Essential Mathematics Units 1 and 2

The Mathematical Association of Western Australia

Buckley - Callaway - Ostberg

Essential Mathematics
Units 1 and 2

Donna Buckley - Jason Callaway - Michelle Ostberg

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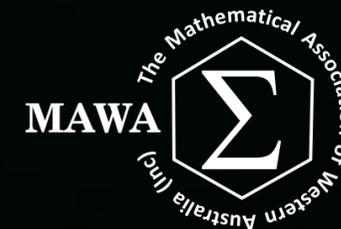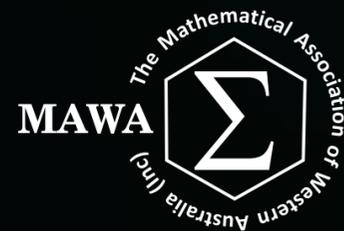
Essential Mathematics
Senior Secondary Course
Units 1 and 2
Revised Edition

Authors

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Editor

Michelle Ostberg

About this book

Sufficient material has been included in this text to provide students with the opportunity to develop their understanding of every item of content in Units 1 and 2 of Essential Mathematics as published by the Australian Curriculum Assessment and Reporting Authority and the content of Mathematics: Essential as published by the School Curriculum and Standards Authority. There are likely to be concepts, however, of which students have already developed a sound understanding. As such, it is expected that teachers will modify the requirement on students to complete every topic as appropriate to their learning needs.

The majority of the content is dealt with through contexts that are relevant and/or interesting to students in this age range. The four units over Years 11 and 12 are explored through contexts that build from a focus on the individual in their immediate environment through their engagement with their community and country to culminate in global contexts and applications of the mathematics.

While the writers have endeavoured to make every topic approximately an hour in length, they have also ensured that the topics are completed at a natural learning end-point. As such, depending on the teacher's choice of activities from those suggested, topics may either extend beyond this time period or require additional material to be added.

The proficiencies as outlined within the Australian Curriculum materials have been fully integrated within the lessons provided. Opportunities are provided for students to develop their *understanding* of the concepts in the course and to build *fluency* in using the mathematics they have learned. In order for students to build their *fluency* further in recognising what mathematics to use for a variety of contexts, the lessons have been interspersed with miscellaneous exercises that build cumulatively so that the final miscellaneous questions cover content from throughout the two units. Students also engage in *reasoning* through questions that require critical analysis and interpretation of information and results, as well as through justification of choices of what mathematics and processes to use. The final proficiency of *problem solving* is an integrated element of the materials as each topic culminates in a range of applications which require students to use their mathematics in either a discrete or integrated manner in order to solve problems.

A Teacher's Guide is also available that contains supplementary materials such as a sample program, sample assessment items, solutions to warm-up questions and blank proformas of relevant tables and activities to help students to complete the tasks in this student text. The sample programs and assessment items are constructed to meet the requirements of the School Curriculum and Standards Authority of Western Australia.

The Mathematics: Essential course for Western Australia also requires that students apply mathematical thinking processes to real-world problems through:

- Interpreting the task and gathering key information;
- Identifying the mathematics which could help to complete the task;
- Analysing information and data from a variety of sources;
- Applying their existing mathematical knowledge and strategies to obtain a solution;
- Verifying the reasonableness of their solutions; and
- Communicating their findings in a systematic and concise manner.

These processes are woven through the text and, where relevant and appropriate, students are given opportunities to perform them, in line with the ACARA proficiencies. Some of these processes, however, are more suitable for the teacher to incorporate as part of lessons and in student interactions, rather than through formal text questions.

Mathematics possesses not only truth, but supreme beauty …

- Bertrand Russell

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Copies of this book can be ordered directly from

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Unit 1

Good mathematics is not about how many answers you know … it's how you behave when you don't know

- Author unknown

Unit 1 syllabus coverage WA curriculum

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1.4.6																							✓	✓	✓

Pure mathematics is, in its way, the poetry of logical ideas

- Albert Einstein

Unit 1 syllabus coverage ACARA curriculum

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ACMEM036				✓	✓		✓		✓							✓		✓	✓	✓					
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ACMEM037																							✓	✓	
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ACMEM039																		✓						✓	✓
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ACMEM041																			✓	✓					
ACMEM042																							✓	✓	✓

Do not worry about your difficulties in mathematics; I can assure you mine are still greater.

It's not that I'm so smart, it's just that I stay with problems longer.

- Albert Einstein

1 Skills review

Materials required: none

Warm-up

1.	$4.50 + $3.50 + $12.85	2.	$14.00 ÷ 4
3.	2^3	4.	$\sqrt{25}$
5.	7 × 8 + 12	6.	½ of 199
7.	10% of 30	8.	1% of 30
9.	367 × 10	10.	3.67 × 10

Unit 1 in Essential Mathematics requires you to have some prior knowledge and skills. This chapter is a brief overview of skills you will need that you would have covered in previous years.

Exercises

When two or more operations are carried out, different answers can result depending on the order in which they are performed. To avoid this problem a **rule of order** was developed (also known as the **order of operations**) which, by convention, is **always used**. This is to ensure everyone will get the same answers to every problem if their arithmetic is correct.

The order in which operations are performed is:
Brackets first (sometimes called parentheses) starting inside and working outwards if brackets within brackets occur;
Indices next, (sometimes called exponents or powers) this includes roots such as square roots or cube roots;
Multiplication and **division** working left to right, as they occur;
Then **addition** and **subtraction** working left to right, as they occur.

Order of operations

1. a) 32 ÷ 8 − 4 =   b) 3 + 27 ÷ 3 =

    c) 10 ÷ 5 × 2 =   d) 4 + 2 × 5^2 =

    e) 3 × (2 + 7) =   f) 4 × $(3 + 2)^2$ =

    g) 40 ÷ 4 − 5 × 2 =   h) 36 - 4 × 5 + 6 =

    i) (4 + 11) ÷ (42 ÷ 14) =   j) 4 × (5 + (56 ÷ 8)) =

2. Insert brackets into each of these number sentences to make the answer correct:
    a)   8 + 3 × 7 = 77

    b)   24 − 8 ÷ 3 − 1 = 20

    c)   36 ÷ 2 × 6 + 6 = 2

    d)   45 − 5 × 16 − 14 × 2 = 25

    e)   144 ÷ 32 − 8 × 3 − 1 = 3

    f)   92 − 2 + 6^2 × 2 = 16

3. What are the answers to the sum below if the following operations replace the asterisks in the order they appear?
        12 * 3 * 4 * 8 =

    a)   + × -

    b)   × - ×

    c)   ÷ + ×

    d)   ÷ × +

    e)   - + ×

Estimation
It is not always possible or convenient to work out or give the exact answer to a problem. Often in these cases an estimation of the actual answer is required. An estimate is not a guess, but a quick approximation of the answer. We use the ≈ symbol to indicate the answer is an approximation.

One method for estimating is to round to the first or leading digit and put zeroes in the other places.

For example:

345 × 277   ≈ 300 × 300
            ≈ 90 000

The actual answer is 95 565 but the estimation gives us a rough idea of the magnitude of the answer we are dealing with.

Use the leading digits to estimate the following:

4. a)   23 × 34   b)   79 × 81   c)   111 × 49
    d)   147 × 67   e)   2 308 × 32   f)   9 858 × 277
    g)   721 ÷ 68   h)   444 ÷ 205   i)   5 799 ÷ 289
    j)   10 006 ÷ 466   k)   8 976 ÷ 933   l)   120 537 ÷ 409

Rounding

Some answers do not require the level of accuracy our calculations give us. In these cases, it is appropriate to round our answers to a given level of accuracy. For example, if measuring in metres, answers rounded to 3 decimal places give the answer to the nearest millimetre, which is not likely to be accurate as the original measurements weren't to the nearest millimetre. When rounding, like answering any question, you should always consider the context first. If the context doesn't give you enough information, then round to the closest number using the required level of accuracy. To find the closest value, a convention we will use is:

decide which is the last digit required;
- leave it the same if the next digit to the right is less than 5 or
- increase it by 1 if the next digit to the right is 5 or more.

e.g.   135.3 = 100 if rounded to the nearest hundred

= 140 if rounded to the nearest ten

= 135 if rounded to the nearest whole number

5.   Round the following to the nearest ten:
a)   346   b)   87   c)   12   d)   995

6.   Round the following to the nearest whole number:
a)   8.6   b)   29.3   c)   1 001.7   d)   99.5

7.   Round the following to one decimal place:
a)   2.345   b)   9.04   c)   11.11   d)   99.4

8.   Round the following to two decimal places:
a)   6.66666   b)   435.0076

c)   208.022   d)   1 999.95

9.   A school hired buses to take students to an interschool sports carnival. Each bus could hold 43 students and there were 138 students attending the carnival. How many buses were hired?

10.   There are 24 small packets of chips in a party pack. They are to be divided evenly amongst 5 groups. How many bags does each group get?

11.   A parent is ordering some ribbon to make costumes for his daughter's upcoming dance performance. He requires 28 metres and the ribbon is sold in rolls of 5 metres. How many rolls of ribbon does he need to buy?

12.   To make a toy box, a carpenter needs 32 counter sunk screws. The screws are sold in packets of 25. How many packets of screws does she need to buy?

Converting fractions

Fractions and decimals are used to represent parts of a whole, as well as being numbers. Percentages are also used to represent a proportion of an amount (divide it into 100 equal parts and then the percentage given is how many of those parts are required).

There are equivalences between each of these forms. For example, the fraction one half ($^1/_2$) is the same as the decimal 0.5 or the percentage 50%.

As a percentage means to divide into 100 equal parts, each one per cent is $^1/_{100}$ as a fraction or 0.01 as a decimal.

To convert from decimals to fractions, use place value to write the decimal as a fraction and then simplify. To convert decimals to percentages, use place value to determine how many hundredths there are.

To convert a fraction to a decimal, convert the fraction to have a denominator that indicates place value (ie a power of 10: 10, 100, 1000 …). The same process is used to convert to a percentage – once the decimal form is found, use place value to decide how many hundredths there are.

13. Complete the table, showing the equivalent fraction, decimal or percentage, as appropriate, for each row.

Fraction	Decimal	Percentage
	0.1	
	0.125	
$^1/_5$		
		25%
$^1/_3$		
	0.5	
$^3/_5$		
		$66.\overline{6}\%$
		75%
	0.8	
$^7/_8$		
$^9/_{10}$		
$^{11}/_{10}$		
		150%
	2.3	

2 Thinking Mathematically

Warm-up

1.	7.5 + 8.23	2.	12.4 + 0.03
3.	What are the next 3 numbers? 96, 97, 98, ___, ___, ___	4.	What are the next 3 numbers? 196, 197, 198, ___, ___, ___
5.	What are the next 3 numbers? 1096, 1097, 1098, ___, ___, ___	6.	8 teams entered a '3-legged race' – how many 'legs' raced?
7.	How many vertices are on a triangular pyramid?	8.	How many rectangular faces are on a triangular prism?
9.	What is the perimeter of a square with sides of 3cm?	10.	What is the area of a square with sides of 3cm?

Discussion

In this course, and in life beyond school, it is important to be able to apply mathematical thinking processes to real-world problems through:

- Interpreting the task and gathering key information;
- Identifying the mathematics which could help to complete the task;
- Analysing information and data from a variety of sources;
- Applying existing mathematical knowledge and strategies to obtain a solution;
- Verifying the reasonableness of solutions; and
- Communicating findings in a systematic and concise manner.

This is the process of thinking mathematically and is an essential component of numeracy.

In this lesson, we will learn about each of these separately, but through the rest of the book and this course you will regularly be asked questions that refine your skills in each of these areas. When solving problems it is expected that you can use this mathematical thinking process independently.

Interpreting the task and gathering key information

When given a question to answer, it is important that you can show you understand what you are trying to find out and what information you need to answer it. For example, 'I want to buy a new TV – which one is the best value for money?'

You will need to establish several things to solve the problem, including:

- What television options are there?
- What do different televisions cost?
- How can they be compared?
- What needs to be considered in judging value for money?
- Are there any restrictions, e.g. budget, available space, wall mounted vs free standing etc.?

Now, as often happens in real life, this question is very broad and general.

To be sure of making the best possible choice you would need to establish criteria to judge value for money and then assess every available television from every store in your city or state or online against the criteria. That's not practical, so you may want to limit it to just those stores that are close to home, or even choose a specific store and compare the models that they stock. In other words, you need to **interpret** the task, make it **simpler** and achievable, and then **gather** the key information.

Discussion:
- What factors influence people's choices of televisions?
- Where can you find out information about the quality of different brands of televisions?
- Where can you buy a television?
- Where can you find out information about the costs of different sizes/models/types of televisions?

Identifying the mathematics

Being numerate means, among other things, that you know what mathematics to use and when, in order to solve real-life problems.

In the television example, an appropriate way to proceed might be to decide on a brand and type of television through research using the internet regarding brand and product reviews. Once a model is chosen, a catalogue or online shopping sites can be used to identify several different sizes and their costs.

Discussion: Television sizes are given as a linear measure. How does this measurement relate to the size of the screen?

Discussion: How could you judge value for money when choosing a television to purchase? What mathematics is used in each of the methods discussed? Which method, of those discussed, would you choose to use and how could you justify it as providing 'the best value for money' to someone else? (Hint: what information is given in supermarkets to enable shoppers to compare value for money when buying groceries and is there a way you could apply that to television costs?)

Analysing information from a variety of sources

Often, someone else will have thought of the same question and done some work to solve it before you, or at least have some information you could use. If using the internet to do your research, you should try to find data from multiple reputable sources, as not all internet sources are accurate and reliable.

When looking to buy a television, if you want to ensure you are getting the best value for money, you should look at information from multiple stores – checking online is an easy way to do this. If you decide to measure value for money as a rate per unit of area, you need to look at the detailed product information in online stores to find the screen dimensions. Most televisions are wide-screen and have a dimensional screen ratio of 16:9. If the screen dimensions aren't given, they can be worked out using several different mathematical methods, but most of these use mathematics that are not part of Unit 1's content. If all the televisions are the same screen ratio, though, then the linear measure is enough to compare value – you would just need to calculate it as a cost per unit of length (e.g. $/cm or $/10cm) rather than as a cost per unit of area (such as $/cm^2).

Applying the mathematics to obtain a solution

Once you have determined how you are going to answer the question, then you can actually do it. More complex questions will require more details and a more extended thinking process. Practising it for simpler questions will make it easier to do for complex ones.

In the television example, in the previous steps you would have carried out the process of choosing a type of television (eg HD), a make or two that are reputable and a range of sizes that you would consider. You also would have identified the stores whose televisions you will compare and sourced the costs and dimensions of the appropriate television sets. With all of this information you can then calculate the value for money as a rate per unit of either length or area for each model of television.

Verifying the reasonableness of your solutions

As you do your calculations or summarise your findings, you should look at your answers to see whether they 'make sense'. Sometimes this is about using your number sense to estimate the answer to a calculation so that you can recognise if your answer is incorrect. More often, it is about looking at the answer you obtained within the context of the original problem, to see if it is logical and meaningful. In the television example, this would be recognising that a cost per square centimetre of $46 is not reasonable if all other rates are between $4 and $10 per square centimetre. You might also find that the best value for money is a television that is 40cm wide and 22.5cm high. If you plan to put the television in a room that is 8m wide and 10m long, it would not be suitable as the screen is too small to be visible for people sitting on the other side of the room.

Communicating your findings

Once you have found the answer to the problem, you need to be prepared to share that answer with others. Part of working like a mathematician means you need to give the final answer with a justification for how you found it. For some problems, it's just a matter of showing your working out; for others, you need to explain the process you followed so that others can judge the validity of your findings. This doesn't mean that you need to write an essay, just a systematic and concise summary of the steps you followed. You could also be expected to justify your answer verbally.

In the example about television recommendations, however, a description of the process that includes your reasons for choosing the makes and sizes, the stores you sourced the information from, a summary of the results for each model and a brief analysis of those results that gives an answer to the question may be required.

You might also want to give a recommendation about how to proceed if you were purchasing. For example, if you gave a recommended make, model and size of television as one appropriate for the available space and recommended negotiating with the seller to achieve a better price, you could give a recommended maximum price to ensure it gives the best value for money of the models examined. The key concern with the mathematical thinking process is that your answer is realistic for the context.

Exercises

Interpreting the task and gathering key information

For the following questions, clarify the information you are being asked to analyse and identify how you could find an answer:

1. A local bakery makes cakes in three sizes – small, medium and large. All of them are square prisms and all have a height of 10cm and a square top face. The small one has a length and width of 10cm, the medium has a length and width of 20cm and the large size has a length and width of 30cm. The chocolate mud cake in the three sizes cost $20, $40 and $60 respectively. Which one is the best value for money?

2. If you were going to change the floor covering in your classroom, list some different options and what they would cost.

3. Design a menu for a day that meets your average daily energy requirements and is healthy. How could it be changed if you wanted to lose weight healthily? What if you wanted to gain weight healthily?

4. As part of the Stephanie Alexander Kitchen Garden project, your school has decided to create an herb and vegetable garden that is 5m by 5m. Choose suitable plants, research their planting requirements and design the layout of the garden.

5. Your class wishes to raise $500 to purchase some equipment for a school in a developing country. How many cupcakes would you need to make and sell to give $500 profit?

6. Choose an area in the grounds of your school and design a layout including outdoor furniture and plants that would make it suitable and attractive for you and your friends to have lunch there.

7. You and a friend have decided to move out of home. You want to set up your kitchen pantry with some basic items. What would you purchase and how much should you set aside as your budget for this purpose?

8. You are organising a family barbecue that you expect 25 people (17 adults and 8 teenagers) to attend. You expect them to be there from 11am until 4pm. Plan a menu that will feed everyone (snacks, lunch and non-alcoholic drinks included) within a budget of $150. If your budget was doubled, how would you change the menu?

9. Plan an appropriate fortnightly budget for a person who rents a property in your suburb who earns minimum wage.

Extended Problem Solving

Complete the television task, following the mathematical thinking process to find and explain an answer to it.

Mathematics is the most beautiful and most powerful creation of the human spirit.

- Stefan Banach

3 Converting units

Materials required: rulers, measuring tapes, trundle wheels, calculators for the final question

Warm-up

1.	24 - 5 × 3	2.	$(4 + 8)^2 -22$
3.	$((4 + 5)^2 ÷ 3)$	4.	$((4 + 5) ÷ 3)^2$
5.	Estimate 222 × 61	6.	Estimate 89 × 171
7.	Round 126 to nearest 10	8.	Round 83 231 to nearest 1000
9.	$35.50 - $17.60	10.	$28.50 + $35 + $102.75

Discussion

Australia, and most parts of the world, uses the Metric system. Discuss what you know of the Metric system, which countries use it and which don't use it.

What metric prefixes do you know and where or when are they used? Try to put them in order from the smallest to largest.

Common metric prefixes

m = milli $\dfrac{1}{1000}$

c = centi $\dfrac{1}{100}$

k =kilo   1 000

M = Mega   1 000 000

G = Giga   1 000 000 000

Length/height	10 mm = 1 cm
	100 cm = 1 m
	1000 m = 1 km

Weight/ mass	1000 mg = 1 g
	1000 g = 1 kg
	1000 kg = 1 tonne

Capacity	1 000 mL = 1 Litre
	1 000 L = 1 kL
	1 000 000kL = 1 GL

The units of measurement used depend on the level of accuracy required and the level of precision of the measuring device.

Discuss the level of accuracy possible using a:

- 30cm ruler;
- trundle wheel;
- measuring tape;
- odometer;
- kitchen scale;
- bathroom scale.

Exercises

1. Convert the following lengths:

    a) 5m = ____ cm

    b) 5mm = ____ cm

    c) 345mm = ____ cm

    d) 12.5m = ____ cm

    e) 1 km = ____ m

    f) 0.2 km = ____ m

    g) 0.23m = ____ mm

    h) 1km 45m = ____ m

    i) 580m = ____ km

    j) 12 050m = ____ km

2. Estimate the following lengths and then measure them to see how accurate you were:

    a) _____

    b) (diagonal line segment)

    c) –

    d) The depth of this book.

    e) The length of your classroom.

    f) The distance from your classroom door to your school canteen.

    g) The distance from your school's administration office to the school oval.

    h) The distance from your home to your school (use an online map to find out the actual distance).

    i) The distance from your town or city to Alice Springs (use an online map to find the actual distance by road).

3. Convert the following weights:

    a) 2kg = _____g

    b) 0.5kg = ____g

    c) 850g = ___kg

    d) 1225g = ____kg

    e) 250kg = ____tonnes

    f) 3350kg = _____tonnes

    g) ½ tonne = ___kg

    h) 13.6 tonnes = ___kg

    i) 1 tonnes = ____g

    j) 89750 g = _____tonnes

4.   Estimate the mass of the following:

   a)   an apple   b)   a can of soft drink

   c)   a dachshund   d)   a horse

   e)   a small car   f)   an elephant

   g)   a feather   h)   a bucket of water

   i)   a suit of armour   j)   a 3-seater couch

5.   A person measures to find the lengths of three planks of wood that
     are required to complete a job. The measurements are 1.5m, 80cm
     and 120cm. He gets to the store and finds that the wood is sold in
     lengths measured in mm.

   a)   What total length of wood in mm does he require?

   b)   The wood is only sold in 1200mm, 1800mm or 2400mm
        lengths. What lengths should he buy to minimise cutting and
        waste?

6.   Jaime is baking apple pies for a school fundraiser. To make one pie,
     the recipe needs 700g of apples, 220g of flour, 25g of sugar and
     110g of butter. Jaime intends to make 5 pies. The flour is sold in
     1kg bags, the sugar in 500g bags and the butter in 250g containers.
     Apples are sold by weight, so decimal amounts can be bought.

   a)   How much of each ingredient does she need to buy (weight)?
        How many bags/containers of each ingredient would this be?

   b)   How much of each ingredient should she have left over?

7.   A carton of 375mL cans of soft drink has 30 cans.

   a)   How many litres is this?

   b)   How many cartons would need to be bought to have at least
        20 litres of soft drink?

8.   In Perth, the average water use is 280 litres per person per day.

   a)   If a household has 4 people, what would be the expected
        water consumption in a year?

   b)   If the household also had a pool which used an additional
        35 000 litres in a year, what would the daily water usage per
        person be?

   c)   If the population of Perth is 1.9 million what would the yearly
        water usage of the population be?

4 About you

Materials required: Measuring tapes or rulers, calculators

Warm-up

1.	3 + 4 × 5	2.	12 ÷ 6 × 2
3.	Round 1256 to nearest 100	4.	Round 1256 to nearest 1000
5.	Round 12.56 to nearest whole number	6.	How many metres in 1 km?
7.	How many metres in 1.6 km?	8.	3 kg = _____ grams
9.	Estimate 21 × 39	10.	Estimate 99 × 121

Discussion

60 seconds = 1 minute   60 minutes = 1 hour

24 hours = 1 day   7 days = 1 week

2 weeks = 1 fortnight   52 weeks = 1 year

365 days = 1 year   366 days = 1 leap year

Discuss why we have numbers like 24 hours in a day and 365 days in a year, whereas most measurements we use in the metric system are base 10 or powers of 10.

Exercises

1. a) Have you been alive for 1 million minutes?
      Calculate how many minutes you have been alive. Don't forget leap years.
   b) What date will you reach a million minutes or the next multiple of a million minutes?
   c) How old were you, or will you be, when you had been alive for one billion seconds?

2. You blink somewhere between 4 and 50 times a minute. On average people will blink about 12 times a minute.

   a) How many seconds between blinks is this on average?

   When reading or concentrating on a computer screen your blink rate decreases to about 4 times a minute.

   b) How many seconds between blinks is this on average?

3. a) Use a measuring tape to measure how tall you are.
        Give your answer in mm, cm and metres.

    As a rule of thumb, the length of the human head is about 1/8 of a person's total height.

    b) Measure the height of your head from your chin to the top of your head, how close is this to 1/8 of your height?

    Measure and check these other rules of thumb, and comment on their accuracy:

    c) Your forearm is about the same length as your foot.

    d) Your arm span is equal to your height.

    e) Your eyes are located halfway down your head.

4. How much do you weigh? If you prefer, use an estimated weight for someone your height and gender.

    a) Give your answer in grams and kilograms.

    b) About 7% of your body weight is blood. What is the weight of the blood in your body?

    c) (i)   Your head is about 8% of your total body weight.

              How heavy would your head be?

        (ii) Would this give a higher or lower percentage if you gained 20kg?

    d) Your skin is the largest organ in the human body and accounts for about 16% of your total weight. How much is this?

    e) (i)   An average brain weighs about 1200 grams. What is this as a percentage of your total weight?

        (ii) Would taller people have bigger brains?

        (iii) Assuming people's brains are basically the same size regardless of height or weight, whose brain would be the larger percentage of their body weight: a tall person or a short person?

Anyone who has never made a mistake has never tried anything new

- Albert Einstein

Extended Problem Solving

On what day of the week were you born?

To calculate the day of the week you were born, we can use the following formula:

$$h = \left(q + \left[\frac{13(m+1)}{5}\right] + K + \left[\frac{K}{4}\right] + 5 - J\right) \bmod 7,$$

where

h is the day of the week (0 = Saturday, 1 = Sunday, 2 = Monday, …)
q is the day of the month
m is the month (3 = March, 4 = April, 5 = May, …, 14 = February)
K is the year of the century (for example, in 1995, the year would be 95)
J is the century (for example, in 1995, the century would be 19, even though it was the 20th century.)

Confused?

Follow these steps:
What month is your birthday (as a number 3-14)?
What day of the month were you born (as a number 1-31)?
What are the last two digits of the year you were born?
e.g. 1999 would be 99

Follow these steps, recording your answer after each line.
• Add one to your month number then multiply by 13
• Divide this number by 5
• Add the day of the month on which you were born
• Add 5
• Multiply the last two digits of the year you were born by 1.25 and add it to your last answer
• Subtract the century you were born
• What is the remainder when this number is divided by 7 (to the nearest whole number)?
• Answer (0 = Saturday, 1 = Sunday, 2 = Monday, …)

On what day of the week were you born?

5 Heart rate

Materials required: A watch or clock that shows seconds, calculators

Warm-up

1.	$5 \times 6 - 3 \times 9$	2.	$5^2 - 4 \div 2$
3.	Round 923 to the nearest 100	4.	Round 923 to the nearest 10
5.	Estimate 32×51 using leading digits	6.	0.6 km = _____ m
7.	0.65 m = _____ mm	8.	What is 10% of 1 200 grams?
9.	Round 99.51 to the nearest 100	10.	Round 99.51 to the nearest whole number

Activity

Your heart rate is the number of times your heart beats every minute. This is measured in beats per minute (bpm) sometimes called your pulse rate. You can measure your heart beat by placing your index finger on the inside of your wrist just below your thumb.
Count the number of beats you feel in 30 seconds.
Double this to record your beats per minute.

_____ bpm

The average resting heart rate is between 60 and 80 beats per minute.

Exercises

1. Based on your (resting) heart rate recorded in the introductory activity, how many times would your heart beat in:

   a) 1 hour?
   b) 1 day?
   c) 1 year?

2. An average of 70 mL are pumped from the heart each heartbeat.

   a) How much blood is pumped by your heart in:
      (i) 1 hour?
      (ii) 1 day?
      (iii) 1 year?

   b) If you lived to the age of 80 how many litres of blood would your heart have pumped?

    c)   Which is the closest to the volume of blood pumped by your heart in your lifetime (assuming an average lifespan)?

        (i)   A 2 litre drink bottle?

        (ii)   A 300 litre bath?

        (iii)   A 2 500 litre rain water tank?

        (iv)   A backyard swimming pool ≈ 180 kilolitres?

        (v)   An Olympic sized (50 m) swimming pool ≈ 2.5 ML?

3.   There are various formulae for calculating your maximum heart rate. Below are two that are widely used.
Maximum Heart Rate = 220 – your age.
Maximum Heart Rate = 208 – 0.7 × your age

    a)   Calculate your maximum heart rate using both formulae.

    b)   What is the difference between the two measurements?

    c)   If a person were 40 years of age what would their maximum heart rate be, using both measurements?

    d)   If a person were 57 years of age what would their maximum heart rate be?

    e)   Describe what happens to a person's maximum heart rate as they get older.

    f)   Comment on the differences between the two measurements. Is there an age at which they give the same result?

4.   For healthy persons, the *Target Heart Rate* is a desired range of heart rate reached during exercise which enables one's heart and lungs to receive the most benefit from a workout. This range varies based mostly on age; however, a person's physical condition, sex, and previous training are also used in the calculation.

    The target heart rate for an aerobic workout varies, but as a guide should be between 50% and 75% of your maximum heart rate. Rounding your answers to the nearest whole number, use the first formula to calculate:

    a)   your target heart rate range.

    b)   the target heart rate range of a 32 year old.

    c)   the target heart rate range of a 45 year old.

    d)   the target heart rate range of a 65 year old.

Extended Problem Solving

Different animals have different heart rates.
1. Complete the table below, comparing different creatures' heart rates.

Creature	Average heart rate (beats per minute)	Average heart rate (beats per hour)	Average heart rate (beats per day)	Average heart rate (beats per year)
Human	70			
Cat	150			
Medium dog		5400		
Rabbit	205		295 200	
Elephant	30		43 200	
Giraffe				34 164 000
Large whales				10 512 000

Often, the larger an animal is, the slower its heart rate.

2. Is this statement true based on the information in the table above?

Larger animals tend to live longer than smaller animals.

3. Copy the average heart rates in beats per year into the table below and use them to calculate the number of heart beats in a lifetime.

Creature	Average heart rate (beats per year)	Average life expectancy	Number of heart beats in a lifetime
Human		80	
Cat		15	
Medium dog		15	
Rabbit		9	
Elephant		70	
Giraffe	34 164 000	25	
Large whales	10 512 000	80	

4. Which of the animals listed has the greatest number of heart beats in their lifetime?

5. Which animal has the fewest heart beats in their lifetime?

6. Calculate the average life expectancy of the above animals if each of the animals had 750 000 000 heart beats in their life.

7. Compare the life expectancies calculated in question 6 to those listed in question 3. What do you notice?

6 Rates

Materials required: Measuring tape, stop watch, calculators

Warm-up

1.	Round $2.79 to the nearest 5 cents	2.	Round $2.79 to the nearest dollar
3.	How many cm in 2.5m?	4.	2.5mm = _____cm
5.	How many minutes in 0.6 of an hour?	6.	What is 10% of $15
7.	$3^3 - 2^3 + 1^3 =$	8.	$2 + 8 \times (14 - 6) =$
9.	Estimate 44×38	10.	$4.5 \div (12.5 - 11) =$

Discussion

A rate compares two related quantities that have different units.
For example, kilometres per hour (km/hr) is a rate that compares distance travelled (kilometres) to time taken (hours).

The following are all types of rates. Discuss where each could be used.

Words per minute   Per cent per annum

Beats per minute   Dollars per kilogram

Megabytes per second   Dollars per hour

Dollars per metre

Exercises

1. A copy machine makes 45 copies in 25 seconds.
    a) Find the unit rate of copies per second.
    b) How many copies could be made in 5 minutes?
    c) How long would it take for the copier to make 100 copies?

2. A car travels at 90km/hr. How far could the car travel in:
    a) 30 minutes?
    b) 3½ hours?

    How long would it take the same car to travel:
    c) 120km?
    d) 225km?
    e) 25km?

3. One kilo of oranges cost $3.99/kg.
    a) How much would 1.5 kilograms cost?
    b) How many kilograms could you buy for $10?
    c) If 3.5kg of limes cost $15.75, how much would 1 kg cost?

4. A communication satellite orbits the earth. In 24 hours it completes one revolution, covering a distance of approximately 26 500km. In km/s, how fast is it travelling?

5. The standard message used to compare texting ability is the long and difficult passage:

    "The razor-toothed piranhas of the genera Serrasalmus and Pygocentrus are the most ferocious freshwater fish in the world. In reality they seldom attack a human."

    The fastest completion of this 160-character text message is 34.65 seconds and was achieved by Frode Ness (Norway) at the Norwegian SMS championships held at the Oslo City shopping centre in Oslo, Østlandet, Norway, on 13 November 2010.

    a) How many characters per second did Frode type?
    b) How many words in the message?
    c) How many words per second was the winning speed?

6. Speed readers can read 500 words per minute. If a book has around 250 words per page and 300 pages, how long would it take a speed reader to read 1 page?

    How long would it take to read the entire book?

7. Measure out 10 metres and time how long it takes you to walk it. Calculate your walking speed in m/s and km/hr. How long would it take you to walk from your house to school at this speed?

8. a) In 2013, the world record for the 1500 metres run was 3 minutes and 26 seconds. How fast is this in km/hr?
    b) Usain Bolt ran the 100 metres in 9.58 seconds. What was his average speed?
    c) A cheetah has been clocked at 120 km/hr for short bursts.
        (i)   If it could maintain this speed, how fast would it run the 100m sprint?
        (ii) If Usain Bolt and a cheetah were to start at the same spot at the same time, how far ahead of Bolt would the Cheetah be by the time it covered 100 metres?
        (iii) How many seconds head start does Bolt need for the race to be a draw?

7 Basic Metabolic Rate (BMR)

Materials required: calculators and includes an opportunity to use a spread sheet application

Warm-up

1. (6 × (5+4)) ÷ 3	2. 48 − (11 − 5)²
3. 23cm = _____ m	4. 4.8 m = _____ mm
5. How many g in ½ kg	6. 10% of $20.50
7. 25% of $120.40	8. Distance travelled at 12km/hr for 4 hours?
9. Distance travelled at 12km/hr for 20 minutes?	10. How long to travel 24km at 80km/hr?

Discussion

Energy is the capacity for a body to do work. Our body gets energy from food. The amount of energy in food is measured in kilojoules or kilocalories. You need enough energy to meet the demands of your Basic Metabolic Rate (BMR) and Physical Activity Level (PAL).

Your **BMR** is the minimum amount of energy, measured in kilojoules, that your body requires to complete its normal functions, such as breathing, breaking down food, and keeping your heart and brain working.

Your **PAL** is the number of kilojoules you use to fuel all of your physical activity.

Bigger people generally need more energy for BMR. Athletes in endurance events and hard training need more energy for their PAL.

BMR

Basic Metabolic Rate (BMR) can be calculated using the following formulae. Gender, age, height (cm) and weight (kg) are important for this calculation.

Female   BMR = 655 + (9.6 × weight) + (1.8 × height) - (4.7 × age)
Male   BMR = 66 + (13.7 × weight) + (5 × height) - (6.8 × age)

This gives a result measured in Calories, so to convert Calories to Kilojoules, we multiply by 4.182.

Example:
An 18 year old male that weighs 75kg and is 180cm tall

BMR = 66 + (13.7 × weight) + (5 × height) – (6.8 × age)
    = 66 + (13.7 × 75) + (5 × 180) – (6.8 × 18)
    = 1 790 Calories
1790 Calories × 4.182 = 7485.78 kilojoules

Exercises

1. From the equation for BMR above what effect does an increase in age have on the BMR of both males and females?

2. Calculate your BMR (in kilojoules).

3. Calculate the BMR (in kilojoules) for the following people:
   a) (i) a 165cm 17 year old female weighing 55kg
      (ii) a 165cm 17 year old male weighing 55kg
   b) (i) a 175cm 56 year old female weighing 70kg
      (ii) a 175cm 56 year old male weighing 70kg
   c) (i) a 155cm 80 year old female weighing 50kg
      (ii) a 155cm 80 year old male weighing 50kg

4. Comment on differences between the BMRs for males and females.

Below is a table showing the average height of males and females across different age groups.

	Male average			Female average	
	Height (cm)	Weight (kg)		Height (cm)	Weight (kg)
1 year	75.7	9.7		74.2	9.3
2 years	86.9	12.5		85.1	12.0
3 years	95.3	14.1		94.0	14.3
4 years	102.4	18.4		100.3	15.4
5 years	109.2	18.4		108.0	17.9
6 years	115.6	20.6		115.6	20.0
7 years	121.9	22.9		121.2	22.5
8 years	128.0	25.6		128.3	25.9
9 years	133.4	28.6		133.4	28.1
10 years	138.4	32.0		138.4	32.0
11 years	143.5	35.6		144.0	37.0
12-13 years	152.4	42.6		154.9	43.5
14-15 years	166.4	53.1		158.8	49.9
16-17 years	174.0	62.6		162.6	54.0
18-20 years	177.8	68.5		162.6	57.6

5.   What happens to a person's weight and height as they get older?

6.   On average, at what age does a male reach maximum height?

7.   On average, at what age does a female reach maximum height?

A rule of thumb says you can predict the adult height of a child if you double their height at age 2.

8.   a)   Double the average height of a male at age two.
      b)   How does this compare to the average adult height?
      c)   Does the rule seem to work? If not, what age would be better to use?

9.   a)   Double the height of a female at age two.
      b)   How does this compare to the average adult height?
      c)   Does the rule seem to work? If not, what age would be better to use?

10.   In what age range do you think a person has the highest BMR? Why?

11.   A breakdown of your body's energy expenditure is detailed below.

      Energy expenditure breakdown

Liver	27%
Brain	19%
Other Organs	19%
Skeletal Muscle	18%
Kidneys	10%
Heart	7%

      How much energy does each of these organs use of your BMR?

Extended Problem Solving

Plot the BMR of males and females on the graph below for each of the age ranges in the table in question 10.

Average BMR at different ages

BMR (y-axis): 0, 500, 1000, 1500, 2000, 2500, 3000

Age (x-axis): 1 year, 2 years, 3 years, 4 years, 5 years, 6 years, 7 years, 8 years, 9 years, 10 years, 11 years, 12-13 years, 14-15 years, 16-17 years, 18-20 years
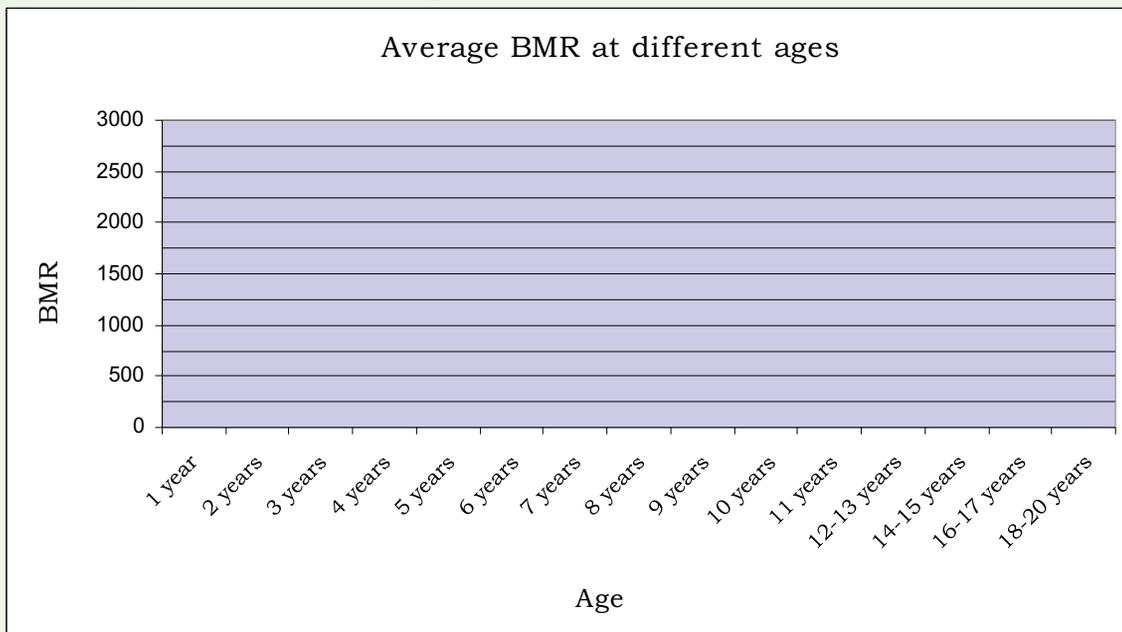
Comment on any trends that you notice.

Q: John has 32 candy bars. He eats 28 of them. What does John have now?

A: Diabetes

8 Total energy expenditure

Materials required: calculators

Warm-up

1.	$16 \div 4 \times 2$	2.	2^6
3.	$4 \div \frac{1}{2}$	4.	$4 \times \frac{1}{2}$
5.	$\frac{1}{2} \div \frac{1}{4}$	6.	10% of 65kg
7.	90% of 65kg	8.	150% of 65kg
9.	A square has a perimeter of 16cm. What's the area?	10.	A square has an area of 25cm². What's the perimeter?

Introductory problem

Your total energy expenditure is the average energy you expend in a 24-hour period. This is equal to your Basic Metabolic Rate (BMR) adjusted for your Physical Activity Level (PAL). Your PAL determines the multiplier you need to use to work out the total energy expended.

In the previous section you calculated your BMR. Divide this quantity by 24 to work out your BMR as an hourly rate.

Record the amount of time you spent doing the following activities over the last 24 hours (you should have all 24 hours accounted for).

Sleep:

Sitting:

Light exercise:

Moderate exercise:

Heavy exercise:

Multiply your hourly BMR by the number of hours spent in each of these activities.
Then multiply sleep by 1 (this means your metabolism is the only energy you use), sitting by 1.5, light exercise by 3.2, moderate exercise by 5.5 and heavy exercise by 30. The Physical Activity Level (PAL) determines the multiplier for the type of exercise.

Add these values together to calculate your Total Energy Expenditure for the last 24 hours.

Exercises

Below is a breakdown of some activities and their energy multipliers:

Physical Activity Level	Multiplier
Light Activities	
Sleep	1.0
Sitting/ standing	1.5
Walking slowly	3.0
Exercise (light)	3.2
Moderate Activities	
Vacuuming, sweeping, or cleaning house	4.0
Walking fast	4.6
Dancing (moderate speed)	5.0
Exercise (moderate)	5.5
Swimming (recreationally)	7.5
Cycling (moderate)	8.0
Jogging	8.6
Heavy Activities	
Dancing (fast)	15.0
Gardening (digging)	19.8
Cycling (racing)	20.0
Horseback riding (hard)	22.0
Swimming (racing)	25.0
Exercise (heavy)	30.0
Football (rugby)	43.8

Calculate the total energy expended by a person who has a daily BMR of 6400kJ who does:
1. 30 minutes digging in the garden
   2 hours slow walking
   20 minutes dancing at moderate speed
   8 hours sleeping and
   the rest of the day sitting or standing

2. 90 minutes hard horseback riding
   1 hour of house cleaning
   3 hours sitting at the computer
   6 hours sleeping and
   the rest of the day doing light exercise

3. 2 hours of rugby
   30 minutes of light exercise
   1 hour moderate cycling
   1 hour vacuuming
   7 hours sleeping and
   the rest of the day sitting or standing

Extended Problem Solving

Record the amount of time you spend doing various activities over a period of a week. Make sure every minute is accounted for by classifying the level of exercise in some way.

Place each of the activities into one of the categories listed in the table above (or approximate the multiplier if it doesn't match exactly).

Using your BMR that you calculated in the previous section, calculate your total energy expenditure for the week.

Music is the pleasure the human mind experiences from counting without being aware that it is counting.

- Gottfried Leibniz

Education is not the learning of facts, but the training of the mind to think.

- Albert Einstein

9 Energy from food

Materials required: calculators, there is an opportunity to use this lesson to look at using formulas in spread sheets

Warm-up

1.	$4 \times ((4 \times 4) - 4)$	2.	$4^3 - 5$
3.	3.2 kg = _____ grams	4.	0.2 tonnes = _____ kg
5.	Round $3.22 to nearest 5c	6.	87g = _____ kg
7.	5800 Joules = _____ kJ	8.	0.05Kg = _____ g
9.	10% of 6500kJ	10.	90% of 6500kJ

Discussion

You get your energy from what you eat and what you drink.

The energy we get from different foods is calculated from the amount of protein, fat and carbohydrates in each.
The energy values are 17 kJ/g for protein, 37 kJ/g for fat and 17 kJ/g for carbohydrates. The Atwater general system also includes alcohol with a rounded value of 29 kJ/g.
This can be written as:

$$E = 17p + 37f + 17c \text{ where}$$

p = protein in grams
f = fat in grams
c = carbohydrates in grams

To calculate the energy available from a food, multiply the number of grams of carbohydrate, protein, and fat by 17, 37, and 17, respectively. Then add the results together.

For example a medium sized apple (125 grams) has 0g protein, 0g fat and 17g of carbohydrates:

$17 \times 0 + 37 \times 0 + 17 \times 17 = 289$ kJ

A guide for the energy from a medium chocolate bar (100g) varies but can have 11g protein, 29.5 g fat and 53 g carbohydrates:

$17 \times 11 + 37 \times 29.5 + 17 \times 53 = 2\ 179.5$ kJ

Discuss which foods and drinks you think give us the most energy. Would you consider these to be good for you?

Exercises

1. Calculate the energy contained in each of these foods/beverages

	Protein (grams)	Fat (grams)	Carbohydrates (grams)	Energy
200mL of coffee, with full cream milk	8.4	9.6	12	
1 teaspoon of sugar	0	0	4	
Banana (110g)	1.9	0.1	21.8	
Hamburger w. cheese, onion, tomato sauce	19.8	23.2	59.2	
Soft drink (330mL)	0	0	35	
Garden salad	1.2	0.1	1.5	
2 slices of toasted white bread	6.2	1.3	32.5	
1 tsp butter	0	5.7	0.1	
1 tsp vegemite	0.8	0	0.6	
fish(grilled)	25	3	0	
chips (110g)	4	15	28	

2   Use the table above to determine the total kilojoules in:
   a)   a cup of coffee with 2 teaspoons of sugar.
   b)   a hamburger, soft drink and a serve of chips
   c)   a garden salad with 600mL of water

3. A suggested breakdown of our daily energy intake is:
   Breakfast   20%
   Morning snack   10%
   Lunch   20%
   Afternoon snack   10%
   Dinner   40%

   a)   If a person requires 8700kJ of energy during the day, how much energy should be in each of the meals?

   b)   Using your daily energy requirements calculated earlier what is the breakdown of your daily energy intake?

   c)   List the foods you have consumed in the last 24 hours and place them into each of the five categories. How closely does your daily energy intake match the one suggested above?

4. Energy is not the only thing that is required to maintain a healthy diet. List some other things you might consider when choosing what to eat.

5. The nutritional value of a standard serve of cooked green peas (1 cup ≈ 137.75 grams) is listed in the table below. The table shows the amount of each nutrient it contains and expresses it as a percentage of the recommended daily intake (RDI).

Nutrient	Amount	RDI (%)	
Vitamin K	35.68 mcg	39.6	
Manganese	0.72 mg	36.0	
Fibre	7.58 g	30.3	
Vitamin B1	0.36 mg	30.0	
Copper	0.24 mg	26.7	
Vitamin C	19.56 mg	26.1	
Phosphorus	161.17 mg	23.0	
Folate	86.78 mcg	21.7	
Vitamin B6	0.30 mg	17.6	
Vitamin B3	2.78 mg	17.4	
Vitamin B2	0.21 mg	16.2	
Molybdenum	6.89 mcg	15.3	
Zinc	1.64 mg	14.9	
Protein	7.38 g	14.8	
Magnesium	53.72 mg	13.4	
Iron	2.12 mg	11.8	
Potassium	373.30 mg	10.7	
Choline	40.91 mg	9.6	

a) Use the information in the table to calculate the full recommended daily intake for each nutrient listed.

b) What is the minimum number of cups of peas you would need to eat each day to get your full RDI of every nutrient listed?

10 Energy

Materials required: calculators

Warm-up

1.	Use leading digits to estimate 325 × 301	2.	Round 1755 to the nearest hundred
3.	Round 3.1415926 to 3 decimal places	4.	Write 37.5% as a fraction
5.	Write 1.001 as a percentage	6.	Calculate the perimeter of a rectangle 23cm by 1.2m
7.	Calculate the area of a rectangle 23cm by 1.2m	8.	How long would it take a car travelling at 50km/hr to travel 2.5 km?
9.	If the distance was increased by 50% how long would it take?	10.	If the speed was increased by 50% how long would it take?

Introductory problem

Appliances use energy in the form of work, which is measured in Watt hours. 3600 Joules is equal to 1 Watt hour.

A hotel in Denmark has a bank of exercise bicycles that are linked to the power supply. The hotel gives guests a free meal if they can generate an additional 10 Watt hours of electricity. That is 10 × 3600 = 36 000 Joules of energy. Power generated by guests through this initiative is stored in a battery and fed back into the main power supply of the hotel.

It is considered that an average healthy non-athlete cycling at 25km/h for an hour will produce around 100 Watts of power.

How long would it take for an average healthy non-athlete to generate enough electricity at this speed to get a free meal at this hotel?

If a 25 Watt compact fluorescent bulb requires 25 Watts to run for 1 hour, how long would the energy generated run 1 light bulb?

Exercises

This table indicates a rough energy guide of various appliances you would find in your home

Appliance	Watts	Appliance	Watts
Central Air Conditioner	5,000	Stereo	60
Laptop	200	Ceiling Fan	100
Oven	3,000	25 watt compact fluor. bulb	28
Hair Dryer	1,500	50 watt DC incandescent	50
Dishwasher	1,500	40 watt DC halogen	40
Microwave	1,200	Mobile Phone Charger	1.5
LCD TV	213		

1. An adult male's average energy requirements in a day, to maintain its normal functions and moderate physical activity is roughly 12 000kJ, how many Watt hours is this?

2. Use the table above to complete the following. The average energy used by a male human body in a day is the same as powering:
   a) An air-conditioner for how long?
   b) A laptop for how long?
   c) A stereo for how long?
   d) A 25 watt compact fluorescent bulb for how long?
   e) A mobile phone charger for how long?

3. a) How many light globes are there in your house?
   b) If each globe was used for an average of 2 hours a day (some will be more, some less) how much energy is required per day if each globe is a:
      (i) 50 watt incandescent bulb
      (ii) 25 watt compact fluorescent bulb
   c) What is the difference in energy consumption between the two bulbs over a year?

4. Cycling is one of the most energy efficient ways of travelling. When riding at about 20km/h, you use about 500kJ to travel 5km.

   a) Using this as a basis, how much energy is required for a person to cycle 12km to work each day?

   b) The Tour de France is around 3 200 kilometres. Cycling it needs about 50% more energy than usual due to the mountain stages and head winds. How much energy is required for the entire race?

5.   Using the same energy as a bike does to travel 5km, a car only travels around 100m.

    a)   Using this as a basis, how much energy is required for a person to drive 12km to work each day?

    b)   Compare this to the energy required for cycling and comment on the difference.

    c)   What would the difference be between the energy requirements of cycling and driving 12km to and from work 5 days a week for 48 weeks of the year?

    d)   What would be some considerations when comparing the energy requirements of the two modes of transport?

Extended Problem Solving

Television size is usually a single measurement like 80cm. This measurement is the diagonal distance across the screen. The length and width measurements for the most common sizes based on a 16:9 screen ratio are:

A 48cm television measures 41.5 cm wide by 23.5 cm high.
An 80cm television measures 70 cm wide by 40cm high.
A 105cm television measures 92 cm wide by 51.5 cm high.
A 115cm television measures 101 cm wide by 56.5 cm high.
A 138cm television measures 120 cm wide by 68 cm high.

A television's energy efficiency is represented using a star rating with one star being the least efficient and 5 stars being the most efficient.

The base energy efficiency for one star is calculated by
    1 star = 90.1 + (0.1168 × screen area) where screen area is in cm^2
For televisions, the Energy Reduction Factor is set as 0.20, which represents a 20% reduction in annual energy consumption per additional star. This means to have a higher star rating requires the television to use 20% less power than the previous star ration, i.e. a 3 star television is 20% more energy efficient than a 2 star television, a 4 star television 20% more efficient than a 3 star and so on.

Calculate the basic energy consumption, and the star ratings of each of the televisions listed.

Size	Base Energy Consumption 1 star	2 star	3 star	4 star	5 star	6 star
48cm						
80cm						
105cm						
115cm						
138cm						

The energy listed is in Watt hours.
If a household had a 105cm, 3 star rated television that they used for five hours a day, how much energy would it use in a year?

There are now provisions for televisions to achieve up to a 10 star efficiency rating.
What would be the energy consumption of a 10 star 138cm television?

How much less energy does it use than a one star television of the same dimensions?

What is the overall decrease as a percentage of its original energy usage?

11 Area

Materials required: calculators

Warm-up

1.	9 + 3 × 8	2.	20 − 14 + 3
3.	10% of 1 400	4.	110% of 1 400
5.	10% of 120	6.	90% of 120
7.	1.3km = _____ m	8.	487m = _____ km
9.	Estimate 315 × 587	10.	Estimate 1 497 ÷ 51

When converting units for measurements involving multiplying more than one dimension, such as area or volume, it is necessary to convert each of the dimensions used.

For example a square with 1cm sides has an area of 1cm^2:
    each of the 1cm long sides is 10mm,
    so the area is 10mm × 10mm = 100 mm^2
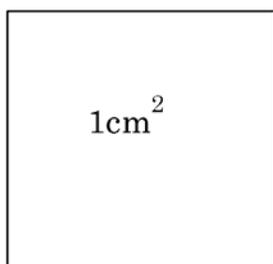
1cm^2

Some other area conversions:

Area	1 cm^2	= 10 mm × 10 mm
		= 100 mm^2
	1 m^2	= 100 cm × 100 cm
		= 10 000 cm^2
	1 hectare	= 100 m × 100 m
		= 10 000 m^2
	1 km^2	= 1000 m × 1000 m
		= 1 000 000 m^2

Exercises

1. Convert the following to the units indicated.
   a) $2cm^2$ = _____mm^2 b) $2m^2$ = _____cm^2

                                                        = _____mm^2

   c) $11.5m^2$ = _____cm^2 d) 2.5 ha = _____m^2
                   = _____mm^2 = _____km^2

2. Estimate the areas of the following:
   a) the top of your school desk.
   b) the front cover of this book.
   c) a basketball court.
   d) the school oval.

3. A room is 270cm by 480cm. The walls are 240 cm high. The room has one door 820mm by 2040 mm and 2 windows 950mm by 1550mm. If you want to paint the room and one litre of paint covers $15m^2$, how many litres are required to give the room two coats of paint?

4. A back lawn has the dimensions shown.
   1 litre of soil wetter covers $60m^2$.
   How many mL are required to cover the whole lawn?
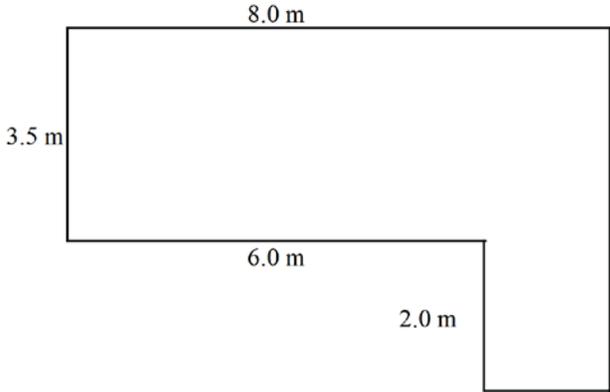
5. Jimmy has a blank wall in his bedroom he wants to fill with posters. The wall is 2.40m high and 2.85m long.
   a) He wants to fit as many 61cm high by 91cm wide posters as possible on his wall. What is the maximum number of posters he can fit?
   b) If he uses only smaller posters of dimensions 46cm by 61cm on the wall, what is the most he can fit?
   c) If he puts a mixture of larger and smaller posters on the wall what is the largest area of the wall he can cover?

6. A person wants to install six solar panels that are 800mm by 1600mm on the roof. How many square metres of roof area are required?

7. Shirley wants to install roof insulation in her shed in the form of ceiling batts. She has chosen batts that are 1160mm × 430mm and they come in packs of 16. The shed is 10m × 4.5m.
   a) What area does one pack of batts cover?
   b) How many packs does she need to insulate the shed?
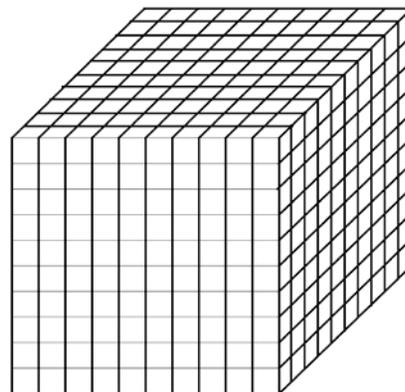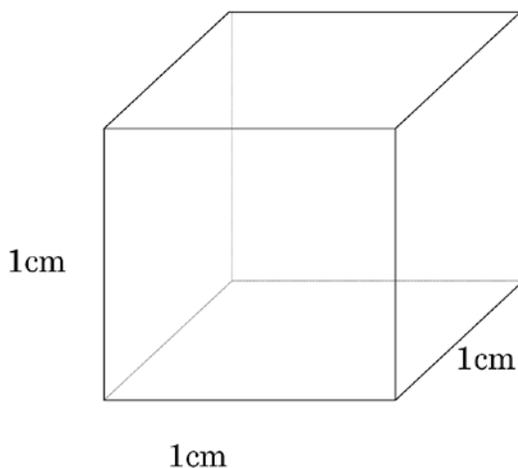
12 Volume

Materials required: calculators

Warm-up

1.	Estimate 23 000 × 71	2.	$15 \times (4 + 6)^2$
3.	Area of rectangle 20mm by 5cm	4.	0.09km = _____ m
5.	Reduce 88 by 10%	6.	25% of 25
7.	$0.1cm^2 =$ _____ mm^2	8.	2.5 hectares = _____ m^2
9.	Write 99.9% as a decimal rounded to 1 decimal place	10.	If apples cost \$4.90/kilo how many kilogram for \$20 to 2 decimal places?

Discussion

When converting units for measurements involving multiplying more than one dimension, such as area or volume, it is necessary to convert each of the dimensions used.

