
a Calculate the size of $\angle S$, to the nearest degree.

b Calculate the size of $\angle G$, to the nearest degree.

c What type of triangle is ΔGST?

d What is the size of $\angle T$?

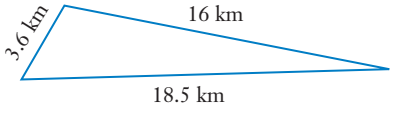
3 A soccer goal is 8 m wide. A player shoots for goal when he is 20 m from one post and 15 m from the other post. Within what angle must the shot be made to score a goal?
Select **A**, **B**, **C** or **D**.



A 10° **B** 21°

C 57° **D** 102°

4 A triathlon has a triangular circuit with three legs: swimming 3.6 km, cycling 18.5 km and running 16 km. What is the size of the angle between the cycling and running legs? Answer to the nearest degree.



5 The 2 legs of a ladder are 2.1 m long and stand on the ground 0.9 m apart. Calculate, to the nearest minute:

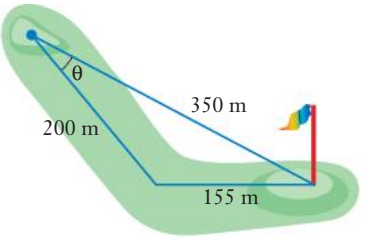
a the angle between the legs

b the angle each leg makes with the ground.

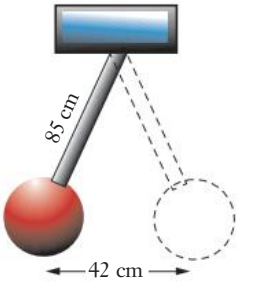
6 A triangle has sides of length 8 cm, 15 cm and 17 cm. Find the size of the smallest angle (correct to the nearest minute).

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7 A golf hole is 350 m in a straight line from the tee. A ball is driven 200 m from the tee but it is off-line. The ball lands 155 m to the right of the hole. By what angle, to the nearest minute, is the shot off-line?



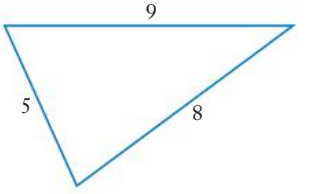
8 A pendulum of length 85 cm swings through a horizontal distance of 42 cm. What is the angle swept by the pendulum? Answer correct to the nearest degree.



9 A triangle has sides of length 8 cm, 10 cm and 16 cm. What is the size of its largest angle? Select **A**, **B**, **C** or **D**.

A 55° **B** 125° **C** 149° **D** 156°

10 What is the size of the smallest angle in this triangle?



8.07 Area of a triangle

The area of a triangle can be found using the formula $A = \frac{1}{2}bh$, where b is the base length and h is the perpendicular height. For a triangle where 2 sides and the **included angle** are known, there is an area formula involving the sine ratio.

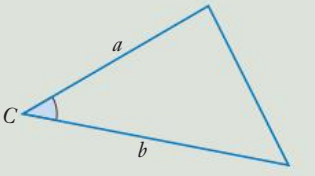
Area of a triangle

In $\triangle ABC$,

$$A = \frac{1}{2}ab \sin C$$

where a and b are 2 known sides

and $\angle C$ is the included angle.

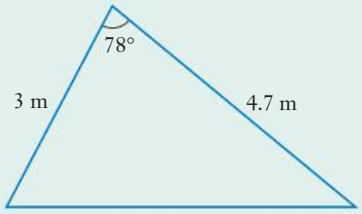


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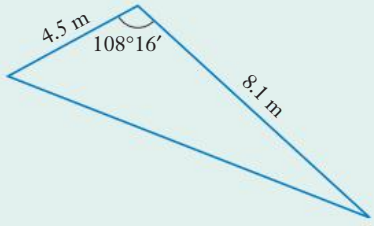
EXAMPLE 14

Calculate the area of each triangle, correct to one decimal place.

a



b



Solution

a $A = \frac{1}{2} \times 3 \times 4.7 \times \sin 78°$

$= 6.8959\ldots$

$\approx 6.9 \text{ m}^2$

$A = \frac{1}{2}ab \sin C$

b $A = \frac{1}{2} \times 4.5 \times 8.1 \times \sin 108° 16'$

$= 17.3066\ldots$

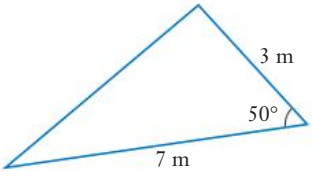
$\approx 17.3 \text{ m}^2$

Exercise 8.07 Area of a triangle

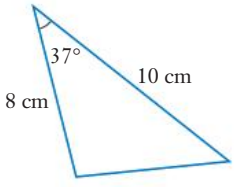
Example 14

1 Calculate the area of each triangle, correct to 2 decimal places.

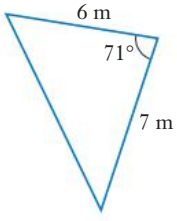
a



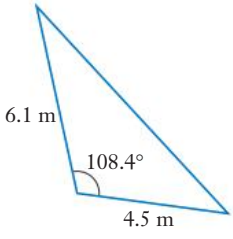
b



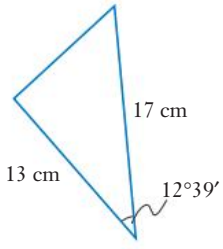
c



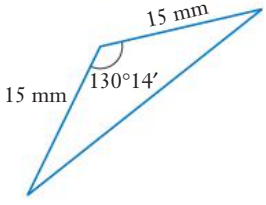
d



e



f



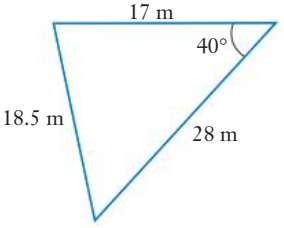
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2 A parallelogram has sides of length 7 cm and 5.2 cm, and the included angle of 130°. Calculate its area, correct to one decimal place.

3 A kite has adjacent sides of length 4 cm and 7 cm, The longer diagonal makes angles of 45° and 23.8° with the sides of the kite. What is the area of this kite? Select **A**, **B**, **C** or **D**.

A 15.5 cm^2 **B** 16.5 cm^2 **C** 24.3 cm^2 **D** 26.1 cm^2

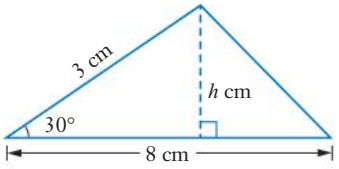
4 Find the area of this triangle, correct to 2 decimal places.



5 Calculate the area of this triangle:

a using the sine area formula

b using $A = \frac{1}{2}bh$ (find h using sin 30° first).

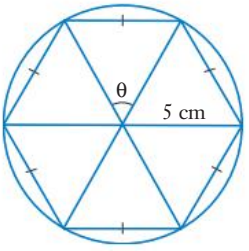


6 A regular hexagon is inscribed in a circle of radius 5 cm.

a What is the size of angle θ?

b Calculate the area of the hexagon, correct to 2 decimal places.

c Show that every side of this hexagon is 5 cm.



7 Calculate, correct to 2 decimal places, the area of an equilateral triangle with side 7 cm.

8 The area of an equilateral triangle with side s can also be found using the formula

$$A = \frac{\sqrt{3}s^2}{4}.$$

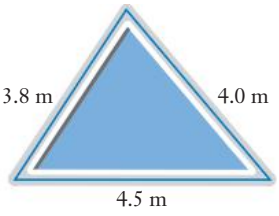
Use this formula to calculate the area of an equilateral triangle of side 7 cm and check that it agrees with the answers calculated in Question **7**.

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9 A traffic island has sides of length 3.8 m, 4.0 m and 4.5 m.

a Use the cosine rule to calculate, to the nearest minute, the size of one angle of the island.

b Use this angle to calculate the area of the island, correct to one decimal place.

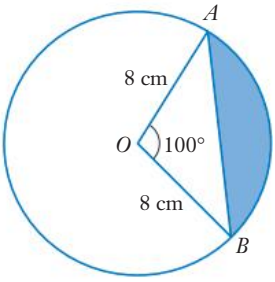


10 O is the centre of a circle with radius 8 cm.

a Calculate, to 2 decimal places, the area of sector OAB using the formula $A = \frac{\theta}{360}\pi r^2$.

b Calculate the area of $\triangle OAB$, correct to 2 decimal places.

c Hence, find the area of the shaded segment.



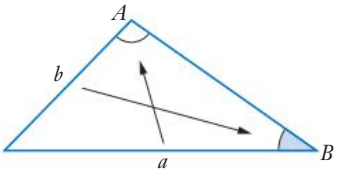
WS Applying trigonometry

WS Finding an unknown side

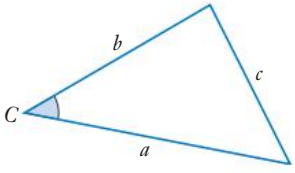
WS Finding an unknown angle

8.08 Problems involving the sine and cosine rules

The **sine rule** $\frac{a}{\sin A} = \frac{b}{\sin B}$ is used for triangle problems involving 2 *sides* and the 2 *angles opposite these sides*.

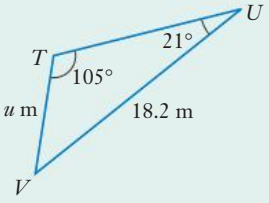


The **cosine rule** $c^2 = a^2 + b^2 - 2ab\cos C$ or $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ is used for triangle problems involving 3 *sides* and 1 *angle*.



EXAMPLE 15

Find u, correct to 3 significant figures.



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Solution

Problem involves 2 sides and 2 angles: use the sine rule.

$$\frac{u}{\sin U} = \frac{t}{\sin T}$$

$$\frac{u}{\sin\ 21°} = \frac{18.2}{\sin\ 105°}$$

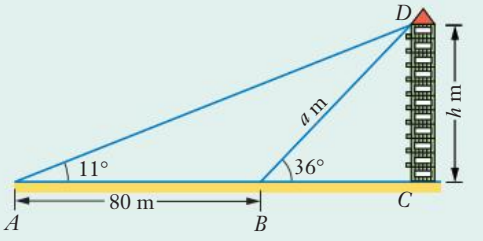
$$u = \frac{18.2\ \sin\ 21°}{\sin\ 105°}$$

$$= 6.7523\ \ldots$$

$$\approx 6.75$$

EXAMPLE 16

Hasaan observes a tower at an angle of elevation of 11°. Walking 80 m towards the tower, he finds that the angle of elevation increases to 36°.



a Show that $a = \frac{80 \sin 11°}{\sin 25°}$.

b Hence evaluate the height, h metres, of the tower, correct to 2 significant figures.

Solution

a First find $\angle ADB$, then use the sine rule to find a in $\triangle DAB$.

$\angle ABD = 180° - 36° = 144°$ (Angles on a straight line)

$\angle ADB = 180° - 11° - 144° = 25°$ (Angle sum of a triangle)

$$\frac{a}{\sin A} = \frac{AB}{\sin \angle ADB}$$

$$\frac{a}{\sin 11°} = \frac{80}{\sin 25°}$$

$$\therefore a = \frac{80 \sin 11°}{\sin 25°}$$

b Using the sine ratio (not the sine rule) in right-angled triangle $\triangle DCB$:

$$\sin 36° = \frac{h}{a}$$

$$h = a \sin 36°$$

$$= \frac{80 \sin 11° \sin 36°}{\sin 25°}$$ Using the answer from part **a**

$$= 21.2304\ldots$$

$$\approx 21$$

The height of the tower is 21 m.

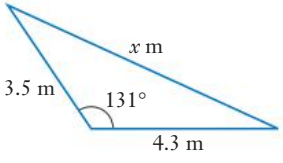
Note: This is an example of a 'Show that…' question, where the answer to the question is given. 'Show that' questions are popular in HSC exams. Do these questions as you would any other question and aim to achieve the given answer at the end. However, you must not use the given answer in your working out.

Exercise 8.08 Problems involving the sine and cosine rules

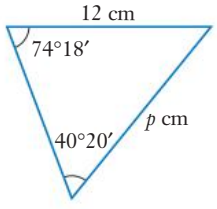
Example 15

1 Find the value of each pronumeral, correct to 2 decimal places.

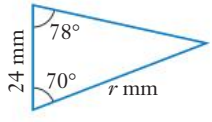
a



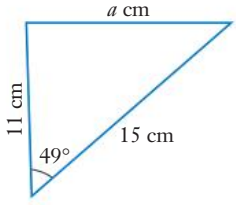
b



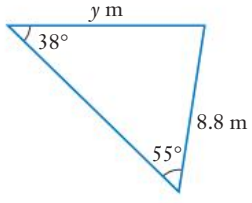
c



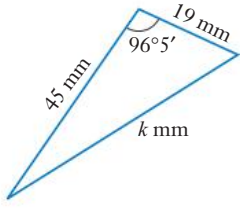
d



e



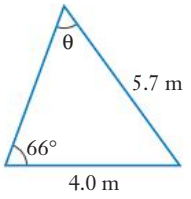
f



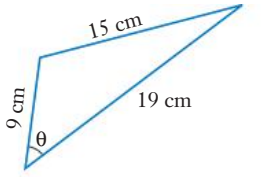
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2 Find the value of θ, to the nearest degree.

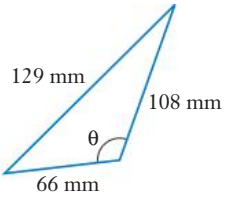
a



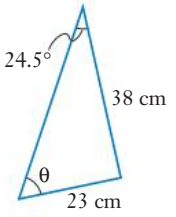
b



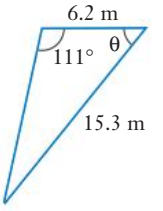
c



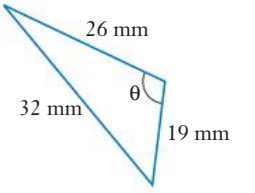
d



e



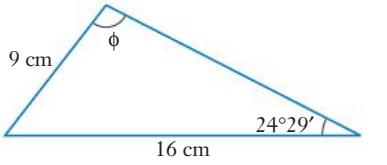
f



3 Find the size of ϕ if it is obtuse. Select **A**, **B**, **C** or **D**.

A 47°27′ **B** 132°33′

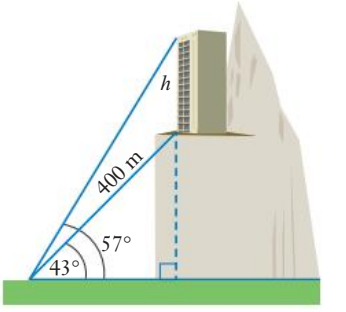
C 135°48′ **D** 166°31′



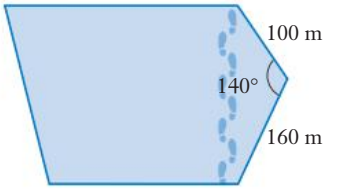
4 Tashi observes the top and bottom of a mountain resort to be at angles of elevation of 57° and 43° respectively. He is 400 m from the bottom of the resort.

a Copy this diagram and find the sizes of all angles.

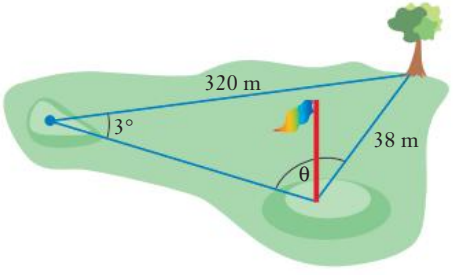
b Find the height of the resort, correct to 2 significant figures.



5 Instead of walking along 2 edges of a park, Megan takes a shortcut through the park as shown. Calculate the distance saved (to the nearest metre) by taking the shortcut.

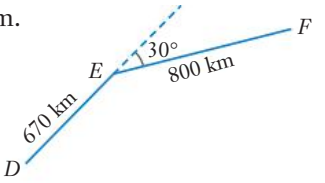


6 A golfer drives the ball 320 m at an angle of 3° off-centre. The ball lands 38 m from the hole. Calculate the obtuse angle θ, to the nearest minute.



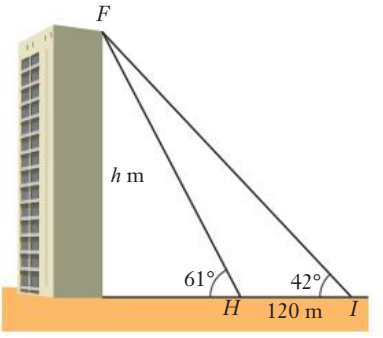
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7 A plane flies from Derby (D) to Eyre (E), a distance of 670 km. It then turns 30° and flies another 800 km to Frankston (F). Calculate the distance between Derby and Frankston to the nearest kilometre.



Example 16

8 At point I, Sunil observes the top, F, of an office tower at an angle of elevation of 42°. He walks 120 m towards the tower to H where the angle of elevation increases to 61°.

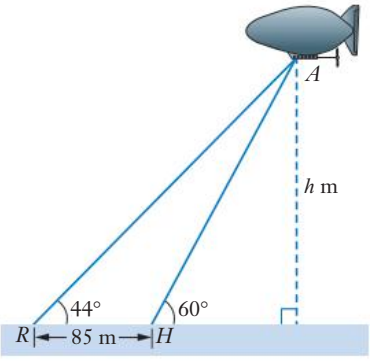


a Explain why $\angle IFH = 19°$.

b Show that $FH = \dfrac{120 \sin 42°}{\sin 19°}$.

c Hence show that $h = \dfrac{120 \sin 42° \sin 61°}{\sin 19°}$ and evaluate h, to the nearest metre.

9 Hayley (H) and Rhonda (R), standing 85 m apart, observe an airship (A) at angles of elevation of 60° and 44° respectively.



a Prove that the distance between Hayley and the airship is given by

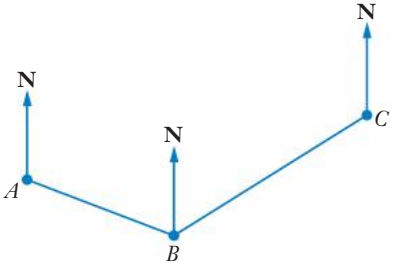
$$HA = \frac{85 \sin 44°}{\sin 16°}$$

b Hence, calculate the height of the airship, h metres, correct to 2 significant figures.

10 A parallelogram has adjacent sides of length 7 cm and 5.2 cm and included angle 130°. Calculate the length of its longer diagonal. Select **A**, **B**, **C** or **D**.

A 5.4 cm **B** 11.1 cm **C** 12.3 cm **D** 29.2 cm

11 A yacht sails 7 km from A to B on a bearing of S 56° E. It then sails 10 km to C. The size of $\angle ABC$ is 116°.



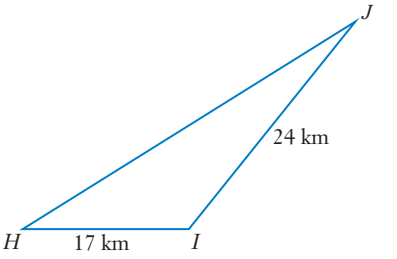
a What is the compass bearing of C from B?

b Calculate the distance from A to C. Answer correct to the nearest kilometre.

c What is the compass bearing of A from C? Answer to the nearest degree.

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12 In the diagram below, Herne (H), Innes (I) and Joytown (J) represent the locations of 3 towns. Innes is due east of Herne and the true bearing of Joytown from Innes is 052°.

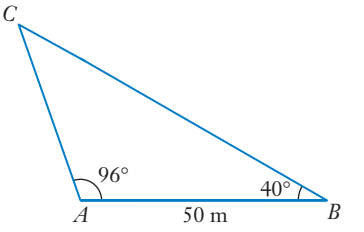


a Find the size of $\angle HIJ$.

b Find the distance from Herne to Joytown, correct to one decimal place.

c Calculate the bearing of Innes from Joytown.

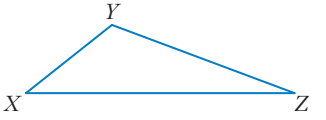
13 This figure shows an example of **triangulation**: surveying using angles of a triangle. A baseline, AB, is chosen and measured. A third point, C, is positioned and the angles from A and B are measured. If $\angle A = 96°$, $\angle B = 40°$ and $AB = 50$ m, calculate the lengths of AC and CB, correct to 2 decimal places.



SAMPLE HSC PROBLEM

An orienteering course is in the shape of a triangle as shown. The first leg of the course XY is 3.2 km long. Z is due east of X and the bearing of Y from Z is 297°. Y is on a bearing of 052° from X.

a Copy and complete this diagram of $\triangle XYZ$, showing all information given.



b Find the length of legs YZ and ZX and hence the length of the course, correct to one decimal place.

c If Marcus ran the course at a speed of 7.3 km/h, how long did it take him to complete the course? Answer to the nearest minute.

d Find the area of $\triangle XYZ$, correct to the nearest square metre.

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Study tip

Keeping it all in perspective

There *is* life after the HSC. Despite what many people will tell you, it's not the most important time in your life. Sure, it is one of the turning points, but 12 months from now the HSC will be a distant memory and you'll be laughing about how much stress it caused you.

Every year when the high achievers of the HSC are interviewed by the media, there are stories of hard work and determination. But these successful students also kept it all in perspective and remembered to maintain balance in their lives. They didn't neglect their other needs: sport, recreation, going out, talking to their families, watching movies, listening to music. So don't forget to have a life. Plan to do nothing once in a while. Breathe, live, exercise, meditate.

When planning your study, be aware that you only have a limited amount of time. It is not possible to do everything, so make the most of the time you do have and be prepared to compromise. Set realistic goals. Spend time on what's important. Don't overcommit yourself. Be reasonable and practical in your expectations.

If you're making up for lost time or past failures remember this: you cannot change the past, so don't dwell on it. What's done is done. You only have control of your present and future. Do the best you can now. Be motivated and confident. Start studying today!

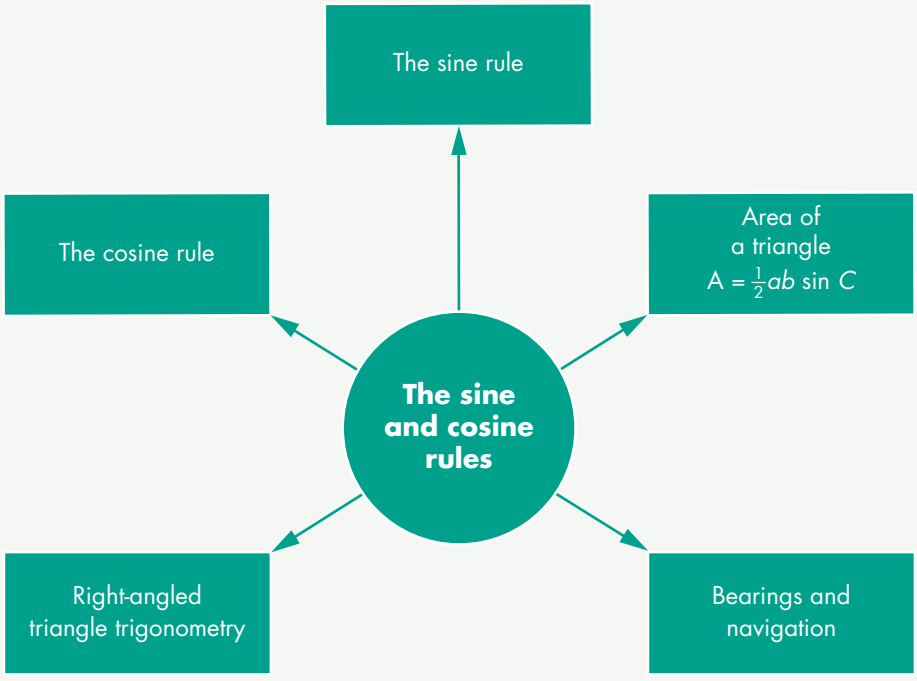
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8. CHAPTER SUMMARY

Trigonometry crossword

This chapter, The sine and cosine rules, extended trigonometry theory to non-right-angled triangles. There were 3 main formulas covered: the sine rule, the cosine rule and the sine formula for the area of a triangle. Aim to become competent and confident in using these formulas, especially when applying them to practical problems. Be sure to differentiate between the basic sine, cosine and tangent ratios for right-angled triangles, and the sine and cosine rules for non-right-angled triangles. Also practise solving problems involving the use of bearings and angles of elevation and depression.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

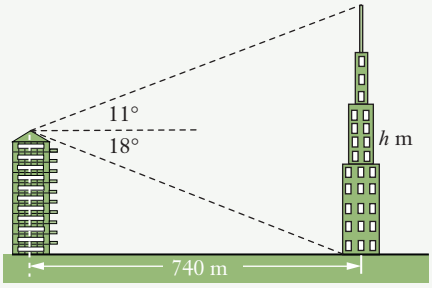


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8. TEST YOURSELF

Exercise 8.01

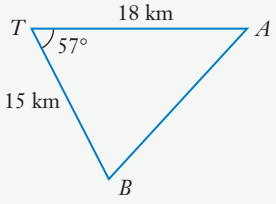
1 Libby stands on the rooftop of her apartment block and observes a tower 740 m away. The angle of elevation of the top of the tower is 11° and the angle of depression of the bottom is 18°. Calculate the height of the tower, to the nearest metre.



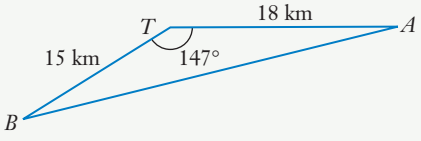
Exercise 8.02

2 Two cars left a town, T, on different roads. Car A travelled 18 km on a bearing of 088° while car B travelled 15 km on a bearing of 145°. Which diagram accurately shows this information? Select **A**, **B**, **C** or **D**.

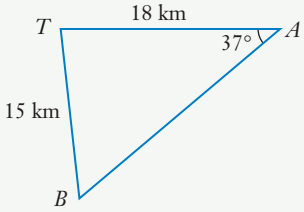
A



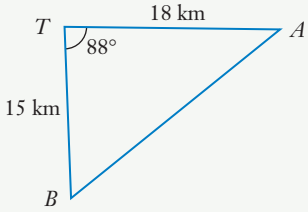
B



C



D

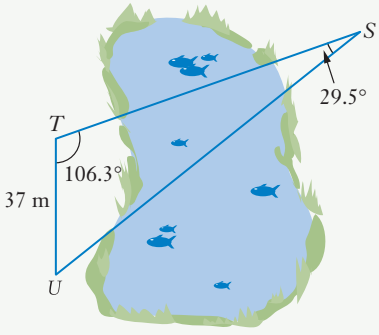


Exercise 8.02

3 Hockeyville is 4.2 km from Footyfield on a bearing of 200°. How far south of Footyfield is Hockeyville? Give your answer correct to 2 decimal places.

Exercise 8.03

4 Sindy (S) notices 2 trees, T and U, 37 m apart on the other side of a lake. If $\angle T = 106.3°$ and $\angle S = 29.5°$, calculate SU, the distance between Sindy and the second tree, correct to 3 significant figures.

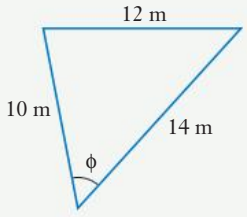


ISBN 9780170413633

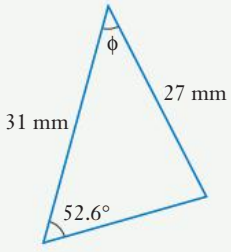
5 Find the size of ϕ in each triangle, correct to the nearest minute.

Exercises 8.04, 8.06

a



b

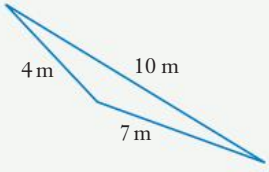


6 Two ships sail from a port. One sails 14 km due south while the other sails 17 km on a bearing of 120°. How far apart are the ships, correct to one decimal place?

Exercise 8.05

7 If the largest angle in this triangle is θ, which of these formulas is correct? Select **A**, **B**, **C** or **D**.

Exercise 8.06



A $\cos\theta = \dfrac{4^2+7^2-10^2}{2\times4\times7}$

B $\cos\theta = \dfrac{4^2+10^2-7^2}{2\times4\times10}$

C $\cos\theta = \dfrac{7^2+4^2-10^2}{2\times7\times10}$

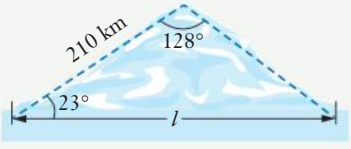
D $\cos\theta = \dfrac{10^2+7^2-4^2}{2\times7\times4}$

8 An isosceles triangle has 2 equal sides of length 5.1 cm and one side of length 8.3 cm. Calculate the size of one of the equal angles, correct to the nearest minute.

Exercise 8.06

9 The largest iceberg ever recorded was found in Antarctica. Its cross-section is roughly the shape of a triangle, illustrated opposite. Calculate:

Exercise 8.07



a its length, l, correct to 2 significant figures

b the area of the cross-section, to the nearest square kilometre.

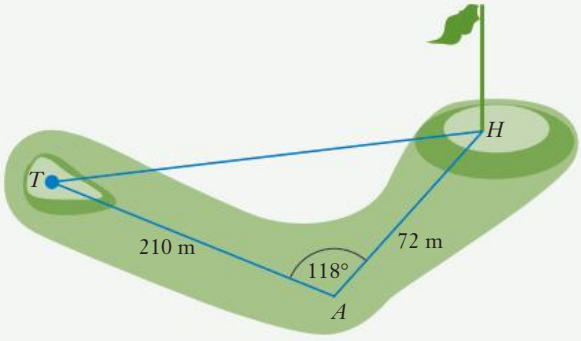
ISBN 9780170413633

Exercise 8.06

10 The towns of Laurieton, Merrylands and Nutbush are linked by three straight roads. Laurieton is 25 km from Merrylands and 20 km from Nutbush. Calculate, to the nearest degree, the size of the angle between the 2 roads coming out of Nutbush if the distance between Merrylands and Nutbush is 11 km.

Exercise 8.05

11 This golf fairway has a dogleg at A.



a How far is the tee, T, from the hole, H, in a direct line? Answer to the nearest metre.

b Find, correct to the nearest square metre, the area of $\triangle TAH$ using the formula $A = \frac{1}{2}ab \sin C$.

Qz Chapter quiz

ISBN 9780170413633

Practice paper 3

Mathematics Standard 2 Reference sheet

Recommended time: One hour

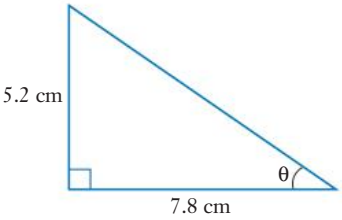
Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Question 1

Exercise 8.01

Find the size of angle θ, correct to the nearest minute.



A 56°19′ **B** 41°49′ **C** 48°11′ **D** 33°41′

Question 2

Exercise 7.03

How much energy does an 18 W compact fluorescent lamp use if it is on for 6 hours per day for one year?

A 6.57 kWh **B** 10.80 kWh **C** 39.42 kWh **D** 584.0 kWh

Question 3

Exercise 7.03

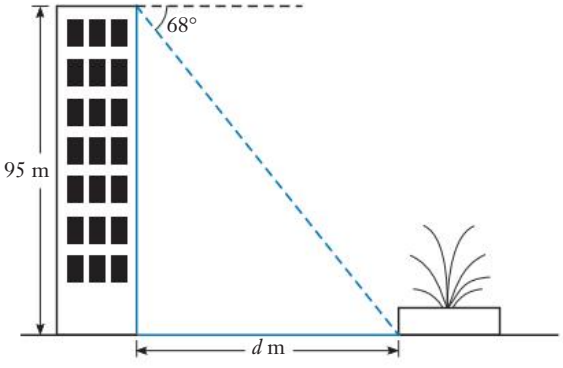
If the cost of electricity is 24.25 cents/kWh, calculate the cost of 780 kWh.

A $18.92 **B** $189.15 **C** $32.16 **D** $321.65

Question 4

Exercise 8.01

From the rooftop of her apartment building 95 m high, Jessie can see the park's fountain at an angle of depression of 68°. Calculate the distance between the fountain and the bottom of the apartment building.



A 36 m **B** 38 m
C 88 m **D** 235 m

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Exercise 7.04

Question 5

The Energy Rating label information on a reverse-cycle air conditioner is shown below.

COOLING	
CAPACITY OUTPUT 4.45 KW	POWER INPUT 1.25 KW
HEATING	
CAPACITY OUTPUT 4.75 KW	POWER INPUT 1.23 KW



Alamy Stock Photo/Global Warming Images

The air conditioner is on heating mode for 5 hours per day for 115 days of the year. How much energy is used by the air conditioner during this period?

A 707 kWh

B 719 kWh

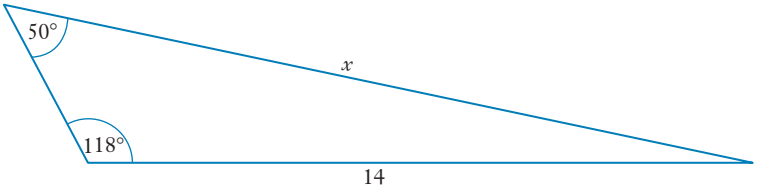
C 2559 kWh

D 2731 kWh

Exercise 8.03

Question 6

Which expression gives the correct value for x?



A $x = \dfrac{14\sin 50°}{\sin 118°}$

B $x = \dfrac{14\sin 118°}{\sin 50°}$

C $x = \dfrac{\sin 50°}{14\sin 118°}$

D $x = \dfrac{\sin 118°}{14\sin 50°}$

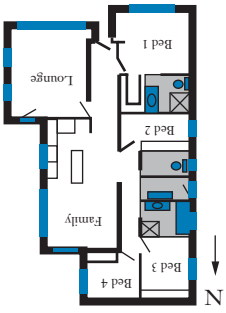
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Question 7

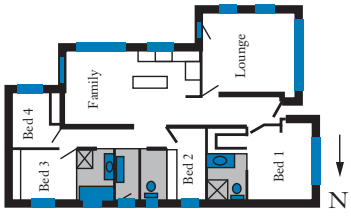
Exercise 7.05

Which diagram shows the most energy-efficient way to position the house on the block of land?

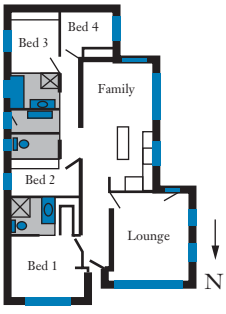
A



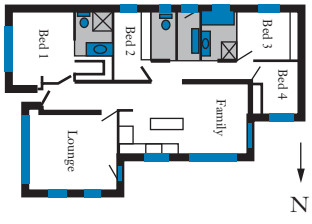
B



C



D



Question 8

Exercise 7.02

For lunch, Alexis has a tuna avocado sushi roll with an energy content of 1272 kJ. When she runs, she burns 1730 kJ of energy per hour. For how many minutes must Alexis run to work off the energy from the sushi roll?

A 23 **B** 44 **C** 52 **D** 74

Question 9

Exercise 7.01

Seth's heart beats 31 times in 24 seconds. What is his pulse rate in beats per minute?

A 37 **B** 78 **C** 86 **D** 144

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Exercise 8.02

Question 10

Minton is due north of Ballington and 54 km from Wellgrove. The bearing of Wellgrove from Ballington is 042° and they are 37 km apart.

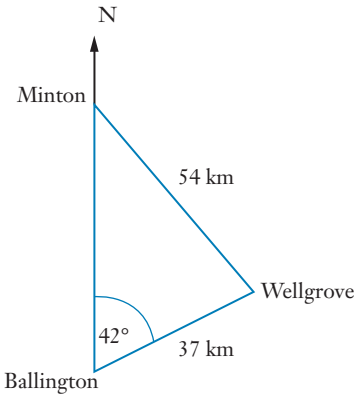
Find the true bearing of Wellgrove from Minton.

A 027°

B 333°

C 146°

D 153°



Section 2

3 questions: 10 marks each

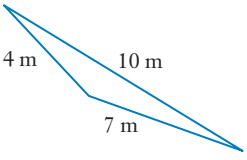
Question 11 (10 marks)

Exercise 7.04

a A family of 4 people uses, on average, 8 minutes of hot water per shower. The shower is connected to a 3.6 kW hot water unit and each person showers once per day. If electricity is charged at 30.8 c/kWh when the hot water unit is being used, what is the weekly cost of the hot water used in showers by this family? **3 marks**

Exercise 8.06

b Find the size of the obtuse angle in this triangle, correct to the nearest minute. **3 marks**



Exercise 8.07

c Hence, find the area of the triangle above, correct to one decimal place. **2 marks**

Exercise 7.02

d Convert 0.8 megajoules to:

i kilojoules **1 mark**

ii calories. **1 mark**

[1 Cal = 4.184 kJ]

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Question 12 (10 marks)

a Paul is aged 19 and has a resting pulse of 75 beats/minute. He does weight training, which is a high intensity exercise. The formulas for maximum heart rate (MHR) and target heart rate (THR) are:

Exercise 7.02

$$\text{MHR for males} = 220 - \text{age in years}$$

$$\text{MHR for females} = 226 - \text{age in years}$$

$$\text{THR} = I \times (\text{MHR} - \text{RHR}) + \text{RHR}$$

where I = exercise intensity as a decimal, according to the table below

RHR = resting heart rate.

Exercise intensity	Low	Medium	High
I	0.4 to 0.5	0.5 to 0.7	0.7 to 0.85

i What is Paul's maximum heart rate? **1 mark**

ii Find Paul's maximum target heart rate for weight training. **1 mark**

b A 3-bedroom house has 20 lights, all of which are compact fluorescent lights (CFLs) of 18 W each. The lights are on for an average of 6 hours per day and the cost of electricity is 24.85 cents/kWh.

Exercise 7.04

i What is the cost of lighting for one year? **2 marks**

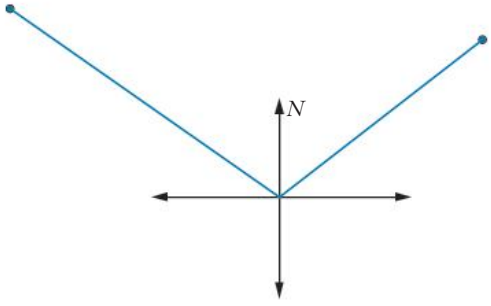
ii If 12 W LED lights are used instead of the CFLs, how much is saved in lighting cost over one year? **2 marks**

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Exercise 8.05

c Caitlin leaves town on a road with a bearing of N 70° W and travels 20 km. Jeremy leaves the same town on a road with a bearing of N 40° E and travels 18 km.

i Copy and complete the diagram below to illustrate the situation, showing all given information. **2 marks**



ii Calculate the distance between Caitlin and Jeremy, correct to the nearest 0.1 km. **2 marks**

Exercise 7.02

Question 13 (10 marks)

a Nikky had a 150 g tub of yoghurt for a snack. The tub contained a label showing the dietary intake information per 60 g serve.



Calculate, in kilojoules, the total amount of energy in this tub of yoghurt. **2 marks**

Exercise 7.03

b Convert 480 MW to GW. **1 mark**

c This table shows the pricing information for an electricity supplier.

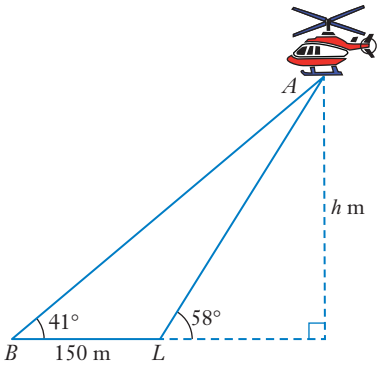
Exercise 7.03

Electricity tariffs	Cost
Supply charge	67.18 c/day
Usage: first 1755 kWh/qtr	24.85 c/kWh
Remaining usage/qtr	27.71 c/kWh

Calculate the amount charged by the supplier for a quarter (92 days) if the usage is 2875 kWh. **3 marks**

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d Bella and Lee, standing 150 m apart, observe a helicopter, A, at angles of elevation of 41° and 58° respectively.



i Explain why $\angle BAL = 17°$. **1 mark**

ii Hence show that $AL = \dfrac{150 \sin 41°}{\sin 17°}$. **1 mark**

iii Hence calculate h, the height of the helicopter above the ground in metres, correct to the nearest whole number. **2 marks**

END OF EXAMINATION

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9.

SCALE MAPS AND PLANS

In this photograph, the ratio of the height of the tyre to the height of the truck is about 5 : 13. The truck in the photograph is a fraction of its real size but without knowing the scale factor, you cannot find the actual size of the truck. Scales can be found on maps, building plans and illustrations.

CHAPTER OUTLINE

M7	9.01	Scale drawings
M7	9.02	Scale maps and plans
M7	9.03	House plans
M6, M7	9.04	Offset and radial surveys
M7	9.05	Volume of tanks and dams



IN THIS CHAPTER YOU WILL:

- apply scale ratios to scale diagrams, maps and plans
- interpret house plans
- calculate perimeters and areas using offset surveys, compass radial surveys and the trapezoidal rule
- calculate volumes of rainfall, water tanks and dams using diagrams and formulas.

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TERMINOLOGY

compass radial survey
offset survey
scale drawing
enlargement
ratio
scale factor
floor plan
reduction
trapezoidal rule

WS
Assignment 9

SkillCheck

1 Simplify each fraction or ratio.

a $\frac{24}{36}$ **b** $\frac{18}{6}$ **c** 12 : 28

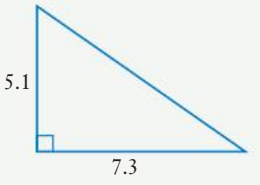
d 27 : 18 **e** $\frac{3}{4} : \frac{5}{8}$ **f** 15 cm : 40 mm

g 1 m : 10 cm **h** 50 m : 500 cm **i** 1 cm : 2 m

2 Find the area of a trapezium with parallel sides 23 m and 42 m and a distance between them of 21 m.

3 Find the area of each triangle, correct to 2 significant figures. Measurements are in metres.

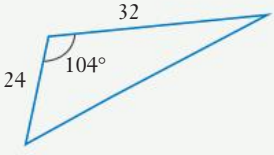
a



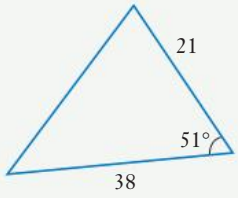
b



c



d



4 Find, correct to 2 significant figures, the length of the unknown side in each triangle in Question **3**.

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9.01 Scale drawings

A **scale drawing** is usually a reduction of the real object it represents but it can also be an enlargement of a small object, such as a computer chip.

The **scale** ratio compares lengths in the scale drawings to actual lengths.

For example, if the scale is 1 : 3, the actual lengths are 3 times the size of the scaled lengths.

WS Scale drawings

Scale drawings

WS Scaled vs actual size

WS Scales and scale diagrams

EXAMPLE 1

By measurement and calculation, find the actual length (in centimetres) of the tennis racquet.



Tennis racquet 1 : 12

Solution

Scaled length of racquet = 5.5 cm — By measurement

Actual length of racquet = 12×5.5 cm

$= 66$ cm

A scale of 1 : 12 means the racquet is 12 times larger than its image.

Scales

The scale on a map, plan or illustration can be represented in several ways.

EXAMPLE 2

Write each scale shown below as a ratio.

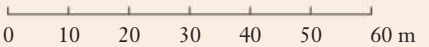
a



b 10 mm to 1 m

c 1 cm = 0.4 mm

d



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Solution

a Scale = 1 cm : 5 km — By measurement, 1 cm represents 5 km.

= 1 cm : 500 000 cm — Converting 5 km to cm.

= 1 : 500 000

b Scale = 10 mm : 1 m

= 10 mm : 1000 mm — Converting 1 m to mm.

= 1 : 100

c Scale = 1 cm : 0.4 mm

= 10 mm : 0.4 mm — Converting 1 cm to mm.

= 10 : 0.4

= 100 : 4 — Multiplying both terms by 10.

= 25 : 1

A scale of 25 : 1 indicates an enlargement as the scale diagram is 25 times larger than the object.

d Scale = 8 mm : 10 m — By measurement, 8 mm represents 10 m.

= 8 mm : 10 000 mm — Convert 10 m to mm.

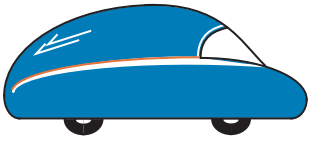
= 1 : 1250

Exercise 9.01 Scale diagrams

Example 1

1 By measurement and calculation, find the real length of each object.

a



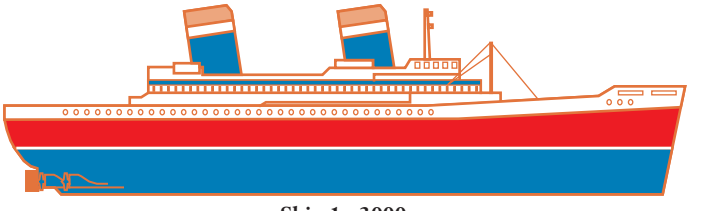
Robocar 1 : 100

b



Ant 4 : 1

c



Ship 1 : 3000

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Example 2

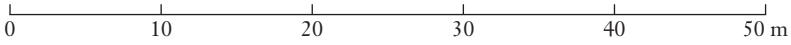
2 Write each scale as a ratio in simplest form.

a



b 2 mm to 1 cm

c



d 10 cm = 1 m

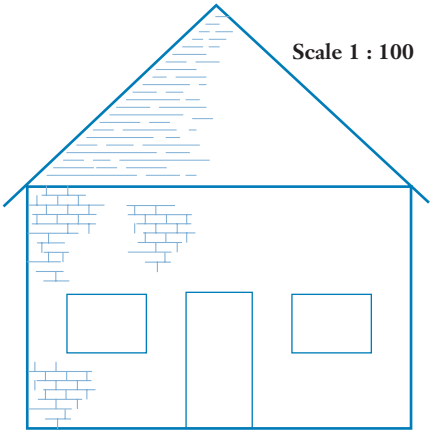
e 5 m : 2 cm

3 Refer to the scales in Question **2**.

a Which has a scale factor of 5?

b Which indicates an enlargement?

4 This diagram of the front of a house is drawn to a scale of 1 : 100.



a By measurement and calculation, find the actual height, in metres, of the door in the diagram.

b What is the scaled length, in millimetres, of the house if it is actually 9.5 m long?

5 The scale on a map of Australia is 1 : 6 000 000. If the distance from Perth to Kalgoorlie on the map is 6 cm, which of the following is the actual distance from Perth to Kalgoorlie? Select **A**, **B**, **C** or **D**.

A 100 km **B** 360 km **C** 1000 km **D** 3600 km

6 A photograph of Tony shows him to be 3.5 cm tall. If the photograph has a scale of 1 : 50, what is Tony's real height?

7 If Nadine is 168 cm tall, what is her height on a photograph with a scale of 1 : 30?

8 The distance from Sydney to Penrith is 55 km. What scaled distance would this be on a map with a scale of 1 : 2 000 000?

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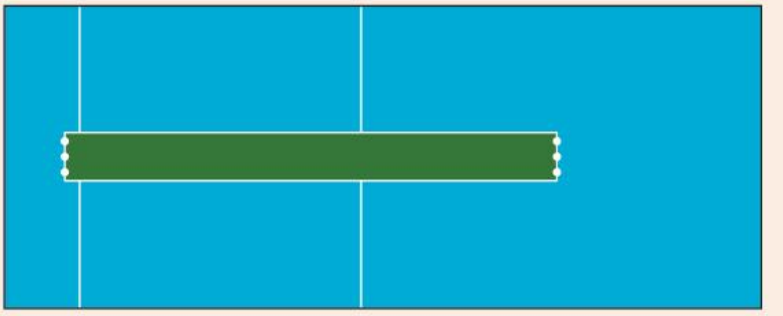
WS
Scale maps

9.02 Scale maps and plans

Scale maps and plans are 2 common examples of scale drawings.

EXAMPLE 3

Here is a scale plan of an indoor cricket area.



Scale: 1 cm to 3 m

a Write the scale as a ratio.

b What is the length of the cricket pitch (shaded green on the plan)?

c What is the size of the playing area?

Solution

a Scale = 1 cm : 3 m

= 1 cm : 300 cm

= 1 : 300

b Scaled length of pitch = 6.5 cm

Actual length = 6.5×3 m

= 19.5 m

$\approx$ 20 m

1 cm on scale plan represents 3 m

c Scaled length of playing area = 10 cm

Actual length = 10×3 m = 30 m

Scaled width of playing area = 4 cm

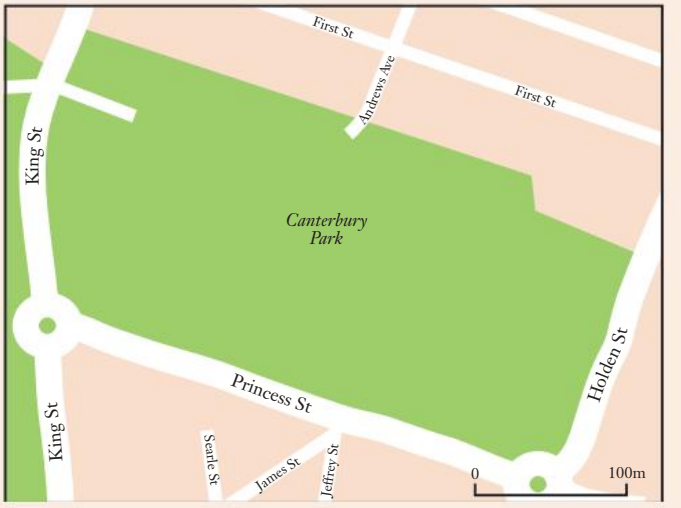
Actual width = 4×3 m = 12 m

$\therefore$ Playing area = $30 \times 12 = 360$ m^2

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EXAMPLE 4

Here is a scale map of Canterbury Park, which is roughly in the shape of a trapezium.



a What does 1 cm represent on this map?

b Estimate the perimeter of Canterbury Park.

c Estimate the area of Canterbury Park.

Solution

a Scale = 2 cm : 100 m

= 1 cm : 50 m

1 cm represents 50 m.

b Using a ruler, measure the length of each boundary of the park on the scale map and convert them to real lengths using the scale 1 cm : 50 m.

Top parallel side = 7.6 cm × 50 = 380 m

Right side = 3.6 cm × 50 = 180 m

Bottom parallel side = 6.8 cm × 50 = 340 m

Left side = 3.7 cm × 50 = 185 m



Perimeter of park = 380 + 180 + 340 + 185

= 1080 m

Area of a trapezium formula

$$\text{Area of park} = \frac{180}{2}(380+340)$$

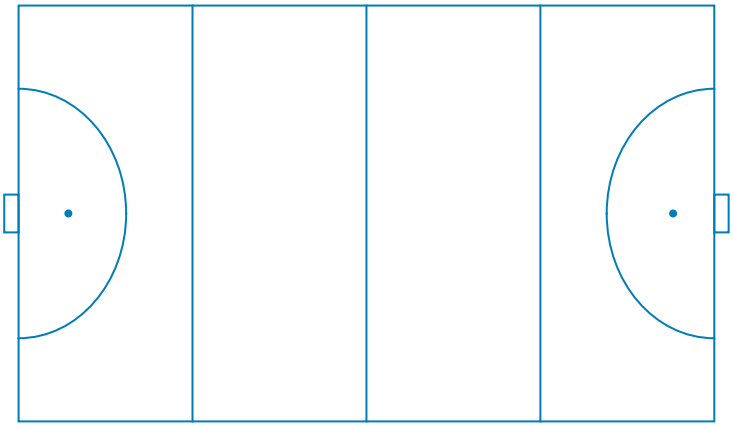
$$= 64\,800 \text{ m}^2$$

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Exercise 9.02 Scale maps and plans

Example 3

1 Here is a scale plan of a hockey field.



Scale: 1 cm = 10 m

a What is the scaled width of the hockey field?

b What is the width of the hockey field?

c What is the length of the hockey field?

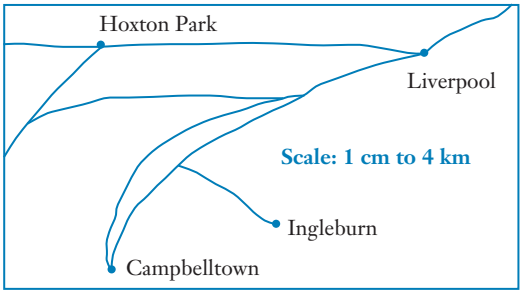
d Hence find the area of the field.

2 Use this scale map to find the actual distance (in a direct line) between these locations.

a Liverpool and Campbelltown

b Hoxton Park and Ingleburn

c Campbelltown and Hoxton Park



3 For this map, find:

a the scale ratio

b the distance (in a straight line) from:

i Sydney to Katoomba

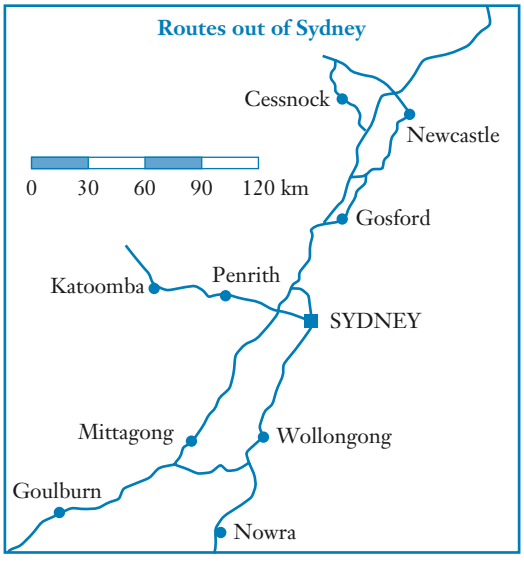
ii Newcastle to Nowra

iii Goulburn to Gosford

iv Sydney to Cessnock.

c the town that is 92 km from Sydney

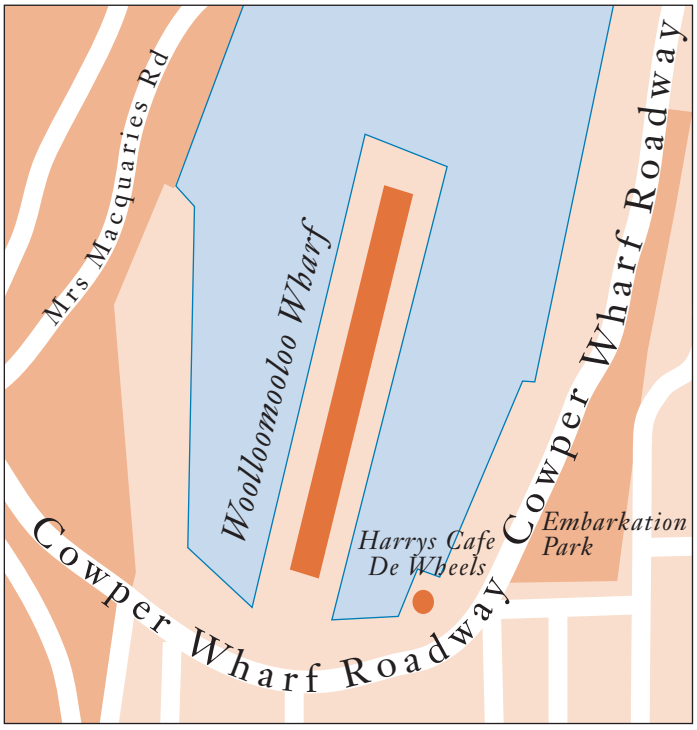
d the towns that are exactly 40 km apart.



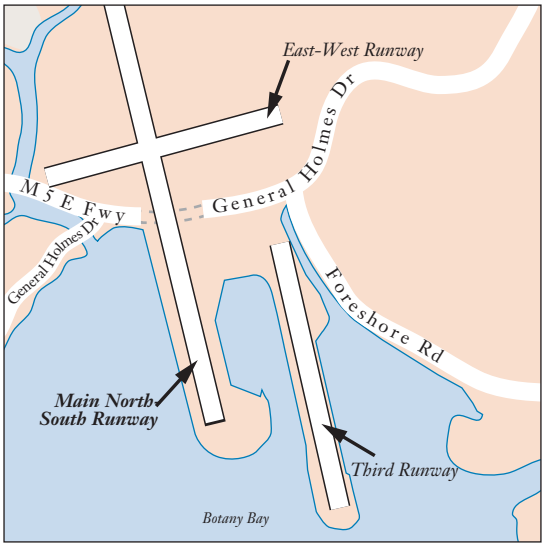
ISBN 9780170413633

4 Measure each feature described to find the scale of each map as a ratio in the form $1 : r$, where r is rounded to the nearest whole number.

a Woolloomooloo Wharf in Sydney is 410 m long.



b The north–south runway at Sydney airport is 3960 m long.



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c Conrod Straight at Mount Panorama in Bathurst is 1.916 km long.



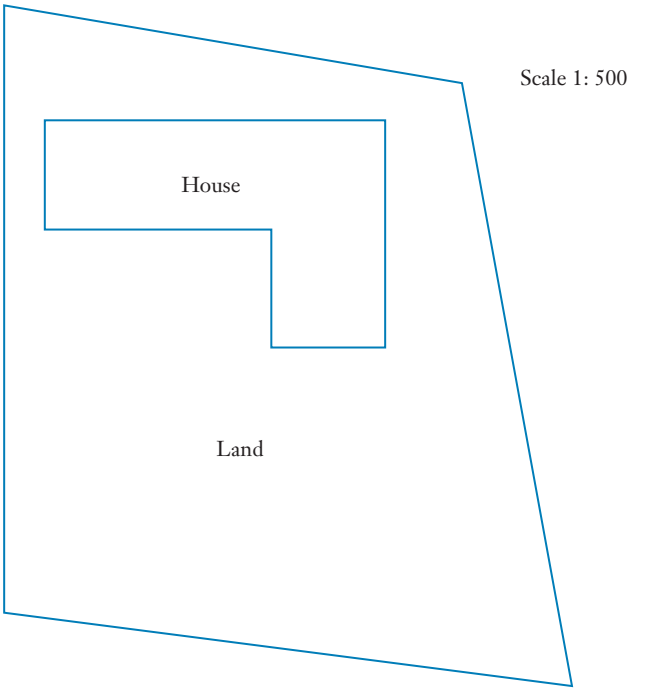
Example 4

5 For each map, use the scale to measure and calculate (to the nearest whole unit):

i the perimeter of the location

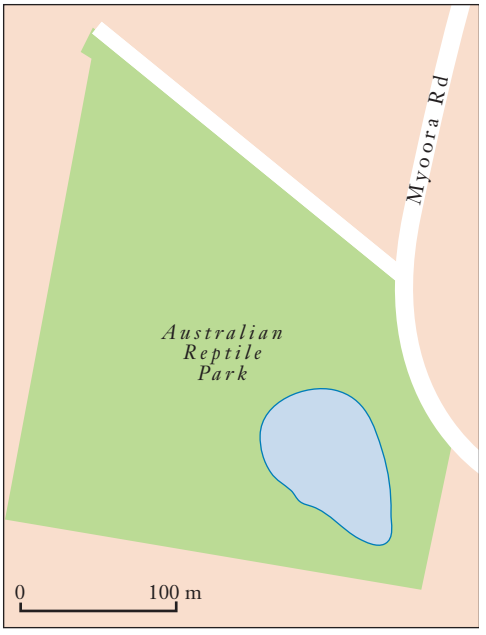
ii the area of the location, which is approximately a trapezium.

a a house block

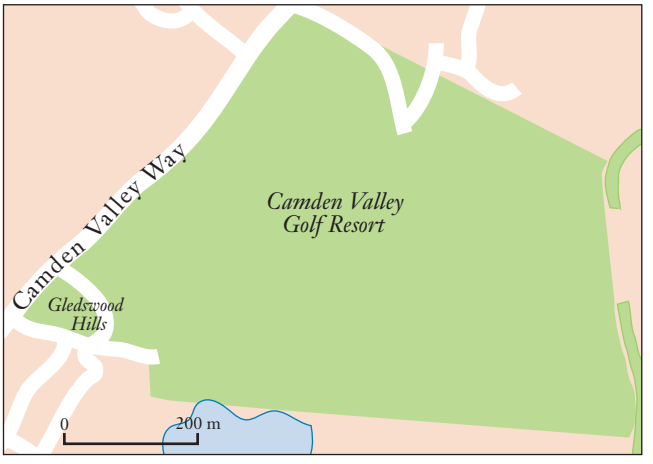


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b Australian Reptile Park



c Camden Golf club and Gledswood Hills



6 Use either method below to measure the length of each location given:

- **i** the **Google Maps** website and use the online tools
- **ii** **Google Earth** and from the menu choose **Tools** and **Ruler** and **Line** (settings should be in km). Click on each end of the location to calculate its length.

a Anzac Parade in Canberra (which leads to the Australian War Memorial)

b Sydney Harbour Bridge

c Queen Victoria Building in George St, Sydney

d a runway at Bankstown Airport or at your closest airport.

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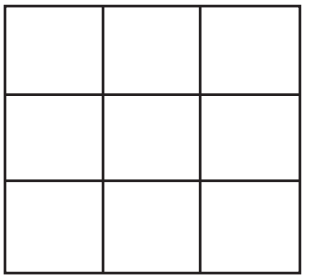
INVESTIGATION

METHODS FOR ESTIMATING LAND AREAS

There are different methods for estimating areas of land and catchment areas such as reservoirs and dams.

The Brickpit is a large pond at Sydney Olympic Park with a raised circular walkway for observing the endangered green and golden bell frog. We will use two approximation methods for calculating its area.

GRID SQUARE METHOD







Map data ©2018 Google

1 Search 'Brickpit Park' at **Google Maps** and find its image in Satellite view.

2 Click '+' zoom until the scale is 100 m.

3 Print out the image or copy-and-paste it into a word processor or graphics file. Create a square grid over the image using the scale given. Each square below has an actual length of 100 m.

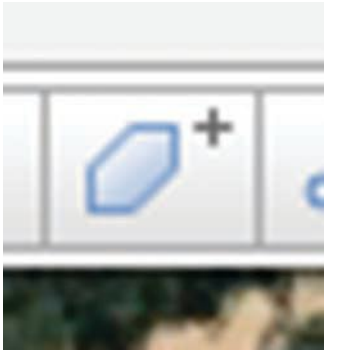
4 Estimate the area of the circle in grid squares, approximately 4.

Each grid square is $100\text{ m} \times 100\text{ m} = 10\,000\text{ m}^2$

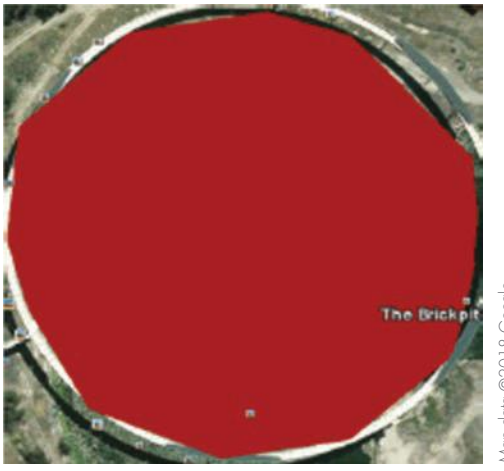
Area of the circle $\approx 10\,000 \times 4 = 40\,000\text{ m}^2$

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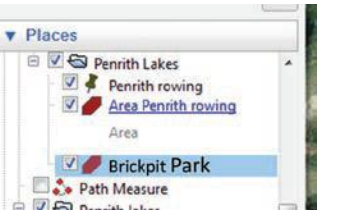
POLYGON METHOD



1. Fly To 'Brickpit Park' using Google Earth Pro software.
2. Zoom in until the scale is 100 m.
3. Right-click **Places** (on left), select **Add** and **Folder**. Call it 'Brickpit Park'.
4. Select **Add Polygon** (top of screen – tools).



5. Click around the perimeter of the circle to create a polygon. The more 'clicks' or vertices you create, the more accurate the area calculation will be.
6. Copy the image to another website to calculate its area. In the left column, right-click on Brickpit Park and **Copy**.



7. Visit the **Earth Point** website, select **Polygon Area** from the menu, and paste the image into the box that appears. Choose **square metres** and **View on Webpage**. A box that shows the approximate area should appear underneath.

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8 For each map below, use the Grid Square or Polygon method to estimate the area of the specified location.

a Sydney International Aquatic Centre, Homebush



b Rowing Centre, Penrith



c Jervis Bay



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9.03 House plans

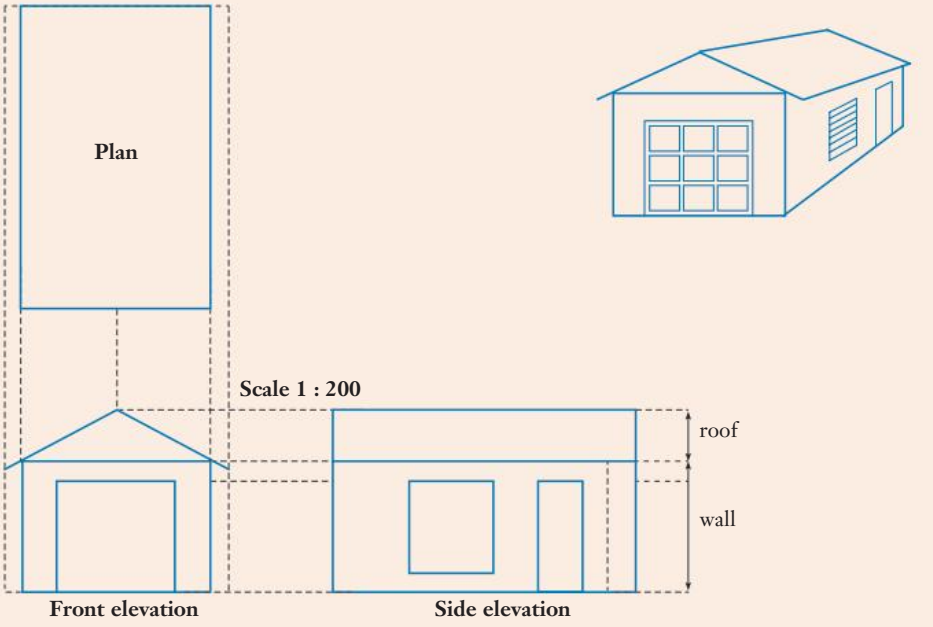
WS
Interpreting an office plan

An architect or draftsperson is employed to construct plans for new buildings and additions to existing buildings. These plans are then used by the builder. Two sets of drawings are drawn to scale:

- the plan: a diagram of the floor (**floor plan**)
- the **elevations**: the views of the front, back and sides of the house.

EXAMPLE 5

This is a drawing of a garage, showing its plan, front and side elevations.



a What is the floor area of the garage in square metres?

b What is height of the highest point of the garage's roof?

c The four inside walls of the garage need to be painted, including the doors but not the window (of dimensions 2.6 m by 2.2 m). How many whole litres of paint would you need to buy to put two coats of paint on the walls if 1 L covers 10 m^2?

Solution

a Scaled width of garage = 2.5 cm — By measurement

Actual width = 2.5 cm × 200 = 5 m — Scale factor is 200

Scaled length of garage = 4 cm

Actual length = 4 cm × 200 = 8 m

Floor area = 5 m × 8 m = 40 m^2

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b Scaled height of highest point = 2.4 cm

Actual height = 2.4 cm × 200 = 4.8 m

c Find the total area of the garage walls.

Scaled height of ceiling = 1.7 cm

Actual height of ceiling = 1.7 cm × 200 = 3.4 m

Width of garage = 5 m — From part **a** above

Area of front and back walls = $2 \times 5 \times 3.4 = 34$ m^2

Length of garage = 8 m — From part **a** above

Area of side walls = $2 \times 8 \times 3.4 = 54.4$ m^2

Area of window = $2.6 \times 2.2 = 5.72$ m^2

Area to be painted = $(34 + 54.4 - 5.72)$ m^2 = 82.68 m^2

But area with two coats = (2×82.68) m^2 = 165.36 m^2

Amount of paint needed = 165.36 ÷ 10 L

= 16.536 L

≈ 17 L

Symbols on house plans

House plans usually contain symbols and abbreviations showing features such as doors and toilets. You should be familiar with these when reading house plans.

	Hinged door	Ens	Ensuite (bathroom)
	Toilet (water closet) or WC		Bath
	Shower		Vanity
	Stove		Kitchen sink
Ptry	Pantry		Laundry tub
	Washing machine		Linen cupboard
Robe	Wardrobe	WIR	Walk-in (ward)robe
FL	Floor level	GL	Ground level
CL	Ceiling level		Window

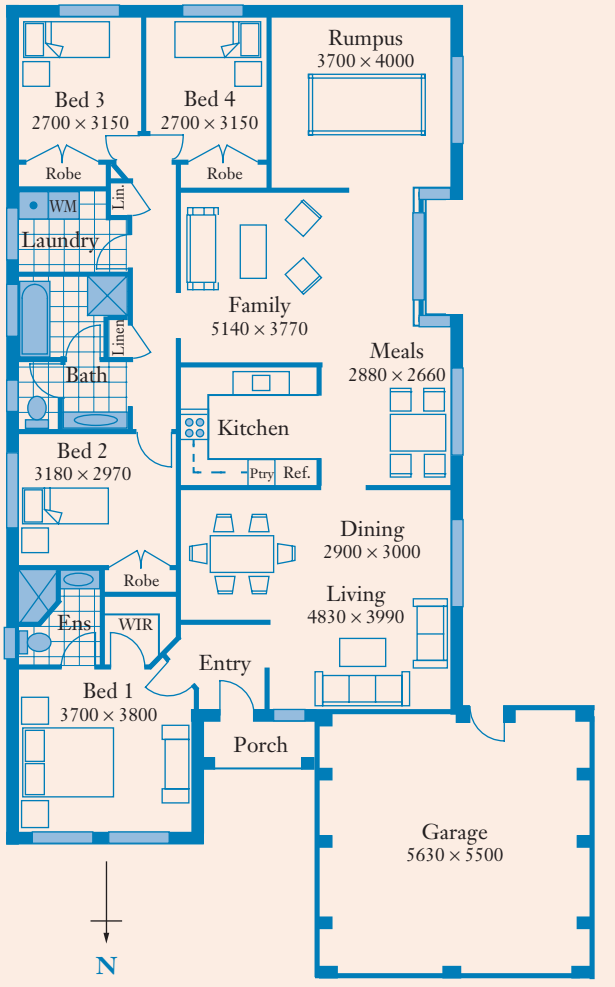
Measurements on plans are always shown in **millimetres** because this avoids the use of decimal points, which can lead to errors in printing and reading.

Not all plans are scale diagrams. If this is the case, 'not to scale' is written on the plan.

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EXAMPLE 6

This house plan, not to scale, shows dimensions in mm.



a Write the symbol for a window and count the number of windows on the plan.

b Count the number of hinged doors on the plan, excluding those on wardrobes (robe, WIR) or linen cupboards.

c How many showers are there in this house?

d What is used to indicate the wall cupboards in the kitchen on this plan?

e What is the area of the family room, correct to the nearest square metre?

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Solution

a There are 16 windows.

b There are 10 hinged doors.

c There are 2 showers.

d Dotted lines.

e Area of the family room = 5140 mm × 3770 mm

= 5.14 m × 3.77 m Converting mm to m

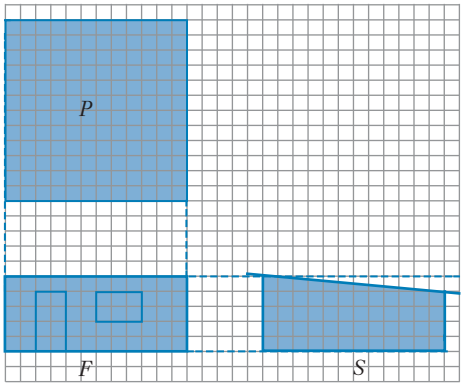
= 19.3778 m^2

≈ 19 m^2

Exercise 9.03 House plans

Example 5

1 The scaled plan for a fishing shack is shown. The floor plan is labelled *P*, the front elevation is labelled *F* and the side elevation is labelled *S*.



Scale 1 cm : 2.5 m

a What is the height of the back wall?

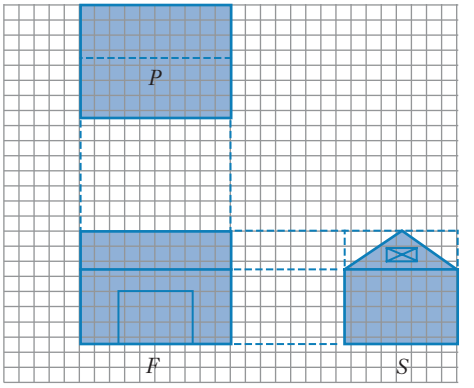
b What is the width of the shack?

c What is the floor area?

d How many whole litres of paint are required to paint the outside walls of the shack, including the door but not the window, if 1 L of paint covers 10 m^2?

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2 The scaled plan for a barn is shown.



Scale 1 : 400

a What is the height of the highest point of the roof from the ground?

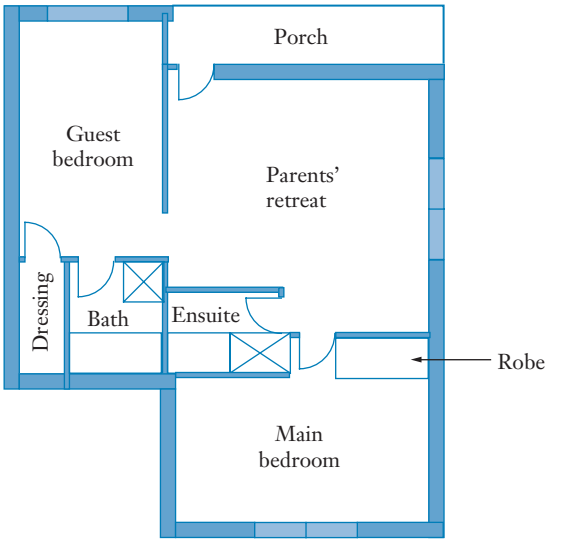
b What is the length of the barn?

c What is the floor area?

d What is the cost of tiling the floor if tiles are \$125.50 per m^2 laid?

3 This is the floor plan of an extension to Tiffany and Pete's house.

New extension



Scale 1 : 200

a What is the internal area of the extension? Answer to the nearest m^2.

b Find (to the nearest \$100) the cost of carpeting the guest bedroom if carpet is \$38 per m^2 laid.

c Find (to the nearest \$100) the cost of tiling the porch if tiles are \$64.50 per m^2 laid.

Example 6

4 Refer to the house plan in Example **6** on page 405.

a How many bedrooms are in the house?

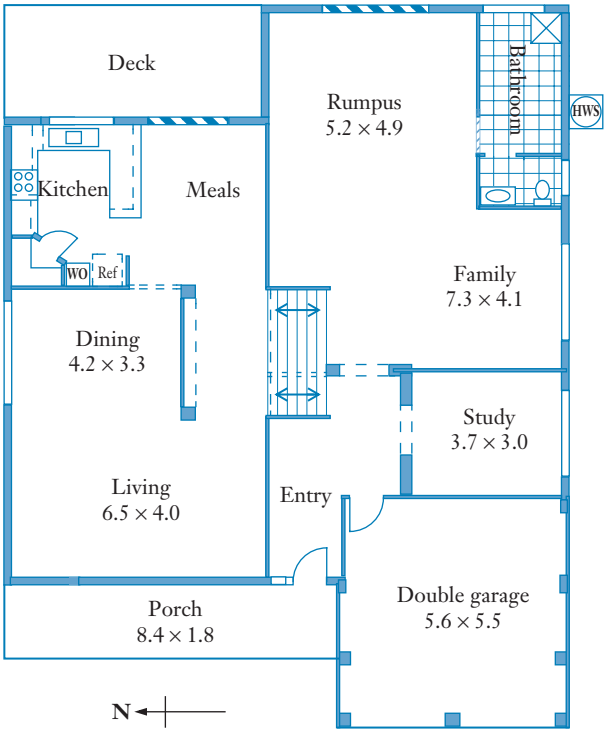
b What does the abbreviation WIR stand for?

c How many baths are in the house?

d In which room is the pantry?

e What is the area of the combined living and dining room, to the nearest m^2?

5 This is the floor plan of a split-level home.



a How many steps do we walk down from the family room to the living room?

b Draw the symbols used on the plan to represent each feature.

i hot water service **ii** hotplates **iii** wall oven

iv front door **v** sliding door onto the deck.

c The porch is to be tiled with terracotta tiles. How many square metres of tiles are needed (rounded up to the nearest 0.5 m^2)?

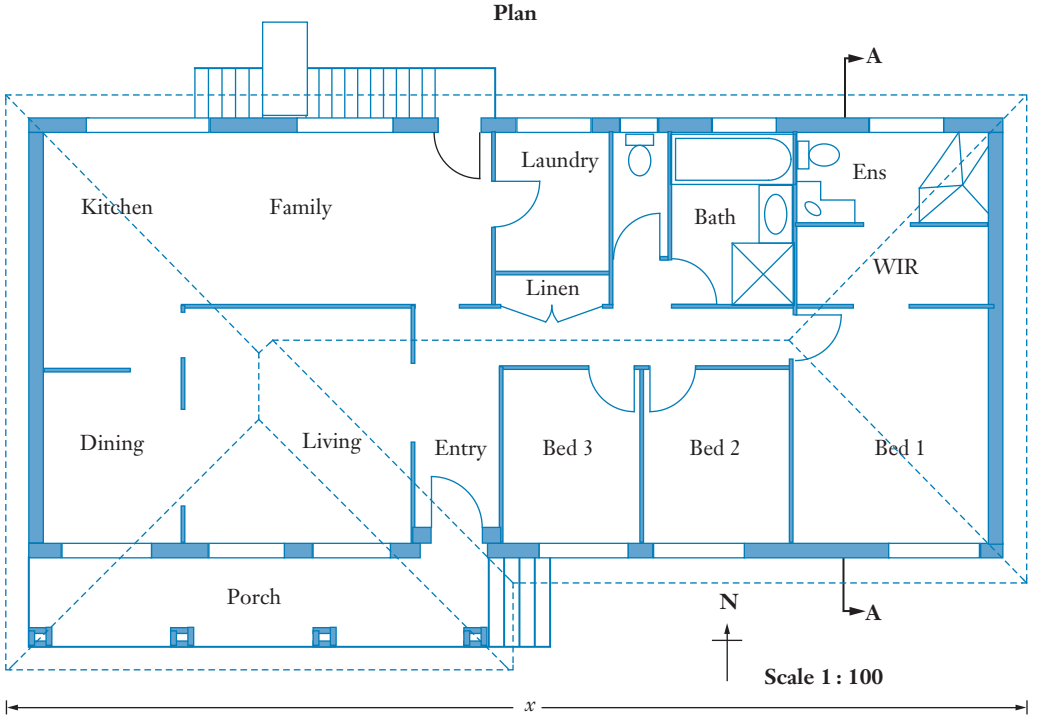
d If each tile measures 30 cm by 30 cm, how many whole tiles are needed to tile the porch?

e The dining and living areas are to be carpeted. How much (correct to the nearest \$100) will it cost to carpet the rooms if carpet costs \$145 per m^2 laid?

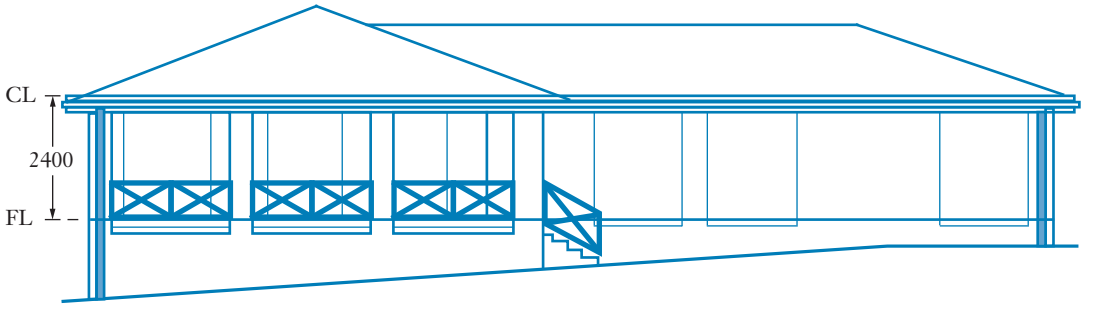
ISBN 9780170413633

6 The floor plan and south elevation of a home built on a sloping block of land are shown.

Plan



South elevation



a What scale is shown on the plan?

b What is the actual length of the house (denoted by x)?

c How many windows are there in the house?

d How many external doors are there in the house?

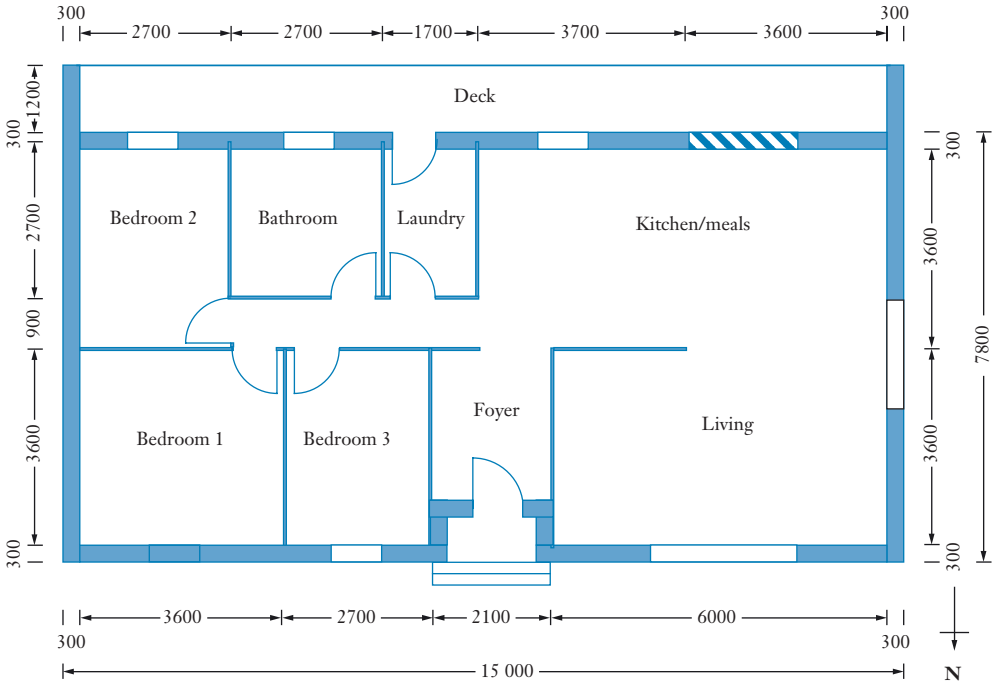
e What is the area of the porch? Give your answer to the nearest m^2.

f What is the ceiling height?

g The entry and hallway are to have ceramic tiles. These cost \$155 per m^2 laid. How much will it cost to have the entry and hallway tiled? Give your answer rounded up to the nearest \$100.

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7 This is the plan for a 3-bedroom house. The dimensions are in millimetres.



a What are the internal dimensions of bedroom 1?

b A wardrobe, 600 mm wide, is to be built along the eastern wall of bedroom 1.

i What is the maximum length the wardrobe can be?

ii What floor area will be left in the bedroom to carpet?

c What is the area of the deck (to the nearest square metre)?

d What is the overall area of the house (to the nearest square metre)?

e How much will it cost to build this house (including the deck) if the current building rate is $1450 per square metre?

f The bathroom floor is to be tiled with 20 cm square ceramic tiles. Find:

i the number of tiles (rounded to the nearest whole number) that will fit along the length of the bathroom floor

ii the number of tiles required.

INVESTIGATION

HOUSE PLANS

1 Collect plans from newspapers, magazines or the Internet. Discuss the various features, the symbols used and the size of rooms.

2 Choose one house plan and say what it is you like about the plan. Is it easy to read? Does it give you a clear idea of what the actual house will look like?

3 Draw a scaled floor plan of your house, apartment or classroom.

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9.04 Offset and radial surveys

WS
Offset surveys

Surveying involves setting out the positions of proposed construction or engineering works. It involves measuring the boundaries of a field and calculating perimeters and areas.

When **right-angled triangles** are involved, surveyors use Pythagoras' theorem, the 3 trigonometric ratios and the area formula $A = \frac{1}{2}bh$.

When **non-right-angled triangles** are involved, surveyors use the sine and cosine rules and the area formula $A = \frac{1}{2}ab\sin C$.

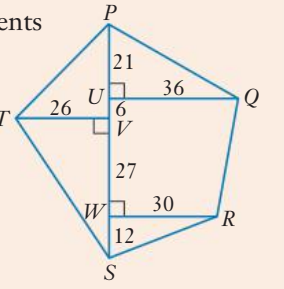
The area of a trapezium formula is also used: $A = \frac{h}{2}(a+b)$.

Offset survey

One method that surveyors use to measure the area of an irregular block of land is an **offset survey**, also called a **traverse survey**. The surveyor chooses a diagonal of the field to be the **traverse line** (traverse means 'to cross'), then measures the offsets (perpendicular distances) to each corner.

EXAMPLE 7

A surveyor drew this diagram of a field, *PQRST*, where measurements are in metres.



a Calculate the area of the field, correct to 2 significant figures.

b Calculate the length, *QR*, correct to the nearest metre.

c Find the perimeter of the field.

Solution

a The area (in m^2) of the field can be found by adding together the areas of 3 triangles and one trapezium.

$$\text{Area } \triangle PUQ = \frac{1}{2} \times 36 \times 21$$
$$= 378$$

$$\text{Area } \triangle RWS = \frac{1}{2} \times 30 \times 12$$
$$= 180$$

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Area $\triangle PTS = \frac{1}{2} \times 66 \times 26$ $PS = 21 + 6 + 27 + 12 = 66$

$= 858$

Area trapezium $UQRW = \frac{33}{2} \times (36 + 30)$ $UW = 6 + 27 = 33$

$= 1089$

Total area of field $= 378 + 1089 + 180 + 858$

$= 2505$

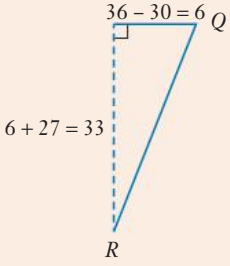
$\approx 2500 \text{ m}^2$

b The length of QR can be found using Pythagoras' theorem by drawing a perpendicular from R to UQ at X.

$QR = \sqrt{33^2 + 6^2}$

$= 33.5410 \ldots$

$\approx 34 \text{ m}$



c To find the perimeter, first use Pythagoras' theorem to find the lengths of the other four sides, correct to the nearest metre.

$PQ = \sqrt{21^2 + 36^2} \approx 42$

$RS = \sqrt{30^2 + 12^2} \approx 32$

$ST = \sqrt{39^2 + 26^2} \approx 47$

$TP = \sqrt{26^2 + 27^2} \approx 37$

Perimeter of the field $= 42 + 32 + 47 + 37 + 34$

$= 192 \text{ m}$

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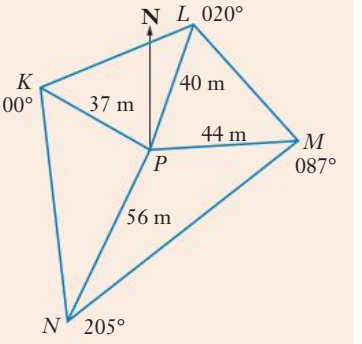
Compass radial survey

A **compass radial survey** involves taking the true bearing of each corner of a field using a compass placed in the middle of the field.

Radial surveys

EXAMPLE 8

This diagram shows the result of a compass radial survey of the area *LMNK*.



a Calculate the size of each of the 4 angles that meet at *P*.

b Hence calculate the area of the field.

c What is the length of side *MN*?

Solution

a $\angle LPM = 87° - 20° = 67°$

$\angle MPN = 205° - 87° = 118°$

$\angle KPN = 300° - 205° = 95°$

$\angle KPL = 360° - 67° - 118° - 95° = 80°$ Angles at a point add to 360°

b The area (in m^2) of field *LMNK* is found by adding the areas of 4 triangles.

Area $\Delta LPM = \frac{1}{2} \times 40 \times 44 \sin 67° = 810.0442\ldots$ $A = \frac{1}{2}ab \sin C$

Area $\Delta MPN = \frac{1}{2} \times 56 \times 44 \sin 118° = 1087.7914\ldots$

Area $\Delta KPN = \frac{1}{2} \times 37 \times 56 \sin 95° = 1032.0577\ldots$

Area $\Delta KPL = \frac{1}{2} \times 37 \times 40 \sin 80° = 728.7577\ldots$

Area $KLMN = 728.7577\ldots + 810.0442\ldots + 1087.7914\ldots + 1032.0577\ldots$

$= 3658.6490\ldots$

$\approx 3700\ m^2$

Round the answer to 2 significant figures as measured values are given to 2 significant figures.

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c Use the cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

where $a = 56$, $b = 44$, $C = 118°$

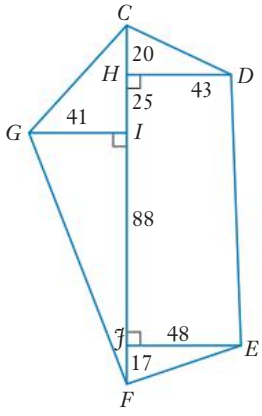
$$MN^2 = 56^2 + 44^2 - 2 \times 56 \times 44 \times \cos 118°$$
$$= 7385.5558 \ldots$$
$$MN = \sqrt{7385.5558}$$
$$= 85.9392 \ldots$$
$$\approx 86 \text{ m}$$

Note: To find the perimeter of the field, use the cosine rule to find the other 3 sides, *NK*, *KL* and *LM*, and then add the 4 sides.

Exercise 9.04 Offset and radial surveys

Example 7

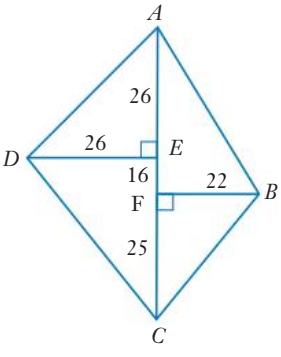
1 Calculate the area of field *CDEFG*, correct to the nearest square metre.



2 This is a surveyor's sketch of a park, *ABCD*.

a Calculate its area, correct to the nearest square metre.

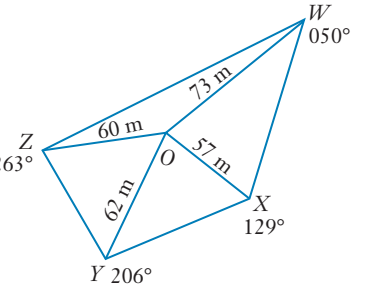
b Callum walks from *A* to *C* via *B*. What distance did he walk? Answer to the nearest 0.1 m.



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Example 8

3 This diagram shows a compass radial survey of the field *WXYZ*.



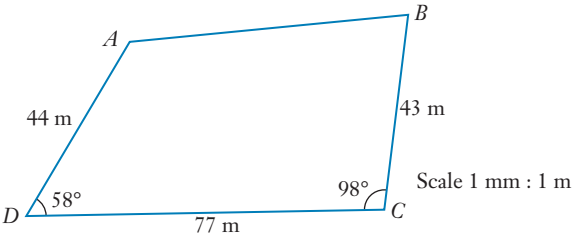
a Find the size of each of the 4 centre angles that meet at *O*.

b Calculate the area of the field *WXYZ*, correct to 2 significant figures.

c Calculate the length of *WX*, correct to the nearest unit.

d Find the other 3 sides and hence the perimeter of the field.

4 Construct accurately on a sheet of paper, this scale diagram of the field *ABCD*.



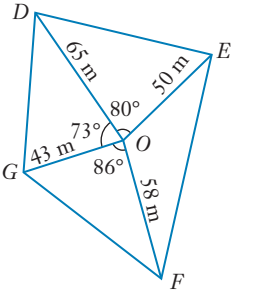
Calculate the area of the field by measurement and calculation using:

a an offset survey along the traverse line *AC*

b the formula $A = \frac{1}{2}ab\sin C$ in triangles *ADC* and *ABC*.

Write the answers correct to 2 significant figures.

5 This is a compass radial survey of a field *DEFG*.



a What is the size of $\angle EOF$?

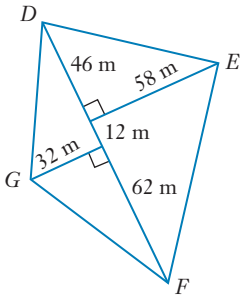
b Calculate the area of the field, correct to the nearest square metre.

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6 This is an offset survey of the same field *DEFG* from Question **5**.

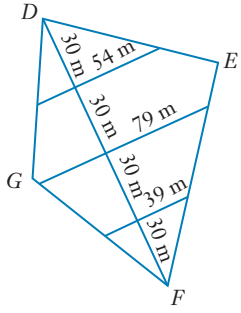
a What is the length of the traverse line *DF*?

b Calculate the area of the field.



7 In this survey of the same field, *DEFG*, offsets are taken at 30 m intervals along *DF*.

Use 4 applications of the trapezoidal rule,
$A = \frac{h}{2}(d_f + d_l)$, to calculate the area of the field *DEFG*.



8 **a** Your answers for the area of the field *DEFG* from Questions **5**, **6** and **7** are similar but not exactly the same. Why?

b Which of the 3 methods do you think is most accurate? Why?

c Which method is the least accurate? Why?



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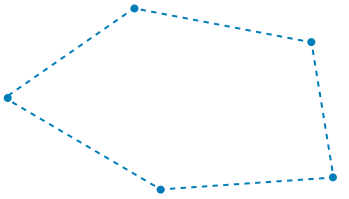
INVESTIGATION

SURVEYING ON SCHOOL GROUNDS

Equipment: Traffic cone markers, teacher's protractor or navigational compass, teacher's set square or T-square, ruler, tape measure and/or trundle wheel, table, art paper, masking tape or Blu-Tack.

a Stake out a field of 5 sides on school grounds, placing the cones or markers on the corners.

b Select a surveying method to find the area and perimeter of the field: offset survey or compass radial survey.



c If using an offset survey, choose a traverse line and walk along it with a trundle wheel or tape measure. Measure a perpendicular offset to each corner of the field.

d If using a compass radial survey, set up a table in the middle of the field and tape a large sheet of blank paper to it. Use a protractor or navigational compass to measure the true bearing of each corner, then measure the distances to each corner.

e Make a scale drawing of the field and mark on it all relevant information.

f Use the scale drawing to measure and calculate the perimeter of the field.

g Use Pythagoras' theorem (offset survey) or the cosine rule (radial survey) to calculate the perimeter of the field. How does this answer compare to your scale drawing answer?

h Calculate the area of the field, to the nearest square metre.

i Choose another surveying method and compare your results.

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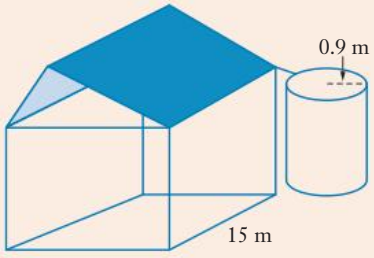
9.05 Volume of tanks and dams

Water tanks

EXAMPLE 9

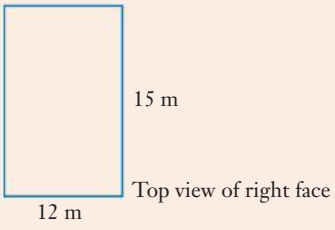
Rainwater falling onto the right face of the roof of a building is collected into an empty cylindrical water tank.

The top view of the right face shows the aerial dimensions of the catchment area.



a After 20 mm of rainfall, how many litres of water are drained into the tank?

b If the radius of the water tank is 90 cm, what was the water level in the tank after 20 mm of rainfall, correct to the nearest 0.1 m?



Top view of right face

Solution

a Area of a rectangle $= 12 \times 15$

$= 180 \text{ m}^2$

Volume of rainfall $= 180 \text{ m}^2 \times 20 \text{ mm}$ $\quad V = Ah$

$= 180 \text{ m}^2 \times 0.02 \text{ m}$ $\quad$ Change 20 mm to m

$= 3.6 \text{ m}^3$

$= 3.6 \times 1000 \text{ L}$ $\quad 1 \text{ m}^3 = 1000 \text{ L}$

$= 3600 \text{ L}$

b Volume of a cylinder is $V = \pi r^2 h$

$V = 3.6 \text{ m}^3$ from part **a** and $r = 90 \text{ cm} = 0.9 \text{ m}$

$3.6 = \pi \times 0.9^2 \times h$

$= 2.5446 \ldots \times h$

$h = \dfrac{3.6}{2.5446\ldots}$

$= 1.4147\ldots$

$\approx 1.4 \text{ m}$

The water level in the tank was 1.4 m.

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The trapezoidal rule for estimating area

$$A \approx \frac{h}{2}(d_f + d_l)$$

where h = distance between successive measurements

d_f = first measurement

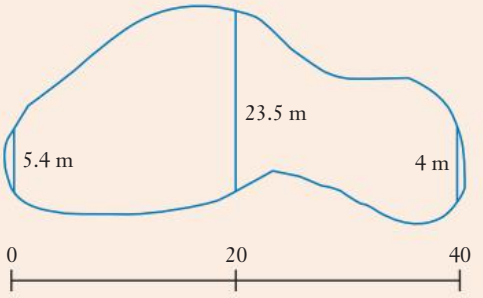
d_l = last measurement

or in words

$$\text{Area} \approx \frac{\text{width of strip}}{2}(\text{first measurement} + \text{last measurement})$$

EXAMPLE 10

Vertical measurements of a lake are taken at 20 m intervals.



a Use 2 applications of the trapezoidal rule to estimate the area of the lake, correct to one decimal place.

b Find the volume of water in the lake if it is a constant depth of 2.5 m.

Solution

a Left trapezium: $d_f = 5.4$, $d_l = 23.5$, $h = 20$

$$A = \frac{h}{2}(d_f + d_l)$$
$$= \frac{20}{2}(5.4 + 23.5)$$
$$= 289$$

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Right trapezium: $d_f = 23.5$, $d_l = 4$, $h = 20$

$$A = \frac{h}{2}(d_f + d_l)$$
$$= \frac{20}{2}(23.5 + 4)$$
$$= 275$$

Total area of lake = 289 + 275 = 564 m^2.

b $V = Ah$

$= 564 \times 2.5$

$= 1410$ m^3

The volume of the lake is 1410 m^3.

Exercise 9.05 Volume of tanks and dams

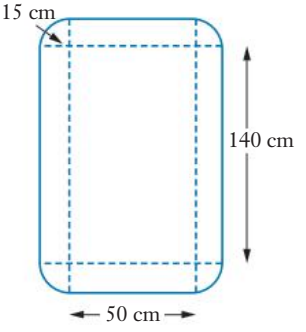
Example 9

1 Calculate the capacity of each tank or pool, correct to the nearest 0.01 kL.

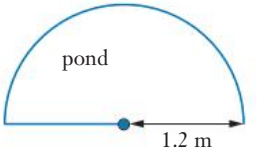
a a cylindrical rainwater tank with diameter 9.36 m and height 2.18 m

b a rectangular swimming pool with dimensions 2.4 by 4.8 m, filled to a depth of 1.2 m

c a water tank that has a height of 1.6 m and a base as shown below



2 An artificial semicircular pond has a radius of 1.2 m and a depth of 0.4 m. Calculate its volume, correct to the nearest litre.

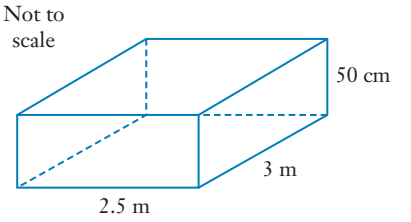


3 During a flood, 1.4 hectares of land was covered by water to a depth of 15 cm. How much water covered the land? Select **A**, **B**, **C** or **D**.

A 2.1 kL **B** 2100 kL **C** 210 000 kL **D** 2 100 000 kL

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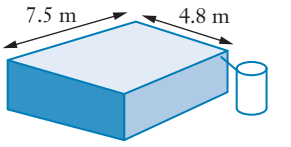
4 Berri has installed 2 water tanks under her new house to collect rain. They are identical and in the shape of a rectangular prism as shown.



a Calculate the combined capacity, in litres, of the water tanks.

b In April, Berri's family uses an average of 570 L/day. If the water tanks are full, how long will it take to use all the water in the tanks (assuming no rainfall)? Answer to the nearest day.

5 Rainwater falls onto the rectangular roof of a shed so that it drains into an empty cylindrical water tank of radius 60 cm and height 2.4 m. The aerial dimensions of the catchment area are 7.5 m by 4.8 m.



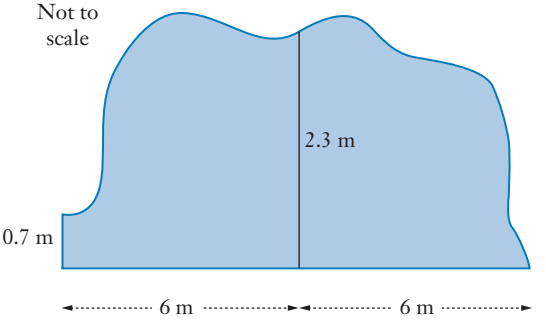
a After 50 mm of rainfall, how many litres of water are drained into the tank?

b What is the water level after 50 mm of rainfall?

c How many millimetres of rain needs to fall onto the roof to completely fill the tank?

6 The diagram shows the outline of a lake in a botanical garden.

Example 10

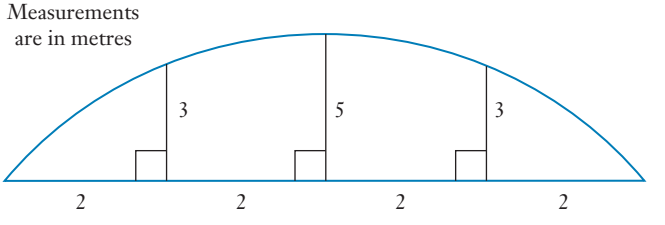


a Use 2 applications of the trapezoidal rule to find the approximate surface area of the lake.

b The average water depth in the lake is 1.6 metres. How many whole litres of water can the lake hold when full?

7 An artificial pond is designed for a new shopping centre.

a Calculate the area of the cross-section using 4 applications of the trapezoidal rule.



b If the pond is 1.6 m deep, calculate its capacity in litres.

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DID YOU KNOW?

Sydney's Tank Stream

In 1788, Sydney was chosen by the first British settlers in Australia because of its deep harbour and freshwater stream. Water was plentiful in rain and scarce in drought so Governor Phillip had his convicts excavate the sandstone banks of this stream to create storage tanks to hold the precious water. There were four tanks and each held about 20 000 litres. This is how Tank Stream got its name. It is now totally underground as part of Sydney's stormwater system.

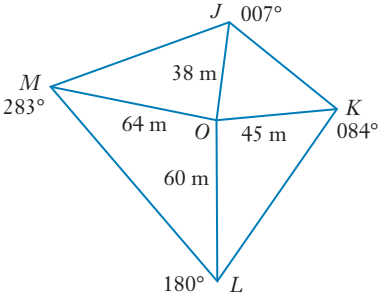


Newspix/Craig Greenhill

Find a map of the original path of the Tank Stream in Sydney. You can also visit the stream when it is open to the public.

SAMPLE HSC PROBLEM

A compass radial survey is conducted on the field *JKLM*.



a What is the size of $\angle MOL$?

b Calculate the distance between M and L, correct to 2 significant figures.

c Calculate the area of the triangle MOL, correct to the nearest square metre.

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Study tip

Ten common errors made in HSC exams

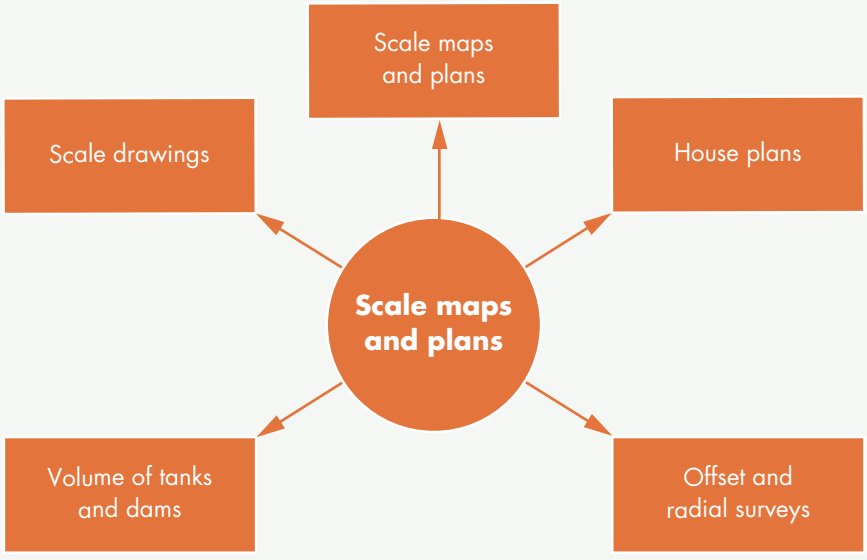
1. Careless setting out of answers or 'bald answers' with no working
2. Careless errors in basic algebra
3. Not choosing the simplest and quickest method
4. Not reading or answering the whole question
5. Poorly-drawn diagrams that are messy, small and without labels or detail
6. Not having a ruler: for drawing, measuring and graph reading
7. Rounding too early in a calculation, so that the final answer is inaccurate
8. Not checking whether a numerical answer makes sense or sounds reasonable
9. Difficulties in explaining answers in words: writing mini-essays rather than 1 or 2 sentences that are to-the-point
10. Not realising that the parts of a question are related and require an answer or hint from a previous part.

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9. CHAPTER SUMMARY

This chapter, Scale maps and plans, looked at scales and scale drawings, especially those used on maps and house plans. It also found areas and perimeters of locations using offset and compass radial surveys and investigated the volume of tanks and dams. You should be able to interpret scales and scale drawings related to actual objects and calculate perimeters and areas using scale maps and plans as well as finding volumes of tanks and dams using formulas including the trapezoidal rule.

Make a summary of this topic. Use the outline at the start of this chapter as a guide.
An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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9. TEST YOURSELF

1 The width of a house is 15.3 m. On a plan of the house, the width was 5.1 cm. What is the scale used on the plan? Select the correct answer **A**, **B**, **C** or **D**.

Exercise 9.01

A 1 : 300 **B** 1 : 3 **C** 1 : 30 **D** 300 : 1

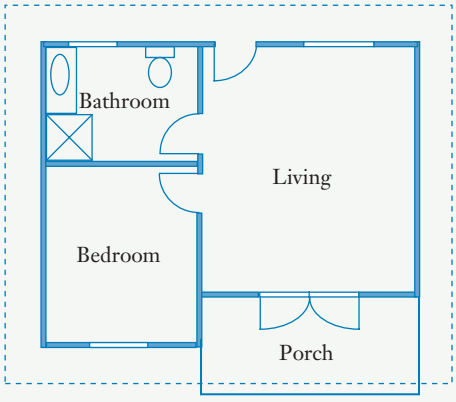
2 The plan of a house is drawn to a scale of 1 : 250. If a bedroom measures 18 mm by 15 mm on the plan, what is the actual area of the bedroom, to the nearest m^2?

Exercise 9.01

3 This is a scale plan of a cabin in a national park. The dotted lines indicate how far the roof extends past the walls.

Exercise 9.02

Plan



Scale 1 : 200

a What are the internal dimensions of the living room?

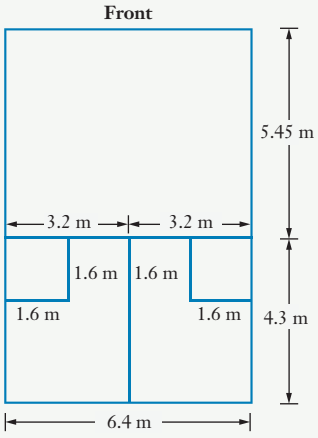
b What is the area of the porch, to the nearest square metre?

c How much would it cost to carpet the bedroom if carpet costs \$65 per m^2, correct to the nearest \$100?

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Exercise 9.02

4 This is a scale drawing of a squash court.



a Find the scale in the form 1 : r, where r is rounded to the nearest whole number.

b What is the area of the front-of-court section? Answer correct to the nearest 0.5 m^2.

c A player stands in the small square when serving. What is the area of the service square?

d How many square metres of timber flooring are needed for a squash court?

Exercise 9.03

5 For this question, refer to the house plan in Example **6** on page 405.

a Draw the symbol used for each feature shown on the plan.

i a washing machine **ii** a stove **iii** a vanity basin

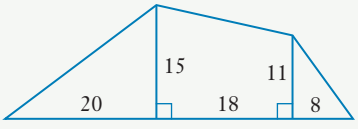
b What is the combined area, correct to the nearest m^2, of the 4 bedrooms?

c The family room floor is to be covered with tiles measuring 30 cm by 30 cm. How many whole tiles are needed?

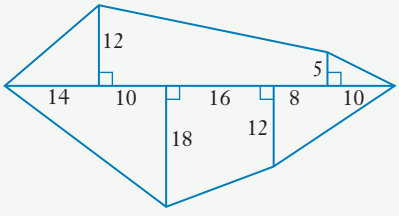
Exercise 9.04

6 Find the area of each field, correct to 2 significant figures. Measurements are in mm.

a

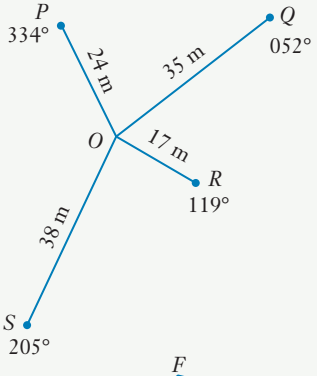


b



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7 This diagram shows the result of a compass radial survey of a field, *PQRS*.



Exercise 9.04

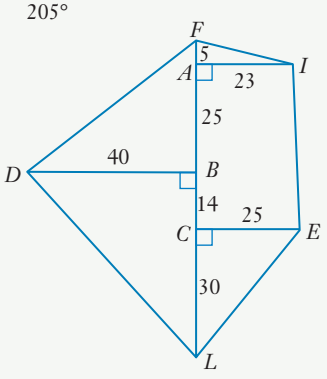
a Find the size of $\angle POQ$.

b Hence, calculate the length of *PQ*, correct to the nearest metre.

c Find the size of $\angle POS$.

d Hence, calculate the area of $\angle POS$, correct to the nearest square metre.

8 A surveyor drew this diagram after taking an offset survey of a field, *FIELD*, with measurements in metres.

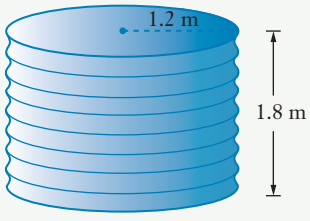


Exercise 9.04

a What is the area of the field, correct to 2 significant figures?

b What length of fencing, to the nearest metre, is required to put a fence around this field?

9 A water tank has the shape of a cylinder with a base radius of 1.2 m. If the tank is 1.8 m high, find, correct to 2 significant figures:



Exercise 9.05

Qz Chapter quiz

a its volume in m^3

b its capacity in litres.

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ALGEBRA

10

NON-LINEAR FUNCTIONS

The graph of a linear function $y = mx + c$ is a straight line. The graph of a non-linear function is a curve and can have many forms of equations. Australia's future population can be modelled by the non-linear function $P = 25(1.016)^n$ where P is the population in millions and n is the number of years after 2020. This model provides the following predictions:

Year	2020	2025	2030	2035	2040	2045	2050
Population (millions)	25	27.07	29.30	31.72	34.34	37.18	40.25

In what year will the population reach 50 million using this model?

CHAPTER OUTLINE

A4.2 10.01 The quadratic function
A4.2 10.02 Maximum and minimum problems
A4.2 10.03 The exponential function
A4.2 10.04 Exponential growth and decay
A4.2 10.05 The reciprocal function
A4.2 10.06 Inverse variation



IN THIS CHAPTER YOU WILL:

- graph the quadratic function $y = ax^2 + bx + c$, recognise the features of a parabola and use the function to model practical situations
- interpret the meaning of the turning point and intercepts of a parabola in a quadratic model
- graph the exponential function $y = b(a^x)$ and $y = b(a^{-x})$, recognise its features and solve problems involving exponential growth and decay
- graph the reciprocal function $y = \frac{k}{x}$, recognise the features of a hyperbola and use the function to model practical situations
- graph and solve problems involving the inverse variation $y = \frac{k}{x}$.

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TERMINOLOGY

asymptote
concave up/down
constant of variation
decreasing
direct variation
exponential decay
exponential function
exponential growth
hyperbola
increasing
intercept
inverse variation
linear function
maximum
minimum
model
non-linear function
parabola
proportional to
quadratic function
reciprocal function
turning point
varies directly as
varies inversely as
vertex
vertical intercept

WS Assignment 10

WS Tables of values

SkillCheck

1 Evaluate, correct to 4 decimal places:

a 1.4^6 **b** 5^{-4} **c** 6000×1.017^5

2 **a** If $y = kx^2$, find k if $x = 4$ and $y = 10$.

b If $p = \frac{k}{t}$, find k if $t = 16$ and $p = 32$.

3 If $x = 3$, evaluate:

a $2x^2 + 5$ **b** $4x^3$ **c** 5^x

d $-x^2 + 9$ **e** $-\frac{6}{x}$ **f** $3(1.25)^{-x}$

4 **a** Copy and complete this table for $y = x^2 + 2$.

x	-2	-1	0	1	2
y					

b Hence graph $y = x^2 + 2$ on a number plane.

5 The maximum distance, d metres, a ball covers if thrown at velocity v m/s is given by the formula $d = \frac{v^2}{9.8}$. Find:

a the maximum distance covered, correct to one decimal place, if the ball is thrown at 25 m/s

b the velocity at which a ball is thrown if it covers a maximum distance of 20 m.

6 **a** If $V = at$ and $V = 3500$ when $t = 24$, what is the value of a, correct to one decimal place?

b If $y = kx$ and $y = 54$ when $k = 3$, what is the value of x?

7 **a** If $y = \frac{a}{x}$ and $y = 12$ when $x = 5$, what is the value of a?

b If $V = \frac{k}{t}$ and $V = 7$ when $k = 56$, what is the value of t?

ISBN 9780170413633

10.01 The quadratic function

Linear functions are of the form $y = mx + c$ and their graphs are **straight lines**.

Non-linear functions take many different forms and their graphs are **curves**.

A **quadratic function** is a non-linear function where the highest power of the independent variable is 2. The general form of a quadratic function is $y = ax^2 + bx + c$ where a, b and c are constants, but $a \neq 0$. Here are some examples of quadratic functions: $y = 5x^2 - 9x + 4$, $y = 2x^2 + x$, $d = -\frac{1}{2}t^2 + 1$, $C = -v^2 + 3v + 7$.

WS Graphing quadratic functions

WS Graphing quadratics

Quadratic functions

WS Features of a parabola

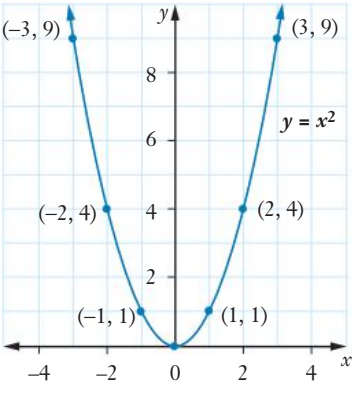
The shape of a parabola

The graph of a quadratic function is a parabola. You can sketch a simple parabola if you know its features or draw up a table of values to create a more accurate graph.

The simplest quadratic function is $y = x^2$ and its graph is drawn here using this table of values. The graph is symmetrical since $(-x)^2 = x^2$ for all values of x.

$y = x^2$

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

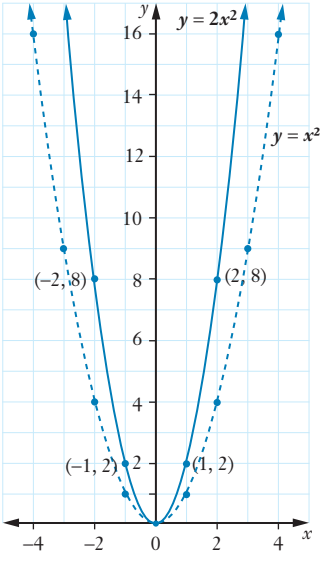


The vertex (turning point) is (0, 0).

The axis of symmetry is the y-axis.

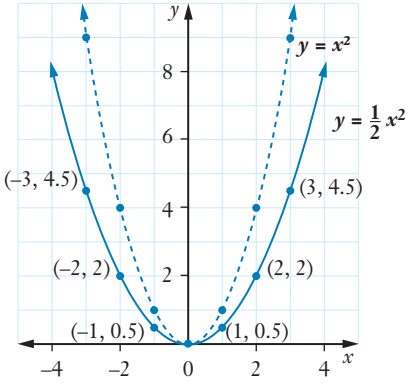
The graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ are shown on the next page. Compare them with the graph of $y = x^2$.

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$y = 2x^2$ is taller and steeper than $y = x^2$ as the y values are twice as large.

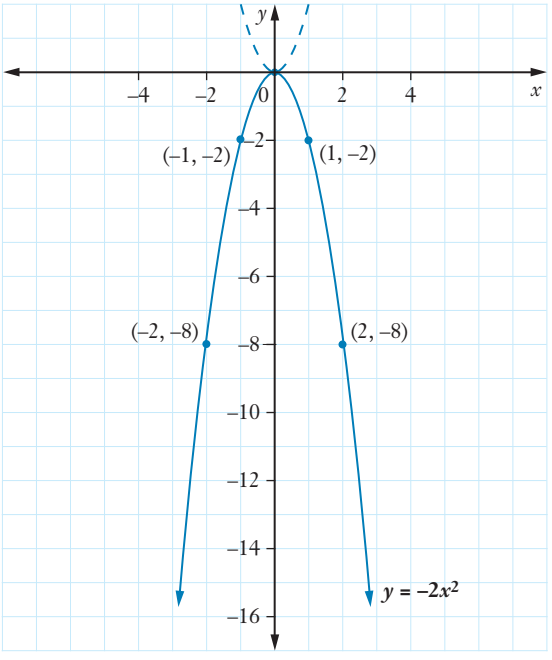
It lies 'inside' the graph of $y = x^2$.



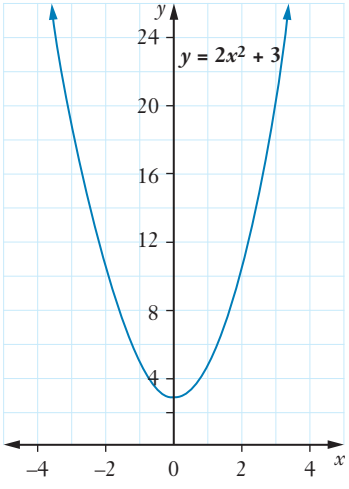
$y = \frac{1}{2}x^2$ is flatter than $y = x^2$ as the y values are half as large.

It lies 'outside' the graph of $y = x^2$.

The graphs of $y = -2x^2$ and $y = 2x^2 + 3$ are shown below. Compare them with the graph of $y = 2x^2$.



$y = -2x^2$ is exactly the same shape as $y = 2x^2$ but it is 'upside-down' as the y values are negative.



$y = 2x^2 + 3$ is exactly the same shape as $y = 2x^2$ but it has been moved up 3 units as the y values have increased by 3. The new vertex is (0, 3).

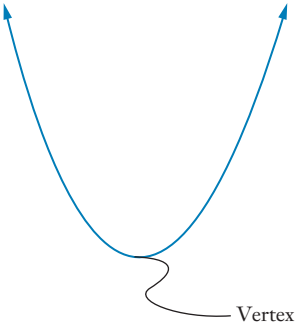
ISBN 9780170413633

The parabola $y = ax^2 + c$

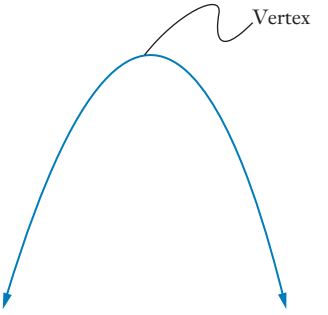
The graph of $y = ax^2 + c$ is a parabola with the y-axis as the axis of symmetry.

- If a is positive, the parabola is **concave up** (shaped like a bowl).
- If a is negative, the parabola is **concave down** (shaped like an upside-down bowl).

The y-intercept is c and the **vertex** is $(0, c)$.



Concave up (a is positive)



Concave down (a is negative)

Positive = 'smiling' parabola
Negative = 'frowning' parabola



EXAMPLE 1

a Graph $y = x^2 - 4x + 1$.

b Write the coordinates of the vertex of the parabola.

Solution

a Complete a table of values.

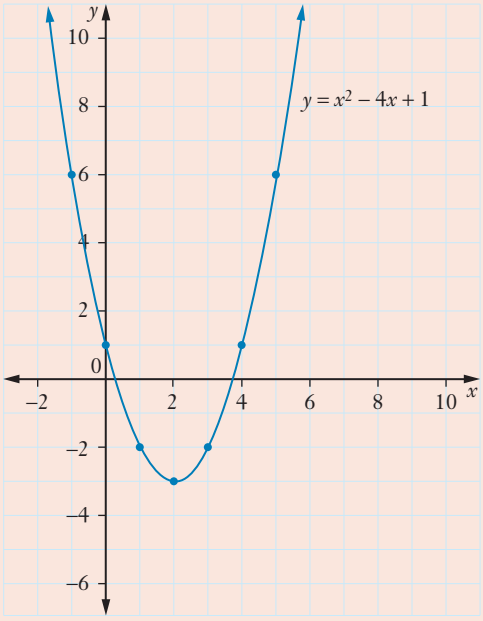
$y = x^2 - 4x + 1$

x	–1	0	1	2	3	4	5
y	6	1	–2	–3	–2	1	6

Then graph the table of values and draw a parabola.

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Solution



b From the graph, the vertex is $(2, -3)$.

The parabola $y = ax^2 + bx + c$

The graph of $y = ax^2 + bx + c$ is a parabola with y-intercept at c.

- If a is positive, the parabola is **concave up** (shaped like a bowl).
- If a is negative, the parabola is **concave down** (shaped like an upside-down bowl).

The x-intercepts are where $y = 0$.

EXAMPLE 2

a Graph $y = -x^2 + 5x - 6$.

b What are the x-intercepts of the parabola?

c What is the y-intercept?

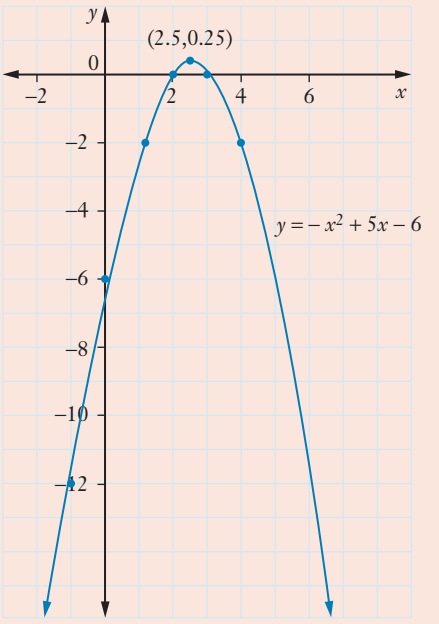
d Find the coordinates of the vertex.