For example a cube with 1cm sides has a volume of $1cm^3$:
    each of the 1cm long sides is 10mm,
    so the volume is 10mm × 10mm × 10mm = $1000 \ mm^3$

Some other volume conversions:

Volume	$1 \ cm^3$	= 10 mm × 10 mm× 10 mm
		= 1 000 mm^3
	$1 \ m^3$	= 100 cm × 100 cm× 100 cm
		= 1 000 000 cm^3
	$1 \ km^3$	= 1 000 m × 1 000 m × 1 000 m
		= 1 000 000 000 m^3

Calculating the volumes of prisms
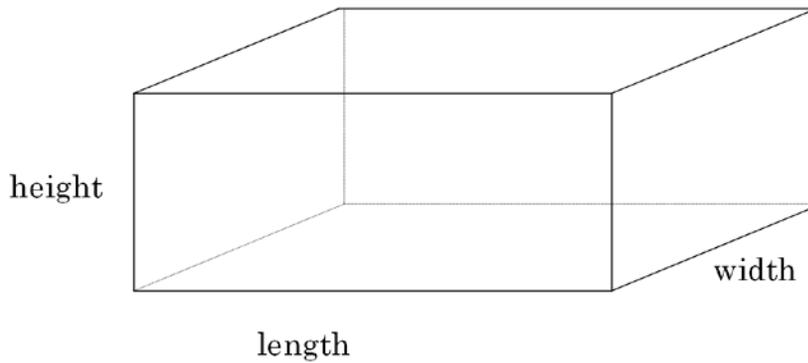
Volume of rectangular prism = length × width × height

Volume of triangular prism =
½ width × height × length
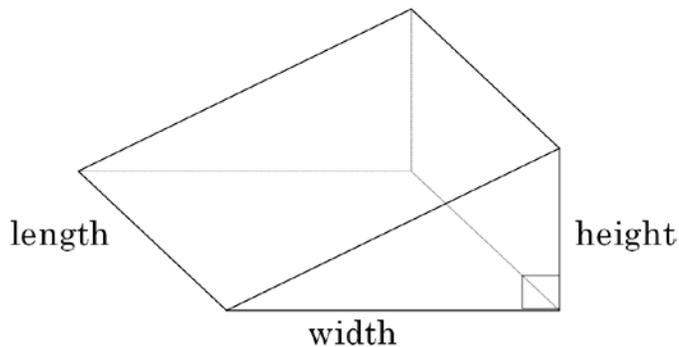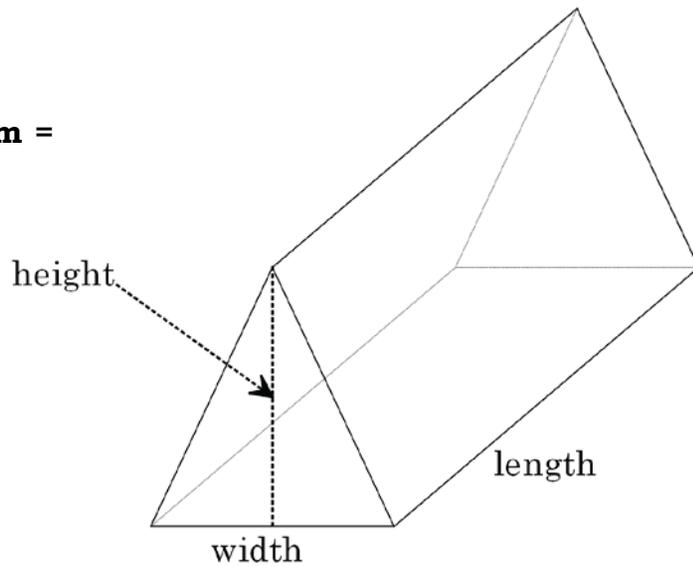
Exercises

1.   Convert the following to the units indicated.

a)   20cm³ = _____mm³

b)   200m³ = _____cm³ = _____mm³

c)   1.115m³ = _____cm³ = _____mm³

d)   0.01125 km³ = _____m³

2. Calculate the volume of the following prisms.

a)

4.5cm

8cm

b)

5 cm

40mm

c)

8cm

1.2cm

5mm

d)

4m

2.5m

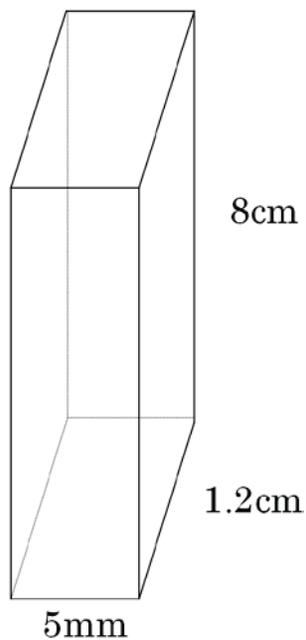10m
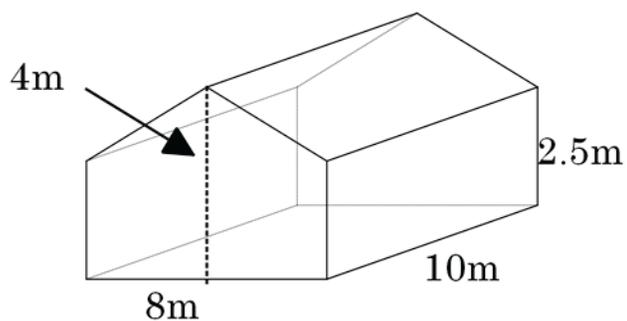
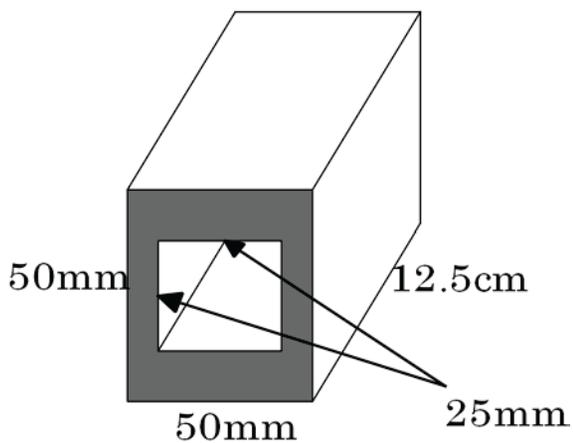8m

e)

50mm

12.5cm

50mm

25mm
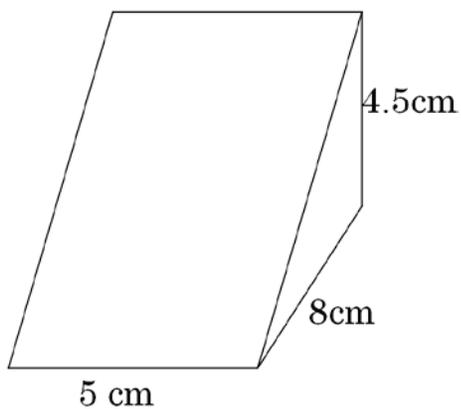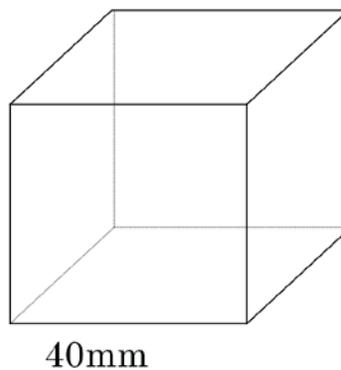
3. Estimate the volume of the following:
    a)   a lunchbox

    b)   your classroom

    c)   a 50m swimming pool

    d)   your school gymnasium

4. The Australian $50 note has the dimensions 151 mm × 65 mm, a ream of paper (500 sheets) is about 5.2cm deep.
    a)   Use this information to estimate how large a container would need to be to hold a million dollars.

    b)   A $5 note is 65mm × 130mm how big would a container be to hold $1 million dollars in $5 notes.

5. A sandwich container has the dimensions:
    - length and width = 119mm
    - height = 68mm
    What is its volume and capacity ($1cm^3$ = 1mL)?

6. A pool has dimensions 2m wide, 6m long and 1.5m deep.
    a)   Calculate its volume.

    b)   Convert its volume in m^3 to cm^3.

    c)   If $1cm^3$ = 1mL, how much water could the pool hold?

Extended Problem Solving

A manufacturer wants to design a special edition 1kg chocolate.

$1cm^3$ of this particular chocolate weighs 1.25grams.
Investigate the dimensions of at least 10 different packages that would hold 1kg of chocolate. Include all of the following shapes in your exploration:
      Cube
      Rectangular prism
      Triangular prism

13 Capacity

Materials required: calculators

Warm-up

1.	$150 – $77.75	2.	28 ÷ 2.5 × 2
3.	Write ¾ as a percentage.	4.	Write 1.001 as a percentage.
5.	Increase 0.1 by 20%.	6.	555cm = ____m
7.	555m = _____km	8.	555cm² = _____m²
9.	2.5 kg of sausages at $8.99/ kilo costs _____	10.	Run at 20km/h for 30seconds. How far would you run?

Discussion

What is the difference between the volume of an object and its capacity? What units are used to measure volume? What units are used to measure capacity? How are they related?

Exercises

1. Convert the following capacities to the units indicated:
   a) 200mL = _____L   b) 1250mL   = _____L
   c) 2.5L   = _____mL   d) 6650L   = _____kL
   e) ¾ kL   = _____L   f) 200 000L = _____GL
   g) 0.2kL   = _____mL   h) 756 000kL = _____GL

2. Estimate the capacity of the following:
   a) a lunchbox
   b) a school bag
   c) the boot space of a medium car
   d) a teaspoon
   e) an esky
   f) a washing machine
   g) a kitchen sink
   h) a large gas bottle
   i) a fuel tank of a large car
   j) a coffee cup

3. A backpack manufacturing company wants to design 3 backpacks with the capacity of 40 litres, 65 litres and 80 litres what dimensions could they be?

4. Calculate the capacity of these two containers and then suggest 3 possible uses for each.

a)

228mm × 122mm × 97mm

b)

265mm × 128mm × 180mm

5. a) A car's rectangular fuel tank has dimensions of 880mm by 610mm by 205mm. What is its capacity?

   b) The actual capacity of a fuel tank can vary by as much as 3 percent of the stated capacity. Calculate the maximum and minimum capacity possible with a 3% variation.

   c) On many cars and bikes the fuel light comes on when the tank is at 15% of the capacity or less. How much fuel remains when the fuel light comes on in this vehicle?

6. A cube has a capacity of 20 litres. What are its dimensions?

Numbers are the highest degree of knowledge. It is knowledge itself.

- Plato

Extended Problem Solving

A rough rule of thumb for stocking aquariums is 1cm of fish for every litre. Here are the approximate lengths of some aquarium fish:

      Neon Tetra 5cm
      Goldfish   20cm
      Oscar   30cm
      Sword Tail 10cm

Calculate how many fish of each type would fit into each of the different sized aquariums in the table below.

Length (cm)	Width (cm)	Height (cm)	Neon Tetra	Goldfish	Oscar	Sword Tail
76	30	30				
122	33	41				
182	46	61				

The owner decides he wants 12 neon tetras, 3 goldfish, 2 oscars and 5 sword tails. Investigate what dimensions the aquarium could be to accommodate the fish using the rule of thumb.

Puzzle:

On what basis have these letters been separated?

AEFHIKLMNTVWXYZ

BCDGJOPQRSU

14 Earning an income

Warm-up

1.	$10.20 × 2	2.	$10.20 × 1.5
3.	10% of $67 000	4.	1% of $67 000
5.	11% of $67 000	6.	Increase $67 000 by 11%.
7.	Write 11% as a decimal.	8.	Write ¾ as a percentage.
9.	$312.4567 rounded to the nearest cent.	10.	$312.4567 rounded to the nearest dollar.

Discussion

You may have some sort of casual/part time employment or be thinking about earning some extra money by gaining employment while you complete your senior schooling. Different types of employment have different types of pay rates and conditions. While many people may work for an employer, others may earn an income from selling goods or services. Discuss with your classmates the different ways you or your family members earn money.

Methods of Payment

Salary: A fixed regular payment for professional or office work.

Wages: A fixed hourly, daily, weekly payment for manual or unskilled labour.

Contract: A contract of employment is an agreement between an employer and employee about the terms and conditions of employment for the employee. It is often used in piece-work for example a concreter, cake maker or fisherman. Employees may need an Australian Business Number (ABN) to participate in contract work.

Commission: Commission is a sum of money that is paid to an employee upon completion of a task, usually selling a certain amount of goods or services. Commission may be paid as a percentage of the sale or as a flat dollar amount based on sales volume. Employers often use sales commissions as incentives to increase worker productivity. A commission may be paid in addition to a salary or instead of a salary.

Fees: A fixed charge for a professional service for example a doctor or dentist.

Profit Sharing: Each person gets a share in the profit of a business or company based on their input.

Minimum wage: An employee's basic rate of pay depends on such things as their age, job classification and the industrial award they're covered by. The national minimum wage acts as a safety net for employees in the national workplace relations system to provide minimum rates of pay for employees not covered by awards or agreements. National minimum wage orders are made by the Minimum Wage Panel of Fair Work Australia.

Australia's minimum casual wage for a 16 year old is $10.20 per hr

(Oct 2013)

Exercises

1. What method of payment do you think each of the following part-time employees receives?

    a) Jodie Green   Mows lawns and general garden clean ups

    b) Harry Holden   Motor mechanic

    c) Brett Brick   Brick-paver, paid per square metre laid

    d) Jemma Gem   Sells jewellery she created in an online store

    e) Amy Mathies   Mathematics teacher

    f) Harry Cutter   Surgeon

    g) Suzi Commando   Paint Ball ticket salesperson

2. The following table shows the time sheet for hours worked by four employees at a fast food outlet. Sierra and Alex are both 16 years old and paid the minimum casual wage of $10.20 per hour. Ryan is 18 and earns $14.28 per hour and Emma is a 21 year old university student who earns $20.40 per hour.

Name	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Sierra	4 hrs			4 hrs		3 hrs	3 hrs
Alex		3.5 hrs	3 hrs		4 hrs	5 hrs	
Ryan	3 hrs		4 hrs			4 hrs	4 hrs
Emma		5 hrs		3 hrs	3hrs		5 hrs

   a) Calculate the weekly wage for each of the employees.

   b) Calculate the total wages paid to the four employees for the week.

   c) On public holidays, the employees are paid time and a half. Calculate the public holiday hourly rate for each employee.

3. For each of the following wage earning employees calculate their hourly rate and hence who earns the best rate per hour.

   a) Wayne earned $5020 last month working 8 hours a day for 25 days.

   b) Kim earns $928 per week working 5 days a week from 9am to 5pm.

   c) Pauline earns $826 for 35 hours work.

4. Psy's You Tube hit "Gangnam Style" reached more than 1 billion hits and he made an estimated $800 000 to $2 million in revenue.

   a) Calculate the average revenue made per hit if Psy earned:

   (i) $800 000   (ii) $2 000 000

   b) For each estimate above calculate the number of hits Psy would have needed to make before he earned a dollar.

   c) Comment on the reliability of your answers in b).

5. For each 16 year old employee below, calculate the percentage change above or below the national minimum casual 16 year old hourly wage.

$$percentage\ change\ =\ \frac{difference}{minimum\ wage}\ \times\ 100$$

   a) Theme Park Employee $12.60 per hour

   b) Hairdresser apprentice $7.95 per hour

6. The number of 15 – 64 year olds in the Australian working population is approximately 15 610 000 people.

   a) If the unemployment rate is 5.8%, calculate the number of people unemployed in Australia.

   b) 34% of the unemployed people from a) are aged between 15 and 24. Calculate the number of unemployed youth.

   c) Youth unemployment figures are considered to be unreliable. What factors could distort youth unemployment figures used in the media?

Extended Problem Solving

Part-time work enables you to gain skills and experiences that can support you to gain future employment. It is important to identify and explore career options, and to find information on occupations, industries, courses, training providers and scholarships so you can make the best decisions for your future. Visit the website www.myfuture.edu.au to help you make career decisions and plan your future career pathway.

15 Measurement contracts

Materials required: tape measures, trundle wheel, measuring equipment, graph paper, rulers and calculators

Warm-up

1.	Perimeter is _____	2.	$13.50 × 4
3.	2m = ____mm	4.	1894mm = _____m
5.	$1m^2$ = _____cm^2	6.	$1m^3$ = _____cm^3
7.	Round 45.7847 to two decimal places	8.	If a square has area of $64cm^2$, calculate the length its sides.
9.	Sum of 35 and 4	10.	Product of 40 and 5

Activity

The basketball/netball court in your school is going to be remodelled. This involves replacing the bitumen and painting new boundary lines. Three different contractors charge the following rates:

Name of contractor	Call Out Fee	Cost of bitumen per square metre	Cost per linear metre painted
Cheap as Chips	No call out fee	$110	$12
Blue Fix	$500	$70	$15
Wajja Contracting	$250	$100	$20

a) Estimate the area and perimeter of your basketball/netball court.
b) Compare your estimate by accurately measuring and drawing the perimeter and area of your school's basketball/netball court.
c) Calculate the amount that will be charged by each contractor.
d) Discuss any other considerations the school finance committee may need to consider before they choose a suitable contractor.

Payment by contract

Contract employees are paid by the amount of work they complete. Many trades require the employee to calculate measurements involving length area and volume to receive payment.

Length is a one dimensional measurement of a path, for example the length of fencing required for a boundary perimeter. Common units used are mm, cm, m and km.

Area is a two dimensional measurement of the amount of surface of an object, for example the area required to paint. Common units used are mm^2, cm^2, m^2, hectares and km^2.

Volume is a three dimensional measurement of the space occupied, for example the volume of concrete or mulch necessary. Common units used are mm^3, cm^3 and m^3.

Exercises

1. a) Calculate the perimeter of the following shapes.

(i)

7cm

12cm

(ii)

15mm

20mm

35mm
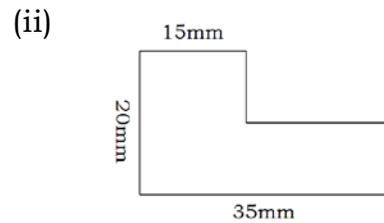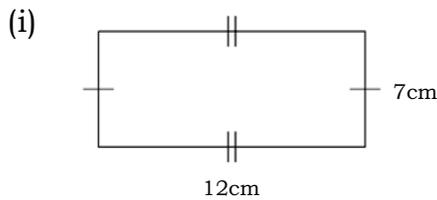
b) The perimeter calculations of two squares are shown below.
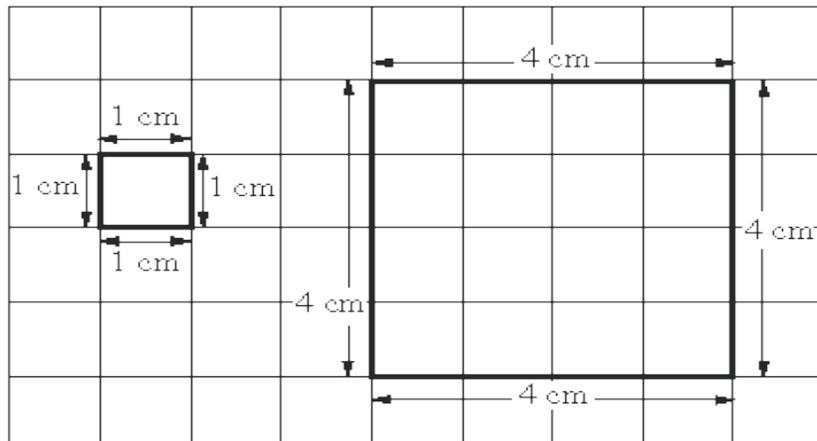
1 cm

1 cm   1 cm

1 cm

4 cm

4 cm

4 cm

4 cm

Perimeter = 1 +1 + 1 + 1   Perimeter = 4 + 4 + 4 + 4
        = 4 cm   = 16 cm

(i) Copy and complete the table:

Length of side (n)	1	4	6		13.1	
Perimeter (P)				44		102

(ii) Write a general formula to find the perimeter of a square from the length of its sides.

2. Rowena has been contracted $35.50 per linear metre to build a wall around some garden beds at a local golf course. Calculate how much Rowena will be paid to the nearest dollar to build the garden bed walls shown below.

a)

6 m

5 m

3 m

10 m

b)

2.75m
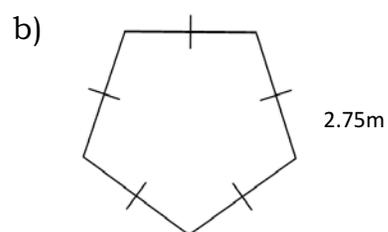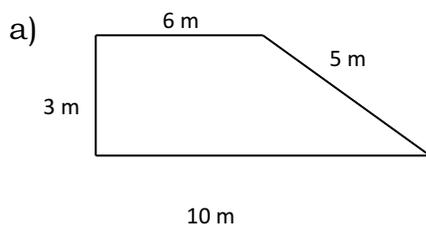
3. a) Calculate the area of this shape:
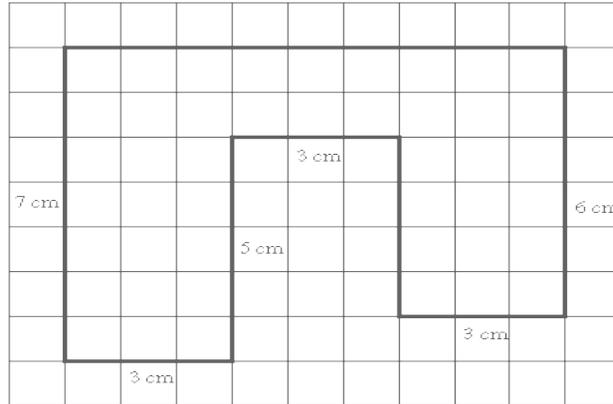
b) Show how the area of the shape in a) could be calculated using three rectangles.

c) The area of a triangle is found by the formula:
A = ½ base × height

Calculate the area of the triangles:

(i)

(ii)
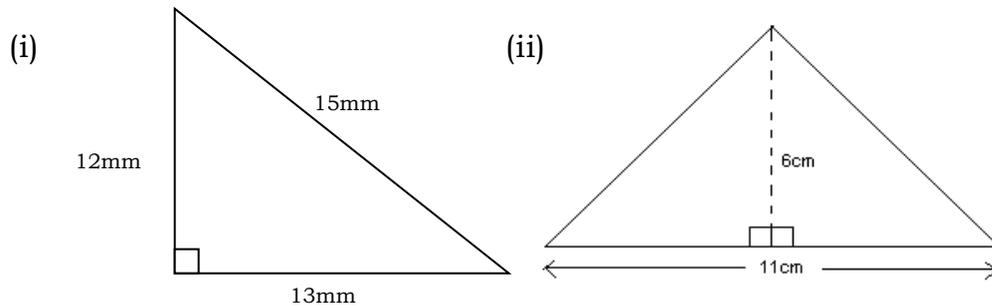
4. Mark builds patios and is paid per square metre of patio built. A copy of his invoice template is shown below.

Type of Patio	Total Area in m²	Payment per m²	Total Payment
GABLES		$35	
DOMES		$35	
PYRAMIDS		$55	
FLATS		$19	
SOLAR SPAN		$40	
TIMBER LINING		$66	

How much will each of the following customers be charged?

MR JONES Rectangular gabled roof patio of dimensions 5.5m × 3.6m	MRS BRIDGE – Timber lined flat patio 	MR CRAB Square based pyramid of length 5500 mm

5. Alex is decorating her rectangular room, which is 4 metres long by 3.5 metres wide and 2.4 metres high. The walls are shown:

2.4m

4m

3.5m

1m 140cm 1m 2.4m

4m

2.4m

2m

90cm

2.4m

3.5m
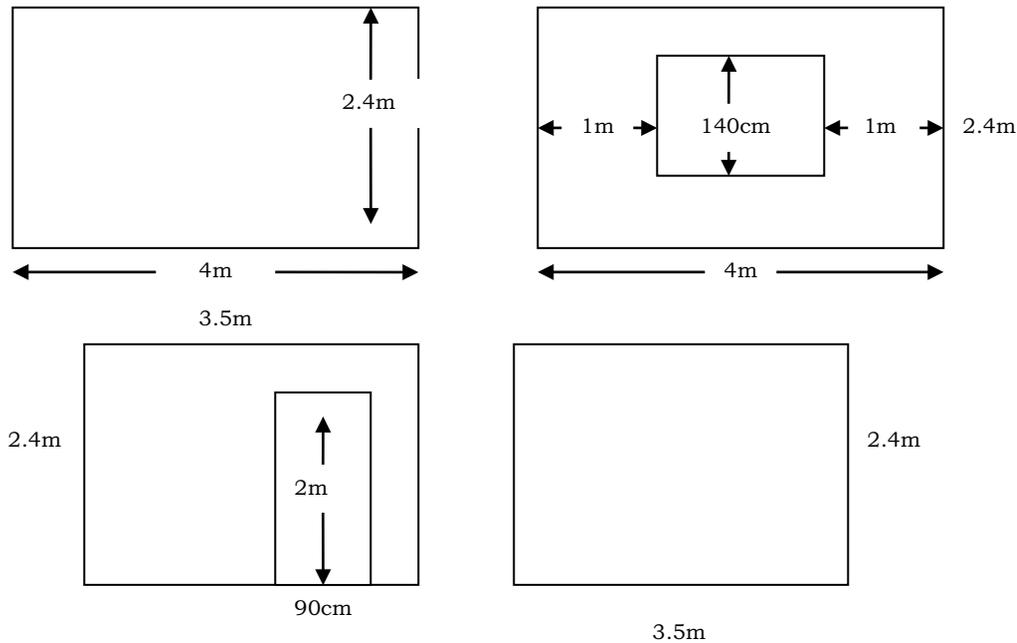
a) Jenna will be paid $12.25 per m² to paint flat paint on the walls and $25.00 per m² to paint gloss paint on one side of the door.
    (i)   Calculate the area of wall to be painted.
    (ii) Calculate the area of door to be painted.
    (iii) Hence calculate how much Jenna will be paid.

b) Finn is to fit blinds at a cost of $55 per m² plus $6 for each linear metre of curtain track across the top of the window. Calculate the amount Finn will charge to fit the blinds.

6. Richard is a furniture removalist and he has a truck with a trailer of storage dimensions 2.5m × 4m × 2.5m.
    a) Calculate the volume of his trailer.

    Richard has a contract for moving fifty queen size mattresses for a store that is relocating. Each mattress has dimensions 153cm × 203cm × 35cm.

    b) Calculate the volume of one mattress.

    c) Calculate the number of mattresses Richard can fit in his truck.

    d) How many trips will Richard need to take to move all 50 mattresses?

    e) Richard estimates that it will take one day to move all 50 mattresses and will cost $20 petrol for each trip. If he likes to earn $650 a day to cover wages and other expenses, calculate the rate to the nearest dollar he should charge to move each mattress.

16 Commission

Materials required: calculators, access to a spread sheet application

Warm-up

1.	1% of $75 000	2.	10% of $75 000
3.	2% of $75 000	4.	12% of $75 000
5.	Calculate the volume of	6.	Calculate the area of

5. Calculate the volume of (box: 3cm, 2cm, 4cm)

6. Calculate the area of (triangle: 3cm, 5cm, 4cm)
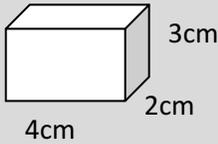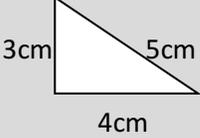
7.	$88 - $78.50	8.	$140 ÷ 7
9.	0.75 as its simplest fraction	10.	3 fortnights = _____ days

Discussion

A commission is a way of earning money based on a percentage of sales or services. Earning money by a commission is very different from having a weekly or fortnightly salary

Commission-based employees are paid a fee or percentage based on how much they sell, instead of a wage based on the number of hours they work. This is called their commission.

Commission-based employees can be paid a commission-only wage or are paid a commission on top of a base wage (called a retainer).

They should always get at least the minimum entitlements in their award or agreement. Normally, this means the commission they get must be at least as much as they would earn from being paid under the award or agreement for every hour they work.

a) What do you think are some of the advantages of being paid on a commission only basis?
b) What do you think are some of the disadvantages of being paid on a commission only basis?
c) What difference would it make if you were paid a retainer and a commission?

Real Estate Commission

Most real estate salespeople receive a commission for selling a house that is a designated percentage of the Agent Selling Fee. The owner of the business takes a percentage and the salesperson takes a percentage. Currently, Real Estate Agent selling fees are calculated according to approximately 2.5% of the selling price of the property. This percentage rate varies across agents, states and territories.

Example: What is the Real Estate Agent Fee on a sale of $425 000

2.5% of $425 000 = $^{2.5}/_{100}$ × 425 000 = $10 625

Exercises

1. Calculate the 2.5% commission earned by a real estate agent on the following property sales:
   a) $ 350 000   b) $720 000   c) $1.2 million

2. Calculate the amount earned by each of the following employees paid by commission and/or retainer.
   a) Paintballing sales representative Kiera earns a commission of 15% on every package she sells. Kiera sets herself up at a shopping centre one day and sells six $95 packages. How much did Kiera earn for the day?
   b) Harry works in a retail store and is paid a base salary of $620 per week and earns 5% commission on his total sales. If he sold $2800 worth of goods for the week how much commission did he earn? Hence calculate his income for the week.
   c) Gavin sells phone contracts and earns 8% of the total contract value. One week he sold five contracts: three contracts valued at $1200; one at $2400; and one valued $3600. How much did Gavin earn for the week?
   d) Events promoter Monique promotes and sells tickets for gigs by local musicians. She charges a retainer of $200 and 3.5% of all ticket sales go to Monique. For the event Harmony Festival, 2800 tickets are sold for $110 each. How much did Monique earn from promoting the event?
   e) Kyle a car sales representative earns a retainer of $400 per week and 1.75% of his sales. In the June end of financial year sales Kyle sold $162 100 worth of cars. How much money did he earn in the month of June?

3. Sophie sells a $120 software package called "Bookkeeper 4U" online to small businesses around the world. She earns 24% commission on her sales. If she earned $72 000 for the year,
   a) Calculate the value of her sales for the year.
   b) Hence calculate the total number of software packages sold by Sophie for the year.

4. The graph below shows the relationship between commission and property values

2.5% Commission on Property Sales
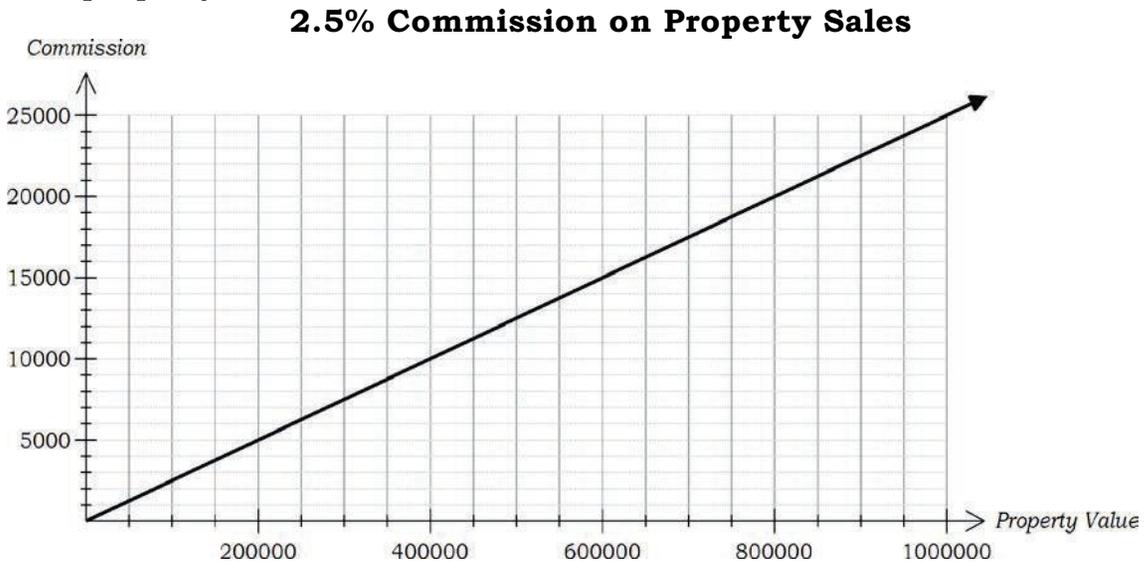
a) Copy and complete the table, using the graph to approximate the commission earned to the nearest 100 on the median house price across each of Australia's capital cities.

Capital City	Median House Price	Commission Earned
Sydney	$673 700	
Melbourne	$545 000	
Brisbane	$430 000	
Adelaide	$395 000	
Perth	$505 000	
Canberra	$482 500	
Hobart	$360 000	
Darwin	$592 000	

House Price Change, Annual (%)

Source: Australian Bureau of Statistics
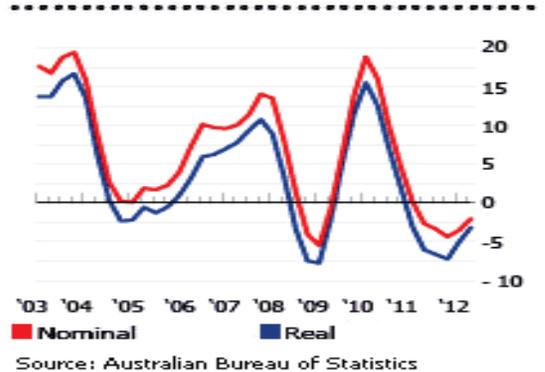
b) Describe how a real estate agent's income may have varied from 2003 – 2012 based on the information displayed on the graph on the left.

5. Charlottes Child Care Centre Supplies pays its employees a varying commission based on the employees sales. The various commission rates are displayed in the spread sheet/two way table below.

	A	B	C	D	E	F	G
1			Total Sales				
2	D		0-200	201-400	401 - 600	601-1000	1000+
3	e p	Crafts	3.50%	4.75%	5.00%	5.50%	6.20%
4	a r	Books	4.50%	5.00%	5.20%	5.25%	5.50%
5	t m	Toys	4.20%	4.60%	5.25%	5.50%	6.00%
6	e n	Swing Sets	3.00%	3.50%	3.75%	4.00%	5.00%
7	t	Furniture	6.50%	6.75%	7.00%	7.20%	7.50%

Calculate the commission earned for the following sales

a) Holly sold $1200 of swing sets and $650 general toys.

b) Marishvan sold $120 of books and $310 of tricycles (toys).

c) Imogen sold $800 of furniture and $150 of craft supplies.

Extended Problem Solving

Spread sheets can be used to calculate the pay of salespeople who are under different commission contracts.

Consider the following arrangements of three sales people:
- Fergus earns a flat rate of 6% on all sales
- Colleen earns 4% of all sales and has a retainer of $175
- Angus earns 5.5% of all sales and has a retainer of $120

Develop a spread sheet that can be used to calculate their pay for different sales amounts.

Puzzle: How many triangles are in the shape below?
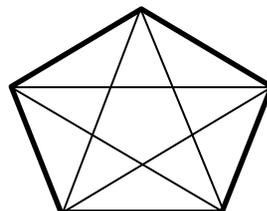
17 Superannuation

Materials required: Two 1000mL beakers, post it notes to label the beakers, bucket of coloured water, container with a lip, towel, graph paper and or graphing calculator/software, rulers, calculators

Warm-up

1.	1 litre = _____mL	2.	500mL = ____L
3.	250mL = _____L	4.	$^1/_5$ as a percentage
5.	If 100 × b = 10, b = ____	6.	If 50 ÷ a = 100, a = ____
7.	9% as a decimal multiplier	8.	9.25% as a decimal multiplier
9.	100% as a decimal multiplier	10.	109% as a decimal multiplier

Activity: Exploring percentages of an amount

Superannuation is a compulsory amount of money paid by an employer for each employee, to be invested into a superannuation fund, for future retirement income. In 2012, the compulsory superannuation rate was 9% of an individual's income, this will increase by gradual increments to 12% in 2019.

1) On graph paper draw a set of axes. Label the x axis 'Income' (0 < x < 2000 mL) and the y – axis 'Superannuation' (0 < y < 300mL)

2) Label the beakers Income and Superannuation

3) Fill the Income beaker to 250mL, calculate 9% of 250mL, fill your Superannuation beaker to this level, plot this point on your axes.

4) Fill the Income beaker to 500mL, calculate 9% of 500mL, fill your Superannuation beaker to this level, plot this point on your axes.

5) Fill the Income beaker to 750mL, calculate 9% of 750mL, fill your Superannuation beaker to this level, plot this point on your axes.

6) Fill the Income beaker to 1000mL, calculate 9% of 1000mL, fill your Superannuation beaker to this level, plot this point on your axes.

7) What do you notice about the points you have plotted? Is it appropriate to join these points? Explain your reasoning.

8) Write a general rule for finding 9% superannuation of an income.

9) Repeat this activity using 10%, 11% and 12% and graph on the same axes. Discuss any patterns you see in the graphs.

10) Why are we using two separate beakers to demonstrate this concept?

Exercises

1. Calculate the superannuation invested each pay period for each of the following employees:

    a) John Baracus – bartender earning $850 per week with 11% superannuation
    b) Suzie Quenton – beautician earning $1475 per fortnight with 9.5% superannuation
    c) Grace Hall – Motor mechanic paid $18.60 p/h for 40 hours/week with 12% superannuation

2. In the activity you would have found that an amount determined as a set percentage of a quantity, increases at a constant rate.

    a) Copy and complete the table below for an 11% superannuation allocation.

Amount	$200	$300	$400	$500	$1000
Superannuation invested (S)					

    b) Plot these points on a set of axes.

    c) Does it make sense to join the points? Justify your answer.

    d) Write a general rule using a decimal multiplier to calculate the 11% superannuation investment (S) from the amount earned (E).

3. Jodie wishes to calculate the total amount of money (A) she needs to pay each employee with 10% invested for superannuation on top of their original income (I). One rule she could use is to find 10% of each employee's income and then add it to their income (I)

$$A = I + \frac{10}{100} \times I$$

    Which rule of those listed below could also be used to calculate this amount? Justify your choice.

    A. $A = 10 \times I$ **B.** $A = 0.1 \times I$

    C. $A = 0.9 \times I$ **D.** $A = 1.10 \times I$

4. Jack has a 10% superannuation contribution and a total of $120 has been invested into his superannuation account, calculate his gross income.

5. Matilda has started a new job and earns $67 000 per year and receives 12% superannuation. Assuming she continues on the same pay scale, calculate the total superannuation paid after five years.

6. Most people work for 30 to 40 years and live for another 25 to 30 years after retiring. You want your super to grow and to keep pace with inflation during this time. For this reason, a growth or balanced strategy may suit a long-term investor who won't be spending their super for more than 10 years, a higher risk strategy may deliver higher returns, but the risk is losses in bad years.

    Historically, over any 20-year period, a growth or balanced strategy has given better returns than more conservative investment options. You must decide if the likely rewards are worth the risk.

    Comment on each type of superannuation investment shown below and describe the type of person who may choose each investment option.

a)
Balanced
■ Around 70% in shares and property
☐ The rest in cash or fixed interest

$14,400 $10,000 after 5 years

Expected return   Medium volatility   Expect a loss
↑ 7.5%*   ∿∿   4 years in 20

b)
Conservative
■ Around 30% in shares and property
☐ The rest in cash or fixed interest

$13,700 $10,000 after 5 years

Expected return   Low volatility   Expect a loss
↑ 6.5%*   ∿∿   0 years in 20

c)
Growth
■ Around 85% in shares and property
☐ The rest in cash or fixed interest

$14,700 $10,000 after 5 years

Expected return   High volatility   Expect a loss
↑ 8%*   ∿∿∿   4-5 years in 20

d)
Cash
■ 100% in deposits with Australian deposit-taking institutions

$12,800 $10,000 after 5 years

Expected return   Very low volatility   Expect a loss
↑ 5%*   ∿∿   0 years in 20

Extended Problem Solving

Research information about how superannuation works
at: https://www.moneysmart.gov.au/superannuation-and-retirement/how-super-works

18 Income tax

Materials required: Tax Payment Forms can be downloaded from https://www.ato.gov.au/Forms/Tax-return-for-individuals, calculator

Warm-up

1.	0.3, 0.6, 0.9 , ___, ___	2.	1050, 1100, 1150, ____, ____
3.	3a = 35, a = ____	4.	5b + 2 = 37, b = ____
5.	10% of $56 000	6.	1% of $56 000
7.	9% of $56 000	8.	9% off $56 000
9.	2600 mm = _____ m	10.	Round 45.8736 to two decimal places

Introductory problem

Marella has worked for the Lulu Linen Company for the last financial year and her pay details are presented in her Payment (PAYG) summary which is printed below. Prepare a sample tax return for Marella by completing the form provided by your teacher.

PAYG payment summary – individual non-business payment summary for year ending 30 June 2014

Payee Details

SURNAME:	JEWEL
Given Names:	Marella Lou
Address:	999 Pretend Street
	NOWHERE WA 6666
Tax File Number:	000666999
Date of Birth:	04/04/1993
Position:	Personal Assistant

Period during which payments were made
01/07/2013 to 30/06/2014

Gross Payments:	$68 173
Uniform Allowance:	$365
Total Tax Withheld:	$15 217
Lump Sum Payments:	Nil

Payer Details

ABN: 0004448882222   Name: Lulu Linen Company

Authorised by Essential Mathematics teacher   Date: 30/06/2014

How did you go with Marella's sample tax return? Filing a tax return can be a complex process with a lot of tax-specific terminology. For this reason many people may use an accountant to assist them in completing their annual tax returns. You may have also heard parents or relatives talking about doing their own tax returns. Why is it important to fill in a tax return?

Income Tax

The Australian Government needs money to fund the support it provides for Australian citizens, e.g. health, education, defence, conservation and roads. The government obtains money by taxing the income of individuals and businesses, this is called *income tax*. People are taxed on their *total income.* Income includes among other things:

- regular wages from the main employer;

- money earned as interest on an investment;

- dividends paid for shares;

- interest from savings in bank accounts;

- money earned from extra work, e.g. second job, babysitting; and

- allowances from employees for tools, clothing and laundry.

Some expenses paid by employees are allowed as *deductions*. This means they can be taken off the *gross* wage so then their *taxable income* is decreased. Deductions include:

- Donations to charity;

- Work uniforms or work specific clothing;

- Work-related travel;

- Union/professional association fees; and

- Accountant fees.

An employer provides an employee with a *payment summary* at the end of each financial year. This includes money earned by the employee and the tax deducted. If the tax deducted is more than the employee is required to pay they may receive a *tax refund*. If insufficient tax has been withheld by the employer then an individual may have to pay more tax to the Australian Taxation Office.

All individuals are required to have their own tax file number for identification. Tax tables are used to work out the tax owed on taxable income.

The Terminology of Tax

Determine which definition on the right matches the word or phrase on the left

	PHRASE		DEFINITION
A	Financial year		Husband or wife
B	Payment Summary		A unique number which identifies a particular bank
C	TFN		Employment Termination Payment – may be paid at the end of a job
D	Australian Resident		All the tax your employer pays to the government of your behalf
E	EFT		Total amount before any is taken away
F	BSB		A statement of the financial year's earnings- supplied by the employer
G	ETP		Extra money earned on an investment
H	Total Tax withheld		A person who will fill in your tax return for you
I	Youth Allowance		Tax File Number – identifying number for each person
J	Austudy Payment		Real and True
K	Interest		Electronics Fund Transfer
L	Gross		A person who lives on a regular benefit paid by the government
M	Dividend		July 1st to June 30th
N	Accountant		A person who lives in Australia
O	Deduction		A gift
P	Donation		An extra amount that a person pays for a service
Q	Spouse		A government benefit paid to students - usually at full time university
R	Pensioner		The extra amount owed for a loan or gained from an investment
S	Levy		A government benefit paid to young people
T	Valid		A true statement that is personally signed
U	Declaration		Money that can be taken off another amount

Individual income tax rates

The table below shows the tax rates for 2013 – 2014 for individuals who are Australian residents.

Taxable Income	Tax on this Income
0 - $18 200	Nil
$18 201 - $37 000	19c for each $1 over $18 200
$37 001 - $80 000	$3 572 plus 32.5c for each $1 over $37 000
$80 001 - $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

This table can be used to calculate the amount of tax required to be paid by an individual.

Examples:

1. What tax will Jono pay if he has earned $12 519 in the past year? He is a student and works in a supermarket as a casual employee.

   Jono will pay no tax at all – he is below the tax free threshold ($18 200)

2. Calculate the tax owed by Russell who had a taxable income of $35 678 last financial year.

   Russell pays tax on $35 678 - $18 200 = $17 478

   Tax = 19c for each dollar of $17 478
   = 0.19 × 17 478   [19c equal to $0.19]
   = $3 320.82

3. What tax is owed on a taxable income of $89 000?

   Tax owed   = $17 547 + 37c for each dollar over $80 000
                = $17 547 + 0.37 × $89 000 - $80 000
                = $20 877

   Note: When you enter this calculation into your calculator as it is written, you get the incorrect answer. Why? What is missing?

Exercises

1. Determine the total annual income for each of the following taxpayers:
    a) Clancy's gross income from his day job was $28 500, from his part-time job was $4 500 and his bank interest was $56.
    b) Geoffrey's gross income was $57 450 and interest earned on his investments was $1020. Geoffrey also received money for travel and uniform to the tune of $700 for the year.
    c) Lucas was in a 50:50 partnership with his brother. Their business income was $156 700 and the interest on the business investments was $32 100.
    d) Silas earned $398 each week from his regular job, $50 each week for looking after his nephew and $150 per fortnight for driving for the local senior citizens centre.

2. Calculate the tax owed by Clancy, Geoffrey, Lucas and Silas.

3. People who work for regular wages have tax deducted on a regular basis.
    a) Calculate the tax deducted over the full year for each of the three fitness instructors Angela, Rebecca and Caitlyn.

Instructor	Pay period	Tax deducted (PAYG)	Total Tax deducted over a full year
Angela	Weekly	$212.80	
Rebecca	Fortnightly	$439.60	
Caitlin	Monthly	$864	

    b) Who paid the highest amount of tax?
    c) When the girls submitted their tax returns, the tax payable as calculated by the taxation office was different in each case. The amounts were:

    Angela $10 987
    Rebecca $8 900
    Caitlin $11 215

    Suggest possible reasons to explain why these amounts are different to the tax taken out of their pays.
    d) Who owes the tax department more money? How much?
    e) Who will get a refund? How much is the refund?

4. Greg works as an auto-electrician and his payment summary shows that he earned $45 682. His two bank accounts show he has earned interest of $84 and $187. He has an income of $3 412 from co-owning a family business and his share dividend was $734.
    a) What was his total income for the year?
    b) If his total deductions are $1 987, what is his taxable income?
    c) How much tax should he pay?
    d) Greg has already paid $6 364 in PAYG tax. Does he owe the tax department any more money? If so, how much?
    e) What percentage of his taxable income did Greg pay in tax?

5. Lani earns a salary of $63 291 and has paid $12 382 in tax instalments over the year. She also paid $520 in union fees and has accumulated $67 in interest in her bank account. Her donations to CARE Australia total $850 and she is claiming 867 kilometres of work travel at 69c per kilometre.
    a) How much does Lani pay for her union fees each week?
    b) Lani is paid fortnightly; how much is deducted for tax each pay?
    c) Determine Lani's total income.
    d) What is the total sum of Lani's deductions?
    e) Calculate Lani's taxable income.
    f) Will Lani get a refund or owe money to the ATO?
    g) Calculate the percentage of total income that is paid by Lani in tax.

6. Copy and complete the table below

Name	Taxable Income	Tax Payable	Possible occupation (job or position)
Seb	$3 500		
David	$32 600		
Sabrina	$53 278		
Fatima	$95 760		
Tony	$163 000		
Julia	$535 000		

Extended Problem Solving: Tax File Numbers and Tax

Some websites that might help you answer these questions are http://www.taxsuperandyou.gov.au/, the Australian Taxation Office and the Commonwealth Bank.

1. How can a student obtain a tax file number?

2. Why do taxpayers need a tax file number?

3. Are tax file numbers compulsory?

4. If an employee does not have a tax file number, what % of tax can the employer take out of the employee's pay?

5. Which of the following are not recommended?

    a) Having your TFN written on a card in your wallet.

    b) Storing your TFN on your mobile.

    c) Giving your TFN to a friend in case you forget it.

6. Name five organisations that are allowed to ask for your tax file number.

7. Can you apply for Newstart without a Tax File Number?

8. Can two people have the same TFN? Explain.

9. For how long does a TFN belong to a particular individual?

10. By what date must individuals lodge a tax return?

11. Can a tax return be completed and lodged online?

12. Which Australian citizens are exempt from paying tax?

13. Do you need to have an ABN before getting a tax refund?

14. What can you say about people who pay no tax?

15. What can you say about people who pay more than $60 000 tax?

19 Miscellaneous

Materials required: A3 paper, calculators

Warm-up

1.	10% as a decimal	2.	10% of $34 000
3.	1% of $34 000	4.	89% of $34 000
5.	¾ as a decimal	6.	18 – 3 × 2 =
7.	125, 25, 5, ___, ___	8.	Round 456 781 to nearest 100
9.	32m = _____ cm	10.	1m^2 = _____ cm^2

Activity: Round Robin

1.   You will require a piece of A3 paper to record ideas. Work in a group of 3-4 students and choose one person from your group to be a scribe.

2.   For 90 seconds, list some of the mathematics and requirements you will need to consider when earning money in Australia. The scribe will record your answers on the A3 paper.

3.   After 90 seconds move your A3 paper to the group on your left. You should now have another group's ideas in front of you.

4.   One member of your group will now read the other group's responses aloud and you will continue to add some more ideas for the next 2 minutes.

5.   After 2 minutes, move the papers to the left and repeat the process.

6.   After a couple of rotations, discuss and record in your own notes what you will need to consider when earning money in Australia.

Exercises

1. Complete the table to show fraction, decimal and percentage equivalents. The first one has been done for you.

Which part of the whole amount?	Fraction	Decimal	Percentage
$50 out of $200	$\dfrac{1}{4}$	0.25	25%
$56 out of $112			
$78 out of $780			
$21 out of $2 100			
$12 out of $60			
$1 out of $8			
$10 065 out of $100 650			
$32 out of $128			
$90 out of $9 000			
$34.55 out of $69.10			
$400 out of $1 000			
$180 out of $200			
$60 out of $500			

2. Bronwyn is paid a retainer of $300 each week and 0.5% of the total value of the cars she has sold. In a particular week Bronwyn sold cars to the value of $150 000. How much has Bronwyn earned in that particular week?

3. Alice is paid a weekly wage for her 38 hours at the hairdressing salon. Her hourly rate is $24.50 and she gets paid fortnightly.
   a) What is Alice's weekly wage?

   b) What is Alice's fortnightly pay?

4. Jenna is the bookkeeper for Jax's Plumbing Service and uses a spread sheet shown below to keep track of the wages to be paid.

E2		f_x =0.12*D2				
	A	B	C	D	E	F
1	**EMPLOYEE**	Hours worked	Hourly Wage	Income	Superannuation	
2	Kurt Kobult	26	$26.50	$689.00	$82.68	
3	Ralph Rocket	38	$31.15	$1,183.70	$142.04	
4	Mandy Mission	15	$18.20	$273.00	$32.76	
5						

   a) Cell E2 = 0.12 * D2, what percentage of superannuation does this represent?

   b) What formula did Jenna use to use to calculate cell D2?

   c) The symbol Σ is shown on the toolbar of the spread sheet. How could Jenna use this tool to assist in her bookkeeping?

5. Lim earned an annual salary of $73 250, calculate
    a) her monthly income.

    b) her fortnightly income.

    c) her weekly income.

6. Molly has had $13 210 taken out as PAYG. She completed her tax return and got a refund of $360 from the taxation department. Which of the following might explain the refund?
    a) She had extra income.

    b) She had received a petrol allowance from her employer.

    c) She made donations to UNICEF.

    d) She was able to claim self-education expenses.

    e) Union fees were paid by Molly from her bank account.

7. Andrew works in hospitality and is paid $66 per hour when he works at double time rates. In one particular week he worked 10 hours at double time and 37.5 hours at the normal rate. Calculate the income Andrew earned that week.

8. Max and Jamie are concreting contractors who are paid per square metre of concreting. The rate of payment will vary according to the depth of the concrete used displayed in the table below.

Type	Driveway	Granny Flat	Verandah	Shed
Recommended Depth	20 – 25cm	15 – 20 cm	12.5 – 15cm	10 –12.5cm
Rate per m²	$45.00	$38.50	$34.30	$32.75

Calculate how much they will earn on each of the concreting jobs below.

a)   SHED

6m

4.2m

b)   DRIVEWAY

12m

3.2m

2.7m

3.2m

c)   VERANDAH

1375mm

6900mm

800mm

d)   GRANNY FLAT

2m

5.2m

4.2m

6.9m
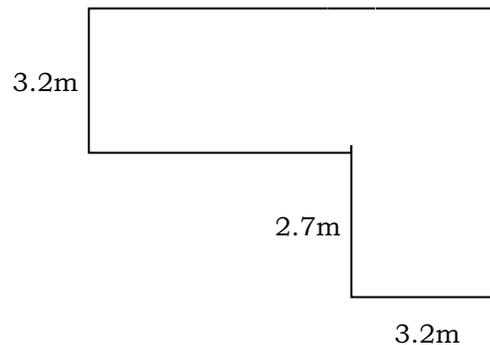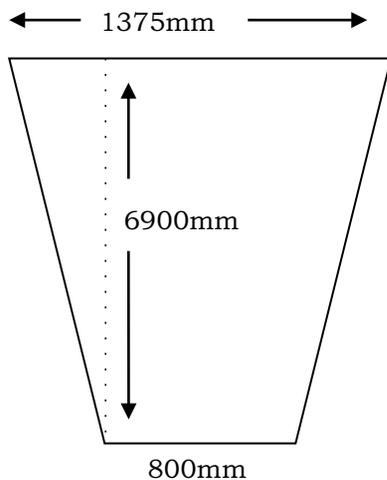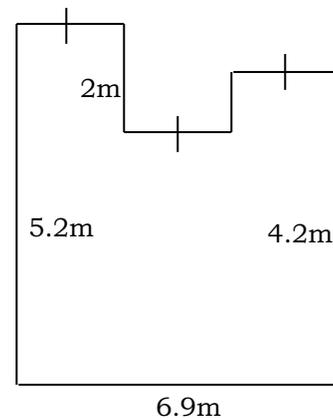
9. If Fiona's employer has made a compulsory 11% contribution to Fiona's superannuation fund and this amounted to $4 800, what was Fiona's annual salary?

10. Andy, a real estate agent, earns 2.25% commission on sales. Calculate the commission earned on house sales of:
a) $423 000   b) $825 000   c) $1.1 million.

20 Bank accounts

Materials required: Photocopy of Silent Card Shuffle from Teacher's Guide

Warm-up

1.	Round $37 378 to the nearest thousand dollars.	2.	Round $1 345.78 to the nearest dollar.
3.	How many twenty cent pieces make $4.60?	4.	How many $2 coins make $72?
5.	$4 \times 15 - 6 =$	6.	$13 - 2 \times 3 =$
7.	$3 \times 5 + 2 \times 6 =$	8.	$(4 + 1)^3 =$
9.	$2.3 \div 100 =$	10.	$2.3 \times 100 =$

Activity – Silent Card Shuffle

- Silent Classification - in groups of four, distribute the cards provided to you by your teacher evenly among your group. With *no talking* to your group members, classify the cards into each category provided in the cards in **bold** print.
- Challenge, Justify and Refine - once directed by your teacher, you can discuss and challenge the classification. With whole group approval make any card classification changes.
- Circle and Observe - once directed by your teacher, leave a person in your group as a group representative to answer any questions, the other members then walk around and observe the other students' efforts.
- Return and Refine – return to your group and make any changes based on what you've seen, again justifying and reaching consensus.
- Solutions – your teacher will give you the solutions and mark your results. Which group achieved the highest score?

Money makes the world go around.

Money is a "medium of exchange" which has an agreed value. In earlier times cattle, shells, gold, silver or grain were used as money. When you gain employment you and your employer have an agreement of payment for goods or services you will provide.

Why do you need a bank account?

For your employer to give you your *income*, you will be required to provide a bank account for the money to be placed into. To open a bank account you will need to provide 100 points of identification.

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Once you start to earn money you will also want to purchase goods and services. To do this you will need to access your money and this can be done 24 hours a day with internet banking, phone banking or at an Automated Teller Machine (ATM). Money is also available from a bank branch or by purchasing goods using Electronic Funds Transfer at Point of Sale (EFTPOS) and asking for cash out with your purchase.

Exercises

1. Pauline needs to pay her TAFE fees and has written a cheque for the first semester's fees below.

a) Why do you think it is necessary to write the amount to be paid in words and numerals?

b) Convert the following written amounts into numerals:
   (i) Three hundred and five dollars
   (ii) One thousand, seven hundred and thirty two dollars
   (iii) Ten thousand, four hundred and fifteen dollars
   (iv) Five hundred and six thousand dollars

c) Write these values in words
   (i) $428.50   (ii) $12 040   (iii) $862 050

2. Jo has been saving money in her piggy bank and has tallied the numbers of each coin in the table below.

Coin	$2	$1	50c	20c	10c	5c
Number	32	28	44	145	134	86

a) Calculate the total amount of money Jo has in her piggy bank.

b) If Jo decides to exchange this for cash at the bank, what will be the minimum notes and coins she will receive?

c) Jo decides she would like to use the money to put in her saving account so she can save for a $1200 holiday to Bali for leavers. How much more money would she need to save to reach her holiday target?

3. To calculate the amount of interest earned in a savings account over a period of time, we can use the formula **I = P × R × T**, where **I** = *interest earned*, **P** = *initial deposit (principal)*, **R** = *interest rate ÷100* and **T** = *time in years*. Copy and complete the table below using the interest formula shown above.

PRINCIPAL	RATE	TIME	INTEREST
$500	3%	3 years	
$800	3.6%	4 years	
$2500	2.75%	5 years	
$1500	3.5%		$210
	4.25%	5 years	$552.50

4. When money is saved in a savings account a special **monthly** rate of interest is often used. The interest earned is calculated on the **minimum monthly balance** and only *truncated* whole numbers of dollars are used.

<div align="center">

Interest = R × minimum monthly balance

12

</div>

Calculate the interest earned for each of the following accounts during June:

a) Sam's minimum monthly balance is $1 526.80 and the interest rate is 3.25%.

b) Rhonda's minimum monthly balance is $2 620.35 and the interest rate is 4.2%.

c) Colin's minimum monthly balance is $5 360 and the interest rate is 3.85%.

d) Margaret's minimum monthly balance is $3 144.10 and the interest rate is 4.35%.

5. Jenny deposited $2000 into a savings account and continued to deposit $100 each month. Tammy deposited $1000 with regular monthly deposits of $200. Assuming no interest is earned and both girls make no withdrawals on the accounts, which girl would have saved the most money after 18 months and by how much? Hint: use a table or a spread sheet to see what happens.

Extended Problem Solving

Visit some banks or their websites to collect information about the everyday banking accounts they have available. In a group, collate the important features of each account. You may want to include things like the identification required to open the account, fees, interest and charges or accessibility to your money.