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Solution

a $y = -x^2 + 5x - 6$.

x	–1	0	1	2	3	4
y	–12	–6	–2	0	0	–2



b From the table or graph, the x-intercepts are 2 and 3.

c From the equation, table or graph, the y-intercept is –6.

d By symmetry, the x-coordinate of the vertex is halfway between the x-intercepts 2 and 3, so $x = 2.5$.

When $x = 2.5$, $y = -(2.5)2 + 5(2.5) - 6 = 0.25$.

So the vertex is (2.5, 0.25).

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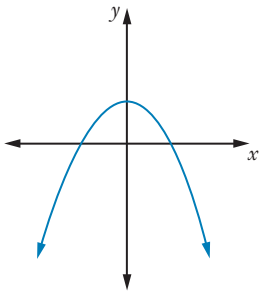
Exercise 10.01 The quadratic function

1 Match each quadratic function with its graph below.

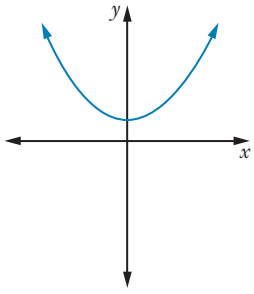
a $y = 2x^2 - 2$ **b** $y = -x^2 + 1$ **c** $y = -7x^2$

d $y = x^2 - 1$ **e** $y = \frac{1}{2}x^2 + 1$ **f** $y = 2x^2$

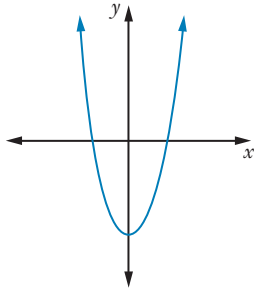
A



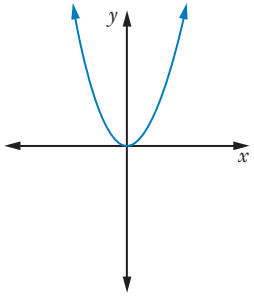
B



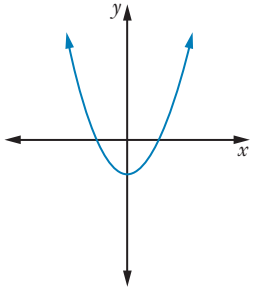
C



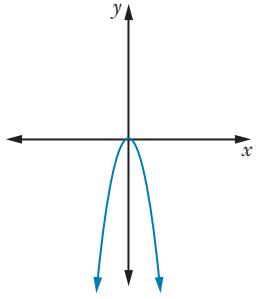
D



E



F



Example 1

2 Graph each quadratic function, showing the vertex and x- and y-intercepts.

a $y = 2x^2 + 1$ **b** $y = \frac{1}{2}x^2 + x + \frac{1}{2}$ **c** $y = -4x^2 + 4$

d $y = x^2 - 2x - 3$ **e** $P = n^2 - 9n + 8$ **f** $w = 2r^2 - 4r + 5$

Example 2

3 A ball is thrown up in the air from the balcony of Oliver's apartment. Its height above ground level after t seconds is given by the quadratic function $h = -5t^2 + 20t + 25$, where h is the height in metres.

a What is the independent variable of this function?

b Graph $h = -5t^2 + 20t + 25$ for values of t from 0 to 6.

c What is the vertical intercept of this graph and what does it represent?

d Find the maximum height of the ball and the time when this occurs.

e When does the ball hit the ground?

4 The braking distance of a car travelling at a speed of v km/h is given by the quadratic function $d = 0.01v^2 + 0.7v$ where d is measured in metres.

a Calculate the braking distance of the car if it is travelling at:

i 86 km/h **ii** 58 km/h.

b By a guess-and-check method, find the speed that allows a stopping distance of 20.24 m.

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5 A railway bridge underpass has an arch in the shape of a parabola with quadratic function $y = 16 - 0.02x^2$, where x and y are in metres.

a Copy and complete this table of values for $y = 16 - 0.02x^2$.

x	−30	−20	−10	0	10	20	30
y							

b Hence graph $y = 16 - 0.02x^2$.

c Estimate the x-intercepts.

d What are the coordinates of the vertex?

e What is the height of the highest point of the arch?

f Estimate how wide the arch is at ground level.

TECHNOLOGY

Graphing functions on a graphics calculator

WS Graphing curves: graphics calculator

Use a graphics calculator, graphing software or a spreadsheet to graph the different functions studied in this chapter.

1 On a Casio calculator, press MENU and select **GRAPH**. On a TI calculator, press STAT PLOT and select **PlotsOff** ENTER, Y= to turn off any statistical graphs.

2 Enter the function to be graphed at Y1 by pressing x, θ, T or x, T, θ, n, then EXE or ENTER.

3 To select the scale on a Casio, press V-Window and select the **STD** (standard) scale of –10 to +10 on both x- and y-axes, then EXIT. On a TI, press **WINDOW**, ZOOM and select **ZStandard**.

4 To graph the function, select **DRAW** on a Casio or GRAPH on a TI.

5 To zoom in or out of the graph, press ZOOM, select **Zoom IN** or **Zoom OUT**, position the cursor where you want to zoom and press EXE or ENTER.

6 Press TRACE and the left/right arrow keys to display the coordinates of points on the graph. Note that the exact coordinates of some points may not be shown due to the scale of the screen window.

For more detailed instructions on graphing functions on a graphics calculator, download the worksheet **Graphing curves: graphics calculator**.

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DID YOU KNOW?

The parabola in headlights and satellite dishes

You will find parabolas in many places.

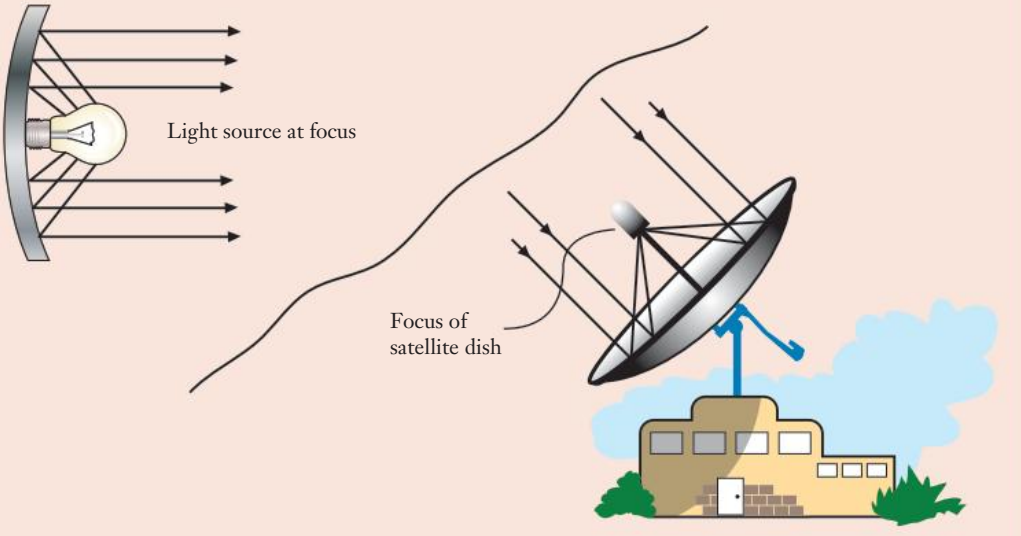




L: Alamy Stock Photo/David Grossman;
R: iStock.com/Dogukanyilmaz

The parabola is used in applications involving the reflection of light, sound and signals. For example, the shape of a car headlight or a satellite dish is a parabola.

In a car headlight, the light source is positioned at the **focus** of the parabola. Light from the focus bounces off the mirror surface and moves parallel to its axis of symmetry.



The reverse happens with a satellite dish. Signals arriving from space reflect off the dish and meet at the focus, where the signals are read.

Before every Olympic Games torch relay, the torch is lit by the rays of the sun caught by a parabolic mirror at the Temple of Hera.

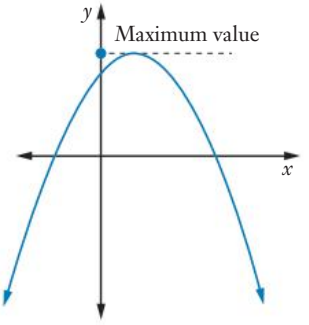
Where is the Temple of Hera?

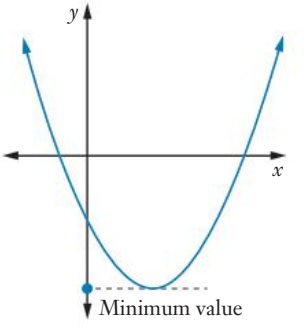
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10.02 Maximum and minimum problems

WS
Maximum/ minimum problems

The **maximum** or **minimum** value of a quadratic function $y = ax^2 + bx + c$ is represented by the vertex of its graph. The maximum or minimum value is the y-value of the vertex.





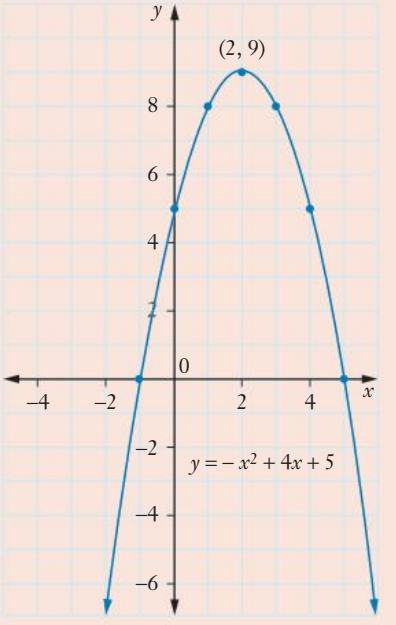
EXAMPLE 3

Find the maximum value of $y = -x^2 + 4x + 5$.

Solution

$a = -1$ is negative, so the parabola is concave down with a vertex that shows a maximum value.

x	-1	0	1	2	3	4
y	0	5	8	9	8	5

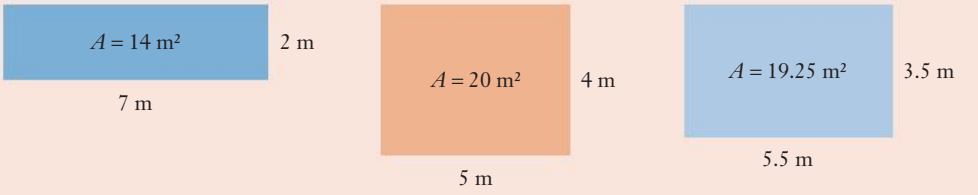


The vertex is (2, 9), so the maximum value of $y = -x^2 + 4x + 5$ is 9.

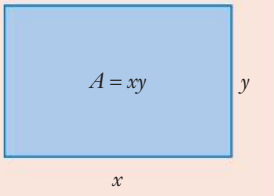
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EXAMPLE 4

Farmer Brad has 18 m of fencing and wants to build a rectangular pig pen with the largest area possible. He sketches some possible pens using his 18 m of fencing:



Let x and y be the length and width of the pen, in metres.



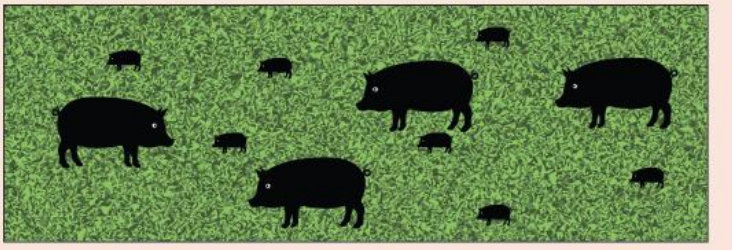
a Explain why $2x + 2y = 18$.

b Hence show that $y = 9 - x$.

c Hence show that the area of the pen, A m^2, is given by the function $A = -x^2 + 9x$.

d Find the largest possible area of the pen.

e What are the dimensions of the pen that give this maximum area?



Solution

a Perimeter of pen $= x + y + x + y = 18$ — Sum of the four sides

$2x + 2y = 18$

b $2y = 18 - 2x$ — Divide both sides by 2 to make y the subject.

$$\frac{2y}{2} = \frac{18 - 2x}{2}$$

$$y = 9 - x$$

c $A = xy$ — Area = length × width

$= x(9 - x)$ — Substitute $y = 9 - x$ from part **b**

$= 9x - x^2$

$= -x^2 + 9x$

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d For $A = -x^2 + 9x$ — x is positive because it refers to a length

x	1	2	3	4	5	6
A	8	14	18	20	20	18

The maximum value of A must occur at $x = 4.5$. — At the vertex of the parabola halfway between 4 and 5

When $x = 4.5, A = -4.5^2 + 9(4.5) = 20.25$.

The largest possible area is 20.25 m^2.

e When $x = 4.5$ m, $y = 9 - 4.5 = 4.5$.

The pen with maximum area has dimensions 4.5 m by 4.5 m (a square).



4.5 m

4.5 m

Exercise 10.02 Maximum and minimum problems

1 Find the maximum value of $y = -5x^2 + 20x + 11$. Select **A**, **B**, **C** or **D**. Example 3

A 11 **B** 26 **C** 31 **D** 45

2 A golf ball is hit into the air and its height, h metres, above the ground after time, t seconds, is given by the quadratic function $h = -5t^2 + 24t$.

a What is the dependent variable?

b Find the height of the ball after 2 seconds.

c Show that the height of the ball after 1.8 and 3 seconds is the same. Why is this so?

d After how many seconds does the ball reach its maximum height? How did you find this value?

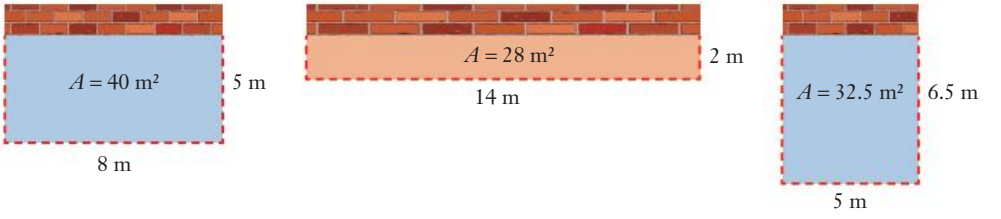
e Find the maximum height reached.

f Why is this quadratic model not useful for values of t greater than 5?

3 Find the minimum value of $y = 2x^2 - 10x + 7$. Select **A**, **B**, **C** or **D**.

A −5.5 **B** −5 **C** −1 **D** 2.5

4 Ivy has 18 metres of wire fencing to border a new baby lamb enclosure. She wants to use an existing brick wall as one side and sketches three possible enclosures. Example 4



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a If x m and y m are the length and width of the enclosure respectively, explain why $x + 2y = 18$.

b Hence show that $y = 9 - \frac{1}{2}x$.

c Hence show that the area, A m^2, of the enclosure is $A = -\frac{1}{2}x^2 + 9x$.

d Find the maximum area of the enclosure and the dimensions that give this area.

5 At an air show, the height (in metres) of a plane in a power dive is given by the quadratic function $h = t^2 - 10t + 80$ where t is the time in seconds.

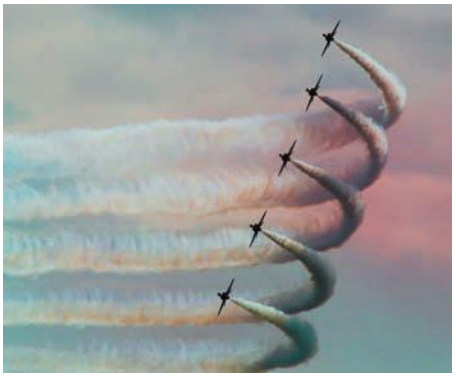
a Use a table of values to graph $h = t^2 - 10t + 80$ for values of t from 0 to 10.

b What are the coordinates of the vertex?

c What is the minimum value of h and what does this represent?

d When is the lowest height the plane reached?

e What is the height of the plane at the start of the power dive?



Getty Images/John Smyth - North Atlantic Skyline

6 The fuel consumption of a car travelling at a speed of s km/h is modelled by the function $C = 0.01s^2 - s + 33$, where C is measured in L/100 km.

a What is the independent variable of this function?

b Copy and complete this table of values.

s	20	30	40	50	60	70	80
C							

c What is the most economical speed, that is, the speed at which the fuel consumption is lowest?

d What is the fuel consumption at the most economical speed?

e This function is a good model only for speeds between 20 km/h and 80 km/h. Why would the function not be useful for $s = 0$?

7 The temperature, T°C, at Whitetown n hours after 6 a.m. is given by the function

$$T = -\frac{1}{6}n^2 + 2n + 14.$$

a Graph this function for values of n from 0 to 15.

b What was the temperature at these times?

i at 6 a.m. **ii** at 2 p.m.

c What was the maximum temperature and when was it reached?

d Estimate to the nearest half-hour the times when the temperature reached 19°C. Why are there 2 answers?

e For how many hours (to the nearest whole number) was the temperature 19°C or higher?

f This quadratic model is only accurate for values of n from 0 to 13. Why do you think the model does not hold for values of n above 13?

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INVESTIGATION

QUADRATIC HANDSHAKES



Getty Images/Klaus Vedfelt

1 How many handshakes take place if three people meet and each person shakes hands with every other person?

2 How many handshakes take place if five people meet?

3 The formula for the number of handshakes when n people meet is $H = \frac{1}{2}n(n-1)$.

4 Use the formula to calculate the number of handshakes that take place when:

a 12 people meet **b** 22 people meet.

5 Use a guess-and-check method to find how many people meet if 666 handshakes take place.

10.03 The exponential function

An **exponential function** is a non-linear function where the independent variable is the **power** (or **exponent**). The general form of the exponential function is $y = b(a^x)$ or $y = b(a^{-x})$ where a and b are constants, and $a > 0$. Here are some examples of exponential functions: $y = 2^x$, $y = 5^{-x}$, $y = 3(10^x)$, $A = 5000(1.08)^n$, $S = 12\,000(0.83)^n$. You may have noticed from the last 2 examples that compound interest and declining-balance depreciation involve exponential functions.

WS Graphing exponentials

The shape of an exponential curve

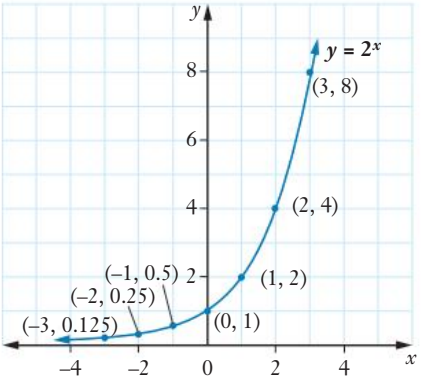
The graph of an exponential function is called an **exponential curve**. You can sketch a simple exponential curve if you know its features or draw up a table of values to create a more accurate graph.

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A simple exponential function is $y = 2^x$ and its graph is drawn here using this table of values.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8} = 0.125$	$\frac{1}{4} = 0.25$	$\frac{1}{2} = 0.5$	1	2	4	8

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

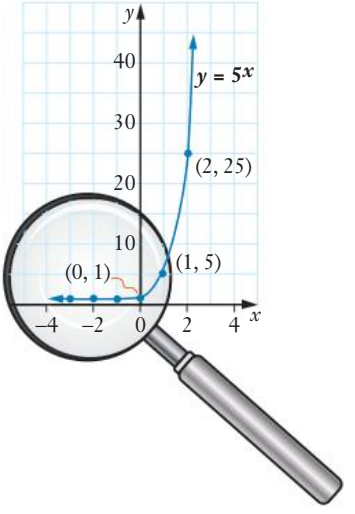


The curve is increasing and becoming steeper because the y-values are doubling (multiplying by 2).

The y-intercept is 1 because $2^0 = 1$.

As the x-values become smaller (moving left on the graph), the y-values are halving and the curve becomes closer and closer to the x-axis without reaching it. We call the x-axis an **asymptote**, a line that the curve approaches but never touches.

The graph of $y = 5^x$ is shown below.



x	-3	-2	-1	0	1	2	3
y	$\frac{1}{125} = 0.008$	$\frac{1}{25} = 0.04$	$\frac{1}{5} = 0.2$	1	5	25	125

The curve is increasing and is steeper than the graph of $y = 2^x$ because the y-values are multiplying by 5.

The y-intercept is 1 because $5^0 = 1$.

As the x-values become smaller, the y-values are dividing by 5 so the curve becomes closer to the x-axis asymptote faster.

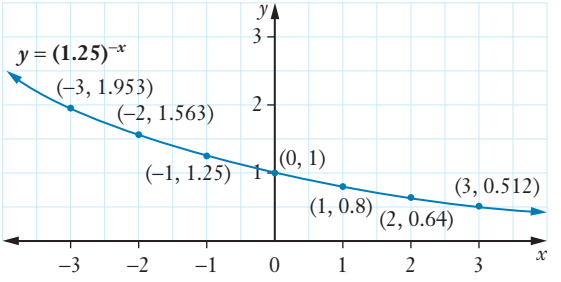
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This is the graph of $y = (1.25)^{-x}$.

x	–3	–2	–1	0	1	2	3
y	1.953	1.563	1.25	1	0.8	0.64	0.512

y-values are rounded to 3 decimal places here.

The curve is **decreasing** and is flatter than the graph of $y = 2^x$ because the *y*-values are being *divided* by 1.25 (or multiplied by $\frac{1}{1.25} = 0.8$). The effect of the negative power in $y = (1.25)^{-x}$ is to make the function decreasing and its graph's properties are the opposite of those of $y = 2^x$.



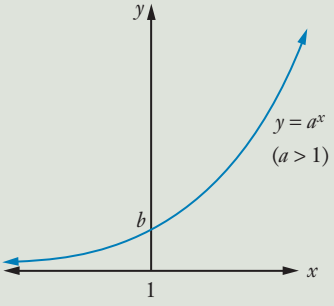
The *y*-intercept is 1 because $(1.25)^0 = 1$.

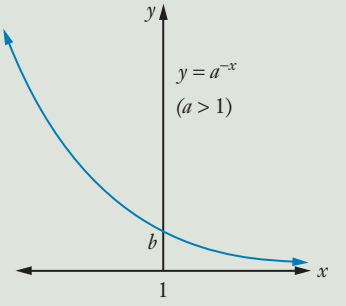
As the *x*-values become *larger*, the *y*-values become smaller so the curve approaches the *x*-axis asymptote.

Furthermore, since the *y*-values are being multiplied by 0.8, the exponential function $y = (1.25)^{-x}$ (with a negative power) can also be written as $y = (0.8)^x$ (with a positive power) where the base, a, is a decimal between 0 and 1.

The exponential curve $y = a^x$ or $y = a^{-x}$

- The exponential curve $y = a^x$ or $y = a^{-x}$ is always above the *x*-axis.
- The *x*-axis is called an **asymptote**: the curve approaches it but never touches it.
- The *y*-intercept is 1.
- $y = a^x$ is **increasing** if $a > 1$ (but decreasing if a is between 0 and 1).
- $y = a^{-x}$ is **decreasing** if $a > 1$ (but increasing if a is between 0 and 1).





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EXAMPLE 5

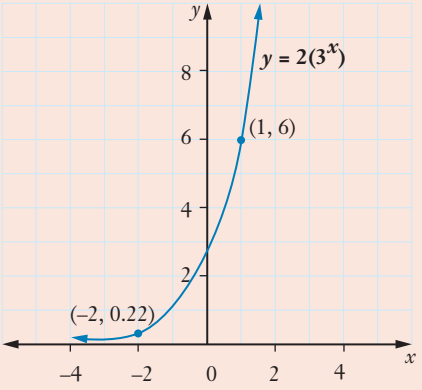
a Graph the function $y = 2(3^x)$.

b State the y-intercept and asymptote of the graph.

Solution

a $y = 2(3^x)$

x	–3	–2	–1	0	1	2	3
y	0.07	0.22	0.67	2	6	18	54

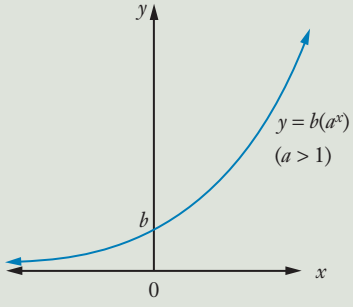


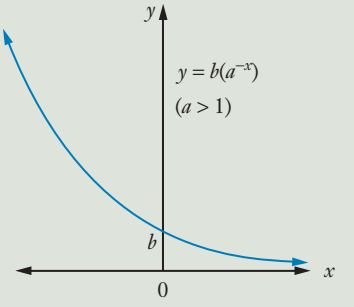
Note: you do not have to plot all points.

b The y-intercept is 2 and the asymptote is the x-axis.

The exponential curve $y = b(a^x)$ or $y = b(a^{-x})$

- The exponential curve $y = b(a^x)$ or $y = b(a^{-x})$ is always above the x-axis.
- The x-axis is an **asymptote**.
- The y-intercept is b.
- $y = b(a^x)$ is **increasing** if $a > 1$ (but decreasing if a is between 0 and 1).
- $y = b(a^{-x})$ is **decreasing** if $a > 1$ (but increasing if a is between 0 and 1).





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Exercise 10.03 The exponential function

1 Which equation represents an exponential function? Select **A**, **B**, **C** or **D**.

A $y = \frac{x}{2}$ **B** $y = 2^x$ **C** $y = \frac{2}{x}$ **D** $y = 2x$

2 Which equation represents a decreasing exponential function? Select **A**, **B**, **C** or **D**.

A $y = 2^{-x}$ **B** $y = 0.5(2^x)$ **C** $y = (0.5)^{-x}$ **D** $y = -2(5^x)$

3 Which equation has a graph with a y-intercept of 3? Select **A**, **B**, **C** or **D**.

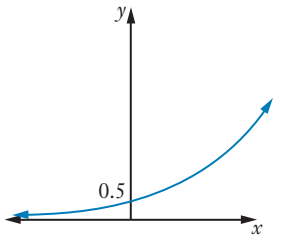
A $y = 3^x$ **B** $y = 3(2^x)$ **C** $y = 2^x + 3$ **D** $y = 2(3^x)$

4 Match each exponential function to its graph below.

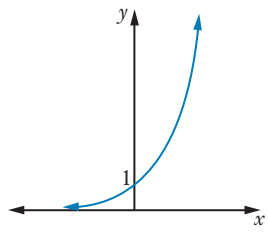
a $y = 2^x$ **b** $y = 10^x$ **c** $y = 10^{-x}$

d $y = 0.5(2^x)$ **e** $y = 8(2^x)$

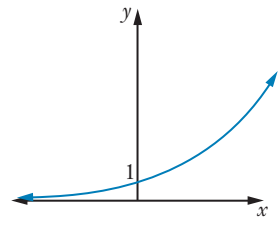
A



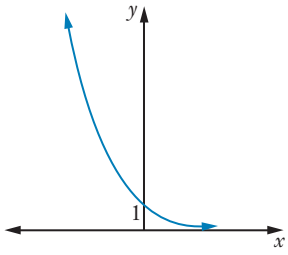
B



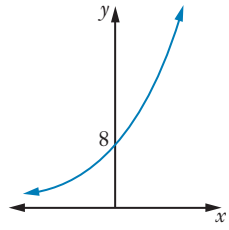
C



D



E



5 **a** Copy and complete this table of values and draw the graph of $y = 3^x$.

Example 5

x	–3	–2	–1	0	1	2	3
y							

b State the y-intercept and asymptote of the graph.

6 **a** Draw the graph of $y = (1.6)^{-x}$.

b At what point does the graph cut the y-axis?

c Is this function increasing or decreasing?

7 **a** Draw the graph of $y = 3(4^x)$.

b What is the y-intercept?

c Explain why there isn't an x-intercept.

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8 **a** Draw the graph of $y = 80(1.3)^x$.

b What is the y-intercept?

c Is this function increasing or decreasing?

9 Which function has a graph that is decreasing? Select **A**, **B**, **C** or **D**.

A $y = 1.4^x$ **B** $y = (0.25)^x$ **C** $y = 0.4(1.1)^x$ **D** $y = 0.2(2.5)^x$

10 A large sheet of paper is 0.1 mm thick and is folded repeatedly. After one fold, the thickness of the sheet will double to 0.2 mm, after two folds to 0.4 mm, after three folds to 0.8 mm, and so on. After n folds, the sheet has a thickness of $T = 0.1(2^n)$ where T is the thickness in millimetres.

a Calculate the thickness of the sheet after:

i 5 folds **ii** 12 folds.

b By a guess-and-check method, find the number of folds required for the sheet to be:

i 400 mm thick **ii** 1 m thick.

c What are the limitations of this model?

WS Population growth

WS Practising growth problems

10.04 Exponential growth and decay

Exponential growth

Any quantity that increases according to the exponential function $y = b(a^x)$, where $a > 1$, is showing **exponential growth**. Something that grows exponentially increases slowly at first, then more quickly. Examples of exponential growth are:

- **population** size (people, animals, bacteria)
- an investment attracting compound interest
- a flu or computer virus
- the size of a bushfire.

EXAMPLE 6

The population of Seaside grows exponentially according to the function $P = 5000(1.08)^t$ where t is the number of years after 2020 and P is the population at the start of that year.

a What was Seaside's population in 2020?

b Calculate the population of Seaside in the year 2026 (to the nearest whole number).

c Graph P for values of t from 0 to 10.

d Use a guess-and-check method to find when the population will be triple that of 2020.

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Solution

a When $t = 0$, 2020 is 0 years after 2020

$$P = 5000(1.08)^0$$
$$= 5000$$

The population was 5000 in 2020.

b When $t = 6$, 2026 is 6 years after 2020

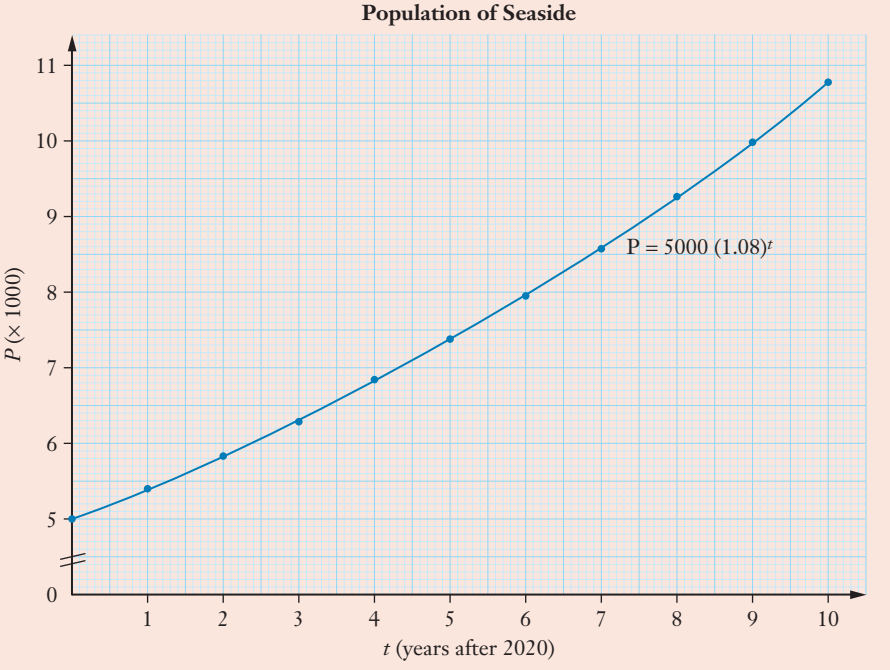
$$P = 5000(1.08)^6$$
$$= 7934.37\ldots$$
$$\approx 7934$$

The population will be 7934 in 2026.

c

t	0	1	2	3	4	5	6	7	8	9	10
P	5000	5400	5832	6299	6802	7347	7934	8569	9255	9995	10795

P-values are rounded to the nearest whole number.



d When does $P = 3 \times 5000 = 15\,000$?

Guess-and-check:

When $t = 15$, $P = 5000(1.08)^{15} = 15\,861$ Too high

When $t = 14$, $P = 5000(1.08)^{14} = 14\,685$ Too low but close

When $t = 14.3$, $P = 5000(1.08)^{14.3} = 15\,029$ Very close

The population will be triple in early 2034 (14.3 years after 2020).

TECHNOLOGY

Exponential growth on a calculator

In Example **6**, each new P-value is 1.08 times larger than the previous one so the population of Seaside is increasing by 8% each year. This can be keyed on the calculator like this:

Calculator keys	Display	
5000 [=]	5000	$t = 0$
[×] 1.08 [=]	5400	$t = 1$
(Then keep pressing repeatedly.)		
[=]	5832	$t = 2$
[=]	6298.56	$t = 3$
[=]	6802.4448	$t = 4$, etc.

Use your calculator to show that the population of Seaside in 2026 will be 7934.

Exponential decay

Any quantity that decreases according to the exponential function $y = b(a^{-x})$ where $a > 1$ or $y = b(a^{x})$ where a is between 0 and 1 is showing **exponential decay**. Something that decays exponentially decreases quickly at first, then more slowly. Examples of exponential decay are:

- radioactive decay
- the cooling of a hot liquid
- declining-balance depreciation
- a person's growth in height.

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EXAMPLE 7

A small rubber ball is dropped from a balcony. Its height reached, h metres, above the ground after each bounce is $h = 12(0.6)^b$ where b is the number of bounces.

a Calculate, to the nearest centimetre, the height of the ball above the ground after the fifth bounce.

b Graph the function for values of b from 0 to 8.

c What is the height from which the ball was first dropped?

d Estimate after which bounce the ball will rise to 12 cm.

e What are the limitations of this exponential model for large values of b?

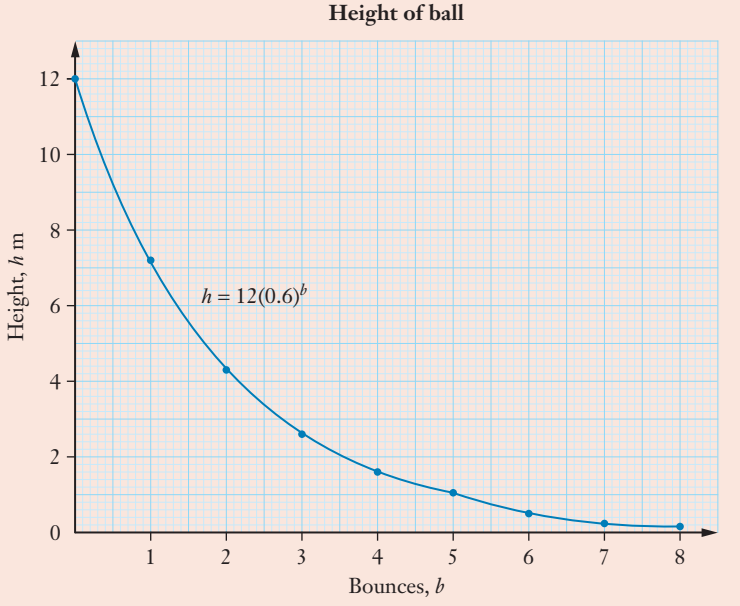
Solution

a When $b = 5$,

$$h = 12(0.6)^5$$
$$= 0.933\,12 \text{ m}$$
$$= 93.312 \text{ cm}$$
$$\approx 93 \text{ cm}$$

b $h = 12(0.6)^b$, h-values rounded to one decimal place.

b	0	1	2	3	4	5	6	7	8
h	12	7.2	4.3	2.6	1.6	0.9	0.6	0.3	0.2



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c When $b = 0$, $h = 12$.

The ball was dropped from a height of 12 m.

d When does $h = 12$ cm $= 0.12$ m?

Guess-and-check:

When $t = 10$, $h = 12\ (0.6)^{10} = 0.07255\ldots$ Too low

When $t = 8$, $h = 12(0.6)^8 = 0.20155\ldots$ Too high

When $t = 9$, $h = 12(0.6)^9 = 0.1209\ldots$ Very close

The ball will reach 12 cm after the 9th bounce.

e The exponential model assumes the ball will continue bouncing forever and never reach a 'height' of 0 m. In reality, it will stop bouncing after some time.

Exponential growth and decay

- Growth occurs when $y = b(a^x)$ and $a > 1$.
- Exponential decay occurs when $y = b(a^{-x})$ and $a > 1$, or when $y = b(a^x)$ and a is between 0 and 1.
- b is the initial value of y (when $x = 0$).
- b is the y-intercept of the exponential curve.

Exercise 10.04 Exponential growth and decay

1 Which function represents exponential growth? Select **A**, **B**, **C** or **D**.

A $y = 1000(0.15)^x$ **B** $y = 6\left(\frac{3}{4}\right)^x$ **C** $y = (0.7)^x$ **D** $y = 0.5(1.07)^x$

Example 6

2 A bushfire is spreading through a forest and the amount of area burnt is modelled by the exponential function $A = 5(1.22)^h$ where A is in hectares and h is the time in hours.

a What was the initial area burnt?

b What area (to the nearest hectare) was burnt after $3\frac{1}{2}$ hours?

c After how many hours will 20 ha of land be burnt?

d Graph the function for values of h from 0 to 6.

e Use your graph to find:

i after how many hours 10 ha of land will be burnt

ii the area burnt after $5\frac{1}{2}$ hours.

f What are the limitations of this exponential model?

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3 The thickness, T cm, of a tree trunk grows according to the function $T = 16(1.11)^n$ where n is the time in years after the first measurement.

a What is the dependent variable of this function?

b Calculate, to the nearest centimetre, the thickness of the trunk:

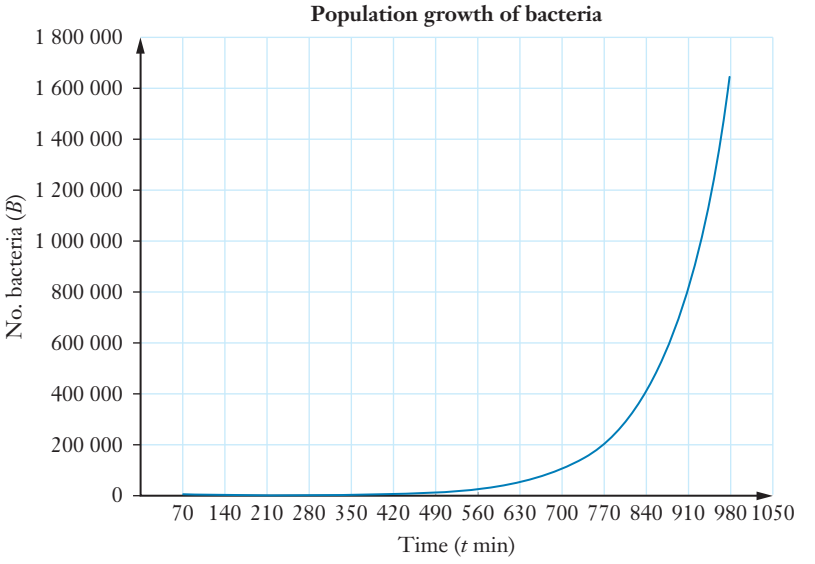
i after 6 years **ii** when first measured.

c Calculate when the tree trunk will reach a thickness of 24.29 cm.

d Graph of the function for values of n from 0 to 5.

4 Alyssa observed bacteria through a microscope, recorded the number of bacteria, B, every 70 minutes and graphed her results as shown.

Time (t min.)	0	70	140	210	280	350	420	490	560	630	700	770	840	910	980
No. bacteria (B)	100	200	400	800	1600	3200	6400	12 800	25 600	51 200	102 400	204 800	409 600	819 200	1 638 400



a How many bacteria are in the initial population?

b What happens in the first 490 minutes?

c Use the graph to estimate the number of bacteria after:

i 805 minutes **ii** 16 hours.

d Use the graph to determine after how many hours (to nearest hour) the bacteria population is:

i 800 000 **ii** 1 500 000.

e Alyssa says that the population is doubling every 70 minutes. Do you agree with her findings? Justify your answer.

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Example 7

5 A bowl of hot soup is cooling on a table. Its temperature, T°C, after t minutes is given by the formula $T = 100(1.075)^{-t}$.

a Is this an example of exponential growth or decay?

b Graph this function for values of t from 0 to 15.

c Calculate the temperature of the soup after 2.5 minutes. Select **A**, **B**, **C** or **D**.

A 79°C **B** 83°C **C** 85°C **D** 91°C

d What is the vertical intercept of this function and what does it represent?

e Determine when the soup's temperature will reach 27°C.

f Why is this model probably inaccurate for values of t greater than 20?

6 The number of germ cells in a particular culture is given by $N = 1000(6.9)^t$ where t is the time in minutes.

a Calculate the number of germ cells in the culture after:

i 10 minutes **ii** 1 hour.

Answer in scientific notation, correct to 2 significant figures.

b What was the initial number of germ cells?

c Estimate when the number of germ cells will pass 2 000 000.

7 According to one theory, the number of days, D, that milk will stay fresh if stored at a temperature of T°C is $D = 18(1.39)^{-T}$.

a What is the number of days milk will stay fresh if stored at:

i 5 °C? **ii** 0 °C?

b What temperature will keep milk fresh for 6.7 days? Select **A**, **B**, **C** or **D**.

A 1°C **B** 2°C **C** 3°C **D** 4°C

8 The number of computers, C, affected by a virus after n hours is $C = 5(3.03)^n$.

a Is this an example of exponential growth or decay?

b What is the vertical intercept and what does it represent?

c How many computers are affected by the virus after 8 hours?

9 Australia's population can be modelled by the function $P = 25(1.016)^n$ where P is the population in millions and n is the number of years after the year 2020.

a Copy and complete this table of values, expressing P correct to one decimal place.

Year	2020	2025	2030	2035	2040	2045	2050	2055	2060
***n* (years)**	0	5	10						
***P* (millions)**				31.7					

b Graph P against n.

c Predict Australia's population in the year 2033 using:

i the graph **ii** the function.

d Predict when Australia's population will reach 40 million using:

i the graph **ii** the function.

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10 A baby gorilla has a mass of M kg at age x months. The function $M = 1.4(1.3)^x$ can be used to calculate the mass of the gorilla up to 12 months of age.

a What is the mass of the gorilla at the following ages?

i 3 months **ii** 6 months **iii** 1 year

b Use a guess-and-check method to find the age of the gorilla with a mass of:

i 5 kg **ii** 25 kg

11 Carbon-14, a radioactive isotope of carbon, decays according to the function $P = 100(0.999\,88)^n$ where P is the percentage of the original mass remaining after n years.

a What percentage, correct to one decimal place, of carbon-14 remains after:

i 200 years? **ii** 500 years?

b Find the **half-life** of the carbon-14 isotope, which is the time it takes to decay to half its original mass. Answer to the nearest 10 years.

c At an archaeological dig, Sam discovered a human bone containing 96% of its original carbon-14. Estimate the age of the bone.

12 The future value (compound interest) formula is an example of exponential growth. If a principal of \$8000 is invested at 6.5% p.a. interest, the future value of the investment after n years is $FV = 8000(1.065)^n$ where A is in dollars.

a What is the independent variable for this exponential function?

b What is the vertical intercept of the graph of the function and what does it represent?

c What is the amount of the investment after 10 years?

d Use a guess-and-check method to find the year in which the principal will be doubled.

DID YOU KNOW?

Population clocks

A **population clock** is a counter that estimates the population of a country at any time using an exponential model. According to the population clocks at the Australian Bureau of Statistics and the US Bureau of Census websites, the populations of Australia and the world on 17 May 2018 were 24 937 711 and 7 473 558 200 respectively.

In 2018, Australia's overall population was increasing by one person every 84 seconds and the world's population was increasing by one person every 14 seconds. The world's population is expected to reach 10 billion in 2060.

China has the largest population in the world and it is said that 'one in every 5 people in the world is Chinese'.

1 Find the current populations of Australia, China and the world, and decide if the '1-in-5' claim is true.

2 Which other country also has a population of over 1 billion like China's?

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WS Graphing $y = \frac{10}{x}$

WS Matching graphs

WS Functions match-up

10.05 The reciprocal function

A **reciprocal function** is a non-linear function in which x is in the denominator. One simple form of a hyperbolic function is $y = \frac{k}{x}$ where k is a constant. It is called a reciprocal function because $\frac{1}{x}$ is the reciprocal of x.

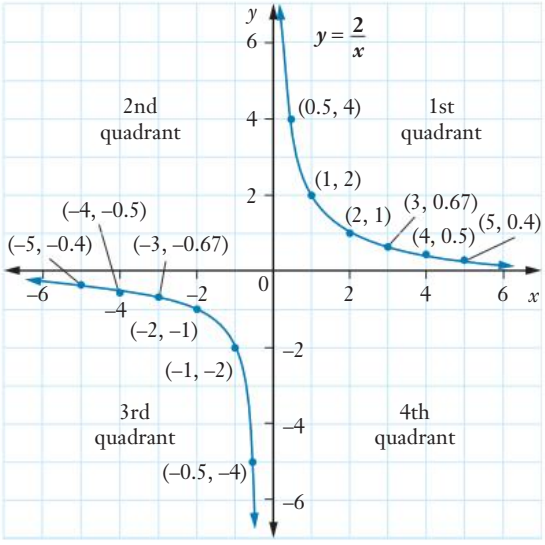
The shape of a hyperbola

The graph of a reciprocal function is called a **hyperbola**. This curve has 2 separate branches and two asymptotes.

A simple reciprocal function is $y = \frac{2}{x}$ and its graph is drawn here using this table of values, where y values are rounded to 2 decimal places.

x	–5	–4	–3	–2	–1	–0.5	–0.2	0	0.2	0.5	1	2	3	4	5
y	–0.4	–0.5	–0.67	–1	–2	–4	–10	—	10	4	2	1	0.67	0.5	0.4

You cannot divide any number by 0.



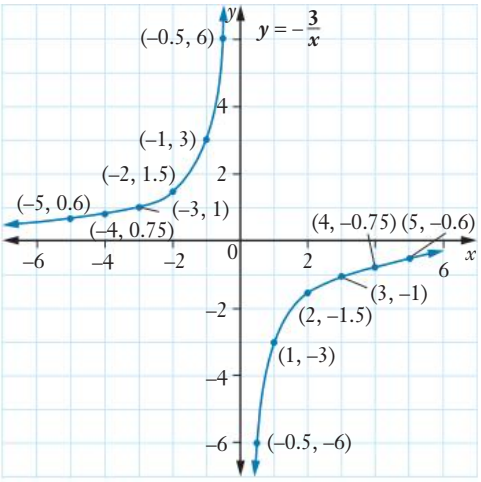
The two branches of this hyperbola are in the 1st and 3rd quadrants.

The x- and y-axes are both asymptotes.

The graph of $y = -\frac{3}{x}$ is shown here using this table of values.

x	–5	–4	–3	–2	–1	–0.5	–0.2	0	0.2	0.5	1	2	3	4	5
y	0.6	0.75	1	1.5	3	6	15	—	–15	–6	–3	–1.5	–1	–0.75	–0.6

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The two branches of this hyperbola are in the 2nd and 4th quadrants.

The x- and y-axes are both asymptotes.

This hyperbola $y = -\frac{3}{x}$ is further away from the x- and y-axes than the graph of $y = \frac{2}{x}$ is.

The hyperbola $y = \frac{k}{x}$

- The hyperbola $y = \frac{k}{x}$ has 2 separate branches in opposite quadrants of the number plane.
- It has **point symmetry** about (0, 0): if you rotate it upside-down, it will map onto itself.
- Its **asymptotes** are the x- and y-axes.
- If k is positive, the hyperbola is **decreasing** and its branches are in the 1st and 3rd quadrants.
- If k is negative, the hyperbola is **increasing** and its branches are in the 2nd and 4th quadrants.
- The higher the value of k, the further the hyperbola is from the x- and y-axes.

EXAMPLE 8

The time, T months, taken to pay off an interest-free loan is related to the size of each monthly repayment, \$$r$, by the reciprocal function $T = \frac{1800}{r}$.

a Graph the function for values of r from 100 to 900.

b Use the graph to estimate the time to pay off the loan if the monthly repayment is \$250.

c Use the formula to find the monthly repayment required to pay off the loan in 5 months.

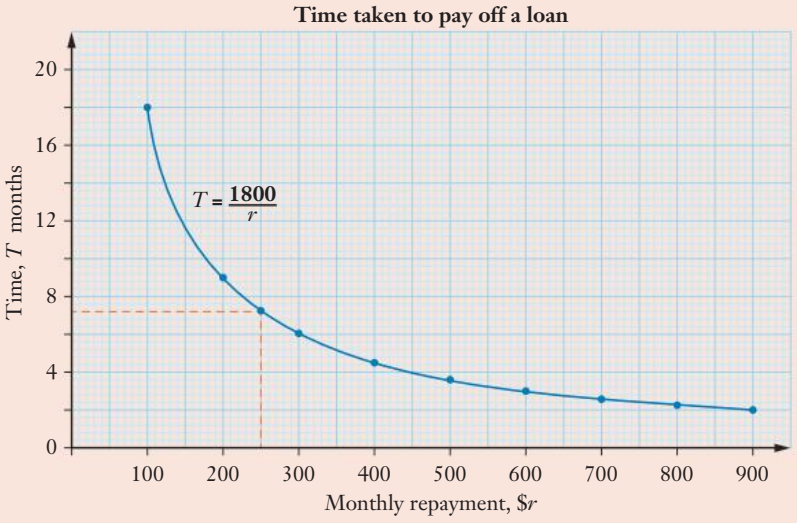
d Why is this model not useful for values of r greater than 1800?

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Solution

a $T = \dfrac{1800}{r}$, T-values rounded to 2 decimal places.

r	100	200	300	400	500	600	700	800	900
T (months)	18	9	6	4.5	3.6	3	2.57	2.25	2



b From the graph, when $r = 250$, $T = 7.2$.

The time taken to pay off the loan is 8 months. Rounded up to the nearest month.

c From the formula, when $T = 5$,

$$5 = \frac{1800}{r}$$
$$5r = 1800$$
$$r = \frac{1800}{5}$$
$$= 360$$

The monthly repayment required is \$360.

d If $r > 1800$, $T < 1$. It is not possible to pay off a loan in less than one month using monthly repayments.

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Exercise 10.05 The reciprocal function

1 Which equation represents a decreasing reciprocal function? Select **A**, **B**, **C** or **D**.

A $y = \frac{x}{2}$ **B** $y = \frac{2}{x}$ **C** $y = -\frac{x}{2}$ **D** $y = -\frac{2}{x}$

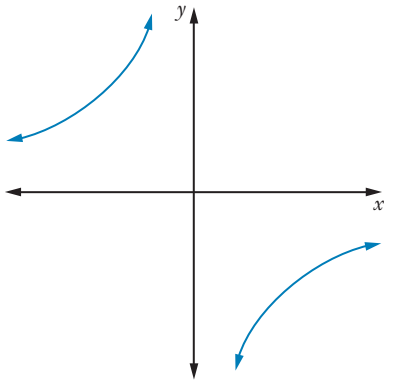
2 Which function's hyperbola lies in the 2nd and 4th quadrants? Select **A**, **B**, **C** or **D**.

A $y = \frac{4}{x}$ **B** $y = \frac{1}{4x}$ **C** $y = -\frac{5}{x}$ **D** $y = \frac{3}{x}$

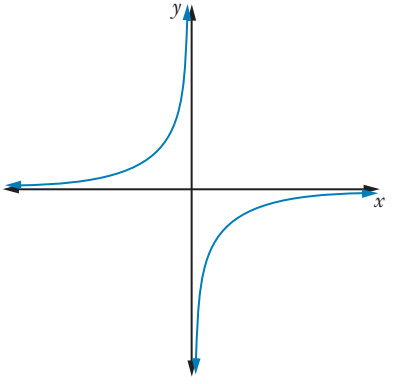
3 Match each reciprocal function with its hyperbola.

a $y = \frac{3}{x}$ **b** $y = -\frac{1}{x}$ **c** $y = -\frac{4}{x}$ **d** $y = \frac{5}{x}$

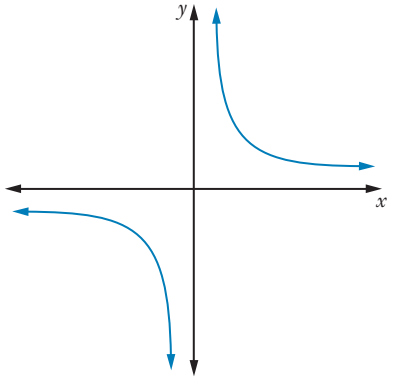
A



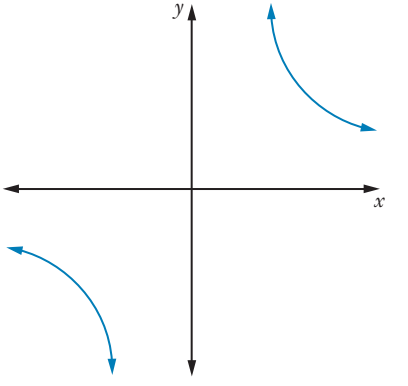
B



C



D



4 The time, T minutes, taken for Trish to drive to Nowra from her home depends on her average speed, s km/h, according to the function $T = \frac{4800}{s}$.

Example 8

a What is the dependent variable?

b Graph T for values of s from 40 to 110.

c Calculate the time in minutes taken to drive to Nowra at a speed of 92 km/h:

i using the graph **ii** using the function.

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d Use the graph to find the speed (to the nearest km/h) at which Trish must drive to reach Nowra in 70 minutes.

e How far is Nowra from Trish's house?

5 Nilshan, standing next to a lightpole, notices that the length of his shadow decreases as he moves further away from the light, according to the formula $h = \frac{56\,000}{d}$ where h cm is the length of his shadow and d cm is his distance from the light.

a Copy and complete this table of values, rounding h to the nearest whole number if needed.

d (cm)	50	100	150	200	250	300	350	400
h (cm)								

b What is the length of Nilshan's shadow, to the nearest cm, when he is 220 cm from the light?

c How far is Nilshan standing from the light if his shadow is 860 cm long?

WS
Variation problems

10.06 Inverse variation

Direct variation occurs when two variables, say x and y, change in the same direction.

If $y = kx$, we say 'y varies directly as x' or 'y is directly proportional to x', where k is a constant (the constant of variation).

Some examples of direct variation are:

- **fuel consumption and size of car:** the bigger the car, the higher the fuel consumption
- **stretch of a spring and the mass supported:** the heavier the mass, the longer the stretch
- **viewing range and height of tower:** the higher up you go, the further you can see.

Inverse variation occurs when two variables change in different directions.

If $y = \frac{k}{x}$, we say 'y **varies inversely as** x' or 'y is **inversely proportional to** x', where k is a constant.

Some examples of inverse variation are:

- **time of journey and average speed:** the faster you go, the less time it takes to get there
- **air temperature and altitude of aeroplane:** the higher up you fly, the lower the temperature
- **speed of Internet downloads and number of users:** the more users, the slower the download speed.

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Solving an inverse variation problem

1 Identify the two variables, say x and y, and form a variation equation $y = \frac{k}{x}$ involving a constant k.

2 Substitute the given values of x and y to find k.

3 Rewrite the equation $y = \frac{k}{x}$ using the value of k.

4 Substitute a value for x or y into the equation to solve the problem.

EXAMPLE 9


Inverse variation

The blood alcohol content (BAC), B, of an adult male after drinking beer varies inversely as his weight, W kg. A 72 kg man has a BAC of 0.059 after drinking beer. Find, to 3 decimal places, the BAC of a 90 kg man who drinks the same amount as the 72 kg man.

Solution

a $B = \frac{k}{W}$ Inverse variation

This equation means that the heavier the man, the lower his BAC.

b Substitute $W = 72$, $B = 0.059$ to find the value of k.

$$0.059 = \frac{k}{72}$$
$$k = 0.059 \times 72$$
$$= 4.248$$
$$\therefore B = \frac{4.248}{W}$$

When $W = 90$,

$$\therefore B = \frac{4.248}{90}$$
$$= 0.0472$$
$$\approx 0.047$$

The BAC of the 90 kg man is 0.047.

Exercise 10.06 Inverse variation

1 What is the missing word in each sentence?

a With direct variation, when one variable increases, the other variable _____.

b With inverse variation, when one variable increases, the other variable _____.

2 Which equation is an example of inverse variation? Select **A**, **B**, **C** or **D**.

A $y = \frac{x}{2}$ **B** $y = 2x^2$ **C** $y = \frac{1}{2}x$ **D** $y = \frac{2}{x}$

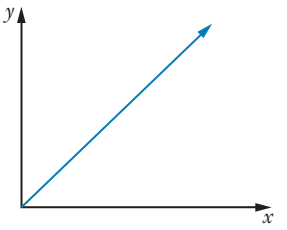
3 **a** If $y = \frac{k}{x}$ and $y = 12$ when $x = 5$, what is the value of k?

b If $V = \frac{k}{t}$ and $V = 7$ when $k = 56$, what is the value of t?

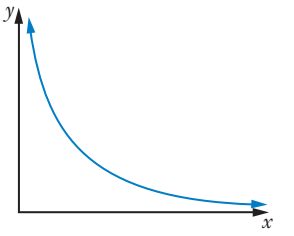
4 Match each equation with its graph if k is positive.

a $y = kx^2$ **b** $y = kx$ **c** $y = \frac{k}{x}$

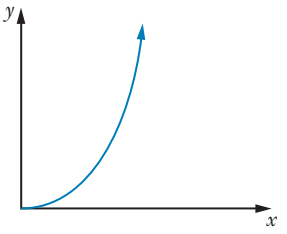
A



B



C



Example 9

5 The frequency, F beats per second, that a bird beats its wings varies inversely as the length, L cm, of its wings. A bird with wings of length 14 cm beats them at a frequency of 8 beats per second.

a Find the variation equation for F in terms of L.

b Calculate, to the nearest whole number, the wingbeat frequency for wings of length 18 cm.

c A bird beats its wings with a frequency of 4.5 beats per second. What is the length of its wings, correct to the nearest centimetre?

6 The temperature, T°C, of the air is inversely proportional to the height, h m, above sea level. At 400 m above sea level, the temperature is 12°C.

a What is the temperature 1000 m above sea level?

b At what height, to the nearest metre, is the temperature 19°C?

7 The journey time varies inversely as the speed. At a speed of 60 km/h, it takes Harley 90 minutes to drive to Windsor. How fast does he need to go to reach Windsor in 50 minutes?

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8 The amount of time it takes Sarah to move house is inversely proportional to the number of friends she has to help her. When she has four friends helping, the job takes $3\frac{3}{4}$ hours.

a How long will it take if she has six friends helping?

b How many friends must she have to help her to move house in 3 hours?

9 The volume of gas in a balloon is inversely proportional to the pressure applied to compress it. Helium has a volume of 20 m^3 at a pressure of 9 kPa.

a What is the volume of helium at a pressure of 45 kPa?

b What pressure is necessary to compress the gas to 8 m^3?

10 Eddy believes that at a train station, the number of people waiting on the platform is inversely proportional to the time until the next train arrives. According to his model, when there are 16 people waiting, the train will arrive in 2.5 minutes.

a When will the train arrive if there are five people waiting?

b How many people are waiting at the station 10 minutes before the train arrives?

11 The rate of vibration of a string varies inversely as its length. A string 8 cm long vibrates at 9375 Hz (hertz). What length of string will vibrate at 6250 Hz? Select **A**, **B**, **C** or **D**.

A 5 cm **B** 7 cm **C** 12 cm **D** 73 cm

12 The average amount of time a teacher can spend helping each student during a lesson varies inversely with the number of students in her class. If there are 24 students in Mrs Ellsmore's class, she can spend an average of 1.9 minutes per student.

a Calculate the average time Mrs Ellsmore can spend with each student if there are 29 students in her class. Answer to the nearest 0.1 minute.

b Calculate the class size that would allow her to spend 2.3 minutes with each student.

13 The number of people who can attend a concert varies inversely with the amount of space allowed per person. If 3000 cm^3 of space is allowed per person, the concert hall can hold 14 000. How many people can be accommodated if 2200 cm^3 of space is allowed per person?

SAMPLE HSC PROBLEM

A certain strain of yeast cell grows exponentially according to the formula $N = 350(1.04)^t$ where N is the number of yeast cells and t is the time in minutes.

a Graph this function for values of t from 0 to 30.

b What is the vertical intercept and what does it represent?

c How many yeast cells are there after 80 minutes?

d Use the graph to estimate when the number of cells reaches 1000.

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Study tip

Read the key words in an exam question

- **Compare:** show how things are similar or different
- **Describe:** provide characteristics or features
- **Estimate:** make an educated guess for a number or measurement, find roughly or approximately
- **Evaluate/calculate:** find the value, the numerical answer
- **Expand:** rewrite the algebraic expression without brackets, for example, $3(2b + 1) = 6b + 3$
- **Explain/Give a reason:** describe why or how
- **Hence:** find or prove a result using the answer(s) from the previous part of the question
- **Justify:** support argument or conclusion
- **Prove that/Show that/Deduce that:** in questions where the answer is given, prove that the result is true using formal mathematics and reasoning
- **Solve:** find the value of the unknown pronumeral in an equation
- **Write down/state:** write the answer, formula or result without working or explanation (usually in a 1-mark question).