Share the information you have collected with your class through a class presentation. Be sure to *plan* your speech and present your material with a *beginning, body* and *conclusion*.

Two important tools that will help you present your talk is your voice (verbal) and your body (non-verbal). You only have one opportunity to make an impression on your listeners when preparing a talk, this is why careful preparation and planning is essential

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21 Spending and discounts

Materials required: calculators

Warm-up

1.	10% of $35	2.	1% of $35
3.	21% of $35	4.	21% off $35
5.	Round $4.1276 to the nearest cent.	6.	Round $22.154 to the nearest cent
7.	25% = ¼, true or false?	8.	$\frac{1}{5}$ = 15% True or False
9.	18% as a decimal	10.	2½% as a decimal

Introductory problem

Julie-Anne wishes to purchase the latest *Kimberley K.0.1 Smart Phone.* She researches prices and found discounted adverts by the following three mobile phone retailers.

Always cheapest *HONEST HARRY'S MOBILE PHONE SPECIALIST STORE* *Kimberley K.0.1 Smart Phone $235*	*Straightforward Steve's Discount Warehouse* *15% off all stock* *Kimberley K.0.1 Smart Phone RRP $265*	**Trustworthy Tess's Electronics** $\frac{1}{5}$ **off the price of all phones in store** **Kimberley K.0.1 normally $295** **Tues and Wed only**

a) Which store has the cheapest price for the smart phone?
b) What other considerations could affect Julie-Anne's choice when purchasing the phone?
c) Each advertisement belonged to an online store, and postage charges for each are as follows:

> Honest Harry $21.50
> Straightforward Steve $15.00
> Trustworthy Tess $18.25

How would this affect the overall price of each of the phones?

Spending Money Wisely

In a very competitive market, with a wide range of products available at numerous outlets, increased availability and a variety of packaging and promotions and add on incentives, it makes sense for consumers to shop around for whatever they want to purchase. This includes comparing prices and service, and checking for good quality products. To summarise shoppers should:

   a) choose purchases wisely,
   b) shop around to compare prices and quality,
   c) avoid unplanned or impulse buying of goods and services,
   d) buy only what is needed – no article is a bargain if it is not needed,
   e) watch for special rates at reputable stores,
   f) buy in quantity if it is sure to be used,
   g) never decide to buy because of the cheap price only; consider quality, quantity and need,
   h) ask for discounts; many stores will oblige,
   i) wait for the end of year or end of season sales if possible.

Discounts

Discounts are an everyday part of retail trading and retailers offer discounts for variety of reasons: to clear stock at the end of the season; to reward regular customers; to encourage commercial traders who buy in large amounts; and to encourage people to pay their accounts promptly.

Exercises

1. There are many reasons why discounts are offered.
    a) List three situations where you have seen a discount offered by a retailer to encourage consumers to part with their money.
    b) Sometimes we need to be careful about buying discounted products in case we are not getting the quality we want. What is an example of this?
    c) Is it worthwhile buying a product that is not top quality? Explain.
    d) List two other ways retailers encourage consumers to purchase products from their stores.

2. Find the discounted price for each of the following:
    a) 20% off fitted sheet sets marked at $32.95
    b) 15% off a set of 4 Super Sport car tyres at $303 each
    c) 8% off a family coffee maker, normally $245
    d) 12½% off a printer/scanner priced at $495

3. Savannah decided to "buy smart" this year when replenishing her summer wardrobe. She waited until the store advertised its summer sale with all clothing reduced by $\frac{1}{5}$ of the original price.

    How much did she pay for these items and how much did she save overall?
    2 skirts $29 and $54, 2 blouses at $16 each, 2 pairs of slacks at $35 and $59, bathers at $49.90 and shoes at $26
    a) What is the discounted price for each of these items?
    b) How much did she pay altogether?
    c) How much did she save by buying them on sale?

4. John runs a painting business and has an account at a major hardware store where he automatically gets a 15% trade discount and a further 3% discount if he pays his account within 14 days. What does he pay on each of these bills if he settles the account within the 14 days?
    a) $560
    b) $1020
    c) $430

5. Calculate the percentage discount that has been given in each of the following sales:
    a) $152 off the regular price of $950
    b) $3.12 off the regular price of $26
    c) $327.60 off the regular price of $2 340
    d) $64.78 off the regular price of $1 580
    e) $30.96 off the regular price of $430

6. The Robinsons shopped around for a plasma TV which they saw advertised in a junk mail brochure for $1 498. They negotiated a price of $1 378 which included delivery of the set. What was the percentage discount obtained?

7. Bradley's book store is offering a 15% discount on all books in the store. To assist Bradley's staff to calculate the discount:
    a) Using algebra, write a formula the staff could use to calculate the discounted price(d) from the original price of the book (b).
    b) Show how you could write this formula in another way.

8. Siobhan is pleased to hear that her favourite accessories store is offering a further 50% discount on their already 50% discounted items. She is excited by the prospect of getting 100% discount, hence free items. Her best friend Paula explains to her that the items will not be free as 50% of 50% is not 100%. Which girl is correct? Justify your answer with reasoning.

Estimating the weekly grocery shopping

When doing the weekly grocery shopping a good way to keep within the budget is to keep a rough running estimate of how much is in the trolley at any time. The tally at the end doesn't have to be exact, but should be within $3 to $4 of the register total.
With practice, the amount can be added up quickly mentally and the estimate can become quite accurate.

a) With a partner, look at the sample list below and discuss the sort of rounding you might do if you were roughly adding the bill up in your head. For example, $9.99 becomes $10, $1.75 becomes $2, and $1.35 becomes $1. Now have a try at estimating the total.

Sample list

Paper towels	2.75
Cordial	3.99
Beef burgers	4.99
6 Bread rolls	2.80
Domestos	4.59
Lettuce	1.99
Tomatoes	4.29
Onions	2.79
Frozen peas	2.25
Paracetamol tablets	3.75
Milk	2.89
Vegetable oil	5.25

Check the total with your calculator. Repeat the above estimating procedure with the following two lists, write down your estimations, then check with a calculator. How well did you go? Did you get within $3 to $4 of the total?

List one		**List two**	
Milk	2.89	Milk	2.89
Ice-cream	4.69	Deodorant	4.99
Potato chips	1.69	Chocolate biscuits	3.24
Cheese biscuit	1.65	Tin peaches	2.75
Cat food	0.99	Potatoes	3.49
Leg ham	4.04	Apples	1.99
Cheese	7.98	Bananas	2.75
Fruit & nut mix	3.99	Broccoli	1.39
Multigrain bread	2.45	Cream 600mL	2.49
King prawns	10.35	Sausages	6.25
Mushrooms	3.98	Dog food	12.99
Baked beans	1.79	Window cleaner	3.60
Puff pastry	3.85	Garlic cloves	2.20

22 Best buys

Materials required: Four samples of different brand and size of a food product e.g. tin fruit or cereal boxes with cost clearly labelled on each

Warm-up

1.	0.3 kg = _____grams	2.	12.6 ÷ 100 =
3.	2.5 L = _____mL	4.	11 × 9 × 2 =
5.	14 035 ÷ 1000 =	6.	$34.25 = _____cents
7.	34cents = $_____	8.	132 ÷ 11 =
9.	3.045 < 3.251, true or false?	10.	Round 4.637634 to two decimal places.

Introductory Problem – Bulk buying

Sophie and Maxine have just moved out of home into a new apartment and are researching the best way to grocery shop in the area. Sophie comes from a big family and suggests that you get better value for money if you shop in bulk, however Maxine thinks you should buy what you are going to use for the week.

a) Comment on the positive and negative factors of both girls' suggestions.

b) Using an online directory, Maxine found that there are two large supermarket chains in the area that also provide online home delivery, a local butcher, a seafood monger and a fruit and vegetable store. Where should Maxine and Sophie do their weekly shopping? Discuss the pros and cons of each of the shopping options.

Activity - Predict, Observe and Explain

1. Copy the table below into your workbooks

PREDICT	OBSERVE	EXPLAIN

2. Your teacher will display four food products for approximately ten seconds. You need to predict which product is the *best buy* and give a simple explanation why in the predict column of your table.
3. Now you will make more detailed observations of each product and record them in the observe column, considering things like the weights, cost and type of packaging.
4. Using the example below calculate the unit price of each item and show your working in the explain column.
5. Compare the differences between your observations and predictions in the explain column.

Getting value for money

We all want to get the best *value for money* and this requires calculations to work out the *unit price*. This enables you as a consumer to make comparisons of value and brand. Large retailers and on-line providers are required by law to display the unit price using the most appropriate unit of measurement; capacity (per 100 millilitres), weight (per 100 grams) or length (per metre) rounded to the nearest cent.

For example, determine the size that gives the best value for money

Cereal $4.78 for 300g or $6.80 for 500g
$4.78 ÷ 3 $6.80 ÷ 5
= $1.59 per 100 grams =$1.36 per 100 grams

The 500g size is the best value.

Exercises

1. For each of the following shopping items, calculate which is the best value for money:

   a) $6.25 for 500g of cheese or $11.80 for 1kg of cheese
   b) $2.75 for 500g of margarine or $5.60 for 1kg of margarine
   c) $22.00 for 30 cans of cola or $12.00 for 15 cans of cola
   d) $1.62 for 75g of chocolate or $3.10 for 250g of chocolate
   e) $3.99 for 4kg of potatoes or $9.99 for 10kg of potatoes
   f) $4.75 for 250g of bacon or $12.20 for 1kg of bacon
   g) $3.20 for 20metres of foil or $5.20 for 60metres of foil
   h) $2.20 for 1litre of choc-milk or $5.45 for 2litres of choc-milk
   i) $6.10 for a 6 × 200g yogurt or $5.15 for 1kg of yogurt
   j) $3.29 for 20pk of painkillers or $4.97 for 32pk of painkillers

2. Try to do these best buy problems without your calculator:

   a)   250g for $1.96   or   1kg for $6.80
   b)   3 tins for $2.97   or $1.05 each tin
   c)   200g for $2.20   or   500g for $5.80
   d)   750mL for $3.15   or   1 litre for $4.10
   e)   apples at $4.69 per kg or   3kg bag for $10.00
   f)   300mL for $6.30   or   500mL for $10.50
   g)   15 cans for $8.50   or   24 cans for $12.00

3. Ronny has a choice between two deals on a Playstation4 and two games he wants to buy.
   Store A: A Playstation4 for $650 plus the two games worth $129 each on a 'buy one get the other half price' basis.
   Store B: A Playstation4 for $720 plus the two games at $130 each with a 15% discount on the Playstation4 and 20% discount on the games.
   a)   Calculate the value of each deal.
   b)   Which is the cheaper option?
   c)   What is the difference in the two prices?

4. A certain brand of jasmine rice can be purchased in five sizes

   10 kg for $23.86   5kg for $14.09   2kg for $5.31
            1kg for$3.24   250g for $1.60

   a)   Calculate the unit price of each bag of rice and rank the bags in order of best buy.
   b)   Explain why rice may be an item that a single person may consider buying in larger quantities in their food shopping.

5. Calculate the cost of each of the following items if the unit price

   a)   1 litre of olive oil at $0.80 per 100mL
   b)   750 grams of muesli at $0.58 per 100g
   c)   440 grams of cake mix at $0.45 per 100g
   d)   1.7kg of BBQ sausages for $4.12 per 1kg

6. Tara works at a large grocery store, and is required to choose and calculate the most appropriate unit price and unit of measurement to use for various products. Calculate the appropriate unit price for each of products below to three decimal places.
   a)   500g of cereal for $5.25   b)   750 g of coffee for $12.60
   c)   2L of orange juice for $3.99   d)   50g of fishfood for $4.20
   e)   3L of milk for $3.75   f)   12m of string for $1.50
   g)   283g of curry paste for $4.15   h)   75mL of vanilla essence for
                                               $3.35

Extended Problem Solving GST

The *Goods and Services Tax (GST)* was introduced into Australia on 1 July 2000. The GST is an extra 10% added to the values of most goods and services provided. Foods such as fruit and vegetables, plain milk and bread are exempt from GST. To calculate the GST of an item, find 10% of the original price. The retail price is then the original price with the GST added to it.

1. The following items need to have a GST component added to them. Copy and complete the table.

Item	Original Cost	GST	Final Cost
Tennis lesson	$50.00		
Club membership	$180.00		
Call out fee for an electrician	$65.00		
1 zone concession SmartRider	$0.94		
Chocolate bar	$1.60		
Strawberry milk	$4.45		

2. From the table, you can see that it is not true to say that the GST can be found by dividing the final cost by 10. Check that you agree with this statement.

   GST = Final Cost ÷ N Use your table and calculator to guess and check the value of N.

3. Use the rule developed in question 3 to calculate the GST component of the following charges. Round your answer to the nearest cent.

   a) Familyrider $11.60   b) Digital camera $98

   c) iPad $429   d) Frying Pan $49.50

   e) Fridge Magnet $5.95   f) Solar Light $32.25

4. The formula from question 2 can be rearranged so that the final cost can be calculated from the GST of the item. Write down this formula and check that it works using the table in question 1.

23 Online shopping

Warm-up

1.	Round 2.7586 to 3 decimal places.	2.	Round 24.098145 to 2 decimal places.
3.	$3 \times 12.1 + 1.6 =$	4.	$1.6 + 12.1 \times 3 =$
5.	Volume of a cube = L × W × ___	6.	1m = _____cm
7.	$1m^2$ = _____cm^2	8.	$1m^3$ = _____cm^3
9.	10% of $55 =	10.	5% of $55 =

Activity: extent barometer

To what extent is online shopping a good practice?

The world of on-line shopping has changed the retail world forever. While some people benefit from on-line shopping others have found the change difficult. In this activity you will give a rating from 0-100% to help you evaluate the question, *to what extent is online shopping good practice?*

a)   Copy the table below and, with a partner, discuss the positive and negative effects that can occur from each perspective and hence decide on a percentage rating

100	Totally
75	Fairly High
50	Medium
25	Low
0	None at All

Perspective: the main events	Data: effects that occur due to online shopping	Rating 0-100
Consumers		
Store owners		
Shopping Centres		
Cost		
Quality		
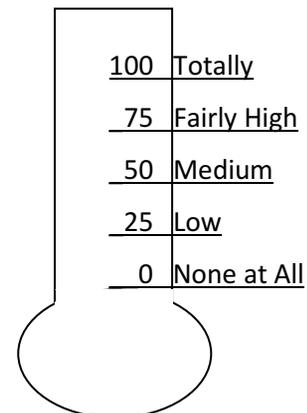
b)   Give an overall rating and a brief summary to describe to what extent online shopping is good practice.

c)   Share your findings with your class and discuss how online shopping has affected you.

Shopping online

Shopping on-line is convenient and has the added benefits of having your product delivered to your front door. It is important, however, that consumers are careful to read the fine print before clicking *buy* on a product and beware of some of the common pitfalls below.

Shipping costs and times need to be considered in the purchase, and may affect the overall price. Some stores charge the same price for all packages while others charge a separate fee for each item shipped. Shipping can also be delayed so be careful of presents not arriving on time.

Inaccurate sizing can affect your shopping experiences as variations of sizes occur between countries and brands. Research the return policy on the store to see if returns are free or can be done at a bricks and mortar store. Read feedback from other shoppers to check the sizing runs true.

Misleading product descriptions are common and the picture shown does not always match the product. Stick with stores that you know and you will avoid the risk of confusing or fraudulent descriptions.

Payment issues occur when a shopper hits the confirm and pay button twice and payment pages are slow to load. Carefully review your orders before pressing submit.

Poor packaging can leave shoppers with a broken or damaged item.

Emotional spending can leave a consumer with products they really don't need. Take your time when shopping and avoid advertisements that may pressure emotional spending.

Exercises

1. Stacey wishes to order some stationery from *Officeonline* and a statement of her purchases is shown below.

Item	Quantity	Individual price
Blue Pen	4	$1.35
Red Pen	2	$1.35
A4 Notebook 60 page	5	$2.95
Ruler	1	75 cents
Stapler	1	$12.60

    a)   Calculate the amount Stacey will need to pay for her purchase.

    b)   10% GST is to be added to the order, and a delivery fee of $8.25. Calculate the total amount Stacey will have to pay.

2. Kim and Suki are getting ready for their Year 11 functions, and have decided to purchase their entire outfits from several online stores.

Kim	Item	Cost	Delivery
	Dress	$129.00	Free for purchases over $80
	Shoes	$ 47.50	$6.25
	Necklace	$ 14.25	$4.05
	Earrings	$ 12.20	$3.45
	Handbag	$ 38.99	$4.90

Suki	Item	Cost	Delivery
	Dress	$143.50	
	Shoes	$ 65.00	Suki shops at a store
	Bracelet	$ 12.75	where delivery is 2.5%
	Earrings	$ 10.05	of purchases
	Handbag	$ 42.00	

a) Calculate the cost of each girl's outfit.
b) Suki has a store card that entitles her to a 5% discount on her purchase, how much discount is she entitled to?
c) Kim has ordered the wrong size dress and wishes to exchange the dress. Unfortunately she has not read the small print and she will have to pay postage costs of 20% of the dress price to return the goods. Calculate how much extra she will need to pay.

3. When shopping, online purchases are converted from one currency to another. It is possible to use the formula:

$$C = p \div r$$

where C = price in Australian dollars
p = price in other currency
r = exchange rate for $AU1

a) Using the currency conversion formula convert each of the following purchases bought overseas into Australian dollars to the nearest ten cents

	Item	Purchase country currency	Price in overseas currency (p)	Exchange rate(r)
(i)	Handbag	British Pound	£16	0.6
(ii)	Mascara	Taiwan dollar	$302.10	27.23
(iii)	Painting	Indonesian Rupiah	1 250 000 IRP	10 533
(iv)	Batman Costume	Chinese Yuan	¥ 41	5.47
(v)	Garlic crusher	US Dollars	$6.15	0.9

b)   The table shows the cost of items bought over the internet from a Chinese online store. Plot these points on graph paper and describe the pattern you see in the points.

Cost in $ Au	1	2	4	6	10	15
Cost in yuan ¥	5.47	10.94	21.88	32.02	54.7	82.05

Australia Post is used by Australian on-line retailers to distribute orders around the country to customers. Information about the cost, sizing and delivery times of various delivery services are shown below.

	Courier Post	Express Post	Parcel Post
Delivery time and network	Same day in metro areas of the same state	Next Business day in same metro areas of the same state	2+ days must be used for deliveries across states
Small satchel Holds up to 500grams		$10.55	$8.25
Medium satchel holds up to 3kg	$34.95	$14.80	$13.40
Large Satchel holds up to 5kg		$23.60	$16.70

4.   Using the Australia Post pricing tables above, estimate the mass of each item and hence calculate the delivery costs:

a)   A gold ring for next day delivery in Perth metropolitan area.
b)   An iPad delivered from a Sydney store to a Darwin suburb.
c)   12 textbooks delivered by Parcel Post in the same capital city.
d)   Some clothing for a four year old child, including a dress, two t-shirts and one pair of denim shorts delivered from a Melbourne store to Canberra.
e)   An important 25 page document that is required for the next business day in the same capital city.
f)   A mobile phone cover delivered interstate.
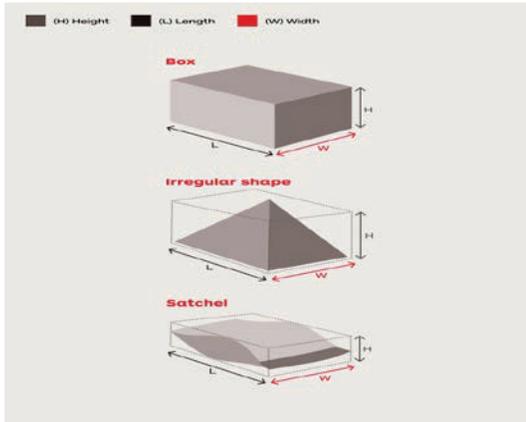
Australia Post uses a parcel's cubic weight to calculate domestic postage costs. This is calculated by the parcel's volume in cubic metres, multiplied by 250. To work this out, use the formula below:

Height x length x width x 250 = cubic weight

5. Graham and Jeet are expanding their car accessories store to include online sales. They have purchased several boxes that will hold the various car accessories.

a) Copy and complete the table below using the cubic weight formula, rounding your answers to four decimal places.

	Box Dimensions			Volume (m³)	Cubic weight (kg)
	Length (m)	Width(m)	Height(m)		
A	0.1	0.1	0.1		
B	0.2	0.2	0.2		
C	0.25	0.2	0.3		
D	0.305	0.2		0.0216	
E		0.3	0.4		10.5

b) The column graph below shows the postage charges for each of the boxes to New Zealand.

(i)   Which option is the cheapest?

(ii) Which option is the most expensive?

(iii) Estimate the price difference between the least and most expensive postage.

(iv) How much is the postage for Box C delivered Express Post?

(v) What box has the biggest difference between Airmail and Express Post charges?

(vi) Which delivery service do you think is faster? Why?

24 Mobile phones

Warm-up

1.	13 – 4 × 2 =	2.	36 – 12 × 3 =
3.	¼ as a percentage	4.	¾ as a decimal
5.	If a square has area 36cm², calculate its side length.	6.	What is the area of a rectangle with dimensions 3cm × 12cm?
7.	12 ÷ 0.5 =	8.	√121
9.	Write eighty two thousand, six hundred and seventy one in numerals.	10.	Write twenty six million, three hundred and four thousand in numerals.

Activity: How do you use your mobile phone?

In this activity you will calculate your phone usage in a typical month. To make it easier, think about how much you use your phone on a typical day. Remember to consider weekdays and weekends when estimating your usage for the week.

Example *2 posts on social media with a photo each week*
= 2 × 4 = 8 posts per month

Data Usage *= 8 × 2MB*
= 16MB

In the table on the next page, estimate your phone usage per week in each category. Convert this to a monthly figure. Also estimate your data usage each month.

Add up all of your estimated data usage for the month and convert to the most appropriate unit below, megabyte, gigabyte or terabyte.

1MB = 1 024 KB and 1GB = 1024 MB and 1TB = 1024 GB

Type of phone usage	Estimate per week	Estimate per month	Data usage
Text Messages			
Minutes of calls to the same network service			
Minutes of calls to other network services			
Data - Downloads (songs, videos, apps) 7MB per download			
Data – social media posts with photo 2MB per post			
Web browsing - how many pages you visit, 400KB per page			
Streaming video 5.8MB per min			
Streaming music 1MB per min			
Emails with attachments 500KB per email			
Video calling 3.2MB per minute			
GPS navigation 83KB per min			
Other needs : rural user international roaming, voice mail			

Discussion

How did you go in the activity? Discuss how you use your phone and how this compares with other students in the class. Discuss some of the specific needs you may have for your phone. Do you access free wi-fi so you use less data? Discuss how you manage your mobile phone account. Do you have a pre-paid mobile or is your phone on a plan? What are some of the benefits and disadvantages of having a mobile phone plan or pre-paid account?

Exercises

1. The graph below shows the responses of 80 people on their mobile phone data usage. How many participants use their mobile to:

    Data usage over mobile phone

    a) send emails?

    b) access Facebook?

    c) play online games?

    d) surf the web?

    e) watch streamed TV?
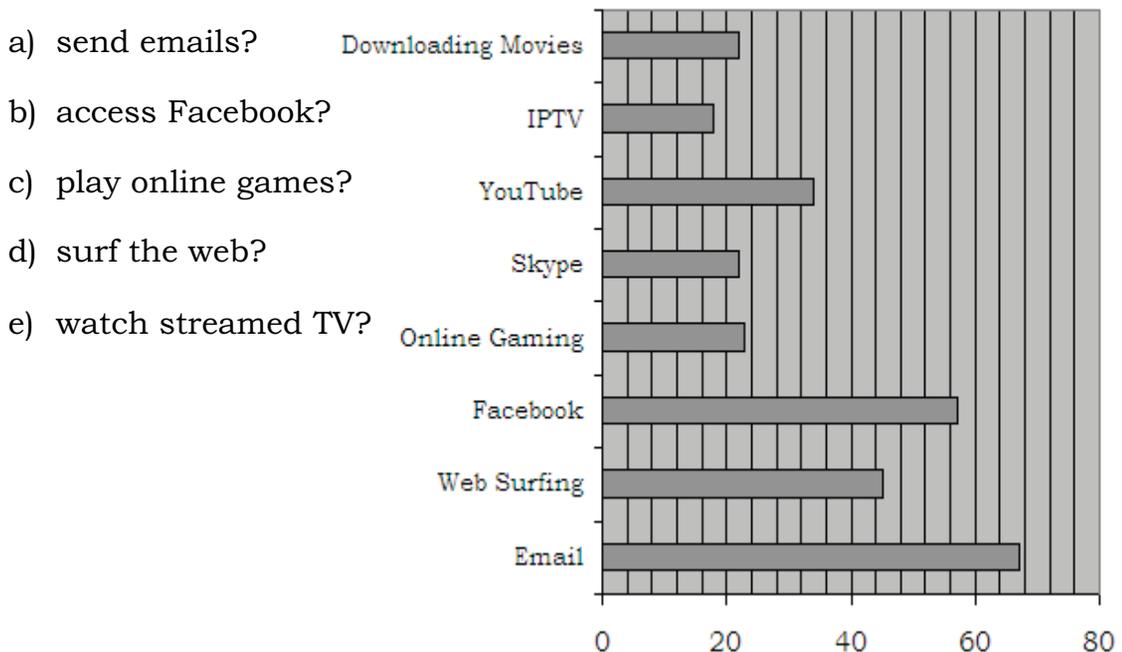
2. In the introductory activity you calculated the way you use your mobile phone in a month. Display this information in an appropriate graph.

3. The following table shows the number of Australian children between 5 and 14 years of age with a mobile phone in 2012.

	5 to 8 year olds	9 to 11 year olds	12 to 14 year olds	Total
Female	13 250	91 300	312 450	421 300
Male	12 450	86 200	303 850	397 200
Total	25 700	177 500	616 300	

    a) How many males between 9 and 11 have a mobile phone?
    b) How many females under 12 years of age have a mobile phone?
    c) Calculate the total number of children between 5 and 14 who own a mobile phone.
    d) Calculate the percentage of male owners of mobile phones.
    e) Explain the trend of mobile phone ownership as children get older.
    f) Would you expect this trend to continue for older age brackets for example 30 – 40, 40 – 50 year olds? Why?
    g) Draw an appropriate graph that represents the information displayed in the table.

4.   The chart below shows how much Australian people use their mobile phones each day. Use the chart to write a paragraph explaining how long Australians use their phones each day.

For how long do you use your mobile phone each day?
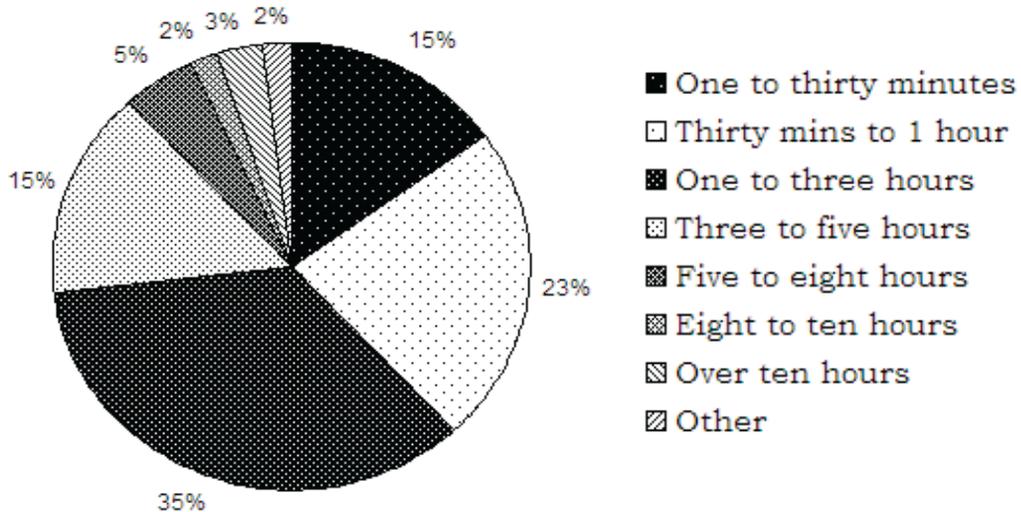
5.   In December 2012, 17.4 million mobile phone users in Australia accessed the internet by mobile phone. By June 2013 this had increased to 19.6 million subscribers. Calculate the percentage increase of mobile internet users over this six month period.

6.   The volume of data downloaded via mobile handsets for the three months ending 30 June 2013 was 19,636 Terabytes, a 43% increase from the three months ending 31 December 2012. Calculate the amount of data downloaded in the three months ending 31 December 2012.

7.   Some of the fastest-growing rates of mobile phone uptake in the world are changing the way Pacific Islanders communicate, learn, engage in political debate, co-ordinate activities and access services. The chart below shows the rate of mobile phone ownership growth per 100 people on eleven pacific islands.

Mobile phone usage (per 100 people)
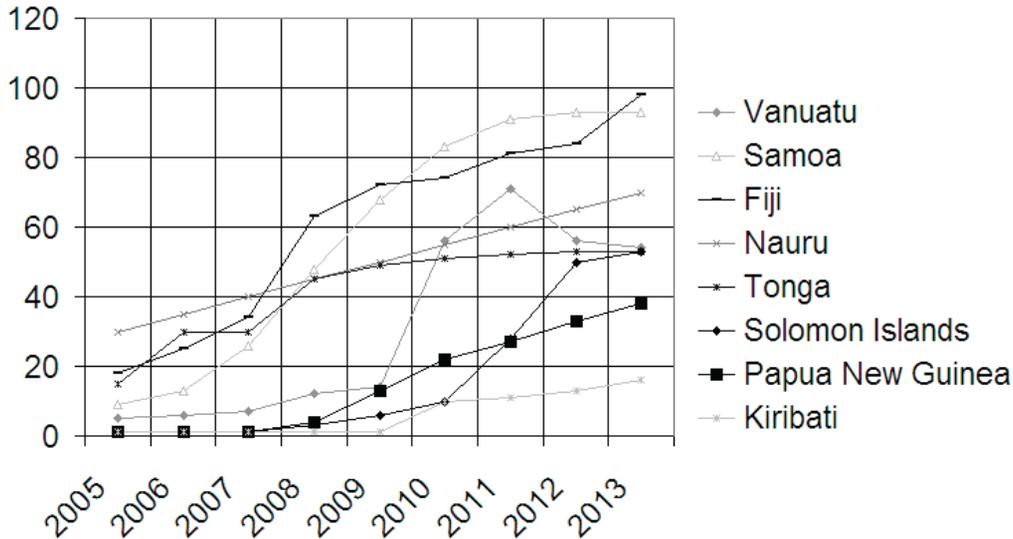
a)   Which island had the lowest number of mobile phones per 100 people in 2013?

b)   What was the mobile phones usage for Fiji in 2012?

c)   What was the mobile phone usage for Papua New Guinea in 2013?

d)   In what period did the Solomon Islands experience its highest rate of increase in mobile phone ownership?

e)   Which island experienced a constant rate of growth in the eight year period?

Extended Problem Solving

Using the results about your phone usage in the introductory activity, research different phone plans to find a plan that would best suit your mobile phone needs. Write a brief report on your findings, commenting on some of the risks associated with signing up to a contract.

25 Budgets

Warm-up

1. $50 - $30.85
2. $50 - $49.05
3. $20 – 35c
4. $20 - $10.45 - $5.10
5. How much is $125 more than $97.80
6. How much is $16.70 more than $9.85
7. If five friends have $5.50 each how much do they have all together?
8. If seven friends share $42.35 how much do they get each?
9. What is the product of 12 and 6 ?
10. What is the sum of 12 and 6?

Introductory problem

Think of something you really want, do you have a plan for getting it? If you don't, stop dreaming and make it happen – with a budget. Budgeting helps you reach your financial goals, especially if there are big things you are planning to buy. Start your budget by recording your income and expenses for a month in the spread sheet below.

	A	B	C	D
1	Monthly Income		Monthy Expenses	
2	Job		Phone	
3	Allowance		Car/Public Transport	
4	Other		Clothing	
5	Total		Beauty	
6			Sports/hobbies	
7			Gifts	
8			Savings	
9			Dining Out	
10			Movies/music	
11			Tuition/lessons	
12			Dining out	
13			Savings	
14			Other	
15			Total	

How did you go listing your expenses and income? Are your expenses less than your income? Discuss with your partner some budgeting strategies you could use.

Creating a budget

Large businesses, small businesses, schools, federal and state governments all prepare yearly budgets. Budgeting helps people to be more disciplined about spending and saving. It helps people develop a clearer idea of what they can afford and how much they can save.

A *budget* is a plan which outlines income, expenses and savings for a set period of time.
Income refers to money received.
Expenses describe the money being spent or going out of the system.
Savings refer to the differences between the income and the expenses.
Losses may occur if expenses exceed income.

An efficient budget divides the expenses into fixed and variable categories and also includes some savings. Examples of these include:

a) *Fixed*: rent, electricity, telephone, car insurance, loan repayments.
b) *Variable*: food, petrol, clothing, entertainment, holidays.
c) *Savings*: money, set aside each week for things which may be wanted in the future.

Exercises

1. From the introductory problem, create an appropriate graph to represent your expenditures. You could use a spread sheet to help you do this.

2. Classify each of the following budget items as fixed or variable expenditures:
    a) Rent
    b) Entertainment
    c) Petrol
    d) Clothing
    e) Food

3. Diana earns $2 814 per month. In her budget she allocates 30% to rent and household expenses, 20% to food, 15% to rates and taxes, 15% to savings, 10% to entertainment and sport and 10% to clothing.
    a) How much does she spend on each budget item per month?
    b) Draw a circle graph to show Diana's budget.

4. When purchasing a car, not only do you need to budget for the purchase of the vehicle but you must consider ongoing running costs. The RAC website can assist consumers to compare running costs of different vehicles. For example, costs for a Ford Festiva 1.6L 5 Speed and Holden Barina 1.6L 5 Speed are shown in the table below.

| Vehicle | Price | Average $ per week | | | Average cents per km | |
		Depreciation	Interest on Loan	Other Costs	Fuel	Tyres
Barina	$15 990	$39.05	$16.14	$37.42	9.86	0.79
Festiva	$15 490	$37.83	$15.71	$38.14	7.54	0.84

a) Calculate the average interest paid on a Barina in a one year period.

b) Wally Westgate bought a Festiva and travels roughly 15 000 km in a year. How much money should he include in his annual budget for fuel and tyres?

c) Other costs include insurance, repairs and maintenance. Calculate the difference between his other costs and his fuel and tyres for an average year.

d) Calculate the total depreciation of both cars in a five year period. Hence how much would each car be worth after five years? Convert this to a percentage of the original cost.

e) What other factors might a consumer consider before purchasing a vehicle?

5. Fixed bills like telephone and electricity do not come every week, but they need to be planned for so there is enough money for them when they arrive. Sam Star lists all of his bills from the previous year:

|	
Telephone/internet	$115 per month
Car Insurance	$276 per year
Car Registration	$535 per year
Electricity	$275 every two months
Gas	$230 every quarter
Water Rates	$930 per year
Water Consumption	$68 every two months
Council rates	$1030 per year
Household Insurance	$585 per year

a) What is the total of Sam's bills for the year?

b) What amount would you suggest that Sam puts aside in his special budget account each week?

c) What percentage of Fred's budget is for utilities, that is telephone, gas, water consumption and electricity?

6. The table below shows part of Claire's budget for 2008.

    a)   Transfer the information below into a spread sheet.
    b)   Calculate the values missing from the shaded cells, using spread sheet formulas such as *CellF12*=SUM(B12:E12).

	A	B	C	D	E	F
1	CLAIRE'S BUDGET 2015					
2						
3	INCOME	MAR	APR	MAY	JUNE	TOTALS
4						
5	WAGES	$3,568.00				
6	GIFTS			$150.00		
7	SHARES	$160.00	$165.00	$156.00	$150.00	
8	LOTTO				$30.00	
9	TOTAL INCOME		$3,945.00	$4,086.00	$3,960.00	
10						
11	EXPENSES					
12	TAX	$532.20	$567.00	$567.00	$567.00	
13	HEALTH INSURANCE	$100.00	$100.00	$100.00	$100.00	$400.00
14	GIFTS	$50.00	$50.00	$50.00	$50.00	$200.00
15	TRANSPORT	$85.00	$85.00	$85.00	$85.00	$340.00
16	FOOD	$400.00	$400.00	$350.00	$400.00	$1,550.00
17	RENT	$280.00	$280.00	$280.00	$280.00	
18	PETROL	$50.00	$50.00	$50.00	$50.00	
19	HECS	$35.00	$37.80	$37.80	$37.80	
20	CLOTHES	$200.00	$200.00	$200.00	$200.00	
21	GENERAL NEEDS	$400.00	$300.00	$350.00	$350.00	
22	CAR REPAYMENT	$335.00	$335.00	$335.00	$335.00	
23	RAC MEMBERSHIP					
24	TOTAL EXPENSES	$2,467.20	$2,404.80	$2,404.80		$9,734.60
25						
26	SAVINGS	$1,257.80		$1,681.20	$1,505.20	
27	CUMULATIVE SAVINGS	$1,257.80		$4,479.20		

    c)   Claire is saving to go overseas. How much should she have saved by the end of June?
    d)   In what month did Claire get a pay rise?
    e)   What was the most expensive item for Claire?
    f)   Name two ways by which Claire can increase the amount of money she saves.
    g)   Were there any months in which Claire spent more than she earned?
    h)   In which month did Claire earn the most? Spend the most?
    i)   In which month did Claire earn the least? Spend the least?

7.   The pie graph below shows the spending patterns of a typical family
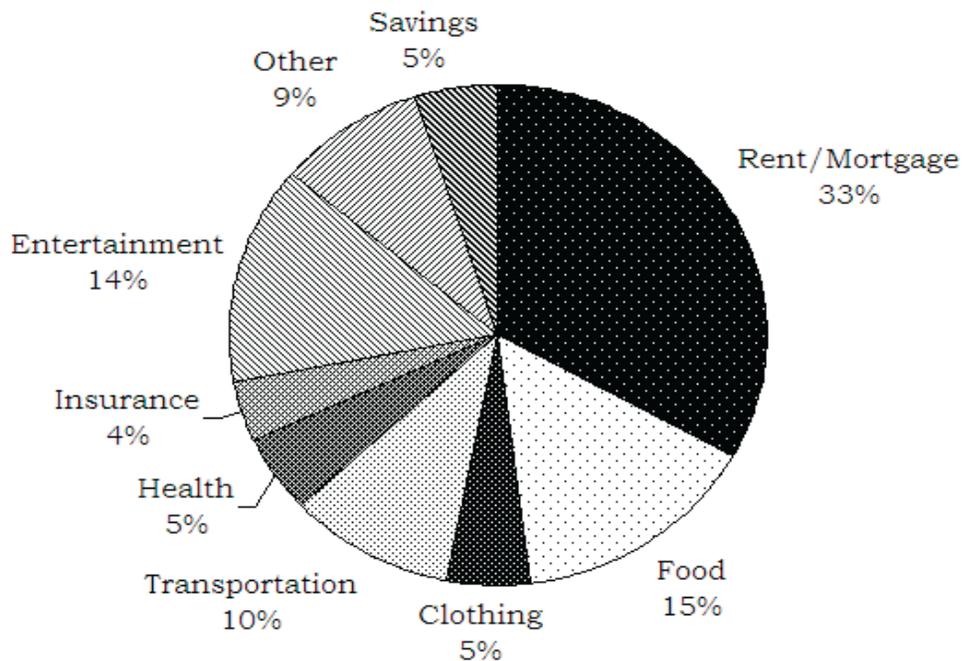
a)   What expenditure takes up the largest portion of a typical family's budget?

b)   If the Smith family has a combined income of $1 650 per week, calculate the amount of income they could expect to spend on each item based on the average percentages.

c)   The Lee family spend on average $250 per week on food. If this is 15% of their weekly income, calculate the income earned each week.

Extended Problem Solving: Planning a Dinner Dance

1. Make a list of activities that are necessary to the organisation and running of a dinner dance for a group of Year 11 students. Examples might include hiring a venue, decorating the venue, selecting the caterers and hiring security.

2. Research likely costs of these activities in your local area.

3. Apart from selling tickets, from where could the students get money to fund the dinner dance?

4. Calculate the cost of tickets so that the dinner dance does not run at a loss.

5. Calculate the personal cost (e.g. outfit, transport etc) for you to attend.

Unit 2

> A mathematician is a machine for turning coffee into theorems.
>
> - Alfred Rényi

Unit 2 content coverage WA curriculum

Content	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Chapter																									
Topic 2.1: Representing and comparing data																									
2.1.1		✓																							
2.1.2		✓																							
2.1.3				✓							✓	✓													
2.1.4					✓		✓	✓				✓													
2.1.5					✓		✓	✓																	
2.1.6		✓										✓													
2.1.7			✓		✓			✓	✓			✓													
2.1.8			✓		✓							✓													
2.1.9			✓				✓																		
2.1.10									✓																
2.1.11					✓	✓	✓																		
2.1.12						✓																			
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2.1.15									✓	✓															
2.1.16										✓															
2.1.17							✓																		
Topic 2.2: Percentages																									
2.2.1				✓							✓	✓			✓										
2.2.2												✓			✓										
2.2.3									✓			✓									✓				
2.2.4															✓										
Topic 2.3: Rates and ratios																									
2.3.1													✓	✓											
2.3.2													✓	✓											
2.3.3													✓	✓											
2.3.4													✓	✓											
2.3.5													✓	✓											
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2.3.7													✓	✓											
2.3.8														✓				✓							
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Content	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Chapter																									
Topic 2.3: Rates and ratios continued																									
2.3.10															✓				✓						
2.3.11															✓				✓						
2.3.12															✓				✓						
Topic 2.4 Time and motion																									
2.4.1												✓					✓			✓					
2.4.2																	✓			✓					
2.4.3																	✓			✓					
2.4.4																			✓						
2.4.5																			✓						
2.4.6																			✓						
2.4.7																				✓					
2.4.8																							✓	✓	✓
2.4.9																							✓	✓	✓
2.4.10																			✓						
2.4.11																			✓						
2.4.12																			✓				✓	✓	✓
2.4.13																					✓	✓			
2.4.14																					✓	✓			

> Mathematics is the alphabet in which God has written the universe.
>
> - Galileo Galilei

Unit 2 content coverage ACARA curriculum

Content	Chapter 26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Topic 1: Representing and comparing data																									
ACMEM043		✓																							
ACMEM044		✓																							
ACMEM045			✓								✓	✓													
ACMEM046					✓		✓	✓				✓													
ACMEM047					✓		✓	✓																	
ACMEM048	✓											✓													
ACMEM049			✓		✓			✓	✓			✓													
ACMEM050			✓		✓			✓	✓			✓													
ACMEM051			✓		✓							✓													
ACMEM052			✓				✓																		
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ACMEM058									✓	✓															
ACMEM059										✓															
ACMEM060							✓																		
Topic 2: Percentages																									
ACMEM061			✓								✓	✓				✓									
ACMEM062												✓				✓									
ACMEM063										✓		✓									✓				
ACMEM064																✓									
Topic 3: Rates and ratios																									
ACMEM065													✓	✓											
ACMEM066													✓	✓											
ACMEM067													✓	✓											
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ACMEM070													✓	✓											
ACMEM071															✓				✓						
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ACMEM075															✓				✓						

Content	Chapter																								
	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Topic 4: Time and Motion																									
ACMEM076												✓					✓			✓					
ACMEM077																	✓			✓					
ACMEM078																	✓			✓					
ACMEM079																			✓						
ACMEM080																			✓						
ACMEM081																			✓						
ACMEM082																				✓					
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ACMEM084																							✓	✓	✓
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ACMEM087																			✓				✓	✓	✓
ACMEM088																					✓	✓			
ACMEM089																					✓	✓			

Life is good for only two things, discovering mathematics and teaching mathematics.

\- Siméon-Denis Poisson

26 Data collection and representation

Warm-up

1.	72 ÷ 6 × 12	2.	Write 0.125 as a fraction.
3.	Write 90% as a decimal.	4.	Reduce $140 by 10%.
5.	450 g = _____ kg	6.	1180mm = _____m
7.	Round 158.5 to nearest 10.	8.	How many mm^2 in $45cm^2$?
9.	What is the area of a 6m by 8.5m rectangle?	10.	What is the perimeter of the same rectangle?

Discussion
Statistics
Statistics are numerical data that have been organised to serve a useful purpose. Data are observations or facts which when collected, organised and evaluated become information or knowledge.

> *"Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write."*
> **H.G. Wells (1866-1946)**

> *"There are three types of lies. Lies, damned lies and statistics"*
> **Benjamin Disraeli (1804 -1881)**

> *"...like veal pies, are good if you know the person that made them, and are sure of the ingredients."*
> **Lawrence Lowell (1909)**

> *"By a small sample, we may judge of the whole piece."*
> **Miguel de Cervantes from Don Quixote**

1. Discuss what you think is meant by each of these four quotes.

2. List then discuss who uses statistics, and for what purpose.

Exercises

1. Suggest reasons why data would be collected on the following topics:
    a) Test scores   b) Causes of death   c) Climate

    d) Forests   e) Immigration   f) Schools

    g) Populations by   (i)   districts/suburb
                         (ii)   country

2. a) What city in Australia or the world do you think is the best?

    b) List what information could be gathered to support your choice.

    c) How could this information be collected?

3. a) Who do you believe is the best band or musical performer?

    b) What statistical information would you gather to convince others to agree with your choice?

4. Sometimes the same statistical information could be used to justify opposite positions. Consider compulsory school uniform:
    a) Make a list of arguments that could be made supporting a compulsory school uniform. What data could be collected to support this view?

    b) Make a list of arguments that could be made against a compulsory school uniform. What data could be collected to support this view?

    c) List other situations where statistics can be used to argue opposing views.

5. You are on the school committee organising an end of year function. What data would you collect to ensure the function is a success in the opinion of the majority of students who will attend? How would you gather this information?

Extended Problem Solving

Research any three of the following occupations. State what the occupation does and how they use statistics.

- actuary
- biometrician
- environmental statistician
- forensic statistician
- government statistician
- health service statistician
- market research statistician
- medical statistician
- pharmaceutical statistician

Watch commercial television for half an hour recording how many advertisements use statistics to endorse their product. How many don't? Express your answers as percentages. Also take note of how statistics are used in these adverts.

Read the newspaper for a week recording how many of the articles in the first 3 pages use statistics in their reporting.
How many articles use multiple statistical references?
How often are graphs or tables used to display statistics?

Geometry will draw the soul toward truth and create the spirit of philosophy.

- Plato

27 Types of data

Warm-up

1.	Write $^7/_{10}$ as a percentage	2.	Write $^7/_8$ as a decimal
3.	$4 \times (11 - 3)^2$	4.	1.25km = _____m
5.	2 kilowatts = _____watts	6.	Round 22.226 to the nearest tenth.
7.	Increase 750g by 15%.	8.	Area of 0.7m by 450cm rectangle?
9.	Estimate 402 × 1900 using leading digits.	10.	Write 12.345 to 2 significant figures.

Discussion

Types of Data

Data are observations or facts which, when collected, organised and evaluated, become information or knowledge. Data can be either categorical or numerical.

Categorical data

This is data whose values are categories. Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other). Categories may have numerical labels, e.g. the numbers worn by players in a sporting team, but these labels have no numerical significance – they merely serve as labels.

Numerical data have values that describe a measurable quantity as a number, like 'how many' or 'how much'. A numeric variable is one that describes a numerically *measured* value.
Numerical data can be either continuous or discrete.

Data is said to be **continuous** if it can take *any value within a certain range.* Examples of continuous data may be distance, age or temperature. The measurement of a continuous variable is restricted by the methods used, or by the accuracy of the measuring instruments. For example, the height of a student is a continuous variable because a student may be 1.6321748755... metres tall. However, when the height of a person is measured, it is usually only measured to the nearest centimetre. Thus, this student's height would be recorded as 1.63m.

Continuous data is usually grouped when recording it due to the likelihood that most values will be different with few values repeated.

NOTE: measurement of a continuous variable is always a discrete approximation.

Discrete data is based on a count from a set of distinct whole values.
It cannot take a fractional value that is between one value and the next closest value. Examples of discrete variables include the number of registered cars, number of business locations, and number of children in a family, all of which are measured as whole numbers (i.e. 1, 2, 3 cars).

Exercises

1. For each of the following, indicate whether it is numerical or categorical data:

    a) People's names

    b) People's ages

    c) Favourite meals

    d) Number of brothers and sisters

    e) Number of pets

    f) Types of pets

    g) Amount of energy used by an appliance

    h) Cost of a house

    i) Time to run 100m

    j) Your Postcode

2. Give two examples of categorical data that use numbers.

3. For each of the following, indicate whether it is discrete or continuous data:

    a) The time taken for you to get to school.

    b) The number of couples who were married last year.

    c) The number of goals scored by a women's hockey team.

    d) The speed of a bicycle.

    e) Your age.

    f) The number of subjects that you can choose to do next year.

    g) The duration of a phone call between two people.

    h) The annual income of an individual.

    i) The number of people working at the Australian Bureau of Statistics.

    j) The number of brothers and sisters you have.

    k) The distance between your house and school.

    l) The number of pages in this book.

4.   Give two examples, different to any of those given previously, of:

    a)   a discrete variable.

    b)   a continuous variable.

5.   Why do you think the measurement of continuous data is always a discrete approximation?

> He who can properly define and divide is to be considered a god.
>
>   - Plato

28 Measures of location

Materials required: measuring tape

Warm-up

1.	3560 mm = _____m	2.	Estimate 2300 × 1990
3.	Write 73% as decimal	4.	Write 7.3% as a decimal
5.	Write the ratio 12: 20 in its simplest form.	6.	Increase $1250 by 10%
7.	Increase $1250 by 11%	8.	What is the area of a 1.2m by 45 cm rectangle?
9.	What is the area of a 5m, 12m, 13m right angled triangle?	10.	If you walk at 4.5km per hour for 80 minutes how far would you travel?

Discussion

Measures of location or measures of central tendency are also known as the averages. An average is (usually) a single figure that gives us a measure, an indication or information on the centre of the data (a typical score).

There are three main types of average:

- Mean (arithmetic mean): $\dfrac{\text{the sum of the scores}}{\text{the number of scores}}$

- Median: middle score when all the scores are arranged (ranked) in order

- Mode: the number that occurs most frequently

Mean (arithmetic mean) is relatively easy to calculate but is affected by extreme values.

The mean might be a score in the original data, but is not required to be.

Example

Four students got test scores of 60%, 55%, 50%, 45%

(60 + 55 +50 +45) ÷ 4 = 210 ÷ 4 = 52.5%

This gives an indication of the centre of the data, but as can be seen the score itself 52.5% is not one of the initial four scores.

A fifth student sits the test later and gets 100%. What is the new mean?

(60 + 55 + 50 + 45 + 100) ÷ 5 = 310 ÷ 5 = 62.5%

This mean gives an indication of a central value but is skewed by the extreme (100%) value which is considerably higher than the other four scores. As a result the new mean is greater than all of the original four scores and is no longer a clear indication of the centre of the data.

The smaller the sample size, and the more extreme the value, the greater the effect the extreme score has on the mean.

- A company has 9 workers who each earn $50 000 a year. What is the mean wage?

    It is easy to determine that the mean would be $50 000 as all the values are the same.

- The same company has a director who earns $1 000 000 a year. What is the new mean?

    (9 × 50 000) + 1 000 000 = 1 450 000
    1 450 000 ÷ 10 = 145 000
    So the new mean of $145 000 is considerably higher and not truly representative of the average or typical wage in this case.

The **median** is unaffected by extreme values, but takes more effort to calculate because the data has to be ordered first. It is generally used when a data set is likely to contain extreme values which need to be included.

Discuss in small groups: list some examples of when the median would be a better indication of average than the mean.
Revise as a think, pair, share activity: how do you calculate the median?

The **mode**, while useful to manufacturers, for example, who want to identify the most common size, is not always typical of a population as a whole.

The mode is easy to calculate for small data sets. If a data set has two values that are equally the most frequent, we state both and say the data is bimodal. If there are more than two values, though, we say there is no mode.

Which average to use? Consider:

- Mean and median always exist and are unique.

- Mode may not be unique or may not exist.

- Mean and median are very common and familiar.

- Mode is used less frequently.

- Only the mean uses the value of every data element.

- The mean, can be distorted by extreme data elements.

- The mode is the only appropriate average for nominal data.

- Median requires the data set to be placed in order, which can be time consuming for large data sets.

Introductory Activity

What is the average height of your class?

Students line up smallest to tallest.

Median

> To find the middle, students are to sit down in pairs – one from each end of the line (the tallest and smallest people standing) until only one or two students remain.
>
> If one student remains, they are the median height. If two remain the median is the height halfway between the two. Notice how the heights of all other students have no effect on the median height.

Mean

> To calculate the mean we need to measure the height of every student in the class. To do so, make sure all your measurements are taken in the same way.
>
> How accurate can you make your measurements?
>
> Should you measure with shoes on or off?
>
> How should you measure a student's height?
>
> Once all measurements are taken calculate the mean.

Mode

> Are there any measurements that occur more than once? If so, the measurements that occur the most will be the mode.

Exercises

1.   What average is best used to measure:
    a)   average house prices in a suburb?
    b)   average shoe size?
    c)   average height in a basketball team?
    d)   average test results of a class?
    e)   average shirt size?
    f)   average yearly rainfall?
    g)   average weekly earnings?
    h)   average people per household?
    i)   average temperature?

2.   Calculate the mean, median and mode of the following:
    a)   7 people aged 7, 18, 9, 12, 3, 27, 31.
    b)   hours of homework done each week by 20 students:
            1, 7, 0, 1, 1, 5, 2, 1, 13, 3, 1, 24, 1, 0, 4, 4, 6, 3, 6, 8.
    c)   heights in cm 162, 165, 150, 148, 188, 183, 166, 164, 172.
    d)   cars per household 1, 1, 2, 3, 5, 0, 0, 2, 2, 1, 2.
    e)   travel time to school in minutes:
            5, 15, 12, 25, 12, 35, 18, 45, 95.

3.   50% of people are below average. Is this true? Explain your answer.

4.   The average speed of speech of most public speakers can vary from 80 up to 160 words per minute.
      In general, we could state that you speak **slowly** when you say 80 to 100 words per minute (wpm).
      The **average** talking frequency of human beings is 120 wpm.
      A **Fast** speaking pace is 140 to 160 wpm.
      Calculate how many words you would need for a 10 minute presentation.
      What factors do you need to consider?

5.   A passenger elevator is rated for 1200 kg and 14 passengers.
      a)   (i)   What is considered the average weight for the purpose of the rating?
            (ii)   The floor space of the elevator is 1.5 × 2 metres. What is the average floor space for each passenger if it is loaded to capacity?
            (iii) Draw a scale diagram and divide the floor space equally between the 14 passengers. Is the result realistic? Comment on your findings.

      b)   Another design has floor dimensions of 1950mm by 1750mm and has a carrying capacity of 1600kgs and 21 passengers.
            (i)   What is considered the average weight for the purpose of the rating?
            (ii)   What is the average floor space for each passenger if it is loaded to capacity?
            (iii) Draw a scale diagram and divide the floor space equally between the 14 passengers. Is the result realistic? Comment on your findings.
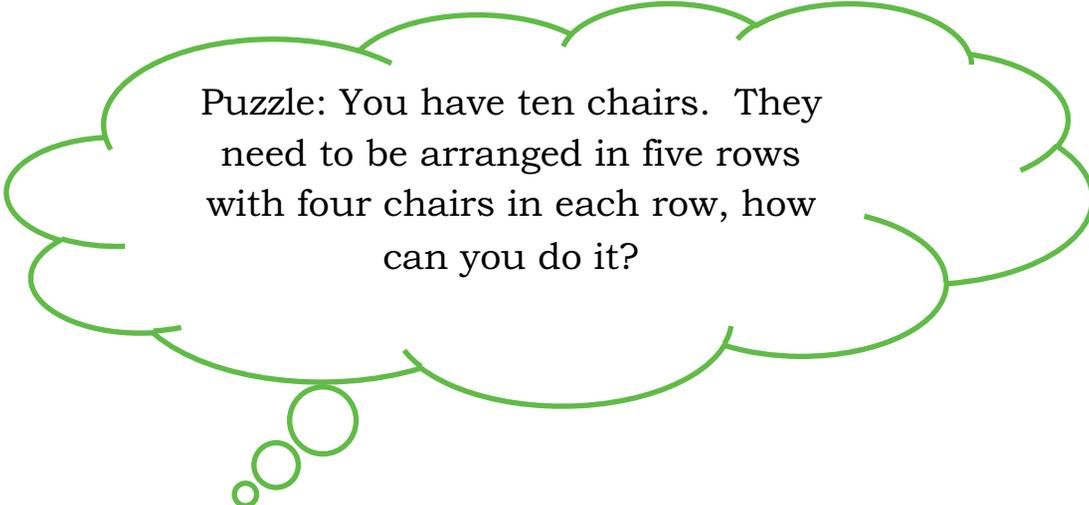
Puzzle: You have ten chairs. They need to be arranged in five rows with four chairs in each row, how can you do it?

29 Representing data

Warm-up

1.	Round 12 454 to the nearest 100.	2.	Increase 55 by 10%.
3.	Write $^2/_3$ as a percentage	4.	Write 0.35 as a fraction.
5.	1200 mL = _____L	6.	12mL = _____L
7.	Area of a 4m by 50cm rectangle?	8.	Area =
			Sides 2cm, 4cm 3.5cm
9.	If it takes 45 minutes to travel 12 km, how fast are you travelling?	10.	If I'm travelling at 90km/h, how long will it take to travel 60km?
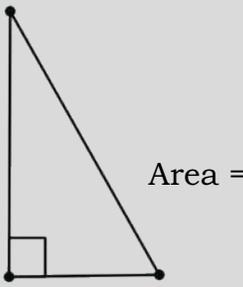
Activity

Information that is presented in a graph can be quick and easy to understand. There are times when information is better presented in a graph than in a table. This is often the case when there is a *trend* or *comparison* to be shown. A graph does this effectively.

Using Graphs

Knowing how to convey information by graph is important in the presentation of statistics. The following is a list of some general rules to keep in mind when preparing graphs. A graph should:

* be simple and not too cluttered,
* show data without changing the data's message,
* clearly show any trend or differences in the data, and
* be accurate in a visual sense (if one chart value is 15 and another 30, then 30 should appear to be twice the size of 15).

It is also important to know what type of graph to use when presenting statistics.

Bar Graphs

Bar graphs are the most common way to display categorical data. A bar graph can be either vertical or horizontal. To distinguish between the two we call vertical a column graph. Column graphs are best used when comparing the amounts of categorical data. In general, column graphs display comparisons between data better than horizontal bar graphs, so you should use them in preference.

One disadvantage of column graphs is the limited space for a written label at the foot of each column; so it is best to use a column graph when the label is short. However, when category labels in the graph are long it is better to display information using a horizontal bar graph.

Exercises

1.

Eye Colour of 100 students

Frequency
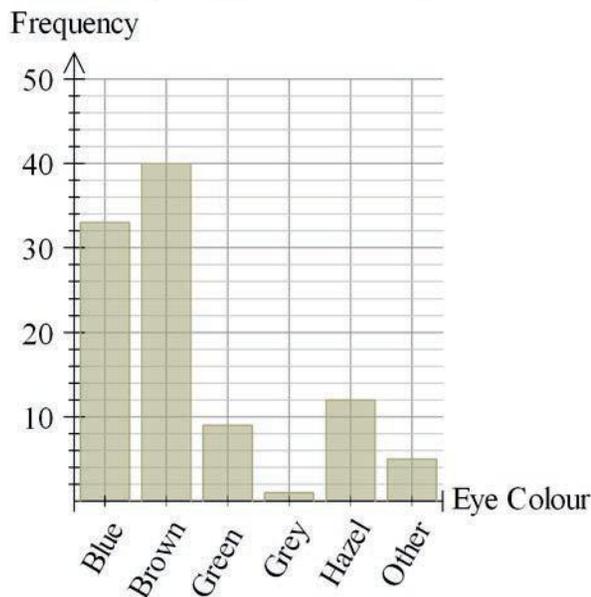
This column graph displays the eye colours of

When drawing graphs it is important to include:
- Title
- Labels on Axes
- Scale

In this example there are 100 students so conversion to a percentage is easy.

a) What percentage of students had green eyes?

b) How many students had blue eyes?

c) What percentage of students had either brown or hazel eyes?

d) What percentage of students had neither blue, brown nor hazel eyes?

e) The graph shows that 5% of the students had *other* eye colour. What do you think this means?

f) Collect the data of the eye colours of students in your class and record the information in a frequency table.

Eye colour	Tally	Frequency	Percentage

g) Calculate the percentage of each colour.

h) How does this compare to the results in the graph above?

2. Students were surveyed to determine how many were participating in selected activities. The results are displayed in the two graphs below.
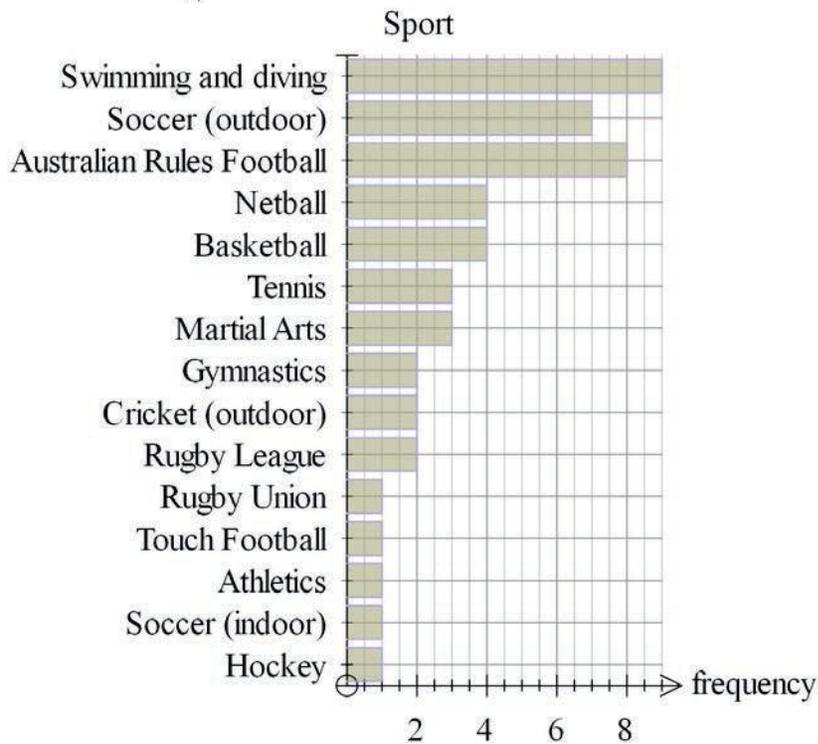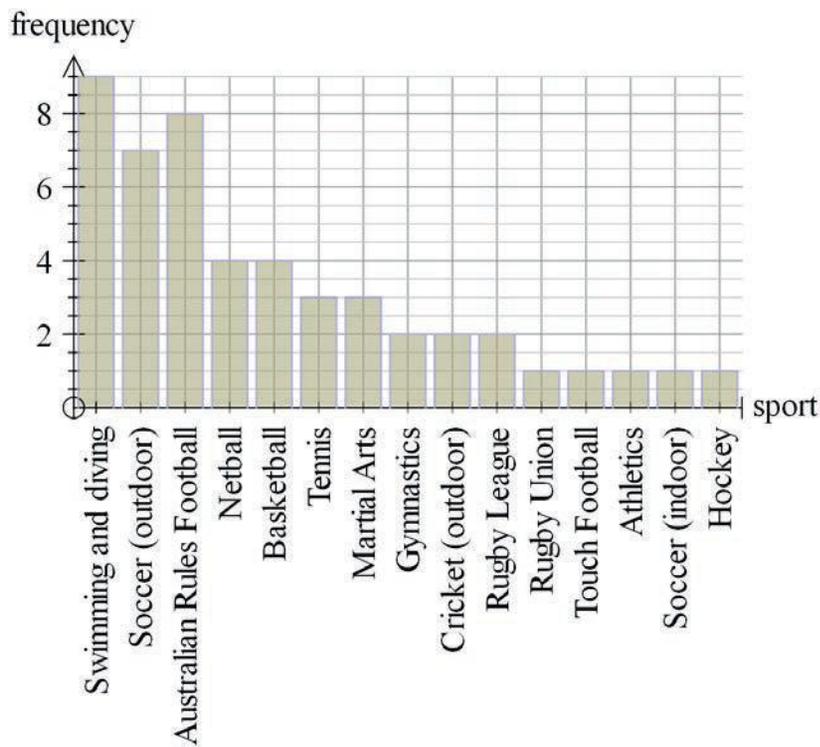
a) Comment on the effectiveness of each.

b) Which three sports did the most students participate in?

c) If 40 students took part in the survey, what percentage participated in:

(i) netball?

(ii) Australian rules football?

d) Is it possible for the percentages playing each sport to add to over 100? If so, explain how.

e) Ruby wanted to know how many students were surveyed. She counted the number of responses and came up with 49 students. Why is this not an accurate result? How many students actually took part in the survey?

f) If this survey was done in different states how do you think it might differ?

g) If this survey was done at different times of the year, do you think the results would vary? If so, how?

h) Survey the sports participation of your class. Record the results and display them in a bar graph. Comment on any similarities and differences you notice.

> The mathematical sciences particularly exhibit order, symmetry and limitation: and these are the greatest forms of the beautiful.
>
> - Aristotle

30 Dot plots

Materials required: calculators

Warm-up

1.	Write 111% as a decimal.	2.	$3.3m^2$ = ____ cm^2
3.	375 mL = _____ cm^3	4.	375 mL = _____ mm^3
5.	If someone ran 1500m in 5 minutes what was their average speed?	6.	Increase $150 by 10% then decrease the result by 10%.
7.	Write the ratio 4:6 in its simplest form.	8.	A 20cm × 20cm × 40cm rectangular prism has a capacity of _____Litres
9.	3, 4, 5, 7, 7, 11 What is the median?	10.	3, 4, 5, 7, 7, 11 What is the mean?

Introductory problem

Dot frequencies (or dot plots) give a quick overview of a distribution: they show clustering, extreme values, and can help to see if data should be grouped. Each dot in a dot plot represents one value.

The dot plot below displays the number of siblings of each student in a class of 26 Year 12 students.

Number of Siblings
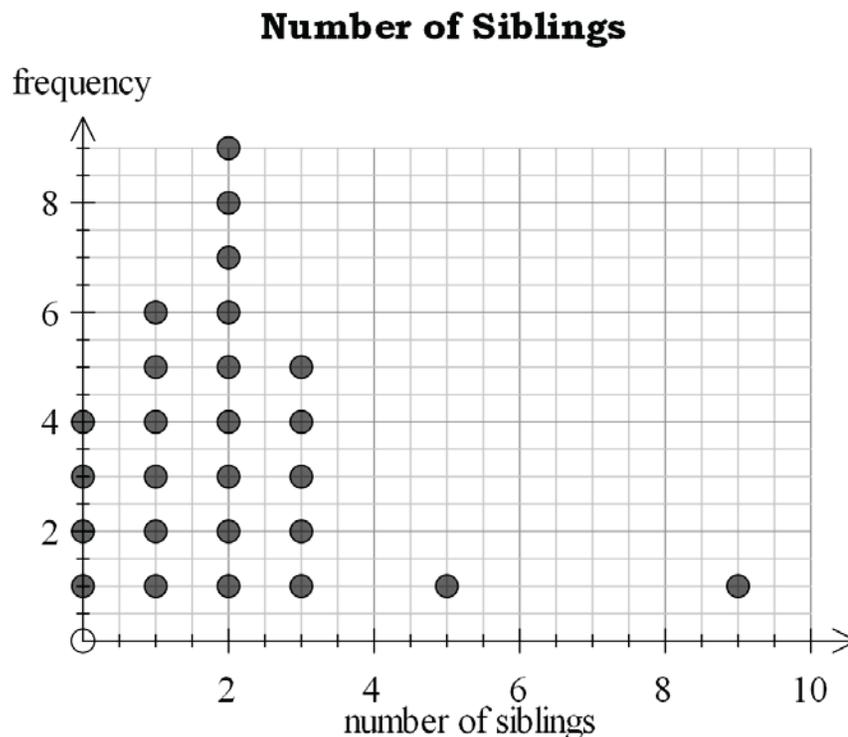
Calculating the mean, mode and median

The median is the middle score. Each dot represents one person so counting the dots tells us there are 26 people. Splitting the dots in half, means the median value will be halfway between the 13th and 14th value. Counting the dots from the left hand side shows 4 × 0 siblings, 6 × 1 sibling so both the 13th and 14th value are in the 2 sibling column so the median number of siblings is 2.

The mode is the most frequently occurring value which is also 2 siblings.

For large quantities of data, the mean is a little more difficult to calculate.

We could add all the individual values e.g.

0 + 0 + 0 + 0 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + etc....

This is time consuming and can also lead to errors

An alternative and more efficient way is to organise the data into a frequency table.

Number of Siblings (x)	Frequency (f)	Number of siblings multiplied by the frequency (xf)
0	4	
1	6	
2	9	
3	5	
4	0	
5	1	
6	0	
7	0	
8	0	
9	1	
10	0	
Total	26	

Adding of the frequency column calculates the number of students (26). Instead of adding the values individually, multiplying the number of siblings by the frequency gives us the same result, i.e. 6 × 1 will give us the same answer as adding 1 six times.

1.   Complete the frequency table.

2.   Calculate the mean by adding the values in the final column and dividing by the total number of students.

Dot plots are also useful for identifying outliers, gaps and clusters in the data. These are useful when describing the distribution of the data.

- Clusters are groupings of data around a certain value or values.
- An outlier in a set of data is a score that is far from the others. It is an observation that is inconsistent with the remainder of that set of data because it is of a significantly different size. On a dot plot, these become obvious.
- Gaps area large spaces between data points, which are obvious when displayed on a dot plot.

3. In the dot plot on siblings describe any clusters, outliers and gaps.

Exercises
1. a) Construct a frequency table and dot plot of the number of siblings of people in your class.
   b) Calculate the mean, median and mode.
   c) How does this compare with the dot plot and averages calculated in the example?
   d) The 2011 Australian Census reported that the average number of children in Australian families was 1.9. Which measure of central tendency was most likely used? Why?

2. 80 Year 11 students responded to the question, "How many hours of sleep do you usually get on a school night?" Their results were placed in a dot frequency below.

Average number of hours sleep on weeknight
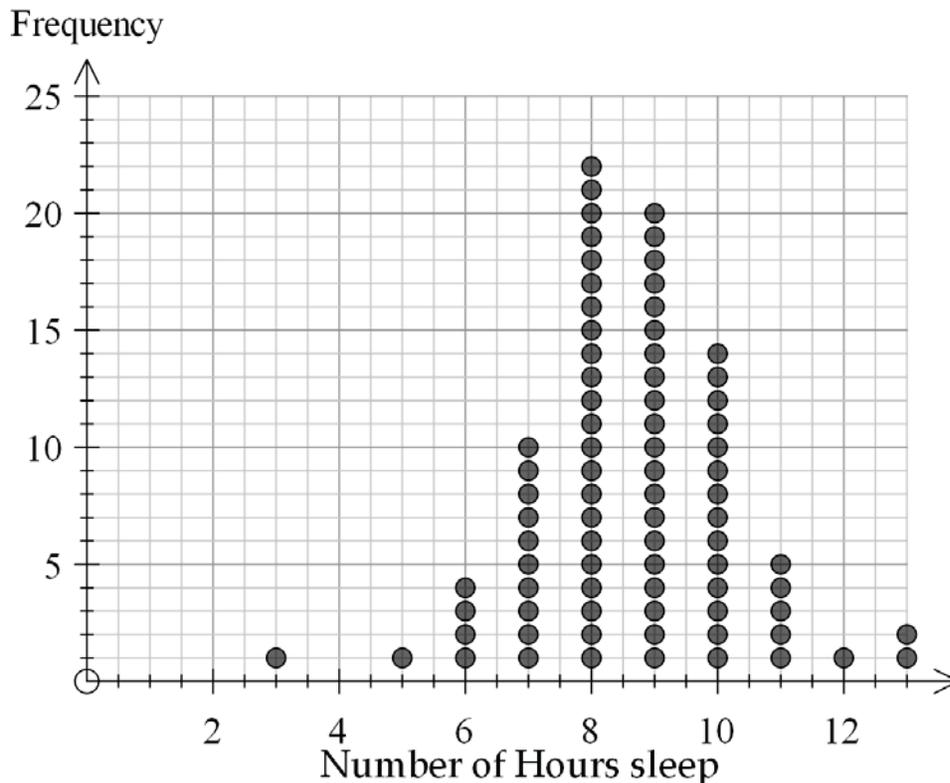
a)   What is the modal number of hours of sleep by the Year 11s surveyed?

b)   What is the median number of hours of sleep by the Year 11s surveyed?

c)   What is the mean number of hours of sleep by the Year 11s surveyed?

d)   What percentage of students averaged 8 hours of sleep or more?

e)   Comment on any noticeable clusters of data and outliers.

f)   A teacher wanted to determine if there were any differences between the average number of hours of sleep Year 11s had on a weeknight compared to weekends.

g)   Plan and complete this question for students in your class.

3.   Students from 3 different countries were surveyed regarding the number of languages they spoke. Their responses are displayed in these dot plots. Use statistical language to comment on the similarities and differences between the data from the different countries.

Number of languages spoken (Australia)

Number of languages spoken (New Zealand)

Number of languages spoken (Canada)
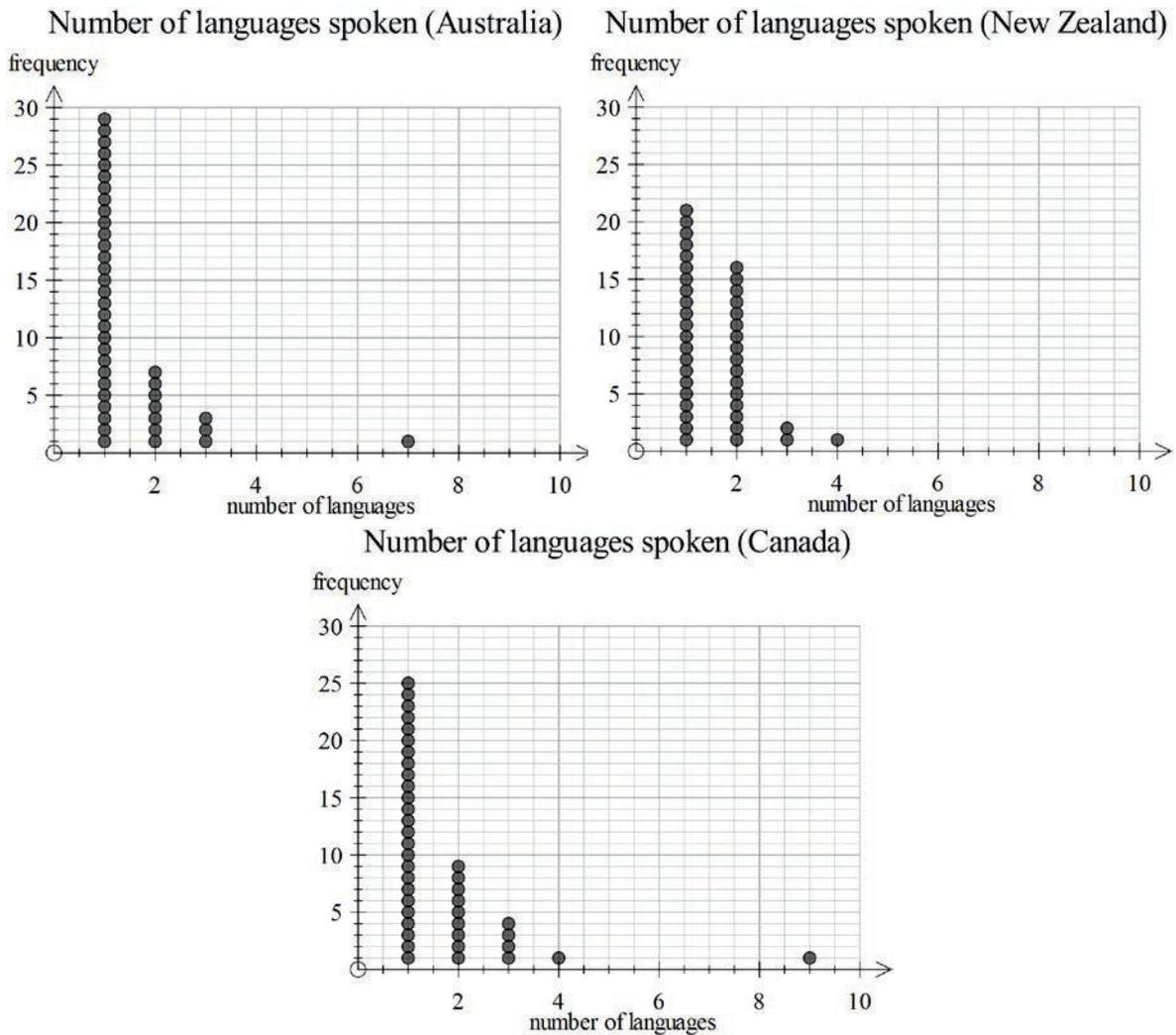
31 Measures of spread

Warm-up

1.	¾ hour = _____seconds	2.	$(8 - 5)^2 \times (3 + 7)^2 =$
3.	2.75 as a percentage	4.	0.001 as a fraction
5.	3.06 kg = ____grams	6.	Capacity of a $1m^3$ cube?
7.	What is the volume of a rectangular prism 2m by 45 cm by 200 mm?	8.	The triangular face of a triangular prism has an area of $6cm^2$. The volume is $150cm^2$. What is its length?
9.	What is 1:17pm in 24 hour time?	10.	How much time between 8:40pm and 1:00am?

Introductory problem

Often calculating the average is not enough.

1. Calculate the mean, median and mode of each of these data sets

    a)   50, 50, 50, 50, 50, 50

    b)   40, 40, 50, 50, 60, 60

    c)   0, 25, 50, 75, 100

    d)   0, 0, 49, 50, 80, 81, 90

    Comment on your results.

2. Work out two sets of data, with 6 different values that have the same mean and median.

3. Do the measures of location give you enough information about a set of scores for you to picture how it looks?

To provide a further and more complete description of a data set it is sometimes useful to also include measures of spread. Two measures of spread are the range and the interquartile range.

Range

The statistical range is the difference between the lowest and highest scores in a set of numbers.
To find the range of a set of numbers:
- Identify the highest value and the lowest value
- Subtract the smallest from the largest

Examples

> The range of 65,73,89,56,73,52,47 is 89 – 47 = 42.
> The range of 50, 50, 50, 50, 50, 50, 100, 150 is 150 – 50 = 100

Considerations
- A great deal of information is ignored when computing the range since only the largest and the smallest data values are considered; the remaining scores are ignored.
- The range of a data set is greatly influenced by the presence of just one unusually large or small value in the sample (outlier).

Interquartile range (IQR)

The interquartile range (IQR) is a measure of statistical spread, equal to the difference between the third and first quartiles.

The IQR is the width of an interval which contains the middle 50% of the sample, so it is smaller than the range and its value is less affected by outliers.

To calculate the interquartile range, arrange the data in ascending order (smallest to largest).

> *Data:*   6 5 2 3 6 8 6 4 9 7
> *Arranged:*   2 3 4 5 6 6 6 7 8 9

Calculate the median: $\boxed{2\ \ 3\ \ 4\ \ 5\ \ 6}\ \uparrow\ \boxed{6\ \ 6\ \ 7\ \ 8\ \ 9}$

Median = 6

Examine the data to the left of the median: 2 3 4 5 6
The lower quartile is midpoint of this set of data
> *Lower quartile* = 4

Examine the data to the right of the median: 6 6 7 8 9
> *Upper quartile* = 7

IQR: 7 – 4 = 3

The IQR together with the range gives an indication of the spread of the data. The larger the range is, the larger the difference between the maximum and minimum values. The larger the IQR the greater the spread of the middle 50% of the data set.

Exercises

1. Calculate the range and interquartile range of the data from the introductory problems.

2. Calculate the median, range and interquartile range of the following.

   a)   3, 4, 5, 6, 7, 7, 7, 10, 25

   b)   1, 2, 2, 2, 2, 3, 3, 5, 8, 9, 10

   c)   48%, 51%, 55%, 60%, 61%, 66%, 71% 98%, 99%

   d)   2.2mm, 3.3mm, 3.4mm, 4.8mm, 8.1mm

3. One class of 20 students had marks in their last maths test with a range of 40%, an interquartile range of 10% and a mean of 52.4%. A second class of 17 students received marks with a range of 70%, an interquartile range of 25.5% and a mean of 60.7%. Compare the performance of the two classes.

4. **Challenge**
   a)   Create a set of 7 numbers with a minimum of 4, maximum of 25, mean of 9, median of 15 and an interquartile range of 12.

   b)   Create a set of 6 numbers with a minimum of 45, a range of 55, mean of 55, median of 65 and a mode of 100.

   c)   Create a set of 10 numbers with a maximum of 26, mean of 14.2, median of 15.9, a mode of 18.9 an upper quartile of 19.1 and an interquartile range of 16.3.

   d)   Create a set of 7 numbers with a minimum of 4, maximum of 25, mean of 9, median of 15 and a interquartile range of 12.

There are three kinds of people in the world – those who can count and those who can't.

31.5 Standard Deviation

Warm-up

1.	Find the mode of Y = {2, 4, 5, 5, 6, 7, 8}	2.	Find the range of X = {1, 3, 4, 7, 10}
3.	34 – 4 x 5 =	4.	Calculate 15% of $300
5.	Convert 25% to a decimal	6.	Continue the pattern 0, 12, 24, 36, ____, ____
7.	Convert 1305 to 12 hour time	8.	Convert 8:34pm to 24 hour time
9.	100 ÷ 0.5 =	10.	Are child birth weights categorical or numerical data

Opening Problem: 6-MP and Leukaemia

The drug 6-MP (mercaptopurine) has been used in therapy to extend remission in leukaemia patients. The lengths of remission (in weeks) for 21 patients receiving this treatment were observed as follows

10, 7, 32, 21, 22, 6, 16, 34, 32, 25, 11, 20, 19, 6, 17, 35, 6, 13, 9, 6, 10

E.J Freireich *Blood 21* (1963)

1. Show that the mean of the remission times for the 21 patients is 17 weeks.
2. You can now calculate the individual deviations from the mean for the 21 patients by subtracting the mean away from each patient's remission time.
   For example 10 – 17 = -7
   $$7 - 17 = -10$$
   $$32 - 17 = 15$$
   Continue this process until you have found the deviation from the mean for each patient.
3. Find the mean of your deviations, by adding them together and dividing by 21. You should get an answer of '0'. Why do you think this may occur? Discuss if this would be a good measure of the average spread from the mean.
4. One way we can counteract this problem is by squaring the deviations before adding all of them and calculating the mean. Use your calculator to square each of your answers from step 2, record them as you go and then find their mean. You should obtain an answer of 94.4. This value is called the variance.
5. Find the square root of the variance. You should obtain an answer of 9.71.

Congratulations you have just calculated the standard deviation (s or σ) of the 21 values. The algebraic formula for this is:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{x})^2}{n}}$$

s = Standard Deviation

Σ = the sum

X_i = each score

$\bar{x}$ = mean of the scores

n = number of scores

Don't worry – you don't have to use this formula to calculate the standard deviation. A scientific calculator will do it for you.

Standard Deviation

In Lesson 31 we defined the range and the interquartile range as a measure of the width or spread of a sample distribution. The standard deviation is another measure used to see how spread out data is from the mean.

The standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around the mean.

Calculation of the standard deviation may be done via the use of appropriate technology such as a scientific calculator or in a computer package.

The larger the standard deviation, the more spread out the data will be from the mean. The standard deviation is measured in the same units used for the original data set.

For example, consider the three data sets below:

SET A = {10, 10, 10, 10, 10} mean = 10, standard deviation = 0

SET B = {9, 10, 11, 7, 13} mean = 10, standard deviation = 2

SET C = {1, 1, 10, 19, 19} mean = 10, standard deviation = 8.05

Their frequency graphs (dot plots) are:
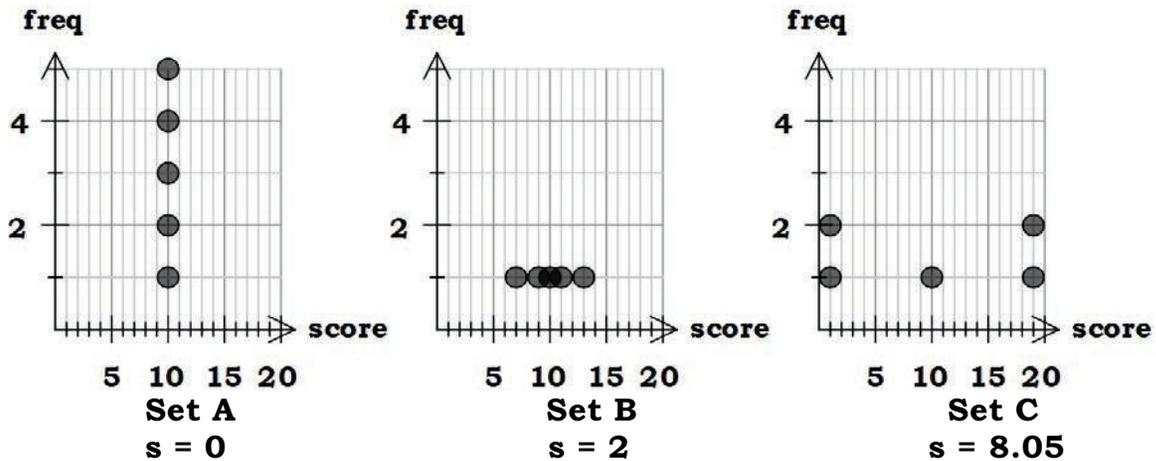
By comparing the scores in each data set, their dot plots and their standard deviation we can see that the higher the standard deviation is, the more spread out the data is.

Exercises

1. Consider the opening problem involving leukaemia patients. Give reasons why medical staff or patients may be interested in the mean and standard deviation of the number of weeks the patients taking 6-MP had been in remission.

2. Without calculating the standard deviation, rank the following sets of scores from the smallest standard deviation to the largest.

    SET A = {20.0, 20.0, 20.1, 19.7, 20.1, 19.8, 20.0, 19.9}

    SET B = {12, 198, 176, 34, 200, 456, 674, 870}

    SET C = {25, 35, 45, 55, 65, 75, 85, 95}

    SET D = {101, 101, 101, 101, 101, 101, 101, 101}

3. Match each of the statements below with the most suitable standard deviation value.

Standard deviation		Data description
a) 0.66	i)	The ages of four siblings
b) 3.03	ii)	The ages of the entire female population
c) 38.08	iii)	The ages of the students in your class

4. Three classes sat a test and their results are shown below

	Mean ($\bar{x}$)	Standard Deviation (σ)
Class A	55%	6.2
Class B	61%	14.5
Class C	57%	8.1

a) Which class had the highest mean?
b) Which class had the greatest spread of marks? Justify your choice.
c) Use the means and the standard deviations to compare the test results of the three classes.

We can use technology such as your scientific calculator to calculate the standard deviation.
*   Put your calculator into statistics [STAT] mode.
*   You may need to clear any old data first by pressing [CA] or [CSR].
*   One at a time, enter each new value of the list, pressing [Σ+] or [M+] depending on your calculator.
*   The data entry number should be displayed for each new value entered – check this matches the number of values in your set.
*   To find the mean press the [$\bar{X}$] key and for the standard deviation use the [σ] key.

5. Use your calculator to determine the mean and standard deviation of the Sets A to D from question 2.

SET A = {20.0, 20.0, 20.1, 19.7, 20.1, 19.8, 20.0, 19.9}

SET B = {12, 198, 176, 34, 200, 456, 674, 870}

SET C = {25, 35, 45, 55, 65, 75, 85, 95}

SET D = {101, 101, 101, 101, 101, 101, 101, 101}

Extended Problem Solving

In this problem solving task you will use your scientific calculator (or an Excel spread sheet) to investigate the changes to the mean and standard deviation of a data set when you add or subtract a constant (c) or multiply each value by a constant ratio (r).

X = 3, 5, 6, 6, 8, 9, 12, 15, 15, 15 18, 21, 25, 29, 30
     Mean = 14.5   Standard deviation = 8.4

1. Predict what would happen to the mean and standard deviation of X if you added three to each value in the set to make Y.

Y = 6, 8, 9, 9, 11, 12, 15, 18, 18, 18, 21, 24, 28, 32, 33

2. Use your calculator or Excel to calculate the new mean and standard deviation, compare it to your prediction
3. Calculate the mean and standard deviation of X if you subtract 4 from each value.
4. Based on your results, generalise what happens to the mean and standard deviation of a data set when you add or subtract a constant c.

A = 0, 1, 1, 3, 5, 6, 6, 8, 12, 13, 15, 17, 18, 20, 20
     Mean = 9.7   Standard deviation = 6.9

5. Predict what would happen to the mean and standard deviation of A if you multiply each value in the set by three to make B.

B = 0, 3, 3, 9, 15, 18, 18, 24, 36, 39, 45, 51, 54, 60, 60

6. Use your calculator or Excel to calculate the new mean and standard deviation, compare it to your prediction
7. Calculate the mean and standard deviation of A if you multiply each value by 2.
8. Based on your results, generalise what happens to the mean and standard deviation of a data set when you multiply by a constant ratio r.

32 Histograms

Materials required: calculators

Warm-up

1.	Increase 17.5 by 10%.	2.	Write $^3/_8$ as a decimal.
3.	Simplify the ratio 8:22.	4.	$56mm^2$ = _____cm^2
5.	What is the area of a rectangle 1.4km by 350m?	6.	Find the volume of a 6cm cube.
7.	750mL = _____Litres	8.	45, 65, 85 mean = _____
9.	Write 7:15pm in 24 hour time.	10.	120km in 75 minutes, what is the speed?

Definition

A histogram has a similar appearance to a column graph but no gaps between the columns. It is used to depict data from the measurement of a continuous variable.

Technically, the difference between column graphs and histograms is:
- in a histogram: frequency is measured by the area of the column.
- in a column graph: frequency is measured by the height of the column.

Histograms visually depict:
- the median or centre of the data,
- the range or spread of the data,
- any skewness in the data,
- the presence of outliers, and
- the presence of multiple modes (or peaks) of the data.

The distribution of a data set can be described by the shape of its histogram.

Symmetry

A symmetrical distribution is one in which the two "halves" of the histogram appear as mirror images of one another. If a distribution is perfectly symmetrical then the mean, mode and median are all equal, and shown by the peak in the middle of the distribution.
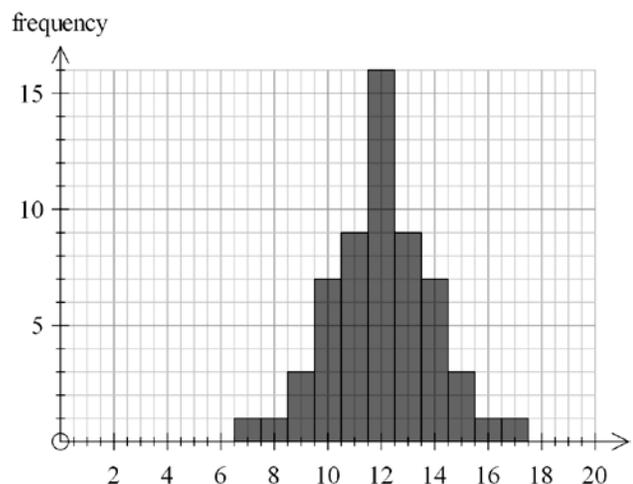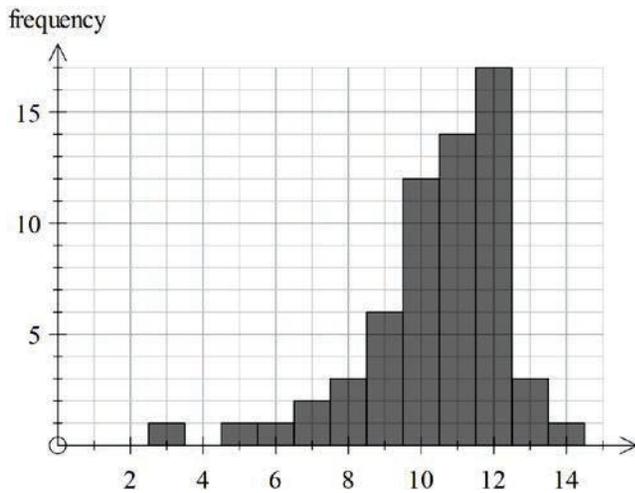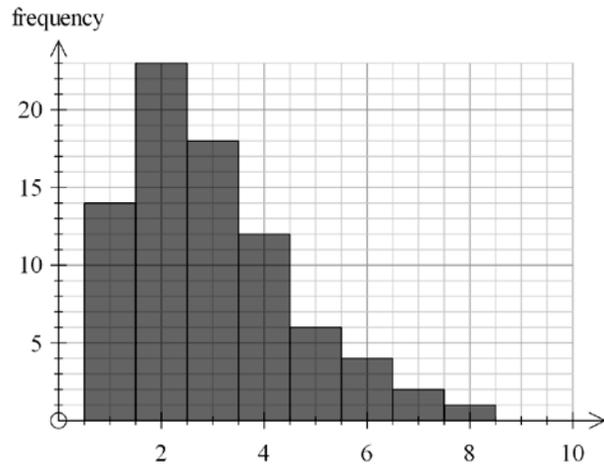
Skewness:

positively skewed

A data set that is skewed to the right, has the most commonly occurring values to the left. This is also called positively skewed.

The median and the mean will be greater than the mode and in most cases the median will be greater than the mean.
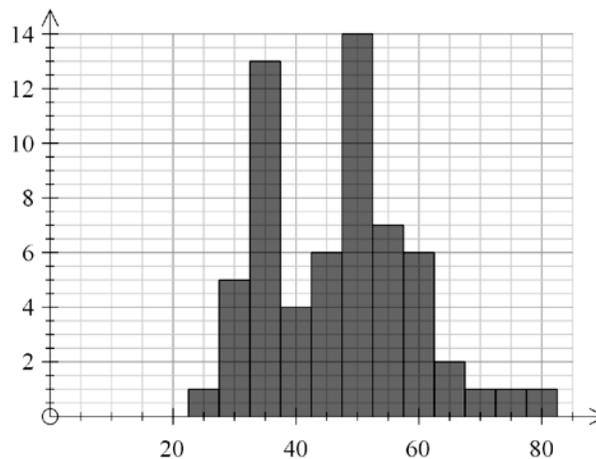

negatively skewed

A data set that is skewed to the left, has the most common values to the right of the graph. This is also called negatively skewed.
The median and the mean will be less than the mode and in most cases the median will be less than the mean.

bimodal

A bimodal shape will have two peaks, that is, two classes of data occurring more frequently than the rest.

Exercises

1. When determining which hours to employ more casual staff, a café owner recorded the average number of customers served throughout the day.

Time of Day	8am - 9am	9am -10am	10am-11am	11am-12pm	12pm-1pm	1pm-2pm	2pm-3pm	3pm-4pm	4pm-5pm	5pm-6pm	6pm-7pm	7pm-8pm	8pm-9pm
Number of customers	17	33	24	48	75	62	26	29	36	56	43	28	19

   a) Display this information as a histogram and comment on the distribution of the number of customers the café gets throughout the day.

   b) How many customers does the café get during the day?

   c) What percentage of daily customers does the café get during the lunch rush (11am-2pm)?

2. 40 Year 11 and 12 students were surveyed regarding the amount of money they earned from their main source of income in the last week. Their responses are contained in the table below.

44	0	30	82	5	180	2	30	25	380
150	40	38	20	100	25	400	10	0	200
10	100	0	200	35	124	0	50	50	30
0	93	0	184	260	200	260	50	35	60

   a) Calculate the mean, median and mode.

   b) Choose an appropriate grouping (equal sized groups) and draw a histogram.

   c) Describe the distribution of the histogram.

3. Students from a secondary school were surveyed regarding how long they took to get to school. Their responses are displayed in the histogram below.

Time taken to get to school

Frequency

Time taken to get to school (in minutes)
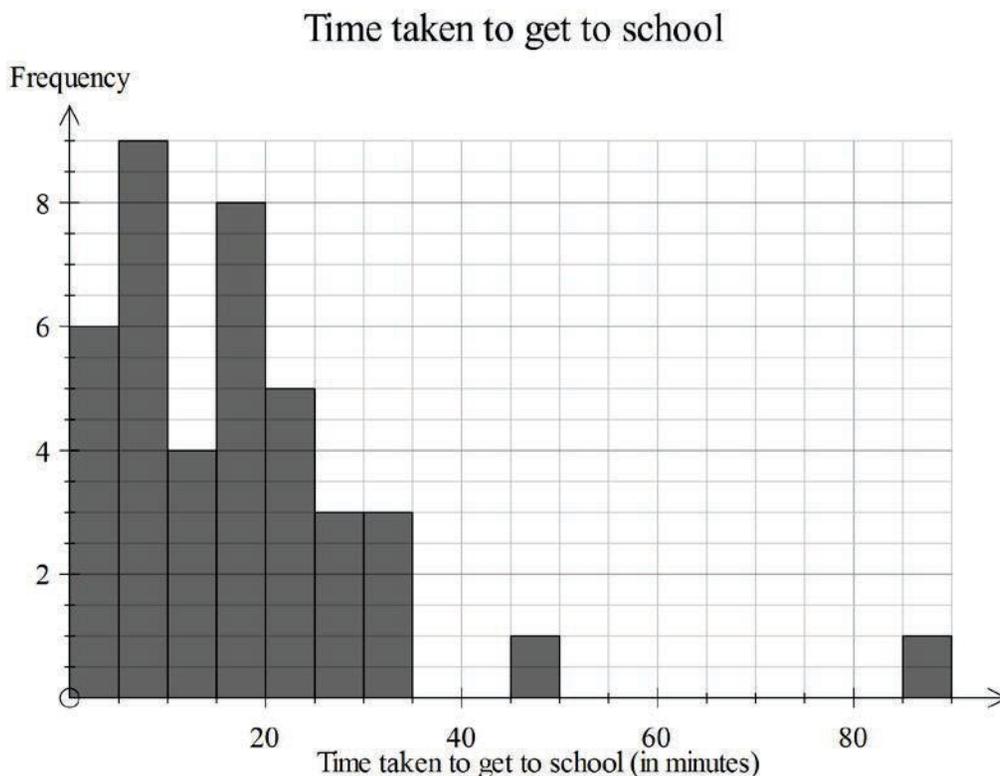
a) How many students were surveyed?

b) Describe the distribution, including skewness, clusters, gaps and the presence of any outliers.

c) Estimate the average time taken by the students to get to school.

d) Comment on the limitations of calculating the average from a histogram.

4. The number of people participating in sports (in 1000s) according to the 2013 Australian Census was:

Age	Male	Female
15–24	125.3	103.4
25–34	134.0	130.2
35–44	124.9	108.9
45–54	109.1	99.4
55–64	86.0	76.6
65 and over	75.1	79.8

Display this information in two separate histograms and compare the two distributions.

5. 100 eggs were collected at a farm and their weights in grams recorded in the table below.

41	59	68	54	54	52	41	60	69	76
53	59	47	44	51	64	50	37	56	59
65	66	69	54	61	48	41	53	60	54
73	55	56	48	63	60	66	70	66	47
44	46	55	73	43	59	55	57	57	72
45	57	39	61	60	43	51	54	70	71
47	51	43	61	55	41	62	58	65	55
63	47	68	79	56	56	51	42	38	57
57	59	71	65	71	54	60	67	41	63
39	76	54	59	54	47	72	71	53	57

Classify the weights above into this frequency table according to the following sizes.

Size	peewee	Small	Medium	Large	X-large	Jumbo	X-jumbo
weight	35-42	42-49	49-56	56-63	63-70	70-77	77-84
Tally							
Frequency							

Use this information to draw up a histogram, commenting on the distribution of the egg sizes.

Note: For eggs whose weights were on the boundaries of the class groups, did you include them in the lower or higher group?
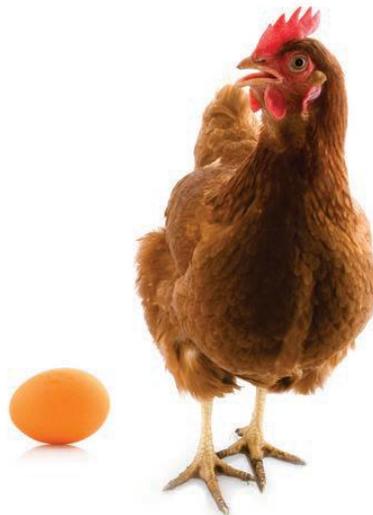
33 Stem and leaf displays

Materials required: calculators

Warm-up

1.	$8 \times 4 \div 2$	2.	Write 110% as a decimal.
3.	Write 55% as a fraction.	4.	How many days in 6 weeks?
5.	How much time between 2:45am and 3:10pm?	6.	Reduce $324 by 10%.
7.	1.35km = _____ m	8.	250mL = _____ cm^2
9.	Distance travelled at 90km/h for 90 minutes?	10.	_____ mm^3 = 400 cm^3

A stem-and-leaf plot is a method of organising and displaying numerical data in which each data value is split into two parts, a 'stem' and a 'leaf'. It is useful to use because it is quick to draw and displays the distribution of the data while retaining the individual data points.

For example:
The numbers of books ten students read in one year were as follows:
$$12, 23, 19, 6, 10, 7, 15, 25, 21, 12$$
The values are split into a stem and a leaf:
　　　12 has a stem of 1 and leaf of 2,
　　　6, written as 06, has a stem of 0 and leaf of 6.

Stem　Leaf
```
  0 | 6 7
  1 | 2 9 0 5 2
  2 | 3 5 1          Key: 2|3 represents 23
```

The stem '0' represents the class interval 0-9,
the stem '1' represents the class interval 10-19, and
the stem '2' represents the class interval 20-29.

When working with integers, the stem is generally all digits before the units place and the leaf is the units. When working with decimals, the numbers must be written with the same number of decimal places, then the stem is usually all digits before the last decimal place and the leaves are the last decimal place. A stem and leaf, though, is often quite impractical when displaying numbers to more than one decimal place due to generally requiring a very long stem with a lot of empty rows.

A key is given when the display shows numbers other than integers, e.g. Key: 21|3 represents 21.3.

To calculate the median from a stem and leaf plot, the data needs to be arranged in order.

Stem Leaf
```
 0  | 6 7
 1  | 0 2 2 5 9
 2  | 1 3 5
```

Count the leaves – here there are 10 scores.

Find where the midpoint is. For 10 values, it is halfway between the 5th and 6th values.

So the median is halfway between 12 and 15, i.e. 13.5.

Split Stem and Leaf
Fifteen people were asked how often they drove to work over ten working days.
Their responses were as follows:
5, 7, 9, 9, 3, 5, 1, 0, 0, 4, 3, 7, 2, 9, 8
The stem plot could be drawn as follows:

Stem   Leaf
```
0          0 0 1 2 3 3 4 5 5 7 7 8 9 9 9
```

This stem and leaf plot's organisation does not give much information about the data. Having only one stem creates an overcrowded leaf. In this case it is useful to split the stem. The stem plot is then displayed as follows:

Stem Leaf
```
0(0)    0 0 1 2 3 3 4
0(5)    5 5 7 7 8 9 9 9
```

- The stem $0^{(0)}$ means all the data within the interval 0-4.
- The stem $0^{(5)}$ means all the data within the interval 5-9.

A more detailed breakdown can be used if required, e.g. $0^{(0)}$ for 0-1, $0^{(2)}$ for 2-3, $0^{(4)}$ for 4-5, $0^{(6)}$ for 6-7, $0^{(8)}$ for 8-9, but they must all be kept the same size.

The use of a stem and leaf plot, can be used to classify either *discrete* or *continuous* variables.

Back to Back Stem and Leaf

To compare two sets of related data, you can use a 'back to back' Stem and Leaf Plot. This graph has the stem in the centre of the display and the leaves for one set of data going to the left and the other to the right.

For example the heights (cm) of female and male students were recorded in a back to back stem and leaf plot:

```
                   19 | 013
                   18 | 012223445566667
     986554432100  17 | 3345555668
     887555543100  16 | 7
           954100  15 |
                4  14 |
```

Female   Male

Exercises

1. For the stem and leaf plot given below, find the highest score, lowest score, median score and modal score.

    ```
    1    1 2 5 7
    2    0 1 3 4 8
    3    2 9
    4    3 3 5 6 8
    5    0 1 6
    ```

2. Place these values into a stem and leaf plot:
    20, 98, 85, 66, 43, 29, 23, 48, 18, 76, 39, 44, 36, 46, 34, 80, 63

3. Two basketball teams at a school scored points in their first ten games as shown below.

    Team A: 56, 84, 82, 71, 112, 24, 66, 38, 100, 41
    Team B: 98, 76, 86, 84, 71, 82, 73, 96, 68, 76

    Display the scores as back to back stem and leaf displays.
    Which is the better team? How did you decide?

4. A class of Year 11 students timed how long it took each of them to run from the teacher's desk to the classroom door and back to the teacher's desk again. The results (in seconds) were:
    7.6, 8.1, 9.2, 6.8, 5.9, 6.2, 6.1, 5.8, 7.3, 8.1, 8.8, 7.4, 7.7, 8.2

    a)   What was the fastest time?

    b)   What was the slowest time?

    c)   What was the median time?

    d)   What was the range of the times?

5. A class of Year 11 students recorded their pulse rates and displayed their results in a stem and leaf plot.

Heart rate

male		female
97	7(5)	5
4332211	7(0)	12224
996	6(5)	689
	6(0)	34
	5(5)	9

male   female
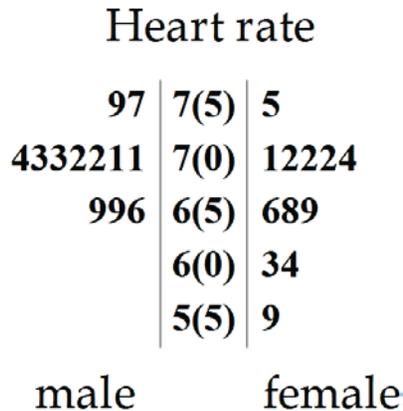
a) Calculate the mean, median and mode of each of the male and female students' resting heart rates.

b) Comment on the similarities and differences between the two groups.

The students then exercised for 5 minutes and recorded their pulse rates as soon as they finished.
After resting for 1 minute they recorded their pulse rates again.

Their results were recorded in the tables below.

Male

After exercise	121	107	127	135	130	129	128	111	117	125	124
After 1 minute	85	72	78	101	80	95	68	67	85	91	65

Female

After exercise	139	130	120	119	114	132	129	109	125	130	126
After 1 minute	89	92	91	68	75	98	79	71	75	81	83

c) In separate back to back stem and leaf plots, display the after exercise and 1 minute after exercise pulse rates for
    (i)   males and
    (ii) females

The recovery rate is the difference between the heart rate after exercising and the heart rate after 1 minute of rest.
d) Calculate the recovery rate for each student.

e) What was the mean, median and range of recovery pulse rates, for both the male and female students tested?

6. The road fatalities of drivers and passengers over a 6 month period were recorded and displayed in a stem and leaf:

Road fatalities - age of driver/passenger

	9	0
8877743322110	8	2248
90	7	02449
66555422	6	00011112224457999
842100	5	0011223344678999
6630	4	00011222233334445555788999
8643221	3	0022222333345556666677899
8765431100	2	0000001112344455566666899
9998887	1	78888899999
	0	05

a) What age group(s) had the most road fatalities?

b) Were there more driver or passenger fatalities?

c) What was the median age for road fatalities for
    (i)   Drivers
    (ii) Passengers

d) Comment on the road fatalities shown in the stem and leaf plot. Include the median age and numbers of both driver and passenger fatalities. Provide a possible reason for these figures.

$\frac{4}{5}$ of people have trouble with fractions. The other 40% are just fine.

34 Data groupings

Materials required: calculators

Warm-up

1.	Number of hours in a week	2.	$28 \div 2 \times 3$
3.	$1 \text{ m}^3 =$ _____mL	4.	$1\text{km}^2 =$ _____ha
5.	Area of a square with a perimeter of 16km?	6.	Perimeter of a square with an area of 49mm²?
7.	Mean of 2, 4, 5, 6, 8?	8.	Mean of 4, 5, 8, 9, 15?
9.	10% of 5 430?	10.	90% of 5 430?

When we have a set of data, we can calculate a range of statistics that tell us some information about it. You are already familiar with some measures of location (e.g. mean, mode and median) and some measures of spread (e.g. range, interquartile range). We can also do some calculations that divide the data set up into roughly equal sections.

If we divide the data into tenths, they are called deciles.

If we divide the data into quarters, they are called quartiles.

If we divide the data into hundredths, they are called percentiles.

The median is also the 50th percentile, and the fifth decile, and the second quartile because no matter how the data is grouped, it is still the middle of the scores.

Deciles, percentiles and quartiles are linked:

Percentiles P_1-P_{100}
Deciles D_1-D_{10}
Quartiles Q_1-Q_4
Full data set

Lowest   Median   Highest
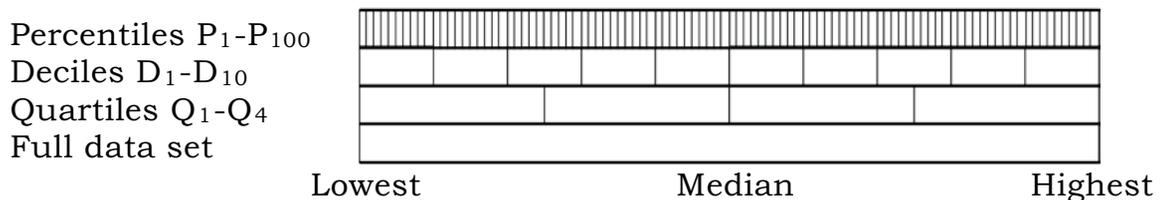
So the equivalences are:

Decile	1	2	3	4	5	6	7	8	9	10
Percentile	10	20	30	40	50	60	70	80	90	100

Quartile	1	2	3	4
Percentile	25	50	75	100

Example Find the quartiles for the data set 5, 1, 7, 8, 3, 6, 3, 6, 8, 4.

Put in order: 1 3 3 4 5 | 6 6 7 8 8

Quartiles: We divide into Q_1 Med Q_3
4 equal sections   =3   =5.5   =7

There are 10 values in the data set. To find the quartiles, we cut into quarters by cutting in half and then half again.

Cutting in half means there are 5 values in each half, so the middle is between 5 and 6.

Taking each half separately (5 values), cutting them in half means 2 scores are in each 'piece' and the 'cuts' are directly on the middle numbers, so the first quartile (or lower quartile) is 3 and the second quartile (or upper quartile) is 7.

Cutting into deciles means creating 10 equally sized groups.

For the data set 1 3 3 4 5 6 6 7 8 8,

10 scores in 10 equal groups places 1 value in each group. This is pointless. We only use deciles and percentiles when there are large quantities of data, otherwise it is not meaningful. It is easiest to estimate these from a graph that shows the cumulative number of scores compared to the attribute being measured.

Example A total of 100 000 people visited the Royal Show on one day in 2014. The total number of people who passed through the gates was recorded throughout the day from the time the gates opened (t=0). The results are displayed on the graph below.
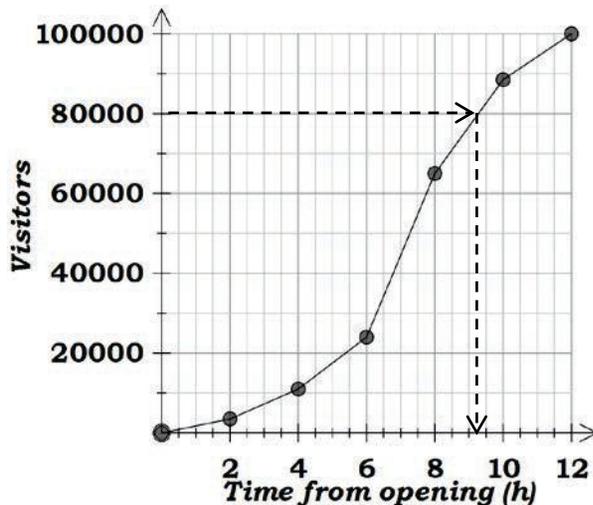
After how many hours had 80% of people arrived?

This is both the 8th decile and the 80th percentile. To calculate it as the 8th decile, we are finding the time at which $8/10$ of visitors had arrived. As a percentage, it is $80/100$.

There were 100 000 visitors, so $8/10$ or $80/100$ is 80 000. As you can see by the arrows on the graph, this shows t ≈ 9.25h, i.e. 9 h 15 mins after opening.

Exercises

1. Find the median, lower quartile and upper quartile for the values:
   5, 7, 1, 9, 11, 22, 15.

2. The number of Carnaby's black cockatoos has been in decline. It is an endangered species that is native to Western Australia. A survey in 2010 showed 6672 birds were sighted during the annual count. The following year only 4222 birds were sighted. Several roosting sites for the birds are in the Beeliar Regional Park. At one of these roosting sites, the numbers of Carnaby's black cockatoos have been tracked since 2001 and are shown below. This count has been done at the end of June each year.

Number of Carnaby's black cockatoos
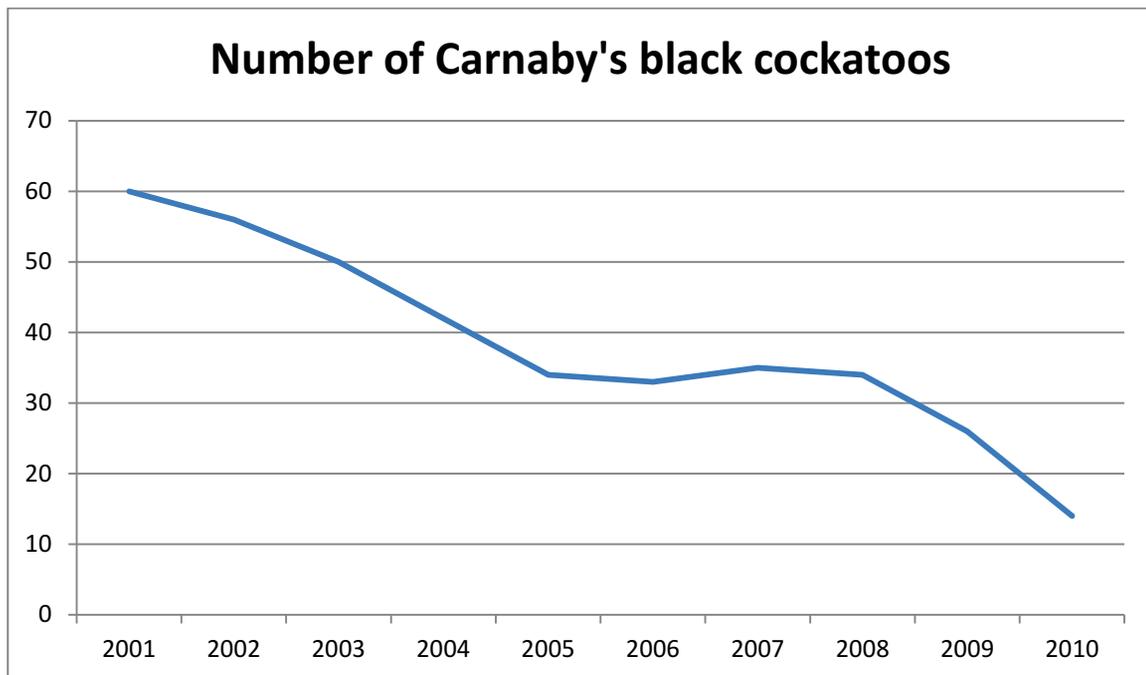
   From the graph, estimate when the population of the roosting site reached:
   a) the 70th percentile
   b) the 7th decile
   c) the 30th percentile
   d) the 56th percentile
   e) the 4th decile
   f) the median.

   From the graph, to the nearest 5 per cent, estimate the percentile of the initial population that was present:
   g) at the start of 2008
   h) in the middle of 2007.

3.   One day in June of the year 2000 a farmer in Denmark stocked the dams on his farm with a small number of marron. He left them to breed over the following years, monitoring their numbers each year at the same time.