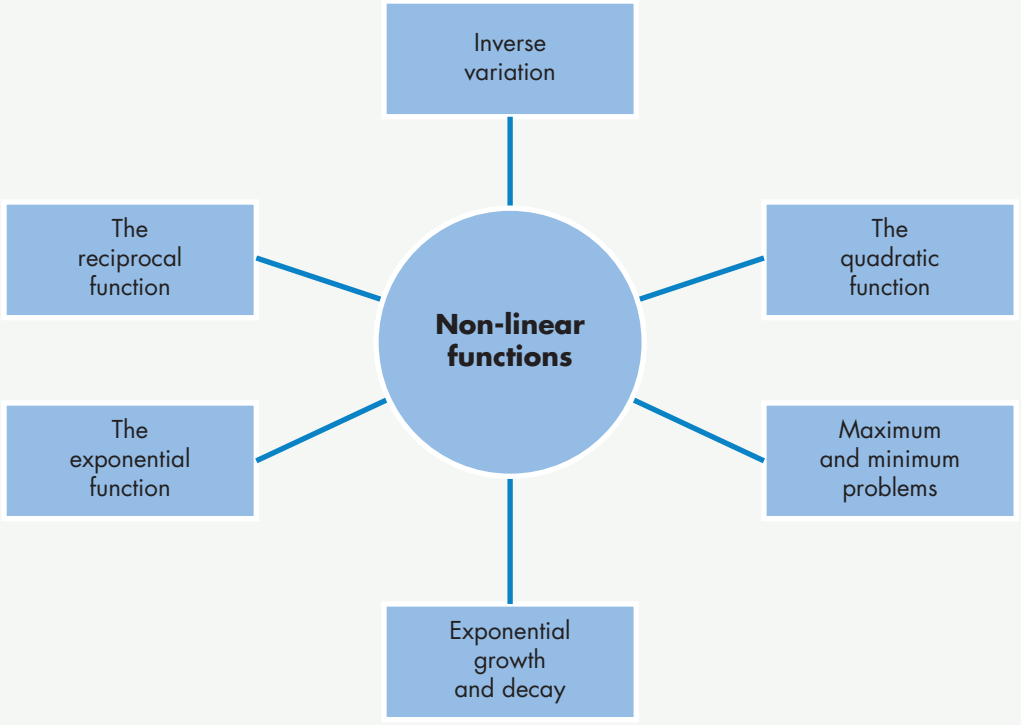
See the **Mathematical verbs** on page xiv at the front of the book for more details.

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10. CHAPTER SUMMARY

This chapter, Non-linear functions, examined quadratic, exponential and reciprocal functions, their graphs and their use as algebraic models. Make sure you know the name of each function, its equation and the properties of its graph, such as y-intercepts and asymptotes. Be aware of how different constants change the shape of a graph. Practise applying the functions to practical problems such as population growth and the braking distance of a car. Also revise the steps involved in solving an inverse variation problem.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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10. TEST YOURSELF

Exercise 10.01

1 **a** What type of function is $y = -x^2 + 4$?

b What is the name of its graph?

c Graph this function and write the coordinates of the vertex.

Exercise 10.02

2 The running cost, $\$C$ per hour, of a ship travelling at an average speed of s knots, is given by the function

$C = 2s^2 - 128s + 2400.$

a Copy and complete this table of values.

s	22	26	30	34	38	42	46	50
C	552		360			552		

b Hence draw the graph for the function $C = = 2s^2 - 128s + 2400$.

c Use your graph to estimate the average speeds that have a running cost of \$480 per hour.

d Find the minimum running cost and the speed that gives this cost.

Exercise 10.03

3 **a** Draw the graph of $y = 5(1.2)^x$ for values of x from 0 to 5.

b What is the y-intercept?

c Is this function increasing or decreasing?

4 **a** Draw the graph of $y = (1.5)^{-x}$ for values of x from –4 to 4.

b What is the y-intercept?

c Is this function increasing or decreasing?

d What is the asymptote?

Exercise 10.04

5 A full water tank is being emptied and the volume of water remaining in the tank is $V = 1400(0.97)^t$ where V is in litres and t is the time in minutes.

a How much water does the tank hold?

b How much water, to the nearest litre, is in the tank after 42 minutes?

c Find when the tank will be half full.

d What are the limitations of this exponential decay model?

6 The population of a town is modelled by the function $P = 65\ 000(1.04)^n$, where P is the population at the end of n years.

a What is the population of the town after 10 years?

b What is the vertical intercept and what does it represent?

c After how many years will the initial population double?

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7 The number of days, D, to build a house is related to the number of builders, b, working on the job, according to the function $D = \frac{240}{b}$.

Exercise 10.05

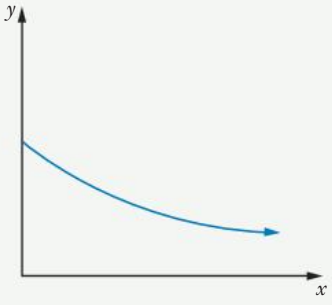
a Graph this function for values of b from 2 to 10.

b How long, to the nearest day, will it take to build the house using nine builders?

c How many builders are needed to build the house in 5 days?

d What are the limitations to this model?

8 What could be the equation of this graph?
Select **A**, **B**, **C** or **D**.

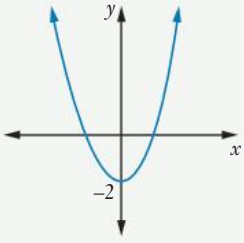


A $y = 2(5^{-x})$ **B** $y = \frac{2}{x}$

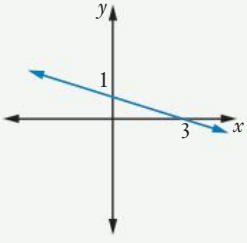
C $y = \frac{1}{2}x^3$ **D** $y = 5(2^x)$

9 Match each graph with its function below.

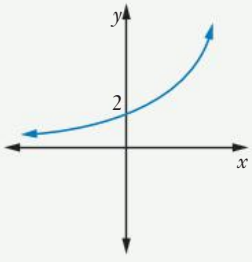
a



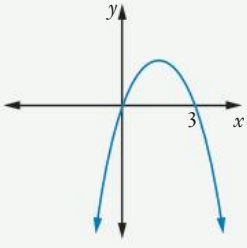
b



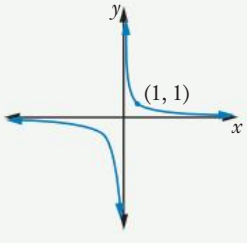
c



d



e



A $y = 2(1.8^x)$ **B** $y = -\frac{1}{3}x + 1$ **C** $y = x^2 - 2$

D $y = -x^2 + 3x$ **E** $y = \frac{1}{x}$

10 The time taken, T hours, for a train trip between Sydney and Melbourne varies inversely as the average speed, S km/h, of the train. It takes 10 hours for the trip travelling at 80 km/h.

Exercise 10.06

a Calculate how long the trip will take travelling at 64 km/h.

b If the trip takes 12 hours, what is the speed of the train, correct to the nearest km/h?

Qz Chapter quiz

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STATISTICAL ANALYSIS

11.

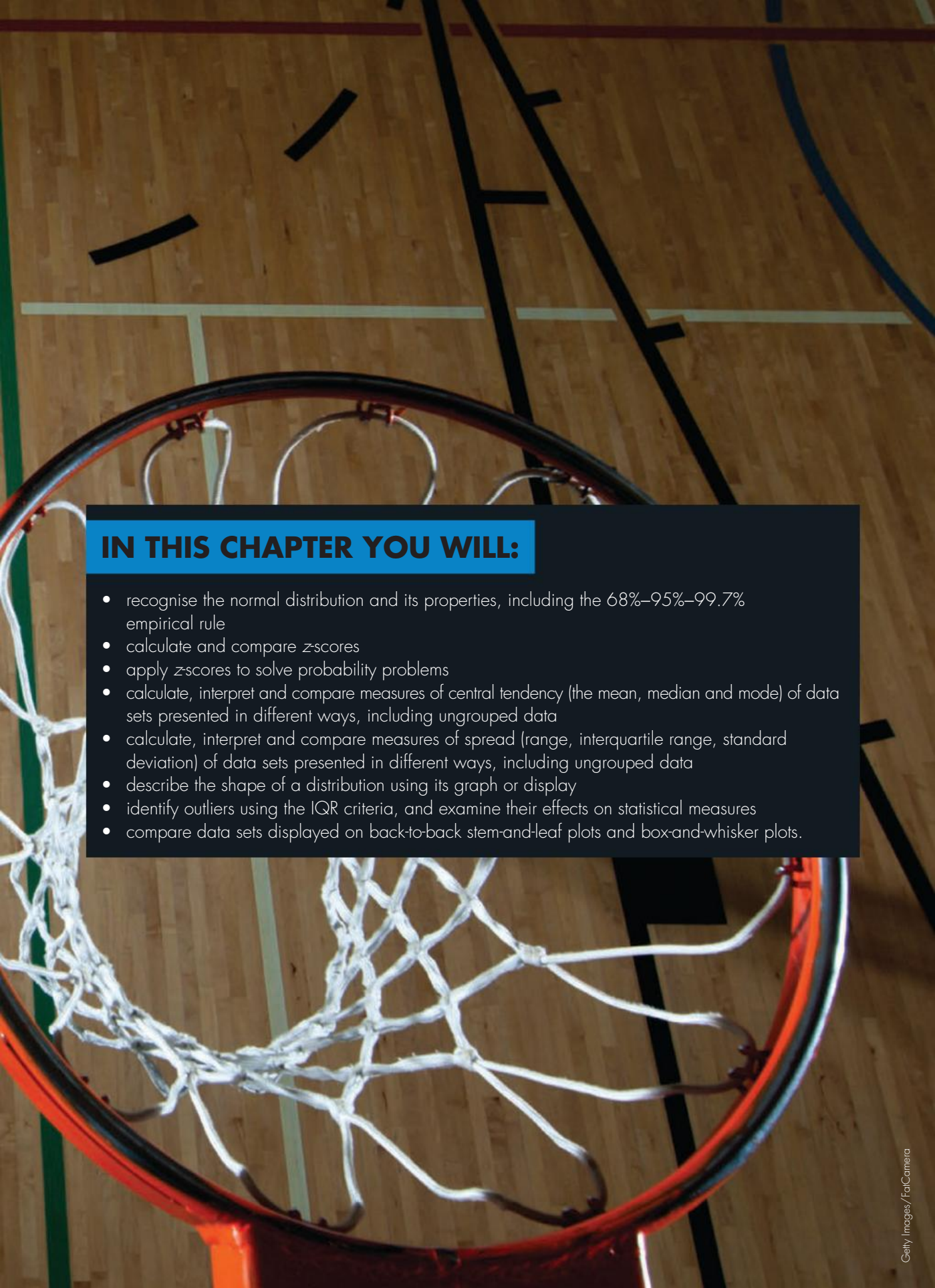
THE NORMAL DISTRIBUTION

The mean height of Australian men is 176 cm, with a standard deviation of 7 cm. Australian basketball player Andrew Bogut is 213 cm tall, which makes him just over 5 standard deviations above the mean. How does this figure compare Andrew to all Australian men? What percentage of the adult male population has a height of 195 cm?

CHAPTER OUTLINE

S5 11.01 The normal distribution
S5 11.02 *z*-scores
S5 11.03 Comparing *z*-scores
S1.2* 11.04 Measures of central tendency and spread
S1.2* 11.05 The shape of a distribution
S1.2* 11.06 The effect of outliers
S1.2* 11.07 Comparing data sets using plots

*Year 11 revision



IN THIS CHAPTER YOU WILL:

- recognise the normal distribution and its properties, including the 68%–95%–99.7% empirical rule
- calculate and compare *z*-scores
- apply *z*-scores to solve probability problems
- calculate, interpret and compare measures of central tendency (the mean, median and mode) of data sets presented in different ways, including ungrouped data
- calculate, interpret and compare measures of spread (range, interquartile range, standard deviation) of data sets presented in different ways, including ungrouped data
- describe the shape of a distribution using its graph or display
- identify outliers using the IQR criteria, and examine their effects on statistical measures
- compare data sets displayed on back-to-back stem-and-leaf plots and box-and-whisker plots.

Getty Images/FatCamera

TERMINOLOGY

back-to-back stem-and-leaf plot
cluster
empirical rule
interquartile range (IQR)
measures of spread
negatively skewed
outlier
range
skewness
symmetry
box plot
cumulative frequency histogram
five-number summary
mean
median
normal curve
positively skewed
raw score
standard deviation
tail
class centre
distribution
frequency histogram
measures of central tendency
mode
normal distribution
population
shape
stem-and-leaf plot
z-score

WS
Assignment 11

SkillCheck

1 Find the following, correct to the nearest whole number.

a 99.7% of 12 600 **b** 2.5% of 320 **c** 0.3% of 5000

2 Of 12 000 university students, 95% passed their final exam. How many passed?

3 A survey of 6000 people found that 68% did not smoke. How many people smoked?

4 In a region in India, 99.7% of days per year have a temperature greater than 25°C. How many days have a temperature less than or equal to 25°C? Answer to the nearest day.

5 Find the value of:

a $\frac{95-68}{2}$ **b** $95+\frac{99.7-95}{2}$ **c** $68+\frac{1}{2}(95-68)$.

6 Given the formula $z=\frac{x-\bar{x}}{s}$, find, correct to one decimal place:

a z when $x=170, \bar{x}=160, s=8$

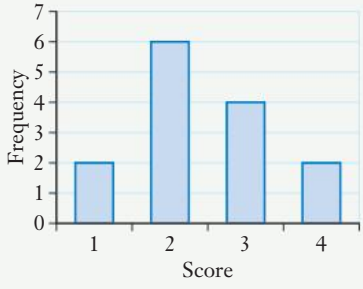
b x when $z=1.7, \bar{x}=90, s=12$.

7 Find the mean of each data set.

a

x	f
10	9
11	8
12	3
13	7

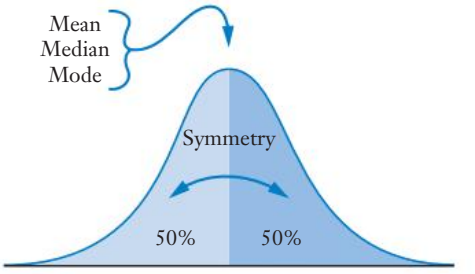
b



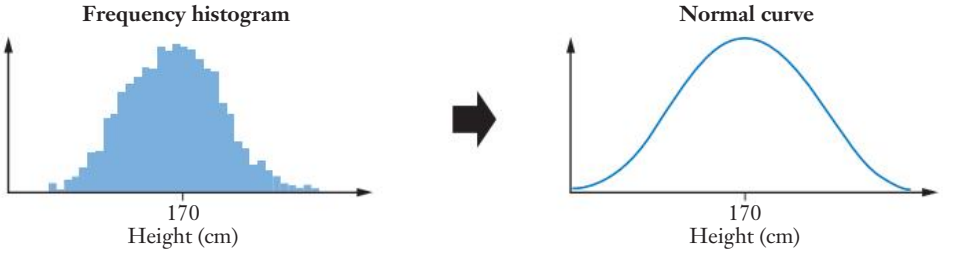
ISBN 9780170413633

11.01 The normal distribution

A **normal distribution** is a statistical distribution where the mean, median and mode are equal. It is a bell-shaped curve that is **smooth** and **symmetrical** about the mean.

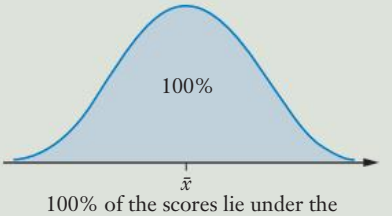


The heights of a large **sample** of adults are represented by this **frequency histogram** with a mean of 170 cm. As the sample becomes larger and resembles the **population** of all adults, the histogram becomes closer to a **normal curve**.

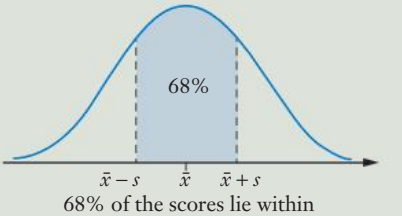


Many natural characteristics, such as height, blood pressure and skull width, are **normally distributed**.

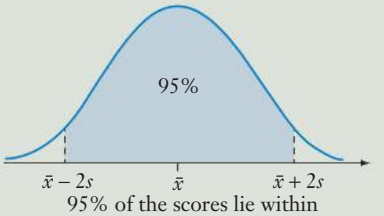
The empirical rule for a normal curve



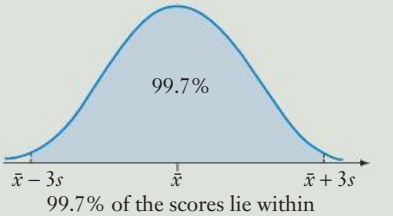
100% of the scores lie under the normal curve.



68% of the scores lie within 1 standard deviation of the mean.



95% of the scores lie within 2 standard deviations of the mean



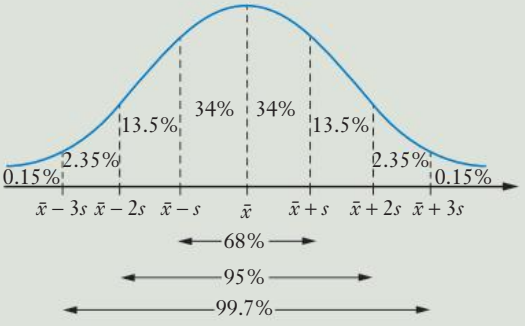
99.7% of the scores lie within 3 standard deviations of the mean.

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WS
The normal curve

This diagram shows the **empirical rule** in more detail. These percentages can be calculated using the **symmetry** of the curve.

A full-page version of this diagram can be downloaded from NelsonNet as the worksheet: The normal curve.



EXAMPLE 1

The heights of a large group of adults are normally distributed with a mean of 170 cm and a standard deviation of 8 cm.

a What percentage of heights:

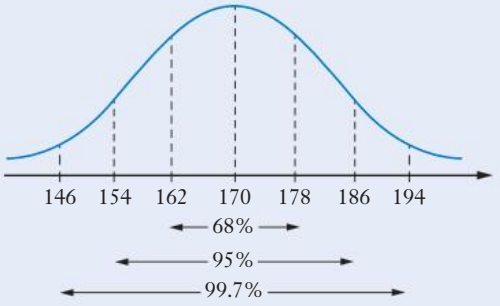
- **i** lie between 162 cm and 178 cm?
- **ii** are more than 178 cm?

b If a random sample contains 6000 adults, how many have a height:

- **i** between 146 cm and 194 cm?
- **ii** between 162 cm and 186 cm?

Solution

Mean $\bar{x} = 170$, standard deviation $s = 8$

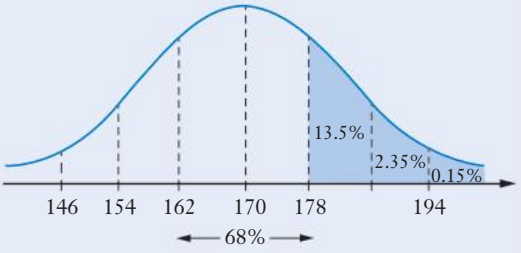


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Draw a diagram with values on the x-axis at different standard deviations from the mean.

a i 68%

Within one standard deviation of the mean



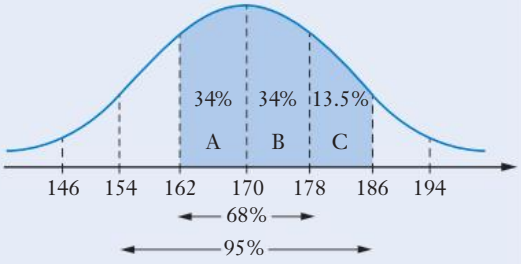
ii Required percentage is represented by the shaded area. By symmetry of curve,

percentage is $\frac{100\% - 68\%}{2} = 16\%$

or $13.5\% + 2.35\% + 0.15\% = 16\%$

b i $99.7\% \times 6000 = 5982$

Within 3 standard deviations of the mean



ii Required percentage is represented by the shaded area A + B + C.

By symmetry, percentage is

$68\% + \frac{95\% - 68\%}{2} = 81.5\%$ Area A and B + Area C

or $\frac{68\%}{2} + \frac{95\%}{2} = 81.5\%$ Area A + Area B and C

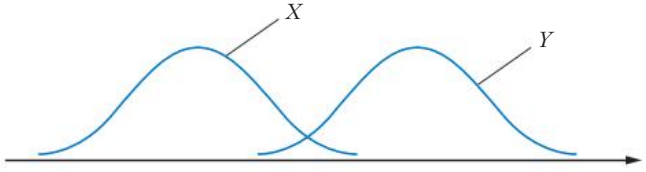
or $34\% + 34\% + 13.5\% = 81.5\%$

$81.5\% \times 6000 = 4890$

Exercise 11.01 The normal distribution

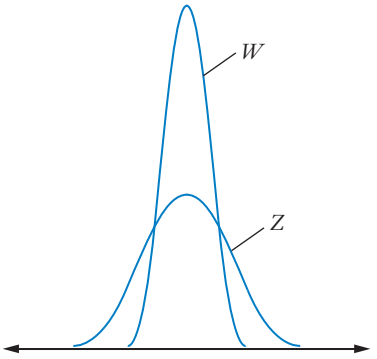
Hint: Draw a diagram.

1 a Which normal curve, X or Y, has the higher mean?



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b Which normal curve, W or Z, has the higher standard deviation?



2 The mean of a normally distributed set of scores is 31 and the standard deviation is 6. What percentage of the scores:

a lie between 25 and 37?
b are less than 19?
c lie between 19 and 43?
d are more than 37?
e are less than 13?
f greater than 49?

3 Roofing nails are produced by a machine and the lengths are normally distributed with a mean of 5.00 cm and a standard deviation of 0.01 cm. Complete the following.

a 68% of nails will have a length between _____ and _____ .
b 95% of nails will have a length between _____ and _____ .
c 99.7% of nails will have a length between _____ and _____ .

4 A factory makes circular brake discs with a mean diameter of 300 mm and a standard deviation of 2 mm. The diameters are normally distributed. If diameters more than 3 standard deviations from the mean are rejected, which of these diameters would be rejected? Select **A**, **B**, **C** or **D**.

A 295 mm **B** 307 mm **C** 306 mm **D** 296 mm

Example 1

5 Soft drink is sold in cans that have a mean capacity of 330 millilitres and a standard deviation of 5 millilitres. The distribution of the soft drink in each can is normal.

a What amount of soft drink is:
i 1 standard deviation above the mean?
ii 2 standard deviations below the mean?

b To fill a soft drink can, what percentage of cans hold:
i between 330 and 335 millilitres?
ii more than 345 millilitres?

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6 The nicotine content of a brand of cigarette is normally distributed with a mean of 14.5 mg and a standard deviation of 1.2 mg. What percentage of the nicotine content is between 13.3 mg and 15.7 mg? Select **A**, **B**, **C** or **D**.

A 34% **B** 95% **C** 68% **D** 99.7%

7 The expected life of light globes is normally distributed with a mean of 15 000 hours.

a What is the standard deviation if 95% of globes are expected to last between 14 500 and 15 500 hours?

b Between what two values does 68% of the lifetimes lie?

c If 12 000 light globes are produced, how many are expected to last less than 14 250 hours?

8 The skull widths of a group of adults are normally distributed with a standard deviation of 49 mm.

a What is the mean width if 2.5% of skull widths are less than 1297 mm?

b If there are 8000 adults in the group, how many have a skull width between 1297 mm and 1542 mm?

9 A machine produces steel pipes with a mean length of 148.4 mm and a standard deviation of 0.03 mm. The lengths are distributed normally.

a If pipes with a length more than 3 standard deviations from the mean are rejected, what percentage of pipes are rejected?

b One customer only accepts pipes with lengths between 148.34 mm and 148.43 mm. What percentage of pipes does she accept?

10 During pregnancy, the period from conception to delivery of a baby is called the gestation period. The gestation period for humans is approximately normally distributed with a mean of 266 days and a standard deviation of 16 days.

a Between what two values will 95% of gestation periods lie?

b What percentage of pregnancies will last more than 282 days?

c Is it possible to have a gestation period of 320 days? Justify your answer.

11 Packets of muesli have an advertised weight of 500 g. The actual weights were found to be normally distributed with a mean of 490 g and a standard deviation of 10 g.

a What percentage of packets would you expect to be below the advertised weight?

b Do you think this product is labelled fairly? Justify your answer.

12 Wedges of cheddar cheese are labelled as weighing 250 g. A survey found the weights to be normally distributed about a mean of 255 g with a standard deviation of 2.5 g.

a What percentage of wedges contained more than the labelled weight?

b If 6000 wedges were included in the survey, how many contained less than the labelled weight?

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13 Packets of sugar are labelled as weighing 1 kg. When the weights of 400 packets were checked, they were found to be normally distributed with a mean of 1.015 kg and a standard deviation of 0.015 kg.

a What percentage of packets weighed less than the labelled weight?

b The packaging machine is to be reset so that the average weight of a packet is 1.030 kg. What percentage of packets will now weigh less than the labelled weight?

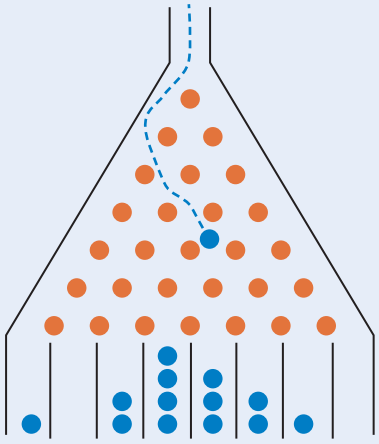
DID YOU KNOW?

Galton board

In 1890, English explorer and scientist Francis Galton designed a probability device called a **Galton board**, probability pinball, or bean machine. Balls fall through a system of pins into slots. Every time a ball hits a pin, it has an even chance of falling to the left or to the right of the pin. The piles of balls that accumulate in the slots at the bottom of the board form a histogram that is approximately a normal curve.

A demonstration of the Galton board can be viewed on YouTube.

What is the probability that a ball lands in the first slot on the left?



INVESTIGATION

IS THIS DATA NORMAL?

The stem-and-leaf plot displays the lifetimes, in hours, of two brands of batteries.

Betta Charge	Stem	Longa Life
5 5 4 2	10	3 4 5
8 7 7 7 7 7 7 4 4 3	11	2 3 3 4 4 5 6
9 9 8 8 7 6 6 6 5 4 4 0	12	1 2 2 3 3 4 5 5 7 9 9 9
8 8 8 7 7 7 6 5 4 3 1	13	0 2 3 3 4 4 4 5 6 6 8 8 9
9 8 8 8 5 5 2 2	14	1 2 2 3 5 5 6 7 8 8
7 7 5 1	15	0 3 3 4 6

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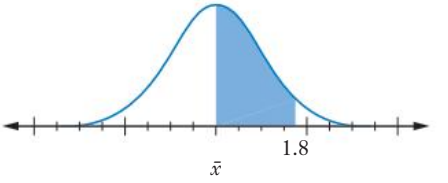
1 Examine the data for Betta Charge batteries.

- **a** Find the mean, median and mode. Are they approximately equal?
- **b** Calculate the standard deviation.
- **c** Find the percentage of batteries with lifetimes within 1 standard deviation of the mean.
- **d** Find the percentage of batteries with lifetimes within 2 standard deviations of the mean.
- **e** Consider the percentages in parts **c** and **d** and decide if the data is normally distributed.

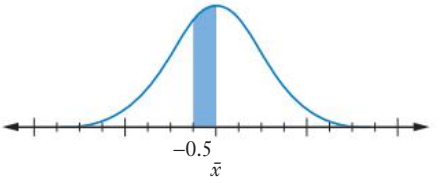
2 Are the lifetimes of Longa Life batteries normally distributed? Justify your answer.

11.02 *z*-scores

A ***z*-score** (or **standardised score**) shows the position of a '**raw**' **score** relative to the mean. For example, a *z*-score of 1.8 means a score is 1.8 standard deviations above the mean.



A *z*-score of –0.5 means a score is half a standard deviation below the mean.

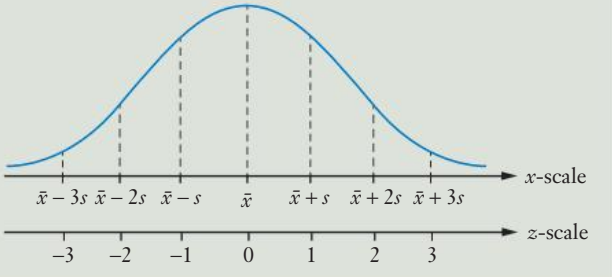


z-scores

The ***z*-score** is the number of standard deviations a score is from the mean.

$$z\text{-score} = \frac{\text{score} - \text{mean}}{\text{standard deviation}}$$

$$z = \frac{x - \bar{x}}{s}$$



The distribution of *z*-scores is a normal distribution with a mean of 0 and a standard deviation of 1.

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z-scores

EXAMPLE 2

The measured amounts of milk in 600 mL cartons were found to be normally distributed with a mean of 600 mL with a standard deviation of 5 mL.

a Calculate the z-scores for these amounts.

i 610 mL **ii** 596 mL

b What amount of milk has a z-score of:

i 1.5? **ii** −2.6?

c What percentage of cartons have less than 590 mL of milk?

Solution

a $\bar{x} = 600, s = 5, z = \dfrac{x - \bar{x}}{s}$

i $z = \dfrac{610 - 600}{5} = 2$

ii $z = \dfrac{596 - 600}{5} = -0.8$

b **i** $1.5 = \dfrac{x - 600}{5}$

$7.5 = x - 600$

$x = 607.5$ mL

ii $-2.6 = \dfrac{x - 600}{5}$

$-13 = x - 600$

$x = 587$ mL

c 590 mL is 2 standard deviations below the mean and has a z-score of −2.

Percentage of scores below $z = -2$ is $\dfrac{100\% - 68\%}{2} = 2.5\%$

EXAMPLE 3

The net weights of packets of muffin mix have a normal distribution with a mean of 432 g and a standard deviation of 2 g.

a Complete the table.

Net weight (g)	428		432	434	
z-score		−1	0		2

b What percentage of packets will have a net weight:

i greater than 432 g? **ii** between 428 g and 434 g?

c If packets are labelled 'net weight 430 g', what percentage of packets will have a net weight less than 430 g?

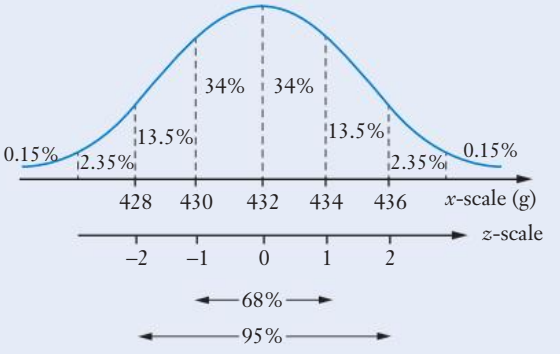
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Solution

a

Net weight (g)	428	430	432	434	436
z-score	-2	-1	0	1	2

b



i 50% — Greater than $z = 0$

ii $68\% + \frac{95\% - 68\%}{2} = 81.5\%$ — Between $z = -2$ and $z = 1$

or $13.5\% + 34\% + 34\% = 81.5\%$

c $\frac{100\% - 68\%}{2} = 16\%$ — Less than $z = -1$

or $13.5\% + 2.35\% + 0.15\% = 16\%$

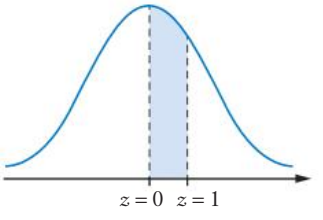
Exercise 11.02 z-scores

1 Phoebe competed in a dance competition. The mean score in the dance competition was 7.5 with a standard deviation of 1.3. Phoebe scored 10.1. What was this as a z-score?

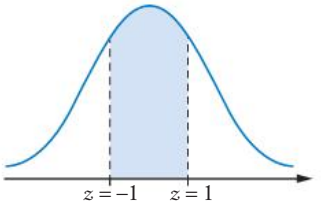
Example 2

2 What proportion of scores is shaded in each diagram?

a

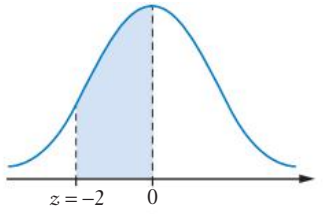


b

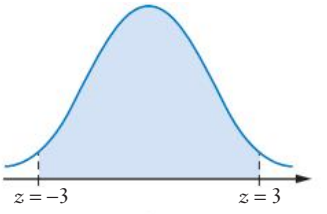


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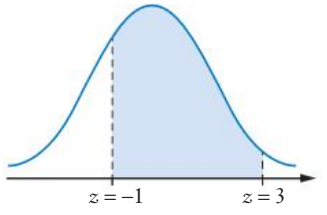
c



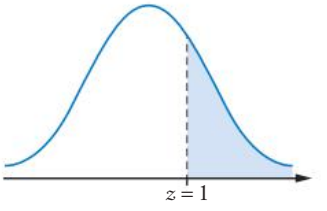
d



e



f



3 In a test, the marks are normally distributed. The average mark is 70 and the standard deviation is 15. If Jordan has a z-score of 1.8, what is his actual mark? Select **A**, **B**, **C** or **D**.

A 97 **B** 85 **C** 27 **D** 87

4 David's results in two geography tests are shown in the table below.

	Class mean	Class standard deviation	David's test marks
Test 1	72	6	84
Test 2	72	12	84

a What is David's z-score in test 2?

b Explain what David's z-score for test 2 means, in terms of his actual mark, the class mean and standard deviation.

c What mark in test 1 would be equivalent to a z-score of –0.5?

5 Blood pressures are normally distributed with a mean of 130 and a standard deviation of 20. Sam has a z-score of –1 when his blood pressure is taken.

a Interpret this z-score in terms of the mean and standard deviation of blood pressures.

b What is Sam's blood pressure?

c If Zac has a blood pressure of 145, what is his z-score?

6 The heights of a group of preschool children are normally distributed with a mean of 90 cm and a standard deviation of 10 cm.

a Find the z-score, to one decimal place if necessary, for:

i Jamal: 86 cm **ii** Maria: 1.25 m.

b If Ann has a z-score of 2.2, how tall is she?

c Drew has a z-score of –1.4. How tall is he?

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d What percentage of children are between 70 cm and 90 cm tall?

e What percentage of children are more than 1 m tall?

7 Jellybeans are put into 100 g packets. The machine that performs this task is set to a mean mass of 100 g with a standard deviation of 2 g.

Example 3

a Complete the table.

Mass (g)	94		98	100		104	
***z*-score**		–2		0	1		3

b What percentage of packets will have a mass:

i more than 100 g? **ii** between 98 g and 102 g? **iii** less than 96 g?

c The masses of 2500 packets are noted. How many are expected to be greater than 104 g?

8 IQs (intelligence test scores) are normally distributed with a mean of 100 and a standard deviation of 16.

a What are the z-scores for IQs of:

i 100? **ii** 132?

b If 30 million people live in Australia, how many are expected to have an IQ above 132?

c To join the Four Sigma Club you need an IQ with a z-score of 4 or more.

i What is meant by a z-score of 4? **ii** What IQ is this?

9 The weights of flathead fish caught in Currumbene Creek are normally distributed with a mean of 1.2 kg and a standard deviation of 0.4 kg.

a Complete the table.

Weight (kg)		0.4		1.2		2.0	
***z*-score**	–3		–1		1		3

b What percentage of flathead weigh between 0.4 kg and 1.6 kg?

c If 4800 flathead were caught one season, how many weighed more than 2 kg?

d If flathead weighing less than 0.8 kg are returned to the creek, how many of the 4800 flathead would be returned?

10 Anthea sat for two Maths tests. The test results of Year 12 are normally distributed for both tests and the means and standard deviations are shown in the table.

	Trigonometry	**Probability**
Mean	64	59
Standard deviation	6.2	6.1

a If Anthea scored 65 in Trigonometry and 70 in Probability, what are her z-scores?

b If 56 students sat for Trigonometry, how many are expected to score less than 70.2?

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WS
z-scores

11.03 Comparing *z*-scores

Since z-scores do not depend on the value of the mean, we can use them to compare scores from different normal distributions.

EXAMPLE 4

Ashleigh's marks for Maths, Science and History as well as the mean and standard deviation of her class's marks for each subject are given in the table.

Subject	Ashleigh's mark	Class mean	Class standard deviation
Maths	80	75	16
Science	60	54	12
History	70	78	10

a Find Ashleigh's z-score for each subject.

b In which subject did she perform best? Justify your answer.

Solution

a

Subject	Ashleigh's mark	Ashleigh's z-score
Maths	80	$\frac{80-75}{16} = 0.3125$
Science	60	$\frac{60-54}{12} = 0.5$
History	70	$\frac{70-78}{10} = -0.8$

b Science, because she had the highest z-score for this subject.

Exercise 11.03 Comparing *z*-scores

Example 4

1 Ziad scored 78% on a Maths test and Nina scored 83% on an English test. Both students gained a z-score of 1.3. Who performed better? Select **A**, **B**, **C** or **D**.

A Nina

B Ziad

C Both performed the same

D Can't tell

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2 Jude's marks in her half-yearly exams are shown in the table, along with the class mean and standard deviation for each subject.

Subject	Mark	Mean	Standard deviation
English	65	75	14
Maths	72	65	10
Science	58	62	16
Art	78	85	8

a Find Jude's z-scores for each subject.

b Rank her subjects in order of her performance compared with the other students.

3 Test scores in 3 subjects are approximately normally distributed with means and standard deviations as shown. The raw scores for 3 students are given.

Subject	Peta	Mel	Joe	Mean	Standard deviation
Visual Arts	48	60	70	66	16
Legal Studies	67	82	74	68	12
Maths Standard 2	70	79	55	70	9

a Calculate the z-scores for each student.

b What is Mel's best subject?

c Which is better: Peta's mark in Maths Standard 2 or Joe's mark in Visual Arts?

d Who achieved the best result overall? What subject was this in?

4 Elizabeth and Adam are students who compared their heights with data collected about almost one million 18–24 year olds. The heights are normally distributed.

	Mean height (cm)	Standard deviation
Male group	178	6.5
Female group	164	5

a If Elizabeth's height is 169 cm and Adam's height is 188.4 cm, who is taller for their age group? Justify your answer.

b If Amy joined the women's group and has a z-score of 2, how tall is she?

c Lleyton joined the men's group. If his z-score is 2, how much taller than Amy is he?

5 Annual rainfall figures are normally distributed. In Downtown the annual rainfall distribution has a mean of 950 mm and a standard deviation of 420 mm. In Uptown the mean annual rainfall is 1325 mm with a standard deviation of 238 mm. Which town is more likely to have an annual rainfall of 1100 mm? Justify your answer.

6 The 2-year-old model of a particular car has an average value of $18 000 with a standard deviation of $1500. The 3-year-old model of the same car has an average value of $15 000 with a standard deviation of $1600. Which is the better car: a 2-year-old model with a value of $17 000 or a 3-year-old model with a value of $13 000?

INVESTIGATION

SCALING OF EXAM MARKS

In order to compare results in different subjects, an examiner uses z-scores to scale raw exam marks to a common mean and standard deviation.

If Ashleigh's exam marks in each subject in Example **4** were scaled to a common mean of 65 and standard deviation of 12, her **scaled mark** in Maths would be:

$$z = \frac{x - \bar{x}}{s}$$

$$0.3125 = \frac{x - 65}{12}$$

$$3.75 = x - 65$$

$$x = 68.75$$

$$\approx 69$$

Show that Ashleigh's other scaled marks are 71 for Science and 55 for History.

Statistical measures

Statistical measures puzzle

Statistical match-up

WS Interquartile range

WS Scoring statistics

WS Statistical calculations

11.04 Measures of central tendency and spread

A **measure of central tendency** (or **measure of location**) is an **average** used to indicate the centre of a **data set**.

Measures of central tendency

- **Mean**, (or average) $\bar{x} = \dfrac{\text{sum of scores}}{\text{number of scores}} = \dfrac{\Sigma x}{n}$ or $\dfrac{\Sigma fx}{\Sigma f}$
- **Median** is the middle score when scores are arranged in order.

  If there is an even number of scores, it is the average of the 2 middle scores.
- **Mode** is the most common score or category in a data set. A data set can have more than one mode, or no mode at all.

The mean or median best describe the centre of **numerical** data whereas the mode is the only central measure of **categorical** data.

Measures of spread are used to indicate the spread of data in a set. They measure where the data lie.

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Measures of spread

- **Range** = highest score – lowest score
- **Interquartile range (IQR)** = upper quartile – lower quartile = $Q_3 - Q_1$

  The IQR is the range of the middle 50% of scores.

- **Standard deviation**, σ, is found using a calculator's statistics mode.


Interquartile range


Interquartile range

The **standard deviation** is the best measure of spread because its value, like the mean, depends on every score in the data set. The larger the standard deviation, the more spread out are the scores.

We usually use the **population standard deviation**, σ. If the set of data is a sample, then use the **sample standard deviation**, s, to estimate the results for a population.

EXAMPLE 5

For each data set below, find:

i the mean (to one decimal place) **ii** the median **iii** the mode

iv the range **v** the interquartile range.

a

Score (x)	Frequency (f)
2	2
3	4
4	1
5	3
6	5

b

Stem	Leaf
0	3 5
1	2 6 7
2	0 3 3
3	1 9
4	2 3

Solution

a Data set has 15 scores

Sum of f column

i Mean $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{2\times2+3\times4+4\times1+5\times3+6\times5}{15}$$

$$= 4.333\ldots$$

$$\approx 4.3$$

ii The scores arranged in order are:

2 2 3 3 3 3 4 5 5 5 6 6 6 6 6

Median

Median $(Q_2) = 5$

There is an odd number of scores. The median is the middle (8th) score.

iii Mode = 6

The most common score

iv Range = highest score – lowest score

$= 6 - 2$

$= 4$

2 2 3 3 3 3 4 5 5 5 6 6 6 6 6

Q_1 Median Q_3

$\text{IQR} = Q_3 - Q_1$

$= 6 - 3$

$= 3$

50% of the data lies between 3 and 6.

b Data set has 12 scores.

i $\text{Mean } \bar{x} = \dfrac{3+5+12+16+\ldots\ldots+42+43}{12}$

$= \dfrac{274}{12}$

$= 22.8333\ldots$

≈ 22.8

ii The scores arranged in order are:

3 5 12 16 17 20 23 23 31 39 42 43

$\text{Median} = \dfrac{20+23}{2}$

$= 21.5$

There is an even number of scores. The median is the average of the 6th and 7th scores.

iii Mode = 23

iv Range = highest score – lowest score

$= 43 - 3$

$= 40$

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v 3 5 12 16 17 20 23 23 31 39 42 43

Q_1 Median Q_3

$$Q_1 = \frac{12+16}{2} = 14 \qquad Q_3 = \frac{31+39}{2} = 35$$

$$\text{IQR} = Q_3 - Q_1 = 35 - 14 = 21$$

EXAMPLE 6

For each data set below, find:

i the range **ii** the interquartile range.

a

Score (x)	Frequency (f)
15	1
16	5
17	4
18	3
19	2

b

Stem	Leaf
0	4 5 8 9
1	2 2 5
2	0 3 8 8 8
3	1 6 7 9
4	0 5

Solution

a Data set has 15 scores.

The scores arranged in order are:

15 16 16 16 16 16 17 17 17 17 18 18 18 19 19

Q_1 Q_2 Q_3

i Range = 19 – 15 = 4

ii Interquartile range = $Q_3 - Q_1$
= 18 – 16
= 2

b Data set has 18 scores.

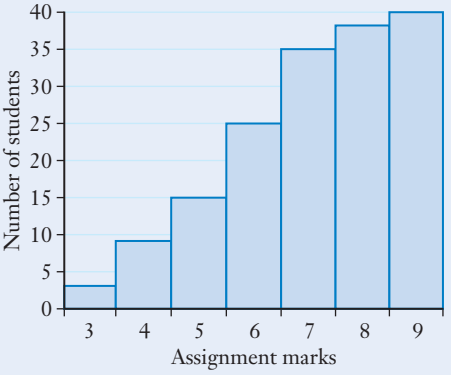
Stem	Leaf	
0	4 5 8 9	Q_1
1	2 2 5	Q_2
2	0 3 8 8 8	Q_3
3	1 6 7 9	
4	0 5	

i Range = 45 − 4 = 41

ii Interquartile range = $Q_3 - Q_1$
= 36 − 12
= 24

EXAMPLE 7

The assignment marks of Year 12 students at Baramvale High are shown in the **cumulative frequency** histogram.



a How many students completed the assignment?

b What was the range?

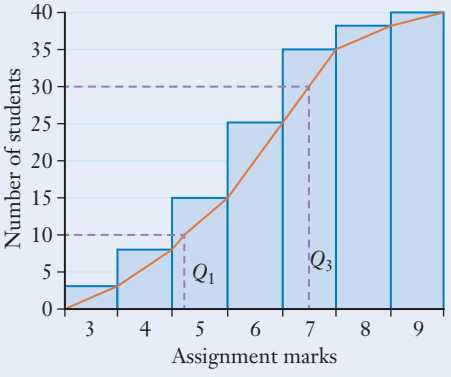
c Find the interquartile range.

Solution

a 40 students completed the assignment.

b Range = 9 − 3 = 6

c Interquartile range = $Q_3 - Q_1$
= 7 − 5
= 2



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Mean and standard deviation from a calculator

EXAMPLE 8

The net weekly wages of 8 labourers are:

\$730 \$490 \$600 \$440 \$490 \$370 \$700 \$580

Use your calculator to show that:

a the mean wage is \$550

b the standard deviation is \$117.26.

Solution

Follow the instructions for the statistics mode (SD or STAT) of your calculator as shown in the tables below.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR This also clears the statistical memory.	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 730 = 490 =, etc. to enter in column AC to leave table	730 M+ 490 M+, etc.
Calculate the mean. ($\overline{x} = 550$)	SHIFT 1 Var $\overline{x}$ =	RCL $\overline{X}$
Calculate the standard deviation. ($\sigma_x = 117.26$)	SHIFT 1 Var σ_x =	RCL σx
Return to normal (COMP) mode.	MODE COMP	MODE 0

WS Statistics mode: graphics calculator

EXAMPLE 9

In a field survey, 20 echidnas from Booderee National Park were tagged and returned to their habitat. Rangers later captured several samples of 10 echidnas over a 2-month period and recorded the number tagged in each sample.

Find, correct to 2 decimal places:

a the mean number of tagged echidnas per sample

b the standard deviation of tagged echidnas.

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No. of echidnas tagged per sample	Frequency (f)
0	8
1	11
2	5
3	4
4	2
5	1

Solution

For data presented in a frequency table, follow the instructions for your calculator as shown in the tables below for the following answers:

a Mean $\bar{x} \approx 1.48$ **b** Standard deviation $\sigma_n \approx 1.34$

Note: You can also find the mean by adding an 'fx' column to the table, calculating the totals of the Frequency and fx columns, and dividing them using the formula $\bar{x} = \frac{\Sigma fx}{\Sigma f}$.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 0 = 1 =, etc. to enter in x column 8 = 11 =, etc. to enter in FREQ column AC to leave table	0 2ndF STO 8 M+ 1 2ndF STO 11 M+ etc.
Calculate the mean ($\bar{x} = 1.4838$ ...)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Calculate the standard deviation ($\sigma_x = 1.3409\ldots$)	SHIFT 1 Var σ_x =	RCL σx

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Exercise 11.04 Measures of central tendency and spread

Example 5

1 Calculate, correct to one decimal place, the mean, median, mode, range and interquartile range of each data set.

a Cost of two concert tickets

$300, $400, $600, $440, $300, $700, $250, $580, $260

b Masses of large rocks

165 kg, 146 kg, 178 kg, 190 kg, 158 kg, 147 kg, 201 kg, 194 kg

2 The test scores of a sample of Biology students are shown in the stem-and-leaf plot.

Stem	Leaf
5	0 1 2
6	3 4
7	1 5
8	6
9	1 3

What is the median of the data? Select **A**, **B**, **C** or **D**.

A 7 **B** 64

C 71 **D** 67.5

3 The hair colour of 75 people was recorded.

Hair colour	No. of people
Brown	16
Blonde	25
Black	14
Red	7
Other	13

a What is the modal hair colour?

b Why is the mode the only valid measure of central tendency here?

4 The households on a street were surveyed on the number of computers owned, and the results are shown in the frequency table. What is the mean number of computers owned per household on that street? Select **A**, **B**, **C** or **D**.

No. of computers	Frequency
2	2
3	4
4	1
5	3
6	5

A 4.3 **B** 4.0

C 3.3 **D** 3.0

5 Joshua swam 1 km each morning for 10 days to prepare for a swimming carnival. His times (in minutes) were:

28 24 22 24 25 24 26 26 24 27

a Which measure of centre (mean, median or mode) best describes this data? Give a reason for your answer.

b Describe the mode for this data.

6 For each data set, find:

i the range **ii** the interquartile range.

a 27 22 20 15 17 17 22 18 26 24 12 16 24 25

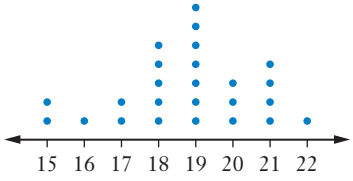
b 15 12 8 9 12 10 7 6 11 14 16

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c

Stem	Leaf
2	0 3 4 5 8
3	3 4 6 8 9 9
4	1 2 7 8
5	3 5 6 7
6	2 7

d



Example 6

7 To determine local parking availability, 30 households were surveyed on the number of cars owned. The results are shown in the table.

Number of cars	Frequency
0	2
1	11
2	12
3	3
4	2

Find:

a the mean number of cars per household

b the median number of cars per household

c the range of this data set

d the interquartile range.

8 The table shows the marks obtained by students in a quiz.

Mark in quiz (x)	Frequency (f)	fx
3	2	
4	4	
5	6	
6	9	
7	14	
8	9	
9	5	
10	1	

a How many students completed the quiz?

b What was the mode?

c Copy and complete the frequency table and then calculate the mean.

d Find the median.

e Find the range.

9 The table shows the number of goals per game scored by Newcastle Jets football team in the 2015–16 season.

Goals scored per game	Cumulative frequency
0	11
1	20
2	24
3	27
4	28

a How many games were played?

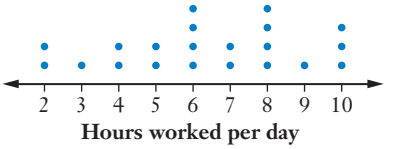
b In how many games did Newcastle Jets score two goals?

c What is the median number of goals per game?

d Calculate the mean number of goals scored per game.

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10 The number of hours per day that Michelle worked for three weeks before Christmas are shown in the **dot plot**.



a What is the mode?

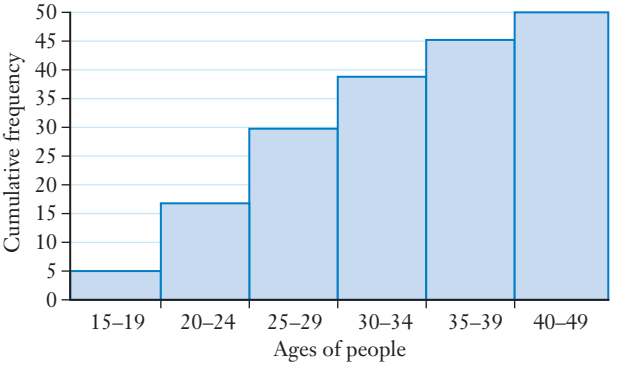
b Find the median.

c Calculate the mean number of hours per day worked by Michelle.

d Calculate the interquartile range for the hours per day worked by Michelle.

11 The ages of people attending the *Keep Fit and Stay Fit* centre from 6 a.m. to 8 a.m. on a Saturday morning are shown in this cumulative frequency histogram.

Example 7



a How many people went to the centre from 6 a.m. to 8 a.m.?

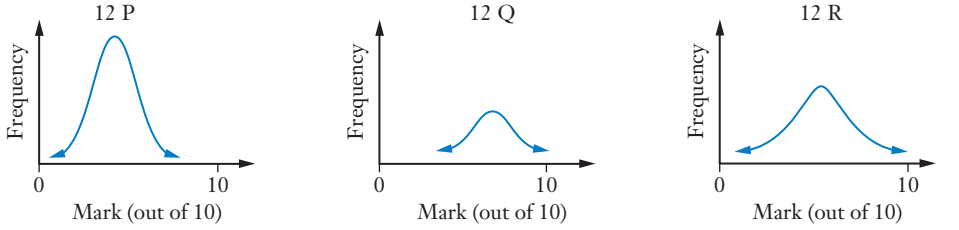
b How many people aged 30–34 attended the centre during this period?

c What was the median age group of the people attending the fitness centre?

d What is the modal age group of people attending?

e Find the interquartile range.

12 The same Maths test was given to 3 different Year 12 classes. Their results are represented graphically below.



Which class had:

a the lowest mean?

b the largest standard deviation?

c the best performance in the Maths test? Explain why.

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13 The annual salaries of employees at Gen Y manufacturing company are shown in the table.

Annual salary (× $1000)	Number of employees
20 – < 30	10
30 – < 40	6
40 – < 50	4
50 – < 60	5
60 – < 70	3
70 – < 80	2

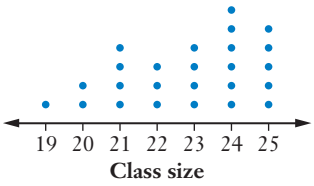
a How many people are employed at the company?

b Using **class centres** 25, 35, 45,…, find the estimated mean salary of the employees, to the nearest $100.

c What is the standard deviation, to the nearest $100?

Example 8

14 The numbers of students in different Year 11 classes at Westvale High School are shown in the dot plot.



a What is the mean number of students per class?

b What is the standard deviation, correct to one decimal place?

Example 9

15 The numbers of stray dogs collected by the RSPCA in several suburbs are shown in the table.

No. of dogs	5	6	7	8	9	10
No. of suburbs	14	18	15	26	20	31

a What is the mean number of dogs collected per suburb?

b What is the sample standard deviation of the number of dogs? Answer to 2 decimal places.

16 The 2016 monthly rainfall figures for Nowra, NSW are as follows:

80.4 95.4 64.8 23.0 24.6 411.8 128.6 49.2 75.4 33.2 29.8 59.8

a What is the median monthly rainfall?

b What is the mean monthly rainfall?

c What is the interquartile range of the monthly rainfall at Nowra?

d What is the standard deviation of the monthly rainfall?

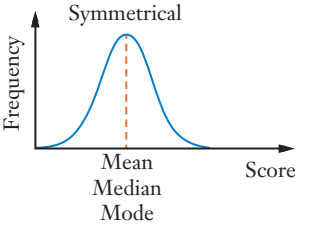
e If you were asked to say to decide the most appropriate measures of central tendency and spread for these figures, which two would you choose?

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11.05 The shape of a distribution

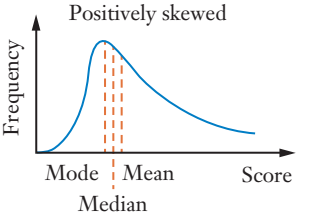
The **shape** of a statistical **distribution** (data set) shows how the data is spread, and can be seen by drawing a curve around its graph or display. For a **frequency distribution** with a large number of scores or if the **class intervals** of a distribution are decreased, the **frequency polygon** will tend to be a curve.

A distribution is **symmetrical** if the data are balanced or evenly spread about the centre of the distribution, with the mean, median and mode being equal. One example of a symmetrical distribution is students' marks in an HSC examination.



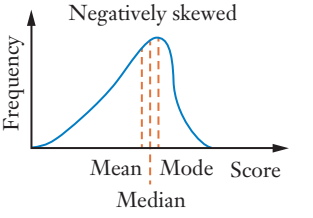
The word 'skewed' means twisted.

A distribution is **positively skewed** if its **tail** points to the right (the positive direction), because the mean is above the mode and median.



An example of a positively skewed distribution is house prices in a large country town.

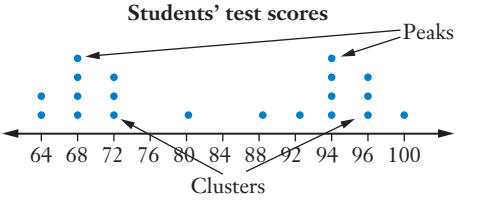
A distribution is **negatively skewed** if its tail points to the left (the negative direction), because the mean is below the mode and median.



One example of a negatively skewed distribution is the heights of the players in a basketball team.

Peaks are the high points of the distribution, the more frequent scores, and the highest peak is the mode.

Clusters are groups of scores bunched or close together.



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EXAMPLE 10

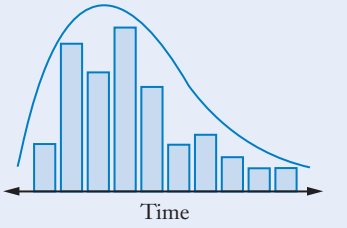
For each distribution shown below:

- **i** describe its shape and smoothness
- **ii** identify any clusters
- **iii** state the number of peaks.

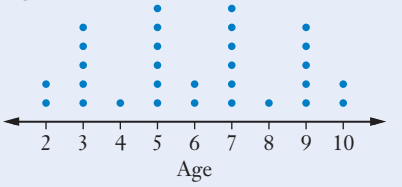
a Marks in a Japanese test

Stem	Leaf
3	1 2
4	3 5 9
5	0 2 6
6	4 5
7	3 5 6 7 7 8 9
8	0 2 4 6 6 8 8 9
9	1 2 4 8 9

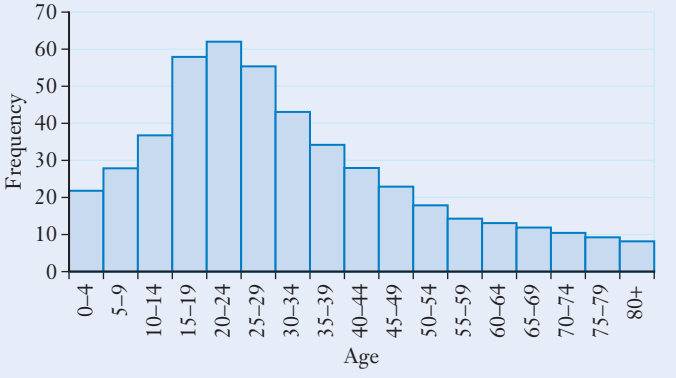
b Amount of traffic on Sydney's roads



c Ages of children at a cinema



d Age of people in a small coastal town



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Solution

a **i** negatively skewed (tail points towards lower scores)
ii clusters in the 70–90s
iii one peak

b **i** positively skewed (tail points towards right)
ii clusters at earlier hours
iii 2 peaks

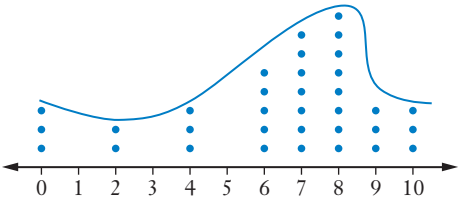
c **i** symmetrical
ii no clusters
iii 4 peaks

d **i** positively skewed and smooth (tail points toward higher ages)
ii cluster from 15 to 29
iii one peak

Exercise 11.05 The shape of a distribution

Example 10

1 The dot plot shows the film reviewers' scores (out of 10) of the action film *The Fat and the Fearless*. Which of the following statements is true about the distribution? Select **A**, **B**, **C** or **D**.



A The data is positively skewed with a cluster around 6 to 8.
B The data is symmetrical with no modes.
C The data is negatively skewed with one mode.
D The data is positively skewed with a cluster around 0 to 4.

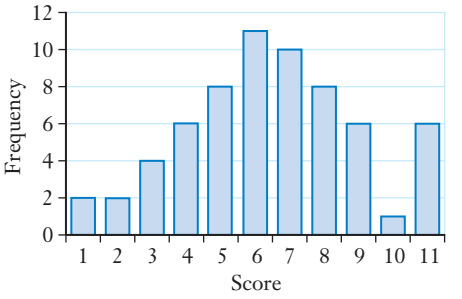
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2 For each distribution:

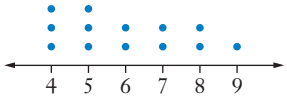
i describe its shape and smoothness

ii state the number of peaks.

a



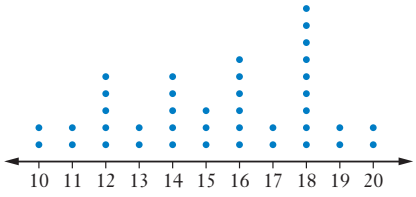
b



c

Stem	Leaf
1	4 4 5 5 6 8 8 9 9
2	0 7 8 8 9
3	1 2 4 5 6 7
4	2 2 3
5	2 4

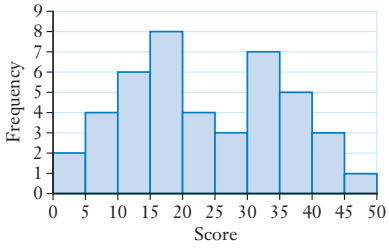
d



e

Stem	Leaf
4	1 3
5	5 5 6
6	0 3 5 5 6 8
7	2 6
8	5 5 8

f



3 This stem-and-leaf plot shows the number of mobile phones sold in January across various OzTel stores in Australia.

a How many OzTel stores were surveyed?

b Describe the shape of the data.

c Where does the clustering occur?

d What is the mode?

Stem	Leaf
2	2 6
3	0 1
4	4 8
5	2 6 9
6	1 3 4 5
7	0 2 3 4 4 5 5 7 7 7 8 9
8	3 5 7 7 8 8 8 8 9
9	2 8

4 The number of visits to the MyFace website was recorded between 1200 (noon) and 2100 (9 p.m.) one day.

Hour	1201–1300	1301–1400	1401–1500	1501–1600	1601–1700	1701–1800	1801–1900	1901–2000	2001–2100
Hits	1300	800	400	2100	2500	4500	3900	5300	2300

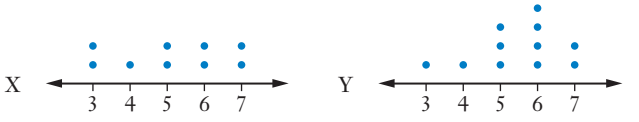
ISBN 9780170413633

a Draw a histogram to represent this data.

b Comment on the shape of your histogram, mentioning clusters and peaks.

c Suggest a possible reason for the skewness of this data.

5 Which of the following statements is true about the data sets below? Select **A**, **B**, **C** or **D**.



A Y is positively skewed.

B X does not have a mode.

C The mean of Y is 5.

D X and Y are both symmetrical.

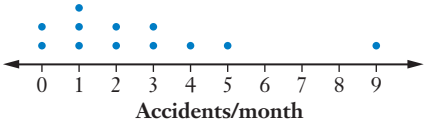
6 These are the ages of employees at the Berry Good Biscuit factory:

16 36 15 16 15 19 55 59 18 20 50 22 21 35 22 19 15 17 43 49

a Draw a stem-and-leaf plot for this data.

b Comment on the shape of the distribution, mentioning skewness, peaks and clusters.

7 This dot plot represents the number of accidents per month at a factory over a year.



a Comment on the shape of the dot plot.

b What is the mode?

c Calculate the mean (correct to one decimal place) and compare it to the mode.

11.06 The effect of outliers

An **outlier** is a very high or very low score in a data set that is clearly separate from the other scores. It can occur for a variety of reasons and should be investigated. If it was obtained through incorrect measurement, it should be excluded.

Outliers

An outlier is a score that is either:

- less than $Q_1 - 1.5 \times \text{IQR}$ or
- greater than $Q_3 + 1.5 \times \text{IQR}$

This is only one of many ways of determining whether a score is an outlier.

where Q_1 (or Q_L) is the lower quartile, Q_3 (or Q_U) is the upper quartile, and IQR is the interquartile range.

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EXAMPLE 11

The following scores are marks obtained by students in a test.

11	8	12	12	15	13	10	25	12	11	7
10	13	16	10	12	16	11	12	16	17	20

Test which scores are outliers.

Solution

The scores arranged in order are:

7 8 10 10 10 11 11 11 12 12 12 12 12 13 13 15 16 16 16 17 20 25

Q_1 or Q_L Q_2 Q_3 or Q_U

$$\text{IQR} = Q_3 - Q_1$$
$$= 16 - 11$$
$$= 5$$
$$\therefore 1.5 \times \text{IQR} = 1.5 \times 5$$
$$= 7.5$$
$$\therefore Q_1 - 1.5 \times \text{IQR} = 11 - 7.5 \qquad Q_3 + 1.5 \times \text{IQR} = 16 + 7.5$$
$$= 3.5 \qquad = 23.5$$

$\therefore$ A score is an outlier if it is less than 3.5 or greater than 23.5.

$\therefore$ 25 is an outlier.

Outliers and measures of central tendency

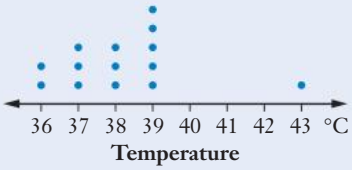
Outliers can affect the measures of central tendency of a data set.

- The mean is most affected by outliers (because its value depends on every score).
- The median can be affected, but not by much.
- The mode is not affected at all.

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EXAMPLE 12

The dot plot shows the temperatures of patients in a hospital ward.



a Calculate the mean, mode and median of this data set.

b What is the outlier temperature?

c Calculate the mean, mode and median of this data set if the outlier is excluded.

d Describe the effect the outlier has on the measures of central tendency of the distribution.

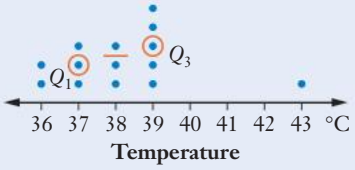
Solution

a $\text{Mean} = \frac{535}{14} = 38.214\ldots \approx 38.2$

$\text{Mode} = 39$

$\text{Median} = \frac{38+38}{2} = 38$

b From the dot plot, $Q_1 = 37$, $Q_3 = 39$



$\text{IQR} = Q_3 - Q_1 = 39 - 37$
$= 2$

$1.5 \times \text{IQR} = 1.5 \times 2$
$= 3$

If 43 is an outlier, it must be greater than $Q_3 + 1.5 \times \text{IQR}$

$\therefore\ Q_3 + 1.5 \times \text{IQR} = 39 + 3$
$= 42$

$\therefore$ the outlier is 43.

c $\text{Mean} = \frac{492}{13} = 37.846\ldots \approx 37.8$

$\text{Mode} = 39$

$\text{Median} = 38$

d The high outlier does not affect the mode and median but it increases the mean.

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Exercise 11.06 The effect of outliers

Example 11

1 The following scores are the number of goals scored by a hockey team during a season.

2 0 0 1 4 3 4 9 2 3 5 2 1 3

a Find the interquartile range.

b Find the value of:

i $Q_1 - 1.5 \times \text{IQR}$ **ii** $Q_3 + 1.5 \times \text{IQR}$.

c Is the score of 9 goals an outlier? Give reasons.

2 Determine whether each data set has outliers.

a 22 23 25 27 27 28 30 35

b 9 13 13 14 14 15 15 15 15 16 16 16 16 16 17 17 18

c

Stem	Leaf
1	2 9
2	0 3 4 4 8
3	4 5 6 7
4	1 4 9
5	0 2
6	8

d

Score (x)	Frequency (f)
4	3
5	12
6	4
7	3
8	0
9	1

Example 12

3 The employees at the Bread and Butter Cafe earned the following wages in a week:

$530 $480 $600 $210 $890 $540 $410

a What is the mean wage?

b What is the median wage?

c Find the interquartile range.

d The manager's wage is an outlier. What is this wage? Verify that it is an outlier.

e If the manager's wage is not included, how does this affect the mean and median wage?

f If each employee receives a 10% pay rise, what will be the new mean and median wage? Is it 10% more than the old mean and median?

4 The number of cups of coffee drunk by a sample of HSC exam markers in one night is shown in the table.

Cups of coffee	No. of markers
2	1
3	3
4	6
5	9
6	0
7	0
8	1

a How many markers were surveyed?

b What is the outlier?

c What is the mean if the outlier:

i is included?

ii is not included?

d If the outlier is included, what effect does this have on the mean number of cups of coffee that were drunk?

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5 A group of friends visits the cinema. The ages of the group are:

13 12 11 14 12 15 14 13.

If Taylor brings her 5-year-old sister as well, what will happen?
Select **A**, **B**, **C** or **D**.

A The median age increases.

B The median age decreases.

C The mean age increases.

D The mean age decreases.

6 In a netball tournament of 5 matches, the points scored by 3 teams each match are shown.

The Wombats	14	28	20	9	25
The Possums	17	12	25	18	19
The Koalas	28	10	16	36	15

a What is the sample standard deviation for each team, correct to 2 decimal places?

b Which team is most consistent? Why?

c An error was made in the scoring for the Wombats—the score of 9 should have been 19. What is its new sample standard deviation?

d Which team is most consistent now? Why?

7 Lisa and Hong sell copiers. The numbers of copiers each sold per week are sorted in ascending order.

Lisa	1	2	3	3	5	6	7	8	12	25
Hong	3	3	7	14	16	18	18	24	32	35

a What is the modal number of copiers sold by each person?

b What could you say about each person if you only knew the mode?

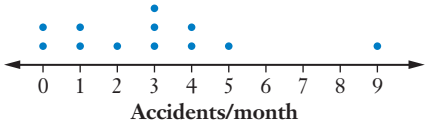
c What is the median number of copiers sold by each person?

d What is the mean number of copiers sold by each person?

e Which measure of central tendency, mean, median or mode, is the best to compare their sales performances?

f Who is the better salesperson? Justify your answer.

8 This dot plot represents the number of accidents per month at a factory over a year.



a Calculate the mean, mode and median of this data set.

b What is the outlier number of accidents? Explain why.

c Calculate the mean, mode and median of this data set if the outlier is excluded.

d Describe the effect the outlier has on the three measures of central tendency.

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9 *NelsonNet Games* store employs the following people with annual wages as shown:

1 store manager	$76 800
2 cashiers	$46 200 each
2 part-time clerical staff	$39 500 each
3 salespeople	$54 600 each
2 part-time cleaners	$26 500 each

a Find the mean, median and modal annual salary for the 10 employees.

b Which measure of central tendency would *NelsonNet Games* use to make the salaries appear higher? Why?

c Which measure best represents the average wage for an employee at *NelsonNet Games*? Why?

WS Comparing city temperatures

WS Comparing word lengths

WS Investigating young drivers

Box plots and outliers

11.07 Comparing data sets using plots

Back-to-back stem-and-leaf plots

A **back-to-back stem-and-leaf plot** compares 2 data sets.

EXAMPLE 13

This back-to-back stem-and-leaf plot compares the amounts spent (in dollars) by a group of boys and girls visiting the Royal Easter show.

Boys		Girls
2 1	1	1 2 3 8 9
3 0	2	0 2 4 4 6 7 8
8 6 6 5 4 4 2 0	3	4
6 6 6 5 5 3	4	9

a What is the modal amount spent for the whole group?

b What is the mean amount, to the nearest dollar, spent by:

i boys? **ii** girls? **iii** the whole group?

c Did boys or girls have the greater range of amounts spent?

d Compare the shapes of the two data sets (boys and girls).

e Who, on average, spent more – boys or girls? Justify your answer.

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Solution

a \$46

b **i** Mean for boys $= \frac{\$612}{18}$

$= \$34$

ii Mean for girls $= \frac{\$327}{14}$

$= \$23.35714\ldots$

$\approx \$23$

iii Mean for group $= \frac{\$939}{32}$

$= \$29.343775$

$\approx \$29$

c Range for boys = \$46 – \$11 = \$35

Range for girls = \$49 – \$11 = \$38

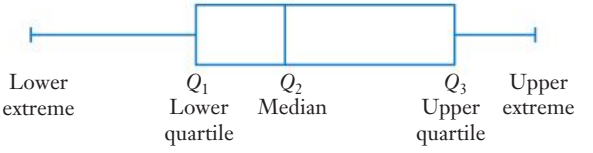
Girls had the greater range.

d The boys' spending was negatively skewed, while the girls' spending was positively skewed.

e The boys spent more, as shown by the greater mean of the boys, and in the shape of the stem-and-leaf plot.

Box plots

A **box plot** (or **box-and-whisker plot**) uses a **five-number summary** to show the spread of data.



The box contains the middle 50% of scores with each whisker representing 25% of the remaining scores.

EXAMPLE 14

Liz and George deliver pamphlets to letterboxes. The number of pamphlets delivered per hour over 12 hours is shown:

Liz	24	25	26	27	28	28	31	32	32	32	35	35
George	15	18	21	24	25	29	31	31	32	38	38	45

a Find the five-number summary for each data set and draw two box plots on the same axis.

b What are the interquartile ranges of pamphlets delivered by each person?

c What is the median number of pamphlets delivered by each person?

d Compare the two data sets, describing their shapes and measures of central tendency and spread.

e Is Liz is a better worker than George? Give reasons for your answer.

Solution

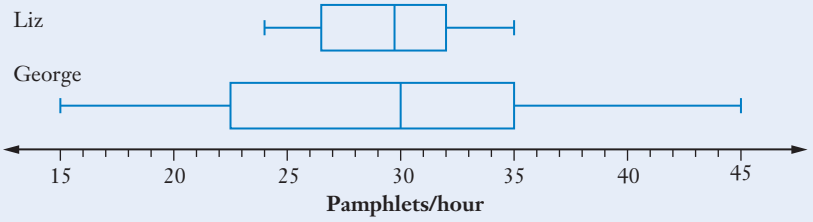
a

Liz: 24 25 26 27 28 28 31 32 32 32 35 35

lower extreme $Q_1 = 26.5$ $Q_2 = 29.5$ $Q_3 = 32$ upper extreme

George: 15 18 21 24 25 29 31 31 32 38 38 45

lower extreme $Q_1 = 22.5$ $Q_2 = 30$ $Q_3 = 35$ upper extreme



b Interquartile range for Liz = 32 – 26.5 = 5.5

Interquartile range for George = 35 – 22.5 = 12.5

c Median for Liz = 29.5

Median for George = 30

d Both distributions are fairly symmetrical and have similar medians. The data set for Liz has a much smaller range and interquartile range so its spread is generally smaller.

e Liz is not necessarily the better worker because the medians of both data sets were similar. Liz is, however, more consistent as there is less spread in her delivery rate.

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Exercise 11.07 Comparing data sets using plots

Example 13

1 A class was given 2 different Science tests and their marks are displayed in the back-to-back stem-and-leaf plot.

Test A		Test B
2	0	3
5	1	
5	2	
	3	8
8	4	5
	5	2 4 7
5	6	2
7	7	
0	8	
2	9	4

a Find the range, interquartile range and median for each test.

b Which Science test was easier? Justify your answer.

2 The weights (in kilograms) of 2 teams of players were recorded.

Netball players	77	72	80	77	62	72	62	79	58	75		
Hockey players	81	86	64	74	92	75	73	81	64	52	82	79

a Display the data in a back-to-back stem-and-leaf plot.

b Comment on the shape of each set of data, mentioning skewness, clusters and peaks.

c What is the mean weight of each team?

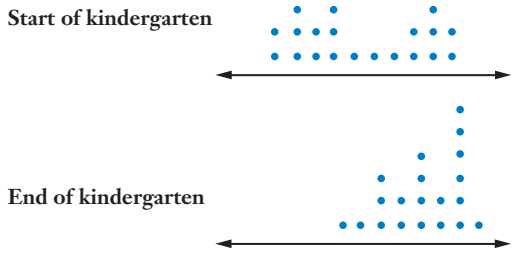
d What is the median weight of each team?

e Which is the better measure of central tendency, the mean or median? State a reason for your answer.



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3 The dot plots show the reading scores for a group of kindergarten students at the start and finish of the year.



Which statement is true about the reading scores of the kindergarten students at the end of the year? Select **A**, **B**, **C** or **D**.

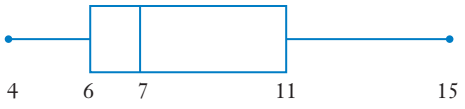
A The mean has increased and the standard deviation has decreased.

B The mean and standard deviation have both increased.

C The mean and standard deviation have both decreased.

D The mean has decreased and the standard deviation has increased.

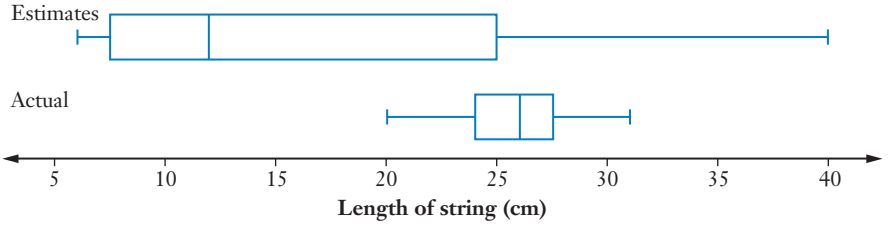
4 What is the interquartile range of the data shown in this box-and-whisker plot? Select **A**, **B**, **C** or **D**.



A 2 **B** 5 **C** 7 **D** 11

Example 14

5 Yashneel decided to investigate his theory that 'People always underestimate the length of a piece of string'. He asked students to estimate the lengths of several pieces of string, measure the actual lengths and display the results in box-and-whisker plots.



a Write down the median of:

i the estimated lengths **ii** the actual lengths.

b Compare the spreads of each data set.

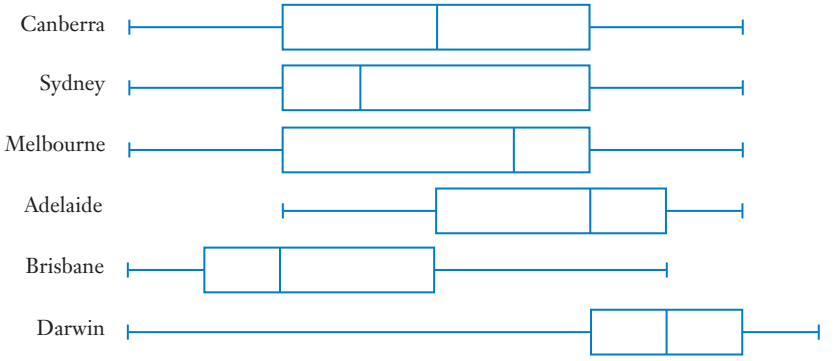
c Do you agree with Yashneel's theory? Justify your answer.

6 **a** Use the data from the back-to-back stem-and-leaf plot in Example **13** on page 505 to create 2 five-number summaries and draw their box-and-whisker plots.

b What information is seen more easily in the box plots than in the stem-and-leaf plots?

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7 The parallel box plots show the distribution of petrol prices in 6 Australian capital cities.



a Which city has:

i the greatest range of prices?

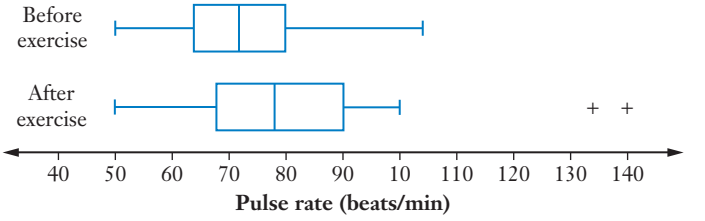
ii the lowest median price?

iii the smallest interquartile range of prices?

iv a symmetrical distribution of prices?

b Is petrol cheaper in Sydney than Melbourne? How can you tell?

8 A group of 20 people had their pulse rates taken before and after an exercise class and the results displayed. Two outlier pulse rates (shown as '+') were excluded from the 'After exercise' data.



a How many people had a pulse rate between 64 and 72 before the exercise class?

b The lower extreme 'before' and 'after' the class did not change. Give a possible reason for this.

c Give a possible reason for the 2 outlier pulse rates.

d What was the interquartile range of pulse rates after the class?

e By how much did the median pulse rate increase?

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9 A group of 12 people took part in the QUIT smoking program. The number of cigarettes smoked per day was recorded before the start of the program and 6 weeks later in the following table.

Before	21	10	36	16	32	42	9	14	21	18	34	45
6 weeks later	6	24	31	21	16	19	16	18	28	32	8	13

a Find the five-number summary for each data set and draw two box-and-whisker plots.

b What is the interquartile range of each data set?

c Is the QUIT program generally working for this group of people? Justify your answer.

10 This table shows the average number of rainy days per month for Sydney and Melbourne.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	12	12	13	12	12	12	10	10	10	12	11	12
Melbourne	8	7	9	12	14	14	15	16	15	14	12	11

a Use a back-to-back stem-and-leaf plot to display the data.

b Draw parallel box plots representing the data.

c Write down one fact that is clearly shown in each display.

d Does Melbourne generally have more rainy days than Sydney? Justify your answer.

TECHNOLOGY

Box plots using GeoGebra

1 To create a box plot, click **VIEW** and **SPREADSHEET VIEW**. Right-click in the **Graphics View** and choose **GRID**.

2 You will display the rainfall data from Question **10** above.
Click on cell A1 in the spreadsheet view (right side of your screen).

3 Type: **Boxplot[]**
Note the use of square brackets (outer brackets) and the 'curvy' brackets inside. The entry below represents the following formula:
Boxplot[width of box, y-scale, {data set}]
Enter the Sydney rainfall data in cell A1 to create the box plot.

4 Find the five-number summary for this data set.

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INVESTIGATION

SELF-SERVE SUPERMARKET CHECKOUTS

One of the main roles of a statistician is to critically analyse related data sets and report on the findings. Businesses often use the results of an analysis for promotional purposes and companies report to their shareholders.

To critically analyse data sets, you need to draw suitable displays, find measures of central tendency and spread and write a report on the relationship between the data sets, mentioning skewness, clusters, peaks and outliers as well as noting any patterns or unusual features. From this report you can draw conclusions and make recommendations.

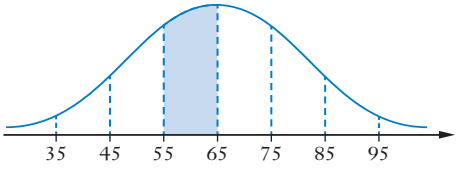
At a supermarket, the times (in seconds) to check a basket of 20 grocery items at 15 self-serve and 15 cashier-assisted checkouts were recorded.

Self-serve	45	58	63	43	75	69	84	65	96	73	90	61	84	72	96
Cashier	95	105	82	110	125	148	136	137	86	99	145	119	101	97	124

Critically analyse the data and report back to the manager of the store on any benefits of self-serve checkouts.

SAMPLE HSC PROBLEM

The HSC marks for Ancient History follow a normal distribution with a mean of 65 and a standard deviation of 10.



a Hayley's mark in Ancient History was 59. What was her z-score?

b What percentage of Ancient History marks lies in the 55 to 65 range?

c Raad achieved a z-score of 2.3 for Ancient History.

i Interpret this z-score in terms of the mean and the standard deviation.

ii What was Raad's actual mark in Ancient History?

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Study tip

When answering an exam question ...

- Make sure you show enough working.
- Note the number of marks allocated to the question: this indicates the amount of working or steps required.
- Choose the simplest and quickest method.
- Check that your answer sounds reasonable, especially if it involves money or measurement.
- If it is a measurement question, don't forget the units (for example, mm, cm^2, hours, m^3).
- If it is a money question, don't forget the dollars and cents.
- If it is an equation, don't forget the pronumeral: write '$x = 7$', not just '7'.
- Don't round answers too early—only round the final answer.
- Read the question again to make sure you have actually answered it.
- When writing worded answers, use correct terminology and write 1 or 2 sentences, not mini-essays.
- Use a ruler to draw large, neat graphs with labels and axes marked.

Good luck in the exam!

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11. CHAPTER SUMMARY

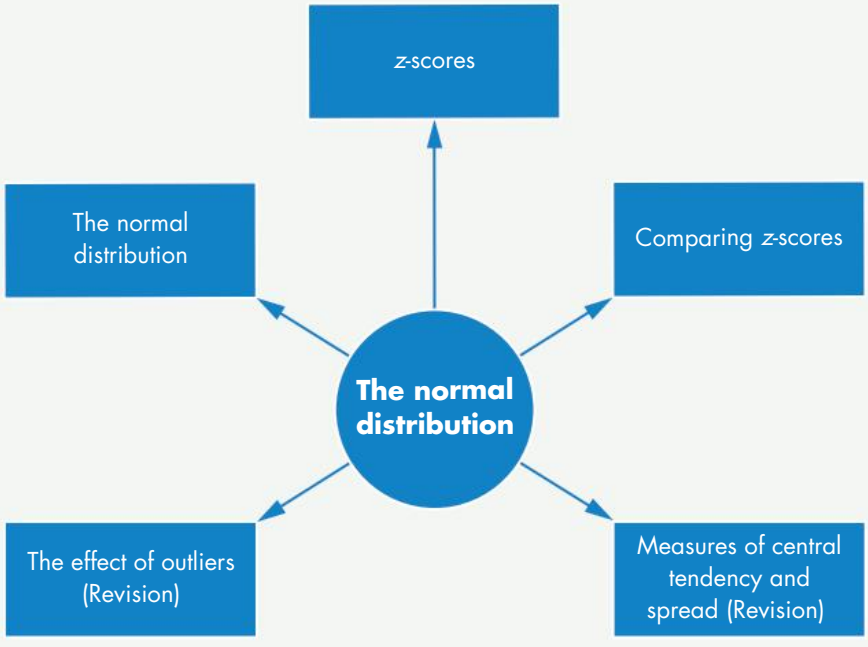
This chapter, The normal distribution, examined the normal distribution and its properties and compared scores from different normal distributions. You should be able to quote the 68%–95%–99.7% limits of the normal curve, calculate z-scores and compare data using z-scores. You revised measures of central tendency and spread to compare data sets, interpreting and analysing information from different displays, including the shape of distributions. Be sure to include skewness and the effect of outliers in your summary as well as a glossary of statistical terms.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

Data and statistics crossword

WS Statistics review

WS Statistics crossword



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11. TEST YOURSELF

Exercise 11.01

1 The mean of a normally distributed set of History test results is 71% and the standard deviation is 8%. What percentage of the scores:

a lie between 63% and 79%?
b are less than 55%?
c lie between 55% and 87%?
d are more than 95%?
e are less than 47%?
f greater than 87%?

Exercise 11.01

2 A machine produces bolts with a mean length of 75 mm and a standard deviation of 2 mm. The lengths are normally distributed. If lengths more than 3 standard deviations from the mean are rejected, which of these bolt lengths would be rejected?
Select **A**, **B**, **C** or **D**.

A 70 mm **B** 79 mm **C** 81 mm **D** 88 mm

Exercise 11.02

3 **a** Leonardo's z-score for his Art assessment task was 2.5. What does this mean?

b If the task mean was 68 and Leonardo's raw score was 82, what was the task standard deviation?

c What was Michaelangelo's z-score if he scored 62 for the task?

d What was Raphael's score if his z-score was 0.8?

Exercise 11.02

4 The time taken to repair computer components was found to be normally distributed with a mean of 8 hours and standard deviation of 2 hours.

Time (*h*)			6	8		12	
z-score	–3	–2		0	1		3

a Complete the table.

b What percentage of repairs took:

i between 10 and 14 hours? **ii** less than 6 hours?
iii more than 12 hours? **iv** between 4 and 10 hours?

Exercise 11.03

5 Tom's test scores plus the class mean and standard deviation are shown in the table. The scores are normally distributed.

Subject	Score	Class mean	Class standard deviation
Drama	80	75	16
Photography	60	54	12
Textiles	70	78	10

a Find Tom's z-score for each subject, correct to one decimal place.

b In which subject did Tom perform best? Justify your answer.

c Show the position of Tom's z-scores on a standard normal curve.

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6 Calculate, correct to one decimal place where necessary, the mean, median, mode, range and interquartile range of each data set.

Exercise 11.04

a Cost of textbooks

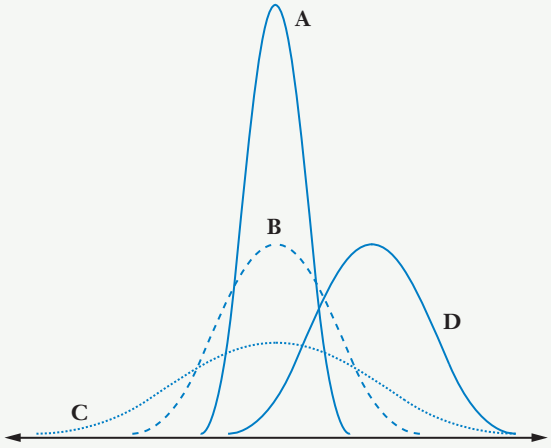
$36, $45, $84, $29, $25, $56, $60, $56, $22, $15

b Capacity of soft drink cans

368 mL, 385 mL, 376 mL, 370 mL, 375 mL, 382 mL, 372 mL

7 The diagram shows curves representing four normal distributions: **A, B, C** and **D**.

Exercise 11.04



a Which distribution has the largest standard deviation?

b Which distribution has the smallest standard deviation?

c Which distributions have the same mean?

d Which distributions have the same standard deviation?

8 A group of 10 competitors entered the KAC cross-country ski race. Their times, to the nearest minute, are:

Exercise 11.04

38 17 42 36 38 41 19 22 54 16

a Find the mean time.

b Find, correct to 2 significant figures, the standard deviation of the times.

c Draw an ordered stem-and-leaf plot for the data.

d Find the median.

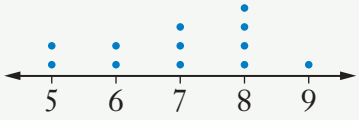
e Find the interquartile range.

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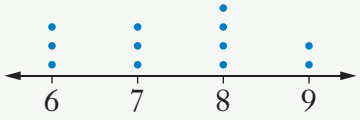
Exercise 11.05

9 A dot plot was created to show the judges' scores for a participant in a skating competition. Which of the following graphs represents a set of scores that is positively skewed? Select **A**, **B**, **C** or **D**.

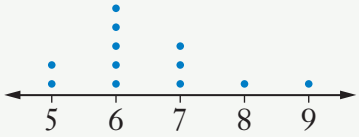
A



B



C



D



Exercise 11.06

10 The September sale prices for units in Bigtown were:

$750 000, $747 000, $842 000, $745 000, $850 000, $835 000 and $828 000.

On October 1, another unit in this suburb sold for $905 000.

If this unit had been sold in September, what have been the effect on the mean and median unit prices for September?

A Neither the mean nor the median would have changed.

B Both the mean and the median would have increased.

C The mean would have increased and the median would remain unchanged.

D The mean would have increased and the median would have decreased.

Exercise 11.07

11 The ages of members of a yoga class at Husky Health Club one morning are shown in the back-to-back stem-and-leaf plot:

Women		Men
4	2	2 6
2	3	2 2 5 6 8
8 3	4	2 4 5 6
9 4 0 0	5	3
8 5 4 3 3	6	3 3 4

a Find the interquartile range of the ages of the women in the class.

b Find the median age of the men in the class.

c Compare the shapes of age distributions of the women and men. Comment on any clusters, skewness, peaks or outliers for each data set.

d Which group was younger on average: women or men? Justify your answer.

Qz Chapter quiz

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Practice paper 4

Mathematics Standard 2 Reference sheet

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Question 1

Exercise 10.01

Which one of the following is a quadratic function?

A $y = 4^x$ **B** $y = 4x^2$ **C** $y = \left(\frac{1}{4}\right)^x$ **D** $y = \frac{4}{x}$

Question 2

Exercise 11.04

The colour of 64 cars was recorded. What is the modal car colour?

A red
B white
C black
D blue

Colour	No. of cars
white	38
black	12
red	8
other	6

Question 3

Exercise 11.01

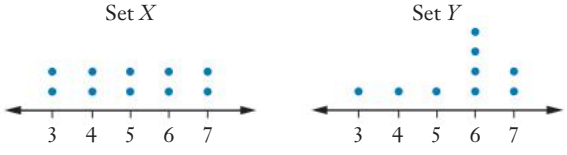
The masses of a population of adults are normally distributed with a mean of 80 kg and a standard deviation of 10 kg. What percentage of the population has a mass between 60 kg and 100 kg?

A 83.35% **B** 81.5% **C** 68% **D** 95%

Question 4

Exercise 11.05

Which statement is true about these 2 data sets?



A The mean of Y is 6.
B X does not have a mode.
C Y is positively skewed.
D X and Y are both symmetrical.

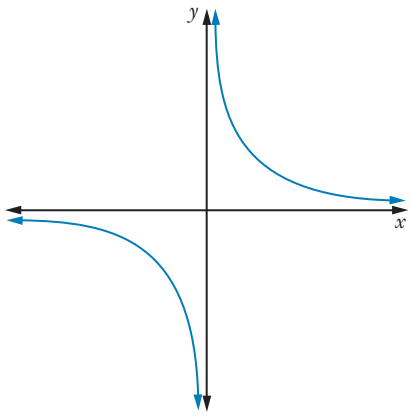
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Exercise 10.05

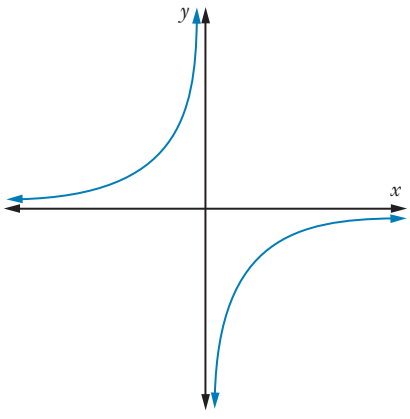
Question 5

Which one of these graphs could be the graph of $y = -\frac{2}{x}$?

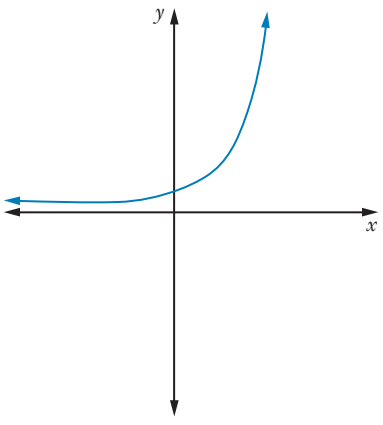
A



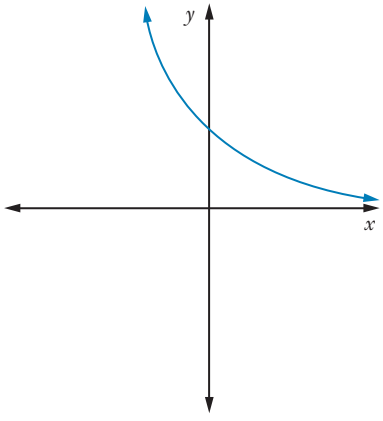
B



C



D



Exercise 11.04

Question 6

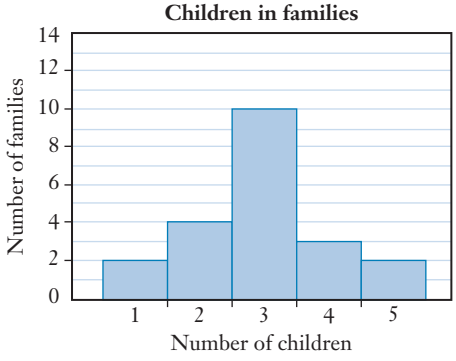
This frequency histogram shows the results of a survey about the number of children in each family. How many families were surveyed?

A 5

B 15

C 21

D 62



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Question 7

Exercise 11.04

The sugar content of a brand of lolly is normally distributed with a mean of 4.5 mg. 2.5% of lollies have a sugar content greater than 4.9 mg. What is the standard deviation of the lolly distribution?

A 0.12 mg **B** 0.4 mg **C** 0.2 mg **D** 0.11 mg

Question 8

Exercise 10.02

A ball is thrown into the air and its height, h m, at time, t s, is given by the equation $h = -4t^2 + 12t$. After how many seconds does the ball reach its maximum height?

A 4 **B** 3 **C** 12 **D** 1.5

Question 9

Exercise 11.04

The cost of driving lessons at 9 driving schools are:

$320, $210, $180, $270, $300, $330, $210, $270, $240

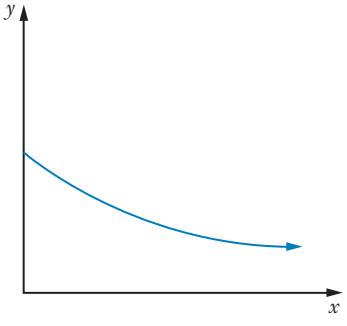
Find the interquartile range.

A $30 **B** $100 **C** $150 **D** $80

Question 10

Exercise 10.03

What could be the equation of this graph?



A $y = 4(0.2)^x$

B $y = \frac{2}{x}$

C $y = 3 - x$

D $y = 3(2^x)$

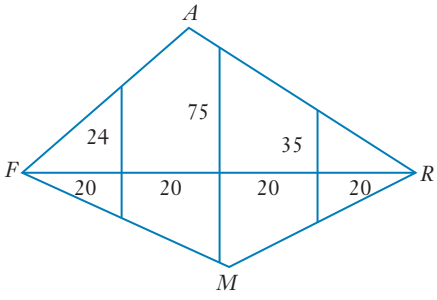
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Section 2

3 questions: 10 marks each

Question 11 (10 marks)

a In the survey of a rural property, *FARM*, measurements are taken at right angles each 20 m interval along *FR*.



Use 4 applications of the trapezoidal rule $A \approx \frac{h}{2}(d_f + d_l)$ to calculate the area of *FARM*. **3 marks**

b The heights of a population of netball players are normally distributed with a mean of 172 cm and a standard deviation of 11.

i If Ivy is a netball player whose height has a z-score of 1.5, what is her height, to the nearest cm? **1 mark**

ii If Rose is in the same netball team and has a height of 165 cm, what is her z-score, to 2 decimal places? **1 mark**

c The population of Bogong grows exponentially according to the function $P = 3500(1.06)^t$ where t is the number of years after 2018 and P is the population at the start of that year.

i Graph P for values of t from 0 to 8. **3 marks**

ii By how much will the population of Bogong increase from 2018 to 2025? Answer to the nearest whole number. **1 mark**

iii Use a guess-and-check method to find when the population will be triple that of 2018. **1 mark**

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Question 12 (10 marks)

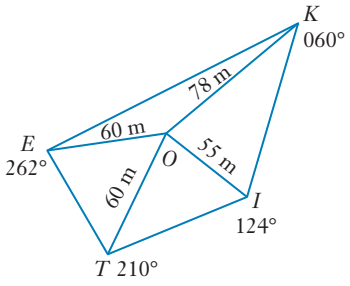
a The time taken, T hours, for a road trip varies inversely as the average speed, S km/h, of the car. It takes 5 hours for the trip travelling at 80 km/h.

i Write an equation for T in terms of S and a constant k and find the value of k. **2 marks**

ii Calculate how long the trip will take travelling at 60 km/h. **1 mark**

iii If the trip takes 7.6 hours, what is the speed of the car, correct to the nearest km/h? **1 mark**

b This diagram shows a compass radial survey of the field *KITE*.



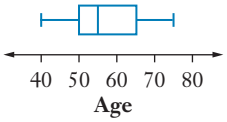
i Find the size of each of the 4 central angles that meet at O. **2 marks**

ii Calculate the area of the field *KITE* to the nearest square metre. **2 marks**

iii Calculate the length of *KE*, correct to the nearest metre. **2 marks**

Question 13 (10 marks)

a This box-and-whisker plot shows the ages of members at a golf club.



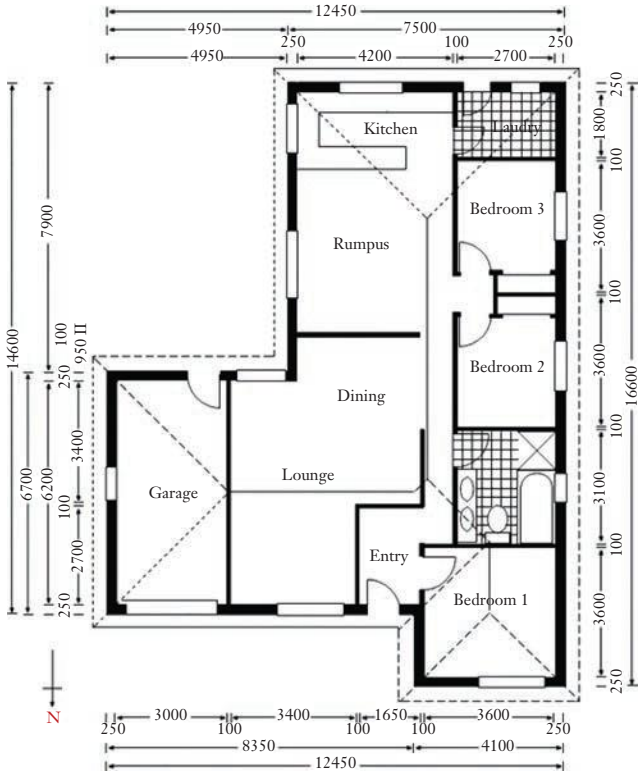
i What is the age of the oldest member? **1 mark**

ii Calculate the interquartile range of ages. **1 mark**

iii If there are 320 members in the club, how many of them are aged between 40 and 55? **1 mark**

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b Here is a house plan.



i What is the internal perimeter, in metres, of the garage? **1 mark**

ii Bedroom 1 is to be carpeted. The carpet is 3.6 m wide and is sold at $220 per metre laid. How much will it cost to carpet bedroom 1? **2 marks**

iii How many windows in this house face west? **1 mark**

c A biologist studying a new strain of bacteria models its growth by the formula $N = 80(1.4)^t$ where N is the number of bacteria and t is the time in hours.

i How many bacteria are present after 5 hours? **1 mark**

ii Show that the original number of bacteria present was 80. **1 mark**

iii By what percentage is the bacteria increasing per hour? **1 mark**

END OF EXAMINATION

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ANSWERS

Chapter 1

SkillCheck

1 **a** $22.80 **b** $2947.50 **c** $520.80
2 **a** 3717.71 **b** 13 302.21 **c** 770.71
3 **a** $16 848 **b** $935.75
c $2147.20 **d** $10 206
4 **a** 1.6 **b** 0.4
c 2.25 **d** 0.75
5 **a** 10 504.85 **b** 13 036.06
6 **a** $7800 **b** 1.73%

Exercise 1.01

1 $2764.80
2 **a** $775.62 **b** $2406.25
c $981.57 **d** $257.46
3 C
4 **a** $113.10 **b** $32 307.69
c $80.89 **d** $1117.25
5 3.5 years **6** 36 weeks **7** 5.89% p.a.
8 $32 448 **9** 0.0138% per day
10 3.24% p.a. **11** $16 520
12 **a** $900 **b** $4500 **c** $187.50
13 **a** $15 382.50 **b** $197.22
14 D **15** 0.8% per month
16 $1424.87

Exercise 1.02

1 $44 246.04
2 **a** **i** $22 196.43 **ii** $3596.43
b **i** $63 567.08 **ii** $17 567.08
c **i** $8456.21 **ii** $1456.21
d **i** $54 930. 21 **ii** $22 430.21
e **i** $115 048.55 **ii** $5048.55
3 A **4** $15 891.66
5 $12 760.54 **6** $30 357
7 **a** $34 522.93 **b** $29 041.24
c $35 041.24 **d** $34 522.93
8 $25 000 **9** 3.5% p.a. **10** 4.4%
11 B **12** 3.8% p.a.
13 **a** $28 736.69 **b** $28 773.20 **c** $28 791.84
d $28 804.42 **e** $28 810.55
14 $13\frac{1}{2}$ years

Exercise 1.03

1 D **2** $67.10
3 $1 **4** C
5 $4.29 **6** $1514.35
7 $25 293 **8** 14.5% p.a.
9 $90 (to the nearest dollar)
10 $6240
11 **a** $1215.50 **b** $1721.57
12 **a** Teacher to check, e.g. paintings, jewellery
b Teacher to check, e.g. computers, cars
13 $14 353.56 **14** $92 650.65
15 $56 000 **16** $87.53
17 $1633 **18** 4.5%
19 $537.79

Exercise 1.04

1 A
2 **a** $71 875 **b** $1437.50
c $2000, 2.93%
3 **a** $2.23 **b** $1070.40
4 **a** $1.43 **b** 8.9c
5 **a** $33 349.20 **b** $11 980.80
6 **a** $11 234 **b** $11 489.18
c $697 **d** 6.2%
7 **a** $12 210 **b** 14c

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8 **a** $92 143 **b** $3478.00

9 **a** $4305 **b** 1.7%

c 173

10 $15 942.17

11 **a** $69 791.63 **b** $1000

c **i** $73 375 **ii** 1.36%

12 **a** $130 944 **b** $8355.50

Exercise 1.05

1 D

2 B

3 **a** $2.81 **b** $2.75 **c** $2.37

d 6 196 800

4 **a** Viva Energy **b** No change

5 **a** BHP **b** –27 c

6 **a** AGL Energy **b** Myer

7 BHP

8 **a** $3.54 **b** $4.15

9 **a** $17.95 **b** JB Hi-Fi

c Company code for the stock exchange

d JB Hi-Fi **e** $42.19

f 64c **g** 9.7%

10 **a** $14 **b** $15 **c** 3 months

d 4 **e** $19 **f** $20

11 **a** $8.20

b



c Price rose to a high at Week 3, then dropped to a low at Week 6

d No

12 **a** $3.40 **b** $2.05 **c** 3

d 95 cents

e Dropped near the end of October. High at the start of November, then dropped to lowest point towards end of November, rose steadily until end of December, then fell steadily during first half of January.

f **i** $2.10 **ii** $3.00

g Yes, price is low. **h** No, price is low.

Exercise 1.06

1 D

2 **a** $2600 **b** $5200

3 **a** $7200 **b** $37 100

4 **a** $180 **b** $810

5 **a** $1370

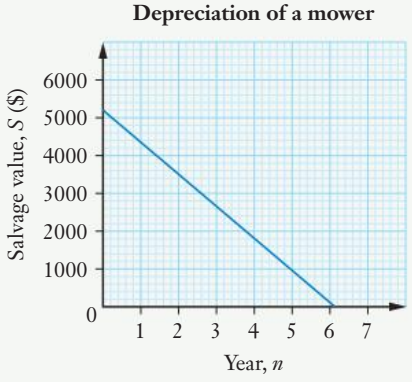
b In the 7th year (after 6.1 years)

6 **a** $37 800 **b** $4200 **c** $25 200

d $21 000 **e** $S = 37\,800 - 4200n$

7 **a** $5200 **b** $1680 **c** $2680

d



e $S = 5200 - 840n$

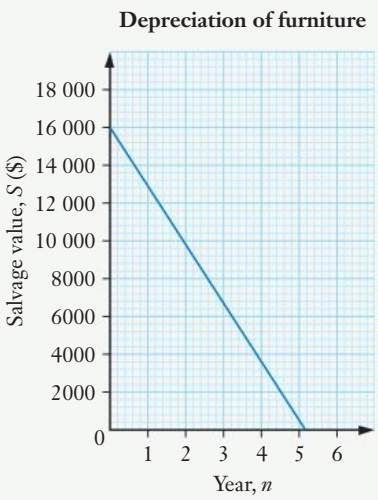
8 **a**

Year	Accumulated depreciation ($)	Value of end of period ($)
0	0	16 000
1	3100	12 900
2	6200	9800
3	9300	6700
4	12 400	3600
5	15 500	500

b $3600 **c** $6200

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d



e In the 6th year (5 years 2 months)

f $S = 16000 - 3100n$

9 **a** \$11 800 **b** \$11 200

c **i** 3 years **ii** 5.5 years

d \$1600 **e** $S = 11800 - 1600n$

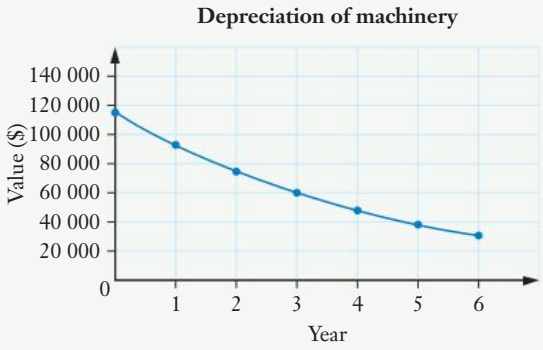
Exercise 1.07

1 **a** \$23 000 **b** \$92 000 **c** \$47 104

d

Year	Depreciation	Value at end of year
0	0	115 000
1	23 000	92 000
2	18 400	73 600
3	14 720	58 880
4	11 776	47 104
5	9420.80	37 683.20
6	7536.64	30 146.56

e



2 A

3 **a** \$8323 **b** \$2663

4 **a** \$24 750 **b** \$16 637.50

c In the 4th year

5 **a** \$50 000

b **i** \$24 000 **ii** \$11 000

c **i** \$9000 **ii** \$5500

d \$6850.57

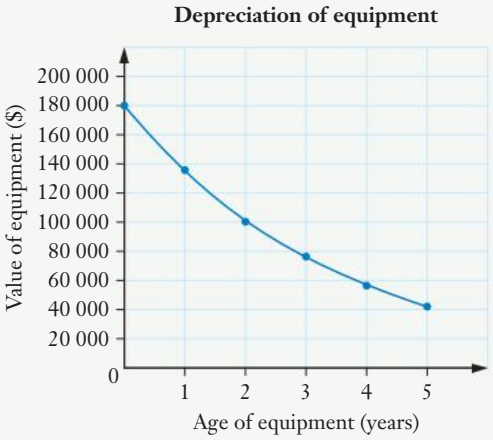
6 **a** **i** \$37 730 **ii** \$6341

b **i** \$16 170 **ii** \$2718

7 **a** \$75 937 **b** 25%

c In the 8th year

d



e \$180 000, cost of equipment

8 **a** \$100 233 **b** \$25 414

9 \$39 669

10 **a** \$150 000

b **i** \$98 000 **ii** \$61000

c **i** \$42 000 **ii** \$54 000

d \$29 531

11 **a** \$12 207.29 **b** \$8845.20

c In the 7th year **d** 30.4%

12 **a** \$2600 **b** \$936

Sample HSC problem

a \$2887.77 **b** \$64.44

c Interest on interest is earned more frequently when compounded monthly.

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Test yourself 1

1 **a** $187.20 **b** $1530 **c** $646.43
2 4.5 years **3** 2.75% **4** C
5 **a** $9569.85, $1569.85
b $10 301.47, $801.47
c $15 408.80, $1408.80
d $54 691, $22 191
6 $14 389.90 **7** 3.6% **8** $9270
9 $1.75 **10** $19 689 **11** $1136.64
12 **a** $15 473.60 **b** $4368.80
13 **a** $400 500 **b** $4950 **c** 1.24%
14 **a** 3 months
b $20.50, mid-November
c Share price has been variable, moving up and down, lowest in mid-November and highest at the start of January.
d 6 **e** $30
f Buy, because price is low
15 **a** $64.30
b SkyNetTV – no change
c AMP, $1.57
d $6.72
16 $1518.75 **17** $580
18 After 4 years **19** D
20 **a** $28 990 **b** $6595.23 **c** 5 years
21 **a** $40 000
b **i** $27 000 **ii** $15 000
c $5000 **d** $8176
22 **A** depreciates to $9000 while **B** depreciates to $10 290.
23 C

Chapter 2

SkillCheck

1 **a** $\frac{7}{3}$ **b** $\frac{1}{4}$ **c** $\frac{5}{12}$
2 **a** $1.28 **b** $2.49 **c** $18.75
3 $500 000
4 $12 800
5 **a** 2000 **b** 457 **c** 60 000
d 0.025 **e** 196 **f** 12

Exercise 2.01

1 **a** 1 : 3 **b** 2 : 1 : 5 **c** 1 : 1
d 7 : 5 **e** 1 : 4 **f** 4 : 12 : 3
g 1 : 10 **h** 6 : 5 **i** 3 : 10
j 1 : 8 **k** 3 : 14 **l** 84 : 19
m 3 : 20 **n** 1 : 500 **o** 1 : 5
p 2 : 3 : 52 **q** 200 : 1 **r** 7 : 48
2 10 : 2 : 1
3 C **4** D **5** 3 : 26
6 **a** 1 : 3 **b** 1 : 3 **c** 3 : 2
7 1 : 4
8 **a** 15 cm; 45 cm **b** 135 cm
9 1150
10 **a** 550 g **b** 2250 g
11 80 teachers
12 C
13 24
14 46 600 books
15 **a** 12 shovelsful **b** 1467 kg
16 **a** 200 g **b** 1000 g or 1 kg
c 2000 g or 2 kg
17 **a** 400 mL or 0.4 L **b** 3.9 L
18 $336 000

Exercise 2.02

1 B
2 **a** $14 000, $17 500 **b** 3250 kg, 500 kg
c 15 692 m^2, 18 308 m^2 **d** $\frac{1}{2}$ cup, 2 cups
e 3000 L , 4500 L
f 62 500 m^2, 37 500 m^2 or 6.25 ha, 3.75 ha
3 470 girls
4 318 kg, 182 kg
5 KJ $33 500; Brad $20 100
6 $75 000, $50 000, $50 000, $25 000
7 0.4 L of oil, 19.6 L of petrol
8 50 kg of cement; 200 kg of sand; 300 kg of aggregate
9 2400 mL of juice; 2400 mL of lemonade; 1200 mL of ginger ale
10 Jas $330, Lisa $220, Ally $110
11 280 mL
12 842 mL

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13 **a** $216 **b** $540
14 **a** 6 : 1 : 19 **b** 808 g
c 520 g **d** 370.5 g

Exercise 2.03

1 **a** $/min **b** $/hour
c words/minute **d** $/g
e $/kg **f** m/s
2 Answers may vary. Suggested answers include:
a Population density
b Cost of a newspaper classified ad
c Fuel consumption
d Internet download speed
e Birth rate
f Cost of carpeting.
3 2.4 cm/week
4 5.25 runs/over
5 2750 persons/year
6 $28.44/kg
7 1.3 MB/s
8 3.21 persons/km^2
9 **a** 336 km **b** 5 h
10 23 min
11 **a** 0.05 mg/g **b** 5 mg
12 12 h 40 min
13 147
14 **a** 63c **b** A$317.46
15 A
16 74.88 L
17 **a** 4.1 kg **b** 6 times
18 **a** 340 055 **b** 187 277

Exercise 2.04

1 **a** 300 g carton
b Perhaps the supermarket had excess stock and sold the 300 g carton at a cheaper price.
2 Regular
3 B
4 B
5 Pack of 4
6 400 g tube
7 380 g jar
8 D
9 **a** 375 g **b** 2 L
c 735 g **d** 115 g
10 Supermarket
11 Coolies
12 24 cans
13 **a** Bullworths **b** GIA
c Baldy **d** Bullworths
14 Supplier C
15 Teacher to check
16 Better quality of toothpaste, smaller and more convenient to use
17 Too many apples to consume; may not be fresh or of good quality; may take up space

Exercise 2.05

1 **a** 8 h 54 min **b** 2 h 48 min **c** 7 h 43 min
2 **a** 16 km/h **b** $50\frac{2}{3}$ km **c** 1 h 23 min
3 C
4 29.7 knots
5 **a** 887.1 km **b** 38.1 L
c **i** 287.8 km **ii** 500 km
d Highway, smoother driving, constant speeds, less start–stop driving
e 6.4 L
6 **a** 43 200 km **b** 75.52 days
7 C
8 Larger vehicles have bigger engines that require more fuel.
9 22.08 L
10 547 km
11 **a** 113 L **b** 478 km
12 **a** 3.5 h **b** 14 km/h
13 1.6 km
14 12.5 min
15 **a** 26.8 L **b** 246 km
16 4 h 23 min
17 **a** 44 924.77 km **b** 1 h 48 min

Exercise 2.06

1 **a** $3851/month **b** 1.25 km/min
c 120 L/day **d** 180 m/min
e 48 cm/h **f** 1.3 words/s
g 0.082 L/km **h** 105 000 mL/min

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2 **a** \$14.40/h **b** 28.8 km/h
c 54 L/h **d** 18 c/min
e $16\frac{2}{3}$ m/s **f** 2.08 kg/min

3 161 km/h

4 C

5 1800 L/h

6 37.6 km/h

7 25.54 km/h

8 **a** 464.3 km/h **b** **i** 7738 m **ii** 129 m

9 104.2 m

10 **a** 15 m **b** 21 m **c** 25 m

Sample HSC problem

a 3

b **i** 17.3 m/s **ii** 62.1 km/h

c 60.4 m

Test yourself 2

1 **a** 1 : 4 **b** 6 : 13
c 3 : 1 **d** 1 : 20

2 **a** 65 mL **b** 416 mL

3 Daniel \$26 610, Anastasia \$17 740

4 \$1092

5 19 h 36 min

6 B

7 4 h 11 min

8 **a** 10.6 L/100km
b 26.5 L

9 25 c/min

10 **a** **i** 2.39 m/s **ii** 8.61 km/h
b 7 min

Chapter 3

SkillCheck

1 **a** $m + 16$ **b** $30\,mn^2$
c $\frac{3h}{5}$ **d** $3x - a$

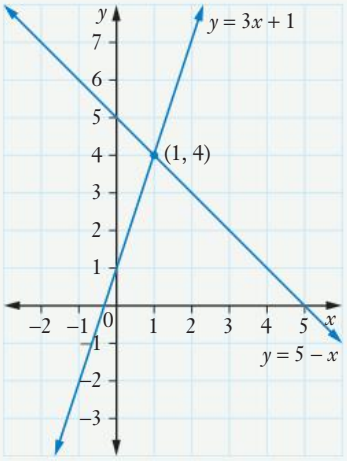
2 **a** $6r - 15$ **b** $-6y - 42$ **c** $-16 + 2k$

3 **a** 54 **b** 4 **c** 108

4 **a** $x = 7$ **b** $d = -1$ **c** $w = 18$

5 **a** y **b** 3
c 1 **d** $x = -5$

6



7 $y = 3x - 5$

8 **a** Gradient $= \frac{1}{2}$, Equation $y = \frac{1}{2}x + 2$
b Gradient $= -2$, Equation $y = -2x + 9$

Exercise 3.01

1 A

2 A

3 **a** $d = 6$ **b** $p = 2.5$ **c** $u = 3.5$
d $a = -3$ **e** $b = -2$ **f** $a = 4.5$
g $m = -5$ **h** $h = 12$ **i** $r = 13$
j $x = -20$ **k** $y = -5$ **l** $n = -1$
m $y = 5$ **n** $c = 7.5$ **o** $z = 60$

4 **a** $h = 2.75$ **b** $r = -8.5$ **c** $y = \frac{2}{3}$
d $k = -28$ **e** $w = 42$ **f** $k = -2.5$
g $m = 16$ **h** $x = -1$ **i** $c = 1$
j $y = \frac{11}{3}$ **k** $d = 21$ **l** $a = -6.4$

5 C

Exercise 3.02

1 100.4°F **2** 0.5°C **3** 4.5 cm

4 43.2 km/h **5** D **6** 8 years

7 **a** 270.6 km **b** 14 h 17 min

8 0.5 years (or 6 months old)

9 **a** \$A16.18 **b** \$A30.74 **c** \$A80.89

10 75 kg

11 **a** 16.5 mL **b** 11%

12 $a = 22$

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13 280 cm **14** 9000 m **15** 0.083 m^3

16 $R_1 = 60$

17 2.2 kg **18** 4.5 cm

Exercise 3.03

1 a $x = \frac{y+5}{3}$ **b** $x = \frac{T-11}{2}$

c $x = 8d - 2$ **d** $x = 22 - p$

e $x = \frac{k-r}{2}$ **f** $x = 3(C-n)$

g $x = \frac{s}{12y}$ **h** $x = 6V + 5$

i $x = \frac{12y-z}{a}$

2 a $S = \frac{D}{T}$ **b** $T = \frac{D}{S}$

3 a 1440°

b $n = \frac{A}{180} + 2$ or $n = \frac{A+360}{180}$ **c** 15

4 D

5 a $h = \frac{2A}{a+b}$ **b** $a = \frac{2A}{h} - b$ or $a = \frac{2A-bh}{h}$

6 a $r = \frac{C}{2\pi}$ **b** $x = \frac{y-c}{m}$ **c** $a = \frac{v^2-u^2}{2s}$

d $s = \frac{x-m}{z}$ **e** $m = \frac{E}{c^2}$ **f** $m = \frac{2K}{v^2}$

g $b = y^2 - 10d$ **h** $y = \frac{x+16}{6}$

i $b = \frac{2A}{h}$ **j** $m = Nw - d$

k $E = IR - V$ **l** $C = \frac{Q}{e-ir}$

m $r = \frac{tv}{m^2}$

n $u = \frac{2s-at^2}{2t}$

7 $m = Bh^2$, $m = 86.29$ kg

8 $h = 16$ cm

9 $F = \frac{9C}{5} + 32$ **a** 107.6 **b** 14

10 $P = \frac{10}{M\pi h}$, $P = 0.20$

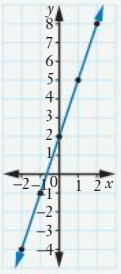
Exercise 3.04

1 B

2 a i x **ii** y

b

x	–2	–1	0	1	2
y	–4	–1	2	5	8

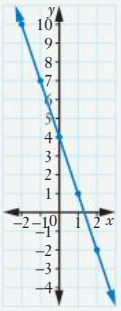


c Gradient = 3, y-intercept = 2

d The gradient, 3, is the coefficient of x in the equation $y = 3x + 2$ and the y-intercept, 2, is the constant term.

3 a

x	–2	–1	0	1	2
y	10	7	4	1	–2



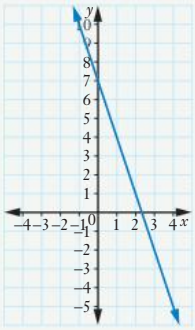
b Gradient = –3, y-intercept = 4

c The gradient, –3, is the coefficient of x in the equation $y = 4 - 3x$ and the y-intercept, 4, is the constant term.

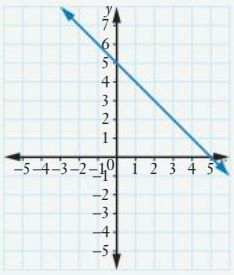
4 a $m = -3$, $c = 7$ **b** $m = -1$, $c = 5$

c $m = \frac{1}{2}$, $c = -3$

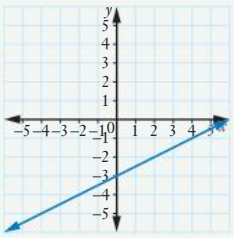
5 a



b



c



6 D

7 **a** $C = -8k + 200$ **b** $v = 6t + 3$

c $P = -3n + 150$ **d** $t = 5x - 12$

8 **a** V, volume of water **b** $C = 1.64V + 142$

c Cost in dollars per kilolitre of water

d Fixed cost of water usage (even when no water is used)

e \$230.56 **f** 75 kL

9 **a** $F = 1.8C + 32$ **b** C

c 32, the °F equivalent to 0°C

d **i** 53.6°F **ii** 30°C

e **i** 54°F **ii** 30°C

f 9°

10 **a** $V = -82m + 3760$ (answers may vary)

b \$3760

c −82, decrease in value of computer in dollars per month

d **i** \$2120 **ii** \$316

e 3 years 10 months

f The model only works until 3 years 10 months. After that, the value of the computer should stay being \$0, not negative amounts.