Growth of marron population
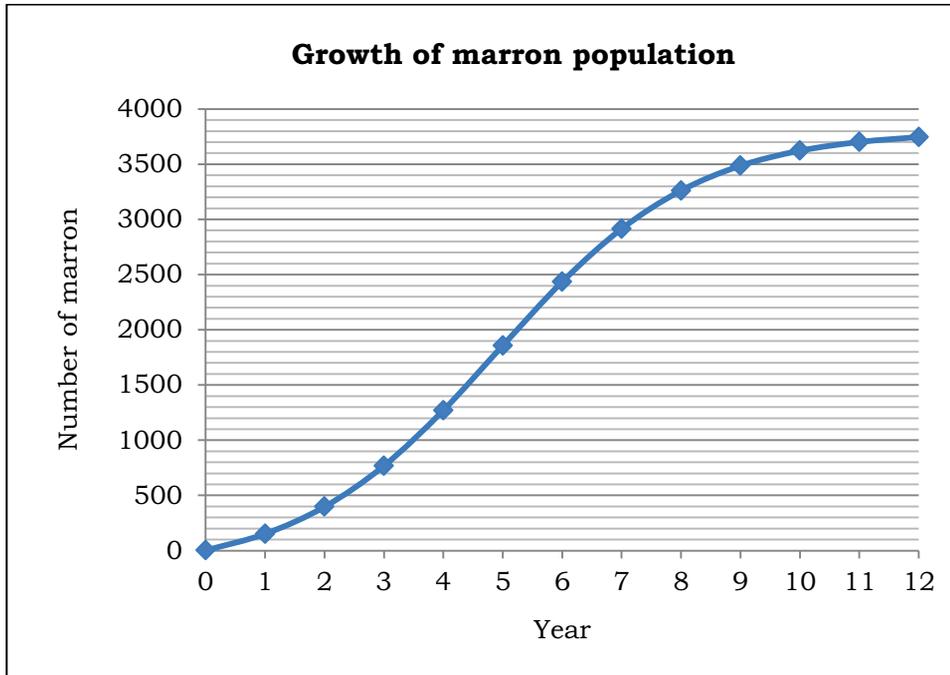
a)   What was the population after 12 years?

b)   After how many years did the population reach the first quartile?

c)   After how many years did the population reach the 7th decile?

d)   After how many years did the population reach the median?

e)   After how many years did the population reach the 67th percentile?

f)   After 10 years, what population percentile had been reached?

Many populations in nature follow this pattern, where population growth starts slowly, increasing in its rate as the breeding population increases, but eventually slowing down as resources become exhausted and the population approaches the maximum that is sustainable. This is called a logistic curve.

g)   Based on this graph, what do you think the maximum sustainable population would be?

4. Australian Bureau of Statistics data on the Australian population are graphed below.

Australian population from 1788
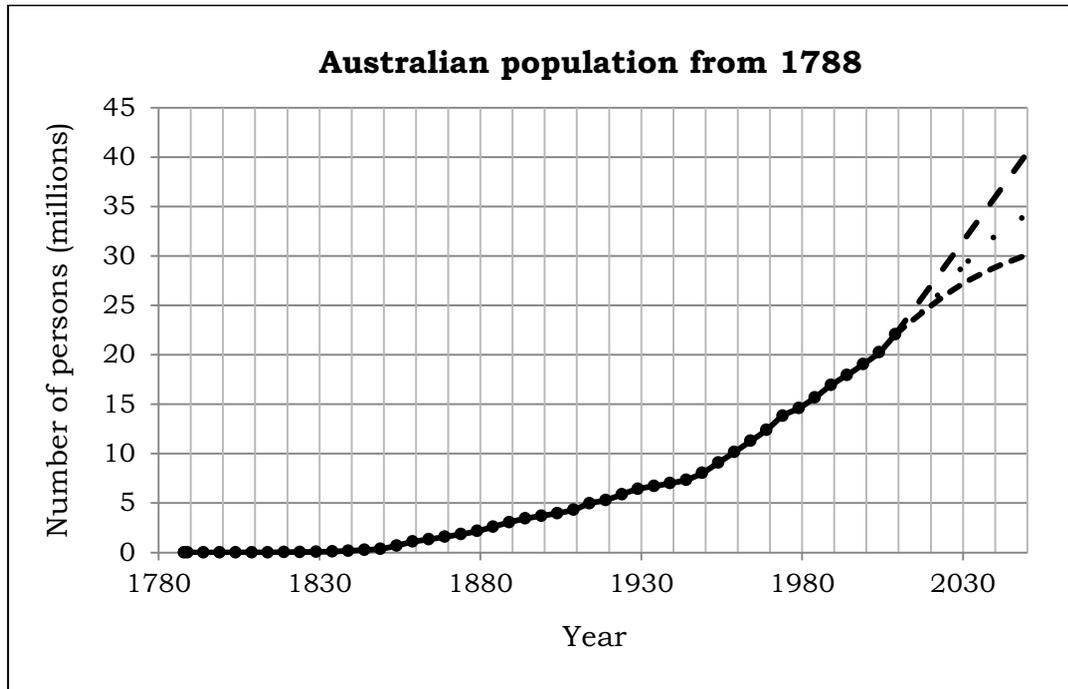
a) What was the population in 2010? Take this to be the 100th percentile.

b) When was the 50th percentile reached?

c) What percentile to the nearest 5% had been reached by 1880?

d) From 1939 to 1945, the growth rate slowed slightly. What event may have caused this?

e) Which decile or deciles showed the fastest population growth?

f) Which decile(s) showed the slowest population growth?

g) How do the lengths of the time periods for the deciles in parts e) and f) compare to each other?

h) What do you think the three curves from 2010 onward represent? Describe what circumstances you think might influence which of these will eventuate.

5. Deciles are also used to describe where in a population a person is positioned according to a particular attribute. In the table below are the scores that corresponded to the deciles in a range of Year 12 courses one year.

	Ranges of Scaled Marks Corresponding to Decile Places in Tertiary Entrance Examination Subjects/WACE Examination Course, 2006									
Subject/Course	Lowest Scaled Mark in Each Decile Place*									
	1	2	3	4	5	6	7	8	9	10
Accounting	71.87	67.25	63.50	60.35	57.09	53.70	50.18	45.62	38.44	0.00
Ancient History	76.58	71.48	66.97	63.04	59.94	56.60	53.02	47.41	40.53	0.00
Applicable Mathematics	80.17	74.25	70.14	66.35	62.92	59.53	55.73	50.48	42.84	0.00
Art	70.82	65.78	61.54	57.03	53.43	50.01	46.32	41.63	34.35	0.00
Aviation	71.33	66.03	58.95	56.74	50.44	46.15	44.80	40.67	31.50	0.00
Biology	73.38	68.09	64.09	60.06	56.52	53.19	49.04	43.85	36.25	0.00
Calculus	81.34	75.92	72.18	68.49	65.57	61.95	58.50	53.06	46.09	0.00
Chemistry	79.99	74.49	70.33	66.68	63.16	59.76	56.13	50.57	43.02	0.00
Chinese: Advanced	66.07	56.27	53.62	47.89	45.26	38.83	32.32	27.71	24.15	0.00
Chinese: Second Language	96.79	74.93	64.95	60.13	56.21	52.00	48.88	42.69	33.36	0.00

Source: School Curriculum and Standards Authority, Table 4.17, 2006 Secondary Education Statistics (Years 11 and 12)

a) A student with a score of 60 would be in which decile for each of the courses listed?

b) Which course had the best results? How do you know?

c) Which course had the worst results? How do you know?

d) A mark in what range would place a student in the second decile for Accounting?

e) 10 students who studied Ancient History received marks of 90.2, 65.3, 63.4, 61.9, 61.7, 61.7, 60.8, 58.3, 57.9, and 56.6. The same 10 students also studied Calculus and received the same marks. In terms of decile positions, which course had the higher results?

35 Box and whisker graphs

Materials required: Tape measures for measuring height

Warm-up

1.	Write 3.05 as a percentage.	2.	2.06m = _____cm
3.	Increase $50.50 by 10%.	4.	Increase $50.50 by 110%.
5.	Increase $50 by 10% and then increase the result by 10%.	6.	8, 8, 8, 8, 8 What is the range?
7.	At 90km/h for 80 minutes, what distance was covered?	8.	Write 30 out of 60 as a percentage.
9.	Write 45 out of 60 as a percentage.	10.	Convert 10 metres/second to km/h.

A common way to graph the 5 number summary of the minimum, maximum, lower quartile, upper quartile and median of a set of data is to use a box and whisker plot.
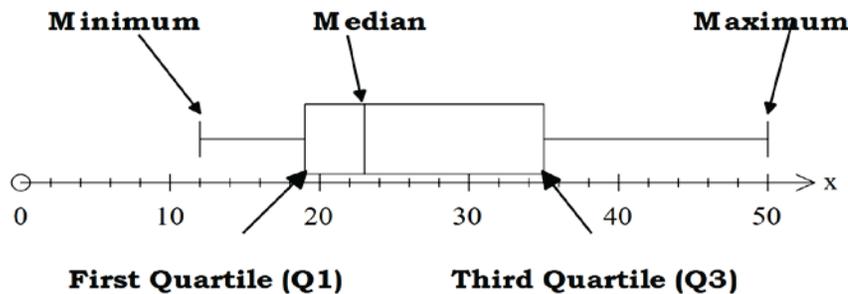
What are the minimum, maximum, lower quartile, upper quartile and median of the data above?

The interquartile range is the difference of the upper and lower quartiles. What is the interquartile range of the data displayed above?

Example Draw a box and whisker graph for 5, 1, 7, 8, 3, 6, 3, 6, 8, 4
Placing them in order, we can find the values we need:

1 3 3 4 5 | 6 6 7 8 8

minimum = 1
Q_1 = 3
median = 5.5
Q_3 = 7
maximum = 8

A box and whisker plot can be drawn either vertically or horizontally
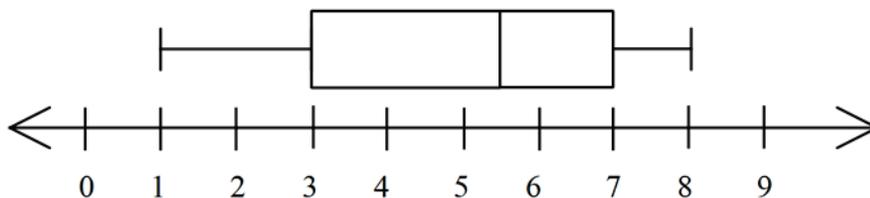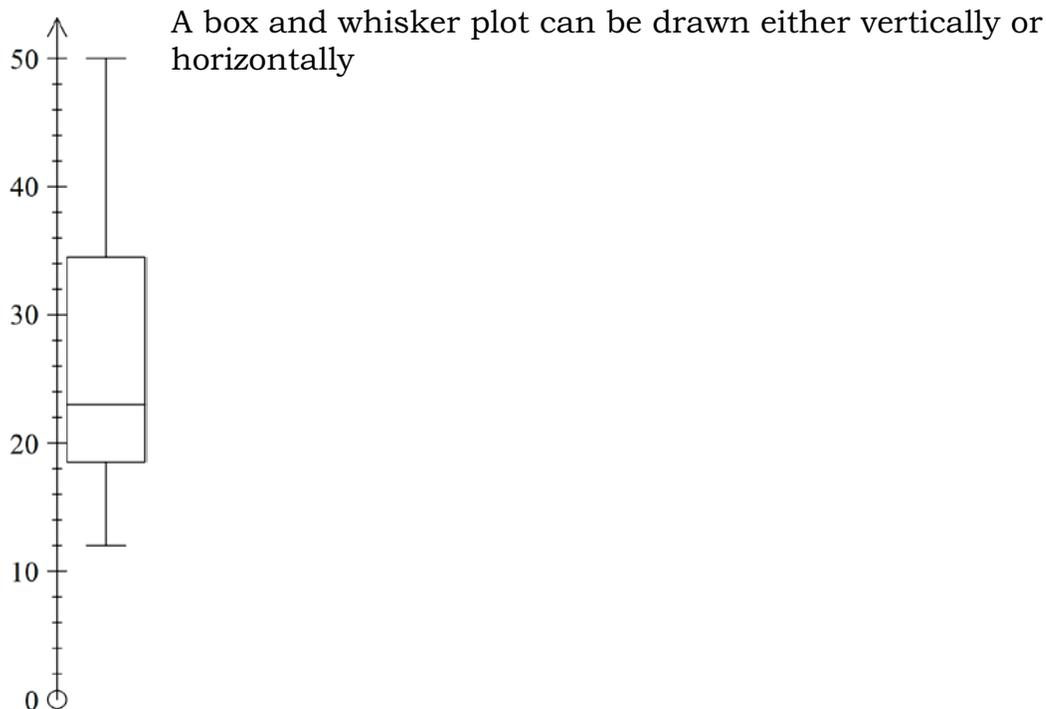
Box plots are often used to compare two sets of data. In these cases two sets of data are displayed on the same axis, allowing the spread, median and distribution to be compared. Discuss with a partner what is shown about these two sets of data:
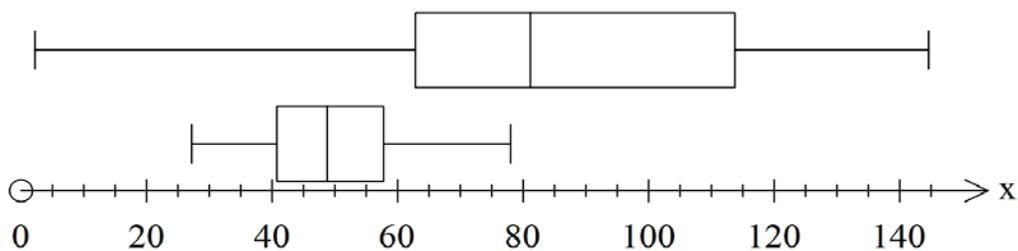
Exercises

1. Draw a box and whisker plot for the following data set:
    5, 7, 1, 9, 11, 22, 15

2. On one set of axes, display the following three data sets:

Data set	Min	Q1	Median	Q3	Max
A	0	10	30	50	60
B	5	10	20	30	35
C	10	25	30	60	65

Describe each of the three distributions individually and then compare them to each other.

3. A large sample of students throughout Australia was asked to rate how important the following environmental issues were to them. They scored each from 0 to 1000. These were the results:
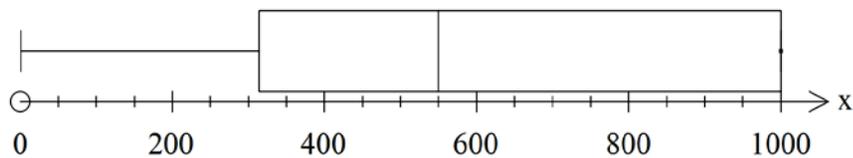
**Reducing pollution**
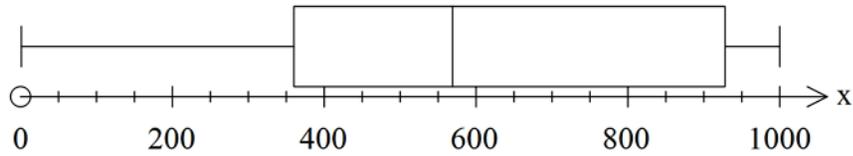
Recycling rubbish
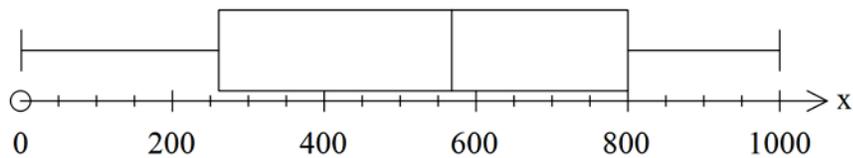
Conserving Water

a) Why is there no right-hand whisker on the last graph?

b) Which of the environmental issues has the greatest median?

c) Which environmental issue has the greatest interquartile range?

d) Using the maximum, minimum, median, range and interquartile range, comment on the how the students surveyed ranked the importance of the environmental issues displayed.

e) Survey your class on the same environmental issues, display the results in a box and whisker plot and then compare the class results to those shown above.

4. Below are two box and whisker plots showing the height of groups of 16 year old male and female students.

Male heights age 16

Female heights age 16
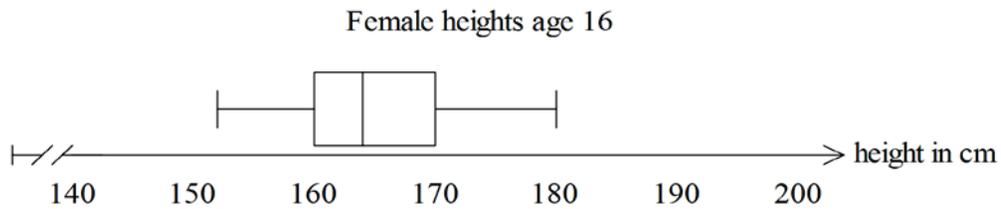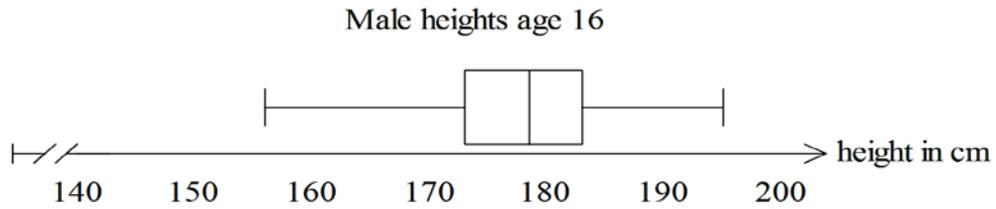
a) Compare the results in the two box and whisker plots noting similarities and differences in the data displayed.

Below is a display of the heights of the same students when they were 12 years old.

Male heights age 12

Female heights age 12
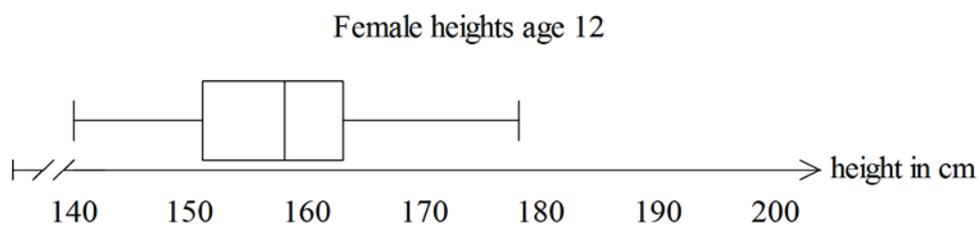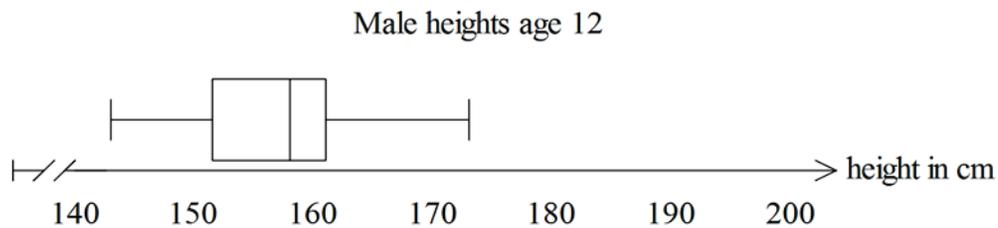
b) Compare the results on these two box and whisker plots noting similarities and differences in the data displayed.

c) Examine the difference between the 12 and 16 year old male heights and 12 and 16 year old female heights. Comment on the changes in the heights of these 2 groups.

d) Comment on the similarities and differences in the changes in height between the ages of 12 and 16 of the males and females sampled above.

5.

Road fatalities male/female drivers

Road fatalities male/female passengers

a)   What do the box and whisker plots above display?

b)   What are the median values for all four box plots?

c)   What are the minima and maxima of all four box plots?

d)   Compare the results for the first two box plots commenting on the similarities and differences displayed.

e)   Compare the results for the second two box plots commenting on the similarities and differences displayed.

f)   How do the results for drivers compare to the results for passengers for each gender?

6.   A community hall is running three activities, X, Y and Z. The ages of the people participating are illustrated in the following box and whisker plots:

a)   Which activity do you think you would not be allowed to take part in? Why? What do you think it could be?

b)   Which activity would you probably enjoy most? Why?

c)   Which activity do you think your parents would probably enjoy most? Why?

Extended Problem Solving

1   Heights

Problem: At what age are the heights of males and females most similar? What happens to the difference in their heights as they get older?

a)   What do you think will happen to male and female heights as they get older?

b)   What will happen to the differences between them?

c)   At what age do you think their heights will be most similar?

d)   What age group will show the greatest difference between males and females? Explain your answer.

e)   Collect data on this and display the data in a box and whisker plot. (You can access relevant data through the random sampler in the ABS census at school website.)

2   Environmental issues

Three environmental issues are displayed in question 3.

a)   List other environmental issues that you think will be of concern to students.

b)   Choose an environmental issue that was not displayed in question one that you believe would be of significant concern to students.

c)   Collect data on student beliefs about this issue through surveying your class. Represent your results in a box plot and comment on your results, comparing them to the other environmental issues displayed.

d)   Do you think young people's concerns about certain environmental issues change over time? What data could you collect, and how could you collect it, that either supports or refutes your opinion?

36 Blood types

Materials required: calculators

Warm-up

1.	$4^2 + 5^2 + \sqrt{16}$	2.	$(13 - 7)^2 \times 5$
3.	Write 25% as a fraction.	4.	Write 2.5% as a fraction.
5.	Write 2.5% as a decimal.	6.	Increase 65m by 15%.
7.	Find the area in m^2 of a rectangle 1.8m by 60cm.	8.	Find the volume of a cube with edges of 2.5m?
9.	Simplify the ratio 6:12:27.	10.	Divide 375mL in the ratio 1:2.

Discussion

Everybody has a particular blood type which is jointly inherited from their mother and their father. There are two major blood type systems – the ABO system and the Rhesus system (Rh factor) - the different combinations of which result in eight major blood types. A person's blood group is described by the appropriate letter (A, B, AB or O) and by whether or not their blood is Rh-positive or Rh-negative.

Blood groups in Australia

According to the Australian Red Cross Blood Service, the percentage of blood group frequency in Australia is:

- O positive – 40 per cent
- O negative – 9 per cent
- A positive – 31 per cent
- A negative – 7 per cent
- B positive – 8 per cent
- B negative – 2 per cent
- AB positive – 2 per cent
- AB negative – 1 per cent.

1. If Australia has a population of 23.5 million, how many people are there of each blood type?

2. If your school has a similar distribution of blood types as above, how many students of each blood type would there be?

3. If your class has a similar distribution of blood types as above, how many students of each blood type would there be?

4. Discuss how many people in the class know their blood types.

The Australian Red Cross Blood Service collects around 1.4 million blood donations each year. Most of this is used to help people with medical conditions that require blood or blood products regularly.

How donated blood is used:
- Cancer and blood diseases 34%
- Other cause of anaemia 19%
- Surgical patients 18%
- Other medical problems 13%
- Orthopaedic problems 10%
- Obstetrics 4%
- Road trauma 2%

5. How many of the 1.4 million blood donations are used for each of these procedures?

6. How many donations are given each week?

7. a) If there are around 1.4 million blood donations and the regular blood donation is 470 mL, how many litres of blood is this a year?

b) If a backyard swimming pool holds an average of 23 000L, would the amount of blood donated in Australia in a year fill it? If so, how many times?

When blood transfusions are required a person can only receive certain blood types. This table illustrates the compatability of the different blood types. This shows that not all blood types can be donated to patients of other blood types, but the blood does not have to be an exact match.

		Donor's Blood Type							
		O-	O+	B-	B+	A-	A+	AB-	AB+
Patients' blood type	AB+	✓	✓	✓	✓	✓	✓	✓	✓
	AB-	✓		✓		✓		✓	
	A+	✓	✓			✓	✓		
	A-	✓			✓	✓			
	B+	✓	✓	✓	✓				
	B-	✓		✓					
	O+	✓	✓						
	O-	✓							

For example the blood of a donor with type A- could be used in a transfusion for a patient with either blood type AB+, AB-, A+ or A- (reading down the column with heading A- under Donor's Blood Type).

8. a) What blood type is called the universal donor (i.e. can donate blood to any other blood type)?

   b) What blood type is called the universal recipient (i.e. can receive blood of any other blood type)?
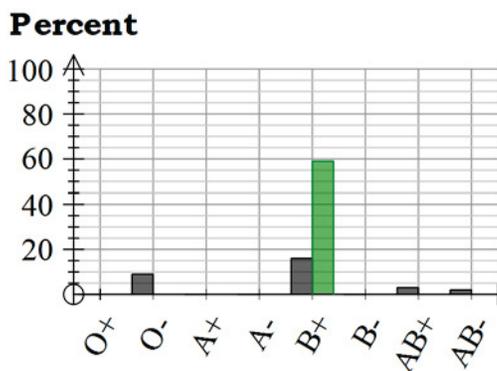
From this table a patient with O- could preferably only receive transfusions from O- (reading across the row titled O- with the title Patient Blood Type).
From the list of blood type distributions in the populations at the beginning of the exercise, this corresponds to 9% of the population.

A patient with B+ could receive blood from O-, O+, B- and B+ this translates to 40 + 9 + 8 + 2% = 59% of the population.

   c) Construct and complete the table below showing the percentage of the population which can donate to and receive blood from each blood type. Some entries have been done for you.

Blood type	____ can donate blood to what percentage of the population	____ can receive blood from what percentage of the population
AB+	2%	
AB-	3%	
A+		
A-		
B+	16%	59%
B-		
O+		
O-		9%

   d) Use the table you have completed above to construct a multiple column graph for each of the blood types showing the percentage of the population each is able to donate blood to and the percentage of the population each is able to receive blood from. An example has been started for you.

9. a) Which blood type has the biggest difference in percentage of population they can give blood to and the percentage of people they can receive blood from?

    b) Which blood type has the smallest difference in percentage of population they can give blood to and the percentage of people they can receive blood from?

10. Different countries have different distributions of blood types. Below is a multiple column graph showing the percentage distribution of the different blood types of Australia, USA, Turkey and Sweden.
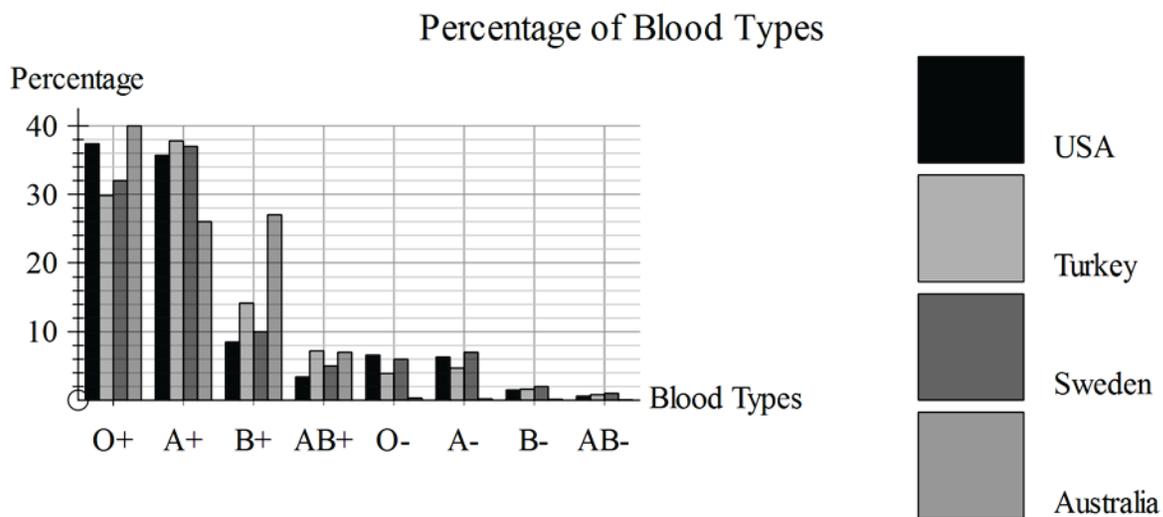
    a) From the graph, what is the percentage of blood type O+ for each of the countries?
    b) What is the difference in the percentage of O- in USA and Australia?
    c) Comment on the similarities and differences in each of the blood types from each of the countries. Which country is significantly different to the others? In what ways?

37 Statistics in the media

Materials required: A week's worth of local newspapers, scissors, internet access

Warm-up

|			
1.	2 hours = _____seconds	2.	10 weeks = _____h
3.	Find the area of a 2km long, 800m wide rectangle.	4.	$800cm^2$ = ____m^2
5.	11, 12, 13, 14, 15 – what is the median?	6.	11, 12, 13, 14, 14, 15 – what is the median?
7.	Increase $100 by 10%.	8.	Decrease the answer to Q7 by 10%.
9.	Find the volume of rectangular prism 3m by 4m by 8m.	10.	A cube has a volume of $8cm^3$. What is the area of each face?

Activity

Data and statistics are regularly used in the media. They are used both correctly and incorrectly; sometimes journalists misinterpret the statistics, but some 'news' outlets use data displays to intentionally mislead the public for political reasons.

1. Collect a week of newspapers and, in groups within your class, go through the papers to identify articles or advertisements where data and statistics have been used.
2. Cut out the articles and, for each use of data, look at how that data has been collected. Has it been collected by an organisation that uses a scientific approach? Or has the collection method potentially introduced bias? How many of the articles fall into each category?
3. Next, each group should take a selection of the articles and reclassify them according to the use of the data. Has the data been represented in an objective manner that makes it clear and easy to interpret or not? In other words, does the representation introduce the possibility of biased interpretation?
4. Are there some newspapers that are more likely to print biased articles than others? What does this tell you about the motivations of the publishers?

Exercises

1. The population of Australia on July 17 2014 was 23 533 910. The Australia Bureau of Statistics population clock indicates that this increases at the rate of 1 person every 1 minute and 20 seconds.

    a) How does it calculate the population change rate?

    b) How can it be sure?

2. The distribution of the Australian population by age and gender is displayed in this graph.
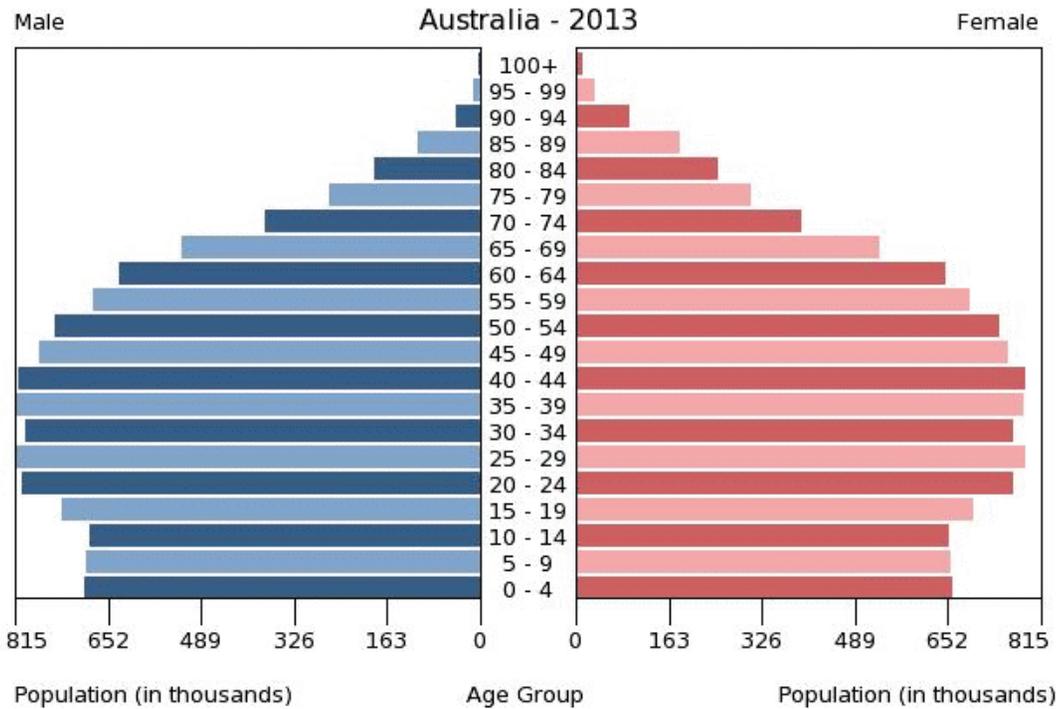
a) What age group(s) has the highest number of people?

b) Are there any particular age groups where there are significantly more males than females? Which ones?

c) Are there any particular age groups where there are significantly more females than males? Which ones?

d) What is the median age group of males and females?

e) If there is a 50% split between male and female why do you think both sides of the graph are unequal?

f) What percentage of the population are under 20?

g) What percentage of the population are over 90?

h) What do you think will happen to the distribution of ages in the next:

    (i)   20 years?

    (ii)   40 years?

    (iii) 60 years?

i) What are some likely outcomes of your predictions?

3. a) According to recent research commissioned by the federal government, one student in every four in Australian schools is affected by bullying. Using the graph from the previous question, estimate how many students this is.

   b) Cyber-bullying is more prevalent in older children with 31% of 14-17 year olds reporting that they have been cyber-bullied compared to 21% of 10-13 year olds (Microsoft/Galaxy Research, 2008). Using the graph from the previous question estimate, how many Australian students this is?

   c) An estimated 200 million children and youth around the world are being bullied by their peers, according to the 2007 Kandersteg Declaration Against Bullying in Children and Youth.

   (i) Using the population of Australia as 23 500 000, how many times the population of Australia is this?

   (ii) How do you think these estimates are gathered?

   (iii) Why do you think they are gathered this way?

4. The Australian Bureau of Statistics released a labour force publication in November 1992 with the following main feature:

   "AN ESTIMATED 25 PER CENT OF ALL FAMILIES HAD NO FAMILY MEMBER EMPLOYED."
   Based on the above, a headline in a leading Australian newspaper read:
   **"UNEMPLOYMENT AFFLICTS ONE IN FOUR FAMILIES."**

   What is wrong with this headline?

5. Using your preferred search engine, enter the phrase "classic and novel graphical techniques to lead with data". A website called Simply Statistics should appear in your list of results. Click on the link and examine the page it takes you to. A pay television station in the United States of America has been profiled there. A range of data displays they have shown on their 'news' programs, that is designed to intentionally mislead viewers, is shown with an explanation of the errors. What are the techniques that they have highlighted on the page? In your own words, describe how each technique creates a false impression or misleads the viewers.

6. On the website you found in question 5, the results of a poll are shown on one slide. The question asked was "Did scientists falsify research to support their own theories on Global Warming?" with the three possible responses being somewhat likely, very likely or not very likely. How does this question create bias? Rewrite it to remove any bias.

7. Use the Australian Bureau of Statistics website to find some data on an issue that is important to you, e.g. an environmental concern, immigration, unemployment, etc. Graph the data to give an accurate representation of the information. Now choose one of the techniques from question 5 to create a misleading display of the data that presents the point of view you would like others to share. Now create a display that presents the same data in a manner that would convince people to hold the opposite point of view.

8. One Australian television station has used a number of incorrect data displays in their news items, although it seems to be more in error than with the intention of misleading. For the following charts, state what is wrong with the display:

a)

25% opt out
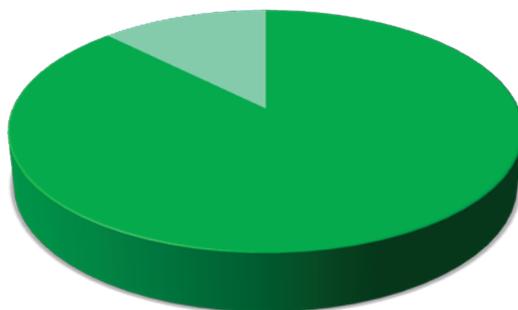
75% of children take part in religion classes

b) ASTHMA SUFFERERS

1 in 10 adults

1 in 5 children

17% by age 3

9. It is often thought and reported that "Monday is the most dangerous day for road accidents".
    a) Why do you think Mondays are often reported as being the most dangerous?

    A sample of 160 road accidents had the day of the week on which they occurred recorded and the results are displayed below.
    b) Record the number of accidents occurring on each of the days of the week in a frequency table.

    c) Determine the percentage of accidents that occurred on each of the days.

    d) Display your results in an appropriate graph. Justify your choice of graph.

    e) Summarise your results using the percentages calculated in part c) and offer possible reasons.

Wednesday	Thursday	Sunday	Thursday	Sunday	Tuesday	Tuesday	Friday
Thursday	Saturday	Sunday	Friday	Sunday	Wednesday	Tuesday	Saturday
Friday	Saturday	Monday	Friday	Sunday	Wednesday	Tuesday	Saturday
Friday	Sunday	Monday	Friday	Friday	Thursday	Tuesday	Saturday
Saturday	Sunday	Monday	Friday	Saturday	Friday	Wednesday	Saturday
Sunday	Sunday	Tuesday	Friday	Saturday	Friday	Wednesday	Saturday
Sunday	Sunday	Tuesday	Friday	Tuesday	Friday	Wednesday	Saturday
Monday	Sunday	Tuesday	Friday	Wednesday	Friday	Thursday	Saturday
Monday	Sunday	Wednesday	Saturday	Wednesday	Saturday	Thursday	Sunday
Monday	Monday	Thursday	Saturday	Wednesday	Saturday	Thursday	Sunday
Monday	Monday	Thursday	Saturday	Thursday	Saturday	Thursday	Monday
Monday	Monday	Thursday	Sunday	Thursday	Saturday	Thursday	Monday
Tuesday	Monday	Friday	Sunday	Saturday	Saturday	Thursday	Tuesday
Tuesday	Monday	Friday	Sunday	Saturday	Saturday	Friday	Tuesday
Thursday	Thursday	Friday	Saturday	Saturday	Sunday	Friday	Tuesday
Saturday	Sunday	Tuesday	Wednesday	Sunday	Tuesday	Wednesday	Thursday
Saturday	Monday	Tuesday	Thursday	Saturday	Tuesday	Wednesday	Tuesday
Sunday	Sunday	Monday	Tuesday	Friday	Friday	Friday	Saturday
Sunday	Monday	Tuesday	Wednesday	Friday	Friday	Friday	Sunday
Monday	Sunday	Tuesday	Friday	Wednesday	Friday	Thursday	Saturday
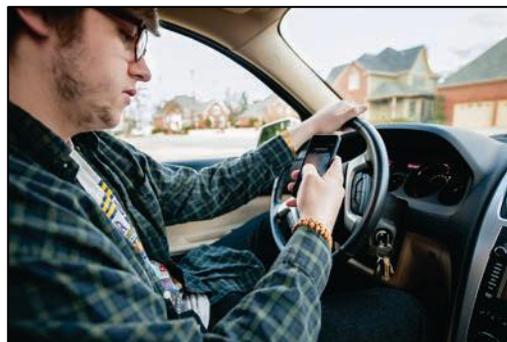
10. Below is a recording of the number of minutes per day people in Australia spent on sport and recreation.

	Males	Females
	Minutes per day	Minutes per day
Socialising	10	10
Visiting entertainment venues	4	5
Attendance at sports event	2	2
Religious activities	4	6
Community participation	8	10
Associated travel	11	13
Social and Community interaction	*40*	*46*
Sport and outdoor activity	25	17
Games, hobbies, arts, crafts	14	12
Reading	21	25
Audio/visual media	154	122
Attendance at recreational courses	-	1
Other free time	23	18
Talking and correspondence	24	36
Associated travel	6	5
Recreation and Leisure	*268*	*237*
Total free time	**308**	**284**

   a) If males and females spent 308 and 284 minutes on sport and recreation respectively, how much time do they spend on other activities?

   b) What could the other activities be?

   c) If this table represents the whole population create a table you think might represent people your age.

   d) How would you determine an indication of the time spent doing each of these activities for people your age?

Extended Problem Solving

Use the internet to research available statistics on a topic of interest to you, e.g. use of social media, participation in or support of a particular sport or team, an environmental or political issue, a social issue affecting teenagers or any other issue you wish. Once you have decided your topic, get your teacher's approval to research it.

Prepare a visual display of the information you found about your topic. You will present and explain your findings to your class. In your presentation include:

•   what topic you decided to research including any particular aspects you were interested in finding out about

•   how you found your data

•   how you verified the data was correct

•   what the data told you about your topic

•   the conclusions you drew from that data.

Supplementary Lesson
Statistical investigations

Warm-up

1.	2.4 + 17.61	2.	21.6 + 0.04
3.	What are the next 3 numbers? 0.6, 0.7, 0.8, ___, ___, ___	4.	What are the next 3 numbers? 1.6, 1.7, 1.8, ___, ___, ___
5.	What are the next 3 numbers? 10.6, 10.7, 10.8, ___, ___, ___	6.	Fruit loses up to 70% of their mass when dried. What would 3kg peaches weigh once dried?
7.	How many vertices are on a triangular prism?	8.	How many triangular faces are on a square pyramid?
9.	What is the perimeter of a square with sides of 6cm?	10.	What is the area of a square with sides of 9cm?

Discussion

In this course, and in life beyond school, it is important that you are able to find answers to real-world problems. This is done through the mathematical thinking process. If the problem requires the use of data, it means applying the statistical investigation process to real-world tasks:

- Clarify the problem and pose one or more questions that can be answered with data;
- Design and implement a plan to collect or obtain appropriate data;
- Select and apply appropriate graphical or numerical techniques to analyse the data;
- Interpret the results of this analysis and relate the interpretation to the original question; and
- Communicate findings in a systematic and concise manner.

This is similar to the mathematical thinking process, but when statistical processes are involved, it looks a little different. That is why they are defined separately.

Here, you will learn about the steps in the statistical investigation process individually, but through the rest of the book and this course you will regularly be asked questions that refine your skills in each of these areas. By the time you complete this course, it is expected that you will be able to apply the statistical investigation process independently so the questions you are asked will gradually become less structured as you progress. You will also learn mathematical skills and processes that will allow you to work with harder mathematical concepts and conduct analysis of more complex data.

Clarify the problem and pose questions

When given a problem to solve, it is important that you can show you understand what you are trying to find out and what information you need to answer it. For example, 'Are males or females better at estimating time?'

You are being asked several things here, including:
- How well do males estimate time? And females?
- Does the length of time being estimated affect the accuracy?
- What are the differences between males and females in the accuracy of their estimates?

Now, as often happens in life, this question is very broad and general.

If you were going to answer the question as it is written, you would need to somehow find out how good every person in the world is at estimating time, summarise the findings for males and the findings for females and then find some differences between these groups. That's not practical, so you may want to limit it to students in your school, or even just students in your class (assuming there are both males and females in your class). In other words, you need to **interpret** and **clarify** the task, make it **simpler** and achievable, and **identify** the key information that you need to focus on in your data collection.

Discussion: What factors do you think affect the accuracy of people's estimates of time for people of all ages and backgrounds?

In the example provided about accuracy of time estimation, it is important to refine the question to something more easily measured. You might choose to reduce it by thinking about the limits you can apply to narrow the scope of the question:
- You could limit it to a certain age range.
- You could limit it to one or more particular lengths of time.

This process changes the question. You could choose to answer any one or more of the following questions:
- Are male or female Year 11 students better at estimating the passage of one minute?
- Does the better estimator change if the length of time changes?
- Are males or females better at estimating the passage of 10 seconds, 30 seconds, 1 minute and 2 minutes?
- Does the better estimator change if the age category changes?

Keep in mind that the more you limit the scope, the further from the original question you get. The more the situation is reduced, the lower the complexity of the problem you are addressing; reducing it too far may limit the mathematics that is used and therefore the mark that can be achieved if it is an assessment item. For example, if there are only 8 people in your class and 5 are male and you choose to just measure their estimation of the passage of an interval of 10 seconds, then your results are not likely to be indicative of the answer to the original question and the analysis is likely to be quite simplistic.

You might also hypothesise about the answer you think you will find, that is, make an educated guess. If you study higher level mathematics at some point in the future, you may learn about statistical analysis techniques such as hypothesis testing which is used in scientific studies in a range of disciplines.

Design and implement a plan to collect or obtain appropriate data

There are several important elements to this:
- What data is appropriate? This has possibly already been addressed in the clarifying step.
- Is it better to obtain secondary data or collect it yourself?
- If collecting it, how can you collect the data in a way that will mean your findings are justifiable and valid?
- Design the plan to collect the data (if required).
- Collect or obtain the data.

When applying this to the question about time estimation, you might choose to look at the factors of gender, age and length of time. In this case, the appropriate data to gather might be:
- estimates of 10 seconds, 30 seconds and 1 minute;
- three age groups: your year group at school, people in their 30s and people in their 50s; and
- males and females in each age group.

This gives us a decent approximation of the original question, even though it is limited to just people in your local area.

It isn't likely that you will be able to find secondary data that will address this question, so collecting the data yourself will be more suitable.

The more data is collected, the more likely your results will reflect that of the broader population. Deciding how many measurements to take, though, must be considered in terms of how easily the data can be gathered and how much time you have available.

In this instance, a sample of 10 males and 10 females in each age group would be sufficient for this purpose. Each time period would need to be assessed for each person at least 5 times. You would also need to decide whether or not you will provide the person with feedback about the actual time they guessed after each trial as this must be consistent for all people involved. Will you have a break between estimates? If so, for how long?
There are other decisions that must be made, such as whether all estimates will be considered or whether the highest and lowest for each person will be discarded. Will each measurement be taken separately or will they be averaged for each person?

Discussion: As a class, discuss the possibilities and together develop a plan for how to gather the data you need.

Activity: As a class, test your process on each other to see if you can identify any potential problems with it that may need to be resolved by changing the process. If required, change your process accordingly.

Discussion: As a class, compare the different ways of recording the results in the activity that were used. Which method was best? Why?

Activity: Working on your own, design a data collection sheet that could be used to collect all of the data for your experiment.

Discussion: As a class, by looking at each other's data collection sheets, determine which one is best and why.

Select and apply appropriate graphical or numerical techniques to analyse the data

Once you have collected your data, you need to decide how you are going to summarise and analyse it. There are several considerations that are part of this process.

- Are you going to use technology to help you? If so, what technology will be most useful?
- What sort of graph or graphs would be appropriate? To determine this, what sort of data have you collected (discrete, continuous, univariate, bivariate etc.)? Also, who is your audience?
- What numerical summaries would be most appropriate (mean, median, mode, range, deciles, quartiles, interquartile range etc.)?

The intention of providing numerical summaries and graphical displays is to enable you to interpret the data more easily. Your choice of graph and summary statistics should make it easier for you to identify any patterns, similarities, differences or relationships that exist and to find an answer to the original problem.

In the time estimation activity, you also need to decide how you will measure the accuracy of the estimates.

Discussion: Is 1 second away for an estimate of 10 seconds better or worse than 5 seconds away when estimating a minute? Looking at the class data, discuss how you could measure accuracy. Is there a need to measure the accuracy with a calculation or is there a way of displaying the information that provides a picture of the accuracy?

Interpret the results of this analysis and relate the interpretation to the original question

Using the summaries you created, the next step is to identify any patterns, similarities, differences or relationships in the data and interpret these in terms of the original problem. Keep in mind, though, that any decisions you make in the first step about reducing the scope of the data you look at will alter how well you can answer the original question. You may also make assumptions about how well your data represents the full situation in the original problem. This needs to be part of your analysis.

Discussion: For the time estimation problem, what difference in performance is enough to say one gender is better at estimating time than another?

Communicate findings in a systematic and concise manner

Once you have found the answer to the problem, you need to be prepared to share that answer with others. For a statistical investigation, this means outlining a summary of the 4 steps you followed in conducting the investigation. This doesn't mean that you need to write an essay, just a systematic and concise summary of the steps you followed. You might also be expected to justify your process verbally.

In the example about estimation of time, however, a description of the data collection method, a summary of the results and a brief analysis of those results that gives an answer to the question may be required.

Exercises

For the following questions, plan how you could find an answer to the question and how you would summarise, display and analyse that information:

1. How often do people blink?

2. Do reaction times slow down with age?

3. Do red cars go faster?

4. Do seagulls prefer to leave their droppings on white cars?

5. What letter of the alphabet is used the most?

6. How well does citronella work to keep mosquitoes away?

7. Are males or females better drivers?

8. How does short term memory vary among different people?

9. Can people really taste the difference between the different brands of cola?

10. At what temperature does water freeze the fastest?

11. What volume is a droplet of water? Does the size change with temperature?

12. Does buttered bread always land buttered side down when you drop it?

Extended Problem Solving

Choose one of the questions posed in the exercises and follow the statistical investigation process to find an answer.

38 Ratio skills

Warm-up

1.	What are the factors of 12?	2.	What are the factors of 9?
3.	What is the highest common factor of 9 and 12?	4.	Simplify the fraction $\dfrac{9}{12}$
5.	Simplify the fraction $\dfrac{12}{9}$.	6.	Express $\dfrac{12}{9}$ as its simplest mixed numeral.
7.	Express $\dfrac{9}{12}$ as a decimal.	8.	Express $\dfrac{12}{9}$ as a decimal.
9.	Express $\dfrac{9}{12}$ as a percentage.	10.	Express $\dfrac{12}{9}$ as a percentage.

Introductory Problem: Winning the Jackpot

Your teacher has won a $4 million jackpot and plans to donate $200 000 to your school. The money will be distributed to learning areas in the same ratio as the popularity of that subject. Your class has been chosen to determine how this money will be distributed.
1. Collect data in your class about the students' favourite learning areas, design an appropriate recording method to record the data.
2. Determine an appropriate way to distribute the money according to subject popularity. Be sure to include your reasons based on your survey findings.

Ratios

A ratio compares quantities of the *same kind* in a definite order, such as 3:5 (read 3 *to* 5), 700g:2.1kg. Ratios are related to fractions and percentages, for example 1:4 means there are 5 parts to the ratio, so it can be expressed as $\dfrac{1}{5}$ to $\dfrac{4}{5}$ or 20% to 80%. Ratios are normally expressed using whole numbers. *Equivalent* ratios have the same values and should be expressed in their simplest form.
For example   3 weeks:6 days = 21 days:6 days = 7:2

The ratio was converted to the same units (days) and then each quantity was divided by 3, the highest common factor.

This can then be converted to a fraction or a percentage by finding the total parts to the ratio.
For the previous example, 7:2 has 7+2=9 parts, so the fraction form is $\dfrac{7}{9}$ to $\dfrac{2}{9}$ which becomes 77.78% to 22.22% (to 2 decimal places).

Exercises

1. Express each of the following diagrams as a ratio of shaded squares to white squares, then convert the ratios to fractions.

   a)

   b)
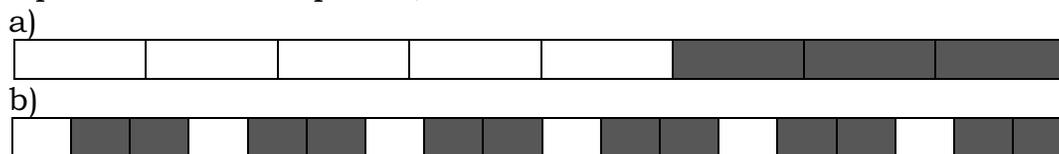
2. Simplify each of the following ratios

   a) 500 : 100   b) 8 : 4   c) 12 : 3

   d) 2 : 6 : 18   e) 4 : 12   f) 10 : 25

   g) 7 : 14 : 35   h) 56 : 8   i) 100 : 20

3. Convert the following ratios to their simplest form

   a) $4 : $12   b) 30cents : $6   c) 10hrs:1 day

   d) 24cm : 1m   e) 30mins : 2hrs   f) 500g : 3kg

   g) 250mL : 1L   h) 400mL : 2L   i) 20kg : 1tonne

4. A ratio expressed in its simplest form should be expressed with whole number values. Express each of the following ratios in their simplest form

   a) ½ : 1   b) 0.3 : 0.9   c) 1 : ¼

   d) $\dfrac{3}{11} : \dfrac{7}{11}$ e) 1.5 : 2   f) 1½ : 1

   g) 2½ : 2   h) $2 : \dfrac{2}{5}$ i) 0.3 : 0.7 : 1.1

5. Share the following amounts of money by calculating a single share then distributing in the ratio stated.

   a) $36.00 shared in the ratio 4 : 5

   b) $40.00 shared in the ratio 3 : 2

   c) $200 shared in the ratio 6 : 4

   d) $9000 shared in the ratio 1 : 4

   e) $21 000 shared in the ratio 2 : 3 : 5

   f) $1 000 000 shared in the ratio 1 : 4 : 5

6. Copy and complete the table below

Ratio	Fractions	Percentages
1 : 4	$\dfrac{1}{5}$ and $\dfrac{4}{5}$	20% and 80%
1 : 3		25% and 75%
2 : 3	$\dfrac{2}{5}$ and $\dfrac{3}{5}$	
4 : 1		
	$\dfrac{3}{10}$ and $\dfrac{7}{10}$	
		$33\dfrac{1}{3}\%$ and $66\dfrac{2}{3}\%$

Extended Problem Solving – Tessellating Art
Exploring Fractions, Percentages and Ratios

Equipment: 10 × 10 2cm grid, various coloured 2cm × 2cm squares, glue

Tessellating patterns are used throughout the world to create artwork. Three or more colours are used to create intricate designs common in Arabic art. In this activity you will use a simple square tessellation with colour repetitions to investigate fractions, decimals, percentages and ratios.

1. Choose at least three colours to use for your art work. White squares can be left blank.
2. Plan a design you could use – repeating patterns of colours works best.
3. Once you are happy with your design you can glue your coloured squares to the 100 chart in your selected design.
4. Count the number of squares for each colour to extend and complete the table below.

Colour	Number	Fraction	Decimal	Percentage

5. Write your colour usage as a ratio and simplify this as much as possible by dividing by the highest common factor.
        Example 20:25:25:30 becomes 4:5:5:6

39 Applications of ratios

Warm-up

1.	What is the formula for the area of a rectangle?	2.	What is the formula for the volume of a rectangular prism?
3.	Simplify 4:12	4.	Simplify 18:4
5.	Calculate the sum of 5 and 6.	6.	Calculate the product of 5 and 6.
7.	For the set {1, 2, 2, 5, 9, 11}, calculate the range.	8.	For the set {1, 2, 2, 5, 9, 11}, calculate the mode.
9.	For the set {1, 2, 2, 5, 9, 11}, calculate the mean.	10.	For the set {1, 2, 2, 5, 9, 11}, calculate the median.

Activity: Determining Ratios

Using information available in your class, determine the following ratios.

1. Count the number of males and females in your class and hence determine the ratio of males to females
2. How many vowels are in your full name? How many consonants? What is the ratio of vowels to consonants in your name?
3. Tabulate and total the class data for the number of vowels and consonants. What is the ratio of vowels to consonants in the names of all class members?
4. What is the ratio of class members wearing watches to those who are not?
5. How many class members have already had their birthday in this calendar year? Determine the ratio of those who have had birthdays to those who have not.
6. Usually in a large population the ratio of people who can roll their tongues to people who cannot roll is 3:1. What is the ratio in your class?

The language of ratios

If the ratio of teachers to students in a school was 1 teacher to 10 students, then

- we can say for every 10 students there is one teacher
- the ratio of students to teachers is 10 : 1
- the ratio of teachers to students is 1 : 10
- we *cannot* say that $\frac{1}{10}$ of the people are teachers because this means that for every 10 people, one of them is a teacher, i.e. 1 teacher and 9 students
- We *can* say that $\frac{1}{11}$ of the people are teachers, i.e. 1 teacher and 10 students

Exercises

1. In Khaynan's class the ratio of males to females is 2 : 3. If there are 12 male students, how many female students are there?

2. A piece of rope is cut into three lengths in the ratio 2:3:5. If the smallest length is 6m, find the length of the original rope.

3. a) Calculate the ratio of the length to the width of the rectangle.

    8cm

    24cm
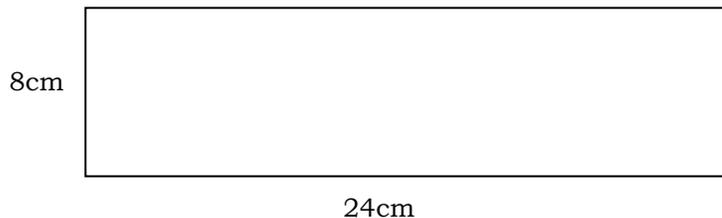
    b) Find the ratio of the areas of the two squares shown

    3cm   5cm
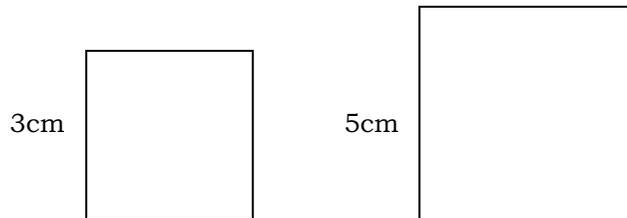
    c) The three angles of a triangle are in the ratio 2:3:4, calculate the sizes of each of the angles.
    d) Two cubes are such that the ratio of their sides is 4:1, find the ratio of their volumes.

4. According to the January 2014 edition of the Western Australian magazine Horizons, for every person killed on the roads, 100 suffer a serious injury in a motor vehicle accident. Research data also indicates that about 20% of all the people involved in accidents had a blood alcohol level more than 3 times the legal limit.
    a) Write down two ratios to express the comparisons described.
    b) Given that there were 162 people killed on the roads in 2013, how many serious injuries probably occurred in motor vehicle accidents?
    c) Of the people killed on the roads, 1 in 4 were not wearing seatbelts. What is the ratio of people wearing seatbelts to those who were not?

5. Two 375mL bottles contain water and orange juice in the ratio 4:1 and 2:1 respectively.
    a) Find the amounts of orange juice and water in each bottle.
    b) If the contents of the two are mixed, what will be the new ratio of orange juice to water?

6. Curious Indiana looked in his shed and discovered some commonly used garden products that use ratios. Indiana's discoveries are shown in the table below. Determine the appropriate ratios and answer Indiana's questions.

Substance	Label	Using simplified ratios, write the ratio of:	Indiana's question
Insecticide	20mL of insecticide in litre of water	Insecticide : water	How much insecticide is needed for 5 litres of water?
Weed Killer	500mL of weed killer to be mixed with 9 litres of water	Weed killer : water	If I only want to use 1 litre of water, how much weed killer should I add?
Paint required for a feature wall	Use 200mL stone paint for each litre of white paint	Stone Paint : white paint	If I only have 500 mL of stone paint left, how much white paint do I need to have the same mixture?
Blackberry Killer	100mL of the chemical for 12 litres of water	Chemical : Water	I will use 6 litres of water. How much chemical should I add?
Eucalypt eradicator	100mL eradicator per 1.1 litres of kerosene.	Eradicator:kerosene	Why do you think they used 1.1 litres rather than 1 litre?

Extended Problem Solving – Golden Ratio

Have you ever heard of the golden ratio? Maybe you have come across it in earlier years. Investigate one of the following and *communicate your findings* by creating a poster, report, video or computer presentation.