11 **a** s **b** $L = \frac{1}{3}s + 8\frac{1}{3}$

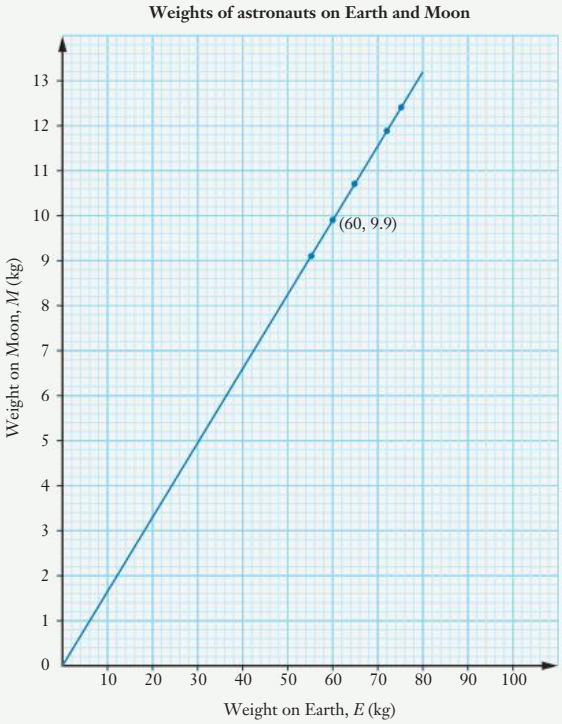
c $10.8\dot{3} = 10\frac{5}{6}$ inches **d** 14

12 **a** $C = 270n + 8400$

b \$62 400 **c** 191 **d** \$11 880

Exercise 3.05

1 **a**

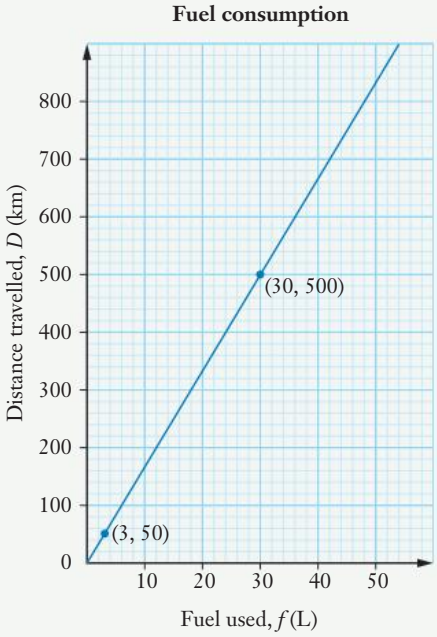


b 0 **c** 0.165

d $M = 0.165E$ **e** 52.1 kg

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2 a



b $D = \frac{50}{3}f$ **c** 40.8 L

3 a $M = 7.8V$ **b** 1404 g

c 1 000 000 cm^3 **d** 7800 kg

4 a $m = \frac{5}{8}K$ **b** 71.875 miles

c 256 km

5 a Ford Mustang: $F = 0.13D$;
Ford Fiesta: $F = 0.058D$

b i 7.2 L **ii** 20.16 L

c 477 km

6 a $T = \frac{1}{50}S$ **b** 60 s **c** 4000 MB

7 C

8 a $P = 5.875D$ **b** 352.5 kPa **c** 141 m

9 \$10 656

10 a $F = \frac{235}{3}x$

b i 27.42 N **ii** 40.73 N

c 0.38 m

11 A

12 a 416 cm **b** 85 cm

Exercise 3.06

1 a $y = 2x + 4, y = -3x - 1, (-1, 2)$

b $y = 1.5x - 3, y = 3, (4, 3)$

2 (2, 3)

3 a CopyCat Express

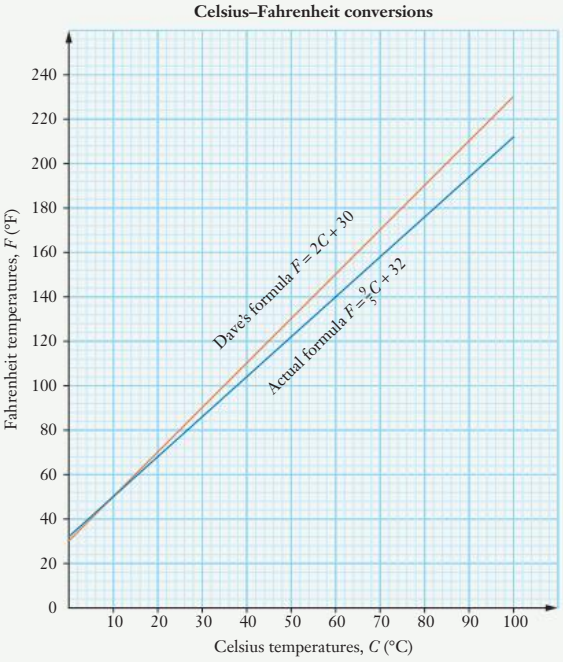
b CopyCat Express \$650, own photocopier \$1450

c Initial cost of buying copier = \$1200

d 15 000

e Cheaper to use own photocopier

4 a



b $C = 10$ **c** $C > 10$

d From $C = 0$ to $C = 35$

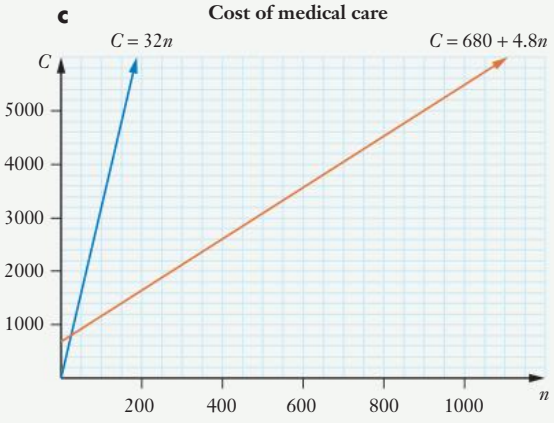
e Yes, for values below 35°C

5 a $C = 32n$

b $0.85 \times 32 = 27.2$, $32 - 27.2 = 4.8$; \$4.80 is the cost of each medical consultation under private health insurance. \$680 is the cost of annual private health insurance
$\therefore C = 4.8n + 680$.

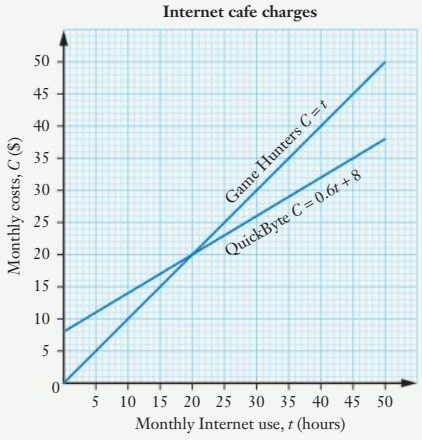
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c



d The point at which the cost of medical expenses is the same with or without private health insurance.

6 a



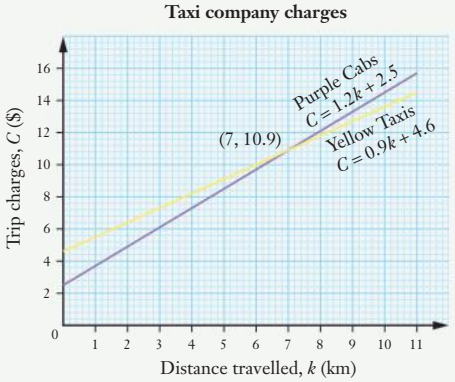
Break-even point is (20, 20).

b QuickByte, Cheaper

c QuickByte $22.40 Game Hunters $24

d Game Hunters is cheaper for users who access the Internet for less than 20 hours per month. QuickByte is Cheaper for users who access the Internet for more than 20 hours per month.

7 a



b The distance travelled, 7 km, for which both companies charge the same, $10.90

c Purple Cabs is cheaper for trips under 7 km. Yellow Taxis is cheaper for trips over 7 km.

d Yellow Taxis, $12

Sample HSC problem

a $C = 500n + 800$ **b** $600

c $R = 600n$ **d** 8

e $700

Test yourself 3

1 a $k = 2.5$ **b** $y = 7$ **c** $m = 10$

d $k = 5.5$ **e** $y = 2$ **f** $x = \frac{22}{3}$

g $p = -9$ **h** $k = \frac{22}{3}$ **i** $d = -\frac{19}{13}$

2 16 years

3 a $y = 2P - 220$ **b** $h = \frac{2(V - 9.9)}{d^2}$

c $I = \frac{V + E}{R}$

4 a $B = 12\,727d + 3000$ (answers may vary)

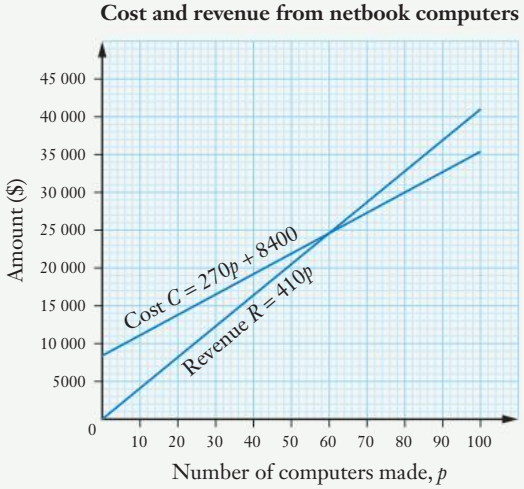
b Gradient = 12 727 and it represents the increase in bacteria per mL per day

c 92 089

d 8th day

5 a 9.57 L/100 km **b** 1082 kg

6 a



b $p = 60$, $C = R = 24\,600$

c Cost $21 900, revenue $20 500

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d It would make a loss.

e 8400, the fixed initial cost in dollars of manufacturing computers

f $410

Practice paper 1

1 C **2** D **3** A **4** A

5 B **6** B **7** C **8** D

9 B **10** B

11 **a** **i** $a = 4$ **ii** $m = 24$ **iii** $x = -1$

b **i** $28.50

ii Range of share price for the 12 months was $17 to $28.50. Higher share value in colder months March to August. Fairly steady increase September to April. After 12 months, the share price is about $5 higher.

c Jack

12 **a** B

b **i** $r = 1 - \frac{S}{700}$ **ii** $r = 0.36$

c **i** $625 000 **ii** $250 000

d **i** $4200 **ii** $337.50

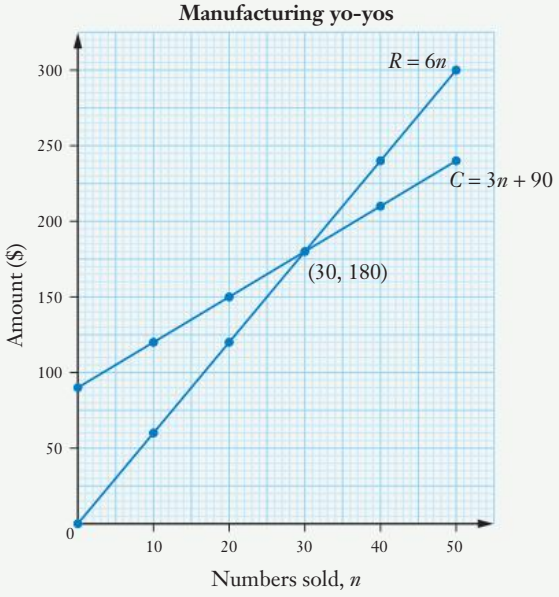
13 **a** **i** $-4 \times (-5) = 20$ (line 2)

$-6A = 9$ (line 4)

$A = \frac{3}{2}$ (line 5)

ii $A = \frac{8}{3}$

b **i**



ii (30, 180) **iii** $165

iv The point is below the break-even point and the factory would lose money if they made 25 yo-yos.

c 7 h 30 min

Chapter 4

SkillCheck

1 **a** 39 **b** 300

c 1040 **d** 26

2 **a** 3 **b** 0.75

c 0.0392 **d** 0.0185

3 **a** 2.93 **b** 50 813.96

4 **a** $31 200 **b** $46 540 **c** $207 000

5 249 395

6 867 638.5

Exercise 4.01

1 C

2 **a**

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment ($P + I$)	Balance ($P + I - R$)
5	$591 669.93	$3081.61	$594 751.54	$590 826.54
6	$590 826.54	$3077.22	$593 903.76	$589 978.76
7	$589 978.76	$3072.81	$593 051.57	$589 126.57
8	$589 126.57	$3068.37	$592 194.94	$588 269.94
9	$588 269.94	$3063.91	$591 333.85	$587 408.85

b $588 269.94 **c** $7591.15

d $27 733.85

3 D

4 **a** $1 187 415.84 **b** $407 415.84

5 **a**

Fortnights (n)	Principal ($P)	Interest (I)	Amount owing ($P + I$)	Balance ($P + I - R$)
1	$875 000	$2423.08	$877 423.08	$873 978.43
2	$873 978.43	$2420.25	$876 398.68	$872 954.03
3	$872 954.03	$2417.41	$875 371.44	$871 926.79
4	$871 926.79	$2414.57	$874 341.36	$870 896.71
5	$870 896.71	$2411.71	$873 308.42	$869 863.77
6	$869 863.77	$2408.85	$872 272.62	$868 827.97

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b $6172.03

c $14 495.87

6 a 0.011 333

b $\frac{0.136}{12} \times 35\,775.27 \approx \$405.45$

c

Month	P	I	$P + I$	$P + I - R$
1	$36 200	$410.27	$36 610.27	$35 775.27
2	$35 775.27	$405.45	$36 180.72	$35 345.72
3	$35 345.72	$400.58	$35 746.30	$34 911.30

d Loan administration fee of $30

e $1216.30

7 a 0.004 81

b

Fortnight	P	I	$P + I$	$P + I - R$
1	$27 800	$133.65	$27 933.65	$27 620.65
2	$27 620.65	$132.79	$27 753.44	$27 440.44

c $266.44

d $27 440.44

Exercise 4.02

1 B

2 C

3 $788.60

4 a 12 days **b** $2067.58

5 a 3256.43 – 3000 + 2195.48 + 3.84 = $2455.75

b $3000 on 15 June

c 1 June – 25 July 2021

d i $0 **ii** $43.85

6 D

7 D

8 $2223.36

Exercise 4.03

1 a 8% p.a. **b** $1298.56

c

5	$22 530.56	$1802.44	$5000	$29 333.00

2 a $42 472.80 **b** $1836.24

c $61 905.06 **d** $9415.98

3 $60 030.50

4 No, short by $48.30.

5 a $3203.49 **b** $788.49 **c** $1920.90

d $3242.24

6 $3937.36

7 $26 620.88

8 $3975.23

9 a $9639 **b** $10 193.20

10 a $6034.49 **b** $1724.05 **c** $33 142.80

11 $1466.67

12 Yes, they will have $61 025.60.

13 a $15 299

b Yes. $1300 \times 12.5076 = \$16\,260$, $\$16\,260 > \$16\,000$

14 a $67 060.50 **b** $59 503.97

15 a $94 178

b No. He would be $11 588 short.

Exercise 4.04

1 B

2 A

3 a i $3416.20 **ii** $614 916 **iii** $234 916

b i $5143 **ii** $1 542 900 **iii** $617 900

c i $8462.40 **ii** $2 030 976 **iii** $800 976

d i $8454.61 **ii** $1 014 553.20

iii $317 553.20

4 a $16 612 **b** $21 494

c $5455.60 **d** $31 464.40

5 a $9.64 **b** $11 731.20

6 a i $6375.24 **ii** $8911.68

b There are other possibilities, e.g.:

1st Option: for $100 000 loan, $954.50; for $80 000 loan, $763.60 Total = $1718.10

2nd Option: for $90 000 loan, $859.05 × 2 = $1718.10

c $93 078 saved

7 a $33 648 **b** save $56 **c** $15 720

Exercise 4.05

1 a 8.55% p.a. **b** $350

c $50 + pro-rata fee

2 a 5.20% p.a. **b** $1012 **c** $60

3 a 12.99% **b** $150 **c** $140

4 a Approx. 20 years

b 9 years 2 months **c** $350 000

d $1 471 680 **e** $561 680

f 11 years 11 months

g 2 years 11 months

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h Approx. $530 000 **i** $950 460
j $40 460 **k** 8 years 1 month
l $521 220

5 a i $330 000 **ii** $140 000
b i 8th **ii** 13th

6 a 4 years **b i** 11 years **ii** 7.5 years
c i $175 000 **ii** $130 000

7 a $600 000 **b** $490 000 **c** 30 years
d $1 186 200 **e** $586 200
f 29 years 6 months **g** $1 176 430
h $576 430 **i** 6 months **j** $9770

Sample HSC problem

a $5641.60 **b** $1 353 984 **c** $533 984
d $156 456 more paid in interest

Test yourself 4

1 a

Month	P	I	$P+I$	$P+I-R$
1	$18 000	$135	$18 135	$17 715
2	$17 715	$132.86	$17 847.86	$17 427.86
3	$17 427.86	$130.71	$17 558.57	$17 138.57
4	$17 138.57	$128.54	$17 267.11	$16 847.11

b $1152.89 **c** $527.11 **d** $1680

2 C

3 $6.86

4 a $56 017 **b** $6017

5 $64 723.20

6 a i $498 **ii** $17 928 **iii** $1928
b $624 saved

7 a $400 000 **b** $290 000
c 21 years 9 months
d 176 months (14 years 8 months)

Chapter 5

SkillCheck

1 a 52.158 **b** 466.18

2 a −1.2 **b** −3.21

3 a $y=\frac{2}{3}x-1\frac{1}{3}$ **b** $y=-\frac{4}{3}x+1$

c $h=-\frac{2}{3}x+55$ **d** $p=1.25t+197.5$

Exercise 5.01

1 a Strong negative **b** Decreases

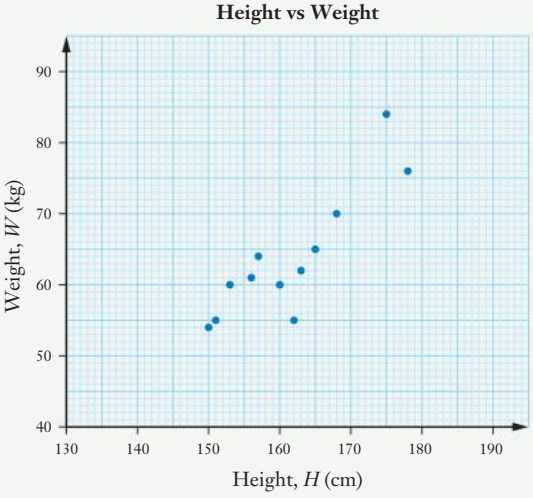
2 All have a linear relationship except **b**.
a Perfect positive
b No relationship
c Moderate negative **d** Moderate positive
e Strong positive **f** Perfect negative

3 e

4 a 99 cm **b** 174 cm **c** yes
d Strong positive as points are in a positive direction and close to a straight line.
e No, waist seems to be slightly more than half the height (about $\frac{3}{5}$).

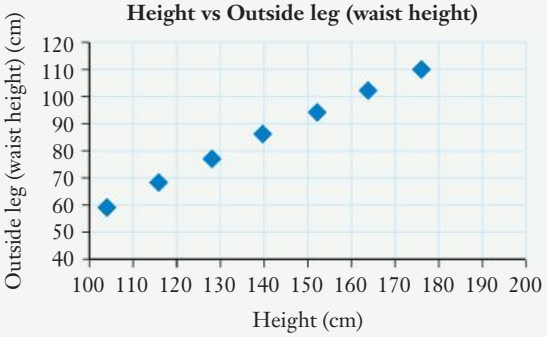
5 a Linear **b** Positive **c** Strong
d Yes, each foot measure is roughly 12 times the corresponding knuckle measure.

6 a



b Linear pattern
c Positive and strong

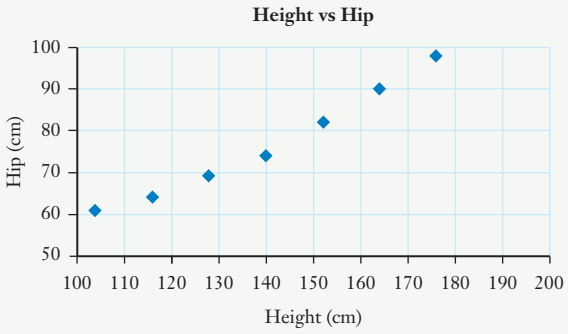
7 a



ISBN 9780170413633

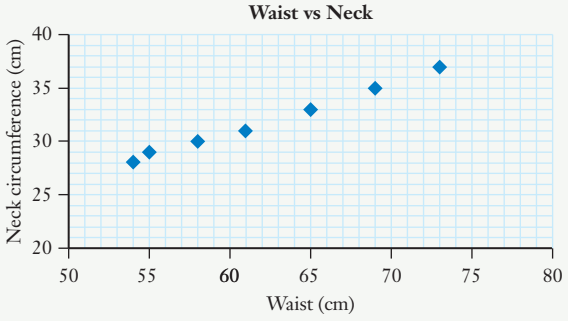
b Strong or perfect positive

c



d Strong positive

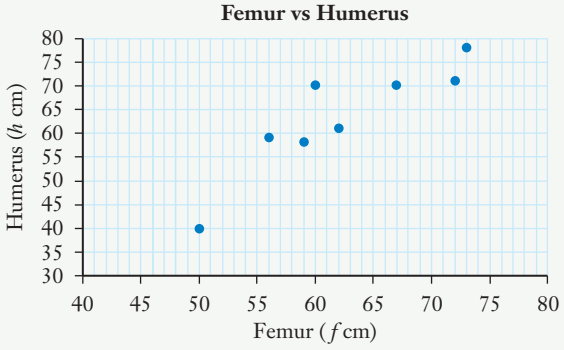
e



f Strong positive

g Yes

8 a



b There is a strong positive relationship between the length of the two bones.

c Yes, by observing humans you can see that the femur and humerus are roughly the same size.

Exercise 5.02

1 a Data relating to 2 variables

b The strength of a relationship between 2 variables

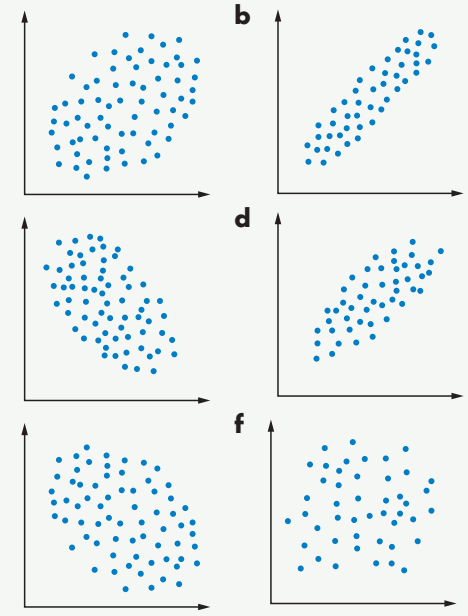
c A value between –1 and +1 describing the strength of the relationship between 2 variables

2 C **3** C

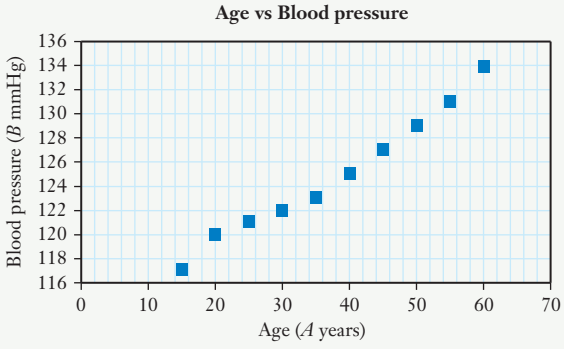
4 a Negative **b** Positive **c** Negative
d Positive **e** Zero **f** Positive
g Positive **h** Negative

5 Causal: d, f, g, i, j

6 a



7 a



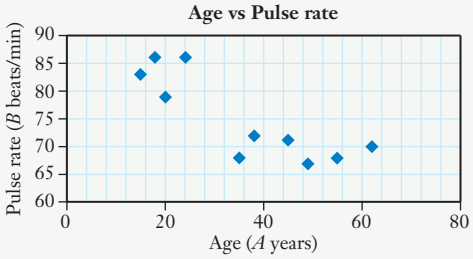
b Strong positive

c 0.9 **d** 0.99

e Teacher to check

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8 a



b Moderate or strong negative

c $r = -0.8$ **d** $r = -0.84$

9 a -0.229 **b** weak

10 a i 0.912 **ii** strong

b i 0.912

ii No, the correlation is the same.

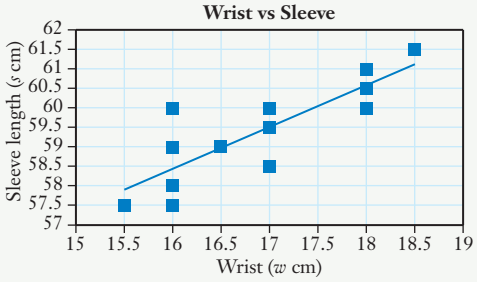
11 a $r = 0.997$ **b** $r = 0.925$ **c** $r = 0.894$

Exercise 5.03

1 a $h = b + 4$ **b** 94 cm **c** 96 cm

d No, this is outside the range of values.

2 a, b



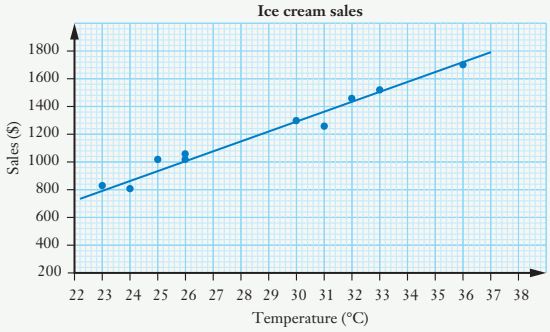
c i 17.5cm **ii** 58 cm

d Equation of line shown in part **b** is $s = w + 42.5$ (answers will vary).

e Using $s = w + 42.5$:

i $w = 16$ cm **ii** $s = 60$ cm

3 a, b

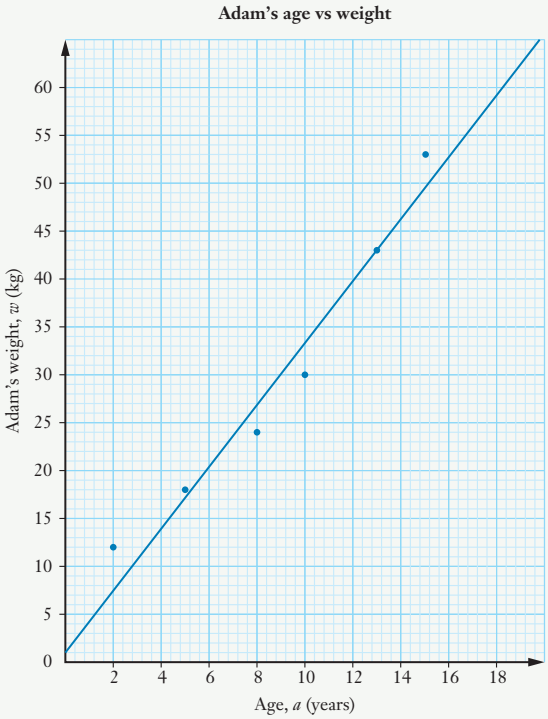


c $S = 71.11T - 837.78$ (answers will vary)

d i \$1160 **ii** \$1580

e It is possible but may not be accurate as both temperatures are outside the data range.

4 a



b $w = 3.27a + 0.49$ **c** 59 kg

d 12.1 years

e Adam's weight will not continue to increase as he gets older.

5 a $C = 73.3a - 73$

b 73.3, increase in circumference in millimetres per year

c i 660 mm **ii** 1393 mm

d 10 years

6 a $E = 9.6w + 468$ **b** 1188 kJ

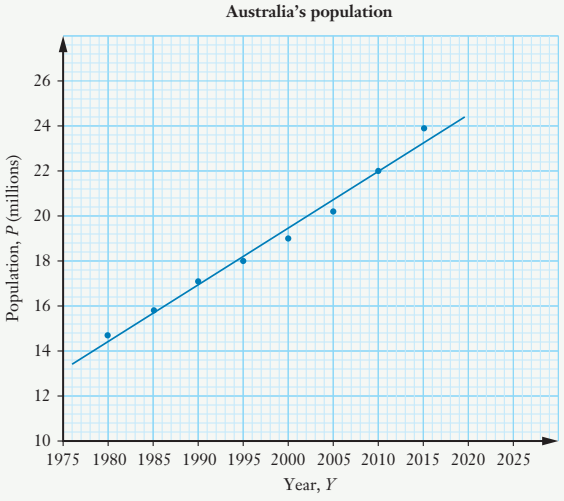
c 86 kg

d Energy used cannot keep decreasing for underweight women; there must be a lower limit.

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7 a Year

b



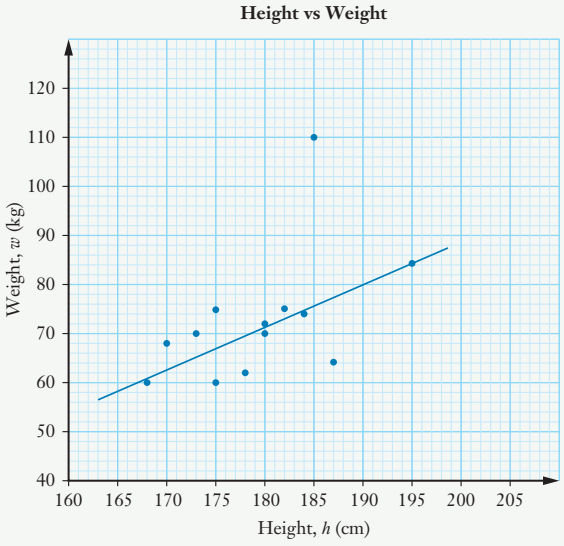
c $P = 0.2514Y - 483.3$ (answers will vary)

d **i** 11.9 million **ii** 21.5 million

e **i** 24.5 million **ii** 27 million

f Factors such as immigration may affect population growth.

8 a



b $w = 0.77h - 68.2$ (answers will vary)

c **i** 64.2 kg **ii** 78.1 kg

d **i** 192 cm **ii** 218 cm

e Tall people don't necessarily weigh more than shorter people.

Exercise 5.04

1 a **i** 0.81

ii 79.2, 9.90

iii 92.4, 8.58

b $c = 0.70w + 36.66$

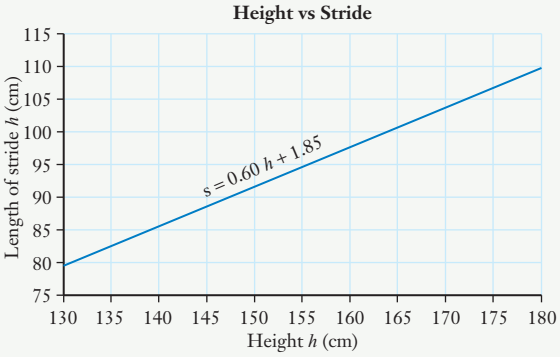
c (72, 73) and (89, 89)

2 a $r = 0.942$

b **i** 162.5, 11.91 **ii** 98.75, 7.54

c $s = 0.60\,h + 1.85$

d

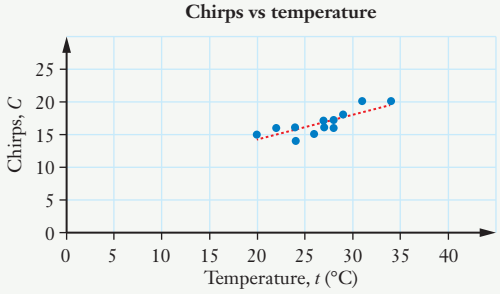


e, f **i** 102 cm **ii** 147 cm

3 a 0.829

b $C = 0.371t + 6.820$

c **i**



ii The points are close to the line, indicating a strong correlation.

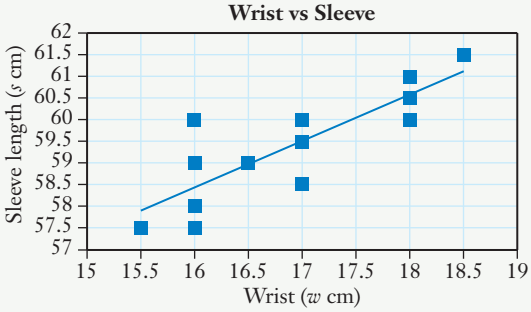
d **i** 18 **ii** 9°C

4 a $W = 0.847H - 82.660$

b **i** 87 kg **ii** 192 cm

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5 a, b



c $s = 1.0773w + 41.204$

d $r = 0.8369$, a strong positive correlation

e The line of best fit from Question 2 had equation $s = w + 42.5$, which is close to that of the line shown here.

6 a 0.875, strong correlation

b $b = 0.139w + 11.775$

c **i** 95 kg **ii** 22.895

d 50 kg is too far removed from the given data.

Exercise 5.05

1 a Very strong positive correlation

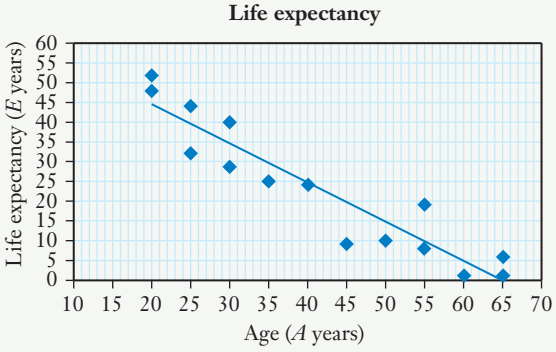
b Teacher to check

c $L = 0.1473Y - 214.32$

d $r = 0.997$

e The gradient 0.1473 is the rate at which the life expectancy is increasing each year.

2 a



b **i** 59 years **ii** 74 years

c About 12 years

d Strong negative **e** –0.94

f The equation of the line through (20, 44) and (50, 14) is $E = -A + 64$, which is very close to $E = -A + 65$

g 19 years. In the data, the 45-year-old is a smoker with a life expectancy lower than that for the average 45-year-old represented by this equation.

h $E = -1.0002A + 64.541$; 64.5 years

3 a **i** Female have a higher life expectancy than males.

ii The life expectancy for males was fairly constant between 1960 and 1976.

b **i** 1910 **ii** 1981

c Increases from 1900 but flattens from 1966 to 1976, then increases again.

d Teacher to check

e $y = 0.2447x - 407.37$ (Answers will vary.)

4 a Teacher to check

b Moderate to strong negative correlation

c $y = -0.4x + 32$ (Answers will vary.)

d –0.78

e **i** 6 **ii** 3.6

5 a Non-linear

b Life expectancy varies mostly between 70 and 80 years.

c About 78 years

d Agree, as the points on the scatterplot show roughly the same life expectancy – around 80 years.

6 a **i** 7.4% **ii** 8.8% **iii** 9%

b **i** 84.8 **ii** 80 **iii** 72

c USA, 16.4%

d Turkey, 5.2%

e UK

f Japan spends more on healthcare (10.2%) than the UK (8.5%) and Hungary (7.4%) and it has a higher life expectancy (84.8) than the UK (80) and Hungary (75.8).

g Yes. Spending more on healthcare is related to higher life expectancy. The scatter plot shows a positive correlation.

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Sample HSC problem

a 0.85

b $h = 0.754w + 37.874$

c **i** 90.6 cm **ii** 82.4 cm

Test yourself 5

1 **a** No correlation

b Linear, strong positive correlation

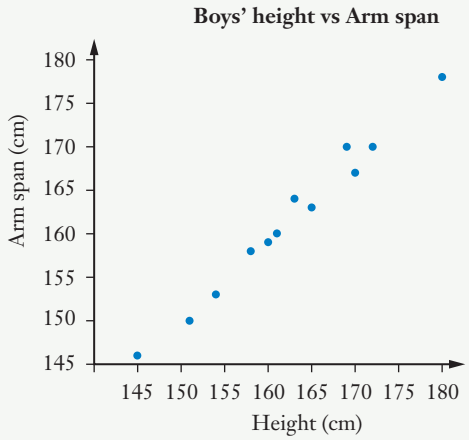
c Linear, moderate to weak negative correlation

d Linear, moderate positive correlation

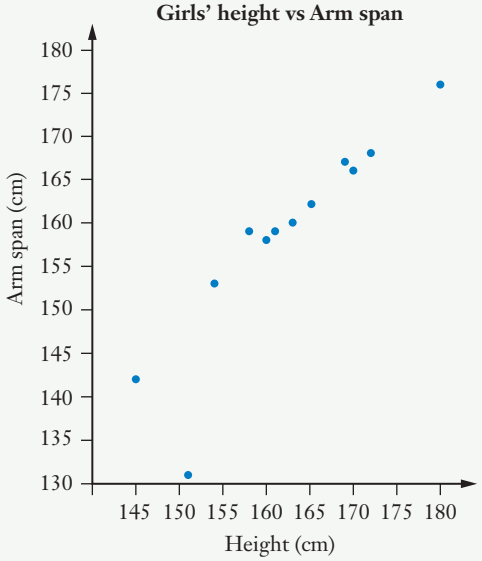
e Linear, strong negative correlation

f Non-linear association

2 **a**



b



c Strong positive

d Weaker (correlation for boys is almost perfectly linear)

e (151, 131), a girl with a height of 151 cm and an arm span of 131 cm.

3 **a** Perfect positive, $r = 1$

b No correlation, $r = 0$

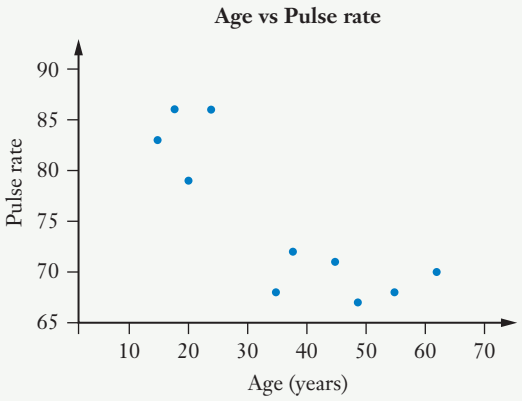
c Moderate negative, $r = -0.7$

d Moderate positive, $r = 0.6$

e Strong positive, $r = 0.95$

f Perfect negative, $r = -1$

4 **a**



b Strong negative **c** –0.8 **d** –0.84

e Teacher to check

5 **a** Teacher to check

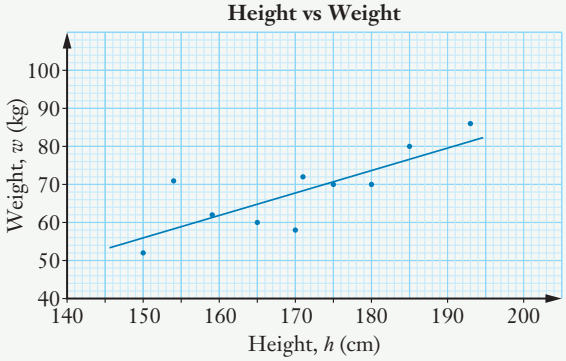
b $D = -0.27P + 111$

c 38% **d** \$410

e 111, it implies a demand of 111% when the price is \$0. Demand cannot be more than 100%, and price cannot be \$0.

6 **a** Height

b



c $w = 0.6h - 34$

d **i** 165 cm **ii** 182 cm

e Whilst it is possible it would not necessarily give an accurate result as 200 cm is outside the range of data given.

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7 a, d



b $r = 0.989$

c $P = 0.536y - 1022.1$

e i 54 000 **ii** 61 000

8 a, b Teacher to check

c $S = 0.3059H - 43.719$

d $r = 0.856$, strong correlation

9 a Teacher to check

b $M = 0.2399y - 402.69$, $F = 0.1524y - 222.26$

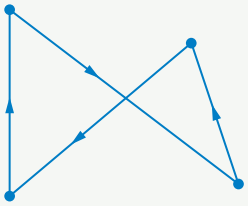
c Males: $r = 0.973$ Females: $r = 0.955$

d Male and female life expectancies are both increasing, with male life expectancy increasing at a faster rate. The correlation coefficient for males is slightly higher than for females, but the correlation for both is very strong.

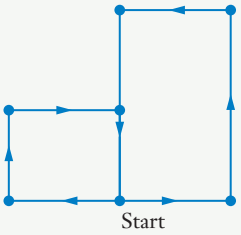
Chapter 6

SkillCheck

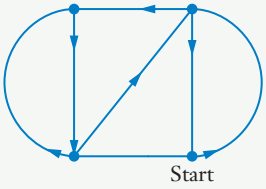
1 a



b



c



2 a *AEBDCA* **b** *AB*

3 78 km

Exercise 6.01

1 a i 5 **ii** 6

b i 8 **ii** 11

c i 5 **ii** 7

d i 7 **ii** 10

e i 4 **ii** 5

f i 4 **ii** 6

2 B

3 D

4 D

5 a i 111 **ii** 42, *abgef*

iii 101, *abgecbdef*

b i 216 **ii** 34, *abcf*

iii 160, *abcdeadf*

6 a i *M*-2, *T*-4, *X*-2, *W*-3, *P*-3

ii 3 **iii** $14 = 2 \times 7$

b i *b*-2, *c*-2, *d*-2, *f*-2 **ii** 4

iii $8 = 2 \times 4$

c i *a*-2, *b*-3, *w*-2, *k*-4, *r*-1, *y*-3, *x*-3, *g*-2

ii 4 **iii** $20 = 2 \times 10$

d i *A*-1, *C*-4, *P*-2, *Y*-1

ii 2 **iii** $8 = 2 \times 4$

7 B

8 B

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9 C

10 **a** **i, v** trail **ii, iii, iv** path

b **i** circuit – **v** **ii** cycle – **iv**

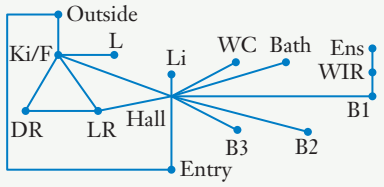
c *gbekmeg* – teacher to check other answers

d *gmkebg*, *bgmeb*

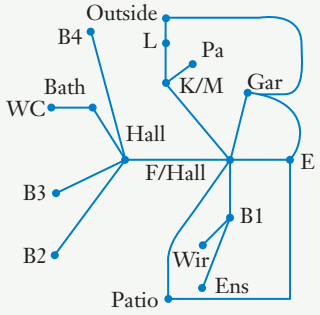
11 D

12 A

13 **a**



b



14



15 C

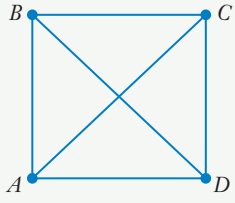
16 **a** There is no edge between the vertices.

b $a = 1, b = 0$

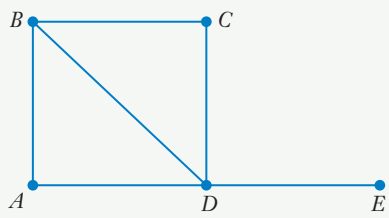
17

	A	*B*	*C*	*D*	*E*
A	0	1	0	0	1
B	1	0	1	1	1
C	0	1	0	1	0
D	0	1	1	0	1
E	1	1	0	1	0

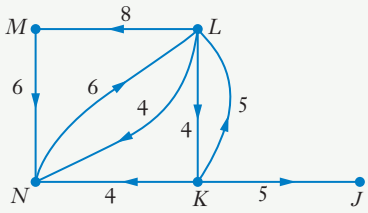
18 **a**



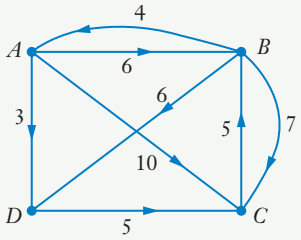
b



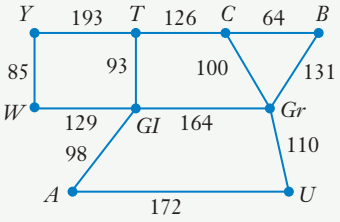
19 **a**



b



20 Teacher to check – one example is:

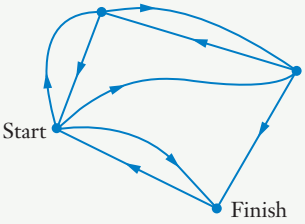


Exercise 6.02

1 A

2 **a** 2 vertices of odd degree

b



3 Yes, 2 vertices (Casino and Tenterfield) have odd degrees.

4 A

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5 **a** All vertices are of even degree.

b Teacher to check – one example is: *FABCAECDEF*

c Teacher to check

6 **a** Yes, since two vertices (Wentworthville and Silverwater) are of odd degree.

b No, since not all vertices are of even degree.

7 **a** Eulerian trail

b Neither – insufficient information

c Eulerian circuit **d** Eulerian trail

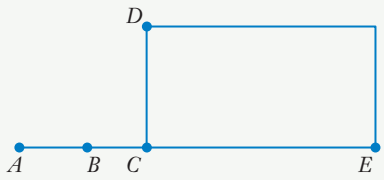
e Neither **f** Eulerian trail

g Eulerian trail **h** Neither

i Neither **j** Eulerian circuit

8 **a** The network cannot start and end at the same vertex since there are odd vertices (Riverstone, Alan St).

b



Riverstone

9 **a** No, as there are more than 2 vertices of odd degree.

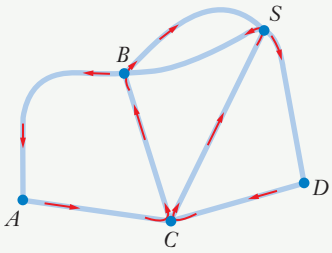
b No, he cannot as there are more than 2 vertices of odd degree.

10 **a** Yes, there are only 2 vertices of odd degree.

b The vertices are not all even so cannot start and end at *S*.

c All vertices are now even so it is possible to start and finish at *S*.

One route is *SDCSBACBS*.



11 **a** All vertices need to be of even degree – *P* and *C* have odd degrees.

b *P* and *C*

c *PFGABGPEBCEDCP* (there are other routes)

12 Teacher to check – some routes are *TYRWPT*, *TPWRYT*, *TPWYRT*

13 **a** 24

b *A* or *E* – the vertices of odd degree.

c 3.66 km

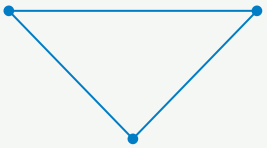
14 C

15 **a** **i** *F*, the other odd vertex.

ii 5

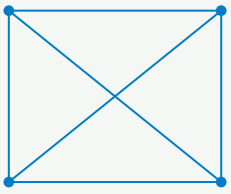
b Teacher to check – 2 routes are *BAHCDEFG*, *BAGFEDCH*

16 **a**



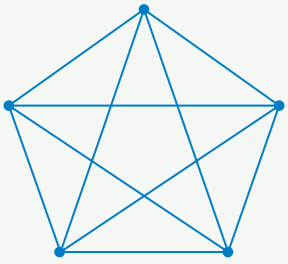
i 2 **ii** 3

b



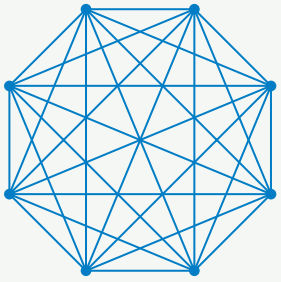
i 3 **ii** 6

c



i 4 **ii** 10

d



i 7 **ii** 28

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Exercise 6.03

1 **a**, **d** are trees as any two vertices are connected by exactly one path.

b, **e** are not trees as they are not connected networks.

Network **c** is not a tree because it contains a cycle.

2 **a** **i** 9 **ii** 8

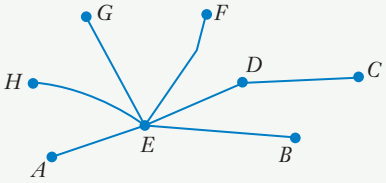
b **i** 12 **ii** 11

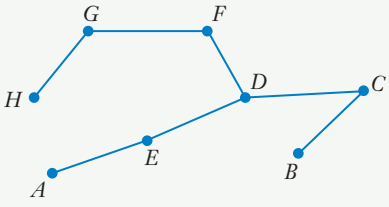
c **i** 8 **ii** 7

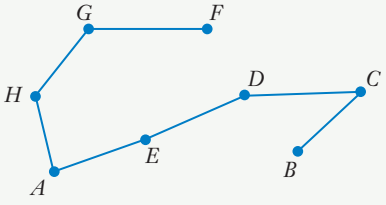
3 B

4 **a**, **d**, **e** and **f** are spanning trees.

5 3 possible spanning trees are:



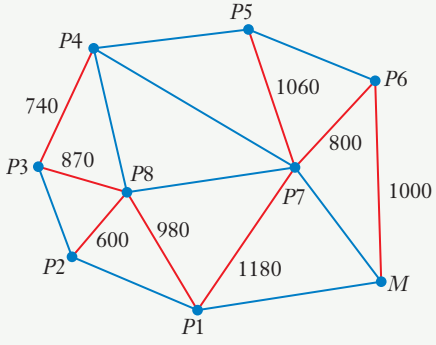




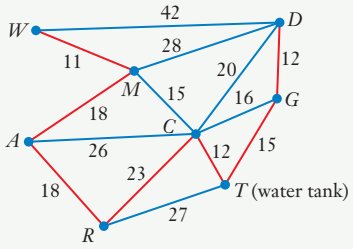
6 **a** **i** \$7960 **ii** \$8090 **iii** \$7530 **iv** \$7840

b **iii** is the best option.

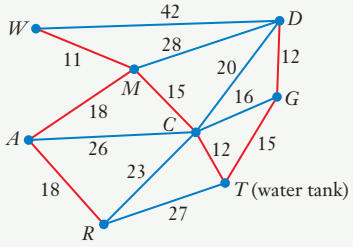
A better option is the spanning tree shown with a weight of \$7230.



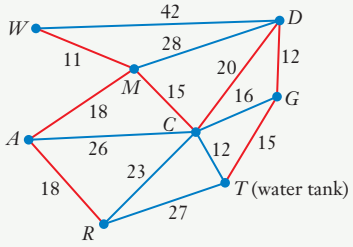
7



Weight = 109



Weight = 101



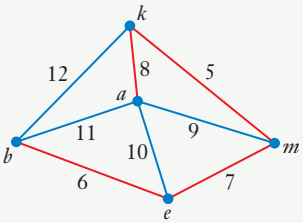
Weight = 109

Minimum length of water pipe required is 101 metres.

8 **a**

Edge	Weight
km	5
be	6
em	7
ak	8
am	9
ae	10
ab	11
bk	12

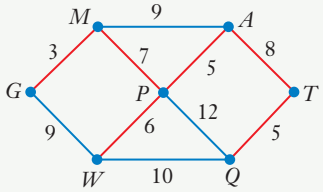
b Edges in order: *km*, *be*, *em*, *ak*



c Weight = 26

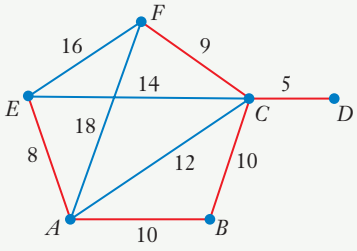
ISBN 9780170413633

9 a i Edges in order are: *GM*, *QT*, *AP*, *PW*, *PM*, *AT*; weight = 34



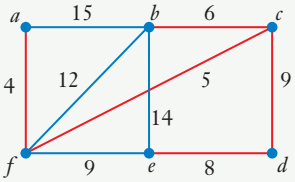
ii Spanning tree has 7 vertices, 6 edges

b i Edges in order are: *CD*, *AE*, *CF*, *BC*, *AB*: weight = 42



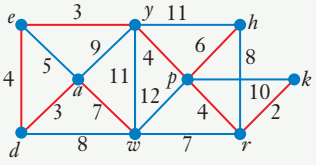
ii 6 vertices, 5 edges

c i Edges in order are: *af*, *cf*, *bc*, *de*, *cd*; weight = 32



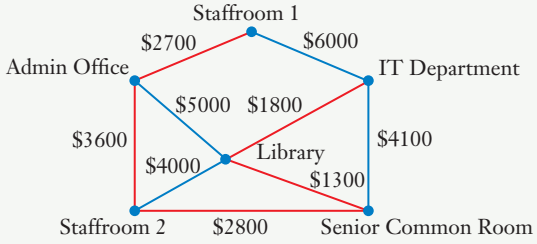
ii 6 vertices, 5 edges

d i Edges in order are: *kr*, *ad*, *ey*, *pr*, *py*, *de*, *hp*, *aw*; weight = 33 (edge *ae* not used as it would have made a cycle)



ii 9 vertices, 8 edges

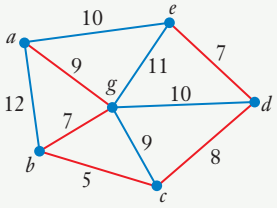
10 a *L SCR*, *L IT*, *A S1*, *S2 SCR*, *A S2*



b Minimum cost = $12 200

11 A

12 a Starting at *d*, edges in order are: *de*, *dc*, *cb*, *bg*, *ga*

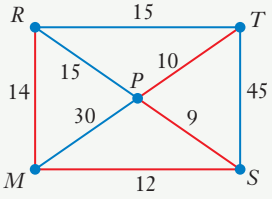


b 36

c No

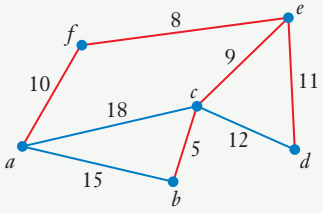
13 a i Starting at *M*, edges in order are: *MS*, *SP*, *PT*, *MR*

ii 45



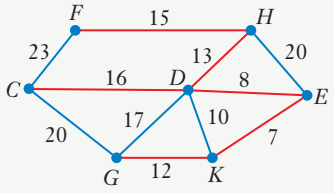
b i Starting at *a*, edges in order are: *af*, *fe*, *ec*, *cb*, *ed*

ii 43



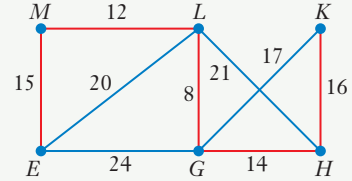
c i Starting at *G*, edges in order are: *GK*, *KE*, *ED*, *DH*, *HF*, *DC*

ii 71



d i Starting at *H*, edges in order are: *HG*, *GL*, *LM*, *ME*, *HK*

ii 65

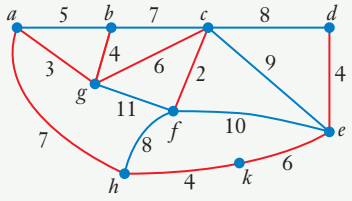


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14 a Edges in order are: *AM*, *MC*, *CS*, *SB*

b 2745 km

15 a i The edges in order are: *cf*, *ag*, *bg*, *hk*, *de*, *cg*, *ek*, *ah*.



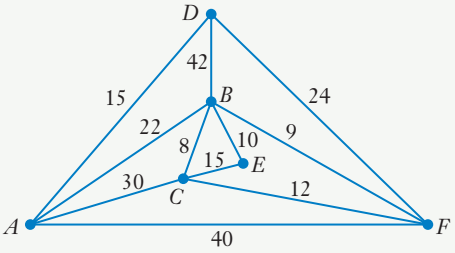
ii The edges in order starting from *e* are: *ed*, *ek*, *kh*, *ha*, *ag*, *gb*, *gc*, *cf*.

b Yes

c 7 + 3 + 4 + 6 + 2 + 4 + 6 + 4 = 36 km

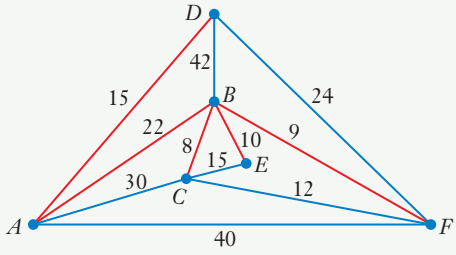
d Shortest route from *a* to *d* is *ag*, *gc*, *cd* and the length is 17 km.

16 a



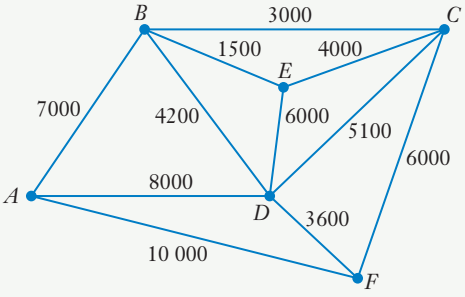
b i Edges in order are: *BC*, *BF*, *BE*, *AD*, *AB*

ii Edges in order starting at *A* are *AD*, *AB*, *BC*, *BF*, *BE*

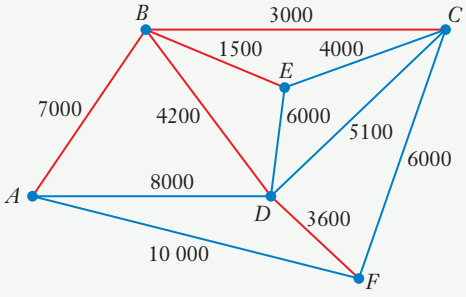


c 64 km

17 a



b Edges in order: *BE*, *BC*, *DF*, *BD*, *BA*



c Least cost = $19 300

Exercise 6.04

1 B

2 a i Mt Pritchard – Cabramatta – Fairfield, 7.9 km

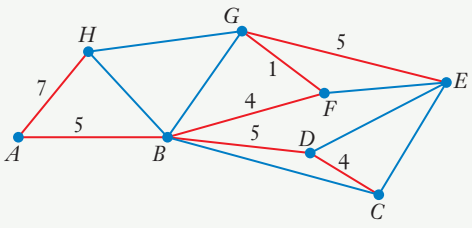
ii Mt Pritchard – Canley Heights – Fairfield, 17 min

b No

3 a

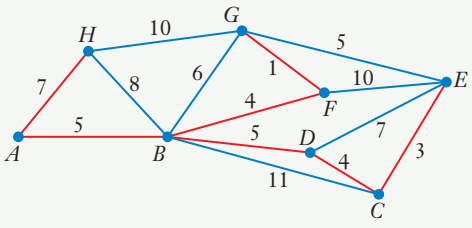
Vertex	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Path	*AB*	*ABDC*	*ABD*	*ABFGE*	*ABF*	*ABFG*	*AH*
Weight	5	14	10	15	9	10	7

b



Weight = 7 + 5 + 4 +1 + 5 + 5 + 4 = 31

c



Weight of minimum spanning tree = 29

d The trees are different and the weight of minimum spanning tree is less than weight of shortest path spanning tree.

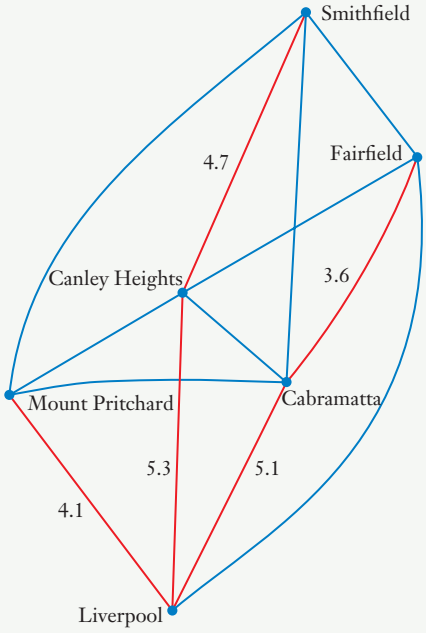
The shortest path from *A* to *E* is not in this minimum spanning tree.