1. *The Golden Ratio in Art* – Examine famous paintings by Leonardo Da Vinci to find how the golden ratio was used.
2. *The Golden Ratio in Architecture* – Research where the golden ratio exists in buildings in ancient civilisations.
3. *Fibonacci's sequence* – Discover the link between Fibonacci's sequence and the Golden Ratio.
4. The Perfect Face – Measuring features of the face to discover if it is "golden".
5. Golden Ratio in Nature – Explore examples of the golden spiral in nature. How is it connected to the golden ratio?

40 Rates

Warm-up

1.	1 year = _____ days	2.	1 fortnight = _____days
3.	0.25 h = _____mins	4.	2.5 hours = _____mins
5.	$3.60 × 2 =	6.	50km × 3 = _____km
7.	450 km ÷ 9 = _____km	8.	450km = _____m
9.	Write 15% as a decimal.	10.	Write 14.5% as a decimal.

Opening Problem: Making sense of rates

Verify whether each of the following statements draws a reasonable connection, giving reasons.

1. If one girl can walk to school in 10 minutes, two girls can walk to school in 20 minutes.
2. If one box of cereal costs $2.80, two boxes of cereal cost $5.60.
3. If one boy makes one model car in 2 hours, then he can make three models in 6 hours.
4. If one girl has two cats, then four girls have 8 cats.
5. If a car travels 50km an hour, it can travel 200km in four hours.
6. If you increase the number of people attending an end of season wind up, you must increase the food you purchase.
7. If your speed is faster, you cover the same distance in less time.
8. If I double the distance travelled and the time remains the same, my speed has been halved.

What about you?

- A hummingbird flaps its wings 3 000 times per minute, how many flaps can you do in a minute?
- Sean Shannon claims to be the fastest talker in the world. He can recite about 660 words per minute, how many can you say in one minute?
- Americans chew on average 300 sticks of chewing gum each year. What do you think your average would be?

A rate is a comparison of two different quantities, with different units. For example

> *Pay rates* – how much money you are paid per hour.
> *Birth rates* – how many babies are born per year.
> *Interest rates* – what percentage of your investment is paid per year.
> *Exchange rates* – what the Australian dollar is worth in foreign currencies.
> *Speed* – what distance you travel over a period of time.

Exercises

1. Jackie earns income by offering a babysitting service to parents in her community. She charges $8.00 per hour, but increases this to $9.50 per hour on Friday and Saturday evenings.
   a) Calculate her income if she babysat:
      (i) Mrs Harris's 3 year old for 3 hours on Tuesday.
      (ii) The Johnson tribe on Thursday night for 2.5 hours.
      (iii) Mike and Julies' 10 year old twins for 5 hours on Friday night.
   b) If Jackie earned $32 on a Monday how many hours did she babysit for?

2. A popular coastal town in Western Australia offers scooter hire to tourists visiting their vibrant community.

	50cc	125/150cc
1 week (7 days)	$200	$300
2 weeks (14 days)	$300	$450
3 weeks (21 days)	$350	$525
Monthly (30 days)	$400	$600

   a) What is happening to the rate as the number of weeks of scooter hire increases?
   b) How much would it cost to hire a scooter for the entire year?
   c) Is the cost of hiring a scooter for two weeks double the cost of hiring a scooter for one week? Explain your answer.

3. Herman travels at an average speed of 55km/h how far does he travel in   a) two hours   b) 30 minutes

4. Savannah was driving home after a long family lunch. If she travelled at an average speed of 80km/h and she lives 60 km away, how long did it take Savannah to return home?

5. The Fiona Stanley Hospital, Murdoch opened in 2014 and has 783 hospital beds. If the cost per day for a hospital bed is approximately $984 calculate:
   a) the total cost for hospital beds per day.
   b) the total cost for hospital beds per year.

6. According to data collected from the ABS, the average birth rate per 1000 residents of Australia has increased from 12.9 in 2002 to 13.6 in 2012.
   a) How many extra births in this time period are in an average population of 100 000 residents?
   b) Is your solution reasonable across different communities, for example an inner city community, a retirement village or a community in the suburbs?

7. The table below shows the exchange rates of popular international holiday destinations for Australian tourists.

Country	Currency	Exchange Rate
New Zealand	Dollar	1.08
USA	Dollar	0.94
Indonesia	Rupiah	10 690
UK	Pound sterling	0.5607
Thailand	Baht	30.33
Singapore	Dollar	1.714
Vietnam	Dong	19 859
China	Renminbi (yuan)	5.8448

Use the exchange rate table above to find the equivalent value in the various currencies:
a)   A$16   in   Singapore dollars
b)   A$230   in   U.S dollars
c)   A$1 200   in   New Zealand dollars
d)   A$50   in   Indonesian rupiah
e)   A$75   in   Thai baht
f)   A$89   in   Chinese yuan
g)   A$65   in   Vietnamese dong
h)   A$100   in   UK Pounds

8. A Chinese tourist in Perth wants to organise a full day trip to the Pinnacles for himself, his wife and his 10 year old son, including a helicopter ride for his son.
a)   How much will this cost in A$?
b)   How much will this cost in Chinese yuan?

Tour cost per person	Adult	Child (u/16)
Includes lunch and entry fees	$157	$97
Optional helicopter flight	$45	$45

Extended Problem Solving –
Julie, Connie and Emma entered a weight loss competition and have started a diet. Their weights over a four week period are shown below.
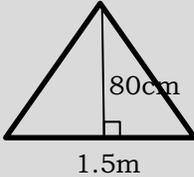
Week	Connie	Emma	Julie
0	95kg	71kg	51kg
1	93kg	69kg	50kg
2	92kg	69kg	48kg
3	90kg	68kg	48kg
4	88kg	67kg	47kg

Using your mathematical knowledge of rates, ratios and percentages, *construct three separate arguments* that each conclude that a different dieter is the winner.

41 Simple interest

Materials required: calculators

Warm-up

1.	Write 0.8 as a percentage	2.	Write 0.08 as a percentage
3.	Write 8.08 as a percentage	4.	What is the area?
5.	A square has a 1m² area, what is its perimeter?	6.	650mm + 65cm + 6.5m =
7.	Decrease 350m by 15%	8.	1 gigabyte = _____ kilobytes
9.	$\sqrt{76 - 24 \div 2}$	10.	Estimate $\sqrt{27}$ to 1 dp

When someone lends money to someone else, the borrower usually pays a fee to the lender. This fee is called 'interest'. Simple interest (or flat rate) is called simple because it is the easiest way of calculating and charging interest. The interest charge is always based on the original amount borrowed or invested. Most loans from banks do not use simple interest. Instead banks charge compound interest.

The simple interest formula is as follows:

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

where:
'Interest' is the total amount of interest paid.
'Principal' is the amount lent or borrowed.
'Rate' is the percentage of the principal charged as interest each year. The rate is expressed as a decimal fraction, so percentages must be divided by 100. For example, if the rate is 15%, then use $^{15}/_{100}$ or 0.15 in the formula.
'Time' is the time of the loan in years.
The simple interest formula is often abbreviated in this form:

$$I = P\,R\,T$$

Whenever money is borrowed, the total amount to be paid back equals the principal borrowed plus the interest charged:

Total repayments = (principal + interest)

Usually the money is paid back in regular instalments, commonly monthly, fortnightly or weekly. To calculate the regular payment amount, you divide the total amount to be repaid by the number of payments over the term of the loan.

Repayment amount = total repayment ÷ number of payments

Example:

$850 is borrowed at 8% p.a. simple interest for 30 months. What are the monthly repayments?

[8% p.a. means 8% per annum (per year)]

$$30 \text{ months } = 30 ÷ 12$$
$$= 2.5 \text{ years}$$

Simple Interest $\quad I = P \times R \times T$
$$= \$850 \times 8\% \times 2.5 \text{ years}$$
$$= 850 \times 0.08 \times 2.5$$
$$= \$170$$

Total repayment $\quad = P + I$
$$= \$850 + \$170$$
$$= \$1\ 020$$

Monthly repayment = Total repayment ÷ number of payments
$$= \$1\ 020 ÷ 30$$
$$= \$34$$

Exercises
1. Calculate the amount of interest and the total repayment when the following amounts are borrowed for one year at the given simple interest rate.
   a) $500 is borrowed at 5% p.a.

   b) An interest rate of 1.8% p.a. is charged on a loan of $4000

   c) $80 000 is borrowed at an interest rate of 3% p.a.

   d) A rate of 9.7% is charged on a loan of $4000.

2. Consider what happens when you borrow $1000 at a flat rate of 10% per year for a number of years. The table below shows the calculations for the first few years. Continue the pattern to complete the table:

Year	Principal	Interest per year	Total owing at the end of the year
1	$1000	10% of $1000 = $100	$1000 + $100 = $1100
2	$1000	10% of $1000 = $100	$1100 + $100 = $1200
3	$1000	10% of $1000 = $100	$1200 + $100 = $1300
4	$1000	10% of $1000 = $100	
5	$1000	10% of $1000 = $100	
6	$1000	10% of $1000 = $100	
7	$1000	10% of $1000 = $100	
8	$1000	10% of $1000 = $100	
9	$1000	10% of $1000 = $100	
10	$1000	10% of $1000 = $100	

a) What is the total owed at the end of the fifth year? The tenth year?

b) How much interest will have been charged after 5 years? After ten years?

c) How much interest will have been charged after 20 years? After 30 years?

d) What is the total owed at the end of 20 years? 30 years?

3. Find the simple interest payable on the following loans:

a) $3750 for 3 years at 3% per annum.

b) $3750 for 3 months at 3% per annum.

c) $8400 for 6 months at 2% per annum.

d) $15 000 for 7 months at 3.5% per annum.

e) $24 000 for 15 months at 3% per annum.

f) $11 500 for 26 weeks at 2% per annum.

4. Claire wanted to buy a TV advertised at $3 291. She already had $1500 in savings and wanted to borrow the rest from her father. He agreed, on the condition she pay him back over 15 months at 8% simple interest per year.

a) How much did Claire borrow?

b) How much interest will Claire pay?

c) What is the total amount that Claire will owe her father?

d) If Claire makes monthly payments to her father, how much does each one need to be to repay the full loan?

5. Natalie and John are planning on moving in together. They are shopping around for a new refrigerator. The model they want is available at three different places and each company offers different financing arrangements:

    A   Wilson White Goods – price $2 250 with 10% paid up front, the remainder covered by a loan at 14%p.a. over two years.

    B   Declan Discounts – price $1 990 with 20% paid up front, the balance borrowed and repaid over 18 months at 16.5%p.a.

    C   Eddie's Electrical – price $1880, will deduct $280 from the cost if they trade-in an old fridge and then borrow the balance over 18 months at 15.4% p.a.

    a) For each of the three options, calculate the amount borrowed, the interest payable on the loan and the total amount they would pay for each fridge (including any up-front payments).

    b) Determine how much the monthly payments will be for each of the three options.

    c) Which option do you think Natalie and John should select? Why?

6. Calculate the time required so that a loan of $8000 at 5% p.a. simple interest will earn $2000 in interest.

7. For how many months would $4000 have to be invested at 12.5% p.a. simple interest to earn $750 in interest?

8. How long would it take before the simple interest owing on a loan of $110 000 reaches $21 450 at an interest rate of 6% per annum? How many months is this?

9. Calculate the interest rate when $1500 is invested for 2 years and returns $135 in simple interest.

10. When $45 000 was borrowed for six and a half years, the simple interest charged was $15 356.25. What was the interest rate?

11. The total amount in an investment account after $75 000 was invested for 8 years was $121 800. At what annual rate did the account earn simple interest?

12. Determine the amount of money invested when $1 395 simple interest is earned over 2.5 years at an annual rate of 9%.

13. How much money would have been borrowed if $5488 simple interest is charged on the loan over 10 years at 5.6% per annum?

14. Calculate the loan taken out over 15 years at an annual rate of 11.5% if the simple interest charged on the loan is $32 085.

Extended Problem Solving

Borrowing money

At the end of 2013, total household debt in Australia reached $1.84 trillion. If there were 23.3 million people in Australia at the time, how much is that debt per person? In 2008, there were approximately 20 million people in Australia and the total household debt was $650 million. Has the debt per person increased or decreased over the five years?

Through research, answer the following questions:

1. Why do people borrow money?

2. From whom do people borrow money?

3. What do people do with the money they borrow?

4. Name the types of goods or services that people buy with the money they borrow. Classify each purchase according to how good an investment you believe each of those items are.

5. Is there such a thing as a good debt? When can this occur?

6. What would constitute a bad debt?

7. What is a credit rating? What can improve your credit rating? What can give you a bad credit rating? What are the consequences of having a bad credit rating?

Mathematics is only the art of saying the same thing in different words.

- Bertrand Russell

42 Time calculations

Materials required: Activity cards from Teacher's Guide, stop watch

Warm-up

1.	½ of 60 =	2.	¼ of 60 =
3.	0.5 × 60 =	4.	Write ¼ as a decimal
5.	2.5 × 60 =	6.	0.25 hours = _____mins
7.	1.25 hours = _____mins	8.	2.75 hours = _____mins
9.	240 mins = _____hrs	10.	45 mins = _____hour

Activity – Race around the Clock

Your class needs to be split into six teams, with each team starting at a workstation. Your teacher will keep track of your time on the stop watch.
In your team calculate the problem provided on the activity card.
Move to the next station as instructed by the solution of your activity.
The first team to visit all six stations with correct solutions wins the 'Race around the Clock'.

Time Calculations

Time calculations are an essential part of our everyday life. We need to arrive on time to places and work out how long it takes us to get there. Days are 24 hours long and are represented in 12 hour and 24 hour form. We use timetables to schedule our school day and to move around our community using public transport.

Exercises

1. Name three different examples of how time calculations are used in your community.

2. Convert the following to 24 hour time:

   a)   1:30am   b)   4:30am

   c)   10:14pm   d)   9:44pm

   e)   11:55am   f)   2:22pm

   g)   3:05pm   h)   12:10am

3.   Convert the following to 12 hour time.

    a)   0230h   b)   1524h

    c)   0342h   d)   1633h

    e)   1111h   f)   0017h

    g)   2328h   h)   1948h

4.   Estimate how long it takes to:

    a)   write an email.   b)   have a shower.

    c)   count to 100.   d)   read a page of a Harry Potter book.

    e)   blink.   f)   boil an egg.

5.   What time would it be

    a)   3 hours after 1pm   b)   30 minutes before 8am

    c)   1½ hours after 3:15am   d)   16 minutes before 11am

    e)   11 minutes before 10:40pm   f)   45 minutes after 11:45am

6.   Calculate the time between

    a)   1:30am and 2:15am   b)   12:10pm to 3:50pm

    c)   10:05am to 12:20pm   d)   8:15am to 3:10pm

    e)   9:55pm to 1:10am   f)   11:26am to 12:08pm

Extended Problem Solving – Once Upon a Time
How people have read the time has changed throughout history. Early man used the sun to keep track of time. Waterclocks, candle clocks and hourglasses were also used in different historical periods and regions of the world. Today's timekeeping devices use atomic readings with a superior level of accuracy.

Gather information on how timekeeping has developed over history and *communicate your findings* on an appropriate timeline.

43 Timetables

Materials required: internet access or timetables available in the Teacher's Guide

Warm-up

1.	1345 in 12 hour time	2.	2:10 PM in 24 hour time
3.	A fortnight = _____weeks	4.	A fortnight = _____days
5.	January has _____ days	6.	September has _____ days
7.	Calculate the time between 11:10am and 1:15pm	8.	Calculate the time between 0845 and 1422
9.	10% of $250	10.	5% of $250

Activity – Catching a Greyhound

In 1905, buses first started running out of Melbourne by A.A. Withers. The American bus company started in 1914 and Greyhound services Australia began fourteen years later when A.R. Penfold began a passenger service between Toowomba and Brisbane.

Australia has a land area of 7 692 024 km², containing vast open spaces between towns and cities. The bus company's ability to transport people between communities played a significant role in Australia's development as a nation.

Your task is to revisit history by planning a trip from *Melbourne* to *Toowomba* using a Greyhound service. Use the timetables provided by your teacher to plan your trip.

- What factors do you need to consider when planning a journey by bus?
- Gather the information you require for your journey and use time calculations to calculate the length of the entire journey.
- Check your answers with those from another group, are your answers reasonable?
- Compare your time calculations to websites http://www.greyhound.com.au/service-info/timetables journey planners. Comment on why they may be different.
- Use an effective method to record your journey (your itinerary).
- How do think your journey would compare to the same journey in 1928? Explain and justify your reasoning.

Exercises

1. The Australind is a train that travels the Bunbury to Perth line
    daily.

Australind

From Perth			103	105
			Daily	Daily
			AM	PM
Perth Station	♿	Dep	9:30	5:55
Armadale Station		Dep	9:56	6:25
Byford		Dep	10:07	6:36
Mundijong		Dep	10:14	6:43
Serpentine		Dep	10:21	6:50
North Dandalup		Dep	10:32	7:01
Pinjarra		Dep	10:42	7:11
Waroona		Dep	11:00	7:29
Yarloop		Dep	11:11	7:40
Cookernup		Dep	11:15	7:44
Harvey		Dep	11:21	7:50
Brunswick Junction		Dep	11:36	8:05
Bunbury Passenger Terminal	♿	Arr	11:55	8:24

From Bunbury			102	108
			Daily	Daily
			AM	PM
Bunbury Passenger Terminal	♿	Dep	6:00	2:45
Brunswick Junction		Dep	6:17	3:02
Harvey		Dep	6:32	3:17
Cookernup		Dep	6:39	3:24
Yarloop		Dep	6:46	3:29
Waroona		Dep	6:56	3:38
Pinjarra		Dep	7:12	3:55
North Dandalup		Dep	7:22	4:07
Serpentine		Dep	7:34	4:18
Mundijong		Dep	7:42	4:24
Byford		Dep	7:49	4:32
Armadale Station		Arr	7:55	4:39
Perth Station	♿	Arr	8:30	5:12

Trains only stop at intermediate stations if advance bookings are made

♿ Wheelchair accessible toilet facilities available

TransBunbury operates a daily bus service between Bunbury Passenger Terminal and Bunbury Visitor Centre. This is subject to change on public holidays. For more information call Transperth InfoLine on 13 62 13 (TIS: 131 450). Hearing or speech impaired? Call via NRS 133 677 and ask for 13 62 13.

The Australind train service connects with the Road Coach service to Albany - see timetable GS3.

Amendments may apply on public holidays. Transwa recommends passengers be ready to board 15 minutes prior to departure.

    a)   If I took the afternoon train from Bunbury, what time would I
arrive in Waroona?

    b)   How long is the morning journey from Perth to Bunbury?
Compare this to the afternoon trip and identify where any
differences occur.

    c)   What station will the train stop at for the evening?

    d)   If Veena travels from Pinjarra to Perth each day for her 9 – 5
job, calculate how much of her day is spent on the train.

2. Jade-Lilly arrived in Fremantle by train at 10:00 am on a Sunday
    and was keen to go on a tour of the Fremantle Prison, attend a
    Bon Scott memorabilia tour and grab some lunch at Cicerellos.
    She also needed to catch the train home at 5:35pm. Using the
    timetables below, create an itinerary for Jade-Lilly that includes
    both tours and some lunch, and allows 10 – 15 mins travelling
    time between each venue (including the train station).

```
Fremantle Prison Tours

Tour Length 1 hour 15 minutes

Daily every 30 minutes
between 10.00am and 5.00pm
```

```
BON SCOTT TOUR (90 MINS)

Sundays and Saturdays only

Departs  9:30am, 11:15am, 2:30pm,
4:30pm
```

3. The ferry timetable for trips to Rottnest Island is shown below.

DEPARTURE			ARRIVAL
Barrack St Perth	B- Shed Fremantle	Northport Fremantle	Thomson Bay
	0715*		0800
0830		1000	1045
0940	1030	1045	1130
1030*			1215
		1345*	1430
1320	1410	1425	1510
	1430	1445	1530

DEPARTURE	ARRIVAL		
Thomson Bay	Northport Fremantle	B- Shed Fremantle	Barrack St Perth
0825*	0900		
0915	0955	1020	1140
1140	1215	1240	
1200*			1340
1410	1445		
1510*			1650
1555	1630	1655	1800
1625	1715	1745	

Cost of adult tickets $30.75, students $26.75
* Direct trip

a) What time is the earliest ferry from Fremantle?

b) Calculate how long a direct trip from Perth to Rottnest takes.

c) If I wish to arrive at the B-Shed by 11:00am, what time should I catch the ferry from Thomson Bay, Rottnest?

d) Harry, Louis, Zayn, Liam and Niall have decided to have a two night getaway at Rottnest Island departing from Northport Fremantle. They have booked a six bed premium view cottage at $315/night. They check in Saturday at 2pm, stay two nights and check out at 9am.

    You have been employed as a travel agent and need to create an itinerary and calculate the individual costs for each of the five boys on their holiday.

4. A tide chart tells the times when high and low tides occur. The tides occur at different times each day and rise and fall by different amounts that depend on your geographical location and the position of the sun and the moon.

    The Jones family are going on a beach holiday for a fortnight in January, the tide chart shows the high and low tides over this two week period.

Day	Date	Tide 1		Tide 2	
Wed	1	4:16 AM	(0.3) L	8:13 PM	(0.9) H
Thur	2	4:57 AM	(0.2) L	8:48 PM	(0.9) H
Fri	3	5:42 AM	(0.2) L	9:25 PM	(1.0) H
Sat	4	6:29 AM	(0.2) L	10:02 PM	(0.9) H
Sun	5	7:19 AM	(0.2) L	10:37 PM	(0.9) H
Mon	6	8:09 AM	(0.2) L	11:07 PM	(0.8) H
Tue	7	8:56 AM	(0.3) L	11:26 PM	(0.8) H
Wed	8	9:35 AM	(0.4) L	11:11 PM	(0.7) H
Thur	9	9:57 AM	(0.5) L	8:38 PM	(0.7) H
Fri	10	7:46 AM	(0.5) L	7:48 PM	(0.8) H
Sat	11	6:07 AM	(0.4) L	6:51 PM	(0.8) H
Sun	12	4:47 AM	(0.4) L	6:45 PM	(0.9) H
Mon	13	4:43 AM	(0.3) L	7:05 PM	(0.9) H
Tue	14	4:57 AM	(0.3) L	7:33 PM	(0.9) H

a) David Jones likes to surf on his holidays and goes half an hour before and after low tide. What time will he need to be at the beach on
    (i)   Monday the 6th?
    (ii) Friday the 10th?

b) Vince and Liesel Jones are planning to go fishing on Saturday the 11th. Fishing conditions are best one hour either side of high tide. What time should they
    (i)   arrive at the beach ready to go fishing?
    (ii) pack up fishing for the night?

c) Donna's family plan on spending at least two days out on their boat. They would prefer to depart 45mins after a 0.5m low tide to ensure they can get past the local reef. What days and times should they depart for their boating outing?

d) Geraldton is +40mins on this tidal chart. Calculate the low and hide times for Geraldton on Thursday the 2nd of January.

44 Speed

Warm-up

1.	2.5 km = _____ m	2.	150m = _____ km
3.	3 hours = _____ minutes	4.	3 minutes = _____ seconds
5.	15 minutes = _____ hour	6.	45 minutes = _____ hour
7.	15 minutes _____ % of an hour	8.	45 minutes = ___ % of an hour
9.	If I travel 30 km in 1 hour, how far do I travel in 2 hours?	10.	If I travel 30 km in 1 hour, how far do I travel in ½ hour?

Introductory problem – A Community Fun Run

Jenna and Aleisha are helping to organise a fun run to raise money for their community cricket club. They are in charge of deciding on the time the fun run should be held, budgeting the expenses and income, and allocating the entry categories for the competitors.

The fun run is held on a 6 kilometre course around a local community lake.

a) It is expected that the slower competitors in the fun run will be walking at a leisurely pace equivalent to 75 metres in one minute. Calculate their speed in km/hour.

b) The faster competitors are known to be able to complete one km of the course in only 3 minutes 30 seconds. Calculate their speed in km/hour.

Jenna and Aleisha decide to hold the fun run at 9am on a Sunday.

c) What time could they expect the first competitor to cross the finish line?

d) What time could they expect the final competitor to cross the finish line?

Calculating Speed

Speed is the rate at which an object covers a certain distance. A faster moving object covers the same distance in a smaller amount of time than a slower moving object. Units of speed are expressed in units of distance per unit of time, for example metres per second or km/h. 100km/h is the speed limit on most highways and the speed of a cheetah running full speed.

The speed of an object can be given as an *instantaneous* value (think of the changing speed on an odometer of a car over a period of time) or we can calculate the *average* speed during a journey.

To calculate the average speed of an object we can use the formula

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

Using the examples from the introductory problem,
a)   If you walk 75 metres every minute what is your speed in km/h?

Speed = 75m/min
          = 4500m/h   *multiplying by 60 to convert the time to hours*
          = 4.5 km/h   *converting metres into km by dividing by 1000*

b)   Faster competitors complete one km of course in only 3 minutes 30
       seconds hence: *distance* = 1 km,
                       time = 3.5 minute = (3.5 ÷ 60) hours = 0.058$\bar{3}$

       Speed = $\frac{\text{distance}}{\text{Time}}$ =   1 ÷ 0.058$\bar{3}$ ≈ 17.15 km/h

Exercises
1.   Match each of the following objects with its relevant speed

	OBJECT		SPEED
a	Aeroplane in full flight	I	36 km/hour
b	Cheetah sprinting	II	50 km/hour
c	Maximum speed of a car in a built up area	III	113 km/hour
d	Champion athletes running over a 100m distance	IV	550 km/hour

2.   If I walk at a speed of 4 km/h, how far do I walk in:
     a)   2 hours   b)   3 hours
     c)   1½ hours   d)   2 ¼ hours
     e)   35 minutes   f)   110 minutes

3.   Now repeat question 2 if I was walking 5½ km/h.

4.   Calculate the speed in km/h of an object going
     a)   12 km in 3 hours   b)   310 km in 4 hours
     c)   20 km in ½ hour   d)   18 km in 10 minutes
     e)   100 m in 2 minutes   f)   8750 m in 15 minutes

5.   Convert these speeds in km/hr to m/s
     a)   10km/hour   b)   5.5 km/hour   c)   0.3 km/hour

6. If I walk at a speed of 4.5km/h how long does it take me to walk:
   a) 9 km   b) 2.7 km   c) 2 km   d) 8½ km?

7. a) Darcy lives in an urban community and catches a bus directly from her home to her school. She lives 5.5 km from the school and it takes her 12 minutes to arrive at school.
      (i) Calculate the average speed of the bus to school.
      (ii) What factors may influence the average speed of the bus compared to the maximum speed limit of 50 km/h on suburban streets?
   b) Ollie lives in a rural community on a farm 135 km from his school. He leaves home at 7am to arrive at school at 8.40am.
      (i) Calculate the average speed of his journey.
      (ii) If he walks 1.5 km to the bus stop each day at a rate of 6km/h, calculate the percentage of his total travelling that he spends walking.

8. On 14th October 2012 Austrian Felix Baumgartner performed the highest sky dive ever of 38 969.3 metres, breaking the speed of sound at one point of the freefall jump (no parachute). If he freefell for 4 minutes and 19 seconds a distance of 36 402.6m, calculate his average speed during freefall in m/s and km/h. You can view videos of this record breaking jump on the internet.

Extended Problem Solving – Wacky races

Equipment: measuring equipment (tape measure/trundle wheel), copy of table below, stop watches, markers for start and finish line, ties for three legged race and any other equipment needed to have different types of movement for the race

1. With the help of your teacher find a grassed area within the school grounds of approximately 30 - 50 metres where you can hold the races.
2. Measure out the track and clearly mark the start and finish lines.
3. Choose two class members to time the races with a stopwatch and a class member to record the results.
4. Individually or in pairs choose a wacky movement to use on the track. Take turns to perform different wacky races, timing and recording the results.
5. Return to your classroom to calculate the speeds of the different movements by class members in the table below.
6. Comment on the speeds of each movement, comparing the fastest and slowest speeds. How do these compare to a typical running or walking speed.

Name	Type of Movement	Time	Speed m/s	Speed km/h

45 Reaction times

Materials required: Rulers, paper, pencil

Warm-up

1.	For the set of four numbers {3, 6, 4, 7}, find the mean.	2.	For the set of four numbers {3, 6, 4, 7}, find the median.
3.	Express $\dfrac{12}{50}$ as a percentage.	4.	Express $\dfrac{10}{25}$ as a percentage.
5.	150 – 96 =	6.	120 – x = 32 x = ?
7.	Find the speed of a car travelling 30km in 15 minutes.	8.	Find the speed of a car travelling 180km in 2 hours.
9.	10 km = _____ m	10.	0.035 m = _____ mm

Activity: How fast is your reaction time?

You will be required to work in pairs for this activity.
1. Ask your partner to hold out their hand in front of them, with their thumb and first finger about one centimetre apart.
2. Hold a ruler above their hand, with the bottom of the ruler between your partner's thumb and finger so that the '0' on the ruler is level with the top edge of their thumb and finger.
3. Drop the ruler without warning your partner, your partner needs to catch the ruler with their thumb and finger as quickly as possible.
4. Record the measurement where they caught the ruler at the top of the thumb (in millimetres).
5. Now repeat the experiment four times recording the results each time.
6. Calculate the mean of the five results.
7. Repeat the experiment again with your partner using their other hand. Record the mean result.

Reaction time – the lapse of time between brain stimulation and the beginning of response

A person's reaction time is influenced by the time taken for three different actions to occur:
1. Information passes from the eyes to the brain.
2. The brain processes the information: first "the ruler is falling", then "catch it!"
3. The brain sends a signal to the fingers to catch the ruler.

Reaction time plays an important role in the stopping time of automobiles in avoiding road accidents. The delay between seeing the brake light in front of you and the time to hit the brake on the car is due to reaction times.

Exercises

1. The ruler in the introduction is falling due to gravity and gives you a reaction *distance* measurement. This can be converted to a reaction *time* using the formula:

    $t = \sqrt{\dfrac{2 \times d}{9.8}}$ where t = time (in seconds),

    d = distance that the ruler fell (in metres, divide your mm results by 1000).

    Using the formula above calculate your mean reaction *time* for each of your hands.

2. Traditionally the average reaction time of a normal driver is 0.5 seconds.
    a) Compare your reaction time to the average reaction time of a normal driver.
    b) Tabulate the reaction time of your class members in an appropriate table.
    c) Calculate the range, mean, median and mode of the class's reaction time.
    d) Write a sentence comparing these results to the average reaction time.
    e) Discuss with your class how reaction times could differ. Can you improve your reaction time with practise? Is your reaction time different at various times of the day? What happens if you're talking to someone when the ruler is dropped?

3. The table below shows the average stopping distance for a car travelling at various speeds in wet weather. The reaction distance is the distance between when you realise you need to stop until you apply the brakes. The braking distance is the distance the vehicle travels from when the brakes are applied to when the vehicle stops.

Speed (km/h)	60	70	80	90	100	110
Reaction distance (m)	11	13	15	17	19	21
Braking distance (m)	21	29	38	48	59	71
Total (m)	32	42	53	65	77	92

    a) Find the total stopping distance for a vehicle at 110km/h?
    b) As a vehicle increases its speed what affect does this have on a car's total stopping distance?
    c) A car travelling at 60km/h has a total stopping distance of 32m. What percentage of this is the reaction distance?
    d) Calculate the percentage of reaction distances in the total stopping distances for the other speed categories in the table.
    e) The speed limit in a school zone is 40km/h during school drop off times. Based on the information in the table, predict the reaction distance of a driver who is travelling 40km/h.

4. The Department of Transport recommends that a driver uses the 2 second rule in fine conditions to ensure a safe driving distance from the vehicle in front. The 2 second rule requires you to choose a landmark that the car in front of you is about to pass, for example a tree or lamp post, then count one thousand two thousand. The car you are driving should then pass the landmark chosen.

    a) If you count fewer than two seconds when driving, should you increase or decrease your speed to maintain a safe stopping distance?
    b) If a car is travelling 50km/h, what is its speed in metres/second?
    c) How many metres away is the car in front of you if you are following the two second rule at a speed of 50 km/h?
    d) As a car increases in speed, how does this affect the distance between the cars if using the two second rule? Explain your reasoning.

5. When a car travelling down a straight road meets a hazard, the time between when the driver hits the brakes and the car comes to a stop is called the *braking distance*. The braking distance, *d* metres, of the car is directly proportional to the square of the speed of the car *u* m/s and this can be given by

$$d = \frac{u^2}{20}$$

    a) Using this formula, complete the missing values in the table below to the nearest metre or metre per second.

u	13	18		25	
d	8.45		18		45

    b) Plot these points on a graph with axes $0 \le u \le 35$ and $0 \le d \le 60$.
    c) Analyse the pattern you see in the graph and hence comment on the relationship between speed and braking distance.

Extended Problem Solving
Create a road safety poster on factors that can affect your stopping distance when faced with a hazard

Factors other than reaction time can affect the stopping distances of vehicles. Braking distance can be affected by:

- your experience and age

- the condition and braking capacity of your car

- the condition of the tyres

- the weather conditions

- road surface

1. Choose one of the factors affecting stopping distances to investigate further.

2. *Gather information* from a range of sources about the factor you have chosen.

3. *Analyse the information* you have found. Use *an appropriate mathematical method* to describe what affect this factor has on stopping distance.

4. Do you think your results are *reasonable*? Discuss your results with the class; adjust your findings if necessary.

5. Write down *the key information* you have found in your research and analysis.

6. Combine your results with other class members to create a road safety poster to *communicate* the risks that can affect your stopping distance when faced with an upcoming hazard.
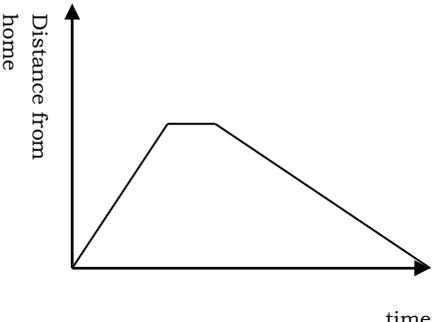
46 Distance-time graphs I

Warm-up

1.	0.25h = _____min	2.	0.75hr = _____min
3.	Simplify 15:20.	4.	Reduce 200kg by 10% then reduce the result by 10%.
5.	Share $20 in the ratio 1:4.	6.	Share $100 in the ratio 2:3.
7.	Represent 8.30am in 24 hour time.	8.	Represent 10.15pm in 24 hour time.
9.	How many minutes are in 3 hours?	10.	How many hours in 210 minutes?

Activity: Matching a graph to a story

Work in pairs to match each graph with its story. Give two reasons to support why you are matching each graph. Be prepared to share these with the class.

| Story	Graph
I walk at a constant speed from my house to the shops, stay there for a while, then return home at the same speed.	
I walk to the bus stop and wait for the bus that then takes me to school.	
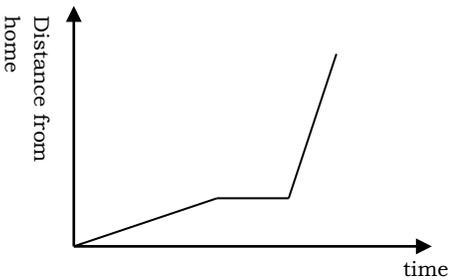
I jog to catch up with my friend at her house, she is not home so I walk home.	
I walk away from home at a steady pace up a hill and then down again.	
I walk to the bus stop and realise that I left my phone at home so I jog back home and then jog back to the bus stop. I realise I have missed the bus so I walk home.	
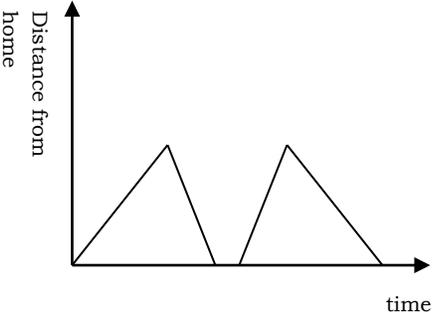
A *distance versus time graph* is a graph used to show how the distance of an object from a specified place varies over time. The slope of the graph gives information about the speed of the object – the steeper the graph, the faster the object is moving. A horizontal section of a travel graph tells us that the object is not moving i.e. is staying at a constant distance as time progresses.

Time is the *independent* variable in a time versus distance graph and hence should always be on the horizontal axis. The *dependent* variable, distance, is placed on the vertical axis as shown in the introductory activity.

Exercises

1.   Match the graph with the appropriate story
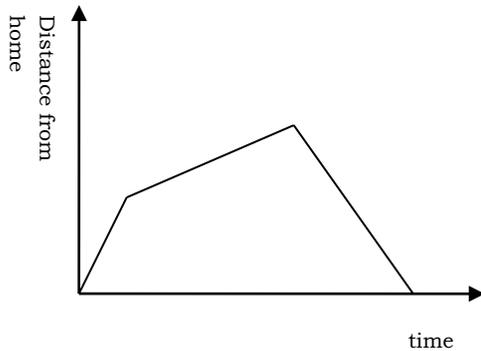
A. Bob took his dog for a walk to the park. He set off slowly and then increased his pace. He then turned around and walked home at a steady pace.

B. Bob went for a jog, he came to the beach and his pace slowed down, then he jogged at a steady pace home.

C. Bob rode his bike East, he came to a steep hill, he slowed down at the top of the hill then rode down the other side.

2.   Match the graph with the appropriate story
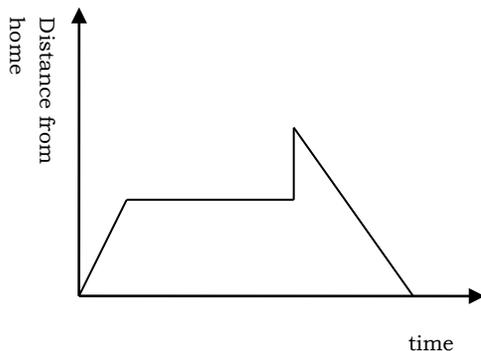
A. Bob took his dog for a walk to the park. He sat down on park bench. He then stood up and walked home at a steady pace.

B. Bob walked to his friend's house where he stayed for a visit. He then borrowed his friend's bike and road the downhill journey home.
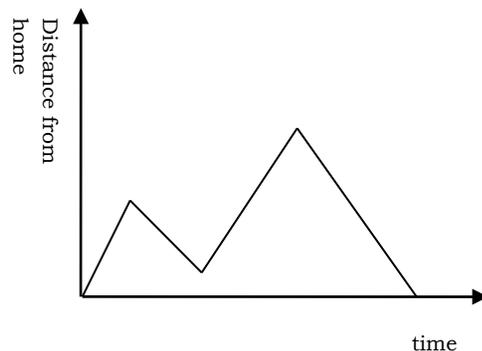
C. This graph is impossible because Bob cannot be in two places at once.

3.   Write a story of your own to match the distance time graphs below:

a)
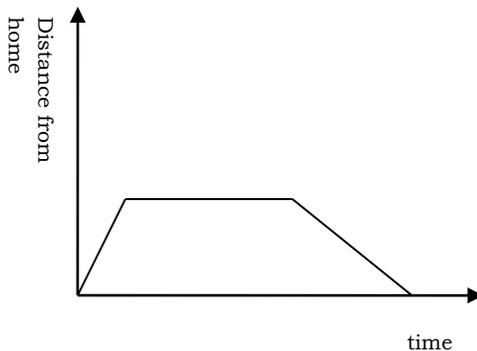
b)

4. Draw a distance time graph to represent:
   a) Jet skateboarded from his house to the skate park, where he stayed for a while. He then walked home afterwards.
   b) Bree walked to the bus stop where she waited for the bus to take her to the city.
   c) Leanne went for a jog away from home and slowly decreased her pace as she got tired.

5. When riding a BMX bike on a race track, the speed of the bike will increase on the straights and decrease around corners. The journey of the winner of a race in a community BMX club has been recorded on the distance-time graph. Use the information in the graph to draw a sketch of the BMX course.
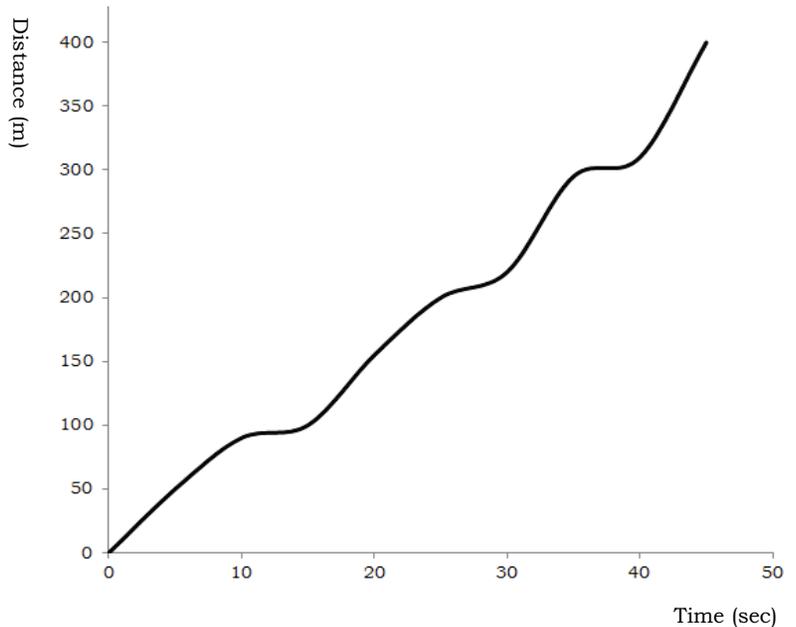
6. Footballers, Jack, Ryan and Sam were required to complete a 5 km race as part of the trials for their community development squad. Use the information in the graph below to describe the journey of each footballer during the race.
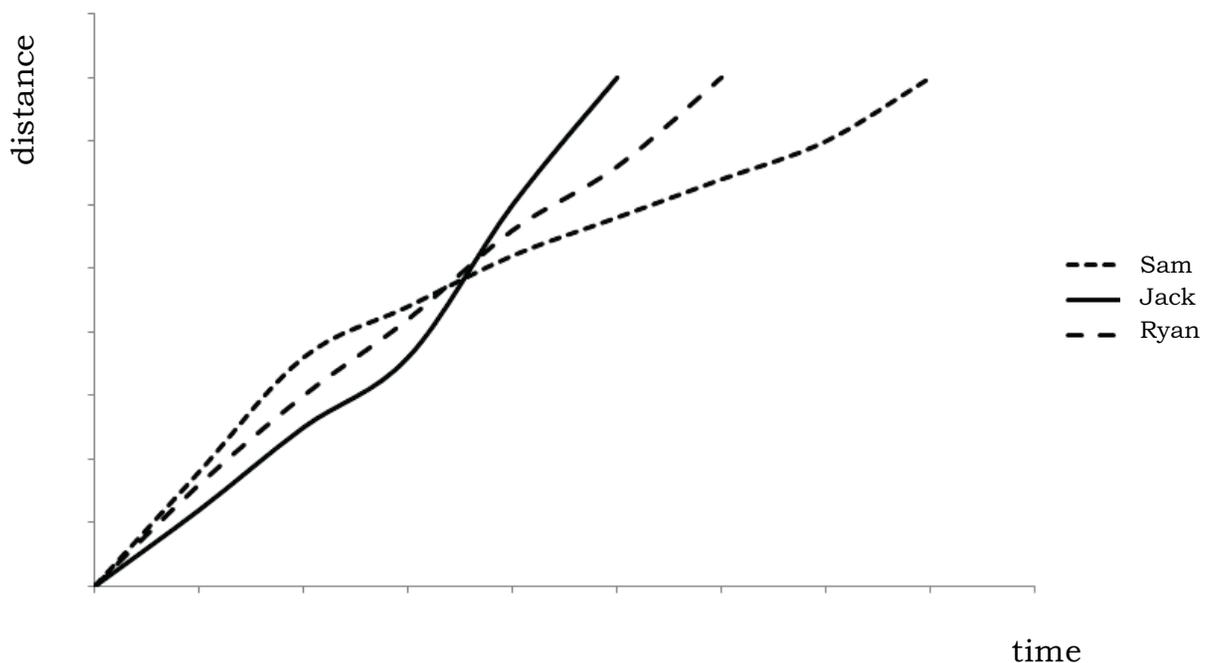
Extended Problem Solving: Formula One Racetracks

The diagrams below show Formula One racetracks throughout Asia and Australia. By assuming that the race cars will travel slower on the bends and faster on the straights, create a distance-time graph to represent the journey of a driver finishing a single lap of each course.

ALBERT PARK, MELBOURNE
Circuit: 3.295 miles / 5.303 km
58 laps

Hill
Ascari
Waite
Senna
Stewart
Clark chicane
Turn 9
Prost
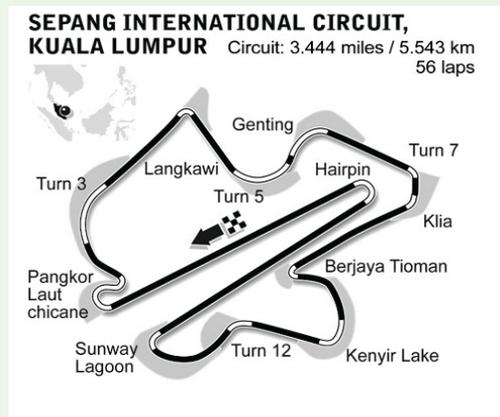Lauda
Marina
Brabham
Hellas corner
Jones
Whiteford
Sports Centre

SUZUKA INTERNATIONAL RACING COURSE, SUZUKA-CITY
Circuit: 5.451 km / 3.387 miles

Spoon
Degner 2
Turn 2
Degner 1
130R
Hairpin
S Curves
Dunlop
Turn 1
Casino Triangle

KOREAN INTERNATIONAL CIRCUIT, YEONGAM
Circuit: 3.49 miles / 5.621 km
55 laps

6
7
8
5
4
3
2
9
Turn 1
10
17
18
12
11
13
14
16
15

SHANGHAI INTERNATIONAL CIRCUIT
Circuit: 3.387 miles / 5.451 km
56 laps

Turn 1
2
3
5
4
7
16
6
8
15
9
12
10
11
14
13

SINGAPORE STREET CIRCUIT
Circuit: 5.067 km / 3.14 miles
61 laps

2
3
Turn 1
4
8
7
6
9
5
10
15
19
20
23
12
14
17
22
21
13
18
11
16

SEPANG INTERNATIONAL CIRCUIT, KUALA LUMPUR   Circuit: 3.444 miles / 5.543 km
56 laps

Genting
Turn 7
Langkawi
Hairpin
Turn 3
Turn 5
Klia
Pangkor Laut chicane
Berjaya Tioman
Sunway Lagoon
Turn 12
Kenyir Lake

47 Distance-time graphs II

Warm-up

1.	If I travel at 60km/h, how far do I travel in 1½ hours?	2.	If I travel at 110km/h, how far do I travel in 3.5 hours?
3.	0.5 minute = _____ seconds	4.	15 seconds = _____ minute
5.	Simplify 10:60	6.	Simplify 10:65
7.	Express 5.5% as a decimal	8.	1/3 hour = _____ minutes
9.	1 fortnight = _____ days	10.	½ year = _____weeks

Introductory problem

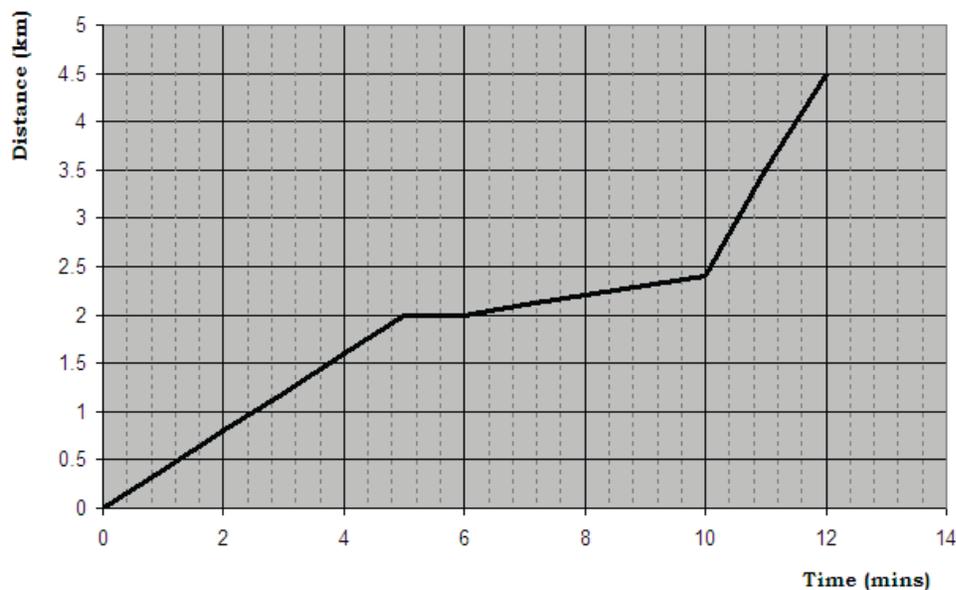The graph below shows Johnny's journey cycling to school. The time in minutes is the time since leaving home and the distance in km is his distance from home.

a)   How long does it take Johnny to get to school?
b)   How far is it from home to school?
c)   What is the average speed of Johnny's journey to school?
d)   It is known that he crosses an intersection where there are traffic lights. How far are these lights from his home? What was the length of time he was delayed at the lights?
e)   If his friend's house in on the way to school and 3km from his own house, how long did it take Johnny to reach his friend's house?
f)   How far did he cycle:
    (i)   in the first 5 minutes?
    (ii)   between the 6th and 10th minutes?
    (iii)   between the 10th and 12th minutes?
g)   Johnny has a hill on his journey to school. When does Johnny reach the top of the hill?

Calculating speed, distance and time from a distance vs time graph

How did you go in the introductory activity? By the inclusion of a scale on the distance time graph axes we can give a more accurate description of an object's journey, for example the duration of time it has travelled, the distance of travel and with this information we can calculate the object's speed.

For example: The graph below shows Jane's journey to the bus stop one day.

Time (sec)
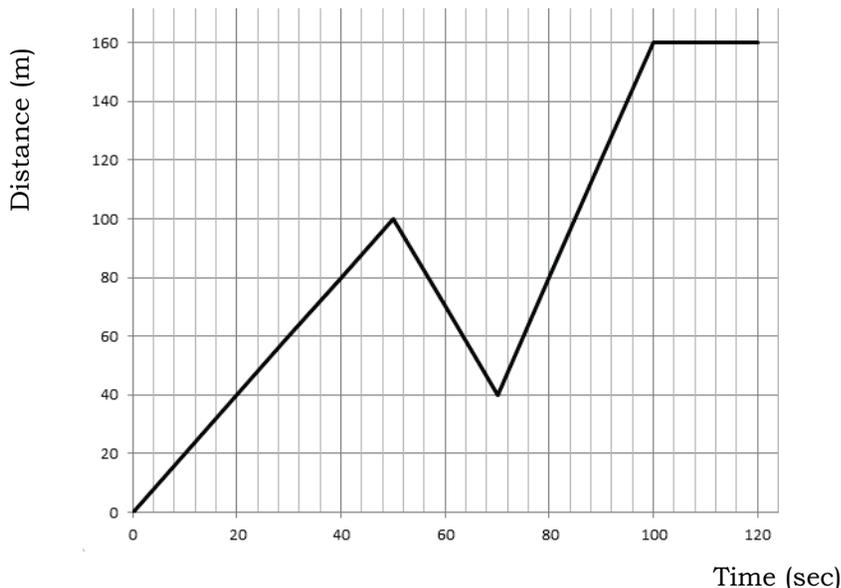
a)   How far is Jane's house from the bus stop?

Reading from the vertical axis Jane's house is 160 metres from the bus stop

b)   How long did Jane take to reach the bus stop?

Reading from the horizontal axis it took Jane 100 seconds to reach the bus stop, 1 minute and 40 seconds.

c)   What is Jane's average speed in m/s for the entire journey?

Average Speed = the total distance travelled ÷ total time taken

= 160m ÷ 100sec

= 1.6 m/s

d)   Describe each section of Jane's journey, including the speed and direction.

Section 1 – Jane walked away from home 100m in 50 seconds, hence a speed of 100 ÷ 50 = 2m/s

Section 2 – Jane walked towards her home 60m in 20 seconds, hence a speed of 60 ÷ 20 = 3m/s

Section 3 – Jane walked away from home to the bus stop a distance of 120m for 30 seconds, hence she travelled at a speed of 120 ÷ 30 = 4m/s

Exercises

1. Copy and complete the tables below for the distance-time graphs shown where distance is in metres and time is in minutes.

a)

Time	Distance
0	
1	
2	
3	
4	
5	
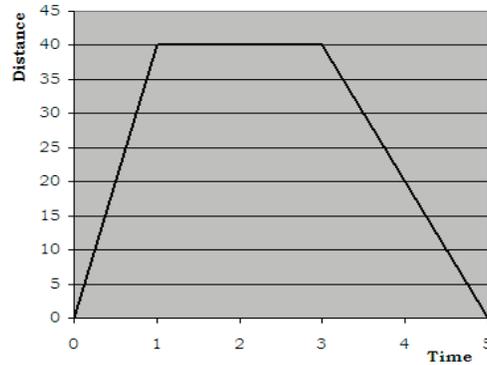
b)

Time	Distance
0	
1	
2	
3	
4	
5	
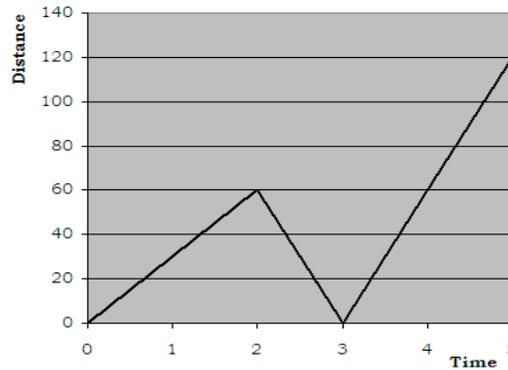
c)

Time	Distance
0	
1	
2	
3	
4	
5	
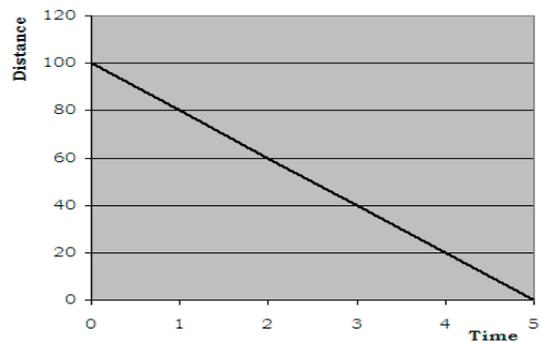
d)

Time	Distance
0	
1	
2	
3	
4	
5	
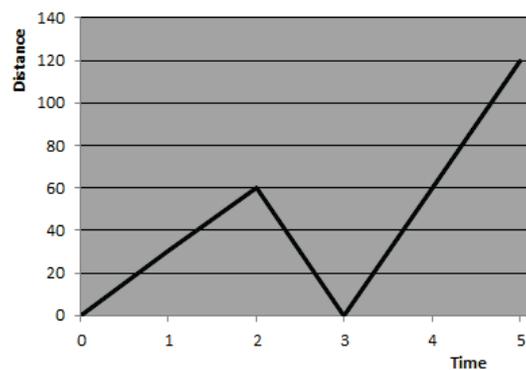
2. The distance-time graph below shows Monica's journey one morning.

    a) How far did Monica travel in total?
    b) If Monica left home at 10.30am, what time period was she at a stop?
    c) If Monica walked for the first twenty minutes, what is her walking speed in km/h?
    d) What time period was she jogging? Explain how you tell this from the graph.
    e) Calculate Monica's average speed.
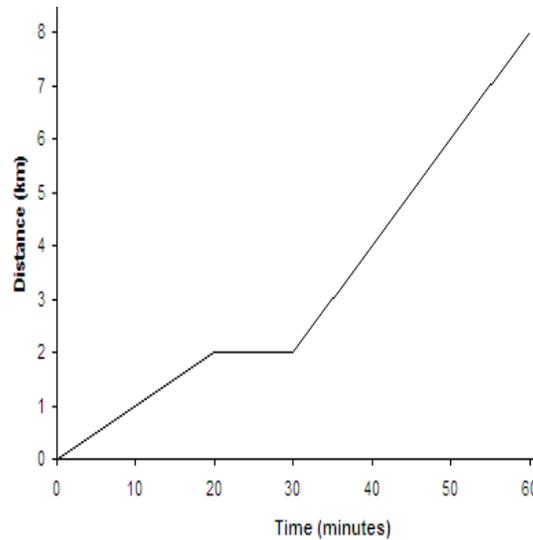
3. In 2009, Usain Bolt broke his own 100 metre world record in a time of 9.58 seconds. The graph of this incredible feat is shown below.
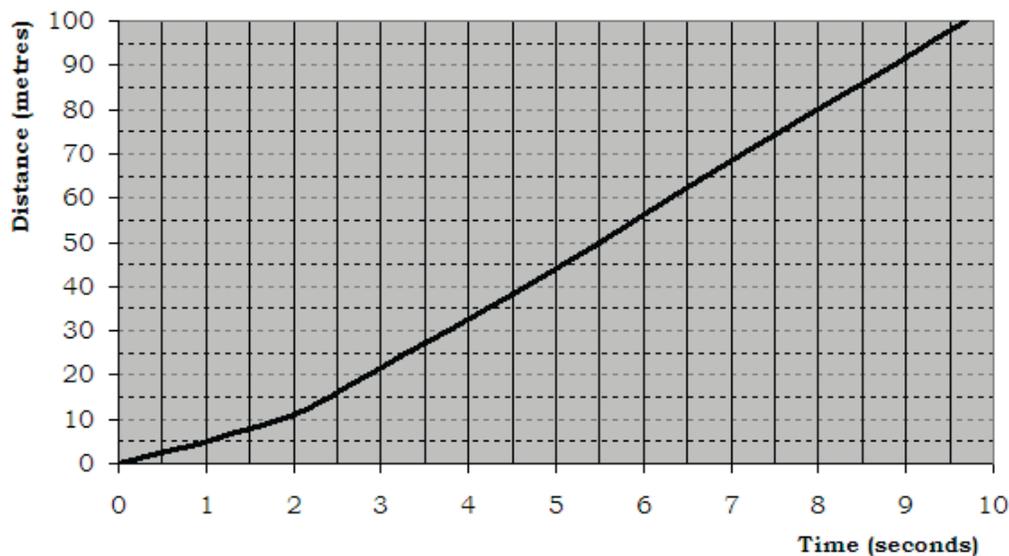
    Use the graph to answer the following:

    a) How far did Bolt travel in the first two seconds?

    b) How long did it take Bolt to reach the halfway point?

    c) Calculate his average speed in metres per second.

    d) Convert this speed to km/h.

4. The graph below shows what happened when three athletes Po, Sue and Fiona ran a 400m hurdle race.
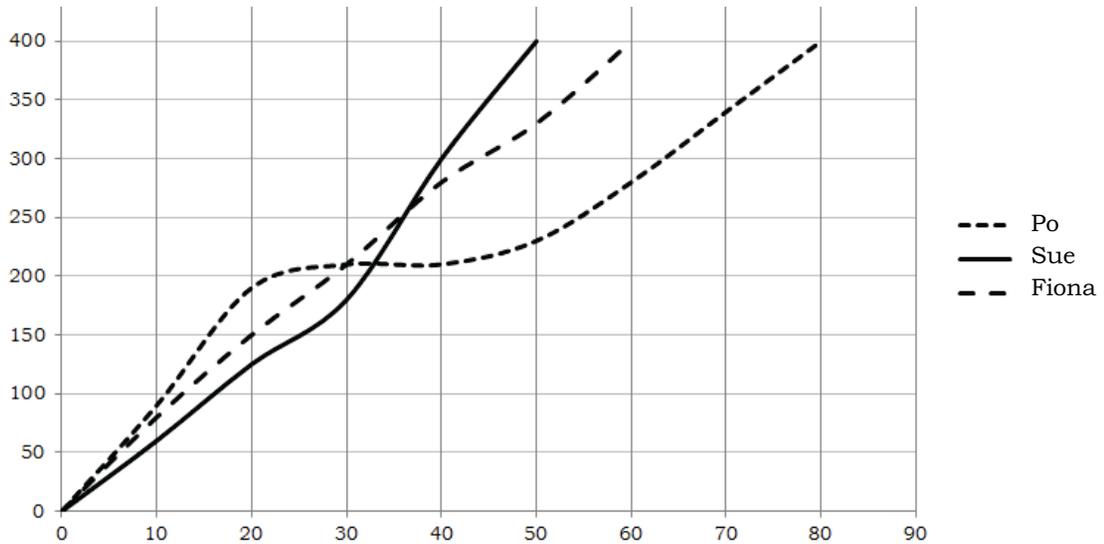
a) Calculate the average speed of each of the athletes.
b) Pretend you are the race commentator and describe what happened during the event.

5. The distance time graph below shows the journey of an aeroplane flying from Perth, Western Australia to Denpasar, Bali, Indonesia.

a) Estimate the distance from Denpasar to Perth.
b) If the flight takes 225 minutes, convert this to hours and minutes.
c) The aeroplane travels at a maximum constant speed between 60 and 180 minutes of the flight. Calculate the speed of the aeroplane during this time period.
d) Describe how the aeroplane's speed may vary throughout the journey.
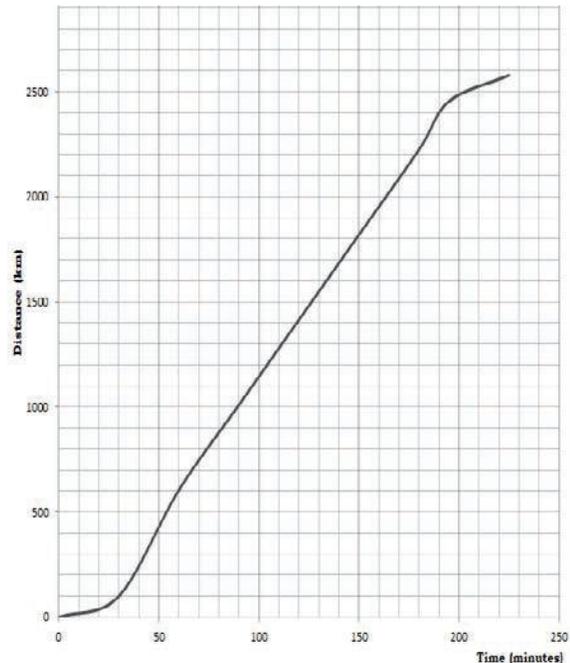
Extended Problem Solving – Wacky Races II
Using the results from your Wacky Races I, draw a distance-time graph for four of the movements on the same set of axes. Comment on your results.

48 Maps

Materials required: Photocopy of Silent Card Shuffle from Teacher's Guide, rulers, lead pencils

Warm-up

1.	1.5 m = _____cm	2.	1.25m = _____mm
3.	3km = _____m	4.	3km =_____mm
5.	2375mm = _____m	6.	320cm = _____m
7.	Round 32.456cm to one decimal place.	8.	Round 0.869 to one decimal place.
9.	Simplify 18:270.	10.	Simplify 14:2.

Activity – Silent Card Shuffle

- Silent Classification - in groups of four, distribute the cards provided to you by your teacher evenly among your group. With *no talking* to your group members, classify the cards into each category provided on the cards with **bold** print.
- Challenge, Justify and Refine - once directed by your teacher, you can discuss and challenge the classification. With whole group approval make any card classification changes.
- Circle and Observe - once directed by your teacher, leave a person in your group as a group representative. The other members then walk around and observe the other students' efforts.
- Return and Refine – return to your group and make any changes
- Solutions – your teacher will give you the solutions and mark your results. Which group achieved the highest score?

Scale Diagrams

A scale defines the ratio of a representation on a diagram, map or plan, to the original object. For example, one unit on a diagram may represent two units on the original object. This means the diagram is *half* the size of the original object. Two units on a diagram may represent one unit of the original object. This means the diagram is *twice* the size of the original object. In the first case, an object has been drawn as a smaller diagram (reduction) and in the second case an object has been drawn as a larger diagram (enlargement).

The ratios would be written in each case as follows:

Scale Diagram : Real Object

1 : 2 *(diagram is smaller)*

2 : 1 *(diagram is bigger)*

Scale ratios can be written in several ways:

i) Fractional scale   **1:1000**   One unit on the scale diagram represents 1000 of the *same* units on the real object.

ii) Written Description   **1cm to 1 km**   *1 cm on the scale diagram represents 1km on the real object.*

iii) Graphic scale
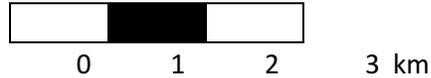
A calibrated bar or line is a graphic representation of scale. This bar measures 4cm; hence 4cm on the diagram represents 3km on the real object.

Exercises

1.   Convert the following written descriptions to a fractional scale in its simplest form.

a)	5 mm to 1 cm	b)	15 cm to 10 mm
c)	4 cm to 1 mm	d)	90 mm to 1 m
e)	4 cm to 1 km	f)	2 cm to 1 km
g)	60 cm to 6 cm	h)	0.5 cm to 2 m

2.
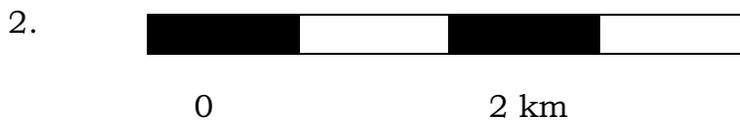
Using the calibrated bar, what is the *real* distance that would have a *map* distance of:

a)   4 cm   b) 1 cm   c) 6.5 cm   d) 12 cm

3.
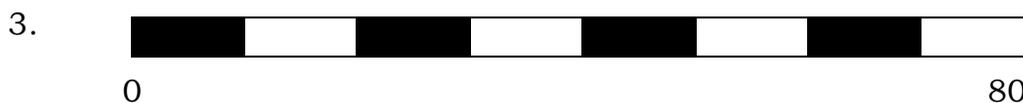
Using the calibrated bar, what is the *map* distance if the *real* distance is:

a)   40 m   b) 10 m   c) 90 m   d) 66 m

4.   A map of Australia has a scale of 1: 20 million. Using this scale what is the *real* distance if the *map* distance is

a)   14 cm   b) 5 cm   c) 8 cm   d) 1.5 cm

5. 1 cm stands for 1km on this map, A and B are two towns near the coast, X and Z are two ships out to sea.
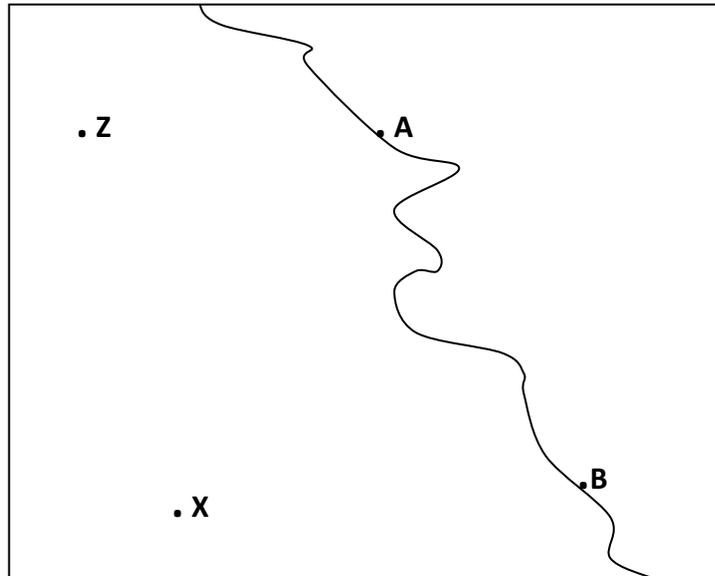
a) Measure the distance to Z and X from A. What is the real distance between each of these ships and town A?

b) Measure the distance to Z and X from B. What is the real distance between each of these ships and town B?

c) Measure the distance from A to B. What is the real distance between these towns?

6. Brad has just returned to Perth after working in Newman 1194km away.

a) Determine a suitable scale for Brad to use so that he can display his journey on an A4 sheet of paper. Justify your choice.

b) West Angelas is an iron ore mine located 128km west of Newman. If Brad travelled an average of 100km/h, how long will it take Brad to go to West Angelas from Newman?

c) Using the same average speed, how long to the nearest hour does it take Brad to travel back to work from Perth?

7.   The scale for this map is shown on the calibrated bar

a)   Measure the distance from Brisbane to Hobart. What is the real distance from Brisbane to Hobart?
b)   Measure the distance from Darwin to Sydney. What is the real distance from Darwin to Sydney?
c)   Measure the distance from Broome to Fremantle. What is the real distance from Broome to Fremantle?
d)   Measure the distance from Canberra to Alice Springs. What is the real distance from Canberra to Alice Springs?

8.   The advertisement right is taken from a real estate agent's listings.

Using a scale of 1:400, draw a diagram of the property for sale. If the property can be broken into house lots measuring 10m x 40m, calculate the number of individual lots that can be created.

FOR SALE BY AUCTION
DEVELOPMENTAL
PROPERTY
located in future city centre
Land area
8180 sq metres

40.7 m street frontage

9.   A map of Fremantle city centre is shown below.

Source: https://archive.org/details/FremantleSocietyNewsletterJune2014
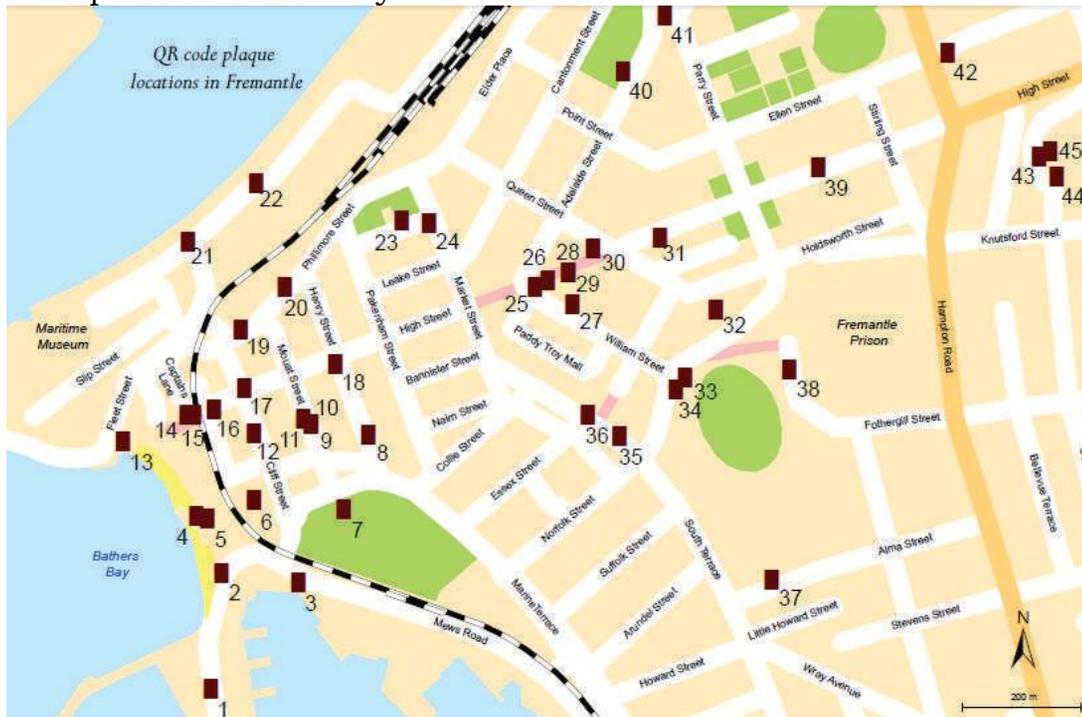
Use the map to calculate the distance from the corner of
Cantonment St and Market St to the Roundhouse (at ■14)
a)   by road.
b)   as the crow flies.
c)   What is the difference between them?

Extended Problem Solving 1 – Street Smart

Materials required: partner, street directory, ruler, pencil

One of your friends (your partner) is going to your house on the
weekend. You are going to see a movie together and so you need to
leave your house by 10am. Your friend hasn't been to your house
before and is a little unsure about how to get there.

Use a street directory or map that includes your street and your
friend's.

Use the map to make your own scale map to show your friend how to
get to your place.

Describe in words the best route to take and show this on your map.

Indicate the time your friend should leave home in order to reach your
house on time.

49 Planning a journey

Materials required: internet access to a mapping application or street directory (photocopy available in Teacher's Guide for introductory problem)

Warm-up

1.	Convert 1400 into 12 hour time.	2.	6 hours is what percentage of a day?
3.	Simplify the ratio 6:24.	4.	Write 7:15pm in 24 hour time.
5.	1cm = 500m represents a fractional scale of 1: ____	6.	1:1000 is a written scale of 1cm = _____m
7.	14 × 1.5 =	8.	18 ÷ 4 =
9.	$5 \times (12 - 8) + 3^2 =$	10.	15% of $300 =

Introductory problem – Helping students get from Central TAFE to a concert in the city

Perth Arena is holding a free concert for students in Western Australia to promote student creativity. The social committee at the Central Institute of Technology is encouraging their students to attend. Your task, as a committee member, is to provide a support service to help students get to the venue. Students may travel by car, walk or catch public transport. The concert is held on a Wednesday night at 6.00pm.

Analyse information from an online mapping service such as Google maps to help you to get the directions from Central Institute of Technology School of Art, Design & Media, 19 Aberdeen St, Perth, WA, to the Perth Arena, 700 Wellington St, Perth, WA.

1. *Travelling by car* – how long does it take to travel by car for this journey? Calculate the distance of this trip and hence calculate the average speed of your journey by car.
2. *Walking this journey* – how long does it take to travel by foot to the Perth Arena? Calculate the distance of this trip and hence calculate the average speed by walking.
3. *The journey by bus* - how long does it take to travel by public transport for this journey? Calculate the distance of this trip and hence calculate the average speed by bus.

Decide how you can communicate your findings to the CIT students. What considerations may be necessary? Would one form of transport be reasonable for all students? Share your findings with the class.

Exercises

1.

Transit Directions to Pristine Station

(A)

Pristine Beach
Marine Parade Australia

(About 13 mins)

1. Head north on Marine Parade toward Magic St   10m

2. Turn right onto Magic St
   Go through 1 roundabout   750m

3. Turn left on Curtin Ave   9m

4. Turn right   140m

5. Turn right   89m

6. Take the pedestrian overpass

(B)

Destination will be on the right   8m

The transit directions left have been taken from a mapping program.

a) Explain what the (A) and the (B) represent in the directions.

b) Calculate the total distance from Pristine Beach to Pristine Station.

c) What is the average walking speed in m/sec used by this application (hint: use 13 mins).

d) If a car travels on average 45km/h, how long will it take to complete this journey by car?

e) Is your answer to d) reasonable? Justify your answer.

2. The table below shows the distance by air between capital cities.

	Perth	Darwin	Melbourne	Hobart	Adelaide
Perth		2890	2708	3325	2118
Darwin	2890		3273	3882	2623
Melbourne	2708	3273		610	643
Hobart	3325	3882	610		1261
Adelaide	2118	2623	643	1261	

a) Plan a journey through each of these capital cities, returning to your starting city and calculate the total distance of travel.

b) If a flight costs the airline 5.5 cents per km per passenger, calculate the cost of a flight from Melbourne to Perth with 128 passengers.

3. A Biology excursion has been planned for the Perth Zoo and Essential Mathematics students have been asked to design the excursion's travel itinerary to arrive by 10am.

    a) Using a mapping application, plan a journey from your school to the zoo. Choose appropriate methods of transport, the time of travel and costs involved.

    b) A map of Perth Zoo is shown below. Use this to plan a trip through the zoo, including ten of the exhibits you will visit in the planned order.

Points of Interest for School Visits
Please photocopy for your students and helpers to use during your Zoo excursion.
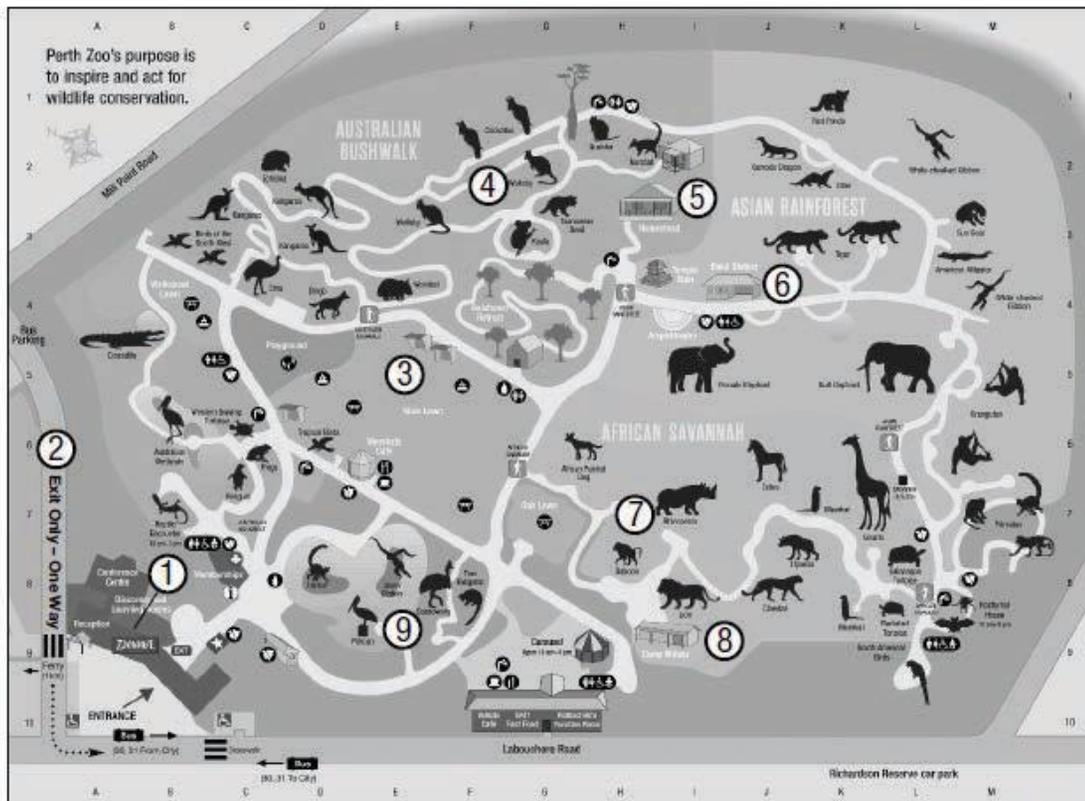
1. Discovery and Learning meeting point
   Meet here 5 mins before your education experience
2. School Bus drop-off/pick-up
3. Picnic Area Shelters
4. Bushwalk Amphitheatre
5. Homestead
6. Bukit Station
7. Painted Dog Hut
8. Camp N'dutu
9. Pelican Pavilion

Extended Problem Solving – A family holiday

Your family is intending to drive around Australia during the summer holidays. Your parents wish to estimate the total distance starting and finishing in your nearest capital, visiting every mainland state's capital city.

They have asked you to determine the direct distance from city to city, by *researching and analysing the information*, and use this to calculate the resulting total distance. *Verify the reasonableness* of your solutions and *communicate your results* in a systematic and concise manner that your parents will easily understand.

50 Shortest path networks

Warm-up

1.	37 + 42 < 72, true or false?	2.	137 > 82 + 34, true or false
3.	2 × 32 × 3 < 3 × 32 × 2, true or false?	4.	0 × 7 = 0 × 12, true or false
5.	0.75 + 0.2 + 1.3 =	6.	18 + 32 + 41 =
7.	2.5 km = _____m	8.	2345mm = _____m
9.	Speed = Distance ÷ ?	10.	Round 45.067 to 2 decimal places.

Activity

Draw a diagram to represent each of the following

1. Hyden is east of Kondinin.
2. Coonana is west of Zanthus.
3. Marchagee is north of Gunyidi
4. Manmanning is south of Cadoux.
5. Highbury is between Narrogin and Wagin.
6. Jazzper's farm house is 600m south of a highway that runs east-west.
7. The Curber Station is 135km east of the Great Northern Highway.
8. A boat sails 30 nautical miles west and then 20 nautical miles south from Sorrento Quay to Rottnest Island.
9. A south west itinerary includes stops at Bunbury, Busselton, Yallingup, Cowaramup, Margaret River and Augusta.
10. A round-the-world itinerary from Perth includes stop-overs at San Francisco, Memphis, Orlando, Birmingham, Paris and Singapore.

Compare your answers with those of a partner.

Networks to represent a shortest path

A network is often used to draw a simplified version of a complicated series of connections, for example the connections between communities or the wiring connecting a group of computers. A network is a *system of points (nodes)* that are connected by *paths (arcs)*.

A *shortest path network* uses the distances and destinations within a map, for example the distance between communities, without the need for a detailed scale. Using a network you can systematically plan a route that contains the *shortest distance* from a *source* (beginning) A to a *destination* B. When we write this route down we call it the *shortest path*.

Example 1. Network Terminology
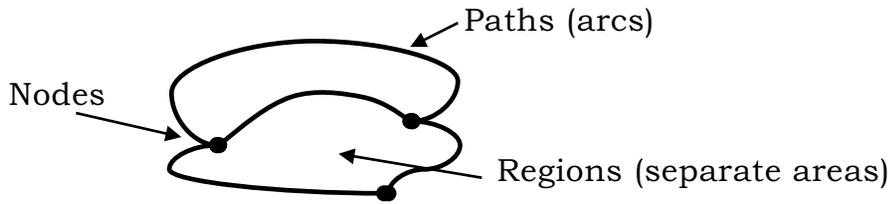
The network above has three nodes, four arcs and three regions.
Note: the outside area is also counted as a region.

Example 2. Using a network to find the shortest path from A to B

Use the network below to find the shortest distance from Bunbury to
Albany. Distances are in kilometres.
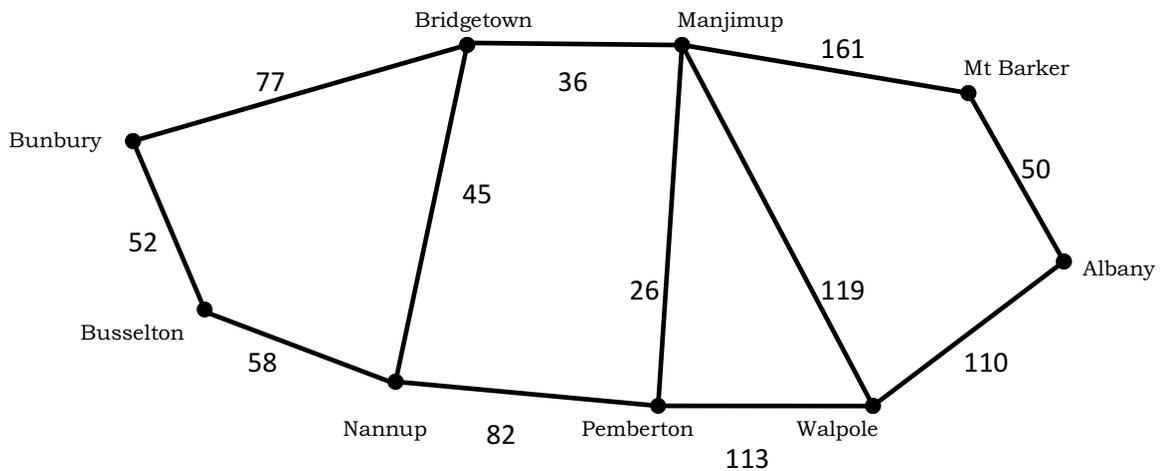
Using a systematic method we find the shortest distance to each node.
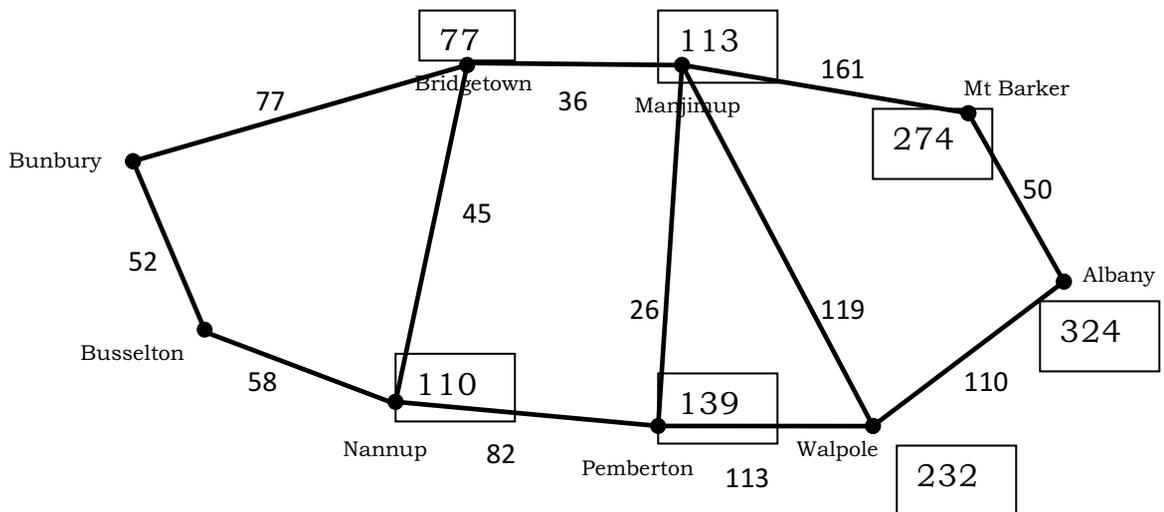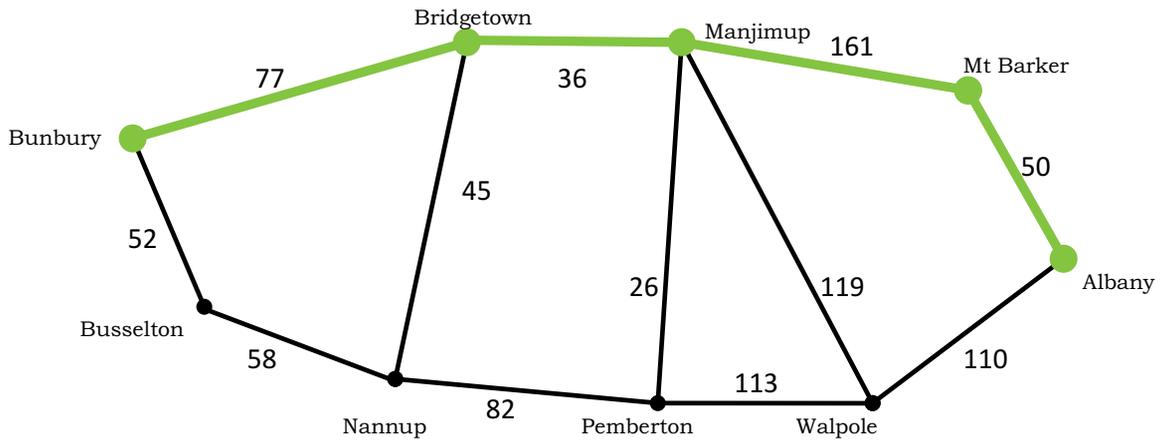
By working systematically through each node in the network, we determine that the shortest path from Bunbury to Albany is: Bunbury, Bridgetown, Manjimup, Mt Barker, Albany and the distance is 324km.
Note: It is not necessary to visit every town
Note: In practice you may use coloured pens to help keep track of your workings on the network.

Exercises

1. The diagrams below represent examples of networks. How many nodes, arcs and regions does each of them have?
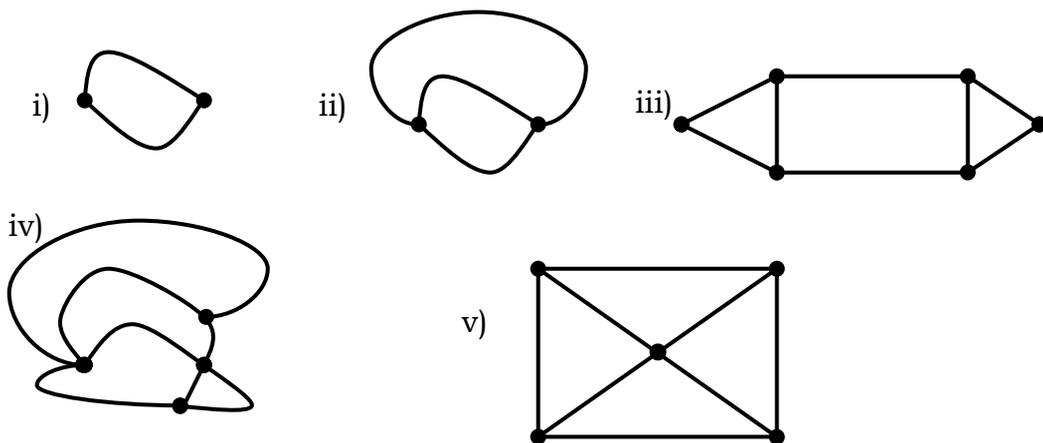
a) Copy and complete the table for the networks above.

Figure	Number of Nodes	Number of Regions	Number of Arcs	Nodes + Regions
(i)				
(ii)				
(iii)				
(iv)				
(v)				

b) From the pattern in the table, determine a rule that generalises the relationship between nodes (n), regions (r) and arcs (a).

2.   The network below shows the possible routes from Melbourne to
      Canberra and from Canberra to Sydney:
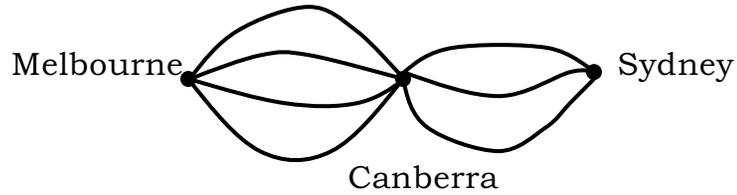
      How many different ways can you travel from:
      a)   Melbourne to Canberra?
      b)   Canberra to Sydney?
      c)   Melbourne to Sydney?

3.   a) What is the shortest
          distance from A to:

          (i) B?   (ii) C?   (iii) D?

      b) List the shortest path from
          A to D.
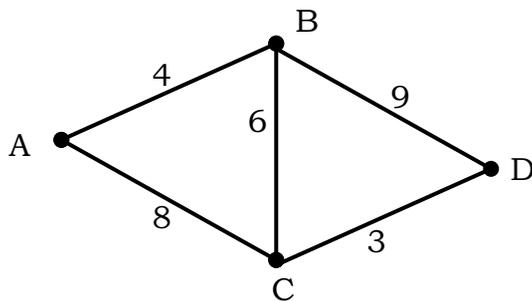
4.
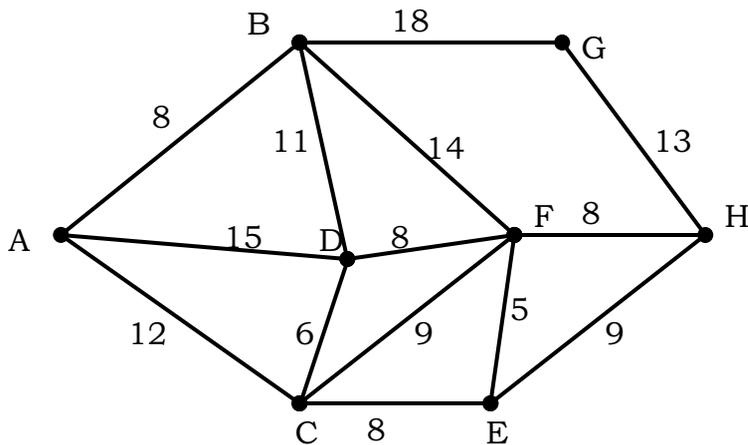
      a)   What is the shortest distance from A to:
            (i)   B?   (ii) C?   (iii) D?
            (iv)   E?   (v)   F?   (vi) G?   (vii) H?

      b)   List the shortest path(s) from A to H

5. Using the systematic method developed in earlier questions, find the shortest distance and path from A to Z for the following networks.

a)

b)

c)

d)

e)
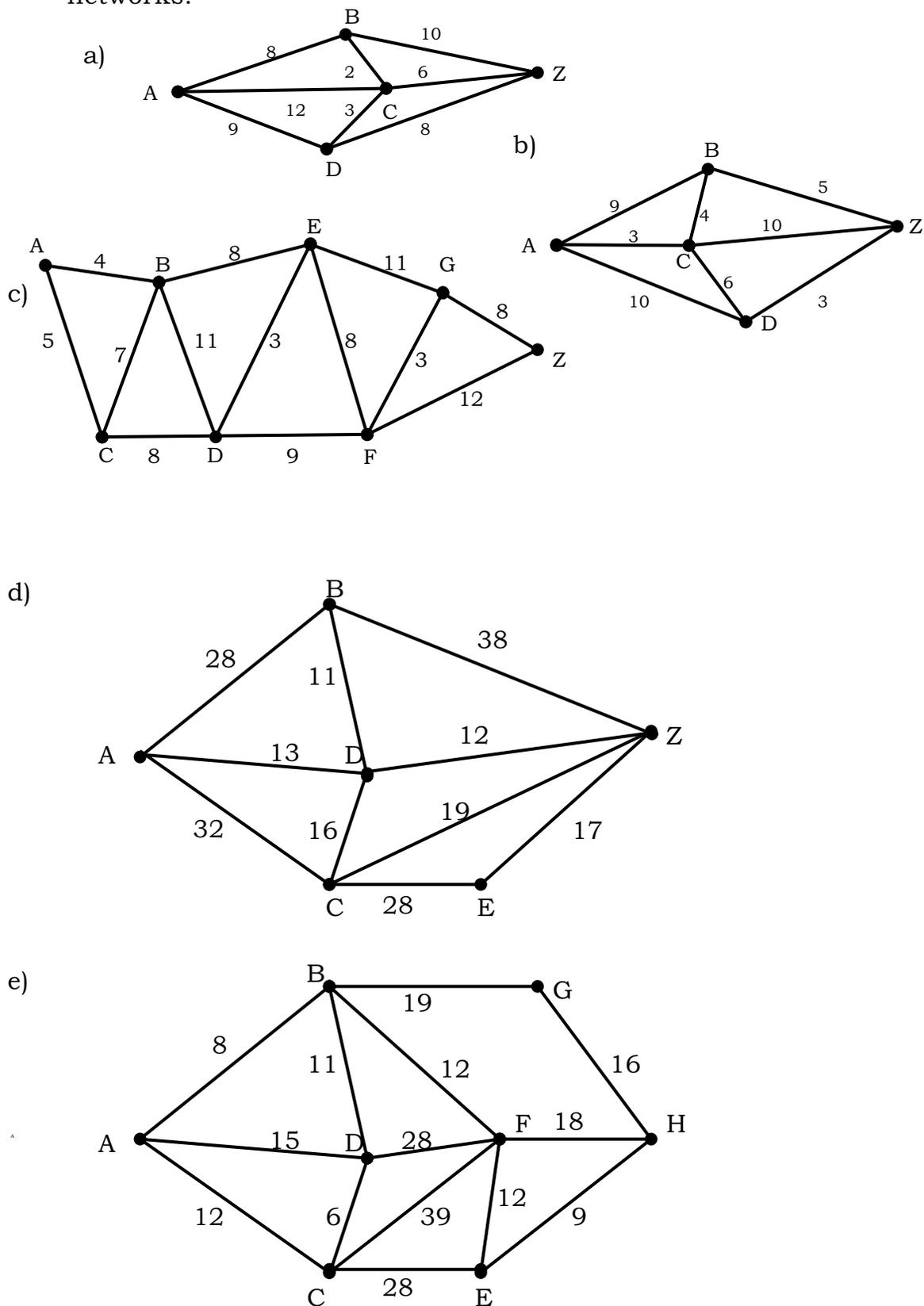
6. Find the shortest path and distance from the community Beacon to Northam. Distances are in kilometres.
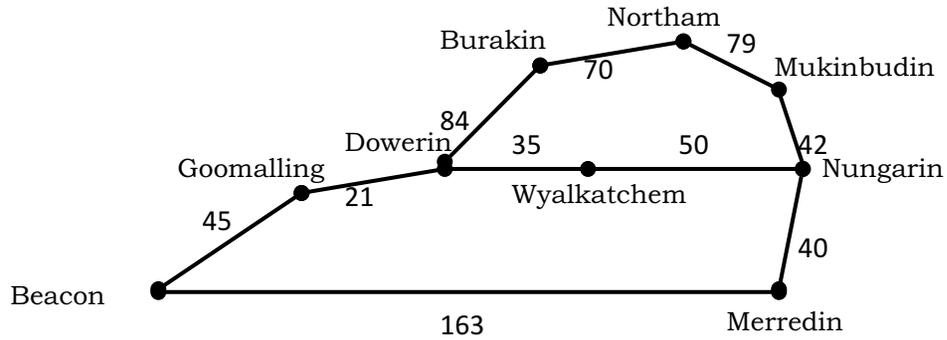
7. The network below represents a network of bike paths on the holiday island community of Rottnest Island.
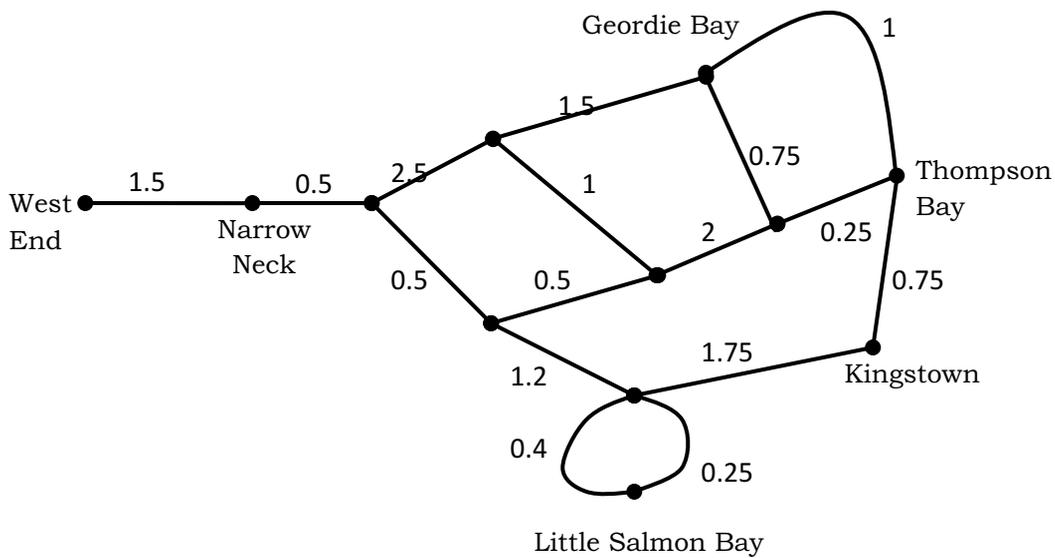
Calculate the shortest distance from West End to:
a) Geordie Bay   b) Thomson Bay

8.
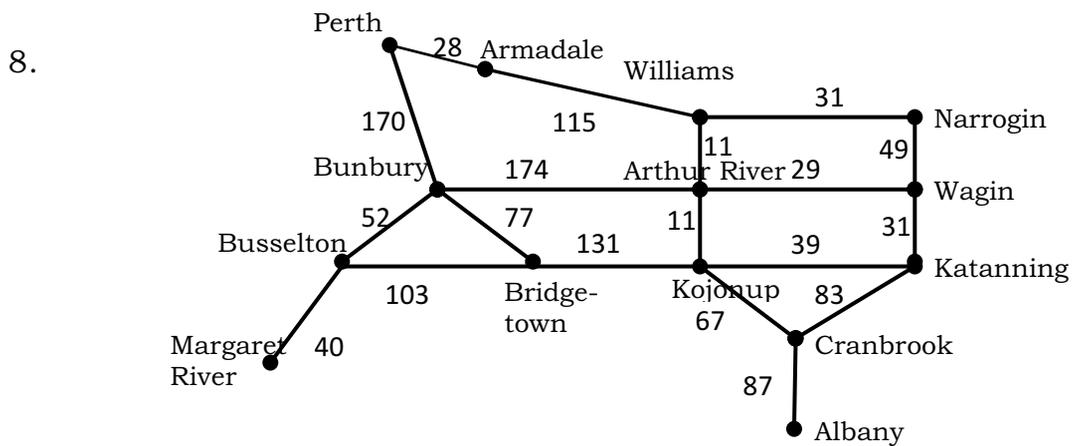
Find the shortest distance and state the shortest route from Narrogin to: a) Perth   b) Margaret River   c) Albany.

Extended Problem Solving – Traversability of Networks

Which of the following networks can you draw without lifting your pen off the paper? Networks that have this property are said to be *traversable.*

i)   ii)   iii)

iv)

v)

vi)   vii)   viii)

ix)

x)
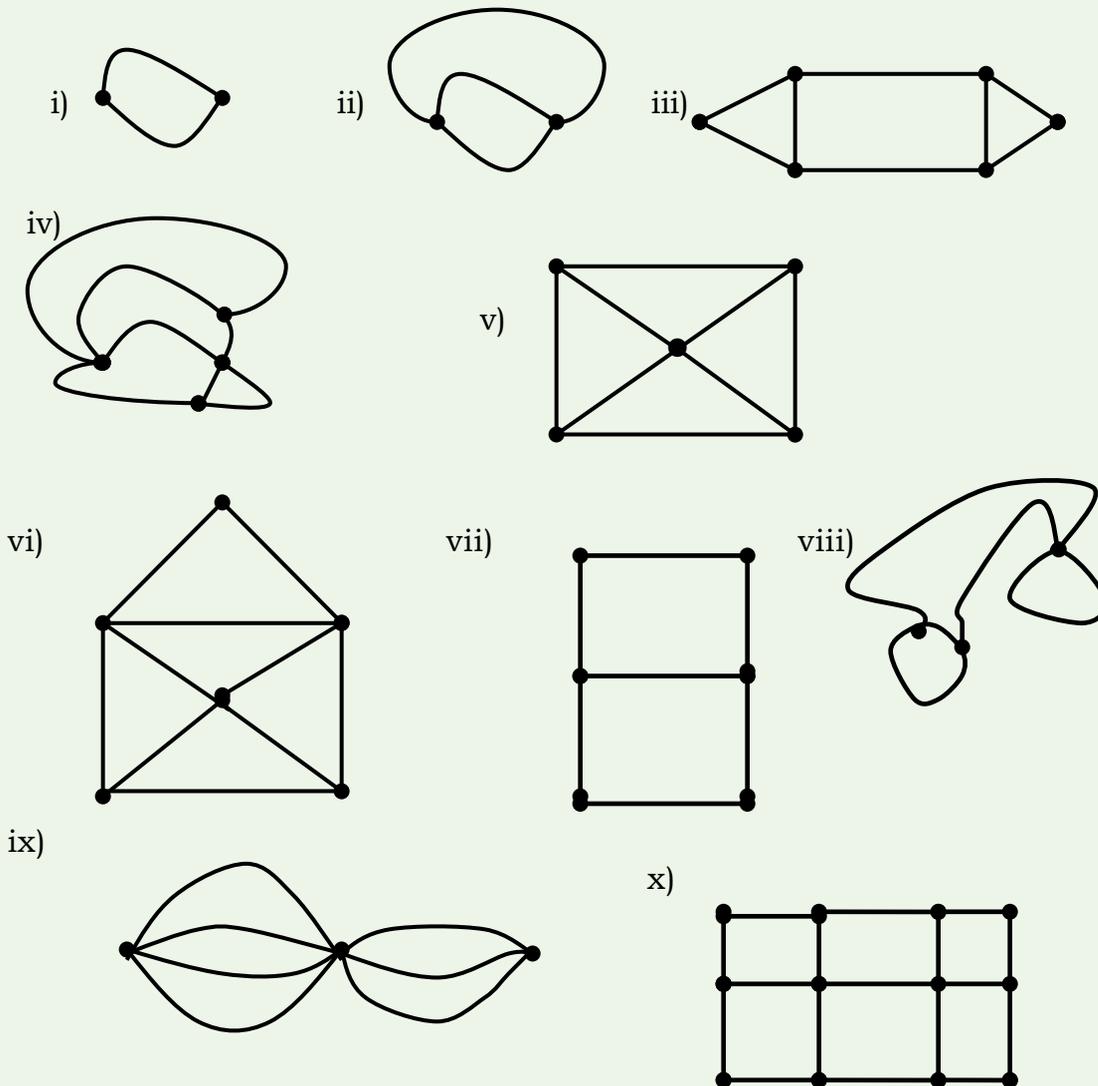
Whether or not a network is traversable depends on the type of nodes that make up the network. Nodes are classified as *odd* or *even* depending on the number of paths entering the node.

Investigate the traversability of networks by counting the number of even and odd nodes. From your findings describe a general rule that could be used to predict the traversability of networks. *Check the reasonableness* of your rule by applying it to other networks.

Answers

What's really neat about mathematics is that even when there's only one right answer, there's never only one right way to do the problem.

- Herb Gross

Unit 1
1 Skills review
1. a) 0   b) 12   c) 4   d) 54   e) 27
       f) 100   g) 0   i) 22   j) 5   k) 48
2. a) $(8 + 3) \times 7 = 77$ b) $24 - 8 \div (3 - 1) = 20$
    c) $36 \div (2 \times 6 + 6) = 2$ d) $45 - 5 \times (16 - 14) \times 2 = 25$
    e) $144 \div ((32 - 8) \times (3 - 1)) = 3$ f) $92 - (2 + 6^2) \times 2 = 16$

3. a) 16   b) 4   c) 36   d) 24   e) 41

4. a) 600   b) 6 400   c) 5 000
    d) 7 000   e) 60 000   f) 3 000 000
    g) 10   h) 20   i) 20
    j) 20   k) 10   l) 300

5. a) 350   b) 90   c) 10   d) 1 000

6. a) 9   b) 29   c) 1 002   d) 100

7. a) 2.3   b) 9.0   c) 11.1   d) 99.4

8. a) 6.67   b) 435.01   c) 208.02   d) 1 999.95

9. 4   10. 4   11. 6   12. 2

13.

Fraction	Decimal	Percentage
$1/10$	0.1	10%
$1/8$	0.125	12.5%
$1/5$	0.2	20%
$1/4$	0.25	25%
$1/3$	$0.\bar{3}$	$33.\bar{3}\%$
$1/2$	0.5	50%
$3/5$	0.6	60%
$2/3$	$0.\bar{6}$	$66.\bar{6}\%$
$3/4$	0.75	75%
$4/5$	0.8	80%
$7/8$	0.875	87.5%
$9/10$	0.9	90%
$11/10$	1.1	110%
$3/2$	1.5	150%
$23/10$ or $2\,3/10$	2.3	230%

2 Thinking mathematically
Answers will vary – discussion should elicit the various responses.

3 Converting units

1. a) 500   b) 0.5   c) 34.5   d) 1 250   e) 1 000
   f) 200   g) 230   h) 1 045   i) 0.58   j) 12.05
2. a) 2000   b) 500   c) 0.85   d) 1.225   e) 0.25
   f) 3.35   g) 500   h) 13 600   i) 1 000 000 j) 0.08975
3. a) 3500mm   b) 1 × 1200mm and 1 × 2400mm
4. a) 3500g apples, 1100g flour = 2 bags, 125g sugar = 1 bag,
      550g butter = 3 containers
   b) 900g flour, 375g sugar, 200g butter
5. a) 11.25L   b) 2 cartons
6. a) 408 800L   b) 304L   c) 194 180 000 000L

4 About you

1. a) Yes (1 000 000 minutes is a little less than 2 years) –
      individual answers will vary
   b) Answers will vary, but will be within two years
   c) Approximately 31 years, 8 months and 9 days approximately
      (days may vary depending on number of leap years included)
2. a) 5 seconds   b) 15 seconds
3. Answers will vary
4. a) b) c)(i) Answers will vary
   c) (ii) lower percentage, although the head would increase
       slightly in weight with general weight gain
   d) Answers will vary
   e) (i) Answers will vary
      (ii) No – there are minor variations, but not in relation to
       height
      (iii) Assuming a similar build, a short person's brain would
       be a greater percentage as they would weigh less.

5 Heart rate

1. Answers will vary, but found by multiplying heart rate by:
   a) 60   b) 60×24   c) 60×24×365
2. a) Answers will vary, but found by multiplying answer to
      question 1 by 70mL.
   b) Answers will vary, 60bpm: 176 722 560L in 80 years
   c) An Olympic sized swimming pool
3. a) 16yo: 204bpm and 197bpm   b) 7bpm
   c) 180bpm and 180bpm   d) 163bpm and 168bpm
   e) As a person ages their maximum heart rate decreases.
   f) Before the age of 40 the first formula gives a higher
      maximum heart rate. After 40, the second formula is higher.
4. a) For a 16 year old, the target range is 102 – 153bpm
   b) 94 – 141bpm   c) 88 – 131bpm
   d) 78 – 116bpm

6 Rates

1. a) 1.8 copies per second   b) 540 copies
   c) 56seconds

2. a) 45km   b) 315km
   c) 1hr 20min   d) 2.5hr   e) ≈17min

3. a) $11.97   b) $5.99   c) ≈2.5kg   d) $4.50

4. 3.07km/sec

5. a) 4.62   b) 26   c) 0.75words/sec

6. 30 seconds and 2.5 hours

7. Answers will vary

8. a) 26.2km/h   b) 10.44m/s = 37.58km/h
   c) (i) 3sec   (ii) 68.7m   (iii) 6.58sec

7 Basic Metabolic Rate (BMR)

1. lowers BMR

2. Answers will vary

3. a) (i) 5855kJ (ii) 6394kJ b) (i) 5766kJ (ii) 6353kJ
   c) (i) 4341kJ (ii) 4107kJ

4. For males, their height, weight and age have a greater impact on their BMR than for females.

5. In general, both increase

6. At or after 18-20 years

7. At 16-17 years

8. a) 173.8cm   b) close – 4cm less than double

9. a) 170.2cm   b) not very close – over by 7.6cm
   Rule of thumb for females is double the height at 18 months

10. 18-20 years as height and weight are at a maximum and these have a greater influence on BMR than age.

11. Answers will vary

8 Total Energy Expenditure

1. 12 084.44kJ

2. 17 666.67kJ

3. 33 853.33kJ

9 Energy from food

1.

	Protein (grams)	Fat (grams)	Carbohydrates (grams)	Energy (kJ)
200mL of coffee, with full cream milk	8.4	9.6	12	702
1 teaspoon of sugar	0	0	4	68
Banana (110g)	1.9	0.1	21.8	406.6
Hamburger w. cheese, onion, tomato sauce	19.8	23.2	59.2	2 201.4
Soft drink (330mL)	0	0	35	595
Garden salad	1.2	0.1	1.5	49.6
2 slices of toasted white bread	6.2	1.3	32.5	706
1 tsp butter	0	5.7	0.1	212.6
1 tsp vegemite	0.8	0	0.6	23.8
fish(grilled)	25	3	0	536
chips (110g)	4	15	28	1 099

2. a)   838kJ   b)   3 895.4kJ   c)   49.6kJ

3. a)   1740kJ, 870kJ, 1740kJ, 870kJ, 3480kJ
    b)   Answers will vary
    c)   Answers will vary

4. Nutrients, balance of food types, allergies and intolerances, health issues (eg diabetes etc) where relevant etc

5. a)

Nutrient	Amount	RDI (%)	RDI	Nutrient	Amount	RDI (%)	RDI
Vitamin K	35.68 mcg	39.6	90.1 mcg	Vitamin B3	2.78 mg	17.4	16mg
Manganese	0.72 mg	36.0	2 mg	Vitamin B2	0.21 mg	16.2	1.3 mg
Fibre	7.58 g	30.3	25.02 g	Molybdenum	6.89 mcg	15.3	45 mcg
Vitamin B1	0.36 mg	30.0	1.2 mg	Zinc	1.64 mg	14.9	11 mg
Copper	0.24 mg	26.7	0.9 mg	Protein	7.38 g	14.8	49.9 g
Vitamin C	19.56 mg	26.1	74.94 mg	Magnesium	53.72 mg	13.4	400.9 mg
Phosphorus	161.17 mg	23.0	700.74 mg	Iron	2.12 mg	11.8	18 mg
Folate	86.78 mcg	21.7	400 mcg	Potassium	373.30 mg	10.7	3 488.8 mg
Vitamin B6	0.30 mg	17.6	1.7 mg	Choline	40.91 mg	9.6	426.1 mg

    b)   10.42 cups of peas

10 Energy

1. 3 333.33 Watt hours

2. a) 40 mins   b) 16 hrs 40 mins
    c) 55 hrs 33 mins 20 sec   d) 119 hrs 3 mins
    e) 2 222hrs 13 mins, 20 sec

3. a) Answers will vary
    b) Answers will vary
    c) Answers will vary

4. a) 1200kJ   b) 480 000kJ

5. a) 60 000kJ
    b) Cycling is much more energy efficient than driving
    c) 28 224 000kJ difference
    d) Where the energy comes from

11 Area

1. a) 200mm^2 b) 20 000cm^2 = 2 000 000mm^2
    c) 115 000cm^2 = 11 500 000mm^2
    d) 25 000m^2 = 0.025km^2

2. a) Answers will vary
    b) 623.7cm^2 c) 436.6m^2 d) ≈20 000m^2

3. 1.8L   4. 533.33mL

5. a) Assuming no rotation or cutting, 9 posters
    b) Assuming no rotation or cutting, 20 posters
    c) Max area covered = 5.8743m^2 (15 small and 3 large)

6. 7.68m^2

7. a) 7.9808m^2 b) 6 packs

12 Volume

1. a) 20 000mm^3
    b) 200 000 000cm^3 = 200 000 000 000mm^3
    c) 1 115 000cm^3 = 1 115 000 000mm^3
    d) 11 250 000m^3

2. a) 90cm^3 b) 64 000mm^3 c) 4.8cm^3
    d) 260m^3 e) 234.375cm^3

3. a) 2L   b) Answers will vary
    c) 1 000m^3 d) Answers will vary

4. a) 20 415.2cm^3 b) 175 760cm^3

5. 963mL

6. a) 18m^3 b) 18 000 000cm^3
    c) 18 000 000mL = 18 000L

13 Capacity

1. a) 0.2L   b) 1.25L   c) 2 500mL
   d) 6.65kL   e) 750L   f) 0.2GL
   g) 200 000mL   h) 756GL

2. a) 2L   b) 50L   c) 380L
   d) 5mL   e) 20-25L medium size
   f) machine 400L, tub 130L   g) 20L
   h) 88L   i) 80L   j) 250mL

3. 40L: 20cm × 40cm × 50cm or others
   65L: 20cm × 50cm × 65cm or others
   80L: 20cm × 50cm × 80cm or others

4. ≈2700mL   anything fairly small can be contained, e.g. chocolate bars, pens and pencils, jewellery etc.
   ≈6100mL   can hold slightly larger items, e.g. small bottles, tools, knitting equipment etc.

5. a) 110L   b) 106.7L to 113.4L
   c) 16.5L

6. 27.14cm (to 2 d.p.)

14 Earning an income

1. a) contract   b) wages   c) contract – piece wise
   d) profit sharing   e) salary   f) fees   g) commission

2. a) Sierra $142.80, Alex $158.10, Ryan $214.20, Emma $326.40
   b) $841.50
   c) Sierra and Alex $15.30, Ryan $21.42, Emma $30.60

3. a) Wayne $25.10   b) Kim $23.20
   c) Pauline $23.60 hence Wayne earns best rate

4. a) (i) $0.0008 or 0.08 cents   (ii) $0.002 or 0.2 cents
   b) (i) 1250 hits   (ii) 500 hits
   c) Unreasonable because you need to have a large amount of hits before You Tube will start to pay you money, rate of money earned on You Tube is not constant. The larger the number of hits, the greater the <u>rate of money</u> earned

5. a) 23.5% above min wage   b) 22% below the min wage

6. a) 905 380   b) 307 830
   c) Youth could be in part-time work, or could be still studying or training to gain a selected career pathway so can distort figures

15 Measurement contracts

1. a) 38cm   b) 110 mm

2. a)

Length of side (n)	1	4	6	11	13.1	25.5
Perimeter (P)	4	16	24	44	52.4	102

   b) P = 4n

3. a) $852   b) $488

4. a) 45cm^2 b) (7x3)+(3x2)+(3x6) = 45cm^2
   c) (i) 78mm^2 (ii) 33cm^2

Type of Patio	Total Area in m^2	Payment per m^2	Total Payment
GABLES	19.8	$35	$693
DOMES		$35	
PYRAMIDS	30.25	$55	$1633.75
FLATS	47.75	$19	$907.25
SOLAR SPAN		$40	
TIMBER LINING	47.75	$66	$3150.84
TOTAL			$6384.84

5. a) (i) 31.4m^2 (ii) 1.8 m^2 (iii) $429.65   b) $166

6. a) 25m^3 b) 1.09 m^3 (2 decimal places)
   c) 22 mattresses   d) 3 trips
   e) Richard should charge $12.00 per mattress.