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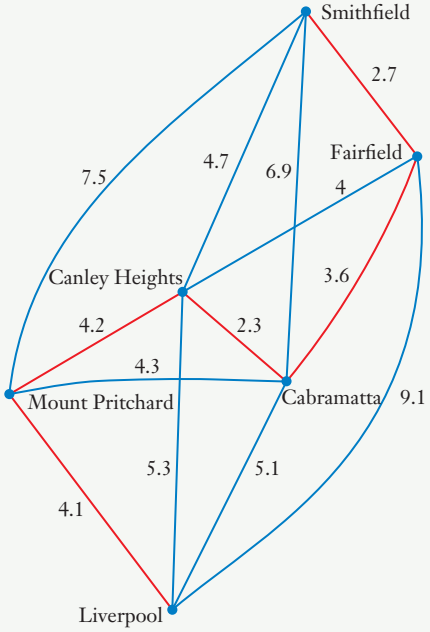
4 a i

Vertex	*MP*	*S*	*CH*	*C*	*F*
Path	*L-MP*	*L-CH-S*	*L-CH*	*L-C*	*L-C-F*
Distance	4.1	10	5.3	5.1	8.7

Weight = 22.8 km



ii Weight of minimum spanning tree = 16.9 km



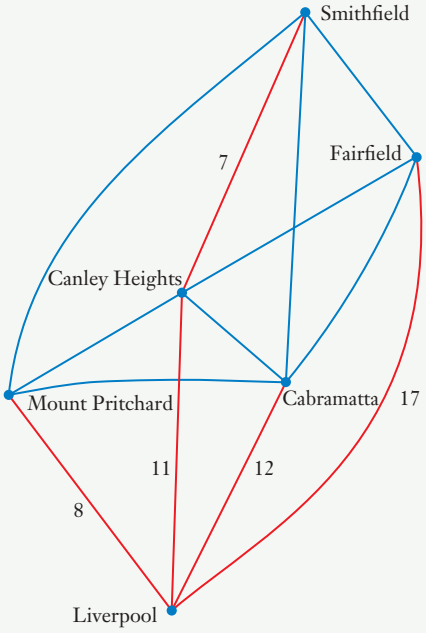
b No

c Liverpool – Canley Heights , Cabramatta, Fairfield

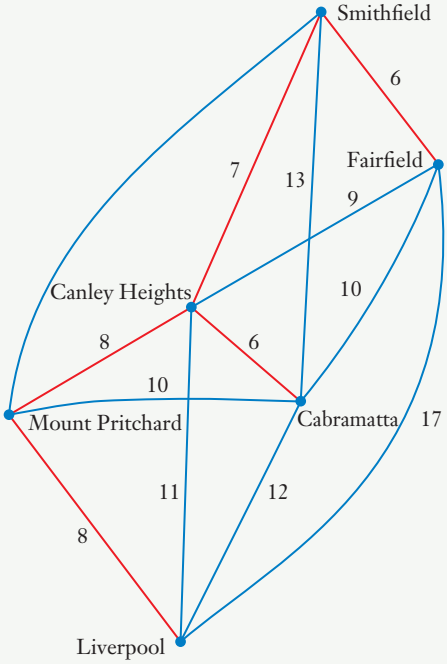
5 a i

Vertex	*MP*	*S*	*CH*	*C*	*F*
Path	*L-MP*	*L-CH-S*	*L-CH*	*L-C*	*L-F*
Distance	8	18	11	12	17

Weight = 55 min



ii Weight of minimum spanning tree = 35 min

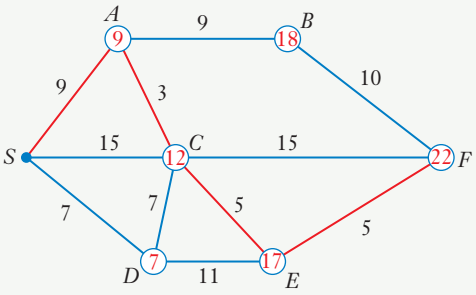


ISBN 9780170413633

b No

c Liverpool – Canley Heights, Cabramatta, Fairfield

6



Shortest path from *S* to *F* is *SACEF* and has weight 22.

7 **a** Shortest path is *SPNF*, weight = 11

b Shortest path is *SADCF*, weight = 14

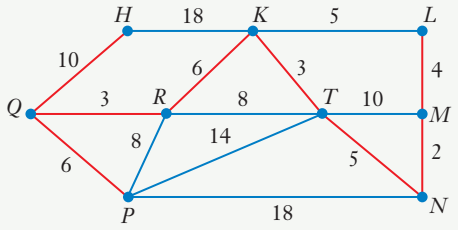
c Shortest path is *SGDEF*, weight = 11

d Shortest path is *SKGEF* or *SAEF*, weight = 46

8 B

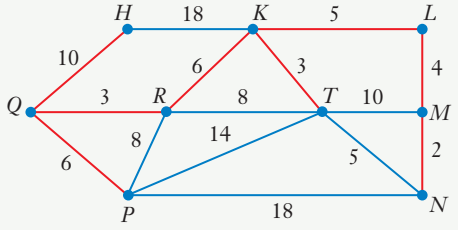
9 The shortest path is *EDBGC*, length = 3.6 km

10 **a**



Weight = 39

b No – the other spanning tree is

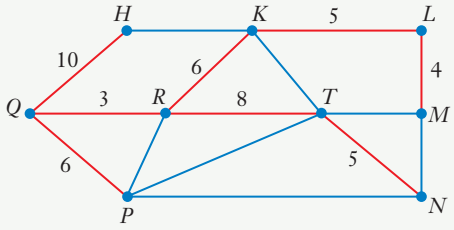


c **i** Shortest path from *Q* to *N* is *QRTN*, weight = 16

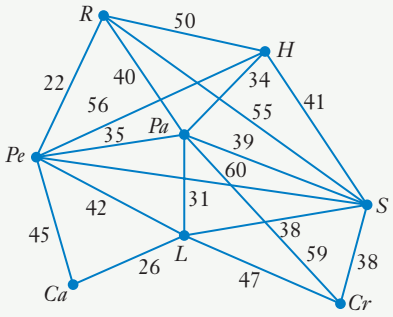
ii No

d

Vertex	*H*	*K*	*L*	*R*	*T*	*M*	*P*	*N*
Path	*QH*	*QRK*	*QRKL*	*QR*	*QRT*	*QRKLM*	*QP*	*QRTN*
Distance	10	9	14	3	11	18	6	16

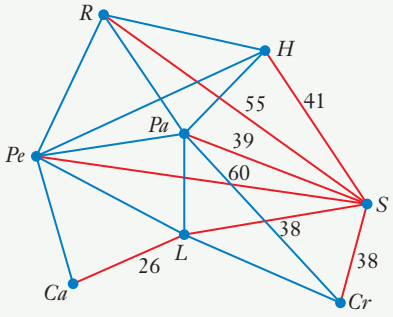


11 **a**



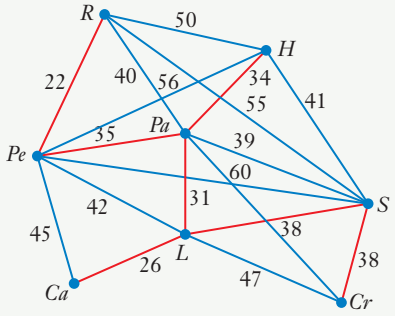
b *Cr*-*S*-*R*, 93 min

c



Weight = 297 min

d **i** Order of edges is: *PeR*, *LCa*, *PaL*, *PaH*, *PaPe*, *SL*, *SCr*



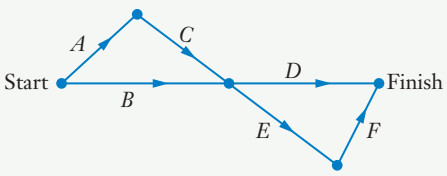
ii weight = 224 min

e No. The minimum spanning tree does not take into account the motorway from Sydney to Richmond as it is not one of the shorter distances.

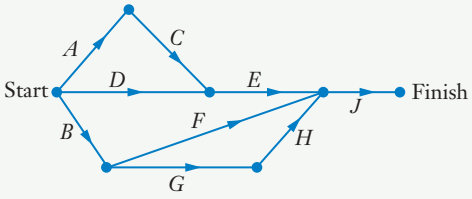
ISBN 9780170413633

Exercise 6.05

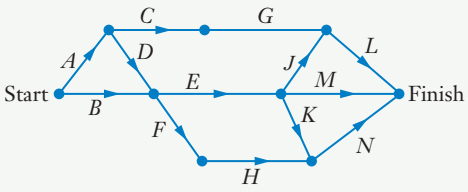
1 a



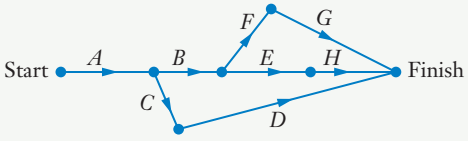
b



c



d



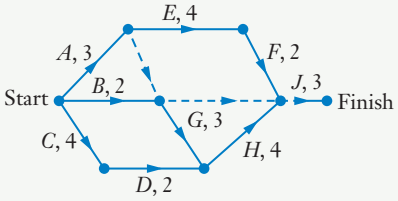
2 B

3 C

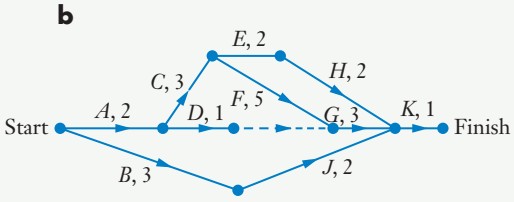
4 D

5 B

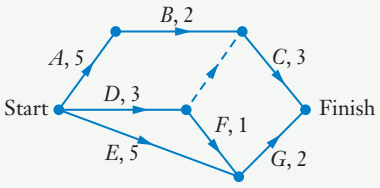
6 a



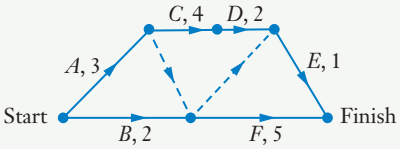
b



c



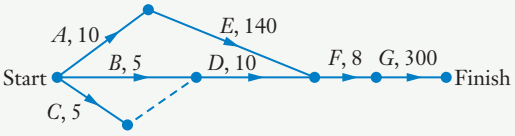
d



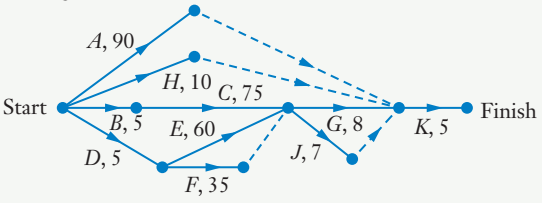
7

Activity	Time	Predecessor(s)
A	8	–
B	3	–
C	5	*A*, *B*
D	3	*C*
E	2	*C*
F	4	*A*, *B*
G	3	*F*
H	1	*E*, *G*
J	2	*E*, *G*
K	7	*F*
L	10	*D*, *J*, *H*

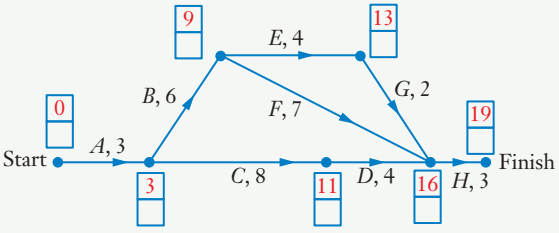
8 a



b

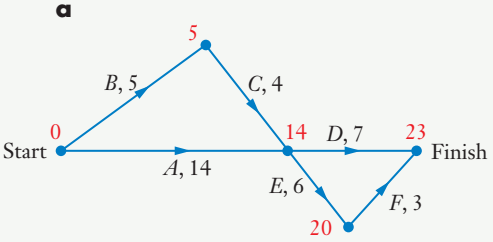


9



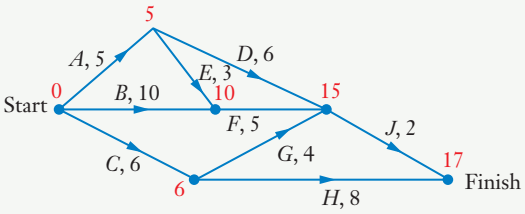
10 Numbers above vertices are ESTs.

a



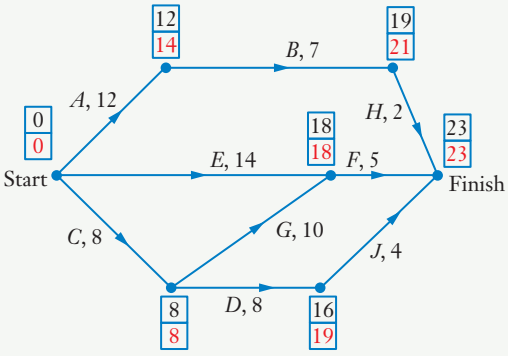
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b

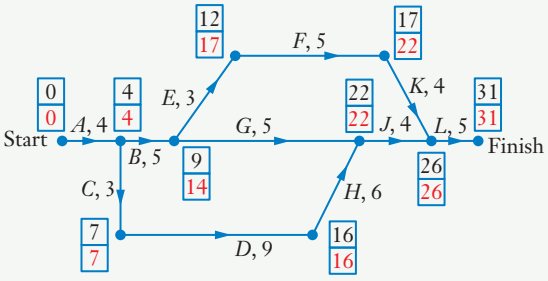


Exercise 6.06

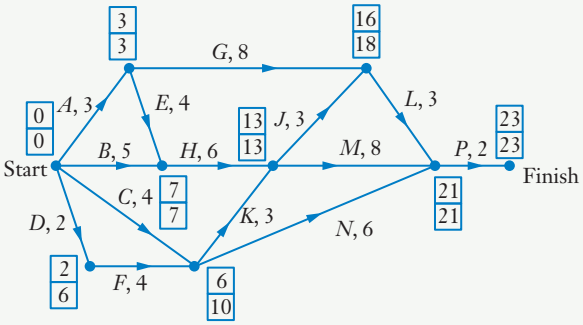
1 a



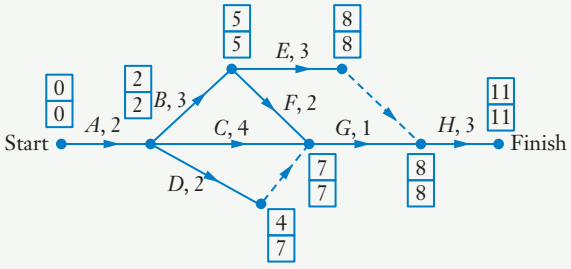
b



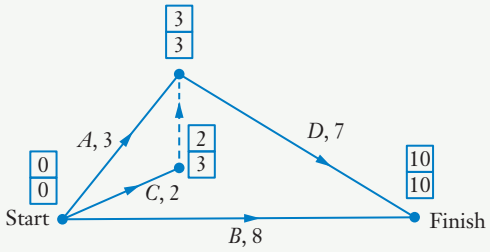
2 a



b

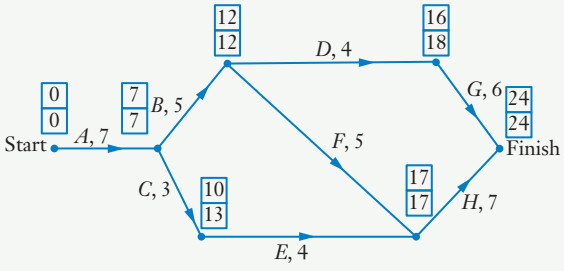


3 a i, ii



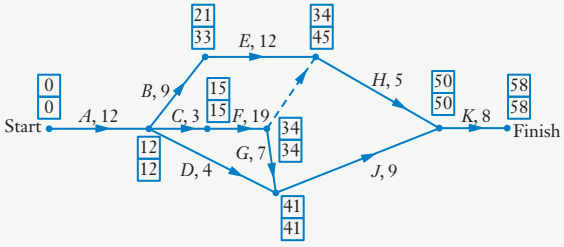
iii *AD*, length = 10

b i, ii



iii *ABFH*, length = 24

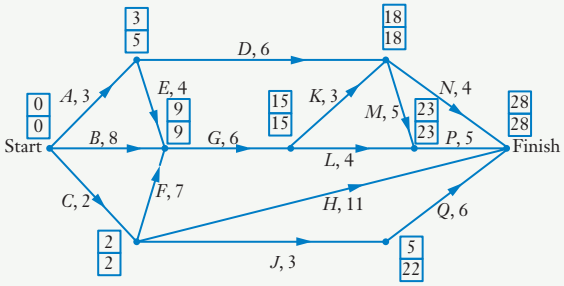
c i, ii



iii *ACFGJK*, length = 58

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d i, ii



iii *CFGKMP*, length = 28

4 B

5 **a** 12 months **b** *ACGFML*, 22 months

c Float time for *K* is 2 months—this means activity *K* can be extended or delayed by 2 months.

6 **a** *C*, float = 1 *B*, float = 2

b *E*, float = 3 *G*, float = 2 *D*, float = 2
C, float = 3

c *B*, float = 12 *E*, float = 12 *D*, float = 25
H, float = 11

d *D*, float = 9 *Q*, float = 17 *A*, float = 2
N, float = 6 *L*, float = 4 *B*, float = 1
E, float = 2 *H*, float = 15 *J*, float = 17

7 **a** *ABCDEF* (or *ABLMPQ*), 22 hours

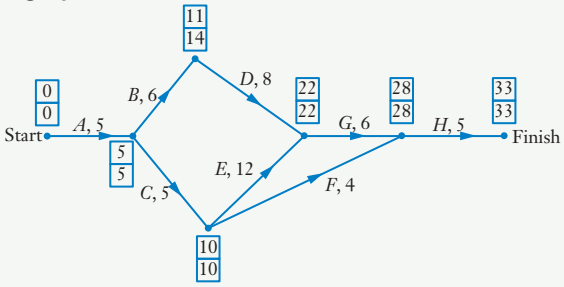
b No, both critical paths have the same completion time 22 hours

c *H*, *N*

d *G*, *H*, *J*, *K*, *N*

e *B*—this will shorten both critical paths by 3 hours.

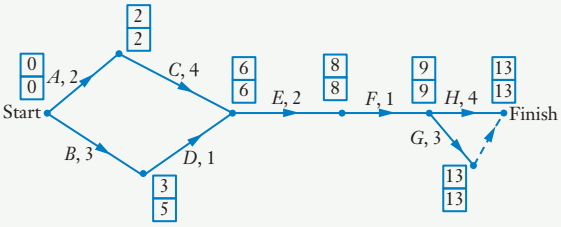
8 **a**



b *ACEGH*, 33 days

c *B*, float = 3 days *D*, float = 3 days
F, float = 14 days

9 **a**

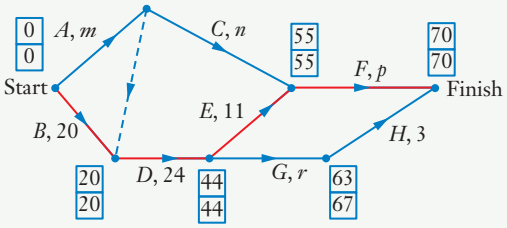


b *ACEFH*

c *B*, float = 2 days *G*, float = 1 day *D*, float = 2 days

d 10 September

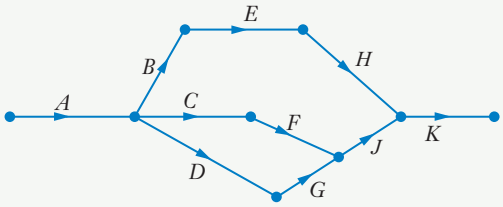
10



$r = 63 - 44 = 19$, $p = 70 - 55 = 15$ (*F* is on CP)

$m + n = 55 - 9 = 46$, and m must be less than 20 (*A* is predecessor of *D*, *A* not on CP) so possible values of m an n are 10 and 36, 15 and 31, 5 and 41.

11 **a**



b For the possible critical paths *ABEHK*, *ACFJK* or *ADGJK* there are 5 non-critical activities.

c **i** 14 days **ii** *ACFJK*
iii 35 – 26 = 9 days **iv** 6 days

Exercise 6.07

1 **a** **i** 800 **ii** 500 **iii** 500
b **i** 450 **ii** 410 **iii** 410
c **i** 450 **ii** 200 **iii** 200
d **i** 260 **ii** 700 **iii** 260

2 **a** **i** 75 **ii** 110 **iii** 115
b **i** 13 **ii** 8 **iii** 15
c **i** 180 **ii** 115 **iii** 155
d **i** 600 **ii** 130 **iii** 540

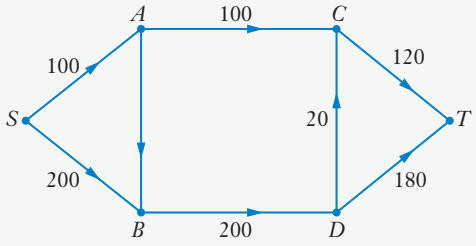
3 **a** 88 **b** 53

4 B

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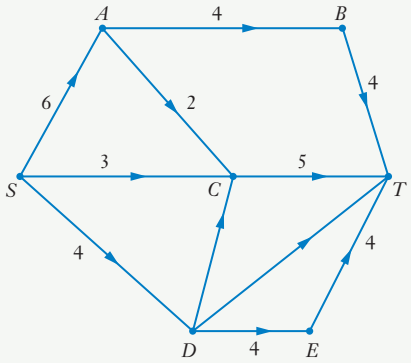
5 a i 300 (*SACT* – 100, *SBDT* – 180, *SBDCT* – 20)

ii



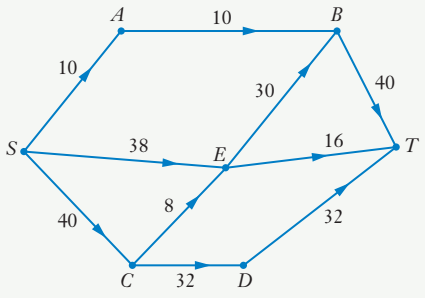
b i 13 (*SABT* – 4, *SDET* – 4, *SACT* – 2, *SCT* – 3)

ii



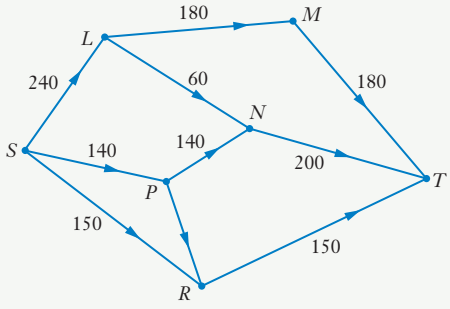
c i 88 (*SCDT* – 32, *SEBT* – 30, *SET* – 8, *SCET* – 8, *SABT* – 10)

ii



d i 530 (*SLMT* – 180, *SPNT* – 140, *SLNT* – 60, *SRT* – 150)

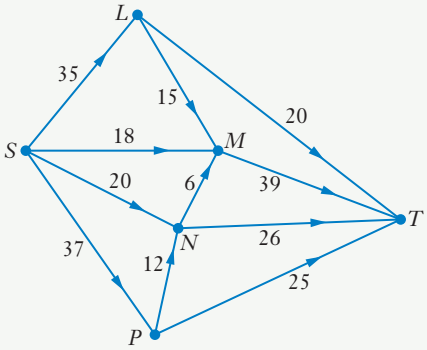
ii



6 a 110 000 L/min

(*SLT* – 20, *SNT* – 20, *SPT* – 25, *SMT* – 18, *SPNMT* – 6, *SLMT* – 15, *SPNT* – 6)

b

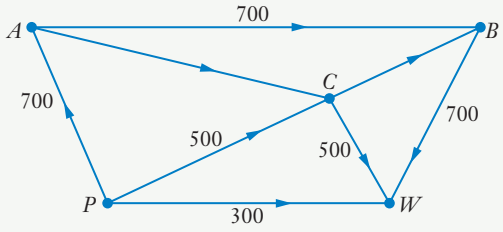


c *L* – 35, *M* – 39, *N* – 32, *P* – 37 (in thousands of litres/min)

d Maximum outflow at *S* is 113 000 L/min. so increase capacity of *PN* by 3000 L/min. The town will then receive 113 000 L/min.

7 a Maximum flow from *P* to *W* is 1500 cars (*PABW* – 700, *PW* – 300, *PCW* – 500)

b



c Capacity of *CW* should be increased by 500 cars/hour so that capacity of traffic flow is increased to 2000 cars/hour (inflow at *W* will be the same as the outflow at *P*).

8 $n + 22 = 12 + 15$, $n = 5$ (inflow = outflow)

$24 + n + 25 + r = 90$, $r = 36$

$27 + q = r$, $q = 9$

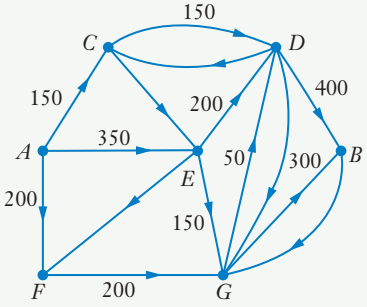
$p + 22 = q + 25$, $p = 12$

$m + 18 = p + 15$, $m = 9$

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9 a 700 cars/hour

(*AEDB* – 200, *ACDB* – 150, *AFGB* – 200, *AEGB* – 100, *AEGDB* – 50)



b Widen road *ED* to increase capacity to 300 cars/h (or increase capacity of *GD* to 150 cars/h). This will maximise traffic flow along road *DB* to 500 cars/h and increase maximum flow to 800 cars/h.

10 a Cut 1 = 37 Cut 2 = 33 Cut 3 = 37

b Cut 1 = 25 Cut 2 = 38 Cut 3 = 35
Cut 4 = 14

c Cut 1 = 54 Cut 2 = 34 Cut 3 = 35
Cut 4 = 17

d Cut 1 = 350 Cut 2 = 300 Cut 3 = 360
Cut 4 = 460

11 a A **b** C

12 a Cut 1 = 45 Cut 2 = 34

a Cut 1 = 33 Cut 2 = 22

13 a Cut 1 as it doesn't separate the source from the sink

b Cut 2 = 35 Cut 3 = 41 Cut 4 = 35

14 C

Exercise 6.08

1 a Maximum flow = 11 (*ABCD* – 4, *AFED* – 5, *ABED* – 2)

b Cut 1 = 17 Cut 2 = 11 Cut 3 = 19
Cut 4 = 12 Cut 5 = 22

c Cut 2 = 11 = Maximum flow

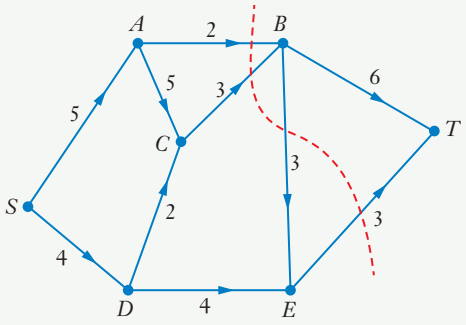
2 a Maximum flow = 12 (*SAT* – 6, *SBT* – 4, *SBAT* – 2)

b Cut 1 = 22 Cut 2 = 15 Cut 3 = 15
Cut 4 = 27 Cut 5 = 12
Minimum cut = cut 5 = 12

c Maximum flow = minimum cut = cut 5

3 a i Maximum flow = 8 (*SABT* – 2, *SACBT* – 3, *SDET* – 3)

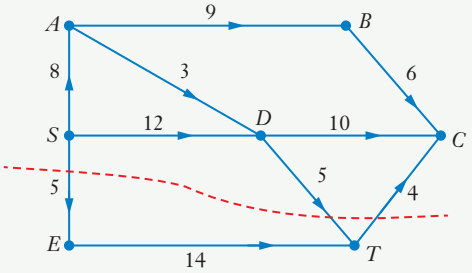
ii



Minimum cut = 2 + 3 +3 = 8 = maximum flow

b i Maximum flow = 14 (*SDT* – 5, *SET* – 5, *SABCT* – 4)

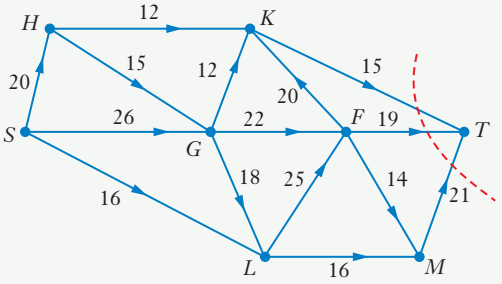
ii



Minimum cut = 5 + 5 + 4 = 14 = maximum flow

c i Maximum flow = 55 (*SHKT* – 12, *SLMT* – 16, *SGFMT* – 5, *SHGFT* – 5, *SGFT* – 12, *SGLMT* – 2)

ii

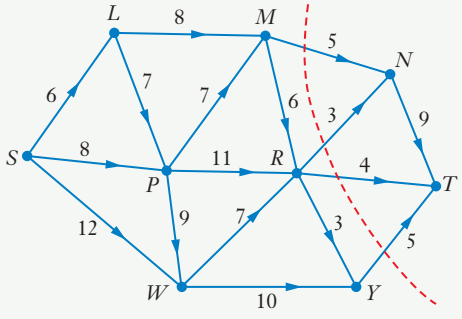


Minimum cut = 15 + 19 + 21 = 55
= maximum flow

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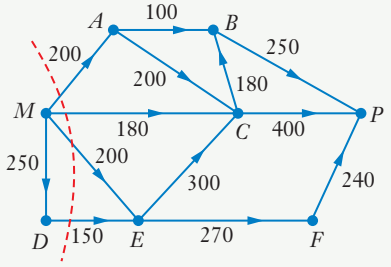
d **i** Maximum flow = 17 (*SLMNT* – 5, *SPRT* – 4, *SWYT* – 5, *SPRNT* – 3)

ii



Minimum cut = 5 + 3 + 4 + 5 = 17
= maximum flow

4 **a**



Minimum cut = 200 + 180 + 200 + 150 = 730

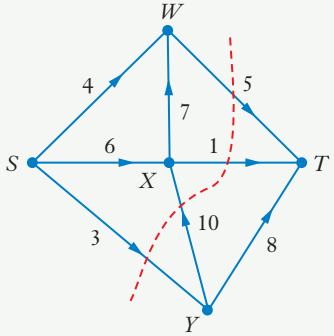
b Max. flow = 730 cars/h = min. cut
(*MACP* – 200, *MCP* – 180, *MECBP* – 180, *MECP* – 20, *MDEFP* – 150)

c 350 cars/h

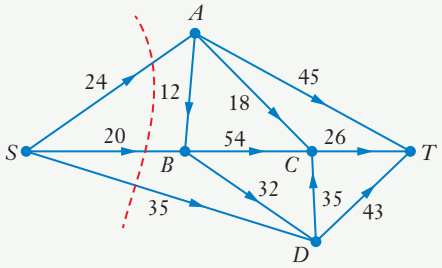
d 220 cars/h

e Increase capacity of *DE* to 240. This will increase the flow to 820 cars/h, which is close to the outflow of 830 cars/h at *M*.

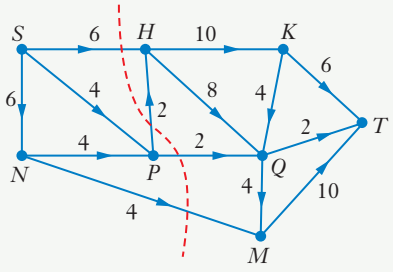
5 **a** Max. flow = min cut = 5 + 1 + 3 = 9



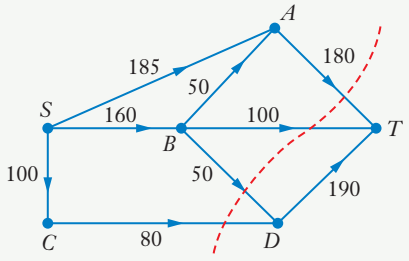
b Max. flow = min cut = 24 + 20 + 35 = 79



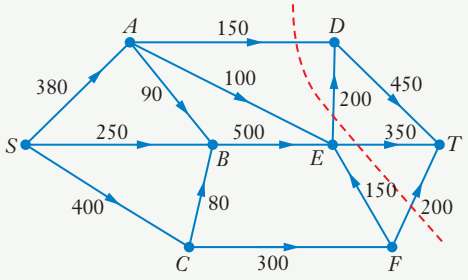
c Max. flow = min. cut = 6 + 2 + 2 + 4 = 14



d Max. flow = min. cut = 180 + 100 + 50 + 80 = 410

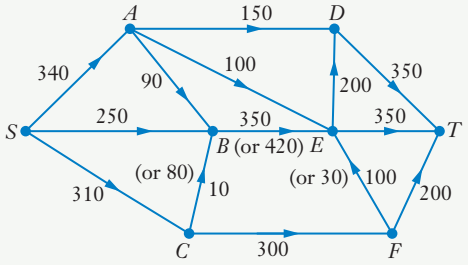


6 **a**



Max. flow = min. cut = 900 L/min

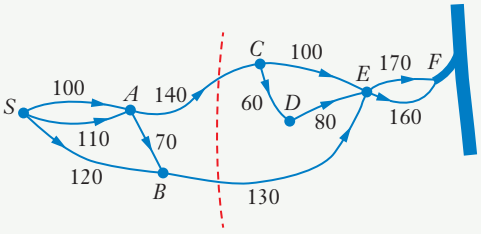
b (alternative solution shown in brackets)



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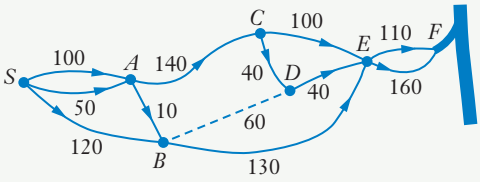
c An extra 90 L/min can still come from pipe *SC*, but pipe *CB* can only take 70 L/min. Pipe *BC* can carry the extra 70 L, but pipe *ET* cannot. So increase capacity of *ET* by 70 L and maximum flow will increase to 970 L/min, which is close to the outflow of 1030 L/min at *S*.

7 a



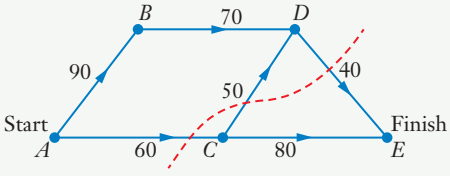
Max. flow = min. cut = 140 + 130
= 270 vehicles/h

b Network diagram shows traffic flow for 270 vehicles/h.



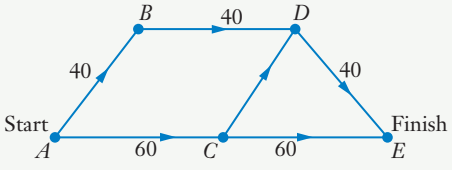
The outflow at *S* is 330 so build a road from *B* to *D* with a capacity of 60 and increase the capacity of *DE* to 100. This will give a traffic flow of 330 through to the freeway.

8 a i, ii



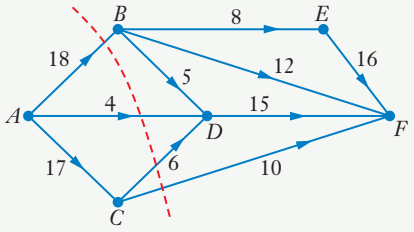
Flow capacity = min. cut = 40 + 60 = 100

iii



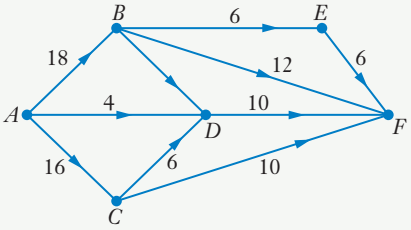
iv Inflow at *E* has a capacity of 120, so flow of 100 is not sufficient.

b i, ii



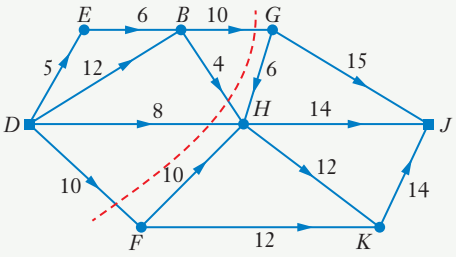
Flow capacity = min. cut = 18 + 4 + 6 + 10 = 38

iii



iv Inflow at *F* has a capacity of 53, but outflow at *A* is 39, while max. flow is 38, so the flow is nearly at its maximum.

9 a



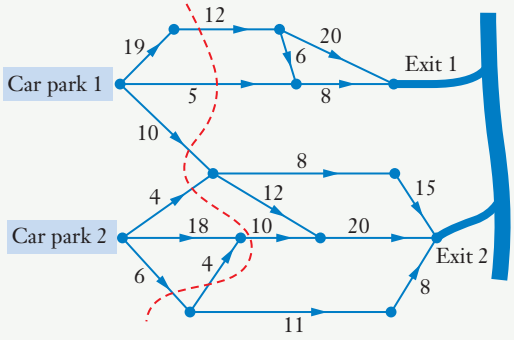
Maximum flow = 3200 vehicles/h

b Widen *DF* or *DH* to increase their capacity by 6 which will increase max flow to 3800 vehicles/h.

c If traffic flow of *GH* is reversed, widen *DH* to carry 1900 vehicles so maximum flow will be 4300 vehicles/h, which is the inflow at *J*.

10 a i 34 cars/min **ii** 28 cars/min

b



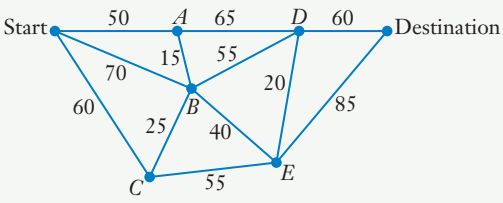
Max. flow = 12 + 5 + 10 + 4 + 10 + 6 = 47 cars/min

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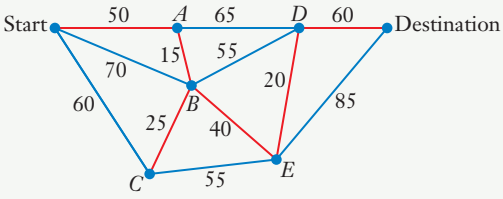
c Increase the flow from:
Car park 1 by increasing flow from 12 to 19
Car park 2 by increasing the flow from 10 to 18 and from 20 to 24
All cars then leaving the car parks will be able to get to the exits without blockages as max. flow = 62 = outflow at carparks.

Sample HSC problem

a



b



c Shortest path is Start-*AD*-Destination.

d No

Test yourself 6

1 **a** **i** $a-2, b-3, c-3, d-2, e-4$

ii number of edges = 7, sum of vertices = $14 = 2 \times 7$

b **i** $A-4, B-3, C-4, D-3, E-4$

ii number of edges = 9, sum of vertices = $18 = 2 \times 9$

2 **a** path **b** trail

c neither, *ab* is repeated

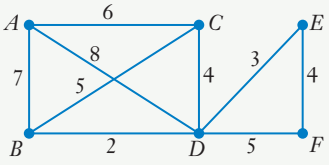
d trail

3 **a** **d** is closed

b **d** is a circuit

c none are cycles

4



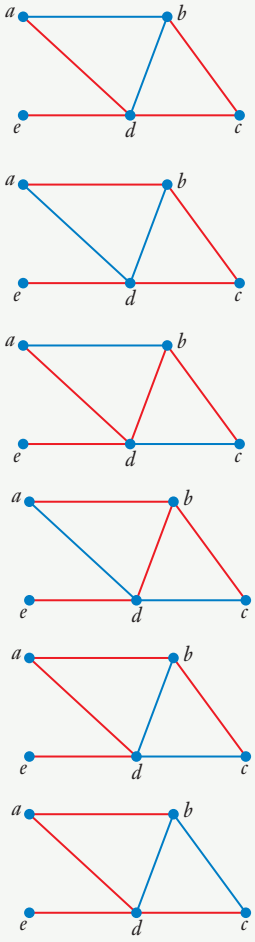
5 **a** *GFEDCBAGBE*

b No, not all vertices are even.

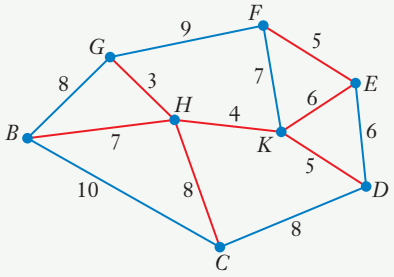
6 **a** *N*

b *RLMNQPNMLQR* 45 km

7



8 Order of edges is: *GH*, *HK*, *FE*, *KD*, *KE*, *BH*, *HC*
weight = 38

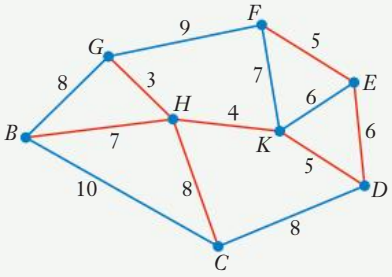


(There is more than one than one spanning tree.)

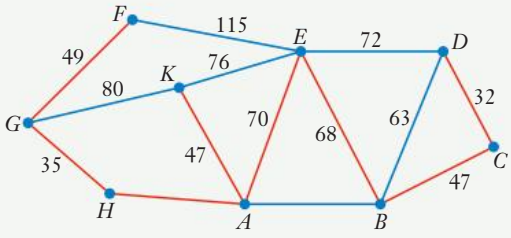
ISBN 9780170413633

9 Order of edges is: *CH*, *HG*, *HK*, *KD*, *DE*, *FE*, *HB*
weight = 38

(The Order of edges depends on starting vertex.)



10 a

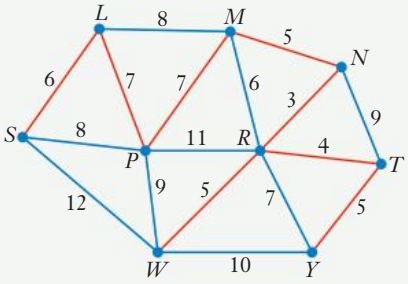


Order of edges is: *EB*, *BC*, *CD*, *EA*, *AK*, *AH*, *HG*, *GF*

b No

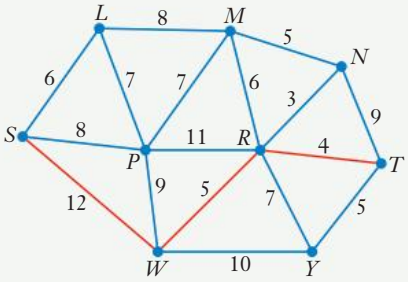
11 $m = 4, p = 2$

12 a



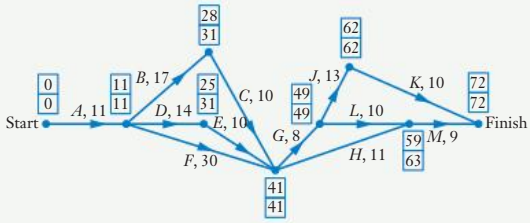
Minimum spanning tree length = 4200 m

b



Shortest path from *S* to *T* is not contained in minimum spanning tree.

13 a

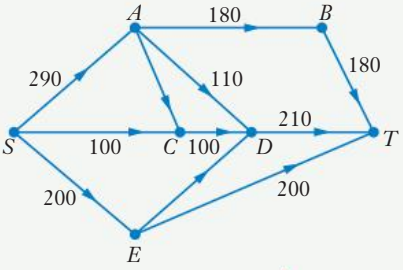


Critical path is *AFGJK*. Minimum time 72 days.

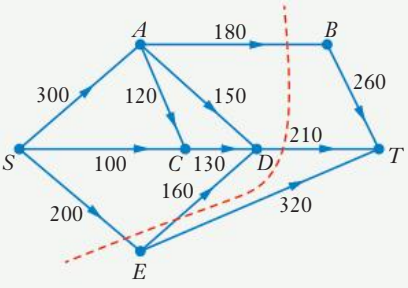
b *C* – 3 days *E* – 6 days *M* – 4 days
B – 3 days *D* – 6 days *H* – 11 days
L – 4 days

14 a Maximum flow = 590 (*SET* – 200, *SCDT* – 100, *SABT* – 180, *SADT* – 110)

b



c



Minimum cut = 180 + 210 + 200 = 590
= maximum flow

d Outflow from *S* is 600 L/min so there is only an excess of 10 L/min. The capacity of *DT* (or *SE* or *AB*) would have to be increased by 10 L/min to achieve a maximum flow of 600 L/min.

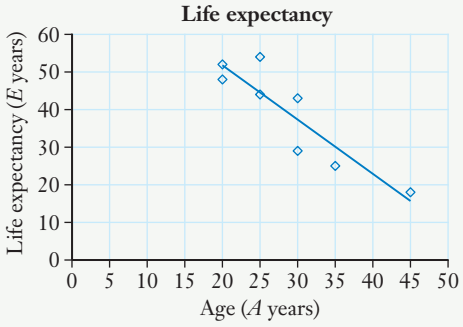
Practice paper 2

1 C **2** A **3** B
4 D **5** B
6 D **7** C **8** B
9 A **10** D

11 a **i** $\$69\,399.53 \times \frac{0.11}{12}$ **ii** $\$69\,195.69$
iii $\$2555.69$
iv $\$68\,989.98$, $\$632.41$, $\$69\,622.39$, $\$68\,782.39$

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b i, ii



iii Teacher to check, approximately 43.

iv Teacher to check, approximately $E = -1.4A + 80$.

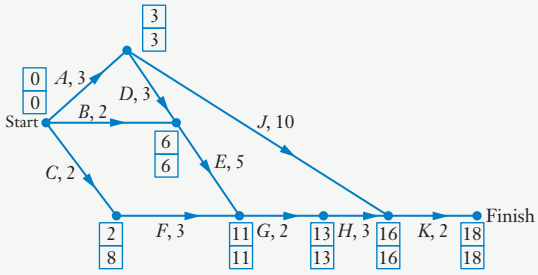
c i *BEFJHK*, 24 weeks

ii 8 weeks

iii *A*, *D* and *G*

iv *F* can be reduced by 5 weeks, resulting in completion time of 19 weeks.

12 a i



ii 18 weeks

b $277.52

c i Cut 1 – 37, Cut 2 – 53, Cut 3 – 50;
Maximum flow = minimum cut = 37 cars/min

ii 61 cars/min

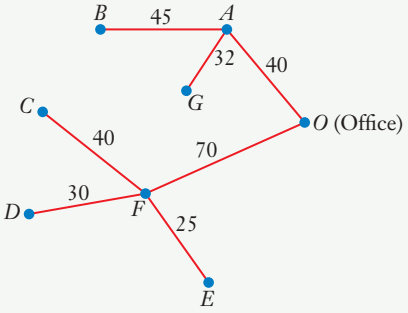
iii The inflow at *A* is 33 cars/min while there is only an outflow of 6 cars/min.
Need to increase capacity of roads from *AB*, *BC* and *CD* to at least 33 cars/min each

13 a i $96 048.80 **ii** $64 887 **iii** $9462

b i

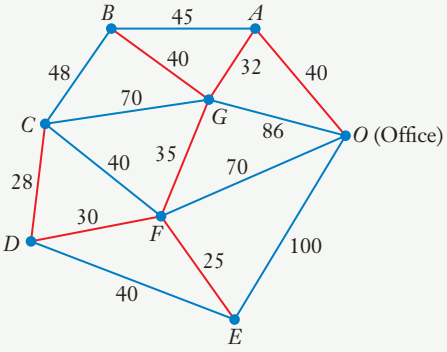
Cabin	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Path	*OA*	*OAB*	*OFC*	*OFD*	*OFE*	*OF*	*OAG*
Weight	40	85	110	100	95	70	72

ii



Weight = 40 + 45 + 40 + 32 + 70 + 30 + 25
= 282 m

iii



Weight = 40 + 32 + 40 + 35 + 30 + 28 + 25
= 230 m

iv $32 200

c i 0.9545 **ii** $y = 0.908x + 15.994$

Chapter 7

SkillCheck

1 a 0.92 beats/s **b** 1650 beats

2 127 c/L

3 a 92 mm **b** 8.6 m **c** 42 L
d 3.16 kL **e** 7500 L **f** 13 min

4 $552.64

5 91 days

6 88.5 c/day

Exercise 7.01

1 123 beats/min

2 B

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3

		Number of heart beats	Time in seconds	Pulse in beats/minute
a	Julia	66	30	132
b	Rodney	30	10	180
c	Amir	45	25	108
d	Serena	39	15	156

4 **a** 174 **b** 186 **c** 202
5 **a** 24 **b** 5
6 **a** 450 min **b** 25 200 beats
7 3330 beats
8 2940 beats
9 D
10 **a** 169.5–189.75
b 4.875 L **c** 160 mL/beat

Exercise 7.02

1 **a** 3 170 000 J **b** 62 000 kJ **c** 3514.56 kJ
d 1.795 MJ **e** 83.65 Cal **f** 262.9 Cal
2 414.216 kJ
3 **a** 3162 kJ **b** 527 kJ **c** 125.96 Cal
4 **a** 14.7 MJ **b** 3513.38 Cal
5 **a** 459 kJ **b** 339 Cal
6 **a** 2797 kJ **b** 32.1%
7 **a** 2186.15 kJ **b** 24%
8 **a** Running/jogging and swimming
b **i** 55 min
ii 24 min
c 67 min, 29 min; Lara needs to exercise longer than Alex.

Exercise 7.03

1 **a** $\frac{3}{50}$ **b** Standby power
c $214.02 **d** $1507.50 **e** 61°
2 **a** $\frac{1}{4}$ **b** 4%
c **i** 350 kWh **ii** 800 kWh
3 **a** 6.5×10^6 MW **b** 4×10^6 kW
c 3.375×10^9 GW
4 D
5 **a** 3.4 kW **b** 38 500 MW
c 3.945 kW
6 C
7 **a** 3250 MW
b 3 250 000 kW or 3.25×10^6 kW
c 3 250 000 000 W or 3.25×10^9 W
8 **a** Quarterly **b** 18.3 kWh
c Increased by $464.08 **d** 1683 kWh
e 92
f 19 May 2019, 60 872 kWh
g Yes, electricity usage for this family lies between typical usage shown on the bill for between 2 and 4 people per household.
9 **a** 1161.84 kWh
b **i** $184.29 **ii** $763.97
c $8.40
10 $695.43

Exercise 7.04

1 **a** 10 kWh **b** 8 kWh
c 0.35 kWh **d** 4.5 kWh
2 D
3 2.6 kWh
4 **a** 16.25 kWh **b** $5.34
5 $4.19
6 $82.05
7 **a** $960.66 **b** $192.13
c $153.71 **d** $512.35
8 $126.01 difference
9 $119.99
10 **a** $477.32 **b** $190.93, $286.39
11 **a** $1695.77 **b** $1618.05
12 **a** $11.18 **b** $27.95
13 **a** 657 kWh **b** $209.60

Exercise 7.05

1 B, Most windows face north.
2 **a** 1 cm : 4 m or 1 : 400
b 190 m^2
c 57 m^2
d The family room/garage side
3 **a** North
b West
c No, as most windows face west.
d There is some cross-ventilation from kitchen through to the family room.

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4 Roof insulation is a material (such as batts, foil backed blankets, loose fill etc.) placed in the ceiling to reduce temperature flow. This will reduce heat coming in during summer and reduce heat going out in winter.

5 **a** **i** 76° **ii** 24°

b Yes, as they don't allow for much summer sun to come through and do allow winter sun to come through.

6 **a** Plant tall trees on the south and south-west to provide shade in summer and provide wind break in cooler months.

b Don't plant tall trees on the north or north-east side of the house because they block the sun in winter when sun is low.

7 **a** 14° **b** 482 mm

c Extend the eaves of the house on that side.

8 **a** 20.8 m^2 **b** 80 W **c** 3.85 W/m^2

d Yes, the lamp power density is less than 5 W/m^2.

9 **a** 9.6 W/m^2 **b** 7.2 W/m^2

c 4.3 W/m^2; The five LED lights comply.

10 **a** Solar hot water system with electric booster with a 5-star rating

b Bathroom, kitchen, laundry

c Location of windows and doors to allow cross-ventilation

d Fluorescent lights or LEDs

e Artificial lighting (such as LEDs) uses electricity; natural lighting allows sunlight to come through windows and skylights.

f A skylight is a 'window' in the ceiling; bathroom, toilet, kitchen; can also be installed in foyers or hallways.

Sample HSC problem

$62.97

Test yourself 7

1 3700 beats

2 **a** 194 **b** 106.7–164.9 beats/min

3 **a** 5230 J **b** 18 000 kJ **c** 38 492.8 kJ

d 0.1105 MJ **e** 174.5 Cal **f** 76.48 Cal

4 **a** 4466 kJ **b** 1067.4 Cal

5 **a** 15.3 MW **b** 5 120 000 kW **c** 39 kW

6 $980.67

7 It is colder in Canberra in winter and hotter in summer. (Sydney is on the coast of NSW and Canberra is inland.)

8 C

9 $122.27

10 $92.82

11 **a** Batts/foil-backed blankets in the ceiling to stop heat coming in during summer, keeping the house cooler.

b To reduce heat from sun entering the house in summer due to high elevation of sun and to increase heat coming into the house from the sun in winter when elevation of sun is lowest.

Chapter 8

SkillCheck

1 **a** 60° **b** 75° **c** 86°

d 36° **e** 110°

2 **a** 37°9′ **b** 48°43′

c 125°41′ **d** 4°32′

3 **a** 7.90 **b** 10.61 **c** 108.06

d 11.79 **e** –0.43 **f** 13.85

4 **a** 4.60 **b** 261.67

5 **a** 63°, angle sum of a triangle

b 132°, angles on a straight line

Exercise 8.01

1 **a** 8.30 m **b** 25.74 cm **c** 3.35 m

d 92.91 mm **e** 0.74 m **f** 21.36 cm

g 6.53 mm **h** 2.05 m **i** 49.20 m

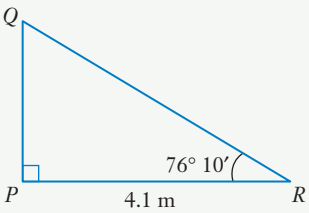
2 **a** 36° **b** 42° **c** 66°

d 56° **e** 35° **f** 8°

3 **a** 74°53′ **b** 32°22′ **c** 48°1′

4 D

5 **a**



b 13°50′ **c** $PQ = 16.65$ m $QR = 17.15$ m

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6 D

7 **a** 5.5 m **b** 1.8 m

8 78°11′ **9** 31.5 m

10 21.7 km **11** 88.9 km

12 B **13** 223 m

14 2.3 m **15** 208 m

16 $\tan 63° = \frac{VT}{145}$, $VT = 145 \tan 63°$; $\tan 49° = \frac{VW}{145}$, $VW = 145 \tan 49°$

$d = VT - VW = 145 \tan 63° - 145 \tan 49°$

$d = 145(\tan 63° - \tan 49°)$

17 19.72 m **18** 4.9°

Exercise 8.02

1 **a** NW **b** SW **c** SE

d NE **e** SW **f** SE

2 **a** 237°; S 57° W

b 059° ; N 59° E

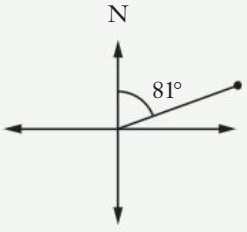
c 294°; N 66° W

d 130°; S 50° E

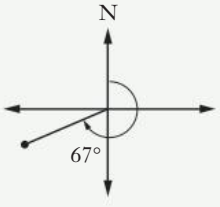
e 196°46′; S 16°46′ W

f 317°45′; N 42°15′ W

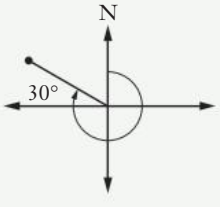
3 **a**



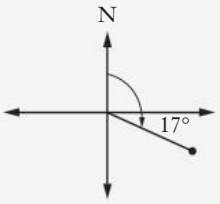
b



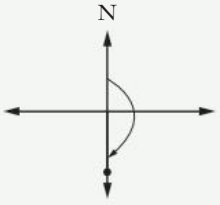
c



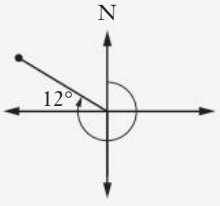
d



e



f



4 **a** 315° **b** 225° **c** 045°

5 **a** 097° **b** 072° **c** 143°

6 C

7 **a** 2.9 km **b** 6.6 km **c** 024°

8 A

9 **a** 14.5 km **b** 18 min

10 **a** 2122 km **b** N 30° W

11 **a** 186° **b** S 6° W

12 **a** 117° **b** S 63° E

13 **a** 024° **b** S 52° E

14 281°

Exercise 8.03

1 **a** 11.39 m **b** 7.47 cm **c** 4.07 m

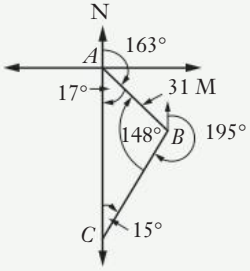
d 9.58 cm **e** 18.69 m **f** 78.46 mm

2 **a** 15.9 km **b** 3.4 km

3 13.9 cm

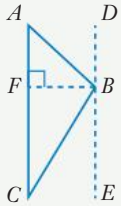
4 101 m

5 **a**



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b



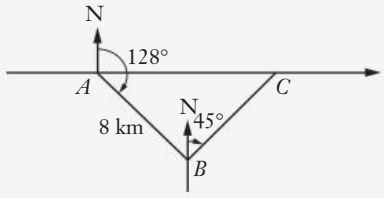
$\angle A = 180° - 163° = 17°$
(angles on a straight line)
$\angle ABF = 180° - 90° - 17°$
$= 73°$
(angle sum of a triangle)
$\angle FBC = 270° - 195° = 75°$
$\angle ABC = 73° + 75°$
$= 148°$

c 63 M

6 **a** 357 m **b** 383 m

c 350 m

7 **a**



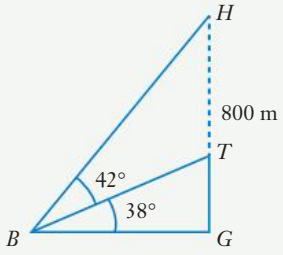
b $\angle A = 38, \angle B = 97, \angle C = 45$

c 11.2 km

8 A

9 7.2 km

10 **a**



b $\angle BTH = 38° + 90°$
$= 128°$ (exterior angle of $\triangle TGB$)
or $\angle BTG = 180° - 90° - 38°$
$= 52°$ (angle sum of $\triangle TGB$)
$\angle BTH = 180° - 52°$
$= 128°$ (angles on a straight line)

c 942 m **d** 208 m **e** 128 m

Exercise 8.04

1 **a** 41°50′ **b** 9°28′ **c** 77°34′
d 28°43′ **e** 32°5′ **f** 64°21′
g 35°6′ **h** 63°29′ **i** 20°57′

2 **a** 120° **b** 111° **c** 131°

3 **a** 47°22′ **b** 132°38′

4 A

5 64°

6 239°

7 Teacher to check

8 125°

9 C

10 43°

Exercise 8.05

1 **a** 5.57 m **b** 6.02 cm **c** 7.59 m
d 8.65 m **e** 5.17 m **f** 27.21 mm

2 0.6 m

3 22.4 km

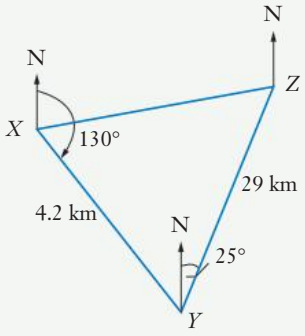
4 **a** $360 - 230 = 130°$

b 59 km

5 9300 m

6 C

7 **a**

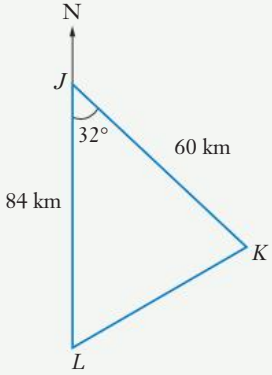


b $180° - 130° = 50°$
(co-interior angles between parallel lines)
$50° + 25° = 75°$

c 4.4 km

8 **a** 360° **b** 72° **c** 47.02 cm

9 **a**



b 46 km

10 **a** 3.14 cm **b** 3.13 cm

c Arc length AB by 0.01 cm

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Exercise 8.06

1 **a** 26° **b** 52° **c** 123°
d 36° **e** 91° **f** 48°

2 **a** 98° **b** 41°
c Isosceles **d** 41°

3 B

4 9°

5 **a** 24°45′ **b** 77°38′

6 28°4′

7 8°28′

8 29°

9 B

10 Smallest angle opposite smallest side (length 5)
33°33′

Exercise 8.07

1 **a** 8.04 m^2 **b** 24.07 cm^2 **c** 19.86 m^2
d 13.02 m^2 **e** 24.20 cm^2 **f** 85.88 mm^2

2 27.9 cm^2

3 D

4 152.98 m^2

5 **a** 6 cm^2 **b** 6 cm^2

6 **a** 60° **b** 64.95 cm^2

c Each triangle has two equal radius sides of 5 cm and an included angle of 60°.

As the triangle is isosceles, the other two unknown angles must be equal.

$$\text{Unknown angle} = \frac{180° - 60°}{2} = \frac{120°}{2} = 60°.$$

All angles are 60°, therefore the triangle is equilateral (all sides equal).

Therefore the side with the dash must also be 5 cm.

Therefore the sides of the hexagon are 5 cm.