16 Commission

1. a) $8750   b) $18 000   c) $30 000

2. a) $85.50   b) $760   c) $768
   d) $10 980   e) $3235

3. (i) $300 000   (ii) 2500 software packages

4. a)

Capital City	Median House Price	Commission Earned
Sydney	$673 700	$16 800
Melbourne	$545 000	$1 400
Brisbane	$430 000	$10 800
Adelaide	$395 000	$9 900
Perth	$505 000	$12 600
Canberra	$482 500	$12 000
Hobart	$360 000	$9 000
Darwin	$592 000	$14 800

b)   2003 and 2004 shows that the percentage house change was positive, hence increasing in value, so the agent would earn more money each year. 2005 house prices remained the same and then steadily increased up to 2009 when the market crashed and house prices decreased. This also occurred in 2011 and 2012. Overall, house prices are generally increasing so the commission earned by a real estate agent over this period will also increase.

5. a) $95.75   b) $19.66   c) $57.60   d) $62.85

17 Superannuation
1. a) $93.50   b) $140.26 c) $89.28

2. a)

Earnings (E)	$200	$300	$400	$500	$1000
Superannuation Invested (S)	$22	$33	$44	$55	$110

b)   Superannuation
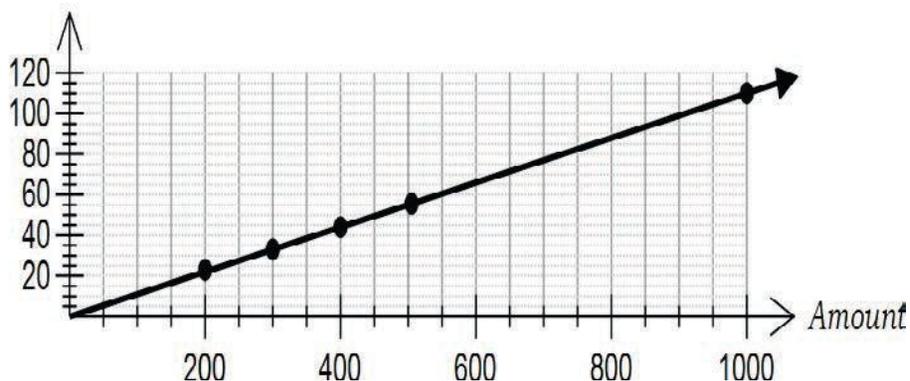
c)   Yes, it is continuous data   d)   S = 0.11 × A

3. D, because if you add 10% you have 110% (1.1) of the original amount

4. $1200   5. $40 200

6. a)   A balanced fund is a medium risk fund suited for most employees not retiring in the near future with expected returns of 7.5% and 4 out of 20 years the fund would expect a loss
    b)   A conservative fund has lower returns of 6.5% and would be better suited for an employee who would be retiring in the near future
    c)   Growth Funds have the most risk associated with them as 85% is invested in shares and properties however can expect a loss every four or five years so suited to an employee who is not retiring in the near future
    d)   A cash based fund is 100% deposited in institutions so is the safest option for a person who does not want to take risk with their superannuation investment

18 Income tax

1. a) $33 056 b) $59 170 c) $94 400 d) $27 196

2. Clancy - $2 822.64   Geoffrey - $10 777.25   Lucas - $22 875 Silas -$1 709

3. a)   Angela - $11 065.60 Rebecca - $11 429.60 Caitlin - $10 368
    b)   Rebecca
    c)   Other income was added from other sources example interest and dividends, deductions were not included
    d)   Caitlin owes $847
    e)   Angela got a refund of $78.60 and Rebecca gets a refund of $2 529.60

4. a) $50 099 b) $48 112 c) $7 183   d)   Yes, he owes $819
    e)   15%

5. a) $10   b) $476.23 c) $63 358 d) $1 968.23
    e)   refund of $883   f) 18%

6.

Name	Taxable Income	Tax Payable	Possible occupation (job or position)
Seb	$3 500	Nil	Student working part time
David	$32 600	$2736	Part-time shop assistant
Sabrina	$53 278	$8 862	Personal assistant
Fatima	$95 760	$23 378	Senior teacher
Tony	$163 000	$48 257	Engineer
Julia	$535 000	$214 297	Bank Manager

19 Miscellaneous

1.

Which part of the whole amount?	Fraction	Decimal	Percentage
$50 out of $200	$\frac{1}{4}$	0.25	25%
$56 out of $112	$\frac{1}{2}$	0.5	50%
$78 out of $780	$\frac{1}{10}$	0.1	10%
$21 out of $2 100	$\frac{1}{100}$	0.01	1%
$12 out of $60	$\frac{1}{5}$	0.2	20%
$1 out of $8	$\frac{1}{8}$	0.125	12.5%
$10 065 out of $100 650	$\frac{1}{10}$	0.1	10%
$32 out of $128	$\frac{1}{4}$	0.25	25%
$90 out of $9 000	$\frac{1}{100}$	0.01	1%
$34.55 out of $69.10	$\frac{1}{2}$	0.5	50%
$400 out of $1 000	$\frac{2}{5}$	0.4	40%
$180 out of $200	$\frac{9}{10}$	0.9	90%
$60 out of $500	$\frac{3}{25}$	0.12	12%

2. $1 050   3. a) $931   b) $1862

4. a) 12%   b) D2 = B2 * C2   c) To sum (total) amounts paid

5. a) $6104   b) $2817.31   c) $1 408.65

6. c) d) and e)   7. $1897.50

8. a) $825.30   b) $2116.80   c) $257.37   d) $1613.54

9. $43 636.36   10. a) $10 575   b) $20 625   c) $27 500

20 Bank accounts

1. a) To check that the amount written is the correct amount.
    b) (i) $305   (ii) $1 732   (iii) $10 415 (iv) $506
    c) (i)   four hundred and twenty eight dollars and fifty cents
        (ii)   twelve thousand and forty dollars
        (iii) eight hundred and sixty two thousand and fifty dollars

2. a) $160.70 b) $100, $50, $10, 50c and 20c   c) $1039.30

3.

PRINCIPAL	RATE	TIME	INTEREST
$500	3%	3 years	$45
$800	3.6%	4 years	$115.20
$2500	2.75%	5 years	$343.75
$1500	3.5%	4 years	$210
$2600	4.25%	5 years	$552.50

4. a) $4.13   b) $9.17   c) $17.20   d) $11.40

5. Tammy has $4 400, Jenny has $3 700 so Tammy has $700 more than Jenny
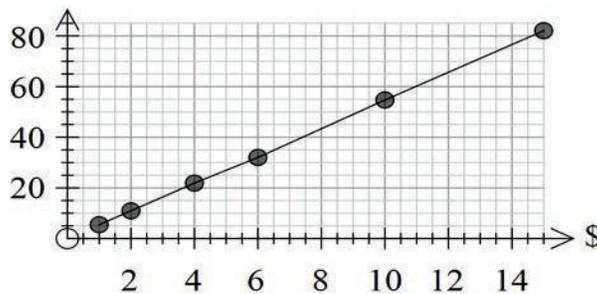
21 Spending and discounts

1. a) End of season sale, staff or loyalty discount, closing down sales
    b) Cheaper car might not have the same features (like leather seats, airbags, tyre quality) as a more expensive car so may not maintain a good resale value; organic produce vs other produce; cheaper clothing could fall apart when worn once whereas better, more expensive, quality clothing can last longer.
    c) Yes it is. If there are cost considerations, it is not always possible to buy the top of the range product, for example with white goods. Consumers should research quality and cost of items and make a choice based on their research.
    d) Locating items in prominent locations, advertising, two for one offers, loyalty point cards

2. a) $26.36   b) $1030.20   c) $225.40   d) $433.13

3. a) skirts $23.20, $43.20, blouses $12.80 each, slacks $28, $47.20 bathers $39.92 shoes $20.80
    b) Total $227.92   c) $56.98

4. a) $461.72 b) $840.99   c) $354.54

5. a) 16%   b)   12%   c)   14%   d)   4.1%
    e) 7.2%

6. 8% discount

7. a) $d = b - 0.15b$ b) $d=0.85b$

8. 50% of 50% = 0.5 x 0.5 = 0.25. Hence 25 % of the original price is the discounted price so a 75% discount altogether not 100%.

22 Best buys

1. a) 1kg of cheese   b) 1kg of margarine   c) 30 cans of soda
   d) 250g of chocolate   e) 4kg of potatoes   f) 1kg of bacon
   g) 60m of foil   h) 1L of choc-milk   i) 1kg of yogurt
   j) 32pack

2. a) 250g   b) 3tins   c) 200g   d) 1litre
   e) 3kg bag f) 24 cans

3. a) Deal A: $843.50 Deal B: $820.00
   b) Deal B is the cheaper option   c) $23.50

4. a) $1.05 per 100 grams   b) $1.68 per 100 grams
   c) $0.200 per 100mL   d) $8.20 per 100grams
   e) $0.125 per 100mL   f) $0.125 per metre
   g) $1.466 per 100grams   h) $4.467 per 100 grams

23 Online shopping

1. a) $36.20   b) $48.07

2. a) Kim $260.59 Suki $280.13
   b) $13.67   c) $25.80

3. a) (i) $25.70   (ii) $11.10   (iii) $118.70 (iv) $7.50
      (v) $6.80
   b)

4. a) $10.55   b) $13.40   c) $16.70   d) $10.55   e) $8.25

5. a)

	Box Dimensions			Volume	Cubic
	Length (m)	Width(m)	Height(m)	(m³)	weight
A	0.1	0.1	0.1	0.001	0.25
B	0.2	0.2	0.2	0.008	2
C	0.25	0.2	0.3	0.015	3.75
D	0.305	0.2	0.35	0.0216	5.4
E	0.35	0.3	0.4	0.042	10.5

   b) (i)   Box A delivered AirMail
       (ii) Box E delivered Express Post International
       (iii) ~ $125 - $10 = $115   (iv) ~ $64.00   (v) Box A
       (vi) Express Post International because it has higher charges
             so you would assume quicker delivery

24 Mobile Phones

1. a) 67   b) 57   c) 23   d) 17
2. Answers will vary
3. a) 86 200   b) 104 550   c) 818 500 d) 49%
   e) Mobile phone ownership increases among children as they get older
   f) Yes you would increase and then should remain fairly constant among the population. It is possible that older people may use their home phone instead of a mobile so may not need one, so mobile phone ownership could start to decrease in older age groups
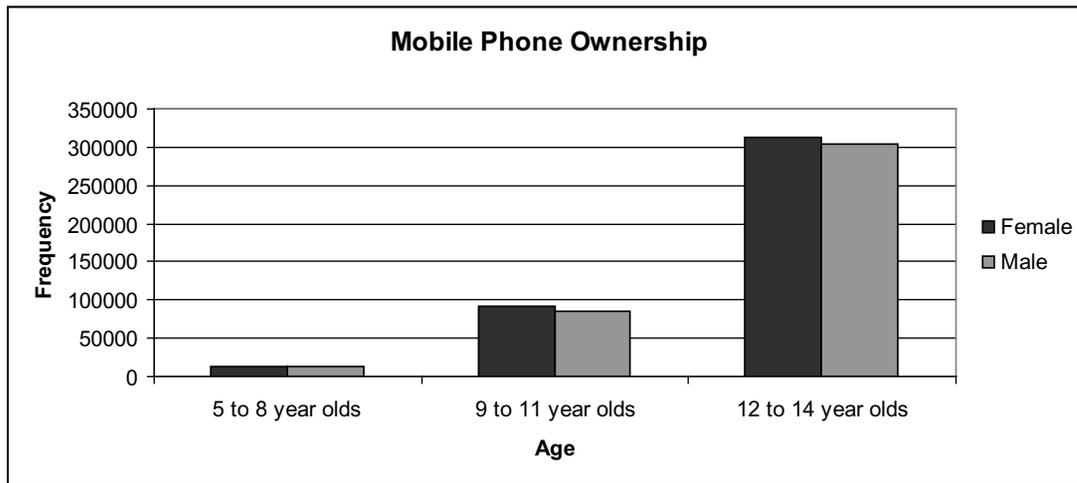

4. The most common amount of phone usage (35%) among Australians is one to three hours per day with a total of 73% of the population using their phone for less than three hours. A small proportion of people (5%) use their mobile phones for more than eight hours a day, 10% use their phone longer than 5 hours. 15% of the population use their phone for three to five hours a day. Other may include people who do not use their phone each day.
5. 12.6%   6. 13731 Terabytes
7. a) Kiribati b) 83 c) 39 d) 2010 – 2012 e) Nauru

25 Budgets

1. Show your graph to your teacher to see if it is appropriate.

2. a) fixed   b) variable c) fixed
   d) variable e) variable

3. a)

Rent & household expenses	Food	Rates & Taxes	Savings	Entertainment	Clothing
$844.20	$562.80	$422.10	$422.10	$281.40	$281.40

b)
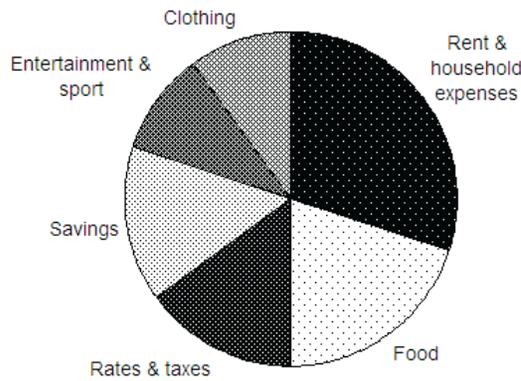
4. a) $839.28   b)   Fuel = $1131 Tyres = $126 Total = $1257
    c) $688.84
    d)   Barina $10153 depreciation so worth $5837 after 5 yrs,
         percentage of original amount = 5837/15990 x100 = 37%
         Festiva $9835.80 depreciation so worth $5654.20 after 5 yrs,
         percentage of original amount = 5624.2/15490 x 100 = 36.3%
    e)   Brand, safety, colour, quality of car…???

5. a)   7714   b) $148.35   c) 56%

6. a) and b)

	A	B	C	D	E	F
1	CLAIRE'S BUDGET 2015					
2						
3	INCOME	MAR	APR	MAY	JUNE	TOTALS
4						
5	WAGES	$3,568.00	$3,780.00	$3,780.00	$3,780.00	$7,560.00
6	GIFTS			$150.00		$150.00
7	SHARES	$160.00	$165.00	$156.00	$150.00	$306.00
8	LOTTO				$30.00	$30.00
9	TOTAL INCOME	$3,728.00	$3,945.00	$4,086.00	$3,960.00	$15,719.00
10						$0.00
11	EXPENSES					$0.00
12	TAX	$532.20	$567.00	$567.00	$567.00	$1,134.00
13	HEALTH INSURANCE	$100.00	$100.00	$100.00	$100.00	$400.00
14	GIFTS	$50.00	$50.00	$50.00	$50.00	$200.00
15	TRANSPORT	$85.00	$85.00	$85.00	$85.00	$340.00
16	FOOD	$400.00	$400.00	$350.00	$400.00	$1,550.00
17	RENT	$280.00	$280.00	$280.00	$280.00	$1,120.00
18	PETROL	$50.00	$50.00	$50.00	$50.00	$200.00
19	HECS	$35.00	$37.80	$37.80	$37.80	$148.40
20	CLOTHES	$200.00	$200.00	$200.00	$200.00	$800.00
21	GENERAL NEEDS	$400.00	$300.00	$350.00	$350.00	$1,400.00
22	CAR REPAYMENT	$335.00	$335.00	$335.00	$335.00	$1,340.00
23	RAC MEMBERSHIP					
24	TOTAL EXPENSES	$2,467.20	$2,404.80	$2,404.80	$2,454.80	$8,632.40
25						
26	SAVINGS	$1,257.80	$1,540.20	$1,681.20	$1,505.20	$7,086.60
27	CUMULATIVE SAVINGS	$1,257.80	$2,798.00	$4,479.20	$5,984.40	

6. continued   c) $7086   d)   April   e)   Food
　　f)   decrease expenses (eg clothes) or increase income
　　g)   No   　　h)   earnings May expenses March
　　i)   earnings $3 728 expenses April and May

7.   a)   Rent/mortgage
　　b)   Rent Mortgage $544.50   Food $247.50   Clothing $82.50
　　　　Transportation $165   Health $82.50   Insurance $66
　　　　Entertainment $231   Other $148.50   Savings $82.50

Unit 2
## 26 Data collection and representation
1.   H.G. Wells predicted over 60 years ago that society would reach a stage where statistics are so prevalent that being able to interpret data and statistics will be a key element of being able to function well in life. Benjamin Disraeli was referring to the misuse of data and statistics and the power of the person presenting the data to manipulate others' perception and beliefs. Lawrence Lowell is referring to the importance of being able to judge the validity and reliability of the statistics being presented. They only have worth if they are a true representation of reality. Miguel de Cervantes is referring to the statistical concept whereby characteristics of a large population can be judged using sampling. In statistics this is a little more complicated than in the novel *Don Quixote*, but the underlying idea is the same.

2.   Answers will vary
3.   a)   Judging student progress, comparing student performance, determining to what extent students have learned and understood the concepts taught.
　　b)   Identifying trends, identifying which parts of a population are more likely to die from different causes.
　　c)   Predicting the future climatic conditions.
　　d)   Determining health of forests, following progress of a disease, animal populations in forests, determining age of a forest, identifying types and ages of wood for primary industries.
　　e)   Determining immigration quotas, identifying services for immigrants that may be required and where to locate them, measuring the contribution to society of immigrant groups.
　　f)   Identifying what factors in schools lead to successful students, predicting future trends and the allocation of resources, determining when and which schools should close and when new schools may be needed.
　　g)   (i)   Providing appropriate and sufficient services, predicting population growth and future population requirements.
　　　　(ii)   Economic predictions, planning future infrastructure needs, evaluating roads and public transport in light of projected growth, evaluating housing supply and demand.
4-7. Answers will vary

27 Types of data

1. Categorical: a, c, f, j   Numerical: b, d, e, g, h, i
2. Examples: year of birth, postcode, year level at school
3. Continuous: a, d, e, g, h, k   Discrete: b, c, f, i, j, l
4. Answers will vary
5. Measuring instruments have a limited degree of accuracy and hence approximate the measurement to that degree. And so have a limited number of possible results.

28 Measures of location

1. a) Median   b) Mode   c) Median
    d) Mean   e) Mode   f) Mean
    g) Median or mean depending on context
    h) Mean or mode   i) Median or mean
2. a) Mean = 17.2   Median = 12   No mode
    b) Mean = 4.55   Median = 3   Mode = 6
    c) Mean = 166.6   Median = 165   Mode = 165
    d) Mean = 1.73   Median = 2   Mode = 2
    e) Mean = 29.1   Median = 18   Mode = 12
3. If the average is the median, then exactly 50% of people are below that average. If the average is the mean, then for large populations it is likely to be approximately 50% of people that are below average, but that may vary, particularly if the data is not symmetrical. If the average is the mode, then the percentage of people below the average will vary quite considerably particularly if the population is relatively small or asymmetrical.
4. Answers will vary, depending on personal speaking rate. Speech rate is the most important factor, but pauses, allowing time in appropriate places for laughter and others are all considerations.
5. a)   (i)   85.7kg   (ii)   1240cm^2/person
        (iii) Each space is 28.47cm wide and 75cm long. People do not stand like this – in a room of strangers, they tend to maximise their distance from everyone near them rather than stand in neat rows. Also, it would be difficult to fit 14 people into the space – the weight allowance is greater than the floor space available. Also, people tend to carry things such as bags which also need to be factored in.
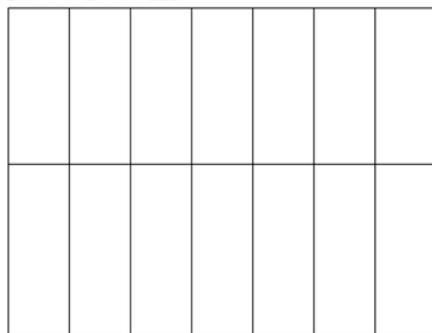
    b)   (i)   76.2kg   (ii)   1625cm2/person

(iii) Each space is 27.86cm wide and 58.33cm long. This is a smaller space than in part a). For the same reasons as in a), it is not realistic.

29 Representing data

1. a) 9%   b) 33%   c) 49%   d) 15%
   e) Their eye colours don't fit into one of the other classifications, e.g. two eyes of different colours, lavender eyes, grey eyes etc
   f)-h) Answers will vary

2. a) Due to the length of the category labels, the horizontal bars are easier to read and interpret.
   b) Swimming, Australian Rules Football and Soccer (outdoor)
   c) (i) 10%   (ii) 20%
   d) It is possible, as some students may play more than one sport so be counted more than once.
   e) We were told previously that 40 students were surveyed. This means that up to 9 students were counted two or more times.
   f) Different states have different popular sports, such as some states are very committed to Rugby League or Rugby Union and less committed to Australian Rules.
   g) Sports are seasonal, so if the survey was done during summer, cricket and swimming may have higher responses.
   h) Answers will vary

30 Dot plots

1. a)-c) Answers will vary
   d) The mean was used. Median would be a whole number or a half, i.e. most likely 1, 1.5 or 2. The mode would be a whole number.

2. a) 8 hours   b) 9 hours
   c) 8.64 hours   d) 80%
   e) Scores are clustered around 8 and 9 hours. 3 hours is an outlier.
   f) Answers will vary

3. The means for each country indicate that Australia on 1.475 seems to have the lowest rate of students speaking more than one language than the other two countries on 1.575 (NZ) and 1.7 (Canada). The medians and modes of all three countries, however, are the same and indicate the responses in all three countries are clustered around one. More than half of the students in each country only speak one language. Australia and Canada both have an outlier of a student who speaks a large number of other languages. Canada has the highest number of students who speak more than one language, as indicated by the mean. New Zealand has the lowest proportion of students who only speak one language.

31 Measures of spread
1. a) Range = 0   IQR = 0   b)   Range = 20   IQR = 20
    c) Range = 100 IQR = 50   d)   Range = 90   IQR = 81
2. a) Median = 7   Range = 22   IQR = 4
    b) Median = 3   Range = 9   IQR = 6
    c) Median = 61%   Range = 51%   IQR = 31.5%
    d) Median = 3.4mm Range = 5.9mm   IQR = 1.5mm
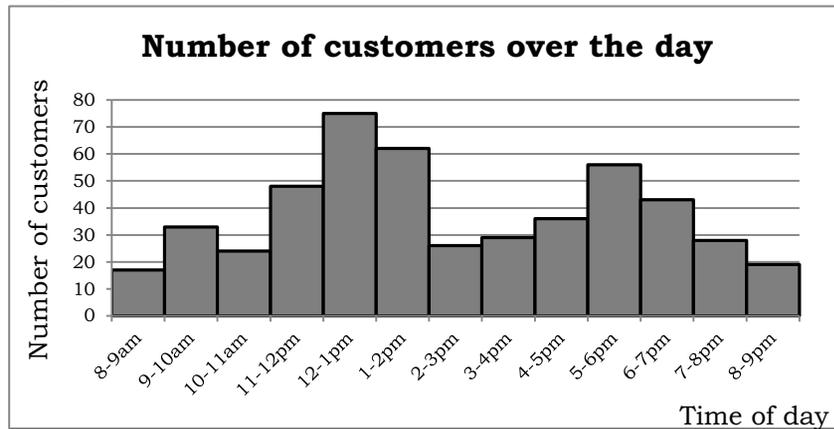
31.5 Standard Deviation
1. The effectiveness of drugs in treating illness needs to be measured in some way to be able to compare their results and determine which is most effective.
2. Set D, Set A, Set C, Set B
3. Match each of the statements below with the most suitable standard deviation value.
    Standard   Data description
    deviation
    a) 0.66   iii)   The ages of the students in your class
    b) 3.03   i)   The ages of four siblings
    c) 38.08   ii)   The ages of the entire female population
4. a) Class B
    b) Class B – highest standard deviation
    c) Class A had the most consistent results, but all were comparatively low (lowest mean and lowest standard deviation)
        Class B probably had the highest scoring students, but also the lowest scoring (high mean and largest standard deviation) – they had the greatest variation in their results.
        Class C had a mean between the other two groups and a standard deviation between them, so likely to completely include the results of Class A and overlap with the results of Class B.
5. SET A σ = 0.132 (to 3dp)   SET B σ = 289.94 (to 2dp)
    SET C σ = 22.91 (to 2 dp)   SET D σ = 0

32 Histograms

1. a)

Number of customers over the day

b)   496   c)   37.3%

2. a)   Mean = $87.55   Median = $42   Mode = $0

b)

Money earned by Year 11 and 12 students
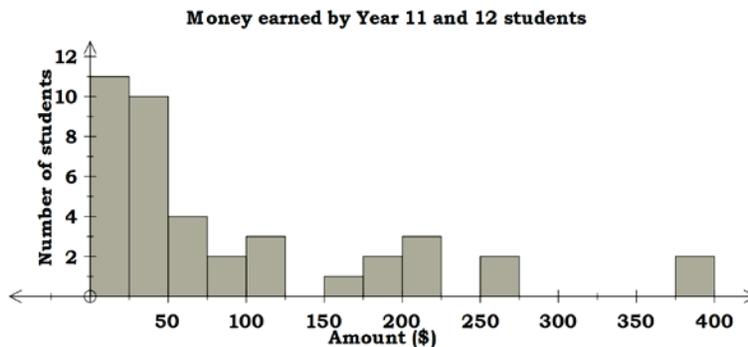
c)   The distribution is positively skewed. It is clustered between 0 and 50 with a gap between $275 and $375; most students earned at most $200; the highest incomes were $375-$400.

3. a)   40 students

b)   The distribution is positively skewed with clusters at 0-10 and 15-25minutes. There is a gap from 35 to 45 and an outlier at 85-90minutes.

c)   The median is after the first score in the 15 to 20 minute range so approximately 17.5 minutes. Using the midpoints of the regions, mean is 17.5 minutes.

d)   Because the data is grouped, we don't have exact scores so can only approximate the values by using the middle of each region.

4.

Males participating in sports

Females participating in sports
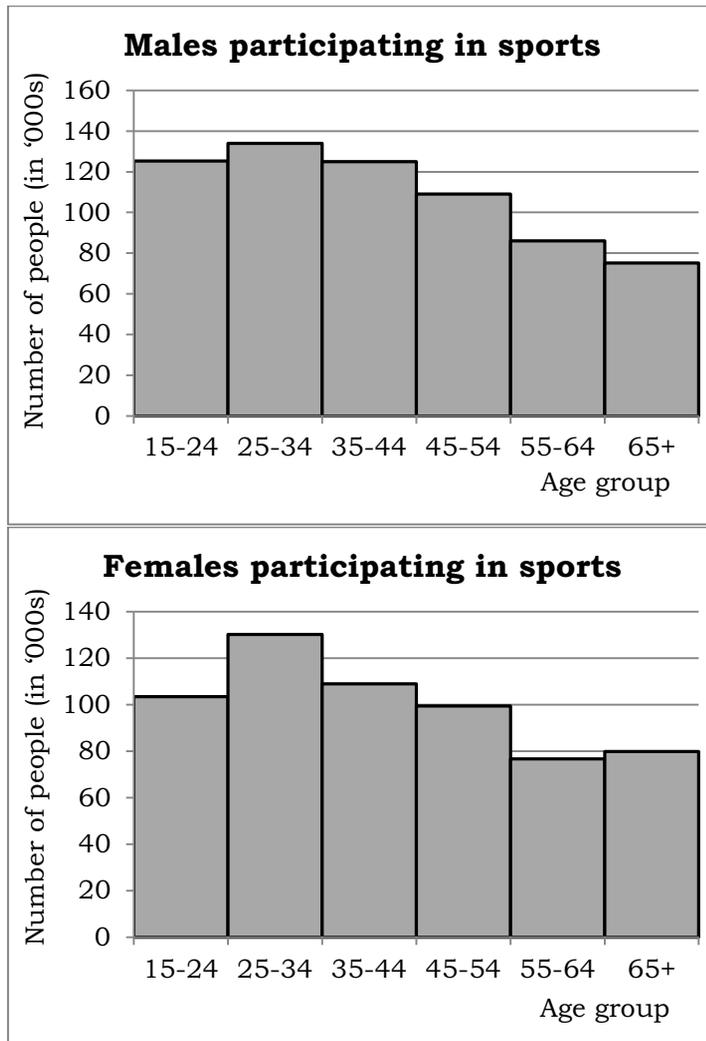
The two distributions have similar trends – they both have peaks in the 25-34 years of age category and fall away after that, being positively skewed. The peak for the females, though, is considerably higher than the other age groups (20 000 above the following age group) whereas for males it is less than 10 000 people more than the next age range. Females have their lowest participation in the 55-64 years of age range, whereas males continue to decline to the 65 and over range. As the 65 and over range is not the same size as the other categories this does not necessarily indicate an increase in female participation.

5.

Size	peewee	Small	Medium	Large	X-large	Jumbo	X-jumbo
weight	35-42	42-49	49-56	56-63	63-70	70-77	77-84
Tally	卌 卌 I	卌 卌 III	卌 卌 卌 卌 卌 I	卌 卌 卌 卌 III	卌 卌 卌 I	卌 卌	I
Frequency	11	13	26	23	16	10	1

Number of eggs by weight range

Number of eggs

30
25
20
15
10
5
0

35 to 42   42 to 49   49 to 56   56 to 63   63 to 70   70 to 77   77 to 84
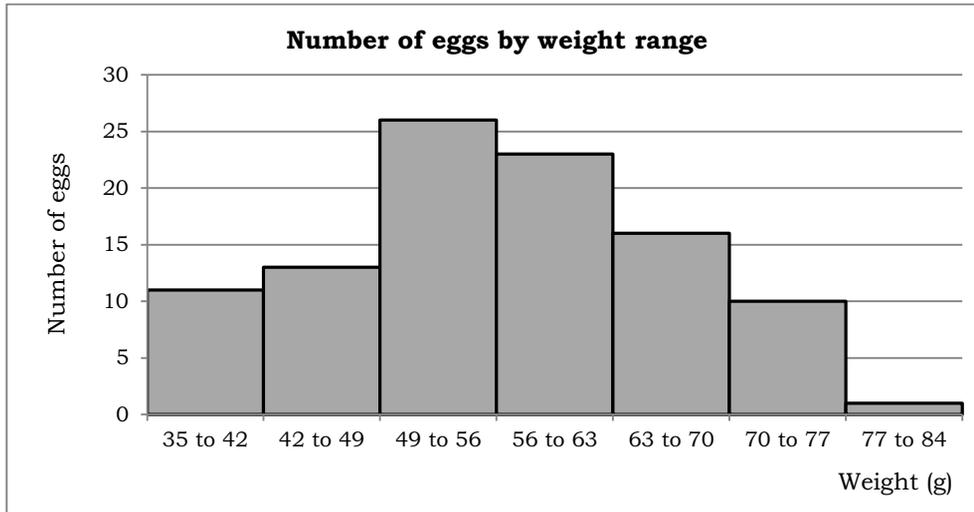
Weight (g)

The graph has a modal weight range of 49g to 56g with approximately a quarter of the eggs collected in this range. Above this, the numbers of eggs in each group gradually decreases to the 70g to 77g range with only one egg in the largest category. Almost a quarter of the eggs collected were below 49g, and were approximately evenly split between the lowest two weight ranges.

33 Stem and leaf displays

1. highest 56   lowest 11   median 32   mode 43

2.

9	8
8	05
7	6
6	36
5	
4	3468
3	469
2	039
1	8

3.

	2	11
	0	10
9	8	68
42	8	246
1	7	1366
6	6	8
6	5	
1	4	
8	3	
4	2	

Team A   Team B

Team B is much more consistent than Team A as their results are all clustered close together on the graph. Team A's results are very varied with two very strong results, but 4 scores that were well below those of Team B.

4. a)   5.8 seconds   b)   9.2 seconds   c)   7.5 seconds
    d)   3.4 seconds

5. a)   Male: mean 72.17   median 72   no mode
        Female: mean 68.75   median 70   mode 72
    b)   The male students' resting heart rates were generally higher than those of the female students. Both have the same modal group of 70-74 beats per minute. The females' results had a larger spread.
    c)   (i)   **Male heart rates**   (ii)   **Female heart rates**

50	13		9200	13	
987541	12		9650	12	
71	11		94	11	
7	10	1	9	10	
	9	15		9	128
	8	055		8	139
	7	28		7	1559
	6	578		6	8

d) Male

After exercise	121	107	127	135	130	129	128	111	117	125	124
After 1 minute	85	72	78	101	80	95	68	67	85	91	65
Recovery rate	36	35	49	34	50	34	60	44	32	34	59

Female

After exercise	139	130	120	119	114	132	129	109	125	130	126
After 1 minute	89	92	91	68	75	98	79	71	75	81	83
Recovery rate	50	38	29	51	39	34	50	38	50	49	43

e) Male: mean 42.45   median 36   range 28
Female: mean 42.82   median 43   range 22

6. a)   20-29 years   b)   passenger
     c)   (i)   50 years   (ii)   40 years
     d)   There were significantly more fatalities for passengers than for drivers at 131 compared to 57. The only age range where there were more driver fatalities than passenger was 80 to 89 years of age. The median age of the deceased drivers (50) was 10 years older than that of the deceased passengers. The largest number of any fatalities was in the 20 to 29 year old age range, which was also the modal group of passengers, but for drivers, the modal group was 80 to 89 year olds.

34 Data groupings

1.   median 9   lower quartile 5   upper quartile 15
2. a)   2004   b)   2004   c)   2010
     d)   2005, 2006 and 2008 e)   2009
     f)   2008   g)   60th   h)   60th
3. a)   3750 marron   b)   3.5 years   c)   6.5 years
     d)   5 years   e)   6 years   f)   96th
     g)   3900 marron
4. a)   22.5 million people b)   1963   c)   10th
     d)   World War II   e)   10th decile   f)   1st decile
     g)   fastest growth in a short time period (5 years), slowest growth is over much longer time frame (90 years)
     h)   Projected population: top one is high birth rate, low death rate, high immigration; middle one is average birth rate, average death rate and average immigration; bottom one has low birth rate, high death rate and low immigration.
5. a)   In order: 5, 5, 6, 4, 3, 5, 7, 6, 2, 5
     b)   Calculus – highest decile cut-offs
     c)   Chinese Advanced – lowest decile cut-offs
     d)   67.25 to 71.87   e)   Ancient History

35 Box and whisker graphs

1.
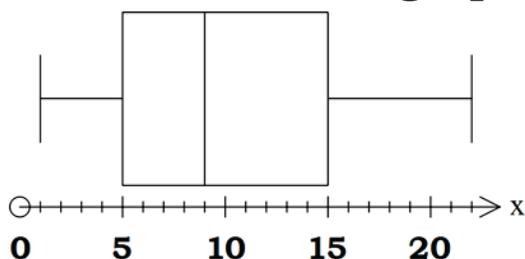
2.
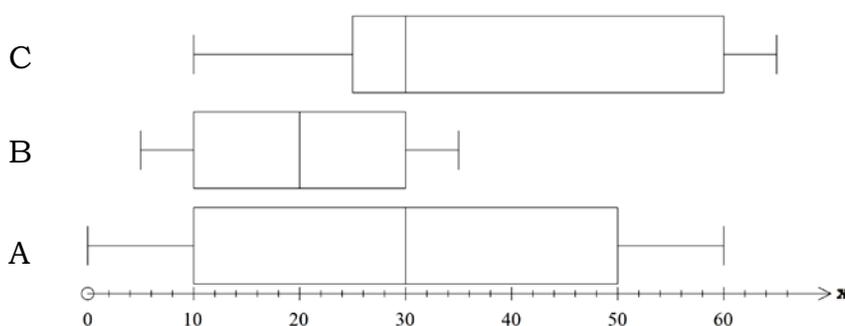
Data set A has the greatest range of scores at 60. Its median of 30 is the same as that of set C and the maximum value is only 5 below the maximum of C. A also has the largest interquartile range of the three groups and it contains the lowest score of all three sets, being 0. Set B has a much narrower range and interquartile range of all three sets. A and B are both symmetrical, but C is positively skewed. The highest value in B is only slightly above the medians of the other two sets.

3.   a)   The third quartile is also the maximum value, i.e. there are several scores of 1000.

b)   Reducing pollution and recycling rubbish have the same median of approximately 570.

c)   Conserving water

d)   All three environmental issues had the same maximum and minimum values of 1000 and 0 respectively giving them all the same range of results. The three medians were very close to each other with water conservation at 550 and the other two issues at 570. Water conservation had the largest interquartile range of the three issues, and its lower median and group of students who rated it at 1000 means that student views are quite diverse on this environmental issue. It is the issue that has the most students strongly rating its importance. Recycling rubbish, however, is the issue that has the lowest ratings in general despite the median being equal to that of pollution.

e)   Answers will vary

4. a)   The 16 year old males are generally taller than the females. The lower quartile of the male students is above the upper quartile of the female students. The two groups have a similar IQR, but the males have a larger range of 38 compared to 22 for the females. The medians are about 15 apart with the male one the higher of the two groups.

    b)   Male and female heights are quite similar at 12 years old. The medians are the same, but the female heights have a larger range and interquartile range, and have both the tallest and shortest student overall in their group.

    c)-d) Between 12 and 16 the males grow a lot more than the females do with the male median increasing by 20cm and the female median increasing 6cm. The shortest female at 12 was 140cm, but at 16 was 152cm. The tallest, however, was 178 at 12 and 180 at 16. The male heights showed a much greater change in the four years.

5. a)   Road fatalities separated by gender and driver versus passenger.

    b)   52, 42, 33, 26   c)   Min: 15, 17, 0, 0   Max: 94, 90, 88, 70

    d)   The male drivers had a slightly wider range of ages than the female drivers. The middle 50% of males were also around 5 years older than the middle 50% of females (same IQR), although the medians were 10 years apart. The female ages show a slight positive skew.

    e)   The ages of male passengers show the same distribution as the females for those at 19 and below – they have the same lowest age and lower quartile. The ages of males, however are much more spread out than the females with males showing an older distribution than females for the remainder of the distribution. The female median is 26 and male is 33, female upper quartile is 43 and male is 80, and oldest female is 70 but oldest male is 88.

    f)   For both genders, the drivers' ages are higher than the passengers' ages for the most part. The male upper quartile being the only exception where the passengers show a very high result.

6. a)   Activity Z – lowest age is 18. Most likely involves alcohol such as wine appreciation, how to make spirits, etc.

    b)   Activity Y – median age is 16.

   c)   Activity X – 75% of the people there are between 34 and 48

36 Blood types

1. In order: 9.4mill, 2.12mill, 7.29mill, 1.65mill, 1.88mill, 0.47mill, 0.47mill, 0.24mill

2-4. Answers will vary

5. In order: 480 000, 270 000, 250 000, 180 000, 140 000, 56 000, 28 000

6. 26 923   7.   a)   658 000L   b)   Yes, 28.6 times

8. a)   O-   b)   AB+
    c)

Blood type	% can donate to	% can receive from
AB+	2%	100%
AB-	3%	19%
A+	33%	87%
A-	41%	24%
B+	16%	59%
B-	13%	11%
O+	81%	49%
O-	100%	9%

    d)

Blood donor and recipient relationships by blood type
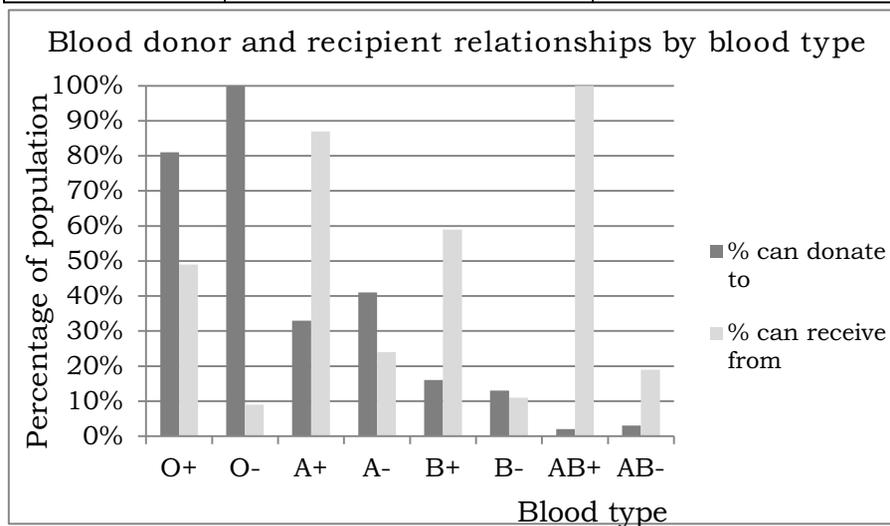
9. a)   AB+   b)   B-
10. a)   37%, 30%, 32%, 40%   b)   6%
    c)   Australia is the country that is significantly different to the
          others with a higher proportion of people with B+ blood and
          lower in A+, O- and A-. We also have the highest proportion
          of O+ people, but the USA is fairly close to Australia on 37-
          38%. Turkey and Sweden are fairly close to each other with
   minor differences in proportions.

37 Statistics in the media

1. a)   It uses the birth, death and immigration rates to estimate the
          changes in population.
    b)   Changes in the rate are gradual so why it may not be precise
          at any one instant, it will be very close over time.
2. a)   25-29
    b)   Yes – all up to 44 years old
    c)   Yes – from about 70 up.
    d)   35-39
    e)   Females have a longer life expectancy than males, so
          naturally comprise a higher proportion of the older
          population. If they are even, then males must make up a
          higher proportion at younger age categories.
    f)   20-25%   g)   1%

h) The most populous age groups will get older. While they will decrease slightly due to the death, the cluster from 20 to 50 will age to:
   (i)   40 to 70   (ii)   60 to 90
   (iii) At this point most of the larger groups from the current graph will have died, so it will depend on what occurs with the birth rate and immigration as to the shape of the graph then.

i) There will be a significant need to support older members of the community through pensions, superannuation, retirement villages, health care etc as the current 20 to 50 year olds move on to retirement and beyond.

3. a) Approximately 1.3 million
   b) Approximately 350 000 14-17 year olds and 220 000 10-13 year olds
   c) (i)   8.5
      (ii)   Through surveys of samples of students which are used to estimate the full population.
      (iii) It is not possible to survey every person involved due to the time it would take – sampling is much more efficient.

4. One in four families have every person unemployed – a lot more families will have at least one person unemployed so a much higher proportion of families will be afflicted by unemployment.

5. Answers will vary

6. Answers will vary – they should remove emotive language such as falsify, and support their own theories.

7. Answers will vary

8. a) The graph shows 12.5%, not 25%. A 3D pie chart is a very poor way to represent data.
   b) All should use the same symbols to be comparative; the last person is larger than the others; they are displaying ratios in a method that is designed to compare quantity; they have 6 children in the image (not 5); a pie chart should not be part of a pictograph.

9. a) Answers will vary
   b) c)

Day	Frequency	Percentage
Monday	19	11.9%
Tuesday	22	13.8%
Wednesday	15	9.4%
Thursday	20	12.5%
Friday	28	17.5%
Saturday	30	18.8%
Sunday	26	16.3%

   d) Categorical data, so pie chart or bar graph is appropriate. Pie charts are harder for people to tell the difference in area for regions that are of a close size, so bar graph is the best choice.

Percentage of accidents on each day of the week
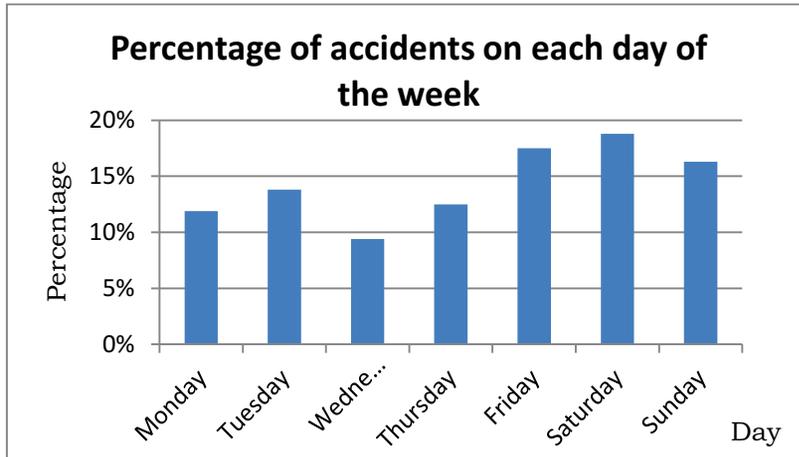
e) Saturday is the day on which most accidents occur with 19%, closely followed by Friday on 17.5% then Sunday on 16%. Wednesday is the least common day for an accident on 9%. Monday has the second lowest proportion of accidents (12%), and is very similar in proportion to Thursday (12.5%).

10. a) 1132 minutes for males and 1156 minutes for females
    b) Working, sleeping, reading, watching television, on the computer, doing homework, …
    c) Answers will vary
    d) Answers will vary

38 Ratio skills

1. a) $3:5$, $\dfrac{3}{8}$ b) $10:6$, $\dfrac{10}{16}$

2. a) $5:1$ b) $2:1$ c) $4:1$ d) $1:3:9$ e) $1:3$
   f) $1:5$ g) $1:2:5$ h) $7:1$ i) $5:1$

3. a) $1:3$ b) $1:20$ c) $5:12$ d) $6:25$ e) $1:4$
   f) $1:6$ g) $1:4$ h) $1:5$ i) $2:50$

4. a) $1:2$ b) $1:3$ c) $4:1$ d) $3:7$ e) $3:4$
   f) $3:2$ g) $5:4$ h) $5:1$ i) $3:7:11$

5. a) $16, 20   b) $24, $16   c) $120, $80
   d) $1800, $7200   e) $4 200, $6 300, $ 10 500

6.

Ratio	Fractions	Percentages
1 : 4	$\dfrac{1}{5}$ and $\dfrac{4}{5}$	20% and 80%
1 : 3	$\dfrac{1}{4}$ and $\dfrac{3}{4}$	25% and 75%
2 : 3	$\dfrac{2}{5}$ and $\dfrac{3}{5}$	40% and 60%
4 : 1	$\dfrac{4}{5}$ and $\dfrac{1}{5}$	80% and 20%
3 : 7	$\dfrac{3}{10}$ and $\dfrac{7}{10}$	30% and 70%
1 : 2	$\dfrac{1}{3}$ and $\dfrac{2}{3}$	$33\dfrac{1}{3}\%$ and $66\dfrac{2}{3}\%$

39 Applications of ratios

1.   18 girls   2.   30m
3.   a)   1:3   b)   9:25   c)   40, 60, 80   d)   64:1
4.   a)   1:100 and 20:80   b)   16 200   c)   1:3
5.   a)   300:75 and 250:125   b)   550: 200 = 11:4
6.   Insecticide 1:50, 100mL   Weed killer 1:18, 56mL Paint
     Paint 1:5, 2.5 litres   Blackberry Killer 1:120, 50mL
     Eucalypt eradicator 1:11, for sufficient dilution

40 Rates

1.   a)   (i) $24 (ii) $20 (iii) $47.50   b) 4 hours
2.   a)   the rate increases   b) $4800
     c)   No, while the rate is increasing this is not a constant increase.
3.   a)   110km   b)   27.5km   4.   45mins
5.   a) $770 472   b) $281 222 280
6.   a)   70 births
     b)   No, the birth rate is an average across Australia, different
     communities may differ
7.   a) $27.42   b) $216.20 c) $1 296   d)   534 500 Rp
     e)   2275 baht   f)   520 ¥   g)   1 290 835 <u>d</u>   h)   ₤ 50.67
8.   a) $456   b)   2665.23 ¥

41 Simple interest

1.   a) $25 $525   b) $72 $4072
     c) $2400 $82400 d) $388 $4388
2.

Year	Principal	Interest per year	Total owing at the end of the year
1	$1000	10% of $1000 = $100	$1000 + $100 = $1100
2	$1000	10% of $1000 = $100	$1100 + $100 = $1200
3	$1000	10% of $1000 = $100	$1200 + $100 = $1300
4	$1000	10% of $1000 = $100	$1300 + $100 = $1400
5	$1000	10% of $1000 = $100	$1400 + $100 = $1500
6	$1000	10% of $1000 = $100	$1500 + $100 = $1600
7	$1000	10% of $1000 = $100	$1600 + $100 = $1700
8	$1000	10% of $1000 = $100	$1700 + $100 = $1800
9	$1000	10% of $1000 = $100	$1800 + $100 = $1900
10	$1000	10% of $1000 = $100	$1900 + $100 = $2000

     a) $1500   b) $500
     c) $2000 $3000   d) $3000 $4000
3.   a) $337.50   b) $28.13   c) $84
     d) $306.25   e) $900   f) $115
4.   a) $1791   b) $179.10   c) $1970.10
     d) $131.34

5. a) A   borrow $2025   interest $567   total $2817
        B   borrow $1592   interest $394.02   total $2384.02
        C   borrow $1600   interest $369.60   total $1969.60
    b) A $117.38   B $132.45   C $109.42
    c) C – no upfront cost, lowest repayments, lowest total cost
6. 5 years   7. 18 months   8. 3.25 years = 39 months
9. 4.5%   10. 5.25%   11. 7.8%
12. $6200   13. $9800   14. $18 600

42 Time calculations
1. Timetables, work schedules, sports, time to get to and from places
2. a) 0130   b) 0430   c) 2214   d) 2144
    e) 1155   f) 1422   g) 1505   h) 0010
3. a) 2:30am b) 3:24pm c) 3:42am d) 4:33pm
    e) 11:11am f) 12:17am g) 11:28pm h) 7:48pm
4. a) ~2- 10 mins (depends on length)   b) ~5mins
    c) ~100 sec d)   ~ 3mins e)   ~ 1sec   f)   ~ 4mins
5. a) 4pm   b) 7:30am   c) 4:45am   d) 10:44am
    e) 9:29pm f) 12:30pm
6. a) 45 mins   b) 3½ h   c) 2hrs 15mins
    d) 6hrs 55 mins   e) 3hrs 15 mins f) 42mins

43 Timetables
1. a) 3:38pm
    b) Morning 2hrs and 25mins Afternoon 2hrs 29 mins hence the afternoon trip is 4 minutes longer
    c) Bunbury Station   d) 2hours 34mins
2. Answers will vary
3. a) 7:15am   b) 1hr 45mins c) 0915
    d) Total Cost is $937.50 so each pay $187.50. Leave on the 1:45pm ferry to arrive at 2:30pm and depart 9:15am to arrive back at 9:55am (alternate times could be possible)
4. a) 7:39am – 8:39am and 7:16am – 8:16am
    b) (i) 5:51pm (ii) 7:51pm
    c) Thur 9th at 9:12am and Fri 10th 7:01am

44 Speed
1. a) IV   b) III   c) II   d) I
2. a) 2km   b) 12km   c) 6km   d) 9km
    e) 2.3km   f) 7.3km
3. a) 11km   b) 16.5km c) 8.25km d) 12.38km
    e) 3.21km f) 10.08km
4. a) 4km/h   b) 77.5 km/h   c) 40km/h
    d) 108km/h   e) 3km/h   f) 35 km/h
5. a) 2.78 m/s   b) 1.52 m/s   c) 0.83 m/s
6. a) 2 hours   b) 36 mins   c) 27mins
    d) 113 mins = 1 hour 53 mins

7. a) (i) 27.5km/h
     (ii) stopping at bus stops to pick up other passengers, heavy traffic slows down speeds, traffic lights stopping and starting so not travelling at max speed.
   b) (i) 84km/h (ii) 15%
8. 140m/s = 504 km/h

45 Reaction times

1. Answers will vary
2. Class activity, discuss results with other students and your teacher
3. a) 92m   b) stopping distance increases
   c) 34%   d) 31%, 28%, 26%, 25%, 23%   e) 7m
4. a) decrease   b) 13.89 m/s   c) 27.78 m
   d) the distance between the cars should increase, because speed = distance ÷ time and this means that as the speed increases the distance must also increase if the time is staying constant.
5. a)

| u	13	18	19	25	30
d	8.45	16.2	18	31.2	45

   b)
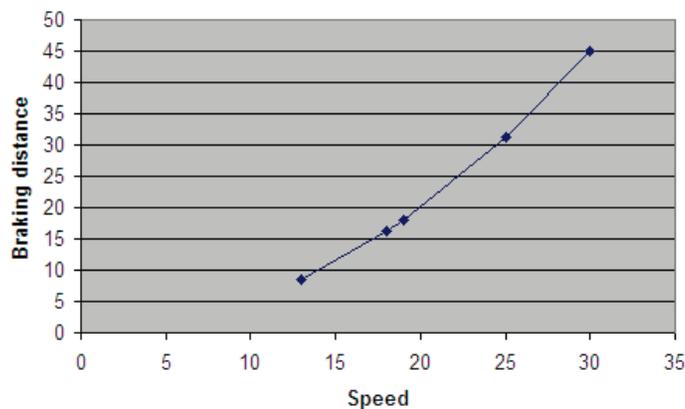
   c) Braking distance increases at an increasing rate as speed increases.
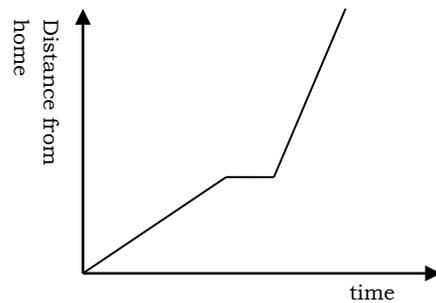
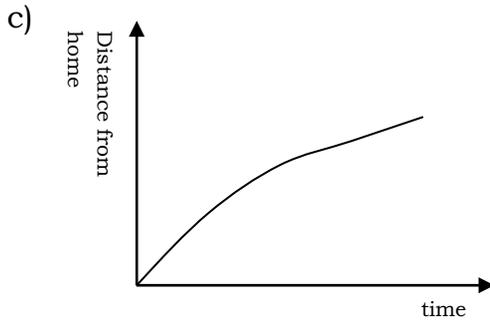## 46 Distance-time graphs I

1. B   2. C
3. a) Jogging to a venue staying there for a period of time before returning home at a walking pace (or similar)
   b) Walking to a venue, realising that you had dropped your wallet so going back to find it, continue walking to the venue at a constant pace then returning home at the same pace
4. a)   b)

c)
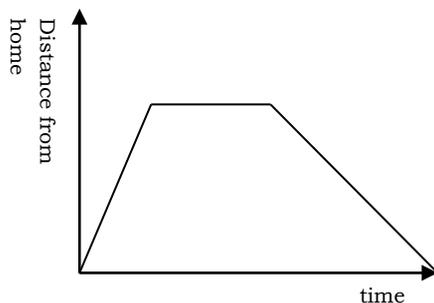
5. Straight, corner, straight, corner, straight, corner, straight
6. Sam started the race in first place, with Ryan second and Jack in third place. Sam begins to tire so his speed decreases, meanwhile Jack increases his speed and Ryan's remains constant. Jack ends up overtaking the others and wins the race, Ryan is second and Sam is in third place.

47 Distance-time graphs II

1. a) b)

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

c) d)

Time	Distance
0	100
1	80
2	60
3	40
4	20
5	0

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

2. a) 8km b) 10.50am – 11.00am
   c) 6km/h d) 11.00am – 11.30am because the graph is steeper
3. a) 10 m b) 5 ½ seconds
   c) 100 ÷ 9.58 = 10.44 m/s d) 37.6 km/h
4. a) Fiona – 6.9 m/s Sue – 8 m/s Po – 5 m/s
   b) Po starts the race in front followed by Fiona and then Sue. After 20 seconds, Po has fallen over a hurdle at the 190m mark and has come to a stop. Meanwhile Fiona takes the lead of the race. At 250m Fiona and Sue are equal, Sue increases her pace to win the race. Po has recovered after 40 seconds and finishes the race in 80 seconds at a constant speed between 50 and 80 seconds. Fiona finishes second in approximately 58 seconds.
5. a) 2580 km b) 3 h 45 mins c) 800km/h
   d) The plane is slowest when it is taking off and landing which takes approximately 30 minutes. The speed mid-flight is the highest.

48 Maps

1. a) 1:2   b) 15:1   c) 40:1   d) 9:100
   e) 1:25000 f) 1:50000 g) 10:1   h) 1:400
2. a) 2km   b) 500m   c) 3.25 km d) 6km
3. a) 6cm   b) 1.5cm   c) 13.5cm d) 10cm
4. a) 2800 km b) 1000km c) 1600km d) 300km
5. a) Z is 3.9km from A and X is 5.2km
   b) Z is 7.7km from B and X is 5km   c) 5km
6. a) Approximately 1: 4800000 (1cm = 48 km) would be a suitable scale for the distance stated or 1cm = 100km if the entire state was included on the page. Various solutions are possible, check with your teacher.
   b) 1 hour and 17 minutes   c) 12 hours

7. a) 4.5cm , 1800 km   b) 8.4cm, 3360km
   c) 4.2cm, 1680km   d) 5.3cm, 2120km
8. a) Diagram will be 5.1cm x 25cm   b) 20 housing lots

49 Planning a journey

1. a) The start and end point of the journey
   b) 1006m   c) 77 m/sec   d) 1.34 mins
   e) No, as a car would not be able to go over such things as a pedestrian overpass and may not take as a direct route as it is when walking
2. a) May differ, depending on order of cities visited
   b) $19 064.32
3. a) and b) check with your teacher

50 Shortest path networks

1. a)

Figure	Number of Nodes	Number of Regions	Number of Arcs	Nodes + Regions
i)	2	2	2	4
ii)	2	3	3	5
iii)	6	4	8	10
iv)	4	5	7	9
v)	5	5	8	10

   b) number of arcs + 2 = number of regions + number of nodes
   i.e. a + 2 = r + n
2. a) 4   b) 3   c) 12
3. a) (i) 4 (ii) 8 (iii) 11   b) ACD
4. a) (i) 8 (ii) 12 (iii) 18 (iv) 20
   (v) 22 (vi) 26 (vii) 29   b) ACEH
5. a) ABZ 16 units   b) ACDZ or ACBZ 12 units
   c) ABEFGZ 30 units d) ADZ 25 units e) ABFH 38 units

6. 220km Beacon, Goomalling, Dowerin, Burakin, Northam
7. a) 5.5km   b)   5.25km
8. a) Narrogin, Williams, Armadale, Perth 174km
    b) Narrogin, Williams, Arthur River, Bunbury, Busselton, Margaret
        River 308km
    c) Narrogin, Williams, Arthur River, Kojonup, Cranbrook, Albany
        207km

> If I were again beginning my studies, I would follow the advice of Plato and start with mathematics.
>
> – Galileo Galilei