7 21.22 cm^2

8 21.22 cm^2

9 **a** 70°25′ or 56°52′ or 52°43′
b 7.2 m^2

10 **a** 55.85 cm^2 **b** 31.51 cm^2 **c** 24.34 cm^2

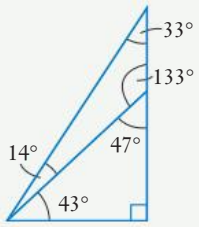
Exercise 8.08

1 **a** 7.11 m **b** 17.85 cm **c** 44.30 mm
d 11.38 cm **e** 11.71 m **f** 50.67 mm

2 **a** 40° **b** 51° **c** 92°
d 43° **e** 47° **f** 89°

3 B

4 **a** **b** 180 m



5 15 m

6 153°51′

7 1420 km

8 **a** $\angle IFH + 42° = 61°$ (exterior angle of $\triangle FIH$)

$\angle IFH = 19°$

or $\angle FHI = 180° - 61°$

$= 119°$ (angles on a straight line)

$\angle IFH = 180°$

$= 119° - 42°$

$= 19°$ (angle sum of $\triangle FIH$)

b $$\frac{FH}{\sin 42°} = \frac{120}{\sin 19°}$$

$$FH = \frac{120 \sin 42°}{\sin 19°}$$

c $$\sin 61° = \frac{h}{FH}$$

$h = FH \times \sin 61°$

$$= \frac{120 \sin 42°}{\sin 19°} \times \sin 61°$$

$h = 216\,\text{m}$

9 **a** $\angle RHA = 180° - 60° = 120°$

$\angle RAH = 180° - 44° - 120° = 16°$

$$\frac{AH}{\sin 44°} = \frac{85}{\sin 16°}$$

$$AH = \frac{85 \sin 44°}{\sin 16°}$$

b 190 m

10 B

11 **a** N 60° E **b** 15 km **c** S 86° W

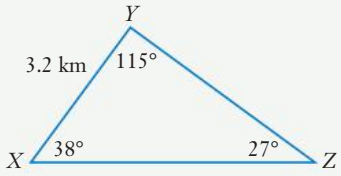
12 **a** 142° **b** 38.8 km **c** 232°

13 $AC = 46.27$ m $CB = 71.58$ m

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Sample HSC problem

a



b $YZ = 4.3$ km and $XZ = 6.4$ km. Course length is 13.9 km.

c 114 min

d 6 km^2

Test yourself 8

1 384 m **2** A

3 3.95 km **4** 72.1 m

5 **a** 57°7′ **b** 61°36′

6 15.7 km **7** A

8 35°32′

9 **a** 340 km **b** 14004 km^2

10 104°

11 **a** 252 m **b** 6675 m^2

Practice paper 3

1 D **2** C **3** B **4** B **5** A

6 B **7** D **8** B **9** B **10** D

11 **a** \$4.14 **b** 128°41′ **c** 10.9 m^2

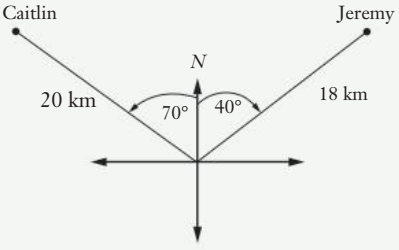
d **i** 800 kJ **ii** 191.2 Cal

12 **a** **i** 201 beats/min

ii 182 beats/min

b **i** \$195.92 **ii** \$65.31

c **i**



ii 31.1 km

13 **a** 2175 kJ **b** 0.48 GW **c** \$808.28

d **i** $\angle BAL = 58° - 41° = 17°$; $58° = 41° + \angle BAL$ (exterior angle of $\triangle BAL$)

ii $\frac{AL}{\sin 41°} = \frac{150}{\sin 17°}$ **iii** 285

Chapter 9

SkillCheck

1 **a** $\frac{2}{3}$ **b** 3 **c** 3 : 7

d 3 : 2 **e** 6 : 5 **f** 15 : 4

g 10:1 **h** 10:1 **i** 1 : 200

2 682.5 m^2

3 **a** 19 m^2 **b** 360 m^2

c 370 m^2 **d** 310 m^2

4 **a** 8.9 m **b** 37 m

c 44 m **d** 30 m

Exercise 9.01

1 **a** 4 m **b** 5 mm **c** 270 m

2 **a** 1 : 100000 **b** 1 : 5 **c** 1 : 500

d 1 : 10 **e** 250 : 1

3 **a** part **b** **b** part **e**

4 **a** 1.8 m **b** 95 mm

5 B

6 175 cm

7 5.6 cm

8 2.75 cm

Exercise 9.02

1 **a** 5.5 cm **b** 55 m

c 92 m **d** 5060 m^2

2 **a** 20 km **b** 13.2 km **c** 11.6 km

3 **a** 1 cm : 40 km = 1 : 4000000

b **i** 84 km **ii** 244 km

iii 216 km **iv** 120 km

c Mittagong

d Mittagong and Wollongong ; Katoomba and Penrith

4 Approximate answers:

a 1 : 6406 **b** 1 : 69474 **c** 1 : 30413

5 **a** **i** 150 m **ii** 1363 m^2

b **i** 1030 m **ii** 63450 m^2

c **i** 2378 m **ii** 256667 m^2

6 Teacher to check
Answers should be approximately:

a 2.5 km **b** 1149 m

c 190 m **d** 1416 m

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Exercise 9.03

1 **a** 2 m **b** 6 m **c** 36 m^2
d 52.5 m^2 to paint; 6 L paint needed
2 **a** 6 m **b** 8 m
c 48 m^2 **d** \$6024
3 **a** 112 m^2 **b** \$900 **c** \$700
4 **a** 4 **b** walk-in robe
c 1 **d** kitchen **e** 28 m^2
5 **a** 7
b **i**  **ii**  **iii** 
iv  **v**
c 15.5 m^2 **d** 168 **e** \$5800
6 **a** 1 : 100 **b** 13.5 m
c 12 windows **d** 2 doors **e** 7 m^2
f 2400 mm (2.4 m) **g** \$1000
7 **a** 3.6 m × 3.6 m
b **i** 3.6 m **ii** 10.8 m^2
c 17 m^2 **d** 135 m^2 **e** \$ 195 750
f **i** 14 **ii** 196

Exercise 9.04

1 9055 m^2
2 **a** 1608 m^2 **b** 80.7 m
3 **a** $WOX = 79°$, $XOY = 77°$, $YOZ = 57°$, $ZOW = 147°$
b 6500 m^2 **c** 84 m
d $XY = 74$ m, $YZ = 58$ m, $WZ = 128$ m, perimeter = 344 m
4 **a, b** 2700 m^2
5 **a** 121° **b** 5424 m^2
6 **a** 120 m **b** 5400 m^2
7 5160 m^2
8 **a** We used three different surveying methods with different accuracies.
b Compass radial survey as it uses more technical equipment
c The trapezoidal rule is least accurate as we are using trapeziums to approximate an area that is not exactly a trapezium.

Exercise 9.05

1 **a** 150.00 kL **b** 13.82 kL **c** 2.15 kL
2 905 L
3 B
4 **a** 7500 L **b** 14 days
5 **a** 1800 L **b** 1.6 m **c** 75.4 mm
6 **a** 15.9 m^2
b 25.44 m^3 = 25.44 kL = 25 440 L
7 **a** 22 m^2 **b** 35 200 L

Sample HSC problem

a 103° **b** 97 m **c** 1871 m^2

Test yourself 9

1 A
2 17 m^2
3 **a** 5.6 m by 6.4 m **b** 15 m^2
c \$1200
4 **a** 1 : 199 **b** 35 m^2
c 2.56 m^2 **d** 62 m^2
5 **a** **i**  **ii**  **iii** 
b 41 m^2 **c** 216
6 **a** 430 m^2 **b** 960 m^2
7 **a** 78° **b** 38 m
c 129° **d** 354 m^2
8 **a** 2800 m^2
b Perimeter = (24 + 39 + 39 + 59 + 50) m = 211 m
9 **a** 8.1 m^3 **b** 8100 L

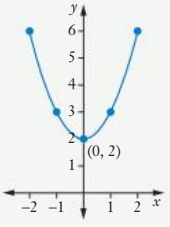
Chapter 10

SkillCheck

1 **a** 7.5295 **b** 0.0016 **c** 6527.6373
2 **a** $\frac{5}{8}$ or 0.625 **b** 512
3 **a** 23 **b** 108 **c** 125
d 0 **e** –2 **f** 1.536
4 **a**

x	–2	–1	0	1	2
y	6	3	2	3	6

b



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5 **a** 63.8 m **b** 14 m/s

6 **a** 145.8 **b** 18

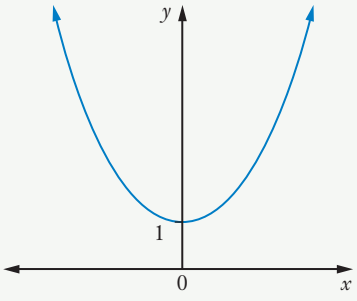
7 **a** 60 **b** 8

Exercise 10.01

1 **a** C **b** A **c** F

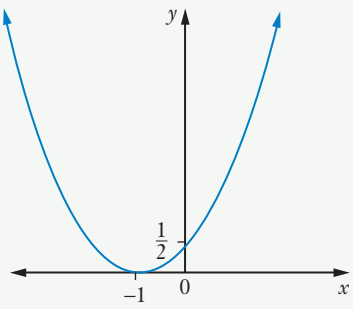
d E **e** B **f** D

2 **a**



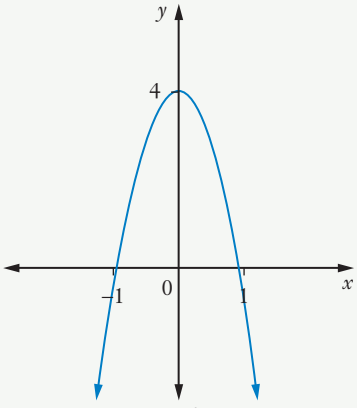
$y = 2x^2 + 1$

b



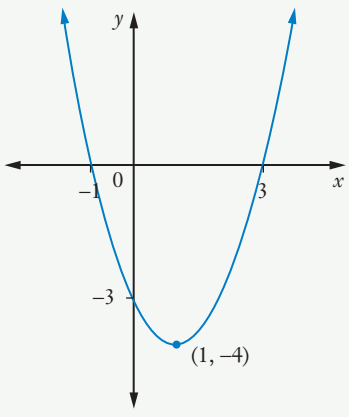
$y = \frac{1}{2}x^2 + x + \frac{1}{2}$

c



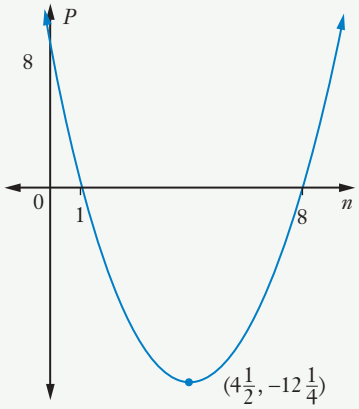
$y = -4x^2 + 4$

d



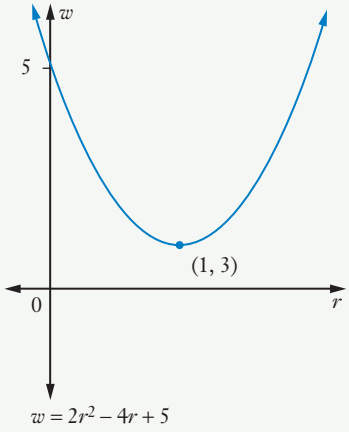
$y = x^2 - 2x - 3$

e



$P = n^2 - 9n + 8$

f

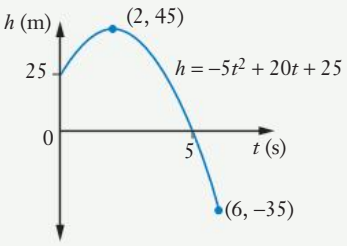


$w = 2r^2 - 4r + 5$

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3 **a** t

b



c 25, height of balcony in metres

d 45 m, 2 s **e** 5 s

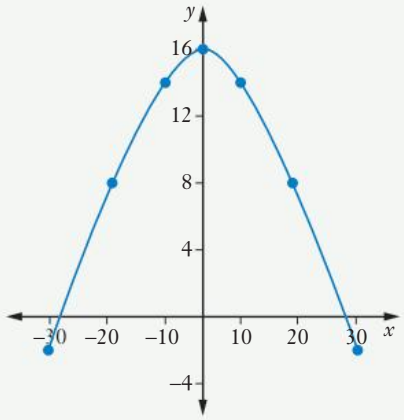
4 **a** **i** 134.16 m **ii** 74.24 m

b 22 km/h

5 **a**

x	−30	−20	−10	0	10	20	30
y	−2	8	14	16	14	8	−2

b



c $x \approx 28, -28$ **d** (0, 16)

e 16 m **f** 56 m

Exercise 10.02

1 C

2 **a** h **b** 28 m

c Going up at 1.8 s, then coming down at 3 s

d 2.4 s (halfway between 1.8 s and 3 s)

e 28.8 m

f h is negative and height cannot be negative.

3 A

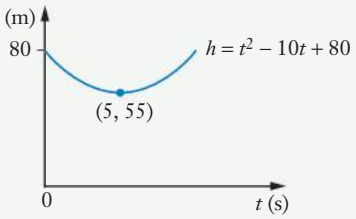
4 **a** Perimeter $= x + y + y = 18$ so $x + 2y = 18$

b Since $2y = 18 - x$, $y = 9 - \frac{1}{2}x$

c $A = xy = x(9 - \frac{1}{2}x) = -\frac{1}{2}x^2 + 9x$

d 40.5 m^2, 9 m by 4.5 m

5 **a**



b (5, 55)

c 55 m, the lowest height of the plane during the dive

d 5 s **e** 80 m

6 **a** s

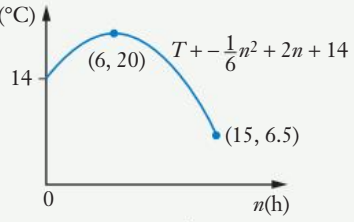
b

s	20	30	40	50	60	70	80
C	17	12	9	8	9	12	17

c 50 km/h **d** 8 L/100 km

e When $s = 0$, $C = 33$. At zero speed, the car is not moving so there is no petrol consumption.

7 **a**



b **i** 14°C **ii** $19\frac{1}{3}$°C

c 20°C at 12 noon.

d 9:30 a.m. and 2:30 p.m.; temperature rising at 9:30 a.m., temperature falling at 2:30 p.m.

e 5 hours

f Temperature will not continue to fall indefinitely after 13 hours but will eventually rise (and fall) again.

Exercise 10.03

1 B **2** A **3** B

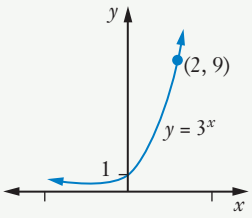
4 **a** C **b** B **c** D

d A **e** E

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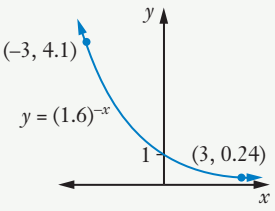
5 **a** $y = 3^x$

x	−3	−2	−1	0	1	2	3
y	0.04	0.11	0.33	1	3	9	27



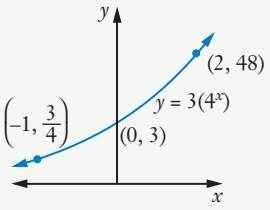
b y-intercept is 1. The asymptote is the x-axis.

6 **a**



b (0, 1) **c** decreasing

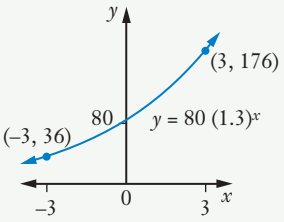
7 **a** $y = 3(4)^x$



b 3

c Because y is always greater than 0, the x-axis is an asymptote.

8 **a** $y = 80(1.3)^x$



b 80 **c** increasing

9 B

10 **a** **i** 3.2 mm **ii** 409.6 mm

b **i** 12 **ii** 14

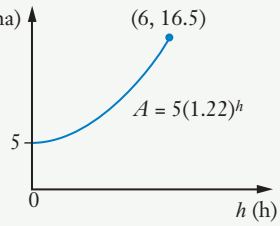
c You cannot keep folding a sheet of paper in half—usually you can only fold it about 7 times.

Exercise 10.04

1 D

2 **a** 5 ha **b** 10 ha **c** 7 h

d



e **i** 3.5 h **ii** 15 ha

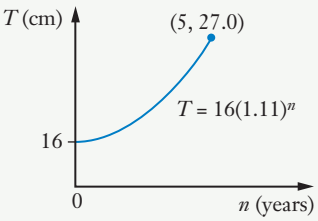
f The model assumes that the area burnt will continue growing quickly and never stop.

3 **a** T **b** **i** 30 cm **ii** 16 cm

c 4 years

d $T = 16(1.11)^n$

n	0	1	2	3	4	5
T	16	17.8	19.7	21.9	24.3	27.0



4 **a** 100

b 100 bacteria multiply to 12 800

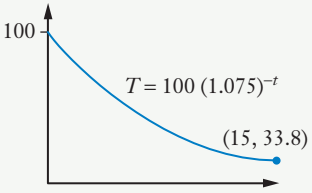
c **i** 300 000 **ii** 1 400 000

d **i** 15 h **ii** 16.2 h

e Yes, approximately doubling every 70 minutes.

5 **a** Decay

b T(°C)



c B

d 100, initial temperature of soup

e 18 min

f This gives values below 23°C, and the temperature of the soup will not decrease below room temperature towards 0°C.

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6 **a** **i** 2.4×10^{11} **ii** 2.1×10^{53}

b 1000 **c** after 3.9 minutes

7 **a** **i** 3 days **ii** 18 days

b C

8 **a** Growth

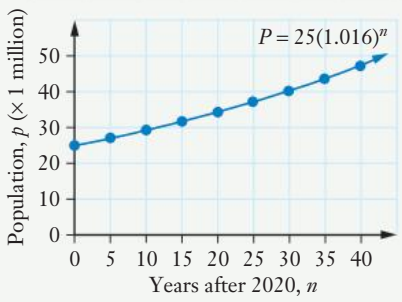
b 5, the initial number of computers affected

c 35 523

9 **a**

Year	2020	2025	2030	2035	2040	2045	2050	2055	2060
n (years)	0	5	10	15	20	25	30	35	40
P (millions)	25	27.1	29.3	31.7	34.3	37.2	40.2	43.6	47.2

b



c **i** About 30 million

ii 30.73 million

d **i, ii** About 2050

10 **a** **i** 3.08 kg **ii** 6.76 kg **iii** 32.62 kg

b **i** 4.85 months **ii** 11 months

11 **a** **i** 97.6% **ii** 94.2%

b 5780 years **c** 340 years

12 **a** n

b 8000, the initial investment of $8000

c $15 017.10

d After 11 years

Exercise 10.05

1 B

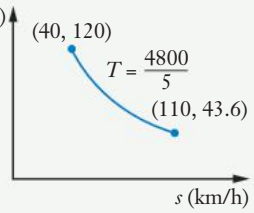
2 C

3 **a** C **b** B

c A **d** D

4 **a** T

b



c **i** 50 min **ii** 52 min

d 69 km/h **e** 80 km

5 **a**

d	50	100	150	200	250	300	350	400
h	1120	560	373	280	224	187	160	140

b 255 cm **c** 65 cm

Exercise 10.06

1 **a** Increases **b** Decreases

2 D

3 **a** 60 **b** 8

4 **a** C **b** A **c** B

5 **a** $F = \frac{112}{L}$ **b** 6 beats/s **c** 25 cm

6 **a** 4.8°C **b** 253 m

7 108 km/h

8 **a** 2.5 h **b** 5

9 **a** 4 m^3 **b** 22.5 kPa

10 **a** 8 min **b** 4

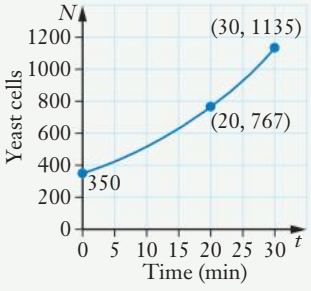
11 C

12 **a** 1.6 min **b** 20 students

13 19 091

Sample HSC problem

a



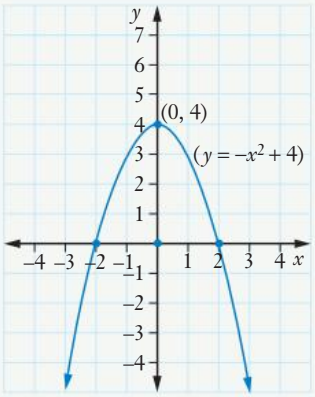
b 350, initial number of yeast cells

c 8067 **d** 27 min

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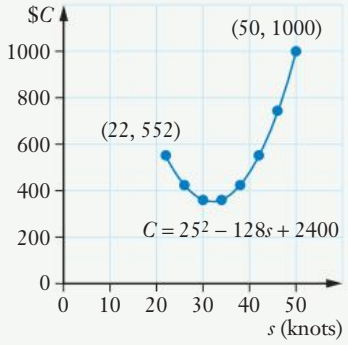
Test yourself 10

1 **a** Quadratic **b** Parabola **c** $y = -x^2 + 4$



2 **a** 424, 360, 424, 744, 1000

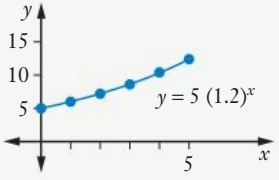
b



c 24 knots, 40 knots **d** $352, at 32 knots

3 **a** $y = 5(1.2)^x$

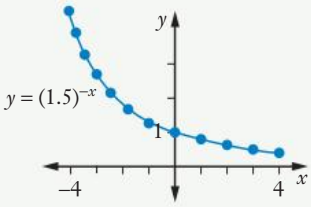
x	0	1	2	3	4	5
y	5	6	7.2	8.6	10.4	12.4



b 5 **c** Increasing

4 **a** $y = (1.5)^{-x}$

x	−4	−3	−2	−1	0	1	2	3	4
y	5.1	3.4	2.3	1.5	1	0.7	0.4	0.3	0.2



b 1 **c** Decreasing

d The x-axis

5 **a** 1400 L **b** 390 L **c** 23 min

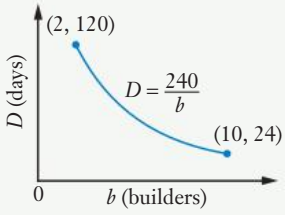
d Assumes volume of water in the tank will never be zero, that is, tank will never be completely empty

6 **a** 96 216

b 65 000, the initial population

c 18 years

7 **a**



b 27 days **c** 48

d Assumes the greater the number of builders, the fewer number of days. Eventually, too many builders will get in each other's way and slow down the completion time.

8 A

9 **a** C **b** B **c** A

d D **e** E

10 **a** 12.5 h **b** 67 km/h

Chapter 11

SkillCheck

1 **a** 12 562 **b** 8 **c** 15

2 11 400

3 1920

4 1 day

5 **a** 13.5 **b** 97.35 **c** 81.5

6 **a** $\frac{5}{4}$ **b** 110.4

7 **a** 11.296 **b** 2.429

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Exercise 11.01

1 **a** Y **b** Z

2 **a** 68% **b** 2.5% **c** 95%
d 16% **e** 0.15% **f** 0.15%

3 **a** 4.99–5.01 **b** 4.98–5.02 **c** 4.97–5.03

4 B

5 **a** **i** 335 mL **ii** 320 mL
b **i** 34% **ii** 0.15%

6 C

7 **a** 250 h
b 14 750–15 250 **c** 18

8 **a** 1395 mm **b** 7788

9 **a** 0.3% **b** 81.5%

10 **a** 234–298 days **b** 16%
c Possible but unlikely as 320 days is more than 3 standard deviations from the mean.

11 **a** 84%
b No. Only 16% of packets are equal to or over the advertised weight.

12 **a** 97.5% **b** 150

13 **a** 16% **b** 2.5%

Exercise 11.02

1 $z = 2$

2 **a** 34% **b** 68% **c** 47.5%
d 99.7% **e** 83.85% **f** 16%

3 A

4 **a** 1
b His mark is 1 standard deviation above the class mean.
c 69

5 **a** $z = -1$ means that Sam's blood pressure is 1 standard deviation below the mean.
b BP = 110
c 0.75

6 **a** **i** –0.4 **ii** 3.5
b 112 cm **c** 76 cm
d 47.5% **e** 16%

7 **a**

Mass (g)	94	96	98	100	102	104	106
z-score	–3	–2	–1	0	1	2	3

b **i** 50% **ii** 68% **iii** 2.5%
c Approx. 62

8 **a** **i** 0 **ii** 2 **b** 750 000
c **i** 4 standard deviations above the mean
ii 164

9 **a**

Weight (kg)	0	0.4	0.8	1.2	1.6	2.0	2.4
z-score	–3	–2	–1	0	1	2	3

b 81.5% **c** 120 **d** 768

10 **a** 0.16, 1.8 **b** 47

Exercise 11.03

1 C

2 **a** English –0.71; Maths 0.7; Science –0.25; Art –0.875
b Maths, Science, English, Art

3 **a**

Subject	Peta	Mel	Joe
Visual Arts	–1.125	–0.375	0.25
Legal Studies	–0.083	1.17	0.5
Maths Standard 2	0	1	–1.67

b Legal Studies
c Joe's mark in Visual Arts
d Mel in Legal Studies

4 **a** Adam is taller; more than 1 standard deviation above the mean.
b 174 cm **c** 17 cm taller

5 Downtown (z-score is 0.357 whereas for Uptown z-score is –0.945).

6 2-year model (z-score is –0.67 whereas 3-year model z-score is –1.25).

Exercise 11.04

1 **a** \$425.56, \$400, \$300, \$450, \$310
b 172.375 kg, 171.5 kg, no mode, 55 kg, 39.5 kg

2 D

3 **a** Blonde
b Because hair colour is described in words (categorical data)

4 A

5 **a** Any of the mean, median or mode as there is no outlier, data is not clustered
b His most common time to swim 1 km is 24 min.

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6 **a** **i** 15 **ii** 7
b **i** 10 **ii** 6
c **i** 47 **ii** 23.5
d **i** 7 **ii** 2

7 **a** 1.73 **b** 2
c 4 **d** 1

8 **a** 50 **b** 7
c fx: 6, 16, 30, 54, 98, 72, 45, 10; $\bar{x} = 6.62$
d 7 **e** 7

9 **a** 28 **b** 4 **c** 1
d 1.07

10 **a** 6 and 8 **b** 6 hours/day
c 6.38 h or 6 h and 23 min
d 3.5 hours/day

11 **a** 50 **b** 9
c 25–29 or 27 **d** 25–29 **e** 11

12 **a** 12P **b** 12R
c 12Q; higher mean and smallest standard deviation out of the three classes.

13 **a** 30 **b** $42 000 **c** $16 200

14 **a** 22.8 **b** 1.8

15 **a** 7.91 **b** 1.70

16 **a** 62.3 mm **b** 89.7 mm
c 56.4 mm **d** 101.7 mm
e Median and IQR as the outliers do not affect the middle (not the median nor the IQR)

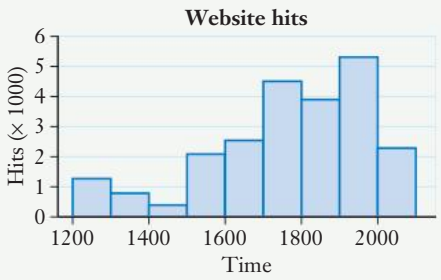
Exercise 11.05

1 C

2 **a** Negative skew; 1 peak
b Positive skew; 2 peaks
c Positive skew; 2 peaks
d Negative skew; 4 peaks
e Symmetrical; 1 peak
f 2 peaks

3 **a** 36
b Negative skew
c 70s–80s
d 88

4 **a**



b Negatively skewed, cluster from 1700 to 2000, peak 1900–2000
c Most visits to Internet sites occur after work hours.

5 C

6 **a**

Stem	Leaf
1	5 5 5 6 6 7 8 9 9
2	0 1 2 2
3	5 6
4	3 9
5	0 5 9

b Positively skewed; cluster 15–19 years; peak mode of 15.

7 **a** Positively skewed
b 1
c 2.6 accidents (higher than the mode)

Exercise 11.06

1 **a** IQR = 3
b **i** −3.5 **ii** 8.5
c Yes, since 9 is greater than 8.5 ($= Q_3 + 1.5 \times$ IQR).

2 **a** No, since outliers must be less than 16.5 or greater than 36.5.
b Yes, 9 is an outlier since it is less than 11 ($= Q_1 - 1.5 \times$ IQR).
c No, since outliers must be less than −11 and greater than 81.
d 9 is the outlier since it is greater than 7.5.

3 **a** $522.86 **b** $530
c IQR = 600 − 410 = 190
d $890, $Q_3 + 1.5 \times$ IQR = 600 + 1.5 × 190 = $885, so $900 is an outlier.
e Mean and median would be lower ($461.67, $505).
f $575.14, $583, Yes

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4 **a** 20

b 8, $8 > 5 + 1.5 \times 1.5$

c **i** 4.35 **ii** 4.16

d The mean increases

5 D

6 **a** Wombats: $s \approx 7.79$

Possums: $s \approx 4.66$

Koalas: $s \approx 10.68$

b Possums as its standard deviation is smallest

c Wombats: $s \approx 5.45$

d Still the Possums as its standard deviation is still smallest

7 **a** Lisa: 3 Hong: 3

b They are equally good salespeople.

c Lisa 5.5 copiers; Hong 17 copiers

d Lisa 7.2 copiers; Hong 16.6 copiers

e Median as Lisa has an outlier of 25

f Hong, as his median sales are much higher than Lisa

8 **a** 2.92, 3, 3

b 9 since $9 > 4 + 1.5 \times 3 = 8.5$ ($Q_3 + 1.5 \times$ IQR)

c 2.3637, 3, 3

d Increases the mean, no effect on the mode, median

9 **a** Mean = \$46 500, median = \$46 200, mode = \$54 600

b Mode as it is the highest

c Median, as it is not affected by the outlier of \$76 800

Exercise 11.07

1 **a** A: 90, 58.5, 56.5 B: 91, 18, 53

b Test B as it has a smaller interquartile range

2 **a**

Netball		Hockey
8	5	2
2 2	6	4 4
9 7 7 5 2 2	7	3 4 5 9
0	8	1 1 2 6
	9	2

b Netball: negative skew, cluster in 70s

Hockey: negative skew, cluster in 70s and 80s

c 71.4, 75.25 **d** 73.5, 77

e Median since data is skewed

3 A

4 B

5 **a** **i** 12 cm **ii** 26 cm

b Estimates: range = 34 cm, IQR = 17.5 cm

Actual: range = 11 cm, IQR = 3.5 cm

c Definitely agree; the box plot for estimates has a much greater spread as well as a much lower median, even though some people did overestimate.

6 **a** Boys: 11 30 35.5 45 46

Girls: 11 18 23 27 49

b Spread of scores, five-number summaries (highest and lowest scores, quartiles, median)

7 **a** **i** Darwin **ii** Brisbane

iii Darwin **iv** Canberra

b Sydney (lower median price)

8 **a** 5

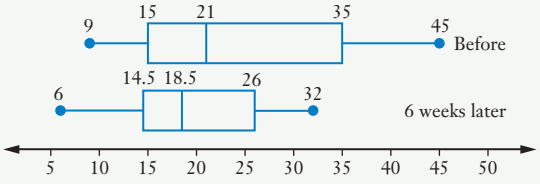
b Those with the lowest pulse did not exercise hard enough and pulse stayed the same; or someone who had a higher pulse due to rushing to class could have reduced their pulse rate by not working hard in the class.

c Two people exercised very hard and got their pulse rates up very high.

d 22 beats **e** 6 beats/min

9 **a** 5 number summaries are:

Before	9	15	21	35	45
6 weeks later	6	14.5	18.5	26	32



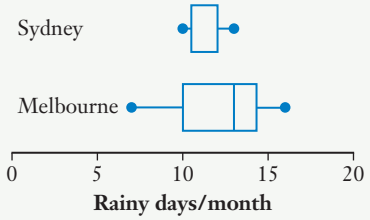
b 20, 11.5

c Yes. Interquartile range and median decreased.

10 **a**

Sydney		Melbourne
	0	
3 2 2 2 2 2 2 2 1 0 0 0	1	1 2 2 4 4 4 5 5 6

b



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c Stem-and-leaf plot: Sydney values cluster around 12 days.

Box plots: Sydney has a much smaller spread than Melbourne; median and upper quartile for Sydney are equal.

d Disagree with 'more'; the median number of rainy days per month is only 1 day greater for Melbourne (13) than for Sydney (12).

Sample HSC problem

a –0.6 **b** 34%

c **i** Raad's mark is 2.3 standard deviations above the mean.

ii 88

Test yourself 11

1 **a** 68% **b** 2.5% **c** 95%
d 0.15% **e** 0.15% **f** 2.5%

2 D

3 **a** 2.5 standard deviations above the mean
b 5.6 **c** –1.07 **d** 72

4 **a**

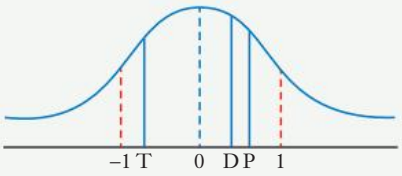
Time (h)	2	4	6	8	10	12	14
z-score	–3	–2	–1	0	1	2	3

b **i** 15.85% **ii** 16%
iii 2.5% **iv** 81.5%

5 **a** Drama 0.3125, Photography 0.5, Textiles –0.8

b Photography. It has largest z-score.

c



6 **a** \$42.80, \$40.50, \$56, \$69, \$31

b 375.4 mL, 375 mL, no mode, 17 mL, 12 mL

7 **a** C **b** A **c** A, B, C
d B, D

8 **a** 32.3 **b** 12

c

Stem	Leaf
1	6 7 9
2	2
3	6 8 8
4	1 2
5	4

d 37 **e** 22

9 C

10 B

11 **a** 18 **b** 42

c Women: negatively skewed, clustered 50s–60s, Men: cluster 30s–40s

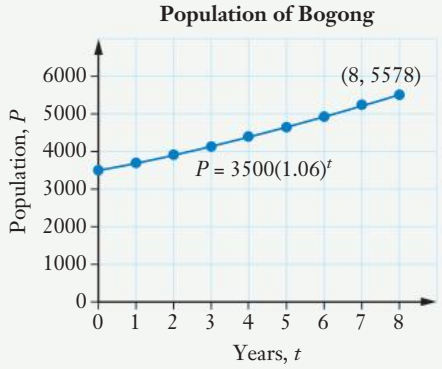
d The men were younger.
Women: mean = 52.5; men: mean = 42.7

Practice paper 4

1 B **2** B **3** D
4 B **5** B **6** C
7 C **8** D **9** B
10 A

11 **a** 2680 m^2 **b** **i** 189 cm **ii** –0.64

c **i**



ii By 1763 people

iii About 2037

12 **a** **i** $T = \frac{k}{S}$, $k = 400$

ii $6\frac{2}{3}$ h or 6 h 40 min

iii 53 km/h

b **i** 64°, 86°, 52°, 158°

ii 5869 m^2 **iii** 136 m

13 **a** **i** 75 **ii** 15 **iii** 160

b **i** 18.4 m **ii** \$792 **iii** 3

c **i** 430

ii When $t = 0$, $N = 80(1.4)^0 = 80$

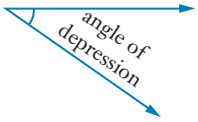
iii 40%

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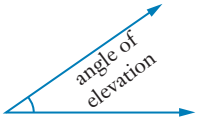
GLOSSARY AND INDEX

activity table: A table that shows the order and completion time for each activity in a task or project. Used in critical path analysis of a network. (p. 244)

angle of depression: The angle between the horizontal and the line of sight, looking down. (p. 340)



angle of elevation: The angle between the horizontal and the line of sight, looking up. (p. 340)



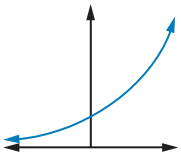
annuity: An investment involving regular contributions (payments) into an account earning compound interest, for example, a superannuation fund. (p. 129)

appreciation: Increase in value of an item or asset over time. The opposite of **depreciation**. (p. 16)

arc: *See* **edge**.

asset: Item or resource, such as machinery or computer equipment, used by a business to earn income. (p. 28)

asymptote: Line that a curve gets very close to but never actually touches. The x-axis is an asymptote for the exponential curve shown here. (p. 444)



back-to-back stem-and-leaf plot: Compares two data sets. (p. 504) *See also* **stem-and-leaf plot**.

backward scanning: A method of finding the latest starting time (LST) for every activity in a directed network. (pp. 247, 254)

balance owing: The amount remaining or owing in a loan. (p. 117)

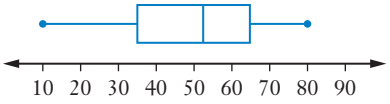
bearing: *See* **compass bearing** and **true bearing**.

best buy: Best value for money. (p. 59)

biometric data: Data obtained by measuring physical characteristics, such as height, arm span, bone length and facial features. (p. 158)

bivariate data: Data that relates two variables measured on the same group, such as height and weight of students, that can be graphed on a scatterplot. (p. 157)

box plot (or **box-and-whisker plot):** A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers. (p. 505)



break-even point: The point or value at which a business stops incurring a loss and starts making a profit. (p. 101)

brokerage: Commission paid to a stockbroker for buying or selling a parcel of shares for a client. (p. 20)

calorie: An older, non-metric unit of energy, equal to 4.184 kilojoules (kJ). (p. 309)

cash advance: Withdrawal of cash from a credit card. (p. 124)

causality: A correlation between two variables when one variable causes the other to occur. (p. 166)

circuit: A closed trail in a network. (p. 201) *See also* **trail**.

class centre: The centre of a class interval. For example, the class centre of the class interval 10–19 is 14.5. (p. 494)

class interval: In statistics, when there are many data scores, they may be grouped into class intervals. For example, ages of people may be grouped into class intervals of 1–10, 11–20, 21–30 and so on. (p. 495)

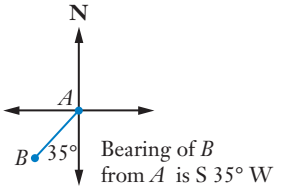
closed walk: A walk in a network that starts and ends at the same vertex. (p. 201)

cluster: A group of data scores that are bunched or close together. (p. 495)

coefficient: A number in front of a variable in a function, for example, the coefficient of x in $y = -2x + 10$ is -2. (p. 87)

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compass bearing: A direction given as an angle from either north and south (whichever is closer) to east or west. For example, the bearing of B from A in the diagram is S 35° W. (p. 346) *See also* **true bearing**.



compass radial survey: Involves taking the true bearing of each corner of a field using a compass placed in the middle of the field. (p. 413)
compound interest: Interest paid on the principal invested as well as on any accumulated interest. (p. 8) Differs from **simple interest**.
compounded: The addition of interest to the principal sum of a loan or deposit, or in other words, interest on interest. (p. 8)
compounding period: How often interest is calculated when using compound interest, for example, monthly, quarterly or yearly. (p. 8)
connected network: A network in which any 2 vertices can be joined by at least one path. No part of the network is isolated. (p. 204)
constant of variation (or **constant of proportionality):** The constant in a variation equation. For example, if y varies as x, the variation equation is $y = kx$ and the constant of variation is k because its value is fixed. (p. 94)
constant term: A number on its own in a function, for example, the constant term in $y = -2x + 10$ is 10. (p. 87)
contribution to an annuity: Regular payment made to an annuity. (p. 129)
correlation: The strength of the relationship between two variables, can be positive or negative, strong (high) or weak (low) or moderate. (p. 162)
correlation coefficient (symbol r): A number between –1 and +1 that measures the correlation between two variables. (p. 163)
cosine: A trigonometric ratio in a right-angled triangle.

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 339) *See also* **sine** and **tangent**.
cosine rule: Trigonometric formula used for triangle problems involving 3 sides and 1 angle. (p. 360)

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

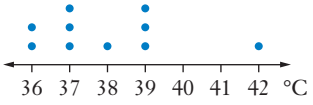
credit card: A card that operates like taking out a short-term loan. It allows you to purchase items and pay for them later when you receive a monthly statement. Interest is charged daily. (p. 122)
critical activity: An activity on a critical path of a network. (p. 257)
critical path: The longest time path between the start and finish vertices in a network, containing activities with equal EST and LST values. (p. 247)
critical path analysis: The process of organising activities to complete a task or project in the shortest time possible. (p. 247)
cumulative frequency: A running total of frequencies. (p. 488)
cut: A line that cuts through the edges of a flow network to block the source from the sink, stopping all flow to the sink. (p. 269)
cycle: A **closed path** in a network. (p. 201)

declining-balance method of depreciation: Method of depreciation in which an item's value decreases by the same percentage each period. (p. 34) Differs from **straight-line method of depreciation**.
degree (of a vertex): The number of edges connected to a vertex. (p. 200)
dependent variable: A variable whose value depends on another variable. It is represented on the vertical axis of a graph. For example, if $y = 3x + 5$, y is the dependent variable because its value depends on the value of x. Differs from an **independent variable**. (pp. 87, 170)
depreciation: Loss in value of an item or asset over time. The opposite of **appreciation**. (p. 28)
direct linear variation (or **direct proportion):** A relationship between two variables (say x and y) represented by the equation $y = kx$, where k is the constant of variation. Differs from **inverse variation**. (p. 460)
directed network: A network with arrows on the edges to indicate that movement can only be in the direction of the arrows. (p. 199)
distribution (or **frequency distribution**)**:** The way the scores of a data set are arranged, especially when graphed. (p. 495). *See also* **shape of a distribution**.
dividend: The share of profits in a company paid to each shareholder. (p. 19).
dividend yield: The percentage return on an investment.

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\% \text{ (p. 19).}$$

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dot plot: A graph that uses dots to show frequencies of data scores (for example, the temperatures in °C of 10 hospital patients). Different from a **stem-and-leaf plot**. (p. 493).



dummy activity: An activity with zero time used in critical path analysis to eliminate repeated edges in a network. (p. 246)

earliest starting time (EST): The earliest starting time for an activity in a directed network. (pp. 247, 248)

edge: A line joining two vertices in a network. (p. 199)

elevation view: On a plan, the front, back or sides view of a building. (p. 403)

empirical rule: The 68%–95%–99.7% rule that gives the percentages of scores that lie under the normal curve, within 1, 2, and 3 standard deviations of the mean. (p.471)

energy-efficient: Use of 'green' design principles to reduce your impact in the environment, such as building orientation and materials, power saving. (p. 325)

energy rating label (ERL): A label on an electrical appliance showing the star rating of how energy efficient it is (the more stars, the better), including the estimated number of kilowatt hours used per year (the lower, the better). (pp. 324, 325)

equation: A mathematical statement that two quantities are equal. An equation contains an equals sign, for example, $4x - 5 = 11$. (p. 77)

Eulerian circuit: A walk in a network that uses each edge only once, and which starts and ends at the same vertex. (p. 215)

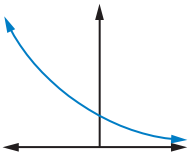
Eulerian trail: A walk in a network that uses each edge only once, and which starts and ends at different vertices. (p. 214)

exercise intensity: The level at which your body works during physical activity. (p. 305)

exponent: *See* **power (index)**.

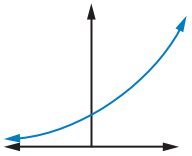
exponential curve: Graph of an **exponential function**. (p. 443)

exponential decay: Decrease of a quantity according to the exponential function $y = b(a^{-x})$ where $a > 1$ and $b > 0$. (p. 450) *See also* **exponential growth**.



exponential function: A function of the form $y = b(a^x)$ where the x occurs as a power, for example $y = 1.7^x$. This function is used to model **exponential growth** (where the quantity is increasing) and **exponential decay** (where the quantity is decreasing). (p. 443)

exponential growth: Increase of a quantity according to the exponential function $y = b(a^x)$ where $a > 1$ and $b > 0$. (p. 448) *See also* **exponential decay**.



extrapolate: To estimate the value of a quantity that is outside the given range of values. Different from **interpolate**. (p. 170)

extremes (upper and lower): The highest and lowest scores in a data set. (p. 505)

five-number summary: The lower extreme, lower quartile, median, upper quartile and upper extreme of a data set. These five numbers are used to draw a **box plot**. (p. 505)

flat-rate interest: *See* **simple interest**.

flat-rate loan: A loan in which simple interest is charged on the principal borrowed for the term of the loan. (p. 117) *See also* **reducing balance loan**.

float time (or slack time): In critical path analysis of a network, the time by which a non-critical activity can be extended or delayed without affecting the time to complete a project. (p. 259)

floor plan: A scale drawing of the floor of a house or other building, often referred to as 'the plan'. (p. 403)

flow capacity: The total flow through a flow network. (p. 265)

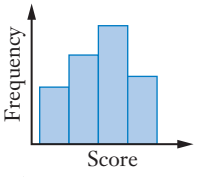
formula: A rule written as an algebraic equation, using variables. For example, the formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 80)

fortnightly: Every two weeks. (p. 8)

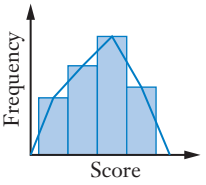
forward scanning: A method of finding the earliest starting time (LST) for every activity in a directed network. (pp. 247, 248)

frequency distribution: *See* **distribution**.

frequency histogram: A bar chart that shows the frequency of data scores. (p. 471)



frequency polygon: A line graph formed by joining the midpoints of the tops of the columns of a frequency histogram. (p. 495)



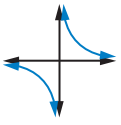
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fuel consumption: The rate at which fuel is used by a vehicle, measured in L/100 km. (p. 64)
function: The relationship between two variables, for example, $y = 3x - 5$. (p. 87)
future value (FV) (or final value): The value of a compound interest investment or annuity at a specific time. (p. 8)
future value of an annuity (*FVA*): The total value of the investment at the end of the term, after the final contribution is made. It is the sum of all contributions plus interest. (p. 130)

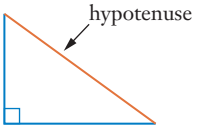
gigawatt (GW): 1000 MW or 1 000 000 000 W. (p. 315)
goods and services tax (GST): The tax (currently at 10%) a consumer pays on any purchased item or service (for example, buying a car or hiring a painter). (p. 302)
gradient (symbol *m*): Slope of a line.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} \text{ (p. 87)}$$

heart rate (or pulse): The frequency of a heartbeat, measured in beats per minute. (p. 56)
histogram: *See* **frequency histogram**.
hyperbola: The graph of a reciprocal function $y = \frac{a}{x}$. It has two branches and the x- and y-axes are asymptotes. (p. 456)



hypotenuse: The longest side of a right-angled triangle, the side opposite the right angle section. (p. 339)



included angle: The angle made by two sides of a triangle. (p. 367)
independent variable: A variable whose value does not depend on another variable. It is represented on the horizontal axis of a graph. For example, if $y = 3x + 5$, x is the independent variable, while y is the **dependent variable**. (pp. 87, 170)
inflation: The increase in the prices of goods and services over time. (p. 14)
instalment: *See* **repayment**.
interest: Money earned on an investment or paid on a loan. (p. 5)
interest-free period: A period of time when interest is not charged on an amount borrowed, for example, 55 days for a credit card account. (p. 122)
interpolate: To estimate the value of a quantity that is within the given range of values. Differs from **extrapolate**. (p. 170)
interquartile range (IQR): The difference between the upper quartile and lower quartile of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data. (p. 485)
inverse variation (or inverse proportion): A relationship between two variables (say x and y) represented by the equation $y = \frac{k}{x}$, where k is the **constant of variation**. Differs from **direct variation**. (p. 460)

joule (J): A unit of energy. (p. 309)

kilo- (k): A prefix meaning 'one thousand'. For example, a kilolitre is 1000 litres. (p. 315)
kilojoule (kJ): A unit of energy equal to 1000 joules. (p. 309)
kilowatt (kW): A unit of power equal to 1000 watts. (p. 315)
kilowatt-hour (kWh): A unit of electrical energy equivalent to that used by one kilowatt of power in one hour. (p. 316)
Kruskal's algorithm: A method for finding a minimum spanning tree for a weighted network. (p. 224) *See also* **Prim's algorithm**.

latest starting time (LST): The latest starting time for an activity in a directed network. (pp. 247, 254)
least-squares regression line: The line of best fit on a scatterplot where distances of each point from the line is as small as possible. (p. 176)
life expectancy: Average number of years of life remaining to a person at a specified age. (p. 183)
line of best fit (or regression line): A straight line that represents a set of points obtained through experiment or observation. (p. 170) *See also* **least-squares regression line**.
linear: A word used to describe something to do with a line. (p. 87)
linear function: A function of the form $y = mx + c$, whose graph is a straight line. (pp. 87, 431)
linear modelling: Using a linear function to approximate a real-life situation. (p. 100)
loan repayment table: Table of calculated repayments for reducing balance loans. (p. 135)
loop: An edge that connects a vertex to itself. (p. 199)

market price (or market value): Current sale value of a share or item. (p. 19)
matrix: A rectangular array of rows and columns, which may contain numbers, symbols or expressions. (p. 204)
maximum/minimum (of a quadratic function): The y-value of the vertex. (p. 439)

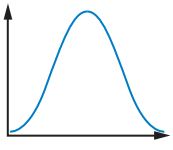
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maximum-flow minimum-cut theorem: A rule for finding the maximum flow of a flow network by cutting through edges of the network: the maximum flow is equal to the total weight of the edges in the minimum cut of the network. (p. 276)
maximum heart rate (MHR): The highest heart rate attained by a person during physical activity. (p. 303)
mean: The average of a set of scores.

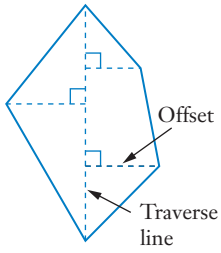
$$\text{mean (or } \bar{x}) = \frac{\text{sum of scores}}{\text{number of scores}} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f} \text{ (p. 484)}$$

measure of spread: A statistical value, such as the range, interquartile range or standard deviation, which describes the spread of a set of data. (p. 484)
measures of central tendency (or measures of location): A statistical value, such as the **mean**, **median** or **mode**, which describes the centre or average of a set of data. (p. 484)
median: The middle score of a data set when scores are arranged in ascending order. If there are two middle scores, the median is the average of them. (p. 484)
mega- (M): A prefix meaning 'one million'. For example, a megatonne is 1 000 000 tonnes. (p. 309)
megajoule (MJ): 1000 kJ or 1 000 000 J. (p. 309)
megawatt (MW): 1000 kW or 1 000 000 W. (p. 315)
milliwatt (mW): One thousandth of a watt. (p. 315)
minimum spanning tree: A spanning tree in a network where the sum of the weights is smallest. (p. 224) *See also* **spanning tree**.
minute: A measure of time or angle size. $\frac{1}{60}$ of an hour or degree. 1 h = 60 min. 1° = 60'. (p. 353)
mode: The most common or frequent score(s) in a set of data. (p. 484)
model: Using mathematics to describe a real-life pattern or relationship. (p. 93)
mortgage: A home loan. (p. 140)

network: A collection of objects connected to each other in some way, represented by a diagram involving vertices and edges. (p. 199)
network flow: The flow of resources, such as traffic, water, electricity and data, across a network. (p. 265)
non-linear function: A function that is not of the linear form $y = mx + c$. Its graph is a curve and can take many forms. (p. 431)
normal distribution: A statistical distribution that is symmetrical about its mean, whose graph is the bell-shaped **normal curve**. The mean, median and mode are all equal, represented by the peak of the curve. (p. 471)

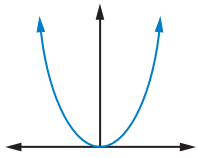


obtuse angle: An angle greater than 90° and less than 180°. (p. 358)
offset survey (or traverse survey): A method of measuring lengths in an irregularly-shaped field, using offsets to each corner of the field from a traverse line. (p. 411)



outlier: An extreme (high or low) score in a data set that is much different than the other scores, either less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$. (p. 499)

parabola: U-shaped graph of a **quadratic function**. (p. 431)



path: A walk or route in a network with no repeated vertices or edges. A **closed** path is a **cycle**. (p. 201)
peak: A high point of a distribution, and the highest peak shows the mode. (p. 495)
per annum (p.a.): Per year, annually. (p. 5)
period: Amount of time (for example month, week, year). (p. 5)
point of intersection: *See* **break-even point**. (p. 101)
population: All of the items under investigation. (p. 471)
predecessor: In critical path analysis of a network, a previous activity that needs to be completed before the current activity can begin. (p. 244)
present value (*PV*): The current value of a sum of money, in contrast to some future value it will have when it has been invested at compound interest. (p. 8)
Prim's algorithm: A method for finding a minimum spanning tree for a weighted network. (p. 224) *See also* **Kruskal's algorithm**.
principal: The original amount of money invested or borrowed. (pp. 5, 117) *See also* **present value.**
proportional to: In the same ratio as. If y is proportional to x, we say $y = kx$ and x and y have a direct linear variation. (p. 94) *See* **direct variation**.
pulse: *See* **heart rate**.

quadratic function: A function of the form $y = ax^2 + bx + c$. Its graph is a **parabola**. (p. 431)
quarterly: 4 times a year, or every 3 months. (p. 9)

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quartiles (upper, middle and lower): The upper quartile is the 3rd quartile (Q_3) that cuts off the top 25% of scores in a data set; the middle quartile (Q_2) is the median, and separates the two middle quarters; the lower quartile is the 1st quartile (Q_1) that cuts off the bottom 25% of scores. (p. 485) *See also* **deciles** and **percentiles**.

radial survey: Survey where a compass is placed in the middle of a field and the true bearing and distance to each corner are measured. (p. 413)

range: The difference between the highest score and the lowest score in a set of data. (p. 485)

rate: A comparison between two quantities expressed in different units, for example, a tree's growth rate can be measured in cm/year. (p. 56) *See also* **ratio**.

ratio: A comparison between two quantities expressed in the same units units, for example, Joe is twice as tall as Jim so their heights are in the ratio 2 : 1. (p. 50) *See also* **rate**.

raw score: A score before it has been standardised into a z-score. *See also* **z-score**. (p. 477)

reciprocal function: A function of the form $y = \frac{a}{x}$. Its graph is a **hyperbola**. (p. 456)

reducing balance loan: A loan where the interest charged is calculated on the balance owing on the loan after each repayment. (p. 117)

regression line: A line of best fit drawn on a **scatterplot**. (p. 176)

repayment (or **instalment):** Amount paid regularly to pay off a loan. (p. 117)

resting heart rate (RHR): The normal heart rate of a person at rest. (p. 305)

rise: The vertical change in position between two points on a line; the number of units going up. Used with the **run** to calculate **gradient**. (p. 87)

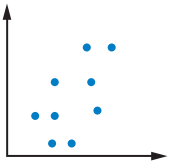
run: The horizontal change in position between two points on a line; the number of units going to the right. Used with the **rise** to calculate **gradient**. (p. 87)

salvage value: The current value of an item after it has depreciated. (p. 28)

scale (on a map or diagram): The ratio of scaled length to actual length, for example, a scale of 1 : 500 means that lengths represented on the map or diagram are actually 500 times larger in real life. (p. 391)

scale drawing: A drawing of an object, usually smaller, whose lengths are in the same ratio as the actual lengths of the object. (p. 391)

scatterplot: A graph of points on a number plane showing a relationship between two variables. (p. 157)



scientific notation: A way of writing very large or very small numbers. For example, $98\,000\,000 = 9.8 \times 10^7$. (p. 316)

second: A measure of time or angle size. $\frac{1}{60}$ of an minute. 1 min = 60 s. $1' = 60''$. (p. 353)

shape of a distribution: The way the data in a frequency distribution is spread, can be symmetrical, positively skewed or negatively skewed. (p. 495) *See also* **symmetrical** and **skewed.**

shareholder: A person who owns shares in a company. (p. 19)

shares (or stock): A portion of a company owned by a shareholder. (p. 19)

shortest path: The path in a weighted network where the sum of the weights is minimised. (p. 235)

significant figures: Meaningful digits in a numeral that tell 'how many'. For example, 98 000 000 has two significant figures—9 and 8. (p. 70)

simple interest (or **flat rate interest):** Interest earned or charged only on the original amount of money (principal) invested or borrowed. Differs from **compound interest**. (p. 5)

simultaneous equations: Two or more equations that must be solved together so that the solution satisfies all equations. For example, the simultaneous equations $y = 3x - 5$ and $y = 2x$ have the solution $x = 5, y = 10$. (p. 100)

sine: A trigonometric ratio in a right-angled triangle:

$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 339) *See also* **cosine** and **tangent**.

sine rule: Trigonometric formula used for triangle problems involving 2 sides and the 2 angles opposite them.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$ (p. 353)

sink: The end of a flow network. The opposite of **source**. (p. 265)

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skewed: The shape of a statistical distribution when most of the data scores are either low (positively skewed) or high (negatively skewed). The tail indicates the direction of the skew. (p. 495)



Negatively skewed Positively skewed

source: The start of a flow network. The opposite of **sink**. (p. 265)

spanning tree: A tree in a network that includes all of the vertices of the network. (p. 222) *See also* **tree** and **minimum spanning tree**.

speed: A rate that compares distance travelled with time taken, often measured in kilometres per hour (km/h) or metres per second (m/s).

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}.$$ (p. 63)

standard deviation (symbol σ): A statistical measure of the spread of a set of scores. (p. 485)

stem-and-leaf plot: A 'number graph' that lists all the data scores, in groups. This stem-and-leaf plot shows 12 test scores, from 42 to 82. Different from a **dot plot**. (p. 485) *See also* **back-to-back stem-and-leaf plot**.

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

stockbroker: Person who buys and sells shares at the current market price, charging a commission called the **brokerage**. (p. 20)

straight-line method of depreciation: Method of depreciation in which an item's value decreases by the same amount each period. Differs from **declining-balance method of depreciation**. (p. 28)

subject of a formula: The variable on the left-hand-side of a formula. For example, the subject of the formula $v = u + at$ is v, but the subject can be changed to u by rewriting the formula as $u = v - at$. (p. 83)

summary statistics: *See* **five-number summary**.

superannuation: A retirement fund for employees. Every payday, an employee and his/her employer invest a part of the employee's pay into a fund in order to provide income for the employee during retirement. (p. 129)

surveying: Involves setting out the positions of proposed construction or engineering works. It involves the measurement of the boundaries of a field and the calculation of perimeters and areas. (p. 411)

sustainability: The practice of maintaining the Earth's natural resources effectively and without waste, to support its long-term ecological balance. (p. 329)

symmetrical (of a curve): The **shape of a distribution** when the data are balanced or evenly spread about the centre of the distribution, with the **mean**, **median** and **mode** being equal. (pp. 471, 496)

tail: The long slope part of a frequency distribution curve. (p. 496)

tangent: A trigonometric ratio in a right-angled triangle.

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. (p. 339) *See also* **sine** and **cosine**.

target heart rate (or training heart rate, THR): The desired heart rate to be achieved during aerobic (cardiovascular) training to provide maximum health for the heart and lungs. (p. 305)

term of a loan: The amount of time it takes a loan to be completely paid off. (p. 8)

three-figure bearing: *See* **true bearing**.

trail: A walk or route in a network with no repeated edges. A **closed** trail is a **circuit.** (p. 201)

trapezoidal rule: Formula for finding the approximate area of an irregular-shaped block using the area of a trapezium. $A \approx \frac{h}{2}\left(d_f + d_l\right)$. (p. 419)

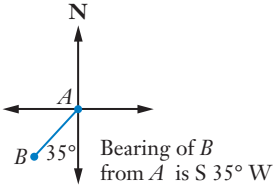
traverse survey: *See* **offset survey**.

tree: A network in which any 2 vertices are connected by exactly one path, which means it is connected with no cycles. (p. 222) *See also* **spanning tree** and **minimum spanning tree**.

trendline: *See* **line of best fit**.

trigonometry: The branch of mathematics dealing with the relationships between the sides and angles of triangles. (p. 339)

true bearing (or three-figure bearing): A direction written as an angle from the north going in a clockwise direction from 000° to 360° (p. 345). *See also* **compass bearing**.

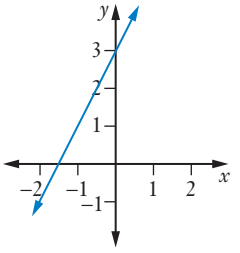


unit price: The price of one unit or item, a unit could be 1 kg or 100 mL. Used to compare the prices of different brands and sizes when shopping. (p. 59)

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unitary method: A way of calculating a value by first finding one unit and then multiplying. (p. 65)

variable: A pronumeral that can take a range of values (that is, it is not a **constant**). (p. 77)
varies directly as: *See* **direct linear variation**.
varies inversely as: *See* **inverse variation.**
vertex (of a network, plural: vertices): A point (or node) in a network where edges meet. A vertex can be odd or even, depending on the number of edges connected to it. (p. 199)
vertex (of a parabola): The turning point of a parabola. (p. 431)
vertical intercept (or ***y*-intercept):** The value at which a straight line graph cuts the vertical axis. For example, the vertical intercept of this line is 3. (p. 87)



walk: Any route through a network, may include repeated vertices or edges. (p. 201)
watt: A unit of power equal to one joule of energy per second. (p. 315)
weight: See **weighted network**.
weighted network: A network with edges labelled with numbers called **weights**. The numbers could represent distances, times, costs or other quantities. (p. 199)

***y*-intercept:** *See* **vertical intercept**.

***z*-score (**or **standardised score):** A calculated statistic that shows the number of standard deviations a **raw score** is above or below the mean. For example, a *z*-score of –2.3 means the raw score is 2.3 standard deviations below the mean.

$z = \frac{x - \overline{x}}{s}$. (p. 477) *See also* **raw score**.

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OVERVIEW

New Century Maths 11 & 12 has been rewritten and updated to cover the new *Mathematics Standard* HSC courses.

Mathematics Standard 2 caters to students heading towards an HSC exam, an ATAR and university studies. The syllabus describes two pathways for Mathematics Standard that start in Year 11. Even though both pathways share a common Year 11 course, students taking each pathway have specific learning needs, so we have published two levels of text for both Years 11 and 12.

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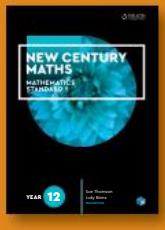
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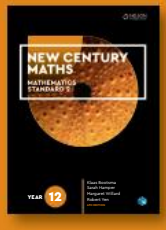
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FEATURES

- Each chapter begins with a chapter table of contents, chapter outcomes, a *Terminology* word list and a *SkillCheck* exercise to review prerequisite skills
- Graded exercises are linked to worked examples and include multiple-choice questions, HSC-style questions and realistic applications
- *Investigations* explore the syllabus in more detail, providing ideas for modelling activities and assessment tasks
- *Technology* promotes ICT in the classroom through the use of spreadsheets, graphics calculators and the Internet
- *Did you know?* sections contain interesting facts and applications of the mathematics learned in a chapter
- Each chapter ends with a *Sample HSC problem*, *Study Tip*, *Chapter Summary* with a mind map exercise and a *Test Yourself* revision set of exercises
- Practice papers after every three chapters revise students' skills and knowledge
- Syllabus grid and codes, answers and glossary/ index included

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